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Robert Gustafson
Major professor

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A TRANSLOG COST FUNCTION STUDY OF PURCHASED INPUTS USED IN U.S. FARM PRODUCTION

By

Hsin-Hui Hsu

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ABSTRACT

A TRANSLOG COST FUNCTION STUDY OF
PURCHASED INPUTS USED IN U.S. FARM PRODUCTION

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Agricultural economists encounter several recurring difficulties when they study the demand structure for purchased farm inputs. A partial list of potential difficulties includes interdependent relationships between inputs, the linear homogeneity condition assumed in Euler's Theorem, the limitations of existing functional forms, and the impracticability of obtaining insight into the input demand structure when a production function is unknown. These problems indicate that an improved analytical framework could lead to resolutions which would enable the public and the private sectors to make better decisions.

This paper presents an empirical demand estimation system by using duality theory and a transcendental logarithmic cost function to measure the interrelationships between U.S. farm inputs. By using time-series data (1910-1981) on five input subgroups and applying Zellner's seemingly unrelated regression technique, a complete set of demand equations can be estimated in quantity dependent form. A comparison of own price demand elasticities and pairwise elasticities of substitution was made between these and other research results.

Capital was found to be a substitute for all other inputs. Labor is a substitute for capital, feed, seed, livestock purchased, and miscellaneous inputs, but there is a weak complemetary relationship between labor and fertilizer.

The specified translog cost function has passed the tests of linear homogenity and monotonicity regularity conditions implied by the duality theory. However, it fails the test of the symmetry condition and the concavity condition is indefinite. Another important conclusion is that factor-augmenting technological change in U.S. agriculture has been mainly labor-saving and capital-using. This confirms previous empirical studies. A 2.9% annual growth rate of agricultural productivity sustained over the last seven decades is quite impressive. Finally, an attempt was made to investigate the question of economies of scale, but the results were inconclusive.

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CHAPTER 1

Introduction

A farm produces output from various combinations of inputs. Inputs are materials and factor services fed in at one end of the production process and used in the process of production. Farmers use land, labor, capital equipment, and other inputs to grow crops and to raise livestock. In economics, it is difficult to describe the characteristics of farm production and it is also not easy to measure farm input demand when interaction between inputs exists, especially if we want to measure the aggregate input demand or describe the nature of production at the industry level.

The agricultural industry is a system consisting of suppliers of farm inputs, farmers, and various businesses which are engaged in buying, processing and distributing farm products. The farm input demand system is part of the agricultural sector and is interlinked with, and affected by, industrial developments in the rest of the economy. The U.S. farming system is also of central importance in an increasingly interdependent world agricultural economy. The combination of scientific industrialization, increasing interdependence and a great deal of production uncertainty creats the need for an improved analytical system, that will help farmers, businessmen and public officials make better judgements on production planning and policy decisions.

This paper broadly investigates the demand for inputs and the research results are potentially valuable for various decision makers. Estimation of the elasticities of substitution between inputs and the own-price elasticities of input demand were of primary interest in this study. The subject-matter of elasticity measurements has many policy implications, such as the following: Is labor a substitute for all other inputs? and to what degree? Is the substitutability between farm labor and machinery declining? As the results emerge from the empirical study, the structure of U.S. farm production can be analyzed. For instance, a slowing down of the rate of mechanization and an intensifying use of agricultural chemicals may change farmers' long-term investment strategies.

I am not suggesting that this paper is the only study which deals with the purchased inputs used in U.S. farm production. Agricultural economists have long recognized the importance of farm input markets. Although there are thousands of economic studies of inputs in U.S. farm production (see references in Dahl and Spinks, 1981), one rarely finds a study which adequately handles the relationships among different groups of inputs and/or within a specific group of inputs. Empirical studies have derived numerous elasticity measures for single farm inputs. However, to the extent that changes in the quantities of inputs occur simultaneously, the estimates of elasticity measures obtained directly from single-equation models are likely to be biased. Furthermore, the basic relationships of demand-supply-price structure, in either an optimization or a behavioral context, need quantitative reestimation as factor and product markets, technology, and institutions change. This is one of the reasons that a new empirical study of the farm input demand system is important.

Let us briefly review U.S. farm input utilization and its economic implications. A major change in the structure of U.S. farm inputs has been the shift from inputs of farm origin (e.g., number of acres and labor), to purchased inputs of nonfarm origin [Table 1.1].

Table 1.1 Index Numbers of Total Farm Inputs and Inputs in Major Subgroups, U.S., Selected Years, 1910-81 (1967=100).

	Total Inputs							
Year	All	Non-purchased	Purchased	Farm Labor	Machinery	Real estate	Farm chemicals	Feed seed
1910	86	158	38	321	20	98	5	19
1920	98	180	43	341	31	102	7	25
1930	101	176	50	326	39	101	10	30
1940	100	159	58	293	42	103	13	42
1950	104	150	70	217	84	105	29	63
1960	101	119	86	145	97	100	49	84
1970	100	97	102	89	100	101	115	104
1980	106	85	127	65	128	96	174	119
1981	105	89	124	63	121	98	183	113

Source: USDA, Economic Indicators of the Farm Sector: Production and Efficiency Statistics, 1980, pp. 64-65, and 1982, p. 59.

This change is quite significant and continuing. Since the volume of farm real estate has remained stable, the major adjustment has been substitution of agricultural chemicals (e.g., fertilizer), machinery, and other capital inputs purchased from the nonfarm sector. Farm machinery, feed, and fertilizer were the most important in value, accounting for more than 60% of total farm inputs (Dahl and Hammond, 1977).

The volume of nonpurchased inputs declined by nearly one-half from 1910 to 1981. Meanwhile, the volume of purchased inputs more than tripled. Labor declined drastically. Part of the decline in labor use is attributable to shorter working hours on the farm, but most is due to outmigration. Farm numbers have declined continuously since 1910, and still appear to be declining.

All other input categories in Table 1.1 increased except real estate. The land resource base is essentially limited, although expansion can be achieved by reclamation of wetlands or irrigation of irrigable land. Both produce only minor changes in cultivatable land. In spite of the possibilities for increasing cropland, the U.S. has actually reduced cropland use between 1930 and the early 1970's. The total acreage of cropland decreased from 382 million in 1930 to 332 million in 1970. (Clearly the number of "real estate" used by the USDA in Table 1.1 is not equivalent to either total cultivatable cropland or to cropland used, but the three are related.) However, the composite measure of all inputs, calculated by the USDA, indicates that the index of total inputs has remained remarkably stable since 1930.

Out-migration from farms and reduced cropland use in agriculture have been offset by substitution of other inputs. Mechanical power has substituted for manual labor and animal power. Fertilizer and lime have substituted for land inputs. But because a given dollar value of purchased inputs was more productive than a given dollar value of farm originated inputs (e.g., labor) which was replaced, output more than tripled from 1910 to 1981 [Table 1.2]. Although the long-term growth of output has been strongly upward, it is somewhat erratic. Output was essentially stable between 1920 and 1930. The largest decennial gains have been made since 1940.

Table 1.2 Index Number of Farm Output, Input, and Productivity, Selected Years, 1910-1981 (1967=100).

	Farm Output	Production Inputs	Productivity (0/I)
1910	43	86	50
1920	51	98	52
1930	52	101	51
1940	60	100	60
1950	74	104	71
1960	91	101	90
1970	101	100	102
1980	122	106	115
1981	142	105	134

Source: USDA, Economic Indicators of the Farm Sector: Production and Efficiency Statistics, 1980, p. 77, and 1982, p. 71.

In the above review of the U.S. farm input structure, we can not avoid the important issue of technological change. Many agricultural production economists speak of efficiency and gains in efficiency. Commonly used measures of farm productivity are output per acre, output per worker-hour, and output per farmer. These measures are somewhat misleading if they ignore the contribution from the nonfarm sector. The total farm productivity index, which is presented in the last column of Table 1.2, probably shows the increase in farm productivity more accurately. The productivity index is defined as the ratio of the total output index to the total input index expressed as a percent. These figures tell us that farm productivity has more than doubled since 1910. However, there is no consensus on the meaning of this farm productivity index. Some argue that the index is basically correct, and some argue that the index underestimates the overall gain of productivity. One of the important issues is how depreciation is calculated.

Whereas the farm productivity index can serve as an indicator of technological change, there are four basic ingredients in the analysis of technological advancement (Yotopoulos and Nugent, 1976): (1) the technical efficiency of production, (2) the scale of operation of production, (3) the bias of technological change, and (4) the elasticity of substitution. An increase in the technical efficiency of production refers to a reduction in the quantity of all factors used in producing the same unit of output, or equivalently, an increase of the quantity of output with inputs held constant. Increasing, decreasing, or constant returns to scale depend on whether total output increases more, less, or equally in proportion to the increase in all inputs. Bias in technological change may be thought of a change in the ratios of marginal products of factor (at given factor levels). The fourth characteristic of a technology -- the elasticity of substitution -- is the ease with which one input can be substituted for another. The combination of these four elements gives us a composite picture of the changes in technology which are reflected in the actual production process.

The increase in farm production can be roughly decomposed into the results of "scale effect", "technological change", and substitution due to exogenous price or input supply effects. Technological change incorporates technical efficiency and bias of technological change, and effects of the third category are determined by the elasticity of substitution. For example, from 1970 to 1980, total input use in agriculture increased approximately 6%. Total output increased by 21%. Consequently, if we assume constant returns to scale, approximately 30% (6/21) of the increase in output was the result of increased input and 70% was attributable to technological change or substitution effects. Agricultural economists are not always in agreement with this explanation. The difference in opinion may stem from disagreement about the mix of input substitutes, scale effect, embodied and disembodied technological change, and the input costs considered.

Time-series econometric models can be useful in predicting real world responses of farmers to input prices and whether, and how, these may be changing with the structure and growing commercialization of agriculture. This research aims at providing an empirical analysis of the structure of demand for U.S. purchased farm inputs. Embodied technological changes and economies of scale are part of the analysis due to the close relationship between input demand and farm production.

In production economic theory, input demand is a derived demand. Usually we consider the marginal revenue product schedule for an input, Xi, to be the firm's demand for Xi, if Xi is the only variable resource employed. There are many ways to derive an input demand function from a given technology and a given endowment of fixed factors of production in the neoclassical framework. The most popular approach is the constrained optimization approach. When a study employs the constrained maximization or minimization approach, a particular underlying production function is assumed. It is difficult to gain insight into the input demand structure when the production function is unknown. The latest developments in duality theory provide a simpler econometric approach and assure us that it is in fact theoretically sound.

The duality theory is important for reasons other than mathematical elegance. One reason for the increasing popularity of the use of duality in applied economic analysis is that it allows flexibility in the specification of input demand equations and permits a close relationship between economic theory and econometric practice.

1.1 Problem Statements

In analyzing demand for farm inputs, researchers have encountered several recurring difficulties. .The first difficulty arises from the desire to measure the interdependence among farm inputs. Simultaneous relationships and interdependence exist for all agricultural inputs. Most of the traditional empirical studies that used production function specifications adopted a form of either the constant elasticity of substitution (CES) or Cobb-Douglas (C-D) variety. This is not to suggest that the traditional functional forms are not appropriate, but only that their use in studying the details of the structure of the input demand system is restrictive. Input demands are difficult to determine when the production technology is complex. For example, the elasticity of substitution between each two factors must be identically one for a C-D production function. Although the CES function accommodates elasticities of substitution different from zero or unity, they remain constant at all levels of inputs. Flexibility of the analytical model is important for measuring the demand interrelationships.

An additional advantage of the duality approach is that by basing it on the cost function instead of the production function, we can evade the problem of requiring linear homogeneity in the production function (as Euler's Theorem implies is required to exhaust the production). Indeed, the cost function will be linearly homogeneous with respect to input price, regardless of the nature of the production function. A more detailed discussion can be found in Chapter 2.

In a large scale modelling work, for instance, the International Institute for Applied System Analysis (IIASA) and the USDA's National Inter-Region Agricultural Projections (NIRAP) model and the MSU Agriculture Model, quantity-dependent input equations are modelled as standard demand equations (Abkin, 1981). In each of these models, there is no attempt to estimate jointly a system of input demand equations. Rather, each demand equation is estimated separately by a single-equation method. For example, in the IIASA-NIRAP2 model, the demand for hired labor is modelled as a function of price received by farmers, the farm wage, the number of farm family workers and the stock value of machinery on farms. Meanwhile, price-dependent input equations also exist in the system, but the prices of input items are determined simply as linear functions of the nonagricultural sector's inflation rate. The primary concern of the estimated price-dependent equations is to generate and to differentiate the total expenditures of farm production rather than to explain the demand behavior.

For these reasons, serious attention has been given to develop new approaches which avoid the above-mentioned difficulties. The following section contains the explanations of why a concerned researcher wants to carry out this study and how to address the relevant issues.

1.2 Research Objectives and Procedures

A model's specification is influenced by the problem definition. Therefore, the intended application of the analysis must be adequately defined. There are many problems that require information on farm inputs for their solution. No model can attempt to produce all of the information required to solve all of these problems. To avoid an oversized research project, my ambition is limited to a modest one of providing only the information required to address a rather well defined set of problems.

In view of the problems and issues presented in the previous section, the objectives of this study are geared to two primary interests:

1) Specify, describe and analyze the demand system for U.S. farm inputs by employing a flexible cost function approach. Emphasis will be placed on the own-price demand elasticities and the pattern of substitution among the inputs.

2) Examine the impact of the technological changes and investigate the economies of scale for the U.S. farm production structure.

To accomplish the objectives of this study, the following research procedures are used:

1) Review and summarize existing empirical studies of agricultural input demand. Particular attention is focused on studies which have employed microeconomic duality theory and the cost function approach. The so-called flexible functional form which is a closely related topic

to the application of the cost function approach will be scrutinized.

2) Identify and define purchased input subgroups, namely hired labor, capital, fertilizer, feed, seed, livestock purchased, and miscellaneous items.

3) Specify the theoretical and statistical models, namely, the farm input demand system derived from a translog cost function.

4) Develop, refine and test the validity of the theoretical regularity restrictions of linear homogeneity and symmetry with respect to factor prices, and also the hypotheses of biased or neutral technological change.

1.3 Summary and dissertation organization

The present study was undertaken in an effort to gain a better understanding of the purchased inputs used in U.S. farm production. The study is different from previous studies on U.S. farm input demand or production structure in four ways: (1) The data were collected over a longer time period, 1910-1981, including the war years, than was used in other studies. Assuming a correctly specified model, then more observations imply more reliable estimates; on the other hand, it is true that using a longer time period makes it somewhat more likely that the model may not be completely and correctly specified. (2) The empirical study includes testing the validity of the regularity conditions implied by the theory, a point that was neglected by most researchers. (3) Long term factor-augmenting technological changes and their impacts on the derived elasticity measures are examined. (4) The study also compares different measures of economies of scale obtained from various specifications.

The plan of the dissertation is as follows. In chapter 2, the input demand relationships are examined by contrasting a primal approach and a dual formulation. The advantages of employing the dual approach are stressed and the properties of the translog cost function and other flexible functional forms are presented. Measures of elasticities of input demand and substitution, returns to scale, and biases in technological change are investigated, based on the neoclassical model of production decisions and assuming the translog cost function is adequately specified.

In chapter 3, an empirical estimation is carried out by using Zellner's seemingly unrelated regression technique. Data sources, input subgroup definition, and hypotheses on the theoretical restrictions are discussed. Special attention is paid to the elasticity of substitution between inputs and the own-price demand elasticity of each input. Also, the results of estimating biased technological changes and economies of scale will be reported.

Chapter 4 summarizes the important conclusions based on the findings of the models and further research needs are suggested.

CHAPTER 2

Input Demand in A Theoretical Setting

Presentation of the theoretical background is limited to relevant studies of demand relationships among U.S. farm input subgroups, particularily those studies which apply the duality approach. Two sets of literature are examined. The first set deals with theoretical development and application of the duality approach. The second set deals mainly with the so-called flexible functional forms.

2.1 Duality theory in application

A firm produces output from various combinations of inputs. The "production possibilities set" of a firm is a convenient way to summarize the set of all feasible production plans. Since the production possibilities set describes all feasible patterns of input and output, this set implies a complete description of the technological possibilities facing the firm. The traditional starting point of production theory is the set of physical technological posssibilities, often described by a production function. The development of production theory then follows the line of a firm's operation, as the firm seeks to achieve its goals subject to limitations of its technology. The results are constructed input demands and output supplies. These demands and supplies are expressed as functions of economic variables, given the technology, and using constrained maximization or minimization.

In practical application, duality theory is different from the earlier form of production theory in two aspects. First, duality theory provides a derived system of input demand equations, consistent with the maximizing or minimizing behavior of a producer, by simply differentiating a function instead of solving for the behavior functions (e.g., by the Lagrange multiplier method) explicitly. Second, duality theory reaches the "comparative static" results (e.g., elasticity of substitution), originally deduced from maximizing behavior, effortlessly (Diewert, 1974a, p. 107). By using duality, the technology implied by an economic model can be tested for compatibility with *a priori* hypotheses.

2.1.1 What is the Duality Theory?

To say there is a duality between the cost and production functions means that there exists an invertible, one-to-one relationship between these two functions. In other words, the mapping that yields the cost function from the production function and the mapping that yields the production function from the cost function are mutual inverses. Diewert (1982, p. 535) describes this relationship as the following:

> Suppose that a production function F is given and that y=F(X), where y is the maximum amount of output that can be produced by the technology during a certain period if the vector of input quantities X = (x1,x2,...,xn) is utilized during the period. Thus, the production function F describes the technology of the given firm. On the other hand, the firm's minimum total cost of producing at least the output level y given the input prices W=(w1,w2,...,wn) is defined as C(W,y), and it is obviously a function of W, y and the given production function F.
>
> ...Thus, there is a duality between cost and production functions in the sense that either of these functions can describe the technology of the firm equally well in certain circumstances.

The production function, y=F(X), referred to as the "primal", describes global output response to all possible combinations of input quantities. The cost function, C(W,y), the "dual" of the production function, describes the minimum cost of producing any level of output given a set of input prices and production technology. Therefore, the existence of a duality between cost and production functions allows a researcher to use either function in analysis since the same information can be obtained from either function.

Duality theory has its roots in the work of Hotelling (1932), Roy (1942), Hicks (1946), and Samuelson (1947), but it is the pioneering work of Shephard (1953) which treats the subject comprehensively. The theoretical background on how to apply duality to empirical studies is rigorously explained and mathematically proven by Shephard (1953, 1970), Diewert (1974a, 1982), Lau (1976, 1978), Fuss and McFadden (1978), Blackorby, Primont, and Russell (1978), and Deaton and Muellbauer (1980). These researchers show that the cost function contains all of the information on production technology that is present in the production function. Therefore, one can proceed with the cost function approach without prior regard to a functional form for production technology.

In brief, assuming that a firm minimizes costs subject to a production function f:

(2.1) $$y = f\ (x1,\ x2,\ \ldots,\ xn)$$

It can be shown that the cost function which corresponds to f has the following form: for $y \geq 0$, $w \geq 0$, (i.e., each component of the vectors y and W is nonnegative)

(2.2) $$C(w,y) = \min_{x} \{W'X:\ f(x) \geq y,\ x \geq 0\}$$

where W'X is $\sum wi.xi$, the inner product of the vectors W and X. Equation (2.2) simply says that the producing unit (e.g., a firm) takes factor prices as given, and attempts to minimize the total cost at a specified level of output. The procedure for deriving input demand functions from constrained minimization of total cost, subject to an output constraint, is commonly known.

(2.3) $$\min_{xi's} L = \sum wixi + \lambda\ [y - f(x1,x2,\ldots,xn)]$$

Solving (2.3) yields the n <u>constant</u> <u>output</u> input demands.

(2.4) $$Xi^{*}\ (w,y) = [x1^{*}\ (w,y),\ \ldots,\ xn^{*}\ (w,y)]$$

The asterisk (*) denotes that the variable is the outcome of an optimization process. In (2.4), Xi^* is the minimum level of input quantity associated with the exogenous input price wi, and an output level y. The substitution of (2.4) into $\sum wixi$ provides an expression for the minimum level of cost in terms of input price and output level, $C(w,y)^*$.

However, by applying duality theory, the producer's system of input demand functions (i.e., equation (2.4)) can be obtained simply by differentiating the cost function with respect to input prices (Shephard Lemma). {Footnote 1} This conceptual simplicity and the ease of generating the farm production expenditure system are the major advantages of adopting a cost function, rather than a production function, to represent production technology.

To represent a rational output constrained minimization of cost given a "well-behaved" {Footnote 2} production technology, a cost function must meet the following regularity conditions:

(2.5) (1) Continuity: continuous with respect to input prices.
(2) Homogeneity: linearly homogeneous in input prices.
(3) Monotonicity: nondecreasing in input prices.
(4) Concavity: concave with respect to input prices.

{Footnote 1} Shephard Lemma: The partial derivative of the cost function with respect to the ith input price yields the constant output demand function for input i. $dC(W,y)/dwi=xi$.

{Footnote 2} "Well-behaved" means that a unique minimum to the cost minimization problem exists.

The empirical validity of these conditions in the context of the present study will be discussed later. The following remarks are intend to explain their meaning and nature

(1) Continuity with respect to factor prices. This condition is possibly true for most factor prices, however, in order to apply the theory, it is assumed true for all of them.

(2) Linear homogeneity in factor prices. For a given level of output total cost must increase proportionally with a proportional increase in all factor prices. This is intuitively plausible if it is assumed that total cost is made up only the cost of purchased inputs. If all factor prices double, one would expect the minimum cost of producing a given output level to double.

(3) Monotonicity with respect to input prices. The cost function must be a non-decreasing function of each factor price. The derivative of the cost function with respect to a factor price, dC/dWi, is expected to be non-negative. If one or more input prices increase and those inputs are used at positive levels, it is necessary to move to a higher isocost line to secure any specified output.

(4) Concavity with respect to factor prices is less intuitively apparent. Mathematically speaking, a cost function C(w,y) is concave if the Hessian matrix {footnote 3} is negative semidefinite within the range of factor prices. The Hessian is negative semidefinite if, and only if, the principal minors obtained from the Hessian alternate in

---

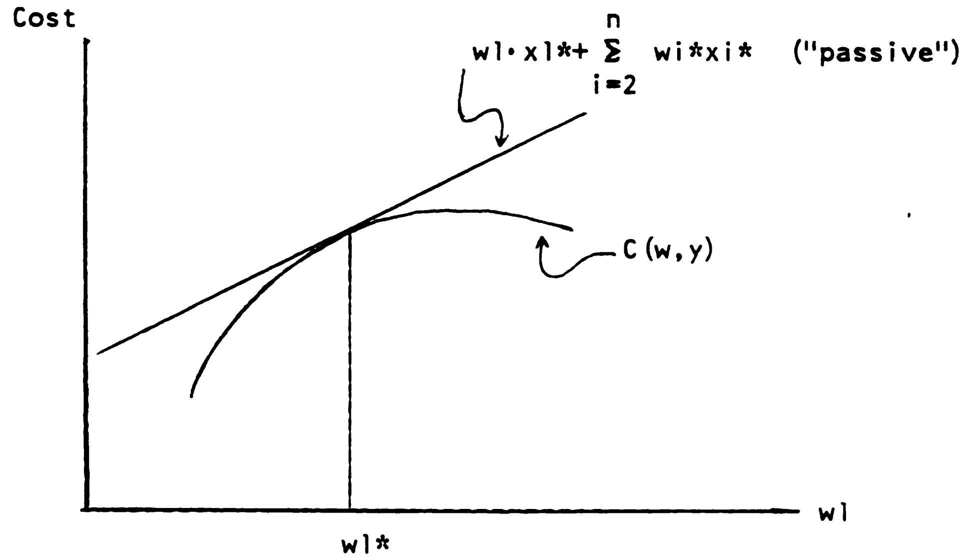
{Footnote 3} The Hessian matrix is the matrix of second-order partial derivatives of a particular function F with respect to its arguments.

sign (so that all odd-numbered principal minors are negative and all even-numbered ones are positive). For estimated empirical functions, one could numerically check for concavity by evaluating the characteristic roots of the Hessian of the cost function at each observation point. The Hessian will be negative semidefinite and the cost function will be concave if, and only if, all characteristic roots are nonpositive (Chiang, 1974, p. 345).

$$H = \begin{vmatrix} H11 & H12 \ldots H1j \ldots H1n \\ H21 & H22 \qquad\qquad\qquad . \\ . & . \qquad Hij \qquad . \\ . & \qquad Hii \qquad\qquad . \\ Hi1 & \qquad\qquad . \qquad . \\ . & Hij \qquad\qquad . \qquad . \\ . & \qquad\qquad\qquad . \quad . \\ Hn1 & \ldots\ldots\ldots\ldots Hnn \end{vmatrix}$$

A graphical presentation may help us to better understand this concept. Suppose we illustrate cost as a function of the price of a single input with all other prices held constant. If the price of a factor rises, cost will never go down (monotonicity property), but the cost will go up at a decreasing rate. Why? Because as this particular factor becomes more expensive and other factor prices stay the same, the cost-minimizing firm will gradually replace this costly input with less-expensive inputs.

Figure 2.1 The Cost Function and the "Passive" Cost Function



Consider Fifure 2.1 (Varian, 1978, p. 29), let x* be a cost-minimizing bundle at price w*. If the price of factor 1 changes from w1* to w1 (w1>w1*), and we behave passively and continue to use x*, the cost curve is the linear line, $C = (w1) \cdot (x1^*) + \sum (wi^*)(xi^*)$. However, the minimal cost of production C(w,y) must be less than this "passive" cost function since the substitution effect is not restricted. Thus, the graph of C(w,y) must lie below the graph of the passive cost function, with both curves coinciding at w1*. Now it is easier to see why the cost function C(w,y) is concave with respect to factor price w1.

For a two dimension coordinate system, the geometrical graph of a concave function always lies below its tangent line. In higher dimensions, we say that the graph of a concave function always lies below its tangent hyperplane. In this cost-minimizing situation, the concavity implies that it is necessary for each isoquant to be strictly convex (to rule out perfect substitutes or perfect complements) for the existence of a unique dual cost function.

As discussed later, it is possible to conduct statistical tests to find out if estimated cost equations meet some of these regularity conditions.

The duality theory can be interpreted either in the producer or the consumer context. I will use producer terminology for the sake of consistency.

2.1.2 To Dual or not to Dual?

One may find neither an absolutely positive nor negative answer to this question. The decision on whether to use the dual (i.e., cost function) or the primal (i.e., production function) is largely a matter of statistical convenience and analytical purpose.

The dual approaches allow estimation of the same information of practical value to policy makers (e.g., elasticity of substitution) that applied economists have supplied by traditional primal approaches for years. In some cases, primal approaches may be superior to dual approaches. However, data availability and the convenience of econometric estimation considerations will allow less difficult and costly analysis of many problems with dual approaches. The cost function $C(W,y)$, is expressed in terms of factor prices and the level of output while the production function, $y=f(X)$, is expressed in terms of input quantities. Silberberg (1978) argues that production functions are largely unobservable. The data points of the production surface represent a sampling of input and output levels that have taken place at different times or places, as factor or output prices changed. An instantaneous adjustment process is implicitly assumed in many production function studies.

The main statistical issue is whether it is safer to treat factor prices and level of output, or the use of inputs, as exogenous to the firm. Direct estimation of the production function is attractive when the level of output is endogenous and the quantity of factor inputs are exogenous. Estimation of the cost function is more attractive, however, if the level of output and factor prices are exogenous.

Neither of these two approaches is completely satisfactory, but I have chosen the latter. In the U.S., output is at least partly exogenous to the farm sector, being determined to some degree by government policy, It may be argue that producers act more like cost minimizers than profit maximizers if they choose to participate in a farm program. Mundlak and Hoch (1965) show that if the firm is a cost minimizer, input choice is necessarily endogenous and direct estimation of the production function will yield inconsistent results.

Furthermore, according to Woodland (1975) and Lopez (1980), farm input prices are determined in the nonagricultural sector. Berndt and Wood (1975, p. 261) also say that "at the level of an individual firm it may be reasonable to assume that the supply of inputs is perfectly elastic and, therefore, the input prices are fixed." Exogeneity of input prices is a convenient assumption, since it permits fairly simple estimation of the cost function and quantity-dependent input demand functions. An alternative would be to assume the farm sector is faced with rising supply curves for its inputs, in practice, this is not usually done.

In conclusion, the dual approach has the advantages of theoretical soundness and statistical convenience, therefore it is the approach I have chosen, rather than direct estimation of the production function, to study the farm input demand.

2.1.3 Cost function vs. profit function

The duality theory is not only applicable to the cost function approach, it is also applicable to the profit function. {Footnote 4} Lopez (1982) makes a distinction between these two approaches. The cost function is used to estimate Hicksian (compensated) input demand while the profit function approach allows researchers to estimate Marshallian (ordinary) input demand. A firm's variable profit function $\Pi(p,w)$ can be simply defined as the maximum revenue minus variable input expenditure.

$$\Pi(p,w) = \max_{x,y} \{p.y - W'X,\ F(x,y) \geq 0\}$$

where W'X is the inner product of factor prices and qunatities.

{Footnote 4} The third method, indirect production function approach, y=f(W,c), can be used to assess own- and cross-price elasticities of input demand for production with constant expenditure (i.e., total budget). We will not discuss this case since it has not been much used.

A common feature of a cost function approach is the assumption that output levels are not affected by factor price changes. Therefore, the indirect effects of factor price changes (via output levels) on factor demands are ignored. On the other hand, using the profit function and Hotelling's lemma, the input demand and output supply equations can be derived by simple differentiation with respect to input price and output price, respectively. However, a profit function requires a stronger behavioral assumption. The profit maximization assumption may be more difficult to support in agriculture than simple cost minimization. This is caused by the risk-related instability of output and product price rather than the costs of production.

The major differences between the two approaches may be briefly summarized as follows: (1) the cost function approach assumes cost minimization, and that output <u>quantities</u> are exogenous; (2) the profit function approach assumes profit maximization, and that output <u>prices</u> are exogenous. (Both assume that <u>input</u> <u>prices</u> are exogenous.) As between these two, I (and most other investigators) have chosen the cost function approach.

2.2 Flexible functional forms

The econometric applications of the new production theory based on the duality relationship between production and variable cost functions are major steps towards generating appropriate empirical estimates for input demand functions. However, to determine the elasticity of demand for fertilizer, elasticity of substitution between labor and capital, and economies of scale (these are examples of some important instruments for addressing policy issues), it is necessary to estimate the parameters of a production function or a cost function.

The development of flexible functional forms permits application of the duality theory to a less restrictive analysis of the nature of production than has been previously possible. A specified flexible cost function suggests a set of derived input demand equations, as indicated by the theory. Flexible functional forms have been developed with two attractive features, namely, they imply derived demand equations which are linear in the parameters, and at the same time, they may represent a very general picture of the production structure even though they are not derived from explicit production functions.

We say f is a "flexible functional form" if it can provide a second-order (differential) approximation to an arbitrary twice continuously differentiable function f* at x* (1982, p. 574). The term "differential approximation" is defined by Lau (1974, p. 183):

> According to Diewert's definition, a function G(y) is a second order approximation to a function F(y) at y0 if the first and second order derivatives of the two functions are equal at y0, that is,
>
> $$G(y0)=F(y0),$$
>
> $$\left.\frac{dG}{dy_i}\right|_{y=y0} = \left.\frac{dF}{dy_i}\right|_{y=y0}, \quad \left.\frac{d^2G}{dy_i\,dy_j}\right|_{y=y0} = \left.\frac{d^2F}{dy_i\,dy_j}\right|_{y=y0}.$$
>
> [for all i and j, and both G and F are assumed to be twice differentiable.]

A flexible functional form should be capable of representing a wide range of technology and be tractable with respect to the ease of computation, estimation, and interpretation. The set of flexible functional forms which are suitable candidates for investigating input demand functions has grown rapidly in the past decade. A partial list of these forms includes the transcendental logarithmic function (translog), the generalized Leontief (GL), the generalized Cobb-Douglas (GCD), the generalized square root quadratic (GSRQ), and the generalized Box-Cox (GBC). The GL, GCD and GSRQ forms were introduced by Diewert (1971, 1973, 1974b). The translog was developed by Christensen, Jorgenson, and Lau (1971, 1975). Berndt and Khaled proposed the GBC form (1979). The advantage of the GBC form is that restrictions on the Box-Cox parameters produce the other flexible functional forms.

(Table 2.1) Common Linear-in-Parameters Flexible Functional Forms

Functional Forms	Formula
Translog	$\ln C = a0 + \Sigma\, ai\, \ln wi + \Sigma\Sigma\, aij\, (\ln wi)(\ln wj)$
Generalized Leontief	$C = y\, .\, [\Sigma\Sigma\, aij\, (wi)^{1/2} (wj)^{1/2}]$
Generalized C-D	$\ln C = a0 + \Sigma\Sigma\, aij\, [\ln(wi+wj)/2]$
Quadratic	$C = a0 + \Sigma\, ai\, wi + \Sigma\Sigma\, aij\, (wi)(wj)$
GSRQ	$C = \Sigma\Sigma\, aij\, [1/2(wi)^2 + 1/2(wj)^2]^{1/2} .\, y$
Generalized Box-Cox	$C = [(2/p)\Sigma\Sigma\, bij\, .\, wi^{p/2} .\, wj^{p/2})]^{1/p} .\, y^{B(y,w)}$
	where $B(y,w) = b + r/2\, .\, \ln y + \Sigma\, c\, \ln wi$, and
	$wi = (wi^{(p/2)} - 1)/(p/2)$.

Selection of the flexible functional form best suited for use in empirical estimation is of primary concern. Mathematically speaking, the generalization of these flexible forms can become a never-ending pursuit. One may construct a variety of flexible functional forms containing the GBC as a limiting case, test for the restrictions of the GBC form, and so forth. The number of flexible functional forms which could be applied to duality theory is very large.

2.2.1 Choice among flexible functional forms

Since, by definition, all previously mentioned flexible functional forms have the same, or similar, attractive properties, it is unclear how the practitioner should choose among them. The choice of specific functional forms for estimating and deriving input demand equations involves compromise. One form cannot serve all analytical purposes. Lau (1974, p. 186) provides two principles for choosing the functional form. The first principle is that the functional form must be capable of approximating an arbitrary function to a desired order of accuracy (flexibility). The second principle is that the functional form must result in estimating forms that are linear in parameters (workability of econometric application). Fuss, McFadden, and Mundlak (1978 p. 224) suggest five more criteria to help distinquish between forms: (1) parsimony in parameters, (2) ease of interpretation, (3) computational ease, (4) interpolative robustness within the sample (e.g., concavity, monotonicity), and (5) extrapolative robustness outside the sample (i.e., forecasting).

Four recent papers discuss the issue of choosing among flexible functional forms. Wales (1977) performed a Monte Carlo study to investigate the ability of the GL and translog forms to represent two-product homothetic preference and exhibit constant elasticity of substitution. He found that in some cases the GL performed better, while in other cases the translog performed better. Wales found the performance of the translog form to deteriorate as the true elasticity of substitution departs from unity in either direction, and found the performance of the GL form to deteriorate as the true elasticity of substitution increases away from zero. Berndt, Darrough, and Diewert (1977) used postwar Canadian expenditure data to estimate three-product nonhomothetic GL, translog, and GCD forms. On the basis of better fit and conformity to neoclassical restrictions, they concluded that the translog form is the preferred form on Bayesian grounds a posteriori. Applebaum (1979) and Berndt and Khaled (1979) showed that the GBC form contains the GL, GSRQ, and translog forms as special or limiting cases. Using 1929-71 U.S. manufacturing data, Applebaum found that the GL and GSRQ forms are the best representations for the primal and dual specifications of technology. Using 1947-71 U.S. manufacturing data, Berndt and Khaled were able to reject the GSRQ model. However, the GL was not rejected and result, regarding the translog were inconclusive. Guilkey, Lovell and Sickles (1983) continue and broaden the line of attack initiated by Wales. The Monte Carlo experiments include the translog, GL, and GCD forms. They conclude that the translog form provides a dependable approximation to reality and outperforms all other flexible functional forms. They also conducted limited experiments on the GBC form and the results were not fruitful.

The translog and the GL are the most popular forms in previous applications. However, the translog is superior to the GL in analyzing technological change. (The translog can incorperate both neutral and factor-augmenting technological changes while the GL can only assume factor-augmenting technological change). Thus, I decided to employ the translog functional form because it is flexible enough to fit my research interests.

2.2.2 The Translog Cost Function

The cost function specified is an adaptation of Christenson, Jorgenson and Lau's (1971,1973) transcendental logarithmic cost function. Expressed as a second-order polynomial in logarithms of input prices and output, it is a generalization of the Cobb-Douglas (which is linear in logarithms) functional form. Note that the translog function places no *a priori* restrictions upon homotheticity, returns to scale, or the elasticity of substitution between pairs of inputs.

$$\text{(2.6)}\quad \ln C(w,y) = a0 + \sum ai \ \ln wi + 1/2 \sum\sum bij \ \ln wi \ \ln wj + ay \ \ln y$$
$$+ \sum di \ \ln wi \ \ln y + 1/2 \ ayy \ (\ln y)^{**}2$$
$$+ \sum ti \ T \ \ln wi + t \ T + 1/2 \ tt \ (T)^{**}2$$

where wi and wj are factor prices, y is output level, T is time, and a0, ai, bij, ay, ayy, di, ti, t, tt are parameters. The following constraints are implied by duality theory. First, in order to correspond to a well-behaved production function, a cost function must be linearly homogeneous in factor price. This implies the following relationships among the parameters:

$$\sum_i ai=1, \quad \sum_j bij=0 \text{ (for all i)}, \quad \sum_i di=0, \text{ and } \sum_i ti=0.$$

These restrictions imply that as input prices rise by a fixed percentage, total cost rises by that same percentage. The bij, di, and ti terms are forced to sum to zero in order to negate any effect they might have on total cost. This leaves ai to exert the only impact on total cost as input prices change, and to maintain the economic meaning of homogeneity.

Second, since the translog function is viewed as a second-order logarithmic approximation, the following symmetry constraint must hold: bij=bji, for all i and j. The symmetry condition is the consequence of the continuity assumption of the parent cost function and Young's Theorem {Footnote 5} from calculus. Combining the symmetry and homogeneity constraints, we have

$$(2.7) \qquad \sum_i ai=1, \quad \sum_i bij= \sum_j bji= \sum_i \sum_j bij=0, \ \sum_i ti = 0 \text{ and } \sum_i di=0.$$

The additional restrictions of ay=1, di=0 for all i, and ayy=0 ensure that C(w,y)=y.C(w,1) (where C(w,1) is the unit cost function) so that the corresponding production function is linearly homogeneous. However, these restrictions are not necessarily always imposed. If ai>0 for all i, $\sum$ ai=1, and bij=0 for all i and j the translog function collapses to a Cobb-Douglas cost function. Most of the CES-like functions may be derived from the translog function as special cases when appropriate restrictions are imposed.

The most interesting feature of the translog function is its flexibility. The translog functional form can serve as a local, second-order approximation to an arbitrary cost function. {Footnote 6}

{Footnote 5} Young's Theorem: Fxy and Fyx, are identical to each other, dF/(dx)(dy)=dF/(dy)(dx), as long as the two cross partial derivatives are both continuous.

{Footnote 6} See appendix A for a mathematical proof.

However, in the econometric model presented in Chapter 3, this translog cost function is assumed as the true data-generating function rather than an approximation to an arbitrary cost function (Berndt and Wood, 1975). This permits additive disturbance terms to be specified for the derived input demand equations and interpreted as random deviations of the endogenous left-hand variables about their cost-minimizing values. The cost minimizing input demand functions $x_i(w,y)$, generated via Shephard's Lemma, are not linear in the unknown parameters. But it is easy to verify that the factor share equations

$$\frac{d \ln C(w,y)}{d \ln w_i} = \frac{d C}{d w_i} \frac{w_i}{C} = \frac{x_i w_i}{C} = s_i$$

(i.e., via logarithmic differentiation using Shephard Lemma) are linear in the unknown parameters and, hence, are in convenient form for estimation.

$$(2.8) \quad s_i = a_i + \sum b_{ij} \ln w_j + d_i \ln y + t_i T \qquad i=1,2,\ldots,n$$

By the monotonicity property the cost function must be an increasing function of input prices, i.e., $s_i > 0$.

Constant-output input demand functions showing quantity demanded (x_i) as a function of prices and output (w and y) could be obtained from the relationship of $x_i = s_i . C / w_i$, where s_i is from (2.8) and C is from (2.6). But it is true that the resulting expression would be highly non-linear in w and y, and not convenient for analytical purposes.

Since $\sum si(w,y)=1$, only n-1 of the n equations defined by (2.8) can be statistically independent. If all n share equations are included in the estimating system, then the singularity of the residual covariance matrix for the factor share equations becomes unavoidable. {Footnote 7} The singularity problem can be overcome by deleting one of the factor share equations, and consequently, the parameters in the share equations become a subset of those in the cost equation.

Now, given data on output (y), input quantity (xi), and input price (wi), all parameters can be statistically determined since si can be observed. Although the cost function could be estimated in isolation from the factor share equations, it is more efficient to estimate the parameters jointly with the factor share equations included in the system. A more detailed discussion can be found in Chapter 3.2.

The transcendental logarithmic functional form has been discussed by Halter, Carter, and Hocking (1957), Christensen et al. (1971, 1973), Griliches and Ringstad (1971), and Sargan (1971). Empirical applications of the translog profit function have been made by Sidhu and Baanante (1981), Weaver (1983), McKay, Lawrence, and Vlastuin (1983). Empirical applications of the translog cost functional form have been made by Christensen et al. (1973), Berndt and Christensen (1973), Binswanger (1974a, 1974b), Burgess (1975), Christensen and Green (1976), Kako (1978), Nadiri and Schankerman (1979), Ray (1982),

{Footnote 7} A singularity problem means the disturbances are linearly dependent, and the covariance matrix cannot be inverted.

and Antle and Aitah (1983).

2.3 Derivation of elasticity measures

The most natural way of measuring how one input is a substitute for another is the cross-elasticity of factor demand (eij):

$$eij = (dxi/dwj)(wj/xi)$$

However, most researchers prefer a related measure known as the the elasticity of substitution. One such elasticity is given by Varian (1978, p. 46),

$$E\ sub = \frac{d(xi/xj)}{d(wi/wj)} \cdot \frac{(wi/wj)}{(xi/xj)}$$

In a two-factor case, this measures the proportionate change in the ratio of the factor quantities per unit change in the ratio of the factor prices when output and other input prices are held constant. This can be pictured as a shift in the input ratio along an isoquant as relative input prices change. When xi/xj responds greatly to change in wi/wj, the elasticity will be high and vice versa. The limiting case of E=0 occurs when the two inputs must be used in a fixed proportion as complements to each other. The other limiting case, with E infinite, occurs when the two inputs are perfect substitutes for each other.

However, an alternative measure is proposed by Allen (1938). The bij parameters in (2.6) and (2.8), where i is not equal to j, can be related to Allen's partial elasticity of substitution (Eij) between inputs i and j. Originally Allen (1938, p. 504) defined the partial elasticity of substitution in terms of the partial derivatives of the production function.

$$E_{ij} = \frac{(\sum x_g f_g) \cdot |F_{ij}|}{x_i \cdot x_j \cdot |F|}$$

where $f_g = df/d(x_g)$,

$$|F| = \begin{vmatrix} 0 & f_1 & \dots & f_n \\ f_1 & f_{11} & \dots & f_{1n} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ f_n & f_{n1} & \dots & f_{nn} \end{vmatrix}$$

and $|F_{ij}|$ is the determinant of (i,j)th cofactor in F. In order to minimize the cost of producing at a specific level of output, a firm must adjust the input such that the ratio of price to marginal product will be the same for each factor, i.e., $f_g = w_g/k$, where k is interpreted as the marginal cost of output. Meanwhile, the rate of change of the independent variables $(x_1, \dots, x_n)$ with respect to changes in factor prices is obtained as (see Samuelson (1947), pp. 63-9),

$$\frac{d x_i}{d w_j} = \frac{|F_{ij}|}{k \cdot |F|}$$

Substituting these relationships in E_{ij} we have

$$E_{ij} = \frac{(\sum w_g x_g)[(d x_i)/(d w_j)]}{x_i x_j}$$

On the other hand, by Shephard's Lemma, $x_i = dC/dw_i$, and utilizing the the fact that $\sum w_g x_g = Cost$, E_{ij} can be defined in terms of the partial derivatives of the total cost function C(w,y) as follows:

$$E_{ij} = \frac{C(w,y)\,\dfrac{d^2 C(w,y)}{d\,w_i\,d\,w_j}}{\dfrac{d\,C(w,y)}{d\,w_i}\;\dfrac{d\,C(w,y)}{d\,w_j}}$$

If $E_{ij}>0$, it indicates a substitution relationship between inputs i and j. When $E_{ij}<0$, we have a complementary relationship. A special feature of this elasticity is that, under the assumption that the unit cost function is weakly separable (Diewert, 1974a, p. 152), E_{ij} does not depend on the specific input m or n in a particular subgroup. The elasticity of substitution between the mth input from subgroup i and the nth input from subgroup j is proved to be the same as the elasticity of substitution between subgroups i and j. E_{ij} is a normalization of the response of input i to a change in the price of input j (i.e., dx_i/dw_j). The normalization process is chosen so that $E_{ij}=E_{ji}$ and E_{ij} is invariant to changes in the scale of measurement of the inputs. A more detailed discussion of Allen's partial elasticity of substitution can be found in Fuss and McFadden (1978, vol.II, part 4.1).

In the translog cost function, the estimated Allen's partial elasticities of substitution between inputs i and j ($\tilde{E}_{ij}$) can be calculated as (see appendix B for the mathematical proof):

$$\tilde{E}ij = \frac{\tilde{b}ij + \tilde{s}i\,\tilde{s}j}{\tilde{s}i\,\tilde{s}j} = \frac{\tilde{b}ij}{\tilde{s}i\,\tilde{s}j} + 1 \qquad (2.9)$$ {Footnote 8}

The special case of unitary elasticity of substitution clearly holds if bij=0, for i not equal to j. If estimated (~bij) is positive, then the elasticity will be greater than one. A negative Eij value implies complementarity. In general, the greater the bij term, the higher the elasticity of substitution between inputs i and j. Also note from this definition, the symmetric relationship Eij=Eji implies that ~bij=~bji.

When i=j, the estimated own-price elasticity (~Eii) can be calculated as (Binswanger, 1974a)

$$\tilde{E}ii = [(\tilde{b}ii + \tilde{s}i(\tilde{s}i-1))/\tilde{s}i^2].\tilde{s}i = (\tilde{b}ii + \tilde{s}i(\tilde{s}i-1))/\tilde{s}i \qquad (2.10)$$ {Footnote 8}

Note that (2.10) cannot be directly derived from (2.9) by simply substituting parameters between i and j.

2.4 Technological change

In empirical work, we need to be specific about the nature and character of technological change. Technological change may be biased with respect to one factor or another, or it may be neutral with regard to all inputs involved. According to Hicks (1932, p. 121), technological changes are classified as labor-saving, neutral, or

---

{Footnote 8} In the above equation, the hats (~) indicate estimated values.

capital-saving respectively, "as their initial effects are to increase, leave unchanged, or diminish the ratio of the marginal product of capital to that of labor". Technological change is defined as Hicksian-neutral if the marginal rates of technical substitution (MRTS=FK/FL) between each pair of factor inputs are independent of technological change. Mathematically this can be expressed as follows:

$$\frac{d}{dt} \text{MRTS} = \frac{d}{dt} \frac{Fk}{FL} = - \left(\frac{d}{dt}\right)\left(\frac{dL}{dk}\right) = 0$$

where Fk and FL stand for the marginal products and the capital-labor ratio is held constant.

An assumption frequently employed in empirical studies of input demand structures has been the absence of technological change. Coupled with constant returns to scale, this implies that all changes in input bundles result from price-induced substitution within a fixed technology. A slightly weaker assumption would be that all technological change was of a "Hicksian neutral" character. Again, in such a specification input mix changes are due to factor price changes.

It is desirable to investigate the input demand structure under a weaker assumption. In particular, previously maintained hypotheses of no technological change and Hicksian-neutral technological change are tested in this study. This will allows us to examine the effect of biased technological change, namely, input mix changes which occur independently of relative price changes over time. The inclusion of time variables in the translog cost function will facilitate the study of technological change biases.

One of the important aspects of this study is to distinguish the movements along a production function from the movements from one production function to another. The former movements are factor substitution, the latter are technological change. There are a number of ways to approach the estimation of technological change. From the standpoint of empirical analysis, Sato (1970) suggested the following two approaches which seem the most appropriate: (1) Assume that the elasticity of substitution is constant and technological change is Hicksian-neutral, and (2) assume that the production function has a variable elasticity of substitution, together with factor-augmenting technological change. The translog cost function allows for a series of specifications on the elasticities of substitution and also allows for factor-augmenting, or biased, technological changes.

Time variables in equation (2.6) are designed to capture the technological changes. The formulation allows for both neutral (t and tt) and biased (ti) technological change. If biased technological change occurs (i.e., ti does not equal zero), the factor share equations (2.8) are affected. Technological change is assumed to be ith factor using if ti>0, and jth factor saving if tj<0.

The overall rate of technological advancement is the partial derivative of equation 2.6 with respect to time, d(lnC)/dT. If the derived measure has a negative sign, this means that costs are saved over time, a technological progress. Taking the partial derivative of the cost function with the assumption of Hicksian-neutral technological change (tt and ti are zeros), the resulting estimate is a constant, t.

However, the overall rate of technological change is variable with respect to observed factor prices and time in a biased technological change model.

(2.11) $d(\ln C)/dT = t + tt*T + \sum ti*\ln(wi)$

2.5 Economies of scale

The theory of production deals with two concepts of returns to scale. The first concept, most widely recognized for defining "returns to scale," is stated as in terms of the change in output as all inputs increase by a scalar multiple.

$$f(tx1, \ldots, txn) = t^k.f(x1, \ldots, xn) = t^k.y$$

This is the relative increase in output that results from a proportional increase in all inputs along a ray through the origin. If the production function is homogeneous in all inputs then the degree of homogeneity is the measure of returns to scale. In particular, constant return to scale is equivalent to homogeneity of degree one.

The second concept is more relevant, when using the cost function approach for studying the input demand structure. This concept holds that the increase in output is relative to the increase in cost for variations along the expansion path as long as input prices are constant and costs are minimized at every level of output. Hanoch (1975) points out that the latter more appropriately represents economies of scale.

One way to express scale economies is as the proportional increase in cost resulting from a proportional increase in the level of output (Christensen and Greene, 1976). The proportional change in cost as a result of proportional change in output is known as the elasticity of total cost with respect to output. We define scale economies (SE) as unity minus this elasticity:

$$(2.12)\quad SE=1-(d\ \ln C/d\ \ln y) = 1 - \left(\frac{dC}{dy}\right)/\left(\frac{C}{y}\right) = 1 - (MC/AC)$$

Thus, SE has a natural interpretation in percentage terms when multiplied by 100. It is the percentage difference between total cost and total revenue, assuming that output is priced at marginal cost, i.e., SE=(C-p.y)/C. A positive SE indicates economies of scale, i.e., marginal cost is less than average cost, and the average cost curve is declining.

$$\frac{d}{dy}\left(\frac{C}{y}\right) < 0, \quad (y*(dC/dy) - C) < 0, \text{ [by the quotient rule of differentiation.]}$$

$$\frac{dC}{dy} - \frac{C}{y} < 0, \text{ that is, } MC-AC<0, \text{ or } MC<AC.$$

This indicates the average cost curve lies above the marginal cost curve. A negative SE indicates diseconomies of scale, i.e., marginal cost is greater than average cost, and the average cost curve is rising.

A well-behaved translog cost function must be homogeneous of degree one in factor prices. This does not necessarily imply the production function is homogeneous or homothetic. Linear homogeneity of a production function means that when the quantities of all the inputs employed are increased by some proportion, say doubled, the output will also be doubled. Given that linear homogeneity exists in a production process, it can be said that the model is homogeneous of degree zero with respect to factor prices. In other words, the relative quantities (or quantity ratios) of the inputs used in production are determined solely by relative prices (or price ratios). If all input prices were to double, there would be no change in the relative quantities of inputs employed.

A production function is said to be "homothetic" if it is a monotonic increasing transformation of a homogeneous function. In other words, a homothetic production function $f(x)$ can be written as $f(x)=g(h(x))$, where g is monotonic and h is homogeneous. Therefore, homogeneity is a special case of homotheticity.

For the translog cost function (2.6), a homothetic production function requires restriction of the cost function parameters as $d_i=0$, for all i; a homogeneous (of degree $1/a_y$) production requires further restrictions on the cost function with $d_i=0$, for all i, and $a_{yy}=0$. [See Diewert (1974a, p. 152) and Christensen and Greene (1976) for formal statements and derivations of the restrictions for homotheticity and homogeneity.] The specified scale economies (2.12) are variable, if we choose not to impose the homogeneity condition on the production function, with the level of output, factor prices, and the state of technology. The estimates of scale economies for the homogeneous production models are constant for all levels of output. In general, there are three corresponding formulas for scale economies:

i) Assuming a non-homothetic production function,

$SE=1-(ay+ayy \ln y+\sum di \ln wi)$.

ii) Imposing the homotheticity condition, di=0, for all i,

$SE=1-(ay+ayy \ln y)$.

iii) Imposing the homogeneity condition, ayy=0, di=0, for all i,

$SE=1-ay$.

2.6 Summary

The foregoing presentation of duality theory and flexible functional forms emphasizes that application of the translog cost function is a feasible and appropriate method for studying the structure of inputs used in U.S. farm production. This conclusion, and the rationale presented in Chapter 1, suggests the following guidelines for the design of the present study.

i) Collect data, define inputs or input subgroups, and prepare necessary price indexes.

ii) Specify the statistical model and estimation procedure.

iii) Test research hypotheses and the statistical model specification for theoretical constraints.

iv) Describe and analyze the structure of U.S. farm input subgroups based on the derived elasticity measurements.

Implementation of these guidelines leads to the next chapter, a detailed discussion of empirical input demand system estimation.

CHAPTER 3

Empirical input demand system estimation

This chapter presents an empirical application of duality theory, an investigation of the input demand structure and production technology of U.S. agriculture. The application illustrates a number of facets of econometric research based on duality theory. These include data availability and variable construction, usage of a flexible functional form, econometric estimation procedures, presentation of empirical results, and hypothesis testing of regularity conditions required for the validity of the dual approach. In particular, the translog cost function approach is employed to obtain information on elasticity of substitution and own-price demand elasticity for U.S. farm inputs.

3.1 Input definition and data sources

The system of input demand equations will be estimated by using annual time-series data over the period 1910-81. All data are gathered and constructed from USDA publications.

3.1.1 Inputs

Five purchased input subgroups considered in this study are: (1) hired labor, (2) farm capital, (3) feed, seed, and livestock purchased, (4) fertilizer and lime, and (5) miscellaneous inputs. The "share" (si) of each input subgroup is simply the proportion of corresponding subgroup expenses to total production expenses, as defined in chapter 2.2.2.

Economic theory indicates that a cost function has two basic components, fixed cost and variable cost. In most applications, fixed factors are such things as farm machinery, buildings, cropland and other capital equipment. Variable factors are labor and raw materials. In this study, the concept of "flow" of resource services is used to measure capital-related inputs as well as other inputs. Capital resources, such as farmland, are not used up in the production process. However, a farmer has to pay to acquire the use of capital. For example, he pays for farmland in the form of rent, interest, or depreciation. Therefore, fixed costs are neither assumed as constants over time, nor considered in this study. As the output level increases, the variable cost will increase more or less proportionally until production approaches a capacity level of output determined by the amount of fixed factors.

Total cost is the sum of dollar amounts spent for the input subgroups.

$$\text{cost} = \sum_i (\text{farm production expenses})_i \qquad i=1,2,\ldots,5$$

Detailed explanations of these five input subgroups can be found in various USDA publications (1970, 1981a, 1981b). In brief, the cost of hired labor includes cash wages, perquisites, and social security tax paid by employers.

Farm capital has three major components, namely, farm real estate, machinery, and tax and interest paid for production purposes. Because flow data (e.g., annual machinery depreciation) are preferred to stock data (e.g., stock value of machinery at the year-end), farm capital is the sum of (1) operation and repair of capital items, (2) depreciation and other farm capital consumption, (3) taxes on farm property, (4) interest on farm mortgage debt, and (5) net rent to non-operator landlords.

Feed, seed, and feeder livestock purchased form another category. These purchases are derived from activities of the nonfarm sector, such as feed and seed processing, transportation, and marketing service charges. Seed expense includes bulbs, plants and trees. Fertilizer and lime include both farm and nonfarm use. Recently, national nonfarm use accounts for only three to four percent of total use.

Finally, miscellaneous inputs include interest on non-real estate debt, pesticides, ginning, electricity and telephone, livestock marketing charges, containers, milk hauling, irrigation, grazing, binding materials, horses and mules, harness and saddlery, hardware, veterinary services and medicines, net insurance premiums (e.g., crop, fire, wind, and hail insurance premiums), machine hire and custom work, other livestock and poultry, dairy supplies, nursery, greenhouse, and apiary.

Total production expense is defined as the total cost incurred by farmers for production. Gustafson (1983), a staff agricultural economist at Economic Research Service (ERS) in the USDA, identifies three farming cost series as reported by the USDA. They are: (1) ERS's costs of production, (2) ERS's farm production expenses used in the calculation of net farm income, and (3) the farm production expenditure survey (FPES) conducted by the Statistical Reporting Service (SRS) in the USDA.

The FPES information is based on a survey of about 3,900 usable responses from farms and ranches. This data and additional ERS information are used to estimate farm production expense for net farm income. On the other hand, costs of production are aggregated at the national and state level on a crop year basis, but costs of production for major crop and livestock commodities are reported per acre and per production unit (bushels, hundredweight (cwt), or pounds). Because of the detailed nature of this survey, estimates are limited to major crop and livestock commodities. Compared to the other two series((1) and

(2) above), ERS's farm production expenses better represent U.S. farming cost. These annual figures not only serve as the dependent variable in the cost equation but can also be used as the total cost to generate the factor shares.

Finally, it should be noted that the USDA farm income accounts are based on the assumption that the U.S. agricultural sector is a single large farm. Therefore, certain transactions among farmers are not measured in income or cost accounts since they cancel themselves out within the farm sector. An example of this situation is rent. Only net rent paid to nonfarm landlords is included as an expense. Rent paid to farm operators is not included in farm production expense because it would offset income and cost for farm operators as a group. In addition, the labor input series excludes unpaid operator and family labor.

3.1.2 Output

Two basic series measure agricultural output. These series are the farm output index and the farm marketing and consumption index. The two series differ primarily in the way inventories are handled. The farm output index includes all production on farms, whether or not the products are sold. The farm marketing and consumption index includes output that is sold regardless of the year of production. This index also excludes feed. Over time, the behavior of the two indexes is the same. However, for any given year behavior may differ substantially. In general, the farm output index most accurately represents annual farm production and is used in this study.

The index of farm output measures yearly changes in the combined volume of crop and livestock production available for human consumption. The output index covers not only production in the year a product is produced, but also changes in farm inventory of livestock. Farm output is a gross measure. However, it excludes production and the use of producer goods. Producer goods are produced on farms and used in further agricultural production. These goods include hatching eggs, livestock feed, seeds, and farm-produced power of horses and mules.

The USDA calculates the index using a two-step weighted aggregate method. First, to arrive at quantity-price aggregates, quantities of each commodity produced each year are multiplied by weighted average prices received by farmers during the weight period. Second, the quantity-price aggregates are expressed as percentages of the average quantity-price aggregates in the reference year (1967=100). This percentage is the index.

3.1.3 Input prices

The five input prices are the following: wage rate of hired labor (w1), user cost of farm capital (w2), a Divisia price index {Footnote 9} for feed, seed, and livestock purchased (w3), fertilizer price index (w4), and the price index for all commodities purchased for

{Footnote 9} Divisia price index is a composite index. See appendix C for a complete explanation.

farm production (w5). A timely USDA publication, <u>Agricultural Statistics</u> (1982 edition), adopts 1977 as the new base year (1977=100). This publication contains a new, revised and updated series of prices paid by farmers during the period 1910-81. The five input price indexes employed in this study are either adopted or constructed from this newly published source.

It is worth elaborating a bit more on the user cost of capital. The cost of using more capital is known as the "user cost of capital," or the "rental cost of capital." To derive the user cost of capital, one assumes that the farming sector finances capital purchases by borrowing at an interest rate, R. To obtain an extra unit of capital in each period, a farmer must pay the interest for each dollar's worth of capital equipment that he buys. Thus, the basic measure of the user cost of capital is the interest rate.

Three major interest rates are summarized annually in the farming industry: (1) Federal Land Bank (FLB) Association rates, (2) Production Credit Association (PCA) rates, and (3) the interest rate payable per acre on farm real estate debt. The federal land bank associations are local farmer-owned organizations through which farmers obtain long-term (up to 40 year) loans on land. In 1982 there were 485 FLB associations. Production Credit Associations, 423 total in 1982, are also owned by farmer-borrowers. They provide short- and intermediate-term operating loans for up to 7 years. Both associations are integral parts of the USDA Farm Credit System which supplies nearly one-third of the borrowed capital used by farmers.

A composite Divisia price index may adequately represent overall interest rate fluctuations. However, it seems unlikely that a Divisia price index can be successfully constructed from the reported series of interest rates. PCA data prior to 1933 and the interest paid to each group are unknown. In option three, one assumes that the interest rate on farm real estate is the true price of farm capital. Using this assumption, the estimate results will be unbiased if the real estate interest rate is significantly highly correlated with the non-real estate rate. The following table indicates that all three interest rates are highly correlated for the 1965-81 period.

Table 3.1 Correlation Coefficients of Interest Rates (1965-81).

	FLB	PCA	R
Federal Land Bank (FLB)	-	.893	.882
Production Credit Association (PCA)		-	.903
Interest Rate on Farm Real Estate (R)	(symmetry)		-

Therefore, the convenient real estate mortgage rate is employed as the user cost of capital in this study.

Econometric applications of duality theory rely heavily on the availability of data. It is essential not only that all quantities and price data are available, but also, that they are relatively accurate and specific to the firm, household, or region under consideration. Otherwise, changes in price and other variables may not be meaningful. This is because these changes may represent errors of measurement rather than changes in opportunity cost faced by the decision-maker. Unfortunately, in modelling the agricultural sector as a whole, there are some losses from data aggregation.

3.2 Statistical model specification and procedure

The following non-homothetic, factor-augmenting technology translog cost function is used for estimation purposes.

```
(3.1)  ln C(w,y)=a0+a1*ln w1+a2*ln w2+a3*ln w3+a4*ln w4+a5*ln w5
                +ay*ln y+.5*ayy*(ln y)**2
                +d1*ln w1*ln y+d2*ln w2*ln y+d3*ln w3*ln y
                +d4*ln w4*ln y+d5*ln w5*ln y
                +t*T+.5*tt*T**2+t1*t*ln w1+t2*T*ln w2
                +t3*T*ln w3+t4*T*ln w4+t5*T*ln w5
                +.5*(b11*ln w1*ln w1+b12*ln w1*ln w2+b13*ln w.*ln w3
                                   +b14*ln w1*ln w4+b15*ln w1*ln w5
                    +b21*ln w2*ln w1+b22*ln w2*ln w2+b23*ln w2*ln w3
                                   +b24*ln w2*ln w4+b25*ln w2*ln w5
                    +b31*ln w3*ln w1+b32*ln w3*ln w2+b33*ln w3*ln w3
                                   +b34*ln w3*ln w4+b35*ln w3*ln w5
                    +b41*ln w4*ln w1+b42*ln w4*ln w2+b43*ln w4*ln w3
                                   +b44*ln w4*ln w4+b45*ln w4*ln w5
                    +b51*ln w5*ln w1+b52*ln w5*ln w2+b53*ln w5*ln w3
                                   +b54*ln w5*ln W4+b55*ln w5*ln w5)
                +u1
```

The derived demand equation for each input, in terms of the factor share, is obtained by partially diferentiating the cost function (3.1) with respect to the factor price.

```
(3.2)   s1=a1+b11*ln w1+b12*ln w2+b13*ln w3+b14*ln w4+b15*ln w5
                  +d1*ln y+t1*T+u2

(3.3)   s2=a2+b21*ln w1+b22*ln w2+b23*ln w3+b24*ln w4+b25*ln w5
                  +d2*ln y+t2*T+u3
```

(3.4) s3=a3+b31*ln w1+b32*ln w2+b33*ln w3+b34*ln w4+b35*ln w5
+d3*ln y+t3*T+u4

(3.5) s4=a4+b41*ln w1+b42*ln w2+b43*ln w3+b44*ln w4+b45*ln w5
+d4*ln y+t4*T+u5

The specified equations (3.1)-(3.5) are the stochastic version of previously defined cost (2.6) and factor share (2.8) equations. It is possible to estimate the parameters of the translog cost function (3.1) alone by using the ordinary least squares (OLS) method (Nerlove, 1963). However, since the translog cost function consists of a large number of regressors, a high degree of multicolinearity may be a problem. Consequently, OLS may yield imprecise estimates. Futhermore, the single-equation OLS method neglects the additional information contained in the factor share equations (3.2)-(3.5). An alternative method is to estimate the factor share equations solely as a multivariate regression system (Binswanger, 1974b; Berndt and Wood, 1975). This method is satisfactory if factor share equations contain all parameters in the translog cost function. But, for a non-homothetic, factor-augmenting technology specification, many parameters do not appear in the factor share equations (e.g., ayy, tt). Therefore, this method is inappropriate for this study.

A better estimation approach combines the cost and factor share equations and treats them as a multivariate regression system (Christensen and Greene, 1976; Ray, 1982). An additive disturbance, "ui", for each of the factor share equations and the cost equation, is assumed on the basis that producers make random errors in adjusting the cost-minimizing input levels. From the mathematical viewpoint, the disturbance term represents the influence of higher-order terms, since the translog form is a second-order local approximation to a true function. Note that factor share equations are derived by differentiation. These additive disturbance terms in the factor share equations do not contain the disturbance term from the cost function. However, disturbances are likely to be correlated across equations because random deviations from cost minimization affect all input prices. When ui is "seemingly unrelated" with uj, where i does not equal j (i.e., the error terms exhibit a non-zero covariance), Zellner's (1962) two-stage estimation technique yields more efficient estimates. Zellner shows that when disturbances across equations are correlated, and the correlation is known, the parameters can be more efficiently estimated by taking this information into account. Furthermore, Zellner (1963) demonstrates that even when the correlation is unknown, it is likely that using an estimate of the correlation in the two-stage technique can improve estimation efficiency.

Therefore, the statistical technique implemented in this five-equation multivariate regression system is the Iterated Zellner Efficient Estimation (IZEF) method. The IZEF procedure transforms the error terms to provide a diagonal variance-covariance matrix of error terms and minimizes the trace of the sum of squared transformed residuals. The estimates are consistent and asymptotically efficient. Kmenta and Gilbert (1968), in a series of Monte Carlo experiments, demonstrate that, if the disturbances are normally distributed, Maximum Likelihood (ML) and IZEF result in identical estimates. Later, Ruble (1968) demonstrates the computational equivalence of IZEF and ML estimators. The equivalence of these two methods is important since the IZEF will produce the estimates, but the methods for testing the applicability of the theoretical constraints are based on ML.

The IZEF method is available in the Time Series Processor (TSP) version 3.5 statistical package. Computational facilities are provided by the CDC 750 computer at Michigan State University, East Lansing.

Recall that the five input subgroups are categorized as (1) hired labor, (2) capital, (3) feed, seed and livestock purchased, (4) fertilizer and (5) miscellaneous items. During the estimation procedure, the last equation (i.e., the miscellaneous, s5) is excluded from the system to avoid a singularity problem (i.e., a singular Variance-Covariance matrix of the estimated disturbances). By deleting one of the share equations from the system the IZEF method becomes operational. Barten (1969) shows that the maximum likelihood estimates of the system equations, with one equation deleted, are invariant regardless of which equation is dropped. Forty-five parameters need to be estimated within the system. However, the assumed symmetry and homogeneity (i.e., equation (2.7)) of this translog function requires the following 18 parameter restrictions.

8 restrictions for homogeneity of degree one:

```
      a1+a2+a3+a4+a5=1, or a1=1-a2-a3-a4-a5.
      d1+d2+d3+d4+d5=0, or d1=-(d2+d3+d4+d5).
      t1+t2+t3+t4+t5=0, or t5=-(t1+t2+t3+t4).
b11+b12+b13+b14+b15=0, or b11=-(b12+b13+b14+b15).
b12+b22+b23+b24+b25=0, or b22=-(b12+b23+b24+b25).
b13+b23+b33+b34+b35=0, or b33=-(b13+b23+b34+b35).
b14+b24+b34+b44+b45=0, or b44=-(b14+b24+b34+b45).
b15+b25+b35+b45+b55=0, or b55=-(b15+b25+b35+b45).
```

10 restrictions for symmetry:

b21=b12,
b31=b32, b32=b23,
b41=b14, b42=b24, b43=b34,
b51=b15, b52=b25, b53=b35, b54=b45.

After imposing these 18 restrictions by substituting parameters, the original translog cost function and four factor share equations appear complex. However, the symmetry condition eliminates 10 parameters while the homogeneity of input price reduces the number of free parameters by eight. Hence, the total number of active parameters is reduced from 45 to 27. The starting value of each parameter can be assigned arbitrarily. The starting values here are based on the results of an initial OLS trial run. The tolerance level was given as 0.001. After several iterations, the convergence of the residual covariance matrix is achieved. The estimated parameters of the system, the associated standard errors and t-statistics are reported in Table 3.2.

Table 3.2 Estimated Coefficients of Translog Function 1910-1981, with Homogeneity and Symmetry Restrictions.

	Estimated Coefficient	Standard Error	t-Statistics
a0 (intercept)	17.2935	6.9773	2.4785
*a1	.8085	-	-
a2	1.1334	.1471	7.7048
a3	-.0937	.1251	-.7490
a4	-.2522	.0500	-5.0399
a5	.0210	.0722	.2910
ay	-5.7742	3.4960	-1.6516
ayy	2.0027	.8724	2.2957
*d1	.0113	-	-
d2	-.1969	.0416	-4.7282
d3	.0554	.0344	1.6134
d4	.0654	.0135	4.8280
d5	.0647	.0191	3.3783
*b11 (labor)	.0117	.0075**	-
b12	-.0101	.0020	-5.0924
b13	.0208	.0093	2.2354
b14	-.0075	.0030	-2.5047
b15	-.0148	.0060	-2.4737
*b22 (capital)	.0013	.0039**	-
b23	-.0367	.0039	-9.4582
b24	.0074	.0012	5.9553
b25	.0381	.0021	18.1947
*b33 (feed, seed)	.0821	.0186**	-
b34	.0247	.0065	3.7989
b35	-.0908	.0133	-6.8092
*b44 (fertilizer)	.0395	.0041**	-
b45	-.0640	.0072	-8.8995
*b55 (misc.)	.1316	-	-
t	.0541	.0083	6.5295
tt	-.0018	.0002	-7.8382
t1	-.0022	.0003	-7.8390
t2	.0029	.0007	4.2908
t3	.0014	.0005	2.4711
t4	.0001	.0002	.6248
*t5	-.0022	-	-

Log of Likelihood Function = 1006.04

* Implied estimates computed using the homogeneity constraints.

** These standard errors are obtained from a different set of restrictions: b15= -(b11+b12+b13+b14), b25= -(b13+b23+b33+b34), b35= -(b13+b23+b33+b34), b45= -(b14+b24+b34+b44), b55= -(b15+b25+b35+b45). The estimates have varied slightly, although nothing significantly changed. The value of the log-likelihood function is 1007.64.

The vast majority of parameter estimates are significantly different from zero at the 99% confidence level in a two-tailed test. The t-statistic, at the 1% significance level with 45 degrees of freedom, is about 2.6. An indication of the goodness-of-fit of the estimated system can be obtained from the correlation coefficients between the actual and fitted values of the dependent variables in the estimated equations. These correlation coefficients are: total cost 0.983, labor (s1) 0.970, capital (s2) 0.700, feed, seed and livestock purchased (s3) 0.938, and fertilizer (s4) 0.831. The correlation coefficient for the calculated fifth factor share equation, miscellaneous items (s5), is 0.940. The correlation for capital is relatively small primarily because there is no clear trend in the share of capital in total farm production expenses. Severe volatility for capital inputs during the 1930's and 40's could be the main reason no trend exists. The highest capital share (0.523) occurred in 1932, and in 1945, only 13 years later, the lowest (0.371) occurred.

Before proceeding to demand and substitution elasticities, it is necessary to discuss whether the estimated translog cost function is well-behaved. If the regularity conditions are met, we can be more confident that the estimated elasticities reflect the actual U.S. farm input demand structure.

Recall that a well-behaved translog cost function must satisfy the following regularity conditions: (1) Continuity with respect to factor prices is a maintained hypothesis in this model. Section 3.3.1. addresses the question of symmetry of the estimated demand functions, i.e., $d(si)/d(wj)=d(sj)/d(wi)$, for all i not equal to j? This question pertains to the integrability of the demand functions, i.e., to the existence of a relevant aggregate industry-wide (or sector-wide) cost function.

(2) Linear homogeneity in factor prices. This implies that the regularity condition in equation (2.7) should be imposed on the estimated parameters. This restriction will be tested in section 3.3.2.

(3) The monotonicity property indicates that the cost function must be increasing with respect to each factor price. In other words, estimated factor shares must be strictly greater than zero at every data point. The plotted factor shares are nonnegative at all observations. Therefore, I conclude that the monotonicity condition is satisfied in this model.

(4) Concavity in factor prices. Recall that we can check the concavity condition by confirming that the characteristic roots of the Hessian matrix are negative.

$$H = \left| \begin{array}{llllll} H11 & H12 \ldots & H1j \ldots & & & H1n \\ H21 & H22 & & & & . \\ . & . & & Hij & & . \\ . & & Hii & & & . \\ Hi1 & & & . & & . \\ . & Hij & & & . & . \\ . & & & & . & . \\ Hn1 & \ldots & \ldots & \ldots & \ldots & Hnn \end{array} \right|$$

The ith diagonal entry, Hii, and the (i,j)th off-diagonal entry, Hij, of the Hessian matrix has the form of

$$H_{ii} = \frac{d^2 C}{(dw_i)(dw_i)} = \frac{d X_i}{d w_i} = [(b_{ii}+s_i^2 - s_i).C]/[(w_i)(w_i)]$$

$$H_{ij} = \frac{d^2 C}{(dw_j)(dw_i)} = \frac{d X_i}{d w_j} = [(b_{ij}+s_i.s_j).C]/[(w_i)(w_j)]$$

In actual calculation, we can cancel the "C" without changing the sign of the computed eigenvalues. So we can rewrite these formulae into the following matrix:

$$\begin{vmatrix} \frac{b11+s1.s1-s1}{w1.w1} & \frac{b12+s1.s2}{w1.w2} & \frac{b13+s1.s3}{w1.w3} & \frac{b14+s1.s4}{w1.w4} & \frac{b15+s1.s5}{w1.w5} \\ & \frac{b22+s2.s2-s2}{w2.w2} & \frac{b23+s2.s3}{w2.w3} & \frac{b24+s2.s4}{w2.w4} & \frac{b25+s2.s5}{w2.w5} \\ & & \frac{b33+s3.s3-s3}{w3.w3} & \frac{b34+s3.s4}{w3.w4} & \frac{b35+s3.s5}{w3.w5} \\ & \text{(SYMMETRY)} & & \frac{b44+s4.s4-s4}{w4.w4} & \frac{b45+s4.s5}{w4.w5} \\ & & & & \frac{b55+s5.s5-s5}{w5.w5} \end{vmatrix}$$

These calculations are extremely burdensome because the characteristic roots of a 5x5 matrix are converted into a five-degree polynomial equation. The results are obtained from the user-written program with the assistance of a FORTRAN subroutine -- EIGRS provided by International Mathematical and Statistical Libraries (IMSL), Inc.

(See appendix D for computer program list and a complete result.) The calculated result, the vector of characteristic roots, for example at 1981, is [-.09 -.08 -.02 .00 .01]. The Hessian matrix is negative semidefinite as long as the characteristic roots are nonpositive. Unfortunately, the results show an "indefinite" situation. Therefore, the estimated translog cost function is not concave in factor prices. It should be emphasized that this is not a statistical test. Our estimates themselves do not satisfy the concavity condition, but we have no way of testing whether the deviation from concavity is statistically significant.

3.3 Testing Hypotheses

The translog cost function does not necessarily exhibit the homogeneity and symmetry conditions. Instead, we can statistically test the validity of these restrictions as implied by the theory.

Since (assuming normally distributed disturbances) maximum-likelihood estimates can be obtained from iterating Zellner's seemingly unrelated regression procedure, we can test hypotheses such as linear homogeneity and symmetry by using the likelihood ratio test. Denoting the determinants of the unrestricted and restricted estimates of the disturbance covariance matrix as $|Vu|$ and $|Vr|$, respectively, we can write the likelihood ratio

$$(3.6) \qquad L = \left(\frac{|Vr|}{|Vu|}\right)^{-T/2} = \left(\frac{\text{value of likelihood function restricted}}{\text{value of likelihood function unrestricted}}\right)$$

where T is the number of observations (i.e., years). Clearly L lies between zero and one because the denominator (unrestricted) has to be greater than or equal to the numerator (restricted). As L approaches one, the restricted and unrestricted regressions have very little difference. Therefore, we reject Ho (the null hypothesis) if L < Ca where Ca is a constant defined so that type 1 error (i.e., Prob(reject Ho given Ho is true)) is a. The likelihood ratio test has several important properties. It is asymptotically unbiased and consistent. Furthermore, we test the hypotheses using the fact that -2 ln L has a chi-square distribution in large samples, with degrees of freedom "r" equal to the number of independent restrictions imposed (Theil, 1971). The smaller the L (in equation (3.6)), the larger the computed chi-square statistics.

By placing different restrictions on the equation system (3.1)-(3.5), we obtain a series of models which allows us to perform several interesting tests that deal with the structure of U.S. farm production. Three groups of models are presented under the different specifications of technological changes: 'No' technological change, 'Hicksian-neutral' and 'Non-neutral' technological change. Table 3.3 summarizes these alternative specifications and their log-likelihood values.

Table 3.3 Results of Alternative Model Specification.

Model	Specification	Parameter restrictions	Log of likelihood function
Group A: No technological change ($t=tt=ti=0$, $\forall$ i)			
1	Homogeneity	$\sum ai=1$, $\sum di=\sum bij=o$	1052.33
2	Symmetry	$bij=bji$	962.36
3	Homo/Sym	both	948.54
Group B: Hicksian-neutral technological change ($tt=ti=0$, $\forall$ i)			
4	Homogeneity	$\sum ai=1$, $\sum di=\sum bij=o$	1052.42
5	Symmetry	$bij=bji$	965.78
6	Homo/Sym	both	944.00
Group C: Non-neutral technological change			
7	Homogeneity	$\sum ai=1$, $\sum di=\sum bij=0$	1094.74
8	Symmetry	$bij=bji$	1012.72
9	Homo/Sym	both	1006.04

The rest of this section is subdivided into three parts, namely, discussions of symmetry, homogeneity, and technological changes respectively.

3.3.1 Symmetry

Previous studies have explicitly derived aggregate input demand functions and estimated them from either a profit maximizing or cost-minimizing scheme. Most researchers have not been generally concerned as to whether or not the estimated demand functions satisfy the integrability condition, with the exception of a few authors such as Binswanger (1974a), Lopez (1980), and Rostamizadeh et al. (forthcoming). The question is important because its answer will indicate whether there exists an aggregate cost function from which the farm input demand system can be derived.

Hurwicz and Uzawa (1971) have shown that a system of demand equations is integrable if, and only if, the Hessian matrix is symmetric. The Hessian matrix consists of the second partial derivatives of the cost function. Therefore, the first job should be the testing (rather than the imposition) of the symmetry condition on the proposed input demand system. Rejection of the symmetry condition implies that an aggregate cost function and an aggregate production function do not exist. On the other hand, if the symmetry condition holds, it implies that the input demand functions are integrable to some macrofunction (Lopez, 1980). Note that the proposed symmetry condition implies mathematical integrability, but not necessarily economic integrability.

The corresponding likelihood ratio test is designed to test the null hypothesis that the symmetry condition holds (i.e., $bij=bji$ for all $i \neq j$), against the alternative hypothesis of no symmetry (i.e., unrestricted values of the bij and bji parameters). The degrees of freedom is 10 because we have 10 upper triangular off-diagonal unrestricted elements in a 5x5 symmetric matrix. The chi-square statistic is twice the difference of the corresponding log-likelihood values between two comparable models. For example, in the first group of unspecified technological change, the test of symmetry is carried out by comparing those two models which have the same restriction of homogeneity but one with symmetry restriction (model 3) and the other one without (model 1). From Table 3.3, the calculated chi-square statistics of testing the symmetry restriction under three alternative technology specifications are 207.58, 216.84, and 177.4 for 'No', 'Hicksian-neutral', and 'Non-neutral' technological change, respectively. Since all three statistics are larger than the critical value (i.e., 20.48) at the 5% significance level, the hypothesis of symmetry is rejected. This implies the nonexistence of a mathematically aggregated cost function and the underlying aggregated production function for U.S. agriculture.

From Table 3.4, we find that other researchers may also face the same problem when working with aggregate data. The rejection of symmetry from the study of Rostamizadeh et al. (forthcoming), which employs four input subgroups of capital, land, labor, and fertilizer over the period of 1960-79, reinforces the inherent difficulty of using aggregate data in demand system estimation. On the consumption side, the possibility of failing the symmetry condition for U.S. aggregated food consumption is not less than that for the agricultural production. A study by Chambers and McConnell (1983) shows a clear rejection of the symmetry restriction, which implies the failure of integrability of the U.S. food demand system. Although the statistical test may exhibit small sample bias, rejection of symmetry may occur because producers' (consumers') behavior does not reflect optimization of a well-behaved cost (utility) function, or because of the aggregation problem.

Table 3.4 Statistical Test Results of Aggregated Cost Function Studies of Agricultural Produciton

Study	Functional Form	Data	Test of Symmetry	Test of Homogeneity
Binswanger 1974 U.S. Ag	translog	Cross-sectioal (CS) and Time-series (TS) 1949,54,59,64.	Accepted	None
Kako, 1978 Japan Rice	translog	TS, 1953-70 and CS	None	None
Lopez, 1980 Canada Ag	GL	TS, 1946-77	Accepted	NA
Chambers 1982, U.S. Meat	GL	TS, 1954-76	None	NA
Rostamizadeh et al. 1982 U.S. Ag	translog	TS, 1960-79	Rejected	Rejected
Ray, 1982 U.S. Ag	translog	TS, 1953-70	None	None
Hsu, 1984 U.S. Ag	translog	TS, 1910-81	Rejected	Accepted

For most empirical studies, a flexible cost function C(w,y) is assumed to be twice differentiable with respect to factor prices. Differentiability is an assumed property of a cost function. The first partial derivative of the cost function is the constant output factor demand function,

$$\left.\frac{d\,C(w,y)}{d\,w_i}\right|_{y=\text{constant}} = x_i \quad \text{(Shephard's Lemma)}$$

The second partial derivatives of the cost function yield the symmetry condition,

$$\frac{d^2 C(w,y)}{dwi\ dwj} = \frac{d^2 C(w,y)}{dwj\ dwi}$$

The above symmetry is valid for an aggregate cost function, only if we assume all outputs are fixed at the aggregate level and also for each individual firm, so that the aggregate cost function is in fact just the sum of the individual cost functions. This assumption of fixity of output may be thought of as the essence of the aggregation problem.

Let us assume there are two firms using two inputs to produce one output. The correponding input prices are w1 and w2 respectively. The cost for each firm is

$$C1 = f\ (w1,\ w2,\ y1)$$

$$C2 = g\ (w1,\ w2,\ y2) \quad \text{and}$$

$$y = y1 + y2$$

where y is the aggregate output of the two firms. Now if y1, y2, and y are all fixed, the aggregate cost function is

$$C = C1+C2 = f(w1,w2,y1) + g(w1,w2,y2) = h(w1,w2,y)$$

Since y1, y2, and y are fixed, the symmetry condition (i.e., the second partial derivatives of functions f, g, and h) holds at the aggregate level (aasuming it holds for each firm).

The normal procedure for taking cross-partial derivatives can be stated as follows: If f and g are separate functions and Dij denotes the cross-partial derivatives with respect to variables i and j, and assuming that the symmetry condition holds for both f and g, that is

$$D12\ f = D21\ f, \text{ and } D12\ g = D21\ g,$$

then

$$D12\ (f+g) = D12\ f + D12\ g = D21\ f + D21\ g = D21\ (f+g)$$

so, "D12 (f+g)" must be equal to "D21 (f+g)", or D12 h = D21 h. However, the assumption of fixed output at the aggregate level does not necessarily restrict the individual firm's output level to be fixed at all times. Now suppose y1 and y2 are <u>not</u> fixed, we still have y1+y2=y and the independent relationship between w1 and w2, but y1 and y2 can change in response to the changes in w1 and w2,

$$\frac{d\ C(w,y)}{d\ w1} = \frac{d\ f}{d\ w1} + \frac{d\ f}{d\ y1}\ \frac{d\ y1}{d\ w1}$$

$$\frac{d^2\ C(w,y)}{(d\ w1)\ (d\ w2)} = \frac{d^2\ f}{(dw1)\ (dw2)} + \frac{d\ f}{d\ y1}\ \frac{d^2\ y1}{(dw2)\ (dw1)} + \frac{d\ y1}{d\ w1}\ \frac{d^2\ f}{(dw2)\ (dy1)}$$

Similarly, we can get the partial derivatives in w2,

$$\frac{d\ C(w,y)}{d\ w2} = \frac{d\ f}{d\ w2} + \frac{d\ f}{d\ y1}\ \frac{d\ y1}{d\ w2}$$

$$\frac{d^2 C(w,y)}{(d\,w2)(d\,w1)} = \frac{d^2 f}{(dw2)(dw1)} + \frac{d\,f}{d\,y1}\,\frac{d^2 y1}{(dw1)(dw2)} + \frac{d\,y1}{d\,w2}\,\frac{d^2 f}{(dw1)(dy1)}$$

Relaxing the assumption of fixed y1, symmetry for firm one requires

$$\frac{d\,y1}{d\,w1}\,\frac{d^2 f}{(dw2)(dy1)} = \frac{d\,y1}{d\,w2}\,\frac{d^2 f}{(dw1)(dy1)}$$

It is hard to interprete the above mathematical equations in economic context. By the same token, the resulting condition of symmetry for firm two is

$$\frac{d\,y2}{d\,w1}\,\frac{d^2 f}{(dw2)(dy1)} = \frac{d\,y2}{d\,w2}\,\frac{d^2 f}{(dw1)(dy1)}$$

In this case, it appears that symmetry in the individual function need not imply symmetry in the aggregate cost function. Therefore, this seems to be the essense of the "aggregation problem" in the present context. It is true that, for estimation purpose, a researcher can assume y (aggregate output) as exogenous, or, in a sense, "fixed." But the output is not really fixed and, even if it were, the individual firm's output (y1, y2, etc.) would not necessarily be.

Is it possible to find two individual cost functions which are internally symmetric within each firm while the aggregated cost function of these two firms is asymmetric?

It is true that if the cost function of one of these two firms is asymmetric, then the aggregated cost function is also asymmetric. Therefore, if there are n firms (e.g., 2.4 million American farms in recent years), and only one firm acts irrationally so as to violate the assumed cost-minimizing behavior, even though the other n-1 firms are well-behaved, the aggregated cost function will be unable to pass the symmetry restriction. Of course, we need more than one firm (e.g., maybe a state, or a particular crop region such as the explosive production of sunflower that appeared in North Dakota in recent years) to show the statistical significance of the failure.

3.3.2 Homogeneity

The corresponding likelihood ratio test for the null hypothesis of homogeneity of degree one which holds against the alternative hypothesis of non-homogeneity or homogeneity of any other degree, is conducted as follows. The chi-square degrees of freedom in the first two cases (i.e., 'No technological change' and 'Hicksian-neutral technological change') is seven. The degrees of freedom in the last case of 'Non-neutral technological change' is eight due to the additional restriction on time parameters. The calculated chi-square statistics for these three models are 27.64, 23.56, and 13.36, respectively. In the first two cases, the chi-square statistics exceeded critical values both at the 1% (i.e., 18.475) and at the 5% (i.e., 14.067) significance levels. However, the homogeneity of degree one passes the test in the third case since the calculated chi-square statistic is 13.36 which is less than the critical values at the 1% (i.e., 20.090) and at the 5% (i.e., 15.507) significance levels. Therefore, the empirical results suggest that the imposition of the homogeneity restriction on the specified 'Non-neutral technological change' model seems plausible.

The study of Rostamizadeh et al. (forthcoming) fails this test, possibly as a result of using a shorter time period, 1960-79. Other published studies which employ the cost function approach to examine agricultural production do not address the test of homogeneity at all.

3.3.3 Technological Change

Does it matter if we impose different specifications on technological change? Again, we can answer this question by comparing the log-values of the log-likelihood function of model 3, 6 and 9. Assuming both symmetry and homogeniety restrictions are necessary to the model system, the computed chi-square statistics between model 3 with 'no technological change' against model 9 with 'Non-neutral technological change' is 115.0 and is 124.08 for model 6 with 'Hicksian-neutral technological change' against model 9. Both model 3 and 6 are rejected against the specified 'Non-neutral technological change' model because both statistics far exceed the critical value of the chi-square test.

One might notice that the log-likelihood values of model 3 and model 6, with and without Hicksian-neutral technological change, are very close to each other. The test statistic between these two models is 9.08. With one restriction on time parameter, we have one degree of freedom. Then, according to the chi-square statistics (i.e., 6.635 at the 1% significance level), we still cannot say there is no difference between these two models. This suggests that it is unrealistic to assume the absence of technological change.

The reported estimates in Table 3.2 of Hicksian-biases, ti, show that three out of four estimates are significantly different from zero at the 2% significance level. The only exceptional case is fertilizer. In particular, both labor and capital inputs are extremely significant. The estimates indicate that biased technological change has been mainly labor-saving, and capital-using. Also, these two coefficients are larger in absolute magnitude than those for any other input. They amount to an annual 0.22 and 0.29 percent change (in opposite direction) in the labor share and the capital share, respectively, not attributable to substitution within a given production possibility set. If the production technology had remained static over the period, then in 1981 the share of labor would have been about 13.51% larger and the share of capital would have been 17.07% smaller.

Ray (1982) uses a translog cost function with Hicksian-neutral technology change specification and concludes that the rate of technical change in U.S. agriculture was 1.8% per year over the period of 1939-77. Schultz (1953) found that during the period 1910-50 the index of productivity, measured by the ratio of output to input indexes in agriculture, increased at a 1.35% annual rate. For the subperiod 1924-50, the annual growth rate was 2%. Model 6 (Hicksian-neutral) shows a 2.3% annual increase in productivity, or equivalently, a 2.3% annual decline in total production expenses over the period of 1910-81. Using the biased technological change model (Model 9) and employing the definition of (2.11), d(ln C)/dt, an average of 2.9% technological advancement measure is obtained (See Table 3.5).

Table 3.5 Estimated Measures of Annual Technological Advancement.

Year	Measure	Year	Measure
1910	.0356	1950	-.0427
1911	.0338	1951	-.0419
1912	.0325	1952	-.0438
1913	.0308	1953	-.0453
1914	.0294	1954	-.0471
1915	.0275	1955	-.0485
1916	.0256	1956	-.0501
1917	.0234	1957	-.0517
1918	.0212	1958	-.0536
1919	.0192	1959	-.0553
1920	.0175	1960	-.0567
1921	.0173	1961	-.0582
1922	.0158	1962	-.0600
1923	.0137	1963	-.0615
1924	.0115	1964	-.0630
1925	.0099	1965	-.0645
1926	.0077	1966	-.0661
1927	.0059	1967	-.0678
1928	.0042	1968	-.0695
1929	.0021	1969	-.0772
1930	.0006	1970	-.0729
1931	-.0011	1971	-.0746
1932	-.0023	1972	-.0763
1933	-.0046	1973	-.0777
1934	-.0067	1974	-.0795
1935	-.0087	1975	-.0812
1936	-.0110	1976	-.0829
1937	-.0131	1977	-.0845
1938	-.0154	1978	-.0860
1939	-.0172	1979	-.0875
1940	-.0189	1980	-.0891
1941	-.0210	1981	-.0907
1942	-.0239		
1943	-.0264		
1944	-.0287		
1945	-.0313		
1946	-.0333		
1947	-.0354		
1948	-.0373		
1949	-.0385		

Using the farm productivity index (as earlier mentioned on p. 5), the calculated exponential trend has an annual growth rate of 1.4%. It seems that both figures from my research are higher than the previous two researchers' results but still comparable in size. The main difference may result from the length of the research period and the inclusion of war-time periods.

In conclusion, the characterization of the U.S. agricultural production structure, by a cost function exhibiting linear homogeneity and factor-augmenting technological change, appears to be justifiable. The validity of the symmetry restriction is questionable. However, in order to investigate the structure of input demand and the interrelationships between pairs of inputs, a flexible translog cost function which exhibits linear homogeneity and non-neutral technological change, and with symmetry imposed, seems plausible.

In the coming section, I will present estimates of two well-known measures of price reponsiveness, namely, own-price elasticity of demand and Allen's elasticity of substitution.

3.4 Elasticities of Demand and Substitution

The estimated parameters in Table 3.2 become more meaningful when we transform them into elasticity measures. There are three steps to obtain the estimated elasticity measures. (1) Retrieve the estimates of bii and bij from Table 3.2, including those implied by the regularity conditions. The bii's will be used for computing the own-price demand elasticities and the bij's will be employed for calculating the Allen's partial elasticities of substitution. (2) Retrieve the fitted dependent variables, i.e., the factor shares, in each year. Then calculate the residual factor share (s5) as the difference between one and the sum of the other four factor shares. (3) Finally, compute the estimated Allen's partial elasticity of substitution by using bij, si, and equation (2.9); compute the estimated own-price elasticity of demand based on (2.10).

The estimated own-price elasticities (Eii) and Allen's partial elasticities of substitution (Eij) are the essence of this study. The effect that changing factor prices have on factor employment in U.S. agriculture is the most important consideration for policy decision-making. Since both elasticities can be derived under various specifications, Table 3.6 presents three different sets of results. The purpose of this comparison is to observe the effect that the alternative specifications have on these elasticities.

Table 3.6 The Own-Price Input Demand Elasticity and Allen's Partial Elasticity of Substitution under Various Technology Specifications.*

Elasticities	No Tech Change Model 3	Hicksian-neutral Model 6	Non-neutral Model 9
E11 (Labor)	-.9078	-.6957	-.7680
	(.0534)	(.0445)	(.0132)
E22 (Capital)	-.5639	-.4943	-.5578
	(.0191)	(.0209)	(.0143)
E33 (FSL)	-.6512	-.4561	-.4058
	(.0279)	(.0162)	(.0273)
E44 (Fert.)	-.1795	-.2432	-.0892
	(.1702)	(.1552)	(.1913)
E55 (Misc.)	-.4131	-.1953	.2062
	(.0957)	(.0942)	(.2693)
E12	.8389	.4388	.8186
	(.0583)	(.2677)	(.0651)
E13	2.3215	1.2291	1.8627
	(.3380)	(.0889)	(.2161)
E14	.6247	.9579	-.1841
	(.0440)	(.0105)	(.1673)
E15	-.4915	1.1993	.0857
	(.4797)	(.0826)	(.2988)
E23	.5810	.7082	.6430
	(.0762)	(.0460)	(.0684)
E24	1.2410	1.1698	1.3731
	(.0512)	(.0364)	(.0785)
E25	1.8158	1.4583	1.7230
	(.2361)	(.1053)	(.2093)
E34	1.6571	2.4273	3.3502
	(.2738)	(.6065)	(.9663)
E35	-.0047	-1.0302	-2.0353
	(.1850)	(.3128)	(.5755)
E45	-6.9536	-7.7735	-10.5783
	(2.8359)	(2.2585)	(4.3885)

* These are sample means and standard deviations. (That is, the estimated elasticities were calculated for each year of the sample period, then the mean and standard deviations of the these estimates calculated.) The latter are expressed in parentheses.

The substitutability of labor and capital is the most important elasticity measure for policy decision making. Take elasticity E12 as an example. Three different measures are 0.8389, 0.4388, and 0.8186 for 'no technological change', 'Hicksian-neutral' and 'non-neutral technological change', respectively. The Eij which we obtained under the specification of "no technological change" are generally larger than those estimates derived under less restrictive specifications. These results are anticipated since some of what is classified as substitution response under one specification is reclassified as technological change under the other. The measure of technological change obtained from Hicksian-neutral specification is the net effect of factor substitution, i.e., the marginal rate of substitution is constant. Therefore the resulting elasticity of substitution is less precise than the biased technology specification where the utilization of each input is considered.

Two empirical studies are closely related to the same topic of elasticity measurements, and they will be used hereafter to make comparisons with this study. Binswanger (1974a) uses a translog approximation for the cost function for U.S. agriculture. He employs a single output cost function and uses pooled cross-section and time-series data for 48 states for the years 1949, 1954, 1959, and 1964. He also assumes biased technological change. However, his definition of inputs is somewhat different from this study. For example, Binswanger treats land and capital as separate inputs, while in this study these two are pooled into one group, farm capital. Ray

(1982) also adopts the translog cost function approach to study U.S. agriculture. He treats crops and livestock as two distinct outputs. He assumes Hicksian-neutral technology and employs time-series data from 1939 to 1977. Ray defines five input subgroups which are quite similar to my subgroups.

I will proceed to discuss computed elasticities, and compare these elasticities with the above-mentioned research results.

3.4.1 Own-Price Demand Elasticities

Assuming biased technological change and imposing symmetry and homogeneity restrictions, Table 3.7 displays own-price elasticities of demand for each input subgroup. Although the estimates for labor and capital seem fairly stable over the period of the study (1910-81), I am presenting the complete results for each observed year.

Table 3.7 Estimated Own-Price Elasticities of Demand for Inputs, 1910-81.

Year	Labor E11	Capital E22	Feed, Seed Purchased livestock E33	Fertilizer E44	Miscellaneous E55
1910	-.7354	-.5653	-.3680	.3179	.0443
1911	-.7373	-.5575	-.3669	.3885	.0599
1912	-.7390	-.5767	-.3707	.0998	.0057
1913	-.7411	-.5576	-.3695	.1826	.0694
1914	-.7427	-.5718	-.3698	.0447	.0112
1915	-.7447	-.5756	-.3748	-.0692	.0214
1916	-.7468	-.5520	-.3682	.0937	.0666
1917	-.7446	-.5656	-.3878	.1583	.0822
1918	-.7463	-.5615	-.3913	.0968	.1264
1919	-.7470	-.5593	-.3916	.1846	.1165
1920	-.7482	-.5729	-.3922	.0443	.0591
1921	-.7591	-.5337	-.3300	.1799	-.0236
1922	-.7581	-.5525	-.3458	-.0268	-.0518
1923	-.7579	-.5558	-.3605	-.0213	-.0317
1924	-.7575	-.5517	-.3697	.0763	-.0037
1925	-.7596	-.5549	-.3748	-.0586	.0096
1926	-.7600	-.5561	-.3820	-.0787	.0326
1927	-.7601	-.5561	-.3841	.0111	.0197
1928	-.7605	-.5626	-.3959	-.1111	.0727
1929	-.7618	-.5551	-.3967	-.0684	.0954
1930	-.7647	-.5457	-.3897	-.0654	.0787
1931	-.7688	-.5519	-.3744	-.1420	-.0276
1932	-.7734	-.5346	-.3404	-.1158	-.0796
1933	-.7731	-.5197	-.3528	.0545	-.0293
1934	-.7734	-.4864	-.3630	.4652	.1086
1935	-.7708	-.5279	-.3961	.0182	.1337
1936	-.7718	-.5047	-.3934	.3404	.1912
1937	-.7701	-.5434	-.4106	-.0205	.1805
1938	-.7720	-.5359	-.4079	.0215	.1623
1939	-.7727	-.5372	-.4104	-.0179	.1833
1940	-.7730	-.5431	-.4145	-.0707	.2091
1941	-.7726	-.5492	-.4184	-.0770	.2367
1942	-.7704	-.5728	-.4251	-.1343	.3048
1943	-.7687	-.5729	-.4268	-.1118	.5120
1944	-.7684	-.5772	-.4270	-.0713	.5139
1945	-.7675	-.5738	-.4264	-.0357	.8121
1946	-.7676	-.5754	-.4258	.0102	.8521

1947	-.7681	-.5671	-.4254	.1320	1.0030
1948	-.7684	-.5832	-.4237	-.0259	1.0016
1949	-.7724	-.5701	-.4263	-.0291	.7130
1950	-.7729	-.5688	-.4259	-.0004	.7581
1951	-.7737	-.5719	-.4260	-.0029	.6571
1952	-.7744	-.5768	-.4253	-.0804	.7134
1953	-.7767	-.5709	-.4264	-.1041	.5619
1954	-.7775	-.5704	-.4263	-.1193	.5808
1955	-.7788	-.5716	-.4266	-.1586	.4861
1956	-.7801	-.5673	-.4269	-.1414	.3957
1957	-.7809	-.5622	-.4269	-.1376	.3964
1958	-.7811	-.5738	-.4267	-.1808	.3590
1959	-.7816	-.5737	-.4268	-.1859	.3331
1960	-.7824	-.5764	-.4269	-.2207	.2532
1961	-.7828	-.5751	-.4268	-.2505	.2981
1962	-.7832	-.5729	-.4269	-.2412	.2464
1963	-.7834	-.5787	-.4269	-.2743	.1997
1964	-.7836	-.5727	-.4269	-.2664	.1706
1965	-.7835	-.5767	-.4269	-.2904	.1408
1966	-.7834	-.5693	-.4268	-.2572	.1400
1967	-.7832	-.5862	-.4268	-.2866	.0981
1968	-.7829	-.5779	-.4267	-.2730	.0554
1969	-.7827	-.5769	-.4265	-.2275	.0225
1970	-.7823	-.5732	-.4265	-.2074	.0243
1971	-.7817	-.5862	-.4267	-.2683	-.1062
1972	-.7811	-.5840	-.4267	-.2641	-.0068
1973	-.7815	-.5902	-.4269	-.2800	.0297
1974	-.7783	-.5715	-.4269	-.3629	.1532
1975	-.7741	-.5766	-.4270	-.4204	.1166
1976	-.7734	-.5801	-.4268	-.3778	.0204
1977	-.7708	-.5801	-.4265	-.3709	-.0173
1978	-.7690	-.5835	-.4265	-.3720	-.0383
1979	-.7672	-.5921	-.4267	-.3869	-.0595
1980	-.7596	-.5756	-.4262	-.3910	-.0376
1981	-.7556	-.5800	-.4263	-.4270	-.0343

Assuming that the aggregated input subgroups are fairly representative of actual farming situations, a general demand structure can be used to describe U.S. agriculture. This structure is derived from the elasticity measures. By looking at the calculated elasticities in Table 3.6, it is clear that own-price demand elasticities are negative as anticipated except for the miscellaneous input subgroup. The negative demand elasticities imply downward sloping demand curves. However, for both the fertilizer and the miscellaneous subgroups, there are no clear implications which can be drawn from the the abnormal positive signs. Although commercial fertilizers were introduced in the early 1820's, the major increase of fertilizer application has occurred since World War II. Ray reports that his elasticity for fertilizer is -.4875 in 1977. I find that the demand elasticity for fertilizer after the 1950's persistently increases (in absolute value) and reaches -.4270 in 1981.

The estimated elasticites for each input over the past 72 years are generally less than one in absolute value. Farm hired labor has the highest price elasticity of demand, in absolute terms. In other words, the percentage change in hired labor responds to the percentage change in farm wage rate more strongly than all other inputs respond to their corresponding prices in the studied period.

The standard deviation (SD) of the elasticity estimates is not the standard error of the estimated bii from the multivariate regression system. Nor can it be derived from the mean of the 72 observations. The standard deviation of these estimates should be derived from the formula which was used to compute the elasticity of demand.

(2.10) $E_{ii} = [(b_{ii} + s_i(s_i-1)]/s_i$

If si were constant, the variance of Eii, or Var(Eii), would be (Kmenta, p. 62, Theorem 7):

$$Var(Eii)\Big|_{si=constant} = Var[(bii+si-1)/si] = (1/si)^2\ Var(bii)$$

Since the square root of the variance is defined as the standard deviation of a distribtution, the above formula can be rewritten as:

$$(3.7) \qquad SD(Eii)\Big|_{si=constant} = (1/si)\ SD(bii)$$

Of course, si is not constant, so the true variance (or standard deviation) of the elasticity estimates would be larger.

After re-estimating the model 9 with a different set of linear homogeneity restrictions (see footnote at Table 3.2), and by employing (3.7), the standard deviation of the own-price demand elasticity estimates are:

(1) Labor: .0582.

(2) Capital: .0090.

(3) Feed, seed, and livestock purchased: .0753.

(4) Fertilizer: .0847.

These estimated standard deviations may be compared with elasticity estimates in Table 3.7. Note that during the calculation, the si is the mean value of the fitted si over the sample period. In repeat, these standard deviations are under-estimated when si's are assumed to be constant.

3.4.2 Elasticities of Substitution

Again, by assuming biased technological change and imposing the symmetry and homogeneity restrictions on the translog cost function, Table 3.8 shows Allen's partial elasticity of substitution between two input subgroups. The results show that except miscellaneous input items, most other inputs are substitutes for each other in varying degrees. Miscellaneous input is not a substitute for fertilizer, feed, seed, and livestock purchased.

I found a complementary relationship between labor and fertilizer (E14). By using cross-section data, Binswanger also discovered the same relationship which challenged Ray's substitution in this case.

Capital appears to be a substitute for all other input subgroups. Capital and labor are substitutable at every observed year. This phenomenon can also be seen in numerous traditional two input (capital-labor) studies. However, the magnitude of this substitutability relationship has declined persistently. The degree of substitution between capital and fertilizer (E24) is greater than that between capital and labor (E12), between capital and seed, and feed and livestock purchased (E23).

The highest substitutability is between fertilizer and feed, seed, and livestock purchased (E34). However, this is not the case if we assume the absence of technological change (see Table 3.5).

In the previous section, we derived the estimates of standard deviation for own-price demand elasticities. For the same reason, the standard deviation of Allen's partial elasticity of substitution can be derived as follows:

(2.9) $Eij = (bij/(si)(sj))+1$

By using (2.9), the derived formula for computing the standard deviation of the elasticity of substitution is,

(3.8) $SD(Eij)\big|_{si,sj=constant} = [1/(si)(sj)]\ SD(bij)$

While doing this computaton, the si and sj are the mean values of the fitted si and sj over the sample period. The SD(bij) is the standard error which is retrieved from the Table 3.2. The standard deviation of these elasticity estimates are:

E12 (labor and capital): .0326.

E13 (labor and feed, seed, and livestock purchased): .2652.

E14 (labor and fertilizer): .4365.

E15 (labor and misc.): .3250.

E23 (capital and feed, seed, and livestock purchased): .3647.

E24 (capital and fertilizer): .0573.

E25 (capital and misc.): .0373.

E34 (feed, seed, and livestock purchased and fertilizer): .5437.

E35 (feed, seed, and livestock purchased and misc.): .4142.

E45 (fertilizer and misc.): 1.1443.

The usefulness of these standard deviations can be referred to the elasticity measures in Table 3.8. The true standard deviations are larger because si and sj are assumed constant here. The complementarity between labor and fertilizer (E14) is weakened by the high standard deviation.

Table 3.8 Estimated Elasticity of Substitution between Inputs, 1910-81.

Year	Labor Capital E12	Labor FSL E13	Labor Fertilizer E14	Labor MISC E15	Capital FSL E23
mean	.8186	1.8627	-.1841	.0857	.6430
1910	.8874	1.7253	-.1768	.5146	.5347
1911	.8880	1.7373	-.2586	.4984	.5413
1912	.8815	1.7351	.0051	.5288	.5269
1913	.8850	1.7494	-.0912	.4788	.5457
1914	.8798	1.7573	.0334	.5123	.5309
1915	.8769	1.7534	.1358	.4982	.5357
1916	.8816	1.7852	-.0466	.4585	.5492
1917	.8799	1.7120	-.0946	.4567	.5712
1918	.8796	1.7090	-.0454	.4200	.5823
1919	.8795	1.7121	-.1415	.4234	.5850
1920	.8745	1.7163	-.0062	.4581	.5729
1921	.8739	1.9873	-.0748	.4702	.5094
1922	.8697	1.9326	-.0140	.4979	.5124
1923	.8690	1.8860	-.0179	.4827	.5323
1924	.8708	1.8532	-.1240	.4625	.5519
1925	.8672	1.8534	.0088	.4410	.5576
1926	.8663	1.8316	.0282	.4204	.5695
1927	.8662	1.8244	-.0768	.4303	.5735
1928	.8637	1.7820	.0623	.3866	.5907
1929	.8643	1.7887	-.0007	.3609	.5995
1930	.8630	1.8421	-.0356	.3544	.5940
1931	.8544	1.9433	.0136	.4162	.5600
1932	.8508	2.1299	-.0864	.4294	.5235
1933	.8562	2.0810	-.3119	.3829	.5563
1934	.8651	2.0479	-.8648	.2534	.5997
1935	.8583	1.8772	-.2233	.2584	.6217
1936	.8631	1.9022	-.6621	.1989	.6345
1937	.8548	1.7957	-.1621	.2260	.6418
1938	.8534	1.8319	-.2481	.2214	.6410
1939	.8515	1.8258	-.2078	.1954	.6462
1940	.8491	1.8033	-.1405	.1704	.6525
1941	.8477	1.7713	-.1265	.1509	.6602
1942	.8440	1.6825	-.0173	.1208	.6727
1943	.8473	1.6308	-.0249	-.0224	.6908
1944	.8464	1.6146	-.0722	-.0193	.6943
1945	.8491	1.5812	-.1070	-.2334	.7105
1946	.8485	1.5707	-.1646	-.2631	.7147
1947	.8504	1.5682	-.3258	-.3842	.7234

1948	.8441	1.5514	-.1299	-.3867	.7217
1949	.8407	1.6158	-.1872	-.2344	.7144
1950	.8401	1.6132	-.2338	-.2787	.7183
1951	.8369	1.6220	-.2465	-.2134	.7159
1952	.8333	1.6166	-.1515	-.2714	.7177
1953	.8291	1.6640	-.1635	-.1927	.7119
1954	.8267	1.6699	-.1589	-.2275	.7141
1955	.8213	1.6978	-.1314	-.1762	.7094
1956	.8179	1.7374	-.1916	-.1258	.7059
1957	.8160	1.7561	-.2249	-.1522	.7074
1958	.8100	1.7442	-.1611	-.1214	.7055
1959	.8067	1.7593	-.1732	-.1155	.7049
1960	.7994	1.7948	-.1499	-.0671	.6983
1961	.7965	1.8007	-.1182	-.1336	.7024
1962	.7919	1.8338	-.1661	-.1089	.7000
1963	.7845	1.8528	-.1297	-.0802	.6954
1964	.7801	1.8995	-.1844	-.0837	.6940
1965	.7732	1.9196	-.1629	-.0712	.6908
1966	.7727	1.9459	-.2528	-.0917	.6938
1967	.7640	1.9637	-.2173	-.0597	.6884
1968	.7585	1.9945	-.2713	-.0254	.6842
1969	.7562	2.0146	-.3816	.0076	.6824
1970	.7539	2.0334	-.4499	-.0134	.6852
1971	.7402	2.0484	-.3504	.0140	.6779
1972	.7364	2.0659	-.3874	-.0119	.6807
1973	.7357	2.0105	-.3343	-.0521	.6886
1974	.7257	2.0980	-.2459	-.3304	.7024
1975	.7011	2.1959	-.1835	-.3732	.6951
1976	.6952	2.2353	-.3155	-.2281	.6860
1977	.6837	2.3029	-.3845	-.2063	.6807
1978	.6737	2.3348	-.4136	-.1949	.6778
1979	.6596	2.3523	-.3990	-.1789	.6734
1980	.6459	2.4953	-.5020	-.3232	.6798
1981	.6286	2.5435	-.4357	-.3805	.6781

(continue of Table 3.8)

Year	Capital Fertilizer E24	Capital MISC E25	FSL Fertilizer E34	FSL MISC E35	Fertilizer MISC E45
mean	1.3731	1.7230	3.3502	-2.0353	-10.5483
1910	1.5611	1.6026	5.4031	-2.3922	-13.2532
1911	1.5820	1.6040	5.6682	-2.4757	-14.3649
1912	1.4759	1.5870	4.5975	-2.1831	-10.1558
1913	1.4917	1.6116	4.9043	-2.4839	-12.1369
1914	1.4450	1.5846	4.4152	-2.2187	-9.6394
1915	1.3959	1.5986	3.9525	-2.2025	-8.5131
1916	1.4466	1.6017	4.6095	-2.4885	-11.0422
1917	1.4899	1.6332	4.5377	-2.2800	-12.0580
1918	1.4577	1.6612	4.2827	-2.4021	-11.9894
1919	1.4944	1.6503	4.5587	-2.3580	-12.9403
1920	1.4460	1.6254	4.1001	-2.1189	-10.3457
1921	1.3969	1.5095	4.7854	-2.4857	-8.8265
1922	1.3940	1.5080	4.4359	-2.1780	-7.9475
1923	1.3994	1.5285	4.2919	-2.1249	-8.2968
1924	1.4387	1.5462	4.5285	-2.1518	-9.7523
1925	1.3819	1.5608	3.9894	-2.1453	-8.4723
1926	1.3739	1.5807	3.8330	-2.1563	-8.5512
1927	1.4141	1.5705	4.1084	-2.0720	-9.3912
1928	1.3648	1.6214	3.5496	-2.1156	-8.6743
1929	1.3777	1.6281	3.6743	-2.1907	-9.4828
1930	1.3711	1.6024	3.7791	-2.2364	-9.3059
1931	1.3421	1.5272	3.7001	-1.9852	-7.0839
1932	1.3405	1.4657	4.1418	-2.0826	-6.6755
1933	1.4002	1.4902	4.6645	-2.2201	-9.1199
1934	1.5294	1.5519	6.0085	-2.7458	-16.2616
1935	1.3920	1.6188	3.9565	-2.3479	-11.0899
1936	1.5005	1.6281	5.0180	-2.6177	-16.2602
1937	1.3888	1.6742	3.5946	-2.2280	-11.2079
1938	1.4003	1.6503	3.7673	-2.2250	-11.5291
1939	1.3846	1.6671	3.6054	-2.2422	-11.2802
1940	1.3666	1.6944	3.3770	-2.2298	-10.8746
1941	1.3689	1.7240	3.2758	-2.2045	-11.1196
1942	1.3626	1.8161	2.9329	-2.1205	-11.0450
1943	1.3733	1.9698	2.8785	-2.5006	-13.7279
1944	1.3964	1.9813	2.9319	-2.4308	-14.4986
1945	1.4098	2.1888	2.9221	-3.0001	-18.7331
1946	1.4327	2.2221	2.9858	-3.0232	-20.2601
1947	1.4798	2.3042	3.2195	-3.3285	-25.1560

1948	1.4238	2.3543	2.8264	-3.1870	-21.2187
1949	1.4093	2.1081	2.9276	-2.7440	-17.6925
1950	1.4212	2.1367	2.9683	-2.8107	-18.8555
1951	1.4231	2.0726	2.9656	-2.5743	-17.5497
1952	1.3917	2.1263	2.7658	-2.6421	-16.6117
1953	1.3752	2.0014	2.7755	-2.3999	-14.4282
1954	1.3676	2.0139	2.7308	-2.4244	-14.3410
1955	1.3502	1.9479	2.6661	-2.2356	-12.5854
1956	1.3547	1.8726	2.7429	-2.0760	-11.9256
1957	1.3523	1.8629	2.7631	-2.0981	-11.9972
1958	1.3414	1.8587	2.6294	-1.9398	-10.8768
1959	1.3389	1.8391	2.6216	-1.8801	-10.5250
1960	1.3244	1.7839	2.5658	-1.7145	-9.1694
1961	1.3089	1.8154	2.4806	-1.8040	-9.1406
1962	1.3118	1.7720	2.5220	-1.7038	-8.7920
1963	1.2999	1.7467	2.4456	-1.5822	-7.8578
1964	1.2994	1.7134	2.4924	-1.5509	-7.6980
1965	1.2906	1.6970	2.4353	-1.4698	-7.0874
1966	1.3015	1.6840	2.5282	-1.4875	-7.5324
1967	1.2925	1.6630	2.4554	-1.3666	-6.7417
1968	1.3000	1.6300	2.5047	-1.2671	-6.5046
1969	1.3215	1.6013	2.6298	-1.1871	-6.7120
1970	1.3282	1.5974	2.6793	-1.1926	-6.9660
1971	1.3084	1.5864	2.5162	-1.0682	-5.8987
1972	1.3089	1.5866	2.5212	-1.0726	-5.9858
1973	1.3056	1.6273	2.4232	-1.0962	-6.1678
1974	1.2510	1.6979	2.2239	-1.4413	-6.1737
1975	1.2243	1.6775	2.0929	-1.3688	-5.0706
1976	1.2486	1.6042	2.2270	-1.1396	-4.9008
1977	1.2522	1.5721	2.2654	-1.0597	-4.6678
1978	1.2536	1.5583	2.2640	-.9959	-4.4738
1979	1.2511	1.5510	2.2152	-.9129	-4.1246
1980	1.2390	1.5484	2.2299	-1.0240	-4.2649
1981	1.2225	1.5572	2.1264	-1.0233	-3.8734

3.5 Economies of Scale

The measurement of scale economies (SE) is derived by evaluating the definition (2.12), one minus the cost elasticity with respect to output.

(2.12) $SE = 1 - (d \ln C)/(d \ln y)$

A positive SE indicates economies of scale and a negative SE implies diseconomies of scale.

If one fitted production functions or cost functions to data for individual farms, one would probably expect to find evidence of economies of scale (SE>0), since over a long period average farm size has increased, presumably because larger farms are more efficient. On the other hand, for a cost function fitted to aggregate data, the case may be not so clear. One might argue that the aggregated cost function in some way is an average of, or representative of, or typifies, the individual farm cost functions, and might therefore be expected to exhibit similar characteristics, in particular positive scale economies. Alternatively, one could view the aggregate cost function as being derived form, or related to, an aggregate production function, and argue that, for U.S. agriculture as a whole, there is no reason to suppose anything other than constant returns to scale, that is, zero scale economies.

Whichever viewpoint is taken, it was felt that it would be worth investigating what the data, as analyzed through the interpretation of our estimated cost function, seem to imply. Using model 9 (the one which assumed to fit the data best, which allowed for factor-augmenting technological change, and on which our estimated elasticities of demand and substitution are based), the estimated economies of scale parameter turns out to be quite highly negative even when a scale trend term is incorporated (Model 10). (See Table 3.9.) Since this result seems quite unreasonable, I decided to explore the possible effects, on economies of scale measure, of imposing restrictions on the cost function derived from assumptions of homotheticity, homogeneity, and adding a "scale trend" parameter on the production function.

The scale trend parameter, q, is defined as the derivative of the scale measure with respect to time.

$$q = \frac{d}{dT}\left(\frac{d \ln C}{d \ln y}\right) = \frac{d}{dT}\left(\frac{MC}{AC}\right)$$

If the estimate of q is zero, this implies that the ratio of marginal cost and average cost is a constant over time. A positive q indicates that the ratio between these two cost is rising. Table 3.9 displays these results.

Table 3.9 Alternative Estimates of Scale Economies, with and without Scale Trend.

Model	Specification	Parameter restrictions	Log of likelihood function	SE*
Group C: Biased technological change and non-homothetic production				
		{Z} { $\Sigma ai=1$, $\Sigma di=\Sigma\Sigma bij=0$, $bij=bji$ }		
9	w/o scale trend	q=0	1006.04	-1.9599
10	w/ scale trend		1007.44	-1.5804
Group D: Homothetic production				
		{Z} and di=0, ∀i.		
11	w/o scale trend	q=0	992.46	-.5134
12	w/ scale trend		992.64	.5483
Group E: Homogeneous production				
		{Z} and di=0, ∀i, ayy=0.		
13	w/o scale trend	q=0	989.61	-.1555
14	w/ scale trend		992.58	1.4215

* These SE measurements are based on the mean value of the series of observed points except in the homogeneity situation where it is a constant.

First of all, one may ask whether it is necessary to impose further restrictions on model 9? Do the impositions of homotheticity, homogeneity, and scale trend make any difference? This is subject to the purpose of the analysis. The justification for these restrictions on the estimated parameters is to address and to check the validity of farm production characteristics. Homothetic production models (model 11 and 12) have four additional restrictions (di=0, i=1,2,3,4.) than model 9 and 10, which imply four degrees of freedom. We reject the imposition of the restrictions on group C (homohteticity) models based on the likelihood-ratio test. These two computed chi-square statistics exceed the critical value at the 1% level of significance (i.e., 13.28). For the same reason, the homogeneous production models (i.e., group E) are also rejected on the basis of the likelihood ratio test with the degrees of freedom is five. The critical value of the chi-square statistics at the 1% level of significance and with five degrees of freedom is 15.09.

As to the inclusion of the scale parameter, q (which stands for the right hand side cross-product term of time and output), it is easy to see there are almost no differences between models 9 and 10, and between models 11 and 12. The test-statistic is 5.936 between models 13 and 14. We do not reject the hypothesis that the scale trend parameter is zero at the 1% level since 5.936<6.635. However, we reject the hypothesis if we lower the significance level to the 5% level because the test statistic 5.936 is greater than the critical value of 3.841. The estimates of q in models 10, 12, and 14 are 0.0059, 0.0235, and 0.0405, respectively.

Although the imposition of these restrictions had been rejected on statistical grounds, the results of models 11, 12, 13, and 14 (in Table 3.9) still indicate negative scale economies when the scale trend term is not included, but positive scale economies when it is included. The derived estimates of scale economies between models 12 and 14 (with scale trend) and models 11 and 13 (without scale trend) are opposite.

One cannot conclude much of anything from this mixed bag of results. They are inconcluded here because they might be of some interest or use to future investigators. One possible hypothesis is that there might be more confounding between our estimates of technological change and scale effect. Our estimate of technological progress on average is more or less higher than those investigtor's have obtianed. If this estimate is in fact biased upward, it may be accompanied by some downward bias in the scale effect estimate.

3.6 Summary

The empirical study of U.S. farm input demand was designed and reported in this chapter with five sections. In the first section, five input subgroups were categorized as follows: (1) hired labor, (2) capital, (3) feed, seed, and livestock purchased, (4) fertilizer, and (5) miscenllaneous items. Annual data (1910-81) of input prices, farm production expenses, and output levels are collected from USDA publications.

Econometric estimation procedure of a translog cost function was discussed in section 3.2. Zellner's seemingly unrelated regression technique, an interative version, was used to estimate the demand system and the cost function. In section 3.3, a series of hypothesis testing of the validity of duality theory implied regularity conditions -- symmetry, linear homogeneity, monotonicity, and concavity -- were carried out. The specified biased technological change model successfully passed the linear homogeneity and monotonicity conditions.

However, the symmetry condition has failed and the local concavity condition was indefinite. To complete the rest of the analyses, the symmetry restriction was imposed as the theory suggested.

The investigation of technological change showed that agricultural industry was characterized by labor-saving, capital-using technology. The annual rate of technological advancement was 2.9 percent.

In section 3.4, the own-price demand elasticity and Allen's partial elasticity of substitution were derived based on the selected factor-augementing technological change model. Each own-price elasticity of demand has the anticipated negative sign except for the miscellaneous items. Farm capital appeares to be a substitute for all other farm inputs.

Finally, the investigation of economies of scale for the U.S. farming industry as implied by our estimated cost functions led to inconclusive.

CHAPTER 4

Summary and Conclusion

This chapter contains a summary of the study, conclusions drawn from the analysis, discussion of the results, and suggestions for further research.

4.1 Summary and Implication of Results

In this study I attempted to analyze the structure of U.S. farm input demand and the technology of farm production. By employing the flexible translog cost functional form and the duality theory, a set of results on elasticity of farm input demand, elasticity of substitution, technological change, and economies of scale was derived.

In view of model specifications, a translog cost function which exhibits factor-augmenting technological change and linear homogeneity with respect to factor prices seems appropriate. The derived Allen's partial elasticity of substitution between input subgroups and the price elasticity of input demand, are the main areas of investigation in this study. Capital was found to be a substitute for all other inputs. Labor is a substitute for capital, feed, seed, livestock purchased, and miscellaneous inputs, but there is a weak complemetary relationship between labor and fertilizer. The results show a decline in the substitutablity between labor and capital.

Another important conclusion is that biased technological change in U.S. agriculture is mainly labor-saving and capital-using technology. This confirms previous empirical studies in the same direction. A 2.9% annual growth rate of agricultural productivity sustained over the last seven decades is quite impressive.

The estimated translog cost function passed the test of linear homogenity and monotonicity regularity conditions implied by the duality theory. However, it fails the test of symmetry condition and the concavity condition is indefinite. This once again implies the difficulty of working with aggregate data. This is an unfortunate limitation imposed by considerations of computational manageability within the scope of this econometric study.

4.2 Future research needs

Another important limitation in this study is the exclusion of farm family labor as an input. This is primarily due to the unavailability of data on the farm family labor component of production expense and the wage rate of family labor on the farm. If we assume that farm family labor were paid as the hired labors, we still have to approximate the farm employment, either in hours or as a portion of the entire population. It is possible to include the farm family labor at least in an approximate way.

Selecting and introducing new flexible functional forms requires the application of many mathematical formulations, statistical tests, and large computer accounts. Berndt and Khaled (1979), Chalfant (1983) and other researchers have put out tremendous amounts of effort to search for a better function. As mentioned in chapter 2.2, pursuing a "perfect" functional form is a never-ending job. However, if a super flexible functional form (e.g., Fourier functional form proposed by Gallant (1981, 1982)) is tested repeatedly and shows its strength in most cases, (such as the translog dominating the past decade) then the form is justified for further investigation.

Is output really exogenous? Sometimes it may not be. Endogenous output leads to profit maximization and other explicit behavioral assumptions. The profit function approach, which assumes both exogenous product and factor prices, is an alternative way to investigate U.S. farm input demand structures. However, one must be very careful to explain those so-called "external shocks" to a producer. A better way to deal with the exogeneity is to incorporate risk and uncertainty into the profit function specification. Pope (1982a) demonstrates that risk aversion biases the certainty (i.e., risk-neutrality) results regarding factor demands and output supplies derived from the profit function. The derivatives of expected profit with respect to input prices are not necessarily negative. Ignoring risk might result in analytical biases, but estimation biases from econometric analyses still remain unsolved.

Finally, neoclassical duality theory was developed to describe a firm's behavior. Ideally, this study should use farm level data and investigate the producer's behavior. Use of aggregate data introduces a measure of aggregation bias. Therefore, my estimates should be regarded only as broad indicators and interpreted with care.

APPENDICES

APPENDIX A

Translog Function as a Second-Order Approximation of any Function

The flexibility property of the translog functional form as a second-order Taylor's approximation is developed as follow:

(A.1) Cost function: $C = g(w1, w2, \ldots, wn, y)$

(A.2) Any function: $y = f(x1, x2, \ldots, xn)$

By taking the log of the arbitrary functional form (A.2), we get

$$\ln y = \ln \{ f [e^{(\ln x1)} \ldots e^{(\ln xn)}] \}$$

(A.3) $$\ln y = f (\ln x1, \ldots, \ln xn)$$

Applying Taylor's expansion method, we expand (A.3) around point x=1, or equivalently, (ln x)=0, and obtain

(A.4) $$\ln y = f(\ln 1, \ldots, \ln 1) + \sum_{i=1}^{n} \left. \frac{df}{d(\ln xi)} \right|_{\ln x=0} \ln xi$$

$$+ \frac{1}{2} \sum_{i} \sum_{j} \left. \frac{d^2 f}{d(\ln xi)\, d(\ln xj)} \right|_{\ln x=0} (\ln xi)(\ln xj) + R$$

where R represents the higher order terms. Since

$$\left. \frac{df}{d(\ln xi)} \right|_{\ln x=0} \quad \text{and} \quad \left. \frac{d^2 f}{d(\ln xi)\, d(\ln xj)} \right|_{\ln x=0}$$

are constant for i,j=1,...n, therefore, we assume the following identities:

$$\text{(A.5)} \quad \left. \frac{df}{d(\ln xi)} \right|_{\ln x=0} = ai$$

$$\left. \frac{d^2 f}{d(\ln xi)\, d(\ln xj)} \right|_{\ln x=0} = bij$$

$$\left. \frac{d^2 f}{d(\ln xj)\, d(\ln xi)} \right|_{\ln x=0} = bji$$

$\longrightarrow$ where $bij = bji$

and $f(\ln 1, \ldots, \ln 1) = a0$.

Then, by substituting (A.5) into (A.4) and omitting the higher order term, R, we get equation (2.6) which is the translog cost function without time variable.

The translog function can be expressed in natural exponential form as:

$$y = a0 \left(\prod_{i=1}^{n} xi^{ai} \right) \left(\prod_{i=1}^{n} xi^{1/2\,(\sum bij \ln xi)} \right)$$

Taking the natural log of both side, we have

$$\ln y = \ln a0 + \sum ai \ln xi + \frac{1}{2} \sum_i \sum_j bij \ln xi \ln xj$$

APPENDIX B

The Second Order Partial Derivatives of the Translog Cost Function

The first partial derivative of the translog cost fucntion (2.6)

$$(2.6)\quad \ln C(w,y) = a0 + \Sigma\, ai\, \ln wi + 1/2\, \Sigma\, \Sigma\, bij\, \ln wi\, \ln wj + ay\, \ln y$$
$$+ \Sigma\, di\, \ln wi\, \ln y + 1/2\, ayy\, (\ln y)^{**}2$$
$$+ \Sigma\, ti\, T\, \ln wi + t\, T + 1/2\, tt\, (T)^{**}2$$

with respect to factor price, d [ln C(w,y)]/d [ln wi], is the factor share equation (2.8).

$$(2.8)\quad si = ai + \Sigma\, bij\, \ln wj + di\, \ln y + ti\, T \qquad i=1,2,\ldots,n$$

By Shephard's Lemma, the first partial derivative of the cost itself with respect to the factor price is the input demand equation in quantity-dependent form, i.e., $\dot{x}i = dC/dwi$. Therefore,

$$\frac{d\, \ln C}{d\, \ln wi} = \frac{(1/C)\, d\, C}{(1/wi)\, d\, wi} = \frac{wi}{C}\, .\, xi = si$$

or equivalently,

$$xi = si\, .\, \frac{C}{wi}$$

The second order derivative of the translog cost function with respect to factor prices can be stated as the partial derivative of the input demand, xi. There are two situations: (1) d xi/d wi and (2) d xi/d wj, where i is not equal to j. First let us derive d xi/d wi.

$$\frac{d^2 C(w,y)}{d\, wi^2} = \frac{d\, xi}{d\, wi} = \frac{d\,[si(wi)\,.\,C/wi]}{d\, wi}$$

$$= \frac{d\, si(wi)}{d\, wi} \,.\, \frac{C}{wi} + \frac{d\,(C/wi)}{d\, wi} \,.\, si \quad \text{[by the product rule.]}$$

Since

$$\frac{d\, si}{d\, \ln wi} = bii, \quad \text{hence} \quad \frac{d\, si}{(1/wi)\, d\, wi} = bii, \text{ and} \quad \frac{d\, si}{d\, wi} = \frac{bii}{wi}\,.$$

Therefore, the complete derivation is

$$\frac{d^2 C(w,y)}{d\, wi^2} = \frac{bii}{wi} \quad \frac{C}{wi} + \left(\frac{wi.xi - C}{wi.wi}\right).si \quad \text{[by the quotient rule]}$$

$$= \frac{bii\,.\,C}{(wi)**2} + \left(\frac{si.C - C}{(wi)**2}\right)$$

$$= \frac{C\,(\,bii + si**2 - si\,)}{(wi)**2}$$

The second case of the partial derivative can be derived as follows:

$$\frac{d^2 C(w,y)}{d\,wj\; d\,wi} = \frac{d\,xi}{d\,wj} = \frac{d\,[\,si(wi)\;.\;C/wi\,]}{d\,wj}$$

$$= \frac{d\,(si(wi))}{d\,wj}\;.\;\frac{C}{wi} + \frac{d\,[\,C(wj)/wi\,]}{d\,wj}\;.\;si$$

$$= \frac{bij}{wj}\;.\;\frac{C}{wi} + \left(\frac{wi.xj}{wi.wi} - 0\right)\;.\;si$$

$$= \frac{bij}{wj}\;.\;\frac{C}{wi} + \left(\frac{xj}{wi}\right)\;.\;si$$

$$= \frac{bij}{wj}\;.\;\frac{C}{wi} + \frac{(xj.wj)\;.\;si}{wi\;.\;wj}$$

$$= \frac{bij\;.\;C + (\,sj\;.\;C\,)\;.\;si}{wi\;.\;wj} \qquad [\text{since } xj.wj = sj.C]$$

$$= \frac{C\,(\,bij + si\;.\;sj\,)}{wi\;.\;wj}$$

These second order partial derivatives are employed in calculating the Allen's partial elasticity of substitution and the own-price elasticity of demand as well as checking the concavity condition.

APPENDIX C

Divisia Index

The Divisia index is named after Francois Divisia (1889-1964). Originally the index was related to a general equation of exchange. Using this equation, Divisia, by differentiation, distinquishes two indexes whose product is always proportional to the total payment of the period to which the equation applies. Instead of comparing two discrete price situations, the constructed Divisia index can analyze the continuous effects of price changes.

The Divisia index is defined in terms of a weighted sum of growth rates. By denoting the proportional rate of change of price level by "d ln P", which is equivalent to $\dot{P}/P=(dP/dt)/P$ (the dot over variable P denotes derivative with respect to time) then, for any fixed production level the Divisia price index, in its continuous form, is defined as:

$$\text{(B.1)} \quad \text{Pindex} = \ln P = \int_t \sum_{i=1}^{n} si \; (d \ln Pi) = \int_t \sum_{i=1}^{n} si \left(\frac{\dot{P}i}{Pi}\right)$$

where $si=Pi*xi/\sum Pi*xi$ is the relative share of the value of ith input in total expenditure.

However, in practice, neither the quantities nor the prices are continuously observable. Most economic data take the form of observations at discrete points in time. Tornquist (1936) and Theil (1965, 1967) constructed an index,

$$\text{(B.2)} \quad \ln \left(\frac{w(t)}{w(t-1)} \right) = \frac{1}{2} \sum [si(t)+si(t-1)] \ln \left(\frac{wi(t)}{wi(t-1)} \right)$$

or

$$\ln w(t) - \ln w(t-1) = 1/2 \sum [si(t)+si(t-1)] [\ln wi(t) - \ln wi(t-1)]$$

as a discrete approximation to the Divisia index in logarithms. It approaches the continuous form as the change of t approaches zero. This composite Divisia price index uses the arithematic averages of the relative shares in two periods as the weights. Obviously, the discrete and continuous index numbers are equal if, and only if, relative shares are constant.

The prevailing usefulness of the Divisia index is due to the fact that Diewert (1976) shows that this index, in view of the second-order approximation property, is consistent with a homogeneous translog aggregator function. Diewert introduces the term "aggregator function" as a neutral term for the underlying production or utility function. Since the index is a line integral, the index is dependent, in general, upon the path on which the integral is taken. Hulten (1973) shows that if the aggregate exists, is homogeneous of degree one in its components, and there exists a corresponding price normal at each point unique up to a scalar multiple, then the Divisia index is path independent and can retrieve the actual values of the aggregated function, subject to an arbitrary normalization at some base period. Given these desirable properties, the Divisia index is the best choice among other index numbers.

Constructed Divisia Price Index

Table C.1 Constructed Divisia Price Index for Feed, Seed, and Livestock Purchased, 1910-1981. (1977=1.0)

Year	Price Index of: Feed	Seed	Livestock Purchased	Divisia Price Index
1910	.25	.15	.15	.205779
1911	.25	.17	.14	.203927
1912	.26	.19	.16	.220042
1913	.24	.15	.18	.213462
1914	.26	.14	.19	.226289
1915	.25	.14	.19	.221030
1916	.27	.20	.20	.242861
1917	.44	.26	.26	.358935
1918	.47	.28	.30	.393595
1919	.52	.32	.30	.423112
1920	.52	.38	.26	.412729
1921	.26	.20	.14	.210901
1922	.28	.19	.18	.234315
1923	.34	.20	.18	.266067
1924	.35	.20	.19	.274844
1925	.36	.23	.20	.286896
1926	.31	.27	.21	.268689
1927	.32	.24	.24	.282096
1928	.35	.21	.30	.316724
1929	.34	.21	.29	.307839
1930	.31	.21	.22	.266762
1931	.22	.17	.15	.189906
1932	.16	.11	.12	.140222
1933	.18	.12	.11	.147541
1934	.26	.17	.11	.193044
1935	.27	.20	.19	.234943
1936	.27	.18	.18	.228401
1937	.31	.25	.21	.270073
1938	.23	.18	.20	.214609
1939	.23	.15	.23	.218952
1940	.25	.16	.23	.231111
1941	.27	.16	.26	.251084
1942	.33	.21	.31	.306764
1943	.39	.26	.35	.359677
1944	.43	.30	.33	.383896

(continue of Table C.1)

1945	.43	.31	.36	.393135
1946	.50	.32	.41	.449497
1947	.59	.36	.51	.535063
1948	.63	.42	.63	.597516
1949	.52	.38	.56	.508149
1950	.53	.37	.66	.540346
1951	.59	.33	.80	.608040
1952	.63	.42	.67	.610752
1953	.57	.39	.48	.521643
1954	.57	.36	.49	.520438
1955	.53	.38	.49	.498849
1956	.52	.33	.45	.475749
1957	.51	.35	.51	.489209
1958	.50	.34	.62	.513821
1959	.50	.33	.62	.512845
1960	.49	.34	.59	.499287
1961	.49	.34	.68	.523618
1962	.50	.35	.61	.511369
1963	.52	.37	.57	.513713
1964	.52	.37	.51	.496899
1965	.52	.38	.57	.514658
1966	.54	.38	.65	.549265
1967	.54	.38	.63	.543747
1968	.50	.40	.66	.529304
1969	.51	.41	.74	.557867
1970	.54	.43	.77	.586840
1971	.56	.47	.79	.608661
1972	.57	.52	.94	.661975
1973	.86	.64	1.21	.926710
1974	1.04	.82	.93	.985021
1975	1.00	.94	.85	.951872
1976	1.03	.92	.97	1.00086
1977	1.00	1.00	1.00	1.00000
1978	.98	1.05	1.40	1.11134
1979	1.10	1.10	1.85	1.31505
1980	1.23	1.18	1.77	1.38631
1981	1.34	1.38	1.64	1.44963

Appendix D

The Computer Program for Calculating the Eigenvalues

This program is written in FORTRAN.

```
*JOBCARD*
FTN5.
HAL.
LGO.
*EOS

      PROGRAM HSU

C
C     This program calculate the eigenvalues of Hessian matrix.
C     If eigenvalues are negative, then the Hessian is negative
C     semidefinite.
C
C     "IORDER" is a parameter which stands for the order of the input
C     Hessian matrix. "NVAR" is another parameter which shows the
C     total number of the upper-triangular elements of a symmetric
C     matrix.
C

      PARAMETER (IORDER=5)
      PARAMETER (NVAR=(IORDER+1)*IORDER/2)

C
C     Variable descriptions:
C           A: Input real matrix of order N, an array.
C           N: Input order of the matrix A.
C        JOBN: Input optional parameter. If jobn=0, the subroutine
C              compute eigenvalues only.
C           D: Output vector of A. The length is N.
C           Z: Output NxN matrix. If jobn=0, Z is not used.
C          WK: Work area. If jobn=0, WK is at least N.
C         IER: Error parameter (for output).
C        NOBS: Number of observations.
C
```

(continue of program HSU)

```
      OPEN (UNIT=1,FILE='TAPE1')
      OPEN (UNIT=6,FILE='OUTPUT')
      IZ=IORDER
      NOBS=0

C----Begin the loop-----------------------------------------------------
100   CONTINUE
      READ (1,900,END=999) (A(I),I=1,NVAR)
      NOBS=NOBS+1
         WRITE (6,910) NOBS
         DO 110 I=1,NVAR
            WRITE (6,920) A(I)
110      CONTINUE
C
C        The following subroutine is called from IMSL, Vol. 2, 9th ed.,
C        June, 1982.
C
C        The subroutine "EIGRS" compute the eigenvalues of a symmetric
C        matrix.
C
         CALL EIGRS(A,IORDER,0,D,Z,WK,IER)
            IF(IER .NE. 0) THEN
               PRINT 20, IER
             ELSE
               WRITE (6,40)
                  DO 120 I=1,IORDER
                     WRITE (6,30) D(I)
120               CONTINUE
            ENDIF
      GO TO 100
C----End of loop--------------------------------------------------------
10    FORMAT (I3,2X,15(F7.3,1X))
20    FORMAT ( 'ERROR NUMBER IS  ',I3,' (SEE THE IMSL DOCUMENTATION FOR A
     &DESCRIPTION OF THE ERROE CODES. ')
30    FORMAT (F12.2)
40    FORMAT ( 'THE EIGEN VALUES ARE: ')
900   FORMAT (15(F8.3,1X))
910   FORMAT ( 'OBSERVATION: ',I3,'  INPUT DATA ARE: ')
920   FORMAT (1X,F9.3)
930   FORMAT ( 'NUMBER OF OBSERVATIONS = ', I3)
999   PRINT 930,NOBS
      STOP
      END
```

(Table D.1) The Computed Characteristic Roots of the Hessian Matrix Based on Model 9.

year	root 1	root 2	root 3	root 4	root 5
1910	-4,502.78	-47.57	-.01	61.02	4,551.16
1911	-4,784.35	-49.02	-.01	62.39	4,830.78
1912	-212.31	-.05	9.16	10.85	264.35
1913	-435.74	-.02	15.12	17.38	482.72
1914	-309.59	-.03	12.89	31.33	358.61
1915	-329.29	.03	13.28	36.10	379.30
1916	-273.59	.01	12.00	29.47	305.72
1917	-583.74	-17.24	-.01	25.01	608.16
1918	-289.54	-12.17	-.03	15.32	304.92
1919	-215.28	-10.46	-.05	11.39	224.45
1920	-99.59	-7.13	-.07	7.94	106.40
1921	-2.80	-.48	2.46	12.82	28.36
1922	-2.03	-1.06	1.66	13.67	23.80
1923	-6.42	.59	2.20	14.04	21.79
1924	-30.77	.02	4.10	9.05	39.60
1925	-22.20	.11	3.66	13.91	35.10
1926	-31.78	-.02	4.17	6.89	40.77
1927	-16.00	-.18	3.03	3.29	24.76
1928	-104.09	-7.34	-.02	11.55	113.68
1929	-128.89	-8.14	-.02	11.78	138.55
1930	-23.12	-.02	3.65	4.73	34.09
1931	-60.03	.07	5.81	25.44	78.66
1932	-133.56	-8.58	.00	37.85	164.81
1933	-5.92	1.57	5.70	35.06	59.05
1934	-246.56	.03	11.47	49.70	282.92
1935	-530.12	-16.49	-.01	27.01	560.20
1936	-1,237.53	-24.99	-.01	32.56	1,262.30
1937	-434.72	-14.93	-.04	16.85	459.62
1938	-944.76	-21.85	-.06	18.39	968.43
1939	-802.77	-20.17	-.07	16.54	827.04
1940	-577.09	-17.14	-.11	13.36	601.77
1941	-472.55	-15.50	-.29	7.82	491.32
1942	-62.80	-14.89	-5.53	4.71	77.40
1943	-160.61	-14.79	-8.90	1.00	165.39
1944	-457.94	-15.08	-14.17	.42	457.05
1945	-2,182.12	-32.95	-18.22	.18	2,163.32
1946	-3,577.74	-42.20	-10.05	.22	3,547.60
1947	-7,991.68	-63.06	-.07	15.05	7,919.60
1948	-805.55	-28.79	-19.98	.23	797.25

1949	-647.23	-17.92	-13.61	.31	641.96
1950	-1,100.42	-23.38	-9.02	.30	1,089.46
1951	-561.29	-16.67	-5.85	.44	553.86
1952	-20.31	-17.94	.73	4.54	20.66
1953	-16.51	-12.41	-1.34	3.35	16.79
1954	-15.25	-13.76	1.38	4.47	15.58
1955	-37.58	-11.01	.68	4.31	41.19
1956	-9.95	-7.26	-.57	2.75	9.74
1957	-9.50	-5.90	-1.41	2.21	9.47
1958	-61.89	-6.69	.12	5.58	64.94
1959	-37.16	-5.56	.18	4.30	39.20
1960	-27.84	-3.59	.13	3.68	28.68
1961	-37.38	-1.98	.06	4.26	37.37
1962	-17.30	-2.57	.16	2.85	17.31
1963	-22.97	-.73	.07	3.29	22.35
1964	-5.94	-1.81	.33	1.47	4.85
1965	-5.35	-1.26	.21	1.36	4.01
1966	-2.10	-1.28	.35	.57	1.04
1967	-2.56	-1.02	.23	.71	1.07
1968	-2.04	-1.10	-.12	.45	.59
1969	-1.30	-.99	-.31	.24	.49
1970	-1.07	-.94	-.46	.15	.47
1971	-1.65	-.51	.07	.48	.52
1972	-1.38	-.40	.05	.44	.56
1973	-1.20	-.50	-.28	.01	1.21
1974	-.76	-.26	.01	.08	.18
1975	-.64	-.26	-.15	.09	.12
1976	-.47	-.18	-.02	.02	.10
1977	-.40	-.16	.00	.01	.09
1978	-.26	-.12	.00	.08	.13
1979	-.19	-.09	-.02	.00	.07
1980	-.14	-.07	.00	.04	.07
1981	-.09	-.08	-.02	.00	.01

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