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THE THEORY AND EMPIRICAL ESTIMATION OF CURRENCY SUBSTITUTION

By

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ABSTRACT

THE THEORY AND EMPIRICAL ESTIMATION
OF CURRENCY SUBSTITUTION

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Currency substitution is said to exist if the following two conditions hold. First, transactors in an economy must hold, as a normal course of events, foreign currency balances. Second, the levels of foreign and domestic balances held in the economy must change in response to changes in other economic variables. That is, substitution between balances must occur on the demand side.

The main purpose of this thesis is to develop a model which explains the degree to which foreign currencies are viewed as substitutes for domestic money balances. Our model differs from previous studies in several ways. First, it bases its formulation on the transactions demand for foreign as well as domestic balances. Second, incorporated into the model is a speculative component which allows for additional holdings of a currency if it is expected to appreciate vis à vis the domestic currency over the holding period.

The model is then tested using quarterly data on the Canadian holdings of their own and United States dollars. Estimates of the elasticity of substitution in demand for these two currencies are found to be very low. Further, the size of these estimates depends upon the definition of the relative holding costs and the definition of foreign balances.

Even after one allows for changes in relative holding costs, additional substitution into holdings of U.S. dollars occurs as transactions levels in Canada rise. This finding yields substantial improvement in the explanatory power of the model.

Considerable evidence is also found that the potential for currency substitution is enhanced during fixed exchange rate periods. This result is in sharp contrast to previous studies and tends to support traditional assumptions about the ability of flexible exchange rates to insulate the monetary policies of an economy.

Questions of separability, lags in adjustment of asset balances, and technological innovations are also addressed in the empirical tests. The main conclusion from the study is that any model of currency substitution which does not model explicitly the role of transactions in the demand for foreign currencies is considerably biased.

To my parents

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LIST OF TABLES

TABLE OF CONTENTS

Chapter Page

CHAPTER ONE

Introduction

The monetary approach to the balance of payments has become, since its modern reformulation, a strong rival within the set of alternative theories of the balance of payments and exchange rate determination. Beginning simply as a statement about domestic money market disequilibrium leading to trade imbalances or exchange rate changes, it has been refined and extended to include additional assets (e.g. capital, bonds, equity, foreign exchange) framed in a simultaneous equilibrium model. This movement to greater realism in the paradigm has spawned one subset of models concerned with a phenomenon known as currency substitution (CS).

Currency substitution is said to exist if the following two conditions hold. First, transactors in an economy must hold, as a normal course of events, foreign currency balances. It is not necessary for all actors in an economy to hold more than one currency. Indeed, it is likely that they will not. However, it is possible to identify certain subgroups within the economy who would have strong motives for holding foreign monies. These would include all individuals and firms regularly engaged in foreign transactions. Thus, importers, exporters, multinational corporations, frequent travelers, and border area residents all would be likely to maintain stocks of foreign monies in order to facilitate transactions. One can also imagine situations where speculative and precautionary motives would encourage holdings of foreign exchange.

The second condition for the existence of CS is that the levels of foreign and domestic balances change in response to changes in other economic variables. That is, substitution must occur on the demand side. We emphasize the question of CS is not one of convertibility, or of the prevailing exchange rate regime. In fact, models employing various exchange rate systems have all been considered. Full convertibility is an implicit (or explicit) assumption of all of these models. Indeed, its existence affords an opportunity to rephrase the question to be considered. Given no interference in foreign exchange markets, why do some groups in an economy prefer to hold foreign exchange at all times rather than to convert local currency for it at the time of transactions? Further, what factors determine at the aggregate level changes in the composition of money portfolios?

The main purpose of this thesis is to develop a model which explains the degree to which foreign currencies are viewed as substitutes for domestic balances. This theory differs from previous theories in that it bases its formulation on the transactions demand for foreign as well as domestic balances. Second, we incorporate into our model a speculative component which allows for additional holdings of foreign exchange with a change in the expected rate of depreciation in the exchange rate. Finally, we test our model using data on Canadian holdings of their own currency and U.S. dollars.

Chapter Two presents an extensive review of the CS literature. Because this literature is so new, our survey is the first attempt to integrate the various papers on the topic. In Chapter Three we develop a model of CS. This model is used to derive a set of asset demand equations based on marginal productivity theory of monies as inputs in a production function for money services. In addition, we allow for a changing level of transactions in an economy to affect the currency mix. We also model factor augmenting technological progress as an additional determinate of money holdings.

Chapters Four and Five concentrate on the empirical side of this thesis. Specifically in Chapter Four estimation equations are developed from the theoretical model. We also discuss estimation of model parameters which must be derived from the regression coefficients. Chapter Five presents the results of various attempts at estimating our model. In Chapter Six, our conclusions and suggestions for future research are provided.

CHAPTER TWO

Survey of the Currency Substitution Literature

Introduction

The purpose of this chapter is to review the literature on currency substitution (CS). The body of this literature is relatively small. Several important papers on this topic remain unpublished. Bilson (1979) is the only author to attempt to integrate any of the ideas on this topic. However, since his sole concern was the role of CS in exchange rate determination, he ignores other important aspects of the CS literature (eg. theory of the substitution process, empirical tests). Most papers on international portfolio balancing allow for CS, since foreign exchange appears as an asset in the portfolio. However, in these models, CS is not afforded any special attention. Therefore, this last set of papers will not be considered here.

Although Eurodollar deposits are considered by many authors as an example of the CS process, we feel that the literature on the Eurodollar market is sufficiently peripheral to our interests to allow us to ignore this large body of literature. Specifically, the Eurodollar literature tends to focus on institutional arrangements (see McKinnon (1979)), and on liability management questions of the banks which supply these foreign currency denominated deposits. In contrast, the CS literature is concerned with substitutability in demand between various currency balances.

In the chapter below we consider the origins of the CS literature, the theory of the substitution process, macroeconomic implications, and finally empirical tests and measurements of CS.

Origins

The theoretical framework for the analysis of CS was laid in several independent articles written about the same time by former University of Chicago students, including Russell Boyer, Rudiger Dornbusch, Chau-Nan Chen, and others. These people have developed models which consider the question of an endogeneous demand for foreign balances.

The central theme of their papers is the "store of value" function of money balances. Domestic and foreign balances are demanded because they pay a rate of return (through price deflation or exchange rate appreciation), and not because they may facilitate transactions differently (so that one would need one "type" of money for one type of transaction and another type of money for another transaction). Various monies are held in aggregate asset portfolios and their relative shares are assumed to change as risk-return combinations change. The special twist that is provided to the analysis of open economies by the assumption of CS is the possible presence of a perverse change in real wealth holdings when the exchange rate changes. That is, changes in the exchange rate lead to an appreciation in the real value of one part of real wealth and a depreciation in the other. The net change in real wealth from any change in the exchange rate will then depend upon the relative shares of domestic and foreign denominated wealth (monies) in the economy's total real wealth. To complete the circle, the shares of domestic and foreign balances in total real wealth depend upon the degree to which these alternative assets are substitutes in demand. A question that should be asked (but often is not) is what is so special about money in these models? Would not bonds denominated in various currencies exhibit the same perverse wealth changes? The answers to these questions must lie with the other characteristics of monies vis à vis interest bearing assets. These characteristics would include, of course, liquidity and the ability of money to facilitate transactions directly.

Russell Boyer presents the first formal treatment of CS. His ideas appear as a chapter in a book edited by Putnam and Wilford (1978) on the monetary approach to the balance of payments. Boyer is concerned with the different solutions to the problem of determining the equilibrium exchange rate under the alternate assumptions of zero and perfect currency mobility (perfect substitutability). Using a simple three goods (two monies, and a composite good) and two countries model, he finds that if currencies are immobile (and therefore not substitutable) between the two countries, then there exists a stable long run equilibrium point where excess demands for real balances in both countries are zero.[1] This of course is a standard result of the monetary approach to the balance of payments. The stability of the long run equilibrium is guaranteed under either a fixed rate system (where disequilibrium in a domestic money market would lead to goods flows which would alter official holdings of foreign money until equilibrium reappeared) and under floating rates (where domestic prices would adjust to restore zero excess demand for real balances.)

As Boyer notes, however, once the assumption about zero currency substitution is relaxed, then it is possible that no unique equilibrium can be established for the desired holdings of real balances.[2] Under flexible exchange rates regime, it is conceivable that price levels and the exchange rate may even be indeterminate. This result will be pursued further below. The basic indeterminary in the equilibrium exchange rate arises because currency substitution implies shifts in demands <u>and</u> supplies of real balances in both countries due to exogenous shocks while the monetary approach emphasizes supply shifts (through trade imbalances under fixed rates and price movements under flexible rates) as the vehicle for returning to equilibrium.

Subsequent papers by Laffer (1976), Evans and

Laffer (1977), Girton and Roper (1976), and Kareken and Wallace (1978) elaborate on this question of equilibrium exchange rate indeterminacy and hence expand upon the ideas of Boyer.

While Dornbusch has never formally considered the question of CS, several of his papers have provided a framework for analyzing the effect of CS on the macroeconomy. In a 1973 article Dornbusch presents an elegant model of the dynamic behavior of the balance of payments. Money in his model is treated as the only marketable asset, but residents are restricted to maintaining balances only in their own currency. Under these conditions, devaluation is found to be a largely monetary effect.[3] Real balances change because prices change, and to the extent that this change in wealth affects expenditures, devaluation will be successful.

Lapan and Enders (1978) extend the Dornbusch analysis by examining the "efficacy of a devaluation when residents of a country hold assets denominated in terms of the foreign unit of account."[4] As the authors use the word assets instead of money, it is not clear whether this paper is an explicit example of the CS literature.[5] Actually, because the authors specify a demand for wealth equation which is identical to Dornbusch's liquidity demand function[6] and assume that the assets in their model pay no interest (or, the government taxes interest payments away),[7] we shall continue to classify this paper as a CS model. Inclusion of holdings of foreign denominated balances as part of domestic wealth leads to a set of circumstances whereby the efficacy of devaluation on the balance of trade is assured. Specifically, the authors derive Marshall-Lerner type conditions (regarding proportions of total wealth held in the form of foreign currency in each country) that are sufficient to guarantee that a devaluation would be successful.

Another article by Dornbusch (1976) provides the basis for the contribution of Calvo and Rodriguez. Their paper concerns itself with exchange rate dynamics in a world of CS and rational expectations.

Other seminal papers on CS focus on specific issues, and emphasize the transactions demand for foreign and domestic balances. Chen (1973) employs a Keynesian, short run, underemployment model of a two country world in order to examine the efficacy of monetary and fiscal policies when CS exists. Chen hypothesized that foreign and domestic balances are used as inputs in a Cobb-Douglas type technology in order to produce money services. Miles in several papers challenges Chen's specification that the elasticity of substitution in demand between domestic and foreign monies is unity and attempts to measure empirically this elasticity using a model suggested in the domestic near money literature.[8] Miles finds that elasticities of substitution within several countries between foreign and domestic balances are significantly greater than unity during periods of flexible exchange rates, and are not significantly different from zero during periods of fixed rates.[9]

King, Putnam and Wilford (1978) attempt to develop a theoretical model of the degree of currency substitution. They consider both transactions and speculative demands for foreign balances. Their model suggests that currencies become closer substitutes as the sets of goods and financial assets that these currencies jointly command grows. Hence "it is the integration of world markets for goods and financial assets that allows different currencies to perform similar monetary services and thus provide the institutional framework within which currency substitution is possible."[10]

The Theory of the Substitution Process

One of the problems in organizing this literature around a central theme is that there is no agreement as to how to specify the actual substitution process. One possible solution is provided by Per Meinich in his comment on a paper by Cooper

> Cooper asks the following question: "But if the public can hold foreign money, what role should that play in the money stock equation?" I would answer that question by introducing separate supply, demand, and market clearing equations for foreign money in the model.[11]

Several of the papers follow this approach to some extent. Most ignore the supply side by assuming long run conditions (i.e. infinitely elastic supplies of foreign exchange). Which is to say that in the long run the process of CS itself will have no effect on the exchange rate. Others posit a short run perfectly inelastic supplies of foreign monies within a country so that intra-country exchange rates between the various monies must respond in order to guarantee that citizens are content to hold existing stocks of assets.

The earliest paper on CS offers a simple model of the substitution process. In his view that foreign monies should be treated as financial assets, Boyer describes the meaning of substitutability as follows:

> As financial assets, the relevant variable that...is crucial to determining the amount of each held is the rate of return on that asset as compared with other substitutable assets. For assets without pecuniary yield, their relative rate of return is just the rate of appreciation of one in terms of the other.[12]

Therefore, according to Boyer, expectations of an appreciation in value of any one currency will lead to accumulation of assets denominated in that currency and short sales of asset denominated in the other. The degree of substitution between any two currencies depends upon the degree to which currencies are viewed as substitutes (which depends upon the nature of expectations of changes in the exchange rate.)[13]

Laffer's model is virtually identical to that of Boyer's. He writes

> one money i is defined as a substitute for another money j if an (percentage) increase in the supply of j, $\Delta MS_j > 0$, leads to an (percentage) increase in the price of goods in terms of money i, $\Delta P_{gli} > 0$. Money i is a perfect substitute for money j if $MS_j > 0 \Rightarrow \Delta P_{gli} = 0$.

And finally, money i is a perfect substitute for money j if $\Delta MS_j > 0 = \Delta P_{gli}$ $(\Delta P_{gli} > 0)$.[14]

The mechanism at work is as follows. Under perfectly fixed rates, monies are perfect substitutes on the supply side. This is because an increase in the money supply in country A leads to an excess supply of money in A and therefore a deficit in the balance of trade. This leads to monetary expansion in B (through a trade surplus) and hence an increase in the price level of goods denominated in B's currency. Similar results obtain if monies are perfect substitutes in demand under flexible exchange rates and can be traded internationally. That is, decreases in the supply of money in A lead to increased demands for holdings in A for B's currency. The only way for equilibrium in B's money market to remain (as demand in B rises), would be for prices in B to fall by an equal percentage (since nominal supply of B's money is unchanged.)

Laffer does not specify the characteristics of the various monies which would make them close substitutes in demand. Presumably, however, they must be related to the ability in one country for actors to "carry on their business using foreign money balances."[15] This implies that "non-substitutability of a country's money for another's occurs when each country's excess demand function (for money balances) is strictly independent of the other's."[16]

The most complete model of the processes described by Boyer and Laffer can be found in the paper by Girton and Roper. They introduce a portfolio balance model which considers a three asset world: two traded currencies and a fixed capital stock. They use this model to consider the currency substitution process in one country.

The authors define the following variables:

L_j = real demand for the j^{th} currency j = d (domestic) and f (foreign) ≡ nominal demand for currency j deflated by the price level of a common set of goods.

F_k = real demand for capital

r_j = real interest rate for j^{th} asset j = d,f

i_j = nominal interest rate for j^{th} asset j = d,f

P_j = price level of a common set of goods in currency j j = d,f

p_j = $d \log \dot{P}_j/dt$ j = d,f

δ = $\dot{P}_d - \dot{P}_f$ = differential inflation rate

W = real wealth

$\dot{p}_j^*$ = anticipated rate of inflation

r_j^* = $i_j - \dot{p}_j^*$ = anticipated real interest rate

δ^* = differential in the expected inflation rate

The real demand for each of the assets in the national portfolio is hypothesized to be a function of the national stock of real wealth and the expected real rates of return of all of the assets.[17] Setting available asset supplies equal to demands then summation across equations determines the assets markets equilibrium condition (i.e. the wealth constraint for the economy.) This condition is described below as (after several simplifying assumptions)[18]

(2.1) $L_d(\delta^*, \dot{P}^*_d, W) + L_f(\delta^*, \dot{p}^*_f, W) + F_k(\dot{p}^*_d, \dot{p}^*_f, W) = W$

The term δ^* represents the anticipated differential in the price of a numeraire good (or common set of goods which either currency commands) denominated in the two currencies. This can be viewed as the expected rate of change in an <u>internal</u> exchange rate: $E = \frac{P_d}{P_f}$ (or a change in an exchange rate derived from purchasing power parity of traded goods prices). Thus, δ^* is a measure of expected relative holding costs (in terms of changes in relative purchasing power) of the two currencies. $\dot{P}^*_j$ represents the "own" opportunity cost of holding currency j.

Partial derivatives of (2.1) with respect to its various arguments take on the following interpretation. $\partial L_j/\partial\delta^*$ (j = d,f) is the substitution effect between the two currencies. That is, the substitution effect measures the change in demand for real balances of one currency when the relative holding cost of that asset is anticipated to rise vis à vis the other type of money. The hypothesized signs of these terms, hence, are $\partial L_d/\partial\delta^* < 0$ and $\partial L_f/\partial\delta^* > 0$. Further, $\partial L_j/\partial p^*_d$ (<0) is the "own" effect on demand for real balances denominated in currency j (j = d,f) of changes in anticipated inflation rates.

Total differentiation of (2.1) yields the following constraints on the partial derivatives:

(2.1a) $\partial L_d/\partial\delta^* + \partial L_f/\partial\delta^* = 0$

That is, the sum of the substitution effects between the two currencies (holding real wealth constant) must be zero. From this we see that decreases in the real demand for one currency due to changes in the anticipated differential rates of inflation will be exactly offset by increases in demand for the other currency.

The second and third constraints implied by the model are:

(2.1b) $\partial L_d/\partial \dot{p}^*_d + \partial F_k/\partial \dot{p}^*_d = 0$

(2.1c) $\partial L_f/\partial \dot{p}^*_f + \partial F_k/\partial \dot{p}^*_f = 0$

These conditions refer to the substitution between real balances and capital due to an "own" change in the expected rate of inflation.

Finally, the fourth constraint is

(2.1d) $\partial L_d/\partial W + \partial L_f/\partial W + \partial F_k/\partial W = 1$

This implies that changes in real asset demands brought on by a change in wealth must exhaust that change in wealth. To reiterate, the variables that cause portfolio holdings of currencies to change are changes in the expected inflation rates and changes in the level of real wealth. How are these expectations formed?

Girton and Roper consider two cases. First, they assume perfect foresight. This allows them to replace the arguments in (2.1) which contain expectations terms with their actual values. When this is done, a dynamic equation describing the movement of prices can be posited. These price movements will be affected strongly by the degree of currency substitutability.[19] Alternatively, expectations can be generated according to an adaptive expectations process. This assumption leads the authors to formulate a system of differential equations to explain price changes. Again the degree of substitutability between currencies influences the properties of the system.[20]

All of this discussion leads us to point out that while the authors consider the substitution process, via equations (2.1) and (2.1a-2.1d) they ignore the important question of what determines the degree of substitutability between currencies. This criticism applies to the next paper as well.

Calvo and Rodriguez (1977) design a two good, two asset model of a "small" open economy under flexible exchange rates. Their two assets are again domestic and foreign monies (hence CS). Further, actors in this economy are assumed to hold rational expectations about future events. The two goods in this model are both produced locally and are composites labeled traded (t) and home (h) goods. The price of the traded good is given in terms of foreign exchange (which is exogenous and here it is assumed to be unity), so that the "small" country price is equal to the exchange rate. This allows the authors to define a "real" exchange rate, $E = {}^{e}/p_h$ (which is nothing more than the domestic relative price ratio ${}^{P}t/p_h$).

According to the authors, CS occurs because of changes in expected asset returns. That is, the ratio of domestic to foreign currency held by the country's residents is a function of the difference in the rates of return of the two monies. These are $-\dot{p}^*_d$ and $\dot{e}^* - \dot{p}^*_d$ for domestic and foreign balances respectively.

Where $\dot{p}^*_d$ = expected change in domestic price level

$\dot{e}^*$ = expected change in the official exchange rate

The differential expected return then is $\dot{e}^*$, the anticipated change in the official exchange rate.[21]

Lapan and Enders are not concerned with substitution processes, since they provide no explicit CS relation. The authors begin their analysis with two CS type variables, m and m^f, where m equals the proportion of total wealth held by domestic residents in the form of foreign assets (exchange) and m^f is an analogous variable for foreigners. These ratios are presented as fixed parameters. A clue as to how the values of these variables are determined is given in a footnote within the paper. Specifically, the ratios are treated as constants because residents of the two countries are "assumed to have static expectations" about changes in the exchange rate.[22] Hence, we have the implicit assumption that CS takes place due to changes in expected rates of return (which has been modeled elsewhere as expected changes in the exchange rate). Since the authors are concerned with the effects of foreign currency denominated assets in domestic portfolios under a fixed exchange rate regime, the assumption of static expectations is not too disconcerting. However, it is clearly incorrect to assume static expectations and fixed values of m and m^f under a flexible rate regime (as the authors do in their appendix).[23] Further, if trade has been unbalanced for some time under fixed rates, then residents of both countries may expect a reallignment of currencies. This of course leads to CS (and further pressure on the currency of the deficit country). Thus, the lack of endogenously determined portfolio of assets weakens this paper.

The papers we have discussed so far deal with at most two countries and two currencies. Two papers extend the CS model to the more general N country N currency case. In an unpublished paper, Evans and Laffer explore the role of CS in the cosmopolitan demand for the several national currencies of the world. The authors specify a set of liquidity demand equations for the world's n + 1 countries (under flexible exchange rates), viz:

(2.2)
$$m_{ot} - p_{ot} = \alpha_o + \beta_o y_{ot} - \delta r_{1t} - \delta r_{2t} - \dots - \delta r_{nt} + u_{ot}$$
$$m_{1t} - p_{it} = \alpha_1 + \beta_1 y_{1t} + \gamma r_{1t} - \delta r_{2t} - \dots - \delta r_{nt} + u_{1t}$$
$$\vdots$$
$$m_{nt} - p_{nt} = \alpha_n + \beta_n y_{nt} - \sigma r_{1t} - \delta r_{2t} - \dots + \gamma r_{nt} + u_{nt}$$

where m_{it} = logarithms of the stocks of liquid assets denominated in the currency of the i^{th} country i = o,1,...,n.

p_{it} = logarithms of the price levels of the i countries i = 0,1,...,n.

y_{it} = logarithms of the outputs i = 0,1,...,n.

r_{jt} = differences between the return on liquid assets in country 0. j = 1,...,n.

α_i, β_i = parameters of the model reflecting constant terms and transactions demands for the i currencies i = 0,...,n.

γ, δ = parameters of the model which measure the degree of substitutability between the n + 1 currencies.

u_{it} = error terms i = 0,...,n.

From the specification of (2.2) and from various assumptions we see the continued hypothesis of previous studies carried on in this paper. Namely, monies are held solely for their store of value function. Thus we see an assumption that "liquid assets of one country do not facilitate production and exchange in another."[24] Further, "the quantity r_{it} is made up of two components...The first component is the difference between the interest rate on liquid assets in country i and in country 0. The second component is the amount of appreciation that the public anticipates between currency i and currency 0 over the holding period for liquid assets...in the short run, only changes in the second are important."[25]

The authors proceed to solve the system of equations described by (2.2) for an expression describing the determination of exchange rates between the j currencies (j = 1,...,n) and the numeraire currency, 0. In the process, Evans and Laffer assume rational expectations and purchasing power parity in order to simplify their analysis. Another simplifying assumption made by the authors removes much of the behavioral content of interest to researchers who would be concerned with CS models. In particular, since the δ (coefficients on the substitute currency expected rates of return) are restricted be constant across currency demand equations, the degree of substitutability between all pairs of monies (δ) is constrained to be equal. Likewise, the term representing the "own" rate of return effect on demand for any of the j (j = 1,...,n) currencies (γ) is the same in every equation. Neither of these assumptions makes sense intuitively and in fact are challenged in a paper by Brillembourg and Schadler.

Brillembourg and Schadler (1979) are concerned as Evans and Laffer had been with the effect of CS on the determination of exchange rates. They consider a model where these exchange rates are "determined by the world excess demand for each particular currency."[26] Since the model they develop is a series of currency demand equations with expected rates of return as exogenous variables, it is very similar to that described by (2.2). But, they allow for the possibility that not all pairs of currencies will display equal degrees of substitutability. In fact, determination of this degree of substitutability between several of the worlds trading currencies is a major goal of their research. The authors write,

> In particular, while the substitution between strong and weak currencies will still be important, complementarity or substitutability with respect to third currencies may be equally influential. In this case, an expansionary monetary policy may not only weaken a country's own currency but also weaken those currencies that have in the past tended to follow the weakening currency and strengthen those that have tended to diverge from it.[27]

It is clear from the passage above, that the coefficients on the various expected rates of return are not even, in the author's version of (2), constrained to have the same signs let alone identical values. In particular, while the coefficients on the own rates of return in the various demand equations are predicted to be positive, the coefficients on the other expected rates of return might be negative (indicating net substitutability between the currencies) or positive (suggesting a net complementarity relationship).

The authors also emphasize that several motives exist for holding various currencies. The relative importance of these various reasons is weighed by economic agents when they decide how to divide their liquid assets into the several monies. Specifically,

> Residents of any country may want to hold a variety of currencies in their portfolios, both to facilitate transactions in different currencies and to earn the rate of appreciation of a particular currency vis à vis others. As any one currency becomes less attractive as a store of value or medium of exchange, it is reasonable for portfolio holders to replace it with other stronger currencies.[28]

Because the authors remove several of the restrictive assumptions of previous papers and because they incorporate the concept of a transactions demand for foreign monies we feel that this is a significant paper in the CS literature.

Returning to the context of the single country model, King, Putnam, and Wilford focus their attention upon a theory of the degree of substitutability between domestic and foreign monies. They begin with a money demand function for domestic balances

$$(2.3)\quad \frac{M^d}{P} = \phi \cdot f(y,i,u)$$

where M^d = quantity of domestic currency demanded

P = domestic price level

ϕ = proportion of monetary services provided by domestic money ($0 < \phi < 1$)

y = permanent income

i = opportunity cost of holding money

u = stochastic disturbance

The function f in (2.3) represents the demand for total money services. This demand is assumed to depend upon the usual arguments; permanent income (transactions demand) and opportunity costs (speculative demand). Further, a fraction of these services (equal to $1 - \phi$) is provided by foreign balances. The authors write

> Residents' allocations of their money holdings between foreign and domestic currencies will depend on the degree of substitutability between monies. In the extreme case of perfect substitutability, transactors will be indifferent between domestic and foreign currencies.[29]

Having established a role for foreign currencies in the demand for money services, the authors then turn their attention to what determines the degree of substitutability between currencies. In particular, "currencies are substitutes in demand to the extent that they provide similar monetary services to any transaction."[30] They conclude that while currencies are largely imperfect substitutes because of the dominant role of domestic currency in internal trade, "increased integration of world markets for goods and services - making foreign goods more readily accessible...to domestic residents - would tend to provide a wider role for foreign currencies in satisfying transactions demand for money."[31] Similarly, increased integration of world capital markets implies a wider role for foreign monies in speculative balances held by the public.

Formalizing the ideas presented above, the authors define a term for the elasticity of substitution between foreign and domestic monies as:

(2.3a) $\sigma = g(I) \qquad g > 0$

where σ = elasticity of substitution

I = the intensity of integration of global goods and capital markets

While the level of I is assumed to be given at any point in time, it will change when various market and political variables change. Specifically, the authors describe this process as follows:

(2.3b) $I = h(T, C, \phi, \psi)$ $h_1 < 0,\ h_2 < 0,\ h_3 < 0,$

$h_4 > 0$

where T = barriers to trade

C = capital controls

ϕ = transportation costs

ψ = information availability

The authors conclude that "the ultimate institutional rigidity making for nonsubstitutability (is) the acceptability on the part of sellers of goods and assets of only the domestic currency in their marketing operations."[32]

While it is the degree of integration of world markets which determines the degree of substitutability between monies, the actual share of foreign balances in desired money services depends upon the existing exchange rate regime. That is, if exchange rates are expected to change during holding periods real capital gains or losses will also be anticipated. Thus we find the authors defining the following relation:

(2.3c) $\phi = j\ (E°, V/I)$ $j_1 < 0,\ j_2 < 0$

where $E°$ = the expected exchange rate, in units of domestic currency per unit of foreign currency, relative to the current spot rate

V/I = the uncertainty associated with exchange rate expectations conditional on the institutional structure which determines the degree of currency substitutability.

Hence we find as authors have previous suggested that expectations of domestic currency depreciation lead to increased holdings of foreign monies. Further increased uncertainty about the future value of domestic currency given the structure of world markets also leads to increased holdings of the less risky foreign monies.

While King, Putnam, and Wilford model the degree of CS as being determined largely by institutional factors, Chen and later Miles adopt a more traditional approach to the role of foreign monies in money services. Specifically, they consider the different monies to be inputs in a production function for money services and then use standard production theory to establish hypotheses about the CS process. We consider the Chen paper first.

Chen builds a two country, two currency, Keynesian under-employment model with perfectly elastic supplies of output at constant prices. He assumes both currencies are allowed to flow freely across borders in order to insure that the internal exchange rate conforms to the external (official) exchange rate. He then specifies a Cobb-Douglas production function for the desired level of money services with domestic and foreign monies as inputs. The desired level of money servides is assumed to be a function of total output and the interest rate. Once this desired level is determined, the ratio of domestic to foreign balances is determined through the minimization of opportunity costs in holding these balances subject to achieving the desired level of services. Mathematically this becomes:

$$(2.4) \quad \min\ c_d M_d + c_f M_f \qquad \text{st. } \bar{Y}/\bar{V} = M_d^{\alpha} (eM_f)^{1-\alpha}$$

where c_j = opportunity cost of holding the j^{th} currency $j = d,f$

M_j = nominal stock of j^{th} currency

Y = domestic output

V = velocity

e = currency exchange rate = price of foreign currency in terms of the domestic currency

The first order conditions for the minimization of the Lagrangian function of (2.4) will yield the following results:

$$(2.4a) \quad \frac{M_d}{eM_f} = \left(\frac{\alpha}{1-\alpha}\right) \cdot \left(\frac{c_f}{c_d}\right)$$

where α the fraction of total holding costs of all money balances accounted for by the holding of domestic currency. Chen assumes that $1/2 < \alpha \leq 1$ so that money services in the home country are always domestic currency intensive.[33]

Chen defines the ratio of opportunity holding costs as follows:

$$(2.4b) \quad \frac{c_f}{c_d} = \frac{r - (i_t + \dot{e}^*)}{r - i_d}$$ [34]

where r = real interest rate

i_j = nominal interest rate on j^{th} balance

$\dot{e}^*$ = expected rate of change in e

He then assumes that money balances yield no nominal returns (i.e. $i_d = i_f = 0$) and that $\dot{e}^* = 0$ (the long run solution).

Therefore ${}^{c}f/c_d = 1$ in equilibrium (since real rates of interest are equal world wide). This specification displays homotheticity in money services and interest rates since neither affect the opportunity costs of holding balances of either currency. If M_d is increased while M_f is held constant, then there will be an equivalent percentage increase (depreciation) in e. To the extent that changes in e lead to changes in e*, CS occurs. The underlying Cobb Douglas production function insures smooth substitution since the elasticity of substitution is constrained by this technology to be unity.[35] Note, CS will continue to occur until equilibrium has been restored (${}^{c}f/c_d = 1$).

Chen does not focus in on the link between e and $\dot{e}^*$. In fact, there is no equation to explain how expectations are determined. Miles in several papers is critical of Chen's paper, but not for this reason. Rather, he considers the value of the elasticity of substitution in demand to be an empirical question rather than a given, predetermined parameter of the model.

Miles considers the problem of maximizing the output of money services faced with an existing asset constraint. He defines a CES production function of money services (of which Chen's Cobb-Douglas formulation is just a special case) with domestic and foreign balances considered as inputs in the production process. This production function is given below as (2.5)

$$(2.5)\quad MS = \{\beta_d M_d^{-p} + \beta_f (eM_f)^{-p}\}^{-1/p}$$

where β_d and β_f = weights reflecting the efficiency of domestic and foreign balances in the production of money services. These parameters are assumed to be fixed and exogenous to the model.

p = substitution parameter between domestic and foreign balances

The asset constraint below (2.5a) represents the size of a one period loan necessary to produce the desired level of money services. Specifically,

$$(2.5a)\quad \overline{M} = M_d(1 + i_d) + e\, M_f\, (1 + i_f).$$

This implies that in the notation of Chen the opportunity costs of holding the various monies can be defined as:

$$(2.5b)\quad c_d = 1 + i_d$$

$$c_f = 1 + i_f$$

Substitution of (2.5b) into (2.5a) and then maximization of (2.5) with respect to (2.5a) yields the following first order conditions for constrained maximization:

$$(2.5c)\quad \frac{M_d}{eM_f} = \left(\frac{\beta_d}{\beta_f}\right)^{\frac{1}{1+p}} \left(\frac{c_f}{c_d}\right)^{\frac{1}{1+p}}$$

We see from differentiation of the log-linear version of (2.5c), that in the case of a CES production function, the elasticity of substitution in demand

$$\left[\frac{\partial \ln \left(\frac{M_d}{eM_f}\right)}{\partial \ln \left(\frac{c_f}{c_d}\right)}\right]$$ is given by the term $\frac{1}{1+p}$. Hence, only in the special case that p = 0 will the production of money services exhibit a Cobb-Douglas technology. Hence, Miles presents a direct vehicle for measuring the degree of substitutability in demand between any two monies.

Macroeconomic Implications of Currency Substitution

Much of the CS literature is devoted to the implications of the presence of CS on various macroeconomic variables or policies. Several papers are concerned with the determination of equilibrium exchange rates in a world of flexible rates. Other papers consider the role of CS in the efficacy of various macroeconomic policies. The general policy conclusion of most of this literature is that as long as monies are going to be substitutes on the demand side, then the optimal international monetary system ought to be one of perfectly fixed exchange rates. We consider each of these issues in the section below.

An argument that is often presented for the optimality of flexible exchange rates as an international monetary system centers around the idea that flexible exchange rates insulate economies from the economic policies of the rest of the world. Expansionary policies abroad lead to changes in assets prices (including a depreciation of foreign exchange rates) until existing asset stocks are again willingly held. Adjustment is very rapid and therefore no net flows of official foreign reserves appear. This result is in sharp contrast to the fixed exchange rate regime whereby official flows of reserves occur because economies find it impossible to insulate themselves from the policies of other countries. The literature on CS suggests that where private demand for foreign money exists; there are no capital controls to prevent international flows of private monies; and these monies are perfect substitutes in demand, then "there is no economic difference between fixed or floating exchange rates".[36] This is because under perfectly fixed rates monies are perfect substitutes on the supply side. If we alter our assumptions so that currencies become perfect substitutes on the demand side, then we witness the same economic phenomena. For instance, consider two countries, A and B, whose monetary authorities attempt independent monetary policies. Suppose these authorities increase the money supply in A by X% and authorities in B reduce the money supply by an equal X%, then initially there will be an excess supply of money in A and an equal excess demand for money in B. With flexible exchange rates, prices would rise by X% in A, fall by X% in B and A's currency would depreciate by 2X%. (We assume throughout a simple quantity theory of money and purchasing power parity). If currencies are perfect substitutes in demand however, excess demand for money by residents of B can be offset by increased holdings of balances in A. In fact, in this example, the excess supply of funds created by expansionary policies in A would be completely eliminated by increased demand from country B. In this case, there would be no effect on prices in either country (because a fall in the money supply of B is offset by a fall in demand) or on the exchange rate. Therefore, "the failure of the exchange rate to change because of the close substitutability of money negates the policy effects of the monetary authorities control over the money supply."[37]

The notion that monetary authorities have no control (in the case of perfect substitutability) over the supply of money within their country leads to the conclusion that the exchange rate is no longer determinate. This result is reached in several papers (Laffer, Evans and Laffer, Boyer, Miles, Girton and Roper). Kareken and Wallace agree with this and point out that the only way for monetary authorities to preserve some control over the impact of their policies is to impose portfolio controls or to manage exchange rates.[38]

Obviously, currencies, in general, are not perfect substitutes in demand. Hence, the exchange rate is then determinate. But, exchange rates may deviate from equilibrium values for long periods of time because forces which would lead to a return to equilibrium values (i.e. differential interest rates) are weaker under any degree of CS.[39]

The dynamic path of exchange rates under CS is considered in the paper by Girton and Roper. Girton and

Roper conclude that the exchange rate could be unstable with high degrees of substitutability. Adjustment paths to equilibrium may or may not be cyclical depending upon how inflationary expectations are formed.[40]

As Brillembourg and Schadler point out, in a world of more than two currencies, these monies might be either substitutes or complements in demand. Hence, "this interdependence among currencies can produce quite interesting patterns of exchange rate movements. For example, policy changes in one country may induce an appreciation of another country's currency, which in turn may induce a third currency to appreciate with it and fourth to depreciate."[41]

Currency substitutition under fixed exchange rates leads to equally perverse results. Miles concludes that because of CS, devaluation will not improve a country's balance of payments position by as much as traditional theory indicates. This is because devaluations may serve to increase the perceived risk of continued holdings of the now devalued currency. Both domestic and foreign holders of this currency will switch to balances in other countries. thus "exacerbating the excess supply of money rather than relieving it."[42]

Lapan and Enders focus on the wealth aspects of devaluation. They too find that when foreign balances are held, the efficacy of devaluation may be undermined. They point out that devaluation works through changes in real wealth. Devaluation lowers real wealth because domestic balances command fewer goods in world markets. Hence, consumption falls in the devaluing country leading to an improvement in the balance of payments. When foreign currency balances are included in asset portfolios, the effect of a devaluation on levels of real wealth in the devaluing country are now unclear (because holders of foreign balances achieve capital gains due to the devaluation). The authors arrive at a condition which must obtain in order to guarantee the efficacy of devaluation.[43]

Because Chen begins with a different assumption about the shape of the aggregate supply curve, (specifically, he assumes a short run less than vertical aggregate supply curve so that output will be affected by changes in aggregate demand), he finds that CS implies an hybrid currency system between a world of fixed rates and one of perfectly flexible rates (but currency immobility). In particular, he discovers a "Rybczynski" effect whereby increased supplies of a currency in one country will lead to increased holdings of both currencies in that country (in a two country world). Hence, "we obtain the surprising result that an increasing stock of the first currency with the second currency held constant, may lower the second country's income."[44]

As we have seen, the implications of CS are that under flexible exchange regimes, a country's exchange rate may be indeterminate or unstable, her monetary authorities may lose control over domestic money holdings and hence monetary policy is ineffective. Under fixed rates, devaluation may be ineffective. The policy conclusion is clear and oft repeated: so long as (and to the degree that) the private market treats monies as substitutes, the optimal exchange rate regime is one of fixed rates. As Karaken and Wallace point out,

> for a feasible international monetary system, governments must make a choice. They could choose not to have capital controls...But that regime is politically feasible only when budget policies are coordinated...We believe there is a stronger case for coordination than we have made here. Indeed, there is a persuasive case for continuing budget balance in all countries and for what is then feasible--cooperatively maintained (fixed and not adjustable) exchange rates.[45]

We continue to stress that the results of CS depend upon the degree to which currencies are viewed as substitutes in demand. We pursue empirical measures of this degree in the section below. Before leaving this section, we point out that we are interested in this thesis in pursuing the modeling of the CS process and measurement of the degree of substitutability. The macroeconomic implications of CS we leave unchallenged.

Empirical Tests and Measurement of CS

Several models have been presented in the literature to test for the presence of currency substitution and the degree of substitutability in demand.

Evans and Laffer develop a model of exchange rate changes based upon purchasing power parity and rational expectations. This equation is presented below as:

$$(2.6) \quad \Delta e_{it} = a_i - b_i \Delta m_{it} + b_0 \Delta m_{ot} + c_i \Delta y_{it} - c_o \Delta y_{ot} + v_{it}$$

where Δe_{it} = % change in the exchange rate between country i (i = 1,...,n) and country O

Δm_{it} = % change in the nominal money supply of country i (i = 1,...,n)

Δm_{ot} = % change in the nominal money supply of country O

Δy_{it} = % change in the real output of country i

Δy_{ot} = % change in the real output of country O

Δv_{it} = stochastic error term

The hypothesis to be tested is that if currencies are not substitutes in demand, then changes in any of the right hand side variables will lead to a proportionate change in the exchange rate. In other words, if we assume zero substitutability, then the probability limits of b_i, b_o, c_i, c_o all equal unity. On the other hand, the presence of perfect CS implies that exchange rates are indeterminate but tend not to change regardless of various policies. Hence, under the alternate assumption of CS, the probability limits of b_i, b_o, c_i, and c_o are all zero in value.[46]

Evans and Laffer estimate equation (2.6) using monthly data for Canada, France, Germany, Italy, Japan and the U.K. The observation period is from January 1968 to December 1975. The United States was taken to be the numeraire country. So that the equations estimated for the six countries represents explanations of changes in U.S. bilateral exchange rates over the period 1968-1975. The authors' results which were obtained through ordinary least squares regressions on (2.6) are presented in Table 1 below. They conclude that because no coefficient is larger than .383 in any of the six equations, during the period of analysis substantial CS must have occurred.

Measures of the degree of substitutability have been attempted in two different models. Brillembourg and Schadler are concerned with determining whether currencies are net substitutes or complements in demand. These authors derive a system of exchange rate equations where the right hand side variables are rates of return on own and (n-1) substitute-monetary assets, viz.

$$(2.7) \quad e_k = \alpha_o + \sum_{i=1}^{n} \alpha_i r_i + v_k$$

where e_k = logarithm of the exchange rate between country k (k = 2,...,n) and the U.S.

r_i = $f_i + \beta_i Y_i + \gamma_T$ = return on the i^{th} currency which is assumed to be a linear function of the forward premium (f_i) of that currency vis a vis the U.S. dollar, that country's real output (Y_i) and a time trend (T).

v_k = error term

A comment should be made here regarding the term which reflects the expected rate of return upon holding balances in currency k, r_k. The authors assume that foreign balances yield two types of returns. The first is a pecuniary return which is proxied by the forward premium (f_k). The

TABLE 1

Summary of Evans - Laffer Results

Country	a_i	b_i	b_o	c_i	c_o	R^2	F	D.W.
Canada	.001	-.002	.117	-.000	.011	.01	.1	1.61
	(.89)	(-.04)	(.59)	(-.01)	(.27)			
France	.001	.132	.149	.020	.082	.04	1.0	1.78
	(.46)	(1.10)	(1.22)	(1.38)	(.82)			
Germany	.005	.149	.059	.076	.032	.04	1.0	1.56
	(1.75)	(1.27)	(.56)	(1.55)	(.25)			
Italy	-.001	.120	.190	-.008	-.074	.09	2.2	1.97
	(-.21)	(1.73)	(2.67)	(-.87)	(-1.14)			
Japan	.001	-.027	-.001	-.002	.158	.06	1.4	1.86
	(.58)	(-.42)	(-.02)	(-.08)	(2.23)			
U.K.	-.001	.383	.107	.007	.135	.11	1.3	1.56
	(-.26)	(2.12)	(.96)	(.15)	(1.19)			

"t" statistic for the test that the coefficient equals zero in parentheses.

second return represents a non pecuniary return which is tied to the volume of transactions in the i^{th} currency. This return is proxied by a function of real income and a time trend.[47]

Using monthly data from eight countries over the period March 1973 through June 1978, the authors estimate the system of equations in (2.7) with a full information maximum likelihood procedure. Their results are presented below as Table 2. The coefficients (α_i) represent semi elasticities on the rates of return on currencies. Semi elasticities on own rates of return (i.e. the first eight diagonal elements of Table 2) are assumed to be positive. In seven out of eight cases the authors found positive (and often significant) "own" semi-elasticities. The only non-negative "own" semi elasticity was for the Japanese Yen. This value was insignificantly different from zero. The authors also had trouble measuring with precision the semi-elasticities of substitute currencies--the off or cross-diagonal elements. Only about one fifth of the estimates of the off-diagonal elements were significantly different from zero at the 95% confidence level, but the authors take comfort in the fact that many of the t statistics of these "insignificant" coefficients were greater than unity. The estimated parameters suggest the following interrelationships

> The continental European currencies exhibit strong complementarity, with half the cross semielasticities being significantly different from zero. Both the U.S. dollar and the Canadian

TABLE 2

Summary of Brillembourg - Schadler Results

Currency	α F	α DM	α Lit	α Swf	α $\$_{CA}$	α ₤	α ¥	α $\$_{US}$	β	γ	Constant
French Franc	.871	.402	.491	.061	.177	.015	.005	-.953	-.643	-.097	2.843
	(2.80)	(1.12)	(3.37)	(.53)	(1.57)	(.018)	(.24)	(2.25)	(2.57)	(.39)	(1.63)
Deutsche Mark	.249	1.717	.153	.304	-.190	-.056	.023	.142	-.731	-.096	-.337
	(1.12)	(2.53)	(1.05)	(2.23)	(1.50)	(.63)	(1.15)	(.22)	(2.33)	(.36)	(.22)
Italian Lira	.516	.259	.441	.136	-.052	.132	.303	.355	-.414	-.118	2.535
	(3.37)	(1.05)	(2.27)	(2.04)	(.83)	(1.73)	(1.32)	(.98)	(.75)	(.38)	(1.51)
Swiss Franc	.158	1.271	.336	.232	.321	.101	.022	-2.756	5.645	.711	8.104
	(1.12)	(2.23)	(2.04)	(.75)	(1.12)	(.88)	(1.09)	(2.17)	(1.71)	(2.52)	(3.16)
Canadian Dollar	.406	-.704	-.113	.285	.206	.094	.019	-.342	5.549	-.486	9.379
	(1.57)	(1.50)	(.83)	(1.12)	(.84)	(.97)	(1.19)	(.33)	(1.40)	(1.53)	(2.66)
Pound Sterling	.031	-.183	.252	.078	.082	.182	.009	2.027	-8.974	-.992	6.875
	(.18)	(.63)	(1.73)	(.88)	(.97)	(1.32)	(.44)	(1.81)	(1.77)	(3.28)	(2.52)
Japanese Yen	.002	.013	.104	.003	.003	.002	-.012	.008	-5.028	1.016	2.921
	.24	(1.15)	(1.32)	(1.09)	(1.19)	(.44)	(.60)	(.52)	(.46)	(3.39)	(3.19)
US Dollar	-.341	.051	.075	-.236	-.033	.224	.005	1.400	.089	-----	-----
	(2.25)	(.22)	(.98)	(2.17)	(.33)	(1.81)	(.52)	(1.96)	(1.95)	-----	-----

1. dependent variable is the logarithm of the US price level, all others are respective exchange rates vis à vis the US dollar.
2. numbers in parentheses represent t statistics.

> dollar are indicated to be substitutes for several of the European currencies. The relationships of the pound sterling with other currencies seem to be difficult to estimate with much precision, although complementarity vis à vis the Italian lira and the U.S. dollar is suggested. The surprising feature of the results for the Japanese yen is the small size of most of the semielasticities estimated.[48]

The authors suggest that one possible measure of the impact of currency substitution would be a comparison of the size of "own" semielasticities with cross semielasticities in the same equation.

Miles in several papers applies the analogy of foreign and domestic monies as inputs in a production function for money services. He estimates the elasticity of substitution between these money inputs and suggests that this might be an appropriate measure of the degree of substitutability in demand between different monies. Specifically he estimates equation (2.8):

$$(2.8) \quad \ell n \left(\frac{M_d}{eM_f}\right) = a_o + a_1 \, \ell n \left(\frac{1 + i_f}{1 + i_d}\right) + \varepsilon_o$$

where M_d = holdings of nominal domestic (d) balances by domestic residents

M_f = holdings of nominal foreign (f) balances by domestic residents

e = exchange rate

i_j = nominal short term interest rate in country j ($j = d,f$)

a_o = ratio of efficiency parameters from production of monetary services

a_1 = elasticity of substitution between domestic and foreign balances

In separate studies, Miles considers the demand for Canadian and U.S. dollars by non-bank, private sector Canadians, substitution by U.S. residents between U.S. dollars and several foreign currencies, and West German substitution between Deutche-Marks and several foreign currencies. His results are summarized in Table 3.

The result that Miles finds striking is that in almost every case, during floating exchange rate periods, foreign monies are substitutes in demand for domestic currencies. Futher, they are strong substitutes since values of a_1 are almost always greater than unity during these periods. During fixed rate periods, the same conclusion does not obtain. Miles attributes this to the fact that during fixed rate regimes monies need not be strong substitutes in demand since central banks guarantee through the process of fixing exchange rates that they will be close substitutes on the supply side.

All of these studies suffer from severe problems. Evans and Laffer suggest that their results indicate that the presence of CS. Yet, they also warn that a number of other situations--such as changes in domestic output and money supplies being perceived as transitory--could also generate the same results. Further, their assumptions about identical substitution parameters may be too strict (e.g. consider the results of the Brillembourg-Schadler paper). Finally, their generally poor results may be due to a basic inability to explain very short run phenomena

TABLE 3

Summary of Miles Results

dependent variable	opportunity cost ratio	period in quarters	a_0 (t statistics)	a_1 (t statistics)	R^2	D.W.	exchange rate regime
$\frac{CAN\ \$}{US\ \$}$	$\frac{1+i_{US}}{1+i_{CA}}$	60IV - 75IV	2.56 (18.0	5.43 (2.59)	.78 [1.]	1.44 [2.]	mixture
$\frac{CAN\ \$}{US\ \$}$	$\frac{1+i_{US}}{1+i_{CA}}$	60IV - 62II	2.78 (50.8)	12.8 (2.54)	.48 [1.]	1.66	floating
$\frac{CAN\ \$}{US\ \$}$	$\frac{1+i_{US}}{1+i_{CA}}$	62III - 70II	2.31 (12.7)	2.66 (.79)	.78 [1.]	1.41 [2.]	fixed
$\frac{CAN\ \$}{US\ \$}$	$\frac{1+i_{US}}{1+i_{CA}}$	70III - 75IV	2.79 (16.1)	5.78 (1.83)	.79 [1.]	1.27 [2.]	floating
$\frac{US\ \$}{(b + DM + SF)}$	several [3.]	67I - 78III	5.68 [4.]	2.15 [4.]	NA [5.]	NA [5.]	mixture
$\frac{US\ \$}{(b + DM + SF)}$	several [3.]	67I - 71II	5.89 [4.] (NA)	1.87 [4.] (NA)	NA [5.]	NA [5.]	fixed
$\frac{US\ \$}{(b + DM + SF)}$	several [3.]	71III - 78III	5.64 [4.] (NA)	3.24 [4.] (NA)	NA [5.]	NA [5.]	floating
$\frac{DM}{\text{total foreign holdings}}$	N/A [6.]	65I - 78III	4.26 (20.66)	1.11 (.87)	.79	2.23 [2.]	mixture
$\frac{DM}{\text{foreign holdings}}$	N/A [6.]	65I - 71II	4.48 (26.71)	1.51 (.51)	.69	2.00 [2.]	fixed
$\frac{DM}{\text{foreign holdings}}$	N/A [6.]	71III - 78III	3.71 (5.71)	2.78 (2.23)	.89	2.28 [2.]	floating

1. adjusted for degrees of freedom
2. after Cochran-Orcutt procedure
3. because holdings of several foreign currencies are summed together, the author considers none different interest rate ratios which repsent different weighting schemes.
4. values presented are averages of the estimated coefficients in the nine regression. Therefore t statistics are not presented.
5. average values are not presented
6. independent variable used in these regressions is never indicated.

(month to month changes in the exchange rate) with a long run theory (purchasing power parity.)

While the Brillembourg-Schadler paper corrects many of the faults of Evans-Laffer study, it fails in its attempt to measure the degree of substitutability between currencies. No clue is offered from the signs or sizes of the estimated parameters as to how strong the degree of substitution (or complementarity) is between any two currencies.

The studies by Miles fail on several grounds. First, the exclusion of the expected rate of depreciation of the exchange rate means that there is no possibility for CS because of expected capital gains. Second, CS is likely to occur (as several authors have pointed out) between countries where their foreign exchange is accepted in domestic transactions. This implies high CS between neighboring countries or close trading partners. In either case, expecially the former, it is likely that one country will be "small" relative to the other. The interest rate in the "small" country will tend then to be dominated by the interest rate in the other country, hence forcing interest rate parity. Mile's measure of the ratio of holding costs will tend to be invarient and therefore no induced change in relative balances. Finally, Miles assumes that transactions elasticity of demand for each of the money balances is identical, hence his specification retains the notion of homothetic isoquants regardless of the level of transactions. All of the omissions, suggest the possibility of severe bias in the estimated coefficients of Miles's model. We will attempt to demonstrate this in a later portion of this thesis.

FOOTNOTES

CHAPTER TWO

[1]Boyer, in Putnam and Wilford (1978), Chapter 13, pgs. 185-188.

[2]op. cit. pg. 196.

[3]Dornbusch (1973), pg. 880.

[4]Lapan and Enders (1978), pg. 601.

[5]Kreinin and Officer (1978), in their survey of the monetary approach to the balance of payments, consider the Lapan and Enders paper as part of the CS literature. See page 6.

[6]See Dornbusch, pg. 872 and Lapan and Enders, pg. 602.

[7]Lapan and Enders, pg. 603 (footnote 4).

[8]See Appendix 1 (at the end of this chapter) for a discussion of the near money literature as it applies to this analysis.

[9]The intuition for this result is that during periods of fixed exchange rates, central banks guarantee that foreign balances are perfect substitutes on the supply side by standing ready to convert currencies at a fixed and known rate. See Miles (1978A), in Putnam and Wilford, Chapter 12, pg. 178.

[10]King, Putnam and Wilford, in Putnam and Wilford, Chapter 14, pp. 203-204.

[11]Meinich, in Herin et. al. (1977), pp. 32-33.

[12]Boyer, pg. 189.

[13]op. cit., pg. 189.

[14]Laffer (1976), pg. 9.

[15]op. cit., pg. 8.

[16]op. cit., pg. 9.

[17]No mention is made of the role played by risk in this model. Persumably, the authors must assume that the overall riskiness of the portfolio as measured by the variances and covariance of the asset returns is constant. Therefore, the risk parameters can be subsumed in the parameters of the model.

[18]Specifically the authors assume: (1) equal changes in all expected returns leave the asset demand functions unchanged; (2) nominal rates of interest are zero (or constant); (3) the anticipated real rate of return on capital is constant.

[19]Girton and Roper (1976), pg. 9.

[20]op. cit., pg. 14.

[21]Calvo and Rodriguez (1977) pp. 619-620. Actually, to simplify their analysis, the authors reverse their casual relation and specify the CS process as

$$\dot{e}^* = G\left(\frac{M_d}{eM_f}\right) \quad G' < 0$$

[22]Lapan and Enders, pg. 604.

[23]op. cit., pp. 612-613.

[24]Evans and Laffer (1977), pg. 8.

[25]op. cit., pg. 9.

[26]Brillenbourg and Schadler (1979), pg. 515.

[27]op. cit., pg. 516.

[28]op. cit., pg. 515.

[29]King, Putnam and Wilford, pg. 202.

[30]op. cit., pg. 203.

[31]op. cit., pg. 203.

[32]op. cit., pp. 204-205.

[33]Chen (1973), pg. 98.

[34]Actually, in a footnote (page 99) Chen defines $C_f = r - i_f + \dot{e}^*$. Clearly this must be incorrect since this suggests that an increase in $\dot{e}^*$ (an increase in the expected rate of depreciation of domestic balances) raises the

holding cost of foreign balances. It seems that the term $i_f + \dot{e}^*$ which represents the expected return from holding foreign balances should have parentheses around it (as we have written in 2.4b).

[35]If we define the elasticity of substitution, σ, as

$$\frac{d \ln \left(\frac{M_d}{eM_f}\right)}{d \ln \left(\frac{C_f}{C_d}\right)}$$

then straight forward differentiation of the logarithmic form of 2.4a demonstrates this point. Specifically, taking logarithms of both sides of 2.4a yields:

$$(2.4a') \quad \ln \left(\frac{M_d}{eM_f}\right) = \ln \left(\frac{\alpha}{1 - \alpha}\right) + \ln \left(\frac{C_f}{C_d}\right)$$

Finally, differentiation of 2.4a' yields

$$\frac{d \ln \left(\frac{M_d}{eM_f}\right)}{d \ln \left(\frac{C_f}{C_d}\right)} = \sigma = 1.$$

[36]Laffer, pg. 3.

[37]op. cit., pg. 8.

[38]Kareken and Wallace (1978), pp. 4-5.

[39]Evans and Laffer, pg. 6.

[40]Girton and Roper, pg. 17.

[41]Brillembourg and Schadler, pg. 517.

[42]Miles (1977), pg. 8.

[43]Their derivation suggests that if $1 - m - m^f < 0$, then a devaluation cannot improve a country's balance of trade unless certain unlikely conditions about relative prices and incomes also obtain. (Recall, m equals the proportion of total wealth held by domestic residents in the form of foreign exchange and m^f is an analogous term for foreigners.)

[44]Chen, pg. 106.

[45]Karaken and Wallace, pp. 6-7.

[46] For complete derivations of these probability limits, see Evans and Laffer, pg. 13 and pp. 23-24.

[47] Brillembourg and Schadler, pg. 521.

[48] op. cit., pg. 527.

APPENDIX 1

Development and Use of the CES Function in the Near Money Literature

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Development and Use of the CES Function in the Near Money Literature

This appendix provides a brief review of several models of the domestic near money literature (NML). In particular, since we imply the CES function in our model of currency substitution we will focus on the development of the CES function as a theoretical paradigm for the measurement of the degree of substitutability between money and domestic near money assets. For a complete review of the near money literature, the reader is directed to the survey by Feige and Pearce (1977).

The near money literature can best be described as a set of empirical papers that began to appear in the early 1960's which analyzed the importance of yields on nonbank intermediary liabilities (e.g. savings and loan shares, treasury bills, certificates of deposit) in explaining the demand for money. The underlying issue is the Gurley-Shaw hypothesis that these non-bank intermediary liabilities have become such close substitutes in demand that the short run efficacy of monetary policy has become imperiled. That is, for instance, whenever the Federal Reserve attempts to restrict monetary expansion, agents will substitute for bank deposits these other financial claims.

The standard approach for measuring the degree of substitutability of any of these near money assets for money has been to specify a demand for money function with the yields on these assets as arguments of the function. The cross price elasticities (determined from the regression coefficients) could then be used to make inferences about whether the near money assets were close substitutes for money balances.

An alternative to this approach was developed in a paper by Chetty (1969). In this paper, he suggested that one could develop a utility maximization model with money and other assets as arguments of the utility function. A measure of the degree of substitutability could then be obtained from the slope of the indifference curves of the utility function.

Chetty chose for the functional form of his utility function (in the two asset) the CES (constant elasticity of substitution) utility function. Then he derived an estimation equation from the first order conditions (F.O.C.) of maximizing the utility function subject to a two period budget constraint, viz.

(A.1) $$\max U = (\beta_1 M^{-\rho} + \beta_2 T^{-\rho})^{-1/\rho} - \lambda \left[\left(M + \frac{T}{1+i}\right) - M_o\right]$$

F.O.C.

(A1.1a) $$\frac{\partial U}{\partial M} = \lambda$$

(A1.1b) $$\frac{\partial U}{\partial T} = \lambda/(1+i)$$

(A1.1c) $\frac{\partial U}{\partial \lambda} = 0$

where M = end of period cash holdings

T = end of period holdings of near money asset

M_o = initial cash holdings

i = interest yield on T

λ = Lagrangian multiplier

β_i = efficiency parameters of the utility function

ρ = substitution parameter of the utility function

Dividing A1.1a by A1.1b, taking logarithms and rearranging terms yields a regression model:

(A1.2) $$\ln \frac{M}{T} = \sigma \ln \frac{\beta_2}{\beta_1} + \sigma \ln \frac{1}{1+i}$$

where $\sigma = \frac{1}{1+\rho}$ is the elasticity of substitution between M and T

Chetty analyzed the substitution between money and several near money assets: commercial bank time deposits, savings and loan shares, and mutual savings bank deposits by estimating equation A1.2 using United States annual data for the period 1945-1966. In each case, he found large, statistically significant estimates of σ.

Because movements of M and T may reflect substitution into other assets, Chetty extended his model to the case where the utility function contained N assets. Here, he chose the generalized CES function of Mukerji (1963) and Dhrymes and Kurz (1964) as his specific utility function.

His regression model was derived from the F.O.C. of the maximization of A1.3,

$$\text{(A1.3)} \quad \max U^* = (\beta M^{-\rho} + \beta_2 X_2^{-\rho_2} + \ldots + \beta_N X_N^{-\rho_N}) - \lambda[(M + \frac{X_2}{1+i_2} + \frac{X_3}{1+i_3} + \ldots + \frac{X_N}{1+i_N}) - M_o]$$

where X_i = ith near money asset i = 2,....,N

i_i = yield on i^{th} near money asset i = 2,....,N

The regression model can be interpreted as a set of i demand equations for the i near money assets. The jth equation is presented below as

$$\text{(A1.4)} \quad \ln X_j = \frac{-1}{\rho_j+1} \ln \frac{\beta\rho}{\beta_i \rho_j} - \frac{1}{\rho_j+1} \ln \frac{1}{1+i_j} + \frac{\rho+1}{\rho_j+1} \ln M$$

Since M is endogenous, each of the i equations in A1.4 was estimated using two stage least squares. (Note, Chetty also estimated this system using ordinary least squares and found little change in the estimated coefficients).

A measure of the degree of substitutability between money balances and any of the near money assets would be the Hicks Allen direct partial elasticity of substitution (σ_{Mi}). This measure can be obtained from the estimated coefficients of the model through the following equation:

$$\text{(A1.5)} \quad \sigma_{Mi} = \frac{d \ln (M/X_i)}{d \ln (\frac{\partial U^*/\partial M}{\partial U^*/\partial X_i})} = \frac{1}{(1+\rho) + (\rho_i - \rho)/[1 + \frac{\beta_i \rho_i X_i^{-\rho_i}}{\beta \rho M^{-\rho}}]}$$

Chetty estimated equation A1.4 with the same data defined above and found large partial elasticities of substitution for each of the near money assets. He used these results to conclude that the near money assets he studied were close substitutes for money and because of their strong substitutability an appropriate definition of the money supply would include these assets.

In a comment on Chetty's paper, Lee (1972) criticized the use of pre-1951 data because that year represented an institutional change which could affect the stability of the substitution parameters (ρ_i). Lee was also critical of the use of the generalized CES function which assumes-because of its formulation-constant ratios of the partial elasticities of substitution. (The reader should see Solow (1967) pp. 45-46 for elaboration of this point.)

Steinhauer and Chang (1972) criticized the asset constraint used by Chetty, suggesting that additional holdings of other assets can come from reduced consumption as well as substitution out of money balances. They also complained that no mention was made in the Chetty model of the non-monetary services provided by financial assets and hence, the monetary services of these assets were assigned too great a weight in the construction of an expanded monetary aggregate.

Edwards (1972) applied Chetty's model to mid year 1962 cross section data for thirty-seven United States metropolitan areas. Edwards found very low partial elasticities of substitution between money and bank time and savings deposits (2.41) and savings and loan and mutual savings bank shares (-1.23). Edwards posited from these results that the degree of substitutability between money and these near monies was, in fact, very low. Edwards cited two reasons why his estimates were much lower than Chetty's. First, he found severe simultaneous equations bias in the ordinary least squares estimates of the asset demand equations. Second, he noted that time series data implies constant asset quality over time. If this constraint is false, then cross section estimation would provide a truer picture (since asset quality varies less between regions than over time) of the degree of substitutability between money and near monies.

Bisignano (1974) considered a broadened set of asset substitution. Specifically he expanded the CES and generalized CES utility functions to include holdings of long term United States securities and consumer durables. Further Bisignano considered utility maximization over a lifetime (rather than two period) horizon, and the effects of taxation and depreciation and capital gains. His findings were similar to those of Edwards. Specifically, while time deposits were the closest substitutes for money balances, the estimated elasticity of demand was less than .1. Hence, the author concluded that his results do not support the conclusions reached by Chetty.

Before considering additional papers, we must make the following observations. In the Chetty formulation, the elasticity of substitution is the coefficient of the logarithm of relative rates of return between money and the near money substitutes. Chetty, in fact, uses as this ratio $\frac{1}{1+i_j}$. In a footnote, Chetty (1969, pg. 273), he asserts but does not prove that one would obtain similar results if the yield ratio were $\frac{1}{i_j}$. It is this latter sort of yield ratio that Bisignano employed. The yield ratio was interpreted as a relative price ratio and took the form $\frac{r}{Z_j}$, where r was the discount rate in the utility maximization and Z_j equaled the discount rate minus the after tax rate of return on asset j. In the conclusion to his paper, Bisignano concluded that he had some doubts about the entry of interest rate variables in the form $\frac{1}{1+i_j}$. If fact, he commented that "it is possible to obtain substantial substitutability between real money and real durable goods if we enter our user cost variable in this manner." (pg. 32.) Thus, to some degree, the results of the studies discussed so far depend crucially on the specification of the model.

In a 1976 paper, Moroney and Wilbratte have revised somewhat the Chetty model. The authors posited that the household sector maximizes wealth subject to a technological constraint (i.e. a production function of the generalized CES form with assets treated as factor inputs in production function) and investigated the possibility that growing income might exert an independent influence on portfolio composition.

The first of these changes in Chetty formulation would not alter the estimating equation (equation A1.4) but would provide an alternative reason for the introduction of interest rates in the form $\frac{1}{1+i_j}$. This is because the objective function of the model becomes:

$$\text{(A1.6)}\quad W_t = M_t + \sum_{j=1}^{n} X_{jt}\,[1 + j_t]$$

where W_t = wealth in period t

X_{jt} = nominal dollar holdings of the j^{th} class of interest bearing asset in period t

i_{jt} = effective yield on asset j at time t

The second change is handled by revising the form of the CES function as follows:

$$\text{(A1.7)}\quad T_t = \{\beta_t M_t^{-\rho} + \sum_{j=1}^{n} \beta_{jt} X_{jt}^{-\rho_j}\}^{-1/\rho}$$

and $\beta_t = \beta Y_t^{\theta}$

$\beta_{jt} = \beta_j Y_t^{\theta_j}$

where Y_t is permanent income

and θ, θ_j are parameters not constrained to be equal.

The authors included this revision in the efficiancy parameter β, for two reasons: first, to reflect the changing characteristics of assets over time, and second, to mirror the changes in the transactions demand for money.

The Moroney and Wilbratte model differs from the Chetty model since an income term has been added. Specifically the demand equation for the jth asset can be written below as:

$$\text{(A1.8)}\quad \ln X_j = \frac{1}{1+\rho_j} \ln \frac{\beta\rho}{\beta_j\rho_j} + \frac{(\theta_j - \theta)}{1+\rho_j} \ln Y_t + \frac{\rho+1}{\rho_j+1} \ln M_t - \frac{1}{1+\rho_j} \ln \left(\frac{1}{1+i_j}\right)$$

The authors estimated equation A1.8 using both a full adjustment and partial adjustment specification with quarterly U.S. data for the period 1956-1970. They found no substantive differences in the substitutability between any of several assets (both long and short term) and money. Also, in most specifications, the income term was significantly below zero indicating that there had been a secular growth in the transactions demand for money over the period.

Finally, in a very detailed paper, Barth, Kraft and Kraft (1977) considered the issue of separability in demand by building a multilevel CES utility function. The hypothesis here was that households may maximize their utility from wealth holdings of several subsets of assets according to some constraint. Then, they would optimize between these subsets according to an overall asset constraint. The functional form for this process can be described as a multilevel CES function, where there is a CES function for each asset subset nested inside an overall CES function. Estimation of such a system is done by a non linear full information maximum likelihood procedure.

The authors considered three different cases. First, they grouped all five assets (money, time deposits, savings and loan shares, mutual savings bank deposits, and treasury bills) into one category. This implies that the marginal rate of substitution between any pair of these assets is not affected by holdings of other assets outside this group. They also considered a case where only money and time deposits were in one group. Finally, they considered a two group case where the only omitted asset in the first group was the stock of Treasury Bills held by the public. The results from this experiment showed that the degree of substitutability between any two assets depends upon the assumptions made about separability. In particular, the estimated Hicks-Allen partial elasticity of substitution between any two assets rose from 1.12 in the first case to a high of 24.7 in the third. The authors conclude that their results were in broad agreement with Chetty's.

Thus, while we see that the methodology of Chetty has been used in several instances to study essentially the same question, the results have been contradictory and inconclusive. Further, there seems to be a direct relationship between the form of the relative yield (cost) ratio and the size of the elasticity of substitution. Moreover, this situation of contradictory results is not limited to NML. In 1967, writing about the use of CES functions to study production, Nerlove (1967, pg. 58) points out, "Even slight variations in the period or concepts tend to produce drastically different estimates of the elasticity." Hence, we feel that comparison of estimates of substitution elasticities from different studies may be less than meaningful. A more interesting question may be whether the elasticity has changed over time.

CHAPTER THREE

A Theoretical Model of Currency Substitution

Introduction

The more that citizens view foreign money balances as substitutes for domestic balances, the more likely the effects of CS described in the previous chapter will obtain. The purpose of this chapter is to develop a model of asset demand that allows us to measure this degree of currency substitution. The model will be used to explain the existence of CS under different exchange rate regimes and in the presence of foreign exchange risk.

The determination of the degree of substitution between any two goods is a lesson in applied demand analysis. We begin our model by specifying the wealth constraint of our economy. Then, we isolate several monetary assets (domestic and foreign money balances) and propose a testable hypothesis as to how these assets are combined in an efficient manner to produce "money services."[1] We then can determine empirically the technology inherent in this process. A byproduct of this method is that the degree of substitutability (namely, the elasticity of substitution) can be measured directly as a parameter of this model.

The Wealth Constraint

Consider an economy where private, nonbank wealth can be held in N different assets. N-1 of these asset forms are financial (monies--domestic and foreign, debt claims, equity) while the N^{th} is a non-depreciating capital stock. Following Tobin (1969), we can specify the aggregate demand for the i^{th} asset as a fraction of total nonbank wealth holdings as:

$$(3.1) \quad \frac{X_i}{W} = X_i(r_1, r_2, \ldots., r_n, T)$$

where X_i = value of i^{th} asset in total private nonbank wealth holdings. $i = 1,2,\ldots.,N$

r_i = rate of return on the i^{th} asset $i = 1,2,\ldots.,N$

T = vector of variables that could affect the transactions demand for the highly liquid assets included in W

W = total private nonbank wealth

We will assume that assets X_i $\{i = 1,\ldots.,N\}$ are all gross substitutes in demand.[2] Further, we assume that for some, highly liquid assets $\frac{\partial X_i}{\partial T} > 0$. For other less liquid assets (e.g., consumer durables, the capital stock, long term government debt, etc.) $\frac{\partial X_i}{\partial T} \leq 0$. These last two assumptions imply that if the level of transactions demand rises, the private sector will adjust their portfolios as to hold a higher proportion of its nominal wealth in the form of liquid (money and near-money) assets.

We assume that whenever substitution between asset types occurs, the following wealth constraint must hold:

$$(3.2) \quad \sum_{i=1}^{N} X_i = W$$

Substitution of the N demand equations into the wealth constraint and partial differentiation with respect to the various functional arguments yields,

$$(3.3a) \quad \sum_{i=1}^{N} \frac{\partial X_i}{\partial r_j} = 0 \qquad \text{for any } j.$$

$$(3.3b) \quad \sum_{i=1}^{N} \frac{\partial X_i}{\partial T} = 0$$

$$(3.3c) \quad \sum_{i=1}^{N} \frac{\partial X_i}{\partial W} = 1 \quad .$$

Relations (3.3a) and (3.3b) imply that trading assets at one point in time (due to changes in rates of return or transactions demand) cannot change the value of wealth held by the nonbank public. Equation (3.3c) means that no new assets can be created in this economy, hence, any increase in private nonbank wealth must be distributed among the N assets.

One measure of the degree of substitution is the cross price elasticity of demand. In our model, the degree of substitutability between assets X_i and X_j may be described by the cross rate of return elasticity, viz:[3]

$$\eta_{X_i X_j} = \frac{\partial X_i}{\partial r_j} \cdot \frac{r_j}{X_i} \quad .$$

In a world of many assets and many rates of return, the analysis of substitution in demand within a particular subset of assets may be unnecessarily complicated if every asset demand equation must be specified and studied. It may be possible to separate the various assets into subsets based upon certain common characteristics, e.g., liquidity, return, riskiness, maturity, etc. If we assume that the assets in these subgroups are held in efficient combinations to their ability to produce desired levels of services, then we can consider the degree of substitutability between the assets within these subsets.[4]

Separation Rule

We begin by assuming that wealth holdings provide both pecuniary and nonpecuniary services to any economy. That is, we should consider both pecuniary attributes such as liquidity, return, riskiness as well as non-pecuniary attributes like status and security that wealth delivers when defining a wealth function. Consider the following production function for wealth services:

$$(3.4) \quad WS = W(X_1, X_2, \ldots, X_N)$$

If we can partition the elements of W into subsets, then it is possible to consider a two stage procedure whereby wealth services are maximized. That is, in the first stage relative input shares are optimized within each subset and then, in the second stage optimal holdings are found by holding input intensities fixed within subsets and optimizing across subsets.

The conditions necessary for such a two step process are well known. They involve the notion of separability of production (or utility) functions.[5] These conditions will be discussed below.[6]

We begin by considering the set of all asset inputs $X = \{X_1, X_2, \ldots, X_N\}$. Then, we partition X into M ($M \leq N$) mutually exclusive and exhaustive subsets ($S_1, S_2, \ldots, S_M$). The production function W(X) is said to be <u>weakly separable</u> with respect to a partition ($S_1, S_2, \ldots, S_M$) if the marginal rate of substitution, $W_i(X)/W_j(X)$, (where $W_i(X) = \partial W/\partial X_i$) between two assets i and j from S_k does not depend upon the quantities of assets held in subsets other than S_k. Mathematically, we have:

$$\frac{\partial}{\partial X_\ell}\left(\frac{\partial W(X)}{\partial X_i} \Big/ \frac{\partial W(X)}{\partial X_j}\right) = 0 \quad \text{for all } i,j \in S_k \text{ and } \ell \notin S_k .$$

It is possible to show that the condition of <u>weak</u> separability with respect to a partition ($S_1, S_2, \ldots, S_M$) is necessary and sufficient for the function W(X) to be of the form $W(X^1, X^2, \ldots, X^M)$, where X^i is a function of the elements of S_i $i = 1, \ldots, M$ only. This last result is known as the fundamental theorem of weak separability.[8]

From the above, it is obvious that if the weak law of separability holds for the partition we have chosen, then we can consider the slope of an isoquant (hence, the degree of substitutability) between two or more members of an asset subset without regard to the quantities of assets held in other subsets of the set of assets.[9]

Asset Partition

We choose to partition the set of all assets, $X = \{X_1, X_2, \ldots, X_n\}$ into two subsets, S_1 and S_2. Let S_1 contain the stocks of domestic and foreign currency denominated money balances held by the private sector of the economy. We choose this partition rule because these balances share the common characteristic that they may be used directly--without conversion--to pay for all (or at least certain) transactions. We assume that assets in S_2 must be converted into money balances before transactions can be affected.[10]

Thus, we see that if our petition rule is correct, then we can consider the question of how efficient combinations of the elements of S_1 are chosen in order to provide money services. That is, we can specify a production function for money services with the elements of S_1 as factor inputs and identify the degree of substitutability between these elements without regard to the levels of other assets held in the economy.

The Model

The model we propose differs from existing models in CS and near money (NM) literature in several important ways. Specifically, we consider the dual role that transactions play in the demand for the several forms of money held in the private sector. This dual role arises when one considers that changes in the level of transactions within an economy will affect both the efficiency in the use of any of these balances and the relative holding costs of these currencies. Second, we incorporate a different objective function for the optimization problem faced by the economy. We posit that residents--rather than seeking to maximize the level of monetary services or the utility of monetary services[11]--subject to a fixed level of holding costs--act to minimize the dual of the problem. That is, we assume that money holding costs are minimized subject to an existing technology (production function) for the production of the desired level of services. This emphasis on cost minimization allows us to explore in depth the nature of costs inherent in the maintenance of the various money balances. Again, our analysis of the holding costs of both domestic and foreign monies goes beyond previous work in either the NM or CS literature.

We begin by assuming a multi-country world where each country has a monetary authority that issues its own money supply. Residents of each country are assumed to hold non-negative amounts of the several currencies. Considering any country in isolation, we can describe the cost minimization problem described above as follows:

$$(3.5) \quad \min_{X_i} \sum_{i=1}^{Q} c_i e_{1i} X_i \quad \text{s.t.} \quad \overline{MS}\,(e_1 X_1, e_{1i} X_i) \quad i=1,\ldots,Q$$

where c_i = expected holding costs (in percent) for currency i $i = 1, \ldots, Q$

X_i = nominal holdings of currency i by residents of country 1, denominated in units of origin. $i = 1, \ldots, Q$

e_{1i} = exchange rate = country 1's currency price for 1 unit of country i's currency $i = 2, \ldots, Q$ $(e_{11} = 1)$

$\overline{MS}$ = desired level of money services[12]

M = production function for money services.

This minimization problem may be solved through the Lagrangian constrained minimization technique. Specifically, we form equation (3.6) and take partial derivatives of that equation with respect to the Q monetary assets (X_i) and the Lagrangian multiplier (λ). A necessary (or first order) condition for constrained minimization is that each of the partial derivatives equal zero.[13] We have:

(3.6) $$\mathcal{L} = \sum_{i=1}^{Q} c_i e_{1i} X_i - \lambda\{M(e_{11}X_1, e_{12}X_2, \ldots, e_{1Q}X_Q) - \overline{MS}\}$$

(3.6a) $$\frac{\partial \mathcal{L}}{\partial e_{11}X_1} = c_1 - \lambda M_1 = 0$$

(3.6b) $$\frac{\partial \mathcal{L}}{\partial e_{12}X_2} = c_2 - \lambda M_2 = 0$$

$$\vdots$$

(3.6q) $$\frac{\partial \mathcal{L}}{\partial e_{1Q}X_Q} = c_Q - \lambda M_Q = 0$$

(3.6r) $$\frac{\partial \mathcal{L}}{\partial \lambda} = M(e_{11}X_1, e_{12}X_2, \ldots, e_{1Q}X_Q) - \overline{MS} = 0$$

(NOTE: $M_i = \dfrac{\partial M}{\partial e_{1i}X_i}$ $i = 1, \ldots, Q$)

Division of (3.6a) by any of the other q-1 first order condition equations, (3.6b) - (3.6q), yields the familiar result that for cost minimization, balances should be held relative to levels of domestic money (X_1) such that the ratio of their marginal products (in the production of money services) equals the ratio of their expected costs.

The Cost Function

There are several costs in holding money balances of any sort. These can be divided into opportunity costs and actual (or perceived) costs of transactions and exposure to risk.

It is clear that the opportunity cost in holding money is the interest foregone on the next best alternative asset. There is no consensus on how this should be modeled. Two interest rates that have been used in the domestic near money literature are the rates on short term treasury bills and on long term (3-5 year) government bonds. The other costs mentioned above are less easy to model and hence have been largely ignored in both the domestic near money and the CS literature.

Actual costs in maintaining domestic balances include checking account charges, minimum balance fees, accounting costs, etc. Foreign balance accounts are likely to be even more costly. In particular, they may be subject to special taxes. Further, it is likely that these balances must be maintained in banks located in major financial centers or in overseas accounts. Hence, the cost of communicating long distances must be added to any standard service charges. Finally, there are likely to be substantial costs to obtaining the requisite information necessary for the efficient use of these balances.[14]

Added to this are the political risks which must be borne by anyone taking an open position in foreign exchange. These risks could be as extreme as government confiscation of foreign owned accounts, the freezing of foreign assets during times of political turmoil, or "currency reforms" which could leave foreign balances essentially worthless. More likely, political risk might entail the imposition of exchange controls after an open position in the "controlled" currency has been assumed. This could lead to the prevention of repatriation of funds or the payment of exorbitant black market exchange rates and thus, substantial capital losses. While these may seem extreme cases, <u>any</u> government imposed impediment to exchange convertibility can be construed as an element of political risk and the degree of likelihood that such actions would be taken <u>ought</u> to be factored into the costs of holding foreign balances.

The greatest cost faced by a money diversifier is the possibility that between the time foreign balances are obtained and the time they are utilized for transactions a change in the exchange rate will have occurred. This will alter the purchasing power of the desired level of money balances and must entail a component of the cost of holding foreign balances.[15] Specifically, if foreign balances were to depreciate vis à vis domestic balances while they are held then the bearer faces increased total costs (in terms of the number of domestic goods his balances command) because of the additional assets that must be liquidated in order to maintain a constant level of transactions. If, on the other hand, these foreign balances are expected to appreciate in the short run, the holder can reduce the size of his domestic money holdings given any desired level of transactions.

We introduce these ideas into our model by defining equations for the holding costs of each of the several monies:

$$(3.7a)\quad c_1 = r_1 + t_1$$

$$(3.7b)\quad c_i = r_i + t_i + \Omega_i - \dot{e}_{1i}^*$$

where r_i = monetary yield on the appropriate alternative asset $i = 1,\ldots,Q$.

t_i = transactions costs (as a percentage of total holdings) of maintaining balances in currency i $i = 1,\ldots,Q$.

Ω_j = expected losses relative to total holdings due to political risk of foreign balances $j = 2,\ldots,\Omega$.

$\dot{e}_{ij}^*$ = expected rate of change in the domestic currency price of country j's currency $j = 2,\ldots,Q$.

Therefore, the ratio of holding costs of the j^{th} currency relative to domestic balances is then defined as:

$$(3.8)\quad \frac{c_j}{c_1} = \frac{r_j + t_j + \Omega_j - \dot{e}_{1j}^*}{r_1 + t_1}$$

We assume that domestic balances are free from political risk and hence, omit that term in equation (3.7a). The inclusion of the expected rate of change in the exchange rate reflects the fact that the realization of capital gains on foreign balances clearly reduces the cost of maintaining these accounts.[16]

Technology for Money Services Production

We assume that the primary reason for maintaining balances in several currencies is to affect various domestic and foreign transactions. That is, from the point of view of certain actors within the economy it may be cheaper (in terms of both money and time) to maintain these several accounts than to convert domestic assets (money or other less liquid assets) into foreign money whenever foreign goods or services are acquired. Thus, just as producers find it expedient to maintain inventories of their output, agents in our economy are assumed to maintain foreign balances.

The decision as to the level of each currency type to be held depends upon relative holding costs of that currency vis à vis domestic (or third currency) balances and the degree to which these balances are viewed as substitutes. That is, our optimality condition implies tangency of the isocost line with an isoquant representing the level of desired money services. The shape of this isoquant reflects the nature of the technology available to the economy for conversion of domestic and foreign balances into the output of money services. The nature of this technology is largely an empirical question for each economy. It is possible to describe various scenarios, however, regarding possible technologies.

If we assume that domestic and foreign trade are conducted in domestic and foreign currencies, solely, and none of these monies is viewed as a substitute for another then the level of holding of each is a function solely of the level of transactions to be conducted with that currency. Relative holding costs would play no part in money holdings decisions. This is a world described by a fixed proportions production function for MS (e.g., $MS = \min\{\frac{X_1}{c_1}, e_{12}\frac{X_2}{c_2}, \ldots, e_{1Q}\frac{X_Q}{c_Q}\}$). In a world with only two currencies, the above function is represented by right angle isoquants in the input space. Efficient combinations of the two currencies would occur at the vertices of these isoquants and remain there for all finite, non-zero cost ratios.

It is also possible to imagine a technology whereby changes in relative cost ratios lead to equal percentage changes in money balance ratios in the opposite direction. This type of smooth substitution between inputs is defined by the Cobb-Douglas function

$$MS = X_1^{\alpha_1}(e_{12}X_2)^{\alpha_2}\ldots(e_{1Q}X_Q)^{\alpha_Q}$$

where $0 < \alpha_i < 1 \quad i = 1,\ldots,Q$ and $\sum_{i=1}^{Q} \alpha_i = 1$.[17] For the two input case, both the fixed proportions and the

Cobb-Douglas production functions can be identified by their elasticity of substitution[18] (σ) since they are special cases of the more general constant elasticity of substitution (CES) production function.

In order to consider the degree of substitutability between any two inputs of a multi-input production function, we introduce the concept of Hicks-Allen (HA) direct partial elasticity of substitution:[19]

$$\sigma_{ij} = \frac{\partial \ln (\frac{X_i}{X_j})}{\partial \ln (\frac{c_j}{c_i})} .$$

We can think of the meaning of σ_{ij} in the following manner. Hold the level of money services constant and fix all other inputs. This allows us to determine a curve in the X_i, X_j space. Along that curve, σ_{ij} measures the elasticity of X_i/X_j with respect to the cost ratio c_j/c_i.

Replacing the functional notation of M in the minimization problem described in (3.5) with a multi-input CES production function yields:

$$(3.9) \quad \min \sum_{i=1}^{Q} c_i e_{1i} X_i \text{ s.t. } \overline{MS} = \{\beta_1 X_1^{-\rho_1} + \beta_2 (e_{12} X_2)^{-\rho_2} + \ldots . \beta_Q (e_{1Q} X_Q)^{-\rho_Q}\}^{-1/\rho}$$

where β_i = efficiency (distribution) parameter associated with the i^{th} money balance i = 1,....,Q

ρ_i = substitution parameter associated with the j^{th} money balance i = 1,....,Q

ρ = overall substitution parameter

Following Dhrymes and Kurz,[20] we can calculate a value for σ_{ij}, viz.

$$(3.10)\quad \sigma_{ij} = \frac{1}{(1+\rho_i) + (\rho_j - \rho_i) \Big/ \left(\dfrac{1 + \beta_j \rho_j X_j^{-\rho_j}}{\beta_i \rho_i X_i^{-\rho_i}} \right)}$$

in the case where $\rho_i = \rho_j \quad i = 2,\ldots,Q$, then (3.10) reduces to the standard CES form described by:

$$(3.11)\quad \sigma_{ij} = \sigma = \frac{1}{1+\rho} > 0 \quad \text{if } \rho_i = \rho \ \forall_i \quad i = 2,\ldots,Q\ .$$

If, however, $\rho_i \neq \rho_j$ for some i then it is possible for certain (but not all) σ_{ij} to take on values anywhere on the real number line. The interpretation one should apply to these values is that inputs (X_i, X_j) are net substitutes (complements) in production as σ_{ij} is > 0 (<0). It is possible to show, but beyond the scope of this study, that not all inputs in a production function can be net complements (as defined above).[21]

Transactions Demand

The criterion we used in separating the set of assets X into S_1 and S_2 was based on the assumption that various forms of money are held in order to pay for the goods. While we do not rule out the speculative motive (desire for realization of capital gains) for holding foreign balances we emphasize in this section the role that transactions demand plays in determining the optimal levels of the various balances.

If we assume that the demand for the several forms of money depends upon the level of transactions faced by the holder, then it is likely the transactions elasticity of demand for each of these balances will differ. If this is so, then the money services function of equation (3.9) is incorrect. Specifically, the function, as written, presumes that the relative input shares (β_i) remain fixed for all levels of money services. But, if transactions elasticities differ, then as transactions levels change, these parameters should also change reflecting substitution among respective currency holdings irrespective of changes in relative holding costs. In other words, increased domestic transactions may lead asset holders to switch out of foreign balances even if relative holding costs have remained unchanged.

There are several reasons for thinking that this process occurs. First, as total transactions in an economy rise, the likelihood of external trade increases. With increased trade comes the possibility of economies in currency conversion costs through the maintenance of foreign balances. Also, it is likely that as trade expands, domestic banks will seek to recapture some of the business lost as domestic balances are converted to foreign by offering to service foreign denominated accounts. This process serves to lower transactions costs and may offer the holder marginally larger rates of return in the form of interest payments than those of foreign banks.[22] In addition, rising levels of transactions (and incomes) mean that more and more people are able to purchase the requisite information for the efficient use of their liquid balances. Because of all these factors, as well as the conclusion of various international agreements which increase currency convertibility and lower trade barriers the scope[23] for CS is broadened.[24]

In addition to the likelihood that relative balances will change (i.e., CS will occur) with varying levels of transactions, it is also possible that factor augmenting technological changes will occur in the banking and financial sectors of the economy over time. These improvements, which are largely associated with advances in computerization of accounting functions and communications facilities, need not have affected the efficiency of the several monies in the provision of money services to the same degree or at the same time. That is, for example, innovations in the banking industry of one country may increase the efficiency (therefore lower the requisite level) of balances in that currency. Since innovations may be country specific (responding to local legal restrictions or requirements) and are dependent upon the prevailing technology they need not be adopted across countries or currencies simultaneously or affect holdings in the same fashion.

To incorporate these ideas into our model we focus on the impact that the level of transactions and the degree of technical innovation have in determining the distribution of monies as inputs in the money service function.

Specifically, the parameters $\beta_i (i = 1,....,Q)$ should not be fixed [25] but should be functions of both the level of transactions and of time,[26] viz

$$(3.12) \quad \beta_i = \bar{\beta}_i T_t^{-\theta_i} \exp(-s_i t)$$

where $\bar{\beta}_i$ = constant term. $i = 1,....,Q$ $\bar{\beta}_i > 0$

T_t = level of transactions at time t

θ_i = transactions related efficiency parameter growth rates $i = 1,....,Q$ $\theta_i > 0$

s_i = rate of factor augmented technological change $i = 1,....,Q$ $s_i > 0$

The specification suggested by equation (3.12) is unique to our model and represents--as we will demonstrate--a significant refinement of the CS literature. Substitution of equation (3.12) into relation (3.9) allows us to write the final form of our theoretical model as:

$$(3.13) \quad \min \sum_{i=1}^{Q} c_i e_{1i} X_i \text{ s.t. } \overline{MS} = \{\bar{\beta}_1 T_t^{\theta_1} \exp(-s_1 t) X_1^{-\rho_1} + ... + \bar{\beta}_Q T_t^{\theta_Q} \exp(-s_Q t) [e_{1Q} X_Q]^{-\rho_Q}\}^{-1/\rho}$$

In order to solve this problem we form equation (3.14), viz:

$$(3.14) \quad L^1 = \sum_{i=1}^{Q} c_i e_{1i} X_i - \lambda\{(\bar{\beta}_1 T_t^{-\theta_1} \exp(-s_1 t) X_1^{-\rho_1} + ... + \bar{\beta}_Q T_t^{-\theta_Q} \exp(-s_Q t) [e_{1Q} X_Q]^{-\rho_Q})^{-1/\rho} - \overline{MS}\}$$

Following the Lagrangian constrained minimization technique, we partially differentiate 3.14 with respect to the local currency values of the Q currencies and the Lagrangian multiplier (λ). Setting these terms equal to zero yields the first order conditions for cost minimization. These conditions are written below,

$$(3.14a)\quad \frac{\partial \mathcal{L}'}{\partial X_1} = \lambda \overline{MS}^{-1}\, \bar{\beta}_1 T^{-\theta_1} \exp(-s_1 t) X_1^{-\rho_1-1} = c_1$$

$$(3.14i)\quad \frac{\partial \mathcal{L}'}{\partial e_{1i} X_i} = \lambda \overline{MS}^{-1}\, \bar{\beta}_i T^{-\theta_i} \exp(-s_i t) X_i^{-\rho_i-1} = c_i$$

$$(3.14r)\quad \frac{\partial \mathcal{L}'}{\partial \lambda} = 0 \qquad i = 2, \ldots, Q$$

Division of equations (3.14a) by (3.14i), determines the optimal relative balances of home and i^{th} currency held by residents in country 1. Several terms cancel when this operation is carried out.

Simplification of the resultant ratio and substitution of equation (3.8) for relative costs yields:

$$(3.15)\quad \frac{X_1}{e_{1i} X_i} = \left(\frac{\bar{\beta}_1}{\bar{\beta}_i}\right)^{\frac{1}{1+\rho_1}} (T_t)^{\frac{\theta_i-\theta_1}{1+\rho_1}} \exp\left\{\left(\frac{s_i - s_1}{1-\rho_1}\right) t\right\} \left(\frac{r_i + t_i + \Omega_i - \dot{e}_{1i}^*}{r_1 + t_1}\right)^{\frac{1}{1+\rho_1}} (e_{1i} X_i)^{\frac{\rho_i - \rho_1}{1+\rho_i}}$$

On the basis of this equation we assert the following testable hypotheses:

1. The optimal relative holdings of various currencies within the asset portfolio of an economy depend upon the relative costs of holding these currencies. An important component of these relative costs is the expected rate of change of the exchange rate, e_{1i}^*.

2. Even if relative costs remain constant, relative balances may change (hence CS occurs) due to changes in the level of transactions or to differences in the rate

of technological change in home or foreign financial industries.

Currency Substitution and Exchange Rate Regimes

The previous chapter pointed out the implications of the presence of CS for the efficacy of monetary policy under different exchange rate regimes. Briefly, under fixed rates, the presence of foreign currency denominated assets in the set of all assets means that perverse changes in domestic wealth holdings would occur with any change in the exchange rate. (Likewise, under a free float, CS implies that national monetary policies are not fully insulated from each other--as had been hypothesized.) While CS has different macroeconomic effects under different exchange rate regimes, these alternative systems have differing impacts on the relative holding cost ratios of the various monies.

Specifically, under a rigidly fixed exchange rate system $\dot{e}_{1i}^{*} \rightarrow 0$. Hence, this term vanishes from equation (3.8) (so long as Central banks are 100% expected to be able to maintain parities). The ratio of costs between the i^{th} currency balance and domestic balances becomes:

$$(3.16) \quad \frac{c_i}{c_1} = \frac{r_i + t_i + \Omega_i}{r_1 + t_1}$$

If, in addition, we assume that $\Omega_i = 0$ and that t_1 and t_i are constant, then changes in relative holding costs would depend totally on relative interest rates in the two countries. In other words, changes in the relative holdings of these two currencies due to changes in relative holding costs would, in this case, depend upon the ability and willingness of domestic and foreign monetary authorities to maintain interest rate differentials.

In the long run, under fixed exchange rates and perfect capital mobility, there is a tendency for interest rate convergence.[27] If this occurs, and there are no changes in transactions costs or political risk, then the ratio of holding costs for any ratio of currencies becomes invariant. Hence, from equation (3.15), if relative balances are to change (i.e., if CS is to occur) under fixed rates, in the long run, given these additional assumptions, it must be due to changes in the level of transactions (either within or between countries) or because of technological innovations in the use of one or both of the two monies.

Under floating exchange rates, the situation is different. The exchange rate varies and thus, $\dot{e}_{1i}^{*} \neq 0$. Further, if floating rates afford even some degree of insulation from the monetary policies of other countries, then divergent internal and external objectives may be pursued in each country--increasing the likelihood of long term deviations in interest rates. The result of all of this is that relative holding costs will vary as monetary policies are implemented in either country which alter rates of return or exchange rates. As costs vary, relative balances will change even if transactions levels or rates of technical change remain constant.

In order to pursue the question of CS under floating rates, we must define how expectation of exchange rate changes are formed. One model of expectations is derived from the "interest rate parity condition" defined below [28]

$$(3.17)\quad \frac{e^f_{1i} - e_{1i}}{e_{1i}} = \frac{r_1 - r_i}{1 + r_i}$$

where e^f_{1i} = forward exchange rate = domestic currency price of one unit of country i's currency to be delivered at a future date $i = 2, \ldots, Q$.

If domestic and foreign assets are perfect substitutes, supplies of arbitrage funds are infinitely elastic, and there are no transactions costs, then equation (3.17) would hold exactly. If it did not, then under these conditions, the existence of riskless profits would induce capital flows until (3.17) is obtained. In the real world, this relation never holds. It serves only to approximate conditions in spot and forward currency markets.

Given that (3.17) is a reasonable but imperfect reflection of foreign exchange market conditions, we make the simplest assumption possible about the formation of exchange rate expectations, viz

$$(3.18)\quad e_{1i}{}^* = e^f_{1i}$$ where $e_{1i}{}^*$ is the expected spot rate for country i's currency one period hence $i = 2 \ldots, Q$.

That is, equation (3.18) asserts that today's forward rate is expected to prevail as tomorrow's spot rate. Insertion of (3.18) into (3.17) yields a relation that defines the expected rate of change in the exchange rate. This relation can be written in several ways:

$$(3.19)\quad \dot{e}_{1i}^* = \frac{e_{1i}^* - e_{1i}}{e_{1i}} = \frac{e_{1i}^f - e_{1i}}{e_{1i}} = \frac{r_1 - r_i}{1 + r_i} \approx r_1 - r_i$$

Any of the last three terms on the right hand side may be used in our model, the question of which is most appropriate seems largely an empirical one.[29]

For the time being, we will focus on the latter two terms because we can analyze directly the CS implications of monetary policies which alter relative interest rates. Recall the cost ratio for the home country currency and the currency of country i defined by equation (3.8).

$$(3.8)\quad \frac{c_i}{c_1} = \frac{r_i + t_i + \Omega_i - \dot{e}_{1i}^*}{r_1 + t_1}$$

For simplicity, we assume that $\Omega_i = 0$ and that t_1 and t_i are constants that can be ignored. Replacing the expectations term with the definitions based on interest rate differentials in relation (3.19) allows us to define two alternative cost ratios which are solely functions of interest rates, viz

$$(3.20a)\quad \frac{c_i}{c_1} = \frac{r_i - \left(\frac{r_1 - r_i}{1 + r_i}\right)}{r_1} = \frac{r_i(1 + r_i) - (r_1 - r_i)}{r_1\ (1 + r_i)} = \frac{r_i^2 + 2r_i - r_1}{r_1\ (1 + r_i)}$$

$r_i^2 + 2r_i$

and

$$(3.20b)\quad \frac{c_i}{c_1} \approx \frac{r_i - (r_1 - r_i)}{r_1} = \frac{2r_i - r_1}{r_1}$$

Consider the impact of contractionary monetary policy on the part of the home country monetary authority through increases in domestic interest rates. Not only with relative opportunity costs of domestic and foreign balances be altered, but expectations about changes in future exchange rates will change, too. Consider the following example, based on differentiation of equation (3.20a). A ceteris paribus increase in the domestic interest rate will affect the relative cost ratio vis à vis country i's currency as follows:

$$(3.21)\quad \frac{\partial(\frac{c_i}{c_1})}{\partial r_1} = \frac{-r_i\ (r_i + 2)}{r_1^{\,2}\ (1 + r_i)} = \frac{-r_i}{r_1^{\,2}} - \frac{r_i}{r_1^{\,2}}\ (\frac{1}{1 + r_i}) < 0$$

Mathematically, the expression in equation (3.21) must be negative because the various interest rates (r_1, r_i) are always positive. Intuitively, this would make sense. Contractionary monetary policy raises domestic interest rates relative to foreign rates, thereby raising the relative opportunity cost of holding domestic money balances. At the same time, the differential between domestic and foreign interest rates increases, which leads to the expectation of a depreciation in the exchange rate.

It is also possible to rewrite equation (3.21) as:

$$(3.21a)\quad \frac{\partial(\frac{c_i}{c_1})}{\partial r_1} = \frac{\partial(\frac{r_i}{r_1})}{\partial r_1} + \frac{\partial(\frac{r_i}{r_1})}{\partial r_1} \cdot \frac{\partial(\dot{e}_{1i}{}^*)}{\partial r_1}$$

As (3.21a) shows, the change in relative holding costs due to an increase in the domestic interest rate can be factored into two terms: the change in relative opportunity costs due to the interest rate change plus the latter change times the expected change in the exchange rate due to the higher domestic interest rate.

A similar result obtains when any foreign rate rises relative to r_1:

$$(3.22)\quad \frac{\partial(\frac{c_i}{c_1})}{\partial r_i} = \frac{(1 + r_i)^2 + 2}{r_1(1 + r_i)^2} = \frac{1}{r_1} + \frac{1}{r_i}\left(\frac{2}{(1 + r_i)^2}\right) > 0$$

and

$$(3.22a)\quad \frac{\partial(\frac{c_i}{c_1})}{\partial r_i} = \frac{\partial(\frac{r_i}{r_1})}{\partial r_i} + \frac{\partial(\frac{r_i}{r_1})}{\partial r_i} \cdot \frac{\partial(\dot{e}_{1i}{}^*)}{\partial r_i}$$

Again this result is intuitive. An increase in r_i leads not only to an increase in the relative opportunity cost of holding currency i vis à vis domestic balances but to the expectation of an appreciation in the exchange rate. The conclusion, then, is straight-forward. Restrictive monetary policies relative to other economies in the home country will lower the relative costs of maintaining foreign balances and may induce, ceteris paribus, larger shares for foreign balances in the money component of wealth, and vice versa.

FOOTNOTES

CHAPTER THREE

[1]Substitutability between similar assets has been the subject of many papers. Feige and Pearce (1977) provide an extensive survey on various studies that attempt to measure the substitutability in demand for various domestic (U.S.) liquid (near money) assets. A subset of this literature was begun when Chetty (1969) proposed a CES type utility function for money services and attempted to measure the elasticity of substitution between money and various domestic near monies (e.g., time deposits, savings and loan shares, etc.). See Appendix 1 (end of Chapter 2).

None of the near money (NM) literature cited in the Feige-Pearce survey considers the role that foreign balances may play in determining optimal money balances. Miles (1978), employing the technique of Chetty, is the first published author to attempt to measure the elasticity of substitution between domestic and foreign balances. We will comment extensively on his methodology and his results in the empirical chapter of this thesis.

[2]Two goods are gross substitutes if an increase in the price of one good leads to increased consumption of the second. Both income and substitution effects are taken into account in the relation between the two goods. This implies

$$\frac{\partial X_i}{\partial r_j} < 0 \text{ for all } i \neq j \text{ and } \frac{\partial X_i}{\partial r_j} > 0 \text{ for } i = j.$$

[3]Note, absolute values are used here, since $\frac{\partial X_i}{\partial r_j} < 0$ for all $j \neq i$. We define the following rule: X_i and X_j are strong (weak) substitutes in demand as

$|\eta_{XiXj}| > (<1)$.

[4]It is also possible to consider the degree of substitutability between subsets of assets. Since currency substitution is limited to the analysis of substitutability between specific highly liquid assets we leave this question for later research.

[5]Moroney and Wilbratte (1976) suggest the issue of separability in the NM context. However, they do not provide a formal treatment of the concept.

[6]The following section borrows freely from a discussion of separability of production functions in E.R. Berndt and L.R. Christensen (1972). Their paper is an excellent summary of various issues critical to production theory, e.g., separability, multifactor elasticities of substitution, translog functions, etc.

[7]The proof of this statement is found in Leontief, W.W., in "A Note on the Interrelation of Subsets of Independent Variables of a Continuous Function with Continuous First Derivatives," Bulletin of the American Mathematical Society, 53, 1947.

[8]A condition of strong separability also can be defined. Berndt and Christensen, op. cit., write

> The production function F(X) is said to be strongly separable with respect to the partition $[N_1, N_2, \ldots, N_r]$ if the marginal rate of substitution $F_i(X)/F_j(X)$ between two inputs from different subsets N_s and N_t, respectively, does not depend upon the quantities of the inputs outside N_s and N_t. (pg. 11)
>
> Strong separability with respect to a partition $[N_1, \ldots, N_r]$ is necessary and sufficient for the production function F(X) to be of the form $F(X^1 + X^2 + \ldots + X^r)$ where X^r is a function of the elements of N_r only. (pg. 12)

Weak separability is both a necessary and sufficient assumption for our analysis to continue.

[9]Actually, we can also exclude the level of wealth from our analysis. That is, since we are interested in the degree of substitutability between any two monetary assets, we begin by taking logarithms of the asset demand equations and subtracting one from the other. In the process, the wealth term vanishes. From equation (3.1) we have:

$$X_i = W \cdot X_i(r_1, r_2, \ldots, r_n, T) \qquad i = 1, \ldots, n$$

Therefore

$$\ln X_i = \ln W + \ln \{X_i(r_i, r_2, \ldots, T)\}$$

and hence

$$\ln X_i - \ln X_j = \ln \{X_i(r_1, r_2, \ldots, T\} - \ln \{X_j$$

$$(r_1, r_2, \ldots, T)\} + \ln W - \ln W.$$

Obviously, the last two terms cancel.

[10]This is equivalent to assuming that the following partitioning rule is used:

$$X_i \text{ is } \varepsilon\ S_1 \text{ iff } \frac{\partial X_i}{\partial T} > 0 \text{ otherwise, } X_i\ \varepsilon\ S_2\ \forall_i\ .$$

and specifying that all transactions must be conducted with domestic or foreign money balances.

[11]Note, because monetary services may include nonpecuniary attributes such as convenience neither the utility nor the level of monetary services is an observable variable. Hence, because we cannot label the particular isoquant that represents the desired level of money services we impose the condition that with respect to the output of money services all isoquants are identical. This condition occurs if and only if the production function is homothetic with respect to the level of output.

[12]We make no assumptions about how the desired levels of money services are determined. One reasonable hypothesis is that a portfolio balancing model (à la Tobin Markowitz) which incorporates risk and return, could be used to explain the relative levels of all assets in the wealth holdings of the economy. So long as the function M is homothetic with respect to the level of MS and the elements of S_1 are functionally separable from the remaining elements in the asset set X, then the two step optimization procedure is valid.

[13]The second order condition for cost minimization is that the Bordered Hessian matrix be positive definite.

[14]There is a reason to believe that for some foreign currencies, the level of transactions costs decreases as transactions increase. This could occur as domestic banks became more familiar with this currency and began to offer banking services for these balances.

[15]We assume that wealth holders in our economy are risk neutral.

[16]The term e_{1i}* is subtracted in the i^{th} cost equation because $e_{1i}* > (<)\ 0$ as the home currency is expected to depreciate (appreciate) against currency i.

[17]This implies constant returns to scale.

[18]

$$\sigma = \frac{\ln(\frac{X_1}{X_2})}{\ln(\frac{c_2}{c_1})} \geq 0$$

Specifically, the production function is fixed proportions if and only if $\sigma = 0$. It is Cobb-Douglas if and only if $\sigma = 1$. Finally, X_1 and X_2 are said to be perfect substitutes as $\sigma \to \infty$.

[19]There are, in fact, several alternative measures of the partial elasticity of substitution. For several descriptions of these see E.R. Berndt and L. Christensen, op. cit., pp. 9-10, C.E. Ferguson, The Neoclassical Theory of Production and Distribution, Cambridge Press, 1969, pp. 107-111 and R.M. Solow, (1967) pp. 41-46. Both V.K. Chetty, op. cit. and Dhrymes and Kurz (1964), pp. 287-315, employ the Hicks-Allen direct partial elasticity measure.

[20]Dhrymes and Kurz, p. 290.

[21]See R.G.D. Allen, Mathematical Analysis for Economists, London: MacMillan & Co., 1953, pp. 503-509.

[22]This would certainly be the case for foreign owned U.S. dollar demand deposits in the United States prior to the invention of interest bearing draft accounts.

[23]We wish to differentiate the notion of the scope of CS from the size of the HA direct partial elasticity. The scope for CS refers to the number of times or ways in a particular period that CS can be efficiently utilized. The size of the elasticity of substitution is determined by the nature of the money services technology faced by residents of the economy.

[24]King, Putnam, and Wilford (1979) present a theoretical model of currency substitution where the proportion of monetary services provided by foreign money is an increasing function of the integration of world goods and capital markets.

[25]Moroney and Wilbratte, op. cit., introduce income terms as proxy for transactions in the efficiency parameters. Again, their analysis is part of the domestic near money literature.

[26]Lieberman (1977) argues forcefully that both a transactions term and a time trend must be included in any theoretical or empirical specification of the demand for money. His analysis, however, does not employ a CES specification as we suggest and he does not consider foreign

balances. Brillembourg and Schadler (1979, pp. 534-535) suggest that the nonpecuniary return for any currency can be described by a linear function of real income and a time trend.

[27]For a discussion of this point as a part of the "global monetarist" approach to the analysis of the balance of payments, see Kreinin and Officer (1978, p. 13).

[28]The term $\frac{r_1 - r_i}{1 + r_i}$ is known as the interest agio and since $1 + r_i$ is usually very close to unity it is often approximated as $r_1 - r_i$.

[29]Note, however, that each term to the right of the second in equation (3.19) represents a slightly more restrictive assumption.

CHAPTER FOUR

Estimating Equations and Empirical Methodology

Introduction

In the last chapter, we developed a theoretical model of currency substitution (CS). We posited that agents in any economy determine their desired holdings of various assets according to a two step process. First, they optimize their holdings according to some objective function within the various subsets of assets and then they optimize between these asset subsets.

Our analysis focuses on one particular asset subset, the set of domestic and foreign monies. We suggest that agents hold foreign and domestic balances because they may directly facilitate transactions. Nonetheless, these balances are costly to hold. Costs would include returns available on alternative assets (opportunity costs), transactions costs (both in maintaining balances and switching between them), and the various risks (political and exchange rate) faced by the holders. Therefore, it is our hypothesis that agents act to minimize the holding costs of these balances subject to obtaining a desired level of "money services" from them.

In the chapter below, we develop a set of asset demand equations for the Q-1 foreign balances from our theoretical model. We discuss how these equations can be estimated based on several alternative hypotheses about the speed of adjustment and the nature of substitutability between the various monies. We also demonstrate how estimates of the several parameters of the model can be obtained from our empirical work.

Production Function for Money Services

Recall from the last chapter that we chose to describe the technology for the production of aggregate money services from the holdings of Q currencies as a generalized CES production function where the various monies serve as inputs in the production function. This is written below as,

$$(4.1) \quad MS = \left\{\beta_1 X_1^{-\rho_1} + \beta_2 (e_{12} X_2)^{-\rho_2} + \ldots + \beta_Q (e_{1Q} X_Q)^{-\rho_Q}\right\}^{-1/\rho}$$

where e_{1j} = domestic currency (1) price of one unit of country j's currency $j = 2, \ldots, Q$

X_i = nominal holdings of currency i by residents of Country 1 denominated in units of origin $i = 1, \ldots, Q$

β_i = efficiency (or share) parameter associated with the ith money balance $i = 1, \ldots, Q$

ρ_i = substitution parameter associated with the ith money balance $i = 1, \ldots, Q$

ρ = overall substitution parameter

We rule out non positive holdings of any money balances by assuming the following inequality constraints. $\beta_i, X_i \geq 0$

($i = 1,\ldots,Q$). Further, we assume that markets for foreign balances are nicely behaved so that exchange rates are always positive ($e_{1j} > 0 \quad j = 2,\ldots,Q$).

In addition to these assumptions we place certain restrictions on the substitution coefficients of the model. Specifically, in order to guarantee that money services are produced within the economic region (i.e. to insure cost minimization) and to rule out inadmissible behavior (i.e. "negative returns to scale") we impose the following constraints: $\rho_i > -1$ and ρ_i (and ρ) must be of the same sign.[1]

The advantage of using the generalized CES functional form is that it places very few constraints on the underlying technology. Rather, the degree of substitutability between any two assets (X_i, X_j) is a byproduct of the estimation process. In particular a measure of the degree of substitutability between X_i and X_j, the Hicks-Allen direct partial elasticity of substitution (σ_{ij}) can be calculated from the estimated parameters of the model.

Specifically, we define σ_{ij} as follows:

$$(4.2) \quad \sigma_{ij} = \frac{d \ln \left(X_i / X_j\right)}{d \ln \left(\dfrac{\partial MS/\partial X_j}{\partial MS/\partial X_i}\right)}$$

This term can be calculated using the following formula,[2]

$$(4.2a) \quad \sigma_{ij} = \frac{1}{(1+\rho_i) + (\rho_j - \rho_i) \Big/ \left(1 + \dfrac{\beta_j \rho_j}{\beta_i \rho_i} \cdot \dfrac{(e_{1j} X_j)^{-\rho_j}}{(e_{1i} X_i)^{-\rho_i}}\right)}$$

If we assume that there is a constant and equal partial elasticity between all pairs of assets, and constant returns to scale then equation (4.2) can be simplified greatly. In this case, $\rho_i = \rho_j = \rho$ for all $i,j=1,\ldots,Q$ and (4.2a) reduces to (4.2b) for all pairs of assets,

$$(4.2b) \quad \sigma_{ij} = \frac{1}{1+\rho} = \sigma$$

Another feature of the generalized CES function is that is allows us to incorporate into our model various assumptions about the role that the volume of transactions plays in the CS process. As we have indicated previously, there are a priori reasons for believing that changing volumes of transactions affect optimum holdings of the various balances differently. That is, it is an empirical question whether the transactions elasticities of demand for the different monies are identical. Likewise, it remains an empirical question whether there have been differential rates of technological change in the utilization of the various monies to produce money services. Our model addresses this last question as well.

In addition to testing these hypotheses, we want a model which examines the following issues:

1. Are all monies equally substitutable between each other and especially with respect to holdings of domestic balances? This would imply that $\rho_i = \rho_j$ for all $i \neq j$. Or, are some foreign balances weaker substitutes for domestic balances (i.e. $\rho_j \neq \rho_1$ for some j)

2. What impact do expectations about exchange rate changes play on the degree of

substitutability between assets? In addition, do we witness different behavior regarding the CS process under different exchange rate regimes?

3. Are domestic and foreign balances separable in demand from other near money assets?

4. How quickly do economic agents react to changes in exogenous variables?

In answering these and other related questions, we seek to develop a complete and consistent model of CS.

The Empirical Model

The estimating questions of our empirical model are derived from the first order conditions (FOC) for cost minimization subject to achieving a desired level of output of money services. This problem is described below:

$$(4.3)\quad \min\ C_1X_1 + C_2e_{12}X_2 + \ldots + C_Qe_{1Q}X_Q \ \text{ s.t. } MS = \left\{\beta_1X_1^{-\rho_1} + \beta_2(e_{12}X_2)^{-\rho_2} + \ldots + \beta_Q(e_{1Q}X_Q)^{-\rho_Q}\right\}^{-1/\rho}$$

where: C_i = per unit holding cost of currency i,i=1,....,Q

In addition, we wish to incorporate the roles played by the level of economic activity (transactions) and technological change in achieving the optimal mix of currency holdings. This we do by allowing the efficiency parameters in equation (4.1) to vary according to the following relation:[3]

$$(4.4)\quad \beta_i = \bar{\beta}_i T_t^{-\theta_i} \exp\{-s_it\} \qquad i=1,\ldots,Q$$

where $\bar{\beta}_i$ = fixed efficiency parameter for the i<u>th</u> money balance i = 1,....,Q

T_t = level of transactions at time t.

θ_i = transactions efficiency parameter growth rate for the ith currency balance, i=1,....,Q. $\theta_i \geq 0$.

s_i = rate of growth of factor argument technological change for input i (i=1,....,Q). $s_i \geq 0$.

exp = exponential operator.

Upon substitution of the relations defined in 4.4 into equation 4.1 the minimization problem described by 4.3 is solved algebraically using the Lagrangian constrained optimization technique. That is, we form equation 4.5 (below) and then take partial derivatives of that equation with respect to the desired domestic value of the Q monetary assets ($e_{1i}X_i$) and the Lagrangian multiplier (λ).[4] A necessary (or first order) condition for constrained optimization is that each of these partial derivatives equal zero.

Specifically, we have

$$(4.5)\quad L = C_1X_1 + C_2e_{12}X_2 +\ldots+ C_Qe_{1Q}X_Q - \lambda\Big\{[\overline{\beta}_1T_t^{-\theta_1} \exp(-s_1t)X_1^{-\rho_1}+\ldots+\overline{\beta}_QT_t^{-\theta_Q} \exp(-s_Qt)(e_{1Q}X_Q)^{-\rho_Q}\Big]^{-1/\rho} - \overline{MS}\Big\}$$

where the first order conditions for constrained minimization are:

$$(4.5a)\quad \frac{\partial L}{\partial X_1} = C_1 - \frac{\lambda}{\rho}\,\overline{MS}^{-1}\,\overline{\beta}_1T_t^{-\theta_1} \exp(-s_1t)\rho_1X_1^{-\rho_1-1} = 0$$

$$(4.5b)\quad \frac{\partial L}{\partial e_{12}x_2} = C_2 - \frac{\lambda}{\rho}\,\overline{MS}^{-1}\,\overline{\beta}_2 T_t^{-\theta_2}\exp(-s_2 t)\,\rho_2 (e_{12}x_2)^{-\rho_2-1}$$

$$= 0$$

.

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.

.

$$(4.5q)\quad \frac{\partial L}{\partial e_{1Q}x_Q} = C_Q - \frac{\lambda}{\rho}\,\overline{MS}^{-1}\,\overline{\beta}_Q T_t^{-\theta_Q}\exp(-s_Q t)\,\rho_Q (e_{1Q}x_Q)^{-\rho_Q-1} = 0$$

$$(4.5r)\quad \frac{\partial L}{\partial \lambda} = 0$$

Dividing the first equation 4.5a above by any of the other first q FOC, then taking logrithms and solving for $\ln(e_{1j}x_j)$ $(j=2,\ldots.,Q)$ yields a demand equation for the jth foreign money balance, viz.

$$(4.6)\quad \ln(e_{ij}x_j) = \frac{1}{1+\rho_j}\ln\left[\frac{\rho_j\overline{\beta}_j}{\rho_1\overline{\beta}_1}\right] + \frac{(\theta_1-\theta_j)}{1+\rho_j}\ln T_t + \left[\frac{s_1-s_j}{1+\rho_j}\right]t + \left[\frac{1+\rho_1}{1+\rho_j}\right]\ln x_1 - \frac{1}{1+\rho_j}\ln\left[\frac{c_j}{c_1}\right]$$

Our estimating equations are derived from the set of Q-1 foreign money demand equations defined by 4.5. The jth equation of this system is presented below:[5]

$$(4.7)\quad \ln(e_{ij}x_j) = \gamma_{0j} + \gamma_{ij}\ln T_t + \gamma_{2j}t + \gamma_{3j}\ln x_1 + \gamma_{4j}\ln\left[\frac{c_j}{c_1}\right] + u_t \qquad j=2,\ldots.,Q$$

where: u_t is a stochastic error term (assumed $\sim N(0,\sigma_u^2)$)

$$\gamma_{0j} = \frac{1}{1+\rho_j} \ln\left(\frac{\rho_j\bar{\beta}_j}{\rho_1\bar{\beta}_1}\right) \qquad \gamma_{3j} = \left(\frac{1+\rho_1}{1+\rho_j}\right)$$

$$\gamma_{1j} = \frac{\theta_1 - \theta_j}{1+\rho_j} \qquad \gamma_{4j} = -\left(\frac{1}{1+\rho_j}\right).$$

$$\gamma_{2j} = \frac{s_1 - s_j}{1+\rho_j}$$

Standard relations from economic theory and the restrictions which we imposed provide us with the signs of several of the terms defined above. One expects, for instance, that the utilization of any input would vary inversely with the price of that input. Here, we expect that holdings of any monetary asset will be inversely related, ceteris paribus, to the holding costs of that asset. Therefore, γ_{4j} is negative. This, of course, establishes a theoretical reason for our initial restriction that ρ_j must be greater than minus unity. Specifically, we see that if:

$$\gamma_{4j} = -\frac{1}{1+\rho_j} < 0 \text{ implies}$$

$$\frac{1}{1+\rho_j} > 0 \quad \text{or}$$

$$\rho_j > -1$$

$$j=2,\ldots,Q$$

From the restriction that ρ and the ρ_i $(i=1,\ldots,Q)$ have the same sign, we assert that γ_{0j} will be positive and so will γ_{3j} (if $\rho > -1$). However, the signs of γ_{1j} and γ_{2j} are indeterminate and depend upon the differentials in the transactions elasticities of demand for and the rates of technological progress in the use of the various balances.

Estimation Techniques

So far we have not discussed methods for estimating our model. Clearly if all of the right side (r.h.s.) variables are exogenous (or predetermined) and the stochastic error terms are contemporaneously uncorrelated, then each equation of the system defined by equation 4.7 can be estimated using ordinary least squares (OLS). However, from the derivation of the estimation equation, it is clear that the holdings of domestic balances (X_1) should not be treated as exogenous to the system. Since these balances are not exogenous, we introduce the possibility of simultaneous equations bias. That is, an element of the r.h.s. variables will be correlated with the stochastic error term. Using OLS to estimate the equations of such a system will yield biased and inconsistent estimates of the regression coefficients. It is necessary therefore to use an estimation technique which purges the r.h.s. of its endogenous elements.

Several estimation methods have been suggested in the literature. Chetty solves his system of FOC for the demand for domestic balances function (again, this is in the context of the domestic near money literature). Included as arguments of this function are all of the interest rates in the model as well as the level of national income. Expansion of this function through its Taylor series allows one to estimate each near money balance demand equation using two stage least squares.[6]

Bisignano uses a slightly different technique to obtain nonrandom estimates of domestic balances. That is, he calculates an independent measure of money demand as a function of real permanent and transitory income, real interest rates, and a measure of the implicit yield on money balances. These independent (of the model) estimates are then substituted for domestic balances as a r.h.s. variable in the other asset demand equations.[7]

The method we would choose is to estimate each foreign money demand equation using two stage least squares. This means that in the first stage domestic money balances will be regressed on all the exogenous variables in the system of demand equations. Included in the set of exogenous variables are all of the Q holding costs (c_i, $i=1,\ldots,Q$), the logarithm of the level of transactions, and a time trend. In the second stage of the estimation process the predicted holdings of domestic balances are inserted into the foreign money demand equations replacing their actual values and OLS is applied to the resultant equations.

Two stage least squares procedure yields estimates of the coefficients of equation 4.7. These can be used to obtain estimates of some of the underlying parameters of the model. That is, from the following relation, we obtain estimates of the j (j=2,....,Q) substitution parameters:

$$\gamma_{4j} \equiv - \frac{1}{1 + \rho_j}$$

implies $\hat{\rho}_j = - (1 + \hat{\gamma}_{4j})/\hat{\gamma}_{4j}$

Since the estimated values $(\hat{\rho}_j)$ are non-linear functions of the $\hat{\gamma}_{4j}$, the variances of the $\hat{\rho}_j$ must be calculated according to some approximation technique. The method we have chosen was first suggested by Klein. (See appendix 2). Here, for instance, we approximate the variance of $\hat{\rho}_j$ as follows:

$$\text{var } \hat{\rho}_j \approx \left(\frac{1}{\hat{\gamma}_{4j}{}^4}\right) \text{ var } (\hat{\gamma}_{4j})$$

Each estimation of the Q-1 foreign money demand equations yields an independent estimate for ρ_1, the substitution parameter associated with domestic money balances. This is, in each equation estimated,

$$\hat{\rho}_1 = - \hat{\gamma}_{3j}/\hat{\gamma}_{4j} - 1$$

Again, because the $\hat{\rho}_1$ are non-linear functions of the estimated regression coefficients, we must approximate the variances of these estimators. The formula is derived, as above, from Klein's technique and is presented below:

$$\text{var}(\hat{\rho}_1) \approx \left(\frac{1}{\gamma_4^2}\right)\text{var}(\hat{\gamma}_3) + [\hat{\gamma}_3^2/\hat{\gamma}_4^4]\ \text{var}(\hat{\gamma}_4)$$

$$-2\ [\hat{\gamma}_3/\hat{\gamma}_4^3]\ \text{cov}(\hat{\gamma}_3,\hat{\gamma}_4)$$

Because we have Q-1 estimates of ρ_1, it is too much to expect (given the stochastic nature of the process) that all of the estimates will be identical. This leaves the problem of which estimate to report. One solution to this dilemma is to incorporate the information from all of the model equations by constructing an estimate of $\hat{\rho}_1$ from a weighted average of the Q-1 point estimates of $\hat{\rho}_1$. The weights would be proportional to the inverse of the estimated variances for $\hat{\rho}_1$ from each equation; would sum to unity; and would thereby give extra weight to the most precise estimates.[8]

It is impossible to identify estimators for the θ_i and the s_i (i=1,....,Q). Nonetheless, we can solve for estimates of the differences between θ_1 (the transactions related growth rate in the efficiency parameter β_1) and its counterpart θ_j for any of the j foreign balances (j=2,...,Q). This estimate is derived below:

$$\theta_1 \hat{-} \theta_j = -\hat{\gamma}_{1j}/\hat{\gamma}_{4j} \qquad j=2,....,Q$$

Following the method described above, the estimated variance for this estimator is approximated by the following term:

$$\text{var } \theta_1 \hat{-} \theta_j \approx \frac{1}{\hat{\gamma}_{4j}^2} \text{var } (\hat{\gamma}_{1j}) + [\hat{\gamma}_{1j}{}^2/\hat{\gamma}_{4j}{}^4] \text{ var } (\hat{\gamma}_{4j})$$

$$-2\ [\hat{\gamma}_{1j}/\hat{\gamma}_{4j}{}^3] \text{ COV } (\hat{\gamma}_{1j}, \hat{\gamma}_{4j})$$

We are interested in measuring this term because it allows us to test the hypothesis that changes in transactions levels affect the demand for the various money balances differently. A rejection of the null hypothesis that the term $\theta_1 \hat{-} \theta_j$ = zero would establish some validity to our argument that this is the way the CS process works.

Similarly, we can examine the question of differential rates of technological change in the use of these balances through testing the value of the estimator derived below

$$s_1 \hat{-} s_j = \hat{\gamma}_{2j}/\hat{\gamma}_{4j}$$

The variance of this estimate is approximated by:

$$\text{var } (s_1 \hat{-} s_j) \approx (1/\hat{\gamma}_{4j}{}^2) \text{ var } (\hat{\gamma}_{2j}) + [\hat{\gamma}_{2j}{}^2/\hat{\gamma}_{4j}{}^4]$$

$$\text{var } (\hat{\gamma}_{4j}) -2\ [\hat{\gamma}_{2j}/\hat{\gamma}_{4j}{}^3] \text{ COV } (\hat{\gamma}_{2j}, \hat{\gamma}_{4j})$$

Finally, the use of two stage least squares guarantees that the estimates of the equation coefficients are consistent. Further, they are also efficient relative to all other single equation estimation techniques.[9]

If all monies are viewed as identical substitutes for domestic balances and if we assume constant returns to scale (ie. $\rho_i = \rho_j = \rho$ for all $i \neq j$, then the problem described above can be handled with a simple transformation of the estimating equation. Specifically, we impose the constraint that the substitution parameters are equal. This implies a theoretical value of unity for the γ_{3j} of each of the j equations (j = 2,....,Q). Empirically this constraint is imposed by setting the coefficient (γ_{3j}) to unity in each equation and rearranging terms so that the logarithm of domestic balances is moved to the left hand side of the equation. This yields a constrained estimating equation for a homothetic CES technology of:

$$(4.8) \quad \ln e_{ij}X_j - \ln X_1 = \Lambda_{0j} + \Lambda_{1j} \ln T_t + \Lambda_{2j} t + \Lambda_{3j} \ln \left(\frac{C_j}{C_1}\right) + \varepsilon_t$$

where ε_t is a random error term

$$\Lambda_{0j} = \sigma \ln \left(\frac{\overline{\beta}_1}{\beta_j}\right) \qquad \Lambda_{2j} = \sigma (s_1 - s_j)$$

$$\Lambda_{1j} = \sigma (\theta_1 - \theta_j) \qquad \Lambda_{3j} = -\frac{1}{1+\rho} = -\sigma$$

and where each pair of assets has a Hicks-Allen partial elasticity of substitution defined below as: $\sigma = \frac{1}{1+\rho}$ Provided the constraint is valid,[10] OLS on equation 4.8 is the appropriate single equation estimation technique since the remaining r.h.s. variables are all assumed exogenous to the system.[11]

Additional parameters of the underlying production function for money services can be estimated from the regression coefficients:

$$\hat{\rho} = \frac{1}{\hat{\Lambda}_{3j}} - 1$$

$$\theta_1 \hat{-} \theta_j = \frac{-\hat{\Lambda}_{1j}}{\hat{\Lambda}_{3j}}$$

$$s_1 \hat{-} s_j = \frac{-\hat{\Lambda}_{2j}}{\hat{\Lambda}_{3j}}$$

Again, the variances of these constructed estimates may be derived using the Klein approximation technique, described above and in the appendix to this chapter.[12]

Lagged Adjustment Models

So far, we have assumed that desired balances are determined and achieved within one period. That is economic agents fully adjust their various money assets to changes in exogenous variables within each period. This assumption may be too restrictive. Rather, it may take several periods for full adjustment to new equilibrium values to occur. If so, our model (implied by either equations 4.7 or 4.8) is mis-specified, and estimates derived from it would be biased due to omitted r.h.s. variables.

One way to model lags in the CS process is to specify a standard partial adjustment model. We can describe this partial adjustment process for the jth foreign balance as follows:

$$(4.9) \quad \left(\frac{e_{1jt}X_{jt}}{e_{1jt-1}\ X_{jt-1}}\right) = \left(\frac{\widetilde{e_{1jt}X_{jt}}}{e_{1jt-1}\ X_{jt-1}}\right)^{\delta_j}$$

where $e_{1jt}X_{jt}$ = the domestic currency value of currency j in time period t j = 2,....,Q

$\widetilde{e_{1jt}X_{jt}}$ = the desired domestic currency value of holdings of currency j in time period t. j = 2,....,Q

δ_j = rate of adjustment of holdings of currency j

$0 \leq \delta_j \leq 1$

If we take natural logrithms of both sides of 4.9, then collect terms involving previous period holdings of the jth foreign balance, we obtain

$$(4.10) \quad \ln\ (e_{1jt}X_{jt}) = \delta_j \ln \widetilde{e_{1jt}X_{jt}} + (1 - \delta_j) \ln e_{1jt-1}\ X_{jt-1}$$

Replacing the term $\ln \widetilde{e_{ijt}X_{jt}}$ with the estimation equation developed previously (equation 4.7) yields an alternative estimating equation which allows us to measure the speed of adjustment, viz.

$$(4.11) \quad \ln e_{1jt}X_{jt} = \pi_{0j} + \pi_{1j} \ln T_t + \pi_{2j}t + \pi_{3j} \ln X_1 + \pi_{4j} \ln \frac{C_j}{C_1} + \pi_{5j} \ln e_{1jt-1}\ X_{jt-1} + v_t$$

where $\pi_{0j} = \delta_j\{\gamma_{0j}\}$

$\pi_{1j} = \delta_j\{\gamma_{1j}\}$

$\pi_{2j} = \delta_j\{\gamma_{2j}\}$

$\pi_{3j} = \delta_j\{\gamma_{3j}\}$

$\pi_{4j} = \delta_j\{\gamma_{4j}\}$

$\pi_{5j} = 1 - \delta_j$

v_t = stochastic error term

Because the simultaneous equations bias described for equation 4.7 still remains in the model defined by 4.11, this partial adjustment model must be estimated using a systems estimator. Again, in this case, we choose to employ the most efficient single equation systems estimator - two stage least squares. Note, the addition of a lagged value of the endogenous variable as a r.h.s. variable adds no new econometric problems so long as the error terms of each demand equation are assumed to have the usual nice properties.[13]

A similar partial adjustment scheme can be thought of in terms of our restricted-homothetic CES model. Again, we begin with the following relation:

$$(4.12) \quad \frac{R_{jt}}{R_{jt-1}} = \left(\frac{\tilde{R}_{jt}}{R_{jt-1}}\right)^{K_j}$$

where $R_{jt} = \dfrac{e_{1jt}X_{jt}}{X_{1t}}$ = the ratio of holdings of foreign balance j to domestic balances at time t.

$\tilde{R}_{jt}$ = the desired ratio of foreign to domestic balances at time t.

K_j = the adjustment coefficient for ratio j ($j = 2,\ldots.,Q$)

After taking logrithms, rearranging terms, and substitution of equation 4.8 for $\tilde{R}_{jt}$, we have

$$(4.13) \quad \ln \left[\frac{e_{1jt} X_{jt}}{X_{1t}}\right] = \psi_{0j} + \psi_{1j} \ln T_t + \psi_{2j} + \psi_{3j} \ln \left(\frac{C_j}{C_1}\right) + \psi_{4j} \ln \frac{e_{1jt-1} X_{jt-1}}{X_{1t-1}} + \omega_t$$

where

$$\psi_{0j} = K_j \Lambda_{0j} \qquad \omega_t = \text{stochastic error}$$

$$\psi_{1j} = K_j \Lambda_{1j}$$

$$\Psi_{2j} = K_j \Lambda_{2j}$$

$$\Psi_{3j} = K_j \Lambda_{3j}$$

$$\psi_{4j} = 1 - K_j$$

Since we assume that the stochastic errors are nicely behaved, 4.13 can be estimated using OLS.[14]

Equations 4.7, 4.8, 4.11 and 4.13 provide us with the basic estimation equations for our model. Each imposes different constraints on the underlying process. Each can be used to test different hypotheses about currency substitution. The next chapter details the results of this process.

FOOTNOTES

[1]For a more complete discussion of these restrictions see Dhrymes and Kurz (1964, pp. 292-293).

[2]Dhrymes and Kurz, pg. 290.

[3]This formulation is similar to the varying efficiency parameter model developed by Moroney and Wilbratte (1976). See pp. 186-187.

[4]The Lagrangian multiplier (λ) can be interpreted as the marginal cost at the equilibrium point. See Dhrymes and Kurz, pg. 293.

[5]It is possible to write the jth first order condition for cost minimization (i.e. equation 4.5) in the form $C_j = \frac{\partial MS}{\partial X_j} \exp(\mu_j)$. Taking logarithms of both sides yields an additive error term. Equation 4.7 is formed in a similar fashion. We assume that the source of error originates from imperfect knowledge about factor prices or imperfections in the cost minimization process.

[6]Chetty (1969) pp. 276-277. Actually, Chetty follows the approach first suggested of Dhrymes and Kurz. See Dhrymes and Kurz, pp. 293-295.

[7]Bisignano (1974) pg. 12 and pp. 36-39.

[8]This is the procedure followed by Dhrymes and Kurz, Chetty and Moroney and Wilbratte.

[9]See Schmidt (1976), pp.151-153 (for consistency) and pp. 164-165 (for efficiency). Note, each equation of 4.7 is overidentified since Q-2 cost terms are excluded and an equality constraint on the coefficients of the included cost terms is also imposed. Obviously, we must also assume that the errors are not comtenporaneously correlated across equations. If this assumption failed, thea a technique that employed this information such as three stage least squares or full information maximum likelihood would be more efficient.

[10]In the case where there are only two minies in the production function for money services, the concept of a CES technology requires that $\rho_1 = \rho_2$. This point is proven in the appendix to the next chapter.

[11]We assume, again, that there is no contemporaneous correlation across equations.

[12]Again, we have Q-1 estimates of $\hat{\rho}$. This necessitates a procedure such as that described above for constructing a weighted estimate of $\hat{\rho}$.

[13]Violations of this assumption would include autocorrelated errors. If this occurs, maximum likelihood (or some other efficient estimation technique) estimation is called for.

[14]We assume that the errors are neither autocorrelated nor contemporaneously correlated across equations.

APPENDIX 2

On Approximation of the Distribution of Derived Parameters

APPENDIX 2

On Approximation of the Distribution of Derived Parameters

Let $\underset{m \times 1}{\xi} = \underset{n \times 1}{f(\theta)}$

Suppose $\hat{\theta}$ is consistent and $\sqrt{T}(\hat{\theta} - \theta) \rightarrow N(0, \psi)$

Then $\hat{\xi} = f(\hat{\theta})$ is consistent and

$$\sqrt{T}(\hat{\xi} - \xi) \rightarrow N(0, D\psi D')$$

$$\text{where} \quad D = \begin{bmatrix} \frac{\partial \xi_1}{\partial \theta_1} & \cdots & \frac{\partial \xi_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial \xi_m}{\partial \theta_1} & \cdots & \frac{\partial \xi_m}{\partial \theta_n} \end{bmatrix} = \frac{d\xi}{d\theta}$$

Example $\quad \hat{\rho}_j = -(1 + \hat{\gamma}_{4j})/\hat{\gamma}_{4j}$

$$\therefore \sqrt{T}(\hat{\rho}_j - \rho_j) \rightarrow N\{0, D^2 \text{ var } (\hat{\gamma}_4)\}$$

$$\text{where} \quad D^2 = \left(\frac{\partial \hat{\rho}_j}{\partial \hat{\gamma}_{4j}}\right)^2 = \left(\frac{1}{\hat{\gamma}_{4j}^2}\right)^2 = \frac{1}{\hat{\gamma}_{4j}^4}$$

CHAPTER FIVE

Estimation Results

Introduction

Using quarterly data on private, nonbank Canadian holdings of their own and United States dollars between 1962 and 1978, we tested the models developed in the previous chapter. We chose Canadian-United States data for several reasons. First, the data are good and very detailed. In addition, most of the data are available over the last twenty years. This allows us to examine the effects of altering exchange rate regimes (the data divide almost exactly in half between periods when the Canadian dollar was fixed and when it floated) upon the CS process.

Second, there is some evidence to suggest that the CS process is limited, for the residents of Canada, to a decision between holding their own and U.S. dollars. That is, over the past twenty years virtually 100 percent of the foreign currency liabilities of Canadian banks to private, non-bank Canadians has been denominated in U.S. dollars. While this is not conclusive evidence (since Canadians could hold foreign balances in third countries) it suggests that Canadian bankers felt that there was insufficient demand for deposit liabilities in these hypothetical third currencies and therefore did not supply them.[1] A more hueristic argument that CS in Canada is limited to Canadian and U.S. dollars would center around the close political ties and economic interdependence of the two countries.

Third, if the proposition we have just suggested is true, then estimation of our models is somewhat easier. Specifically, as we show in Appendix 3 (at the end of this chapter), if there are only two inputs in a generalized CES production function, then the substitution parameters (ρ_i, i=1,2) must be constrained to be equal in order to preserve the idea that along any isoquant of money services there is a constant elasticity of substitution. Therefore, in the two currency input case, we can limit our analysis to the constrained CES estimating equations defined in the last chapter by equation 4.8 (full adjustment) and equation 4.13 (partial adjustment).

Finally, the case of Canadian-U.S. currency substitution has been considered in several papers by Miles (1978A, 1978B, 1979). As we have pointed out, we feel that there is a possibility that his model is mis-specified. Hence, the results of his empirical work provide a convenient basis for comparison with the results we obtain.[2]

In this chapter, we present the results of our empirical work. We hope to demonstrate that our models provide a plausible solution to the problem of measuring the degree of currency substitution. Further, as our results will indicate, we feel there is considerable evidence for a transactions demand based theory of the CS process.

This chapter continues with a description of the data used in our study. Then the estimation results from the full adjustment model are presented. These results are compared to those of Miles. Next, the partial adjustment model is considered. Finally, we investigate questions of separability and discuss other unresolved issues.

The Data

In order to test our empirical model and derive a measure of the degree of substitutability in demand between currencies, data were gathered from a variety of published sources. The data are presented in Appendix 4. None of the data used in our study were adjusted for seasonal variation. Much of the data on Canadian money holdings (both Canadian and U.S. dollars), interest rates, and exchange rates were taken from various issues of the Bank of Canada Review and Statistical Summary. In addition, several time series on Canadian economic activity were found in issues of the Canadian Statistical Review and National Income and Expenditure Accounts published by Statistics Canada. Information on Canadian holdings of U.S. dollar accounts in the United States was located in the Treasury Bulletin of the United States Department of the Treasury. The specific time series used in our study are described more completely below.

Canadian holdings of their own currency were derived from the time series "currency and privately held deposits" estimated monthly by the Bank of Canada. This time series includes currency, non-interest bearing demand deposits, and time deposits. In excludes deposits in government owned accounts.

A time series of Canadian holdings of United States dollars was constructed using information from several sources. This series was formed by adding U.S. dollar liabilities of Canadian banks to private non-bank Canadians (less "swapped" deposits outstanding)[3] to the Canadian dollar value of short term liabilities of the U.S. banking system payable in U.S. dollars to all non-bank, non-official Canadians.

We used as measures of opportunity costs several different interest rates. These included the weighted averages of tender rates on three month Canadian and U.S. treasury bills. These rates were obtained from Bank of Canada publications. Consequently, the U.S. rates had been adjusted to conform to Canadian reporting standards (i.e. the U.S. rates were converted to reflect an equivalent yield basis to Canadian rates).[4] Data on other interest rates in Canada such as long term government bond yields and "swapped" deposit rates also come from various issues of the Bank of Canada Review and from International Financial Statistics (published by the International Monetary Fund).

Two different measures of Canadian economic activity (i.e. transactions) were located. One series was constructed from monthly observations on the value of checks cashed in Canadian clearing centers. The other transactions series was quarterly observations of Canadian gross national product valued at market prices.

Observations on spot and three-month forward exchange rates (expressed as Canadian dollars per one U.S. dollar) were derived from monthly closing values of the different rates and in several cases from the closing three-month forward spread.

Estimation of the Full Adjustment Model

The full adjustment constrained CES model (equation 4.8) - which we denote as Model 1 - is presented below

$$(4.8)\quad \ln\left[\frac{x_1}{e_{12}x_2}\right] = \Lambda_{01} + \Lambda_{11}\ln T_t + \Lambda_{21}t + \Lambda_{31}\ln\left[\frac{C_2}{C_1}\right] + u_t$$

It was estimated with quarterly data for the period 1962II-1978IV using three different definitions of the relative cost ratio $(\frac{C_2}{C_1})$. Table 4 presents the results from these experiments. Cost ratios 1 and 2 (defined below) represent attempts to incorporate into the holding costs of foreign balances a term representing expected changes in the exchange rate.[5]

Specifically, we define the following two cost ratios as:

TABLE 4 [1.]

Estimation Results with Full Adjustment Model
Estimation Period 1962II – 1978IV

COST RATIO [2.]	$\hat{A}_0$ CONSTANT (1)	$\hat{A}_1$ TRANSACTIONS TERM (2)	$\hat{A}_2$ TIME TREND (3)	$\hat{A}_3$ $\hat{\alpha}$ (4)	R^2 (5)	QUINN [3.] STATISTIC (Q) (6)	S.E.R. (7)	$\hat{\mu}$ [4.] (8)	F (9)	$\hat{\rho}$ (10)	$\hat{\theta}_{US} - \hat{\theta}_{CAN}$ (11)	$s_{US} - s_{CAN}$ (12)
1 [5.]	10.433	-.6290	.0086	.0612	.888	1.641 [6.]	.100	.920 [6.]	159.2	15.34	-10.28	.141
	(5.139)**	(.1855)**	(.0112)	(.0472)						(11.27)	(7.10)	(.190)
1 [5.]	10.180	-.5719	----	.0576	.888	1.466 [6.]	.0998	.940 [6.]	241.1	16.37	9.93	NA
	(1.915)**	(.1476)**	----	(.0406)						(12.26)	(7.20)	---
1 [5.]	3.624	----	-.0156	.0303	.867	.3915 [6.]	.109	.901 [6.]	199.2	32.00	NA	-.515
	(.3797)**	----	(.0072)**	(.0451)						(49.08)	---	(.828)
1 [5.]	2.732	----	----	.0406	.861	.1403 [6.]	.110	.961 [6.]	381.0	21.63	NA	NA
	(.315)**	----	----	(.0441)						(26.72)	----	----
2	9.696	-.5698	.0076	.0663	.887	1.040	.101	.908	162.4	14.08	-8.59	.115
	(2.009)**	(.1835)**	(.0101)	(.0342)*						(7.789)*	(5.068)*	(.126)
2	9.256	-.5044	----	.0625	.886	.8939	.101	.927	245.8	15	-8.07	NA
	(1.801)**	(.1396)**	----	(.0329)*						(8.416)*	(2.561)**	---
2	3.541	----	-.0145	.0592	.870	-.0090	.108	.898	210.1	15.89	NA	-.245
	(.363)**	----	(.0069)**	(.0367)						(10.48)	----	(.207)
2	2.664	----	----	.0695	.864	-.2379	.109	.956	406	13.39	NA	NA
	(.286)**	----	----	(.0348)*						(7.21)*	----	----
3	9.631	-.5573	.0060	2.528	.884	1.417	.102	.900	158.1	-.604	-.220	.002
	(2.036)**	(.1862)**	(.0097)	(1.721)						(.269)**	(.137)	(.004)
3	9.183	-.4979	----	2.436	.884	1.270	.102	.915	239.8	-.589	-.204	NA
	(1.699)**	(.1326)**	----	(1.690)						(.285)**	(.155)	---
3	3.580	----	-.015	2.339	.868	.3339	.109	.888	206.5	-.572	NA	-.006
	(.316)**	----	(.006)**	(1.832)						(.335)*	----	(.006)
3	2.690	----	----	2.558	.860	.1482	.111	.959	392.5	-.609**	NA	NA
	(.314)**	----	----	(1.809)						(.277)	----	---

NOTES: 1. Source: Statistical Summary, Treasury Bulletin
2. All equations were estimated using Cochrane Orcutt technique
3. This statistic is a measure of latent 1st order serial correlation after a Cochrane Orcutt estimation procedure has been applied. It is calculated with the following formula:

$$Q = \frac{\sqrt{N}(1 - \frac{1}{2}D.W.)}{|\hat{\mu}|} \sim N(0,1)$$

with H_0: $\mu = 0$
H_A: $\mu \neq 0$

where N = sample size

$\hat{\mu}$ = estimated serial correlation coefficient

D.W. = Durbin Watson statistic on the Cochrane Orcutt equation. See Quinn (1979)

4. estimated serial correlation coefficient
5. observation #62 (1976I) omitted from the sample
6. adjusted for 1 gap in the time series

N = 66
mean of dependent variable = 2.936
standard errors in parentheses
* significant at 95% level
** significant at 99% level

$$\text{COST RATIO 1} = \frac{C_{US}}{C_{CAN}} = \frac{r_{US} - \left(\frac{e^f - e^s}{e^s}\right)\left(\frac{365}{90}\right)}{r_{CAN}}$$

(5.1)

$$\text{COST RATIO 2} = \frac{C_{US}}{C_{CAN}} = \frac{r_{US} - \left(\frac{r_{CAN} - r_{US}}{1 + r_{US}}\right)}{r_{CAN}}$$

where: r_{US} = 90 day Treasury bill yield

r_{CAN} = 90 day Canadian Treasury bill yield

e^f = 90 day forward exchange rate Canadian dollars per one U.S. dollar

e^s = current spot exchange rate. Canadian dollars per one U.S. dollar

As we noted in Chapter 3, the choice between these two variables is largely an empirical question, although the use of cost ratio 2 implies a more restrictive assumption about the process of expectations formation.[6] The third cost ratio is one suggested by Miles, viz.

$$(5.2) \quad \text{COST RATIO 3} = \frac{C_{US}}{C_{CAN}} = \frac{1 + r_{US}}{1 + r_{CAN}}$$

The rationale offered by Miles for this cost ratio is that it represents the ratio of borrowing costs of the two currencies in each period.[7]

Our analysis differs from that of Miles in another important aspect. That is, Miles defines the Canadian holdings of U.S. dollars in each period as the sum of U.S. dollar liabilities booked in Canada to private non-bank Canadians (C $ CL) plus short term deposit liabilities of

U.S. banks (payable in U.S. dollars) to all private, nonbank Canadians (UCDL). There is a problem in defining Canadian holdings of U.S. dollars in this fashion. Specifically, the term C $ CL (defined above) includes as one of its components "swapped" deposits.[8] As Freedman points out:

> Swapped deposits are funds converted into a foreign currency, usually U.S. dollars, that have been placed on term deposit with a bank and that the bank has undertaken to convert back [emphasis added] into Canadian dollars at maturity.[9]

Further, the rate paid on these deposits is determined by both the rate paid on uncovered U.S. dollar deposits in Canada and on the forward spread on the Canadian-U.S. exchange rate. Therefore, holders of these "swapped" deposits are fully hedged against exchange rate risk during the holding period. The implication of this fully hedged position for the holders of "swapped" accounts is that the stock of these accounts should not be treated as part of the Canadian demand for U.S. dollars. Shearer supports this position. He bases his argument on interviews with Canadian business persons. Specifically:

> treasurers of companies holding swapped deposits generally indicated that they did not consider them to be "international" investments. Indeed, the by-laws of some of these companies prohibit investment in foreign currency securities but permit investments in swapped deposits. This is logical, of course, since swapped deposits are in effect Canadian dollar liabilities of Canadian banks.[10] [emphasis added]

In order to correct our data, we subtracted the value of "swapped" deposit liabilities (which is published as a separate time series by the Bank of Canada and denoted here as SWAPS) from the term C $ CL. The resultant ratio of Canadian holding of their own to U.S. dollars differs from the ratio of money holdings defined by Miles as follows. (Time series for both of these variables are presented in Appendix 4)

$$\text{MRATIO} = \frac{\text{CM2}}{\text{C\$CL} - \text{SWAPS} + r^{s} \cdot \text{UCDL}}$$

(5.3)

$$\text{MILESY} = \frac{\text{CM2}}{\text{C\$CL} + r^{s} \cdot \text{UCDL}}$$

where CM2 = the sum of currency and privately held deposits of nonbank Canadians denominated in Canadian dollars.

<u>In the results presented below we describe the outcome of regressions of several alternative specifications on values of MRATIO.</u>

The most surprising result from Table 4 is the striking low values of the elasticity of substitution in demand (column 4) between Canadian and U.S. dollar deposits over the estimation period. Even when COST RATIO 3 is employed, the estimated elasticity ($\hat{\sigma}$) is roughly one half of the elasticity reported by Miles. For the period 1960IV - 1975IV, using the logarithm of MILESY as his dependent variable (i.e. inclusion of SWAPS as part of Canadian holdings of U.S, dollars) Miles found the elasticity of substitution in demand to be 5.43. Our experiments suggest a value of $\hat{\sigma}$ of 2.56 using an identical specification of the right hand side variables, the logarithm of MRATIO as the dependent variable, over a slightly different time period.

When the logarithm of either cost ratio defined by 5.1 (i.e. COST RATIO 1 and COST RATIO 2) was used as an explanatory variable, the estimated values of $\hat{\sigma}$ approach zero. This suggests that when one incorporates into the holding costs of foreign monies expected changes in the exchange rate little or no substitution occurs between holdings of domestic and foreign balances when relative holding costs change. Hence, this implies that U.S. dollar balances are <u>not</u> viewed by Canadians as <u>close</u> substitutes (or even substitutes at all) for balances in their own currency. This is a very striking result and it bears further examination.

Using the first definition of relative holding costs, we found that it was necessary to eliminate one observation from our sample period. Specifically, the calculated relative holding cost for the first quarter of 1976 (observation #62) was -1.454. This is troublesome because it is mathematically impossible to take the logarithm of a negative number. The negative value arose because the expected return from holding uncovered balances in U.S. dollars (as calculated from the forward spread) exceeded in that period the opportunity cost of holding U.S. dollars in a non-interest bearing account. This result apparently arose because of considerable speculative pressure that saw Canadian dollars selling forward at a considerable discount at the same time that Canadian interest rates were almost double comparable United States rates. In fact, during this period in Canada, interest rates were rising, an expanded wage and price controls program was announced, and unemployment levels were very high.[12] Fortunately, for us, within one quarter, the forward rate returned to levels predicted by interest rate parity.[13]

None of the estimated elasticities of substitution in the regressions for the full period using COSTRATIO 1 were significantly different from zero at the 95% level. However, several estimates of $\hat{\sigma}$ had t values exceeding unity suggesting that Canadians viewed U.S. dollars to some extent as substitutes (albeit weak substitutes) for balances in their currency. Further proof of the existence of this relationship lies in the very small range of the estimated values of $\hat{\sigma}$ (.0303 - .0612).

If we replace COSTRATIO 1 with COSTRATIO 2 as a right hand side variable, then the results we described above are substantiated. That is, we find virtually identical estimates for the estimates of $\hat{\sigma}$. Further, in three of four specifications, the estimates of $\hat{\sigma}$ are significantly different from zero at the 95% level.[14] Also, we find an even smaller range on the values of $\hat{\sigma}$ (.0592 to .0695), suggesting again a very stable relationship between relative money holdings and relative holding costs.

The inference we draw from these results (as we noted above) is that at least during the sample period, Canadians did not view U.S. dollars as close substitutes to balances in their own currencies. This is a very plausible result since the role of $\hat{\sigma}$ is to indicate the degree that relative balances change when relative costs change. But, as we indicated above, relative balances can (and do) change even when relative costs are held constant. We posit that these changes occur because of different transactions demand for the different currencies or perhaps because of differing technological innovations in the use of these monies. The different specifications presented in Table 4 for each of the cost ratios represent our attempts to establish differential transactions or technological effect on money demand.

From Table 4, column 2, we see that the transactions term is highly significant in the determination of relative holdings. In every case, the estimated coefficient is significant at the 99% level. The specific transactions term used for these results was the total value of checks cleared through Canadian clearing centers in that quarter. We also estimated these results using quarterly data on Canadian GNP. Similar results obtained, but we rejected these latter data as being a less precise measure of total transactions within the period.[15] Recall from the last chapter that the coefficient on the transactions term is the product of the elasticity of substitution (σ) and the difference between transactions related coefficients in the efficiency parameter specification, viz

(5.4) $\Lambda_{1j} = \sigma(\theta_j - \theta_1)$.

Hence, our estimate of this coefficient is the product of two estimates. In order to determine if there is a differential transactions effect, we must divide the estimated coefficient ($\hat{\Lambda}_{1j}$) by our estimate of the elasticity of substitution ($\hat{\sigma}$). The dividend from this process is our estimate of the differential transactions parameter (here $\theta_{US} \hat{-} \theta_{CAN}$). The values of these terms (column 11) under alternate specifications are presented in Table 5. Standard errors constructed using Klein's approximation technique are presented below the values. Note, the similar values obtained using either COSTRATIO 1 or COSTRATIO 2. The values of this term when COSTRATIO 3 is used are smaller. This is due to the substantially larger estimate of $\hat{\sigma}$ obtained using this definition of relative holding costs.

In every case, the estimated values of $(\theta_{US} - \theta_{CAN})$ were negative. In two cases the values were significantly below zero at the 95% level. In all of the cases, the t values were less than minus one. Hence, we conclude that the transactions related coefficients are not equal. Further the coefficient for the Canadians' own currency is larger (in absolute value) than the comparable coefficient for foreign (U.S.) balances over the period of estimation.

We conducted a similar experiment to test for the existence of a differential in technological change over the entire estimation period. Specifically the coefficient on the time trend (Λ_2) represents the product of the elasticity of substitution and the differential rates of technological change, viz

$$(5.5) \quad \Lambda_2 = \sigma(s_{US} - s_{CAN})$$

Estimates of $\hat{\Lambda}_2$ (column 3) were significantly different from zero only when the transactions term was excluded from the equation. This indicates that the transactions series and the time trend are colinear (a less than surprising result). Therefore, it is only when the trend serves as a proxy for transactions that a significant estimate results. When we divided the estimate of $\hat{\Lambda}_2$ by the estimated elasticity of substitution we obtained values for the estimated differential in technological change ($s_{US} \hat{-} s_{CAN}$). We constructed standard errors of these estimates using Klein's approximation technique. Tests (t tests) of the hypothesis of the existence of differential failed in every case. Since we failed to detect the presence of any differential technological change effect over the estimation period, we removed that term from experiments conducted on subsamples of our data.

In summary, then, the evidence in Table 4 suggests that Canadians do not view U.S. dollar balances as being a close substitute for holding their own currency. They will, however, to a very limited extent, switch their currency holdings when relative costs change. A stronger effect seems to be the role of transactions demand for the separate balances. In particular, the estimated coefficient on the transactions term is an elasticity, which measures the percentage change in relative balances associated with a percentage change in transactions. Partial differentiation of equation 4.8 of the last chapter yields (after identification of the relevant subscripts)

$$(5.6) \qquad \frac{\partial \ln(\frac{X_{CAN}}{X_{US}})}{\partial \ln T} = \sigma(\theta_{US} - \theta_{CAN})$$

In every specification of our equation which incorporated a transactions term, this elasticity was negative and significant. Hence, as transactions rise, Canadians will hold more U.S. balances relative to balances in their own currency.

Over the entire estimation period, approximately one half of the observations came from a time when the exchange value of the Canadian dollar was fixed via á vis the U.S. dollar (1962IV - 1970II). The remainder of the observations come from periods when the Canadian dollar was allowed to float by the Bank of Canada (1970III - 1978IV). A natural experiment would then be to divide the data into observations from fixed regimes and observations from floating, to examine whether or not the differing exchange regimes lead to different behavior. Tables 5 and 6 present the results from this experiment.

Table 5 offers estimated parameters from estimation of equation 4.8 over the fixed exchange rate period of the Canadian dollar (1962III - 1970II). Again, we find that when the first two definitions of relative costs are used, the estimates of $\hat{\sigma}$ (column 3) are near zero. Only when expected changes in the exchange rate are ignored (i.e. when COSTRATIO 3 is used) does the estimated elasticity rise above unity.

Further, note the substantial improvement in the "goodness of fit" of the equations (regardless of the cost definition) when a transactions term is included. Not only are the R^2 terms higher and the standard errors reduced, but the t values on the estimates of $\hat{\sigma}$ rise above unity in all three cases. In the case of the COSTRATIO 1 specification, the value of $\hat{\sigma}$ becomes significantly different from zero at the 95% level. The highly significant values of the Quinn statistic imply, however, the presence of second order autocorrelation. This suggests, to us, perhaps the omission of a second trended term. Therefore, a partial adjustment model may be applicable. We will test this hypothesis later.

Again, we note that the estimated transactions elasticities (column 2) are negative and significantly different from zero regardless of specification. This suggests that during the fixed rate period, Canadians increased their holdings of U.S. dollars relative to Canadian dollars as the level of economic activity increased.

TABLE 5 [1.]

Estimation Results: Full Adjustment Model
Fixed Exchange Rate Period 1962III - 1970II

COST RATIO [2.]	$\hat{\Lambda}_0$ (1)	$\hat{\Lambda}_1$ (2)	$\hat{\Lambda}_3$ $\hat{\sigma}$ (3)	R^2 (4)	S.E.R. (5)	Q (QUINN STATISTIC) (6)	$\hat{\mu}$ [4.] (7)	F (8)	$\hat{\rho}$ (9)	$\theta_{US} \hat{-} \theta_{CAN}$ (10)
1	13.71	-.9058	.1502	.922	.070	3.263**	.401	164.4	5.658	-6.031
	(.9529)**	(.0805)**	(.0752)*						(3.333)*	(2.989)*
1	2.844	-----	.0336	.829	.102	-1.208	.906	140.1	28.76	-----
	(.1748)**	-----	(.0748)						(66.26)	-----
2	13.49	-.8870	.0924	.915	.073	3.335**	.394	151.3	9.822	-9.600
	(.9696)**	(.0820)**	(.0717)						(8.398)	(7.447)
2	2.840	-----	.0553	.832	.101	-1.160	.900	143.5	17.083	-----
	(.1780)**	-----	(.0598)						(19.55)	-----
3	13.574	-.894	4.83	.919	.071	3.128**	.388	157.7	-.793	-.185
	(.9460)**	(.0700)**	(2.880)						(.123)**	(.110)
3	2.837	-----	2.675	.831	.101	-1.184	.910	142.6	-.626	
	(.1955)**	-----	(3.298)						(.461)	

NOTES:
1. Source: Statistical Summary, Treasury Bulletin
2. All equations were estimated using the Cochrane-Orcutt Estimation Technique
3. Test statistic for remaining autocorrelation after the Cochrane-Orcutt estimation
4. Estimated first order serial correlation coefficient

N = 31

Mean dependent variable = 3.0028

* significantly different from zero at 95% level

** significantly different from zero at 99% level

When we tested for the presence of a differential effect we found that the term, $\hat{\theta_{US} - \theta_{CAN}}$ (column 10), was significantly different from zero at the 95% level. The other two specifications yielded estimates with t values greater than unity.

In Miles' study of currency substitution, he was unable to isolate a statistically significant measure of the elasticity of substitution during a fixed exchange rate regime. He concluded that this was due to the readiness of central banks to supply foreign balances at a fixed exchange rate. As we show in Table 5, the inclusion of a transactions term represents a significant contribution and it allows us to separate out the effects of both transactions and changing holding costs. Specifically, it seems to us that during fixed exchange rate periods agents in an economy would still be willing to hold foreign balances in order to facilitate transactions and economize on transactions costs and would substitute between various balances as costs change. Further, during these fixed rate periods, the exposure to exchange risk incurred from maintaining foreign balances is substantially reduced vis à vis flexible exchange rate periods. Hence, holders of foreign exchange face less uncertainty during fixed rate period. Thus, our findings do not confirm the conclusions reached by Miles.

During the flexible rate period after 1970III our results become somewhat less precise. These results are presented in Table 6. Using two specifications of equation

TABLE 6 [1]

Estimation Results: Full Adjustment Model
Flexible Exchange Rate Period 1970III - 1978IV

COST RATIO	$\hat{\Lambda}_0$ (1)	$\hat{\Lambda}_1$ (2)	$\hat{\Lambda}_3$ $(\hat{\sigma})$ (3)	R^2 (4)	S.E.R. (5)	Q QUINN STATISTIC (6)	$\hat{\mu}$ (7)	F (8)	$\hat{\rho}$ (9)	$\theta_{US} - \hat{\theta}_{CAN}$ (10)
1 [5.]	12.773	-.7573	.0330	.920	.097	.258	.723	160.7	29.30	-22.9
	(1.772)	(.1355)**	(.0512)						(47.00)	(36.7)
1 [5.]	2.213	-----	.0587	.887	.114	1.052	.987	227.3	16.04	-----
	(1.403)	-----	(.0581)						(16.85)	-----
2	12.53	-.7396	.0323	.912	.101	-.120	.694	156.3	29.96	-22.9
	(1.709)**	(.1308)**	(.0434)						(27.54)	(29.7)
2	2.371	-----	.0489	.878	.117	.656	.977	222.9	19.45	-----
	(.771)**	-----	(.0487)						(20.37)	-----
3	12.46	-.7349	1.507	.912	.101	-.0623	.693	156.3	-.336	-.488
	(1.73)**	(.1324)**	(2.015)						(.887)	(.688)
3	2.470	-----	2.220	.878	.117	.7203	.973	222.5	-.550	-----
	(.640)**	-----	(2.229)						(.452)	-----

NOTES:
1. Source, Statistical Summary, Treasury Bulletin
2. All equations were estimated using the Cochrane-Orcutt estimation technique
3. Test statistic for remaining autocorrelation after Cochrane-Orcutt estimation
4. Estimated first order serial correlation coefficient
5. Observation #62 (1976I) omitted from sample

N = 33
mean of dependent variable = 2.871
* significantly different from zero at 95% level
** significantly different from zero at 99% level

4.8, three separate cost ratios and data from the period 1970III - 1978IV, we were unable to identify a statistically significant measure of the elasticity of substitution in demand (column 3). This is hardly surprising given the political and economic turmoil of Canada and the world during those years. Several of the t values for estimates of σ are close to unity. Likewise, we failed in our attempt to establish a statistically significant measure of the differential in transactions demand parameters $\widehat{\theta_{US} - \theta_{CAN}}$ (column 10).[16]

Despite our inability to measure either of the effects independently, the product of the elasticity of substitution and the transactions differential transactions - the transactions elasticity of relative balances (column two)—again was significantly negative at the 99% level in every specification. Further, inclusion of this term as an exogenous variable in the specification improved most "goodness of fit" measures.[17]

The conclusion we draw from these last results is that because of the increased fluctuations in exchange rates and therefore increased risk, Canadians were less likely to substitute foreign balances for holdings of their own simply because of expected changes in relative holding costs. Rather, the dominant influence leading to changes in relative holdings appears to be transactions related. We turn now to a discussion of the partial adjustment model.

Estimation of the Partial Adjustment Model (Model 2)

Regardless of specification or estimation period, we could never reject the hypothesis of the presence of first order autocorrelation. Hence, in every case we estimated Model 1 using the Cochrane Orcutt procedure in order to eliminate the first order autocorrelation. The presence of autocorrelation in our model could suggest the omission of an important variable as a explanator of current relative money holdings. Suppose for instance that economic agents are not fully able to adjust their various money holdings in one period. The actual holdings they achieve in the current period will affect behavior next period. Such behavior implies the partial adjustment model that we developed as equation 4.13 in the last chapter.

$$(4.13)\quad \ln \frac{X_1}{e_{12}X_2} = \psi_0 + \psi_1 \ln T_t + \psi_2 t + \psi_3 \ln \left(\frac{C_2}{C_1}\right) + \psi_4 \ln \left(\frac{X_1}{e_{12}X_2}\right)_{t-1} + u_t$$

Table 7 presents the results from the estimation of equation 13 over the entire estimation period. The most striking result of this experiment is the extremely large (and significant) values of $\hat{\psi}_4$ $(=1-\hat{K})$, (see column 5). These estimates imply very small values for the speed of adjustment, $\hat{K}$ (column 6). Hence, relative balances are very slow to adjust to <u>any</u> economic shocks (e.g. changes in relative holding costs, level of transactions, etc). For instance,

TABLE 7 [1.,2.,5.]

Estimation Results: Partial Adjustment Model - Full Estimation Period
1962II - 1978IV

COST RATIO	$\hat{\psi}_0$ (1)	$\hat{\psi}_1$ (2)	$\hat{\psi}_2$ (3)	$\hat{\psi}_3$ (4)	$\hat{\psi}_4$ (5)	$\hat{K}$ (6)	R^2 (7)	Durbin h 3. (8)	S.E.R. (9)	F (10)	$\hat{\sigma}$ (11)	$\hat{\theta}_{US} - \hat{\theta}_{CAN}$ (12)	$\hat{a}_{US} - \hat{a}_{CAN}$ (13)	$\hat{\omega} \equiv \sigma(\hat{\theta}_{US} - \hat{\theta}_{CAN})$ (14)
1 [4.]	3.99	-.329	.011	-.010	.881	.119	.862	.430	.111	95.4	-.084	32.9	-1.10	-2.76
	(1.70)*	(.155)*	(.006)*	(.027)	(.063)**	(.063)*					(.243)	(83.3)	(2.78)	(1.99)
1 [4.]	.923	-.043	-----	.015	.866	.134	.854	.146	.114	121.2	.112	-2.87	-----	-.321
	(.450)*	(.025)*	-----	(.024)	(.064)**	(.064)*					(.177)	(4.89)	-----	(.156)*
1 [4.]	.406	-----	-.0013	.018	.877	.123	.852	.022	.115	119.0	.146	-----	-.072	-----
	(.217)*	-----	(.0009)	(.074)	(.065)**	(.065)*					(.178)	-----	(.106)	-----
1 [4.]	.190	-----	-----	.014	.932	.068	.847	-.185	.115	174.8	.206	-----	-----	-----
	(.156)	-----	-----	(.025)	(.052)**	(.052)					(.356)	-----	-----	-----
2	3.49	-.277	.008	.014	.862	.138	.862	.467	.112	96.4	.101	-19.8	.57	-2.01
	(1.73)*	(.158)*	(.006)	(.022)	(.063)**	(.063)*					(.160)	(32.0)	(1.18)	(1.50)
2	1.07	-.051	-----	.032	.849	.151	.857	.238	.113	125.7	.212	-1.59	-----	-.338
	(.45)**	(.024)*	-----	(.020)	(.063)**	(.063)**					(.140)	(1.21)	-----	(.137)**
2	.483	-----	-.0016	.035	.857	.143	.855	.120	.114	123.5	.245	-----	-.046	-----
	(.216)*	-----	(.0009)*	(.020)*	(.064)**	(.064)*					(.152)	-----	(.034)	-----
2	.206	-----	-----	.030	.928	.072	.847	-.133	.116	177.4	.417	-----	-----	-----
	(.154)	-----	-----	(.020)	(.052)**	(.052)					(.357)	-----	-----	-----
3	3.81	-.308	.010	.282	.867	.133	.861	.497	.112	95.8	2.12	-1.09	.035	-2.32
	(1.81)*	(.167)*	(.006)	(1.43)	(.067)**	(.067)*					(10.4)	(5.90)	(.186)	(1.84)
3	1.05	-.048	-----	1.59	.843	.157	.843	.157	.114	124.0	10.13	-.030	-----	-.306
	(.45)*	(.025)*	-----	(1.17)	(.066)**	(.066)*					(7.33)	(.027)	-----	(.140)*
3	.499	-----	-.0015	1.81	.849	.151	.853	.200	.114	121.9	11.99	-----	-.0008	-----
	(.224)*	-----	(.0009)	(1.19)	(.067)**	(.067)*					(7.66)	-----	(.0007)	-----
3	.246	-----	-----	1.67	.914	.086	.847	-.052	.116	176.7	19.42	-----	-----	-----
	(.166)	-----	-----	(1.20)	(.055)**	(.055)					(14.52)	-----	-----	-----

NOTES:
1. Source: Statistical Summary, Treasury Bulletin
2. All equations were estimated using ordinary least squares.
3. This is a measure of serial correlation when there are lagged endogenous variables on the right hand side of the equation. It is calculated with the following formula:

$$\text{Durbin h} = \hat{\rho}\sqrt{\frac{N}{1 - N \cdot V(1 - \hat{K})}}$$

where N = sample size

$\hat{\rho}$ = estimated first order serial correlation computed from OLS residuals

$V(1 - \hat{K})$ = estimated variance of the OLS estimate of $1 - \hat{K}$.

The statistic is distributed standard normal under the null hypothesis that $\rho = 0$

4. observation #62 omitted from the sample
5. standard errors in parenthesis

* significant at 95% level.
** significant at 99% level.

using the COSTRATIO 1 and the full specification of the model, we see that only about 12% of the difference between the desired and actual relative balances is eliminated within a single period. These results seem even more surprising when one considers that assets markets are usually assumed to be very quick to clear.[18]

In the case of Model 2, the estimated coefficients of the transactions term (column 2),[19] the time trend, (column 3), and the relative holding costs (column 4) are now all functions of the rate of adjustment, K. Therefore, in order to derive coefficients which have the same interpretation as those derived from estimating Model 1, it is necessary to deflate each estimated by the estimated value of K. Again, we find that because the parameters we construct are nonlinear functions of the estimated coefficients, we must approximate the variances of these parameters according to some approximation technique.

We find, for instance, that depending upon the particular definition of the cost ratio, the estimated elasticities of substitution (column 11) are either very low (when COSTRATIO 1 or COSTRATIO 2 is used) or very high (corresponding to the use of COSTRATIO 3). In all cases, we were unable to find values of $\hat{\sigma}$ significantly different from zero at the 95% level.[20]

The interpretation of the estimated coefficients is that they represent short run elasticities, since they are all multiplied by the adjustment speed (K). The parameters obtained by dividing these coefficients by $\hat{K}$ are then estimates of the corresponding variables long run elasticities. Following this interpretation the column $\hat{\sigma}$ provides estimates of the long run elasticities of substitution. We see that these long run elasticities are (in most cases) approximately twice the size of the estimates of σ from Model 1. This suggests that in the long run, the percentage change in relative balances due to a one percent change in relative costs, will be almost double the short run change.

In this model, we again see the strong influence of the level of transactions on the determination of relative balances. In particular, the short run transactions elasticities - determined by the estimated coefficients of the transactions terms (column 2) - were all significantly below zero at the 95% level. This confirms our findings under the previous model that Canadians increased their holdings of U.S. dollars during periods of rising transactions.

Estimates of $\omega(=\sigma(\theta_{US} - \theta_{CAN}))$(column 14), the long run transactions elasticity vary substantially, according to whether or not a time trend is included as a right hand side variable. In all cases, where the trend was omitted the estimates of ω were significant at the 95% level and between zero and minus one. When the trend is included, then the $\hat{\omega}$ become statistically insignificant (although the t values are all greater than one) but below minus two in value. We are unable to explain these results, but the answer must lie in the strong collinearity between the level of transactions and the time trend.

Because we were unable to estimate the coefficient of the relative holding costs term with very much accuracy in any of the specifications, we could not identify statistically significant estimates of the transactions parameter differentials ($\theta_{US} - \theta_{CAN}$) (column 12) or the technological change differential ($s_{US} - s_{CAN}$) (column 3). The estimates of ω substantiate the role of the transactions term in our specification. However, since we were unable to verify the existence of a difference in the rates of technological change, we excluded the time trend from estimates during the different subperiods.

Finally, we note that, in general, the specification of Model 2 seemed to be slightly worse than Model 1 in explaining the demand for relative balances. Specifically, in every case, the standard error of the regression was higher for equations in Model 2 then the corresponding equations from Model 1. Other goodness of fit measures (R^2,F) were also lower when Model 2 was estimated relative to Model 1. Values of the Durbin-h statistic kept us from rejecting in every case the hypothesis of nonautocorrelated errors. We turn now to the results of estimating Model 2 during the fixed and flexible rate periods.

Table 8 presents the regression results from the fixed rate period. Two different specifications of equation

TABLE 8 [1.,2.,3.]

Estimation Results: Partial Adjustment Model
Fixed Exchange Rate Period 1962III - 1970II

COST RATIO	$\hat{\psi}_0$ (1)	$\hat{\psi}_1$ (2)	$\hat{\psi}_3$ (3)	$\hat{K}$ (4)	$\hat{\psi}_4$ (5)	S.E.R. (6)	R^2 (7)	Durbin h (8)	F (9)	$\hat{\sigma}$ (10)	$\hat{\omega}$ (11)	$(\hat{\theta}_{US} - \theta_{CAN})$ (12)
1	10.80	-.720	.097	.762	.238	.071	.921	2.08*	108.8	.127	-.945	-7.42
	(1.79)**	(.122)**	(.049)*	(.124)**	(.124)*					(.055)*	(.066)**	(3.07)*
1	.286	-----	-.109	.103	.897	.105	.823	-.467	67.4	-1.06	-----	-----
	(.241)	-----	(.052)*	(.080)	(.080)**					(.937)	-----	-----
2	10.42	-.695	.076	.735	.265	.072	.920	1.90*	107.8	.103	-.946	-9.14
	(1.69)**	(.116)**	(.041)*	(.117)**	(.117)*					(.048)*	(.084)**	(3.96)*
2	.296	-----	-.083	.106	.894	.107	.817	-.685	64.9	-.783	-----	-----
	(.245)	-----	(.046)*	(.081)	(.081)**					(.694)	-----	-----
3	9.82	-.654	5.62	.698	.302	.070	.925	1.34	114.8	8.05	-.937	-.116
	(1.40)**	(.095)**	(2.42)*	(.103)**	(.103)**					(3.26)*	(.079)**	(.045)**
3	.275	-----	-1.84	.096	.904	.112	.799	-.138	57.6	-19.17	-----	-----
	(.275)	-----	(3.47)	(.085)	(.085)**					(38.7)	-----	-----

NOTES:
1. Source: Statistical Summary, Treasury Bulletin
2. Standard errors in parentheses
3. All equations are estimated using ordinary least squares

* significant at the 95% level

** significant at the 99% level

4.13, using each cost ratio were estimated. First, we included a transactions term (the value of checks cleared in the quarter) in the partial adjustment specification. In the second regressions, we omitted the transactions term. In every case incorporation of the transactions term led to substantial improvement in the results.

First, when we included the check clearing variable, regardless of specification, every parameter of the model was significantly different from zero <u>at least</u> at the 95% level. Second, every term had the expected sign and plausible values. That is, the values of the adjustment coefficient, $\hat{K}$ (column 4) were approximately equal to .7. This indicates rather rapid adjustment of actual to desired balances within one period. The values of $\hat{\sigma}$ (column 10) were several times larger than the estimates of $\hat{\sigma}$ for the entire period. This suggests that as exchange risk diminishes, currencies become closer substitutes in demand. Finally, the estimates of the transactions elasticity, $\hat{\omega}$ (column 11), were negative and about three times the values discovered over the whole period. This supports our hypothesis of the strong role that transactions demand (especially when exchange risk is low) plays in achieving desired money balances.

In contrast, when the transactions variable is omitted, the estimates of the adjustment coefficient, $\hat{K}$, become implausibly low. This suggests considerable inertia exists in the asset market. Likewise, the estimates of $\hat{\sigma}$

are in each version of this second case insignificant and of the wrong sign.

Summary statistics of "goodness of fit" substantiate the considerable improvement in explanatory power given to the model through the inclusion of a transactions term. In each case, after inclusion, R^2 and F standard errors fell (by a third!) Unfortunately, the high values of the Durbin h statistic in the first two cost ratio versions suggest that presence of autocorrelated errors. This is a very troublesome result since it is well known that ordinary least squares applied to an autoregressive model with autocorrelated errors produces inconsistent estimates. The reason that the estimates are inconsistent is that the lagged endogenous variable on the right hand side of the equation is correlated with the lagged portion of the error term.

In this situation, maximum likelihood estimation of the model is called for. Since such estimation involves grid search procedure which can be computationally costly we chose an alternative technique developed by Hatanaka. His two step technique involves obtaining a consistent estimate of the autocorrelation term through instrumental variables estimation of the initial model. Then in the second stage, an autoregressive transformation is performed using the estimate derived of the autocorrelation coefficient previously obtained. The second stage estimates are consistent and asymptotically efficient (and therefore converge maximum likelihood estimates in large samples).[21] Table 9 presents these second stage estimates.

Comparison of the values presented in Table 9 with the corresponding results from Table 8 shows that little information is gained from the consistent (and asymptotically efficient) Hatanaka estimates. The signs and relative magnitudes of these estimates are virtually unchanged from the ordinary least squares estimates presented in Table 8. Note, however, using this alternative technique that the estimates of $\hat{K}$ (Table 9, column 4) are all very close to unity. In fact, in none of the specifications can we reject at the 99% level the hypothesis that agents fully adjust relative holdings in one period.[22] Hence, in this case, the full adjustment model appears to be appropriate.

Table 10 presents regression results for Model 2 over the flexible exchange rate period. Again, as in the case of the full adjustment model, we see that we have strong measures of "goodness of fit", but imprecise measures of $\hat{\sigma}$ (column 10). In fact, none of the six estimates of $\hat{\sigma}$ are significantly different from zero. Further, in several cases, these estimates exhibit the wrong sign.

Estimates of the adjustment coefficient ($\hat{K}$) (column 4) are small, especially in the equations where the transactions term is omitted. Low values of $\hat{K}$ suggest very slow adjustment of actual to desired balances and tend to confirm our hypothesis that the degree of substitutability between

TABLE 9 [1,2]

Hatanaka Two Stage Estimates: Partial Adjustment Model
Fixed Exchange Rate Period

COST RATIO	$\hat{\psi}_0$ (1)	$\hat{\psi}_1$ (2)	$\hat{\psi}_3$ (3)	$\hat{K}$ (4)	$\hat{\psi}_4$ (5)	S.E.R. (6)	R^2 (7)	D.W. (8)	$\hat{\alpha}$ (9)	$\hat{\omega}$ (10)	$\hat{\theta}_{US} - \theta_{CAN}$ (11)
1	13.60	-.903	.182	.973	.027	.070	.927	1.676	.187	-.928	-4.96
	(2.69)**	(.179)**	(.075)**	(.196)**	(.196)				(.101)*	(.367)**	(1.74)**
2	12.74	-.846	.159	.909	.091	.070	.927	1.837	.175	-.931	-5.32
	(2.51)**	(.167)**	(.066)**	(.185)**	(.185)				(.092)*	(.367)**	(2.01)**
3	12.02	-.797	7.06	.865	.135	.070	.927	1.855	8.162	-.913	-.113
	(2.46)**	(.164)**	(2.85)**	(.182)**	(.182)				(4.133)*	(.377)**	(.044)**

NOTES: 1. Source, Statistical Summary, Treasury Bulletin
2. Standard errors in parentheses
* significant at 95% level
** significant at 99% level

TABLE 10 [1.,2]

Estimation Results: Partial Adjustment Model - Flexible Exchange Rate Period
1970III - 1978IV

COST RATIO	$\hat{\psi}_0$ (1)	$\hat{\psi}_1$ (2)	$\hat{\psi}_3$ (3)	$\hat{K}$ (4)	$\hat{\psi}_4$ (5)	S.E.R. (6)	R^2 (7)	Durbin h (8)	F (9)	$\hat{\sigma}$ (10)	$\hat{\omega}$ (11)	$(\hat{\theta}_{US} - \hat{\theta}_{CAN})$ (12)
1 [3.]	3.09	-.195	-.026	.200	.800	.100	.914	-.062	103.1	-.130	-.975	7.50
	(.977)**	(.062)**	(.030)	(.084)*	(.084)**					(.165)	(.321)**	(8.10)
1 [3.]	.081	-----	.011	.031	.969	.114	.885	.843	114.9	.355	-----	-----
	(.217)	-----	(.032)	(.075)	(.075)					(.974)	-----	-----
2	3.15	-.197	-.011	.212	.788	.104	.907	.088	97.5	-.052	-.929	17.9
	(1.00)**	(.064)**	(.030)	(.087)*	(.087)**					(.146)	(.132)**	(47.5)
2	.148	-----	.028	.054	.946	.117	.878	.862	111.4	.519	-----	-----
	(.234)	-----	(.031)	(.080)	(.080)					(.652)	-----	-----
3	3.26	-.208	-1.01	.203	.797	.103	.908	.030	98.6	4.98	-1.02	.206
	(1.00)**	(.065)**	(1.49)	(.088)**						(8.11)	(.386)**	(.284)
3	.143	-----	1.14	.026	.974	.118	.877	.919	110.3	43.8	-----	-----
	(.249)	-----	(1.51)	(.085)	(.085)**					(115.6)	-----	-----

1. Source: Statistical Summary, Treasury Bulletin
2. standard errors in parentheses
3. observation #62 omitted

* significant at 95% level
** significant at 99% level

domestic and foreign balances is quite low during flexible exchange rate periods.

Again, the major cause of substitution between monies appears directed by differentials in the transactions elasticities of demand. Each time the transactions term was included its estimated coefficient (column 2) was significantly different from zero at the 99% level. Likewise, estimates of the transactions elasticity $(\hat{\omega})$ (column 11) were also highly significant and virtually identical regardless of specification.

A comparison of "goodness of fit" measures between the results from equations which included the transactions term and those which did not suggest that the partial adjustment model performs somewhat better when the transactions term is included. Further, in these cases and during this time period, there appears to be no autocorrelated errors. Hence, there was no need to reestimate these equations using the Hatanaka technique.

Other Issues

One of the assumptions we made when we developed our theoretical model was that the determination of the holdings of various currency balances could be considered apart from the determination of the holdings of non-currency assets. Hence, one could imagine the allocation of wealth among various assets as a two-step procedure, whereby one would optimize within the set of monies according to some objective function, optimize within the set of non-currency assets, and then optimize between the subsets.

Therefore, in restricting our analysis to the issue of Canadian substitution in demand between their own and U.S. dollar balances we have imposed the restriction that the marginal rate of technical substitution (the slope of an isoquant of money services) - and therefore the elasticity of substitution - is unaffected by the quantities of non money assets held by Canadians.

One way to test this restriction has been suggested in the near money literature. Moroney and Wilbratte (1976) regress the residuals from their ordinary least squares estimate on the yields of assets not included in their model. If these assets were not separable from the assets in the model, the authors expected to find significant correlation coefficients between the residuals and the yields on the excluded assets.

We tried a similar experiment. We obtained the ordinary least squares residuals from two versions of each of the three equations (corresponding to COSTRATIO 1, COSTRATIO 2, COSTRATIO 3) for the whole and each of the sub-periods. Then in separate regressions we correlated these residuals with yields on two alternative assets available to Canadians over the time period: long term Canadian government bonds (RLONG) and the yield on swapped deposits in Canadian banks (SWAPRATE). These two yields represent two ends of the asset spectrum for Canadians, (in terms of maturity) and hence, offer the greatest test of our specification. Table 11 summarizes our findings.

The values of the correlation coefficients (R) are uniformly small confirming our initial restriction. Note however, in every possible pairwise comparison, specifications which included the transactions term were less correlated with either of the yields than those specifications which omitted this term. We view this as additional support for our hypothesis of a transactions based demand for relative balances.

Another issue that we have suggested but have not confirmed is whether there is any evidence of an effect of changing the international monetary system from fixed to flexible exchange rates. One test of this would be to conduct a Chow test for the equality of coefficients from the fixed and flexible rate periods. The appropriate statistic for this test is the F statistic calculated from the sum of squared residuals of the regression on the pooled data and the data from each sub period.[23] Table 12 presents the results of this test.

As the results clearly show, there is considerable evidence pointing to a structural change in the process of currency substitution for Canadians with changes in the exchange rate regime.[24] This change is most apparent in the case of the models incorporating the transactions term. Hence the conclusions we reached in earlier sections of this paper are confirmed. Specifically, the movement from

TABLE 11

Separability Tests [1.,2.]

Residuals calculated from OLS Regression of MRATIO on	Exchange Rate Regime	Coefficient on RLONG (t value)	R	Coefficient on SWAPRATE (t value)	R
Transactions, COSTRATIO 1	Mixed	-.08 (.22)	.032	-.354 (.915)	.10
CRATIO 1	Mixed	-.52 (1.12)	.14	-.859 (1.78)	.22
Transactions, COSTRATIO 2	Mixed	-.08 (.22)	.032	-.33 (.89)	.10
CRATIO 2	Mixed	-.53 (1.15)	.14	-.86 (1.81)	.22
Transactions, COSTRATIO 3	Mixed	0.07 (.21)	.032	0.32 (.88)	.10
CRATIO 3	Mixed	-.46 (1.05)	.14	-.75 (1.68)	.2
Transactions, COSTRATIO 1	Fixed	-.02 (.09)	.017	-.06 (.24)	.04
CRATIO 1	Fixed	-.54 (.80)	.14	-.95 (1.40)	.24
Transactions, COSTRATIO 2	Fixed	-.02 (.08)	.014	-.06 (.27)	.04
CRATIO 2	Fixed	-.54 (.80)	.14	-.14 (1.39)	.24
Transactions, COSTRATIO 3	Fixed	-.03 (.12)	.032	-.07 (.33)	.063
CRATIO 3	Fixed	-.56 (.82)	.14	-.97 (1.40)	.24

TABLE 11 (Continued)

Residuals calculated from OLS Regression on MRATIO on	Exchange Rate Regime	Coefficient on RLONG (t value)	R	Coefficient on SWAPRATE (t value)	R
Transactions, COSTRATIO 1	Flexible	-.04 (.11)	.02	-.173 (.41)	.071
CRATIO 1	Flexible	-.23 (.38)	.063	-.51 (.81)	.14
Transactions, COSTRATIO 2	Flexible	-.04 (.10)	.017	-.166 (.41)	.070
CRATIO 2	Flexible	-.18 (.33)	.054	-.41 (.72)	.14
Transactions, COSTRATIO 3	Flexible	-.04 (.10)	.017	-.164 (.40)	.070
CRATIO 3	Flexible	-.15 (.28)	.04	-.37 (.67)	.10

NOTES: 1. Sources: Bank of Canada Review, International Financial Statistics
2. all equations estimated with ordinary least squares
* significant at 95% level
** significant at 99% level

TABLE 12

F - Statistics for H_0: Regressions are Equal in Fixed and Floating Subperiods

	Regression with Cost Ratio Only	Regression with Cost Ratio and Transactions Term
COST RATIO 1	2.042	8.154**
COST RATIO 2	2.931	9.084**
COST RATIO 3	3.412*	9.762**

* significant at 95% level
** significant at 99% level

fixed to floating rates seems to have made various monies <u>less</u> substitutable in demand.[25] This result is not surprising when one considers the increased risk of foreign exchange exposure under floating rates.[26]

FOOTNOTES

Chapter Five

[1]For an excellent discussion of the determinants of demand for foreign currency deposits at Canadian banks and an analysis of decision by Canadian banks to supply these liabilities, see Freedman (1974) Chapters 2 and 3.

[2]See Miles (1978) pp. 432-436 for a complete discussion of his model, data and results.

[3]In the next section we discuss the reason for subtracting this term.

[4]The yield on United States Treasury bills is quoted in the United States on a 360 day discount basis. Canadian Treasury bill yields are quoted on a 365 day true yield basis. Hence, the Canadian authorities when presenting the yield on U.S. Treasury bills convert these yields to conform to Canadian reporting practices. See Federal Reserve Bulletin, October 1964, pp. 1253-1254 for further details.

[5]Through our empirical work we assume that transactions costs and political risk are constants (or zero) and hence, can be subsumed in the constant term. It can be argued that over the period of analysis little or no political risk existed for Canadians who held U.S. dollar deposits (except perhaps in 1971 when President Nixon closed the gold window). In fact this risk may have been negative at times. Specifically, during the late - 1970's the rise of the Quebec separatist movement in Canada may lead to tightened government controls - both political and economic. Using information from a chronology of events in Canada over this period published in various issues of the Canada Yearbook, we attempted to construct a dummy variable to reflect the effect of separatist movement might have on Canadian holdings of U.S. dollars. We failed to detect any significant effects (perhaps because of the admittedly ad hoc nature of the variable).

[6]In order to use COST RATIO 1, we assume that the expected future spot rate equals the current forward rate. COST RATIO 2 retains this assumption as well as the assumption that the relationship between current and forward exchange rates are determined by interest rate parity.

[7] Miles (1978) pp. 433-434.

[8] We are grateful to William Gasser of the Federal Reserve Bank of New York for pointing out this problem to us.

[9] Charles Freedman (1974) pg. 6.

[10] Ronald Shearer (1965) pg. 344.

[11] We ignore the optimization problem implied by the fact that Canadians can hold their own currency in several forms in order to focus our attention on the CS issue. Nonetheless, it would be a simple matter to nest a CES type relation which could model the allocation between currency and other Canadian liquid deposits into the larger CES relation describing the allocation between Canadian and U.S. deposits. For a discussion of the allocation problem between currency and deposit holdings see Offenbacher (1978). For an example of nested CES functions as a model of several allocation problems, see Barth, et. al. (1977).

[12] Canada Yearbook 1976 - 1977, pp. 1082-1083.

[13] Interest rate parity would predict the following relationships viz.

$$\frac{e^f - e^s}{e^s} = \frac{i_{US} - i_{CAN}}{1 + i_{US}}$$

we tested for the existence of interest rate parity by regressing the exchange agio $(\frac{e^f - e^s}{e^s})$ on the interest rate agio $(\frac{i_{US} - i_{CAN}}{1 + i_{US}})$ over our sample periods. We found the following results:

PERIOD: 1962II - 1978IV

$$\frac{e^f - e^s}{e^s} = \underset{(.0025)}{-.0007} + \underset{(.179)}{1.27} \left[\frac{i_{US} - i_{CAN}}{1 + i_{US}}\right]$$

R^2 = .438 D.W. = 1.88 SER = .019 F = 50.7

PERIOD: 1962II - 1978IV (1976I excluded)

$$\frac{e^f - e^s}{e^s} = \underset{(.0009)}{-.0009} + \underset{(.070)}{.824} \left[\frac{i_{US} - i_{CAN}}{1 + i_{US}}\right]$$

R^2 = .684 D.W. = .132 SER = .007 F = 138.5

PERIOD: 1962II - 1970II

$$\frac{e^f - e^s}{e^s} = \underset{(.0011)}{-.0011} + \underset{(.167)}{.776} \left[\frac{i_{US} - i_{CAN}}{1 + i_{US}}\right]$$

R^2 = .418 D.W. = 1.51 SER = .005 F = 21.6

PERIOD: 1970III - 1978IV

$$\frac{e^f - e^s}{e^s} = \underset{(.004)}{.001} + \underset{(.264)}{1.34} \left[\frac{i_{US} - i_{CAN}}{1 + i_{US}}\right]$$

R^2 = .448 D.W. = 1.934 SER = 0.26 F = 25.9

PERIOD: 1970III - 1978IV (1976I excluded)

$$\frac{e^f - e^s}{e^s} = \underset{(.0015)}{-.0006} + \underset{(.093)}{.833} \left[\frac{i_{US} - i_{CAN}}{1 + i_{US}}\right]$$

R^2 = .723 D.W. = 1.217 SER = .009 F = 80.8

(standard errors)

As the results above clearly point out, the inclusion of the observation for the first quarter of 1976 leads to considerable bias.

[14]The fourth is significant at the 90% level.

[15]Note, the value of checks cleared is an under estimate of transactions since it ignores all transactions conducted with currency.

[16]This is most likely due to the basic lack of precision in our measurement of $\hat{\sigma}$. Recall the Klein approximation of the variance of $\theta_{US} - \theta_{CAN}$, viz.

$$\mathrm{var}\,(\hat{\theta}_{US} - \theta_{CAN}) = \frac{1}{\hat{\sigma}^2}\,\mathrm{var}\,(\sigma(\hat{\theta}_{US} - \theta_{CAN}))$$

$$+ \frac{[\sigma(\hat{\theta}_{US} - \theta_{CAN})]^2}{\hat{\sigma}^4}\,\mathrm{var}\,(\hat{\sigma}) - 2\,\frac{[\sigma(\hat{\theta}_{US} - \theta_{CAN})]}{\hat{\sigma}^3}$$

$$\mathrm{COV}\,[\hat{\sigma},\, \sigma(\hat{\theta}_{US} - \theta_{CAN})]$$

[17]Note that the Quinn statistics for second order autocorrelation during this period are statistically insignificant. The missing trend effect that we noted during the fixed rate period seems to have disappeared during the floating rate period.

[18]The values of $\hat{K}$ are in line with the values of adjustment coefficients in studies of the short run demand for money and on relative asset balances in the domestic near money literature. See Hafer and Hein (1980) for a discussion of the former studies and Bisignano, (1974) for estimates of adjustment coefficients in the later case.

[19]We continue to proxy the level of transactions as the quarterly value of checks cleared in Canada.

[20]In several cases, however, t values exceeded unity.

[21]See Hatanaka (1974). The instruments we chose for estimation of the first stage were the constant term, the transactions term, the transactions term lagged one period, the relative cost term, and the relative cost term lagged one period.

[22]t values of the null hypothesis that $\hat{K}$ equals unity for COSTRATIO's 1,2, and 3 respectively are -.788, -1.45, -2.13. The 99% critical value of t with 30 degrees of freedom is 2.457.

[23]See Fisher (1970) pp. 361-364.

[24]The sum of squared residuals was obtained in each case from the Cochrane-Orcutt estimations. Since some have questioned the applicability of the Chow Test in this context (See comment in Hafer and Hein, pg. 31) we reran the test using statistics from the ordinary least squares estimation of the model. The results were even more striking, since in every case the null hypothesis of identical coefficients was rejected.

[25]In the sense that in every case the elasticities of substitution are no longer significantly different from zero and the transactions elasticities are larger.

[26]This result is directly the opposite of the conclusion reached by Miles.

APPENDIX 3

Exponent Rule for a Two-Input CES Function

APPENDIX 3

Exponent Rule for a Two-Input CES Function

Consider a CES function with two arguments X_1 and X_2 and parameters $\rho_1, \rho_2, \beta_1, \beta_2, \rho$:

$$(1) \qquad Q = \{\beta_1 X_1^{-\rho_1} + \beta_2 X_2^{-\rho_2}\}^{-1/\rho}$$

Consider the process of minimizing a linear combination of X_1 and X_2 subject to achieving a desired level of $Q(\bar{Q})$. This problem, written in the Lagrangian constrained minimization form would look as follows,

$$(2) \qquad \min \mathcal{L} = C_1 X_1 + C_2 X_2 - \lambda[\{\beta_1 X_1^{-\rho_1} + \beta_2 X_2^{-\rho_2}\}^{-1/\rho} - \bar{Q}]$$

Theorem: Given the model above, then in order for there to be a constant elasticity of substitution along an isoquant (or indifference curve) of Q, ρ_1 must equal ρ_2.

Proof: The elasticity of substitution, σ, can be determined according to the following formula:

$$\sigma = \frac{d \ln (\frac{X_i}{X_j})}{d \ln (\frac{C_j}{C_i})} \qquad i,j = 1,2$$

Take first order conditions for the minimization of 2.

$$(2a) \qquad \frac{\partial \mathcal{L}}{\partial x_1} = C_1 - \frac{\lambda}{\rho} \cdot \rho_1 Q^{-1} \beta_1 X_1^{-\rho_1 - 1} = 0$$

(2b) $\frac{\partial L}{\partial x_2} = C_2 - \frac{\lambda}{\rho} \rho_2 Q^{-1} \beta_2 X_2^{-\rho_2 - 1} = 0$

(2c) $\frac{\partial L}{\partial \lambda} = 0$

Divide 2a by 2b and take logarithms: This yields 3.

(3) $\ln \left(\frac{C_1}{C_2}\right) = \ln \left(\frac{\rho_1}{\rho_2}\right) \ln \left(\frac{\beta_1}{\beta_2}\right) - (\rho_1 + 1) \ln X_1 + (\rho_2 + 1) \ln X_2$

If we move $\ln X_1$ to the left hand side of 3 and the $\ln \left(\frac{C_1}{C_2}\right)$ to the right hand side, divide both sides by $(\rho_1 + 1)$, and subtract $\ln X_2$ from both sides of the equation, we have:

(4) $\ln X_1 - \ln X_2 = \frac{1}{1 + \rho_1} \ln \left(\frac{\rho_1 \beta_1}{\rho_2 \beta_2}\right) + \frac{1}{1 + \rho_1} \ln \left(\frac{C_2}{C_1}\right) + \frac{\rho_2 - \rho_1}{1 + \rho_1} \ln X_2$

Recognizing that $\ln X_1 - \ln X_2 = \ln \left(\frac{X_1}{X_2}\right)$ and then taking the appropriate derivative, we have our first measure of σ, σ_1, viz:

(5) $\sigma_1 = \frac{d \ln \left(\frac{X_1}{X_2}\right)}{d \ln \left(\frac{C_2}{C_1}\right)} = \frac{1}{1 + \rho_1}$

Suppose now, we revise the procedure described after 3 and move $\ln X_2$ to the left side, $\ln \left(\frac{C_1}{C_2}\right)$ to the right, divide by $1 + \rho_2$ and subtract $\ln X_1$ from both sides. This yields after rearrangement 6:

(6) $$\ln\left(\frac{X_2}{X_1}\right) = \frac{1}{\rho_2 + 1} \ln\left(\frac{\rho_2\beta_2}{\rho_1\beta_1}\right) + \frac{1}{\rho_2 + 1} \ln\left(\frac{C_1}{C_2}\right) + \frac{\rho_1 - \rho_2}{1+\rho_2}$$

$$\ln X_1$$

Here, obviously

$$\sigma_2 = \frac{d\ln\left(\frac{X_2}{X_1}\right)}{d\ln\left(\frac{C_1}{C_2}\right)} = \frac{1}{1 + \rho_2}$$

Hence, the only case where there is a unique elasticity of substitution is the case where $\sigma_1 = \sigma_2$. This can only occur when $\rho_1 = \rho_2$. Finally, note that this is true regardless of the value of the exterior exponent $^{-1/\rho}$.

CHAPTER SIX

CONCLUSIONS

In this thesis a model of an endogenous demand for foreign balances was developed. This demand is based on several factors: holding costs, transactions, and technological innovations in the use of these balances to produce money services. Expansion of the CS model in this fashion allows us to explain changes in the currency composition of the aggregate asset portfolio of an economy even when holding costs are constant.

Further, we tested our model using Canadian data in order to answer a set of questions described in Chapter Four pertaining to the CS process. In particular, we found the following results:

1. Estimates of the elasticity of substitution between Canadian demand for their own and U.S. currencies are remarkably low. As has been the case in other contexts (see Appendix 1) the sizes of the estimates of these elasticities depend upon the form of the relative cost term. They also depend upon the definition of foreign balances.

2. Even after we allow for changes in relative holding costs, additional substitution into holdings of U.S. dollars occurs as transactions levels in Canada rise. Hence, substantial improvement in the explanatory power of the model occurs-in every specification and period of analysis-when a transactions term is included.

3. We were unable to uncover any evidence of factor augmenting technological change in the use of either foreign or domestic currencies. Hence, any innovations in the banking industry of Canada during this time period must have been of the Hicks-neutral variety.

4. Evidence from regressions on sub periods of the data suggest that the potential for CS is enhanced during fixed rate periods. We reach this conclusion because regardless of the definition of the holding cost ratio, the estimated elasticity of substitution is greater and the estimated transactions elasticity is smaller in the fixed rate period than the flexible rate period. Also, Chow tests allow us to reject this hypothesis that (at least in the case where both relative holding costs and transactions are regressors) the coefficients in the two periods are identical. Further, partial adjustment model estimates during these two sub periods suggest that adjustment speeds are several times greater during fixed exchange rate periods. All of these results are in sharp contrast to the conclusion that the degree of substitutability between domestic and foreign currencies rose during flexible exchange rate periods (which is the result of Miles).

5. Estimation of the partial adjustment model suggests that long run elasticities of substitution are somewhat larger than short run elasticities. Therefore, changes in current holdings may be due in part to previous changes in holding costs. This is especially true during flexible exchange rate periods and it may explain in part our inability to obtain precise measures of this elasticity.

We conclude then that any model of CS which does not recognize the important role of transactions demand for foreign currencies is significantly biased. There are several policy conclusions that one should derive from this result. First, there is little evidence to suggest that CS under flexible exchange rates will lead to emasculation of monetary policy. Further, the more flexible are the rates, the less predictable they become. Hence, the greater the risk attached to holding open positions in foreign exchange. Second, the more expansionary are domestic programs, the greater the accumulation of foreign relative to domestic monies. Thus, the increased risk of the potential perverse effects of CS.

We note in closing that our conclusions are based on Canadian demand for U.S. dollars. Because of the special role of the U.S. dollar as a vehicle currency in the world economy, we may be over emphasizing the importance of the transactions motive in foreign currency demand. Tests of our model on the private demand for other currencies would allow us to examine this question more closely.

More work needs to be conducted, as well, on an expanded model of asset choice. This perhaps could involve the nested CES model of Barth, et. al., and would include domestic and foreign interest bearing assets. Another significant contribution would be to develop empirical definitions of transactions costs and political risk. Finally, we point out that since future exchange rates are not known with certainty, a risk factor should be incorporated in the cost term reflecting this risk. These expansions are left to future research.

APPENDIX 4

The Numerical Data

APPENDIX 4

The Numerical Data

(For the explanations and sources of the variables, see Chapter Five)

Year/Quarter		CM2	CDCL	UCDL	SWAPS	GNE	CC
1962	II	14617	1018	211	516	73.2	79.4
	III	14567	828	168	427	73.5	78.7
	IV	15014	776	195	414	73.9	92.3
1963	I	15117	791	162	403	74.2	81.3
	II	15485	818	170	390	74.7	93.0
	III	15894	881	171	551	74.8	86.1
	IV	15848	796	195	473	75.4	99.7
1964	I	16018	746	160	410	75.8	93.5
	II	16390	915	185	453	76.3	108.0
	III	17016	1100	173	611	77.0	103.5
	IV	17209	1322	217	735	77.4	122.0
1965	I	17795	1161	198	609	78.0	114.9
	II	18533	1026	191	411	78.6	124.0
	III	19014	1155	187	440	79.7	120.5
	IV	19111	1211	225	543	80.0	131.4
1966	I	19302	1394	211	745	81.4	126.3
	II	19773	1454	240	735	82.3	134.3
	III	20220	1664	216	885	83.3	132.3
	IV	20413	1623	234	797	83.5	144.8
1967	I	21115	1432	249	648	84.9	143.2
	II	21947	1396	259	548	85.8	150.8
	III	23375	1531	250	626	86.0	136.7
	IV	23577	1949	284	894	86.7	154.2
1968	I	23530	1893	268	842	87.8	147.4
	II	25152	1895	261	450	88.3	159.1
	III	26237	1993	266	715	89.0	157.8
	IV	26751	2036	281	845	89.5	172.2
1969	I	27481	2196	272	929	90.9	168.9
	II	27528	2993	287	1409	92.4	186.8
	III	27751	3366	369	1650	93.1	182.3
	IV	27906	3260	440	1592	93.8	197.2
1970	I	27665	3279	338	1702	95.6	183.5
	II	29056	2801	357	1344	96.3	203.3
	III	30135	3244	336	1653	97.4	206.9
	IV	30795	3184	388	1771	98.4	224.0
1971	I	32187	2507	320	1351	98.3	209.5
	II	33775	2269	370	1091	99.7	224.1
	III	34970	2025	284	953	100.2	224.7
	IV	35818	1688	260	758	101.8	261.0
1972	I	37519	1471	249	495	103.0	244.6
	II	39476	1268	284	243	103.9	263.6
	III	40775	1161	295	171	105.4	266.2
	IV	41046	1573	316	270	107.3	298.8
1973	I	42257	1634	249	314	109.3	303.1
	II	44235	2052	278	491	111.8	340.1
	III	45894	2730	278	774	115.7	348.6
	IV	48842	2984	582	880	120.0	378.0
1974	I	50835	3848	501	1275	124.7	365.1
	II	52850	5822	487	2635	130.4	424.4
	III	55328	5957	391	2865	135.2	428.8
	IV	56521	4726	353	1787	138.0	479.7
1975	I	59473	4013	312	1143	140.9	488.7
	II	61782	4299	325	1144	143.8	532.8
	III	65173	3971	339	988	148.3	535.0
	IV	66274	4403	427	848	152.1	581.9
1976	I	68841	5792	482	1129	155.1	580.6
	II	73579	5785	355	948	159.7	612.8
	III	76024	6715	397	1179	161.7	619.5
	IV	78169	6183	472	1281	165.3	656.5
1977	I	80405	6695	506	1287	167.2	647.0
	II	85145	6775	590	1384	170.8	684.6
	III	87713	7781	593	1979	173.4	681.4
	IV	88636	7393	559	1545	175.3	750.0
1978	I	89567	8671	538	1747	178.0	697.4
	II	93761	9085	641	1655	181.8	783.8
	III	97794	10564	635	1831	184.1	773.0
	IV	101155	11129	710	1538	186.6	883.9

APPENDIX (cont.)

Year/Quarter		USTB	CTB	SWAPRATE	RLONG	CDSR	CDFR
1962	II	2.79	5.45	5.50	4.91	1.0816	1.0866
	III	2.75	4.99	5.31	5.40	1.0766	1.0817
	IV	2.89	3.91	4.00	5.11	1.0778	1.0789
1963	I	2.92	3.62	4.10	5.08	1.0781	1.0798
	II	2.98	3.24	3.50	4.93	1.0761	1.0780
	III	3.38	3.56	3.88	5.15	1.0778	1.0780
	IV	3.52	3.78	3.94	5.11	1.0809	1.0809
1964	I	3.55	3.88	4.21	5.20	1.0806	1.0805
	II	3.48	3.59	3.77	5.22	1.0812	1.0804
	III	3.56	3.73	4.30	5.22	1.0750	1.0756
	IV	3.87	3.82	4.59	5.13	1.0741	1.0745
1965	I	3.92	3.62	4.02	5.04	1.0794	1.0785
	II	3.78	3.93	4.26	5.10	1.0834	1.0826
	III	3.98	4.23	4.98	5.30	1.0762	1.0778
	IV	4.46	4.54	5.75	5.46	1.0750	1.0768
1966	I	4.56	5.06	5.30	5.59	1.0772	1.0773
	II	4.44	5.00	5.62	5.68	1.0756	1.0758
	III	5.50	5.01	5.81	5.84	1.0778	1.0769
	IV	4.75	4.96	6.26	5.86	1.0838	1.0834
1967	I	4.15	4.13	5.06	5.59	1.0825	1.0820
	II	3.46	4.28	5.63	5.70	1.0797	1.0803
	III	4.63	4.76	6.49	6.03	1.0741	1.0773
	IV	5.12	5.95	6.41	6.52	1.0809	1.0824
1968	I	5.33	6.98	6.83	6.83	1.0825	1.0867
	II	5.38	6.56	7.49	6.74	1.0759	1.0784
	III	5.29	5.66	6.54	6.56	1.0728	1.0749
	IV	6.39	6.24	6.54	7.16	1.0728	1.0738
1969	I	6.12	6.58	7.21	7.35	1.0762	1.0744
	II	6.73	7.13	7.89	7.52	1.0809	1.0770
	III	7.45	7.77	8.34	7.62	1.0791	1.0775
	IV	8.38	7.81	9.34	8.12	1.0731	1.0732
1970	I	6.45	7.00	8.24	8.16	1.0728	1.0727
	II	6.83	5.94	7.67	7.97	1.0341	1.0311
	III	5.98	5.39	7.30	7.75	1.0191	1.0168
	IV	4.96	4.44	6.09	7.40	1.0103	1.0107
1971	I	3.60	3.16	4.13	6.76	1.0081	1.0077
	II	5.22	3.37	4.46	7.22	1.0234	1.0202
	III	4.80	4.06	5.22	7.20	1.0091	1.0072
	IV	3.82	3.21	4.69	6.61	1.0022	1.0006
1972	I	3.94	3.57	6.07	6.96	.9969	.9988
	II	4.24	3.50	5.66	7.35	.9853	.9856
	III	4.76	3.62	5.32	7.46	.9834	.9829
	IV	5.25	3.65	5.24	7.15	.9956	.9945
1973	I	6.44	4.46	5.19	7.22	.9990	.9928
	II	7.47	5.48	6.96	7.62	.9984	.9944
	III	7.57	6.50	8.96	7.76	1.0058	1.0022
	IV	7.65	6.35	9.68	7.65	.9958	.9950
1974	I	8.59	6.51	9.07	7.89	.9724	.9706
	II	8.11	8.75	11.52	9.06	.9722	.9683
	III	6.58	8.94	11.10	9.71	.9858	.9848
	IV	7.34	7.12	9.43	8.95	.9906	.9900
1975	I	5.70	6.33	6.70	8.31	1.0018	1.0011
	II	5.83	6.99	7.37	8.88	1.0298	1.0313
	III	7.34	8.41	9.36	9.48	1.0252	1.0286
	IV	5.34	8.64	9.45	9.47	1.0160	1.0247
1976	I	5.06	9.07	10.53	9.32	.9844	1.0287
	II	5.52	8.98	9.61	9.34	.9690	.9782
	III	5.21	9.11	9.52	9.26	.9714	.9811
	IV	4.40	8.14	7.80	8.79	1.0088	1.0168
1977	I	4.73	7.54	7.92	8.66	1.0539	1.0611
	II	5.10	7.07	7.26	8.78	1.0593	1.0629
	III	6.16	7.10	7.33	8.63	1.0746	1.0754
	IV	6.34	7.17	7.13	8.74	1.0940	1.0938
1978	I	6.50	7.73	8.11	9.13	1.1339	1.1359
	II	7.19	8.26	8.52	9.20	1.1226	1.1236
	III	8.39	9.17	9.44	9.15	1.1844	1.1846
	IV	9.70	10.46	10.72	9.73	1.1858	1.1831

APPENDIX (cont.')

Year/Quarter		CRATIO1	CRATIO2	CRATIO3	MRATIO	MILESY	RER1
1962	II	.16792	.03710	.97477	20.01	11.72	1.8747
	III	.16609	.11421	.97866	25.55	14.73	1.9211
	IV	.63327	.48558	.99018	26.24	15.22	.41390
1963	I	.62997	.61874	.99324	26.86	15.65	.63950
	II	.93136	.84182	.99748	25.33	15.46	-.03761
	III	.92829	.90053	.99826	30.90	14.91	.07525
	IV	.93121	.86477	.99749	29.69	15.74	0.
1964	I	.92462	.83281	.99682	31.47	17.43	-.03753
	II	1.0529	.93974	.99893	24.75	14.69	-.30007
	III	.89373	.91041	.99836	25.20	13.23	.22635
	IV	.97355	1.0256	1.0004	20.98	11.06	.15103
1965	I	1.1762	1.1626	1.0029	23.23	12.94	-.33815
	II	1.0380	.92505	.99855	22.54	15.03	-.29946
	III	.81768	.92875	.99855	20.75	14.01	.60294
	IV	.83280	.96551	.99923	21.00	13.15	.67907
1966	I	.69374	.80663	.99524	22.02	11.90	.03764
	II	.87291	.78076	.99466	20.23	11.54	.07541
	III	1.1654	1.1905	1.0046	19.98	10.66	-.33865
	IV	.98783	.91724	.99799	18.90	10.87	-.14967
1967	I	1.0502	1.0094	1.0001	20.04	12.40	-.15732
	II	.75575	.62323	.99213	19.46	13.09	.22537
	III	.71385	.94653	.99875	19.91	12.98	1.2082
	IV	.76591	.72780	.99216	17.31	10.45	.56280
1968	I	.53817	.53918	.98457	17.54	10.77	1.5735
	II	.67646	.64942	.98892	14.57	11.55	.94236
	III	.79436	.87254	.99649	16.78	11.51	.79387
	IV	.96345	1.0466	1.0014	17.92	11.44	.37803
1969	I	1.0331	.86421	.99563	17.61	11.04	-.67831
	II	1.1491	.89133	.99626	14.53	8.333	-1.4632
	III	1.0362	.92048	.99703	13.12	7.372	-.60132
	IV	1.0681	1.1403	1.0052	13.03	7.477	.03779
1970	I	.92682	.84761	.99436	14.26	7.59	-.03780
	II	1.3479	1.2900	1.0034	15.91	9.165	-1.1765
	III	1.2792	1.2127	1.0056	15.58	8.402	-.91529
	IV	1.0809	1.2287	1.0049	17.06	8.611	.16056
1971	I	1.1901	1.2736	1.0042	21.76	11.37	-.16091
	II	1.9252	2.0706	1.0179	21.69	12.75	-1.2681
	III	1.3703	1.3561	1.0071	25.74	15.12	-.76360
	IV	1.3917	1.3730	1.0059	30.08	18.38	-.64746
1972	I	.88712	1.2033	1.0035	30.64	21.82	.77295
	II	1.1761	1.4142	1.0071	30.25	25.50	.12348
	III	1.3718	1.6155	1.0120	31.35	28.09	-.20620
	IV	1.5611	1.8548	1.0154	25.37	21.74	-.44808
1973	I	2.0082	1.8610	1.0139	26.93	22.44	-2.5169
	II	1.6596	1.7010	1.0188	24.05	18.98	-1.6248
	III	1.3879	1.3176	1.0100	20.52	15.24	-1.4515
	IV	1.2560	1.3949	1.0122	18.20	13.70	-.32581
1974	I	1.4348	1.6137	1.0195	16.61	11.72	-.75072
	II	1.1127	.85920	.99411	14.43	8.394	-1.6266
	III	.78203	.48833	.97833	15.91	8.723	-.41139
	IV	1.0654	1.0596	1.0020	17.18	11.13	-.24564
1975	I	.94524	.80631	.99407	18.68	13.74	-.28337
	II	.74953	.67723	.98915	17.70	13.33	.59073
	III	.71284	.75424	.99013	19.56	15.09	1.3450
	IV	.21611	.25547	.96962	16.61	13.70	3.4727
1976	I	-1.4543	.13706	.96323	13.39	10.98	18.250
	II	.18591	.24955	.96825	14.20	12.00	3.3504
	III	.12736	.16499	.96425	12.33	10.70	4.0497
	IV	.14543	.10044	.96541	14.53	11.73	3.2161
1977	I	.25985	.27147	.97387	13.53	11.12	2.7706
	II	.52641	.45623	.98160	14.15	11.50	1.3782
	III	.82508	.74239	.99122	13.62	10.41	.30192
	IV	.89458	.77538	.99225	13.72	11.07	-.07414
1978	I	.74834	.69147	.98858	11.88	9.650	.71532
	II	.82672	.74960	.99011	11.50	9.562	.36126
	III	.90747	.83646	.99285	10.31	8.642	.06846
	IV	1.0156	.86110	.99312	9.69	8.450	-.92342

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