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PARAMETER ESTIMATION AND MODEL CONSTRUCTION FOR RECURSIVE CAUSAL MODELS WITH UNIDIMENSIONAL MEASUREMENT

By

David Wayne Gerbing

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ABSTRACT

PARAMETER ESTIMATION AND MODEL CONSTRUCTION
FOR RECURSIVE CAUSAL MODELS WITH
UNIDIMENSIONAL MEASUREMENT

By

David Wayne Gerbing

Two kinds of linear models often used in the social sciences are measurement models and causal models. Measurement models specify the relation of latent variables to observed variables and causal models relate latent variables to each other so that their ordering in the model specifies the outcome of an underlying causal process. The purpose of this paper is to compare the utility of two distinct estimation procedures which are embodied in two separate computer programs: PACKAGE and LISREL. The comparison is limited to recursive models with unidimensional measurement models.

LISREL is based on the more recently developed procedures of "full information maximum likelihood". LISREL simultaneously estimates the parameters of both the measurement and causal models. PACKAGE is based on least squares techniques. The parameters of the measurement model are estimated with centroid factor analysis with communalities in the diagonal, and then, in a separate step, the estimated correlations among the latent variables are subjected to an ordinary least squares (OLS) path analysis.

The results of this paper show PACKAGE to be a superior technique.

If the model is correctly specified, then both PACKAGE and LISREL recover the underlying structure, although LISREL does so at a much greater cost in computer time than does PACKAGE. If the model is misspecified, then LISREL spreads the errors related to the misspecified equations throughout the entire system so that all of the parameter estimates tend to be affected by the misspecification. In particular, even if the measurement model is correctly specified, a misspecification of the causal model precludes the correct recovery of the correlations among the latent variables. On the other hand, PACKAGE not only separates the estimation of the parameters of the two models, it is based on single equation techniques which localize the errors.

Some authors have claimed that the use of the LISREL first derivatives for detecting misspecification is superior to the use of residuals. Instead, the use of derivatives was shown to be misleading in certain cases, and under none of the circumstances investigated in this paper did the derivatives provide more information than the residuals.

One of the claimed advantages for LISREL was the capability of allowing for correlated disturbance terms and correlated measurement errors. Yet counter-examples were constructed in which the use of correlated errors was misleading. In particular, these examples demonstrate that, contrary to the current literature, omitted variables do not lead to correlated errors. False indication of correlated errors can be produced by (a) ad-hoc composites and (b) missing paths. In certain cases the covariance of "correlated disturbances" is exactly the OLS residual for a misspecified model defined by the deletion of a path.

Dedicated to my best friend

Monica Gerbing

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CHAPTER I

INTRODUCTION

Causal Models

The primary goal of science is theory construction. Most theories are descriptions of the causal processes among a set of variables. The direct study of the underlying causal processes, i.e., the study of the system dynamics, requires the construction of mathematical models of these processes expressed in terms of differential or difference equations (e.g., Hunter & Cohen, Note 1; Hunter, Nicol & Gerbing, Note 2). A model of dynamics allows one to construct a developmental sequence of the variables in the system--a trajectory--which can be compared to the actual behavior of the system.

Within the social sciences, however, the causal processes are usually not studied directly. Instead a causal theory, most often expressed qualitatively, is used to predict the relations among the variables. The model is tested by observing these relations after the operation of the causal processes. Thus the distinction is drawn between (a) the underlying causal processes and (b) the effect these processes have on the relations between the system variables. And the outcome of the process, not the process per se, is usually the object of study within the social sciences.

The statistical methodology employed to test a causal theory by studying the outcome of the causal processes is, in part, provided by

regression analysis. For example, if a model identifies variable X as a causal antecedent of variable Y, then it follows that variables X and Y are related. If this relation is linear, then (a) the variables should be correlated, and (b) the slope parameter of the regression of Y on X, which indicates the linear relation between Y and X with the other predictor variables in the equation held constant, is interpreted as the indicator of the causal impact X has on Y.

A "complete" system involves a set of variables such that many of the variables are consequent to some variables and antecedent to other variables. A causal theory which predicts such a network of relations can be tested by the analysis of a set of simultaneous regression equations. But the distinction should always be maintained between this model of relations among the variables following the outcome of the causal processes and the model of dynamics which is tested by the outcome model, even if the process model exists only in the form of verbal relations.

An example of a causal model. Consider a theory which predicts that X_1 causes X_2 and that X_2 causes X_3 but X_1 does not directly influence X_3. The implications of this theory for the relations among the variables can be represented in either diagrammatic or equation form. If the relations among the variables are linear, the equations which define the model are:

$$X_2 = p_{21}X_1 + U_2$$

$$X_3 = p_{32}X_2 + U_3$$

The equivalent representation of the model in diagrammatic form appears in Figure 1. X_1 is the single exogenous variable of the system (i.e., no antecedents specified) and X_2 and X_3 are the two endogenous

variables of the system. No constant term appears in the equations since the variables are mean deviates. The error terms, also called residuals or disturbances and denoted by U_i, indicate that the specification of the causal influences of the variables X_2 and X_3 is incomplete. Each disturbance is also specified to be uncorrelated with the predictor variables in the corresponding equation and with the other disturbances. That is, the disturbances are specified as random influences.

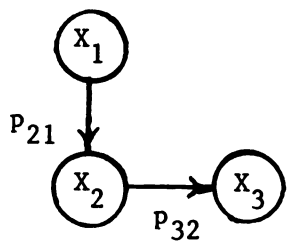


Figure 1. A path diagram.

The model illustrated above is called a recursive model. For some authors (e.g., Duncan, 1975) a model is recursive if all of the causal linkages or paths form a hierarchy, i.e., the paths flow so that a path never leads back to the same variable from which the path began. Other authors (e.g., Heise, 1975) require that a recursive model be both hierarchical and that the disturbance terms across equations be independent. In this paper, the latter definition will be used to aid the distinction between hierarchical models with correlated disturbances and recursive models.

Structural equations. The regression equations are called structural equations and the regression weights are called structural parameters or path coefficients. The diagram is called a path diagram. The structural equations and/or the path diagram define a causal model

which is a specific interpretation of the underlying causal processes described by the theory. Since the relations between the variables are linear, the expected differences (denoted by "Δ") would be given by:

$$\Delta X_2 = p_{21}\Delta X_1 \text{ and } \Delta X_3 = p_{32}\Delta X_2$$

For example, if manipulation or natural processes changed the value of X_1 by one unit for all of the individuals in the population, then the theory specifies that the mean difference in X_2 would be:

$$\Delta X_2 = p_{21}(1) = p_{21}$$

and predicted mean difference in X_3 would be:

$$\Delta X_3 = p_{32}\Delta X_2 = p_{32}p_{21}$$

Thus the path coefficients indicated the amount of change that can be expected in the system variables given a change in antecedent variables following the operation of the underlying causal processes.

The distinction between structural equations and regression equations which are not structural equations is the distinction between theory and blind prediction. If the variables can be measured, any variable can be regressed on any other variable or set of linearly independent variables without consideration of causality. For example, consider the situation in which two variables, Y_2 and Y_3, share a common antecedent Y_1 but are not causally related in any other way. The variable Y_3 can be mathematically expressed as a function of Y_2 even though this relationship does not mirror an existing causal relationship. From a purely mathematical perspective, the equation $Y_3 = \gamma Y_2 + U$ implies $\Delta Y_3 = \gamma \Delta Y_2$, but this derived relationship among the changes in the two variables has no counterpart in physical processes. That is, a manipulation of a unit change in Y_2 independent

of Y_1 will not cause a change in Y_3.

The covariance structure of a causal model: An example. The key to the evaluation of causal models is the calculation of the covariances among the variables predicted by the model. The central idea is that the model imposes constraints on the covariances among the variables and it is these constraints on which the test of the model is based. For example, consider the causal model presented at the beginning of this paper in which X_1 causes X_2 and X_2 causes X_3. Multiplying both sides of the equation for X_3 by X_1 yields:

$$X_3X_1 = p_{32}X_2X_1 + X_1U_3$$

Taking expected values,

$$E(X_3X_1) = p_{32}E(X_2X_1) + E(X_1U_3)$$

Since the variables are mean deviates,

$$\sigma(X_3X_1) = p_{32}\sigma(X_2X_1) + \sigma(X_1U_3)$$

But $\sigma(X_1U_3) = 0$. And if the variables are standardized,

$$r_{31} = p_{32}r_{21}$$

Finally, $p_{32} = r_{32}$ since there is only a single predictor in the equation for X_3, so

$$r_{31} = r_{32}r_{21}$$

Thus the model imposes a structure upon the correlations among the variables. In this simple example it is this single constraint which provides a test of the model by comparing the observed value of r_{31} with the predicted value $\hat{r}_{31}$ obtained by multiplying the observed values of r_{32} and r_{21}.

This test of the model that $r_{31} = \hat{r}_{31}$ is equivalent to tests based on partial correlation coefficients and multiple regression

coefficients. Since

$$r_{31\cdot 2} = \frac{r_{31} - r_{32}r_{21}}{\left(1 - r_{32}^2\right)^{\frac{1}{2}}\left(1 - r_{21}^2\right)^{\frac{1}{2}}} = \frac{r_{31} - \hat{r}_{31}}{\left(1 - r_{32}^2\right)^{\frac{1}{2}}\left(1 - r_{21}^2\right)^{\frac{1}{2}}}$$

the model implies $r_{31\cdot 2} = 0$. If X_3 is regressed on X_1 and X_2 and $\beta_{31\cdot 2}$ is the corresponding multiple regression weight, then:

$$\beta_{31\cdot 2} = \frac{r_{31} - r_{32}r_{21}}{1 - r_{21}^2} = \frac{r_{31} - \hat{r}_{31}}{1 - r_{21}^2}$$

Thus, the model implies $\beta_{31\cdot 2} = 0$. The variables X_3 and X_1 are related since $r_{31} \neq 0$, but only through the intervening variable X_2.

The general principle of constructing a model defined by regresaion equations and then using this model to derive the constraints imposed by the model on the covariances among the variables has been known for over 50 years. In 1921, Wright introduced "path analysis" as a test of recursive causal models, though it was not until Simon's (1957) work, followed by Duncan (1966) and Blalock (1964) that path analysis began to be used by social scientists other than econometricians. The computation of the predicted correlations of a causal model is covered extensively in the literature on causal models. Standard references are the texts by Heise (1975), Duncan (1975), Asher (1976), and the article by Lewis-Beck (1974).

Measurement Models

Latent variables. Before the variables can be related by a set of regression equations, they must be measured. The variables of interest, the variables in the causal model, are called the latent variables since they are not directly observed. The observed variables are the indicators of the latent variables. The relation between the

indicators and the latent variables is specified by the measurement or factor model. The measurement model is a set of simultaneous regression equations of the indicators regressed on the latent variables.

Each measure is an imperfect indicator of a latent variable because of random response error and invalidity. The problem of measurement error for the analysis of causal models is more serious than simply the lack of precise measures of the latent variables. The analysis of a causal model based on fallible data yields biased parameter estimates (e.g., Wiley, 1973). For a regression equation with a single predictor, measurement error in the independent variable attenuates the slope coefficient in proportion to the reliability of the independent variable. However, for multiple regression equations the effect of measurement error of the independent variables on the estimates of the regression coefficients is usually unpredictable. Wiley (1973) presented an example of a two-predictor multiple regression equation in which the presence of measurement error reversed the magnitude of the sample parameter estimates from the values of the population parameters. The problem of measurement error can be countered by providing multiple indicators of each latent variable. The use of multiple indicators improves the precision of the estimates of the latent variables and allows reliability estimates of the composite scores to be computed. Traditionally, these reliability estimates are used to correct for attenuation the correlations among the latent variables computed from the observed composites.

The covariance structure of unidimensional measurement models. The testing of measurement models in which the measures are partitioned into clusters such that the measures in each cluster are postulated to

be alternate indicators of only a single common latent variable is called Spearman factor analysis or oblique multiple groups analysis (Tryon, 1939; Holzinger, 1944). The original conceptualizations were provided by Spearman in 1904. The computations of a multiple groups analysis involve any factor analytic method by which a single factor is extracted from each group. The computations provide estimates of the factor loadings of each indicator on the group factors.

An extensive discussion of the construction and evaluation of unidimensional measurement models, an example of a procedure called confirmatory factor analysis, is provided by Hunter (Note 3) and Hunter and Gerbing (Note 4). Measurement models are evaluated according to the same principles which underlie the evaluations of causal models. The basic idea is to derive the covariances among the observed variables predicted by the model. The covariance structure of unidimensional measurement models is given below.

As presented in Figure 2, let the observed variables X_1 and X_2 be indicators of latent variable (or true score or factor) F and let Y be an indicator of G. The errors of measurement are specified as random. The curved double-headed arrow in Figure 2 represents a correlation without the specification of causality.

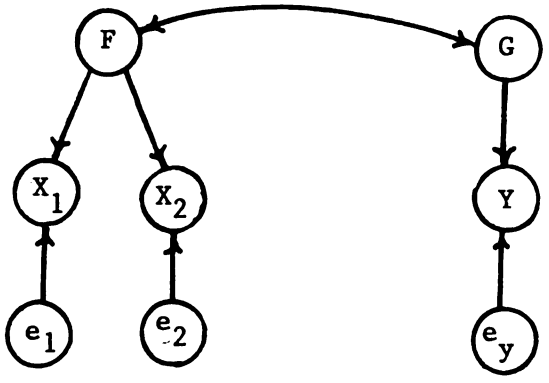


Figure 2. A two-factor multiple groups measurement model.

In equation form,

$$X_1 = \beta_1 F + e_1$$

$$X_2 = \beta_2 F + e_2$$

and $$Y = \beta_Y G + e_Y$$

where $r(F,e_i) = r(G,e_i) = r(e_i,e_j) = 0$.

The covariance structure of the indicators can be described by considering (a) the covariances among indicators of different factors, and (b) the covariances among indicators of the same factor. Consider first the relation between X_1 and Y. The model implies:

$$X_1 Y = \beta_1 \beta_Y FG + \beta_1 F e_1 + \beta_Y G e_Y + e_1 e_Y$$

$$E(X_1 Y) = \beta_1 \beta_Y E(FG) + \beta_1 E(F e_1) + \beta_Y E(G e_Y) + E(e_1 e_Y)$$

$$\sigma_{1Y} = \beta_1 \beta_Y \sigma_{FG}$$

Standardizing the observed and latent variables,

$$r_{1Y} = \beta_1 \beta_Y r_{FG}$$

where β_1 and β_2 are standardized regressions weights. Since

$$\beta_1 = r_{1F} \quad \text{and} \quad \beta_Y = r_{YG}$$

by substitution,

$$r_{1Y} = r_{1F} r_{FG} r_{GY}$$

Thus one test of the measurement model is the comparison of the observed value of r_{1Y} with the value predicted by the model r_{1Y}, which is the product of the observed values of r_{1F}, r_{FG}, and r_{GY}.

Hunter and Gerbing (Note 4) call the expression $r_{1Y} = r_{1F} r_{FG} r_{GY}$ the product rule for external consistency of items. By similar logic, they also derive the product rule for external consistency of items and

factors, which is,

$$r_{1G} = r_{1F}r_{FG}$$

If X_1 and X_2 are both indicators of the same factor F, an equivalent test based on these covariance restrictions can be used to test the unidimensional measurement model. The product rules imply that the correlations of X_1 and X_2 with other items or other factors are proportional to their respective factor loadings on their own group factors. That is, for some other factor G,

$$\frac{r_{1G}}{r_{2G}} = \frac{r_{1F}r_{FG}}{r_{2F}r_{FG}} = \frac{r_{1F}}{r_{2F}}$$

and, if Y is an indicator of factor G,

$$\frac{r_{1Y}}{r_{2Y}} = \frac{r_{1F}r_{FG}r_{GY}}{r_{2F}r_{FG}r_{GY}} = \frac{r_{1F}}{r_{2F}}$$

These proportionality rules are equivalent to the tetrad difference criterion, which is usually written as

$$r_{1G}r_{2F} = r_{2G}r_{1F} \quad \text{or} \quad r_{1Y}r_{2F} = r_{2Y}r_{1F}$$

They were presented in the context of factor analysis by Spearman in 1907 and first illustrated in the context of data analysis by Burt (1909) and Spearman (1914).

The stability of this proportionality of the correlations of two indicators of the same factor across other indicators and/or factors is indexed by the following formula, originally used as a measure of the similarity of two factors.

$$\phi(X_iX_j) = \frac{\sum_{k=1}^{n} r(X_iX_k)\, r(X_jY_k)}{\left(\sum_{k=1}^{n} \{r(X_iX_k)\}^2\right)^{\frac{1}{2}} \left(\sum_{k=1}^{n} \{r(X_jX_k)\}^2\right)^{\frac{1}{2}}}$$

Hunter (1973) called this index a "similarity coefficient". Conceptually, the "intercolumnar criterion" is an "unadjusted correlation . . . a product-moment coefficient based on absolute deviations, instead of on the deviations about the means" (Burt, 1940, p. 343). If communalities are placed in the diagonal of the correlation matrix, the value of ϕ ranges from -1 to 1 with ϕ equaling 1 or -1 if the two items, X_i and X_j, have correlations that are perfectly proportional.

The product rules for external consistency and the implied proportionality rules specify the relations between indicators of a factor with variables external to the factor. The constraint imposed on the correlations between indicators of the same factor is called the rule for internal consistency. The internal consistency product rule is a special case of the external consistency product rule in which $r_{FG} = 1$. That is,

$$r_{12} = r_{1F}r_{F2}$$

where X_1 and X_2 are both indicators of F. This product rule is Spearman's (1904) general factor equation and is a special case of Thurstone's (1931) "fundamental theorem of factor analysis" for the single factor solution.

Other authors have independently discovered these properties. For example, Tryon called indicators which meet these constraints "collinear" (Tryon & Bailey, 1970). Jöreskog (1971) called the latent variables "congeneric measures" and Burt (1976) called the latent variables "point variables" if the indicators of the latent variables satisfy these constraints. Kenny (1979) called the product rules for internal and external consistency the rules for "homogeneity within constructs" and "homogeneity between constructs".

Evaluation of the Full Causal Model

The analyses of the regression equations which define the measurement and causal models are similar. The analysis begins with the construction of the model, either a measurement model or a causal model or both. Estimates of the parameters of the regression equations which define the model are computed from the observed sample correlation matrix. Given the model and the parameter estimates, the covariance matrix among the variables implied by the model can be computed. The comparison of implied and actual covariance matrices provides an evaluation of the fit of the model to the data. Since the fit of the model can usually be improved by constructing a revised model, the previous steps are repeated until the investigator has obtained a good fitting model or abandons the project. The steps involved in such an analysis are illustrated in the flow chart in Figure 3.

Competing statistical procedures exist for computing parameter estimates. Should the measurement and causal models be analyzed separately or simultaneously? Should least squares or maximum likelihood methods be used? Should single equation or full information methods be used? The central issue of this paper is a comparison of these competing techniques.

OLS and centroid factor multiple groups analysis. Hunter has argued that testing the full causal model can be accomplished in two separate steps: test the multiple indicator measurement model using a multiple groups analysis, and then submit the estimated correlation matrix between latent variables from the multiple groups analysis to a path analysis. Examples of this procedure are found in Hunter, Hunter and Lopis (in press) and Hunter, Gerbing, and Boster (Note 5).

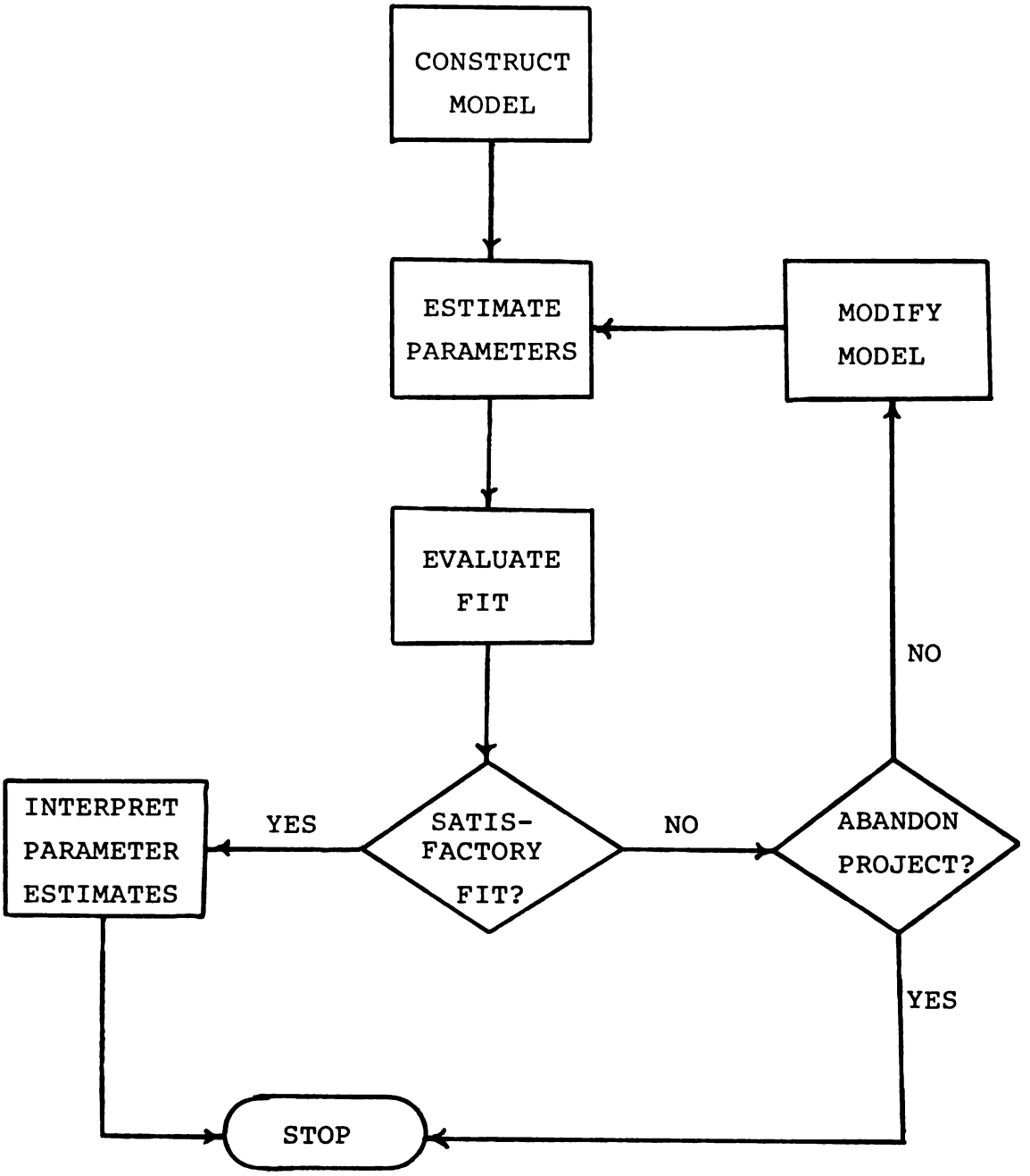


Figure 3. A flow chart of the model building process.

The initial computations of a multiple groups analysis are presented in Holzinger (1944), Hunter (Note 3), and Hunter and Gerbing (Note 4). The method consists of extracting a first centroid factor from each cluster of indicators. The extraction of this group factor is accomplished by computing the correlations of the indicators with the factors and the correlations among the factors. If communalities are not used in the analysis, then the first centroid factor is simply the sum of the indicators which define the group, i.e.,

$$F = X_1 + X_2 + X_3 + \ldots + X_n$$

where X_1, X_2, ..., X_n are the indicators of factor F. The multiple groups analysis is based on the repeated application of the rule "the covariance of a sum is the sum of the covariances" to the following correlational formulas:

$$r(X_i,F_j) = \frac{\sigma(X_i,F_j)}{\sigma(X_i)\sigma(F_j)}$$

$$r(F_i,F_j) = \frac{\sigma(F_i,F_j)}{\sigma(F_i)\sigma(F_j)}$$

The remaining step in the analysis is the computation of the communality estimates which is accomplished with an iterative procedure outlined in Hunter (Note 3) and Hunter and Gerbing (Note 4). The communality of interest for each observed variable is the communality of the indicator across the remaining indicators in the corresponding group. As Hunter (Note 3) demonstrated, if these communalities are inserted in the diagonal of the correlation matrix, then the observed correlation matrix among the indicators is transformed to the covariance matrix of true scores. Hunter (Note 3) also demonstrated, by example, that factoring

a covariance matrix of true scores implies that the computed factor loadings and factor-factor covariances are corrected for attenuation (Spearman, 1907) due to the measurement error in the composite (i.e., factor) scores. At the same time, the use of communalities also eliminates the spuriously large "part-whole" correlations between indicator and factor, i.e., communalities eliminate the upward bias of correlating an indicator with a composite of which it is a part.

However, if the indicators can not be measured without error, communalities must be used in the multiple groups analysis if parameter values are to be correctly estimated. The convergence of this iterative process for these data is discussed in Appendix A.

The input to the multiple groups analysis is the correlation matrix of the observed variables. The input to the path analysis is the correlation matrix among the latent variables. Parameter estimates of recursive models have traditionally been accomplished with ordinary least squares (OLS). That is, the estimated parameter values are those values that minimize the sum of the squared errors of the sample data points about the sample regression surface for each equation in the model.

Both centroid factor multiple groups analysis and OLS path analysis are called <u>single equation</u> estimation procedures. That is, the computations of these analyses are accomplished "equation by equation"; only the variances and covariances of the variables which appear in a particular equation are used in the computations of the estimated parameters of that equation. The general equation for a multiple indicator measurement model is

$$X_i = \lambda_i F + e_i$$

since each indicator or observed variable is a function of only a single factor plus measurement error. In a recursive model, the factors can be ordered so that all variables causally antecedent to a given factor are listed ahead of it. If the variables are so listed, then the general equation for a recursive path analysis is

$$F_i = \sum_{k=1}^{i-1} P_{ik} F_k + U_i$$

The computation of both the centroid factor multiple groups analysis and the OLS path analysis may be accomplished with PACKAGE (Hunter & Cohen, 1969). The use of the multiple groups subprogram is explained by Hunter and Cohen (Note 6) and Hunter and Gerbing (Note 4). The path analysis subprogram is explained by Hunter (Note 7).

LISREL. Based on the work of Lawley (1943) and Bock and Bargmann (1966), Jöreskog (e.g., 1967, 1978) has developed an alternative analysis of causal and measurement models which differs from the OLS with correction for attenuation strategy along two different dimensions. Jöreskog has also developed a computer program, LISREL, which contains the computational algorithms (Jöreskog & Sörbom, 1978). Both analytic methods, OLS with correction for attenuation and LISREL were developed to account for the biasing effects of measurement error. However, LISREL simultaneously estimates the parameters of the measurement and causal models. LISREL also is not restricted to testing multiple indicator measurement models in which each indicator is an indicator of only a single factor, i.e., multiple groups measurement models. Nor does LISREL require the measurement errors to be uncorrelated. In terms of the causal model, LISREL is not restricted to the analysis of recursive path models. The causal model may be nonrecursive in that the

disturbances may be correlated and/or "feedback loops" may appear in the model.

The complexity of the general LISREL model requires the use of matrix algebra in its representation. Since Jöreskog's parameterization of the model includes an explicit distinction between exogenous and endogenous variables, there is also a much greater notational complexity. Many of these symbols are defined in Table 1. Other symbols include Σ_{xy} for the covariance matrix of the exogenous and endogenous observed variables and Σ for the covariance matrix of all of the indicators.

The LISREL model. Jöreskog has parameterized the general measurement model in separate equations for the exogenous latent variables and a model for the endogenous latent variables.

$$\underset{pxl}{\underline{y}} = \underset{pxm}{\Lambda_y} \underset{mxl}{\underline{\eta}} + \underset{pxl}{\underline{\varepsilon}} \quad \text{and} \quad E(\underline{\eta}\,\underline{\varepsilon}') = 0$$

$$\underset{qxl}{\underline{X}} = \underset{qxn}{\Lambda_x} \underset{nxl}{\underline{\xi}} + \underset{qxl}{\underline{\delta}} \quad \text{and} \quad E(\underline{\xi}\,\underline{\delta}') = 0$$

The i^{th} row of the factor pattern matrices Λ_y and Λ_x corresponds to the i^{th} endogenous or exogenous indicator. The j^{th} column of each of the matrices refers to the j^{th} endogenous or exogenous factor. Thus if the scalar equations take the form of

$$y_i = \lambda_{y_i} \eta + \varepsilon_i \quad \text{or} \quad x_i = \lambda_{x_i} \xi + \delta_i$$

as is the case for the multiple indicator models with one indicator per factor, then each row of Λ_y and Λ_x will contain a single λ with the remainder of the elements in the row equal to zero.

The parameterization of the causal model is best understood by beginning closer to the more traditional path analytic parameterization,

Table 1

LISREL Notation

Variables

		Variable symbol	Number of such variables	Covariance matrix
Exogenous	Observed	x	q	Σ_{xx}
	Unobserved	ξ	n	ϕ
	Measurement error	δ	q	Θ_{δ}
Endogenous	Observed	y	p	Σ_{yy}
	Unobserved	η	m	C
	Measurement error	ε	p	Θ_{ε}
	Disturbance	ζ	m	ψ

Parameters

	Scalar	Matrix	Function
Measurement model	λ_x	Λ_x	$\xi \xrightarrow{\lambda_{x_i}} x_i$
	λ_y	Λ_y	$\xi \xrightarrow{\lambda_{y_i}} y_i$
Causal model	γ	Γ	$\xi \xrightarrow{\gamma} \eta$
	β	B	$\eta_i \xrightarrow{B_{ji}} \eta_j$

$$F_i = \sum_{k=1}^{N} P_{ik} F_k + U_i$$

where there are N latent variables, designated by F. In matrix form,

$$\underset{Nx1}{\underline{F}} = \underset{NxN}{P}\ \underset{Nx1}{\underline{F}} + \underset{Nx1}{\underline{U}}$$

If a distinction is made between exogenous and endogenous factors, then the model can be rewritten as

$$\underset{mx1}{\underline{\eta}} = \underset{mxm}{A}\ \underset{mx1}{\underline{\eta}} + \underset{mxn}{\underline{\Gamma}}\ \underset{nx1}{\underline{\xi}} + \underset{mx1}{\underline{\zeta}}$$

For recursive models, A is a lower triangular matrix with 0's down the main diagonal and $\psi \equiv E(\underline{\zeta}\,\underline{\zeta}')$ is a diagonal matrix with the disturbance variances down the main diagonal.

However, Jöreskog's parameterization of the causal model is based on the following transformation of the above equation.

$$\underline{\eta} - A\,\underline{\eta} = \Gamma\,\underline{\xi} + \underline{\zeta}$$

$$(I - A)\underline{\eta} = \Gamma\,\underline{\xi} + \underline{\zeta}$$

$$B\,\underline{\eta} = \Gamma\,\underline{\xi} + \underline{\zeta} \quad \text{and} \quad E(\underline{\xi}\,\underline{\zeta}') = 0$$

where $B \equiv I - A$. Thus B has 1's down the main diagonal and the path coefficients are reversed in sign. For recursive models, B is still lower triangular.

The LISREL covariance structure. Given the three equations which define the model, it is now possible to derive the covariances among the indicators predicted by the model. The derivation is based on the partitioning of Σ shown in Figure 4.

$$\Sigma = \left[\begin{array}{c|c} \Sigma_{yy} & \\ \hline \Sigma_{yx} & \Sigma_{xx} \end{array}\right]$$

Figure 4. The LISREL partitioning of Σ.

For example, to compute the implied covariances among the indicators of the exogenous variables,

$$\Sigma_{xx} \equiv E(\underline{x}\,\underline{x}') = E\{(\Lambda_x \underline{\xi} + \underline{\delta})(\Lambda_x \underline{\xi} + \underline{\delta})'\}$$

$$= E(\Lambda_x \underline{\xi}\,\underline{\xi}'\Lambda_x') + E(\Lambda_x \underline{\xi}\,\underline{\delta}') + E(\underline{\delta}\,\underline{\xi}'\Lambda_x') + E(\underline{\delta}\,\underline{\delta}')$$

$$= \Lambda_x E(\underline{\xi}\,\underline{\xi}')\Lambda_x' + \Lambda_x E(\underline{\xi}\,\underline{\delta}') + E(\underline{\delta}\,\underline{\xi}')\Lambda_x' + E(\underline{\delta}\,\underline{\delta})'$$

$$= \Lambda_x \phi \Lambda_x' + \underline{\Theta}_\delta$$

which is the usual expression for the correlation of the observed variables expressed as a function of the factor pattern matrix, the correlation matrix among the oblique factors, and the communalities.

Similar derivations lead to

$$\Sigma_{yx} \equiv E(\underline{y}\,\underline{x}') = E\{(\Lambda_y \underline{\eta} + \underline{\varepsilon})(\Lambda_x \underline{\xi} + \underline{\delta})'\}$$

$$= \Lambda_y E(\underline{\eta}\,\underline{\xi}')\,\Lambda_x'$$

and

$$\Sigma_{yy} \equiv E(\underline{y}\,\underline{y}') = E\{(\Lambda_y \underline{\eta} + \underline{\varepsilon})(\Lambda_y \underline{\eta} + \underline{\varepsilon})'\}$$

$$= \Lambda_y C \Lambda_y' + \underline{\Theta}_\varepsilon$$

But, the covariances among the endogenous factors, $C \equiv E(\underline{\eta}\,\underline{\eta}')$, can be decomposed according to the causal model. Since

$$B\underline{\eta} = \Gamma\,\underline{\xi} + \underline{\zeta},$$

$$\underline{\eta} = B^{-1}\underline{\Gamma}\,\underline{\xi} + B^{-1}\underline{\zeta}$$

So

$$E(\underline{\eta}\,\underline{\eta}') = E\{(B^{-1}\Gamma\,\underline{\xi} + B^{-1}\underline{\zeta})(B^{-1}\underline{\Gamma}\,\underline{\xi} + B^{-1}\underline{\zeta})'\}$$

$$= B^{-1}\Gamma\,\phi\,B'^{-1} + B^{-1}\psi\,B'^{-1}$$

Similarly,

$$E(\underline{\eta}\,\underline{\xi}') = B^{-1}\Gamma\,\phi$$

So

$$\Sigma_{yx} = \Lambda_y B^{-1}\Gamma\,\phi\,\Lambda_x'$$

$$\Sigma_{yy} = \Lambda_y(B^{-1}\Gamma\phi B'^{-1} + B^{-1}\psi B'^{-1})\Lambda_y' + \underline{\Theta}_\varepsilon$$

And, from before,

$$\Sigma_{xx} = \Lambda_x\phi\Lambda_x' + \underline{\Theta}_\delta$$

Thus the predicted covariance matrix of the indicators, $\hat{\Sigma}$, has been parameterized in terms of the estimators of the four covariance matrices, $\hat{\phi}$, $\hat{\psi}$, $\hat{\Theta}_\varepsilon$, $\hat{\Theta}_\delta$, and the four parameter matrices, $\hat{\Gamma}$, $\hat{B}$, $\hat{\Lambda}_y$, $\hat{\Lambda}_x$.

In the two-step strategy of OLS preceded by centroid factor analysis, (a) the correlations among the indicators are parameterized only in terms of the factor pattern, the factor correlations, and the communalities, and (b) the covariances among the factor correlations are parameterized only in terms of the causal model. In contrast, Jöreskog simultaneously parameterizes the covariances among the indicators in terms of both the measurement and causal models. The complexity of the algebra varies greatly across the two strategies, but the key ideas remain the same: (a) construct a model, (b) derive the implications of this model for the covariance structure of the variables, i.e., compute $\hat{\Sigma}$, and (c) test the model by comparing $\hat{\Sigma}$ to the observed sample covariance matrix S.

The metric of the latent variables in a simultaneous LISREL analysis. A potential source of confusion in the interpretation of the simultaneous LISREL analysis is the metric of the latent variables. If the variance of each latent variable is not set by the user, the model is underidentified. The classic solution to this problem is to set the variance of the factor to 1, i.e., to standardize the factors. Instead, Jöreskog and Sörbom (1978) chose to fix the factor loading of one of

the indicators of each latent variable at 1.00. Jöreskog does not clearly indicate the effect of this specification on the parameter estimates other than that "the scales for [the latent variables] have been chosen to be the same as for [the corresponding indicators whose factor loadings were set at 1.0]" (1978, p. 468). The phrase "the scales [are] the same as" is often interpreted to imply that the metric of the latent variable is set at the metric of the corresponding indicator with a fixed factor loading of 1.0, though this is not true.

The implications of this practice can be derived by beginning with the basic relationship

$$y = \lambda \eta + \varepsilon \quad \text{or} \quad \sigma_y^2 = \lambda^2 \sigma_\eta^2 + \sigma_\varepsilon^2 \quad \text{or} \quad \sigma_\eta^2 = \frac{\sigma_y^2 - \sigma_\varepsilon^2}{\lambda^2}$$

It follows that, for the specific case $\sigma_y^2 = \sigma_\eta^2 = 1$ in which the true value of λ is used,

$$1 = \lambda_{TRUE}^2 + \sigma_\varepsilon^2 \quad \text{or} \quad \sigma_\varepsilon^2 = 1 - \lambda_{TRUE}^2$$

That is, the value of σ_ε^2 computed by LISREL is determined by the actual value of λ if the value of σ_ε^2 is not fixed a priori by the user. But if λ is fixed at 1.0 while σ_ε^2 remains free,

$$\sigma_\eta^2 = \frac{\sigma_y^2 - \sigma_\varepsilon^2}{\lambda^2} = \frac{1 - (1 - \lambda_{TRUE}^2)}{1} = \lambda_{TRUE}^2$$

Fixing the value of a factor loading at 1.0 determines the variance of the corresponding latent variable for computational purposes, but this variance is an arbitrary and meaningless metric which is neither the variance of the observed variable nor is it the actual variance of the latent variable. Consequently, the computed covariance matrices of the latent variables and of the latent and observed variables are expressed in a metric which does not correspond

to the parameters of the model used to generate the data.

Moreover, the maximum likelihood estimates (and the corresponding standard errors) of the regression parameters of the measurement model do not correspond to the parameters of the model used to generate the data because the relation

$$\sigma^2_{y_i} = \lambda^2_i \sigma^2 + \sigma^2_{\varepsilon_i}$$

must be satisfied. If the factor loading λ of the first indicator of η is fixed at 1.0, then obviously $\lambda_{1(\text{TRUE})} \neq \lambda_{1(\text{LISREL})}$. But consider the i^{th} indicator. The value of $\sigma^2_y = 1$ is given. LISREL computes the values of σ^2_η and $\sigma^2_{\varepsilon_i}$ as

$$\sigma^2_\eta = \lambda^2_{1(\text{TRUE})}$$

$$\sigma^2_{\varepsilon_i} = 1 - \lambda_{i(\text{TRUE})}$$

Thus,

$$\lambda_{i(\text{LISREL})} = \frac{\sigma^2_{y_i} - \sigma^2_{\varepsilon_i}}{2}$$

$$= \frac{1 - \left(1 - \lambda^2_{i(\text{TRUE})}\right)}{\lambda^2_{1(\text{TRUE})}}$$

$$= \frac{\lambda^2_{i(\text{TRUE})}}{\lambda^2_{1(\text{TRUE})}}$$

For example, if the true or actual factor loadings of the indicators of a given latent variable were .80, .60, and .40, and if the factor loading of the first variable was set at 1.0, then the computed factor loadings would be 1.00, .75, and .50. The sets of factor loadings are equivalent except for the constant of proportionality, .80.

If the variance of the latent variables is 1.0, the covariance matrices and the measurement model parameters happen to correspond to the correct values in the standardized solution. Unfortunately, LISREL prints only the factor pattern matrix and the covariance matrix of the endogenous variables (in addition to the causal parameter matrices) in the standardized solution. If the full factor loading matrix, the full matrix of covariances among latent variables, or the covariances among latent and observed variances is desired in which the latent variables are expressed in variances chosen by the user, then the following procedure is suggested.

Use an arbitrary metric such as that obtained with fixing an indicator of each latent variable at 1.0. Take the value of σ_ϵ^2 provided by LISREL and substitute it into the following formula in which σ_y^2 is given by the data and the value of σ_η^2 is chosen by the user.

$$\lambda = \frac{\sigma_y^2 - \sigma_\epsilon^2}{\sigma_\eta^2}$$

The LISREL program can then be rerun with a single factor loading for each latent variable set at the corresponding value of λ as determined by the formula. If the metric of the observed variable is to match the metric of the latent variable, the formula simplifies to

$$\lambda = 1 - \frac{\sigma_\epsilon^2}{\sigma_y^2}$$

If the observed variables have variance 1.0 and the latent variables are chosen to have variance 1.0 also, then

$$\lambda = 1 - \sigma_\epsilon^2$$

which is just the value of λ computed by LISREL in the standardized solution.

LISREL parameter estimation. LISREL estimation is based on a method called "full information maximum likelihood" (FIML). Instead of estimating the value of each parameter from a different subset of the covariances, LISREL uses all of the data in the computation of each parameter estimate (unless particular restrictions are placed on the model).

LISREL assumes that the latent variables and the measurement errors are generated by a multivariate normal distribution with a mean vector $\underline{\mu}$ and covariance matrix Σ. Wishart (1928) has shown that the observed sample covariance matrix S has what is now called a Wishart density, which is expressed in terms of the covariance matrix Σ and the sample size n. The parameter estimates of LISREL are given by the set of values which maximizes the value of the corresponding multivariate likelihood function. The effective log likelihood function of the Wishart density, i.e., the logarithm of the density function with constant terms deleted, is

$$\log L(\Sigma) = -1/2\ n \left[\log |\Sigma| + tr(\Sigma^{-1}S)\right]$$

In practice, however, LISREL minimizes a transformation of log L called F where

$$F(\Sigma) = \log |\Sigma| + tr(S\Sigma^{-1}) - \log |S| - (p + q)$$

Since $\log |S|$ and p and q are constants for a given S, and since the sign of F is the opposite of the sign of log L, maximizing log L is equivalent to minimizing F. In terms of computations, Jöreskog has chosen to minimize F since this transformation of log L is directly related to the likelihood ratio test of the fit of the model--a topic

which will be examined in more detail later.

F is minimized by computing the partial first derivatives of each of the free parameters with respect to F, setting the resulting set of simultaneous equations to zero, and solving for the values of the parameters. Unfortunately, an analytic solution is not available for most models, so the function is minimized with an iterative procedure from numerical analysis. The method of Fletcher and Powell (1963) is "a rapidly converging iterative procedure for minimizing a function of several variables when analytical expressions for the first-order derivatives are available" (Jöreskog, 1969, p. 187). The procedure is based on an approximation of the inverse of the matrix of second derivatives. As the program iterates, the approximation improves until it converges to the best approximation of the inverse at the minimum of F. Details of the method as applied to LISREL are provided by Gruvaeus and Jöreskog (Note 8). As with most iterative procedures, there is the problem of local minima: "If there are several minima of F there is no guarantee that the method will converge to the absolute minimum" (Jöreskog & Sörbom, 1978).

LISREL two-step analysis. Although most studies in the literature have used LISREL to simultaneously estimate the parameters of the causal and measurement models and provide the test of fit for the model as a whole, LISREL could also be used in a two-step analysis (e.g., Kohn & Schooler, 1978). LISREL could be used instead of centroid factor multiple groups analysis to perform a FIML multiple groups analysis to test the measurement model and estimate the factor correlations. LISREL could then be used instead of OLS to do the path analysis on the estimated factor correlations. However, for a

recursive model with independent disturbances, Land (1973) has demonstrated that "FIML reduces to equation by equation least squares regression provided that there are no cross-equation constraints on the coefficients" (p. 42).

Detecting Misspecification

The actual vs. the specified model. The goal of the investigator is to build a properly specified model. Ordinarily, the interpretation of the parameter estimates would not be of interest unless the model was a valid representation of the outcome of the underlying causal processes. The problem is that typically the initial model specified by the investigator does not fit the data as well as a competing model composed from the same set of variables. That is, the initial specified model is usually not the actual model of the relations between the variables which exist following the operation of the underlying causal processes.

The most important task facing the researcher after a model has been constructed is the detecting and subsequent correction of misspecification. Thus the first two questions the researcher usually asks after the first computer run are (a) how badly did the model fit? and (b) how can the model be revised to improve the fit? Both OLS and LISREL offer several indices of the fit of both the model as a whole and for specific equations of the model.

The residual matrix (OLS and LISREL). The residual matrix is defined by the difference of the predicted covariances implied by the equations of the model and the observed covariances, i.e., $\hat{\Sigma} - S$ in the notation of LISREL. The OLS program contained within PACKAGE (i.e., PATHPAC) prints the residual matrix and the sum of the squared

residuals which indicates the fit of the model as a whole. If desired, this sum may be tested for significance. Although LISREL does not print the sum of the squared residuals, the residual matrix is printed from which the sum of squared residuals could be computed.

The residual matrix is also used to pinpoint specification errors. Residuals which exceed the boundaries of sampling error indicate lack of fit. In terms of a causal model, large residuals may indicate the addition or deletion of a path or a reordering of the variables. The residuals of a unidimensional measurement model may indicate the necessity of repartitioning the variables into a new set of clusters. The residuals from a LISREL simultaneous analysis of measurement and causal models are less interpretable since a large residual may imply misspecification in either or both models. An assessment of LISREL in this regard is one of the goals of this study.

Confidence intervals about parameter estimates (OLS and LISREL). In path models, a proposed path may be deleted if the corresponding structural parameter is small. The usual standard error for a regression weight underestimates the correct standard error for an OLS estimated parameter since the factor correlations have been implicitly corrected for attenuation. The usual test for the significance of a correlation coefficient may be applied to the factor loadings uncorrected for attenuation. LISREL prints the asymptotic standard error of each parameter estimate which can be used to construct a confidence interval about the parameter estimate. Since these standard errors are based on the large sample properties of maximum likelihood estimators, their validity for small samples is in question.

First derivatives (LISREL). LISREL prints the values of the first

partial derivatives of F with respect to all the parameters--fixed, free or constrained. The purported usefulness of these derivatives for detecting a specific source of misspecification is based on their relation to the iterative procedure used in the search for the minimum of $F(\hat{\Sigma})$. At convergence, the values of the partial derivatives for the free parameters will be zero by construction. The derivatives of the free parameters should be approximately zero according to the model assumptions. If LISREL does not converge, the non-zero derivatives should indicate the misspecified free and constrained parameters.

The test for locating a potential misspecification for a solution which has converged is based on the values of the derivatives of the fixed parameters. If the model does not fit the data well, some of the derivatives of the parameters fixed at constant values should differ from zero. LISREL prints the first derivatives of all the elements in the eight matrices which define the parameterization of $\hat{\Sigma}$. Even if the model is specified as a recursive model with uncorrelated measurement errors, LISREL still prints the values of the complete B matrix, i.e., the derivatives of all the possible paths including those in the "opposite" direction, and the values of the complete ψ matrix and the Θ_{ε} and Θ_{δ} matrices including the covariances of the disturbances and the covariances of the measurement errors respectively.

Sörbom (1975) recommends the use of the first derivatives of the parameters over the use of the residual matrix (i.e., $\hat{\Sigma} - S$) in detecting misspecification. He advises that "we should relax the . . . restriction for that element which gives the largest decrease in $F(\hat{\Sigma})$" (p. 143). That element is the element with the largest partial derivative. However, the use of the partial derivatives for detecting

misspecification has not been adequately studied nor has the usefulness of this information been compared to the usefulness of the information supplied by the residual matrix.

Likelihood ratio test (LISREL). The likelihood ratio test can be used as a test of the fit of the model as a whole. The null hypothesis H_0 is that the observed covariances S conform to the constraints imposed on the covariances of the observed variables by the model, i.e., that $S = \hat{\Sigma}$ to within sampling error. The alternative hypothesis H_1 is that Σ is not described by $\hat{\Sigma}$, i.e., that "Σ is any positive definite matrix" (Jöreskog & Sörbom, 1978, p. 14) with no restrictions placed on its structure.

The test is based on the comparison of the value of the likelihood function. Since S is the unconstrained maximum likelihood estimator of log L, log L_1 is obtained by the substitution of S for $\hat{\Sigma}$. The value of Log L_0 becomes the value of log L with $\hat{\Sigma}$ substituted for Σ. The values of log L evaluated at S and $\hat{\Sigma}$ are compared by the statistical test based on the likelihood ratio or its logarithm, $L_1 - L_0$. That is, under H_0,

$$\chi^2 \sim -2 \log \frac{L_0}{L_1} \quad \text{with df} = \tfrac{1}{2}(p + q)(p + q + 1) - t$$

where t is the number of independent parameters to be estimated.

Since

$$\log L_0 < \log L_1 \text{ unless } \hat{\Sigma} - S = 0$$

the question addressed by the likelihood ratio test is, how large a discrepancy between the observed S and the predicted $\hat{\Sigma}$ is required for a given sample size to indicate that the model which was used to generate $\hat{\Sigma}$ is false?

The χ^2 statistic (Long, 1976) can be expressed as

$$-2 \log \frac{L_0}{L_1} = -2 \log L_0 + 2 \log L_1$$

$$= n\left[\log |S| + \mathrm{tr}(S\hat{\Sigma}^{-1}) - \log |S| - (p + q)\right]$$

$$= n\, F(\hat{\Sigma})$$

In order to directly obtain the χ^2 value, Jöreskog has chosen to find the value of the parameters which maximize the value of the likelihood function by minimizing F (since $F(\hat{\Sigma})$ and log L differ only by a constant).

Jöreskog warns that "the values of χ^2 should be interpreted very cautiously because of the sensitivity of χ^2 to various model assumptions such as linearity, additivity, multinormality, etc., but also for other reasons" (1978, p. 448). The primary "other reason" is the sensitivity of any significance test to sample size. No model is perfect, so any model will be rejected by the χ^2 test given sufficient sample size.

A potentially more useful application of the χ^2 statistic is the test of specific parameters by comparing the χ^2 values for nested models. A model M_1 is nested within the more general model M_2 if M_1 is defined by "constraining one or more of the free parameters in M_2 to be fixed" (Long, 1976, p. 170). Under the null hypothesis that the parameter is zero, the difference in the χ^2 values of the two models is also distributed approximately as χ^2 with one degree of freedom for large sample sizes. Jöreskog (1978) does not discuss the formal comparison of values in terms of significance tests. He suggests that a model may be "relaxed" by introducing more constraints. "If the drop in χ^2 is large compared to the difference in degrees of freedom, this

is an indication that the change made in the model represents a real improvement. If . . . the drop in χ^2 is close to the difference in number of degrees of freedom, this is an indication that the improvement in fit is obtained by capitalizing on chance and the added parameters may not have any real significance or meaning" (p. 448).

"Reliability" of the model (LISREL). Tucker and Lewis (1973) have noted some of the problems with the χ^2 statistic as a test of fit of the model and have proposed an alternative indicator of global fit which was originally developed for maximum likelihood confirmatory factor analysis.

$$\rho = \frac{C_0 - C_q}{C_0 - E(C_q)} = \frac{\text{"amount of covariation explained by the proposed model"}}{\text{"total amount of covariation available to be explained"}}$$

C_0 is the sum of squares of the off-diagonal elements of the sample covariance matrix divided by the degrees of freedom--the number of unique elements in the matrix, $(n)(n + 1)/2$ where n is the number of variables. C_q is the sum of squared covariances not explained by the model and is approximately equal to the ratio of sum of squares of the partial correlations of the variables with the group factors partialed out to the degrees of freedom of the model--the same degrees of freedom for the χ^2 test of fit. $E(C_q)$ is the expected value of the sum of squared covariances not explained by the model.

This "reliability" coefficient indicates the extent that a measurement model explains or accounts for the covariation among the observed variables. Burt (1973) notes that "Tucker and Lewis's statistic is sensitive to sample size in its computation of the expected value of explained covariance (sampling variability), but it is much less subject to large sample sizes than is the likelihood ratio

since it focuses on covariation rather than on total variation (p. 148). The sampling distribution of the coefficient is unknown, but Tucker and Lewis (1973) indicate that "any accepted solution should have a high coefficient of reliability" (p. 9) which, from their examples, apparently means at least somewhere about .9.

Tucker and Lewis (1973) also noted that the sum of the correlations among the observed variables with the factors partialed out is approximately equal to the value of the function F minimized by LISREL. Since the value of $F(\hat{\Sigma})$ at its minimum given the model and the observed covariances is simply the obtained χ^2 statistic multiplied by the sample size, Burt (1973) recommends using this "reliability" coefficient for the more general simultaneous analysis of the causal and measurement models.

Miscellaneous oddities (LISREL). Anecdotal evidence gathered from previous experience with LISREL indicates that certain misspecified models never converge, even after hundreds of (expensive) iterations. Concomitant with the lack of convergence is the existence of some "strange" parameter values which continue to become stranger as the program continues to iterate. Also, as the program iterates, the residual matrix can become almost completely flat at zero even though the program never converges and some of the parameter estimates are nonsense values.

CHAPTER II

OVERVIEW AND PROCEDURE

PACKAGE vs. LISREL

There are at least three competing strategies for the analysis of fully recursive causal models: (a) the PACKAGE analysis, i.e., a centroid factor multiple groups analysis with communalities followed by an OLS path analysis, (b) the LISREL analysis, a FIML simultaneous analysis of the measurement and causal models, and (c) a separate FIML confirmatory factor analysis (Jöreskog, 1966, 1967, 1979), followed by a FIML analysis of the factor correlations to test the causal model. However, Land (1973) has noted that in a recursive model, the FIML analysis of the factor correlations is identical to OLS. Thus the only difference between methods (a) and (c) is in the method of factor analysis for the measurement model. The contrast of greatest interest is between (b), the one-step LISREL procedure, and (a), the two-step analysis. That is, one main objective of this paper is to assess the utility of analyzing the measurement and causal models separately.

Hunter's strategy of separately analyzing the measurement and causal models has not received much attention. Some social scientists still seem content with the more traditional OLS estimation procedure without considering the need for good measurement. Those who recognize the biasing effect of measurement error on the parameter estimates of the causal model have used the one-step LISREL analysis. Jöreskog has

never suggested the possibility of a separate analysis in his writings even though he has used LISREL for confirmatory factor analyses (e.g., Jöreskog & Sörbom, 1978) in other contexts. Duncan (1975) has written chapters on specification error, measurement error, and multiple indicator models (which include the biasing effects of measurement error on OLS estimators), but the possibility of preceding OLS with correction for attenuation is not discussed. Duncan even concludes that "the contribution of multiple indicators should not be exaggerated" (1975, p. 137) in reference to some of the problems involved in estimating the parameters of both models simultaneously. Burt (1976) identified some of the undesirable properties of the effects of misspecification of the measurement model on the estimation of the parameters of the causal model, but he never considered the possibility of a separate analysis.

Does the additional cost, the potential problems with local minima characteristic of such iterative procedures, and the potential sensitivity to misspecification justify the use of LISREL over the strategies embodied in PACKAGE? In what situations do both approaches do equally well and in what situations do the approaches give poor results, or worse yet, results which look good but are, in fact, wrong? Which combination of strategies best leads the investigator to uncover the original structure with the most accurate set of parameter estimates given the inevitable sampling error and initial misspecification of the causal and measurement models?

The analysis strategies are evaluated in terms of both the validity of the statistical estimates and the usefulness of the program in providing information for the construction of revised, better-fitting models. Parameter estimates are usually not interpretable

unless the model fits the data. But if the model is misspecified, what is important is not the validity of the parameter estimates, but the information provided on how to correctly specify the model. It is precisely this kind of information which rarely appears in the literature.

What patterns of errors are associated with various specification errors such as variable deletion? Which set of misspecification indices allow one to most accurately pinpoint specification error: OLS and centroid factor multiple group residuals, LISREL simultaneous analysis residuals, or LISREL first derivatives? Which is the better indicator of the overall fit of the model: sum of the squared OLS residuals, the χ^2 test of fit, or the "reliability" of the model?

The issue is this: One reason for the recent popularity of LISREL is its ability to account for the influence of measurement error, to analyze nonrecursive models, and to allow for the possibility of specifying constraints on the equality of parameter estimates. However, <u>if</u> measurement error is accounted for in an independent confirmatory factor analysis, <u>if</u> only recursive models are considered, and <u>if</u> no constraints are placed on the parameter equalities, is there still an advantage to using LISREL? And how can either approach, LISREL or OLS, best be used to recover underlying structure?

<u>The Equivalence of MGRP and OLS with LISREL Given Correct Models</u>

If the specified model is correct, then the OLS preceded by correction for attenuation strategy, the one-step LISREL analysis, and the two-step LISREL analysis all provide correct parameter estimates and indicate a perfect fit of the model. The primary difference between the approaches is cost. The total execution time in seconds is

approximately one second for the PACKAGE subprograms MGRP and PARTIAL for the analysis of the measurement model and PATH for the OLS parameter estimates of the causal model. For the simultaneous LISREL analyses, all free parameters were initialized at 0.6 except the causal parameters relating the latent endogenous variables which were initialized at -0.5. For the LISREL confirmatory factor analyses, the start values for the factor loadings, error variances and factor correlations were arbitrarily set at 0.5, 0.8, and 0.3 respectively. Either of the LISREL analyses used 64 seconds of execution time for a 18 x 18 correlation matrix. Thus for models of the size considered in this paper, LISREL is 64 times as expensive as PACKAGE. For larger models, the difference increases more than proportionately.

The MGRP analysis is equivalent to a LISREL confirmatory factor analysis when the measurement model is defined by the partitioning of the observed variables into mutually exclusive clusters. This is somewhat surprising in view of the attention the maximum likelihood approach has recently received. For example, a chapter on confirmatory factor analysis in a factor analysis text by Mulaik (1972) is a chapter on Jöreskog's work. The centroid factor analysis multiple groups alternative is not mentioned. Moreover, in an early section of the text on the history of factor analysis, Spearman (1904) is given credit for developing "the first common-factor-analysis model" (p. 6), but the section closes with "more recently Bock and Bargmann (1966) and Jöreskog (1969) considered hypothesis testing from the point of view of fitting a hypothetical model to the data . . . . The author expects, as a consequence, factor analysis will be more fruitfully used in the future in the development of structural theories in psychology and in

other areas of study as well" (p. 10). What Mulaik (1972) has perceived as linear development is in actuality circular! Finally, Jöreskog (1966, 1967, 1969, 1970, 1971, 1978) has never mentioned the multiple groups method. He has even introduced the terms "congeneric measurement" and "the hypothesis of congenerism" (1971, p. 132) to describe what traditionally has been called "unidimensional measurement" or "the hypothesis of unidimensionality" which has been tested empirically with Spearman (1904) factor analysis using the centroid method. There is no necessary reason for tying the conceptual approach of a confirmatory factor analysis to the computational method of FIML.

The critical differences between OLS and LISREL emerge if the model is not correctly specified. The two methods will be shown to diverge widely in response to such errors. In summary, the OLS and multiple groups method localizes errors whereas LISREL spreads the error over the entire matrix. Thus LISREL is much harder to use in generating a more appropriate revised model.

The focus of this paper is on specification error, so sampling error is ignored throughout. However computations done along these lines suggest that sampling error adds further problems for LISREL since it tends to capitalize more on chance than do the OLS methods.

Procedure

It seems reasonable to expect that some consistent patterns can be captured and described if either PACKAGE or LISREL are valid approaches for data analysis and theory construction. The chief restrictions in this paper are that only recursive, causal models and unidimensional measurement models free from correlated errors of measurement are considered. All variables, latent and observed, are standardized.

(1) "Arbitrarily" construct a model. Begin with a full model which is complete with all parameter values in both the causal and measurement models. Four causal models are illustrated in Figure 5. If necessary, the regression coefficients can be varied in either the causal or the measurement models, but in this paper, all latent variables will have three indicators with the following factor loadings: .8, .6, and .4.

(2) Compute Σ. Given the rules of path analysis, the causal and measurement models and the parameter values of both models imply the correlations among the observed variables. The correlations among the latent variables are also computed as a prerequisite to computing the correlations among the observed variables.

(3) Model misspecification for observed data. Three kinds of misspecifications are possible, depending on what part of the model, if any, is misspecified.

(a) Misspecify causal model only.

(b) Misspecify measurement model only.

(c) Misspecify both measurement and causal models.

(4) Estimate parameters and evaluate fit. Estimate parameters with:

(a) a LISREL simultaneous measurement and causal analysis;

(b) separate measurement and causal analyses:

(i) a PACKAGE analysis with centroid factor multiple groups followed by OLS path analysis.

(ii) a LISREL analysis with FIML multiple groups analysis followed by a OLS path analysis with LISREL and/or a FIML path analysis.

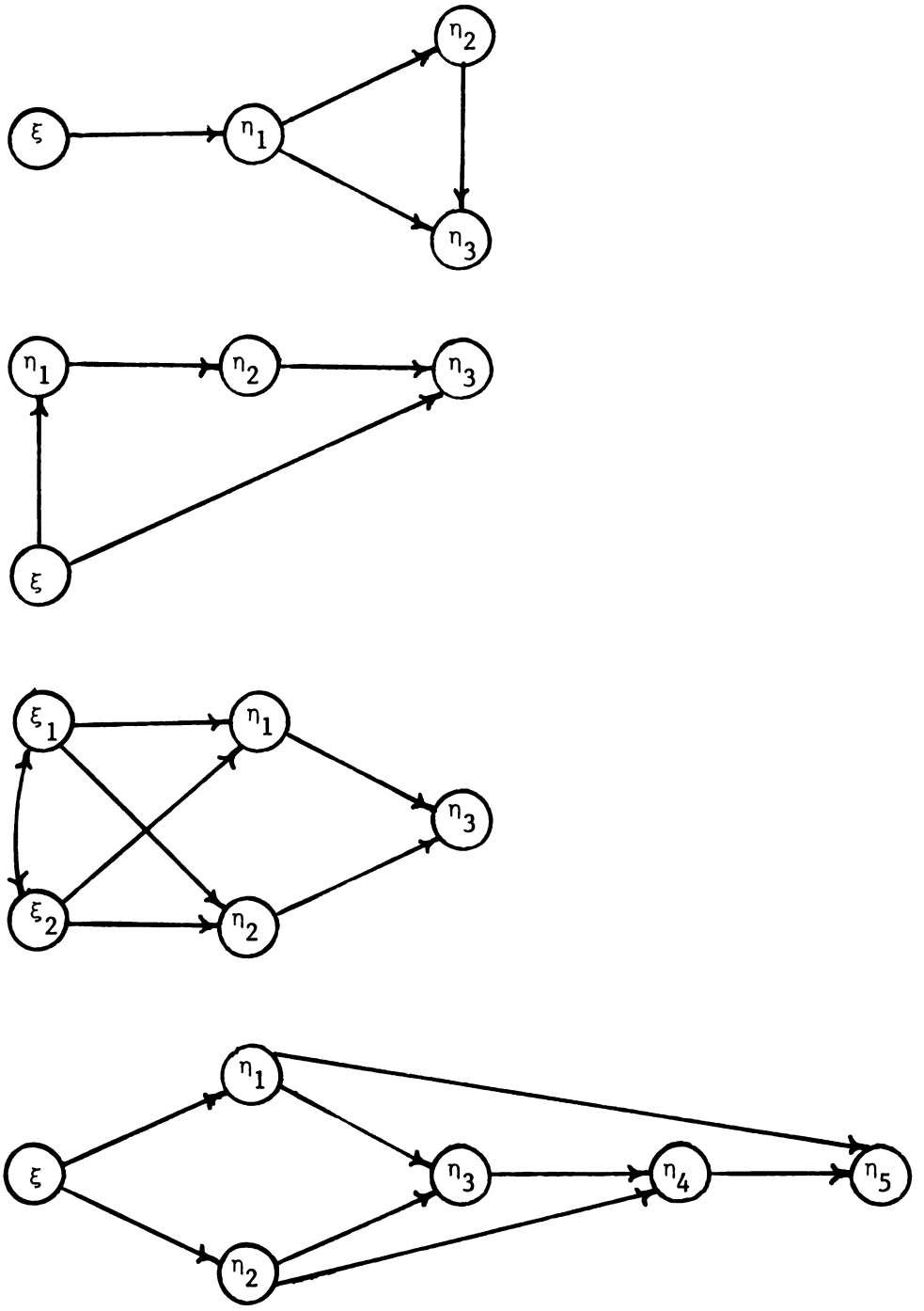


Figure 5. Four actual causal models used in this study.

The fit of the model can be evaluated with $\hat{\Sigma} - S$, the χ^2 test of fit, and the "reliability" coefficient.

(5) Revise the model. Describe how the model would be modified given the output of the programs and compare the revision to the correct model.

CHAPTER III

TWO-STEP vs. ONE-STEP ESTIMATION: PACKAGE vs. LISREL

Single Equation vs. Full Information Methods

Since a single equation method such as MGRP or OLS estimates the parameters of each equation separately, the effect of the misspecification of a single equation or just a few equations would be confined to those equations. The source of the misspecification should thus be relatively easy to locate. Errors in estimating one parameter are somewhat independent of errors in estimating parameters in other equations. However, the use of a full information procedure such as LISREL implies that the effects of a misspecification in one part of the model will be absorbed by the entire set of computations so that the errors in different parameters may be intimately interwoven. A full information procedure assumes that the entire specified model is the correctly specified model and simultaneously computes all the parameter estimates given this assumption. If the model is misspecified, the diffusion of the specification error throughout the model may hinder the location of the source of misspecification. Worse yet, a detected misspecification may appear in the wrong place. The use of LISREL for the simultaneous estimation of both measurement and causal models would presumably exacerbate these potential problems since misspecification in the measurement model would influence the computations of the causal model and vice versa.

Burt (1976) is one of the few authors who has studied the effects of misspecification and the resulting consequences for the interpretation of the parameter estimates computed by LISREL. He is particularly concerned with the effects of misspecified measurement models on the estimated causal parameters. He cites problems in interpretation that result from the possible confusion of two potential sources of "empirical meaning" of a latent variable or factor. The empirical meaning of a factor is the meaning assigned to that variable in terms of its relations to the observed variables. Since each parameter estimate in a full information technique is based on all of the observed covariances, empirical meaning is potentially assigned to an unobserved variable both in terms of its own indicators ("epistemic criteria") and the indicators of the remaining unobserved variables ("structural criteria"). "Interpretational confounding occurs when an individual assumes that an unobserved variable is assigned empirical meaning in terms of epistemic criteria when in fact it is assigned empirical meaning in terms of structural criteria" (Burt, 1976, p. 14).

In computational language, what Burt has stated is that the correlations of a given factor with other variables are not only based on the correlations of its indicators with those other variables (as in multiple groups), but depend also on the correlations between the other variables. In multiple groups analysis, the estimated correlation between factors A and B depends only on the observed correlations between the indicators of A and B. But in LISREL, the estimated correlation between A and B also depends on the correlations between indicators of A and C, of B and C, and even C and D! This problem is compounded in simultaneous LISREL by dependence on the postulated path

model.

The problem results from the characteristic of a FIML method to force the parameter estimates to maximize the fit of the entire model to the data. If a unidimensional measurement model is specified, then the program will attempt to satisfy the product rules of internal and external consistency in the computation of the parameter estimates. But if the measurement model is incorrectly specified, the lack of external consistency in the observed correlations implies that the misspecified indicators will differentially correlate with other factors and the indicators of those factors (e.g., Hunter, Gerbing & Boster, Note 5). The estimated correlations of the misspecified indicators vary as a function of the strength and type of causal relations in the particular model.

Consider the problem opposite to that considered by Burt (1976), i.e., a perfectly specified measurement model but a misspecified path model. How might errors in the causal model affect the parameter estimates of the measurement model? Two analytic strategies will be compared, a one-step and a two-step analysis:

(a) a LISREL simultaneous analysis of the indicator correlations.

(b) an OLS path analysis of the estimated factor correlations from a multiple groups factor analysis or from a LISREL confirmatory factor analysis.

The comparison is evaluated in terms of the accuracy of the estimated parameters and the usefulness of the information provided for detecting the misspecification in the causal model. The strategy is to examine a particular model in detail and then to determine how well the results from the comparison applied to a particular model generalize to a

variety of models and misspecifications.

An Example of a Misspecified Causal Model

Consider the following causal model and the associated misspecified causal model defined by the deletion of the path from η_2 to η_3. These models appear in Figure 6. The misspecified model was subjected to MGRP, LISREL CFA, OLS, and a simultaneous LISREL analysis.

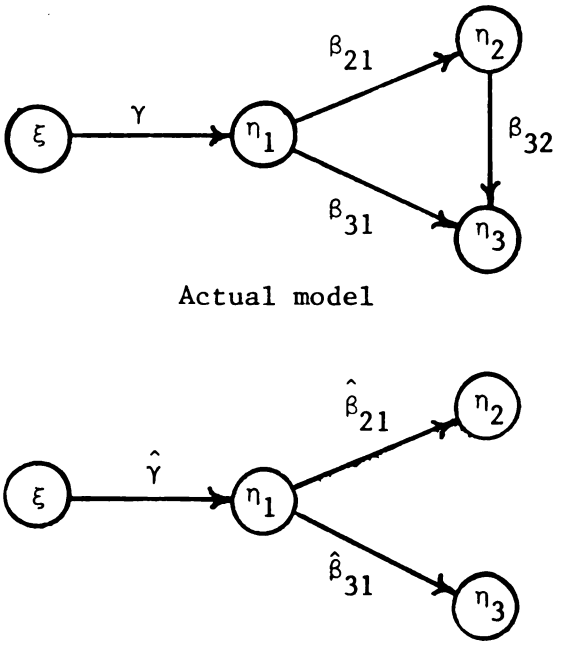


Figure 6. Actual model (a) from Figure 5 and an accompanying misspecified model.

The parameters of the causal model in Figure 6 were set at $\gamma = .30$, $\beta_{21} = \beta_{31} = .35$, and $\beta_{32} = .40$. The four variables of the model were given three indicators apiece with factor loadings of .80, .60, and .40. Using the product rules for internal and external consistency and the implied correlations among the factors, the Σ matrix was generated for the actual model and analyzed under the constraints of the misspecified model.

Analysis of the measurement model. Since the measurement model

was perfectly specified, both the multiple groups confirmatory factor analysis with the PACKAGE subprogram MGRP and the full information maximum likelihood confirmatory factor analysis with LISREL recovered the factor loadings and factor correlations perfectly. Thus the OLS path analysis was performed on the actual factor correlations.

OLS path analysis. As calculated in Appendix B, the OLS estimates of the parameters of the misspecified model are:

$$\hat{\gamma} = \gamma$$

$$\hat{\beta}_{21} = \beta_{21}$$

$$\hat{\beta}_{31} = \beta_{31} + \beta_{21}\beta_{32}$$

Since $\gamma = r(\eta_1, \xi)$ and $\beta_{21} = r(\eta_1, \eta_2)$, the values of $\hat{\gamma}$ and $\hat{\beta}_{21}$ are equal to the corresponding parameter values in the properly specified model. The value of $\hat{\beta}_{31}$ differs from β_{31} since the actual correlation between η_1 and η_3 is determined by the direct effect $DE_{31} = \beta_{31}$ and the indirect effect $IE_{31} = \beta_{21}\beta_{32}$ (see Lewis-Beck, 1974). In the misspecified model, the indirect influence of η_1 on η_3 has been eliminated by the deletion of the path, i.e., by fixing $\beta_{32} = 0$. Thus the parameter estimate for β_{31} must "absorb" this indirect effect. Although the value of β_{31} is biased since $\hat{\beta}_{31} \neq \beta_{31}$, OLS made the "rational" adjustment to the misspecification since $\hat{\beta}_{31}$ is the total effect of η_1 on η_3 in the actual model. That is, $\hat{\beta}_{31} = TE_{31}$. Given this "adjustment", although the paths from ξ to η_3 and from η_1 to η_3 are different in the actual and misspecified models, the total influence of ξ on η_3 and η_1 on η_3 remains unchanged.

The OLS residuals of the misspecified model are calculated in Appendix B. Since the decomposition of $r(\xi, \eta_1)$, $r(\xi, \eta_2)$, and

$r(\eta_1,\eta_2)$ according to the misspecified model involved only terms consisting of $\hat{\gamma}$ and $\hat{\beta}_{31}$, the predicted values of these correlations equaled their actual values. Since the total effect of ξ on η_3 and η_1 on η_3 remained unchanged, their residuals were also zero. Thus all the residuals were zero except for:

$$\begin{aligned} \text{Res}(\eta_2,\eta_3) &\equiv r(\eta_2,\eta_3) - \hat{r}(\eta_2,\eta_3) \\ &= \beta_{32}(1 - \beta_{21}^2) \end{aligned}$$

The residual matrix of the OLS solution provided useful information regarding the correct specification of the model. The positive residual between η_2 and η_3 correctly indicated that the relation between η_2 and η_3 had been underpredicted by the model, although it does not indicate the direction of the missing path. Since the remaining residuals were equal to zero, they correctly indicated that the rest of the model had been correctly specified.

LISREL simultaneous analysis. The factor loadings of the measurement model were almost perfectly recovered despite misspecification of the causal model. The only error larger than .003 was for the first indicator of η_1 where an estimate of .76 was obtained instead of the correct .80.

However, the simultaneous LISREL analysis has a very damaging problem as shown in Table 2: the estimated factor correlations are wrong. Even though this run was made without sampling error, the estimated correlation between η_2 and η_3 is off by .29. Thus even though the measurement model was correctly specified, the simultaneous LISREL analysis generated large errors in the estimated correlations of the latent variables. Moreover, there is nothing in the LISREL

Table 2

Correct and Estimated Factor Correlations
from OLS and LISREL and the Residuals

	η_1	η_2	η_3	ξ
η_1	1.00	.35	.49	.30
η_2	.35	1.00	.52	.11
η_3	.49	.52	1.00	.15
ξ	.30	.11	.15	1.00

a) The correct factor correlations.

	η_1	η_2	η_3	ξ
η_1	1.00	.43	.55	.30
η_2	.43	1.00	.23	.13
η_3	.55	.23	1.00	.17
ξ	.30	.13	.17	1.00

b) The factor correlations computed from a LISREL simultaneous analysis.

	η_1	η_2	η_3	ξ
η_1	.00	-.08	-.06	.00
η_2	-.08	.00	.29	-.02
η_3	-.06	.29	.00	-.02
ξ	.00	-.02	-.02	.00

c) The error in the estimated factor correlations from the simultaneous analysis.

	η_1	η_2	η_3	ξ
η_1	1.00	.35	.49	.30
η_2	.35	1.00	.17	.11
η_3	.49	.17	1.00	.15
ξ	.30	.11	.15	1.00

d) The factor correlations computed from an OLS analysis.

	η_1	η_2	η_3	ξ
η_1	.00	.00	.00	.00
η_2	.00	.00	.35	.00
η_3	.00	.35	.00	.00
ξ	.00	.00	.00	.00

e) The error in the estimated factor correlations from an OLS analysis.

printout to suggest that these estimates are wrong.

The estimated factor correlations from LISREL have values such that an OLS path analysis of the estimated correlations would result in an indication of perfect fit. That is, LISREL placed all of the error into the estimated factor correlations and maintained perfect consistency between the factor correlations and the path coefficients. Since the misspecification occurred in the causal model only, this is the exact reverse of reality. Thus the residual correlations in LISREL suggest that the location of error is in the measurement model even though the measurement model is perfect. The residual correlations are zero for the causal model even though it is quite wrong.

Thus most of the estimates of the causal parameters of the misspecified causal model were different from the corresponding OLS estimates. The parameter estimates for the OLS and the simultaneous LISREL analyses are presented in Table 3 along with the actual values of the parameters from the properly specified model. Computational details are presented in Appendix C. The OLS estimates are closer than the LISREL estimates; the LISREL estimate of β_{21} deviates from the correct value by .08 whereas OLS deviates by 0, and the LISREL deviation of β_{31} is .20 while the OLS deviation is only .14.

Table 3

Parameter Values and Estimates

	Actual value	OLS	LISREL
γ	.30	.30	.30
β_{21}	.35	.35	.43
β_{31}	.35	.49	.55

In terms of pinpointing misspecification, the information provided by the LISREL analysis added nothing to the information inferred from the OLS residuals. Analysis of the full causal model yielded nine times as many residuals, i.e., a 12 x 12 matrix of indicator residuals instead of a 4 x 4 matrix of factor residuals. Each factor correlation in the 4 x 4 matrix is replaced by a 3 x 3 block of nine residuals in the 12 x 12 matrix. The six blocks of residuals formed approximately the same pattern as the six OLS factor residuals except that the pattern in the LISREL analysis was more blurred than the pattern of the OLS residuals. All of the residuals between the indicators of the single exogenous factor and the indicators of the endogenous variables, except between η_2 and η_3, were only approximately zero since they varied in magnitude from .002 to .037. The residuals between the indicators of η_2 and η_3, which replaced the single OLS factor residual of .352, are presented in Table 4.

Table 4

The Residuals of the Indicators of η_2 and η_3

		Indicators of η_2		
		1	2	3
Indicators of η_3	1	.18	.14	.09
	2	.14	.10	.07
	3	.09	.07	.05

In Appendix C, it is demonstrated that the indicator residuals from the simultaneous analysis are a function of the estimated factor correlations. The sizes of the indicator residuals in each block of

nine residuals correspond to the size of the corresponding factor residual except that each indicator residual is proportionately diminished by the measurement error in the corresponding indicators.

A comparison of OLS and LISREL. The similarities and differences between OLS and LISREL observed in the analyses of the model presented in Figure 6 are presented in Table 5. The crucial difference between OLS and LISREL is that, for the OLS path analysis, the factor correlations were estimated at their correct values by the confirmatory factor analysis. However, for the LISREL simultaneous analysis, the factor correlations were adjusted to match the estimated path coefficients and yield a model of apparent perfect fit--regardless of the amount of error in the estimated path coefficients.

Analysis of a variety of models and specifications. The conclusions above were supported in many other models. These additional examples misspecified the given model in at least four distinct ways:

(a) deletion of a causal path;

(b) deletion of a latent variable;

(c) addition of a causal path;

(d) reversal of the direction of a causal path.

The correct models were those of Figure 5 and Figure 6. The factor correlations and the factor loadings were perfectly reproduced by multiple groups analysis in each misspecified model. The information provided by the derivatives was redundant with the information provided by the residuals in either the LISREL analysis of the path model or the full simultaneous analysis. The full LISREL analysis always correctly recovered the factor loadings of the measurement model to within a reasonable degree of error, but the parameter estimates of the causal

Table 5

Comparison of PACKAGE vs. LISREL

Criterion	MGRP + OLS	LISREL
Factor loadings	No error	No error (after transformation to standard metric)
Factor correlations	No error	Error in most factor correlations (some large)
Path coefficients	Error in only one path coefficient, β_{31}	Error in most path coefficients (some large)
Detecting misspecification	Visible discrepancy between the predicted and actual factor correlations	False consistency in relation of actual factor correlations to the estimated path coefficients

model were farther off than the estimates provided by the OLS analysis. In all cases, the one-step LISREL procedure matched its estimated factor correlations to the erroneous path coefficients. Thus whereas the multiple groups analysis always correctly reproduced the factor correlations, the LISREL one-step procedure was always way off. This is a crucial difference since it is discrepancies between observed and reproduced correlation coefficients which provide the best guide to the detection and remediation of errors in the causal model. LISREL always falsely suppressed these discrepancies.

In summary, if the measurement model is correct, then the two-step procedure will correctly estimate the correlations between the latent variables. However the LISREL one-step analysis will be correct only if the causal model is <u>also</u> correct. Thus for LISREL, a correct measurement model provides no clues as to the correct causal model.

CHAPTER IV

SPECIFICATION ERROR IN BOTH CAUSAL AND MEASUREMENT MODELS: AD-HOC COMPOSITES

A composite score may be defined as the sum over any set of component measures, irrespective of the content or statistical behavior of the components. If all of the component measures are indicators of the same latent variable, though perhaps with varying degrees of reliability, then the observed composite score is an estimate of the score for that construct. Whether or not the component measures are indicators of a construct is determined by the procedures of construct validity. The correlations of the indicators of a construct satisfy one product rule for internal consistency in terms of their correlations with each other, and a second product rule for external consistency in terms of their relations with other variables.

A composite defined by a set of measures which are not indicators of a construct is called an "ad-hoc composite", i.e., an ad-hoc composite is an aggregation of related but conceptually and statistically distinct constructs. Such composites are usually constructed according to the procedures of content validity and have been useful in applications in education and industrial psychology where empirical prediction is the primary objective. As Cronbach (1970, p. 44) wrote, "Nothing in the logic of content validation requires that the universe or test be homogenous in content." However, a misspecification occurs when the indicators of distinct constructs are collapsed together and

specified as alternate indicators of a single latent variable.

The embedding of such an amorphous measure in a causal model insures that both the measurement and causal models are misspecified. The lack of external consistency of the indicators of the misspecified construct implies that the component constructs of the ad-hoc composite tend to differentially relate with the other latent variables in the model. Thus the component constructs would occupy different positions in a properly specified model. This is illustrated in the Hunter, Gerbing and Boster (Note 5) analysis of the Christie and Geis (1970) Mach IV measure of Machiavellianism.

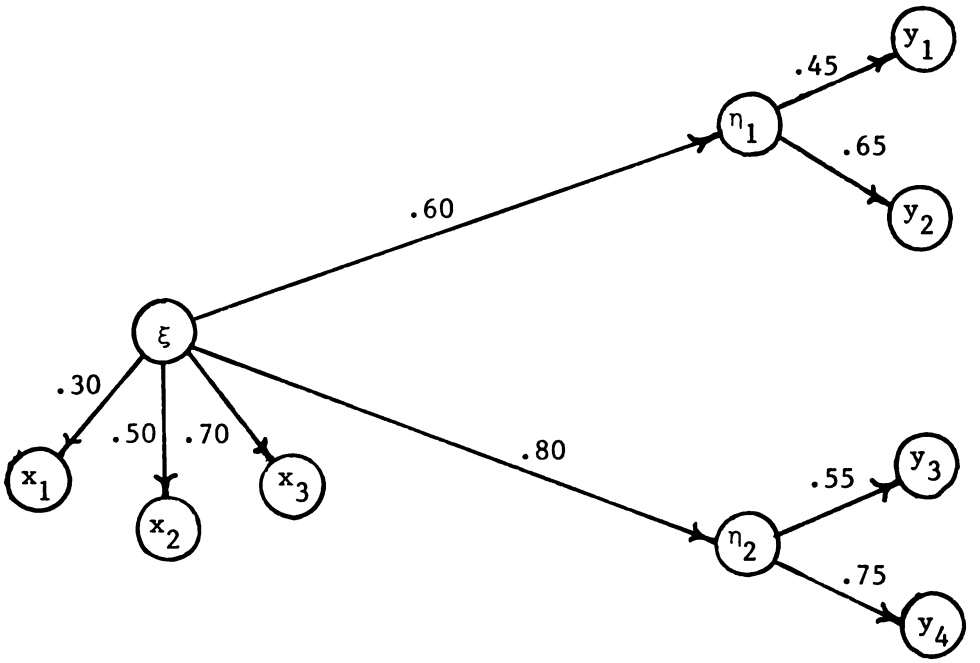
Unfortunately, the most common statistical analysis for the construction and evaluation of measurement scales is not a confirmatory factor analysis. Instead, the usual analysis, such as that provided by the SPSS subprogram RELIABILITY, is based primarily on (a) item-total correlations, and (b) coefficient alpha or some other index of "internal consistency" reliability. The use of item-total and reliability statistics as indices of unidimensionality can lead to crude, nebulous measures. Following Hunter (Note 3), consider a scale defined by 10 items which breaks into two clusters of five items each. Items within each cluster intercorrelate .7 with each other, but all correlations between items in different clusters are only .2. Although the scale is clearly two dimensional, coefficient alpha is .88. The uncorrected item-total correlations are .69, and item-total correlations calculated with communalities are .67. Thus an examination of only item-total correlations or coefficient alpha can lead to the acceptance of a scale as unidimensional when it is actually an ad-hoc composite.

The problem with coefficient alpha as an indicator of dimensionality was noted by Green, Lissitz and Mulaik (1977) and by Hunter (Note 3). The amount of measurement error in a composite score is a concept which is logically independent of the dimensionality of the scale. If each component is perfectly measured, then the reliability of the composite score is also 1.0 no matter what the relations between the components might be. The problem with item-total correlations (even if corrected for upward bias by the use of communalities) is that although an indicator will correlate most highly with its own properly specified construct, it can have a substantial correlation with any other correlated factor. Thus the item-total correlation is only meaningful on a comparative basis; there is no absolute size criterion for unidimensionality.

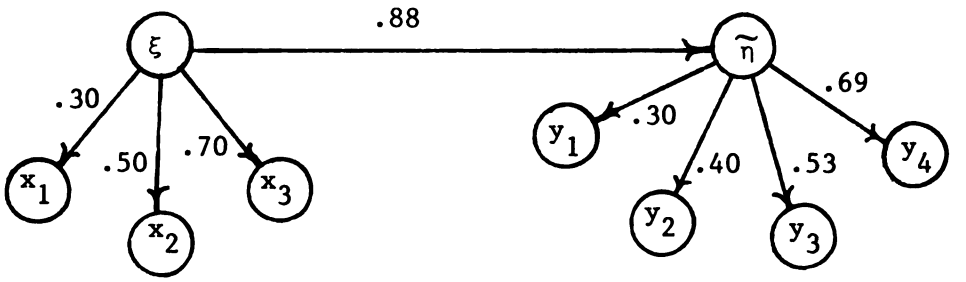
The Pattern of Residuals from an Ad-hoc Composite Misspecified as a Construct

Consider the actual and misspecified models illustrated in Figure 7. The misspecification was defined by the pooling of the indicators of η_1 and η_2 into a composite which is misspecified as the "construct" $\widetilde{\eta}$. The purpose of this section is to identify a characteristic pattern of the residuals of such a misspecification. The residuals among the four indicators as computed by LISREL are presented in Table 6. The dotted lines in Table 6 indicate the partitioning of the indicators into their respective sub-clusters based on the actual model.

The first consideration is the relative sizes of the factor loadings in the actual and misspecified models. The actual construct of an indicator is causally antecedent to the indicator, so the correlation of the indicator with other variables is mediated by the construct. That is, according to the product rule for external



Actual model



Misspecified model

Figure 7. The actual and misspecified models from Costner and Schoenberg (1973) with parameter estimates computed by LISREL.

Table 6

Residuals from the Endogenous Indicators in Figure 7

	y_1	y_2	y_3	y_4
y_1	.00	.17	-.04	-.04
y_2	.17	.00	-.04	-.04
y_3	-.04	-.04	.00	.05
y_4	-.04	-.04	.05	.00

consistency,

$$r(i,z) = r(i,\eta)\, r(\eta,z)$$

where η is the actual construct of indicator i and z is some other variable. In classical reliability theory, this result is expressed as "the maximum value of the validity coefficient . . . is equal to the reliability index . . ." (Magnusson, 1967, p. 150) where the reliability index is the correlation between the indicator and the construct. If $z \equiv \tilde{\eta}$, i.e., if

$$r(i,\tilde{\eta}) = r(i,\eta)\, r(\eta,\tilde{\eta})$$

then

$$r(i,\eta) > r(i,\tilde{\eta}) \text{ or } \lambda_{\eta_i} > \hat{\lambda}_{\tilde{\eta}_i}$$

For example, $\lambda_{11} = .45$ and $\hat{\lambda}_{\tilde{\eta}_1} = .30$, as shown in Figure 7.

In terms of predicting the pattern of residuals for the misspecified model, consider first the correlations among the indicators of the same properly specified construct. The correlation among these indicators adheres to the product rule for internal consistency, i.e., for the i^{th} and j^{th} indicators of η,

$$r(i,j) = r(i,\eta)\, r(\eta,j)$$

But for the misspecified model,

$$\hat{r}(i,j) = \hat{r}(i,\tilde{\eta})\, \hat{r}(\tilde{\eta},j)$$

And, from above,

$$|\lambda_{\eta_i} \lambda_{\eta_j}| > |\hat{\lambda}_{\tilde{\eta}_i} \hat{\lambda}_{\tilde{\eta}_j}|$$

If the residual is defined as

$$\text{Res}(i,j) \equiv r(i,j) - \hat{r}(i,j)$$

then the expected residuals of an ad-hoc composite misspecified as a construct should be positive for the indicators of the <u>same</u> construct

in the correctly specified model, as they are in this example in which $\text{Res}(y_1,y_2) = .09$ and $\text{Res}(y_3,y_4) = .11$.

Now consider the residuals among the indicators of different constructs in the ad-hoc composite. The observed correlations for the indicators were computed with the product rule for external consistency. For example,

$$r(y_1,y_3) = r(y_1,\eta_1)\, r(\eta_1,\eta_2)\, r(\eta_2,y_2)$$

$$= (.45)\ (.48)\ (.55) = .119$$

But the predicted correlations among these indicators which have been falsely placed in the same cluster will again be computed using the product rule for internal consistency. For example,

$$\hat{r}(y_1,y_3) = \hat{r}(y_1,\eta)\, \hat{r}(y_3,\eta)$$

$$= (.30)\ (.53) = .159$$

So the residual is $.119 - .159 = .040$.

Thus there are two competing influences on the relative sizes of $r(i,j)$ and $\hat{r}(i,j)$, where i and j are indicators of different constructs in the actual model. As shown before, the actual factor loadings are larger than the computed factor loadings. This discrepancy is large and the residual is positive, to the extent $r(\eta_1,\eta_2)$ is small. What is new for this situation is the presence of $r(\eta_1,\eta_2)$ in the expression for the actual correlation $r(i,j)$. The smaller $r(\eta_1,\eta_2)$, the smaller the product $r(i,\eta_1)\, r(\eta_1,\eta_2)\, r(\eta_2,j)$. Let the value of $r(\eta_1,\eta_2)$ decrease, but let the remainder of the actual model be unchanged. If the value of this product decreases faster than the discrepancies between the actual and computed factor loadings, the residuals could even become negative, as they are in this example. But

these residuals should at least tend to be smaller than the residuals of indicators of the same construct. This result is proved in Appendix D for the special case of equal factor loadings in the actual model.

A Re-interpretation of the Costner and Schoenberg Analysis

Costner and Schoenberg (1973) investigated the effects of misspecification with the strategy utilized in this paper. One of their models, which is presented in Figure 7, was misspecified to form what is here labelled an ad-hoc composite. Costner and Schoenberg (1973) predicted the pattern of residuals resulting from the analysis of this model without any formal justification. They simply indicated that "our intuition is that . . . we might expect large residuals between . . . the indicators of [ξ] on the one hand and the indicators of [η_1] on the other, or between indicators of [ξ] and [η_2]" (p. 175). Following an analysis of the misspecified model with LISREL, the authors noted that "the actual pattern of residuals . . . does not conform to this pattern at all" (p. 176).

How should the model be respecified given the residual matrix? Costner and Schoenberg (1973) believed that the respecification of a model when confronted with a matrix of nonzero residuals among the indicators should be to allow the error variables for indicators with the largest residual to be correlated. They respecified the model by relaxing the $r(\varepsilon_1,\varepsilon_2) = 0$ constraint and noted that (a) the respecified model fits relatively well, and (b) this result was misleading since the respecified model was not the correct model. The authors then concluded on the basis of this example and others that "the respecification suggested by an intuitive appraisal of the pattern of residuals may be grossly misleading" (p. 177). Rejecting the

information provided by the residuals, the authors go on to devise a laborious procedure for detecting misspecification which involves separately testing all possible combinations of two-indicator models and then testing specified combinations of three-indicator models.

However, the obtained pattern of residuals does conform to the predictions of the previous section. The residuals between indicators of the same construct in the correctly specified model are positive. The residuals between indicators from different constructs are not only smaller but negative in this example. Contrary to the conclusion of Costner and Schoenberg (1973), the residuals do appear to provide useful information for the respecification of this misspecified model. The incorrect assumption is that positive residuals imply correlated errors.

Does the use of PACKAGE provide information not provided by LISREL? The application of the multiple groups analysis to the Costner and Schoenberg (1973) example generated the residuals and partial correlations presented in Table 7.

Table 7

MGRP Residuals and Partial Correlations

	Residuals					Partial Correlations			
	y_1	y_2	y_3	y_4		y_1	y_2	y_3	y_4
y_1	.00	.09	-.06	-.07	y_1	1.00	.14	-.07	-.06
y_2	.09	.00	-.06	-.01	y_2	.14	1.00	-.08	-.07
y_3	-.06	-.06	.00	.11	y_3	-.07	-.08	1.00	.17
y_4	-.07	-.01	.11	.00	y_4	-.06	-.07	.17	1.00

The residuals derived from PACKAGE follow exactly the same pattern as the LISREL residuals. However, since the specified model contains only two factors, the LISREL simultaneous analysis is equivalent to a confirmatory factor analysis.

Alternate residuals can be obtained by partialling out a single factor at a time--a straightforward operation with the PACKAGE subprogram PARTIAL following the use of the MGRP subprogram. In the actual model $r(\eta_1,\eta_2)$ can be decomposed entirely into the spurious influence of an exogenous variable ξ, i.e., $r(\eta_1,\eta_2 \cdot \xi) = 0$. This same relationship among the factors is mirrored by the indicators of the factors, as illustrated in Table 8. Again, contrary to the conclusion of Costner and Schoenberg (1973), the residuals provide directly usable information for locating specification errors.

Table 8

Correlations of the Endogenous Indicators with ξ Partialled Out

	y_1	y_2	y_3	y_4
y_1	1.00	.21	.00	.00
y_2	.21	1.00	.00	.00
y_3	.00	.00	1.00	.21
y_4	.00	.00	.21	1.00

Finally, Costner and Schoenberg (1973) could have applied a test suggested by Spearman in 1914 that would also have unambiguously determined that the misspecified model contained an ad-hoc composite. Spearman noted that the proportionality constraint which follows from the product rule for external consistency implies that the correlations

of two indicators of the same construct across other variables are "perfectly correlated" (1914, p. 109). These "intercolumnar" or "second order" correlations may be computed by correlating the correlations of each pair of indicators with the diagonal value of 1.00 defined as missing data. The resulting matrix of second order correlations is presented in Table 9. Actually, similarity coefficients (e.g., Hunter, 1973) should be used instead of the second order correlation coefficients since proportionality is a stricter criterion than linearity. Second order correlations were computed because of the availability of the computer program with a missing data provision.

Table 9

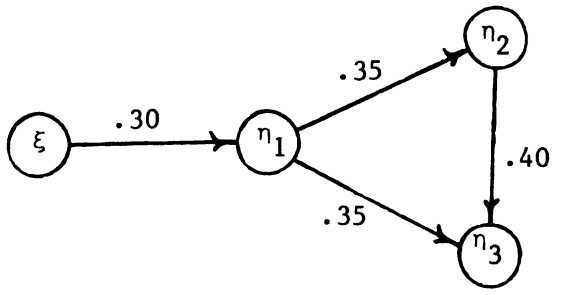
The Second Order Correlations of the Endogenous Indicators

	y_1	y_2	y_3	y_4
y_1	1.00	1.00	.05	-.02
y_2	1.00	1.00	.26	.08
y_3	.05	.26	1.00	1.00
y_4	-.02	.08	1.00	1.00

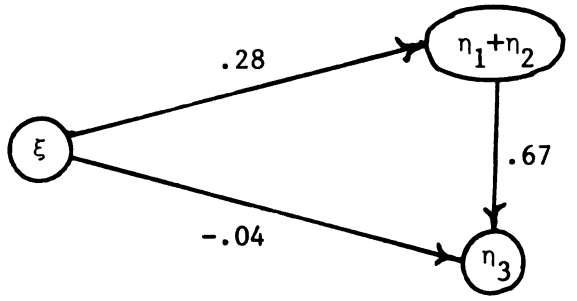
A More Complicated Model

Since the misspecified model analyzed by Costner and Schoenberg (1973) contained only two latent variables, a simultaneous LISREL analysis is equivalent to a LISREL CFA analysis. Consider the actual and misspecified models in Figure 8. The factor loadings of the correctly specified factors were correctly recovered while the factor loadings of the ad-hoc composite were attenuated from their true values. The simultaneous and CFA solutions in terms of factor loadings,

residuals, and derivatives were identical. The LISREL residuals among the six indicators and the derivatives for the measurement error correlations between the six indicators of the ad-hoc composite are presented in Table 10. The pattern is redundant between the two matrices. Contrary to the assertion of Costner and Schoenberg (1973), the residuals clearly indicate subdivision of the indicators into two sets. Contrary to Sörbom (1975), the derivatives falsely suggest correlated errors.



Actual model



Misspecified model

Figure 8. Actual model (a) from Figure 5 and a second misspecified model.

The partial correlations of the indicators of the ad-hoc composite from the PACKAGE analysis are presented in Table 11. This pattern

Table 10

LISREL Residuals and Derivatives

Residuals

	y_1	y_2	y_3	y_4	y_5	y_6
y_1	.00	.23	.14	-.11	-.11	-.08
y_2	.23	.00	.09	-.11	-.11	-.08
y_3	.14	.09	.00	-.08	-.08	-.06
y_4	-.11	-.11	-.08	.00	.17	.11
y_5	-.11	-.11	-.08	.17	.00	.06
y_6	-.08	-.08	-.06	.11	.06	.00

Derivatives

	y_1	y_2	y_3	y_4	y_5	y_6
y_1	.00	-.41	-.23	.24	.21	.13
y_2	-.41	.00	-.13	.22	.18	.11
y_3	-.23	-.13	.00	.14	.12	.07
y_4	.24	.22	.14	.00	-.37	-.20
y_5	.21	.18	.12	-.37	.00	-.09
y_6	.13	.11	.07	-.20	-.09	.00

Table 11

Partial Correlations

	y_1	y_2	y_3	y_4	y_5	y_6
y_1	.00	.28	.16	-.19	-.15	-.11
y_2	.28	.00	.10	-.15	-.12	-.08
y_3	.16	.10	.00	-.11	-.08	-.06
y_4	-.19	-.15	-.11	.00	.28	.16
y_5	-.15	-.12	-.08	.28	.00	.10
y_6	-.11	-.08	-.06	.16	.10	.00

Table 12

Second Order Correlations

	y_1	y_2	y_3	y_4	y_5	y_6
y_1	1.00	1.00	1.00	.04	.11	.08
y_2	1.00	1.00	1.00	.11	.14	.10
y_3	1.00	1.00	1.00	.10	.11	.07
y_4	.04	.11	.10	1.00	1.00	1.00
y_5	.11	.14	.11	1.00	1.00	1.00
y_6	.08	.10	.07	1.00	1.00	1.00

the partial correlations was again similar to the pattern of the residuals and/or derivatives from the LISREL analysis. The matrix of second order correlations is presented in Table 12.

In practice, the indicators of the ad-hoc composite might not be listed in subclusters as they are in the examples presented here. Use of the PACKAGE subprogram ORDER on the matrix of partial or second order correlations would reorder the variables so that the indicators which define the component constructs are listed consecutively.

The Use of Derivatives in Detecting Misspecification in Ad-hoc Composites

A re-analysis of previous studies. Sörbom (1975) was interested in models with correlated errors. He accepted Costner and Schoenberg's (1973) conclusion regarding the problems with the residual matrix for detecting misspecification, but he sought a less troublesome alternative than the procedure outlined by Costner and Schoenberg. Sörbom (1975) advocated the use of the first derivatives as an alternative to the residuals for locating misspecification. "We should relax the zero-restriction for that element which gives the largest decrease in F" (p. 143), i.e., the element with the largest first derivative. He demonstrated that the procedure worked for an actual model with correlated errors which was misspecified as a model with uncorrelated errors.

However, Sörbom's (1975) procedure would lead to false conclusions in the case of the collapsed indicator model or ad-hoc composite. For the Costner and Schoenberg (1973) example, the matrix of first derivatives for the covariance matrix of measurement errors is given in Table 13. The pattern of first derivatives is redundant with the

Table 13

Derivatives of the Measurement Errors of the Endogenous Indicators

	y_1	y_2	y_3	y_4
y_1	.00	-.23	.07	.09
y_2	-.23	.00	.07	.10
y_3	.07	.07	.00	-.12
y_4	.09	.10	-.12	.00

information supplied by the matrix of residuals. The only difference between the patterning of the derivatives and the residuals is that the derivatives have opposite algebraic signs. If the model were respecified by falsely assuming correlated measurement errors for y_1 and y_2 and for y_3 and y_4, then the actual structure would not be recovered, as Costner and Schoenberg (1973) have already noted.

Saris, Pijper and Zegwaart (1978) also noted that the Costner and Schoenberg (1973) procedure for detecting misspecification was "quite time-consuming and not completely clear as to how one should proceed in all circumstances" (p. 152). Following Sörbom (1975), they were concerned only with first derivatives, but they sought to improve Sörbom's procedure by considering the correlations among the derivatives. For each of the m fixed parameters, they computed the function

$$w_i = \sum_{j=1}^{m} \hat{r}_{ij}\hat{v}_i$$

where $\hat{v}_i$ is the estimated first derivative of the i^{th} parameter, and $\hat{r}_{ij}$ is the estimated correlation between the derivatives of the i^{th} and j^{th} parameters. They proposed "a stepwise procedure . . . where at

each step the restriction is dropped with the highest $\underline{w}$ in absolute value" (Saris et al., 1978, p. 158).

Saris et al. (1978) began with two-factor models which contained (a) correlated measurement errors and/or (b) observed variables which were indicators of both factors. The misspecified model was always the corresponding two-factor model with unidimensional measurement, i.e., no correlated error variances and all indicators were indicators of only a single factor. They concluded that "the Saris procedure performs better than Sörbom's" (p. 163). However, like Sörbom, these authors never began with a recursive model with unidimensional measurement. Like Sorbom, they never considered residuals.

Thus the "improvement" suggested by Saris et al. (1978) leads to the same error of the Sörbom (1975) method when applied to the Costner and Schoenberg (1973) example. Since all derivatives except the derivatives of the measurement errors of the endogenous indicators shown in Table 13 were zero, the application of the Saris et al. method to these data would also lead to a false respecified model which contains correlated measurement errors. The only potential difference between the procedures is that the correlated measurement errors might be added in a different order.

An extrapolation of the Sörbom procedure. As constraints of the model are relaxed, the apparent fit of the model continually improves. In the ad-hoc composite example, the suggestion of Sörbom (1975) to assume correlated measurement errors does improve the apparent fit of the model though the respecified model is false. The question addressed in this section is, what is the end result of allowing increasingly more measurement error covariances?

The largest residual from the LISREL analysis of the misspecified model in Costner and Schoenberg (1973) was between the indicators of y_1 and y_2. The largest derivative was between the measurement error covariances of the corresponding error terms. If this error covariance is freed, it assumes the value of .198, the fit of the model improves since $F(\hat{\Sigma})$ decreases from .588 to .142, and the factor loadings of the two indicators decrease about .04. The largest residual is now between y_3 and y_4 and the largest derivative is between the corresponding covariance between their error terms. If the model is respecified again by adding this second measurement error covariance, the model fits perfectly, i.e., $F(\hat{\Sigma}) = 0.000$. The estimated regression parameters continue to change with each respecification since the factor loadings of y_3 and y_4 decrease about .09 and, more interestinly, the coefficient relating the latent variables becomes equal to 1.00.

The generality of this result can be checked by examining the more complicated model which appears in Figure 8. The misspecified model was successively respecified by freeing a new measurement error covariance on each respecification. The chosen covariance on each round corresponded to the largest derivative or, equivalently, the largest residual. The model was respecified until perfect fit was obtained.

In general, as more measurement error covariances are added, the factor loadings decrease in magnitude while the estimated regression parameters of the causal model increase in magnitude. The fit of the model continues to improve until six covariances have been included, at which point $F(\hat{\Sigma}) = .014$. Yet in this "perfect" model, $\hat{\beta}$ is equal to the nonsense value of .911. And this "perfect" model is, in actuality, a misspecified ad-hoc composite.

The pattern of correlated errors reveals the nature of the misspecification. The misspecified model fit the data perfectly if all the covariances between measurement errors of indicators of the same construct were unconstrained. For the Costner and Schoenberg (1973) model this criterion was achieved after only two measurement error covariances were freed since the ad-hoc composite contained the indicators from only two factors and each factor had only two indicators. The present model required six free error covariances since there were three indicators for each factor.

The apparent fit of a model with correlated measurement errors means only that the model is misspecified. If the model fits poorly without correlated errors, but fits very well with correlated errors, and if the indicators can be partitioned into clusters with positively correlated errors within clusters and zero or negative covariances between clusters, then at least one of the latent variables is an ad-hoc composite. The indicators should be partitioned accordingly. However as was shown earlier, the same analysis can be performed directly on the original model residuals. There is no need to obtain an intermediate solution with "correlated errors".

Summary

In contemporary path models, a construct is defined as a latent variable whose indicators form a unidimensional set in the sense of Spearman (1904). To use such models, one must "reduce" composites which are measured by a conglomerate of constructs into component variables. Such an ad-hoc composite can be regarded as a model which has been misspecified by collapsing the indicators of distinct constructs into a single latent variable. Moreover, the recognition of

such a misspecification can be accomplished by the examination of the residuals--although previous work has claimed that the residuals were not helpful in detecting misspecification. Contrary to previous claims, the derivatives from a LISREL analysis were shown not to add any more information than the residuals. Indeed the recommendations of Sörbom (1975) and Saris et al. (1978) were shown to lead to the false assertion of correlated errors.

CHAPTER V

MISSING VARIABLES, MISSING PATHS, AND CORRELATED ERRORS

No existing path model contains all the relevant causal factors. If only complete models were capable of analysis using OLS, then there would be no situations in which OLS could be used. This is precisely the claim of a number of contemporary authors. They argue that missing variables always result in correlated disturbances and hence the use of OLS is never justified in real data sets. The following examples show this claim to be false. On the contrary, these examples suggest that most recommendations for correlated disturbances are misguided responses to data sets which call for the addition of missing paths in the misspecified models.

The Effect of a Missing Variable on the Fit of the Model

Previous assumptions. The traditional estimation procedure used in path analysis is OLS, which assumes that the disturbance terms across equations are uncorrelated. However, several authors have argued that correlated disturbances could be produced by missing variables. These arguments are important since the complete specification of all of the variables in a causal process is impossible.

Presumably, if a common antecedent of some of the endogenous variables is omitted, the omitted variable will "appear" in the corresponding disturbance terms since the disturbance represents the influences which operate on the "dependent variable" of the equation

which are not described in the set of predictor variables in the equation. For example,

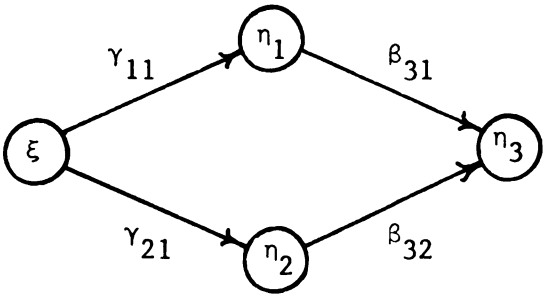
> "[The realism of] the assumption that the disturbance terms are mutually uncorrelated in any given instance will depend upon the completeness of the causal system . . . . Whenever common causes of the disturbance terms for two or more equations can be located or measured, they should be explicitly introduced into the equations as additional variables" (Namboodiri, Carter, & Blalock, 1975, pp. 446 and 448).

> "If the same explanatory factor is excluded from more than one equation, the effect of that factor will be present in more than one error term and will cause the error terms to be somewhat correlated . . ." (Hanushek & Jackson, 1977, pp. 230 and 231).

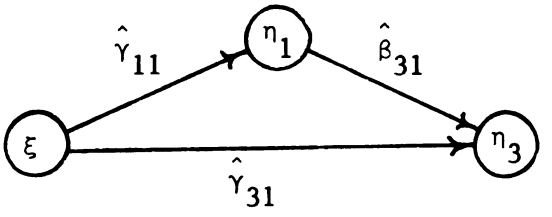
> "The uncorrelated residuals assumption is basically equivalent to the assertion that there is no confounding variable impinging upon both X_1 and X_2 where by a confounding variable is meant any unmeasured factor that directly influences two or more of the measured variables" (Asher, 1976, p. 16).

> "Correlated error terms arise when omitted variables simultaneously influence different observed variables" (Saris, Pijper, & Zegwaart, 1978, p. 161).

A test of the assumptions: Deletion of an endogenous variable. The authors cited above believe that a missing variable results in correlated errors. The example in Figure 9, which was presented by Duncan (1975), appears to contradict this principle, although this example did not appear in a discussion of correlated errors. Duncan (1975) noted that "the OLS estimator of γ_{31} in [the respecified model] estimates $\gamma_{21}\beta_{32}$ without bias. The principle is that insertion of an "intervening variable" into one path of an initial model does not invalidate that model, but merely elaborates it" (p. 109). The following discussion is an elaboration and extension of these principles.



Actual model



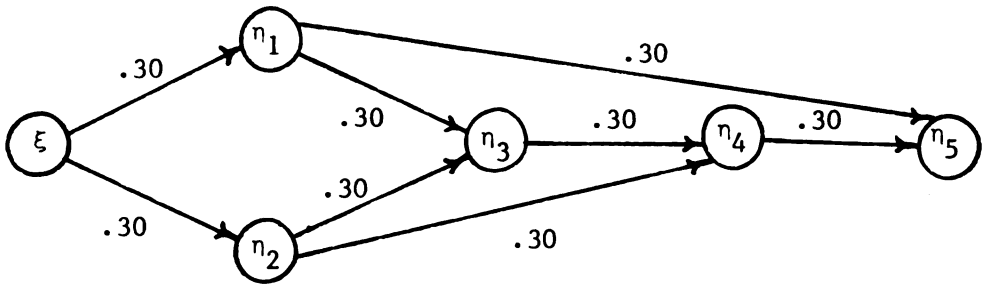
Respecified model

Figure 9. The actual and respecified models presented by Duncan (1975).

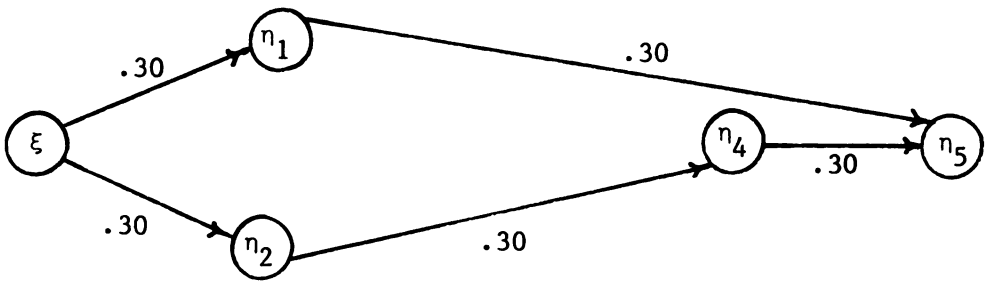
Consider the more complex model presented in Figure 10 where the misspecification is defined by the deletion of η_3. The parameter estimates presented in the misspecified model were computed with OLS. The residuals of the misspecified model are presented in Table 14.

The pattern of residuals is that which would be produced by the deletion of a path. If the misspecified model is respecified by the inclusion of a path from η_1 to η_4 with $\hat{\beta}_{41} = .09$, then (a) the other parameter estimates remain unchanged and (b) the respecified model fits perfectly, i.e., all residuals are zero. The difference between the two models is that in the misspecified model, the elimination of the path from η_1 to η_3 to η_4 was not replaced by a direct path from η_1 to η_4. Thus the causal impact of η_1 on η_4 was not accounted for in the misspecified model.

Any model is incomplete in the sense that there are missing variables. Otherwise all multiple correlations in the model would be 1.00. What is crucial is not that variables are missing, but that the causal effects of the missing variables are represented by paths connecting the variables which are observed. If paths corresponding to indirect causation are present, then any subset of a recursive model can also be fit by a recursive model. Deleted endogenous variables will not affect the fit of the model if the direct antecedents of the deleted variable directly influence the direct consequents of the deleted variable. If this condition is met, the total effects of the antecedent variables on the consequent variables are the same in both models. That is, deletion of an endogenous variable does not lead to a misspecification unless paths are incorrectly deleted in the reduced model. Thus the deletion of an endogenous variable does not imply that



Actual model



Misspecified model

Figure 10. Actual model (d) from Figure 5 and an accompanying misspecified model.

Table 14

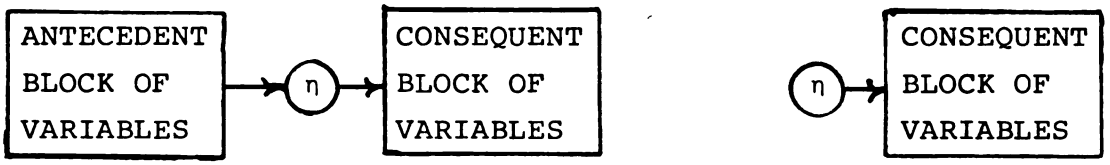
Residuals of the Misspecified Model of Figure 10

	ξ	η_1	η_2	η_4	η_5
ξ	.00	.00	.00	.03	.01
η_1	.00	.00	.00	.09	.03
η_2	.00	.00	.00	.00	.00
η_4	.03	.09	.00	.00	.03
η_5	.01	.03	.00	.03	.00

the disturbance terms of any of the variables remaining in the model should be correlated.

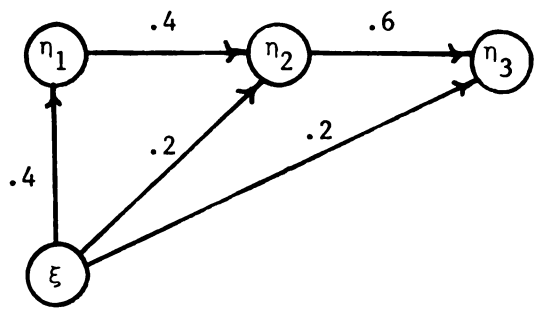
A test of the assumptions: Deletion of an exogenous variable. If a single variable is the only consequent of a block of variables and this variable is antecedent to another block of variables, then the deletion of the first block of variables implies that the specified model will fit the data perfectly. This general situation is illustrated in Figure 11. The variables in the deleted block of variables do not directly influence the correlations of the variables in the consequent block of variables. Thus, the result of deleting these variables is similar to the result from deleting an endogenous variable. The deletion of the variable(s) does not affect the fit of the model and so does not lead to correlated disturbance terms between any of the remaining variables in the model.

The deletion of an antecedent variable or block of antecedent variables becomes salient, however, if the deleted variable(s) directly influence variables within the consequent block of variables. For example, the misspecified model presented in Figure 12 which is defined by the deletion of ξ does not fit the data generated by the correct model because ξ directly influences η_2 and η_3 as well as η_1. The misspecified model could be "fixed" by adding a path from η_1 to η_3, but this respecification amounts to replacing the spurious effect of ξ on η_1 and η_3 by a direct effect. However, even though the respecified model is conceptually incorrect, it fits the data perfectly, so there are no correlated disturbance terms in the respecified model.

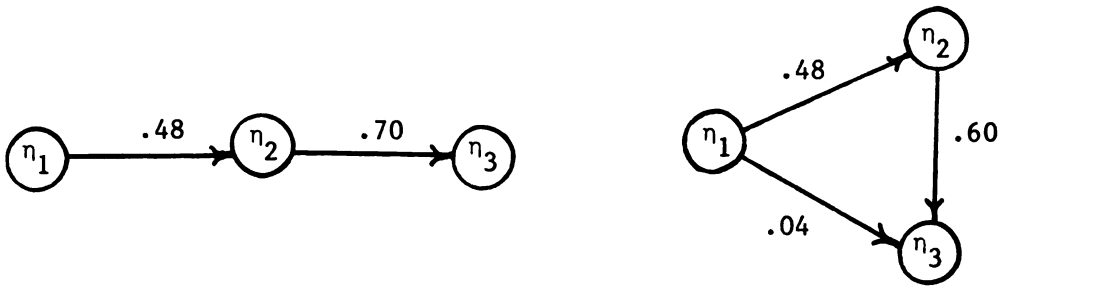


Actual model Specified model

Figure 11. Deletion of a block of exogenous variables which affects only a single endogenous variable.



Actual model



Misspecified model Respecified model

Figure 12. Actual model (d) from Figure 5 and misspecified and respecified versions of this model.

Correlated Errors in Practice: The Use of Derivatives to Respecify a Model

Given a maximum likelihood estimation procedure such as that used by LISREL, a test for locating a potential misspecification for a solution which has converged is based on the values of the first partial derivatives of the likelihood function with respect to each of the fixed parameters. If the model fits the data poorly, some of the derivatives of the parameters fixed at constant values should differ from zero. Sörbom (1975) recommends that a misspecified model may be respecified by using these derivatives. He advises that "we should relax the . . . restriction for the element which gives the largest decrease in $F(\hat{\Sigma})$" (p. 143). That element is the element with the largest partial derivative.

Consider the actual and misspecified model in Figure 6. Since β_{32} was falsely fixed at zero in the misspecified model, the derivative with respect to β_{32} should not be zero in a LISREL solution of the misspecified model given the data generated by the actual model. However there were three nonzero first derivatives in the LISREL analysis of the path model in Figure 6 and these derivatives appeared in both the B and ψ matrices. The corresponding derivatives were also nonzero in the LISREL simultaneous analysis of the model in Figure 6 in which each of the latent variables was measured with three indicators with factor loadings of .80, .60, and .40. These derivatives appear in Table 15.

In both the two-step and simultaneous LISREL solutions, the largest derivative was for $\sigma(\zeta_2,\zeta_3)$. In the simultaneous analysis the value of this derivative was almost twice the size of the next largest derivative over all the parameters, which is the derivative for β_{32}.

Table 15

Derivatives for the Model in Figure 6 Computed by LISREL

Parameter	Two-step analysis	Simultaneous analysis
$\sigma(\zeta_2,\zeta_3)$	-.53	-.34
β_{32}	-.46	-.18
β_{23}	-.40	-.15

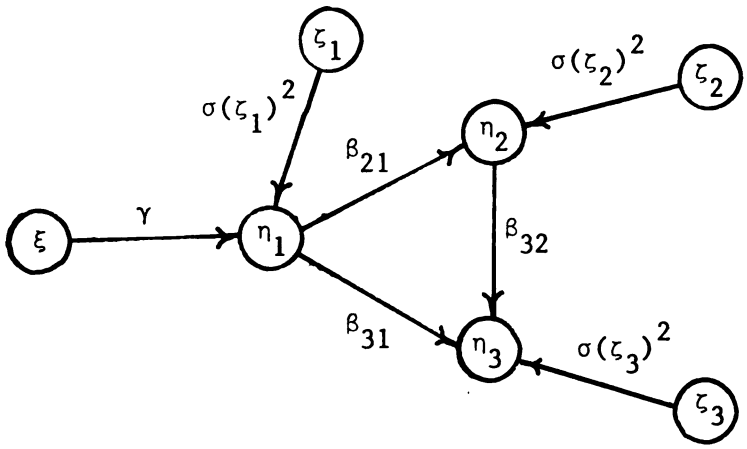
If the model were respecified by relaxing the constraint which yielded the largest derivative, the fit of the respecified model would be improved, but the respecified model would not be the model which generated the data. Thus for this model, Sörbom's (1975) advice is false. Moreover, it was the deletion of a path--not the deletion of a variable--that led to the generation of correlated disturbances. The rules for models with correlated disturbances are just the rules for models with missing paths.

The equivalence between correlated disturbances and a misspecification defined by a missing path is even clearer in the following example. The actual and misspecified models in Figure 13 are the same models presented in Figure 6 except that the three disturbance covariances are <u>not</u> constrained at zero in the misspecified model. These disturbance terms are included in Figure 13 for heuristic purposes only. The parameter estimates of the misspecified model are computed in Appendix B.

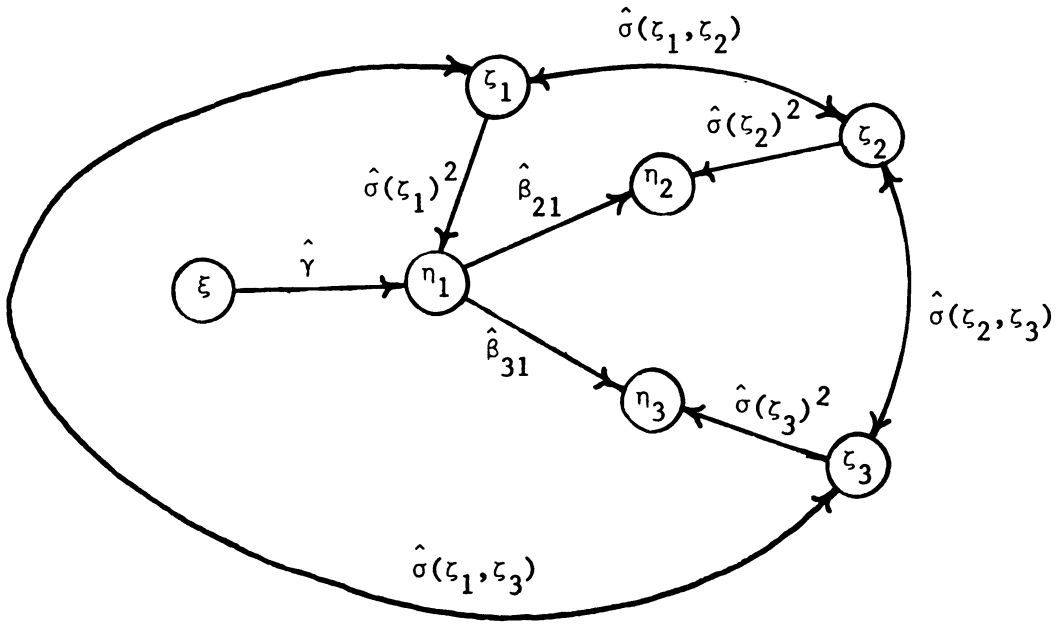
The estimated regression parameters of the two misspecified models are equal. Of particular interest, however, is the comparison between the single nonzero OLS residual and the single nonzero disturbance covariance: they are equal. That is,

$$\text{OLS}[\text{Res}(\eta_2,\eta_3)] = \text{LISREL}[\hat{\sigma}(\zeta_2,\zeta_3)] = \beta_{32}(1 - \beta_{21}^2)$$

The correlated disturbance term in the nonrecursive misspecified model is simply the lack of fit in the corresponding recursive misspecified model. The use of correlated disturbances provides no more information than was available from the information provided by the traditional OLS solution. If the disturbance covariances are fixed at zero as they are for the OLS solution, the model does not fit. If the disturbance



Actual model



Misspecified model

Figure 13. Actual model (a) from Figure 5 and a third misspecified model with correlated disturbance terms.

covariances are free parameters whose values are to be computed from the data, then the model fits the data perfectly. But to say that this model with free disturbance covariances fits the data is to say nothing at all given that the OLS solution does not fit the data.

Correlated Errors: Summary

There are situations such as the analysis of longitudinal data (Hunter, Coggin, & Gerbing, Note 9) in which an analysis with correlated errors is appropriate. But the results of this paper suggest that there are at least two situations in which the use of correlated errors is inappropriate or at best misleading: (a) correlated measurement errors as a substitute for decomposing ad-hoc composites, and (b) correlated disturbances as a substitute for addition of a path. To propose a model with correlated errors in either of these two circumstances is to do nothing more than propose a misspecified nonrecursive, unidimensional model. To claim that "OLS is biased if there are correlated disturbances in the presence of a deleted path" is to say nothing more than that the parameter estimates from OLS are wrong because the model is false. If LISREL with correlated disturbances is used instead of OLS to estimate the parameters, the model is still false. The parameter estimates are different but equally misleading. Although the apparent fit of the respecified model with correlated errors may be dramatically improved, the model is still wrong.

Most of the simulation studies which examine the properties of the statistical procedures for simultaneous equation models begin with a model or models which contain correlated errors, e.g., Hanushek and Jackson (1977) or Cragg (1968). Hanushek and Jackson (1977) concluded that "there is a noticeable increase in the bias in the ordinary least

squares estimator as the correlation between the error terms in the two equations increases" (p. 237), which is only to say that as the model becomes more and more false, the OLS estimators become less and less useful. Or consider the studies by Costner and Schoenberg (1973), Sörbom (1975), or Saris et al. (1978) in which they attempted to determine useful indices for locating misspecification. These authors, with but the one exception in Figure 7, used a given model with correlated errors to generate the data and then attempted to fit a recursive, unidimensional model to this data. Under these conditions it is not surprising that all of these authors have failed to realize that the residuals may contain some very useful information for detecting misspecification.

Contemporary use of correlated errors and correlated disturbances has led to erroneous models in most cases. Perhaps the fact that LISREL permits correlated errors should be regarded as a flaw in the program instead of an advantage.

APPENDICES

APPENDIX A

COMMUNALITIES IN A MULTIPLE GROUPS ANALYSIS

Unlike most contemporary authors, Kenny (1979) recognizes that the classical multiple groups analysis is a confirmatory factor analysis. However he completely abandons multiple groups in favor of LISREL. A "factor analytic solution to [a multiple indicator model] . . . is the multiple group solution . . . which is rarely used. Like most factor analytic solutions, it . . . suffers from the communality problem; that is, communalities must be specified in advance" (p. 138). This is a serious misunderstanding of the factor analysis literature since communalities, like any other parameter, must be estimated. This is true of LISREL as well. The LISREL estimate of the communality of the i^{th} observed variable is λ_i^2.

Nunnally (1978) is one of the strongest contemporary advocates of the classical multiple groups approach, but he argues that "the diagonal unities in a centroid factor multiple groups analysis can make the loadings seem spuriously high . . . . In a sense, there is nothing wrong with this, because it is the correct mathematical solution. The illusory appearance of large loadings could be reduced by . . . the use of SMCs as communalities . . . . However, this is not really necessary" (p. 420). Nunnally (1978) not only fails to recognize the possibility of iterating for the communality values, he

does not recognize the need.

However, Hunter (Note 7) has shown that except in certain unusual cases, the communalities in a multiple groups analysis will converge to their correct values on successive iterations. The convergence for each indicator of the measurement model used in this study is graphed in Figure 14.

The 0^{th} iteration for each indicator is the initial computation obtained before the first iteration, i.e., the value Nunnally (1978) recommends. If "the correct mathematical solution" implies that the underlying structure is recovered with perfect accuracy in the absence of sampling or specification error, and if the indicators are measured with error, then communalities must be inserted into the diagonal of the correlation matrix.

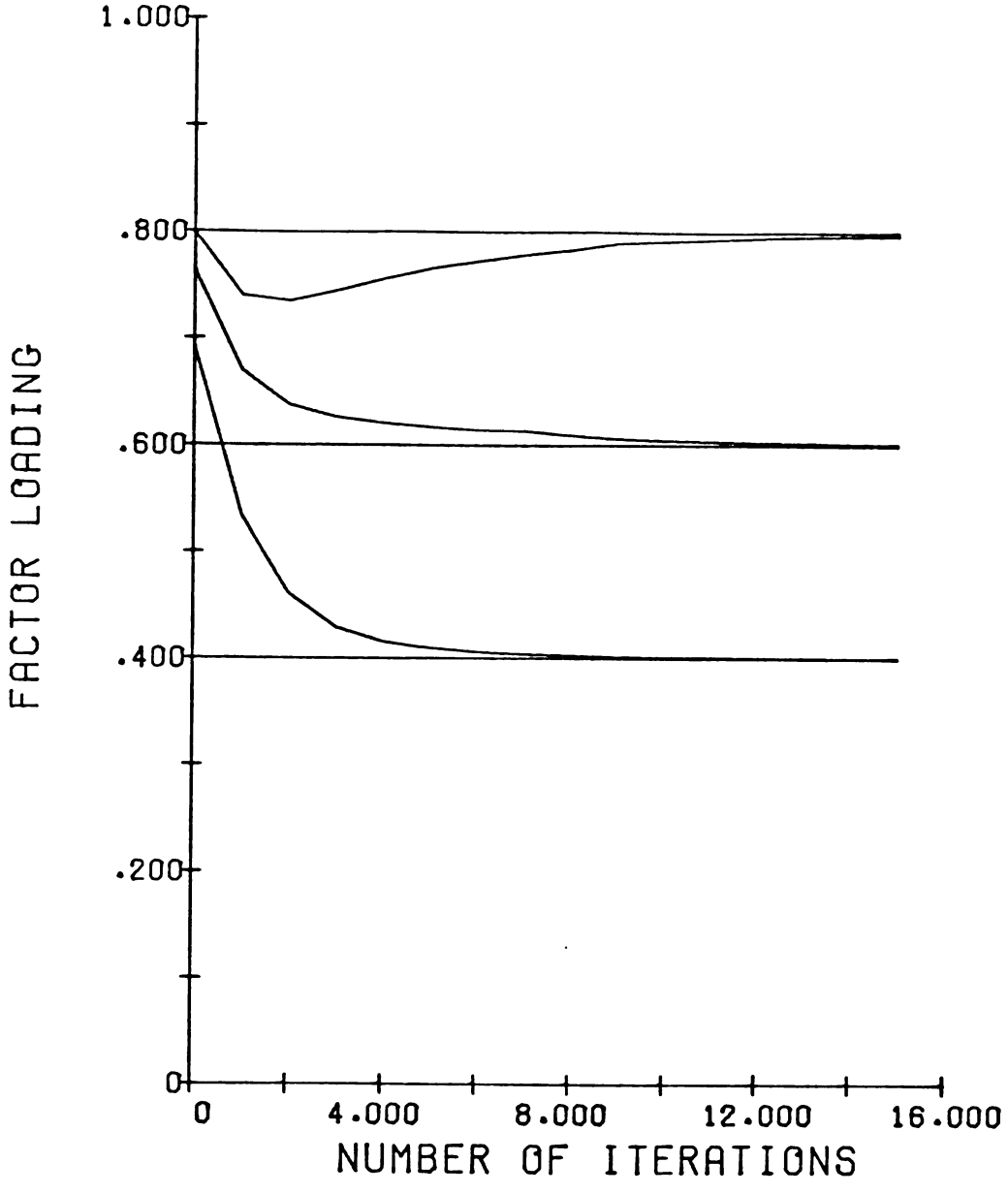


Figure 14. An example of MGRP iterations.

APPENDIX B

ANALYTIC SOLUTION FOR THE PARAMETER ESTIMATES OF MISSPECIFIED MODELS

The parameter estimates and consequently the residuals of the misspecified causal models can be solved analytically by the following algorithm developed for this paper:

(1) Generate the predicted correlations of the correctly specified model, Σ.

(2) Generate the predicted correlations of the misspecified model, $\hat{\Sigma}$.

(3) Solve for the parameters of the misspecified model in terms of Σ.

(4) Express the solution of the parameters of the misspecified model in terms of the parameters of the correctly specified model.

This procedure is illustrated for the misspecified models which appear in Figure 6 and Figure 13 and the corresponding actual model, which is the same in both figures. For the models in Figure 6,

$r(\xi,\eta_1) = \gamma$ $\qquad$ $\hat{r}(\xi,\eta_1) = \hat{\gamma}$

$r(\xi,\eta_2) = \gamma\beta_{21}$ $\qquad$ $\hat{r}(\xi,\eta_2) = \hat{\gamma}\hat{\beta}_{21}$

$r(\xi,\eta_3) = \gamma\beta_{31} + \gamma\beta_{32}\beta_{21}$ $\qquad$ $\hat{r}(\xi,\eta_3) = \hat{\gamma}\hat{\beta}_{31}$

$$r(\eta_1,\eta_2) = \beta_{21} \qquad \hat{r}(\eta_1,\eta_2) = \hat{\beta}_{21}$$

$$r(\eta_1,\eta_3) = \beta_{31} + \beta_{21}\beta_{32} \qquad \hat{r}(\eta_1,\eta_3) = \hat{\beta}_{31}$$

$$r(\eta_2,\eta_3) = \beta_{32} + \beta_{31}\beta_{21} \qquad \hat{r}(\eta_2,\eta_3) = \hat{\beta}_{31}\hat{\beta}_{21}$$

Given these correlations, the OLS parameter estimates and the residuals of the misspecified model may be computed. The OLS estimates of the parameters of the misspecified model are

$$\gamma = r(\eta_1,\xi) \qquad \hat{\beta}_{21} = r(\eta_2,\eta_1) \qquad \hat{\beta}_{31} = r(\eta_3,\eta_1)$$

which can be expressed in terms of the parameters of the actual model.

$$\hat{\gamma} = r(\eta_1,\xi) = \gamma$$

$$\hat{\beta}_{21} = r(\eta_2,\eta_1) = \beta_{21}$$

$$\hat{\beta}_{31} = r(\eta_3,\eta_1) = \beta_{31} + \beta_{21}\beta_{32}$$

The OLS residuals of the misspecified model are defined by $S - \hat{\Sigma}$. Since the Σ matrix is the observed matrix for these examples, the residuals are defined by $\Sigma - \hat{\Sigma}$.

$$r(\xi,\eta_1) - \hat{r}(\xi,\eta_1) = \gamma - \hat{\gamma} = \gamma - \gamma = 0$$

$$r(\xi,\eta_2) - \hat{r}(\xi,\eta_2) = \gamma\beta_{21} - \hat{\gamma}\hat{\beta}_{21} = \gamma\beta_{21} - \gamma\beta_{21} = 0$$

$$r(\xi,\eta_3) - \hat{r}(\xi,\eta_3) = \gamma\beta_{31} + \gamma\beta_{32}\beta_{21} - \hat{\gamma}\hat{\beta}_{31}$$

$$= \gamma\beta_{31} + \gamma\beta_{32}\beta_{21} - \gamma\beta_{31} - \gamma\beta_{21}\beta_{32} = 0$$

$$r(\eta_1,\eta_2) - \hat{r}(\eta_1,\eta_2) = \beta_{31} + \beta_{21}\beta_{32} - \hat{\beta}_{31}$$

$$= \beta_{31} + \beta_{21}\beta_{32} - \beta_{31} - \beta_{21}\beta_{32} = 0$$

$$r(\eta_2,\eta_3) - \hat{r}(\eta_2,\eta_3) = \beta_{32} + \beta_{31}\beta_{21} - \hat{\beta}_{31}\hat{\beta}_{21}$$

$$= \beta_{32} + \beta_{31}\beta_{21} - \beta_{31}\beta_{21} - \beta_{21}^2\beta_{32}$$

$$= \beta_{32}(1 - \beta_{21}^2)$$

Consider the misspecified model in Figure 13 which is identical to the misspecified model in Figure 6 except that the disturbance covariances are unconstrained. The covariance structure of the misspecified model is presented below.

$$r(\xi,\eta_1) = \hat{\gamma}$$

$$r(\xi,\eta_2) = \hat{\gamma}\hat{\beta}_{21}$$

$$r(\xi,\eta_3) = \hat{\gamma}\hat{\beta}_{31}$$

$$r(\eta_1,\eta_2) = \hat{\beta}_{21} + \hat{\sigma}(\zeta_1,\zeta_2)$$

$$r(\eta_1,\eta_3) = \hat{\beta}_{31} + \hat{\sigma}(\zeta_1,\zeta_3)$$

$$r(\eta_2,\eta_3) = \hat{\beta}_{31}\hat{\beta}_{21} + \hat{\beta}_{31}\hat{\sigma}(\zeta_1,\zeta_2) + \hat{\beta}_{21}\hat{\sigma}(\zeta_1,\zeta_3) + \hat{\sigma}(\zeta_2,\zeta_3)$$

The parameters of the misspecified model appear to be just identified since there are six equations in six unknowns, $\hat{\gamma}$, $\hat{\beta}_{21}$, $\hat{\beta}_{31}$, $\hat{\sigma}(\zeta_1,\zeta_2)$, $\hat{\sigma}(\zeta_1,\zeta_3)$, and $\hat{\sigma}(\zeta_2,\zeta_3)$. (The disturbance variances are functions of these six parameters.)

Since there are as many equations as there are unknowns, the parameters of the misspecified model can be expressed in terms of the predicted correlations. Moreover, a just identified model fits any data set perfectly since there are no overidentifying restrictions or degrees of freedom to test the fit of the model. So each predicted correlation $\hat{r}_{ij}$ equals the corresponding actual correlation r_{ij}. Since the actual correlations can be expressed in terms of the parameters of the actual model, solving for the parameters of the misspecified model in terms of the correlations implied by the misspecified model is equivalent to expressing the parameters of the misspecified model in terms of the parameters of the actual model for a just identified model. That is, the FIML solution computed by LISREL can be obtained

heuristically for the just identified model. These computations for this example are presented below.

$$\hat{\gamma} = \hat{r}(\xi,\eta) = r(\xi,\eta_1) = \gamma$$

$$\hat{\beta}_{21} = \frac{r(\eta_2,\xi)}{\hat{\gamma}} = \frac{r(\eta_2,\xi)}{\gamma} = \frac{\beta_{21}\gamma}{\gamma} = \beta_{21}$$

$$\hat{\beta}_{31} = \frac{r(\eta_3,\xi)}{\hat{\gamma}} = \frac{r(\eta_3,\xi)}{\gamma} = \frac{\gamma\beta_{31} + \gamma\beta_{32}\beta_{21}}{\gamma} = \beta_{31} + \beta_{32}\beta_{21}$$

$$\begin{aligned}\hat{\sigma}(\zeta_1,\zeta_2) &= \hat{r}(\eta_1,\eta_2) - \hat{\beta}_{21} \\ &= r(\eta_1,\eta_2) - \beta_{21} \\ &= \beta_{21} - \beta_{21} \\ &= 0\end{aligned}$$

$$\begin{aligned}\hat{\sigma}(\zeta_1,\zeta_3) &= \hat{r}(\eta_1,\eta_3) - \hat{\beta}_{31} \\ &= r(\eta_1,\eta_3) - \hat{\beta}_{31} \\ &= (\beta_{31} + \beta_{32}\beta_{21}) - (\beta_{31} + \beta_{32}\beta_{21}) \\ &= 0\end{aligned}$$

$$\begin{aligned}\hat{\sigma}(\zeta_2,\zeta_3) &= \hat{r}(\eta_2,\eta_3) - \hat{\beta}_{31}\hat{\beta}_{21} \\ &= r(\eta_2,\eta_3) - \hat{\beta}_{31}\hat{\beta}_{21} \\ &= (\beta_{32} + \beta_{31}\beta_{21}) - (\beta_{31} + \beta_{32}\beta_{21})(\beta_{21}) \\ &= \beta_{32}(1 - \beta_{21}^2)\end{aligned}$$

APPENDIX C

COMPUTATIONAL DETAILS OF A SIMULTANEOUS LISREL ANALYSIS

The Relation Between the Indicator Residuals and Factor Residuals

Consider the model in Figure 6 such that each latent variable has three indicators with factor loadings .80, .60, and .40, and $\Upsilon = .30$, $\beta_{21} = \beta_{31} = .35$, and $\beta_{32} = .40$. Also consider the residuals from the simultaneous LISREL analysis between the indicators of η_2 and η_3 which appear in Table 4. Since the residuals are defined as the difference of the corresponding observed and predicted correlations, these residuals may be computed by first computing the observed and predicted correlations.

The observed indicator correlations were generated by the product rule for external consistency. For example, since the actual correlation between η_2 and η_3 reported in Table 2 is .523,

$$r(y_{21},y_{31}) = \lambda_{21} r(\eta_2,\eta_3) \lambda_{31}$$
$$= (.8)\ (.523)\ (.8)$$
$$= .334$$

LISREL computed the corresponding predicted correlations according to the same product rule. The difference is that the estimated values of the factor correlations were substituted for the actual values. That is,

$$r(y_{21},y_{31}) = \hat{\lambda}_{21} r(\eta_2,\eta_3) \hat{\lambda}_{31}$$

Since the factor loadings were correctly recovered by LISREL,

$$\hat{\lambda}_{21} = \hat{\lambda}_{31} = .80$$

The LISREL estimate of $r(\eta_2,\eta_3)$, reported in Table 2, was .234. So,

$$\begin{aligned}\hat{r}(y_{21},y_{31}) &= (.8)\ (.234)\ (.8)\\ &= .150\end{aligned}$$

Thus the residual of the indicators y_{21} and y_{31} is

$$\begin{aligned}\text{Res}(y_{21},y_{31}) &\equiv r(y_{21},y_{31}) - \hat{r}(y_{21},y_{31})\\ &= .334 - .150\\ &= .184\end{aligned}$$

And .184 is the same value computed by LISREL as listed in Table 4.

The keys to reconstructing the residuals between indicator correlations are the residuals among the factor correlations and the respective factor loadings. This can be more explicitly stated by first recomputing the residual between y_{21} and y_{31}.

$$\begin{aligned}\text{Res}(y_{21},y_{31}) &\equiv r(y_{21},y_{31}) - \hat{r}(y_{21},y_{31})\\ &= \lambda_{21} r(\eta_2,\eta_3)\lambda_{31} - \lambda_{21}\hat{r}(\eta_2,\eta_3)\lambda_{31}\\ &= \left[r(\eta_2,\eta_3) - \hat{r}(\eta_2,\eta_3)\right](\lambda_{21}\lambda_{31})\\ &\equiv \left[\text{Res}(\eta_2,\eta_3)\right](\lambda_{21}\lambda_{31})\end{aligned}$$

That is,

$$\begin{aligned}\text{Res}(y_{21},y_{31}) &= (.523 - .234)\ (.8)\ (.8)\\ &= (.289)\ (.8)\ (.8)\\ &= .184\end{aligned}$$

This result can be generalized to any Factors F and G with indicators i and j respectively,

$$\text{Res}(y_{Fi},y_{Gi}) = \text{Res}(F,G)\ \lambda_{Fi}\ \lambda_{Gj}$$

The entire block of residuals for the indicators of factors F and G can be described as

$$(r_{Fi} - \hat{r}_{FG})\ \underline{\lambda}_F \underline{\lambda}'_G$$

where $\underline{\lambda}_F$ is the vector of factor loadings for the N_F indicators of F and $\underline{\lambda}_G$ is the vector of factor loadings for the N_G factor loadings of G. The resulting "block" of residuals is a matrix of order N_F x N_G. For example, let $\eta_2 \equiv$ F, $\eta_3 \equiv$ G, $\underline{\lambda}'_F = \underline{\lambda}'_G \equiv \begin{bmatrix} .8 & .6 & .4 \end{bmatrix}$. Then

$$\begin{bmatrix} .18 & .14 & .09 \\ .14 & .10 & .07 \\ .04 & .07 & .05 \end{bmatrix} = (.289) \begin{bmatrix} .64 & .43 & .32 \\ .48 & .36 & .24 \\ .32 & .24 & .16 \end{bmatrix}$$

For this example in which each factor in the model had the same number of indicators with an identical pattern of factor loadings, the entire residual matrix of indicator correlations can be expressed as a Kronecker product of (a) the entire residual matrix of the factor correlations and (b) the factor loadings. That is,

$$\underset{12x12}{(\Sigma_{IND} - \hat{\Sigma}_{IND})} = \underset{4x4}{(\Sigma_{FAC} - \hat{\Sigma}_{FAC})} \otimes \underset{3x3}{(\underline{\lambda}\,\underline{\lambda}')}$$

The Kronecker product, designated by $\otimes$, is defined as the matrix formed by justaposing the scalar products of every element of the first matrix with the entire second matrix. That is, every element of $(\Sigma_{FAC} - \hat{\Sigma}_{FAC})$ is "replaced" by the multiple of that element with the matrix $(\underline{\lambda}\,\underline{\lambda}')$.

Thus the complete 12 x 12 matrix of residuals defined by $\Sigma - \hat{\Sigma}$ where Σ is computed in a LISREL simultaneous analysis can be expressed as a function of the actual and estimated factor loadings and factor correlations. The actual factor loadings and factor correlations are given and the estimated factor loadings are approximately equal to the

actual factor loadings. The problem of reconstructing the residuals of the indicator correlations has been reduced to reconstructing the residuals of the factor correlations.

Computation of the Parameter Estimates of the Causal Model

LISREL selects those parameter estimates which minimize a function $F(\hat{\Sigma})$ of the residuals among the indicator correlations. But minimizing the residuals among the indicator correlations is just minimizing the residuals among the factor correlations. Since LISREL is a full information technique, the parameter estimates are computed to simultaneously minimize the factor correlation residuals across the entire model.

Given the present example in which the misspecification is defined by the deletion of the path from η_2 to η_3, there are two conflicting "pressures" which must be simultaneously resolved. First, the only source of the correlation between η_2 and η_3 in the misspecified model is the common antecedent η_1, i.e.,

$$\hat{r}(\eta_2,\eta_3) = \hat{\beta}_{21}\hat{\beta}_{31}$$

but the actual correlation between η_2 and η_3 is decomposed as

$$r(\eta_2,\eta_3) = \beta_{32} + \beta_{21}\beta_{31}$$

To the extent β_{32} is large, $\hat{\beta}_{31}$ and $\hat{\beta}_{32}$ must be biased upward to minimize $\text{Res}(\eta_2,\eta_3)$. That is, LISREL "would like" to increase the values of $\hat{\beta}_{31}$ and $\hat{\beta}_{32}$ until

$$\hat{\beta}_{31}\hat{\beta}_{32} = r(\eta_2,\eta_3)$$

But, as a second consideration, LISREL cannot adjust both $\hat{\beta}_{31}$ and $\hat{\beta}_{32}$ without affecting the residuals $\text{Res}(\eta_1,\eta_2)$ and $\text{Res}(\eta_1,\eta_3)$. To the

extent that

$$\hat{\beta}_{31} > \beta_{31} \quad \text{and} \quad \hat{\beta}_{32} > \beta_{32}$$

the values of $\text{Res}(\eta_1,\eta_2)$ and $\text{Res}(\eta_2,\eta_3)$ will increase.

The resulting parameter values computed by LISREL are a compromise. The values of $\hat{\beta}_{31}$ and $\hat{\beta}_{32}$ are <u>increased</u> over their OLS counterparts, which reduces $\text{Res}(\eta_2,\eta_3)$ for the LISREL solution compared to the corresponding OLS residual. However, this increase in $\hat{\beta}_{31}$ and $\hat{\beta}_{32}$ increases $\text{Res}(\eta_1,\eta_3)$ and $\text{Res}(\eta_2,\eta_3)$ over their OLS counterparts, which are equal to zero. Thus the simultaneous LISREL solution provided incorrect factor correlations and obscured the detection of the misspecification in terms of the residuals.

APPENDIX D

RELATIVE SIZES OF THE INDICATOR RESIDUALS WITHIN AND BETWEEN CONSTRUCTS IN AN AD-HOC COMPOSITE IF ALL THE INDICATORS HAVE EQUAL FACTOR LOADINGS

The examination of the residuals from a misspecified ad-hoc composite should reveal a pattern in which the largest residuals were between the indicators of the same construct. The residuals of the indicators of different constructs may be negative, but they should at least tend to be smaller than the residuals of indicators of the same construct. For example, let y_i and y_j be indicators of η_F and y_k be an indicator of η_G. Let all of the actual factor loadings be equal. If y_i, y_j, and y_k are pooled together and misspecified as a construct $\widetilde{\eta}$, then

$$\begin{aligned} Res_{ij} - Res_{ik} &= (r_{ij} - \hat{r}_{ij}) - (r_{ik} - \hat{r}_{ik}) \\ &= r_{ij} - \hat{r}_{ij} - r_{ik} + \hat{r}_{ik} \\ &= r_{iF}r_{jF} - r_{i\widetilde{\eta}}r_{j\widetilde{\eta}} - r_{iF}r_{FG}r_{Gk} + r_{i\widetilde{\eta}}r_{k\widetilde{\eta}} \end{aligned}$$

If $r_{iF} = r_{jF} = r_{kG}$,

$$\begin{aligned} Res_{ij} - Res_{ik} &= r_{iF}^2 - r_{i\widetilde{\eta}}r_{j\widetilde{\eta}} - r_{iF}^2 r_{FG} + r_{i\widetilde{\eta}}r_{k\widetilde{\eta}} \\ &= r_{iF}^2 - r_{iF}^2 r_{FG} \end{aligned}$$

So if $r_{FG} < 1$ and if the actual factor loadings are equal, then the residual for y_i and y_j, indicators of the same construct, will always be larger than the residual for y_i and y_k, indicators of different constructs.

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