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WHAT MAKES TEACHING DIFFICULT?

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# CRITICAL MOMENTS IN THE
# TEACHING OF MATHEMATICS:
# WHAT MAKES TEACHING DIFFICULT?

By

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ABSTRACT

CRITICAL MOMENTS IN THE TEACHING OF MATHEMATICS:
WHAT MAKES TEACHING DIFFICULT?

By

Janet Carolyn Shroyer

This study investigates how teachers cope with unpredictable student performance which is inevitable in classrooms, particularly when teaching emphasizes problem solving. Prior research on teacher thought indicated that teachers experience problems, but this is the first use of student difficulties and insights as a strategic research site. The focus is on understanding the relationship between student performance, teacher thought and behavior and the subject -- mathematics -- being taught at critical moments.

Using a process-tracing technique and stimulated recall to gather teachers' thoughts and feelings, case studies were conducted of three teachers as they taught a six- or seven-day unit on rational numbers. Teachers from the upper elementary grades were selected because of their experience with the Unified Science and Mathematics Elementary School project and their good reputations.

Teachers exhibited variation in the content emphasized, instructional materials, strategies and style which were indicative of their different instructional goals and conceptions of learning mathematics. Teaching

ranged from traditional mathematics instruction to a problem-solving approach recommended by the National Council of Teachers of Mathematics.

The impact of the instructional environments was evident in the variations in frequency, proportion, and density of student difficulties and insights. Unsolicited student contributions were more prevalent with the less traditional instructional approach, and student difficulties outnumbered student insights. Teachers' elective actions also varied. Those used with student difficulties were similar when classified according to the broad categories of exploiting, alleviating, and avoiding moves, but techniques for alleviating student difficulties varied. Teachers also responded differently to student insight. Elective actions varied in accordance with teachers' understanding of mathematics, instructional goals, conceptions of learning and attributions for student difficulties. Likewise, their negative emotional reactions varied in intensity and type.

A distinction had to be made between critical moments of a short-term nature and those based on pervasive problems. Critical moments arose over specific incidents of student difficulty or insight, while what was eventually labelled critical discrepancies reflected a more pervasive pattern of student performance. There were four types of conditions that produced cognitive difficulties and emotional discomfort for teachers: student difficulties, student insights, instructional pace, and unanticipated success. Further subdivisions according to the particular difficulties teachers reported suggest a taxonomy of critical moments.

Although teachers processed student occlusions somewhat differently than pervasive patterns of student performance, the same processes were involved: interpreting, goal setting, searching for elective actions and evaluating the effectiveness of the actions. Once the goal was set, teachers attempted to find, execute and test the results.

Critical moments typically occurred with more distinctive student occlusions. Teaching became difficult when teachers had trouble achieving their goals which happened by reason of default -- being unable to think of something else to try -- or by interference from antigoals -- situations to be avoided. Some goal had to be satisfied; if not the initial one, then a secondary one. When the threat was to the teacher's activity goal, the switch was more difficult. The primary use of teachers' critical moments,however, was their limited knowledge of mathematics and occasionally their limited repertoire of pedagogical techniques.

To help cope more effectively with critical moments, they need task- and topic-specific information about the types of student occlusions to expect and prescriptions for elective actions. Recommendations were made for the use of critical moments in training teachers and for preparing curricular materials by activities and units.

I dedicate this dissertation to my parents, John and Claire Shroyer. Throughout my educational endeavors, they have been steadfast in their love, support, and encouragement. In addition, they have been model educators: effective and dedicated. No one could have asked for more.

## ACKNOWLEDGEMENTS

I wish first to express my admiration and appreciation to the three teachers who participated in this study. Without their courage, honesty, and cooperation in divulging their thoughts and feelings and the time they willingly gave, this study could not have been done. Nor could so much have been learned about the mental life of the teacher.

The members of my doctoral committee have been supportive throughout the study, even when progress was slow. Special thanks go to Perry E. Lanier for serving as my doctoral advisor, to William M. Fitzgerald for continuing our work on the development of problem-solving units, to Edward L. Smith for giving advice, and to John Wagner for helping me to maintain a historical perspective. I am particularly indebted to my dissertation director, Lee S. Shulman, for his repeated efforts to focus my attempts to complete this project. His insistence on quality has been both an inspiration and a challenge.

This study was planned and the data collected while serving as a Research Intern at the Institute for Research on Teaching at Michigan State University. The Institute was funded primarily by the Teaching Division of the National Institute of Education, United States Department of Health, Education, and Welfare (Contract No. 400-76-0073). I am grateful for the support, resources, and research environment which the Institute provided.

My friend and typist, Barbara Reeves, has helped and encouraged me to complete this paper. To her, no expression of appreciation is adequate. My thanks also to Sandra Gross and Susan Battenfield for their contributions.

To my husband, Joe Byers, I offer my appreciation for his endurance and support; and to my good friends, I thank you for your understanding. Lastly, I thank my special friend as well as my oftentimes distraction, Tasha.

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# INTRODUCTION

## Problem

The National Council of Teachers of Mathematics' recommendations for school mathematics in the 1980s call for teachers to "create classroom environments in which problem solving can flourish" (NCTM, 1980, p. 4). Students need experience in applying mathematics which includes formulating questions, analyzing and conceptualizing problems, discovering patterns and similarities, suggesting explanations, experimenting and transferring skills and strategies to new situations. These recommendations come at a time when the council has more information than ever before about what students have learned about mathematics and how teachers teach mathematics (Suydam & Osborn, 1977; Easley, 1978; Stake & Easley, 1978; Weiss, 1978; Fey, 1980).

Despite the curricular projects of the sixties and teacher training efforts over the last twenty years, little change has occurred in the way mathematics is taught. The data indicate that mathematics instruction limits students to "...routine computation at the expense of understanding, applications and problem solving...[leaving] little hope of developing the functionally competent student that all desire" (NCTM, 1980, p. 5). In addition, the use of manipulatives to model and apply concepts is rare. The Council acknowledges

that "...explanation, practice and directive teaching are important but should not diminish the time necessary to achieve this priority [emphasis on problem solving]" (NCTM, 1980, p. 11).

To assist teachers in making the change, NCTM calls for the development of appropriate curricular materials to teach problem solving, identification and analysis of effective teaching strategies and techniques, and improvement of teacher training programs. Past failures to produce substantive changes in the teaching of mathematics through similar efforts indicate that something has been omitted. It is the teacher and how s/he actually functions in the classroom. Efforts to improve the teaching of mathematics must be based on a much greater understanding of teachers and the teaching process if they are to have any chance of success.

One aspect of teaching which needs to be considered is how teachers process and respond to unexpected student performance. The NCTM recommendations call for learning environments which foster unpredictable student performance. Errors, misconceived conjectures, and insightful ones should be commonplace in activities where questioning, conceptualizing, experimenting, and discovering are the desired learner behaviors. Teachers need to be flexible and responsive to these student difficulties and insights, but how much is known about how teachers actually cope with unexpected student performance? How teachers behave, think, and feel about such unpredictable events is not really understood.

It has only been in recent years that the "mental lives"[1] of teachers, not only classifications and tabulations of their behavior, have received attention. Interest in the mental life of the teacher was spurred by the National Conference on Studies of Teaching (1974) and the panel report on Teaching as Clinical Information Processing (Shulman, 1975). The goal of this panel was

> ...to develop an understanding of the mental life of teachers, a research-based conception of the cognitive processes that characterize that mental life, their antecedents and their consequences to teaching and student performance (p. 1).

The report encouraged the view of

> ...teacher as agent, rather than as a passive employer of teaching skills or techniques, and a commitment to understanding the ways in which teachers cope with the demands of the classroom (p. 2).

It is the relationship between teachers' thoughts and actions which become crucial in the formulation rather than thoughts or actions studied in isolation.

The few studies which have been conducted on the mental life of teachers during interactive teaching have focused on identifying, classifying, and quantifying teachers' decisions and on identifying principles or "implicit theories" by which teachers operate. Work by Peterson and Clark (1978) suggests that much of what teachers do is habituated "business as usual." Teachers often have little conscious thought to report during stimulated recall sessions.[2]

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[1]It was William James who apparently first referred to psychology as "the science of mental life."

[2]Stimulated recall is a research method in which audio or video tape replay is employed to assist participants to recall their thoughts and feelings while they were involved in the taped occurrences.

Teachers do report thoughts about unexpected student performance, particularly if their lessons are going badly. Morine-Dershimer (1979) found that teachers were able to handle some unexpected student performance with "inflight" decisions, but that other situations were far more discomfiting. Teachers rarely made changes even when they felt a situation demanded that something be done.

Expectations produced by considering teaching as decision making (Shavelson, 1973) have not been supported. Not only were teachers making decisions infrequently, but they were rarely considering alternatives. This was particularly evident in the quantitative data of Peterson and Clark (1978). Peterson, Marx, and Clark (1978) found that teachers considered alternative strategies only when teaching was going poorly as determined by student participation and involvement. Teachers did not report thinking of alternatives when things were going well which is not surprising unless teachers are expected to continually strive to improve the learning experience.

Research which has been conducted on teacher thought provides little understanding of teachers as they deal with unpredictable student performance. Part of the problem is the way in which the research has been conducted. In the current infancy of research, teacher thought has typically been examined independently of both teacher and student behavior. The researchers have paid little attention to the context in which these thoughts occurred. Studies have been conducted in both laboratory and natural settings with small and large group instruction. Investigation of teacher decision making was even limited to small segments of lessons, although other studies have traced teacher thought for entire lessons. If the

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the antecedents as well as the consequences of teachers' cognitive processing are to be understood and the relationship between teachers' thoughts and actions are to be found, greater attention must be paid to the instructional environment and the behaviors of students and teachers. To obtain sufficient numbers of incidents of unexpected student performance and to be able to isolate antecedents and consequences, instruction also needs to be traced over longer periods of time.

One thing research on teacher thought does is to support unpredictable student performance as a strategic research site. The concept of a strategic research site was first articulated by Merton (1959). In discussing the history of sociology, he indicated that "...problems could be brought to life and developed by investigating them in situations that strategically exhibited the nature of the problem" (p. xxvii). Merton cited multiple discoveries in science as one example. Shulman (1978) elaborated the notion of a strategic research site for understanding the process of teaching. Discontinuity or incongruence is the primary criterion for identifying these sites. Striking discontinuities which are perceived by the teacher were recommended as appropriate sites for understanding not only the behaviors of students and teachers, but their judgments and intentions. Incidents which interrupt the teacher's internal stream of consciousness and the action they take to recover it also provide an opportunity to examine what occurred before and what follows. Research on teacher thought does indicate that such discontinuities occur as teachers face unpredictable student performance.

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Thus the student difficulties and insights which occur during instruction provide a research site both as occasions to investigate teachers' thoughts and feelings and as desirable events in the teaching-learning experience which teachers sometimes find problematic.

## Purpose

The purpose of this study is to investigate incidents of unexpected student performance during teaching of a mathematics unit and to identify and describe those which prove troublesome for teachers of mathematics which we label "critical moments." From an examination of these critical moments, it is anticipated that much will be learned about how and why they make the teaching of mathematics difficult for teachers. In addition, a more general characterization of teachers' mental processing of critical moments will be sought. Before examining the questions of interest to this study, two assumption on which this study is based need to be examined: one is the assumption of teacher as information processor, and the other has to do with a particular theoretical conception of instruction.

## Assumptions

### Teacher as Information Processor

The information processing perspective has been adopted for this study of teacher thought and behavior. According to Newell (1973), to predict or even understand the behavior of a teacher coping with unexpected student difficulties and insights would require knowledge of three things: the first is the goals of the teacher, the second is the structure of the task environment in which the teacher is operating, and the third is the invariant structure of the teacher's processing mechanisms. There is much that has been learned about the invariant limitations of human mental processing capacities. According to Simon (1969),

> ...evidence is overwhelming that the system is basically serial in its operation: that is can produce only a few symbols at a time and that the symbols being processed must be held in special, limited memory structures whose content can be changed rapidly. The most striking limits on subject's capacities to employ efficient strategies arise from the very small capacity of the short-term memory structure and from the relatively long time required to transfer a chunk of information from long-term memory to short-term memory (p. 53).

The consequence of these processing limitations is that the teacher must function with less than full awareness and knowledge of all that might be known or recognized about the student's performance. The teacher must make choices on the basis of his/her own simplified representation of the situation. Simon (1957) refers to this as "bounded rationality" which he explains in the following manner:

> The capacity of the human mind for formulating and solving complex problems is very small compared with the size of the problems whose solution is required for objectively rational behavior in the real world--or even a reasonable approximation to such objective rationality--the first consequence of the principle of bounded rationality is that the intended rationality of an actor requires him to construct a simplified model of the real situation in order to deal with it. He behaves rationally with respect to this model, and such behavior is not even approximately optimal with respect to the real world (Simon, 1957, p. 198).

Similarly, to understand and even to model what teachers do when confronted with unexpected student difficulties and insights, it will be necessary to try to reconstruct their simplified representations of events, the "problem spaces" in which their choices were made. To do so information will come primarily from the thoughts teachers are able to report th

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report through the process of stimulated recall. Inferences might also be drawn from their behavior patterns and from other information they supply about their beliefs, goals, and knowledge.

The task environment of interactive teaching is the most difficult to characterize. It is substantially more complex than the relatively simple tasks which have been used in other process tracing studies. The task environments, even when extensive are more definable for some tasks such as chess (de Groot, 1965) and the Missionary-Cannibals problem (Jeffr Polson, Razran, & Atwood, 1977). Teaching also has a more complex task environment than required by the investment broker managing a portfolio (Clarkson, 1962) or a physician diagnosing patient illness (Elstein, Shulman, & Sprafka, 1978) because of the number of people who form part of the task environment in the classroom. The complexity of classroom teaching is largely a consequence of the number of people involved; students and their actions confound the task environment because of their diversity and because of the dynamic nature of teacher-student interactions.

Concept of Instruction

This study of critical moments is based on a model of instruction stimulated by discussion with Schwab during a staff session at the Institute for Research on Teaching at Michigan State University. In this mode, the teacher is viewed as an information processor. Interactive teaching is assumed to be a purposeful activity, one which a teacher approaches with some plan in mind. According to Schwab, the teacher seeks to generate an instructional flow for an activity, one which provides

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the momentum to both attract and maintain student involvement. Instructional flow denotes more than the pace of an activity; it also refers to the routines for performing the task and for presenting the mathe matics. Instructional flow includes the teacher's style, strategy, questioning pattern and the content. In the interactive phase of teaching, the orchestration of an activity and the dynamic interactions between teacher and students sometimes require a teacher to respond in ways that connot always be planned or even routinized. The instructional flow, then, is susceptable to occlusions, unpredictable events which can impede or even stop the intended flow.

The primary source of occlusions is unexpected student performance, unexpected in the sense that student performance does not fall within the expected limits of the planned instructional flow. Student difficulties and insights represent unexpected student performance; they are observable student occlusions. An overt student occlusion is not necessarily viewed by the teacher as interrupting the instructional flow. Thus, a distinction is needed between observable student occlusions and the student occlusions which make teaching difficult, the critical moments. Critical moments are student occlusions for which teachers report some cognitive difficulty or emotional discomfort; to classify an event as a critical moment requires both objective and subjective evidence that the instructional flow has been interrupted. Student occlusions constitute potential critical moments. Occlusions of student insight or difficulty are not necessarily signals of undesirable events in the interactive classroom. Occlusions also provide teachers with "teachable moments." The purpose of identifying and describing teachers

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teachers' critical moments is an attempt to understand how coping with student occlusions makes teaching difficult.

A similar distinction could be made between the potential opportunities for and actual "sacking" of the quarterback that the game of football affords the defensive linebacker. The opportunity for a "sack" does not ensure its occurring as most fans know. The linebacker can only try to capitalize on those opportunities which he encounters and recognizes. In the meantime, like the teacher, he will function with the habitual behaviors he has learned for his position. If this analogy has a flaw, it is that teachers might not be looking for unexpected student performance to exploit as much as defensive linebackers are looking for a chance to down the quarterback.

The teacher's role is not only to establish the initial instructional flow for an activity, but to maintain it. When confronted with a student occlusion, a teacher elects to respond with some action. The term <u>elective action</u> was chosen as an alternative to <u>decision</u> because decision has traditionally implied the consideration of alternatives, a process for which research on interactive teacher thought has found little support (Peterson and Clark, 1978). For teacher behavior to be labeled as an elective action requires only than an outside observer would judge that different action could have been taken, regardless of whether the teacher considered such alternatives. Elective actions can vary from little or no shift in the prevailing flow to a radical change in activity. However, it is assumed that after the elective action has been completed, the teacher would either return to the same instructional flow which preceded the student occlusion or establish a new one.

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Thus, the critical moment is followed by a subsequent flow, unless the cycle repeats itself with yet another student occlusion.

A proposed model of critical moments is shown in Figure 1.1. It begins with the <u>prevailing flow</u> of instruction which is interrupted by a <u>student occlusion</u>. Some mental processing is assumed to take place followed by the teacher's <u>elective action</u> and the <u>consequences</u> of the <u>critical moment</u>. Consequences can be overt or covert. They may be evident in the subsequent flow of instruction when changes are made or they may be evident in the emotional reactions and thoughts of the teachers.

With the limitations in the human processing capacity, it would be unrealistic to expect teachers to process each student occlusion with the same degree of awareness. Research on teacher thought suggests this to be the case. The construct of a critical moment, however, refers to a psychological state in which the teacher senses a discontinuity or occlusion in the instructional flow. For critical moments, teachers should be more likely to have some thoughts and feelings to report. Since cognitive processing takes place within the teacher's mental "problem space"--a simplified version of reality--an investigation of critical moments should provide some evidence of the constraints and influences under which teachers operate. For example, if teaching is a purposeful act, then evidence of the teacher's goals and plans influencing his/her thoughts and actions should be detected in the examination of critical moments.

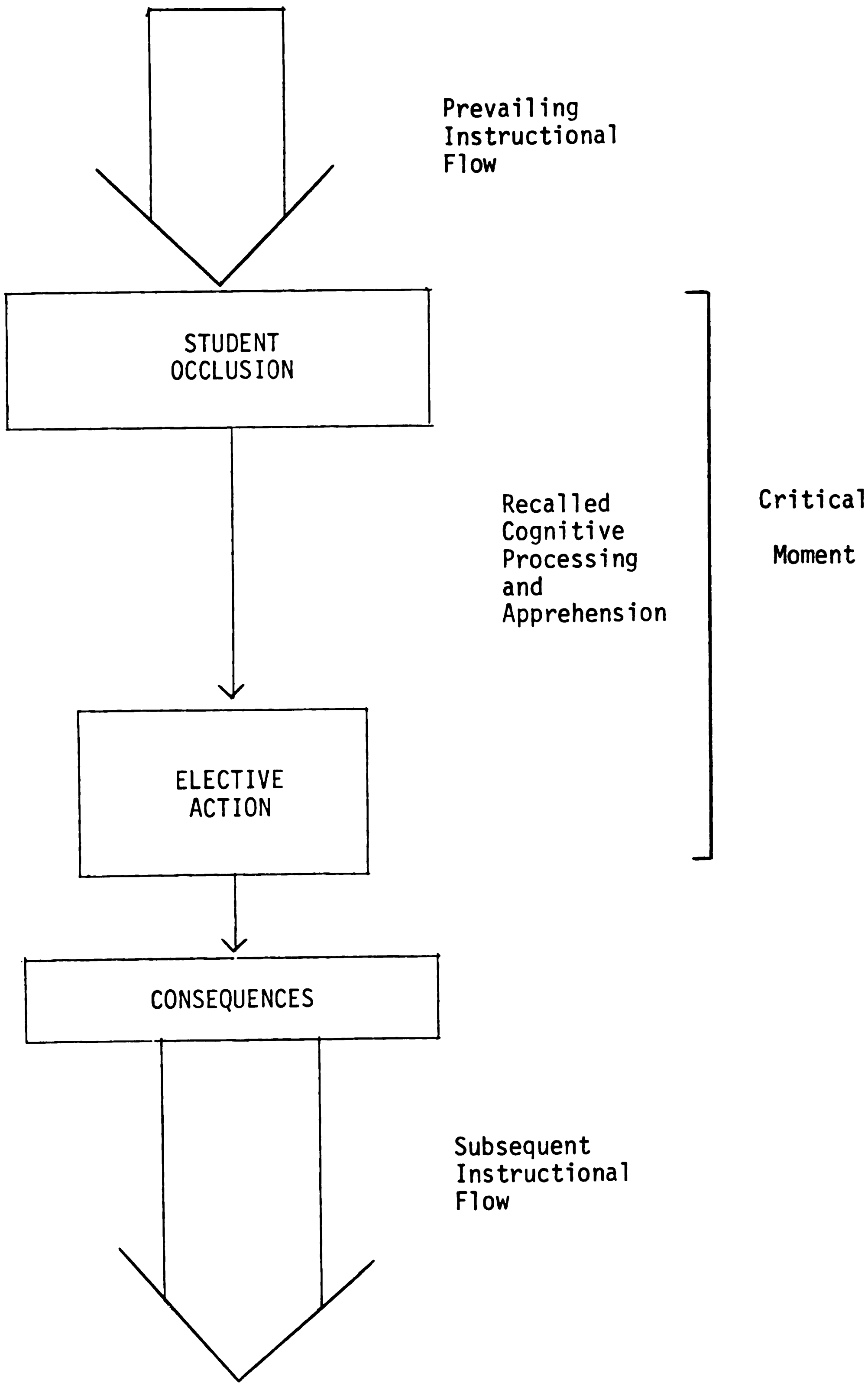


Figure 1.1. Model for teaching: a Critical Moment.

## Study and Questions

This study of the critical moments associated with unexpected student performance calls for rich descriptions, interpretation, and theoretical conjecture about the instructional environment, the unpredictable student performance, and the teacher's reactions, both overt and covert, if the phenomenon is to be understood. Critical moments will be investigated through observations, videotaping of teachers engaged in instruction of the total class, the use of stimulated recall to obtain teachers recalled thoughts and feelings, and an analysis of all these data. Case studies of three teachers will be conducted.

In order to provide adequate opportunity for natural variations in the amount and types of unpredictable student occlusions and for critical moments to occur, these teachers will be followed as they teach a unit of mathematics. The set of related lessons in a unit provide a cohesiveness not otherwise available in the brief "snapshots" of teaching one lesson or segments of a lesson. The sequence of lessons also makes it possible to establish behavioral patterns and to search for antecedents, causes, and consequences of critical moments.

The questions to be addressed in this study fall into four categories: instructional environment, critical moments, mental processing, and implications.

I. What is the nature of the instructional environment?

   A. What is the nature of the instructional flow?

      1. The mathematical content of the units and how they are organized

      2. The instructional style and strategies of the teachers

      3. Teachers' instructional goals and conceptions of the mathematics to be learned

II.

III.

IV.

B. What is the nature and distribution of the unpredictable student performance?

1. Student occlusions of difficulty and insight

2. Teacher elective actions

II. What is the nature of the critical moments?

A. What are the overt conditions which produced them?

1. Characteristics of the prevailing flow

2. Student occlusions and teacher elective actions

3. Characteristics of subsequent flow

B. What are teachers' covert reactions to them?

1. Cognitive thoughts

2. Factors influencing their mental processing

3. Emotional consequences

III. What is the nature and function of teachers' mental processing of unexpected student performance during critical moments?

A. What model(s) best describe teachers' cognitive processing?

B. What are the characteristics of each process?

C. What factors influence each process?

IV. What has been learned about and what are the implications for the methodology of research on critical moments? What are the implications of the findings on critical moments for teachers, teacher training, and preparation of curricular materials for mathematics?

Understanding critical moments--what precipitates them, how teachers process them, and what consequences they have for teachers--will provide evidence about what makes teaching difficult. It is only a first step in trying to find ways to help mathematics teachers be more

effective in coping with unexepcted student performance. Nevertheless, this study of critical moments will have implications for research on the teaching of mathematics and research on teacher thought.

## Overview of the Study

Chapter II consists of a review of the pertinent literature on interactive teacher thought with attention to both the methodology and the results of the research. In Chapter III, the methods employed in this study will be described, including the selection of the teachers, descriptions of their classrooms and schools, and the manner in which data were collected and analyzed. Chapter IV offers a description of the instructional environment in which the teachers were teaching. The focus is on the nature and characteristics of the prevailing instructional flow and the nature and distribution of student occlusions and teachers' elective actions. In Chapter V, the critical moments which have been identified from teacher reports of cognitive difficulty and emotional discomfort will be described in detail. In Chapter VI, teachers' mental processing of unexpected student performance and models of their processing will be discussed. A discussion of the implications for methodology, teachers, teacher training, and the preparation of curricular materials will follow in Chapter VII. A summary and conclusions are presented in Chapter VIII.

## CHAPTER II

## REVIEW OF THE LITERATURE

### Overview

In proposing a research paradigm for studying teacher decision making, Clark and Joyce (1975) were able to report that

> ...there are virtually no reported studies of teachers' interactive decision making. That is to say, we know very little in any scientific sense about the kinds of information or cues that teachers use when making decisions "on the fly." We have not even developed major categories for classifying the acts of decision making that we know must go on regularly in the course of teaching (p. 3).

Since that time, there have been few studies of interactive teacher thought processes. By 1977, when he addressed a conference about research on mathematics teaching, Kilpatrick made the following observation:

> ...research on the teacher's thoughts and behavior in teaching mathematics to elementary school pupils is such a wilderness! Researchers have spent a lot of time during the past two decades sitting in classrooms and watching teachers teach, and yet almost no one has considered how teachers approach elementary school mathematics (p. 1).

Interest in teacher as decision maker was spurred by the adoption of this view in educational literature (e.g., Shavelson, 1973; Whitfield, 1971; Bishop, 1970). The decision making paradigm is based on a rational model which assumes (a) goals are set, (b) alternatives are formulated, (c) outcomes are predicted for each outcome, and (d) alternatives are evaluated in relation to goals. Shavelson (1976) elaborated on this paradigm for teacher decision making by incorporating teachers' judgment of the states of nature and the utility (value) of an alternative. Research on decision making sought evidence of the cues teachers used to make their judgments, alternatives they considered, and reasons for their choices. To their disappointment, the researchers found little evidence that teachers performed as decision makers--at least not rational decision makers.

The contrast between the types of decisions teachers were imagined to make and those that are revealed by research exemplifies two different views of humans. Simon (1957) dispenses with Barnard's (1938) conception of "economic man" as a rational decision maker who "...selects the best alternative from among all those available to him." Instead, Simon substitutes "administrative man" as a "satisficer" who "looks for a course of action that is satisfactory or 'good enough'..." (pp. xxv-xxvi). Thus, as the evidence accumulated, Peterson and Clark (1978) were led to characterize teachers as satisficers rather than optimizers in the classroom. Elstein, Shulman, and Sprafka (1978) came to the same conclusion about the diagnostic process of physicians.

In this chapter the field of research on teaching which has focused on interactive teacher thought and decision making will be reviewed. In the first section, a portrait of the teacher in the interactive classroom is presented. Findings of these studies provide information about how teachers function; the nature, substance, and types of decisions; their lessons; their mental problem spaces; their principles; and their instructional approaches. Methods for studying teacher thought are also reviewed with details on the manner in which stimulated recall sessions were conducted. Mention is also made of some of the classification systems used to code teacher thought and behavior.

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## Research Findings

Marx and Peterson (1975), Clark and Peterson (1976) and Peterson and Clark (1978) in their Stanford study examined teacher decision making in a laboratory setting using social studies units with junior high students. Morine and Vallance (1975), in their Beginning Teacher Effectiveness Study (BTES), conducted by the Far West Laboratory, investigated teacher and pupil perceptions of classroom interaction using reading materials. They compared teachers with varying effectiveness determined by their pupil gain scores and their teaching styles. Sutcliffe and Whitfield (1975) used physiological data in England to study the process of teacher decision making. Joyce, Morine, and McNair (1977), Joyce and McNair (1979), and Morine-Dershimer (1979) in their South Bay study examined interactive teaching thought using elementary school teachers and reading lessons. [These reports may be found in McNair and Joyce (1978-79) and Morine-Dershimer (1978-79) in a shortened form.]

Marland (1977), conducting research at the University of Alberta's Center for Research on Teaching, used elementary school students to investigate the kind of information teachers processed and the ways in which they processed it during language arts and mathematics lessons. A year later, Conners (1978) replicated Marland's study of teacher thought processes, beliefs, and principles in the same academic areas. In 1980 Heymann did research on the modeling and conceptualizing of teaching, learning, and thinking processes related to mathematics. His study from Bielefeld's Institute for Didactics in Mathematics is the only study which has the teaching of mathematics as its prime concern.

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Findings from these research efforts provide the following portrait of teachers' decisions while engaged in classroom instruction.

Researchers found that teachers operate in a relatively habituated manner for which they are often unable to recall any conscious thoughts or feelings. Teachers often report doing what they planned to do or what they usually did. This led Clark and Peterson (1976) to characterize much of what teachers do as business as usual.

Nature of Decisions

Teachers rarely mention having considered alternative actions or strategies for their tactical moves (Clark & Peterson, 1976). Instead, as Marland (1977) indicated, alternatives seemed to "pop up" spontaneously as if they were ready-made habits. Whitfield (1971) came to the same conclusion, saying, "Our explorations so far suggest that teachers do not generate all the options open to them before making their choice." He speculates that either "they are insensitive or inflexible...or they have well-developed value systems as a result of experience which enables them to filter out options which they might have generated in early career" (p. 164).

Teachers reported considering alternatives only when their lessons were going poorly (e.g., Marland, 1977). There was little evidence of conscious thought when lessons were going smoothly. Marland indicated that little of teachers' interactive thinking conformed to the problem-solving paradigm. Teachers encountered problems when students failed to grasp a point, when teachers were unable to find ways to help a student understand, and when the teacher's expectation for the individual did not match his/her performance. Heymann (1980) found that teachers emphasize behavior when they are confronted with learning c

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learning difficulties as well as when introducing mathematical concepts. Marland added that teachers did not usually seek explanations for unexpected student performance. Morine-Dershimer (1979) remarked about a related problem. Teachers sometimes postponed taking action, but she was unable to follow through to see if postponement meant a delayed response or the absence of a response.

Teachers' judgments about the effectiveness of their lessons were made primarily on student involvement or attention and not the quality of the classroom discussions (Clark and Peterson, 1976). Teachers in the Stanford study did not mention the performance of individual students since they were in a laboratory setting and they had not met their students prior to the study.

Teachers did not appear to be very flexible with regard to their teaching. Few instances were noted when teachers made changes in their plans (Morine and Vallance, 1975; Marland, 1977). Evidently teachers teach the lessons they plan. Marland (1977), however, focused on evidence of customized teaching behaviors in the protocols of all six of his teachers. Teachers do alter their behavior for some unpredictable events, whether they do so consciously or not.

## Substance of Teacher Thought

The substance of teacher's interactive decisions and thoughts were related to instructional moves. Most had to do with pupils and what they were doing and saying. Morine and Vallance (1975) indicate the bulk of teachers' decisions had to do with their interchanges with students and the activities they planned. Joyce, and McNair (1979) and Whitfield (1971) discuss how teachers constantly make "inflight

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"inflight" or "on-the-spot" decisions about who to call on, how to provide corrective feedback, when to discipline a student, and how fast to proceed through a lesson. More than anything else, teachers focused on what students were learning. Much less concern was shown for students' attitudes or teachers' affective goals. Teachers also mentioned selecting, organizing, presenting, and reviewing content (Morine & Vallance, 1975; Joyce & McNair, 1979; and Conners, 1978).

In the South Bay study, Joyce and McNair (1979) determined that teachers of less successful students were more concerned with student behavior, the effectiveness of their own directions, the appropriateness of the content for the students, the content being learned, and pupils' attitudes than were their counterparts with more successful students. Marland (1977) reports that teachers' lesson plans indicate little attention was given to teacher-student interactions; teachers do not anticipate unpredictable student performance before teaching a lesson. Teachers did not, as had been anticipated, engage in frequent or systematic consideration of

> ...their own teaching style, its effectiveness and impact on students....Teachers seldom checked the accuracy of their interpretations...[or]...inferences about the covert cognitions of students or their affective states. They operated on the basis of hunches and intuitions (Marland, 1977)

<u>Types of Decisions</u>

The fact that teachers did not consider alternatives as often as the decision making models predicted led to different ways of classifying decisions. Interactive decisions in the Marland (1977) study were designated as decisions which require conscious choice of alternatives, designat

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designated as decisions which require conscious choice of alternatives, deliberate acts which signify choice without considering alternatives, and proactive teaching which indicates interaction that reflects deliberate planning and control, but not conscious choice. Principles which signify working hypotheses or fundamental laws held and exercised by teachers were also identified from their comments. Teachers reported few (less than ten; average about seven) decisions per lesson, and the number of alternatives rarely exceeded two with no more than three reported. Deliberate acts were most common, averaging 21.5 per lesson. These data suggested about 30 potential choice points per lesson. Decisions in the South Bay study averaged about 21 decision points per lesson, although the productivity varied across teachers.

Peterson and Clark (1978) used Snow's (1973) model of teachers' cognitive processing to re-analyze the Stanford data. They classified decisions as one of four types to correspond with the model which is shown in Figure 2.1. A path 1 decision indicates student performance was within the teacher's level of tolerance and that the teacher was able to continue; other decisions indicate it was not. A decision was designated as path 2 when no alternative action was available. When the second decision point was reached, the teacher found that a path 3 decision came when alternative actions were considered but not taken. A path 4 decision meant an alternative action was taken.

Path 1 decisions were most frequently made by teachers which Peterson and Clark took as an indication of business as usual. With path 2 decisions, teachers expressed feelings of helplessness or surprise; there was no difference among teachers based on their experence. Path 3 was reported least often which supports the

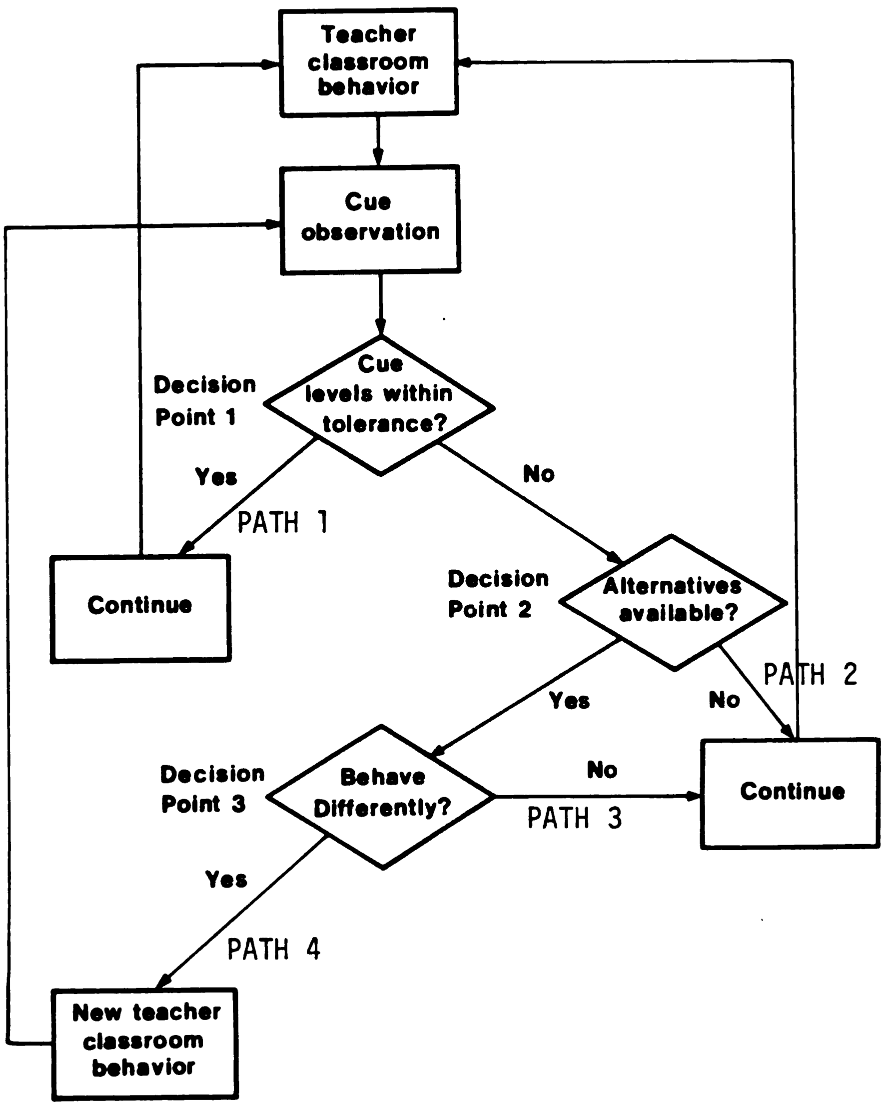


Figure 2.1. Model of a teacher's cognitive processes during teaching (Peterson and Clark, 1978) (labeling of paths added).

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conclusion that teachers make path 4 decisions if they know of an alternative action. Teachers who focused more on instructional process in planning had more path 4 decisions than those who focused on lower order subject matter in their planning. This lends some support to the notion that teachers who focus more on the learning of facts and skill are flexible only within the boundaries of the lessons they are teaching.

When frequencies of teacher decisions and concerns were compared across teachers with different pupil gain scores, Morine and Vallance (1975) found that teachers with low pupil gain scores tended to mention larger numbers of items. They interpreted this to mean the teachers were trying to process more information, but they offered no information as to whether, in fact, there was more unexpected student performance to process. Joyce and McNair (1979) found less variation between the two types of teachers in the South Bay study.

Marx and Peterson (1975) obtained a negative correlation when they compared the productivity (frequency) of preactive decision making with that of interactive decision making. They thought perhaps "teachers who respond well in one mode do not respond productively in the other." Teachers who generated more preactive decisions on the first day of instruction also asked more questions and in general focused more on subject matter than on pupils when compared to other teachers.

## Teachers' Lessons

Morine-Dershimer (1979) focused on the discrepancy between a teacher's plan and the reality of the class presentation. The amount of discr

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of discrepancy was measured by the proportions of decision points at which teachers registered "surprise" and reported being disturbed or bothered by the event. Lessons were classified into three categories: (a) lessons with little or no discrepancy between plans and classroom reality, (b) lessons with minor discrepancies, and (c) lessons with critical discrepancies.

For the first category, lessons with little or no discrepancy, less than 25% of the teacher-identified decision points were described as "non-expected" events. Morine-Dershimer (1979) described the teachers' information processing as "image-oriented." Decision points were handled by established teaching routines. For lessons with minor discrepancies, 50% or more of the teachers' decision points were reported as non-expected events with less than one-fourth of these events described as disturbing or bothersome. She characterized teachers' information processing in this category "reality oriented" where a fairly narrow range of pupil behavior was observed. Decision points in these lessons were handled primarily by "inflight" decisions. When 50% of the decision points were both non-expected and bothersome, the lessons were termed critical discrepancies. For these lessons, Morine-Dershimer described teachers' information processing as being "problem-oriented" with teachers' tapping a broader spectrum of information about their students. Decisions about how to handle these situations were postponed.

These three categories of lessons reflect differences in teachers' decisions and in students' behavior. Morine-Dershimer's analysis points to the importance of unpredictable student performance in teachers' mental processing. A more extensive examination of teachers' thoughts

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thoughts, actions, and feelings with regard to student difficulties and insights, as planned for this study, might reveal even more information. The teaching of mathematics could be different than the teaching of reading.

Teachers' Mental Problem Space

The substance of teachers' decisions led Joyce et al. (1977) to hypothesize that "...selection of materials and subsequent activity flow establish the 'problem frame'--the boundaries within which decision-making will be carried on" (p. 13). They even went so far as to suggest that the instructional model based on the materials used and content taught had a greater influence on teachers' information processing than the stylistic variation among them. In other words, they suggest that task and instructional goals control teachers' thinking more than their individual characteristics of students. Marland (1977) also indicated that the teachers' decisions about the nature of specific actions with students fit within their lessons plans.

The importance of the activity to teaching was highlighted by Whitfield (1971). He argued that teaching should not be judged exclusively by student learning since the most important part of teaching is "the intention that pupils shall learn through various activities." The problem for the teacher--if there is one--is whether the content selected is appropriate for the learners. Therefore, Whitfield encouraged that teachers' intentions be examined to establish a link between their intentions and actions, but acknowledged that their actions can be the result of inappropriate decisions.

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Marx and Peterson (1975) suggested that teacher decision making could be related to their interactive behavior through some organizational tactic such as goal setting or summarizing during a lesson. A clear and consistent relationship was found between teacher thought and action in that "teachers who dealt with subject matter in their decision making also tended to focus on subject matter in their teaching." There was also some indication that teachers who focused on subject matter concerns during the stimulated recall interviews were really focusing on lower-order subject matter, namely facts and figures. Their students tended to do less well on essay tests which required abstract reasoning.

Teachers' Principles

Clark and Peterson (1976) reported that "teachers were able to describe in general terms what they were doing...but less able to articulate why." Marland (1977) and Connors (1978) did find evidence to explain why teachers behaved as they did, even if teachers did not always express them in specific terms. Each sought evidence of teaching principles from the recall data. Marland (1977) was able to identify five principles: (a) compensation, (b) strategic leniency, (c) power sharing, (d) progressive checking, and (e) suppressing emotions. Principles of compensation and strategic leniency referred to teachers' discriminating behavior in favor of the shy students who had low ability or who were culturally impoverished, particularly by teachers of first and third graders. Power sharing referred to one teacher's theory that peer influence could be used as an instrument for dispensing her own influence in the class. Progressive checking referred to a techn

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a technique of intermittent interruptions while students worked at their desks, used especially by teachers with students of low ability. Teachers considered this method necessary for enhancing learning. All teachers suppressed their emotions at some time, particularly when under duress. In his study, Conners (1978) replicated and extended

Conners (1978) replicated and extended Marland's study by drawing inductive inferences from teachers' behavior as well as their recall data. He classified teachers' principles into overarching principles of instruction, general pedagogical principles, and more specific principles derived from learning theory, motivation, and human growth and development. Teachers were also influenced by their beliefs, classroom rules, and other factors including role conceptions, idiosyncratic intrusion, ecological influences, objectives, and information concerning pupils and expectations. Three <u>overarching principles</u> were teacher <u>authenticity</u> which is a desire to be approachable and to be able to make mistakes, <u>suppressing emotions</u> which is identical to what Marland found, and <u>teacher self-monitoring</u> which is awareness of influence of teaching behavior on pupils. The last category contradicts Clark and Peterson's (1976) findings in which teachers are characterized as not engaging in any self-monitoring behavior or thought.

About <u>general pedagogy</u>, Conners (1978) found teachers relied on processes of <u>cognitive linking</u>--relating new knowledge with what pupils already possess, particularly during introductory or review phases of lessons; <u>integration</u>--crossing subject area boundaries; <u>general involvement</u>--involving pupils in lessons to develop their personalities, socialize, or minimize teacher role in discussion;

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Principles of learning included specific learning principles: repetition, reinforcement, motivation, feedback, and so forth; and transfer of learning. The major learning processes teachers cited were problem solving, association and discrimination, and a variety of modes of presentation. Miscellaneous principles had to do with the self concept of the class or class atmosphere and development principles. Teachers' beliefs were largely linked to their principles. For example, a general belief was that a teacher should be supportive and approachable. Teachers thought about instructional moves, feedback, structuring, organizing, controlling, disciplining, presenting, and reviewing. Interpretations were made about the students, and both lesson-specific and lesson-facilitating objectives were reported.

## Instruction

Most characterizations of teachers' instructional goals, their subject matter concerns, and their actions indicate a preoccupation with factual and skill learning; there is much less evidence of teachers' teaching for understanding of concepts and relationships. This description of the content being taught is very much in agreement with descriptions of content taught in the typical mathematics class.

Instructional styles of the teachers in the South Bay study were quite similar. The general pattern of classroom interaction made heavy use of factual information, and teachers rarely mentioned learning beyond factual mastery (McNair & Joyce, 1979). There was an emphasis on implementing instruction, directive procedures, and positiv

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positive feedback. The practice of giving positive feedback has been shown to occur more at the lower grades and with special education teachers than in the upper elementary grades (Evertson, Anderson, Anderson, & Brophy, 1980). The researchers concluded:

> Apparently, most of these teachers share a common set of concerns and attribute to those concerns the same priorities. Teachers do not differ a great deal between themselves with respect to what they attend to as the lesson proceeds. These teachers appear to be working out of a standard framework that emphasizes learning rather than thinking, task-directedness, and teacher dependencies (p. 59).

McNair and Joyce (1979) did report some variation with one teacher who was "...mostly concerned with the individual needs of each of his/her students [so that] management or group concerns declined in importance" (p. 60).

## Methods of Studying Teacher Thought

### Stimulated Recall

Research on teacher thought has relied almost exclusively on the technique of stimulated recall to obtain teachers' thoughts and feelings from their interactive classroom teaching. From Bloom's (1954) pioneering research using stimulated recall to facilitate retrospective verbalization of thought processes during lectures and discussions to the more recent uses of stimulated recall in the studies of interactive teacher thought, much has been learned about the process. Kagan, Krathwohl, and Miller (1963) first introduced videotape replays in research on counselor-education as a means of maximizing cues. Interpersonal Process Recall (IPR), as Kagan's technique is known, has been refined and is still used today. (Kagan, 1975). Drawing from both Bloom and Kagan's work, Elstein, Shulman, and Sprafka (1978) conducted medical inquiry studies of physicians' diagnosing of patient illness.

As Clark and Yinger (1979) have observed, studies of interactive teacher decision making

> ...reflect a series of refinements in the method of collecting data about these decisions. The method of conducting stimulated recall interviews has changed from using very short, randomly selected videotaped segments and asking a standard set of questions about each segment to reviewing a videotape in its entirety and giving the teacher control over when to stop the tape and over the kinds of mental processes on which to focus. Another trend has been to move from the laboratory to the classroom. Both developments have made the problems of data reduction and analysis more challenging, but they have also increased the representativeness of the situations being studied (p. 259).

The change is reflected in the following descriptions of the methods used in these studies to gather teachers' thoughts and feelings.

In the Stanford study, where teacher decision making was examined in a laboratory setting, twelve experienced teachers taught the same social studies unit to three "classes," each consisting of eight junior high school students who had volunteered to participate in the experiment. Teachers' plans were obtained during "think aloud" planning sessions which preceded the actual teaching of the lessons, and interactive teacher thought was obtained by stimulated recall process. Teachers were shown the first five minutes and three randomly selected segments (one to three minutes) and asked to respond to structured interviews.

Data collection was planned with the expectation of teachers' acting as decision makers as is apparent from the questions used for the structured interviews for use in the stimulated recall sessions: (a) What were you doing and why? (b) Were you thinking of any alternative actions and strategies? (c) What were you noticing about the students?

(d) How were students responding? (e) Did student reactions cause you to act differently than planned? (f) Did you have any particular objectives in mind in this segment (of the lesson)? and (g) Do you remember any aspects of the situation that might have affected what you did in this segment?

The objective of the BTES study was to identify types of observations teachers made about pupils and to compare teachers who differed in pupil gain scores (effectiveness criteria) and in their style of teaching. The study was done under more natural conditions than was the Stanford study, but there were similarities.

Using reading materials given to them, forty 2nd and 5th grade teachers were videotaped as they taught a twenty-minute reading lesson to a small group of their own classes. Unlike the Stanford study in which teachers were shown only brief segments of their lessons, teachers in this study were asked to stop the videotape at points where they recalled making a specific decision. If the teacher did not stop the tape after an incorrect response or when there was a shift in pupil activity, the interviewer did. Teachers were asked to respond to a set of questions: (a) What were you thinking? (b) What did you notice that made you stop and think? (c) What did you decide to do? (d) Was there anything else you thought of doing but decided against? What? The questions used in the BTES study, like the questions in the Stanford study, were formulated on the assumption that teachers are decision makers as the models hypothesized (Shavelson, 1973).

In the South Bay study, the interactive teacher thought of ten teachers in one elementary school in a large metropolitan area was examined over a one-year period. Data on both behavior and thinking were gathered in hopes of finding relationships that would "serve to illuminate the process of teaching." Reports were prepared on the teaching styles of the teachers, their thoughts while teaching, their conceptions of pupils, and the relationship betwen a teacher's plan and the classroom reality of its implementation.

Two reading lessons for each teacher, one each for higher and lower ability groups of students, were videotaped and observed three times during the year. Techniques used to collect teacher thought in the Stanford and BTES studies were used with some modification. In the South Bay study, teachers taught reading lessons as part of their normal curriculum. At the end of the day, both lessons were played back to the teacher, first using two random stops for each lesson, a Stanford study technique; and then playing the entire lesson back, stopping the tape at teacher-identified decision points, a BTES study technique. At each decision point, the teacher was asked the same questions used in the BTES study.

Information was also gathered on teachers' plans. Although teachers typically recorded only minimal information about their lesson plans in their weekly plan books, they participated in planning interviews prior to each of the lessons being observed and for which teachers were asked to participate in stimulated recall. They answered questions about their general plan, specific objectives and teaching strategies, their diagnoses of pupil needs, the use of instructional materials, and seating arrangements. Through this extensive probing, it was found that teachers have a "mental image" of their plan which is far more detailed than the plans they write.

Sutcliffe and Whitfield (1975) relied on physiological data on heart rate rather than teacher self-report data to identify decisions because they had become disillusioned with the effectiveness of the stimulated recall method:

> Apart from the reliability of verbal responses when collecting data by way of introspection, there are other problems associated with self-analysis of videorecords. Individuals respond somewhat differently to seeing themselves in particular roles....Thus, to ask teachers to introspect whilst watching a playback of a recent lesson is likely to lead to greater unreliability of data so collected, without first acclimatising the teacher to the process. Hence a structured interview with the teacher was preferred to a wholly introspective technique, with the interview taking place as soon as practicable after the recorded lesson but without any viewing of the lesson by the teacher (p. 10).

They were concerned that videotaping procedures had an inhibiting effect on a proportion of the students within each class since it had been reported by the classroom teachers. Interestingly, they found that teachers seldom admitted the videotaping had any effect on themselves although some heart rates during the videotaped teaching sessions were quite high. To reduce the possibility of interference by the videotaping, teachers were taped on four occasions with only one chosen for analysis.

Studies of teacher decision making conducted in the United States and Canada have not reported the same difficulties with the videotaping and stimulated recall process as did Whitfield and Sutcliffe. This could be attributed to several reasons, including the selection of teachers, since Sutcliffe and Whitfield involved far more teachers in the process than did other researchers. Differences were noted in teachers' responsiveness to the stimulated recall process by Joyce et al. (1977). Some teachers were more shy than others. They also noted a difference in teachers' ability to articulate their thoughts and reasons and in their awareness of their routine actions.

In a study of forty-two teachers, experienced and inexperienced, Sutcliffe and Whitfield (1975) found that

> When introspecting, teachers tend to describe their lessons by recalling their own set of lesson markers, rather than in terms of any prior aims and intentions they may have had for the lesson... (p. 16).

They also found that teachers could easily give examples of decisions made on the spur of the moment but without any consciousness of the decision process itself. Some teachers even regarded this ability as a measure of experience. Data on the effectiveness of using heart rate to determine decision points did not prove to be as informative as anticipated. Thus, an effective alternative to the stimulated recall method to obtain teacher thought has not been found.

In his study of six teachers, Marland (1977) sought to investigate the kinds of information teachers processed and the ways in which they were processed. As part of his study, observational data were taken on teacher behavior using the Brophy-Good Dyadic Interaction coding system.

Six teachers, two each from grades one, three, and six in a Canadian urban school district volunteered to participate. Two one-hour lessons were videotaped with the stimulated recall sessions held after school. Except for the sixth grade classes, who were viewed only during language arts lessons, one lesson was videotaped for language arts and one for mathematics. The interviewer viewed the tapes ahead of time to identify segments which appeared to be most likely to produce interactive teacher thought. Segments used for a recall session ranged between 20 and 30 minutes. Unlike previous studies using stimulated recall to obtain teacher thought, this one did not ask teachers to respond to a structured set of questions. Instead, they were invited to provide detailed accounts about (a) their thoughts, feelings, and moment to moment reactions; and (b) their conscious choices, alternatives considered before making a choice, and the reasons for making a choice. The teacher had primary control over the events that would receive comment. The interviewer role was more that of facilitator, although the interviewer could also stop the tape and ask what the teacher had been thinking.

Modifying and extending Marland's (1977) work, Conners (1978) conducted a study of teacher thought processes, beliefs, and principles during instruction. Using a similar research strategy to Marland's, Conners did stimulated recall with nine teachers as they taught mathematics and language arts.

In the stimulated recall sessions for the Conners study the role of interviewer was not just a facilitator. The interviewer was encouraging, receptive, and interested, though not critical. A discussion atmosphere rather than segmented and highly structured sessions was sought. If the teacher stopped the videotape at a stimulus point, the interviewer following up with open-ended, probing questions when reasons were not volunteered or if the teacher's comments needed clarifying or confirming. When the interviewer stopped the tape and initiated verbal exchange, open-ended questions were asked. This was done "to give the respondent freedom of response and to avoid any biasing tendencies on the part of the interviewer." Leading questions were avoided as were evaluative statements and digressions. Probes had a single focus. Once the recall had been conducted, each thought unit was categorized into a single classification and the transcript was organized by segments which centered around a particular stimulus point. Thus, there was a behavioral indicant of what the recalled thoughts were about.

Further refinement of the stimulated recall method has been conducted by Tuckwell (1980) at the University of Alberta. Drawing on the experiences of Marland and Conners, Tuckwell has written a technical report on the application of content analysis to stimulated recall interview data. It offers guidelines for the complex task of differentiating interactive from non-interactive thoughts and for calculating the reliability of the classifications. Tuckwell also provided a discussion of the theoretical perspective of stimulated recall.

In a study at the Bielefeld Institute for Diadactics in Mathematics in Germany, Heymann (1980) was involved in modeling and conceptualizing the teaching, learning, and thinking processes related to mathematics. This is the only study mentioned concerned solely with the teaching of mathematics. Using a technique similar to the more open questioning of teachers during stimulated recall used in the Conners study, Heyman questioned teachers as to the reasons for their behavior. He was seeking considerations, concepts, experience rules, and general evaluations. In essence, Heymann tried to reconstruct individuals' subjective theories as he explored teachers' values and value conflicts, their knowledge gained from experience (principles) and preferences for certain activity patterns. It is similar to the work of Conners, but done in the context and from the perspective of mathematics' teaching.

The use of stimulated recall for this study has been influenced by Kagan's work and the early studies of interactive teacher thought conducted at Stanford and BTES. At the time this study was planned and carried out, the Marland, Conners, and Tuckwell studies had not yet been reported. Nevertheless, many of the same procedures recommended by Conners and Tuckwell were followed in this study.

## Coding Teacher Thought

Several methods of coding stimulated recall data were used in these studies to classify teachers' thoughts or decisions. In the Stanford study, Marx and Peterson (1975) first coded the teachers' comments according to four facets of the communication: the source and type (student question), focus (person or subject matter), process (cognitive level), and function (goals or implementation). The coding system was not found to be very representative of how teachers responded to the questions in the structured interview. Frequencies from a number of subcategories were combined as the counts were so low.

Marland (1977) generated the System of Analysis of Teachers' Interactive Thought (SATIT) to classify each thought unit obtained through stimulated recall. The SATIT system classifies thought units according to perception, interpretations, prospective tactical deliberations, retrospective tactical deliberations, reflections, anticipations, information about pupil or other, goal statements, fantasies, and feelings.

Another coding system was also developed for coding stimulated recall data in the South Bay study. At each teacher-designated decision point, teachers' comments were classified according to the type of decision, the instructional concern mentioned, and the sources of information referred to. Subcategories were defined for each. As with other category systems, each decision point or thought unit results in the reporting of quantitative data (McNair & Joyce, 1979; Morine-Dershimer, 1979). Frequencies are reported without much regard for what is occurring in the classroom at the time. The present study was planned to counter some of the preoccupation with quantifying and classifying. Instead, specific events will be identified and described from the perspective of the teachers and the observer with attention to the unpredictable performance of the students and the instructional environment.

## Conclusion

As was evident in these findings, the dominant research theme has been to quantify and classify interactive teacher thought. Relationships between teacher decision points and the unpredictable events in a classroom provide only enough information to support the use of student difficulties and insights as a strategic research site for investigating critical moments. Clark and Yinger (1978) indicate a need for continued efforts to find out more about the relationships among student performance, teacher thought, teacher action, and the curriculum.

> Researchers have made a promising start toward understanding why teachers behave as they do. This understanding should grow and develop as more of this kind of research

> is done. But the most exciting possibility is that the research may bring research on instruction and the the behavior of teachers together with that on curriculum and materials. All of these concerns come together in the minds of teachers as they make the plans, judgments, and decisions that guide their behavior. Indeed, the first practical theory of instruction may evolve from research on the thinking of teachers (p. 259).

Brophy (1980) is also impressed with the potential value of research on interactive teacher thought. He views it as a fertile source of hypotheses about effective teaching, particularly issues about individualization and optimizing responses to unpredictable student behavior. He contends this cannot be done with attention to quality as well as description. Brophy stresses the need to be more selective in recruiting teachers to study, for providing more information about their backgrounds and characteristics, and being more diligent in assessing and reporting their levels of effectiveness. "I maintain that information form and about certain teachers is of much more value than that from and about other teachers" (Brophy, 1980, p. 50). It is not surprising, then, that Brophy recommends studying teachers who are both experienced and effective according to objective criteria. Teachers for this study will be selected from those with some experience in teaching curricular materials based on solving real problems. The reason for doing so is to seek diversity in how mathematics is taught.

This study hopes to build on the research just described, but in greater depth and from a perspective of teaching mathematics. It is an attempt to search beyond the surface structure to a deeper structure where clearly understandable patterns emerge as they did in the medical inquiry studies (Shulman, 1974). Mirroring Brownell's complaints about the neglect of research on the psychology of school subjects, Shulman also argued that the "...critical missing element may well be the character of the task, subject matter or problem to be mastered" (p. 32).

## Summary

The research literature on interactive teacher decision making and thought has been reviewed. A portrait of the teacher, as revealed by these studies, indicates that the teacher is not a rational decision maker. Much of what teachers do is routine; it can be done with little, if any, conscious thought. Choices of actions are primarily made within the problem space determined by the selection of materials and instructional flow of an activity. Even the more conscious decisions are usually made without considering alternative options. Alternatives are mentioned more when teachers' lessons are not going well. Judgments about the lessons are based primarily on student performances and most teaching decisions have to do with unexpected events and student performance. All this suggests that student occlusions for which teachers experience some momentary crisis will sufficiently disrupt the teacher's more automated processing to quantify as a strategic research site.

Inferences about the principles influencing teacher behavior have been made from teacher action and self-report data. Classifications of lessons have also been made using the proportion of unexpected events which teachers find bothersome. Teachers are mainly concerned with teaching facts or skills at the mastery level, so they emphasize implementing plans, giving directives, and giving feedback. In this regard there are many similarities between the characteristics of a typical mathematics teacher. A careful selection of teachers is planned for this study to find at least one teacher who approaches the teaching of mathematics with a somewhat different orientation, preferably one in which problem solving is encouraged.

Methods of studying teacher thought were also examined. The procedures for conducting the stimulated recall process ranged from structured interview sessions to a more relaxed posture with questions to elicit or clarify teacher thought, and from viewing brief segments to entire lessons. Coding systems have been used to clarify and quantify each decision point, thought unit, or teaching move. Methods for this investigation of teachers' critical moments, which are based on these findings and methods, will be described in the next chapter.

# CHAPTER III

## METHOD

This study of critical moments in the teaching of mathematics traces the cognitive processes of three teachers attempting to cope with unpredictable student performance in a natural setting. It is what Shulman and Elstein (1975) would characterize as a high-fidelity or total-task investigation with the thoughts and feelings of teachers obtained through a procedure known as "stimulated recall." Stimulated recall was used by Bloom (1954) to recapture students' thoughts during college lectures and by Kagan, Krathwohl, and Miller (1963) to elicit and examine thoughts and feelings of an individual interacting with a counselor. It is a technique which is continued to be used to obtain thoughts that cannot be provided through think-aloud protocols during the experience itself. Elstein, Shulman, and Sprafka (1978) used it to gather information from physicians about their diagnoses of patient illness. Kagan has also used it to stimulate recollections of students' thoughts and feelings in elementary classrooms.

Stimulated recall procedures have been used in all but one of the studies of teacher decision making and interactive thought described in the previous chapter. The procedures used in the Stanford and BTES studies of teacher decision making were conducted in laboratory or semi-controlled settings with teachers' using designated curricular materials to teach small groups of students, not necessarily their own.

Teachers were shown portions of videotaped lessons and asked to respond to a prescribed set of questions. These questions had been designed with the expectation that teachers' cognitive processing would approximate that of the decision making model (Shavelson, 1976). Little evidence was found to support that expectation. Teachers did not respond in a rational manner, generating alternatives and selecting that with the best chance of succeeding. Instead, they were reactors to stimuli rather than contemplators; they were satisficers, not optimizers.

This study was designed to study teachers' critical moments in as natural an environment as was possible within the constraints of teaching new mathematical concepts and skills to their own classes. The natural environment was taken to mean more than merely the physical setting and students in a teacher's own classroom. Since teachers teach mathematical topics or units which continue for more than one day's lesson or activity, it was an instructional unit that defined the duration of the study. To study teachers' teaching entire units would provide variation, both in instruction and in student response. The unit would also provide cohesiveness with which to search for the antecedents and consequences of teachers' actions and thoughts, their instructional goals and beliefs about the teaching and learning of mathematics.

Teachers were asked to teach a unit which would continue for more than one week but no more than two. They were to plan a unit on a similar topic. Rational numbers was selected. Two teachers taught a unit on fractions, and one taught a unit on decimals having already taught his fraction unit earlier in the year. The teachers' approach to teaching both units was quite similar, however. The units were taught on consecutive days with exceptions caused by altered school schedules or teacher absences. The units were introductory in nature with new concepts and skills' being taught for the first time in the current school year or the first time in any student's experience.

The study was conducted during the school year of 1976-77 with the support of the Institute for Research on Teaching at Michigan State University. Three teachers participated by teaching a mathematics unit to their own classes for a period of six or seven days during January and February of 1977. Teachers were both observed and videotaped during each day's lesson. Afterward, teachers were asked to reveal their thoughts and feelings during a stimulated recall session. The purpose of this chapter is to explain the procedures used in selecting teachers and in collecting and analyzing the data.

## Teacher Selection

In selecting teachers to participate in this study, several criteria were used. Teachers were to be experienced, have reputations for being "good" teachers, have had past experience with the Unified Science and Mathematics Program for Elementary School (USMES), and teach in similar grade levels in the same school district. The reason teachers were to be experienced and have good reputations was to reduce the likelihood of discipline problems' detracting from their teaching experience. Affiliation with the USMES program was used as a screening device for several reasons. First, past experience with the program was intended to increase the likelihood of finding teachers who valued a problem-solving experience for students. USMES activities were structured around real problems and the learning experience to foster problem solving. Second, teachers in the USMES program were often observed and even videotaped, so the presence of an observer and camera might have been less threatening to teachers with these prior experiences. Selecting teachers from the same district and similar grade levels was done to provide some similarity in the teaching environments.

Three teachers from three different elementary schools were found to satisfy all the criteria and were willing to participate in this study. The school district was that of a large Midwestern city adjacent to a community with a large state university. Students in the schools where the three teachers taught came primarily from lower-middle and middle class homes. Classes were racially integrated, and a number of students were bussed to school. The three teachers, hereafter designated as Martha, Zelda, and Ralph, each had at least four years of classroom experience. Martha was the most experienced, having taught twelve years. Their participation in the USMES program varied. Zelda had been a part of the evaluation team. She had received extensive training and did observation and videotaping in different USMES classrooms. Martha had been one of the USMES classroom teachers and received some training. Ralph had had the least exposure to the USMES project, having become interested because of his spouse's involvement. He attended some USMES workshops, but did not participate as a USMES teacher.

All three teachers expressed an interest in and a willingness to participate in the study. Ralph was also hopeful that the experience would help him to become a better teacher. Martha and Zelda had both been videotaped before and were accustomed to having visitors in their classes. They indicated no concern about being videotaped and observed again. Ralph had never been videotaped while teaching class and was mildly apprehensive about the experience. He had just completed a course at the university in which interactions between himself and another student/colleague had been videotaped and the tapes used to stimulate recollections of his thoughts and feelings. It was the same method known as interpersonal process recall (IPR) developed by Kagan for counseling purposes that was used in designing this study (Kagan 1975).

All three teachers participating in the study knew the investigator was a mathematics educator with a background and interest in teacher training. Teachers had been informed that the purpose of the study was not to evaluate their teaching but to learn more about their decisions, thoughts and feelings while teaching mathematics. Teachers were encouraged to think of themselves as colleagues with something of interest to reveal.

Prior to the actual study, teachers and their classes participated in a pilot study conducted for two mathematics lessons. The pilot phase of the study served several functions: (a) to acquaint the participants with the procedures, (b) to weed out ineffective techniques and equipment, (c) to acclimate the teachers and students to the presence of an observer and camera, and (d) to provide the teachers with some training in the process of stimulated recall. Students were not shown any videotaped portions of the lessons until the entire project was completed so they were not reinforced for any "silly" antics displayed in front of the camera.

## Data Collection

Data from this study came from several sources. The teachers' lessons were both videotaped and observed. Due to the dual role of the investigator, however, there was little time and opportunity for field notes to be taken. To capitalize on what the investigator could recall about the lesson which might not have been visible on the videotape or written into the field notes, a transcript was drafted for each lesson using audio- and videotapes after the stimulated recall session had been completed. The investigator added remembered details that did not appear on the screen. The second major source of data came from the stimulated recall sessions held following the teaching of the lesson. These sessions were audiotaped and later transcribed by a typist as were the written dialogues of the lessons. Prior to the teaching of the unit, teachers were questioned about their plans for the unit. Teachers' plans were also checked on a daily basis unless no new plans had been made. Following the last stimulated recall session for the unit, teachers were also given a pupil card sort to determine their judgments about their students. More detailed descriptions of the videotape and stimulated recall procedures follow as do some details about the pupil sorts.

### Videotaping

When a unit was taught, the equipment was moved to the school where it remained until the unit had been completed. The videotaping equipment consisted of a 3/4" Sony cassette videotape deck capable of taping an hour's worth of instruction with one cassette. Sound was picked up by a wireless microphone on the teacher and a second microphone placed nearer the tape deck. An auxilliary audio tape recorder was also used in case of mechanical failure and to possibly pick up student comments not audible through the other microphones. The two-track recording capability of the videotape recorder was under one-control sound which meant the tracks could not be balanced accordingly. Sound from the teacher's microphone dominated as it was more sensitive to audio signals. The recording system and a small Sony monitor were placed on a movable cart to make it easier to remove the equipment immediately following a mathematics lesson and to move it to the location where the stimulated recall process was to take place.

In each classroom, the camera was placed as much out of the visual range of the students as was possible, given the constraints of the physical setting. In Martha and Zelda's classrooms, there was additional space to place the camera, but not in Ralph's. Martha's classroom was one-half of a large area which had originally housed four open classrooms. There was ample room behind students' desks to locate the camera. Zelda's classroom was fairly small, but there was a large alcove which also served as a hallway to an inner classroom. The camera was placed at the edge of this alcove to the side of the class. Ralph's classroom was small and crowded with no additional space, so the camera was placed in a back corner. The proximity to the students in both Zelda and Ralph's classrooms meant that camera movement could be more easily detected by the students. Consequently, the camera was usually left in a stationary position, focused on the teacher with a wide lens opening; the zoom capability of the camera was infrequently used. Quality of the video pictures was not valued as much as being unobtrusive while filming the lessons.

Stimulated Recall

The stimulated recall sessions were held as soon after the instruction as was possible. With few exceptions, this was done immediately following the lessons or one class period later. A substitute or student teacher taught one class so that teachers were freed to participate in the stimulated recall process. Most recall sessions lasted more than one hour, so the sessions were conducted during lunch period or after school as needed. On one occasion, a teacher returned in the evening to complete the viewing of a lesson. As this was the most concentrated and extensive use of stimulated recall ever reported, flexibility was valued more than consistency, and procedures were so designed.

Teachers viewed videotapes of their entire lessons. They were instructed to stop the tape whenever they could recall thoughts, feelings, or decisions that occurred while teaching the mathematics' lesson. The investigator could also initiate questions about particular events. Because the cassette video machine was not equipped with a pause control, it was somewhat cumbersome to stop and start the tape at desired points. Teachers and the investigator sometimes preferred talking while the tape continued to play. When the comments were obviously not going to be brief, the machine was stopped. Conners (1978) and Tuckwell (1980) recommend using a reel-to-reel tape deck instead of cassettes because of this feature, but the newer 3/4" cassette machines are equipped with pause buttons which eliminate the difficulty.

Teachers were not asked to respond to a fixed set of questions as was the case in the Stanford and BTES studies. In this manner, teachers would not be constrained or guided in their comments; teachers had more control over what they wanted to report. A more relaxed atmosphere was desired, one where teachers were treated more like collaborators. The goal was to help teachers feel comfortable and not to be threatened by the process. The role of the investigator was to act as a facilitator of the teachers' recollections of their thoughts and feelings, to clarify meaning, and to probe for more information. It was also the task of the facilitator to check to see if teachers were recalling past thoughts or giving additional information. The role of the facilitator was a nonevaluative, but supportive one. Probing techniques were adapted from those recommended by Kagan (1975). His techniques were based on years of experience in conducting interpersonal recall sessions with individuals in counseling settings and with students in classroom settings. The important criterion was to stimulate recollection, not to influence teacher thought. Although Conner's (1978) procedures had not been published at the time this study was conducted, those used were similar to the ones he described.

The investigator-facilitator refrained from initiating questions about specific events unless the teacher failed to do so. There was no prescribed rule as to when the facilitator would initiate probes. It was left to the discretion of the facilitator for two reasons, both relating to the need for flexibility stressed earlier. The first was to allow the facilitator to exercise judgment as to which student occlusions and teacher elective actions would be worth inquiring about either because of the nature of the mathematical comments or the context in which they occurred. The second was so that teachers were not repeatedly asked about incidents for which they consistently offered no recollections, particularly important in view of the extensive use of stimulated recall over a fairly short period of time. Investigator-initiated questions and probes were open-ended. Teachers were asked questions such as, "Do you recall thinking anything at this point?" When teachers said no, further probes were avoided.

Each recall session was concluded with two questions: How did you feel about today's lesson? and If you were able to reteach the lesson, is there anything you would do differently? These provided the teachers with one more opportunity to make known their thoughts and feelings after having taught the lessons and viewed them on tape. If time had not been available to check on teachers' plans prior to the lesson, questions about teachers' plans were asked before the playing of the videotapes.

### Card Sort

At the conclusion of their units, teachers were asked to do card sorts on individual students from their classes. Teachers were asked to sort the students according to ability in mathematics and on how they had performed during the unit. Teachers were also asked to identify five students they would most like to keep if their classes were to be changed and the five they would most like to have removed from their classes. Once these classifications had been made, teachers were encouraged to sort the cards on as many dimensions as they could. Card sort data were used to obtain information about a teacher's perceptions of students involved in the critical moments.

## Data Analysis

Data collected during this study were analyzed in several ways and phases. First the lessons teachers taught were broken down into activities and coded according to the nature of the tasks, content, materials, and roles of the teachers and students. A major shift in any of these categories resulted in the coding of a different activity. Only activities for which teachers were engaged in interactive instruction were analyzed for this study, eliminating seatwork, tests, and games. Descriptions of the content that was taught and the instructional styles and strategies exhibited during the teaching of these activities are provided in Chapter IV. Also provided are the instructional goals of each teacher and his/her conceptions of mathematics and the teaching or learning of mathematics. The latter was obtained by searching the audiotape and transcripts of the stimulated recall sessions of teachers' daily lesson plans for appropriate comments.

Once the activities had been categorized, student occlusions and teachers' elective actions had to be identified and classified. Student occlusions were incorrect responses to teachers' questions and unsolicited student contributions that had to do with the mathematical content of the activity or the unit. Student occlusions were further classified as student difficulties or insights according to the mathematical validity of student response or input. Not included were correct responses to teacher questions unless a response was offered as an alternative to one already given and the alternative had not been requested by the teacher. Student comments which did not pertain to the mathematical content of the lessons were not coded. Teachers' elective actions were actions taken in response to the student occlusions. Details of the types and distributions of student occlusions and elective actions are given in Chapter IV.

Next, the stimulated recall data were coordinated with the potential sites for critical moments, the student occlusions. Teachers' comments which suggested cognitive difficulties or emotional discomfort with student occlusions or their elective actions were used to identify teachers' critical moments. Descriptions of teachers' critical moments are given in Chapter V. Included in the descriptions are relevant characteristics of the student occlusions and elective actions and of the instructional flow in which they occurred. Also included was information supplied by the teachers regarding their thoughts during these events.

Throughout this phase of the study, it was necessary to make distinctions between teachers' recollections of their interactive thoughts and noninteractive comments. Guidelines used to make these distinctions were essentially those used by Marland (1977), Conners (1978), and generalized by Tuckwell (1980). Teachers' comments which showed awareness of actions taken, engaged in a general discussion about teaching, or provided a reason, explanation, or rationale for his/her behavior were considered to be noninteractive thoughts unless the teacher clearly recalled thinking them at the time. The same was true for comments in which the teacher summarized or reviewed what had been said or when the teacher reflected on what had occurred or thought about what might be done in future lessons. In addition to cues from the teachers' comments, context or interviewer probes were used to distinguish between interactive and noninteractive thoughts and feelings when necessary. The intent of this study was to examine teachers' critical moments and not to classify and quantify teachers' recalled thoughts and feelings. Consequently, the procedure did not call for the coding of every thought unit as was done in the Marland (1977) study.

Once the identification and description phases of the analysis had been completed, the data and findings were examined in order to determine the nature of teachers' mental processing of unexpected student input. The primary source of information about teachers' mental processing came from the critical moments. Instances of student difficulty or insight for which teachers reported their thoughts or feelings but which did not prove troublesome for them were also examined. Generalizations about teachers' mental processing of unpredictable student performance based on these data can be found in Chapter VI.

It is important to note that the investigator served in all roles: observer, cameraperson, facilitator of the stimulated recall process, and analyzer of the data. Thus, the investigator was always knowledgeable about the lessons and the context in which they occurred when viewing or listening to tapes or reading transcripts. With multiple responsibilities of videotaping and observing during actual instruction, few observer's field notes were taken.

While the investigator attempted to maintain an unbiased and inquisitive attitude thoughout the data collection and analysis phases of this study, it is unlikely that this report is entirely free of personal prejudices and convictions. Having been involved in mathematics education for about twenty years, the investigator has a strong commitment to improving mathematics teaching and learning and has a definite preference for the type of instruction which NCTM has recommended.

At the same time this investigator is not unrealistically idealistic about the possibility of teachers' becoming mathematicians or even adequately knowledgeable about what they teach. The goal, therefore, is to seek ways of improving instruction within these constraints. Teacher proof programs are not the answer as teachers are mediators between the curriculum and the learners; they must be recognized and respected for their role.

## Summary

In this chapter, the procedures for conducting the study have been described. Data collection included observing and videotaping teachers teaching a unit on rational numbers and conducting and audiotaping teachers' recalled thoughts, feelings, and other pertinent information through a process known as stimulated recall. The results of the analysis will be given in the next three chapters. Chapter IV will describe the instructional environment in which critical moments occurred, and Chapter V the critical moments. In Chapter VI, the teachers' mental processing of unexpected student academic performance will be described. A critical appraisal of the method will be given in the discussion chapter, Chapter VII.

# CHAPTER IV

# INSTRUCTIONAL ENVIRONMENTS

## Introduction

Understanding the instructional environments in which Martha, Zelda, and Ralph were teaching is a necessary step toward understanding the critical moments which they experienced. The information can also be used in interpreting the influences on teachers' mental processing of critical moments. Schwab (1978) has criticized research conducted on the phenomena of education, arguing the need to give equal status to each of four commonplaces: teacher, learner, curriculum, and milieu. It is difficult to give equal attention to all four areas in a study which seeks to understand the mental life of the teacher. It is less difficult, however, to provide a rich description in which some attention is given to the learner, curriculum, and milieu as well as the teacher. The purpose of this chapter is to offer such descriptions of the instructional environments for each of the teachers in this study.

Descriptions of the instructional environment will center on the instructional flow and the behaviors which represent a potential threat to that flow and, therefore, a potential critical moment: student occlusions and the teacher elective actions taken to re-establish the flow. The chapter is subdivided accordingly into the following sections: (a) nature of the instructional flow, (b) nature and distribution of student occlusions, and (c) nature and distribution of elective actions.

Instructional flow is generated by the interaction of the teacher with the students and the curriculum. This interaction occurs within and helps to create a particular climate. To capture the essence of this instructional flow, descriptions are presented under the corresponding headings: teaching students and teaching mathematics. The two are definitely not separate tasks; they are intrinsically related. The separation is an artificial one done to organize the information. Included under the heading of teaching students are descriptions of the setting, instructional style, and student involvement for each teacher. Descriptions for teaching mathematics focus on how teachers organized their units and how they planned and taught the content. Information is also given about teachers' instructional goals as revealed through the process of stimulated recall.

For student occlusions and teacher elective actions, the behaviors observed during this study will be identified and described. There are two major categories of student occlusions: difficulties and insights; and there are three for teacher elective actions: exploiting, alleviating, and avoiding moves. Distributions of the different times of occlusions and elective actions help to characterize the instructional environments and to highlight the similarities and differences in those environments across the three teachers.

## Nature of Instructional Flow

### Setting

The teachers in this study taught mathematics to students in the upper elementary grades although the student populations of the three classes were not all the same. Classes were located in different school buildings within the same school district. Because of the busing programs, each school had a diverse population of students. Martha taught a split fourth-fifth grade in which only four students were fourth graders. Zelda taught a split fifth-sixth grade in which almost equal numbers of students were fifth and sixth graders. Ralph's students were slightly older and more capable in mathematics. In Ralph's school, students from grades four through six were grouped by ability for both reading and mathematics. The year this study was conducted, Ralph was teaching the top group in mathematics as determined by achievement test data. Many of the students were sixth graders, with some fifth and no fourth grade students.

Martha. Martha's class of 25 fourth and fifth graders was reduced to 22 students for this study since three left to participate in a reading program for advanced readers during the time of the lessons. This was Martha's second mathematics class of the day because she participated in a team situation with two other classes three mornings a week. In the morning class, students worked on the mathematics objectives provided by the school district.

The present unit was taught in the afternoon to enable the lessons to be presented in sequence rather than two days a week for three weeks. So as not to draw attention to the fact that this was the second mathematics lesson of the day, Martha listed this period as "videotaping" on the daily schedule. Class was held after lunch and before gym class. Because she sometimes had to wait for some of her students to return from safety patrol, Martha had to delay the start of the lessons.

During this time she either allowed students to play with the Cuisenaire rods, or she began with a review activity.

Martha's room was located in half a larger, open classroom originally designed for four classes. The other half was occupied by a reading specialist. In her half of the room, students' desks were arranged at one end in a modified semi-circle or horseshoe, three rows deep. The open part of the semi-circle was exposed to the chalkboard and the projection screen. She conducted class from the front and center of the room where she had access to the chalkboard and overhead projector. During work periods and a few activities, Martha moved about the room, giving assistance where needed.

Zelda. Zelda's class was made up of 29 fifth and sixth graders, so she had a full-time aide to assist her. The aide was not involved in the teaching of mathematics lessons, but she did move about, providing assistance during work periods. Zelda's students behaved much like Martha's.

Mathematics class was held in the morning after spelling and before reading to accommodate the study. This meant that gym period was delayed until after reading. The length of the class period could vary as Zelda had to coordinate her schedule with only one other teacher.

Students sat in clusters of two, three, or four. Most arrangements were based on student choice; exceptions were due to students' inabilities to concentrate on their school work when sitting with their friends. Zelda conducted her lessons from the end of the room nearest the black board. When not writing on the board, she tended to move off to the side, but still near the front. During work periods, Zelda moved about the class answering questions.

Ralph. Ralph's class was made up of 34 fifth and sixth graders. With a few exceptions, students had been assigned to his mathematics class on the basis of their high scores on a placement test. As a result, there was a predominance of sixth graders, and the make up of this class was distinctly different from those of Martha and Zelda. They behaved more like middle or junior high students than elementary students.

Class was held in the morning after homeroom and before reading and lunch. Students changed teachers for reading, so there was little variability in the amount of time allotted for math class. As the room was relatively small, there was little space left to move about the desks and tables. Students sat in clusters of three or four, much as they did in Zelda's room.

Ralph taught the lessons from the front of the room where he had access to the blackboard and a lectern on which he placed his textbook. During work periods he moved about the room to offer assistance to those having difficulty and to keep students on task. Occasionally, he would settle somewhere and let the students come to him. At times there would be a number of students clustered around him waiting to receive help.

## Instructional Style

One way to characterize a teacher's instructional style is to use a variable from other studies. Two process-product studies which investigated the teaching and learning of mathematics in similar grades were the junior high school study conducted by Evertson, Anderson, Anderson, and Brophy (1980) and the Missouri

Mathematics Effectiveness Project conducted in fourth grade classes by Good and Grouws (1979). In these studies, the more effective teachers could be characterized by a number of variables. Many of these same variables could also be used to characterize Martha, Zelda, and Ralph's teaching of mathematics. They were active, well organized, strongly academically oriented, task-focused, and nonevaluative. In addition they asked many questions, particularly "lower order" product or application questions, generated a relaxed learning environment, had high achievement expectations, managed their classes effectively, and were able to "nip trouble in the bud," thereby minimizing behavioral disorders. And, as required for participation in this study, they emphasized whole class instruction. In addition, Ralph and Zelda provided time for seatwork; Martha did not. Instead, she tried to provide students with enough practice on the tasks through whole class instruction. Although they varied in intensity, the teachers were also enthusiastic and nurturant.

While these variables are helpful in characterizing the instructional flow in Martha, Zelda, and Ralph's classes, they are not adequate. They emphasize only general, common characteristics. Each of these teachers also conducted her/his lessons in a distinctive manner. For this reason, a description of each teacher's instructional style will follow addressing the roles teachers assumed, their questioning patterns and strategies, their feedback techniques, and the overall pace of their lessons. Integrated into these descriptions is an indication of student involvement in the lessons as well as the types of behavioral problems the teachers occasionally encountered.

Martha. Martha's basic role was that of a director conducting a group in an extemporaneous performance. As director, she provided the necessary cues by posing the problems and asking the questions. The students performed by using the concrete materials, finding solutions, answering questions, and volunteering information. Teacher talk was minimal; Martha did not lecture. She was soft-spoken and deliberate in her speech. At the same time, she exhibited an enthusiasm and cheerfulness that was compatible with her mild manner. There was an easy give-and-take between teacher and students in Martha's class. Occasionally, she would make joking remarks, but never so that it interrupted or distracted from the thrust of the lessons.

Martha's tendency was to pose a problem and let the students respond rather than tell or show them how it should be done. This was particularly true in lessons when she was presenting concepts. When answers were given, she would ask for some form of explanation from the students, drawing out the important characteristics of the concept. When the tasks became more complex, she still began with a problem rather than a demonstration, but she focused on one subprocedure at a time, guiding students through the solution of the problems with her questions. Her questioning techniques often changed once a routine had been established and the students were able to perform a task. The distinction was between introducing new concepts or tasks and practicing them. Thus, Martha's questioning routines varied in accordance with the nature of the tasks and students' familiarity with them. Martha made an effort to have all the students contribute during the unit. When she was not selecting among volunteers, she could call on different students. In some cases, she called on students who were reticent to volunteer, posing simpler questions to those she described as being less capable.

Martha expected students to wait for recognition before speaking and, with few exceptions, they did. If students impetuously called out answers to problems, she reminded them with a comment such as, "Please let me call on people." When one student impulsively offered a mathematical explanation to another, however, she seemed more appreciative than bothered.

Students were never criticized in Martha's class, but praise was used sparingly. She praised the class as well as individuals when she was particularly pleased with what they had said or done. Martha did not always indicate unambiguously whether a student response was correct. Particularly when new ideas or tasks were being presented, she would seek opinions from other students instead. They may have occasionally made it a bit more difficult for students to know whether or not their answers were correct than if she had clearly told them. She was prepared to trade off this type of clarity for increased student involvement.

Student involvement was high. There were only a few moments during the unit when Martha made overt efforts to regain student attention. Student participation was high in both concrete tasks and group discussions. Some tasks proved more difficult than others, as the distribution of student occlusions will indicate; but student participation remained high throughout the unit. Students frequently raised their hands to respond to questions or to volunteer information. Every student participated in the group discussions, though some much more frequently than others.

Instructional pace was fairly rapid and consistent during most of Martha's lessons. Exceptions were the times when she moved about the room, giving individual assistance to a few students who were having difficulty with a concrete task.

There were very few occasions in Martha's class when she expressed displeasure with student behavior. Discipline problems were almost non-existent while she was teaching. Some minor off-task behavior occurred when her attention was directed to individuals rather than the class. Student attention was easily returned to the task on those occasions.

Zelda. Zelda's instructional role was similar to Martha's. She performed more like a director than a lecturer or demonstrator. Zelda was more animated in her teaching than Martha; she was also cheerful, patient, and in control.

In a form of guided discovery, Zelda often posed problems and questions which encouraged the recognition of a new pattern or rule. Unlike Martha, however, Zelda not only posed problems to exemplify the concepts, but to stimulate the recognition of relationships. Students' recognition of a pattern was evident in their responses to her questions; the number of correct responses increased as did the students' eagerness to give the answers.

Zelda's style of questioning sometimes prompted guessing and resulted in students' calling out answers. Unlike Martha, Zelda allowed them to do so, particularly during the initial phase of a lesson before patterns had been established. However, once Zelda did call on a student, she would ask that others give him/her a chance to respond. Students who sat closer to the front of the room demonstrated more willingness to participate than did those sitting farther away, and Zelda called on them frequently. Sometimes she called on students in the back of the room or those who were less likely to volunteer.

Student participation varied enormously. During some lessons, students were so eager to respond that some literally rose out of their seats, straining to be recognized. In her excitement, one student even dashed to the board to make her point. This eagerness to respond was most characteristic of activities when students were recognizing number patterns. Zelda held her students accountable for asking questions; and when none were being asked, she was not willing to continue with the presentation. One day when Zelda's attempts to elicit student questions were not successful, she ended this phase of the lesson and set the students to work on their assignments.

Zelda provided feedback to students by accepting a correct response out of a number that were being offered or called out. Praise was given to the class when students were successful. This was an indication of her nurturing behavior, a trait she shared with Martha. Criticism did not exist.

The instructional pace varied with the enthusiasm of student response and the routine Zelda was using with an activity. Her "mania" for writing things on the board--a residue of her USMES experience, she believed--sometimes slowed the pace of an activity.

When individuals in class needed to be reminded to pay attention, Zelda would do so with remarks such as, "Tom, are you with me or against me?" At times she called on students to focus their attention on the lesson. One of Zelda's students was disturbed. He became a severe behavior problem at times, but never during the class presentations. The eruptions, when they occurred, came during work periods. With an aide to assist her, Zelda was able to monitor this individual's behavior during work periods, though it bothered her when he required too much of her attention.

Ralph. Teacher talk was much more prevalent in Ralph's presentations as he attempted to motivate, explain, and demonstrate the types of problems students were to be assigned. He was the authority figure and leader who "delivered pages" from the textbook. He was noticably more stiff and formal then either Martha or Zelda. He spoke in a rather loud voice which sometimes sounded strained. In contrast, his demeanor during work periods was quieter, more relaxed and friendly.

Ralph began each lesson by demonstrating the sample problems shown in the textbook; sometimes he paraphrased or read from the book or asked a student to read. Students were called on to answer questions about the simpler subroutines. Most of his questions called for students to apply their skills with fractions--a topic which had been covered earlier in the year--or a definition of values for which a chart was always available for reference. Ralph would go through several problems in order to give the students an opportunity to learn how to do the types of problems he was about to assign. Representative problems were selected from the different problem sets. Occasionally, Ralph would ask the students to select the problems. Ralph's questioning pattern did not vary appreciably within or across activities, but neither did the tasks.

Ralph sometimes told the class he was going to call on students at random so as to encourage their attention, but he seldom did. He wanted to get correct answers; consequently, he often called on

students who would be more likely to give him the correct response. He preferred to handle student questions during the work periods. At the same time he did not wish to be deluged with student questions, so he tried to provide students with enough information to prevent that. If a number of students immediately began to ask the same questions once the work period had begun, he would return to the chalkboard to work another problem with the class.

Ralph acknowledged many correct answers with brief comments such as "good" and "right!" As these words were spoken with enthusiasm, they were a form of praise.

Student participation in Ralph's class varied, but there was less variation than in Zelda's class. Overall, his students were less willing to participate than were students in Martha's class. As was the case in the other classrooms, students were more responsive to questions they could answer easily. They were also more responsive on several occasions when Ralph physically separated himself from the textbook to work problems on the chalkboard; students could no longer simply follow the book--they had to see what he was doing. Students were less attentive when Ralph gave longer explanations.

To regain or maintain student attention, Ralph sometimes directed comments about the importance of listening and learning to the class or to individuals. Such remarks were sprinkled throughout his lessons, but did not disrupt the instructional pace.

Ralph was particularly interested in maintaining a fairly rapid pace because of the capabilities of the students in his class:

> There are enough kids in class that are really accelerated that I really can't slow down too much for the ones farther back...I must have about five or six kids that could do

> with a slower pace than what I go, but I think the bulk of the kids can stay up with me...Maybe I am still a little bit too slow for the very rapid learners.

The instructional pace of during Ralph's lessons was fairly rapid. Slow times were, understandably, associated with a decrease in student participation.

## Teaching Mathematics

In this section of the chapter, the focus is on the mathematics taught during the three units under investigation. Included is a general description of the way in which teachers organized their units into activities and the amount of time spent in class and in actual instruction. Following this general description of the organization, there will be a description of the content that was presented. Included under content are descriptions of (a) how the lessons were prepared--planning, (b) how the lessons were taught--teaching, and (c) what goals and conceptions the teachers professed--mathematical goals. Separate descriptions of these three areas are provided for each teacher. This section will close with a brief summary and comparison across teachers of the nature of mathematics that was taught and the amount of time allowed for teaching it.

Organization. Martha, Zelda, and Ralph each taught a six- or seven-day unit on rational numbers. Their lessons were analyzed into activities according to the nature of the tasks, materials, and modes of instruction. Each activity was associated with a specific task, problem type(s), materials, questioning routines, and method(s) for performing the task or solving the problems. Variations within an activity, when they occurred, were not judged sufficiently distinctive to warrant their designation as new activities.

The number of activities and amount of time, both total class time and instructional time, varied across units and teachers, as can be seen in Table 4.1.

Table 4.1. Instructional Time and Activity Data Per Teacher.

| | Martha | Zelda | Ralph |
|---|---|---|---|
| Number of days in unit | 6 | 7 | 6 |
| Total class time (minutes) | 269 | 299.5 | 277 |
| Total instructional time | 232.5 | 166.5 | 139.5 |
| Average class time/day | 45 | 43 | 46 |
| Average instructional time per day | 39 | 24 | 23 |
| Total number of activities | 25 | 18 | 14 |
| Number of instructional activities | 20 | 13 | 8 |
| Average length of instructional activity (min) | 11.5 | 13 | 17 |

The class time was much the same for each teacher, but Zelda's extra day provided her with an additional half hour. Instructional time, however, was quite varied. Martha had at least an hour more of instructional time because she provided little class time for work periods; she incorporated the practice into her instructional activities. Of her five non-instructional activities, three were brief diagnostic tests, one was like a game, and only one was a practice period. In contrast, Zelda's five non-instructional activities were work periods as were Ralph's six.

In synthesizing the reports of the national surveys and observational studies, Fey (1980) characterized the typical mathematics lesson as being the "...very common explanations and questioning ...followed by students' seatwork on paper and pencil assignments..." (p. 12). Ralph's unit was, therefore, most representative of mathematics teaching today. The same could be said for the organization of most of Zelda's lessons. Martha's teaching was atypical in this regard.

The reason that the average activity was so much shorter than average instructional time was because there was more than one instructional activity. This was particularly true for Martha and far less so for Ralph. When daily figures for total class time and instructional time are examined (see Table 4.2), the consistencies and variations are apparent. Ralph's class periods varied the least, as the time was fixed by a school schedule.

Table 4.2. Minutes of Class and Instructional Time Across Days and Teachers.

| Day | Martha | Zelda | Ralph |
|---|---|---|---|
| 1 | 28/23 | 23.5/14.5 | 45.5/23.5 |
| 2 | 44/30 | 48.5/41.5 | 46/28 |
| 3 | 38/31.5 | 26.5/12.5 | 45.5/15.5 |
| 4 | 53.5/53.5 | 45.5/45.5 | 48.5/28 |
| 5 | 51.5/51.5 | 51/30 | 45.5/25.5 |
| 6 | 54/43 | 50.5/5.5 | 46/19 |
| 7 | --------- | 54/17 | --------- |

The lengths of Martha and Zelda's classes were also consistent on most days except for those shortened for school events or emergencies. Zelda also had more control over the time class was held, having only to coordinate one other class period with another teacher. Daily instructional time showed much more variation. In this case Martha and Ralph exhibited less variation; Ralph taught about half the class period, allotting 20 minutes a day for students to work on their assignments. This was the same time Good (1979) and Good and Grouws (undated M.S.) recommended in their model of instruction. Zelda's instructional time ranged from almost no time (5.5) to an entire class period (45.5).

Content. In preparing their units on rational numbers, all three teachers engaged in unit and daily planning. The unit plans were preliminary plans about what the teachers thought they would be able to cover. These plans were always revised by the daily plans as the timing of student performance could not be made more than a day at a time or for several days at a time. Daily plans were not written out in great detail, but neither did teachers know exactly what they were to do at all times. Some of the fine tuning, the interactive planning, took place while the lessons were actually being taught. Teachers occasionally made minor deviations from their plans, or they had to supplement them due to circumstances which arose during the lessons.

Concepts, relationships, and operations of rational numbers were taught, although the teachers did so with somewhat different emphases using some similar and some different tasks. Concepts were modeled and named using concrete or picture representations, and they were defined and translated from one form to another at the symbolic level. Relationships of equality and inequality were investigated by comparing numbers from models and symbols, by generating sets of equivalent values from models and with continuing patterns, and by finding values

such as the simplest name or the missing number in two equal fractions. The operations taught were addition and subtraction; combining was done with and without the aid of models. Rational numbers included the simplest form of unit fractions (1/a), proper fractions ($b/a \leq 1$), improper fractions ($b/a > 1$), mixed fractions, and decimals.

Teachers in this study had somewhat different conceptions of the mathematics they wanted their students to learn. For this reason they expressed somewhat different mathematical goals and beliefs during the stimulated recall sessions. Their goals ranged from what Skemp (1979) has termed instrumental understanding--a knowledge for doing--to a rational understanding in which understanding of concepts is essential.

Descriptions of the planning and teaching of each unit and of the mathematical goals each teacher professed are given in the next section. These descriptions offer an overview of what each teacher did. Detailed information about the time and content of each activity can be found in the Appendix (pp. 253-257).

Martha: Planning. In preparing to teach her fraction unit, Martha first listed the required and supplemental objectives from the list for her grade level provided by the school district. Of the twenty-one she found, nine were required. Students had already passed an average of four objectives, though not necessarily the same four; the range was from zero to seven. These objectives provided Martha with some topics and tasks for her lessons, but she did not plan activities to address the specific objectives. Instead, she searched through supplementary materials and drew upon her past experiences teaching fractions to plan activities. She did not use the textbook as a resource, nor did she plan a unit which was in any way like the fraction unit in the textbook.

Martha wrote out plans for each day's lesson which covered more than half of her twenty activities; the rest were spontaneously conceived to fill remaining class time, to respond to student suggestions, or to satisfy her perceived need for a change in routine. Plans for a couple of activities were quite sketchy and left a good deal to be decided once they were in progress. Her written plans were brief comments about the tasks students were to perform. In most cases example problems, procedures, or questions were indicated, but not all problems were listed ahead of time, even for her diagnostic tests. Instead, she usually relied on her ability to generate different fractions or models while conducting the activities. No unit test was planned or given as testing was to be done on an individual basis when students took the posttests for the fraction objectives.

Once the unit had begun, Martha also used student performance from one lesson in planning for the next day. Evidence came from observed student behavior and from short, diagnostic tests which she gave on the first three days. She stopped giving these tests when she realized she knew as much about students' progress from what she had observed them doing. There was more evidence that Martha modified her plans because of students' performance than there was for either of the other two teachers. Perhaps this was because of her expectations when planning a lesson. "As I lay out a lesson, I'm thinking <u>all</u> kids are going to do the same thing the same way. They should be able to do all the objectives [tasks] I've set. I want to reach it!" If she felt this as keenly as she reported it, her surprise would understandably increase her sensitivity to students. Concern for their learning was also reflected in her instructional goals which follow the content description.

<u>Martha: Teaching</u>. To teach fraction concepts, Martha relied on models. Fractions models were based on measurement: length with Cuisenaire rods or number lines and area with pictures of rectangles or circles. To represent fractions with the rods, a unit rod or train was matched with one or more single colortrains, as shown in Figure 4.1.

<table>
<tr><td colspan="6">dark green--unit</td></tr>
<tr><td colspan="3">light green</td><td colspan="3">light green</td></tr>
<tr><td colspan="2">red</td><td colspan="2">red</td><td colspan="2">red</td></tr>
<tr><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td><td>w</td></tr>
</table>

Figure 4.1. Cuisenaire rod key.

In the above key, one red rod represents 1/3 of a dark green unit, two reds represent 2/3, and so on. To illustrate 5/3 requires five reds, and six reds represent 6/3 which can easily be shown to equal two unit rods. A key of all possible single color trains was formed to find different fractional parts of the same unit and to work with fractions having different denominators. From this same key, students could identify 1/2 as being equal to 3/6, compare the values of 1/2 and 2/3, or add 1/2 and 1/3.

Cuisenaire rods were used to introduce students to fractions and to draw rectangles, thereby helping to make the transition from the rods to pictures. Then the concrete interpretations of fractions were applied to other tasks. Martha began with unit fractions (1/a) and then moved to other proper fractions ($b/a \leq 1$). Improper fractions were introduced accidently on the third day and were explored during the next lesson. Students built or drew models of fractions and identified fractions from models. They were told which unit rod to use in all but two activities when rods were in use. In these two activities, students were asked to represent 1/3 or 6/4 which meant finding an appropriate unit to model the fractions.

The use of a key built with Cuisenaire rods had several advantages. Martha could ask a number of questions without having to pause for students to stop and build a different model which meant the activity could proceed at a fairly rapid rate. The use of the key also meant that students could identify equivalent fractions, make comparisons, and add fractions more easily. These tasks were easier to perform when a key was left intact, which Martha encouraged her students to do.

It was near the end of the third day when students were first asked to compare and order fractions, using fractions with like denominators and numerators. Then students used the rods to model and compare all types of fractions. Equivalent fractions were found with keys of rods and, for some students, without rods.

Addition of fractions was modeled with the Cuisenaire rods during the last two days. When the fractions being added had the same denominator, students were able to answer quickly without using the rods; but when the fractions had unlike denominators, the procedure was more complex. For addition of fractions with unlike denominators, Martha found equivalent fractions with the same denominator so that the sum could be easily computed. Her approach was a concrete demonstration of the algorithm $\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$. Some students, still using the rods, found solutions without finding like denominator equivalents. Using the key shown in Figure 4.1, the two methods for finding $\frac{1}{2} + \frac{1}{3}$ are as follows:

> Martha's approach: First model 1/2 and 1/3 with red and light green rods. Then, using the white rods, find equivalent values of 3/6 and 2/6. Rewrite the problem as $\frac{3}{6} + \frac{2}{6}$ and add to get 5/6.
>
> Students' approach: Use the red and light green rods to form a train and compare to the key. The answer of 5/6 can be read directly.

The specific amount of time and the number of activities Martha used to teach these concepts, relationships, and operations as well as the tasks will be listed at the end of this section along with similar data about Zelda and Ralph's instruction. At this time some comparisons will be made, noting the similarities and differences in the nature of tasks teachers chose to teach rational numbers and the time they spent doing so. For specific details about each activity, see the Appendix (pp. 253-257 ).

Martha: Mathematical Goals. During the stimulated recall sessions, Martha expressed her instructional goal as wanting students "to experience fractions." She believed that a concrete introduction to fractions might eliminate or prevent some of the more predictable confusions with fractions:

> I think they need to build the concept in their heads, concretely, and then you can do the numbers because you know what you are doing...if you shift this number here and put this number there and that one down there without understanding why, you'll always keep forgetting where to shift what.

Martha was less articulate as to how students were to make the transition from the concrete to the symbolic. She seemed to feel it would happen automatically as she said, "If you can use the concrete enough until they can see it, their going to numbers doesn't phase them."

Paper and pencil activities, Martha contended, got in the way of seeing the fraction concept, of dividing something into equal parts, an everyday concept. Instead, they seem to result in students' thinking too narrowly about the concept. Nevertheless, she spoke of feeling "...obligated to give them at least exposure to tests they were going to have...to show them how that relates to those tests on paper."

The sad thing, according to Martha, was that not all the curriculum was like fractions. "A lot of times all you have is a test, the paper and pencil, and you don't get into giving them a concrete base." She also admitted that, while she tended to teach fractions with concrete experience, the unit was not typical of her mathematics' instruction. She did not always have the time needed to develop understanding, by having students model and apply concepts with concrete materials.

Students were expected to learn the concepts through practice with varied tasks and materials. Verbalizing fractions was also important. "The more they hear them and the more they say them, the better off they will be; you can't get enough experiences!"

Martha believed the difficulty of the tasks were appropriate for younger students. "This is really a second and third grade type thing, this experiencing fractions." Nevertheless, she also believed the tasks were appropriate for her students. "I don't think they get this in second and third grades, so I start way back there [referring to the tasks] with them." This was her opinion even though she had taught a similar unit to some of the same students two years earlier. Before starting the first activity, she asked her class what "fraction" meant. Their responses reinforced this belief as she concluded that students did not really know "what fractions are, what the concept is."

Martha admitted she was not all that comfortable about her ability to design and teach the fraction unit. "I don't know enough about fractions," she said on the fourth day. "I feel inadequate to structure experiences for them to grasp some of the concepts." About the activities on the second day, Martha said, "I don't know if they are getting the meaning, the concepts of fractions this way. I just have to do it and try to see if they have got it." To the observer, however, Martha's demeanor was one of confidence about what she was teaching throughout most of the unit. Exceptions will be reported in the descriptions of her critical moments.

Zelda: Planning. Zelda prepared a unit plan by listing the topics she intended to cover. The topics came from the fifth and sixth grade textbooks from the Elementary School Mathamtics series by Eicholz and O'Daffer (1964). She listed the models she intended to use to provide students "experience with fraction concepts and relationships," and she wrote suggestions as to which methods might be used to perform some tasks. During the first week of the unit, Zelda did not use the textbook in planning her lessons:

> I rejected the source available, the book. It has those little pictures, shapes divided into regions. And there is enough terminology with fractions anyway without throwing in regions, shaded areas and stuff. I think that tends to confuse them.

Zelda's daily plans or at least those for the first week were written in greater detail than were Martha's. For some of her activities, Zelda wrote out all the problems she intended to cover. She even drew pictures of all the different Cuisenaire rod keys that displayed the equivalent fractions. Once Zelda began to use the textbooks as the source of problems to assign students, she no longer wrote such elaborate plans. Instead, she merely listed topics and page references. Evidently, the care with which she wrote her plans was based on her own perceptions of her ability. On the fifth day after teaching a lesson in which she had written out the problems for her presentation ahead of time she said,

> I thought mentally I was better organized today...When I think about teaching fractions, I find I can confuse myself very easily with fractions...I was going through something and I thought...I confuse myself between the least common multiples and greatest common factors. So I wrote those notes very carefully, and I had them in my hand and I thought, "If you say this wrong!"

Zelda did not always plan with such great care, however. For the same lesson she had failed to coordinate the student's assignment with her presentation. The two had very little to do with each other, and the students encountered many difficulties.

Zelda: Teaching. Zelda's unit, like Martha's, was planned as an introduction to fractions. The first activity was designed to motivate a need for fractions. Using the familiar number-line model, Martha asked students to imagine it's being stretched like bubble gum and then having to name points between 0 and 1. Cutting the segment into different numbers of equal parts, she asked the students to name the different points and to name the points found by repeated halving of a segment. Zelda then switched to using the Cuisenaire rods. Students formed all the single color trains (keys) for each of the rods and named the different unit fractions. Afterwords, students named successive fractions by counting the pieces. For a model like the one in Figure 4.1, students counted the whites as 1/6, 2/6, 3/6, 4/6, 5/6, and 6/6. A special point was made of the fact that 6/6 was equal to one.

On the second day, equivalent fractions were identified from a rod key much as they had been in Martha's class, but Zelda took a very different approach from that point. Once students had found 1/2, 2/4, 3/6, and 6/12 to be equal from the orange-red key, they were asked to fill in the missing values of 4/8 and 5/10. From this list, a counting pattern emerged: numerators went up 1, 2, 3, 4,... and denominators 2, 4, 6, 8,.... Similar patterns were established for other unit fractions (e.g., 1/3 or 1/4) and much later for fractions such as 2/3. There was no mention of the general rule: $a/b = ak/bk$.

From equivalent fractions, Zelda moved to a comparison task. Students modeled the fractions with rods to make the comparisons. This task activity was followed by another in which fractions with like denominators were added. Again, Zelda introduced the task with the rods, but her students quickly abandoned the rods as they recognized the simple rule, much as they had in Martha's class. Zelda did not use the rods to add fractions with unlike denominators. Instead, she had students generate sets of equivalent fractions, select those with like denominators, use them to replace the fractions, and then add. This was done on the fourth day of the unit.

On the last three days of the unit, students were asked to identify the simplest name of a fraction and to find the missing number in two equal fractions (e.g., 3/8 = 9/?). Students were shown an alternative method to working the set of equivalent fractions based on division. The simplest name was found by finding common factors of the numerator and denominator and then dividing each by a common factor. Using this method to find the missing value in 5/6 = 15/?, 15 is three times five which means the unknown (?) is three times the six or 18. The problems for these activities came from the textbook as did some of the solution methods. Generating sets and using rods were not illustrated in the text.

Zelda: Mathematical Goals. From her remarks, it was clear that Zelda's goal was to bridge the gap between concept learning and textbook problems. She hoped to introduce students to the basic fraction concepts through concrete experiences. Zelda believed that Cuisenaire rods involved students more than chalkboard presentations:

> If you get everybody involved in the manipulating of rods and they have to agree.,.then I feel confident that they are going to have an idea of the relationship of one rod to another and be able to label fractions.

One advantage she saw to introducing students to fractions with the rods was that it would help students once they began working problems from the textbook or any kind of pencil and paper work:

> If a student has a problem with, say 1/3 and 1/3, we can go back to the rods, sometimes just be talking about them. Or, if they are really hung up, you can go back to the rods and try it with them. Not for everybody, but for whoever needs it. It gives them something to relate to.

Zelda was not interested in having students become proficient in using the rods to solve problems; her concern with the transfer from rods to paper tasks was the following:

> I am not sure I want to teach kids to be super efficient with rods when they are going to be tested with paper and pencil. Somewhere you have got to transfer...I would like to stay with rods all the rest of this week and go through these concepts and perhaps next week transfer to pencil and paper...I think I will only go through addition and subtraction and then will transfer to pencil and paper and later on get into division and multiplication.

Zelda sought to introduce everything first and then to return to "go over one major concept a day with practice work." Through this spiral approach, she envisioned having an opportunity to help individual students.

Part of Zelda's concern with the transfer from the concrete to the pencil and paper tasks and with providing alternative approaches came from her past experiences with mathematics. About her presentation on the fifth day (Activity 13), Zelda said,

> There are all kinds of different ways you can do fractions. I want to make sure I am really going slow with this because I wouldn't want to confuse anybody. I can remember when I was learning fractions, I got all mixed up. So I'll try showing you something else you could do.

In this regard, Zelda felt much the same as Martha.

Zelda believed it was important to offer students alternatives, "Particularly when we've got the total group working on the same activity." Alternative procedures were needed to accommodate students who, for example, were able to multiply and those who would be more comfortable counting. When trying to find the lowest common multiplier, students were allowed to think of rather than write out all the factors. "A few of them could handle it that way. Fine. There is no reason to make them feel that when they are getting to lowest terms that they have to write out all the factors."

Ralph: Planning. Ralph's decimal unit was based on the first portion of the decimal chapter in the sixth grade Addison Wesley text: Elementary School Mathematics (1964) by Eicholz and O'Daffer. Activities were primarily planned and structured around pages and problem sets in the book, and every activity was planned. On a daily basis, Ralph needed only to decide how many pages and problem sets he expected to cover or which of the supplementary materials found in his initial search for the unit to include. Prior to teaching each lesson, he read over comments and examples in the text and also glanced through the problem sets to be assigned. A few times he wrote comments in the margin or underlined things he wanted to emphasize, but this did not always ensure he remembered to do so.

Ralph's students had not yet started to work on the decimal objectives which meant students had not taken the tests to determine if they could pass any of them. No unit test was planned as students would be tested on the objectives. Ralph expressed confidence that all the students would complete the decimal objectives by the end of the year. Knowing the unit was only an introduction did not diminish Ralph's eagerness for the students to do well. As he put it, "I am really hoping they get a good start and get momentum going!"

Ralph never assigned the starred exercises in the text which were designated as more challenging problems by the authors. About the starred problems on one page, he admitted, "I do not understand why those exercises are there." His reason for avoiding them was that they required "almost another preparation for me to understand them," and he did not believe he should assign something without being prepared to help students with it.

Ralph: Teaching. Having already completed a fraction unit earlier in the year, Ralph taught a unit on decimals. Decimal values were defined in terms of fractions which were listed in a chart. The initial task consisted of identifying values of digits in a decimal number (e.g.,"value of five in .576 means?"). After this, students were asked to express decimals in expanded fraction notations and vice versa. Extending this one more step, students were able to convert the number to the alternate form.

Ralph transformed decimals into fractions by writing expanded sums as illustrated in the textbook. To change $\frac{234}{100}$ to $2\frac{34}{100}$, he made a position-by-position translation of the decimal number to fractions (2 + 3/10 + 4/100). Equivalent fractions were then found (2 = 30/100 + 4/100) so they could be added. The procedure was reversed when a fraction was to be transformed into a decimal. It was only after a number of problems had been solved in this manner that a direct conversion from decimal to fraction equivalent was allowed. Equivalent fractions were found by a conventional method, dividing out or introducing a common factor for both numerator and denominator.

Initially, students were given fractions for which denominators were powers of ten, but fractions with other denominators were also given. The textbook method was to change $\frac{1}{8}$ to $\frac{125}{1000}$ before writing the decimal, and Ralph indicated the answer could be found by dividing the numerator by the denominator.

In one activity students were asked to give decimal names for points on a number line. Comparisons were only required for one problem set which was assigned without a corresponding explanation. Addition/subtraction was the topic for the last two days of the unit. Fractional justifications were used to introduce the operations. After a number of problems had been solved, Ralph elicited the common rule: line up the decimal points and add or subtract.

Ralph: Mathematical Goals. Ralph's primary instructional goals were for the students to understand decimals, by which he meant they would be able to do problems:

> I am using the word "understand" strictly within the framework of the textbook. I want them to be able to do the activities [problems] that are in the textbook and be able to perform the pretests and posttests of the objectives.

Beyond this goal of understanding, Ralph did not appear to have very well-formed ideas about what students were to learn. When probed about his mathematical goals for one lesson, he replied, "Well, quite honestly, I don't consider that question."

For the most part, he believed students were learning a collection of skills acquired through repetitive drill:

> Learning mathematics is memorizing a few basic skills... it's just a matter of repeating and repeating and repeating how to do it before they finally grasp how to do it ...students will forget the whys. What students remember is how to do things with decimals [i.e., adding] probably because they receive a lot of drill.

Ralph's preference for doing rather than understanding mathematics was rooted in his own knowledge and experience. In addition to believing that he had "a pretty good understanding in my own mind of the basics of math" and was "uncomfortable" with the "why you do things" in mathematics. He claimed not to be "heavy in math," and said he had not had "any of that modern math." Yet, believing that "as a teacher, you have to relate to your own experiences in mathematics," Ralph had concluded:

> Maybe it is a good approach to just teach how to first and then maybe the why will make sense later. That is true for me personally, I feel...that is what it was like when I got into math in high school.
>
> I don't worry about [trying] to explain why this is done this way, but just to do it this way so that after you do several...it will make sense.

In spite of his belief, Ralph was teaching from a text he saw as having "a lot of whyness" to it.

As for the topic of decimals, Ralph did not personally consider them to be basic to the mathematics curriculum. Putting decimals in a class with "fractions and things like that," he argued, "is the kind of thing if you need on a job, you will learn to do it." In his presentation, however, he made reference to ways in which decimals would be useful and needed by students; they were applications mentioned in the text.

## Nature and Distribution of Student Occlusions

A student occlusion is an unpredictable student response or contribution which arises during the teaching of a lesson. Occlusions are indicated by students' failure to provide correct answers to teachers' questions and to students' unsolicited questions, suggestions, or comments. These unexpected incidents of student performance can disrupt the instructional flow of an activity and cause teachers problems. For this reason, student occlusions are considered to be potential sites for critical moments. Before examining the episodes identified as critical moments for teachers in this study, it is important to know something about the nature of all the student occlusions which occurred during the teaching of mathematics under investigation. The following is a description of the nature of the student occlusions identified and of the distribution of these occlusions across types and teachers. The emphasis will be on characterizing the patterns of student occlusions for each teacher and some possible causes for the differences.

Nature of Student Occlusions

Student occlusions were classified as student insights or difficulties according to whether they suggested student understanding or the lack of it. To be classified as student insight, student questions, or suggestions had to be mathematically correct. They could only be signaled by unsolicited statements or queries; they were not simply correct responses to teachers' questions. Student insight came in one of three forms: pattern detection, explanations, or alternative responses. Students may not always have understood what they volunteered, but their offers had mathematical validity. Both student difficulties and insights were further classified in order to reflect the different types of things students did. Incidents of student difficulty were primarily failures on the parts of students to give the desired responses to teacher-solicited responses. They were coded as errors. Other types of student difficulties came from student-initiated expressions of confusion and from teacher-initiated moves which signaled student difficulties (see Table 4.3).

Table 4.3. Types of Student Occlusions.

| | Difficulties | Insights |
|---|---|---|
| TEACHER-SOLICITED OR INITIATED | Errors<br>Patterned<br>Nonpatterned<br>Teacher Signaled | Pattern Detection |
| STUDENT-INITIATED | Expressions of Confusion | Pattern Detection<br>Explanations<br>Alternative Responses |

Errors were further classified as patterned and nonpatterned errors. Patterned errors were responses from which a misconception or inappropriate rule could easily be inferred. For example, an answer of 2/5 to 1/2 + 1/3 was coded as a patterned error as was the answer of 9/10 instead of 9/100 in the decimal 34.09. Also designated as patterned errors were answers such as 11/11 for 11/10, 6/10 for 6/5 when two circles were shown divided into fifths, and a model for 4/6 when one for 6/4 had been requested. A patterned error was also indicated when students ignored the unit and continued labeling successive pieces of a rectangle as 1/5, 1/4, and 1/3.

Nonpatterned errors, by their very nature, did not readily suggest specific misconceptions or rules. Some had the appearance of being guesses. "No response" was also classified as a nonpatterned error. If a teacher continued talking after a question had been asked without giving any overt sign of waiting for or seeking a response, no coding was made of an error. If there were some sign that the answer was being sought from another student, a nonpatterned error was coded. When more than one response could be heard at once because students were calling out their answers, the mixture of student responses was designated as a nonpatterned error.

Expressions of confusion were most often student-initiated admissions of confusion. Some were forthcoming after a teacher had asked the students if they understood a task of concept. It was a general question and not directed to anyone in particular. Expressions of confusion were sometimes general as "I'm really lost!" Some indicated specific difficulties such as, "I don't see what 12ths are!" Also included in this category were inappropriate, unsolicited student explanations or challenges. When 6/5 appeared for the first time in Martha's class, several students clamored, "You can't have 6/5!" And after 3/4 had been shown to be larger than 3/5, one student volunteered it was because there were fewer pieces of pie left. What confused students most in Martha's class were improper fractions and the addition of fractions with unlike denominators. In Zelda's class, it was the use of the rods to model and compare fractions. In Ralph's class, students did not volunteer their confusions as will be apparent in the distribution of student occlusions by teachers.

The last category was teacher-signaled student difficulties. This category was included to account for the times when a teacher stopped to give individual assistance to a student even though there had been no evidence from class discussion that the student was experiencing difficulty. In most cases, the teacher was able to spot difficulties as students were performing tasks with the Cuisenaire rods.

Pattern detection was usually an unrequested offer of rule or relationship based on available examples or information. To what extent a pattern recognition was spontaneously conceived or recalled from past experiences cannot be stated. It was, however, the first time the idea had been mentioned during the unit without the teacher's overtly attempting to elicit it. Solicited pattern recognition was only included when the result spontaneously occurred during the lesson.

Students' descriptions of patterns were sometimes general, as was a rule for determining which of two fractions without the same numerator was larger: "The smaller the bottom number gets, the more you got!" To explain the addition of two fractions with like denominators, "You add the two numbers on top" was offered. Sometimes numerical examples were used to express a pattern as students did to ask about adding fractions with unlike denominators, "How do you add 6/12 and 2/6?" To describe what fractions were equivalent to the number two, one student tried, "It's just like 4+4, 3+3, 2+2, like two numbers..." In this case, a more general rule was offered as he concluded, "You double it" [referring to doubling the denominator to get the numerator].

Student explanations were typically brief attempts to clarify why certain things could or could not be. Some were offered to counter an inappropriate answer or explanation. To counter the incorrect justification for 3/4's being larger than 3/5, mentioned earlier, another student reported:

> Yeah, but the pieces are bigger for 3/4. There are only four pieces. In order to make five, you have to do it a bit smaller to get five pieces [Martha's class].

Alternative responses were offers of equivalent values or procedures that had not already been given. In Martha's class, for example, students were quick to point out fractions equal to 1/2. As a correct response had already been given, the 1/2 was labeled as an alternative response. One alternative procedure was a student's attempt to eliminate the intermediate steps being used to convert decimals to fractional equivalents. Instead of progressing 3/25 = 3 + 2/10 + 5/100 = 3 + 20/100 + 5/100 = $3\frac{25}{100}$, the student wanted to know why they couldn't just go directly from 3.25 to $3\frac{25}{100}$. Another example occurred in the first activity in which Martha did not specify the unit rod students were to use. After 6/4 had been modeled with one unit (purple), another student volunteered a different unit (brown).

Distribution of Student Occlusions

Different patterns of unexpected student performance emerge from the distribution of student occlusions across teachers. Frequencies range from 49 for Ralph to 155 for Zelda and 217 for Martha, but the figures are somewhat unrepresentative of the students' performance because of the different amounts of time each teacher was engaged in total class instruction. For this reason, comparisons across teachers will be made on the basis of proportions and densities rather than frequencies. Proportions are the percent of student occlusions of the total for a teacher, and densities are the number of student occlusions per minute of instruction. The proportions and frequencies of student occlusions are shown by type and teacher in Table 4.4.

As might be expected when teacher questions are the primary means of eliciting student participation, student difficulties were the dominant source of student occlusions. Upon closer examination of the data, however, different patterns of student occlusions can be observed across teachers. One difference is the amount of variation in the types of occlusions a teacher encountered during the teaching of his/her unit. (For details about each activity see Appendix, (p. 257).)

The least amount of variation occurred in Ralph's class. With errors accounting for 88% of the student occlusions, he had only to cope with errors. Expressions of confusion and insightful contributions were extremely rare. Had the students exhibited no difficulties during the work periods, their lack of confusion might have been indicative of understanding, but that was not always the case. Students often asked for help during these periods, but, of course, it is possible they never realized how confused they were until they had begun to

Table 4.4. Distribution of Proportions and Frequencies of Student Occlusions by Types and Teachers.

| | Martha | Zelda | Ralph |
|---|---|---|---|
| **Student Difficulties** | | | |
| Errors | 41 ( 88)[a] | 64 (100) | 88 (43) |
| Patterned | 14 ( 30) | 9 ( 14) | 31 (15) |
| Nonpatterned | 27 ( 58) | 55 ( 86) | 57 (28) |
| Confusion | 19 ( 41) | 24 ( 37) | 4 ( 2) |
| Teacher-Signaled | 18 ( 39) | 3 ( 4) | -- -- |
| TOTAL: | 77 (168) | 64 (100) | 92 (45) |
| **Student Insights** | | | |
| Pattern Recognition | 4 ( 9) | 5 ( 7) | 2 ( 1) |
| Explanations | 6 ( 14) | 0 ( 1) | -- -- |
| Alternatives | 12 ( 26) | 4 ( 6) | 6 ( 3) |
| TOTAL: | 23 ( 49) | 9 ( 14) | 8 ( 4) |
| TOTAL OCCLUSIONS: | 100 (217) | 100 (155) | 100 (49) |

[a]Percent of total (frequency of occlusions during unit of instruction)

work on their assignments. The lack of insights is not necessarily a negative reflection on the ability of the students as the nature of the tasks did not afford them with many opportunities to be insightful.

Student occlusions in Martha's class showed the greatest diversity. Her proportion of insights was three times that of the other classes, and the occlusions were more evenly distributed across types. The frequency and proportion of insights were inflated by the large number of alternative responses from students volunteering equivalent fractions. Representing fractions with concrete models afforded students with opportunities for noticing alternative names for the fractions. Most teacher-signaled occlusions came during two activities when students were using the rods. Martha was able to move about the room monitoring student progress while continuing to pose problems and ask questions. It was her way of providing help in the absence of work periods. Some of this variation in occlusions was due to the nature of the tasks and Martha's instructional style, as well as to the student contributions. Students in Martha's class were also vocal about what they did not understand. Confusion was another category well represented in Martha's class (one-fourth of all occlusions), but not Ralph's class (one-twentieth).

The pattern of student occlusions in Zelda's class showed some similarities with each of the other classes. Her high proportion of student difficulties was almost identical to Ralph's, but her proportion of errors was lower. Zelda's students, like Martha's, expressed their confusion. Like Ralph, Zelda used the work periods to help individual students and so had very few teacher-signaled student difficulties. The few she did have occurred under circumstances similar to the ones in which Martha's occurred--when students were working with Cuisenaire rods.

When types of errors are considered, the dominance of nonpatterned errors is most apparent. Nevertheless, there were differences in proportions of nonpatterned errors. Nonpatterned errors outnumbered patterned errors almost two-to-one in Martha and Ralph's classes. The ratio was six-to-one in Zelda's. By allowing students to call out their answers frequently rather than having them wait to be recognized, more guessing and multiple responses occurred, both of which contributed to the larger share of nonpatterned errors in Zelda's class.

Differences in students' willingness to volunteer their ideas or to express their confusion are even more apparent when student occlusions are categorized as solicited or unsolicited. Teacher-signaled difficulties were omitted because they were teacher-initiated. Table 4.5 displays the gradations across teachers. The proportion of unsolicited student occlusions are 12% for Ralph, 33% for Zelda, and 42% for Martha. One possible reason for this difference, which has already been suggested, is the difference in tasks: students performed different tasks in Ralph's decimal unit than they did in the fractions units. Another possibility could be the difference in the classroom climates. A more relaxed atmosphere prevailed when Martha and Zelda were teaching; Ralph's presentations were more formal. Also, as will be apparent from his choice of elective actions, Ralph preferred not to face student questions during the presentation, but to deal with them during the work periods.

Table 4.5. Proportions and Frequencies of Student Occlusions by Initiator and Teacher.[a]

| Student Occlusions | Martha | Zelda | Ralph |
|---|---|---|---|
| Solicited | 41 (88)[b] | 64 (100) | 88 (43) |
| Unsolicited | 42 (90) | 33 ( 51) | 12 ( 6) |

[a]Teacher-signaled occlusions omitted
[b]Percent (frequency)

Densities offer yet another view of student occlusions as they are figured on the basis of time rather than total number of occlusions. Data on total student difficulties and insights, colapsed across categories, are shown in Table 4.6. These data reveal the similarity in the rates at which Martha and Zelda confronted student occlusions, close to one per minute. Ralph, on the other hand, processed student occlusions at about one every three minutes. A similar pattern existed for student difficulties.

Table 4.6. Density[a] of Student Occlusions by Types and Teachers.

| | Martha | Zelda | Ralph |
|---|---|---|---|
| All Student Occlusions | .93 | .93 | .35 |
| Student Difficulties | .72 | .85 | .32 |
| (Errors) | (.38) | (.60) | (.31) |
| Student Insights | .21 | .08 | .03 |

[a]Number of occlusions per minute of instruction

One possible reason for this difference is that Ralph was far more verbal than were the other two teachers. He spent more time explaining and, therefore, asked questions at a slower rate. Also, as was prevously indicated, students in Ralph's class expressed confusion less often. Differences in the content may also have been significant factors, both in producing the similarities in Martha and Zelda's experiences and the difference in Ralph's. The error rate is entirely different. Both Martha and Ralph were confronted with errors at about the same rate; Zelda's rate was almost twice as fast.

Densities for each activity within a teacher's unit provide yet a different view of the rate at which teachers encountered student occlusions. The range and value of activity densities is illustrated in Figure 4.1. Once again, little variance is shown for Ralph's unit as the densities of his eight activities ranged between one every five minutes and one every two minutes. Martha's activities showed the greatest variation ranging from no student occlusions to about two per minute. Zelda's activities also varied in their rate of student occlusions. Her data differed from Martha's only in that she taught fewer activities and that she had no activities with zero densities. More detailed information on the data for each activity can be found in the appendix, but some general remarks will help to distinguish between the tasks that produced the extreme density values in Martha and Zelda's units.

Martha's two zero density activities included a review of modeling unit fractions while waiting for all the students to arrive. The other task was comparing and ordering fractions with like denominators which the students found to be a very easy task the first time it was introduced. At the opposite end of the density measures (1.3 or above) were eight different activities. Students had more difficulty representing fractions when the unit rod was not specified, when naming fractions from a rectangle model which had been drawn on grid paper (their own division lines were confused with the existing grid lines), when representing and naming improper fractions, and when adding fractions with unlike denominators. The activity in which students were adding fractions with like denominators also had a high density measure, not because students were having difficulty, but because of their insightful occlusions. Insightful occlusions also were very apparent in the activities in which fractions with unlike denominators were being added.

In Zelda's class the lowest density measures came when students were being introduced to and when they were reporting all the different unit fractions that could be represented with the rods. There were only three activities in which the rate of occlusions was 1.3 or higher. One was the first activity of the unit when she was asking students to name points on the number line. Another fell on a Monday when a method for finding simplest names of a fraction was being reviewed. It was on this day that Zelda put the students to work on their assignment when they would not ask questions. Another activity the students found difficult was the first time they had been asked to generate sets of equivalent fractions with a counting pattern. This was also the activity in which the most insightful occlusions occurred; most of the others came with adding fractions and generating sets of equivalent fractions for non-unit fractions.

These distributions of student occlusions have provided a look at the similarities and differences in the number and proportion of student occlusions confronting the teachers in this study as well as the rate at which they occurred. The importance of task, instructional style, and climate in influencing the differences in patterns have also been stressed. In the next section, teachers' reactions to the student occlusions will be examined.

## Nature and Distribution of Elective Actions

### Nature of Elective Actions

Elective actions teachers chose in response to the student occlusions were classified as exploiting, alleviating, or avoiding moves. The three categories can better be defined in terms of the specific behaviors teachers used to execute them. Before doing so, however, their general characteristics need to be clarified.

Exploiting moves refer to those actions which take advantage of the opportunities afforded by student occlusions to explore some mathematics which might not otherwise have been covered, at least not at the time when the occlusion occurred. Such actions pursue a different concept, task, or even routine than what was called for by the ongoing activity. Avoiding moves do just the opposite; they are actions that do not take advantage of the opportunities created by the student occlusions.

Alleviating moves are taken to mend the break in the instructional flow without giving up what the teacher was striving to accomplish through a problem, question, or task. Alleviating actions are also taken for the benefit of the individual student more than the entire class even though there may be consequences to the group.

These descriptions of the categories of elective actions suggest intent on the part of the teacher. To classify an elective action by the teacher's intent, however, would require more information than teachers were able to divulge through the process of stimulated recall. To classify intent without this information would require high-inference ratings on the part of the investigator. Instead, the categories were made more operational by specifying the behaviors or techniques teachers used, thereby reducing the level of inference required. The techniques teachers used for exploiting, alleviating, and avoiding moves are listed in Table 4.7.

Table 4.7. Elective Actions.

| Category | Technique |
|---|---|
| Exploit | Display and seek |
| | Exemplify and try |
| Alleviate | Redirect/re-ask |
| | Probe/guide/explain |
| | Tell |
| | Restate |
| | Individual assist |
| Avoid | Acknowledge (only) |
| | Modify (to correct) |
| | Reject |
| | Delay |

Teachers exploited student occlusions by integrating new or varied content into the ongoing activity, by taking a momentary detour to explore a different concept or task, or by switching to a different activity altogether. Integration was most common; there were few detours or changes made in activities as will be apparent in the distribution of elective actions. In exploiting a student occlusion teachers used one of two general strategies: "display-and-seek" or "exemplify-and-try." With the display-and-seek strategy, a teacher would bring a student's unsolicited contribution or solicited response to the attention of the class and then request the opinions of students or ask specific questions. With the exemplify-and-try technique, a sample problem was given for the class to solve, several examples were presented, and a pattern sought. Exploiting moves were highly public as teachers made attempts to involve other students or to communicate directly to the whole class.

Alleviating moves were also public in the sense that the class could hear what was being said, but the comments were directed more to an individual than to a class. Simple techniques for alleviating a student occlusion included redirecting the question to another student or re-asking the same question of the same student, sometimes with additional clues. Some redirects specified the individual who was to answer the question while others were open invitations for anyone else to try. Zelda, for example, sometimes gave negative feedback and then looked for another student to call on. Ralph sometimes asked who wanted to respond; it was a technique which increased the likelihood of his finding a student who could give the correct response. Martha avoided giving the feedback herself by asking if others agreed with a student's answer. If a student volunteered a response before the teacher had made any overt move to seek additional input, this, too, was coded as a redirect, albeit an indirect one.

Another technique teachers used as an alleviating move was a probing, guiding, or explaining move. The three were clustered into one subcategory because teachers used them relatively infrequently and because distinctions were sometimes difficult to make. Telling the answer or giving information was another technique. It was kept separate from the previous cluster because it was the teacher who was giving the information and not the student.

Two other alleviating moves were restating and individual assists. Restating what an individual had said was a means of making information known to the class and/or giving credit to that individual. Martha would either repeat what had been said or write it on the board and move on. Had the teacher or students followed up on this restate idea, the incident might have become an exploit. Individual assists were personalized moves that occurred when a teacher stopped to help a student. The techniques used are not coded because this is a private interaction between teacher and student. Teacher-signaled occlusions were identified by the use of the technique. An individual assist, however, could be used with any student occlusion.

Avoiding moves were less public than alleviating moves, except for individual assists. Teachers' avoiding moves tended to be brief, often spoken rather quietly and directed to a particular student. Teachers managed to avoid having a student occlusion interrupt the instructional flow by one of several techniques. Some student contributions were acknowledged as being worthwhile with no other comment: "Yes, that's another way to do it." Some comments were modified to make them conform to what the teacher had wanted to hear: (S) "Two green." (T) "Yes, two-fourths." Some student ideas were rejected: "You can't do it that way." Others were delayed for a moment, a lesson, or indefinitely: "We will get to that next."

## Distribution of Elective Actions

Alleviating moves were the most frequent means of responding to student occlusions; they accounted for at least three-fourths of each teacher's actions. Both frequencies and proportions of teachers' elective actions are given in Table 4.8. Exploiting and avoiding moves

Table 4.8. Proportions and Frequencies of Elective Actions by Type and Teacher.[a]

| | Martha | Zelda | Ralph |
|---|---|---|---|
| Exploiting | 12 ( 27) | 9 ( 14) | 2 ( 1) |
| Alleviating | 74 (161) | 79 (122) | 82 (40) |
| Redirect/reask | 29 ( 64) | 45 ( 69) | 65 (32) |
| Probe/explain/guide | 9 ( 20) | 18 ( 28) | 10 ( 5) |
| Tell | 6 ( 12) | 12 ( 18) | 6 ( 3) |
| Restate | 9 ( 20) | 0 ( 1) | -- -- |
| Individual assists | 21 ( 45) | 4 ( 6) | -- -- |
| Avoiding | 13 ( 29) | 12 ( 19) | 16 ( 8) |
| TOTALS | 99 (217) | 100 (155) | 100 (49) |

[a]Proportion (frequency of elective actions during unit of instruction)

were, therefore, less frequently used with a slight preference shown for avoiding moves. Because of the low frequencies, data are collapsed across subcategories for both exploiting and avoiding moves. A breakdown according to techniques is shown for the alleviating moves.

Even though the proportions of elective actions were substantially different across the major categories, there was a remarkable similarity in the proportions across teachers. To find variation, it is necessary to look at the different techniques within the alleviating category. Once again it is Ralph who shows the least variation, Martha the most, and Zelda somewhere in between. Redirects or re-asks were most popular with Ralph as they accounted for almost three-fourths of his alleviating moves. The proportion decreased for Zelda and even more so for Martha. Martha was the only one to use restate techniques, and her large number of individual assists corresponds with her teacher-signaled student difficulties.

In light of the elective actions just reported, it is worth noting what Evertson, Anderson, Anderson, and Brophy (1980) concluded about students' reactions to teachers' treatment of their input.

> In general, teachers who tended to respond to student questions, comments or answers by ignoring them [avoid], asking another student [redirect], repeating the question [re-ask], or giving the answer [tell] often received lower ratings than teachers who asked new questions [probe], simplified the original question [guide], or integrated the students' contributions into the class discussion [exploit]. In other words, students liked teachers who treated their contributions with respect (p. 57, underlining and bracketed items added).

If the data on the corresponding elective actions in this study are categorized in light of students' preferences, there were substantial differences across teachers. The unpreferred elective actions of avoiding, re-asking, and telling were used most often by Ralph (87%), somewhat less by Zelda (69%), and least by Martha (48%). The preferred actions of probing, guiding, and exploiting, were used about 20-30% by Martha and Zelda, but only 12% of the time by Ralph. These differences reflect variation in teachers' instructional styles and goals. Ralph was most intent on seeking correct responses to his questions and the least willing to attend to or be diverted by student occlusions. Martha was the most intent on students' understanding concepts and the most sensitive to student occlusions. The Evertson et al. (1980) study was unable to establish similar effects with regard to student achievements. The current study suggests that not only do teachers' choices of elective actions influence students' affective states, but so many teachers' goals and instructional style.

A more realistic way to view the distribution of elective actions would be in terms of the student occlusions to which they responded. Proportions of elective actions conditioned on the type of student occlusions--difficulty or insight--are pictured for each teacher in Figure 4.2. The similar manner in which each teacher responded to student difficulties is most apparent, as is the dissimilar manner in which they reacted to student insights. The pattern of conditional probabilities for student difficulties closely resembles the proportions of all elective actions.

It was for student insights that exploits were most likely, at least for Martha and Zelda. They exploited about 40% of the insightful occlusions. Ralph was the least responsive to student insights, having avoided each of the four that occurred. Martha's large proportion of alleviating moves was due to her use of restating to bring information to the attention of others.

Despite the apparent similarity in teachers' reactions to student difficulties, variation did occur. It can be seen in the teachers' choice of alleviating techniques for the different types of student difficulty. Probabilities conditioned on the total number of student occlusions in each category are shown in Table 4.9. The probabilities of alleviating moves

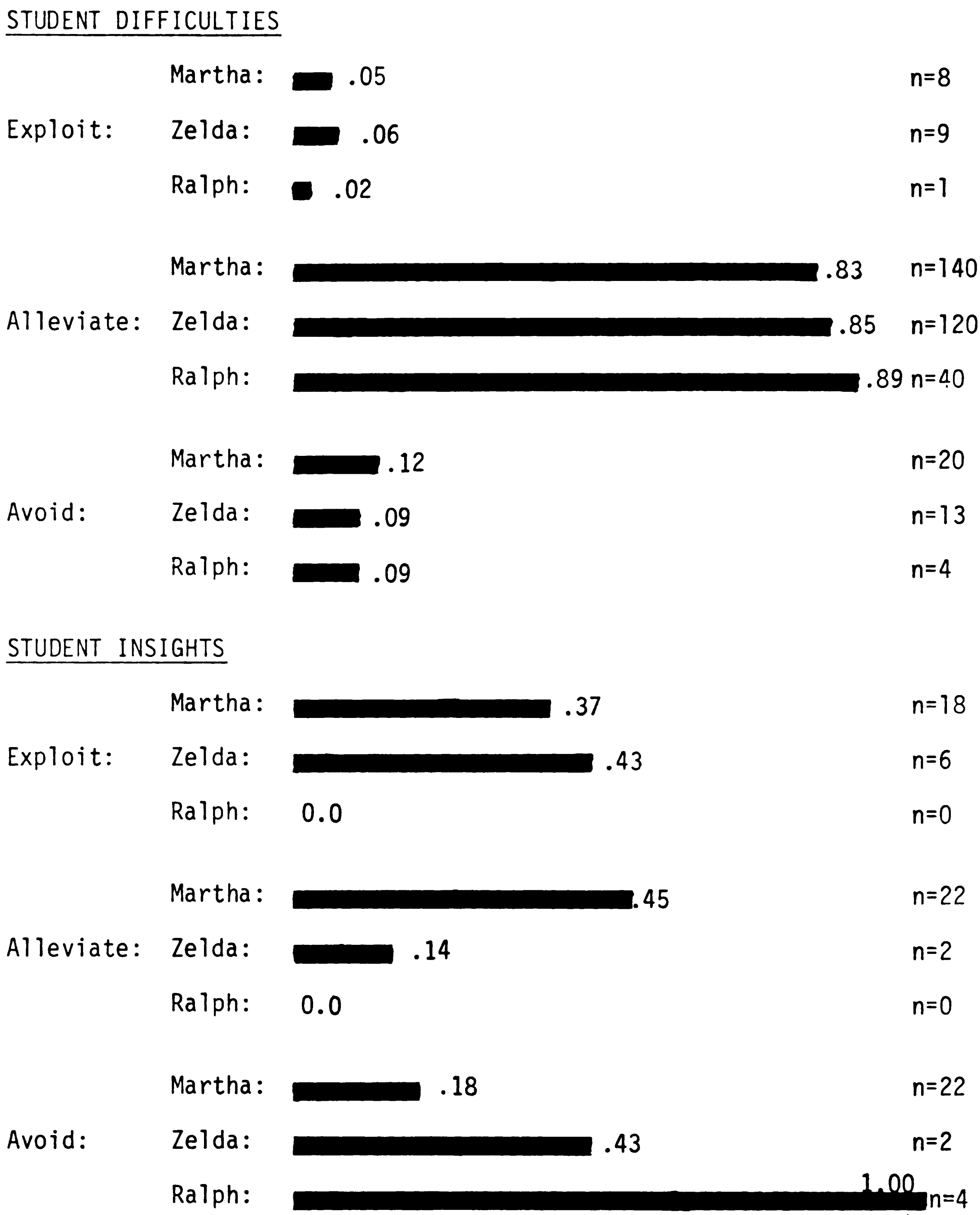


Figure 4.2. Conditional probabilities of elective actions for student difficulties and insights by teacher.

Table 4.9. Conditional Probabilities of Alleviating Moves by Types of Student Difficulties and by Teachers.

| | Patterned Errors | | | Nonpatterned Errors | | | Confusion | | |
|---|---|---|---|---|---|---|---|---|---|
| | Martha | Zelda | Ralph | Martha | Zelda | Ralph | Martha | Zelda | Ralph |
| Alleviating moves | | | | | | | | | |
| Redirect/reask | .56 | .93 | .33 | .58 | .64 | .89 | .12 | .05 | 1.00 |
| Probe/guide/explain | .10 | -- | .27 | .05 | .15 | .04 | .30 | .35 | -- |
| Tell | .13 | .07 | .07 | .05 | .10 | .07 | .07 | .22 | -- |
| Restate | .07 | -- | -- | .10 | -- | -- | .02 | -- | -- |
| Individual assists | -- | -- | -- | .05 | .01 | -- | .07 | .03 | -- |
| Totals | .86 | 1.00 | .67 | .83 | .90 | 1.00 | .58 | .65 | 1.00 |

sum to 100 in those categories for which teachers used avoiding or exploiting moves. Most important is how the conditional probabilities for the same elective actions varied from all student occlusions (Table 4.3) to the different types of student difficulties (Figure 4.2).

Although Martha used the redirect/re-ask technique for only a third of the student occlusions, she was more selective when the nature of the student difficulty is taken into account. She used this technique for more than half the errors, but not for student confusion. Similar contrasts can be noted for Zelda and Ralph with the same elective action. The relatively small frequencies in other categories did not lend themselves to analysis of such internal variation for a given teacher. The predictability of teacher behavior is indicated when these data are examined by type of student occlusions.

When confronted with patterned errors, Zelda was most predictable using redirects and re-asks for almost every error. Martha was less predictable, although she used these same moves for approximately one-half of the students' patterned errors. Ralph's behavior was more difficult to predict because he used redirects and re-asks with only one-third of the patterned errors. He used some form of probing, guiding, or explaining almost as often. For nonpatterned errors, redirects and re-asks were most likely to be used by all three teachers, but with varied predictability. For nonpatterned errors, Ralph chose redirects or re-asks three times as often as he had for patterned errors. Zelda was less likely to use the same techniques as she had for patterned errors while Martha chose these techniques with the same probability as she had patterned errors. Confusion stimulated a tendency to probe, guide, or explain for Martha and Zelda, while Ralph continued to re-ask.

The varied distribution of elective actions by types and teachers demonstrates the selectivity of teachers in choosing their actions. The nature of the student occlusion does matter, although to a greater or lesser extent according to the variability of the actions used. Ralph's behavior was the most predictable because he varied his options so little. For patterned errors, however, he was less consistent. The range of Martha's and Zelda's elective actions was more extensive which meant their choices were not as predictable as Ralph's, at least not for all types of student occlusions.

## Conclusions

The key argument for investigating teachers' critical moments was the fact that instruction of the type recommended by NCTM would necessitate teachers' having to contend with more unpredictable student occlusions. The concern was that typical instruction in mathematics as described by Fey (1980) and Easley (1978) does not "create classroom environments in which problem solving can flourish" (NCTM, 1980, p. 4). In other studies about interactive teacher thought, the teachers have been more like the "typical" mathematics teacher. For this reason, it is important to make some assessment about the nature of the instructional focus for each of the teachers in the study. A general characterization would be inappropriate as teachers in this study did not teach in the same manner nor did they encourage the same type of learning. Instead, they provided examples of different types of teaching which are located at different points on the continuum between the typical mathematics teacher and the one characterized by NCTM.

Ralph represents the typical mathematics teacher using typical methods. He focused on skill-oriented and symbolic tasks using explaining and questioning techniques to teach decimals. Students were asked to supply numerical answers for the various subroutines in converting decimals to fractions or vice versa. Lessons were planned and taught in accordance with the procedures and problems in the textbook. Ralph's goal was for students to be able to do the problems; it was a goal for instrumental understanding. There was less variation in student occlusions and in the types of critical moments than in the other classes. Student errors and alleviating moves dominated; Ralph's behavior was also the most predictable.

Martha's instructional approach would take her in the opposite direction from Ralph. Her teaching came closest to examplifying the NCTM recommendations. Martha's focus was on developing concepts and applying them through the use of models (Cuisenaire rods and pictorial regions). Posing problems and asking questions, she relied on student answers and their unsolicited contributions to convey the ideas. Her instructional strategy was one of exemplify and try. As the tasks became more complex, however, she did more to direct her students through the problems. Her goal was understanding, which was a goal for relational understanding. She developed her own unit and did not follow any textbook. Her instructional approach to teaching fractions was also more in keeping with research findings (Suydam, 1978). Student occlusions were more plentiful and diverse with students' offering a number of suggestions.

Zelda taught in ways which had some of the characteristics of the traditional and the NCTM-type instruction. Her intent was to help students span the gap between the concrete models and the more skill-oriented tasks of the textbook. She did this by briefly introducing concepts and relationships through concrete tasks and then shifting to more symbolic tasks. First, students developed procedures based on patterns exhibited through example and then more traditional algorithms were introduced. In this manner she encouraged a relational understanding while also fostering instrumental understanding. The nature and distribution of student occlusions and teacher elective actions in Zelda's class were similar to but less varied than those in Martha's.

## Summary

In this chapter, the instructional environments in which three teachers experienced critical moments have been described. Descriptions were given for the instructional flow that was generated and the student occlusions and elective actions that occurred during the flow. Despite the similarities in the fraction and decimal units planned for and taught by these experienced teachers for a period of six or seven days, there were numerous differences in the instructional flow: students, nature of the topics and tasks, and teachers' instructional styles. Many of the differences were a reflection of the teachers' goals for how and what mathematics students were to learn.

Differences in the instructional flow also produced some differences in how students performed. Distributions based on the frequencies, proportions, and densities of student occlusions indicated differences across the three classes, both in the types of occlusions and the rate at which they occurred. Student difficulties--errors in particular--were far more prevalent than insights, just as there were more solicited responses than unsolicited contributions or expressions of confusion. Teacher responses to student occlusions also varied. In keeping with the patterns of student occlusions, teachers more frequently used alleviating actions to handle the difficulties, most often with redirects or re-asks. Exploiting and avoiding moves were used more sparingly. The following brief characterization of each teacher's instructional environment will illustrate some of these differences.

The varied instructional environments which resulted provide the backgrounds in which to search for critical moments for each teacher. To understand what precipitated the cognitive and emotional discomfort that teachers report with their critical moments, it is necessary to be able to view the events in perspective. Knowledge of the patterns in the instructional flow, student occlusions, and teacher elective actions will provide some clues to what caused and what influenced teachers' critical moments. In Chapter V, the critical moments that were identified will be described in detail.

# CHAPTER V

# CRITICAL MOMENTS

## Introduction

The three teachers in this study experienced critical moments while teaching mathematics in the instructional environments described in Chapter IV. The model of critical moments (Chapter I) assumed they would arise over unpredictable student difficulties or insights. Identification of these teaching difficulties, therefore, necessitated that teachers report, through the process of stimulated recall, the problems and discomfort they experienced while coping with the unpredictable student behavior. The purpose of this chapter is to describe and exemplify the nature of the critical moments revealed by the teachers. Descriptions will include information about the relevant characteristics of the prevailing flow of instruction, the student behavior, teacher moves, the thoughts and feelings of the teacher, consequences of teacher behavior, and possible causes of critical moments.

Written transcripts and audiotapes of the stimulated recall sessions were examined for cognitive or emotional evidence of teaching difficulties. When appropriate, these teaching problems were identified with student occlusions. However, not all critical moments arose over specific incidents of student difficulties or insights. Some arose over more general patterns of student performance. These patterns created pervasive teaching problems which extended over time and

and across students; they characterized student performance for activities, not for specific student occlusions. Even when patterns of student performance were set by student occlusions, it was not any specific occlusion that troubled the teacher; it was the overall pattern. Teachers in this study did not report teaching difficulties in the absence of external cues from students. Critical moments described in this chapter arose over two different sources of unpredictable student performance: isolated and momentary student occlusions and pervasive patterns of student performance.

In all, only twenty critical moments were identified during the six or seven days of instruction by the three teachers in this study. Fourteen occurred over specific student occlusions and six over pervasive patterns of student performance. Compared with the 421 student occlusions and 41 activities listed in the previous chapter, a rather small proportion resulted in critical moments. Fifteen percent of all the activities became pervasive problems for the teachers, but 85% did not. The number of critical moments associated with specific student occlusions represents three percent of the total number of student occlusions, but this figure is somewhat misleading. Critical moments about specific student difficulties or insights often occurred over episodes involving several related occlusions rather than a single interchange which suppresses the actual percentage. Furthermore, for a pervasive teaching problem, teachers reported recurring thoughts about their students' performance during an activity; it was not a single experience. Also, unpredictable student performance for which teachers recalled thoughts and feelings about their successes were not included.

To qualify as a critical moment, there had to be some evidence the teacher was experiencing cognitive difficulty or emotional discomfort.

Why so few of the student occlusions and activities resulted in critical moments is, at least in part, explained by what has already been established in other studies of teacher thought (Clark and Yinger, 1979). Much of what teachers do is "business as usual." Teachers implement their plans and rely on routine behaviors for much of what they do unless they find things are not going as expected. Routine behaviors include not only the typical responses to students, but routines associated with the task and content of an activity. The advantage of routine behavior is that it enables teachers to reduce the complexity of their tasks and to be more selective over what they do give their attention to. It was the more distinctive student occlusions, performance patterns, and teacher elective actions that teachers tended to report through stimulated recall. Such things as student errors which signaled clear misconceptions or recurring incidents of the same difficulty were noted. These did not occur often. The same was true for teachers' elective actions. The less frequently used exploiting and avoiding moves were quite prevalent in critical moments involving student occlusions, as were moves which demanded mathematical knowledge not being used during an activity (see Appendix for time and content of activities). These events were distinctive; they were not common.

The scarcity of these twenty critical moments in no way diminishes the need or importance of studying them. If anything, it increases the possibility of learning something of value about teachers and their teaching of mathematics. Tracing three teachers' units in mathematics provided the background necessary for viewing their critical moments in perspective. Critical moments provided the focal points around which to organize and interpret the otherwise massive amounts of data. Distinctive features of critical moments were more apparent when viewed in relation to the patterns of student and teacher behavior and characteristics of the activities. Furthermore, with access to so much data, information is incorporated from all four of Schwab's common places--teacher, learner, curriculum, and milieu (Schwab, 1978)--although the perspective of the teacher is central.

Situations which produced critical moments for the teachers in this study are universal teaching problems and, therefore, offered no surprises. Critical moments arose over essentially four different situations: student difficulties and insights, pacing dilemmas, and unexpected success. For purposes of simplification, the author labeled these situations Types A, B, C, and D. A brief description of what caused the teacher trouble in each type of situation follows:

Type A: <u>Student Difficulty</u>: Teacher was unable to correct misconception, diagnose difficulty, or elicit desired response

Type B: <u>Student Insight</u>: Teacher was unable to accept or satisfactorily execute the elective action

Type C: <u>Pacing Dilemma</u>: Teacher had problems maintaining the instructional pace while trying to meet the needs of one or more students who had difficulty

Type D: <u>Unexpected Success</u>: Teacher was confronted with unexpected student success

This is not intended as an exhaustive listing of all possible teaching problems, although it is a beginning of such a taxonomy.

The distribution of critical moments across these four situations and three teachers is shown in Table 5.1. Most student difficulties and insights arose over single student occlusions, or clusters of related occlusions while pacing dilemmas and unexpected success were most frequently pervasive teaching problems associated with activities. More critical moments were reported by Martha partly because of her four pervasive teaching problems in contrast to one for each of the others. Pervasive critical moments accounted for 20% of her activities, eight percent of Zelda's, and about 12% of Ralph's. Martha also gave total class instruction for more time and with more activities. She provided only one separate work period for the students during her introductory unit on fractions. Although more student occlusions occurred during Martha's instruction, it was Ralph who had the highest proportion of critical moments involving specific student occlusions--12% compared to an average of three percent.

Table 5.1. Distribution of
Critical Moments by Type and Teacher.

| Type | Martha | Zelda | Ralph | Total |
|---|---|---|---|---|
| A: Student Difficulty | 4 | 2 (1P)[a] | 3 (1P) | 9 (2P) |
| B: Student Insight | 2 | 1 | 3 | 6 |
| C: Pacing Dilemma | 2 (2P) | - | 1 | 3 (2P) |
| D: Unexpected Success | 2 (2P) | - | - | 2 (2P) |
| | 10 | 3 | 7 | 20 |

[a](#P) refers to the number of pervasive critical moments

Experiences teachers reported and circumstances which led to their critical moments revealed similarities as well as differences within and across types. To highlight these similarities and differences, the critical moments are described by types and, when appropriate, by subcategories within types.

## Type A: Student Difficulty

The student difficulties which resulted in critical moments were correcting misconceptions (A-1), eliciting desired responses (A-2), or diagnosing difficulties (A-3). The somewhat different nature of these tasks define the subcategories into which these critical moments are clustered and described. In the first subcategory (A-1), Martha proved to be the most consistent, reporting four similar experiences. Each was an isolated episode in which her attempts to help a student understand a concept or procedure failed. There was only one other critical moment in this cluster; it was reported by Ralph. Related, but quite different with regard to the covert teacher experiences, was the second subcategory of student difficulty (A-2). Zelda and Ralph found they could not elicit the desired response to a question. The third subcategory (A-3) contained two critical moments; both were pervasive problems caused when teachers could not determine why students were exhibiting difficulties with an activity. Zelda noticed student difficulties with the task, and Ralph had trouble figuring out why the students were not more willing to volunteer.

### Type A-1: Teacher Unable to Correct Student Misunderstanding

So similar were the experiences Martha reported about four critical moments in this first subcategory that one general description and one detailed example characterize them quite well. Brief remarks about the other three critical moments are offered only to point out variations in circumstances, their impact on Martha, and causal agents.

The four critical moments occurred when Martha found herself unable to help a student understand a concept or procedure despite her efforts to do so. A chain of interactions began with what Martha believed to be a signal of student difficulty, a patterned error, or a specific question about a procedure. The initial indicators of student difficulty included student-initiated confusions and responses to teacher questions. One initial speculation was coded as insight instead of difficulty because the question raised had mathematical validity. Whether the student had intuitively recognized this or not was never certain, but Martha's interpretation was. She reported thinking the student was confused and had behaved accordingly. These initial signals presented Martha with tangible evidence of a particular misconception or concern over the task. The initial student difficulties in these four critical moments were precipitated by some new aspect of the content or task. Students were being asked to perform a new variation of a previously introduced task, work with a new fraction concept, or both.

In response to these distinctive signals of misunderstanding, Martha chose elective actions which offered some form of assistance through an exploiting or alleviating move. She illustrated an erroneous interpretation of an improper fraction in contrast to the correct one already in evidence, she asked the class to build an appropriate representation of an improper fraction before asking what fraction was modeled, she gave the class another problem to work so that a student could try to figure out why it was that white rods were used to identify the sum of two fractions (teacher error as it depends on the choice of unit), and she probed to try to uncover how a student was interpreting the fractional pieces. Both the student signals which precipitated the critical moments and Martha's techniques for dealing with the student difficulties were relatively unusual behaviors.

Each of Martha's Type A-1 critical moments was preceded with a related incident which may have sensitized her for what was to follow. In these related student difficulties, she may have prepared herself or her prior elective actions may have set a precedent which was easily followed. As with the prevailing flows in which these critical moments took place, Martha never offered these prior incidents as reasons for her actions or covert responses. These critical moments occurred at the end of the lesson during activities that had not been planned. She never mentioned time as having influenced her, nor did she seem terribly aware of it at that moment. The lack of preparation, on the other hand, may have contributed to Martha's difficulties.

By themselves, the initial student occlusion and elective action did not produce sufficient evidence of a critical moment. It took evidence that the student misunderstandings persisted. Recurring signals were made possible as Martha provided additional opportunities for the individuals to respond. She did so by continuing to question the student or by checking back with the individual after a demonstration or by picking up on a less direct cue indicating a student was still struggling. In the face of the second student difficulty indicating that her efforts had not been successful, Martha continued to try in three of her four critical moments. She used some of the same techniques already described and the more direct one of telling, a move she seldom used. She never continued after four inappropriate responses from the same student.

Martha broke off her attempts to help the students with an avoiding move, knowing that the students still did not understand. First, she indicated that the idea or task would be dealt with again, a positive gesture. Then, she either suggested that the student continue to "puzzle it out" or indicated her belief that the student would eventually "get it." Her second remark suggests the consequences of differential expectations for her students. The challenge to continue "puzzling" was given to the two students for whom she had high expectations, and her supportive comments went to two low expectation students.

A significant critical moment, both in its duration and how it affected the teacher, occurred during the third day of the unit during the last activity (Activity 14). Students had been so successful with the planned activities that Martha felt another change was indicated. Switching the task from offering fractions with like denominators, she began giving two fractions with like numerators for students to compare. For the second problem Martha inadvertently wrote 6/5 before the class had been introduced to improper fractions. As might have been foreseen, students objected to this number, and Martha exploited the opportunity to demonstrate how the improper fraction could be represented. After two "pies" had been drawn and divided into fifths with six of them shaded that, theinitial student occlusion for this critical moment occurred.

Max wanted to label the pictorial representation for 6/5 as 6/10. Martha's choice of action was to counter by illustrating 6/10. There was ample precedent for this action during the activity, as she had used the picture presentation to justify the comparison of 3/4 and 3/5, the first problem, and then again to demonstrate 6/5. Cutting the circle into tenths and shading six of them, Martha said, "Tenths are a lot smaller than fifths." Unfortunately, she had drawn the circle for tenths much larger than she had the circles for fifths which made a realistic comparison of the two models impossible. The distortion of the circles did not appear to contribute to Max's argument, however, as he was focused on the number of pieces into which the circles had been divided rather than the relative sizes of the pieces. The distorted units did suggest Martha may not have thoroughly understood the necessary conditions for representing and comparing fractions.

After having illustrated 6/10, she asked the class if 6/10 and 6/5 were the same. As "no" came from a number of students, she erased both representations and asked if anyone were confused. Student response was mixed. Martha responded with, "We'll try a few more and maybe that will help you." The antecedent to "few more" was not clear, however, as she gave another comparison problem, one with no improper fraction.

At this point Martha was not bothered and, had Max not been so persistent, the critical moment might never have occurred. She was not "let off the hook" so easily as Max continued to wonder and, evidently, mumble about the picture for 6/5 being 6/10. Noticing this, Martha provided Max with another opportunity to use an improper fraction with a somewhat easier task. Writing down two fractions with like denominators, including one improper fraction, she asked which was larger, 5/4 or 3/4. At first, Max said he didn't know, although he later gave the correct response. In the meantime and after waiting for him to respond, Martha tried once again to demonstrate the concept with another picture, taking suggestions from the class.

After several student comments (unfortunately inaudible on the tapes) Martha went on to explain, "I have to have 4/4 and 1/4 to make 5/4." Once again, she turned to Max to probe, "What about that puzzles you?" Still struggling over the picture for 6/5, he repeated his belief that it represented 6/10. Sounding exasperated she replied, "You can't do it that way!" and proceeded to recapitulate what had already been said. Her last remark pointed to future chances to deal with the concept and encouraged him to "keep puzzling over it."

Her own recollection indicates why Martha discontinued the discussion:

> I was feeling inadequate here because I couldn't explain it to him! I wanted to make it clear. I've got to come at it in a new way, and I'm really getting frustrated here! ...All I can think to say at one point is that you just don't do it that way! [Activity 14]

Several things became clear here in addition to her feelings of inadequacy and frustration. It substantiates her conscious desire to help the student, a goal consistent with her learning and instructional goals for the unit. She valued and wanted to be flexible and responsive to her students as they developed an understanding of the fraction concepts. Martha's inability to think of a different explanation because of her limited knowledge forced her to abandon her goals.

Peterson and Clark (1978) found approximately 40% of their teachers' decisions demonstrated an inability to think of an alternative.

In Martha's other three critical moments from this category similar consequences were reported. Frustration was the most commonly reported emotion, although the intensity of the emotion varied as did the amount of effort expended. She also reported having difficulty in figuring out what a student was thinking or what could be done to help to explain the ideas. One source of her difficulty was her own knowledge of fractions and the concrete approaches she was using to teach them.

Evidence of her limited knowledge of fractions came from Martha's own admission and several mistakes she made during the unit. She was asked if she realized she had drawn unit circles of different sizes for comparing tenths and fifths to see if it were accidental or that she had insufficient understanding. She had not realized she had drawn different sized circles, nor did she immediately grasp the significance of that fact. Martha appeared confused as she first said it didn't matter and then that it did. It was only after the investigator interfered, posing an analogous question about cutting different sized pizzas, that Martha was certain she needed to use the same sized units for comparing fractions. It was not that she had never known the concept, but that she had not thought about it ahead of time and was not familiar enough with it to have been able to remember it under pressure. Similar difficulties were mentioned by all the teachers at one time or another as they talked of needing more time to think some things through or to come up with another idea. They did not, however, talk about the need to think something through ahead of time or to plan more meticulously. Research on teacher planning demonstrates that teachers do not typically prepare their lessons with such care (Clark & Yinger, 1979; Smith & Sendlebach, 1979).

The reason Martha was so interested in trying to find out what students were thinking in these critical moments can be attributed to several things. It was compatible with her instructional goals. She also expected her students could learn the concepts. This was undoubtedly enhanced by her perception of student effort; several times she mentioned they were trying. For these same reasons, Martha also saw the student difficulties as indications they did not understand, and she felt that could be corrected with some effort on her part. In addition, Martha believed the fraction concepts were "hard for students to understand." She was not attributing blame to the students, but offering a plausible reason for their difficulties.

Martha may have persisted in trying to help Max as long as she did because of her expectations or perceptions as much as her desire to help him. "I don't know quite why he is not seeing it. That's why I keep questioning him. I want to see where he was coming from. He is on the verge of getting it!" Her last comment may have been due to a perceived change or an anticipated one. Research on the effects of teacher expectations have shown a greater willingness on the part of teachers to pursue an idea with high-expectation than with low-expectation students, and Max was rated in the top group of mathematics students.

A third suggestion of teacher expectation effects in Martha's Type A critical moments was her reported fear of confusing the students, a fear expressed for the students for whom she had lower expectations. This mention of fear was confounded with Martha's apparent understanding of the mathematics in these critical moments. She seemed to know what the representations and concepts were even though she had difficulty figuring out what the student was thinking or in finding a way to explain it. When dealing with the better math students, Martha appeared to have trouble understanding the concepts, and she was aware of it in one of the two cases.

Martha's sensitivity to confusion students might be experiencing was apparent in the following comments from the two critical moments with her less able math students.

> Boy, this is screwing up her head [improper fractions] I can just tell that! I can just feel her confusion! [Activity 17]

> [What] I worry about here is that he has his way of understanding it [unit fractions] and by forcing him to say other things, I might be confusing him...I just wanted to get off it so it didn't shake him up. [Activity 6]

Her fear of confusing students was based her own experiences in learning mathematics which were not always satisfying. "Math is like that for me [confusing]. And when someone tried to explain it to me, it just got worse because I was so upset I wasn't getting it! My whole mind was turned like an eggbeater!" Martha had good reason for backing off when she sensed or anticipated that students were or might be getting confused.

Ralph experienced the only other Type A critical moment which was similar in the evidence of student misconception, the teacher's impulse to offer some assistance, recurring evidence of student difficulty, and admission of being unable to think of an alternative approach. Both teachers mentioned not wanting to confuse students as supporting their choice to offer no more help, but Ralph's experience was quite different from Martha's. He never really attempted to examine the student's misconception or to provide additional opportunities for the student's confusion to surface. Instead, he merely told the student what the correct response was and hoped that it was sufficient. A nonverbal cue indicated it was not. The critical moment occurred during a review of the first day's assignment during the first activity of the second day (Activity 3). The content, then, was still relatively new, and the problem which triggered the patterned error was a decimal in which a zero was in the last place and after the decimal. It was the first time such an example had been covered in class, although Ralph had selected another problem with a zero between two other non-zero digits.

In expressing 5.490 as the sum of whole and fractional numerals, Bill answered incorrectly saying 90/100. Ralph responded by asking what place the nine was in and was given the correct response. At this point, he told Bill how he would have responded (9/100 + 0/1000 or, simply, 9/100) and then moved right on to the next problem. For this critical moment, Ralph detected the student's continuing confusion from his behavior. "He just kind of gave me a blank expression on that. I could see he wasn't clear on that, but I hoped I made it clear when I said don't forget the zero in the thousandth's place."

This was the only occasion for which Ralph revealed having wanted to offer an explanation and found himself unable to do so. Knowing what he had done to assist the student was insufficient and that he was unable to think of anything else to say clearly made an impact on him:

> I was very conscious of whether I should speak of [student's] mistake and try to clear that up in his mind... but then I thought, because I was on very shaky ground there, I am going to confuse others by doing that. I had the tendency to want to [clear it up on the spot] real quick and then I caught myself. I thought, I don't know what I would do on the board or what I could say... [so] I am just going to give the correct answer.

Unlike Martha, Ralph did not label his emotions, but his reactions seemed to be a mixture of fear and frustration. Ralph's fear of confusing students was directed not at the student, whose confusion he was trying to alleviate, but at the rest of the class, which is similar to his pacing dilemma. Also, there was no preceding incident of a related nature, though something else may have served to heighten his sensitivity to student difficulty with the problem. As it was a review of the previous day's assignment, Ralph deliberately selected problems with greater potential for error.

<u>Type A-2: Teacher Unable to Elicit Desired Responses</u>

A somewhat different situation led to Zelda's only Type A-2 critical moment. She was unable to elicit a desired response to a planned question from anyone in the class, even after repeated attempts to do so. As with the Type A-1 critical moments, students were experiencing difficulty with new content. There was recurring evidence that the difficulty had not been resolved, and the final teaching move of this episode was to leave the question using a delay tactic.

The episode occurred during the first activity of the unit before the clear pattern of student response or teacher questioning had been established. Zelda was using the numberline to motivate the students to think about numbers between 0 and 1. She first divided the segment

(0,1) in half and obtained 1/2 as the name of that point. Then, after marking the other two quarter points and getting 1/4 as the name of the first one, she sought to obtain 2/4 as the name of the next point. It was over this question that the student difficulty and critical moment occurred.

Zelda agreed to the first response of 1/2 and asked if it could not be something else. One-fourth was offered next, and Zelda countered by pointing to that position on the numberline. By this time, more students were trying to call out answers, but no one called out 2/4. Again, she acknowledged that 1/2 was correct, reemphasized where 1/4 was located, and asked for the value of the next point. A repeat of the inappropriate callouts was rebuffed as Zelda encouraged them to think about it and not just call out answers. One-fourth was offered again, and this time she agreed that the second segment was another fourth and that two 1/4s reached the middle. At this stage, Zelda altered her question somewhat, asking what 1/4 and 1/4 would be. An error of 1/3 was essentially ignored as she asked how the answer should be written. After seven unsuccessful attempts, Zelda finally aborted her quest, saying to the class, "Let's not worry about that right now; I don't want it to get confusing."

Differences between this and the previous critical moments are apparent--some due to differences in instructional style. It was not one individual exhibiting this difficulty; it was the class. A number of different students tried unsuccessfully to give Zelda the response she was seeking. Some she solicited from individual students, and others were callouts. While this was a fairly common situation for Zelda, in all the other cases there was at least one correct response for her to select. In this critical moment, this was not true, and her covert reaction was different. Zelda seemed to be more annoyed than frustrated that the students had failed to come through on this question. She was not bothered by the recurring evidence of student difficulty, although the number was large, but that no one was able to give her the response she was seeking. Her discomfort was aggravated by two things: she had planned to pursue this line of questioning throughout the activity, and she had reason to believe that the students were capable of giving her the answer.

Her choice to abandon seeking the answer of 2/4 was total. She did this recognizing that she had other things to do and there would be other opportunities to deal with the concept:

> I really didn't consider anything else because I knew I wanted to do something with Cuisenaire rods today. I thought it really wasn't worth it, and we are going to get there anyway. It wasn't a matter of I'll do this, this, and this; it was a matter of I want to get to these things; and if I'm not going to, I'll just dump it.

Another comment made nearer the end of the recall session, however, suggested that the incident had made a more lasting impression on Zelda and that she may have been more disappointed than she had realized at the time, "That really limited what we accomplished today!" She made it clear why she preferred not to spend too much time on this concept: "There are so many things to learn about fractions that it seems a shame to drag it out forever."

Zelda's surprise over the inability of the students to respond was evident when she spoke of her plan. "I was thinking about going 1/4, 2/4, and 3/4; but the next thing that happened is I [could not] because no one could rename 1/2!" The expectation that students should have been able to answer her question was based on a previous year's learning experience with some of the students in her class:

> I was thinking I had a hard core [about six or seven students] that had done fractions before, and we could start building equivalent fractions which would lead us right into that the next day; so it would be very easy then to deal with equivalent fractions.

Unlike Martha who attributed her students' difficulties to a lack of understanding and difficult concepts, Zelda suggested an inability to remember--a mental block--and environmental conditions as the inhibiting influence of large group instruction. Neither required a change in her expectations about the capability of her students:

> ...obviously they don't remember it. Also, we had already named it 1/2, and they probably thought that you can't name it anything else because a half is a half is a half!

> ...the ability range is interesting because it stops kids that can handle it in small groups. [Student] could handle it last year...but she couldn't today.

Both of these conditions, one internal and one external to the students, were unstable. She did not mention the task as a possible source of difficulty as did Martha, even though the numberline task was distinctively different from what the students had experienced before. With Cuisenaire rods, individual rods can be counted as 1/4, 2/4, and so on, while the numberline requires that a value be assigned to a position. Whether or not the task difference contributed to the students' difficulty cannot be stated, but the distinction existed.

Zelda's reports of her cognitive experience during this time were in conflict. She spoke of trying both to think of an alternative way to ask the question and about not thinking of any alternatives. If both reports were valid, it would be reasonable to assume that the search took place during the time she was engaged in her alleviating actions, still seeking the desired response, and that the absence of search was concurrent with her decision to abandon the question. In much the same manner as Martha did, she described her search as "kind of spinning my wheels trying to think if there really was some other way to do it!"

Like Martha and Ralph, Zelda was unable to think of another alternative, but this was not central in Zelda's mind. Rather it was her discomfort at not being able to continue with what she had planned. She tentatively mentioned her limited knowledge of mathematics after being questioned about what bothered her, which suggests that she may have been trying to please the investigator:

> I guess I wish I knew more mathematics so I had more different ways at my disposal to deal with the concept. Very often I have to go to the book and check things out. This is not my best content area...[social studies was her best] ...I don't feel inadequate or incompetent; I feel somewhat limited.

Her last remark points to a difference as to how Martha and Zelda responded to a Type A critical moment. Unable to help the students, Martha said she felt inadequate and frustrated. Zelda, on the other hand, claimed only to feel "somewhat limited," and her emotion was more annoyance than frustration, although she did not so label it. The difference in emotion suggests a difference in acceptance of responsibility (Weiner, 1979).

Ralph also faced a situation in which he could not get the desired response. Getting no visible sign of effort, response, or even eye contact from the student bothered him. No response to a direct question was not all that unusual in Ralph's classroom, but there was one incident that produced a strong emotional reaction from him. On the fourth day of the unit (Activity 8) when students were being asked to identify rational numbers for unlabeled points on numberlines, the incident occurred. The question which resulted in this critical moment was similar to those the students had been answering for three days. It was not new material as was the case in the other Type A critical moments described so far.

When asked how to write the common fraction 96/100 as a decimal, Judy gave no immediate response. Ralph re-asked the question and waited a bit longer before redirecting the question to another, unsuspecting student who also gave no response. At this point, Ralph admonished the group to pay attention, saying,

> Now, listen. I think that the people who are having difficulties changing common fractions to decimals might want to listen closely because it is not that difficult, and you can pick it up easy.

He then called on another student who gave the correct response. Before moving on to the next problem, Ralph followed with a brief rule, "Decimals for [a fraction written in] hundredths have to end in the hundredths' position."

Ralph was exasperated trying to reconcile Judy's behavior with what he believed to be the simple nature of the task:

> You know, when a student just sits there and doesn't respond at all...you have to think. I don't know how anybody could not [respond]. But you just wonder what is her problem? I mean, we have gone over this stuff, and I think a person is much farther ahead if they have enough confidence to respond, even if they get it wrong. At least you know maybe they are listening and trying this stuff out. But when she doesn't answer at all, then you ask yourself: has she just not been paying attention?

His only explanation for Judy's failure to respond was her lack of attention. "I think some kids do that. You surprise them, and they are not ready for it. So they don't even hear your question, even if you repeat it." Even when he re-asked the question, she made no move to respond, and this really affected him: "She just kept looking down at her book and would not move her head up or anything! So, finally, I called on someone else, but I was really..." Having made a move (re-ask) to get her attention, Ralph was left without a plausible explanation to justify Judy's lack of effort. When probed, he did admit there might be reasons why she had not been attentive. "There probably could be a lot of reasons, myself not excluded. I might just have bored her to death or problems at home or anything."

Ralph's explanation for Judy's not responding was consistent with his belief that students could answer the questions he asked in class if they paid attention. Furthermore, he expected students to pay attention while he provided them with information and an opportunity to practice so they would be able to work on their assignments without having to swamp him with questions during the work periods to follow. His goal was not to have students understand the concepts, but to have them be able to do the problems. Thus, he equated Judy's lack of response with inattentiveness and a lack of effort which were unstable, internal conditions under the control of the student, another documented phenomenon in attribution research (Weiner, 1980).

This critical moment over Judy's response came during an activity which he found most enjoyable to teach. Ralph had expected to have difficulty in presenting the lesson, but found instead that it was easier and more successful than anticipated. He gave a great deal of attention to the delivery of the activity, particularly during the first problem or so, until he had a workable routine for approaching the problems. He physically moved away from the textbook during this activity. He put pictures of numberlines on the board and asked a variety of questions, questions which students could not have anticipated by following the text. Student attention and participation was good. When someone was unable to answer a question, Ralph easily found another who could. He felt the "whole group was attentive," that he "didn't feel the need to repeat" and "belabor all the little mistakes." He "didn't feel a tenseness or pressure" to be sure they understood. It was in this environment that Judy failed to even react to the simple question Ralph had directed to her. Ralph's reaction to this incident under these conditions became a critical moment.

Several features of Ralph's Type A-1 critical moment, described earlier, provide an interesting contrast to his Type A-2 critical moment. The Type A-1 critical moment came about when he felt the desire to but could not think of a way to help Bill understand, and the Type A-2 came when Judy could not answer a question and made no attempt to do so. Differences included familiarity of the task, prevailing flow, his causal attributions, and expectations. In the first case, the task was relatively new, and the problem was selected because it was hard. In the second, it was essentially a review question, within a more extensive and satisfactory exploration of other ideas, and considered to be easy. Despite the nonverbal signals which Ralph saw as a blank expression or looking down at the book, he attributed Bill's reaction to a lack of understanding and Judy's to a lack of attention or effort. Bill was also a student Ralph enjoyed socially and considered to be in the highest group with regard to mathematics. Judy, on the other hand, was one of five students Ralph would have been willing to remove from his class. He did not enjoy talking to her and ranked her in the low-middle group with regard to ability. Differences in the circumstances and participants led to Ralph's experiencing different critical moments.

### Type A-3: Teacher Unable to Diagnose Student Activities During Performance

In this last group, dealing with student difficulties, the two critical moments were based on pervasive problems. Zelda and Ralph referred to what they perceived to be student difficulties existing throughout activities. The pervasive pattern of student behavior manifested itself in different ways. Zelda had both visual and verbal cues of their difficulty from several students as the class worked with Cuisenaire rods. Ralph noticed that students were not volunteering to respond; they appeared unwilling to pay attention to or participate in learning decimals.

Activity 11 on the fifth day was a particularly frustrating activity for Ralph to teach; it was his only pervasive Type A critical moment, and it was similar in many ways to his experience with Judy the day before. Ralph found the students unresponsive and inattentive: "I get constant feedback that people are not listening...or they are not really tuned into what I am saying." There were few errors made during the activity; the density of student occlusions was the lowest of any activity in the unit.

This was a particularly frustrating situation for Ralph. Had there been a reason for the students' behavior that he could have recognized and controlled, his discomfort might have only been momentary.

Instead, he had difficulty in reconciling the reality of the passive student response with his expectations.

> The questions I ask I think are not that difficult. I just feel that three-quarters at least, maybe more, know the answers to the questions I will ask...but, yet, very few hands go up, and kids like to raise their hands and give right answers.

Consequently, Ralph attributed the students' behavior to their inattentiveness, "I just assume they are either so disinterested, bored, or whatever, or just not tuned in, so distracted that it is not getting to them." As was typical for Ralph, he did not attribute this inattentiveness to the ability of his students. This might have been an excuse he denied himself, knowing his was the top math group in the school. Later he offered the day of the week as an explanation, saying, "Monday is a bad day for kids!"

Nor did he attribute the students' inattention to his instruction; but, by the time he had viewed the lesson again on videotape, his frustration was so great that he made the following plea for help. "It brings up the question of what am I hoping to achieve by standing there and doing that kind of thing! Or, even a more important question is, '<u>What can you do to improve the situation?</u>'" Unable to come up with any suggestions as to how he might have improved this situation, Ralph continued in a way which further expressed his helplessness:

> I feel limited in that situation of being able to involve students in some sort of feedback. It is not like other subjects where they have a background in what you are talking about; this is new material!

It would be unfair to attribute all Ralph's discomfort to the diminished attention of his students. As he indicated, the content was new. It was the first time that the students were introduced to adding and subtracting decimals. Most important was what happened to Ralph during this presentation. When preparing for the lesson, he had not anticipated any difficulties: "I went through those six boxes, and I thought it was relatively simple," referring to two examples of addition and subtraction in the text. The sum or difference of the digits in each position from thousandths to tenths was highlighted in three boxes as well as a fractional justification. His presentation did not proceed as he had anticipated. Once into his delivery, Ralph found the content more complex: "As you go through them [the examples], you want to verbalize!" The dilemmas he faced explaining the regrouping of the fractions follow:

> I was torn between whether I should explain why when you add twelve thousandths, you can carry the one to the tenths ...I had thought about that when I looked this over...and I kind of decided against it then. But, when I started doing it all of a sudden, I thought, "If I am going to do it, I'm going to do it now."
>
> I didn't really think that one out either...I hadn't really planned ahead how I was even going to demonstrate that on the board or make them realize that even though there is no number in the ones' column [of the numbers being added] that you will end up with one there anyway because of regrouping...When I came to it, it kind of gave me some doubt.

As he said after the activity, "Realizations like that, as you go along, kind of surprise you!"

Some of his comments made about the course of the activity reflected not only his surprise, but a lack of confidence, an inadequate knowledge of mathematics as well as planning:

> This isn't as easy as I thought it was. I'm clumsy here ...that wasn't necessary...I jumped ahead for no reason at all...wished I hadn't said that; I thought at that point, it would do nothing but confuse them unless I wanted to stop there and do this [a later problem]. I don't think that would have been very coherent either. I got confused in a few other places here, too.

Despite his own confusion, Ralph wanted very much not to confuse the students, "I'm not going to say something that is going to confuse them; that's going to take more time to undo the mistake I made." This was a common fear of all the teachers.

Ralph's concentration on what he was explaining also affected his delivery. It slowed him down and had a definite effect on him. "When I slow down, I am aware of my slowing down and my pauses and my ho-hums; I become almost bored myself!" Nowhere was this boredom more evident than in the following recollection about his hands: "I was aware of my hands' being in my pockets here in a minute, and I will jerk them out...it made me realize that I am not being very enthusiastic about what I am doing." Thus, the prevailing flow of instruction was a direct contrast to Activity 8 in which Judy was judged to be inattentive.

Throughout the activity, Ralph also had trouble in finding someone to give a rule for adding decimals. After asking the question and indicating the need for a rule several times without any student response, Ralph's brightest student finally raised his hand to say, "... line up the decimal points." The rule was not expressed in the text, but it was highlighted on the ditto Ralph intended to use the next day.

Not all Ralph's discomfort, therefore, was brought about by the lack of student attention. He was having his own difficulties in communicating the mathematics, in maintaining his own interest, and in trying to prevent the students from becoming confused. In being more cautious with his explanations, Ralph did most of the talking and asked fewer questions. Thus, students were given fewer opportunities to participate, a fact which he also acknowledged along with its effect on their attention: "There is not much participation by the students, and I feel that is something that really aids them in attending." This comment appeared to be a reflection rather than recalled thought. From the observer's perspective, the students were usually more attentive when the pace was faster and more unpredictable questions were asked. They were less attentive when Ralph was more talkative, more of a lecturer.

Despite all his discomfort with his presentation and the lack of student attention, he indicated his reluctance to terminate this activity:

> But when I was done with the explanation, I still had the feeling that something is lacking if I should let the kids loose on these [problems]. There is more information I should provide that might help them so they [will] have fewer questions.

This may have contributed to Ralph's eagerness to return to the board to work yet another problem once the work period had begun.

Zelda's Type A-3 critical moment was brought about by the overall difficulty she saw students having with the task of the first activity on the fourth day of the unit (Activity 9). Her awareness of the students' difficulty was not entirely based on student occlusions during verbal interaction although the density of occlusions was in the mid-to-high range, but on her observations of her students' use of Cuisenaire rods to compare fractions. She came into even closer contact with how students were using the rods to find the answers by giving individual assists on three occasions. It was somewhat unusual for her to use this technique during a presentation, but the task was also new and took some time for students to perform. Instead of being at the board recording results or drawing pictures, she was able to move about and see what students were doing.

Zelda mentioned the difficulties of five individuals who were important indicators of the performance of others. They all sat near the front of the room and were frequent contributors to the lessons. As she admitted, "I notice mostly kids that are right next to me." Three were boys whom she had once identified as her "barometers," saying, "I figure how things are going because if they get it, then all the other people I worry about in the room probably got it, too." The other two were girls who "...don't normally have problems!" Whether by inference or observation, she realized that the difficulties were not limited to these five individuals as was apparent in this comment, "I get bothered because quite a few people really looked puzzled."

Zelda had not anticipated that students would have so many problems with what she believed was not that complex a task, and she was having trouble figuring out why:

> I thought it shouldn't be causing them that many problems! Maybe there is something else. Maybe I am going too fast for them. But I didn't think I really was going fast at all! I am sort of puzzled!

As the activity continued, she continued to puzzle over why the students were having so much trouble with the task.

Comparison of fractions was a new task, but the source of the students' difficulty was a more fundamental one, relating to using the rods. They had trouble in naming and representing the fractions to be compared. Once this part of the task had been completed, students had little or no trouble in making the comparisons. One source of the difficulty was that students were destroying their keys as they tried to

form the fractions. Without being able to see the entire key, the relationships between fractions and rods was not evident.

They were having trouble with the prerequisite skill of representing fractions. Although students had used the rods to represent and name fractions in four activities prior to this, two activities dealt exclusively with unit fractions. Proper fractions had been named by counting out pieces in an already existing key. The students had <u>never</u> been asked to <u>identify</u> or <u>model</u> proper fractions directly. The task of this activity, therefore, required the students to perform two new skills, one of which should have been familiar.

When Zelda was unable to come up with a good rationale for why the students were having so much trouble, she finally asked the class if they understood. This was not an unusual move for her; it was one she often used near the end of an activity. According to Zelda's recollection, her question had not been an automatic reaction, but one made with purpose:

> I don't recall being tense at all. I just thought, "Well, we better stop and figure this out because if they are having difficulty at this level, we better fix it! What I am doing is not that complex!"

Students immediately voiced their confusion, but it was clear that Zelda did not understand what was causing their difficulty. She reminded them they had been getting the answers; and, indeed, there had been many correct answers.

Zelda's next reaction was to ask if the students were wondering why they were being asked to perform the task. Then, when one girl blamed their difficulties on the rods, "...it finally clicked that she doesn't remember which rod is which fraction!" Now, Zelda had an explanation for the difficulty she had been seeing, and she immediately

assumed it to be true for others, "I think that was going through a lot of peoples' minds. 'What does 1/2 have to do with the rod? And if she just puts the number up there, how can I remember which one is which?'" It was only later in what appeared to be a reflective moment that Zelda alluded to the complexity of the task as being a possible cause and then only in general terms. "I guess they were not into it because I asked them to do more difficult things in a way," she responded when asked if she had any other thoughts as to why the students were reacting as they did. Again, this seemed more a concession than a recalled thought.

Why should Zelda assume that students could remember the rod-fraction relationships in the first place? One of the reasons the Cuisenaire rods are used is to enable students to acquire and apply a concept of fractions rather than operating by rote. Greater familiarity with the rods and the task would have made it possible for some students to operate from memory. Zelda, it would seem, focused on the wrong task difficulty.

With the realization that "they can't transfer the fractions to the rods," she opted to eliminate the task difficulty by removing any need for students to have to recall or recognize the rod-fraction relationships. She wrote a code on the board which listed fractional values for each color of rods. Unfortunately, the code was only valid for the unit rod being used at the moment. After a brief check to see if the students thought this code would be helpful, she ended the activity. No more comparison problems were given so that students could use the code. Instead, she simply referred to the code at the start of the next activity. With the source of the student difficulty resolved, Zelda evidently was no longer bothered.

Zelda reported feeling "not too peppy" and having a "definite don't-mess-around attitude" on the day this critical moment occurred. The lack of "pep" was not apparent to the observer, but she did not tolerate off-task behavior and made comments to several students about their behavior. Neither seemed to have any direct influence on her response to the difficulty students were experiencing with the task. If there were any other features of the prevailing flow that might have influenced Zelda, it was the slow pace, a function of the task. She did not typically conduct activities at a slow pace.

If this critical moment had further consequences for Zelda's teaching, it was over another decision involving the use of rods. In the middle of the next activity, when students were offering rules for adding fractions with like denominators, she collected the rods before continuing with more problems. The impact of prior student difficulty with the rods may have made it easier for her to make this decision.

The Type A critical moments described above occurred over student difficulties and could be broken into three subcategories. Most were isolated incidents of a teaching difficulty, but some were more pervasive, lasting for much of an activity and observed across students. The first type (A-1) occurred when teachers' attempts to correct students' misunderstandings proved ineffective. The second type (A-2) occurred when teachers had trouble eliciting a desired response to a question. The last type (A-3) occurred when teachers could not satisfactorily diagnose difficulties students were having with activities. A summary of the characteristics of Type A critical moments is given

in Table 5.2. Brief descriptions are given for prevailing flow, student occlusions, elective actions, and consequences.

## Type B: Student Insights

The six critical moments of the second type, Type B, were a result of the teachers' elective actions to incidents of student insight. Their discomfort was due to one of two situations. Either the teacher became confused while exploiting the student's suggestion (B-1) or was uncomfortable about avoiding one (B-2). Three critical moments for Martha and Zelda were of the first type (B-1) while Ralph's three critical moments were the second reaction (B-2).

### Type B-1: Teacher Confusion During Exploit

Martha and Zelda's critical moments for both the circumstances which produced them and their emotional reactions were almost identical. They became confused with the mathematical explanations or procedures being used to exploit insightful student suggestions or questions. This confusion was both overt and covert. The teachers had not anticipated the students' ideas which came up during the lesson. Some had been considered in the unit plans; some had not. Contributing to the teachers' choice to exploit the students' ideas were some unusual circumstances or prior, related events in the instructional flow. In each case, the teacher resolved her own confusion by changing her approach. The emotional reaction was anger for having "goofed" and some fear that the mistakes may have confused the students. Immediate emotional reactions did vary. Martha reported an intense--though brief--anger; and Zelda reported having been only "kind of mad."

Table 5.2. Summary of Type A Critical Moments: Student Difficulties.

Type A-1: Teacher unable to correct student misunderstanding:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Martha (4 CMs) | Content: new concept task (pictorial/concrete). Student participation: good. Instructor's pace: good. Prior related incident: same difficulty/elective action. | Initial: patterned errors/specific question (unsolicited). Recurring: errors, questions, no response. (Teacher expectations: 2 high, 2 low.) | Initial: exploit or alleviate (probe/demonstrate). Subsequent moves: (similar or tell). Concluding: delay (with challenge to high ex., support to low). | Attributions: S not understand; task/concept hard; teacher inadequate. Emotions: frustration, fear, confusion, low ex. students. Cognition: no more alternatives. Behavior: plan activity, check on individuals. |
| Ralph (1 CM) | Content: new concept (defitional). Student participation: OK. Instructional pace: OK. Problem selected for potential difficuity. | Initial: patterned error. Recurring: nonverbal. (Teacher expectations: high.) | Initial: alleviating (probe/tell). | Attributions: S not understand. Emotion: frustration, fear, confusion. Cognition: no other alternatives, hope enough. Behavior: none. |

Type A-2: Teacher unable to elicit desired response:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Zelda (1 CM) | Content: new concept/introduction (pictorial). Student participation: lots volunteering. Pace: good. Planned question. | Multiple errors from number of students. (Teacher expectations: some S would remember.) | Multiple alleviating: (reasking/varying question). Concluded: delay. | Attributions: S forget/mental block, large group inherent. Emotion: surprise, not too bothered. Cognition: limited alternatives. Behavior: altered plan. |
| Ralph (1 CM) | Content: new application of concept. Student participation: good. Pace: good. Question was reviewed. | Initial: no response. Second: no response. (Teacher expectations: low.) | Initial: reask, then redirect (twice). | Attributions: S not paying attention, no effort. Emotion: frustration, anger. Cognition: incomprehensible. Behavior: none. |

Type A-3: (Pervasive Problems) Teacher unable to diagnose student performance during activity:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Zelda (1 CS) | Content: new task. No prior activity on prerequisite task Pace: slower (task time consuming). Student participation: OK. | Teacher observed task difficulties from individuals. (Teacher expectations: 2 high, 3 barometers, moderate to low.) | Three individual assists. Continues activity. Asked students trouble. Exploit (give table). | Attributions: pace too fast, not remember relationships. Emotion: not tense. Cognition: puzzled. Behavior: switched activity. |
| Ralph (1 CS) | Content: new procedure for adding. Teacher consciously focusing on presentation. Teacher highly verbal, asked fewer questions. Pace: slow (teacher bored). Student participation: down. | Teacher perceived class inattention. | Continues activity. | Attribution: S not attentive. Day of week: Monday. Emotion: frustration. Cognition: reluctance to stop. Recall: plea for help. |

Zelda's critical moment came as she attempted to exploit Diane's question about how two fractions with unlike denominators (3/6 and 6/12) might be added. The question was raised during the first activity in which fractions with like denominators were being added. Initially, Zelda had asked the students to use the Cuisenaire rods to find answers, but the students had quickly picked up the number pattern and were answering without them. Zelda momentarily delayed her exploit of the student's suggestion so she could give the students a bit more practice in applying their newly acquired rule before shifting into this new and more difficult task. It was not long after Diane's question had been raised and after several students had urged her to answer the question that Zelda switched to the new task. Using the fractions Diane suggested proved to be a "fatal error" as Zelda later termed it.

To understand what may have prompted Zelda to respond in this manner requires some knowledge of the students' prior experiences. Equivalent fractions had been the topic of two previous activities in which the only technique for finding equivalent fractions was based on a counting pattern. Students took a unit fraction such as 1/3 and generated a set of equivalent numbers by counting. The set of 1/3, 2/6, 3/9...was obtained by counting 1, 2, 3,... for the top numbers and 3, 6, 9,... for the bottom numbers. Students had not yet been asked to simplify fractions or to generate sets of equivalents with a non-unit fraction.

In order to try to help the students solve the problem of 3/6 + 6/12, Zelda first tried generating a set of equivalent fractions for 1/2. She first exhibited confusion by writing 1/2, 1/3, 1/4, and 1/5.

Realizing her mistake and chuckling about it, she switched to generating equivalent sets for 1/6 and 1/12, which proved to be a big mistake, although she did not realize it at the time. Students were eagerly contributing fractions to these sets with their counting strategy. The sets were soon filled with at least five equivalent fractions when Zelda realized she had made a mistake. Instead of trying to correct herself again, she switched to a different problem saying to Diane, "I wish you hadn't given me that problem!" The problem she chose instead was 1/2 + 1/3, a problem that could be solved by generating the equivalent sets, selecting two fractions with the same denominator, and adding. This procedure proved somewhat difficult for the students to understand as evidenced by their questions.

Zelda seemed to have mixed emotions about this experience as she talked of "laughing" about getting off on the wrong track, being "glad" it didn't go farther, and "feeling uncomfortable" when it came time to give up on the problem. She did not report the sudden flood of negative emotion, but as she said of another incident, "I'm not going to be bothered by anything today." Zelda thought unpleasant feelings were brought about by trying to decide what to do and worrying what Diane might feel as though her question had not been a good one. Diane was a girl Zelda described as being in the top math group, one who is "always ready with answers," but one who "gets turned off easily if I don't let her participate right away." Thus, Zelda's opinions about Diane may have contributed to her willingness to exploit and risk the confusion she experienced.

Speculating as to what might have produced her confusion, Zelda suggested a momentary lapse of attention, a reason she had given on other occasions:

> I think I was thinking about something else, and this is whenever I get in trouble teaching--it's usually because I am mentally distracted. I would guess I was thinking about something totally unconnected about what we were doing.

Although she was unable to recall what thoughts might have interfered this explanation may well have accounted for Zelda's initial confusion. During recall, she concluded that this "interesting" question should have been saved as a topic for the next day. "Maybe thirty minutes is all I can handle before I become distracted in a situation where I am demanding their attention all the time." She was referring to the fact that this day's lesson was the longest presentation of the unit, and there was no work period.

Plausible explanations for Zelda's confusion and mistakes include insufficient knowledge or recognition of relationships of options in her working memory. Zelda's initial idea to generate a set of equivalent numbers for 1/2 would have enabled her to satisfactorily complete the problem as both 3/6 and 6/12 are in the set. She did report recognizing that these two fractions were equivalent, but that did not help her to correct her first mistake and correctly build a set of fractions equivalent to 1/2. She did not realize until later in the activity this would have been a viable option. Thus, switching to 1/6 and 1/12 and generating equivalent fractions for them was her first substantive error. Disoriented by her first careless mistake, Zelda evidently switched to whatever came to mind without pausing to think it through, a consequence of the pressure to perform.

Another alternative for solving 3/6 + 6/12 would have been to use the Cuisenaire rods. The key from which all the necessary representations could be found was the same one that students had been using at the beginning of the two previous activities, including her Type A-3 critical moment. Zelda had the rods picked up and put away during the previous activity, knowing she was about to exploit this addition problem. At the time, she believed it necessary to remove the rods to work the problem. Before starting to add 3/6 and 6/12, however, Zelda told her class the problem could be worked out with the rods and admitted "maybe we should have kept them for a few more minutes." Evidently, she was having second thoughts, although she did not so indicate during recall. Zelda's first thought, an erroneous one, was that the fractions needed to be simplified before they could be added. Any equivalent fractions with the same denominator could have been added with the rule the students had already found. Had Zelda been more aware of the mathematical possibilities for solving this problem, the critical moment might never have occurred. The same could be said for two similar experiences of Martha's. Nevertheless, both teachers were motivated to exploit their students' suggestions despite their limited understanding.

Martha's Type B-1 critical moments were much like Zelda's as she, too, found herself confused in trying to follow through on her choices to exploit insightful student contributions. After having "goofed" in her explanations to the class, Martha reported experiencing immediate, emotional reactions. Despite her professed belief that "making mistakes in front of the students and admitting them makes for easier rapport with the kids," she was unable to suppress her "gut" reactions to her own mistakes:

> I really goofed! God, that was bad!...It goofs up everything I have done so far! It gives them the wrong idea! [Activity 11] I really got screwed up! Somehow I couldn't convert 2/2 to 6/6! [Activity 21]

Fortunately, her emotional reactions were only momentary. In discussing how she felt once the mixup over the equivalent fracitons was over, Martha said, "I got a little angry that I made such a silly mistake, but I didn't feel super-embarrassed in front of the children." Her secondary comment did not display the same emotional intensity as did her initial exclamation.

Martha's Type B-1 critical moments were caused by student's unsolicited ideas about equivalent fractions and the addition of fractions. In the first case she resolved her confusion quickly, but the second led to the longest and most verbal of her elective actions or any presentation in this unit. The incident involving equivalent fractions occurred on the third day when Rodney offered 1/2 as another name for 2/4. This offer came before equivalent fractions had been introduced to the class and the next day after the same student had volunteered 1/2 as another name for 3/6. Martha had suppressed her impulse to exploit the first offer, which she recalled with Rodney's second offer.

In her exploitive move, Martha tried to justify the equivalence of the two fractions by referring to their Cuisenaire rod equivalents and became confused over which rods represented which fractions. Without any rods available and fearing she might confuse the students by continuing, Martha reported thinking, "The more I explain, the worse it is going to be. I am just going on to the next thing." Shifting tactics, she asked, "Who would have more?" When the students emphatically replied "Nobody!" she went on to the next problem.

A bit farther into the lesson, opportunity to elicit another equivalent fraction arose but passed unnoticed. This suggests Martha's earlier exploit was done more to respond to Rodney than to explore equivalent fractions. Rodney was the youngest member of the class, and Martha thought him adorable; but she rated him in the lower half of the class on his mathematical performance.

Martha also became confused while trying to respond to Max's question about how two fractions with unlike denominators could be added without the use of the rods. She became confused as she started her explanation. It suddenly occurred to her that the algorithm required division; it was an unnecessary assumption, but one consistent with her understanding. She believed division was not appropriate to use with her class as some students had not yet learned how to divide. Consequently, she tried to use Cuisenaire rods to explain how the fractions could be changed, but she became confused as to which rods represented which fractions. Admitting the obvious, Martha exclaimed, "Wait a minute. I'm mixed up! You mixed me up!" A moment later she said, "I've got it now! I was in the wrong ball park there." At this point she directed the students' attention to the rods already on their desks and completed the explanation using the same approach the student had been trying to avoid. Realizing what she had done, Martha apologized, provided an explanation, and indicated the task would be dealt with eventually. She did not seem bothered that she had failed to give Max the answer he wanted. Her opinion of his approach to learning was not in agreement with her instructional goals. "He is one that pushes through objectives, not too concerned with understanding concepts. He doesn't want to get slowed down with that. Just, how do I get the answer?" She enjoyed Max and had high expectations for him.

Type B-2. Teacher Unable to Accept/Exploit

Ralph's three Type B critical moments differed from those described for Martha and Zelda. It was not confusion in trying to exploit student insight and anger over having made mistakes that resulted in his discomfort. Instead, Ralph indicated surprise, uncertainty, fear, and resistance to students' suggestions; he avoided them. Each student suggested something he had not noticed or considered in preparing the lesson or was not able to comprehend. His avoiding moves were attempts to adhere to the approach taken by the textbook, at least as he was able to interpret it from the examples and problem sets. Just how dependent Ralph was on the textbook and his interpretation of its instructional approach is best illustrated by the following description.

He was demonstrating a procedure for converting fractions to decimals on the third day of the unit (Activity 6). As illustrated in the text, he was taking fractions having denominators with powers of ten, breaking them into their component parts, and simplifying as $15/10 = 10/10 + 5/10 = 1 + 5/10 = 1.5$. It is an artificial approach once students recognize the pattern for converting fractions to decimals. After several fractions had been changed into decimals using the expanded approach, Ralph offered to let someone else pick the next problem. One student, evidently noting the denominator was not a power of ten, selected 7/8. Ralph forthrightly responded, "Let me think this one out; I'm not prepared for this one." There was a fairly long pause as he attempted to figure out how to do the problem. His response was to eliminate this and similar problems from the assignment; he did not change 7/8 to a decimal until the next class period.

Ralph chose to avoid the student's question about converting 7/8 to a decimal for two reasons. First, despite the printed hint in the text (7/8 = ?/1000 = ?), he failed to recognize the method being suggested was to change the fraction into thousandths before giving the decimal. It was a natural extension of the equivalent fraction approach already used. At the time Ralph thought:

> I don't see how that transfers over at this point. I don't see how that makes a smooth transition from common fractions [having denominators in powers of ten] to this [without powers of ten in the denominator]...When you say you are going to divide eight into seven, I don't see how they [completed the problem].

His last comment indicates the second reason for having avoided the problem. Ralph viewed 7/8 as a division problem. This so dominated his thinking that when the answer to the problem was given on the next day, he simply indicated that the solution could be obtained by dividing. The problem was not worked out on the board, and Ralph still did not seem to realize what method the text had suggested.

When asked why he had not pursued this problem, Ralph replied, "I was frightened of the directions it would take." It was a fear based on past experiences:

> I would take something I was not familiar with and start working it out on the board trying to get lots of feedback from the kids...More often than not it has turned out to be something that has taken a long time to do and [was] more confusing. I think my thinking now is...I do feel I have to know how to do it in advance...I never have just written something on the board or presented the kids with a problem I did not know the answer to or did not know

> how to figure out...I might say let's try to figure this out together; but if I do, I know how they would [should] figure it out.

When asked to fantasize about what might happen if he did attempt something he did not already know how to do, he talked about not wanting to present information in a way that "[the] students misinterpreted or I misinformed the students to the point where they would actually work out a problem and get the wrong answer and think they did it correctly." With such unpleasant expectations, it was not surprising that, even after thinking about the event, Ralph supported his choice to delay the problem. "I am kind of <u>secure</u>, satisfied that I did make the choice because if [one student] wanted to know how to do them, then it wasn't making others [do it] too." He made it sound as though one student's curiosity had not been imposed on the rest of the class, as though his action had been taken to benefit the group. It is interesting to compare the above comments with what Ralph said when asked earlier if he would be bothered by making a mistake in front of the class:

> Usually, never at all, but maybe with the camera and everything, I kind of think about it more...I think I have generated a feeling in there [class] that I am not afraid or ashamed to make mistakes. It is an atmosphere I enjoy having; it removes a lot of fears in making a mistake. I used to be afraid of looking foolish and having kids laugh. [Now] I don't have that fear at all.

It appears that Ralph was not being consistent in what he professed to believe his reaction would be and what it was. If the camera were an interference, it is doubtful that it so inhibited Ralph that he avoided every insightful student suggestion. He might have accepted mistakes made through carelessness without experiencing discomfort, but not confusions he was able to anticipate. Having been alerted by his lack of understanding, Ralph chose to prevent further confusion by avoiding the ideas altogether.

The extent of his annoyance when a valid suggestion was offered by a student is vividly portrayed in the following quote:

> This is very awkward for me when someone gives the wrong answer and I am not prepared for it. It was something I didn't want to hear. I have to take into account my inadequacies in presenting this material.

He was simply not open to anything other than what he was prepared to present and was unlike Martha who voiced her desire to be flexible in responding to the students.

There was no hint in any of Ralph's comments that the individual students influenced his actions or feelings in any way. Nevertheless, the students who offered these suggestions were described "...as being in the top group with regard to their mathematical ability." And he had mentioned having students who "know they are mathematically wiser than I am." While this knowledge may have increased his discomfort to some extent, it was certainly not Ralph's primary reason for avoiding their ideas.

Ralph's other Type B-2 critical moments fit the above pattern. Students suggested alternative procedures for completing problems, simpler and more direct procedures than had been demonstrated; yet he still relied on what he believed the textbook required. He even mentioned this to the class as justification for his having rejected an idea. In one instance (Activity 11), when a student wanted to add decimals written horizontally without rewriting them in the standard array, he was stunned. "I couldn't believe it.! I was thinking, 'Are you kidding me?' I looked at the problem and thought, 'I'm not going to add that in my head!'" Actually, the numbers might easily have been added mentally, but Ralph did not seem to realize it at the time. His conjecture was that the student might intend to use a calculator.

The other Type B-2 critical moment came when a student anticipated how to change decimals to fractions without all the intermediary steps. Even though he admitted he would have used the shorter method to do the problem, Ralph interpreted the student's suggestion to mean, "Why not just memorize it...!" This also illustrates his dependency on the textbook for guiding his presentations. The year before, Ralph had found that going through all the intermediary steps had caused the lessons to drag out and left the students confused, but he was optimistic about this class. "I think this group can handle it; and as long as they can, I am hoping it is appropriate!"

The characteristics of these Type B critical moments are summarized in Table 5.3. Martha and Zelda both became confused and made mistakes when exploiting insightful student suggestions (Type B-1). Ralph's Type B-2 critical moments were a consequence of his not knowing how to respond to the students' suggestions.

## Type C: Pacing Dilemma

In the third category of critical moments, teacher discomfort occurred in relation to pacing dilemmas. Teachers reported feeling both the pressure to maintain the instructional pace so as not to lose the attention of the class and a desire to help one or more individuals with the task. It is the recurring problem of whether to attend to the needs of many or one. Three such critical moments and two subcategories were identified. An isolated pacing dilemma (Type C-1) was

Table 5.3. Summary of Type B Critical Moments: Student Insights.

Type B-1: Teacher confusion during exploit:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Zelda (1 CM) | Content: new. Student participation: very good. Pace: rapid. Prior activity (A-3 CM). | Question: more complex task (unsolicited). (Teacher expectation: high.) | Exploit: new activity. Teacher (mistakes/confusion. Change problem. | Attributions: T's momentary loss of attention. Emotions: amused, anger (mild). Cognition: confusion, wished had delayed. Behavior: return to problem next day. |
| Martha (2 CM) | Content: new or extension. Student participation: good. Pace: good. Prior related incident: same student, same att. or none. | Question: more complex procedure or offer. Alternative: (unsolicited). (Teacher expectations: 1 high, 1 low.) | Exploit: explanation. Change procedure or explanation. Delay. | Attribution: T not remember. Emotion: anger, fear, confused, low expectation S. Cognition: grading inappropriate for class. Behavior: none. |

Type B-2: Teacher unable to accept/exploit.

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Ralph (3 CM) | Content: new/varied tasks. Student participation: OK. Pace: OK. | Alternative procedure/problem (unsolicited or volunteered). (Teacher expectations: 3 high.) | Avoid (delay/reject). | Attribution: T not able to own inadequacies. Emotions: surprise, fear, confuse, awkward. Cognition: adhere to textbook, unprepared/no recognition. Behavior: none. |

reported by Ralph as he attempted to elicit the correct response from one of his students. The other two, designated as Type C-2, were pervasive critical moments. They occurred when Martha tried to keep the instructional flow at a reasonable pace and offer individual assists during two different activities.

Type C-1: Teacher Unable to Give Desired Assistance

Ralph's Type C-1 critical moment was, as he described it, "One of the greatest problems [for teachers]...[the reason] it is hard to keep kids' attention!" His pacing dilemma occurred on the first day of the unit. A student, Sheri, could not correctly identify the value of two in the numeral 537.29. Having first said it was in the hundredths' place, she was told to look at her book where the positions were defined, but she didn't have one. The teacher then asked if the two was in the first or second place after the decimal point. When she didn't respond, he wrote the number on the board, repeated the question, and indicated that, according to the book and the chart he pointed to at the front of the room, hundredths was in the second position. He asked what two in the first decimal place meant. This time Sheri responded "ten." Reacting as though he heard "tenths," he again asked for the value of two. To her next response of "twenty," Ralph replied that two 10s would be 20, but this (pointing to the position with two) was "tenths, a fraction, a part of a whole." He concluded the interchange by telling her than two in 537.29 means 2/10.

This interchange was similar to the Type A-1 critical moments already described for Martha when a student's response suggested an inappropriate interpretation of a concept when it was first being taught.

Like Martha, Ralph recalled a conscious urge to help the student. "I feel the urge to belabor that until she finds it [the answer] on her own or at least until she feels that she has found the answer to that problem on her own with my urging and coaxing!" Ralph's momentary goal was qualitatively different from Martha's: he wanted to help the student say the correct answer while she wanted to help the student understand the concept. He did not report feeling inadequate or having to struggle to think of a way to reach Sheri, though it appeared he had exhausted his supply of ideas. Instead, he referred to the problem caused by the interruption of the instructional flow.

He felt spending so much time with one student caused the rest of the class "to sit there just bored!" He elaborated:

> I mean I don't know how they [the class] could possibly get into that kind of conversation between myself and another student about some particular problem she is having...going all over this stuff. And, probably three-fourths of the class...I was losing their attention, maybe even more than that. Then, when I am done with her, I think, OK I have got to work again at getting this group back...they are going to be into different thoughts.

Despite all that he said about this incident during the stimulated recall session, he made only one attention-getting remark to another student during this interchange. He was able to resume the lesson with less difficulty than he had anticipated or feared, suggesting that his emotional reaction was caused, at least in part, by a history of experiencing this dilemma and a sense of helplessness which accompanies it:

> I have to be constantly deciding how long I am going to deal with this individual before I finally just say, "Ok, you helped her out." I don't know if I was consciously thinking about it right at that point, but I do think about it several times throughout any lesson period when I am giving a lecture of that nature. I think maybe the most

> sensitive part of teaching is awareness of whether the kids are listening or not. I try to pick up on that by looking at their faces and what they are doing.

He was annoyed at having to gain the attention of the class once the instructional flow had been interrupted by helping a student:

> It seems to almost totally direct what you say and what you do in terms of how often you repeat yourself or how often you have to stop what you are doing and say, "Now, is everybody listening up here?" or "Look down at your books!" Just constantly going through that because I am not comfortable with going on and having the feeling that only a few kids are paying attention!

Ralph believed that student attention was critical if learning were to take place and if he were to prevent students from deluging him with questions once the work period had begun.

Ralph's negative reaction to this incident might have been intensified by the length of the interaction. Probably for all the above reasons, Ralph rarely made so many attempts to elicit an answer from one student. Some of the unusual circumstances in which the event took place may have led him to persist as long as he did in this instance. Because it was the first activity of the unit, Ralph may have made a greater effort to help this student. He admitted his discomfort in having to make this presentation involving new concepts. He also admitted his anxiety about being filmed despite earlier experiences during the pilot study.

During the initial phase of this activity students had not been too eager to respond. When Ralph asked who would like to volunteer for a problem in this exercise set, no one volunteered. This interchange with Sheri came shortly thereafter. He described Sheri and her seat partner as

> ...quite bright kids but, yet, they have a hard time in math, and I am not sure why [he rated them much higher in other subjects], whether it is because they don't listen well or because they are just not inclined...[I don't know].

When questioned about it, he acknowledged that these two girls might have been good indicators of what others in the class were learning; but he emphasized it was not something he consciously thought about at the time.

### Type C-2: Teacher Unable to Maintain Instructional Pace While Providing Individual Assistance

The two pacing dilemmas (Type C-2) reported by Martha were passive critical moments. Martha offered individual assistance to students she could see were having difficulty performing certain tasks with their Cuisenaire rods, a situation not unlike that faced by Zelda in her A-3 critical moment. Unlike Zelda whose problem had been in figuring out why the students were having difficulty with the task, Martha had negative feelings over trying to meet the contradictory needs of two groups of students.

Martha was essentially conducting practice sessions with the whole class. Students were performing tasks which had been introduced earlier, possibly without the aid of the Cuisenaire rods. Appropriate "keys" were built from which the fractions could be represented. Students were to apply their concrete interpretations and compare fractions (Activity 15) or combine them (Activity 22). These tasks were somewhat time-consuming so that Martha had time between problems to circulate around the room giving assistance where it was needed. She was also freer to move about because of the highly routinized manner in which the activities were conducted.

Activity 15, the fourth activity to compare fractions, was taught like a game. Students went to the board to spin for two fractions, and comparisons were to be made with the rods before anyone was selected to give the answer. During Activity 22, the second activity in which fractions with unlike denominators were being added, Martha had to think up fractions for students to combine as well as pose the questions. Participation was good in both activities as students sought to contribute their results. Moving about the class, Martha was able to spot students' having difficulty with the task. Relatively few were, but their difficulties often persisted. "It got to where I was only watching about four students and doing a quick scan of the rest."

She made extensive use of individual assists during these activities. She gave about thirty-eight assists in each, three-fourths of all the ones used during the unit. The others were scattered over several activities. Some were quite brief, lasting only a matter of seconds, while others lasted for as long as one to three minutes. The combination of her elective actions and the concrete tasks slowed the overall pace of these activities and made them the two longest activities in the unit.

As she continued in her efforts to help the few students, Martha was aware of the need to press on:

> I kept being uneasy about those that this is really easy for, too. They are just sitting there wasting their time. I should have some other things for them to do [Activity 15].
>
> I get frustrated! I know that others are sitting there waiting. It's always on my back!...Oh, my, he needs this; they need that!...What bothers me is the length of time they [successful students] have to wait for the next problem [Activity 21].

Her comments reveal an interesting difference between her concern over pacing dilemmas and that of Ralph. Martha was concerned about wasting students' time, while Ralph was concerned with having to regain student attention. Martha had little difficulty in regaining student attention. Students were continually having to switch their attention from teacher or student talk to the task and back again or wait while Martha gave individual assistance. Kounin and Doyle (1975) have suggested that such an inconsistent signal system is conducive to student inattention. During the delays when Martha was working with one student, the noise level of the class increased as there was more talk and some minor off-task behavior. A couple of times, students even moved in front of the camera to display their silly antics, an unobtrusive measure of their off-task behavior. Once Martha was ready to resume where she had left off, however, the students gave her their attention. Even the longest of her individual assists (three minutes) did not result in any worse student behavior.

Helping the same students repeatedly and feeling frustrated over her neglect of others in the class only served to annoy Martha:

> I get impatient here! I have to keep going over the same things with those students. I want to get to something else...I've got twenty kids sitting there knowing how to do it! I know they are going to get off [task]. But, I have to be patient and show them again! [Activity 15]

Her impatience did not please her:

> I did a lot of showing today. It kept bothering me because he wasn't doing it. But I couldn't wait for him; I had the rest of the class waiting [Activity 15].
>
> Initially, I want to hurry up and tell that kid to find a way to catch him up with everybody else! [Activity 22]

It was not clear, however, whether Martha had been dissatisfied with her actions during the lesson or only after watching herself on the videotape.

As with the other critical moments in this study, Martha had been caught unprepared for the divergent performances of her students during these activities. She had not considered the possibility of individual differences as was apparent when she characterized her planning practices:

> As I lay out a lesson, I am thinking all the kids are going to do the same thing the same way. A lot of times I get into the lesson before I remember for the nine millionth time, "Oh, yeah, they learn at different rates!" which means that tomorrow I'm going to have to plan for some different pace of activities. As often as I have gotten hung up on that, I'll still lay it out for the "ideal child," thinking, of course, mine are going to be ideal and learn at the same rate.

After some reflection she mentioned an alternative strategy of splitting the class into two groups. This strategy, however, was not in keeping with the focus of this study or large group instruction.

Once all the students had been given a chance to spin for a fraction during Activity 15, a technique which slowed the pace even more, Martha quickened the pace by taking charge and giving the rest of the problems herself. It was a visible consequence of the discomfort of her critical moment. She was no longer free to move about as answers were given at a fairly rapid rate. Her tension was removed; and at the end of the recall session, she was able to comment rather favorably about the lesson:

> They did more of the thinking than I today. I always feel better if I feel they have had more of that exercise in doing it than me. I was much happier with this lesson because it was a whole experiencing thing.

## Type D: Unexpected Success

Martha was the only teacher to experience a Type D critical moment. As were her pacing dilemmas (C-2), these were pervasive teaching dilemmas associated with activities rather than with specific student occlusions. The problem was not caused by student difficulties, but by the lack of them. She had planned the activities with the expectation that the students needed more opportunities to learn or review the concepts, and the students were successful. What caused her discomfort was not their success, but the feeling that she was wasting their time giving them something they already knew how to do.

The two activities which became Type D critical moments were taught on the third and fifth days of the unit. The task was a review of proper fractions with students' representing or naming fractions using rectangular picture models (Activity 11) or Cuisenaire rods (Activity 18). She had reason to believe the students needed another opportunity to strengthen their concept of fractions. For Activity 11, her assumption was based on misconceptions the students had demonstrated the day before, including a Type A-1 critical moment and on a short test. Activity 18 had been planned as a review in case students had forgotten over a long weekend. The instructional pace was rapid with many questions' being asked, thereby providing numerous opportunities for student difficulties to surface, but the students did not demonstrate any need for help. There were few or no occlusions of student difficulty in either activity; they seemed to find the tasks relatively easy.

Martha's surprise and discomfort over the situation was mentioned repeatedly as she viewed videotapes of these activities and recalled her thoughts and feelings.

> It became obvious to me they knew how to do it...I kept thinking it [continuing the activity] is ridiculous...They were getting this; this is redundant!...I'm beginning to run this into the ground...I'm really at war with myself now. I remember thinking I should not be doing this, but why did I keep doing it? [Activity 11]

> This is too easy; it is not helping them. I've got to do something else. But I didn't quite know where to go or how...feeling we are going over stuff they already know. I'm trying to think where to go. [Activity 18]

While it might appear that Martha's comments were made to impress the observer-investigator, there was much evidence to suggest otherwise. These were the only two activities in which these conditions occurred, and she offered similar thoughts and feelings for both. Her anxiety over wasting students' time was also established with the pacing dilemma. She had some basis for being concerned that students might become bored; and, since she was operating on such a routine level, she had ample opportunity to note how well the students were doing. Not only were students exhibiting successful learning, but there was no unsolicited student insights which required attention.

On several occasions Martha mentioned the need to switch from one activity to another before students became bored or began to anticipate what she was going to do next. "The real secret," she said, "is you have to move quickly or they get bored." Perhaps she had detected some cues to indicate some students were losing interest. When she was switching from Activity 18 to the next one, she reported "feeling ill at ease...I noticed [student] playing with his rods." Why did she continue the activities for as long as she did? The primary reason appeared to be the nature of her plans.

Martha had not been at all sure what to do following Activity 18. She had first planned this lesson several days earlier by looking through some activity cards. There had been no opportunity to review or expand on her initial plans and sketches. After asking all the questions on the activity card, she began to pose problems of her own as she struggled to think what to do next. Her train of thought was, "I'm trying to think where I go from here! I'm confused. I want them to think!" Her own lack of direction was clear as she wrote first one problem and then erased it. "When I saw what I wrote, I knew it wasn't what I wanted." Once she had started, the activity progressed smoothly.

For Activity 11, her plans were fairly extensive and included a dittoed page of picture problems. Martha did not switch to the next activity, a test, until all the pictures had been used. In the previous case, she was slow to change activities because she was unsure of what to do next and, in this one, she may have been inhibited by the availability of the prepared ditto. In trying to justify why she did continue for as long as she did, she spoke about the value of practice and success:

> [I did it] for the practice with different kinds of fractions, I guess. I thought as long as I am doing this, I will get them super-saturated.
>
> The only saving part of it is I feel it is good to do things with kids somethings that they can succeed at very quickly. It gives them a feeling of "I've got it!" and maybe at this point with the fractions they need some, so they get a good feeling about fractions, that these things they can do.

Because of her students' success during Activity 11, Martha reported making the test shorter than she had intended. It was the last of these tests she gave during the unit, although it was doubtful that this critical moment was the only cause. Following each of these critical moments, she exploited all student suggestions; the (unexpected) success of these activities may have increased her willingness to do so.

Both Type C and D critical moments are summarized in Table 5.4. Type C critical moments were pacing dilemmas in which the teacher had to choose between the needs of an individual and the needs to maintain the pact of the instructional flow for the good of the class. There were both isolated and pervasive pacing dilemmas. Type D critical moments were pervasive problems caused by unexpected student success.

## Summary

This chapter provided detailed descriptions of the critical moments teachers reported in teaching their six-day units on fractions or decimals. A distinction was made between critical moments which occurred in relation to isolated student occlusions and more pervasive patterns of student performance during activities. For critical moments which occurred as a result of occlusions caused by student difficulties or insights, teachers had trouble overcoming student difficulties as they tried to help students to understand (Types A-1 and C-1) and to elicit correct responses (Type A-2). Student insights precipitated teachers' confusion which was apparent when they either exploited (Type B-1) or avoided valid suggestions (Type B-2). Pervasive critical moments arose from problems teachers had in diagnosing student difficulties with the task of an activity (Type A-3), over pacing dilemmas caused by diverse student performance--success versus difficulties (Type C-2), and in adjusting to unexpected student success (Type D).

Table 5.4. Summary of Types C and D
Critical Moments: Pacing Difficulties and Unexpected Success.

Type C-1: Teacher unable to continue individual assistance:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Ralph (1 CM) | Content: new, first activity. Student participation. Pace: not yet established. | Initial: patterned error. Recurring: errors. (Teacher expectations: low, math; high, other.) | Initial: alleviate, reask. Subsequent moves: coax/question. Conclude: tell. | Emotion: aggrevated. Concern: lose attention of class. |

Type C-2: (Pervasive Problem) Teacher unable to satisfactorily maintain instructional pace:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Martha (2 CS) | Content: review, practice concept, application (concrete). Student participation: very good. Pace: intermittent, fast/slow. | Divergent performance: few students having difficulty with task. Recurring incidents. | Individual assists (vary in length). | Emotion: tense, uneasy, frustrated. Behavior: alter routine of activity. Concern: waste students' time waiting. |

Type D: (Pervasive Problem) Teacher confronted with unexpected student success:

| | Prevailing Flow | Student Difficulties | Elective Actions | Consequences |
|---|---|---|---|---|
| Martha (2 CS) | Conent: review concept application (concrete). Student participation: good. Pace: rapid. Expectation: students need more practice. | Success, few if any occlusions | Routine. | Emotion: surprise. Concern: not helping, wasting students' time. Behavior: shorten activity (same or next). Justification: success builds positive affect. |

Descriptions and summary tables provided information about the nature of the student occlusions or performance patterns, the manner in which teachers chose to respond (elective actions), and the relevant characteristics of the environment in which it all took place (the prevailing flow). Descriptions also included teachers' thoughts: the ways in which they viewed the events, their reasons for their actions, and their feelings. Differences and similarities were noted across teachers and circumstances.

The critical moments revealed teachers' propensity to respond to distinctive student difficulties and insights. Student errors were usually indicative of definite misconceptions or were incomprehensible to the teachers. Unsolicited student contributions consisted of suggestions and questions about mathematical ideas not yet explored. If the initial student occlusion of a more extended interchange was not distinctive, then the recurring incidents from the same individual or over the same question were unusual. When students were familiar with the content and task, the absence of a legitimate excuse for student difficulty attracted teacher attention.

For the most part, distinctive student occlusions arose because the opportunity existed. Student difficulties and insights arose primarily out of new and varied tasks with new concepts which were still being developed. In some instances there were prior events which may have helped to alert teachers, but teachers did not report any awareness of their impact. Critical moments occurred during activities in which student participation and instructional pace were good; the activities were not pervasive problems. Teachers did not generally have discipline problems which interfered significantly with their teaching.

Teachers' reactions to these distinctive student occlusions were themselves somewhat distinctive. Elective actions varied, but most often they deviated from the routine of the activity or the typical response of the teacher. Exploiting and avoiding moves were quite prevalent in critical moments. With regard to the content of their actions, there were also differences. Teachers had to deal with mathematical ideas or explanations which were not part of the routine of the activity or which had not been presented earlier in the same manner. Exploiting and avoiding moves were often involved, with avoiding used to end interactions between a teacher and student. Alleviating moves were used to probe students' thinking and to coax students into giving the desired response. Although telling was classified as an alleviating move, it was used in much the same way as avoiding moves were, to terminate the interchange. Avoiding or telling mvoes came immediately after a student occlusion or after one or more attempts to follow through on what teachers immediately tried to accomplish. Both avoiding and telling moves were perceived by teachers as admissions of defeat rather than desired choices. Critical moments were brought about by difficulties teachers experienced in trying to achieve what they desired to do -- whether it was to help a student, elicit a particular response, exploit an idea, or alter an activity.

The distribution of student occlusions and elective actions in the previous chapter supports the claim that these are distinctive forms of behavior. While teachers varied in their choices of actions and the extent to which they pursued their goals, they were flexible within the range of options demonstrated throughout the teaching of their units. Variations reflected differences in teachers' typical behavior patterns, their understanding of the mathematics, and their conceptions about learning and goals for instruction. Pervasive teaching problems were the result of unexpected student performance during an activity. What made the patterns of student behavior distinctive was that they countered teachers' expectations. Teachers planned activities with the expectation that they were within the range of student capability; to find otherwise was disconcerting. When activities were planned with the expectation that students needed additional practice, success was unexpected and, therefore, distinctive. Teachers continued to teach the activity as planned, at least for a while. When change occurred, it was made in the routine of an activity or in the duration of the same or subsequent activities.

Teachers' negative emotional reactions to these critical moments and situations varied in intensity and in type. They ranged from surprise to frustration and anger. They included strong and immediate "gut" reactions as well as acknowledgements of mild sensations. Teachers offered varied reasons for the difficulties they encountered. However, most often they were at least partly due to their own limited understanding of the mathematics which affected not only their choice of actions but their understanding of students' difficulties and insights. Causal attributions for students or their own behavior also varied. When teachers extracted themselves from critical moments, they sometimes alluded to future opportunities, "postponing" action, as Morine-Dershimer (1979) reported. Teachers sometimes followed up on these incidents by returning to the same problem the next day or by planning an activity to address a particular idea, task, or difficulty. Thus, the effect of a critical moment or situation was sometimes apparent in teachers' plans, their alterations of ongoing or upcoming activities. There were also incidents for which no follow-up was carried out. For these critical moments, individual difficulties or interests were no longer thought about, and they did not seem to influence teacher planning.

The detailed descriptions of these critical moments and situations are, however, of little use other than to add to the collection of what teachers do, think, and feel in classrooms while teaching mathematics. From these personal episodes, a more general characterization of the covert and overt experiences of teachers as they confront unexpected student performance is necessary. A more general characterization of teachers' mental processing of unexpected student performance will be given in the next chapter.

# CHAPTER VI

# TEACHERS' MENTAL PROCESSING

## Introduction

This chapter will go beyond the descriptions of critical moments in the previous chapter in order to characterize teachers' mental processing of unexpected student performance. As a strategic research site, critical moments provided the necessary discontinuities in teachers' mental processing to enable them to reveal something about the nature and function of their cognitive processes as well as their emotional reactions. From their attempts to recall conscious thoughts about student behavior, it was found that teachers were generally unable to describe their cognitive processes in any great detail and that they were unaware of much of what was processed. Nevertheless, sufficient information was obtained to be able to identify and characterize different processes and to build models of teacher cognitive processing.

An examination of the teachers' self-report data along with information about the instructional environments and behavior patterns of both teachers and students produced evidence on which factors influenced teachers' mental processes. These have been incorporated in the descriptions of the functions of the cognitive processes. In discussing the genre of research that is becoming known by such varied labels as educational ethnography, participant observation, qualitative observation case study, or field study, Smith (1978) characterizes these levels of analys s: descriptive, theoretical-analytical-interpretative, and metatheoretical. In the previous two chapters, description was the level of analysis. The theoretical interpretations which follow represent the second level of analysis in qualitative case studies as described by Smith.

From this investigation it was found that teachers engaged in four cognitive processes: interpreting student occlusions or performance patterns, setting goals for responding to these student cues, searching for the means to accomplish their goals--elective actions, and evaluating the success of their actions with respect to their goals. These four processes are similar to the mechanisms Skemp (1979)[1] asserts are responsible for directing the mental processing of a goal-directed organism. These processes also show a similarity to the well-known steps in problem solving first suggested by Polya (195 ).[2] (See also, Shulman, Loupe, and Piper, 1968; and Elstein, Shulman, and Sprafka, 1978.)

Before describing the nature and function of each of these four processes, an overview of the total process is needed. This can best be accomplished through an examination of a model. A model of teachers' cognitive processing of critical moments should reflect the way in which teachers actually function, but the decision making models

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[1]Skemp's mechanisms include four components in the mental capacities of any goal-directed organism. They include a sensor which is sensitive to the relevant aspect of the present state of the object, a another by which the goal state can be set, the comparator which determines the difference between the present and goal states, and a plan which determined what to do when the object is not in its goal state and even when it is (p. 42, underlining added).

[2]Polya's problem solving steps include understanding the problem, devising a plan, carrying out the plan, and looking back. The presence of the problem and goal to solve it are assumed.

(Shavelson, 1976; Peterson and Clark, 1978) used to plan and analyze research on teacher thought do not represent how teachers in this study processed actual moments. Even more important, teachers' cognitive processing of isolated student occlusions was distinct from their processing of the pervasive patterns of student performance. The two situations placed different demands on the immediacy with which teachers had to respond, and they occurred in relation to different types of goals. Consequently, separate characterization of teachers' cognitive processing for student occlusions and performance patterns are presented.

This chapter is divided into three main sections: (a) teachers' cognitive processing of student occlusions, (b) teachers' cognitive processing of pervasive patterns of student performance, and (c) teachers' emotional reactions to unexpected student performance. In the first two sections, models of teachers' cognitive processing precede the descriptions of the nature and function of the processes. The descriptions of the processes include a description of the factors found to influence the cognitive processes. Since the four cognitive processes will be described in detail for student occlusions, the description for pervasive performance patterns will highlight the differences between the two types of critical moments: student occlusions and pervasive, unexpected student performance. The chapter will end with a brief summary.

## Teachers' Cognitive Processing of Student Occlusions

### Model

A model of teachers' cognitive processing of student occlusions derived from the investigation of critical moments is shown in Figure 6.1. The cognitive processes of interpreting, goal setting, searching, and evaluating are shown in relationship to the model of critical moments presented in Chapter I (p. 6). After the instructional flow is interrupted by a student occlusion--an unpredictable student response or contribution--the teacher responds with an elective action which may or may not be followed with an overt response from the student. A cycle of student and teacher interaction could be repeated any number of times before instructional flow is reestablished. The covert mental processes are shown on a different, horizontal level than the overt behaviors to emphasize the distinction.

Arrows have been drawn to indicate the direction of the mental flow through the questions and cognitive processes. Awareness of some processes seemed to produce an increased awareness for others as indicated by arrows between processes. For example, when a teacher consciously interpreted a novel student occlusion, awareness of a goal was often experienced as well. Elective actions could be executed without conscious thought, but the evaluation process seemed to monitor that execution at some level. The teachers were alerted to discrepancies in their own elective actions and to failure in achieving the desired student response. When such failures were detected, the teacher either searched for another elective action or changed the current goal. When a search failed to produce an elective action, teachers changed their

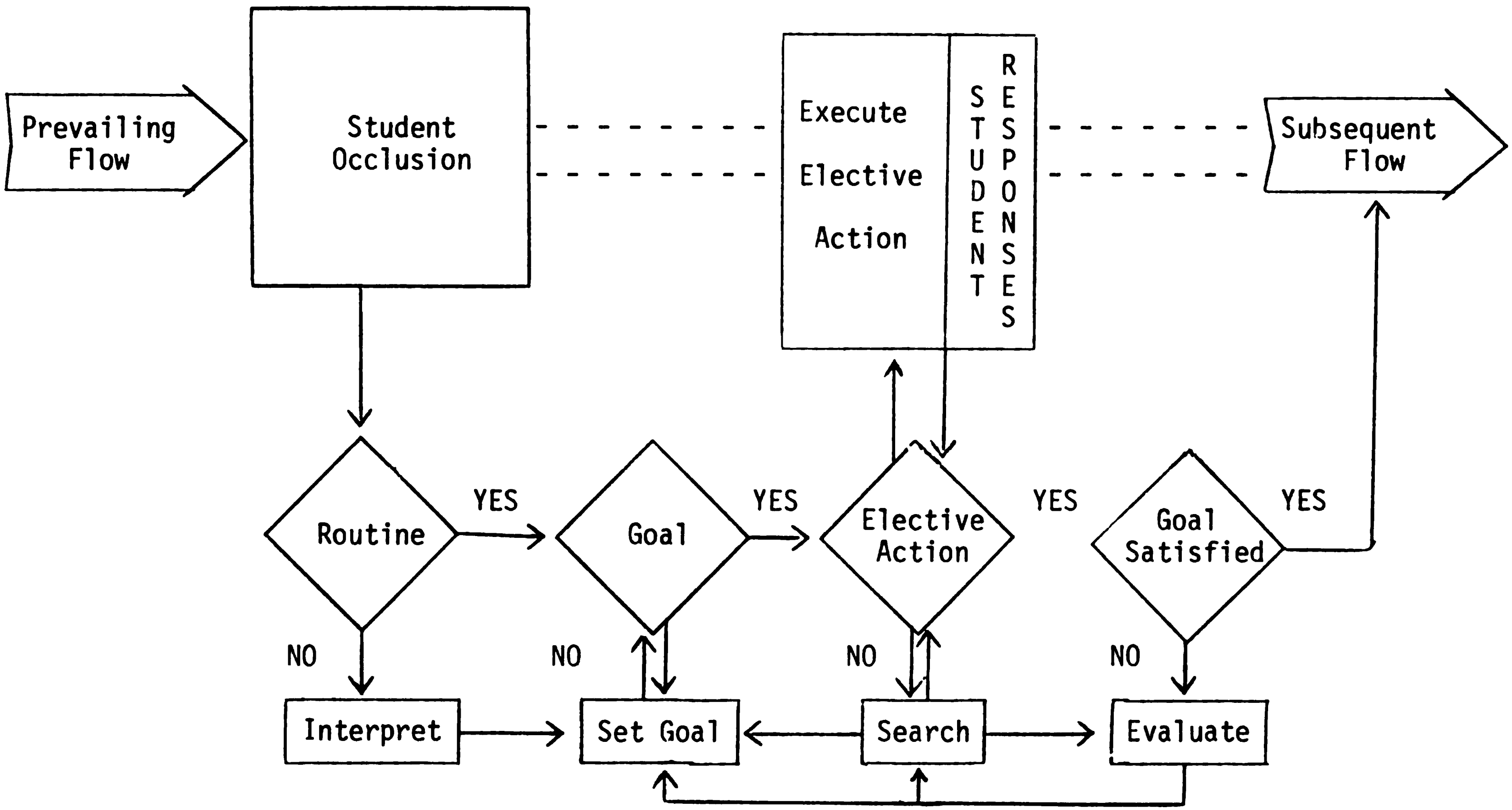


Figure 6.1. A model of a teacher's mental processing of student occlusions and critical moments.

goals. In this model, critical moments would occur when teachers changed their initial goals for responding to a student occlusion.

Not all student occlusions were processed in exactly the same manner. The small number of critical moments was an indication that teachers cope with numerous incidents of student difficulty and even insights without having any conscious thoughts worth retelling. Consequently, in modeling these events, it was necessary to incorporate a mechanism by which teachers could process student occlusions automatically as well as with increased awareness. The following questions, one for each of the cognitive processes, provide such a mechansim:

1. Is the student occlusion routine (not novel)?
2. Is a goal available to handle the occlusion?
3. Is an elective action available?
4. Is the goal satisfied?

Affirmative responses to all the above questions indicate the teacher was able to process the event with a low level of consciousness using routine behaviors. A negative response to any of these questions, however, indicates the corresponding process would be performed with an increased awareness by the teacher.

## Nature and Function of Cognitive Processing

Interpretation Process. Teachers' interpretation of student occlusions were basically interpretations about the content of student occlusions. Of interest to teachers were the validity of student responses or contributions and, in some cases, a diagnosis of their misconceptions. For the most part, teachers were correct in their judgments about the validity of student difficulties or insights with some notable exceptions. Teachers' judgments, regardless of their correctness, served as a filtering system by which they exercised control over the direction their lessons would take.

For the first few problems in which fractions with unlike denominators were being added, Zelda used only unit fractions, thus exemplifying the pattern $\frac{1}{a} + \frac{1}{b} = \frac{a + b}{ab}$. When one of the students tried to express this rule, Zelda was not receptive. She did not seem to realize the validity of the rule or that her choice of fractions had precipitated the offer. Her rejection of the offer was not in keeping with her usual strategy of selecting problems for the purpose of stimulating and rewarding pattern recognition, but this was also not a rule she had intended to elicit. In this instance, Zelda's lack of expectation and unfamiliarity with the pattern biased her interpretation of the student's insightful offer.

Student occlusions for which teachers were more aware of their interpretive thoughts had distinctive characteristics. Patterned errors and unsolicited contributions were perceived as being novel, particularly if they were being offered for the first time. So, too, were student occlusions which ran counter to the overall pattern of student response. Zelda reported her conscious thoughts about the rule being offered in the previous example just as Ralph did about Judy's inability to answer a simple review question. Less distinctive student occlusions were processed more routinely. Judging the correctness of a student response was typically done with little conscious thought unless the teacher had trouble determining the correctness of the response or the response proved interesting.

Repetition of student occlusions influenced teachers' judgments about their distinctiveness; it either enhanced or suppressed teachers' awareness of some occlusions. Repetition made offers of equivalent fractions seem more commonplace to Martha until she no longer mentioned them during the recall process. Skemp explains this process in the following way:

> ...when these [student occlusions] are encountered for the first time...consciousness is heightened. As a director system gradually becomes more adept in a new situation, its functioning becomes more routine and less conscious...If the initally novel components of a situation are enountered repeatedly, they gradually cease to be experienced as novel. Less conscious attention is then needed for the director system to function in what has now become a familiar situation (p. 15, 1979, bracketed words added).

Repetition could also heighten teacher awareness. During one of Martha's lessons, the same error pattern occurred in responses from several students over several problems. Martha did not recognize the pattern immediately; it was the repetition of the same error pattern which alerted her to the misconception. Similarly, it was not Max's initial error with the improper fraction that produced a critical moment for Martha; it was his persistent misinterpretation of the concept which caused her problems.

Another factor influencing teachers' perceptions of the novelty of student occlusions was interactive planning, which refers to thoughts about the teaching of an activity while it is in progress. Concentration on the presentation of content and the formulation of a questioning pattern for an activity were two forms of interactive planning which either suppressed or enhanced teachers' awareness of student occlusions. How interactive planning could prevent a teacher from consciously interpreting a student response is easily imagined, but an example is needed to illustrate how interactive planning could heighten a teacher's awareness of an occlusion.

In planning the first activity of her unit, Martha had been uncertain how students would grasp the concept of fractions. She was still not sure when the first question was asked. Students were to hold up a Cuisenaire rod which represented one-third of the rod she displayed. Out of numerous correct displays, Martha noticed an incorrect one. It suddenly occurred to her to exploit this error in order to clarify the concept, and the move proved most effective. Martha's incomplete plan caused her to notice and exploit an incorrect response that could easily have been overlooked. It was an error that definitely would have been ignored if the correct answer had been all that she was seeking.

Goal Setting Process. Goals which directed teachers' responses to student occlusions were momentary goals. Teachers were conscious of having chosen some goals while others were revealed through their actions. Conscious goal setting occurred with distinctive student occlusions: patterned errors, insightful suggestions, and student responses which deviated from the overall class performance or the expected difficulty of a question.

Goal setting was an automatic process by which teachers relied on general operating goals such as seeking correct responses to their questions. The process was also simplified by repetition of an occlusion, when a teacher continued to operate with a goal set for an earlier occlusion. This was true for Martha as students continued to volunteer equivalent responses. Using a goal set for one of the earlier instances, she responded to subsequent offers in much the same manner. Some goals deliberately incorporated subsequent occlusions. Martha chose to delay feedback until different answers had been given for the first addition problem involving fractions with unlike denominators. Of the nine answers given, she indicated having only thought about one or two.

Teachers reported goals were either impulsive or reasoned choices. Impulsive goals reflected their interpretations of distinctive student occlusions. Students' exhibiting difficulty were to be helped, and insightful ideas were to be used. Reasons were offered for goals which did not coincide with these interpretations. For example, teachers were reluctant to explore mathematics they considered inappropriate for their classes. When a student tried to justify the representation of a fraction by division, Martha simply acknowledged the suggestion and moved on to the next problem. Some students, she explained, had not yet started to work on learning how to divide and were unfamiliar with the operation.

Teachers' momentary goals were not complete plans of action. They merely indicated the directions teachers wanted or believed they ought to follow. Similarities in teachers' goal statements did not reflect the full nature of their intent, nor did they indicate the specific moves teachers would use. Simple goal statements, such as wanting to help a student, had different meanings. For Ralph it meant providing hints and clues until the student could come up with the correct response, while for Martha it meant assisting the student to acquire a proper understanding of a concept. These substantially different goals reflected teachers' more stable instructional goals (being able to work problems in the text versus developing and applying concepts) and their different conceptions of learning (paying attention versus constructing concepts through concrete experiences).

Teachers sometimes functioned with more than one goal in mind, although one eventually dominated. On several occasions, Martha responded to patterned errors by probing and asking related questions. She seemed to be trying both to find out what caused the students to make their errors and to help them understand. For some episodes which ended without any clear evidence that the students understood, Martha expressed pleasure over having figured out possible causes of their errors. Her desire to know what the students were thinking may have overshadowed her desire to help them and, possibly, her awareness of whether she had. In this manner, goals influenced her judgments.

Once teachers were able to satisfy their initial momentary goals, they reestablished the instructional flow. When they were unable to satisfy these goals, however, they abandoned or converted them to secondary goals. Secondary goals were reported in all critical moments involving student occlusions. This was a form of retreat as it was typically followed by an avoiding move. Skemp describes the changing or unsetting of a goal as a defense mechanism by which a person is released from the obligation of an existing goal. While this mechanism had immediate benefits for teachers, it also had emotional consequences as will be discussed in a later section.

Secondary goals were set by default, interference, or some combination of the two. To default meant that a teacher either had to anticipate being unable to effect a desired change or to feel unable to retrieve the necessary knowledge, mathematical or pedagogical. The switch in goals could occur before any action had been taken or after numerous attempts had been made to satisfy the initial goal. It was by default that Zelda finally gave up trying to elicit 2/4 as the name of a point. After seven attempts she could no longer think of anything to try. In this case the change was for more than a momentary goal. Zelda abandoned all questions dealing with non-unit, proper fractions for the rest of the lesson, thereby limiting the amount of practice students received on this task. When Martha finally broke off her attempts to help Max understand 6/5, she was exasperated at not being able to think of yet another way to approach improper fractions. Having tried and failed caused Martha to report feeling inadequate, but not Zelda. She admitted feeling limited, but not inadequate.

To describe how secondary goals came to be set by interference, it is necessary to make a distinction between desired and undesired goals. Skemp refers to them as goals and antigoals. The assumption is that teachers would actively seek to effect movement in the direction of desired goals and away from undesired or antigoals. Teachers learned to avoid certain situations from unpleasant teaching experiences which they had translated into principles or "rules of thumb." Martha had learned it was best to back off once she sensed students were becoming confused because excessive teacher explanations had only confused her when she was learning mathematics. Similarly, Ralph had found that when attempting to explain something for which he was unprepared, his lesson deteriorated. Fear of these antigoal situations interfered with teachers' willingness to try to satisfy their initial momentary goals. Fear prompted teachers to switch to secondary goals. Teachers did not begin their lessons thinking about these antigoals; their awareness of antigoals was activated by their recognition of the potential for such situations to develop.

Search Process. Once a momentary goal for coping with a student occlusion was set, an elective action was chosen for carrying out that goal. This was the function of the search process. Most elective actions were chosen and executed with little conscious thought on the part of the teachers. These were commonly used behaviors which could be handled with content-free moves such as a redirect or with no more knowledge than required by the mathematical task and routine of an activity. At most, teachers had to rely on knowledge needed for recently completed activities. These were relatively simple plans for action, many of which could be attributed to habit. Habits, according to Skemp, "are learnt, not innate, but with repetition they become so automatic that one might almost think of them as 'wired-in...'" (1979, p. 168). This is not to say that all such teaching behaviors were found without conscious thought, but that most were processed automatically, thereby enabling teachers to press on to the next question or problematic situation.

Just as distinctive student occlusions received more conscious attention from the teachers, so did distinctive elective actions. Teachers reported thoughts about moves which did not conform to their pedagogical or mathematical routines. This occurred before the routine of an activity had been established or when exploiting and avoiding moves were being used in a new situation. Unusual elective actions simply received more conscious effort than routine moves.

Teachers reported their unexpected successes as well as their problems. They were acutely aware of an unproductive search for an elective action. In some instances they were aware of this immediately following the student occlusion, while in others several attempts to satisfy a goal were made before they realized their options were being depleted if not exhausted. Ralph, for example, realized immediately that he did not know how to respond to an insightful suggestion or how to explain the decimal concept. On the other hand, Zelda talked of the difficulties she had in trying to think of ways to elicit the name of 2/4 as she continued to ask one question after another.

The nature of the search process was relatively unexplained by what teachers described. Their plans seemed to be formulated from scattered bits of information they were able to retrieve. Teachers reported sudden recognitions of what they might do with comments such as, "Oh, I know what I can do!" There was little description of how or even why they chose the actions, nor was there much evidence of their having considered alternatives. Whatever plan of action their search produced was executed. In fact, some moves were initiated before the teachers actually knew what they were doing. This was apparent when

Martha and Zelda became confused while trying to exploit students' insights.

The effectiveness of their actions was determined after the fact. Once a momentary goal had been set, teachers sought to find, execute, and test an elective action. This was similar to Miller, Galanter, and Pribram's TOTE (1960). The processes were performed in rapid-fire succession with the sequence repeated until the teachers satisfied their momentary goals or until they switched to a secondary goal by default or interference. The largest number of moves used to pursue a goal was seven, which occurred when Zelda failed to elicit 2/4 as the name of a midpoint. Teachers usually made two to four attempts when students did not understand or respond.

The search process just described resembles what Skemp terms "intuitive path finding." It is a form of planning in which connections between concepts are established. The process itself might not be conscious, but the products tend to "erupt into consciousness." A more conscious process is reflective planning which requires a more comprehensive structure in which the connections have already been established. If teachers were to engage in reflective planning, they would be able to examine these relationships. Very little of what teachers described, however, suggests reflective planning or even competent problem solving (de Groot, 1965). There was none of the conditional "trying out"of plans prior to taking action that de Groot found with experienced chess players; however, these teachers were not experts in mathematics. All three claimed that social studies was their best subject.

Teachers encountered difficulties with the search process, primarily because of their insufficient understanding of mathematics and a limited repetoire of pedagogical moves.

> ...it is hard to have a good idea if we have little knowledge of the subject, and impossible to have it if we have no knowledge. Good ideas are based on past experience and formally acquired knowledge (Polya, 1957, p. 9).

Teachers freely admitted when they had difficulty thinking of something to try and when they knew they lacked understanding. Inadequate knowledge was also apparent when teachers made mistakes or became confused. Another indication of their limited knowledge was the extent to which they were able to draw upon mathematical knowledge not already being used with the task and routine of the activity. Sometimes teachers were only able to repeat what they had already said; other times they incorporated very different mathematical approaches.

The variety of pedagogical moves, established in Chapter IV, showed Martha was the most versatile, and Ralph was the most consistent. When faced with similar situations in which neither knew how to offer a mathematical explanation, they relied on different pedagogical techniques. Ralph told how he would have answered the question, while Martha posed another problem to give the student another opportunity to figure it out.

Another example illustrates the differences of teachers' goals and knowledge as well as their pedagogical preferences. In almost identical circumstances, students in Martha and Zelda's classes asked how two fractions with unlike denominators could be added. The questions were asked during activities in which addition of like-denominator fractions had been introduced with Cuisenaire rods, and students were solving problems with great success. Both teachers chose to exploit the opportunities, but in somewhat different ways. Martha immediately switched to a simpler problem than the student had been given and used the rods to find the solution. Zelda had students put away the rods before attempting the fractions. It was only after she made several mistakes that she substituted simpler fractions and solved the same problem Martha had posed.

Evaluation Process. The function of the evaluation process was to determine the effectiveness of the teacher's elective actions by monitoring change with respect to the goal in effect. Teachers made judgments about their own actions, about students' reactions to their elective actions, and about causes of student difficulties. These judgments were used to determine what teachers would do next. When it was determined that a goal had not been reached, teachers' mental processing was recycled to search for a different move or to change goals.

The evaluation process was relatively automatic for many incidents involving student occlusions which had already been processed with little awareness or which had ended satisfactorily. Teachers' comments suggested a more conscious awareness of the evaluation process and indicated their surprise or disappointment. They reported their own errors and confusion when they were aware of it. They also reported their judgments about students' reactions in those incidents when they had already begun a more conscious monitoring of a student's difficulties or when the student continued to have trouble understanding. An example of how a student's reaction to an elective action could capture the teacher's attention, even when all that preceding it was handled routinely, comes from one of Ralph's critical moments. When Judy did not respond after Ralph had repeated a question, he was stunned because the question had been a simple review task.

Teacher evaluation of student reactions to their elective actions was not unlike their interpretation of student occlusions. Mathematical knowledge was a major factor influencing the teacher's judgment about the content of the student's response. It was also important in determining how long teachers were able to search for other approaches as was their pedagogical knowledge. Differences were evident in the interpretation and evaluation processes. Evaluations were based on more information than the interpretation of initial student occlusions. Knowledge of the subsequent interchange between students(s) and teacher was also available, although teachers did not always request or receive overt student reactions to their elective actions. In some instances teachers told of noticing nonverbal cues or of simply hoping what they had done would be helpful. To avoid this uncertainty about the student's understanding when other student input had intervened, Martha had adopted a habit of checking back with the student whose difficulty she was trying to resolve. Thus, the history of the event was another factor influencing the evaluation process.

Teachers' expectations also played a role in determining how long teachers would persist in their efforts to help a student. Past experiences and lesson plans contributed to Zelda's tenacity (seven attempts and failures) in trying for the elusive 2/4. She believed some students were capable of answering the question because of a prior learning experience, and she had planned to ask this and similar questions. Expectations about individual student ability were also found to have some influence. When Martha was trying to help Max understand improper fractions, for example, she kept thinking he was "just about to get it." This expectation undoubtedly contributed to her willingness to keep trying just as her fear of confusing the less capable students contributed to her reluctance to continue. Research on teacher expectations has also shown that teachers are more likely to persist in their efforts with students they regard as being more capable (Brophy and Good, 1974).

Although they were not formally asked to do so, teachers spontaneously offered their opinions of what caused their students' difficulties, attributing most student difficulties to unstable causes internal to the students: students did not understand, they were not paying attention, or they were forgetting--experiencing a mental block. External conditions mentioned by teachers were the inhibiting influence of a large group and the difficulty of the content. All these causes except the temporary state of not understanding could be found in attributional literature (Cooper and Burger, 1980). Teachers did not cite themselves as causes of student difficulties, at least not without prodding. This was consistent with Cooper and Burger's finding that participants report playing a larger role in success than failure. To designate teachers as causes required investigator influence.

Attributing cause was not exclusively the function of the evaluation process, but of the interpretation process as well. When Martha first diagnosed Max's error, she attributed his difficulty to a lack of understanding. At this stage in the processing of student occlusions, her judgment undoubtedly had some impact on her choice of goal and actions.

Teachers in this study attributed student difficulties to causes which conformed with their own conceptions of how students learn and their own instructional goals. Ralph, who believed students would learn as long as they paid attention and followed what he was saying and demonstrating in class, attributed student difficulties to a failure to pay attention. When he did mention student understanding, his concern was that the student could not do the problem. Of the three teachers in this study, Ralph was the most concerned about student attention. Zelda attributed student difficulties to "forgetting," "not remembering," and "mental blocks." This reflected what she believed about learning--once presented it was retained--and her belief that students had already been given opportunities to learn. Martha, who wanted students to develop their own understanding through concrete experiences, attributed students' difficulties to a lack of understanding. To her this meant that a correct or complete concept had not yet been constructed or means of its application understood. Martha's assessments were made with the interpretation of the student occlusions and did not change when students continued to exhibit difficulty.

Attributions cited in this study also differed from those reported by Cooper and Burger (1980). In trying to link high and low expectations with attributions, they found that

> ...unexpected events led to greater use of internal and unstable causes, whereas expected events led to greater use of internal stable causes. Bright student failure was more often attributed to immediate effort while slow student failure was perceived more often as ability causes (1980, p. 108).

Since teachers in this study offered internal and unstable causes for both high and low expectation students, this expectation effect was not supported. As previously mentioned, some relationships were indicated between teachers' expectations and their willingness to persist in helping students.

Summary. In summary, then, teachers were able to process student occlusions by interpreting, goal setting, searching for and executing elective actions, and evaluating the effects. When teaching problems were encountered, the processing was recycled for another search or to replace the present goal. The same four processes were used in teachers' cognitive processing of critical moments for pervasive student behavior patterns associated with activities. A model and description will indicate how this deviated from teachers' processing of student occlusions.

## Teachers' Cognitive Processing of Pervasive Patterns of Student Performance

### Model

Teachers' cognitive processing of critical moments involving pervasive patterns of student performance was similar in many ways to what has just been described for student occlusions, but there were significant differences. A student occlusion caused momentary interruption in the instructional flow of an activity. Teachers had only to respond to the individual(s) and content of the occlusion by setting a momentary goal and taking action. If they had problems satisfying their goals, they could disengage by switching to secondary goals. The choice was irreversible on only three occasions when the momentary goal was to change activities and explore a new concept or task.

Pervasive student behavior patterns associated with critical moments challenged the appropriateness of ongoing activities. They represented threats to teachers' goals, plans, and expectations for ongoing activities. Even when change was indicated, activities were not easily altered or discarded, particularly without something to suggest direction for change. When teachers became conscious of a pervasive pattern of student performance, they had to evaluate it in relation to the goals of the activity. If it were determined that change was needed, the means to do so still had to be found. The sequencing of cognitive processes for pervasive patterns of student performance, therefore, required a somewhat different processing model than the one already proposed for student occlusions. A model of teachers' cognitive processing of pervasive patterns of student performance is shown in Figure 6.2.

According to this model, no teaching problems were reported when teachers' responses to the first two questions in the model were affirmative:

1. Is the pattern of student performance routine (not novel)?
2. Does it satisfy the activity goals?
3. Is a change indicated?
4. Is an alternative activity or routine available?

Recognition that students were not behaving as expected forced an evaluation of the pattern. Thus, when teachers acknowledged the students' performance was not acceptable, a search was automatically initiated, with change dependent on finding an alternative. This processing sequence was repeated several times during an activity.

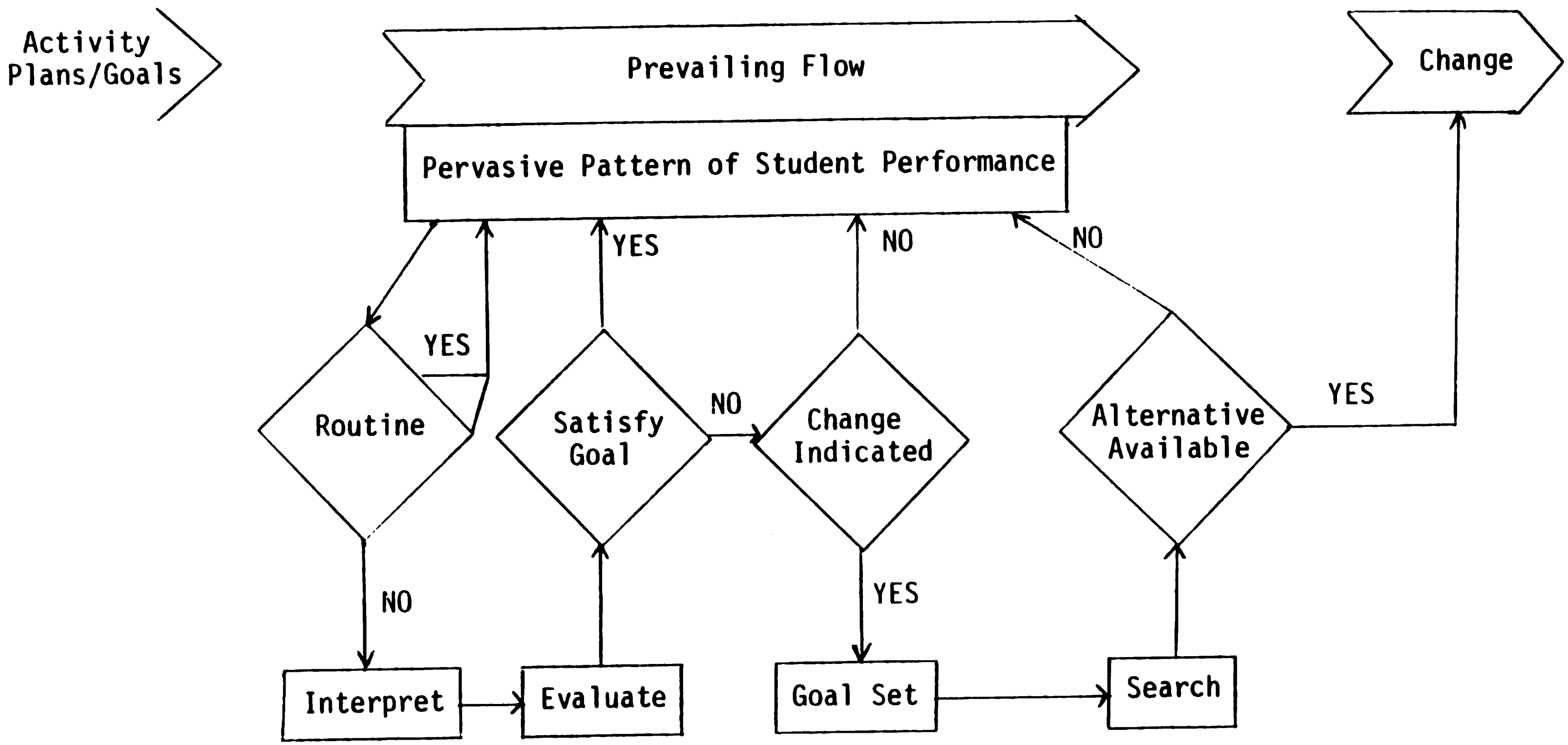


Figure 6.2. Model of a teacher's cognitive processing of pervasive patterns of student performance.

The proposed model is actually a modified version of the Peterson and Clark (1978) model shown in Figure 2.1 (p. 23). The major difference is the reversal of two processes: goal setting and searching. Peterson and Clark's model assumed that teachers would search for an alternative action before deciding to make a change. Instead, teachers concluded a change was needed before searching for the means to bring about the change. In all probability, the two processes are not separate and sequential, but interwoven. Nevertheless, the distinction seems to be important. Teachers implemented whatever actions they thought of, be that responding to student occlusions or modifying activities.

## Nature and Function of Cognitive Processes

When teachers became aware of these unanticipated and undesired but pervasive patterns of student performance, they exhibited one of two coping mechanisms. They suppressed conscious awareness while continuing the activity or they took action to alter the condition. They did not institute change immediately as recognition of these behavior patterns seemed to fade in and out of their consciousness until they were ready to act. Teachers allowed an activity to come to a natural conclusion, modified the routine of an activity, or terminated it prematurely. Each activity in which critical moments arose over pervasive student behavior was followed by another activity. Although these activities had been planned, at least partially, there were reports of further planning or alterations being made to implement them.

Change in the routine of an activity could be obvious or subtle. By specifying which fractions to compare rather than having students come to the board to spin for them, Martha was able to quicken the pace and avoid giving more individual assists. It was a minor but obvious change. During an activity in which his students were not being very responsive, Ralph increased the amount of explanation. It was a subtle change made apparent only by his own report.

Teachers found it difficult to make changes in their activities. This can be explained by their planning practices and by the scarcity of available options. Teachers planned their lessons anticipating student success. They did not consider or plan for excessive difficulty or remarkable success. No alternatives were available unless teachers planned for other activities. Spontaneous plans were not easily formulated with the teachers' limited knowledge of how to teach the content.

## Summary of the Factors Which Influenced Teachers' Cognitive Processes

The more stable factors found to influence teachers' cognitive processing of critical moments included teachers' mathematical knowledge, pedagogical options, instructional goals, conceptions of mathematics and learning, beliefs about teaching, antigoals and expectations. Somewhat less stable influences were teachers perceived ability, attribution for student performance, and the history of the event itself. The complexity of the interaction between these factors and teachers' mental processing is difficult to untangle. As Clark and Peterson (1976) have indicated, teachers are not able to explain just why they do what they do. Nor is circumstantial evidence enough to establish the extent of influence by any factor. Nevertheless, some relative measure of the importance of the factors can be recognized by considering which processes these factors were noted to impact. Table 6.1 helps to emphasize which factors influence more processes and the complexity of some processes.

Table 6.1. Factors Influencing Teachers' Cognitive Processing of Student Occlusions

| Interpreting | Goal Setting | Searching | Evaluating |
|---|---|---|---|
| Mathematical Knowledge | Mathematical and Pedagogical Knowledge Interpretation | Mathematical and Pedagogical Knowledge | Mathematical Knowledge |
| Repetition (history of prior events) | Instructional Goals and Conceptions of Learning | | Instructional Goals and Conceptions of Learning |
| | Perceived Ability | | |
| | Principles (Antigoals) | | Expectations |
| | | | Attributions |
| | History of Event | | History of Event |

The importance of teachers' mathematical knowledge is more apparent as it has some relationship with each of the four processes. Teachers' repertoire of pedagogical options and their instructional goals are also important factors. They influence two processes: goal setting and evaluation.

Goal setting and evaluating are influenced by more factors than are the interpreting and searching processes. Goal setting is particularly complex during critical moments when teachers' goals are challenged or changed to secondary goals. Similarly, the evaluation process is influenced by more factors when teachers find it necessary to reprocess an event because their goals are not being satisfied.

## Teachers' Emotional Responses to Unexpected Student Performance

Critical moments were characterized by their negative affect as well as the cognitive difficulties teacher were having. When teachers did not verbalize their emotions during recall, their remarks were often indicative of the discomfort they felt. Skemp regards emotions as signals of the progress an individual is making in relation to goal and antigoal states. In an effort to begin building a theoretical framework in which emotions are incorporated, Skemp designated relationships between emotions and movement with respect to the two types of goals. In Table 6.1 emotions of pleasure and relief are linked with movement towards a desired goal state and away from an antigoal state, respectively. A general state of displeasure is indicated when movement is away from a desired goal and when there is fear of movement towards an antigoal.

Teachers in this study reacted as Skemp's model predicts. However, the model in which only change and emotion are related to goal states does not account for differences in the intensity and nature of teachers' emotional reactions. Martha's frustration was qualitatively different from Zelda's. Zelda was only a little bothered while Martha was very frustrated. Yet, both emotional reactions occurred when they had failed to satisfy a momentary goal after multiple attempts. Skemp anticipated this variation:

> But even more important to an organism than whether it is, at a given time, moving towards or away from goal or antigoal states, is whether it is able by its own efforts to bring about these changes (1979, p. 12.)

Table 6.2. Skemp's Model of Emotions as
as Signals of Change with Regard to Goal States
and as a Consequence of
Perceived Ability to Effect Change.

| Emotion | Perceived Ability | Goal States | Change | Emotion |
|---|---|---|---|---|
| Confidence | Capable | | Towards | Pleasure |
| | | Desired Goal | | |
| Frustration | Incapable | | Away From | Displeasure |
| Anxiety | Incapable | | Towards | Fear |
| | | Anti-goal | | |
| Security | Capable | | Away From | Relief |

By incorporating the teacher's perceived ability into his model, Skemp was able to describe the emotions more definitively. These relationships to goal states and perceived ability are also shown in Table 6.2.

Different emotional reactions associated with movement away from a desired goal can be explained by differences in the teachers' perceived abilities. Martha reported frustration came with the admission that she felt incapable, while Zelda, who was only slightly bothered, made a point of saying that she did not feel incapable, only limited in her knowledge of mathematics. When antigoals interferred with teachers' willingness to pursue their goals, emotions were more difficult to identify. Perhaps teachers failed to label their emotions because of mixed reactions: relief at having been able to prevent or terminate an antigoal situation and displeasure at not being able to do as they set to do so. When Ralph broke off efforts to help a student because he feared the loss of attention from his class, he appeared to be comfortable with his choice. However, when students were not responding as he felt they should, he appeared anxious. It was more of a helpless reaction, an indication of his lack of confidence or ability.

Zelda was the teacher least bothered by her critical moments. According to Skemp's theory, this was because she was able to maintain her perception of being capable. This suggests a further relationship between teachers' attributions and their perceived abilities. When students were having trouble answering a question or performing a task or when Zelda became confused while exploiting a student suggestion, she attributed it to forgetting something which neither she nor the students could control. With forgetting as her causal explanation, Zelda's capability was not threatened, nor was she pressed to seek other explanations for the student difficulties.

Attention, the attribution most used by Ralph, is generally viewed as being under the control of the student. Nevertheless, when Ralph attempted to gain student attention and was not successful, he seemed to feel helpless to do otherwise. From his comments it was clear that Ralph accepted responsibility for restoring attention when he could not teach in a manner which maintained it. Thus, failure to control what he believed to be a controllable condition produced a negative reaction as was the case with Martha. She attributed student difficulties to a lack of understanding and accepted the responsibility for correcting their misconceptions and confusions. Like Ralph, this left her little room to escape blame or to deny her incapability. Quite predictably, Martha was frustrated when she was unable to help students and angry when she made the mistakes. Attributions teachers offered were also indications of their beliefs about the controllability of the causes they cited. Those causes over which teachers had little control resulted in less negative emotions, while those for which they judged themselves to be responsible and able to control produced the most negative emotions.

The relationships between attributions and emotions for both success and failure have also been investigated by Weiner (1980). He found a link between the attributions about ability and the emotional reactions of different outcomes associated with confidence or incompetence. The same relationship was indicated for the general ascription of internal locus of control and confidence. Martha and Zelda, however, were most explicit about the anger they felt when they made mistakes exploiting student insights; and Weiner linked anger with attributions to others and external conditions.

While the relationships among emotions, attributions, and perceived ability, efficacy, or control is evident, the direction of these relationships and their effect on teacher behavior is not. What does seem clear is the relationship of emotions to the outcomes with regard to goals and antigoals, as Skemp proposed. It is further evident that some factor(s) influence the degree of comfort or discomfort a teacher feels in relation to any of the four goal states. Each of the factors mentioned can be shown to have some possible influence, but they cannot yet be ordered if, indeed, ordering is even appropriate.

Weiner (1980) has been examining the role of affect in attributional approaches to human motivation and suggests that (a) emotions are responses to particular attributions; (b) emotions, rather than causal ascriptions, are motivators of actions; and (c) effects can function as cues guiding self perception (p. 4). The verification of this model, however, has yet to be completed. It should be remembered that attributions in this study were also shown to reflect the teacher's instructional goals and conceptions of learning, so the circle of influences may be ever widening. For now, the relationships between emotions and the cognitive aspects of teaching are still being established.

For critical moments over pervasive patterns of student performance, one of the defense mechanisms which enabled teachers to continue the activity was the suppression of their conscious thoughts. Skemp calls this "withdrawal of consciousness" which he offers as a defense mechanism for escaping the negative emotions. While feelings such as frustration and anxiety are still present, the teachers are no longer fully aware of them. Skemp (1979) suggests that "a residual awareness may persist and be experienced as 'tension.'" That was evident in Martha's remarks about several of her experiences; she mentioned feeling tense. Skemp admits he can offer no explanation for what he believes to be an involuntary process, but he does describe the consequences:

> ...no further adaptation takes place in a director system from which consciousness is withheld. So, once established, such a situation may be self-perpetuating, since a return of consciousness to this area is inseparable from a return of the experienced frustration and anxiety (1979, p. 17).

It was the teachers' recurring awareness of the undesired performance patterns of their classes that produced the critical moments.

## Summary

Through the investigation of critical moments, much was learned about teachers' mental processing of unexpected student performance. In this chapter the focus has been on the nature and function of the cognitive processes including factors which influenced them, on models of the cognitive processing, and on the emotional component. Because of differences in the ways teachers processed the two types of critical moments, separate models and descriptions were required for their cognitive processing of student occlusions and pervasive patterns of student performance. The corresponding models were based on four cognitive processes: (a) interpreting student performance, (b) goal setting, (c) searching for and executing elective actions, and (d) evaluating the effectiveness of their actions.

Teachers' cognitive processing of student occlusions proceeded in sequence. Teachers set momentary goals to cope with student difficulties or insights and followed with a fairly rapid find-execute-test routine. Elective actions were found by habit or intuitive planning with little evidence of a more rational approach. Teachers were more aware of their thoughts and feelings for distinctive student occlusions and elective actions. Distinctiveness was determined by deviation from the norm, either for content or student performance, and by repetition of similar occlusions.

Teachers experienced critical moments over student occlusions when they were unable to satisfy their momentary goals. Failure to do so was due to default, being unable to come up with an effective elective action, or to interference of an antigoal. Antigoals were associated with perceived or anticipated situations which teachers preferred to avoid. To disengage themselves from an existing goal, teachers changed their initial, momentary goals to secondary goals which could be more easily satisfied by avoiding, telling, or even modifying an action.

With pervasive patterns of student performance, the processing order was somewhat different as was the duration of the cognitive experience. Critical moments began with teachers' recognition of an undesirable pattern of student performance. These patterns were observed over time and across students. An evaluation of the pattern was made in relation to activity goals. Once a need to change was recognized, the teacher's problem was what to do. Overt changes came slowly as teachers' awareness of these undesired situations faded in and out of consciousness. Eventually, some change was made either in the routine of the activity or with the ending of it.

Relationships were noted between teachers' cognitive processes and the factors influencing them: mathematical and pedagogical knowledge, conceptions of mathematics learning, expectations, principles and instructional goals. Relationships were also indicated between teachers' emotional reactions, their progress with respect to their goals, their perceived abilities to produce change, and their attributions about student difficulties.

Emotions teachers reported with critical moments reflected the progress that was being made with respect to their goals and antigoals. The variation in emotions further reflected teachers' perceptions of their ability to bring about change--an example of Skemp's theoretical model. Also indicative of teachers' emotional reactions to these incidents were their attributions for student difficulties.

Knowledge about how teachers' cognitive processing of student occlusions and pervasive student performance patterns should suggest ways in which teachers might be helped to improve their teaching of mathematics. Implications from these findings will be discussed in the next chapter.

# CHAPTER VII

# DISCUSSION

This study of critical moments has investigated teachers' reactions to unexpected student performance which occurs during instruction of the total class. The goal has been to understand teachers when they are faced with incidents of unexpected student performance and critical moments. Much has been learned about the three teachers in this study and what made teaching difficult. The purpose of this chapter is to discuss some of the findings in relation to existing research, highlight the significant findings, and to critique the method of research. As there are some differences in the points of interest, this chapter is divided into three sections: teacher thought and teaching, methodology for studying critical moments, and teaching mathematics.

## Teacher Thought and Teaching

### Research Findings

Much of what was learned in investigating teachers' critical moments lends support for the portrait of the teacher given in Chapter II which reviewed research on teacher decision making and teacher thought. Rather than repeat the same results, attention will be given to a few of the more significant findings:

1. Different types of critical moments and teachers' cognitive processing

2. Distinctive and observable characteristics of student occlusions associated with critical moments interrelatedness goals of teachers' activities, actions, goals, conceptions, and attributions

### 3. Variation in teaching and student occlusions

Critical moments were initially conceived as arising over incidents of unpredictable student performance which were designated as student occlusions. Student occlusions were described as momentary but observable interruptions in the instructional flow of an activity identified either by students' inability to provide correct responses to teachers' questions or by students' unsolicited contributions. While such behavior might observably occlude the instructional flow of a student, it did not always intrude on the teachers' mental life. For this reason, stimulated recall was used as the means of obtaining evidence of which events teachers consciously processed. Student occlusions for which teachers indicated having cognitive difficulties and emotional discomfort were designated as critical moments. Critical moments, therefore, represented momentary crises which were experienced by the teacher as problematic or disruptive.

In analyzing the data, another form of unexpected student performance was found to produce similar reactions among the teachers. In addition to reporting their subjective problems with isolated student occlusions, teachers also reported their difficulties and discomforts over more pervasive patterns of student performance which were observed across students and over time. These pervasive patterns of student performance were designated and reported as critical moments. The two experiences, however, were so significantly different that two models of teachers' cognitive processing of unpredictable student performance were proposed: one for isolated student occlusions and another for pervasive patterns of student performance. Although

the cognitive processes were the same, the sequences in which they were arranged differed. Different, too, were the immediacy with which teachers had to respond to the two types of critical moments, their goals which were in jeopardy, and the types of action required to resolve the teachers' problems.

Teachers reported having only fleeting thoughts and discomfort over undesirable patterns of student performance, but the recurrence of their thoughts over time belies the meaning of a momentary experience. In the future, then, it would be helpful to distinquish between the two types of problems teachers experience with different labels. The term critical moment should be reserved to indicate teachers' momentary crises over student occlusions. Another term to refer to teachers' more enduring problems regarding the pattern of student performance associated with an activity might be critical discrepancy. Morine-Dershimer (1979) labeled lessons in which more than 50% of teachers' decision points were both unexpected and bothersome with this term. Her criterion does not apply to all the teachers' critical moments associated with the pervasive patterns of student performances as will be discussed later.

Two models of teachers' cognitive processing were given. One was for teachers' processing of pervasive patterns of student performance associated with activities and another for their processing of student occlusions. Neither model had been used before to investigate or classify teacher decisions, although the one for pervasive patterns of student performance was quite similar to that used by Peterson and Clark (1978). The model for student occlusions was sufficiently different from the model for pervasive patterns of student performances. Similarly, teachers' experiences of the two types of critical moments were different enough to have implications for future research and analyses of teachers' decisions.

Student performance, associated with critical moments, was quite distinct from student performance associated with student occlusions. As this point will be repeated in several sections throughout the chapter, these differences will not be reviewed here other than to say that the observable characteristics for student occlusions were much more apparent than were those for pervasive patterns of student performance.

A good deal of variation was evident in teachers' activities, actions, intentions, and attributions. There was also noticeable variation in student performance across activities and teachers. As much of the variation was already detailed in Chapter IV, only a brief description of the teachers will be given.

Ralph represented a more typical mathematics teacher, and Martha one teaching more in the style recommended by NCTM. Ralph's goal was for students to be able to do the problems, and Martha's was for them to be able to experience or understand the concepts. The difference in their goals was essentially the difference between what Skemp (1978) has termed instrumental and relational understanding. There is also a difference in the mode of learning they encouraged (Skemp, 1981). Differences were apparent in the causes to which teachers attributed student difficulties. Ralph cited lack of attention for students' difficulties, while Martha talked more about their lack of understanding.

Ralph was also teaching more in accordance with the direct instruction model for which teachers are encouraged to seek low error

rates. The direct instruction model has most frequently been applied to the teaching of facts and skills rather than discovery learning or problem solving. The benefits of direct instruction have been demonstrated by a number of researchers including Stallings and Kaskowitz (1974). In reviewing research on direct instruction, Peterson (1979) and Rosenshine (1979) were able to report that a direct or traditional teaching approach was somewhat more effective in increasing student achievement than an open approach. Nevertheless, Peterson does not support direct instruction as the sole model of instruction.

> To me, the picture of direct instruction seems not only grim, but unidimensional as well. It assumes that the only important educational objective is to increase measurable achievement and that all students learn in the same way and thus should be taught in the same way.... Educators should provide opportunities for students to be exposed to both [direct and open instruction] (1979, pp. 66-67).

## Implications and Directions for Research

Implications of the findings from this study and directions for future research will be discussed in relation to the (a) role of the activity, (b) role of student performance, and (c) teachers' responses to critical moments.

Role of activity. The activity--the task, questioning routine and instructional style of the teacher--plays an important role in determining student performance and teachers' mental problem space. The variance of student occlusions could well be attributed to task differences in the activities. Most incidents of unexpected student occlusions came during activities in which new content or tasks were being presented while review and slightly modified tasks resulted in fewer student occlusions. Thus, the nature of the task and student's familiarity with it were two important characteristics of the activity.

A teacher's questioning routine and instructional style for an activity were related to the nature of the tasks and, therefore, also influential in determining student performance. Ralph's emphasis on skills and questions seeking numerical responses to subprocedures may have had as much to do with the lower frequencies and densities (rates) of student occlusions as the tasks themselves. In contrast, the concrete tasks for which students were asked to model, identify, and apply concepts and the symbolic tasks for which students were to recognize patterns from examples produced more occlusions, particularly more insightful suggestions. Ralph's avoidance of insightful student suggestions may have discouraged others from offering their ideas while Martha's support and exploits of unsolicited student contributions probably encouraged others to try. Similarly, by allowing students to call out their responses to some questions, Zelda increased the number of student guesses.

The importance of the task in influencing student behavior is central in the ecological perspective (Doyle, 1979). This view has already begun to influence research on teaching. After extensive research on classroom instruction, Soar and Soar (1979) acknowledged the importance of the learning task and what might be termed the instructional goal of an activity as influencing student behavior and thought. Their acknowledgement of the importance of task parallels Kounin's shift from the more general characteristics such as "withitness" (Kounin, 1970) to the more specific aspects of the task signals (Kounin & Doyle, 1975; Kounin & Gump, 1974). Doyle (1979) acknowledges the teachers' role in selecting tasks, saying they "are not simply directors of activities or contingency managers, but rather organizers of task systems." So far, the researchers have characterized classroom tasks into fairly general categories. This study suggests a need to classify activities on several dimensions because of the interdependence of the task and the teachers' questioning routine and instructional style.

Joyce, Morine, and McNair (1977) characterized the activity as defining the teachers' problem space with which they processed unexpected student performance. This notion will be discussed further in relation to the findings about teaching mathematics. At this point, it need only be said that teachers were able to process more routinely or more successfully the student occlusions for which the mathematical content was within the task, numerical domain, and questioning routine of the activity. Teachers were more likely to experience critical moments when the mathematics they needed was not within this space. They also had more difficulties when needed pedagogical techniques were not within their repetoire . When teachers did not know mathematically or pedagogically how to respond to students, they often cited their fears as to what might happen if they tried to respond.

Role of student performance. The nature of student performance had a definite effect on the manner in which teachers processed student behavior and the actions they took. Different models of teachers' cognitive processing were required for different types of unpredictable student performances. The type of student performance also had implications for teachers' choice of elective action. When frequencies of teachers' elective actions were examined as conditional probabilities according to the nature of occlusions--difficulties, insights, and their subcategories--patterns of teacher preference were quite apparent. If the data were reanalyzed so that routinely processed student occlusions were separated from the more consciously processed, the patterns might become even clearer.

The distinctiveness of the episodes involving student occlusions for critical moments suggests that a reasonably accurate list of potential critical moments can be identified from a more extensive list of student occlusions. The routinely processed events are identifiably different than the more consciously processed events. The critical moments described in this study offer the beginnings of a taxonomy for student occlusions with the potential for producing critical moments. The completeness of this taxonomy and its ability to be generalized across teachers and situations, however, has yet to be determined. Characteristics of critical moments which arose over more pervasive patterns of student performance associated with activities rather than student occlusions, are not as easily recognized from observable cues. Research is needed both to confirm and refine the criteria for identifying potential critical moments and critical discrepancies. Some uses for the taxonomy will be indicated in the sections that follow.

<u>Teachers' responses to critical moments</u>. Teachers varied in their responses to critical moments. Ralph and Martha's behavior represents qualitatively different responses to their critical moments. Martha found ways to use them to her advantage; they became <u>benchmarks</u> by which to monitor the effectiveness of her activities and to tailor her plans to the needs of the students as she perceived them. Student difficulties were often explored while she attempted to resolve them, and student insights were often exploited as she attempted to capitalize on appropriate ideas. This is not to suggest that a critical moment was not discomforting for Martha, but that she responded to them differently than Ralph by using them more constructively.

Ralph did not want students to exhibit any difficulties or contribute any insightful ideas during his presentations. Instead, he wanted this phase of the lesson to progress according to plan with little or no disruption from student occlusions; he preferred to deal with individual student difficulties during the work periods. Ralph was simply not receptive to and did not value the unpredictable student occlusions which occurred during his lessons. Nor did his teaching benefit from the existence of critical moments. He viewed the critical moments as threats and problems to be overcome or avoided.

The difference between these teachers was not so much a matter of one's having a much better understanding of mathematics than the other, as both Marth and Ralph exhibited a limited understanding of the mathematics they were teaching. Differences were also apparent in their instructional goals and beliefs about the teaching of mathematics and in the manner in which they taught.

The third teacher in this study, Zelda reacted to the critical moments more like Martha than Ralph, but she did not use the critical moments as benchmarks for planning subsequent lessons. She was so tolerant and unperturbed by most unexpected student performance that she reported few critical moments. In two of her three critical moments, Zelda terminated a question or activity, which suggests her acceptance or constructive use of critical moments was less than Martha's. Zelda was also of two minds about her instructional objectives focusing on both relational and instrumental learning. She attributed learning difficulties to forgetting or mental blocks, which was similar to but not the same as Ralph.

With regard to the differences in their activities, instructional approaches and goals, Ralph and Martha support what Bussis, Chittenden, and Amarel (1976) reported. They found that as teachers' curricular constructs progressed from a total focus on grade-level facts and skills to an orientation toward more comprehensive priorities, their actions demonstrated an increasing willingness to experiment or change. Ralph and Martha's different responses to critical moments may have been exaggerated by the particular units they were teaching. Martha lamented being unable to teach other mathematics topics in the same manner as fractions. Ralph offered verbal support for problem solving experiences, but admitted he was unable to and did not have the time to plan such activities. Thus, the differences in these two teachers' responses to the critical moments may have been a function of the activities--task and instructional approach.

Zahorik (1975) and Peterson and Clark (1978) suggest that teachers' planning can be counterproductive to their sensitivity to student ideas if it is too extensive. During this study, five factors were observed to reduce teacher responsiveness to student ideas and needs within the ongoing lesson: availability of a detailed plan, provision for time in which students were to work on their assignments, difficulty in instantly formulating a new plan, instructional goals which stressed doing and not understanding mathematics, and teachers' limited knowledge and understanding of mathematics they were teaching. Planning, which was one of the factors, would also reflect differences in teachers' goals.

A teacher like Ralph becomes more stressed as the frequency or intensity of critical moments increases, whereas a teacher like Martha is able to constructively use a number of critical moments without their becoming too stressful. For Martha, stress became greater when critical moments were too infrequent or when the amount became quite excessive. On the other hand, Ralph was upset by most of his critical moments, particularly the one in which he was displeased with a pervasive pattern indicating less eagerness on the part of the students to participate. Teachers of the same type as Martha and Ralph might be expected to respond in a similar manner unless differences could be achieved by having them teach activities of a different nature. The question of stability of a teacher's instructional style and goals, conceptions of how students learn and even their attributions have yet to be tested across different types of tasks and content.

If teachers are to teach in a problem solving mode which fosters unpredictable student performance, it may make a difference as to how they respond to critical moments. A teacher with the same goals and beliefs that Ralph held during the unit investigated might be expected to respond in a similar manner. An increase in the number of critical moments would expose a teacher to more difficult and stressful situations, making it unlikely that the teacher would willingly continue to function in such an environment. If the teacher could not adjust to the increase in the number or intensity of critical moments, then s/he would have to modify the type of instruction to reduce the stress. It is easier to imagine a teacher like Martha being able to implement the NCTM recommendations, than it is one like Ralph.

These remarks are not meant to imply that teachers like Ralph can not learn to use critical moments in a more constructive manner. Before any generalizations about flexibility and constructive use of critical moments can be determined, however, the same and different teachers must be examined across situations. One study to show how the same teachers from this study responded while teaching a problem solving unit is already underway. If it should turn out that the teachers do, in fact, respond in accordance with the materials being taught, there can be some optimism about the possibility of implementing the NTCM recommendations with more traditional teachers. If, instead, the teacher who is resistant to unpredictable student performance is unable to adjust for different types of instructional materials, the possibility is slim and the future for improving mathematical teaching is bleak.

## Methodology

For this study, a great deal of time was spent observing teachers both in their classrooms and repeatedly through the use of videotapes. Time was also spent in gathering teachers' thoughts through the process of stimulated recall and then using these data to identify and describe the behavior of students and teachers and teachers' critical moments. From these experiences, the investigator has formed some impressions and drawn some conclusions about the methodology for studying critical moments and some implications of these conclusions.

This section is a critical analysis of the methodology for studying teachers' critical moments. Attention will be given to (a) identifying criteria for critical moments, (b) classification of student and teacher behavior, (c) method and effect of stimulated recall, and (d) investigator bias. This will be followed by some remarks as to the implications and directions for future research on critical moments.

### Criteria for Identifying Critical Moments

In order for a student occlusion to be designated a critical moment, it had to be associated with teachers' reports of cognitive difficulties and felt discomfort. These criteria made it possible to identify and describe critical moments, although the choice was not always clear. The task was made simpler by the indication of secondary goals being set to replace teachers' initial goals or impulses, even when they took no action to implement them. Separate criteria for recognizing critical discrepancies had not been established prior to the analysis. So, when teachers indicated similar reactions to more pervasive patterns of student performance instead of student occlusions, the occasions were also designated as critical moments.

Excluded by these criteria were incidents in which teachers faced momentary crises over what to do, but were pleased when their actions led to satisfactory results. Examples of successfully resolved student occlusions proved useful in characterizing and modeling teachers' cognitive processes. The question is whether or not the concept of a critical moment should exclude these experiences. To extend the notion of critical moment to include both satisfactorily and unsatisfactorily resolved student occlusions would require a revised set of identifying criteria. Although the choice should depend on the purpose of an investigation, in the opinion of this investigator, the modification is necessary if teachers' subjective classroom experiences are to be captured adequately.

## Classification of Student and Teacher Behavior

All student occlusions were identified and coded,as were teachers' elective actions from the videotapes and transcripts of each lesson. Stimulated recall data about these occlusions were used to provide evidence of teachers' critical moments. In Chapter IV the data on student occlusions and teacher elective actions were not reported with reliability measures; there was no systematic cross validation of the classification of student and teacher behaviors. As some subjective judgment was required in identifying and coding these behaviors, some comments about the process and the appropriateness of the classification scheme is in order.

Student occlusions were easily recognized by inaccurate responses to teacher questions and unsolicited student comments. It was more difficult to identify students by name from the videotape and transcripts. When available, this information was used in describing critical moments because teachers' opinions of the individual students were considered. The distinction between student difficulty and insight was not difficult to make since the criterion was mathematical validity which did not require any inference about the student or teachers' interpretations. There was some concern, however, over what should be included as student insight. Unsolicited and valid contributions were included so that alternative names for fractions were coded as insights in Martha's class. There were few instances when teachers asked students to describe patterns. Instances for which teachers asked for new patterns and students could describe them were designated as student insights. Subcategories, which were determined after the occlusions had been identified, were also reasonably apparent. The most subjective choice was whether student errors were patterned or nonpatterned.

Classification of teachers' elective actions was more difficult as a pre-existing scheme was not used, at least not systematically. In retrospect, a modified version of the Brophy-Good Dyadic System would have been satisfactory. Initially, an attempt was made to define a new coding scheme. Eventually, the categories of exploiting, alleviating, and avoiding moves emerged, but classification was highly subjective until specific behaviors were designated and assigned a category. The coding task was still somewhat difficult, however, when teachers used several behavioral techniques in response to the same student occlusion. Distinguishing between alleviating and exploiting moves was sometimes difficult since it required a judgment about the task and questioning routine of an activity. Some student occlusions occurred early in an activity before routines were well established. This made it impossible to determine what the routines would have been had the occlusions not occurred. If this coding system is to be used in other studies, reliability measures need to be established.

A few remarks are also in order about the teachers' stimulated recall data. Each thought unit and decision point was not coded even though distinctions were made between recalled thoughts and feelings or other information. The emphasis of this study was on the substance of teachers' remarks and not on quantifying or classifying them. The loss of such detail was compensated for by the interpretations of teachers' intentions, the relevant antecedents to and causes of their critical moments, and the consequences.

### Method and Effect of Stimulated Recall on Teachers

The method of conducting stimulated recall proved to be a reasonably effective means of obtaining teachers' thoughts and feeings. However, asking teachers to view the entire lesson for roughly two hours on a daily basis had both advantages and disadvantages. One advantage was that most incidents of unpredictable student performance which teachers processed at a more conscious level were reported. Had segments of the lessons been preselected for viewing, some might have been missed as so few actually occurred. The sustained use of stimulated recall also reduced the amount of verbal discourse that teachers sometimes use to explain their actions (e.g., Conners, 1978). Teachers realized what things had been said before and avoided repetition. They also became conditioned to the process as they focused on recalling their covert experiences. Nevertheless, teachers found it difficult to restrict themselves to reporting recalled thoughts and feelings, as Marland's (1977) analysis of teachers' comments showed. Interfering with teachers' suspended state of recall were interactive diversions, retroactive thoughts, fatigue or boredom, and the probes of the investigator.

The process was an interactive experience for teachers. Viewing the lessons again provided them with opportunities to notice things they had missed and to plan what they would do next or what they would do differently another time. Retroactive interference occurred when knowledge of a future event interfered with a teacher's recollection of the incident being shown. Some events were mentioned earlier than they appeared on the screen so that recalled thoughts did not always occur in juxtaposition with what was being observed on tape. Nor were teachers always certain just when their thoughts did occur.

Fatigue or boredom was particularly noticeable during long stretches of a lesson when there was little of interest for teachers to recall or examine. During these periods, teachers either remained quiet or began to talk about other things. Fatigue was evident as a consequence of participating in the daily stimulated recall sessions after about a week. All three teachers reached a point where they were no longer interested in continuing, so they ended the units. The lengthy recall sessions interfered with their planning time. They were tired of preparing lessons for the substitute teacher, who relieved them for one class, and of student complaints about the substitute and changes in their schedules.

Investigative probes to elicit teachers' thoughts may have distracted teachers on occasion, particularly when teachers continued to talk as the videotape played and when the investigator sought to learn more about the teachers' understanding of mathematics. On a couple of occasions, the probes bordered on intervention or helping moves, particularly when teachers appeared to be at a loss for what to do or when they had made glaring errors. These probes may have been perceived as a threat. In such instances, the problem for the investigator was in maintaining a detached and nonevaluative or noninstructive role. The investigator tried to remain supportive of the teachers--they seemed to need it--in much the same manner as described by Conners (1978). From the perspective of the investigator, the relationship between the investigator and teachers was friendly and nonthreatening.

Teachers exhibited different styles in response to the stimulated recall process. While all the teachers cooperated, some found it easier to recall thoughts and feelings. The same is apparent in the quantitative measures of teachers' comments in Marland's (1977) study. Zelda reported the fewest thoughts; little seemed to bother her during her lessons. As a result, the investigator's repeated inquiries about her thoughts and feelings may have been irritating. Her primary distraction during recall was with what she might have done to improve the activities. Martha was particularly vocal about her thoughts and feelings so that probes were made more to clarify than to elicit. She used the opportunity afforded by viewing the lesson again to examine her students' behavior and thinking. Both Martha and Zelda were comfortable in talking while the videotape continued to play, particularly if their remarks were brief. Ralph willingly reported his thoughts and feelings, but in a manner which indicated resistance to interference by the investigator. He retained more control over stopping and starting the videotape than the other teachers. While the tape continued to run, he concentrated on remembering the experience, but once he began talking, he wanted the tape stopped. He was more verbal about his concerns and his own teaching than were the other two teachers, which was most apparent in his long passages in the typed transcripts of the recall session.

## Investigator Bias

Observing a teacher in action is quite different from combining the observation with the experience of and data from stimulated recall. At the time the classroom observations were made, Zelda's classes were the most exciting, Ralph's seemed dull and boring, and Martha's were good. In the process of analyzing and reporting data, however, the investigator's opinions underwent some change. Martha emerged as the most sensitive, caring, and competent teacher, despite her own reservations about her ability and knowledge of mathematics. Zelda came across as a confident teacher, responsive to the students, but always to their mathematical needs. Ralph remained the least interesting with his emphasis on skills for doing problems. If he is a true representative of teachers in typical mathematics classes and research on teacher thought, then it is no wonder that so little variation is found in these classes. One of the distinct advantages in the choice of teachers for this study was the variation in their teaching and goals for instruction which helped to highlight characteristics of teachers' actions and mental processing of unpredictable student performance.

## Implications and Directions for Future Research on Critical Moments

The coding system used in this study for describing student occlusions and teachers' elective actions needs to be checked for reliability, modified if necessary, and compared with other classification schemes. Separate, but nevertheless similar, coding systems emerge from different studies because of different researchers' goals. Reanalyses of the same data set by various coding and analysis procedures would provide more direction about the differences of each.

In future studies of teacher thought where stimulated recall is to be used, some modifications may simplify the procedure without loss of relevant data. First, because some episodes were so salient in teachers' minds, they wanted to begin talking about them immediately following the lesson. Some of their spontaneity may have been lost or recollections distorted by asking them to wait until the episodes appeared on the screen. A debriefing interview prior to the showing of the tape might be advisable, although this procedure could alter the recall process once the videotapes are shown. A debriefing interview might even be adequate for gathering teachers' recollections without viewing a videotape, although that remains to be investigated. Second, if an entire lesson is to be shown, fast forward and pause controls on the videotape deck would be helpful. The tape could be speeded up when there was little of interest to report and stops could be made easily when teachers were speaking. A third suggestion is to preselect portions of the lesson for viewing based on the criteria for identifying potential critical moments discussed earlier in the chapter.

After teachers' critical moments and the models of cognitive processing had been identified and described, it became clear that the separate coding and counting of each student occlusion did not reflect the teachers' subjective experiences during critical moments. The behaviors which occur while a teacher is attempting to satisfy a momentary goal should be considered as one event. Thus, the event can consist of only one student occlusion and teacher elective action or it can include a chain of several such interactions. As long as the teacher is attempting to elicit a particular response, help an individual overcome a misunderstanding,or use a student suggestion, the sequence of student and teacher behaviors should be considered as a whole unit. The coding of these episodes should reflect the number and nature of the student occlusions and the sequence of teaching moves--the teacher's strategy. The goal state of the teacher is a natural umbrella under which to cluster student and teacher behavior. Episodes involving more than one student occlusion were recalled as a single experience, since teachers were not always able to distinguish when particular thoughts and feelings occurred.

A benefit of coding the unpredictable student behavior in this manner is that potential sites for critical moments can be more readily identified. Disappointed in the lack of consistency which he found between teachers' differential expectations for student performance and their behavior, Cooper (1979) recommended a similar approach for analyzing teacher behavior:

> Undeniably, classroom interactions are not independent events; any exchange is highly dependent on what has gone before. Closer concentration on behavioral sequences, as opposed to aggregated behavioral frequencies, would undoubtedly produce a wealth of new information. Models which predict behavioral sequences ought to be viewed as more powerful than models dealing with total or averaged behavioral occurrences (p. 405).

If proportions or frequencies are to be used, the number of critical moments should not be compared with the total number of student occlusions; instead, the number of episodes involving student occlusions should be clustered according to the teachers' immediate goals. The identification of these episodes through a taxonomy would help in distinguishing routine student occlusions from those with greater potential for being more consciously processed and for producing critical moments. Distinctions might also be made and ratios formed of the satisfactorily or unsatisfactorily resolved student occlusions relative to the total. A neutral category might need to be included as well. A comparison of different quantitative measures is needed to see which best characterizes teachers reported experiences or their lessons.

## Teaching Mathematics

The primary argument used to support the need for this study of critical moments grew out of recommendations for teaching mathematics proposed recently by the National Council of Teachers of Mathematics. The Council urged that students be given more problem solving experiences and that teachers focus more on developing concepts, using concrete materials, and solving real world problems rather than limiting instruction to computational skills. The concern of the investigator is that in carrying out these recommendations, teachers may have to cope with more unpredictable student performance or student occlusions of a somewhat different nature. Either could cause teachers to resist implementing the recommended style. Whatever verbal support teachers have given for providing problem solving experiences and encouraging understanding of mathematics, they have made little progress in changing how they teach mathematics.

If teachers do find it difficult to respond satisfactorily to unpredictable student performance and critical moments, then every effort should be made to find ways to help them. In this section the findings and implications for teaching mathematics will be discussed, after which recommendations for training teachers and preparing curricular materials will be offered.

## Findings and Implications

Findings which had particular significance for the teaching of mathematics dealt with

1. The nature of student occlusions which produced critical moments;
2. Causes of teachers' critical moments;
3. The role of activities with regard to student performance and teachers' cognitive processing and planning; and
4. The role of teacher antecedents--mathematical goals, conceptions of learning, and instructional style--in teachers' responses to critical moments.

## Student Occlusions in Critical Moments

Student occlusions did not have equal potential to cause teachers to consciously process these events and to experience critical moments. The distinctive features of the student occlusions and teacher elective actions associated with critical moments were due primarily to one or more of the following: (a) the mathematical content of a student response or contribution in comparison with the task and questions of an activity, (b) the response of a student in comparison to the norm of responses to similar questions or in comparison to the plans and expectations of the teacher, and (c) recurring incidents of the same difficulty or insight from the same student or over the same questions. Recurring incidents were presented as distinctive until the teacher managed to satisfactorily resolve the problem or set a goal which encompassed student repetitions. For the most part the student errors were patterns indicative of misconceptions or an inability to respond to a particular question or task. Student insights were suggestions of new rules or variations in the tasks--usually more complex ones.

The implication is that teachers' critical moments come close to matching student occlusions which have greater potential for flexible teacher actions and constructive use of the information. Two are possible advantages to finding teachers conscious of and interested in responding to distinctive student difficulties and insights. One is that teachers may fail to respond or may respond inappropriately to critical moments because they lack sufficient knowledge, not because they fail to note the potential for action. The second advantage is that if teachers are provided with appropriate information, they might improve their responses to critical moments and the student occlusions associated with them. The question, then, is what do teachers need in the way of assistance? The answer must be found in the causes of teachers' critical moments.

Causes of Critical Moments

The primary causes of teachers' critical moments were their limited knowledge of mathematics and, to a lesser extent, their limited repetoire of pedagogical options. These deficiencies sometimes interfered with teachers' interpreting student occlusions, searching for elective actions, and evaluating the effectiveness of their actions. Since the teachers did not receive help in these areas, they could not have been more effective in coping with their critical moments.

Teachers did not anticipate the types of student difficulties or insights which might occur nor did they plan any responses to them. The teachers in this study were usually quite surprised by the students' performance at critical moments. Whether this is due to a failure of teachers' planning practices or their knowledge is not totally clear, but there was evidence to suggest that the teachers in this study were incapable of anticipating many of the more distinctive student occlusions. They simply did not understand the mathematics well enough and had not taught the same unit enough times to be able to draw upon their past experiences.

Prescriptions given to teachers for actions in specific situations for specific content would suggest an approach similar to the diagnostic-prescriptive programs in mathematics. The difference is not in theory, but in practice--the way diagnoses are to be made, prescriptions implemented, and for what purpose. The diagnostic-prescriptive materials have been designed primarily for testing and interviewing students on an individual basis with the emphasis on helping students who are having difficulties achieving mastery. The recommendations in this section have to do with total class instruction which introduces students to new mathematical ideas and tasks through activities. Teachers must respond to both student difficulties and insights immediately. Much of the information which has been learned from the diagnostic clinicians (e.g., Ashlock, 1979), however, would undoubtedly be of some use to the teacher engaged in total class instruction. Similarly, information about student understanding of mathematics could be useful in identifying the types of occlusions teachers might encounter (e.g., Behr & Post, 1971; Pulos, Stage, Karplus & Karplus, 1981; Erlwanger, 1973; and Moser, 1981). Research on the effectiveness of specific actions in particular settings, however, is still greatly needed. The problem with past research efforts has been that attention was focused on general pedagogical moves without sufficient regard to the details of the instructional setting and the mathematical content.

Teachers' experiences with critical moments seem to indicate that they do not need general information as much as they need task-and topic specific mathematical knowledge and situation-specific pedagogical advice. Teachers need to be made aware of the student difficulties and insights that could arise, what they might indicate, and possible courses of action. In order to be able to prescribe specific actions, however, more research is needed to find what Whitfield (1971) termed the "temporal best" decisions. He was concerned with the mechanisms teachers might use to make their choices as well as the information available for them to use. If experiences of teachers in this study are any indication, efforts to train teachers in more general instructional strategies might be less effective.

Teachers did not have a clear understanding of abstractions. Terms such as concept are often used indiscriminately to describe whatever ideas were being taught. Similarly, without the specific mathematical knowledge these strategies require, it is doubtful that teachers could be successful in implementing them. Teaching strategies, at least as they have been characterized by Cooney, Kansky, and Retzer (1975), also do not provide pedagogical techniques for using the strategies with an entire class or even with the different types of materials teachers are expected to use. Herscovics (1980) may come to similar conclusions in training teachers to conduct clinical interviews. Instead, teachers may need to learn their general tactics for responding to pedagogical situations or to mathematics through examples as Resnick (1981) has indicated. Teachers, like students, develop their heuristics by performing in a more algorithmic or prescriptive manner. Shulman (1977) reminds us, "One of the inexorable blessings of human cognition is the process of invention and construction. We do not merely imitate; we always construct" (p.271)

Role of Activities for Students and Teachers

The importance of activities to student performance and to teachers' cognitive processing and planning must not be overlooked. In this study different frequencies and distributions of student occlusions could be noted across different activities and teachers. Some of the more salient features of the activities which seemed to influence the amount and types of unpredictable student performance were (a) the task and student's familiarity with it, (b) the teacher's questioning routine, instructional strategy and the type of learning encouraged, and (c) the instructional style and goals of the teacher.

The activity is also an important determinant of the problem space in which teachers are attempting to process the student occlusions including the more distinctive ones associated with critical moments. Researchers from the South Bay study (Joyce et al., 1977) came to similar conclusions about the importance of the selection of the materials and instructional flow of an activity in determing the boundaries of the teachers' mental processing of unpredictable events. Once an activity is underway and a teacher has established a routine for questioning the students about the task, most of the in-flight decisions or choices of elective actions are made within the same frame.

When the mathematical knowledge required exceeds that being used in an ongoing activity, teachers experience difficulties. Similarly, as long as teachers are able to rely on pedagogical routines which they have acquired as habit they are able to select and use these techniques without much conscious effort. However, when the cognitive demands for interpreting a student remark or for searching for an elective action exceed the available frame, teachers are not able to process them. Instead, they have to draw upon knowledge they may not be as familiar with or may not even have. The difference is essentially the same as that between cognitive processing within the short-term memory or having to search the long-term memory where information is not as readily accessible.

An activity is also the product of a teacher's planning, albeit not always a very complete product. Teachers plan and implement activities as research on teacher planning has shown (Yinger, 1977; Clark & Yinger, 1979; Shulman, 1981) which this study has supported.

## Role of Teacher Antecedents in Their Responses to Critical Moments

The diversity with which teachers in this study responded to their critical moments has already been discussed. The range was from simple discomfort and little or no flexibility to discomfort with more flexibility and constructive use of critical moments. The question which has yet to be answered is whether teachers can respond differently to critical moments--even with assistance--or if their responses are a reflection of more stable characteristics. The less flexible teacher taught with goals for instrumental understanding and in a manner similar to what Easley (1978) described in his depiction of the typical mathematics teacher. The more flexible teacher was the one who sought relational understanding and who taught in a manner encouraged by the National Council of Teachers of Mathematics (1980). To what extent these differences can be altered by different curricular materials or by different training practices has yet to be learned.

Teachers' goals are another indication of their responses to their critical moments. Because of the interrelatedness of teachers' goals, conceptions of how students learn mathematics, and attributions of student difficulties, it may be possible to predict how teachers respond to distinctive student occlusions and the constructive use to which they put the critical moments using one or a combination of factors.

## Directions for Training Teachers and Development of Curricular Materials

In their recommendations for the '80s, the National Council of Teachers of Mathematics encourages teachers to provide problem-solving experiences for their students and calls for the improvement of teacher training and curricular materials. Because instruction based on these recommendations tends to stimulate unpredictable student performance, findings on teachers' critical moments offer some directions for improving teacher training and the preparation of curricular materials.

### Critical Moments as Training Sites

Critical moments provide a natural research and training site. Because critical moments are particularly salient events, they provide opportunities for teachers to investigate probable causes of the student difficulties and insights and elective actions which might be tried. For teachers to be trained with their own critical moments in a personalized manner has disadvantages due to time and personal constraints. There is also the possibility that sufficient or desired examples might not occur. Consequently, there is a need to consider training by simulation. A library of critical moments, gathered from videotapes and transcripts of actual lessons, could be used in simulation experiences for training teachers or conducting research. Responding to critical moments in the context of the activity being taught would provide a more realistic teaching situation than the descriptive sketches written by Bishop and Whitfield (1972). Critical moments would provide case studies as already used in the study of medicine and business.

In a critical review of research in mathematics education conducted in the United States, Kilpatrick (1981) calls for more research conducted in the classroom. Integrating the findings of research conducted during actual instruction with research conducted on students' understanding of mathematics outside the classroom is one way of lending more credibility to both. For example, in their study of students' knowledge of rational numbers, Behr and Post (1981) found that visual-perceptual distractors affect students' understanding of fractions. One of Martha's critical moments vividly illustrates this same difficulty. Student difficulties or insights which occur could also point to needed areas for research on student understanding and the effects of teachers' actions.

Preparation of Curricular Materials

Alerting teachers to the potential student difficulties and insights would not necessarily diminish the distinctiveness of these incidents in teachers' minds. Instead, the advanced information could alert them to signals that otherwise might be overlooked or misinterpreted. Similarly, prescriptions or recommendations for action would be useful in cases in which teachers were not so sure of how they might respond. Teachers could also benefit from suggestions for follow-up activities to be used with individual students or to use when they encountered difficulties with their activities.

Supplying teachers with this information along with the activities to be taught would serve five functions. First, it would assist the teachers in what they claimed was a difficult task--planning introductory activities--particularly when more than just basic facts and skills are to be learned. Since teachers process unpredictable student occlusions in terms of the mathematical task and questioning routine of an activity, the prepared activity would help to define the parameters. Thirdly, activities could be designed to illustrate different kinds of instruction such as direct instruction of skills or guided discovery with problem solving. This leads to the fourth function of the activity, the impact of the task and teaching method on the type and number of student occlusions. If, for example, instruction is to encourage student conjecture, explanation, and experimentation, the activity must be taught so as to stimulate this type of student behavior. Lastly, for information about probable student occlusions and prescriptions for elective actions to be credible, it needs to be gathered in learning environments which are similar to what teachers will create. Prepared activities would help stabilize these learning environments.

The author's recommendation, then, calls for the development of curricular materials packaged in activities for which teachers' actions can be tested for their effectiveness. An exemplary unit of this type has been developed by Fitzgerald and Shroyer (1979). Additional units of the same type are in the process of being developed by the Middle Grades Mathematics Project (Lappan, 1980). These units consist of a carefully sequenced set of activities designed to teach mathematical concepts and relationships in a problem solving mode. Teachers are given general information about the content and how it is to be taught as well as scripts for the activities. The scripts provide teachers with specific information about what to say and do, how students might respond, and what actions might be taken. Scripts are the algorithms from which it is hoped that teachers will achieve successful teaching experiences Hawkins (1966) believes are necessary:

> The good teaching I have observed, teaching by teachers who are accustomed to major success, owes little to modern theories of learning and cognition and much to apprenticeship, on-the-job inquiry, discussion, trial--ceaseless trial--within a common-sense psychological framework; a framework that is not all unsophisticated, however, and is able to accept individual insights from psychological sources but without jargon or dogma: keeping the practice dry (p. 5).

The availability of such curricular materials—even with the additional information about student performances and possible perceptions for responding--may still not be sufficient. There is ample evidence that curricular materials are not always implemented as intended. Smith and Sendelbach (1979), for example, found that teachers misinterpret, alter, and ignore suggestions in the teachers' guides for science programs. They selectively choose and plan according to their goals and conceptions. The extent to which teachers' personalized versions of curricular materials influence the type of student performance, however, has yet to be documented. Similarly, the effect of the recommended way of preparing curricular materials on teachers' responses to critical moments also has to be investigated.

Instructional units of the type recommended and just described could also be used in preservice teacher education. This would provide trainees with examples of teachable units which illustrate different topics, grade levels, materials, and instructional styles. Too often teachers are left to discover what constitutes an activity when they begin teaching. Case studies of units of activities could help to overcome this deficiency.

## Summary

This discussion has focused on the findings, implications, and directions for research and development about critical moments. The important findings about teacher thought and teaching had to do with (a) the different types of critical moments and teachers' cognitive processing; (b) the distinctive and observable characteristics, of student occlusions; (c) interrelatedness of teachers' activities, actions, goals, conceptions, and attributions; and (d) the variation in teaching and student occlusions.

Implications were indicated (a) the role of an activity in determining student performance and teachers' mental problem space, (b) the role of student performance in determining teachers' actions and level of processing, and (c) the variation of teacher responses to critical moments.

Methodological concerns and recommendations had to do with the (a) criteria for identifying critical moments, (b) the classification of student and teacher behavior, and (c) the method and effect of stimulated recall on teachers. The possibility of investigator bias was also indicated.

The important findings with regard to the teaching of mathematics dealth with (a) nature of student occlusions in critical moments, (b) causes of critical moments, (c) role of activities, and (d) role of teacher antecedents in their responses to critical moments.

Directions for teacher training had to do with using critical moments as training sites. Recommendations for improving the preparation of curricular materials were also given.

# CHAPTER VIII

## SUMMARY AND CONCLUSIONS

The purpose of this study was to investigate how teachers cope with unpredictable student performance which is inevitable in the problem-solving mode of teaching. Problem solving has long been a desired goal of instruction in mathematics, and the recommendations of the National Council of Teachers of Mathematics for the 1980s declared it a number one priority in the teaching of mathematics. Admittedly this is not an easy task for teachers to undertake, and it makes their instruction more difficult. Addressing the teaching of problem solving with curricular changes and training programs leaves out the vital role of the teachers, their thoughts and actions, and their interaction with the behavior of their students. Several studies have established the failure of curricular projects and training programs over the last twenty years to bring about any change in the teaching of elementary school mathematics. If this is, indeed, because the teacher as vital element has been ignored (Shulman, 1975), the need to explore teacher behavior and thought processes is apparent.

The initial chapter raised questions about four areas: the nature of the instructional environment dealing with how teachers organized units, their teaching styles, their goals, student performance and teachers' elective actions. The second question introduced the concept of critical moments and teacher reaction to it. The third proposed creating models of how teachers mentally process student performance during critical moments. Emanating from these three areas of inquiry, the fourth set of questions proposed to consider the implications for further research, for developing appropriate teacher training and preparation of curricular materials. This study attempts to understand more about the mental processing and behavior of teachers in relation to unexpected student performance; it studies critical moments--a source of difficulties for teachers.

A review of the literature reported several studies on interactive teacher thought and decision making. After a brief description of the research and an extensive analysis of the findings, the methods of stimulated recall, coding, and data collection were detailed as a preamble to this study.

Prior research conducted over the last decade indicates that teachers do experience difficulties in coping with unexpected student performance, but the use of student difficulties and insights as a strategic research site had not been explored. Similarly, only general relationships between content taught, teacher behavior and teacher thought have been examined. This study proposed to explore in depth the relationship between student performance, teacher thought and behavior, and the subject--mathematics--being taught at critical moments.

The teachers selected for study were three experienced teachers who had had some affiliation with the teaching and philosophy of the Unified Science and Mathematics Program in Elementary School, a curriculum project committed to solving real problems. The teachers were teaching upper elementary grades to predominantly fifth and sixth graders and a few fourth grade students. Case studies were conducted using the process-tracing technique and stimulated recall to gather teachers' thoughts and feelings while teaching mathematics to supplement the observable behaviors. The procedure was similar to the one used by Conners (1978). Each teacher taught a six- or seven-day unit on rational numbers, fractions, or decimals. These units were introduced so that teachers were engaged in interactive teaching with their whole class for at least a portion of each day's lesson.

The focus was on studying the incidents of unexpected student difficulties or insights which caused the teachers some cognitive or emotional difficulties. Behavioral data were broken down into activities according to the content, task, and mode of instruction. Potential teacher difficulties were identified from student insights and difficulties--the unexpected student contribution. Both student occlusions and teachers' elective actions were categorized and then compared with the teachers' thoughts and feelings to identify the critical moments--those occasions for which teachers experienced cognitive difficulty and emotional discomfort. As this was both a descriptive and theoretical investigation of critical moments, descriptions from both observable and self-report data were used to answer the questions posed in Chapter I.

Teachers in this study, as shown in Chapter IV, bore many similarities to the characteristics of more effective teachers identified in the process/product studies of Evertson et al. (1980) and of Good and Grouws (1979). Teachers also exhibited variation in the content emphasized, instructional materials, strategies, and style. These overt variations were manifestations of different instructional goals of the teachers and of their conceptions of the teaching and learning of mathematics. They were also consequences of the differences in students' ages, past learning experiences, and capabilities.

Using a thumbnail description, the teachers could be characterized as follows. One was a typical teacher using traditional methods--exemplification and questioning. With an emphasis on paper-and-pencil tasks and symbolic reasoning, he was encouraging instrumental understanding. At the other end of the continuum was a teacher who was atypical, but close to the NCTM recommendations. Wanting to provide students with a solid base, she gave them concrete manipulatives for concept development and relational understanding. The third teacher was a combination of both. She used concrete tasks to introduce the concepts and then shifted to symbolic skills. She also provided alternative symbolic approaches. Her approach was an attempt to bridge the gap between concrete experiences and textbook problems.

The impact of the instructional environments was evident in the variations in frequency, proportion, and density of student difficulties and insights. Unsolicited student contributions were more prevalent with the less traditional instructional approach, and student difficulties outnumbered student insights. Teachers' elective actions for student difficulties were similar when classified according to the broad categories of exploiting, alleviating, and avoiding moves. Their techniques for alleviating student difficulties varied. Teachers responded differently to student insight; both exploits and avoids were more common. One teacher avoided every instance of student insight.

A distinction had to be made between critical moments of a short-term nature and those based on pervasive problems. Critical moments arose over specific incidents of student difficulty or insight, while what was eventually labelled critical discrepancies reflected a more general pattern of student performance. There were four types of conditions that produced cognitive difficulties and emotional discomfort for teachers. The resulting critical moments were labelled as follows: (Type A) student difficulties, (B) student insights, (C) instructional pace, and (D) unanticipated success. All but the last type were further subdivided according to the particular difficulties teachers reported. Of the three Type A teaching difficulties, two were critical moments which occurred when teachers were trying to correct student misconceptions or to elicit desired responses to teacher-initiated questions. The third occurred when teachers had trouble diagnosing student difficulties with understanding a task. This constituted a critical discrepancy. Difficulties teachers had with student insights were in carrying out their exploits and in having to avoid student suggestions. Pacing dilemmas were produced by trying to meet the divergent needs of two groups--students having difficulty and students having to wait while assistance was given to others. This difficulty occurred in relation to student occlusions and to a recurring pattern of giving individual assists during a group activity.

Critical moments occurred in relation to student occlusions that were distinctive: student errors indicative of misconception; student performance distinct from the norm; unsolicited student contributions--insightful or inappropriate--about mathematics not covered in class. Typically, elective actions associated with critical moments called for the use of mathematics not routinely used within the activity which placed a greater cognitive demand on the teacher.

Elective actions varied in accordance with the teacher's understanding of math, their conceptions of learning and their goals for instruction. Likewise, their negative emotional reactions which varied in intensity and type were found to be partially a function of their limited understanding of math which affected not only their choice of actions, but their understanding of their students difficulties and insights. At times, teachers altered their plans and activities which showed that critical moments had an effect; however, when these critical moments were no longer thought about, they did not influence the planning process.

The nature and function of teacher mental processing of unexpected student performance was examined using research on cognitive processing. Using the theoretical writing of Skemp (1979) to interpret much of what was found from examining critical moments, four cognitive processes were identified and described in the processing of student occlusions: <u>interpreting</u>, <u>goal setting</u>, <u>searching</u> for elective actions, and evaluating the effectiveness of the actions. Typically, it was the more distinctive student occlusions and elective actions which teachers processed at a more conscious level--a necessary condition for more adaptive behavior. Action was initiated to set a goal to act, then find, execute, and test the results. Teachers failed to achieve their goals by reason of default--being unable to think of something else to try--or by interference from antigoals--situations to be avoided.

When teachers encountered difficulties in achieving their goals, they could either continue to search for another elective action or change their goals to secondary goals that could be satisfied more easily. A goal had to be satisfied; if not the initial one, then a secondary one. A relationship was found among teachers' affective reactions, the movement towards or away from goals, and teachers' perceived abilities to bring about change. Teacher attributions for student difficulties were found to reflect their instructional goals and conceptions of learning more than their expectations for students.

Critical discrepancies were threats to teachers' more enduring goals for the activities. A switch to secondary goals could provide relief from the discomfort of critical situations, but this was not easily done. Instead, teachers would endure the discomfort as these discrepancies faded in and out of consciousness--at least for a while. Awareness of the situation recurred as long as the condition remained. Escape came from changing the routine of the activity or ending it.

The model of cognitive process used by Peterson and Clark (1978) to analyze interactive teacher thought was more appropriate for critical discrepancies than critical moments. Different models were proposed for teachers' cognitive processing of unpredictable student performance--one for student occlusions and critical moments and another for pervasive patterns of student performance and critical discrepancies.

Factors found to influence teacher thought and behavior included the more stable factors of mathematical and pedagogical knowledge, conceptions of learning, principles of teaching, expectations, and instructional goals. The less stable factors identified were the event itself and the intermediate consequences of the stable factors, such as the teacher's perceived ability to effect change. Of particular importance was mathematical knowledge which influences all four cognitive processes, and pedagogical knowledge which influences goal setting and searching for elective actions. A second point of interest was the complexity of the goal setting process when teachers' initial goals were being threatened.

Teaching was made difficult by critical moments because teachers could not achieve their goals--momentary goals for responding to student occlusions or activity goals. The primary cause of teachers' difficulties was their limited knowledge of mathematics. In some cases, teachers' actions were constrained by their repetoire of pedagogical techniques or fears of consequences.

Teacher thought, as revealed in this study, was similar to the findings of other researchers investigating teacher decisions and thought. Implications of particular importance include (a) the role of the activity in influencing unpredictable student performance and teachers' problem space for processing the same, (b) the influence of student performance on teachers' cognitive processing and actions, and (c) teachers' varied responses to critical moments. Teachers' flexibility over and constructive use of critical moments was consistent with their choice of activities.

In the critique of methodology for studying teachers' critical moments, recommendations were made about the coding of student and teacher behavior and the techniques to obtain teacher thought with and without the use of stimulated recall.

Implications for the teaching of mathematics had to do with the nature of student occlusions as a determining factor of critical moments, the importance of teachers' knowledge of mathematics, and somewhat less their pedagogical routines as causes of critical moments.

For teachers to cope more effectively with critical moments, they need task-and topic-specific information about the types of student occlusions to expect and prescriptions for elective actions. Recommendations were given for the use of critical moments in training teachers and for preparing curricular materials by activities and units.

Directions for future research and development included the following:

1. Investigate the effects of an activity--task, questioning, routine--on the number and type of unpredictable student performance and on the teachers' instructional approach and response to critical moments.

2. Investigate the consistency of the relationships between teachers' goals, conceptions of learning, attributions, and emotional responses to critical moments.

3. Refine and validate taxonomies for identifying potential critical moments from observable characteristics.

4. Compare methods for coding student occlusions and teacher elective actions, for gathering teachers' thoughts, and for identifying teachers' critical moments and discrepancies.

5. Compile cases of critical moments to illustrate different types of student difficulties and insights, teacher elective actions for different mathematical tasks, and types of instruction for use in training by simulation.

6. Prepare instructional units of the type recommended and use to identify types of unpredicted student difficulties or insights by tasks and topics and determine effective actions for use by teachers.

7. Investigate the effects of training teachers with simulated critical moments and with improved preparation of curricular materials.

APPENDIX

Table A.1 Martha's Fraction Unit: Time in Minutes and Content by Activity

| Day | Activity | Time | Topic | Content Task | Domain | Materials |
|---|---|---|---|---|---|---|
| 1 | 1 | 14 | Concept | name→model | 1/a | rods[a] |
| | 2 | 5 | Concept | model→name | 1/a | pictures |
| | (3) | (5) | (test) | | | |
| | 4 | 4 | Concept | name→model[b] | 1/3 | rods |
| 2 | 5 | 3 | Concept | name→model | 1/a | rods |
| | 6 | 20.5 | Concept | model→name | $b/a\leq1$ | pictures |
| | 7 | 3.5 | Concept | name→model | $b/a\leq1$ | pictures |
| | (8/9) | (14) | (test/game) | | | |
| | 10 | 3.0 | Relationship | compare | $b/a\leq1$ like denom. | symbols |
| 3 | 11 | 17 | Concept | model→name | $b/a\leq1$ | pictures |
| | (12) | (6.5) | (test) | | | |
| | 13 | 5.5 | Relationship | compare/order | $b/a\leq1$ like denom. | symbols |
| | 14 | 9 | Relationship | compare | like numerators | symbols |
| 4 | 15 | 38.5 | Relationship | compare | $b/a\leq1$ | rods |
| | 16 | 9.5 | Concept | model→name | $b/a\geq1$ | rods |
| | 17 | 5.5 | Concept | name→model[b] | 6/4 | rods |
| 5 | 18 | 7.5 | Concept | model→name | $b/a\leq1$ | rods |
| | 19 | 14 | Relationship | find equivalent | $b/a\leq1$ | rods/symbols |
| | 20 | 5.5 | Operation | add | $b/a\leq1$ like denom. | rods/symbols |
| | 21 | 24.5 | Operation | add | $b/a\leq1$ | rods |
| 6 | 22 | 27.5 | Operation | add | $b/a\leq1$ | rods |
| | 23 | 4.5 | Concept | define | all types | words/symbols |
| | (24) | (11) | (Operation) | (add) | ($b/a\leq1$) | (rods) |
| | 25 | 11 | Operation | add | $b/a\leq1$ | rods/symbols |

a rods = Cuisenaire rods

b no unit rod specified

Table A.2 Zelda's Fraction Unit: Time in Minutes and Content by Activity

| | | | | Content | | |
|---|---|---|---|---|---|---|
| Day | Activity | Time | Topic | Task | Domain | Materials |
| 1 | 1 | 10 | Concept | name | 1/a | picture |
| | 2 | 4.5 | Concept | model name | 1/a | rods |
| | (3) | (9) | (work period) | | | |
| 2 | (3) | (7) | (work period) | | | |
| | 4 | 12.5 | Concept | (report activity 3) | | |
| | 5 | 14 | Concept | name model | | |
| | 6 | 15 | Relationship | name equivalents | $b/a \leq 1$ | rods |
| 3 | 7 | 12.5 | Relationship | generate set of equivalents | 1/a | rods/symbols |
| | (8) | 14 | (work period) | | | |
| 4 | 9 | 19 | Relationship | compare name model | $b/a \leq 1$ | rods |
| | 10 | 8 | Operation | add | $b/a \leq 1$ like denom. | rods/symbols |
| | 11 | 18.5 | Operation | add | unlike denom. | |
| 5 | 12 | 11 | Relationship | find simplest equivalent | $b/a \leq 1$ | symbols |
| | 13 | 19 | Relationship | find simplest name | $b/a \leq 1$ | symbols |
| | (14) | (21) | (work period) | | | |
| 6 | 15 | 5.5 | Relationship | find missing value | $b/a \leq 1$ | symbols |
| | (16) | (45) | (work period) | | | |
| 7 | 17 | 17 | Relationship | generate set equivalent | $b/a \leq 1$ | symbols |
| | (18) | (37) | (work period) | | | |

Table A.3 Ralph's Decimal Unit: Time in Minutes and Content by Activity

| Day | Activity | Time | Topic | Content: Task | Content: Domain* | Content: Materials |
|---|---|---|---|---|---|---|
| 1 | 1 | 23.5 | Concept | translate decimal to fraction | decimal | symbols |
| | (2) | (22) | (work period) | | | |
| 2 | 3 | 6.5 | Concept | (report answers from activity 1) | | |
| | 4 | 21.5 | Concept | translate decimal to fraction | decimal or $10^k$ | symbols |
| | (5) | (18) | | | | |
| 3 | 6 | 15.5 | Concept | translate fraction to decimal | $10^k$ | symbols |
| | (7) | (30) | (work period) | | | |
| 4 | 8 | 19.5 | Relationship | order and equate | $10^k$ | symbols |
| | 9 | 8.5 | Concept | translate frac- | $\neq 10^k$ | symbols |
| | (10) | (20.5) | (work period) | | | |
| 5 | 11 | 25.5 | Operation | add/sub. fraction explanation | decimal | symbols |
| | (12) | (20) | (work period) | | | |
| 6 | 13 | 19 | Operation | add/subtract | decimal | symbols |
| | (14) | (27) | | | | |

*Fraction domains are in terms of denominator: $10^k$ or $\neq 10^k$.

Table A.4 Distribution of Instructional Time in Minutes By Content and Teacher

| Content | Teacher | | |
|---|---|---|---|
| | Martha | Zelda | Ralph |
| Concepts | 94 (40)* | 41 (25) | 75.5 (54) |
| model/name | | | |
| unit (1/a) | 26 | 27 | |
| proper ($b/a \leq 1$) | 48.5 | 14 | |
| improper ($b/a > 1$) | 15 | | |
| define/translate | | | |
| all fractions | 4.5 | | |
| denom $= 10^k$ | | | 67 |
| denom $\neq 10^k$ | | | 8.5 |
| Relationships | 70 (30) | 88 (53) | 19.5 (14) |
| compare/order | | | |
| rods/pictures | 38.5 | 19 | |
| symbol | | | |
| like denom | 8.5 | | |
| like num. | 9 | | |
| equate | | | |
| rods/picture | 14 | 15 | 19.5 |
| generate set | | 12.5 | |
| find missing value | | 14.5 | |
| Operations | 68.5 (30) | 37.5 (22) | 44.5 (32) |
| add (subt) | | | |
| like denom rods/symbols | 5.5 | 8 | 44.4 |
| unlike denom | | | |
| rods | 52 | 9 | |
| symbols | 11 | 29.5 | |
| | 232.5 | 166.5 | 139.5 |

*(percent of instructional time)

Table A.5 Frequency and Density* of Student Occlusions by Teacher and Activity.

| | Martha | | | | Zelda | | | | Ralph | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Day | Activity | Student Diff. | Student Insight | Density | Activity | Student Diff. | Student Insight | Density | Activity | Student Diff. | Student Insight | Density |
| 1 | 1 | 7 | 4 | .8 | 1 | 16 | 0 | 1.6 | 1 | 12 | 0 | .5 |
| | 2 | 3 | 0 | .6 | 2 | 2 | 0 | .4 | | | | |
| | 4 | 6 | 1 | 1.8 | | | | | | | | |
| 2 | 5 | 0 | 0 | 0.0 | | | | | | | | |
| | 6 | 10 | 1 | .5 | 4 | 7 | 0 | .6 | 3 | 3 | 0 | .5 |
| | 7 | 5 | 0 | 1.4 | 5 | 6 | 0 | .4 | 4 | 8 | 1 | .4 |
| | 10 | 3 | 0 | 1.0 | 6 | 0 | 0 | .5 | | | | |
| 3 | 11 | 5 | 3 | .5 | 7 | 12 | 4 | 1.3 | 6 | 3 | 2 | .3 |
| | 13 | 0 | 0 | 0.0 | | | | | | | | |
| | 14 | 10 | 3 | 1.4 | | | | | | | | |
| 4 | 15 | 26 | 0 | .7 | 9 | 18 | 0 | 1.0 | 8 | 7 | 0 | .4 |
| | 16 | 14 | 0 | 1.5 | 10 | 3 | 3 | .8 | 9 | 3 | 0 | .4 |
| | 17 | 8 | 1 | 1.6 | 11 | 16 | 2 | 1.0 | | | | |
| 5 | 18 | 3 | 2 | .7 | 12 | 10 | 2 | 1.1 | 11 | 5 | 1 | .2 |
| | 19 | 10 | 3 | .9 | 13 | 19 | 0 | 1.0 | | | | |
| | 20 | 2 | 7 | 1.6 | | | | | | | | |
| | 21 | 21 | 12 | 1.3 | | | | | | | | |
| 6 | 22 | 26 | 9 | 1.3 | 15 | 9 | 0 | 1.6 | 13 | 4 | 0 | .2 |
| | 23 | 4 | 1 | 1.1 | | | | | | | | |
| | 25 | 5 | 2 | .6 | | | | | | | | |
| 7 | | | | | 17 | 16 | 3 | 1.1 | | | | |

*Density is measured in number of occlusions per minute of instruction.

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