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OPTIMUM ECONOMIC TUBE DIAMETER
FOR PUMPING HERSCHEL-BULKLEY
FLUIDS

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Edgardo Jose Garcia Caes

has been accepted towards fulfillment
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James F. Steffe
Major professor

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# OPTIMUM ECONOMIC TUBE DIAMETER
# FOR PUMPING HERSCHEL-BULKLEY
# FLUIDS

By

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IN MEMORY OF

Willie and Jaime

DEDICATED TO

My Family and Makito

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## LIST OF SYMBOLS

$a$ = annual fixed cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr, Equation (2.16)

$a'$ = annual fixed cost of the pump station expressed as a fraction of the initial installed cost of the pump station, 1/yr, Equation (2.16)

$B$ = empirical wall effect parameter in mixing length theory, Equation (3.28)

$b$ = annual maintenance cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr

$b'$ = annual maintenance cost of the pump station expressed as a fraction of the initial installed cost of the pump station, 1/yr

$C$ = total installed cost of the tube system including the cost of fittings, valves, installation, etc., \$/m, Equation (2.17)

$C_1$ = unknown cost of equipment of size $Q_2$, Equation (2.18) or (2.19)

$C_2$ = known cost of equipment of size $Q_2$

$C'$ = purchase cost of a new pipe, \$/m, Equation (2.14)

$C_D$ = empirical constant for the pump station cost, \$/$W^{s'}$

$C_D'$ = total direct cost of equipment of size $Q_2$

$C_e$ = cost of electrical energy, \$/W hr

$C_I$ = empirical constant for the pump station cost, \$

$C_I'$ = total indirect cost of equipment of size $Q_2$

$C_{op}$ = total annual operating cost per unit length of tube, \$/yr m, Equation (2.20)

$C_p$ = empirical constant for the tube system cost, $/m^{1+s}$

$C_{pi}$ = total annual cost of installed tube system per unit length of tube, \$/yr m, Equation (2.15) or (3.36)

$C_{ps}$ = total cost of installed pump station, \$, Equation (3.37)

$C_{pu}$ = total annual cost of installed pump station per unit length of tube, \$/yr m, Equation (3.38)

$C_T$ = total annual cost of a pumping system per unit length of tube \$/yr m, Equations (3.7), (3.39), or (3.40)

$C_{T_{min}}$ = minimum total cost of a pumping system per unit length of pipe, \$/yr m, Equation (3.39) or (3.40)

D = tube/pipe inside diameter, m

$D_{est}$ = best estimate of $D_{opt}$ to start numerical search, m

$D_{opt}$ = value of D at $C_{T_{min}}$

E = combined fractional efficiency of pump and motor

$E_f$ = energy loss due to friction, J/kg, Equation (2.10), (2.11), and (2.12)

F = ratio the total cost for fittings and installation of pipe and fittings to purchase cost of new pipe

f = Fanning friction factor, Equation (3.15)

$f_c$ = laminar-turbulent transition valve of f

$f_{est}$ = best estimate of f to start numerical search

g = acceleration due to gravity (9.8 $m/s^2$)

h = hours of operation per year

He = generalized Hedstrom number, Equation (3.19)

i = interest rate (fraction)

K = consistency coefficient, Pa $s^n$

$K_f$ = dimensionless fittings resistance coefficient

k = Prandtl's universal mixing length constant = 0.36

L = tube/pipe length, m

$L_f$ = equivalent length of pipe for fittings frictional loss

$\ell$ = modified Prandtl's mixing length

$\dot{M}$ = mass flow rate, kg/s

N = life-time of equipment, yr

n = flow behavior index, dimensionless

P = power, Watts, Equation (2.13)

p' = constant for purchase cost of pipe dependent on the pipe material, dimensionless

$p_1$ = pressure at point 1, Pa

$p_2$ = pressure at point 2, Pa

$Q_1$ = size of equipment with unknown cost

$Q_2$ = size of equipment with known cost

q = cost capacity factor

R = turbulance parameter, Equation (3.27)

r = radial coordinate

$R_c$ = laminar-turbulent transition valve of R

Re = generalized Reynolds number, Equation (3.18)

$Re_c$ = laminar-turbulent transition valve of Re, Equation (3.21)

$r_w$ = tube/pipe radius, m

s = exponent of tube system cost equation, dimensionless

s' = exponent of pump station cost equation, dimensionless

u = dimensionless velocity, $v/\bar{v}$

$u_o$ = dimensionless plug velocity, $v_o/\bar{v}$

v = axial velocity component, m/s

$v_o$ = plug velocity, m/s

$\bar{v}$ = mass average velocity, m/s

$\bar{v}_1$ = value of $\bar{v}$ at point 1, m/s

$\bar{v}_2$ = value of $\bar{v}$ at point 2, m/s

W = work per unit mass, J/kg, Equations (2.9) (3.33) or (3.35)

X = purchase cost of one inch diameter pipe per unit meter of pipe length, \$/m $in^{p'}$

x = a small positive number

$z_1$ = elevation at point 1, m

$z_2$ = elevation at point 2, m

z = axial coordinate

## Greek Letters

$\alpha_1$ = kinetic energy correction factor at point 1

$\alpha_2$ = kinetic energy correction factor at point 2

$\Gamma$ = $-du/d\xi$

$\Gamma_w$ = value of $\Gamma$ at the pipe wall ($r = r_w$)

$\dot{\gamma}$ = rate of shear $(-dv/dr)$, $s^{-1}$

$\dot{\gamma}_w$ = value of $\dot{\gamma}$ at the pipe wall

$\Delta p$ = pressure change between points 1 and 2 $(p_2 - p_1)$, Pa

$\Delta P_f$ = pressure drop due to friction, Pa

$\Delta z$ = elevation change between point 1 and 2, $(z_2 - z_1)$, m

$\zeta$ = dimensionless rate of shear, $\Gamma/\Gamma_w$

$\zeta_j$ = value of $\zeta$ calculated with $\lambda_j$ and $\xi_o \leq \xi_j \leq 1$ for $j = 1, 2, 3 \ldots$

$\eta$ = pastic viscoisty, Pa s

$\kappa$ = laminar stability parameter, Equation (3.20)

$\bar{\kappa}$ = maximum value of $\kappa$ at $\xi = \bar{\xi}$, 404

$\lambda$ = dimensionless mixing length, $\ell/r_w$

$\lambda_j$ = value of $\lambda$ calculate for $\xi_o \leq \xi_j \leq 1$ for j = 1, 2, 3 . . .

$\mu$ = Newtonian viscosity, Pa s

$\mu_a$ = apparent viscosity, Pa s, Equation (2.6)

$\xi$ = dimensionless radial coordinate, $r/r_w$

$\bar{\xi}$ = value of $\xi$ where $\kappa = \bar{\kappa}$

$\xi_j$ = value of $\xi$ for $\xi_o \leq \xi_j \leq 1$ j = 1, 2, 3 . . ., use in to evaluate the integral in Equation (3.31)

$\xi_o$ = dimensionless unsheared plug radius, $\tau_o / \tau_w$

$\xi_{oc}$ = laminar-turbulent transition valve of $\xi_o$

$\pi$ = pi (3.1415)

$\rho$ = fluid density, $kg/m^3$

$\sigma$ = parameter in the df/dD Equation for laminar flow, Equation (3.45)

$\tau_o$ = yield stress, Pa

$\tau_{rz}$ = shear stress, Pa

$\bar{\tau}_{rz}$ = time average momentum flux, Equation (3.23)

$\tau_{rz}^L$ = molecular flux, Equation (2.1)

$\tau_{rz}^T$ = turbulent flux or Reynolds stress, Equation (3.24)

$\tau_w$ = value of $\tau_{rz}$ at $r = r_w$, Equation (3.4)

$\phi$ = parameter in mixing length, Equation (3.26)

$\psi$ = laminar flow function, Equation (3.14)

$\psi_c$ = laminar-turbulent transition value of $\psi$

ABSTRACT

OPTIMUM ECONOMIC TUBE DIAMETER
FOR PUMPING HERSCHEL-BULKLEY
FLUIDS

By

Edgardo Jose Garcia Caes

The optimum tube diameter, for which the total cost of a pumping system is a minimum, has been derived for the case of Herschel-Bulkley fluids in both laminar and turbulent flow. The method accounts for the tube system cost as a function of diameter, as well as the pump station and operating costs as a function of the power requirements. The optimum diameter can be estimated given the rheological properties, density of the fluid, mass flow rate, and economic parameters. The elevation and pressure difference in the system are irrelevant when a linear relationship is used for the pump station cost. The friction loss in fittings can be ignored when the tube length is much greater than the tube diameter. The pump station cost has less influence than the operating cost in determining the optimum diameter. The use of apparent viscosity and Newtonian flow behavior for non-Newtonian fluids may lead to severe errors in pipe sizing.

# 1. INTRODUCTION

## 1.1 General Remarks

A problem associated with the design of fluid handling systems is the selection of tube or pipe size. The installed cost of a process piping system can vary between 7% and 60% of the total fixed investment (Wright, 1950). It is, therefore, important to choose the tube size that would result in the greatest economy while maintaining the designated operating conditions and performance requirements. Three criteria often control the selection of tube size; the pressure drop available, velocity allowable, and least annual cost. The first criterion is usually used when a given pressure drop must be absorbed by the tube. Limits in velocity may be encountered in the handling of slurries in which a minimum velocity must be maintained to keep the particles in suspension. Conversely, quality degradation of the product may restrict high velocities. The least annual cost applies when a given amount of fluid is to be pumped through a tube system. It is based on an economic balance of the capital and operating cost to give a tube size that will result in the least annual charge (Nolte, 1978; Kent, 1978).

In this study, techniques to estimate the optimum tube diameter based on the least annual cost are developed for tube systems transporting non-Newtonian fluids. The Herschel-Bulkley model was selected due to its generality and wide application in fluid foods, as well as other fluid materials (Holdsworth, 1971; Higgs and Norrington, 1971; Steffe et al., 1983; Boger and Tiu, 1974).

Non-Newtonian characteristics must be considered in the design of pumping systems when handling fluids of this type (Cheng, 1975; Johnson, 1982). Failure to do so may lead to under or over sizing, resulting in a system inefficient to operate or more costly to erect as suggested by Steffe (1983) and Nolte (1978).

## 1.2 Objectives

The specific objectives of this study are as follows.

Objective 1: Develop an equation to predict the total annual cost of a pumping system as a function of the tube diameter.

Objective 2: Develop an equation to estimate the optimum economic tube diameter for pumping systems handling Herschel-Bulkley fluids.

Objective 3: Demonstrate the design errors caused by using apparent viscosity and Newtonian flow behavior to design pumping systems handling non-Newtonian fluids.

# 2. LITERATURE REVIEW

## 2.1 Herschel-Bulkley (H-B) Model

The flow behavior of many fluid foods and other industrially important fluids may be described by the H-B model which can be written as (Herschel and Bulkley, 1926).

$$\tau_{rz} = \tau_o + K \dot{\gamma}^n \tag{2.1}$$

where

$\tau_{rz}$ = shear stress, Pa

$\tau_o$ = yield stress, Pa

$K$ = consistency coefficient, Pa $s^n$

$n$ = flow behavior index, dimensionless

$\dot{\gamma}$ = rate of shear (-dv/dr), $s^{-1}$

This model simplifies to other well-known models. The power law or Ostwald-de Waele model is written as

$$\tau_{rz} = K \dot{\gamma}^n \tag{2.2}$$

where

$\tau_o = 0$

A power law fluid is called pseudoplastic when $0 < n < 1$ and dilatant when $n > 1$. Equation (2.1) reduces to the Bingham plastic model when $n = 1$ and $\eta = K$ as

$$\tau_{rz} = \tau_o + \eta \dot{\gamma} \qquad (2.3)$$

where

$\eta$ = plastic viscosity, Pa s

Newtonian fluids are described by Equation (2.1) when $\tau_o = 0$, $n = 1$, and $\mu = K$ as

$$\tau_{rz} = \mu \dot{\gamma} \qquad (2.4)$$

where

$\mu$ = Newtonian viscosity, Pa s

The shear stress-shear rate relationships for the above models are shown graphically in Figure 1.

It is common practice to use an apparent viscosity ($\mu_a$) and assume Newtonian fluid behavior to estimate the frictional pressure losses for the flow of non-Newtonian fluids in tubes. Apparent viscosity is defined as

$$\tau_{rz} = \mu_a \dot{\gamma} \qquad (2.5)$$

Since equation (2.5) is used to describe H-B fluids, $\mu_a$ may be written in terms of the H-B parameters using Equation (2.1) and (2.5) as

$$\mu_a = \tau_o \dot{\gamma}^{-1} + K \dot{\gamma}^{n-1} \qquad (2.6)$$

From Equation (2.6), it is evident that $\mu_a$ is defined at a particular rate of shear. Therefore, the use of an apparent viscosity may lead to over- or under-estimation of the pressure losses and power

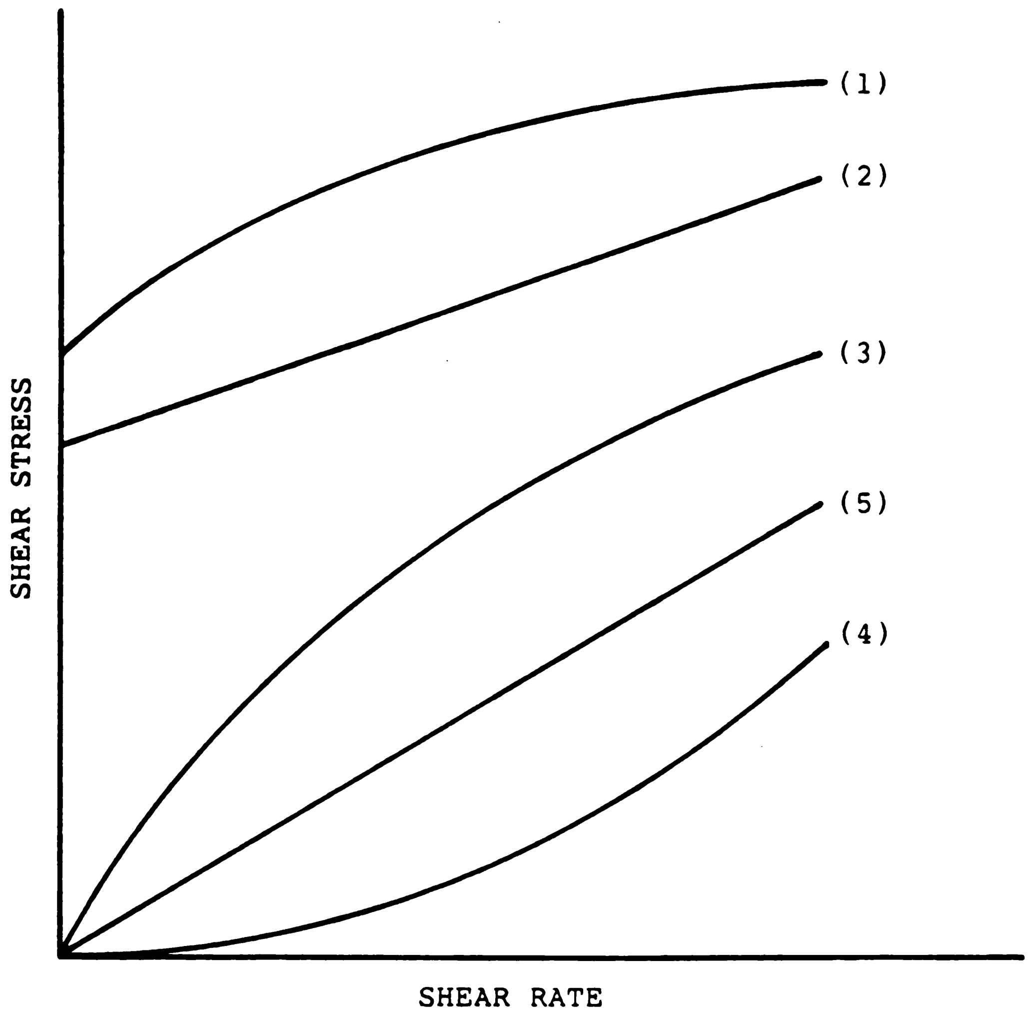


Figure 1. Shear stress - shear rate relationship for time-independent non-Newtonian and Newtonian fluids: (1) Herschel-Bulkley model, (2) Bingham plastic model, (3) Pseudoplastic model, (4) Dilatant model, (5) Newtonian model.

requirements depending the rate of shear at which the apparent viscosity is measured. This, in turn, may lead to improper sizing of pipe, pump, and motor (Steffe, 1983).

## 2.2 Optimization

The selection of a value for a given design variable to minimize the total cost of a project is possible whenever a change in this variable causes some costs to increase while other costs decreases (Skelland, 1967). For a pumping system, the total cost can be divided into three components: the tube system cost, the pump station cost, and the operating cost (Darby and Melson, 1982). The tube system cost primarily consist of the installed cost of tube, fittings, and values. This increases with increasing tube diameter (Skelland, 1967; Darby and Melson, 1982; Jelen, 1970). The pump station cost mainly consist of the installed cost of pump and motor while the operating cost parimarily consists of the cost of electrical power required to pump the fluid through the system. Both of these costs are directly proportional to the power requirements which decrease with increasing tube diameter since the pressure drop due to friction decreases with increasing tube diameter. Consequently, the pump station cost and operating cost decrease with increasing tube diameter (Skelland, 1967; Darby and Melson, 1982; Downs and Tait, 1953). This is shown graphically in Figure 2. Clearly, the optimum value for the diameter can be obtained when the sum of these costs is at a minimum.

Mathematically, the total cost $C_T$ can be expressed as function at the tube diameter (D) with the following algebraic equation (Skelland, 1967).

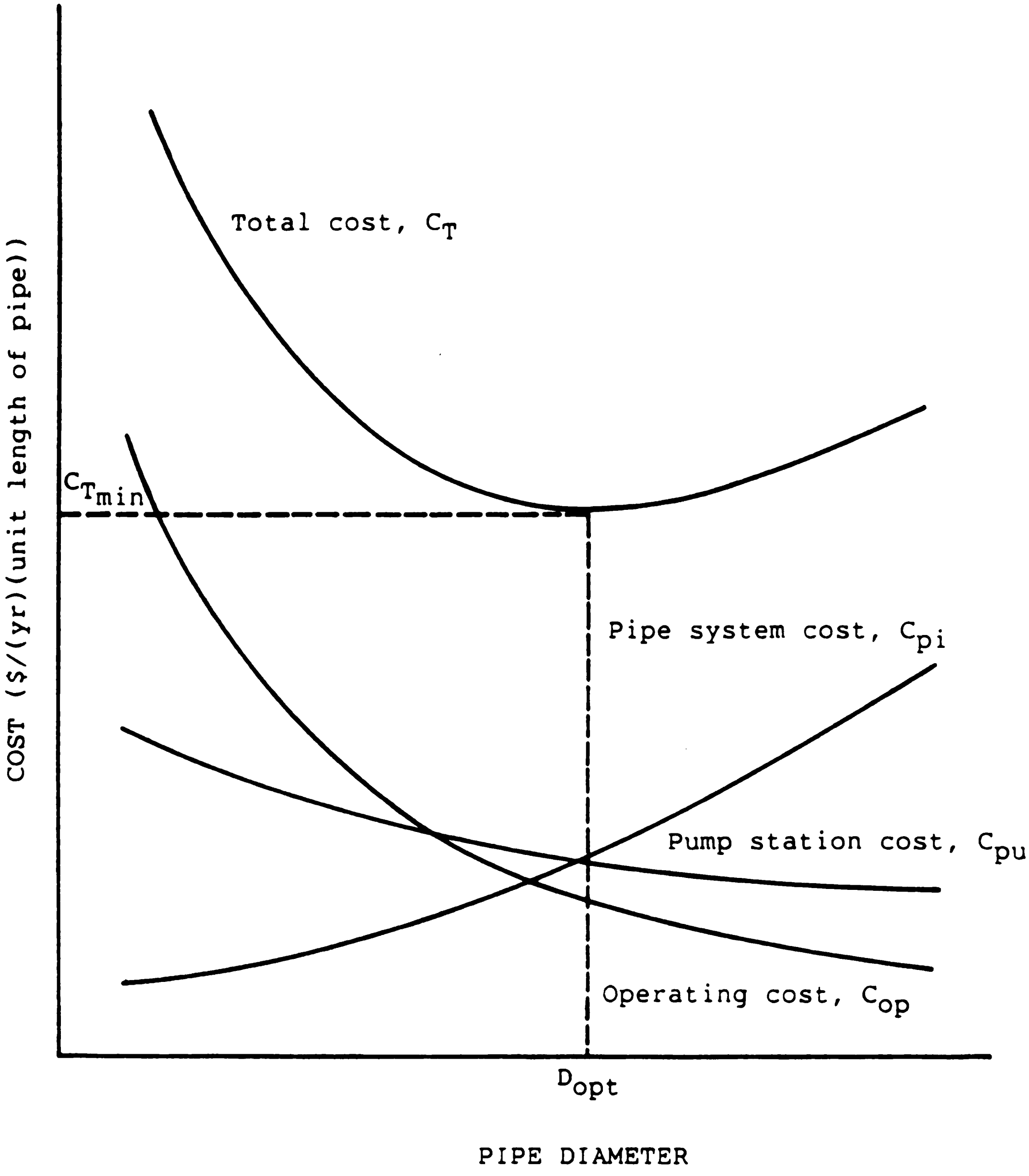


Figure 2. Optimum economic pipe diameter for minimum total cost at a fixed mass flow rate.

$$C_T(D) = C_{pi}(D) + C_{pu}(D) + C_{op}(D) \tag{2.7}$$

where

$C_T$ = total annual cost of pumping system per unit length of tube, \$/yr m

$C_{pi}$ = total annual cost of installed tube system per unit length of tube, \$/yr m

$C_{pu}$ = total annual cost of installed pump station per unit length of tube, \$/yr m

$C_{op}$ = total annual operating cost per unit length of tube, \$/yr m

The analytical method for optimization of a function of a single variable involves differentiating with respect to the variable and equating the result to zero. So the result, for D, in the total cost equation is

$$\frac{d}{dD} C_T = \frac{d}{dD} C_{pi}(D) + \frac{d}{dD} C_{pu}(D) + \frac{d}{dD} C_{op}(D) = 0 \tag{2.8}$$

Solving Equation (2.8) for D gives the optimum diameter for which the total cost is at minimum (Skelland, 1967; Jelen, 1970; Reklaitis et al., 1983).

## 2.3 Power Requirements

The work per unit mass required to pump an incompressible isothermal fluid through a tube system from point 1 to point 2 under steady state conditions is given by the mechanical energy balance equation (Heldman and Singh, 1981) written as

$$W = E_f + \frac{\bar{v}_2^2}{\alpha_2} - \frac{\bar{v}_1^2}{\alpha_1} + \frac{p_2 - p_1}{\rho} + g(z_2 - z_1) \tag{2.9}$$

where

$W$ = work per unit mass, J/kg

$E_f$ = energy loss due to friction, J/kg

$p$ = pressue, Pa

$\rho$ = fluid density, kg/m$^3$

$g$ = acceleration due to gravity (9.8 m/s$^2$)

$z$ = elevation, m

$\bar{v}$ = mass average velocity, m/s

$\alpha$ = kinetic energy correction factor

1,2 = subscripts referring to points 1 and 2, respectively

Osorio and Steffe (1984) developed an equation for the kinetic energy correction factor ($\alpha$) for Hershel-Buikley fluids in laminar flow. $\alpha$ is equal to two for turbulent flow. For the purpose of tube/pipe selection, however, the change in kinetic energy can be assumed to be zero since the tube has a constant diameter ($\bar{v}_1 = \bar{v}_2$) and point one and two have been located far enough from any entrance, bend, or fitting to have the same velocity profile ($\alpha_1 = \alpha_2$) (Skelland, 1967).

The energy loss due to friction in a straight pipe can be written in terms of the Fanning equation as (Govier and Azis, 1972)

$$E_f = \frac{2f\bar{v}^2L}{D} \qquad (2.10)$$

where

$f$ = fanning friction factor

$L$ = tube/pipe length, m

$D$ = tube/pipe inside diameter, m

The pressure drop due to friction depends on the flow characteristic, as well as the fluid properties. At slow flow, the fluid velocity is parallel to the tube axis and the pattern is smooth. This condition is known as laminar or streamline flow. As the velocity of flow increases, there is a point where the flow becomes unstable, eddies develop, and cause the fluid to swirl in all directions to the line of flow. The flow is then turbulent. The region from the end of laminar to fully turbulent flow is known as transitional region.

The theoretical relationship between pressure drop due to friction and flow rate for a H-B fluid in laminar flow can be optained by integrating Equation (2-1) as shown by Cheng (1970), Charm (1978), Skelland (1967), and Govier and Azis (1972). This relationship can be rewritten in term of the friction factor and generalized Reynolds number (Hands, 1978; Heywood and Cheng, 1982) and will be outlined later in this study.

The transitional flow of non-Newtonian fluids has been subject to research for many years. Various criteria of transition has been developed based on the end of the laminar flow regime (Metzner and Reed, 1957; Ryan and Johnson, 1959; and Mishra and Tripathi, 1973). Le Fur and Martin (1967) applied the Ryan and Johnson criterion for

Bingham and power law fluids. This criterion was also used by Hanks and Christiansen (1962) for nonisothermal flow of pseudoplastic fluids and by Cheng (1970) for H-B fluids. Hanks (1963) developed a more general stability criterion and applied it to the transitional flow of Bingham plastic fluids (Hanks, 1963). More recently Hanks and Ricks (1974) presented the transition flow behavior of H-B fluids based on his theory of laminar flow stability (Hanks, 1969).

Numerous equations have been developed to calculate the friction factor of power law (Dodge and Metzner, 1959; Shaver and Merrill, 1959; Kemblowski and Kolodziejski, 1973; Tomita, 1959; Szilas et al., 1981; Clapp, 1961; Hanks and Ricks, 1975), Bingham plastic (Tomita, 1959; Thomas, 1962; Hanks and Dadia, 1971; Darby and Melson, 1981) and H-B (Torrance, 1963; Hanks, 1978) fluids in turbulent flow. Good reviews of these equations are found in articles by Heywood and Cheng (1982), Cheng (1975), Kenshington (1974), Govier and Azis (1972), and Skelland (1967). Unlike Newtonian flow, the friction factor prediction for non-Newtonian fluids varies greatly, depending on the equation used. This deviation increases with decreasing flow behavior index, but is not very sensitive to the yield stress, up to a Hedstrom number of $10^4$. This is the motivation for using equations based on the power law model to predict friction factor for H-B fluids (Heywood and Cheng, 1982). However, this may lead to over estimation of the friction factor (Cheng, 1970). For all the methods developed for transitional and turbulent flow, the work of Hanks and Ricks (1974) and Hanks (1978) are the most comprehensive in describing the flow behavior of H-B fluids in laminar, transition, and turbulent flow. This work will be presented later in this study.

So far only the friction loss in a straight tube has been considered. However, to determine the total pressure drop in a tube system, one must add the friction loss arrising from any fittings, valves, and any other devices in the line. The total energy loss due to friction then can be written in terms of Equation (2.8) and the summation of the energy loss in fitting and other devices (Steffe et al., 1984) as

$$E_f = \frac{2f\bar{v}^2 L}{D} + \sum K_f \frac{\bar{v}^2}{2} \quad (2.11)$$

where

$K_f$ = dimensionless fittings resistance coefficient

An alternative way to account for the friction loss in fittings is by means of an equivalent length $L_f/D$ or $L_e = L + L_f$, where $L_f$ is the equivalent length of pipe for the fittings, valves, and other devices. Then, Equation (2.8) can be rewritten as (Govier and Azis, 1972).

$$E_f = \frac{2f\ \bar{v}^2\ (L + L_f)}{D} \quad (2.12)$$

Numerical data for the equivalent length and resistance coefficients for turbulent flow of Newtonian fluids through valves, bends, fittings, and other devices is available in standard reference books (Crane, 1982; Perry and Chilton, 1973; Govier and Azis, 1972). These values can be used as an approximation for non-Newtonian fluids since the friction loss in fittings does not depend significantly on the non-Newtonian character of the fluid during turbulent flow (Cheng, 1970, 1975). Although rather limited, some information on friction loss in fittings, valves, and entrances of non-Newtonian fluids in laminar flow is given by Wilkinson (1960), Skelland (1967), and Ury (1966). Unlike turbulent flow, the friction loss coefficient in laminar flow depends on the fluid properties and increases significantly with decreasing Reynolds number. This was shown by the data of Kittredge and Rowley (1957) for Newtonian fluids and Steffe et al. (1984) for a power law fluid. Iwanami and Suu (1970) considered the pressure drop in right-angle fittings for various slurries. Steffe et al. (1984) used a Blasius type equation to correlate the friction loss coefficient to a generalized Reynolds number for the laminar flow of a power law fluid through a tee (used as elbow), 90° elbow and a three-way plug valve. The pressure drop in entrances under laminar flow conditions has been considered by Michiyoski et al. (1966) for a Bingham plastic fluid, Collins and Schowalter (1963) for a power law fluid, and Soto and Shah (1976) for a H-B fluid. Cheng (1970) presented a technique to approximate frictional fittings loss for non-Newtonian fluids using the tabulated Newtonian losses.

Once the total energy loss per unit mass is known, the power requirement is given by

$$P = \frac{\dot{M} \cdot W}{E} \tag{2.13}$$

where

$P$ = power, Watts

$E$ = combined fractional efficiency of pump and motor

$\dot{M}$ = mass flow rate, kg/s

## 2.4 Economic Considerations

In most cases, the purchase cost per unit length of a pipe may be written in terms of the pipe diameter with the following empirical relationship (Skelland, 1967; Peters and Timmerhaus, 1968)

$$C' = X\,(39.37\ D)^{P'} \tag{2.14}$$

where

$C'$ = purchase cost of a new pipe, \$/m

$X$ = purchase cost of one inch diameter pipe per unit meter of pipe length, \$/m in $^{P'}$

$p'$ = constant for purchase cost of pipe dependent on the pipe material, dimensionless

$D$ = tube/pipe inside diameter, m

Typical valve of p' for different pipe materials is given by Nolte (1978), Skelland (1967), and Darby and Melson (1982). This relation permits estimation of the cost of any size pipe from the cost of a specific size pipe. Nolte (1978) used 2 inch diameter as a reference because of the greater availability of purchase cost data at this size. Based on Equation (2.14), the total annual cost of installed pipe system can be expressed as

$$C_{pi} = (a + b)\ (F + 1)\ X\ (39.37\ D)^{p'} \qquad (2.15)$$

where

$C_{pi}$ = total annual cost of installed tube system per unit length of tube, \$/yr m

a = annual fixed cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr

b = annual maintenance cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr

F = ratio the total cost for fittings and installation of pipe/tube and fittings to purchase cost of new pipe/tube

The ratio F is estimated at the reference size taken for the purchase cost of the pipe. That is, the pipe size used to estimate X in Equation (2.14). For Equation (2.15) the reference size is one inch. Notice that a, b, and F are assumed to be invariant with tube diameter, and p' only depends of the tube material. The maintenance cost (b) is generally taken as 4% per year of the new equipment cost. For corrosive processes or highly instrumented equipment, this figure may be as high as 7 to 10% of the investment (Perry and Chilton, 1973). The annual fixed cost, a, can be estimated, assuming zero salvage valve, from the uniform recovery factor (Newnan, 1983) as

$$a = \frac{i\ (1 + i)^N}{(1 + i)^N - 1} \qquad (2.16)$$

where

N = life-time of equipment, yr

i = interest rate (fraction)

Alternative methods to estimate the annual fixed cost are discussed by Newnan (1983), Jelen (1970), Peters and Timmerhaus (1968), and Perry and Chilton (1973).

The installed cost of a pipe system can also be correlated to the tube size with a logarithmic plot of the total installed cost, including fittings, valves, installation, etc., versus the tube diameter (Jelen, 1970). The installed cost-diameter relationship can then be written as

$$C = C_p D^s \tag{2.17}$$

where

$C$ = total installed cost of the tube system including the cost of fittings, valves, installation, etc., \$/m

$C_p$ = empirical constant for the tube system cost, $\$/m^{1+s}$

$s$ = exponent constant in the tube system cost equation, dimensionless

The annual cost of a pump station can be written in terms of power capacbility. Darby and Melson (1982) gave a linear relationship between the cost and horsepower for large size pump stations. An alternative method is to estimate the pump station cost from the cost data of a different pump size with the following logarithmic relationship (Jelen, 1970; Perry and Chilton, 1973).

$$C_1 = C_2 \left(\frac{Q_1}{Q_2}\right)^q \qquad (2.18)$$

where

$C_1$ = unknown cost of equipment of size $Q_1$

$C_2$ = known cost of equipment of size $Q_2$

$q$ = cost capacity factor

Values of q for different pump types and power ranges are given by Jelen (1970), Peters and Timmerhaus (1968), and Perry and Chilton (1973). When q = 0.6, this relationship is known as the six-tenths-factor rule. A closer approximation of this relationship has been found to be (Perry and Chilton, 1973)

$$C_1 = C_D' \left(\frac{Q_1}{Q_2}\right)^q + C_I' \qquad (2.19)$$

where

$C_D'$ = total direct cost of equipment of size $Q_2$

$C_I'$ = total indirect cost of equipment of size $Q_2$

Cost data for pipes, pumps, and fittings are presented by Peters and Timmerhaus (1968), Jelen (1970), Marshall and Brandt (1970), and Barrett (1981). This data could be updated with cost indexes, however, current data should be used whenever possible (Jelen, 1970).

The annual operating cost is primarily the annual electrical energy consumption and is given as (Skelland, 1967)

$$C_{op} = \frac{C_e \; h \; P}{L} \tag{2.20}$$

where

$C_{op}$ = total annual operating cost per unit length of tube, \$/yr m

$C_e$ = cost of electrical energy, \$/W hr

h = hours of operation per year

L = tube/pipe length, m

P = power, Watts, Equation (2.13)

## 2.5 Optimum Economic Pipe Diameter

Various relationships have been developed for the optimum economic diameter of Newtonian fluids under laminar and turbulent flow conditions. Genereaux (1937) was probably the first to present pipe diameter optimization methods based on the economic balance of pipe and operating costs. Further details of Genereuax's work are given by Peters and Timmerhaus (1968). Downs and Tait (1953) based their analysis on the economic balance of pipe and pump costs and provided corrections to account for the operating cost. Perry and Chilton (1973), and Peter and Timmerhaus (1968) presented optimum diameter relationships based on the concept or return on incremental investment. Other methods for determining economic pipe diameter for Newtonian fluids are discussed by Wright (1950), Sarchet and Colburn (1940), Nolte (1978), Dickson (1950), Braca and Happel (1953), and Nebeker (1979).

Optimum economic diameter relationship are more limited for non-Newtonian fluids. Duckham (1972) gave general guidelines to estimate the optimum diameter of non-Newtonian fluids. Skelland (1967) developed optimum diameter equations based on Metzner and Reed (1955), and Dodge and Metzner (1959) friction factor relationships for non-Newtonian fluids in laminar and turbulent flow, respectively. For laminar flow his relationships may be written in terms of the power law model [as defined by Equation (2.2)]. The analysis is based on the economic balance of pipe and operating costs assuming a pump was already available or its cost was invariant with pipe diameter. Skelland (1967) also developed a relationship to estimate the optimum pumping temperature based on the economic balance of heating cost and operating cost. The latter decreases with increasing temperature due to the decrease of consistency coefficient with increasing temperature. Application of Skelland's relationships for the food processing industry was presented by Boger and Tiu (1974). More recently, Darby and Melson (1982) applied dimensionless analysis to developed graphs from which the optimum diameter can be obtained directly for Newtonian, Bingham plastic, and power law fluids. In their analysis, they assumed the friction factor to be constant in the differentiation of the total cost [Equation (2.7]. The friction factor relationships of Churchill (1977) and Darby and Melson (1981) were used for Newtonian and Bingham plastic fluids, respectively. These relationships span all flow regimes. The equation of Dodge and Metzner (1959) was used for the turbulent flow of power law fluids. Unlike Skelland, their economic analysis includes the pump station cost for which they developed a linear relationship with pump power.

# 3. THEORETICAL DEVELOPMENT

## 3.1 Flow Behavior of Herschel-Bulkley Fluids

The theoretical pressure-drop/flow rate relationship for H-B fluids in laminar flow, in terms of the fanning friction factor, has been derived by Hanks (1978) and Heywood and Cheng (1982). To date, Torrance (1967) and Hanks (1978) have presented theoretical analysis of turbulent flow for H-B fluids. Hanks' analysis is the most comprehensive method. Unlike the Torrance equation, the Hank's relationship deals with transitional flow and includes the laminar-turbulent transition criterion developed by Hanks and Ricks (1974). In addition, Hanks' analysis accounts the viscous dampening effect of the wall on eddy properties near the wall and radial variation of shear stress, and retained the molecular flux term. His relationship along with the laminar-turbulent transition of Hanks and Ricks (1974) and the laminar flow relationship will be presented in this section. The Torrance equation is given in Appendix A. The Hanks relationship is particularly suitable for determining the optimum diameter because it provides a continuous function of friction factor with tube or pipe diameter.

### 3.1.1 Laminar Flow

Consider a tube of length L and radius $r_w (D = 2r_w)$ with frictional pressure drop between points 1 and 2 of $\Delta P_f$ (Figure 3). A force balance on the core of the fluid gives

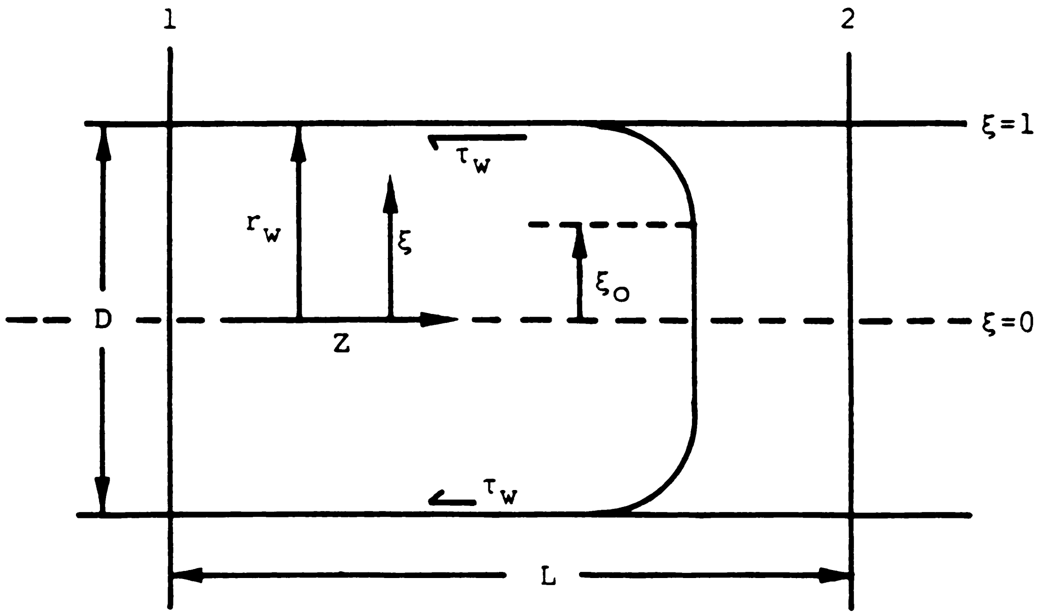


Figure 3. Velocity profile for Herchel-Bulkley fluid in a tube.

$$\pi r^2 \ \Delta P_f = 2\pi r L \tau_{rz} \tag{3.1}$$

or

$$\tau_{rz} = \frac{r \quad \Delta P_f}{2L} \tag{3.2}$$

At the wall, Equation (3.2) becomes

$$\tau_w = \frac{r_w \quad \Delta P_f}{2L} \tag{3.3}$$

Combing Equations (3.2) and (3.3) yields

$$\tau_{rz} = \frac{r}{r_w} \quad \tau_w = \xi \ \tau_w \tag{3.4}$$

Defining additional dimensionless variable as $u = v/\bar{v}$, $\xi_o = \tau_o/\tau_w$, $\Gamma = -du/d\xi$ and $\zeta = \Gamma/\Gamma_w$, Equation (2.1) can be written in dimensionless form as

$$\xi = \xi_o + \frac{K\bar{v}^n \ \Gamma_w^n}{\tau_w \ r_w^n} \ \zeta^n \tag{3.5}$$

Since $\zeta = 1$ when $\xi = 1$, it follows from Equation (3.5) that

$$\Gamma_w^n = (1 - \xi_o) \ \frac{\tau_w \ r_w^n}{K\bar{v}^n} \tag{3.6}$$

Substituting Equation (3.6) into (3.5) gives the result

$$\xi = \xi_o + (1 - \xi_o) \ \zeta^n \tag{3.7}$$

Assuming no slip at the wall, the velocity distribution can be obtained from

$$u = \int_{\xi}^{1} \left(- \frac{du}{d\xi'}\right) d\xi' = \Gamma_w \int_{\xi}^{1} \zeta(\xi', \xi_0)\, d\xi' \qquad (3.8)$$

where $\xi'$ is a dummy variable.

Upon integration, Equation (3.8) gives the relationship for the velocity distribution as

$$u = \frac{\Gamma_w}{(1 - \xi_0)^{1/n}} \left( \frac{n}{n + 1} \right) \left[ (1-\xi_0)^{1/n+1} - (\xi - \xi_0)^{1/n+1} \right] \qquad (3.9a)$$

for

$\xi > \xi_0$

$$u_0 = \frac{\Gamma_w}{(1 - \xi_0)^{1/n}} \left( \frac{n}{n + 1} \right) (1 - \xi_0)^{1/n + 1} \qquad (3\text{-}9b)$$

for $\xi \leq \xi_0$

In terms of the defined dimensionless variable, the expression for the flow rate is given by

$$2 \int_{0}^{\xi_0} \xi u_0 \, d\xi + 2 \int_{\xi_0}^{1} \xi u d\xi = 1 \qquad (3.10)$$

Substituting Equation (3.8) into Equation (3.10) yields, since $u_o$ is constant for $0 \le \xi \le \xi_o$

$$\Gamma_w \left[ \xi_o^2 \int_{\xi_o}^{1} \zeta\,(\xi',\xi_o)\; d\xi' + 2 \int_{\xi_o}^{1} \xi \int_{\xi}^{1} \zeta\,(\xi',\xi_o)\; d\xi'\; d\xi \right] = 1 \tag{3.11}$$

Integrating the double integral in Equation (3.11) by parts, using Leibnitz' rule (Hanks and Ricks, 1974) gives

$$\Gamma_w \int_{\xi_o}^{1} \xi^2\, \zeta\,(\xi,\, \xi_o) \quad d\xi = 1 \tag{3.12}$$

Substituting Equations (3.7) into (3.12) and integrating results in

$$\Gamma_w^n = \frac{(1 - \xi_o)}{\psi\, \left(\frac{n}{1 + 3n}\right)^n} \tag{3.13}$$

where

$$\psi = (1 + 3n)^n (1 - \xi_o)^{1+n} \left[ \frac{(1 - \xi_o)^2}{(1 + 3n)} + \frac{2\xi_o\,(1 - \xi_o)}{(1 + 2n)} + \frac{\xi_o^2}{(1 + n)} \right]^n \tag{3.14}$$

By combining Equation (3.6) and the definition of the Fanning friction factor as

$$f = \frac{2\tau_w}{\rho \bar{v}^2} = \frac{2\tau_o}{\xi_o \rho \ \bar{v}^2} \tag{3.15}$$

Equation (3.13) can be written in terms of the friction factor as

$$f = \frac{16}{\psi Re} \tag{3.16}$$

where $\psi$ is given by Equation (3.14) and Re, the generalized Reynolds number, is (by definition)

$$Re = 8 \left(\frac{n}{1 + 3n}\right)^n \frac{\rho \ r_w^n \ \ \bar{v}^{2-n}}{K} \tag{3.17}$$

If one eliminates $\bar{v}$ using equation (3.6), and the definitions of f and Re, Equations (3.13) may be rearranged (Hanks, 1978) to give

$$Re = 2 \ He \left(\frac{n}{1 + 3n}\right)^2 \left(\frac{\psi}{\xi_o}\right)^{\frac{2-n}{n}} \tag{3.18}$$

where $\psi$ is given by Equation (3.14) and

$$He = \frac{D^2 \rho}{K} \left(\frac{\tau_o}{K}\right)^{\frac{2-n}{n}} \tag{3.19}$$

Equation (3.19) is a generalization of the Hedstrom number. Equation (3.18) defines $\xi_o$ as an implicit function of Re and He for He > 0. $\xi_o = 0$ when He = 0, i.e., $\tau_o = 0$.

### 3.1.2 Laminar-Turbulent Transition

Laminar instability starts when the ratio $\kappa$, the rate of change of angular momentum of a deforming fluid element to its rate of loss of frictional drag momentum, exceeds a critical valve $\bar{\kappa}$ (Hanks, 1969). For rectilinear pipe flow, the stability parameter can be written as

$$\kappa = \frac{\rho L}{2\Delta P_f} \frac{d}{dr}(v^2) = \frac{Re\ \psi \Gamma_w u \zeta}{16} \tag{3.20}$$

where $\Gamma_w$, $\zeta$, u, and $\psi$ are given by equations (3.6), (3.7), (3.9a), and (3.14), respectively. $\kappa$ is a function of the radial position $\xi$ having the value of zero at $\xi = 1$ and $\xi = \xi_o$, and a maximum value at some point in the field ($\xi = \bar{\xi}$ , $\kappa = \bar{\kappa}$) where maximum instability occurs. The transitional critical Reynolds number ($Re_c$) is obtained from Equation (3.20) when one sets $\xi = \bar{\xi}$ and $\bar{\kappa} = 404$ (Hanks and Ricks, 1974). This valve will give $Re_c = 2100$ for Newtonian pipe flow. The radial position of maximum instability $\bar{\xi}$ is found by setting $d\kappa/d\xi = 0$. For H-B fluids, the critical Reynalds number is then given by the following expression (Hanks and Ricks, 1978)

$$Re_c = \frac{6464\ n\ \psi_c^{2/n-1}\ (2+n)^{\frac{2+n}{1+n}}}{(1+3n)^2\ (1-\xi_{oc})^{1+2/n}} \tag{3.21}$$

where $\psi_c$ is given by Equation (3.14) with $\xi_o = \xi_{oc}$ Equation (3.18) is also valid at $Re = Re_c$. Now, by eliminating $Re_c$ with Equations (3.18) and (3.21), the relationship between He and $\xi_{oc}$ can be obtained as

$$\mathrm{He} = \frac{3232\ (2 + n)^{\frac{2+n}{1+n}}\ \xi_{oc}^{\ 2/n-1}}{n\ (1 - \xi_{oc})^{2/n+1}} \tag{3.22}$$

which defines $\xi_{oc}$ as in implicit function of He (Equation (3.19)) and n.

The critical friction factor, $f_c$, can be estimated from Equations (3.16) with $\psi = \psi_c$ and Re = $Re_c$.

### 3.1.3 Transitional and Turbulent Flow

For transitional and turbulent flow, the time average momentum flux can be expressed as (Hanks, 1968)

$$\bar{\tau}_{rz} = \tau^{L}_{rz} + \bar{\tau}^{T}_{rz} \tag{3.23}$$

where $\tau^{L}_{rz}$ is the molecular flux, given by Equation (2.1), and $\bar{\tau}^{T}_{rz}$ is the turbulent flux (or Reynolds stress). This latter flux is given by Hanks and Dadia (1971), Hanks and Ricks (1975), Hanks (1978), as

$$\bar{\tau}^{T}_{tz} = \rho \ell \dot{\gamma}^{2} \tag{3.24}$$

where $\ell$ is a modified Prandtl's mixing length (Hanks, 1968) and is given in terms of the dimensionless variable $\lambda = \ell/r_w$ as (Hanks and Ricks, 1975; Hanks, 1978)

$$\lambda = k\ (1 - \xi)\{1 - \exp\ [-\phi(1 - \xi)]\} \tag{3.25}$$

where

$$\phi = \frac{R - R_c}{\sqrt{8}\ B} \tag{3.26}$$

k = Prandtl's universal mixing length constant = 0.36

and

$$R = \left(\frac{1 + 3n}{n}\right) \left[ Re \left(\frac{f}{16}\right)^{\frac{2-n}{2}} \right]^{1/n} \tag{3.27}$$

R is a working parameter and reduces to $R = Re\sqrt{f}$ for Newtonain fluids (Hanks, 1968). The parameter $R_c$ is estimated from Equation (3.27) with $Re = Re_c$ and $f = f_c$.

The parameter B is given by the following empirical relationship for the H-B model (Hanks, 1978).

$$B = \frac{22}{n} \left[ 1 + \frac{0.00352\ He}{(1 + 0.000504\ He)^2} \right] \tag{3.28}$$

Substituting equations (2.1) and (3.24) into (3.23), Equation (3.23) can be rewritten in dimensionless form as

$$\xi = \xi_o + \frac{K\bar{v}^n\ \Gamma_w^n}{\tau_w \Gamma_w^n}\ \zeta^n + \frac{\rho\bar{v}^2\lambda^2\Gamma_w^2}{\tau_w}\ \zeta^2 \tag{3.29}$$

where $\Gamma_w$ is given by Equation (3.6) since $\xi = 1$ and $\lambda = 0$ when $\xi = 1$.

Using Equation (3.6), (3.15), (3.17), and (3.27), Equation (3.29) can be written as

$$\xi = \xi_o + (1 - \xi_o)\ \zeta^n + \frac{R^2}{8}(1 - \xi_o)^{2/n}\ \lambda^2 \zeta^2 \qquad (3.30)$$

Combining equations (3.6), (3.12), (3.15), (3.17), and (3.27), Equations (3.12) can be written in equivalent form

$$(1 - \xi_o)^{\frac{2-n}{n}} \left(\frac{n}{1 + 3n}\right)^n \frac{R^2}{Re} \left[\int_{\xi_o}^{1} \xi^2\ \zeta\ (\xi) d\xi\right]^{2-n} = 1 \qquad (3.31)$$

where $\zeta(\xi)$ is given implicity by Equation (3.30).

Finally, from the definitions of f, Re, R and He, it can be shown that

$$R^2 = \frac{2\ He}{\xi_o^{\frac{2-n}{n}}} \qquad (3.32)$$

The methodology to estimate the friction factor is outlined in Figure 4. A computer program (Appendix D) written in FORTRAN 77 as developed to accomplish these calculations.

## 3.2 Total Annual Cost of a Pumping System

Assuming negligible kinetic energy change and substituting Equation (2.11), the work per unit mass, Equation (2.9), can be written as

$$W = \frac{2f\bar{v}^2 L}{D} + \sum K_f \frac{\bar{v}^2}{2} + \frac{\Delta p}{\rho} + g\Delta z \qquad (3.33)$$

| | Input Variables $K, n, \tau_o, \rho\ \dot{M}$ $D, D_{est}$ or $D_{opt}$ | Fluid Properties, Mass Flor Rate Pipe/Tube Inside Diameter |
|---|---|---|
| 1. | $\bar{v}$ | Calculate $\bar{v}$ from Equation (3.34) |
| 2. | Re | Calculate Re from Equation (3.17) |
| 3. | He | Calculate He from Equation (3.19) |
| 4. | $\xi_{oc}$ | Calculate $\xi_{oc}$ from Equation (3.22) through iteration $0 \leq \xi_{oc} < 1$ |
| 5. | $\psi_c$ | Calculate $\psi_c$ from Equation (3.14) with $\xi_o = \xi_{oc}$ |
| 6. | $Re_c$ | Calculate $Re_c$ from Equation (3.21) |
| 7. | $f_c$ | Calculate $f_c$ from Equation (3.16) with $\psi_c = \psi_c$ and $Re = Re_c$ |
| If $Re < Re_c$ then laminar flow. | | If transition or turbulent go to Step 11. |
| 8.-a | $\xi_o$ | Calculate $\xi_o$ from Equation (3.18) through interation. ($\xi_o = 0$ if $\tau_o = 0$ (He = 0)). $\xi_{oc} \leq \xi_o < 1$ |
| 9.-a | $\psi$ | Calculate $\psi$ from Equation (3.14) |
| 10.-a | f | Calculate f from Equation (3.16) |
| Alternative for laminar flow | | |
| 8.-b | $\xi_o$ | Calculate $\xi_o$ from Equation (3.15) guessing $f > 2\ \tau_o / (\rho\ \bar{v}^2)$. $\xi_{oc} \leq \xi_o < 1$ |
| 9.-b | $\psi$ | Calculate $\psi$ from Equation (3.14) |
| 10.-b | f | Calculate f from Equation (3.16) and compare with the guess value in Step 8-b |
| If $Re > Re_c$, then transitional/turbulent flow | | |
| 11. | $f_{est}$ | Guess a value for $f > 2\tau_o/(\rho\bar{v}^2)$ |

Figure 4. Calculation scheme to estimate the friction factor.

| | | |
|---|---|---|
| 12. | $R_c$ | Calculate $R_c$ from Equation (3.27) with $Re = Re_c$ and $f = f_c$ |
| 13. | $R$ | Calculate R from Equation (3.27) |
| If $R < R_c$, then go to step 11. | | Guess a higher valve for $f_{est}$ |
| 14. | $\xi_o$ | Calculate $\xi_o$ from Equation (3.32) or (3.15). $0 \leq \xi_o < \xi_{oc}$ |
| 15. | B | Calculate B from Equation (3.28) |
| 16. | $\phi$ | Calculate $\phi$ from Equation (3.26) |
| 17. | $\lambda_j$ | Generate values of $\lambda_j$ from Equation (3.25) with $\xi_o \leq \xi \leq 1$ (j = 1, 2, 3 ...) |
| 18. | $\zeta_j$ | Generate values of $\zeta_j$ from Equation (3.30) with values of $\lambda_j$ and $\xi_o \leq \xi_j \leq 1$ (j = 1,2,3 ...) |
| 19. | | Evaluate the integral of Equation (3.31) by numerical methods with $\zeta_j$ and $\xi_o \leq \xi_j \leq 1$ (j = 1,2,3 ...) |
| 20. | | Calculate Equation (3.31). If result $\neq 1$, then go to Step 11. |
| 21. | f | If Equation (3.31) is equal to one, then $f = f_{est}$ |

Figure 4. Continued.

where the friction factor f is obtained using Hanks' method described in the previous section and the friction loss coefficients, $K_f$, for fittings can be approximated with the Newtonian data for turbulent flow (Crane, 1982; Perry and Chilton, 1973; Govier and Azis, 1972) and the relationship given by Iwanami and Suu (1970), Steffe et al. (1984), and Soto and Shah (1976) for laminar flow.

The mass average velocity may be written as

$$\bar{v} = \frac{4\dot{M}}{\pi \rho D^2} \tag{3.34}$$

Substituting Equation (3.34) into (3.33) gives the result

$$W = \frac{32 f \dot{M}^2 L}{\pi^2 \rho^2 D^5} + \frac{8\dot{M}^2}{\pi^2 \rho^2 D^4} \sum K_f + \frac{\Delta p}{\rho} + g\Delta z \tag{3.35}$$

The annual cost of a pipe system can be estimated using Equation (2.17) as

$$C_{pi} = (a + b) C_p D^s \tag{3.36}$$

where $C_{pi}$ = total annual cost of installed tube system per unit length of tube, \$/yr m

$a$ = annual fixed cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr

$b$ = annual maintenance cost of the tube system expressed as a fraction of the initial installed cost of the tube system, 1/yr

$C_p$ = empirical constant for the tube system cost, $\$/m^{1+s}$

$s$ = exponent of tube system cost equation, dimensionless

As stated before, $C_p$ and s can be estimated from a log-log plot of the installed csot of the tube system (tube, fittings, valves, etc.) versus the tube inside diameter. Notice that Equation (3.36) can also be interpreted as Equation (2.15) if one lets s = p' and $C_p = (F + 1) \times (39.37)^{P'}$. This permits one to obtain the installed annual cost of the tube system as a function of the diameter from the knowledge of the costs of one-inch tube and fittings. However, some error may be introduced by assuming F to be independent of D and extrapolating from the cost of one tube size. Therefore, this method should only be used in preliminary tube sizing when data and knowledge of the system are limited. More accurate results can be obtained if the variables $C_p$ and s are calculated from the installed cost of the tube system for various tube diameters. Even in this case, extrapolating beyond the diameter range used should be avoided. That is, Equation (3.36) should be estimate using a range of diameters where the optimum diameter is expected. Notice that the annual fixed cost and the annual maintenance cost ratios are assumed to be independent of tube diameter. However, these costs, as well as other costs associated with the tube system which may depend on the tube diameter, may be included in the estimation of the installed cost of the tube system. Then the fixed and maintenance cost will be included in the varialbes $C_p$ and S. If this is done, the term (a + b) in Equation (3.36) can be set equal to one.

The cost of a pump station can be written, in a manner similar to Equation (2.19) as

$$C_{ps} = C_D\, P^{s'} + C_I \tag{3.37}$$

where

$C_{ps}$ = total cost of installed pump station, \$

$C_D$ = empirical constant for the pump station cost, $\$/W^{s'}$

$C_I$ = empirical constant for the pump station cost, \$

$s'$ = exponent of the pump station cost equation, dimensionless

The value of $C_D$, $C_I$, and s' can be obtained from a plot of the installed cost of the pump versus the power requirements. The installed pump station cost includes the purchase cost of pump, motor, and other costs dependent on the size of the pump. Equation (3.37) permits the use of a linear (s' = 1) or power ($C_I$ = 0) relationship for $C_{ps}$ versus P. Notice also that this equation can be interpreted as Equation (2.19) if one lets $C_I = C_I'$, $P = Q_1$, $s' = q$, and $C_D = C_D' / Q_2{}^q$. The annual pump station cost per unit length of tube can then be expressed as

$$C_{pu} = (a' + b')(C_D P^{s'} + C_I)/L \tag{3.38}$$

where

a' = annual fixed cost of the pump station expressed as a fraction of the initial installed cost of the pump station, 1/yr

b' = annual maintenance cost of the pump station expressed as a fraction of the initial installed cost of the pump station, 1/yr

Again, the fixed cost and the maintenance costs may be included in the estimation of the installed cost of the pump station for the different pump sizes accounting for these costs in the variables $C_I$, $C_D$, and $s'$. Then, the term (a' + b') in Equation (3.38) could be set equal to one.

The total annual cost of a pumping system per unit length of tube can be obtained by adding Equations (2.20), (3.36), and (3.38) which gives, after rearrangement,

$$C_T = (a + b)\ C_p D^s + \frac{C_e h P}{L}\left[\frac{(a' + b')\ (C_D P^{s'} + C_I)}{C_e h\ P} + 1\right] \quad (3.39)$$

Substituting Equation (2.13) into (3.39) yields

$$C_T = (a + b) C_p D^s + \frac{C_e h \dot{M} W}{LE}\left[\frac{E(a' + b')(C_D \dot{M}^{s'} W^{s'}\ E^{-s'} + C_I)}{C_e h\ \dot{M}\ W} + 1\right]$$

(3.40)

where

$W$ = work per unit mass, J/kg, Equation (3.35)

The procedure to estimate the pumping system costs is outlined in Figure 5. The computer program developed to accomplish these calculations is given in Appendix D.

## 3.3 Optimum Economic Tube Diameter

As stated before, the optimum tube diameter, $D_{opt}$, can be obtained by setting $dC_T/dD = 0$ assuming that $C_T$ is only a function of D, i.e., Imput Variables

| | |
|---|---|
| $\dot{M}$, L, $\Delta p, \Delta z$, $\Sigma K_f$ | Pumping system parameters |
| n, K, $\tau_o$, $\rho$ | Fluid properties |
| $C_p$,s, a, b | Tube system cost parameters |
| $C_I$, $C_D$, s', a', b' | Pump station cost parameters |
| $C_e$, h, E | Operating cost parameters |
| D or $D_{opt}$ | Tube/pipe inside diameter |

| | | |
|---|---|---|
| 1. | $\bar{v}$ | Calculate $\bar{v}$ from Equation (3.34) |
| 2. | f | Calculate f from scheme in Figure 4 |
| 3. | W | Calculate W from Equation (3.35) |
| 4. | P | Calculate P from Equation (2.13) |
| 5. | $C_{pi}$ | Calculate $C_{pi}$ from Equation (3.36) |
| 6. | $C_{pu}$ | Calculate $C_{pu}$ from Equation (3.38) |
| 7. | $C_{op}$ | Calculate $C_{op}$ from Equation (2.20) |
| 8. | $C_T$ | Calculate $C_T$ from Equation (3.39) or (3.40) |

Figure 5. Calculation scheme to estimate the annual costs of a pumping system.

$$\frac{d}{dD} C_T = (a + b)sC_p D_{opt}^{s-1} - \frac{32\dot{M}^3 C_e h}{\pi^2 \rho^2 D^6 E} \left[ \frac{(a' + b')s' C_D M^{s'-1}}{C_e h \; E^{s'-1}} + 1 \right] \cdot$$

$$\left[ 5f - D_{opt} \frac{d}{dD} f + \frac{D_{opt}}{L} \Sigma K_f - \frac{D_{opt}^2}{4L} \frac{d}{dD} \Sigma K_f \right] = 0 \qquad (3.41)$$

For laminar flow, df/dD can be obtained from the derivative of Equation (3.16) with respect to the diameter which gives

$$\frac{d}{dD} f = - \frac{16}{\psi Re^2} \frac{dRe}{dD} - \frac{16}{Re \psi^2} \frac{d\psi}{dD} \qquad (3.24)$$

Replacing $\bar{v}$ with Equation (3.34) in Equation (3.17) and taking the derivative of Re with respect to D gives

$$\frac{dRe}{dD} = \frac{(3n - 4)}{D} Re \qquad (3.43)$$

Similarly, substituting $\xi_o$ with Equation (3.15) in Equation (3.14) and taking the derivative of $\psi$ with respect to D gives.

$$\frac{d\Psi}{dD} = \frac{\psi \sigma \xi_o}{f} \frac{d}{dD} f - \frac{4 \psi \sigma \; \xi_o}{D} \qquad (3.44)$$

where

$$\sigma = \left[ \frac{(1 + 3n)(1 + n)(1 - \xi_o)^2 + 2(1 + 2n)(1 + 3n)\xi_o (1 - \xi_o) + (1 + 3n)(1 + 2n)(1 + n)\xi_o^2}{(1 + 2n)(1 + n)(1 - \xi_o)^3 + 2(1 + 3n)(1 + n)\xi_o \, (1 - \xi_o)^2 + (1 + 3n)(1 + 2n)\xi_o^2 \, (1 - \xi_o)} \right]$$

(3.45)

Substituting Equations (3.43) and (3.44) into Equation (3.42) and solving for df/dD gives

$$\frac{d}{dD} f = \frac{64\xi_o\sigma + 16\ (4 - 3n)}{Re\psi D\ (1 + \sigma\xi_o)} \tag{3.46}$$

or

$$\frac{d}{dD} f = \frac{4f\xi_o\sigma + (4 - 3n)f}{D\ (1 + \sigma\xi_o)} \tag{3.47}$$

where f, $\xi_o$ and $\sigma$ are given by equations (3.16), (3.18), and (3.45), respectively.

Equation (3.46) was confirmed for the special cases of the power law, Bingham plastic and Newtonian fluids by comparing it to independent analytical solutions for these fluids. It was also confirmed numerically for two examples of a H-B fluid (Appendix B).

A numerical integration was required to estimate the friction factor for turbulent flow as seen in Equation (3.31); hence, the derivative of the friction factor with respect to the diameter must be approximated numerically for this flow condition by

$$\frac{d}{dD} f(D) = \frac{f(D + x) - f(D-x)}{2x} \tag{3.48}$$

where

f(D) = the friction factor expressed as a function of D

x = a small positive number

The backward difference method is used to evaluate the derivative for diameters just below the critical diameter where turbulent flow starts

(Appendix C). Examples of f(D) versus D are shown in Appendix B. An alternative equation for df/dD for turbulent flow is given in Appendix A when the friction factor is estimated with the relationship developed by Torrance (1967).

Since no general equation exists for the fitting resistance coefficient ($K_f$), it must be assumed to be independent of the diameter. That is, $dK_f/dD = 0$. In addition, when $L >> D$, $L/D$ and $D^2/4L$ will be small numbers having a small influence on $D_{opt}$. Hence, constant $K_f$ values for Newtonian fluids for turbulent flow can be used as an approximation to evaluate $D_{opt}$. Then, the $D_{opt}$ is given implicity by eliminating the $dK_f/dD$ term and solving for $D_{opt}$ from Equation (3.41) as

$$D_{opt}^{s+5} = \left[\frac{32\dot{M}^3 C_e h}{(a+b)sC_p \pi^2 \rho^2 E}\right]\left[\frac{(a'+b')s'C_D\ P^{s'-1}}{C_e h} + 1\right]$$

$$\left[5f - D_{opt}\frac{df}{dD} + \frac{D_{opt}}{L}\Sigma K_f\right] \qquad (3.49)$$

where

P = power, watts, Equation (2.13)

By letting $C_D = 0$, $\Sigma K_f = 0$, $\xi_o = 0$ and $C_p = (1 + F)(39.37)^s X$, and substituting df/dD for Equation (3.46) or (3.47), Equation (3.49) reduces to

$$D_{opt}\left[\left(\frac{4(1+3n)C_e\dot{M}hK}{s(a+b)(F+1)(39.37)^s \; X \; \rho E}\right)\left(\frac{8\dot{M}(1+3n)}{\pi\rho n}\right)^n\right]^{\left(\frac{1}{s+3n+1}\right)} \tag{3.50}$$

which is an equivalent form of Skelland's equation (Skelland, 1967; p. 245) for power law fluids in laminar flow, but with the variables expressed in SI units.

The procedure to estimate the optimum diameter is shown in Figure 6. The computer program to do these calculations is given in Appendix D.

## 3.4 Limitation of Design Method

Some assumptions are inherent in the design method presented. Even though some of these assumptions were stated previously, they will be summaried here:

1. Use of Newtonian values for the fittings resistance coefficients
2. Constant fluid density (incompressible fluid)
3. Homogeneous or pseudohomogeneous fluids
4. No slip or apparent slip at the wall
5. No elastic or time-dependent behavior
6. Smooth wall (for turbulent flow only)
7. Isothermal flow
8. Steady state flow
9. Negligible kinetic energy change

| | | |
|---|---|---|
| | Imput variables | |
| | $\dot{M}$, L, $\Delta p$, $\Delta z$, $\Sigma K_f$ | Pumping system parameters |
| | n, K, $\tau_o$, $\rho$ | Fluid properties |
| | $C_p$, s, a, b | Tube system cost parameters |
| | $C_I$, $C_L$, s', s', b' | Pump station cost parameters |
| | $C_e$, h, E | Operating cost parameters |
| 1. | $D_{est}$ | Guess $D_{opt}$ |
| 2. | $\bar{v}$ | Calculate $\bar{v}$ from Equation (3.34) |
| 3. | f | Calculate f from scheme in Figure 4 |
| 4. | df/dD | Calculate df/dD from Equation (3.46) or (3.47) if the flow is laminar ($Re < Re_c$) or from Equation (3.48) if the flow is turbulent ($Re > Re_c$) |
| 5. | W | Calculate W from Equation (3.35) |
| 6. | P | Calculate P from Equation (2.13) |
| 7. | $D_{opt}$ | If Equation (3.49) is true then $D_{opt}$ = $D_{est}$, otherwise go to Step 1 |

Figure 6. Calculation scheme to estimate the optimum diameter.

The first assumption may be violated when handling non-Newtonian fluids under laminar flow. Under this flow condition, the fittings resistance coefficient increases with decreasing Reynolds number (Steffe et al., 1984). The frictional loss in fittings may become significant in a complex tube system with a great number of fittings. This may cause errors in estimating the optimum diameter, particularly with short tube systems.

The next three conditions may be violated in the handling of heterogeneous or multiphase fluids. In these systems, a particle-free layer may form at the pipe wall creating a variation of solids concentration. The lubricating action of this liquid layer is known as effective slip. These systems cannot be accurately described by the H-B fluid model. More complex models are also required to describe the flow behavior of viscoelastic and time-dependent fluids. Viscoelastic fluids show partial elastic recovery on removal of deforming shear stresses. Such materials exhibit both viscous and elastic properties. Time-dependent fluids exhibit reversible decrease (thixotropic), irreversible decrease (rheomalaxis) or reversible increase (rheopectic) in shear stress with time at constant rate of shear (Skelland, 1967). These and various time-independent rheological models not described by Equation (2.1), such as the Ellis and Casson models, are not considered in this study.

For turbulent flow, wall roughness leads to increased pressure drop (Cheng, 1975). Therefore, the power requirements will be underestimated for this condition since the pressure-drop/flow-rate relation used in this study for turbulent flow (Sections 3.1.3)

is applicable only for smooth walls. This, in turn, will lead to under estimation of $D_{opt}$. Nonisothermal conditions will cause errors in the design method since the consistency coefficient (K) depends on temperature. It decreases with increasing temperature according to the Arrhenius relationship (Cheng, 1975). Nonisothermal conditions may be caused by changes in environmental temperature or by mixing of various streams at different temperature. In addition, unsteady flow conditions may be encountered in start-up operations. Also, pressure surge waves may develop in long pipeline due to fluid inertia and compressibility (Cheng, 1975). Finally, appreciable kinetic energy changes may be found in complex tube systems with variation of tube diameter, entrances, fittings, etc. The current design method is not applicable for such systems.

# 4. RESULT AND DISCUSSION

## 4.1 Model Verification

To validate the model, the optimum diameter ($D_{opt}$) was first estimated for the example given by Skelland (1967) (Illustration 7.1 (c) pp. 253) for a power law fluid. His data, in terms of the variables and units of the model developed in this study, are given in Table 1. The $D_{opt}$ using this model was found to be 0.1653 (0.5425 ft) which is the same as the one obtained from Skelland's optimum diameter equation for power law fluids (as defined by Equation (2.2)). Notice that the answer in his illustration is different and is due to round-off error in his numerical constants. Direct calculation using his original equation (pp. 245) gave the same result. This is to be expected since Equation (3.49) is a general form of Equation (3.50) which is an equivalent form of Skelland's equation. The model was also tested using the example from Darby and Melson (1982) for a Bingham plastic fluid in turbulent flow. Their data are given in Table 2. The $D_{opt}$ for this case was found to be 0.865m which is 0.82% higher than their value. This small deviation is probably due to the fact that Darby and Melson assumed $df/dD = 0$ and used an approximation (Darby and Melson, 1981) of the friction factor relationship of Hanks and Dadia (1971) in the derivation of their model. Even though the result of Darby and Melson was close to the one obtained using the model of this study, their assumption of $df/dD = 0$ may introduce

Table 1. Fluid properties and other pertinent data for the optimum diameter example problem given by Skelland (1967)

Fluid Properties

$n = 0.5$; $K = 3.02$ Pa $s^n$; $\rho = 977.29$ kg/m$^3$

Pipe Cost Parameters

$C_p = 354.58$ \$/m$^{2.5}$; $s = 1.5$; $a = 0.14$; $b = 0.06$

Power Cost Parameters

$C_e = 2.0 \times 10^5$ \$/W hr; $h = 6570$ hrs/yr

Other Pertinent Data

$\dot{M} = 13.83$ kg/s; $L = 1523.93$ m; $E = 0.3$

Optimum Pipe Diameter

$D_{opt} = 0.1653$ m

Table 2. Fluid properties and other pertinent data for the optimum diameter example problem given by Darby and Melson (1982)

Fluid Properties

$n = 1.0$; $\eta = 0.03$ Pa s; $\tau_o = 4.2$ Pa; $\rho = 1400$ kg/m$^3$

Pipe Cost Parameters

$C_p = 409.58$ \$/m$^{2.2}$; $s = 1.2$; $a - 0.05$

Pump Station Cost Parameters

$C_I = 173800$ \$; $C_D - 0.6$ \$/W; $s' = 1.0$; $a' = 0.05$

Power Cost Parameters

$C_e = 4.0 \times 10^{-5}$ \$/W hr; $h = 8640$ hrs/yr

Other Pertinent Data

$\dot{M} = 729.16$ kg/s; $L = 1$ m; $E = 0.6$

Optimum Pipe Diameter

$D_{opt} = 0.858$ m

significant error for other fluid properties and flow conditions. This assumption is based on df/dD being much smaller than $5f/D_{opt}$ for the term $5f/D_{opt}$-df/dD which appears in Equation (3.49) if the equation is divided by $D_{opt}$. However, for power law fluids in laminar flow, df/dD can vary from 20% to 5f/D for n = 1 to 74% of 5f/D for n = 0.1. For H-B fluids, df/dD was found to be as much as 68% of 5f/D. Therefore, the assumption of df/dD << 5f/D is questionable.

In addition, the model was verified by comparing the $D_{opt}$ obtained analytically (Equation (3.49)) and graphically (Figure 2) as will be shown later.

## 4.2 Cost Parameters and Other Pertinent Data for a Pumping System

Consider a pumping system consisting of 100m of 304 stainless steel tubing with both ends at the same pressure and elevation. The tube system includes three tees (used as elbow), three 90° elbows, twenty-one union couplings, and two plug valves giving an overall fittings resistance coefficient of 10. A close coupled sanitary centrifugal pump is to be used with pump and motor (combined) efficiency of 70%. The variation of the installed tube system costs per meter length of tube with tube inside diameter are shown in Table 3. These are plotted on log-log coordinates in Figure 7. As seen, a straight line described by Equation (2.17) gives the constant $C_p$ and s shown in the figure and a regression coefficient of 0.95. The values of $C_p$ and s are also given in Table 4. In addition, the fixed (a) and maintenance (b) annual cost ratios for the tube system are presented. The values of a was estimated from Equation (2.16) assuming

Table 3. Variation of the installed cost of a tube system per meter length of tube with tube diameter. Estimated from the purchase cost (January 1985) of 100 m of Tri-Clover 304 Stainless Steel Tubes (1-3 in tubes are gauge 16, 4 in tube in gauge 14), polished ID + OD; 3 tees (7MP); 3 90° Elbows (2CMP); 3 Caps (16AMP); 2 plug valves (DIOMP); 30 Gaskets (40MP-U); 30 Clamps (13MHHM); 36 Furrales (14RMP). Ladish Company, Tri-Clover Div., Kenosha, Wisconsin. Installation costs approximated with 1.5 man-hr/joint-diameter (in) relation (Jeler, 1970), and labor cost of $35/man hr.

| Diameter | | Installed Cost |
|---|---|---|
| OD (in) | ID (m) | $/M |
| 1 | 0.0221 | 36.53 |
| 1½ | 0.0348 | 44.24 |
| 2 | 0.0475 | 56.57 |
| 2½ | 0.0602 | 76.35 |
| 3 | 0.0729 | 94.65 |
| 4 | 0.0974 | 144.71 |

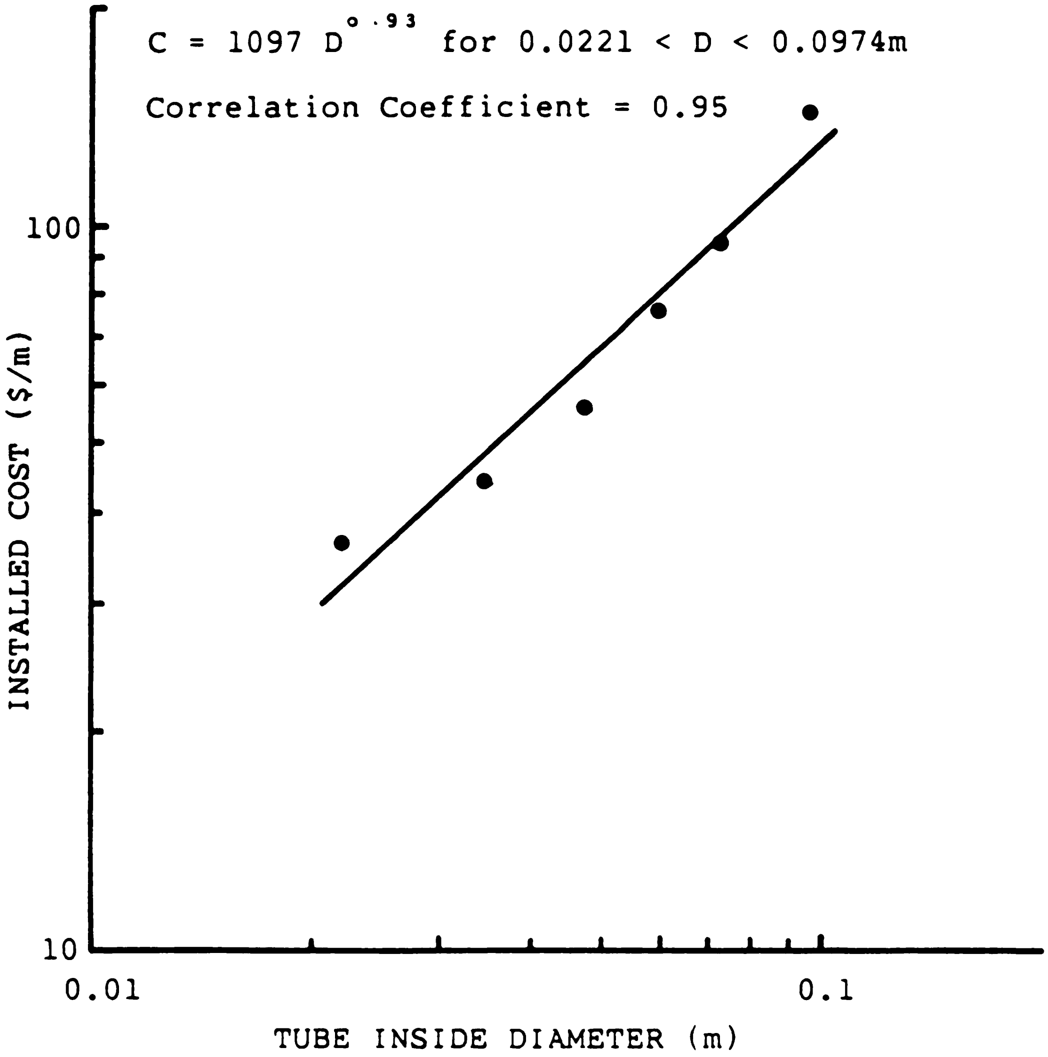


Figure 7. Variation of the installed cost of the tube system (Table 3) with tube inside diameter.

Table 4. Cost parameters for the tube system presented in Table 3. Based on Figure 7 and Equations (2.18) and (3.36)

| | | |
|---|---|---|
| $C_p$ | ($\$/m^{s+1}$) | 1097 |
| s | | 0.93 |
| a | (1/yr) | 0.18 |
| b | (1/yr) | 0.10 |

an interest rate of 12% and lifetime of 10 years. The value of b was taken as 10% of the installed cost of the tube system.

The variation of the installed costs of the pump station with pump size are shown in Table 5. These are plotted in Figure 8. As seen, a straight line (s' = 1), described by Equation (3.37) gives the constants $C_I$ and $C_D$ shown in the figure with a regression coefficient of 0.96. The valves of $C_I$, $C_D$, and s' are also shown in Table 6 along with fixed (a') and maintenance (b') annual cost ratios for the pump station. An interest rate of 12% and lifetime of 5 years were used to estimate a'. As for the tube system, b' was taken as 10% of the installed pump station. In addition, the system is to be operated 75% of the year (6,570 hrs/yr) and the electrical energy cost if 0.06 \$/kW hr. These and other pertinent data are tabulated in Table 7.

## 4.3 Optimum Diameter for a System Handling Tomato Ketchup

It is desired to determine the most economical diameter ($D_{opt}$) for transport of tomato ketchup at a mass flow rate of 4.0 kg/s. This fluid can be considered to be a homogeneous non-Newtonian fluid described by the H-B model (Higgs and Norrington, 1971). The fluid properties at 25°C are given in Table 8.

Using given variables (Tables 4, 6, 7, and 8), the $D_{opt}$ minimum cost ($C_{T_{min}}$) power (P), work (W), and pumping system costs ($C_{pi}$, $C_{pu}$, $C_{op}$) at optimum were estimated using the procedure outlined in Figures 5 and 6. The results are summarized in Table 9. Figure 9

Table 5. Variation of the installed pump station cost with power requirements. Estimated from the purchase cost (January, 1985) of Tri-Flo close-coupled sanitrary centrifugal pumps, C216 (with water cooled rotary seal) and electric motor (60 cycle 230/460 volt-3 phase), 1750 rpm for ½-2 Hp pumps and 3500 rpm for 1-15 Hp pumps. ("Easy-Clean" totally-enclosed motor), Ladish Company, Tri-Clover Division, Denosha, Wisconsin. Installation cost taken as 25% of the total purchase cost (Peters and Timmerhaus, 1968).

| Power | | Installed Cost |
|---|---|---|
| Hp | Watts | ($) |
| 1/2 | 372.9 | 1313 |
| 3/4 | 559.3 | 1348 |
| 1 | 745.7 | 1366 |
| 1 1/2 | 1118.6 | 1384 |
| 2 | 1491.4 | 1484 |
| 2 | 1491.4 | 1414 |
| 3 | 2237.1 | 1791 |
| 5 | 3728.5 | 1876 |
| 7 1/2 | 5592.8 | 2175 |
| 10 | 7457.0 | 2225 |
| 15 | 11185.5 | 2686 |

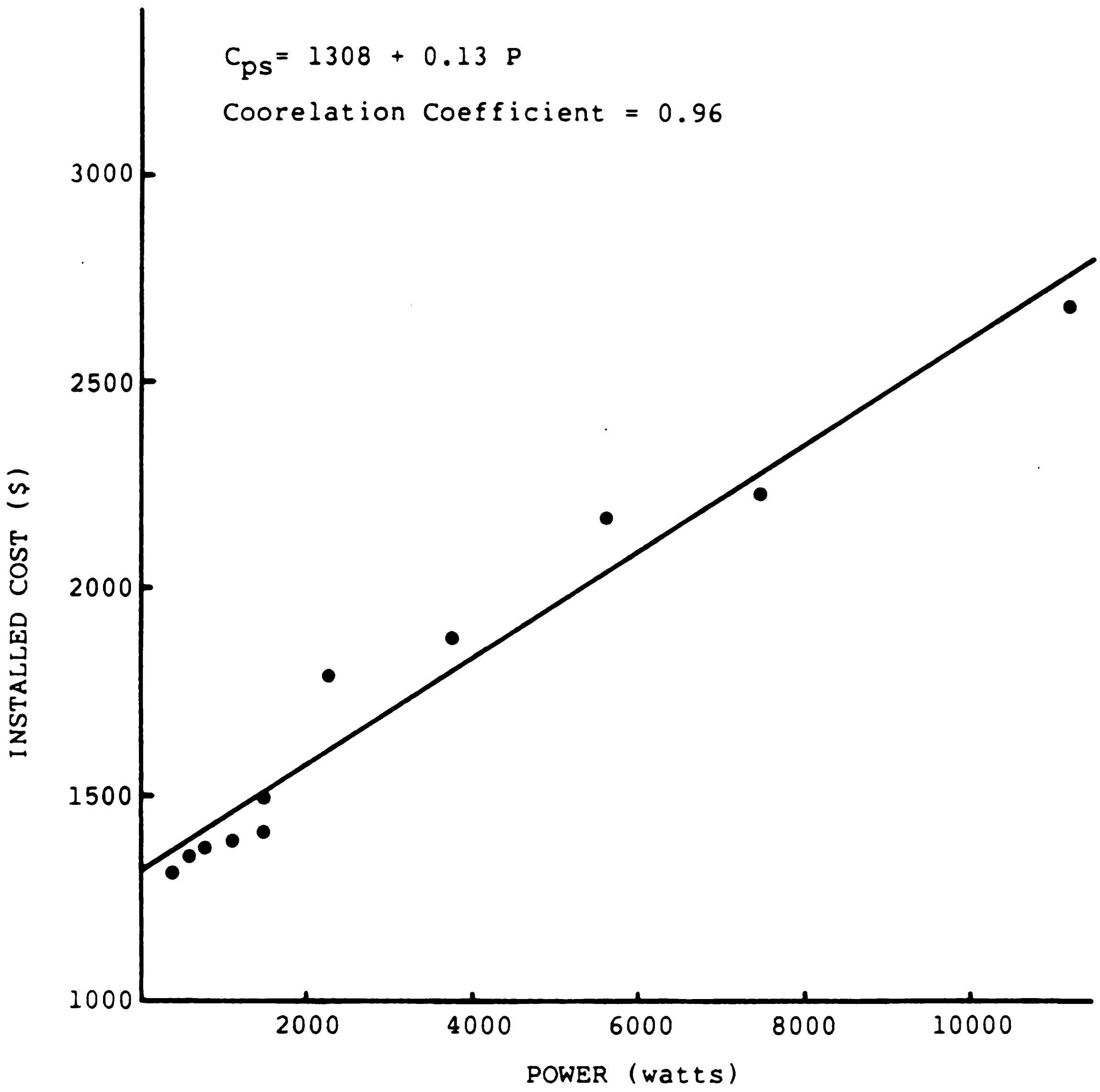


Figure 8. Variation of the installed cost of the pump station (Table 5) with power requirements (linear relationship).

Table 6. Cost parameters for the pump station presented in Table 5. Based on Figure 8 and Equations (3.37) and (3.38)

| | | |
|---|---|---|
| $C_I$ | (\$) | 1308 |
| $C_D$ | \$/W | 0.13 |
| s' | | 1.0 |
| a' | (1/yr) | 0.28 |
| b' | (1/yr) | 0.10 |

Table 7. Electrical energy cost, hours of operations per year, combined pump and motor efficiency, summation of the fittings resistance coefficients, tube length, pressure and elevation change, and mass flow rate used to estimate the costs, and optimum diameter for the pumping system presented in Table 3 and 5

| | | |
|---|---|---|
| $C_e$ | (\$/W hr) | $6.0 \times 10^{-5}$ |
| h | (hrs/yr) | 6570 |
| E | | 0.70 |
| $\Sigma K_f$ | | 10.0 |
| L | (m) | 100.0 |
| $\Delta p$ | (Pa) | 0 |
| $\Delta z$ | (m) | 0 |
| $\dot{M}$ | (kg/s) | 4.0 |

Table 8. Rheological properties (Higgs and Norrington, 1971) and density (Lopez, 1981) for tomato ketchup at 25°C

| | | |
|---|---|---|
| $n$ | | 0.27 |
| $K$ | (Pa $s^n$) | 18.7 |
| $\tau_o$ | (Pa) | 32.0 |
| $\rho$ | ($kg/m^3$) | 1110.0 |

Table 9. Optimum economic tube diameters, pumping system costs, and work and power requirements, at optimum for a system (Tables 4, 6, and 7) transporting tomato ketchup with properties given in Table 8

| | | |
|---|---|---|
| $D_{opt}$ | (m) | 0.06907 |
| $C_{Tmin}$ | (\$/yr m) | 45.70 |
| $C_{pi}$ | (\$/yr m) | 25.58 |
| $C_{pu}$ | (\$/yr m) | 6.66 |
| $C_{op}$ | (\$/yr m) | 13.46 |
| W | (J/kg) | 597.81 |
| p | (k Watts) | 3.42 |

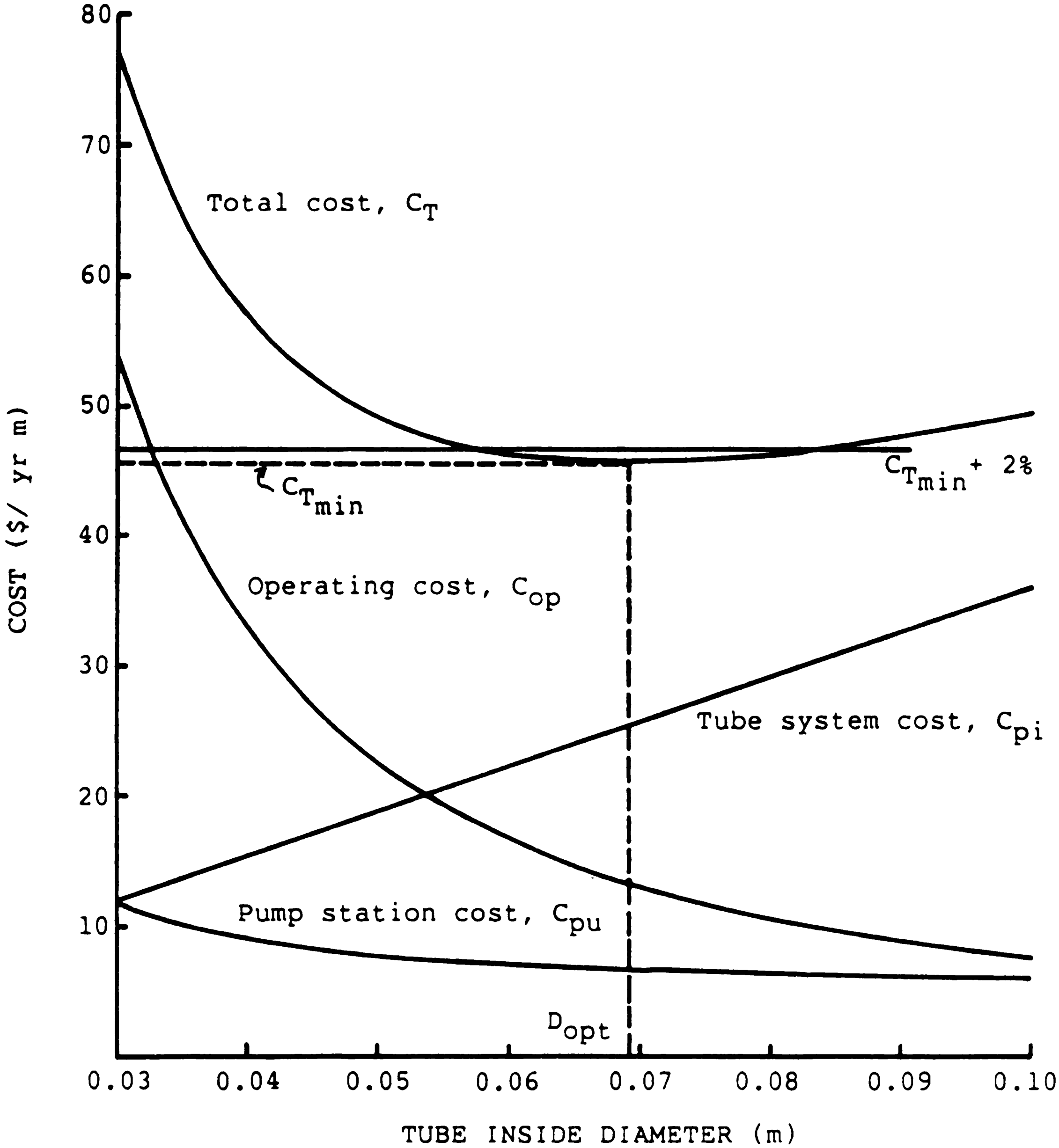


Figure 9. Variation of tube system cost, pump station cost, operating cost, and total cost with tube inside diameter for a system (Table 4, 6, and 7) transporting tomato ketchup with properties given in Table 8.

shows the variation of the costs with tube inside diameter. As seen, $D_{opt}$ obtained graphically and analytically was 0.06907m for a $C_{T_{min}}$ of 45.70 \$/yr m or 5¢/ton of tomato ketchup pumped annually. P was found to be 3.42 kW (4.58Hp). It can also be seen that tube diameter between 0.0585m and 0.0835m results in total costs which does not exceed the minimum value by more than 2% and that the deviation from minimum increases more rapidly as the diameter decreased. The Reynolds number was found to be less than the critical Reynolds number for $D_{opt}$, hence the flow was laminar as seen in Table 10. These values (Table 10) were estimated following the scheme shown in Figure 4 with the program given in Appendix 7.4.

## 4.4 Sensitivity Analysis

This section is devoted to study the sensitivity of $D_{opt}$ on the various input variables shown in Figure 6. This was done by estimation the percent change of $D_{opt}$ obtained using a ±10% value of each variable. Even though the analysis is mostly based on the example of Section 4.3, some general insight can be obtained on the relative importance of the cost components of the pumping system other variables in determining $D_{opt}$. The percent changes of $D_{opt}$ for each of the variables are shown in Table 11. The change in the $C_{pu}$ variables, with the excpetion of s', resulted in small changes of $D_{opt}$. This is due to the small variation of $C_{ps}$ with P obtained in Figure 8. Changing s' just changed the nature of this relationship. The variation of the variables of $C_{pi}$ and $C_{op}$ resulted in considerable changes of $D_{opt}$. The greater influence of $C_{pi}$ and $C_{op}$ on the $D_{opt}$ can also be

Table 10. Flow condition of $D_{opt}$ = 0.06907 m for the pumping system handling 4.0 kg/s of tomato ketchup with properties given in Table 8

| | |
|---|---|
| Re | 107 |
| He | 8.85 |
| f | 0.2214 |
| $\xi_o$ | 0.2815 |
| $Re_c$ | 2754.0 |
| $f_c$ | 0.007506 |

Table 11. Percent change of $D_{opt}$ with ±10% change of imput variable for the example presented in Section 4.3

| Percent change of variable | | -10% | +10% |
|---|---|---|---|
| | Variables | Percent Change of $D_{opt}$ | |
| Fluid Properties | $n$ | -5.11 | +5.17 |
| | $K$ | -3.04 | +2.88 |
| | $\tau_o$ | -0.80 | +0.78 |
| | $\rho$ | +5.07 | -4.39 |
| Pumping System | $\dot{M}$ | -4.91 | +4.65 |
| | $L$ | +0.07 | -0.07 |
| | $\Sigma K_f$ | -0.09 | +0.07 |
| | $\Delta p$ | -- | -- |
| | $\Delta z$ | -- | -- |
| Tube system cost ($C_{pi}$) | $C_p$ | +4.11 | -3.58 |
| | $s$ | -5.52 | +5.82 |
| | $a$ | +2.56 | -2.36 |
| | $b$ | +1.39 | -1.33 |
| Pump Station Cost ($C_{pu}$) | $C_I$ | -- | -- |
| | $C_D$ | -0.43 | +0.42 |
| | $s'$ | -2.62 | +5.94 |
| | $a'$ | -0.32 | +0.30 |
| | $b'$ | -0.12 | +0.10 |
| Operating Cost ($C_{op}$) | $E$ | +4.11 | -3.58 |
| | $C_e$ | -3.49 | +3.30 |
| | $h$ | -3.49 | +3.30 |

noticed in Figure 9. $C_{pu}$ has less influence on $D_{opt}$ compared to $C_{op}$. An economic balance on $C_{pi}$ and $C_{op}$ along gave a $D_{opt}$ of 0.06602 m which is 4.42% lower than the value found when $C_{pu}$ was considered. For the example to Table 2, $D_{opt}$ excluding $C_{pu}$ was found to be 0.855 m, 1.17% lower than the value found when $C_{pu}$ was included. The changes of a and b did not effect the results as much as $C_p$ and s. A change of ±10% in a represents an appropriate change of ±20% in the life-time (N) or ±25% change in interest rate (i). These variations on N and i result in less than ±3% change on $D_{opt}$. From Equation (3.49), it can be observed that where using a linear relationship (s' = 1) for $C_{ps}$ vs. P, the $D_{opt}$ is independent of the pressure energy change ($\Delta p$) and the elevation change ($\Delta z$). Even if s' is not equal to one, $D_{opt}$ can be assumed to be independent of $\Delta p$ and $\Delta z$, since $C_{pu}$ generally varies little with P. To show this, the data in Table 5 were fitted to the curve shown in Figure 10. The value of $C_D$, $C_I$, and s' for this curve are given in Table 12. The $D_{opt}$ using these constants and $\Delta z = \Delta p = 0$ was found to be 0.06902 m which is only 0.07% lower than the value found using the linear relationship of Figure 8. For $\Delta z$ = 20 m and $\Delta p$ = 0, the $D_{opt}$ was found to be 0.46% lower. This variation was also obtained for $\Delta z$ = 0 and $\Delta p$ = 217.78 kPa (2.15 atm) which shown the small influence of $\Delta p$ and $\Delta z$ have on $D_{opt}$.

The changes on L and $\Sigma K_f$ also produced small changes on $D_{opt}$; hence, using the constant $K_f$ value of Newtonian fluids in turbulent flow for approximating non-Newtonian fluid behavior will introduce negligible error. As seen from Equation (3.49), $D_{opt}$ can also be

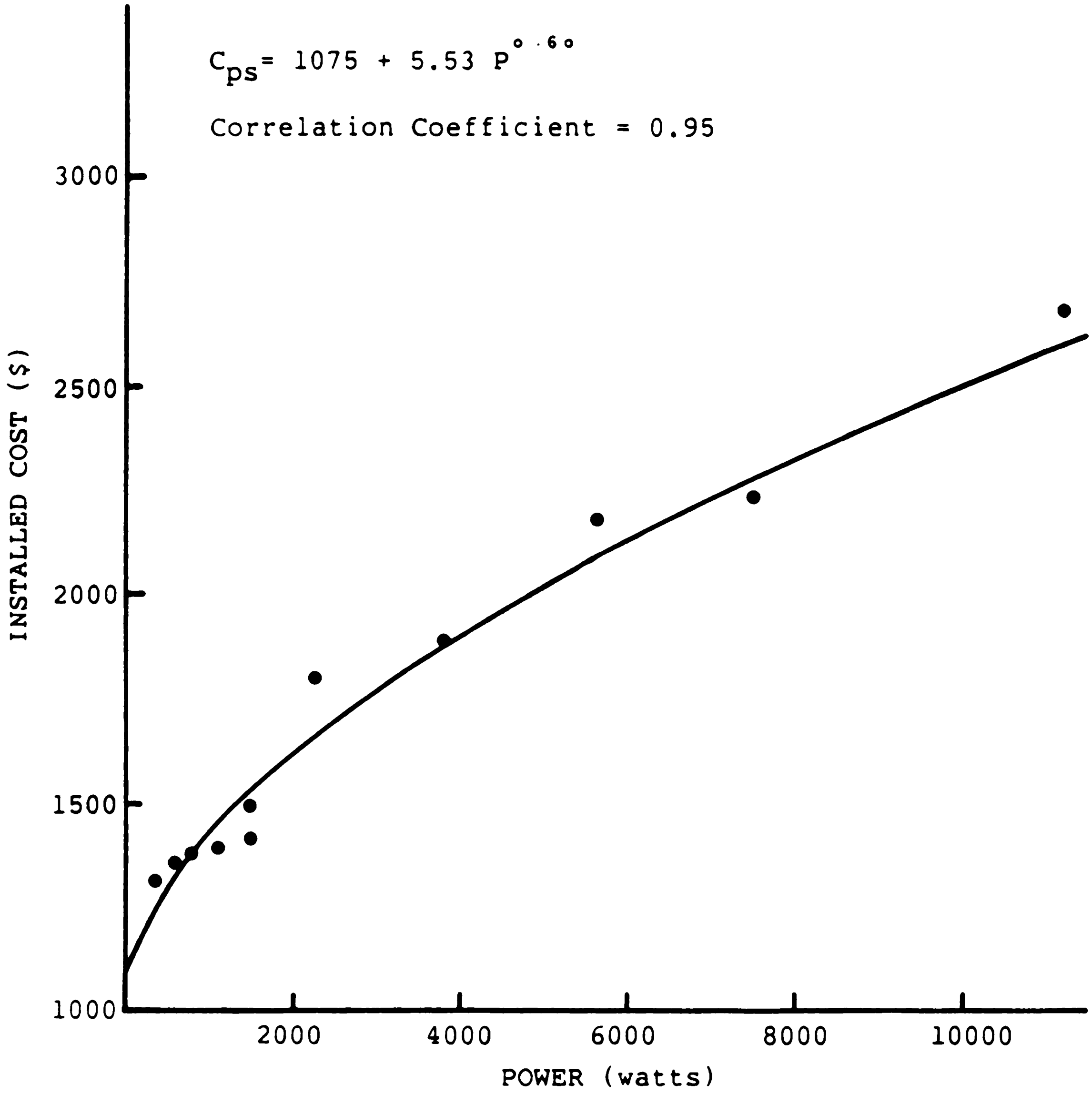


Figure 10. Variation of the installed cost of the pump station (Table 5) with power requirements (non-linear relationship).

Table 12. Cost constants of Equation (3.37) for the pump station presented in Table 5 based on Figure 10

| | | |
|---|---|---|
| $C_I$ | (\$) | 1075 |
| $C_D$ | (\$/$W^{s'}$) | 5.53 |
| s' | | 0.60 |

assumed to be independent of $K_f$ if the tube length is much greater than the $D_{opt}$ expected. Otherwise, the summation of the resistance coefficient per unit length of pipe ($\Sigma K_f/L$) can be used as an approximation without greatly effecting the results. Notice also that if $s' = 1$ and $L \gg D_{opt}$ or an approximation of $\Sigma K_f/L$ is used, $D_{opt}$ is also independent of L. In other words, $D_{opt}$ can be estimated from the costs of a unit length of tube/pipe (e.g., one meter). The small effect of the error of $\Delta p$, $\Delta z$, $\Sigma K_f$, and L on $D_{opt}$ is of great value since these variables are usually not well known in preliminary sizing of a pipe system. Finally, as seen in Table 11, ±10% change in the fluid properties and mass flow rate, except for the yield stress, resulted in considerable change on $D_{opt}$.

## 4.5 Optimum Diameter Using Apparent Viscosities

The optimum diameter ($D_{opt}$), pumping system costs, power requirements and work requirements to transport tomato ketchup were estimated assuming Newtonian flow behavior and apparent viscosities ($\mu_a$) of 4.723 Pa s and 1.715 Pa s to show the problems that may arise from this practice. The value of $\mu_a$ were calculated from Equation (2.6) using a rates of shear of $15s^{-1}$ and $50\ s^{-1}$, respectively, to simulate point measurement (such as those which might be made with a Brookfield viscometer) at these shear rates. The results, using $\mu_a$ = 4.723 Pa s, are shown in Table 13 and compared to the results of Table 9 (Section 4.3). As seen, as $D_{opt}$ was over estimated significantly given a $C_{T_{min}}$ 20.63% higher and a P value 37.56% lower. However, this $D_{opt}$ does not give the actual minimum as seen in Table 14 which illustrates the actual pumping system costs, and work and power requirements estimated using the H-B model at D = 0.1138 m (the $D_{opt}$ obtained using $\mu_a$ = 4.723 Pa s). As seen, D = 0.1138 m gives a total cost which deviates from the minimum by 15.4%. In addition, the actual power requirement is 33.78% lower than the one estimated using the apparent viscosity. If the pumping system was designed using this apparent viscosity (4.723 Pa s), the tube system cost ($C_{pi}$) and pump station cost ($C_{pu}$) would be estimated to be 40.7 \$/yr m and 6.02 \$/yr m for a tube size and pump size of 0.1138 m and 2.13 kWatts, respectively (Table 13). However, the operating cost would be 6.28 \$/yr m since the actual power requirement at D = 0.1138m is 1.59 kWatts (Table 14). So the total cost for this system would be 53.0 \$/yr m which is 15.97% higher than the one in Table 9. In addition, the system would have a oversized (hence, less efficient) pump.

Table 15 shows the results obtained using $\mu_a$ = 1.715 Pa s and a comparison with the values of Table 9. The $D_{opt}$ for this case was found to be 0.09271 m, 34.23% higher than the one obtained in Table 9.

Table 13. Optimum economic tube diameter, pumping system costs, and work and power requirements at optimum estimated assuming Newtonian flow behavior and an apparent viscosity of 4.723 Pa s

| | Results for $\mu_a$ = 4.723 Pa-s | % Difference with the results of Table 9 |
|---|---|---|
| $D_{opt}$ (m) | 0.1138 | +64.76 |
| $C_{T_{min}}$ (\$/yr m) | 55.13 | +20.63 |
| $C_{pi}$ (\$/yr m) | 40.7 | +59.11 |
| $C_{pu}$ (\$/yr m) | 6.02 | - 9.61 |
| $C_{op}$ (\$/yr m) | 8.41 | -37.56 |
| W (J/kg) | 373.25 | -37.56 |
| p (kWatts) | 2.13 | -37.56 |

Table 14. Pumping system costs, and work and power requirements estimated using the H-B model for D = 0.1138 m

| | | |
|---|---|---|
| $C_T$ | (\$/yr m) | 52.74 |
| $C_{pi}$ | (\$/yr m) | 40.7 |
| $C_{pu}$ | (\$/yr m) | 5.76 |
| $C_{op}$ | (\$/yr m) | 6.28 |
| W | (J/kg) | 279.0 |
| P | (kWatts) | 1.59 |

Table 15. Optimum economic tube, diameter, pumping system costs, and work and power requirements at optimum estimated assuming Newtonian flow behavior and an apparent viscosity of 1.715 Pa-s

| | | Results for $\mu_a$ = 1.715 Pa-s | % Difference with Results of Table 9 |
|---|---|---|---|
| $D_{opt}$ | (m) | 0.09271 | +34.23 |
| $C_{T_{min}}$ | (\$/yr m) | 46.42 | + 1.58 |
| $C_{pi}$ | (\$/yr m) | 33.63 | +31.47 |
| $C_{pu}$ | (\$/yr m) | 5.84 | -12.31 |
| $C_{op}$ | (\$/yr m) | 6.95 | -48.40 |
| W | (J/kg) | 308.49 | -48.40 |
| P | (kWatts) | 1.6 | -48.40 |

Again, this $D_{opt}$ does not give the actual minimum as seen in Table 16. In this case, the total cost deviated from minimum by 5.51%. From Tables 15 and 16, it can also be seen that the actual power requirement at D = 0.09271 m is 18.7% higher than the one estimated using the apparent viscosity. If the tube size and pump size were to be selected, base or Table 15, the pumping system would have a undersized pump uncapable of meeting the actual operating conditions. Therefore, the pump would have to be replaced or the operating time would have to be increased resulting in a more expensive system.

As these two examples show, the use of $\mu_a$ and Newtonian flow behavior to design non-Newtonian handling systems may lead to errors depending on the rate of shear at which $\mu_a$ was measured.

### 4.6 Optimum Diameter for a System Handling a Herschel-Bulkley Fluid in Turbulent Flow

A problem was selected to test the optimum diameter model for a H-B fluid that resulted in a $D_{opt}$ for which the flow was turbulent. For this purpose, the $D_{opt}$ was estimated for a hypthetical H-B fluid with properties given in Table 17 and the cost data shown in Tables 4, 6, and 7. The results at $C_{T_{min}}$ are shown in Table 18. The flow condition for $D_{opt}$ was found to be turbulent (Table 19) and the variation of the costs with D is shown in Figure 11. The $D_{opt}$ obtained graphically and analytically was found to be 0.04065 m for a $C_{T_{min}}$ of 24.16 \$/yr m confirming Equation (3.49) for turbulent flow. The diameter range, which $C_T$ did not exceed the $C_{T_{min}}$ by more

Table 16. Pumping system costs, and work and power requirements estimated using the H-B model for D = 0.09271 m

| | | |
|---|---|---|
| $C_T$ | (\$/yr m) | 48.22 |
| $C_{pi}$ | (\$/yr m) | 33.63 |
| $C_{pu}$ | (\$/yr m) | 6.04 |
| $C_{op}$ | (\$/yr m) | 8.55 |
| W | (J/kg) | 379.46 |
| P | (kWatts) | 2.17 |

Table 17. Rheological properties and density for a hypothetical Herschel-Bulkley fluid

| | |
|---|---|
| n | 0.70 |
| K (Pa $s^n$) | 0.03 |
| $\tau_o$ (Pa) | 2.0 |
| $\rho$ (kg/$m^3$) | 1400.0 |

Table 18. Optimum economic tube diameter pumping system costs and work and power requirements, power at optimum for a system (Table 4, 6, and 7) transporting a H-B fluid with properties given in Table 17

| | | |
|---|---|---|
| $D_{opy}$ | (m) | 0.04065 |
| $C_{T_{min}}$ | ($/yr m) | 24.16 |
| $C_{pi}$ | ($/yr m) | 15.62 |
| $C_{pu}$ | ($1/yr m) | 5.37 |
| $C_{op}$ | ($/yr m) | 3.17 |
| W | (J/kg) | 140.68 |
| P | kWatts | 0.80 |

Table 19. Flow condition at $D_{opt}$ = 0.04065 m for the pumping system handling 4.0 kg/s of a H-B fluid with properties given in Table 17

| | |
|---|---|
| Re | $2.40 \times 10^4$ |
| He | $1.88 \times 10^5$ |
| f | 0.004883 |
| $\xi_o$ | 0.1207 |
| $Re_c$ | $6.34 \times 10^4$ |
| $f_c$ | 0.08200 |

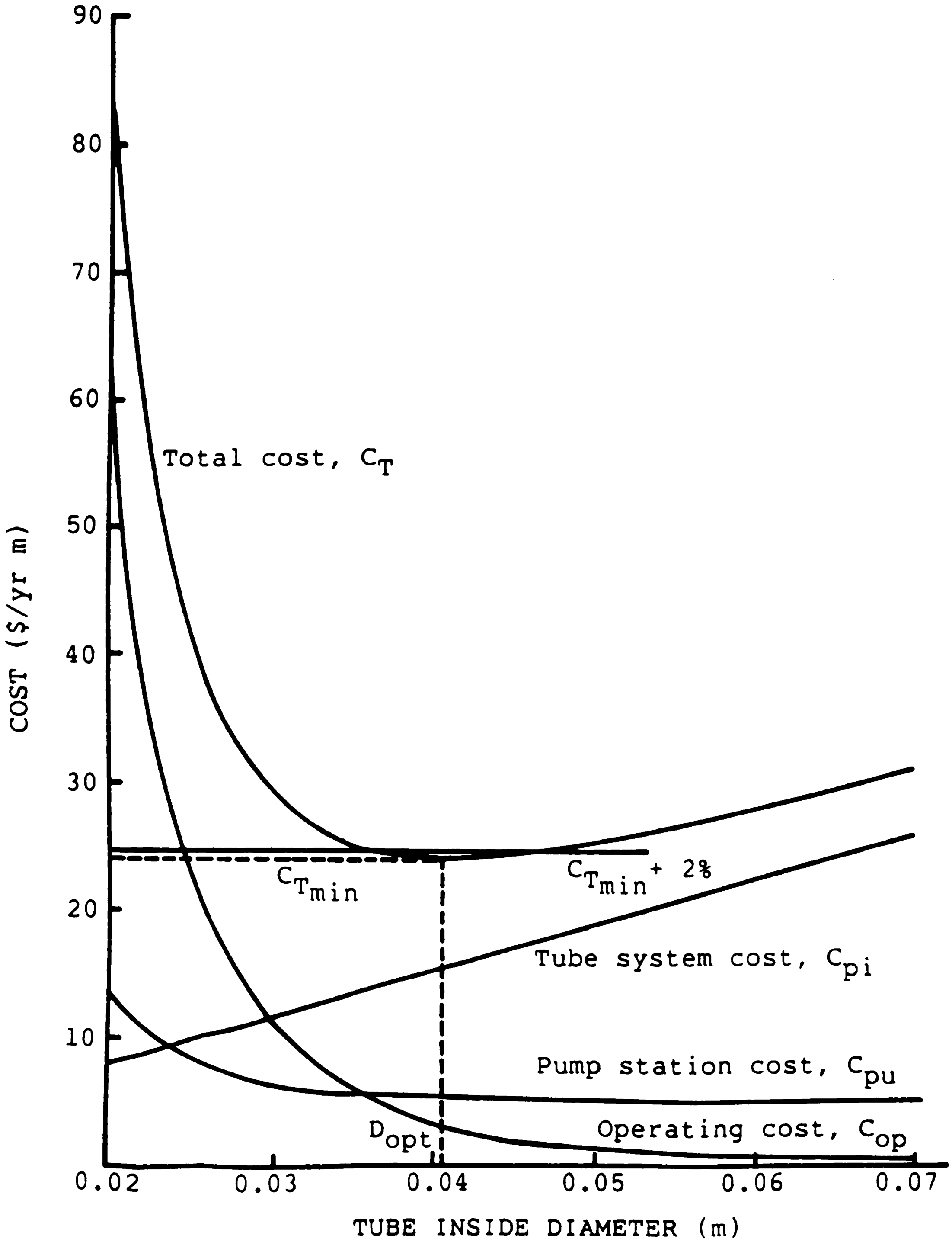


Figure 11. Variation of tube system cost, pump station cost, operating cost, and total cost with tube inside diameter for a system (Tables 4, 6, and 7) transporting a H-B fluid with properties given in Table 17.

than 2%, was 0.0365 m to 0.046 m, which is smaller than the example of Section 4.3. As observed in the previous example, the total cost deviated from minimum most slowly as the diameter increased (Figure 11). This rate of increase is practically given by $C_{pi}$ as seen from the similarity of the slopes of the $C_T$ and $C_{pi}$ curves for $D > D_{opt}$.

# 5. SUMMARY AND CONCLUSIONS

1. An equation to determine the total annual cost of a pumping system as a function of tube diameter (based on the costs of the tube system, pump station, and operation) has been developed for system handling Herschel-Bulkley fluids under laminar, transitional, or turbulent flow condition.

2. An equation to determine the optimum economic tube diameter has been developed for pumping systems handling Herschel-Bulkley fluids under laminar, transitional, or turbulent flow conditions.

3. The pump station cost had less influence than the operating cost in determining the optimum economic tube diameter.

4. The optimum economic tube diameter is independent of any elevation difference ($\Delta z$) and pressure energy difference ($\Delta p$) in the system if a linear relationship ($s' = 1$) is used to correlate the pump station cost to power requirements. In addition, $\Delta z$ and $\Delta p$ do not have to be known accurately if the variation of the pump station cost with power is small.

5. The optimum ecnomic tube diameter can be obtained from the pumping system costs of a unit length of tube if a linear relationship is used to correlate the pump station cost to the power requirements ($s' = 1$) and the length of the tube system is much

greater than the tube diameter ($L >> D_{opt}$) or the frictional loss in fittings and values is approximated as the summation of the fittings resistance coefficient per unit length of tube.

6. The use of apparent viscosity and Newtonian flow behavior for non-Newtonian fluids caused significant errors in the estimation of the optimum economic tube diameter, total annual cost, and power requirements of the pumping system.

APPENDICES

# APPENDIX A

## ALTERNATIVE EQUATIONS FOR f AND df/dD FOR H-B FLUIDS IN TURBULENT FLOW

## APPENDIX A

## ALTERNATIVE EQUATIONS FOR f AND df/dD FOR H-B FLUIDS IN TURUBLENT FLOW

Torrance (1967) developed a friction factor relationship for H-B fluids in turbulent flow as

$$\frac{1}{\sqrt{f}} = 0.45 - \frac{2.275}{n} + \frac{1.97}{n} \ln (1 - \xi_0)$$

$$+ \frac{1.97}{n} \ln \left[ Re \; {\frac{1+3n}{n}}^{n} \; f^{1-n/2} \right] \qquad (A.1)$$

where

Re and $\xi_0$ are given by Equations (3.17) and (3.15), respectively

Combining the definition of f, Re, and He, Equation (3.15) can be rewritten as

$$\xi_0 = \frac{16 \, (2He)^{\frac{2}{2-n}} \left(\frac{n}{1+3n}\right)^{\frac{2n}{2-n}}}{Re^{\frac{2}{2-n}} \; f} \qquad (A.2)$$

Equations (A.1) and (A.2) gives the friction factor as a function of Re, He, and n. Replacing $\bar{v}$ with Equation (3.34) in Equation (3.17) and $\xi_0$ with Equation (3.15) in Equation (7.1), the derivative of f with respect to D from Equation (7.1) gives

$$\frac{df}{dD} = \frac{3.94\ [4 - 3n\ (1 - \xi_0)]\, f^{3/2}}{\left[3.94 f^{\frac{1}{2}} + (1.-1.97 f^{\frac{1}{2}}) n (1-\xi_0)\right] D} \tag{A.3}$$

This equation can be used instead of Equation (3.48) if the Torrance relationship (Equation (A.1)) is used to estimate the fanning friction factor in turbulent flow. However, it is not clear what laminar-turbulent criterium should be used with the Torrance equation. Equation (A.3) was confirmed in the same manner as Equation (3.46).

# APPENDIX B

# VERIFICATION OF df/dD EQUATION FOR LAMINAR FLOW

APPENDIX B

VERIFICATION OF df/dD EQUATION FOR LIMINAR FLOW

The friction factor relation for power law fluids in laminar flow is given by Equation (3.16) with $\psi|_{\xi_{o=1}} = 1$ as

$$f = \frac{16}{Re} \tag{B.1}$$

where

Re is defined by Equation (3.17)

When Equation (B.1) is differentiated with respect to D, it yields

$$\frac{df}{dD} = \frac{16\ (4-3n)}{ReD} \tag{B.2}$$

When a value of zero for $\xi_o$ ($\tau_o = 0$) is substituted into Equation (3.46), it reduces to Equation (B.2), indicating that Equation (3.46) is correct for the special case of the power law fluid. Equation (B.2) was also obtained by Darby and Melson (1982), and indirectly by Skelland (1967).

For the Bingham plastic fluid, the friction factor is given by Equation (3.16) with $K = \eta$, and Re and $\psi$ evaluated at $n = 1$ as

$$f = \frac{16}{Re\big|_{n=1}\ \psi\big|_{n=1}} \tag{B.3}$$

where

$$Re\Big|_{n=1} = \frac{D\bar{v}\rho}{\eta} \tag{B.4}$$

and

$$\psi\big|_{n=1} = 1 - \frac{4}{3}\,\xi_o + \frac{1}{3}\,\xi_o^4 \tag{B.5}$$

When Equation B.6) is differentiated with respect to D, it gives

$$\frac{df}{dD} = \frac{64\xi_o\left[\dfrac{4\ (1 - \xi_o^3)}{3\ \psi\big|_{n=1}}\right] + 16}{Re\big|_{n=1} \cdot \psi\big|_{n=1} \cdot D \cdot \left[1 + \left[\dfrac{4\ (1-\xi_o^3)}{3\ \psi\big|_{n=1}}\right]\xi_o\right]} \tag{B.6}$$

If one evaluates Equation (3.45) at n=1, it can be shown that

$$\sigma\big|_{n=1} = \frac{4\ (1 - \xi_o^3)}{3\ \psi\big|_{n=1}} \tag{B.7}$$

Then if n=1 is substituted in Equation (3.46), it reduces to Equation (B.6) which shows that Equation (3.46) is correct for the special case of the Bingham plastic. Similar results are found when considering the solution for a Newtonian fluid. In addition to the method just outlined, Equation (3.46) was confirmed numerically using Equation (3.48). The properties of tomato ketchup (Table 8) and the H-B fluid given in Table 17 along with mass flow rate of 4.0 Kg/s were considered. These fluid properties and flow condition were also used for the examples given in Section 4.3 and 4.6, respectively. The variations of the friction factor with diameter for tomato ketchup and the H-B fluid are given in Figures 12 and 13, respectively. These curves can be obtained using the scheme shown in Figure 4. The analytical value obtained for df/dD at D = 0.06907m (the $D_{opt}$ found in Section 4.3) was found to be 11.0585. Using x = 0.0001 in Equation (3.48), the numerical value was found to be 11.0586 which is only 0.001% higher than the analytical one. For the H-B fluid (Table 17), the analytical value of df/dD at D = 0.1m was found to be 1.045, 0.02% lower than the numerical value (1.0452) obtained using x = 0.001. It is clear that the analytical results are very close to the numercial results and small differences can be attributed to the limitations associated with the numerical solution technique.

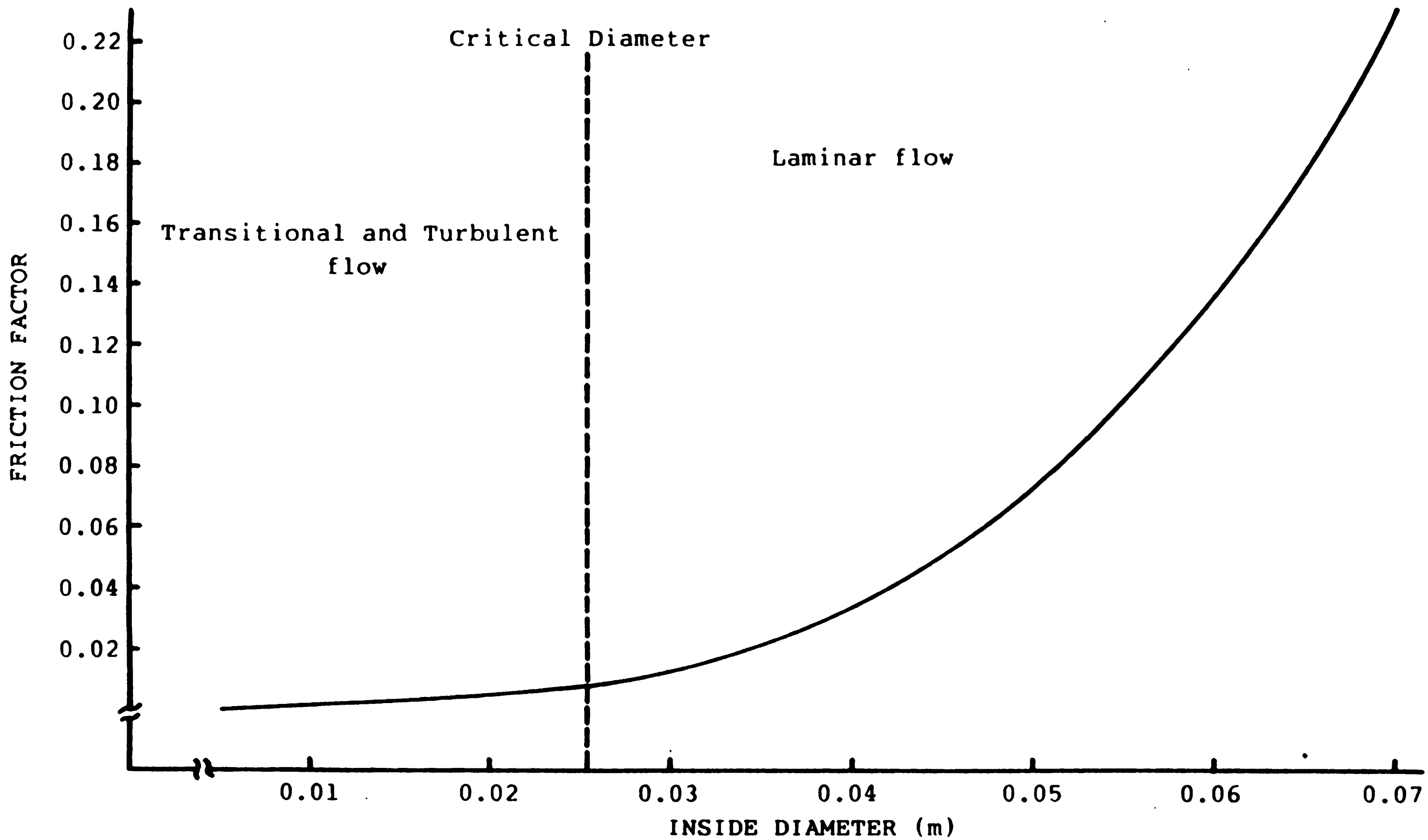


Figure 12. Friction factor versus tube inside diameter for the fluid properties given in Table 8 and a mass flow rate of 4.0 kg/s.

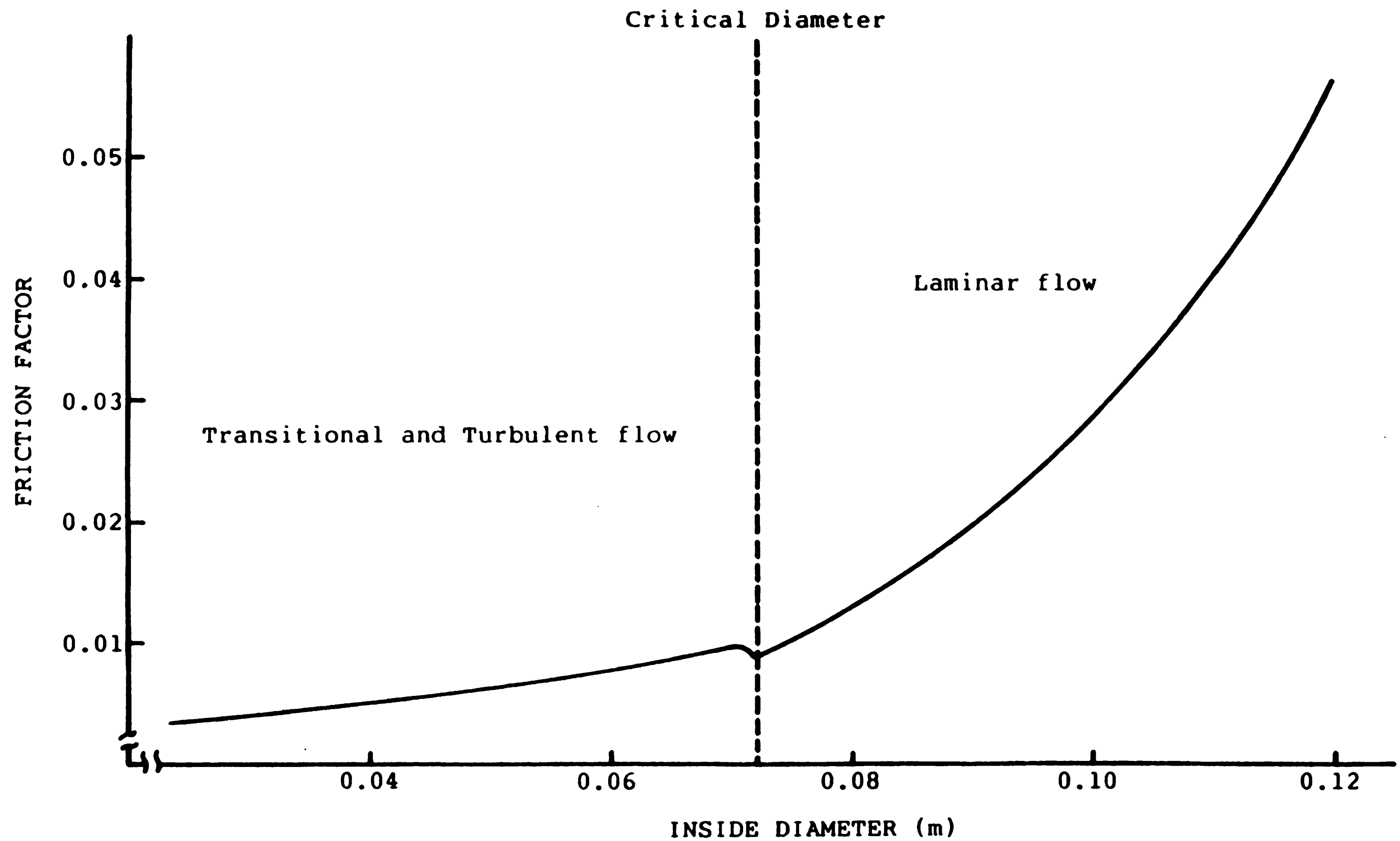


Figure 13. Friction factor versus tube inside diameter for the fluid properties given in Table 17 and a mass flow rate of 4.0 kg/s.

APPENDIX C

APPROXIMATION OF df/dD FOR TURBULENT FLOW

## APPENDIX C

### APPROXIMATION OF df/dD FOR TURBULENT FLOW

As seen from Figure 14 and 15, the variation of the friction factor in the turbulent region is small compared to the laminar region for these examples. However, higher variation, similar to the laminar region, may be found for higher values of He. Since the numerical solutions [using Equation (3.48)] of df/dD for the x values used gave a good approximation of the analytical solution in the laminar region, it is expected that the same will be true for the turbulent region. The following values of x are therefore suggested:

| Diameter (m) | x |
|---|---|
| $0.001 \leq D < 0.01$ | 0.00001 |
| $0.01 \leq D < 0.10$ | 0.0001 |
| $0.10 \leq D < 1.00$ | 0.001 |
| $1.00 \leq D < 10.00$ | 0.01 |

The value of df/dD for D = 0.04065m, the $D_{opt}$ obtained in the example of Section 4.6, was found to be 0.10929. The friction factor may increase and then decrease for the region just below the diameter where turbulent flow start (Figure 15). The backward difference method with the above values of x can be used to evaluate df/dD in

this region. However, designing so close to the laminar-turbulent transition is not recommended due to the unstability of the flow and variation in frictional pressure losses.

APPENDIX D

LISTING OF COMPUTER PROGRAM

APPENDIX D

LISTING OF COMPUTER PROGRAM

```
      PROGRAM FFTCOD(INPUT,OUTPUT,TAPE10=INPUT,TAPE20=OUTPUT)
C
C     WRITTEN BY   EDGARDO J. GARCIA-CAES
C     COMPLETED    MARCH, 1985
C
C
C THIS PROGRAM PERFORMS THE FOLLOWING FUNCTIONS:
C
C   1.  CALCULATES THE FRICTION FACTOR IN LAMINAR, TRANSITIONAL OR
C       TURBULENT FLOW GIVEN DE,DIM,K,MFR,N AND YS
C   2.  CALCULATES THE PUMPING SYSTEM COSTS AND WORK AS A FUNCTION
C       OF DIM GIVEN API,APP,BPI,BPP,CD,CEP,CHEL,CHPS,CI,CP,DE,EFF,
C       HR,K,LEGT,MFR,N,PPI,PPP,SUFFC AND YS
C   3.  CALCULATES THE OPTIMUM ECONOMIC DIAMETER GIVEN API,APP,
C       BPI,BPP,CD,CEP,CHEL,CHPS,CI,CP,DE,EFF,HR,LEGT,MFR,N,PPI,PPP,
C       SUFFC AND YS
C
C   THE PROGRAM FIRST GIVES THE FOLLOWING OPTIONS: 1- TO
C   ESTIMATE THE FANNING FRICTION FACTOR, 2- TO GENERATE COSTS
C   V.S. DIAMETER DATA.  TO ESTIMATE THE OPTIMUM PIPE DIAMETER,
C   ONE MUST FIRST ESTIMATE THE COSTS FOR VARIOUS DIAMETERS(OPTION 2) IN
C   ORDER TO SELECT THE RANGE OF THE DIAMETER WHERE THE OPTIMUM DIAMETER
C   IS LOCATED. THAT IS, A RANGE OF DIAMETERS THAT CONTAIN THE MINIMUM
C   TOTAL COST. AFTER OPTION 2 IS COMPLETED, THE PROGRAM GIVES THE
C   FOLLOWING OPTIONS: 1- TO START THE PROGRAM, 2- TO ESTIMATE THE
C   OPTIMUM DIAMETER,3- TO CONTINUE WITH A NEW RANGE OF DIAMETERS (E.G.
C   IF THE RANGE WHERE THE OPTIMUM DIAMETER IS LOCATED HAS NOT BEEN
C   FOUND), 4- TO EXIT THE PROGRAM. AFTER THE OPTIMUM DIAMETER HAS BEEN
C   FOUND, THE PROGRAM GIVES THE FOLLOWING OPTIONS: 1- TO PRINT THE COSTS
C   V.S. DIAMETER DATA GENERATED DURING THE OPTIMUM DIAMETER
C   ITERATION, 2- TO EXIT THE PROGRAM.  TO ESTIMATE THE PUMPING
C   SYSTEM COSTS AND WORK FOR A GIVEN DIM, START THE PROGRAM
C   AND ENTER OPTION 2.  THEN, WHEN ASKED FOR THE DIAMETER RANGE,
C   ENTER THE SAME VALUES FOR THE DIM LOWER & UPPER BOUNDS AND ONE
C   FOR THE NUMBER OF DATA POINTS.
C   N MUST BE GREATER THAN 0. AND LESS THAN 2.0 FOR TURBULENT FLOW
C   ALL EQUATION NUMBERS IN COMMENT STATEMENTS REFER TO THE ONES IN
C   THE THESIS.
C
C   LIST OF VARIABLES:
C
C   API= ANNUAL FIXED COST OF THE PIPE SYSTEM EXPRESSED AS A
C        FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C        SYSTEM,1/YR,EQ.(2-16)
C   APP= ANNUAL FIXED COST OF THE PUMP STATION EXPRESSED AS
C        FRACTION OF THE INITIAL COST OF THE PUMP STATION,1/YR,EQ.(2-16)
C   BPI= ANNUAL MAINTAINANCE COST OF THE PIPE SYSTEM EXPRESSED
C        AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C        SYSTEM,1/YR
C   BPP= ANNUAL MAINTAINANCE COST OF THE PUMP STATION EXPRESSED
C        AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PUMP
C        STATION,1/YR
C   CD= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$/W**PPP
C   CEP= COST OF ELECTRICAL ENERGY,$/W HR
C   CHEL= ELEVATION CHANGE,M
C   CHPS= PRESSURE CHANGE,PA
C   CI= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$
C   CONDIT= FLOW CONDITION I.E. LAMINAR,TURBULENT OR CRITICAL
C   CP= EMPIRICAL CONSTANT FOR THE PIPE SYSTEM COST,$/M**(1+PPI)
C   CPCT1= TOTAL ANNUAL COST OF INSTALLED PIPE/TUBE SYSTEM
C          PER UNIT LENGTH OF PIPE/TUBE,$/YR M,EQ.(3-36)
C   CPCT2= TOTAL ANNUAL COST OF INSTALLED PUMP STATION
C          PER UNIT LENGTH OF PIPE/TUBE,$/YR M,EQ. (3-38)
C   DE= FLUID DENSITY,KG/M**3
C   DELTA= DIAMETER INCREMENT FOR THE GENERATION OF COSTS V.S.
```

```
C          DIAMETER DATA
C     DIM= TUBE/PIPE INSIDE DIAMETER,M
C     DIC= LAMINAR-TURBULENT TRANSITION VALUE OF DIM,M
C     DIC1= WORKING VARIABLE TO CALCULATE DIC4
C     DIC2= WORKING VARIABLE TO CALCULATE DIC5
C     DIC3= WORKING VARIABLE TO CALCULATE  DIC4 & DIC5
C     DIC4= LOWER BOUND GUESS TO CALCULATE DIC
C     DIC5= UPPER BOUND GUESS TO CALCULATE DIC
C     DIX= DIAMETER ARRAY GENERATED DURING THE OPTIMUM DIAMETER
C          ITERATION
C     DI1= OPTIMUM DIAMETER LOWER BOUND,M
C     DI2= OPTIMUM DIAMETER UPPER BOUND,M
C     EFF= COMBINED FRACTIONAL EFFICIENCY OF PUMP AND MOTOR
C     EO= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     EOC= LAMINAR-TURBULENT TRANSITION VALUE OF EO
C     FC= LAMINAR-TURBULENT TRANSITION VALUE OF FFX
C     FFX= FANNING FRICTION FACTOR
C     FFY= FRICTION FACTOR ARRAY GENERATED DURING THE OPTIMUM
C          DIAMETER ITERATION
C     HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C     HR= HOURS OF OPERATION PER YEAR
C     K= CONSISTENCY COEFFICIENT,PA S**N
C     LEGT= TUBE/PIPE LENGTH,M
C     MFR= MASS FLOW RATE,KG/S
C     N= FLOW BEHAVIOR INDEX
C     NANS= MENU POINTER
C     NO= NUMBER OF DATA POINTS GENERATED DURING THE OPTIMUM
C         DIAMETER ITERATION
C     NOROOT= ROOT INDICATOR:  0-YES; 1-NO; 2-TOO MANY ITERATION
C     NP= COUNTER
C     OPCT= TATAL ANNUAL OPERATING COST PER UNIT LENGTH OF
C           PIPE/TUBE,$/YR M,EQ.(2-20)
C     OPDIAM= OPTIMUM ECONOMIC TUBE/PIPE INSIDE DIAMETER,M
C     PI= 3.141593
C     POINTS= NUMBER OF COSTS & WORK V.S. DIAMETER DATA POINTS
C     PPI= EXPONENT IN THE PIPE SYSTEM COST EQUATION
C     PPP= EXPONENT IN THE PUMP STATION COST EQUATION
C     RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C     REC= LAMINAR-TURBULENT TRANSITION VALUE OF RE,EQ.(3-21)
C     SUFFC= SUMMATION OF THE FITTINGS RESISTANCE COEFFICIENT
C     TOCT= TOTAL ANNUAL COST OF A PUMPING SYSTEM PER UNIT
C           LENGTH OF PIPE/TUBE,$/YR M,EQ.(2-7)
C     TOLA= TOLERANCE ERROR TO FIND DIC
C     TOLC= TOLERANCE ERROR FOR EQ.(3-22)
C     TOLD= TOLERANCE ERROR FOR EQ.(3-49)
C     TOLI= TOLERANCE ERROR FOR THE INTEGRAL OF EQ.(3-31)
C     TOLL= TOLERANCE ERROR FOR EQ.(3-18)
C     TOLT= TOLERANCE ERROR FOR EQ.(3-31)
C     TOLV= TOLERANCE ERROR FOR EQ.(3-30)
C     U= MASS AVERAGE VELOCITY,M/S
C     WORK= WORK PER UNIT MASS,J/KG,EQ.(3-35)
C     YS= YIELD STRESS,PA
C
C     LIST OF SUBROUTINES:
C
C     BISECT1= ROOT FINDING SUBROUTINE: BISECTION-SECANT METHODS
C     BISECT2= ROOT FINDING SUBROUTINE: BISECTION-SECANT METHODS
C     TOTCOST= COMPUTE THE PUMPING SYSTEM COSTS & THEORETICAL
C              WORK REQUIREMENTS.
C     SORTING= SORT ARRAYS DIX & FFY
C     SWAP= INTERCHANGE THE VALUE OF TWO VARIABLES
C
C     LIST OF FUNCTIONS:
C
C     DIMCRIT= CRITICAL DIAMETER FUNCTION
C     FRICFAC= FRICTION FACTOR FUNCTION
C
C
      REAL MFR,K,N,PI,FFY(100),DIX(100),LEGT
      PARAMETER (PI=3.141593)
      CHARACTER CONDIT*10
      EXTERNAL DIMCRIT
      COMMON/MFRCCF/MFR,K/YSTDEN/YS,DE/FLWIDX/N/AVEVEL/U
      COMMON/CRICON/REC,FC,EOC/FLWCON/RE,HE
      COMMON/UNSPLG/EO/CRITDI/DIC
      COMMON/PIPECT/API,BPI,CP,PPI,LEGT
      COMMON/PUMPCT/APP,BPP,CI,CD,PPP,EFF/ELECTS/CEP,HR
      COMMON/CHPSEL/CHPS,CHEL/FRICFC/FFX
      COMMON/FFVSDI/FFY,DIX,NO
      COMMON/TOLER1/TOLV,TOLI/TOLER2/TOLC/TOLER3/TOLL,TOLT
      COMMON/ROOTNO/NOROOT/CODFLW/CONDIT/FLCFT/SUFFC
      DATA TOLA,TOLC,TOLL,TOLT,TOLV,TOLI,TOLD/5*1.E-4,2*1.E-3/
      NOROOT=0
```

```
C
5     WRITE(20,100)
      PRINT*
      PRINT*
      PRINT*
      WRITE(20,110)
      WRITE(20,330)
      WRITE(20,110)
      PRINT*
      PRINT*
      PRINT*
      PRINT*,' 1.  ENTER 1 TO ESTIMATE THE FANNING FRICTION FACTOR'
      PRINT*
      PRINT*,' 2.  ENTER 2 TO GENERATE COST V.S.  DIAMETER DATA'
      PRINT*
      PRINT*,'     ANSWER?...........'
      READ*,NANS
      PRINT*
      PRINT*
      IF(NANS.NE.1.AND.NANS.NE.2) GO TO 5
C
C          INPUT OF FLUID PROPERTIES
C
      PRINT*
      WRITE(20,110)
      WRITE(20,110)
      PRINT*
      PRINT*
      PRINT*,' ENTER FLOW INDEX.................................'
      READ*,N
      PRINT*
      PRINT*,' ENTER CONSISTENCY COEFFICIENT, (PA SEC**N)........'
      READ*,K
      PRINT*
      PRINT*,' ENTER YIELD STRESS, (PA)..........................'
      READ*,YS
      PRINT*
      PRINT*,' ENTER FLUID DENSITY, (KG/M**3)....................'
      READ*,DE
      PRINT*
      PRINT*
C
C          INPUT OF PUMPING SYSTEM VARIABLES
C
      WRITE(20,340)
      PRINT*
      PRINT*
      PRINT*,' ENTER MASS FLOW RATE, (KG/SEC)....................'
      READ*,MFR
      IF(NANS.EQ.1) GO TO 10
      GO TO 15
10    PRINT*
      PRINT*,' ENTER PIPE DIAMETER, (M)..........................'
      READ*,DIM
C
C          ESTIMATION OF THE FRICTION FACTOR
C
      FFX=FRICFAC(DIM)
C
C          OUTPUT OF THE RESULTS OF THE FRICTION FACTOR
C
      PRINT*
      PRINT*
      WRITE(20,350)
      WRITE(20,350)
      PRINT*
      WRITE(20,150) CONDIT
      PRINT*
      WRITE(20,160) RE
      WRITE(20,170) HE
      WRITE(20,180) FFX
      WRITE(20,380) EO
      WRITE(20,370)
      PRINT*
      WRITE(20,120)
      PRINT*
      WRITE(20,130) REC
      WRITE(20,140) FC
      WRITE(20,360) EOC
      WRITE(20,370)
      GO TO 75
C
```

```
C            MORE INPUT OF PUMPING SYSTEM VARIABLES
C
15      PRINT*
        PRINT*,' ENTER PIPE LENGTH, (M)............................'
        READ*,LEGT
        PRINT*
        PRINT*,' ENTER PRESSURE CHANGE, (PA)........................'
        READ*,CHPS
        PRINT*
        PRINT*,' ENTER ELEVATION CHANGE, (M)........................'
        READ*,CHEL
        PRINT*
        PRINT*,' ENTER SUMMATION OF THE FITTINGS RESISTANCE'
        PRINT*,'       COEFFICIENTS.................................'
        READ*,SUFFC
        PRINT*
        PRINT*,' ENTER COMBINED FRACIONAL EFFICIENCY'
        PRINT*,'       OF PUMP AND MOTOR............................'
        READ*,EFF
        PRINT*
        PRINT*
C
C            INPUT OF TUBE SYSTEM COST VARIABLES
C
        WRITE(20,340)
        PRINT*
        PRINT*
        PRINT*,' ENTER EMPIRICAL CONSTANT FOR THE TUBE SYSTEM'
        PRINT*,'       COST, CP, ($/M**(1+S))......................'
        READ*,CP
        PRINT*
        PRINT*,' ENTER EXPONENT FOR THE TUBE SYSTEM COST, S.........'
        READ*,PPI
        PRINT*
        PRINT*,' ENTER ANNUAL FIXED COST OF THE TUBE SYSTEM'
        PRINT*,'       EXPRESSES AS A FRACTION OF THE INITIAL'
        PRINT*,'       INSTALLED COST OF THE TUBE SYSTEM...........'
        READ*,API
        PRINT*
        PRINT*,' ENTER ANNUAL MAINTAINANCE COST OF THE TUBE'
        PRINT*,'       SYSTEM EXPRESSED AS A FRACTION OF THE'
        PRINT*,'       INITIAL INSTALLED COST OF THE TUBE SYSTEM...'
        READ*,BPI
        PRINT*
        PRINT*
C
C            INPUT OF PUMP STATION COST VARIABLES
C
        WRITE(20,340)
        PRINT*
        PRINT*
        PRINT*,' ENTER EMPIRICAL CONSTANT FOR THE PUMP STATION'
        PRINT*,'       COST, CI, ($)...............................'
        READ*,CI
        PRINT*
        PRINT*,' ENTER EMPIRICAL CONSTANT FOR THE PUMP STATION'
        PRINT*,'       COST, CD, ($/WATTS**S-PRIME)................'
        READ*,CD
        PRINT*
        PRINT*,' ENTER EXPONENT FOR THE PUMP STATION COST,S-PRIME..'
        READ*,PPP
        PRINT*
        PRINT*,' ENTER ANNUAL FIXED COST OF THE PUMP STATION'
        PRINT*,'       EXPRESSED AS A FRACTION OF THE INITIAL'
        PRINT*,'       INSTALLED COST OF THE PUMP STATION..........'
        READ*,APP
        PRINT*
        PRINT*,' ENTER ANNUAL MAINTAINANCE COST OF THE PUMP'
        PRINT*,'       STATION EXPRESSED AS A FRACTION OF THE'
        PRINT*,'       INITIAL INSTALLED COST OF THE PUMP STATION..'
        READ*,BPP
        PRINT*
        PRINT*
C
C            INPUT OF OPERATING COST VARIABLES
C
        WRITE(20,340)
        PRINT*
        PRINT*
        PRINT*,' ENTER COST OF ELECTRICAL ENERGY, ($/WATTS HR).....'
        READ*,CEP
        PRINT*
```

```
      PRINT*,' ENTER HOURS OF OPERATION PER YEAR.................'
      READ*,HR
      PRINT*
      PRINT*
      WRITE(20,340)
      IF(NANS.EQ.2) GO TO 60
      GO TO 5
20    CONTINUE
C
C          ESTIMATION  OF CRITICAL DIAMETER THROUGH
C          ITERATION FROM EQ.(3-17) & (3-21)
C
C          THE INITIAL DIC RANGE IS OBTAINED FROM EQ.(3-17) & (3-34)
C          WITH RE=1.E3 & RE=1.E6 FOR DIC4 & DIC5,RESPECTIVELY
C
      DIC1=1.E3*PI**(2.-N)*K*((3.*N+1.)/(4.*N))**N
      DIC2=1.E6*PI**(2.-N)*K*((3.*N+1.)/(4.*N))**N
      DIC3=2.**(7.-5.*N)*DE**(N-1.)*MFR**(2.-N)
      DIC4=(DIC1/DIC3)**(1./(3.*N-4.))
      DIC5=(DIC2/DIC3)**(1./(3.*N-4.))
25    CALL BISECT2(DIC4,DIC5,40,TOLA,DIMCRIT,DIC)
      IF (NOROOT.EQ.0) GO TO 30
      PRINT*
      PRINT*
      PRINT*,' THE CRITICAL DIAMETER, DIC WAS NOT FOUND'
      PRINT*
      PRINT*,' IN THE RANGE   ',DIC4,' <= DIC <= ',DIC5
      PRINT*
      PRINT*,' ENTER A WIDER RANGE FOR DIC'
      READ*,DIC4,DIC5
      NOROOT=0
      GO TO 25
30    CONTINUE
C
C          ESTIMATION OF THE OPTIMUM TUBE DIAMETER
C          THROUGH ITERATION FROM EQ.(3-49)
C
      PRINT*
      PRINT*
      PRINT*,' ENTER RANGE FOR THE OPTIMUM PIPE DIAMETER, (M)....'
35    READ*,DI1,DI2
      CALL BISECT1(DI1,DI2,50,TOLD,OPDIAM)
      IF(NOROOT.EQ.2) GO TO 40
      IF(NOROOT.EQ.0) GO TO 45
      PRINT*
      PRINT*
      PRINT*,' THE OPTIMUM PIPE DIAMETER WAS NOT FOUND IN'
      PRINT*,' THE RANGE GIVEN. ENTER A NEW RANGE, (M)...........'
      NOROOT=0
      GO TO 35
40    PRINT*
      PRINT*
      PRINT*,' TOO MANY INTERACTION TO FIND THE OPTIMUM PIPE'
      PRINT*,' DIAMETER.  ENTER A SMALLER RANGE, (M).............'
      NOROOT=0
      GO TO 35
C
C          ESTIMATION OF THE PUMPING SYSTEM COSTS AND
C          WORK AT THE OPTIMUM DIAMETER
C
45    CALL TOTCOST(OPDIAM,WORK,TOCT,OPCT,CPCT1,CPCT2)
C
C          OUTPUT OF RESULTS AT THE OPTIMUM DIAMETER
C
      PRINT*
      PRINT*
      PRINT*
      WRITE(20,350)
      WRITE(20,350)
      PRINT*
      WRITE(20,190) OPDIAM
      WRITE(20,200) TOCT
      WRITE(20,210) OPCT
      WRITE(20,220) CPCT1
      WRITE(20,230) CPCT2
      WRITE(20,240) WORK
      PRINT*
      WRITE(20,150) CONDIT
      PRINT*
      WRITE(20,160) RE
      WRITE(20,170) HE
      WRITE(20,180) FFX
      WRITE(20,380) EO
```

```
      WRITE(20,370)
      PRINT*
      WRITE(20,120)
      PRINT*
      WRITE(20,130) REC
      WRITE(20,140) FC
      WRITE(20,260) DIC
      WRITE(20,360) EOC
      WRITE(20,370)
      WRITE(20,250)
C
C
      PRINT*
      PRINT*
      WRITE(20,110)
      PRINT*
      PRINT*
      PRINT*
      PRINT*,' 1.  ENTER 1 TO PRINT COST V.S. DIAMETER DATA GENERATED'
      PRINT*,'                DURING THE ROOT FINDING '
      PRINT*
      PRINT*,' 2.  ENTER 2 TO EXIT'
      PRINT*
      PRINT*,'     ANSWER?............'
      READ*,NANS
      PRINT*
      IF(NANS.EQ.1) GO TO 50
      GO TO 75
50    CONTINUE
C
C           ESTIMATION OF THE PUMPING SYSTEM COSTS AND
C           WORK FOR THE FRICTION FACTOR V.S. DIAMETER
C           GENERATED DURING THE OPTIMUM DIAMETER ITERATION
C
      PRINT*
      PRINT*
      WRITE(20,110)
      PRINT*
      PRINT*
      WRITE(20,270)
      WRITE(20,280)
      WRITE(20,290)
      WRITE(20,300)
      WRITE(20,310)
      PRINT*
      CALL SORTING(DIX,FFY,NO)
      DO 55 J=1,NO
      FFX=FFY(J)
      CALL TOTCOST(DIX(J),WORK,TOCT,OPCT,CPCT1,CPCT2)
      WRITE(20,320) DIX(J),WORK,CPCT2,CPCT1,OPCT,TOCT
55    CONTINUE
      WRITE(20,250)
      GO TO 75
60    PRINT*
C
C           GENERATION OF PUMPING SYSTEM COSTS AND WORK
C           V.S. DIAMETER
C
      PRINT*
      PRINT*,' ENTER RANGE OF PIPE DIAMETER, (M)................'
      READ*,DI1,DI2
      PRINT*
      PRINT*,' ENTER NUMBER OF DATA POINTS......................'
      READ*,POINTS
      IF(DI1.GT.DI2) THEN
      CALL SWAP(DI1,DI2)
      END IF
      IF(POINTS.LE.1.) THEN
      DI2=DI1
      ELSE
      DELTA=(DI2-DI1)/(POINTS-1.)
      END IF
      PRINT*
      PRINT*
      WRITE(20,350)
      WRITE(20,350)
      PRINT*
      PRINT*
      WRITE(20,270)
      WRITE(20,280)
      WRITE(20,290)
      WRITE(20,300)
      WRITE(20,310)
```

```
      PRINT*
      DIM=DI1
      NP=1
65    CONTINUE
      FFX=FRICFAC(DIM)
      CALL TOTCOST(DIM,WORK,TOCT,OPCT,CPCT1,CPCT2)
      WRITE(20,320) DIM,WORK,CPCT2,CPCT1,OPCT,TOCT
      IF(NP.GE.POINTS) GO TO 70
      DIM=DIM+DELTA
      NP=NP+1
      GO TO 65
70    CONTINUE
C
C
      WRITE(20,250)
      PRINT*
      PRINT*
      WRITE(20,110)
      PRINT*
      PRINT*
      PRINT*
      PRINT*,' 1.  ENTER 1 TO START THE PROGRAM'
      PRINT*
      PRINT*,' 2.  ENTER 2 TO ESTIMATE THE OPTIMUM PIPE DIAMETER'
      PRINT*
      PRINT*,' 3.  ENTER 3 TO CONTINUE WITH A NEW RANGE'
      PRINT*
      PRINT*,' 4.  ENTER 4 TO EXIT'
      PRINT*
      PRINT*,'     ANSWER?............'
      READ*,NANS
C
C
      PRINT*
      IF(NANS.EQ.1) GO TO 5
      IF(NANS.EQ.2.OR.NANS.EQ.3) THEN
      PRINT*
      PRINT*
      WRITE(20,110)
      PRINT*
      END IF
      IF(NANS.EQ.2) GO TO 20
      IF(NANS.EQ.3) GO TO 60
75    CONTINUE
      PRINT*
      PRINT*
      WRITE(20,110)
      WRITE(20,330)
      WRITE(20,110)
      WRITE(20,100)
C
100   FORMAT('1')
110   FORMAT(' ',90('*'))
120   FORMAT('0',10X,'LAMINAR-TURBULENT TRANSITION')
130   FORMAT('0',2X,'CRITICAL REYNOLDS NUMBER',11('.'),4X,E12.6)
140   FORMAT(' ',2X,'CRITICAL FANNING FRICTION FACTOR...',2X,F10.6)
150   FORMAT('0',10X,'FLOW CONDITION : ',A10)
160   FORMAT('0',2X,'REYNOLDS NUMBER',20('.'),4X,E12.6)
170   FORMAT(' ',2X,'HEDSTROM NUMBER',20('.'),4X,E12.6)
180   FORMAT(' ',2X,'FANNING FRICTION FACTOR',12('.'),2X,F10.6)
190   FORMAT('0',2X,'OPTIMUM PIPE DIAMETER',14('.'),3X,F8.5,' M')
200   FORMAT(' ',2X,'MINIMUM COST',23('.'),4X,E12.6,' $/YR M')
210   FORMAT(' ',2X,'ELECTRICAL ENERGY COST',13('.'),4X,E12.6,' $/YR M')
220   FORMAT(' ',2X,'PIPE SYSTEM COST',19('.'),4X,E12.6,' $/YR M')
230   FORMAT(' ',2X,'PUMP STATION COST',18('.'),4X,E12.6,' $/YR M')
240   FORMAT(' ',2X,'*WORK',30('.'),4X,E12.6,' J/KG')
250   FORMAT('0',2X,'* NEGLECT KINETIC ENERGY CHANGE')
260   FORMAT(' ',2X,'CRITICAL DIAMETER',18('.'),3X,F8.5,' M')
270   FORMAT('0',2X,'DIAMETER',6X,'* W O R K',5X,'PUMP  STATION')
280   FORMAT('+',47X,'PIPE  SYSTEM',5X,'ELECTRICAL',6X,'T O T A L')
290   FORMAT (' ',4X,'(M)',11X,'(J/KG)',2(9X,'C O S T'))
300   FORMAT('+',63X,'ENERGY COST',7X,'C O S T')
310   FORMAT(' ',24X,4(8X,'($/YR M)'))
320   FORMAT(' ',2X,F8.5,5(4X,E12.6))
330   FORMAT(' ',90('$'))
340   FORMAT(' ',90('-'))
350   FORMAT(' ',90('='))
360   FORMAT(' ',2X,'CRITICAL UNSHEARED PLUG RADIUS.....',5X,E9.4)
370   FORMAT(' ',2X,'(DIMENSIONLESS)')
380   FORMAT(' ',2X,'UNSHEARED PLUG RADIUS',14('.'),5X,E9.4)
C
      END
C
```

```
      SUBROUTINE TOTCOST(DI,WORK,TOCT,OPCT,CPCT1,CPCT2)
C
C   THIS SUBROUTINE CALCULATES THE FOLLOWING:
C
C     1. WORK PER UNIT MASS,EQ.(3-35)
C     2. POWER REQUIREMENTS,EQ.(2-13)
C     3. PUMPING SYSTEM COSTS, EQ.(2-20),(3-36)
C        (3-38) & (2-7)
C
C   NOTE: POWER IS ONLY USED INTERNALLY
C
C   LIST OF VARIABLES:
C
C   API= ANNUAL FIXED COST OF THE PIPE SYSTEM EXPRESSED AS A
C        FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C        SYSTEM,1/YR,EQ.(2-16)
C   APP= ANNUAL FIXED COST OF THE PUMP STATION EXPRESSED AS
C        FRACTION OF THE INITIAL COST OF THE PUMP STATION,1/YR,EQ.(2-16)
C   BPI= ANNUAL MAINTAINANCE COST OF THE PIPE SYSTEM EXPRESSED
C        AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C        SYSTEM,1/YR
C   BPP= ANNUAL MAINTAINANCE COST OF THE PUMP STATION EXPRESSED
C        AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PUMP
C        STATION,1/YR
C   CD= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$/W**PPP
C   CEP= COST OF ELECTRICAL ENERGY,$/W HR
C   CHEL= ELEVATION CHANGE,M
C   CHPS= PRESSURE CHANGE,PA
C   CI= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$
C   CP= EMPIRICAL CONSTANT FOR THE PIPE SYSTEM COST,$/M**(1+PPI)
C   CPCT= TOTAL ANNUAL CAPITAL COST OF INSTALLED EQUIPMENT
C          PER UNIT LENGTH OF PIPE/TUBE,$/YR M
C   CPCT1= TOTAL ANNUAL COST OF INSTALLED PIPE/TUBE SYSTEM
C           PER UNIT LENGTH OF PIPE/TUBE,$/YR M,EQ.(3-36)
C   CPCT2= TOTAL ANNUAL COST OF INSTALLED PUMP STATION
C           PER UNIT LENGTH OF PIPE/TUBE,$/YR M,EQ. (3-38)
C   DE= FLUID DENSITY,KG/M**3
C   DI= TUBE/PIPE INSIDE DIAMETER,M
C   EFF= COMBINED FRACTIONAL EFFICIENCY OF PUMP AND MOTOR
C   FFX= FANNING FRICTION FACTOR
C   HR= HOURS OF OPERATION PER YEAR
C   K= CONSISTENCY COEFFICIENT,PA S**N
C   LEGT= TUBE/PIPE LENGTH,M
C   MFR= MASS FLOW RATE,KG/S
C   OPCT= TATAL ANNUAL OPERATING COST PER UNIT LENGTH OF
C          PIPE/TUBE,$/YR M,EQ.(2-20)
C   PI= 3.141593
C   POWER= POWER REQUIREMENTS,EQ.(2-13)
C   PPI= EXPONENT IN THE PIPE SYSTEM COST EQUATION
C   PPP= EXPONENT IN THE PUMP STATION COST EQUATION
C   SUFFC= SUMMATION OF THE FITTINGS RESISTANCE COEFFICIENT
C   TOCT= TOTAL ANNUAL COST OF A PUMPING SYSTEM PER UNIT
C          LENGTH OF PIPE/TUBE,$/YR M,EQ.(2-7)
C   WORK= WORK PER UNIT MASS,J/KG,EQ.(3-35)
C   WORK1= WORK REQUIRED DUE TO PIPE FRICTION
C   WORK2= WORK REQUIRED DUE TO FITTINGS FRICTION
C   WORK3= WORK DUE TO PRESSURE AND ELEVATION DIFFERENCE IN
C           THE SYSTEM
C   YS= YIELD STRESS,PA
C
C
```

```
      REAL MFR,K,PI,LEGT
      PARAMETER (PI=3.141593)
      COMMON/MFRCCF/MFR,K/YSTDEN/YS,DE
      COMMON/PIPECT/API,BPI,CP,PPI,LEGT
      COMMON/PUMPCT/APP,BPP,CI,CD,PPP,EFF/ELECTS/CEP,HR
      COMMON/CHPSEL/CHPS,CHEL/FRICFC/FFX
      COMMON/FLCFT/SUFFC
C
C          CALCULATION OF WORK FROM EQ.(3-35)
C
      WORK1=32.*LEGT*MFR*MFR*FFX/(PI*PI*DE*DE*DI**5)
      WORK2=8.*MFR*MFR*SUFFC/(PI*PI*DE*DE*DI**4)
      WORK3=CHPS/DE+9.8*CHEL
      WORK=WORK1+WORK2+WORK3
C
C          CALCULATION OF POWER FROM EQ.(2-13)
C
      POWER=MFR*WORK/EFF
C
C          CALCULATION OF OPERATING COST FROM EQ.(2-20)
C
      OPCT=CEP*HR*POWER/LEGT
C
C          CALCULATION OF TUBE SYSTEM COST FROM EQ.(3-36)
C
      CPCT1=(API+BPI)*CP*DI**PPI
C
C          CALCULATION OF PUMP STATION COST FROM EQ.(3-38)
C
      CPCT2=(APP+BPP)*(CI+CD*POWER**PPP)/LEGT
C
C          TOTAL ANNUAL CAPITAL COST
C
      CPCT=CPCT1+CPCT2
C
C          CALCULATION OF TOTAL COST FROM EQ.(2-7)
C
      TOCT=OPCT+CPCT
      RETURN
      END
C
```

```
      FUNCTION DIMCRIT(DI)
C
C     THIS FUNCTION EQUATE THE GENERALIZED REYNOLDS NUMBER,
C     EQ.(3-17), WITH THE CRITICAL REYNOLDS NUBER,EQ.(3-21) TO GIVE
C     A FUNCTION IN TERM OF DIC FOR A GIVE MASS FLOW RATE.
C
C     LIST OF VARIABLES:
C
C     A1= LOWER BOUND GUESS OF EOC
C     A2= UPPER BOUND GUESS OF EOC
C     DE= FLUID DENSITY,KG/M**3
C     DI= TUBE/PIPE INSIDE DIAMETER,M
C     EOC= LAMINAR-TURBULENT TRANSITION VALUE OF EO
C     HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C     HX=HE
C     K= CONSISTENCY COEFFICIENT,PA S**N
C     MFR= MASS FLOW RATE,KG/S
C     N= FLOW BEHAVIOR INDEX
C     NOROOT= ROOT INDICATOR:  0-YES; 1-NO; 2-TOO MANY ITERATION
C     PI= 3.141593
C     PSIC= LAMINAR-TURBULENT TRANSITION VALUE OF Y
C     RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C     REC= LAMINAR-TURBULENT TRANSITION VALUE OF RE,EQ.(3-21)
C     RECP= HERSCHEL-BULKLEY GENERALIZED CRITICAL REYNOLDS NUMBER= REC*PSIC
C     REC1,REC2,REC3= WORKING VARIABLES TO CALCULATE REC
C     RE1,RE2,RE3= WORKING VARIABLES TO CALCULATE RE
C     TOLC= TOLERANCE ERROR FOR EQ.(3-22)
C     U= MASS AVERAGE VELOCITY,M/S
C     YS= YIELD STRESS,PA
C
C     LIST OF SUBROUTINES:
C
C     BISNEWT= ROOT FINDING SUBROUTINE: BISECTION-NEWTON METHODS
C
C     LIST OF FUNCTIONS:
C
C     DFUN1= DERIVATIVE OF FUN1 WITH RESPECT TO EOC
C     FUN1= EQ.(3-22) REWRITTEN AS FUN1(EOC)=0.
C     Y= LAMINAR FLOW FUNCTION (PSI),EQ.(3-14)
C
C
      REAL MFR,K,N,PI
      PARAMETER (PI=3.141593)
      EXTERNAL FUN1,DFUN1
      COMMON/MFRCCF/MFR,K/YSTDEN/YS,DE/FLWIDX/N
      COMMON/HEDSTR/HX/TOLER2/TOLC
      COMMON/ROOTNO/NOROOT
C
C           CALCULATION OF U FROM EQ.(3-34)
C
      U=4.*MFR/(PI*DE*DI**2)
C
C           CALCULATION OF RE FROM EQ.(3-17)
C
      RE1=(N/(3.*N+1.))**N
      RE2=(DI/2.)**N
      RE3=U**(2.-N)
      RE=8.*DE*RE1*RE2*RE3/K
```

```
C
C           CALCULATION OF HE FROM EQ.(3-19)
C
      IF(YS.EQ.0.) THEN
      HE=0.
      EOC=0.
      ELSE
      HE=(DE/YS)*DI**2*(YS/K)**(2./N)
      END IF
C
      IF(HE.EQ.0.) GO TO 10
C
C           CALCULATION OF EOC THROUGH ITERATION FROM EQ.(3-22)
C
      HX=HE
      A1=0.
      A2=.999999999
5     CALL BISNEWT(A1,A2,40,TOLC,FUN1,DFUN1,EOC)
      IF(NOROOT.EQ.0) GO TO 10
      PRINT*
      PRINT*
      PRINT*,' THE DIMENSIONLESS UNSHEARED PLUG RADIUS,EOC,WAS NOT'
      PRINT*
      PRINT*,' FOUND IN THE RANGE  ',A1,' <= EOC <= ',A2
      PRINT*
      PRINT*,' ENTER A NEW RANGE FOR EOC: 0. <= EOC < 1.0 ......'
      READ*,A1,A2
      NOROOT=0
      GO TO 5
C
10    CONTINUE
C
C           CALCULATION OF REC FROM EQ.(3-21)
C
      REC1=33600.*SQRT(1./27.)*N/(1.+3.*N)**2
      REC2=(2.+N)**((2.+N)/(1.+N))
      REC3=(1.-EOC)**(1.+2./N)
C
C           CALCULATION OF PSI-CRITICAL FROM EQ.(3-14) WITH EO=EOC
C
      PSIC=Y(EOC)
C
C           CONTINUE CALCULATION OF REC
C
      RECP=REC1*REC2*PSIC**(2./N)/REC3
      REC=RECP/PSIC
C
      DIMCRIT=1.-REC/RE
      RETURN
      END
C
```

```
      SUBROUTINE BISECT1(XA,XB,MAX,ERROR,NEWX)
C
C     THIS SUBROUTINE COMPUTES THE OPTIMUM DIAMETER FROM
C     EQ.(3-49) (COMPUTES THE ROOT OF THE FUNCTION OPTDIAM). IT
C     IS A COMBINATION OF THE BISECTION AND SECANT ITERATION
C     METHOD.  THE BISECTION INTERVAL IS USED TO START THE
C     SECANT ITERATION.  THE PROGRAM CONTINUES WITH THIS METHOD
C     UNTIL THE SOLUTION IS FOUND OR THE FOLLOWING SITUATIONS
C     OCCUR: 1- X FALLS OUTSIDE THE INTERVAL KNOWN TO CONTAIN THE
C     SOLUTION; 2- X IS OUT OF RANGE OR INDEFINITE; 3- TOO
C     FAR AWAY FROM THE SOLUTION; 4- THE NUMBER OF ITERATIONS
C     EXCEEDS MAX.  IF THESE SITUATIONS OCCUR, THE PROGRAM
C     SWITCH TO THE BISECTION METHOD TO OBTAIN A SMALLER
C     INTERVAL.  REFERENCE: MOORE,E.  1982.  "INTRODUCTION TO
C     FORTRAN AND ITS APPLICATION". ALLYN AND BACON, INC.,
C     BOSTON,MASS.
C
C     LIST OF VARIABLES:
C
C     DIFF= DIFFERENCE BETWEEN TWO ITERATION POINTS
C     DIX= DIAMETER ARRAY GENERATED DURING THE OPTIMUM DIAMETER
C          ITERATION
C     ERROR= TOLERANCE ERROR
C     FA= VALUE OF OPTDIAM AT XA
C     FB= VALUE OF OPTDIAM AT XB
C     FFX= FANNING FRICTION FACTOR
C     FFY= FRICTION FACTOR ARRAY GENERATED DURING THE OPTIMUM
C          DIAMETER ITERATION
C     FM= VALUE OF OPTDIAM AT XM
C     FO= VALUE OF OPTDIAM AT XO
C     F1= VALUE OF OPTDIAM AT X1
C     LM= -1 IF X IS INDEFINITE; +1 IF OUT OF RANGE; 0 OTHERWISE
C     MAX= MAXIMUM NUMBER OF SECANT ITERATION
C     NEWX= ROOT OF OPTDIAM
C     NO= NUMBER OF DATA POINTS GENERATED DURING THE OPTIMUM
C         DIAMETER ITERATION
C     NOROOT= ROOT INDICATOR:  0-YES; 1-NO; 2-TOO MANY ITERATION
C     X= POINT FROM THE SECANT ITERATION EQUATION
C     XA= LOWER BOUND POINT USED IN THE BISECTION METHOD
C     XB= UPPER BOUND POINT USED IN THE BISECTION METHOD
C     XM= MIDPOINT BETWEEN XA & XB
C     XO= SECOND POINT REQUIRED FOR THE SECANT PROCESS
C     X1= FOCUS POINT FOR THE SECANT ITERATION
C
C     LIST OF SUBROUTINES:
C
C     SWAP= INTERCHANGE THE VALUE OF TWO VARIABLES
C
C     LIST OF FUNCTIONS:
C
C     FRICFAC= FRICTION FACTOR FUNCTION
C     OPTDIAM= OPTIMUM DIAMETER FUNCTION,EQ.(3-49) REWRITTEN AS
C              OPTDIAM(X)=0.
C
C
      REAL NEWX,FFY(100),DIX(100)
      COMMON/ROOTNO/NOROOT
      COMMON/FRICFC/FFX
      COMMON/FFVSDI/FFY,DIX,NO
      NO=2
      IF(XA.GT.XB) THEN
      CALL SWAP(XA,XB)
      END IF
      FFX=FRICFAC(XA)
      FFY(1)=FFX
      DIX(1)=XA
      FA=OPTDIAM(XA)
      FFX=FRICFAC(XB)
      FFY(2)=FFX
      DIX(2)=XB
      FB=OPTDIAM(XB)
      XO=XA
      FO=FA
      X1=XB
      F1=FB
      XM=(XA+XB)/2.
      IF (FO*F1.GT.O.) GO TO 90
   10 IF(NO.GT.99) GO TO 95
      IF(ABS(F1/FO).GT.5..OR.ABS(FO/F1).GT.5.) GO TO 40
      IF(ALOG(ABS(F1)).GT.O..OR.ALOG(ABS(FO)).GT.O.) GO TO 40
```

```
C
C          SECANT ITERATION
C
      DO 30 J=1,MAX
      IF(ABS(F1).GE.ABS(FO)) THEN
      CALL SWAP(FO,F1)
      CALL SWAP(XO,X1)
      END IF
      X=X1-F1*(X1-XO)/(F1-FO)
      LM=LEGVAR(X)
      IF (LM.NE.O) GO TO 40
      IF(X.LT.XA.OR.X.GT.XB) GO TO 40
      DIFF=ABS(X-X1)
      IF(DIFF.LE.ABS(X*ERROR)) GO TO 80
      XO=X1
      FO=F1
      X1=X
      FFX=FRICFAC(X)
      NO=NO+1
      FFY(NO)=FFX
      DIX(NO)=X
      F1=OPTDIAM(X)
30    CONTINUE
C
C          BISECTION ITERATION
C
40    FFX=FRICFAC(XM)
      NO=NO+1
      FFY(NO)=FFX
      DIX(NO)=XM
      FM=OPTDIAM(XM)
      IF (FM.EQ.O.) GO TO 70
      IF (FA*FM.LE.O.) GO TO 50
      XA=XM
      FA=FM
      FO=FA
      XO=XA
      F1=FB
      X1=XB
      GO TO 60
50    XB=XM
      FB=FM
      X1=XB
      F1=FB
      XO=XA
      FO=FA
60    XM=(XA+XB)/2.
      IF (ABS(XA-XB).GT.ABS(XM*ERROR)) GO TO 10
70    X=XM
80    NEWX=X
      FFX=FRICFAC(NEWX)
      NO=NO+1
      FFY(NO)=FFX
      DIX(NO)=NEWX
      RETURN
90    NEWX=(XA+XB)/2.
      NOROOT=1
      RETURN
95    NEWX=(XA+XB)/2.
      NOROOT=2
      RETURN
      END
C
```

```
      FUNCTION OPTDIAM(DI)
C
C  THIS FUNCTION EXECUTES THE OPTIMUM DIAMETER EQUATION,
C  EQ.(3-49), REWRITTEN AS OPTDIAM(DI)=0.
C
C  LIST OF VARIABLES:
C
C  API= ANNUAL FIXED COST OF THE PIPE SYSTEM EXPRESSED AS A
C       FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C       SYSTEM,1/YR,EQ.(2-16)
C  APP= ANNUAL FIXED COST OF THE PUMP STATION EXPRESSED AS
C       FRACTION OF THE INITIAL COST OF THE PUMP STATION,1/YR,EQ.(2-16)
C  BPI= ANNUAL MAINTAINANCE COST OF THE PIPE SYSTEM EXPRESSED
C       AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PIPE
C       SYSTEM,1/YR
C  BPP= ANNUAL MAINTAINANCE COST OF THE PUMP STATION EXPRESSED
C       AS A FRACTION OF THE INITIAL INSTALLED COST OF THE PUMP
C       STATION,1/YR
C  CD= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$/W**PPP
C  CEP= COST OF ELECTRICAL ENERGY,$/W HR
C  CHEL= ELEVATION CHANGE,M
C  CHPS= PRESSURE CHANGE,PA
C  CI= EMPIRICAL CONSTANT FOR THE PUMP STATION COST,$
C  CP= EMPIRICAL CONSTANT FOR THE PIPE SYSTEM COST,$/M**(1+PPI)
C  DE= FLUID DENSITY,KG/M**3
C  DI= TUBE/PIPE INSIDE DIAMETER,M
C  EFF= COMBINED FRACTIONAL EFFICIENCY OF PUMP AND MOTOR
C  FFX= FANNING FRICTION FACTOR
C  HR= HOURS OF OPERATION PER YEAR
C  K= CONSISTENCY COEFFICIENT,PA S**N
C  LEGT= TUBE/PIPE LENGTH,M
C  MFR= MASS FLOW RATE,KG/S
C  OP1,OP2,OP3,OP4= WORKING CALCULATIONAL PARAMETERS
C  PI= 3.141593
C  POWER= POWER REQUIREMENTS,EQ.(2-13)
C  PPI= EXPONENT IN THE PIPE SYSTEM COST EQUATION
C  PPP= EXPONENT IN THE PUMP STATION COST EQUATION
C  SUFFC= SUMMATION OF THE FITTINGS RESISTANCE COEFFICIENT
C  WORK= WORK PER UNIT MASS,J/KG,EQ.(3-35)
C  WORK1= WORK REQUIRED DUE TO PIPE FRICTION
C  WORK2= WORK REQUIRED DUE TO FITTINGS FRICTION
C  WORK3= WORK DUE TO PRESSURE AND ELEVATION DIFFERENCE IN
C         THE SYSTEM
C  YS= YIELD STRESS,PA
C
C  LIST OF FUNCTIONS:
C
C  DFFWD= DERIVATIVE OF FFX WITH RESPECT TO DI
C
C
      REAL MFR,K,LEGT
      PARAMETER (PI=3.141593)
      COMMON/MFRCCF/MFR,K/YSTDEN/YS,DE
      COMMON/PIPECT/API,BPI,CP,PPI,LEGT
      COMMON/PUMPCT/APP,BPP,CI,CD,PPP,EFF/ELECTS/CEP,HR
      COMMON/FRICFC/FFX/FLCFT/SUFFC/CHPSEL/CHPS,CHEL
C
C           CALCULATION OF WORK FROM EQ.(3-35)
C
      WORK1=32.*LEGT*MFR*MFR*FFX/(PI*PI*DE*DE*DI**5)
      WORK2=8.*MFR*MFR*SUFFC/(PI*PI*DE*DE*DI**4)
      WORK3=CHPS/DE+9.8*CHEL
      WORK=WORK1+WORK2+WORK3
C
C           CALCULATION OF POWER FROM EQ.(2-13)
C
      POWER=MFR*WORK/EFF
C
C           OPTIMUM DIAMETER EQUATION
C
      OP1=(API+BPI)*PPI*CP*PI*PI*DE*DE*EFF*DI**(PPI+5.)
      OP2=32.*CEP*HR*MFR**3
      OP3=(APP+BPP)*PPP*CD*POWER**(PPP-1.)/(CEP*HR)
      OP4=5.*FFX-DI*DFFWD(DI)+DI*SUFFC/LEGT
      OPTDIAM=1.-OP2*OP4*(OP3+1.)/OP1
      RETURN
      END
C
```

```
      FUNCTION DFFWD(DI)
C
C     THIS FUNCTION COMPUTES THE DERIVATIVE OF THE FRICTION
C     FACTOR WITH RESPECT TO THE TUBE/PIPE INSIDE DIAMETER
C     FOR LAMINAR OR TURBULENT FLOW
C
C     LIST OF VARIABLES:
C
C     DI= TUBE/PIPE INSIDE DIAMETER,M
C     DIC= LAMINAR-TURBULENT TRANSITION VALUE OF DIM,M
C     DFF= DERIVATIVE OF THE FRICTION FACTOR WITH RESPECT TO
C          DIAMETER(DI)
C     DF1,DF2= WORKING VARIABLES TO CALCULATE DFF-LAMINAR
C     EO= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     FFX= FANNING FRICTION FACTOR
C     FH1= VALUE OF THE FRICTION FACTOR AT DI+H
C     FH2= VALUE OF THE FRICTION FACTOR AT DI-H
C     H= SMALL POSITIVE NUMBER
C     I= -1 IF DFF IS INDEFINITE; +1 IF OUT OF RANGE; 0 OTHERWISE
C     N= FLOW BEHAVIOR INDEX
C     SIGMA= PARAMETER IN THE DFFX/DD EQUATION FOR LAMINAR
C            FLOW,EQ.(3-47)
C
C     LIST OF FUNCTIONS:
C
C     FFL1= EQ.(3-45)
C     FRICFAC= FRICTION FACTOR FUNCTION
C
C
      REAL N
      COMMON/FLWIDX/N/FRICFC/FFX
      COMMON/UNSPLG/EO/CRITDI/DIC
      IF(DI.LT.DIC) GO TO 10
C
C          DERIVATIVE FOR LAMINAR FLOW,EQ.(3-47)
C
      SIGMA=FFL1(EO)
      DF1=4.*FFX*SIGMA*EO-FFX*(3.*N-4.)
      DF2=DI*(1.+SIGMA*EO)
      DFF=DF1/DF2
      GO TO 20
   10 CONTINUE
C
C          NUMERICAL APPROXIMATION FOR TURBULENT FLOW
C
      H=0.1
      IF(DI.LT.0.01) THEN
      H=0.0001*H
      ELSE IF(DI.LT.0.1) THEN
      H=0.001*H
      ELSE IF(DI.LT.1.) THEN
      H=0.01*H
      ELSE IF(DI.LT.10.) THEN
      H=0.1*H
      END IF
      FH1=FRICFAC(DI+H)
      FH2=FRICFAC(DI-H)
```

```
C
C           BACKWARD OR FORWARD DIFFERENCE ARE USED NEAR
C           DIC FOR DI < DIC
C
      IF(FH1.GT.FFX.AND.FH2.GT.FFX) THEN
      IF(ABS(FFX-FH1).LE.ABS(FFX-FH2)) THEN
C
C           FORWARD DIFFERENCE
C
      DFF=(FH1-FFX)/H
      ELSE
C
C           BACKWARD DIFFERENCE
C
      DFF=(FFX-FH2)/H
      END IF
      ELSE
C
C           QUADRATIIC APPROXIMATION.EQ.(3-48)
C
      DFF=(FH1-FH2)/(2.*H)
      END IF
      I=LEGVAR(DFF)
C
C           IF THE DERIVATIVE IS INDEFINITE OR OUT OF
C           RANGE, THEN IT IS NEGLECTED. THIS MAY ONLY HAPPEN NEAR
C           DIC FOR DI < DIC WHERE THERE MAY BE AN ABRUPT
C           INCREASE OF FFX
C
      IF(I.NE.0) THEN
      DFF=0.
      END IF
20    CONTINUE
      DFFWD=DFF
      RETURN
      END
C
```

```
      FUNCTION FRICFAC(DI)
C
C     THIS FUNCTION CALCULATES THE FRICTION FACTOR  ACCORDING TO
C     THE SCHEME OF FIGURE 4. TORRANCE RELATIONSHIP, EQ.(A-1),
C     IS USED TO OBTAIN THE INITIAL GUESSES OF THE FRICTION
C     FACTOR FOR TURBULENT FLOW.
C
C     LIST OF VARIABLES:
C
C     A1= LOWER BOUND GUESS OF EO OR EOC
C     A2= UPPER BOUND GUESS OF EO OR EOC
C     CONDIT= FLOW CONDITION I.E. LAMINAR,TURBULENT OR CRITICAL
C     DE= FLUID DENSITY,KG/M**3
C     DI= TUBE/PIPE INSIDE DIAMETER,M
C     EO= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     EOC= LAMINAR-TURBULENT TRANSITION VALUE OF EO
C     FC= LAMINAR-TURBULENT TRANSITION VALUE OF FF
C     FF= FANNING FRICTION FACTOR
C     FTORR= TORRANCE'S FRICTION FACTOR FOR TURBULENT FLOW,EQ.(A-1)
C     FFO= LOWER BOUND FOR FF OR FTORR, EQ.(3-15) WITH EO=1.0
C     FF1= LOWER BOUND GUESS FOR THE CALCULATION OF THE TURBULENT
C           FF AND FTORR
C     FF2= FINAL LOWER BOUND GUESS FOR THE CALCULATION OF FTORR
C     FF3= FINAL UPPER BOUND GUESS FOR THE CALCULATION OF FTORR
C     FF4= LOWER BOUND GUESS FOR THE CALCULATION OF THE TURBULENT FF,
C           EQ..(3-27) WITH R=RC
C     FF4P= WORKING VARIABLE TO CALCULATE FF4
C     FF5,FF7= UPPER BOUND GUESS FOR THE CALCULATION OF TURBULENT FF
C     FF6= LOWER BOUND GUESS FOR THE CALCULATION OF TURBULENT FF
C     FF8= FINAL LOWER BOUND GUESS FOR THE CALCULATION OF THE
C           TURBULENT FF
C     FF9= FINAL UPPER BOUND GUESS FOR THE CALCULATION OF THE
C           TURBULENT FF
C     HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C     HX=HE
C     K= CONSISTENCY COEFFICIENT,PA S**N
C     LIMLOW= COUNTER
C     LIMUP= COUNTER
C     MFR= MASS FLOW RATE,KG/S
C     N= FLOW BEHAVIOR INDEX
C     NOROOT= ROOT INDICATOR:  0-YES; 1-NO; 2-TOO MANY ITERATION
C     NOTIME= COUNTER
C     PI= 3.141593
C     PSIC= LAMINAR-TURBULENT TRANSITION VALUE OF Y
C     RC= LAMINAR-TURBULENT TRANSITION VALUE OF R
C     RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C     REC= LAMINAR-TURBULENT TRANSITION VALUE OF RE,EQ.(3-21)
C     RECP= HERSCHEL-BULKLEY GENERALIZED CRITICAL REYNOLDS NUMBER= REC*PSIC
C     REC1,REC2,REC3= WORKING VARIABLES TO CALCULATE REC
C     RE1,RE2,RE3= WORKING VARIABLES TO CALCULATE RE
C     TOLC= TOLERANCE ERROR FOR EQ.(3-22)
C     TOLI= TOLERANCE ERROR FOR THE INTEGRAL OF EQ.(3-31)
C     TOLL= TOLERANCE ERROR FOR EQ.(3-18)
C     TOLT= TOLERANCE ERROR FOR EQ.(3-31)
C     TOLV= TOLERANCE ERROR FOR EQ.(3-30)
C     U= MASS AVERAGE VELOCITY,M/S
C     YS= YIELD STRESS,PA
C
C     LIST OF SUBROUTINES:
C
C     BISECT2= ROOT FINDING SUBROUTINE: BISECTION-SECANT METHODS
C     BISNEWT= ROOT FINDING SUBROUTINE: BISECTION-NEWTON METHODS
C
C     LIST OF FUNCTIONS:
C
C     DFUN1= DERIVATIVE OF FUN1 WITH RESPECT TO EOC
C     DTORREN= DERIVATIVE OF TORREN WITH RESPECT TO FTORR
C     DUNPGRA= DERIVATIVE OF USPGRA WITH RESPECT TO EO
C     FFTM= EQ.(3-31) REWRITTEN AS FFTM(FF)=0.
C     FUN1= EQ.(3-22) REWRITTEN AS FUN1(EOC)=0.
C     R= TURBULENT PARAMETER, EQ.(3-27)
C     TORREN= TORRANCE EQUATION, EQ.(A-1) REWRITTEN AS
C              TORREN(FTORR)=0.
C     UNPGRA= EQ.(3-18) REWITTEN AS UNPGRA(EO)=0.
C     Y= LAMINAR FLOW FUNCTION (PSI),EQ.(3-14)
C
C
```

```
      REAL MFR,K,N,PI
      PARAMETER(PI=3.141593)
      CHARACTER CONDIT*10
      EXTERNAL FFTM
      EXTERNAL FUN1,DFUN1
      EXTERNAL USPGRA,DUSPGRA
      EXTERNAL FFT2,DFFT2
      EXTERNAL TORREN,DTORREN
      COMMON/MFRCCF/MFR,K/YSTDEN/YS,DE/FLWIDX/N/AVEVEL/U
      COMMON/CRICON/REC,FC,EOC/FLWCON/RE,HE
      COMMON/UNSPLG/EO
      COMMON/TOLER1/TOLV,TOLI/TOLER2/TOLC/TOLER3/TOLL,TOLT
      COMMON/ROOTNO/NOROOT/BLOCK1/RC
      COMMON/CODFLW/CONDIT
      COMMON/HEDSTR/HX
C
C           CALCULATION OF U FROM EQ.(3-34)
C
      U=4.*MFR/(PI*DE*DI**2)
C
C           CALCULATION OF RE FROM EQ.(3-17)
C
      RE1=(N/(3.*N+1.))**N
      RE2=(DI/2.)**N
      RE3=U**(2.-N)
      RE=8.*DE*RE1*RE2*RE3/K
C
C           CALCULATION OF HE FROM EQ.(3-19)
C
      IF(YS.EQ.O.) THEN
      HE=O.
      EOC=O.
      EO=O.
      ELSE
      HE=(DE/YS)*DI**2*(YS/K)**(2./N)
      END IF
C
      IF(HE.EQ.O.) GO TO 10
C
C           CALCULATION OF EOC THROUGH ITERATION FROM EQ.(3-22)
C
      HX=HE
      A1=O.
      A2=.999999999
5     CALL BISNEWT(A1,A2,40,TOLC,FUN1,DFUN1,EOC)
      IF(NOROOT.EQ.O) GO TO 10
      PRINT*
      PRINT*
      PRINT*,' THE DIMENSIONLESS UNSHEARED PLUG RADIUS,EOC,WAS NOT'
      PRINT*
      PRINT*,' FOUND IN THE RANGE  ',A1,' <= EOC <= ',A2
      PRINT*
      PRINT*,' ENTER A NEW RANGE FOR EOC: O. <= EOC < 1.0 ......'
      READ*,A1,A2
      NOROOT=O
      GO TO 5
C
10    CONTINUE
C
C           CALCULATION OF REC FROM EQ.(3-21)
C
      REC1=33600.*SQRT(1./27.)*N/(1.+3.*N)**2
      REC2=(2.+N)**((2.+N)/(1.+N))
      REC3=(1.-EOC)**(1.+2./N)
C
C           CALCULATION OF PSI-CRITICAL FROM EQ.(3-14) WITH EO=EOC
C
      PSIC=Y(EOC)
C
C           CONTINUE CALCULATION OF REC
C
      RECP=REC1*REC2*PSIC**(2./N)/REC3
      REC=RECP/PSIC
C
C           CALCULATION OF FC FROM EQ.(3-16) WITH PSI=PSI-C & RE=REC
C
      FC=16./RECP
```

```
C
      IF(RE.NE.REC) GO TO 15
C
C          THE FLOW IS CRITICAL
C
      CONDIT='CRITICAL'
      FF=FC
      EO=EOC
      GO TO 60
C
15    CONTINUE
      IF(RE.GT.REC) GO TO 30
C
C          THE FLOW IS LAMINAR
C
      CONDIT='LAMINAR'
C
C          ESTIMATION OF EO THROUGH ITERATION FROM EQ.(2-18)
C
      IF(HE.EQ.O.) GO TO 25
      A1=EOC
      A2=0.999999999
20    CALL BISNEWT(A1,A2,50,TOLL,USPGRA,DUSPGRA,EO)
      IF(NOROOT.EQ.O) GO TO 25
      PRINT*
      PRINT*
      PRINT*,' THE DIMENSIONLESS UNSHEARED PLUG RADIUS,EO,WAS NOT'
      PRINT*
      PRINT*,' FOUND IN THE RANGE  ',A1,' <= EO <= ',A2
      PRINT*
      PRINT*,' ENTER A NEW RANGE FOR EO: O. <= EO < 1.0 ........'
      READ*,A1,A2
      NOROOT=O
      GO TO 20
25    CONTINUE
C
C          ESTIMATION OF FF FROM EQ.(3-14) & (3-16)
C
      FF=16/(RE*Y(EO))
      GO TO 60
C
30    CONTINUE
C
C          THE FLOW IS TURBULENT
C
      CONDIT='TURBULENT'
C
C          ESTIMATION OF LOWER & UPPER BOUND FOR FTORR. FFO IS
C          CALCULATED FROM EQ.(3-15) WITH EO = 1.0. THEN THIS IS
C          USED TO CALCULATE THE LOWER BOUND GUESS FOR THE FRICTION
C          FACTOR FOR WHICH THE CONDITION EO < 1.0 IS VALID. THIS
C          IS ALSO USED TO ESTIMATE THE FINAL TURBULENT FF
C
      FFO=2.*YS/(DE*U*U)
      FF1=FFO+1.E-5
      FF2=FF1
      FF3=1.0
C
C          CALCULATION  OF FTORR THROUGH ITERATION FROM EQ.(A-1)
C          THIS VALUE IS USED TO GET A RANGE FOR THE FF
C
35    CALL BISNEWT(FF2,FF3,50,TOLT,TORREN,DTORREN,FTORR)
      IF(NOROOT.EQ.O) GO TO 40
      PRINT*
      PRINT*
      PRINT*,' THE INITIAL GUESS FOR THE FRICTION FACTOR (FTORR) '
      PRINT*
      PRINT*,' WAS NOT FOUND IN THE RANGE  ',FF2,' <= FTORR <= ',FF3
      PRINT*
      PRINT*,' ENTER A NEW RANGE FOR FTORR. > ',FFO,' ......'
      READ*,FF2,FF3
      NOROOT=O
      GO TO 35
40    CONTINUE
```

```
C
C          CALCULATION  OF RC FROM EQ.(3-27) WITH RE=REC & FF=FC
C
      RC=R(REC,FC)
C
C          ESTIMATION OF A LOWER & UPPER BOUND FOR FF IN TURBULENT FLOW.
C          FF4 IS ESTIMATED FROM EQ.(3-27) WITH R=RC. THIS WILL GIVES
C          A FRICTION FACTOR GUESS FOR WHICH R > RC OR EO < EOC,
C          A CONDITION FOR TURBULENT FLOW
C
      FF4P=(N/(1.+3.*N))**N
      FF4=16.*(FF4P*RC**N/RE)**(2./(2.-N))
      FF5=1.35*FC
      IF(N.GE.1.0) THEN
      FF6=.4*FTORR
      FF7=1.2*FTORR
      ELSE IF(N.GE.0.7) THEN
      FF6=.5*FTORR
      FF7=1.5*FTORR
      ELSE IF(N.GE.0.6) THEN
      FF6=.6*FTORR
      FF7=1.6*FTORR
      ELSE IF(N.GE.0.5) THEN
      FF6=.7*FTORR
      FF7=1.7*FTORR
      ELSE IF(N.GE.0.4) THEN
      FF6=.8*FTORR
      FF7=1.9*FTORR
      ELSE IF(N.GE.0.3) THEN
      FF6=.9*FTORR
      FF7=2.*FTORR
      ELSE IF(N.GE.0.2) THEN
      FF6=1.1*FTORR
      FF7=2.2*FTORR
      ELSE
      FF6=1.4*FTORR
      FF7=2.8*FTORR
      END IF
      NOTIME=1
      FF8=FF1
      FF9=FF5
      LIMLOW=1
      LIMUP=1
      IF(FF4.GT.FF8) THEN
      FF8=FF4
      LIMLOW=2
      END IF
      IF(FF6.GT.FF8) THEN
      FF8=FF6
      LIMLOW=3
      END IF
      IF(FF7.LT.FF9) THEN
      FF9=FF7
      LIMUP=2
      END IF
45    CONTINUE
C
C          ESTIMATION OF FF THROUGH ITERATION FROM EQ.(3-31)
C
      CALL BISECT2(FF8,FF9,20,TOLT,FFTM,FF)
      IF (NOROOT.EQ.0.OR.NOROOT.EQ.2) GO TO 55
      IF(LIMLOW.EQ.1.AND.LIMUP.EQ.1) GO TO 50
```

```
C
C          IF THE ROOT IS NOT FOUND, IT MAKES SEVERAL ATTEMPS WITH A
C          WIDER FF RANGE  BEFORE IT ASKS THE USER TO ENTER A NEW RANGE
C
      NOTIME=NOTIME+1
      IF(NOTIME.EQ.2) THEN
      IF(LIMLOW.EQ.2) THEN
      IF(FF6.GT.FF1) THEN
      FF8=FF6
      LIMLOW=3
      ELSE
      FF8=FF1
      LIMLOW=1
      END IF
      ELSE IF(LIMLOW.EQ.3) THEN
      IF(FF4.GT.FF1) THEN
      FF8=FF4
      LIMLOW=2
      ELSE
      FF8=FF1
      LIMLOW=1
      END IF
      ELSE
      FF9=FF5
      LIMUP=1
      END IF
      ELSE IF(NOTIME.EQ.3.AND.LIMUP.EQ.2) THEN
      FF9=FF5
      LIMUP=1
      ELSE
      FF8=FF1
      FF9=FF5
      LIMLOW=1
      LIMUP=1
      END IF
      NOROOT=0
      GO TO 45
50    CONTINUE
      PRINT*
      PRINT*
      PRINT*,' THE FRICTION FACTOR (F.F.) WAS NOT FOUND IN THE RANGE'
      PRINT*
      PRINT*,' ',FF8,' <= F.F. <= ',FF9
      PRINT*
      PRINT*,' ENTER A NEW RANGE FOR F.F. > ',FFO,' ......'
      READ*,FF8,FF9
      NOROOT=0
      GO TO 45
C
55    EO=2.*YS/(FF*DE*U*U)
60    CONTINUE
      FRICFAC=FF
      RETURN
      END
C
C
```

```
      FUNCTION FUN1(EC)
C
C  THIS FUNCTION IS EQ.(3-22) REWRITTEN AS FUN1(EC)=O.
C
C  LIST OF VARIABLES:
C
C  EC= DIMENSIONLESS UNSHEARED PLUG RADIUS AT THE
C      LAMINAR-TURBULENT TRANSITION
C  HX= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C  N= FLOW BEHAVIOR INDEX
C  P1,P2,P3= WORKING CALCULATIONAL PARAMETERS
C
C
      REAL N
      COMMON/FLWIDX/N/HEDSTR/HX
      P1=(16800.*SQRT(1./27.)/N)*(2.+N)**((2.+N)/(1.+N))
      P2=(EC/(1.-EC)**(1.+N))**((2.-N)/N)
      P3=1./(1.-EC)**N
      FUN1=HX-P1*P2*P3
      RETURN
      END
C
```

```
      FUNCTION DFUN1(EC)
C
C  THIS FUNCTION IS THE DERIVATIVE OF FUN1, EQ.(3-22),
C  WITH RESPECT TO EC. I.E. DFUN1(EC) = D FUN1(EC)/ D EC.
C  IT IS USED IN THE NEWTON'S ITERATION METHOD IN
C  SUBROUTINE BISNEWT.
C
C  LIST OF VARIABLES:
C
C  EC= DIMENSIONLESS UNSHEARED PLUG RADIUS AT THE
C      LAMINAR-TURBULENT TRANSITION
C  N= FLOW BEHAVIOR INDEX
C  P1,P2,P3,P4,P5= WORKING CALCULATIONAL PARAMETERS
C
C
      REAL N
      COMMON/FLWIDX/N
      P1=-(16800.*SQRT(1./27.)/N)*(2.+N)**((2.+N)/(1.+N))
      P2=(2.-N)*EC**((2.-2.*N)/N)/N
      P3=(1.-EC)**((2.+N)/N)
      P4=(2.+N)*EC**((2.-N)/N)/N
      P5=(1.-EC)**((2.+2.*N)/N)
      DFUN1=P1*(P2/P3+P4/P5)
      RETURN
      END
C
```

```
      FUNCTION USPGRA(EX)
C
C  THIS FUNCTION IS EQ.(3-18) REWITTEN AS USPGRA(EX)=0.
C
C  LIST OF VARABLES:
C
C  EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C  N= FLOW BEHAVIOR INDEX
C  P1,P2= WORKING CALCULATIONAL PARAMETERS
C  RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C
C  LIST OF FUNCTIONS:
C
C  Y= LAMINAR FLOW FUNCTION (PSI),EQ.(3-14)
C
C
      REAL N
      COMMON/FLWCON/RE,HE/FLWIDX/N
      P1=RE*EX**((2.-N)/N)
      P2=2.*HE*Y(EX)**((2.-N)/N)
      P3=(N/(1.+3.*N))**2.
      USPGRA=P1-P2*P3
      RETURN
      END
C
```

```
      FUNCTION Y(EX)
C
C  THIS FUNCTION EXECUTES EQ.(3-14). I.E. PSI=Y(EX)
C
C  LIST OF VARIABLES:
C
C  EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  N= FLOW BEHAVIOR INDEX
C  P1,P2,P3,P4= WORKING CALCULATIONAL PARAMETERS
C
C
      REAL N
      COMMON/FLWIDX/N
      P1=(1.-EX)**2
      P2=2.*EX*(1.-EX)*(1.+3.*N)/(1.+2.*N)
      P3=EX**2*(1.+3.*N)/(1.+N)
      P4=(1.-EX)**(1.+N)
      Y=P4*(P1+P2+P3)**N
      RETURN
      END
C
```

```
      FUNCTION DUSPGRA(EX)
C
C  THIS FUNCTION IS THE DERIVATIVE OF FUNCTION USPGRA,
C  EQ.(3-18), WITH RESPECT TO EX. I.E. DUSPGRA(EX) =
C  D USPGRA(EX)/D EX. IT IS USED IN THE NEWTON'S ITERATION
C  METHOD IN SUBROUTINE BISNEWT.
C
C  LIST OF VARABLES:
C
C  EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  FFL1= SIGMA, EQ.(3-45)
C  HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C  N= FLOW BEHAVIOR INDEX
C  P1,P2,P3,P4= WORKING CALCULATIONAL PARAMETERS
C  RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C
C  LIST OF FUNCTIONS:
C
C  FFL1= EQ.(3-45)
C  Y= LAMINAR FLOW FUNCTION (PSI),EQ.(3-14)
C
C
      REAL N
      COMMON/FLWCON/RE,HE/FLWIDX/N
      P1=(2.-N)/N
      P2=(N/(1.+3.*N))**2.
      P3=2.*HE*P2*P1*FFL1(EX)*Y(EX)**P1
      P4=P1*RE*EX**((2.-2.*N)/N)
      DUSPGRA=P3+P4
      RETURN
      END
C
```

```
      FUNCTION FFL1(EX)
C
C  THIS FUNCTION EXECUTES EQ. (3-45). I.E. SIGMA=FFL1(EX)
C
C  LIST OF VARIABLES:
C
C  EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  FFL1= PARAMETER IN THE DF/DD EQUATION FOR LAMINAR FLOW
C        (EQ.(3-47)),(SIGMA),EQ.(3-45). ALSO USED IN
C        USED IN FUNCTION DUSPGRA.
C  N= FLOW BEHAVIOR INDEX
C  P1,P2,P3,P4,P5,P6= WORKING CALCULATIONAL PARAMETERS
C
C
      REAL N
      COMMON/FLWIDX/N
      P1=1.+3.*N
      P2=1.+2.*N
      P3=1.+N
      P4=1.-EX
      P5=P1*P3*P4**2+2.*P2*P1*EX*P4+P1*P2*P3*EX**2
      P6=P2*P3*P4**3+2.*P1*P3*EX*P4**2+P1*P2*P4*EX**2
      FFL1=P5/P6
      RETURN
      END
C
```

```
      FUNCTION TORREN(FX)
C
C     THIS FUNCTION EXECUTES THE FRICTION FACTOR EQUATION
C     OF TORRANCE FOR TURBULENT FLOW,EQ.(A-1),
C     REWRITTEN AS TORREN(FX)=0.
C
C     LIST OF VARIABLES:
C
C     EO= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     EO1,EO2,EO3= WORKING VARIABLES TO CALCULATE EO
C     FX= FANNING FRICTION FACTOR
C     HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C     N= FLOW BEHAVIOR INDEX
C     P1,P2,P3= WORKING CALCULATIONAL PARAMETERS
C     RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C
C
      REAL N
      COMMON/FLWIDX/N/FLWCON/RE,HE
C
C           CALCULATION OF EO FROM EQ.(A-2)
C
      EO1=(N/(1.+3.*N))**(2.*N/(2.-N))
      EO2=16.*(2.*HE)**(N/(2.-N))
      EO3=FX*RE**(2./(2.-N))
      EO=EO1*EO2/EO3
C
C           TORRANCE EQUATION
C
      P1=0.45-2.75/N+1.97*ALOG(1.-EO)/N
      P2=((1.+3.*N)/(4.*N))**N
      P3=1.97*ALOG(RE*P2*FX**(1.-N/2.))/N
      TORREN=P1+P3-1./SQRT(FX)
      RETURN
      END
C
```

```
      FUNCTION DTORREN(FX)
C
C     THIS FUNCTION IS THE DERIVATIVE OF FUNCTION TORREN,EQ.(A-1),
C     WITH RESPECT TO FX. I.E. DTORREN(FX)=D TORREN(FX)/D FX. IT
C     IS USED IN THE NEWTON'S ITERATION METHOD IN SUBROUTINE
C     BISNEWT.
C
C     LIST OF VARIABLES:
C
C     EO= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     EO1,EO2,EO3= WORKING VARIABLES TO CALCULATE EO
C     FX= FANNING FRICTION FACTOR
C     HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C     N= FLOW BEHAVIOR INDEX
C     P1,P2,P3= WORKING CALCULATIONAL PARAMETERS
C     RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C
C
      REAL N
      COMMON/FLWIDX/N/FLWCON/RE,HE
C
C           CALCULATION OF EO FROM EQ.(A-2)
C
      EO1=(N/(1.+3.*N))**(2.*N/(2.-N))
      EO2=16.*(2.*HE)**(N/(2.-N))
      EO3=FX*RE**(2./(2.-N))
      EO=EO1*EO2/EO3
C
C           THE DERIVATIVE OF TORREN WITH RESPECT TO FX
C
      P1=3.94*EO*SQRT(FX)+N*(1.-EO)
      P2=3.94*(1.-N/2.)*(1.-EO)*SQRT(FX)
      P3=2.*N*(1.-EO)*FX**1.5
      DTORREN=(P1+P2)/P3
      RETURN
      END
C
```

```
      FUNCTION FFTM(FT)
C
C  THIS FUNCTION EXECUTES EQ.(3-31) REWRITTEN AS FFTM(FT)=0.
C
C  LIST OF VARIABLES:
C
C  E0= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  FT= FANNING FRICTION FACTOR
C  HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C  N= FLOW BEHAVIOR INDEX
C  PR =R
C  P2,P3= WORKING CALCULATIONAL PARAMETERS
C  RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C
C  LIST OF FUNCTIONS:
C
C  R= TURBULENT PARAMETER, EQ.(3-27)
C  ROMBIN= INTEGRAL IN EQ.(3-31)
C
C
      REAL N
      COMMON/FLWIDX/N/FLWCON/RE,HE
C
C          CALCULATION OF RP FROM EQ.(3-27)
C
      PR=R(RE,FT)
C
C          CALCULATION OF E0 FROM EQ.(3-32)
C
      E0=(2.*HE/PR**2)**(N/(2.-N))
      P2=(1.-E0)**((2.-N)/N)
      P3=(N/(1.+3.*N))**N
C
C          CALCULATION OF EQ.(3-31)
C
      FFTM=1.-PR**2*P2*P3*ROMBIN(E0,PR)**(2.-N)/RE
      RETURN
      END
C
```

```
      FUNCTION R(RX,FX)
C
C  THIS FUNCTION EXECUTES EQ.(3-27). I.E. 'R'=R(RX,FX)
C
C  LIST OF VARIABLES:
C
C  FX= FANNING FRICTION FACTOR
C  N= FLOW BEHAVIOR INDEX
C  RX= GENERALIZED REYNOLDS NUMBER
C  P1,P2= WORKING CALCULATIONAL PARAMETERS
C
C
      REAL N
      COMMON/FLWIDX/N
      P1=(RX*(FX/16.)**((2.-N)/2.))**(1./N)
      P2=(1.+3.*N)/N
      R=P1*P2
      RETURN
      END
C
```

```
      FUNCTION ROMBIN(EX,RY)
C
C     THIS FUNCTION COMPUTES THE INTEGRAL IN EQ.(3-31)
C     BY THE ROMBERG METHOD. THE INTEGRATION IS DIVIDED
C     IN SEVERAL SECTIONS DEPENDING THE FLOW BEHAVIOR
C     INDEX AND THE INTEGRAL IS INDEPENDENTLY COMPUTED
C     FOR EACH SECTION. THE VALUES OF THE DIMENSIONLESS
C     SHEAR RATE ARE SAVED IN ARRAY FS1 TO BE USED AS
C     LOWER & AND UPPER BOUND GUESSES IN CONSECUTIVE
C     ROOT ITERATION OF FUNCTION FFT2(EQ.(3-30).REFERENCE:
C     MILLER, A.R.  1982.  "FORTRAN PROGRAMS". SYBEX,
C     INC.,BERKELEY,CA.
C
C     LIST OF VARIABLES:
C
C     DELTA = INTERVAL VALUE
C     EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C     FS1,FS2,FS3,FS4= DIMENSIONLESS  SHEAR RATE ARRAYS
C     FU1= VALUE OF FFT1 AT EX OR 0
C     FU2= VALUE OF FFT1 AT UPPER
C     FU3= VALUE OF FFT1 AT X
C     LOWER= LOWER LIMIT OF INTEGRATION
C     N= FLOW BEHAVIOR INDEX
C     PIECES= NUMBER OF INTERVALS
C     RY= TURBULENCE PARAMETER,EQ.(3-27)
C     T= VALUES OF ROMBERG TABLEAU
C     TOLI= TLERANCE ERROR FOR THE INTEGRATION
C     TOSUM= FINAL VALUE OF THE INTEGRATION IN EQ.(3-31)
C     UPPER= UPPER LIMIT OF INTEGRATION
C     X= TRAPEZOIDAL  POINTS
C
C     LIST OF FUNCTIONS:
C
C     FFT1= FUNCTION INSIDE THE INTEGRAL IN EQ(3-31)
C
C
      INTEGER PIECES,NX(13)
      REAL N
      REAL LOWER,T(92)
      REAL FS1(4097),FS3(4097),FS2(2049)
      COMMON/FLWIDX/N
      COMMON/TOLER1/TOLV,TOLI
      DO 5 KV=1,4097
      FS1(KV)=0.
      FS3(KV)=0.
5     CONTINUE
      DO 10 KV=1,2049
      FS2(KV)=0.
10    CONTINUE
      TOSUM=0.
      FU1=0.
      FS1(1)=0.
      LOWER=EX
      IF(N.LE.0.1) THEN
      V=.8
      ELSE IF(N.LE.0.2) THEN
      V=.7
      ELSE IF(N.LE.0.3) THEN
      V=.7
      ELSE
      V=.5
      END IF
      UPPER=V*(1.-LOWER)+LOWER
C
15    CONTINUE
      FS1(2)=1.
      FU2=FFT1(UPPER,FS1(1),FS1(2),EX,RY)
      FS1(2)=FU2/UPPER**2
      FS4=FS1(2)
20    PIECES=1
      NX(1)=1
      DELTA=(UPPER-LOWER)/PIECES
      NZ=2
      C=(FU1+FU2)/2.
      SUM=C
      T(1)=DELTA*C
      NM=1
      NN=2
      L1=1
```

```
25    NM=NM+1
      FOTOM=4.
      NX(NM)=NN
      PIECES=PIECES*2
      LL=PIECES-1
      L2=(LL+1)/2
      DELTA=(UPPER-LOWER)/PIECES
C
C          COMPUTE TRAPEZOIDAL SUM FOR 2**(NM-1)+1 POINTS
C
      DO 30 II=L1,L2
      I=II*2-1
      X=LOWER+DELTA*I
      FU3=FFT1(X,FS1(II),FS1(II+1),EX,RY)
      SUM=SUM+FU3
      FS2(II)=FU3/X**2
      FS3(II*2)=FS2(II)
30    CONTINUE
      NZ=NZ+L2
      DO 35 KV=1,L2+1
      FS3(KV*2-1)=FS1(KV)
35    CONTINUE
      T(NN)=SUM*DELTA
      NTRA=NX(NM-1)
      KK=NM-1
C
C          COMPUTE T ARRAY
C
      DO 40 MM=1,KK
      J=NN+MM
      NT=NX(NM-1)+MM-1
      T(J)=(FOTOM*T(J-1)-T(NT))/(FOTOM-1.)
      FOTOM=FOTOM*4.
40    CONTINUE
C
C          NEW ORDERED VALUES OF THE DIMENSIONLESS SHEAR RATE
C
      DO 45 KV=1,NZ
      FS1(KV)=FS3(KV)
45    CONTINUE
      IF (NM.LT.5)GO TO 50
      IF(T(NN+1).EQ.0.0) GO TO 50
      IF(ABS(T(NTRA+1)-T(NN+1)).LE.ABS(T(NN+1)*TOLI)) GO TO 60
      IF(ABS(T(NN-1)-T(J)).LE.ABS(T(J)*TOLI)) GO TO 60
      IF(NM.GT.12) GO TO 55
50    NN=J+1
      GO TO 25
55    CONTINUE
C
60    TOSUM=TOSUM+T(J)
      LOWER=UPPER
      FU1=FU2
      FS1(1)=FS4
      IF(UPPER.EQ.1.) GO TO 75
      IF(UPPER.GE..9999999999999) GO TO 70
      IF(1.-FU2.GT.0.9) GO TO 65
      FS1(2)=1.
      FU2=1.
      UPPER=1.
      GO TO 20
65    CONTINUE
      UPPER=(1.-LOWER)/2.+LOWER
      GO TO 15
70    CONTINUE
      TOSUM=TOSUM+(1.-UPPER)*(1.+FU2)/2.
75    CONTINUE
      ROMBIN=TOSUM
      RETURN
      END
C
```

```
      FUNCTION FFT1(EY,C1,C2,EX,RY)
C
C  THIS FUNCTION EXECUTES THE FUNCTION INSIDE THE
C  INTEGRAL IN EQ.(3-31). I.E.  FFT1=(ZETA)*(XI)**2
C
C  LIST OF VARIABLES:
C
C  B= EMPIRICAL WALL EFFECT PARAMETER IN MIXING LENGTH
C     THEORY,EQ.(3-28)
C  CV= DIMENSIONLESS RATE OF SHEAR (ZETA)
C  C1= LOWER BOUND GUESS FOR CV
C  C2= UPPER BOUND GUESS FOR CV
C  E= DIMENSIONLESS RADIAL COORDINATE (XI)
C  EX= DIMENSIONLESS UNSHEARED PLUG RADIUS
C  EY= E
C  EZ= EX
C  HE= GENERALIZED HEDSTROM NUMBER,EQ.(3-19)
C  L= DIMENSIONLESS MIXING LENGTH (LAMBDA),EQ.(3-25)
C  N= FLOW BEHAVIOR INDEX
C  NOROOT= ROOT INDICATOR:  0-YES; 1-NO; 2-TOO MANY ITERATION
C  Q= PARAMETER IN MIXING LENGTH (PHI), EQ.(3-26)
C  RC= LAMINAR-TURBULENT TRANSITION VALUE OF RY
C  RE= GENERALIZED REYNOLDS NUMBER,EQ.(3-17)
C  RY= TURBULANCE PARAMETER, EQ.(3-27)
C  RZ= RY
C  TOLI= TOLERANCE ERROR FOR THE INTEGRAL OF EQ.(3-31)
C  TOLV= TOLERANCE ERROR FOR EQ.(3-30)
C
C  LIST OF SUBROUTINES:
C
C  BISNEWT= ROOT FINDING SUBROUTINE: BISECTION-NEWTON METHODS
C
C  LIST OF FUNCTIONS:
C
C  DFFT2= DERIVATIVE OF EFFT2 WITH RESPECT TO CV
C  FFT2= EQ.(3-30) REWRITTEN AS FFT2(CV)=0.
C
C
      REAL N,L
      EXTERNAL FFT2,DFFT2
      COMMON/FLWIDX/N/FLWCON/RE,HE
      COMMON/BLOCK1/RC/BLOCK2/E,EZ,RZ,L
      COMMON/TOLER1/TOLV,TOLI/ROOTNO/NOROOT
      E=EY
      EZ=EX
      RZ=RY
C
C          CALCULATION OF B FROM EQ.(3-28)
C
      B=22.*(1.+.00352*HE/(1.+.000504*HE)**2)/N
C
C          CALCULATION OF Q (PHI) FROM EQ.(3-26)
C
      Q=(RZ-RC)/(B*SQRT(8.))
C
C          CALCULATION OF L (LAMBDA) FROM EQ.(3-25)
C
      L=.36*(1.-E)*(1.-EXP(-Q*(1.-E)))
C
C          CALCULATION OF CV (ZETA) THROUGH ITERATION
C          FROM EQ.(3-30)
C
      CALL BISNEWT(C1,C2,100,TOLV,FFT2,DFFT2,CV)
      NOROOT=0
C
C          FUNCTION INSIDE THE INTEGRAL OF EQ.(3-31)
C
      FFT1=CV*E**2
      RETURN
      END
C
```

```
      FUNCTION FFT2(CX)
C
C   THIS FUNCTION IS EQ.(3-30) REWRITTEN AS FFT2(CX)=0.
C
C   LIST OF VARIABLES:
C
C   CX= DIMENSIONLESS RATE OF SHEAR (ZETA)
C   E= DIMENSIONLESS RADIAL COORDINATE (XI)
C   EZ= DIMENSIONLESS UNSHEARED PLUG RADIUS
C   L= DIMENSIONLESS MIXING LENGTH (LAMBDA),EQ.(3-25)
C   N= FLOW BEHAVIOR INDEX
C   P1,P2= WORKING CALCULATIONAL PARAMETERS
C   RZ= TURBULANCE PARAMETER,EQ.(3-27)
C
C
      REAL N,L
      COMMON/FLWIDX/N/BLOCK2/E,EZ,RZ,L
      P1=EZ-E+(1.-EZ)*CX**N
      P2=RZ**2*L**2*CX**2*(1.-EZ)**(2./N)/8.
      FFT2=P1+P2
      RETURN
      END
C
```

```
      FUNCTION DFFT2(CX)
C
C   THIS FUNCTION IS THE DERIVATIVE OF FUNCTION FFT2(CX),
C   EQ.(3-30), WITH RESPECT TO CX. I.E. DFFT2(CX)=
C   D FFT2(CX)/D CX. IT IS USED IN THE NEWTON'S ITERATION
C   METHOD IN SUBROUTINE BISNEWT.
C
C   LIST OF VARIABLES:
C
C   CX= DIMENSIONLESS RATE OF SHEAR (ZETA)
C   E= DIMENSIONLESS RADIAL COORDINATE (XI)
C   EZ= DIMENSIONLESS UNSHEARED PLUG RADIUS
C   L= DIMENSIONLESS MIXING LENGTH (LAMBDA),EQ.(3-25)
C   N= FLOW BEHAVIOR INDEX
C   P1,P2= WORKING CALCULATIONAL PARAMETERS
C   RZ= TURBULANCE PARAMETER,EQ.(3-27)
C
C
      REAL N,L
      COMMON/FLWIDX/N/BLOCK2/E,EZ,RZ,L
      P1=N*(1.-EZ)*CX**(N-1.)
      P2=RZ**2*L**2*CX*(1.-EZ)**(2./N)/4.
      DFFT2=P1+P2
      RETURN
      END
C
```

```
      SUBROUTINE BISNEWT(XA,XB,MAX,ERROR,FUN,DFUN,NEWX)
C
C  THIS SUBROUTINE COMPUTES THE ROOT OF THE FUNCTION FUN. IT
C  IS A COMBINATION OF THE BISECTION AND NEWTON'S ITERATION
C  METHOD. THE MIDPOINT OF THE INTERVAL (PART OF THE BISECTION
C  PROCESS) IS USED TO START THE NEWTON'S ITERATION. THE
C  PROGRAM CONTINUES WITH THIS METHOD UNTIL THE SOLUTION IS
C  FOUND OR THE FOLLOWING SITUATIONS OCCUR: 1- DFUN=0.
C  2- X FALLS OUTSIDE THE INTERVAL KNOWN TO CONTAIN THE
C  SOLUTION; 3- THE DIFFERENCE IN SUCCESIVE APPROXIMATION DOES
C  NOT DECRASED; 4- THE NUMBER OF ITERATIONS EXCEEDS MAX. IF
C  THESE SITUATIONS OCCUR, THE PROGRAM SWITCH TO THE
C  BISECTION METHOD TO OBTAIN A SMALLER INTERVAL. REFERENCE:
C  MOORE,E.  1982. "INTRODUCTION TO FORTRAN AND ITS
C  APPLICATION". ALLYN AND BACON, INC.,BOSTON,MASS.
C
C  LIST OF VARIABLES:
C
C  DIFF= DIFFERENCE BETWEEN TWO ITERATION POINTS
C  ERROR= TOLERANCE ERROR
C  FA= VALUE OF FUN AT XA
C  FB= VALUE OF FUN AT XB
C  FM= VALUE OF FUN AT XM
C  FUNC= VALUE OF FUN AT X
C  FPRIME= VALUE OF DFUN AT X
C  M= -1 IF X IS INDEFINITE; +1 IF OUT OF RANGE; 0 OTHERWISE
C  MAX= MAXIMUM NUMBER OF NEWTON'S ITERATION
C  NEWX= ROOT OF FUN
C  NOROOT= ROOT INDICATOR:  0-YES; 1-NO
C  OLDDIFF= DIFFERENCE OF PREVIOUS ITERATION
C  OLDX= PREVIOUS VALUE OF X
C  X= POINT FROM NEWTON'S ITERATION EQUATION
C  XA= LOWER BOUND POINT USED IN THE BISECTION METHOD
C  XB= UPPER BOUND POINT USED IN THE BISECTION METHOD
C  XM= MIDPOINT BETWEEN XA & XB
C
C  LIST OF SUBROUTINES:
C
C  SWAP= INTERCHANGE THE VALUE OF TWO VARIABLES
C
C  LIST OF FUNCTIONS:
C
C  FUN= FUNCTION WHOSE ROOT IS COMPUTED
C  DFUN= DERIVATIVE OF FUN WITH RESPECT TO THE ROOT VARIABLE
C
C
      REAL NEWX
      COMMON/ROOTNO/NOROOT
      IF(XA.GT.XB)THEN
      CALL SWAP(XA,XB)
      END IF
      FA=FUN(XA)
      IF(FA.EQ.0.) GO TO 90
      FB=FUN(XB)
      IF(FB.EQ.0.) GO TO 100
      IF(FA*FB.GT.0.)GO TO 80
      XM=(XA+XB)/2.
10    OLDDIF=ABS(XA-XB)/2.
      X=XM
```

```
C
C          NEWTON'S ITERATION
C
      DO 20 J=1,MAX
      OLDX=X
      FPRIME=DFUN(X)
      M=LEGVAR(FPRIME)
      IF(M.NE.O) GO TO 30
      IF(FPRIME.EQ.O.)GO TO 30
      FUNC=FUN(X)
      X=X-FUNC/FPRIME
      M=LEGVAR(X)
      IF(M.NE.O) GO TO 30
      IF (ABS(X-XM).GT.ABS(XA-XB)/2.) GO TO 30
      DIFF=ABS(X-OLDX)
      IF(DIFF.LE.ABS(X*ERROR)) GO TO 70
      IF(DIFF.GE.OLDDIF) GO TO 30
      OLDDIF=DIFF
20    CONTINUE
C
C          BISECTION ITERATION
C
30    FM=FUN(XM)
      IF(FM.EQ.O.)GO TO 60
      IF(FA*FM.LE.O.)GO TO 40
      XA=XM
      FA=FM
      GO TO 50
40    XB=XM
50    XM=(XA+XB)/2.
      IF(ABS(XA-XB).GT.ABS(XM*ERROR)) GO TO 10
60    NEWX=XM
      RETURN
70    NEWX=X
      RETURN
80    NEWX=(XA+XB)/2.
      NOROOT=1
      RETURN
90    NEWX=XA
      RETURN
100   NEWX=XB
      RETURN
      END
C
```

```
      SUBROUTINE BISECT2(XA,XB,MAX,ERROR,FUN,NEWX)
C
C     THIS SUBROUTINE COMPUTES THE ROOT OF THE FUNCTION FUN. IT
C     IS A COMBINATION OF THE BISECTION AND SECANT ITERATION
C     METHOD.  THE BISECTION INTERVAL IS USED TO START THE
C     SECANT ITERATION.  THE PROGRAM CONTINUES WITH THIS METHOD
C     UNTIL THE SOLUTION IS FOUND OR THE FOLLOWING SITUATIONS
C     OCCUR: 1- X FALLS OUTSIDE THE INTERVAL KNOWN TO CONTAIN THE
C     SOLUTION; 2- X IS OUT OF RANGE OR INDEFINITE; 3- TOO
C     FAR AWAY FROM THE SOLUTION; 4- THE NUMBER OF ITERATIONS
C     EXCEEDS MAX. IF THESE SITUATIONS OCCUR, THE PROGRAM
C     SWITCH TO THE BISECTION METHOD TO OBTAIN A SMALLER
C     INTERVAL.  REFERENCE: MOORE,E.  1982.  "INTRODUCTION TO
C     FORTRAN AND ITS APPLICATION". ALLYN AND BACON, INC.,
C     BOSTON,MASS.
C
C     LIST OF VARIABLES:
C
C     DIFF= DIFFERENCE BETWEEN TWO ITERATION POINTS
C     ERROR= TOLERANCE ERROR
C     FA= VALUE OF FUN AT XA
C     FB= VALUE OF FUN AT XB
C     FM= VALUE OF FUN AT XM
C     FO= VALUE OF FUN AT XO
C     F1= VALUE OF FUN AT X1
C     LM= -1 IF X IS INDEFINITE; +1 IF OUT OF RANGE; 0 OTHERWISE
C     MAX= MAXIMUM NUMBER OF SECANT ITERATION
C     NEWX= ROOT OF FUN
C     NOROOT= ROOT INDICATOR:  0-YES; 1-NO
C     X= POINT FROM THE SECANT ITERATION EQUATION
C     XA= LOWER BOUND POINT USED IN THE BISECTION METHOD
C     XB= UPPER BOUND POINT USED IN THE BISECTION METHOD
C     XM= MIDPOINT BETWEEN XA & XB
C     XO= SECOND POINT REQUIRED FOR THE SECANT PROCESS
C     X1= FOCUS POINT FOR THE SECANT ITERATION
C
C     LIST OF SUBROUTINES:
C
C     SWAP= INTERCHANGE THE VALUE OF TWO VARIABLES
C
C     LIST OF FUNCTIONS:
C
C     FUN= FUNCTION WHOSE ROOT IS COMPUTED
C
      REAL NEWX
      COMMON/ROOTNO/NOROOT
      IF(XA.GT.XB) THEN
      CALL SWAP(XA,XB)
      END IF
      FA=FUN(XA)
      FB=FUN(XB)
      XO=XA
      FO=FA
      X1=XB
      F1=FB
      XM=(XA+XB)/2.
      IF (FO*F1.GT.O.) GO TO 90
10    CONTINUE
      IF(ABS(F1/FO).GT.5..OR.ABS(FO/F1).GT.5.) GO TO 40
      IF(ALOG(ABS(F1)).GT.-2..OR.ALOG(ABS(FO)).GT.-2.) GO TO 40
```

```
C
C           SECANT ITERATION
C
      DO 30 J=1,MAX
      IF(ABS(F1).GE.ABS(FO)) THEN
      CALL SWAP(FO,F1)
      CALL SWAP(XO,X1)
      END IF
      X=X1-F1*(X1-XO)/(F1-FO)
      LM=LEGVAR(X)
      IF (LM.NE.O) GO TO 40
      IF(X.LT.XA.OR.X.GT.XB) GO TO 40
      DIFF=ABS(X-X1)
      IF(DIFF.LE.ABS(X*ERROR)) GO TO 80
      XO=X1
      FO=F1
      X1=X
      F1=FUN(X)
30    CONTINUE
C
C           BISECTION ITERATION
C
40    FM=FUN(XM)
      IF (FM.EQ.O.) GO TO 70
      IF (FA*FM.LE.O.) GO TO 50
      XA=XM
      FA=FM
      FO=FA
      XO=XA
      F1=FB
      X1=XB
      GO TO 60
50    XB=XM
      FB=FM
      X1=XB
      F1=FB
      XO=XA
      FO=FA
60    XM=(XA+XB)/2.
      IF (ABS(XA-XB).GT.ABS(XM*ERROR)) GO TO 10
70    X=XM
80    NEWX=X
      RETURN
90    NEWX=(XA+XB)/2.
      NOROOT=1
      RETURN
      END
C
```

```
      SUBROUTINE SORTING(X,Y,NU)
C
C  SHELL-METZNER SORT FOR ARRAYS X & Y OF SIZE NU, IN
C  INCREASING ORDER OF X. MAX. SIZE = 100. REFERENCE:
C  MILLER, A.R.  1982. "FORTRAN PROGRAMS". SYBEX INC.
C  BERKELEY, CA.
C
C
      REAL X(100),Y(100)
      JUMP=NU
10    JUMP=JUMP/2
      IF(JUMP.EQ.0) GO TO 99
      J2=NU-JUMP
      DO 30 J=1,J2
      I=J
20    J3=I+JUMP
      IF(X(I).LE.X(J3)) GO TO 30
      CALL SWAP(X(I),X(J3))
      CALL SWAP(Y(I),Y(J3))
      I=I-JUMP
      IF(I.GT.0) GO TO 20
30    CONTINUE
      GO TO 10
99    RETURN
      END
C
```

```
      SUBROUTINE SWAP(AA,BB)
C
C  THIS SUBROUTINE INTERCHANGE THE VALUE OF TWO VARIABLES
C
C
      HOLD=AA
      AA=BB
      BB=HOLD
      RETURN
      END
C
```

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