

ABSTRACT

MATHEMATICAL MODELING OF SUPERCRITICAL ONCE THROUGH BOILERS

By

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The installation of supercritical once through boilers in modern power plants has presented problems of design and control, due to the high steam pressures and temperatures of operation. Sudden changes of the electrical load that occur during normal operation may cause fluctuations of the steam conditions, which in turn may result in excessive wear of metal parts and in losses of thermal efficiency.

The highly non linear and inter-related processes that take place in supercritical once through boilers require a non linear mathematical model in order that the system's dynamic response to various changes in the operating conditions may be adequately described.

The objective of this study is to develop a mathematical model of general applicability to all supercritical once through boilers.

The model includes a mathematical formulation derived from the physical Laws of Conservation and includes thermodynamic relationships and transport properties of the flue

gas and of the working fluid (water or steam).

The system of non linear partial differential equations, together with a non linear algebraic formulation of the Equation of State for water and steam, is solved numerically by the method of Finite Differences with the aid of a digital computer.

The model includes a computer program which solves for the variation with respect to both time and space of the fluid pressure, temperature, velocity, and specific volume, and of the gas temperature.

The Equations of State, which are presented in this report as subroutines of the computer program, are based on the 1967 IFC Formulation of Thermodynamic Properties of Steam for Industrial Use.

The open-loop, dynamic response of the system to variation of the fluid flow rate, temperature and pressure, and to variation of the fuel firing rate and of the burner tilt are described. No limitations on the magnitude of the disturbances that may be solved for, or on their functional form, were found. A CDC-6500 digital computer required 80 seconds for the computation of 100 seconds of response time. Thus, the capability of the model to provide rapid solutions of the system's dynamic was demonstrated.

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OF SUPERCRITICAL ONCE THROUGH
BOILERS

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To my wife

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INTRODUCTION

In the normal operation of power plants, variations of load and other types of perturbations may result in undesirable fluctuations of the power output and of the steam conditions at the boiler outlet. This requires effective means for controlling the power generation process, particularly where the systems are designed to operate at high steam pressures and temperatures.

The development of once through boilers for central power station, operating at supercritical pressures and high temperatures, has brought about improved thermal efficiencies and reduced costs. The trend to operate at still higher temperatures and pressures is subject to limitations of present materials of construction. Fluctuations of the steam temperature during a transient state, resulting from some perturbation of the operating conditions, may cause excessive wear in the tube circuitry and in the turbine, as well as reduced efficiencies.

The purpose of this work is to develop a mathematical model of a supercritical once through boiler. The model should provide a description of the dynamics of the boiler which is

the most important and the least well understood part of the power plant.

The model is to be of general applicability to all supercritical once through boilers, rather than to a specific design. The model should include: a) A mathematical formulation of the process dynamics; b) A numerical method for solving dynamic problems, involving various types of disturbances; c) A computer program which can perform the numerical solution.

The mathematical formulation is to be derived from the physical laws of conservation, from the thermodynamic equation of state for water and steam, and from correlations of transport properties of the working fluid and of the gas.

Considering the highly non-linear nature of the equation of state, particularly in the neighborhood of the critical point, a linearized approach is not expected to provide a useful tool for control. A linearized model is applicable to small perturbations only, whereas in practice large scale disturbances are normally encountered. Therefore it is required that the equations should not be linearized, and that the numerical method for solving them would allow rapid computation, so that the model may be used for control purposes.

CHAPTER 1: BACKGROUND

The subject of power generation has been covered extensively in the literature. The reader may find that general information, describing aspects of design, construction, operation, and control of power plants¹², is a useful background for this work.

This Chapter contains a description of some basic features of once through boilers (§ 1.1), and a discussion of factors affecting the rate of combustion in the boiler furnace (§ 1.2).

The various modes of heat transfer, and the laws governing its rate, are discussed in § 1.3.

The laws of conservation of mass, momentum, and energy are stated in §1.4 in the form of the differential equations of change. Previous work on modeling of once through boilers is reviewed in § 1.5. In each case, the major simplifying assumptions involved in the simulation are given, and the applicability of the model is discussed.

A qualitative description of the equations of state for water and steam is given in § 1.6. The explicit formulation is given in Appendix C, as computer routines.

§ 1.1 The Once Through Boiler

In a once through boiler there is no recirculation of the working fluid (water or steam) within the unit. In elemental form, the boiler is merely a length of tubing through which the fluid is pumped. Heat is applied, and the water flowing through the tube is converted to steam, superheated to the desired temperature at the outlet. In practice, a single tube is replaced by a multiplicity of small tubes, arranged to provide effective heat transfer.¹⁷

In modern central station boilers, most, if not all, of the furnace enclosure consists of waterwalls, which are exposed to high flame and gas temperatures. The walls are made up of panels with parallel tube circuits, all arranged in a single upward pass. They are fed from furnace wall inlet headers at the lower end, and terminate in the outlet headers at the upper end.¹⁷

The increased thermal efficiencies obtained by operation at higher steam pressures and temperatures, made the elimination of the steam drum a necessity, and enhanced the development of the once through boiler. Rising fuel costs provide further incentive for operation at still higher temperatures and at supercritical pressures. This trend is subject to limitations of tube materials and costs. Thus, operation at 3500 psi and 1000°F (steam pressure and temperature at the

superheater outlet) is widely accepted today in the design of central station boilers.

When steam is generated above the critical pressure (3208 psi), there is no boiling, and the change of phase occurs in a continuous manner. The corresponding changes in the fluid properties (e.g., density, heat capacity) are more moderate than in subcritical operation. As a result, the distribution of flow in a bank of parallel tubes is not as strongly influenced by variation in heat absorption. This provides for further simplification of design by eliminating the requirement of flow distribution devices.¹⁷ Significant losses in plant efficiency result if the steam temperatures fall below turbine nominal admission design values.¹⁶ Steam temperature is controlled by desuperheating, gas recirculation, regulation of firing rate, and burner tilting.¹⁶

The first two methods involve mixing of fluid or of gas streams of different temperatures. Thermodynamically, this results in a net "degradation" of energy, with subsequent loss in thermal efficiency.

The nature of the combustion process is discussed in § 1.2. The flame propagation is considered rapid enough, so that the dynamics of the fire is not important, and the heat release in the furnace is taken as proportional to the firing rate.¹⁶

The change of heat absorption rate along the height of

the enclosure wall, in a typical pulverized-coal-fired furnace, as affected by burner tilt, is shown in Figure 1.1.

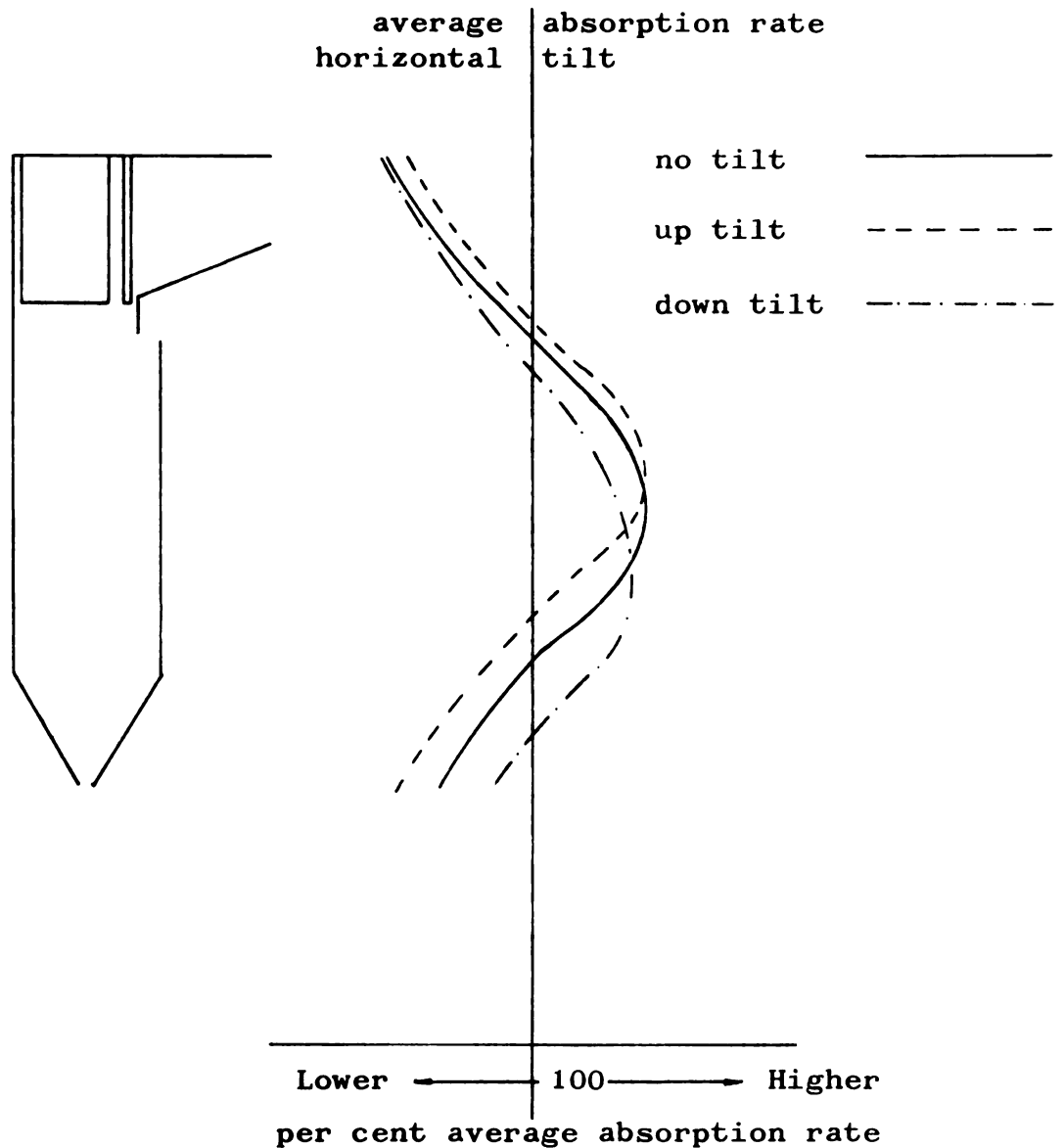


Figure 1.1: Furnace Heat Absorption Pattern, Tangential Firing

§ 1.2 The Combustion Process

Combustion is an exothermic oxidation reaction. In boilers, the reaction vessel is the furnace, which is enclosed by

heat absorbing surfaces, and is provided with means for continuous discharge of the reaction products; namely, flue gases and ash.

Fuel and air enter the furnace and are subjected to rapid heating until the mixture ignites. The heat released by combustion also serves to ignite the incoming fuel, and the process sustains itself.

In the case of powdered-coal-fired furnaces, the reaction mixture is heterogeneous. Considering a coal particle suspended in the gaseous reaction mixture, the reaction may be described as a three stage process:¹¹

1) Oxygen from the gas phase diffuses towards the surface of the coal particle.

2) The oxygen adsorbs chemically on the particle's surface and reaction occurs.

3) The reaction products desorb from the solid surface and diffuse towards the bulk of the gas phase.

Some of the factors affecting the kinetics of this process are:^{11,13} particle size; concentration of oxygen and other gases (nitrogen, reaction products), in the bulk of the gas phase and at the surface of the burning particle; temperature variation from the bulk of the gas towards the particle's surface, and within the particle; total pressure; fuel quality and composition; local velocities of the particle

and of the gas; diffusivities; heat capacities; viscosities; etc.

The overall rate of combustion in the furnace depends on some additional factors:^{11,15,32} the geometrical arrangement of the combustion chamber and of the heat absorbing surfaces; total fuel to air ratio in the feed; fuel quality and particle size distribution; angle of firing (burner tilt), etc.

Some of the factors enumerated above are strongly interdependent. For example, the temperature field in the furnace depends on the relative rates of heat generation by combustion and of heat transfer within and out of the furnace.

This temperature distribution also affects the rates of heat transfer phenomena, as well as reaction kinetics parameters, and physical properties of the reaction components, thus determining the rate of heat generation.

The flow characteristics in the furnace exert a profound effect on the overall rate of reaction. While it is improbable that a rigorous detailed treatment of the complex flows of practical furnaces can be carried out, it is essential that the main features of real flows be taken into account.

Attempts to describe a furnace using "stirred tank" models (thus neglecting transport resistances), have been made.²⁶ It was suggested that real furnaces might be represented by a combination of "stirred tank" and "plug flow" reactors.²¹ This could also serve as a model for the radiative

heat transfer.¹⁹

Experimental methods, such as smoke table tests and 3-dimensional water and air models, have been found useful for visualizing flows in furnaces.¹⁴

In conclusion, a rigorous analytical treatment of the rate of reaction in a boiler furnace is not feasible at the present time.

§ 1.3 Heat Transfer

Heat transfer in general, and in boilers in particular, is the subject of many books and articles.^{22,23,28,34} The basic laws expressing the rate of heat transfer are:

1) Newton's law of convection

$$q = hA \Delta T \quad \dots\dots\dots (1-1)$$

2) Fourier's law of conduction in the x-direction

$$q = - kA \frac{\partial T}{\partial x} \quad \dots\dots\dots (1-2)$$

3) Stefan-Boltzmann law of radiation

$$q = \epsilon \sigma AT^4 \quad \dots\dots\dots (1-3)$$

In the boiler, heat is transferred by some or all of these mechanisms. Heat generated at the surface of a burning coal particle is transferred by conduction towards the center of the particle, by conduction and convection to the gas surrounding the particle, and by radiation in all directions.

Radiative heat flux is absorbed by gases, suspended fuel and ash particles, and metal surfaces. Heat is carried by the hot gas flowing in gross circulation, as well as in small eddies due to turbulence. Heat absorbed at the tube walls' surfaces, exposed to the hot gas, is transferred through the metal wall by conduction and into the fluid stream by conduction and convection. Fresh fuel and air entering the furnace are heated to the point of ignition by the three modes of heat transfer.

In all cases, local temperatures and velocities play a dominant role in determining the rate of heat transfer.

For the purpose of studying the temperature distribution in the gas, conduction is usually considered negligible. The flow in the furnace being highly turbulent, gaseous thermal conductivity is replaced by eddy thermal conductivity, which is due to the mixing of portions of gas of different temperatures, by small scale eddies.¹³

Heat transfer in the furnace is primarily by radiation.^{13,24} The radiative heat flux from the bulk of the hot gas to the absorbing walls, may be approximated by:¹³

$$q = CA \epsilon (T_g^4 - T_w^4) \dots\dots\dots (1-4)$$

where A is the absorbing surface area, normal to the radiative flux; ϵ is the combined average emissivity of the cold surface and of the flame. The emissivity of the flame depends on

temperature, flame luminosity, and composition of the medium. The emissivity of the metal walls depends on surface properties, the degree of slag accumulation, and the temperature. C is the Stefan-Boltzmann constant. The wall temperature, T_w , may be taken as that of the fluid inside the tube, without introducing an appreciable error.

Whereas the most important radiative effect occurs between the gas mass and the boundary walls, a significant interchange of radiant heat occurs between the suspended particles and the absorbing components of the gas. This may be accounted for in terms of absorption coefficients of the gas.¹³

A treatment of heat transfer within a volume element δV of the gas, which will take into account (i) heat generation by combustion, and (ii) heat transport by radiation, conduction, and convection, will result in the equivalent of a radiation field. An effective method for treating radiative heat exchange for such elementary volumes is not available.

Approximations with varying degree of validity have been attempted. It is claimed that furnace performance can be calculated with fair accuracy by assuming a single uniform furnace temperature.¹⁹ An equation expressing the variation of furnace temperature along the furnace height is developed by dimensional analysis, using average overall furnace

emissivities and gas heat capacities.²⁴

Whereas radiation accounts for about 99% of the total heat transferred from the gas to the walls in the furnace,²⁴ convection becomes relatively important in the boiler sections where gas temperatures are lower. Value of gas-to-metal convective transfer coefficients range between 4 and 13 Btu/(hr.)(sq.ft.)(°F), for mass velocity of 6000 lb./ (sq.ft.)(hr.).¹³

A simplified dimensional equation for gases flowing normal to a bank of staggered tubes describes the variation of h_g with the 0.6 power of the mass velocity of the gas.³⁰

The metal-fluid convective film coefficient may be calculated by the Dittus-Boelter or by the Sieder-Tate correlations for convection inside tubes:²⁹

$$\text{Dittus-Boelter} \quad \frac{hD}{k_b} = 0.023(Re)_b^{0.8} (Pr)_b^{0.4} \quad \dots\dots\dots (1-5)$$

$$\text{Sieder-Tate} \quad \frac{h}{(C_p v \rho)_b} (Pr)_b^{2/3} (\mu_w / \mu_b)^{0.14} = 0.023(Re)_b^{0.2} \quad (1-6)$$

$$\text{where} \quad Re = \frac{Dv\rho}{\mu} \quad \text{and} \quad Pr = \frac{C_p \mu}{k}$$

a subscript b indicates evaluation of the subscripted variable at the fluid bulk temperature; a subscript w indicates evaluation of the subscripted variable at the wall temperature.

These correlations are good for: $0.7 < Pr < 120$,

$10^4 < Re < 1.2 \cdot 10^5$, for moderate temperature differences, and for long tubes.

If the temperature differences are high, a good correlation is obtained by the modified expression:

$$Y = 0.023X^{0.8} \quad \dots\dots\dots (1-7)$$

$$\text{where } Y = \frac{hD}{k_f} (Pr)_f^{-0.4} \quad X = \frac{Dv_b \rho_f}{\mu_f}$$

a subscript f indicates evaluation of the subscripted variable at an average temperature

$$T_f = \frac{T_b + T_w}{2}$$

§ 1.4 The Equations of Change

The physical laws of conservation of mass, momentum, and energy, are formulated mathematically as differential equations, referred to as the equations of change. Their explicit form is given below, with time t , and a single space variable z , being the independent variables.

The continuity equation ("C. Egn.") is an expression of the principle of mass conservation.⁴

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial z} + v \frac{\partial \rho}{\partial z} = 0 \quad \dots\dots\dots (1-8)$$

The velocity v is in the z -direction.

The motion equation ("M. Eqn.") is an expression of the principle of momentum conservation.⁴

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \frac{\partial \mathbf{v}}{\partial z} \right) = - \frac{\partial p}{\partial z} - \frac{\partial \tau}{\partial z} + \rho \vec{g} \quad \dots\dots\dots (1-9)$$

The term $\frac{\partial \tau}{\partial z}$ expresses the rate of momentum transfer across the boundaries of the volume element, by a molecular motion mechanism. τ is the normal stress, which is related to the velocity gradient.

The energy equations are expressions of the principle of energy conservation. The two forms given below are equivalent.⁶

The "H. Eqn."

$$\rho \left(\frac{\partial H}{\partial t} + \mathbf{v} \frac{\partial H}{\partial z} \right) = - (\nabla \cdot \vec{q}) - (\tau : \nabla \mathbf{v}) + \left(\frac{\partial p}{\partial t} + \mathbf{v} \frac{\partial p}{\partial z} \right) \dots (1-10)$$

The "T. Eqn."

$$\rho C_p \left(\frac{\partial T}{\partial t} + \mathbf{v} \frac{\partial T}{\partial z} \right) = - (\nabla \cdot \vec{q}) - (\tau : \nabla \mathbf{v}) + \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_p \left(\frac{\partial p}{\partial t} + \mathbf{v} \frac{\partial p}{\partial z} \right) \dots \dots (1-11)$$

Equations 1-8 through 1-11 may be derived by applying the laws of conservation to an infinitesimal volume element of fluid. In the case of a 1-dimensional, fully developed, turbulent flow in a tube, it is possible to compute friction losses from the expression of the friction force F .

$$F = (\pi D \Delta z) \left(\frac{1}{2} \rho v^2 \right) f \quad \dots\dots\dots (1-12)$$

The empirical factor f is a dimensionless quantity, defined by equation 1-12. It is found to vary with the Reynolds Number, and is strongly influenced by the "smoothness" of the tube wall. As the Reynolds Number increases, f approaches a constant value, which depends on the "smoothness" of the tube wall. For example,⁵ at $Re = 10^6$, $f = .003$ for "hydraulically smooth" tube, and increases several fold as the "roughness" of the tube wall increases.

The term $(\nabla \cdot \vec{q})$, appearing in the energy equation, is an expression in vector notation of the heat flowing across the boundaries of the volume element by conduction.

The term $(\tau : \nabla v)$ in tensor notation, is an expression of the heat produced in the volume element by internal friction.

§ 1.5 Mathematical Modeling

The representation of the once through boiler by a set of non-linear differential equations, and the solution of the equations, requires simplifying assumptions. Most attempts at simulation of the steam generation dynamics that have been reported to date have resorted to solution of the problem using lumped parameter approximations and linearized forms of the process equations.

The need for the linearized approach is a result of the

limitations of analog computers, which were used to solve the equations. Even in the linearized form, the magnitude of the problem reaches the upper bound of capability of large analog facilities.

Three attempts at simulation of once through boilers will be described below, in some detail.

1.5-1 Charles P. Crane Unit No. 1

The modeling of a subcritical crushed-coal-fired unit, with 2475 psig steam pressure, and 1050°F steam temperature at the superheater outlet, is described.²

The main assumptions are:

- 1) Fluid properties are uniform at any given cross-section.
- 2) Axial conduction of heat in the fluid, gas, and tube walls is not significant.
- 3) Dynamic effects of gas pressure changes are negligible.
- 4) Balanced flow and uniform heat flux exist at any cross-section in multitube heat exchange boiler sections.
- 5) The system's dynamic behavior is adequately described by small excursions from a series of operating points.
- 6) The portion of the feedwater loop comprised of high and low pressure heaters, condenser, condensate pump, and booster pump, is considered to affect the system only as a slow disturbance to feedwater temperature.
- 7) The turbine-generator and air-heater dynamics are assumed

to be insignificant compared with boiler dynamics.

- 8) Feed pump dynamics are much faster than those of the boiler and therefore are not included in the simulation.

The system, which includes the boiler, the boiler feed-pump, and the turbine-generator, was divided into 14 main sections. In each section, the parameters were lumped. The process equations were linearized, and the dynamics were viewed as small excursions about an operating point. If wider changes occurred, the solution required shifting to another operating point. A digital computer was used to compute the parameters for each operating point, and an analog computer to solve for the dynamic behavior of the system, using those parameters. The model was field tested for step changes in throttle-valve position, combustion rate, and fluid flow rate. The authors report a good fit with the experimental results.

1.5-2 Simulation of Bull Run Super-critical Unit

The modeling of a coal-fired, twin-furnace, supercritical once through unit, with 3500 psi and 1000°F at the superheater outlet, is described.²⁵

The main assumptions are:

- 1) Fluid flow is assumed to be a one-dimensional, fully developed turbulent flow.
- 2) All gravitational and kinetic energy terms are neglected.

- 3) Heat transfer from the tube wall to the working fluid may be expressed by the Nusselt equation, which theoretically applies only to constant flow. (This is the Dittus-Boelter correlation, or Equation 1-5, given in §1.4 above).
- 4) There is no heat transfer along the length of the tube, or along the length of the fluid.
- 5) Heat storage of the tube is concentrated at the center of the tube wall.
- 6) Fluid properties are defined by average values over the cross-sectional area.
- 7) Uniform distribution of tubes in parallel is assumed. Thus, a single fluid flow path was used to represent the multiple flow paths of the unit.
- 8) Fuel flow to air flow ratio is constant.
- 9) The dynamics of the gas-side equations are neglected in the simulation, because they are fast compared to the rest of the process.
- 10) For convective heat transfer on the gas-side, the film coefficients were assumed to vary as the 0.6 power of gas flow, corresponding to the usual assumption of cross flow.
- 11) For both steam-side and gas-side convective film coefficients, variation with temperature was neglected.
- 12) Gas-side radiative heat transfer was represented by the usual Stefan-Boltzmann law, with the assumption of a

constant overall interchange factor.

- 13) Volumes and metal weights of headers and connecting piping were considered to be lumped with their adjacent sections.
- 14) Several of the dynamic equations for each lump are based on the steam properties at the outlet of the lump. During a transient disturbance, the steam properties at the location will change, but this variation in properties was neglected.
- 15) For all of the work, the steady state operating point has been the 75% load condition.
- 16) Equations of state for water and steam were expressed in linearized form around operating values.

The system was divided into a total of 36 sections. In the boiler, the smaller sections were subdivided into two "lumps" each, and the larger sections into four "lumps" each. Fewer "lumps" were used in pressure representation, than in the temperature representation. In each "lump", the equations were linearized by the standard method (neglecting products of increments).

Furnace Dynamics

It was assumed that the products of combustion enter the furnace at a temperature which is below the theoretical (adiabatic) flame temperature, but above the furnace exit temperature. This temperature, referred to as the flame temperature,

was an artifice to develop a dynamic model, and there was no expectation that it could be measured in the actual boiler.

The assumed furnace model was intermediate between two limiting cases which have been proposed: 1) The constant furnace temperature model, which assumes that combustion within the furnace and heat transfer from the furnace are so related, that there is a uniform furnace temperature; and 2) The theoretical flame temperature model, which assumes that fuel and air are perfectly mixed and burned completely, without energy loss, before entering the furnace, and that the products of combustion enter the furnace at the theoretical flame temperature.

The value selected in the present model was based on design data supplied by the manufacturer. It was one which gave the predicted heat transfer rate for reasonable values of emissivities and overall interchange factors.

It was further assumed that changes in flame temperatures were proportional to changes in firing rate and not affected by changes in other manipulated variables.

The model was field-tested, and the authors report a good agreement between the experimental and the computed results.

1.5-3 Canada's Subcritical Once Through Unit No. 3

The modeling of a 220 megawatt unit, with 2400 psi, 1050°F throttle steam conditions, is described.^{3,8} A linearized approach is considered inadequate due to the very non-linear dynamics of once through boilers. Also, for the purpose of designing a control system, large scale disturbances must be considered in the model.

Some of the assumptions are:

- 1) The dynamics of the feedwater heating system are relatively slow, compared to the remaining parts of the fluid circuit, and therefore these components were not considered in detail, in the model simulation.
- 2) Equations describing pressure and mass flow phenomena, may be solved independently from the equations describing the temperature phenomena.
- 3) In each subsection, the continuous flow process is visualized as a sequence of pulsations, whereby the volume of the subsection is filled up instantaneously with fresh fluid at inlet conditions; after residing in that volume for a time equal to the residence time, the fluid is expelled instantaneously to next subsection.

The fluid path is divided into 30 sections. The differential equations are solved numerically by a digital computer, for each solution time interval.

§ 1.6 The Equations of State for Water and Steam

A set of formulations relating the various thermodynamic properties was adopted by the International Formulation Committee (IFC) of the Sixth International Conference on the Properties of Steam. The formulations, relating enthalpy, entropy, specific heat, Gibbs function, Helmholtz function, pressure, temperature and specific volume are contained in the "1967 ASME Steam Tables".³¹

There are four basic sets of functions. Each represents one subregion on the pressure-temperature diagram shown in Figure 1.2. Other formulations define the saturation line, and the boundary between Regions 2 and 3. The two remaining boundaries are constant temperature lines.

Region 1 extends from 32°F to 662°F, and from the saturation pressure to 14,500 psia.

Region 2 covers the entire vapor region, with the exception of a portion near the critical point.

In both regions 1 and 2, the independent variables, or arguments, are pressure and temperature.

A small area near the critical point is covered by the two additional sets of formulations, with specific volume and temperature being the independent variables.

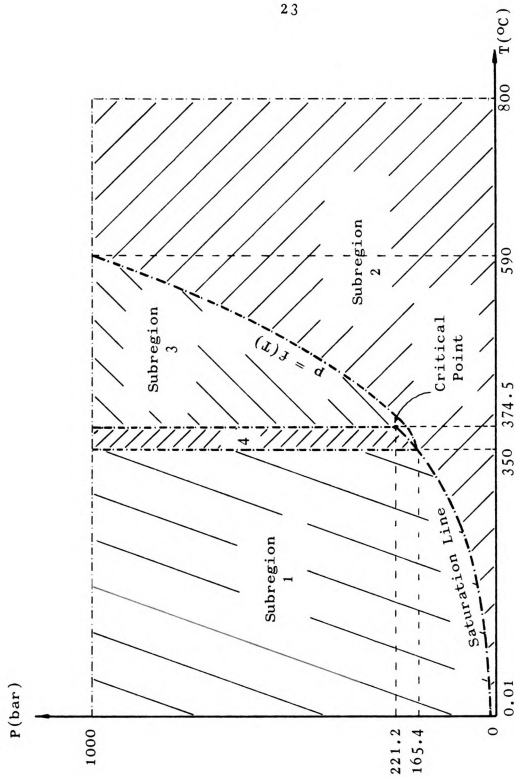


Figure 1.2: Subregions of the Equation of State for Water and Steam

CHAPTER 2: DESCRIPTION OF THE SYSTEM

In this Chapter the system to be modeled will be described, and the major assumptions will be stated. Some of the assumptions will be discussed in detail, whereas those of a generally accepted nature (See § 1.5) will merely be listed. It is usually the gas side which is the least well understood and requires more simplifying assumptions and a detailed discussion.

§ 2.1 The System

The system includes the basic elements of a boiler: a furnace, a waterwall tubing system, and horizontal superheater tube banks.

The fluid path begins at the bottom of the waterwalls. It flows upwards and receives heat from the gas side through the tube's wall. From the upper end of the waterwalls, the fluid discharges into headers and flows into the superheater tube banks. The fluid path is shown in Figure 2.1, where a single tube represents a multiplicity of parallel tubes.

The fluid in the superheater flows along 4 horizontal passes, the first of which being the uppermost. This produces a countercurrent effect in the Superheater Section, where the hottest fluid is in contact with the hottest gas. The

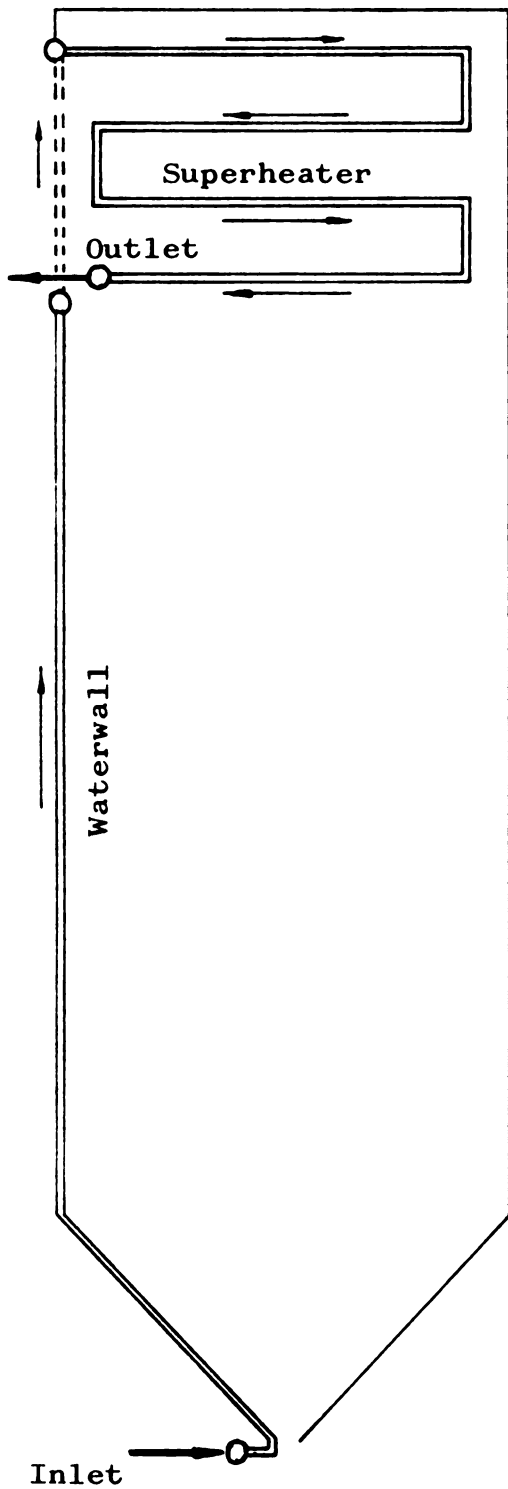


Figure 2.1: The Fluid Path

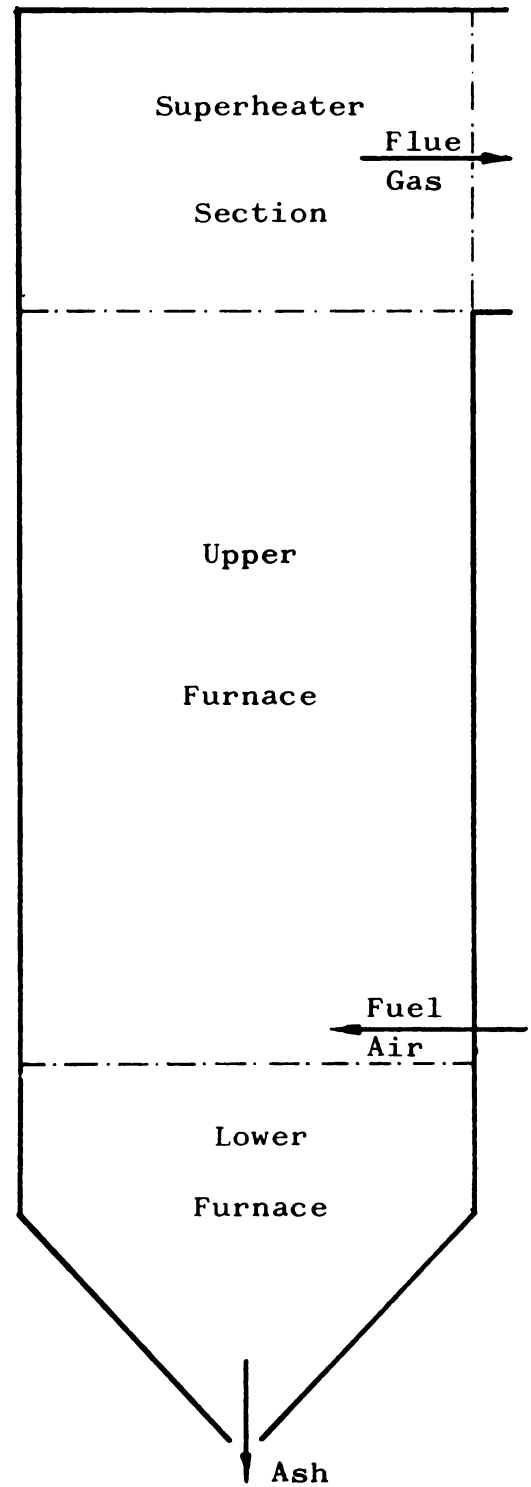


Figure 2.2: The gas Side

outlet from the fourth superheat pass is the end point of the fluid path, or the boiler outlet. The fluid properties (e.g. temperature, pressure, velocity, etc.) at this point are regarded as the outputs of the system. In a power station, the fluid leaves the boiler outlet as a superheated steam and discharges through a regulating valve (throttle valve) into the High Pressure Turbine.

The fuel-air mixture is fired into the furnace at a point approximately $1/3$ of the overall height. It undergoes combustion, and the flue gases flow upwards, transferring heat to the fluid in the tubes. Ash particles flow downwards and are removed at the bottom of the boiler.

The gas enclosure may be divided conceptually into three parts, as shown in Figure 2.2: i) The lower furnace, extending from the bottom up to a (variable) level somewhat below the firing nozzles; ii) The upper furnace, which extends up to the waterwall outlet headers; iii) The superheater section, which encloses the superheater tube banks.

§ 2.2 The Main Assumptions

2.2-1 The Fluid Side

- 1) Fluid flow is assumed to be a one-dimensional, fully developed turbulent flow. Flow properties are defined by average

values over the cross-sectional area of the tube.^{2,25}

- 2) Uniform distribution of tubes in parallel is assumed.

Thus, a single tube is used to represent the multiple tube system.^{2,25}

- 3) The predominant mode of heat transfer in the fluid is by convection. Heat transfer by conduction may be neglected.^{2,25}
- 4) Resistances to heat transfer of the tube wall, and of the fluid-side film, are small in comparison to the resistance of the gas-side film. Therefore, the overall gas to fluid resistance may be approximated as equal to the gas-side film resistance. (See Appendix D, § D.3).

2.2-2 The Gas Side

- 1) The variation of kinetic and potential energy in an element of volume δV of the gas is small compared to energy generated within δV by combustion, and to heat transported into and out of δV by convection and radiation. (See Appendix D, § D.4).
- 2) Dynamic pressure effects in the gas are negligible.^{2,25} A result of this and of the previous assumption is, that the energy phenomena in the gas are independent of pressure effects. The modeling of the gas may, therefore, be limited to the energy equation.
- 3) Terms of energy input and output, into and out of a volume element δV of the gas (including heat generation by

combustion), are large in comparison to energy contained in, or accumulated in δV . Therefore, the variation of energy in δV with time, may be described as a sequence of steady states, over small time intervals. (See Appendix D, § D.4).

- 4) Gas properties are defined by average values over the cross-sectional areas of the boiler.^{2,25}
- 5) Radiation, convection, and turbulent conduction are considered the predominant modes of heat transfer within the gas, and from the gas to the cold walls. Molecular conduction is assumed negligible.^{2,25}

§ 2.3 Heat Transfer and Generation in the Gas

2.3-1 Heat Generation

Exact determination of the rate of heat generated by combustion would require formulation of the chemical kinetics expression, and of its functional relationship to the flow pattern, boiler geometry, cooling rate, etc. (See § 1.1). Such treatment is not considered feasible at the present time. Therefore, additional simplifying assumptions are necessary.

The combustion process occurs within the volume of the flame. The location and the boundaries of the flame vary with the fuel firing rate, burner tilt, and cooling rate. It is assumed that combustion rate at any cross-sectional area of

the boiler may be defined as a uniform average value. The rate of heat generation may be derived from the reaction rate, and expressed in terms of: (heat generated)/(unit time)(unit length of furnace). The available data is not sufficient for formulation of such a function. It may be asserted, however, that it would reach a maximum at some point near the geometrical center of the flame and would diminish in the direction of the flame boundaries. This variation is approximated by a linear function $f(z)$ in Figure 2.3. The point of maximum rate will vary with burner tilt.

2.3-2 Absorption of Radiation within the Gas Volume

Heat transfer from the bulk of the gas to the cold walls is assumed to occur by convection and by radiation. At a given height z , the gas bulk temperature is T_g , and the cold wall temperature is T . Heat is exchanged, according to the Stefan-Boltzmann law of radiation, between the gas envelope "surface" at T_g , and the walls' surface at T . Such description assumes a radiative flux in radial (perpendicular to the cold surface) direction. The radiative flux vector field is, however, very complex. Hot particles emit in all directions; the medium, comprising of gases and solid particles, absorbs and scatters some parts of the radiation that passes through it; the boundary walls reflect part of the incident flux.

A rigorous treatment of the radiation field is not considered feasible (See § 1.3). It is necessary, however, to compensate for the fact that some of the heat generated within the flame is transported instantaneously to colder regions, where it is absorbed by the gases and by ash particles.

A pseudo-generation profile, which describes the absorbed heat as heat "generated" within the gas volume, is proposed.

Qualitatively, radiation absorption is proportional to the flux intensity, the absorptivity of the medium, the volume and the density of the absorbing medium. Thus, the absorption will decrease in the colder parts of the boiler, where the radiation intensities are low.

In the Lower Furnace, where the medium is a heavy suspension of ash particles and the fluid in the waterwalls is at its lowest temperature, the radiation intensities may be assumed to decrease rapidly, as we move away from the flame.

Assuming a linear variation of the absorption along the height of the boiler, and superimposing it over the actual heat-generation function, the resulting effective heat generation function, $f(z)$, is as shown in Figure 2.4. By the definition of $f(z)$, the integral

$$\int_a^b f(z)dz$$

is equal to the net rate of heat generation within the gas volume bounded by $z = a$ and $z = b$. Therefore, the shaded area in Figure 2.4 is equal to the total heat released by combustion, minus heat losses.

2.3-3 Heat Transfer in the Lower Furnace

In the Lower Furnace there is no net flow of gas. However, the gas cannot be considered stagnant. A certain degree of turbulence is known to occur, as well as a temperature gradient in the z (vertical) direction.

It is assumed that there exists a downward heat flow as a result of the temperature gradient, and that the mechanism is governed by the turbulence, or eddies, rather than by molecular motion. Such a description is known as a Dispersion Model. It should be noted that the gas in the Lower Furnace is a rather heavy suspension of ash particles, the physical properties of which cannot be easily defined. An estimate of the Dispersion Coefficient would require a better understanding of the flow pattern in the Lower Furnace, as well as a definition of the physical properties of this suspension. Therefore, the Dispersion Coefficient is taken to be an empirical parameter, to be determined when the model is made to fit an actual boiler.

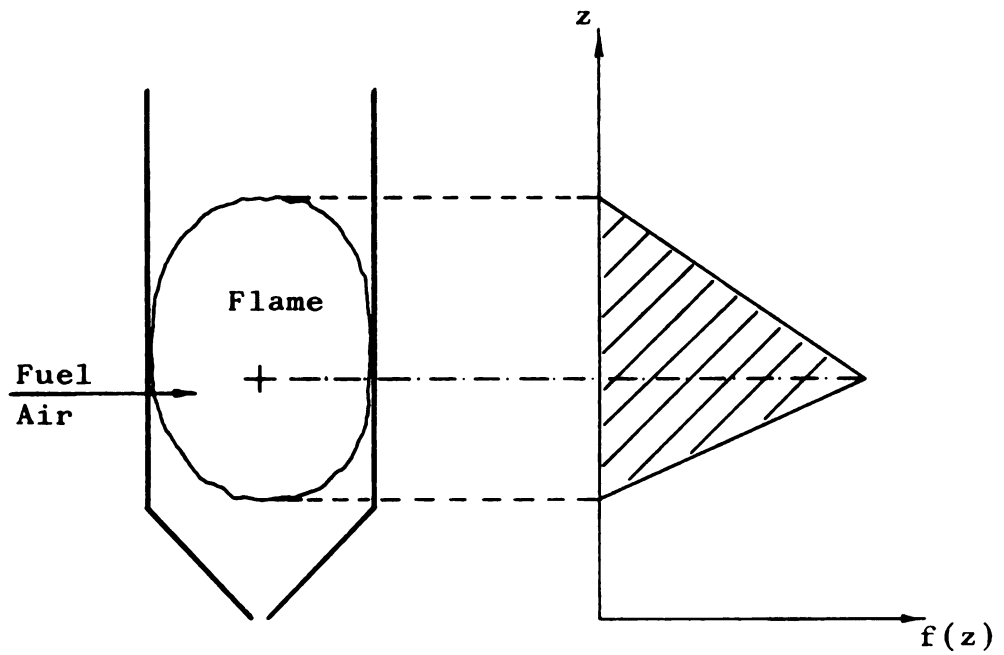


Figure 2.3: Flame Location and Heat Generation

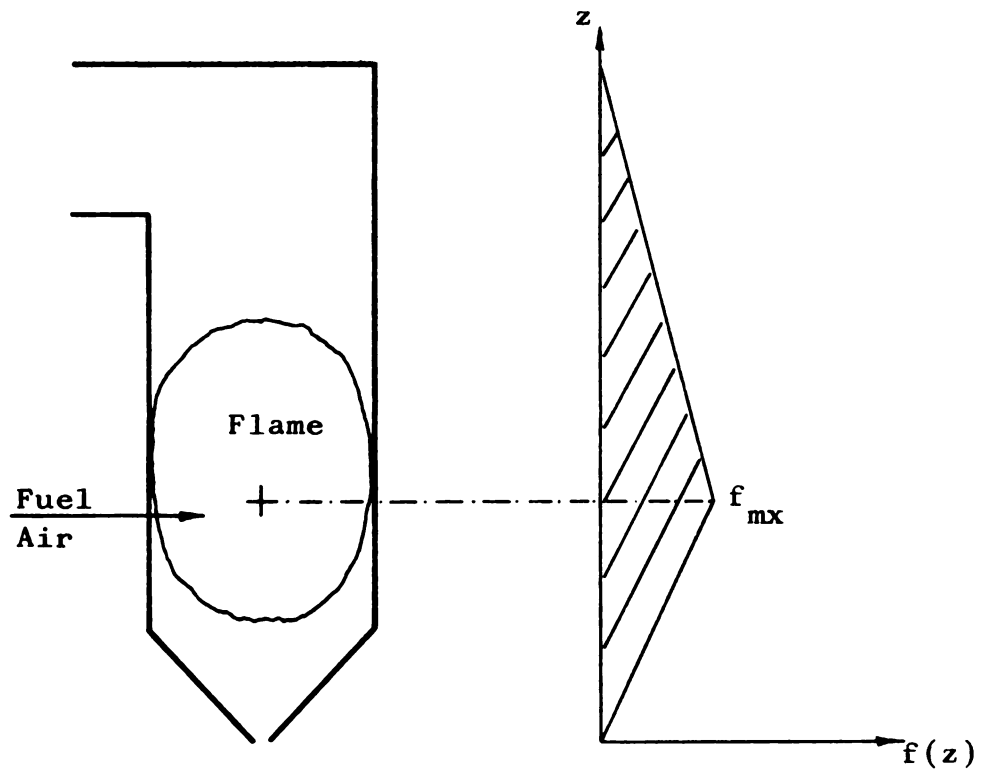


Figure 2.4: Combined Effect of Heat Generation and Absorption of Radiation in the Gas

CHAPTER 3: EQUATIONS AND BOUNDARY CONDITIONS

In this chapter, the equations describing the behavior of the system will be formulated. These include differential equations (the Eqns. of Change), based on physical laws of conservation, equations of state for water and steam, and other relationships between physical properties of the fluid and the gas.

A detailed discussion of the equations is given in Appendix A. The equations of state of the fluid and of heat capacity of the gas are given in Appendix C as computer routines.

The statement of the mathematical problem is completed with a discussion of the boundary conditions.

§ 3.1 Equations of the Fluid Side

The equation of continuity ("C. Eqn."):

$$\frac{\partial \rho}{\partial t} + v \frac{\partial \rho}{\partial z} + \rho \frac{\partial v}{\partial z} = 0 \quad \dots\dots\dots (3-1)$$

The equation of motion ("M. Eqn."):

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) = - 10^6 \frac{\partial p}{\partial z} - \rho \cdot g - \frac{2f}{D} \cdot \rho v^2 \quad \dots\dots\dots (3-2)$$

The equation of energy ("T. Eqn."):

$$\rho C_p \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial z} \right) = q_{tr} + 0.1 \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_p \left(\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} \right) \dots\dots (3-3)$$

where

$$q_{tr} = U \frac{\Delta A_c}{\Delta V} (T_g - T) + \epsilon \sigma \frac{\Delta A_r}{\Delta V} [(T_g/1000)^4 - (T/1000)^4] \dots (3-4)$$

Remarks:

- 1) The units used are in the CGS system (cm, gram, sec), unless otherwise stated. Temperature is given in degrees Kelvin. Energy, heat, and power are given in Joule and Watt; pressure in bars (1 bar = 14,503 psi = 10^6 dyn/sq.cm).
- 2) The acceleration of gravity g in Eqn. 3-2 is taken as zero in horizontal segments of the tube.
- 3) The friction factor f (see definition in § 1.4), depends on the Reynolds Number. At turbulent flow it approaches a constant value which depends on the smoothness of the tube wall. In this work, it is taken to be a constant $f = 0.01$.
- 4) The overall convective heat transfer coefficient, U , is assumed approximately equal to the gas side film coefficient. This latter quantity is generally assumed to vary with the 0.6 power of the gas flow rate, W_g . In this work, W_g is assumed constant along the gas path. Therefore, U is taken to be a constant.

- 5) The convective heat transfer area, ΔA_c , is the surface area of a tube, of length Δz . In the waterwalls, where only one side of the tube's surface is exposed to the gas, ΔA_c is one half of the surface area. The radiative heat transfer area, ΔA_r , is the area of a tube of length Δz , normal to the radiation flux. In the waterwalls, $\Delta A_r = D \cdot \Delta z$. In the superheater tubes, $\Delta A_r = 2D \cdot \Delta z$.
- 6) The emissivity ϵ of the cold surfaces, is taken to be a constant and equal to 1.
- 7) The inside diameter of the waterwall tube in this model is different from that of the superheater tube. There is, however, no expansion or contraction in the fluid path. The equations are written for one superheater tube, or for an equivalent number of waterwall tubes, on the basis of an equal cross-sectional area.
- 8) The effect of piping bends and connections on the equation of motion is neglected.
- 9) The effect of heat content of the metal parts is neglected.

The Equation of State for Steam and Water ("S. Eqn.")

This is a set of equations covering the range:

$0 < T \leq 800^\circ\text{C}$, $0 < p \leq 1000$ bar. The region is subdivided into 4 subregions, with separate formulations for each subregion. The specific volume, $\bar{v}$, enthalpy, H , and entropy, s , are given as explicit functions of the pressure, p , and

temperature, T , in subregions 1 and 2. In subregions 3 and 4, p , H , and s , are given as explicit functions of $\tilde{v}$ and T .

The expressions of enthalpy and entropy are not directly used in this work. However, the derived thermodynamic properties C_p and $(\frac{\partial \ln \tilde{v}}{\partial \ln T})_p$, are also referred to as equations of state and are given in Appendix C as computer routines.

Thus, we have:

In subregions 1 and 2:

$$\begin{aligned}\tilde{v} &= f(p, T) && \dots\dots\dots (3-5_a) \\ C_p &= f(p, T) && \dots\dots\dots (3-5_b) \\ (\frac{\partial \ln \tilde{v}}{\partial \ln T})_p &= f(p, T) && \dots\dots\dots (3-5_c)\end{aligned}$$

And in subregions 3 and 4:

$$\begin{aligned}p &= f(\tilde{v}, T) && \dots\dots\dots (3-5_d) \\ C_p &= f(\tilde{v}, T) && \dots\dots\dots (3-5_e) \\ (\frac{\partial \ln \tilde{v}}{\partial \ln T})_p &= f(\tilde{v}, T) && \dots\dots\dots (3-5_f)\end{aligned}$$

§ 3.2 Equations of the Gas Side

The value of the gas temperature, T_g , is required in the fluid side "T. Eqn." (Eqns. 3-3,4). Therefore, the treatment of the gas side is intended to provide the variation of T_g with respect to time and space. The assumptions inherent in

the gas side model effectively decouple the gas energy equation from the other equations of change. It is, thus, sufficient to solve the energy equation, in the form of heat balances, in order to obtain the gas temperature variation as required.

In the Upper Furnace and in the Superheater Section, the gas is assumed to be flowing in the "axial" (along the z axis) direction.

$$q_i - q_o + W_g C_g T_{gi} - W_g C_g T_{go} = 0 \quad \dots\dots\dots (3-6)$$

q_i is the rate of heat "generation" (See § 2.3).

q_o is the rate of heat transfer from the gas into the fluid.

C_g is the heat capacity of the gas. (Computer routine SCG,

Appendix C; Derivation in Appendix A, § A.4).

$$C_g = f(T_g) \quad \dots\dots\dots (3-7)$$

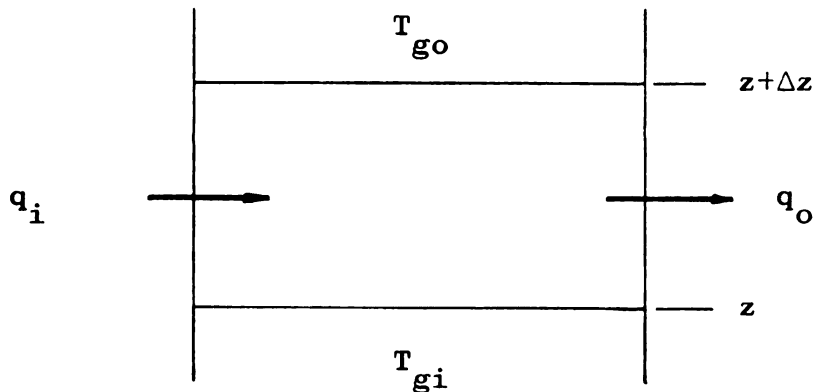


Figure 3-1: Gas Energy Balance in the Upper Furnace

In the Lower Furnace, there is no net gas flow in the axial direction. Axial heat transport is expressed by the Dispersion Model.

$$q_i - q_o + q_{ai} - q_{ao} = 0 \quad \dots\dots\dots (3-8)$$

$$q_{ao} = - D_c \left. \frac{dT_g}{dz} \right|_{z+\Delta z} \quad \dots\dots\dots (3-9_a)$$

$$q_{ai} = - D_c \left. \frac{dT_g}{dz} \right|_z \quad \dots\dots\dots (3-9_b)$$

q_i , q_o are defined as in the Upper Furnace.

D_c is the heat Dispersion coefficient; an empirical parameter to be determined when the model is made to fit a real system. In this work, it is taken to be a constant.

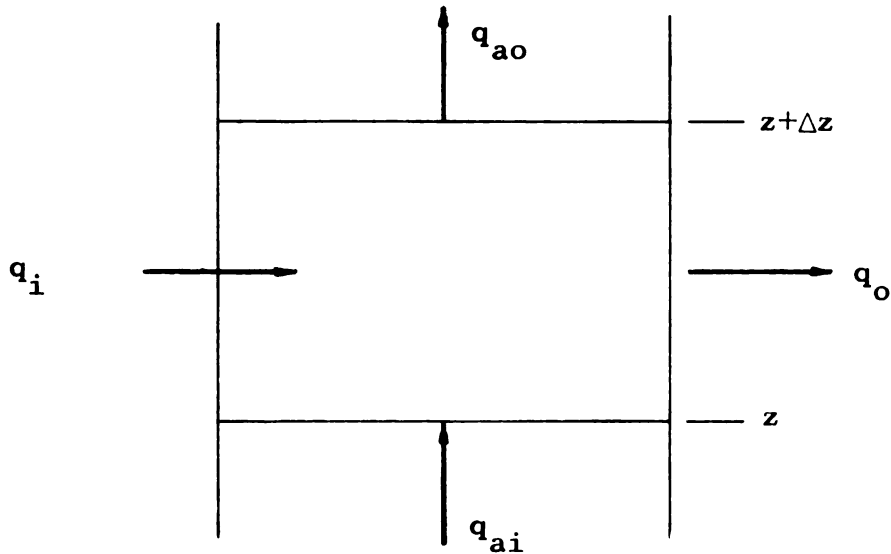


Figure 3.2: Gas Energy Balance in the Lower Furnace

The explicit form of Equations 3-6 and 3-8 depends on the numerical method of solution and will be given in § 3.4.

§ 3.3 The Boundary Conditions

At steady state operation, the energy content of the fluid at the boiler's outlet must be adequate for the required electrical load. This energy equals the sum of enthalpy and kinetic energy of the fluid and is proportional to the mass flow rate.

The behavior of the system at steady operation can be described by the temperature, pressure, density, and velocity profiles of the fluid, and by the temperature profile of the gas. The term "profile" refers to the variation of the quantity along the fluid's (or gas') path. These profiles can be seen as solutions to a set of ordinary differential equations, plus the equations of state of the fluid, and properties of the gas. The steady state differential equations are obtained from the equations of change (Eqns. 3-1,2,3), by omitting the time derivatives:

$$\text{C. Equation} \quad \rho v = w = \text{constant} \quad \dots\dots\dots (3-10)$$

$$\text{M. Equation} \quad w \frac{dv}{dz} = - 10^6 \frac{dp}{dz} - \rho g - \frac{2f}{D} wv \quad \dots\dots\dots (3-11)$$

$$\text{T. Equation} \quad wC_p \frac{dT}{dz} = q_{tr} + 0.1 \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_p v \frac{dp}{dz} \quad \dots\dots\dots (3-12)$$

To solve these equations, three boundary conditions are

required. One boundary condition would be the fluid temperature at the Superheater outlet (which is also the inlet to the turbine). This value is usually included in the boiler's specifications. Similar consideration applies to the fluid pressure at that point. The fluid velocity determines the mass flow rate. Thus, these three variables specify the energy input into the turbine. The pressure and temperature at steady state operation are those given in the boiler rating, and the velocity varies with the electrical load. The relationship between the load and the velocity is outside the scope of this work. Deviations from steady state, caused by load variations, are viewed as changes in fluid velocity at the inlet to the boiler. Similarly, changes of fluid pressure and temperature at the boiler inlet are considered "inputs" to the system, in the sense used in Systems Analysis. The values of the fluid pressure, temperature, and velocity at the boiler's outlet are the most interesting outputs, in the same sense, although the method of solution described in Chapter 4 provides those quantities at all points along the tube.

It is, thus, possible to solve the steady state equations, using the boundary conditions at the boiler outlet, and obtain steady state profiles. Then, the response of the system to some disturbance, or input, may be studied. The steady state

profiles constitute initial conditions for the dynamic problem, and fluid pressure, temperature, and velocity at the inlet of the boiler are its boundary conditions.

The gas temperature profile is a result of the interaction between the rates of heat generation and heat transfer. At unsteady state, the gas temperature variation will be treated as a sequence of steady states. (This point is discussed in § 4.4-5). The value of the gas temperature at the bottom of the boiler is taken to be a constant.

CHAPTER 4: THE NUMERICAL SOLUTION

§ 4.1 The Numerical Method

The fluid dynamics is described by a system of 3 differential equation (Eqns. 3-1,2,3), referred to as the Equations of Change. The equations are first-order, nonlinear, partial with respect to time t and distance z , and include 4 variables. These variables are the fluid temperature T , pressure p , velocity v , and density ρ .

Using the equations of state, the specific volume $\tilde{v} = 1/\rho$, may be expressed in terms of p and T (Eqn. 3-5_a), or p in terms of $\tilde{v}$ and T (Eqn. 3-5_d). The number of independent functions is, thus, reduced (implicitly) to three, and the system is consistent. The highly complex form of the state equations does not permit elimination by substitution of any function from the equations of change. Neither is it deemed feasible to obtain an analytical solution of the system. Linearization of the equations will impose serious limitations on the model. Most of the variables and parameters show marked nonlinear behavior in the vicinity of the critical point. As a result, the accuracy of linearization will be limited to very small deviations from steady state.

The technical limitations of analog computers make it necessary to use a numerical method which a digital computer

can handle in reasonable computing time. The method selected for this work is the method of Finite Differences, whereby first-order derivatives are approximated by ratios of finite increments.

$$\frac{\partial F}{\partial z} \approx \frac{\Delta F}{\Delta z} \quad , \quad \frac{\partial F}{\partial t} \approx \frac{\Delta F}{\Delta t} \quad \dots\dots\dots (4-1)$$

The t and z coordinate axes are divided into equal increments of time and distance, respectively. The z - t space may be mapped by a rectangular grid, as shown in Figure 4.1.

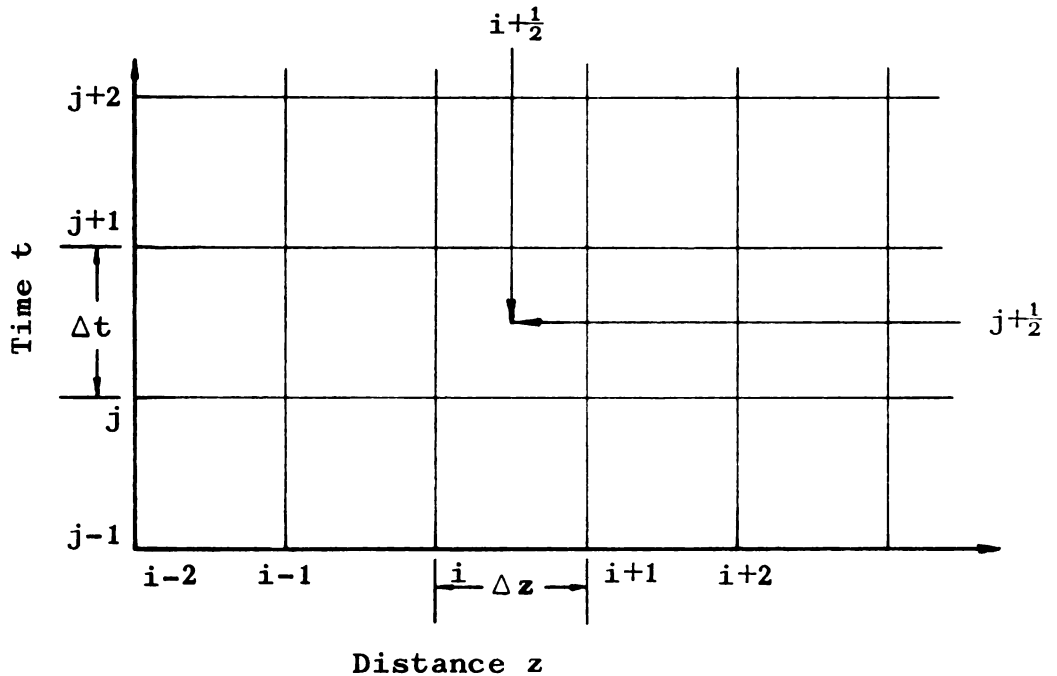


Figure 4.1: Time-distance Space in a Finite-differences Rectangular Grid

A point in the z - t space is specified by an index number i , for the distance z , and by an index number j , for the time t . Using this notation, Equations 4-1 become:

$$\frac{\partial F}{\partial z} = \frac{F_{i+1,j+1} - F_{i,j+1}}{\Delta z}, \quad \frac{\partial F}{\partial t} = \frac{F_{i,j+1} - F_{i,j}}{\Delta t} \dots (4-2)$$

The differential equations can be written in the Finite Differences form, which lends itself to algebraic solution.

For example, the C. Equation (Eqn. 3-1):

$$\frac{\rho_{i,j+1} - \rho_{i,j}}{\Delta t} + v_{i+\frac{1}{2},j+\frac{1}{2}} \frac{\rho_{i+1,j+1} - \rho_{i,j+1}}{\Delta z} + \rho_{i+\frac{1}{2},j+\frac{1}{2}} \frac{v_{i+1,j+1} - v_{i,j+1}}{\Delta z} = 0 \dots (4-3)$$

For brevity, a different notation, shown in Figure 4.2, will be adopted.

$$F_{i,j} = F_y, \quad F_{i,j+1} = F_x$$

$$F_{i+1,j} = F_z, \quad F_{i+1,j+1} = F_m$$

$$F_{i+\frac{1}{2},j+\frac{1}{2}} = F_j$$

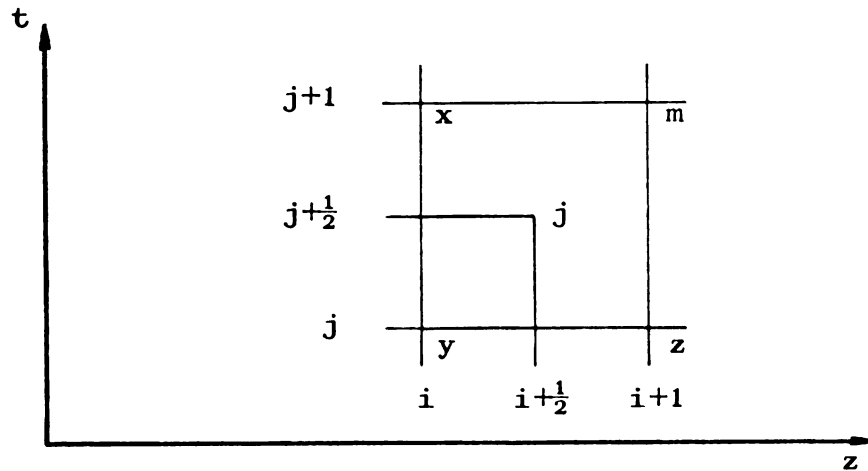


Figure 4.2: Notation for the Time-distance Grid

Given the values F_y , F_z , and F_x , the intermediate value F_j is approximated by linear interpolation:

$$F_j = (F_x + F_z)/2 \quad \dots\dots\dots (4-4)$$

F_m may be expressed in terms of F_x , F_y , F_z , and F_j by rearranging the differential equation. The physical properties of the fluid appearing in the differential equation are evaluated at the point "j".

It may be noted that the equations of change are "coupled"; i.e., each of them involves more than one function. Therefore, the whole system of equations, which includes: i) The Equations of Change (Eqns. 3-1,2,3), ii) the S. Equation (Eqns. 3-5), and iii) the gas energy balances (Eqns. 3-6 and 3-8), must be solved simultaneously. The thermodynamic and transport properties are computed at each point along the fluid and gas paths by theoretical or empirical formulae, given as computer sub-routines.

Initial conditions for the dynamic problem will be the steady state profiles. Their computation is described in §4.2. The solution of the dynamic problem is described in §4.3, and the computation of the gas temperature profiles is discussed in §4.4.

§ 4.2 The Steady State Solution

4.2-1 The Equations of Change

The steady state equations (Eqns. 3-10,11,12), are first-order, nonlinear, ordinary differential equations. By the Finite-differences method, a derivative is approximated as follows:

$$\frac{dF}{dz} \approx \frac{\Delta F}{\Delta z} = \frac{F_{i+1} - F_i}{\Delta z} = \frac{F_m - F_x}{\Delta z}$$

Using the above notation and rearranging, the Equations of Change become:

C. Equation

$$\rho_m v_m = w = \text{constant} \quad \dots\dots\dots (4-5)$$

M. Equation

$$v_m = v_x - \frac{10^6}{w} (p_m - p_x) - \Delta z [g/v_x + (2f/D)v_x] \quad \dots\dots (4-6_a)$$

$$p_m = p_x - 10^{-6} \left\{ w(v_m - v_x) + \Delta z [\rho g + (2f/D)wv_x] \right\} \dots\dots (4-6_b)$$

T. Equation

$$T_m = T_x + \frac{\Delta z}{wC_{px}} q_{tr} + 0.1 \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_{px} \frac{\tilde{v}_x}{C_{px}} (p_m - p_x) \quad \dots\dots (4-7)$$

The term q_{tr} is the combined radiative and convective heat transfer rate across the tube wall per unit of fluid

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volume. It may be recalled (§ 3.1) that the superheater tube is different from the waterwall tube. As a result, the ratios $\frac{\Delta A_r}{\Delta V}$ and $\frac{\Delta A_c}{\Delta V}$, which appear in the expression of q_{tr} (Eqn. 3-4), are different. It is desired, however, to maintain the expression of q_{tr} unchanged throughout the boiler. This is done by defining D_{eq} as the equivalent diameter (see Appendix A, § A.3), and we obtain:

$$q_{tr} = \frac{D_{eq}}{A_{cx}} \left\{ \sigma [(T_{gx}/1000)^4 - (T_x/1000)^4] + \frac{\pi}{2} U(T_{gx} - T_x) \right\} \dots \dots (4-8)$$

Remarks:

- 1) By the notation used in Equations 4-5 through 4-8, $F_x = F_i$, $F_m = F_{i+1}$; i being the index number of the z (distance) coordinate.
- 2) Physical properties of the fluid and the gas temperature T_g , are evaluated at point i , the "entrance" to the volume element.
- 3) The emissivity, ϵ , taken to be equal to 1, is omitted from the expression of q_{tr} .

4.2-2 The Computational Procedure

The simultaneous solution of Equations 4-5 through 4-7, together with the Equations of State (Eqns. 3-5), requires an iterative computational procedure. This procedure is described

below for a single volume element of the tube.

- 1) Given v_x , T_x , p_x , T_{gx} , the values of $\tilde{v}_x$, C_{px} , $\left(\frac{\partial \ln \tilde{v}}{\partial \ln T}\right)_{px}$, are computed from the S. Equation (Eqns. 3-5_{a,b,c}).
- 2) $\rho_x = 1/\tilde{v}_x$
- 3) The iterative process:
 - i) An initial guess is made on the value of p_m .
 - ii) v_m is computed from Eqn. 4-6_a.
 - iii) ρ_m is computed from Eqn. 4-5.
 - iv) $\tilde{v}_m = 1/\rho_m$
 - v) T_m is computed from Eqn. 4-7.
 - vi) $\tilde{v}_s$ (sp. volume) is computed from the S. Equation, (Eqn. 3-5_a), using the values of p_m and T_m .
 - vii) If $\tilde{v}_m \neq \tilde{v}_s$, the difference $\tilde{v}_m - \tilde{v}_s$ is used to correct the initial guess of p_m , by linear interpolation or extrapolation. This is repeated until $\tilde{v}_m = \tilde{v}_s$.

The values p_m , T_m , $\tilde{v}_m$, and v_m are then recorded and used in the computation of the next point along the tube.

When the end of the tube is reached, the steady state profiles of p , T , $\tilde{v}$, and v are given in tabulated form (i.e., as numerical values at discrete and equidistant points along the tube).

An alternative stagewise computation may be used, whereby the initial guess is made on the value of v_m . The value of p_m is then computed from Eqn. 4-6_b, ρ_m and $\tilde{v}_m$ from Eqn. 4-5, and

T_m from Eqn. 4-7. Using T_m and $\tilde{v}_m$, the pressure p_s is computed from the S. Equation (Eqn. 3-5_d). p_s is compared to p_m , and the initial guess of v_m is corrected as in the former method until $p_m = p_s$. This procedure is used in subregions 3 and 4 of the state equations, where pressure is expressed as a function of $\tilde{v}$ and T .

4.2-3 The Gas Temperature

The solution of the gas-side energy balances provides the gas temperature profiles. This will be described in §4.4.

4.2-4 The Boundary Conditions

Boundary conditions have to be assigned for the problem to be mathematically defined. The values of fluid pressure and temperature are specified at the boiler outlet, and the value of fluid velocity is relative to the electrical load (see § 3.3).

It is, therefore, convenient to start the computation at the boiler outlet, and proceed "backwards" along the tube. This does not affect the form of the equations, as Δz is given a negative value.

§ 4.3 The Unsteady State Solution

4.3-1 The Equations of Change

Using the notation described in §4.1, the Equations of Change (Eqns. 3-1,2,3) may be written as follows:

C. Equation

$$\rho_m = \rho_x - \frac{\Delta z}{v_j} \cdot \frac{\rho_x - \rho_y}{\Delta t} - \frac{\rho_j}{v_j} (v_m - v_x) \quad (4-9)$$

M. Equation

$$v_m = v_x - \Delta z \left\{ \frac{1}{v_j} \left[\frac{v_x - v_y}{\Delta t} + g \right] + (2f/D)v_j \right\} - \frac{10^6}{\rho_j v_j} (p_m - p_x) \dots$$

..... (4-10_a)

alternatively:

$$p_m = p_x - \frac{\rho_j v_j}{10^6} \left\{ \Delta z \left[\frac{1}{v_j} \left(\frac{v_x - v_y}{\Delta t} + g \right) + (2f/D)v_j \right] - (v_m - v_x) \right\} \dots$$

..... (4-10_b)

T. Equation

$$T_m = T_x + \frac{\Delta z}{v_j} \left[\frac{T_x - T_y}{\Delta t} + \frac{q_{tr}}{\rho_j C_{pj}} \right] +$$

$$0.1 \Delta z \frac{\tilde{v}_j}{C_{pj} v_j} \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_{pj} \left[\frac{p_x - p_y}{\Delta t} + v_j \frac{p_m - p_x}{\Delta z} \right] \dots \quad (4-11)$$

where $\tilde{v}_j = 1/\rho_j$

and

$$q_{tr} = \frac{D_{eq}}{A_{cx}} \left\{ \sigma [(T_{gy}/1000)^4 - (T_j/1000)^4] + \frac{\pi}{2} U (T_{gy} - T_j) \right\} \dots \quad (4-12)$$

The gas temperature is evaluated at point "y" (point i, j of the grid). This, and the computation of T_g , will be discussed in §4.4.

4.3-2 The Computational Procedure

The simultaneous solution of Equations 4-9 through 4-12, together with the Equations of State (Eqns. 3-5), is carried out in a similar manner to the steady state problem. The procedure for a single volume element of the tube, of length Δz , is described as follows:

- 1) Given the values $T_x, T_y, T_z, p_x, p_y, p_z, v_x, v_y, v_z$, the intermediate values T_j, p_j, v_j , are computed from Equation 4-4.
- 2) $\tilde{v}_j, C_{pj}, \left(\frac{\partial \ln \tilde{v}}{\partial \ln T}\right)_{pj}$, are computed from the S. Equation (Eqns. 3-5_{a,b,c}), using the values T_j , and p_j .
- 3) $\rho_j = 1/\tilde{v}_j$ by definition.
- 4) The iterative process:
 - i) An initial guess is made on the value of p_m .
 - ii) v_m is computed from Eqn. 4-10_a.
 - iii) ρ_m is computed from Eqn. 4-9.
 - iv) $\tilde{v}_m = 1/\rho_m$
 - v) T_m is computed from Eqns. 4-11 and 4-12.
 - vi) $\tilde{v}_s$ (sp. volume) is computed from the S. Equation (Eqn. 3-5_a), using the values T_m and p_m .

vii) If $\tilde{v}_m \neq \tilde{v}_s$, the difference $\tilde{v}_m - \tilde{v}_s$ is used (by linear interpolation or extrapolation), to correct the initial guess of p_m . This process is repeated until $\tilde{v}_m = \tilde{v}_s$.

The values of p_m , T_m , v_m , and $\tilde{v}_m$ are then recorded and used in the computation of the next point along the fluid path. This is continued until the end of the tube is reached, and the unsteady state profiles of p , T , v , and $\tilde{v}$, at time t , are given in tabulated form. The profiles thus obtained, and the boundary conditions (see § 4.3-4), will be used to compute the profiles at time $t + \Delta t$ in the same way.

In subregions 3 and 4, where the pressure p is given as a function of $\tilde{v}$ and T , an alternative computational procedure is used. The initial guess is made on the value of the velocity v_m . p_m is then computed from Eqn. 4-10_b, ρ_m and $\tilde{v}_m$ from Eqn. 4-9, and T_m from Eqn. 4-11. Using T_m and $\tilde{v}_m$, the pressure p_s is computed from the State Equation (Eqn. 3-5_d). p_s is compared to p_m , and the initial guess is corrected as in the former method, until $p_m = p_s$. The process is continued as described above.

4.3-3 The Gas Temperature

The gas temperature profile is computed simultaneously with the equations of change, from the gas energy balances

(Eqns. 3-6,7,8,9), made over each volume element. The method is described in §4.4.

4.3-4 The Boundary Conditions

The steady state profiles constitute the initial conditions of the unsteady state problem. The variation with respect to time of v , T , and p at the boiler inlet ($z = 0$) is chosen to be the boundary conditions of the unsteady state problem, and this completes the mathematical definition of the problem.

The physical significance of the boundary conditions has been discussed in § 3.3. Accordingly, the variations with time of v , T , and p at the boiler inlet are considered "inputs" to the system. The dynamic, open-loop, response of the system to various types of inputs will be studied. In addition to the inputs described above, changes in fuel firing-rate and in burner tilt, are considered to be inputs. This will be discussed in §4.4-5.

§ 4.4 Gas-side Energy Balances

The gas side is divided into 3 parts (see § 2.1), the Lower Furnace, the Upper Furnace, and the Superheater Section. The differences in gas flow and in tube geometry, result in different expressions for the energy balances in each of these

parts. In all cases, it is assumed that heat accumulation within a volume element of gas, over a period Δt , is small relative to heat "generation" (see §2.3) and to heat transport.

As with the fluid, where one tube is taken to represent the multiple tube system, so in the gas side, a part of the gas stream, corresponding to a single tube, is taken to represent the entire stream.

4.4-1 The Upper Furnace

The gas in the Upper Furnace is assumed to flow axially upwards, in parallel to the fluid flowing in the surrounding waterwall tubes. On the basis of a volume of gas, corresponding to a tube of length Δz , the energy balance equations may be written as follows:

$$H_{gi} - H_{go} + \Delta z(q_s - q_{rc}) = 0 \quad (4-13)$$

where

$$H_{gi} = W_g \cdot C_g \cdot T_g \Big|_z \quad \text{is the enthalpy of the gas evaluated at point } z$$

$$H_{go} = W_g \cdot C_g \cdot T_g \Big|_{z+\Delta z} \quad \text{is the gas enthalpy evaluated at point } z + \Delta z$$

$$q_s = \frac{f_{mx}}{L_t - z_{fu}} (L_t - z) \quad \text{is the rate of heat generation within the gas, per unit length of the gas column. (See$$

Appendix A, § A.5 for definition of the parameters f_{mx} , z_{fu} , and L_t , and for the derivation of the expression).

$q_{rc} = q_{tr} \cdot A_{cx}$ is the rate of heat transport from the gas into the fluid, per unit length of the gas column. Equations 4-8 and 4-12 give q_{tr} for steady and unsteady states, respectively. A_{cx} is the cross-sectional area of the tube. (Note that in the Upper Furnace, the length of the gas column coincides with the length of the tube. This also is correct in the Lower Furnace but not in the Superheater Section.)

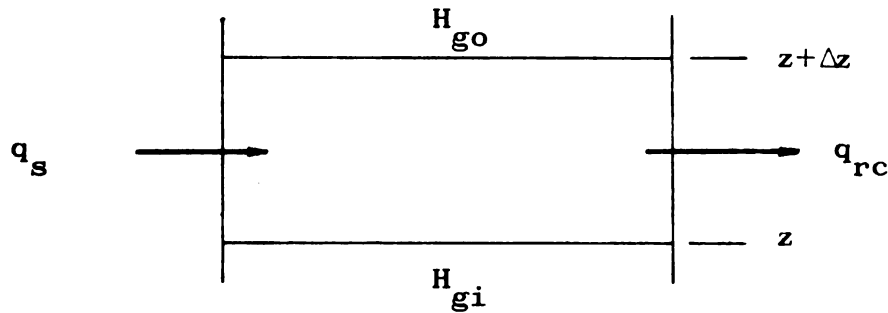


Figure 4.3: Energy Balance in the Upper Furnace

Substituting T_g into Equation 4-13 and rearranging, we have:

$$T_g \Big|_{z+\Delta z} = T_g \Big|_z + \Delta z (q_s - q_{rc}) / (W_g C_g) \quad \dots\dots\dots (4-14)$$

C_g is the specific heat of the gas, given as a function of T_g , (Appendix A, § A.4).

The average slope of the gas temperature profile may also be computed:

$$\Delta T_g / \Delta z = (q_s - q_{rc}) / (W_g C_g) \quad \dots\dots\dots (4-15)$$

4.4-2 The Superheater Section

The Superheater Section contains four horizontal passes of the Superheater tube. The fluid path in these passes is shown schematically in Figure 4.4. The gas temperature is assumed constant at each pass. Four energy balances are made: one for each pass.

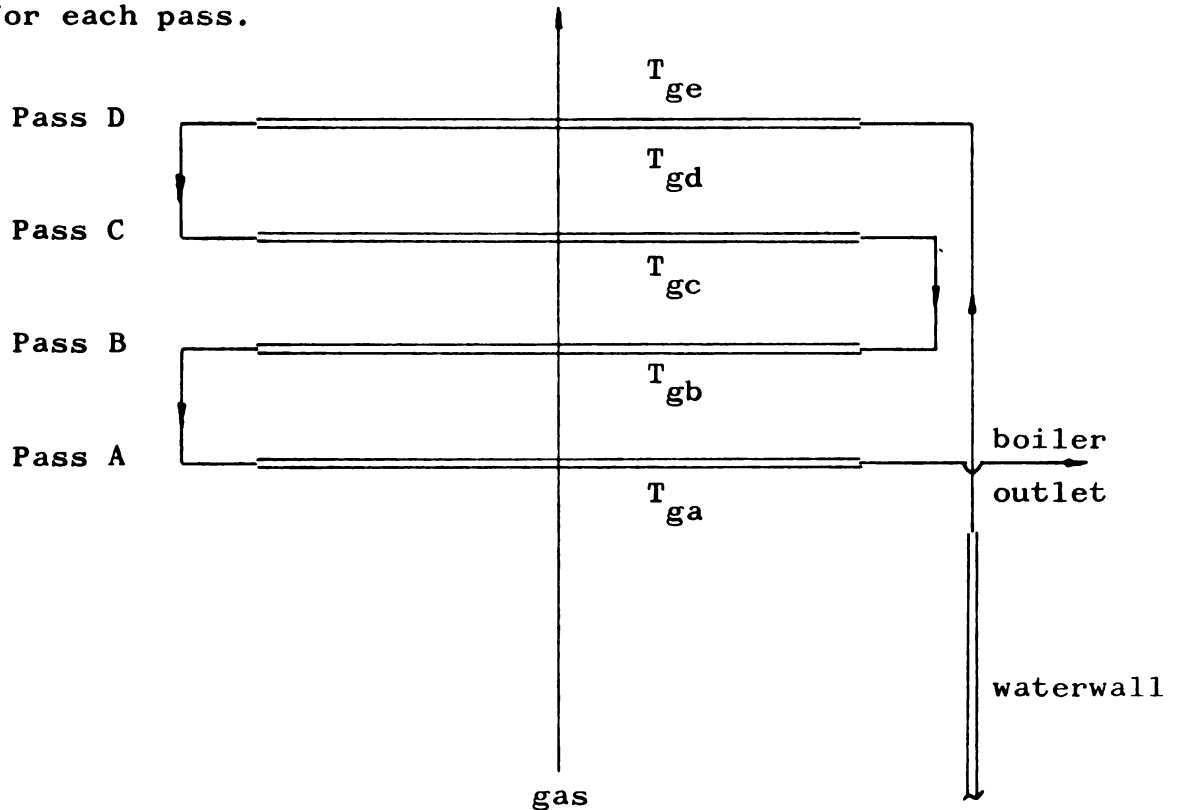


Figure 4.4: Schematic View of the Superheater Section

The gas energy balance is expressed as in Equation 4-13,

$$\text{with: } H_{gi} = W_g C_g T_{ga} \quad , \quad H_{go} = W_g C_g T_{gb} \quad , \quad T_{gi} = T_{ga} \quad ,$$

$$T_{go} = T_{gb} \quad \text{for superheat pass A, with similar expressions}$$

for superheat passes B, C, and D, respectively.

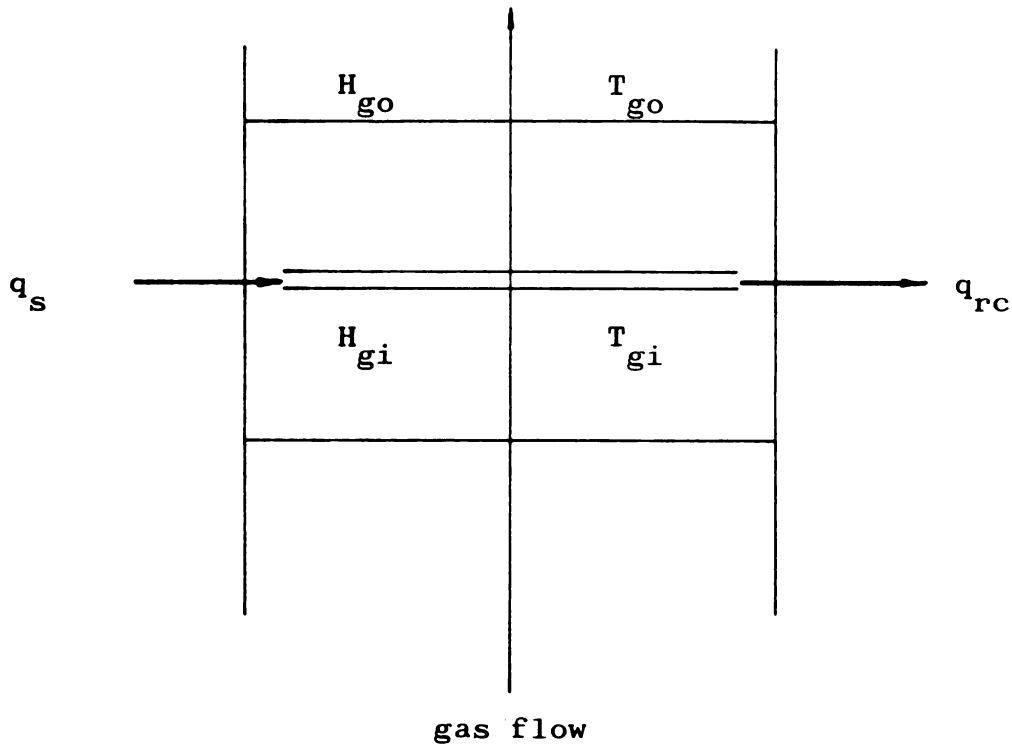


Figure 4.5: Energy Balance in the Superheater Section

The value of q_{rc} is obtained by numerical integration of the terms $q_{tr} \cdot A_{cx} \cdot \Delta_z$ over the entire length of the pass.

The value of q_s is obtained by integration of the heat generation function over the length of the gas column, containing the superheat pass. Denote this length by l_s , and the length of the entire Superheater Section by L_s , then $l_s = L_s/4$, 4 being the number of superheat passes. The value of q_s is, thus, the heat generated within a gas column of length l_s . The resulting expressions are:

$$q_{sa} = \frac{7}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2 \quad \dots\dots\dots (4-16_a)$$

$$q_{sb} = \frac{5}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2 \quad \dots\dots\dots (4-16_b)$$

$$q_{sc} = \frac{3}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2 \quad \dots\dots\dots (4-16_c)$$

$$q_{sd} = \frac{1}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2 \quad \dots\dots\dots (4-16_d)$$

The derivation of Equations 4-16 is given in Appendix A, §A.5.

4.4-3 The Lower Furnace

The axial heat transfer in the Lower Furnace is described by the Dispersion Model (see §2.3 and §3.2).

Consider a volume of gas, corresponding to a waterwall tube of length Δz . The energy balance may be written as follows:

$$q_a \Big|_z - q_a \Big|_{z+\Delta z} + \Delta z(q_s - q_{rc}) = 0 \quad \dots\dots\dots (4-17)$$

where q_a is the rate of axial heat flow.

$$q_a \Big|_z = - D_c \cdot \frac{dT_g}{dz} \Big|_z \quad \dots\dots\dots (4-18_a)$$

$$q_a \Big|_{z+\Delta z} = - D_c \cdot \frac{dT_g}{dz} \Big|_{z+\Delta z} \quad \dots\dots\dots (4-18_b)$$

and

$$q_s = \frac{f_{mx}}{z_{fu}} \cdot z \quad \text{is the rate of heat generation per unit length of the gas column} \quad \dots\dots\dots (4-19)$$

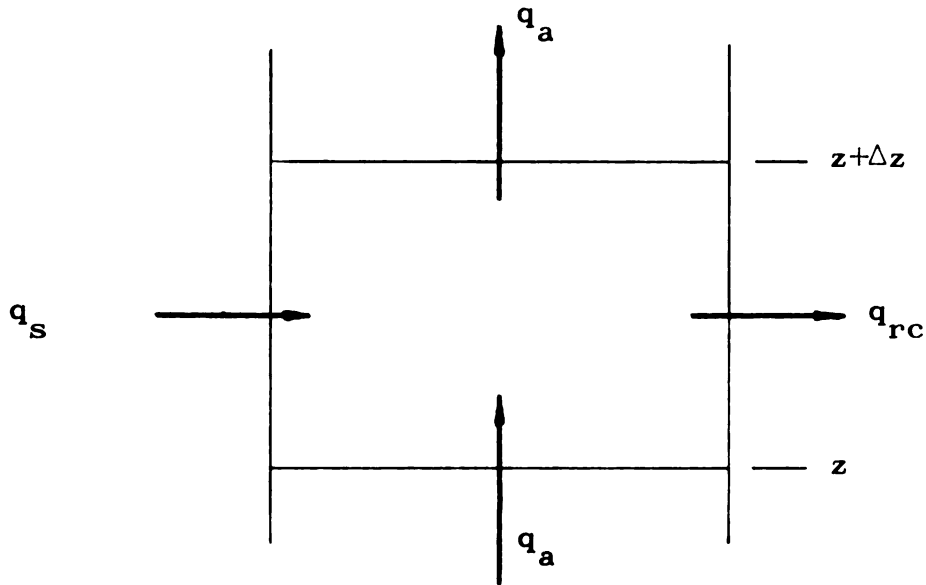


Figure 4.6: Energy Balance in the Lower Furnace

The variation of T_g with z over the length Δz can be expressed by an ordinary differential equation:

$$D_c \cdot \frac{d^2 T_g}{dz^2} = q_{rc} - \frac{f_{mx}}{z_{fu}} z \quad \dots\dots\dots (4-20)$$

Assuming that q_{rc} may be taken as a constant over Δz , it is possible to carry out the integration of eqn. (5-20) and obtain:

$$\left. \frac{dT_g}{dz} \right|_{z+\Delta z} = \left. \frac{dT_g}{dz} \right|_z + \frac{\Delta z}{D_c} \left[q_{rc} - \frac{f_{mx}}{2z_{fu}} (2z + \Delta z) \right] \quad \dots\dots\dots (4-21)$$

$$T_g \Big|_{z+\Delta z} = T_g \Big|_z + \left. \frac{dT_g}{dz} \right|_z \Delta z + \frac{(\Delta z)^2}{2D_c} \left[q_{rc} - \frac{f_{mx}}{3z_{fu}} (3z + \Delta z) \right] \dots\dots (4-22)$$

The derivation of Equations 4-19 through 4-22 and the definition of the parameters appearing in the equations are given

in Appendix A, § A.6.

4.4-4 The Steady State Gas Temperature Profile

The steady state gas temperature profile is a numerical solution of an ordinary differential equation. It is necessary to provide a boundary condition. For example, the value of T_g at some fixed point along the gas path. This, however, is considered undesirable due to the definition of T_g . It has been assumed that T_g can be taken as an average over the cross-sectional area of the gas column. An experimental determination of this average value would be difficult, both technically and conceptually. It is possible to circumvent this difficulty in a way compatible with the general concept which considers the effects of T_g on heat transfer rather than its "real" physical significance. Noting that the fluid temperature at the boiler inlet is a fairly constant value (ca. 600°F), we can use this value, denoted by T_{in} , as a boundary condition for the gas equation.

An initial guess is made of T_{ga} , the gas temperature at the fourth superheat pass, and the steady state profiles are computed. Denote the computed value of the fluid temperature at the boiler inlet by T_{ic} and compare the values of T_{in} , and T_{ic} . If $T_{ic} \neq T_{in}$, then the difference $T_{ic} - T_{in}$ is used to correct the initial guess of T_{ga} by linear interpolation or

extrapolation. This is repeated until $T_{ic} = T_{in}$. The steady state profiles, including the gas temperature profile, are then recorded.

4-4.5 The Unsteady State Gas Temperature Profiles

The gas temperature at any instant is a result of both the heat generation rate and the cooling rate. At the same time, T_g determines the cooling rate (Eqn. 4-12). This presents some difficulty which could be overcome by trial and error computation. A more serious difficulty arises from the discontinuity of the gas path with respect to the fluid path (or vice versa) at the passage from the waterwalls to the Superheater.

In real systems there are many such discontinuities, and it is desired to avoid compounded trial and error computations, such as would be required for the gas temperature profile at each discontinuity. Therefore, a simplifying assumption is made, whereby the variation of T_g with time is seen as a sequence of steady states. (See Appendix D, § D.4).

Two boundary conditions are required for the Lower Furnace. The values $T_g \Big|_{z=0}$ and $\frac{dT_g}{dz} \Big|_{z=0}$ are considered to be boundary conditions, and, in general, "inputs" to the system.

The computation of the unsteady state profile is, thus, essentially the same as of the steady state. We start at

$z = 0$, and proceed along the positive direction of z (i.e., upwards), following the fluid path. The cooling rate q_{rc} is based on q_{tr} as defined in Equation 4-12.

Additional "inputs", or disturbances, to the system are changes in fuel firing rate, and changes in burner tilt.

A change in firing rate will be modeled as a change in the values f_{mx} and W_g , and a change in burner tilt will be modeled as a change in the value of z_{fu} . (See Appendix A, §A.5).

CHAPTER 5: STABILITY AND CONVERGENCE

A numerical solution to a differential equation is an approximation of its exact solution. It is obtained by neglecting high order terms in the Taylor Series expansion. An additional source of inaccuracy is the truncation error which depends on the machine's precision and on the amount of computation involved.

Errors of the first type can be made smaller by reducing the numerical mesh size, i.e., the magnitudes of the distance increment, Δz , and of the time increment, Δt . It is necessary to establish that the numerical solutions converge to the exact solution as the mesh size is reduced. In many cases of practical importance, as in this case, an exact solution is not possible. It is proposed, therefore, to test the numerical solution for convergence by obtaining several solutions of the problem, using a different mesh size for each solution. If the solution curves tend to come closer together as the mesh size is reduced, this would indicate convergence to the exact solution.

The repetitive computation process may tend to compound errors of both types. This results in an unstable solution which tends to oscillate and, eventually, blow up. Error propagation can be treated analytically in some simple cases, but this is not considered feasible for the problem in this work.

From both the theory and the practice of numerical analysis we know that stability depends on the increments' sizes. It is proposed, therefore, to establish by experiment the mesh size that would yield a stable solution.

Preliminary tests of stability and convergence were conducted on a simplified system, comprised of an horizontal superheat pass, in order to study the major effects. The results are described in §5.1. Further study of the complete model, as described in Chapters 1 and 2, and the conclusions are given in §5.2 and §5.3, respectively.

§ 5.1 Preliminary Tests

The system under study is a horizontal tube, representing a superheat pass. At steady state, the fluid mass velocity is $60 \text{ g}/(\text{cm}^2)(\text{sec})$, the outlet pressure is 240.0 bar, and the outlet temperature is 560°C .

The inputs are step changes of the fluid inlet pressure and velocity, and a ramp change of the fluid inlet temperature.

The fixed parameters are:

Pressure step change	-0.2 bar
Velocity step change	-40.0 cm/sec
Temperature ramp change	0.2 $^\circ\text{C}/\text{sec}$
Tube length	1200. cm
Tube inside diameter	4.0 cm

Gas temperature	1200.	°C
Overall convective heat transfer coefficient	6.0	Btu/(hr)(ft ²)(°F)
Friction factor	0.01	

5.1-1 Tests of Convergence

Four computer solutions to the problem were obtained, with different mesh sizes. The number of the distance increments is denoted by n .

- a) $\Delta z = 60.0$ cm $\Delta t = 5.00$ sec $n = 20$
- b) $\Delta z = 30.0$ cm $\Delta t = 2.50$ sec $n = 40$
- c) $\Delta z = 15.0$ cm $\Delta t = 1.250$ sec $n = 80$
- d) $\Delta z = 7.5$ cm $\Delta t = 0.625$ sec $n = 160$

The results are shown in the Figures 5.1 through 5.5. In Figure 5.1, fluid temperature profiles at various response times, with mesh size (a), are shown. The profiles are nearly linear for the system under study, and the variation with respect to time is observed to be largest during the initial 10 seconds of response time. The rate of temperature change levels off shortly afterwards, thus following the input ramp change. The effects of mesh size variation on the temperature profiles is shown in Figure 5.2. The profiles, at response time $t = 5$ seconds, are markedly convergent.

The effects of mesh size on the outlet steam conditions

are shown in Figures 5.3, 5.4, and 5.5. The results indicate poor convergence during the initial 10 seconds of response time, and good convergence thereafter.

5.1-2 Test of Stability

Seven computer solutions to the problem were obtained, with a fixed value $\Delta z = 60$ cm, and with different values of Δt . The results are presented in Figures 5.6 and 5.7, where the variation of outlet temperature with time is plotted for different values of Δt . In these graphs the time scale is different for each curve, and therefore the abscissa was chosen to be j , the time increment index number. The curves are observed individually for indications of instability, such as oscillations.

The curves appear to be stable for $\Delta t = 5$ seconds and for $\Delta t = 1$ second. Signs of instability are first noticeable when $\Delta t = 0.2$ seconds and become more pronounced as Δt is decreased.

The responses of velocity and specific volume are similar to those of the temperature. The pressure response reveals no signs of instability in the range of values of Δt under study.

§5.2 Determination of the Mesh Size

The results of the preliminary tests indicate that for $\Delta z = 60$ cm and $\Delta t = 5$ seconds adequate stability and convergence are obtained, except for the initial 10 seconds of response time. Another conclusion is that stability may be improved by increasing the ratio $\Delta t/\Delta z$.

In order to establish the adequate mesh size for the complete model, which includes the interaction of the gas temperature with the steam conditions, further studies were made with the system as described in Chapters 2 and 3, using the numerical method described in Chapter 4.

The system was disturbed from steady state by a -22% step input to the fluid inlet velocity. Different combinations of increment sizes were tried, and the variations of the fluid outlet temperature T_o with time were plotted in Figures 5.8 through 5.10.

The effects of varying Δz , with Δt being kept constant at 5 seconds, are shown in Figure 5.8. When $\Delta z = 60$ cm, some oscillation occurs during the initial 30 seconds of response time. The oscillation is reduced when Δz is made smaller, and none can be observed when the values $\Delta z = 15$ cm and $\Delta z = 7.5$ cm are used. Convergence is also improved as Δz is decreased.

The effects of varying Δt , with Δz being kept constant at

15 cm, are shown in Figure 5.9. Slight oscillation during the initial 10 seconds is discernible for $\Delta t = 1.25$ seconds. All three curves converge at the end of the transient period at response time 90 seconds and beyond.

The effects of the magnitudes of both Δz and Δt on convergence are shown in Figure 5.10. The ratio $\Delta t/\Delta z$ was kept constant, and the increments' sizes used were $\Delta t/\Delta z = 5/15$, $2.5/7.5$, and $1.25/3.75$ sec/cm, respectively. In general, higher values were obtained for finer mesh sizes with a maximum deviation of 3.5°C .

§ 5.3 Conclusions

For a fixed ratio $\Delta t/\Delta z$, improved convergence is obtained for finer mesh sizes as seen from Figures 5.2 through 5.5, and from Figure 5.10.

From the results shown in Figures 5.6 through 5.9 it appears that stability is improved as the ratio $\Delta t/\Delta z$ is increased.

Comparing the response curves corresponding to $\Delta t/\Delta z = 1.25/15$ in Figure 5.9, and $\Delta t/\Delta z = 5/60$ in Figure 5.8, we see that stability is improved in the finer mesh, (the ratio $\Delta t/\Delta z$ being the same in both cases). One may attribute the improvement to the reduction of either Δt , or Δz , or both. However, the observed stability of some response curves with

$\Delta t = 5$ seconds, and the observed instability whenever $\Delta z = 60$ cm, imply that stability is affected by the magnitude of Δz , and not of Δt , in the range of values under study.

When Δt is kept constant, the choice of Δz affects the final value of the variable (reached after approximately 90 seconds of response time), whereas when Δz is kept constant, the final value is not affected by the magnitude of Δt .

Both stability and convergence are considered satisfactory for $\Delta z = 15$ cm and $\Delta t = 5$ seconds. With this mesh size, 80 seconds of computation (not including compilation time) were required for a CDC-6500 digital computer to solve the problem and provide the system's response during the 100 seconds after the step change was introduced.

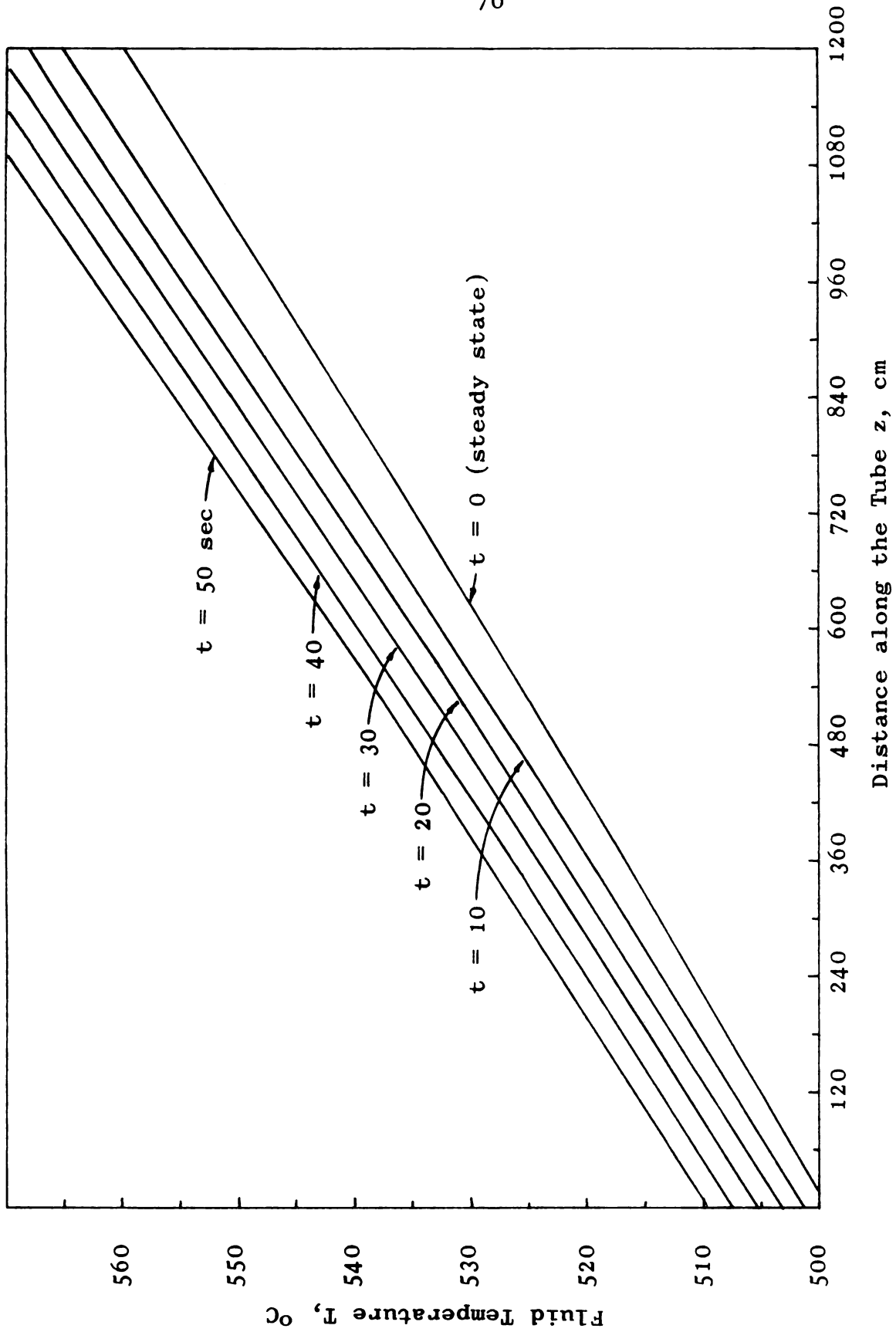


Figure 5.1: Fluid Temperature Profiles; $\Delta t = 5$ seconds and $\Delta z = 60$ cm

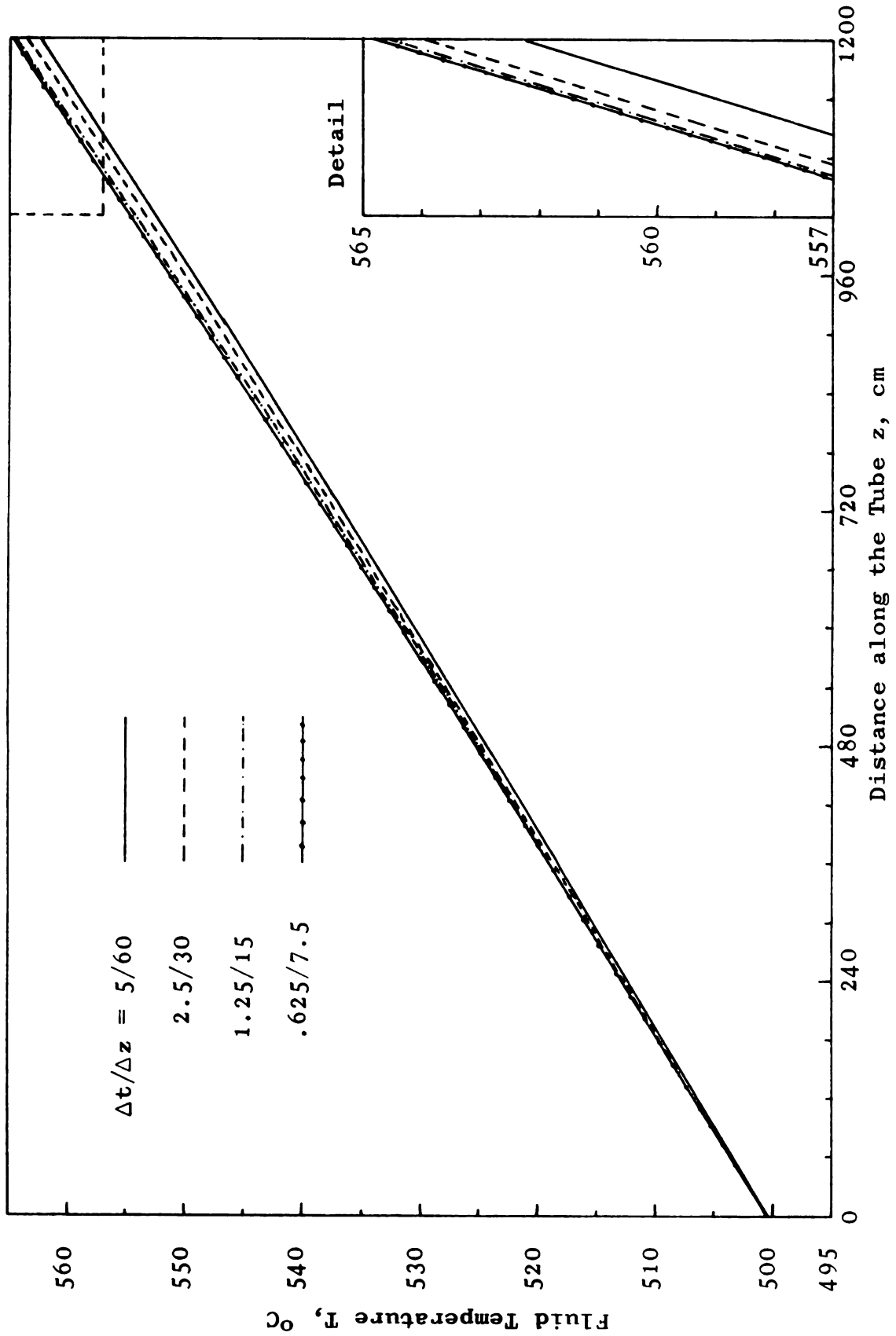


Figure 5.2: The Effects of Mesh Size Variation on The Fluid Temperature Profiles at $t = 5$ seconds

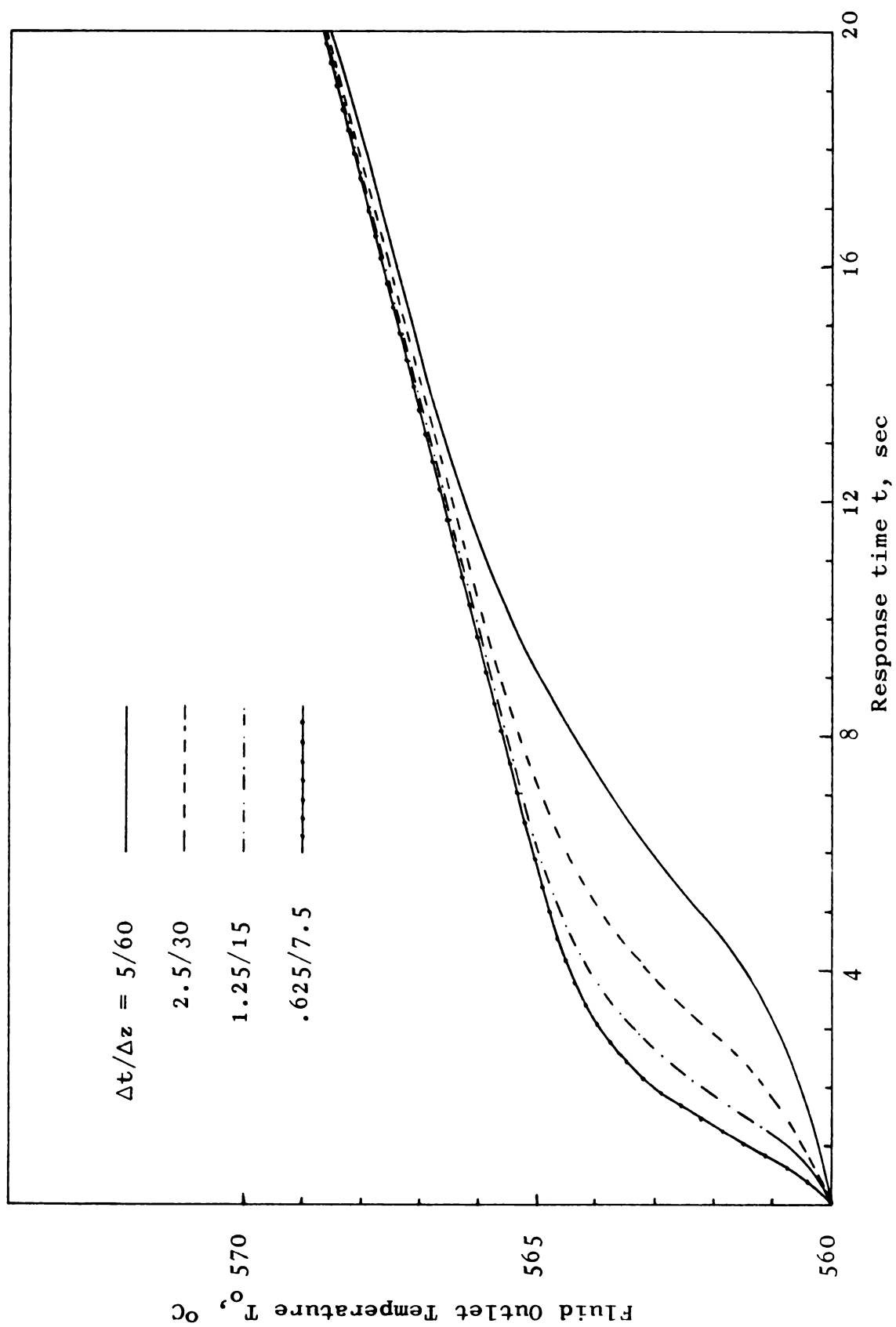


Figure 5.3: The Effects of Mesh Size Variation on the Outlet Temperature Response

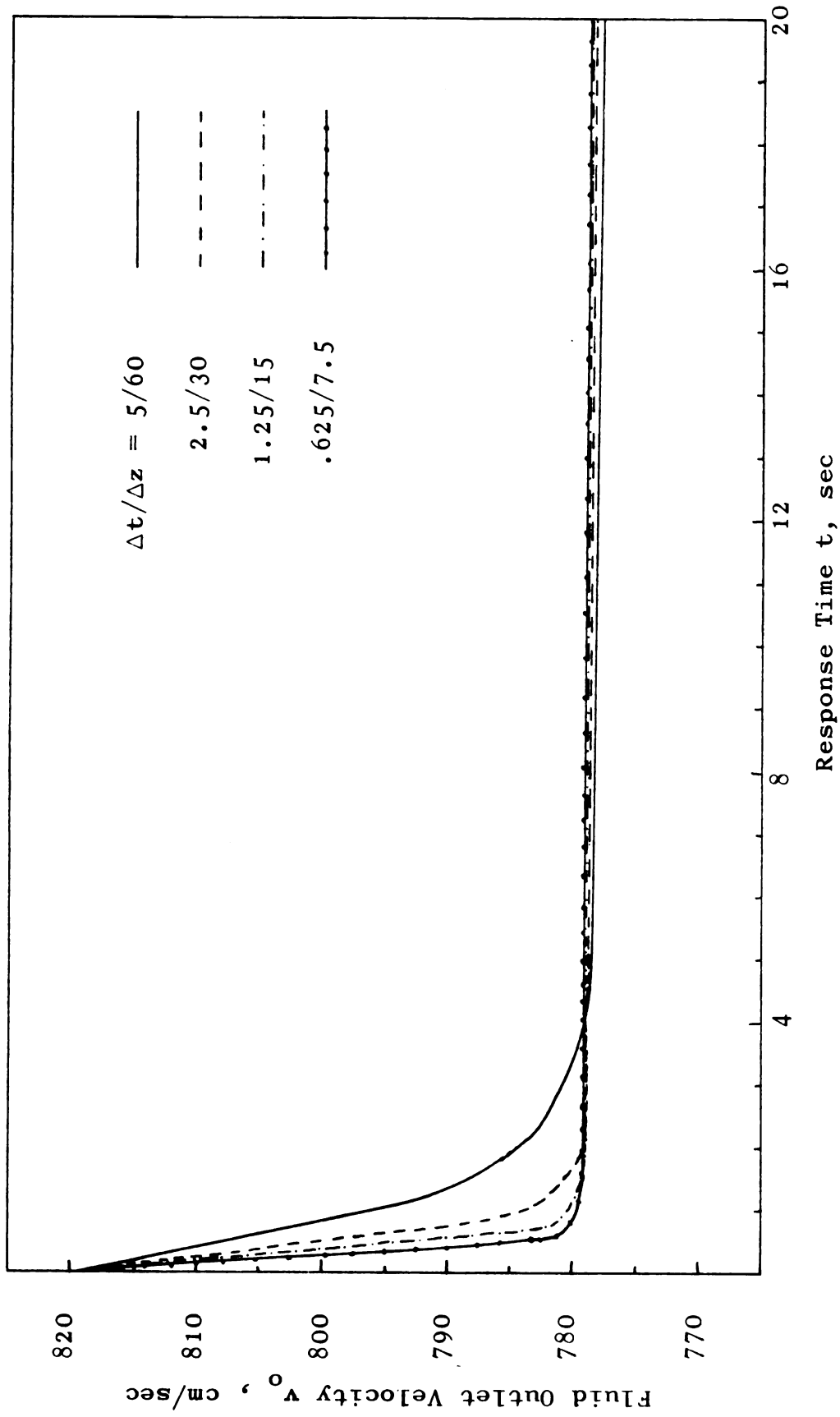


Figure 5.4: The Effects of Mesh Size Variation on the Outlet Velocity Response

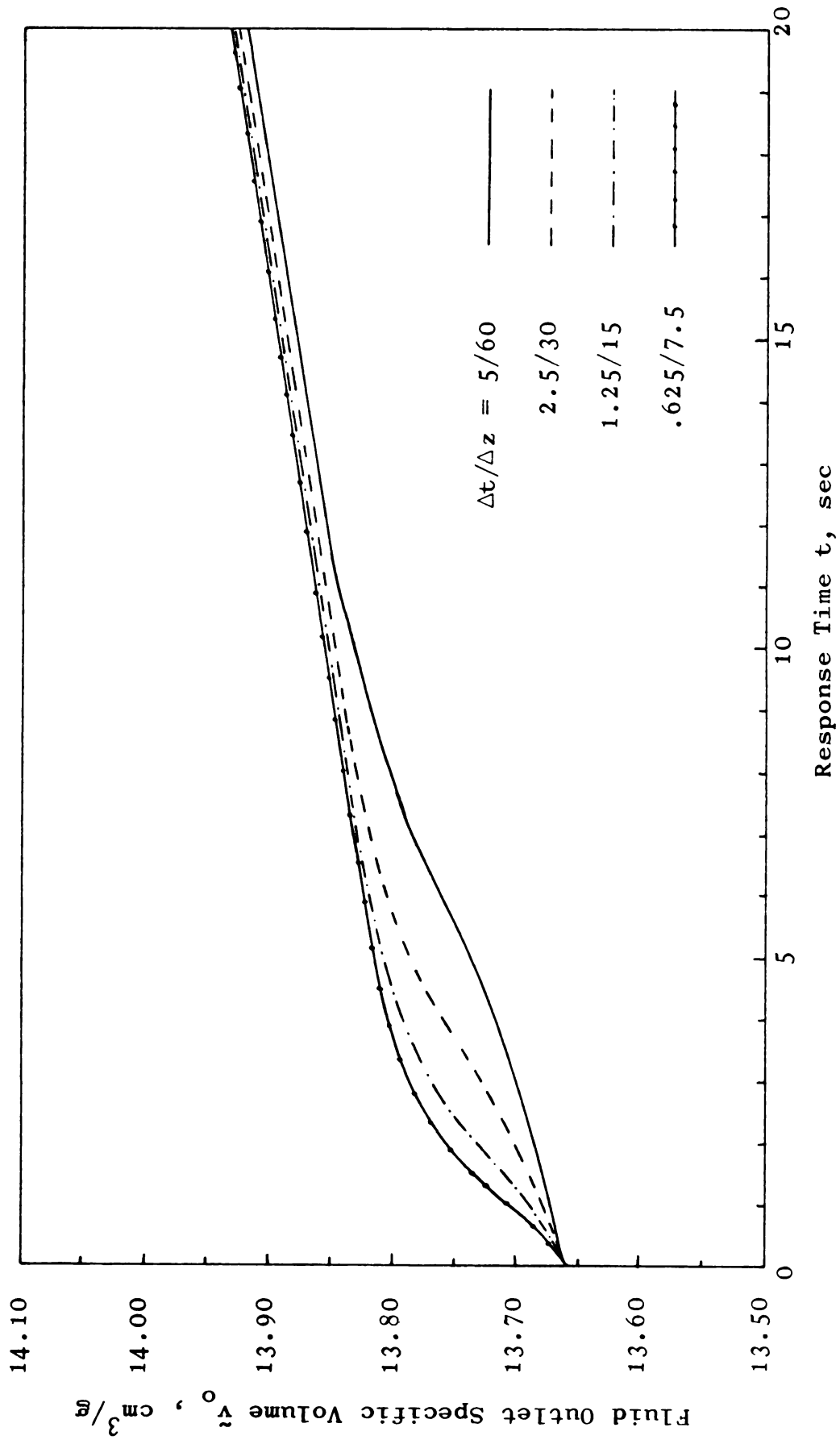


Figure 5.5: The Effects of Mesh Size Variation on the Outlet Specific Volume Response

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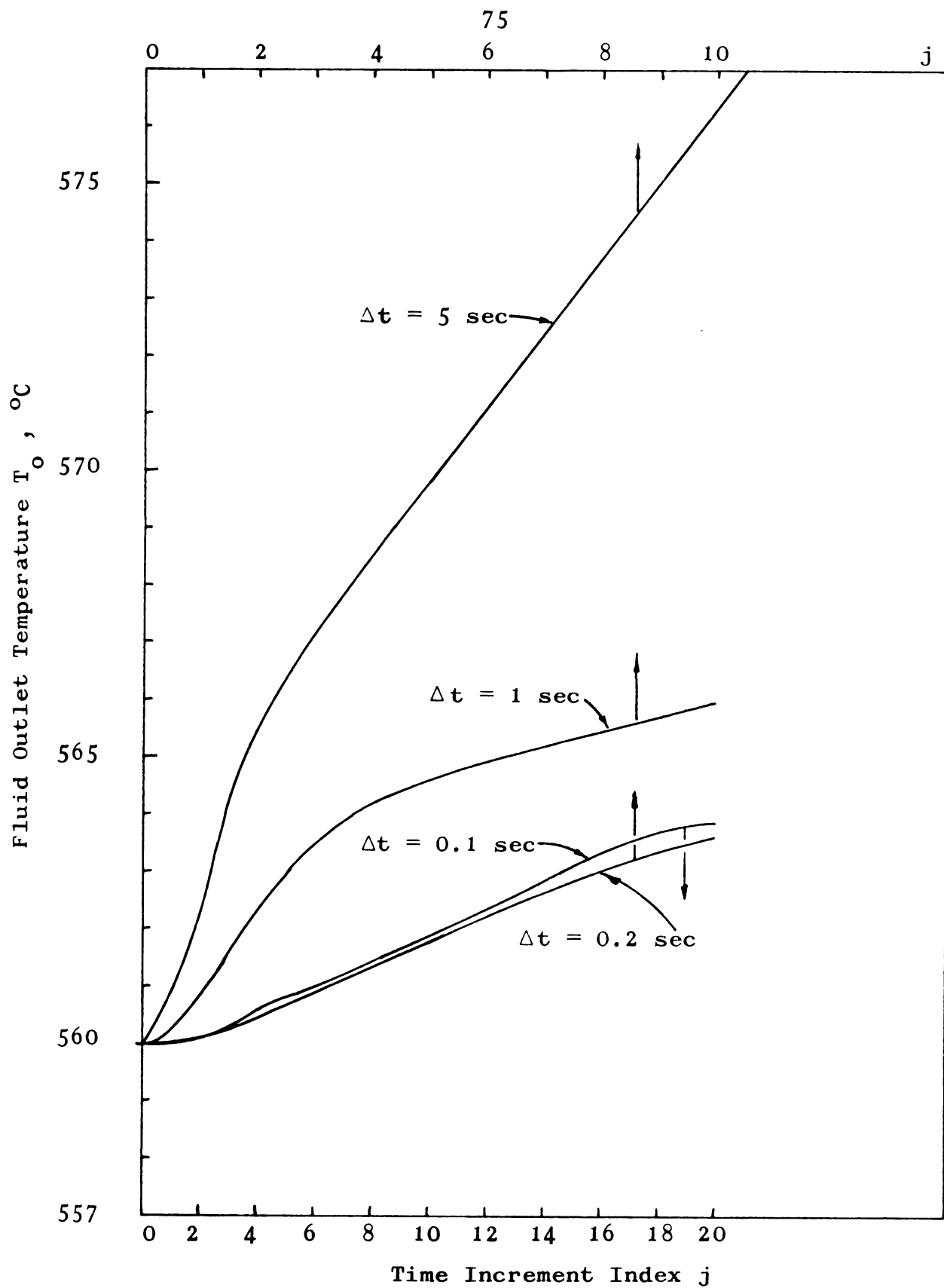


Figure 5.6: The Effects of Variation of Δt on the Outlet Temperature Response, $\Delta z = 60 \text{ cm}$

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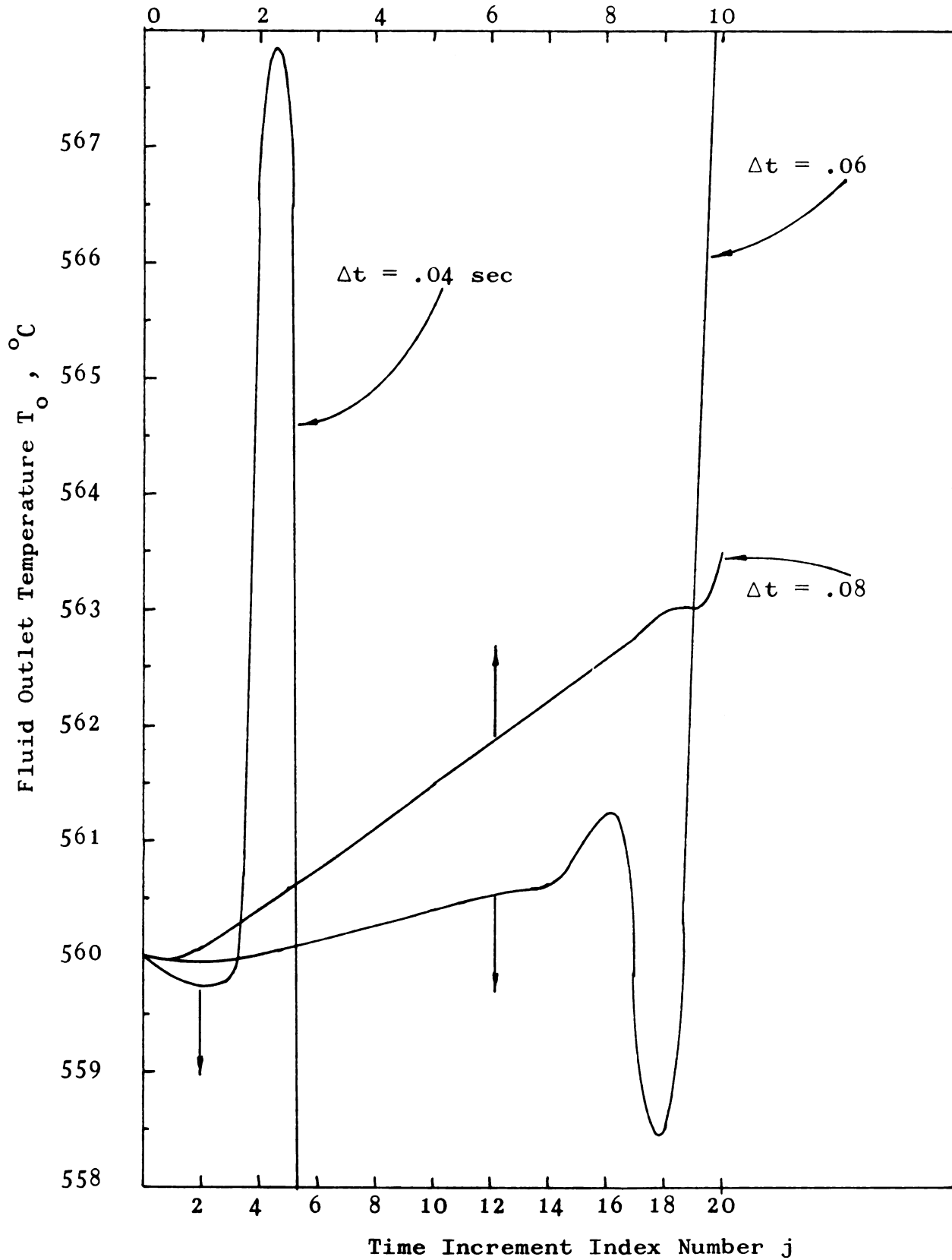


Figure 5.7: The Effects of Variation of Δt on the Outlet Temperature Response, $\Delta z = 60$ cm

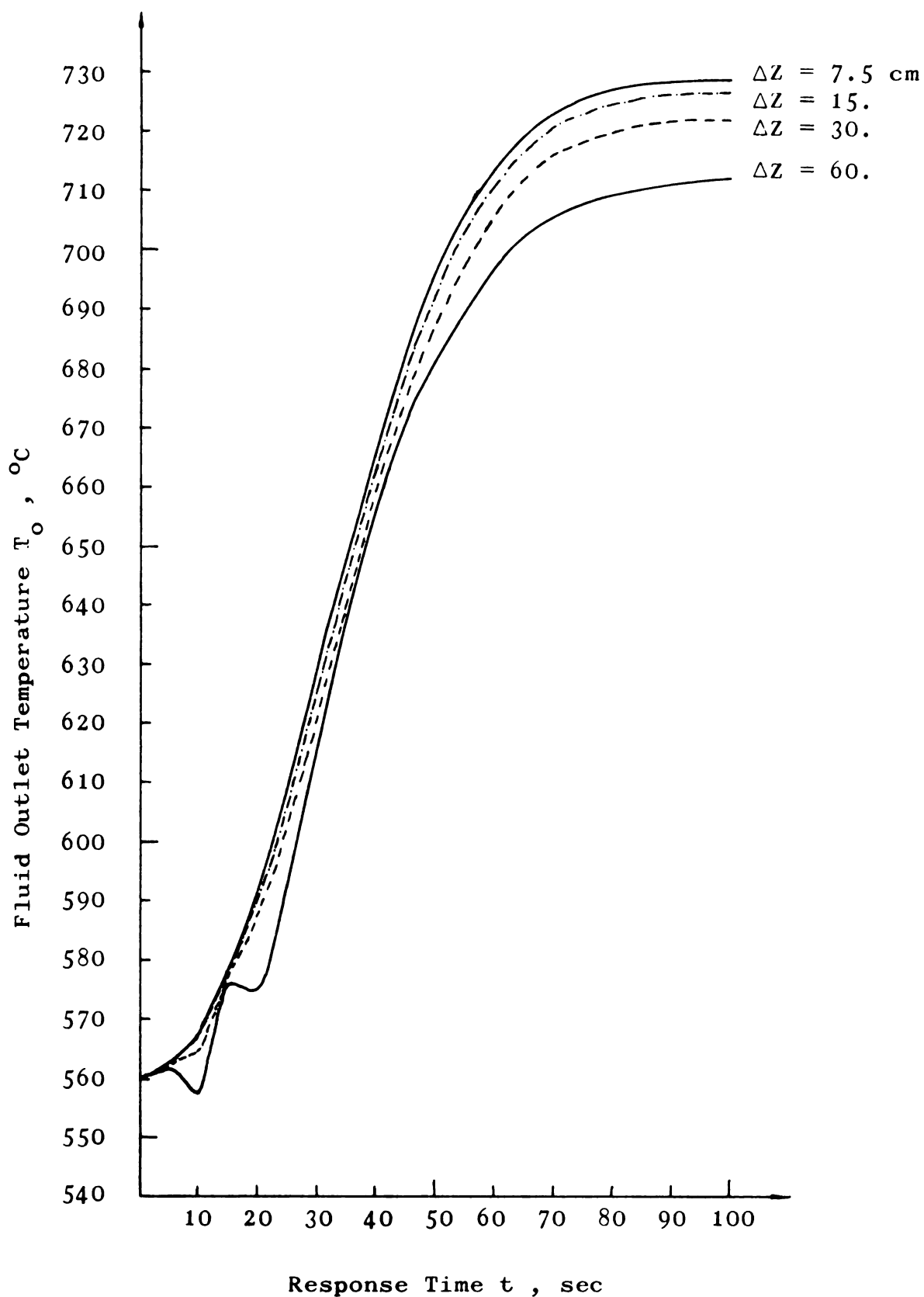


Figure 5.8 The Effects of Variation of Δz on the Outlet Temperature Response, $\Delta t = 5$ seconds

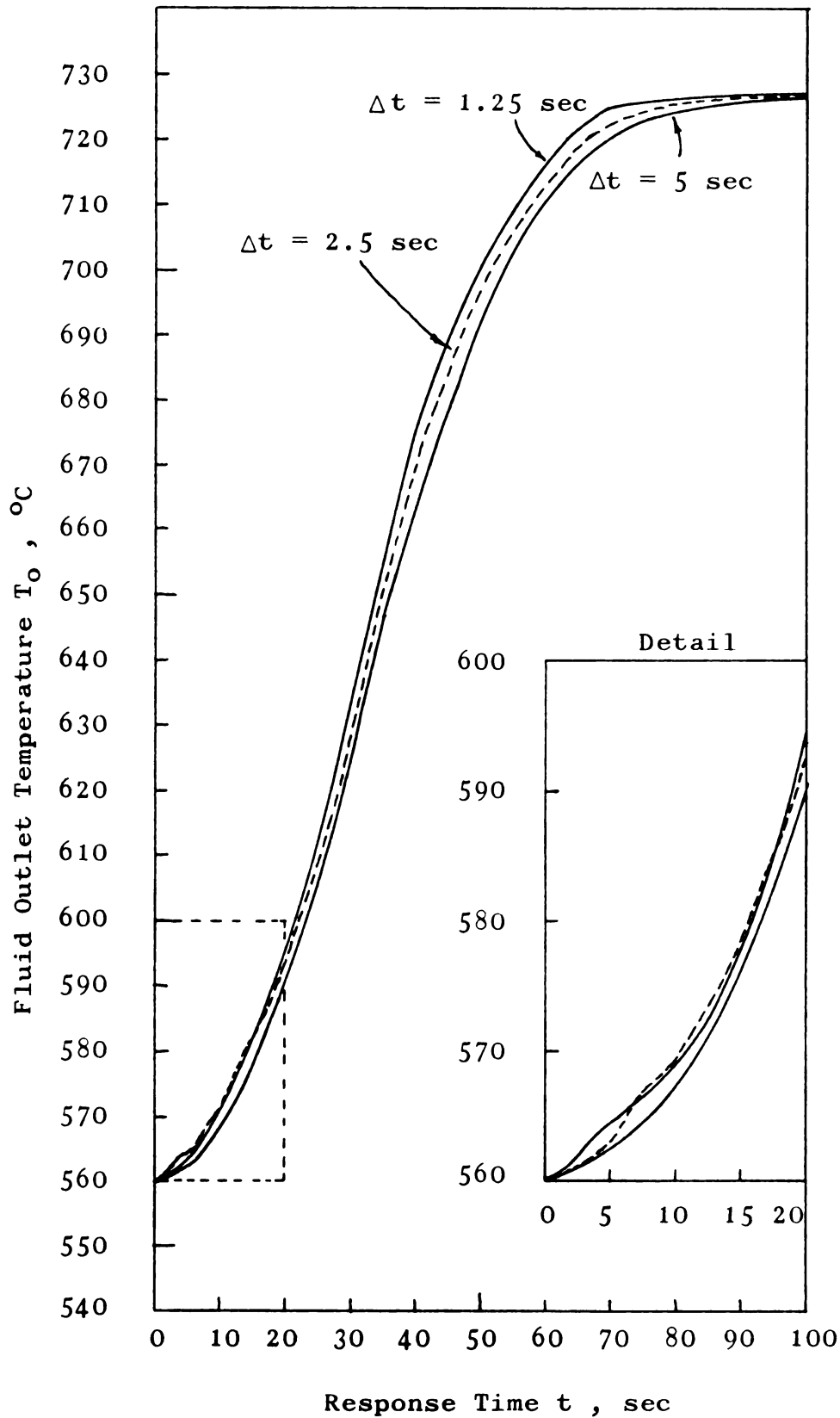


Figure 5.9 The Effects of Variation of Δt on the Outlet Temperature Response, $\Delta z = 15$ cm

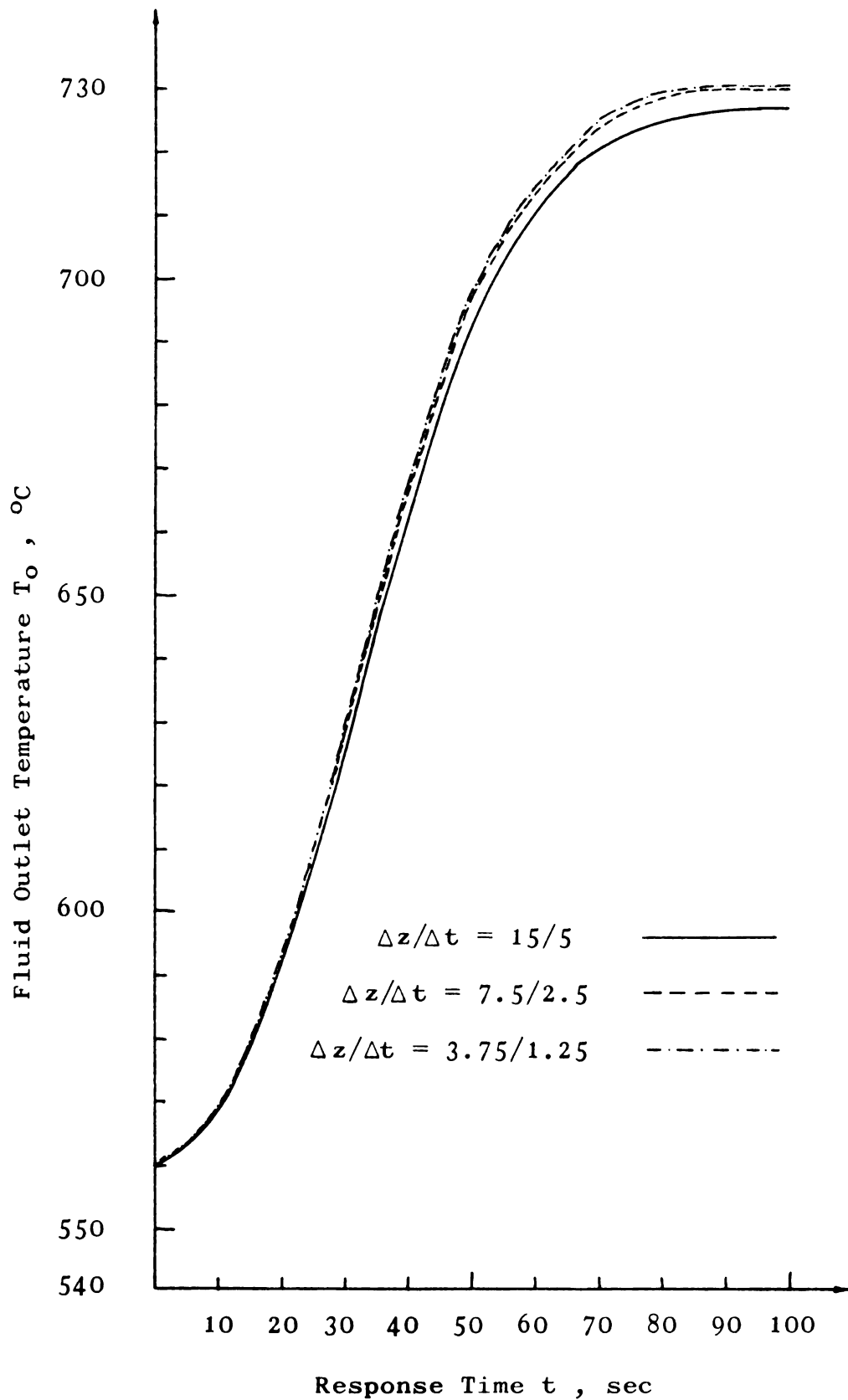


Figure 5.10 The Effects of Mesh Size Variation on the Outlet Temperature Response

CHAPTER 6: RESULTS AND DISCUSSION

The mathematical model is designed to provide the dynamic response of the system to various disturbances or inputs.

The computer solution includes the steady state profiles (variation along the fluid or gas path) of the fluid temperature T , the fluid pressure p , the fluid velocity v , the fluid specific volume $\tilde{v}$, and the gas temperature T_g in tabulated form. Similar profiles, at time intervals of 5 seconds, describe the dynamics of the system in response to specified inputs. Thus, the computer solution gives the variation of T , p , v , $\tilde{v}$, and T_g with respect to both time and distance.

The fluid path is divided into 360 equal distance increments, $\Delta z = 15$ cm. Thus, we have 361 values of each of the aforementioned variables describing the profile. Up to 26 time increments, $\Delta t = 5$ seconds, for a total of 130 seconds of response time, were found to be needed to describe the transient response. To include all that information in this report would require many volumes of tabulated data, and therefore this was not done. Rather, selected portions of the data recorded by the computer were presented in graphical form.

The most important information is the variation with respect to time of the steam conditions at the boiler outlet. In some cases profiles were also included in order to describe the variation along the fluid or gas path.

The steady state solution is shown in Table 6.1. The apparent discontinuity in the T_g profile at $z = 3000$ cm is due to the fact that the T_g data follows the fluid path which runs countercurrent to the gas path in the Superheater Section.

§ 6.1 Variation of Fluid Inlet Velocity

Changes in the fluid inlet velocity result in changes in the fluid flow rate, and consequently in the energy output of the boiler. Such changes are made by manipulating the throttle valve and the boiler feedpump. The effects of step changes of -11%, -22%, and +22%, respectively, are shown in Figures 6.1 through 6.4.

The fluid temperature profiles at various response times are shown in Figure 6.1 for the -11% step input. The slope of the curve depends on the local rate of heat transfer as well as on the heat capacity of the fluid at that point. Thus, in the neighborhood of the critical point ($T = 370$ to 390°C), the slope is almost zero because the specific heat is high. The variation of the slope at $z = 3000$ cm corresponds to the beginning of the Superheater Section where the fluid and the gas paths are countercurrent. The slope increases in this Section because the gas temperature increases along the fluid path.

The fluid velocity profiles for the -22% step input are

shown in Figure 6.2. The velocity increases sharply in the zone of transition from a liquid to a vapor state. In the liquid region the profile is almost flat, and in the vapor region (above the critical temperature) the velocity is roughly proportional to the temperature.

The dynamics of the fluid outlet temperature T_o , pressure p_o , and velocity v_o , in response to the -22% step input, are shown in Figure 6.3. The temperature rises to a new steady state at a higher level, $T = 727^{\circ}\text{C}$. This is to be expected, since fluid flow in the tube is decreased while the heat absorption remains essentially the same.

The initial drop of the outlet velocity v_o in response to the reduced inlet velocity is later offset by the decrease in the fluid density, associated with the rise in temperature.

The outlet pressure p_o also rises, presumably as a result of the reduced flow rate.

The responses to the +22% step input, shown in Figure 6.4 are inversely similar to the former case.

§6.2 Variation of Fluid Inlet Pressure

Fluid inlet pressure changes are associated with the boiler feedpump operating conditions. The effects of a -1 bar (ca. 14.5 psi) step input to the fluid pressure at the boiler inlet are shown in Figures 6.5 and 6.6.

The pressure profiles at various response times are shown in Figure 6.5. The slope of the curve is steeper along the vertical section of the tube. A steady state is reached after 5 seconds throughout the length of the tube.

The dynamics of T_o , p_o , and v_o , are shown in Figure 6.6. The transient response of the fluid pressure is terminated after 5 seconds. The outlet temperature undergoes slow fluctuations before reaching the final steady state value.

§6.3 Variation of Fluid Inlet Temperature

The fluid inlet temperature depends on the feedwater heating system. This system's dynamics are reported to be slower than the boiler dynamics, and therefore the disturbances were described as ramp functions.

The effects of a $0.2^\circ\text{C}/\text{sec}$ ramp input to the fluid temperature at the boiler inlet are shown in Figures 6.7 and 6.8. Fluid temperature profiles at various response times are shown in Figure 6.7. The general shape of the profile at $t = 50$ seconds is different from that of the steady state profile, indicating transient changes within the system. The resulting lag in the T_o response can be seen in Figure 6.8 in which the dynamics of T_o and v_o are shown. It appears that the outlet temperature response lags approximately 70 seconds behind the ramp input at the boiler inlet. At $t = 100$ seconds, the

rate of change of T_o becomes steady, and the shape of the profiles becomes similar to that of the steady state profile.

§6.4 Variation of Firing Rate

Firing rate is a manipulated parameter in the operation of power plants. This is simulated as a change in the value of f_{mx} , with a proportional change of the gas flow rate (See Appendix A, §A.5).

The effects of a 20% decrease of the firing rate are shown in Figure 6.9. As expected, the outlet temperature drops to a lower level. The outlet velocity drops as a result of the increase in the fluid density, associated with the temperature drop.

§6.5 Variation of Burner Tilt

Tilting the burner affects the flow pattern within the furnace and thus serves as a control parameter in the operation of power plants. This is simulated in this work as a change in the value of z_{fu} (See Appendix A, § A.5).

The steady state no-tilt gas temperature profile is compared with the steady state uptilt and downtilt profiles obtained at $t = 100$ seconds, as shown in Figure 6.10. An upward tilt is represented by increasing z_{fu} by 240 cm, and a downward tilt by decreasing z_{fu} by 240 cm. The upward tilt

resulted in an upward "shift" of the gas temperature profile, and vice versa.

The dynamics of T_o and v_o are shown in Figure 6.11. The response curves appear to be antisymmetric. The transient response lasts approximately 70 seconds after which a new steady state is reached.

§6.6 Combination of Inputs

A combination of a -22% step input to the inlet velocity and a 20% decrease of the firing rate may be viewed as a simulated control action, following a reduction in the electrical load. The dynamics of T_o and v_o are shown in Figure 6.12. The initial temperature drop appears to be a result of the faster dynamics of the gas side. Thus, the effects of the decrease in firing rate result in the initial temperature drop. After 30 seconds, the reduced fluid flow rate causes the temperature to rise.

Table 6.1 : Steady State Profiles

L	Z(CM)	T(DEG.C)	P(BAR)	VOL(CC/G)	VE(CM/SEC)	TG(DEG.K)
=====	=====	=====	=====	=====	=====	=====
1	-0.	320.04	241.3516	1.4254	45.614	1177.94
5	60.	322.34	241.3098	1.4360	45.951	1192.51
9	120.	324.71	241.2683	1.4473	46.313	1207.66
13	180.	327.15	241.2271	1.4595	46.703	1222.91
17	240.	329.67	241.1862	1.4726	47.123	1237.75
21	300.	332.25	241.1457	1.4867	47.576	1251.70
25	360.	334.88	241.1057	1.5019	48.062	1264.25
29	420.	337.54	241.0660	1.5182	48.583	1274.91
33	480.	340.21	241.0267	1.5356	49.138	1283.17
37	540.	342.87	240.9879	1.5539	49.726	1288.50
41	600.	345.48	240.9495	1.5732	50.344	1290.51
45	660.	348.15	240.9116	1.5944	51.022	1317.86
49	720.	350.90	240.8742	1.6181	51.780	1343.49
53	780.	353.64	240.8373	1.6438	52.601	1367.43
57	840.	356.55	240.8011	1.6740	53.569	1389.72
61	900.	359.56	240.7655	1.7095	54.704	1410.40
65	960.	362.58	240.7307	1.7506	56.018	1429.52
69	1020.	365.52	240.6968	1.7976	57.522	1447.11
73	1080.	368.28	240.6637	1.8508	59.227	1463.23
77	1140.	370.78	240.6316	1.9108	61.147	1477.92
81	1200.	372.99	240.6005	1.9783	63.306	1491.23
85	1260.	374.87	240.5704	2.0547	65.749	1503.21
89	1320.	376.43	240.5415	2.1419	68.539	1513.91
93	1380.	377.67	240.5137	2.2411	71.716	1523.38
97	1440.	378.62	240.4871	2.3535	75.310	1531.68
101	1500.	379.34	240.4616	2.4795	79.345	1538.85
105	1560.	379.91	240.4372	2.6200	83.839	1544.94
109	1620.	380.37	240.4141	2.7750	88.800	1550.01
113	1680.	380.78	240.3928	2.9442	94.216	1554.10

Table 6.1 (cont'd)

L	Z(CM)	T(DEG.C)	P(BAR)	VOL(CC/G)	VE(CM/SEC)	TG(DEG.K)
==	=====	=====	=====	=====	=====	=====
117	1740.	381.17	240.3727	3.1272	100.070	1557.26
121	1800.	381.56	240.3532	3.3235	106.351	1559.53
125	1860.	381.98	240.3344	3.5326	113.044	1560.96
129	1920.	382.44	240.3163	3.7542	120.135	1561.59
133	1980.	382.99	240.2994	3.9872	127.592	1561.46
137	2040.	383.65	240.2829	4.2305	135.375	1560.61
141	2100.	384.46	240.2672	4.4822	143.431	1559.07
145	2160.	385.45	240.2521	4.7410	151.711	1556.88
149	2220.	386.64	240.2375	5.0052	160.167	1554.07
153	2280.	388.07	240.2234	5.2737	168.758	1550.68
157	2340.	389.74	240.2097	5.5452	177.446	1546.72
161	2400.	391.67	240.1963	5.8186	186.196	1542.24
165	2460.	393.86	240.1833	6.0931	194.978	1537.24
169	2520.	396.32	240.1709	6.3676	203.762	1531.76
173	2580.	399.06	240.1583	6.6428	212.571	1525.82
177	2640.	402.16	240.1461	6.9173	221.355	1519.44
181	2700.	405.49	240.1340	7.1907	230.104	1512.63
185	2760.	409.05	240.1221	7.4626	238.804	1505.41
189	2820.	412.84	240.1104	7.7325	247.441	1497.80
193	2880.	416.85	240.0988	7.9999	255.998	1489.81
197	2940.	421.07	240.0873	8.2644	264.460	1481.45
201	3000.	425.49	240.0759	8.5253	272.810	1472.73
205	3060.	427.08	240.0744	8.6156	275.698	1249.75
209	3120.	428.71	240.0729	8.7060	278.593	1249.75
213	3180.	430.36	240.0713	8.7967	281.493	1249.75
217	3240.	432.06	240.0698	8.8875	284.399	1249.75
221	3300.	433.70	240.0682	8.9784	287.310	1249.75
225	3360.	435.55	240.0666	9.0695	290.225	1249.75
229	3420.	437.34	240.0651	9.1608	293.145	1249.75
233	3480.	439.17	240.0635	9.2522	296.069	1249.75
237	3540.	441.03	240.0618	9.3436	298.996	1249.75

Table 6.1 (cont'd)

L	Z (CM)	T (DEG.C)	P (BAR)	VOL (CC/G)	VE (CM/SFC)	TG (DEG.K)
==	=====	=====	=====	=====	=====	=====
241	3600.	442.93	240.0602	9.4352	301.926	1249.75
245	3660.	445.28	240.0585	9.5470	305.505	1312.62
249	3720.	447.68	240.0568	9.6590	309.088	1312.62
253	3780.	450.14	240.0551	9.7711	312.675	1312.62
257	3840.	452.62	240.0534	9.8832	316.263	1312.62
261	3900.	455.18	240.0517	9.9954	319.854	1312.62
265	3960.	457.77	240.0499	10.1076	323.445	1312.62
269	4020.	460.40	240.0481	10.2199	327.036	1312.62
273	4080.	463.07	240.0463	10.3320	330.626	1312.62
277	4140.	465.70	240.0445	10.4442	334.214	1312.62
281	4200.	468.54	240.0427	10.5562	337.800	1312.62
285	4260.	472.03	240.0408	10.6955	342.256	1385.06
289	4320.	475.57	240.0389	10.8347	346.709	1385.06
293	4380.	479.17	240.0369	10.9736	351.156	1385.06
297	4440.	482.83	240.0350	11.1124	355.596	1385.06
301	4500.	486.54	240.0330	11.2509	360.029	1385.06
305	4560.	490.31	240.0310	11.3892	364.453	1385.06
309	4620.	494.12	240.0290	11.5271	368.867	1385.06
313	4680.	497.98	240.0269	11.6647	373.270	1385.06
317	4740.	501.89	240.0249	11.8019	377.660	1385.06
321	4800.	505.84	240.0228	11.9387	382.039	1385.06
325	4860.	510.99	240.0206	12.1140	387.648	1472.73
329	4920.	516.20	240.0184	12.2887	393.239	1472.73
333	4980.	521.49	240.0162	12.4627	398.808	1472.73
337	5040.	526.83	240.0140	12.6361	404.354	1472.73
341	5100.	532.23	240.0117	12.8086	409.877	1472.73
345	5160.	537.69	240.0094	12.9804	415.374	1472.73
349	5220.	543.20	240.0071	13.1514	420.845	1472.73
353	5280.	548.76	240.0048	13.3215	426.288	1472.73
357	5340.	554.26	240.0024	13.4907	431.702	1472.73
361	5400.	560.00	240.0000	13.6589	437.086	1472.73

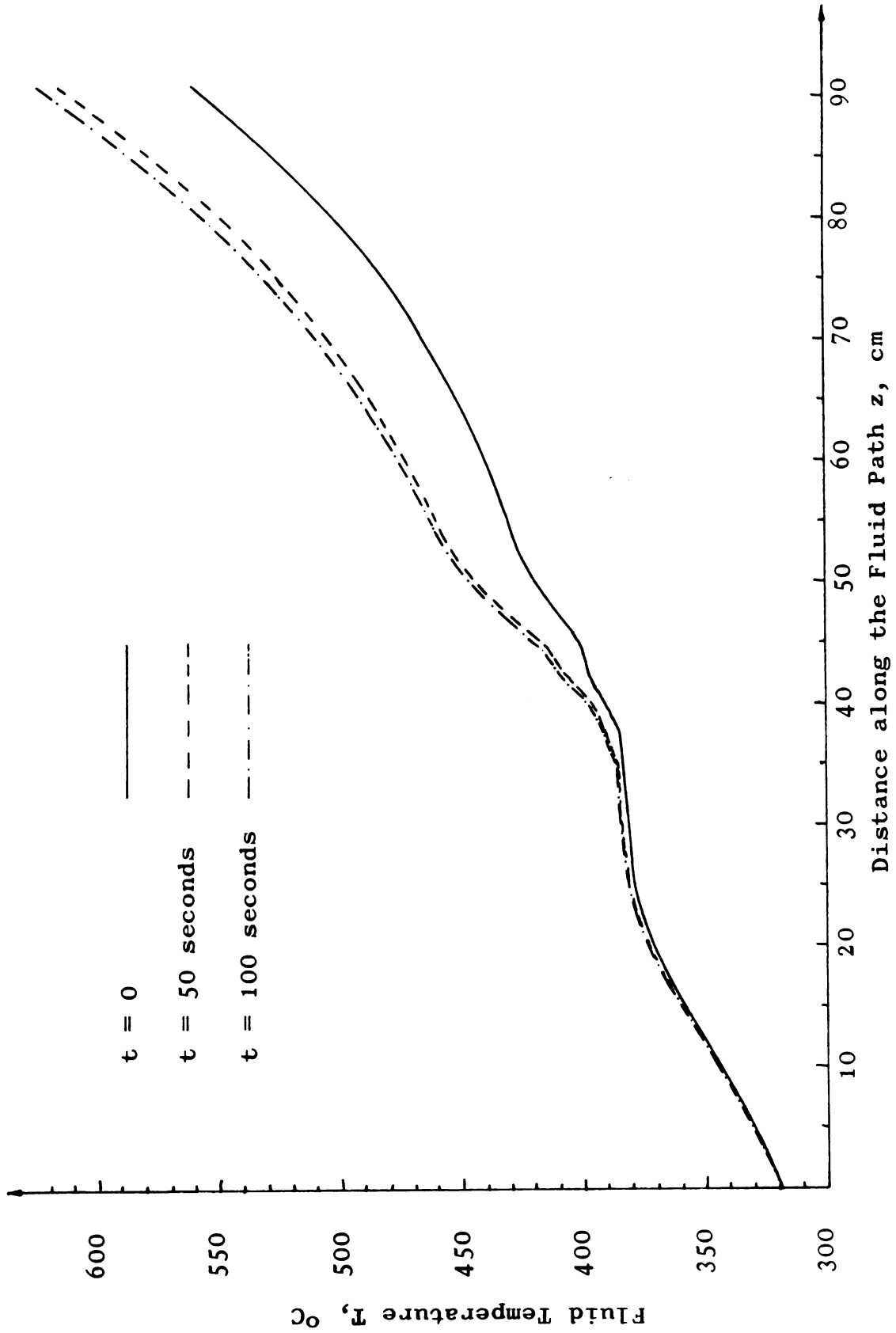


Figure 6.1: Fluid Temperature Profiles, -11% Velocity Step Input

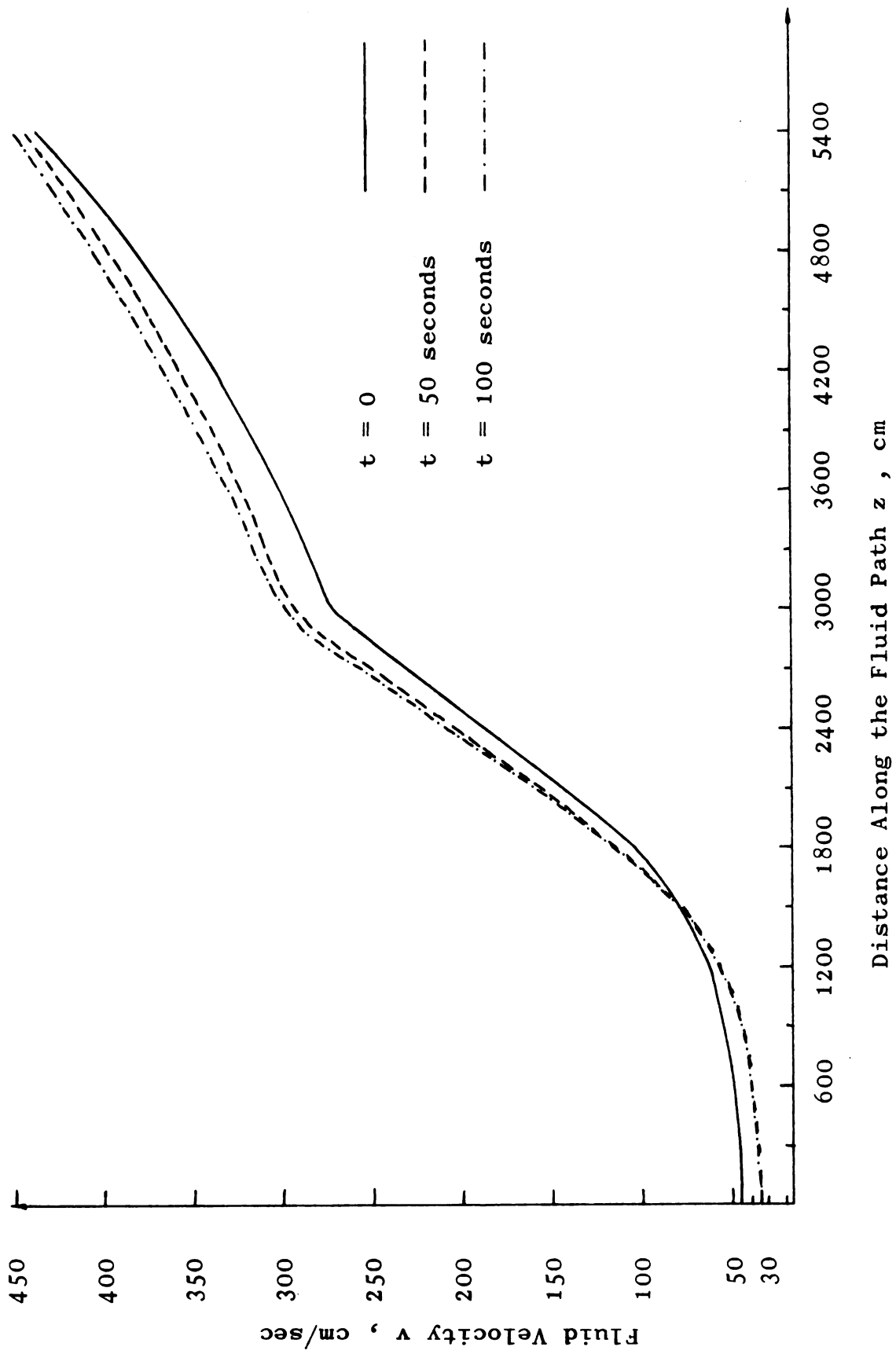


Figure 6.2: Fluid Velocity Profiles, -22% Velocity Step Input

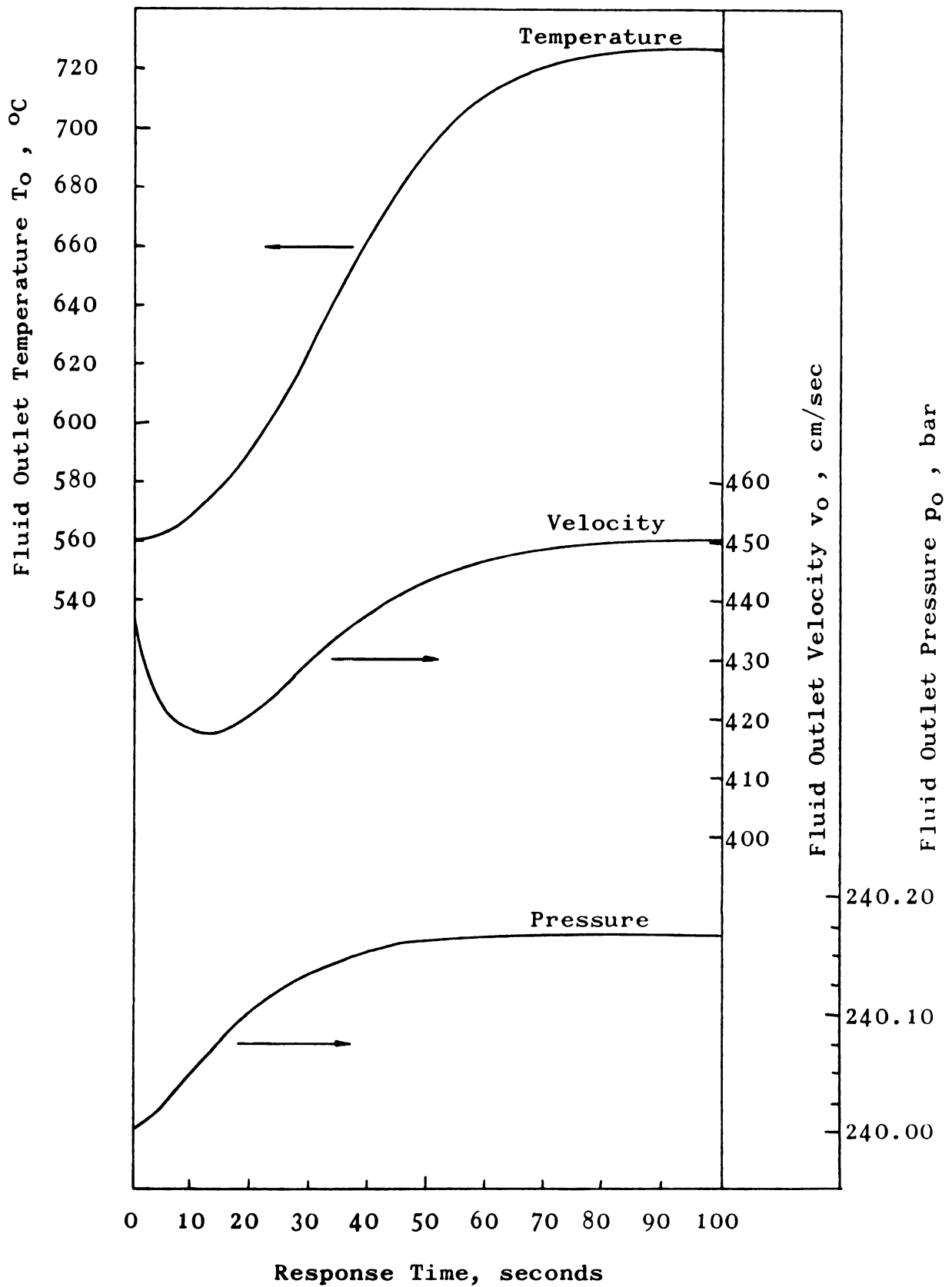


Figure 6.3: Fluid Dynamics at Boiler Outlet, -22% Velocity Step Input

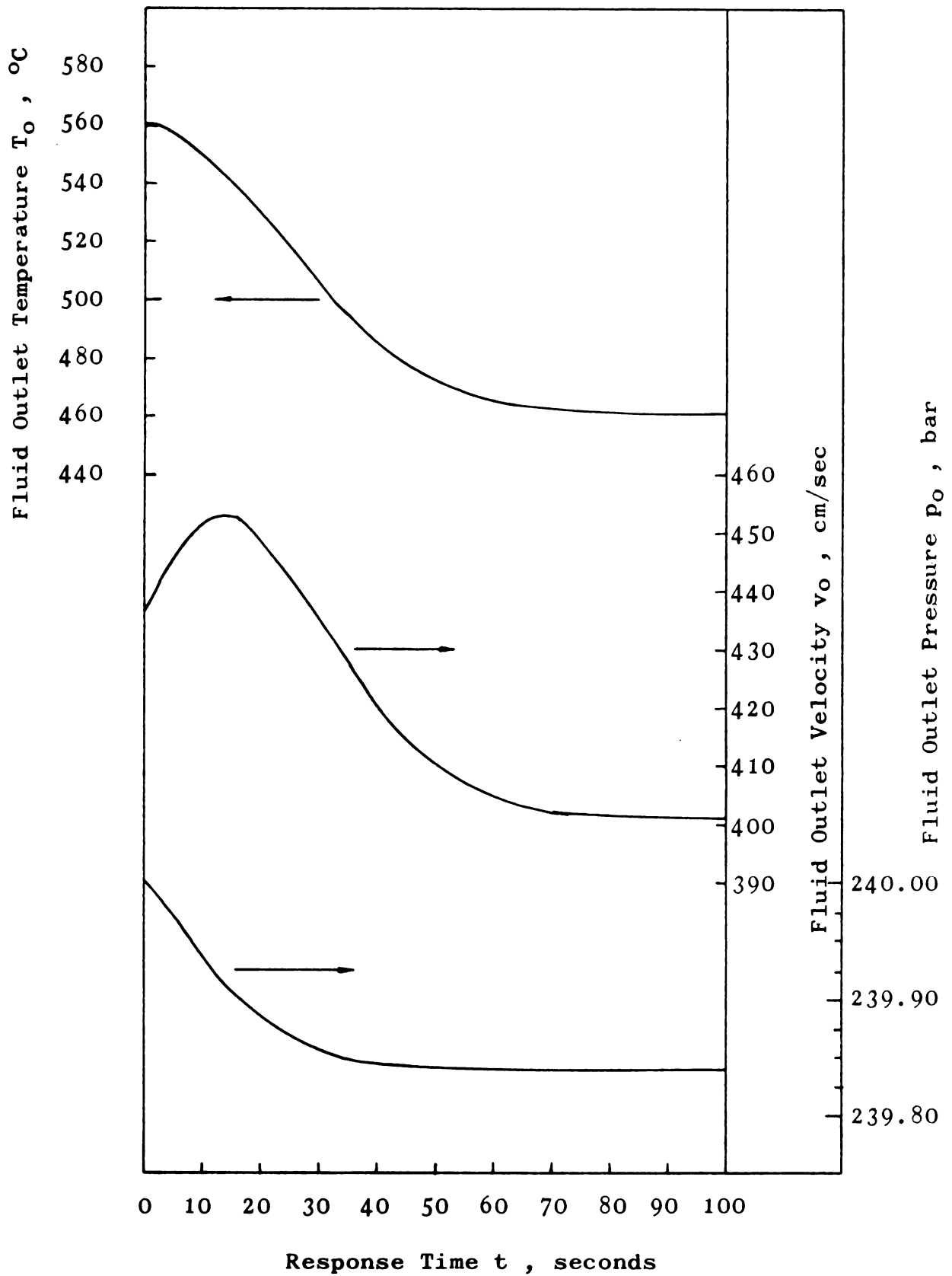


Figure 6.4: Fluid Dynamics at Boiler Outlet, +22% Velocity Step Input

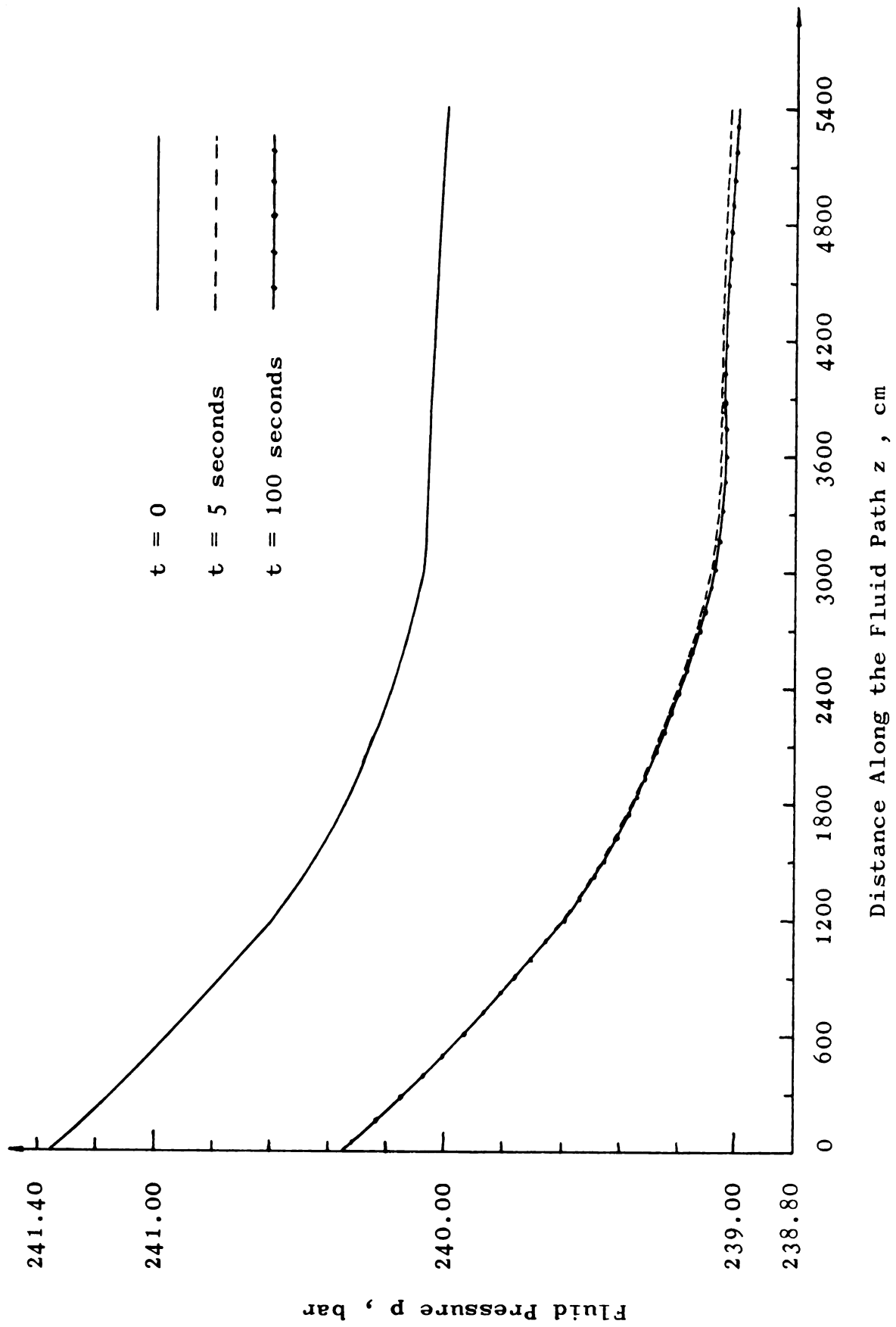


Figure 6.5: Fluid Pressure Profiles, -1. bar Pressure Step Input

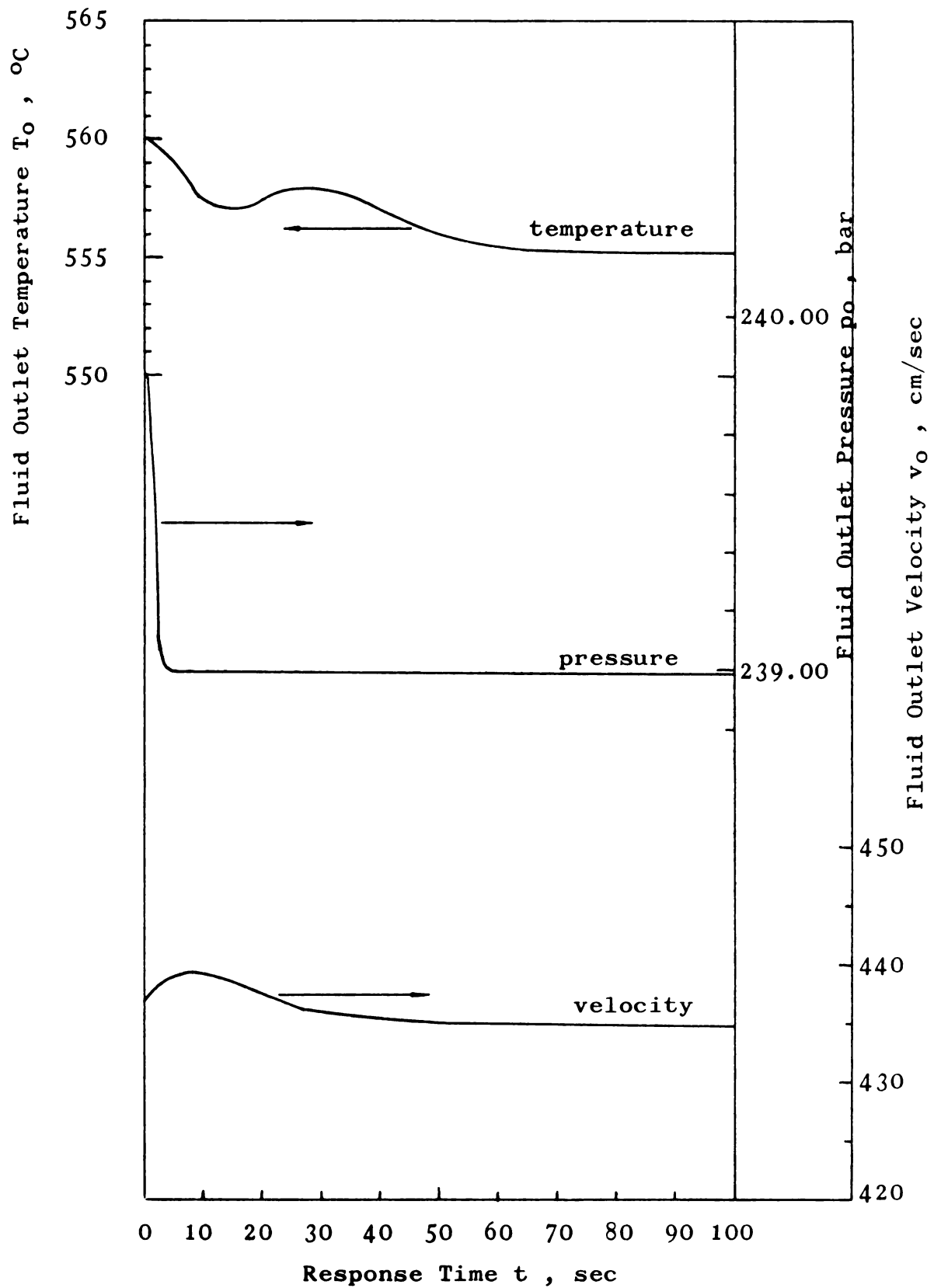


Figure 6.6: Fluid Dynamic at Boiler Outlet, -1. bar Pressure Step Input

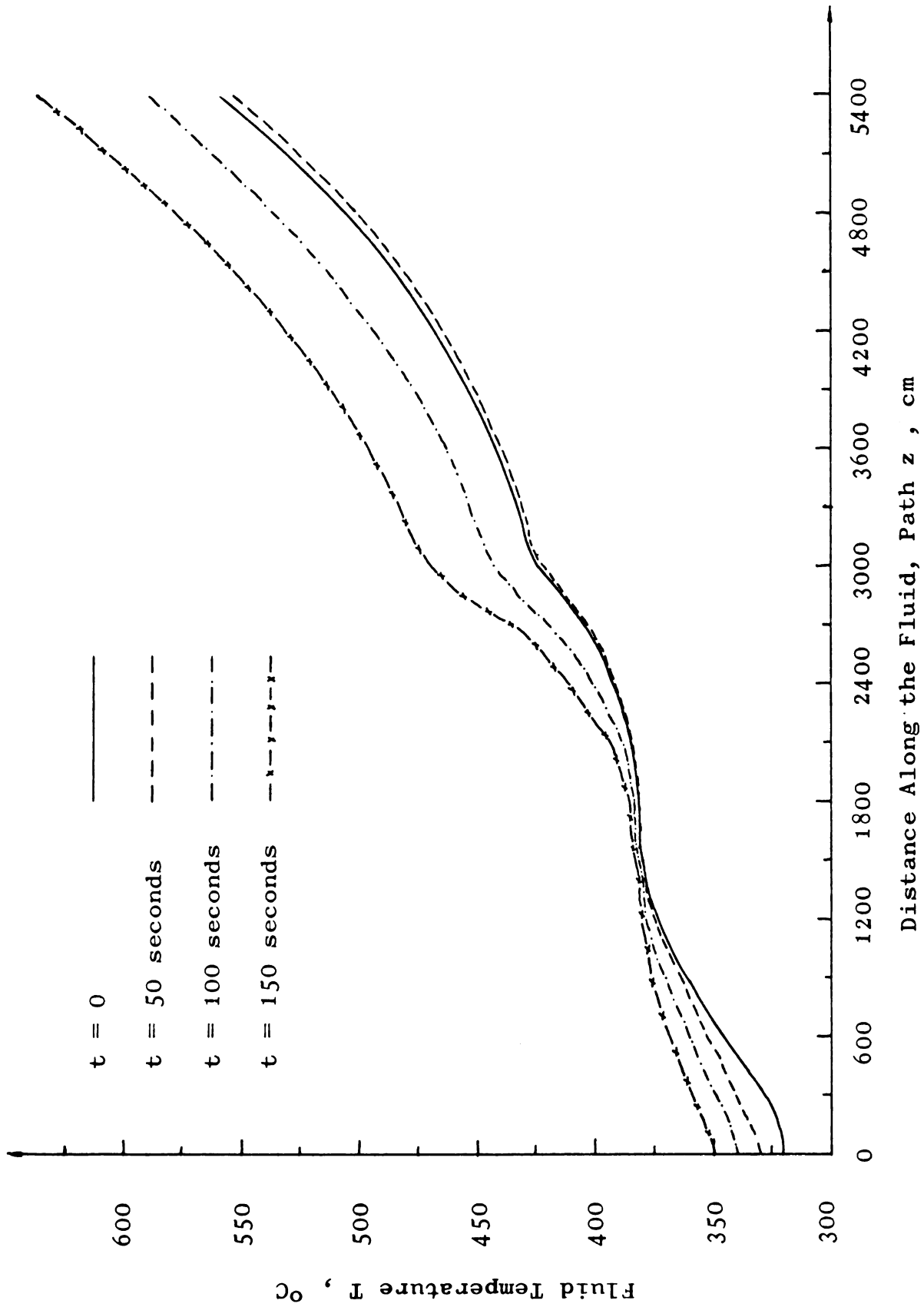


Figure 6.7: Fluid Temperature Profiles, Temperature Ramp Input

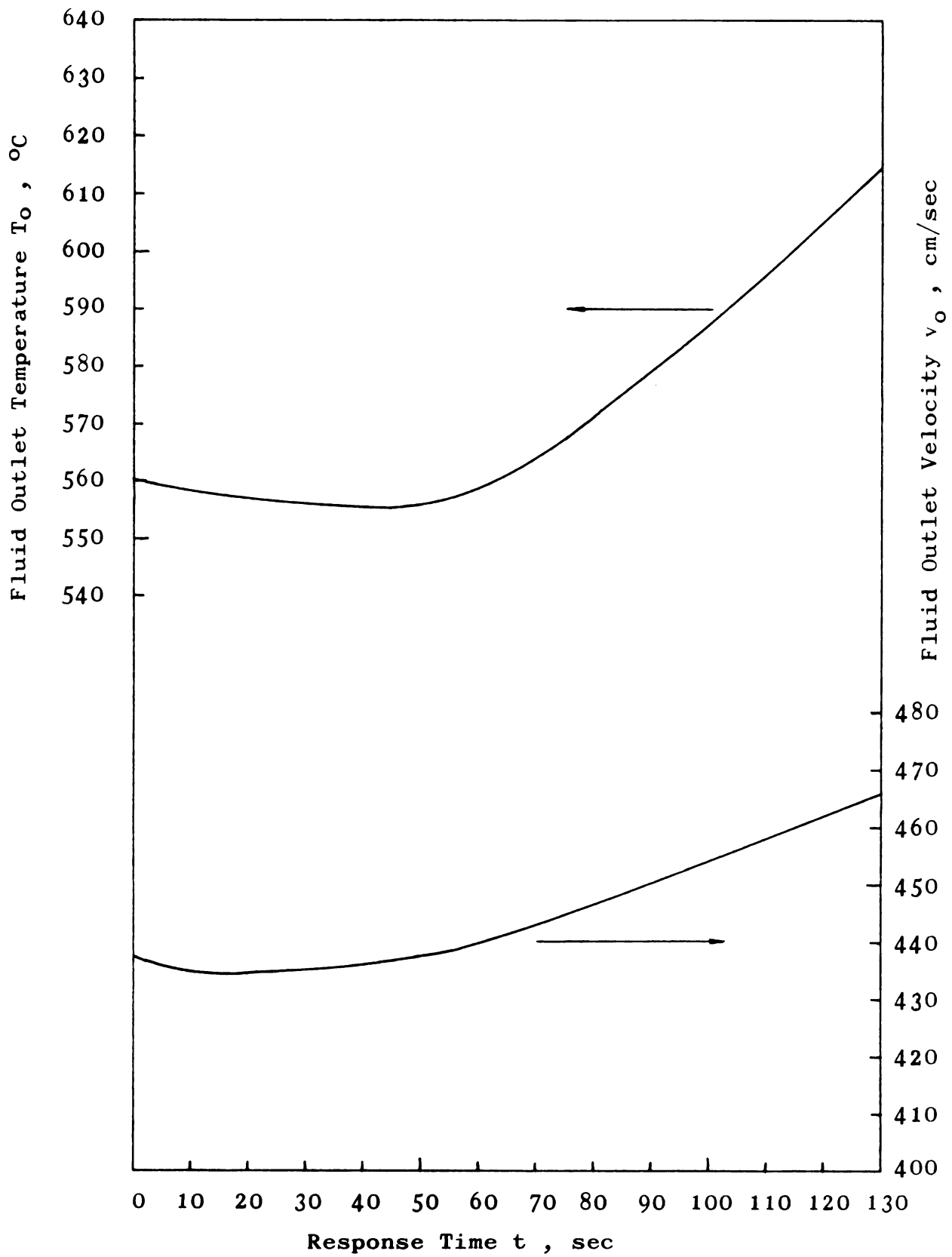


Figure 6.8: Fluid Dynamics at Boiler Outlet, Temperature Ramp Input

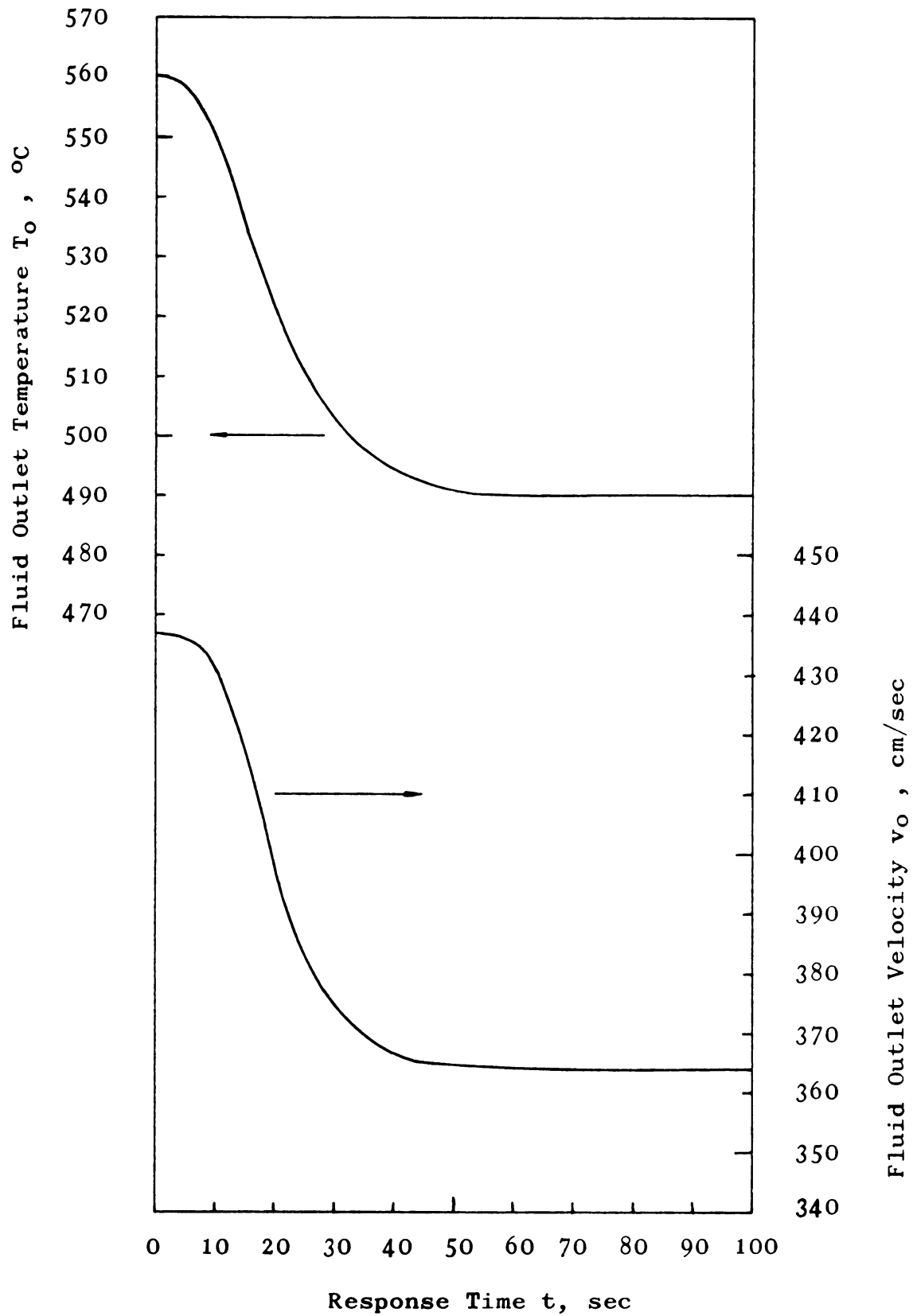


Figure 6.9: Fluid Dynamics at Boiler Outlet, -20% Firing Rate Step Input

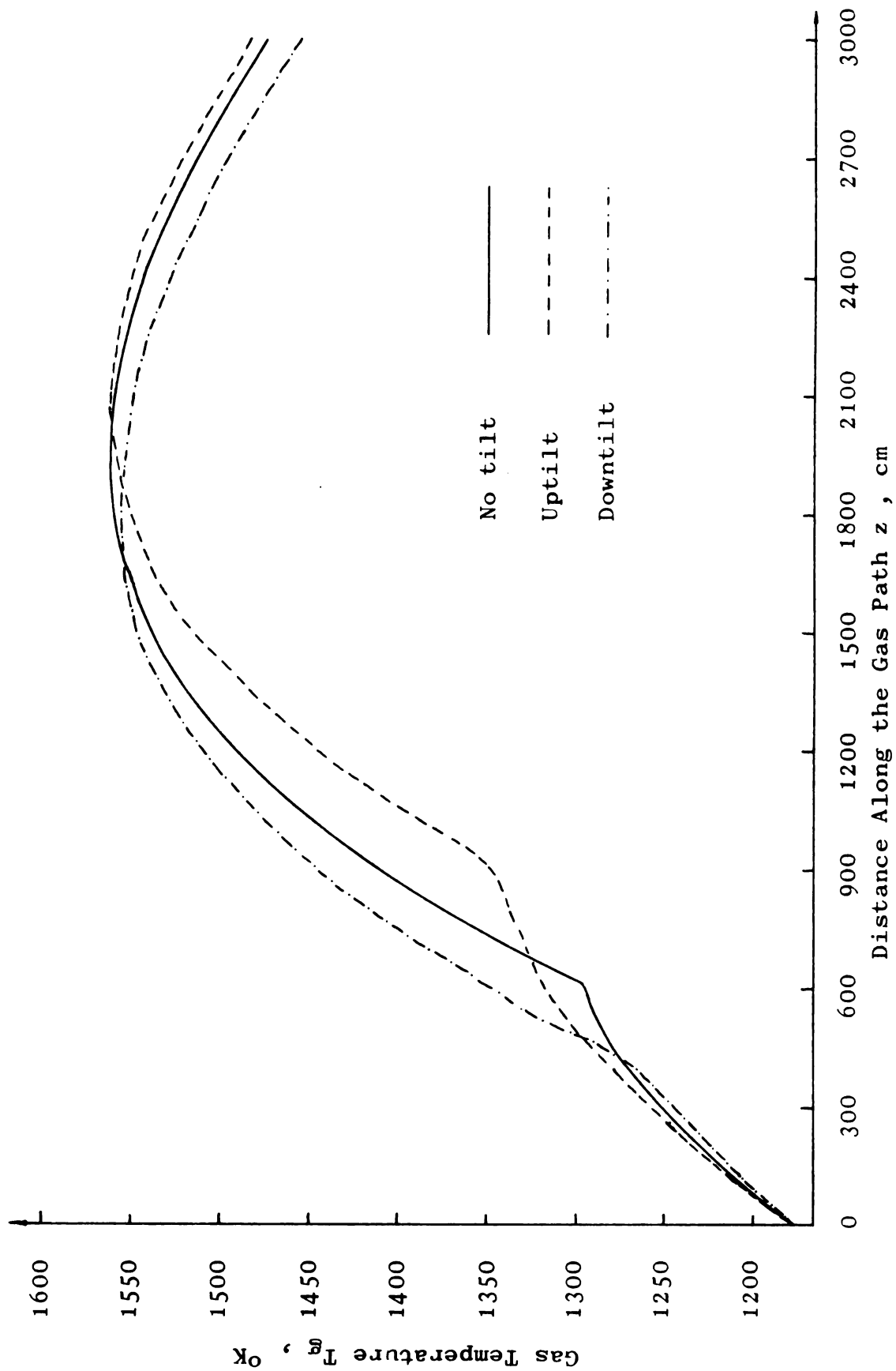


Figure 6.10: Gas Temperature Profiles, Burner Tilt Inputs

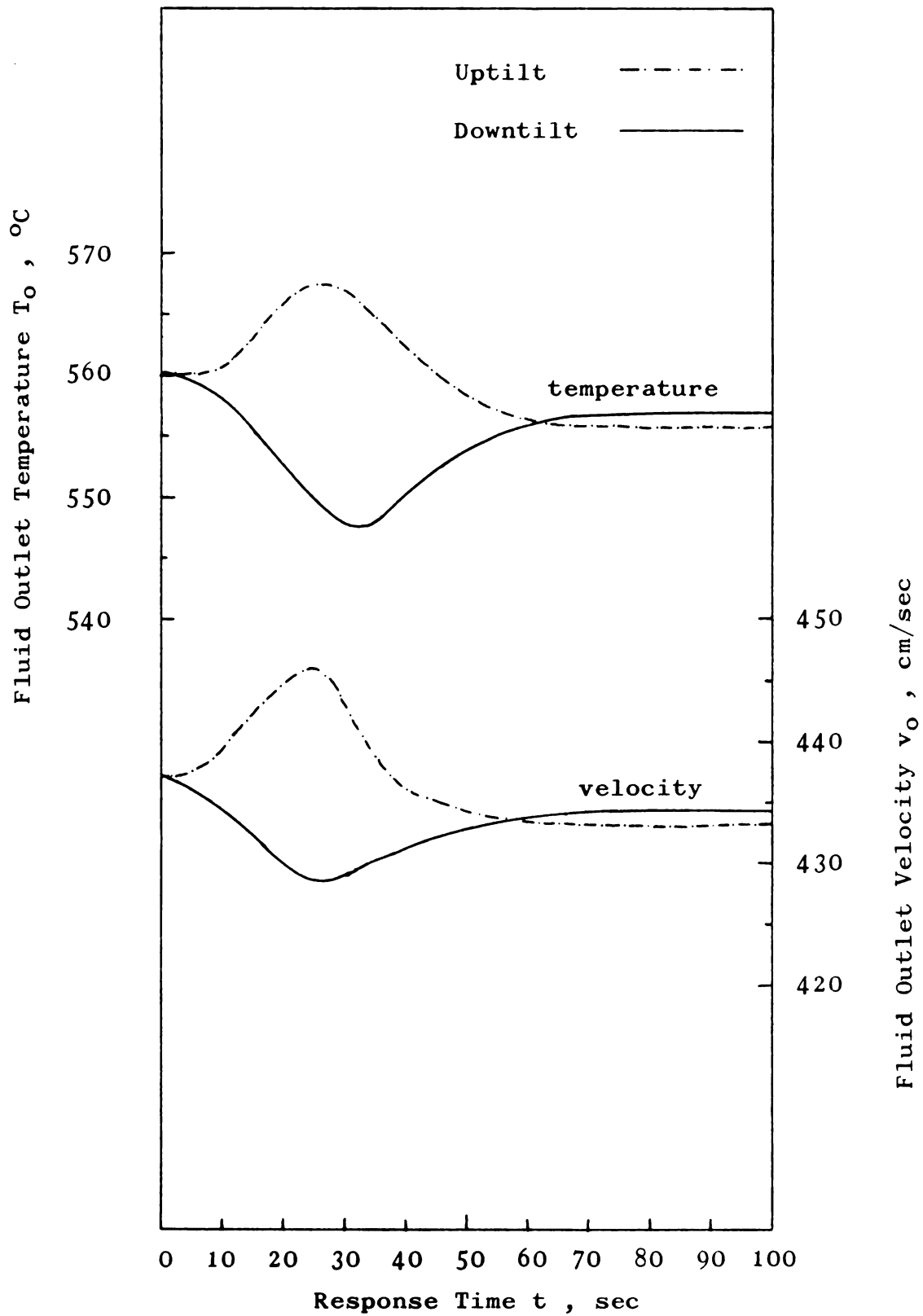


Figure 6.11: Fluid Dynamics at Boiler Outlet, Burner Tilt Inputs

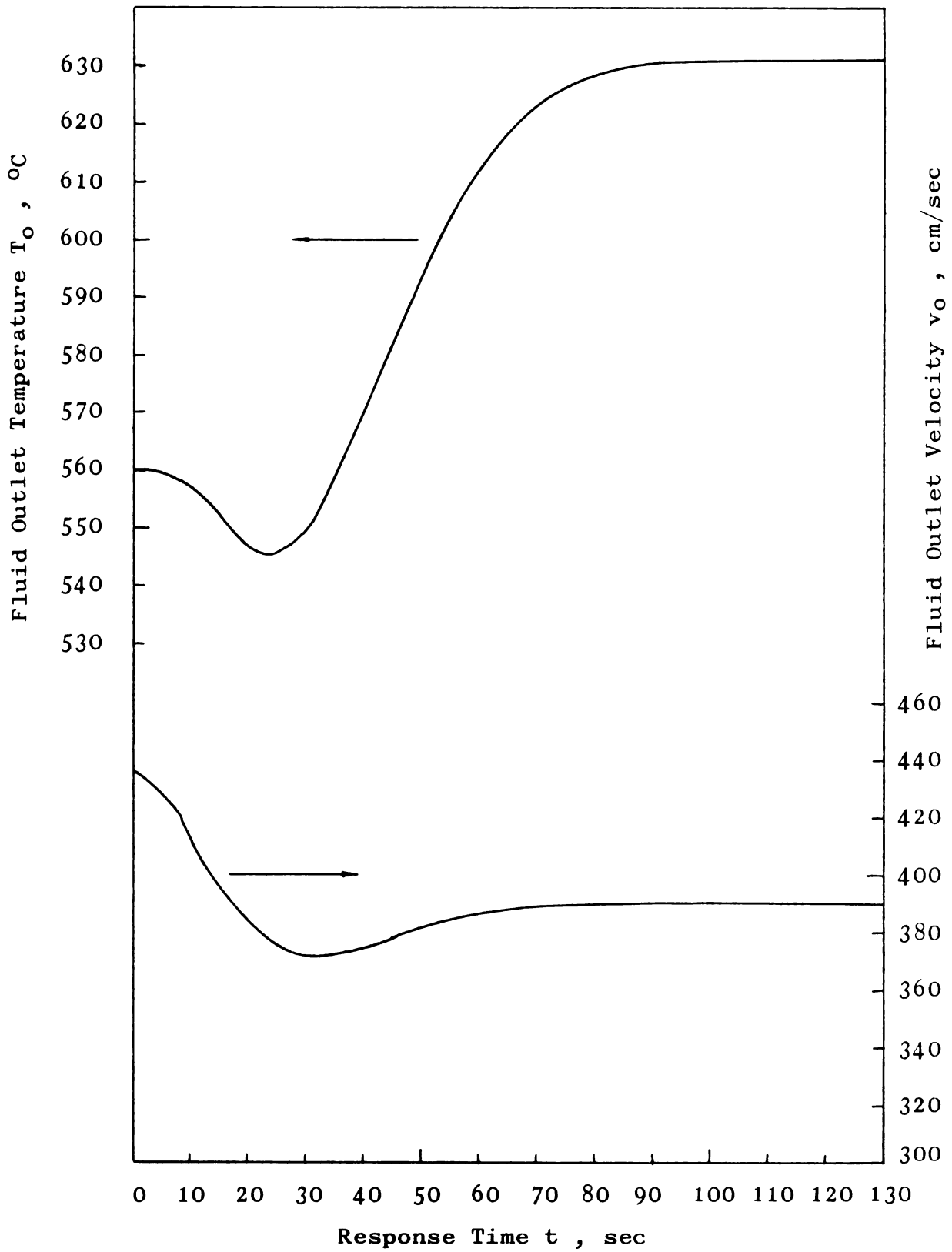


Figure 6.12: Fluid Dynamics at Boiler Outlet, Fluid Velocity and Firing Rate Step Inputs

CHAPTER 7: CONCLUSIONS AND RECOMMENDATIONS

The results described in Chapter 6 demonstrate the importance of mathematical modeling for understanding and controlling the power generation process. It is unlikely that a system of such complexity, represented in this work by 3 non-linear partial differential equations, and by the highly non-linear State Equations, may be approximated by assuming overall average values for any of the variables.

The response curves provide the necessary information for both design and control; namely: the final steady state values in response to step inputs, the final rates of change in response to ramp inputs, response time lags, fluctuations, etc. Of special interest is the observed response of the system to pressure inputs. The results (§6.2) indicate that a pressure step input is transmitted very rapidly and that steady state is attained within 5 seconds. Similar results have also been reported in the literature.^{3,8} It may be concluded that pressure inputs at the boiler outlet, associated with variation of the throttle valve position, may be simulated by pressure inputs at the boiler inlet, and thus a time consuming iterative solution of a split boundary condition problem will not be necessary.

Also noteworthy is the considerable time lag (ca. 70 seconds) associated with the response to the fluid temperature

input (§6.3).

The apparent difference in the response time lags of the gas and of the fluid sides (§6.6) is significant for the design of a control system that will eliminate the resulting fluctuations.

An important feature of the model is its versatility with respect to the nature and functional form of the input. Solutions for inputs of fluid flow rate, temperature, and pressure and of firing rate and burner tilt were described in Chapter 6. Both step and ramp inputs were tried, and other functional forms can be treated in a similar manner.

A source of variation in a power plant operation which has not been accounted for in this work is the changing quality and conditions of the fuel and air mixture. When the fuel is pulverized coal, the quality of the coal, the fuel to air ratio and the thermal conditions of the mixture may be expected to vary under normal operating conditions. It is proposed to further develop the model by including an additional section in the gas side. This section, designated the Middle Furnace and located in the burners area between the Lower and the Upper Furnace sections, may be modeled as a stirred tank reactor. The fuel-air mixture may then be treated as a variable input to the Middle Furnace.

The capability of the model to solve for large inputs, in

excess of 20%, is a marked improvement over linearized models. It should be noted that disturbances of such magnitude are to be expected under normal operating conditions. The availability of complete and thermodynamically consistent State Equations for water and steam has been a major contribution to this work in facilitating the development of the non-linearized model and in the derivation of additional thermodynamic relationships as required in the Equations of Change.

In the design of central station boilers, some sections of the gas often contain more than one section of the fluid circuitry. For example, the Superheater Section may be enclosed in a waterwall type circuitry, in addition to the superheater tube banks contained in it. This would raise difficulties in the procedure of the numerical solution where the sequence of computations follows the fluid path and the gas temperature is calculated from the energy balance.

In such situations, the following iterative procedure is proposed: let the two overlapping fluid section be designated Heat Exchanger A and B, respectively. Let the gas stream be conceptually divided into two streams, with flow rates W_{ga} and W_{gb} , respectively. When computing along Heat Exchanger A, the gas temperature is calculated from the energy balance using the value of W_{ga} for the gas flow rate. Denote the gas temperature at the end point of A by T_{ga} . The computation then

proceeds along the fluid path until Heat Exchanger B is reached. In similar manner to A, the gas temperature is computed using the value of W_{gb} for the gas flow rate. Let the gas temperature at the end point of B thus computed be denoted by T_{gb} . As both T_{ga} and T_{gb} refer to the same location in the gas path, they should be equal. If not, then the difference between them may be used in an iterative procedure to correct the initial guess of the relative magnitudes of W_{ga} and W_{gb} (the sum of which is the actual gas flow rate). This may be repeated until $T_{ga} = T_{gb}$.

Whereas a rigorous test for the validity of the model can be made only by applying it to a real system, we may conclude that the observed results are qualitatively compatible with known or predictable behavior. This applies to the general shape of the response curves, to the pressure dynamics, to the observed fast response of the gas side, relative to that of the fluid side, and to the effects of burner tilting on the gas temperature profiles.

In summary, the major accomplishments of the proposed model, compared to previous models, are: i) Absorption of radiation by the gas and by dust particles is accounted for; ii) The interaction between the fluid conditions and the gas temperature in the furnace is expressed by the modeling of the heat generation function. Thus, it is not necessary

to assume that a constant gas temperature exists in the furnace, and that variation of the fluid flow has no effect on the gas temperature; iii) The proposed model can solve for inputs that are up to an order of magnitude larger than in any previously reported work; iv) The computing time required is well within the practical limits for industrial use; v) the accuracy of the results, as seen from the convergence tests (Chapter 5), is highly satisfactory; vi) The introduction of an accurate and thermodynamically consistent formulation of the equations of state for water and steam.

APPENDIX A

APPENDIX A

§A.1 The Equation of Continuity

The general form of the equation of continuity in rectangular coordinates is:⁴

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x} (\rho v_x) + \frac{\partial}{\partial y} (\rho v_y) + \frac{\partial}{\partial z} (\rho v_z) = 0 \quad \dots\dots\dots (A-1)$$

Assuming variation in the z-direction only, we have

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial z} (\rho v_z) = 0 \quad \dots\dots\dots (A-2)$$

write $v_z = v$

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v}{\partial z} + v \frac{\partial \rho}{\partial z} = 0 \quad \dots\dots\dots (A-3)$$

Equation A-3 is identical with Equation 3-1.

§ A.2 The Equation of Motion

Assuming variation in the z-direction only, the equation of motion is:

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial z} - \frac{\partial \tau_{zz}}{\partial z} + \rho \vec{g} \quad \dots\dots\dots (1-9)$$

In a horizontal tube, the gravitational acceleration $\vec{g} = 0$;
in a vertical tube with upwards flow $\vec{g} = -g = -981 \text{ cm/sec}^2$.

The normal stress τ_{zz} is related to the velocity gradient.

For Newtonian fluids with constant viscosity μ , we have

$$-\frac{\partial \tau_{zz}}{\partial z} = \mu \frac{\partial^2 v_z}{\partial z^2} \dots\dots\dots (A-4)$$

This term is small, compared to the other terms in Equation 1-9, and is neglected.

Each term in Equation 1-9 expresses rate of momentum transfer per unit volume or, equivalently, force per unit volume.

Consider a fluid volume element ΔV , contained in a tube segment of length Δz and inside diameter D . We have:

$$\Delta V = \frac{\pi D^2}{4} \cdot \Delta z, \quad \text{and cross sectional area} \quad A_{cx} = \frac{\pi D^2}{4}$$

$$\text{Area in contact with the tube wall} \quad A_f = \pi D \Delta z.$$

The friction force acting on the fluid in ΔV is given by:

$$F = f A_f (\rho v^2 / 2) = f (\pi D \Delta z) (\rho v^2 / 2) \dots\dots\dots (A-5)$$

Dividing F by ΔV and including it in the force balance, Equation 1-9 becomes:

$$\rho \left(\frac{\partial v}{\partial t} + v \frac{\partial v}{\partial z} \right) = - \frac{\partial p}{\partial z} - \rho g - (2f/D) \rho v^2 \dots\dots\dots (A-6)$$

A factor of 10^6 multiplies the pressure term in Equation 3-2 in order that the units will be consistent ($1 \text{ bar} = 10^6 \text{ dyn/cm}^2$).

§A.3 The Equation of Energy

Assuming variation in the z-direction only, the equation of energy is:

$$\rho C_p \left(\frac{\partial T}{\partial t} + v \frac{\partial T}{\partial z} \right) = - (\nabla \cdot \vec{q}) - (\tau : \nabla v) + \left(\frac{\partial \ln \tilde{v}}{\partial \ln T} \right)_p \left(\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} \right) \dots \quad (1-11)$$

Each term in Equation 1-11 expresses rate of energy transfer or interchange, per unit volume of the fluid.

The term $(\tau : \nabla v)$ is a tensor notation representation of the irreversible transformation of mechanical energy to internal energy by viscous dissipation. This effect is small, unless high velocity gradients are encountered.

The term $-(\nabla \cdot \vec{q})$ is the rate of heat transfer by conduction across the boundaries of ΔV . In this work, this term is replaced by an expression of heat transfer from the gas into the fluid through convection and radiation.

Consider a segment of the tube as in §A.2. Convective heat transfer is given by $U(T_g - T)\Delta A_c$, where ΔA_c is the area of convective heat transfer. Radiative heat transfer is given by $\epsilon \sigma' (T_g^4 - T^4) \Delta A_r$, where ΔA_r is the area normal to the radiative flux. The combined rate, per unit volume of the fluid is:

$$q_{tr} = U(T_g - T)\Delta A_c / \Delta V + \epsilon \sigma' (T_g^4 - T^4) \Delta A_r / \Delta V \quad \dots \dots \dots (A-7)$$

Equation 3-4 is obtained from Equation A-7 by using $\sigma = \sigma' \cdot 10^{12}$.

In the Superheater Section, we have:

D_h = the inside diameter of the tube

A_{cx} = the cross sectional area of the tube

$$\Delta V = \pi D_h^2 \Delta z / 4, \quad \Delta A_c = \pi D_h \Delta z, \quad \Delta A_r = 2 D_h \Delta z,$$

$$\text{hence } \Delta A_c / \Delta V = 4 / D_h \quad \text{and} \quad \Delta A_r / \Delta V = 8 / (\pi D_h)$$

With $\epsilon = 1$, Equation A-7 becomes:

$$q_{tr} = \frac{8}{\pi D_h} \left[\frac{\pi}{2} U(T_g - T) + \sigma' (T_g^4 - T^4) \right] \dots\dots\dots (A-8)$$

The waterwall tube's inside diameter is D_v . There are n waterwall tubes per one superheater tube, but the total cross sectional area remains unchanged. Thus, $A_{cx} = \pi D_h^2 / 4 = n \pi D_v^2 / 4$, and $n = (D_h / D_v)^2$. Noting that only one side of the waterwall tubes is exposed to the gas, we have:

$$\Delta V = n \pi D_v^2 \Delta z / 4, \quad \Delta A_c = n \pi D_v \Delta z / 2, \quad \Delta A_r = n D_v \Delta z,$$

$$\Delta A_c / \Delta V = 2 / D_v, \quad \Delta A_r / \Delta V = 4 / \pi D_v,$$

and the rate expression becomes:

$$q_{tr} = \frac{4}{\pi D_v} \left[\frac{\pi}{2} U(T_g - T) + \sigma' (T_g^4 - T^4) \right] \dots\dots\dots (A-9)$$

In order to have a uniform formulation throughout the boiler in the computer program, D_{eq} is defined as follows:

$D_{eq} / A_{cx} = 8 / \pi D_h$ for the superheater tube, and $D_{eq} / A_{cx} = 4 / \pi D_v$ for the waterwall tube. With $A_{cx} = \pi D_h^2 / 4$, we get $D_{eq} = 2 D_h$ and $D_{eq} = D_h^2 / D_v$ for the superheater and waterwall tubes,

respectively.

The term $\left(\frac{\partial \ln \tilde{v}}{\partial \ln T}\right)_p$ was obtained explicitly from the other equations of state as follows:

$$\left(\frac{\partial \ln \tilde{v}}{\partial \ln T}\right)_p = \frac{T}{\tilde{v}} \cdot \left(\frac{\partial \tilde{v}}{\partial T}\right)_p \quad \dots\dots\dots (A-10)$$

When p is given as an explicit function of $\tilde{v}$ and T , as in Equation 4-5_d, then we can use the relation:

$$\left(\frac{\partial \tilde{v}}{\partial T}\right)_p = - \frac{(\partial p / \partial T)_{\tilde{v}}}{(\partial p / \partial \tilde{v})_T} \quad \dots\dots\dots (A-11)$$

The resulting expressions appear in the computer subroutines (Appendix C), under the variable name DLNT.

§ A.4 The Specific Heat of the Gas

Data on the variation of C_g vs. T_g for various fuels is given in the literature¹⁸ in the form of a chart. The curve corresponding to Bituminous Midwestern coal with 20% excess air was divided into segments, each of which was approximated by a straight line. The equations of these line segments are of the form:

$$C_g = a T_g + b \quad \dots\dots\dots (A-12)$$

The values a , b were obtained from the chart, and given as data in Subroutine SCG, Appendix C.

§ A.5 The Heat Generation Function

Qualitatively, the form of the heat generation function $f(z)$ is given in Figure 2.4. The area under the curve is equal to the total heat absorbed by the fluid in the boiler, under steady state conditions:

$$Q_f = (H_{\text{outlet}} - H_{\text{inlet}})w \cdot A_{\text{cx}} \quad \dots\dots\dots (A-13)$$

Denote the height of the boiler by L_t . Then the area under the curve $f(z)$ is $L_t \cdot f_{\text{mx}}/2$, where f_{mx} is the maximum value of $f(z)$, occurring at $z = z_{\text{fu}}$. Equating Q_f to the area under the curve we obtain:

$$f_{\text{mx}} = \frac{2w \cdot A_{\text{cx}}}{L_t} (H_{\text{outlet}} - H_{\text{inlet}}) \quad \dots\dots\dots (A-14)$$

The function $f(z)$ is given by two straight line equations:

$$f(z) = \begin{cases} (f_{\text{mx}}/z_{\text{fu}})z & \text{for } 0 \leq z \leq z_{\text{fu}} \dots (A-15_a) \\ \frac{f_{\text{mx}}}{L_t - z_{\text{fu}}} \cdot (L_t - z) & \text{for } z_{\text{fu}} \leq z \leq L_t \dots (A-15_b) \end{cases}$$

In the Superheater Section of the boiler we have 4 superheat passes, designated as pass A, pass B, pass C, and pass D, respectively. Each pass occupies a volume of gas, corresponding to a length l_s of the gas column. The amount of heat generated within a gas volume of a superheat pass was calculated

by integration of Equation A-15_b along an interval l_s . Thus:

$$q_{sd} = \frac{l_s}{2} \left[f(z) \Big|_{z=L_t} + f(z) \Big|_{z=L_t-l_s} \right] = \frac{l_s}{2} \left(0 + \frac{f_{mx}}{L_t - z_{fu}} l_s \right)$$

$$= \frac{1}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2$$

$$q_{sc} = \frac{l_s}{2} \left[f(z) \Big|_{z=L_t-l_s} + f(z) \Big|_{z=L_t-2l_s} \right] = \frac{l_s}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} (l_s + 2l_s)$$

$$= \frac{3}{2} \cdot \frac{f_{mx}}{L_t - z_{fu}} l_s^2$$

The expressions of q_{sb} and q_{sa} are obtained in a similar manner. The results are given in Equations 4-16.

The numerical values that were used are:

$$L_t = 3600 \text{ cm} \quad l_s = 150 \text{ cm} \quad \text{hence} \quad l_s^2/2 = 11,250 \text{ cm}^2$$

The gas flow rate W_g is estimated from empirical data,^{15a} correlating the heat of combustion, heat losses, and the ratio of air to fuel in the feed.

When the firing rate is changed, then both f_{mx} and W_g are changed, in the same proportion.

The value of z_{fu} depends on the burner tilt. When the tilt is downwards, z_{fu} is reduced, and vice versa.

§ A.6 Energy Equations in the Lower Furnace

The heat balance in the Lower Furnace is given by the equations:

$$q_a \Big|_z - q_a \Big|_{z+\Delta z} + \Delta z (q_s - q_{rc}) = 0 \quad \dots\dots\dots (4-17)$$

$$q_a \Big|_z = -D_c \frac{dT_g}{dz} \Big|_z, \quad q_a \Big|_{z+\Delta z} = -D_c \frac{dT_g}{dz} \Big|_{z+\Delta z} \quad \dots\dots (4-18)$$

$$q_s = (f_{mx}/z_{fu})z \quad \dots\dots\dots (4-19)$$

In order to obtain the variation of T_g within the gas volume element of length Δz , replace Δz in Equations 4-17 and 4-18 by δz and rearrange Equation 4-17:

$$-\frac{q_a \Big|_{z+\Delta z} - q_a \Big|_z}{\delta z} = -q_s + q_{rc} \quad \dots\dots\dots (A-16)$$

taking limits as $\delta z \rightarrow 0$, we have

$$-dq_a/dz = -q_s + q_{rc} \quad \dots\dots\dots (A-17)$$

Substituting Equations 4-18 and 4-19, obtain:

$$D_c \frac{d^2 T_g}{dz^2} = q_{rc} - \frac{f_{mx}}{z_{fu}} \cdot z \quad \dots\dots\dots (A-18)$$

Equation A-18 is identical with Equation 4-20.

Integrating Equation A-18 twice between z and $z+\Delta z$, taking q_{rc} to be constant along the interval Δz , we obtain

Equations 4-21 and 4-22.

At $z = z_{fu}$, the transition from the Lower Furnace to the Upper Furnace is accounted for by assuming eddy conductance at z and gas flow upwards at $z + \Delta z$. The resulting heat balance is:

$$q_a \Big|_z + W_g \cdot C_g \cdot (T_g \Big|_z - T_g \Big|_{z+\Delta z}) = (q_{rc} - q_s) \Delta z \quad \dots\dots (A-19)$$

Substituting Equations 4-18 and 4-19, and dividing by Δz , we have:

$$-\frac{D_c}{\Delta z} \cdot \frac{dT_g}{dz} - W_g \cdot C_g \cdot \frac{T_g \Big|_{z+\Delta z} - T_g \Big|_z}{\Delta z} = q_{rc} - f_{mx} \quad \dots\dots (A-20)$$

Assuming

$$\frac{dT_g}{dz} \approx \frac{T_g \Big|_{z+\Delta z} - T_g \Big|_z}{\Delta z}, \quad \text{and rearranging, obtain:}$$

$$\frac{dT_g}{dz} = \frac{f_{mx} - q_{rc}}{W_g C_g + D_c / \Delta z} \quad \dots\dots\dots (A-21)$$

and

$$T_g \Big|_{z+\Delta z} = T_g \Big|_z + \frac{dT_g}{dz} \Delta z \quad \dots\dots\dots (A-22)$$

APPENDIX B

APPENDIX B

The Main Program

```

PROGRAM ROILER      (INPUT,OUTPUT,TAPE 2=INPUT,TAPE 3=OUTPUT)
DIMENSION T(2,361),P(2,361),VOL(2,361),VE(2,361),TG(2,361)
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),      TR,PR,VR,CPR,DLNT,
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT

      NOMENCLATURE
      =====
BEL  BOUNDARY BETWEEN SUBREGIONS 2 AND 3 IN THE STATE EQUATIONS.
---
CLEV,CLEP,CLET  ARE THE TOLERANCES FOR SPECIFIC VOLUME, PRESSURE,
AND TEMPERATURE, RESPECTIVELY, IN THE ITERATIVE
COMPUTATIONS.
DIP, DIVE  STEP INPUTS OF PRESSURE AND VELOCITY, RESPECTIVELY.
-----
DTOT  RAMP INPUT  (RATE OF CHANGE) OF TEMPERATURE.
-----
DTGZ  THE DERIVATIVE OF TG WITH RESPECT TO Z .
-----
DTG1  DTGZ AT THE ROILER INLET
-----
EDR  THE VALUE OF DEO IN THE WATERWALLS
-----
DTGO, DVEO, DPO  INCREMENTS OF TG, VE, AND P, RESPECTIVELY, IN
THE ITERATIVE COMPUTATIONS.
L  INDEX NUMBER OF DISTANCE ALONG THE TUBE.
--
N  THE TOTAL NUMBER OF DISTANCE INCREMENTS ALONG THE FLUID PATH.
--

```

```

C LE = N-1
C --
C NJL THE NUMBER OF DISTANCE INCREMENTS IN EACH OF THE SUPERHEAT
C -- PASSES.
C OS THE RATE OF HEAT GENERATION WITHIN A VOLUME OF GAS CONTAINING
C -- A SUPERHEAT PASS.
C SUMO THE TOTAL HEAT ABSORBED BY THE FLUID, PER UNIT TIME. IN
C ---- A SUPERHEAT PASS.
C TCEN FLUID TEMPERATURE (DEGREE C) .
C ----
C TLB FLUID TEMPERATURE AT THE BOILER INLET (DEGREE K) .
C ---
C READ ( 2,11) CA,SA,CB,SB,CO,C1,C2,C3,C40,C41,C5,C6,C7,D,V,C
11 FORMAT (4E18,0)
C WRITE ( 3,21) CA,SA
21 FORMAT (7H CA-SA=.4E2R,10/)
C WRITE ( 3,22) CB,SB
22 FORMAT (7H CB-SB=.4F2R,10/)
C WRITE ( 3,23) CO,C1,C2,C3,C40,C41,C5,C6,C7,D
23 FORMAT (7H CO-D=.4F2R,10/)
C WRITE ( 3,24) VC
24 FORMAT (7H VC=.4F2R,10/)
C READ (2,13) Z,TCEN,PX,W,DH,EDR,F,SIG,PAI,WAG,FMX,ZFU,DIC,DV
13 FORMAT (SE15,0)
C WRITE (3,27) Z,TCEN,PX,W,DH,EDR,F,SIG,PAI,WAG,FMX,ZFU,DIC,DV
27 FORMAT (9H2 Z=.E24,9/9H TCEN=.E24,9/9H PX=.E24,9/
1 9H W=.E24,9/9H DH=.E24,9/9H EDR=.E24,9/9H F=
2, E24,9/9H SIG=.E24,9/9H PAI=.E24,9/9H WAG=.E24,9/
3 9H FMX=.E24,9/9H ZFU=.E24,9/9H DIC=.E24,9/
4 9H DV=.F24,9///)
C WRITE (3,30)
30 FORMAT (RX,1HL,6X,=HZ(CM),12X,4HTCEN,16X,6MP(BAR),14X,8HVOL(CGS),
1 12X,7HVE(CGS),11X,8HTG(DEGK))
C WRITE (3,31)
31 FORMAT (7X,3H=====,4X,7H=====,11X,6H=====,14X,8H=====,12X,

```

```

1 10H=====,10X,9H=====, 9X,10H=====)
EPS=-15,
LA=-3000./EPS +.1
LB=-3600./EPS +.1
LC=-4200./EPS +.1
LD=-4800./EPS +.1
LE=-5400./EPS +.1
NJL=-600./EPS +.1
ACX=PA1*DH**2/4.
TGA=1475.
UB=6.
U=5.6782E-4*UR
FOW=1.F+6/W
CLFV=1.F-2
CLFP=5.F-4
CLFT=1.
DTGO=1.
TLR=593.15
DO 405 ITG=1.50
DX=DH
DFQ=2.*DH
TGX=TGA
GAC=0.
TFD=2.*F/DX
SUMQ=0.
*****COMPUTATION OF R.C.
Z=5400.
TCFN=560.
PX=240.
TX=TCFN+273.15
TR=TX/VC(5)
PR=PX/VC(6)
CALL SV2
VOX=VR*VC(7)
VEX=VOX*W

```

C *****

```

CALL CP2
CPX=CPR*VC(9)
DLNTX=DLNT
BEL=VC(1)+VC(2)*TR+VC(3)*TR*TR
N=361
DVEO=1.*EPS/7.*
DPO=-.0000*EPS/7.*
DO 399 I=1,N
STV=EPS*(TFD*VFX+GAC/VFX)
FELT=.1*VOX*DLNTX/CPX
TQ=(TX/1000.)*(TX/1000.)
TQ=TO*TQ
TGO=(TGX/1000.)*(TGX/1000.)
TGO=TGO*TGO
GRAC=DEQ*(PAI/2.*U*(TGX-TX)+SIG*(TGO-TQ))
GOWV=GRAC/(W*ACX)
GET=EPS*GOWV/CPX
CALL SCG
C *****DISTRIBUTION TO SURREGIONS OF S. EQN. *****
100 IF (TR-VC(11)) 105,105,100
101 IF (TR-1.) 201,101,101
101 IF (PR-REL) 105,105,201
C *****SURREGIONS 1 AND 2 *****
105 CONTINUE
DPO=DPO/2.
PO=PX+DPO
PR=PO/VC(6)
DVFM=-FOW*(PO-PX)-STV
VFM=VFX+DVFM
VOM=VFM/W
TM=TX+GET+FFLT*(PO-PX)
TR=TM/VC(5)
C BETWEEN SURG1 AND SURG2
IF (TR-1.) 107,100,100
107 CALL SV1
GO TO 121
100 CALL SV2
C ADJUST VOLUMES
121 VST=VR*VC(7)

```

```

ZTO=VOM-VST
IF (ABS(ZTO)-CLFV) 301,301,131
131 PN=PO+DP6
PR=PN/VC(6)
DVFM=-FOW*(PN-PX)-STV
VEM=VEX+DVEW
VOM=VEM/W
TM=TX+QET+FFLT*(PN-PX)
TR=TM/VC(5)
C BETWEEN SURG1 AND SURG2
IF (TR-1.) 137,139,139
137 CALL SV1
GO TO 141
139 CALL SV2
C ADJUST VOLUMES
141 VST=VR*VC(7)
ZTN=VOM-VST
IF (ABS(ZTN)-CLFV) 301,301,151
151 DPO=ZTN*(PO-PN)/(ZTN-ZTO)
PO=PN
ZTO=ZTN
GO TO 131
C *****SURREGIONS 3 AND 4
201 CONTINUE
DVEO=DVFO/2.
VEO=VEX+DVEO
VOM=VEO/W
VR=VOM/VC(7)
DPM=-1./FOW*(VEO-VFX+STV )
PM=PX+DPM
TM=TX+QET+FFLT*(PM-PX)
TR=TM/VC(5)
C BETWEEN SURG3 AND SURG4
CALL SP3
IF (TR-1.) 205,207,207

```

```

205 PR3=PR
   CALL SP4
C   ADJUST PRESSURES
207 PST=PR*VC(6)
   ETO=PM-PST
   IF (ARS(ETO)-CLFP) 301.301.211
211 VEN=VEO+DVFO
   VOM=VEN/W
   VP=VOM/VC(7)
   DPM=-1./FOW*(VFN-VEX+STV )
   PM=PX+DPM
   TM=TX+GET+FELT*(PM-PX)
   TR=TM/VC(5)
C   BETWEEN SURG3 AND SURG4
   CALL SP3
   IF (TR-1.) 215.217.217
215 PR4=PR
   CALL SP4
C   ADJUST PRESSURES
217 PST=PR*VC(6)
   ETN=PM-PST
   IF (ABS(ETN)-CLEP) 301.301.221
221 DVEO=ETN*(VEO-VEN)/(ETN-ETO)
   VEO=VEN
   ETO=ETN
   GO TO 211
301 L=N+1-I
   T(1.0L)=TX
   P(1.0L)=PX
   VOL(1.0L)=VOX
   VF(1.0L)=VFX
   TG(1.0L)=TGX
   IF (L-1) 400.400.309
309 TX=TM
   PX=PR*VC(6)

```

```

VOX=VOM
VEX=W*VOM
C *****COMPUTATION OF TG *****
  IF (L-(LA+2)) 351,350,311
  311 IF (L-(LB+2)) 317,320,313
  313 IF (L-(LC+2)) 317,320,315
  315 IF (L-(LD+2)) 317,320,317
  317 SUMQ=SUMQ-QRAC*EPS
      GO TO 361
  320 SUMQ=SUMQ-QRAC*EPS
      20L=(L-LA-2)/NJL
      OS=(2.*PCL+1.)*1.125F+4*FMX/(3600.-ZFU)
      TGX=TGX+(OS-SUMQ)/(WAG*CGOT)
      SUMQ=0.
      GO TO 361
  350 TGX=TGA
      GAC=981.
      DX=DV
      DEO=EOR
      TFD=2.*F/DX
      GO TO 361
  351 IF (Z-ZFU) 357,355,353
  353 DTGZ=(FMX/(ZFU-3600.))*(Z+EPS/2.-3600.)-QRAC)/(WAG*CGOT)
      TGX=TGX+FPS*DTGZ
      GO TO 361
  355 DTGZ=(FMX-QRAC)/(WAG*CGOT-DIC/EPS)
      TGX=TGX+FPS*DTGZ
      GO TO 361
  357 TGX=TGX+EPS*DTGZ+FPS**2/(6.*DIC)*(3.*QRAC-FMX/ZFU*(3.*Z+EPS))
      DTGZ=DTGZ+EPS/(2.*DIC)*(2.*QRAC-FMX/ZFU*(2.*Z+EPS))
  361 CONTINUE
C *****COMPUTATION OF CPX,DLNTX *****
      TR=TX/VC(5)
      PR=PX/VC(6)
      VP=VOX/VC(7)

```



```

REL=VC(1)+VC(2)*TR+VC(3)*TR*TR
IF (TR-VC(11)) 377.377.373
373 IF (TR-1.) 385.375.375
375 IF (PR-BEL) 377.377.385
377 IF (TR-1.) 379.381.381
379 CALL CP1
GO TO 391
381 CALL CP2
GO TO 391
385 CALL CP3
IF (TR-1.) 387.391.391
387 CALL CP4
391 CPX=CPR*VC(9)
DLNTX=DLNT
Z=Z+EPS
399 CONTINUE
400 CONTINUE
IF (ITG-1) 401.401.403
401 TIN=T(1.1)-TLR
IF (ARS(TIO)-CLET) 409.409.402
402 TGO=TGA
DTGO=TIO/ABS(TIO)*DTGO
TGN=TGO+DTGO
TGA=TGN
GO TO 405
403 TIN=T(1.1)-TLR
IF (ABS(TIN)-CLET) 409.409.404
404 DTGO=TIN*(TGO-TGN)/(TIN-TIO)
TIO=TIN
TGN=TGN
TGN=TGO+DTGO
TGA=TGN
405 CONTINUE
409 CONTINUE
JAD=-60./EPS

```

```

DO 411 L=1,N,JAD
TCEN=T(1,L)-273.15
AL=L
Z=-EPS*(AL-1.)
411 WRITE ( 3,40) L,Z,TCEN,P(1,L),VOL(1,L),VE(1,L),TG(1,L)
40 FORMAT (110,F10.2,5F20.5)
WRITE ( 3,42) ITG,TGA,DTGO
42 FORMAT (7H3 ITG=,110/7H TGA=,F10.5/8H DTGO=,F10.5)

C
C
C *****
C * UNSTEADY STATE *
C *****
OSD=1.125F+4*FMX/(1600.-ZFU)
OSC=3.*OSD
OSR=5.*OSD
OSA=7.*OSD
C *** M TIME INCREMENTS *****
M=20
EPS=-EPS
DTGI=DTGZ
C *** STEP CHANGES OF VELOCITY AND PRESSURE *****
DIP =0.
DIVE=-10.
P(2,1)=P(1,1)+DIP
VF(2,1)=VE(1,1)+DIVE
C ***** PAMP CHANGE OF TAMP. *****
TIM=5.
DTOT=0.
DO 801 K=1,M
=====
AK=K
TIMC=AK*TIM
WRITE ( 3,51) K,TIMC
51 FORMAT (25H1 TIME INCREMENT NO. ,12/16H TIME(SEC)=,F10.5)

```

```

1  ///
```

```

WRITE ( 3,30)
WRITE ( 3,31)
DX=DV
DEQ=EDR
GAC=981.
TFD=2.*F/DX
DVFO=.04*EPS/7.5

C
C
C      BOUNDARY CONDITIONS
C      =====
      IF (K-1) 473,473,471
471 P(2,1)=P(1,1)
      VE(2,1)=VE(1,1)
473 T(2,1)=T(1,1)+TIM*DTOT
      TG(2,1)=TG(1,1)
      DTGZ=DTG1
      PR=P(2,1)/VC(6)
      TR=T(2,1)/VC(5)
      CALL SV1
      VOL(2,1)=VR+VC(7)

C
C
C      =====
C      COMPUTATION UP THE TUBE
C      =====

      NM1=N-1
      DO 701 L=1,NM1
C      =====
      IF (L-(LA+1)) 480,500,500
C      Nomenclature
C      =====
480 TGX=TG(2,L)
      TGY=TG(1,L)
      PXX=P(2,L)
      TX= T(2,L)
      VOX=VOL(2,L)
      POX=1./VOX

```

```

VFX= VE(2,L)
PY= P(1,L)
TY= T(1,L)
VOY=VOL(1,L)
POY=1./VOY
VEY= VF(1,L)
PZ= P(1,L+1)
TZ= T(1,L+1)
VOZ=VOL(1,L+1)
POZ=1./VOZ
VFZ= VF(1,L+1)
PJ=(PX+PZ)/2.
TJ=(TX+TZ)/2.
VOJ=(VOX+VOZ)/2.
VFJ=(VFX+VEZ)/2.
TQ=(TJ/1000.)*(TJ/1000.)
TQ=TQ*TO
TGO=(TGY/1000.)*(TGY/1000.)
TGO=TGO*TGO
ORAC=DEQ*(PAI/2.*U*(TGY-TJ)+SIG*(TGO-TQ))
ROTE=-EPS/VFJ*(ROX-ROY)/TIM
PCT=.1*VEX
TR=TJ/VC(5)
PR=PJ/VC(6)
REL=VC(1)+VC(2)*TR+VC(3)*TR*TR
C ***** SURREGIONS OF THE STATE EQUATION *****
IF (TR-VC(11)) 507,507,507
507 IF (TR-1.) 551,505,505
505 IF (PR-BFL) 507,507,551
507 CONTINUE
C
C
C ***** SURREGIONS 1 AND 2 *****
IF (TR-1.) 509,511,511
509 CALL SV1

```

```

CALL CP1
GO TO 513
511 CALL SV2
CALL CP2
513 VOJ=VR*VC(7)
ROJ=1./VOJ
CPJ=CPR*VC(9)
DLNTJ=DLNT
C *** FUNCTIONS IN THE EQUATIONS OF CHANGE EVALUATED AT J *****
FOW=1.F+6/(VEJ*ROJ)
VOGAF=-FPS*((VFX-VFY)/TIM+GAC)/VEJ+IFN*VFJ)
FELT=0.1*DLNTJ*VOJ/CPJ
QFT=EPS/VEJ*(-(TX-TY)/TIM+ORAC/(ACX*ROJ*CPJ)+FELT*(PX-PY)/TIM)
C
C FIRST ITERATION
C =====
DPO=VOGAF/EOW*(1.+5.F-5)
PO=PX+DPO
PM=PO
PD=PM/VC(6)
C ***** EQUATIONS OF CHANGE *****
VFM=VEX+VOGAF-FOW*(PM-PX)
POM=POX+ROTE-ROJ/VEJ*(VFM-VFX)
VOM=1./POM
TM=TX+QFT+FELT*(PM-PX)
C ***** COMPARE VOM TO VST, OBTAINED FROM STATE EQUATION *****
TP=TM/VC(5)
IF (TP-1.) 515,517,517
515 CALL SV1
GO TO 519
517 CALL SV2
519 VST=VR*VC(7)
ZTO=VOM-VST
IF (ARS(ZTO)-CLEV) 601,601,521
521 PN=PO+DPO
C =====

```

```

C
C
C      ITERATIONS
C      =====
C      PM=PN
C      PD=PM/VC(6)
C      ***** EQUATIONS OF CHANGE *****
C      VEM=VFX+VOGAF-FOW*(PM-PX)
C      COR=ABS(VEM-VEX)
C      IF (COR-PCT) 522,522,514
C      514 DPO=DPO/(COR-PCT+1.)
C      GO TO 521
C      522 POM=POX+ROTF-DQJ/VFJ*(VEM-VEX)
C      VOM=1./POM
C      TM=TX+QET+FFLT*(PM-PX)
C      *****COMPARE VOM TO VST, OBTAINED FROM STATE EQUATION *****
C      TR=TM/VC(5)
C      IF (TR-1.) 523,525,525
C      523 CALL SV1
C      GO TO 527
C      525 CALL SV2
C      527 VST=VR*VC(7)
C      ZTN=VOM-VST
C      IF (ABS(ZTN)-CLFV) 601,601,529
C      529 DPO=ZTN*(PO-PN)/(ZTN-ZTO)
C      PO=PN
C      ZTO=ZTN
C      GO TO 521
C
C      SURREGIONS 3 AND 4
C      =====
C      551 DVFO=DVFO/2.
C      VP=VOJ/VC(7)
C      20J=1./VOJ
C      CALL CP3
C      IF (TR-1.) 553,555,555

```

```

553 CALL CPA
555 CPJ=CPR*VC(9)
DLNTJ=DLNT
C ***** FUNCTIONS IN THE EQUATIONS OF CHANGE EVALUATED AT J *****
EOW=1.0F+6/(VEJ*ROJ)
POGAF=-FPS/EOW*((VFX-VEY)/TIM+GAC)/VEJ+TFD*VEJ)
FFLT=0.1*DLNTJ*VOJ/CPJ
QFT=EPS/VEJ*(-(TX-TY)/TIM+GRAC/(ACX*ROJ*CPJ)+FELT*(PX-PY)/TIM)
C
C FIRST ITERATION
C =====
VFO=VEX+DVEO
VFM=VFO
C ***** EQUATIONS OF CHANGE *****
PM=PX+POGAF-(VFM-VFX)/EOW
ROM=ROX+ROTF-ROJ/VFJ*(VFM-VFX)
VOM=1.0/ROM
TM=TX+QFT+FELT*(PM-PX)
C *****COMPARE PM TO PST. OBTAINED FROM STATE EQUATION *****
TP=TM/VC(5)
VP=VOM/VC(7)
CALL SP3
IF (TP-1.0) 557,559,559
557 PR3=PR
CALL SP4
559 PST=PR*VC(6)
FTO=PM-PST
IF (ARS(FTO)-CLFP) 601,601,561
561 VEN=VFO+DVFO
C =====
C
C ITERATIONS
C =====
VFM=VEN
C ***** EQUATIONS OF CHANGE *****
PM=PX+POGAF-(VFM-VFX)/EOW

```

```

ROM=ROX+ROTF-ROJ/VFJ*(VEM-VEX)
VOM=1./ROM
TM=TX+OFT+FELT*(PM-PX)
*****COMPARE PM TO PST. OBTAINED FROM STATE EQUATION *****
TR=TM/VC(5)
VR=VOM/VC(7)
CALL SP3
IF (TR-1.) 563,565,565
563 PD3=PR
CALL SP4
565 PST=PR*VC(6)
FTN=PM-PST
IF (ARS(FTN)-CLFP) 601,601,567
567 DVFO=ETN*(VEO-VEN)/(FTN-ETO)
VEO=VEN
ETO=ETN
GO TO 561
=====
***** ASSIGN THE COMPUTED VALUES *****
601 T(2*L+1)=TM
P(2*L+1)=PM
VOL(2*L+1)=VOM
VF(2*L+1)=VEM
=====
COMPUTATION OF TG
=====
AL=L
Z=(AL-1.)*EPS
IF (L-LA) 621,620,603
603 IF (L-LR) 611,613,605
605 IF (L-LC) 611,615,607
607 IF (L-LN) 611,617,609
609 IF (L-LF) 611,619,619

```

```

611 SUMQ=SUMQ+QRAC*EPS
   GO TO 701
613 SUMQD=SUMQ+QRAC*EPS
   TGY=TG(1,LR+2)
   SUMQ=0.
   GO TO 701
615 SUMQC=SUMQ+QRAC*EPS
   TGY=TG(1,LC+2)
   SUMQ=0.
   GO TO 701
617 SUMQB=SUMQ+QRAC*EPS
   TGY=TG(1,LD+2)
   SUMQ=0.
   GO TO 701
619 SUMQA=SUMQ+QRAC*EPS
   GO TO 701
620 CALL SCG
   DTGZ=(FMX/(ZFU-3600.))*(Z+EPS/2.-3600.)-QRAC)/(WAG*CGOT)
   TGA=TX+EPS*DTGZ
   TG(2,L+1)=TGA
   TGY=TG(1,LA+2)
   GAC=0.
   DX=DH
   DFO=2.*DH
   TFD=2.*F/DX
   DPO=-.001
   SUMQ=0.
   GO TO 701
621 IF (Z-ZFU) 625,622,623
622 CALL SCG
   DTGZ=(FMX-QRAC)/(WAG*CGOT+DIC/EPS)
   TGM=TX+EPS*DTGZ
   GO TO 631
623 CALL SCG
   DTGZ=(FMX/(ZFU-3600.))*(Z+EPS/2.-3600.)-QRAC)/(WAG*CGOT)

```

```

TGM=TGX+EPS*DTGZ
GO TO 631
625 TGM=TGX+EPS*DTGZ+EPS*EPS/(6.*DIC)*(3.*QRAC-FMX/ZFU*(3.*Z+EPS))
    DTGZ=DTGZ+EPS/(2.*DIC)*(2.*QRAC-FMX/ZFU*(2.*Z+EPS))
631 CONTINUE
    TG(2,L+1)=TGM
701 CONTINUE
    TGX=TGA
    CALL SCG
    TGR=TGA+(QSA-SUMQA)/(WAG*CGOT)
    TGX=TGR
    CALL SCG
    TGC=TGR+(QSB-SUMQB)/(WAG*CGOT)
    TGX=TGC
    CALL SCG
    TGD=TGC+(QSC-SUMQC)/(WAG*CGOT)
    TGX=TGD
    CALL SCG
    TGE=TGD+(QSD-SUMQD)/(WAG*CGOT)
    DO 721 JL=1,NJL
        JLD=JL+LA+1
        JLC=JL+LR+1
        JLR=JL+LC+1
        JLA=JL+LD+1
        TG(2,JLA)=TGA
        TG(2,JLR)=TGB
        TG(2,JLC)=TGC
        TG(2,JLD)=TGD
721 =====
C
    JAQ=1
    DO 791 L=1,N*JAQ
        TCEN=T(2,L)-273.15
        AL=L
        Z=(AL-1.)*EPS
        WRITE ( 3,40) L,Z,TCEN,P(2,L),VOL(2,L),VE(2,L),TG(2,L)

```

```

701 CONTINUE
DO 795 L=1,N
  T (1,L)=T (2,L)
  P (1,L)=P (2,L)
  VOL(1,L)=VOL(2,L)
  TG (1,L)=TG (2,L)
  VE (1,L)=VE (2,L)
  WRITE ( 3,60) TGE
60 FORMAT (10H TGE=,F10.5)
801 CONTINUE
CALL EXIT
END

```

Data

6.824687746E 03	-5.422063674F 02	-2.096666206E 04	3.941286790E 04	A1
-6.733277740E 04	9.902381033E 04	-1.093911774E 05	8.590841668E 04	A2
-4.511168745E 04	1.418138927E 04	-2.017271113E 03	7.982692718E 00	A3
-2.616571843E-02	1.522411791E-03	2.284279056E-02	2.421647003E 02	A4
1.269716088E-10	2.074838329E-07	2.174020351E-08	1.105710499E-09	A5
1.293441934E 01	1.308119072E-05	6.047626339E-14	8.438375408E-01	A6
5.362162166E-04	1.720000001E 00	7.342278494E-02	4.975858873E-02	A7
6.537154305E-01	1.150000000E-06	1.510800000E-05	1.418800000E-01	A8
7.002753170E 00	2.905284926E-04	2.040000000E-01	0.00000000E 00	A9
1.683599274E 01	2.856067796F 01	-5.438923329E 01	4.330662834E-01	CB-1
-6.547711697E-01	8.565182058F-02	6.670375918F-02	1.388983801E 00	CB-2
8.390104328F-02	2.614670893F-02	-3.373439453F-02	4.520918904F-01	CB-3
1.069036614F-01	-5.975336707F-01	-8.847535804F-02	5.958051609E-01	CB-4
-5.159303373E-01	2.075021122F-01	1.190610271E-01	-9.867174132E-02	CB-5
1.683998803F-01	-5.809438001F-02	6.552390126F-03	5.710218649F-04	CB-6
1.936587558F 02	-1.388522425F 03	4.126607219F 03	-6.508211677E 03	CB-7
5.745984054E 03	-2.693088365F 03	5.235718623E 02	7.633333333E-01	CB-8

4.00607394PF-01	8.636081627F-02	-8.532322921F-01	3.460208861F-01	CR-2
-6.839900000F 00	-1.722604200F-02	-7.771750390F 00	4.204607520E 00	C1
-2.768070380F 00	2.104197070F 00	-1.146495880E 00	2.231380850F-01	C2
1.162503630E-01	-8.209005440F-02	1.941292390F-02	-1.694705760E-03	C3
-4.311577033E 00	7.086360850F-01	1.236794550E 01	-1.203890040F 01	C4
5.404374220E 00	-9.938650430E-01	6.275231820E-02	-7.747430160E 00	C5
-4.298850920E 00	4.314305380E 01	-1.416193130E 01	4.041724590E 00	C6
1.555463260E 00	-1.665689350E 00	3.248811580E-01	2.936553250E 01	C7
7.948418420F-06	8.088597470E 01	-8.361533800E 01	3.586365170E 01	C8
7.518959540E 00	-1.261606400E 01	1.097174620E 00	2.121454920E 00	C9
-5.465295660F-01	8.328754130F 00	2.759717760E-06	-5.090739850E-04	C10
2.106363320E 02	5.528935335F-02	-2.336365955E-01	3.697071420E-01	C11
-2.536415470E-01	6.828087013E-02	-2.571600553E 02	-1.518783715E 02	C12
2.220723208F 01	-1.802039570F 02	2.357096220E 03	-1.462335698E 04	C13
4.542916630E 04	-7.053556432E 04	4.381571428E 04	-1.717616747E 00	C14-D1
3.526389875E 00	-2.690899373E 00	9.070982605E-01	-1.138791156E-01	D2
1.301023613F 00	-2.64277743F 00	1.996765362E 00	-6.661557013E-01	D3
8.270860589E-02	3.426663535E-04	-1.236521258E-03	1.155018309E-03	D4
1.574373327E 01	-3.417061978F 01	1.931380707E 01	4.260321148E 00	VC-1
6.473000000F 02	2.212000000F 02	3.170000000E 00	7.012040000E 01	VC-2
1.083275143F-01	0.000000000F 00	9.626911787E-01	1.333462073E 00	VC-3
1.657886606E 00	7.475191707F-01	4.520795660E 00	0.000000000E 00	VC-4
5.40E+3	5.6E+2	2.4E+2	3.20E+1	7.50E+0
2.25E+1	1.0E-2	5.67E+0	3.1415927E+0	1.80E+3
1.525E+3	6.0E+2	1.00E+6	2.50E+0	4.73E+2
				VD3

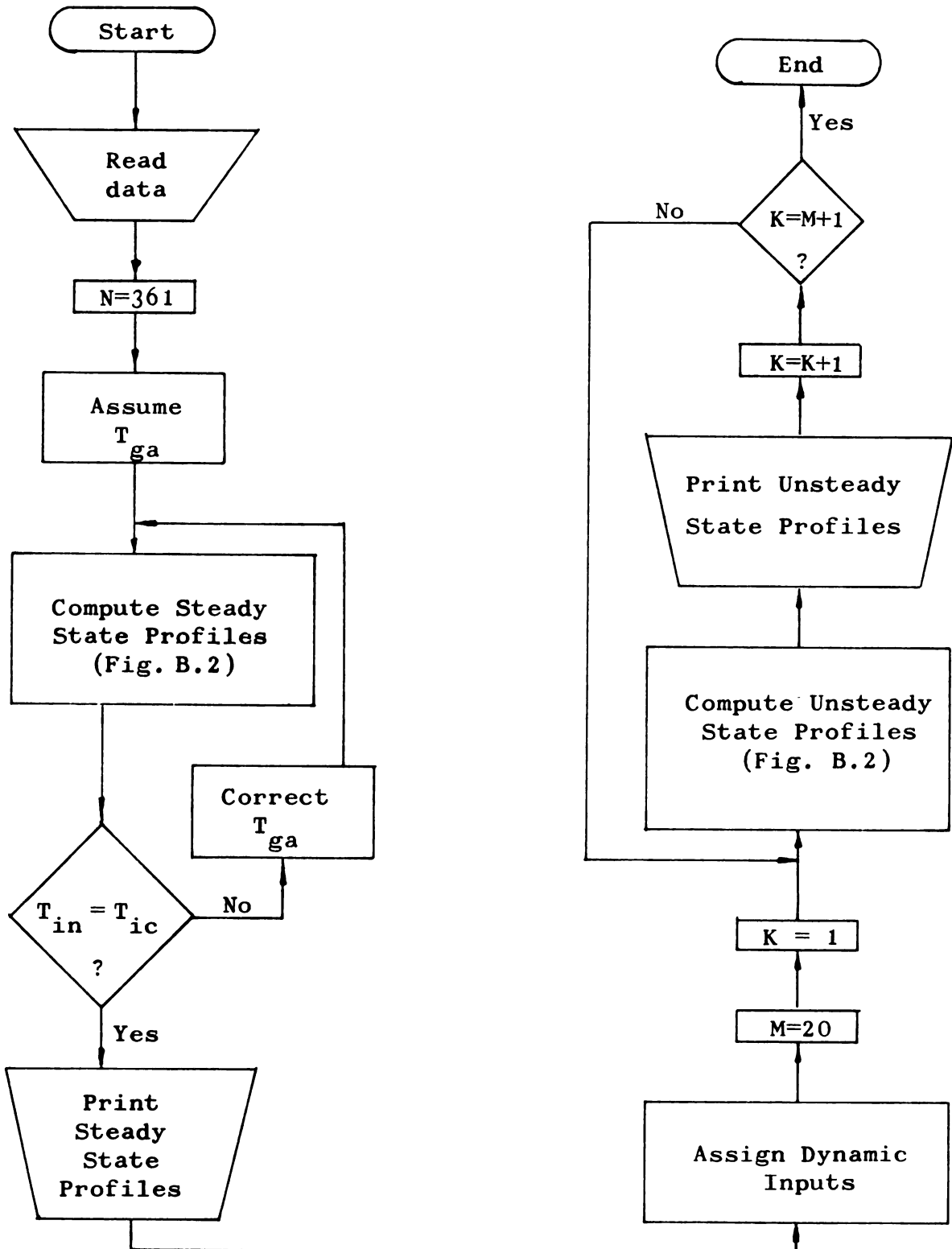
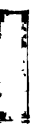


Figure B.1: A Flowchart of the Main Program



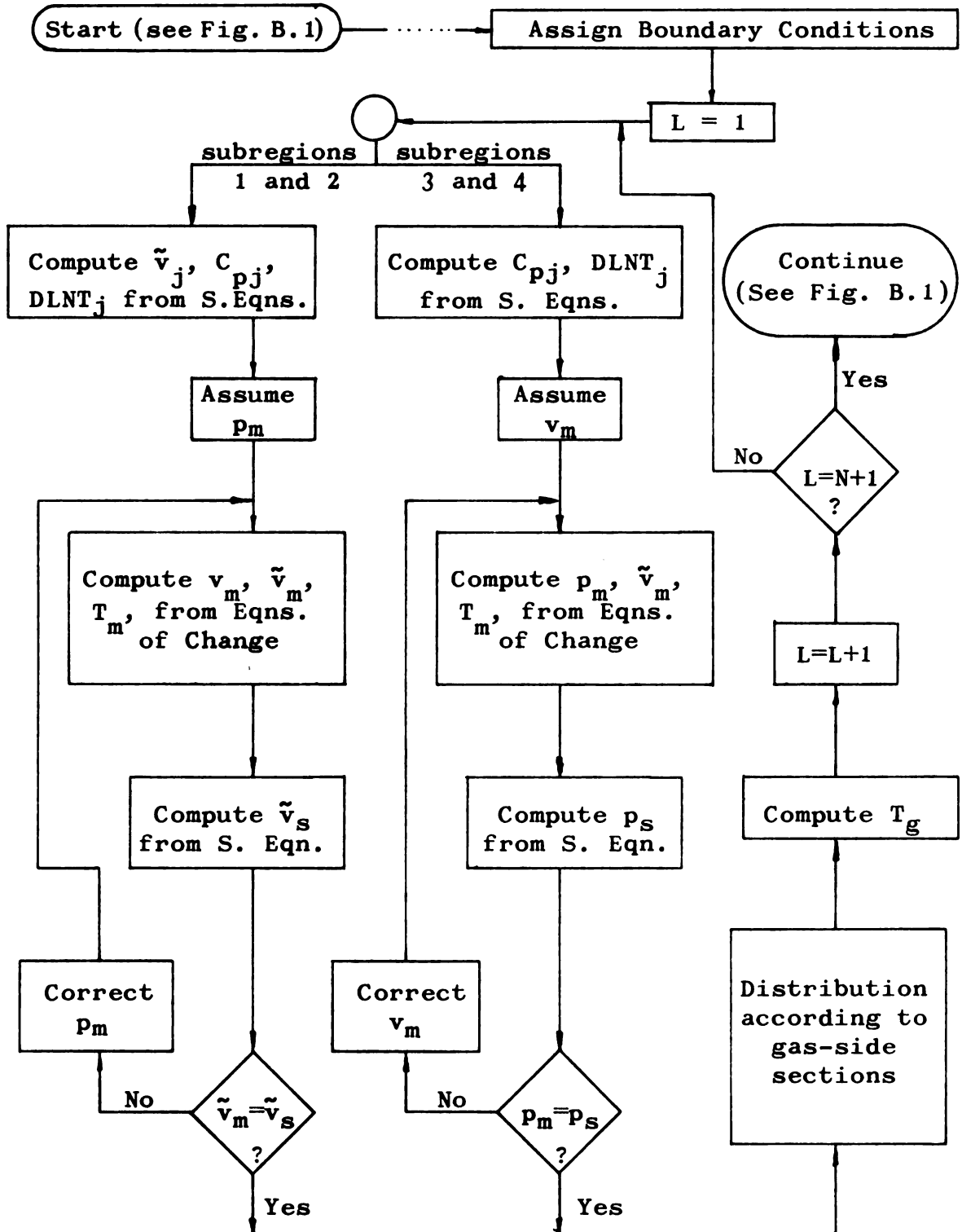


Figure B.2: A Flowchart of the Profiles Computation

APPENDIX C

§ C.1 Subroutine SCG - The Specific Heat of the Gas

```

C      SUBROUTINE SCG
C      EQUATION 3-7
C      =====
      DIMENSION A(12),R(12)
      COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),      TR,PR,VR,CPR,DLNT,
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
      DATA A/28.E-6,36.E-6,40.E-6,40.E-6,36.E-6,36.E-6,28.E-6,24.E-6,
1 20.E-6,16.E-6,12.E-6,12.E-6/
      DATA B/.248,.246,.244,.244,.248,.248,.260,.261,.275,.284,.294,
1 .294/
      TGF=(TGX-273.15)*1.8+32.
      JR=TGF/250.+1.
      IF (JR-12) 25,25,23
23 JR=12
25 CGR=A(JR)*TGF+R(JR)
      CGOT=4.1840*CGR
      RETURN
      END

```

§ C.2 Subroutines SV1 and CP1 - State Equations for Subregion 1

```

C      SUBROUTINE SV1
C      EQUATION 3-5A
C      =====
      COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),      TR,PR,VR,CPR,DLNT,
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT

```



```

T2=TR*TR
T3=T2*TR
T6=T3*T3
T18=T6*T6*T6
YF=1.-SA(1)*T2-SA(2)/T6
FAA=SORT(SA(3)*YF*YF-2.*SA(4)*TR+2.*SA(5)*PR)
ZF=YF+FAA
FZR5=CA(12)*SA(5)/(ZF*(5./17.))
FA19=SA(7)+T18*TR
FA11=SA(8)+T6*T3*T2
FSP4=(SA(10)+PR)*(SA(10)+PR)
FSP4=-3./(FSP4+FSP4)+SA(11)
FST10=(SA(6)-TR)*(SA(6)-TR)
FST10=CA(16)*FST10+FST10*FST10*FST10
VR=FZR5+(CA(13)+CA(14)*TR+CA(15)*T2+FST10
1 +CA(17)/FA10)-(CA(18)+2.*CA(19)*PR+3.*CA(20)*PR*PR)/FA11
2 -CA(21)*T18*(SA(9)+T2)*FSP4
3 (SA(12)-TR)*PR*PR+4.*CA(23)/(T18*T2)*PR*PR*PR
RETURN
END

```

```

C
C
SUBROUTINE CPI
EQUATIONS 3-5R AND 3-5C
=====
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
T2=TR*TR
T3=T2*TR
T6=T3*T3
T18=T6*T6*T6

```

```

P1=PR*PP*PP
YF=1.-SA(1)*T2-SA(2)/T6
YFP=-2.*SA(1)*TR+6.*SA(2)/(T6*TR)
YPP=-2.*SA(1)-42.*SA(2)/(T6*T2)
FAA=SQRT(SA(3)*YF*YF-2.*SA(4)*TR+2.*SA(5)*PR)
ZF=YF+FAA
ZFP=YFP+(SA(3)*YF*YFP-SA(4))/FAA
ZPP=YFP+SA(3)/FAA*(YFP*YFP+YF*YFP)-(SA(3)*YF*YFP-SA(4))*
1 (SA(3)*YF*YFP-SA(4))/(FAA*FAA*FAA)
Z017=ZF*(1./17.)
ZR5=1./(Z017*Z017*Z017*Z017*Z017*Z017)
Z12=Z017*Z017/(ZR5*ZR5)
FZR22=Z017*Z017/(Z12*Z12)*ZFP
FA19=SA(7)+T18*TR
FA192=FA19*FA19
FA11=SA(8)+T6*T3*T2
FA112=FA11*FA11
FSP=SA(10)+PR
FSP3=FSP*FSP*FSP
FST=SA(6)-TR
FST8=FST*FST*FST*FST*FST
FST8=FST8*FST8
SUM=2.*CA(4)
PRO=1.
DO 35 J=1.7
AJ=J
PRO=PRO*TR
35 SUM=SUM+(AJ+1.)*(AJ+2.)*CA(J+4)*PRO
SUMA=SUM
SUMB=CA(12)*((12./29.*ZF-YF)*(ZR5*ZPP-5./17.*FZR22*ZFP
1 )+(24./29.*ZFP-2.*YFP)*ZR5*ZFP+(17./29.*ZPP-17./12.*
2 YPP)*Z12)
SUMC=PR*(2.*CA(15)+90.*CA(16)*FST8+722.*CA(17)*T18*T18/(FA192*FA19
1 )-342.*CA(17)*T18/TR/FA192)-(242.*T18*T2/(FA112*FA11))-110.*
2 T6*T3/FA112 )*(CA(18)*PR+CA(19)*PR*PR+CA(20)*P3) -CA(21)*T18

```

```

3 /T2*(1306.*SA(9)+190.*T2)*(1./FSP3      +SA(11)*PR)
4 +420.*CA(23)*T2/T18/T6*PR*P3
CPR=CA(1)-TR*(SUMA+SUMB+SUMC)
SUM1=-5./17.*CA(12)*SA(5)*FZR22      +(CA(14)+2.*CA(15))*TR
1 -10.*CA(16)*FST*FSTR      -19.*CA(17)*T18/FA192      )+11.*T6*T2*T2
2 /FA112 *(CA(18)+2.*CA(19)*PR+3.*CA(20)*PR*PR)
SUM2=-CA(21)*(-3./(FSP*FSP3)      +SA(11))*2.*T18*(9.*(SA(9)+T2)
1 /TR+TR)-3.*CA(22)*PR*PR-80.*CA(23)/(T18*T3)*P3
DLNT=TR/VR*(SUM1+SUM2)
RETURN
END

```

§ C.3 Subroutines SV2 and CP2 - State Equations for Subregion 2

```

C      SUBROUTINE SV2
C      EQUATION 3-5A
C      =====
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),      TR,PR,VR,CPR,DLNT,
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
XF=EXP(SB(1))*(1.-TR)
BEL=VC(1)+VC(2)*TR+VC(3)*TR*TR
BLP=VC(2)+2.*VC(3)*TR
X4=XF*XF*XF*XF
X12=X4*X4*X4
X14=X12*XF*XF
X18=X14*X4
X24=X12*X12
FP3=PR*PR*PR
FPS=FP3*FP3
FP6=1./FPS
FP08=(PR/BEL)*(PR/REL)
FP08=FP08*FP08*FP08*FP08*FP08*FP08

```

```

QF1=FP6*PR*PR+SB(2)*X14
QF2=FP6*PR+SB(3)*X18*XF
QF3=FP6+SB(4)*X18*X18+SA(5)*X14*X12*XF
SUM1=VC(4)*TR/PR-(CB(7)*X12*XF+CB(8)*X4/XF)-2.*PR*(CB(9)*X18+
1 CB(10)*XF*XF+CB(11)*XF)-3.*FP3/PR*(CB(12)*X18+CB(13)*X14/X4)
2 -4.*FP3*(CB(14)*X24*XF+CB(15)*X14)-5.*FP3*PR*(CB(16)*X18*X14+
3 CB(17)*X14*X14+CB(18)*X24)
SUM2=-4.*FP6*PR*(CB(19)*X12+CB(20)*X12/XF)/QF1/QF1-5.*FP6*(CB(21)
1 *X24+CB(22)*X18)/QF2**2-6.*FP6/PR*(CB(23)*X24+CB(24)*X14)/QF3/QF3
SUM=CB(25)
PROD=1.
DO 36 J=1.6
PROD=PROD*XF
36 SUM=SUM+CB(J+25)*PROD
SUM3=SUM
VR=SUM1+SUM2+11.*FPOA*SUM3
RETURN
END

```

```

C
C
SUBROUTINE CP2
EQUATIONS 3-5R AND 3-5C
=====
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
XF=EXP(SB(1)*(1.-TR))
REL=VC(1)+VC(2)*TR+VC(3)*TR*TR
BLP=VC(2)+2.*VC(3)*TR
X2=XF*XF
X4=X2*X2
X12=X4*X4*X4

```

```

X14=X12*X2
X18=X14*X4
X24=X12*X12
FP3=PR*PR*PR
FPS=FP3*FP3
FP6=1./FPS
FPOB=(PR/BEL)*(PR/BFL)
FPOB=FPOB*FPOB*FPOB*FPOB*FPOB*FPOB
SB12=SB(1)*SB(1)
FB2=2.*CB(10)*X2
*FB3=3.*CB(8)*XF*X2
FA10=10.*CB(13)*X12/X2
FB11=CB(20)*X12/XF
FA12=CB(19)*X12
FB13=13.*CB(7)*X12*XF
FB19=SB(3)*X18*XF
FB25=25.*CB(14)*X24*XF
FB27=SB(5)*X24*XF*X2
FA28=28.*CB(17)*X14*X14
FA32=32.*CB(16)*X18*X14
FA54=SB(4)*X18*X18*X18
QF1=FP6*PR*PR+SB(2)*X14
QF2=FP6*PR+FB19
QF3=FP6+FB54+FB27
Q13=QF1*QF1*QF1
Q23=QF2*QF2*QF2
Q33=QF3*QF3*QF3
SUM1=VC(4)/PR+SB(1)*(FB13+FB3+2.*PR*(18.*CB(9)*X18+FB2+CB(11)*XF)
1 +3.*PR*PR*(18.*CB(12)*X18+FB10)+4.*FP3*(FB25+14.*CB(15)*X14)
2 +5.*FP3*PR*(FA32+FA28+24.*CB(18)*X24))
SUM2=FP6*SB(1)*(4.*PR*(QF1*(12.*FB12+11.*FA11)-28.*SB(2)*X14*
1 (FB12+FB11))/Q13 +5.*(QF2*(24.*CB(21)*X24+18.*CB(22)*X18)
2 -38.*FB19*(CB(21)*X24+CB(22)*X18))/Q23 +6./PR*(OF3*(24.*CB(23)
3 *X24+14.*CB(24)*X14)-54.*(2.*FB54+FB27)*(CB(23)*X24+CB(24)*X14))
4 /Q33)

```

```

SUM=CB(25)
PROD=1.
DO 36 J=1,6
  PROD=PROD*XF
36 SUM=SUM+CB(J+25)*PROD
SUM3=SUM
SUM=0.0
PROD=1.
DO 37 J=1,6
  AJ=J
  PROD=PROD*XF
37 SUM=SUM+AJ*CB(J+25)*PROD
SUM4=SUM
DLNT=TR/VR*(SUM1+SUM2+11.*FPOB      *(-10.*BLP/BEL*SUM3-SB(1))*
1 SUM4))
F60=FP3*PR*(FR12+FR11)
F61=-SB(1)*FP3*PR*(12.*FR12+11.*FB11)
F62=SB12      *FP3*PR*(14.*FR12+121.*FR11)
F70=FPS/PR*(CB(21)*X24+CB(22)*X18)
F71=-SB(1)*FPS/PR*(24.*CB(21)*X24+18.*CB(22)*X18)
F72=SB12      *FPS/PR*(576.*CB(21)*X24+324.*CB(22)*X18)
F80=FPS*(CB(23)*X24+CB(24)*X14)
FR1=-SB(1)*FPS*(24.*CB(23)*X24+14.*CB(24)*X14)
FR2=SB12      *FPS*(576.*CB(23)*X24+196.*CB(24)*X14)
G60=1.+SB(2)*FP3*PR*X14
G61=-14.*SB(1)*(G60-1.)
G62=-14.*SB(1)*G61
G70=1.+FPS/PR*FR19
371=-19.*SB(1)*(G70-1.)
G72=-19.*SB(1)*G71
GA0=1.+FPS*(FB54+FB27)
GA1=-27.*SB(1)*FPS*(2.*FB54+FB27)
GB2=729.*SB12      *FPS*(4.*FB54+FB27)
G602=G60*G60
G702=G70*G70

```

```

GA02=GA0*GA0
G612=G61*G61
G712=G71*G71
GA12=GA1*GA1
SUMA=-CB(1)/TR+2.*CB(4)      +6.*CB(5)*TR
1  +3.*FB3+PR*(324.*CB(9)*X1A+2.*FR2+CB(11)*XF)+FP3/PR*(324.*CB(12)
2  *X1B+10.*FB10)+FP3*(25.*FB25+196.*CB(15)*X14)+FP3*PR*(32.*FB32
3  +28.*FB2B+576.*CB(1A)*X24))
SUMB=-(F62/G60-(2.*F61*G61+F60*G62)/G602+2.*F60*G612/(G602*G60))
1  -(F72/G70-(2.*F71*G71+F70*G72)/G702+2.*F70*G712/(G702*G70))
2  -(F82/G80-(2.*F81*G81+F80*G82)/G802+2.*F80*G812/(G802*G80))
SUM=0.0
PROD=1.
DO 3A J=1.6
AJ=J
PROD=PROD*XF
3A SUM=SUM+AJ*AJ*CB(J+25)*PROD
SUM5=SUM
SUMC=FPOB*PR/BEL*(SUM3*(110.*BLP*BLP/BEL-20.*VC(3))+20.*SB(1))*BLP
1  *SUM4+SB12      *BEL*SUM5)
CPR=-TR*(SUMA+SUMB+SUMC)
RETURN
END

```

§ C.4 Subroutines SP3 and CP3 - State Equations for Subregion 3

```

SUBROUTINE SP3
C  EQUATION 3-5D
C  =====
COMMON CA(23)*SA(13)*CB(31)*SB(5)*CO(13)*C1(7)*C2(8)*C3(10)*
1  C40*C41*C5*C6(5)*C7(9)*D(13)*VC(16)      TR*PR*VR*CPR*DLNT.
2  PR3*DVT*DV2*DT2*CPR3*TX*CGOT

```

```

T3=TP*TP*TR
T12=T3*T3*T3*T3
T24=T12*T12
VM2=1./(VR*VR)
V6=1./(VM2*VM2*VM2)
TM1=TR-1.
SUM=0.0
PRO=1./VR
DO 39 J=1.5
AJ=-J
PRO=PRO/VR
39 SUM=SUM+AJ*C1(J+1)*PRO
SUM5 =SUM
SUM=0.0
PRO=1./VR
DO 41 J=1.6
AJ=-J
PRO=PRO/VR
41 SUM=SUM+AJ*C2(J+1)*PRO
SUM6 =SUM
SUM=0.0
PRO=1./VR
DO 43 J=1.8
AJ=-J
PRO=PRO/VR
43 SUM=SUM+AJ*C3(J+1)*PRO
SUM7 =SUM
SUM=0.0
PRO=1./TR
DO 55 J=1.5
PRO=PRO/TR
55 SUM=SUM+C6(J)*PRO
SUM13=SUM
SUM=0.0
PRO=1./VR

```

```

DO 57 J=1.10
AJ=-J
PRO=PRO/VR
57 SUM=SUM+AJ*CO(J+2)*PRO
SUM14=SUM
PR=- (CO(2)+SUM14+CO(13)/VR)-(C1(1)+SUM5+C1(7)/VR)*TM1
1 -(C2(1)+SUM6+C2(8)/VR)*TM1*TM1-(C3(1)+SUM7+C3(10)/VR)*TM1*TM1
2 +5.*C41/V6*TR/T24*TM1 -6.*V6/VR*SUM13
PR3=PR
RETURN
END

```

```

C
C
SUBROUTINE CP3
EQUATIONS 3-5F AND 3-5F
=====
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
T3=TR*TR*TR
T12=T3*T3*T3*T3
T24=T12*T12
VM2=1./(VR*VR)
V6=1./(VM2*VM2*VM2)
TM1=TR-1.
PRO=1.
DO 31 J=1.6
PRO=PRO/VR
31 SUM=SUM+C2(J+1)*PRO
SUM1 =SUM
SUM=0.0
PRO=1.

```

```

DO 33 J=1.8
PRO=PRO/VR
33 SUM=SUM+C3(J+1)*PRO
SUM2 =SUM
SUM=0.0
PRO=1.0/T3
DO 35 J=1.5
AJ=J
PRO=PRO/TR
35 SUM=SUM+(AJ+1.0)*(AJ+2.0)*C6(J)*PRO
SUM3 =SUM
SUM=2.0*C7(2)
PRO=1.0
DO 37 J=1.7
AJ=J
PRO=PRO*TM1
37 SUM=SUM+(AJ+1.0)*(AJ+2.0)*C7(J+2)*PRO
SUM4 =SUM
SUM=0.0
PRO=1.0/VR
DO 39 J=1.5
AJ=-J
PRO=PRO/VR
39 SUM=SUM+AJ*C1(J+1)*PRO
SUM5 =SUM
SUM=0.0
PRO=1.0/VR
DO 41 J=1.6
AJ=-J
PRO=PRO/VR
41 SUM=SUM+AJ*C2(J+1)*PRO
SUM6 =SUM
SUM=0.0
PRO=1.0/VR
DO 43 J=1.8

```

```

AJ=-J
PRO=PRO/VR
43 SUM=SUM+AJ*C3(J+1)*PRO
SUM7 =SUM
SUM=0.0
PRO=1.0/(TR*TR)
DO 45 J=1.5
AJ=J+1
PRO=PRO/TR
45 SUM=SUM+AJ*C6(J)*PRO
SUM8 =SUM
SUM=0.0
PRO=VM2
DO 47 J=1.10
AJ=J
PRO=PRO/VR
47 SUM=SUM+AJ*(AJ+1.)*C0(J+2)*PRO
SUM9 =SUM
SUM=0.0
PRO=VM2
DO 49 J=1.5
AJ=J
PRO=PRO/VR
49 SUM=SUM+AJ*(AJ+1.)*C1(J+1)*PRO
SUM10=SUM
SUM=0.0
PRO=VM2
DO 51 J=1.6
AJ=J
PRO=PRO/VR
51 SUM=SUM+AJ*(AJ+1.)*C2(J+1)*PRO
SUM11=SUM
SUM=0.0
PRO=VM2
DO 53 J=1.8

```

```

AJ=J
PRO=PRO/VR
53 SUM=SUM+AJ*(AJ+1.)*C3(J+1)*PRO
SUM12=SUM
SUM=0.0
PRO=1./TR
DO 55 J=1.5
PRO=PRO/TR
55 SUM=SUM+C6(J)*PRO
SUM13=SUM
DT2=2.*(C2(1)*VR+SUM1+C2(8)*ALOG(VR))+6.*(C3(1)*VR+SUM2+C3(10)*
1 ALOG(VR))*TM1 +(C40+C41*VR/V6)*(506.*TR-552.)/(T24*TR)+
2 C5/TR+V6*SUM3+SUM4
DVT=C1(1)+SUM5+C1(7)/VR+2.*(C2(1)+SUM6+C2(8)/VR)*TM1
1 +3.*(C3(1)+SUM7+C3(10)/VR)*(TM1*TM1) -5.*C41/V6*(23.-22.*TR)
2 /T24-6.*V6/VR*SUM8
DV2=SUM9-CO(13)*VM2+(SUM10-C1(7)*VM2)*TM1 +(SUM11-C2(8)*VM2)
1 *(TM1*TM1) +(SUM12-C3(10)*VM2)*(TM1*TM1*TM1)+30.*C41/(V6*VR)
2 *TM1 /T24*TR+30.*V6*VM2*SUM13
CPR3=TR*(-DT2+DVT*DVT/DV2)
CPR=CPR3
DLNT=-TR/VR*DVT/DV2
RETURN
END

```

§ C.5 Subroutines SP4 and CP4 - State Equations for Subregion 4

```

C SUBROUTINE SP4
C EQUATION 3-5D
C *****
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16),
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
TR,PR,VR,CPR,DLNT,

```

```

YF=(1.-TR)/(1.-VC(11))
Y2=YF#YF
Y4=Y2#Y2
Y32=Y4#Y4#Y4#Y4
Y32=Y32#Y32
SUM1=0.0
SUM2=0.0
PROD=1./VR
DO 39 J=1.4
AJ=J
PROD=PROD/VR
SUM1=SUM1+AJ#D(J+1)*PROD
SUM2=SUM2+AJ#D(J+6)*PROD
PR=PR3+Y2#YF#SUM1+Y4#SUM2-Y32*(D(12)+2.*D(13)*VR)
RETURN
END
39

```

```

C
C
SUBROUTINE CP4
EQUATIONS 3-5E AND 3-5F
=====
DIMENSION SUM(8),PROD(3)
COMMON CA(23),SA(13),CB(31),SB(5),CO(13),C1(7),C2(8),C3(10),
1 C40,C41,C5,C6(5),C7(9),D(13),VC(16), TR,PR,VR,CPR,DLNT,
2 PR3,DVT,DV2,DT2,CPR3,TGX,CGOT
YF=(1.-TR)/(1.-VC(11))
Y2=YF#YF
Y4=Y2#Y2
Y32=Y4#Y4#Y4#Y4
Y32=Y32#Y32
OMVC=1.-VC(11)
DO 31 K=3.6

```

```

31 SUM(K)=0.0
   SUM(1)=D(1)
   SUM(2)=D(6)
   PROD(1)=1.
   PROD(2)=1./VR
   PROD(3)=PROD(2)/VR
DO 35 J=1.4
  AJ=J
DO 33 K=1.3
  33 PROD(K)=PROD(K)/VR
     SUM(1)=SUM(1)+D(J+1)*PROD(1)
     SUM(2)=SUM(2)+D(J+6)*PROD(1)
     SUM(3)=SUM(3)+AJ*D(J+1)*PROD(2)
     SUM(4)=SUM(4)+AJ*D(J+6)*PROD(2)
     SUM(5)=SUM(5)+AJ*(AJ+1.)*D(J+1)*PROD(3)
     SUM(6)=SUM(6)+AJ*(AJ+1.)*D(J+6)*PROD(3)
     SUM(7)=D(11)+D(12)*VR+D(13)*VR*VR
     SUM(8)=D(12)+D(13)*VR
     DT24=1./(OMVC*OMVC)  *(6.*YF*SUM(1)+12.*Y2*SUM(2)+992.*Y32/
1  Y2*SUM(7))
     DVT4=1./OMVC        *(3.*Y2*SUM(3)+4.*Y2*YF*SUM(4)-32.*Y32/
1  YF*SUM(8))
     DV24=Y2*YF*SUM(5)+Y4*SUM(6)+Y32*D(13)
     CPR=TR*(-DT2-DT24+(DVT+DVT4)*(DVT+DVT4)/(DV2+DV24))
     DLNT=-TR/VR*(DVT+DVT4)/(DV2+DV24)
     RETURN
     END

```

APPENDIX D

APPENDIX D

§D.1 The Equation of Motion

The term $-\partial\tau_{zz}/\partial z = \mu(\partial^2 v_z/\partial z^2)$, appearing in Equation 1-9, is considered negligible. An estimate of the maximum possible value of this term is presented to justify the assumption.

From Table 6.1, the average variation of v along the tube is $(\partial v/\partial z)_{\text{ave.}} = (437 - 45)/(90 \cdot 60) = 0.0727 \text{ (cm/sec)/cm}$. Using the approximation

$$\frac{\partial^2 v_z}{\partial z^2} \approx \frac{(\partial v/\partial z)|_{z+\Delta z} - (\partial v/\partial z)|_z}{\Delta z}$$

and assuming $(\partial v/\partial z)|_{z+\Delta z} = (\partial v/\partial z)_{\text{ave.}}$, $(\partial v/\partial z)|_z = 0$, we have $\partial^2 v_z/\partial z^2 = .0727/60 = 1.21 \cdot 10^{-3} \text{ (cm} \cdot \text{sec)}^{-1}$.

The maximum value of the viscosity encountered is:³¹
 $\mu = 8.90 \cdot 10^{-4} \text{ g/(cm)(sec)}$. Thus, $-\partial\tau_{zz}/\partial z = 1.08 \cdot 10^{-6} \text{ (dyn/cm}^3\text{)}$. In comparison, the friction force per unit volume is (from Table 6.1, at $z = 600 \text{ cm}$) 12.8 dyn/cm^3 , and the gravity force per unit volume is 612 dyn/cm^3 .

§D.2 Heat Conduction in the Fluid

Heat conduction in the z -direction $k(\partial^2 T/\partial z^2)$ is considered negligible in the fluid energy equation. We shall

use the approximation

$$\frac{\partial^2 T}{\partial z^2} \approx \frac{(\partial T/\partial z) \Big|_{z+\Delta z} - (\partial T/\partial z) \Big|_z}{\Delta z}$$

The average variation of T along z is (from Table 6.1),

$$(\partial T/\partial z)_{\text{ave.}} = (560 - 320)/(60 \cdot 90) = .0444 \text{ } ^\circ\text{K/cm.}$$

Taking $(\partial T/\partial z) \Big|_z = 0$ and $(\partial T/\partial z) \Big|_{z+\Delta z} = (\partial T/\partial z)_{\text{ave.}}$, we have $\partial^2 T/\partial z^2 = .0444/60 = 7.41 \cdot 10^{-4} \text{ } ^\circ\text{K/cm}^2$. The maximum value of the thermal conductivity encountered is:³¹

$$k = 0.318 \text{ Btu}/(\text{hr.})(\text{ft.})(^\circ\text{F}) = 0.0055 \text{ Joule}/(\text{sec.})(\text{cm})(^\circ\text{K})$$

$$\text{Hence } k(\partial^2 T/\partial z^2) = 4.07 \cdot 10^{-6} \text{ Joule}/(\text{sec.})(\text{cm}^3).$$

In comparison, heat transfer by radiation is:

$$\begin{aligned} \frac{D_{\text{eq}}}{A_{\text{cx}}} \cdot \sigma \cdot [(T_g/1000)^4 - (T/1000)^4] &= 0.34 \cdot 5.67 \cdot (1.2^4 - .83^4) \\ &= 3.08 \text{ Joule}/(\text{sec.})(\text{cm}^3) \end{aligned}$$

§ D.3 Gas-to-fluid Convective Heat Transfer

The overall resistance to heat transfer from the gas to the fluid is composed of 3 resistances in series: i) The gas-to-metal film resistance h_{gm} ; ii) The metal wall resistance; iii) The metal-to-fluid resistance h_{mf} .

Experimental data^{15a} for h_{gm} ranges from 2 to 20 Btu/(hr.)(ft.²)(°F). The minimum resistance is $1/h_{gm} = 0.05$.

The thermal conductivity of the tube metal (steel) is:^{30a}
 $k = 20 \text{ Btu}/(\text{hr.})(\text{ft.}^2)(^\circ\text{F})$. Assuming a wall thickness $\Delta x = 1/4''$
 we have the wall resistance $\Delta x/k = 1.04 \cdot 10^{-3}$.

The fluid film resistance may be calculated from the Dittus-Boelter correlation (Equation 1-5), the Sieder-Tate correlation (Equation 1-6), and from the modified correlation for high temperature gradients (Equation 1-7). The results are given in Table D.1. The data³¹ corresponds to 3500 psi. Values of h_{mf} calculated by Equation 1-7 are invariably higher than those obtained from Equation 1-5, and therefore were not included in the table.

The results show a maximum resistance of: $1/(h_{mf})_{\min.} = 1/278 = 3.6 \cdot 10^{-3}$. We see that the maximum resistance of the fluid film is 7% of the minimum resistance of the gas film, and that the tube wall resistance is smaller yet. It is, therefore, permissible to ignore the variations of the fluid and of the wall resistances, and to lump them together with the gas film resistance as the overall convective coefficient U .

Table D.1: Calculated Value of Metal-to-fluid Convective Heat Transfer Coefficient

T (°F)	600	700	800	900	1000	1100	1200
$\mu \cdot 10^4$ (poise)	8.90	4.84	3.35	3.21	3.35	3.49	3.69
k [Btu/(hr)(ft)(°F)]	.318	.155	.088	.063	.060	.062	.064
C_p [Btu/(lb)(°F)]	1.326	4.19	1.563	.941	.775	.701	.666
Pr Number	0.92	1.35	1.43	1.23	1.06	0.98	0.93
$Re \cdot 10^{-4}$	9.0	16.53	23.9	24.9	23.9	22.9	21.7
1) h_{mf} [Btu/(hr)(ft ²)(°F)]	775	718	560	390	338	327	318
2) h_{mf} [Btu/(hr)(ft ²)(°F)]	618	1335	486	325	290	280	278

1) Calculated by Equation 1-5

2) Calculated by Equation 1-6

§D.4 The Gas Energy Equation

The kinetic and potential energy terms were neglected in the gas energy balances. Furthermore, an assumption was made that the energy dynamics may be modeled as a sequence of steady states.

To demonstrate the validity of these assumptions, consider a segment of the gas column, corresponding to a tube of length Δz . The volume of the gas is ΔV . The energy balance over ΔV may be written in the following general form:

$$\Delta E / \Delta t = Q_s - Q_{rc} + \Delta (W_g C_g T_g) + \Delta E_k + \Delta E_p \quad \text{..... (D-1)}$$

where,

$$\Delta E / \Delta t = (\rho_g \Delta V) C_g \Delta T_g / \Delta t = \text{the rate of energy accumulation within } \Delta V$$

$$Q_s = q_s \cdot \Delta z = \text{the rate of heat generation within } \Delta V$$

$$Q_{rc} = q_{rc} \cdot \Delta z = \text{the rate of heat transfer out of } \Delta V$$

$$\Delta (W_g C_g T_g) = W_g C_g \cdot (T_{g,in} - T_{g,out}) = \text{the net heat carried by the gas stream}$$

$$\Delta E_k = \frac{1}{2} W_g \cdot (v_{g,in}^2 - v_{g,out}^2) = \text{the kinetic energy change}$$

$$\Delta E_p = W_g \cdot g \cdot \Delta z = \text{the potential energy change}$$

Using the values:

$$W_g = 1800 \text{ g/sec} \quad \Delta z = 60 \text{ cm} \quad \rho_g = .001 \text{ g/cm}^3$$

$$\Delta V = 2.16 \cdot 10^5 \text{ cm}^3 \quad v_g = 500 \text{ cm/sec}$$

$$q_s = 100 \text{ Joule/(cm)(sec)} \quad q_{rc} = 50 \text{ Joule/(cm)(sec)}$$

$$C_g = 1.5 \text{ Joule/(g)(}^\circ\text{K)}$$

We have:

$$\Delta (W_g C_g T_g) = -5040 \text{ Joule/sec, assuming a variation of } -2^\circ\text{K over } \Delta z$$

$$Q_s = 6000 \text{ Joule/sec}$$

$$Q_{rc} = 3000 \text{ Joule/sec}$$

$$\Delta E_k = 11.25 \text{ Joule/sec, assuming a 50\% velocity change over } \Delta z$$

$$\Delta E_p = 10.6 \text{ Joule/sec}$$

$$\Delta E / \Delta t = 302 \Delta T_g / \Delta t$$

The results indicate that kinetic and potential energy effects are, indeed, negligible.

The rate of change of T_g , as obtained from the energy balance is $\Delta T_g / \Delta t = -6.7 \text{ }^\circ\text{K/sec}$. Such a rapid change would result in a new steady state within a short period of time. This justifies the modeling of the gas dynamic as a sequence of steady states.

NOMENCLATURE

<u>Text</u>		<u>FORTRAN</u>	
a	-		a constant parameter
A	-		area
A _{cx}	-	ACX	- cross sectional area of a superheater tube
b	-		a constant parameter
C _g	-	CGOT	- specific heat of the gas
C _p	-		specific heat at constant pressure
C _{pr}	-	CPR	- reduced (dimensionless) specific heat
d, ∂	-		prefix, indicating differentiation
D	-		diameter
D _c	-	DIC	- dispersion coefficient (See § 3.2)
D _{eq}	-	DEQ	- equivalent diameter (See § A.3)
D _h	-	DH	- inside diameter of the superheater tube
D _v	-	DV	- inside diameter of the waterwall tube
E	-		energy
f	-		function
f	-	F	- friction factor
F	-		function; friction force
f _{mx}	-	FMX	- maximum value of the heat generation function (See § A.5)

<u>Text</u>	<u>FORTRAN</u>	
$g, \vec{g}$ -	GAC	- gravitational acceleration
h -		convective heat transfer coefficient (film coefficient)
H -		enthalpy
h_{gm} -		gas to tube wall film coefficient
h_{mf} -		tube wall to fluid film coefficient
i -		distance index number in the Finite Differences grid
j -		time index number in the Finite Differences grid
j -		location in the Finite Differences grid (See Figure 4.2)
k -		thermal conductivity
l_s -		length of gas column containing one superheat pass
L_s -		length of the Superheater Section
L_t -		total length (height) of the boiler
n -		number of tubes
p -	P	- pressure
p_r -	PR	- reduced (dimensionless) pressure
Pr -		Prandtl Number
p_s -	PST	- pressure as computed from the State Equation (See § 4.3)
$q, \vec{q}$ -		heat flux
Q_f -		rate of heat absorption by the fluid in the whole boiler

<u>Text</u>		<u>FORTTRAN</u>	
q_{rc}	-	QRAC	- rate of heat transfer per unit length of the tube
Q_{rc}	-		rate of heat transfer by convection and radiation
q_s	-		rate of heat generation per unit length of the tube
Q_s	-		rate of heat generation
q_{tr}	-		rate of heat transfer per unit volume of the tube
Re	-		Reynolds Number
s	-		entropy
t	-		time
T	-	T	- temperature
Δt	-	TIM	- time increment
T_{ic}	-		computed fluid temperature at boiler inlet (See § 4.4-4)
T_{in}	-		fluid temperature at the boiler inlet (See § 4.4-4)
T_r	-	TR	- reduced (dimensionless) temperature
U	-	U, UB	- overall convective heat transfer coefficient
v	-	VE	- velocity
$\tilde{v}$	-	VOL	- specific volume
v_r	-	VR	- reduced (dimensionless) specific volume
V	-		volume
$\tilde{v}_s$	-	VST	- specific volume as computed from the State Equation (See § 4.3)

<u>Text</u>		<u>FORTTRAN</u>	
w	-	W	- mass velocity of the fluid
W_g	-	WAG	- mass flow rate of the gas
x	-		distance coordinate
z	-		distance coordinate; distance along the fluid or gas path
Δz	-	EPS	- distance increment
z_{fu}	-	ZFU	- location of the boundary of the Lower Furnace (See § A.5)

Subscripts

a	-		axial
a	-	suffix A	- for superheat pass A
b	-		bulk
b	-	suffix B	- for superheat pass B
c	-		convective; computed
c	-	suffix C	- for superheat pass C
d	-	suffix D	- for superheat pass D
f	-		film; fluid; friction
g	-		gas
i	-		in; inlet; distance index number in the Finite Differences grid
j	-		time index number in the Finite Differences grid
j	-	suffix J	- location in the Finite Differences grid (See Figure 4.2)

<u>Text</u>		<u>FORTRAN</u>	
k	-		kinetic
m	-	suffix M	- location in the Finite Differences grid (See Figure 4.2)
o	-		outlet; out
p	-		potential
r	-		radiative; reduced
w	-		wall
x	-		distance coordinate
x	-	suffix X	- location in the Finite Differences grid (See Figure 4.2)
y	-		distance coordinate
y	-	suffix Y	- location in the Finite Differences grid (See Figure 4.2)
z	-		distance coordinate
z	-	suffix Z	- location in the Finite Differences grid (See Figure 4.2)

Greek

Δ, δ	-		increment; interval; difference
∇	-		Del operator
ϵ	-		emissivity
μ	-		viscosity
π	-	PAI	- a number 3.1416...
ρ	-	R0	- density
σ, σ'	-	SIG	- the Stefan-Boltzmann constant
τ	-		stress

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