A STOCHASTIC APPROACH FOR EVALUATING SHORT-TERM FINANCING ALTERNATIVES UNDER CONDITIONS OF RISK

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ABSTRACT

A STOCHASTIC APPROACH FOR EVALUATING SHORT-TERM FINANCING ALTERNATIVES UNDER CONDITIONS OF RISK

By

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At least some facet of the cash management problem is faced by every manufacturing firm. In the decision process, management must decide upon an optimal minimum cash balance and a policy for handling cash shortages. A model to aid management in selecting an optimal financing strategy from a given set of alternatives is presented in this study. The objective is the maximization of the utility of the unrestricted ending cash balance at the end of a specific planning period.

A sequential, multi-period approach that considers risk is incorporated in the simulation model. The net cash flow values that serve as input to the model are generated by a simulation of the stochastic processes of the input variables which make up the net cash flow. The condition of stochastic dominance is utilized to evaluate

the cumulative probability distributions which were generated from the model output.

In summary, two quantitative techniques, simulation and the condition of stochastic dominance have been integrated and applied to analyzing and selecting an optimal financing strategy from a given set of alternatives.

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CHAPTER I

INTRODUCTION

Statement of the Problem

Cash management and short-term financial planning have become increasingly complex and important in the past decade. As the cost of borrowing and yields on marketable securities have increased, firms have begun to recognize the advantages of economizing on cash holdings, speeding up cash inflows, controlling cash outflows, and investing excess cash.[1] The rapid expansion of business activities has required increasing quantities of working capital.

At least some facet of the cash balance problem is faced by every manufacturing firm. Only recently have firms become cognizant of the importance of economizing cash holdings, however. The need for cash stems from a lack of balance between cash inflows and outflows, and the difficulty of accurately predicting some of these flows. An adequate amount of cash must be maintained to perform regular transactions and to meet unexpected cash requirements. Management must decide on an optimal minimum cash

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[1] Yair E. Orgler, Cash Management--Methods and Models (Belmont, California: Wadsworth Publishing Company, Inc., 1970), p. 25.

balance and a policy for handling cash shortages. It is the intent of this study to develop a model that is capable of assisting management in selecting an optimal short-term financing alternative under conditions of risk, when a net cash drain is anticipated.

Plan of the Study

The model presented in this study is designed to assist management in determining an optimal short-term financing strategy. The criterion for decision will be the maximization of the utility of the unrestricted ending cash balance[1] at the end of a specific planning period.

The financing requirement of a firm in a given planning horizon is determined by the initial cash balance and the projected net cash flow. The cash balance is affected by the stochastic nature of the net cash flow for each period. The cash balance may be decreased by a number of methods including investing surplus cash in marketable securities and/or paying off short-term debt. Conversely, in situations demanding more cash, the cash balance may be increased by reducing the firm's holdings of

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[1]The unrestricted ending cash balance is defined as the ending cash balance less that portion of cash that is restricted to the future payments of principal and interest specific to each financing alternative, plus the amount of interest and principal due from invested surplus cash.

marketable securities and/or undertaking some short-term financing. The cash demands must be satisfied at a reasonable cost, and a minimal level of cash must be maintained. It will be assumed in this study that management specifies the minimum required cash balance for each period. The financing alternatives will be evaluated considering this minimum required balance.

A model will be introduced to evaluate four short-term financing strategies--line of credit, term loan, commercial paper, and accounts receivable loan. Input data to the model will include the following information for each period: (1) the distribution of the interest rates of the alternative financing strategies; (2) the net cash flow distribution; (3) the minimum required cash balance; and (4) constraints on the financing alternatives. The net cash flow data and the data for the cumulative probability distributions of the unrestricted ending cash balances for each financing alternative will be generated by Monte Carlo simulation. The latter cumulative probability distributions will be compared by the condition of stochastic dominance[1] to determine an optimal alternative.

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[1]See page 86, Chapter V for a discussion of the condition of stochastic dominance.

Certainty, Risk and Uncertainty

Models currently being utilized can be divided into two basic types: (1) those that are restricted to variables known with certainty--deterministic models; and (2) those that allow probabilistic values for certain variables, thus incorporating risk--stochastic models. Certainty is defined as the condition that exists when all parameters are completely known and the variability associated with these parameters is zero. Thus, each action is known and leads invariably to a specific outcome.

The classical distinction between risk and uncertainty was proposed by F. H. Knight (Risk, Uncertainty and Profit, Houghton-Mifflin Company, 1921).[1] Accordingly, risk is defined as the situation in which the outcomes are not completely known but objective probabilities can be associated with possible outcomes. Uncertainty refers to situations for which probabilistic results cannot even be predicted in probabilistic terms. The distinction between risk and uncertainty in an organizational context cannot be so clearly delineated, however. Decision-makers generally have some feelings about the probabilities of future events. These feelings will affect judgments made by management.[2]

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[1] Alfred Rappaport, "Sensitivity Analysis in Decision-Making," The Accounting Review, LXII (July, 1967), 441.

[2] Ibid.

Thus, although these terms have been used interchangeably in the literature, the term risk will be used in this study.

Organization of Remainder of the Thesis

The cash management problem has been established as significant and worthy of further study. A model has been outlined in this chapter which represents an approach to aid management in solving some facets of this problem.

Chapter II presents and discusses some models in the literature related to the cash management problem. The deterministic and stochastic approaches to solving cash management problems are delineated. The models are discussed in terms of their relevance to the development of problem-solving techniques and to the model in this study.

Chapter III is devoted to the development of the model in decision-theory framework. The single-stage decision problem under certainty is extended to include several stages and incorporate risk. The model in this study is presented as a multi-stage decision problem, and the assumptions of the model are outlined. Finally, the simulation approach to the model is discussed.

Chapter IV develops the model itself. The financing alternatives to be evaluated are discussed in detail, and the method of computer analysis presented. The input data to the model are defined.

Chapter V consists of the presentation and evaluation of the results. The inputs to the model and the model output are presented. The cumulative probability distributions for each financing alternative at the specified minimum required cash balances, are graphed. The distributions are then compared using the condition of stochastic dominance. An optimal financing alternative is then selected for each minimum required cash balance. A final chapter consists of the summary and conclusions.

CHAPTER II

SURVEY OF THE RELATED LITERATURE--
METHODS AND MODELS

Introduction

In recent years mathematical programming techniques and computers have been applied to the solution of various problems in finance. In this chapter some general models discussed in the literature will be reviewed. The primary intent of this discussion is to indicate references that relate to cash management problems. Several capital investment decision-making models will also be presented since the approaches may be applicable to short-term financing problems.

Two basic approaches are used to solve cash management problems--the deterministic approach in which risk is not considered, and the stochastic approach which incorporates risk. The deterministic method is based upon the assumption that the values of the variables and parameters in any specific problem are known with certainty. The objective of these models is to maximize or minimize some deterministically defined function. The assumption of complete information, essential to these models, has been thought to impair their validity, since variability of conditions is an observable fact. Proponents of the approach contend, however, that no existing method is capable of adequately incorporating this variability, and, in the attempt to do so, often introduce more error than they resolve. The stochastic approach is becoming more widely accepted, however, as methods are developed which more closely approximate real world conditions.

The stochastic approach allows some of the variables and parameters in any specific problem to be defined as random. The random values are described by some probability distribution. The degree to which risk is incorporated varies with the specific model under consideration. The stochastic approach allows management to make decisions with regard to some of the variable input data. Their personal experience may be invaluable in enabling the model to reflect the specific conditions of the firm under study. The development of the deterministic and stochastic models which relate to the cash management problem in this study will be described in this chapter.

Deterministic Approaches

Inventory Models.--The cash management problem is very similar to the problem of inventory management.[1] Baumol[2] formulated one of the first models to solve cash balance problems by utilizing a deterministic approach incorporating the classical economic lot size model of inventory management. His objective was to minimize the total cost of the borrowing transactions over some planning horizon. The model determined how much cash, C*, is obtained, how often this cash is to be obtained, L*, and the average cash balance, $\bar{C}$*, as follows:

$$\text{Transfer } C^* = \sqrt{\frac{2bm}{i}} \qquad L = \sqrt{\frac{2b}{mi}}$$

$$\text{with an average cash balance of: } \bar{C}^* = \sqrt{\frac{bm}{2i}}$$

where: b = fixed cost for borrowing or withdrawal

i = opportunity interest rate per day

m = constant dollar expenditure per day

L = number of days in a period

Graphically, the model can be expressed as follows:

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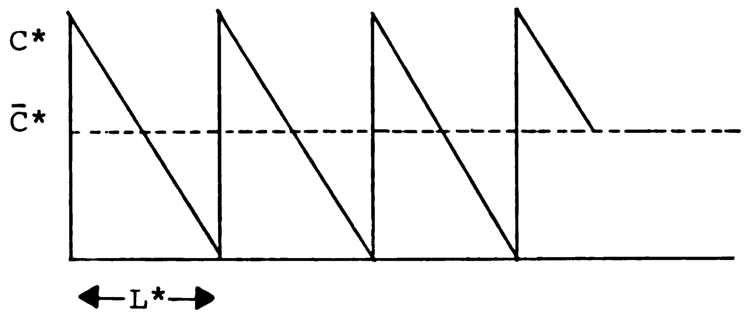
[1]Edwin J. Elton, and Martin J. Gruber, "Dynamic Programming Applications in Finance," The Journal of Finance, XXVI, No. 2 (May, 1971), 499.

[2]William J. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach," Quarterly Journal of Economics, LXVI (November, 1952), 545-556.



The Baumol model held that the cost of borrowing or withdrawal, the opportunity interest rate, and disbursements over time were constant and known. Only the transactions demand for cash was actually considered. Despite these recognized limitations the model provided a definite contribution to the literature and was a basis for considerable further work in this area.

Tobin[1] developed a model similar to Baumol's. Tobin's model determined the optimal average cash and bond holdings to maximize interest earnings net of transactions costs. Conversely, as has been shown, Baumol was interested in minimizing the total cost of the transactions irrespective of the financial efficacy of investing in bonds as opposed to holding cash.

Linear Programming.--An article by Charnes, Cooper and Miller[2] presented the first application of linear

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[1] James Tobin, "The Interest-Elasticity of Transactions Demand for Cash," The Review of Economics and Statistics, XXXVIII, No. 3 (August, 1956), 241-247.

[2] A. Charnes, W. W. Cooper, and M. H. Miller, "Application of Linear Programming to Financial Budgeting and Costing of Funds," The Journal of Business, XXXII, No. 1 (January, 1959), 20-46.

programming to finance. Their approach considered the optimal allocation of funds within the firm which included the operating environment and the existing organization of financial and physical resources. The objective was the maximization of profits, subject to capacity, buying and selling constraints, and a liquidity requirement. Charnes, Cooper and Miller extended the basic approach to include lending and borrowing. The use of linear programming was a significant contribution to the solution of finance problems. The technique presented in this article found later application in solving short-term cash management problems even though their specific model could not be used in this regard.

An extension of the model by Charnes et al. was developed by Ijiri, Levy, and Lyon.[1] It covers marketable securities transactions and is limited to a single-period approach. The model was further amplified in later work by Charnes, Cooper, and Ijiri.[2] Modifications and extensions of the models using linear programming led to experimentation in its application. Although the specific emphases of the models may have differed, the basic techniques, which led

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[1] Y. Ijiri, F. K. Levy, and R. C. Lyon, "A Linear Programming Model for Budgeting and Financial Planning," Journal of Accounting Research, I, No. 2 (Autumn, 1963), 198-212.

[2] A. Charnes, W. W. Cooper and Y. Ijiri, "Break-Even Budgeting and Programming to Goals," Journal of Accounting Research, I, No. 1 (Spring, 1963), 16-43.

to the development of increasingly comprehensive approaches, were established.

Robichek, Teichroew and Jones[1] developed a multiple-period linear programming model for optimizing short-term financing decisions. Their discussion was concerned with the decision of how much and when money should be acquired from specific sources of short-term funds to minimize cost to the firm. The model made the following assumptions: (1) the minimum cash balance required at all times was specified; (2) the cash inflows and/or outflows were known with certainty; and (3) the costs and limitations of all of the financial alternatives were known. With these assumptions and given cash balance requirements, the model was developed.

The Robichek, Teichroew and Jones (RTJ) model is particularly significant with regard to the model in this study. The basic approaches are similar except that the RTJ model utilizes linear programming and thus solves the problem of selecting an optimal financing alternative, simultaneously. The model in this study is probabilistic and considers the selection of an optimal financing alternative, sequentially. Similarities of the two models include the consideration of multiple periods, the

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[1]A. A. Robichek, D. Teichroew, and J. M. Jones, "Optimal Short-Term Financing Decision," Management Science, XII, No. 1 (September, 1965), 1-36.

assumption of known constraints on the financing alternatives, and the certainty with which the minimum cash balance is known.

Orgler *et al.*[1] developed a linear programming model differing only slightly from the RTJ approach. D. E. Peterson[2] presented a model, also utilizing linear programming, but which was more comprehensive in that it incorporated the following sources of short-term financing: accounts payable, line of credit, accounts receivable and sale of commercial paper.[3] The discussion of selecting an optimal financing method from specific alternatives in this article assumed that the needs of a firm were known with complete certainty. The certainty constraints necessary in any linear programming approach are its major short-coming. Linear programming provides a useful and valid tool for problem-solving as long as its limitations are recognized.

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[1]Orgler, pp. 70-118.

[2]D. E. Peterson, *A Quantitative Framework for Financial Management* (Homewood, Illinois: Richard D. Irwin, Inc., 1969), Chapter 7.

[3]The model in this study, although stochastic in nature, offers four similar financing alternatives.

Dynamic Programming.--Mao[1] presented an approach for managing the cash balance during a finite planning horizon through the use of a two-step systematic search procedure using dynamic programming. The cash balance decision was viewed as a problem involving a sequence of interrelated decisions. These decisions included whether to retain cash holdings in the form of securities or in the form of cash, and which quantitites of securities should be held or sold by the firm.

The dynamic programming approach is one of the valid methods for solving cash management problems sequentially. The importance of the inter-temporal relationships lends credibility to dynamic programming (or recursive optimization) as a technique of solving these problems. This approach will be discussed more fully in the section on probabilistic dynamic programming.

Stochastic Approaches

Many of the techniques utilized in deterministic models have been extended to allow them to incorporate risk. A gradual refinement of the models has taken place, as computer techniques have been increasingly applied and accepted for use in solving business problems. Computers have made feasible the solution to heretofore complex

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[1] James C. T. Mao, Quantitative Analysis of Financial Decisions (Toronto: The Macmillan Company, Collier-Macmillan Canada, Ltd., 1969), pp. 505-514.

mathematical problems, particularly problems involved in the probabilistic dynamic programming and simulation approaches. Various approaches are discussed in the following section which assisted the author in selecting the components of the model presented in this study.

Inventory Models.--Miller and Orr[1] extended the Baumol model to allow the net cash flows to be stochastic. The model was based on the assumption that the cash balance varies irregularly and unpredictably over time. The random behavior of the cash flows was generated in a sequence of independent Bernoulli trials. The model attempted to minimize the expected cost per day of managing the firm's cash balance over a finite planning horizon.

The cash balance was allowed to fluctuate freely within predetermined maximum, h, and minimum, 0, values. When either of these values was reached, a transfer of funds or liquidation of assets was effected to achieve an "optimal" level, z. A fixed transfer cost independent of the amount of funds transferred was assumed. Optimal values of z and h were found as follows:

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[1]Merton H. Miller and Daniel Orr, "A Model of the Demand for Money by Firms," Quarterly Journal of Economics, LXXX (August, 1966), 413-435.

$$z^* = \sqrt[3]{\frac{3\gamma m^2 t}{4v}}$$

$$h^* = 3z^*$$

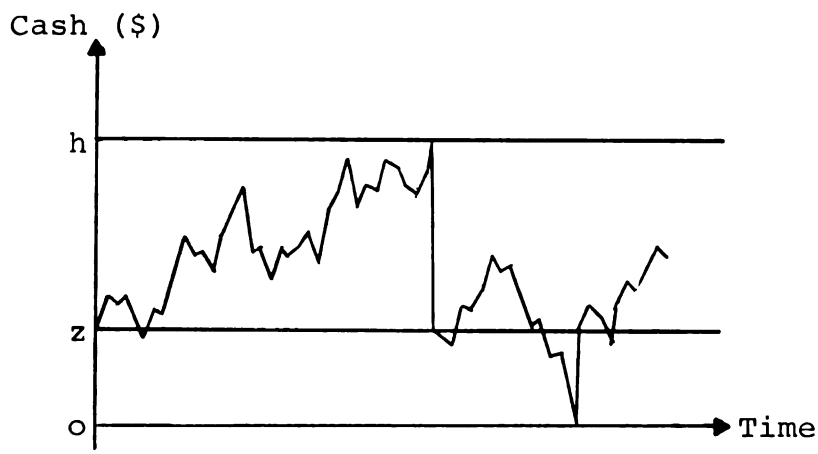
where: γ = cost per transfer

m = constant expenditure in dollars per day

v = daily rate of interest earned on the portfolio

t = number of operating cash transactions per day

The model is expressed graphically as follows:



Linear Programming.--Stochastic linear programming models in which either coefficients of the objective function or technical coefficients of the constraints are treated as random variables have been employed under conditions of risk. Since the origination of this linear programming application with the works of Tintner[1] and Babbar,[2] there have been numerous theoretical extensions. Frequently the linear stochastic model is transformed into a consistent non-linear programming model which is solved by non-linear techniques resulting in an optimal feasible solution to the original model. Application of stochastic linear programming has been limited because of the computational burdens involved.[3]

Chance-Constrained Programming.--Näslund[4] incorporated risk into his model through the use of chance-constrained programming. The model involved allocating a limited fund between several investment projects (classified as independent) over a finite horizon. Borrowing and lending were added as decision variables. Risk was introduced into the model through probabilistic constraints, one for each year, reflecting the fraction of the time that the firm's original financial plan was allowed to be violated.

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[1]G. Tintner, "Stochastic Linear Programming with Applications to Agricultural Economics," Second Symposium in Linear Programming, Washington (Washington, D.C.: National Bureau of Standards, 1955).

[2]M. M. Babbar, "Distributions of Solution of a Set of Linear Equations with Applications to Linear Programming," Journal of the Statistical Association, L (1955), 155-164.

[3]Rappaport, p. 443.

[4]Bertil Näslund, "A Model of Capital Budgeting Under Risk," The Journal of Business, XXXIX, No. 2 (April, 1966), 257-271.

The decision-maker had to be able and willing to specify the probability level at which the constraints must be met. These probabilistic constraints were transformed into deterministic equivalents by inverting the probability constraints and replacing the technical coefficients by their means and variances. The variables were assumed to be normally distributed. The objective function of Näslund's model was the maximization of the expected value of the firm at the end of the planning period.

Chance-constrained programming is significant as a conceptually sound approach of considering risk. However, the constraints introduced by defining all probability distributions as normal and the inability to handle multiperiod, interrelated data sequentially discount its application to the model in this study.[1]

Dynamic Programming.--Eppen and Fama[2] coauthored three related articles concerned with the cash balance problem. Initially, a linear-programming model was developed to indicate the properties of optimal operating policies for stochastic cash-balance and simple dynamic portfolio problems. The objective of the model was to

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[1]For a further discussion of chance-constrained programming see A. Charnes and W. W. Cooper, "Chance-Constrained Programming," Management Science, October, 1959.

[2]Gary D. Eppen, and Eugene F. Fama, "Solutions for Cash-Balance and Simple Dynamic-Portfolio Problems," Journal of Business, XLI, No. 1 (January, 1968), 94-112.

minimize the discounted expected costs over some infinite horizon.

A later article[1] included a special adaptation of this model using a dynamic programming approach for establishing general properties of cash balance problems.

In their latest work, Eppen and Fama[2] discussed the determination of an optimal operating policy for a three asset stochastic cash balance problem. They retained the dynamic programming approach used in the above mentioned models, however. In contrast to these models, which considered holding cash in one of two forms--cash or earning assets, the model proposed in this article divided the non-cash holdings into two earning assets categories--"stocks" and "bonds." "Stocks" assets comprised the major portion of the firm's earnings, and were assumed to have higher expected returns per period than "bonds." "Stocks" also had higher transactions costs. Thus, it could pay the firm to hold some "bonds" as protection against fluctuations in the cash account. The article discussed this last point in some detail.

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[1]Gary D. Eppen, and Eugene F. Fama, "Cash Balance and Simple Dynamic Portfolio Problems with Proportional Costs," International Economic Review, X, No. 2 (June, 1969), 119-133.

[2]Gary D. Eppen, and Eugene F. Fama, "Three Asset Cash Balance and Dynamic Portfolio Problems," Management Science, XVII, No. 5 (January, 1971), 311-319.

Eppen and Fama presented a relatively sophisticated and realistic general approach to the cash management problem through utilization of the inventory method. Their approach is a recent, advanced and comprehensive presentation of the recursive relationships involved in the probabilistic dynamic programming solution to the cash management problem. The relationships were "solved" to develop a set of rules or policies specific to the conditions which exist in any particular situation.

The use of probabilistic dynamic programming in the above articles by Eppen and Fama led the author to explore this approach for possible use in his study. Their approach was theoretical, however. Practical application of probabilistic dynamic programming to the multi-stage, sequential cash management problem which incorporates risk will be possible only when a practical computer approach for solving dynamic programming problems is developed. Simulation was subsequently selected. Several models which utilize simulation will be presented in the following section.

Simulation.--Donaldson[1] developed a probabilistic approach to the evaluation of corporate debt capacity. The purpose involved determining the probability that interest

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[1]Gordon Donaldson, Corporate Debt Capacity (Boston: Harvard Business School, 1961), Chapter 7 and Appendix B.

on a given amount of debt could force a company into cash insolvency should a severe recession occur. The approach incorporated some random variables and some constants to determine the probability of an ending cash balance of less than zero. Certain cash outflows were deemed mandatory. The level of sales and average collection period on receivables were assumed to be statistically independent with a known combined probability distribution. Utilizing the model, management could ascertain the probability distribution of its ending cash balance during a recession.

Mao[1] adopted Donaldson's model and utilized it in a computer simulation experiment. By a simulation of the cash flows, the probability of cash insolvency was determined. The method involved the use of probability distributions of sales, resulting in the generation of a random sales level. A collection period was then determined for that level of sales and the ending cash balance was calculated. As a result of the simulation, a probability distribution of the ending cash balance was produced and the risk of insolvency determined.

Mao also developed a model to determine the pattern of financing to minimize the net interest cost in view of an anticipated net cash drain in the subsequent twelve month period. Mao introduced uncertainty into the model by randomizing the sales variable. All other variables

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[1]Mao, pp. 566-577.

were assumed to be known with certainty. Three alternative financing strategies were studied in the stated time horizon resulting in a frequency distribution of the net interest cost for each financing strategy. The probability distributions, when compared, would assist management in deciding upon the preferred financing strategy for their individual firm.

The basic model presented by Mao was considered in the preparation of the model in this study. The author elected to allow more of the variables to be defined as probabilistic. These variables were selected to approximate real world conditions. Further, whereas Mao did not elaborate on a recommended method for comparing the financing alternatives, this author suggests such a technique. The objective functions of the models also differ, but the basic approach remains similar.

Problems resulting from the significant degree of risk in the economic environment and the increasing complexity involved in financial policy decisions demand sophisticated methods of analysis. Simulation, as a technique of incorporating risk, has become increasingly appropriate.

CHAPTER III

A DECISION-THEORY STATEMENT OF THE PROBLEM

Objective

The objective of this study is to solve the multi-stage decision problem of determining an optimal short-term financing strategy under conditions of risk by optimizing the decision-maker's utility function. The decision problem is presented in this study in terms of a multi-stage decision model (DM). Initially, in order to explain and clarify the relationships involved in the multi-stage model, a single-stage deterministic model will be presented.[1] It will be subsequently amplified to include several stages and incorporate risk.

Single-Stage Decision Problem (DP) Under Certainty

Any situation where a decision is needed can be considered as a decision problem. This problem can be stated in terms of a decision model (DM). The model is composed of a decision structure and a search problem (SP).

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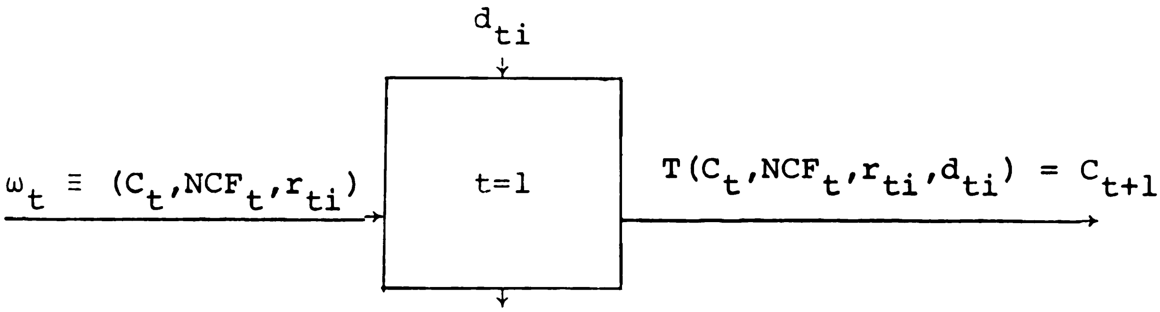
[1]The method of Carr and Howe [Charles R. Carr, and Charles W. Howe, Quantitative Decision Procedures in Management and Economics (New York: McGraw-Hill Book Company, 1964] was utilized in this discussion (pp. 38-40).

The decision structure[1] of this model can be expressed schematically as follows:



$$U(C_{t+1}) \equiv U[T(C_t, NCF_t, r_{ti}, d_{ti})]$$

$$\equiv W(C_t, NCF_t, r_{ti}, d_{ti})$$

where: C_t = set of input variables to stage t describing the relevant aspects of the decision-maker's environment prior tc making a decision at point t.[2] For example: cash balance at the beginning of stage t and interest on invested surplus cash that will be received at the beginning of stage t.

i = the financing alternative under consideration.

NCF_t = net cash flow in stage t, i.e., for the period of time from time point t to time point t+1 at which the next decision is to be made.

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[1]Although t, defining the number of periods, is actually superfluous in the single-stage model, the author elected to include it to familiarize the reader with the notation to be used in the subsequent multi-stage model descriptions. For the single-stage model t=1.

[2]It should be noted that stage t is a period of time and that the decision is to be made at the beginning of stage t, i.e., at point t.

r_{ti} = interest rate per appropriate length of time under alternative i for stage t.

d_{ti} = the amount of the minimum required cash balance specified at the beginning of stage t for the end of stage t.

T = transformation function

U = utility function

W = payoff function

<u>Decision Structure</u>.--The decision structure is composed of a state space, Ω_t, an action space, P_t, a transformation function, T, a utility function, U, and a payoff function, W, where:

State space $\Omega_t := \{\omega_t \equiv (C_t, r_{ti}, NCF_t)\}$

Under certainty, Ω_t is a single-element set containing the non-controllable values involved in the particular decision situation.

Action space P_t[1] $= \{d_{ti} \varepsilon R\}$[2]

P_t represents the set of all feasible actions $d_{ti} \varepsilon P_t$ (specifying minimum required cash balance values) and the particular decision situation. The action space states the feasible actions for the particular decision situation. As the stage progresses, d_{ti} will influence the amount the firm borrows or reinvests.

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[1]Although it is possible that the action space, P_t, could be different for each financing alternative, i, inferring that P_{ti} should be used, P_t will be used to simplify this discussion.

[2]R is defined as the set of real numbers.

Transformation function T

$T: (\Omega_t \times P_t) \rightarrow C_{t+1}$ expresses the consequences of the non-controllable value $\omega_t \varepsilon \Omega_t$ and the decision $d_{ti} \varepsilon P_t$. $T(\omega_t, d_{ti}) = c_{t+1} \varepsilon C_{t+1}$ where C_{t+1} is the set of all possible consequences of the interaction of the state space Ω_t and the action space P_t, and c_{t+1} is a single consequence. The transformation function, $T(C_t, NCF_t, r_{ti}, d_{ti}) = C_{t+1}$, transforms the inputs to stage t and the decision in stage t into the output state.[1]

Utility function U

$U: C_{t+1} \rightarrow R$ is a mapping such that $(U(C_{t+1})$ is the utility of a consequence of $c_{t+1} \varepsilon C_{t+1}$ to the decision-maker. Alternatively, $U(C_{t+1}) \equiv U[T(C_t, NCF_t, r_{ti}, d_{ti})]$ measures the utility of the transformation of the input state to the output state.

Payoff function W

$W: (\Omega_t \times P_t) \rightarrow R$ expresses the result of the interaction of the state space and the action space in terms of a payoff to the decision-maker. The payoff function, $W(C_t, NCF_t, r_{ti}, d_{ti})$, is defined in terms of utility as follows: $W: (\Omega_t \times P_t) \rightarrow R$, such that $U(C_{t+1}) \equiv U[T(C_t, NCF_t, r_{ti}, d_{ti})] \equiv W(C_t, NCF_t, r_{ti}, d_{ti})$.

SP.-- The search problem is to optimize the utility function, U, over all of the decisions, d_{ti}, in the action space, P_t. Thus:

For a given i, i=1,...,4

$$\max_{d_{ti}} U[T(C_t, NCF_t, r_{ti}, d_{ti})].$$

---

[1]The output state becomes the input state for the subsequent period t+1.

The optimal[1] decision for each i is denoted by d^*_{ti} and the optimum utility as $U[T(C_t, NCF_t, r_{ti}, d^*_{ti})]$. Then the $U[T(C_t, NCF_t, r_{ti}, d^*_{ti})]$ for each i are compared so as to:

$$\max_i U[T(C_t, NCF_t, r_{ti}, d^*_{ti})].$$

Result.--The SP solution gives the following results :

1. The optimal decision, $d^*_{ti^*}$, with respect to the utility function, U.
2. The optimal value, $U(C_{t+1}) \equiv U[T(C_t, NCF_t, r_{ti}, d^*_{ti^*})]$, of the utility function with respect to all possible actions in the action space, P_t.

Model of the Single-Stage Problem Under Risk

The single-stage deterministic model will now be modified to incorporate risk. The net cash flow and the interest rates will be assumed to be unknown prior to making the decision in stage t, and defined as random (stochastic) variables. The model of this decision structure is represented schematically in the following diagram:

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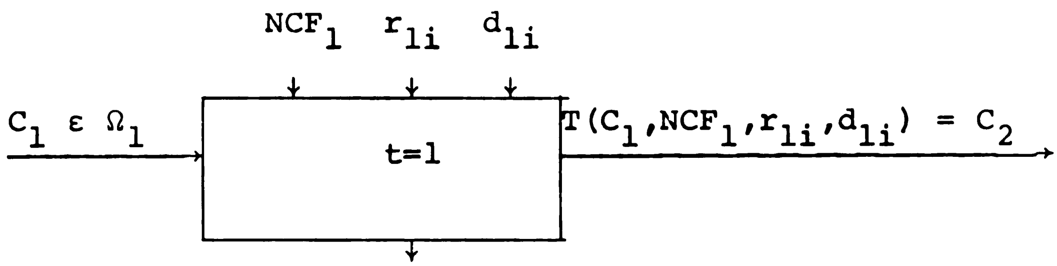
[1]Asterisks denote optimal values.



$$U(C_2) \equiv U[T(C_1,\ NCF_1,\ r_{1i},\ d_{1i})]$$

$$\equiv W(C_1,\ NCF_1,\ r_{1i},\ d_{1i})$$

Decision Structure.--The state space is modified to indicate that the net cash flow, and the interest rates are unknown. The action space remains the same

SP.--The SP is now concerned with the *expected* value of the utility function. Mathematically, these modifications can be incorporated to result in the following relationships:

For a given i, i=1,...,4

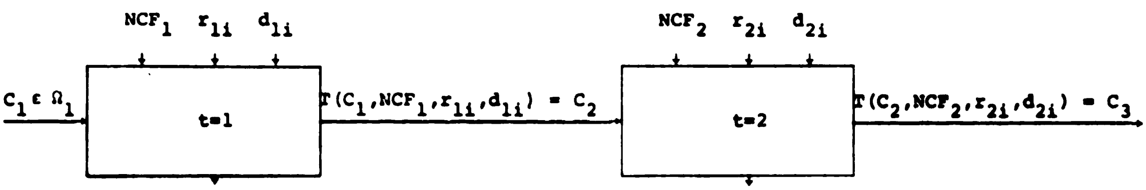
$$\max_{d_{1i}} E[U[T(C_1,\ NCF_1,\ r_{1i},\ d_{1i})]\,|\,i].$$

The optimal decision for each i is denoted by d^*_{1i} and the optimal expected utility as $E[U[T(C_1,\ NCF_1,\ r_{1i},\ d^*_{1i})]\,|\,i]$. Then the $E[U[T(C_1,\ NCF_1,\ r_{1i},\ d^*_{1i})]\,|\,i]$ for each i are compared so as to $\max_i E[U[T(C_1,\ NCF_1,\ r_{1i},\ d^*_{1i})]\,|\,i]$.

Result.--The result is the optimal decision, d^*_{1i*}, and the optimum expected utility, $E[U[T(C_1,\ NCF_1,\ r_{1i},\ d^*_{1i*})]]$, for that decision structure.

Multi-Stage Decision Problem Under Risk

The model is now expanded to include more than one stage. The resulting model will incorporate random (stochastic) variables and multiple stages. The decision structure is represented schematically for two stages as follows:



$U(C_2) = U[T(C_1,NCF_1,r_{1i},d_{1i})]$ $\qquad$ $U(C_3) = U[T(C_2,NCF_2,r_{2i},d_{2i})]$

$= W(C_1,NCF_1,r_{1i},d_{1i})$ $\qquad$ $= W(C_2,NCF_2,r_{2i},d_{2i})$

where: $\Omega_t = \{C_t\}$; C_t = multiple-element set containing the non-controllable values indicating the state of the system for stage t. Ω_t is known immediately prior to making the decision in stage t.

C_1 = the input state to stage one, where $C_1 \varepsilon \Omega_1$

C_2 = the output from stage one which serves as the input state to stage two, where $C_2 \varepsilon \Omega_2$

C_3 = the output from stage two that will serve as the input state to stage three, where $C_3 \varepsilon \Omega_3$.

Decision Structure.--The state space is expanded to include the variables in stage two as well as those in stage one. The action space is similarly expanded to include both stages. The utility function also incorporates both stages. The payoff function for each stage is related to only the variables within the stage to which it applies.

SP.--The SP is to maximize the expected utility for the final stage in the multi-stage problem. The utility of the final stage is expressed as $U(C_3) \equiv U[T(C_2, NCF_2, r_{2i}, d_{2i})]$. Since the intermediate value $C_2 = T(C_1, NCF_1, r_{1i}, d_{1i})$, the latter expression can be entered into the preceding expression in place of C_2:

$$U(C_3) \equiv U[T[T(C_1, NCF_1, r_{1i}, d_{1i}), NCF_2, r_{2i}, d_{2i}]].$$

In general, T* may be used to denote a composite of multiple transformation functions. In the preceding two-stage problem, this composite transformation function is described as follows:

$$T^*[C_1, NCF_1, r_{1i}, d_{1i}, NCF_2, r_{2i}, d_{2i}] =$$

$$T[T(C_1, NCF_1, r_{1i}, d_{1i}), NCF_2, r_{2i}, d_{2i}].$$

For a given i, i=1,...,4

$$\max_{d_{1i}, d_{2i}} E[U[T^*(C_1, NCF_1, r_{1i}, d_{1i}, NCF_2, r_{2i}, d_{2i})] | i].$$

Then:

$$\max_i E[U[T^*(C_1, NCF_1, r_{1i}, d^*_{1i}, NCF_2, r_{2i}, d^*_{2i})] | i].$$

Result.--The result is the optimal set of decisions, $d^*_{i^*}$, where $d^*_{i^*} = (d^*_{1i^*}, d^*_{2i^*})$, and the optimum expected utility of the decision structure,
$E[U[T^*(C_1, NCF_1, r_{1i}, d^*_{1i^*}, NCF_2, r_{2i}, d^*_{2i^*})]]$.

The N-Stage Decision Problem

The preceding description of the two-stage problem may be expanded to include any N number of stages:

$$\text{Let } NCF = (NCF_1, \ldots, NCF_N) \in Y \subset R^N$$

$$d_i = (d_{1i}, \ldots, d_{Ni}) \in P_1 \times P_2 \times \ldots \times P_N \equiv P \subset R^N$$

$$r_i = (r_{1i}, \ldots, r_{Ni}) \in Z \subset R^N$$

The transformation function can also be restated as:

$$T: (\Omega_1 \times Y \times Z \times P) \rightarrow \Omega_{N+1} \text{ where}$$

$$T(C_1, NCF, r_i, d_i) = C_{N+1} \in \Omega_{N+1} \text{ for all}$$

$$(C_1, NCF, r_i, d_i) \in (\Omega_1 \times Y \times Z \times P)$$

The solution to the N-stage problem is the set of decisions, $d^*_{i^*}$, where $d^*_{i^*} = (d^*_{1i^*}, \ldots, d^*_{Ni^*})$, and the optimum expected utility of the decision structure,

$E[U[T^*(C_1, NCF, r_i, d^*_{i*})]]$, where T^* is the composite of the N individual transformation functions.

The Decision Problem to be Analyzed in This Study

The objective of the model in this study is to evaluate short-term financing under conditions of risk. The criterion for decision is the expected utility of the unrestricted ending cash balance of the firm at the end of the final stage in the planning period.[1]

It is assumed that the minimum required cash balance has been specified. Given this required minimum, the model determines that financing alternative which maximizes the expected utility. Hence, the decision for this study is a subset of the total decision problem of determining the optimal minimum required balance level and optimal financing alternative. This total multi-stage decision problem is discussed in the following section. Subsequently, the decision problem in this study will be presented in terms of that total problem.

The multi-stage probabilistic decision structure is represented schematically as follows:

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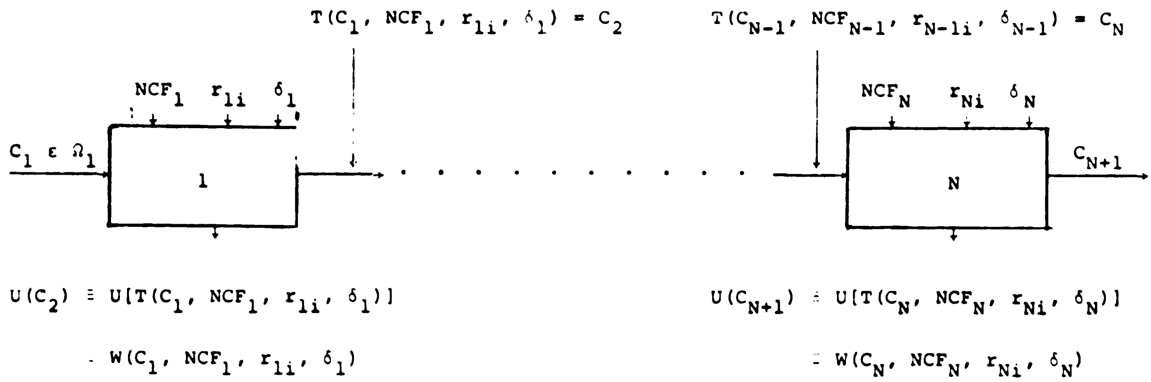
[1]The unrestricted ending cash balance is defined as the ending cash balance less that portion of cash that is restricted to the future payments of principal and interest specific to each financing alternative, plus the amount of interest and principal due from invested surplus cash.



where the following definitions hold for t=1,...,N:

$C_t = \{C_{t1i}, C_{t2i}, C_{t3i}, C_{t4i}\}$

C_{t1i} = cash balance at the beginning of stage t under financing alternative i.

C_{t2i} = amount of interest and principal payable at the beginning of stage t under financing alternative i.

C_{t3i} = amount of interest and principal receivable from invested surplus cash at the beginning of stage t under financing alternative i.

C_{t4i} = maximum amount available for borrowing during stage t under financing alternative i.

NCF_t = net cash flow random variable for stage t.

r_{ti} = interest rate random variable for stage t under financing alternative i.

δ_{ti} = a strategy function such that:

$$\delta_{ti}\underbrace{(C_1, d_{1i}, NCF_1, \ldots, d_{t-1i}, NCF_{t-1})}_{h_{ti}} \equiv d_{ti} \varepsilon P_t$$

This δ_{ti} is a strategy function for determining the value of the minimum required cash balance in stage t for financing alternative i. Thus, all essential prior history of the process, h_{ti}, is known immediately before the decision is made.

T = transformation function

U = utility function

W = payoff function

and let:

$$NCF \equiv (NCF_1, \ldots, NCF_N)$$

$$r_i \equiv (r_{1i}, \ldots, r_N)$$

$$\delta_i \equiv \{\delta_{1i}, \ldots, \delta_{Ni}\}$$

where $\delta_i(h_i, \ldots, h_t, \ldots, h_N) \equiv$

$(d_{1i}, \ldots, d_{ti}, \ldots, d_{Ni}) \equiv d_i$ and,

$h_t \equiv (C_1, d_{1i}, NCF_1, \ldots, d_{t-1i}, NCF_{t-1})$

Decision Structure.--

State space $\Omega_t = \{C_t\}$

Action space[1] $\Delta = \{\delta_i\}$

Utility function U, where $U(C_{N+1}) \equiv$

$U[T(C_N, NCF_N, r_{Ni}, \delta_{Ni})] \equiv$

$U[T^*(C_1, NCF, r_i, \delta_i)]$.

SP.--

For a given i, i=1,...,4

$$\max_{\delta_i} E[U(C_{N+1})|i] = E[U(T^*(C_1, NCF, r_i, \delta_i))|i].$$

[1]The action space is now a set of strategies, Δ, as opposed to the earlier definition as a set of actions, P.

The optimal strategy for a given alternative i is denoted by δ^*_i.

Then:

$$\max_i \{\max_{\delta_i} \{E[U(C_{N+1})|i]\}\}$$

$$= \max_i E[U[T^*(C_1, NCF, r_i, \delta^*_i)]|i].$$

Result.--The result is the optimal strategy function, $\delta^*_{i^*}$, where $\delta^*_{i^*}(h_1,\ldots,h_t,\ldots,h_N) \equiv d^*_{i^*}$, and $d^*_{i^*} = (d^*_{1i^*},\ldots,d^*_{Ni^*})$, and the optimum expected utility of the decision structure, $E[U[T^*(C_1, NCF, r_i, \delta^*_{i^*})]]$.

The Model in This Study as An Analysis of a Subset, Δ^g_i, of the Strategies δ_i in Δ

In this study, certain minimum required cash balance strategy functions, δ^g_i, were evaluated. These δ^g_i are elements of a set of minimum required cash balance strategy functions, Δ^g, where $\Delta^g_i = \{\delta^g_i\}$[1] and Δ^g is a subset of Δ. The evaluation procedure utilized in this study consisted of evaluating the financing alternatives for a set of strategy functions which resulted from the selected set of minimum required cash balance values.[2]

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[1]The value of a strategy function is a set of minimum required cash balances. For example: {($3,000,000),...,($3,000,000)}. This indicates that the minimum required cash balance is $3,000,000 for each period.

[2]See page 83, Chapter V for a discussion of the minimum required cash balances selected for evaluation and presentation in this study.

The output state of the final stage was determined for each financing alternative. Then the alternatives were compared and an optimal alternative selected. This process was repeated for each set of minimum required cash balances considered in the study. In notational form the following procedure was followed for each set of minimum required cash balances:

For each i, i=1,...,4, determine $E[U(C_{N+1})|\Delta_i^g]$.

Then:

$$\max_i E[U(C_{N+1})|\Delta_i^g].$$

Result.--The result is the optimal financing alternative i* and the optimum utility $E[U(C_{N+1})|\Delta_i^{g}*]$.

The results may be suboptimal since: $E[U(C_{N+1})|\Delta_i^{g}*] \leq E[U(C_{N+1})|\Delta_i*] \leq$ the expected value that would result from an optimal combination of financing alternatives and minimum required cash balance strategies $\leq$ the expected value that would result if the firm were allowed to utilize combinations of financing alternatives or change alternatives from period to period.

Assumptions

1. The beginning balances of C_{t1i}, C_{t2i}, C_{t3i}, C_{t4i}, are known with certainty by management.
2. Management specifies the minimum amount of cash that the firm desires to hold in each stage.

3. Borrowing is effected at a given annual rate of interest, different for the different types of borrowing. The interest rates are probabilistically defined, and management will decide the type of distribution which best describes the interest rates for each stage.
4. Borrowing constraints will be imposed, specific to the type borrowing.
5. Borrowing shall be accomplished as needed during the stage in which the need arises, the amounts of which will follow from the specification of the minimum required cash balance.
6. Funds acquired through borrowing are repayable, including interest owed, at the beginning of the stage due unless otherwise stated.
7. All surplus cash will be invested in an earning asset at the beginning of the stage available.
8. Management will project the type of distribution for the interest rate on reinvested cash.
9. All invested surplus cash will be available, plus interest, at the beginning of the stage following the stage in which it was invested.

10. The model considers the same financing alternative for all thirteen periods. The alternatives are handled separately and no combinations are allowed. The periods are examined sequentially.

Probabilistic Dynamic Programming[1] as a Possible Approach to Solving the General Stochastic Multi-Stage Model

The general model, presented in the preceding section, is defined as stochastic in nature in that it contains variables that are described as random. These variables are expressed in the form of probability distributions. The effect of these random variables makes the solution to a multi-stage problem appreciably more difficult when the variables are considered simultaneously. Thus, methods which allow for sequential solution are preferable to those that do not. Two methods have been generally discussed in the literature for use in solving this type of problem--probabilistic dynamic programming and simulation. Probabilistic dynamic programming, an optimization technique, is worthy of discussion even though, without a practical computer approach for solving complex dynamic programming problems, it is not suitable for routine use.

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[1]George L. Nemhauser, Introduction to Dynamic Programming (New York: John Wiley and Sons, Inc., 1966), pp. 153-158.

The stochastic, multi-stage model consists of an N-stage stochastic system with random variables that affect the utility function and transformation function at each stage. The following definitions are made:

h_t = input state to stage t (history of the process to stage t)

δ_{ti} = decision strategies in stage t

$$\left.\begin{array}{l} NCF_t = \text{net cash flow} \\ r_{ti} = \text{interest rates} \end{array}\right\} \text{random variables in stage } t$$

$U_t = U_t(h_t, NCF_t, r_{ti}, \delta_{ti})$ - utility function for stage t

$h_{t-1} = T_t(h_t, NCF_t, r_{ti}, \delta_{ti})$ - transformation function for stage t

The subscript t refers to that stage where there are t stages left in the process. NCF_t and r_{ti} are assumed to be independently distributed with probability distributions $P_t(NCF_t)$ and $P_t(r_{ti})$. Probabilistic dynamic programming also assumes that the total utility (R_N) for the N stage problem is equal to the sum of the utility of each stage.

$$R_N(C_N,\ldots,C_1, NCF_N,\ldots,NCF_1, r_{Ni},\ldots,r_{1i}, \delta_{Ni},\ldots,\delta_{1i})$$

$$= \sum_{t=1}^{N} U_t(C_t, NCF_t, r_{ti}, \delta_{ti})$$

$f_t(C_t)$ = the maximum expected utility as a function of the input state C_t

$$f_t(C_t) = \max \Sigma \Sigma P_t(NCF_t) \cdot P_t(r_{ti})[U_t(C_t, NCF_t, r_{ti}, \delta_{ti})$$

$$+ f_{t-1}(T_t(C_t, NCF_t, r_{ti}, \delta_{ti}))], \text{ where } t=2,\ldots,N$$

and

$$f_1(C_1) = U_1(C_1, NCF_1, r_{1i}, \delta_{1i}), \text{ where } t=1.$$

Probabilistic dynamic programming presents a feasible approach to the stochastic multi-stage problem. Some probabilistically defined variables in this study exhibit interdependence between stages, however. This interdependence does not necessarily discount solution of the model by probabilistic dynamic programming. Rather, the interdependence of the net cash flows between periods necessitates the redefinition of the probability distributions, $P_t(NCF_t)$, as conditional probabilities, $P_t(NCF_t \mid \overline{NCF}_{t-1})$, where $\overline{NCF}_{t-1}$ denotes the net cash flows prior to period t.

Since additional state variables must be added to account for the interdependence, the problem becomes more difficult and time-consuming to solve.[1] Similarly, the financing alternatives affect more than one stage and the additional state variables necessary again complicate solution. Until a practical computer approach for solving

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[1]Each additional state variable expands the recursive relationships and causes the problem to become more complex.

large-scale probabilistic dynamic programming models is developed, the problems remain computationally complex and expensive.[1]

Simulation

Simulation provides another method for solving the multi-stage probabilistic problem. Simulation is defined as ". . . a numerical technique for conducting experiments with certain types of mathematical models which describe the behavior of a complex system on a digital computer over extended periods of time."[2] As in probabilistic dynamic programming, simulation allows the multi-stage stochastic process to reveal itself sequentially. A solution may be achieved by maximizing the final stage utility function.

Since the net cash flow of the model in question involves random variables as well as interdependence of those variables (lag structures), analytical solutions are difficult. Naylor notes that even though an analytical solution may exist for a particular model, it may be less costly in terms of analyst and computer time to run a simulation. That is, the additional information gained from an analytical solution may not be sufficient to justify

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[1] Orgler, p. 41.

[2] Thomas H. Naylor, Computer Simulation Experiments with Models of Economic Systems (New York: John Wiley and Sons, Inc., 1971), p. 2.

the search time involved.[1] In this particular model, the net cash flow determination provided a major point of discussion. The author explored various techniques and elected to run a simulation. The simulation inputs were obtained from real-world data as discussed in Chapter IV.

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[1] *Ibid.*, p. 5.

CHAPTER IV

MODEL DESCRIPTION

Introduction

The model to be presented in this study is designed to assist management in the evaluation of alternative short-term financing strategies. The criterion for decision is the maximization of the expected utility of the unrestricted ending cash balance.[1] The maximization of the unrestricted ending cash balance is assumed in this study to be synonymous with minimizing the net financing cost to the firm.

The financing requirement of a firm in a given planning horizon[2] is determined by the initial cash balance and the projected net cash flows. Given a need to effect financing, four alternative strategies are evaluated by the model, and an optimal strategy selected. Input to the model for each period consists of the constraints on the financing alternatives, the distribution of the interest rate for each financing alternative, the net cash

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[1]See p. 32, Chapter III for a definition of the unrestricted ending cash balance.

[2]The planning horizon in this study will consist of thirteen periods (or stages) with each period being four weeks in length.

flow distribution, and the minimum cash balance specified by management.

The effect on the cash balance for each period specific to the individual financing alternative is calculated utilizing a simulation program. The cash balance is affected, for example, by any principal and interest payments made in the period, by the investment of surplus cash in marketable securities or by the conversion of marketable securities into cash. At the end of the thirteenth period the unrestricted ending cash balance for each financing alternative is determined. The unrestricted ending cash balance must be calculated for many different values of the variables associated with each financing alternative because of the probabilistic nature of the cash flows and interest rates. Ultimately, two-hundred values of the unrestricted ending cash balances for each financing alternative are generated. These two-hundred values are punched on cards by the computer. The cards are then sorted and a cumulative probability distribution drawn for each alternative. The four resulting distributions are then compared through the use of stochastic dominance, and an optimal short-term financing alternative selected. A general diagram of the model can be seen in Figure 1. The following definitions correspond to the diagram:

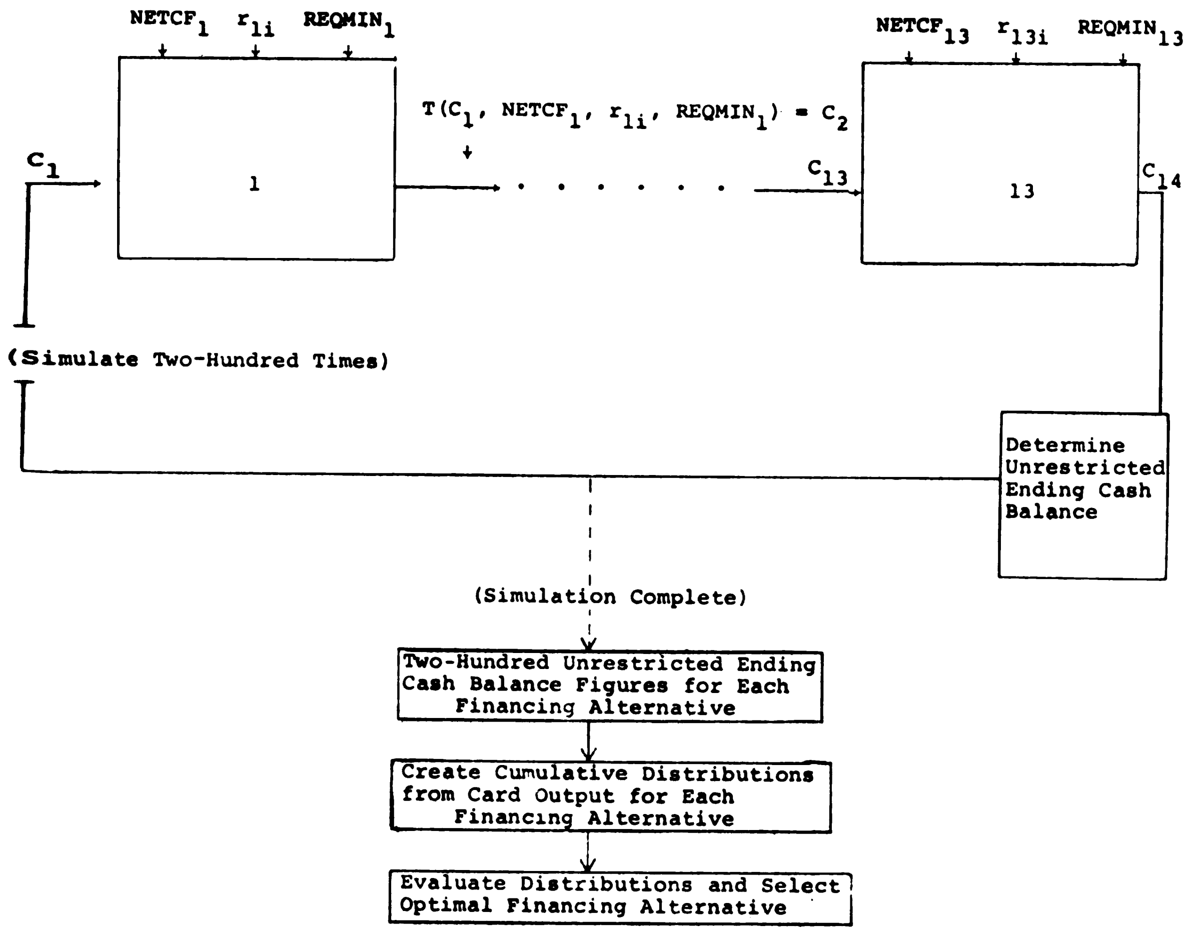

Figure 1.--Diagram of the model.

i = the financing alternative being evaluated

$NETCF_t$ = the net cash flow in period t

r_{ti} = the interest rate in period t for financing alternative i

$REQMIN_t$ = the minimum required cash balance for period t

The following designations have been changed to correspond to the notation in the computer program:

$NETCF_t$ = the net cash flow in period t, and replaces NCF in the preceding models.[1]

$REQMIN_t = \delta_{ti}(h_t) \equiv d_{ti}$ = the minimum required cash balance. $REQMIN_t$ replaces δ_t in the preceding models.

Methodology

The model considers the same financing alternative for all thirteen periods in the planning horizon. The periods are examined sequentially. The alternatives are handled separately and no combinations are allowed. For the first period, the input data are considered to determine whether excess cash exists to be invested, or whether the short-term financing alternative being evaluated should be implemented. If short-term financing is needed, the constraints applied to that alternative are initiated. The output consists of the cash balance, interest and principal

[1]See Chapter III.

payable in future periods, any surplus cash receivable in future periods under that alternative and any interest on that invested surplus cash.

The alternative i=1 is applied to each of the thirteen periods. At the end of the thirteenth period, the unrestricted ending cash balance under i=1 is determined. This process is repeated for all four financing alternatives (i=1,...,4). Four unrestricted ending cash balances, one for each alternative, result. If all values for the variables in the model were deterministic, the analysis would be complete at this point. The four values would be compared and an optimal financing alternative selected.

However, a number of the variables are probabilistic and, as such, are defined by probability distributions. Therefore, the entire analysis must be repeated a number of times. In this model, the analysis is run two-hundred times for each financing alternative resulting in a probability distribution which is an empirical estimate of the theoretical distribution for each alternative. These distributions are then compared, and an optimal alternative selected by stochastic dominance.

The Transformation Function

The transformation function is responsible for converting the input state of period t, C_t, and any transactions in that period into the output state for period t, C_{t+1}. The transformation function states the relationships between the input state, the net cash flow, the amount of money borrowed or invested, the interest rate and the minimum required cash balance for the period to which it applies. Thus, the transformation function is stated as $T(C_t, NETCF_t, r_{ti}, REQMIN_t) = C_{t+1}$. The actions of the transformation function can be divided into four steps, as follows:

1. The transformation function determines the potential cash balance[1] from the input state for period t, C_t, and the value of the net cash flow and minimum required cash balance[2] at the beginning of period t.

PCASBAL(t) = CASH(t-1) + NCF(t)

[NCF(t) = NETCF(t) + INVINC(t) + INVSPC(t-1) - FINTRN(t)]

where:

NCF(t) = adjusted net cash flow for period t

PCASBAL(t) = potential cash balance for period t prior to any short-term financing in period t

NETCF(t) = net cash flow in period t, a random variable

INVINC(t) = interest received at beginning of period t on surplus cash invested in the prior period t-1

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[1]The potential cash balance for a period consists of the ending cash balance of the previous period plus the net cash flow for that period and any adjustments necessary as a result of prior financing arrangements.

[2]See p. 76 of this chapter for a discussion of the minimum required cash balance.

INVSPC(t-1) = surplus cash invested in period t-1

FINTRN(t) = payment of principal and interest at the beginning of period t

CASH(t-1) = ending cash balance for period t-1

C_t = input state consisting of: beginning cash balance in period t; any interest on invested surplus cash that will be received at the beginning of period t; invested surplus cash that will be converted back to cash at the beginning of period t; the amount of interest and principal payments at the beginning of period t for the financing alternative under consideration; and the amount available for borrowing in period t under financing alternative i.

2. The transformation function then determines the difference between the potential cash balance and the minimum required cash balance:

DIF(t) = PCASBAL(t) - REQMIN(t)

where:

DIF(t) = difference between the potential cash balance and the minimum required cash balance

REQMIN(t) = minimum required cash balance for period t.

3. The DIF(t) is then evaluated. If the DIF(t) is positive, the excess surplus cash is invested. This invested surplus is converted to cash at the beginning of the subsequent period. The interest on this invested cash is assumed to be received at the beginning of the subsequent period.[1] If the DIF(t) is negative, the short-term

[1]See Figure 2.

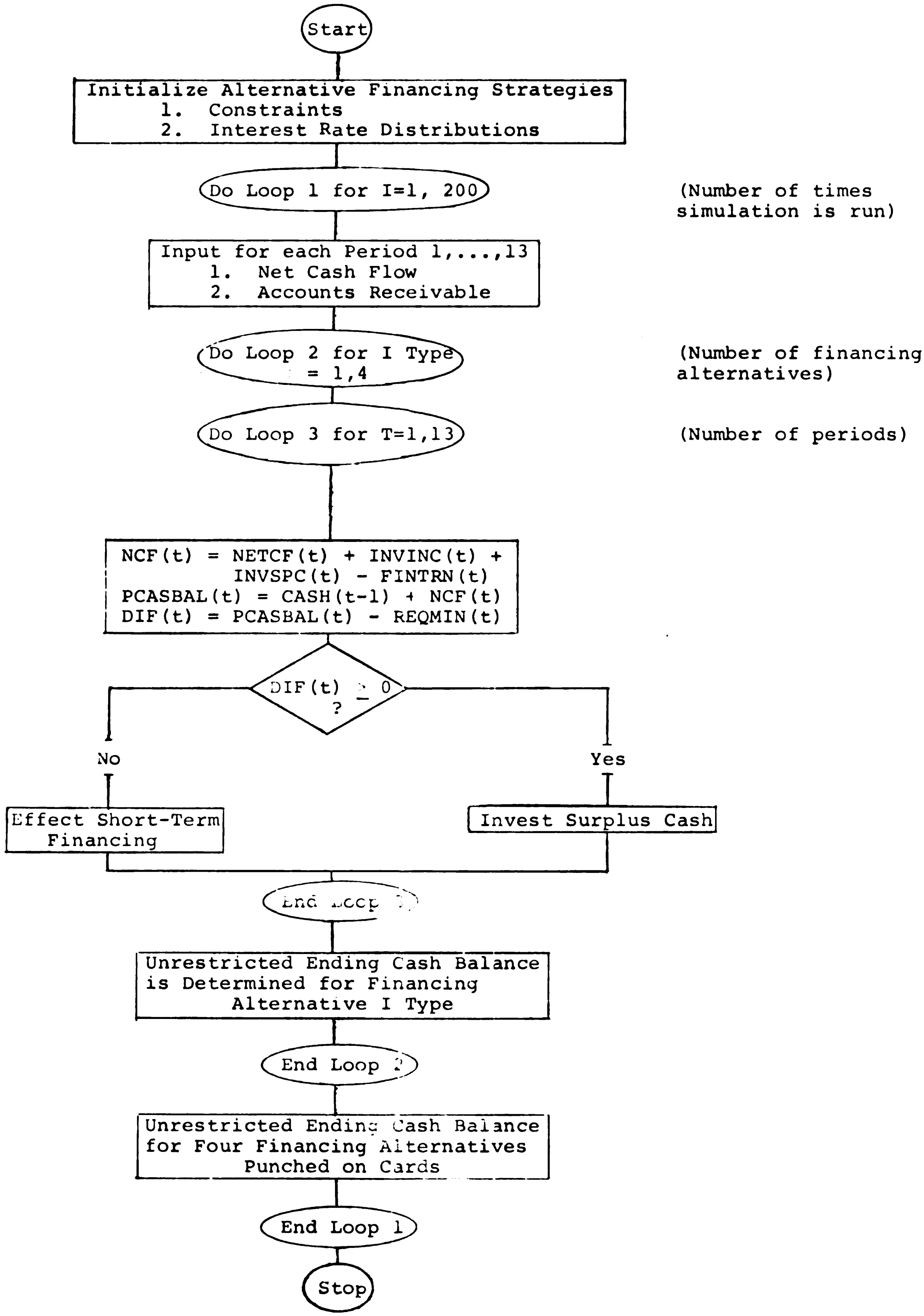

Figure 2.--A general flow chart of the financing alternatives simulation.

financing alternative under consideration is implemented. The amount to be borrowed is determined, as well as the payment schedule for the principal and interest.

4. The output state for period t, C_{t+1}, is then determined. It consists of the cash balance at the beginning of the period t+1, the amount of surplus cash invested in period t and interest on that surplus cash (both of which will be received at the beginning of the period t+1), and the amount of interest and principal resulting from previous short-term financing payable at the start of period t+1.

Four financing alternatives are included in the model. In the subsequent section, these short-term financing alternatives will be discussed. Further, the techniques of evaluating these alternatives using the computer will be presented.

The Short-Term Financing Alternatives

Four financing alternatives are considered by the model--line of credit, term loan, commercial paper, and accounts receivable loan.

Line of Credit.--A line of credit consists of an informal agreement between a commercial bank and the customer regarding the maximum amount of unsecured credit the bank will allow the customer at any one time. The bank requires that the customer maintain a cash balance at the bank, the amount of which is directly proportional to either the amount of funds borrowed, or the amount of the commitment.

Term Loan.--A term loan is a formal loan agreement effected at a commercial bank. Repayment of the loan and interest payments are made in regular periodic installments. The duration of the loan may be up to ten years but the minimum and maximum limits are arbitrary. For the purposes of this study, the loan will extend over six periods.

Commercial Paper.--Commercial paper is issued by a firm in the form of unsecured short-term negotiable promissory notes which are sold in the money market. Unlike the prime rate or bank loan, commercial paper rates fluctuate considerably reflecting money market conditions. Issuance of commercial paper is generally a less expensive means of obtaining funds than a short-term loan from a commercial bank. However, only the most credit-worthy firms can issue it.

Accounts Receivable Loan.--The accounts receivable constitute one of the most liquid assets of a firm. The commercial bank will allow a firm to borrow utilizing their accounts receivable as security for the loan. The bank assesses the quality and size of the receivables to determine the percentage of the face value that can be borrowed. The accounts receivable loan is a more or less continuous financing arrangement with the maximum amount of the loan allowable varying with the amount and quality of the accounts receivable of the firm.

Computer Evaluation of the Financing Alternatives

Line of Credit.--The maximum amount that can be borrowed on a line of credit is by definition a deterministic value at any point in time. For the purposes of this study, that value is assumed to remain constant for each period in the planning horizon. The maximum amount that can be borrowed with this financing alternative, and the distribution of interest rates for each period form the input data to the model. The model then determines if the maximum amount that can be borrowed is sufficient to cover the financing needs of the firm. If it is sufficient, the amount needed is borrowed. It it is not sufficient, the maximum available is borrowed and the remainder is classified as a stockout.[1,2] The cash balance for each period then is equal to the potential cash balance

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[1]A stockout indicates that a less than optimal quantity of cash can be obtained and results in an unrestricted ending cash balance that is below that achievable when the desired amount can be borrowed.

[2]See Figure 3.

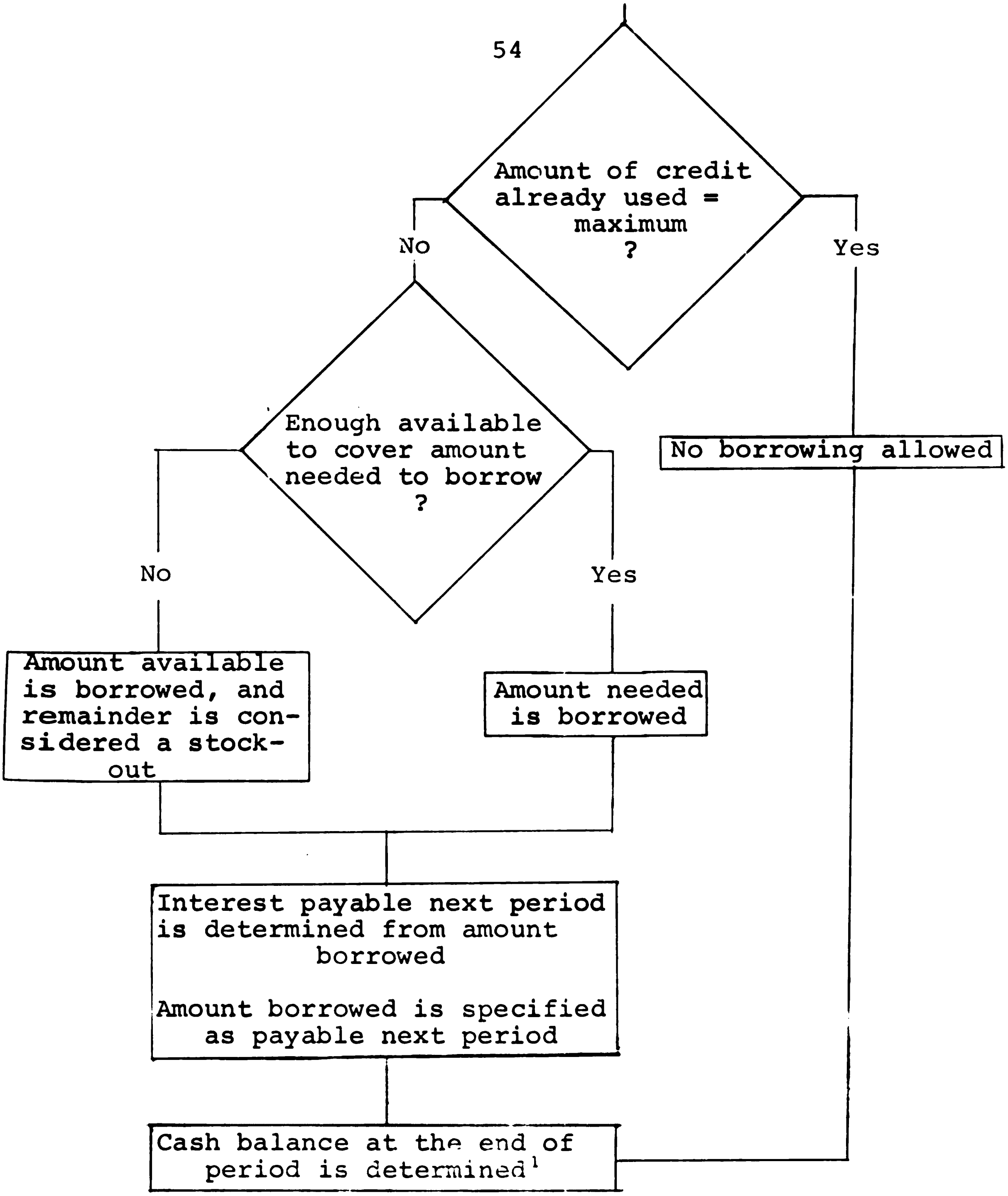

Figure 3.--Line of credit.

[1]CASH(t) = PCASBAL(t) + DIF(t)

[2]The flow chart is for each period.

plus the amount borrowed. At the end of the thirteenth period, the unrestricted ending cash balance is equal to the cash balance in the thirteenth period, plus interest on invested surplus cash not received, plus invested surplus cash, less principal and interest due the subsequent period.

Term Loan.--Input data for the term loan financing alternative include: (1) the maximum and minimum amounts that can be borrowed each period; (2) the maximum amount that can be outstanding at any time; and (3) the distribution of interest rates for each period.[1]

The model evaluates the amount needed in terms of the maximum and minimum constraints on the amount that can be borrowed as follows: If the amount available is greater than the amount needed and:

1. The amount needed is less than the minimum, then nothing can be borrowed and the needed amount is considered a stockout.
2. The amount needed is greater than the maximum, then the maximum is borrowed and the remainder is considered a stockout.
3. The amount needed is between the maximum and the minimum, then the amount needed is borrowed.

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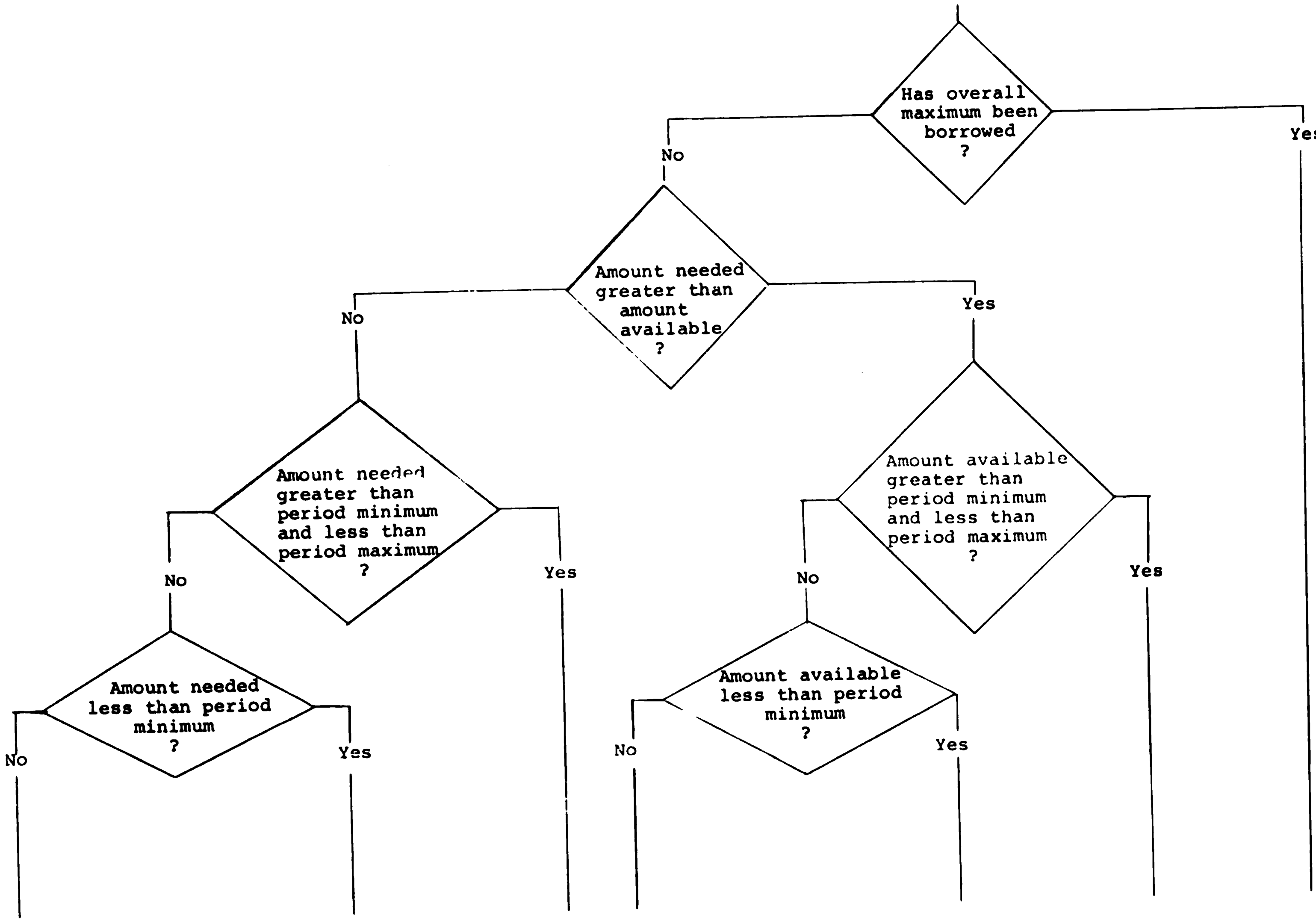
[1]See Figure 4.


Has overall maximum been borrowed ?
Yes
No
Amount needed greater than amount available ?
Yes
No
Amount available greater than period minimum and less than period maximum ?
Yes
No
Amount available less than period minimum ?
Yes
No
Amount needed greater than period minimum and less than period maximum ?
Yes
No
Amount needed less than period minimum ?
Yes
No

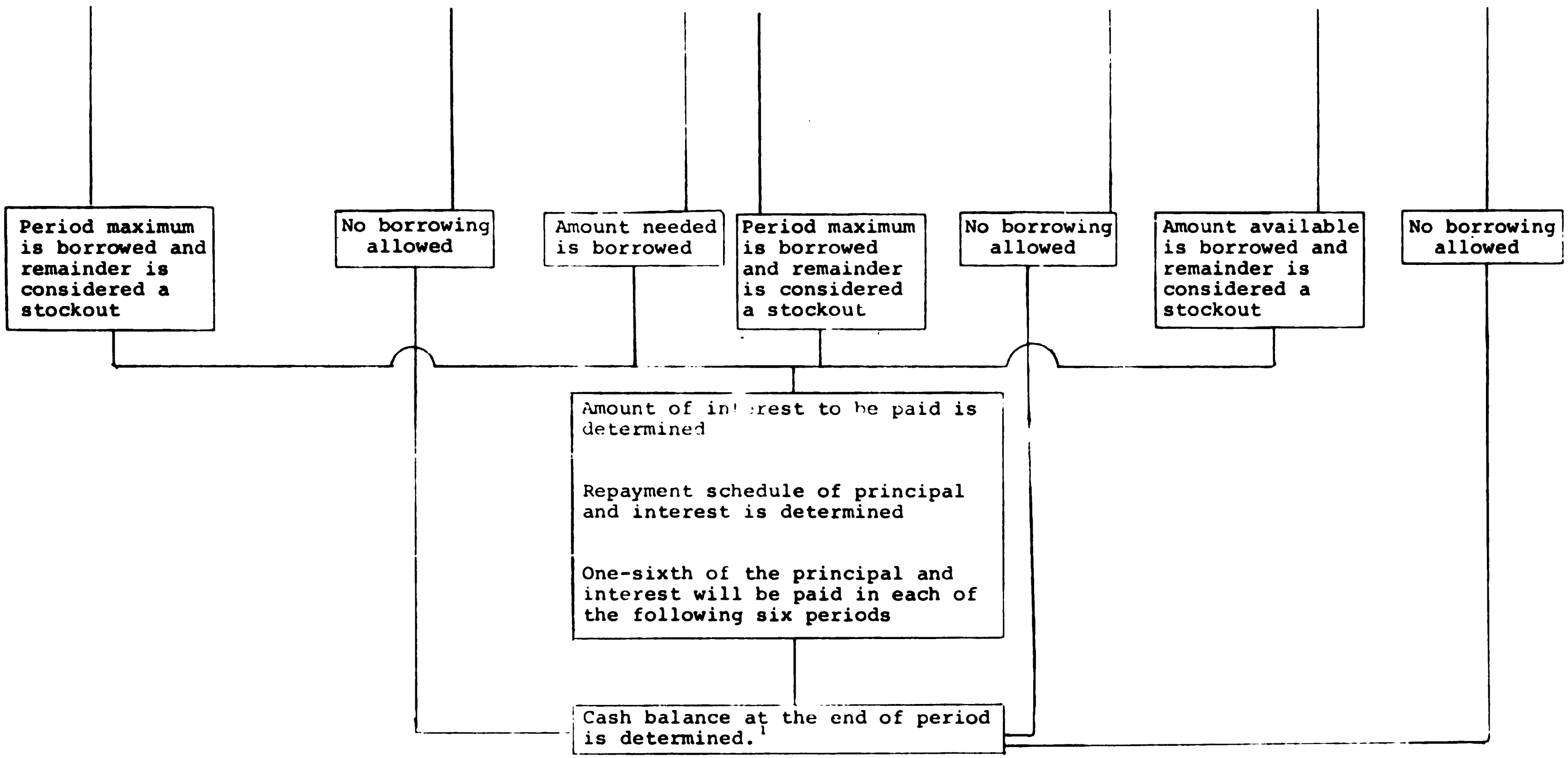


Figure 4.--Term loan.[2]

[1]CASH = PCASBAL(t) + DIF(t)

[2]The flow chart is for each period.

If the amount available is less than the amount needed and:

1. The amount needed is less than the minimum, then nothing can be borrowed and the needed amount is considered a stockout.
2. The amount available is greater than the maximum, then the maximum is borrowed and the remainder is considered a stockout.
3. The amount available is between the maximum and the minimum, the amount available is borrowed and the remainder is considered a stockout.

One-sixth of the principal and interest due is to be paid each period. The program determines the dollar amount to be paid as principal and interest. The cash balance for each period is equal to the potential cash balance plus the amount borrowed. At the end of the thirteenth period the unrestricted ending cash balance equals the cash balance plus the interest on invested cash not yet received, plus invested surplus cash, less principal and interest payments for the next six periods.

<u>Commercial Paper</u>.--In theory, the quantity of commercial paper that can be issued by a firm has no static maximum. However, management generally sets a maximum issuance which is considered to be in the best interests of the financial well-being of the firm. This maximum

amount of commercial paper that is allowed to be outstanding at any one time, and the interest rate to be paid on that outstanding paper each period serve as input data for the model.

The model then determines if the amount needed exceeds the maximum amount available with this alternative. If the amount needed is available it is borrowed. If the amount needed exceeds that available, what is available is borrowed and the remainder is considered a stockout. The model also determines the interest due on the amount borrowed. The interest must be paid in the period during which it was borrowed. Thus, the cash balance at the end of the period is equal to the potential cash balance, plus the amount borrowed, less any interest paid. At the end of the thirteenth period the unrestricted ending cash balance is equal to the cash balance at the end of the final period, plus invested surplus cash and any interest on those funds, less the amount of principal due for the next three periods.[1]

Accounts Receivable Loan.--The accounts receivable balance, the maximum amount in dollars that can be borrowed, and the maximum percentage of accounts receivable that can be used as collateral for the loan are entered as input to

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[1]See Figure 5.

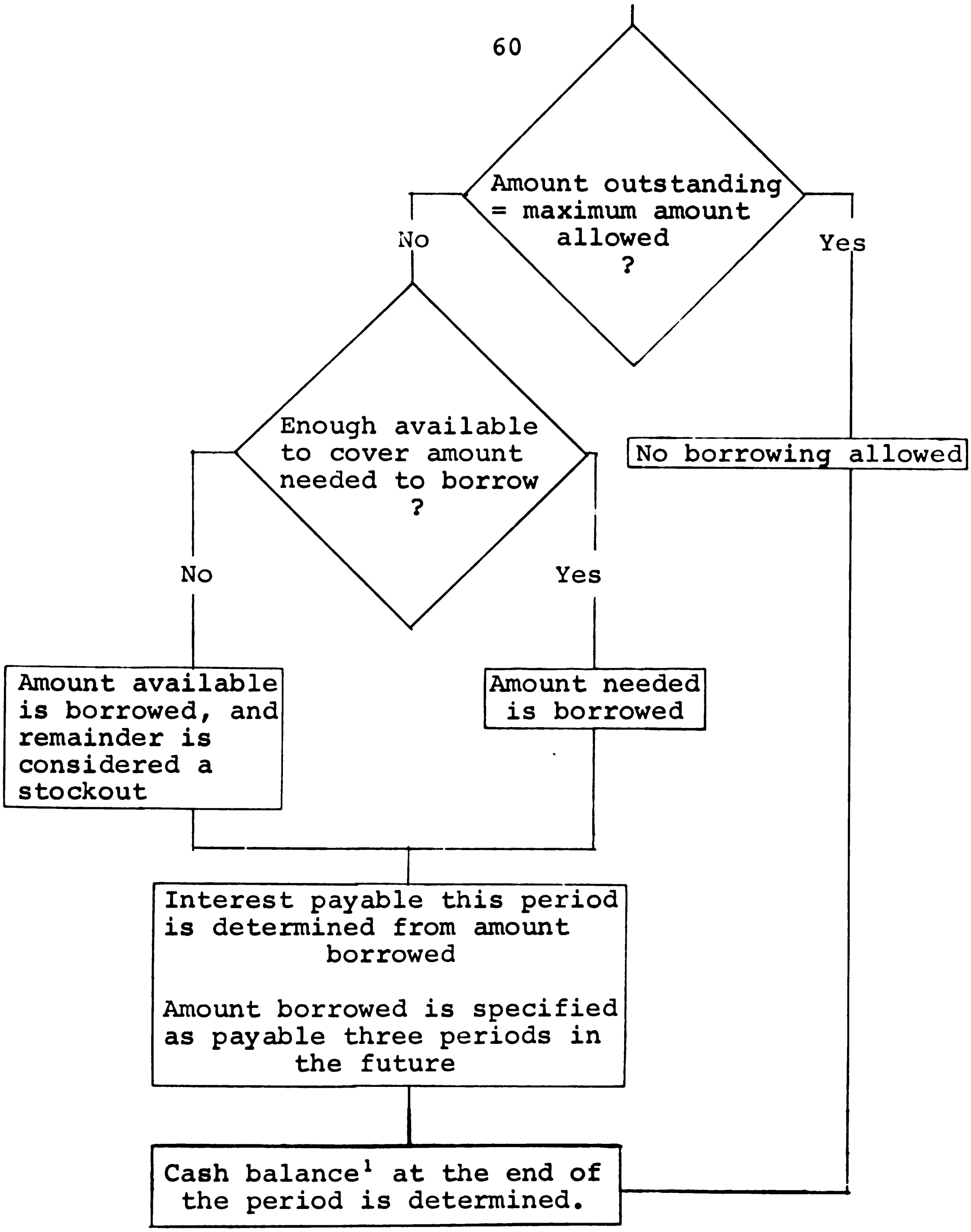

Figure 5.--Commercial paper.[2]

[1]The cash balance CASH(t) = PCASBAL(t) + DIF(t) - INTPAY(t).

[2]The flow chart is for each period.

the model. The interest rate to be applied each period is also entered.

The model then determines if the maximum amount that may be borrowed on the accounts receivable has been exceeded. If it has, the amount needed is declared a stockout. If the maximum amount available has not been exceeded, the program considers the total dollar value of the accounts receivable and applies to that value the percentage upon which money can be borrowed. If the resultant value has been exceeded, the amount needed is considered a stockout. If the amount available has not been exceeded, the amount needed is borrowed assuming the full amount is available. If the full amount needed is not available, what can be borrowed, is, and the remainder is a stockout.

Interest on the loan is to be paid at the beginning of the period subsequent to the one in which the loan was effected. The amount borrowed is repayable at the beginning of the next period. The cash balance for each period is equal to the potential cash balance plus the amount borrowed. The unrestricted ending cash balance at the end of the thirteenth period is equal to the invested surplus cash and interest on those funds, less principal and interest payable the subsequent period.[1]

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[1]See Figure 6.

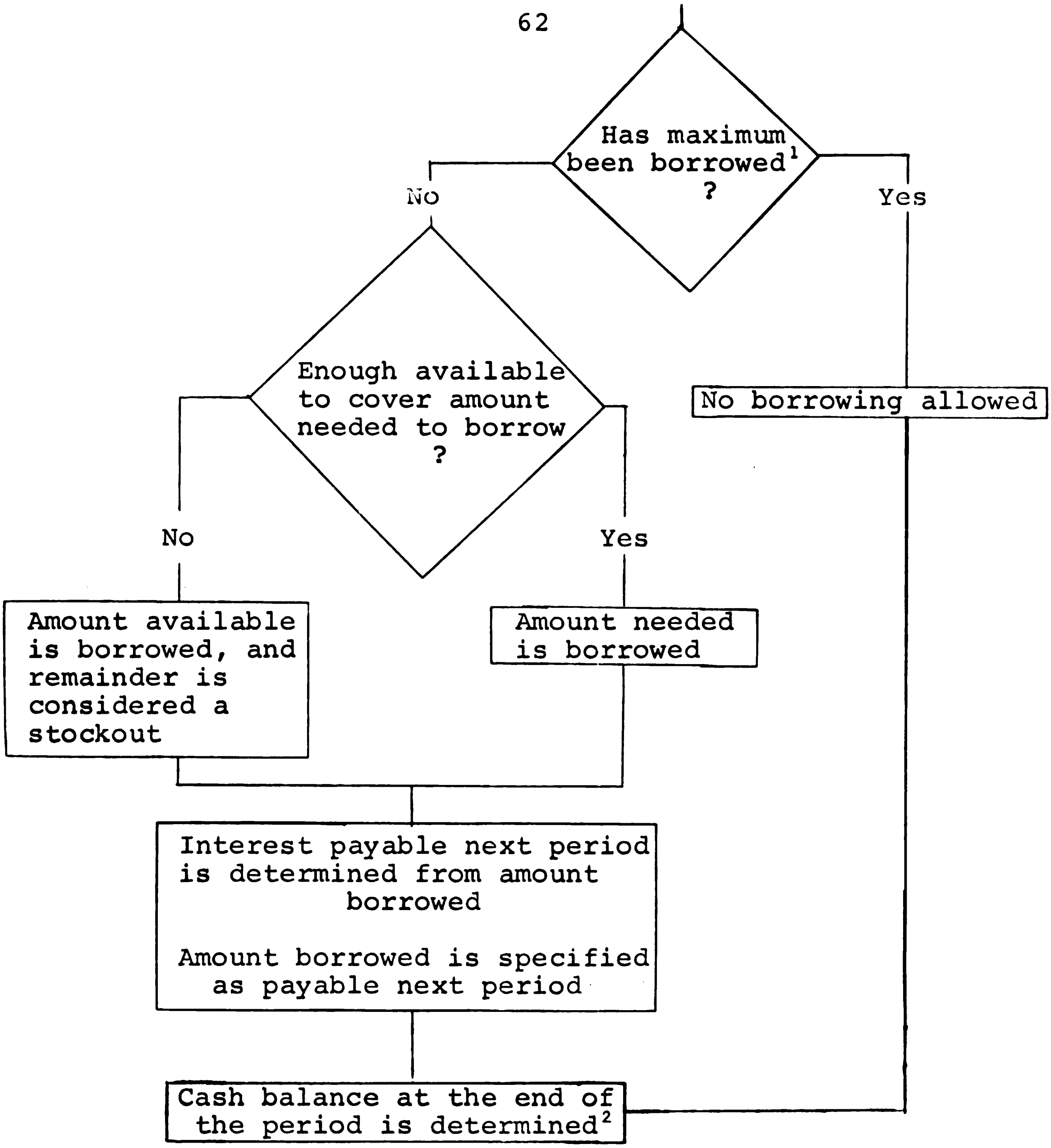

Figure 6.--Accounts receivable loan.[3]

[1]Maximum = minimum (specified minimum dollar amount or the percentage of the dollar amount of the accounts receivable).

[2]CASH(t) = PCASBAL(t) + DIF(t)

[3]The flow chart is for each period.

Model Input Data

There are four basic categories of input data to the model. These include, for each period, the constraints on the financing alternatives, the interest rates or distributions specific to the financing alternatives, the net cash flow distribution and the minimum required cash balance. The constraints on the financing alternatives were presented in the preceding section. The other three categories will be discussed in detail in the following section.

Interest Rates on the Financing Alternatives

The interest rate for each financing alternative is entered into the model as input data for each period. The interest rates may be submitted either as deterministic values or as probabilistic values with normal or uniform distributions.[1] Management specifies the value that is used according to the certainty with which the value can be described at the point in time when the model is to be run. When the interest rate is a deterministic value, only that figure is necessary as input. However, when the interest rate is projected to be a probabilistic value with a normal distribution, the expected value of the

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[1]Computer subroutines for handling the deterministically or probabilistically defined interest rates may be found in Appendix A.

interest rate and the standard deviation for each period must be entered. Similarly, when a uniform distribution is projected, the maximum and minimum interest rate values for each period must be included.

The interest rates are subject to the constraints applied to the financing alternatives with regard to the number of periods for which the loan is made and the repayment schedule.

Methods of Determining the Net Cash Flow.--In theory, there are a number of approaches which could be used to determine the random values of the net cash flow. Three methods will be briefly discussed in the following section: (1) time series analysis; (2) the net cash flow as some function of one or more random variables; and (3) simulation. A time series analysis of the historical net cash flow values could be made to estimate the stochastic process that generated those values.[1] The stochastic process would then be applied to generate the input net cash flow values for the model. Another approach could consist of analytically developing the stochastic process of the net cash flow as some function of the random variable, sales, which itself is generated by a stochastic

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[1]Paul E. Pfeiffer, Concepts of Probability Theory (New York: McGraw-Hill Book Company, 1965), pp. 294-295.

process.[1] A similar technique consists of using the stochastic processes of more than one random variable. The net cash flow is then some function of a finite number of random variables. The stochastic process thus determined is then used to generate the input net cash flow data. Finally, the stochastic processes of the input variables may be sampled using simulation to generate the net cash flow values which can be used in the model. The analytically complex derivation of the stochastic process of the net cash flow is then eliminated.

1. Time series analysis.[2] A stochastic process can be defined as a random phenomenon which arises through a process which is developing in time and in a manner controlled by probabilistic laws.[3] In time series analysis observations are made at a constant interval over time. The statistical theory of time series analysis attempts to determine the mechanism generating an observed time series. The stochastic process, $\{X(t), t \in T\}$, can

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[1] *Ibid.*, pp. 164-170.

[2] For a further discussion of time series analysis see: E. J. Hannan, *Time Series Analysis* (London: Butler and Tanner, Ltd., 1967); Andrew H. Jazwinski, *Stochastic Processes and Filtering Theory* (New York: Academic Press, Inc., 1970); and The Research Section of the Royal Statistical Society, "Symposium on Stochastic Processes," *The Journal of the Royal Statistical Society, Series B*, XI, No. 2, 1949 (reprint).

[3] Emanuel Parzen, *Stochastic Processes* (San Francisco: Holden-Day, Inc., 1962), p. 22.

be described by defining the joint probability law of the n random variables $X(t_1),\ldots,X(t_n)$ for all intergers n and all n points $t_1,\ldots,t_n \varepsilon T$. This joint probability law can be expressed by the joint distribution function,[1] F, for all real numbers $x_1,\ldots,x_n$ as follows:

$$F_{X(t_1),\ldots,X(t_n)}(x_1,\ldots,x_n) = Pr\{X(t_1) \leq x_1,\ldots,X(t_n) \leq x_n\}$$

where $F_{X(t_1),\ldots,X(t_n)}$ is a finite dimensional distribution of the process. The joint distribution function must be determined. Once the distribution of the stochastic process is estimated in the time series analysis, that distribution is utilized to generate the values of the net cash flow. The problem rapidly becomes involved and time-consuming to solve.

2. The net cash flow distribution as a function of one or more random variables. The stochastic process of the net cash flow can be determined if the distribution of some random variable, usually sales, and the function relating the net cash flow to this random variable are known. However, in this study, there are several random variables as well as some deterministic variables which enter into the calculation of the net cash flow. In order

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[1]A distribution function indicates the cumulative probability of the random variable having a value $\leq$ some real value.

to estimate the stochastic process of the net cash flow, the distributions of the random variables must be known. Further, the function relating these variables to the net cash flow must be known.

The function which relates the random variables to the net cash flow is not a linear function nor are all the variables defined by the same type of distribution. The Central Limit Theorem[1] states that "under very general conditions, the sum (or average) of a large number of independent random variables is a random variable which is approximately normally distributed." However, in this study, where only a few random variables are considered and the variables are not independent, the Central Limit Theorem fails to aid in achieving a solution. An analytical solution becomes necessary which may be difficult to solve. Thus, the author has discounted the application of this approach of determining the net cash flow distribution since it appears that the problems involved belie practical use by the layman.

An exploration of the two above approaches may well reveal valid methods for determining the stochastic process of the net cash flow. The approaches are both statistically complex, however, and as such are significant studies in themselves. They are perhaps more suitable

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[1]Pfeiffer, p. 134.

for those pursuing expertise in mathematics or statistical analysis.

3. Simulation.[1] Simulation is another approach that can be used to generate net cash flow data. The stochastic processes of the random input variables are sampled to generate the net cash flow values. The method eliminates the problems associated with deriving the stochastic process of the net cash flow. The variables, both deterministic and stochastic, are defined and the program written and tested. Management can then simply supply the input variable values and derive the net cash flow values.

The author concedes that this approach may not be a panacea for all the problems involved in determining the net cash flow information necessary for the model. Nevertheless, it is an entirely adequate approach. It has a distinct advantage over the other approaches in that it requires less complex input from the firm.[2] Experience has shown that this is an important consideration for management. Further, while the importance of the net cash flow data should not be understated, a method which generates

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[1]See Chapter III, page 41 for a further discussion of simulation.

[2]That is to say, that no analytical or time series analysis must be made before the model can be used.

data representing a reasonable approximation of the net cash flow and yet is not unduly complex, is entirely sufficient.

Monte Carlo simulation will be used to generate values for the cash inflow and cash outflow variables and parameters for each period.[1] There are two basic categories of variables.[2] Some have their values determined from the values of other variables and the parameter value which relates them. Others are not specifically dependent on the values of any other variable. The variables will be defined as either probabilistic or deterministic. Parameter values, which relate one variable to another numerically, will be handled deterministically or stochastically depending upon whether risk is involved. Management decision will influence some of the variables when a statement of policy can affect the cash flow. Figures 8 and 9 present a detailed outline of the net cash flow variables and how they are defined.

Probability Distribution Determination.--Three types of probability distributions are used in this study to define the probabilistic variables--normal, uniform and general. Each distribution is contained in a subroutine in the program and can be accessed according to the type of

[1]See Figure 7.

[2]See Table 1.

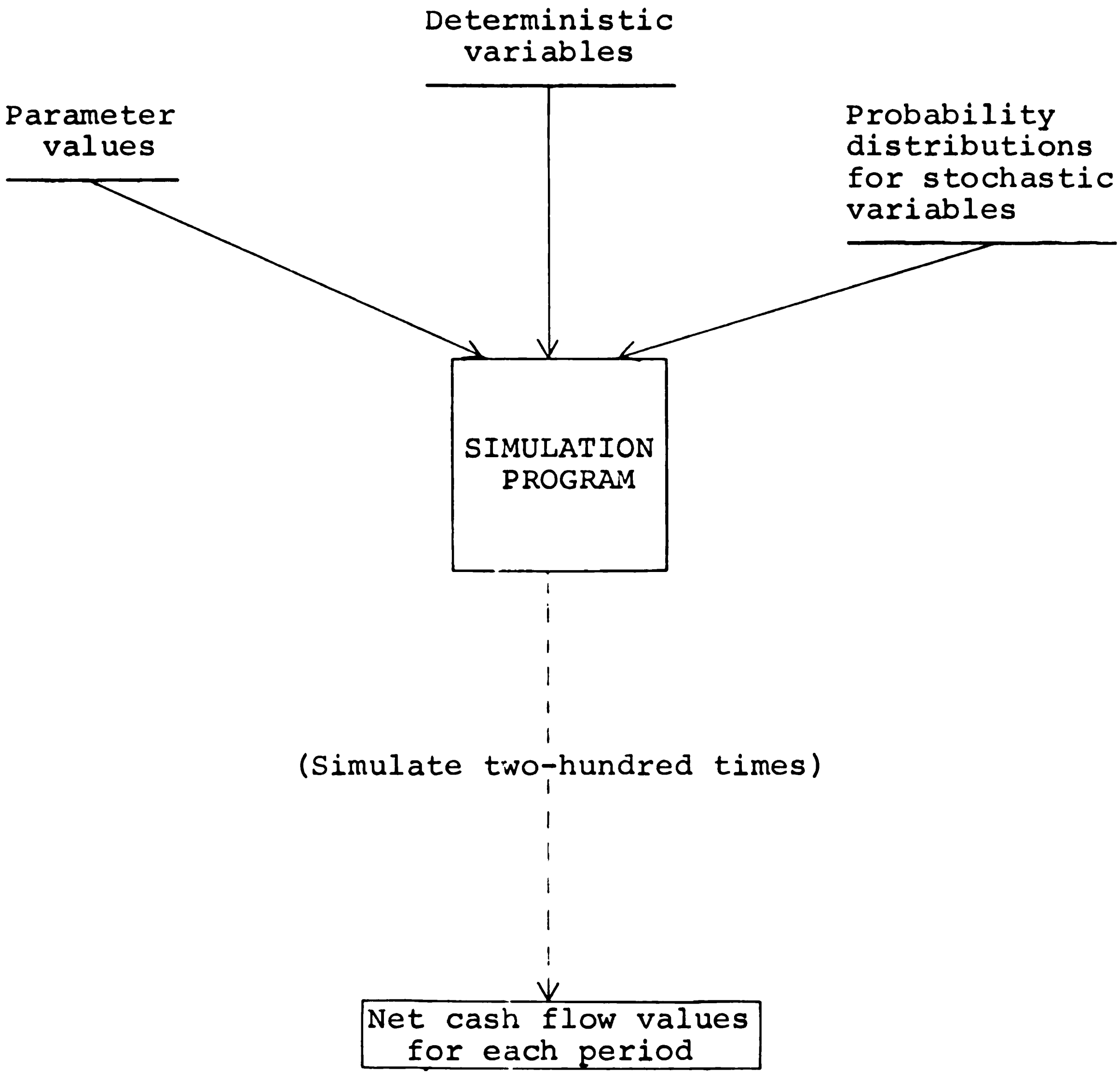

Figure 7.--General diagram of the net cash flow.

TABLE 1.--The net cash flow simulation variables.

Variables	Type	Determination	Symbol
TOTAL SALES	S		SALES
Cash Receipts			
Cash Sales	S	P	CS
Collection of Accounts Receivable	D	P	CAR
New Debentures	D		ND
Common Stock Issued	D		COMSI
Sale of Fixed Assets	D		SFA
Cash Disbursements			
Direct Wages	S	P	DW
Raw Materials	S	P	RM
Factory Overhead*			FOH
Selling Costs*			SELL
General and Administrative Costs	D/S	P	GA
Research and Development Costs*			RD
Interest Payments on Long-Term Debt	D		DIP
Tax Payments	D/S		TXP
Miscellaneous Costs	D/S	P	MSCL
Debenture Repayment	D		DRP
Debenture Retirement Fund	D		DRF
Capital Expenditures	D/S		CAP
Dividends--Preferred	D		DIVP
Dividends--Common	D		DIVC

*Combined in general and administrative costs.

D - Deterministic Variable
S - Stochastic Variable
P - Parameter

Net Cash Flow for Each Period (T)

A. Total sales (used as a basis for determining the values of some of the variables) (SALES)

B. Cash inflows (CR)

CR(T) = CS(T) + CAR(T) + ND(T) + COMSI(T) + SFA(T)

1. Cash sales (CS)
2. Collection of accounts receivable (CAR)
3. Issuance of new long-term debt (ND)
4. Issuance of common or preferred stock (COMSI)
5. Sale of fixed assets (SFA)

C. Cash outflows (CD)

CD(T) = DW(T) + RM(T) + FOH(T) + SELL(T) + GA(T) + RD(T) + DIP(T) + TXP(T) + DRP(T) + DRF(T) + CAP(T) + DIVP(T) + DIVC(T) + MSCL(T) + SPMISC(T)

1. Direct wages (DW)
2. Raw material (RM)
3. Factory overhead (FOH)
4. Selling cost (SELL)
5. General and administrative cost (GA)
6. Research and development cost (RD)
7. Interest on long-term debt (DIP)
8. Tax payments (TXP)
9. Repayment of long-term debt (DRP)
10. Sinking fund for long-term debt (DRF)
11. Capital expenditure (CAP)
12. Dividends--common (DIVC)
13. Dividends--preferred (DIVP)
14. Miscellaneous (MSCL) and special miscellaneous (SPMISC)

D. Net cash flow calculation (NETCF)

NETCF(T) = CR(T) - CD(T)

E. Additional input data

Value of outstanding accounts receivable and timing of their collection during initial periods of the simulation.

Figure 8.--Net cash flow variables--basic outline.

A. Total Sales

Variable: deterministic or probabilistic
Probability distribution: uniform, normal or general

B. Cash Inflows

1. Cash sales

Parameter Alpha(t): probabilistic
Probability distribution: uniform or normal
CS(T) = ALPHA(T) * SALES
CRS(T) = SALES * (1-ALPHA(T)) where CRS(T) is the credit sales in period T

2. Collection of accounts receivable

Variable: deterministic
The following data must be entered:
  a. The number of periods necessary to complete collection of each credit sale
  b. The percentage of outstanding accounts receivable to be collected each period
The program calculates the amount of cash receivables to be collected as cash at the beginning of each period.

3. Issuance of new long-term debt

Variable: deterministic
Management indicates the timing and amount of any issuance of new long-term debt

4. Issuance of common or preferred stock

Variable: deterministic
Management indicates the timing and amount of any issuance of common or preferred stock

5. Sale of fixed assets

Variable: deterministic
Management indicates the timing and amount of any sale of fixed assets

C. Cash Outflows

1. Direct wages

Parameter A(T): probabilistic
Probability distribution: uniform or normal
DW(T) = A(T) * SALES

2. Raw material

Parameter B(T): probabilistic
Probability distribution: uniform or normal
RM(T) = B(T) * SALES

3,4,6. Factory overhead, selling cost, and research and development. In the test company, these data were combined and added to the general-and-administrative category. The program can easily be modified to allow them to be individually considered, however.

5. General and administrative

Parameter THETA(T): probabilistic
Probability distribution: uniform or normal
GA(T) = FDGA(T) + THETA(T) * SALES where FDGA(t) is the fixed portion of this cost determined through regression analysis

7. Interest on long-term debt

   Variable: deterministic
   Management indicates the timing and amount of interest on long-term debt

8. Tax payments

   Variable: deterministic or probabilistic
   Deterministic--management indicates the timing and amount of tax payments
   Probabilistic--management enters the timing, amounts and probabilities of the amounts of the tax payments.
   Probability distribution: general

9. Repayment of long-term debt

   Variable: deterministic
   Management indicates the timing and amount of any repayment of long-term debt

10. Sinking fund for long-term debt

   Variable: deterministic
   Management indicates the timing and amount of any sinking fund payments

11. Capital expenditure

   Variable: deterministic or probabilistic
   Deterministic--management indicates the timing and amount of capital expenditures
   Probabilistic--management enters the timing, amounts and probabilities of the amounts of the capital expenditures
   Probability distribution: general

12. Dividends--common

   Variable: deterministic
   Management indicates the timing and amount of common stock dividends paid

13. Dividends--preferred

   Variable: deterministic
   Management indicates the timing and amount of preferred stock dividends paid

14. Miscellaneous

   Parameter DELTA(T): probabilistic
   Probability distribution: uniform or normal
   MSCL(T) = MISC(T) + DELTA(T) * SALES where MISC(T) is the fixed portion of this cost determined through regression analysis

   Special miscellaneous

      Variable: deterministic or probabilistic
      Deterministic--management indicates the timing and amount of the special miscellaneous costs
      Probabilistic--management enters the timing, amounts and probabilities of the amounts of the special miscellaneous costs
      Probability distribution: general

Figure 9.--Net cash flow variables--detailed outline.

distribution that most closely describes each probabilistic variable. The normal distribution requires as input, the mean and standard deviation of the parameter or variable. The upper and lower bounds of the parameter or variable value must be submitted when the uniform distribution is to be used. The general distribution allows management to submit possible values for the variable in question and indicate for each value the probability of occurrence associated with it.

Data for the Net Cash Flow.--The financial information necessary to simulate the net cash flow and for subsequent use in determining the unrestricted ending cash balances for the financing alternatives, was collected from real-world data. A Mid-west-based firm in the Motor Carrier Industry (Company Z) agreed to provide the data. The firm has an annual revenue of approximately $50 million and extends its operations over a multi-state area.

The basic financial planning structure for the firm is divided into periods of four weeks, each period ending on a Saturday, with thirteen periods per year. Data for each period of the calendar years 1969 and 1970 were collected as well as data from the first four periods of 1971. The accounting information was then converted to a cash basis.

<u>The Minimum Required Cash Balance</u>

The minimum required cash balance, $REQMIN_t \equiv \delta_{ti}(h_{ti}) \equiv d_{ti}$, for each period is assumed in this study to be a deterministic value specified by management. The amount of funds borrowed or reinvested, DIF_t, is directly related to the interaction between this required minimum, and the values of the input state variables, C_t, and the net cash flow, $NETCF_t$.[1] As previously discussed,[2] the results obtained may be suboptimal because only certain minimum required cash balance values are considered in the analysis. Ideally, the selection of an optimal financing alternative would require an analysis including all minimum required cash balances.

The minimum required cash balance value specified by management serves as an indicator of the feeling of management regarding the necessary balance between liquidity and profitability. A higher liquidity requirement indicates a more conservative attitude toward cash management. Conversely, a lower liquidity requirement may reveal more willingness on the part of management to risk a potential cash insolvency in order to gain higher profitability.[3]

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[1]See pages 48-51.

[2]See Chapter III, page 36.

[3]Higher profitability may be achieved by investing the funds rather than holding them as cash.

CHAPTER V

PRESENTATION AND EVALUATION OF RESULTS

In the preceding chapters, the short-term financing decision has been discussed and a model to assist management in making that decision has been proposed. The model is designed to evaluate four specific financing alternatives--line of credit, term loan, commercial paper and accounts receivable loan. Input data to the model for each period included: (1) the distribution of the interest rates of the financing alternatives; (2) the net cash flow distribution; (3) the minimum required cash balance; and (4) the constraints on the financing alternatives. Management received as output from the model a set of cumulative empirical probability distributions, one for each financing alternative, which were evaluated to select the optimal financing alternative for any specified short-term financing decision.

The application of the model to actual data will be presented and discussed in this chapter. The results will be evaluated using the condition of stochastic dominance. Finally, the model itself will be evaluated in terms of its applicability, effectiveness and practicability for use in the solution of the short-term financing problem.

The input data will be presented in the following sections.

The Net Cash Flow Simulation

Input Data.--The probability distributions for the stochastic input variables[1] to the net cash flow simulation were obtained through discussion with the management of the test company, Company Z. The distributions[2] of the stochastic variables were defined as follows:

1. Revenue had a uniform distribution
2. Tax payments had a uniform distribution
3. Special miscellaneous costs had a general distribution
4. The breakdown of credit and cash revenue had a uniform distribution
5. The parameter relating wages to total revenue had a uniform distribution
6. The parameter relating raw materials to total revenue had a normal distribution
7. The parameter relating general and administrative costs to total revenue had a normal distribution
8. The parameter relating miscellaneous costs to total revenue had a uniform distribution.

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[1]See pp. 73-74 of Chapter IV for a full discussion of both the stochastic and the deterministic input variables.

[2]The probability distributions which are used in this study are discussed on pp. 69, 75 of Chapter IV.

All other input variables were deterministically defined. The specific values for the input data to the net cash flow simulation are listed in Appendix B as they appear in the program.

Output Data.--The output data consisted of the two-hundred net cash flow values for each of the thirteen periods in the planning horizon considered in this study. A cumulative net cash flow for each of the two-hundred simulations over the planning horizon was also given as printed output. In addition, the amount of the outstanding accounts receivable at the beginning of each of the thirteen periods was generated by this program. The latter data functioned as part of the input to the model in order to facilitate the evaluation of the accounts receivable loan alternative. An example of the printed net cash flow output is included in Appendix C.

Constraints

The financing alternatives will be designated by type in the discussion in this chapter as follows:

Type 1 refers to Line of Credit

Type 2 refers to Term Loan

Type 3 refers to Commercial Paper

Type 4 refers to Accounts Receivable Loan

Type 1 was divided into two categories, 1A and 1B, since the compensating balance required with a line of credit may be determined in two ways. The method of calculating the compensating balance influenced the evaluation of the alternative.

A number of constraints were imposed specific to each financing alternative. A line of credit is an informal agreement between a commercial bank and the customer regarding the maximum amount of unsecured credit the bank will allow the customer at any one time. For the purposes of this study, that maximum amount was $5 million. The bank requires that the customer maintain a compensating balance at the bank. This amount may be either directly proportional to the amount of the funds borrowed, Type 1A, or to the amount of the commitment, Type 1B. In this study, 15 percent of the funds borrowed equaled the compensating balance for Type 1A. Type 1B had a required compensating balance which was equal to 15 percent of the $5 million commitment, or $750 thousand.

A term loan is a formal loan agreement effected at a commercial bank. Repayment of the loan and interest payments are made in regular periodic installments. For the purpose of this study the loan extended over six periods. The total maximum amount that could be borrowed with the term loan was $5 million. Further, no more than $1 million or less than $50 thousand could be borrowed in any one period.

Commercial paper is issued by a firm in the form of unsecured short-term negotiable promissory notes which are sold in the money market. The maximum amount that a firm was allowed to borrow with this alternative in this study was $5 million. The only additional constraint placed on this alternative was the ability of the firm to successfully issue commercial paper.

The accounts receivable loan is effected at a bank and is a loan for which the firm's accounts receivable are accepted as collateral. The percentage of the outstanding accounts receivable that may be borrowed against is a highly individual figure depending upon the amount and quality of the receivables and the confidence of the bank. This value generally varies considerably. Since this percentage is so variable and individual, and since it may readily be changed in the model specific to any particular firm, the author elected to make it a deterministic value in this study. An arbitrary value of 45 percent was used. The maximum amount that could be borrowed was $5 million or 45 percent of the outstanding accounts receivable, whichever was less.

Stockout Penalties

A stockout occurred when the amount a firm needed to borrow was not available. This resulted in a cash balance at the end of the period that was less than the minimum required cash balance. A value had to be placed on the stockout since a stockout implied that the amount of funds unavailable from the selected alternative must be obtained from another, presumably less desirable, alternative. This resulted in an alternative that was less attractive than it would have been had a stockout not occurred.

The value, or penalty, placed on the alternative when a stockout occurred was a difficult value to select. The stockout penalty significantly affected the evaluation of the alternative upon which it was imposed. Since the constraints on the alternatives affected the probability that a stockout would occur and since the constraints were specific to the individual alternatives, the stockout penalty did not affect all the alternatives equally. The magnitude of the effect of the stockout penalty was a function of the interaction of the constraints and the amount the firm needed to borrow. The greater the need the more significant the constraints on the alternative became and the greater the likelihood that a stockout would occur. Therefore, it was important to attempt to select a value for the stockout penalty that would be relatively realistic and yet not disproportionately penalize the alternatives that would be most likely to experience a stockout.

Two values for the stockout penalty were tested. The alternatives most frequently affected by stockouts were evaluated three ways: (1) when no stockout penalty was imposed; (2) when a 1 percent stockout penalty was applied; and (3) when a 5 percent penalty was imposed. The dollar value of the penalty was calculated as a percentage of the dollar value of the stockout and subtracted from the unrestricted ending cash balance of the alternative. Figures 13, 14 and 17, 18 show the results of the comparative tests. The magnitude of the effect of the 5 percent penalty on the alternatives suggested that no higher percentage needed to be imposed. The 5 percent penalty caused the effected alternatives to be clearly inferior to the other alternatives.

Interest Rates

The interest rates that were applied to the financing alternatives were derived from Company Z management projections of the prime rate. The author modified these projections to obtain the appropriate ranges for the interest rate distributions for each alternative. The rates were defined as uniformly distributed.

The Minimum Required Cash Balance

The minimum required cash balance was defined in this study to be a deterministic value specified by management.[1] The value of this required minimum influenced the amount of borrowing that must be effected. Further, the degree to which the constraints were imposed upon the alternatives was related to the amount of borrowing effected. The more funds required by the firm, the more each alternative was effected by the constraint on the maximum amount available for that alternative. Since the level of the minimum required cash balance significantly affected the model's output data, the author attempted to select several values for this required minimum for use in demonstrating the model results. A number of values were considered and three were finally selected. At a value of $3 million, a relatively low level of borrowing was experienced. The constraint on maximum funds available with each alternative was infrequently imposed. The results reflected these conditions. Higher minimum required cash balance levels, $4.5 million and $6 million, were also explored to demonstrate the effects of the constraints at these levels. It can be seen from the results presented in the following section that, as the amount of funds borrowed increased, evaluation of the alternatives through the

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[1]As previously discussed on pages 34 and 46. $REQMIN_t \equiv \delta_{ti}(h_{ti})$. Thus, when a value for $REQMIN_t$ for $t=1,\ldots,13$ is specified, this means that the strategy function, $\delta_i = \{\delta_{1i},\ldots,\delta_{13i}\}$, is implied but not explicitly stated. Each vector of the required minimum, $\{REQMIN_1,\ldots,REQMIN_{13}\}$, implies a different strategy function.

condition of stochastic dominance was facilitated since stochastic dominance was more distinctly demonstrated.

The author also explored the possibility of defining the minimum required cash balance as a stochastic variable. The results of this trial are presented following the discussion of the other results.

The Computer Programs Involved in the Model

A computer program was used to generate the net cash flow data using simulation.[1] These data were then submitted as input to the model program. The latter program[2] generated output which was used to estimate the empirical cumulative probability distributions of the alternatives. These programs were written by the author in FORTRAN IV and were run on the CDC 3600.

Model Output

The objective of the model was to determine that alternative that maximized the expected utility of the unrestricted ending cash balance of the firm at the end of the planning horizon.[3] The output of the model[4] consisted

[1]See Appendix B.

[2]See Appendix D.

[3]At the end of the planning horizon (the thirteenth period), the unrestricted ending cash balance is equal to the cash balance in the thirteenth period, plus interest on invested surplus cash not received, plus invested surplus cash, less principal and interest due in subsequent periods.

[4]See Appendix E.

of a set of unrestricted ending cash balances for each financing alternative from which a cumulative probability distribution for that alternative was estimated. The distributions were plotted on graphs. The horizontal axis indicated the values of the unrestricted ending cash balance in intervals of $50 thousand with a lower bound of $1.65 million and an upper bound of $4.1 million. The right-hand vertical axis indicated the number of times a value greater than the corresponding value on the horizontal axis was achieved. The left-hand vertical axis gave the same value as a percentage of the total occurrences achieved in the simulation. A table for each graph giving the specific numerical data which have been plotted for each financing alternative is included in Appendix F. The cumulative probability distributions for the four financing alternatives were then evaluated using the condition of stochastic dominance.

Stochastic Dominance[1]

The condition of stochastic dominance relates pairs of probability distributions. Stochastic dominance may be useful for the analysis of a variety of decision problems

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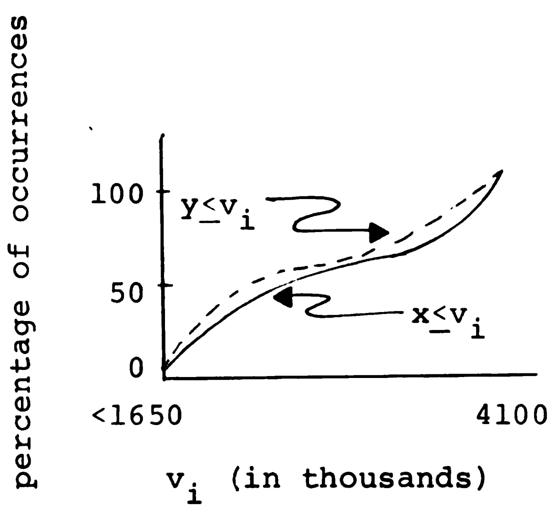
[1] Josef Hadar, Mathematical Theory of Economic Behavior (Reading, Massachusetts: Addison-Wesley Publishing Company, 1971), pp. 261-262; and Josef Hadar and William R. Russell, "Rules for Ordering Uncertain Prospects," The American Economic Review, LIX (1969), 25-34.

under risk. The condition of stochastic dominance makes possible a prediction about preference for probability distributions without requiring any knowledge of the utility function of the decision-maker. Thus, consider two uncertain prospects, X and Y, and X is larger than Y in the sense of first degree stochastic dominance (FSD). Then X is preferred to Y if it is desirable to maximize the expected utility. X is larger than Y in the sense of FSD when the cumulative probability of X being less than or equal to v_i is less than or equal to that of Y for all unrestricted ending cash balance values, v_i. This can be stated as follows:

$$P(X \leq v_i) \leq P(Y \leq v_i).$$

This condition is depicted in the following graph. The lower cumulative probability curve exhibits stochastic dominance as defined in the preceding section.

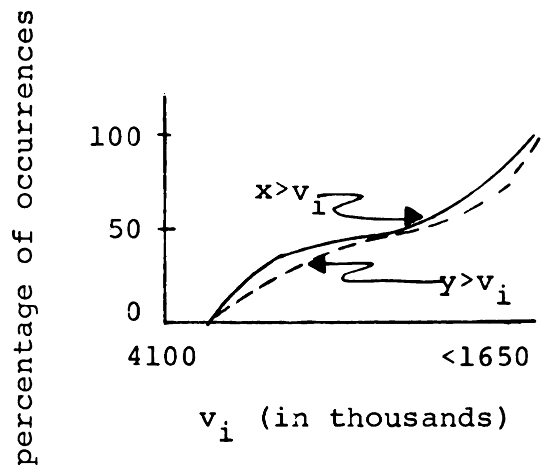


In this study, $P(X \leq v_i) \leq P(Y \leq v_i)$ has been restated as:

$$P(X > v_i) \geq P(Y > v_i). \quad \text{Thus,}$$

the cumulative probability of X being greater than v_i is greater than or equal to that of Y for all v_i. This is the statement of stochastic dominance that will be used in this study.

The format of the graphs used in the following sections to explain the results of the model is presented below:



In the above graph, X exhibits the condition of first degree stochastic dominance over Y.

The following sections will be devoted to an analysis and evaluation of the output of the model.

Evaluation at a Minimum Required Cash Balance of $3 Million

The four financing alternatives were evaluated at three different minimum required cash balance levels. At a minimum required balance of $3 million, Type 1A exhibited stochastic dominance over all other alternatives.[1] Type 4

[1] See Figure 10.

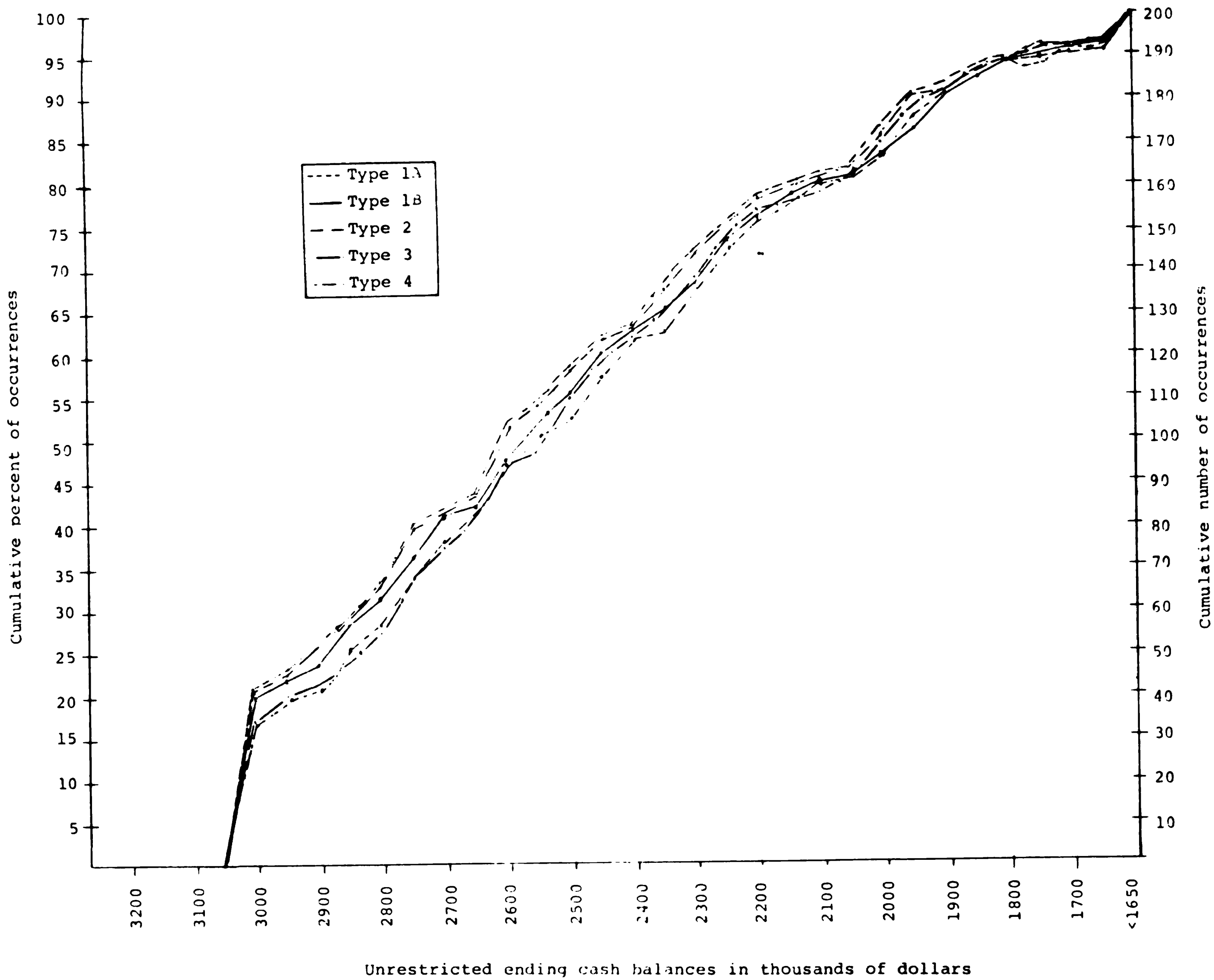


Figure 10. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $3 million and a stockout penalty of 1 percent.
Source: Table II.

had a cumulative probability distribution similar to and nearly as optimal as that of Type 1A. This degree of similarity can be expected since the only significant difference between the two alternatives at this particular minimum required cash balance was the interest rate paid in each period. Type 4 was not affected by the constraint governing the maximum amount of funds available with this alternative since this amount of borrowing was not needed. Type 4 experienced a stockout only once in the two-hundred simulations at this level.

Type 3 was dominated by Types 1A and 4 because of the requirement that the firm borrow the funds for three periods. Thus, interest had to be paid for the three periods. Alternatives 1B, 2, and 3 failed to exhibit stochastic dominance among themselves. These alternatives were clearly inferior to Types 1A and 4, however. In summary, the rankings of the alternatives at this minimum required cash balance can be explained by the low level of borrowing which eliminated some of the constraints on several of the alternatives and caused the interest rates and the duration of interest payments to be highly significant.

Types 2 and 4 were evaluated both with and without stockout penalties. Neither the 1 percent nor the 5 percent stockout penalty caused any significant change in the cumulative probability distribution of Type 4 because essentially no stockouts occurred. Type 2 experienced 102 stockouts in the simulation and was affected by the 1 percent stockout to some degree. It was not the least desirable alternative consistently, even with the stockout penalty, however. When a 5 percent penalty was imposed, the desirability of this alternative was significantly reduced.

Evaluation at a Minimum Required Cash Balance of $4.5 Million

At a minimum cash balance of $4.5 million the rankings of the alternatives began to reflect the effects of the constraints. The data will first be evaluated considering a 1 percent stockout penalty, and then re-evaluated when a 5 percent stockout penalty was imposed.

Stockout Penalty at 1 Percent[1].--Type 3 and Type 1A exhibited stochastic dominance over the other alternatives. These two types had similar cumulative probability distributions.[2] Type 3 did not exhibit stochastic dominance over Type 1A since, at three points on the curve,[3] the cumulative probability distribution for Type 3 failed

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[1]See Figure 11.

[2]The additional interest payment on the compensating balance for Type 1A was nearly equal to the three-period interest payment required for Type 3.

[3]The stochastic dominance failed at the unrestricted ending cash balance levels of $2.3 million, $3.7 million and $3.65 million (see Table 3).

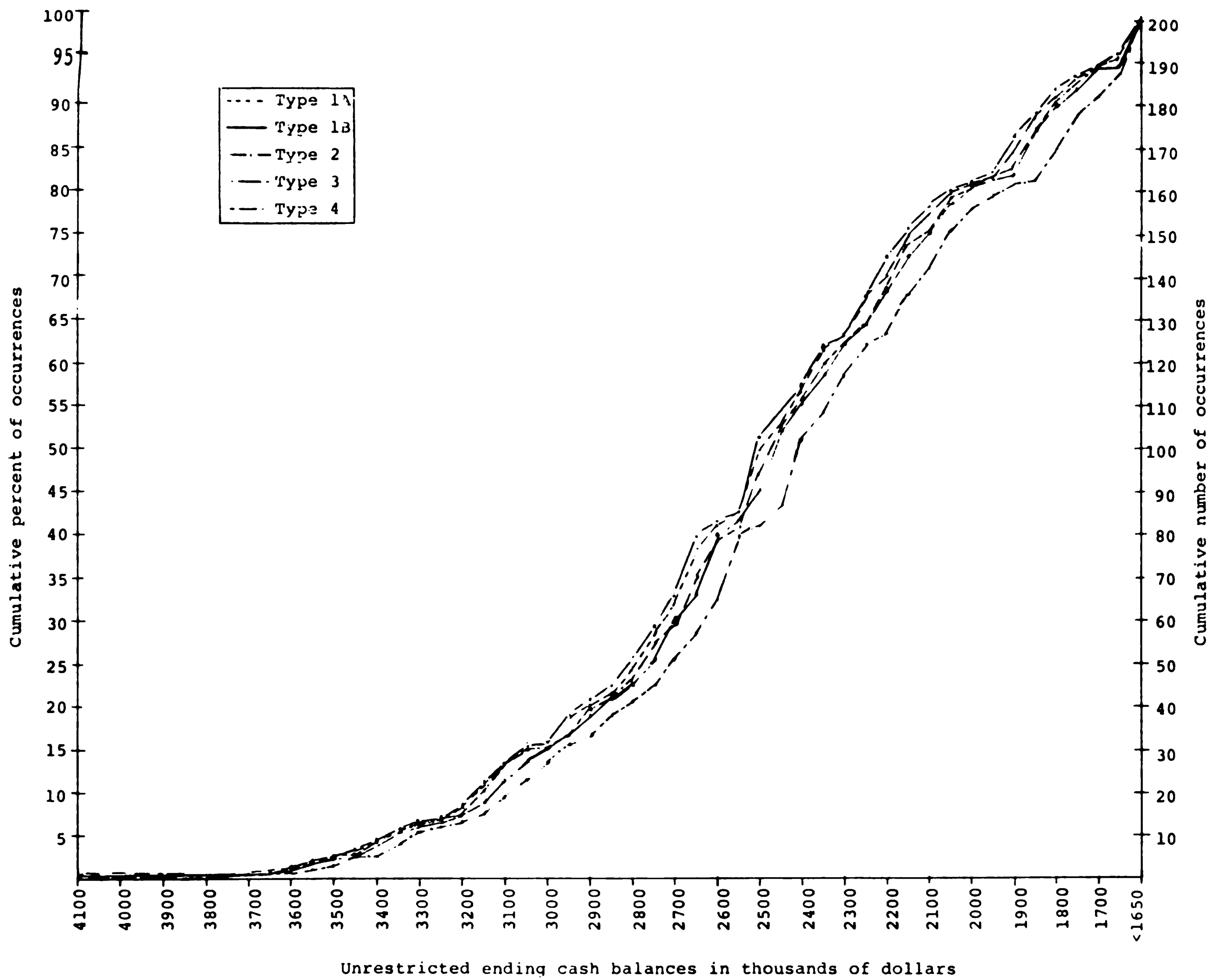

Figure 11. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $4.5 million and a stockout penalty of 1 percent. Source: Table III.

to be equal to or greater than that for Type 1A at those points. It is possible that if more simulations were run, a stochastic dominance would be exhibited. It should be noted, however, that a large number of firms are unable to issue commercial paper. Type 3 would then be eliminated leaving 1A clearly dominant over the remaining alternatives. Firms which are able to utilize Type 3 often attempt to combine Type 3 and Type 1A when borrowing.[1] The data shown in Figure 11 tend to justify this approach.

Type 2 is the least desirable alternative since it is dominated by all of the other alternatives. The duration of the term loan in this study was six periods. Interest must be paid for each of the six periods and the total interest paid was greater than that paid on the alternatives which have a single-period repayment schedule. The lower percent interest rate available with Type 2 was not sufficient to make it a more desirable alternative in this case. In addition, Type 2 experienced stockouts in each of the two-hundred simulations. The stockouts increased the divergence between the cumulative probability distributions.

No stochastic dominance could be established between Types 1B and 4. Type 4 was less desirable than it was at the \$3 million level because the alternative

[1] James C. Van Horne, Fundamentals of Financial Management (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1971), pp. 235-236.

experienced 178 stockouts at the $4.5 million level. Thus, the stockout penalty was imposed and caused the alternative to become less attractive.

Stockout Penalty at 5 Percent[1].--The data for alternatives 1A, 1B, and 3 were identical to those discussed in the preceding section since these alternatives did not experience stockouts. Type 2 was markedly affected by the 5 percent stockout penalty because stockouts occurred 200 times in the same number of simulations. Type 4 was also significantly affected by the stockout penalty although to a lesser degree than Type 2. Type 4 became noticeably less desirable in the regions of the graph where larger sums were being borrowed. The differences between Type 4 and Type 1B became less marked in the lower regions of the graph where less borrowing was effected. No stochastic dominance was exhibited by Type 1B over Type 4.

Type 2 and the effects of the stockout penalties on that alternative are presented graphically in Figure 13. Stockout penalties of both 1 and 5 percent were imposed. Figure 14 presents Type 4 and the effects of the stockout penalties on the cumulative probability distributions of that alternative.

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[1]See Figure 12.

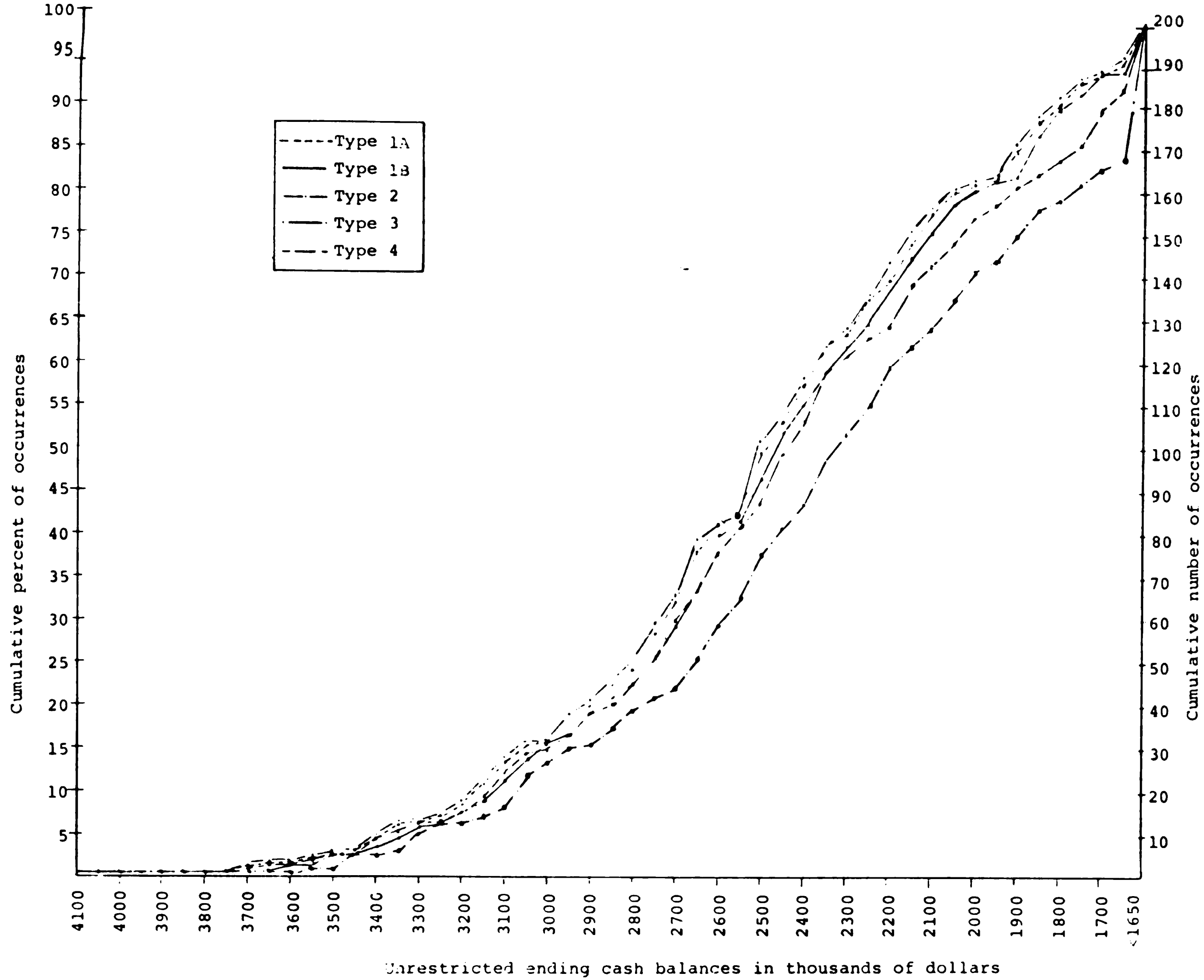

Figure 12. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $4.5 million and a stockout penalty of 5 percent. Source: Table IV.

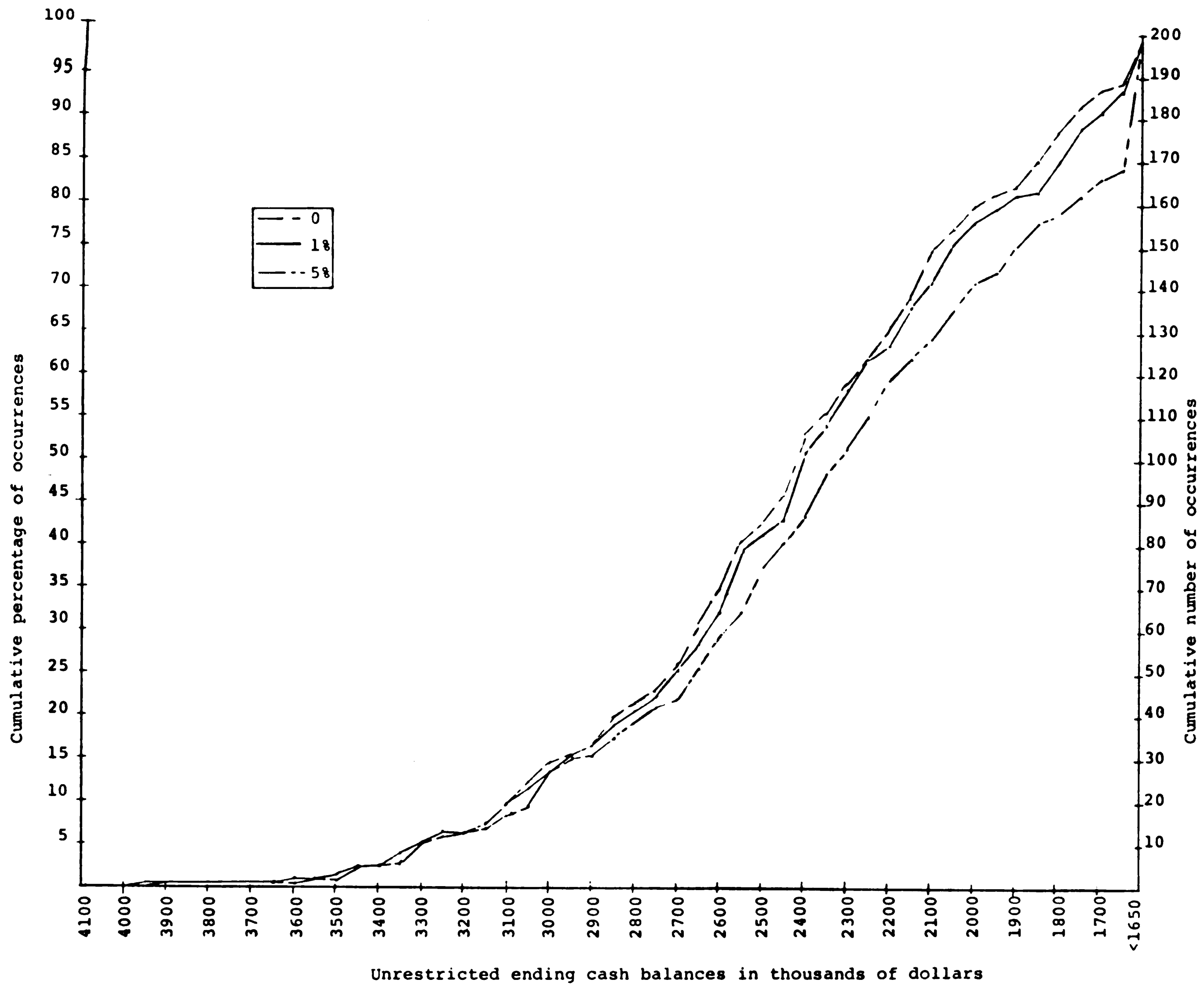

Figure 13. Comparative cumulative probability distributions for Type 2 at stockout penalties of 0, 1, and 5 percent with a minimum required cash balance of $4.5 million. Source: Table V.

Figure 14. Comparative cumulative probability distributions for Type 4 at stockout penalties of 0, 1, and 5 percent with a minimum required cash balance of $4.5 million. Source: Table VI.

Evaluation at a Minimum Required Cash Balance of $6 Million

Stockout Penalty at 1 Percent[1].--At a minimum required cash balance of $6 million Type 3 exhibited stochastic dominance over all other alternatives. Type 3 dominated Type 1A because the dollar value of the interest rate for Type 3, even for three periods, was less than the interest for Type 1A for one period because of the large compensating balance required with Type 1A.[2] At this particular minimum required cash balance, every alternative except for Type 2 exhibited stochastic dominance over at least one other alternative. The alternatives ranked as follows: Type 3, Type 1A, Type 1B, Type 4, Type 2. When sufficient funds were borrowed the constraints on the alternatives were fully effective. Types 2 and 4 were clearly dominated by the other alternatives since both of these types experienced stockouts for every simulation.

Although Type 1A dominated Type 1B, the difference between the desirability of the two alternatives was diminishing. At higher levels of borrowing, the actual dollar values of the compensating balances converge. Thus, if a simulation were run for a minimum required cash

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[1]See Figure 15.

[2]The difference between the two interest rates times the amount needed by the firm equals that amount of interest that is less than the interest rate of Type 1A times the amount of the compensating balance.

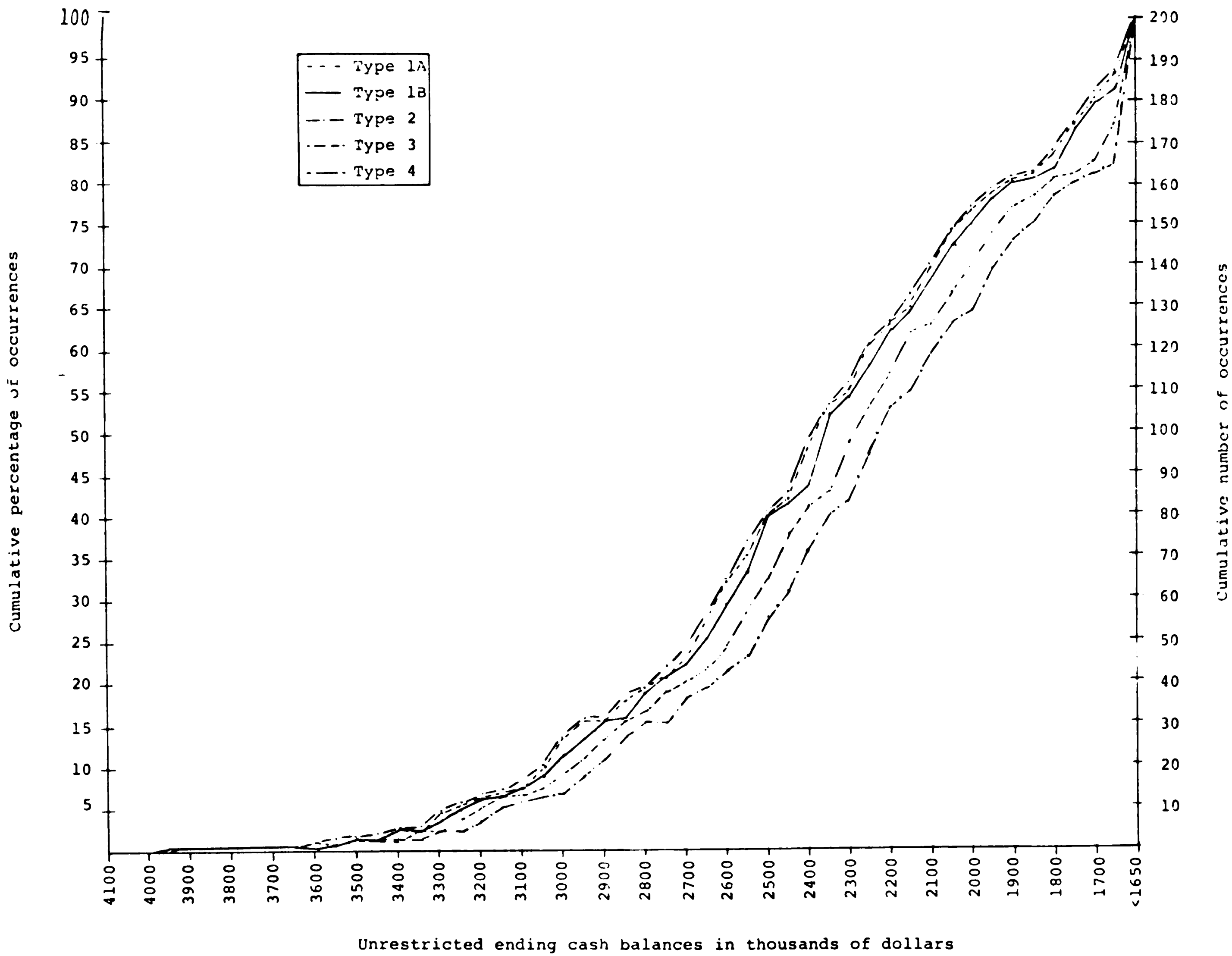

Figure 15. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $6 million and a stockout penalty of 1 percent.
Source: Table VII.

balance of $8 million, the curves of Types 1A and 1B would be approximately the same.

Stockout Penalty at 5 Percent.--The data for alternatives 1A, 1B, and 3 presented in Figure 16 were identical to those plotted in Figure 15 and were included merely for comparative purposes. The 5 percent stockout penalty with a minimum required cash balance of $6 million significantly affected Types 2 and 4. Type 4 was less severely effected by the stockout penalty than Type 2 because the maximum amount that could be borrowed with Type 4 was greater than with Type 2. Thus, even though both alternatives experienced extensive stockouts, Type 2 was the less desirable alternative.

Figures 17 and 18 present Types 2 and 4 and the changes effected by the stockout penalties at 1 and 5 percent. The significant impact of the stockout penalties at the $6 million level is demonstrated.

Evaluation When the Minimum Required Cash Balance is a Stochastic Variable

In an attempt to explore further the effects of the minimum required cash balance on the rankings of the financing alternatives, this value was allowed to be probabilistically defined.

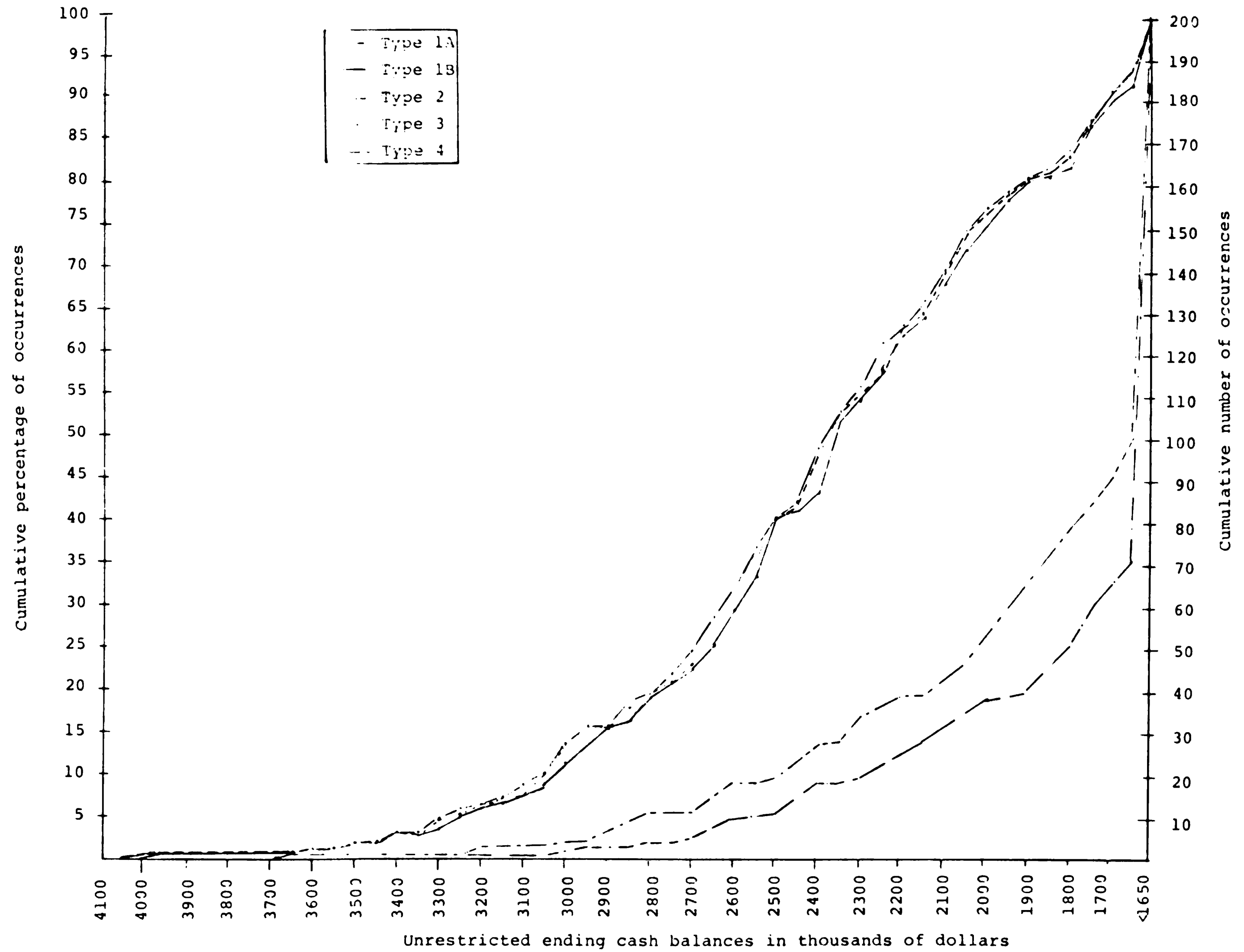

Figure 16. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $6 million and a stockout penalty of 5 percent. Source: Table VIII.

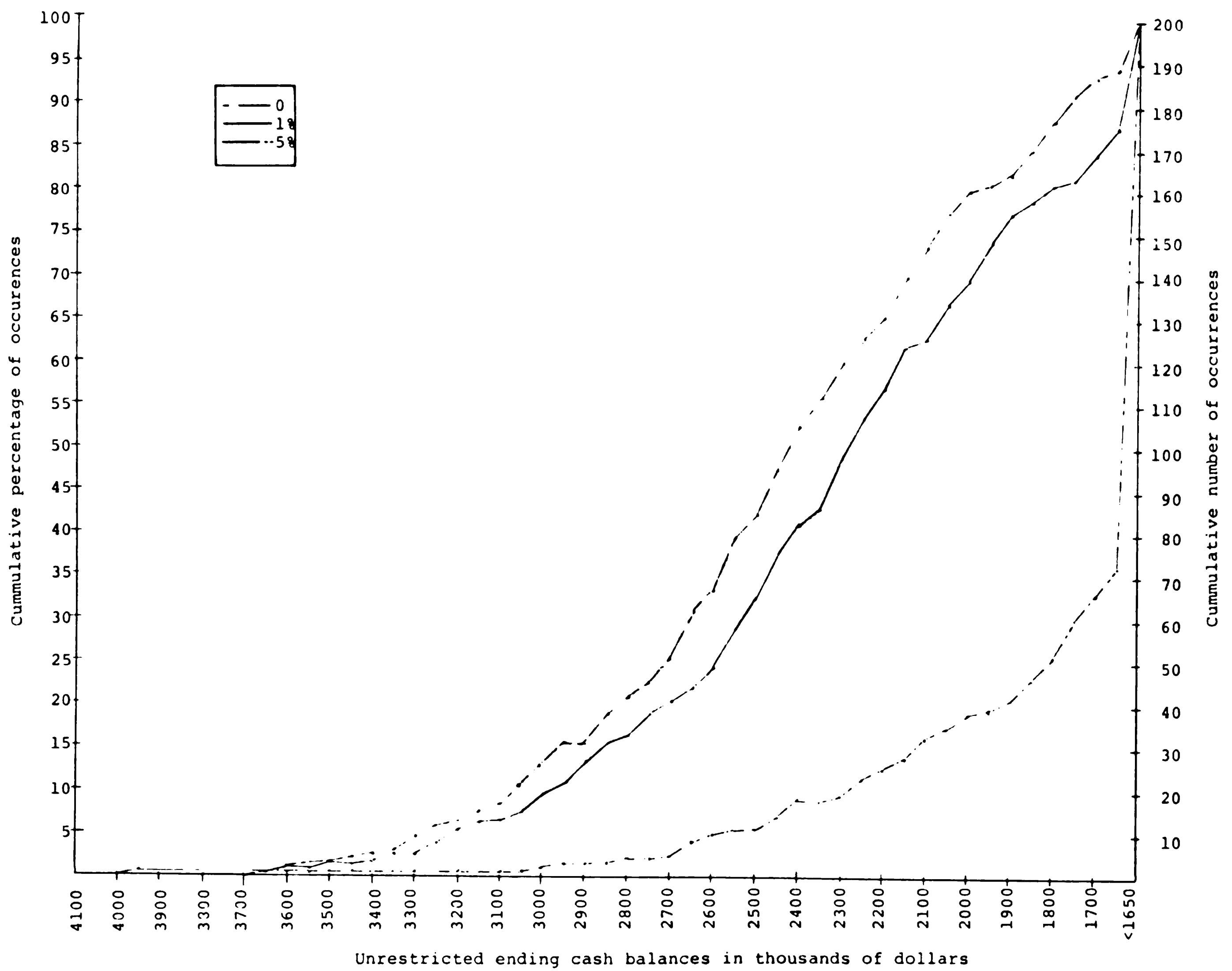

Figure 17. Comparative cumulative probability distributions for Type 2 at stockout penalties of 0, 1, and 5 percent with a minimum required cash balance of \$6 million. Source: Table IX.

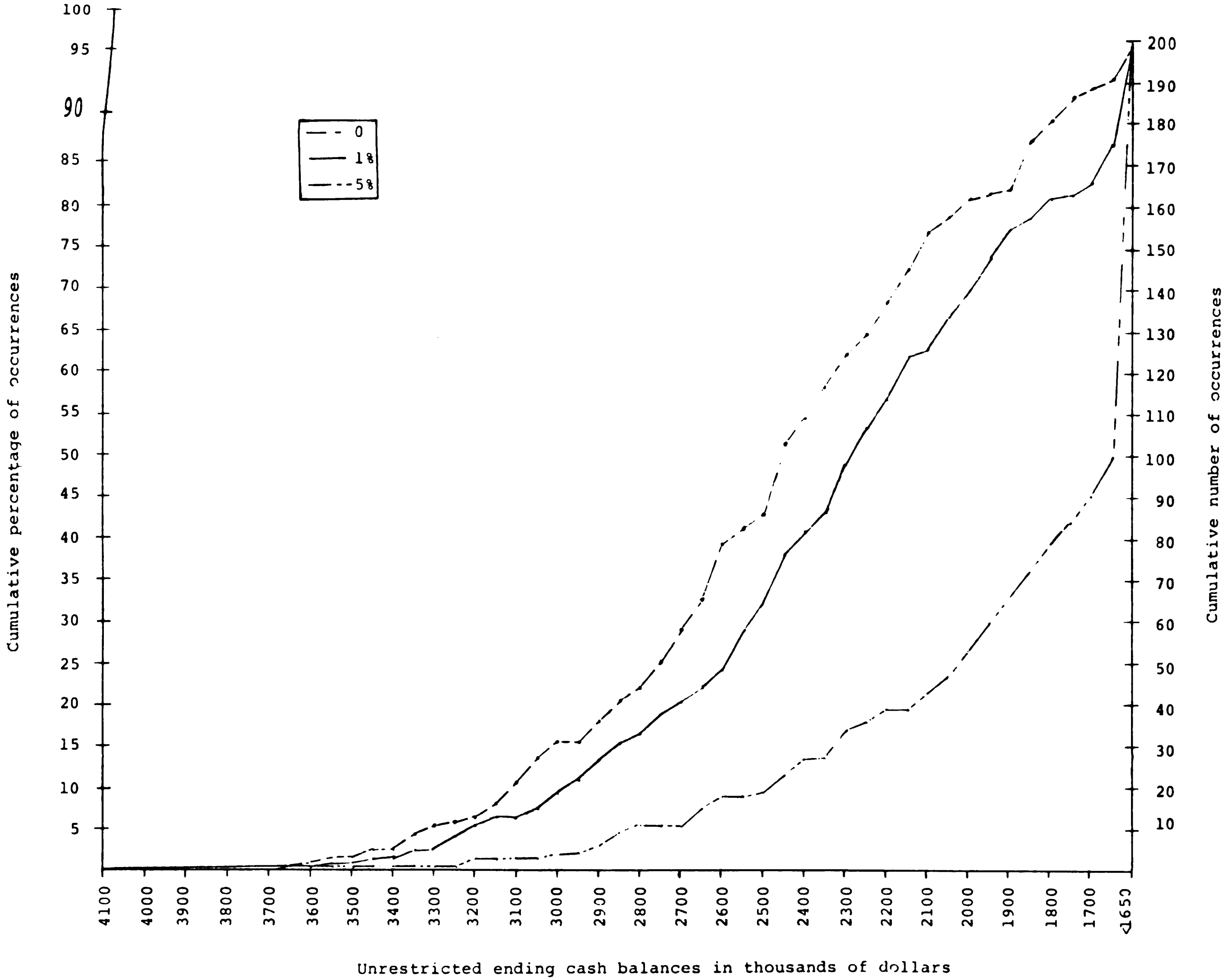

Figure 18. Comparative cumulative probability distributions for Type 4 at stockout penalties of 0, 1, and 5 percent with a minimum required cash balance of $6 million. Source: Table X.

The $4-$5 Million Range with a 1 Percent Stockout Penalty[1].--When a uniform distribution of $4-$5 million was selected and the results of the simulation were tabulated and graphed, it appeared that in allowing this one additional value to be stochastic, the cumulative probability distributions of the financing alternatives could no longer be evaluated using stochastic dominance. The most conclusive statement that could be made was that Type 2 was clearly the inferior alternative. Types 1A, 1B, 3 and 4 exhibited no significant dominance with regard to each other.

The $4-$5 Million Range with a 5 Percent Stockout Penalty[2].--When a 5 percent stockout penalty was imposed, Type 4 became a less desirable alternative. Type 4 still failed to become inferior to Types 1A, 1B, and 3 at all points, however. Figures 19 and 20 can be compared with the deterministic graph at the $4.5 million level.[3] In the deterministic graph alternative 3 had a cumulative probability distribution which ranked above the other alternatives. Although Type 3 did not exhibit true stochastic dominance,[4] it was close to being dominant over

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[1]See Figure 19.

[2]See Figure 20.

[3]See Figure 11.

[4]Type 3 failed to exhibit stochastic dominance at three points on the curve.

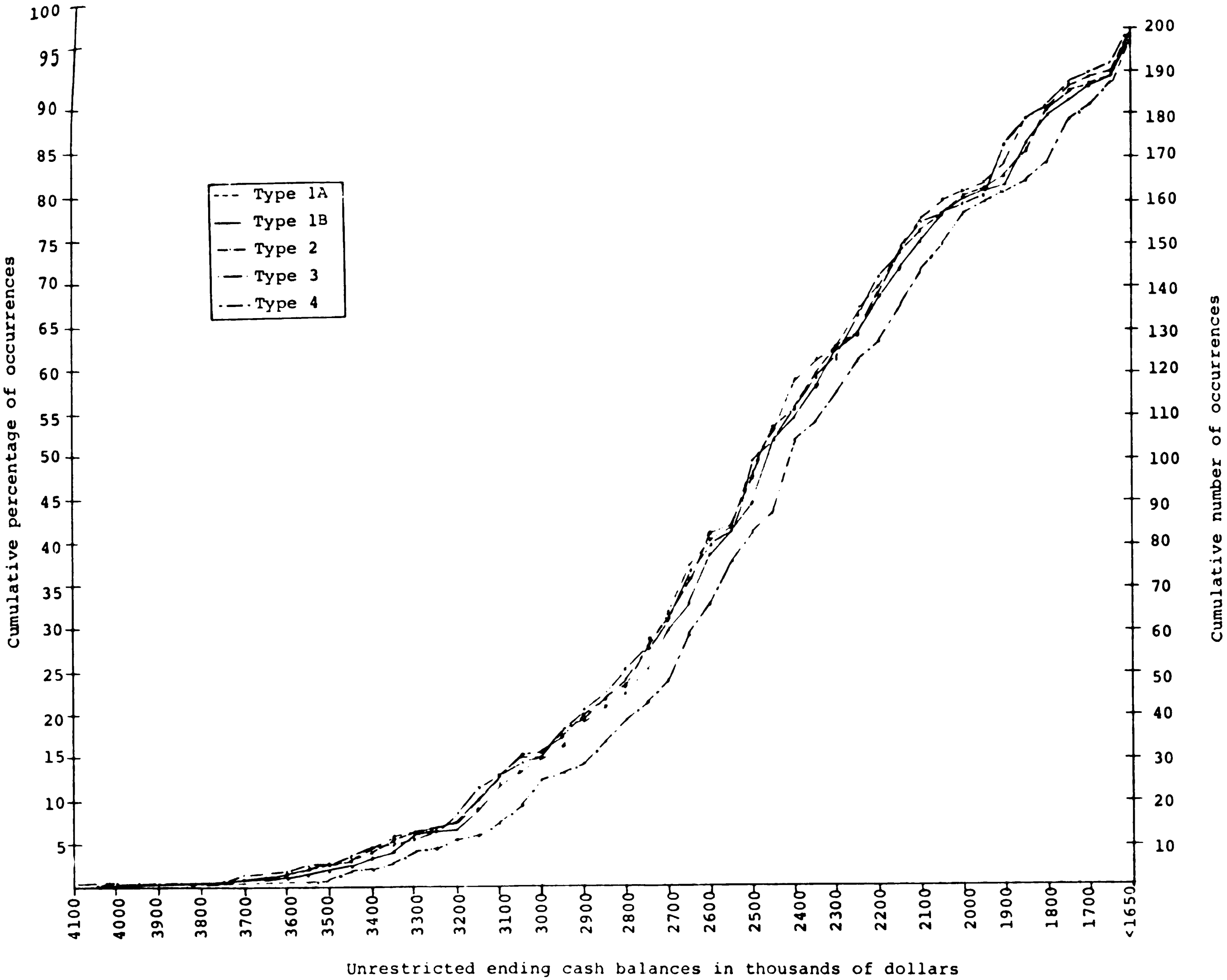

Figure 19. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of \$4-\$5 million and a stockout penalty of 1 percent. Source: Table XI.

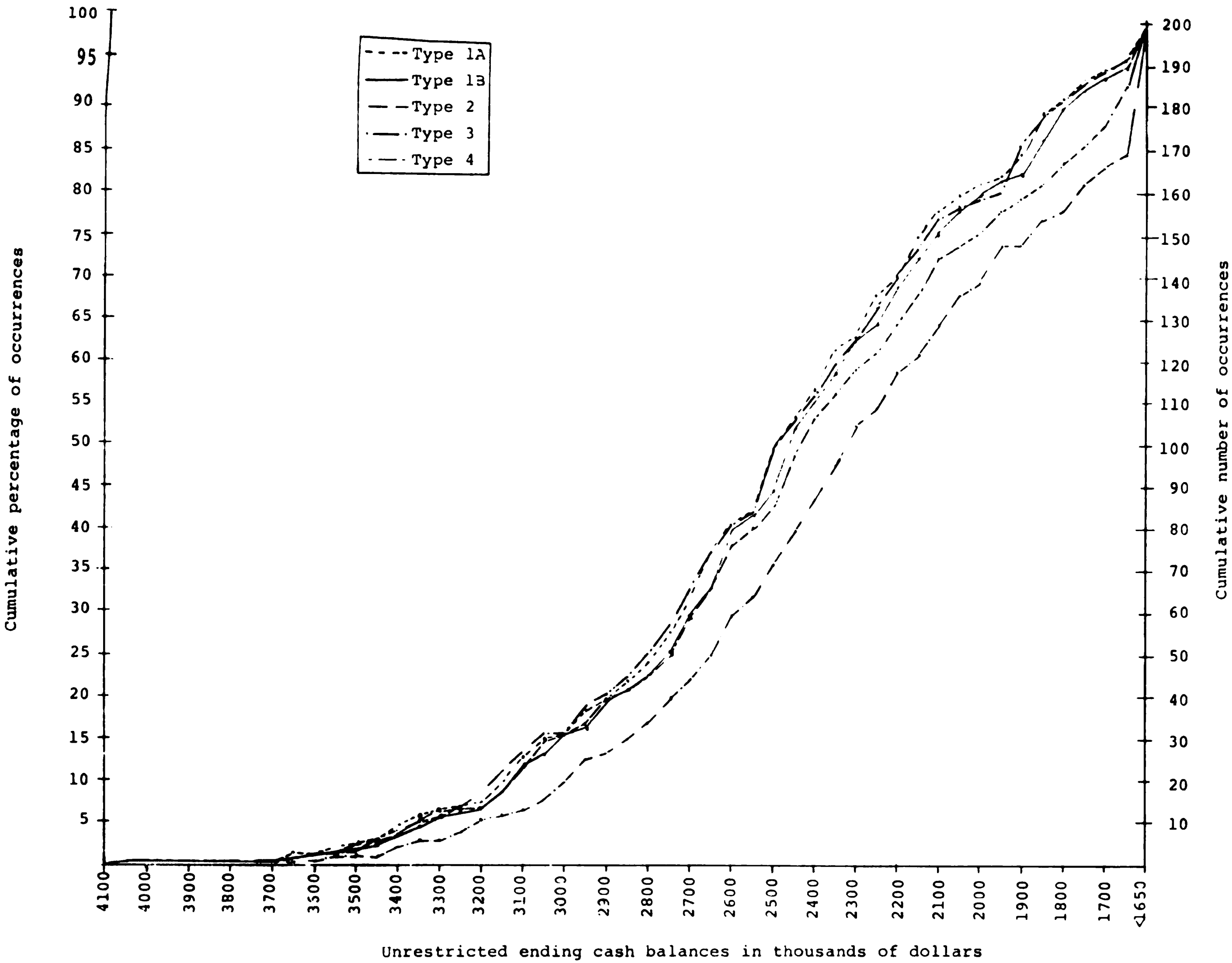

Figure 20. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $4-$5 million and a stockout penalty of 5 percent.
Source: Table XII.

Type 1A, and these two alternatives exhibited dominance over the other alternatives. In the stochastic graph not even this degree of dominance was exhibited. The relatively low level of borrowing caused ranking of the alternatives to be less distinct than they became when higher levels of borrowing were effected. When the minimum required cash balance was allowed to be stochastic, interpretation of the data became even more difficult and no advantage could be seen in this approach at this level.

The $5.5-$6.5 Million Range.--A stochastic minimum required cash balance was also tested at a higher range to determine if stochastic dominance could be established in ranking the alternatives at another level. The results of this simulation, as seen in Figure 21, not only exhibited stochastic dominance between all of the alternatives, but presented a graph remarkably similar to that of the deterministic level of $6 million.[1] The constraints on the alternatives were fully effective at this level, and definitive rankings were more likely to be achieved with the stochastic as well as the deterministic approach.

Summary

The results of the simulations examining the four financing alternatives have been presented in the preceding

[1]See Figure 15.

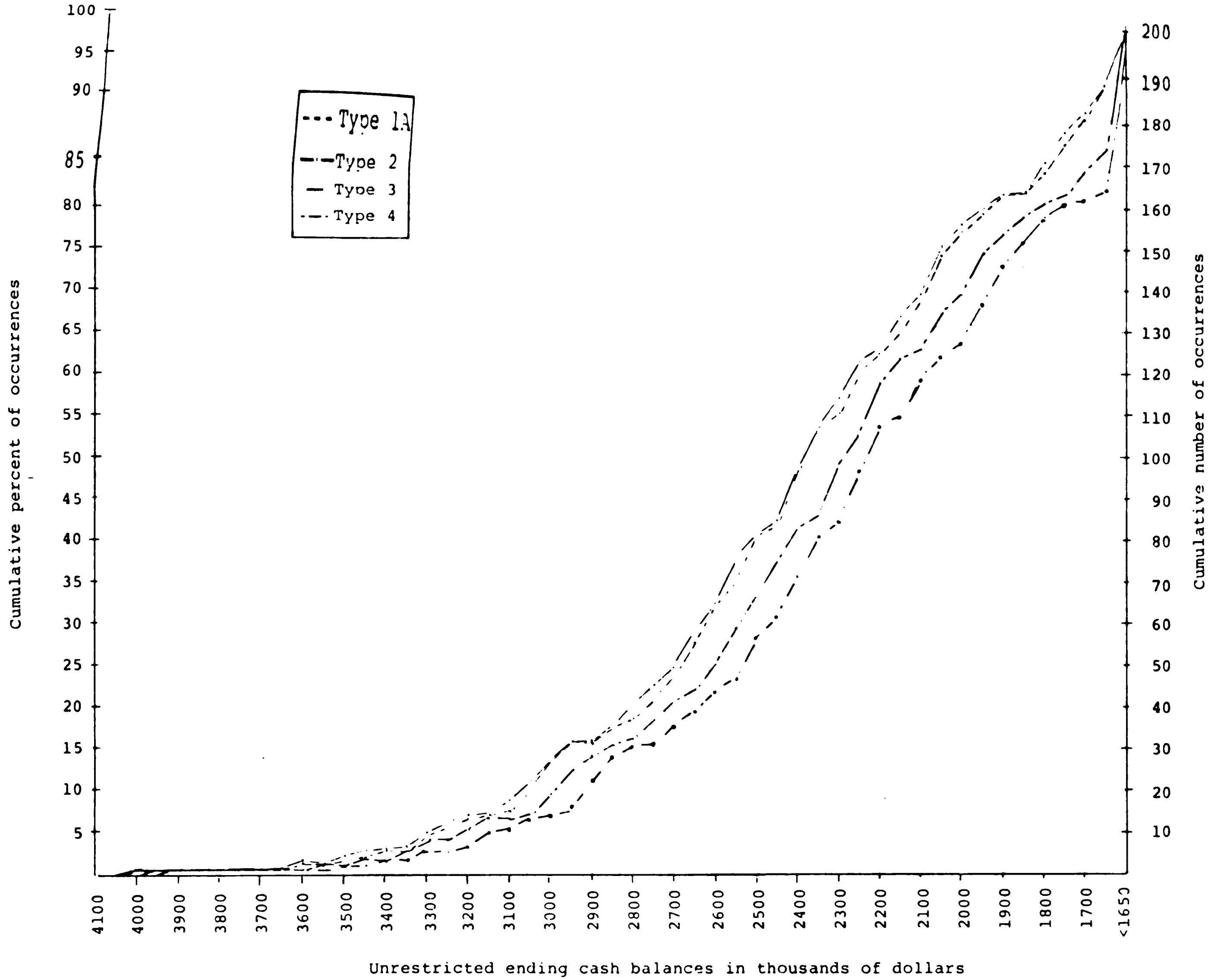

Figure 21. The cumulative probability distributions for the financing alternatives at a minimum required cash balance of $5.5-$6.5 million and a stockout penalty of 1 percent. Source: Table XIII.

section. The net cash flow data and the constraints on the alternatives remained constant while different levels of the minimum required cash balance and stockout penalties were tested. The alternatives were evaluated using stochastic dominance to select an optimal alternative specific to a certain minimum required cash balance. The stockout penalties affected the rankings of the alternatives but in no case changed the optimal alternative. The following table presents a summary of the results.

Minimum Required Cash Balance	Optimal Alternative
Deterministic	
$3 million	Type 1A
$4.5 million	Type 3/Type 1A
$6 million	Type 3
Stochastic	
$4-$5 million	None
$5.5-$6.5 million	Type 3

The Results of the Model in Terms of the Short-Term Financing Problem

Introduction.--The short-term financing problem is complex because of the difficulty of forecasting cash needs. This difficulty arises from the dynamic nature of the individual firm, the industry of which it is a part, and the economy as a whole. The model presented in this study represents an attempt to facilitate management in making that short-term financing decision.

The results of the application of the model to actual data have been presented in the preceding section. The primary objective of this section was to demonstrate the flexibility of the model with regard to input data, to evaluate the effectiveness of the model at various levels of borrowing, and to assess, insofar as possible, the practicability and applicability of the model to the short-term financing problem. In addition, the condition of stochastic dominance was explored as a technique for evaluating the results of the model simulations.

Evaluation of the Model.--The results of the application of the model to actual data suggest that the model may be of significant value to management in the role for which it was designed. The model allows management to evaluate financial information necessary for the short-term financing decision, rapidly and relatively inexpensively. The model quantifies risk which is incorporated into the model through probabilistically defined net cash flow variables and interest rates. Risk is an important consideration since variability of conditions is an observable fact and not recognizing it as such and failing to include it would impair the validity of the model. Further, subjective judgments of management can be considered in the model with regard to some of the variable input data.

Management may utilize the model in the decision-making process without having to define the utility function when stochastic dominance is exhibited. This capability alleviates the problem of deciding upon an appropriate utility function. The model is flexible, facilitating the changes necessary to allow reasonable approximation of conditions specific to the firm employing it. Further, management may vary the input data to the model, or to the net cash flow simulation, to evaluate the results of potential future actions being considered.

The model indicates, if the condition of stochastic dominance is exhibited, that an optimal financing alternative exists, and which alternative is optimal. If there is no stochastic dominance, management will need to specify a utility function in order to maximize the expected value of that function. The results of this study suggest, however, that even if clear stochastic dominance is not demonstrated between the alternatives, those alternatives that are inferior are quite definitely established.

Attributes of the Model.--The model considers the sequential, multiple-period nature of the cash management problem. The incorporation of risk into the model allows it to be relatively realistic, and the simulation approach moderates its computational complexity. In contrast to other studies, the model in this study attempts to broaden its application to consider the options of holding, borrowing, and reinvesting cash. Further, this thesis presents an exploration of the applicability of the condition of stochastic dominance to the evaluation of short-term financing alternatives. This technique was successful in this study particularly at the higher levels of borrowing.

A full understanding of the results of this study cannot be achieved without recognizing the limitations of the model. Four specific financing alternatives are studied. Whereas other alternatives could be added to the model, reprogramming would be required. Further, some firms may want to evaluate combinations of some of the alternatives. The model is not designed to consider combinations.

It is also possible that some of the assumptions of the model may impair its ability to simulate real-world conditions.[1] It is recognized, however, that with nearly every model, certain assumptions must be made. It

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[1]See pages 36 and 37, Chapter III for a list of the assumptions of the model.

can only be hoped that those necessary assumptions do not measurably distort the results.

One further limitation exists which may, to some extent, inhibit full evaluation of the model. As with any heuristic device, the only true test of its effectiveness and predictive powers is a long-term study of the effects of the model recommendations assuming that these recommendations were completely implemented by a test firm or firms. The difficulty of such a study in terms of time and cooperation of a test firm is enormous, and constitutes a significant study in itself. The author in this study elected to utilize real-world data for net cash flow input to the model, and to attempt to select realistic values and distributions for the remaining required input data. The care exercised in the selection of the input data suggests that the results represent a relatively accurate reflection of these data. The author maintains that the model can be evaluated and reasonable predictions about its effectiveness made even though a true test for validity has not been conducted.

In summary, the model in this study has been applied to test data and the results presented. The model has been evaluated in terms of the results, the advantages associated with its use, and its limitations. The model appears to present a useful and realistic approach to the evaluation of short-term financing alternatives.

CHAPTER VI

SUMMARY AND CONCLUSIONS

Summary

Chapter I established the importance of the short-term financing problem, and the need for economizing cash holdings. A model was proposed which was designed to assist management in evaluating four short-term financing alternatives. The basic cash management decision process was discussed and some important terms defined.

In Chapter II, models in the literature which related to the cash management problem were presented and briefly discussed. The major differences between the deterministic and the stochastic approaches were outlined. Methods, including those utilizing linear programming, dynamic programming and simulation, were summarized. The significance of each method in terms of the model in this study was elucidated.

Chapters III and IV were primarily devoted to a thorough explanation of the model. In Chapter III the model was described in decision-theory framework. The single-stage decision problem under certainty was extended to incorporate risk and include multiple stages. The reader was guided through each step of the development of the final multi-stage decision problem under risk specific to the model in this study. The assumptions of the model were also presented. Finally, the author discussed alternative methods of solving the model and indicated the rationale for utilizing simulation. Chapter IV presented a further discussion of the model. The short-term financing alternatives to be evaluated were defined and the method of computer analysis of each alternative was discussed. Finally, the input data to the model were presented in detail.

The results of the application of the model to actual data were presented in Chapter V. Input data in terms of the net cash flow simulation, the constraints on the financing alternatives, the interest rates and the minimum required cash balance were specifically analyzed. The computer programs were briefly explained, and the model output stated. The condition of stochastic dominance was defined, and its role in the evaluation of the financing alternatives delineated. The results of the model were discussed and graphs of the resultant cumulative probability distributions for the financing alternatives included. The model was then discussed in terms of its applicability to the short-term financing problem and its usefulness to management.

Conclusions

The objective of the model in this study was to select an optimal financing strategy from a given set of alternatives. The model included the capability of assessing the options of holding, borrowing or reinvesting cash. The condition of stochastic dominance was employed successfully to evaluate the cumulative probability distributions of the financing alternatives.

The results of the application of the model to actual data suggested its potential value to management. The model was able to produce results that could be interpreted in terms of an optimal financing alternative. The model was flexible in terms of input data, and relatively inexpensive to run. Risk was incorporated into the model and a sequential, multiple-period approach was used. The model was designed to approximate real-world conditions without becoming unduly complex computationally. Finally, the model programs could be implemented by a firm with a minimum amount of effort and computer knowledge.

Suggestions for Further Research

In the process of developing the model, the author encountered the problem of determining net cash flow values to use as input to the model. Several approaches to this problem are mentioned in Chapter IV. It would appear that a thorough exploration of the problem would be a valuable contribution to the literature.

An interesting related problem also involves the net cash flow. It may be possible that the stochastic process that describes the unrestricted ending cash balances for each alternative could be analytically derived from the stochastic process of the net cash flow, assuming that the latter process were known. In such a study, the distribution of the interest rates and the constraints must also be known and/or defined. If the stochastic process of the net cash flow was not known and could not be defined, an alternative approach would be the utilization of the stochastic processes of the variables of which the net cash flow is composed. If such a technique were successful, computer facilities for solving a model would be unnecessary. This would allow very small businesses to benefit from the approach suggested in this study.

Another aspect of the study which provides a fertile area for further research is the empirical verification of the model. A sufficient number of firms should be analyzed, ideally, to establish the statistical significance of the predictive powers of the model. Firms in different industries could also be included.

Contributions of the Study

It is felt that this study makes a number of contributions which supplement the existing literature. A sequential, multi-period approach that considered risk was incorporated in the simulation model that was presented in this thesis. The net cash flow values that served as input to the model were generated by a simulation of the stochastic processes of the input variables which made up the net cash flow. Finally, the condition of stochastic dominance was utilized to evaluate the cumulative probability distributions which were generated from the model output. Two quantitative techniques, simulation and the condition of stochastic dominance have been integrated and applied to analyzing and selecting an optimal financing strategy from a given set of alternatives.

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APPENDICES

APPENDIX A

COMPUTER SUBROUTINES FOR PROBABILITY

DISTRIBUTIONS

```
      SUBROUTINE UNIFRMI(A,B,X)
      TYPE INTEGER A,B,X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
```

```
      SUBROUTINE UNIFRMR(A,B,X)
      TYPE REAL A, B, X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
```

```
      SUBROUTINE NORMALI(EX,STDX,X)
      TYPE INTEGER EX,STDX,X
      SUM=0.0
      DO 4 I=1,12
      R = RANF(-1)
    4 SUM=SUM + R
      X=STDX *(SUM- 6.) + EX
      RETURN
      END
```

```
      SUBROUTINE NORMALR(EX,STDX,X)
      TYPE REAL EX,STDX, X
      SUM=0.0
      DO 4 I=1,12
      R = RANF(-1)
    4 SUM=SUM + R
      X=STDX *(SUM- 6.) + EX
      RETURN
      END
```

```
      SUBROUTINE GENERAL (A,P,N,X)
      DIMENSION P(5),CP(5),A(5)
      TYPE REAL P,CP
      TYPE INTEGER X,N ,A
C     DETERMINE CUMULATIVE DISTRIBUTION
      CP(1)= P(1)
      IF(N .GT. 1)GO TO 20
C     DETERMINISTIC VALUE
      X= A(1)
      GO TO 200
   20 DO 1 I= 2,N
      CP(I)= CP(I-1) + P(I)
    1 CONTINUE
      R = RANF(-1)
C     PICKS POINT ON THE CUMULATIVE DISTRIBUTION
      IF (R .LE. CP(1))10,100
   10 X =A(1)
      GO TO 200
  100 I=1
  101 I = I + 1
      IF(CP(I) .LT. 1.00)GO TO 105
      X = A(I-1)
      WRITE(61,1)I
    1 FORMAT(24H CP(I) GT 1.00 ERROR I= ,I4)
      GO TO 200
  105 IF(R .LE. CP(I))110,101
  110 X = A(I)
  200 CONTINUE
      RETURN
      END
```

APPENDIX B

NET CASH FLOW SIMULATION PROGRAM

AND INPUT DATA

```
'JOB,466696,543379,3,0,ELLIS,JOHN
'FOR,L,X,D
      PROGRAM ELLIS1
   85 DIMENSION C2(52),         CRS(52), CAR(52), ND(52), COMSI(52),
     1 SFA(52),CR(52)
   90 DIMENSION DW(52),RM(52),                                  GA(52),
     1 FDGA(52), CD(52),         DIP(52),            DRP(52), DRF(52),
     2          DIVP(52), DIVC(52)
   95 DIMENSION NETCF(52), NCF(52)
  110 DIMENSION SF1(52), SF2(52), SF3(52), SF4(52)  ,MSCL(52)
      DIMENSION CRSPER(52),ACTSRV(52),XACTSRV(200,13),XNETCF(200,14)
  125 DIMENSION CRSOLD(6),INTSPC1(52), SF2RP(56),OLDOUT(6),MISC(52)
      DIMENSION IVAL(5),PCNT(5)
      DIMENSION  S1(52),S2(52),NS(52),S3(52,5),PS(52,5)
      DIMENSION  CAP1(52),NCAP(52),CAP3(52,5),PCAP(52,5)
      DIMENSION  TXP1(52),NTXP(52),TXP3(52,5),PTXP(52,5)
      DIMENSION  SPMISC1(52),NMISC(52),SPMISC3(52,5),PMISC(52,5)
      TYPE REAL INTSPC1
      TYPE INTEGER S1,S2,S3,CAP1,CAP3,TXP1,TXP3,SPMISC1,SPMISC3
      TYPE INTEGER CAP,TXP,SPMISC
      TYPE INTEGER CS,     CRS, CAR, ND, COMSI, SFA,OUT2,CR
      TYPE INTEGER DW, RM,                    GA, FDGA
      TYPE INTEGER CD,     DIP,      DRP, DRF,      DIVP, DIVC
      TYPE INTEGER NETCF, NCF
      TYPE INTEGER CRSOLD ,OLDOUT,MISC, SALES,SS1,SS2
      TYPE INTEGER K, T, KK, COLCRT, J
      TYPE INTEGER          ACTSRV, XACTSRV, XNETCF
      COMMON/ELLIS1/IVAL,PCNT
C     BEGINNING
C     N IS NUMBER OF PERIODS
      IN1=60
      OUT2=61
      N=13
      DO 200 J=1,N
      ND(J)=0
      COMSI(J)=0
      SFA(J)=0
      DIP(J)=0
      DRP(J)=0
      DRF(J)=0
      DIVP(J)=0
      DIVC(J)=0
      MISC(J)=0
  200 CONTINUE
C     CASHFLOW DATA
```

```
C        INDXXXX INDICATES TYPE OF DATA
C        1     DETERMINISTIC
C        2     UNIFORN DISTRIBUTION
C        3     NORMAL DISTRIBUTION
C        4     GENERAL DISTRIBUTION
C        REVENUE CAN BE DETERMINISTIC, UNIFORM,NORMAL OR GENERAL
         READ(IN1,1008)IND
         IF(IND .GT. 1)GO TO 201
C         DETERMINISTIC
         READ (IN1,1000)(S1(J),J=1,N)
         INDS=1
         GO TO 210
  201    IF(IND .EQ. 2)GO TO 202
         IF(IND .EQ. 3)GO TO 203
         IF(IND .EQ. 4)GO TO 204
C        SALES HAS UNIFROM DISTRIBUTION
  202    READ(IN1,1000)(S1(J),J=1,N)
         READ(IN1,1000)(S2(J),J=1,N)
         INDS=2
         GO TO 210
C        SALES HAS NORMAL DISTRIBUTION
  203    READ(IN1,1000)(S1(J),J=1,N)
         READ(IN1,1000)(S2(J),J=1,N)
         INDS=3
         GO TO 210
C        SALES HAS GENERAL DISTRIBUTION
  204    DO 205 J=1,N
         READ(IN1,1009)NS(J)
         II= NS(J)
         DO 206 I=1,II
         READ(IN1,1010)S3(J,I),PS(J,I)
  206    CONTINUE
  205    CONTINUE
         INDS=4
  210    CONTINUE
C        CAPITAL EXPENDITURES CAN BE DETERMINISTIC OR GENERAL
         READ(IN1,1008)IND
         IF(IND .GT. 1)GO TO 221
C         DETERMINISTIC
         READ(IN1,1000)(CAP1(J),J=1,N)
         INDCAP=1
         GO TO 230
  221    CONTINUE
C        GENERAL DISTRIBUTION
         DO 222 J=1,N
```

```
      READ(IN1,1009)NCAP(J)
      II=NCAP(J)
      DO 223 I=1,II
      READ(IN1,1010)CAP3(J,I),PCAP(J,I)
  223 CONTINUE
  222 CONTINUE
      INDCAP=4
  230 CONTINUE
C     TAX PAYMENTS CAN BE DETERMINISTIC OR GENERAL
      READ(IN1,1008)IND
      IF(IND .GT. 1)GO TO 231
C      DETERMINISTIC
      READ(IN1,1000)(TXP1(J),J=1,N)
      INDTXP=1
      GO TO 240
  231 CONTINUE
C     GENERAL DISTRIBUTION
      DO 232 J=1,N
      READ(IN1,1009)NTXP(J)
      II=NTXP(J)
      DO 233 I=1,II
      READ(IN1,1010)TXP3(J,I),PTXP(J,I)
  233 CONTINUE
  232 CONTINUE
      INDTXP=4
  240 CONTINUE
C     SPECIAL MISCELLANEOUS COSTS CAN BE DETERMINISTIC OR GENERAL
      READ(IN1,1008)IND
      IF(IND .GT. 1)GO TO 241
C     DETERMINISTIC
      READ(IN1,1000)(SPMISC1(J),J=1,N)
      INDMISC=1
      GO TO 250
  241 CONTINUE
C     GENERAL DISTRIBUTION
      DO 242 J=1,N
      READ(IN1,1009)NMISC(J)
      II=NMISC(J)
      DO 243 I=1,II
      READ(IN1,1010)SPMISC3(J,I),PMISC(J,I)
  243 CONTINUE
  242 CONTINUE
      INDMISC=4
  250 CONTINUE
C     READ IN DETERMINISTIC DATA
```

```
C       NEW DEBT ISSUED  ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(ND(J),J=1,N)
C       COMMON STOCK ISSUED  ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(COMSI(J),J=1,N)
C       SALE OF FIXED ASSET  ASSUMED DETERMINISTIC
        READ(IN1,1000)(SFA(J),J=1,N)
C       INTEREST ON LONG TERM DEBT ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(DIP(J),J=1,N)
C       DEBT RETIREMENT  ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(DRP(J),J=1,N)
C       DEBT SINKING FUND  ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(DPF(J),J=1,N)
C       PREFERRED STOCK DIVIDENDS  ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(DIVP(J),J=1,N)
C       COMMON STOCK DIVIDENDS   ASSUMED TO BE DETERMINISTIC
        READ(IN1,1000)(DIVC(J),J=1,N)
C        GENERAL AND ADMINISTRATIVE FIXED PORTION OF COST
        READ (IN1,1000)(FDGA(J),J=1,N)
C       FIXED PORTION OF MISCELLANEOUS
        READ(IN1,1000)(MISC(J),J=1,N)
C       RESEARCH AND DEVELOPMENT
C       READ(IN1,10000)(RD(J),J=1,N)
C       FIXED SELLING COSTS
C       READ (IN1,1000)(FDSELL(J),J=1,N)
C       THE FOLLOWING PARAMETERS USE EITHER UNIFORM OR NORMAL DISTRIBUTIONS
C       PARAMETER ALPHA
        READ(IN1,1008)IND
        INDALPHA=2
C       FOR UNIFORM DISTRIBUTION
C       ALPHA1 IS LOWER VALUE, ALPHA2 IS UPPER VALUE
C       FOR NORMAL DISTRIBUTION
C       ALPHA1 IS EXPECTED VALUE, ALPHA2 IS STD DEVIATION
        READ(IN1,1030)ALPHA1,ALPHA2
        IF(IND .GT. 2)INDALPHA=3
C       FOR PARAMETER A
        READ(IN1,1008)IND
        INDA=2
        READ(IN1,1030)A1,A2
        IF(IND .GT. 2)INDA=3
C       FOR PARAMETER B
        READ(IN1,1008)IND
        READ(IN1,1030)B1,B2
        INDB=2
        IF(IND .GT. 2)INDB=3
C       FOR PARAMETER THETA
```

```
      READ(IN1,1008)IND
      INDTHETA=2
      READ(IN1,1030)THETA1,THETA2
      IF(IND .GT. 2)INDTHETA=3
C     FOR PARAMETER DELTA
      READ(IN1,10 8)IND
      INDDELTA=2
      READ(IN1,1030)DELTA1,DELTA2
      IF(IND .GT. 2)INDDELTA=3
C     KK IS NUMBER OF PERIODS THAT CREDIT SALES TAKES TO BE COMPLETELY
C         COLLECTED
      KK = 4
      READ (IN1,1002)(CRSPER(J),J=1,KK)
C     OUTSTANDING CREDIT SALES
      READ (IN1,1005)(CRSOLD(J),J=1,KK)
      OLDOUT(1)=0
      DO 370 J=1,KK
      OLDOUT(1)= OLDOUT(1) + CRSOLD(J)
  370 CONTINUE
      DO 371 J=2,KK
      OLDOUT(J)= OLDOUT(J-1) - CRSOLD(J-1)
  371 CONTINUE
    1 FORMAT (8H SALES   ,I8 ,8H CS(T)= ,I8 ,9H CRS(T)= ,I8 ,
     1     12H ALPHA    =  ,F5.3,9H ACTSRV= ,I8)
    2 FORMAT (10H COLCRT=  ,I8 ,9H CRS(J)= ,I8 ,12H CRSPER(I)= ,F5.3)
    3 FORMAT (8H CR(T)= ,I8 ,9H CAR(T)= ,I8 ,8H ND(T)= ,I8,
     1     12H COMSI(T)=  ,I8,9H SFA(T)= ,I8)
    4 FORMAT ( 8H WAGES= ,I8,7H A    = ,F5.3,15H RAW MATERIAL= ,I8,
     1     7H B    = ,F5.3)
    5 FORMAT (                                      8H GA(T)= ,I8,
     1 11H THETA    = ,F5.3,10H MSCL(T)= ,I8,11H DELTA    = ,F5.3)
    6 FORMAT (9H DIP(T)= ,I8,6H TXP= ,I8,6H DRP= ,I8 ,6H DRF= ,I8,
     1     6H CAP= ,I8,7H DIVP= ,I8,7H DIVC= ,I8,9H SPMISC= ,I8)
   16 FORMAT(1H0,20H START OF PERIOD T= ,I3)
   20 FORMAT(12H CRSPER(J)= ,4F5.3)
 1000 FORMAT(9I8/4I8)
 1001 FORMAT(13F5.3)
 1002 FORMAT (4F5.3)
 1004 FORMAT (I8)
 1005 FORMAT (4I8)
 1006 FORMAT(I2)
 1007 FORMAT(I2,I8)
 1008 FORMAT(I2)
 1009 FORMAT(I2)
 1010 FORMAT(I8,F5.3)
```

```
1011 FORMAT (10F6.4/10F6.4/10F6.4/10F6.4/10F6.4/2F6.4)
1012 FORMAT (3I8)
1013 FORMAT (I8,F6.4)
1030 FORMAT(2F5.3)
1400 FORMAT (17H DEBUG INPUT DATA)
1401 FORMAT (1H ,11(1X,I8))
1405 FORMAT (1H ,4(3X,F5.3))
1408 FORMAT (1H ,5(3X,F6.4))
1411 FORMAT (1H ,6(3X,F5.3))
1412 FORMAT(1H0,6H MAX1 ,I8,6H MAX2 ,I8,8H MAX2PA ,I8,8H MIN2PA ,I8,
    1 6H MAX3 ,I8,6H MAX4 ,I8,7H PER4= ,F6.4)
1413 FORMAT (1H0,10H CASHBAL  ,I8)
1414 FORMAT(1H0,12H CRSOLD(T)  ,4I8)
1415 FORMAT(2X,7I10,2X,I3,1HA)
1416 FORMAT(2X,6I10,12X,I3,1HB)
1417 FORMAT(1H ,I3,1X,14I9)
1418 FORMAT(2X,7I10,2X,I3,1HC)
1419 FORMAT(2X,6I10,12X,I3,1HD)
1420 FORMAT(1H ,I3,1X,13I9)
1426 FORMAT(1H1,54X,15H NET CASH FLOW )
1427 FORMAT(1H1,60X,21H ACCOUNTS RECEIVABLE )
1428 FORMAT(1H ,4H NO ,13(9H  PERIOD ),11H CUMULATIVE)
1429 FORMAT(1H ,4H NO ,13(9H  PERIOD ))
1430 FORMAT(5H     ,4X,2H 1,7X,2H 2,7X,2H 3,7X,2H 4,7X,2H 5,7X,2H 6,
    1                     7X,2H 7,7X,2H 8,7X,2H 9,7X,2H10,7X,2H11,
    2                     7X,2H12,7X,2H13,7X,4H NCF)
1431 FORMAT(5H     ,4X,2H 1,7X,2H 2,7X,2H 3,7X,2H 4,7X,2H 5,7X,2H 6,
    1                     7X,2H 7,7X,2H 8,7X,2H 9,7X,2H10,7X,2H11,
    2                     7X,2H12,7X,2H13)
     WRITE (OUT2,1400)
     WRITE(OUT2,20)(CRSPER(J),J=1,KK)
     DO 1502 J = 1, N
     WRITE (OUT2,1401)ND(J),COMSI(J),SFA(J),     DIP(J),TXP1(J),DRP(J),
    1 DRF(J),CAP1(J),DIVP(J),DIVC(J),FDGA(J)
1502 CONTINUE
     WRITE(OUT2,1405)A1,A2,B1,B2
     WRITE(OUT2,1411)ALPHA1,ALPHA2,THETA1,THETA2,DELTA1,DELTA2
     WRITE (OUT2,1414)(CRSOLD(J),J=1,KK)
C
C    SIMULATION DO LOOP FOR NUMBER OF TIMES RUN
     DO 1720 ITIS = 1,200
C    DETERMINE CASH FLOW FOR PERIOD T
     DO 1800 T = 1,13
C    BASIC EQUATIONS OF CASH INFLOWS
C    CR (T) = CS (T)  +    CAR (T) +   ND (T)  + COMSI (T) + SFA (T)
```

```
C       CASH SALES AND CREDIT SALES IN PERIOD T
 2000   CONTINUE
        IF(INDS .GT. 1)GO TO 2001
        SALES=S1(T)
        GO TO 2010
 2001   IF(INDS - 3)2003,2005,2007
 2003   SS1=S1(T)
        SS2=S2(T)
        CALL UNIFRMI(SS1   ,SS2   ,SALES)
        GO TO 2010
 2005   SS1=S1(T)
        SS2=S2(T)
        CALL NORMALI(SS1   ,SS2   ,SALES)
        GO TO 2010
 2007   II=NS(T)
        DO 2008 I=1,II
        IVAL(I)=S3(T,I)
        PCNT(I)=PS (T,I)
 2008   CONTINUE
        CALL GENERAL(IVAL,PCNT,II ,SALES)
 2010   CONTINUE
C       DETERMINE VALUE OF PARAMETER ALPHA
        IF(INDALPHA .EQ. 2)CALL UNIFRMR(ALPHA1    ,ALPHA2    ,ALPHA)
        IF(INDALPHA .EQ. 3)CALL NORMALR(ALPHA1    ,ALPHA2    ,ALPHA)
        CS (T)= ALPHA      * SALES
        CRS (T) = SALES * (1 - ALPHA)
C       CALCULATE ACCOUNTS RECEIVABLE AT START OF PERIOD T
C       AND
C       COLLECTION OF CREDIT SALES IN PERIOD T, KK IS MAX NO. PERIODS BACK
C       ACTSRV(1) = OLDOUT(1)
C       ACTSRV(T + 1)=ACTSRV(T) + CRS(T) - CAR(T)
        COLCRT = 0
        IF (T .LE. KK)2011,2025
 2011   IF(T .EQ. 1)2012,2015
 2012   COLCRT=CRSOLD(1)
        ACTSRV(T)= OLDOUT(1)
        GO TO 2035
 2015   COLCRT=CRSOLD(T)
        L=T - 1
        DO 2017 J=1,L
        I=L - J + 1
        COLCRT= CRS(J)*CRSPER(I) + COLCRT
        IF(ITIS .LE. 5 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
       1WRITE(OUT2,2)COLCRT,CRS(J),CRSPER(I)
 2017   CONTINUE
```

```
      GO TO 2035
 2025 COLCRT=0
      K = T - KK
      L= T - 1
      DO 2030 J = K, L
      I = L - J + 1
      COLCRT = CRS (J) * CRSPER (I) + COLCRT
      IF(ITIS .LE. 5 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,2)COLCRT,CRS(J),CRSPER(I)
 2030 CONTINUE
 2035 CAR (T) = COLCRT
      ACTSRV(T + 1) =ACTSRV(T) + CRS(T) - CAR(T)
C     CASH RECEIPTS
      CR (T) = CS (T) + CAR (T) + ND (T) + COMSI (T) + SFA (T)
C     BASIC EQUATIONS OF CASH OUTFLOWS
C     CD (T) = DW (T) + RM (T) + FOH (T) + SELL (T) + GA (T) +
C              RD (T) + DIP (T) + TXP (T) + DRP (T) + DRF (T) +
C              CAP (T) + DIVP (T) + DIVC (T)
C     DIRECT WAGES
      IF(INDA .EQ.2)CALL UNIFRMR(A1    ,A2    ,A)
      IF(INDA .EQ.3)CALL NORMALR(A1    ,A2    ,A)
      DW (T) = A      * SALES
C     RAW MATERIAL
      IF(INDB .EQ. 2)CALL UNIFRMR(B1    ,B2    ,B)
      IF(INDB .EQ. 3)CALL NORMALR(B1    ,B2    ,B)
      RM (T) = B      * SALES
C     GENERAL AND ADMINISTRATIVE
      IF(INDTHETA .EQ. 2)CALL UNIFRMR(THETA1    ,THETA2    ,THETA)
      IF(INDTHETA .EQ. 3)CALL NORMALR(THETA1    ,THETA2    ,THETA)
      GA (T) = FDGA (T) + THETA      * SALES
C     MISCELLANEOUS
      IF(INDDELTA .EQ. 2)CALL UNIFRMR(DELTA1    ,DELTA2    ,DELTA)
      IF(INDDELTA .EQ. 3)CALL NORMALR(DELTA1    ,DELTA2    ,DELTA)
      MSCL(T) = MISC(T) + DELTA    * SALES
C     OVERHEAD
C     FOH (T) = BETA      * SALES
C     SELLING
C     SELL (T) = FDSELL (T) + GAMMA      * SALES
C     DETERMINE VALUE OF CAPITAL EXPENDITURES
      IF(INDCAP .GT. 1)GO TO 2105
      CAP    = CAP1(T)
      GO TO 2115
 2105 II=NCAP(T)
      DO 2110 I=1,II
      IVAL(I)=CAP3(T,I)
```

```
      PCNT(I)=PCAP(T,I)
 2110 CONTINUE
      CALL GENERAL(IVAL,PCNT,II ,CAP)
C     DETERMINE TAX PAYMENTS
 2115 IF(INDTXP .GT. 1)GO TO 2125
      TXP = TXP1(T)
      GO TO 2135
 2125 II=NTXP(T)
      DO 2130 I=1,II
      IVAL(I)= TXP3(T,I)
      PCNT(I)= PTXP(T,I)
 2130 CONTINUE
      CALL GENERAL(IVAL,PCNT,II,TXP)
C     DETERMINE SPECIAL MISC VALUES
 2135 IF(INDMISC .GT. 1)GO TO 2145
      SPMISC = SPMISC1(T)
      GO TO 2155
 2145 II=NMISC(T)
      DO 2150 I=1,II
      IVAL(I)=SPMISC3(T,I)
      PCNT(I)= PMISC(T,I)
 2150 CONTINUE
      CALL GENERAL(IVAL,PCNT,II,SPMISC)
 2155 CONTINUE
C     CASH DISBURSEMENTS
      CD (T) = DW (T)  + RM (T)                              + GA (T)  +
     1                   DIP (T)  + TXP        + DRP (T)   + DRF (T) +
     2          CAP     + DIVP (T) + DIVC (T) + MSCL(T)   + SPMISC
      IF(ITIS .LE. 5 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )1798,1799
 1798 CONTINUE
      WRITE (OUT2,16)T
      WRITE (OUT2,1) SALES, CS (T), CRS (T), ALPHA ,ACTSRV(T)
      WRITE (OUT2,3) CR (T), CAR (T), ND (T), COMSI (T), SFA (T)
      WRITE (OUT2,4) DW (T), A     , RM (T), B
      WRITE(OUT2,5)                 GA(T),THETA   ,MSCL(T),DELTA
      WRITE (OUT2,6) DIP (T), TXP    , DRP (T), DRF (T), CAP    ,
     1    DIVP (T), DIVC (T) , SPMISC
 1799 CONTINUE
C     NET CASH FLOW CALCULATIONS
      NETCF (T) = CR (T) - CD (T)
      XNETCF(ITIS,T) = NETCF(T)
      XACTSRV(ITIS,T)= ACTSRV(T)
 1800 CONTINUE
C     PUNCH OUT NET CASH FLOW RESULTS
      WRITE(62,1415)((NETCF(J),J=1,7),ITIS)
```

```
      WRITE(62,1416)((NETCF(J),J=8,13),ITIS)
      WRITE(62,1418)((ACTSRV(J),J=1,7),ITIS)
      WRITE(62,1419)((ACTSRV(J),J=8,13),ITIS)
C     WRITE OUT NETCASH FLOW RESULTS AT END OF EACH ITIS FOR
C     13 PERIODS AND CUMULATIVE NET CASH FLOW FOR TOTAL
      NETCF(14) =0
C     DETERMINE CUMULATIVE NET CASH FLOW
      DO 2200 I=1,13
      NETCF(14)= NETCF(14) + NETCF(I)
 2200 CONTINUE
      XNETCF(ITIS,14)= NETCF(14)
      IF(ITIS .LE. 5 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )2210,1720
 2210 CONTINUE
      WRITE(OUT2,1417)ITIS,(NETCF(J),J=1,14)
      WRITE(OUT2,1420) ITIS,(ACTSRV(J),J=1,13)
 1720 CONTINUE
C     WRITE OUT NETCASH FLOW AS ARRAY FOR ALL SIMULATIONS
      WRITE(OUT2,1426)
      WRITE(OUT2,1428)
      WRITE(OUT2,1430)
      DO 1721 I=1,200
      WRITE(OUT2,1417)I,(XNETCF(I,J),J=1,14)
 1721 CONTINUE
C     WRITE OUT ACTSRV ARRAY FOR ALL SIMULATION
      WRITE(OUT2,1427)
      WRITE(OUT2,1429)
      WRITE(OUT2,1431)
      DO 1722 I=1,200
      WRITE(OUT2,1420)I,(XACTSRV(I,J),J=1,13)
 1722 CONTINUE
      END
      SUBROUTINE UNIFRMI(A,B,X)
      TYPE INTEGER A,B,X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
      SUBROUTINE UNIFRMR(A,B,X)
      TYPE REAL A, B, X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
      SUBROUTINE NORMALI(EX,STDX,X)
      TYPE INTEGER EX,STDX,X
```

```
      SUM=0.0
      DO 4 I=1,12
      R = RANF(-1)
    4 SUM=SUM + R
      X=STDX *(SUM- 6.) + EX
      RETURN
      END
      SUBROUTINE NORMALR(EX,STDX,X)
      TYPE REAL EX,STDX, X
      SUM=0.0
      DO 4 I=1,12
      R = RANF(-1)
    4 SUM=SUM + R
      X=STDX *(SUM- 6.) + EX
      RETURN
      END
      SUBROUTINE GENERAL (A,P,N,X)
      DIMENSION P(5),CP(5),A(5)
      TYPE REAL P,CP
      TYPE INTEGER X,N ,A
C     DETERMINE CUMULATIVE DISTRIBUTION
      CP(1)= P(1)
      IF(N .GT. 1)GO TO 20
C     DETERMINISTIC VALUE
      X= A(1)
      GO TO 200
   20 DO 1 I= 2,N
      CP(I)= CP(I-1) + P(I)
    1 CONTINUE
      R = RANF(-1)
C     PICKS POINT ON THE CUMULATIVE DISTRIBUTION
      IF (R .LE. CP(1))10,100
   10 X =A(1)
      GO TO 200
  100 I=1
  101 I = I + 1
      IF(CP(I) .LE. 1.00)GO TO 105
      X = A(I-1)
      WRITE(61,1)I
    1 FORMAT(24H CP(I) GT 1.00 ERROR I= ,I4)
      GO TO 200
  105 IF(R .LE. CP(I))110,101
  110 X = A(I)
  200 CONTINUE
      RETURN
```

```
     END
'RUN,3,0,3000
02         SALES DISTRIBUTION IS UNIFORM
 6598890 6598890 6598890 6684570 6684570 6684570 6541770 6541770 6541770 S171
 6394253 6394253 6394253 6394253                                         S171
 8065310 8065310 8065310 8170030 8170030 8170030 7995496 7995496 7995496 S171
 7815197 7815197 7815197 7815197                                         S171
04    CAPTIAL EXPENDITURES HAS GENERAL DISTRIBUTION
03    T=1
  100000 .300
  135000 .350
  150000 .350
03   T=2
  100000 .300
  130000 .350
  150000 .350
03   T=3
  100000 .300
  130000 .350
  150000 .350
02   T=4
  250000 .500
  300000 .500
02   T=5
  250000 .500
  300000 .500
02 T=6
  250000 .500
  300000 .500
03 T=7
  230000 .200
  250000 .400
  300000 .400
03   T=8
  230000 .200
  250000 .400
  300000 .400
02   T=9
  350000 .400
  400000 .600
02   T=10
  350000 .500
  400000 .500
02   T=11
  350000 .400
```

```
   400000 .600
02 T=12
   300000 .450
   400000 .550
01 T=13
          01.000
04        TAX PAYMENTS HAS GENERAL DISTRIBUTION
01        T=1
          01.000
01        T=2
          01.000
01        T=3
          01.000
03        T=4
   150000 .200
   195000 .600
   220000 .200
01        T=5
          01.000
04        T=6
   550000 .200
   575000 .200
   600000 .300
   620000 .300
01        T=7
          01.000
01        T=8
          01.000
01        T=9
          01.000
03        T=10
   430000 .300
   450000 .400
   480000 .300
01        T=11
          01.000
01        T=12
          01.000
03        T=13
   600000 .250
   650000 .500
   685000 .250
04       SPECIAL MISCELLANEOUS HAS GENERAL DISTRIBUTION
03        T=1
   600000 .100
```

```
       650000 .500
       700000 .400
     02        T=2
       150000 .500
       175000 .500
     03        T=3
       200000 .200
       240000 .600
       270000 .200
     02        T=4
        80000 .450
       100000 .550
     02        T=5
        33000 .500
        35000 .500
     01        T=6
               01.000
     01        T=7
               01.000
     03        T=8
       100000 .200
       125000 .600
       150000 .200
     01        T=9
               01.000
     01        T=10
               01.000
     02        T=11
       115000 .500
       135000 .500
     01        T=12
               01.000
     01        T=13
               01.000
                0          0          0          0          0          0          0          0          0 ND71
                0          0          0          0                                                       ND71
                0          0          0          0          0          0          0          0          0 COMSI71
                0          0          0          0                                                       COMSI71
                0          0          0          0          0          0          0          0          0 SFA71
                0          0          0          0                                                       SFA71
             6075       6075      19810       6075     123880       6075      19810       6075       6075 DIP71
            19810     123880       6075      19810                                                       DIP71
                0          0      34000          0          0          0      34000          0          0 DRP71
            34000          0     127500      34000                                                       DRP71
                0          0          0          0          0          0          0          0          0 DRF71
```

```
      0       0       0       0                                                       DRF71
  10691       0       0   10691       0       0   10691       0       0 DIVP71
      0   10691       0       0                                                       DIVP71
 173995       0       0  173995       0       0  173995       0       0 DIVC71
      0  173995       0       0                                                       DIVC71
1195000 1195000 1195000 1195000 1195000 1195000 1195000 1195000 1195000 FDGA71
1195000 1195000 1195000 1195000                                                       FDGA71
  45000   45000   45000   45000   45000   45000   45000   45000   45000 MISC71
  45000   45000   45000   45000                                                       MISC71
02      ALPHA IND  UNIFORM DIST
 .450 .500   ALPHA12
02      A IND UNIFORM
 .490 .505  A12
03      B IND NORMAL
 .205 .010 B12
02      THETA IND NORMAL
 .043 .002   THETA12
02      DELTA IND UNIFORM
 .009 .014  DELTA12
 .952 .025 .012 .013           CRSPER(KK)
 3800000  100000   50000   50000          CRSOLD(KK) 1971
 ''
```

APPENDIX C

NET CASH FLOW OUTPUT

NET CASH FLOW

NO.	PERIOD 1	PERIOD 2	PERIOD 3	PERIOD 4	PERIOD 5	PERIOD 6	PERIOD 7	PERIOD 8	PERIOD 9	PERIOD 10	PERIOD 11	PERIOD 12	PERIOD 13	CUMULATIVE NCF
1	-25173	-152185	-130288	49093	492195	-264434	78191	-42861	180811	-56910	-156346	-632519	117585	-429041
2	-701922	410476	204830	-504139	-49222	40336	-80926	288636	340032	-411463	-218305	434300	-383607	-630974
3	-258369	-20941	222846	-950164	864083	-713421	142642	-30214	280892	-224428	-371092	79064	-288557	-1267559
4	-424104	47692	-64658	-86485	714327	-727835	109766	138894	60234	-494947	160853	-254004	-570682	-1390949
5	-325020	232970	164976	-378968	-470916	34479	68818	292741	-179933	-400873	388777	-308428	-470695	-1352072
6	-404203	351823	131120	-143759	242767	-543373	527516	-596147	528468	57613	-486464	153581	-776316	-957374
7	-226918	27889	196167	-610310	336642	-557128	125638	-336927	542119	-347462	-309516	16922	133117	-1009767
8	-520480	466927	229716	-152852	649019	-734482	570264	514489	-578703	-89845	178062	-168620	70178	433673
9	-216244	-74082	505438	-536934	-268966	-143690	423553	-181486	199736	-187490	301370	-72885	-532255	-784135
10	-386680	-90111	626230	-108188	274022	-347703	60891	-33202	98551	44916	-497610	-202729	-101289	-662842
11	-496478	87604	-257473	-2614	70091	-385168	490519	143734	-159868	-152435	-483948	18049	-6088	-1134075
12	-346000	250142	-93648	302381	154026	-607062	336740	263569	732156	-814383	49980	-112779	91656	206778
13	-514110	325828	-223954	286805	116294	-444290	-204519	394504	221004	84499	-459404	65187	-475658	-827814
14	-472605	697209	79045	-295448	462530	-514531	-111076	5106	283901	-377160	-128442	-195003	-83351	-649125
15	-547472	554804	-662204	-149159	515144	-443309	614586	130600	-203771	-270011	-961990	177401	362080	-883301
16	-589509	106348	415086	-622689	385611	227270	-604458	854543	-28849	-278118	183083	-135389	69541	-17530
17	-277121	-401547	396119	-73158	-310313	284463	-195333	-160677	152485	-361472	-292795	97218	-543364	-1685495
18	-466394	210748	-594239	-378409	488242	83614	-66121	9344	430454	-122282	-106757	101085	-185376	-596081
19	-276222	-549371	673307	-281777	340338	-453977	67723	160097	511606	-284529	-458802	-404374	40501	-915480
20	-939337	466612	178889	-212209	-98564	-372255	-310397	412482	567705	-443404	-288083	94010	-300598	-1245149
21	-224214	-54243	403473	426080	-283597	-7510	282236	118864	287335	-392885	-122492	-69550	-207248	156249
22	-433349	219791	279775	-443210	-338730	-67096	-279390	-147684	311691	-384391	-529252	268661	-303533	-1846717
23	-538761	82371	395542	-387301	372669	-422122	269458	610642	-64987	-213717	-615713	15053	-167758	-664624
24	-433896	332297	288840	-44581	143032	-264987	-604992	497070	246428	-227850	-629973	101752	172043	-424817
25	-311081	-26063	11126	-880619	497692	-297893	131425	646797	-67618	-443733	122752	410749	-308549	-521015
26	-770431	704265	-77338	-575173	398630	-907566	358853	317140	46749	194719	-127627	-365633	431132	-372280
27	-725671	808031	-357002	-11756	221517	-518491	226269	-451429	178190	-201029	94388	446618	-235585	-525950
28	-589907	612112	171602	-824672	300229	-719468	499448	-207216	468598	-528812	48906	-218050	323618	-663612
29	-850579	811879	-115234	-673413	434960	-324390	-77196	457260	-170116	80520	-460155	-62401	-93960	-1042825
30	-398445	131950	-39037	-311539	449910	-718819	-24042	-56197	-152825	-373384	75836	390467	-173088	-1199313
31	-418851	-95125	-120985	405577	-145603	-103004	-195252	325896	548134	-440426	-202939	-258889	-688357	-1389824
32	-374309	64059	226263	62961	-185382	-308954	567875	-407953	96080	-448907	-111651	-158075	-271668	-1249661
33	-345355	242473	-412317	679606	-244837	-552354	287045	-77192	559250	-100161	-193493	250841	-386797	-293251
34	-345917	246567	107447	102462	-217849	-622815	131608	370237	587655	-677425	-84141	-251941	-852	-654964
35	-313133	207002	394480	-324886	450791	29224	425743	-140862	16408	241359	-784256	426482	-545900	82452
36	-270080	-335802	592485	-316556	54981	-191600	427866	361319	388523	173905	-645682	24628	-210076	53911
37	-555421	306877	-123543	264154	262837	-677476	353717	-143530	227022	-747287	213861	-29111	-680521	-1328421
38	-443644	116619	376644	-668466	342870	-333205	-34963	29468	112748	-519952	79685	-124770	-41131	-1108097
39	-678020	370664	231386	-186675	403812	121057	180608	263827	216029	-184061	-781696	-71345	105604	-8810
40	-395569	367805	-210315	-92209	-225806	95860	-1830	-191546	127918	-246985	-291451	-352269	24092	-1392305
41	-454105	36041	558913	-311535	693415	-671956	371036	-246273	413291	-221881	-195542	160147	-261394	-129843
42	-265968	220819	319618	-467702	192956	-154856	57442	325961	-208015	-392906	156265	49491	-99019	-265914
43	-572507	-14695	89428	109103	35623	-675763	310771	444380	-160932	61831	-985201	254423	-172110	-1275649
44	-432028	22200	-74866	-174910	381848	-354687	693326	195937	53355	-839294	-299268	264928	43456	-520003
45	-362622	-168614	141202	-148768	-58802	307485	-354902	506001	98067	-83578	-500067	137969	26046	-460583
46	-455785	7165	523986	-578668	99970	-357650	-11119	379133	-158338	160240	-524511	24404	-702071	-1593244
47	-455950	82880	112906	93868	-160261	-75295	95463	112180	424708	-193965	-122292	50935	-347275	-382098
48	-428308	247870	286706	-250696	305555	-214853	-26071	269559	270955	-227293	-203718	157038	57813	244577
49	-305761	-206082	743531	-884090	381241	-705132	455271	-173713	570979	18409	-419002	-294411	-101019	-919779
50	-688335	687987	95409	70709	-39807	-512324	308011	497670	232338	-414536	-534153	-312010	82328	-526713
51	-364285	415558	-12155	-1010258	889467	-919550	520894	-74485	610769	-233525	-375718	-209515	239417	-523386
52	-438998	172635	-48521	-130820	91115	-421303	-130874	-190045	172412	-108066	-147897	-72437	-426852	-1679641
53	-918929	734370	277745	-323163	-139374	-358665	-87441	621663	-165378	-339891	-248621	344578	-466012	-1069138
54	-168363	-170316	67960	61383	-24570	-635516	279186	306329	163618	-555236	143475	121081	-78526	-489495
55	-553794	37277	63621	11659	588938	-170557	330668	-10763	-140763	-361786	363757	-147847	349621	360031
56	-530016	370749	54527	-172680	101051	-825310	317002	207841	-182000	-178161	-451367	-349704	-454660	-2092728
57	-320012	290377	-112129	-790277	215354	-203219	2636	-495316	916806	-580405	-81207	159315	40326	-957754
58	-864607	650675	-525252	279990	-293343	186246	194431	-146520	526668	-963882	-287977	352090	-232976	-1124457

59	-376382	147655	352369	-643694	186279	-503419	366736	-165350	139720	-53260	5582	28442	-132325	-647647
60	-397437	-361433	468219	-170045	173701	-186415	394727	191729	159084	-206514	-372992	543194	-62540	167278
61	-515592	-105057	186696	-144290	497648	-141051	106648	398728	673097	-549948	-852286	311306	-293474	-427575
62	-254834	-457119	441964	-224239	9312	47261	437419	61807	-104465	7490	-120419	-341554	118303	-379074
63	-592600	185972	480994	-552131	197155	-423203	123054	220841	-119837	-209819	-193810	-234052	217848	-899588
64	-377426	-107342	-13568	-79106	294733	-470277	-17695	271783	324625	-410734	-744458	370941	82601	-875923
65	-600087	448119	-51114	-263957	-89794	-144835	20444	750261	-347083	-389779	-574909	566031	-716400	-1393103
66	-312476	-78257	544603	-514546	650389	-330017	-237220	505951	-102798	-585816	-298749	153513	-326541	-931964
67	-501751	341234	247071	-13105	311758	-528438	112099	834046	208676	-366619	-214662	203403	-674150	-40438
68	-646959	227979	454162	23598	-97175	-153461	-352721	271279	169735	346182	-375257	141514	-351241	-342365
69	-468022	111368	-68461	45742	-475625	2916	285149	-130590	14688	177533	-558145	-213627	-88205	-1365279
70	-510527	499534	176131	-57159	92634	34119	-220321	400254	-147594	-149063	-675826	626732	-289018	-220104
71	-105051	-377730	222188	-143420	-272740	-329862	777687	-484069	508501	-135783	-300382	-115738	-303649	-1060048
72	-435736	286185	-43725	-380978	-375164	175071	-61125	215361	50112	-1031731	151759	-580437	536620	-1493588
73	-266051	65810	269340	-468544	556855	-813628	655350	75353	-73922	11093	-390629	-22236	-29660	-430869
74	-323688	463987	16479	-210376	363646	-542849	71620	-316169	494618	-420284	-271894	17737	-636413	-1293586
75	-230834	-94146	596380	-405941	-109955	57748	-141944	-124858	263163	-150254	-442939	15691	198221	-569668
76	-457907	134635	-198291	-303620	159512	108664	324811	105792	11293	-1823	-749306	473339	-289520	-682421
77	-477543	136792	574746	-642415	270096	73949	-294268	431038	-256288	-511456	-288762	326665	-204506	-861952
78	-240500	-324016	647618	-252897	-198002	-514775	19631	194207	116392	-32282	-129246	-279324	270485	-722709
79	-627359	480038	-36490	-413377	-177845	-329465	-511485	251007	405265	-170927	-74159	-90090	-367767	-1662654
80	-402286	408170	-57084	-55368	-186728	-380674	649823	144063	516222	-280753	-522874	-36936	-590025	-793950
81	-387030	727071	-421350	40459	87685	-25238	-136930	-175199	-288100	-76431	-466650	620223	-181766	-683256
82	-567742	772880	-186136	87272	-291605	46994	69974	122471	409442	-1131837	-197441	640884	-308775	-533619
83	-444290	205094	-122192	31665	293431	-348132	462594	218149	-367201	22485	-253024	-318931	255418	-364934
84	-764880	236831	128691	-288568	-99118	19843	-45202	340976	-52239	-763877	-75287	332959	9304	-1020567
85	-408738	55566	878807	-480466	80441	-48452	175915	-146896	734837	-121353	-175972	-292857	67401	318233
86	-510746	189648	318273	-207924	107112	-396408	539836	358491	10022	-551765	-706344	-206752	71916	-984141
87	-502991	-125333	81189	109880	-198607	-44161	266882	-517803	646973	-207566	-855908	357124	-118621	-1108942
88	-215290	-103564	-39923	-340964	206688	1648	188595	291266	-64029	-307091	-232803	279321	-602336	-938482
89	-520287	478997	-483332	243266	501507	-698633	227713	644117	-421010	-132677	263189	57617	-293430	-132963
90	-286477	226721	-183486	605312	-140897	-220215	7397	-201868	231609	251690	-805759	-296054	59912	-752115
91	-246487	-42743	-260515	-281917	747869	-315194	-80427	154224	431977	-472574	-304315	-161986	95974	-736114
92	-505066	89276	115157	5312	320002	394710	-149868	328502	-49110	-499985	-214568	549154	-519363	-135847
93	-285183	-195913	-287686	126162	-15306	130801	98319	230495	413063	-105588	-708809	-96585	-145629	-841859
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95	-725375	531532	392059	-650673	305732	-583933	516195	-14057	-2106	224803	-934341	663639	-637814	-914339
96	-428774	86402	382850	-571267	158666	-387045	184574	-101947	289874	-70778	-93190	-396269	281152	-665752
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104	-436124	607144	-333056	-592037	334912	82690	-222248	50253	151222	176709	-472223	-83873	-474688	-1251299
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109	-671821	501754	-432589	309905	26502	-377453	512157	104808	424896	-379982	25243	-168990	-402718	-528288
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116	-480564	430031	-198285	-366424	228410	-736843	377413	364507	295808	-276809	50608	-147126	-306810	-766084
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118	-799850	496692	150394	-428264	281488	-71830	51440	298249	687620	-712153	250327	-68030	-633351	-497268
119	-214066	408141	-153153	-516984	699571	-564126	461438	-80265	302348	-394154	-545630	30709	-449551	-1015722

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121	-420613	-172265	171085	-218994	-429661	-183268	263761	297876	392542	-100168	-108688	-335720	408235	-435878
122	-346229	92001	480796	68144	155642	-234304	108220	-104740	184380	-518408	145806	176617	-891501	-683576
123	-358166	-120997	376023	-362907	13339	-448681	257147	-162769	147834	-120077	-560149	-477930	-418771	-1280244
124	-613159	290774	137533	314765	258694	-153761	179797	137394	272426	-306816	131536	-80143	317807	886847
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127	-414028	92103	-122020	-534617	238481	-12180	289248	275045	-5912	-77458	-495545	628723	-154455	-292615
128	-555813	587680	150783	-554006	44528	469401	-520073	682491	-96455	-708688	-393861	285643	-52275	-660645
129	-349576	-429125	569312	-206502	224405	-529348	-80350	644549	-161387	-487644	57453	-141225	85713	-803725
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133	-507548	-152589	59803	318546	-20922	-143057	159736	100006	545868	-635440	231346	-262431	19172	-287510
134	-693916	914698	-473478	-159167	134670	-100684	234767	58299	-31459	-136625	-553570	226398	-302404	-882471
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143	-614720	485662	-144499	-100893	359513	-356828	204599	-311390	188770	-175593	-460118	-96683	-307401	-1329581
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155	-781945	666599	18339	42877	-112094	203851	-160921	382687	-81669	-545370	-201066	-341775	-342089	-1252516
156	-722147	665816	-246677	-104364	-67039	139100	287752	532639	28676	-191437	-329719	-117586	-114688	-239674
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163	-368375	-239390	409187	-91173	633548	-424387	256636	-214033	260618	198946	-375454	133911	-399949	-219915
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165	-606405	358925	-81448	-449027	362940	-283656	-3203	-73969	401448	-78795	-345581	-523597	-98481	-1420849
166	-765998	672975	-146905	9829	-438802	-466726	600630	15879	80635	-555540	-286808	-20507	38549	-1262789
167	-901475	912220	265656	-155525	-326463	145847	-6920	-130504	674034	-482318	-490162	-56925	70777	-481758
168	-51559	-449459	254374	-167819	223579	-305161	310995	381141	-100655	-149904	-265297	380065	166267	226567
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171	-304263	-297206	590520	-596787	318795	-676786	-97109	827237	-131567	-354360	8796	-340947	18369	-1035308
172	-261129	-173977	113171	-251645	93854	-264453	341612	77493	-310780	149690	-567072	-152016	268008	-937244
173	-532398	-405496	72286	278447	71668	-364705	454288	150182	214092	-391905	-427692	65137	-153755	-969851
174	-697888	415543	-433687	-322036	599182	-498226	462166	166892	86689	-361494	-240833	70493	-242565	-995764
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177	-526193	-10149	333180	-535845	228093	43103	-591514	444570	241159	103423	-335865	203037	-282518	-685519
178	-494751	243549	84054	-78604	67615	15538	-400159	819041	-51563	-123940	112631	-491666	50699	-247556
179	-487339	364848	-91744	269996	-350812	-215367	69329	547714	175300	-556620	-335740	420405	-284708	-474738
180	-447743	404707	64726	-476415	613081	-954841	709442	-88466	-90243	174352	-377511	-608211	392648	-684474

181	-811200	321297	431994	-375512	4079	-118323	250090	486967	6547	-224852	-280561	-163628	-48758	-521860
182	-447459	-9549	191044	-214641	262053	-41511	-269218	7569	576787	-512865	-91259	165715	-196150	-579484
183	-607931	565657	-418052	146542	170782	-744452	592875	80260	-72234	189658	-161741	-291289	9921	-540004
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185	-455684	440359	-269135	-3775	118498	-374325	282296	-734858	570968	-187643	-459169	266277	11905	-794286
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187	-709493	315842	243260	-431027	-208278	-140242	366268	455946	-42887	-230305	-552182	-21283	-225658	-1180039
188	-102658	-332700	567901	-164190	214864	4485	331051	237915	-46336	-200165	-519854	218769	-240257	-31175
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190	-213480	-38211	158486	-25027	-31945	-435561	185988	-35553	511375	-584757	-554098	510416	-429955	-982322
191	-482546	278720	479941	-287709	-355628	-66026	193788	178642	-15046	3984	-398488	288386	-263314	-445296
192	-201744	-243200	735072	-640105	-32397	-355286	-23720	-162643	314170	99011	-880438	577780	-205706	-1019206
193	-634826	266384	600442	-390925	557629	-638686	412694	-442059	487713	152806	-431426	-355787	376293	-39748
194	-279443	356223	-501855	-153941	316676	-112747	180692	337228	-48298	78373	-711261	250788	-425209	-712774
195	-404996	224018	-19334	37215	12795	-344711	772484	123110	197748	-585458	161947	-378086	-29966	-233236
196	-531378	145772	88947	37676	565574	-475784	66567	50787	254708	-151413	-163264	166116	-14581	39727
197	-508370	467496	319277	-102690	325301	-672916	948596	-528067	645211	-549034	-604203	226650	-541584	-574333
198	-438892	-129987	300487	66148	-142457	-333226	32524	16623	103638	-88766	-349618	171445	-295079	-1085160
199	-350501	139740	-203068	78517	-390771	379397	95984	712708	-17907	-244596	-488367	97003	69279	-172632
200	-585576	382271	99379	-286635	-69499	-513788	478992	176028	-362815	58639	-487955	440425	-307429	-977963

APPENDIX D

SIMULATION OF THE FINANCING ALTERNATIVES--
COMPUTER PROGRAM AND INPUT DATA

```
'JOB,466696,543379,3,0,ELLIS,JOHN
'EQP,L,X,D
      PROGRAM ELLIS2
   95 DIMENSION NETCF(52), NCF(52), INVINC(55), INVSPC(55), FINTRN(55),
     1 STFRP(55), INTPAY(55)
  100 DIMENSION PCASBAL(52), CASH(52), DIF(52), REQMIN(52)
      DIMENSION REQMIN1(13),REQMIN2(13),INTSPC(13)
  105 DIMENSION INVSPC1(55), INVINC1(55), ACTSRV(52)
      DIMENSION NETINTA(5)
      DIMENSION SF1(52),SF2(52),SF3(52),SF4(52),SF2RP(52)
      DIMENSION INTSPC1(52),INTSPC2(52),STOCK(200,4)
      DIMENSION INT11(52), INT12(52)
      DIMENSION INT21(52), INT22(52)
      DIMENSION INT31(52), INT32(52)
      DIMENSION INT41(52), INT42(52)
      DIMENSION CASH14(4)
      DIMENSION NETA(200,4),CASHA(200,4)
      TYPE INTEGER NETA,CASHA
      TYPE REAL INTSPC1, INTSPC2,INTSPC
      TYPE REAL INT11,INT12,INT21,INT22,INT31,INT32,INT41,INT42
      TYPE REAL PER4 ,PER1
      TYPE INTEGER NETINTA,ACTSRV,FINNP,SPCNR
      TYPE INTEGER OUT2,STOCK
      TYPE INTEGER NETCF, NCF, INVINC, INVSPC, FINTRN, STFRP, INTPAY
      TYPE INTEGER K, T, KK,          J, CASHBAL
      TYPE INTEGER CASH14
      TYPE INTEGER PCASBAL, CASH, DIF, REQMIN
      TYPE INTEGER INVSPC1, INVINC1,REQMIN1,REQMIN2
      TYPE INTEGER SF1, SF2, SF3, SF4
      TYPE INTEGER INTDMY, STDMY, TLP, TEST1, TEST2
      TYPE INTEGER TOTINC, TOTPAID
      TYPE INTEGER MAX1, STF1, SW1, STKOUT1, AVAIL1
      TYPE INTEGER MAX2, STF2,                AVAIL2, MIN2PA, MAX2PA
      TYPE INTEGER MAX3, STF3,                AVAIL3 ,SF2RP
      TYPE INTEGER MAX4, STF4,                AVAIL4, MAX4PER
C     BEGINNING
      IN1=60
      OUT2=61
      N=13
      KK= 4
      INTNOPD=0
      INTNORC=0
      FINNP=0
      SPCNR= 0
C     MINIMUM REQUIRED CASH BALANCE
```

```
      READ (IN1,1000)(REQMIN1(J),J=1,13)
      READ (IN1,1000)(REQMIN2(J),J=1,13)
C     CASH
      READ (IN1,1004)CASHBAL
C     INTEREST RATE ON SURPLUS CASH REINVESTED
C     CAN BE UNIFORM OR NORMAL
C     TRANSACTION COST
      TCSPC= 0.0
      READ(IN1,1008)IND
      IF (IND  .EQ. 2) GO TO 261
      IF (IND  .EQ. 3) GO TO 262
C     UNIFORM DISTRIBUTION
  261 READ(IN1,1040)(INTSPC1(J),J=1,N)
      READ(IN1,1040)(INTSPC2(J),J=1,N)
      INDSPC= 2
      GO TO 265
C     NORMAL DISTRIBUTION
  262 READ(IN1,1040)(INTSPC1(J),J=1,N)
      READ(IN1,1040)(INTSPC2(J),J=1,N)
      INDSPC=3
      GO TO 265
C     INPUT CONSTRAINTS TO SHORT TERM FINANCING
C     TYPE 1 - LINE OF CREDIT
  265 CONTINUE
      READ (IN1,1013)MAX1  ,PER1
C     TRANSACTION COST
      TXC1 = 0.0
C     COMPENSATING BALANCE
      ICOMB= MAX1 * PER1
      READ(IN1,1008)IND
C     CAN BE UNIFORM OR NORMAL
      IF (IND  .EQ. 2) GO TO 266
      IF (IND  .EQ. 3) GO TO 267
  266 CONTINUE
C     UNIFORM DISTRIBUTION
      READ(IN1,1040)(INT11(J),J=1,N)
      READ(IN1,1040)(INT12(J),J=1,N)
      INDINT1 = 2
      GO TO 270
C     NORMAL DISTRIBUTION
  267 CONTINUE
      READ(IN1,1040)(INT11(J),J=1,N)
      READ(IN1,1040)(INT12(J),J=1,N)
      INDINT1 = 3
      GO TO 270
```

```
C       TYPE 2 - TERM LOAN
   270  CONTINUE
        READ (IN1,1012)MAX2,MAX2PA,MIN2PA
C       TRANSACTION COST
        TXC2 = 0.0
        TLP = 6
        READ(IN1,1008)IND
        IF (IND  .EQ. 2) GO TO 271
        IF (IND  .EQ. 3) GO TO 272
   271  CONTINUE
C       UNIFORM DISTRIBUTION
        READ(IN1,1040)(INT21(J),J=1,N)
        READ(IN1,1040)(INT22(J),J=1,N)
        INDINT2 = 2
        GO TO 275
   272  CONTINUE
C       NORMAL DISTRIBUTION
        READ(IN1,1040)(INT21(J),J=1,N)
        READ(IN1,1040)(INT22(J),J=1,N)
        INDINT2 = 3
        GO TO 275
C       TYPE 3 - COMMERCIAL PAPER
   275  CONTINUE
        READ (IN1,1004)MAX3
C       TRANSACTION COST
        TXC3 = 0.0
        READ(IN1,10 8)IND
C       CAN BE UNIFORM OR NORMAL
        IF (IND  .EQ. 2) GO TO 276
        IF (IND  .EQ. 3) GO TO 277
   276  CONTINUE
C       UNIFORM DISTRIBUTION
        READ(IN1,1040)(INT31(J),J=1,N)
        READ(IN1,1040)(INT32(J),J=1,N)
        INDINT3 = 2
        GO TO 280
   277  CONTINUE
C       NORMAL DISTRIBUTION
        READ(IN1,1040)(INT31(J),J=1,N)
        READ(IN1,1040)(INT32(J),J=1,N)
        INDINT3 = 3
        GO TO 280
C       TYPE 4 - ACCOUNTS RECEIVABLE LOAN
   280  CONTINUE
        READ (IN1,1013)MAX4,PER4
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C       TRANSACTION COST
        TXC4 = 0.0
        READ(IN1,1008)IND
C       CAN BE UNIFORM OR NORMAL
        IF (IND  .EQ. 2) GO TO 281
        IF (IND  .EQ. 3) GO TO 282
  281   CONTINUE
C       UNIFORM DISTRIBUTION
        READ(IN1,1040)(INT41(J),J=1,N)
        READ(IN1,1040)(INT42(J),J=1,N)
        INDINT4 = 2
        GO TO 285
  282   CONTINUE
C       NORMAL DISTRIBUTION
        READ(IN1,1040)(INT41(J),J=1,N)
        READ(IN1,1040)(INT42(J),J=1,N)
        INDINT4 = 3
        GO TO 285
  285   CONTINUE
C       CONVERT ANNUAL PERCENT INTO APPROPRIATE LENGTH PERCENT
        DO 290 J=1,N
        INTSPC1(J)= INTSPC1(J)/13
        INTSPC2(J)= INTSPC2(J)/13
        INT11(J) = INT11(J)/13
        INT12(J) = INT12(J)/13
        INT21(J)= (INT21(J)/13) * 6
        INT22(J)= (INT22(J)/13) * 6
        INT31(J)= (INT31(J)/13) * 3
        INT32(J)= (INT32(J)/13) * 3
        INT41(J) = INT41(J)/13
        INT42(J) = INT42(J)/13
  290   CONTINUE
    7   FORMAT(4H T= ,I3,8H ITYPE= ,I3,
     1          11H NETCF(T)= ,I8,12H INVINC(T)= ,I8,
     2         12H FINTRN(T)= ,I8,9H NCF(T)= ,I8)
    9   FORMAT (10H DIF(T)=  ,I8,14H PCASBAL(T)=  ,I8,13H REQMIN(T)=  ,
     1     I8,
     2          11H STFRP(T)= ,I8,12H INTPAY(T)= ,I8)
   10   FORMAT(15H INVINC1(T+1)= ,I8,9H DIF(T)= ,I8,13H INTSPC (T)= ,F6.4,
     1 13H INVSPC1(T)= ,I8)
   11   FORMAT(4H T= ,I2,  10H CASH(T)= ,I8,            12H INVINC(T)= ,I8,
     112H INTPAY(T)= I8,9H TOTINC= ,I8,10H TOTPAID= I8)
   13   FORMAT(1H0,20H START OF PERIOD T= ,I3,12H FOR ITYPE= ,I3)
   15   FORMAT(20H NET INTEREST COST= ,I8,
     1 9H INTNOR= ,I8,8H SPCNR= ,I8,10H INTNOPD= ,I8,8H FINNP= ,I8,
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```
    2 11H CASH(14)= ,I8,2X,I3)
   31 FORMAT(11H STOCKOUT= ,I8)
   40 FORMAT(9H SF1(T)= ,I8,14H INTPAY(T+1)= ,I8,7H STF1= ,I8,
     1 8H XINT1= ,F6.4)
   41 FORMAT(9H INTDMY= ,I8,8H STDMY= ,I8,7H STF2= ,I8,9H SF2(T)= ,I8,
     1 8H XINT2= ,F6.4)
   42 FORMAT(7H STF2= ,I8,11H SF2RP(T)= ,I8)
   50 FORMAT(7H STF3= ,I8,11H SF3(T-3)= ,I8)
   51 FORMAT(9H SF3(T)= ,I8,10H INTPAY3= ,I8,7H STF3= ,I8,
     1 8H XINT3= ,F6.4)
   60 FORMAT(7H S3F4= ,I8,11H SF4(T-1)= ,I8)
   61 FORMAT(8H TEST1= ,I8,9H ACTSRV= ,I8,7H PER4= ,F6.4,8H TEST2= ,I8)
   62 FORMAT(9H SF4(T)= ,I8,13H STFRP(T+1)= ,I8,14H INTPAY(T+1)= ,I8,
     1 8H XINT4= ,F6.4)
 1000 FORMAT(9I8/4I8)
 1001 FORMAT(13F5.3)
 1002 FORMAT (4F5.3)
 1004 FORMAT (I8)
 1005 FORMAT (4I8)
 1006 FORMAT(I2)
 1007 FORMAT(I2,I8)
 1008 FORMAT(I2)
 1009 FORMAT(I2)
 1010 FORMAT(I8,F5.3)
 1012 FORMAT (3I8)
 1013 FORMAT (I8,F6.4)
 1030 FORMAT(2F5.3)
 1040 FORMAT(13F6.4)
 1400 FORMAT (17H DEBUG INPUT DATA)
 1401 FORMAT (1H ,12(1X,I8))
 1405 FORMAT (1H ,4(3X,F5.3))
 1408 FORMAT (1H ,10(3X,F6.4))
 1411 FORMAT (1H ,6(3X,F5.3))
 1412 FORMAT(1H0,6H MAX1 ,I8,6H MAX2 ,I8,8H MAX2PA ,I8,8H MIN2PA ,I8,
     1 6H MAX3 ,I8,6H MAX4 ,I8,7H PER4= ,F6.4)
 1413 FORMAT (1H0=10H CASHBAL  ,I8)
 1414 FORMAT(1H0,12H CPSOLD(T)  ,4I8)
 1418 FORMAT(2X,7I10)
 1419 FORMAT(2X,6I10)
 1420 FORMAT(1H1,22X,18H NET INTEREST COST,11X,
     1 33H UNRESTRICTED ENDING CASH BALANCE)
 1421 FORMAT(1H ,3X,2(5X,6HTYPE-1,5X,6HTYPE-2,5X,6HTYPE-3,5X,6HTYPE-4))
 1422 FORMAT(1H ,3X,8(3X,I8))
 1423 FORMAT(1H1,14X,18H NET INTEREST COST,12X,
     1 33H UNRESTRICTED ENDING CASH BALANCE,
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     2 18X, 9HSTOCK OUT)
 1424 FORMAT(1H ,2X,3(4X,6HTYPE-1,4X,6HTYPE-2,4X,6HTYPE-3,4X,6HTYPE-4))
 1425 FORMAT(48X,27H INCLUDING COST OF STOCKOUT)
 1426 FORMAT(1H ,12(2X,I8))
 1450 FORMAT(8(2X,I8))
 1452 FORMAT(2X,6'I2,2X,I8))
C     TEST INITIALIZATION OF INTEREST RATES
      DO 1507 J=1,N
      WRITE(OUT2,1408) INT11(J),INT12(J),INT21(J),INT22(J),INT31(J),
     1 INT32(J), INT41(J), INT42(J),INTSPC1(J),INTSPC2(J)
 1507 CONTINUE
      WRITE (OUT2,1412) MAX1,MAX2,MAX2PA,MIN2PA,MAX3,MAX4,PER4
      WRITE (OUT2,1413) CASHBAL
C
C
C     START OF PROCESSING CASH FLOW THROUGH FINANCING ALTERNATIVES
      DO 7720 ITIS=1,200
      READ(IN1,1418)(NETCF(J),J=1,7)
      READ(IN1,1419)(NETCF(J),J=8,13)
      READ(IN1,1418)(ACTSRV(J),J=1,7)
      READ(IN1,1419)(ACTSRV(J),J=8,13)
      NETINTA(1)= 0
      NETINTA(2)= 0
      NETINTA(3)= 0
      NETINTA(4)= 0
      DO 2405 J=1,13
      IREQ1= REQMIN1(J)
      IREQ2=REQMIN2(J)
      CALL UNIFRMI(IREQ1,IREQ2,IREQ)
      REQMIN(J)=IREQ
 2405 CONTINUE
      DO 2407 J=1,13
      XSPC1 = INTSPC1(J)
      XSPC2 = INTSPC2(J)
      IF(INDSPC .EQ. 2) CALL UNIFRMF(XSPC1, XSPC2, XINTSPC)
      IF(INDSPC .EQ. 3) CALL NORMALR(XSPC1, XSPC2, XINTSPC)
      INTSPC(J)= XINTSPC
 2407 CONTINUE
      DO 1700 ITYPE = 1,4
C     INITIALIZATION OF ARRAYS
      DO 2410 J=1,19
      INVINC (J) = 0
      INVSPC (J) = 0
      FINTPN (J) = 0
      INTPAY (J) = 0
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      STFRP   (J) = 0
      CASH (J)    = 0
      INVSPC1 (J) = 0
      INVINC1 (J) = 0
      SF1 (J) = 0
      SF2 (J) = 0
      SF3 (J) = 0
      SF2RP(J)=0
      SF4 (J) = 0
 2410 CONTINUE
      STKOUT1 = 0
C     STKOUT2 = 0
C     STKOUT3 = 0
C     STKOUT4 = 0
      SW1 =0
C     SW2 = 0
C     SW3 = 0
C     SW4 = 0
      STF1 = 0
      STF2 = 0
      STF3 = 0
      STF4 = 0
      TOTINC = 0
      TOTPAID = 0
      INTPAY3=0
      DO 1701 T=1,13
C     REPAYMENT PLUS INTEREST AT BEGINNING OF PERIOD T
      FINTRN (T) = STFRP (T) +  INTPAY (T)
      IF(T .EQ. 1)2420,2430
 2420 NCF(T)=NETCF(T)  + INVINC(T)
      PCASBAL (T) = CASHBAL + NCF (T)
      GO TO 2501
 2430 NCF (T) = NETCF (T) + INVINC (T)  + INVSPC (T-1) - FINTRN (T)
C     POTENTIAL CASH BALANCE PRIOR TO FINANCING ACTIVITY FOR PERIOD T
      PCASBAL (T) = CASH (T-1) + NCF (T)
C     DIFFERENCE BETWEEN POTENTIAL CASH BALANCE AND REQUIRED MINIMUM
      IF(ITYPE .EQ. 1)STF1= STF1 -SF1(T-1)
      IF(ITYPE .EQ. 2)STF2= STF2 - SF2RP(T)
      IF(ITYPE .EQ. 3 .AND. T .GT. 3)STF3= STF3 -SF3(T-3)
      IF(ITYPE .EQ. 4)STF4= STF4 -SF4(T-1)
 2501 DIF (T) = PCASBAL (T) - REQMIN (T)
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )2431,2432
 2431 CONTINUE
      WRITE(OUT2,7)T,ITYPE,NETCF(T),INVINC(T),FINTRN(T),NCF(T)
      WRITE(OUT2,9)DIF(T),PCASBAL(T),REQMIN(T),STFRP(T),INTPAY(T)
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 2432 CONTINUE
C       DECISION FOR PERIOD T
        IF ( DIF (T) .GT. 0)4901,4920
C       INVEST SURPLUS CASH
 4901   INVSPC1 (T) = DIF (T)   * (1 - TCSPC)
        INVINC1 (T+1) = DIF (T) *  INTSPC(T)
        IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
      1WRITE(OUT2,10)INVINC1(T+1),DIF(T),INTSPC(T),INVSPC1(T)
        GO TO 5999
 4920   IF ( DIF (T) .EQ. 0)4921,4930
C       NO ACTION TAKEN
 4921   INVSPC1 (T) =0
        INVINC1 (T+1) = 0
        GO TO 5999
C       DIF(T) IS LESS THAN ZERO
C       MUST USE SHORT TERM FINANCING
C       NEED TO BOOROW DIF(T)
 4930   DIF(T)= -DIF(T)
        GO TO (5000,5100,5200,5300),ITYPE
C       TYPE 1 - LINE OF CREDIT
 5000   IF (MAX1 .EQ. STF1) 5011,5020
C       TOTAL DOLLAR MAXIMUM ALREADY USED UP  SW1 =1 INDICATES STOCKOUT
 5011   SW1 = 1
        STKOUT1 = DIF (T)
        IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
        GO TO 5999
C       HOW DO AMOUNT NEEDED AND AMOUNT AVAILABLE COMPARE
 5020   AVAIL1 = MAX1 - STF1
C5021   IF(ICOMB  .GT. REQMIN(T))
C     1 DIF(T) = DIF(T) + (ICOMB - REQMIN(T))
C       INCLUDE COMPENSATING BALANCE OF 15 PER CENT OF AMOUNT NEEDED
C5021   DIF(T) = DIF(T) * 1.15
C       COMPENSATING BALANCE IS 15 PERCENT OF LINE OF CREDIT
C       DIF(T) = DIF(T) + ICOMB
 5021   DIF(T) = DIF(T) * 1.15
        IF (AVAIL1 .LT. DIF(T))5022,5030
C       NOT ENOUGH AVAILABLE
 5022   SF1 (T) = AVAIL1
        SW1 = 1
        STKOUT1 = DIF (T) - SF1 (T)
        IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
        GO TO 5031
C       ENOUGH AVAILABLE
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C       CALCULATIONS OF INTEREST AND REPAYMENT
 5030   SF1 (T) = DIF (T)
 5031   CONTINUE
        X11 = INT11(T)
        X12 = INT12(T)
        IF (INDINT1 .EQ. 2) CALL UNIFRMR(X11, X12, XINT1)
        IF (INDINT1 .EQ. 3) CALL NORMALR(X11, X12, XINT1)
        SF1(T) = SF1(T)/(1 - TXC1)
        INTPAY (T+1) = SF1 (T) * XINT1   + INTPAY (T+1)
        STFRP (T+1)  = STFRP (T+1) + SF1 (T)
        STF1 = STF1 + SF1 (T)
        IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
       1WRITE(OUT2,40)SF1(T),INTPAY(T+1),STF1,XINT1
        GO TO 5999
C       TYPE 2 -  TERM LOAN
 5100   CONTINUE
        IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
       1WRITE(OUT2,42)STF2,SF2RP(T)
        IF(MAX2 .LE. STF2)5101,5120
C       TOTAL DOLLAR MAXIMUM ALREADY USED UP
 5101   SW1 = 1
        STKOUT1 = DIF (T)
        IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
        GO TO 5999
 5120   AVAIL2 = MAX2 - STF2
        IF (DIF (T) .GE. AVAIL2)5122,5140
 5122   IF (AVAIL2 .GE. MIN2PA )5123,5101
 5123   IF (AVAIL2 .LE. MAX2PA )5124,5130
C       BORROW AMOUNT AVAILABLE
 5124   SF2 (T) = AVAIL2
        SW1 = 1
        STKOUT1 = DIF (T) - SF2 (T)
        IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
        GO TO 5150
C       BORROW PERIOD MAXIMUM
 5130   SF2 (T) = MAX2PA
        SW1 = 1
        STKOUT1 = DIF (T) - SF2 (T)
        IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
        GO TO 5150
 5140   IF ( DIF (T) .GE. MIN2PA )5141,5101
 5141   IF (DIF (T) .LE. MAX2PA )5142,5130
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C       BORROW AMOUNT NEEDED
 5142 SF2 (T) = DIF (T)
      GO TO 5150
C       DETAILS OF TYPE 2 FINANCING
C       SPLIT REPAYMENT OF PRINCIPAL AND INTEREST AMONG FUTURE PERIODS
 5150 CONTINUE
      X21 = INT21(T)
      X22 = INT22(T)
      IF (INDINT2 .EQ. 2) CALL UNIFRMR(X21, X22, XINT2)
      IF (INDINT2 .EQ. 3) CALL NORMALR(X21, X22, XINT2)
      SF2(T) = SF2(T)/(1 - TXC2)
      STF2 = STF2 + SF2 (T)
      INTDMY = SF2 (T) * XINT2
      STDMY  = SF2 (T)
      INTDMY = INTDMY / TLP
      STDMY  = STDMY / TLP
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,41)INTDMY,STDMY,STF2,SF2(T),XINT2
      K = T + TLP
      L= T + 1
      DO 5160 J = L, K
      SF2RP(J)= STDMY
      INTPAY (J) = INTDMY + INTPAY (J)
      STFRP (J)  = STFRP (J) + STDMY
 5160 CONTINUE
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,1452)((I,INTPAY(I)) ,I=L,K)
      GO TO 5999
C       TYPE 3 - COMMERCIAL PAPER
 5200 IF (T .LE. 3)5203,5201
 5201 CONTINUE
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,50)STF3,SF3(T-3)
 5203 IF(MAX3 .LE. STF3)5210,5220
C       TOTAL DOLLAR MAXIMUM ALREADY OUTSTANDING
 5210 SW1 = 1
      STKOUT1 = DIF (T)
      IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C       POSSIBLE STOCKOUT COST
      GO TO 5993
 5220 AVAIL3 = MAX3 - STF3
      IF ( DIF (T) .GT. AVAIL3)5225,5230
C       HOW DO AMOUNT NEEDED AND AMOUNT AVAILABLE COMPARE
C       NOT ENOUGH AVAILABLE
 5225 SF3 (T) = AVAIL3
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      SW1 = 1
      STKOUT1 = DIF (T) - SF3 (T)
      IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C     POSSIBLE STOCKOUT COST
      GO TO 5231
C     ENOUGH AVAILABLE
 5230 SF3 (T) = DIF (T)
 5231 CONTINUE
      X31 = INT31(T)
      X32 = INT32(T)
      IF (INDINT3 .EQ. 2) CALL UNIFRMR(X31, X32, XINT3)
      IF (INDINT3 .EQ. 3) CALL NORMALR(X31, X32, XINT3)
C     TO MAINTAIN MINIMUM CASH BALANCE IN PERIOD T
      SF3(T) = SF3(T)/(1 - XINT3 - TXC3)
      INTPAY3    = SF3 (T) * XINT3
      INTPAY(T) = INTPAY(T) + INTPAY3
      STFRP (T + 3)   = STFRP (T + 3)   + SF3 (T)
      STF3 = STF3 + SF3 (T)
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,51)SF3(T),INTPAY3,STF3,XINT3
      GO TO 5999
C     TYPE  4 -  ACCOUNTS RECEIVABLE LOAN
 5300 IF (T .LE . 1)5302,5301
 5301 CONTINUE
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,60)STF4,SF4(T-1)
 5302 TEST1 = MAX4 - STF4
      IF(TEST1 .LE. 0)5303,5305
C     TOTAL DOLLAR MAXIMUM ALREADY USED UP    SW4 = 1 INDICATES STOCKOUT
 5303 SW1 = 1
      STKOUT1 = DIF (T)
      IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C     POSSIBLE STOCKOUT COST
      GO TO 5999
 5305 CONTINUE
       MAX4PER = PER4 * ACTSRV(T)
      TEST2 = MAX4PER - STF4
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,61)TEST1,ACTSRV(T),PER4,TEST2
      IF(TEST2 .LE. 0)5303,5320
C     HOW DO AMOUNT AVAILABLE AND AMOUNT NEEDED COMPARE
 5320 AVAIL4 = MIN0(TEST1, TEST2)
      IF ( DIF (T) .GT. AVAIL4)5322,5330
C     NOT ENOUGH AVAILABLE
 5322 SF4 (T) = AVAIL4
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      SW1 = 1
      STKOUT1 = DIF (T) - AVAIL4
      IF(ITIS .LE. 3) WRITE(OUT2,31)STKOUT1
C     POSSIBLE STOCKOUT COST
      GO TO 5331
C     ENOUGH AVAILABLE
 5332 SF4 (T) = DIF (T)
 5331 CONTINUE
      X41 = INT41(T)
      X42 = INT42(T)
      IF (INDINT4 .EQ. 2) CALL UNIFRMR(X41, X42, XINT4)
      IF (INDINT4 .EQ. 3) CALL NORMALR(X41, X42, XINT4)
      SF4(T) = SF4(T)/(1 - TXC4)
      INTPAY (T+1) = SF4 (T) * XINT4      + INTPAY (T+1)
      STFRP (T+1)  =  STFRP (T+1) + SF4 (T)
      STF4 = STF4 + SF4 (T)
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,62)SF4(T),STFRP(T+1),INTPAY(T+1),XINT4
      GO TO 5999
C     FINANCING HAS BEEN FINISHED FOR PERIOD T
C     NOW MUST MADE ADJUSTMENTS TO PREPARE FOR PERIOD T+1
 5999 CONTINUE
      IF(SW1 .EQ. 0) GO TO 6000
      STOCK(ITIS,ITYPE) = STOCK(ITIS,ITYPE) + STKOUT1
      SW1=0
 6000 CONTINUE
      INVSPC (T) = INVSPC1 (T)
      INVINC (T+1) = INVINC1 (T+1)
      CASH (T) = PCASBAL (T) + SF1 (T) + SF2 (T) + SF3 (T) + SF4 (T)
     1           - INTPAY3 - INVSPC  (T)
      INTPAY3 =0
C     TOTAL INCOME FROM INTEREST ON SURPLUS CASH
      TOTINC  = TOTINC + INVINC (T)
C     TOTAL INTEREST PAID ON SHORT TERM FINANCING
      TOTPAID = TOTPAID + INTPAY (T)
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,11)T, CASH(T),      INVINC(T),INTPAY(T),TOTINC,TOTPAID
 1701 CONTINUE
C     NET INTEREST COST FOR ITYPE
C     INTNORC IS INTEREST NOT RECEIVED AT END OF LAST PERIOD
C     SPCNR IS INVESTED SURPLUS CASH NOT RECEIVED BY END OF LAST PERIOD
C     INTNOPD IS INTEREST NOT PAID AT END OF LAST PERIOD
C     FINNP IS FINANCING PRINCIPLE NOT PAID AT END OF LAST PERIOD
      INTNORC = INVINC(14)
      IF(ITYPE . EQ. 1 .OR. ITYPE .EQ. 4)1702,1703
```

```
 1702 CONTINUE
      INTNOPD= INTPAY(14)
      FINNP= STERP(14)
      GO TO 1710
 1703 IF(ITYPE .EQ. 3)1704,1705
 1704 J=13 + 1
      K = 13 + 3
      DO 1707 I=J,K
      FINNP = FINNP + STERP(I)
 1707 CONTINUE
      GO TO 1710
 1705 IF(ITYPE .NE. 2) GO TO 1710
      J = 13 + 1
      K = 13 + TLP
      DO 1706 I=J,K
      INTNOPD = INTNOPD + INTPAY(I)
      FINNP= FINNP + STERP(I)
 1706 CONTINUE
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,1452)((I,INTPAY(I)),I=J,K)
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,1452) ((I,STERP(I)),I=J,K)
 1710 CONTINUE
      NETINT = TOTPAID - TOTINC
      NETINTA(ITYPE) = NETINT
      CASH14(ITYPE)= CASH(13) + INTNORC + SPCNR - INTNOPD - FINNP
      IF(ITIS .LE. 3 .OR. ITIS .EQ. 50 .OR. ITIS .EQ. 100 )
     1WRITE(OUT2,15)NETINT,INTNORC,SPCNR,INTNOPD,FINNP,CASH14(ITYPE),
     2 ITYPE
      CASHA(ITIS,ITYPE)= CASH14(ITYPE)
      NETA(ITIS,ITYPE)= NETINTA(ITYPE)
      INTNORC=0
      INTNOPD=0
      FINNP= 0
      SPCNR= 0
 1700 CONTINUE
C     PUNCH OUT NET INTEREST COST
      WRITE(62,1450)(NETINTA(I),I=1,4)(CASH14(I),I=1,4)
      NETINTA(1)=0
      NETINTA(2)=0
      NETINTA(3)=0
      NETINTA(4)=0
   10 CONTINUE
C     WRITE OUT ARRAY OF NET INTEREST COST AND
C     UNRESTRICTED ENDING CASH BALANCE
```

```
      WRITE(OUT2,1420)
      WRITE(OUT2,1421)
      DO 7730 I=1,200
      WRITE(OUT2,1422)((NETA(I,J),J=1,4),(CASHA(I,J),J=1,4))
 7730 CONTINUE
      DO 7735 I=1,200
      DO 7736 J=1,4
      NETA(I,J)= NETA(I,J) + .01 * STOCK(I,J)
      CASHA(I,J) = CASHA(I,J) - .01* STOCK(I,J)
 7736 CONTINUE
 7735 CONTINUE
      DO 7738 I=1,200
      WRITE(62,1450)(NETA(I,J),J=1,4),(CASHA(I,J),J=1,4)
 7738 CONTINUE
      WRITE(OUT2,1423)
      WRITE(OUT2,1425)
      WRITE(OUT2,1424)
      DO 7739 I=1,200
      WRITE(OUT2,1426) (NETA(I,J),J=1,4),(CASHA(I,J),J=1,4),
     1(STOCK(I,J)=J=1,4)
 7739 CONTINUE
      END
      SUBROUTINE UNIFRMR(A,B,X)
      TYPE REAL A, B, X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
      SUBROUTINE NORMALR(EX,STDX,X)
      TYPE REAL EX,STDX, X
      SUM=0.0
      DO 4 I=1,12
      R = RANF(-1)
    4 SUM=SUM + R
      X=STDX *(SUM- 6.) + EX
      RETURN
      END
      SUBROUTINE UNIFRMI(A,B,X)
      TYPE INTEGER A,B,X
      R= RANF(-1)
      X = A + (B-A) * R
      RETURN
      END
RUN,3,0,3000
000000 4000000 4000000 4000000 4000000 4000000 4000000 4000000 4000000 REQMIN
```

```
000000 4000000 4000000 4000000                                                         REQMIN
000000 5000000 5000000 5000000 5000000 5000000 5000000 5000000 5000000 REQMIN
000000 5000000 5000000 5000000                                                         REQMIN
300000    CASH BALANCE AT START
    INDSPC IS UNIFORM
0525 .0525 .0475 .0475 .0425 .0425 .0475 .0475 .0475 .0525 .0525 .0525 .052511
0625 .0625 .0575 .0575 .0525 .0525 .0575 .0575 .0575 .0625 .0625 .0625 .062512
000000 .1500      MAX1, PER1
    INDINT1 IS UNIFORM
0625 .0625 .0575 .0575 .0525 .0525 .0575 .0575 .0575 .0625 .0625 .0625 .062511
0725 .0725 .0675 .0675 .0625 .0625 .0675 .0675 .0675 .0725 .0725 .0725 .072512
000000 1000000    50000          MAX2,MAX2PA,MIN2PA
    INDINT2 IS UNIFORM
0650 .0650 .060  .0600 .0550 .0550 .0600 .0600 .0600 .0650 .0650 .0650 .065021
0775 .0775 .0725 .0725 .0675 .0675 .0725 .0725 .0725 .0775 .0775 .0775 .077522
000000    MAX3
    INDINT3 IS UNIFORM
0575 .0575 .0525 .0525 .0475 .0475 .0525 .0525 .0525 .0575 .0575 .0575 .057531
0675 .0675 .0625 .0625 .0575 .0575 .0625 .0625 .0625 .0675 .0675 .0675 .067532
000000 .4500      MAX4,PER4
    INDINT4 IS UNIFORM
0825 .0825 .0775 .0775 .0725 .0725 .0775 .0775 .0775 .0825 .0825 .0825 .082541
0925 .0925 .0875 .0875 .0825 .0825 .0875 .0875 .0875 .0925 .0925 .0925 .092542
```

APPENDIX E

OUTPUT FROM THE SIMULATION OF THE FINANCING ALTERNATIVES

UNRESTRICTED ENDING CASH BALANCE

TYPE-1	TYPE-2	TYPE-3	TYPE-4
2781271	2691574	2551124	2771012
2538860	2480480	2540241	2531662
1892345	1843516	1902913	1893314
1779553	1712207	1774878	1778126
1817862	1750921	1815532	1806458
2237585	2151734	2239539	2229895
2149889	2105024	2162510	2144701
3657269	3593518	3667641	3649160
2397100	2325433	2409113	2386876
2536460	2447188	2264271	2524416
2024100	1965361	2043957	2025214
3415705	3255559	3424803	3405964
2348079	2275468	2343935	2342231
2545285	2454385	2563160	2538088
2292404	2218140	2310437	2285707
3173182	2844570	3180138	3162363
1475519	1404325	1496996	1480184
2564984	2515666	2569153	2557502
2261468	2187971	2278860	2257790
1895324	1868022	1902889	1908055
3382186	3319708	3396441	3371340
1292310	1273042	1317127	1315291
2530304	2440050	2525173	2520842
2772286	2689264	2780466	2757805
2651160	2583428	2657157	2643613
2805241	2730336	2817969	2795301
2638583	2577185	2649696	2636651
2502300	2455064	2503108	2500085
2121614	2064053	2139268	2120851
1952164	1914595	1961414	1957930
1784123	1717730	1784114	1781185
1932894	1848294	1946959	1929984
2903427	2814183	2909762	2890897
2537892	2455041	2541750	2525061
3316457	3251896	3316240	3305295
3268724	3187876	3278246	3258939
1858833	1774807	1862967	1846427
2053661	1998519	2050929	2053852
3191464	3122911	3200128	3182589
1776011	1712179	1740925	1773316
3083023	3011490	3089574	3071988
2939232	2847365	2940518	2925950
1875641	1835293	1883609	1877896
2660322	2596364	2658524	2645196
2717013	2638039	2723663	2703803
1575458	1518937	1570954	1565818

2811192	2728945	2809960	2797352
3469112	2975463	3475817	3456762
2265571	2178012	2264468	2256415
2674525	2574165	2685301	2664522
2649830	2582153	2661171	2644103
1461442	1428340	1467993	1473285
2094547	2039861	2111001	2097078
2692337	2619296	2700397	2678951
3572491	3401885	3585035	3558895
1049947	1014455	1047824	1063111
2189244	2157371	2200108	2197912
2028896	1972051	2048107	2035118
2521429	2454501	2528681	2515630
3358196	3283548	3187235	3349004
2786528	2704277	2788887	2773820
2801962	2723653	2814396	2789905
2259241	2214380	2263704	2255021
2283206	2237677	2287823	2286723
1783169	1707039	1786611	1772768
2261729	2169781	2266442	2257827
3184619	3123860	3185764	3171516
2850830	2772990	2865310	2838732
1782205	1745309	1798756	1787156
2994983	2917626	2999395	2982407
2108792	2055375	2127669	2109676
1648413	1612257	1667170	1665819
2765259	2678916	2763919	2753489
1896008	1812975	1901768	1884605
2619423	2535561	2628473	2603877
2489976	2426847	2503346	2478033
2314578	2244552	2317067	2302358
2445655	2374026	2453088	2439071
1467090	1452593	1477259	1491436
2395830	2313864	2400852	2382929
2496069	2416635	2498351	2489041
2658907	2567169	2650263	2650302
2825809	2740571	2836606	2810552
2125735	2084283	2130286	2136713
3538885	3463618	3540801	3529949
2207979	2116662	2207836	2201408
2051818	1998442	[illegible]916108	2051377
2240917	2173641	2248741	2234542
3061947	2989233	3070267	3051492
2460371	2366247	2470544	2452133
2438851	2367871	2444896	2437626
3086173	3011766	3083158	3074651
2334199	2256439	2343254	2327608
3721536	3471116	2729775	3712172
2262082	2188383	2275963	2258196
2508491	2431209	2521397	2496979
2868451	2777985	2883142	2858361
2356127	2278198	2361080	2341992

1726172	1667563	1723314	1726256
3126530	3041551	3136212	3110863
2222428	2143083	2236261	2210005
1865833	1814247	1868878	1856930
2181416	2097302	1920095	2173117
1902198	1851141	1906783	1897350
3203204	2874939	3220193	3190881
2500306	2420954	2512043	2491581
2955217	2876410	2960815	2947003
2109446	2020428	2115054	2096559
2665469	2583188	2654948	2651999
1669621	1613138	1670669	1671421
2381651	2294252	2394083	2363441
2365531	2287890	2374472	2356441
1905458	1847623	1921451	1900973
2744318	2619718	2284698	2729198
3447729	3383249	3456464	3434647
2413689	2344585	2411113	2405531
3142076	3073552	3156711	3127581
2705748	2617521	2697765	2695458
2175848	2089858	2180621	2170676
2442821	2382029	2112994	2433478
2724564	2673421	2735836	2718358
2536355	2457155	2536118	2524794
1867259	1830444	1887161	1877601
4118733	4075754	4025757	4110603
2960604	2685865	2961827	2946800
2487764	2415647	2500411	2475185
2895319	2817498	2907026	2880386
2534203	2452129	2540568	2521174
2358488	2299143	2373745	2360331
2403942	2321581	2409575	2391518
2864246	2790055	2880400	2852005
2796350	2704578	2800468	2782491
2898554	2817932	2905377	2882517
2292931	2226065	2300524	2279009
2399959	2321442	2398944	2389160
2749202	2674285	2762025	2737474
1920085	1854206	1918829	1923537
2264171	2187650	2270991	2255313
3141486	3087814	3150635	3132257
2396743	2339083	2405008	2392920
3384239	2790063	3255714	3374272
1852506	1791110	1870794	1851135
1833677	1773536	1827520	1829464
3139738	3047149	2879381	3129672
3133326	3044857	3134239	3117500
2481474	2407197	2496016	2468180
2410583	2344151	2421668	2399895
2199286	2146254	2208600	2190334
2678430	2592778	2688581	2659982
2664421	2581169	2672041	2651294

2745779	2662619	2748728	2732573
2681802	2601073	2684021	2672133
2679083	2577609	2693441	2670458
1830197	1763888	1836225	1829868
1925772	1849638	1932169	1921933
2963251	2883716	2969395	2953202
2520418	2440764	2520250	2505876
2618386	2538411	2614620	2609684
2362921	2280224	2370511	2353316
1058425	1007918	1049672	1074436
1398758	1340494	1396864	1399907
2525718	2447225	2531456	2512565
2992073	2919023	3005222	2980419
1857936	1818741	1860046	1856991
1742781	1683635	1753703	1739085
1884080	1852797	1887438	1893334
2717271	2640104	2736800	2707585
3418521	3344659	3435287	3404917
2797189	2707728	2816131	2784267
2559302	2459494	2565191	2549802
2127953	2061270	2136812	2130930
2230676	2171730	1915956	2224923
2195316	2135763	2207340	2185054
2161722	2105031	2166404	2157340
3109342	3024482	3109202	3099348
2510008	2439505	2528781	2501373
2490554	2427305	2496105	2473890
2951778	2874320	2960746	2938128
2722227	2640808	2727550	2708073
2503620	2409500	2515192	2498236
2657301	2586804	2665993	2640632
2596239	2529934	2605033	2582607
2639154	2562939	2649287	2625926
1674030	1596913	1695296	1671710
2375131	2307177	2390223	2372670
2765766	2694495	2778113	2756162
1981179	1922047	1983679	1975207
3198672	3137424	3203106	3190441
2169022	2086166	2186983	2167118
2197052	2128381	2213256	2187077
2757815	2683327	2247304	2744567
2138698	2091998	2147485	2145242
3157830	3072079	3166120	3144582
2471090	2389509	2488025	2461219
2963316	2878879	2970189	2948492
3230433	3150018	3235508	3217183
2649121	2552672	2649713	2641894
2069548	2026169	2077919	2070487
3076990	2660644	2991648	3064859
2191638	2138522	2199352	2186930

APPENDIX F

TABLES OF DATA FOR THE CUMULATIVE PROBABILITY DISTRIBUTIONS OF THE FINANCING ALTERNATIVES

TABLE 2.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $3 million and a stockout penalty of 1 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	193	192	191	193	193
1700	192	192	191	192	192
1750	192	190	189	192	192
1800	189	189	189	189	189
1850	187	185	185	187	187
1900	183	181	181	183	182
1950	181	173	175	179	180
2000	173	166	166	170	171
2050	164	163	163	161	164
2100	162	161	161	158	162
2150	160	158	157	156	160
2200	157	152	152	154	157
2250	151	147	145	144	151
2300	144	137	135	138	144
2350	136	130	125	130	136
1400	127	126	124	123	127
2450	124	120	115	118	124
2500	117	111	104	110	117
2550	109	106	100	97	108
2600	103	95	93	93	103
2650	87	84	80	81	87
2700	83	82	75	73	83
2750	79	72	67	66	79
2800	66	62	56	53	66
2850	58	56	50	46	58
2900	51	47	41	41	51
2950	45	43	38	38	45
3000	41	39	33	34	41
3050	0	0	0	0	0

*Unrestricted ending cash balances.

TABLE 3.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $4.5 million and a stockout penalty of 1 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	191	189	187	192	189
1700	189	189	182	189	189
1750	187	184	178	187	185
1800	182	181	170	183	181
1850	177	174	163	178	174
1900	169	164	162	172	165
1950	163	163	159	164	163
2000	162	161	156	162	161
2050	160	157	151	160	158
2100	155	151	142	156	151
2150	149	145	136	151	147
2200	139	137	127	144	138
2250	135	129	124	135	129
2300	127	125	117	126	125
2350	124	117	108	124	119
2400	114	110	102	114	110
2450	106	104	86	108	106
2500	99	90	82	102	94
2550	85	83	79	85	82
2600	83	80	65	82	79
2650	76	66	57	79	69
2700	64	60	51	65	59
2750	57	51	45	58	54
2800	48	45	41	51	46
2850	45	42	38	45	42
2900	40	38	33	41	39
2950	37	33	31	38	33
3000	31	31	27	31	30
3050	31	27	23	31	30
3100	27	23	19	27	27
3150	22	18	15	22	20
3200	17	15	13	17	15
3250	14	13	13	14	14
3300	13	12	11	13	13
3350	12	9	8	12	11
3400	9	7	5	9	9
3450	6	5	5	6	5
3500	5	5	3	5	5
3550	4	3	2	4	4
3600	3	3	2	3	3
3650	3	2	1	2	2
3700	2	1	1	1	1
3750	1	1	1	1	1
3800	1	1	1	1	1
3850	1	1	1	1	1
3900	1	1	1	1	1
3950	1	1		1	1
4000	1	1		1	1
4050	1	1		1	1
4100	1			1	1

*Unrestricted ending cash balances.

TABLE 4.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $4.5 million and a stockout penalty of 5 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	191	189	168	192	185
1700	189	189	166	189	180
1750	187	184	162	187	171
1800	182	181	158	183	167
1850	177	174	156	178	164
1900	169	164	150	172	161
1950	163	163	144	164	157
2000	162	161	142	162	154
2050	160	157	135	160	148
2100	155	151	128	156	143
2150	149	145	124	151	138
2200	139	137	119	144	129
2250	135	129	110	135	126
2300	127	125	103	126	122
2350	124	117	97	124	113
2400	114	110	87	114	106
2450	106	104	81	108	99
2500	99	90	75	102	87
2550	85	83	65	85	82
2600	83	80	59	82	76
2650	76	66	51	79	67
2700	64	60	44	65	59
2750	57	51	42	58	51
2800	48	45	39	51	45
2850	45	42	35	45	40
2900	40	38	31	41	38
2950	37	33	30	38	33
3000	31	31	27	31	30
3050	31	27	19	31	29
3100	27	23	17	27	25
3150	22	18	14	22	19
3200	17	15	13	17	15
3250	14	13	12	14	13
3300	13	12	10	13	13
3350	12	9	6	12	11
3400	9	7	5	9	9
3450	6	5	5	6	5
3500	5	5	2	5	5
3550	4	3	2	4	4
3600	3	3	1	3	3
3650	3	2	1	2	3
3700	2	1	1	1	2
3750	1	1	1	1	1
3800	1	1	1	1	1
3850	1	1	1	1	1
3900	1	1	1	1	1
3950	1	1		1	1
4000	1	1		1	1
4050	1	1		1	1
4100	1			1	1

*Unrestricted ending cash balances.

TABLE 5.--Data for graphs of the comparative effects of stockout penalties for Type 2 at a minimum required cash balance of $4.5 million.

URECB* Dollars in Thousands	0 Percent	1 Percent	5 Percent
<1650	200	200	200
1650	189	187	168
1700	187	182	166
1750	183	178	162
1800	177	170	158
1850	170	163	156
1900	164	162	150
1950	162	159	144
2000	159	156	142
2050	154	151	135
2100	149	142	128
2150	138	136	124
2200	131	127	119
2250	124	124	110
2300	118	117	103
2350	111	108	97
2400	106	102	87
2450	92	86	81
2500	85	82	75
2550	81	79	65
2600	69	65	59
2650	62	57	51
2700	52	51	44
2750	46	45	42
2800	43	41	39
2850	40	38	35
2900	33	33	31
2950	31	31	30
3000	29	27	27
3050	25	23	19
3100	19	19	17
3150	15	15	14
3200	13	13	13
3250	13	13	12
3300	11	11	10
3350	8	8	6
3400	5	5	5
3450	5	5	5
3500	3	3	2
3550	2	2	2
3600	2	2	1
3650	1	1	1
3700	1	1	1
3750	1	1	1
3800	1	1	1
3850	1	1	1
3900	1	1	1
3950	1		

*Unrestricted ending cash balances.

TABLE 6.--Data for graphs of the comparative effects of stockout penalties for Type 4 at a minimum required cash balance of $4.5 million.

URECB* Dollars in Thousands	0 Percent	1 Percent	5 Percent
<1650	200	200	200
1650	192	189	185
1700	189	189	180
1750	187	185	171
1800	182	181	167
1850	177	174	164
1900	170	165	161
1950	164	163	157
2000	162	161	154
2050	150	158	148
2100	155	151	143
2150	149	147	138
2200	139	138	129
2250	131	129	126
2300	125	125	122
2350	121	119	113
2400	110	110	106
2450	106	106	99
2500	94	95	87
2550	84	82	82
2600	80	79	76
2650	70	69	67
2700	64	59	59
2750	55	54	51
2800	46	46	45
2850	43	42	40
2900	39	39	38
3950	34	33	33
3000	31	30	30
3050	31	30	29
3100	27	27	25
3150	20	20	19
3200	15	15	15
3250	14	14	13
3300	13	13	13
3350	11	11	11
3400	9	9	9
3450	5	5	5
3500	5	5	5
3550	4	4	4
3600	3	3	3
3650	3	3	3
3700	2	2	2
3750	1	1	1
3800	1	1	1
3850	1	1	1
3900	1	1	1
3950	1	1	1
4000	1	1	1
4050	1	1	1
4100	1	1	1

*Unrestricted ending cash balances.

TABLE 7.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $6 million and a stockout penalty of 1 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	187	184	165	187	175
1700	181	181	163	182	166
1750	175	174	161	175	163
1800	167	164	158	168	162
1850	163	162	151	163	158
1900	162	161	146	162	155
1950	158	157	139	159	148
2000	154	151	129	155	140
2050	149	145	126	129	134
2100	139	137	119	140	126
2150	130	129	110	133	124
2200	126	125	106	126	114
2250	121	117	95	123	107
2300	110	109	84	112	98
2350	106	104	80	106	86
2400	96	87	72	98	82
2450	84	83	63	85	76
2500	80	80	56	81	65
2550	71	67	47	74	58
2600	64	59	43	64	49
2650	55	51	39	57	44
2700	46	45	38	49	41
2750	42	42	31	44	38
2800	39	38	31	39	33
2850	36	33	27	38	31
2900	31	31	22	31	27
2950	31	27	18	31	22
3000	27	23	14	27	19
3050	20	18	13	21	15
3100	15	15	13	17	13
3150	14	13	10	14	13
3200	13	12	7	13	11
3250	11	10	5	12	8
3300	9	7	5	9	5
3350	5	5	3	6	5
3400	5	5	3	5	3
3450	3	3	2	4	3
3550	3	3	1	3	2
3600	2	1	1	2	1
3650	1	1	1	1	1
3700	1	1	1	1	1
3750	1	1	1	1	1
3800	1	1	1	1	1
3850	1	1	1	1	1
3900	1	1		1	1
3950	1	1		1	1
4000	1			1	

*Unrestricted ending cash balances.

TABLE 8.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $6 million and a stockout penalty of 5 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	187	184	72	187	100
1700	181	181	66	182	91
1750	175	174	60	175	84
1800	167	164	51	168	79
1850	163	162	46	163	72
1900	162	161	41	162	66
1950	158	157	39	159	60
2000	154	151	38	155	53
2050	149	145	35	149	47
2100	139	137	32	140	43
2150	130	129	28	133	39
2200	126	125	25	126	39
2250	121	116	23	123	36
2300	110	109	19	112	34
2350	106	104	18	106	28
2400	96	87	18	98	27
2450	84	83	14	85	23
2500	80	80	11	81	19
2550	71	67	11	74	18
2600	64	59	10	64	18
2650	55	51	8	57	15
2700	46	45	5	49	11
2750	42	42	5	44	11
2800	39	38	4	39	11
2850	36	33	3	37	9
2900	31	31	3	31	6
2950	31	27	3	31	4
3000	27	23	2	27	4
3050	20	18	1	21	3
3100	15	15	1	17	3
3150	14	13	1	14	3
3200	13	12	1	13	3
3250	11	10	1	12	1
3300	9	7	1	9	1
3350	5	5	1	6	1
3400	5	5	1	5	1
3450	3	3		4	1
3500	3	3		3	1
3550	2	2		3	1
3600	2	1		2	1
3650	1	1		1	1
3700	1	1		1	
3750	1	1		1	
3800	1	1		1	
3850	1	1		1	
3900	1	1		1	
3950	1	1		1	
4000	1			1	

*Unrestricted ending cash balances.

TABLE 9.--Data for graphs of the comparative effects of stockout penalties for Type 2 at a minimum required cash balance of $6.0 million.

URECB* Dollars in Thousands	0 Percent	1 Percent	5 Percent
<1650	200	200	200
1650	189	165	72
1700	187	163	66
1750	185	161	60
1800	177	158	51
1850	170	151	46
1900	164	146	41
1950	162	139	39
2000	160	129	38
2050	155	126	35
2100	147	119	32
2150	140	110	28
2200	131	106	25
2250	126	95	23
2300	120	84	19
2350	112	80	18
2400	105	72	18
2450	95	63	14
2500	84	56	11
2550	79	47	11
2600	67	43	10
2650	62	39	8
2700	51	37	5
2750	45	31	4
2800	42	31	4
2850	38	27	3
2900	31	22	3
2950	31	18	3
3000	27	14	2
3050	22	13	1
3100	17	13	1
3150	15	10	1
3200	13	7	1
3250	12	5	1
3300	9	5	1
3350	6	3	1
3400	5	3	1
3450	4	2	
3500	3	1	
3550	2	1	
3600	2	1	
3650	1	1	
3700	1	1	
3750	1	1	
3800	1	1	
3850	1	1	
3900	1		
3950	1		

*Unrestricted ending cash balances.

TABLE 10.--Data for graphs of the comparative effects of stockout penalties for Type 4 at a minimum required cash balance of $6.0 million.

URECB* Dollars in Thousands	0 Percent	1 Percent	5 Percent
<1650	200	200	200
1650	191	175	100
1700	189	166	91
1750	187	163	84
1800	181	162	79
1850	176	158	72
1900	164	155	66
1950	163	148	60
2000	162	140	53
2050	158	134	47
2100	154	126	43
2150	145	124	39
2200	137	114	39
2250	129	107	36
2300	125	98	34
2350	117	86	28
2400	109	82	27
2450	103	76	23
2500	86	65	19
2550	83	58	18
2600	79	49	18
2650	65	44	15
2700	58	41	11
2750	50	38	11
2800	44	33	11
2850	41	31	9
2900	36	27	6
2950	31	22	4
3000	31	19	4
3050	27	15	3
3100	21	13	3
3150	16	13	3
3200	13	11	3
3250	12	8	1
3300	11	5	1
3350	9	5	1
2400	5	3	1
3450	5	3	1
3500	3	2	1
3550	3	2	1
3600	2	1	1
3650	1	1	1
3700	1	1	
3750	1	1	
3800	1	1	
3850	1	1	
3900	1	1	
3950	1	1	
4000	1		

*Unrestricted ending cash balances.

TABLE 11.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $4-$5 million and a stockout penalty of 1 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	191	189	188	192	189
1700	189	188	182	189	188
1750	187	184	179	187	186
1800	182	180	169	182	181
1850	179	173	164	179	172
1900	169	164	162	172	166
1950	164	163	160	161	163
2000	162	160	157	159	161
2050	160	157	150	157	158
2100	156	151	144	155	153
2150	149	145	136	147	148
2200	140	138	127	142	139
2250	135	129	123	133	129
2300	126	125	115	124	126
2350	123	117	108	119	119
2400	113	110	104	112	111
2450	106	104	87	105	107
2500	99	89	83	99	95
2550	84	83	76	84	84
2600	81	80	66	80	82
2650	75	66	59	74	72
2700	64	60	48	62	63
2750	56	51	43	56	56
2800	48	45	39	51	47
2850	44	42	34	45	44
2900	39	39	28	41	40
2950	37	33	27	37	35
3000	30	31	25	30	31
3050	30	27	19	29	30
3100	26	24	15	26	26
3150	20	18	12	23	20
3200	15	14	11	17	15
3250	14	13	9	13	14
3300	13	12	8	11	13
3350	12	9	6	10	11
3400	9	7	4	9	8
3450	6	5	4	7	6
3500	5	4	2	5	5
3550	4	3	2	5	4
3600	3	2	1	3	3
3650	3	2	1	3	2
3700	2	1	1	2	2
3750	1	1	1	1	1
3800	1	1	1	1	1
3850	1	1	1	1	1
3900	1	1	1	1	1
3950	1	1	1	1	1
4000	1	1	1	1	1
4050	1	1	1		1
4100	1				

*Unrestricted ending cash balances.

TABLE 12.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of $4-$5 million and a stockout penalty of 5 percent.

URECB* Dollars in Thousands	Type 1-A	Type 1-B	Type 2	Type 3	Type 4
<1650	200	200	200	200	200
1650	191	189	169	192	185
1700	189	188	166	189	176
1750	187	184	162	189	176
1800	182	180	156	182	167
1850	179	173	154	179	162
1900	169	164	148	172	159
1950	164	163	148	161	156
2000	162	160	139	159	151
2050	160	157	136	157	148
2100	156	151	129	155	145
2150	149	145	122	147	136
2200	140	138	118	142	129
2250	135	129	109	133	122
2300	126	125	105	124	118
2350	123	117	95	119	112
2400	113	110	87	112	106
2450	106	104	79	105	97
2500	99	89	72	99	85
2550	84	83	64	84	80
2600	81	80	59	80	76
2650	75	66	50	74	66
2700	64	60	44	62	60
2750	56	51	40	56	50
2800	48	45	34	51	45
2850	44	42	30	45	42
2900	39	39	27	41	39
2950	37	33	26	37	34
3000	30	31	20	30	31
3050	30	27	16	29	30
3100	26	24	13	26	24
3150	20	18	12	23	18
3200	15	14	11	17	14
3250	14	13	8	13	14
3300	13	12	6	11	13
3350	12	9	6	10	11
3400	9	7	4	9	8
3450	6	5	2	7	6
3500	5	4	2	5	5
3550	4	3	2	5	3
3600	3	3	1	3	3
3650	3	2	1	3	2
3700	2	1	1	2	1
3750	1	1	1	1	1
3800	1	1	1	1	1
3850	1	1	1	1	1
3900	1	1	1	1	1
3950	1	1	1	1	1
4000	1	1	1	1	1
4050	1	1			1
4100	1				

*Unrestricted ending cash balances.

TABLE 13.--Data for the cumulative probability distributions of the financing alternatives at a minimum required cash balance of \$5.5-\$6.5 million and a stockout penalty of 1 percent.

URECB* Dollars in Thousands	Type 1-A	Type 2	Type 3	Type 4
<1650	200	200	200	200
1650	187	164	187	174
1700	182	162	182	169
1750	175	161	177	163
1800	168	158	169	161
1850	163	152	163	158
1900	163	146	162	145
1950	158	137	160	149
2000	154	128	156	139
2050	148	125	151	135
2100	138	119	139	126
2150	130	110	134	124
2200	125	108	126	113
2250	120	97	123	106
2300	110	84	114	98
2350	106	81	107	86
2400	96	72	97	83
2450	83	62	84	75
2500	81	57	81	66
2550	70	46	76	59
2600	64	44	64	50
2650	55	39	58	44
2700	47	36	49	41
2750	42	31	45	37
2800	39	31	41	32
2850	35	28	35	31
2900	31	22	31	28
2950	31	16	31	25
3000	27	14	27	19
3050	20	13	22	14
3100	15	11	17	13
3150	14	10	14	13
3200	13	6	13	11
3250	9	5	9	8
3350	5	3	6	5
3400	5	3	5	3
3450	4	2	5	3
3500	3	2	3	2
3550	2	1	3	2
3600	2	1	2	1
3650	1	1	1	1
3700	1	1	1	1
3750	1	1	1	1
3800	1	1	1	1
3850	1	1	1	1
3900	1	1	1	1
3950	1		1	1
4000	1		1	

*Unrestricted ending cash balances.