LEADING INDICATORS IN STRUCTURAL ECONOMETRIC MODELS WITH APPLICATIONS IN MULTIVARIATE TIME SERIES ANALYSIS ABOUT THE COMMERCE DEPARTMENT LEADING INDICATORS AND A PROPOSED MONETARY LEADING INDICATOR

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ABSTRACT

LEADING INDICATORS IN STRUCTURAL ECONOMETRIC MODELS WITH APPLICATIONS IN MULTIVARIATE TIME SERIES ANALYSIS ABOUT THE COMMERCE DEPARTMENT LEADING INDICATORS AND A PROPOSED MONETARY LEADING INDICATOR

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The Commerce Department leading indicator approach has been criticized as being void of economic theory.

In this study a leading indicator approach is formulated which is firmly embedded in an economic theoretical framework expressed as a dynamic, structural econometric model. A time series model in which leading indicators play a special role is derived directly from this structural model. In this context forecasts of the objective variable can be made with the current information provided by the leading indicator. The variance of the forecast errors can also be obtained in the analysis.

The current state of the art of forecasting with econometric models uses the Final Form approach. The forecasting ability of this approach is compared with that of the proposed leading indicator approach.

In light of the proposed approach, the Commerce

Department's leading indicators are evaluated. Bivariate time series models are built, describing the empirical relationships between economic activity and certain economic time series which the Commerce Department deems as useful leading indicators (components of their Composite Index of Leading Indicators). This examination reveals some possible flaws with the Commerce Department approach. Most of the Commerce Department leading indicators examined display no significant lead over economic activity. Furthermore, one of the few Commerce Department leading indicators which displays a considerable lead, is seen to have a relationship with economic activity that is contrary to the way it is employed by the Commerce Department.

These flaws cast more doubt on the usefulness of the Commerce Department leading indicator approach, and possibly provide some insight as to why the approach has performed so poorly in the past.

Finally, Money is considered as an alternative leading indicator. A multivariate time series model is developed, describing the empirical, dynamic relationship between Money and economic activity. This model is expanded at length to account for various problems with the sample period reviewed. The empirical results are discussed with their implications toward some considerations in Monetary Theory.

To my family

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CHAPTER I

INTRODUCTION

With the advent of the Great Depression in the 1930's came the assigned task of the NBER of developing a leading indicator approach to forecasting, in the hope of helping to prevent another such catastrophe. Wesley Claire Mitchell and Arthur Burns collected data on various economic time series and set up criteria for choosing leading indicators from among these. Over the years the Commerce Department leading indicator approach has evolved into its present state, the current Composite Index of Leading Indicators (CLI).

This approach has been criticized as being void of economic theory (Koopmans, 1947). It is argued that theory should be used in choosing leading indicators, or the underlying structural relationships are unknown, and thus the leading indicators cannot be used for policy decisions. Promoters of the Commerce Department approach have responded to this criticism by claiming that a theoretical foundation is present, since many of the series in the CLI reflect either direct or indirect measures of demand for various components of output, or reflect factors which have an impact on demand. In any case, it is further argued that if a method without underlying theory predicts better, then for certain uses it should be preferred.

The record of the Commerce Department approach, however, has been less than satisfactory. The CLI has displayed two major faults: it has indicated many false downturns, and has displayed a highly variable lead time at true turns. These faults cast much doubt on the usefulness of this approach in the role of forecasting.

This study develops a leading indicator approach which is built upon an economic theoretical foundation, as expressed in a dynamic, structural econometric model. Time series models in which leading indicators play a special role are derived directly from this theoretical framework. In this context, current observations of the leading indicators can be used to forecast the objective variable, and the variance of the forecast errors can be obtained.

The current state of the art of forecasting uses the Final Form of an econometric model. The forecasting ability of this approach is compared with that of our leading indicator approach. Two examples are presented illustrating the approach, and this comparison of forecasting abilities.

In light of our proposed leading indicator approach, we evaluate the Commerce Department leading indicators by building bivariate time series models describing the empirical relationships between the level of economic activity and eight of the components of the CLI. Five of the "leading indicators" examined display no significant lead over economic activity. The other three components exhibit the kind of relationships with economic activity that a good leading indicator is expected to have. However, the component which shows the greatest lead (the Producer Price Index of Crude Materials) is seen to have a negative relationship with economic activity, while it is used in a positive role in the CLI. This analysis sheds light on some potential reasons for the poor record of the Commerce Department approach.

Finally, Money is considered as an alternative leading indicator. The dynamic, empirical relationship between Money and real GNP is examined in the context of a multivariate time series model. The model is expanded to account for two kinds of supply shocks occurring in the sample period: the energy price shocks of the early 1970's, and strikes in the Labor Force. At all stages of its development, the model indicates a stable relationship between Money and real GNP, suggesting that Money may be quite useful in the role of leading indicator.

CHAPTER II

LEADING INDICATORS IN STRUCTURAL ECONOMETRIC MODELS

Introduction

A leading indicator can be defined loosely as an economic time series whose movements in some sense consistently lead economic activity. More formally, a leading indicator can be defined in the following context. We have some presumed knowledge of the joint distribution of IP_{t+k} and LI_t, $f(IP_{t+k}, LI_t)$, where

IP_{t+k} = some measure of economic activity in period t+k (e.g. the index of industrial production),

LI_t = some leading indicator which we define, in period t.

In period t we know the value of LI_t. The leading indicator approach to forecasting suggests that we can use this knowledge of LI_t to tell us more about the distribution of IP_{t+k}. Thus we are interested in:

$$(2.1) \quad f(IP_{t+k} | LI_t) = \frac{f(IP_{t+k}, LI_t)}{f_2(LI_t)}$$

This is the context in which leading indicators can be useful. We presumably know more about the distribution of IP_{t+k} given LI_t, than without that information. That is, we can provide better forecasts of IP_{t+k} by using the conditional distribution, $f(IP_{t+k}|LI_t)$, than by using the unconditional distribution, $f(IP_{t+k})$. Box and Jenkins present an example showing the improvement in forecasting using the conditional distribution over that using the unconditional distribution.[1] The amount of uncertainty in forecasting a time series using their leading indicator (measured as the standard error of the forecasts) is substantially less than the uncertainty present in the appropriate model without the information provided by the leading indicator. This, as well as the record of the leading indicator approach, establishes the usefulness of leading indicators in forecasting.

In their pioneering work with leading indicators, Mitchell and Burns originally analyzed 487 different time series and finally selected 71 by the criteria listed below.[2] This collection has been updated periodically, and now contains 70 indicators: 30 leading, 15 coincident, and 7 lagging (and 28 of less importance). In general the series considered must lead at no less than 2/3 of the reference cycle turning points as defined by the NBER, to be considered a leading indicator.[3]

Mitchell and Burns listed the following criteria to select the better indicators (the NBER uses roughly the same criteria).[4] A series is a better leading indicator:

1. The longer are its average leads at past revivals.

2. The more uniform are these leads in occurrence and length.

3. The closer its specific cycles come to having a one-to-one correspondence to the reference cycles.

4. The more clearly defined are its specific cycles.

5. The less intense are its erratic movements in comparison with the amplitude of its specific cycles.

6. The fewer are the changes in the direction of its month-to-month movements.

7. The smaller and more regular are the seasonal variations that have to be eliminated before the specific cycles can be studied.

8. The larger is the number of past revivals covered by the series.

9. The farther back in time any irregularities in conformity to business cycle revivals have occurred.

10. The broader is the range of activities represented by the series.

11. The more stable is the economic significance of the process represented.

Koopmans[5] criticized the work of Burns and Mitchell as choosing indicators to predict business cycle peaks and troughs without any apparent economic theory behind their methods. He argued that economic theory is useful in choosing those indicators which will best predict, and if theory is not used, the findings and results can not be used for policy decisions or other useful tasks because the underlying structural relationships are unknown.

Vining[6] later argued against Koopman's criticism. Vining suggested that the usefulness of alternative methods should be evaluated by the results achieved by each. If a method without underlying theory (i.e. pure forecasting) predicts better, then for certain uses it should be preferred.

Since this exchange, promoters of the leading indicator approach have been concerned with its theoretical background. Today there is general agreement that there is in fact a theoretical framework underlying the leading indicator approach.[7] The series included in the Composite Index of Leading Indicators reflect either direct or indirect measures of demand for various components of output, or reflect factors which have an impact on demand. Changes in these components of demand tend to lead changes in output in the near future.

To date, this appears to be the main argument of proponents of the leading indicator approach in defending their use of leading indicators.

The Framework

We wish to explicitly formulate an economic theoretical background for the leading indicator approach to forecasting.

Consider a dynamic simultaneous equation model incorporating leading indicators, in the context of a general linear multiple time series process. As Zellner and Palm[8] indicate, a multiple time series process can be represented as:

$$(2.2)\quad \underset{p\times p}{H(L)}\ \underset{p\times 1}{z_t} = \underset{p\times p}{F(L)}\ \underset{p\times 1}{e_t} \qquad t = 1, \ldots, T$$

where z_t = a vector of random variables measured as deviations from their means:

H(L) and F(L) are matrices whose elements are polynomials in L, the lag operator ($h_{ij} = \sum_{\ell=0}^{r_{ij}} h_{ij\ell}L^{\ell}$ and $f_{ij} = \sum_{\ell=0}^{q_{ij}} f_{ij\ell}L^{\ell}$);

e_t = a vector of disturbances with $E(e_t) = 0$ and $E(e_t e_t') = I_p$.

Given prior information suggesting that some elements of z_t are endogenous and some are exogenous, the above system can be rewritten:

$$(2.3)\quad \begin{bmatrix} H_{11}(L) & H_{12}(L) \\ H_{21}(L) & H_{22}(L) \end{bmatrix} \begin{bmatrix} y_t \\ x_t \end{bmatrix} = \begin{bmatrix} F_{11}(L) & F_{12}(L) \\ F_{21}(L) & F_{22}(L) \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \end{bmatrix}$$

where y_t and e_{1t} are of dimension $p_1 x1$,

x_t and e_{2t} are of dimension $p_2 x1$, with $p_1 + p_2 = p$;

and the H_{ij} and F_{ij} submatrices are of the appropriate dimensions.

If y_t is endogenous and x_t is exogenous, these restrictions are implied:

$$H_{21}(L) = 0,\ F_{12}(L) = 0, \text{ and } F_{21}(L) = 0.$$

Hence the above system becomes:

$$(2.4)\quad \begin{aligned} &\underset{p_1 x p_1}{H_{11}(L)}\ \underset{p_1 x 1}{y_t} + \underset{p_1 x p_2}{H_{12}(L)}\ \underset{p_2 x 1}{x_t} = \underset{p_1 x p_1}{F_{11}(L)}\ \underset{p_1 x 1}{e_{1t}} \\ &\underset{p_2 x p_2}{H_{22}(L)}\ \underset{p_2 x 1}{x_t} = \underset{p_2 x p_2}{F_{22}(L)}\ \underset{p_2 x 1}{e_{2t}} \end{aligned}$$

This is in the form of a dynamic structural system of simultaneous equations, with the exogenous variables, x_t, generated by an ARMA process.

In our model, both IP_t and LI_t will be endogenous to the system. Let $LI_t = y_{1t}$, $IP_t = y_{2t}$, and $y_t^* =$ the $(p_1-2)x1$ vector of remaining endogenous variables. Then $[H_{11}(L)\ y_t]$ in (2.4) can be rewritten:

$$(2.5)\quad \begin{bmatrix} h_{11} & | & \\ h_{21} & | & H_{11}^*(L) \\ \vdots & & \\ h_{p_1 1} & | & \end{bmatrix} \begin{bmatrix} y_{1t} \\ y_{2t} \\ \hline y_t^* \end{bmatrix}$$

where $H_{11}^*(L)$ is the last p_1-1 columns of $H_{11}(L)$.

Write h_{i1} as $h_{i1} = h_{i1}^* + h_{i1}^{**}$,

where h_{i1}^* is of order k (highest exponent is L^{k-1}),

and $h_{i1}^{**} = L^k\ h_{i1}^{***}$,

$$\text{with } h_{i1}^{***} = \sum_{\ell=k}^{r_{ij}} h_{i1\ell} L^{\ell-k} ;$$

$i = 1, \ldots , p_1$.

Consider the first column of $H_{11}(L)$:

$$(2.6)\quad \begin{bmatrix} h_{11} \\ h_{21} \\ \vdots \\ h_{p_1 1} \end{bmatrix} = \begin{bmatrix} h_{11}^* + h_{11}^{**} \\ h_{21}^* + h_{21}^{**} \\ \vdots \\ h_{p_1 1}^* + h_{p_1 1}^{**} \end{bmatrix}$$

We can now rewrite our structural system in (2.4).

$$\left(\begin{array}{c|c} h_{11} & \\ h_{21} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1} & \end{array}\right) y_t + H_{12}(L)\, x_t = F_{11}(L)\, e_{1t}$$

$$(2.7) \quad \left(\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right) \left(\begin{array}{c} y_{1t} \\ y_{2t} \\ \hline y_t^{*} \end{array}\right) + \left(\begin{array}{c|c} h_{11}^{**} & \\ h_{21}^{**} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{**} & \end{array}\right) \left(\begin{array}{c} y_{1t} \\ \hline x_t \end{array}\right) = F_{11}(L)\, e_{1t}$$

$$(2.8) \quad \left(\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right) y_t + \left(\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right) \left(\begin{array}{c} L^k\, y_{1t} \\ \hline x_t \end{array}\right) = F_{11}(L) e_{1t}$$

This expresses our structural system in terms of our endogenous variables y_t, and a vector of exogenous variables x_t and a lagged endogenous (predetermined) variable $L^k\, y_{1t}$, $\left(\begin{array}{c} L^k\, y_{1t} \\ \hline x_t \end{array}\right) = x_t^{*}$.

Solving for y_t in terms of x_t^{*}, we get a system of equations which we could call the "partial" final form.[9]

$$(2.9)\quad y_t = -\left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right] x_t^{*}$$

$$+ \left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} F_{11}(L)\, e_{1t}$$

This expresses the current endogenous variables in terms of the current and lagged exogenous and one lagged endogenous (predetermined) variable. This is in the form of a transfer function with input x_t^* and output y_t.

We have now derived a set of p_1 transfer functions from our economic theoretical foundation, expressed in our dynamic structural system of simultaneous equations. Given our assumptions regarding e_t, we can fit a transfer function model about the specific time series we wish to examine.

The second equation in the set of p_1 equations in (2.9) gives us the transfer function model for IP_t implied by our structural system:

$$(2.9)\quad y_t = -\left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right] x_t^{*}$$

$$+ \left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} F_{11}(L)\, e_{1t}$$

This expresses the current endogenous variables in terms of the current and lagged exogenous and one lagged endogenous (predetermined) variable. This is in the form of a transfer function with input x_t^* and output y_t.

We have now derived a set of p_1 transfer functions from our economic theoretical foundation, expressed in our dynamic structural system of simultaneous equations. Given our assumptions regarding e_t, we can fit a transfer function model about the specific time series we wish to examine.

The second equation in the set of p_1 equations in (2.9) gives us the transfer function model for IP_t implied by our structural system:

$$(2.10)\quad y_{2t} = IP_t = \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}_{\substack{2. \\ 1x(p_2+1)}} \underset{(p_2+1)x1}{x_t^{*}}$$

$$+ \left\{ \underset{1xp_1}{\left[\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1}} F_{11}(L) \right\}_{2.} \underset{p_1 x1}{e_{1t}}$$

We have thus derived a transfer function model expressing the relationship between our leading indicator of economic activity and our measure of economic activity. This model is firmly embedded in economic theory, as it is derived from a dynamic structural system of simultaneous equations describing the world. It is also amenable to empirical testing, using Box and Jenkins time series methods.[10]

After identifying and fitting the model, we can obtain optimal (minimum mean square error) forecasts for IP_{t+k}, given LI_t, and the variance of the forecasts errors.[11] That is, we can extract the first two moments describing the conditional distribution of IP_{t+k} given LI_t, $f(IP_{t+k}|LI_t)$. This is the object of our analysis of the leading indicator approach.

Hence we have a theoretically sound and empirically testable framework in which we can study and use leading indicators.

Comparing the Forecasting Abilities of the Final Form and the Proposed Leading Indicator Approach

The Final Form

The current state of the art of forecasting uses the Final Form of the structural model expressed in equation (2.4):

$$(2.15)\quad y_t = -H_{11}^{-1}(L)\,H_{12}(L)\,x_t + H_{11}^{-1}(L)\,F_{11}(L)\,e_{1t}$$

To forecast y_{t+k}, the appropriate ARIMA models for the x_t are fitted, and x_t is projected k periods into the future. Then the forecast for y_{t+k} is produced as follows:

$$y_{t+k} = -H_{11}^{-1}(L)\,H_{12}(L)\,x_{t+k} + H_{11}^{-1}(L)\,F_{11}(L)\,e_{1t+k}$$

$$(2.16)\quad y_{t+k} = -[H_{11}^{-1}(L)\,H_{12}(L)]^{*}x_{t+k} - [H_{11}^{-1}(L)\,H_{12}(L)]^{**}\,x_{t+k}$$

$$+ H_{11}^{-1}(L)\,F_{11}(L)\,e_{1t+k}$$

where the $(i,j)^{th}$ element of $[H_{11}^{-1}(L)\,H_{12}(L)]^{*}$ is of order < k (highest term is L^{k-1}); and the $(i,j)^{th}$ element of $[H_{11}^{-1}(L)\,H_{12}(L)]^{**}$ is of order $\geq$ k, and is equivalent to the $(i,j)^{th}$ element of $[H_{11}^{-1}(L)\,H_{12}(L)]^{***}\cdot(L^k)$. That is; $[H_{11}^{-1}(L)\,H_{12}(L)]^{*}_{ij} = \sum_{\ell=0}^{k-1} h_{ij\ell}\,L^{\ell}$; and

$$[H_{11}^{-1}(L)\,H_{12}(L)]^{**}_{ij} = L^k[H_{11}^{-1}(L)\,H_{12}(L)]_{ij}^{***}$$

$$= L^k \sum_{\ell=k}^{z_{ij}} h_{ij\ell}L^{\ell-k}$$

Using this notation, we can work with (2.16):

$$y_{t+k} = - [H_{11}^{-1}(L)\ H_{12}(L)]^{*} x_{t+k}$$

$$- [H_{11}^{-1}(L)H_{12}(L)]^{***} L^{k} x_{t+k} + H_{11}^{-1}(L)F_{11}(L)e_{1t+k}$$

$$= - [H_{11}^{-1}(L)\ H_{12}(L)]^{*} x_{t+k} - [H_{11}^{-1}(L)H_{12}(L)]^{***} x_{t}$$

$$+ H_{11}^{-1}(L)\ F_{11}(L)\ e_{1t+k}$$

Forecasts of x_t for periods t+1 through t+k are used in the first term of the above expression, and the past history of x_t is used in the second term, to get the forecast for y_{t+k}:

$$(2.17)\quad \hat{y}_t(k) = - [H_{11}^{-1}(L)\ H_{12}(L)]^{*} \hat{x}_t(k) - [H_{11}^{-1}(L)H_{12}(L)]^{***}\ x_t$$

$$+ H_{11}^{-1}(L)\ F_{11}(L)\ \{\hat{e}_{1t+k}\}$$

$$= - [H_{11}^{-1}(L)\ H_{12}(L)]^{*} \hat{x}_t(k)$$

$$- [H_{11}^{-1}(L)H_{12}(L)]^{***}\ x_t + [H_{11}^{-1}(L)F_{11}(L)]^{***}\ e_{1t}$$

Here $\hat{y}_t(k)$ is the vector of p_1 forecasts in period t, of y_{t+k}. Note that $\hat{x}_t(k)$ refers to the vector of forecasts for the p_2 inputs, $\hat{x}_{jt}(k)$, j = 1, ... , p_2. It is understood that in this context, $[L^b \hat{x}_{jt}(k)] = \begin{cases} \hat{x}_{jt}(k-b) & \text{if } b < k \\ x_{j(t+k-b)} & \text{if } b \geq k \end{cases}$

Further note that in this forecast, $H_{11}^{-1}(L)\ F_{11}(L)\ e_{1t+k}$ effectively reduces to $[H_{11}^{-1}(1)\ F_{11}(L)]^{***} e_{1t}$ since $E(e_{1t+k}) = 0$ for any k = 1, 2, ... Here the three asterisks imply the same reconstruction of the matrix $[H_{11}^{-1}(L)\ F_{11}(L)]$ as is used regarding other matrices throughout the paper.

After identifying and fitting the appropriate ARIMA models for the p_2 inputs in x_t, minimum mean square error forecasts, $\hat{x}_t(i)$, are made for $i = 1, 2, \ldots, k$.

The forecast error for the j^{th} input is:

$$(2.18)\quad \hat{e}_{x_{jt}}(i) = \hat{x}_{jt}(i) - x_{j(t+i)} = \sum_{n=0}^{i-1} \Psi_{jn}\, a_{j(t+i-n)}$$

since $$x_{j(t+i)} = a_{t+i} + \Psi_{j1} a_{t+i-1} + \Psi_{j2}\, a_{t+i-2} + \cdots$$

$$\hat{x}_{jt}(i) = E_t(x_{j(t+i)} | \text{past } x_t]$$

$$= \Psi_{ji}\, a_t + \Psi_{j(i+1)}\, a_{t-1} + \Psi_{j(i+2)}\, a_{t-2} + \cdots$$

where the Ψ_{jn} are the weights of the ARIMA process for the j^{th} input, written in pure moving average form;

and where a_{jt} is white noise;

i.e. $E(a_{jt}) = 0$, $E(a_{jt})^2 = \sigma_{ja}^2$, and $E(a_{jt}a_{jt-n}) = 0$, for any input j, for any t, and for any $n \neq 0$.

Note that $$E[\hat{e}_{x_{jt}}(i)] = E\left\{\sum_{n=0}^{i-1} \Psi_{jn}\, a_{j(t+i-n)}\right\} = \sum_{n=0}^{i-1} \Psi_{jn} E(a_{j(t+i-n)}) = 0 .$$

The variance of the forecast error is:

$$(2.19)\quad \text{Var}\,[\hat{e}_{x_{jt}}(i)] = E[\hat{e}_{x_{jt}}(i)]^2 = \sum_{n=0}^{i=1} \Psi_{jn}^{\;2}\, \sigma_{ja}^{\;2}$$

Now consider the forecast error of the Final Form model, from equations (2.16) and (2.17):

$$\begin{aligned}
(2.20)\quad \hat{e}_{y_t}(k) &= \hat{y}_t(k) - y_{t+k} \\
&= -[H_{11}^{-1}(L)\,H_{12}(L)]\,\hat{x}_t(k) + [H_{11}^{-1}(L)\,F_{11}(L)]^{***}e_{1t} \\
&\quad + [H_{11}^{-1}(L)\,H_{12}(L)]\,x_{t+k} - H_{11}^{-1}(L)\,F_{11}(L)\,e_{1t+k} \\
&= -[H_{11}^{-1}(L)\,H_{12}(L)]^{*}\hat{x}_t(k) - [H_{11}^{-1}(L)\,H_{12}(L)]^{***}x_t \\
&\quad + [H_{11}^{-1}(L)\,F_{11}(L)]^{***}e_{1t} \\
&\quad + [H_{11}^{-1}(L)\,H_{12}(L)]^{*}x_{t+k} + [H_{11}^{-1}(L)\,H_{12}(L)]^{***}x_t \\
&\quad - [H_{11}^{-1}(L)\,F_{11}(L)]^{*}e_{1t+k} - [H_{11}^{-1}(L)\,F_{11}(L)]^{***}e_{1t} \\
&= -[H_{11}^{-1}(L)\,H_{12}(L)]^{*}[\hat{x}_t(k)-x_{t+k}] \\
&\quad - [H_{11}^{-1}(L)H_{12}(L)]^{***}(x_t-x_t) \\
&\quad - [H_{11}^{-1}(L)\,F_{11}(L)]^{*}e_{1t+k} \\
&\quad + [H_{11}^{-1}(L)F_{11}(L)]^{***}(e_{1t}-e_{1t}) \\
&= -[H_{11}^{-1}(L)\,H_{12}(L)]^{*}[\hat{e}_{x_t}(k)] \\
&\quad - [H_{11}^{-1}(L)F_{11}(L)]^{*}e_{1t+k}
\end{aligned}$$

Note that $E[\hat{e}_{y_t}(k)] = -[H_{11}^{-1}(L)\,H_{12}(L)]^{*}E[\hat{e}_{x_t}(k)]$

$$\begin{aligned}
&\quad - [H_{11}^{-1}(L)F_{11}(L)]^{*}E(e_{1t+k}) \\
&= 0
\end{aligned}$$

since the parameters in our matrices are fixed;
since $E[\hat{e}_{x_t}(k)] = 0$ as shown in (2.18); and
since $E(e_{1t+k}) = 0$ from our assumptions.

It should be clear that in this context,

$$[L^b\hat{e}_{x_t}(k)] = \begin{cases} \hat{e}_{x_t}(k-b) = \hat{x}_t(k-b) - x_{t+k-b} & \text{if } b < k \\ 0 = x_{t+k-b} - x_{t+k-b} & \text{if } b \geq k. \end{cases}$$

Observe that the forecast error of each endogenous variable reduces to a function of the forecast errors of all inputs, and the noise associated with all p_1 endogenous variables occurring over the forecast period.

Assuming that

(i) the input series are mutually independent,

(ii) each input is independent of each disturbance term, and

(iii) the parameters of our model are known with certainty;

consider the variance of the forecast errors.

$$(2.21)\quad \begin{aligned} \text{Var}\,[\hat{e}_{y_t}(k)] &= E[\hat{e}_{y_t}(k)]^2 \\ &= E\{-[H_{11}^{-1}(L)\,H_{12}(L)]^*[\hat{e}_{x_t}(k)] \\ &\quad - [H_{11}^{-1}(L)F_{11}(L)]^* e_{1t+k}\}^2 \\ &= E\{-[H_{11}^{-1}(L)\,H_{12}(L)]^*[\hat{e}_{x_t}(k)]\}^2 \\ &\quad + E\left([H_{11}^{-1}(L)F_{11}(L)]^* e_{1t+k}\right)^2 \\ &= E\{-[H_{11}^{-1}(L)\;H_{12}(L)]^*[\hat{e}_{x_t}(k)]\}^2 \\ &\quad + \left([H_{11}^{-1}(L)\;F_{11}(L)]\right)^* \text{Var}\,(e_{1t+k}) \end{aligned}$$

by (ii) and (iii) above and our assumptions as to e_{1t}.

Here $\left([H_{11}^{-1}(L)\;F_{11}(L)]\right)^*$ is the transformation made by squaring each element of the matrix $[H_{11}^{-1}(L)\;F_{11}(L)]^*$. See footnote (12) for a convincing argument that this is the appropriate transformation.

In this context, the variance of the forecast error of each endogenous variable is a function of the variances of the forecast errors of <u>all</u> inputs, the various covariances between forecast errors of different time horizons implied by $[H_{11}^{-1}(L)\ H_{12}(L)]^*$ (see footnote (12)), and the variances of the disturbances associated with <u>all</u> endogenous variables.

From our initial assumptions regarding e_{1t} on page 4, we have $\mathrm{Var}(e_{1t}) = \mathrm{Var}(e_{1t+k}) = 1$. Thus equation (2.21) can be rewritten:

$$(2.22)\quad \mathrm{Var}[\hat{e}_{y_t}(k)] = E\{-[H_{11}^{-1}(L)\ H_{12}(L)]^*\ [\hat{e}_{x_t}(k)]\}^2 + \left([H_{11}^{-1}(L)\ F_{11}(L)]\right)^* \begin{pmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{pmatrix}$$

The Proposed Leading Indicator Approach

Consider the forecast errors of our leading indicator approach.

We can rewrite equation (2.9) from our previous work on the leading indicator approach, in order to examine y_{t+k}.

$$(2.23)\quad y_{t+k} = -\left(\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L)\right)^{-1} \left(\begin{matrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{matrix} \middle| \; H_{12}(L)\right) x_{t+k}^{*} + \left(\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L)\right)^{-1} F_{11}(L)\ e_{1t+k}$$

$$\text{But } x^*_{t+k} = \begin{pmatrix} L^k y_{1t+k} \\ \hline x_{t+k} \end{pmatrix} = \begin{pmatrix} y_{1t} \\ \hline x_{t+k} \end{pmatrix}$$

$$\text{Thus, } y_{t+k} = -\left[\begin{pmatrix} h^*_{11} \\ \vdots \\ h^*_{p_1 1} \end{pmatrix} \Bigg| \; H^*_{11}(L)\right]^{-1} \left[\begin{pmatrix} h^{***}_{11} \\ \vdots \\ h^{***}_{p_1 1} \end{pmatrix} \Bigg| \; H_{12}(L)\right] \begin{pmatrix} y_{1t} \\ \hline x_{t+k} \end{pmatrix}$$

$$+ \left\{ \left[\begin{pmatrix} h^*_{11} \\ \vdots \\ h^*_{p_1 1} \end{pmatrix} \Bigg| \; H^*_{11}(L)\right]^{-1} F_{11}(L) \right\} e_{1t+k}$$

Hence our forecasts are:

$$(2.24)\quad \hat{y}_t(k) = -\left[\begin{pmatrix} h^*_{11} \\ \vdots \\ h^*_{p_1 1} \end{pmatrix} \Bigg| \; H^*_{11}(L)\right]^{-1} \left[\begin{pmatrix} h^{***}_{11} \\ \vdots \\ h^{***}_{p_1 1} \end{pmatrix} \Bigg| \; H_{12}(L)\right] \begin{pmatrix} y_{1t} \\ \hline \hat{x}_t(k) \end{pmatrix}$$

$$+ \left\{ \left[\begin{pmatrix} h^*_{11} \\ \vdots \\ h^*_{p_1 1} \end{pmatrix} \Bigg| \; H^*_{11}(L)\right]^{-1} F_{11}(L) \right\}^{***} e_{1t}$$

Here again, $\hat{x}_t(k)$ refers to the vector of forecasts for the p_2 exogenous inputs, $\hat{x}_{jt}(k)$, $j = 1, 2, \ldots, p_2$. Further, the simplification of the disturbance structure in this forecast is analagous to that in the Final Form forecast in equation (2.17)

After identifying and fitting the appropriate ARIMA models for the p_2 inputs in x_t, minimum mean square error forecasts, $\hat{x}_t(i)$, are made for $i = 1, 2, \ldots, k$. The related forecast errors and their variances are expressed in equations (2.18) and (2.19). From equations (2.23) and (2.24) we get the forecast error of our model:

$$(2.25)\quad \hat{e}_{y_t}(k) = \hat{y}_t(k) - y_{t+k}$$

$$= -\left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1} \left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right) \right\} \left(\begin{array}{c} y_{1t} \\ \hline \hat{x}_t(k) \end{array}\right)$$

$$+ \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1} F_{11}(L) \right\}^{***} e_{1t}$$

$$+ \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1} \left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right) \right\} \left(\begin{array}{c} y_{1t} \\ \hline x_{t+k} \end{array}\right)$$

$$- \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1} F_{11}(L) \right\} e_{1t+k}$$

To work with this, we will again use our conventional notation (*, **, ***) to break up these matrices into those containing parameters with lags < k, and those containing parameters with lags $\geq$ k. Rewriting (2.25):

$$
\begin{aligned}
\hat{e}_{y_t}(k) = & - \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}^{*} \left[\frac{y_{1t}}{\hat{x}_t(k)} \right] \\
& - \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}^{***} \left[\frac{y_{1t-k}}{x_t} \right] \\
& + \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}^{*} \left[\frac{y_{1t}}{x_{t+k}} \right] \\
& + \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1}^{*} & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}^{***} \left[\frac{y_{1t-k}}{x_t} \right] \\
& + \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} F_{11}(L) \right\}^{***} e_{1t} \\
& - \left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right]^{-1} F_{11}(L) \right\}^{*} e_{1t+k} +
\end{aligned}
$$

$$- \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} F_{11}(L) \right\}^{***} e_{1t}$$

Simplifying the above:

$$\hat{e}_{y_t}(k) = - \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} \left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right) \right\}^{*} \left(\begin{array}{c} y_{1t}-y_{1t} \\ \hline \hat{e}_{x_t}(k) \end{array} \right)$$

$$- \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} \left(\begin{array}{c|c} h_{11}^{**} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right) \right\}^{***} \left(\begin{array}{c} y_{1t-k}-y_{1t-k} \\ \hline x_t - x_t \end{array} \right)$$

$$- \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} F_{11}(L) \right\}^{*} e_{1t+k}$$

$$+ \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} F_{11}(L) \right\}^{***} (e_{1t}-e_{1t})$$

$$= - \left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} \left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right) \right\}^{*} \left(\begin{array}{c} 0 \\ \hline \hat{e}_{x_t}(k) \end{array} \right) + \tag{2.26}$$

$$- \left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} F_{11}(L) \right\}^{*} e_{1t+k}$$

Observe that in this model, the forecast error of <u>each</u> endogenous variable is a function of the forecast errors of <u>all</u> inputs, and the noise associated with <u>all</u> p_1 endogenous variables occurring over the forecast period. Compare equation (2.26) with equation (2.20).

Again, we wish to consider the variance of the forecast errors, given these assumptions:

(i) the exogenous input series are mutually independent,

(ii) each exogenous input is independent of each disturbance, and

(iii) the model parameters are known with certainty.

$$\mathrm{Var}[\hat{e}_{y_t}(k)] = E[\hat{e}_{y_t}(k)]^2$$

$$= E \left[- \left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} \left[\begin{pmatrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{pmatrix} \Bigg| H_{12}(L) \right] \right\}^{*} \left(\frac{0}{\hat{e}_{x_t}(k)} \right) \right.$$

$$\left. - \left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} F_{11}(L) \right\}^{*} e_{1t+k} \right]^2$$

$$= E\left[-\left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right] \right\}^{*} \left[\begin{array}{c} 0 \\ \hline \hat{e}_{x_t}(k) \end{array}\right] \right]^2$$

$$+ E\left[\left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} F_{11}(L) \right\}^{*} e_{1t+k} \right]^2$$

[by (ii) above]

$$= E\left[-\left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right] \right\}^{*} \left[\begin{array}{c} 0 \\ \hline \hat{e}_{x_t}(k) \end{array}\right] \right]$$

$$+ \left\{\left\{ \left[\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right]^{-1} F_{11}(L) \right\}\right\}^{*} E(e_{1t+k})^2$$

[by (iii) above and our assumptions as to e_{1t}]

Note that as before, double brackets around a matrix, {{ }}*, refer to the transformation of the original matrix with single brackets, in which <u>each</u> <u>element</u> is squared. Again refer to footnote (12).

Thus our forecast error variance finally reduces to the following:

$$= E\left[-\left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} \left[\begin{pmatrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{pmatrix} \Bigg| H_{12}(L) \right] \right\}^{*} \left[\frac{0}{\hat{e}_{x_t}(k)} \right] \right]^{2}$$

$$+ E\left[\left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} F_{11}(L) \right\}^{*} e_{1t+k} \right]^{2}$$

[by (ii) above]

$$= E\left[-\left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} \left[\begin{pmatrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{pmatrix} \Bigg| H_{12}(L) \right] \right\}^{*} \left[\frac{0}{\hat{e}_{x_t}(k)} \right] \right]$$

$$+ \left\{ \left\{ \left[\begin{pmatrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{pmatrix} \Bigg| H_{11}^{*}(L) \right]^{-1} F_{11}(L) \right\} \right\}^{*} E(e_{1t+k})^{2}$$

[by (iii) above and our assumptions as to e_{1t}]

Note that as before, double brackets around a matrix, {{ }}*, refer to the transformation of the original matrix with single brackets, in which <u>each element</u> is squared. Again refer to footnote (12).

Thus our forecast error variance finally reduces to the following:

$$(2.27)\quad V[\hat{e}_{y_t}(k)] = E\left[-\left\{\left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1}\left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right)\right\}^{*}\left(\frac{0}{\hat{e}_{x_t}(k)}\right)\right]^{2}$$

$$\qquad\qquad p_1 x(p_2+1) \qquad\qquad (p_2+1)x1$$

$$+\left\{\left\{\left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1} F_{11}(L)\right\}\right\}^{*} V(e_{1t+k})$$

$$\qquad\qquad p_1 x p_1 \qquad\qquad p_1 x1$$

In this context, the variance of the forecast error of <u>each</u> endogenous variable is a function of the variances of the forecast errors of <u>all</u> exogenous inputs, the appropriate covariances between forecast errors of different time horizons implied by

$$\left\{\left(\begin{array}{c|c} h_{11}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array}\right)^{-1}\left(\begin{array}{c|c} h_{11}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right)\right\}^{*}$$

(again see footnote (12)), and the variances of the disturbances associated with <u>all</u> endogenous variables. Compare equation (2.27) with equation (2.21).

Example 1

Consider an IS-LM model.

Commodity Market

(1) $C_t = a + b_1 Y_{t-1} + b_2 r_{t-1} + e_{1t}$

(2) $I_t = \bar{I}_t + b_3 r_{t-1} + e_{2t}$

(3) $Y_t = C_t + I_t$

Money Market

(4) $\frac{M^d_t}{P_t} = b_4 Y_{t-1} + b_5 r_{t-1} + e_{3t}$

(5) $M^s_t = M_o + b_6 r_t + e_{4t}$

(6) $M^s_t = M^d_t$

(7) $P_t = \bar{P}_t$

where C_t = consumption, I_t = investment, Y_t = output,

r_t = "the" interest rate, P_t = commodity price level

a = autonomous consumption, $\bar{I}_t$ = autonomous investment

and the e_{it} are disturbances with $E(e_{it}) = 0$,

$E(e_{it}e_{jt}) = 0$, and $E(e_{it})^2 = \sigma_i^2$, for $i = 1,2,3,4$; $i \neq j$.

The model consists of seven equations and seven unknowns: Y_t, r_t, P_t, M^d_t, M^s_t, C_t, and I_t.

Note that our dynamic formulation simply says each right hand side endogenous variable affects the left hand side variable with a lag of one period, with one exception. The exception is the Money Supply equation. This equation reflects the likelihood that banks will react quickly and efficiently in adjusting their excess reserve positions, in response to changes in the interest rate.

I wish to assume that a Keynesian aggregate supply curve corresponds to this world. That is, assume:

(i) whatever output is demanded can be produced, and

(ii) $P_t = \overline{P}_t$, as expressed in equation (7).

Note that we can express the system as two equations in two unknowns. These are the IS and LM relationships.

$$\text{IS:}\quad Y_t = C_t + I_t$$

$$= a + b_1 Y_{t-1} + b_2 r_{t-1} + e_{1t} + \overline{I}_t + b_3 r_{t-1} + e_{2t}$$

$$(7)\qquad (1-b_1 B)\, Y_t = a + \overline{I}_t + (b_2+b_3)B\, r_t + (e_{1t}+e_{2t})$$

where B = the backshift operator, or lag operator (previously specified as L).

$$\text{LM:}\quad M_t^d = M_t^s$$

$$b_4 \overline{P}\, Y_{t-1} + b_5 \overline{P}\, r_{t-1} + \overline{P}\, e_{3t} = M_0 + b_6\, r_t + e_{4t}$$

$$(8)\qquad (-b_6+b_5\overline{P}B)\, r_t = M_0 - b_4\overline{P}B\, Y_t + (e_{4t}-\overline{P}e_{3t})$$

Let $e_{1t}+e_{2t} = e_{5t}$, and $e_{4t}-\overline{P}e_{3t} = e_{6t}$, noting that $E(e_{5t}) = E(e_{6t}) = 0$, and $E(e_{5t} \cdot e_{6t}) = 0$.

We now have two equations [(7) and (8)] in two unknowns: Y_t and r_t. This is the classic IS-LM problem, in a linear structural model framework.

The candidate for a leading indicator of Y_t in this context is r_t. Observe that r_t affects Y_{t+1} through its effect on C_{t+1} and I_{t+1} in the commodity market, and affects Y_{t+2} through its effect on Y_{t+1}, which is involved in the IS equation for Y_{t+2}. Further, r_t affects Y_{t+1} through its effect on the LM relationship in period t+1. In short, r_t is a factor in the determination of the locations of both the IS and LM relationships in period t+1.

Consider equations (7) and (8) in matrix form.

$$\begin{bmatrix} -(b_2+b_3)B & (1-b_1B) \\ (-b_6+b_5\bar{P}B) & b_4\bar{P}B \end{bmatrix} \begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \begin{bmatrix} a & 1 \\ M_0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix} \tag{9}$$

This is our structural model with endogenous variables r_t and Y_t, and exogenous variable, $\bar{I}_t$. Applying our transformation, we'll separate the polynomials in B multiplying r_t, into a component with lags, $k < 1$, and a component with lags, $k \geq 1$.

$$\begin{bmatrix} 0 & (1-b_1B) \\ -b_6 & b_4\bar{P}B \end{bmatrix} \begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \begin{bmatrix} (b_2+b_3)B & a & 1 \\ -b_5\bar{P}B & M_0 & 0 \end{bmatrix} \begin{bmatrix} r_t \\ 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

or:

$$\begin{bmatrix} 0 & (1-b_1B) \\ -b_6 & b_4\bar{P}B \end{bmatrix} \begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \begin{bmatrix} (b_2+b_3) & a & 1 \\ -b_5\bar{P} & M_0 & 0 \end{bmatrix} \begin{bmatrix} Br_t \\ 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

Now we can solve for our endogenous variables, $\begin{bmatrix} r_t \\ Y_t \end{bmatrix}$, in terms of our predetermined variables, $\begin{bmatrix} Br_t \\ 1 \\ \bar{I}_t \end{bmatrix}$.

$$\begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \begin{bmatrix} 0 & (1-b_1B) \\ -b_6 & b_4\bar{P}B \end{bmatrix}^{-1} \begin{bmatrix} (b_2+b_3) & a & 1 \\ -b_5\bar{P} & M_0 & 0 \end{bmatrix} \begin{bmatrix} Br_t \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & (1-b_1B) \\ -b_6 & b_4\bar{P}B \end{bmatrix}^{-1} \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

Substitute: $\begin{bmatrix} 0 & (1-b_1B) \\ -b_6 & b_4\bar{P}B \end{bmatrix}^{-1} = \frac{1}{b_6(1-b_1B)} \begin{bmatrix} b_4\bar{P}B & -(1-b_1B) \\ b_6 & 0 \end{bmatrix}$

$$\begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \frac{1}{b_6(1-b_1B)} \begin{bmatrix} b_4\bar{P}B & -(1-b_1B) \\ b_6 & 0 \end{bmatrix} \begin{bmatrix} (b_2+b_3) & a & 1 \\ -b_5\bar{P} & M_0 & 0 \end{bmatrix} \begin{bmatrix} Br_t \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \frac{1}{b_6(1-b_1B)} \begin{bmatrix} b_4\bar{P}B & -(1-b_1B) \\ b_6 & 0 \end{bmatrix} \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

Multiplying through the matrices:

$$\begin{bmatrix} r_t \\ Y_t \end{bmatrix} =$$

$$\frac{1}{b_6(1-b_1B)} \begin{bmatrix} [b_4\bar{P}B(b_2+b_3)+b_5\bar{P}(1-b_1B)] & [ab_4\bar{P}B-M_0(1-b_1B)] & b_4\bar{P}B \\ b_6(b_2+b_3) & ab_6 & b_6 \end{bmatrix} \begin{bmatrix} Brt \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \frac{1}{b_6(1-b_1B)} \begin{pmatrix} b_4\bar{P}Be_{5t} - (1-b_1B)e_{6t} \\ \\ b_6e_{5t} \end{pmatrix}$$

This is the transfer function model implied by our dynamic structural system of simultaneous equations. Explicitly, the two transfer functions are:

$$r_t = (\text{const})_{r_t} + \frac{[b_4\bar{P}B(b_2+b_3)+b_5\bar{P}(1-b_1B)]}{b_6(1-b_1B)} (Br_t)$$

$$+ \frac{b_4\bar{P}B}{b_6(1-b_1B)} (\bar{I}_t)$$

$$+ \frac{1}{b_6(1-b_1B)} [b_4\bar{P}Be_{5t} - (1-b_1B)e_{6t}]$$

$$Y_t = (\text{const})_{Y_t} + \frac{b_6(b_2+b_3)}{b_6(1-b_1B)} (Br_t) + \frac{b_6}{b_6(1-b_1B)} (\bar{I}_t)$$

$$+ \frac{1}{b_6(1-b_1B)} [b_6e_{5t}]$$

From the second transfer function, it is clear that Y_t is a function of lagged values of our leading indicator, r_t. Thus we can fit this transfer function for Y_t, and come up with the estimated mean and variance of Y_t given past r_t. And more importantly, we can come up with an estimate of Y_{t+1} given r_t. That is, we can estimate the mean and variance of the conditional distribution, $f(Y_{t+1}|r_t)$. This is the object of our analysis of the leading indicator approach to forecasting. Consider the relative forecasting abilities of the Final Form (FF) approach and our leading indicator (LI) approach, in the context of example 1.

In the form of equation (2.4), we have:

$$H_{11}(B)\, y_t + H_{12}(B)\, x_t = F_{11}(B)\, e_{1t}$$

$$\begin{bmatrix} -(b_2+b_3)B & (1-b_1B) \\ (-b_6+b_5\overline{P}B) & b_4\overline{P}B \end{bmatrix} \begin{bmatrix} r_t \\ Y_t \end{bmatrix} + \begin{bmatrix} -a & -1 \\ -M_0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \overline{I}_t \end{bmatrix} = \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

The Final Form:

$$\begin{bmatrix} r_t \\ Y_t \end{bmatrix} = \begin{bmatrix} -(b_2+b_3)B & (1-b_1B) \\ (-b_6+b_5\overline{P}B) & b_4\overline{P}B \end{bmatrix}^{-1} \begin{bmatrix} a & 1 \\ M_0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \overline{I}_t \end{bmatrix}$$

$$+ \begin{bmatrix} -(b_2+b_3)B & (1-b_1B) \\ (-b_6+b_5\overline{P}B) & b_4\overline{P}B \end{bmatrix}^{-1} \begin{bmatrix} e_{5t} \\ e_{6t} \end{bmatrix}$$

Call the matrix to be inverted, A.

$$\begin{aligned} \det A &= -b_4\overline{P}(b_2+b_3)B^2 + (1-b_1B)(b_6-b_5\overline{P}B) \\ &= b_6 - b_5\overline{P}B - b_1b_6B + b_1b_5\overline{P}B^2 - b_4\overline{P}(b_2+b_3)B^2 \\ &= b_6 - (b_5\overline{P}+b_1b_6)B + \overline{P}(b_1b_5-b_4(b_2+b_3))B^2 \end{aligned}$$

$$\text{adjoint } A = \begin{bmatrix} b_4\overline{P}B & -(1-b_1B) \\ (b_6-b_5\overline{P}B) & -(b_2+b_3)B \end{bmatrix}$$

$$A^{-1} = \frac{1}{b_6-(b_5\overline{P}+b_1b_6)B+\overline{P}(b_1b_5-b_4(b_2+b_3))B^2} \begin{bmatrix} b_4\overline{P}B & -(1-b_1B) \\ (b_6-b_5\overline{P}B\,) & -(b_2+b_3)B \end{bmatrix}$$

Substituting this into the FF and moving the determinant to the left hand side, we get the following.

$$[b_6-(b_5\overline{P}+b_1b_6)B+\overline{P}(b_1b_5-b_4(b_2+b_3))B^2]\begin{pmatrix} r_t \\ Y_t \end{pmatrix} =$$

$$= \begin{pmatrix} b_4\overline{P}B & -(1-b_1B) \\ b_6-b_5\overline{P}B & -(b_2+b_3)B \end{pmatrix} \begin{pmatrix} a & 1 \\ M_0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ \overline{I}_t \end{pmatrix}$$

$$+ \begin{pmatrix} b_4\overline{P}B & -(1-b_1B) \\ b_6-b_5\overline{P}B & -(b_2+b_3)B \end{pmatrix} \begin{pmatrix} e_{5t} \\ e_{6t} \end{pmatrix}$$

$$[\det A]\begin{pmatrix} r_t \\ Y_t \end{pmatrix} = \begin{pmatrix} [ab_4\overline{P}B-M_0(1-b_1B)] & b_4\overline{P}B \\ [a(b_6-b_5\overline{P}B)-M_0(b_2+b_3)B] & b_6-b_5\overline{P}B \end{pmatrix} \begin{pmatrix} 1 \\ \overline{I}_t \end{pmatrix}$$

$$+ \begin{pmatrix} b_4\overline{P}Be_{5t} - (1-b_1B)e_{6t} \\ (b_6-b_5\overline{P}B)e_{5t} - (b_2+b_3)Be_{6t} \end{pmatrix}$$

Thus we have the Final Form transfer functions:

$$[\det A]\ r_t = (\text{const})_{r_t} + b_4\overline{P}B[\overline{I}_t] + b_4\overline{P}Be_{5t} - (1-b_1B)e_{6t}$$

(i) $$[\det A]\ Y_t = (\text{const})_{Y_t} + (b_6-b_5\overline{P}B)[\overline{I}_t] + (b_6-b_5\overline{P}B)e_{5t} - (b_2+b_3)Be_{6t}$$

These are to be compared with the transfer functions of our leading indicator approach which we derived in the example:

$$\text{(ii)}\quad b_6(1-b_1B)r_t = (\text{const})_{r_t} + [b_4\bar{P}B(b_2+b_3)+b_5\bar{P}(1-b_1B)[Br_t]$$

$$+ \; b_4\bar{P}B[\bar{I}_t] + b_4\bar{P}Be_{5t} - (1-b_1B)e_{6t}$$

$$\text{(iii)}\quad b_6(1-b_1B)Y_t = (\text{const})_{Y_t} + b_6(b_2+b_3)[Br_t] + b_6[\bar{I}_t] + b_6e_{5t}$$

We are interested in the models for Y_t. Consider the forecasts of Y_{t+1} implied by each of the approaches above, in turn.

The Final Form:

From equation (i), we get the model for period t+1:

$$[b_6-(b_5\bar{P}+b_1b_6)B+\bar{P}(b_1b_5-b_4(b_2+b_3))B^2]\,Y_{t+1}$$

$$= (\text{const})_{Y_{t+1}} + (b_6-b_5\bar{P}B)[\bar{I}_{t+1}] + (b_6-b_5\bar{P}B)e_{5t+1}$$

$$- \; (b_2+b_3)Be_{6t+1}$$

$$Y_{t+1} = (\text{const})_{Y_{t+1}} + \frac{1}{b_6}(b_5\bar{P}+b_1b_6)\,Y_t$$

$$- \frac{1}{b_6}\bar{P}(b_1b_5-b_4(b_2+b_3))\,Y_{t-1} + \frac{1}{b_6}(b_6-b_5\bar{P}B)[\bar{I}_{t+1}]$$

$$+ \frac{1}{b_6}(b_6-b_5\bar{P}B)\,e_{5t+1} - \frac{1}{b_6}(b_2+b_3)\,Be_{6t+1}$$

Now expand the remaining Final Form coefficients on the right hand side into the values corresponding to the various lags of the right hand side variables and disturbances.

$$Y_{t+1} = (\text{const}) + \frac{1}{b_6}(b_5\overline{P}+b_1b_6)\,Y_t$$
$$- \frac{1}{b_6}\,\overline{P}\,(b_1b_5-b_4(b_2+b_3))Y_{t-1} + [\overline{I}_{t+1}]$$
$$- \frac{b_5\overline{P}}{b_6}[\overline{I}_t] + e_{5t+1} - \frac{b_5\overline{P}}{b_6}e_{5t} - \frac{b_2+b_3}{b_6}e_{6t}$$

From this, the Final Form forecast is made:

$$\hat{Y}_t(1) = (\text{const}) + \frac{1}{b_6}(b_5\overline{P} + b_1b_6)\,Y_t$$
$$- \frac{1}{b_6}\,\overline{P}\,(b_1b_5-b_4(b_2+b_3))\,Y_{t-1} + [\hat{\overline{I}}_t(1)]$$
$$- \frac{b_5\overline{P}}{b_6}[\overline{I}_t] - \frac{b_5\overline{P}}{b_6}e_{5t} - \frac{b_2+b_3}{b_6}e_{6t}$$

From the above two expressions we get the Final Form forecast error:

$$\text{(iv)} \quad \hat{e}_{Y_t}(1) = \hat{Y}_t(1) - Y_{t+1}$$
$$= [\hat{\overline{I}}_t(1) - \overline{I}_{t+1}] - e_{5t+1} = [\hat{e}_{\overline{I}_t}(1)] - e_{5t+1}$$

The variance of the Final Form forecast error follows:

$$\text{Var}[\hat{e}_{Y_t}(1)] = E[\hat{e}_{Y_t}(1)]^2$$
$$= E\{[\hat{e}_{\overline{I}_t}(1)] - e_{5t+1}\}^2$$
$$= E[\hat{e}_{\overline{I}_t}(1)]^2 + E(e_{5t+1})^2$$

(given the assumption that $\overline{I}_t$ and 3_{5t} are uncorrelated)

The Leading Indicator Approach:

From equation (iii) we get the model for period t+1:

$$(1-b_1B)\, Y_{t+1} = (\text{const}) + (b_2+b_3)[Br_{t+1}] + [\bar{I}_{t+1}] + e_{5t+1}$$

$$Y_{t+1} = (\text{const}) + b_1Y_t + (b_2+b_3)[r_t] + [\bar{I}_{t+1}] + e_{5t+1}$$

From this model our forecast is made:

$$\hat{Y}_t(1) = (\text{const}) + b_1Y_t + (b_2+b_3)[r_t] + [\hat{\bar{I}}_t(1)]$$

And from these two expressions we get the forecast error:

$$\text{(v)} \quad \hat{e}_{Y_t}(1) = \hat{Y}_t(1) - Y_{t+1}$$
$$= [\hat{\bar{I}}_t(1) - \bar{I}_{t+1}] - e_{5t+1} = [\hat{e}_{\bar{I}_t}(1)] - e_{5t+1}$$

Comparing equations (iv) and (v), we see that our leading indicator approach and the Final Form approach yield the <u>same</u> forecast errors in this example. Our leading indicator model, however, appears in a much simplier form in this case.

<u>Example 2</u>

Consider a stochastic model which is an extension of Samuelson's Multiplier-Accelerator Model combined with Metzler's Inventory Model.

(1) $C_t = a + bY_{t-1} + e_{1t}$

(a short run consumption function)

(2) $I_t = \bar{I}_t + c(C_t - C_{t-1}) + e_{2t}$

(an investment model with c = the accelerator coefficient, and $\bar{I}_t$ = autonomous investment)

(3) $V_t = d(C_{t-1} - U_{t-1}) + e_{3t}$

(V_t = production for inventory purposes)

Identities:

(4) $U_t = C_{t-1}$

(U_t = production to meet anticipated sales in period t)

(5) $Y_t = U_t + V_t + I_t$

(income accounting identity)

$= C_{t-1} + V_t + I_t$

(e_1, e_2, and e_3 are disturbances with $E(e_i) = 0$ and $E(e_i e_j) = 0$ for $i \neq j$.)

Equation (3) says that inventories are rebuilt according to the difference between previous consumption and sales.

Substituting for Y_t in (1), and for U_t in (3); the system becomes:

$$C_t = a + b(C_{t-2} + V_{t-1} + I_{t-1}) + e_{1t}$$

(6) $$I_t = \bar{I}_t + c(C_t - C_{t-1}) + e_{2t}$$

$$V_t = d(C_{t-1} - C_{t-2}) + e_{3t}$$

Rewriting this system, with B = the backshift operator:

$$(1-bB^2)\ C_t = a + bB\ V_t + bB\ I_t + e_{1t}$$

$$(7) \qquad I_t = \bar{I}_t + c(1-B)\ C_t + e_{2t}$$

$$V_t = d(B-B^2)\ C_t + e_{3t}$$

In matrix form:

$$(8) \qquad \begin{bmatrix} -bB & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix} \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} a & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Note that this system consists of three equations and three endogenous variables. $\bar{I}_t$ is exogenous.

Production for inventory purposes, V_t, is the candidate for a leading indicator. Increases in V_t affect increases in Y_t directly, as production for inventory purposes is included in Y_t. Increases in V_t also affect increases in C_{t+1}; and thus affect increases in Y_{t+1} through the effect of C_{t+1} on I_{t+1}; and affect increases in Y_{t+2} through the effect of C_{t+1} on U_{t+2}. Hence movements in V_t should be consistently followed by movements in C_{t+1}, I_{t+1}, and Y_{t+1}.

We can apply our transformation to the system in (8) by separating the polynomials in B which multiply V_t, into a component with lags $k < 1$, and a component with lags $k \geq 1$. Note that since the period under consideration in this model is one year, a lead of one period is relatively substantial.

$$(9)\quad \begin{bmatrix} 0 & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix} \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} bB & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} V_t \\ 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

or equivalently:

$$\begin{bmatrix} 0 & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix} \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} b & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} B\ V_t \\ 1 \\ \bar{I}_t \end{bmatrix} + \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Solving the system for the endogenous variables in terms of the predetermined variable, $[B\ V_t]$, and the exogenous variable, $\bar{I}_t$:

$$(10)\quad \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} 0 & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix}^{-1} \begin{bmatrix} b & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} B\ V_t \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \begin{bmatrix} 0 & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & o \end{bmatrix}^{-1} \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Call the matrix we wish to invert, A.

$$\det A = (1-bB^2) - bc(B-B^2) = 1 - bcB + b(c-1)B^2$$

$$[\text{adjoint } A] = \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix}$$

Thus,

$$A^{-1} = \frac{1}{1-bcB+b(c-1)B^2}\begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix}$$

With this substitution, (10) becomes:

$$\begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} =$$

$$\frac{1}{1-bcB+b(c-1)B^2}\begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix}\begin{bmatrix} b & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} BV_t \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \frac{1}{1-bcB+b(c-1)B^2}\begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix}\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Multiplying out the coefficient matrices, and moving the determinant to the left hand side:

$$(11) \qquad [1-bcB+b(c-1)B^2]\begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} =$$

$$\begin{bmatrix} bd(B-B^2) & ad(B-B^2) & bd(B^2-B^3) \\ b & a & bB \\ bc(1-B) & ac(1-B) & (1-bB^2) \end{bmatrix}\begin{bmatrix} BV_t \\ 1 \\ \bar{I}_t \end{bmatrix} +$$

Thus,

$$A^{-1} = \frac{1}{1-bcB+b(c-1)B^2} \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix}$$

With this substitution, (10) becomes:

$$\begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} =$$

$$\frac{1}{1-bcB+b(c-1)B^2} \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix} \begin{bmatrix} b & a & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} BV_t \\ 1 \\ \bar{I}_t \end{bmatrix}$$

$$+ \frac{1}{1-bcB+b(c-1)B^2} \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & [1-bcB+b(c-1)B^2] \\ 1 & bB & 0 \\ c(1-B) & (1-bB^2) & 0 \end{bmatrix} \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Multiplying out the coefficient matrices, and moving the determinant to the left hand side:

$$(11) \qquad [1-bcB+b(c-1)B^2] \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} =$$

$$\begin{bmatrix} bd(B-B^2) & ad(B-B^2) & bd(B^2-B^3) \\ b & a & bB \\ bc(1-B) & ac(1-B) & (1-bB^2) \end{bmatrix} \begin{bmatrix} BV_t \\ 1 \\ \bar{I}_t \end{bmatrix} +$$

(11) (cont'd.)

$$\begin{bmatrix} de_{1t-1}-de_{1t-2}+bde_{2t-2}-bde_{2t-3}+e_{3t}-bce_{3t-1}+b(c-1)e_{3t-2} \\ e_{1t} + be_{2t-1} \\ ce_{1t} - ce_{1t-1} + e_{2t} - be_{2t-2} \end{bmatrix}$$

This is the transfer function model implied by our dynamic structural system of simultaneous equations. Our inputs are the leading indicator, V_t, and the exogenous variable, $\overline{I}_t$. Explicitly, the three transfer functions are:

(12)

$$\begin{aligned} [1-bcB+b(c-1)B^2]\; V_t \;&=\; (\text{const})_{V_t} + bd(B-B^2)(BV_t) \\ &\quad + bd(B^2-B^3)(\overline{I}_t) + [de_{1t-1} - de_{1t-2} + bde_{2t-2} \\ &\quad - bde_{2t-3} + e_{3t} - bce_{3t-1} + b(c-1)\; e_{3t-2}] \\ [1-bcB+b(c-1)B^2]\; C_t \;&=\; (\text{const})_{C_t} + b(BV_t) + bB(\overline{I}_t) \\ &\quad + [e_{1t} + be_{2t-1}] \\ [1-bcB+b(c-1)B^2]\; I_t \;&=\; (\text{const})_{I_t} + bc(1-B)(BV_t) \\ &\quad + (1-bB^2)(\overline{I}_t) + [ce_{1t} - ce_{1t-1} \\ &\quad + e_{2t} - be_{2t-2}] \end{aligned}$$

Finally, we can aggregate our transfer function models to yield the time series model for Y_t, implied by our dynamic structural system of simultaneous equations.

From equation (5) we have:

$$Y_t = U_t + V_t + I_t$$

$$= C_{t-1} + V_t + I_t$$

From this identity and the transfer function models for the components in (12), it is clear that Y_t is a function of lagged values of our leading indicator, V_t.

Hence, using time series methods, we can fit these transfer functions and come up with the estimated mean and variance of Y_{t+1} given values of our leading indicator in previous periods. That is, we can estimate the first two moments of the conditional distribution of Y_{t+1} given V_t, $f(Y_{t+1}|V_t)$. This is the object of our analysis of the leading indicator approach to forecasting.

With this knowledge we can produce optimal forecasts of Y_t, which are presumably better than forecasts produced without the incorporation of our knowledge of the structural relationships between our leading indicator and the other variables in our model.

Consider the relative forecasting abilities of the Final Form approach and our leading indicator approach, in the context of example 2.

In the form of equation (2.4), we have:

$$H_{11}(B)\ y_t + H_{12}(B)\ x_t = F_{11}(B)\ e_{1t}$$

$$\begin{bmatrix} -bB & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix} \begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} + \begin{bmatrix} -a & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \overline{I}_t \end{bmatrix} = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

The Final Form:

$$\begin{bmatrix} V_t \\ C_t \\ I_t \end{bmatrix} = \begin{bmatrix} -bB & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix}^{-1} \begin{bmatrix} a & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \overline{I}_t \end{bmatrix}$$

$$+ \begin{bmatrix} -bB & (1-bB^2) & -bB \\ 0 & -c(1-B) & 1 \\ 1 & -d(B-B^2) & 0 \end{bmatrix}^{-1} \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Call the matrix to be inverted, A.

$$\begin{aligned} \det A &= (1-bB^2) - bc(B-B^2) - bd(B^2-B^3) \\ &= 1 - bB^2 - bcB + bcB^2 + bdB^2 + bdB^3 \\ &= 1 - bcB + (bc-b-bd)B^2 + bdB^3 \end{aligned}$$

$$\text{adjoint } A = \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & (1-bB^2)-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & (1-bB^2)-bd(B^2-B^3) & bc(B-B^2) \end{bmatrix}$$

$$A^{-1} = \frac{1}{\det}\,[\text{adjoint } A]$$

Substituting this into the Final Form and moving the determinant to the left hand side, we get the following:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} =$$

$$= \begin{pmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{pmatrix}\begin{pmatrix} a & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}1\\ \bar{I}_t\end{pmatrix}$$

$$+ \begin{pmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{pmatrix}\begin{pmatrix}e_{1t}\\ e_{2t}\\ e_{3t}\end{pmatrix}$$

Multiplying through the matrices:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} = \begin{pmatrix} ad(B-B^2) & bd(B^2-B^3) \\ a & bB \\ ac(1-B) & 1-bB^2-bd(B^2-B^3)\end{pmatrix}\begin{pmatrix}1\\ \bar{I}_t\end{pmatrix}$$

$$+ \begin{pmatrix} d(B-B^2)e_{1t}+bd(B^2-B^3)e_{2t}+[1-bB^2-bc(B-B^2)]e_{3t} \\ e_{1t} + bBe_{2t} + bBe_{3t} \\ c(1-B)e_{1t}+[1-bB^2-bd(B^2-B^3)]e_{2t}+bc(B-B^2)e_{3t}\end{pmatrix}$$

Consider each of the three Final Form transfer functions in turn. First;

$$[1-bcB+(bc-b-bd)B^2+bdB^3]\ V_t = (\text{const})_{V_t}$$
$$+ \quad [bd(B^2-B^3)\ [\bar{I}_t] + d(B-B^2)e_{1t} + bd(B^2-B^3)e_{2t}$$
$$+ \quad [1-bB^2-bc(B-B^2)]e_{3t}$$

Substituting this into the Final Form and moving the determinant to the left hand side, we get the following:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} =$$

$$= \begin{pmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{pmatrix}\begin{pmatrix} a & 0\\ 0 & 1\\ 0 & 0\end{pmatrix}\begin{pmatrix}1\\ \bar{I}_t\end{pmatrix}$$

$$+ \begin{pmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{pmatrix}\begin{pmatrix} e_{1t}\\ e_{2t}\\ e_{3t}\end{pmatrix}$$

Multiplying through the matrices:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} = \begin{pmatrix} ad(B-B^2) & bd(B^2-B^3) \\ a & bB \\ ac(1-B) & 1-bB^2-bd(B^2-B^3)\end{pmatrix}\begin{pmatrix}1\\ \bar{I}_t\end{pmatrix}$$

$$+ \begin{pmatrix} d(B-B^2)e_{1t}+bd(B^2-B^3)e_{2t}+[1-bB^2-bc(B-B^2)]e_{3t} \\ e_{1t} + bBe_{2t} + bBe_{3t} \\ c(1-B)e_{1t}+[1-bB^2-bd(B^2-B^3)]e_{2t}+bc(B-B^2)e_{3t}\end{pmatrix}$$

Consider each of the three Final Form transfer functions in turn. First;

$$\begin{aligned}[1-bcB+(bc-b-bd)B^2+bdB^3]\ V_t &= (\text{const})_{V_t} \\ &+ [bd(B^2-B^3)\ [\bar{I}_t] + d(B-B^2)e_{1t} + bd(B^2-B^3)e_{2t} \\ &+ [1-bB^2-bc(B-B^2)]e_{3t}\end{aligned}$$

Substituting this into the Final Form and moving the determinant to the left hand side, we get the following:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} =$$

$$= \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{bmatrix}\begin{bmatrix} a & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{pmatrix}1\\ \bar{I}_t\end{pmatrix}$$

$$+ \begin{bmatrix} d(B-B^2) & bd(B^2-B^3) & 1-bB^2-bc(B-B^2) \\ 1 & bB & bB \\ c(1-B) & 1-bB^2-bd(B^2-B^3) & bc(B-B^2) \end{bmatrix}\begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \end{bmatrix}$$

Multiplying through the matrices:

$$[\det A]\begin{pmatrix}V_t\\C_t\\I_t\end{pmatrix} = \begin{bmatrix} ad(B-B^2) & bd(B^2-B^3) \\ a & bB \\ ac(1-B) & 1-bB^2-bd(B^2-B^3) \end{bmatrix}\begin{bmatrix}1\\ \bar{I}_t\end{bmatrix}$$

$$+ \begin{bmatrix} d(B-B^2)e_{1t}+bd(B^2-B^3)e_{2t}+[1-bB^2-bc(B-B^2)]e_{3t} \\ e_{1t} + bBe_{2t} + bBe_{3t} \\ c(1-B)e_{1t}+[1-bB^2-bd(B^2-B^3)]e_{2t}+bc(B-B^2)e_{3t} \end{bmatrix}$$

Consider each of the three Final Form transfer functions in turn. First;

$$[1-bcB+(bc-b-bd)B^2+bdB^3]\ V_t = (\text{const})_{V_t}$$
$$+ \quad [bd(B^2-B^3)\ [\bar{I}_t] + d(B-B^2)e_{1t} + bd(B^2-B^3)e_{2t}$$
$$+ \quad [1-bB^2-bc(B-B^2)]e_{3t}$$

$$\text{(i)} \quad V_t = (\text{const})_{V_t} + bcV_{t-1} + (b+bd-bc)V_{t-2} - bdV_{t-3}$$

$$+ bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2} + bde_{2t-2}$$

$$- bde_{2t-3} + e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}]$$

Second;

$$[1-bcB + (bc-b-bd)B^2 + bdB^3]\, C_t = (\text{const})_{C_t} + bB[\bar{I}_t]$$

$$+ e_{1t} + bBe_{2t} + bBe_{3t}$$

$$\text{(ii)} \quad C_t = (\text{const})_{C_t} + bcC_{t-1} + (b+bd-bc)C_{t-2} - bdC_{t-3} + b[\bar{I}_{t-1}]$$

$$+ [e_{1t} + be_{2t-1} + be_{3t-1}]$$

Third;

$$[1-bcB + (bc-b-bd)B^2 + bdB^3]\, I_t = (\text{const})_{I_t}$$

$$+ [1-bB^2 - bd(B^2-B^3)][\bar{I}_t] + c(1-B)e_{1t}$$

$$+ [1-bB^2 - bd(B^2-B^3)]e_{2t} + bc(B-B^2)e_{3t}$$

$$\text{(iii)} \quad I_t = (\text{const})_{I_t} + bcI_{t-1} + (b+bd-bc)I_{t-2} - bdI_{t-3} + [\bar{I}_t]$$

$$- b(1+d)[\bar{I}_{t-2}] + bd[\bar{I}_{t-3}] + [ce_{1t} - ce_{1t-1} + e_{2t}$$

$$- b(1+d)e_{2t-2} + bde_{2t-3} + bce_{3t-1} - bce_{3t-2}]$$

Note here that I_t is endogenous and $\bar{I}_t$ is exogenous.

The aggregation of equations (i), (ii), and (iii) yields the Final Form transfer function of $Y_t = V_t + U_t + I_t$
$= V_t + C_{t-1} + I_t$.
This is done on the following page.

$$
\begin{aligned}
Y_t \;=\;& (\text{const})_{V_t} + bcV_{t-1} + (b+bd-bc)V_{t-2} - bdV_{t-3} \\
& + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2} \\
& + bde_{2t-2} - bde_{2t-3} + e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}] \\
+\;& (\text{const})_{C_{t-1}} + bcC_{t-2} + (b+bd-bc)C_{t-3} - bdC_{t-4} \\
& + b[\bar{I}_{t-2}] + [e_{1t-1} + be_{2t-2} + be_{3t-2}] \\
+\;& (\text{const})_{I_t} + bcI_{t-1} + (b+bd-bc)I_{t-2} - bdI_{t-3} \\
& + [\bar{I}_t] - b(1+d)[\bar{I}_{t-2}] + bd[\bar{I}_{t-3}] \\
& + [ce_{1t} - ce_{1t-1} + e_{2t} - b(1+d)e_{2t-2} + bde_{2t-3} + bce_{3t-1} \\
& - bce_{3t-2}] \\
=\;& (\text{const}) + bc[V_{t-1} + C_{t-2} + I_{t-1}] \\
& + (b+bd-bc)[V_{t-2} + C_{t-3} + I_{t-2}] - bd[V_{t-3} + C_{t-4} + I_{t-3}] \\
& + [\bar{I}_t] + [ce_{1t} + (1+d-c)e_{1t-1} - de_{1t-2} + e_{2t} + e_{3t}]
\end{aligned}
$$

(iv)
$$
\begin{aligned}
Y_t \;=\;& (\text{const}) + bc[Y_{t-1}] + (b+bd-bc)[Y_{t-2}] - bd[Y_{t-3}] \\
& + [\bar{I}_t] + [ce_{1t} + e_{2t} + e_{3t} + (1+d-c)e_{1t-1} - de_{1t-2}]
\end{aligned}
$$

Note that (const) = $(\text{const})_{V_t} + (\text{const})_{C_{t-1}} + (\text{const})_{I_t}$. Our initial assumptions as to these disturbances were:

$E(e_{it}) = 0$, $E(e_{it})^2 = \sigma_i^2$, $E(e_{it}e_{it-k}) = 0 \;\; \forall\, k \neq 0$,

and $E(e_{it}e_{jt}) = 0$ for $i, j = 1,2,3$ and $i \neq j$.

We see that the disturbance structure of our aggregation of Y_t is a second order autoregressive model about white noise.

From equation (iv) we get the Final Form forecast.

$$Y_{t+1} = (\text{const}) + bc[Y_t] + (b+bd-bc)[Y_{t-1}] - bd[Y_{t-2}]$$
$$+ [\bar{I}_{t+1}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}+(1+d-c)e_{1t}-de_{1t-1}]$$

$$\text{(v)} \qquad \hat{Y}_t(1) = (\text{const}) + bc[Y_t] + (b+bd-bc)[Y_{t-1}] - bd[Y_{t-2}]$$
$$+ [\hat{\bar{I}}_t(1)] + (1+d-c)e_{1t} - de_{1t-1}$$

$$\hat{e}_{Y_t}(1) = \hat{Y}_t(1) - Y_{t+1}$$
$$= [\hat{\bar{I}}_t(1) - \bar{I}_{t+1}] - ce_{1t+1} - e_{2t+1} - e_{3t+1}$$
$$= [\hat{e}_{\bar{I}_t}(1)] - ce_{1t+1} - e_{2t+1} - e_{3t+1}$$

$$\text{Var}[\hat{e}_{Y_t}(1)] = E[\hat{e}_{Y_t}(1)]^2$$
$$= \text{Var}[\hat{e}_{\bar{I}_t}(1)] + c^2\text{Var}(e_{1t+1}) + \text{Var}(e_{2t+1})$$
$$+ \text{Var}(e_{3t+1})$$

Now consider our leading indicator approach. Our leading indicator is V_t, and we have our three transfer functions from equation (12) in the example. Consider each in turn.

First:

$$[1-bcB+b(c-1)B^2]\,V_t = (\text{const})_{V_t} + bd(B-B^2)[V_{t-1}]$$
$$+ bd(B^2-B^3)[\bar{I}_t] + [de_{1t-1}-de_{1t-2}+bde_{2t-2}-bde_{2t-3}$$
$$+ e_{3t} - bce_{3t-1} + b(c-1)\, e_{3t-2}]$$

(vi) $$V_t = (\text{const})_{V_t} + bcV_{t-1} - b(c-1)V_{t-2} + bd[V_{t-2}]$$
$$- bd[V_{t-3}] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2}$$
$$+ bde_{2t-2} - bde_{2t-3} + e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}]$$

Second;

$$[1-bcB+b(c-1)B^2]\ C_t = (\text{const})_{C_t} + b[V_{t-1}] + b[\bar{I}_{t-1}]$$
$$+ [e_{1t} + be_{2t-1}]$$

(vii) $$C_t = (\text{const})_{C_t} + bcC_{t-1} - b(c-1)C_{t-2} + b[V_{t-1}]$$
$$+ b[\bar{I}_{t-1}] + [e_{1t} + be_{2t-1}]$$

Third;

$$[1-bcB+b(c-1)B^2]\ I_t = (\text{const})_{I_t} + bc[V_{t-1}] - bc[V_{t-2}]$$
$$+ [\bar{I}_t] - b[\bar{I}_{t-2}] + [ce_{1t} - ce_{1t-1} + e_{2t} - be_{2t-2}]$$

(viii) $$I_t = (\text{const})_{I_t} + bcI_{t-1} - b(c-1)I_{t-2} + bc[V_{t-1}]$$
$$- bc[V_{t-2}] + [\bar{I}_t] - b[\bar{I}_{t-2}] + [ce_{1t} - ce_{1t-1} + e_{2t}$$
$$- be_{2t-2}]$$

The aggregation of equations (vi), (vii), and (viii) yields the transfer function for Y_t implied by our leading indicator approach.

$$Y_t = V_t + C_{t-1} + I_t$$

$$= (\text{const})_{V_t} + bcV_{t-1} - b(c-1)V_{t-2} + bd[V_{t-2}]$$

$$- bd[V_{t-3}] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2}$$

$$+ bde_{2t-2} - bde_{2t-3} + e_{3t} - bce_{3t-1}$$

$$+ b(c-1)\ e_{3t-2}] +$$

$$+ (\text{const})_{C_{t-1}} + bcC_{t-2} - b(c-1)C_{t-3} + b[V_{t-2}] + b[\bar{I}_{t-2}]$$

$$+ [e_{1t-1} + be_{2t-2}] +$$

$$+ (\text{const})_{I_t} + bcI_{t-1} - b(c-1)I_{t-2} + bc[V_{t-1}] - bc[V_{t-2}]$$

$$+ [\bar{I}_t] - b[\bar{I}_{t-2}] + [ce_{1t} - ce_{1t-1} + e_{2t} - be_{2t-2}]$$

$$Y_t = (\text{const}) + bc[V_{t-1} + C_{t-2} + I_{t-1}] - b(c-1)[V_{t-2} + C_{t-3}$$

$$+ I_{t-2}] + bc[V_{t-1}] + b(1+d-c)[V_{t-2}] - bd[V_{t-3}]$$

$$+ [\bar{I}_t] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [ce_{1t}$$

$$+ (1+d-c)e_{1t-1} - de_{1t-2} + e_{2t} + bde_{2t-2} - bde_{2t-3}$$

$$+ e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}]$$

(ix) $$Y_t = (\text{const}) + bcY_{t-1} - b(c-1)Y_{t-2} + bc[V_{t-1}]$$

$$+ b(1+d-c)\ [V_{t-2}] - bd[V_{t-3}] + [\bar{I}_t] + bd[\bar{I}_{t-2}]$$

$$- bd[\bar{I}_{t-3}] + [ce_{1t} + e_{2t} + e_{3t}]$$

$$+ [(1+d-c)e_{1t-1} - bce_{3t-1}]$$

$$+ [-de_{1t-2} + bde_{2t-2} + b(c-1)e_{3t-2}] + [-bde_{2t-3}]$$

Note that $(\text{const}) = (\text{const})_{V_t} + (\text{const})_{C_{t-1}} + (\text{const})_{I_t}$.

$$= (\text{const})_{V_t} + bcV_{t-1} - b(c-1)V_{t-2} + bd[V_{t-2}]$$

$$- bd[V_{t-3}] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2}$$

$$+ bde_{2t-2} - bde_{2t-3} + e_{3t} - bce_{3t-1}$$

$$+ b(c-1)\, e_{3t-2}] +$$

$$+ (\text{const})_{C_{t-1}} + bcC_{t-2} - b(c-1)C_{t-3} + b[V_{t-2}] + b[\bar{I}_{t-2}]$$

$$+ [e_{1t-1} + be_{2t-2}] +$$

$$+ (\text{const})_{I_t} + bcI_{t-1} - b(c-1)I_{t-2} + bc[V_{t-1}] - bc[V_{t-2}]$$

$$+ [\bar{I}_t] - b[\bar{I}_{t-2}] + [ce_{1t} - ce_{1t-1} + e_{2t} - be_{2t-2}]$$

$$Y_t = (\text{const}) + bc[V_{t-1} + C_{t-2} + I_{t-1}] - b(c-1)[V_{t-2} + C_{t-3}$$

$$+ I_{t-2}] + bc[V_{t-1}] + b(1+d-c)[V_{t-2}] - bd[V_{t-3}]$$

$$+ [\bar{I}_t] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [ce_{1t}$$

$$+ (1+d-c)e_{1t-1} - de_{1t-2} + e_{2t} + bde_{2t-2} - bde_{2t-3}$$

$$+ e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}]$$

(ix) $$Y_t = (\text{const}) + bcY_{t-1} - b(c-1)Y_{t-2} + bc[V_{t-1}]$$

$$+ b(1+d-c)\,[V_{t-2}] - bd[V_{t-3}] + [\bar{I}_t] + bd[\bar{I}_{t-2}]$$

$$- bd[\bar{I}_{t-3}] + [ce_{1t} + e_{2t} + e_{3t}]$$

$$+ [(1+d-c)e_{1t-1} - bce_{3t-1}]$$

$$+ [-de_{1t-2} + bde_{2t-2} + b(c-1)e_{3t-2}] + [-bde_{2t-3}]$$

Note that $(\text{const}) = (\text{const})_{V_t} + (\text{const})_{C_{t-1}} + (\text{const})_{I_t}$.

$$= (\text{const})_{V_t} + bcV_{t-1} - b(c-1)V_{t-2} + bd[V_{t-2}]$$

$$- bd[V_{t-3}] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [de_{1t-1} - de_{1t-2}$$

$$+ bde_{2t-2} - bde_{2t-3} + e_{3t} - bce_{3t-1}$$

$$+ b(c-1)\ e_{3t-2}] +$$

$$+ \quad (\text{const})_{C_{t-1}} + bcC_{t-2} - b(c-1)C_{t-3} + b[V_{t-2}] + b[\bar{I}_{t-2}]$$

$$+ [e_{1t-1} + be_{2t-2}] +$$

$$+ \quad (\text{const})_{I_t} + bcI_{t-1} - b(c-1)I_{t-2} + bc[V_{t-1}] - bc[V_{t-2}]$$

$$+ [\bar{I}_t] - b[\bar{I}_{t-2}] + [ce_{1t} - ce_{1t-1} + e_{2t} - be_{2t-2}]$$

$$Y_t = (\text{const}) + bc[V_{t-1} + C_{t-2} + I_{t-1}] - b(c-1)[V_{t-2} + C_{t-3}$$

$$+ I_{t-2}] + bc[V_{t-1}] + b(1+d-c)[V_{t-2}] - bd[V_{t-3}]$$

$$+ [\bar{I}_t] + bd[\bar{I}_{t-2}] - bd[\bar{I}_{t-3}] + [ce_{1t}$$

$$+ (1+d-c)e_{1t-1} - de_{1t-2} + e_{2t} + bde_{2t-2} - bde_{2t-3}$$

$$+ e_{3t} - bce_{3t-1} + b(c-1)e_{3t-2}]$$

(ix) $$Y_t = (\text{const}) + bcY_{t-1} - b(c-1)Y_{t-2} + bc[V_{t-1}]$$

$$+ b(1+d-c)\ [V_{t-2}] - bd[V_{t-3}] + [\bar{I}_t] + bd[\bar{I}_{t-2}]$$

$$- bd[\bar{I}_{t-3}] + [ce_{1t} + e_{2t} + e_{3t}]$$

$$+ [(1+d-c)e_{1t-1} - bce_{3t-1}]$$

$$+ [-de_{1t-2} + bde_{2t-2} + b(c-1)e_{3t-2}] + [-bde_{2t-3}]$$

Note that (const) = $(\text{const})_{V_t} + (\text{const})_{C_{t-1}} + (\text{const})_{I_t}$.

Also, given our assumptions regarding the disturbances, e_{it}, the noise structure of Y_t is seen to be a third order auto-regressive scheme.

From equation (ix) we get the forecast of our leading indicator approach.

$$
\begin{aligned}
Y_{t+1} = {} & (\text{const}) + bcY_t - b(c-1)\, Y_{t-1} + bc[V_t] \\
& + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\bar{I}_{t+1}] + bd[\bar{I}_{t-1}] \\
& - bd[\bar{I}_{t-2}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\
& + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1}+bde_{2t-1} \\
& + b(c-1)e_{3t-1}] + [-bde_{2t-2}]
\end{aligned}
$$

$$
\begin{aligned}
\text{(x)} \qquad \hat{Y}_t(1) = {} & (\text{const}) + bcY_t - b(c-1)Y_{t-1} + bc[V_t] \\
& + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\hat{\bar{I}}_t(1)] + bd[\bar{I}_{t-1}] \\
& - bd[\bar{I}_{t-2}] + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1} \\
& + bde_{2t-1} + b(c-1)e_{3t-1}] + [-bde_{2t-2}]
\end{aligned}
$$

$$
\begin{aligned}
\hat{e}_{Y_t}(1) & = \hat{Y}_t(1) - Y_{t+1} \\
& = [\hat{\bar{I}}_t(1) - \bar{I}_{t+1}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\
& = [\hat{e}_{\bar{I}_t}(1)] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}]
\end{aligned}
$$

$$
\begin{aligned}
\text{Var}[\hat{e}_{Y_t}(1)] & = E[\hat{e}_{Y_t}(1)]^2 \\
& = \text{Var}[\hat{e}_{\bar{I}_t}(1)] + c^2\text{Var}[e_{1t+1}] + \text{Var}[e_{2t+1}] \\
& \quad + \text{Var}[e_{3t+1}]
\end{aligned}
$$

Also, given our assumptions regarding the disturbances, e_{it}, the noise structure of Y_t is seen to be a third order autoregressive scheme.

From equation (ix) we get the forecast of our leading indicator approach.

$$\begin{aligned} Y_{t+1} = {} & (\text{const}) + bcY_t - b(c-1)\,Y_{t-1} + bc[V_t] \\ & + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\bar{I}_{t+1}] + bd[\bar{I}_{t-1}] \\ & - bd[\bar{I}_{t-2}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\ & + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1}+bde_{2t-1} \\ & + b(c-1)e_{3t-1}] + [-bde_{2t-2}] \end{aligned}$$

$$\text{(x)} \quad \begin{aligned} \hat{Y}_t(1) = {} & (\text{const}) + bcY_t - b(c-1)Y_{t-1} + bc[V_t] \\ & + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\hat{\bar{I}}_t(1)] + bd[\bar{I}_{t-1}] \\ & - bd[\bar{I}_{t-2}] + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1} \\ & + bde_{2t-1} + b(c-1)e_{3t-1}] + [-bde_{2t-2}] \end{aligned}$$

$$\begin{aligned} \hat{e}_{Y_t}(1) &= \hat{Y}_t(1) - Y_{t+1} \\ &= [\hat{\bar{I}}_t(1) - \bar{I}_{t+1}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\ &= [\hat{e}_{\bar{I}_t}(1)] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \end{aligned}$$

$$\begin{aligned} \text{Var}[\hat{e}_{Y_t}(1)] &= E[\hat{e}_{Y_t}(1)]^2 \\ &= \text{Var}[\hat{e}_{\bar{I}_t}(1)] + c^2\text{Var}[e_{1t+1}] + \text{Var}[e_{2t+1}] \\ &\quad + \text{Var}[e_{3t+1}] \end{aligned}$$

Also, given our assumptions regarding the disturbances, e_{it}, the noise structure of Y_t is seen to be a third order autoregressive scheme.

From equation (ix) we get the forecast of our leading indicator approach.

$$\begin{aligned} Y_{t+1} = {} & (\text{const}) + bcY_t - b(c-1)\, Y_{t-1} + bc[V_t] \\ & + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\bar{I}_{t+1}] + bd[\bar{I}_{t-1}] \\ & - bd[\bar{I}_{t-2}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\ & + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1}+bde_{2t-1} \\ & + b(c-1)e_{3t-1}] + [-bde_{2t-2}] \end{aligned}$$

$$\text{(x)} \qquad \begin{aligned} \hat{Y}_t(1) = {} & (\text{const}) + bcY_t - b(c-1)Y_{t-1} + bc[V_t] \\ & + b(1+d-c)[V_{t-1}] - bd[V_{t-2}] + [\hat{\bar{I}}_t(1)] + bd[\bar{I}_{t-1}] \\ & - bd[\bar{I}_{t-2}] + [(1+d-c)e_{1t}-bce_{3t}] + [-de_{1t-1} \\ & + bde_{2t-1} + b(c-1)e_{3t-1}] + [-bde_{2t-2}] \end{aligned}$$

$$\begin{aligned} \hat{e}_{Y_t}(1) & = \hat{Y}_t(1) - Y_{t+1} \\ & = [\hat{\bar{I}}_t(1) - \bar{I}_{t+1}] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \\ & = [\hat{e}_{\bar{I}_t}(1)] + [ce_{1t+1}+e_{2t+1}+e_{3t+1}] \end{aligned}$$

$$\begin{aligned} \text{Var}[\hat{e}_{Y_t}(1)] & = E[\hat{e}_{Y_t}(1)]^2 \\ & = \text{Var}[\hat{e}_{\bar{I}_t}(1)] + c^2\text{Var}[e_{1t+1}] + \text{Var}[e_{2t+1}] \\ & + \text{Var}[e_{3t+1}] \end{aligned}$$

Note that the one step ahead forecast error for the leading indicator approach is identical with that of the Final Form approach.

Hence we have outlined an approach with an explicit theoretical background in which leading indicators can be studied and used, and which performs as well as the Final Form approach.

It is not surprising that the two approaches yield the same forecast errors for forecasts within the horizon of our leading indicators' lead. They are obtained from essentially the same information set. They are just different algebraic manipulations of the same model.

FOOTNOTES

CHAPTER II

[1] Box and Jenkins, Time Series Analysis, Holden Day, Inc., 1977, pp. 404-410.

[2] Evans, M.K., Macroeconomic Activity, New York: Harper and Row, 1969, Chapter 16.

[3] The Handbook of Cyclical Indicators, A Supplement to the BCD, May, 1977, pp. 170-185.

[4] Evans, op. cit.

[5] Koopmans, T.C., "Measurement Without Theory," Review of Economics and Statistics, August, 1947.

[6] Vining, R., "Koopmans on the Choice of Variables to be Studied and of Methods of Measurement," Review of Economics and Statistics, May, 1949.

[7] Harris, M.N. and Jamroz, D., "Evaluating the Leading Indicators," Monthly Review of the Federal Reserve Bank of New York, June, 1976.

[8] Zellner and Palm, "Time Series Analysis and Simultaneous Equation Econometric Systems," Journal of Econometrics, 2, 1974, pp. 17-54.

[9] Theil and Boot, "The Final Form of Econometric Equation Systems," Review of the International Statistical Institute, Volume 30:2; 1962, pp. 136-152.

[10] Box and Jenkins, op. cit.

[11] Box and Jenkins, op. cit., p. 404.

[12] Well known fact:

If z_t is a vector of mutually independent random variables with $E(z_t) = 0$ and $E(z_t\ z_{t-i}) = 0$, $i = 1,2, \dots$, and if H is a matrix with known coefficients, then

$$\text{Var}\{[H]z_t\} = E\{[H]z_t\}^2 = [[H]]\ \text{Var}(z_t);$$

where [[H]] is the transformation of [H] made by squaring <u>each</u> <u>element</u> in [H].

A simple example:

Let

$$[H] = \begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix} \quad ; \quad z_t = \begin{bmatrix} e_{11t} \\ e_{21t} \end{bmatrix}$$

with the e's having properties identical to those in example 1.

$$\text{Var}\{[H]z_t\} = \text{Var}\left\{\begin{bmatrix} 1 & -2 \\ 3 & 0 \end{bmatrix} \begin{bmatrix} e_{11t} \\ e_{21t} \end{bmatrix}\right\}$$

First, with our transformation;

$$= \begin{bmatrix} 1 & 4 \\ 9 & 0 \end{bmatrix} \text{Var} \begin{bmatrix} e_{11t} \\ e_{21t} \end{bmatrix}$$

$$= \begin{bmatrix} \text{Var}(e_{11t})+4\text{Var}(e_{21t}) \\ 9\text{Var}(e_{11t}) \end{bmatrix}$$

Second, the straightforward development;

$$= \text{Var} \begin{bmatrix} e_{11t}-2e_{21t} \\ 3e_{11t} \end{bmatrix}$$

$$= \begin{bmatrix} \text{Var}\ [e_{11t}-2e_{21t}] \\ \text{Var}[3e_{11t}] \end{bmatrix}$$

$$= \begin{bmatrix} \text{Var}(e_{11t})+4\text{Var}(e_{21t}) \\ 9\text{Var}(e_{11t}) \end{bmatrix}$$

It should be noted that in <u>our</u> model, each element in [H] is a polynomial in L, the lag operator. In this context, to transform [H] into [[H]] we need to square the coefficient of each different power of L appearing in every polynomial comprising an element in [H].

A less simple example:

$$\text{Let} \quad [H] = \begin{pmatrix} (B-3B^2) & 2B^2 \\ 3B^3 & 0 \end{pmatrix}; \qquad z_t = \begin{pmatrix} e_{11t} \\ e_{21t} \end{pmatrix}$$

$$\mathrm{Var}\{[H]z_t\} = \mathrm{Var}\left\{\begin{pmatrix} B-3B^2 & 2B^2 \\ 3B^3 & 0 \end{pmatrix} \begin{pmatrix} e_{11t} \\ e_{21t} \end{pmatrix}\right\}$$

First, with our transformation;

$$= \begin{pmatrix} (1)^2B+(-3)^2B^2 & (2)^2B^2 \\ (3)^2B^3 & 0 \end{pmatrix} V \begin{pmatrix} e_{11t} \\ e_{21t} \end{pmatrix}$$

$$= \begin{pmatrix} (B+9B^2) & 4B^2 \\ 9B^3 & 0 \end{pmatrix} V \begin{pmatrix} e_{11t} \\ e_{21t} \end{pmatrix}$$

$$= \begin{pmatrix} BV(e_{11t})+9B^2V(e_{11t})+4B^2V(e_{21t}) \\ 9B^3V(e_{11t}) \end{pmatrix}$$

$$= \begin{pmatrix} V(e_{11t-1})+9\ V(e_{11t-2})+4\ V(e_{21t-2}) \\ 9\ V(e_{11t-3}) \end{pmatrix}$$

Second, the straightforward development;

$$= V\begin{pmatrix} (B-3B^2)e_{11t} + 2B^2e_{31t} \\ 3B^3e_{11t} \end{pmatrix}$$

$$= V\begin{pmatrix} Be_{11t} - 3B^2e_{11t} + 2B^2e_{21t} \\ 3B^3e_{11t} \end{pmatrix}$$

$$= V\begin{bmatrix} e_{11t-1} - 3e_{11t-2} + 2e_{21t-2} \\ \\ 3e_{11t-3} \end{bmatrix}$$

$$= \begin{bmatrix} V(e_{11t-1} - 3e_{11t-2} + 2e_{21t-2} \\ \\ V(3e_{11t-3}) \end{bmatrix}$$

$$= \begin{bmatrix} V(e_{11t-1}) + 9V(e_{11t-2}) + 4V(e_{21t-2}) \\ \\ 9V(e_{11t-3}) \end{bmatrix}$$

Clearly, for this to be a valid transformation, not only must our disturbances be mutually independent, but they must not be autocorrelated as well. i.e. $E(z_t \; z_{t-i}) = 0$, $i = 1,2, \ldots$

This assumption holds concerning our error structure, e_{1t}, but it does <u>not</u> hold in general concerning our inputs, x_t. In particular, consider equation (2.21). We cannot apply the same transformation to the matrix $[H_{11}^{-1}(L)\,H_{12}(L)]^*$ as we apply to $[H_{11}^{-1}(L)\; F_{11}(L)]^*$ because the elements in the vector of forecast errors, $\hat{e}_{x_t}(k)$, are autocorrelated.

e.g. $E[\hat{e}_{x_{1t}}(k-1) \cdot \hat{e}_{x_{1t}}(k-2)]$

$$= E\left\{\left[\sum_{n=0}^{k-2} \psi_{1n} a_{t+(k-1)-n}\right]\left[\sum_{n=0}^{k-3} \psi_{1n} a_{t+(k-2)-n}\right]\right\}$$

(by substituting from equation (2.18))

$$= \sum_{n=1}^{k-3} \psi_{1n}\psi_{1(n-1)}\sigma_{1a}^2 \;\neq 0 .$$

This point becomes obvious in our less simple example, if we replace $z_t = \begin{bmatrix} e_{11t} \\ e_{21t} \end{bmatrix}$ with $z_t' = \begin{bmatrix} \hat{e}_{x_{1t}}(k) \\ \hat{e}_{x_{2t}}(k) \end{bmatrix}$.

We will find in taking the variance that this transformation of [H] will be inappropriate since $\hat{e}_{x_{1t}}(k)$ is autocorrelated with past forecast errors.

Let
$$[H] = \begin{pmatrix} B-3B^2 & 2B^2 \\ 3B^3 & 0 \end{pmatrix} ; \quad z_t' = \begin{pmatrix} \hat{e}_{x_{1t}}(k) \\ \hat{e}_{x_{2t}}(k) \end{pmatrix}$$

with the forecast errors having properties like those in equation (2.18).

$$\mathrm{Var}\{[H]z_t'\} = V \begin{pmatrix} B-3B^2 & 2B^2 \\ 3B^3 & 0 \end{pmatrix} \begin{pmatrix} \hat{e}_{x_{1t}}(k) \\ \hat{e}_{x_{2t}}(k) \end{pmatrix}$$

First, with our transformation;

$$= \begin{pmatrix} B+9B^2 & 4B^2 \\ 9B^3 & 0 \end{pmatrix} V \begin{pmatrix} \hat{e}_{x_{1t}}(k) \\ \hat{e}_{x_{2t}}(k) \end{pmatrix}$$

$$= \begin{pmatrix} V[\hat{e}_{x_{1t}}(k-1)+9V[\hat{e}_{x_{1t}}(k-2)]+4V[\hat{e}_{x_{2t}}(k-2)] \\ 9V[\hat{e}_{x_{1t}}(k-3)] \end{pmatrix}$$

(inappropriate)

Second, the straightforward development;

$$\mathrm{Var}\{[H]z_t'\} = \begin{pmatrix} V[(B-3B^2)\hat{e}_{x_{1t}}(k) + 2B^2\hat{e}_{x_{2t}}(k)] \\ V[3B^3\hat{e}_{x_{1t}}(k)] \end{pmatrix}$$

$$= \begin{pmatrix} V[\hat{e}_{x_{1t}}(k-1)-3\hat{e}_{x_{1t}}(k-2)+2\hat{e}_{x_{2t}}(k-2)] \\ V[3\hat{e}_{x_{1t}}(k-3)] \end{pmatrix}$$

$$= \begin{bmatrix} \begin{Bmatrix} V[\hat{e}_{x_{1t}}(k-1)] + 9V[\hat{e}_{x_{1t}}(k-2) \\ + 4V[\hat{e}_{x_{2t}}(k-2)] - 6E[\hat{e}_{x_{1t}}(k-1)\cdot\hat{e}_{x_{1t}}(k-2)] \end{Bmatrix} \\ 9V[\hat{e}_{x_{1t}}(k-3)] \end{bmatrix}$$

The difficulty is due to more than one power of B appearing in any polynomial which is a single element in [H]. In the variance, the presence of these different powers of B requires consideration of the given variable's autocorrelation.

Although we cannot make a transformation of $[H_{11}^{-1}(L)\ H_{12}(L)]^*$, we can still calculate the variance of the forecast errors as expressed in equation (2.21) in a relatively straightforward manner. See the examples for applications.

APPENDIX 1

FITTING THE TRANSFER FUNCTION IN EQUATION (2.10)

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FITTING THE TRANSFER FUNCTION IN EQUATION (2.10)

How do we fit the transfer function in equation (2.10)? We can prewhiten the input, $x_t^* = \begin{pmatrix} L^k y_{1t} \\ \hline x_t \end{pmatrix}$.

To do this, first consider the bottom p_2 elements of x_t^*; namely x_t. From (2.4) we have:

$$H_{22}(L)x_t = F_{22}(L)\, e_{2t}$$

or

$$x_t = H_{22}^{-1}(L)\, F_{22}(L)\, e_{2t}$$

This is the appropriate ARIMA model for x_t. We can substitute this into x_t^* for the bottom p_2 elements, and these inputs will be "prewhitened." Then the transfer function will be in terms of the predetermined variable input, $L^k\, y_{1t}$, and the p_2 prewhitened inputs, $x_t = H_{22}^{-1}(L)\, F_{22}(L)\, e_{2t}$. That is,

$$x_t^* = \begin{pmatrix} L^k\, y_{1t} \\ \hline H_{22}^{-1}(L)\, F_{22}(L)\, e_{2t} \end{pmatrix}$$

We still need to prewhiten y_{1t}, our leading indicator, in order to work with the system in (2.10) in terms of <u>all</u> prewhitened inputs, We must therefore fit y_{1t} with its appropriate ARIMA model.

The first equation in the set of p_1 equations in

(2.9) gives us the time series model for y_{1t} implied by our dynamic structural system of simultaneous equations:

$$(2.11)\quad y_{1t} = -\left\{ \underset{1x(p_2+1)}{\left(\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1}} \left(\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right) \right\}_{1\cdot} \underset{(p_2+1)x1}{\left[\frac{L^k y_{1t}}{x_t} \right]}$$

$$+ \left\{ \underset{1xp_1}{\left(\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1}} F_{11}(L) \right\}_{1\cdot} \underset{p_1 x1}{e_{1t}}$$

This is a transfer function for y_{1t} with input x_t^*. This equation is interesting in that lagged y_{1t} is an input to the transfer function model for y_{1t}.

The first element of the $1x(p_2+1)$ vector,

$$\left\{ \left(\begin{array}{c|c} h_{11}^{*} & \\ h_{21}^{*} & \\ \vdots & H_{11}^{*}(L) \\ h_{p_1 1}^{*} & \end{array} \right)^{-1} \left(\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{21}(L) \\ h_{p_1 1}^{***} & \end{array} \right) \right\}_{1\cdot}$$

multiplies $L^k\, y_{1t}$ in (2.11). Call this first element of the vector, g(L). The first term in the sum on the right hand side of (2.11) can thus be written: $[g(L)\, L^k\, y_{1t}]$. This can be moved to the left hand side of (2.11) in order to collect all the terms in y_{1t}.

$$(2.12) \quad y_{1t} + g(L)\, L^k\, y_{1t} = (1 + g(L)\, L^k)\, y_{1t}$$

$$= -\left\{ \left[\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}_{1\cdot -} x_t$$

$$+ \left\{ \left[\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array} \right]^{-1} F_{11}(L) \right\}_{1\cdot} e_{1t}$$

Note that this vector multiplying x_t is the same vector as in (2.11), without the first element, $g(L)$. This can now be solved for y_{1t}:

$$(2.13) \quad y_{1t} = -(1+g(L)L^k)^{-1} \left\{ \left[\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array} \right]^{-1} \left[\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array} \right] \right\}_{1\cdot -} x_t$$

$$+ (1 + g(L)L^k)^{-1} \left\{ \left[\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array} \right]^{-1} F_{11}(L) \right\}_{1\cdot} e_{1t}$$

This gives us the transfer function for our leading indicator, y_{1t}, implied by our structural model, in terms of exogenous variables only. This can be fitted by prewhitening the input, substituting $[H_{22}^{-1}(L)\ F_{22}(L)\ e_{2t}]$ for x_t:

$$(2.14)\quad y_{1t} = -(1+g(L)L^k)^{-1}\left\{\quad\right\}_{1\cdot-}[H_{22}^{-1}(L)\ F_{22}(L)e_{2t}]$$

$$+\ (1+g(L)L^k)^{-1}\left\{\quad\right\}_{1\cdot}\ e_{1t}$$

We can now substitute this model for y_{1t} into $x_t^* = \left(\frac{L^k\ y_{1t}}{x_t}\right)$.
Hence we have x_t^* entirely prewhitened, and can now work with our model for the measure of economic activity in (2.10):

$$x_t^* = \left(\frac{\left(-L^k(1+g(L)L^k)^{-1}\left\{\right\}_{1.-}H_{22}^{-1}(L)F_{22}(L)e_{2t} + L^k(1+g(L)L^k)^{-1}\left\{\right\}_{1.}e_{1t}\right)}{H_{22}^{-1}(L)\ F_{22}(L)\ e_{2t}}\right)$$

$$(2.10)\quad y_{2t} = -\left\{\left(\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array}\right)^{-1}\left(\begin{array}{c|c} h_{11}^{***} & \\ h_{21}^{***} & \\ \vdots & H_{12}(L) \\ h_{p_1 1}^{***} & \end{array}\right)\right\}_{2\cdot}\left(x_t^*\right)$$

$$+\left\{\left(\begin{array}{c|c} h_{11}^* & \\ h_{21}^* & \\ \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \end{array}\right)^{-1} F_{11}(L)\right\}_{2\cdot} e_{1t}$$

Note that equation (2.13) is the same equation as is implied in the structural system in equation (2.4).

The Final Form for the system is:

$$(2.15)\ y_t = -H_{11}^{-1}(L)\ H_{12}(L)\ x_t + H_{11}^{-1}(L)\ F_{11}(L)\ e_{1t}$$

The first equation of the Final Form is the same as equation (2.13).

APPENDIX 2

A USEFUL CONVENTION OF ZELLNER AND PALM

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A USEFUL CONVENTION OF ZELLNER AND PALM

At this point let me interject a useful convention which Zellner and Palm point out, when working with this kind of model transformation (see footnote 8).

In both the Final Form and leading indicator approaches, our model is expressed as a vector of endogenous variables in terms of a set of linear combinations of predetermined variables and disturbances, in a dynamic framework. In matrix form, our coefficient matrix is the product of two known matrices (say A and B), one of which is in inverse form. That is, our model is of the form; $y_t = [A^{-1}B]x_t + [A^{-1}C]e_t$.

From our presumed knowledge of A, B, and C, we can compute $A^{-1} = \frac{1}{\det A}$ [adjoint A], and thus we know $[A^{-1}B]$ and $[A^{-1}C]$. Note that in our context, each element of $[A^{-1}B]$ will be the ratio of two polynomials in L, with the denominator being the determinant of A; i.e. $[A^{-1}B] = \frac{1}{\det A}$ [adj A][B]. A distributed lag which is the ratio of two polynomials in L implies a lag of infinite order. Hence we have a quite complicated system.

We can simplify this system by multiplying both sides of the equation of our model by [det A];

i.e. $[\det A]y_t = [\text{adj } A][B]x_t + [\text{adj } A][C]e_t$

Here our system is in the form of a transfer function with current and lagged y_t's in terms of current and lagged x_t's and disturbances. An interesting aspect of this system is that the order and parameters of the autoregressive part of <u>each</u> equation will be the same. This is true because the determinant multiplying the vector, y_t, is a single polynomial in L.

Note that with this manipulation of our models, equations (2.16), (2.17), (2.23), and (2.24) will be changed as follows.

Final Form approach;

$$(2.16)' \quad \det[H_{11}(L)]\, y_{t+k} = -[\text{adj } H_{11}(L)][H_{12}(L)]\, x_{t+k} + [\text{adj } H_{11}(L)][F_{11}(L)]\, e_{1t+k}$$

$$(2.17)' \quad \det[H_{11}(L)]\hat{y}_t(k) = -[\text{adj } H_{11}(L)][H_{12}(L)]\, \hat{x}_t(k) + \{[\text{adj } H_{11}(L)][F_{11}(L)]\}^{***} e_{1t}$$

Leading Indicator approach;

$$(2.23)' \quad \det\begin{vmatrix} h_{11}^* & \vdots & \\ \vdots & \vdots & H_{11}^*(L) \\ h_{p_1 1}^* & \vdots & \end{vmatrix} y_{t+k} =$$

$$= -\left\{ \operatorname{adj} \left[\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L) \right] \left[\begin{matrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{matrix} \middle| H_{12}(L) \right] \right\} \left(\frac{y_{1t}}{x_{t+k}} \right)$$

$$+ \left\{ \operatorname{adj} \left[\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L) \right] F_{11}(L) \right\} e_{1t+k}$$

$$(2.24)' \quad \det \left| \begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L) \right| \hat{y}_t(k) =$$

$$= -\left\{ \operatorname{adj} \left[\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L) \right] \left[\begin{matrix} h_{11}^{***} \\ \vdots \\ h_{p_1 1}^{***} \end{matrix} \middle| H_{12}(L) \right] \right\} \left(\frac{y_{1t}}{\hat{x}_t(k)} \right)$$

$$+ \left\{ \operatorname{adj} \left[\begin{matrix} h_{11}^{*} \\ \vdots \\ h_{p_1 1}^{*} \end{matrix} \middle| \; H_{11}^{*}(L) \right] F_{11}(L) \right\}^{***} e_{1t}$$

In this form the forecasts of $\hat{y}_t(k)$ in equations (2.17)' and (2.24)' will not only be in terms of the history and forecast profile of x_t, but will also depend on the past history and forecast profile of y_t itself. This is due to the determinant multiplying the vector of endogenous variables. Furthermore, the autoregressive part of all of the p_1 forecasts in $\hat{y}_t(k)$ will be identical.

The presence of these "lagged" forecasts of y_t will cause complications when we consider the variance of the forecast errors, since forecasts of $\hat{y}_t(1)$, $\hat{y}_t(2)$, ... , $\hat{y}_t(k-1)$ will be correlated with each other, with forecasts of $\hat{x}_t(1)$, $\hat{x}_t(2)$, ... , $\hat{x}_t(k)$, and with e_{1t}.

This is obvious in the examples, in which this convention is used.

CHAPTER III

THE PROBLEM OF SPECIFICATION ERROR

Introduction

In Chapter II we proposed a framework for studying and using leading indicators. We outlined a procedure for building transfer function models by setting up a structural model, and deriving the time series models directly from this explicit theoretical background.

In Chapters IV and V we will consider multivariate time series models which describe various empirical relationships between "established" leading indicators and economic activity. We will build the transfer functions empirically by following the procedures outlined in Box and Jenkins' Time Series Analysis.[1] This procedure is chosen over the framework formulated in Chapter II.

The transfer functions we examine in Chapter IV consist of economic activity (Industrial Production) as the output, and as a single input, the leading indicator under consideration. In light of Chapter II, it may be argued that there is an econometric problem of omitted variables in this approach. At the outset, our single-input transfer functions will appear to reflect a belief that the level of economic activity is adequately "explained" by the use of just one input. The framework in Chapter II shows that the number of inputs in the transfer function implied by any given econometric model will be equal to the number of exogenous variables plus the number of leading indicators in that model. Even the simplest structural econometric model will imply a transfer function with more than one input.

It is important to emphasize that our work is not done with the presumption that a single input is sufficient to explain the movements in economic activity. We follow the Box and Jenkins procedure because we are interested in the dynamic relationships which exist empirically between economic activity and each of the leading indicators under consideration.

Furthermore we argue in this chapter that the bias introduced in the parameter estimates of our single input transfer functions through the omission of variables, does <u>not</u> present a serious problem if the following conditions characterize the model being studied.

(i) The main objective of building the model is <u>forecasting</u>.

(ii) The variables in the underlying econometric model are drawn from a joint distribution which is <u>covariance stationary</u>.

<u>A Discussion of the Problem</u>

Suppose that the true model describing the world we are examining is

$$(3.1)\quad y_t = \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_K x_{Kt} + \varepsilon_t$$

or

$$(3.2) \quad Y = X\beta + \varepsilon$$

where Y is a Tx1 vector of observations on the endogenous variable, expressed as deviations from the mean; X is a TxK matrix of explanatory variables expressed as deviations from their respective means, which are drawn from a multivariate Normal distribution[2]; β is a Kx1 vector of true model parameters; and ε is a Tx1 vector of disturbances, with $\varepsilon \sim N[0, \sigma^2 I_T]$.

We use only x_{1t}. Thus

$$(3.3) \quad E(y_t | x_{1t}) = \beta_1 x_{1t} + \beta_2 E(x_{2t} | x_{1t}) + \ldots + \beta_K E(x_{Kt} | x_{1t})$$

Under our assumptions,

$$(3.4) \quad \begin{aligned} E(x_{2t} | x_{1t}) &= \frac{\sigma_{12}}{\sigma_1^2} x_{1t} = c_2 x_{1t} \\ E(x_{3t} | x_{1t}) &= \frac{\sigma_{13}}{\sigma_1^2} x_{1t} = c_3 x_{1t} \\ &\vdots \\ E(x_{Kt} | x_{1t}) &= \frac{\sigma_{1K}}{\sigma_1^2} x_{1t} = c_K x_{1t} \end{aligned}$$

where $\sigma_{1j} = \text{Cov}(x_{1t}, x_{jt})$ for $j = 2, 3, \ldots, K$.

and $\sigma_1^2 = \text{Var}(x_{1t})$.[3]

Note that the c_j are constants, for $j = 2, 3, \ldots, K$.

Thus

$$(3.5)\quad E(y_t|x_{1t}) = \beta_1 x_{1t} + \beta_2 c_2 x_{1t} + \dots + \beta_K c_K x_{1t}$$

$$= (\beta_1 + \beta_2 c_2 + \dots + \beta_K c_K)\ x_{1t}$$

$$= \beta^*\ x_{1t}$$

Hence we can write the model as

$$(3.6)\quad y_t = \beta^*\ x_{1t} + v_t \qquad \text{where } E(v_t|x_{1t}) = 0.$$

Ordinary Least Squares will yield a consistent estimate of β^*. Furthermore, given stationarity, v_t has other nice properties like homoskedasticity (since the diagonal of the covariance matrix of the conditional density given in footnote (3) does not depend on t).

It is obvious that bias is present since $\beta^* \neq \beta_1$ unless x_{1t} is uncorrelated with other x_{jt} (in which case the σ_{1j} would be zero for $j = 2,3, \dots ,K$). However, if condition (i) on page 2 characterizes our study, we do not care about the economic interpretation of β_1. In this case, we are concerned with <u>forecasting</u>, and β^* is appropriate for that. Indeed, $[\beta^*\ x_{1t}]$ is a much better forecast of y_t than $[\beta_1\ x_{1t}]$, since it is exactly $E(y_t|x_{1t})$.

The second condition of page 2 is important because it implies that the c_j; $j = 2, 3, \dots, K$, will remain stable over time. If all the variables in the underlying econometric are drawn from a joint distribution which is covariance stationary,[4] then the correlation between the

Thus

$$(3.5)\quad E(y_t|x_{1t}) = \beta_1 x_{1t} + \beta_2 c_2 x_{1t} + \dots + \beta_K c_K x_{1t}$$
$$= (\beta_1 + \beta_2 c_2 + \dots + \beta_K c_K)\, x_{1t}$$
$$= \beta^* x_{1t}$$

Hence we can write the model as

$$(3.6)\quad y_t = \beta^* x_{1t} + v_t \qquad \text{where } E(v_t|x_{1t}) = 0.$$

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The second condition of page 2 is important because it implies that the c_j; $j = 2, 3, \dots, K$, will remain stable over time. If all the variables in the underlying econometric are drawn from a joint distribution which is covariance stationary,[4] then the correlation between the

the single input included in our model and each variable which is omitted, will remain stable in the future. That is, the variances and covariances in the constants, c_j, will not vary over time. Hence, in $E(x_{jt}|x_{1t})$ $(j = 2, \ldots, K)$ we have incorporated the way that the x_{jt} $(j = 2, \ldots, K)$ change <u>on average</u> when x_{1t} changes. We still miss the information in $[x_{jt} - E(x_{jt}|x_{1t})]$, but given condition (ii), this is not a serious problem over time.

FOOTNOTES

CHAPTER III

[1]Box, George and Jenkins, Gwilym, Time Series Analysis, Holden Day, 1976.

[2]This assumption that the x_{it}; i = 1, ... ,K are drawn from a multivariate Normal distribution, is consistent with the notion expressed in Chapter II that the explanatory variables follow ARIMA processes. That is, each explanatory variable can be expressed as an infinite order Moving Average process about a white noise series which is assumed to be Normally distributed.

[3]Dhrymes, Phoebus J., Introductory Econometrics, Springer-Verlag, New York Inc., 1978, pp. 364-66.

Consider his Propositions 7 and 8, restated: Given the set of K random variables, X (Kxl), expressed as deviations from their means.

Let $X \sim N[0,\Sigma]$, or equivalently, $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \quad N\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \Sigma_{11} & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{pmatrix}\right]$

where X_1 is (1x1), X_2 is (K-1)x1, $\Sigma_{11} = \sigma_1{}^2$ is (1x1),

Σ_{22} is (K-1)x(K-1), and $\Sigma_{12} = \Sigma_{21}{}'$ is 1x(K-1).

Note that $\Sigma_{12} = \Sigma_{21}{}'$ is the vector of (K-1) covariances:

$\sigma_{1j} = \text{Cov}(x_{1t}, x_{jt})$; j = 2, 3, ... ,K.

Under these conditions,

$$X_1 \sim N[0, \Sigma_{11}]; \qquad X_2 \sim N[0, \Sigma_{22}];$$

and the conditional density of X_2 given X_1 is

$$N[\Sigma_{21}\Sigma_{11}{}^{-1}X_1, \ \Sigma_{22} - \Sigma_{21}\Sigma_{11}{}^{-1}\Sigma_{12}] .$$

The first moment of this conditional distribution is the set of K-1 conditional expectations:

$$E(x_{2t}|x_{1t}) = \frac{\sigma_{12}}{\sigma_1^2} x_{1t} = c_2 x_{1t}$$

$$E(x_{3t}|x_{1t}) = \frac{\sigma_{13}}{\sigma_1^2} x_{1t} = c_3 x_{1t}$$

$$\vdots$$

$$E(x_{Kt}|x_{1t}) = \frac{\sigma_{1K}}{\sigma_1^2} x_{1t} = c_K x_{1t} .$$

[4] If the joint distribution is Normal, then covariance stationarity implies strict stationarity.

APPENDIX 3

PROOF THAT ORDINARY LEAST SQUARES ESTIMATION YIELDS THE APPROPRIATE ESTIMATE

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Here we will show that Ordinary Least Squares estimation of the misspecified system yields a parameter estimate with an expected value identical to β^* in equations (3.5) and (3.6).

Let X and β be partitioned as follows.

$$\underset{T\times K}{X} = [\underset{T\times 1}{X_1} \ \underset{T\times(K-1)}{X_2}] \qquad \underset{K\times 1}{\beta} = \begin{pmatrix} \beta_1 \\ \overline{\beta}_2 \end{pmatrix} \begin{matrix} 1\times 1 \\ (K-1)\times 1 \end{matrix}$$

In this case, equation (3.2) becomes

$$\begin{aligned} Y &= X\beta + \varepsilon \\ &= [X_1 \ X_2] \begin{pmatrix} \beta_1 \\ \overline{\beta}_2 \end{pmatrix} + \varepsilon \\ &= X_1\beta_1 + X_2\overline{\beta}_2 + \varepsilon \end{aligned}$$

We estimate the following misspecified system.

$$Y = X_1\beta_1 + \varepsilon^* \qquad \text{where } \varepsilon^* = X_2\overline{\beta}_2 + \varepsilon$$

Ordinary Least Squares estimation yields the following.

$$\begin{aligned} \hat{\beta}_1 &= (X_1'X_1)^{-1}X_1'Y \\ &= (X_1'X_1)^{-1}X_1'[X_1\beta_1 + X_2\overline{\beta}_2 + \varepsilon] \end{aligned}$$

$$= \beta_1 + (X_1'X_1)^{-1}X_1'X_2\overline{\beta}_2 + (X_1'X_1)^{-1}X_1'\varepsilon$$

$$E[\hat{\beta}_1] = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\overline{\beta}_2$$

Note that $\sigma_1{}^2 = \frac{1}{T}\sum_{t=1}^{T} x_{1t}{}^2$

$$= \frac{1}{T}(X_1'X_1) .$$

Thus $(X_1'X_1)^{-1} = [T\sigma_1{}^2]^{-1} = \frac{1}{T\sigma_1^2}$.

Furthermore, $\sigma_{1j} = \mathrm{Cov}(x_{1t},x_{jt}) = \frac{1}{T}\sum_{t=1}^{T} x_{1t}x_{jt}$

$$= \frac{1}{T}[X_1'X_2]_{1j} .$$

Thus $X_1'X_2 = T[\sigma_{12}\ \sigma_{13}\ \ldots\ \sigma_{1K}]$.

Substituting these into the expected value of our least squares estimate, we get the following.

$$E[\hat{\beta}_1] = \beta_1 + (X_1'X_1)^{-1}X_1'X_2\overline{\beta}_2$$

$$= \beta_1 + \left(\frac{\sigma_{12}}{\sigma_1{}^2}\ \frac{\sigma_{13}}{\sigma_1{}^2}\ \ldots\ \frac{\sigma_{1K}}{\sigma_1{}^2}\right)[\overline{\beta}_2]$$

$$= \beta_1 + c_2\beta_2 + c_3\beta_3 + \ldots + c_K\beta_K$$

$$= \beta^*$$

CHAPTER IV

AN EVALUATION OF THE COMMERCE DEPARTMENT LEADING INDICATORS

Introduction

The huge amount of effort exerted toward developing the Commerce Department leading indicators has resulted in the current Composite Index of Leading Indicators (CLI). This index represents the fifth complete reworking of the indicator selection. The first was compiled by Burns and Mitchell in 1938, the second and third by G.H. Moore in 1956 and 1960, the fourth by Moore and Shiskin in 1967, and the fifth by the Commerce Department in 1975 (see references [17], [47], and [75].

The CLI is constructed from the following twelve series which have been evaluated as the "most useful" leading indicators, given the Commerce Department's scoring procedure.

Business Conditions Digest Series 1 (BCD1): Average Workweek of Production Workers, Manufacturing

BCD3: Layoff Rate, Manufacturing

BCD8: Value of Manufacturers' New Orders for Consumer Goods and Materials

BCD12: Index of Net Business Formation

BCD19: Index of Stock Prices

BCD20: Contracts and Orders for Plant and Equipment

BCD29: Index of Housing Starts

BCD32: Vendor Performance

BCD36: Change in Inventories on Hand and on Order

BCD92: Percent Change in Sensitive Prices (PPI of Crude Materials)

BCD104: Percentage Change in Total Liquid Assets (M_7)

BCD105: Real Money Supply, M_1

First the monthly data is standardized so that each series displays the same average absolute monthly change. This makes it possible to combine series with different units of measurement.

Next these standardized series are combined into a weighted average, with the weights reflecting the overall performance scores of each component series as a cyclical indicator. The score for a given series depends on its economic significance, statistical adequacy, cyclical timing, conformity to business cycles, smoothness, and currency (how quickly the data are available). The weight given to each series is the ratio of the performance score of that series to the average of the scores of all series in the CLI ([5], [33]).

Finally this weighted average is subjected to a "reverse trend adjustment." Here the trend of the weighted average is made to equal the trend of the Composite Index of Coincident Indicators (CCI). This trend can be viewed as a linear approximation to the secular movement in economic activity. The rationale for this reverse trend adjustment is as follows. Many of the twelve series listed above relate to changes or percent changes in output, prices, or monetary aggregates rather than levels, and thus display no significant trend. Hence the weighted average of leading indicators constructed in the first two steps displays many small declines which are not indicative of a coming drop in the CCI. Reverse trend adjustment is intended to overcome this difficulty ([33], [34], [37], [68]).

The series resulting from these three steps is the CLI.

Critique of the Commerce Department Approach to Leading Indicators

In Chapter II we developed a theoretical framework for leading indicators. It is important to examine whether the leading indicator approach of the Commerce Department outlined above, is appropriate in the context of our work in Chapter II.

Is there a theoretical framework justifying the above construction of the CLI?

This point was examined empirically as early as 1957 by John E. Maher [40] and again in 1973 by Saul Hymans [37]. They each ran a multiple regression of the CCI on the components of the CLI, to see if the resulting coefficients resembled the weights imposed on the twelve series in the Commerce Department construction. Their results showed three leading indicator series with insignificant coefficients, and five more with coefficients of the wrong sign.

In all fairness to the Commerce Department approach, it should be noted that this regression equation does not represent a behavioral relation. Hymans clearly states that it is "merely an exercise in curve-fitting, or - at best - some kind of pseudo reduced form equation." Hence, this exercise is not an appropriate test for the existence of a theoretical framework underlying the Commerce Department approach.

A further criticism which has been used in defense of the Commerce Department approach, is that such a regression equation examines the relationship between leading indicators and the measure of economic activity at all points in time. It is claimed that the Commerce Department is only concerned about the lead of an indicator just prior to true turning points in the economy, and does not worry about the interim periods ([30], [37]).

Hence, "the CLI is alleged to be constructed so as to maximize the use of the turning point information contained in the component leading indicator series. The statement that a turning point in economic activity is typically preceeded by a turning point in many of the component series of the CLI, is not meant to imply direct causality. If it did, one would attempt to estimate a behavioral or technological relation that could be expected to hold outside the sample, that would have directly interpretable coefficients, and so on. Rather, the statement implies something about the process through which those forces that do lead to turning points operate within the structure of the U.S. economy" ([37)].

If these statements constitute the best defense of the Commerce Department leading indicator approach, then it seems that there is no explicit, quantifiable theoretical framework underlying the approach. Instead, promoters of the approach rely on stories about the trade cycle which suggest reasons why many of the component leading indicator series might turn down before economic activity.

These arguments would <u>not</u> justify the Commerce Department approach in the light of our Chapter II. There must be an explicit theoretical framework, expressable in terms of a system of structural dynamic equations, for our approach to be applicable.

Furthermore, the common argument that these series should only be expected to consistently lead economic activity at true turning points, suggests a significant weakness in their value as forecasters. <u>If</u> we had an indicator or an index of indicators which was successful at predicting <u>true</u> turning points and of no predictive value at any other time, it would be useful. However, it is difficult to imagine relying on such an indicator to <u>predict</u> a downturn which has not yet occurred! It seems that at best, this indicator would be useful in an expost role, verifying that a downturn has already occurred. In fact, Julius Shiskin and G.H. Moore have admitted that the Commerce Department approach has only been useful in such an ex post role ([17], [24], [37], [48], [49], [50], [52], [65], [68]).

In any case the existing CLI has not met with great success, nor has any of its components alone, nor has any of the other eighteen series classified as leaders by the Commerce Department ([12], [17], [18], [21], [22], [24], [28], [29], [30], [31], [33], [34], [37], [39], [43], [59], [67], [72]). The leading indicators have displayed small average leads (only two of the thirty leading indicators have an average lead at true turns of more than five months [22]), and their lead times have varied greatly from cycle to cycle. In addition, they have often signalled false downturns. Understandably, the CLI has exhibited the same problems. Though it has not failed to signal a true downturn since World War II, it has displayed lead times which have varied greatly, and it has signalled many false downturns ([3], [22], [34], [37], [44], [70]). These problems make the Commerce Department leading indicator approach quite unreliable as a useful predictor of business cycle turns.

Empirical Evaluation

Given the above arguments in the ongoing debate concerning the Commerce Department leading indicators, a more extensive examination of the relationships between these indicators and the level of economic activity is in order.

Here we consider eight single-input transfer function models with the Industrial Production Index (I.P.) as output, and with one of the leading indicator series used in the CLI as input. This technique allows the data to inform us about the extent and the form of the relationship between each leading indicator and I.P., as well as the average lead involved in the relationship at each point in time.

The first step in this analysis is to identify and fit the univariate ARIMA model appropriate for each leading indicator. These models are used to prewhiten the time series in each transfer function. This prewhitening procedure transforms each input into a series of innovations which presumably contains the information relevant to that leading indicator that is not "explained" by previous observations of the indicator. This series of innovations is compared with the corresponding series of innovations created by applying the same prewhitening transformation to the output, I.P. The cross-correlation between these two series provides us with information describing the relationship between the input and the output, and hence, the form of the transfer function.

In working with this technique, data which is not seasonally adjusted is required on all the time series involved. The time series methods applied, effectively do theor own seasonal adjustment in building the transfer function models.

This has presented a problem in collecting our data on the leading indicators. Though the numbers are readily available in the Business Conditions Digest, all but two of the twelve series we need appear in seasonally adjusted form. The search for the numbers in unadjusted form has produced just eight of the twelve leading indicators. Consequently, our analysis centers on these eight leading indicators.

The univariate models for these eight leading indicators are reproduced in Table IV-1.

To examine the stability of these models, we split the sample of each model into two relatively equal subsamples, and re-estimate. In most cases the new parameter estimates remain within one standard deviation of the original estimate from the model with the entire sample. The new χ^2 statistics and Residual Standard Errors (RSE's) also generally remain close to the corresponding figures from the model with the entire sample. Furthermore, the sum of squared residuals (SSE) for the estimated models in each subsample, generally amount to approximately 50 percent of the SSE of the model with the entire sample. All these observations suggest a high degree of stability!

TABLE IV-1

UNIVARIATE MODELS FOR THE COMMERCE DEPARTMENT LEADING INDICATORS

[The number below each parameter estimate is its standard error.]

BCD1: Average Workweek

Sample: 1947 - September, 1979: n = 393.

$$(1-B^{12})(1-B)\log(z_t) = (1-\underset{(.05)}{.2118}B)(1-\underset{(.033)}{.82089}B^{12})a_t \tag{4.1}$$

$\chi^2_{28} = 34.7$ RSE = .0074

$$\bar{R}^2 = 1 - \frac{(RSE)^2}{\hat{\sigma}^2_{output}} = 1 - \frac{(.0074)^2}{(.010046)^2} = .4574$$

where $\hat{\sigma}_{output}$ is the standard error of $(1-B^{12})(1-B)\log(z_t)$.

BCD3: Layoff Rate

Sample: 1947 - August, 1979: n = 392.

$$(1-B^{12})(1-B)\log(z_t) = (1-\underset{(.037)}{.71096}B^{12})\,a_t \tag{4.2}$$

$\chi^2_{29} = 33.5$ RSE = .1674

$$\bar{R}^2 = 1 - \frac{(.1674)^2}{(.20236)^2} = .3157$$

TABLE IV-1 (cont'd.)

BCD8: Value of Manufacturers' New Orders

Sample: 1958 - May, 1979: n = 257

$$(1-B^{12})(1-B)\log(z_t) = (1-\underset{(.048)}{.69594}B^{12})a_t \tag{4.3}$$

$\chi^2_{29} = 26.3$ RSE = .0356

$$\bar{R}^2 = 1 - \frac{(.0356)^2}{(.042878)^2} = .3107$$

BCD19: Index of Stock Prices

Sample: 1947 - October, 1979: n = 394.

$$(1-\underset{(.051)}{.2186}B)(1-B^{12})(1-B)\log(z_t) = (1-\underset{(.030)}{.8216}B^{12})a_t \tag{4.4}$$

$\chi^2_{28} = 30.7$ RSE = .0353

$$\bar{R}^2 = 1 - \frac{(.0353)^2}{(.046007)^2} = .4113$$

BCD29: Index of Housing Starts

Sample: 1959 - August, 1979: n = 248.

$$(1-B^{12})(1-B)\log(z_t) = (1-\underset{(.046)}{.24912}B+\underset{(.047)}{.30478}B^2 + \underset{(.045)}{.19372}B^5 - \underset{(.048)}{.5952}B^{12})a_t \tag{4.5}$$

$\chi^2_{26} = 47.1$ RSE = .0895

$$\bar{R}^2 = 1 - \frac{(.0895)^2}{(.11213)^2} = .3629$$

TABLE IV-1 (cont'd.)

BCD32: Vendor Performance

Sample: 1947 - October, 1979: n = 394.

(4.6)
$$\underset{(.050)}{(1-.2992B} + \underset{(.050)}{.0692B^7} + \underset{(.048)}{.1559B^{14}})(1-B^{12})(1-B)\log(z_t)$$

$$= \underset{(.040)}{(1-.6640B^{12}})a_t$$

χ^2_{26} = 39.2 RSE = .125

$$\bar{R}^2 = 1 - \frac{(.125)^2}{(.17211)^2} = .4725$$

BCD92: Percent Change in PPI of Crude Materials

Sample: February, 1947 - August, 1979: n = 391.

(4.7)
$$\underset{(.049)}{(1-.3756B} - \underset{(.046)}{.2405B^3})(1-B^{12})(1-B)\ \log(z_t)$$

$$= \underset{(.056)}{(1-.1468B^2} - \underset{(.037)}{.7331B^{12}} + \underset{(.055)}{.099B^{14}})a_t$$

χ^2_{25} = 38.5 RSE = .0157

$$\bar{R}^2 = 1 - \frac{(.0157)^2}{(.021631)^2} = .4732$$

BCD105: Real Money Supply (M1)

Sample: 1947 - September, 1979: n = 393.

(4.8)
$$\underset{(.052)}{(1-.2035B)}(1-B^{12})(1-B)\ \log(z_t)$$

$$= (1 + \underset{(.043)}{.19058B^3} - \underset{(.044)}{.56297B^{12}})a_t$$

χ^2_{27} = 33.4 RSE = .0059

$$\bar{R}^2 = 1 - \frac{(.0059)^2}{(.00702)^2} = .2936$$

Two mild exceptions to the stability described above appear in the models for Housing Starts (BCD29) and Producer Prices (BCD92). Observe that these models are more complicated than the surprisingly simple models which describe the other leading indicators. The problem with both of these series is that the model parameter estimates vary somewhat more in the subsamples. Though some of the parameter estimates remain within one standard deviation of the estimate for the entire sample, others move outside this band of one standard deviation, and a few move outside the $(2\hat{\sigma})$ confidence band. However, both of these models have the redeeming qualities that the χ^2 statistics and the RSE's are quite robust, and the SSE's for the subsamples constitute close to 50% of the SSE for the entire sample. Furthermore, the <u>form</u> of each of these models remains appropriate in all subsamples considered.

It is not surprising that these two models are the least stable. Housing starts have been subject to the whims of Regulation Q enforcement; and the PPI since 1973 has been subject to some degree to the whims of OPEC pricing. In light of these facts, it is remarkable that these series behave as well as they do.

The extent of the stability of these eight models is fairly amazing, given their simplicity, the length of the sample period, and the volatility of many of the leading indicator series.

With these univariate models established, we can examine the cross-correlation function between the prewhitened input and the prewhitened output of each of our eight transfer functions. In so doing we obtain information about the form of the transfer functions under consideration. With this information, we proceed to the estimation stage and finish building our eight models, which are displayed as equations (4.9) through (4.16) in Table IV-2.

To examine the stability of these models, we split the sample of each into two relatively equal subsamples and re-estimate. The resulting parameter estimates appear directly beneath those for the entire sample, for each of our eight models in Table IV-2. Examination of the parameter estimates of these subsamples indicates that our models are quite stable. A study of the diagnostic checks for each model supports this finding.

We conclude that the models adequately represent the bivariate relationships between each of the eight leading indicators under consideration and Industrial Production.

We are interested in the impulse response function implied by each of our models. These eight functions are listed and plotted in Figures (4.1) through (4.8). We would like to compare these eight functions in order to make some evaluation about the strengths and weaknesses of each in the role of leading indicator. However since the inputs are not all measured in the same units, and since each input behaves differently (in particular, since each input has a different standard error), this comparison

TABLE IV-2

BIVARIATE MODELS FOR THE COMMERCE DEPARTMENT LEADING INDICATORS

y_t = BCD47 Index of Industrial Production

(i) x_t = BCD1: Average Workweek

Sample: January, 1947 - September, 1979 (n = 393)

$$(1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B}(1-B^{12})(1-B)\log(x_t) + (1-\theta_{12}B^{12})a_t \tag{4.9}$$

$\hat{\omega}_o = \underset{(.079)}{1.0260}$ $\hat{\delta}_1 = \underset{(.036)}{.6777}$ $\hat{\theta}_{12} = \underset{(.035)}{.7462}$

$\chi^2_{47} = 64.7$ RSE = .0117

$$\left[\begin{aligned} \bar{R}^2 &= 1 - \frac{(RSE)^2}{\hat{\sigma}^2_{output}} \\ &= 1 - \frac{(.0117)^2}{(.019846)^2} = .6524 \end{aligned}\right]$$

Sample: January, 1947 - April, 1963 (n = 196)

$\hat{\omega}_o = \underset{(.133)}{1.2599}$ $\hat{\delta}_1 = \underset{(.056)}{.6098}$ $\hat{\theta}_{12} = \underset{(.053)}{.7670}$

$\chi^2_{47} = 40.0$ RSE = .0136

$$\left[\bar{R}^2 = 1 - \frac{(.0136)^2}{(.019846)^2} = .5304\right]$$

TABLE IV-2 (cont'd.)

Sample: May, 1963 - September, 1979 (n = 197)

$\hat{\omega}_o = \underset{(.092)}{.7493}$ $\hat{\delta}_1 = \underset{(.048)}{.7499}$ $\hat{\theta}_{12} = \underset{(.055)}{.7012}$

$\chi^2_{47} = 57.8$ RSE = .0096

$$\left[\bar{R}^2 = 1 - \frac{(.0096)^2}{(.019846)^2} = .7660\right]$$

(ii) x_t = BCD3 Layoff Rate

Sample: January, 1947 - August, 1979 (n = 392)

$$(4.10)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o - \omega_1 B}{1-\delta_2 B^2}(1-B^{12})(1-B)\log(x_t)$$

$$+ (1-\theta_{12}B^{12})a_t$$

$\hat{\omega}_o = \underset{(.003)}{-.0344}$ $\hat{\omega}_1 = \underset{(.003)}{.0315}$ $\hat{\delta}_2 = \underset{(.045)}{.5927}$ $\hat{\theta}_{12} = \underset{(.036)}{.7565}$

$\chi^2_{47} = 56.4$ RSE = .0116

$\bar{R}^2 = .6584$

Sample: January, 1947 - April, 1963 (n = 196)

$\hat{\omega}_o = \underset{(.005)}{-.0381}$ $\hat{\omega}_1 = \underset{(.005)}{.0331}$ $\hat{\delta}_2 = \underset{(.065)}{.5569}$ $\hat{\theta}_{12} = \underset{(.053)}{.7649}$

$\chi^2_{47} = 48.5$ RSE = .0139

$\bar{R}^2 = .5094$

Sample: May, 1963 - August, 1979 (n = 196)

$\hat{\omega}_o = \underset{(.004)}{-.0299}$ $\hat{\omega}_1 = \underset{(.004)}{.0261}$ $\hat{\delta}_2 = \underset{(.062)}{.6597}$ $\hat{\theta}_{12} = \underset{(.058)}{.7096}$

$\chi^2_{47} = 42.9$ RSE = .0096

$\bar{R}^2 = .7660$

TABLE IV-2 (cont'd.)

(iii) x_t = BCD8 Value of Manufacturers' New Orders

Sample: January, 1958 - May, 1979 (n = 257)

$$(4.11)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o-\omega_1 B}{1-\delta_2 B^2}(1-B^{12})(1-B)\log(x_t) + (1-\theta_{12}B^{12})a_t$$

$\hat{\omega}_o = .2396$ (.019) $\hat{\omega}_1 = -.0865$ (.018) $\hat{\delta}_2 = .3471$ (.062) $\hat{\theta}_{12} = .6954$ (.049)

$\chi^2_{29} = 39.1$ RSE = .0103

$\bar{R}^2 = .7306$

Sample: January, 1958 - December, 1969 (n = 144)

$\hat{\omega}_o = .2306$ (.027) $\hat{\omega}_1 = -.0731$ (.026) $\hat{\delta}_2 = .3175$ (.102) $\hat{\theta}_{12} = .6494$ (.072)

$\chi^2_{29} = 42.2$ RSE = .0116

$\bar{R}^2 = .6584$

Sample: January, 1970 - May, 1979 (n = 113)

$\hat{\omega}_o = .2616$ (.026) $\hat{\omega}_1 = -.1024$ (.025) $\hat{\delta}_2 = .3852$ (.076) $\hat{\theta}_{12} = .7353$ (.079)

$\chi^2_{29} = 25.2$ RSE = .0091

$\bar{R}^2 = .7897$

(iv) x_t = BCD19 Index of Stock Prices

Sample: January, 1947 - October, 1979 (n = 394)

$$(4.12)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B}B^2(1-B^{12})(1-B)\log(x_t) + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B}a_t$$

$\hat{\omega}_o = .0674$ (.015) $\hat{\delta}_1 = .8155$ (.059) $\hat{\theta}_{12} = .7942$ (.032) $\hat{\phi}_1 = .2739$ (.051)

$\chi^2_{46} = 45.6$ RSE = .0132

$\bar{R}^2 = .4424$

TABLE IV-2 (cont'd.)

Sample: January, 1947 - May, 1963 (n = 197)

$\hat{\omega}_o = .0961$ (.031) $\hat{\delta}_1 = .6943$ (.141) $\hat{\theta}_{12} = .8407$ (.046) $\hat{\phi}_1 = .2931$ (.074)

$\chi^2_{46} = 35.8$ RSE = .0159

$\bar{R}^2 = .3581$

Sample: June, 1963 - October, 1979 (n = 197)

$\hat{\omega}_o = .0537$ (.012) $\hat{\delta}_1 = .8919$ (.036) $\hat{\theta}_{12} = .7834$ (.051) $\hat{\phi}_1 = .1831$ (.076)

$\chi^2_{46} = 38.6$ RSE = .0103

$\bar{R}^2 = .7306$

(v) x_t = BCD29 Index of Housing Starts

Sample: January, 1959 - August, 1979 (n = 248)

$$(4.13)\quad (1-B^{12})(1-B)\log(y_t) = \omega_o B^9(1-B^{12})(1-B)\log(x_t) + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = .0172$ (.007) $\hat{\theta}_{12} = .5908$ (.056) $\hat{\phi}_1 = .3797$ (.064)

$\chi^2_{46} = 37.2$ RSE = .0124

$\bar{R}^2 = .6096$

Sample: January, 1959 - April 1969 (n = 124)

$\hat{\omega}_o = .0278$ (.012) $\hat{\theta}_{12} = .4452$ (.094) $\hat{\phi}_1 = .2569$ (.100)

$\chi^2_{46} = 32.7$ RSE = .0134

$\bar{R}^2 = .5441$

TABLE IV-2 (cont'd.)

Sample: May, 1969 - August, 1979 (n = 124)

$\hat{\omega}_o = .0125$ (.096) $\quad \hat{\theta}_{12} = .6539$ (.082) $\quad \hat{\phi}_1 = .5031$ (.090)

$\chi^2_{46} = 39.0 \qquad RSE = .0121$

$\bar{R}^2 = .6283$

(vi) x_t = BCD32 Vendor Performance

Sample: January, 1947 - October, 1979 (n = 394)

$$(4.14) \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B} B^2(1-B^{12})(1-B)\log(x_t) + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = .0180$ (.004) $\quad \hat{\delta}_1 = .7651$ (.078) $\quad \hat{\theta}_{12} = .7811$ (.033) $\quad \hat{\phi}_1 = .2774$ (.052)

$\chi^2_{46} = 45.4 \qquad RSE = .0133$

$R^2 = .5509$

Sample: January, 1947 - May, 1963 (n = 197)

$\hat{\omega}_o = .0163$ (.005) $\quad \hat{\delta}_1 = .8625$ (.072) $\quad \hat{\theta}_{12} = .8061$ (.050) $\quad \hat{\phi}_1 = .2844$ (.074)

$\chi^2_{46} = 34.9 \qquad RSE = .0160$

$\bar{R}^2 = .3500$

Sample: June, 1963 - October, 1979 (n = 197)

$\hat{\omega}_o = .0279$ (.008) $\quad \hat{\delta}_1 = .4870$ (.176) $\quad \hat{\theta}_{12} = .7775$ (.051) $\quad \hat{\phi}_1 = .2138$ (.079)

$\chi^2_{46} = 43.2 \qquad RSE = .0106$

$\bar{R}^2 = .7147$

TABLE IV-2 (cont'd.)

(vii) x_t = BCD92 % Change in PPI of Crude Materials

Sample: January, 1947 - August, 1979 (n = 392)

$$(4.15)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B} B^{10}(1-B^{12})[x_t] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = -.1062$ (.033) $\hat{\delta}_1 = .7792$ (.083) $\hat{\theta}_{12} = .7747$ (.034) $\hat{\phi}_1 = .2886$ (.051)

$\chi^2_{46} = 37.7$ RSE = .0136

$\bar{R}^2 = .5304$

Sample: January, 1947 - April, 1963 (n = 196)

$\hat{\omega}_o = -.0700$ (.040) $\hat{\delta}_1 = .8596$ (.096) $\hat{\theta}_{12} = .8061$ (.052) $\hat{\phi}_1 = .3101$ (.074)

$\chi^2_{46} = 30.5$ RSE = .0168

$\bar{R}^2 = .2834$

Sample: May, 1963 - August, 1979 (n = 196)

$\hat{\omega}_o = -.1965$ (.049) $\hat{\delta}_1 = .6452$ (.105) $\hat{\theta}_{12} = .6689$ (.060) $\hat{\phi}_1 = .1595$ (.079)

$\chi^2_{46} = 44.7$ RSE = .0107

$\bar{R}^2 = .7093$

(viii) x_t = BCD105 Real Money Supply - M_1

Sample: January, 1947 - September, 1979 (n = 393)

$$(4.16)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B} B^5(1-B^{12})(1-B)\log(x_t) + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = .3267$ (.109) $\hat{\delta}_1 = .6511$ (.153) $\hat{\theta}_{12} = .7960$ (.033) $\hat{\phi}_1 = .3429$ (.050)

$\chi^2_{46} = 46.8$ RSE = .0136

$\bar{R}^2 = .5304$

TABLE IV-2 (cont'd.)

Sample: January 1947 - May, 1963 (n = 197)

$\hat{\omega}_o$ = .3455 (.177)	$\hat{\delta}_1$ = .5018 (.333)	$\hat{\theta}_{12}$ = .8455 (.048)	$\hat{\phi}_1$ = .3717 (.072)

χ^2_{46} = 36.8 RSE = .0164

$\bar{R}^2$ = .3171

Sample: June, 1963 - September, 1979 (n = 196)

$\hat{\omega}_o$ = .3378 (.143)	$\hat{\delta}_1$ = .7123 (.151)	$\hat{\theta}_{12}$ = .6881 (.059)	$\hat{\phi}_1$ = .2578 (.076)

χ^2_{46} = 35.5 RSE = .0111

$\bar{R}^2$ = .6872

FIGURE 4.1

GRAPH OF IMPULSE RESPONSE WEIGHTS $[v_k]$

x_t = BCD1 Average Workweek

```
           0.        .25       .5        .75       1.
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++       VALUES [v_k]
```

(k)		VALUES $[v_k]$
0	XXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX	.102605E+01
1	XXXXXXXXXXXXXXXXXXXXXXXXX	.695345E+00
2	XXXXXXXXXXXXXXXXXXX	.471231E+00
3	XXXXXXXXXXXXXX	.319351E+00
4	XXXXXXXXXX	.216422E+00
5	XXXXXXX	.146668E+00
6	XXXXX	.993961E-01
7	XXXX	.673602E-01
8	XXX	.456496E-01
9	XXX	.309365E-01
10	XX	.209655E-01
11	XX	.142082E-01
12	X	.962880E-02
13	X	.652538E-02
14	X	.442222E-02
15	X	.299691E-02
16	X	.203099E-02
17	X	.137639E-02
18	X	.932771E-03
19	X	.632134E-03
20	X	.428393E-03
21	X	.290320E-03
22	X	.196748E-03
23	X	.133335E-03
24	X	.903604E-04
25	X	.612367E-04
26	X	.414998E-04
27	X	.281242E-04
28	X	.190596E-04
29	X	.129166E-04
30	X	.875348E-05
31	X	.593215E-05
32	X	.402021E-05
33	X	.272447E-05
34	X	.184636E-05
35	X	.125127E-05
36	X	.847977E-06
37	X	.574669E-06
38	X	.389450E-06
39	X	.263928E-06
40	X	.178862E-06
41	X	.121214E-06
42	X	.821461E-07
43	X	.556699E-07
44	X	.377272E-07
45	X	.255675E-07
46	X	.173269E-07
47	X	.1174[illegible]
48	X	.795[illegible]

FIGURE 4.2

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD3 Layoff Rate

-.5 -.25 0. .25

(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES[v_k]

(k)		VALUES[v_k]
0	XXX	-.343560E-01
1	XXX	-.315098E-01
2	XX	-.203628E-01
3	XX	-.186759E-01
4	X	-.120691E-01
5	X	-.110692E-01
6	X	-.715335E-02
7	X	-.656075E-02
8	X	-.423980E-02
9	X	-.388856E-02
10	X	-.251294E-02
11	X	-.230476E-02
12	X	-.148942E-02
13	X	-.136603E-02
14	X	-.882781E-03
15	X	-.809649E-03
16	X	-.523226E-03
17	X	-.479880E-03
18	X	-.310116E-03
19	X	-.284425E-03
20	X	-.183806E-03
21	X	-.168579E-03
22	X	-.108942E-03
23	X	-.999172E-04
24	X	-.645702E-04
25	X	-.592210E-04
26	X	-.382709E-04
27	X	-.351004E-04
28	X	-.226832E-04
29	X	-.208040E-04
30	X	-.134444E-04
31	X	-.123306E-04
32	X	-.796849E-05
33	X	-.730835E-05
34	X	-.472293E-05
35	X	-.433167E-05
36	X	-.279929E-05
37	X	-.256739E-05
38	X	-.165914E-05
39	X	-.152169E-05
40	X	-.983375E-06
41	X	-.901909E-06
42	X	-.582848E-06
43	X	-.534563E-06
44	X	-.345455E-06
45	X	-.316836E-06
46	X	-.204751E-06
47	X	-.187789E-06
48	X	-.121356E-06

FIGURE 4.3

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD8 Value of Manufacturers' New Orders

```
            -.25      0.         .25       .5
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++VALUES [vk]
 0                    XXXXXXXXXX                   .239582E+00
 1                    XXXXX                        .864858E-01
 2                    XXXXX                        .831554E-01
 3                    XXX                          .300180E-01
 4                    XXX                          .288621E-01
 5                    X                            .104188E-01
 6                    X                            .100176E-01
 7                    X                            .361623E-02
 8                    X                            .347697E-02
 9                    X                            .125514E-02
10                    X                            .120681E-02
11                    X                            .435642E-03
12                    X                            .418866E-03
13                    X                            .151205E-03
14                    X                            .145383E-03
15                    X                            .524812E-04
16                    X                            .504602E-04
17                    X                            .182155E-04
18                    X                            .175140E-04
19                    X                            .632233E-05
20                    X                            .607877E-05
21                    X                            .219439E-05
22                    X                            .210989E-05
23                    X                            .761643E-06
24                    X                            .732313E-06
25                    X                            .264355E-06
26                    X                            .254176E-06
27                    X                            .917540E-07
28                    X                            .882208E-07
29                    X                            .318465E-07
30                    X                            .306202E-07
31                    X                            .110535E-07
32                    X                            .106278E-07
33                    X                            .383650E-08
34                    X                            .368876E-08
35                    X                            .133159E-08
36                    X                            .128031E-08
37                    X                            .462175E-09
38                    X                            .444377E-09
39                    X                            .160414E-09
40                    X                            .154237E-09
41                    X                            .556773E-10
42                    X                            .535332E-10
43                    X                            .193248E-10
44                    X                            .185806E-10
45                    X                            .670734E-11
46                    X                            .644904E-11
47                    X                            .232802E-11
48                    X                            .223837E-11
```

FIGURE 4.4

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD19 Index of Stock Prices

```
                -.25      0.        .25       .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++VALUES [vk]
 0                    X                            0.
 1                    X                            0.
 2                    XXXXX                        .673801E-01
 3                    XXXX                         .549504E-01
 4                    XXXX                         .448136E-01
 5                    XXX                          .365468E-01
 6                    XXX                          .298049E-01
 7                    XX                           .243068E-01
 8                    XX                           .198229E-01
 9                    XX                           .161661E-01
10                    X                            .131839E-01
11                    X                            .107519E-01
12                    X                            .876844E-02
13                    X                            .715091E-02
14                    X                            .583177E-02
15                    X                            .475598E-02
16                    X                            .387863E-02
17                    X                            .316313E-02
18                    X                            .257963E-02
19                    X                            .210376E-02
20                    X                            .171567E-02
21                    X                            .139918E-02
22                    X                            .114107E-02
23                    X                            .930576E-03
24                    X                            .758911E-03
25                    X                            .618914E-03
26                    X                            .504742E-03
27                    X                            .411631E-03
28                    X                            .335697E-03
29                    X                            .273770E-03
30                    X                            .223267E-03
31                    X                            .182081E-03
32                    X                            .148492E-03
33                    X                            .121100E-03
34                    X                            .987601E-04
35                    X                            .805417E-04
36                    X                            .656840E-04
37                    X                            .535672E-04
38                    X                            .436856E-04
39                    X                            .356268E-04
40                    X                            .290547E-04
41                    X                            .236949E-04
42                    X                            .193239E-04
43                    X                            .157592E-04
44                    X                            .128520E-04
45                    X                            .104812E-04
46                    X                            .854772E-05
47                    X                            .697091E-05
48                    X                            .568497E-05
```

FIGURE 4.5

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD29 Index of Housing Starts

```
           -.25        0.        .25       .5
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++VALUES[v_k]
  0                    X                             0.
  1                    X                             0.
  2                    X                             0.
  3                    X                             0.
  4                    X                             0.
  5                    X                             0.
  6                    X                             0.
  7                    X                             0.
  8                    X                             0.
  9                    XX                             .172268E-01
 10                    X                             0.
 11                    X                             0.
 12                    X                             0.
 13                    X                             0.
 14                    X                             0.
 15                    X                             0.
 16                    X                             0.
 17                    X                             0.
 18                    X                             0.
 19                    X                             0.
 20                    X                             0.
 21                    X                             0.
 22                    X                             0.
 23                    X                             0.
 24                    X                             0.
 25                    X                             0.
 26                    X                             0.
 27                    X                             0.
 28                    X                             0.
 29                    X                             0.
 30                    X                             0.
 31                    X                             0.
 32                    X                             0.
 33                    X                             0.
 34                    X                             0.
 35                    X                             0.
 36                    X                             0.
 37                    X                             0.
 38                    X                             0.
 39                    X                             0.
 40                    X                             0.
 41                    X                             0.
 42                    X                             0.
 43                    X                             0.
 44                    X                             0.
 45                    X                             0.
 46                    X                             0.
 47                    X                             0.
 48                    X                             0.
```

FIGURE 4.6

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD32 Vendor Performance

```
                 -.25         0.           .25          .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES[v_k]

  0                           X                     0.
  1                           X                     0.
  2                           XX                     .180084E-01
  3                           X                      .137775E-01
  4                           X                      .105405E-01
  5                           X                      .806408E-02
  6                           X                      .616947E-02
  7                           X                      .471998E-02
  8                           X                      .361105E-02
  9                           X                      .276265E-02
 10                           X                      .211358E-02
 11                           X                      .161701E-02
 12                           X                      .123710E-02
 13                           X                      .946459E-03
 14                           X                      .724087E-03
 15                           X                      .553967E-03
 16                           X                      .423815E-03
 17                           X                      .324242E-03
 18                           X                      .248063E-03
 19                           X                      .189782E-03
 20                           X                      .145194E-03
 21                           X                      .111081E-03
 22                           X                      .849833E-04
 23                           X                      .650169E-04
 24                           X                      .497415E-04
 25                           X                      .380550E-04
 26                           X                      .291142E-04
 27                           X                      .222740E-04
 28                           X                      .170408E-04
 29                           X                      .130372E-04
 30                           X                      .997416E-05
 31                           X                      .763079E-05
 32                           X                      .583797E-05
 33                           X                      .446637E-05
 34                           X                      .341702E-05
 35                           X                      .261421E-05
 36                           X                      .200002E-05
 37                           X                      .153012E-05
 38                           X                      .117063E-05
 39                           X                      .895596E-06
 40                           X                      .685181E-06
 41                           X                      .524201E-06
 42                           X                      .401043E-06
 43                           X                      .306820E-06
 44                           X                      .234734E-06
 45                           X                      .179585E-06
 46                           X                      .137392E-06
 47                           X                      .105113E-06
 48                           X                      .804170E-07
```

FIGURE 4.7

GRAPH OF IMPULSE RESPONSE WEIGHTS [v_k]

x_t = BCD92 % Change in PPI of Crude Materials

```
           -.5       -.25      0.        .25
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES[v_k]
  0                             X                 0.
  1                             X                 0.
  2                             X                 0.
  3                             X                 0.
  4                             X                 0.
  5                             X                 0.
  6                             X                 0.
  7                             X                 0.
  8                             X                 0.
  9                             X                 0.
 10                         XXXXX                 -.106213E+00
 11                          XXXX                 -.827565E-01
 12                          XXXX                 -.644800E-01
 13                           XXX                 -.502397E-01
 14                           XXX                 -.391444E-01
 15                            XX                 -.304995E-01
 16                            XX                 -.237638E-01
 17                            XX                 -.185156E-01
 18                             X                 -.144265E-01
 19                             X                 -.112405E-01
 20                             X                 -.875803E-02
 21                             X                 -.682384E-02
 22                             X                 -.531682E-02
 23                             X                 -.414262E-02
 24                             X                 -.322773E-02
 25                             X                 -.251490E-02
 26                             X                 -.195949E-02
 27                             X                 -.152674E-02
 28                             X                 -.118956E-02
 29                             X                 -.926853E-03
 30                             X                 -.722160E-03
 31                             X                 -.562673E-03
 32                             X                 -.438409E-03
 33                             X                 -.341587E-03
 34                             X                 -.266149E-03
 35                             X                 -.207371E-03
 36                             X                 -.161573E-03
 37                             X                 -.125890E-03
 38                             X                 -.980879E-04
 39                             X                 -.764254E-04
 40                             X                 -.595471E-04
 41                             X                 -.463963E-04
 42                             X                 -.361498E-04
 43                             X                 -.281662E-04
 44                             X                 -.219458E-04
 45                             X                 -.170991E-04
 46                             X                 -.133228E-04
 47                             X                 -.103805E-04
 48                             X                 -.808802E-05
```

FIGURE 4.8

GRAPH OF IMPULSE RESPONSE WEIGHTS $[v_k]$

x_t = BCD105 Real Money Supply - M_1

```
                -.25       0.         .25        .5
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES[v_k]
  0                    X                                  0.
  1                    X                                  0.
  2                    X                                  0.
  3                    X                                  0.
  4                    X                                  0.
  5                    XXXXXXXXXXXXXX                     .326655E+00
  6                    XXXXXXXXX                          .212692E+00
  7                    XXXXXX                             .138488E+00
  8                    XXXXX                              .901725E-01
  9                    XXXX                               .587132E-01
 10                    XXX                                .382294E-01
 11                    XX                                 .248920E-01
 12                    XX                                 .162077E-01
 13                    X                                  .105532E-01
 14                    X                                  .687138E-02
 15                    X                                  .447410E-02
 16                    X                                  .291318E-02
 17                    X                                  .189683E-02
 18                    X                                  .123507E-02
 19                    X                                  .804177E-03
 20                    X                                  .523616E-03
 21                    X                                  .340937E-03
 22                    X                                  .221991E-03
 23                    X                                  .144543E-03
 24                    X                                  .941151E-04
 25                    X                                  .612803E-04
 26                    X                                  .399009E-04
 27                    X                                  .259803E-04
 28                    X                                  .169163E-04
 29                    X                                  .110146E-04
 30                    X                                  .717181E-05
 31                    X                                  .466971E-05
 32                    X                                  .304055E-05
 33                    X                                  .197976E-05
 34                    X                                  .128907E-05
 35                    X                                  .839337E-06
 36                    X                                  .546510E-06
 37                    X                                  .355844E-06
 38                    X                                  .231697E-06
 39                    X                                  .150863E-06
 40                    X                                  .982301E-07
 41                    X                                  .639596E-07
 42                    X                                  .416455E-07
 43                    X                                  .271162E-07
 44                    X                                  .176559E-07
 45                    X                                  .114961E-07
 46                    X                                  .748538E-08
 47                    X                                  .487389E-08
 48                    X                                  .317349E-08
```

cannot be made with the functions listed in Figures (4.1) through (4.8). We need to transform the impulse response weights in each function into beta coefficients in order to make this comparison.

Comparison With Beta Coefficients

To illuminate this requirement, consider an illustration. The impulse response weights in these figures are analagous to regression coefficients, such as "$\hat{b}$" in the following model.

Suppose the true model is

$$Y = a + bX + \varepsilon$$

Then suppose the line minimizing the sum of squared errors (the Ordinary Least Squares regression line) is

$$\hat{Y} = \hat{a} + \hat{b}X$$

Then $\quad Y = \hat{a} + \hat{b}X + \hat{e}$

where $\hat{e}$ represents the residuals of the regression.

The regressed line will pass through the point, $(\overline{X},\overline{Y})$.

i.e. $\quad \overline{Y} = \hat{a} + \hat{b}\overline{X}$

From this it follows that

$$Y - \overline{Y} = \hat{b}(X - \overline{X}) + \hat{e}$$

or

$$\hat{\sigma}_y\left(\frac{Y - \overline{Y}}{\hat{\sigma}_y}\right) = \hat{b}\hat{\sigma}_x\left(\frac{X - \overline{X}}{\hat{\sigma}_x}\right) + \hat{e}$$

or

$$\left(\frac{Y - \overline{Y}}{\hat{\sigma}_y}\right) = \hat{b}\,\frac{\hat{\sigma}_x}{\hat{\sigma}_y}\left(\frac{X - \overline{X}}{\hat{\sigma}_x}\right) + \frac{\hat{e}}{\hat{\sigma}_y}$$

In this last equation the input and the output are standardized. $\left(\hat{b}\,\frac{\hat{\sigma}_x}{\hat{\sigma}_y}\right)$ is the beta coefficient, which measures the relationship between the standardized input and the standardized output. Observe that it is obtained by multiplying the regression coefficient, $\hat{b}$, by the ratio of the standard deviation of the input to the standard deviation of the output. It can readily be compared with the beta coefficient of any other similar regression in which the input and output are standardized.

We can transform the impulse response weights in each of our functions listed in Figures 4.1 through 4.8 into beta coefficients, by simply multiplying each coefficient in each impulse response function by the ratio, $\frac{\hat{\sigma}_x}{\hat{\sigma}_y}$, relevant to that particular model. This is done in Figures 4.9 through 4.16. In these eight figures the magnitude of the relationship between each leading indicator and Industrial Production can be readily compared.

Implications

Upon examining these figures, we are immediately struck by a distinct fault which characterizes five of the relationships: the lack of any substantial lead. These series are hailed by the Commerce Department as leaders, which suggests that current movements in each indicator should be consistently followed by movements in economic activity after some lag. They are developed with the expressed purpose of providing information about future

FIGURE 4.9

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD1 Average Workweek

The figures are obtained by multiplying the 49 weights in Figure 4.1 by $\frac{.010046}{.019846}$.

The standard error of the series, $(1-B^{12})(1-B)\log(x_t)$, is .010046.

The standard error of the series, $(1-B^{12})(1-B)\log(y_t)$, is .019846.

```
         - .25      0.         .25        .5
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 0                  XXXXXXXXXXXXXXXXXXXXXX            .519384E+00
 1                  XXXXXXXXXXXXXXX                   .351982E+00
 2                  XXXXXXXXXX                        .238536E+00
 3                  XXXXXXX                           .161655E+00
 4                  XXXXX                             .109522E+00
 5                  XXX                               .742430E-01
 6                  XXX                               .503141E-01
 7                  XX                                .340976E-01
 8                  XX                                .231077E-01
 9                  XX                                .156600E-01
10                  X                                 .106127E-01
11                  X                                 .712916E-02
12                  X                                 .487408E-02
13                  X                                 .330263E-02
14                  X                                 .223852E-02
15                  X                                 .151703E-02
16                  X                                 .102808E-02
17                  X                                 .696725E-03
18                  X                                 .472167E-03
19                  X                                 .319985E-03
20                  X                                 .216852E-03
21                  X                                 .146959E-03
22                  X                                 .995934E-04
23                  X                                 .674939E-04
24                  X                                 .457402E-04
25                  X                                 .309979E-04
26                  X                                 .210071E-04
27                  X                                 .142364E-04
28                  X                                 .964793E-05
29                  X                                 .653835E-05
30                  X                                 .443099E-05
31                  X                                 .300284E-05
32                  X                                 .203502E-05
33                  X                                 .137912E-05
```

FIGURE 4.9 (cont'd.)

```
             -.25      0.        .25       .5
(k) +++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 34                     X                              .934623E-06
 35                     X                              .633390E-06
 36                     X                              .429244E-06
 37                     X                              .290896E-06
 38                     X                              .197139E-06
 39                     X                              .133600E-06
 40                     X                              .905395E-07
 41                     X                              .613583E-07
 42                     X                              .415822E-07
 43                     X                              .281800E-07
 44                     X                              .190974E-07
 45                     X                              .129422E-07
 46                     X                              .877084E-08
 47                     X                              .594398E-08
 48                     X                              .402819E-08
```

FIGURE 4.10

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD3 Layoff Rate

The figures are obtained by multiplying the 49 weights in Figure 4.2 by $\frac{.20236}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .20236, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
              -.5       -.25       0.        .25
(k)+++++++++.+++++++++.+++++++++.+++++++++.++++++++++ VALUES
 0                XXXXXXXXXXXXXXX                    -.350311E+00
 1                 XXXXXXXXXXXXXX                    -.321290E+00
 2                      XXXXXXXXX                    -.207630E+00
 3                       XXXXXXXX                    -.190429E+00
 4                          XXXXX                    -.123063E+00
 5                          XXXXX                    -.112867E+00
 6                           XXXX                    -.729392E-01
 7                           XXXX                    -.668968E-01
 8                            XXX                    -.432312E-01
 9                            XXX                    -.396498E-01
10                             XX                    -.256232E-01
11                             XX                    -.235005E-01
12                             XX                    -.151869E-01
13                              X                    -.139287E-01
14                              X                    -.900129E-02
15                              X                    -.825560E-02
16                              X                    -.533508E-02
17                              X                    -.489310E-02
18                              X                    -.316210E-02
19                              X                    -.290014E-02
20                              X                    -.187418E-02
21                              X                    -.171892E-02
22                              X                    -.111083E-02
23                              X                    -.101881E-02
24                              X                    -.658391E-03
25                              X                    -.603848E-03
26                              X                    -.390230E-03
27                              X                    -.357902E-03
28                              X                    -.231290E-03
29                              X                    -.212128E-03
30                              X                    -.137086E-03
31                              X                    -.125729E-03
32                              X                    -.812508E-04
33                              X                    -.745197E-04
34                              X                    -.481574E-04
35                              X                    -.441679E-04
36                              X                    -.285430E-04
37                              X                    -.261784E-04
38                              X                    -.169174E-04
```

FIGURE 4.10 (cont'd.)

```
               -.5                 0.
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 39                                  X                -.155159E-04
 40                                  X                -.100270E-04
 41                                  X                -.919633E-05
 42                                  X                -.594302E-05
 43                                  X                -.545068E-05
 44                                  X                -.352244E-05
 45                                  X                -.323062E-05
 46                                  X                -.208775E-05
 47                                  X                -.191479E-05
 48                                  X                -.123741E-05
```

FIGURE 4.10 (cont'd.)

```
             -.5                 0.
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 39                               X                -.155159E-04
 40                               X                -.100270E-04
 41                               X                -.919633E-05
 42                               X                -.594302E-05
 43                               X                -.545068E-05
 44                               X                -.352244E-05
 45                               X                -.323062E-05
 46                               X                -.208775E-05
 47                               X                -.191479E-05
 48                               X                -.123741E-05
```

FIGURE 4.11

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD8 Value of Manufacturers' New Orders

These figures are obtained by multiplying the 49 weights in Figure 4.3 by $\frac{.042878}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .042878, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
           -.25       0.         .25         .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 0                    XXXXXXXXXXXXXXXXXXXXXX        .517626E+00
 1                    XXXXXXXX                      .186856E+00
 2                    XXXXXXXX                      .179660E+00
 3                    XXXX                          .648550E-01
 4                    XXX                           .623576E-01
 5                    XX                            .225102E-01
 6                    XX                            .216434E-01
 7                    X                             .781300E-02
 8                    X                             .751212E-02
 9                    X                             .271178E-02
10                    X                             .260736E-02
11                    X                             .941220E-03
12                    X                             .904975E-03
13                    X                             .326684E-03
14                    X                             .314105E-03
15                    X                             .113388E-03
16                    X                             .109021E-03
17                    X                             .393552E-04
18                    X                             .378396E-04
19                    X                             .136596E-04
20                    X                             .131334E-04
21                    X                             .474106E-05
22                    X                             .455849E-05
23                    X                             .164556E-05
24                    X                             .158219E-05
25                    X                             .571149E-06
26                    X                             .549156E-06
27                    X                             .198238E-06
28                    X                             .190604E-06
29                    X                             .688055E-07
30                    X                             .661560E-07
31                    X                             .238815E-07
32                    X                             .229617E-07
33                    X                             .828890E-08
34                    X                             .796970E-08
35                    X                             .287695E-08
36                    X                             .276616E-08
37                    X                             .998546E-09
```

FIGURE 4.11 (cont'd.)

```
              -.25       0.         .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 38                     X                               .960093E-09
 39                     X                               .346580E-09
 40                     X                               .333235E-09
 41                     X                               .120293E-09
 42                     X                               .115660E-09
 43                     X                               .417519E-10
 44                     X                               .401441E-10
 45                     X                               .144915E-10
 46                     X                               .139334E-10
 47                     X                               .502977E-11
 48                     X                               .483608E-11
```

FIGURE 4.12

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD19 Index of Stock Prices

These figures are obtained by multiplying the 49 weights in Figure 4.4 by $\frac{.046007}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .046007, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
             -.25      0.        .25       .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES
```

(k)		VALUES
0	X	0.
1	X	0.
2	XXXXXXX	.156201E+00
3	XXXXXX	.127386E+00
4	XXXXX	.103887E+00
5	XXXX	.847228E-01
6	XXXX	.690937E-01
7	XXX	.563480E-01
8	XXX	.459534E-01
9	XX	.347672E-01
10	XX	.305629E-01
11	XX	.249251E-01
12	XX	.203269E-01
13	XX	.165772E-01
14	XX	.135192E-01
15	X	.110253E-01
16	X	.899144E-02
17	X	.733277E-02
18	X	.598010E-02
19	X	.487694E-02
20	X	.397727E-02
21	X	.324358E-02
22	X	.264523E-02
23	X	.215726E-02
24	X	.175931E-02
25	X	.143477E-02
26	X	.117009E-02
27	X	.954243E-03
28	X	.778213E-03
29	X	.634654E-03
30	X	.517578E-03
31	X	.422100E-03
32	X	.344234E-03
33	X	.280734E-03
34	X	.228946E-03
35	X	.186712E-03
36	X	.152269E-03
37	X	.124179E-03

FIGURE 4.12 (cont'd.)

```
          -.25       0.        .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

38                    X                           .101272E-03
39                    X                           .825901E-04
40                    X                           .673546E-04
41                    X                           .549295E-04
42                    X                           .447967E-04
43                    X                           .365330E-04
44                    X                           .297935E-04
45                    X                           .242975E-04
46                    X                           .198153E-04
47                    X                           .161600E-04
48                    X                           .131789E-04
```

FIGURE 4.13

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD29 Index of Housing Starts

These figures are obtained by multiplying the 49 weights in Figure 4.5 by $\frac{.11213}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .11213, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
          -.25       0.         .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 0                    X                            0.
 1                    X                            0.
 2                    X                            0.
 3                    X                            0.
 4                    X                            0.
 5                    X                            0.
 6                    X                            0.
 7                    X                            0.
 8                    X                            0.
 9                    XXXXX                         .973315E-01
10                    X                            0.
11                    X                            0.
12                    X                            0.
13                    X                            0.
14                    X                            0.
15                    X                            0.
16                    X                            0.
17                    X                            0.
18                    X                            0.
19                    X                            0.
20                    X                            0.
21                    X                            0.
22                    X                            0.
23                    X                            0.
24                    X                            0.
25                    X                            0.
26                    X                            0.
27                    X                            0.
28                    X                            0.
29                    X                            0.
30                    X                            0.
31                    X                            0.
32                    X                            0.
33                    X                            0.
34                    X                            0.
35                    X                            0.
36                    X                            0.
37                    X                            0.
```

FIGURE 4.13 (cont'd.)

```
            -.25      0.         .25       .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

38                    X                          0.
39                    X                          0.
40                    X                          0.
41                    X                          0.
42                    X                          0.
43                    X                          0.
44                    X                          0.
45                    X                          0.
46                    X                          0.
47                    X                          0.
48                    X                          0.
```

FIGURE 4.14

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD32 Vendor Performance

These figures are obtained by multiplying the 49 weights in Figure 4.6 by $\frac{.17211}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .17211, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
          -.25       0.        .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 0                   X                          0.
 1                   X                          0.
 2                   XXXXXXX                     .156174E+00
 3                   XXXXXX                      .119482E+00
 4                   XXXXX                       .914101E-01
 5                   XXXX                        .699339E-01
 6                   XXX                         .535033E-01
 7                   XXX                         .409330E-01
 8                   XX                          .313160E-01
 9                   XX                          .239585E-01
10                   XX                          .183296E-01
11                   XX                          .140232E-01
12                   X                           .107285E-01
13                   X                           .820759E-02
14                   X                           .627948E-02
15                   X                           .480416E-02
16                   X                           .367544E-02
17                   X                           .281192E-02
18                   X                           .215127E-02
19                   X                           .164584E-02
20                   X                           .125916E-02
21                   X                           .963325E-03
22                   X                           .736999E-03
23                   X                           .563845E-03
24                   X                           .431312E-03
25                   X                           .330023E-03
26                   X                           .252486E-03
27                   X                           .193166E-03
28                   X                           .147783E-03
29                   X                           .113062E-03
30                   X                           .864987E-04
31                   X                           .661763E-04
32                   X                           .506285E-04
33                   X                           .387336E-04
34                   X                           .296333E-04
35                   X                           .226712E-04
36                   X                           .173447E-04
37                   X                           .132696E-04
```

FIGURE 4.14 (cont'd)

```
          -.25       0.         .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

38                    X                                .101520E-04
39                    X                                .776686E-05
40                    X                                .594208E-05
41                    X                                .454602E-05
42                    X                                .347796E-05
43                    X                                .266083E-05
44                    X                                .203568E-05
45                    X                                .155741E-05
46                    X                                .119050E-05
47                    X                                .911569E-06
48                    X                                .693398E-06
```

FIGURE 4.15

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta coefficients]

x_t = BCD92 % Change in PPI of Crude Materials

These figures are obtained by multiplying the 49 weights in Figure 4.7 by $\frac{.021631}{.019846}$.

The standard error of $(1-B^{12})[x_t]$ is .021631, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846 .

```
          -.5      -.25       0.        .25
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

  0                             X                   0.
  1                             X                   0.
  2                             X                   0.
  3                             X                   0.
  4                             X                   0.
  5                             X                   0.
  6                             X                   0.
  7                             X                   0.
  8                             X                   0.
  9                             X                   0.
 10                        XXXXXX                   -.115766E+00
 11                         XXXXX                   -.901998E-01
 12                          XXXX                   -.702795E-01
 13                           XXX                   -.547584E-01
 14                           XXX                   -.426651E-01
 15                            XX                   -.332427E-01
 16                            XX                   -.259012E-01
 17                            XX                   -.201809E-01
 18                            XX                   -.157241E-01
 19                             X                   -.122515E-01
 20                             X                   -.945575E-02
 21                             X                   -.743759E-02
 22                             X                   -.579503E-02
 23                             X                   -.451522E-02
 24                             X                   -.351804E-02
 25                             X                   -.274110E-02
 26                             X                   -.213573E-02
 27                             X                   -.166406E-02
 28                             X                   -.129655E-02
 29                             X                   -.101022E-02
 30                             X                   -.787113E-03
 31                             X                   -.613281E-03
 32                             X                   -.477841E-03
 33                             X                   -.372310E-03
 34                             X                   -.290087E-03
 35                             X                   -.226022E-03
 36                             X                   -.176105E-03
 37                             X                   -.137213E-03
```

FIGURE 4.15 (cont'd)

```
             -.5       -.25       0.         .25
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES
38                              X                     -.106910E-03
39                              X                     -.832993E-04
40                              X                     -.649029E-04
41                              X                     -.505693E-04
42                              X                     -.394012E-04
43                              X                     -.306995E-04
44                              X                     -.239197E-04
45                              X                     -.186370E-04
46                              X                     -.145211E-04
47                              X                     -.113141E-04
48                              X                     -.881548E-05
```

FIGURE 4.16

GRAPH OF IMPULSE RESPONSE WEIGHTS [Beta Coefficients]

x_t = BCD105 Real Money Supply - M_1

These figures are obtained by multiplying the 49 weights in Figure 4.8 by $\frac{.00702}{.019846}$.

The standard error of $(1-B^{12})(1-B)\log(x_t)$ is .00702, and the standard error of $(1-B^{12})(1-B)\log(y_t)$ is .019846.

```
          -.25       0.         .25         .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 0                    X                              0.
 1                    X                              0.
 2                    X                              0.
 3                    X                              0.
 4                    X                              0.
 5                    XXXXXX                          .115546E+00
 6                    XXXX                            .752342E-01
 7                    XXX                             .489865E-01
 8                    XX                              .318961E-01
 9                    XX                              .207682E-01
10                    XX                              .135226E-01
11                    X                               .880489E-02
12                    X                               .573305E-02
13                    X                               .373292E-02
14                    X                               .243057E-02
15                    X                               .158260E-02
16                    X                               .103046E-02
17                    X                               .670954E-03
18                    X                               .436873E-03
19                    X                               .284456E-03
20                    X                               .185215E-03
21                    X                               .120597E-03
22                    X                               .785235E-04
23                    X                               .511283E-04
24                    X                               .332907E-04
25                    X                               .216763E-04
26                    X                               .141138E-04
27                    X                               .918935E-05
28                    X                               .598370E-05
29                    X                               .389612E-05
30                    X                               .253684E-05
31                    X                               .165179E-05
32                    X                               .107551E-05
33                    X                               .700288E-06
34                    X                               .455975E-06
35                    X                               .296893E-06
36                    X                               .193314E-06
37                    X                               .125870E-06
```

FIGURE 4.16 (cont'd.)

```
             -.25      0.         .25        .5
(k)+++++++++.+++++++++.+++++++++.+++++++++.+++++++++ VALUES

 38                    X                             .819567E-07
 39                    X                             .533638E-07
 40                    X                             .347463E-07
 41                    X                             .226240E-07
 42                    X                             .147310E-07
 43                    X                             .959164E-08
 44                    X                             .624531E-08
 45                    X                             .406644E-08
 46                    X                             .264776E-08
 47                    X                             .172401E-08
 48                    X                             .112254E-08
```

movements in economic activity. Furthermore, they are deemed useful in forming and implementing monetary and fiscal policy decisions aimed at stabilizing economic activity.

To be truly useful in such a role, a leading indicator should show a consistently long lead over Industrial Production. How long should this lead be? There are various decision rules about evaluating when a turning point occurs in any of these series. A common rule "accepted" at this time states that a series has experienced a turning point if a succession of increases (or decreases) is followed by three successive decreases (or increases). With this decision rule, a leading indicator must certainly show a consistent lead of more than three months over economic activity, if it is to be of any value in <u>forecasting</u> a turning point in economic activity.

According to this rule, the recognition lag of a need for stabilization policy action will be three months (provided that the decision rule is followed). After this lag there will be an action lag and an outside lag before policy actions actually have a desired effect on economic activity. The action lag could be extremely long itself if fiscal policy is the desired tool, given the inertia of the Congressional decision-making process. The action lag could be relatively short if monetary policy is implemented. There has been much debate about the length of the outside lag in our economic system, though there is general agreement that it extends at least over several months.

In this light we see the need for leading indicators to display leads which are several months longer than the three months necessary to recognize a turning point. From Figures 4.9 through 4.16 we see that BCD1, BCD3, and BCD8 show no lead at all over Industrial Production. BCD19 and BCD32 display leads of just two months each. BCD105, the Real money supply, has a lead of five months, which may not be long enough to be very useful in the role desired, though it is much better than zero or two months. BCD29, the Index of Housing Starts, has a lead of nine months, which suggests that it may supply useful information as a leading indicator.

Finally, the leading indicator with the longest lead of the eight considered is BCD92, the % change in the PPI of Crude Materials. Its impulse response function implies that a sustained 1% increase in (the seasonal difference of) the growth rate of the PPI will be followed by a decrease of about one tenth of 1% in (the seasonal difference of) the growth rate of Industrial Production after ten months, and further decreases in the following months. This negative relationship could reflect a movement along some demand curve, and thus follows our economic intuition. It is remarkable to note, however, that the Commerce Department uses this leading indicator in a positive role in the CLI [See the Handbook of Cyclical Indicators, pages 2, 3, 61.]. That is, BCD92 is used by the Commerce Department as if an increase in the PPI were consistently followed by increases in Industrial Production. Our model in equation (4.15) and Figures 4.7 and 4.15 suggests that this is an inappropriate use of BCD92!

It is important to note that this same negative pattern is implied by the estimated coefficients for the sample period ending in 1963 [See Table IV-2, (vii)]. This indicates that the negative relationship is quite stable, rather than just a phenomenon of the "supply shocks" in the early 1970's.

Given all these considerations, we are left with just two of these eight leading indicators which should contribute positively to the CLI, as constructed by the Commerce Department: BCD29, the Index of Housing Starts, and BCD105, the Real money supply - M_1.

Conclusions

The Commerce Department construction of the CLI is not backed by an appropriate theoretical framework, as outlined in Chapter II.

Furthermore, our study indicates that five of the eight leading indicators considered display empirical relationships with Industrial Production which do not reflect the characteristics of a good leading indicator. They show no significant lead time. This suggests that these five series may not merit the status given them by the Commerce Department. Their qualifications for the role of leading indicator appear to be lacking.

Three of the eight series considered display empirical relationships with Industrial Production which do reflect the characteristics of a good leading indicator: BCD29, BCD92, and BCD105. However, BCD92, the % change in the PPI of Crude Materials, proves to have a negative empirical relationship with Industrial Production, while the Commerce Department uses it in a positive role in the CLI. This leaves BCD29 and BCD105 which appear to be the only leading indicators of the eight considered, which might contribute positively to the Commerce Department's CLI.

Given these observations, it is not surprising that the Commerce Department's leading indicator approach has been so unreliable. The question that remains is why so much effort has been, and continues to be, spent on its development and use in a predictive role. It seems clear that it will continue to be relied upon in its ex post role of <u>verifying</u> that turning points in economic activity have already taken place. Our study suggests that at best, it should be limited to this role.

CHAPTER V

MONEY AS A LEADING INDICATOR

Introduction

A huge literature exists on the role of Money in an economy, and its relationship to real GNP. Under the current state of thought toward Monetary Theory, what kind of relationship might we expect to see between Money and real GNP?

Friedman describes the adjustment of nominal income to Monetary shocks in the context of a system of simultaneous differential equations.[1] This system is an attempt to explain (a) the short run adjustment of nominal income to a change in autonomous variables; (b) the short run division of a change in nominal income between prices and real output; and (c) the transition between this short run situation and long run equilibrium.

In this framework it is suggested that anything which produces a discrepancy between the nominal quantity of Money demanded and the quantity supplied, or between their rates of change, will cause the rate of change in nominal income to depart from its anticipated (permanent) value. In general form,

$$(a) \qquad \frac{dY}{dt} = f[(\frac{dY}{dt})^{*}, \frac{dM^s}{dt}, \frac{dM^d}{dt}, M^s, M^d]$$

where $Y = Py =$ nominal income,

$P =$ the price level, $y =$ real GNP,

$M^s =$ Money Supply,

$M^d =$ nominal amount of Money demanded,

and a * denotes the anticipated (or "permanent") value of that variable.

A linearized version of (a) might be:

$$(a)' \qquad \frac{d\log Y}{dt} = (\frac{d\log Y}{dt})^{*} + \psi[\frac{d\log M^s}{dt} - \frac{d\log M^d}{dt}] + \phi(\log M^s - \log M^d)$$

Next, the division of a change in nominal income between prices and output depends on two major factors: anticipations about the behavior of prices, and the current level of output compared with its full employment (permanent) level. We can express this in general form as:

$$(b) \qquad \begin{aligned} \frac{dP}{dt} &= g[\frac{dY}{dt}, (\frac{dP}{dt})^{*}, (\frac{dy}{dt})^{*}, y, y^{*}] \\ \frac{dy}{dt} &= h[\frac{dY}{dt}, (\frac{dP}{dt})^{*}, (\frac{dy}{dt})^{*}, y, y^{*}] \end{aligned}$$

where the form of g and h must be consistent with the identity, $Y = Py$.

A linearized version of (b) might be:

$$\frac{d\log P}{dt} = \left(\frac{d\log P}{dt}\right)^* + \alpha\left[\frac{d\log Y}{dt} - \left(\frac{d\log Y}{dt}\right)^*\right]$$

$$+ \gamma\,[\log y - \log y^*]$$

(b)'

$$\frac{d\log y}{dt} = \left(\frac{d\log y}{dt}\right)^* + (1-\alpha)\left[\frac{d\log Y}{dt} - \left(\frac{d\log Y}{dt}\right)^*\right]$$

$$- \gamma\,[\log y - \log y^*]$$

In their general form, the equations in (b) do not by themselves specify the path of prices or output beginning with any initial position. In addition we need to know how anticipated values are formed. Presumably these are affected by the course of events so that, in response to a disturbance which produces a discrepancy between actual and anticipated values of the variables, there is a feedback effect that brings the actual and anticipated values together again. To put this in general terms, we must have:

(c)

$$\left[\frac{d\log P}{dt}(t)\right]^* = j\left[\frac{d\log P}{dt}(T)\right],$$

$$\left[\frac{d\log Y}{dt}(t)\right]^* = k\left[\frac{d\log Y}{dt}(T)\right],$$

$$y^*(t) = m[y(T)],$$

$$P^*(t) = n[P(T)],$$

where t stands for a particular point in time, and T for a vector of all dates prior to t.

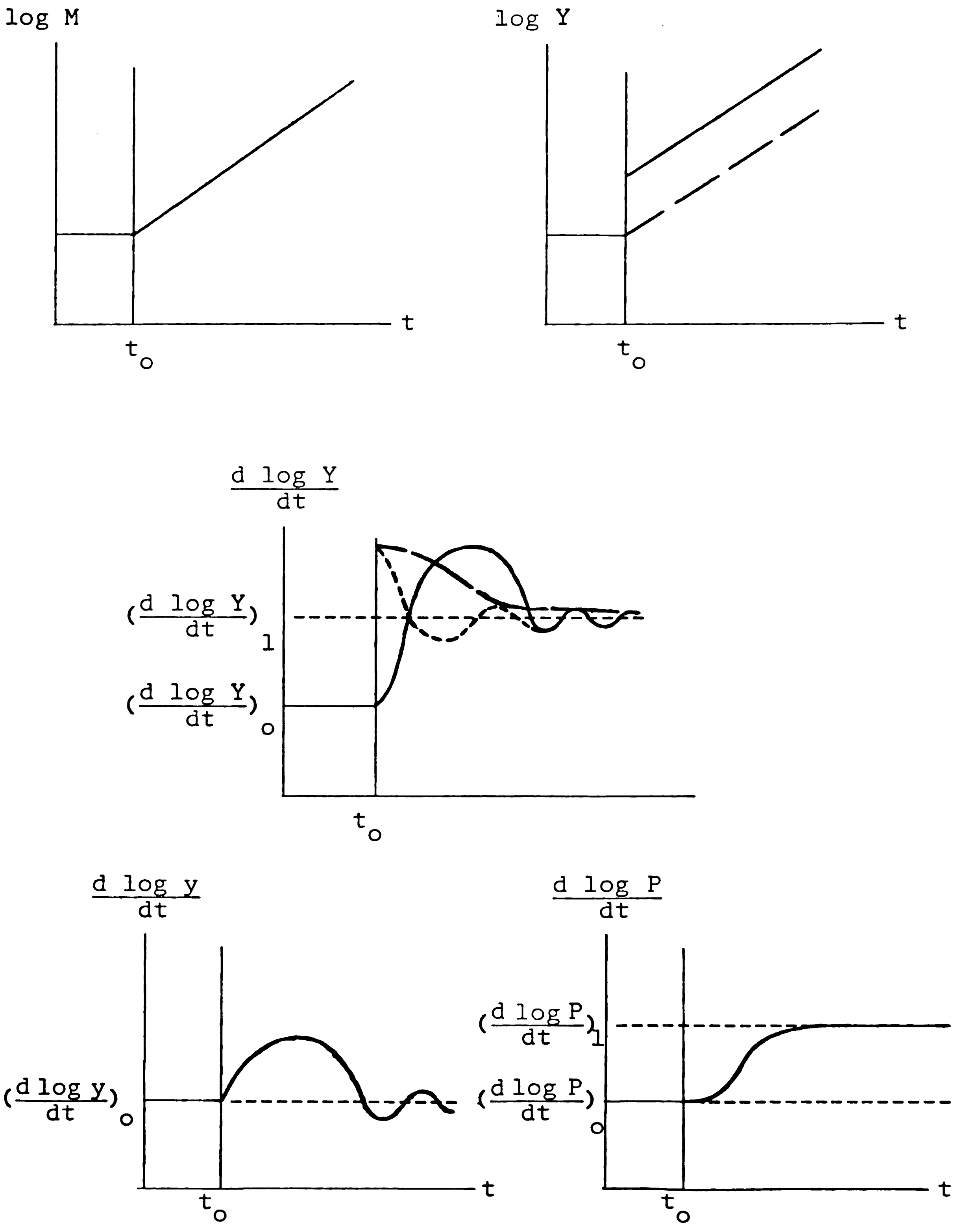
A disturbance of long run equilibrium introduces discrepancies in the two final terms in parentheses on the right hand side of equation (a)'. This will cause the rate of change in nominal income to deviate from its permanent value, which through the equations in (b)', produces deviations in the rate of price change and output change from their permanent values. These will, through the equations in (c), produce changes in the anticipated values that will eventually eliminate the discrepancies between measured and permanent values.

In the context of the above system, consider as such a disturbance of long run equilibrium, a permanent increase in $\frac{d\log M^S}{dt}$, the growth rate of the Money Supply. The first frame in Figure 5.1 shows the time path of the money stock before and after such a shock. The second frame shows the equilibrium path of nominal income.

The slopes of the time paths in these two frames must be equal, since in equilibrium nominal income will grow at the same rate as the money stock, given the framework of Friedman's model. However, the equilibrium path of Y after this shock will be at a higher level than that of the Money Stock. This is because part of the increase in $\frac{d\log Y}{dt}$ will consist of an increase in $\frac{d\log P}{dt}$. With this increase in inflation fully anticipated in equilibrium, it is now more costly to hold money. As a result there will be a decline in the real quantity of money demanded relative to income; i.e. a rise in desired velocity. This rise will be achieved by a rise in nominal income over and above that required to match the rise in the nominal quantity of money.

FIGURE 5.1

TIME PATHS OF NOMINAL INCOME AND ITS COMPONENTS, AFTER A MONETARY SHOCK



The equilibrium path of nominal income will be like the solid line in the second frame of Figure 5.1 rather than the dashed line.

We are interested in the adjustment process involved in the above scenario. It is apparent that in order to produce the shift in the equilibrium path of nominal income from the dashed line to the solid line, nominal income must rise over some period at a faster rate than the final equilibrium rate. That is, there must be an overshooting, or a cyclical reaction in the rate of change in nominal income. The third frame in Figure 5.1 summarizes the various possible adjustment paths of $\frac{d\log Y}{dt}$ consistent with the theory presented above. The one common feature of all possibilities is that the area above the $(\frac{d\log Y}{dt})_1$ line must exceed the area below.

This chapter is concerned with the composition of the path of $\frac{d\log Y}{dt}$ describing the adjustment to a change in $(\frac{d\log M^s}{dt})$. We want to know how this time path is broken up into the time paths of $\frac{d\log y}{dt}$ and $\frac{d\log P}{dt}$, as expressed in the equations in (b)'. The time path of $\frac{d\log y}{dt}$ will reflect the usefulness of the nominal quantity of Money as an indicator of real GNP.

One such possible set of time paths consistent with the equations in (b)' is displayed in the last two frames of Figure 5.1. Note that the vertical sum of these two time paths is the resulting path of $\frac{d\log Y}{dt}$. Further note that in this picture, the time path of $\frac{d\log y}{dt}$ initially rises, following an increase in $\frac{d\log M}{dt}$, but eventually this rise is crowded out so that in the long run there is no rise in $\frac{d\log y}{dt}$. This reflects a situation in which money is neutral in the long run.

The remainder of this chapter will examine the empirical relationship existing between $\frac{d\log y}{dt}$ and $\frac{d\log M}{dt}$.

Empirical Examination of the Relationship Between Money and Industrial Production

The Money Stock Data

In this role, we are concerned with the ability of money to promote spending in the economy. Hence, an appropriate definition of money to consider is:

M_1B = Currency + Demand Deposits at commercial banks + Other Checkable Deposits at all depository institutions including NOW accounts, ATS, Credit Union draft shares, and Demand Deposits at Mutual Savings banks.

It is worth noting that this is not the definition which Friedman would choose, since it excludes most Time Deposits. However, we feel it is appropriate for the work in this chapter.

Data on M_1 (= Currency + Demand Deposits) is available beginning with January, 1947. In 1960 and again in 1962 the Fed changed the definition of Demand Deposits at commercial banks. In 1960 the data were altered to include Demand Deposits due to mutual savings banks and foreign banks, and to exclude float as well as CIPC.[2] These numbers were published from 1947 to 1962, when the data were further amended to include foreign Demand Deposits with Federal Reserve Banks and Demand Deposits that banks in U.S. territories and possessions have at U.S. commercial banks.[3] The data on this definition of M_1 were then published beginning with 1947, and continued to be published until 1980, when the definition was changed once again. The data on this latest definition of M_1 (namely M_1B) have been published beginning with January, 1959.

There is obviously a discrepancy between the old and new definitions, since they measure different things. Table V-1 displays the components of the old M_1 series as published in the 1960 definition, and the new M_1B series as published in the 1980 definition. The last two columns show the discrepancy for the twelve monthly observations in 1959. We are interested in comparing these two definitions since the definition of Demand Deposits in 1960 is closer to the definition of Demand Deposits in 1980, than is the 1962 definition. Note that the currency component is identical in the two definitions in Table V-1. The discrepancy arises in the Demand Deposit component (noting that Other Checkable Deposits are zero for the observations in 1959). The Demand Deposit component in the 1960 definition exceeds the Demand Deposit component in the 1980 definition by the amount of Demand Deposits due to foreign official institutions.

TABLE V-1

MONEY STOCK SERIES - 1959

M_1 = 1960 series, available in January, 1947

M_1B = 1980 series, available in January, 1959

	[Old Definition](1960)			[New Definition](1980)				
Yr./Mo.	Currency	D.D.	Total M_1 (billions)	Currency	D.D. + O.C.D.	Total M_1B (billions)	Diff. (M_1-M_1B)	Ratio (M_1/M_1B)
59.1	28.6	115.8	144.4	28.6	114.2	142.8	1.6	1.011
59.2	28.4	113.0	141.4	28.4	111.5	139.9	1.5	1.011
59.3	28.5	112.3	140.8	28.5	110.5	139.0	1.8	1.013
59.4	28.5	113.3	141.8	28.5	111.8	140.3	1.5	1.011
59.5	28.7	112.0	140.7	28.7	110.5	139.2	1.5	1.011
59.6	28.9	112.5	141.4	28.9	111.0	139.9	1.5	1.011
59.7	29.1	113.0	142.1	29.1	111.6	140.7	1.4	1.010
59.8	29.1	112.6	141.7	29.1	111.4	140.5	1.2	1.009
59.9	29.1	113.0	142.1	29.1	111.8	140.9	1.2	1.009
59.10	29.0	113.2	142.2	29.0	112.3	141.3	0.9	1.006
59.11	29.2	114.1	143.3	29.2	113.2	142.4	0.9	1.006
59.12	29.5	115.7	145.2	29.5	114.7	144.2	1.0	1.007

We can construct a "complete" time series on the money supply by using the 1960 definition of M_1 from January, 1947 through December, 1958, and the 1980 definition of M_1B from January, 1959 to the present. Call this series M_1B^*.

M_1B^* still contains the discrepancy shown in Table V-1, due to the change in the definition of the Demand Deposit component of M_1. This will appear as a jump of approximately 1% in M_1B^*, between December, 1958 and January, 1959. We can correct this fault by building an intervention model for M_1B^*. The intervention term will be defined as follows.

$$I_t = \begin{cases} 1.0 & \text{for January, 1947 - December, 1958} \\ 0.0 & \text{for January, 1959 - December, 1979} \end{cases}$$

This will operate as a dummy variable which should account for the change in the definition of the Demand Deposit component beginning in January, 1959. We anticipate a coefficient of about (.01) for this intervention term, reflecting the 1% jump in the series shown in Table V-1.

We proceed by first building the model for M_1B^* for the sample, January, 1947 - December, 1979 (n = 396).

$$(5.1)\quad (1-B^{12})(1-B)\log(M_1B^*_t) = (1-\theta_3B^3-\theta_{12}B^{12}-\theta_{13}B^{13})a_t$$

$$\hat{\theta}_3 = \underset{(.044)}{-.1688} \quad \hat{\theta}_{12} = \underset{(.046)}{.4697} \quad \hat{\theta}_{13} = \underset{(.046)}{.1478} \quad \bar{R}^2 = 1-\frac{(.0043)^2}{(.004774)^2} = .1887$$

$$\chi^2_{27} = 32.4 \qquad\qquad \text{RSE} = .0043$$

From the form of this model, we build the intervention model for M_1B^*.

$$(5.2)\quad (1-B^{12})(1-B)[\log(M_1B^*_t)-p_oI_t] = (1-\theta_3B^3-\theta_{12}B^{12}-\theta_{13}B^{13})a_t$$

$$\hat{p}_o = \underset{(.0036)}{.0110} \quad \hat{\theta}_3 = \underset{(.044)}{-.1709} \quad \hat{\theta}_{12} = \underset{(.046)}{.4633} \quad \hat{\theta}_{13} = \underset{(.046)}{.1499}$$

$$\chi^2_{46} = 51.3 \qquad RSE = .0042$$

$$\bar{R}^2 = 1 - \frac{(.0042)^2}{(.00477)^2} = .2247$$

It is interesting to note that the coefficient of our intervention term follows the information in Table V-1 in suggesting a shift of about 1% in M_1B^*, in January, 1959.

We can now utilize the information provided by this intervention model to splice our data and obtain a "continuous" series from January, 1947 through December, 1979. The transformation of M_1B^* indicated in the intervention model is following.

$$\log(M_t) = \log(M_1B^*_t) - p_oI_t$$

We are interested in levels of the money supply. That is, we are really interested in M_t, which we obtain by exponentiating the above expression.

$$e^{\log(M_t)} = e^{[\log(M_1B^*_t) + (-p_o)I_t]}$$

$$= e^{[\log(M_1B^*_t)]}e^{(-p_o)I_t}$$

$$(5.3)\qquad M_t = (M_1B^*_t)e^{(\hat{p}_o)I_t}$$

where $\hat{p}_o = .0110$

and $I_t = \begin{cases} 1.0 \text{ for January, 1947 - December, 1958} \\ 0.0 \text{ for January, 1959 - December, 1979} \end{cases}$

Note that

$$M_t = \begin{cases} (M_1 B^*_t)e^{-.0110} & \text{for January, 1947 - December, 1958} \\ (M_1 B^*_t) & \text{for January, 1959 - December, 1979} \end{cases}$$

This spliced series simply shifts the first 12 year segment by 1.1%, to eliminate the jump in the series.

The Identification Stage

We can now proceed to examine the relationship between the growth rate of Money, $\hat{M}_t$, and the growth rate of GNP, $\hat{y}_t$, discussed earlier. We wish to build a transfer function of the following form.

$$(5.4) \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_1(B)}{\delta_1(B)}(1-B^{12})(1-B)\log(M_t) + N_{1t}$$

Note: $(1-B^{12})(1-B)\log(y_t) = (1-B^{12})\hat{y}_t$ is stationary, and

$(1-B^{12})(1-B)\log(M_t) = (1-B^{12})\hat{M}_t$ is stationary,

where y_t = the Index of Industrial production,

and M_t is as defined in equation (5.3).

This relationship will give us the impulse and step response functions describing the reaction of $\hat{y}_t$ to changes in $\hat{M}_t$.

The first step in building this transfer function model is to establish the univariate model for the input, M_t. This model is given in equation (5.2).

We now proceed to the second step, and move to identify the form of the impulse response function,

$v_1(B) = \omega_1(B)/\delta_1(B)$, by calculating the cross-correlation function between the prewhitened input and the prewhitened output in the transfer function, (5.4). This function is listed and plotted in Table V-2 and Figure (5.2). Note that the prewhitening model used is that in equation (5.2).

Note that at the identification stage, the impulse response weights are calculated directly from the cross-correlation coefficients as follows.[4]

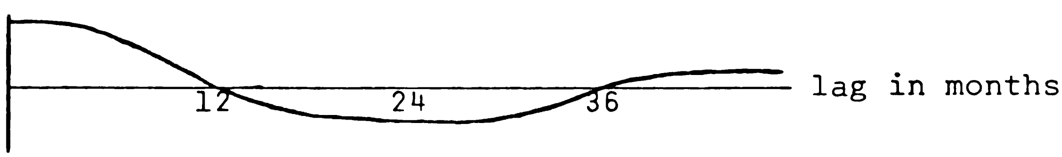
$$\hat{v}_k = \frac{r_{\alpha\beta}(k)\ s_\beta}{s_\alpha} \qquad k = 0, 1, 2, \ldots$$

e.g.

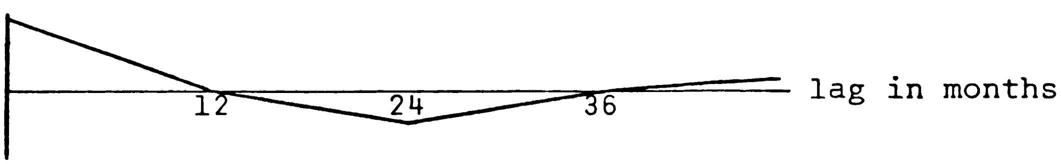
$$\hat{v}_o = \frac{(.149)(.015823)}{.0041893} = .5628$$

Note the remarkably smooth pattern of this unrestricted cross-correlation function. This suggests an impulse response function between $\hat{M}_t$ and $\hat{y}_t$ which might follow either of the following patterns:

(i) a damped cosine wave with a period of 48 months.



(ii) a damped "V" pattern which reverses according to a similar time period.



What does $v_1(B)$ suggest about the monetarist proposition outlined at the beginning of the chapter? To

TABLE V-2

Sample: January, 1947 - November, 1979; n = 395

Cross Correlations [$r_{\alpha\beta}(B)$]

Series 1 - Prewhitened Money Stock{α_t}
Series 2 - Prehitened BCD47 Index of Industrial Production{β_t}

Mean of Series 1 = .17720E-03
St. Dev. of Series 1 = .41893E-02 = S_α
Mean of Series 2 = -.72954E-04
St. Dev. of Series 2 = .15823E-01 = S_β

Number of Lags On Series 1 (k)	Cross Correlation $r_{\alpha\beta}(k)$	Number of Lags On Series 2 (k)	Cross Correlation $r_{\beta\alpha}(k)$
0	.149	0	.149
1	.119	1	.194
2	.082	2	.052
3	.154	3	.051
4	.052	4	-.062
5	.120	5	-.024
6	.093	6	-.080
7	.021	7	.017
8	.096	8	.062
9	.048	9	.007
10	.020	10	.014
11	.003	11	-.090
12	.023	12	-.047
13	-.049	13	-.098
14	-.018	14	.003
15	-.086	15	-.085
16	-.076	16	-.000
17	-.090	17	-.065
18	-.036	18	.022
19	-.072	19	-.020
20	.019	20	.057
21	-.083	21	.089
22	-.065	22	.043
23	-.091	23	-.018
24	-.086	24	.025
25	-.058	25	-.064
26	-.102	26	.004
27	-.104	27	.001
28	-.126	28	.006
29	.039	29	.070
30	-.073	30	.008
31	.079	31	-.051
32	-.048	32	-.057
33	-.024	33	.014
34	-.088	34	-.037
35	-.013	35	.007
36	-.001	36	-.104

TABLE V-2 (cont'd.)

Number of Lags on Series 1 (k)	Cross Correlation $r_{\alpha\beta}(k)$	Number of Lags on Series 2 (k)	Cross Correlation $r_{\beta\alpha}(k)$
37	.097	37	.108
38	.108	38	-.038
39	.050	39	.028
40	-.003	40	-.010
41	.003	41	.048
42	.057	42	-.068
43	-.069	43	.036
44	-.082	44	-.071
45	-.040	45	-.037
46	.018	46	.048
47	.032	47	.023
48	.032	48	.005

FIGURE 5.2

GRAPH OF IMPULSE RESPONSE WEIGHTS ($v_1(B)$)

```
   -1.5      -.75       0.        .75       1.5
(k)  .+++++++++.+++++++++.+++++++++.+++++++++.   VALUES (v_k)
 0                       XXXXXXXX                .56397E+00
 1                       XXXXXXX                 .44946E+00
 2                       XXXXX                   .30840E+00
 3                       XXXXXXXXX               .58296E+00
 4                       XXXX                    .19459E+00
 5                       XXXXXXX                 .45406E+00
 6                       XXXXXX                  .34995E+00
 7                       XX                      .79714E-01
 8                       XXXXXX                  .36108E+00
 9                       XXX                     .17989E+00
10                       XX                      .76150E-01
11                       X                       .12550E-01
12                       XX                      .88438E-01
13                     XXX                      -.18333E+00
14                      XX                      -.67643E-01
15                   XXXXX                      -.32399E+00
16                   XXXXX                      -.28546E+00
17                   XXXXX                      -.33941E+00
18                     XXX                      -.13598E+00
19                   XXXXX                      -.27075E+00
20                       XX                      .71382E-01
21                   XXXXX                      -.31407E+00
22                    XXXX                      -.24658E+00
23                  XXXXXX                      -.34273E+00
24                   XXXXX                      -.32583E+00
25                    XXXX                      -.21741E+00
26                  XXXXXX                      -.38682E+00
27                  XXXXXX                      -.39283E+00
28                 XXXXXXX                      -.47486E+00
29                       XXX                     .14836E+00
30                   XXXXX                      -.27446E+00
31                       XXXXX                   .29997E+00
32                     XXX                      -.18314E+00
33                      XX                      -.91287E-01
34                   XXXXX                      -.33277E+00
35                      XX                      -.47375E-01
36                       X                      -.53849E-02
37                       XXXXXX                  .36524E+00
38                       XXXXXX                  .40658E+00
39                       XXXX                    .19046E+00
40                       X                      -.10152E-01
41                       X                       .12349E-01
42                       XXXX                    .21611E+00
43                    XXXX                      -.26086E+00
44                   XXXXX                      -.30944E+00
```

FIGURE 5.2 (cont'd.)

```
   -1.5       -.75      0.         .75      1.5
(k)  .+++++++++.+++++++++.+++++++++.+++++++++.  VALUES (v_k)

45                     XXX                      -.15067E+00
46                       XX                      .67507E-01
47                       XXX                     .12172E+00
48                       XXX                     .12231E+00
```

discuss this question, we must consider the Step Response Function, $V_1(B)$, implied by this impulse response function, since the monetarist proposition illustrated in Figure 5.1 shows the presumed reaction of $\hat{y}_t$ to a sustained (step) change in $\hat{M}_t$.

We obtain this Step Response Function by summing over the impulse response function:

$$\begin{aligned} V_k &= k^{th} \text{ element of } V(B) \\ &= \sum_{i=0}^{k} v_i \quad \text{where } v_i = i^{th} \text{ element of } v(B). \end{aligned}$$

$V_1(B)$ is listed and plotted in Figure (5.3).

Note the strong resemblance between Figure 5.3 and the bottom left frame of Figure 5.1. We emphasize that $V_1(B)$ is obtained from the identification stage of our time series model building, with no restrictions imposed.

Immediately the question arises as to what $V_1(B)$ might converge to, if allowed to follow the pattern shown. In particular, will $V_1(B)$ converge to zero, as suggested by the monetarist proposition? To answer this question, we must proceed to the estimation stage and finish building this transfer function model.

Estimation of the Single Input Transfer Function

Consider equation (5.4) and Figure (5.2), describing the transfer function between $\hat{M}_t$ and $\hat{y}_t$. This presents a problem in that a damped cosine wave with such a long period

FIGURE 5.3

GRAPH OF STEP RESPONSE WEIGHTS $V_1(B)$

```
          (-2.0)    0.        (2.0)
(k).+++++++++.+++++++++.+++++++++.+++++++++.      VALUES [V_k]

  0                    XXXX                          .56397
  1                    XXXXXX                       1.01343
  2                    XXXXXXXX                     1.32183
  3                    XXXXXXXXXXX                  1.90479
  4                    XXXXXXXXXXX                  2.09938
  5                    XXXXXXXXXXXXXX               2.55344
  6                    XXXXXXXXXXXXXXXX             2.90339
  7                    XXXXXXXXXXXXXXXX             2.98310
  8                    XXXXXXXXXXXXXXXXXX           3.34418
  9                    XXXXXXXXXXXXXXXXXXX          3.52407
 10                    XXXXXXXXXXXXXXXXXXX          3.60022
 11                    XXXXXXXXXXXXXXXXXXX          3.61277
 12                    XXXXXXXXXXXXXXXXXXXX         3.70121
 13                    XXXXXXXXXXXXXXXXXXX          3.51788
 14                    XXXXXXXXXXXXXXXXXX           3.45024
 15                    XXXXXXXXXXXXXXXXX            3.12625
 16                    XXXXXXXXXXXXXXX              2.84079
 17                    XXXXXXXXXXXXXX               2.50138
 18                    XXXXXXXXXXXXX                2.36540
 19                    XXXXXXXXXXX                  2.09465
 20                    XXXXXXXXXXXX                 2.16603
 21                    XXXXXXXXXX                   1.85196
 22                    XXXXXXXXX                    1.60538
 23                    XXXXXXX                      1.26265
 24                    XXXXXX                        .93682
 25                    XXXXX                         .71941
 26                    XXX                           .33259
 27                    X                           -.06024
 28                 XXXX                           -.53510
 29                  XXX                           -.38674
 30                 XXXX                           -.66120
 31                  XXX                           -.36123
 32                 XXXX                           -.54437
 33                 XXXX                           -.63566
 34               XXXXXX                           -.96843
 35               XXXXXX                          -1.01581
 36               XXXXXX                          -1.02119
 37                 XXXX                           -.65595
 38                   XX                           -.24937
 39                    X                           -.05891
 40                    X                           -.06906
 41                    X                           -.05671
 42                    XX                            .15940
 43                   XX                           -.10146
 44                  XXX                           -.41090
 45                 XXXX                           -.56157
```

FIGURE 5.3 (cont'd.)

```
              (-2.0)      0.        (2.0)
(k).+++++++++.+++++++++.+++++++++.+++++++++.      VALUES [V_k]

46                       XXX                      -.49406
47                       XXX                      -.37234
48                        XX                      -.25003
```

cannot be represented parsimoniously as a ratio of two polynomials in B. Furthermore, we must operate within an upperbound of eight or nine total parameters in our model, due to the limitations of our time series computer program.

We have overcome these difficulties by filtering the data on M_t prior to estimation, in a 24-month moving average which follows one half of the period of the cosine function. This is done as follows.

$$1 \text{ period} = 48 \text{ months} = 2\pi \text{ radians}$$

$$1 \text{ month} = \frac{\pi}{24} \text{ radians}$$

$$\text{Filtered } M_t = FM_t = \left[\sum_{i=0}^{23} \cos\left(\frac{i\pi}{24}\right)B^i\right](1-B^{12})(1-B)\log(M_t)$$

We then use FM_t as our input series in the following transfer function.

$$(5.5) \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[FM_t] + N_{1t}$$

The 24^{th} order polynomial in the denominator will work with our 24-month moving average of M_t to make an impulse response function, $v_1(B)$, which resembles the damped cosine wave we desire. If $-1 < \delta_{24} < 0$, then this impulse response function will appear as follows.

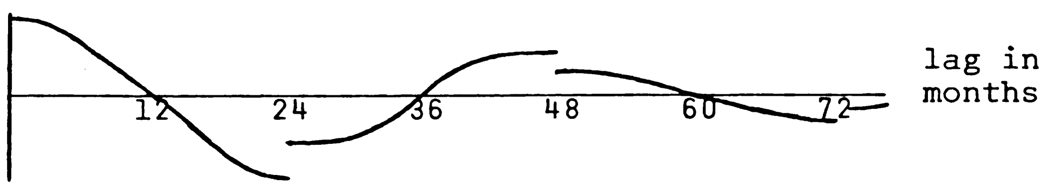


FIGURE 5.4
($\delta_{24} = -.5$)

This picture is still not ideal, as it implies an impulse response function which is discontinuous at all lags which are multiples of 24. We can eliminate these discontinuities by altering our filter slightly:

$$\text{(5.6)}\quad \text{Filtered } M_t = FM_t = \left[\sum_{i=0}^{23} [A]\cos\left(\frac{i\pi}{24}\right)B^i\right](1-B^{12})(1-B)\log(M_t)$$

where $A = \begin{cases} 1.0 \text{ for } i=0, 1, \ldots, 12 \\ -\delta_{24} \text{ for } i=13, \ldots, 23 \end{cases}$

Note that <u>this</u> filter will change the appearance of Figure 5.4 as follows.

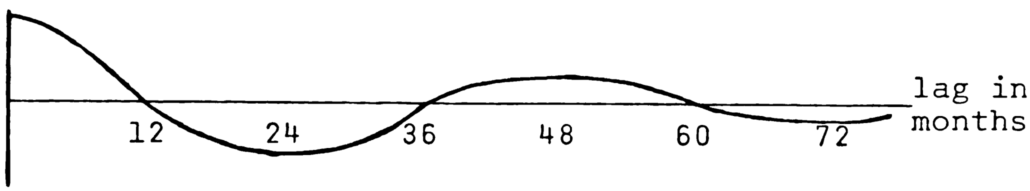


FIGURE 5.5
($\delta_{24} = -.5$)

We can arrive at a model of this form by first choosing a value of δ_{24}, and filtering M_t according to (5.6) with this value. Then we can estimate the transfer function, (5.5), and check the value of $\hat{\delta}_{24}$, to see if it differs substantially from our initial choice used to filter M_t. If it does, we can use the new value of δ_{24} to filter M_t again with (5.6), and then re-estimate (5.5). We can continue this iterative procedure until the estimate of δ_{24} does not vary appreciably from the value used to filter M_t. Using this procedure will give us a transfer function between $\hat{M}_t$ and $\hat{y}_t$ with a "smooth" impulse response function, as in Figure 5.5.

In the following work we iterate on δ_{24} until the estimate remains within a band of ($\pm$.005) from the value used to filter M_t. Since the estimate of ω_o is in all cases less than (.2), this will result in a discontinuity at lag 24 in Figure 5.5 of less than (.001)[=(.2)*(.005)].

We are now ready to estimate this model. But we must first consider some problems with our sample period: January, 1947 - December, 1979. Does the transfer function, (5.5), adequately describe a stable relationship throughout this entire period? We suspect that the oil crunch of 1973 represents an episode for which (5.5) is inadequate. There is a substantial literature on this topic, concerning the supply shock to the economy resulting from the increase in energy prices.[5] This literature dwells on the change in the structure of the world economy after this shock, and the presumed impact on real and potential output. Tatom states that "the large increase in the cost of energy resources from 1972 to 1977 has had profound effects on productivity, investment, and the long term growth path of the U.S. economy." The study of Rasche and Tatom produced empirical results which "support the argument that the new energy regime imposed in 1974 permanently reduced potential output," and suggests that "failure to account for energy prior to 1973 is not critical, but that serious inconsistencies arise when the sample period is extended to include recent years."

It is apparent that the supply shock of 1973 changed the world we are studying. This phenomenon enters our model as outlined in the beginning of the chapter in equation (b), as a sudden change in anticipated potential output, y*. For example in equation (b)', (log y*) will have changed drastically during this episode. This will result in alterations in the time paths of prices and real output.

These considerations move us to believe that the presumed stable relationship between $\hat{M}_t$ and $\hat{y}_t$ should <u>not</u> be expected to hold during this oil crunch of 1973, and should <u>not</u> be expected to account for the change in the world since then. The transfer function in equation (5.5) would likely overpredict y_t during the oil crunch, and subsequently prove to be inadequate.

We can examine this possibility by first estimating the single-input transfer function in (5.5) over the sample period May, 1950 - September, 1973, the period prior to the oil crunch. The following model is the result.[6]

y_t = BCD47 Index of Industrial Production

x_t = FM_t = Filtered M_t as in equation (5.6) with $\delta_{24} = -.5207$

$$(5.7) \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_t] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B}a_t$$

$$\hat{\omega}_0 = \underset{(.059)}{.1877} \quad \hat{\delta}_{24} = \underset{(.252)}{-.5251} \quad \hat{\theta}_{12} = \underset{(.039)}{.8046} \quad \hat{\phi}_1 = \underset{(.062)}{.2776}$$

$$\chi^2_{46} = 37.7 \qquad RSE = .0135$$

$$\bar{R}^2 = .5273$$

We are interested in how this model will forecast over the next 18 months outside the sample, during the oil crunch. Table V-3 shows the 18 one-step-ahead forecasts obtained from this model by including an additional observation on FM_t and y_t at each step. The forecast errors show that the model drastically overpredicts y_t in late 1974, though it performs fairly well for most of the other 18 months considered. Thus our suspicions as to the adequacy of this single-input transfer function during the oil crunch are possibly well-founded.

Expanding the Model to Account for the Energy Price Supply Shocks of the Early 1970's - BCD92

We can correct this situation by considering a second input which might capture the effect of the oil crunch in 1973, and thus enhance our model's ability to predict y_t during this period.

One such possible input is BCD92: the percent change in the PPI of Crude Materials. Recall that this series represents one of the three Commerce Department leading indicators examined in Chapter IV, which displays the characteristics a good leading indicator is expected to have. The second to last column of Table V-3 shows the monthly observations of BCD92. This series displays a noticeable increase in late 1973 and early 1974. Recall from Chapter IV that this leading indicator has a lead of approximately ten months over Industrial Production. This suggests that

TABLE V-3

FORECASTING I.P. WITH $x_t = FM_t$

with the model estimated over the sample: 1950.5 - 1973.9 (n = 281) ; Equation (5.7)

T.O.	Yr/Mo	1-Step-Ah. Forecast	(95% Conf. Int) Lower	Upper	Actual [T.O. + 1]	Forecast Error -(forecast- actual)	% of Actual	BCD92 (T.O. + 1)	% change in Fuel PPI (T.O. + 1)
281	73.9	134.843	131.326	138.454	135.3	.457	0.34	.023	1.373
282	73.10	132.785	129.328	136.335	132.9	.115	0.09	.056	3.388
283	73.11	129.542	126.176	132.998	126.7	-2.842	-2.24	.032	5.008
284	73.12	126.599	123.298	129.987	126.3	-.299	-0.24	.046	7.009
285	74.1	129.293	125.929	132.747	129.8	.507	0.39	.074	8.773
286	74.2	130.452	127.063	133.931	130.8	.348	0.27	.046	6.334
287	74.3	130.749	127.358	134.229	129.9	-.849	-0.65	.058	4.601
288	74.4	129.677	126.320	133.124	131.7	2.023	1.54	-.038	3.183
289	74.5	135.165	131.663	138.760	135.3	.135	0.1	.050	2.990
290	74.6	127.229	123.938	130.606	127.3	.071	0.06	.051	5.184
291	74.7	131.983	128.576	135.480	131.4	-.583	-0.44	.003	1.921
292	74.8	135.479	131.988	139.063	135.5	.021	0.02	.001	-.443
293	74.9	135.562	132.075	139.140	133.1	-2.062	-1.55	-.003	1.544
294	74.10	129.952	126.604	133.388	125.5	-4.452	-3.55	-.001	-.483
295	74.11	120.434	117.299	123.653	114.9	-5.534	-4.82	-.033	.701
296	74.12	113.455	110.443	116.550	111.8	-1.655	-1.48	-.008	1.388
297	75.1	113.766	110.745	116.870	113.0	-.766	-0.68	.007	.043
298	75.2	113.230	110.227	116.314	111.8	-1.43	-1.28	-.012	.301

Theil Decomposition:

Mean Error = ME = .955278
Mean Absolute Error = MAE = 1.363833
Mean Square Error = MSE = 4.212284
Root Mean Square Error = RMSE = 2.052385

Bias Proportion = UM = .216642 = 5.1% of MSE
Variance Proportion = US = .125752 = 3.0% of MSE
Covariance Proportion = UC = 3.869890 = 91.9% of MSE

$\frac{RMSE}{\bar{y}} = \frac{2.052385}{126.8333} = .016$ = proportion of the mean of the dependent variable which the RMSE comprises. [$\bar{y}$ = mean of Actuals]

BCD92 may be successful in capturing the effect of the oil crunch, and thus improve the poor forecasting performance of our model in equation (5.7) during late 1974.

The Identification Stage of Building the Two Input Transfer Function

In building this two-input transfer function, it is interesting to consider the cross-correlation function between the two inputs, FM_t and BCD92. In particular, we are interested in the cross-correlation coefficients between certain transformations of the two inputs. If these coefficients are extremely small, then it will imply that the cross-correlation function between each prewhitened input and the similarly transformed output, can be used separately to identify the respective impulse response function. This implication is developed below.

After differencing to achieve stationarity, our two-input transfer function can be written as:

$$(5.8)\quad y_t = v_{10}x_{1t} + v_{11}x_{1t-1} + v_{12}x_{1t-2} + v_{13}x_{1t-3} + \dots + v_{20}x_{2t} + v_{21}x_{2t-1} + v_{22}x_{2t-2} + v_{23}x_{2t-3} + \dots + n_t$$

Then, on multiplying throughout in (5.8) by x_{1t-k} for $k \geq 0$, we obtain

$$(5.9)\quad x_{1t-k}y_t = v_{10}x_{1t-k}x_{1t} + v_{11}x_{1t-k}x_{1t-1} + v_{12}x_{1t-k}x_{1t-2} + \dots + v_{20}x_{1t-k}x_{2t} + v_{21}x_{1t-k}x_{2t-1} +$$

$$+ v_{22}x_{1t-k}x_{2t-2} + \ldots + x_{1t-k}n_t$$

If we make the assumption that x_{1t-k} is uncorrelated with n_t for all k, taking expectations in (5.9) yields the set of equations

$$(5.10)\quad E[x_{1t-k}y_t] = v_{10}E[x_{1t-k}x_{1t}] + v_{11}E[x_{1t-k}x_{1t-1}] + \ldots + v_{20}E[x_{1t-k}x_{2t}] + v_{21}E[x_{1t-k}x_{2t-1}] + \ldots$$

or

$$\gamma_{x_1y}(k) = v_{10}\gamma_{x_1x_1}(k) + v_{11}\gamma_{x_1x_1}(k-1) + v_{12}\gamma_{x_1x_1}(k-2) + \ldots + v_{20}\gamma_{x_1x_2}(k) + v_{21}\gamma_{x_1x_2}(k-1) + v_{22}\gamma_{x_1x_2}(k-2) + \ldots$$

for k = 0, 1, 2, ...

where $\gamma_{ab}(k)$ is the cross-covariance at lag k between series a and b.

This is the set of cross-covariances between our first input, x_{1t}, and the output, y_t. Suppose that the weights, v_{1j} and v_{2j}, are effectively zero beyond some lag, k=K. Then the first K+1 equations in (5.10) can be written

$$\gamma_{x_1y} = \Gamma_{x_1x_1,x_1x_2}\nu$$

where

$$\gamma_{x_1y} = \underset{(K+1)x1}{\begin{pmatrix} \gamma_{x_1y}(0) \\ \gamma_{x_1y}(1) \\ \vdots \\ \gamma_{x_1y}(K) \end{pmatrix}}$$

$$\nu = \begin{pmatrix} v_{10} \\ v_{11} \\ \vdots \\ v_{1K} \\ v_{20} \\ v_{21} \\ \vdots \\ v_{2K} \end{pmatrix}_{(2k+2)\times 1}$$

$$\Gamma_{x_1x_1,x_1x_2} =$$

$$\begin{pmatrix} \gamma_{x_1x_1}(0) & \gamma_{x_1x_1}(1) & \cdots & \gamma_{x_1x_1}(K) & \gamma_{x_1x_2}(0) & \gamma_{x_1x_2}(1) & \cdots & \gamma_{x_1x_2}(K) \\ \gamma_{x_1x_1}(1) & \gamma_{x_1x_1}(0) & \cdots & \gamma_{x_1x_1}(K-1) & \gamma_{x_1x_2}(1) & \gamma_{x_1x_2}(0) & \cdots & \gamma_{x_1x_2}(K-1) \\ \vdots & & & \vdots & & & & \\ \gamma_{x_1x_1}(K) & \gamma_{x_1x_1}(K-1) & \cdots & \gamma_{x_1x_1}(0) & \gamma_{x_1x_2}(K) & \gamma_{x_1x_2}(K-1) & \cdots & \gamma_{x_1x_2}(0) \end{pmatrix}_{(K+1)\times(2K+2)}$$

This system of equations can normally be used at the identification stage of the Box and Jenkins modeling procedure.[7] If x_1 and x_2 are <u>not</u> significantly cross-correlated, then the $\gamma_{x_1x_2}(k)$ terms drop out, and we can substitute estimates of the autocorrelation function of

x_1, $\gamma_{x_1x_1}(k)$, and the cross-correlations between x_1 and y, $\gamma_{x_1y}(k)$, and solve the system for the top half of ν, which is the transfer function between x_1 and y, $v_1(B)$.

However, it will be a rare case when the two inputs are not significantly cross-correlated! Since the $\gamma_{x_1x_2}(k)$ are not generally zero, the system in (5.10) cannot be solved. We have K+1 equations in 2K+2 unknowns in ν, and the system is not identified.

The problem is changed slightly when we first prewhiten the series in our model. Suppose that the univariate models for x_{1t} and x_{2t} appear as follows.

(5.11) $\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[x_{1t}] = \alpha_{11t}$

(5.12) $\phi_{x_2}(B)\theta_{x_2}^{-1}(B)[x_{2t}] = \alpha_{22t}$

where α_{11t} and α_{22t} are white noise series, with standard deviations s_{α_1} and s_{α_2}, respectively.

Further, define the following.

(5.13) $\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[x_{2t}] = \alpha_{12t}$

(5.14) $\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[y_t] = \beta_{1t}$

(5.15) $\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[n_t] = \varepsilon_{1t}$

(5.16) $\phi_{x_2}(B)\theta_{x_2}^{-1}(B)[x_{1t}] = \alpha_{21t}$

(5.17) $\phi_{x_2}(B)\theta_{x_2}^{-1}(B)[y_t] = \beta_{2t}$

$$(5.18)\quad \phi_{x_2}(B)\theta_{x_2}^{-1}(B)[n_t] \;=\; \varepsilon_{2t}$$

Note that equations (5.13) through (5.15) are just transformations of x_{2t}, y_t, and n_t where the transformation is the prewhitening model for x_{1t}. Likewise equations (5.16) through (5.18) are transformations of x_{1t}, y_t, and n_t with the prewhitening model for x_{2t}.

In order to identify the impulse response function between x_{1t} and y_t, $v_1(B)$, we apply the prewhitening transformation of our first input to the model in equation (5.8). This yields the following model.

$$\begin{aligned}(5.19)\quad \phi_{x_1}(B)\theta_{x_1}^{-1}(B)[y_t] \;=\;& v_1(B)\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[x_{1t}]\\ &+ v_2(B)\phi_{x_1}(B)\theta_{x_1}^{-1}(B)[x_{2t}]\\ &+ \phi_{x_1}(B)\theta_{x_1}^{-1}(B)[n_t]\end{aligned}$$

or $\qquad \beta_{1t} = v_1(B)\alpha_{11t} + v_2(B)\alpha_{12t} + \varepsilon_{1t}$

On multiplying through this by α_{11t-k}, we obtain

$$\begin{aligned}(5.20)\quad \alpha_{11t-k}\beta_{1t} = {}& v_1(B)\alpha_{11t-k}\alpha_{11t} + v_2(B)\alpha_{11t-k}\alpha_{12t}\\ &+ \alpha_{11t-k}\varepsilon_{1t}\end{aligned}$$

If we make the assumption that α_{11t-k} is uncorrelated with ε_{1t} for all k, taking expectations in equation (5.20) yields the following.

$$(5.21)\quad E[\alpha_{11t-k}\beta_{1t}] = v_1(B)E[\alpha_{11t-k}\alpha_{11t}] + v_2(B)E[\alpha_{11t-K}\alpha_{12t}]$$

$$\text{or}\qquad \gamma_{\alpha_{11}\beta_1}(k) = v_1(B)\gamma_{\alpha_{11}\alpha_{11}}(k) + v_2(B)\gamma_{\alpha_{11}\alpha_{12}}(k)$$

Since α_{11t} is white noise, the first term on the right hand side of equation (5.21) reduces to $[v_k\sigma_{\alpha_{11}}^2]$. Finally, if $\gamma_{\alpha_{11}\alpha_{12}}(k)$ is zero for all k, then equation (5.21) reduces to the following.

$$(5.22)\quad \gamma_{\alpha_{11}\beta_1}(k) = v_k\sigma_{\alpha_{11}}^2 \qquad \text{or } v_k = \frac{\gamma_{\alpha_{11}\beta_1}(k)}{\sigma_{\alpha_{11}}^2}$$

where $\gamma_{\alpha_{11}\beta_1}(k) = E[\alpha_{11t-k}\beta_{1t}]$ is the cross-covariance at lag k between α_{11t} and β_{1t}.

Therefore,

$$(5.23)\quad v_k = \frac{\rho_{\alpha_{11}\beta_1}(k)\sigma_{\beta_1}}{\sigma_{\alpha_{11}}} \qquad \text{since} \left(\rho_{\alpha_{11}\beta_1}(k) = \frac{\gamma_{\alpha_{11}\beta_1}(k)}{\sigma_{\alpha_{11}}\sigma_{\beta_1}}\right)$$

Hence the cross-correlation function between the prewhitened first input and the correspondingly transformed output is directly proportional to the impulse response function, $v_1(B)$. We can thus identify the form of the first impulse response function by <u>estimating</u> the cross-correlation function, $r_{\alpha_{11}\beta_1}(k)$, and the standard errors of the prewhitened first input and the similarly transformed output, and then substituting into equation (5.23).

$$(5.24)\quad \hat{v}_k = \frac{r_{\alpha_{11}\beta_1}(k)s_{\beta_1}}{s_{\alpha_{11}}}$$

It is important to emphasize that this procedure rests on the assumption that $\gamma_{\alpha_{11}\alpha_{12}}(k)$ is zero for all k. We can estimate this cross-correlation function between the prewhitened first input and the correspondingly transformed second input, in order to evaluate the applicability of this assumption. If the coefficients of $r_{\alpha_{11}\alpha_{12}}(k)$ are not significantly different from zero, then we cannot reject the hypothesis that the assumption holds.

Note that to identify $v_2(B)$, we will use the cross-correlation function between the prewhitened second input and the similarly transformed output, $\gamma_{\alpha_{22}\beta_2}(k)$. The use of this procedure will rest on the assumption that $\gamma_{\alpha_{22}\alpha_{21}}(k)$ is zero for all k. The applicability of this assumption is testable in the same way we test the assumption regarding the identification of $v_1(B)$.

In summary, this analysis shows us that if $r_{\alpha_{11}\alpha_{12}}(k)$ and $r_{\alpha_{22}\alpha_{21}}(k)$ display coefficients which are not significantly different from zero, then the cross-correlation function between each prewhitened input and the correspondingly transformed output can be used separately to identify the respective impulse response functions.

In continuing our empirical analysis, we now wish to build the two-input transfer function, with FM_t and BCD92 as our two inputs. In order to be able to identify the two impulse response functions separately, we are interested in the two cross-correlation functions, $r_{\alpha_{11}\alpha_{12}}(k)$

and $r_{\alpha_{22}\alpha_{21}}(k)$. These are presented in Table V-4.

The standard error of any given cross-correlation coefficient, $r_{ab}(k)$, is approximated by $1/\sqrt{n-k}$.[8] With n = 352 in the sample of the transfer function we are building, the standard error of $r_{\alpha_{11}\alpha_{12}}(0)$ = the standard error of $r_{\alpha_{22}\alpha_{21}}(0) = 1/\sqrt{352} = .0533$. To test the hypothesis that the estimated coefficients are not significantly different from zero, each coefficient (at lag zero) should be compared with $(1.96)*(.0533) = .1045$.

In the estimated cross-correlation function, $r_{\alpha_{11}\alpha_{12}}(k)$, all the coefficients are less than .1045 except the coefficient at lag 16; $r_{\alpha_{11}\alpha_{12}}(16) = -.111$.

The standard error of this coefficient $= \dfrac{1}{\sqrt{n-k}}$

$$= \frac{1}{\sqrt{352-16}} = .0546.$$

and $(1.96)*(.0546) = 1.07$.

Therefore, $r_{\alpha_{11}\alpha_{12}}(16)$ is "significantly different from zero" at the 95% confidence level. However, if we consider the whole set of 49 coefficients in the cross-correlation function, we would expect about 2 1/2 coefficients to be "significantly different from zero" at the 95% confidence level. Hence the cross-correlation function, $r_{\alpha_{11}\alpha_{12}}(k)$, supports our assumption which implies that we can use the cross-correlation function between prewhitened x_{1t} (= Filtered Money Supply) and the similarly transformed y_t (= Industrial Production Index), to identify $v_1(B)$.

TABLE V-4

Cross Correlations

Series α_{11} - Prewhitened Filtered Money Supply (δ_{24} = -.5600)

Series α_{12} - Prewhitened BCD92 Percent Change in PPI of Crude Materials

n = 352

Mean of Series α_{11} = .42260E-03

St. Dev. of Series α_{11} = .44205E-02

Mean of Series α_{12} = .93668E-04

St. Dev. of Series α_{12} = .21824E-01

Number of Lags on Series α_{11} (k)	Cross Correlation $r_{\alpha_{11}\alpha_{12}}(k)$	Number of Lags on Series α_{12} (k)	Cross Correlation $r_{\alpha_{12}\alpha_{11}}(k)$
0	-.023	0	-.023
1	.018	1	.023
2	-.042	2	.008
3	-.026	3	-.025
4	.090	4	.048
5	.010	5	-.004
6	-.064	6	.042
7	-.005	7	-.055
8	-.027	8	-.064
9	.077	9	.060
10	-.063	10	-.041
11	-.004	11	.059
12	-.051	12	.019
13	.057	13	-.005
14	.080	14	-.068
15	-.020	15	-.016
16	-.111	16	-.021
17	.062	17	.020
18	.030	18	-.047
19	-.022	19	.126
20	-.019	20	-.048
21	.021	21	-.060
22	.003	22	.032
23	.045	23	-.038
24	-.079	24	-.032
25	-.005	25	.060
26	.005	26	.068
27	-.037	27	-.030
28	-.002	28	-.051
29	-.035	29	.021
30	-.003	30	-.016
31	.030	31	-.056
32	.015	32	.095
33	-.035	33	.015

TABLE V-4 (cont'd.)

Number of Lags on Series α_{11} (k)	Cross Correlations $r_{\alpha_{11}\alpha_{12}}(k)$	Number of Lags on Series α_{12} (k)	Cross Correlations $r_{\alpha_{12}\alpha_{11}}(k)$
34	.006	34	-.048
35	-.012	35	-.027
36	.077	36	.052
37	-.039	37	-.018
38	-.046	38	-.031
39	-.032	39	.030
40	.070	40	.093
41	.004	41	-.105
42	-.025	42	.073
43	-.033	43	-.029
44	.025	44	-.020
45	.012	45	-.007
46	-.010	46	.048
47	-.037	47	.031
48	-.016	48	-.055

Prewhitening:

$$(1-.8673B)(1-.2197B-.3052B^{3})(1+.6109B^{12})[FM_t] = (1+.1173B-.1343B^{10}-.1819B^{13})a_t$$

- same model on $(1-B^{12})[BCD92]$.

TABLE V-4 (cont'd.)

Cross Correlations:

Series α_{22} - Prewhitened BCD92 Percent Change in PPI of Crude Materials

Series α_{21} - Prewhitened Filtered Money Supply ($\delta_{24} = -.5600$)

n = 352

Mean of Series α_{22} = .80105E-03

St. Dev. of Series α_{22} = .14037E-01

Mean of Series α_{21} = .39340E-02

St. Dev. of Series α_{21} = .93074E-02

Number of Lags on Series α_{22} (k)	Cross Correlation $r_{\alpha_{22}\alpha_{21}}(k)$	Number of Lags on Series α_{21} (k)	Cross Correlation $r_{\alpha_{21}\alpha_{22}}(k)$
0	-.015	0	-.015
1	.049	1	.029
2	.061	2	.024
3	.026	3	.081
4	.021	4	.046
5	-.002	5	.064
6	.004	6	.061
7	-.043	7	.037
8	-.055	8	.025
9	.001	9	.018
10	-.069	10	.008
11	-.050	11	.006
12	-.107	12	.006
13	-.122	13	.053
14	-.139	14	.030
15	-.131	15	.058
16	-.117	16	-.021
17	-.078	17	.012
18	-.031	18	.011
19	-.015	19	-.002
20	-.036	20	-.015
21	-.003	21	-.023
22	-.029	22	-.007
23	-.029	23	-.015
24	-.011	24	-.054
25	.041	25	-.004
26	.080	26	-.019
27	.029	27	.043
28	-.005	28	-.002
29	.007	29	.017
30	.029	30	.034
31	.002	31	.021
32	.033	32	.008

TABLE V-4 (cont'd.)

Number of Lags on Series α_{22} (k)	Cross Correlation $r_{\alpha_{22}\alpha_{21}}(k)$	Number of Lags on Series α_{21} (k)	Cross Correlation $r_{\alpha_{21}\alpha_{22}}(k)$
33	.083	33	-.002
34	.024	34	.042
35	-.006	35	.009
36	.019	36	.027
37	-.002	37	-.005
38	.030	38	-.038
39	.034	39	.010
40	.053	40	-.029
41	.016	41	-.035
42	.069	42	-.009
43	.027	43	-.027
44	.011	44	-.042
45	.056	45	-.045
46	.048	46	-.025
47	.004	47	-.045
48	.011	48	-.034

Prewhitening:

$$(1-.3850B-.2658B^3)(1-B^{12})[BCD92] = (1-.1542B^2-.8120B^{12} + .1975B^{14})\, a_t$$

- same model on the levels of FM_t.

The estimated cross-correlation function, $r_{\alpha_{22}\alpha_{21}}(k)$ displays five coefficients which are "significantly different from zero." Furthermore the tendency of positive coefficients to be followed by positive coefficients and of negative coefficients to be followed by negative coefficients, seems to indicate that there is some correlation inherent in the relationship between these two series which is not effectively eliminated by the prewhitening model for BCD92. This is somewhat disturbing. However we are reassured by the fact that 43 of the 48 coefficients are <u>not</u> significantly different from zero. <u>This</u> is the key characteristic in our analysis of the applicability of the assumption that all coefficients in $\gamma_{\alpha_{22}\alpha_{21}}(k)$ <u>are</u> zero. Hence we anticipate that the cross-correlation function between the prewhitened input, x_{2t} (= BCD92), and the correspondingly transformed output, y_t (=Industrial Production Index), will be instrumental in identifying $v_2(B)$.

<u>Estimating the Two Input Transfer Function</u>

We have already identified and estimated separately, the two transfer functions with each of these inputs as the single input. Thus we know what to expect as the form of the two impulse response functions, $v_1(B)$ and $v_2(B)$, in our two-input transfer function.

With this in mind, we build the following model.[9]

y_t = BCD47 Index of Industrial Production

$x_{1t} = FM_t$ = Filtered M_t as in (5.6), with $\delta_{24} = -.5231$

x_{2t} = BCD92 % change in PPI of Crude Materials

Sample: May, 1950 - September, 1973 (n = 281)

$$(5.25)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega_o'}{1-\delta_1'B}B^{10}(1-B^{12})[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B}a_t$$

$\hat{\omega}_o = \underset{(.058)}{.1835}$ $\hat{\delta}_{24} = \underset{(.260)}{-.5208}$ $\hat{\omega}_o' = \underset{(.045)}{-.0614}$ $\hat{\delta}_1' = \underset{(.176)}{.7805}$

$\hat{\theta}_{12} = \underset{(.040)}{.7926}$ $\hat{\phi}_1 = \underset{(.061)}{.2540}$

$\chi^2_{46} = 40.4$ RSE = .0134 $\bar{R}^2 = .5441$

A comparison of the coefficients in this equation with those of equation (5.7) on page 148 shows that the addition of the second input, BCD92, does not change the appearance of the first transfer function much. In particular, the coefficients $\hat{\omega}_o$, $\hat{\delta}_{24}$, $\hat{\theta}_{12}$, and $\hat{\phi}_1$ are quite insensitive to the addition of this second input. It is also interesting to compare the coefficients $\hat{\omega}_o'$ and $\hat{\delta}_1'$ in equation (5.25) with the coefficients $\hat{\omega}_o$ and $\hat{\delta}_1$ of the single-input transfer function with BCD92 as the input, which appears in Table IV-2 in Chapter IV. These coefficients are also quite insensitive to the addition of the second variable, FM_t.

We are now interested in how <u>this</u> model will forecast over the next 18 periods. Table V-5 shows the 18 resulting one-step-ahead forecasts. We see improvement in reducing the large forecast errors in late 1974 that appear in Table V-3. Furthermore, the Theil Decomposition statistics show marked improvement. In particular, the RMSE is reduced to 1.75846 from 2.053285. These characteristics suggest that our second input, BCD92, is useful in the role desired.

Our third step in building this model is to re-estimate the two-input transfer function in equation (5.25) over the sample period May, 1950 - March, 1975, the period through the oil crunch. The resulting model follows.[10]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with δ_{24} = -.5520

x_{2t} = BCD92 % change in PPI of Crude Materials

Sample: May, 1950 - March, 1975 (n = 299)

$$(5.26)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega_o'}{1-\delta_1'B}B^{10}(1-B^{12})[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B}a_t$$

$$\hat{\omega}_o = \underset{(.060)}{.1772} \quad \hat{\delta}_{24} = \underset{(.240)}{-.5568} \quad \hat{\omega}_o' = \underset{(.043)}{-.0943} \quad \hat{\delta}_1' = \underset{(.123)}{.7707}$$

$$\hat{\theta}_{12} = \underset{(.042)}{.7699} \quad \hat{\phi}_1 = \underset{(.061)}{.2827}$$

$$\chi^2_{46} = 39.6 \qquad RSE = .0134 \qquad \bar{R}^2 = .5441$$

TABLE V-5

Forecasting I.P. with $x_1 = FM_t$ (= -.5231) and x_2 = BCD92

with model estimated over the sample: 1950.5 - 1973.9 (n = 281) in equation (5.25)

T.O.	Yr./Mo.	Forecast	(95% of Conf. Int.) Lower	Upper	Actual (T.O. + 1)	(Actual-forecast) forecast error	% of Actual
281	73.9	134.680	131.2	138.3	135.3	.620	.46
282	73.10	132.576	129.1	136.1	132.9	.324	.24
283	73.11	129.272	125.9	132.7	126.7	-2.572	-2.03
284	73.12	126.551	123.3	129.9	126.3	-.251	-.20
285	74.1	129.066	125.7	132.5	129.8	.734	.57
286	74.2	130.072	126.7	133.5	130.8	.728	.56
287	74.3	130.338	126.9	133.8	129.9	-.438	-.34
288	74.4	129.378	126.0	132.8	131.7	2.322	1.76
289	74.5	134.807	131.3	138.4	135.3	.493	.36
290	74.6	126.781	123.5	130.2	127.3	.519	.41
291	74.7	131.504	128.1	135.0	131.4	-.104	-.08
292	74.8	134.760	131.3	138.4	135.5	.740	.55
293	74.9	134.850	131.3	138.5	133.1	-1.750	-1.31
294	74.10	129.238	125.9	132.7	125.5	-3.738	-2.98
295	74.11	119.554	116.4	122.7	114.9	-4.654	-4.05
296	74.12	112.743	109.8	115.8	111.8	-.943	-.84
297	75.1	112.946	110.0	116.0	113.0	.054	.05
298	75.2	112.887	110.0	115.9	111.8	-1.087	-.97

Theil Decomposition:

ME = .500167
MAE = 1.226167
MSE = 3.092181
RMSE = 1.758460
UM = .080903 = 2.6% of MSE
US = .133685 = 4.3% of MSE

UC = 2.877593 = 93.1% of MSE

$$\frac{RMSE}{\bar{y}} = \frac{1.75846}{126.8333} = .014$$

Note that the coefficient estimates are extremely stable as we increase the sample size from n = 281 to n = 299, in moving from equation (5.25) to (5.26), even though these eighteen additional periods represent a shock to the economy.

We are now interested in how <u>this</u> model forecasts over the next 4 1/2 years. Table V-6 displays 53 one-step-ahead forecasts for the 4 1/2 years after the sample used to estimate equation (5.26).

Examination of the Theil Decomposition statistics in Table V-6 indicates that our model in equation (5.26) performs remarkably well over this extremely long forecast horizon. The RMSE is reduced to 1.334221 from 1.75846 in Table V-5, and from 2.053285 in Table V-3. We conclude that our model is appropriate, and proceed to the last step of estimating over the entire sample period.[11]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.5600$

x_{2t} = BCD92 % change in PPI of Crude Materials

Sample: May, 1950 - August, 1979 (n = 352)

$$(5.27)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega_o'}{1-\delta_1'B}\, B^{10}(1-B^{12})[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B}\, a_t$$

TABLE V-6

Forecasting I.P. with $x_1 = FM_t$ (δ = -.5520) and x_2 = BCD92

with model estimated thru the sample: 1950.5 - 1975.3 (n = 299) in equation (5.26)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O.+1) Actual	(Actual-forecast) forecast error	% of Actual
299	75.3	110.693	107.9	113.5	113.0	2.307	2.04
300	75.4	113.210	110.4	116.1	113.8	.590	.52
301	75.5	116.669	113.8	119.7	119.2	2.531	2.12
302	75.6	112.952	110.1	115.8	114.5	1.548	1.35
303	75.7	119.729	116.7	122.8	121.4	1.671	1.38
304	75.8	126.873	123.7	130.1	125.9	-.973	-.77
305	75.9	126.662	123.5	129.9	125.4	-1.262	-1.01
306	75.10	123.060	120.0	126.2	123.8	.740	.60
307	75.11	119.964	117.0	123.0	119.8	-.164	-.14
308	75.12	120.955	117.9	124.0	122.2	1.245	1.02
309	76.1	126.982	123.8	130.2	128.3	1.318	1.03
310	76.2	130.106	126.9	133.4	128.6	-1.506	-1.17
311	76.3	129.677	126.4	133.0	128.7	-.977	-.76
312	76.4	130.409	127.2	133.7	130.0	-.409	-.31
313	76.5	134.796	131.4	138.2	133.2	-1.596	-1.20
314	76.6	126.203	123.1	129.4	126.5	.297	.23
315	76.7	132.761	129.4	136.2	131.7	-1.061	-.81
316	76.8	137.115	133.7	140.6	134.3	-2.815	-2.10
317	76.9	134.017	130.7	137.4	133.8	-.217	-.16
318	76.10	131.066	127.8	134.4	132.1	1.034	.78
319	76.11	128.117	124.9	131.4	128.3	.183	.14
320	76.12	129.582	126.3	132.9	128.8	-.782	-.61
321	77.1	133.132	129.8	136.5	133.6	.468	.35
322	77.2	134.556	131.2	138.0	135.7	1.144	.84
323	77.3	136.924	133.5	140.4	136.2	-.724	-.53
324	77.4	137.258	133.8	140.8	137.2	-.058	-.04
325	77.5	141.468	137.9	145.1	141.5	.032	.02

TABLE V-6 (cont'd.)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O.+1) Actual	(Actual-forecast) forecast error	% of Actual
326	77.6	134.575	131.2	138.0	134.1	-.475	-.35
327	77.7	139.698	136.2	143.3	138.2	-1.498	-1.08
328	77.8	141.959	138.4	145.6	142.4	.441	.31
329	77.9	142.491	138.9	146.1	142.7	.209	.15
330	77.10	140.378	136.9	144.0	139.5	-.878	-.63
331	77.11	134.033	130.7	137.5	134.9	.867	.64
332	77.12	135.445	132.1	138.9	134.8	-.645	-.48
333	78.1	138.959	135.5	142.5	139.6	.641	.46
334	78.2	140.366	136.9	144.0	141.4	1.034	.73
335	78.3	142.471	138.9	146.1	144.2	1.729	1.20
336	78.4	146.324	142.7	150.1	144.2	-2.124	-1.47
337	78.5	148.404	144.7	152.2	149.8	1.396	.93
338	78.6	142.504	138.9	146.2	142.9	.396	.28
339	78.7	149.239	145.5	153.1	148.2	-1.039	-.70
340	78.8	153.215	149.4	157.1	153.0	-.215	-.14
341	78.9	152.645	148.8	156.6	153.4	.755	.49
342	78.10	150.132	146.4	154.0	150.5	.368	.24
343	78.11	145.242	141.6	149.0	147.1	1.858	1.26
344	78.12	147.777	144.1	151.6	146.6	-1.177	-.80
345	79.1	150.932	147.2	154.8	152.3	1.368	.90
346	79.2	153.495	149.7	157.4	154.0	.505	.33
347	79.3	155.094	151.2	159.1	151.1	-3.994	-2.64
348	79.4	150.522	146.8	154.4	152.5	1.978	1.30
349	79.5	157.587	153.7	161.6	156.5	-1.087	-.69
350	79.6	148.385	144.7	152.2	148.7	.315	.21
351	79.7	154.773	150.9	158.7	152.3	-2.473	-1.62

Theil Decomposition:

ME = -.015453
MAE = 1.077679
MSE = 1.780146

RMSE = 1.334221
UM = .000134 = 0.0% of MSE
US = .068819 = 3.9% of MSE
UC = 1.711193 = 96.1% of MSE

$$\frac{RMSE}{\bar{y}} = \frac{1.334221}{136.64906} = .010$$

TABLE V-6 (cont'd.)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O.+1) Actual	(Actual-forecast) forecast error	% of Actual
326	77.6	134.575	131.2	138.0	134.1	-.475	-.35
327	77.7	139.698	136.2	143.3	138.2	-1.498	-1.08
328	77.8	141.959	138.4	145.6	142.4	.441	.31
329	77.9	142.491	138.9	146.1	142.7	.209	.15
330	77.10	140.378	136.9	144.0	139.5	-.878	-.63
331	77.11	134.033	130.7	137.5	134.9	.867	.64
332	77.12	135.445	132.1	138.9	134.8	-.645	-.48
333	78.1	138.959	135.5	142.5	139.6	.641	.46
334	78.2	140.366	136.9	144.0	141.4	1.034	.73
335	78.3	142.471	138.9	146.1	144.2	1.729	1.20
336	78.4	146.324	142.7	150.1	144.2	-2.124	-1.47
337	78.5	148.404	144.7	152.2	149.8	1.396	.93
338	78.6	142.504	138.9	146.2	142.9	.396	.28
339	78.7	149.239	145.5	153.1	148.2	-1.039	-.70
340	78.8	153.215	149.4	157.1	153.0	-.215	-.14
341	78.9	152.645	148.8	156.6	153.4	.755	.49
342	78.10	150.132	146.4	154.0	150.5	.368	.24
343	78.11	145.242	141.6	149.0	147.1	1.858	1.26
344	78.12	147.777	144.1	151.6	146.6	-1.177	-.80
345	79.1	150.932	147.2	154.8	152.3	1.368	.90
346	79.2	153.495	149.7	157.4	154.0	.505	.33
347	79.3	155.094	151.2	159.1	151.1	-3.994	-2.64
348	79.4	150.522	146.8	154.4	152.5	1.978	1.30
349	79.5	157.587	153.7	161.6	156.5	-1.087	-.69
350	79.6	148.385	144.7	152.2	148.7	.315	.21
351	79.7	154.773	150.9	158.7	152.3	-2.473	-1.62

Theil Decomposition:

ME = -.015453
MAE = 1.077679
MSE = 1.780146

RMSE = 1.334221
UM = .000134 = 0.0% of MSE
US = .068819 = 3.9% of MSE
UC = 1.711193 = 96.1% of MSE

$$\frac{\text{RMSE}}{\bar{y}} = \frac{1.334221}{136.64906} = .010$$

TABLE V-6 (cont'd.)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O.+1) Actual	(Actual-forecast) forecast error	% of Actual
326	77.6	134.575	131.2	138.0	134.1	-.475	-.35
327	77.7	139.698	136.2	143.3	138.2	-1.498	-1.08
328	77.8	141.959	138.4	145.6	142.4	.441	.31
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330	77.10	140.378	136.9	144.0	139.5	-.878	-.63
331	77.11	134.033	130.7	137.5	134.9	.867	.64
332	77.12	135.445	132.1	138.9	134.8	-.645	-.48
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334	78.2	140.366	136.9	144.0	141.4	1.034	.73
335	78.3	142.471	138.9	146.1	144.2	1.729	1.20
336	78.4	146.324	142.7	150.1	144.2	-2.124	-1.47
337	78.5	148.404	144.7	152.2	149.8	1.396	.93
338	78.6	142.504	138.9	146.2	142.9	.396	.28
339	78.7	149.239	145.5	153.1	148.2	-1.039	-.70
340	78.8	153.215	149.4	157.1	153.0	-.215	-.14
341	78.9	152.645	148.8	156.6	153.4	.755	.49
342	78.10	150.132	146.4	154.0	150.5	.368	.24
343	78.11	145.242	141.6	149.0	147.1	1.858	1.26
344	78.12	147.777	144.1	151.6	146.6	-1.177	-.80
345	79.1	150.932	147.2	154.8	152.3	1.368	.90
346	79.2	153.495	149.7	157.4	154.0	.505	.33
347	79.3	155.094	151.2	159.1	151.1	-3.994	-2.64
348	79.4	150.522	146.8	154.4	152.5	1.978	1.30
349	79.5	157.587	153.7	161.6	156.5	-1.087	-.69
350	79.6	148.385	144.7	152.2	148.7	.315	.21
351	79.7	154.773	150.9	158.7	152.3	-2.473	-1.62

Theil Decomposition:

ME = -.015453
MAE = 1.077679
MSE = 1.780146

RMSE = 1.334221
UM = .000134 = 0.0% of MSE
US = .068819 = 3.9% of MSE
UC = 1.711193 = 96.1% of MSE

$$\frac{RMSE}{\bar{y}} = \frac{1.334221}{136.64906} = .010$$

$\hat{\omega}_o = \underset{(.052)}{.1726}$ $\hat{\delta}_{24} = \underset{(.196)}{-.5583}$ $\hat{\omega}'_o = \underset{(.042)}{-.1036}$ $\hat{\delta}'_1 = \underset{(.148)}{.6842}$

$\hat{\theta}_{12} = \underset{(.037)}{.7776}$ $\hat{\phi}_1 = \underset{(.056)}{.2777}$

$\chi^2_{46} = 44.1$ RSE $= .0129$ $\bar{R}^2 = .5775$

Again, note that the coefficient estimates are extremely stable as we increase the sample size from n = 299 to n = 352, from equation (5.26) to (5.27).

Implications of the Final Model

This is our model for the entire sample period, describing the relationships between M_t and y_t and between BCD92 and y_t. We are especially interested in the impulse and step response functions between $\hat{M}_t$ and $\hat{y}_t$, in order to examine in more detail the empirical evidence regarding the monetarist proposition outlined at the beginning of this chapter. This impulse response function is developed below.

Let $m_t = (1-B^{12})(1-B)\log(M_t)$.

Then the impulse response function is:

$$(5.28)\quad v_1(B)[m_t] = \frac{\omega_1(B)}{\delta_1(B)}[m_t] = \frac{\omega_o}{1-\delta_{24}B^{24}}[FM_t]$$

where FM_t is defined as in equation (5.6), as follows:

$$FM_t = \left(\sum_{i=0}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i\right)m_t; \quad \text{with } A = \begin{cases} 1.0 \text{ for } i = 0, \ldots ,12 \\ -\delta_{24} \text{ for } i = 13, \ldots ,23 \end{cases}$$

$$
\begin{aligned}
FM_t = \; & [1.0 + .99B + .966B^2 + .924B^3 + .866B^4 + .793B^5 \\
& + .707B^6 + .609B^7 + .5B^8 + .383B^9 + .259B^{10} + .13B^{11} \\
& - (-\delta_{24})\ .13B^{13} - (-\delta_{24}).259B^{14} - (-\delta_{24})\ .383B^{15} \\
& - (-\delta_{24})\ .5B^{16} - (-\delta_{24}).609B^{17} - (-\delta_{24})\ .707B^{18} \\
& - (-\delta_{24})\ .793B^{19} - (-\delta_{24}).866B^{20} - (-\delta_{24})\ .924B^{21} \\
& - (-\delta_{24})\ .966B^{22} - (-\delta_{24}).99B^{23}]\ m_t
\end{aligned}
$$

Thus $\omega_1(B)$, the polynomial in B comprising the numerator of $v_1(B)$, is simply $\hat{\omega}_o$ multiplying the above twenty-third order polynomial in B.

$$(5.29)\quad \omega_1(B) = \hat{\omega}_o \left[\sum_{i=o}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i \right]$$

When combined with $\delta_1(B)$, the denominator of $v_1(B)$, the resulting impulse response function is of the following form.

$$
\begin{aligned}
(5.30)\quad v_1(B) = \; & \hat{\omega}_o \left[\sum_{i=0}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i \right] + (\hat{\delta}_{24})\hat{\omega}_o \left[\sum_{i=0}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i \right] [B^{24}] \\
& + (\hat{\delta}_{24})^2\hat{\omega}_o \left[\sum_{i=0}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i \right] [B^{48}] \\
& + (\hat{\delta}_{24})^3\hat{\omega}_o \left[\sum_{i=0}^{23} (A)\cos\left(\frac{i\pi}{24}\right)B^i \right] [B^{72}] + \ldots
\end{aligned}
$$

From equation (5.27) we have $\hat{\omega}_o = .1726$, $-\delta_{24} = .5600$, and $\hat{\delta}_{24} = -.5583$. Using these estimates in equation (5.30) yields the infinite order polynomial in B comprising $v_1(B)$. The first 48 coefficients of this impulse response function are listed and plotted in Figure 5.6, as well as those of the associated step response function.

FIGURE 5.6

GRAPH OF IMPULSE RESPONSE WEIGHTS [$v_1(B)$]

(k)	-.5 0. .5 1. .+++++++++.+++++++++.+++++++++.+++++++++.	VALUES [v_k]	
0	XXXX	.1726	$= \hat{\omega}_o(1.0)$
1	XXXX	.1709	$= \hat{\omega}_o(.99)$
2	XXXX	.1667	$= \hat{\omega}_o(.966)$
3	XXXX	.1595	$= \hat{\omega}_o(.924)$
4	XXXX	.1495	$= \hat{\omega}_o(.866)$
5	XXXX	.1369	$= \hat{\omega}_o(.793)$
6	XXX	.1220	$= \hat{\omega}_o(.707)$
7	XXX	.1051	$= \hat{\omega}_o(.609)$
8	XXX	.0863	$= \hat{\omega}_o(.5)$
9	XX	.0661	$= \hat{\omega}_o(.383)$
10	XX	.0447	$= \hat{\omega}_o(.259)$
11	X	.0224	$= \hat{\omega}_o(.13)$
12	X	0.	$= \hat{\omega}_o(0.0)$
13	X	-.0126	$= \hat{\omega}_o(-\delta_{24})(-.13)$
14	XX	-.0250	$= \hat{\omega}_o(-\delta_{24})(-.259)$
15	XX	-.0370	$= \hat{\omega}_o(-\delta_{24})(-.383)$
16	XX	-.0483	$= \hat{\omega}_o(-\delta_{24})(-.5)$
17	XX	-.0589	$= \hat{\omega}_o(-\delta_{24})(-.609)$
18	XX	-.0683	$= \hat{\omega}_o(-\delta_{24})(-.707)$
19	XXX	-.0766	$= \hat{\omega}_o(-\delta_{24})(-.793)$

FIGURE 5.6 (cont'd.)

(k)	-.5 ... 0. ... .5 ... 1.	VALUES $[v_k]$	
20	XXX	-.0837	$= \hat{\omega}_o(-\delta_{24})(-.866)$
21	XXX	-.0893	$= \hat{\omega}_o(-\delta_{24})(-.924)$
22	XXX	-.0934	$= \hat{\omega}_o(-\delta_{24})(-.966)$
23	XXX	-.0957	$= \hat{\omega}_o(-\hat{\delta}_{24})(-.99)$
24	XXX	-.0964	$= \hat{\omega}_o(\hat{\delta}_{24})(1.0)$
25	XXX	-.0954	$= \hat{\omega}_o(\hat{\delta}_{24})(.99)$
26	XXX	-.0931	$= \hat{\omega}_o(\hat{\delta}_{24})(.966)$
27	XXX	-.0890	$= \hat{\omega}_o(\hat{\delta}_{24})(.924)$
28	XXX	-.0834	$= \hat{\omega}_o(\hat{\delta}_{24})(.866)$
29	XXX	-.0764	$= \hat{\omega}_o(\hat{\delta}_{24})(.793)$
30	XX	-.0681	$= \hat{\omega}_o(\hat{\delta}_{24})(.707)$
31	XX	-.0587	$= \hat{\omega}_o(\hat{\delta}_{24})(.609)$
32	XX	-.0482	$= \hat{\omega}_o(\hat{\delta}_{24})(.5)$
33	XX	-.0369	$= \hat{\omega}_o(\hat{\delta}_{24})(.383)$
34	XX	-.0250	$= \hat{\omega}_o(\hat{\delta}_{24})(.259)$
35	X	-.0125	$= \hat{\omega}_o(\hat{\delta}_{24})(.13)$
36	X	0.	$= \hat{\omega}_o(\hat{\delta}_{24})(.0.0)$
37	X	.0070	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.13)$
38	X	.0140	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.259)$
39	X	.0207	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.383)$
40	XX	.0270	$= \omega_o(\delta_{24})(-\delta_{24})(-.5)$

FIGURE 5.6 (cont'd.)

```
               -.5       0.        .5        1.
(k).+++++++++.+++++++++.+++++++++.+++++++++.
41                       XX
42                       XX
43                       XX
44                       XX
45                       XX
46                       XX
47                       XX
48                       XX
```

(k)	VALUES $[v_k]$	
41	.0329	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.609)$
42	.0382	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.707)$
43	.0428	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.793)$
44	.0467	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.866)$
45	.0499	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.924)$
46	.0521	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.966)$
47	.0534	$= \hat{\omega}_o(\hat{\delta}_{24})(-\delta_{24})(-.99)$
48	.0540	$= \hat{\omega}_o(\hat{\delta}_{24})(\hat{\delta}_{24})(1.0)$

FIGURE 5.6 (cont'd.)

GRAPH OF STEP RESPONSE WEIGHTS [$V_1(B)$]

```
            -.5       0.        .5        1.0       1.5
(k).+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.          VALUES [V_k]
  0                    XXXX                                          .1726
  1                    XXXXXXX                                       .3435
  2                    XXXXXXXXXXX                                   .5102
  3                    XXXXXXXXXXXXXX                                .6697
  4                    XXXXXXXXXXXXXXXXX                             .8192
  5                    XXXXXXXXXXXXXXXXXXXX                          .9561
  6                    XXXXXXXXXXXXXXXXXXXXXXX                      1.0781
  7                    XXXXXXXXXXXXXXXXXXXXXXXXX                    1.1832
  8                    XXXXXXXXXXXXXXXXXXXXXXXXXXX                  1.2695
  9                    XXXXXXXXXXXXXXXXXXXXXXXXXXXX                 1.3356
 10                    XXXXXXXXXXXXXXXXXXXXXXXXXXXXX                1.3803
 11                    XXXXXXXXXXXXXXXXXXXXXXXXXXXXX                1.4027
 12                    XXXXXXXXXXXXXXXXXXXXXXXXXXXXX                1.4027
 13                    XXXXXXXXXXXXXXXXXXXXXXXXXXXXX                1.3901
 14                    XXXXXXXXXXXXXXXXXXXXXXXXXXXX                 1.3651
 15                    XXXXXXXXXXXXXXXXXXXXXXXXXXXX                 1.3281
 16                    XXXXXXXXXXXXXXXXXXXXXXXXXXX                  1.2798
 17                    XXXXXXXXXXXXXXXXXXXXXXXXX                    1.2209
 18                    XXXXXXXXXXXXXXXXXXXXXXXX                     1.1526
 19                    XXXXXXXXXXXXXXXXXXXXXXX                      1.0760
 20                    XXXXXXXXXXXXXXXXXXXXX                         .9923
 21                    XXXXXXXXXXXXXXXXXXX                           .9030
 22                    XXXXXXXXXXXXXXXXX                             .8096
 23                    XXXXXXXXXXXXXXX                               .7139
 24                    XXXXXXXXXXXXX                                 .6175
 25                    XXXXXXXXXXX                                   .5221
 26                    XXXXXXXXXX                                    .4290
 27                    XXXXXXXX                                      .3400
 28                    XXXXXX                                        .2566
 29                    XXXXX                                         .1802
```

```
                                   FIGURE 5.6 (cont'd.)
            -.5       0.        .5        1.0       1.5
(k).+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.                        VALUES [V_k]
30                     XXX                                                    .1121
31                     XX                                                     .0534
32                     X                                                      .0052
33                    XX                                                     -.0317
34                    XX                                                     -.0567
35                    XX                                                     -.0692
36                    XX                                                     -.0692
37                    XX                                                     -.0622
38                    XX                                                     -.0482
39                    XX                                                     -.0275
40                     X                                                     -.0005
4o                     XX                                                     .0324
42                     XX                                                     .0706
43                     XXX                                                    .1134
44                     XXXX                                                   .1601
45                     XXXXX                                                  .2100
46                     XXXXXX                                                 .2621
47                     XXXXXXX                                                .3155
48                     XXXXXXXX                                               .3695
```

We know that the impulse response function will converge to zero (see Figure 5.5), and thus that the step response function will also converge. In light of the Monetarist proposition under consideration, we are interested in what this step response function will converge to. We can pinpoint this number as follows.

Beginning with the sum of the first twelve impulse response weights, we can sum over the next 24 weights to get a single figure which will be subtracted in the step response function during the next 24 periods. We can then multiply this same figure by $(-\delta_{24})$ to get another figure describing the total amount which will be added to the step response function in the following 24 months. Multiplying this figure by $(-\hat{\delta}_{24})^2$ will then yield the total amount to be subtracted again in the following 24 months.

Continuing this procedure indefinitely would give us the exact number to which $V_1(B)$ converges. Continuing for a few iterations will closely approximate this number.

The sum of the first 12 impulse response weights $= V_{12} = 1.4027$.

The sum of the next 24 impulse response weights $= 1.4719$.

Thus, $V_{36} = 1.4027 - 1.4719 = -.0692$.

The sum of the following 24 impulse response weights $= (1.4719)(.5583)$

$= .8218$.

Thus, $V_{60} = -.0692 + .8218 = .7526$.

The sum of the following 24 impulse response weights
$= (1.4719)(.5583)^2$

$= .4588.$

Thus, $V_{84} = .7527 - .4588 = .2938$.

Continuing:

$(1.4719)(.5583)^3 = .2561$

$V_{108} = .2938 + .2561 = .5499$

$(1.4719)(.5583)^4 = .1430$

$V_{132} = .5499 - .1430 = .4069$

$(1.4719)(.5583)^5 = .0798$

$V_{156} = .4069 + .0798 = .4867$

$(1.4719)(.5583)^6 = .0446$

$V_{180} = .4867 - .0446 = .4421$

$(1.4719)(.5583)^7 = .0249$

$V_{204} = .4421 + .0249 = .4670$

$(1.4719)(.5583)^8 = .0139$

$V_{228} = .4670 - .0139 = .4531$

$(1.4719)(.5583)^9 = .0078$

$V_{252} = .4531 + .0078 = .4609$

Hence we see that the step response function converges to approximately .4570. This suggests that money is <u>not</u> neutral, but that a sustained increase in the growth rate of the money stock will produce an increase in the growth rate of Real GNP in the long run.

Finally we are interested in the impulse and step response functions for the second input, BCD92, implied by our model in equation (5.27). These are listed and plotted in Figure (5.7).

Expanding the Model to Account for the Energy Price Supply Shocks of the Early 1970's - Fuel Prices

In equations (5.25), (5.26), and (5.27), we included a second input to account for the effect of the oil crunch in 1973, and the subsequent change in the world. We used as our second input, BCD92, the percent change in the PPI of Crude Materials. This is interesting, since BCD92 is one of the leading indicators which constitute the subject of discussion of Chapter IV. However, much of the supply shock literature listed in footnote 5 uses Fuel prices in this role. Thus we now re-examine the relationships discussed in the last section, with x_{2t}=Fuel prices.

We must first consider the relationship between Fuel prices and Industrial Production. The bivariate model describing this relationship is presented below.[12]

y_t = BCD47 Index of Industrial Production

x_t = PPI of Fuel, Power, and Related Products

Sample: May, 1950 - November, 1979 (n = 355)

$$(1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_1 B} B^8(1-B^{12})(1-B)\log(x_t) + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$$\hat{\omega}_o = \underset{(.065)}{-.1573} \qquad \hat{\delta}_1 = \underset{(.169)}{.6410} \qquad \hat{\theta}_{12} = \underset{(.033)}{.8159} \qquad \hat{\phi}_1 = \underset{(.053)}{.3204}$$

$$\chi^2_{46} = 41.1 \qquad RSE = .0129 \qquad \bar{R}^2 = .5775$$

FIGURE 5.7

GRAPH OF IMPULSE RESPONSE WEIGHTS [$v_2(B)$]
for x_{2t} = BCD92 in equation (5.27)

```
            -.1       0.         .1
(k).+++++++++.+++++++++.+++++++++.+++++++++.   VALUES [v_k]

 0                     X                      0.
 1                     X                      0.
 2                     X                      0.
 3                     X                      0.
 4                     X                      0.
 5                     X                      0.
 6                     X                      0.
 7                     X                      0.
 8                     X                      0.
 9                     X                      0.
10           XXXXXXXXXXX                      -.103629E+00
11              XXXXXXXX                      -.709052E-01
12                XXXXXX                      -.485147E-01
13                  XXXX                      -.331947E-01
14                   XXX                      -.227125E-01
15                   XXX                      -.155403E-01
16                    XX                      -.106330E-01
17                    XX                      -.727529E-02
18                     X                      -.497789E-02
19                     X                      -.340597E-02
20                     X                      -.233043E-02
21                     X                      -.159453E-02
22                     X                      -.109101E-02
23                     X                      -.746487E-03
24                     X                      -.510761E-03
25                     X                      -.349472E-03
26                     X                      -.239116E-03
27                     X                      -.163608E-03
28                     X                      -.111944E-03
29                     X                      -.765939E-04
30                     X                      -.524070E-04
31                     X                      -.358579E-04
32                     X                      -.245347E-04
33                     X                      -.167871E-04
34                     X                      -.114861E-04
35                     X                      -.785898E-05
36                     X                      -.537727E-05
37                     X                      -.367923E-05
38                     X                      -.251740E-05
39                     X                      -.172245E-05
40                     X                      -.117854E-05
41                     X                      -.806377E-06
42                     X                      -.551739E-06
43                     X                      -.377510E-06
44                     X                      -.258300E-06
45                     X                      -.176734E-06
46                     X                      -.128925E-06
```

FIGURE 5.7 (cont'd.)

```
             -.1         0.          .1
(k).+++++++++.+++++++++.+++++++++.+++++++++. VALUES [v_k]

47                       X                    -.827390E-07
48                       X                    -.566116E-07
```

FIGURE 5.7 (cont'd.)

GRAPH OF STEP RESPONSE WEIGHTS [$V_2(B)$] for x_{2t} = BCD92

```
            -.5       -.25       0.        .25
(k).+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.VALUES
  0                              X                   0.
  1                              X                   0.
  2                              X                   0.
  3                              X                   0.
  4                              X                   0.
  5                              X                   0.
  6                              X                   0.
  7                              X                   0.
  8                              X                   0.
  9                              X                   0.
 10                          XXXXX                   -.103629
 11                       XXXXXXXX                   -.174534
 12                     XXXXXXXXXX                   -.223049
 13                    XXXXXXXXXXX                   -.256244
 14                   XXXXXXXXXXXX                   -.278956
 15                  XXXXXXXXXXXXX                   -.294497
 16                  XXXXXXXXXXXXX                   -.305130
 17                  XXXXXXXXXXXXX                   -.312405
 18                 XXXXXXXXXXXXXX                   -.317383
 19                 XXXXXXXXXXXXXX                   -.320788
 20                 XXXXXXXXXXXXXX                   -.323119
 21                 XXXXXXXXXXXXXX                   -.324713
 22                 XXXXXXXXXXXXXX                   -.325804
 23                 XXXXXXXXXXXXXX                   -.326551
 24                 XXXXXXXXXXXXXX                   -.327062
 25                 XXXXXXXXXXXXXX                   -.327411
 26                 XXXXXXXXXXXXXX                   -.327650
 27                 XXXXXXXXXXXXXX                   -.327814
 28                 XXXXXXXXXXXXXX                   -.327926
 29                 XXXXXXXXXXXXXX                   -.328002
 30                 XXXXXXXXXXXXXX                   -.328055
 31                 XXXXXXXXXXXXXX                   -.328091
 32                 XXXXXXXXXXXXXX                   -.328115
 33                 XXXXXXXXXXXXXX                   -.328132
 34                 XXXXXXXXXXXXXX                   -.328143
 35                 XXXXXXXXXXXXXX                   -.328151
 36                 XXXXXXXXXXXXXX                   -.328157
 37                 XXXXXXXXXXXXXX                   -.328161
 38                 XXXXXXXXXXXXXX                   -.328163
 39                 XXXXXXXXXXXXXX                   -.328165
 40                 XXXXXXXXXXXXXX                   -.328166
 41                 XXXXXXXXXXXXXX                   -.328167
 42                 XXXXXXXXXXXXXX                   -.328168
 43                 XXXXXXXXXXXXXX                   -.328168
 44                 XXXXXXXXXXXXXX                   -.328169
 45                 XXXXXXXXXXXXXX                   -.328169
 46                 XXXXXXXXXXXXXX                   -.328169
 47                 XXXXXXXXXXXXXX                   -.328169
 48                 XXXXXXXXXXXXXX                   -.328170
```

FIGURE 5.7 (cont'd.)

GRAPH OF STEP RESPONSE WEIGHTS [$V_2(B)$] for x_{2t} = BCD92

```
          -.5        -.25       0.         .25
(k).+++++++++.+++++++++.+++++++++.+++++++++.+++++++++.VALUES
 0                               X                   0.
 1                               X                   0.
 2                               X                   0.
 3                               X                   0.
 4                               X                   0.
 5                               X                   0.
 6                               X                   0.
 7                               X                   0.
 8                               X                   0.
 9                               X                   0.
10                           XXXXX                   -.103629
11                        XXXXXXXX                   -.174534
12                      XXXXXXXXXX                   -.223049
13                     XXXXXXXXXXX                   -.256244
14                    XXXXXXXXXXXX                   -.278956
15                   XXXXXXXXXXXXX                   -.294497
16                   XXXXXXXXXXXXX                   -.305130
17                   XXXXXXXXXXXXX                   -.312405
18                  XXXXXXXXXXXXXX                   -.317383
19                  XXXXXXXXXXXXXX                   -.320788
20                  XXXXXXXXXXXXXX                   -.323119
21                  XXXXXXXXXXXXXX                   -.324713
22                  XXXXXXXXXXXXXX                   -.325804
23                  XXXXXXXXXXXXXX                   -.326551
24                  XXXXXXXXXXXXXX                   -.327062
25                  XXXXXXXXXXXXXX                   -.327411
26                  XXXXXXXXXXXXXX                   -.327650
27                  XXXXXXXXXXXXXX                   -.327814
28                  XXXXXXXXXXXXXX                   -.327926
29                  XXXXXXXXXXXXXX                   -.328002
30                  XXXXXXXXXXXXXX                   -.328055
31                  XXXXXXXXXXXXXX                   -.328091
32                  XXXXXXXXXXXXXX                   -.328115
33                  XXXXXXXXXXXXXX                   -.328132
34                  XXXXXXXXXXXXXX                   -.328143
35                  XXXXXXXXXXXXXX                   -.328151
36                  XXXXXXXXXXXXXX                   -.328157
37                  XXXXXXXXXXXXXX                   -.328161
38                  XXXXXXXXXXXXXX                   -.328163
39                  XXXXXXXXXXXXXX                   -.328165
40                  XXXXXXXXXXXXXX                   -.328166
41                  XXXXXXXXXXXXXX                   -.328167
42                  XXXXXXXXXXXXXX                   -.328168
43                  XXXXXXXXXXXXXX                   -.328168
44                  XXXXXXXXXXXXXX                   -.328169
45                  XXXXXXXXXXXXXX                   -.328169
46                  XXXXXXXXXXXXXX                   -.328169
47                  XXXXXXXXXXXXXX                   -.328169
48                  XXXXXXXXXXXXXX                   -.328170
```

This model shows that the Fuel PPI displays a substantial lead over Industrial Production. The last column of Table V-3 shows the monthly observations of the first log difference of the Fuel PPI, during the oil crunch of 1973. This series displays a large increase in late 1973 and early 1974. Since the Fuel PPI has a lead of eight months over Industrial Production, it may be successful in capturing the effect of the oil crunch, and thus improving the poor forecasting performance of our model in equation (5.7) during late 1974.

The Identification Stage

As before, with our two inputs, FM_t and BCD92, we are now interested in the cross-correlation functions between the two inputs, FM_t and Fuel Prices; first transformed by the prewhitening model for FM_t, and second, transformed by the prewhitening model for Fuel prices. These two cross-correlation functions, $r_{\alpha_{11}\alpha_{21}}(k)$ and $r_{\alpha_{22}\alpha_{21}}(k)$, are listed in Table V-7.

Examination of $r_{\alpha_{11}\alpha_{21}}(k)$ shows that two coefficients are "significantly different from zero." Since we expect about 2 1/2 coefficients to vary from zero at the 95% confidence level, this cross-correlation function supports the assumption that these cross-correlations <u>are</u> zero. Hence the cross-correlation function between prewhitened FM_t and the similarly transformed output series can be used to identify the first impulse response function in this

TABLE V-7

Cross Correlations:

Series α_{11} - Prewhitened Filtered Money Supply (δ_{24} = -.5520)

Series α_{12} - Prewhitened PPI of Fuel Power and Related Products

n = 355

Mean of Series α_{11} = .38159E-03

St. Dev. of Series α_{11} = .44258E-02

Mean of Series α_{12} = .78262E-04

St. Dev. of Series α_{12} = .13782E-01

Number of Lags on Series α_{11} (k)	Cross Correlation $r_{\alpha_{11}\alpha_{12}}(k)$	Number of Lags of Series α_{12} (k)	Cross Correlation $r_{\alpha_{12}\alpha_{22}}(k)$
0	.046	0	.046
1	.031	1	.054
2	-.053	2	-.067
3	-.037	3	-.016
4	.003	4	-.061
5	.013	5	.057
6	-.095	6	.044
7	.064	7	-.055
8	-.054	8	.036
9	.040	9	-.197
10	-.058	10	.046
11	.142	11	-.004
12	-.035	12	-.003
13	.023	13	-.020
14	-.008	14	-.042
15	.021	15	-.102
16	-.059	16	.168
17	.056	17	-.061
18	.005	18	.021
19	-.066	19	.041
20	.109	20	-.041
21	-.072	21	.023
22	.090	22	-.021
23	-.059	23	.018
24	.099	24	-.048
25	-.096	25	.010
26	.002	26	.048
27	.010	27	-.005
28	.018	28	-.027
29	-.012	29	.107
30	.004	30	-.080
31	.062	31	-.023
32	-.025	32	-.004
33	.026	33	.058
34	-.044	34	.002

TABLE V-7 (cont'd.)

Number of Lags on Series α_{11} (k)	Cross Correlation $r_{\alpha_{11}\alpha_{12}}(k)$	Number of Lags on Series α_{12} (k)	Correlation $r_{\alpha_{12}\alpha_{11}}(k)$
35	.033	35	-.053
36	-.060	36	.054
37	.071	37	-.002
38	-.013	38	-.020
39	-.021	39	.085
40	.004	40	-.026
41	.034	41	-.074
42	-.019	42	.055
43	-.068	43	-.023
44	.015	44	.017
45	.021	45	-.062
46	.018	46	.082
47	-.051	47	-.039
48	-.079	48	.018

Prewhitening:

$$(1-.8681B)(1-.2190B-.3041B^3)(1+.6101B^{12})[FM_t] = (1+.1169B-.1347B^{10}-.1810B^{14})a_t$$

- same model on $(1-B^{12})(1-B)\log[\text{Fuel PPI}]$.

TABLE V-7 (cont'd.)

Cross Correlations:

Series α_{22} - Prewhitened PPI of Fuel Power and Related Products

Series α_{21} - Prewhitened Filtered Money Supply ($\delta_{24} = -.5520$)

Mean of Series α_{22} = .77038E-03

St. Dev. of Series α_{22} = .96624E-02

Mean of Series α_{21} = .56007E-02

St. Dev. of Series α_{21} = .78974E-02

Number of Lags of Series α_{22} (k)	Cross Correlation $r_{\alpha_{22}\alpha_{21}}(k)$	Number of Lags on Series α_{21} (k)	Cross Correlation $r_{\alpha_{21}\alpha_{22}}(k)$
0	-.000	0	-.000
1	.045	1	-.029
2	-.005	2	-.042
3	-.078	3	-.069
4	.041	4	-.018
5	.005	5	-.021
6	-.041	6	-.031
7	-.032	7	.009
8	.001	8	.070
9	-.088	9	.005
10	-.020	10	.042
11	-.048	11	.166
12	-.038	12	.057
13	.034	13	.086
14	-.048	14	.086
15	-.070	15	.048
16	.111	16	.069
17	-.029	17	.096
18	.002	18	.055
19	.022	19	.057
20	.014	20	.141
21	-.009	21	.000
22	.018	22	.119
23	-.017	23	.079
24	.024	24	.064
25	.066	25	.031
26	.070	26	.061
27	-.007	27	.039
28	.099	28	.029
29	.050	29	.038
30	.003	30	.059
31	.003	31	.064
32	.063	32	.053
33	.026	33	.022
34	.003	34	.066

TABLE V-7 (cont'd.)

Number of Lags of Series α_{22} (k)	Cross Correlation $r_{\alpha_{22}\alpha_{21}}(k)$	Number of Lags of Series α_{21} (k)	Cross Correlation $r_{\alpha_{21}\alpha_{22}}(k)$
35	-.009	35	.094
36	.064	36	.027
37	.048	37	.075
38	-.006	38	.040
39	.029	39	.013
40	.105	40	.008
41	-.045	41	.003
42	.033	42	.003
43	.018	43	-.034
44	.021	44	-.003
45	-.000	45	-.039
46	.011	46	.035
47	-.039	47	-.036
48	.048	48	-.048

Prewhitening:

$(1-.6606B)(1-B^{12})(1-B)\log[\text{Fuel PPI}] = (1-.8423B^{12})a_t$

- same model on levels of FM_t.

two-input transfer function.

An examination of $r_{\alpha_{22}\alpha_{21}}(k)$ also shows two coefficients which vary from zero. Analagously, we can use the cross-correlations between prewhitened Fuel prices and the similarly transformed output series to identify the second impulse response function in this two-input transfer function.

The Estimation Stage

We are now ready to redevelop equations (5.25), (5.26), and (5.27), with Fuel prices as our second input.[13]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with δ_{24}=-.4920

x_{2t} = PPI of Fuel, Power, and Related Products

Sample: May, 1950 - September, 1973 (n = 281)

$$(5.25)' \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega'_o}{1-\delta'_1 B} B^8(1-B^{12})(1-B)\log[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = \underset{(.061)}{.1928}$ $\quad \hat{\delta}_{24} = \underset{(.260)}{-.4917}$ $\quad \hat{\omega}'_o = \underset{(.073)}{-.0840}$ $\quad \hat{\delta}'_1 = \underset{(.131)}{.8882}$

$\hat{\theta}_{12} = \underset{(.040)}{.8059}$ $\quad \hat{\phi}_1 = \underset{(.063)}{.2637}$

$\chi^2_{46} = 39.2$ $\quad$ RSE = .0135 $\quad$ $\bar{R}^2$ = .5373

A comparison of these parameter estimates with those in equations (5.25) through (5.27) shows that the parameters,

$\hat{\omega}_0'$, $\hat{\delta}_{24}$, $\hat{\theta}_{12}$, and $\hat{\phi}_1$ are quite stable as we change from BCD92 as our second input, to Fuel prices. A comparison of the parameters $\hat{\omega}_0'$ and $\hat{\delta}_1'$ with the coefficients in the single-input transfer function with input, Fuel prices (on page 179), shows that the form of this impulse response function is also quite stable when the second input, FM_t, is added.

We are interested in how this model will forecast over the next 18 months, through the oil crunch. Table V-8 shows these 18 one-step-ahead forecasts. Comparison with Table V-3 shows much improvement in reducing the forecast errors appearing in late 1974 in our single-input model. Comparison with Table V-5 shows that Fuel prices are more successful in reducing these forecast errors in late 1974 than is BCD92. The Theil Decomposition statistics support this finding.

We now proceed to re-estimate equation (5.26) with Fuel prices as our second input.[14]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.4600$

x_{2t} = PPI of Fuel, Power, and Related Products

Sample: May, 1950 - March, 1975 (n = 299)

$$(5.26)' \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_0}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega_0'}{1-\delta_1'B} B^8(1-B^{12})(1-B)\log[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

TABLE V-8

Forecasting IP with x_1 = FM_t (δ = -.4920) and x_2 = Fuel PPI with the model estimated through the sample: 1950.5 - 1973.9 (n = 281) in equation (5.25)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	Actual (T.O. + 1)	(Actual-forecast) Forecast Error	% of Actual
281	73.9	134.433	131.0	138.0	135.3	.867	.64
282	73.10	132.396	129.0	135.9	132.9	.504	.38
283	73.11	129.110	125.8	132.5	126.7	-2.410	-1.90
284	73.12	126.189	122.9	129.5	126.3	.111	.09
285	74.1	128.782	125.5	132.2	129.8	1.018	.78
286	74.2	129.942	126.6	133.4	130.8	.858	.66
287	74.3	130.309	126.9	133.8	129.9	-.409	-.32
288	74.4	129.166	125.8	132.6	131.7	2.534	1.92
289	74.5	134.557	131.1	138.1	135.3	.743	.55
290	74.6	126.441	123.2	129.8	127.3	.859	.67
291	74.7	130.881	127.5	134.4	131.4	.519	.39
292	74.8	133.282	130.4	137.4	135.5	1.672	1.23
293	74.9	133.391	129.9	136.9	133.1	-.291	-.22
294	74.10	127.708	124.4	131.1	125.5	-2.208	-1.76
295	74.11	118.367	115.3	121.5	114.9	-3.467	-3.02
296	74.12	111.613	108.7	114.6	111.8	.187	.17
297	75.1	111.924	109.0	114.9	113.0	1.076	.95
298	75.2	111.189	108.3	114.1	111.8	.611	.55

Theil Decomposition:

ME = -.153556
MAE = 1.129667
MSE = 2.115044
RMSE = 1.454315
UM = .011148 = 0.5% of MSE
US = .058625 = 2.8% of MSE
UC = 2.045270 = 96.7% of MSE

$$\frac{\text{RMSE}}{\bar{y}} = \frac{1.454315}{126.8333} = .011$$

$$\hat{\omega}_o = \underset{(.060)}{.1814} \quad \hat{\delta}_{24} = \underset{(.280)}{-.4582} \quad \hat{\omega}'_o = \underset{(.060)}{-.1132} \quad \hat{\delta}'_1 = \underset{(.141)}{.8332}$$

$$\hat{\theta}_{12} = \underset{(.040)}{.7943} \quad \hat{\phi}_1 = \underset{(.060)}{.2729}$$

$$\chi^2_{46} = 37.4 \qquad RSE = .0133 \qquad \overline{R}^2 = .5509$$

Note again that the parameter estimates are extremely stable as we increase the sample size from equation (5.25)' to (5.26)'.

We want to examine this model's ability to forecast over the next 4 1/2 years. These 56 one-step-ahead forecasts appear in Table V-9. The forecast errors indicate that this model performs quite well over this long forecast horizon, and the Theil Decomposition statistics show marked improvement over previous models. Hence we proceed to the last step and re-estimate equation (5.27) with our new second input, Fuel prices.[15]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.5520$

x_{2t} = PPI of Fuel, Power, and Related Products

Sample: May, 1950 - November, 1979 (n = 355)

$$(5.27)' \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega'_o}{1-\delta'_1 B} B^8(1-B^{12})(1-B)\log[x_{2t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

TABLE V-9

Forecasting I.P. with $x_1 = FM_t$ ($\delta = -.4600$) and x_2 = Fuel PPI

with the model estimated thru sample: 1950.5 - 1975.3 (n = 299) in equation (5.26)'

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O. + 1) Actual	(Actual-forecast) Forecast Error	% of Actual
299	75.3	109.416	105.7	112.2	113.0	3.584	3.17
300	75.4	112.721	109.9	115.6	113.8	1.079	.95
301	75.5	115.804	112.9	118.8	119.2	3.396	2.85
302	75.6	112.329	109.5	115.2	114.5	2.171	1.90
303	75.7	119.160	116.2	122.2	121.4	2.240	1.85
304	75.8	126.100	122.9	129.4	125.9	-.200	-.16
305	75.9	125.973	122.8	129.2	125.4	-.573	-.46
306	75.10	122.506	119.4	125.7	123.8	1.294	1.05
307	75.11	119.424	116.4	122.5	119.8	.376	.31
308	75.12	120.189	117.2	123.3	122.2	2.011	1.65
309	76.1	126.204	123.0	129.5	128.3	2.096	1.63
310	76.2	129.806	126.5	133.2	128.6	-1.206	-.94
311	76.3	129.028	125.8	132.4	128.7	-.328	-.25
312	76.4	129.552	126.3	132.9	130.0	.448	.34
313	76.5	134.177	130.8	137.6	133.2	-.977	-.73
314	76.6	126.101	122.8	129.3	126.5	.490	.39
315	76.7	132.446	129.1	135.9	131.7	-.746	-.57
316	76.8	136.910	133.5	140.4	134.3	-2.610	-1.94
317	76.9	134.613	131.2	138.1	133.8	-.813	-.61
318	76.10	131.530	128.2	134.9	132.1	.570	.43
319	76.11	128.319	125.1	131.6	128.3	-.019	-.01
320	76.12	130.076	126.8	133.4	128.8	-1.276	-.98
321	77.1	133.548	130.2	137.0	133.6	.052	.04
322	77.2	134.995	131.6	138.5	135.7	.705	.52
323	77.3	137.236	133.8	140.8	136.2	-1.036	-.76
324	77.4	137.693	134.2	141.2	137.2	-.493	-.36
325	77.5	141.485	137.9	145.1	141.5	.015	.01
326	77.6	134.245	130.9	137.7	134.1	-.145	-.11

TABLE V-9 (cont'd.)

T.O.	Yr./Mo.	Forecast	(95% Conf. Int.) Lower	Upper	(T.O. + 1) Actual	(Actual-forecast) Forecast Error	% of Actual
327	77.7	140.374	136.8	144.0	138.2	-2.174	-1.57
328	77.8	142.905	139.3	146.6	142.4	-.505	-.35
329	77.9	142.339	138.8	146.0	142.7	.361	.25
330	77.10	140.076	136.6	143.7	139.5	-.576	-.41
331	77.11	134.457	131.1	137.9	134.9	.443	.33
332	77.12	135.649	132.2	139.1	134.8	-.849	-.63
333	78.1	138.986	135.5	142.6	139.6	.614	.44
334	78.2	140.689	137.2	144.3	141.4	.711	.50
335	78.3	142.422	138.8	146.1	144.2	1.778	1.23
336	78.4	146.282	142.6	150.1	144.2	-2.082	-1.44
337	78.5	148.679	144.9	152.5	149.8	1.121	.75
338	78.6	142.698	139.1	146.4	142.9	.202	.14
339	78.7	149.350	145.6	153.2	148.2	-1.150	-.78
340	78.8	153.469	149.6	157.4	153.0	-.469	-.31
341	78.9	153.605	149.7	157.6	153.4	-.205	-.13
342	78.10	150.871	147.1	154.8	150.5	-.371	-.25
343	78.11	145.761	142.1	149.5	147.1	1.339	.91
344	78.12	148.280	144.6	152.1	146.6	-1.680	-1.15
345	79.1	151.269	147.5	155.2	152.3	1.031	.68
346	79.2	154.012	150.1	158.0	154.0	-.012	-.01
347	79.3	155.823	151.9	159.8	151.1	-4.723	-3.13
348	79.4	151.168	147.4	155.1	152.5	1.332	.87
349	79.5	157.985	154.0	162.1	156.5	-1.485	-.95
350	79.6	148.811	145.1	152.6	148.7	-.111	-.07
351	79.7	155.089	151.2	159.1	152.3	-2.789	-1.83
352	79.8	156.951	153.0	161.0	156.8	-.151	-.10
353	79.9	157.062	153.1	161.1	155.7	-1.362	-.87
354	79.10	152.193	148.4	156.1	152.2	.007	.00

Theil Decomposition:

ME = .029464   RMSE = 1.464241   UC = 1.894165 = 88.4% of MSE

MAE = 1.081821   UM = .000405 = 0.0% of MSE $\frac{RMSE}{\bar{y}} = \frac{1.464241}{137.62678} = .011$

MSE = 2.144000   US = .249430 = 11.6% of MSE

$\hat{\omega}_o = \underset{(.051)}{.1659}$ $\hat{\delta}_{24} = \underset{(.209)}{-.5510}$ $\hat{\omega}'_o = \underset{(.056)}{-.1160}$ $\hat{\delta}'_1 = \underset{(.155)}{.7145}$

$\hat{\theta}_{12} = \underset{(.035)}{.7984}$ $\hat{\phi}_1 = \underset{(.055)}{.2804}$

$\chi^2_{46} = 41.5$ RSE $= .0128$ $\bar{R}^2 = .5840$

Again note the stability of the parameter estimates as we increase the sample size, moving from equation (5.26)' to equation (5.27)'. This is our model for the entire sample period with these inputs. We are again interested in the implied impulse and step response functions between $\hat{M}_t$ and $\hat{y}_t$.

<u>Implications of the Final Model</u>

Again equations (5.28), (5.29), and (5.30) are relevant in the development of these functions. From equation (5.27)' we have $\hat{\omega}_o = .1659$, $-\delta_{24} = .552$, and $\hat{\delta}_{24} = -.551$. Using these estimates in equation (5.30) gives us the impulse response function of this model, $v_1(B)$. Figure 5.8 shows the impulse and step response functions.

It is not surprising that this figure bears much resemblance to Figure 5.6.

To examine the convergence of this step response function we follow the same procedure as before.

The sum of the first 12 impulse response weights $= V_{12} = 1.3483$.

The sum of the next 24 impulse response weights $= 1.3959$.

Thus, $V_{36} = 1.3483 - 1.3959 = -.0475$.

FIGURE 5.8

GRAPH OF IMPULSE RESPONSE WEIGHTS [$v_1(B)$]

```
            -.5       0.        .5        1.
(k).+++++++++.+++++++++.+++++++++.+++++++++.     VALUES [vk]
 0                     XXXX                      .1659
 1                     XXXX                      .1642
 2                     XXXX                      .1603
 3                     XXXX                      .1533
 4                     XXXX                      .1437
 5                     XXXX                      .1316
 6                     XXX                       .1173
 7                     XXX                       .1010
 8                     XXX                       .0830
 9                     XX                        .0635
10                     XX                        .0430
11                     X                         .0216
12                     X                        0.
13                     X                        -.0119
14                     X                        -.0237
15                    XX                        -.0351
16                    XX                        -.0458
17                    XX                        -.0558
18                    XX                        -.0648
19                    XX                        -.0726
20                   XXX                        -.0793
21                   XXX                        -.0846
22                   XXX                        -.0885
23                   XXX                        -.0907
24                   XXX                        -.0914
25                   XXX                        -.0905
26                   XXX                        -.0883
27                   XXX                        -.0845
28                   XXX                        -.0792
29                    XX                        -.0725
30                    XX                        -.0647
31                    XX                        -.0557
32                    XX                        -.0457
33                    XX                        -.0350
34                     X                        -.0237
35                     X                        -.0119
36                     X                        0.
37                     X                         .0066
38                     X                         .0131
39                     X                         .0193
40                     XX                        .0252
41                     XX                        .0307
42                     XX                        .0357
43                     XX                        .0400
44                     XX                        .0437
45                     XX                        .0466
46                     XX                        .0487
47                     XX                        .0500
48                     XX                        .0505
```

FIGURE 5.8 (cont'd.)

GRAPH OF STEP RESPONSE WEIGHTS $[V_1(B)]$

```
            0.        .5        1.0       1.5
(k).+++++++++.+++++++++.+++++++++.+++++++++.       VALUES [V_k]
  0          XXXX                                 .1659
  1          XXXXXXXX                             .3301
  2          XXXXXXXXXXX                          .4908
  3          XXXXXXXXXXXXXX                       .6437
  4          XXXXXXXXXXXXXXXXX                    .7874
  5          XXXXXXXXXXXXXXXXXXX                  .9190
  6          XXXXXXXXXXXXXXXXXXXXXX              1.0363
  7          XXXXXXXXXXXXXXXXXXXXXXXX            1.1373
  8          XXXXXXXXXXXXXXXXXXXXXXXXX           1.2203
  9          XXXXXXXXXXXXXXXXXXXXXXXXXXX         1.2838
 10          XXXXXXXXXXXXXXXXXXXXXXXXXXXX        1.3268
 11          XXXXXXXXXXXXXXXXXXXXXXXXXXXX        1.3484
 12          XXXXXXXXXXXXXXXXXXXXXXXXXXXX        1.3484
 13          XXXXXXXXXXXXXXXXXXXXXXXXXXXX        1.3365
 14          XXXXXXXXXXXXXXXXXXXXXXXXXXX         1.3128
 15          XXXXXXXXXXXXXXXXXXXXXXXXXXX         1.2777
 16          XXXXXXXXXXXXXXXXXXXXXXXXXX          1.2319
 17          XXXXXXXXXXXXXXXXXXXXXXXXX           1.1761
 18          XXXXXXXXXXXXXXXXXXXXXXX             1.1113
 19          XXXXXXXXXXXXXXXXXXXXXX              1.0387
 20          XXXXXXXXXXXXXXXXXXXXX                .9594
 21          XXXXXXXXXXXXXXXXXXX                  .8748
 22          XXXXXXXXXXXXXXXXX                    .7863
 23          XXXXXXXXXXXXXXX                      .6956
 24          XXXXXXXXXXXXX                        .6042
 25          XXXXXXXXXXX                          .5137
 26          XXXXXXXXXX                           .4254
 27          XXXXXXXX                             .3409
 28          XXXXXX                               .2617
 29          XXXXX                                .1892
 30          XXX                                  .1245
 31          XX                                   .0688
 32          X                                    .0231
 33          X                                   -.0119
 34         XX                                   -.0356
 35         XX                                   -.0475
 36         XX                                   -.0475
 37         XX                                   -.0409
 38         XX                                   -.0278
 39          X                                   -.0085
 40          X                                    .0157
 41          XX                                   .0474
 42          XXX                                  .0831
 43          XXX                                  .1231
 44          XXXX                                 .1668
 45          XXXXX                                .2134
 46          XXXXXX                               .2621
 47          XXXXXXX                              .3121
 48          XXXXXXXX                             .3626
```

The sum of the following 24 impulse response weights

$= (1.3959)(.551)$

$= .7691.$

Thus, $V_{60} = -.0475 + .7691 = .7216.$

The sum of the following 24 impulse response weights

$= (1.3959)(.551)^2$

$= .4238$.

Thus, $V_{84} = .7216 - .4238 = .2978$.

Continuing:

$(1.3959)(.551)^3 = .2335$

$V_{108} = .2978 + .2335 = .5313$

$(1.3959)(.551)^4 = .1287$

$V_{132} = .5313 - .1287 = .4026$

$(1.3959)(.551)^5 = .0709$

$V_{156} = .4026 + .0709 = .4735$

$(1.3959)(.551)^6 = .0391$

$V_{180} = .4735 - .0391 = .4344$

$(1.3959)(.551)^7 = .0215$

$V_{204} = .4344 + .0215 = .4559$

$(1.3959)(.551)^8 = .0118$

$V_{228} = .4559 - .0118 = .4441$

$(1.3959)(.551)^9 = .0065$

$V_{252} = .4441 + .0065 = .4506$

Hence the step response function will converge to approximately .4480. Of course the implications are analagous to those of our previous model in equation (5.27), with x_{2t} = BCD92.

Again, we are also interested in the impulse and step response functions for the second input, Fuel prices, implied by our model in equation (5.27)'. These are listed and plotted in Figure 5.9.

Expanding the Model to Account for Supply Shocks Due to Strikes in the Labor Force

At this point we need to consider another potential fault with our model in equations (5.27) and (5.27)'. In the 33 year sample period reviewed, the Index of Industrial Production was substantially influenced at various times by strikes in the Labor Force. We are concerned with the performance of our models during these times. In this regard, a list of specific strikes, their dates, and the sectors of the economy affected, is presented below.

Late 1969: Steel Strike (116 days)

February, 1959: Coal

1970: Teamsters

General Motors (Fall, 134 days)

1974: Coal

1977: Longshoremen

An examination of the residuals of the models in equations (5.27) and (5.27)' reveals very few outlyers in all of the time periods listed above. Further, the few outlyers which do appear near any of these time periods are not extremely large. This suggests that strikes may not present a serious problem in our models. However, we are interested in the possibility of improving the models by including a third input which accounts for these strike episodes. In this role we use the number of hours of work stoppage due to

FIGURE 5.9

GRAPH OF IMPULSE RESPONSE WEIGHTS [$v_2(B)$]
for x_{2t} = Fuel PPI in equation (5.27)'

```
                  -.1        0.        .1
(k).+++++++++.+++++++++.+++++++++.+++++++++.      VALUES [v_k]
 0                      X                         0.
 1                      X                         0.
 2                      X                         0.
 3                      X                         0.
 4                      X                         0.
 5                      X                         0.
 6                      X                         0.
 7                      X                         0.
 8          XXXXXXXXXXXXX                         -.116038E+00
 9              XXXXXXXXX                         -.829026E-01
10                XXXXXXX                         -.592249E-01
11                  XXXXX                         -.423162E-01
12                   XXXX                         -.302326E-01
13                    XXX                         -.215996E-01
14                    XXX                         -.154317E-01
15                     XX                         -.110251E-01
16                     XX                         -.787686E-02
17                     XX                         -.562759E-02
18                      X                         -.402060E-02
19                      X                         -.287250E-02
20                      X                         -.205225E-02
21                      X                         -.146622E-02
22                      X                         -.104753E-02
23                      X                         -.748407E-03
24                      X                         -.534696E-03
25                      X                         -.382011E-03
26                      X                         -.272926E-03
27                      X                         -.194991E-03
28                      X                         -.139311E-03
29                      X                         -.995298E-04
30                      X                         -.711087E-04
31                      X                         -.508033E-04
32                      X                         -.362962E-04
33                      X                         -.259317E-04
34                      X                         -.185268E-04
35                      X                         -.132364E-04
36                      X                         -.945666E-05
37                      X                         -.675627E-05
38                      X                         -.482699E-05
39                      X                         -.344862E-05
40                      X                         -.246385E-05
41                      X                         -.176029E-05
42                      X                         -.125763E-05
43                      X                         -.898510E-06
44                      X                         -.641936E-06
45                      X                         -.458629E-06
46                      X                         -.327665E-06
```

FIGURE 5.9 (cont'd.)

```
           -.1        0.         .1
(k).+++++++++.+++++++++.+++++++++.+++++++++.     VALUES [v_k]
47                      X                        -.234099E-06
48                      X                        -.167251E-06
```

FIGURE 5.9 (cont'd.)

GRAPH OF STEP RESPONSE WEIGHTS [$V_2(B)$] for x_{2t} = Fuel PPI

```
            -.5       -.25      0.         .25
(k).+++++++++.+++++++++.+++++++++.++++++++++.+++++++++.VALUES
  0                              X                   0.
  1                              X                   0.
  2                              X                   0.
  3                              X                   0.
  4                              X                   0.
  5                              X                   0.
  6                              X                   0.
  7                              X                   0.
  8                         XXXXXX                   -.116038
  9                      XXXXXXXXX                   -.198941
 10                    XXXXXXXXXXX                   -.258170
 11                  XXXXXXXXXXXXX                   -.300486
 12                 XXXXXXXXXXXXXX                   -.330719
 13                XXXXXXXXXXXXXXX                   -.352319
 14                XXXXXXXXXXXXXXX                   -.367751
 15               XXXXXXXXXXXXXXXX                   -.378776
 16               XXXXXXXXXXXXXXXX                   -.386652
 17              XXXXXXXXXXXXXXXXX                   -.392280
 18              XXXXXXXXXXXXXXXXX                   -.396310
 19              XXXXXXXXXXXXXXXXX                   -.399173
 20              XXXXXXXXXXXXXXXXX                   -.401225
 21              XXXXXXXXXXXXXXXXX                   -.402692
 22              XXXXXXXXXXXXXXXXX                   -.403739
 23              XXXXXXXXXXXXXXXXX                   -.404488
 24              XXXXXXXXXXXXXXXXX                   -.405022
 25              XXXXXXXXXXXXXXXXX                   -.405404
 26              XXXXXXXXXXXXXXXXX                   -.405677
 27              XXXXXXXXXXXXXXXXX                   -.405872
 28              XXXXXXXXXXXXXXXXX                   -.406011
 29              XXXXXXXXXXXXXXXXX                   -.406111
 30              XXXXXXXXXXXXXXXXX                   -.406182
 31              XXXXXXXXXXXXXXXXX                   -.406232
 32              XXXXXXXXXXXXXXXXX                   -.406269
 33              XXXXXXXXXXXXXXXXX                   -.406295
 34              XXXXXXXXXXXXXXXXX                   -.406313
 35              XXXXXXXXXXXXXXXXX                   -.406326
 36              XXXXXXXXXXXXXXXXX                   -.406336
 37              XXXXXXXXXXXXXXXXX                   -.406343
 38              XXXXXXXXXXXXXXXXX                   -.406347
 39              XXXXXXXXXXXXXXXXX                   -.406350
 40              XXXXXXXXXXXXXXXXX                   -.406353
 41              XXXXXXXXXXXXXXXXX                   -.406355
 42              XXXXXXXXXXXXXXXXX                   -.406356
 43              XXXXXXXXXXXXXXXXX                   -.406357
 44              XXXXXXXXXXXXXXXXX                   -.406358
 45              XXXXXXXXXXXXXXXXX                   -.306358
 46              XXXXXXXXXXXXXXXXX                   -.406358
 47              XXXXXXXXXXXXXXXXX                   -.406359
 48              XXXXXXXXXXXXXXXXX                   -.406359
```

strikes. The resulting models are shown below; first with x_{2t} = BCD92, and following, with x_{2t} = Fuel prices.[16,17]

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.5360$

x_{2t} = BCD92 % change in PPI of Crude Materials

x_{3t} = Work Stoppage due to Strikes

Sample: May, 1950 - July, 1979 (n = 351)

$$(5.31)\quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}] + \frac{\omega'_o}{1-\delta'_1 B} B^{10}(1-B^{12})[x_{2t}] + \omega''_o (1-B^{12})(1-B)\log[x_{3t}] + \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = .1969$ (.051) $\hat{\delta}_{24} = -.5409$ (.171) $\hat{\omega}'_o = -.0870$ (.038) $\hat{\delta}'_1 = .7100$ (.146)

$\hat{\omega}''_o = -.0118$ (.001) $\hat{\theta}_{12} = .7578$ (.039) $\hat{\phi}_1 = .3105$ (.055)

$\chi^2_{46} = 56.7$ RSE = .0117 $\bar{R}^2 = .6524$

y_t = BCD47 Index of Industrial Production

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.5231$

x_{2t} = PPI of Fuel, Power, and Related Products

x_{3t} = Work Stoppage due to Strikes

Sample: May, 1950 - July, 1979 (n = 351)

$$(5.31)' \quad (1-B^{12})(1-B)\log(y_t) = \frac{\omega_o}{1-\delta_{24}B^{24}}[x_{1t}]$$

$$+ \frac{\omega_o'}{1-\delta_1'B} B^8(1-B^{12})(1-B)\log[x_{2t}]$$

$$+ \omega_o''(1-B^{12})(1-B)\log[x_{3t}]$$

$$+ \frac{1-\theta_{12}B^{12}}{1-\phi_1 B} a_t$$

$\hat{\omega}_o = \underset{(.050)}{.1881}$ $\hat{\delta}_{24} = \underset{(.187)}{-.5189}$ $\hat{\omega}_o' = \underset{(.058)}{-.1314}$ $\hat{\delta}_1' = \underset{(.168)}{.6600}$

$\hat{\omega}_o'' = \underset{(.001)}{-.0122}$ $\hat{\theta}_{12} = \underset{(.036)}{.7781}$ $\hat{\phi}_1 = \underset{(.054)}{.3098}$

$\chi^2_{46} = 59.2$ RSE $= .0117$ $\bar{R}^2 = .6524$

A comparison of equations (5.31) and (5.31)' with equations (5.27) and (5.27)' reveals an apparently substantial reduction in the RSE when x_{3t} is included, although some reduction is expected with the addition of any third variable.

We are especially concerned with the performance of our new models during the years in which strikes substantially influenced GNP. The residuals of the models in equations (5.31) and (5.31)' during these strike periods, show no noteworthy improvement over those in equations (5.27) and (5.27)'. This suggests that our models in equations (5.27) and (5.27)' may be considered adequate with regard to the problem of strikes.

FOOTNOTES

CHAPTER V

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[6]Note that we must supply the univariate model for our input, $x_t = FM_t$, as the prewhitening model for the estimation of this transfer function in equation (5.7). Since our input consists of a 24-month moving average of M_t, it is not surprising that we find the complicated univariate model below.

$$x_t = FM_t = \text{Filtered } M_t \text{ as in (5.6), with } \delta_{24} = -.5207$$

Sample: May, 1950-September, 1973: n = 281

$$(1-\phi_1'B)(1-\phi_1B-\phi_3B^3)(1-\phi_{12}B^{12})[x_t] = (1-\theta_2B^2-\theta_{10}B^{10}-\theta_{13}B^{13})a_t$$

$$\hat{\phi}_1' = \underset{(.190)}{.8618} \quad \hat{\phi}_1 = \underset{(.216)}{.2633} \quad \hat{\phi}_3 = \underset{(.147)}{.3303} \quad \hat{\phi}_{12} = \underset{(.055)}{-.6333}$$

$$\hat{\theta}_2 = \underset{(.116)}{-.1683} \quad \hat{\theta}_{10} = \underset{(.067)}{.1452} \quad \hat{\theta}_{13} = \underset{(.065)}{.1493}$$

$$\chi^2_{40} = 61.9 \qquad RSE = .0043$$

[7]op. cit., Box, G. and Jenkins, G., Chapter 11.

[8]op. cit., Box, G. and Jenkins, G., Chapter 11, p. 382.

[9]For use in the estimation of equation (5.25), the univariate models for the two inputs over the sample, May, 1950-September, 1973 (n = 281), are the following.

$$x_{1t} = FM_t = \text{Filtered } M_t \text{ as in (5.6), with}$$

$$\delta_{24} = -.5231$$

Same model as in footnote 6, with:

$$\hat{\phi}_1' = \underset{(.194)}{.8669} \quad \hat{\phi}_1 = \underset{(.220)}{.2574} \quad \hat{\phi}_3 = \underset{(.149)}{.3267} \quad \hat{\phi}_{12} = \underset{(.055)}{-.6338}$$

$$\hat{\theta}_2 = \underset{(.117)}{-.1652} \quad \hat{\theta}_{10} = \underset{(.066)}{.1451} \quad \hat{\theta}_{13} = \underset{(.065)}{.1492}$$

$$\chi^2_{40} = 62.1 \qquad RSE = .0043$$

x_{2t} = BCD92 % change in PPI of Crude Materials

$$(1-\phi_1B-\phi_3B^3)(1-B^{12})[x_{2t}] = (1-\theta_2B^2-\theta_{12}B^{10}-\theta_{14}B^{13})a_t$$

$$\hat{\phi}_1 = \underset{(.063)}{.4133} \quad \hat{\phi}_3 = \underset{(.056)}{.1856} \quad \hat{\theta}_2 = \underset{(.066)}{.0977} \quad \hat{\theta}_{12} = \underset{(.041)}{.7680}$$

$$\hat{\theta}_{14} = \underset{(.065)}{-.1586} \quad \chi^2_{25} = 19.1 \quad RSE = .0125$$

[10]For use in the estimation of equation (5.26), the univariate models for the two inputs over the sample, May, 1950-March, 1975 (n = 299), are the following.

x_{1t} = FM_t = Filtered M_t as in (5.6), with

$$\delta_{24} = -.552$$

Same model as in footnote 6, with:

$$\hat{\phi}'_1 = \underset{(.223)}{.8563} \quad \hat{\phi}_1 = \underset{(.248)}{.2752} \quad \hat{\phi}_3 = \underset{(.162)}{.3303} \quad \hat{\phi}_{12} = \underset{(.051)}{-.6080}$$

$$\hat{\theta}_2 = \underset{(.128)}{-.1219} \quad \hat{\theta}_{10} = \underset{(.062)}{.1419} \quad \hat{\theta}_{13} = \underset{(.061)}{.1638}$$

$$\chi^2_{40} = 69.2 \quad RSE = .0043$$

x_{2t} = BCD92 % change in PPI of Crude Materials

Same model as in footnote 9, with:

$$\hat{\phi}_1 = \underset{(.060)}{.4290} \quad \hat{\phi}_3 = \underset{(.055)}{.2181} \quad \hat{\theta}_2 = \underset{(.065)}{.0739} \quad \hat{\theta}_{12} = \underset{(.042)}{.7739} \quad \hat{\theta}_{14} = \underset{(.067)}{-.1317}$$

$$\chi^2_{25} = 25.1 \quad RSE = .0130$$

[11]For use in the estimation of equation (5.27), the univariate models for the two inputs over the sample, May, 1950 - August, 1979 (n = 352), are the following.

x_{1t} = FM_t = Filtered M_t as in (5.6), with $\delta_{24} = -.5600$

Same model as in footnote 6, with:

$$\hat{\phi}'_1 = \underset{(.118)}{.8673} \quad \hat{\phi}_1 = \underset{(.141)}{.2197} \quad \hat{\phi}_3 = \underset{(.105)}{.3052} \quad \hat{\phi}_{12} = \underset{(.046)}{-.6109}$$

$$\hat{\theta}_2 = \underset{(.086)}{-.1173} \quad \hat{\theta}_{10} = \underset{(.058)}{.1342} \quad \hat{\theta}_{13} = \underset{(.057)}{.1819}$$

$$\chi^2_{40} = 67.5 \quad RSE = .0045$$

x_{2t} = BCD92 % change in PPI of Crude Materials

Same model as in footnote 9, with:

$$\hat{\phi}_1 = \underset{(.054)}{.3850} \quad \hat{\phi}_3 = \underset{(.049)}{.2658} \quad \hat{\theta}_2 = \underset{(.058)}{.1542} \quad \hat{\theta}_{12} = \underset{(.034)}{.8120} \quad \hat{\theta}_{14} = \underset{(.057)}{-.1975}$$

$$\chi^2_{25} = 42.6 \qquad RSE = .0142$$

[12]The univariate model for the input, the Fuel PPI, is needed in the estimation of this single-input transfer function. This model is presented below.

$$(1-\phi_1 B)(1-B^{12})(1-B) \log [x_t] = (1-\theta_{12}B^{12}) a_t$$

$$\hat{\phi}_1 = \underset{(.042)}{.6606} \qquad \hat{\theta}_{12} = \underset{(.031)}{.8423}$$

$$\chi^2_{46} = 52.8 \qquad RSE = .0098$$

where x_t = PPI of Fuel, Power, and Related Products, and the Sample period is May, 1950-November, 1979 (n = 355).

[13]For use in the estimation of equation (5.25)', the univariate models for the two inputs over the sample, May, 1950-September, 1973 (n = 281), are the following.

x_{1t} = FM_t = Filtered M_t as in (5.6), with

$$\delta_{24} = -.492$$

Same model as in footnote 6, with:

$$\hat{\phi}'_1 = \underset{(.195)}{.8664} \quad \hat{\phi}_1 = \underset{(.221)}{.2594} \quad \hat{\phi}_3 = \underset{(.149)}{.3230} \quad \hat{\phi}_{12} = \underset{(.055)}{-.6239}$$

$$\hat{\theta}_2 = \underset{(.118)}{-.1633} \quad \hat{\theta}_{10} = \underset{(.067)}{.1467} \quad \hat{\theta}_{13} = \underset{(.065)}{.1450}$$

$$\chi^2_{40} = 59.9 \qquad RSE = .0043$$

x_{2t} = PPI of Fuel, Power, and Related Products

Same model as in footnote 12, with:

$$\hat{\phi}_1 = \underset{(.061)}{.2377} \qquad \hat{\theta}_{12} = \underset{(.040)}{.8086}$$

$$\chi^2_{46} = 43.4 \qquad RSE = .0079$$

[14] For use in the estimation of equation (5.26)', the univariate models for the two inputs over the sample, May, 1950-March, 1975 (n = 299), are the following.

x_{1t} = FM_t = Filtered M_t as in (5.6), with

$$\delta_{24} = -.4600$$

Same model as in footnote 6, with:

$$\hat{\phi}'_1 = \underset{(.249)}{.8539} \quad \hat{\phi}_1 = \underset{(.273)}{.2833} \quad \hat{\phi}_3 = \underset{(.174)}{.3206} \quad \hat{\phi}_{12} = \underset{(.052)}{-.5945}$$

$$\hat{\theta}_2 = \underset{(.137)}{-.1181} \quad \hat{\theta}_{10} = \underset{(.062)}{.1461} \quad \hat{\theta}_{13} = \underset{(.061)}{.1537}$$

$$\chi^2_{40} = 62.0 \qquad RSE = .0042$$

x_{2t} = PPI of Fuel, Power, and Related Products

Same model as in footnote 12, with:

$$\hat{\phi}_1 = \underset{(.048)}{.6089} \qquad \hat{\theta}_{12} = \underset{(.037)}{.8318}$$

$$\chi^2_{46} = 55.3 \qquad RSE = .0097$$

[15] For use in the estimation of equation (5.27)', the univariate models for the two inputs over the sample: May, 1950-November, 1979 (n = 355), are the following.

x_{1t} = FM_t = Filtered M_t as in (5.6), with

$$\delta_{24} = -.5520$$

Same model as in footnote 6, with

$$\hat{\phi}'_1 = \underset{(.119)}{.8681} \quad \hat{\phi}_1 = \underset{(.143)}{.2190} \quad \hat{\phi}_3 = \underset{(.106)}{.3041} \quad \hat{\phi}_{12} = \underset{(.046)}{-.6101}$$

$$\hat{\theta}_2 = \underset{(.087)}{-.1169} \quad \hat{\theta}_{10} = \underset{(.055)}{.1347} \quad \hat{\theta}_{13} = \underset{(.057)}{.1810}$$

$$\chi^2_{40} = 66.6 \qquad RSE = .0045$$

x_{2t} = PPI of Fuel, Power, and Related Products

Same model as in footnote 12.

[16]For use in the estimation of equation (5.31), the univariate models for the three inputs over the sample, May, 1950-July, 1979 (n = 351), are the following.

$$x_{1t} = FM_t = \text{Filtered } M_t \text{ as in (5.6), with}$$

$$\delta_{24} = -.5360$$

Same model as in footnote 6, with:

$$\hat{\phi}_1' = \underset{(.107)}{.8627} \quad \hat{\phi}_1 = \underset{(.131)}{.2238} \quad \hat{\phi}_3 = \underset{(.101)}{.3132} \quad \hat{\phi}_{12} = \underset{(.046)}{-.6151}$$

$$\hat{\theta}_2 = \underset{(.083)}{-.1147} \quad \hat{\theta}_{10} = \underset{(.059)}{.1497} \quad \hat{\theta}_{13} = \underset{(.057)}{.1806}$$

$$\chi^2_{40} = 60.5 \qquad RSE = .0044$$

$$x_{2t} = \text{BCD92 \% change in PPI of Crude Materials}$$

Same model as in footnote 11.

$$x_{3t} = \text{Work Stoppage due to Strikes}$$

$$(1-B^{12})(1-B)\log[x_{3t}] = (1-\theta_{12}B^{12})\,a_t$$

$$\hat{\theta}_{12} = \underset{(.029)}{.8597}$$

$$\chi^2_{47} = 61.5 \qquad RSE = .404$$

[17]For use in the estimation of equation (5.31)', the univariate models for the three inputs over the sample, May, 1950-July, 1979 (n = 351), are the following.

$$x_{1t} = FM_t = \text{Filtered } M_t \text{ as in (5.6), with}$$

$$\delta_{24} = -.5231$$

Same model as in footnote 6, with:

$$\hat{\phi}_1' = \underset{(.109)}{.8637} \quad \hat{\phi}_1 = \underset{(.133)}{.2227} \quad \hat{\phi}_3 = \underset{(.102)}{.3116} \quad \hat{\phi}_{12} = \underset{(.047)}{-.6136}$$

$$\hat{\theta}_2 = \underset{(.084)}{-.1140} \quad \hat{\theta}_{10} = \underset{(.059)}{.1503} \quad \hat{\theta}_{13} = \underset{(.057)}{.1792}$$

$$\chi^2_{40} = 59.2 \qquad RSE = .0044$$

x_{2t} = PPI of Fuel, Power, and Related Products

Same model as in footnote 12.

x_{3t} = Work Stoppage due to Strikes

Same model as in footnote 16.

CHAPTER VI

CONCLUSION

The proposed leading indicator approach is supported by an appropriate theoretical framework in the form of a dynamic, structural econometric model. In this context, information is obtained about the first two moments of the conditional distribution, $f(y_{t+k}|LI_t)$. This information is the object of our analysis.

A comparison of the forecasting abilities of the proposed approach with the Final Form, reveals essentially that both approaches forecast equally well. That is, both approaches imply the same forecast error variance for forecasts within the lead of the leading indicator.

These observations suggest that the proposed leading indicator approach may deserve more attention as an alternative to the Commerce Department approach.

As we move from this proposed theoretical approach to the empirical evaluation of the Commerce Department leading indicators, justification is needed for the consideration of only one input in the transfer functions developed. It is argued that the bias introduced in the parameter estimates through the omission of relevant variables, is not a problem when forecasting is the main objective. In fact the biased estimates are truly appropriate for forecasting since they contribute to an exact representation of the expectation of the objective variable, conditional on the single input utilized.

With this justification established, bivariate time series models are built describing the empirical relationships between economic activity and eight of the component series in the Commerce Department's CLI. Five of these models show a lack of any significant lead time in the relationship. The other three models suggest relationships with a lead worthy of a "leading indicator". However, the leading indicator which displays the most significant lead over economic activity is the Producer Price Index of Crude Materials, which is seen to have a negative relationship with economic activity, while the Commerce Department uses it in a positive role. These revelations present some possible reasons for the poor performance record of the Commerce Department approach.

Finally, Money is proposed to fill the role of leading indicator. The cross-correlation functions between the prewhitened Money series and the correspondingly transformed Industrial Production series implies an impulse response function which might follow a damped cosine wave with a period of four years. The associated step response at the identification stage bears much resemblance to that implied by the framework suggested by Friedman (1974). This step response function supports a proposition that Money is neutral after 36 months. The step response function approaches zero in that time frame, and the impulse response weights after 36 months, appear to be quite erratic, breaking from the previous appearance of a smooth cosine wave. This may indicate that they reflect only random behavior after that point.

In this light, a bivariate time series model is built, describing the empirical, dynamic relationship between economic activity and Money. An infinite lag model is estimated about the pattern indicated at the identification stage discussed above. This model is expanded to account for two different kinds of supply shocks occurring in the sample period reviewed: the energy price shocks of the early 1970's, and strikes in the Labor Force. At each stage of the expansion of this model, a stable dynamic relationship is observed between Money and economic activity.

After 36 months, the estimated step response function is seen to approach zero, supporting the observations regarding the relationship at the identification stage. However, since an infinite lag relationship is estimated, the impulse response weights after 36 months are constrained to continue to follow the damped cosine wave, even if they truly represent only random movements about zero. These impulse response weights after 36 months must be considered in finding the steady state gain, the convergence of the step response function. These weights are seen to converge to approximately (.45) in the various stages of expansion of the model.

This suggests a nonneutral long run relationship between Money and real GNP, which is somewhat troubling in light of the concepts of current Monetary Theory. Nevertheless, the model provides useful insight into the empirical relationship between Money and real GNP, which is so important in economic theory. In particular, the stable relationship found implies that Money may provide useful information as a leading indicator of economic activity.

APPENDIX 4

DATA SOURCES

APPENDIX 4

DATA SOURCES

The economic indicator series are available in the Business Conditions Digest (BCD), but most are seasonally adjusted.

The Index of Industrial Production (BCD47) appears not seasonally adjusted in The Survey of Current Business, and is available from 1947 to the present.

The Composite Index of Leading Indicators consists of 12 components: BCD series 1, 3, 8, 12, 19, 20, 29, 32, 36, 92, 104, and 105. Series 19, the Index of Stock Prices, and series 32, Vendor Performance, are available not seasonally adjusted in the Business Conditions Digest from 1947 to the present.

BCD series 1, the Average Workweek of Production Workers, appears not seasonally adjusted in The Survey of Current Business, and is available from 1947 to the present.

BCD series 3, the Layoff Rate, is also available not seasonally adjusted in The Survey of Current Business from 1947 to the present.

BCD series 8, the Value of Manufacturers' New Orders for Consumer Goods and Materials, consists of the aggregation of three series: (1) new orders for durable goods industries plus (2) new orders for nondurable goods industries with unfilled orders minus (3) new orders for capital goods and defense products. New orders for durable goods industries is available not seasonally adjusted in The Survey of Current Business from 1947 to the present. The latter two series are both available not seasonally adjusted in the Census Bureau publication, Current Industrial Reports - Manufacturers' Shipments, Inventories, and Orders, from 1958 to the present (see call number C3.158/M3-1.6). These categories are unavailable in this publication for earlier years. Thus our constructed series BCD8 is available from 1958 to the present.

BCD series 29, the Index of New Private Housing Units Authorized by Local Building Permits (not seasonally adjusted), can be constructed from the month to month changes in series HS6BR in the Citibank Data Base. This is available from 1959 to the present.

BCD series 92, the % Change in Sensitive Prices (the PPI of Crude Materials, not seasonally adjusted), can be constructed from series PWCMPX in the Citibank Data Base.

BCD series 105, the Real Money Stock, M_1, not seasonally adjusted, can be constructed by dividing the Nominal Money Stock, M_1 (available as series FZM1 in the Citibank Data Base), by the CPI for All Items (available in The Survey of Current Business). This constructed series is available from 1947 to the present.

We have been unable to locate data for BCD series 12, 20, 36, and 104, on a not seasonally adjusted basis.

Data for the unadjusted Nominal Money Stock series employed in Chapter V is obtained by splicing two different definitions of the Money Stock. This splicing technique is developed extensively in the Chapter. The two Money Stock series used are:

M_1 from 1947.1 - 1958.12: published in the Federal Reserve Bulletin, October, 1960.

M_1B from 1959.1 - present: published in "Redefined Money Stock Measures, Liquid Assets, and Related Measures," Federal Reserve Release, March 24, 1980, with recent updates available in "Federal Reserve Statistical Release H.6," June 20, 1980.

The Producer Price Index for Fuel, Power, and Related Products is published in The Survey of Current Business, from 1947 to the present. However, the numbers appear under three different base periods:

1947.1 - 58.12 with base, 1947-49 = 100;

1959.1 - 66.12 with base, 1957-59 = 100;

1967.1 - present with base, 1967 = 100.

These segments are spliced into a complete series with base, 1967 = 100, as follows. The mean of the observations for 1957.1 - 59.12 with base, 1947-49 = 100, is calculated [mean = 114.2]. Then the mean of the observations for 1967.1 - 67.12 with base, 1957-59 = 100, is calculated [mean = 103.6]. Next, the observations for 1947.1 - 58.12

with base, 1947-49 = 100, are divided by (1.1142*1.036). And finally, the observations for 1959.1 - 66.12 with base, 1957-59 = 100, are divided by (1.036). These two segments, together with the remaining segment from 1967.1 to the present, represent a complete series for Fuel Prices, with base, 1967 = 100.

Finally, the Number of Hours of Work Stoppage Due to Strikes in the Labor Force is obtained from the series, LHSTOP, in the Citibank Data Base.

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