PROBABILITY: SEX AND GRADE LEVEL DIFFERENCES AND THE EFFECT OF INSTRUCTION ON THE PERFORMANCE AND ATTITUDES OF MIDDLE SCHOOL BOYS AND GIRLS

By

Zacchaeus Kunle Oguntebi

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

College of Education

1983

142-3885

ABSTRACT

PROBABILITY: SEX AND GRADE LEVEL DIFFERENCES AND THE EFFECT OF INSTRUCTION ON THE PERFORMANCE AND ATTITUDES OF MIDDLE SCHOOL BOYS AND GIRLS

By

Zacchaeus Kunle Oguntebi

Purpose

This study had four related purposes. The first purpose was to determine existing differences in probability knowledge and in attitudes toward mathematics of grades six through eight students by sex and grade prior to probability intervention. The other purposes were to analyze the effects of instruction on probability skill development, on attitudes toward mathematics, and toward probability, by sex and grade.

Methodology

The probability intervention and data collection took place during Fall 1982 and Winter 1983. About 1460 sixth through eighth graders, from three sites (urban, suburban and rural) in and around Lansing and Pontiac, Michigan, participated in the entire study.

The instruments used included the Mathematics Attitude Scale (MAS), Probability Attitude Scale (PAS) and a Probability Test (PT). MAS and PT were pre and posttest measures while PAS was posttest only. PT was a 25-item test while MAS and PAS were similar six-item bipolar semantic differentials, with high Cronbach α reliability coefficients. The probability instruction material contained ten sequenced activities requiring about three weeks to cover. The statistical analyses included multivariate and univariate analysis of variance and repeated measures.

Major Results

Prior to instruction, there were (1) no sex or site differences in attitudes toward mathematics, but boys outperformed girls in probability performance. (2) grade differences in probability performance (increasing with age) and in mathematics attitudes (decreasing with age), with slight variations.

After instruction: (1) In all grade levels and sites, boys and girls benefited significantly from the intervention. (2) While seventh graders topped the grades, there were no site or sex differences in probability knowledge gains, (in spite of boys' slight superiority in both pretest and posttest scores). In the suburban site, girls slightly but consistently outgained boys. (3) Attitudes to mathematics declined slightly over the period but these were not meaningfully significant. (4) There were no site, sex, or grade differences in attitude change toward mathematics. (5) Boys and girls did not disagree in attitudes toward probability and mathematics. (6) Seventh graders had more favorable attitudes to probability than the other grades.

This Thesis is Dedicated

to

Lawrence Tayo Oguntebi

(my late brother)

to

Rachel Jibike Oguntebi

(my beloved wife)

to

All my Children

to

All my Friends

ACKNOWLEDGEMENTS

My heartfelt appreciation goes to my doctoral committee members: to Professor Perry Lanier, my chairman, whose thoughtful guadance and cooperation helped me throughout my academic experience; to Professor Glenda Lappan, my dissertation director, for her profound help and understanding. Without her relentless readiness to cooperate, the successful completion of this dissertation would have remained a problem for a much longer period; to Prefessor William Fitzgerald whose interest and efforts in seeing me through were very consistent; to Professor James Buschman, who consistently enhanced my other ways of seeing and practicing disciplined inquiry.

My acknowledgement also goes to Dr. David Ben-Haim and Alex Friedlander whose work and advice helped in various ways. My typist Paula Moan also deserves mention for her full cooperation.

My special gratitude goes to my wife Rachel and to my children Blessing and Joy for their assistance and perseverance throughout my graduate studies.

TABLE OF CONTENTS

Chapter Page

APPENDICES

Appendix

Appendix	Page

LIST OF TABLES

Table		Page

Table		Page

Table		Page

LIST OF FIGURES

Figure		Page

CHAPTER I

THE PROBLEM

Introduction and Rationale

Probability has enormous importance in modern society. In a world of uncertainty, we must make choices, take chances and live by the consequences of our judgments. Probabilistic thinking is frequently involved, directly or indirectly, when choosing between alternative courses of action. Many and diverse daily activities and realities depend heavily on probabilistic thinking. Decision making in scientific and educational research, weather forecasts, military operations, business predictions, insurance calculations, design and quality control of consumer products, genetics, politics, computer technology and social science, are a few areas of application of aspects of probability.

Many researchers, scholars and organizations have emphasized the importance of probability and statistics. Shulte (1981) points out that statistics and probability provide methods for dealing with uncertainty and are inherently interesting, exciting and motivating topics for students. The National Council for Teachers of Mathematics (NCTM) has long recognized the importance of statistics and probability in school mathematics. In "An Agenda for Action: Recommendations for School Mathematics of the 1980's," developed by the NCTM, probability and statistics were emphasized as topics deserving attention in school mathematics (1980). Over eighty percent of the scientific (mathematics) community surveyed by the NCTM in a "Priorities in School Mathematics" (PRISM) project (1981, 11-12) strongly support the inclusion of statistics and probability topics in school mathematics for all secondary school students.

In writing the preface of the 1981 Yearbook of the National Council of Teachers of Mathematics (NCTM), the editor, Shulte (1981) observes that the 1981 Yearbook theme of statistics and probability was selected cognizant of the importance and appropriateness of probability and statistics in the school mathematics curriculum. Shulte (1981, ix) asserts:

> All major curriculum groups in this century -- including the NCTM in its recommendations for the curriculum of the 1980's -- have stressed the importance of statistics and probability... We hope the material in this yearbook will capture your interest and give you a springboard for beginning the teaching of statistics and probability.

In an overview and analysis of school mathematics in the secondary school, the Conference Board of the Mathematical Sciences - National Advisory Committee on Mathematical

Education (NACOME) submits that probability and statistics are "indispensable for the solution of policy questions" and other facets of life. The NACOME report laments the little understanding and interest that teachers in general show in probability and statistics, as revealed in an NCTM exploratory survey. Shulte (1981) also comments on the relatively little instructional time that teachers and most school systems give to these topics. Both Shulte (1981) and NACOME (1975) advocate the provision of curriculum materials for teachers in order to encourage teachers to teach probability and statistics.* Even though the NCTM considers these topics important in upper elementary grades and junior high school (NCTM, 1983), not a single topic in the NCTM (1982) Yearbook titled "Mathematics for the Middle Grades (5-9)", is devoted to statistics and probability.

Other writers or groups who stress the importance of probability include Shaughnessy (1976), Wilks (1958), the Cambridge Conference on School Mathematics - Goals for School Mathematics (1963), Johnson (1980), Kass (1964), Lee and Hoban (1975), White (1980) and Huff (1954). Huff and Geis (1959) powerfully sum up the importance of probability this way:

> Probability theory is the underpinning of the modern world. Current research in both the physical and social sciences cannot be understood without it. Today's politics, tomorrow's weather, and next weeks' satellite all depend on it.

---

* The present study includes an evaluation of one such set of curriculum materials. This material was developed by the Middle Grade Mathematics Project (MGMP) and will be described in detail later in the study.

If probability is so important and useful in our modern society, it is worth treating as such in the school mathematics curriculum.

The literature, however, shows that schools and teachers for the most part do not teach probability. Causes include the teachers' lack of knowledge in the subject and the nonavailability of well organized materials to help teachers manage the teaching of the subject.

The objective of this study is to consider questions that will have implications on teaching probability at the middle grades level. For example, what is the level of understanding of middle grade students in probability prior to any curriculum intervention? A similar question has been investigated by a number of researchers. Among them are Jones (1974), Leake, Jr. (1965), Doherty (1965) and Mcleod (1971), who conducted their studies respectively on grade levels (1-3), (7-9), (4-6), and on selected elementary grades. These researchers conclude that elementary and junior high school boys and girls in general possess considerable knowledge of some probability concepts prior to formal instruction. In particular, Jones reports that grades one through three pupils already have some concept of outcomes of a sample space. Doherty concludes that by grades four through six, children have already acquired some familiarity with the probabilities of a sample space, sample events and the union of two or more mutually exclusive events. Leake concludes that in grades seven through nine, students already possess considerable knowledge of the same probability concepts.

Of interest also are the questions of sex differences in probabilistic thinking prior to any instruction, and how these sex differences change with grade level. In other words, do boys and girls develop probabilistic concepts differently or equally without any systematic probability curriculum?

The Comprehensive School Mathematics Project (CSMP) has developed a curriculum which introduces considerable probability in grades 1-6. Evidence obtained during the national evaluation of this project indicates that sex differences which seemed apparent prior to instruction vanished as a result of instruction.

The issue of sex differences in mathematics, of which probability is a part, is widely addressed in the literature. However, there seems to be little consensus on sex differences in mathematics in research studies. Investigations and findings including those of Benbow and Stanley (1980) tend to conclude that boys naturally have a higher mathematical ability than girls. Wilson (1972), Flanagan et al. (1964) and others claim evidence to support this position. On the other hand, others, especially Fennema (1977) and Senk and Usiskin (1982), claim that when one controls for experiences both in course work and informally outside school, there are no sex differences in mathematics achievement. They therefore conclude that differences are largely environmental. More research is therefore desirable on this issue.

In a study involving grades five through eight boys and girls on the concept of spatial visualization, Ben-Haim (1982) reports significant sex differences in the concept prior to instruction but no sex differences in gains were observed from pretest to posttest. This raises the related questions of any sex differences in achievement as a result of probability instruction. Even if boys and girls differ in their knowledge of probability prior to instruction, another important question is whether they gain differently or equally from probability instruction. At what grade level are any differences minimal or maximal? These are questions that have important curriculum implications in mathematics education.

Leake (1965) and Armstrong (1972) all report achievement gains resulting from probability interventions. More research is needed to determine the nature and magnitude of these gains and in which grade levels intervention has the best chances of success.

Attitude is another issue frequently studied in mathematics education. Of particular interest in this study is both an investigation of students' attitudes toward mathematics prior to and after studying a unit on probability and the relationship between their attitudes toward mathematics in general and toward probability in particular after a given probability intervention. Do boys and girls differ in their attitudes toward mathematics and toward probability? How do any differences change after instruction, and from grade to grade?

Many studies involving attitudes to mathematics and probability activities are reported in the literature. Shulte (1967), Clemente (1982), Moliver (1977), Lee (1975) and Moyer (1974) all report little or no gain in attitude toward mathematics as a result of probability instruction. Clemente (1982) and Fennema (1977) report that generally middle grade boys tend to have more positive attitude than girls. On attitude in general, Fennema (1977) gives what seems to be representative of most literature:

1. There is a positive relationship between attitude and mathematics achievement which seems to increase as learners progress in school.
2. Attitudes towards mathematics are fairly stable particularly above the sixth grade, although one longitudinal study showed a marked decrease from sixth grade to twelfth grade (Anttonen, 1969).
3. Grades six through eight seem to be critical in the development of attitudes.
4. Extremely positive or negative attitudes appear to be better predictors of achievement than more neutral feelings.
5. There are sex-related differences in attitudes toward mathematics (p. 104).

Although the above seems to be the general belief, some reports on attitudes toward mathematics still leave us questions about the magnitude and nature of attitudes to mathematics, sex differences, grade level differences and site differences.

Purpose of the Study

There are four purposes of this study. The first is to determine any existing differences in probability knowledge and attitudes toward mathematics of grades six through eight students by sex, by grade level and by school setting, prior to formal instruction.

The second purpose is to examine the effect of instruction on the probability achievement and attitudes towards mathematics of the students by sex, by grade level and by school setting.

The third purpose is to compare attitudes towards mathematics with attitudes toward probability by sex across these grade levels.

The fourth purpose of the study is to compare attitudes toward probability by grade and by sex.

Research Questions

There are two types of questions for consideration in this study. The first set of questions deals with the existing differences in probability skills and attitudes toward mathematics of grades six, seven and eight students by sex, by grade level and by school setting, prior to instruction. These will be called type A questions. The second set of questions, type B, focuses on the effects of instruction on the probability skills of the same students. These questions also concern the effects of instruction on differences in attitudes toward mathematics and probability by sex, by grade level and by school setting after instruction.

Type A Questions

Prior to instructional intervention:

1. What effect, if any, does grade level have on knowledge of probability and/or on attitudes toward mathematics?
2. What effect, if any, does sex have on knowledge of probability and/or on attitudes toward mathematics?
3. Do differences between boys and girls in knowledge of probability skills and/or in attitudes toward mathematics change with grade level?
4. What effect, if any, does school setting have on knowledge of probability and/or on attitudes toward mathematics?

Type B Questions

After instructional intervention:

1. What effect, if any, will probability instructional intervention have on achievement in probability tasks and/or on attitudes toward mathematics of sixth, seventh and eight grade students? Will these effects be different for boys and girls? Will these effects differ by grade level? Will the effects differ by school setting?

2. Do differences exist between students' attitudes toward mathematics in general and the probability activities in particular? Will these differences exist for both sexes? Will these differences exist for each grade level in the study? Will these differences exist for each of the sites 1, 2, and 3?
3. Do differences exist between the sexes in attitudes toward probability activities? Will these differences exist for each grade level? For each site?
4. Do differences exist among the three school settings (site 1, site 2, and site 3) in their attitudes toward probability activities? Will these differences exist for each grade level? Will they exist for each sex?

Research Hypotheses

The hypotheses that will be tested in the investigation of the research questions will be in two parts: Type A, and Type B.

Type A Hypotheses:

These hypotheses are designed to test for differences among a sample of sixth through eight grade boys and girls in their knowledge prior to instruction in probability activities and their attitudes toward mathematics by sex, grade level and school setting.

H_{01}: There will be no difference among the mean scores for each of the three grade levels (six, seven and eight) tested, on both the Middle Grades Mathematics Project Probability Test (MGMPPT) and on the Mathematics Attitude Scale (MAS).

H_{02}: There will be no difference between the mean scores for boys and for girls in grades 6 through 8 on both the MGMPPT and MAS.

H_{03}: There will be no interaction of grade by sex among the mean scores for 6th through 8th graders on both the MGMPPT and MAS.

<u>Type B Hypotheses</u>:

These research hypotheses are designed to test for differences in two major areas among the sample of students after instructional intervention. Some of these hypotheses are to test for any effects of the instruction on the probability skills and on attitudes toward mathematics of the middle grades students by sex, grade level and school setting. Others are to compare the same students' attitudes toward probability, as well as examine sex and grade level differences in attitudes toward probability.

H_{04}: There will be no difference between the posttest means and pretest means of the sixth, seventh, and eighth grade students on both the MGMP probability test and Mathematics Attitudes Scale (MAS).

H_{05}: There will be no difference between the mean <u>gain</u> scores for <u>each</u> of the three grades levels tested in both the MGMPPT and MAS.

H_{06}: There will be no difference between the mean gain scores (posttest minus pretest) for boys and for girls in grades six, seven, and eight on both the MGMPPT and MAS.

H_{07}: There will be no difference between students' mean scores on the Mathematics Attitude Scale (MAS) and students' mean scores on the Probability Attitude Scale (PAS).

H_{08}: There will be no interaction of grade by sex among the mean difference scores--MAS score minus PAS score.

H_{09}: There will be no significant difference between the mean scores for boys and for girls in grades six, seven, and eight on the Probability Attitude Scale.

H_{10}: There will be no difference between the mean scores for each of the three grade levels (six, seven and eight) on the Probability Attitude Scale.

Assumptions of the Study

For the purpose of this study, the following assumptions are made:

1. It is assumed that a paper-pencil, multiple-choice response instrument is a valid means of assessing student's ability in probability skill.
2. It is assumed that the sample does not differ significantly from the population with respect to the variables being measured in the study.
3. It is assumed that all the testing conditions (pretest and posttest) do not differ significantly from school setting to school setting, and from class to class within a setting. Examples of such testing conditions are place, timing, length of testing, the explanation of testing instructions and other administration conditions.

4. It is assumed that teacher effect will not differ significantly from setting to setting and from grade to grade.

Scope and Delimitations of the Study

This study concerns itself with sex and grade level differences in attitudes toward and achievement in probability as contained in the Middle Grades Mathematics Project Probability Unit (MGMPPU). The MGMPPU is implemented by teachers who are most probably of varying mathematical backgrounds and teaching experiences. This study cannot control effects due to these. However, the teachers use the same specified probability unit, activity by activity. The teacher is expected to follow these daily activities as closely as possible, including materials to use, questions to ask and assignments to give. The teachers attended a workshop before teaching the unit. The study does not attempt to compare teachers' attitudes and their students' achievements to others', neither does it attempt to examine the effect of the MGMPPU on the teachers' attitudes toward mathematics or probability or on their knowledge in probability.

The generalizability of the findings of this study is limited to the participating school sites and students during the period of data collection. However, with a large sample, over 1440 students, a case can be made that the sample is representative of grades six through eight students. Moreover, these students were drawn from a wide variety of schools and were instructed by a diverse group of teachers.

CHAPTER II

REVIEW OF RELATED LITERATURE

Introduction

Literature and arguments for the importance and inclusion of probability in any contemporary school mathematics curriculum were briefly presented in the previous chapter. Also discussed briefly were the issues of sex and grade level differences in mathematics and probability. In this chapter, the same aspects of probability and mathematics will be reviewed in detail. Precisely, the following will constitute the focus of the review of the related literature in this study.

1. Development of Probabilistic Thinking in Children and Adolescents.
2. Studies on Children's Understanding of Probability concepts prior to instruction.
3. Curriculum Innovations in Probability.
4. Studies on Achievement and Attitudes Toward Mathematics and Probability.
5. Sex Differences in Achievement and Attitudes Toward Mathematics and Probability.

6. Summary of the Literature Review.

Development of Probabilistic Thinking in Children and Adolescents

We will now turn to the fields of developmental psychology and mathematics education to cite literature concerning the development of probabilistic thinking in children and adolescents.

The work Piaget and Inhelder reported in their book (Piaget and Inhelder, 1951) is the source of much of the research in the development of the probability concept in young children. Piaget presents clinical evidence from interviews with children and concludes that the learning of probability concepts proceeds in stages, in accord with his theory of the development of thought in children. There are three stages in Piaget's theory of the development of the probability concept in children.

In the first stage, generally characteristic of children under seven years of age, the child is unable to distinguish betwen the necessary and the possible. In this stage, uncertainty means only unpredictability of events in the near future. The child does not possess a concept of logical uncertainty, and so does not understand the true nature of a random mixture. Piaget found that children in this first stage of development tried to superimpose an order or discover a pattern amid the chaos of a random mixture.

Two behaviors that Piaget observed in children in the first stage are worth noting in connection with the present study.

In the first place, if a subject was shown instances of events A and B, and if A appeared more frequently than B, the subject would tend to bet on B because it had been skipped too often. This type of behaviour, sometimes referred to as the gambler's fallacy, exemplifies a subject's use of the representativeness heuristic, in the language of Kahneman and Tversky (1972). A truly representative sequence of instances of A's and B's should not favor one or the other (provided, of course, that the probability of instance A is the same as that of instance B.

In the second place, Piaget's subjects tended to predict those events which had been observed most frequently, with total disregard for the population distribution. This type of behaviour is characteristic of the availability heuristic (Tversky and Kahneman, 1973), wherein events are predicted based upon constructible instances.

In the second stage of the development of the probability concept (up to about 14 years), Piaget claims that a child recognizes the distinction between the necessary and the possible, but has no systematic approach to generating a list of the possibles. The present study is concerned with pupils in this stage of probability development. It therefore suggests, if Piaget is correct, that a pupil in the middle grades has no systematic approach to combinational analysis, thus lacking the ability to list the sample space for a probability experiment. For example, it may be too much for a grade six pupil to be required to understand that there are 36 sample points in a single throw of two dice. This child, to Piaget, does not as yet possess the formal operations needed to perform such tasks.

In the third stage, a child begins to develop a combinatorial analysis, understands probability as the limit or relative frequency (law of large numbers), and can deal with the probability of isolated instances as a function of the whole distribution.

Piaget's interview technique requires a high degree of verbalization for the subjects. Some studies have been conducted to see if very young children indicate an understanding of some probability concepts when their decisions are made in a nonverbal format. Davis (1965), and Yost, Siegal, and Andrews (1962) present evidence for the existence of some concepts of probability in children age 3 and 4. The children were permitted to determine probability or frequency by utilizing a non-verbal decision process. Yost et al. claim that the amount of reinforcement in a probability learning experiment with four-year-olds had a significant effect upon the accuracy of the children's predictions.

Smock and Belovicz (1968) claim that the children in Yost's experiment really learned about reinforcement, and not about probability. They present substantial evidence that subjects of junior high school age have a very poor conception of the laws of probability. Smock's subjects could not consistently generate correct sample spaces, and did not recognize or utilize the concept of independence when predicting outcomes.

Cohen and Hansel (1956) identify four stages that children go through in the development of the idea of a probability distribution. At first there is just a "glimmering belief" that the numbers in a distribution will really vary. This corresponds somewhat to recognizing the distinction between the necessary and the possible in Piaget's theory. Secondly, a child feels that the category of exactly equal proportions will occur most often, that is, that every probability distribution is a uniform distribution. In the third stage, likelihoods are assigned to outcomes based upon their similar structure. For example, the outcome one blue and four yellow beads is judged as likely to occur as the outcome one yellow and four blue beads, regardless of the population composition. In this stage, the child applies the principle of symmetry universally. Finally, Cohen and Hansel claim, a child is able to assign a greater probability to the event "one blue and three yellow beads" than the event "four blue beads" in a 50-50 distribution. Cohen and Hansel attribute the stages of mental development both to maturation and physical experience, and say that a child is ordinarily in the fourth stage of development around the age of 15. This theory is very much in accord with that of Piaget.

Studies on Children's Understanding of Probability Concepts Prior to Instruction

In the problem statement in Chapter One, it was shown that mathematics associations (the NCTM being foremost) and mathematics educators have strongly advocated the inclusion of probability curriculum in elementary and secondary school mathematics (NCTM; 1973, 1960, 1975, 1980-1982). Several mathematics educators came to such a conviction following the challenge that resulted from Piaget's theory of probability development and the controversy surrounding the level of probability concept attainment in children at various ages. The College Entrance Examination Board (1959) and the Cambridge Conference on School Mathematics (1963) were motivated by these psychological findings to advocate the teaching of probability and statistics in school mathematics. Until recently (NCTM, 1981 Yearbook), most efforts undertaken in mathematics education were in the words of Shaughnessy:

> ...either feasibility studies undertaken to determine the teachability of probability and statistics in the elementary or secondary schools, or experimental and correlational studies which attempted to measure the effects of teaching a unit of probability (1977).

Next is a review of some recent studies or reports on children's understanding of probability concepts prior to any formal instruction.

Jones (1974) used taped interviews with first, second, and third graders, and embodiments of set and measure to investigate the status of five concepts of probability among early elementary school children. The embodiments were spinners with equal and unequal area divisions, and jars containing discrete objects. Interviews were taped in order to gain insight into the errors made by the subjects. The concepts were sample space; comparison (P_1) of the probability of two events within a fixed sample space; comparison (P_2) of the probability of a given event across three sample spaces with the number of total outcomes held constant, $\frac{a}{n}$, $\frac{b}{n}$, $\frac{c}{n}$; identification of (P_3) uniform probability distribution; and comparison (P_4) of one event across three sample spaces in which the frequency of that event was constant but the total number of outcomes was varied, $\frac{a}{x}$, $\frac{a}{y}$, $\frac{a}{z}$. Jones found evidence in support of the children's understanding of P_2, P_4, and of sample space. He suggests that for primary children, an apparent understanding of probability in one situation does not guarantee understanding will be evidenced in another situation. There is also further evidence in Jones' study that I.Q. predicts the extent of the development of probabilistic thinking in young children, in accord with the findings of Leake, Doherty, and Lefrin (discussed later). The use of embodiments seemed to help the children understand probability although Jones reports that the use of manipulatives to perform an experiment sometimes interferred with the children's ability to list the outcomes of a sample space. Color biases and individual preferences prevented some children from making accurate responses to questions involving the spinners.

Mullenex (1969) investigated the relationships between understanding of probability in grades 3 - 6, and the variables of sex, age, grade level, and skill in other school subjects. His test was based upon the questions that Piaget asked children in interviews. Multiple linear regression techniques indicated a tendency for arithmetic computational skills and reading skills to be relevant predictors of performance on probability measures. Mullenex found sufficient evidence for the understanding of probability in children to warrant inclusion of probability topics in grades 3 - 6.

Doherty (1965) carried out a similar study with fourth, fifth, and sixth graders. An investigation of children's understanding of independent events was added to the three concepts of sample space, simple probability, and mutually exclusive events of Leake's study. Doherty found that children in grades 4 - 6 possess considerable familiarity with these concepts prior to formal instruction. Age, mental age, and achievement were found to be significantly related to the level of understanding of probability concepts. Doherty interprets her results as indicative of the feasibility of teaching probability in the elementary school. She recommends that topics from probability be included in elementary school curricula, and that teacher training programs make provisions for informing prospective elementary teachers about probability topics that would be suitable for elementary school children.

In a study of probability concepts possessed by children in grades 4 - 7 prior to formal instruction, Leffin (1971) reports that children have considerable knowledge of the concepts of finite sample space, probability of a simple event, and quantification of probability. I.Q., sex, and grade level were all found to be significantly related to the understanding of probability. I.Q. was found to be the most accurate predictor of performance on probability tests. In analyzing the children's errors, Leffin mentions that the concept of combinations was very difficult for them to comprehend or to use. When Leffin's subjects could list all the outcomes in a sample space that counted combinations, 92% of them could not use the information from the sample space to calculate a probability. This evidence appears to support Piaget's position that children of this age are in the stage of concrete operations. Leffin's subjects could successfully handle probability in simple situations like drawing balls out of a box given the number of balls of each color that are in the box. However, the more complicated combinatorially-generated sample spaces were not understood by these children. This finding caused Leffin to speculate on how early children can be taught a systematic method of counting. He recommends taped interviews and the use of manipulatives with children in order to obtain more information about children's readiness to learn counting principles.

In an investigation of the development of the notions of chance and probability, Kelsey (1980) concluded that adolescents had a poor understanding of these notions. Thus, Kelsey's findings agreed with those of Beyth-Maron and Shaughnessy.

Leake (1962) found that seventh, eighth, and ninth graders had some understanding of sample space, probability of a simple event, and probability of the union of two disjoint (mutually exclusive) events. As in Doherty's study, mental age and achievement both correlated significantly with understanding of probability. Leake recommends the inclusion of probability topics, in grade levels seven to nine, based on the results of his investigation.

The above literature reviews refer chronologically to elementary and middle grades probability concepts prior to instruction. The studies all recommend that topics in probability be included in elementary through middle grades mathematics curricula. Quite a few probability studies or curriculum developments have been carried out - involving all levels of learners - elementary, intermediate, middle grade, high school and college levels. Some of these studies will now be reviewed in turn. The extent of the review depends on the relevance or relationship of the particular study or intervention to the present investigation.

Curriculum Innovations in Probability

Attempts at developing curriculum materials in probability have been made by a number of innovators. Some of these attempts will next be reviewed.

Studies were carried out by Gipson (1972) to determine what materials would be appropriate for introducing probability concepts to third graders. In one study, children received instruction in small groups and in another instruction was individualized. The instructional sequence dealt with the concept of sample space and the probability of a simple event. Audio and video tapes of the subjects were made to gain deeper insight into the process through which children learn about probability concepts. Gipson, like Shepler, reports that the children had difficulty specifying estimated probability from an experiment. Gipson also adds that the use of the interview-type procedure (clinical interview) would give a deeper insight into how children develop probabilistic concepts. Gipson concludes with a recommendation that the third grade level is an appropriate school stage to introduce selected concepts of probability.

Armstrong (1981) describes the probability included as an integral part of the elementary mathematics curriculum developed by the Comprehensive School Mathematics Program

(CSMP). The CSMP developed stories and games for the second and third grades that introduce such concepts as expected frequency, equally likely events, and prediction. Armstrong reports that third grade CSMP students "considered the thirty-six equally likely outcomes when two dice are thrown and determined that there are six ways for a sum of seven to occur". Armstrong further describes the area model* technique for solving probability problems. In this model, a unit square is divided into regions so that the areas of the regions are proportional to the probabilities involved in the situation. The area model, Armstrong continued, "is a geometric model that satisfies the CSMP criteria for solving probability problems". To be appropriate for students in the intermediate grades, Armstrong claims the model should:

- be sufficiently powerful to handle fairly sophisitcated probability problems;
- rely primarily on mathematical skills that students already have acquired;
- be consistent with the students' current understanding of probabilistic concepts;
- support the eventual development of more advanced solution techniques (1981).

Shepler (1970) developed a unit on probability dealing with sample spaces of one, two, and three dimensions, and necessary counting techniques. The unit was taught to a class of 25 specially selected sixth graders of above average ability. The unit was taught using a mastery learning model that incorporated self-correcting exercises,

---

* The area model technique was also used in the probability instruction implemented for the present study.

specific prescriptions to diagnose and remedy errors, extra help sessions, and extra group instruction when mastery was not satisfactorily attained by a large majority of the class. Objectives included counting outcomes, probability of a simple event, probability of a compound event, equally likely versus unequally likely probability models, and estimating the probability of an event from data in an experiment. A criterion level of 90% correct by 90% of the students was set for mastery of the objectives. All the behavioral objectives were mastered at this level by the students except those dealing with counting the number of outcomes and estimating probability from data. Shepler's results agree with those of Leffin (1971), and suggest that sixth graders do not yet possess the formal operations that Piaget claims are necessary to count all the outcomes systematically. A follow up study (Shepler and Romberg, 1973) indicated that after four weeks the subjects were able to retain most of what they had acquired at the mastery level.

Beyth-Maron (1980) innovated a probability curriculum entitled "Thinking under Uncertainty: A Curriculum". Beyth-Maron's work has a lot in common with the present study. Hence her work will be reviewed in some detail.

Prompted with concerns similar to those already expressed in this study, Beyth-Maron conducted a five-year study which culminated in a workable curriculum in "Thinking under Uncertainty". Beyth-Maron, in her study, reviewed several scholarly studies on thought processes (Miller, 1956; Bruner's concept formation, 1956; Slovic et al., 1977; and Tversky & Kahneman's 'thinking and uncertainty,' 1974). "These studies," claims Beyth-Maron, "have demonstrated cognitive limitations in perceiving, memorizing and processing information." How then do people perceive uncertainty, assess probabilities, evaluate risks and judge the quality of their own and others' decisions? Noting several limitations associated with probabilistic thinking, Beyth-Maron sought in her curriculum to help correct some of these limitations. She asked these leading questions. "Can corrective procedures be devised? Can we show people when and how their judgments are wrong and how they can be improved?" She also remarked on the usual difficult and inapplicable way in which statistical and probability concepts were taught to students and made these observations:

> In teaching, it can be difficult to convince students that probability is relevant to life events and not just the science of coins and playing cards. Even experts who appreciate the relevance of probability to daily matters are prone to the same mistakes as lay people. This may occur because most daily problems are not formulated as neat, textbook probability problems and experts often fail to make the reformulation intuitively. In addition, most courses in statistics, probability are taught without taking account of cognitive processes (Beyth-Maron, 1980).

In her probability curriculum, Beyth-Maron demonstrated five stages of teaching that she considered appropriate for teaching probability to middle grades students. These stages were: (1) Demonstrating by example(s); (2) Analyzing thought processes (introspection); (3) Strengthening good intuitions and showing absurdities by considering alternative thinking and nonexamples; (4) Analyzing the pupils' answers and arguments with them, making them understand how similarity rules do not obey probability rules; and (5) Deciding what the pupil should learn and use specifically.

Junior high school was chosen for Beyth-Maron's curriculum development. Three reasons were given for this choice. First, junior high school students already have a grasp of the minimal mathematics demanded by the probability activities. Second, these students are mature enough for introspection ability, and may even enjoy doing so; and third, they have the time and willingness to accept new experimental areas.

The proability unit included:

(1) General framework for thinking under uncertainty,

(2) Some tools for judgment, and

(3) Probability Instruction.

In the questionnaire evaluation that followed, curriculum participants and non-curriculum participants were compared in 20 items. Beyth-Maron concluded that the program recipients did significantly better than the control group. It was also found that children from high academic schools gained more from the program than those from low academic schools. However, every participant gained significantly from the program.

In another probability curriculum, White (1974) developed and taught some concepts to seventh and eighth grade students. On comparing pretest and posttest results, White found that the subjects benefited significantly from the program. Achievement in probability was correlated significantly with concept attainment, computational ability, and reading ability. McKinley (1960) developed a probability unit for twelfth grade students. McKinley reports that intelligence, language skills, reading comprehension, and mathematics achievement, all correlate significantly with achievement in the unit.

An experimental probability curriculum was developed and implemented by Shaughnessy (1977). Using college students as his subjects Shaughnessy, like Beyth-Maron, attempted to correct certain probabilistic errors in young people. His objective was to provide a probability intervention that would maximize the students' chances of overcoming certain misconceptions of probability and statistics. He argued, like Beyth-Maron, that a conventional lecture approach to the teaching of probability may not be the best way to overcome students' misconceptions about probability. He therefore developed an experimental approach that used small-group, activity based strategies in teaching probability.

The misconceptions that were investigated were those that arise from reliance upon heuristics of representativeness and availability. These heuristics "enable human beings to decode complex probabilistic situations" (Shaughnessy, 1977).

According to the representativeness heuristic, people tend to make decisions about the likelihood of an event based upon how similar the event is to the distribution from which it was drawn. For example, a nursing mother whose six children are boys would see this as not being representative of the random process of child bearing, and would tend to expect the seventh child (if any) to be female, even though she might know that these are independent outcomes. According to the heuristic of availability, subjects tend to base their judgments upon the relative likelihood of the events based upon the ease with which instances of that event can be constructed or called to mind (Tversky & Kahneman, 1973). For example, subjects employing the availablity heuristic tend to favour the misconception that out of a group of 11 people, these are more distinct 4-person committees than there are distinct 7-person committees. It is easier to call to mind more examples of 4-person committees than 7, even though the number of distinct committees is the same (330) in each case. It was found that the experimental course was more effective in overcoming some misconceptions that are attributable to the use of representativeness and availability than the control course.

The experimental activity-based course was constructed as an alternative to the lecture method for an undergraduate course in finite mathematics. A series of nine activities in probability, combinatories, game theory, expected values, and elementary statistics were developed by Shaughnessy. Students in the experimental course worked together in class on the activities in small groups of four or five members. Each activity required the groups to perform experiments, gather data, organize and analyze the data, and finally reach some conclusions which could be stated in the form of a mathematical principle or mathematical model. The students were strongly encouraged to cooperate with one another, to solve problems as a group rather than individually, and to help all the members of their group to understand the concepts and problems of each activity. The groups were changed often so that everyone had a chance to work with everyone else during the course.

Shaughnessy also remarks that the manner in which college students learn probability makes a difference in their ability to overcome misconceptions that arise from availability and representativeness. He concluded his study with this implication for the mathematics teacher (1977, p. 314):

> Peoples' intuition of probabilistic thinking is distorted by science education's emphasis on the necessary, and neglect of the possible. This experiment suggests that the course methodology and the teaching model used in an elementary probability course can help develop peoples' intuition for probabilistic thinking. A course in which students carry out experiments, work through activities to build their own probability models, and discover counting principles for themselves can help students to overcome their misconceptions about probability, and can help restore the synthesis between the necessary the the possible which is essential to probabilistic thinking.

From Beyth-Maron's, Shaughnessy's and other curriculum studies, one sees a lot of similar concerns and experimental results regarding the use of probabilistic thinking and learning that will be examined in the present study.

Studies in Achievement and Attitudes Toward Mathematics and Probability

Studies involving both mathematics and probability at the elementary grades level, with respect to achievement and attitudes, are not common in the research literature. However, several studies on these topics, involving adolescents and adults, are reported.

Shulte (1968) investigated the effects of a probability and statistics unit on the achievements and attitudes of ninth grade general mathematics students. Shulte concluded that the probability and statistics presented in his unit, "The Mathematics of Uncertainty", did not effectively promote student attitude or achievement in computational skills. The intervention however effectively increased proficiency in other mathematics areas.

Moyer (1975) designed and conducted a study to "test the claim that probability has the potential to improve arithmetic computation skill, arithmetic reasoning, and attitudes toward mathematics". Moyer whose subjects were ninth grade general mathematics students, did not find any significant difference in attitudes toward mathematics, but the experimental group outperformed ($P < .05$) the comparison group in knowledge about probability. However, like Shulte, Moyer's study:

> does not support the contention that probability, at least that part of probability contained in the unit taught in this study, can be used in the ninth grade general mathematics classes to improve arithmetic computational skill, arithmetic reasoning, or attitude toward mathematics. However, the study indicated that while gaining knowledge about probability, the experimental group showed equivalent improvement with the comparison group in the ordinary general mathematics areas of arithmetic computation skill and arithmetic reasoning.

Lee (1975) worked with low-achieving junior college students in a study on developing basic mathematics skills through elementary probability and statistics. Lee had three goals.

1. To improve students' mastery of some basic mathematics skills.
2. To help obtain some understanding of probability and statistics and their uses in real world situations.
3. To improve students' attitudes toward mathematics.

In the summative evaluation of the 191 subjects involved, Lee reported that students' attitudes toward mathematics remained unchanged, but over 80 percent of the subjects claimed to have acquired a better understanding of probability and statistics and their applications.

Shevokas (1974) carried out a study using, as subjects, the students taking the general mathematics course in a community college. In the study in which computer oriented and manual Monte Carlo approaches were employed, Shevokas had two purposes. The first was to investigate the effects of the Monte Carlo approach on achievement in and attitude toward mathematics. The second purpose was to examine similar effects in probability and statistics. No significant difference was found in the measures of attitudes toward mathematics. However, both experimental groups (one with computer and the other with manual Monte Carlo procedures) achieved higher ($P < .01$) than the control group which used analytic methods only. Shevokas concluded with the assertion that the non-computer Monte Carlo approach was an optimal method for introducing a probability unit to community college students.

Kipp (1975) investigated the effects of integrating topics from probability with those of elementary algebra in an experiment with college students. She compared experimental and control groups on achievement, retention, and attitude. Greater retention and improved attitude towards mathematics were found in the groups receiving the algebra integrated with probability. Kipp recommends that experimentation be introduced before college students are taught probability formally. She suggests that college students should encounter physical models of both uniform and nonuniform probability distributions.

In an experimental study involving graduate students in the behavioral sciences, Monroe (1980) developed, taught, and evaluated two probability concepts. Monroe reached the following conclusions:

> the relationship between age and performance on the probability test was strong and negative among students with a poor mathematics background; student attitude toward mathematics was not related to performance on the probability test; and, important probability concepts can be taught using a non-traditional curriculum (Monroe, 1980).

Crouse (1977) in his study investigated the effect of the teacher's probability knowledge and mathematics attitudes on student probability achievement. Crouse found that higher student achievement in the selected probability tasks taught was significantly associated with higher teacher knowledge in these tasks. Crouse concluded that teacher attitude toward mathematics had no effect on student achievement in the probability tasks taught.

Achievement in and Attitudes Toward Probability

The studies reviewed above each dealt with probability and mathematics. Quite a few investigations have addressed probability alone, with respect to student achievements and attitudes. Some of these studies are now reviewed.

In a study on first grade children's understanding of probability, Dunlap (1980) indicates that even children with limited or no understanding of probability can be trained to evidence such understanding. He reached this conclusion from the result of pretest-posttest data involving seven groups. Dunlap however suggested that the "rule training" (tutorial) method was more successful with first graders than the "self-discovery training" method.

Armstrong (1972) investigated the ability of fifth and sixth graders to learn selected topics in probability. Armstrong concluded that while sixth graders possessed the ability to learn all the concepts of probability taught, fifth graders were unable to learn the concept of outcome space. The concepts taught in the study were outcome, outcome space, even, probability of a finite event, and mutually exclusive events. Sixth graders gained significantly on all of these concepts and on the total probability test.

In another study, Smith (1966) developed and taught a unit on probability and statistics for seventeen days to three groups of seventh graders. The three groups were low, middle, and high experimental groups. Smith found that all three groups learned significantly ($P < .01$) from the instruction. Smith concluded that the seventh grade was an appropriate level to introduce "at least some topics in probability and statistics."

McClenahan (1974) carried out a study involving an application of Piagetian research to the growth of chance and probability concepts. McClenahan's subjects were low achievers in secondary school mathematics. His conclusions included

> There is a strong indication that the low achiever in mathematics may not have attained the formal operational stage, at least as far as the topic of probability is concerned (McClenahan, 1974).

This study tends to suggest that probability is a relatively abstract topic in mathematics. As such, children's level of development ought to be taken into consideration when introducing probability in school mathematics programs.

Arehart (1978) explored the relationship between ninth and tenth grade student achievement on a probability unit and student opportunity to learn the unit objectives. Twenty-three teachers taught the unit to twenty-six classes. In the analysis of the pretest-posttest scores, Arehart found the following:

1. Student achievement in probability is related to the amount of exposure or opportunity he has to learn that objective.
2. The study also supports the tenet that amount of student work is related to student achievement.
3. Teacher information turns out to be as important as teacher questioning behaviour.
4. The amount of teacher information and teacher questioning about objectives of a lesson relate positively to the achievement of them.

Sex Differences in Probability

Research findings with respect to sex differences in probability seem to concur, irrespective of grade level, that little or no sex differences exist. Studies by Mullenex (1968), Doherty (1965), Smith (1966), and Wavering (1979) were conducted respectively with grades levels 3-6, 4-6, 7, and (8, 10, and 12); and in varied settings. Yet all conclusions were unanimously in favour of no significant sex differences in probability. Doherty's study, reviewed much earlier in this chapter, was carried out prior to instruction. Also McLeod (1972) found no sex differences in his own study. Three treatments in a unit on probability were administered to second and fourth grade children. The treatments were laboratory experience, a teacher demonstration, and a control in which no probability was taught. The unit on probability covered the law of large numbers, prediction of a set of outcomes from an experiment involving repeated trials, and uses of probabilistic terms such as "certain," "impossible", "likely", and "unlikely". McLeod also found no differences among the three treatments in probability achievement.

In an evaluation of the Comprehensive School Mathematics Project (CSMP) probability curriculum, sex differences were reported prior to instruction in both CSMP and non-CSMP students. However, after instruction, according to Dougherty (1981),* sex differences were not found with CSMP students, but sex differences persisted with non-CSMP students.

However, Kass (1964) reported sex differences in probability achievement in favour of boys. Kass, in his study, found that boys outperformed girls in binomial probability tasks. Kass' study is one of the very few

* This report was given by Dougherty of the CSMP at the NCTM (1981) Annual Conference, at St. Louis, Missouri, U.S.A.

studies that found any sex differences in probability. More research is needed with respect to sex differences in probability.

Achievement in and Attitudes Toward Mathematics

So far, reviewed in this chapter are studies in which probability and mathematics are the focus, or in which probability alone is the concern, with respect to achievement and attitudes. Other studies have addressed achievements in and attitudes toward mathematics alone. This section contains a review of attitudes and achievement, and the relation between them, with respect to mathematics.

The general question asked by current researchers is "What is the strength of relationship between attitudes toward mathematics and achievement in mathematics?" Affective variables are believed by many educators to be as important contributors to the learning of mathematics as cognitive variables. Evidently, research is needed to verify or nullify the common sense feeling of heavy dependence, or even causality, between attitudes and achievement with respect to mathematics.

Malcolm (1971) reviewed the question of attitude formation through a ten-month longitudinal study. Malcolm used a sample of 858 students from a large suburban school district, in grades three to four, five to six, and six to seven. The purpose of the study, among other concerns, was to determine if attitudes do decline with age, and if any grade level would emerge as producing the greatest amount of attitude change. Two arithmetic attitude scales were employed. The first scale was the Hoyt Minnesota Pupil Opinion, a 28-item yes-no instrument. The second scale was a semantic differential with fifteen bipolar adjective pairs. Both scales were proved reliable and found to have acceptable internal reliability, with the Hoyt instrument yielding the highest correlations. Like in the present study, sex and grade were the independent variables. Inconsistent results were obtained. With the Hoyt posttest scores, fourth graders had the highest attitudes toward arithmetic and the sixth graders had the lowest. Sex differences were found on the semantic differential scale only. Malcolm submitted these conclusions:

1. Attitudes do decline as one proceeds through school.
2. The later (elementary) grades; i.e., grades five to six and grades six to seven, appear to be important in attitude formation.
3. Girls tend to register more negative attitude change than boys across the grades.

Malcolm concluded his study with a recommendation for longitudinal studies dealing with the identification of factors influencing attitude formation.

As if in response to Malcolm's recommendation, Shaughnessy, Haladyna and Shaughnessy (1983) conducted a study on factors that influence attitude toward mathematics. Admitting that "poor attitudes may be behind a decreased enrollment in advanced mathematics classes in high school, especially on the part of females", Shaughnessy et al. examined the relations of student, teacher, and learning environment variables to attitude toward mathematics. They argued that attitude studies need not be designed in relationship with achievement all the time. They asserted, "Improvement of student attitude has been regarded as a valuable end product in and of itself".

In the study, the research questions examined were:

1. To what extent do student, teacher, and learning environment variables of both types (exogenous and endogenous) account for the variance of a measure of students' attitude toward mathematics?
2. Are these patterns consistent across three different grade levels?
3. Is gender a significant variable in the study of these relationships?

Grades four, seven, and nine students participated in this study. The aspects of affective components measured included student motivation, teacher quality, social-psychological aspects, management and organization, and attitudes toward mathematics. In the attitude toward mathematics questionnaire, items included the composite question: "How do you feel...

1. when it is time for mathematics?
2. during mathematics?
3. when mathematics is over?

4. if you knew you would never go to mathematics again?

Shaughnessy et al. submitted the following conclusions:

1. Exogenous student variables (e.g. gender and socioeconomic status) showed little direct relationship to attitude.
2. Endogenous student variables (e.g. teacher quality and class cohesiveness) showed consistently notable correlations with attitudes toward mathematics.
3. Fatalism (students' perception of their ability to affect school success), and teacher quality indicated the strongest relationships toward attitudes across all three goal levels.
4. The strength of fatalism grows steadily with grade level, and it is higher for girls than for boys.
5. The teacher quality effect is higher for girls in grade level seven, but reversely true in grade level nine.

Shaughnessy et al. concluded their study with some implications for mathematics education. First, there is a need for good teacher quality in order to enhance more positive attitudes toward mathematics. Thus, attention is called to more comprehensive mathematics teacher education programs. Second, student, teacher, and learning environment variables are importantly related to mathematics attitude. These variables must be adequately recognized and taken into consideration in mathematics staff development. Third, more investigation on student fatalism is needed. There seems to be a significant correlation between a student's perception of his ability to affect his school success and his attitude toward mathematics. While this relationship does not necessarily imply causality, more knowledge about its strength is desirable.

Thus, from studies by Malcolm (1971), Shaughnessy et al. (1983), and by others, Knaupp (1973), Epstein (1981), Shaughnessy et al. (1982), and Suydam and Weaver (1975), research evidence abounds that tend to suggest that achievement is not the only variable positively related to attitude toward mathematics. Suydam and Weaver (1975) made the following observation with respect to elementary school studies:

> There is no consistent body of research evidence to support the popular believ that there is a significant positive relationship between pupil attitudes toward mathematics and pupil achievement in mathematics...We have little research basis for believing that these two things are causally related (p. 1-3).

Callahan and Glennon (1975) are also in agreement with Suydam and Weaver. Also reviewing elementary school studies for the same age, they conclude that the state of the art "makes it difficult to present compelling research evidence...that positive attitudes play an important role in contributing to mathematics achievement" (p. 80). Aiken (1976) argues that "when attitudes scores are used as predictors of achievement in mathematics, a low but significant positive correlation is usually found" (p. 295) at the elementary, secondary, college undergraduate and postgraduate levels.

The above studies seem to deemphasize positive relationships between attitude and achievement. Other studies by Anttonen (1969), Malcolm (1971) and Norman (1977) report a decline in attitudes occurring with grade level. However, other equally valid studies, Fennema (1981) in particular, have reported opposite findings.

Fennema (1977) suggests that part of the contradictory conclusions can be explained by the age of the subjects being considered in the reviews. Two reviews, Suydam and Weaver (1975), and Callahan and Glennon (1975), were concerned basically with children in grades one through six. Problems of assessing attitude in these grades have not been addressed adequately and lack of carefully designed measuring instruments may have caused reviewers to seriously question any significant differences reported. Aiken, in his 1976 review, was concerned with a much broader age spectrum. Even while recognizing the serious problems connected with the studies of young children, he was willing to accept the evidence as having some validity because the results coincided with studies having older subjects.

Fennema (1977) summarized the conclusions most often reported in the literature, but which are now being contended:

1. There is a positive relationship between attitude and achievement which seems to increase as learners progress in school.
2. Attitudes toward mathematics are fairly stable - particularly after about the sixth grade, although one longitudinal study showed a marked decrease from 6th to 12th grade (Antonnen, 1969).

3. Grades 6-8 seem to be critical in the development of attitudes.
4. Extremely positive or negative attitudes appear to be better predictors of achievement than more neutral feelings (p. 104).

Fennema indicates there is a fifth conclusion related to sex differences in attitudes toward mathematics which will be discussed in the succeeding paragraphs.

Sex Differences in Attitudes Toward Mathematics

Although it was not explicitly emphasized in the works of Malcolm (1971), and Shaughnessy et al. (1983) reviewed earlier, sex differences were indicated in attitudes toward mathematics.

On the whole Fennema (1977) concludes that "there are sex-related differences in attitudes toward mathematics (p. 104)." But, even though there is consensus that sex-related differences in mathematics attitude exist, the magnitude and specific dimensions of these differences are unclear. Although denoting some studies which failed to find significant sex differences in attitudes and achievement in mathematics, Aiken (1976) indicates that "differences in both attitudes and achievement in mathematics are frequently found to favor boys over girls at junior-high level and beyond" (p. 296). With regard to sex differences in attitudes, Suydam and Weaver (1975) quote studies with contradictory results and say that in other studies no significant sex-related differences were found. Aiken (1976) states that the correlation between attitude and achievement varies not only with grade level but also with the sex of the student and is generally somewhat higher for girls than for boys.

Basic agreement with the conclusion that significant differences in attitudes are frequently found to favor males over females, was reported in the Fennema and Sherman study (1977, 1978) with learners in grades six through eleven. It has also been reported that mathematics test anxiety is significantly higher for eight grade girls than for eighth grade boys (Szetela, 1973). Finally, Ben-Haim (1982) sums up these investigations with this quote from Aiken (1976). This is a summary of some tentative findings of these kinds of investigations in mathematics education:

> 1. Modern mathematics programs do not improve attitudes more than traditional programs.
> 2. Compared to regular classes, "continuous progress" classes do not have a different effect on attitudes toward mathematics.
> 3. Discovery methods are not superior to expository methods in their effects on attitudes toward mathematics.
> 4. Neither follow-up instruction nor flexible scheduling improves attitudes more than traditional instruction.
> 5. An individual approach to instruction in elementary and junior high mathematics sometimes has a more positive effect on attitude than a traditional approach; other times no difference in the effects of the two types of programs is found.
> 6. Certain units or topics in mathematics have a more positive or a more negative effect on attitudes than other units or topics (p. 300-301).

Contemporary Controversy Regarding Sex Differences

A number of studies have identified sex differences in mathematics achievement (Flanagan et al., 1964; NAEP, 1975;

Wilson, 1972, Clemente, 1982; Benbow and Stanley, 1980). Other studies challenge the notion and argue that recent studies tend to prove otherwise (Senk & Usiskin, 1982; Fennema, 1982; Armstrong, 1981; and Becker, 1981). Salient among the proponents of the existence of sex differences in mathematics achievement are Benbow and Stanley (1980) who became strong proponents as a result of a controlled longitudinal study involving high achieving boys and girls. They conclude the following on finding significant differences in favour of boys:

> It is therefore obvious that differential course-taking in mathematics cannot alone explain the sex differences we observed...Sex differences in achievement in and attitude toward mathematics result from superior male mathematical ability.

Thus Benbow and Stanley tend to advocate that boys naturally do better in mathematics than girls. The above study was conducted to investigate Fennema's assertion (Fennema, 1972) that any sex differences in mathematics achievement are due to differential course-taking, especially at high school level, since sex differences are not apparent prior to high school.

In another study on mathematics achievement involving general and high achieving boys and girls, Senk and Usiskin (1982) report findings quite contradictory to those of Benbow and Stanley. In their extensive investigation of sex differences in achievement in geometry proof, Senk and Usiskin report that the more an instrument directly measures a student's formal educational experiences in mathematics, the less the likelihood of sex differences. They concede that boys perform better than girls in tests of problem solving, consumer applications and the Scholastic Aptitude Test-Measure (SAT-M). However, they insist that these are not a measure of students' formal educational experiences in which mathematical ability should be tested, but a measure of students' experiences outside classroom mathematics. Hence, boys tend to out-perform girls in those tasks because they tend to have more experiences than girls in those tasks. Thus, Senk and Usiskin continue their argument that achievement in geometry proof is achievement in complex and high level cognitive reasoning. Senk and Usiskin (1982) conclude:

> Our results with proof, together with our analysis of other studies, lead us to believe that boys and girls are of equal mathematical ability...We have found that when male and female students are tested on geometry proof, a high level cognitive task with spatial requirements that is encountered almost exclusively in the classroom, no sex differences in performance exist. Our results hold for both our national sample of mixed ability students and for select high-scoring samples...Girls and boys perform equally well.

There is therefore some controversy as to the existence of sex differences in mathematics achievement.

Summary

There appears to be a good deal of support and agreement in the literature that the development of the probability concept in children does proceed in stages in accord with the theory of Piaget. However, there is considerable disagreement among investigators as to which probability concepts are actually known by children, and at what age levels. However, most of them are in favour of the introduction of probability in elementary school mathematics. Curriculum innovators in probability tend to suggest that the child's level of probability development, teacher quality, and certain probabilistic errors are the major concerns in any probability instruction. Grade level differences exist in probability achievement but sex differences are rare. There is no consensus in the literature with respect to the nature of sex differences in mathematics achievement and attitudes. Results are conflicting. While some researchers argue that achievement differences are innate and unchangeable, others insist they are environmental and correctable with appropriate instructional procedures. There is also some disagreement in the literature with respect to mathematics attitude change with grade level. While many assert these attitudes are developed early and decline with grade level, others submit that mathematics attitudes increase with grade level. It is thus apparent that more research is needed on these issues that have so much implication on mathematics education. Even when agreement exists with respect to existence of differences, it is desirable to know the extent of these differences.

Finally, the present study will compare achievements in probabilistic skills and attitudes to mathematics and probability activities across settings. An underlying assumption is that students from urban, suburban and rural areas differ in socio-economic status and background. One objective is to investigate how differences in setting affect achievement in and attitudes toward mathematics and probability activities. Studies on attitudes toward and achievement in mathematics, in which setting is one of the independent variables, are hard to come by. In the only similar study available to this investigator, the effects of race, sex, and grade level on change in mathematics achievement were investigated. In that study, Rule (1981) concluded that grade level, when used with sex, race and teaching method, significantly contributed to the prediction of change in student attitudes toward and achievement in mathematics. The next chapter gives a detailed description of the procedures followed in this investigation.

CHAPTER III

METHODOLOGY

Introduction

In this chapter a detailed description of the probability intervention is presented. Also included are descriptions of the population and sample, the procedure and data collection, the instrumentation of the study, the hypotheses to be tested and the statistical design of the study. A summary of these aspects of the study concludes the chapter.

The Philosophy of MGMP Materials

With the major goal of developing units of high quality mathematics curriculum for middle grades students, the MGMP staff developed four mathematics units. Among these is the unit entitled Probability, which is the focus of this study. The other three MGMP units are called Factors and Multiples, Spatial Visualization and Similarity.

Utilizing an instructional model developed by Shroyer and Fitzgerald (1979), MGMP attempts to help students develop a deep, lasting understanding of the mathematical concepts and strategies studied. The model consists of three phases: launching, exploring, and summarizing (Appendix A), and clearly describes what is expected from the teacher and students during each instructional phase in this way:

> During the Launching the teacher follows the script very closely posing the questions and challenges in the sequence they are intended and presented. This sequence allows each student to be engaged in the task at his/her appropriate level with some degree of success.
>
> After the major challenge has been posed, the class can begin working individually or in small groups. The teacher can float around the class to keep abreast of developments. Some children will need additional help beginning the task as one presentation of the challenge is often not sufficient. Other children will need help maintaining progress toward the challenge. The teacher may spot errors the students have made and help the children correct their error. Still other children will finish the task and will need to be presented with an extra challenge to keep them working productively.
>
> Such a work period will result in the children being more different from each other than before. While all children have made progress, some have made much more than others. This is as it should be.
>
> However, it is desirable to bring the class together again to summarize the results of the activity. The orderly tabulation of results will allow children to recognize patterns and generate rules. Again, one should expect great differences among the children, but all can profit from a discussion of the generalizations which might surface from the group (Fitzgerald and Shroyer, 1979).

Simply put, the model is designed to present important, related mathematical concepts to children, using activity-oriented lessons. Children are provided with manipulative experiences and multiple embodiments.

A detailed instructional guide is provided to enhance easy implementation of the teaching model described. It was developed to provide specific suggestions for important questions to be asked at appropriate stages of the activities. Additional questions which involve generalizations and further challenges for high ability students are also included.

The Probability Unit

The probability instructional material used in this investigation was first developed during the 1981/82 school year by the staff of the Middle Grades Mathematics Project (MGMP), Department of Mathematics, Michigan State University, East Lansing, Michigan.

The MGMP is a curriculum development project jointly funded by the National Science Foundation-Development in Science Education (NSF-DISE) and Michigan State University (MSU).

Pilot Testing

Before the Probability Unit was implemented for the purpose of this study, it had been through several phases of pilot testing and modification. One of the later stages of pilot testing took place in Summer 1982 when the MGMP staff taught the unit to forty middle grade students. Eight affiliated middle school teachers participated in this summer teaching institute. The affiliated teachers observed the classes taught by the staff. With suggestions and criticisms from these teachers, the MGMP probability unit was modified. This modified version was retested as schools reopened in September 1982. At this time, it was taught in one school (which later did not participate in this study) by one of the teachers who had watched the summer demonstration. Minor changes were made in the unit in preparation for the present study.

The Probability Unit Activities

The Probability Unit includes ten sequentially developed activities requiring about three weeks of instructional time. The activities of the unit include the following: State lottery, three activities on fair and unfair games, surveys, area models, expected value, newspapers pay, Jonesville families and Pascal's Triangle.

The first five of these activities strictly involve determining probabilities of independent events. These are probabilistic conditions in which the outcome of one event does not depend on another. Simple examples of independent events are observed in the repeated tossing of a die or coins. The remaining five acitivites deal with compound events (for example the probability of a 60% free throw shooter in basketball hitting two in a row) and binomial probabilities.

The unit assumes that the students are being exposed to probability instructions for the first time. Hence in the first activity, the definition of probability as a fraction or ratio is given as follows:

The probability that an event A will happen is the number of times A occurs divided by the total number of possible events. That is,

$$P(A) = \frac{\text{number of A}}{\text{total number of outcomes}}$$

Activities two to four introduce the probabilistic thinking involved in deciding if a game is fair or unfair. Playing fair and unfair games in pairs, the students are introduced to experimental and theoretical probability. Simple tree diagrams are also used to explain theoretical probability. Students are introduced to various ways of conceptualizing probabilities rather than given an abstract definition. Through such experimental approaches as coin and die tossing, and spinner activities, students have experiences both with fractions and decimals, and with identifying a relationship between geometry and probability. For example through the use of spinners as an experimental tool (activity 4), area models (activity 6, 7) students are exposed to such concepts as circles and angles, rectangles and squares.

Activity 5 exposes the students to experimental probability through useful and practical survey activities.

Examples of surveys introduced are traffic patterns, weather predictions, political voting and rating.

Dividing geometrical shapes, especially squares and circles, into equal units of area, and calculating probabilities from these areas, are the focus of Activity 6.

In Activities 7 and 8, the use of probabilities to make predictions, and calculate expected values are introduced. Area models are also used to analyze compound situations.

In Activity 8, students are given an opportunity to plan a simulation of a problem, to carry out the simulation, to analyze the problem theoretically and to compare the results.

The last two activities, 9 and 10, deal with binomial probabilities. Students are introduced to the calculation of probabilities involving dichotomous situations in which two, and only two, possible responses exist at a time. Examples are yes or no, boy or girl, true or false, heads or tails events.

Activity 9 introduces these concepts through a "boy or girl" activity, entitled Jonesville families. Activity 10, with the introduction of the Pascal's Triangle, leads the students to understand and appreciate the theoretical basis of dichotomous probabilities.

Each activity is followed by practice questions. Also at the end of the probability unit are fourteen comprehensive review problems on all the activities in the entire package.

Appendix B contains the test used to evaluate student performance on the unit.

Population and Sample

The subjects of the study are grade six, seven and eight students from six different schools, situated in three distinct sites, two schools per site. One of these schools is an elementary school, three are middle schools and two are junior high schools. The three sites are categorized into urban, suburban and rural settings and are respectively referred to as site 1, site 2 and site 3 throughout this study.

Site 1 comprises two inner city schools. One of these is a junior high school situated in Pontiac, Michigan. The other is an elementary school in the inner city of Lansing, the state capital of Michigan. Their distance apart not withstanding (about 80 miles), these two schools are similar in socioeconomic, racial and demographic distributions. The Lansing district demographic data for 1980-81 shows 65 percent white, 23 percent Black, and 10 percent Latino. The site 1 Lansing inner city school demographic data for the same year shows 56 percent White, 17 percent Black, and 23 percent Latino. These distributions are presumed stable till the time of this study. Site 2, the suburban site, comprises one middle school and one junior high school. Though about fifteen miles apart, both are schools situated in metropolitan Lansing, Michigan, and are also similar in social, economic and racial characteristics. Children attending these schools come from upper-middle class populations. These site 2 schools are situated in a predominantly white domain. Site 3, the urban site, also comprises two schools, a middle school and a junior high school. These schools are situated in rural areas in the suburbs of Lansing and serve middle class, predominantly White communities. In each of these three sites, several sixth, seventh and eight grade classes participated in the study. However, not all sixth, seventh, or eighth grade classes in each site participated in the study.

The entire sample comprised about 1460 boys and girls. These students were from 66 classes taught by 30 different teachers. Some teachers taught more than one class, and a few taught classes in more than one grade level.

Tables 3.1 and 3.2 show descriptive information on the subjects. Table 3.1 shows the distribution of the entire sample by grade level (six through eight) and by sex in each of the three sites. Table 3.2 shows the distributions of each site by the number of students, number of classes, number of teachers and by the average number of students per class.

From the two tables presented, the information shows that a large number of subjects were involved in each site in the study. Any differences observed were therefore pressumably due to factors other than the size of the sample.

TABLE 3.1

DISTRIBUTION OF THE WHOLE SAMPLE BY GRADE AND BY SEX IN EACH SITE

	Site 1				Site 2				Site 3			
	N Classes	N Boys	N Girls	N Total	N Classes	N Boys	N Girls	N Total	N Classes	N Boys	N Girls	N Total
Grade 6	6	76	73	149	15	162	178	340	5	70	63	133
Grade 7	7	66	72	138	6	53	48	101	6	65	75	140
Grade 8	7	74	83	157	9	108	78	180	5	57	51	108
Total	20	216	228	444	30	317	304	621	16	192	189	381

TABLE 3.2

DISTRIBUTION OF THE WHOLE SAMPLE BY SUBJECT BY CLASS BY TEACHER BY SUBJECT AND CLASS

	N_S[a]	N_C[b]	N_T[c]	$N_{S/C}$[d]
Site 1	444	20	12	22
Grade 6	149	6	6	25
Grade 7	138	7	3	20
Grade 8	157	7	3	22
Site 2	621	30	11	21
Grade 6	340	15	6	23
Grade 7	101	6	2	17
Grade 8	180	9	3	20
Site 3	381	16	13	25
Grade 6	133	5	5	27
Grade 7	140	6	6	25
Grade 8	108	5	2	22
All	1446	66	36*	24
Grade 6	622	26	17	20
Grade 7	379	19	11	21
Grade 8	445	21	8	22

[a] N_S-Number of subjects
[b] N_C-Number of classes
[c] N_T-Numkber of Teachers
[d] $N_{S/C}$-Average number of subjects per class to the nearest whole N.
* Teachers who taught two grade levels were counted twice. A total of 30 distinct teachers.

Instrumentation

The instruments used in this study were two semantic differential attitude scales and a performance test on probability ability. The two attitude scales were called the Mathematics Attitude Scale (MAS) and the Probability Attitude Scale (PAS). Throughout the study these two attitude scales are referred to as MAS and PAS, while the performance test instrument is called the Middle Grades Mathematics Project Probability Test (MGMPPT).

Attitudes

The instrument to measure student attitudes toward mathematics, MAS, was developed by Shumway and White (1981). The same instrument was used in this study to measure the subjects' attitudes toward the probability activities treated in the instruction. The MAS was administered as a pretest while both the MAS and PAS were administered as posttest attitude measures. The Mathematics Attitudes Scale and the Probability Attitude Scales appear in Apendix C. Both scales are a six-item semantic differential with five options each. The scales were scored by assigning scores of 5 through 1 to the subject responses. 5 and 1 signified the most favorable and least favorable response respectively, while the in between responses were accordingly scored 4, 3 and 2. The average, which marked the subject's attitude, the subject's attitude, was then calculated on an attitude score of a maximum of 5 and a minimum of 1.

According to Shumway and White (1981), the internal consistency reliability estimates for the MAS ranged from 0.82 to 0.92. In this study, this MAS will be used with all the subjects and the Cronbach α reliability estimates will be computed by sex, by grade level and by site.

In a study in which these attitude scales were used, Ben-Haim (1982) confirmed the reliability of the instruments. Ben-Haim computed Cronbach α reliability estimates for the MAS and these estimates ranged from 0.79 to 0.91 in each site, by time, by grade and by sex. For the Pearson Correlation coefficients between the pretest and posttest scores on the MAS, Ben-Haim also found these to range from 0.64 to 0.82 which was significantly high ($P < 0.01$).

The stability of the Mathematics Attitude Scale instrument was supported by the high correlation between the pre- and post-administration of the instrument. In this study, the Pearson correlation coefficients between the math attitudes pretest and posttest (a time period of about three to four weeks) ranged mostly from 0.60 to 0.76. Tables C.1 to C.6 Appendix C contain the Pearson correlation coefficients between the pretest and posttest scores on the Math Attitude Scale, for the entire subsample per site by grade by sex. Most of the correlations were significant ($P < .01$).

The Cronbach α reliability coefficients for the Probability Attitude Scale in this investigation, ranged from 0.80 to 0.89. Table 3.3 contains the reliability coefficients of the Probability Attitudes Scale for the post testing by site by grade by sex.

Measuring Probability Knowledge

The subject performance in probability was measured by the Middle Grades Mathematics Project Probability Test (MGMPPT). This test was developed by the MGMP staff, including this investigator, in the Mathematics Department of Michigan State University. The test was pilot tested with hundreds of middle grades boys and girls in the Lansing area during development. Its final form contains 25 multiple choice items with 5 options each. Before administering the test, the teacher was instructed to discuss the two sample items on the sample sheet that preceded the test. This writer and members of the project staff supervised the teachers to ensure that each teacher carefully follow the instructions provided for the administration of the test. The same probability test was given for both the pretest and posttest measures, keeping all the testing conditions as much the same as possible. The test was not timed but the average time taken during the pilot trials was about twenty-five minutes. The time lapse between the pretest and posttest was three to four weeks. This varied slightly from school to school. The items were

TABLE 3.3

RELIABILITY COEFFICIENTS-CRONBACH α FOR MGMP PT, MAS AND PAS BY SITE BY TIME BY GRADE BY SEX

	N	PT[a] PRETEST	PT[a] POSTTEST	MAS[b] PRETEST	MAS[b] POSTTEST	PAS[c]
Site 1						
Grade 6	149	.68	.67	.84	.83	.80
Grade 7	139	.61	.81	.83	.82	.82
Grade 8	157	.75	.82	.77	.84	.84
Total	435	.70	.79	.81	.83	.82
Boys	217	.73	.80	.80	.80	.82
Girls	228	.66	.77	.83	.85	.83
Site 2						
Grade 6	340	.58	.74	.85	.86	.86
Grade 7	101	.73	.83	.88	.88	.89
Grade 8	180	.67	.73	.84	.85	.86
Total	621	.69	.78	.88	.88	.87
Boys	317	.74	.81	.86	.89	.88
Girls	306	.59	.73	.89	.87	.87
Site 3						
Grade 6	133	.45	.58	.77	.83	.84
Grade 7	150	.59	.79	.84	.86	.86
Grade 8	109	.58	.73	.82	.83	.82
Total	392	.56	.75	.82	.86	.85
Boys	203	.54	.77	.82	.86	.86
Girls	189	.56	.72	.82	.86	.84
All						
Grade 6	622	.61	.71	.83	.85	.85
Grade 7	380	.70	.83	.84	.85	.87
Grade 8	444	.70	.77	.83	.86	.88
Total	1446	.68	.79	.84	.86	.87
Boys	731	.72	.80	.84	.86	.86
Girls	717	.75	.62	.85	.86	.87

[a] PT - Probability Test (Range 0-25).
[b] MAS - Mathematics Attitudes Scale (Range 1-5).
[c] PAS - Probability Attitudes Scale (Range 1-5).

scored by assigning 1 point for each correct item. Thus it was possible for each subject to score a total between 0 and 25 (inclusive). Appendix B contains the 25 items that constitute the MGMP PT.

The MGMP Probability Test served two purposes. First, it served as a measure of probability knowledge in order to assess existing differences by grade level and by sex prior to the intervention. Second, it served as a pretest-posttest on probability in order to evaluate the effects of instruction in activities involving probability tasks.

The Cronbach α reliability coefficients calculated for the MGMP PT ranged largely between 0.65 and 0.83. With the exception of the value of 0.58 for site 3 grade 6 pretest, each of the 18 Cronbach α reliability coefficients calculated for the MGMP PT posttest was well above 0.60. Table 3.3 includes the reliability coefficients for the MGMP PT for each site, by time by grade and by sex.

Test validity and reliability were based upon scholarly analyses of test items by researchers, mathematics educators and mathematicians. Another indicator of the quality of the instrument was the significantly high correlations between the pre-post test scores. In this study, the Pearson correlation coefficients ranged from 0.53 to 0.76. Tables C.1 to C.6 in Appendix C contain the Pearson correlation coefficients on the MGMP PT scores for each site, by grade and by sex. Almost all the correlations were significant at $P < .01$.

Procedure and Data Collection

The study was conducted during Fall 1982 and Winter 1983 and the duration of the data collection was between October 1982 and January 1983. Each teacher gave the Mathematics Attitude Scale (MAS) and MGMP Probability Test on the day prior to the beginning of the probability intervention.[1]

The administration of the tests was restricted to the regular mathematics hour. The unit had not been taught previously by any of the teachers involved with the unit. The probability instruction continued through every mathematics lesson until the completion and administration of the two posttests and the Probability Attitude Scale (PAS).

Data Collection

The following data were collected:

1. General Information Such information on the subjects and the schools participating in the study as the size, type, and setting.

---

[1] Due to factors beyond the control of the researcher, each teacher started the instruction from any Monday of his or her convenience and completed it in three to four weeks.

2. Pretesting Pretest scores on the Mathematics Attitude Scale and MGMP Probability Test from the whole sample in all three sites.
3. Observations During the instruction in the probability unit which required about three weeks of instructional time, the investigator made some observations in many of the participating classrooms. The purpose of these observations was to supervise as well as to document the progress of the activities.
4. Posttesting At the end of the instructional intervention all subjects were given the Mathematics Attitude Scale, the Probability Attitude Scale (PAS) and the MGMP Probability Test.

The Design of the Study

Several statistical procedures were selected to analyze the data collected during the study. Particular statistics were chosen to test the hypotheses given in the following paragraphs. The analyses included the following: means, standard deviations, correlations, multivariate and univariate analyses of variance and repeated measures. Planned comparisons and Scheffe's Post Hoc comparison were also selected. All these analyses were carried out on the 3600 Computer at the Michigan State University Computer Center, using the Statistical Package for the Social Sciences (SPSS).

The multivariate model with a two-way fixed effects analysis of variance (MANOVA) was used to assess the differences in probability knowledge and attitudes toward mathematics by grade level and sex prior to the instruction.

As described earlier in this chapter, the populations of the three sites (1, 2, and 3) were different in their characteristics. Hence the data from each site was analyzed separately. For each site, the design was 3 x 2 completely crossed with two criterion measures and with unequal numbers of subjects per cell. The multivariate design for each site appears in Table 3.4. The table shows that the data matrix is completely crossed since in each site there were three grade levels (6, 7 and 8) with boys and girls in each grade level. The grade level and sex main effects constituted the independent variables, while the MGMP probability test and attitude scores constituted the dependent variables. A total of five measures were taken on each subject. Two of these were administered prior to instruction. These two were the pretest in probability and pretest in mathematics attitude. The other three were given immediately after the instruction. These were the Posttest in Probability (POSTPTOT), Posttest Mathematics Attitude Scale (POSTMAS) and the Probability Attitude Scale (PAS).

TABLE 3.4

THE 3 X 2 MULTIVARIATE CROSSED DESIGN DATA MATRIX

Indepedent Variables		Dependent Variables	
Grade	Sex	MGMP PT[a] Scores	MAS[b] Scores
6	Boys	X_1	X_2
	Girls	X_1	X_2
7	Boys	X_1	X_2
	Girls	X_1	X_2
8	boys	X_1	X_2
	Girls	X_1	X_2

[a] MGMP PT - Middle Grades Mathematics Project Probability Test.
[b] MAS - Mathematics Attitude Scale.

The hypotheses tested within this design, given in null form, were:

H_{01}: There will be no difference among the mean scores for each of the three grade levels (six, seven, and eight) tested, on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitudes Scale.

H_{02}: There will be no difference between the mean scores for boys and for girls in grades six through eight on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

H_{03}: There will be no interaction of grade by sex among the mean scores for sixth through eighth grade students on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

For the tests of significance, the assumptions of independence and normality were met. In other words, the characteristics of the measures of all the dependent variables in the study included the fact that subjects responded independently of each other, and that any measurement errors within each group were normally distributed with mean zero and general population variance. Another assumption met was that measures of the dependent variable must not be linear functions of one another for each subject. For example, in the measure of each pair of dependent variables in this study, not every subject had the same subset scores or total score for both dependent variables. Also, since the model had only six (3 x 2) subclass means, and since assumptions of independence of these subclass means was met, it was permissible to test or estimate the significance of these six means. However, because there were only six degrees of freedom among means, not more than six independent significance estimations was allowed.

To analyze the effects of instruction on the probability skills, differences in probability and attitudes toward mathematics, by sex and grade level, the Multivariate Analysis of Repeated Measures Model was selected.

The design for the subsample from each site was a Two-way six-group design, with four measures per subject. The design appears in Table 3.5.

The hypotheses tested within this design, given in null form were:

H_{04}: There will be no difference between the posttest means and the pretest means of sixth, seventh, and eighth grade students on both the Middle Grade Mathematics Project Probability Test and on the Mathematics Attitudes Scale.

H_{05}: There will be no difference between the mean gain scores (posttest minus pretest) for each of the three grade levels tested, six, seven, and eight on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

H_{06}: There will be no difference between the mean gain scores (posttest minus pretest) for boys and for girls in grades six, seven, and eight on the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale

To analyze the effects of instruction on probability, the Multivariate Analysis of Repeated Measures was used. Both the Finn and the SPSS methods were available for the repeated measures analysis but the latter was used to keep uniformity with the other statistical analyses tested in the study.

TABLE 3.5

THE MULTIVARIATE ANALYSIS OF REPEATED MEASURES DESIGN FOR THE SUBSAMPLE FOR EACH SITE

Grade	Sex	MGMPPT Scores[a]		MAS Scores[b]	
		Pretest	Posttest	Pretest	Posttest
6	Boys	X_1	X_2	Y_1	Y_2
	Girls	X_1	X_2	Y_1	Y_2
7	Boys	X_1	X_2	Y_1	Y_2
	Girls	X_1	X_2	Y_1	Y_2
8	Boys	X_1	X_2	Y_1	Y_2
	Girls	X_1	X_2	Y_1	Y_2

[a] MGMP PT - Middle Grades Mathematics Project Probability Test.

[b] MAS - Mathematics Attitude Scale.

To compare the attitudes toward mathematics and probability by sex and by grade after instruction, a Two-Way ANOVA design was used. The hypotheses tested within this design for each site were:

H_{07}: There will be no difference between students' mean scores on the Mathematics Attitude Scale (MAS) and students' mean scores on the Probability Attitude Scale (PAS).

H_{08}: There will be no interaction of grade by sex among the mean difference scores--MAS score minus PAS score.

A Two-Way ANOVA design was also applied in order to examine sex and grade level differences in attitudes toward probability. The hypotheses tested within this design in each site were:

H_{09}: There will be no significant difference between the mean scores for boys and for girls in grades six, seven, and eight on the Probability Attitude Scale.

H_{10}: There will be no difference between the mean scores for each of the three grade levels (six, seven and eight) on the Probability Attitude Scale.

Planned comparisons for the grade effect and Scheffe's Post Hoc comparisons were used to identify the sources of significant main effects or interactions. The .05 level of significance was the limit accepted in testing all the hypotheses in the study.

Summary

This chapter described in detail the entire research methodology of this study. Specifically, the procedures followed in the investigation of probability knowledge and attitudes toward mathematics per site by sex and by grade level were presented.

This chapter also described in detail the population and sample of the study including the distribution by Site (1, 2, and 3), by grade level (6, 7, and 8). The unit of analysis chosen for the study was the student. From Site 1, the urban setting, 444 subjects took part. From the suburban setting, Site 2, 621 subjects participated while 381 subjects took part in the study from Site 3, the rural setting. Of the 1460 subjects involved in the study, 1446 took part consistently in the data analyses. The difference of 14, which was highly insignificant, was due to random omission. There were 622 sixth graders, 379 seventh graders, 445 eighth graders, 725 boys and 721 girls who took part in the entire study.

Also described in this chapter was the Middle Grades Mathematics Project (MGMP) Probability Unit which constituted the instructional intervention of the study. The sequence of activities and the instructional model were described. The instrumentation included two semantic differential scales (MAS and PAS) for measuring attitudes toward mathematics and toward probability. It also included the MGMP probability test for measuring the probability performance of the subjects. Reported reliability coefficients for the two attitude scales ranged from .80 to .89, while those of the probability test ranged from .54 to .83 except for the .45 for site 3 grade 6. The Pearson correlation coefficients between successive administration of the instruments were reported to be mostly significant at $P < .01$.

The statistical design, hypotheses and methods of analyses were presented. These included means, standard deviations, correlations, univariate and multivariate analysis of variance, and repeated measures, planned comparisons, and Sheffe's Post Hoc comparisons for significance. The significance level chosen was at least .05 for all hypotheses.

Results obtained from the different analyses and their interpretation are presented and discussed in the next chapter.

CHAPTER IV

PRESENTATION AND ANALYSIS OF DATA

This chapter presents a summary of the data collected during this investigation, the analysis of data, and findings based on this analysis. It consists of seven sections:

1. Sex and Grade level differences in probability performance and in attitudes toward mathematics
2. Comparison of results from the three sites.
3. The effects of instruction.
4. Comparison of effects of instruction among the three sites.
5. Comparison of attitudes toward mathematics with attitudes toward probability.
6. Sex and grade level differences in attitudes toward probability.
7. The chapter concludes with a summary of results.

Sex and Grade Level Differences in Probability Knowledge and in Attitudes Toward Mathematics

For the data for Site 1, Site 2 and Site 3, the Multivariate model with a Two-Way fixed effect analysis of variance (MANOVA) was used to determine differences in probability knowledge and in attitudes toward mathematics by grade level and by sex prior to the intervention. As explained earlier (Chapter 3), the data from each site was analyzed separately since the populations from the three sites were different in their characteristics.

The analysis for each site includes the hypotheses to be tested, means and standard deviations, profiles by grade and by sex for each measure, a summary table of multivariate and univariate analysis of variance, results of post hoc comparisons, and the results of the significance tests for each null hypothesis. The level of significance for each test was ($P < .05$) and the design was 3 x 2 crossed with two criterion measures.

Site 1: The Urban Site

The multivariate null hypotheses tested within this design were:

H_{01}: There will be no difference between the mean scores of the three grade levels (6, 7 and 8) tested, on both the MGMP Probability Test and on the Mathematics Attitude Scale.

H_{02}: There will be no difference between the mean scores for boys and for girls in grades six, seven and eight on both the MGMP Probability Test and on the Mathematics Attitude Scale.

H_{03}: There will be no interaction of grade by sex between the mean scores for sixth, seventh and eighth grade students, on both the MGMP Probability Test and on the Mathematics Attitude Scale.

Table 4.1 provides means and standard deviations for the Middle Grades Mathematics Project Probability Test (MGMPPT) and the Mathematics Attitude Scale (MAS) scores for

TABLE 4.1

MEANS AND STANDARD DEVIATIONS OF MGMP PT AND MAS PRETEST SCORES FOR SITE 1 BY GRADE AND BY SEX

		MGMP PT[a]		MAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	149	6.63	3.44	3.607	.947
Boys	76	6.97	4.04	3.568	.964
Girls	73	6.27	2.65	3.648	.934
Grade 7	138	7.62	3.30	3.457	.920
Boys	66	7.45	3.25	3.548	.871
Girls	72	7.78	3.35	3.373	.962
Grade 8	157	7.92	4.09	3.462	.818
Boys	74	8.16	4.20	3.532	.807
Girls	83	7.70	3.99	3.400	.828
Total	444	7.39	3.67	3.509	.896
Boys	216	7.53	3.89	3.549	.880
Girls	228	7.27	3.46	3.471	.911

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).

[b] MAS - Mathematics Attitude Scale (Range 1-5).

site 1 by grade and by sex. The mean scores from this table were used in the profiles shown in Figures 4.1 and 4.2 for grade levels by sex for both the MGMP PT and MAS measures. The same means in Table 4.1 were also used in Figures 4.1 and 4.2 to display the sex by grade level profiles for the same two measures.

A summary of the Multivariate and Univariate analysis of variance for the 3 x 2 crossed design for this site is presented in Table 4.2. The multivariate test indicated that the grade by sex interaction and sex main effects were not significant but were significant for the grade level main effect ($P < .05$). The univariate tests showed the significance to be confined to the pretest probability test (PREPTOT) scores, and not to the pretest attitude scale (PREMAS) scores. This means that in Site 1, there was no significant difference among grades 6, 7 and 8 in attitudes toward mathematics prior to instruction.

In order to determine in which of the three grade levels significant differences occured in the MGMP Probability test scores prior to instruction, Scheffe's Post Hoc tests were used. As can be observed in Table 4.2 and confirmed statistically with the Post Hoc tests, the significant grade level differences in the PREPTOT was confined to between sixth and seventh graders and not between seventh and eighth graders. Both the profiles of grade levels in Figure 4.1 and the mean PREPTOT scores in Table 4.1 indicate a difference on the side of seventh

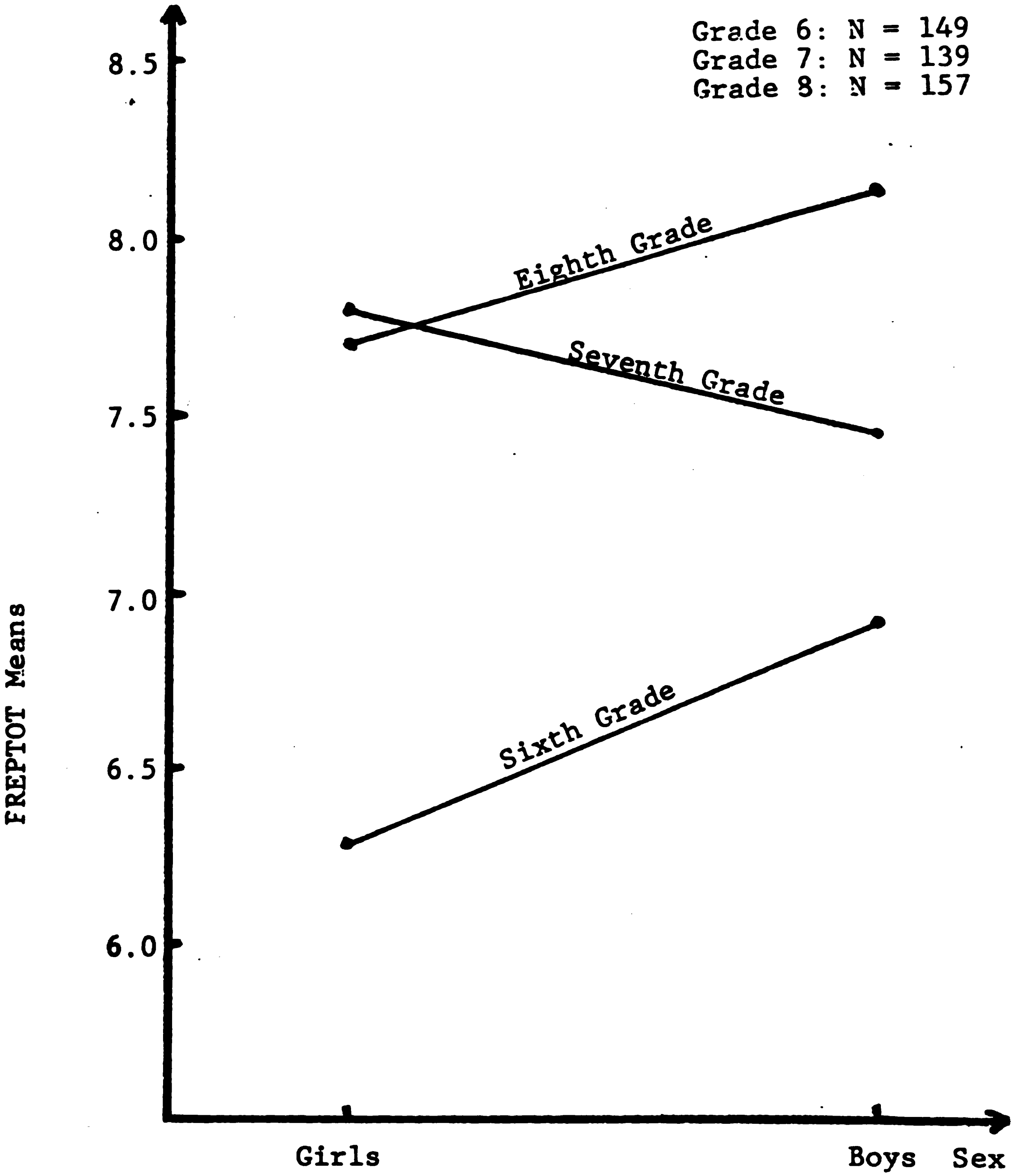


Fig. 4.1 PREPTOT Means--Profiles of Sex by Grade Level at Site 1.

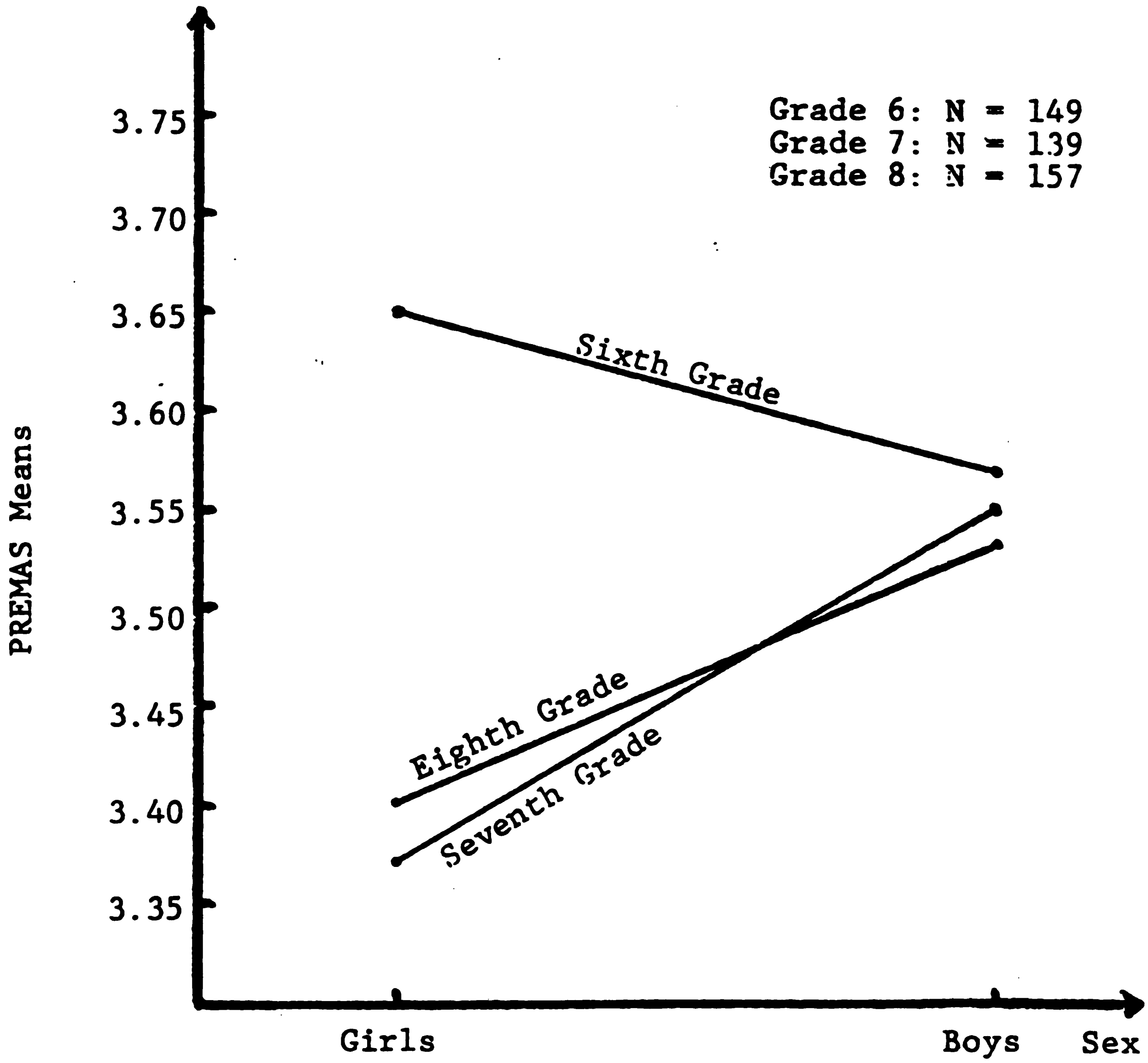


Fig. 4.2 PREMAS Means--Profiles of Sex by Grade Levels at Site 1.

TABLE 4.2

A SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE FOR THE 3 x 2 DESIGN FOR SITE 1

Source of		Multivariate[a]			Univariate		
Variation	D.F.	F	P<		MS	F	P<
Constant (C)	1	3907.11	.0001	PREPTOT[b]	24274.98	1829.14	.0001
				PREMAS[c]	5467.04	6816.06	.0001
Grade (G)	2						
G_1[d]	(1)	4.35	.013	PREPTOT	73.67	5.55	.019
				PREMAS	1.67	2.09	.149
G_2[e]	(1)	2.98	.052	PREPTOT	63.33	4.77	.029
				PREMAS	.499	.623	.430
Sex	1	.659	.518	PREPTOT	9.860	.743	.389
				PREMAS	.6070	.757	.385
G by Sex	2	.910	.456	PREPTOT	10.18	.767	.465
				PREMAS	24.71	.856	.426
Between Groups	6			PREPTOT (MSe) =	13.271		
Within Group	438			PREMAS (MSe) =	.802		
Total	444						

[a] Multivariate D.F. = 2 and 437, except for the Grade by Sex interactions for which D.F. = 4 and 874.
[b] PREPTOT - Pretest Probability Test.
[c] PREMAS - Pretest Mathematics Attitude Scale.
[d] G_1 - Contrast of Grade 6 vs. Grade 7.
[e] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.

graders. Appendix E (Table E.1) includes the summary table of Scheffe's posteriori comparisons of the probability pretest means for boys and for girls in grade levels six and seven. These comparisons were found to be statistically different in favour of grade level seven girls over grade level six girls at .05 level. However, an examination of the two means showed a difference of only 1.50. Hence this statistical difference was detected due to the high precision level of the test. In other words, because of the large sample size used, the test was powerful enough to detect small differences which were not necessarily meaningful. Thus, the statistical difference identified notwithstanding, the grade seven girls were not necessarily superior to grade six girls in MGMP probability knowledge prior to instruction.

To summarize, based on the statistical analysis of the data from Site 1, the following decisions were made with respect to the hypotheses H_{01}-H_{03} (starting with the interaction):

1. The multivariate null hypothesis (H_{03}) of no interaction of grade by sex was retained.
2. The multivariate null hypothesis (H_{02}) of no difference between boys and girls in grades six through eight was also retained.
3. The multivariate null hypothesis (H_{01}) of no difference between grade levels six through eight was rejected.

The ensuing post hoc contrast between grade levels eight versus six and seven was not significant. However, the contrast between grade level six versus seven was significant ($P < .05$). The univariate analyses for these two grades for the MGMP PT and MAS scores showed that the statistical significance was limited to the MGMP PT scores only. Lastly, the Scheffe's Post Hoc comparison of the probability test scores between grade levels six and seven was statistically but not meaningfully significant between grade six girls versus grade seven girls.

Site 2: The Suburban Site

The same three multivariate null hypotheses (H_{01}-H_{03}) were tested in the analysis of the data from this site.

Table 4.3 presents the means and standard deviations of both the MGMP PT and the MAS criterion measures by grade and by sex. These means were used in Figures 4.3 to 4.5 to illustrate various profiles for both criterion measures by grade level and by sex.

The summary of the Multivariate and Univariate analysis of variance for the 3 x 2 design of Site 2 is presented in Table 4.4 No interaction was found of grade by sex but the multivariate analysis of sex and grade main effects were significant. The test for sex main effects was significant ($P < .001$) and the corresponding univariate test showed only the probability test scores to be significant ($P < .001$).

TABLE 4.3

MEANS AND STANDARD DEVIATIONS OF MGMP PT AND MAS PRETEST SCORES FOR SITE 2 BY GRADE AND BY SEX

		MGMP PT[a]		MAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	340	7.80	3.11	3.510	.908
Boys	162	8.14	3.41	3.422	.930
Girls	178	7.50	2.78	3.590	.883
Grade 7	101	11.71	3.86	3.406	.856
Boys	53	12.68	4.09	3.582	.947
Girls	48	10.65	3.31	3.212	.702
Grade 8	180	9.66	3.65	2.622	.866
Boys	102	10.13	4.02	2.613	.850
Girls	78	9.05	3.03	2.635	.892
Total	621	8.98	3.70	3.236	.970
Boys	317	9.54	4.08	3.188	.990
Girls	304	8.40	3.15	3.285	.948

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).

[b] MAS - Mathematics Attitude Scale (1-5).

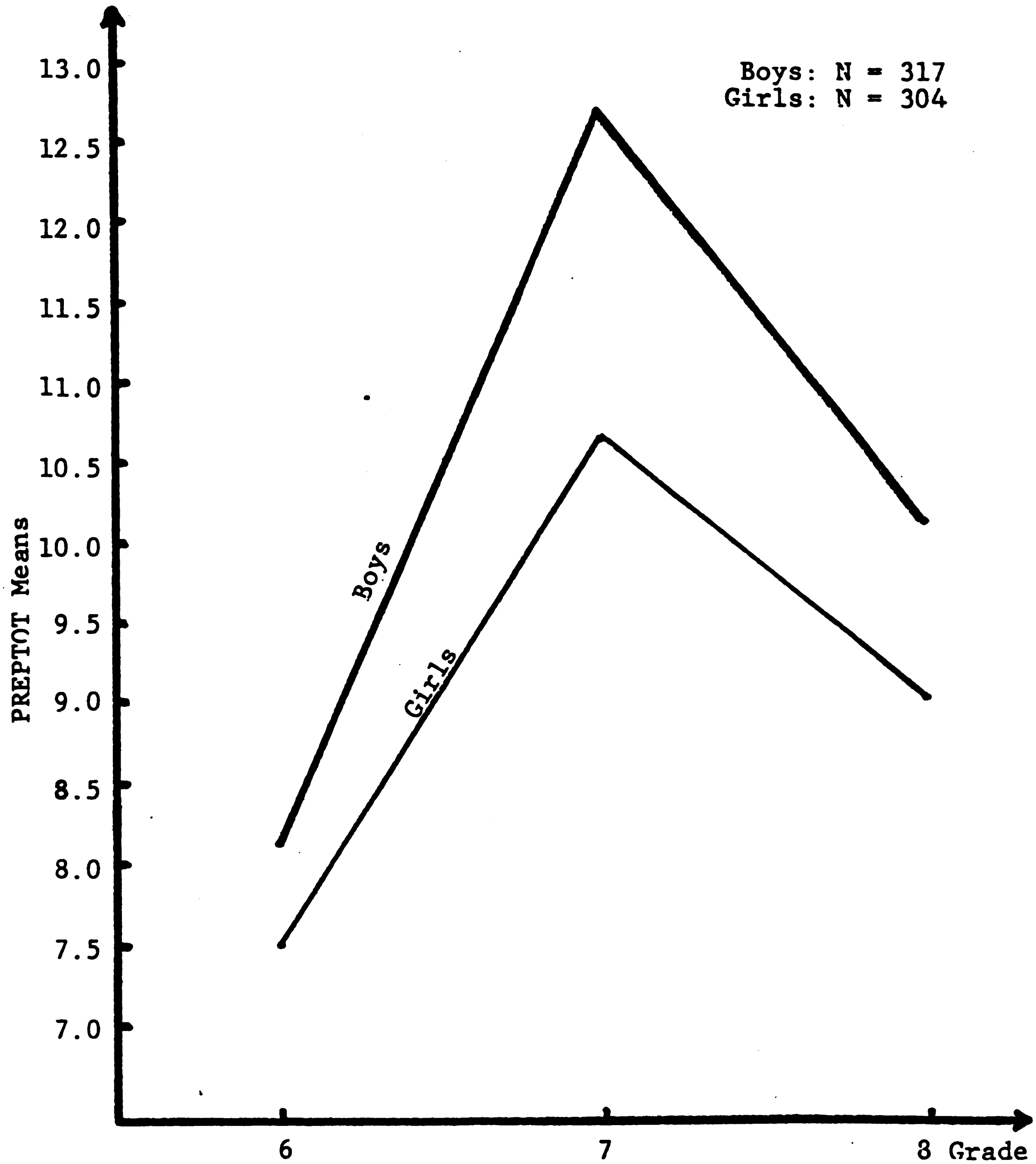


Fig. 4.3 PREPTOT Means--Profiles of Grade Levels by Sex at Site 2.

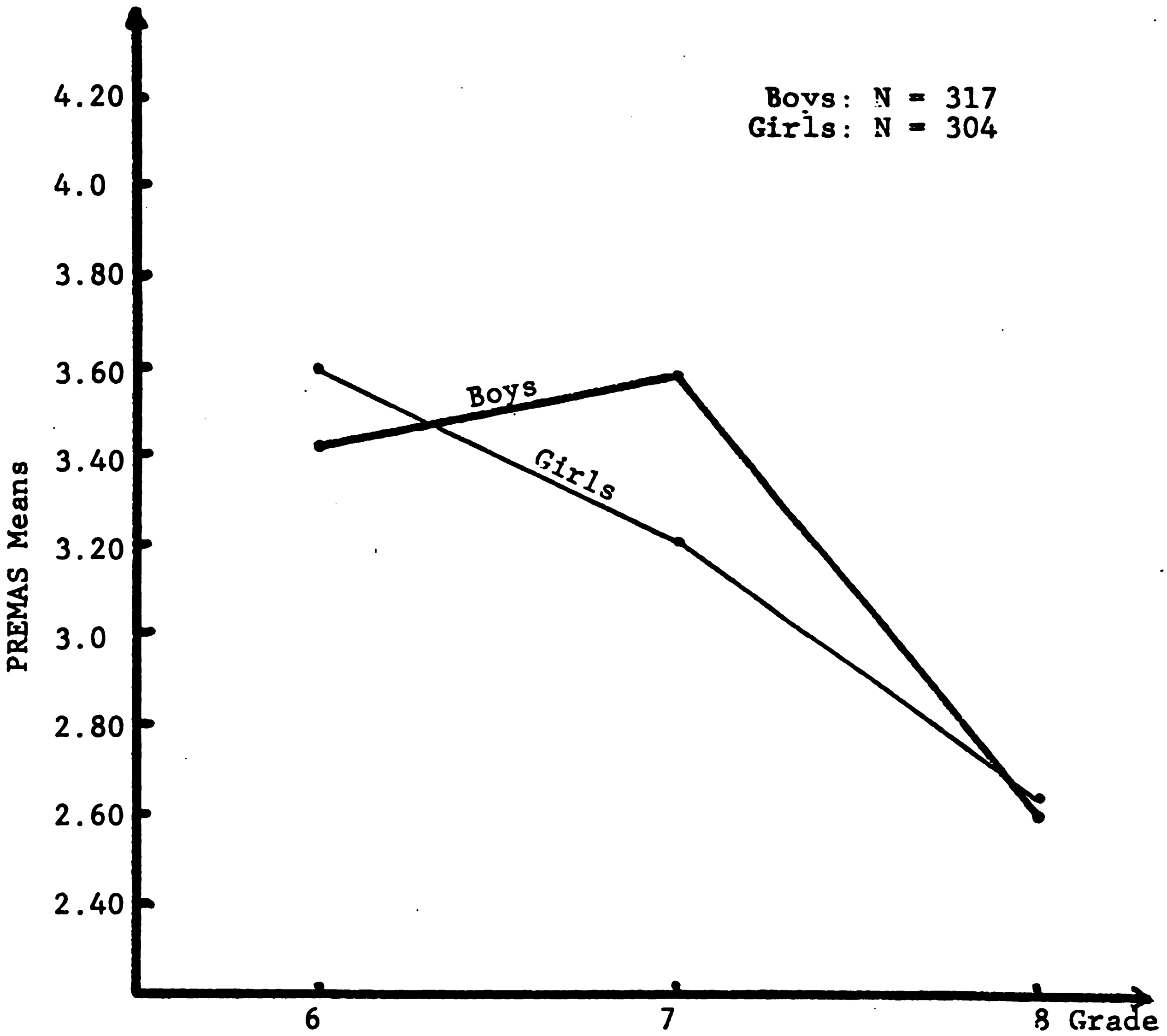


Fig. 4.4 PREMAS Means--Profiles of Grade Levels by Sex at Site 2.

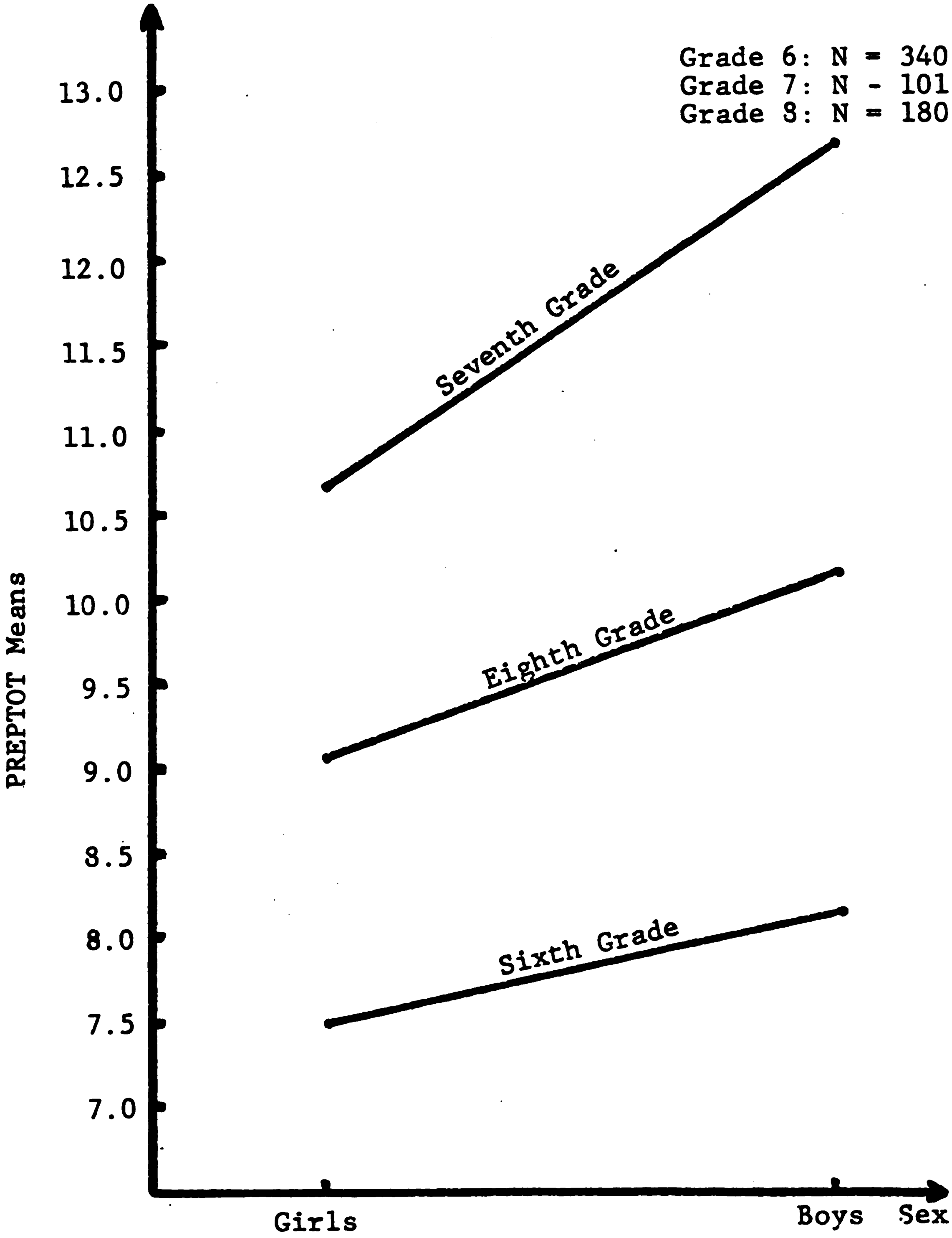


Fig. 4.5 PREPTOT Means--Profiles of Grade Levels by Sex at Site 2.

TABLE 4.4

SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE FOR THE 3 x 2 DESIGN FOR SITE 2

Source of Variation	D.F.	Multivariate[a] F	Multivariate[a] P<		Univariate MS	Univariate F	Univariate P<
Constant (C)	1	5579.94	.0001	PREPTOT[b]	50049.32	4419.17	.0001
				PREMAS[c]	6501.48	8313.79	.0001
Grade (G)[d]	2						
G_1[e]	(1)	74.29	.0001	PREPTOT	1254.90	110.80	.0001
				PREMAS	16.06	10.54	.0001
G_2[f]	(1)	50.87	.0001	PREPTOT	2.99	.26	.607
				PREMAS	78.92	100.92	.0001
Sex	1	7.26	.001	PREPTOT	151.23	13.35	.0003
				PREMAS	.23	.29	.591
G by Sex	2	2.322	.055	PREPTOT	19.20	1.69	.184
				PREMAS	2.82	3.61	.028
Between Groups	6			PREPTOT (MSe)	= 11.79		
Within Group	615			PREMAS (MSe)	= .782		
Total	621						

[a] Multivariate D.F. = 2 and 614, except for the Grade by Sex interactions for which D.F. = 4 and 1228.
[b] PREPTOT - Pretest Probability Test.
[c] PREMAS - Pretest Mathematics Attitude Scale.
[d] A reordering of the main effects testing the Grade (G) main effect before the sex main effect resulted in multivariate F values of 77.26 for G_1, and 51.58 for G_2 and exactly the same P values as before.
[e] G_1 - Contrast of Grade 7 vs. Grade 6.
[f] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.

Scheffe's posteriori sex comparisons, presented in Appendix E (Table E.2) showed no significant difference between boys and girls in grade six. However, these comparisons showed seventh and eighth grade boys to be significantly higher ($P < .05$) than the seventh and eighth grade girls. The profiles of grade levels by sex in Fig. 4.3 also verify this. No statistically significant sex differences were observed in MAS scores.

For the grade main effects, both planned comparisons, grades seven versus grade six and grade eight versus grades six and seven, were significant ($P < .0001$ and $P < .001$ respectively). The corresponding univariate tests showed grade seven to be significantly higher than grade six in the probability test ($P < .0001$) but no such difference was observed when grade eight was compared to grades six and seven. However, both Fig. 4.5 and the Table E.2A in Appendix E, showing the summary table of the Scheffe's posteriori comparisons of grade levels for boys and for girls in the MGMP PT, show the mean of grade seven to be significantly higher than that of grade eight. The Post Hoc tests, and the profiles of grade levels by sex in Fig. 4.3, show the superiority of grade seven boys and girls over their corresponding sex in grades six and eight in the MGMP PT. The Post Hoc posteriori comparison of grade levels for boys and for girls on the MAS is presented in Table E.2B of Appendix E. Mathematics attitude differences were not observed between grade levels six and seven but eighth graders significantly ($P < .05$) demonstrated lower mathematics attitude on the MAS than either the sixth or seventh graders.

To summarize, based on the statistical analysis of the data from Site 2, the following decisions were made with respect to hypothesis H_{03}, H_{02} and H_{01} in that order:

1. The multivariate null hypothesis (H_{03}) of no interaction of grade by sex was retained.
2. The multivariate null hypothesis (H_{02}), of no sex differences in each of grades six, seven and eight, was rejected ($P < .001$). The corresponding univariate test for the probability test mean scores was also rejected ($P < .001$) while that of the MAS mean scores was retained.
3. The multivariate null hypothesis (H_{01}), of no difference among mean scores for each of the three grade levels six through eight, was rejected ($P < .001$). The two planned comparisons, between grade level seven versus six, and grade levels eight versus six and seven, were both significant ($P < .0001$). The corresponding univariate null hypotheses for the MGMP PT mean scores was rejected for the contrast between grade seven and six ($P < .0001$), but retained for that between grade eight versus grades six and seven. The univariate null hypotheses for the MAS mean scores was also rejected ($P < .0001$).

Site 3: The Rural Site

Again, the three multivariate null hypotheses (H_{01}-H_{03}) were tested in the analysis of the data from this site.

The means and standard deviations of both the MGMP PT and the MAS criterion measures are presented in Table 4.5 by grade and by sex. These means were used in the profiles of grade level and of sex in Figures 4.6 and 4.7.

A summary of the Multivariate and Univariate analysis of variance for the 3 x 2 design of site 3 is presented in Table 4.6. The multivariate analysis showed no interaction of grade by sex, but showed the sex main effects to be significant ($P < .01$), and the two planned comparisons for the grade main effects to be significant; ($P < .0001$) for G_2, the contrast of grade eight versus grades six and seven, and ($P < .01$) for G_1, the contrast of grade seven versus grade six.

The univariate test for the sex main effects was significant ($P < .01$) for the MGMP PT mean scores but not significant for the MAS mean scores. Employing Scheffe's Post Hoc posteriori comparisons to determine in which grade levels these sex differences existed, the differences were observed to be significant ($P < .05$) only at grade level six, in favour of boys. Table E.3 in Appendix E includes a summary of these comparisons.

The univariate tests for the contrast G_2 (grade 8 versus grades 6 and 7) were significant for both the MGMP PT

TABLE 4.5

MEANS AND STANDARD DEVIATIONS OF MGMP PT AND MAS PRETEST SCORES FOR SITE 3 BY GRADE AND BY SEX

		MGMP PT[a]		MAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	133	8.64	2.69	3.450	.752
Boys	70	9.26	2.83	3.419	.756
Girls	63	7.95	2.36	3.484	.752
Grade 7	140	9.13	3.20	3.207	.819
Boys	65	9.40	3.12	3.177	.706
Girls	75	8.89	3.27	3.233	.910
Grade 8	108	9.99	3.16	3.005	.804
Boys	57	10.27	3.16	2.842	.902
Girls	51	9.69	3.15	2.186	.640
Total	381	9.20	3.03	3.236	.810
Boys	192	9.60	3.04	3.166	.817
Girls	189	8.79	3.03	3.304	.797

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).
[b] MAS - Mathematics Attitude Scale (Range 1-5).

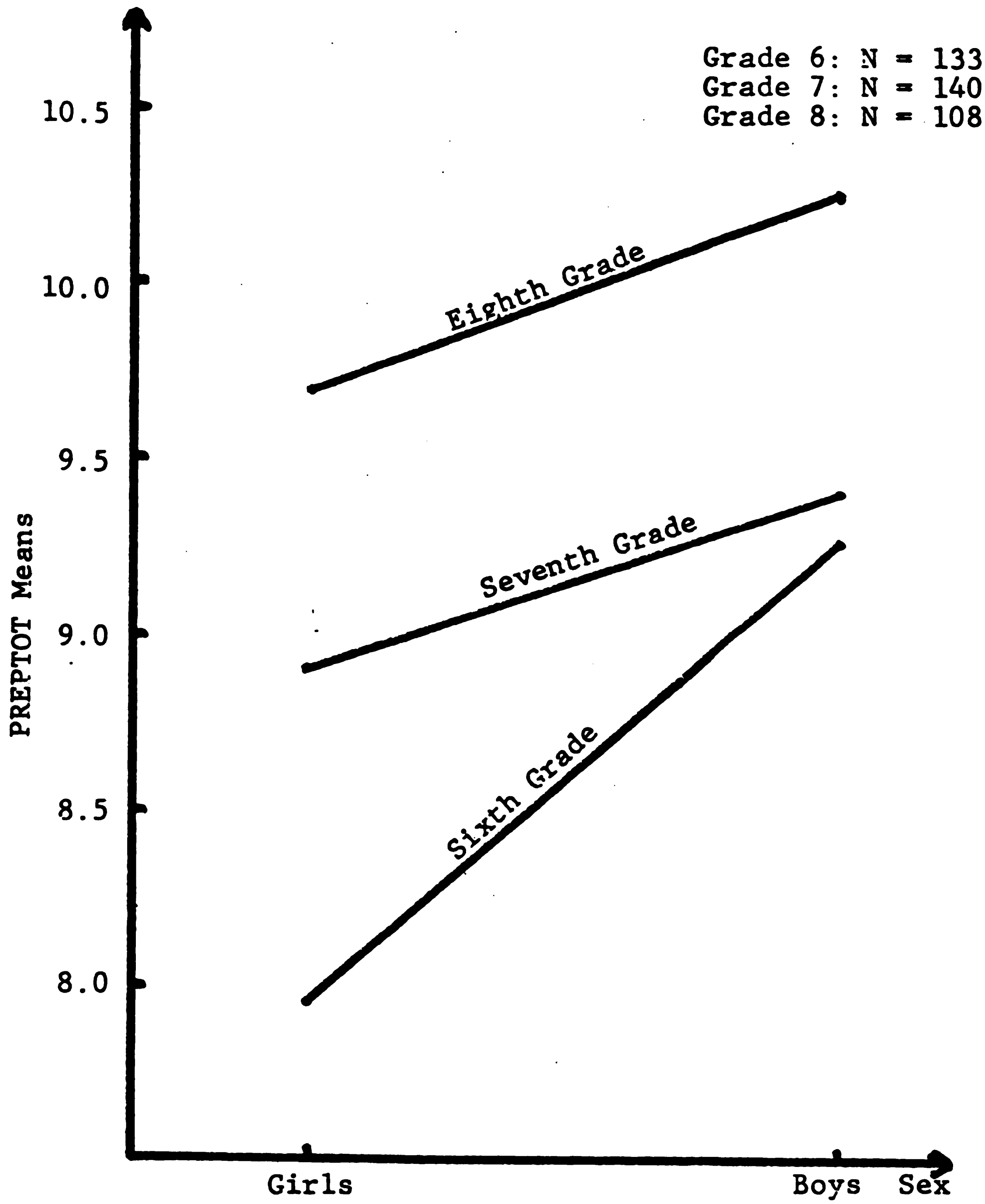


Fig. 4.6 PREPTOT Means--Profiles of Sex by Grade Level at Site 3.

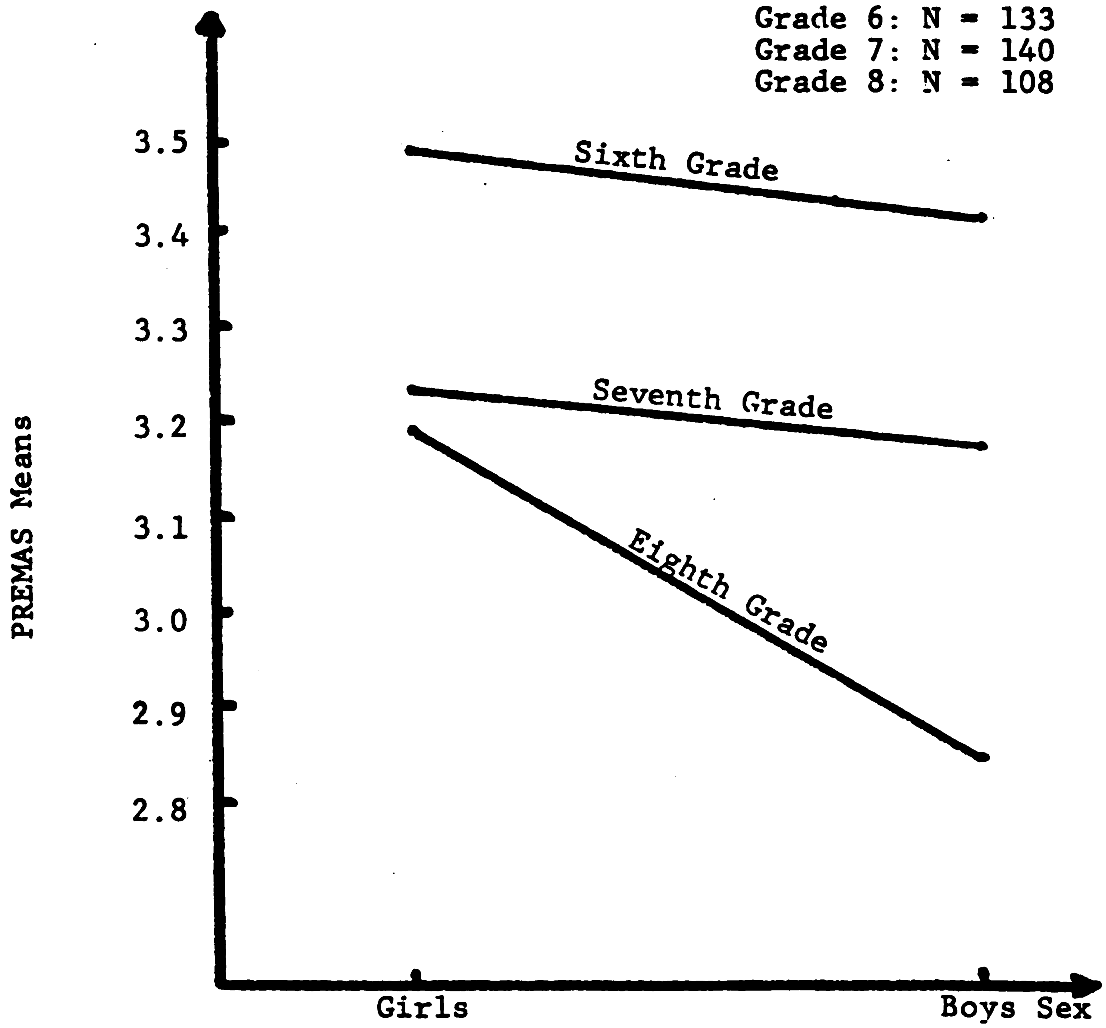


Fig 4.7 PREMAS Means--Profiles of Sex by Grade Level at Site 3.

TABLE 4.6

SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF VARIANCE FOR THE 3 x 2 DESIGN FOR SITE 3

Source of Variation	D.F.	Multivariate[a] F	Multivariate[a] P<		Univariate MS	Univariate F	Univariate P<
Constant (C)	1	4170.12	.0001	PREPTOT[b]	32262.56	3591.10	.0001
				PREMAS[c]	3985.95	6394.71	.0001
Grade (G)[d]	2						
G_1[e]	(1)	5.37	.005	PREPTOT	18.79	2.09	.149
				PREMAS	4.16	6.68	.010
G_2[f]	(1)	14.25	.0001	PREPTOT	90.29	2.09	.002
				PREMAS	7.88	12.64	.0004
Sex	1	6.13	.002	PREPTOT	61.56	6.85	.009
				PREMAS	1.89	3.03	.083
G by Sex	2	.95	.438	PREPTOT	6.39	.71	.492
				PREMAS	.776	1.24	.289
Between Groups	6			PREPTOT (MSe) =	8.98		
Within Group	375			PREMAS (MSe) =	.62		
Total	381						

[a] Multivariate D.F. = 2 and 374, except for the Grade by Sex interactions for which D.F. = 4 and 748.

[b] PREPTOT - Pretest Probability Test.

[c] PREMAS - Pretest Mathematics Attitude Scale.

[d] A reordering of the main effects testing the Grade (G) main effect before the sex main effect resulted in multivariate F values of 5.04 for G_1, and 14.57 for G_2 and exactly the same P values as before.

[e] G_1 - Contrast of Grade 7 vs. Grade 6.

[f] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.

mean scores and the MAS mean scores ($P < .01$ and $P < .001$ respectively). For the contrast G_1 (grade 7 versus grade 6), these tests were not significant for the MGMP PT mean scores but were significant ($P < .01$) for the MAS mean scores as observed from Table 4.6. Using the Scheffe's Post Hoc comparison, significant grade level differences were observed only between the boys in grades eight and six for the MAS mean scores, and only between the girls in the same grades (eight and six) for the MGMP PT mean scores. The profiles of sex by grade level in Fig. 4.7 shows a drop in the MAS mean scores from grade level six to eight. However, as mentioned already, the post hoc tests showed the significant difference on the MAS scores ($P < .05$) was limited to between eight grade boys versus sixth grade boys, and in favour of the latter. No significant contrasts were found in the other grade levels.

As a summary, based on the statistical analysis of the data from Site 3, the following decisions were made concerning the hypotheses (H_{03}, H_{02}, H_{01}) in that order:

1. The multivariate null hypothesis (H_{03}), of no interaction of grade by sex among the mean scores for grades six through eight, was retained.
2. The multivariate null hypothesis (H_{02}), of no difference between the mean scores for boys and for girls six, seven and eight, was rejected ($P < .01$). The univariate test for the MGMP PT mean scores was also

rejected ($P < .01$), but retained for the MAS scores. Scheffe's Post Hoc posteriori comparisons showed that these significant sex differences, observed in the MGMP PT mean scores, were found between boys and girls in the sixth grade, but in no other grade level.

3. The multivariate null hypothesis (H_{01}) of no difference among mean scores for each of the grade levels six to eight was rejected ($P < .01$). The two planned comparisons, between grade levels eight versus six and seven, and between grade levels seven versus six, were significant ($P < .0001$ and $P < .01$ respectively). With the exception of the MGMP PT mean scores for the contrast of grades seven and six, the corresponding univariate hypotheses for both criterion measures were each significant for each contrast.

Comparison of Pretest Results Among Sites 1, 2, and 3

Although the data from each site was analysed separately on the assumption that the three subsamples were systematically different and representative of three different populations, it was desirable and statistically permissible to compare differences and similarities of the results across the three sites.

The multivariate null hypothesis (H_{03}) of no interaction of grade by sex was retained for the data in each site. The multivariate null hypothesis (H_{02}) of no sex effects was retained for the data in Site 1, but rejected in Sites 2 and 3, and similar conclusions were reached with respect to the univariate analyses. Significant grade level differences were concluded for both criterion measures in all three sites with respect to the multivariate analyses, but results differed from site to site in the corresponding univariate analyses. In Site 1, no significant grade differences were found in the MAS mean scores and the only MGMP probability test significant differences were found between girls in grades seven and six. No significant sex differences were found in any site with respect to the MAS mean scores. In each of the sites, graph profiles showed that grade six performed lower than grades seven and eight on the MGMP PT mean scores. In Site 2, grade seven significantly outperformed grades six and eight in the MGMP PT mean scores. Also, a comparison of the grade seven MGMP PT mean scores for all sites from Tables 4.1, 4.3, and 4.5 showed that Site 2 grade seven recorded the highest mean scores of all grades of all sites. Fig. 4.8 includes the MGMP PT profiles of all sites by grade levels. Table 4.7 includes the means and standard deviations of totals for each site, for each grade level and for each sex for all the five measures of the dependent variables for the entire study.

TABLE 4.7

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR THE ENTIRE SAMPLE BY TIME BY GRADE BY SEX

		PREPTOT[a]	POSTPTOT[b]	PREMAS[c]	POSTMAS[d]	PAS[e]
		M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)
Site 1	444	7.394 (3.674)	10.712 (4.432)	3.509 (.896)	3.351 (.811)	3.315 (.840)
Site 2	621	8.977 (3.695)	12.034 (4.309)	3.236 (.970)	3.161 (.954)	2.654 (.939)
Site 3	381	9.202 (3.058)	12.465 (3.935)	3.236 (.810)	3.164 (.841)	3.081 (.889)
Grade 6	622	7.701 (3.177)	10.963 (3.750)	3.520 (.887)	3.389 (.878)	3.027 (.043)
Grade 7	379	9.269 (3.775)	13.193 (4.838)	3.351 (.872)	3.258 (.855)	3.048 (.897)
Grade 8	445	9.126 (3.802)	11.593 (4.259)	3.011 (.909)	2.952 (.862)	2.823 (.960)
Boys	737	9.959 (3.871)	12.104 (4.551)	2.292 (.926)	3.209 (.910)	2.965 (.961)
Girls	721	9.143 (3.274)	11.447 (4.055)	3.349 (.902)	3.230 (.860)	2.969 (.919)
All	1446	8.551 (3.613)	11.741 (4.309)	3.319 (.015)	3.220 (.886)	2.070 (.941)

[a] Probability Pretest (Range 0-25).
[b] Probability Posttest (Range 0-25).
[c] Pretest Mathematics Attitude Scale (Range 1-5).
[d] Posttest Mathematics Attitude Scale (Range 1-5).
[e] Probability Attitude Scale (Range 1-5).

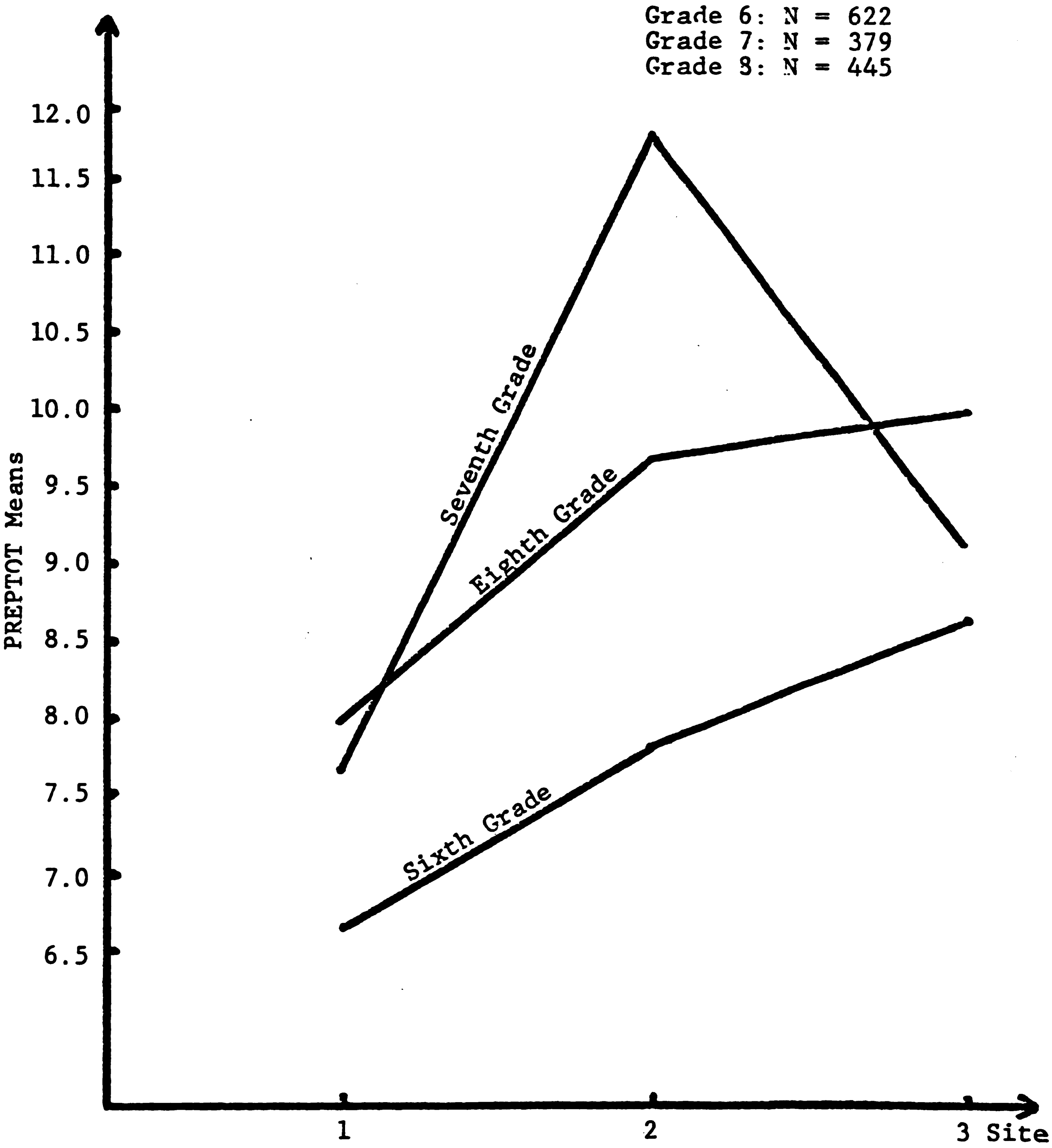


Fig 4.8 Pretest Probability Means--Profiles of All Sites by Grade Level.

The Effects of Instruction

The analysis of the effects of instruction on probability skills and on differences in probability and attitudes towards mathematics, by sex and by grade level, was conducted separately for each site. The design for the subsample from each site was a Two-Way 3 x 2 group design, with four measures on each subject. The Multivariate and Univariate Analysis of Repeated Measures was used for the data from each site. The following three multivariate null hypotheses were tested for each site:

H_{04}: There will be no difference between the posttest means and the pretest means of sixth, seventh, and eighth grade students, on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

H_{05}: There will be no difference between the mean gain scores (posttest minus pretest) for each of the three grade levels tested, six, seven, and eight, on both the Middle Grades Mathematics Project Test and on the Mathematics Attitude Scale.

H_{06}: There will be no difference between the mean gain scores (posttest minus pretest) for boys and for girls in grades six, seven, and eight, on the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

Site 1: The Urban Site

The pretest and posttest means and standard deviations of the MGMP Probability Test (PT) scores and the Mathematics Attitudes Scale (MAS) scores for the data from Site 1 are presented by grade and by sex in Table 4.8.

A Summary of Multivariate and Univariate Analysis of Repeated Measures for the data from Site 1 is presented in

TABLE 4.8

PRE AND POSTTEST MEANS* OF MGMPT PT AND MAS SCORES FOR SITE 1 BY GRADE AND BY SEX

Grade	N	MGMP PT[a] Pretest M	MGMP PT[a] Posttest M	MAS[b] Pretest M	MAS[b] Posttest M
Grade 6	149	6.62	9.79	3.607	3.259
Boys	76	6.97	10.16	3.568	3.241
Girls	73	6.27	9.41	3.648	3.276
Grade 7	138	7.62	11.07	3.450	3.379
Boys	66	7.45	10.38	3.548	3.407
Girls	72	7.78	11.75	3.373	3.350
Grade 8	157	7.93	11.29	3.466	3.419
Boys	74	8.16	11.91	3.532	3.453
Girls	83	7.70	10.66	3.400	3.384
Total	444	7.39	10.71	3.509	3.351
Boys	216	7.53	10.82	3.549	3.364
Girls	228	7.27	10.61	3.471	3.339

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).

[b] MAS - Mathematics Attitude Scale (Range 1-5).

* The corresponding standard deviations are included in Appendix D, Table D.2.

Table 4.9, following the format used by Winer (1962). The table is in two parts. The first part shows averages of PT and MAS mean scores over time, respectively abbreviated as AVGPTOT and AVGMAS in Table 4.9. AVGPTOT can be taken as a measure of the average amount of MGMP probability knowledge possessed by subjects over time from pretest through posttest. AVGMAS is the corresponding average of the Mathematics Attitude Scale scores. Although AVGPTOT and AVGMAS indicate some measure of the effect of instruction over time, they were not the major concern of the null hypotheses tested on the effect of instruction in this study. Hence these two measures were of little interest in these analyses.

The second part of Table 4.9 shows differences of PT and MAS mean scores over time, respectively abbreviated as DIFPTOT and DIFMAS. DIFPTOT and DIFMAS are a measure of time effect (differences or changes) over subjects from pretest to posttest. Since the multivariate hypotheses (H_{04}-H_{06}) tested in this study were concerned with differences between posttest means and pretest means, DIFPTOT and DIFMAS were the major focus of the analyses of these hypotheses.

In all these analyses of repeated measures, each DIFPTOT turned out to be a gain regardless of the independent variable considered over time. DIFMAS generally stayed the same over the given time interaction. The MGMP PT pre-posttest means from Table 4.8 were used to draw the profiles for pre-post test scores or gains by grade and by sex in Figures 4.9, 4.12 and 4.13 (pages 57 and 58).

The Multivariate Analysis of Repeated Measures for Site 1 showed no significant interaction of grade by sex by time, nor of sex by time. The test however showed significant interaction of one of the contrasts (G_1) by time ($P < .05$). The univariate test was not significant for DIFPTOT but was significant ($P < .01$) for DIFMAS. This means that in the G_1 contrast (of grade 7 versus grade 6), grade 7 was significantly different from grade 6 in attitude change to mathematics over the period of instruction, but not significantly different in probability knowledge gain from pretest to posttest as measured by the MGMP PT.

Indeed, from the table of pre-posttest means, averages and differences (Table 4.16, page 56), the attitude change was negative for both grades six and seven. Scheffe's Post Hoc posteriori contrasts (Appendix E.4) showed grade six girls to have significantly changed more in mathematics attitude than grade seven girls at the .05 level. However, a comparison of their actual mean losses in the MAS scores showed that the difference was -.35. This difference was therefore concluded to be non meaningfully significant, given the high power level of the statistical test as a result of the large sample size characterizing the study. The other contrast G_2 by time (grade 8 versus grades 6 and 7 over time) was confounded in the G_1 by time test and hence could not be tested. However, from a close look at

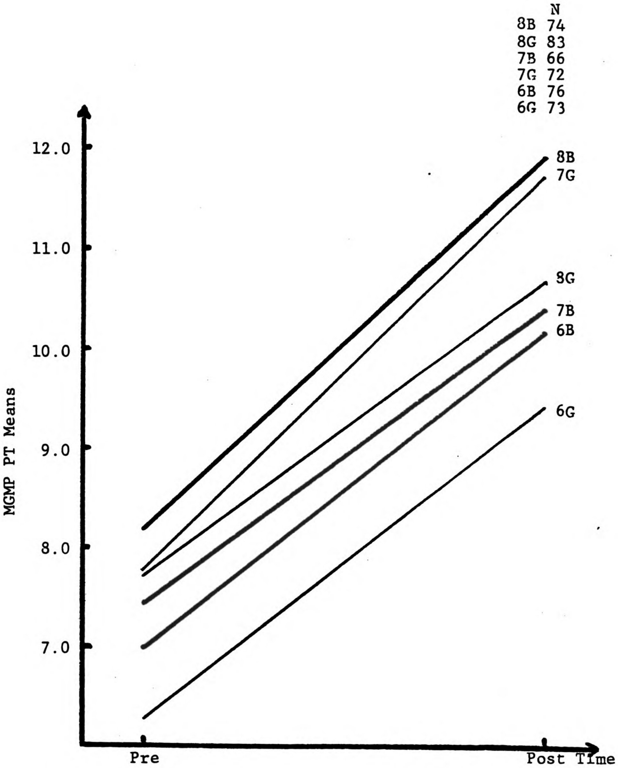


Fig. 4.9 Profile of Means on the MGMP PT by Grade by Sex by Time in Site 1.

TABLE 4.9

A SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF REPEATED MEASURES FOR DATA FROM SITE 1

Source of Variation	D.F.	Multivariate[a] F	Multivariate[a] P<		Univariate F	Univariate P<
Grand Mean	1	5131.61	.0001	AVGPTOT[b]	2692.99	.0001
				AVGMAS[c]	9030.23	.0001
Grade	2					
G_2[d]	(1)	.15	.862	AVGPTOT	.27	.602
				AVGMAS	.06	.802
G_1[e]	(1)	6.10	.002	AVGPTOT	11.75	.001
				AVGMAS	.003	.955
Sex	1	.42	.657	AVGPTOT	.47	.493
				AVGMAS	.52	.470
Grade by Sex	2	1.63	.166	AVGPTOT	2.37	.095
				AVGMAS	.59	.555
Time[f]	1	239.06	.0001	DIFPTOT[g]	457.00	.0001
				DIFMAS[h]	18.16	.0001
Grade[i] by time	2					
G_2 by time	(1)	2.47	.086	DIFPTOT	.004	.951
				DIFMAS	4.95	.027
G_1 by time	(1)	4.47	.012	DIFPTOT	.64	.425
				DIFMAS	8.41	.004
Sex by time	1	.26	.771	DIFPTOT	.02	.894
				DIFMAS	.51	.477
Grade by Sex by time	2	1.63	.165	DIFPTOT	2.88	.057
				DIFMAS	.41	.664
Between Groups	6					
Within Group	438					

Univariate MS error
AVGPTOT: MSe = 27.02
AVGMAS: MSe = 1.16
DIFPTOT: MSe = 5.35
DIFMAS: MSe = .31

Table 4.9 (cont'd.).

[a] Multivariate D.F. = (2,437) except for the Grade by Sex interactions for which D.F. = (4,874).
[b] AVGPTOT - Averaging Probability Test scores (Pre + Posttest).
[c] AVGMAS - Averaging MAS scores (Pre + Posttest).
[d] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.
[e] G_1 - Contrast of Grade 7 vs. Grade 6.
[f] Time - Overall (Pre-posttest) effect over all subjects.
[g] DIFPTOT - Differences in Probability Test scores (time effect) over subjects.
[h] DIFMAS - Differences in MAS scores (time effect) over subjects.
[i] A reordering of the grade contrasts, testing the G_2 by Time contrast before G_1 by Time resulted in multivariate F values of .14 for G_2 by Time and 6.88 for G_1 by Time, and about the same P values as before.

the table and profiles of mean differences, Tables 4.16 and 4.17 and Fig. 4.9, it was apparent that boys and girls did not differ significantly within and between grade levels in MGMP probability knowledge gains over time. Table E.4 in Appendix E also shows the post hoc contrast of a significant .36 MAS mean difference between girls in grades six and eight. The test of the overall time effect over the subjects was highly statistically significant ($P < .0001$). Although this test was also confounded in the G_1 by Time interaction (Table 4.9), the profiles (Fig. 4.9) and Table (4.16) of mean gains show systematic mean gain scores in the MGMP PT, ranging from 3.97 (by grade 7 girls) to 1.92 (by grade 8 boys), from pretest to posttest, in each of the six mean (difference) cells (Table 4.15). Thus, the significant difference of overall time effect on the subjects in the MGMP PT scores, showed in Table 4.9, was retained. However, since the DIFMAS changes showed a range of magnitiude of .35 across the six mean (difference) cells, Table E.4, Appendix E, it was concluded that there was no meaningfully significant change of attitudes toward mathematics from pretest to posttest, the post hoc result between girls in grades eight and six notwithstanding.

To summarize, based on the statistical analysis of the pretest-posttest data from Site 1, the following decisions were made with respect to hypotheses (H_{04}-H_{06}):

1. The multivariate null hypothesis (H_{04}) of no difference between the posttest means and pretest means was rejected ($P < .0001$).
2. The only statistically significant difference in grade levels (six through eight) in mean gain scores was in the mathematics attitude change. Further tests showed this difference to be between grades six and eight girls. This was considered logically small and non-meaningfully significant. The multivariate null hypothesis (H_{05}) of no difference between the mean gain scores for each of the three grade levels was however rejected ($P < .05$) to avoid the risk of a Type II error.
3. The hypothesis (H_{06}) was retained, and the conclusion was in favour of no significant difference between the mean gain scores for boys and for girls in each of the grade levels six, seven and eight, on the MGMP PT and on the MAS.

The summary of the Multivariate Analysis of Repeated Measures for Site 1 (Table 4.9) and the graphical representation in Figs. 4.13 and 4.17 (pages 58, 62) show that results with respect to AVGPTOT were similar to those of DIFPTOT. There was no significant grade by sex interaction, neither was the sex main effect significant. Boys and girls in Site 1 therefore did not differ in their overall knowledge (averages) in the probability measured by the MGMP Probability Test. The grade levels seven and eight did not differ statistically from each other in their knowledge of MGMP probability, but each differed statistially from grade level six as showed by Scheffe's Post Hoc posteriori comparison. There was a mean average difference of only 1.37 between grades eight and six, and only 1.3 between grades seven and six out of a maximum mean difference of 25. Hence, these statistically significant differences were not considered meaningful. Finally although grades seven and eight knew more MGMP Probability than grade six (statistically higher AVGPTOT) they did not gain more (DIFPTOT). One might conclude that, in Site 1, the MGMP instruction was more effective in grade six than in grades 7 and 8.

Site 2: The Suburban Site

The pretest and posttest means and standard deviations of the MGMP PT and MAS scores for the data from Site 2 are presented by grade and by sex in Table 4.10.

A summary of Multivariate and Univariate Analysis of Repeated Measures for the data from Site 2 is included in Table 4.11. The layout and the interpretation format of the table are similar to the corresponding Table (4.8) in Site 1. AVGPTOT, AVGMAS, DIFPTOT and DIFMAS all refer to the same measures in each site.

The results of the Multivariate Analysis of Repeated Measures for Site 2 (Table 4.11) showed no significant interactions of grade by sex by time, nor of sex by time. The grade by time interaction was significant. However, in

TABLE 4.10

PRE AND POSTTEST MEANS* OF MGMP PT AND MAS SCORES FOR SITE 2 BY GRADE AND BY SEX

		MGMP PT[a]		MAS[b]	
		Pretest	Posttest	Pretest	Posttest
Grade	N	M	M	M	M
Grade 6	340	7.80	11.27	3.510	3.404
Boys	162	8.14	11.40	3.422	3.360
Girls	178	7.50	11.15	3.590	3.443
Grade 7	101	11.71	14.97	3.406	3.320
Boys	53	12.68	15.91	3.582	3.547
Girls	48	10.65	13.94	3.212	3.069
Grade 8	180	9.66	11.83	2.622	2.613
Boys	102	10.13	12.05	2.613	2.601
Girls	78	9.05	11.55	2.635	2.628
Total	621	8.98	12.03	3.236	3.161
Boys	317	9.54	12.36	3.188	3.147
Girls	304	8.40	11.69	3.285	3.175

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).
[b] MAS - Mathematics Attitude Scale (Range 1-5).
* The corresponding standard deviations are given in Appendix D. Table D.3.

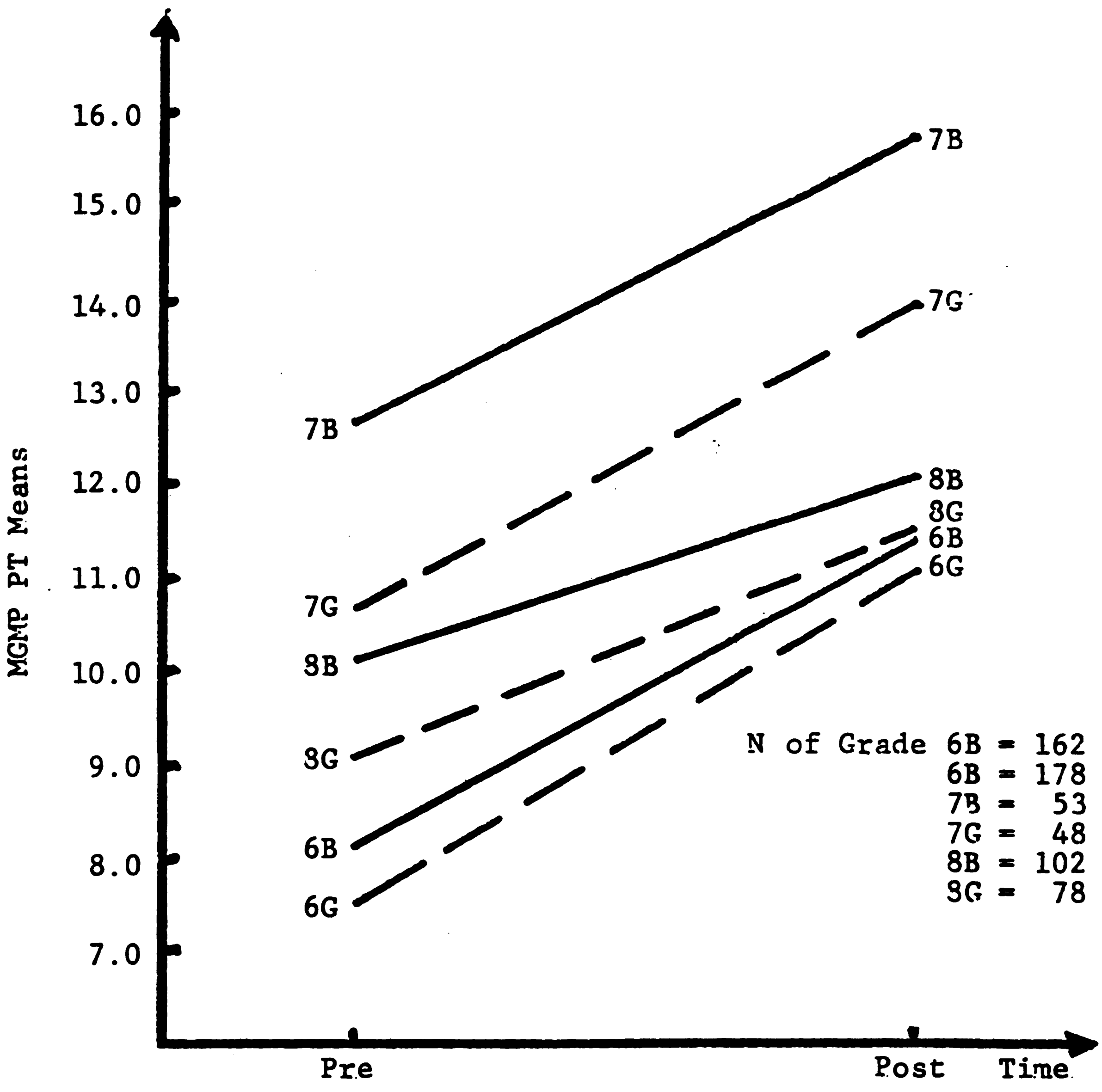


Fig. 4.10 Profiles of MGMP PT Means by Grades by Sex by Time in Site 2.

TABLE 4.11

A SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF REPEATED MEASURES FOR DATA FROM SITE 2

Source of Variation	D.F.	Multivariate[a] F	Multivariate[a] P		Univariate F	Univariate P
Grand Mean	1	6446.93	.0001	AVGPTOT[b]	6028.29	.0001
				AVGMAS[c]	9489.32	.0001
Grade	2					
G_2[d]	(1)	68.86	.0001	AVGPTOT	.81	.367
				AVGMAS	126.82	.0001
G_1[e]	(1)	53.41	.0001	AVGPTOT	97.11	.0001
				AVGMAS	1.02	.314
Sex	1	7.03	.001	AVGPTOT	11.22	.001
				AVGMAS	.91	.341
Grade by Sex	2	2.70	.029	AVGPTOT	2.07	.128
				AVGMAS	4.40	.013
Time[f]	1	272.12	.0001	DIFPTOT[g]	540.35	.0001
				DIFMAS[h]	7.68	.006
Grade by time	2					
G_2 by time	(1)	9.64	.0001	DIFPTOT	17.46	.0001
				DIFMAS	2.14	.144
G_1 by time	(1)	.15	.860	DIFPTOT	.26	.612
				DIFMAS	.05	.823
Sex by time	1	2.33	.097	DIFPTOT	3.19	.075
				DIFMAS	1.62	.203
Grade by Sex by time	2	.27	.899	DIFPTOT	.20	.821
				DIFMAS	.33	.722
Between Groups	6					
Within Group	615					

Univariate MS error	AVGPTOT: MSe = 22.74
	AVGMAS: MSe = 1.34
	DIFPTOT: MSe = 5.37
	DIFMAS: MSe = .23

Table 4.11 (cont'd.).

[a] Multivariate D.F. = (2,614) except for the Grade by Sex interactions for which D.F. = (41,228).
[b] AVGPTOT - Averaging Probability Test scores (Pre + Posttest).
[c] AVGMAS - Averaging MAS scores (Pre + Posttest).
[d] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.
[e] G_1 - Contrast of Grade 7 vs. Grade 6.
[f] Time - Overall (Pre-posttest) effect over all subjects.
[g] DIFPTOT - Differences in Probability Test scores (time effect) over subjects.
[h] DIFMAS - Differences in MAS scores (time effect) over subjects.

the planned comparison of the contrasts G_1 (grade 7 versus grade 6), and G_2 (grade 8 versus grades 6 and 7), the former (G_1) was not significant. This means that there were no significant differences in MGMP PT and MAS gains due to interaction between grade and sex over the time. Also, there were no significant differences between boys and girls in each grade level in MGMP Probability mean gains or mean attitude changes. Grade 7 also did not differ significantly from grade 6 in these criterion measures. However, the contrast of grade 8 versus grades 6 and 7 was significant ($P < .0001$). The corresponding univariate analysis confined these differences to ($P < .0001$) to DIFPTOT and not DIFMAS (Table 4.11). That is, differences between grade 8 versus grades 6 and 7 were found in mean probability achievement gains (DIFPTOT), but not in change in attitude to mathematics (DIFMAS). In short, in the entire analysis of the data from Site 2, no significant differences were observed among both independent variables (sex and grade levels) in attitude change toward mathematics (Table 4.11), as measured by the Mathematics Attitude Scale. However, Table 4.10 of pretest and posttest means showed that boys kept more steadily, on the positive side than girls, and grades eight and seven than grade six. However, all differences in attitude change toward mathematics over time were small and non statistically significant. Hence, it was considered unecessary to display tables and profiles of attitude change towards mathematics.

To determine which grade level and sex cells were responsible for the MGMP PT gain differences observed in the contrast of grade 8 versus grades 6 and 7, Scheffe's Post Hoc posteriori comparisons were employed. These pairwise comparisons showed that significant probability gain differences occurred between boys in grades 7 and 8, between boys in grades 6 and 8, and between girls in grades 6 and 8 (Table E.5, Appendix E). These probability gain differences were unexpectedly always in favour of the lower grade! Similar conclusions are also observable from the table of MGMP Probability gains and averages (Table 4.15), and from the profiles of Site 2 mean gains and averages in Figure 4.12. The same profiles show an ordinal interaction between boys and girls across all the grade level factors in both the gains and averages in the MGMPPT. In other words, in each grade level in Site 2, girls gained more, but knew less, probability than boys, as measured by the MGMP Probability Test. Similar conclusions followed a close examination of the profiles by grade, by sex, and by time (Fig. 4.10), of the MGMP PT pre-posttest means.

The multivariate test showed the over all time effect to be significant ($P < .0001$, Table 4.11). This test was however confounded in the G_2 by time contrast and the result cannot be reported as significant with absolute certainty. However, the table of probability achievement gains (DIFPTOT), in Table 4.15, show some substantial gains by each grade and sex with the exception of boys in grade 8

TABLE 4.12

PRE AND POSTTEST MEANS* OF MGMP PT AND MAS SCORES FOR SITE 3 BY GRADE AND BY SEX

		MGMP PT[a]		MAS[b]	
		Pretest	Posttest	Pretest	Posttest
Grade	N	M	M	M	M
Grade 6	133	8.64	11.50	3.450	3.496
Boys	70	9.26	12.06	3.419	3.502
Girls	63	7.95	10.87	3.484	3.489
Grade 7	140	9.13	13.98	3.207	3.096
Boys	65	9.40	14.43	3.177	3.085
Girls	75	8.89	13.59	3.233	3.107
Grade 8	108	9.99	11.69	3.005	2.843
Boys	57	10.26	12.04	2.842	2.766
Girls	51	9.69	11.32	3.186	2.928
Total	381	9.20	12.47	3.236	3.164
Boys	192	9.66	12.85	3.166	3.142
Girls	189	8.79	12.07	3.304	3.186

[a] MGMP PT - Middle Grade Mathematics Project Probability Test (Range 0-25).
[b] MAS - Mathematics Attitude Scale (Range 1-5).
* The corresponding Standard Deviations are given in Appendix D, Table D.4.

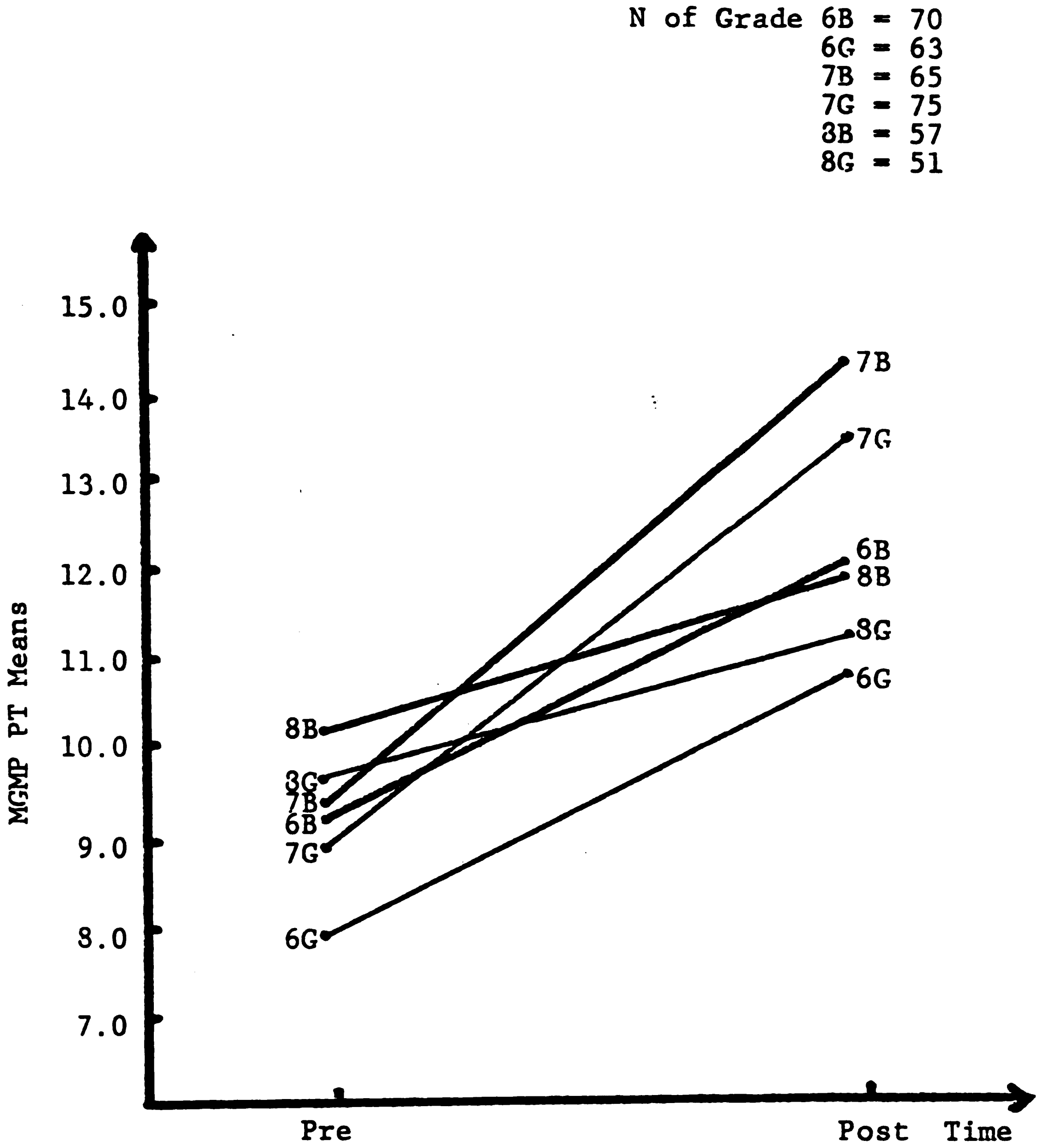


Fig. 4.11 Profiles of MGMP PT Means by Grade by Sex by Time in Site 3.

(with only a gain of 1.92). Hence the overall time effect hypothesis was not considered tenable. Moreover, the risk of a Type I error involved was small since both the multivariate and univariate tests were highly significant ($P < .0001$) in favour of MGMP Probability gains. Hence, the conclusion was in favour of significant overall gains in the MGMP PT scores by Site 2 subjects.

In summary, based on the Multivariate Analysis of Repeated Measures of the data from Site 2, the following decisions were made with respect to the null hypotheses (H_{04}-H_{06}):

1. The null hypothesis (H_{04}) of no difference between the posttest means and pretest means of the three grade levels studied was rejected. However, no meaningfully significant difference was found between the MAS mean gain scores. This means that there was an overall significant time effect on the subjects due to the MGMP Probability instruction.
2. The null hypothesis (H_{05}) of no difference between the mean gain scores for the three grade levels was rejected. The tests showed evidence of significantly different mean gain scores among the three grade levels studied on the MGMP PT scores, but not on the MAS scores. Gain differences were found between grades 6 and 8, between grades 7 and 8, always in favour of the lower grade; but not between grades 6 and 7.

3. The null hypothesis (H_{06}) was tenable and retained. The conclusion was that there was no significant difference in the mean gain scores for boys and for girls in grades six, seven, and eight on both the MGMP PT and MAS.

Although the hypotheses tested did not involve averages of probability achievement means and mathematics attitude means, AVGPTOT and AVGMAS, it was deemed desirable to comment on the Multivariate and Univariate results involving these measures. The multivariate grade by sex interaction test was significant ($P < .05$), and the univariate test showed this significance to be in AVGMAS only, ($P < .05$, Table 4.11). This means that there was a grade by sex interaction in the subjects' overall attitudes to mathematics as measured by the MAS. Because of this significant grade by sex interaction the sex and grade level, main effects could not be tested separately. Hence, no statistical conclusions were feasible. However, the profiles of mean averages by time by sex (Fig. 4.13) and by grade level (Fig. 4.12) showed an ordinal interaction between boys and girls across the three levels (6, 7 and 8) of the grade factor. From the same two profiles and from the Table 4.15, it was observed that boys consistently scored higher averages than girls in the MGMP PT, most conspicously in the seventh grade. A conclusion therefore was that while girls consistently gained more probability than boys across the three grade levels (as measured by the

MGMP PT), boys consistently knew more (Figures 4.12 and 4.13).

Site 3: The Rural Site

The pretest and posttest means and standard deviations of the MGMPPT and MAS scores for the data from Site 3 are presented by grade and by sex in Table 4.12.

Table 4.13 presents a summary of Multivariate and Univariate Analysis of Repeated measures for the data from Site 3. There was no significant interaction of grade by sex by time, nor of sex by time. The profiles of mean gains by sex in Fig. 4.13 and by grade level in Fig. 4.12 both attest to the fact that boys and girls gained statistically equally from pretest to posttest. Both the G_1 by Time and G_2 by Time planned comparisons were significant ($P < .0001$) according to the multivariate tests, even when the grade and sex main effects were reordered (Table 4.13). The corresponding univariate results showed that DIFMAS was not significant in each case. Hence, in the Multivariate and Univariate Analysis of the data from Site 3, there were no significant differences in attitude change to mathematics (as measured by the MAS) by sex, or by grade, or by the interaction of both. Hence as before, tables and profiles of mathematics attitude changes from pretest to posttest were not necessary. However, from a survey of the pretest-posttest means in Table 4.12, it was observed that

TABLE 4.13

A SUMMARY OF MULTIVARIATE AND UNIVARIATE ANALYSIS OF REPEATED MEASURES FOR DATA FROM SITE 3

Source of Variation	D.F.	Multivariate[a] F	Multivariate[a] P<		Univariate F	Univariate P<
Grand Mean	1	5082.3	.0001	AVGPTOT[b]	4958.02	.0001
				AVGMAS[c]	7347.35	.0001
Grade	2					
G_2[d]	(1)	11.07	.0001	AVGPTOT	.002	.968
				AVGMAS	21.26	.0001
G_1[e]	(1)	20.01	.0001	AVGPTOT	17.88	.0001
				AVGMAS	13.72	.0002
Sex	1	5.00	.007	AVGPTOT	6.72	.010
				AVGMAS	1.49	.223
Grade by Sex	2	.60	.663	AVGPTOT	.40	.668
				AVGMAS	.88	.414
Time[f]	1	207.71	.0001	DIFPTOT[g]	401.28	.0001
				DIFMAS[h]	4.60	.033
Grade[i] by time	2					
G_2 by time	(1)	18.89	.0001	DIFPTOT	36.24	.0001
				DIFMAS	3.24	.073
G_1 by time	(1)	16.33	.0001	DIFPTOT	26.93	.0001
				DIFMAS	3.80	.052
Sex by time	1	1.06	.348	DIFPTOT	.01	.939
				DIFMAS	2.08	.150
Grade by Sex by time	2	.30	.876	DIFPTOT	.18	.838
				DIFMAS	.41	.661
Between Groups	6					
Within Group	375					

Univariate MS error
AVGPTOT: MSe = 18.04
AVGMAS: MSe = 1.06
DIFPTOT: MSe = 5.05
DIFMAS: MSe = .21

Table 4.13 (cont'd.).

[a] Multivariate D.F. = (2,374) except for the Grade by Sex interactions for which D.F. = (4,748).
[b] AVGPTOT - Averaging Probability Test scores (Pre + Posttest).
[c] AVGMAS - Averaging MAS scores (Pre + Posttest).
[d] G_2 - Contrast of Grade 8 vs. Grades 6 and 7.
[e] G_1 - Contrast of Grade 7 vs. Grade 6.
[f] Time - Overall (Pre-posttest) effect over all subjects.
[g] DIFPTOT - Differences in Probability Test scores (time effect) over subjects.
[h] DIFMAS - Differences in MAS scores (time effect) over subjects.
[i] A reordering of the grade contrasts, testing the G_2 by Time contrast before G_1 by Time resulted in multivariate F values of 29.78 for G_2 by Time and 5.49 for G_1 by Time, and about the same P values as before.

the sixth graders had and gained more positive attitudes toward mathematics than seventh and eighth graders.

The univariate analysis did show the contrasts (G_1) of grades seven versus six, and (G_2) of grade eight versus grades seven and six to be significant ($P < .0001$) with respect to DIFPTOT. That is, the three grade levels differed significantly in their gains in the MGMP PT scores from pretest to posttest.

Scheffe's Post Hoc posteriori comparisons were conducted in order to determine details of these differences. Table E.6 in Appendix E includes a summary of these pairwise contrasts. Significant grade level differences in MGMP Probability gains were found ($P < .05$) between boys in grade six versus grade seven, and between girls in grade six versus grade seven; always in favor of grade seven. Similar results, also in favour of grade seven, were found between grades seven and eight. Differences were also found ($P < .05$) between boys in grades six and eight, and between girls in these two grade levels - both in favour of grade six. The contrast differences between grades six and eight were however considered small and non meaningful - judging from the high precision of the tests, due to the large sample size used in the study. From these post hoc results and from the profiles of mean gains in Fig. 4.12, the conclusion was that grade seven gained most while grade eight gained least by time in the MGMP Probability Test scores. Similar conclusions were observed from the profiles of means by grade, by sex, and by time in Fig. 4.11 and from the AVGPTOT and DIFPTOT of MGMP PT mean scores in Table 4.15.

With respect to the overall time effect over the subjects, the multivariate and univariate analyses were each highly significant ($P < .0001$) in favour of gains in the MGMP PT scores. Hence, with arguments congruent to those used previously with respect to time effect in Site 2, it was concluded that there was significant overall gain in the MGMP Probability by the subjects.

In summary, based on the Multivariate Analysis of Repeated Measures of the data from Site 3, the following decisions were made with respect to the null hypotheses (H_{04}-H_{06}):

1. The null hypothesis (H_{04}), of no differene between the posttest means and pretest means of the three grade levels studied was rejected. However, no meaningfully significant difference was found between the MAS mean gain scores. This means that there was an overall significant time effect on the subjects due to the MGMP Probability instruction.
2. The null hypothesis (H_{05}), of no difference between the mean gain scores for the three grade levels, was rejected. The tests showed evidence of significantly different mean gain scores among the three grade levels studied in the MGMP PT scores, but not on the MAS scores. Gain differences were found between grades six and eight, between grades seven and eight, always in

favour of the lower grade, and between grades six and seven, in favour of grade seven.

3. The null hypothesis (H_{06}) was retained. The conclusion was that there were no significant differences in the mean gain scores for boys and for girls in grades six, seven, and eight, on both the MGMP PT and MAS.

With respect to results involving MGMP PT mean averages, AVGPTOT, the Multivariate and Univariate tests were significant with respect to sex main effects, to the G_1 contrast, and to the G_2 contrast ($P < .01$, $P < .0001$ repectively). The profiles of mean averages by sex (Fig. 4.13) and by grade level (Fig. 4.12) indicate that boys scored significantly higher averages than girls across the three grade levels - a highly ordinal interaction, as opposed to the disordinal (crossing) interaction of sex with grade levels in the profiles of gains (Fig. 4.12). As in Site 2, grade seven recorded the highest averages in the probability means from pretest to posttest. Thus in Site 3, seventh graders significantly gained and knew more of the MGMP Probability than sixth or eighth graders. While boys significantly knew more of the MGMP Probability than girls, they did not gain more.

Comparison of the Effect of Instruction Among Sites 1, 2, and 3

The same three null hypotheses were tested separately for each site using Multivariate and Univariate Analysis of Repeated Measures.

The null hypothesis (H_{04}), of no difference between the posttest means and pretest means of the three grade levels (6, 7, and 8) studied, was rejected, ($P < .0001$) for the data in each site. This means that, in the entire study, there was evidence of a significant overall time effect on the subjects. In each site, the effect of instruction was found to cause significant gains from pretest to posttest in the MGMP PT scores. All attitude changes were small and non-meaningfully significant. The profiles of pre-posttest means of the MAS of the entire sample by grade (Fig. 4.18), and, by sex and by site (Fig 4.19), all show slight mathematics attitude changes from pretest to posttest, none of which was meaningfully significant. Fig 4.19 shows that on the whole, boys were more steady than girls in their attitudes toward mathematics as measured by the MAS. Although girls in general dropped more in attitudes than boys, they still scored higher on average attitude (AVGMAS) in the pretest and posttest measures (Tables 4.14 and 4.15).

The null hypothesis (H_{05}) was rejected ($P < .05$) for the data from Site 1. The rejection was due to change differences found between girls in grades six and seven

TABLE 4.14

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR THE ENTIRE SAMPLE BY TIME BY GRADE BY SEX

		PREPTOT[a]	POSTPTOT[b]	PREMAS[c]	POSTMAS[d]	PAS[e]
		M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)
Site 1	444	7.394 (3.674)	10.712 (4.432)	3.509 (.896)	3.351 (.811)	3.315 (.840)
Site 2	621	8.977 (3.695)	12.034 (4.309)	3.236 (.970)	3.161 (.954)	2.654 (.939)
Site 3	381	9.202 (3.058)	12.465 (3.935)	3.236 (.810)	3.164 (.841)	3.081 (.889)
Grade 6	622	7.701 (3.177)	10.963 (3.750)	3.520 (.887)	3.389 (.878)	3.027 (.043)
Grade 7	379	9.269 (3.775)	13.193 (4.838)	3.351 (.872)	3.258 (.855)	3.048 (.897)
Grade 8	445	9.126 (3.802)	11.593 (4.259)	3.011 (.909)	2.952 (.862)	2.823 (.960)
Boys	737	9.959 (3.871)	12.104 (4.551)	2.292 (.926)	3.209 (.910)	2.965 (.961)
Girls	721	9.143 (3.274)	11.447 (4.055)	3.349 (.902)	3.230 (.860)	2.969 (.919)
All	1446	8.551 (3.613)	11.741 (4.309)	3.319 (.015)	3.220 (.886)	2.070 (.941)

[a] Probability Pretest (Range 0-25).
[b] Probability Posttest (Range 0-25).
[c] Pretest Mathematics Attitude Scale (Range 1-5).
[d] Posttest Mathematics Attitude Scale (Range 1-5).
[e] Probability Attitude Scale (Range 1-5).

TABLE 4.15

AVGPTOT AND DIFPTOT MEANS OF THE MGMP PT[a] SCORES BY GRADE BY SEX PER SITE

	Site 1			Site 2			Site 3		
	N	AVGPTOT[b]	DIFPTOT[c]	N	AVGPTOT	DIFPTOT	N	AVGPTOT	DIFPTOT
Grade 6	149	8.21	3.16	340	9.54	3.46	133	10.07	2.86
Boys	76	8.57	3.18	162	9.77	3.27	70	10.66	2.80
Girls	73	7.84	3.14	178	9.32	3.65	63	9.41	2.92
Grade 7	138	9.36	3.47	101	13.34	3.26	140	11.55	4.85
Boys	66	8.92	3.92	53	14.29	3.23	65	11.92	5.03
Girls	72	9.76	3.97	48	12.29	3.29	75	11.24	4.69
Grade 8	157	9.58	3.33	180	10.75	2.17	108	10.84	1.70
Boys	74	10.03	3.74	102	11.09	1.92	57	11.15	1.77
Girls	83	9.18	2.96	78	10.30	2.50	51	10.50	1.63
All Grades	444	9.05	3.32	621	10.51	3.06	381	10.83	3.26
Boys	216	9.18	3.30	317	10.95	2.83	192	11.23	3.25
Girls	228	8.94	3.34	304	10.04	3.30	189	10.43	3.28

[a] MGMPPT -- Middle Grades Mathematics Project Probability Test Scores (Range 0-25).
[b] AVGPTOT -- Averaging Probability Test Mean Scores (Pre + Posttest).
[c] DIFPTOT -- Differences in Probability Test Mean Scores (time effect) over subjects.

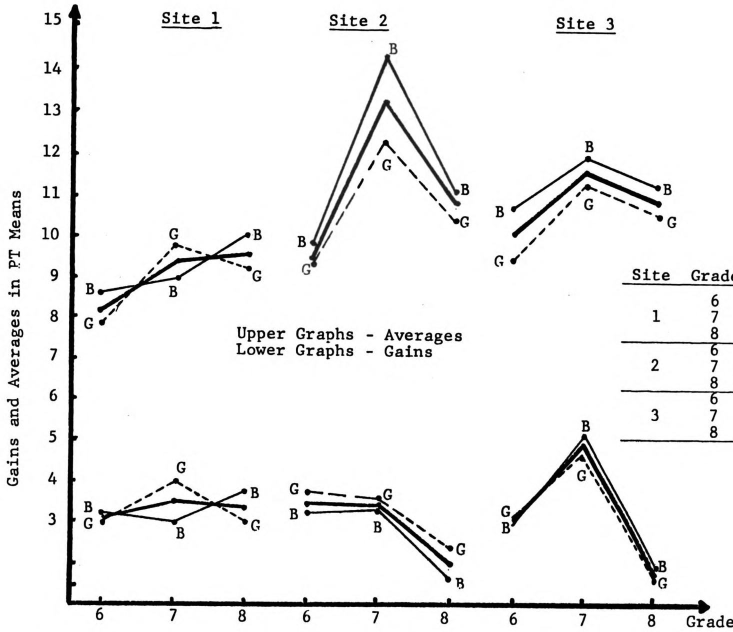
TABLE 4.16

PRE-POSTTEST MEAN DIFFERENCES AND AVERAGES OF THE MGMP PT SCORES BY GRADE BY SEX PER SITE

	N	AVGPTOT[a]	DIFPTOT[b]
Site 1	444	9.05	3.32
Boys	216	9.18	3.30
Girls	228	8.94	3.34
6	149	8.21	3.16
7	138	9.36	3.47
8	157	9.58	3.33
Site 2	621	10.51	3.06
Boys	317	10.95	2.83
Girls	304	10.04	3.30
6	340	9.54	3.46
7	101	13.34	3.26
8	180	10.75	2.17
Site 3	381	10.83	3.26
Boys	192	11.23	3.25
Girls	189	10.43	3.28
6	133	10.07	2.86
7	140	11.55	4.85
8	108	10.84	1.70
All Sites	1446	10.15	3.19
Boys	725	10.50	3.08
Girls	721	9.79	3.30
6	622	9.33	3.26
7	389	11.23	3.92
8	445	10.36	2.48

[a] AVGPTOT - Averaging Probability Test Mean Scores (Pre + Posttest).

[b] DIFPTOT - Differences in Probability Test Mean Scores (time effect) over subjects.



Site	Grade	N
1	6	149
	7	138
	8	157
2	6	340
	7	101
	8	180
3	6	133
	7	140
	8	108

Fig. 4.12 Profiles of MGMP PT Mean Gains and Averages by Time in Each Site by Grade Level.

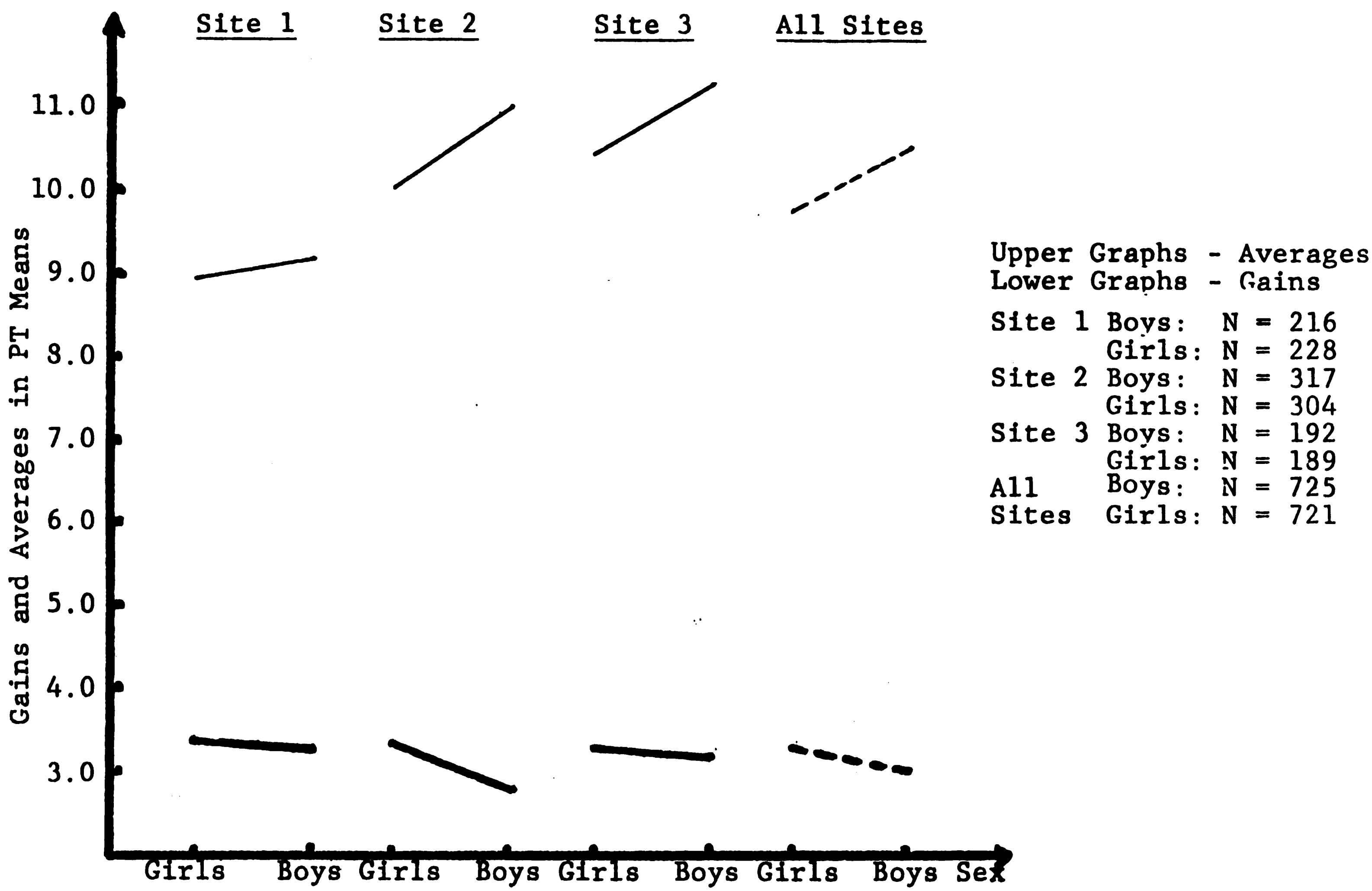


Fig. 4.13 Profiles of Mean Gains and Averages by Time by Sex,

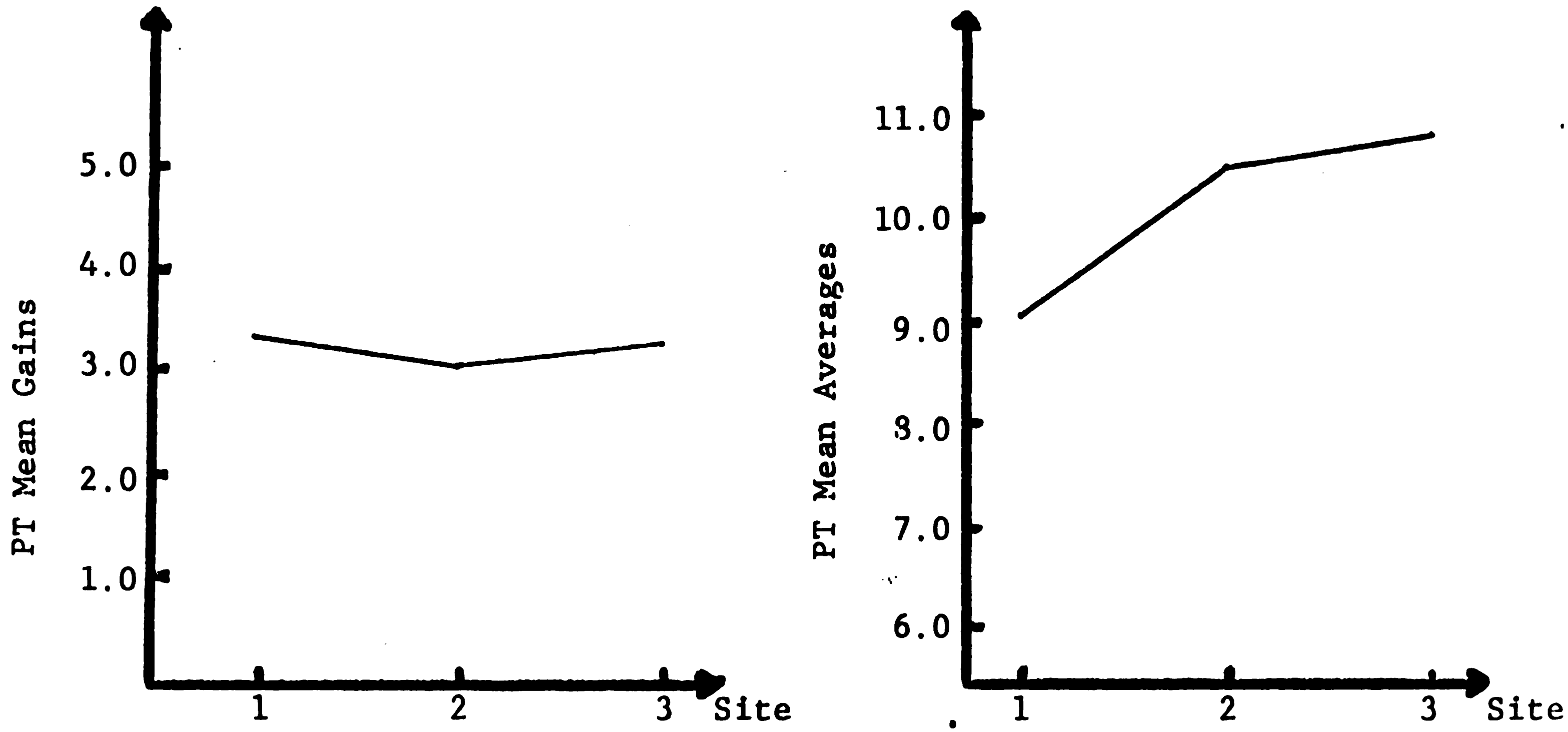


Fig. 4.14 Profiles of Mean Gains and Averages by Site by Time for the Entire Sample.

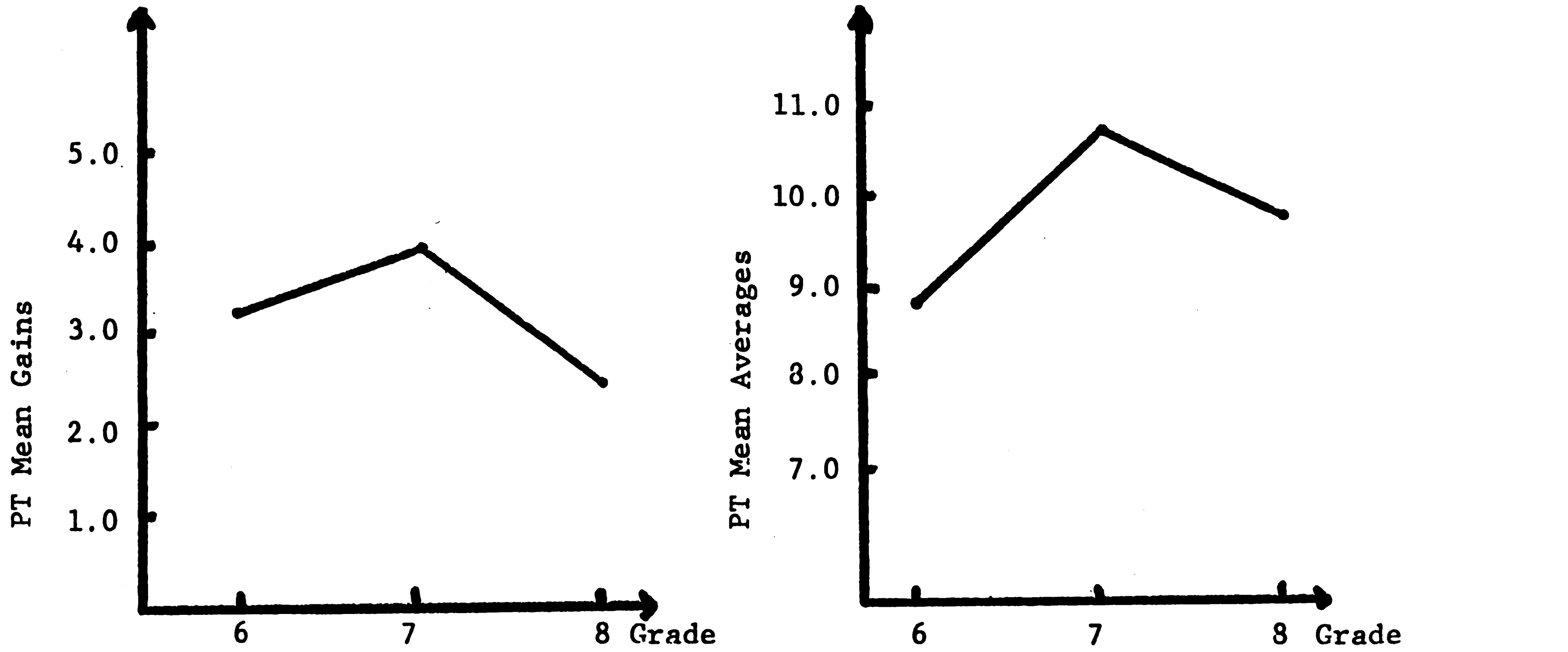


Fig. 4.15 Profiles of Mean Gains and Averages by Grade Level by Time for Entire Sample.

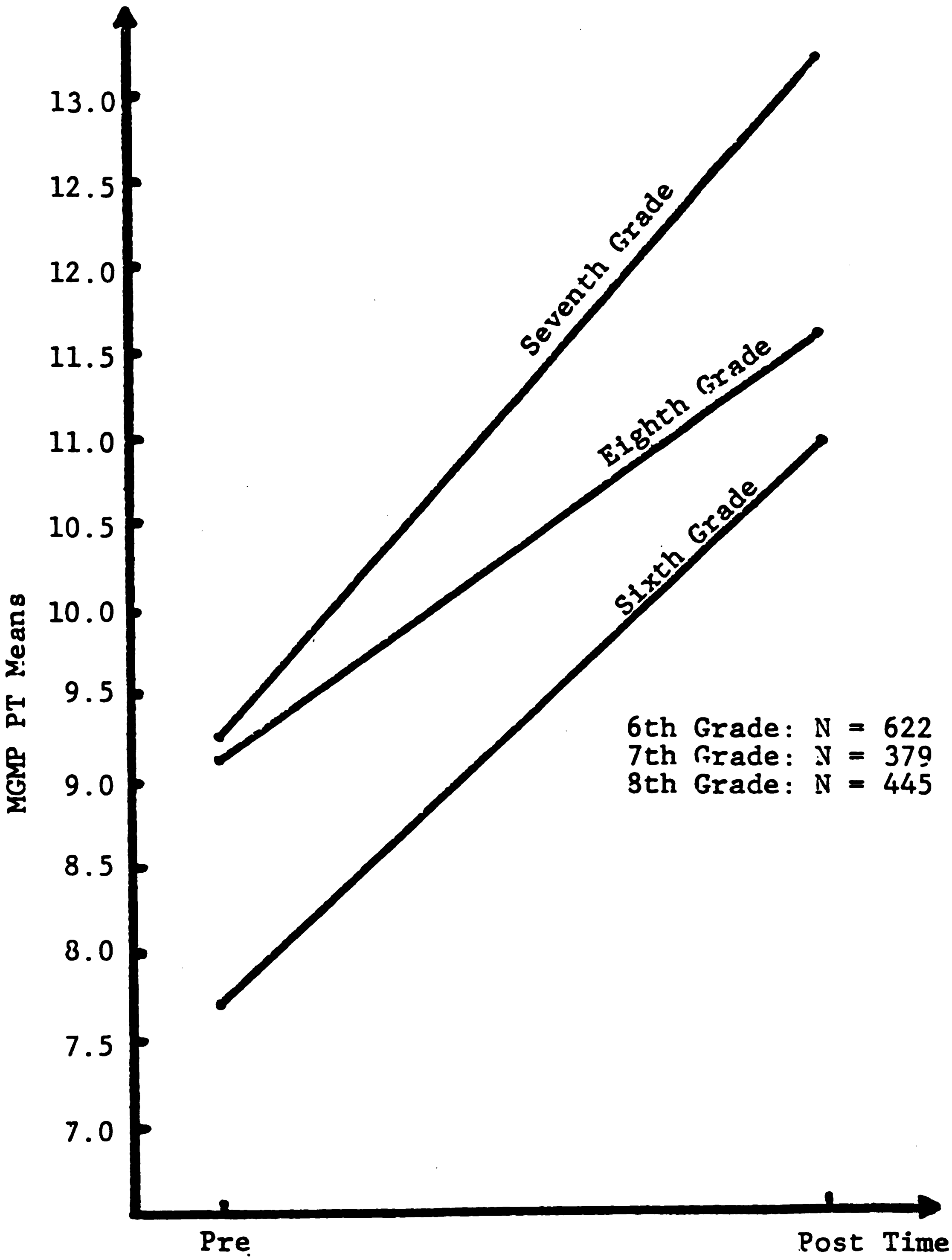


Fig. 4.16 Profiles of Pre-Post MGMP PT Means of Entire Sample by Grade.

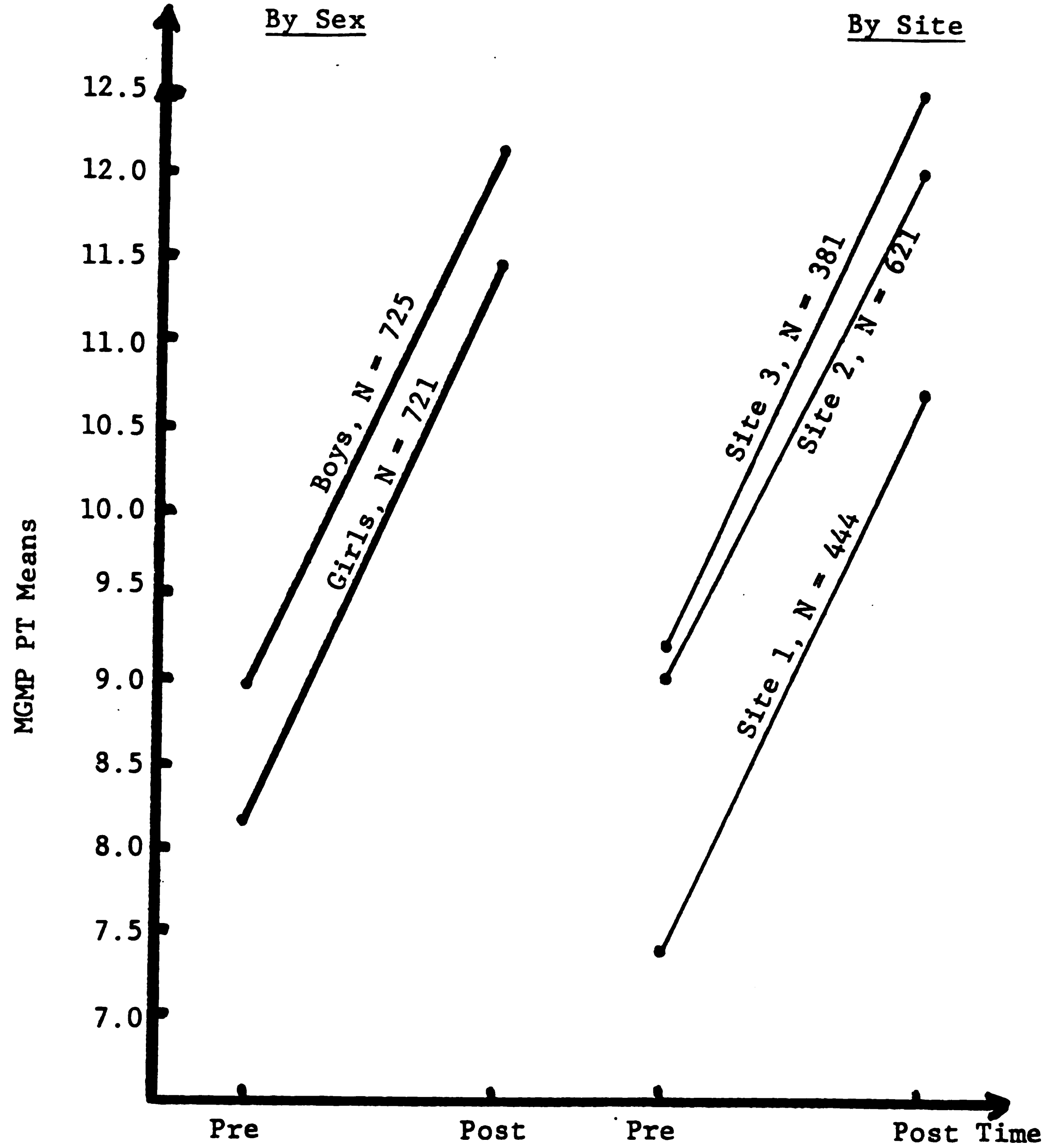


Fig. 4.17 Profiles of Pre-Post MGMP PT Means of Entire Sample by Sex by Site.

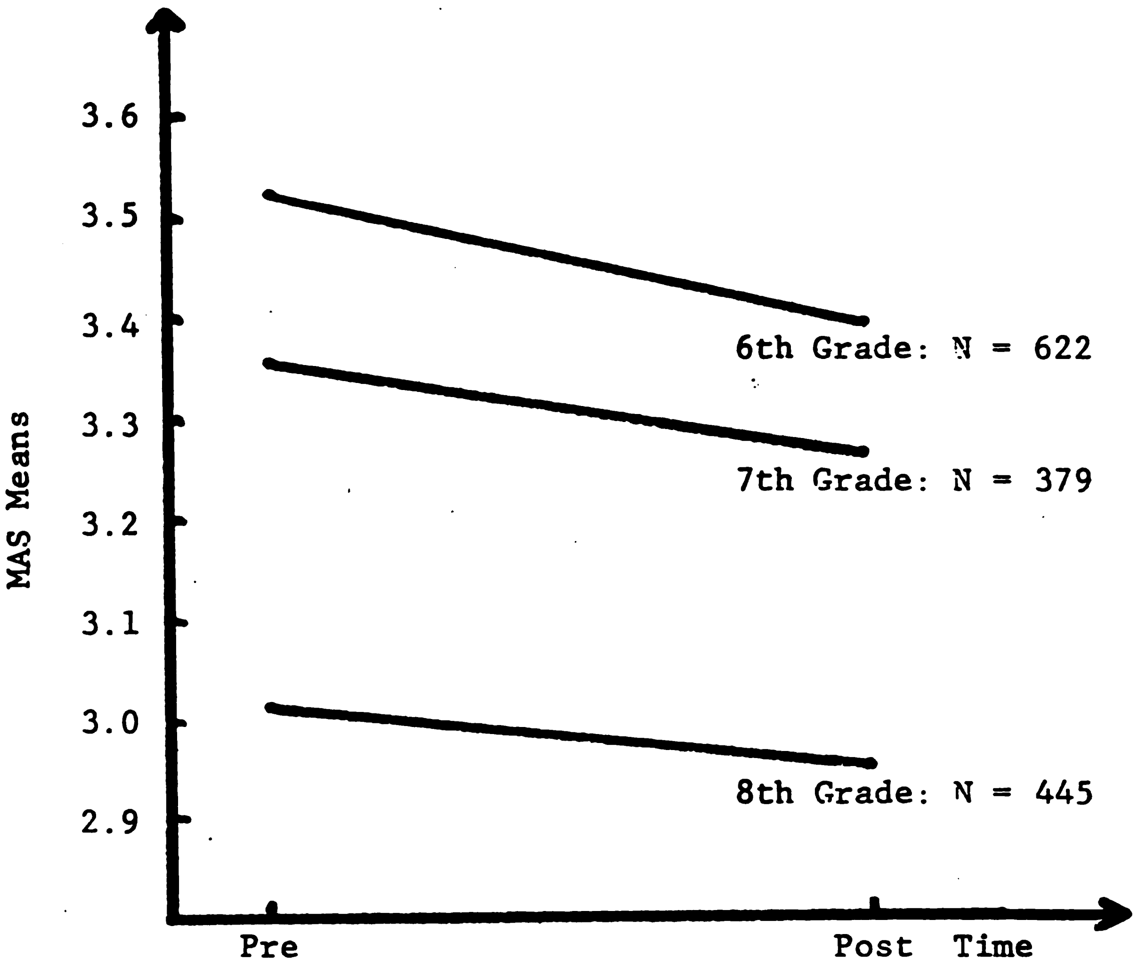


Fig. 4.18 Profiles of Pre-Post Mas Means of Entire Sample by Grade.

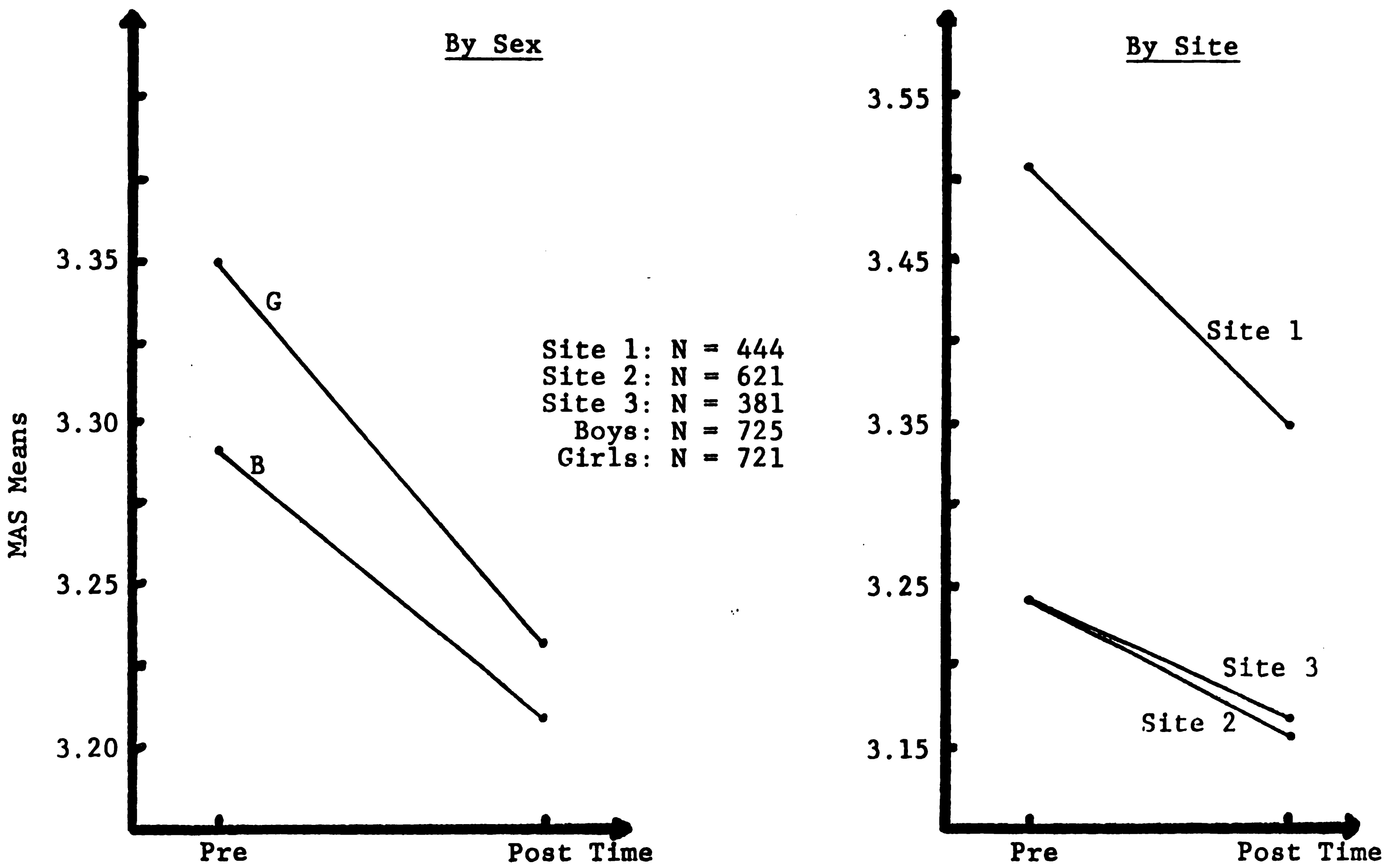


Fig. 4.19 Profiles of Pre-Post MAS Means of Entire Sample by Sex and by Site.

in attitudes toward mathematics. That is, in Site 1, no significant grade level differences were found in the MGMP PT mean gain scores, but were in the MAS mean scores. In Sites 2 and 3, significant grade level differences were found but in the MGMP PT mean gain scores only. In Site 2, these differences were detected between boys in grade levels seven and eight, between girls in grade levels six and eight, and between boys in grade levels six and eight, always in favour of the lower grade. In Site 3, these same grade level pairwise differences were also found to be statistically significant ($P < .05$), but gain differences between grades six and eight were considered not meaningfully significant. In addition, significant grade level MGMP Probability gain differences were found between boys and girls in grades six and seven, and in grades seven and eight always in favour of grade seven. Thus, in Site 3, the seventh graders gained more in MGMP PT scores than grades six and eight. In Sites 2 and 3, the least MGMP mean gain scores were recorded by eighth graders (Fig. 4.12).

Although average scores by time were not the focus of the null hypotheses tested in the study, interesting results were found in Sites 2 and 3. In both sites, seventh graders averaged significantly higher by time ($P < .05$) than sixth and eight graders in the MGMP Probability Test. In Site 3, seventh graders gained more from pretest to posttest and had higher average scores on the MGMP PT than sixth and eighth graders.

These results can be observed from the profiles in Figures 4.16, and 4.12, and also from Table 4.15.

From the profiles for the entire sample of mean gains and averages (Figs. 4.14, 4.15) in MGMP PT scores, and of pre-posttest means (Figs. 4.16 and 4.17), more comparisons can be made among the three sites, with respect to grade level differences in mean gains and averages. For the entire study, there were no site by time interactions in the MGMP PT mean scores (Fig. 4.17). In other words, the three sites differed in the same order in both the pretest and the posttest. However, Site 1 gained slightly more than Sites 2 and 3 (Fig. 4.14 and Table 4.12). These gain differences were small and therefore considered nonmeaningful. On the whole, grade seven gained more and averaged higher than grades six and eight (Figs. 4.15, 4.16, and Table 4.12).

The null hypothesis (H_{06}) of no difference between the mean gain scores for boys and for girls in grades six, seven and eight in both criterion measures was not rejected for the data from any site. This means that no significant sex differences per grade level were found in any site, with respect to gains in the MGMP PT scores and in Mathematics Attitude Scale. In every site, girls gained slightly higher than boys in the MGMP PT scores, but none of these was found to be significant (Table 4.12 and Fig. 4.13). In MGMP PT knowledge (averages) by time, boys significantly outperformed girls in Sites 2 and 3 but not in Site 1.

In conclusion, no significant differences were found between boys and girls, on the whole, with respect to gains in the MGMP PT and MAS mean scores. No meaningfully significant differences were found in the entire study in change in MAS scores by sex, by grade, or by site. Girls changed (dropped) more than boys in the MAS scores (Fig. 4.19), but these were not statistically significant.

Grade level differences in MGMP mean gains were found in Sites 2 and 3, but not in Site 1. In Sites 2 and 3, grade seven averaged significantly more than grades 6 and 8 in the MGMP PT scores. In Sites 2 and 3, grades six and seven gained more than grade eight, while in Site 3, grade seven gained significantly more than grades six and eight.

Finally, in the entire study, no meaningfully significant differences were found by sex, by grade, and by site, in attitude change toward mathematics. With respect to knowledge gain in the MGMP PT, no overall differences were found between boys and girls, or among the three sites, but grade seven significantly outperformed the other grades.

Comparison of Attitudes Toward Mathematics With Attitudes Toward Probability

Attitudes toward mathematics were compared by sex and by grade level with attitudes toward probability, after the instruction. A Two-way Analysis of Variance (ANOVA) design was employed in these comparisons and the data from each site was analysed separately for reasons earlier explained.

The null hypotheses tested within this design were:

H_{07}: There will be no difference between students' mean scores on the Mathematics Attitude Scale (MAS) and students' mean scores on the Probability Attitude Scale (PAS).

H_{08}: There will be no interaction of grade by sex among the mean difference scores--MAS score minus PAS score.

Site 1: The Urban Site

The means and standard deviations of posttest scores in Mathematics Attitude Scale (POSTMAS) and scores in the Probability Attitude Scale (PAS)* for Site 1 are included in Table 4.17. These statistics are tabulated by grade and by sex. Table 4.18 presents a summary of the Analysis of Variance (ANOVA) for the results of the test on the mean difference between POSTMAS and PAS scores.

From the ANOVA results, it was found that the interaction of grade by sex was not significant. The F values of the sex and grade main effects were also not significant.

Hence, both null hypotheses H_{07} and H_{08} were retained. This means that neither boys nor girls, nor the grade levels six, seven, and eight studied in Site 1 significantly differed in their attitudes toward mathematics (MAS) versus attitudes toward probability (PAS) after the probability instruction.

---

* PAS - The Probability Attitude Scale test had no pretest.

TABLE 4.17

MEANS AND STANDARD DEVIATIONS OF POSTMAS AND PAS SCORES FOR SITE 1 BY GRADE AND BY SEX

		POSTMAS[a]		PAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	149	3.258	.816	3.156	.869
Boys	76	3.241	.826	3.118	.937
Girls	73	3.276	.810	3.194	.796
Grade 7	138	3.377	.858	3.460	.797
Boys	66	3.407	.744	3.288	.787
Girls	72	3.350	.933	3.818	.778
Grade 8	157	3.416	.760	3.339	.828
Boys	74	3.453	.776	3.426	.735
Girls	83	3.384	.749	3.261	.901
Total	444	3.351	.811	3.315	.840
Boys	216	3.364	.791	3.276	.833
Girls	228	3.339	.828	3.352	.847

[a] POSTMAS - Posttest Mathematics Attitude Scale (Range 1-5).

[b] PAS - Probability Attitude Scale - no pretest for PAS (Range 1-5).

TABLE 4.18

ANALYSIS OF VARIANCE SUMMARY FOR MEAN DIFFERENCE BETWEEN THE POSTMAS[a] AND PAS[b] SCORES FOR SITE 1

Source of Variation	D.F.	MS	F	P
Grade	2	1.424	1.489	.227
Sex	1	1.114	1.165	.281
Grade x sex	2	2.233	2.335	.098
Between Groups	5			
Within Groups	438	.956		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
[b] PAS - Probability Attitude Scale.

Site 2: The Suburban Site

Table 4.19 includes the means and standard deviations of the POSTMAS and PAS scores for the data from Site 2 by grade and by sex. Table 4.20 presents a summary of the ANOVA results for the mean difference between the POSTMAS and PAS scores. These (ANOVA) results indicated that both the interaction of grade by sex and the sex main effects were not significant. Hence, the null hypothesis (H_{08}) was not rejected. However, the grade level main effect was significant. The results of the planned comparisons, G_2 (grade 8 versus grades 6 and 7), and G_1 (grade 7 versus grade 6), were significant ($P < .05$) in favour of G_2. The null hypothesis (H_{07}) was therefore rejected. The conclusion was that grades 6 and 7 did not differ significantly from each other in their attitudes to mathematics and probability but together differed from grade 8 in these attitudes. Further, Table 4.19 indicates that, in Site 2, sixth and seventh graders preferred mathematics to probability, but eighth graders did not.

Site 3: The Rural Site

The means and standard deviations of the POSTMAS and PAS scores for Site 3 are presented by grade and by sex in Table 4.21. A summary of the ANOVA results for the mean difference between the POSTMAS and PAS scores is contained in Table 4.22. The results of the ANOVA were similar to

TABLE 4.19

MEANS AND STANDARD DEVIATIONS OF POSTMAS AND PAS SCORES FOR SITE 2 BY GRADE AND BY SEX

		POSTMAS[a]		PAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	340	3.404	.930	2.808	.943
Boys	162	3.360	.995	2.840	1.004
Girls	178	3.444	.869	2.780	.886
Grade 7	101	3.320	.850	2.790	.923
Boys	53	3.547	.945	2.937	1.011
Girls	48	3.069	.653	2.629	.793
Grade 8	180	2.613	.824	2.285	.839
Boys	102	2.601	.819	2.335	.849
Girls	78	2.628	.836	2.220	.827
Total	621	3.161	.954	2.654	.939
Boys	317	3.147	1.006	2.694	.987
Girls	304	3.175	.898	2.612	.887

[a] POSTMAS - Posttest Mathematics Attitude Scale (Range 1-5).

[b] PAS - Probability Attitude Scale - no pretest for PAS (Range 1-5).

TABLE 4.20

ANALYSIS OF VARIANCE SUMMARY FOR MEAN DIFFERENCE[a] BETWEEN THE POSTMAS[b] AND PAS[c] SCORES FOR SITE 2

Source of Variation	D.F.	MS	F	P
Grade**	2			
G_2[d]	(1)	3.15	5.83*	.016
G_1[e]	(1)	1.10	2.05	.153
Sex	1	1.30	1.20	.273
Grade x sex	2	1.03	.95	.388
Between Groups	5			
Within Groups	615	.540		

a The corresponding ANOVA Table of Averages is in Appendix D.9.
b POSTMAS - Posttest Mathematics Attitude Scale.
c PAS - Probability Attitude Scale.
d G_2 - Contrast of Grade 8 versus Grades 6 and 7.
e G_1 - Contrast of Grade 7 versus Grade 6.
* Significant P < .05.
** Table D.6 in Appendix D includes the alternative ANOVA test without grade level contrasts.

TABLE 4.21

MEANS AND STANDARD DEVIATIONS OF POSTMAS AND PAS SCORES FOR SITE 3 BY GRADE AND BY SEX

		POSTMAS[a]		PAS[b]	
Grade	N	M	S.D.	M	S.D.
Grade 6	133	3.496	.792	3.440	.863
Boys	70	3.507	.805	3.469	.885
Girls	63	3.489	.783	3.407	.843
Grade 7	140	3.096	.837	2.828	.826
Boys	65	3.085	.782	2.805	.825
Girls	75	33.107	.886	2.849	.831
Grade 8	108	2.843	.761	2.968	.866
Boys	57	2.766	.827	2.927	.977
Girls	51	2.928	.678	3.013	.729
Total	381	3.164	.841	3.081	.889
Boys	192	3.142	.855	3.083	.938
Girls	189	3.186	.828	3.079	.840

[a] POSTMAS - Posttest Mathematics Attitude Scale (Range 1-5).

[b] PAS - Probability Attitude Scale - no pretest for PAS (Range 1-5).

TABLE 4.22

ANALYSIS OF VARIANCE SUMMARY FOR MEAN DIFFERENCE[a] BETWEEN THE POSTMAS[b] AND PAS[c] SCORES FOR SITE 3

Source of Variation	D.F.	MS	F	P
Grade**	2			
G_2[d]	(1)	3.25	6.91*	.009
G_1[e]	(1)	1.53	3.24	.073
Sex	1	.09	.09	.759
Grade x sex	2	.08	.09	.918
Between Groups	5			
Within Groups	375	.470		

a The corresponding ANOVA Table of Averages is in Appendix D.9.
b POSTMAS - Posttest Mathematics Attitude Scale.
c PAS - Probability Attitude Scale.
d G_2 - Contrast of Grade 8 versus Grades 6 and 7.
e G_1 - Contrast of Grade 7 versus Grade 6.
* Significant $P < .01$.
** Table D.7 in Appendix D includes the alternative ANOVA test without grade level contrasts.

those of Site 2. No significant interaction of grade by sex was found. Hence the null hypothesis (H_{08}), of no interaction of grade by sex, was retained. However, grade level differences were found ($P < .01$) between grade 8 versus grades 6 and 7 (contrast G_2). Thus, the null hypothesis (H_{07}) was rejected. Hence, statistically, attitudes to mathematics differed from attitudes to probability among the three grade levels. However, from a close comparison of the PASTMAS and PAS means (Table 4.21), one might conclude that these grade level differences in attitudes toward mathematics versus attitudes toward probability were not meaningful.

In conclusion, the null hypothesis (H_{08}) was retained for the data in Site 2 and 3. There was no evidence of significant interactions of grade by sex in attitudes toward mathematics and probability in Site 2 and 3, but interactions were present in Site 1. In all sites, boys and girls did not differ in their attitudes toward mathematics versus attitudes toward probability. Hence, the hypothesis (H_{07}) was retained.* In Sites 2 and 3, the three grade levels studied differed in these comparisons. However, in Site 3, these differences were considered nonmeaningful. In Site 2, sixth and seventh graders preferred mathematics to probability, while eighth graders did not (as measured by the Mathematics and Probability Attitude Scales.

* Table D.5 in Appendix D contains the ANOVA table for mean difference between the POSTMAS and PAS for the entire sample.

Sex and Grade Level Differences in Attitudes Toward Probability

The attitudes of boys were compared with attitudes of girls toward probabilty after instruction. The attitudes of the grade levels (six, seven, and eight) studied were similarly compared with one another. To examine these differences in attitudes toward probability after instruction a Two-Way ANOVA design was used. The following two null hypotheses were tested separately for the data from each site:

H_{09}: There will be no difference between the mean scores for boys and for girls in grades six, seven, and eight on the Probability Attitude Scale.

H_{10}: There will be no difference between the mean scores for each of the three grade levels (six, seven and eight) on the Probability Attitude Scale.

Table 4.23 includes the means and standard deviations of the PAS scores for Site 1, Site 2, and Site 3, by grade and by sex. Tables 4.24, 4.25 and 4.26 respectively present a summary of the ANOVA for Sites 1, 2, and 3, by grade and by sex, for the results of the tests on attitudes toward probability.

Site 1: The Urban Site

The results of the ANOVA indicated that the interaction of grade by sex was significant ($P < .05$). Hence, the tests for the grade and sex main effects were confounded in the interaction between the two effects. However, from the profiles of PAS mean scores by grade and by sex (Fig. 4.20),

TABLE 4.23

MEANS AND STANDARD DEVIATIONS OF PAS[a] SCORES
BY SITE BY SEX BY GRADE

Grade	Site 1			Site 2			Site 3		
	N	M	S.D.	N	M	S.D.	N	M.	S.D.
Grade 6	149	3.156	.869	340	2.808	.943	133	3.440	.863
Boys	76	3.118	.937	162	2.840	1.004	70	3.469	.885
Girls	73	3.194	.796	178	2.780	.886	63	3.407	.843
Grade 7	138	3.460	.797	101	2.790	.923	140	2.828	.826
Boys	66	3.288	.787	53	2.937	1.011	65	2.805	.825
Girls	72	3.618	.778	48	2.629	.793	75	2.849	.831
Grade 8	157	3.339	.828	180	2.285	.839	108	2.968	.866
Boys	74	3.426	.735	102	2.335	.849	57	2.927	.977
Girls	83	3.261	.901	78	2.220	.827	51	3.013	.729
Total	444	3.315	.840	621	2.654	.939	381	3.081	.889
Boys	216	3.276	.833	317	2.694	.987	192	3.083	.938
Girls	228	3.352	.847	304	2.612	.887	189	3.079	.840

[a] PAS - Probability Attitude Scale (Range 1-5).

TABLE 4.24

ANALYSIS OF VARIANCE SUMMARY TABLE FOR PAS[a] SCORES IN SITE 1

Source of Variation	D.F.	MS	F	P
Grade	2	3.336	4.861*	.008
Sex	1	.542	.789	.375
Grade x sex	2	2.243	3.268*	.039
Between Groups	5			
Within Groups	438	.686		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
* Significant P < .01.

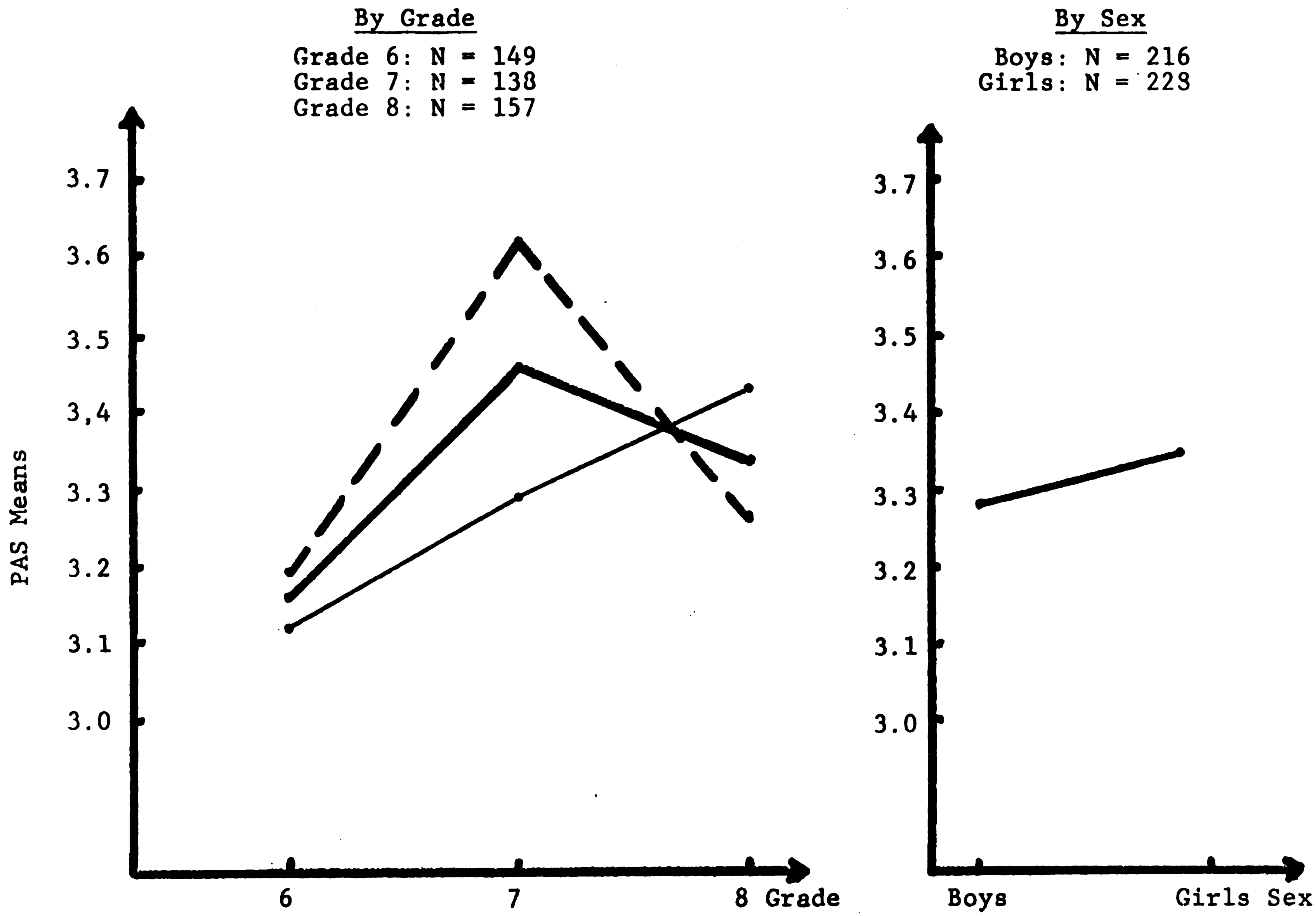


Fig. 4.20 Profiles of Means of PAS Scores by Grade and by Sex in Site 1.

TABLE 4.25

ANALYSIS OF VARIANCE SUMMARY TABLE FOR PAS[a] SCORES IN SITE 2

Source of Variation	D.F.	MS	F	P
Grade	2	17.760	21.445*	.001
Sex	1	2.078	2.510	.114
Grade x sex	2	.603	.728	.484
Between Groups	5			
Within Groups	615	.828		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
* Significant $P < .01$.

TABLE 4.26

ANALYSIS OF VARIANCE SUMMARY TABLE FOR PAS[a] SCORES IN SITE 3

Source of Variation	D.F.	MS	F	P
Grade	2	13.735	18.883*	.001
Sex	1	.034	.047	.829
Grade x sex	2	.179	.246	.782
Between Groups	5			
Within Groups	375	.727		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
* Significant $P < .01$.

it was observed that sex differences were not significantly plausible. Hence, the null hypothesis H_{09} was tenable and retained. The conclusion was that, in Site 1, boys and girls did not differ in their attitudes to the MGMP Probability as measured by the PAS.

Differences were observed among the means for grades six, seven, and eight (Fig. 4.20), with grade seven recording the highest mean. Scheffe's Post Hoc posteriori comparisons (Appendix E, Table E.7) showed the significant difference to be confined to between grades six and seven girls, in favour of the latter. Hence, the null hypothesis H_{10} was rejected ($P < .01$). The conclusion was that in Site 1, seventh graders differed significantly from sixth graders in their attitudes toward the MGMP Probabilty as measured by the PAS.

Site 2: The Suburban Site

The results of the ANOVA indicated no significant interaction of grade by sex. The sex main effects were also not significant (Table 4.25). The null hypothesis H_{09} was therefore retained. Only the grade main effects were significant ($P < .01$). Hence, the null hypothesis H_{10} was rejected. An examination of the PAS means by grade (Table E.8, Appendix E, and Table 4.23) indicated grade six to be significantly higher than grade eight. This means that, in Site 2, eighth graders had less positive attitudes to the MGMP Probability than sixth graders, as measured by the PAS.

Site 3: The Rural Site

The results of the ANOVA for this site were similar to those of Site 2. No significant interaction of grade by sex, or of the sex main effects were indicated. Hence, the null hypothesis H_{09} was retained. However, grade level differences in attitudes toward the MGMP Probabilty were significant ($P < .01$) among grade levels six, seven, and eight. Table 4.23 and the post hoc tests (Appendix E, Table E.9) show that in Site 3, sixth graders were significantly higher than seventh and eighth graders in attitudes toward probability. The highest differences were observed between boys in grades six and seven, in favour of the former.

In conclusion, in each of the three sites studied, the two null hypotheses (H_{09} and H_{10}) were retained and rejected respectively. The interpretation of this was that, in the entire study, boys and girls did not differ in their attitudes toward probability, but the grade levels did. Table D.9 in Appendix D shows the ANOVA table for the three sites conbined. In Site 1, the urban site, grade seven recorded the highest attitudes to probability. On the contrary, the same grade level recorded the lowest attitudes to probability in Site 3. In general, sites 1 and 3 appeared to have demonstrated more favorable attitudes to probability than Site 2.

Summary of Findings

The statistical analysis of the data collected in this investigation was presented in this chapter. The following were the major findings.

Prior to Instruction: Using MANOVA on sex and grade level differences in probability knowledge and attitudes toward mathematics, the following results were found.

1. The null hypothesis (H_{01}) was rejected in each site. In Site 1, significant differences were found between girls in grade levels six and seven in their existing knowledge in probability prior to instruction. Attitude differences to mathematics between boys and girls, or among the grade levels (six, seven, and eight), were not found in Site 1.
2. In Site 2, grade seven was found to be significantly different from grade six in both probability knowledge and mathematics attitudes. While the probability knowledge favoured the seventh graders, the reverse was the case in attitudes toward mathematics. Seventh graders were also found to be significantly superior to eighth graders in both probability knowledge and attitudes toward mathematics.
3. In Site 3, grade level differences in probability were significant between girls in grades six and eight, favouring grade eight. However, the differences

between the girls were considered nonmeaningful.

4. Sex differences were not found in Site 1 in either probability knowledge or mathematics attitudes.
5. In Site 2, seventh and eighth grade boys were superior to the respective grade level girls in probability knowledge, prior to instruction. No sex differences were found in Site 2 in attitudes toward mathematics.
6. Sex differences were not found in Site 3 with respect to mathematics attitudes. In probability knowledge, boys in grade six outperformed girls.
7. In all these sites, interactions of grade by sex were not significant. Also, sex differences were not found in attitudes toward mathematics.

After Instruction: The Multivariate Analysis of Repeated Measures was used to test the effect of instruction as stated in the null hypotheses (H_{04}-H_{06}). To compare attitudes toward mathematics with attitudes toward probability by grade and by sex, (H_{07} and H_{08}), ANOVA was used. Lastly, to compare attitudes to probability by grade and by sex, (H_{09-10}), ANOVA was also used. The following were the results found after the instruction.

8. In all sites, time effect was found to be significant ($P < .0001$) for all subjects. In other words, a significant effect of the instruction was found from

pretest to posttest, in both probability and attitudes toward mathematics. Gains over time in the MGMP Probability Test and change over time in attitudes toward mathematics (as measured by the MAS) were, on the whole, significant. However, all attitude changes were not considered meaningful.

9. Sex differences were also not found in probability gains from the instruction and in attitude change toward mathematics over the time period.
10. In general, grade level differences were found in probability knowledge over time (pre + posttest), in probability gains over time (posttest - pretest), and in attitude change to mathematics over time.
11. In Site 1, grade level differences in probability gains were not found, but significant differences were found between girls in grades six and seven in favour of grade seven. Due to inordinal interactions of probability knowledge (over time) between boys and girls among the three grade levels, significant grade level differences were not found in probability knowledge over time.
12. In Site 2, sixth and seventh graders gained equally from the probability instruction, but each outgained the eighth graders. Boys consistently outscored girls in probability knowledge (over time) across all grade levels, but this was significant only in grade seven. On the other hand, girls consistently outgained boys

in scores on the pre-post probability test, but these were not statistically significant.

13. While grade eight gained least from the probability instruction in Site 3, grade seven gained most. Grade seven also demonstrated more probability knowledge (over time) than grades six and eight.
14. In the entire study, all three sites did not differ significantly from one another in either probability gains over time or in attitude change toward mathematics. Boys and girls, and all grade levels, changed equally in mathematics attitudes. Boys and girls gained equally, and grade seven outgained the other grades, on the probability unit test pre-post.
15. With respect to attitude comparison to mathematics and to probability, no sex differences were found by site and by grade level.
16. Grade level differences were found in Sites 2 and 3 in attitudes to mathematics versus attitudes to probability. In Site 2, sixth and seventh graders preferred mathematics to probability, while the eighth graders had no preference.
17. In the comparison of attitudes to probability, boys and girls did not differ in any grade or site.
18. In Site 1, grade seven had significantly higher attitudes to probability than grades six and eight. On the contrary, the same grade level recorded the lowest attitude to probability in Site 3. On the whole,

attitudes to probability were more favourable in Sites 1 and 3 than in Site 2, while grade seven, on the whole, recorded higher liking of probability than any other grade level.

CHAPTER V

SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

The problem addressed in this investigation, the rationale, and delimitations of the study, were all introduced in the first chapter. In the second chapter, the review of related literature was given. The third chapter described the methodology of the study. In the fourth chapter, the analysis of the data and the interpretation of the findings were reported.

This final chapter, the fifth, contains the following:

1. Summary and Findings.
2. Conclusions and Discussion.
3. Implications for Mathematics Education.
4. Recommendations for Future Research.

Summary and Findings

This section contains a summary of the purpose of the study, research questions, related literature, methodology, hypotheses, and findings.

Purpose of the Study

This study had four related purposes. The first purpose was to determine any existing differences in probability knowledge and in attitudes toward mathematics of grades six through eight students by sex and by grade level prior to a formal probability instruction. The second purpose was to examine the effect of instruction on the probability achievement and on attitudes toward mathematics of these students. The third purpose was to compare attitudes toward mathematics with attitudes toward probability by sex and by grade level. The fourth purpose was to compare attitudes toward probability by sex and by grade level.

Research Questions

Two sets of related questions were considered in this study. The first set of questions, called type A, dealt with the existing (prior to instruction) differences in probability knowledge and attitudes toward mathematics of grades six, seven, and eight students, by sex, by grade level and by school setting. The second set of questions, type B, dealt with effects of instruction in three parts. The first part focussed on the effects of instruction on the probability knowledge and on attitudes toward mathematics of the same students. The second part concerned differences, by grade level and by sex, in attitudes toward mathematics versus attitudes toward probability, after instruction. The last part was a comparison of attitudes toward probability after instruction, by grade level and by sex. These questions were all examined for each of three school settings (Sites 1, 2, and 3). Type A questions included the following:

1. What effect, if any, does grade level have on knowledge of probability and/or on attitudes toward mathematics?
2. What effect, if any, does sex have on knowledge of probability and/or on attitudes toward mathematics?
3. Do differences between boys and girls in knowledge of probability skills and/or in attitudes toward mathematics change with grade level?
4. What effect, if any, does school setting have on knowledge of probability and/or on attiudes toward mathmatics?

Type B Questions

After instructional intervention:

1. What effect, if any, will probability instructional intervention have on achievement in probability tasks and/or on attitudes toward mathematics of sixth, seventh and eighth grade students? Will these effects be different for boys and girls? Will these effects differ by grade level? Will the effects differ by school setting?
2. Do differences exist between students' attitudes toward mathematics in general and the probability activities in

particular? Will these differences exist for both sexes? Will these differences exist for each grade level in the study? Will these differences exist for each of the sites 1, 2, and 3?

3. Do differences exist between the sexes in attitudes toward probability activities? Will these differences exist for each grade level? For each site?
4. Do differences exist among the three school settings in their attitudes toward probability activities? Will these differences exist for each grade level? Will they exist for each sex?

Related Literature

The review of the literature revealed how a number of investigators have concluded scientifically that elementary and middle grades boys and girls possess some knowledge of probability concepts prior to formal instruction. Further review indicated that Piaget propounded the existence of three stages of probability development just as his widely known four stages of mental maturation. Other researchers have also concurred with Piaget's assertion. Thus, there is a popular suggestion in the literature that the amount of probability concepts possessed by a pupil, and the level of probability sophistication the pupil is ready to learn, are a function of the child's stage of probability development. A corrolary to this assertion is the expectation of grade level differences in probability gains, in favour of the higher grade level. Also, probability achievement in favour of higher I.Q., is another popular assertion made frequently in the literature.

Only a relatively few investigations on sex differences in probability were detected by the investigator. In general, the literature reviewed tended to conclude that there are no significant sex differences in existing probability ability or gains from instruction. However, a few studies have suggested sex differences in favour of boys, prior to instruction. Further, girls tended to gain more than boys from probability interventions. Consequently, only one study (reviewed) reported the persistence of any sex differences from posttest measures.

Quite a few probability curriculum innovations were identified in the literature. Of these, many claimed that people of all ages tend to commit certain errors in probabilistic thinking. These errors, according to these investigators, tend to interact with the learning of probabilistic concepts. However, research evidence abounds that boys and girls do benefit significantly from probability intervention.

The issue of sex differences in mathematics is widely addressed in the literature. However, although most of these studies tend to conclude in favour of boys, there is disagreement as to the nature or extent of these differences. Renown researchers have explained mathematics achievement differences in terms of environmental conditions, while equally disciplined investigators have adduced heredity for their explanation. Thus, the issue of the cause of sex differences in mathematics achievements is essentially the issue of nurture versus nature.

The review of literature related to attitude studies suggested that earlier investigators tended to conclude in favour of a positive relationship between attitude and mathematics achievement. However, more contemporary studies have challenged this assertion. They have suggested that such other factors as teacher quality, classroom cohesiveness and student fatalism correlate as strongly as attitude, with mathematics achievement.

Methodology

The probability intervention employed in the study took place during Fall 1982 and Winter 1983; while data collection took place between October 1982 and January 1983. The information collected included pretest and posttest measures in probability performance and in attitudes toward mathematics, and attitudes toward probability (posttest measures only). All posttest measures were the same as the corresponding pretest measures.

The subjects of the study were grade six, seven and eight students from six different schools, situated in three distinct sites; two schools per site. One of these schools was an elementary school, three were middle schools and two were junior high schools. The three sites were categorized into urban, suburban and rural settings and were respectively referred to as Site 1, Site 2 and Site 3 throughout this study. From Site 1, the urban setting, 444 subjects took part. From the suburban setting, Site 2, 621 subjects participated while 381 subjects took part in the study from Site 3, the rural setting. Of the 1460 subjects involved in the study, 1446 took part consistently in the data analyses. The difference of 14, which was highly insignificant, was due to random omission. There were 622 sixth graders, 379 seventh graders, 445 eighth graders, 725 boys, and 721 girls who took part in the entire study.

Instrumentation

The instruments used in this study were two semantic differential attitude scales and a performance test on probability ability. The two attitude scales were called the Mathematics Attitude Scale (MAS) and the Probability Attitude Scale (PAS). Throughout the study these two attitude scales were referred to as MAS and PAS, while the performance test instrument was called the Middle Grades Mathematics Project Probability Test (MGMP PT). Both attitude scales were simple six-item bipolar semantic differentials, with five response options. The MGMP PT consisted of 25 multiple choice items. Appropriate test instructions and conditions were provided and the teachers involved invigilated all tests. Appendix B includes these tests and test instructions.

The Probability Unit included ten sequentially developed activities requiring about three weeks of instructional time. The activities of the unit included the following: State lottery, three activities of fair and unfair games, surveys, area models, expected value, newspapers pay, Jonesville families and Pascal's Triangle. The Probability Unit employed an instructional model consisting of three phases: launching, exploring, and summarizing. Each teacher was provided with a detailed instructional guide which was presumably followed throughout each probability activity.

The first five activities strictly involved determining probabilities of independent events. The last five activities treated a variety of probabilty concepts. These activities included the calculation of probabilities from geometric shapes, the use of probabilities to make predictions, the calculation of expected values, and simulation and analysis of theoretical problems. Binomial probabilities were also introduced.

Hypotheses and Design

Several statistical procedures were selected to analyze the data collected during the study. Particular statistics were chosen to test the hypotheses given in the following paragraphs. The analyses included the following: means, standard deviations, correlations, multivariate and univariate analyses of variance and repeated measures.

Planned comparisons and Scheffe's Post Hoc comparison were also selected. All these analyses were carried out on the 3600 Computer at the Michigan State University Computer Center, using the Statistical Package for the Social Sciences (SPSS).

The multivariate model with a two-way fixed effects analysis of variance (MANOVA) was used to assess the differences in probability knowledge and attitudes toward mathematics, by grade level and sex, prior to the instruction.

The hypotheses tested within this design, given in null form, were:

H_{01}: There will be no difference among the mean scores for each of the three grade levels (six, seven, and eight) tested, on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitudes Scale.

H_{02}: There will be no difference between the mean scores for boys and for girls in grades six through eight on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

H_{03}: There will be no interaction of grade by sex among the mean scores for sixth through eighth grade students on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

To analyze the effects of instruction on the probability skills, differences in probability and attitudes toward mathematics, by sex and grade level, the Multivariate Analysis of Repeated Measures Model was selected.

The hypotheses tested with this design, given in null form were:

H_{04}: There will be no difference between the posttest means and the pretest means of sixth, seventh, and eighth grade students on both the Middle Grade Mathematics Project Probability Test and on the Mathematics Attitudes Scale.

H_{05}: There will be no difference between the mean gain scores (posttest minus pretest) for each of the three grade levels tested, six, seven, and eight on both the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

H_{06}: There will be no difference between the mean gain scores (posttest minus pretest) for boys and for girls in grades six, seven, and eight on the Middle Grades Mathematics Project Probability Test and on the Mathematics Attitude Scale.

To compare the attitudes toward mathematics and probability by sex and by grade after instruction, a Two-Way ANOVA design was used. The hypotheses tested within this design for each site were:

H_{07}: There will be no difference between students' mean scores on the Mathematics Attitude Scale (MAS) and students' mean scores on the Probability Attitude Scale (PAS).

H_{08}: There will be no interaction of grade by sex among the mean difference scores--Mas score munus PAS score.

A Two-Way ANOVA design was also applied in order to examine sex and grade level differences in attitudes toward probability. The hypotheses tested within this design in each site were:

H_{09}: There will be no significant difference between the mean scores for boys and for girls in grades six, seven, and eight on the Probability Attitude Scale.

H_{10}: There will no difference between the mean scores for each of the three grade levels (six, seven and eight) on the Probability Attitude Scale.

Planned comparisons for the grade effect and Scheffe's Post Hoc comparisons were used to identify the sources of significant main effects or interactions. The .05 level of significant was the limit accepted in testing all the hypotheses in the study.

Findings and Conclusions

The statistical analysis of the data collected in this investigation was presented in Chapter IV. The following were the major findings.

Prior to Instruction: Using MANOVA on sex and grade level differences in probability knowledge and attitudes toward mathematics, the following results were found.

1. The null hypothesis (H_{01}) was rejected in each site. In Site 1, significant differences were found between girls in grade levels six and seven in their existing knowledge in probability prior to instruction. Attitude differences to mathematics between boys and girls, or among the grade levels (six, seven, and eight) were not found in Site 1. The MGMP PT means for grades six and seven girls were 6.27 and 7.78 respectively. (All pairwise mean differences can be estimated from the profiles of probability pretest means of all sites by grade level by sex in Fig. 5.1).
2. In Site 2, grade seven was found to be significantly different from grade six in both probability knowledge and mathematics attitudes. While the probability knowledge favoured the seventh graders, the reverse was

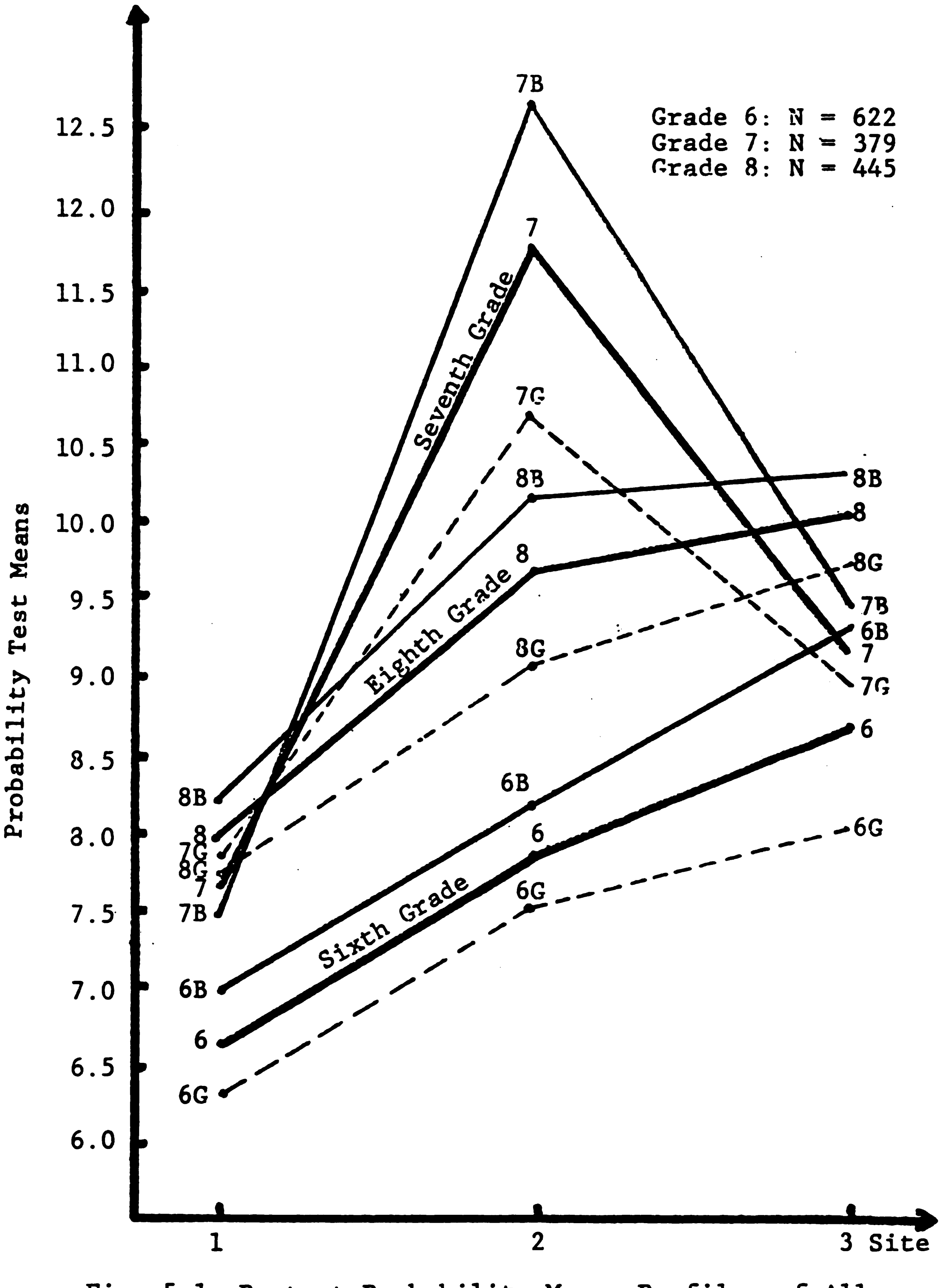


Fig. 5.1 Pretest Probability Means-Profiles of All Sites by Grade Level by Sex.

the case in attitudes toward mathematics. Seventh graders were also found to be significantly superior to eight graders in both probability knowledge and attitudes toward mathematics. The means for grades six, seven, and eight were respectively 7.80, 11.71, and 9.66 for the PT; and 3.51, 3.41, and 2.62 for the PREMAS scores. The post hoc comparison restricted the attitude differences to differences between girls in grades six and seven, favouring grade six. As measured by the MAS, these attitude means were 3.59 and 3.21. Also in this site, the suburban setting, boys and girls in grade seven outperformed boys and girls in grade eight in both probability knowledge and mathematics attitude. Hence, in the suburban site, attitudes toward mathematics dropped with grade level or with age. With respect to probability knowledge prior to instruction, seventh graders unexpectedly outscored eighth graders.

3. In Site 3, the rural site, grade level differences in probability were significant between girls in grades six and eight, favouring grade eight. The mean scores were 7.95 and 9.69. The difference was, however, only 1.73, and was considered nonmeaningful. With respect to mathematics attitudes in Site 3, the means for grades 6, 7, and 8 were respectively 3.45, 3.21, and 3.01. Hence, although these differences were not significant, a gradual drop in mathematics attitudes with grade level

was observed like in Site 2. Fig. 5.2 shows an ordinal interaction by grade level.

4. No significant sex differences were found in all sites in attitudes toward mathematics, as measured by the MAS, prior to instruction. However, sex differences in probability knowledge were found in Sites 2 and 3, but not in Site 1. From further tests, all these significant differences favoured boys. In Site 2, the suburban site, these differences occurred in grades seven and eight. In Site 3, the rural site, they occurred in grade six only. These findings are observable from the profiles of probability means by grade level and by sex in Fig. 5.1.
5. In all the sites, interactions of grade by sex were not significant. Thus, the sex and grade level main effect tests were plausible.

After Instruction: The Multivariate Analysis of Repeated Measures was used to test the effect of instruction as stated in the null hypotheses (H_{04}-H_{06}). To compare attitudes toward mathematics with attitudes toward probability by grade and by sex, (H_{07} and H_{08}), ANOVA was used. Lastly, to compare attitudes to probability by grade and by sex, (H_{09}-H_{10}), ANOVA was also used. The following were the results found to be significant.

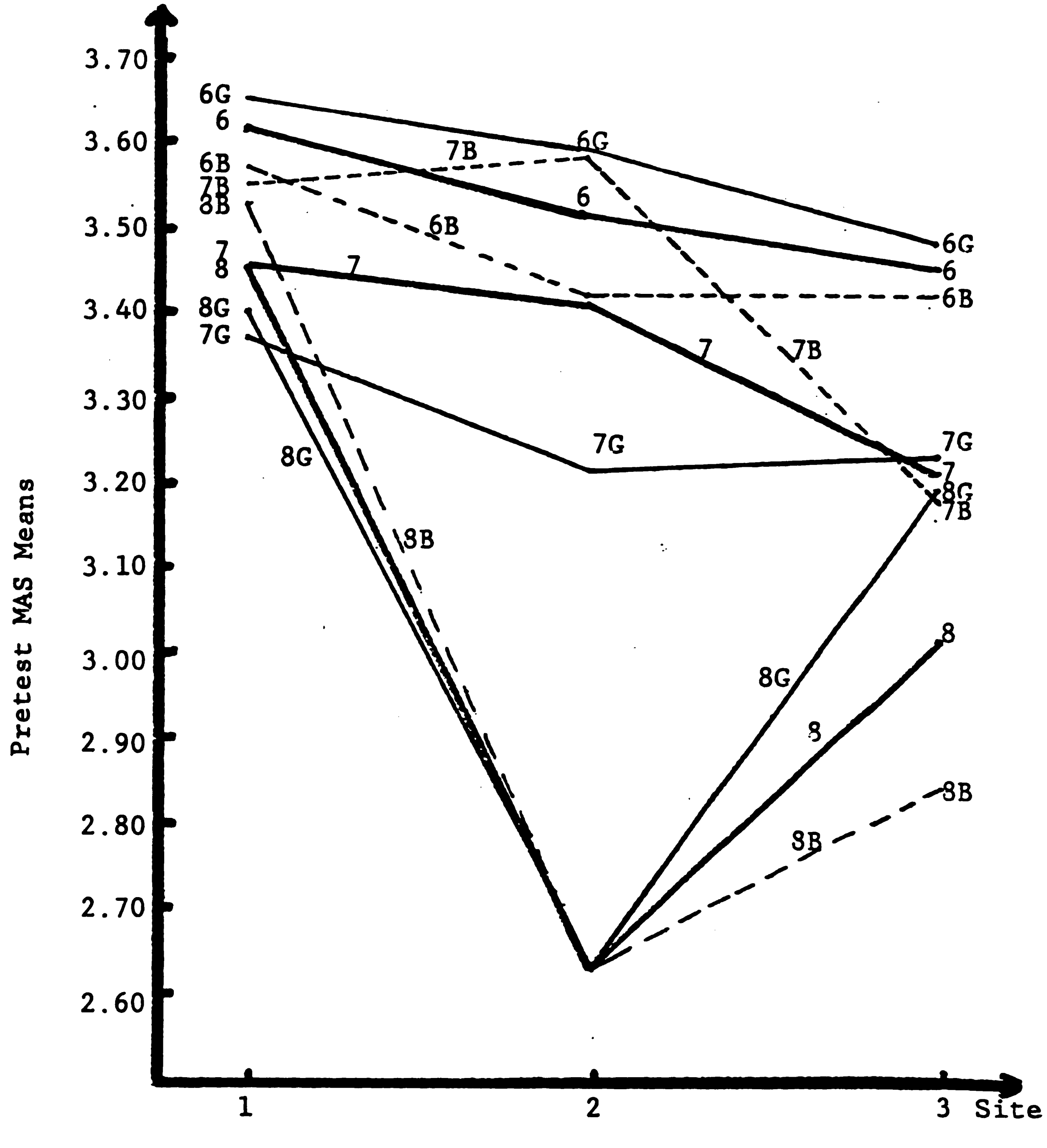
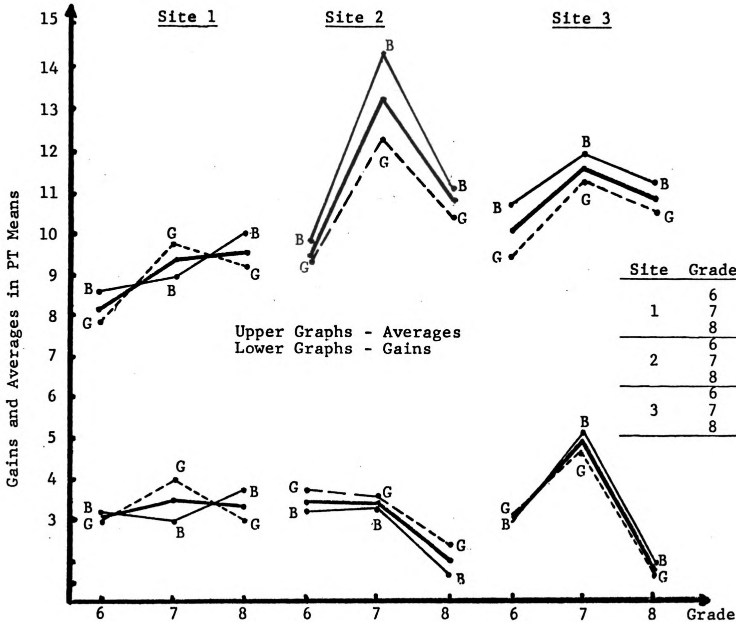


Fig. 5.2 Profiles of PREMAS Means by Site by Sex and by Grade Level.

6. In all sites, time effect was found to be significant ($P < .0001$) for all subjects. In other words, a significant effect of the instruction was found from pretest to posttest, in both probability and attitudes toward mathematics. Gains over time in the MGMP Probability Test and change over time in attitudes toward mathematics (as measured by the MAS) were, on the whole, significant. Fig. 5.3 summarizes all probability gains and averages. However, all attitude changes were not considered meaningful. The probability mean gains over time were 3.32, 3.06, and 3.26, in Sites 1, 2, and 3 respectively, while the averages over time were respectively 9.05, 10.51, and 10.83. Hence, although Site 1 averaged (knew) less probability than Sites 2 and 3, it outgained them. Attitude change to mathematics was also significant over time. However, as discussed in Chapter IV, the differences between pretest and posttest attitudes were small. The significant difference was most likely due to the high precision level of the test. In other words, because the sample used was large, the degrees of freedom used were large. Hence, the test was powerful enough to detect differences which were not necessarily meaningful. The pretest and posttest attitude means were 3.51 and 3.35 for Site 1, 3.24 and 3.16 for Site 2, and 3.24 and 3.16 for Site 3. Because of these small differences, the conclusion was that, on the whole, middle grades (six, seven, and eight) boys



Site	Grade	N
1	6	149
	7	138
	8	157
2	6	340
	7	101
	8	180
3	6	133
	7	140
	8	108

Fig. 5.3 Profiles of MGMP PT Mean Gains and Averages Over Time in Each Site by Grade Level and by Sex.

and girls did not change their attitudes appreciably toward mathematics during the study.

7. Sex differences were also not found in probability gains from the instruction and in attitude changes toward mathematics over the time period. Thus, in the entire study, boys and girls gained equally from the probability intervention. In Site 2, there was an ordinal interaction of sex by grade level in probability gains over time (Fig. 5.3). This means that girls gained more than boys in each grade level, but these differences were not statistically significant. Although attitude changes by sex were not significant, girls in general dropped slightly more than boys from pretest to posttest in attitudes toward mathematics. The pretest and posttest means were 3.29 and 3.21 for boys, while these means were 3.35 and 3.23 for girls. These attitude changes by sex are observable graphically in Fig. 4.19.

8. In general, grade level differences were found in probability knowledge over time (pre + posttest), in probability gains over time (posttest-pretest), and in attitude change to mathematics over time. Both probability knowledge and gains over time were found to be in favour of the seventh graders (Fig. 4.15). The overall pretest and posttest probability means were 7.70 and 10.96 for grade six, 9.27 and 13.19 for grade seven, and 9.13 and 11.59 for grade eight. Although attitude changes by grade were not significant as measured by the

MAS, attitude decline by grade level was observed in both the pretest and posttest scores (Fig. 4.18, Table 4.14). That is, the higher the grade level, the lower the attitudes toward mathematics.

9. In Site 1, grade level differences in probability gains were not found, but significant differences were found between girls in grades six and seven in favour of grade seven. Due to inordinal interactions of probability knowledge (over time) between boys and girls among the three grade levels, significant grade level differences were not found in probability knowledge over time. In Site 1, the mean gains were 3.16, 3.47, and 3.33 for grades 6, 7, and 8 respectively.

10. In Site 2, sixth and seventh graders gained equally from the probability instruction, but each outgained the eighth graders. Boys consistently outscored girls in probability knowledge (over time) across all grade levels, but this was significant only in grade seven. On the other hand, girls consistently outgained boys in scores on the pre-post probability test, but these were not statistically significant.

11. While grade eight gained least from the probability instruction in Site 3, grade seven gained most. Grade seven also demonstrated more probability knowledge (over time) than grades six and eight.

12. In the entire study, all three sites did not differ significantly from one another in either probability gains over time or in attitude change toward mathematics. Boys, and girls, and all grade levels, changed equally in mathematics attitudes. Boys and girls gained equally, and grade seven outgained the other grades, on the probability unit test pre-post. Fig. 5.3 presents all gains and averages by site, by grade, and by sex.
13. With respect to attitude comparison to mathematics and to probability, no sex differences were found by site and by grade level. This means boys and girls did not disagree in their attitudes toward probability and mathematics.
14. Grade level differences were found in Sites 2 and 3 in attitudes toward mathematics versus attitudes toward probability. In Site 2, sixth and seventh graders preferred mathematics to probability, while the eighth graders had no preference.
15. In the comparison of attitudes to probability, boys and girls did not differ in any grade or site.
16. In Site 1, grade seven had significantly higher attitude to probability than grades six and eight. On the contrary, the same grade level recorded the lowest attitude to probability in Site 3. On the whole, attitudes to probability were more favourable in Sites 1 and 3 than in Site 2, while grade seven, on the

whole, recorded more liking of probability than any other grade level.

Discussion

The findings and conclusions reached in this study were presented in the previous section. A number of observations are made about middle grades (six, seven, and eight) boys and girls, with respect to their probability knowledge and attitudes toward mathematics prior to a probability intervention.

Perhaps a major observation from this study is the presence of sex differences in probability prior to instruction among middle school pupils. In general, boys seem to outperform girls in probabilistic concepts in the absence of instruction. However, although boys may still outperform girls after instruction, differences may no longer be tangible. In fact, in the present study, girls benefited more than boys from the probability intervention, as demonstrated in the pretest-posttest scores. Evidence of such sex differences in research studies exists. Kass (1964) found sex differences in favour of boys in binomial probability tasks. The explanation Kass adduced for this was that boys have a natural tendency to interact with out-of-class probabilistic events that involve dichotomous choices. For example, boys tend to be associated more than girls in gambling activities involving head or tail, win or lose situations.

Similar to the present study, in the evaluation of the Comprehensive School Mathematics Project (CSMP), sex differences which were found in the pretest measures, vanished after the program, for CSMP participants, but sex differences persisted among non-CSMP participants. On the whole, the present finding is in agreement with most studies in the literature. For example, in studies by Mullenex (1968), Doherty (1965), Wavering (1979), Smith (1966) and McLeod (1972), sex differences were not found in probability achievement.

That sex differences in attitudes to mathematics were not found in this study either prior to, or sequel to instruction, may perhaps surprise some. Notably, investigators such as Fennema (1977), Fennema and Sherman (1978), Malcolm (1971), and Shaughnessy et al. (1983), all found sex differences in attitudes toward mathematics. However, this finding agrees with observations by Suydam and Weaver (1975) that sex differences were not found in attitudes to mathematics in some studies. It is, however, worth remarking that although sex differences were not significant in the present study, boys had a tendency to remain more steady than girls. For example, while girls on the whole had more positive attitudes toward mathematics, they dropped more than boys from pretest to posttest, but again, these were negligible differences.

The findings from the present study suggest that grades six, seven, and eight boys and girls do differ in probability knowledge prior to instruction. Although these three grade levels are all within Piaget's second stage of probability development, from seven to fourteen years of age, higher grades would be expected to show more maturity in mathematical ability, of which probability is a part. Hence, it was not a surprise that grade six performed less well than the other grades in both pretest and posttest probability scores. However, the most important grade level question is which grade benefited (gained) most from the probability intervention. Interestingly, the seventh graders gained more probability knowledge than sixth and eighth graders, in the present study. This important result is similar to the conclusion reached by Smith (1966) in a study in which grade seven constituted the subjects used. Smith reported that seventh graders gained significantly from a 17-day probability intervention. Several questions arise as to why grade seven should do better in probability than grades six and eight, especially grade eight. First, this superiority might be due, by chance, to higher teacher and/or student quality. Or the reverse might be true of grade eight, especially in the suburban setting. But, the seventh graders outperformed the others in the rural setting, and did not significantly differ from the eighth graders in the urban setting. Hence, grade seven may possess some interesting characteristics with respect to probability. Another explanation might be found in the nature of mathematics topics that were covered just before the probability intervention. These topics may tend to promote the learning of the probability activities in this study.

According to the present study, grades six, seven, and eight do not differ appreciably in their attitude change as a result of probability instruction. This result agrees with Fennema (1977) who reported that attitudes toward mathematics remain fairly stable between grades six and twelve. Although attitude differences were not significant in the present study, distribution showed that attitudes tended to decrease with grade level, with girls decreasing more than boys. Malcolm (1974) reached very similar conclusions.

Finally, a major result of this study was that irrespective of sex, grade level or site, middle school students benefited significantly from the training program in probability tasks. Similar conclusions were arrived by such investigators as Beyth-Maron (1980), Shaughnessy (1977), and White (1974). Perhaps a partial explanation for overall significant student gains from the Middle Grades Probability Project Unit is the experimental nature of the activities. The activities employed the strategies of launching, exploration, and summarization. Moreover, concrete operations and multiple embodiments, proved to be effective by Piaget and Inhelder (1951) and Jones (1974), were utilized in all the probability activities in this study.

Implications for Mathematics Education

In the present investigation, it was demonstrated that middle school students can respond very well to probability insruction. Although grade seven appears optimal for the introduction of the (MGMP) Probability Unit, it has worked very well for grades six and eight too.

Not only do grade levels six, seven, and eight respond well to probability instruction, it was found, in this study, that boys and girls respond equally and favorable well to the probability instruction.

Also, probability instruction among middle school students was demonstrated in this investigation to work equally well in urban, suburban, and rural areas despite socioeconomic and other background differences. Although pretest and posttest measures indicated that students in urban settings performed less well, findings revealed that they benefited equally from the probability training program as students from the other settings.

Another implication is for mathematics teachers. Regardless of grade level, sex, or school setting, all teachers, when supplied with well-sequenced instructional activities, successfully taught a unit on probability. The test and unit materials are easy and handy, and almost all the manipulatives can be improvised locally.

Another implication is for mathematics teacher education. The importance and use of probability knowledge are being emphasized by contemporary mathematics educators

(Shulte 1981). However, for teachers to be encouraged to take the topic more seriously, adequate staff development is necessary for preservice and inservice teachers. Undergraduate and graduate mathematics teacher education programs should include the teaching of probability.

Recommendation for Future Research

The following recommendations are based on the investigator's findings and conclusions in the present study.

1. It is recommended that this study be replicated with similar subjects and extended to include a test of their retention span in probability knowledge.
2. It is recommended that the study be replicated among grade levels nine through twelve students to complete the investigation through all postprimary grades. In this case, efforts should be made to reduce teacher differences as much as possible. For example, design the study so that teacher participants have a mathematics teaching certificate. This should limit variability in teacher content knowledge.
3. It is recommended that the study be replicated in same grade levels using identified high, middle, and low ability students. This would afford the information on how various ability levels respond to probability, and to the Middle Grades Mathematics Project materials in particular.

4. The initial question of interest to the investigator was an analysis of students' patterns of errors in probabilistic thinking. An appropriate question to investigate might be an examination of the heuristics of availability and representativeness (reviewed in Chapter II). In addition, analysis of students' errors could help identify at what grade level certain probability concepts are not amenable to instruction. This would have implications for mathematics curriculum development. A corollary advantage would be availability of findings that speak critically to Piaget's three stages of probability development.
5. With the use of the same Probability Unit, it is recommended that the teaching of probability be studied by qualitative methods, in particular those associated with ethnographic research. Especially in conjunction with the statistical methods employed in the present study, ethnographic perspectives could provide theoretical explanations of pretest-posttest results which would not be feasible otherwise. The researcher could document, through systematic participant observations of the teacher-student class interactions, behavioral patterns that lead to certain results. Had such methods been available to complement the present study, such perplexing questions as why seventh grade girls tended to gain more than boys in the probability instruction even though they knew less could have been investigated.

Other questions are to what extent the teacher followed the curriculum materials, felt comfortable in responding to student questions, or treated boys and girls differentially. These could also be observed and analyzed through the use of ethnographic methods. Such methods can aid in providing a more complete description of what took place (or did not take place) during the teaching-learning process. They also make it possible for more relevant and useful questions to be raised as the study progresses; questions which can have far reaching implications for curriculum development and research in mathematics education.

APPENDICES

APPENDIX A

Brochure of Middle Grades Mathematics Project (MGMP)
Department of Mathematics
Michigan State University

MIDDLE GRADES
MATHEMATICS PROJECT
DEPARTMENT OF MATHEMATICS
MICHIGAN STATE UNIVERSITY

The MGMP is a curriculum development project funded by NSF - DISE, to develop units of high quality mathematics instruction for grades 5 through 8. Each unit

* is based on a related collection of important mathematical ideas,
* provides a carefully sequenced set of activities which lead to an understanding of the mathematical challenges,
* helps the teacher foster a problem-solving atmosphere in the classroom,
* uses concrete manipulatives where appropriate to help provide the transition from concrete to abstract thinking,
* utilizes an instructional model which consists of three phases...launching, exploring, and summarizing,
* provides a carefully developed instructional guide for the teacher,
* requires two to three weeks of instructional time.

The goal of the MGMP materials is to help students develop a deep, lasting understanding of the mathematical concepts and strategies studied. Rather than attempting to break the curriculum into small bits to be learned in isolation from each other, MGMP materials concentrate on a cluster of important ideas and the relationships which exist among these ideas. Where possible the ideas are embedded in concrete models to assist the students in moving from this concrete stage to more abstract reasoning.

Many of the activities are built around a specific mathematical challenge. The instructional model used in the units focuses on helping the students solve the mathematical challenge. The instruction is divided into three phases. During the first phase the teacher launches the challenge. The launching consists of introducing new concepts, clarifying definitions, reviewing old concepts, and issuing the challenge.

The second phase of instruction is the class exploration. During the exploration the students work individually or in small groups. The students may be gathering data, sharing ideas, looking for patterns, making conjectures, or developing other types of problem-solving strategies. The teacher's role during exploration is to encourage the students to persevere in seeking a solution to the challenge. The teacher does this by asking appropriate questions, encouraging and redirecting where needed. For the more able students, the teacher provides extra challenges related to the ideas being studied.

When most of the children have gathered sufficient data, the class returns to a whole class mode (often beginning the next day) for the final phase of instruction, summarizing. Here the teacher has an opportunity to demonstrate ways to organize data so that patterns and related rules become more obvious. Discussing the strategies used by the children helps the teacher to guide the students in refining these strategies into efficient, effective problem solving techniques.

The teacher plays a central role in this instructional model. First the teacher provides and motivates the challenge and then <u>joins</u> the students in exploring the problem. The teacher asks appropriate questions, encouraging and redirecting where needed. Finally, through the summary, the teacher helps the students to deepen their understanding of both the mathematical ideas involved in the challenge and the strategies used to solve it.

To aid the teacher in using the teaching model described, a detailed instructional guide is provided. This guide was developed as a result of many classroom trials of the materials. It provides help with both the mathematics content and the classroom management of the activities. Specific suggestions for important questions to be asked at appropriate stages of the activities are included. Extension questions and challenges for the more able students are provided along with suggestions for helping those students who are having difficulty. The units developed include:

SPATIAL VISUALIZATION

FACTORS AND MULTIPLES

PROBABILITY

SIMILARITY

STAFF

Glenda Lappan, Director

William M. Fitzgerlad

Elizabeth Phillips

Mary Jean Winter

Pat Yarbrough

David Ben-Haim

Alex Friedlander

Zacchaeus Oguntebi

CONSULTANTS

Janet Shroyer (Development)

Aquinas College, Grand Rapids, MI

Richard Shumway (Evaluation)

Ohio State University

APPENDIX B

MGMP Probability Test
Mathematics Attitude Scale (MAS)
Probability Attitude Scale (PAS)
Now It's your Turn

PROBABILITY TEST

DO NOT WRITE ON THIS TEST BOOKLET.

YOU MAY USE A SHEET OF SCRATCH PAPER.

READ QUESTIONS CAREFULLY.

SELECT THE ANSWER TO THE QUESTION.

MARK YOUR ANSWER ON THE ANSWER SHEET.

	A	B	C	D	E
EXAMPLE	1	2	3	●	5

BE SURE TO FILL THE CIRCLE COMPLETELY.

ERASE COMPLETELY WHEN NECESSARY.

MARK ONLY IN THE RESPONSE CIRCLES PROVIDED.

MAKE NO STRAY MARKS ON THE ANSWER SHEET.

STOP: WAIT UNTIL YOU ARE TOLD TO BEGIN.

PROBABILITY

PRETEST

Materials:

A) Somantic Differential Test about Mathematics

B) Probability Test Booklet

C) Answer Sheet

D) #2 Pencils

Instructions:

A) Give Semantic Differential Test first. 5-10 minutes should be sufficient. Collect this paper before distributing next test. Students should PRINT their name and circle girl or boy.

B) Distribute answer sheets and #2 pencils. Complete only the name and sex sections.

C) Distribute Probability Test Booklets.
Provide scrap paper.
Review cover sheet instructions.
Allow as much time as needed for the 25 questions.
(Calculators are not allowed)

Note: Please keep the classes separated and provide a class list with each class set of Semantic Differential and Probability Test.

The packages of materials will be collected as soon as the test is completed

Thank you very much for your cooperation.

PROBABILITY

POSTTEST

Materials:

1. Semantic Differential Test on Attitudes Toward Math.
2. Semantic Differential Test on Attitudes Toward Probability Activities.
3. Now It's Your Turn.
4. Answer Sheets.
5. Probability Test booklets (25 questions).
6. #2 Pencils.

Instructions:

1. Administer attitude test in following order: Have students print their name and circle their sex.

   a) Attitudes Toward Math (about 3 minutes)
   b) Attitudes Toward Probability Activities (about 3 minutes)
   c) Now it's Your Turn (about 10 minutes)

2. Distribute answer sheets.
   Students complete name and sex only.
   Use #2 pencils.

3. Distribute scrap paper.

4. Distribute Probability Test Booklets.
   Allow as much time as needed for the 25 questions.
   About 25 minutes is average. (calculators are not allowed)

5. Please have marks erased from booklets before passing them to another teacher, or administering the test to another class.

   Note: Please keep the classes separated and provide a class list with each class set of Semantic Differential and Probability Tests.

The packages of materials will be collected as soon as the testing is completed.

Thank you very much for your cooperation.

1. A spinner is divided into 15 sections of equal size. Five of these sections are red, four are blue, three are green, and three are yellow. If the spinner is spun, what is the probability that it will stop on a blue section?

(A) $\frac{1}{3}$ (B) $\frac{4}{15}$ (C) $\frac{4}{11}$ (D) $\frac{11}{15}$ (E) 4

2. The probability of an event happening is $\frac{3}{8}$. What is the probability that the event will <u>not</u> happen?

(A) 0 (B) $\frac{3}{8}$ (C) $\frac{5}{8}$ (D) $\frac{3}{4}$ (E) 1

3. A bowl contains 3 red marbles, 5 green marbles, and 4 blue marbles. A blue marble is drawn and not replaced. Then the contents of the bowl are thoroughly mixed. After this, you are asked to draw a marble from the bowl without looking. What is the probability that you will draw a blue marble?

(A) $\frac{3}{12}$ (B) $\frac{3}{11}$ (C) 12 (D) $\frac{4}{11}$ (E) $\frac{1}{3}$

4. Which of the following numbers could <u>not</u> be a probability?

(A) 1 (B) $\frac{3}{7}$ (C) $\frac{8}{9}$ (D) $\frac{5}{4}$ (E) 0

5. A fair coin has been tossed 10 times and has come up heads each time. Which of the following statements is <u>true</u>:

(A) The coin will come up heads on the next toss.
(B) The coin will come up tails on the next toss.
(C) There is an equal chance of coming up heads or tails on the next toss.
(D) The coin is more likely to come up heads on the next toss than tails.
(E) The coin is more likely to come up tails on the next toss than heads.

6. The probability of getting exactly one head and one tail when two fair coins are tossed is:

(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) $\frac{1}{2}$

7. If two dice are tossed over and over again, which sum would you expect to occur most often?

(A) 6 (B) 7 (C) 8 (D) 9 (E) 12

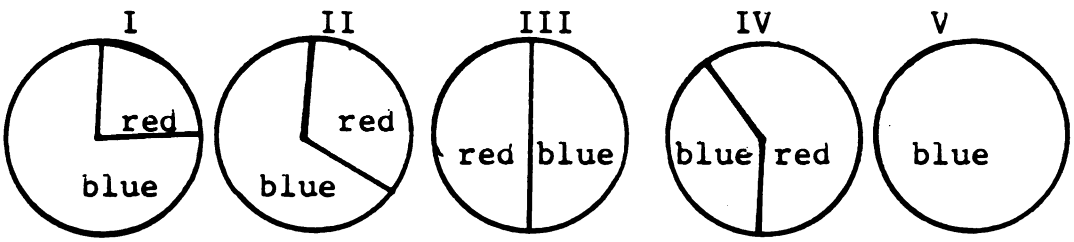
8. The probability of getting a sum of 12 when two dice are thrown is:

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$ (E) $\frac{1}{36}$

9. Bill Bailey tossed a thumbtack 50 times. It landed point up 22 times. If he tossed the same thumbtack 250 times, about how many times would you expect it to land point up?

(A) 88 (B) 110 (C) 125 (D) 200 (E) 250

Questions 10-12 relate to the 5 spinners shown below.



10. Which spinner is the most likely to stop on red?

(A) I (B) II (C) III (D) IV (E) V

11. Kim spun a spinner 100 times and made a record of her results.

Outcome	Blue	Red
Number of times	86	14

Which spinner is most likely the one Kim used?

(A) I (B) II (C) III (D) IV (E) V

12. If spinner III is to be spun twice, what is the probability of getting red - red?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $\frac{3}{4}$ (E) 1

13. A bag contains only red and blue marbles.

The probability of drawing a red marble is $\frac{3}{5}$. What is the probability of drawing a blue marble?

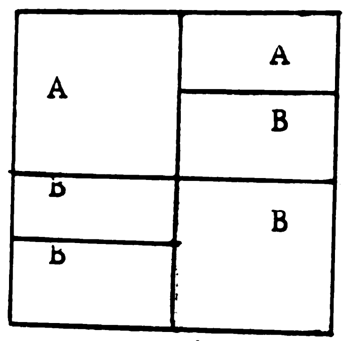
(A) $\frac{5}{3}$ (B) $\frac{1}{5}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$ (E) 1

14. A bag contains 2 yellow, 2 blue, and 4 red marbles. How many blue marbles must be added to the bag to make the probability of drawing a blue marble $\frac{1}{2}$.

15. Three pennies are tossed. What is the probability of getting 2 heads and 1 tail?

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{3}{8}$

16. John is tossing bean bags randomly onto the mat below. What is the probability of a bean bag landing in an area marked B?

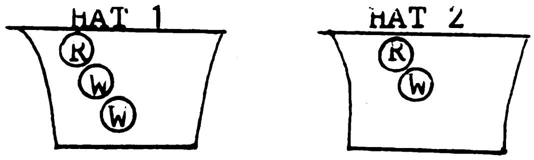


(A) $\frac{1}{4}$ (B) $\frac{3}{8}$ (C) $\frac{1}{2}$ (D) $\frac{5}{8}$ (E) $\frac{2}{3}$

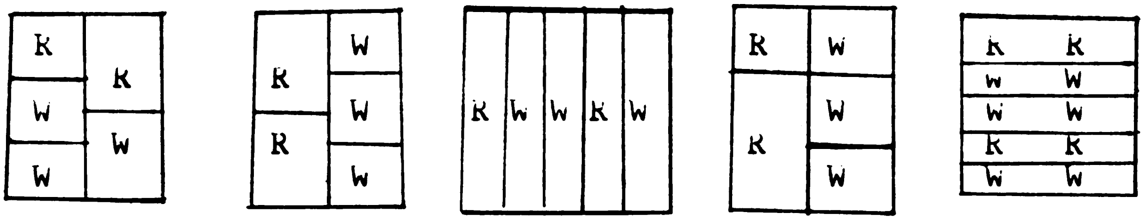
17. Sally has a 50% free throw shooting average in basketball. She goes to the line to take two shots. What is the probability that she will make both shots?

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) $\frac{3}{4}$ (E) 1

18. Hat 1 and Hat 2 contain red and white marbles as shown below. A hat is chosen at random and a marble drawn from it.

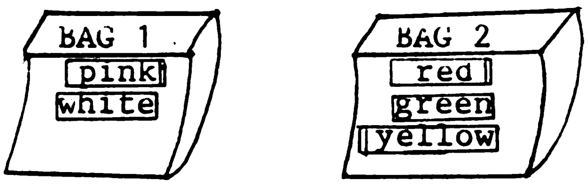


Which area model can be used to find the probability of drawing a white marble?



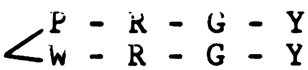
(A) (B) (C) (D) (E)

19. Bag 1 and Bag 2 contain blocks as shown below.

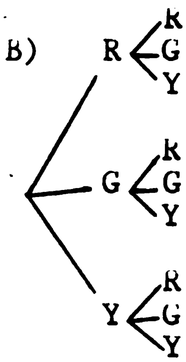


Which of the following is a tree diagram showing the possible combined results of drawing a block from bag 1, and then, a block from bag 2?

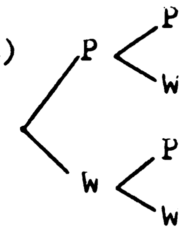
(A)



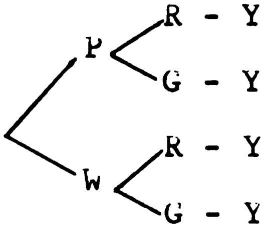
(B)



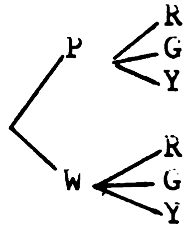
(C)



(D)



(E)



20. Two coins are flipped. Player A gets a point if the coins match and player B gets a point if there is no match. In this game

(A) A is more likely to win.
(B) B is more likely to win.
(C) A and B have the same chances of winning.
(D) There is not enough information to decide.
(E) B can never win.

21. Two bills are drawn from a bag containing a five dollar bill and 3 one dollar bills. If the experiment is repeated many times, what would you expect the average amount of money drawn per time to be?

(A) \$2 (B) \$3 (C) \$4 (D) \$5 (E) \$6

22. What is the probability that a family of three children will have 2 girls and 1 boy?

(A) $\frac{1}{8}$ (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\frac{1}{2}$ (E) $\frac{3}{8}$

23. How many different ways could you answer a 4 item true-false test?

(A) 1 (B) 2 (C) 4 (D) 8 (E) 16

24-25 In order to determine what ice cream flavors to have in the cafeteria a random poll is taken of 30 students on their favorite ice cream flaveo. The results are:

Vanilla	8	Butter Pecan	4
Chocolate	10	Peppermint	3
Strawberry	5		

24. If there are 600 students in the school, about how many would you expect to prefer chocolate?

(A) 10 (B) 20 (C) 200 (D) 250 (E) 300

25. If a student is chosen at random, what is the probability that the student favors either chocolate or butter pecan?

(A) $\frac{10}{30}$ (B) $\frac{14}{30}$ (C) $\frac{4}{30}$ (D) $\frac{6}{30}$ (E) $\frac{16}{30}$

MATHEMATICS ATTITUDE SCALE (MAS)

NAME ______________________ BOY/GIRL

EXAMPLE: FOR EACH PAIR OF WORDS BELOW PLACE A X ON THE BLANK THAT BEST TELLS HOW YOU FEEL ABOUT--

SNOW

LIKE _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ HATE

UGLY _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ PRETTY

WORK _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ : _ _ _ _ _ PLAY

DIRECTIONS: FOR EACH PAIR OF WORDS BELOW PLACE AN X ON THE BLANK THAT BEST TELLS HOW YOU FEEL ABOUT--

MATHEMATICS

BAD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ GOOD

SAD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ HAPPY

BORING _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ EXCITING

JUMP IN _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ HOLD BACK

HARD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ EASY

MORE _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ LESS

PROBABILITY ATTITUDE SCALE (PAS)

NAME ______________________ BOY/GIRL

EXAMPLE: FOR EACH PAIR OF WORDS BELOW PLACE A X ON THE BLANK THAT BEST TELLS HOW YOU FEEL ABOUT--

SNOW

LIKE _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ HATE

UGLY _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ PRETTY

WORK _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ : _ _ _ _ PLAY

DIRECTIONS: FOR EACH PAIR OF WORDS BELOW PLACE AN X ON THE BLANK THAT BEST TELLS HOW YOU FEEL ABOUT--

PROBABILITY ACITIVIES

BAD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ GOOD

SAD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ HAPPY

BORING _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ EXCITING

JUMP IN _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ HOLD BACK

HARD _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ EASY

MORE _ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _:_ _ _ _ LESS

NOW IT'S YOUR TURN

THINK ABOUT THE PROBABILITY ACTIVITIES WE HAVE EXPLORED, SOME OF THE ACTIVITIES WERE TAKE SURVEYS, AREA MODELS FOR PROBABILITY, FAIR AND UNFAIR GAMES, EXPECTED VALUE, PASCAL'S TRIANGLE.

(1) DID THESE ACTIVITIES MAKE YOU THINK? EXPLAIN.

(2) WHAT IS THE MOST INTERESTING IDEA THAT YOU LEARNED FROM THESE ACTIVITIES?

(3) WHAT DID YOU FIND HARD ABOUT THESE ACTIVITIES?

(4) WHAT DID YOU LIKE MOST ABOUT THESE ACTIVITIES? WHY?

(5) WHAT DID YOU LIKE LEAST ABOUT THESE ACTIVITIES? WHY?

(6) OTHER COMMENTS?

APPENDIX C

Pearson Correlation Matrices and Reliability Coefficients

TABLE C.1

PEARSON CORRELATION MATRIX
FOR SITE 1 BY GRADE[a]

	Grade	(1)	(2)	(3)	(4)
PT[b]	6	1.000			
Pretest (1)	7	1.000			
	8	1.000			
PT	6	.574*	1.000		
Pretest (2)	7	.631*	1.000		
	8	.786*	1.000		
MAS[c]	6	.188*	.243*	1.000	
Pretest (3)	7	.123	.122	1.000	
	8	.105	.091	1.000	
MAS	6	.162	.210*	.526*	1.000
Posttest (4)	7	.143	.192*	.639*	1.000
	8	.112	.103	.604*	1.000
PAS[d]	6	.206*	.220*	.507*	.267*
Posttest (5)	7	.171	.249*	.228*	.289*
	8	.143	.255*	.383*	.786*

[a] N of 6th Graders = 149, N of 7th Graders = 139, N of 8th Graders = 157.
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant P < .01.

TABLE C.2

PEARSON CORRELATION MATRIX
FOR SITE 2 BY GRADE[a]

	Grade	(1)	(2)	(3)	(4)
PT[b]	6	1.000			
Pretest (1)	7	1.000			
	8	1.000			
PT	6	.595*	1.000		
Pretest (2)	7	.626*	1.000		
	8	.708*	1.000		
MAS[c]	6	.136*	.205*	1.000	
Pretest (3)	7	.125	.369*	1.000	
	8	.103*	.195*	1.000	
MAS	6	.129*	.212*	.705*	1.000
Posttest (4)	7	.064	.214*	.670*	1.000
	8	.171*	.154*	.764*	1.000
PAS[d]	6	.149*	.168*	.184*	.256*
Posttest (5)	7	.107	.210*	.502*	.473*
	8	.234	.073*	.298*	.428*

[a] N of 6th Graders = 340, N of 7th Graders = 101, N of 8th Graders = 180.
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant P < .01.

TABLE C.3

PEARSON CORRELATION MATRIX
FOR SITE 3 BY GRADE[a]

	Grade	(1)	(2)	(3)	(4)
PT[b]	6	1.000			
Pretest (1)	7	1.000			
	8	1.000			
PT	6	.569*	1.000		
Pretest (2)	7	.565*	1.000		
	8	.579*	1.000		
MAS[c]	6	.233*	.174	1.000	
Pretest (3)	7	.230*	.116	1.000	
	8	.147*	.150*	1.000	
MAS	6	.182*	.153	.671*	1.000
Posttest (4)	7	.137	.134	.715*	1.000
	8	.104	.162*	.693*	1.000
PAS[d]	6	.086	.089	.174	.382*
Posttest (5)	7	.070	.144	.396*	.372*
	8	-.106	-.062	.242*	.324*

[a] N of 6th Graders = 133, N of 7th Graders = 109, N of 8th Graders = 203.
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant P < .01.

TABLE C.4

PEARSON CORRELATION MATRIX
FOR SITE 1 BY SEX[a]

		(1)	(2)	(3)	(4)
PT[b]	Total	1.000			
Pretest (1)	Boys	1.000			
	Girls	1.000			
PT	Total	.686*	1.000		
Pretest (2)	Boys	.712*	1.000		
	Girls	.657*	1.000		
MAS[c]	Total	.123*	.130*	1.000	
Pretest (3)	Boys	.134	.125	1.000	
	Girls	.110	.133	1.000	
MAS	Total	.146*	.172*	.578*	1.000
Posttest (4)	Boys	.152*	.216*	.626*	1.000
	Girls	.141*	.129*	.535*	1.000
PAS[d]	Total	.184*	.254*	.364*	.2961*
Posttest (5)	Boys	.226*	.314*	.509*	.456*
	Girls	.144*	.197*	.238*	.153*

[a] Total Sample N = 445, #Boys = 217, #Girls = 228
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant P < .01.

TABLE C.5

PEARSON CORRELATION MATRIX
FOR SITE 2 BY SEX[a]

		(1)	(2)	(3)	(4)
PT[b]	Total	1.000			
Pretest (1)	Boys	1.000			
	Girls	1.000			
PT	Total	.666*	1.000		
Pretest (2)	Boys	.717*	1.000		
	Girls	.582*	1.000		
MAS[c]	Total	.066	.202*	1.000	
Pretest (3)	Boys	.133*	.255*	1.000	
	Girls	-.008	.147*	1.000	
MAS	Total	.057	.176*	.755*	1.000
Posttest (4)	Boys	.119*	.244*	.728*	1.000
	Girls	-.030	.087	.790*	1.000
PAS[d]	Total	.066	.142*	.330*	.388*
Posttest (5)	Boys	.100	.202*	.286*	.339*
	Girls	.000	.055	.387*	.452*

[a] Total Sample N = 621, #Boys = 317, #Girls = 304
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant $P < .01$.

TABLE C.6

PEARSON CORRELATION MATRIX
FOR SITE 3 BY SEX[a]

		(1)	(2)	(3)	(4)
PT[b]	Total	1.000			
Pretest (1)	Boys	1.000			
	Girls	1.000			
PT	Total	.546*	1.000		
Pretest (2)	Boys	.579*	1.000		
	Girls	.492*	1.000		
MAS[c]	Total	.148*	.114*	1.000	
Pretest (3)	Boys	.147*	.150*	1.000	
	Girls	.173*	.094	1.000	
MAS	Total	.112*	.134*	.670*	1.000
Posttest (4)	Boys	.104	.162*	.693*	1.000
	Girls	.130	.109	.709*	1.000
PAS[d]	Total	-.001	-.032	.262*	.368*
Posttest (5)	Boys	-.011	-.062	.242*	.324*
	Girls	.014	.011	.287*	.422*

[a] Total Sample N = 392, #Boys = 203, #Girls = 189
[b] PT - Probability Test.
[c] MAS - Mathematics Attitude Scale.
[d] PAS - Probability Attitude Scale.
* Significant P < .01.

TABLE C.7

RELIABILITY COEFFICIENTS-CRONBACH α FOR MGMP PT, MAS AND PAS BY SITE BY TIME BY GRADE BY SEX

	N	PT[a] PRETEST	PT[a] POSTTEST	MAS[b] PRETEST	MAS[b] POSTTEST	PAS[c]
Site 1						
Grade 6	149	.68	.67	.84	.83	.80
Grade 7	139	.61	.81	.83	.82	.82
Grade 8	157	.75	.82	.77	.84	.84
Total	435	.70	.79	.81	.83	.82
Boys	217	.73	.80	.80	.80	.82
Girls	228	.66	.77	.83	.85	.83
Site 2						
Grade 6	340	.58	.74	.85	.86	.86
Grade 7	101	.73	.83	.88	.88	.89
Grade 8	180	.67	.73	.84	.85	.86
Total	621	.69	.78	.88	.88	.87
Boys	317	.74	.81	.86	.89	.88
Girls	306	.59	.73	.89	.87	.87
Site 3						
Grade 6	133	.45	.58	.77	.83	.84
Grade 7	150	.59	.79	.84	.86	.86
Grade 8	109	.58	.73	.82	.83	.82
Total	392	.56	.75	.82	.86	.85
Boys	203	.54	.77	.82	.86	.86
Girls	189	.56	.72	.82	.86	.84
All						
Grade 6	622	.61	.71	.83	.85	.85
Grade 7	380	.70	.83	.84	.85	.87
Grade 8	444	.70	.77	.83	.86	.88
Total	1446	.68	.79	.84	.86	.87
Boys	731	.72	.80	.84	.86	.86
Girls	717	.75	.62	.85	.86	.87

[a] PT - Probability Test (Range 0-25).
[b] MAS - Mathematics Attitudes Scale (Range 1-5).
[c] PAS - Probability Attitudes Scale (Range 1-5).

APPENDIX D

Mean, Standard Deviations and ANOVA Tables

TABLE D.1

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR THE ENTIRE SAMPLE BY TIME BY GRADE BY SEX

		PREPTOT[a]	POSTPTOT[b]	PREMAS[c]	POSTMAS[d]	PAS[e]
		M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)
Site 1	444	7.394	10.712	3.509	3.351	3.315
		(3.674)	(4.432)	(.896)	(.811)	(.840)
Site 2	621	8.977	12.034	3.236	3.161	2.654
		(3.695)	(4.309)	(.970)	(.954)	(.939)
Site 3	381	9.202	12.465	3.236	3.164	3.081
		(3.058)	(3.935)	(.810)	(.841)	(.889)
Grade 6	622	7.701	10.963	3.520	3.389	3.027
		(3.177)	(3.750)	(.887)	(.878)	(.043)
Grade 7	379	9.269	13.193	3.351	3.258	3.048
		(3.775)	(4.838)	(.872)	(.855)	(.897)
Grade 8	445	9.126	11.593	3.011	2.952	2.823
		(3.802)	(4.259)	(.909)	(.862)	(.960)
Boys	725	9.959	12.104	3.292	3.209	2.965
		(3.871)	(4.551)	(.926)	(.910)	(.961)
Girls	721	9.143	11.447	3.349	3.230	2.969
		(3.274)	(4.055)	(.902)	(.860)	(.919)
All	1446	8.551	11.741	3.319	3.220	2.070
		(3.613)	(4.309)	(.015)	(.886)	(.941)

[a] Probability Pretest (Range 0-25).
[b] Probability Posttest (Range 0-25).
[c] Pretest Mathematics Attitude Scale (Range 1-5).
[d] Posttest Mathematics Attitude Scale (Range 1-5).
[e] Probability Attitude Scale (Range 1-5).

TABLE D.2

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR SITE 1 BY TIME BY GRADE BY SEX

		PT[a]		MAS[b]		PAS[c]
	N	Pretest	Posttest	Pretest	Posttest	
		M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)
Grade 6	149	6.631	9.792	3.607	3.258	3.156
		(3.347)	(3.548)	(.947)	(.816)	(.869)
Boys	76	6.974	10.16	3.568	3.241	3.118
		(4.043)	(3.92)	(.964)	(.826)	(.937)
Girls	73	6.274	9.41	3.648	3.276	3.194
		(2.647)	(3.09)	(.934)	(.810)	(.796)
Grade 7	138	7.623	11.094	3.456	3.377	3.460
		(3.298)	(4.698)	(.920)	(.858)	(.797)
Boys	66	7.455	10.379	3.548	3.407	3.268
		(3.254)	(4.706)	(.871)	(.774)	(.787)
Girls	72	7.778	11.750	3.373	3.350	3.618
		(3.354)	(4.626)	(.962)	(.933)	(.778)
Grade 8	157	7.917	11.248	3.462	3.416	3.339
		(4.086)	(4.821)	(.808)	(.760)	(.828)
Boys	74	8.162	11.905	3.537	3.453	3.426
		(4.204)	(4.916)	(.807)	(.776)	(.735)
Girls	83	7.700	10.663	3.400	3.384	3.261
		(3.990)	(4.686)	(.828)	(.749)	(.901)
Total	444	7.394	10.712	3.509	3.351	3.315
		(3.674)	(4.432)	(.896)	(.811)	(.840)
Boys	216	7.528	10.824	3.549	3.364	3.276
		(3.893)	(4.570)	(.880)	(.795)	(.833)
Girls	228	7.268	10.605	3.471	3.339	3.352
		(3.458)	(4.306)	(.911)	(.828)	(.847)

[a] MGMP Probability test (Range 0-25).
[b] Mathematics Attitude Scale (Range 1-5).
[c] Probability Attitude Scale (Range 1-5).

TABLE D.3

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR SITE 2 BY TIME BY GRADE BY SEX

		PT[a]		MAS[b]		PAS[c]
	N	Pretest	Posttest	Pretest	Posttest	
		M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)	M (S.D.)
Grade 6	340	7.803	11.268	3.510	3.404	2.808
		(3.109)	(3.974)	(.908)	(.930)	(.943)
Boys	162	8.136	11.401	3.422	3.360	2.840
		(3.411)	(4.096)	(.930)	(.995)	(1.004)
Girls	178	7.500	11.146	3.590	3.444	2.78
		(2.781)	(3.867)	(.883)	(.869)	(.886)
Grade 7	101	11.713	14.970	3.406	3.320	2.937
		(3.861)	(4.757)	(.856)	(.850)	(.923)
Boys	53	12.679	15.906	3.583	3.547	2.937
		(4.094)	(5.208)	(.947)	(.945)	(1.011)
Girls	48	10.646	13.938	3.212	3.069	2.629
		(3.310)	(4.008)	(.702)	(.653)	(.793)
Grade 8	180	9.661	11.833	2.622	2.613	2.285
		(3.653)	(3.969)	(.866)	(.824)	(.839)
Boys	102	10.128	12.049	2.613	2.601	2.335
		(4.019)	(4.225)	(.850)	(.819)	(.849)
Girls	78	9.051	11.551	2.635	2.628	2.220
		(3.028)	(3.613)	(.892)	(.836)	(.827)
Total	621	8.978	12.034	3.236	3.161	2.654
		(3.695)	(4.309)	(.970)	(.954)	(.939)
Boys	317	9.536	12.363	3.188	3.147	2.694
		(4.077)	(4.618)	(.990)	(1.006)	(.987)
Girls	304	8.395	11.691	3.285	3.175	2.612
		(3.151)	(3.940)	(.948)	(.898)	(.887)

[a] MGMP Probability test (Range 0-25).
[b] Mathematics Attitude Scale (Range 1-5).
[c] Probability Attitude Scale (Range 1.5).

TABLE D.4

MEANS AND STANDARD DEVIATIONS OF MGMP PT, MAS, AND PAS SCORES FOR SITE 3 BY TIME BY GRADE BY SEX

	N	PT[a] Pretest M (S.D.)	PT[a] Posttest M (S.D.)	MAS[b] Pretest M (S.D.)	MAS[b] Posttest M (S.D.)	PAS[c] M (S.D.)
Grade 6	133	8.639	11.496	3.450	3.496	3.440
		(2.687)	(3.071)	(.752)	(.792)	(.863)
Boys	70	9.257	12.057	3.419	3.502	3.469
		(2.827)	(3.261)	(.757)	(.805)	(.885)
Girls	63	7.952	10.873	2.480	3.489	3.407
		(2.358)	(2.739)	(.752)	(.783)	(.843)
Grade 7	140	9.129	13.979	3.207	3.096	2.828
		(3.203)	(4.282)	(.819)	(.837)	(.826)
Boys	65	9.400	14.431	3.177	3.085	2.805
		(2.122)	(4.448)	(.706)	(.782)	(.825)
Girls	75	8.893	13.587	3.233	3.107	2.849
		(3.274)	(4.123)	(.910)	(.886)	(.831)
Grade 8	108	9.991	11.694	3.005	2.843	2.968
		(3.155)	(3.844)	(.804)	(.761)	(.866)
Boys	57	10.263	12.035	2.842	2.766	2.927
		(3.160)	(4.031)	(.902)	(.827)	(.977)
Girls	51	9.686	11.314	3.186	2.928	3.013
		(3.153)	(3.625)	(.640)	(.678)	(.729)
Total	381	9.202	12.465	3.236	3.164	3.081
		(3.058)	(3.935)	(.810)	(.841)	(.889
Boys	192	9.604	12.854	3.166	3.142	3.083
		(3.044)	(4.064)	(.817)	(.855)	(.938)
Girls	189	8.794	12.069	3.304	3.186	3.079
		(3.026)	(3.770)	(.799)	(.828)	(.840)

[a] MGMP Probability test (Range 0-25).
[b] Mathematics Attitude Scale (Range 1-5).
[c] Probability Attitude Scale (Range 1-5).

TABLE D.5

ANALYSIS OF VARIANCE SUMMARY FOR MEAN DIFFERENCE BETWEEN THE POSTMAS[a] AND PAS[b] SCORES FOR THE ENTIRE SAMPLE

Source of Variation	D.F.	MS	F	P<
Grade	2	7.427	7.052*	.001
Sex	1	.101	.096	.757
Grade x sex	2	3.256	3.091	.046
Between Groups	5			
Within Groups	1440	1.053		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
[b] PAS - Probability Attitude Scale.
* Significant P < .001.

TABLE D.6

ANALYSIS OF VARIANCE SUMMARY FOR MAIN DIFFERENCE BETWEEN THE POSTMAS[a] AND PAS[b] SCORES FOR SITE 2

Source of Variation	D.F.	MS	F	P<
Grade**	2	3.974	3.680*	.026
Sex	1	1.300	1.204	.273
Grade x sex	2	1.025	.949	.388
Between Groups	5			
Within Groups	615	1.080		

[a] POSTMAS - Posttest Mathematics Attitude Scale.
[b] PAS - Probability Attitude Scale.
* Significant P < .05.
** Alternative to the test in Table 4.20. Grade level contrasts are omitted from this test.

TABLE D.7

ANALYSIS OF VARIANCE SUMMARY FOR MAIN DIFFERENCE BETWEEN THE POSTMAS[a] AND PAS[b] SCORES FOR SITE 2

Source of Variation	D.F.	MS	F	P<
Grade**	2	7.711	5.008*	.007
Sex	1	.089	.094	.759
Grade x sex	2	.081	.086	.918
Between Groups	5			
Within Groups	375	.941		

a POSTMAS - Posttest Mathematics Attitude Scale.
b PAS - Probability Attitude Scale.
* Significant P < .01.
** Alternative to the test in Table 4.22. Grade level contrasts are omitted from this test.

TABLE D.8

ANALYSIS OF VARIANCE SUMMARY FOR PAS[a]
FOR SITE 3

Source of Variation	D.F.	MS	F	P<
Grade	2	7.09	7.979*	.001
Sex	1	.19	.022	.883
Grade x sex	2	.67	.417	.659
Between Groups	5			
Within Groups	1440	.78		

[a] PAS - Probability Attitude Scale.
* Significant P < .001.

TABLE D.9

ANALYSIS OF VARIANCE SUMMARY FOR MEAN AVERAGES BETWEEN THE POSTMAS AND PAS SCORES FOR EACH SITE

Site 1: Source of Variation	D.F.	MS	F	P<
Grade	2			
G1 (7 vs. 6)	(1)	6.52	7.51*	.006
G2 (8 vs 6 & 7)	(1)	.85	.98	.323
Sex	1	.09	.10	.752
Grade x sex	2	1.25	1.44	.239
Between Groups	5			
Within Groups	438	.868		

Site 2: Source of Variation	D.F.	MS	F	P<
Grade	2	53.45	49.94*	.0001
Sex	1	1.52	1.42	.234
Grade x Sex	2	3.23	3.02	.050
	5			
	615	1.07		

Site 3: Source of Variation	D.F.	MS	F	P<
Grade	2	24.57	27.28*	.0001
Sex	1	.22	.25	.619
Grade x Sex	3	.39	.43	.651
Between Groups	5			
Within Groups	375	.90		

* Significant P < .01.

APPENDIX E

Scheffé's Post Hoc Comparisons

Of interest to the investigator was an attempt to find the exact testmanship (Hays, 1973)* and location of differences between each pairwise comparison. Thus, after a significant F was found for a given factor (grade level or sex), Scheffé's posteriori comparisons were employed to contrast relevant pairs of means. The method due to Scheffé (1959) was used for its advantages of simplicity, applicability to unequal group sizes, and robustness to violation of normality and homogeneity of variance. Moreover, there is no limit, unlike planned comparisons, as to the number of such pairwise comparisons that can be made. Finally, Scheffé's Post Hoc contrast permits comparisons for both within and between group cells.

According to Scheffé's (1959, P.71), a contrast is significantly different from zero at the .05 level, (and in favour of the grade level or sex written first), if and only if:

$|\hat{\Psi}| > S\hat{\sigma}_{\hat{\Psi}}$, where

$|\hat{\Psi}|$ = the magnitude of the difference between the two compared means, and

$\hat{\sigma}_{\hat{\Psi}} = \sqrt{MSe\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$, where n_1, n_2 are the cell sizes compared, and

* According to Hays (1973, p. 413), testmanship means "how big is a difference?"

MSe = the mean square error of the relevant test (MAS or MGMP PT);

$S = \sqrt{(J-1)\ F_{0.95,\ J-1,\ N-J}}$, where

J = 3 for grade level contrasts

J = 2 for sex contrasts, and

N = the total number of (within group) subjects

involved in the original multivariate and univariate analysis.*

The following Tables E.1 to E.9, of various kinds of posteriori contrasts, are constructed, using the formulae described above. Each significant contrast is in favour of the grade level or sex factor written first in each contrast.

* More discussion on Scheffé's posteriori contrast was given by Glass and Stanley (1970, Chapter 16).

TABLE E.1

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] ON THE PROBABILITY PRETEST OF GRADE LEVEL BOYS AND GIRLS

Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
Grade 7 Boys versus		
Grade 6 Boys	1.50	.48
Grade 7 Girls versus		
Grade 6 Girls	1.48	1.50*

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.2.
* Significant at the .05 level, in favour of grade 7 girls.

TABLE E.2A

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF MGMP PROBABILITY PRETEST MEANS FOR BOYS AND FOR GIRLS IN SITE 2

Contrast	Effect	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}^{b}$
Grade 6 Boys vs. Grade 6 Girls	Sex	.73	.64
Grade 7 Boys vs. Grade 7 Girls	Sex	1.01	2.03*
Grade 8 Boys vs. Grade 8 Girls	Sex	1.01	1.08*
Grade 7 Boys vs. Grade 6 Boys	Grade	1.33	4.54*
Grade 7 Girls vs. Grade 6 Girls	Grade	1.43	3.15*
Grade 7 Boys vs. Grade 8 Boys	Grade	1.42	2.55*
Grade 7 Girls vs. Grade 8 Girls	Grade	1.54	1.60*

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.3.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.2B

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF MATHEMATICS ATTITUDE PRETEST MEANS OF GRADE LEVEL BOYS AND GIRLS IN SITE 2

Contrast	Effect	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}^b$
Grade 6 Boys vs. Grade 7 Boys	Grade	.34	.16
Grade 6 Girls vs. Grade 7 Girls	Grade	.35	.38*
Grade 7 Boys vs. Grade 8 Boys	Grade	.37	.97*
Grade 7 Girls vs. Grade 8 Girls	Grade	.39	.78*
Grade 6 Boyss vs. Grade 8 Boys	Grade	.27	.81*
Grade 6 Girls vs. Grade 8 Girls	Grade	.29	.96*

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.3.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.3

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF BOTH PROBABILITY AND MATHEMATICS ATTITUDE SCALE PRETEST MEANS OF SEX AND GRADE LEVEL EFFECTS IN SITE 3

Contrast	Effect	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
Grade 6 Boys vs. Grade 6 Girls	Sex	1.02	1.31*
Grade 7 Boys vs. Grade 7 Girls	Sex	1.0	.51
Grade 8 Boys vs. Grade 8 Girls	Sex	1.13	.58
Grade 7 Boys vs. Grade 6 Boys	Grade	1.26	.14
Grade 7 Girls vs. Grade 6 Girls	Grade	1.25	.94
Grade 8 Boys vs. Grade 7 Boys	Grade	1.33	.86
Grade 8 Girls vs. Grade 7 Girls	Grade	1.33	.79
Grade 8 Boys vs. Grade 6 Boys	Grade	1.33	1.01
Grade 8 Girls vs. Grade 6 Girls	Grade	1.38	1.73*
MAS:			
Grade 6 Boys vs. Grade 8 Boys	Grade	.35	.58*
Grade 6 Girls vs. Grade 8 Girls	Grade	.36	.30

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.4.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.4

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF PT AND MAS MEAN DIFFERENCES AND AVERAGES FROM PRETEST TO POSTTEST IN SITE 1

Criterion Measure	Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
DIFMAS[c]	Grade 6 Boys vs. Grade 7 Boys	.23	-.19
	Grade 6 Girls vs. Grade 7 Girls	.23	-.35**
AVGPTOT[d]	Grade 7 Boys vs. Grade 6 Boys	1.87	.35
	Grade 7 Girls vs. Grade 6 Girls	2.12	1.92
	Grade 8 Boys vs. Grade 6 Boys	.38	.25
	Grade 8 Girls vs. Grade 6 Girls	.27	.36*

a A description of these comparisons is found on page 225.

b $\hat{\psi}$ indicates the difference between appropriate means from Table D.2.

c DIFMAS - Mathematics Attitudes mean differences (Posttest-Pretest).

d AVGPTOT - Probability Attitudes mean averages (Pretest + Posttest).

* Significant at the .05 level, in favour of the sex or grade level weritten first in each contrast.

** The negative sign indicates that grade 6 girls significantly lost more mathematics attitude than grade 7 girls.

TABLE E.5

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF PROBABILITY TEST MEAN GAINS AND AVERAGES FROM PRETEST TO POSTTEST IN SITE 2

Criterion Measure	Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
DIFMAS[c]	Grade 7 Boys vs. Grade 6 Boys	.90	.04
(Gains in	Grade 7 Girls vs. Grade 6 Girls	.92	.84
probability	Grade 7 Boys vs. Grade 8 Boys	.96	1.30*
mean scores)	Grade 7 Girls vs. Grade 8 Girls	1.04	.79
	Grade 6 Boys vs. Grade 8 Boys	.72	1.34*
	Grade 6 Girls vs. Grade 8 Girls	.77	1.15*
AVGPTOT[d]	Grade 6 Boys vs. Grade 6 Girls	1.01	.45
(Averages in	Grade 7 Boys vs. Grade 7 Girls	1.86	2.00*
probability	Grade 8 Boys vs. Grade 8 Girls	1.41	.79
mean scores)	Grade 7 Boys vs. Grade 6 Boys	1.85	4.52*
	Grade 7 Girls vs. Grade 6 Girls	1.90	2.97*

[a] A description of these comparisons is found on page 225.

[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.3.

[c] DIFMAS - Mathematics Attitudes mean differences (Posttest-Pretest).

[d] AVGPTOT - Probability Attitudes mean averages (Pretest + Posttest).

* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.6

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] OF PROBABILITY TEST MEAN GAINS FROM PRETEST TO POSTTEST IN SITE 3

Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
Grade 7 Boys vs. Grade 6 Boys	.95	2.23*
Grade 7 Girls vs. Grade 6 Girls	.94	1.77*
Grade 7 Boys vs. Grade 8 Boys	1.00	3.26*
Grade 7 Girls vs. Grade 8 Girls	1.00	3.07*
Grade 6 Boys vs. Grade 8 Boys	.98	1.03*
Grade 6 Girls vs. Grade 8 Girls	1.04	1.29*

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate means from Table D.4.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.7

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a] FOR PROBABILITY ATTITUDE SCALE BY GRADE LEVEL FOR SITE 1

Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]	P<
Grade 7 to Grade 6	.24	.30*	.05
Grade 7 Boys to Grade 6 Boys	.34	.17	ns
Grade 7 Girls to Grade 6 Girls	.34	.42*	.05
Grade 8 to Grade 6	.23	.18	ns
Grade 7 to Grade 8	.24	.12	ns

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate PAS means from Table 4.23.
* Significant at the .05 level, in favour of the grade level written first in each contrast.

TABLE E.8

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a]
FOR PROBABILITY ATTITUDES SCALE MEANS
OF GRADE LEVELS FOR SITE 2

Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
Grade 6 vs. Grade 7	.25	.02
Grade 6 vs. Grade 8	.21	.52*
Grade 6 Boys vs. Grade 8 Boys	.28	.51*
Grade 6 Girls vs. Grade 8 Girls	.30	.56*
Grade 7 vs. Grade 8	.28	.51*
Grade 7 Boys vs. Grade 8 Boys	.38	.60*
Grade 7 Girls vs. Grade 8 Girls	.41	.41

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate PAS means from Table 4.23.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

TABLE E.9

SUMMARY OF SCHEFFÉ'S POSTERIORI COMPARISONS[a]
FOR PROBABILITY ATTITUDES SCALE MEANS
OF GRADE LEVELS FOR SITE 3

Contrast	$S\hat{\sigma}_{\hat{\psi}}$	$\hat{\psi}$[b]
Grade 6 vs. Grade 7	.25	.61*
Grade 6 Boys vs. Grade 7 Boys	.36	.66*
Grade 6 Girls vs. Grade 7 Girls	.36	.57*
Grade 6 vs. Grade 8	.27	.47*
Grade 6 Boys vs. Grade 8 Boys	.39	.54*
Grade 6 Girls vs. Grade 8 Girls	.39	.39
Grade 8 vs. Grade 7	.27	.14
Grade 8 Boys vs. Grade 7 Boys	.38	.12
Grade 8 Girls vs. Grade 7 Girls	.38	.16

[a] A description of these comparisons is found on page 225.
[b] $\hat{\psi}$ indicates the difference between appropriate PAS means from Table 4.23.
* Significant at the .05 level, in favour of the sex or grade level written first in each contrast.

BIBLIOGRAPHY

BIBLIOGRAPHY

Aiken, L.R. "Update on Attitudes and Other Affective Variables in Learning Mathematics." Review of Educational Research 46 (1976):293-311.

Anttonen, R.G. "A Longitudinal Study in Mathematics Attitude." Journal of Educational Research 62 (1969):467-77.

Arehart, J.E. The Relationship Between Ninth and Tenth Grade Student Achievement On a Probability Unit and Student Opportunity to Learn the Unit Objectives University of Virginia, (1978):186.

Armstrong, P.W. The Ability of Fifth and Sixth Graders to Learn Selected Topics in Probability. The University of Oklahoma, 1972.

Armstrong, Jane M. "Achievement and Participation of Women in Mathematics: Results of Two National Surveys." Published in Journal for Research in Mathematics Education. The National Council of Teachers of Mathematics, Vol. 12, No. 1, Jan. 1981.

Becker, J.R. "Differential Treatment of Females and Males in Mathematics Classes." Published in Journal for Research in Mathematics Education. The National Council of Teachers of Mathematics; Vol. 12, No. 1, Jan. 1981.

Benbow, C.P. & Stanley, J.C. "Sex Differences in Mathematical Ability: Fact or Artifact?" Science (December 12, 1980):1262-64.

Ben-Haim, D. Spatial Visualization: Sex Differences, Grade Level Differences and the Effect of Instruction on the Performance and Attitudes of Middle School Boys and Girls. Unpublished doctoral dissertation. Michigan State University, 1982.

Bruner, J.S., Goodnow, J.J., and Austin, G.A. A Study in Thinking. New York: Wiley, 1956.

Callahan, L.G., and Glennon, V.J. Elementary School Mathematics: A Guide to Current Research. Washington, D.C.: Association for Supervision and Curriculum Development, 1975.

Cambridge Conference on School Mathematics. Goals for School Mathematics. Boston: Houghton-Mifflin, 1963.

College Entrance Examination Board. Commission on Mathematics. Introductory Probability and Statistical Inference for Secondary Schools: An Experimental Course. York: New York, 1959.

Clemente, J. A Comparison of Two Mathematics Curricula for Seventh Grade Metropolitan Caracas Students. Boston Universilty School of Education, 1982.

Cohen, J., and Hansel, M. Risk and Gambling. New York: Philosophical Library Incorporated, 1956.

Crouse, R.J. An Investigation of the Relationship Between Teacher Knowledge and Student Achievement on Selected Probability Tasks. University of Delaware, 1977.

Davis, C.M. "Development of the Probability Concept in Children." Child Development, 1965, 36, 779-788.

Doherty, J., Level of Four Concepts of Probability Possessed by Children of the Fourth, Fifth, and Sixth Grade Before Formal Education. Unpublished doctoral dissertation. Missouri, 1965.

Dunlap, L.L. First Grade Children's Understanding of Probability. The University of Iowa, 1980.

Epstein, J.L. The Quality of School Life. Lexington, Massachusetts: D.C. Health, 1981.

Fennema, E. "Girls and Mathematics: The Crucial Middle Grades." Mathematics for the Middle Grades (5-9). National Council of Teachers of Mathematics, 1982 Yearbook.

________. "The Sex Factor." In Mathematics Education Research: Implications for the 80's. Association for Supervision and Curriculum Development' National Council of Teachers of Mathematics, 1981.

________. "Influences of Selected Cognitive, Affective and Educational Variables on Sex-Related Differences in Mathematics Learning and Studying." In Woman and Mathematics: Research Perspectives for Change, pp. 79-135. Edited by L.. Fox, E. Fennema, and J. Sherman. Washington, D.C.: National Institute of Education, 1977.

Fennema, E. and Sherman, J. "Sex-Related Differences in Mathematics Achievement, Spacial Visualization and Affective Factors." American Educational Research Journal 14 (Winter 1977): 51-71.

________. "Sex-Related Differences in Mathematics Achievement and Related Factors: A Further Study." Journal for Research in Mathematics Education 9 (1978):189-203.

Fennema, E., Wolleat, P.L., Pedro, J.D., and Becker A.D. "Increasing Women's Participation in Mathematics: An Intervention Study." Journal for Research in Mathematics Education, 12, No. 1, 1981.

Fitzgerald, W. and Shroyer, J. "The Mouse and the Elephant." Oregon Mathematics Teacher (February 1979): 10-13.

Flanagan, J.C., Davis, F.B., Daily, J.T., Shaycroft, M.F., Orr, D.B., Goldberg, I., and Neyman, C.A., Jr. The American High School Student. (Cooperative Research Project No. 635), Universilty of Pittsburgh, Project TALENT Office, 1964.

Gipson, J.G. Teaching Probability in Elementary School: An Experimental Study. Illinois, 1971.

Glass, G.V. and Stanley, J.C. Statistical Methods in Education and Psychology. Englewood Cliffs, New Jersey, Prentice-Hall, 1970.

Hays, W.L. Statistics for Social Sciences. Second Edition. Holt, Rinehart and Winston, Inc. 1973.

Huff, D. How to Lie with Statistics. London: W.W. Norton, 1954.

Huff, D. and Geis, I. How to Take a Chance. New York. W.W. Norton and Co., 1959.

Jones, G.A. The Performances of First, Second, and Third Grade Children on Five Concepts of Probability and the Effects of Grade, I.Q., and Embodiments on Their Performances. Unpublished doctoral dissertation. Indiana, 1974.

Kahneman, D., and Tversky, A., Subjective probability: A Judgment of representativeness. Cognitive Psychology, 1972, 3, 3, 430-454.

Kahneman, D., and Tversky, A., On the Psychology of Prediction. Psychological Review, 1973, 80, 4, 237-251.

Kass, N., Risk and Decision-making as a Function of Age, Sex, and Probability Preference. Child Development, 1964, 35, 577-582.

Kelsey, L.A. An Investigation of the Development of the Notions of Chance and Probability in Adolescents. The University of Iowa, 1980, 110 pp.

Kipp, W.E. Anvestigation of the Effects of Integrating Topics of Elementary Algebra with Those of Elementary Probability within a Unit of Mathematics Prepared for College Basic Mathematics Studetns. Unpublished doctoral dissertation. Florida State, 1975.

Knaup, J. Are Children's Attitudes toward Learning Arithmetic Really Important. School Science and Mathematics, 1973, 73, 9-15.

Lappan, G. (Diector). Middle Grades Mathematics Project Brochure. Department of Mathematics, Michigan State University, 1982.

Leake, L., The Status of Three Concepts of Probability in Children of the Seventh, Eighth, and Ninth Grades. The Journal of Experimental Education, Vol. 34, No., 1, Fall 1965.

Lee, C.S. Developing Basic Mathematicsl Skills Through Elementary Probability and Statistics for Low-Achieving Junior College Students. Columbia Universilty, 1975.

Lee, C.S. and Hoban, M. Probability: An Approach to Basic Mathematics. Harper's College Press, Harper & Roe, Publishers, Inc., 1975.

Leffin, W.W., A Study of Three Concepts of Probability Possessed by Children in Grades Four-Seven. ERIC Document ED 070 657, 1971.

Malcolm, S.V., A Longitudinal Study of Attitudes Toward Arithmetic in Grades Four, Six, and Seven. Case Western Reserve University, 1971.

McClenahan, M.D. An Application of Piagetian Research to the Growth of Chance and Probability Concepts with Low Achievers in Secondary School Mathematics. University of Kansas, 1974.

McKinley, J.E. Relationship Between Selected Factors and Achievement in a Unit on Probability and Statistics for Twelfth Grade Students. Unpublished doctoral dissertation. Stanford, 1971.

McLeod, G.K. An Experiment in the Teaching of Selected Concepts of Probability to Elementary School Children. Unpublished doctoral dissertation. Stanford, 1971.

Miller, G.A. "The Magical Number 7, Plus or Minus 2: Some Limits on Our Capacity for Processing Information." Psychological Review, 1956, 63, 81-97.

Moliver, M. A Program in Probability for Non-College Bound Students in the Tenth Grade General Mathematics. Temple University, 1977.

Monroe, J.A. An Experimental Model for Teaching Two Probability Concepts to Graduate Students in the Behavioral Sciences. Columbia University Teachers College, 1980.

Moyer, R.E. Effects of a Unit on Probability on Ninth Grade General Mathematics Students' Arithmetic Computation Skills, Reasoning, and Attitudes. Unpublished doctoral dissertation. Illinois, 1974.

Mullenex, J.L. A Study of the Understanding of Probability Concepts by Selected Elementary School Children. Unpublished doctoral dissertation. Virginia, 1968.

National Advisory Committee on Mathematical Education, Overview and Analysis of School Mathematics, Grades K-12. Washington, D.C.: NACOME, Conference Board of the Mathematical Sciences, 1975.

National Council of Teachers of Mathematics. An Agenda for Action: Recommendations for School Mathematics of the 1980's. Reston, Va.: The Council, 1980.

________. Priorities in School Mathematics (PRSM) Executive Summary of the PRISM Project. Reston, Va., 1981.

Norman, R.D. "Differences in Attitudes toward Arithmetic-Mathematics from Early Elementary School to College Levels." The Journal of Psychology, 1977, 97, 247-56.

Piaget, J. and Inhelder, B. Origin of Idea of Chance in Children. Translated (from French) by Leake, Burnell, and Fishbein. London: W.W. Norton and Company Incorporated, 1975.

Priorities in School Mathematics (PRISM): Executive Summary of the PRISM Project. NCTM, 1981.

Roland, L.H. Use of a Multidimensional Attitude Scale to Measure Grade and Sex Differences in Attitude Toward Mathematics in Second Through Sixth Grade Students. University of Washington, 1979. 163 pp.

Rule, A.M. A Study of the Diagnostic/Prescriptive Process of Teaching Mathematics with Respect to Change in Attitude Toward Mathematics and Change in Achievement in Mathematics for Fourth and Sixth Grade Inner-City School Students. Kent State University, 1981, 189 pp.

Scheffe, H. The Analysis of Variance. New York: John Wiley & Sons, 1959.

Senk, S., and Usiskin, Z. Geometery Proof Writing: A New View of Sex Differences in Math Ability. University of Chicago, 1982.

Shaughnessy, J., Haladyna, T. and Shaughnessy, J.M. "Relations of Student, Teacher, and Learning Environment Variables to Attitude Toward Mathematics." School Science and Mathematics: Vol. 83, (1) January, 1983.

Shaughnessy, J.M. A Clinical Investigation of College Students' Reliance Upon the Heuristics of Availability and Representativeness in Estimating the Likelihood of Probabalistic Events. Unpublished doctoral dissertation, Michigan State University, 1976.

Shaughnessy, J.M. "Misconceptions of Probability: An Experiment with a Small Group, Activity-Based Model Building Approach to Introductory Probability." Educational Studies in Mathematics 8, 1977.

Shepler, J. "Parts of a Systems Approach to the Development of a Unit in Probability and Statistics for the Elementary School." Journal of Research in Mathematics Education, 1970, 1, 4, 197-205.

Shepler, J., and Romberg, T. "Retention of Probability Concepts: A Pilot Study into the Effects of Mastery Learning with Sixth Grade Students." Journal of Research in Mathematics Education, 1973, 4, 1, 26-32.

Shevokas, C. Using a Computer-Oriented Monte Carlo Approach to Teach Probability and Statices in a Community College General Mathematics Course. University of Illinois at Urbana-Champaign, 1974.

Shulte, A.P. Effect of a Unit in Probability and Statistics on Students and Teachers of a Ninth Grade General Mathematics Class. Unpublished doctoral dissertation. Michigan, 1967.

________. (Ed). Teaching Statistics and Proability. National Council of Teachers of Mathematics, 1981 Yearbook.

Shumway, R.J. (Editor). Research in Mathematics Education. National Council of Teachers of Mathematics, Inc., 1906 Association Drive, Reston, Virginia 22091, 1980.

Shumway, R.J., White, A.L., Wheatley, G.H., Keys, R.E., Coburn, T.G., and Schoen, H.L. "Initial Effect of Calculators in Elementary School Mathematics. Journal for Research in Mathematics Education. 12, (1981):119-41.

Slovic, P., Kahneman, D., and Tversky, A. Judgement Under Uncertainty. Cambridge University Press, 1982.

Smith, M.A. Development and Preliminary Evaluation of a Unit on Probability and Statistics at the Junior High School Level. University of Georgia, 1966.

Smock, C., and Belovicz, G. Understanding of Concepts of Probability Theory by Junior High School Children. Final Report. ERIC Document ED 020 147, 1968.

Suydam, M.N., and Weaver, J.F. "Using Research: A Key to Elementary School Mathematics." Columbus, Ohio: ERIC Center for Science, Matheamtics, and Environmental Education, 1975.

Swift, J. "Challenges for Enriching the Curriculum: Statistics and Probability." The Mathematics Teacher. National Council of Teacher of Mathematics, Vol. 76, No. 4, 1983.

Szetela, W. "The Effects of Test Anxiety and Success-Failure on Mathematics Performance in Grade Eight." Journal for Research in Mathematics Education 4 (1973):152-60.

Tversky, A., and Kahneman, D. Judgement Under Uncertainty: Heuristics and Biases. Science, 1974, 185.

Wavering, M.J. The Interrelationshps of Piaget's Formal Operational Schemata: Proportions, Probability, and Correlations. The University of Iowa, 1979.

White, C.W. A Study of the Ability of First and Eighth Grade Students to Learn Basic Concepts of Probability and the Relationship Between Achievement in Probability and Selected Factors. Unpublished doctoral dissertation. Pittsburgh, 1974.

Wilks, S.S. Mathematical Statistics. New York, Wiley, 1963.

Winer, B.J. Statistical Principles in Experimental Design. New York: McGraw-Hill Book Co. 1962.

Yost, P., Siegal, A., and Andrews, J. Nonverbal Probability Judgments by Young Children. Child Development, 1962, 33, 769-780.