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RISK REDUCTION CAPABILITIES OF HEDGING TECHNIQUES IN THE FINANCIAL FUTURES MARKET: A COMPARISON TEST

By

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ABSTRACT

RISK REDUCTION CAPABILITIES OF HEDGING TECHNIQUES IN THE FINANCIAL FUTURES MARKET: A COMPARISON TEST

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Hedging of interest rate risk has been an important goal of financial managers and investors who must leave their funds at risk for a specific period of time. A number of approaches have been attempted to determine the best method of reducing interest rate risk. The advent of active futures markets for government and quasi government securities has opened another avenue for hedging activities that attempt to immunize the investment portfolio against interest rate changes.

Prior to the advent of futures markets in securities, immunization was accomplished by changing the composition of the portfolio, altering both its overall risk and its expected rate of return. Hedging by taking positions in contracts for future delivery of government securities allows the investor to change the risk of the portfolio without changing the expected return.

Research in the area of hedging effectiveness has developed along two lines.

1. Take positions in futures based on the correlation between spot and futures prices of the same security. The correlation approach has also been used for cross-hedging.
2. Take futures positions based on the relative durations of the investment security and the hedging vehicle.

Both of these approaches require restrictive assumptions about the shape and movements in the yield curve.

This study develops hedge positions based on investor expectations of future yields. No assumptions are made about the dynamics or future shape of yield curves. Since it deals with changes in asset prices as a result of changes in forward interest rates, this approach should be equally useful in direct and cross-hedging. The tests were done as cross hedges between government bonds and Treasury Bill futures contracts.

When risk was measured by variance of wealth change, there was no significant difference between the expectations hedge method and the duration method or the correlation adjusted method. When risk was measured as the possibility of earning below-target returns ($R < 0$) the expectations hedge provided better protection against interest rate changes.

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CHAPTER I

INTRODUCTION

Investors who must put funds at risk for a known period of time are faced with a number of alternatives that would allow them to mitigate that risk. The investor who is averse to risk is expected to take such action that will reduce the risk of the investment portfolio until the expected return of the investment portfolio is at the appropriate level given the level of risk accepted. One component of risk that is faced by the investor in such a situation is interest rate risk. This is the risk that the market value of the investment will have changed over the investment period because of changes in the interest rate structure and levels that will have taken place over the planning horizon or time the funds are at risk. The other component of risk is reinvestment risk. This is the risk that an investor faces when investing in an asset whose maturity is shorter than the planning horizon. The principal will be reinvested at a rate that is unknown at the time of the original commitment.

This interest rate risk can be mitigated by matching the duration of the investment to the planning horizon. An example of this might be a corporate treasurer investing corporate funds that will be needed for operations in three months. If these funds are invested in 91 day Treasury Bills the expected value of each bill is $1,000,000 and it is default free. The expected return is a function of current interest rates. There is no interest rate risk because the receipt of $1,000,000 per bill is assured at the end of the planning horizon which corresponds to the maturity of the T-Bill.

Contrast this with the investment behavior of the corporate treasurer in the same situation of having funds available for a short time who invests those funds in a Treasury Bond with 24 months to maturity. This investment might be undertaken to take advantage of higher expected returns offered by the longer maturity investment. The interest rate risk in this position derives from the stochastic nature of the interest rate structure in the future. The value of the T-Bonds three months hence is a function of the interest rates that will prevail at that future time for the period from that point to the date of maturity of the bonds.

The investor does not have to accept that risk. The longer maturity bonds could be replaced by securities whose durations match the planning horizon. This would defeat the goal of higher expected returns. Alternatively, the investor could take a position in the futures contracts of a financial instrument such that changes in the value of the futures position would offset changes in the value of the investment position.[1] Both values are affected by changes in forward rates between the inception of the position and offsetting hedge and the end of the planning horizon. This activity has the benefit of changing the duration of the investment position without changing the expected return on the assets since the present value of the futures contract is zero. So, hedging allows the investor to reduce the risk of the investment position without reducing the expected return. This is optimizing behavior for the risk-averse investor.

In a perfect market hedging would not be necessary. The valuation of an asset would appropriately reflect the expected cash flows and the risk of those cash flows for each asset in the market. Investors would be able to provide their own hedging combinations so it would not be incumbent upon the firm to do so. Even in the situation where the firm could use futures contracts to immunize its investment portfolio against interest rate risk without changing the expected return, there would be no incentive for the firm to do so. Where there are imperfections in the market, such as indivisibility or differential taxes or differential access, there would be a need for hedging. Where there are inefficiencies such as differential information availability hedging might also be beneficial.

Hedging would be a trivial activity if all assets

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[1] Bankers attempt to immunize their assets and liabilities by some similar form of matching.

experienced the same price changes as a result of interest rate changes. An investor would take a position in the futures contracts calling for delivery of securities of the same face value as the value of the original investment. Because of differences in duration, coupon and default risk there is not a one-to-one relationship of those price changes. Even with default risk differences abstracted there is still considerable difference in the price effect of interest rate changes. Hedges that are different in this way are referred to as cross-hedges.

The determination of the number of futures contracts to acquire in order to hedge a unit of investment is the hedge ratio. Hedge ratios calculated using the durations of the investment and the asset underlying the hedging vehicle (futures contract) have been shown to be superior to the naive or one-to-one hedge in reducing wealth changes over the planning horizon. The present study improves upon the hedging activity suggested by the duration-based hedges by taking into consideration that yield curves will be changed by any number of additive and multiplicative shocks that will change forward rates independently of one another. The duration-based hedges are limited to allowing a single additive shock to the entire yield curve.

The tests of effectiveness of the hedges are tests of the variance and semi - variance of the wealth changes resulting from each of the hedging behaviors tested.

CHAPTER II

REVIEW OF LITERATURE

This study of hedging effectiveness of alternative hedging strategies is based on developments in three areas of research:

1. Duration and immunization;
2. Hedging methods and strategies; and
3. Term structure of interest rates.

Duration and Immunization

The concept of duration is central to hedging positions suggested by those whose hedging activities and hedge ratio calculations will be compared with the approach suggested in this study. Duration is a measure of the sensitivity of asset prices with respect to interest rate changes. In that way it is a measure similar to that measure referred to as maturity, however duration is defined as dimensionals. Both are measured in units of time.

The term duration was coined by Macaulay (1938). The measure takes into consideration that the receipt of coupon payments or other periodic distributions and their subsequent reinvestment, reduce the average amount of the investment. Hicks (1946) developed the same measure independently. He referred to it as the "average period" of investment. Duration calculations use averages of expected flows weighted by the length of time until receipt of those flows. Maturity measures do not consider the timing of expected intermediate cash flows. Rather, all expected flows are equally weighted so the maturity becomes the time to the last expected cash flow with no consideration of the effects of the intermediate expected flows.

Duration, as expressed by Macaulay, is

$$D = \frac{\sum_{t=1}^{n} \frac{CF_t(t)}{(1+R)^t}}{\sum_{t=1}^{n} \frac{CF_t}{(1+R)^t}}$$

where CF = cash flow at time t
n = maturity
R = yield to maturity.

The duration of a pure discount, zero coupon, bond is equal to its maturity because there are no intermediate cash flows. When the duration of a coupon bond is calculated, that duration will be shorter than the maturity of the bond. Macaulay has shown that for long maturity bonds selling below par, the duration decreases with maturity in some cases. He also shows that all durations reach a finite limit even though the maturities may be limited.

Investors with an identifiable planning horizon have an interest in assuring an expected return over that planning horizon. Fisher and Weil (1971) define an immunized portfolio as one whose

> ...value at the end of the holding period, regardless of the course of interest rates during the holding period, must be at least as large as it would have been had the interest rate function been consistent throughout the holding period. (pg. 415)

Redington (1952) provided an early statement of this idea. Fisher and Weil relax some of Redington's restrictive assumptions about interest rates to make their model more general.

One obvious way that an investor can insure an expected return over a planning horizon would be for the investor to purchase pure discount instruments that mature at the end of the planning horizon. There is no problem with reinvestment of coupons because there are no coupons. The expected value of the bonds at maturity is the amount the borrower is expected to repay. In the case of the U.S. government as borrower, that value is the face value of the bonds. A policy of purchasing zero coupon bonds to match the planning horizon is difficult to implement because of a dearth of such zero coupon securities over a range of maturities.[2]

An investor who chooses a coupon bond is subject to two different risks because of interest rate fluctuations.

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[2]Lately, securities brokers have been marketing U.S. government bonds with the right to receive coupon payments stripped from the security. The right to receive coupon payments is marketed separately. If this new type of security enjoys wide market acceptance in the future, investors may be able to make more use of the direct matching policy.

First is the reinvestment risk. Yield-to-maturity calculations assume reinvestment of coupon payments such that the average return will be the yield-to-maturity. Interest rates change so the risk of reinvestment is always present. The second risk is a price risk. Because of interest rate changes, the price received for a bond whose maturity is beyond some planning horizon will not be the expected price at maturity. Interest rate changes affect the value of coupon bonds in opposite directions through the reinvestment and price effects. Immunization occurs when the effects are equalized, leaving the wealth of the investor unchanged as a result of the interest rate changes experienced.

Hopewell and Kaufman (1973) provide an explanation of the observation that bonds selling at a discount behave differently from what would be expected as a result of the belief that the market prices of longer term bonds exhibit greater changes with a change in interest rates than do shorter term bonds. They show that the relationship is better described using duration as a measure of price sensitivity than it is using maturity. They point out that the duration of a discount bond increases with maturity but then reaches a maximum and actually begins to decrease. This confirmed the earlier work of Macaulay.

Fisher and Weil (1971) provide a review of the development of the relationship between duration and immunization. Redington (1952) was the first to use the term

<u>immunization</u> to refer to protection of the value of a portfolio from the ravages of interest rate changes. Indeed, Redington's <u>mean term</u> is duration.

Hicks (1946) showed that the change in present value of one investment relative to another as the interest rate changes is a function of the duration of the payment streams of the investments. Samuelson (1945) extended this argument by looking at an asset and a liability.[3] He noted that the effect on the net investment value will be different depending on the duration of the two positions. If the duration of the liability is greater than that of the asset, an increase in interest rates will result in an increase in the present value of the position. The effect on the longer duration liability will be greater than that on the shorter duration asset. The implication of this for planning is that a position can be fully hedged (immunized) by matching the durations of the assets and the liabilities. An investor can take a speculative position by adjusting durations according to expectations of the direction of interest rate changes.

Redington's immunization is the banker's or asset-liability approach to immunization. The banker is seen as managing the size and duration of both the assets and the liabilities of the institution. The banker intentionally can leave a gap between durations of assets and

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[3]"Positive flow" and "negative flow" in the Hicks context.

liabilities, the size and sign of which are a function of the banker's expectations of interest rate fluctuations. In order to immunize the bank's assets, their duration should equal the duration of the liabilities. This is accomplished by adjusting the asset and liability holdings.

An approach to immunization that is similar to asset-liability matching is the matching of asset duration to a planning horizon. When asset durations do not match the investor's planning horizon, that investor is subject to reinvestment risk and price risk as a result of the effect of interest rate changes until the planning horizon is reached. The hedging activity considered in the present study is a method of adjusting the duration of assets so is especially useful in the planning horizon matching goal. This planning horizon goal strategy is assumed in this study.

Fisher and Weil (1971) test an immunization strategy based on duration that implies matching duration to planning horizon and adjusting the duration to the impending planning horizon through appropriate reinvestment of the coupon payments. Earlier immunization strategies assumed that there could be only parallel shifts in a flat yield curve for each investment. A yield curve demonstrates a parallel shift when a single shock affects all yields in the same magnitude of change. Thus, if the one month yield increases by 100 basis points, so do each of the longer yields increase by 100 basis points. The shape of the yield curve does not change, rather the whole term structure is displaced. This is Redington's approach. Fisher and Weil show that a modified immunization strategy can be effective when there are multiple shocks to the term structure. However, they require periodic rebalancing of the portfolio over the planning horizon to achieve maximum benefit from duration-based immunization.

Grove (1966) points out that an immunization approach is valid for small changes in the interest rate. If interest rate changes are large, immunization based on duration is less effective. He also put duration in a Pratt-Arrow risk analysis context (1974) and argues that for utility functions increasing in wealth with decreasing absolute risk aversion, the investor will maximize utility by attempting a perfect hedge only if no change is expected in interest rate variability. Otherwise the investor will take some risk by exposing either a net asset or net liability position.

Fisher and Weil also show in their proof of the immunization theorem that immunizing with a discount bond whose maturity is equal to the planning horizon is inferior to immunizing with coupon bonds of appropriate duration because coupon bonds provide an expected return that is no less than that required but may be more. Bierwag and Kaufman (1977) extend the work of Fisher and Weil to show that the measure of duration will be more complex when the random effects on the yield curve are

multiplicative rather than additive.

Bierwag (1977) provides proofs of these theorems which also point up the differences for immunizing choices depending upon the effect of the shocks that change interest rates. He shows that an immunization strategy in the case of combined additive and multiplicative effects requires a more complex approach than a linear combination of assets for immunization.

Kaufman (1978) shows duration as an important component of bond risk. Risk measures that have been developed for the equity market are not appropriate for measuring bond risk. Such measures would only be appropriate when there is a completely immunized portfolio. An expected duration can be calculated for equity securities. The implication of this is that risk of bond portfolios cannot be thought of in the same context as equity risk. Duration comes into play as risk and expected return must be considered in terms of both the terminal price of the bond and of the income from reinvestment of the coupons. Appropriate use of duration to immunize the portfolio will offset the risks.

Bierwag and Khang (1979) show that immunization provides for maximization of the minimum expected return. They see immunization as an optimization technique. They also show that an immunized portfolio is a zero-beta portfolio since it is riskless with respect to interest rate changes. Included in their paper is a capsule history of the immunization-duration relationships beginning with the contributions of Macaulay and continuing through portfolio approaches.

Bierwag (1979) develops an immunization policy that takes into account successive changes in the yield curve rather than just one shock. This is an extension of his earlier work. In it, immunization is achieved by adjusting the portfolio to reflect changes in the length of the planning horizon and changes in the yield curve which derive from changes in the forward rates from the new decision point forward. This is a multi-period approach. It extends the single-period analysis done for prior tests. However, it requires that portfolio decisions be made periodically rather than allowing immunization to be achieved with a single decision. The single decision could be expected to be the goal of a multi-period approach to the investment decision.

Cox, Ingersoll, and Ross (1979) have developed a measure of duration from their continuous time model of the term structure of interest rates. Their method allows the yield curve to attain any shape and sustain more than one shock or change. The immunization procedures developed with this model could be more generally applicable for risk reduction because of the less restricted nature of the interest rate generating process.

Weil (1973) presents an early history of duration. Bierwag, Kaufman, and Khang (1978) have written a complete description of the uses of duration. Ingersoll, Skelton, and Weil (1978) have done a survey of the properties and uses of duration for immunization of portfolios. These papers survey the field of duration. Bierwag, Kaufman, and Khang in their criticism of the uses of duration, say that duration is not a measure of risk because the way risk is usually defined implies the use of a measure of utility. Duration measures do not, in themselves, use utility measures. They also note the limitations imposed by the restrictive assumption of flat yield curves and the single additive shock. They discuss the relationship between duration and beta as developed by Kaufman (1978). Beta relates the duration of the portfolio and the market portfolio and the period over which beta is measured. If that period is equal to the planning horizon and the portfolio is default-free, the portfolio beta will be equal to zero.

In light of the restrictive nature of the assumptions and the conflicting conclusions of previous studies, a more generally useful approach to immunization is needed. The shape of the yield curve will be irrelevant in this approach. No assumptions need be made about the allowable number or effects of shifts in the yield curve. Other, earlier immunization strategies based on duration were not expected to be completely effective because of the duration assumptions that were necessarily of a simplifying nature.

Hedging Methods and Strategies

In order to immunize the bond portfolio, it is necessary to match its duration to the planning horizon. This would apparently require that a portfolio of coupon bonds, or for that matter, one coupon bond, be chosen such that the combination of coupon payments and maturity provide a duration that matches the planning horizon. In the case where the investments are already held, some adjustment would be necessary that could alter the expected return of the investment because bonds would have to be replaced by others with durations that would allow for the matching. The investor is forced to give up his chosen level of expected return in order to reduce the risk of the investment.

The existence of an organized futures market for U.S. Treasury securities, GNMA securities, and CDs provides the investor with another alternative. For small transactions cost[4] the investor can take a position in futures contracts that will hedge the position and provide the interest rate risk protection that comes with immunization. Portfolio immunization can be achieved without material effect upon its expected return.

Hedging strategies have been proposed by a number of authors in the past few years. The strategies are distinguished by their underlying assumptions about the

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[4]Round-turn (buy and sell) commissions are less than $25 per contract.

shape and movement of yield curves and whether they take a portfolio or a duration approach.

The commodity futures markets have long provided hedging opportunities for participants in cash markets for commodities. From the commodity trading standpoint, the holder of a position in the cash market who wants to hedge that position takes a position in the futures market that is opposite to and equal in volume to the cash position. This is the naive hedge. As long as the futures price and the spot or cash price move in the same direction and by the same amount, the position is effectively hedged. The spot price and the futures price are not expected to be equal until the delivery date. The difference between spot and futures price is called basis. That is,

$$B = FP - P.$$

If a market participant has a position in the cash market, the gain or loss realized is the price change of the commodity from the time the position is initiated until the time the position is liquidated. Symbolically,

$$G = A(P_1 - P_0)$$

where A is the size of the position and
P is price.

If the position is hedged in the futures market, the gain or loss on the overall position, cash and futures, is the net of the gains and losses in the two offsetting positions.

Letting GH represent the net gain,

$$GH = A(P_1 - P_0) - A(FP_1 - FP_0).$$

A perfect hedge occurs when the change in basis is zero so

$$(P_1 - FP_1) - (P_0 - FP_0) = 0.$$

In the commodity markets both the change in basis, known as basis risk, and the basis itself are considered small. Where this is the case, most of the risk associated with the position can be hedged naively.

Ederington (1979) provides an analysis of three hedging theories.

1. Traditional Theory. Hedgers take futures market positions equal and opposite to their cash positions. This begs the question of basis change. The traditional theory holds that basis and basis change are small because of the possibility of delivery on the futures contract. Traditional theory provides an approximation of an optimal hedge ratio. As noted above, the hedger attempts to reduce the variance of the position through hedging.

2. Working's Hedging Hypothesis. Working (1953, 1962), considers investors to be speculators

who will hedge only if they expect the basis change to be unfavorable to their position. Thus, he deals directly with the problem of basis change. Investors are considered to be, at best, risk-neutral profit maximizers. Working argues that futures trading is not done primarily to reduce risk but in expectation of a favorable basis change. He calls this selection or anticipatory hedging (1962).

3. Hedging in a Portfolio Context. Johnson (1960) and Stein (1961) see futures transactions as another investment in the portfolio. When analyzed in this manner, futures contracts are included in the portfolio to the extent that the expected change in the value of the portfolio is greater than zero with the addition of the futures contracts. Ederington (1979) shows that for risk minimization the proportion of the portfolio to be hedged is a function of the covariance of the spot and futures prices and the variance of the futures price.

Hill and Schneeweis (1982) use the portfolio risk and expected return analysis to develop a hedging strategy. Their hedge is a cross-hedge. The investment portfolio is represented by a corporate bond index. The hedging vehicles are GNMA futures contracts and Treasury Bond futures contracts. Their hedge ratio is Johnson's (1960) proportion of the portfolio to be hedged. Letting HR represent the hedge ratio, number of futures contracts bought or sold per unit of investment.

$$HR = cov\ (Cs,Cf)/Var(Cf)$$

where Cs and Cf are price changes over the planning horizon for spot and future prices. The measure of effectiveness they use is the reduction in portfolio variance. They also show that risk can be reduced more effectively if there is a higher correlation between futures and spot prices. They found that this hedge strategy gave greater risk reduction than did a naive hedging strategy.

D'Antonio and Howard (1982) also used a portfolio approach to test the effectiveness of financial futures for hedging. Their analysis fits futures positions into the classical CAPM framework. Their optimal hedge ratio is a function of a risk and expected return relationship between the expected price change for futures contracts and the risk premium for the risky security. Their measure of hedging effectiveness is the change in expected return of the hedged portfolio over the expected return of the unhedged. They used a direct hedge. The assumed investment was in T-Bills and T-bill futures were the hedging vehicles. D'Antonio and Howard found only moderate improvement in the portfolio risk and return relationship by

using futures hedges. This is to be expected because the portfolio concepts they use assume that the financial markets are in equilibrium. The T-Bill and T-Bill futures markets are apparently efficient and themselves in equilibrium.

Their work is an extension of the analysis provided by Fischer Black (1976). Black, working with commodity futures contracts in a CAPM model, finds that the change in futures price is related to the beta of the futures contract. But the investor makes no investment in a futures contract so the beta cannot be measured in terms of a rate of return, rather it is measured in dollars. D'Antonio and Howard point out that their model is derived from Black's CAPM. To do this they must assume that their investment portfolio (T-Bills) is the market portfolio. They also need to be able to place Black's model in terms of returns so they compare their model to Black's by dividing through by the futures price. The difficulty with this comparison is that Black begins by setting the initial price at zero because the value of a futures contract at its inception is zero and the contract is revalued to zero each trading day.[5]

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[5]The process is called mark-to-market. At the end of each day's trading the delivery (settlement) price of every contract is adjusted to the market closing price. Holders of contracts make adjustments with the clearing corporation, usually through their brokers, in cash. Thus, any net change in the settlement price is offset by a net change in cash position.

D'Antonio and Howard's finding of little or no effectiveness of their hedge is not surprising. For the hedge to be effective, there would have to be significant covariance. Dusak (1973) in a study of commodity futures contracts for wheat, corn, and soybeans found a zero beta, that is, covariances between futures price changes and the return on the Standard and Poor's Stock Index were found to be close to zero. While portfolio theory hedges and their effectiveness is an appropriate area for further study, the present study does not consider hedges based on portfolio concepts.

Two duration based hedge ratios will be included in this study. Kolb and Chiang (1981) have developed a duration based hedge ratio. They recognize that duration is affected by the asset's interest rate sensitivity. Their hedge ratio is expressed as,

$$N = -\bar{R}_j \; P_i \; D_i / \bar{R}_i \; FP_j \; D_j$$

where,

R_j = 1 + the yield to maturity expected for the asset underlying the futures contract

R_i = 1 + the yield to maturity expected for the asset being hedged

P_i = Price at the termination of the hedge of the asset being hedged

FP_j = Price at which title to the underlying asset will pass when the futures contract matures

D_i = Expected duration at termination of the hedge of the asset being hedged

D_j = Expected duration at termination of the hedge of the underlying asset.

In the derivation of this hedge they find that the hedge ratio will be a function of an elasticity of the expected rate of return on the asset being hedged with respect to the expected rate of return on the asset underlying the futures contract. They assume this elasticity is unity for the purpose of developing their hedge ratio. They assume a flat yield curve for each instrument and parallel shifts in the yield curve to test their hedge ratio. In a later paper, Kolb and Chiang (1982) develop hedge ratios while relaxing the assumption of a flat yield curve for each instrument. The parallel shift assumption is still in force. They also develop a hedge ratio for risky assets.

Kolb (1982) combines the duration approach and the portfolio approach by regressing asset yields on the yields on the assets underlying the futures contract. He then includes that regression coefficient in the duration based hedge ratio replacing the assumed relation of unit elasticity:

$$N = (-\bar{R}_j\ P_i\ D_i/\bar{R}_i\ FP_j\ D_j)\hat{r}_{ij}$$

where r = the regression coefficient estimated. That regression coefficient is a measure of the elasticity dR_i/dR_j. For the present study, each of the yields over the investment periods were regressed on the three month yield to determine the appropriate regression coefficients for incorporation in the hedge ratio calculation.

The Term Structure of Interest Rates

As has already been noted, hedging activities in the eyes of many analysts are determined by the relationship among yields over a number of relevant maturities and by how those relationships are expected to change. Hedging activities are an attempt to protect the investor from the effects of those changes. There have been a number of explanations of the term structure that have had some acceptance. These are attempts to explain how interest rates on securities of one maturity relate to rates for securities of other maturities.

The expectations theory was developed by Irving Fisher (1930). In simplest form it states that any long-term rate of interest is an average of expected future short-term rates and the current spot rate. The pure expectations theory requires the assumptions of perfect markets with no differential taxes, no transactions costs, and homogeneous expectations. With these assumptions and expected return maximizing behavior on the part of investors, the current forward rates equal expected future short-term rates. A forward rate is usually defined as a one period rate on an investment made in the future. Thus

$$(1 + {}_tR_n)^n = (1 + {}_tR_1)(1 + {}_{t+1}r_{1,t})(1 + {}_{t+2}r_{1,t})(1 + {}_{t+n-1}r_{1,t}),$$

where R = spot rate
r = forward rate

The forward rate under the certainty and perfect market conditions noted above is the future spot rate.

If, in a two period case, the investor's expectation of the second period spot rate was lower than the forward rate, the two-period investment would be undertaken. If the expected future rate was higher it would behoove the investor to make a one period investment and reinvest at the higher expected rate, which under the assumptions, is certain. In equilibrium the forward rate would equal the future spot rate.

$$E\ (1 + {}_1R_{2,t}) = (1 + {}_1r_{1,t}) = (1 + R_2)^2 \ / \ (1 + R_1)$$

Under this expectations theory we would conclude that bonds of any maturity will have the same expected return over a given holding period. The expected return will also be the realized return. Any one bond with any maturity is a perfect substitute for any other over that holding period. The ability of investors to make costless transactions will allow the arbitrage mechanism to work as investors attempt to maximize returns over their holding periods. With perfect markets and costless arbitrage opportunities, bond prices would reflect all relevant information in an unbiased manner. When new information becomes available expectations adjust instantaneously as do prices.

Liquidity preference theory allows for uncertainty. Instead of the future short-term rate being equal to the forward rates, those future rates are uncertain. Hicks (1946) developed the liquidity preference theory.[6] He sees investors forming expectations of the uncertain future rates. Borrowers are assumed to desire to borrow long to avoid interest rate risk, that is, the need to refinance at a higher rate. Lenders want to lend short-term in order to maintain the stability of their wealth. Short-term investments are less sensitive to interest rate fluctuations than are long-term investments.

If risk aversion is assumed on the part of investors, they must be offered a premium in return if they are to invest in the less desirable long-term bonds. Forward rates are no longer unbiased estimates of the future short-term rates. They are biased and exceed the the expected future rates by this premium for liquidity. For any forward rate:

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[6]While Hicks is the developer of liquidity preference theory, its roots can be traced back to Keynes' "normal backwardation."

$$(1 + {}_{t+n}r_{1,t}) = E_t\,(1 + {}_{t+n}R_{1,t}) + {}_{t+n}L_{1,t}.$$

Thus, the perfect substitutability from the expectations theory is lost. A long-term investment would have a higher expected return than would a series of short-term investments. Empirical tests of the liquidity preference theory show that it is more consistent with the efficient markets hypothesis than is the pure expectations theory. Liquidity preference and risk aversion are part of the information set that is available to the market and, as such, are impounded in security prices instantaneously and in an unbiased manner.

An argument against liquidity preference and supportive of the expectations hypothesis is that there are speculators who seek risk and investors who are risk indifferent (expected return is their sole decision criterion) in the market. This would imply that there are investors who would pay a premium for the longer maturity investments. Through their arbitrage activities, these investors are seen to eliminate the liquidity premium. Where these investors are present, all maturities of bonds would provide the same liquidity because of investor indifference to risk (maturity) and because of risk seeking investors, so liquidity premiums disappear.

Market segmentation is a third approach. It is presented in opposition to expectations theory.

Proponents of this theory argue that the investor is risk averse. There are structural or institutional reasons that limit the participation of certain investors to certain segments, by maturity, of the market. Investors with long planning horizons, such as life insurance firms, would invest primarily in long maturity assets since they are concerned with maintaining a level of income and not exposing themselves to reinvestment risk. Similarly, short-term securities are attractive to investors with short planning horizons. This theory recognizes that investors try to match planning horizon and portfolio maturity. Bonds of different maturities are not substitutes under this theory. Culbertson (1957) is recognized as having promulgated the concept of market segmentation. In such a segmented market, the term structure results from the supply and demand for securities within each segment. Changing the maturity structure of government securities, for example, would change the yield curve for government securities. Proponents of the expectations hypothesis would not accept this conclusion.

Modigliani and Sutch (1966) developed another approach to segmentation. This is their <u>preferred habitat</u> theory. The theory is a synthesis of liquidity preference and market segmentation. The habitat referred to is maturity that equals a planning horizon or perceived need for funds in the future. Unlike Culbertson, though, they allow for investors with a preferred habitat to be willing to invest in securities of other maturities to take advantage of higher expected returns. There is not, then, only one precise maturity which will be acceptable to an investor. This theory provides for a continuity in the term structure that is not present under the market segmentation approach. It relates to the liquidity preference theory under the circumstance where all investors' preferred habitat is the shortest maturity possible. Like the market segmentation hypothesis, this theory would lead to the conclusion that a change in the relative supply of bonds in a specific maturity range would cause shifts in the yield curve.

Tests of Term Structure Theories

Two of the theories discussed relate observed forward rates to expected future rates. Tests of the expectations theory were based on the market activity that forced forward rates and future rates to be equal. In order to perform such a test, assumptions need to be made about the forming of expectations on the part of the investors. Usually this is done by making market efficiency assumptions that allow the investigator to accept past levels of interest rates to be used as the only information necessary to derive expectations of future rates. Then the hypothesis as usually tested,

$$_{t+1}r_{1,t} = {}_{t+1}R_1 + e_{t+1} \quad ,$$

is a test of a joint hypothesis. The first argument is that market expectations are formed as assumed above. The second is that the term structure comes about from a martingale model of prices as hypothesized. The test of the joint hypothesis falls prey to an inability to distinguish which statement is being rejected. A rejection of the hypothesis of the expectations theory tested in this manner may just as well be due to a misspecification of the way expectations are formed or to the term structure not being developed from the expectations hypothesis.

Jarrow (1981) notes that testing of term structure theories generally begins with the expectations hypothesis as the null. He points out that there have been three statements of the expectations hypothesis. They are:

1. Forward rates equal expected future spot rates;
2. Yield equals average expected future spot rates; and
3. Over a given holding period, bonds with different maturities have the same expected return.

Macaulay (1938), Meiselman (1962), and McCulloch (1975) use the first approach. Indeed, Meiselman develops an error learning hypothesis to explain the way expectations are formed. He argues that expectations do not have to be correct to form yield curves. Meiselman determined that forecast errors could be used to explain significant portions of changes in forward rates. According to Meiselman, investors adjust their expectations based on the new information contained in the realized short-term interest rates being different from those which were forecasted. This adjustment causes the entire yield curve to shift.

The second approach has been used by Modigliani and Shiller (1973) and by Dobson, Sutch, and Vanderford (1976) in their distributed lag average model. Tests of the expectations hypothesis using the third statement form were done by Santomero (1976) and by Fama (1976).

These three statements of the expectations hypothesis appear to be consistent. In a later work Cox, Ingersoll, and Ross (1978) demonstrate certain inconsistencies. They use a continuous time model based on contingent claims in a rational expectations framework. They show that the only statement of the expectations hypothesis that is consistent with their model is that of equality of expected returns of bonds of differing maturities. The other two are shown to be inconsistent with this one. Theirs is an equilibrium model which they use in a later work (1981) to evaluate the liquidity preference, expectations and preferred habitat models. They find that all statements of the expectations hypothesis except on instantaneous returns on bonds are the same for all maturities which they call "Local Expectations" imply a term premium that is inconsistent with expectations theory. Cox, Ingersoll, and

Ross also conclude that the preferred habitat theory which encompasses the liquidity preference as a special case requires that risk aversion rather than consumption plans as used by Modigliani and Sutch be the determinant of habitat. No theoretical specification of the term structure is necessary for the development of an immunization strategy for an investment portfolio. A portfolio is seen to be immunized when its expected return is made secure from the effects of interest rate changes over the investor's planning horizon.

Estimation of Yield Curves

Estimation of yield curves has been done frequently, using regression analysis. Cohen, Kramer, and Waugh (1966) found that although there are a number of government bonds with the same maturity because of continual issuing of new securities, the yields are different because of coupon differences, tax effects, and preference and institutional differences. A group of regressions were run on yields as they existed on certain specific days. They found that there was a model regressing before-tax yield and one regressing after-tax yield on maturity and log of maturity that provided significant coefficients and explained a large proportion of the variability. They concluded that OLS regression methods were useful for estimating yield curves. McCulloch (1971) argues that linear regression on direct observations is inappropriate because the yield calculations are a weighted average of the principal and coupon weights and tend to generate errors at the longer maturities. This makes precise estimation of that part of the yield curve impossible. Another difficulty he sees is that errors in fitting will be magnified when forward rates are calculated from the yield curve. McCulloch, instead, develops a discounting or present value function first that gives instantaneous forward rates. The average of these rates is then used in the regression analysis. He contends that this method eliminates the difficulties encountered in using direct measurements. He found the best fit to be in segments, that is, discontinuous, and quadratic in form.

Echols and Elliott (1976) point out that the data (bond prices) that are available for term structure analysis are incomplete since not all bonds are always traded nor are the data homogeneous because of coupon differences or call provisions. Their regression equation includes a term for the coupon yield. They find the coefficient of this term to be significant.

Because of continuing difficulty in the estimation of yield curves by econometric techniques, hand plotted curve-fitting, or merely "eyeball" fits statistical measurement has not been particularly useful for reproduction or evaluation. Yield curves will not need to be estimated for the present study. As will be explained in the methodology section of this paper, we need only calculate an average of current prices plus accrued interest for all government bonds extant at any point in time of interest to the study.

Durand (1942) provided yield curves based on corporate bond yields. He fit the curves by hand through the lower portion of a scatter diagram in an attempt to adjust for risk. The U.S. Treasury Department publishes yield curves that are fit by hand through the middle of the scatter of yield data for Treasury securities. Carleton and Cooper (1976) add another approach to term structure estimation using regression techniques. Instead of an average at each maturity, they use all bond prices. Their model uses periodic rather than continuous interest payments and does not require the discount rate to be the same over all periods.

CHAPTER III

THE HEDGING MODELS

The goal of the hedging activity is to minimize the variability of wealth over the investor's planning horizon. An unhedged position is subject to all the risk inherent in changes in the term structure and level of interest rates during the planning horizon. Hedging strategies that have been proposed earlier have been direct hedges where the investment asset position has been hedged by taking positions in futures contracts for delivery of those same securities. U.S. Treasury Bills would be hedged with T-Bill futures contracts. This approach limits the types of assets that could be hedged. Active futures markets exist for only a few securities. They are:

1. U.S. Treasury Bills,
2. U.S. Treasury Notes,
3. U.S. Treasury Bonds,
4. GNMA securities, and
5. Bank CDs.

There are also active futures markets in a number of foreign currencies and a fairly recent development of futures contracts for common stock indexes such as the S & P 500, New York Stock Exchange Composite Index, and

Value Line Index. An investor who wanted to be directly hedged would be limited to three U.S. Government obligations. The other two debt related futures contracts are based on groups of securities.

A cross hedge approach to hedging provides a broader range of possibilities. A cross-hedge exists where a position in one asset is hedged with a futures position in contracts of a different asset.[7] The hedging activity tested in this study can be characterized as cross-hedging. Positions in U.S. Treasury securities are hedged with T-Bill Futures.

The model presented here is an extension of the cross-hedging models developed by Kolb and Chiang (1981) and by Kolb (1982). The difference between the proposed Expectations Hedge Model and those developed by Kolb and Chiang and by Kolb rests in how the investor is seen to judge the future shape of the yield curve. The two duration based hedge ratios assume the investor sees a fixed structural relationship that exists at the time of the institution of the hedge. That relationship is described by flat and/or parallel shifting yield curves over the holding period. These are simplifying assumptions that are unrealistic both in their view of how investors use information and in how interest rates have changed in the past.

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[7]Those differences could be in type of asset, maturity, delivery date, or risk.

An Expectations Hedging Model

The expectations hedge model uses a stochastic statistical model of the formation of interest rates for developing hedge ratios. The hedge ratio calculated is an expected hedge ratio rather than an exact ratio developed from a relationship between expected price changes of an asset and a corresponding hedging instrument. The expectations hedge uses the term structure at the time of the hedge as one piece of information in an information set that includes prior realizations of and changes in forward rates. The term structure that is relevant to the decision maker is the unknown term structure that will exist at the end of the planning horizon.

The duration based hedges project the yield curve that exist at the time of the initiation of the hedge to the end of the planning horizon. When the yield curve is allowed to shift in a more realistic manner, it is expected that the duration based hedges would be less effective than would be an expectations hedge that recognizes that expectations are formed by risk averse investors using more information.

A model for an expectations hedge ratio was developed that allows for multiple random shocks to the term structure of interest rates. The investor uses the expectations hedge:

$$E(H) = -E\left[\frac{\left(\frac{dP_i}{d\tilde{R}_N} \,\middle|\, \Delta\tilde{R}_p\right)}{\left(\frac{dP_j}{d\tilde{R}_N} \,\middle|\, \Delta\tilde{R}_p\right)}\right]$$

where,

P_i = Price of the investment at termination of the hedge,

R_N = 1 + forward rate N-periods ahead,

P_j = Price of the asset underlying the futures contract at delivery, and

r_p = 1 + forward rate in period p

to try to ensure that

$$\Delta\tilde{P}_i = -H\Delta\tilde{P}_j$$

so

$$H = \Delta\tilde{P}_i/\Delta\tilde{P}_j \quad .$$

The expectations hedge is derived in the following manner:

$$P_i = \sum_{t=s+1}^{I} \frac{C_{it}}{(R_i)^{t-s}} \quad , \qquad P_j = \sum_{t=s+1}^{s+J} \frac{C_{jt}}{(R_j)^{t-s}}$$

$$N = \frac{dP_i}{dR_N} \bigg/ \frac{dP_j}{dR_N}$$

(Note that this measure has no time dimension and is similar to Macaulay's original conception.)

$$dP_i/dR_N = \left[\sum_{t=s+1}^{s+J} \left\{ \frac{C_{it}}{\left(\prod_{p=s+1}^{t} \tilde{R}_p \right)^2} \left[\sum_{N=s+1}^{t} \frac{\prod_{p=s+1}^{t} \tilde{R}_p}{\tilde{R}_N} \right] \right\} + \right.$$

$$\left. \sum_{t=s+J+1}^{I} \left\{ \frac{C_{it}}{\left(\prod_{p=s+1}^{t} \tilde{R}_p \right)^2} \left[\sum_{N=s+1}^{s+J} \frac{\prod_{p=s+1}^{t} \tilde{R}_p}{\tilde{R}_N} \right] \right\} \right]$$

$$d\,P_j/dR_N = \sum_{t=s+1}^{s+J} \left\{ \frac{C_{jt}}{\left(\prod_{p=s+1}^{t} \tilde{R}_p \right)^2} \left[\sum_{N=s+1}^{t} \frac{\prod_{p=s+1}^{t} \tilde{R}_p}{\tilde{R}_N} \right] \right\}$$

where,

s = planning horizon,

I = maturity of investment,

J = number of periods to be hedged,

R_p = 1 + forward rate in period p,

R_N = 1 + forward rate N-periods ahead,

R_i = 1 + yield to maturity of the investment, and

R_j = 1 + yield to maturity of the asset underlying the futures contract.

Then,

$$E(H) = -E \left[\frac{\left(\dfrac{dP_i}{d\tilde{R}_N} \,\middle|\, \Delta \tilde{R}_p \right)}{\left(\dfrac{d\tilde{P}_j}{d\tilde{R}_N} \,\middle|\, \Delta \tilde{R}_p \right)} \right]$$

R_i and R_j are yield calculations that are the product of a series of forward rates. The change in any one forward rate that is part of the product will change the expected yield, but obviously will not change forward rates for periods prior to the change. The yield curve can be shifted or twisted[8] or both and this expectations based hedge will still be appropriate.

Santoni (1984) relates the elasticity measure of interest rate sensitivity used in the expectations hedge to the duration measure. He shows that the duration of a portfolio of assets and liabilities (a firm) taken together is not simply some weighted linear combination of the durations of each of the assets and liabilities. Rather, the duration of the portfolio can be a value outside the range of durations of the assets and liabilities and may even be negative. He concludes that duration is not as good a measure of interest rate sensitivity as is the elasticity measure.

Comparison to the Kolb and Chiang Hedge

Kolb and Chiang (1981) calculate a hedge ratio as:

$$H = -\tilde{R}_j P_i D_i / \tilde{R}_i P_j D_j , \quad E(\Delta\tilde{R}_i / \Delta\tilde{R}_j) = 1 ,$$

where all symbols are as above and D = Duration.

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[8] A yield curve shifts when the yields over all the observed maturities maintain their relative sizes. Twisting of the yield curve occurs when the relative sizes are not maintained.

The yield to maturity of the investment, R_i, and the yield to maturity of the asset underlying the futures contract, R_j, are expected to change in the same proportional manner over the planning horizon. That is, $E(\Delta R_i/\Delta R_j) = 1$. This is the parallel shift in the yield curve that is assumed. This also implies that the shape of the current yield curve is preserved, although the level of rates may change.

Kolb (1982) recognizes that a complex yield curve assumption

> ...is important in making a conceptual advance over the ordinary bond pricing equation (but) the attempt to apply it to all aspects of bond pricing generates more heat than light. (pg. 58)

It is hoped that the tests in this study will show that the amount of light generated will justify the attendant heat.

The expectation hedge ratio,

$$E(H) = -E\left[\frac{\left(\frac{dP_i}{d\tilde{R}_N} \,\Big|\, \Delta\tilde{R}_p\right)}{\left(\frac{dP_j}{d\tilde{R}_N} \,\Big|\, \Delta\tilde{R}_p\right)}\right],$$

in contrast, uses expectations and variances of the expected yield curves. The investor considers the first two moments of the distribution of hedge ratios calculated from the stochastic process generating forward rates. The hedge ratio is an expected hedge ratio in the sense that it is the mean of a distribution of possible hedge ratios that could occur under various combinations of forward rates that might occur. The investor is not constrained to assuming only a single additive shock to the term structure, a parallel shift.

Comparison to the Kolb Hedge

Kolb (1982) expands on the Kolb and Chiang hedge to allow a measured relationship between $\Delta\tilde{R}_i$ and $\Delta\tilde{R}_j$. He regresses $\tilde{R}_i$ on $\tilde{R}_j$. This results in the hedge ratio:

$$H = -\frac{\bar{R}_j P_i D_i}{\bar{R}_i P_j D_j}\hat{r}_{ij} \; ; \quad \hat{r}_{ij} = \frac{E\left((\Delta\tilde{R}_i \mid (\Delta R_i,\ \Delta R_j)_{t-1}^{t-n}\right)}{E\left((\Delta\tilde{R}_j \mid (\Delta R_i,\ \Delta R_j)_{t-1}^{t-n}\right)}$$

where,

t = point where hedge is instituted and
1→n = previous observation period for estimating r.

So the yield curve expected at the termination of the hedge is the yield curve that exists at the inception of the hedge.

The two duration based hedges are calculated from the extrapolation of the existing yield curve. No consideration is given to the variability of possible yield curves in the future. Indeed that variability is assumed way. The expectations hedge reflects a distribution of hedge ratios. Variability is a factor in the development of this hedge ratio. If the variance of forward rates forecast further in the future is greater than the variance of near term forecasts, a hedge ratio that considers variance should be more effective than one that does not.

The hedge ratios based on duration explicitly measure the duration of the assets and the assets underlying the futures contracts. This duration measure requires the assumption of flat or parallel shifting yield curves. No such assumptions are necessary for the expectations hedge because duration, while implicit in the calculation of the hedge ratios, need not be measured explicitly.[9]

Both of the hedges based on duration use a calculation of duration that has a time dimension. This is true to Macaulay's measurement of duration but not to the definition. Macaulay's definition would use spot and forward rates rather than yield to maturity. When spot and forward rates are used in the calculation, the time dimension disappears.

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[9]The duration based hedges can be calculated without the explicit measure of duration, but they were developed from the theoretical duration concepts.

CHAPTER IV

HYPOTHESES

The goal of hedging is risk reduction. It is useful to consider the ways risk is perceived. The usual surrogate for risk that is used to judge risk changes or risk reduction is the variance of returns or, in the case of hedging, the variance of wealth changes. Kolb and Chiang (1981) and Kolb (1982) both use the variance of wealth changes as a result of hedging activities to measure the effectiveness of their hedges. The present study also uses variance of wealth change as a measure of effectiveness.

Variance as a measure of risk has been accepted in the economics and finance literature at least as far back as the work of Irving Fisher. Markowitz (1952) uses variance of returns as the risk measure in the E - V approach to portfolio selection and Hirshleifer (1965), attributing early statement of a mean - variability analysis to Fisher in 1912 says,

> The mean, variability approach to investment decision under uncertainty selects as the objects of choice expected returns and variability of returns from investments. In accordance with the common beliefs of observers of financial markets, the assumption is made that investors desire high values of the former and low values of the latter -

> as usually measured by the mean (μ) and standard deviation (σ) respectively of the probability distribution of returns.... (pg. 518)

Fisher and Weil (1971) use the standard deviation of wealth changes as a measure of hedging effectiveness. There is much to commend the use of variance as a measure for risk.

On the other hand, researchers have also noted that investors see risk as the possibility of loss or of returns below some expected level. Markowitz (1959) states that the semi-variance would be a more appropriate measure to use in the mean-variance analysis as the measure of risk. He then returns to the use of variance because of the computational problems he sees in using the semi-variance.

The idea of risk as some below - target variability is a pleasing one. Domar and Musgrave (1944) in a paper discussing the effect of an income tax on risk-taking behavior recognized risk as

> probability of actual yield (from an investment) being less than zero.... (pg. 396)

And they defined risk more specifically as the sum of each possible loss weighted by the probability of occurrence of that loss.

Subsequently, Grayson (1960) studied attitudes toward risk among managers engaged in the business of oil drilling which is an endeavor characterized by small probabilities of large returns and large probabilities of losses. He described decison-making processes and inferred utility curves for the individuals who were interviewed. The utility curves developed were all steeper for losses than for gains. The consequences of losses were seen to be much greater than the benefits for similar sized gains. Halter and Dean (1971) found similar derived utility functions in agricultural pursuits.

Kahneman and Tversky (1979) studied choices of risky gambles and found risk averse behavior for gains and risk seeking behavior for losses. This was interpreted as the possibility of loss being a more appropriate measure of risk than other measures. They developed a theory of risk-taking behavior that they call "Prospect Theory."

Mao (1970a) calculated semi-variances and compared the investment decisions made under expected return - variance to those made under expected return semi-variance and concluded that expected return - semi-variance choice objects are more consistent with utility functions that are not concave at all levels of wealth. He found that these utility functions are more descriptive of investor behavior than are those that are concave downward over the entire range of wealth.

Mao (1970b) has also surveyed executives responsible for capital budgeting about their attitudes toward risk. He learned that the executives surveyed see risk as the possibility of not meeting some target or required return. Mao concluded that although variance is the measure of risk most used in capital budgeting analysis, risk, in the eyes of decision-makers, emphasizes downside possibilities. This is more consistent with the semivariance measure.

An approach to calculating a measure of the risk of below-target returns was developed by Fishburn (1977). It is a two parameter model which incorporates a factor for differing attitudes toward risk with risk measured as the probability of not reaching a targeted return. The Fishburn measure is:

$$F_{\alpha}(t) = \int_{-\infty}^{t} (t - Y)^{\alpha} dF(Y)$$

where,

t = target return,
$F_{\alpha}(t)$ = probability of return below t,
Y = observed return < t, and
α = risk aversion measure.

This measure will be used in judging the efficacy of the hedge ratios in mitigating risk.

The Hypotheses

1. The variance of the net wealth change is smaller using the expectations based hedge ratio than is the net wealth change using either of the duration based hedges.
2. The Fishburn measure of below-target wealth change is smaller for the expectations hedge ratio than it is for either of the duration based hedges.

In order to develop the forecasts of forward rates necessary for this study, the independence of adjacent forward rate series needed to be established. This step was required so forward rates could be forecasted independently. A secondary hypothesis was tested:

S1. Forward rates are independent across time.

CHAPTER V

METHODOLOGY

The effectiveness of the hedging methods used in this study is tested by testing the hypotheses relating to wealth change. The risk that is being hedged is interest rate risk. In order to control for default risk, both the investment and the assets underlying the hedging vehicles are U.S. Treasury securities.

Data

The source of the bond price data is the CRSP[10] Government Bond File. The data used for this study are the month-end bid and asked prices and accrued interest. Bond yields were calculated over two overlapping five-year periods:

December 1971 - November 1976 and

December 1973 - November 1978,

for which the asked prices were the source of price and accrued interest data for the simulation of purchase and subsequent sale of assets at the end of the planning horizon. The investor's asset position was based on the

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[10] Center for Research in Security Prices, University of Chicago.

asked price plus accrued interest at the inception of the hedge. The bonds were "sold" at the bid price plus accrued interest at the end of the planning horizon.

The hedging instrument is the 91-day U.S. Treasury Bill Futures Contract. Trading in T-Bill futures began on the International Monetary Market (IMM) of the Chicago Mercantile Exchange in January of 1976. Prices used to calculate the wealth changes resulting from the hedge positions are the closing prices of relevant contracts on the last day of each month that the contracts are traded. The data is published daily in The Wall Street Journal.

The hedges being tested are generated as a result of investors' expectations of the term structure of interest rates. The duration based hedges and the expectations hedge differ in their recognition of the way in which investors form those expectations. The duration based hedges are a function of the term structure that exists at the inception of the hedge. The term structure at inception is the hedger's forecast of the term structure that will exist at the end of the planning horizon.

The expectations hedge comes about from the investor's explicit forecast of the term structure. The investor is faced with an efficient market and forms expectations rationally. Thus, the rational investor will act as if all past information and all relevant current information and expectations are impounded in current bond and futures prices.

In order to forecast future yields, the investor will forecast the single-period forward rates, which are independent for adjacent months, based on past forward rate information contained in past yields. The forecasted forward rates and forecast variances are used to simulate possible realizations of the forecasted forward rates.

Hacket (1978) used a simulation approach to develop yield curves. Rather than simulate individual forward rates from time series model, though, Hackett used both multiplicative and log models for the term structure and simulated single shocks to the yield curve. His asset portfolio is restructured after each shock. In essence, the same shock value is applied to each forward rate that is inferred from a yield structure at a specific point in time.

Yield-to-Maturity

Yield curves are calculated from the CRSP Government Bond Price data. For each five year calculation period, monthly annualized yields-to-maturity are calculated for holding periods from one month to 24 months in monthly increments. A new yield curve was calculated at the end of each of sixty months represented by each of the two overlapping five-year periods. This results in sixty yield curves for each five-year period. These calculated yield curves are used to calculate a series of sixty single-period forward rates of from one to 23 months forward.

Each yield that is calculated represents an average of the yields implied in the prices of all the U.S. Treasury securities that were outstanding over each yield period.[11] The yields are calculated as:

$$\sum_i P_0 = \sum_i \sum_{t=1}^{m} \frac{C_t}{\left(1 + \frac{R}{n}\right)^t}$$

where,

i = number of securities outstanding,
m = number of cash flows to maturity,
n = coupon payment frequency, and
R = yield-to-maturity.

A computer program for calculating the yields is found in Appendix A, Table A1.

The term structures at each inception period for the hedge tests are listed in Table 1. The yields are those used to develop the duration based hedges. They are the yields that are projected forward as the forecasted yields at the end of the planning horizons.

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[11]Callable bonds, flower bonds and other non-standard instruments are excluded as the yields on these bonds are not amenable to direct calculation but require simplifying assumptions to make a calculation of their yields.

Table 1

Holding Period	Annualized Yields as of November 30, 1976	1978
1	.04165	.09079
2	.04443	.09212
3	.03910	.09584
4	.04555	.09662
5	.04677	.09792
6	.04247	.09992
7	.04775	.10079
8	.04764	.10309
9	.04836	.10218
10	.04903	.10149
11	.04926	.10229
12	.04961	.10009
13	.04956	.10143
14	.04993	.10140
15	.05075	.09781
16	.05184	.09678
17	.05273	.09971
18	.05264	.09975
19	.05364	.09784
20	.05396	.09849
21	.05483	.09538
22	.05728	.09449
23	.05416	.09766
24.	.05425	.09507

Forward Rates

In order to calculate the hedge ratios under the expectations method, it is necessary to calculate the forward rates implied in each of the yield curves. This calculation results in sixty forward rates in each of the two five year calculation periods for each of the 23 months forward. The single period forward rates are calculated as:

$$F = \frac{\left(1 + R_{n+1}\right)^{\frac{n+1}{12}}}{\left(1 + R_n\right)^{\frac{n}{12}}} - 1.$$

where,

R = Yield to Maturity and

n = number of months forward

These forward rate series are used to forecast forward rates. These forecasts and the forecast variances are used to simulate the investor's forecasting process for hedging interest rate risk.

The series of forward rates that are calculated represent a separate time series for each of the 23 months forward. For example, there is a six-month forward rate implied in the yield curve calculated as of December 31, 1971, and another six-month forward rate implied in the yield curve that exists on January 31, 1972, and 58 more six-month forward rates covering the yield curves from February 28, 1972 through November 30, 1976. A computer program to calculate the forward rates is included in Appendix A, Table A2.

Forecasting Forward Rates

The time series of forward rates provide the investor with information necessary to forecast succeeding forward rates. One method of forecasting is that developed by Box and Jenkins (1970) where a variable observed is described in terms of previous values of the variable and a series of random shocks occurring at previous times. Box and Jenkins show a method of estimating values for coefficients of previous values and previous and current random shocks that expresses autocorrelated time series in terms of autoregressive and/or moving average components. The models so expressed can be used to produce forecasts whose variances are minimized. The models of forward rates are generally referred to as ARIMA (Autoregressive, Integrated, Moving Average) models and take the general form

$$\phi(B)(1-B)^d z_t = \theta_0 + \theta_1(B)a_t$$

where,

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \ldots \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \ldots \theta_8 B^q$$

B = Backshift operator,

d = number of observation periods of difference; $z_t - z_{t-d}$,

a_t = random shock at time t, and

z_t = observation at time t,

Box and Jenkins (1970).

This model can also be expressed in terms of the random shocks which is useful for modeling and forecasting because the procedure assays the current observation of a time series as the result of a series of uncorrelated shocks, each shock carrying a weighted value.

The mean of the previous shocks is zero and the variance is constant.

Each of the two sets of 23 forward rates is modeled using the ARIMA process. Table 2 lists the models.

Table 2

Month Forward	ARIMA Models	
	Period 1	Period 2
1	$(1-B)z_t=(1-.375B^6)a_t$ (2.75)	$(1-B)(1+.458B^2)z_t=(1-.298B)a_t$ (-3.82) (2.27)
2	$(1-B)z_t=(1-.502B)a_t$ (4.15)	$(1-B)z_t=(1-.579B)a_t$ (5.16)
3	$(1-B)z_t=(1-.453B)a_t$ (3.88)	$(1-B)z_t=(1-.545B)a_t$ (4.91)
4	$(1-B)z_t=(1-.395B)a_t$ (3.28)	$(1-B)z_t=a_t$
5	$(1-B)z_t=(1-.459B)a_t$ (3.72)	$(1-B)z_t=(1-.579B)a_t$ (5.28)
6	$(1-.453B)(z_t-2.897)=a_t$ (3.93) (4.53)	$(1-B)z_t=(1-.527B)a_t$ (4.74)
7	$z_t-5.335=a_t$ (26.93)	$z_t-5.634=a_t$ (28.14)
8	$z_t-5.141=a_t$ (21.29)	$z_t-5.509=a_t$ (23.97)
9	$z_t-5.628=(1+.330B+.413B^3)a_t$ (18.06) (-2.99) (-3.71)	$z_t-5.923=a_t$ (33.20)
10	$z_t-5.569=a_t$ (26.55)	$(1-.346B^2)(z_t-3.850)=a_t$ (2.74) (5.08)
11	$z_t-5.640=a_t$ (20.11)	$z_t-5.960=a_t$ (24.51)

Table 2--Continued

Month Forward	ARIMA Models	
	Period 1	Period 2
12	$(1-.313B^6)(z_t-5.166)=(1+.289B^2)a_t$ (2.43) (13.65) (-2.21)	$z_t-5.792=a_t$ (34.65)
13	$z_t-5.419=a_t$ (24.96)	$z_t-6.040=a_t$ (32.36)
14	$z_t-5.419=a_t$ (25.61)	$z_t-5.884=a_t$ (32.63)
15	$(1-.312B)(z_t-3.947)=a_t$ (2.56) (5.50)	$z_t-6.181=a_t$ (24.95)
16	$z_t-5.886=a_t$ (37.26)	$z_t-6.171=a_t$ (30.29)
17	$(1-B)z_t=(1-.741B)a_t$ (8.11)	$z_t-5.899=(1+.438B^5)a_t$ (23.87) (-3.02)
18	$(1-B)z_t=(1-.757B)a_t$ (8.21)	$z_t-6.271=a_t$ (28.94)
19	$z_t-5.918=(1+.358B^6)a_t$ (22.81) (-2.71)	$z_t-6.391=(1+.323B^3)a_t$ (21.99) (-2.38)
20	$z_t-5.842=(1+.401B)a_t$ (23.16) (-3.35)	$z_t-5.913=a_t$ (34.21)
21	$z_t-5.888=a_t$ (29.71)	$z_t-5.811=a_t$ (29.11)
22	$z_t-6.040=a_t$ (31.76)	$z_t-6.034=a_t$ (28.70)
23	$z_t-4.823=a_t$ (12.06)	$z_t-5.288=a_t$ (16.17)

t - values in parentheses.

It is of interest to note that 25 of the series are simply randomly distributed about some constant value. There are no significant autocorrelations so the best forecast of these processes is the constant. These random processes are consistent with the notion of market efficiency.

As will be explained below, the planning horizons for the hedge calculations will be limited to 24 months. Thus it will be necessary to allow for forecasting of forward rates up to 24 months or steps ahead. The forecast variances are also calculated since these variances are used to develop the simulations from which the expectations hedge ratios are calculated.

The ARIMA modeling process can be extended to the forecasting step. Box and Jenkins show that the forecasts can be seen as a weighted sum of past and current random shocks,

$$\hat{z}_t(\ell) = \sum_{j=0}^{\infty} \psi_j a_{t+\ell-j}, \quad \psi_0 = 1$$

where ℓ = periods ahead to be forecast.

The ψ-weights need to be calculated. They are a function of the ϕ-values and Θ-values estimated in the original modeling steps for the series:

$$(1+\psi_1 B+\psi_2 B^2+\psi_3 B^3 \ldots .) = \frac{1-\phi_1 B-\phi_2 B^2-\ldots-\phi_{p+d} B^{p+d}}{1-\Theta_1 B-\Theta_2 B^2-\ldots-\Theta_q B^q} .$$

The forecast variance at any lead (ℓ) is calculated as:

$$v(\ell) = (1 + \sum_{j=1}^{\ell-1} \psi_j^2)\ \sigma_a^2 .$$

The forecasts and forecast variances used in this study were generated by an SPSSx program which uses this Ψ-weight method of forecasting. The original modeling of the forward rate series was also accomplished using SPSSx.

Before the forecasts of the individual forward rates could be attempted, it was necessary to determine the independence of adjacent forward rates. If adjacent forward rates, i.e. the two month forward rate and the three month forward rate, were not independent they would have to be forecasted jointly. Thus, the secondary hypothesis of independence of forward rates.

The test of independence of adjacent forward rates is the test suggested by Haugh (1976). It is a test of the lagged cross-correlations of the white-noise residuals of two time series modeled by the ARIMA method. In this case, the time series in question were series of adjacent forward rates. Haugh shows that the cross-correlation function over a given number of lags for the residual series from two ARIMA models are normally distributed. This makes a test statistic available which can be tested as distributed chi-square.

The Haugh statistic is calculated:

$$H_M = N^2 \sum_{k=-M}^{M} (N - |k|)^{-1} \hat{r}_{ij}(k)^2, \text{ d.f.} = 2M + 1 ,$$

where,

N = length of the series,
$|k| < M$, and
M = maximum lags.

The lagged cross-correlation functions for the residuals from the ARIMA models of the series of forward rates were calculated using BMDP (1981). The program used to calculate the Haugh statistic is found in Appendix A, Table A3. The forecasts were made independently. The two sets of forecasts and the variances are included as Appendix B, Tables B1 and B2.

Simulation

The goal of the simulation is to provide a description of the investor's expectations-forming process. For each of the two sets of 24 months that make up the possible ends of the randomly chosen planning horizons, one hundred realizations of the combination of 23 forward rates are generated. Each of the sets of 100 realizations represents an n-step ahead forecast of the forward rates that combine to form 100 yield curves. The simulation is accomplished by adding to each forecast value a value selected at random from the range bounded by the forecast variance as suggested in Naylor et al. (1968). The simulation program is included in Appendix A, Table A4. The resultant set of data is 100 yield curves for each of 24 months ahead for each of the two investment periods.

The next step in the analysis calls for the selecting of 100 combinations of specific U.S. Treasury securities from the CRSP Government Bond files along with a specific planning horizon, in months, for each bond chosen. Any individual security can be and is selected more than once but each combination of security and planning horizon is unique and randomly chosen. The limits for selection are:

1. Maturity $\leq$ 24 months and
2. Planning horizon $<$ maturity.

The first limit is required because there is a maximum of eight futures contracts representing a possible hedge coverage of 24 months. Each futures contract calls for delivery of a 91-day T-Bill at successive three month intervals. The value of each T-Bill is affected by forward rate changes over successive three month periods. The three months of forward rates for a specific bill would be those months during which the bill would be outstanding.

The second limit ensures that there will be a period of time during which the investor will need to be hedged against interest rate risk. The bonds will be sold at a market price that is a function of forward rates (expected yields) for the period between the end of the planning horizon and the maturity of the bond. At inception of the hedge

$$P_0 = f \prod_{t=1}^{M} (1 + R_t)$$

and at the end of the planning horizon

$$P_H = f \prod_{t=H+1}^{M} (1 + R_t) \ .$$

Thus at t_0, the inception of the hedge, the investor makes a forecast of ${}_HR_t$, $t|_{H+1}^{M}$. It is this forecast that determines the number of futures contracts that are sold to offset a change in value of the original investment.

Wealth Change

The two inception points for the hedge tests were chosen as November 30, 1976 and November 30, 1978. An investment of \$10,000,000 was assumed as the beginning position. The number of bonds represented in the portfolio is determined as:

$$B_0 = \$10{,}000{,}000/(AP_0 + AC_0)$$

where,

AP is the asked price for the bond and

AC is accrued interest.

The gain or loss on the investment over the planning horizon is determined by:

$$G_I = B_0 \ (BP_H + AC_H) - \$10{,}000{,}000$$

where,

H is the planning horizon in months and

BP is the bid price for the bond.

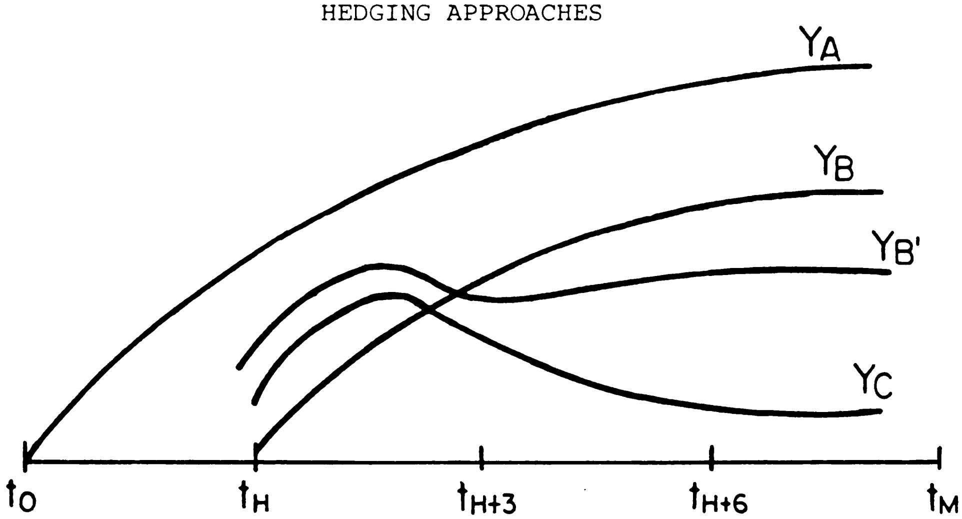
The purpose of the hedging activity is to reduce this gain or loss to a minimum or, if the alternative view of risk is accepted, to reduce the losses to a minimum while letting gains accrue. The hedge ratios or number of contracts sold to hedge each of the 100 realizations of future yield curves in the simulation are determined using the hedge relationships developed in Chapter III.

There is a difference between the duration based hedges and the expectations hedges that relates to the futures contracts bought. The duration based hedges allow positions to be taken in only one contract. It is the contract that calls for delivery of T-bills in the first delivery month after the planning horizon. This procedure leaves the investor at risk of interest rate changes between the maturity of those T-bills and the maturity of the original investment. This risk is partly offset by the comparing of durations of the investment and the underlying T-bills in order to develop the hedge ratio. That is, the difference in time is considered but not the explicit time periods. As is shown in Santoni (1984) this approach to duration matching may not provide the hedging effect desired.

The user of the expectations hedge is free to include any futures contract available at the time of the inception of the hedge. Thus, if there is a nine-month period between the end of the planning horizon and the maturity of the investment, positions are taken in three different futures contracts. Each contract provides a hedge against changes in forward rates over the three-month period represented by the life of the T-bill underlying that contract. The diagrams in Figure 1 are useful in illuminating this difference between the duration hedge and the expectations hedge.

Figure 1

ILLUSTRATION OF DIFFERENCES BETWEEN DURATION AND EXPECTATIONS HEDGING APPROACHES



It is the interest rate changes between t_H and t_M (maturity) that need to be hedged against.

- Y_A - Yield curve at inception of hedge.
- Y_B - Yield curve assumed at planning horizon. For duration based hedges it is identical to Y_A.
- Y_C - Actual yield curve at end of planning horizon.
- $Y_{B'}$ - One of the possible realizations of the yield curve resulting from the forecasting of forward rates under the expectations hedge approach.

Since the duration based hedges require a position at t_0 only in the contract calling for delivery at t_{H+3}, those investors are not hedged against fluctuations in forward rates from t_{H+3} to t_M. They do short a large number of contracts reflecting the duration difference between the 91-day T-bill and the longer duration asset.

The expectations hedge calculation allows the investor to consider explicitly interest rate changes over the entire period t_H to t_M. Futures positions would be taken in all three contracts, at t_0, representing forward rate changes over the entire nine-month period that requires hedging. The expectations hedge uses positions in

- F_1 - hedging forward rate changes for the period t_H to t_{H+3},
- F_2 - hedging forward rate changes for the period t_{H+3} to t_{H+6}, and
- F_3 - hedging forward rate changes for the period t_{H+6} to t_M.

The number of contracts for each period is determined by the relationship expressed in Chapter III. The forward rate changes during the period from t_{H+3} to t_M are considered explicitly in the expectations hedge ratio calculations but not in the duration based hedge ratio calculations.

The curve Y_B, represents one of the 100 realizations of the yield curve resulting from the simulation of forward rates. Using 100 iterations simulates the expectations forming activities of investors. The programs used to calculate hedge ratios and number of contracts are included in Appendix A, Tables A5 to A7.

For the expectations hedge, each iteration of forward rate forecasting results in a different hedge ratio and number of contracts sold. The hedging behavior is simulated by using the average number of contracts sold for calculating the wealth change for each planning horizon-investment combination. Thus, the average number of contracts sold represents the best expectation. There is no similar average for the duration based hedges because there is only one possible forecast of the yield curve. That forecast is the current yield curve.

The two duration based hedges differ in their estimation of dR_i/dR_j

where,

R_i = Yield to maturity of the investment and

R_j = Yield to maturity of the asset underlying the futures contract used for hedging.

In the present study, R_j is the three-month yield to maturity, representing the 91-day T-bill yield. The Kolb and Chiang model assumes $dR_i/dR_j = 1$, which is the parallel shift in the yield curve. In the later expression of this hedging approach Kolb relaxes the assumption and regresses the investment asset yields on the yield to the asset underlying the futures contract. The regression coefficient becomes the estimate of dR_i/dR_j. Kolb, Corgel, and Chiang (1982) calculated the effectiveness of GNMA futures for hedging mortgage interest rate risk using this method.

In order to provide an estimate of dR_i/dR_j for this study, yields of from one to 24 months are regressed on the three month yields for the seven year period December 1971 to November 1978. The appropriate regression coefficient is applied for each bond investment for which a hedge ratio is calculated. The regression coefficients are listed in Table 3.

Table 3

Regression Coefficients Estimated From

$$R_i = \alpha_i + \beta_i R_j + \varepsilon_t$$

R_j = 3-month yield to maturity

i	β
1	.98225
2	.96799
3	1.00000
4	.97325
5	.93734
6	.94139
7	.91335
8	.88203
9	.88555
10	.86332
11	.84995
12	.81894
13	.80128
14	.79067
15	.73640
16	.71712
17	.66422
18	.71925
19	.70443
20	.65792
21	.62933
22	.62105
23	.61151
24	.59557

Tests of the Hypotheses

Two investment periods were chosen from which to draw maturity and planning horizon combinations. They provide a comparison between shorter and longer term hedging effectiveness. In the first period covering December 1976 to November 1978 there were fewer futures contract delivery dates being traded than there were in the second period. The first period investment choices were thus constrained to shorter maturities.

The gain or loss resulting from the futures trading activities is found by:

$$G_F = \sum_i C_{i0} (FP_{iH} - FP_{i0})$$

where,

C = number of contracts,
i = futures contract, and
FP = futures price.

Then, the net wealth change is:

$$W = G_I - G_F .$$

A program to calculate wealth changes and mean wealth change over all iterations is included in Appendix A, Table A8. Hypothesis 1, the variance of wealth change comparison is made by using an F-test where the observed variances of wealth changes for the 100 sample maturity and planning horizon combinations for each of the test periods are used to calculate a statistic that is distributed according to the F-distribution.

Hypothesis 2 is a test of negative wealth changes since the target wealth change is zero for a hedge. The Fishburn measure is estimated as:

$$F_{\alpha}(t) = 1/N \sum_{j=1}^{N} (R_j)^{\alpha}$$

where,

R = negative wealth changes,

N = number of negative wealth changes observed, and

α = risk aversion measure with higher values for α representing stronger antipathy toward large negative wealth changes.

The estimate of comparative effectiveness of hedging techniques is:

$$\hat{M} = F_{\alpha}^{D}(t)/F_{\alpha}^{E}(t)$$

where,

D = duration hedge and

E = expectations hedge.

There is no theoretical distribution that will allow parametric testing of the significance of this statistic, however, values of $M > 1$ would indicate greater effectiveness of the expectations hedge in reducing risk of negative returns. This test is performed for levels of risk aversion, $\alpha = 1$ to $\alpha = 4$. A similar test was used by Johnson and Walther (1984) to determine hedging effectiveness in the foreign exchange market. A program to calculate the Fishburn measure is included in Appendix A, Table A9.

Summary of Steps in Analysis

1. Calculate bond yields from CRSP Government Bond Files.
    a) This is an average yield assuming equal investment in each bond.
    b) Two overlapping 5-year periods were chosen:
        1) December 1971 - November 1976
        2) December 1973 - November 1978
    c) Each month in each 5-year period was used as a starting point and a 24 month yield curve was calculated for each. The yield calculations were made using average bid prices + accrued interest. The number of securities used for each calculation ranged from one to nine for any month.
2. Calculate forward rates from the calculated bond yields.
    a) This results in two sets of forward rates.
    b) There are 60 calculated forward rates for each of one to 23 months forward.
3. Model each series of forward rates using ARIMA process.
    a) This results in two sets of ARIMA models for each of from one to 23 months forward.
    b) The residuals from these models are used to test for independence of adjacent month forward rates.
4. Adjacent forward rates are tested for independence.
    a) The Haugh test is used which measures

cross-correlation functions of white noise residuals.

b) This test is based on the significance of a chi-square like statistic.

c) Most of the adjacent forward rates were shown to be independent.

5. Forecast forward rates.

   a) The ARIMA models are used to develop forecasts of from one to 24 months (steps) ahead for each of the starting months. There are 24 starting months for each testing period.

   b) Forecasts and forecast variances are generated from the ARIMA models of forward rates.

6. Simulate yield curves.

   a) Simulation of yield curves for each of 24 steps ahead is done by randomly selecting a value from the range of the forecast variance for each forward rate (one month forward to 23 months forward) and adding that value to the forecast value. This process is repeated 100 times for each step ahead yield curve.

   b) These simulated yields form the basis for calculating the forward rates used in the hedge ratio calculations.

7. Select bond maturity - planning horizon combinations.

   a) One hundred combinations (maturity < planning horizon) are selected at random from the bonds available for each of the two five-year periods.

b) The first set of combinations represents shorter periods of exposure to interest rate risk than does the second set. During the first testing period there were only five or six futures contracts traded. Only 15 - 18 months could be hedged with the available contracts using the expectations hedge.

8. Calculate hedge ratios for each bond - planning horizon combination for each of the 100 realizations of the yield curve simulation for the number of months ahead represented by the planning horizon.

    a) Duration hedge.

    b) Expectations hedge.

    c) Adjusted duration hedge.

        1) Duration hedge ratio is adjusted for the correlation between the three-month rate and the appropriate rate for the maturity of the bond.

        2) Each of the 24 month's annualized yields is regressed on the three-month yield and the regression coefficients are used in the hedge ratio calculation.

9. Calculate the average number of futures contracts entered into for the 100 realizations of the yield curve simulation.

10. Calculate the mean wealth change and variance of wealth change based on a $10,000,000 investment and sale and closeout at the end of the planning horizon. An F-test is used to measure the difference in variance.

11. Calculate the Fishburn statistic which considers the probability of earning below-target returns as the risk. For a hedge, this means that negative wealth changes are to be minimized. There is no parametric test.

CHAPTER VI

RESULTS AND CONCLUSIONS

Independence of Forward Rates

The Haugh statistics,

$$H = N^2 \sum_{K=-M}^{M} (N - |k|)^{-1} \hat{r}_{ij} (k)^2$$

$N = 60$

$M = 24$

$i|_1^{23}, \; j|_2^{24}$

$d.f. = 49$

are shown in Table 4 for each of the two five-year periods used as a base for forecasting. For the first five-year period, December 1971 - November 1976, independence can be rejected for four sets of adjacent forward rates. The Haugh statistics for 7 and 8 months forward and 8 and 9 months forward show strong evidence of dependence. Two others are barely significant. For the second five-year period, December 1973 - November 1978, four different sets of forward rates cannot be considered independent. There are two that show a strong dependence measure 18 and 19 months forward and 19 and 20 months forward.

Table 4

Adjacent Months	Haugh Statistic Period 1	Period 2
1,2	62.446	46.245
2,3	54.813	43.286
3,4	46.314	35.050
4,5	53.025	40.600
5,6	60.184	63.218
6,7	69,865*	53.197
7,8	87.290*	51.050
8,9	73.350*	62.678
9,10	52.890	55.433
10,11	60.642	65.633
11,12	48.138	36.010
12,13	62.177	29.398
13,14	50.824	38.786
14,15	51.055	41.698
15,16	52.466	51.731
16,17	57.673	37.583
17,18	47.045	68.064*
18,19	47.335	74.689*
19,20	56.332	78.594*
20,21	72.951*	59.790
21,22	52.134	65.246
22,23	37.659	69.856*

* - significant at $\alpha = .05$

Because of the small number of significant dependencies, the forward rate forecasting was done independently. The independence of adjacent forward rates is of secondary importance to this study since the forward rates are used to provide a number of realizations of yield curves which represent simulations of investor perceptions. These simulations are meant to provide a range of possible perceived outcomes and not to be precise forecasts. The forecasts of forward rates and the attendant forecast variances are listed in Appendix B. It can be seen that the investor is making forecasts up to 24 steps ahead. The forecast variances increase with increasing leads.

Hedging Activity

Each of the hedging calculations determines a hedge ratio which is converted into a number of futures contracts sold to hedge the $10,000,000 initial investment. As was noted earlier, the duration based hedges use contracts for only one delivery month while the expectations approach allows use of contracts calling for delivery in a number of months. Under the expectations method as many as eight different delivery months may be used in the hedge. The number of contracts sold for each of the maturity/planning horizon combinations for each of the inception points is included in Appendix C, Tables C1 to C6.

Each individual hedging decision results in a change of wealth over the period until the end of the planning horizons. The mean wealth changes for each of the hedging methods are listed in Table 5.

Table 5

WEALTH CHANGES

Inception 11/30/76	Expectations Hedge	Duration Hedge	Regression Adjusted Hedge
Mean Wealth Change	\$-52,363	\$-73,307	\$-74,227
Wealth Change Variance	2.297×10^{10}	2.330×10^{10}	2.361×10^{10}

Inception 11/30/78	Expectations Hedge	Duration Hedge	Regression Adjusted Hedge
Mean Wealth Change	\$91,723	\$74,199	\$71,936
Wealth Change Variance	2.784×10^{10}	3.315×10^{10}	3.268×10^{10}

The F-ratios in Table 6 show the comparisons of the variances of wealth change for the tested hedging procedures.

Table 6

F - Ratios
F(100,100)

Inception		Duration Hedge	Regression Adjusted Hedge
11/30/76	Expectations Hedge	0.986	0.973
11/30/78	Expectations Hedge	0.840	0.852

There is no evidence that the expectations hedge reduces the wealth change or its variance any more than do the other hedging approaches. The variance of wealth changes was somewhat less under the expectations hedges than either of the others but not significantly so.

Test of Lower Partial Moments

The test statistic M is calculated for each of the three hedges for each of the inception points. Table 7 illustrates the results of those calculations.

At all levels of risk aversion measured, represented by $\alpha = 1$ to $\alpha = 4$ it can be seen that $M > 1$ for comparisons between duration based hedges and the expectations hedge. Increasing values of α represent increasing levels of risk aversion with $\alpha = 1$ corresponding to risk neutrality and higher values describing risk aversion.[12]

[12] An $\alpha = 2$ corresponds to the semi-variance measure used in earlier analysis, such as that of Markowitz (1959).

Table 7

Fishburn α - t
Ratios

	Inception 11/30/76		
	Dur/Exp	Regr.Adj/Exp	Regr.Adj/Dur
α = 1	1.09361	1.10389	1.00939
α = 2	1.18563	1.20625	1.01739
α = 3	1.24886	1.27739	1.02284
α = 4	1.28212	1.31663	1.02691
	Inception 11/30/78		
	Dur/Exp	Regr.Adj/Exp	Dur/Regr.Adj
α = 1	1.13208	1.15594	1.02107
α = 2	1.32001	1.28691	0.97492
α = 3	1.66940	1.49204	0.89376
α = 4	2.25932	1.80195	0.79756

Conclusions

The results obtained from the simulation of investment and hedging behavior over a large number of maturity/planning horizon combinations leads to the conclusion that hedging activity based on forecasts of forward rates that consider the possibility of complex shifts in the term structure of interest rates over a planning horizon provide better hedges if not better immunization. Such hedging activity reduces the interest rate risk of an investment position of the type described in this study to a greater extent than does hedging activity based on the duration models. This is so when risk is measured in terms of the achievement of a stated goal. The stated goal in this study is a minimum wealth change of zero over the planning horizon. The risk is one of not achieving this minimum goal. When risk is measured as variance of wealth change over the planning horizon there is no support for the dominance of the expectations hedge over the duration hedges.

The investor whose attitude toward risk is described by aversion to the possibility of earning below-target returns is able to reduce that risk best by making forecasts of the course of interest rates using all available information and including those forecasts in the calculation of the hedge ratios. The simplifying assumption of parallel shifts in the yield curve that is used for the duration hedge calculations appears to be so restrictive as to prevent the investor from realizing the most effective risk reduction.

Further Research

The current study does not consider transactions costs. Negotiated commission structures have resulted in round-turn commissions of less than $25 per contract for financial futures contracts. This study attempts to compare different hedging strategies where a similar number of contracts are used for each hedging method. Thus, the differential effects of transactions cost would be small. They were disregarded. Further study could test the absolute effects of transactions costs on the hedging results.

This study considers the use of 91-day T-bill futures contracts as vehicles for hedging interest rate risk over periods up to two years. This could represent an extreme example of cross-hedging. Similar studies could be undertaken using futures contracts in longer maturity instruments to hedge positions over longer planning horizons. The hedging vehicle could be T-bond futures or GNMA futures which call for delivery of securities of maturities of eight years or longer. If nothing else, use of these contracts which will closer match the durations or maturities of the investment being hedged should simplify the hedge ratio calculation and reduce the variety of contracts that need to be bought or sold to hedge a position. An added difficulty, though, in using T-bond futures contracts is that there are a number of Treasury bonds that are deliverable on any contract. Each of these bonds represents a different coupon and duration combination. The optimal bond to deliver on a contract needs to be determined. This cannot always be accomplished as a unique solution at the time of the inception of the hedge, so the hedge ratio is not determined either. Further analysis of the relationship between bond duration and the optimum bond for delivery on a futures contract might mitigate this difficulty.

Further study could also include differential levels of default risk between the investment and the asset underlying the hedging vehicle. The default risk cross-hedge would consider the variability of the interest rate spread for assets of differing levels of perceived risk. It could be possible to develop appropriate hedging behavior to offset the interest rate risk in the risky bond portfolio of a financial institution using futures positions in default risk-free U. S. Treasury securities.

The entire area of risk measures has been opened to question by the works of Kahneman and Tversky (1979), Coombs (1975), Swalm (1966), and Williams (1966) to name a few. There is reason to expect that future studies will move away from utility maximization as the accepted goal of investors. The new approaches may well change some long-held ideas of normative investment behavior.

APPENDIX A

APPENDIX A

Table A1

```
20 Calculates YTM and counts number cf times each is entered.
30 OPTION BASE 1
40 DIM YTM(84,25)
50 DIM C(84,25)
60 DIM PI(25)
70 DIM AI(25)
80 DIM SU(25)
90 DIM YM(84,25)
100 OPEN "ytm.sum" FOR INPUT AS #1
110 IF EOF(1) THEN CLOSE: GOTO 190
120 INPUT #1, YMT,I,J,CC,AVYLD
130 GOTO 160
140 PRINT "AVYLD ("I","J") is" AVYLD;
150 PRINT "COUNT ("I","J") is" CC
160 YTM(I,J) = YMT
170 C(I,J) = CC
180 GOTO 110
190 OPEN "BME.CMS" FOR INPUT AS #2
200 INPUT #2,BME,CMS
210 CLOSE #2
220 PRINT "  HIGHEST BME IS  ",BME
230 INPUT "  session start maturity  ",IM
240 INPUT "  session start start  ",IS
250 IF BME > 0 THEN GOTO 280
260 BME = IS - 552
270 CMS = IM - IS
280 MT = IM
290 SM = IS
300 GOTO 360
310 INPUT "  next bond maturity  ",MT
320 IF MT = 0 THEN GOTO 1180
330 INPUT "  next bond start month  ",SM
340 IF(MT=MTT) AND (SM=SMT) THEN TQ = 1 ELSE TQ = 0
350 IF MT > SM THEN GOTO 360 ELSE GOTO 310
360 INPUT "  T-Bill = 1  ",BQ
370 IF BQ = 0 THEN GOTO 420
380 IP = 0
390 AC = 0
400 CF = 1
410 GOTO 490
420 INPUT "  interest payment  ",IP
430 INPUT "  INITIAL CASH FLOWS  ",ICF
```

```
440 CF = ICF
450 IF CF = 1 THEN GOTO 490
460 FOR D = 1 TO CF
470 INPUT "  BEGINNING DAYS  ",N(D)
480 NEXT D
490 CM = MT - SM
500 Z = MT - SM
510 BMT = SM - 552
520 GOSUB 1460
530 MTT = MT
540 SMT = SM
550 IF CM > CMS THEN CMS = CM
560 BM = SM - 552
570 IF BM > BME THEN BME = BM
580 IF CF > 1 GOTO 610
590 INPUT "  BEGINNING DAYS  ",J
600 IF Z = 1 THEN GOTO 630
610 FOR G = 1 TO Z
620 IF BMT + G > 85 THEN G = Z: GOTO 1160
630 PP = PI(G)
640 IF BQ = 1 THEN GOTO 800
650 AC = AI(G)
660 IF CF = 1 THEN GOTO 800
670 S = SU(G)
680 IF N(1) - S > 0 THEN GOTO 760
690 IF CF > 1 THEN CF = CF - 1
700 IF CF = 1 THEN J = N(2): GOTO 810
710 FOR T = 1 TO CF
720 W = T + 1
730 N(T) = N(W) - S
740 NEXT T
750 GOTO 820
760 FOR T = 1 TO CF
770 N(T) = N(T) - S
780 NEXT T
790 GOTO 820
800 S = SU(G)
810 J = J - S
820 PV=PP+AC
830 PFS=0
840 LP=IP+100
850 V=365*((((IP*CF)+100)/PV)-1)
860 IF CF=1 THEN R=V/J ELSE R = V/N(CF)
870 IF CF>1 THEN GOTO 910
880 P1=J/365
890 PFS=LP/((1+R)**P1)
900 GOTO 990
910 FOR A=1 TO CF
920 M(A)=N(A)/365
930 IF A=CF THEN GOTO 960
940 PF(A)=IP/((1+R)**M(A))
```

```
950 GOTO 970
960 PF(A)=LP/((1+R)**M(A))
970 PFS=PFS+PF(A)
980 NEXT A
990 IF ABS(PFS-PV) <= .010001 THEN GOTO 1070
1000 IF ABS(PFS - PV) < .05 THEN GOTO 1030
1010 IF PFS > PV THEN R = R + .0001
1020 IF PFS < PV THEN R = R - .0001
1030 IF PFS > PV THEN R=R+.00001
1040 IF PFS < PV THEN R=R-.00001
1050 PFS=0
1060 GOTO 870
1070 YM(BM,CM) = R
1080 YTM(BM,CM) = YTM(BM,CM) + YM(BM,CM)
1090 C(BM,CM) = C(BM,CM) + 1
1100 IF Z = 1 THEN GOTO 1170
1110 IF MT - (BM + 552) = 1 THEN GOTO 1160
1120 IF BM = 84 THEN GOTO 1170
1130 BM = BM + 1
1140 IF BM > BME THEN BME = BM
1150 CM = CM - 1
1160 NEXT G
1170 GOTO 310
1180 OPEN "ytm.sum" FOR OUTPUT AS #1
1190 FOR I = 1 TO BME
1200 FOR J = 1 TO CMS
1210 YMT = 0
1220 YMT = YTM(I,J)
1230 CC = C(I,J)
1240 IF YMT = 0 THEN GOTO 1310
1250 AVYLD = YMT/CC
1260 WRITE #1, YMT,I,J,CC,AVYLD
1270 GOTO 1310
1280 PRINT "AVYLD("I","J")is" AVYLD;
1290 PRINT "("I","J")count is" CC;
1300 PRINT "YTM is" YMT
1310 NEXT J,I
1320 OPEN "BME.CMS" FOR OUTPUT AS #2
1330 WRITE #2, BME, CMS
1340 PRINT "  BME WRITE  " BME "  CMS WRITE  " CMS
1350 CLOSE #1,#2
1360 END
1370 INPUT "  F  ",F
1380 INPUT "  Q  ",Q
1390 IF F = 10 THEN GOTO 1430
1400 C(F,Q) = C(F,Q) - 1
1410 YTM(F,Q) = YTM(F,Q) - YM(F,Q)
1420 GOTO 1370
1430 INPUT "  bm  ",BM
1440 INPUT "  cm  ",CM
1450 STOP
```

```
1460 FOR B = 1 TO Z
1470 IF BMT + B > 85 THEN GOTO 1500
1480 INPUT "  pp  ", PI(B)
1490 NEXT B
1500 IF BQ = 1 THEN GOTO 1550
1510 FOR H = 1 TO Z
1520 IF BMT + H > 85 THEN GOTO 1550
1530 INPUT "  ac  ", AI(H)
1540 NEXT H
1550 IF TQ = 1 THEN GOTO 1600
1560 FOR F = 1 TO Z
1570 IF BMT + F > 85 THEN GOTO 1600
1580 INPUT "  subt  ", SU(F)
1590 NEXT F
1600 RETURN
```

APPENDIX A

Table A2

```
10 FORWARD RATE CALCULATION
20 OPTION BASE 1
30 DIM FWD(24,84)
40 DIM MYL(84,25)
50 DIM YLD(84,25)
60 open "ytm.sum" for input as #1
70 IF EOF(1) THEN CLOSE: GOTO 110
80 input #1, ymt,i,j,cc,avyld
90 YLD(I,J) = AVYLD
100 GOTO 70
110 I = 0
120 J = 0
130 I = I + 1
140 J = J + 1
150 IF YLD(I,J) = 0 THEN A = J - 1: GOTO 480
160 IF J = 1 THEN GOTO 200
170 D = J - 1
180 IF YLD(I,D) = 0 THEN TY2 = YLD(I,J): GOSUB 540
190 IF SW = 1 THEN GOTO 290
200 MYL(I,J) = ((1 + YLD(I,J))**(J/12)) - 1
210 IF (J = 25) AND (I = 84) THEN GOTO 230
220 IF J = 25 THEN GOTO 120 ELSE GOTO 140
230 FOR F = 1 TO 84
240 FOR H = 1 TO 25
250 IF H = 1 THEN GOTO 340
260 K = H - 1
270 N = F - 1
280 IF F = 1 THEN N = F + 1
290 FWD(K,F) = ((1 + MYL(F,H))/(1 + MYL(F,K))) - 1
300 IF FWD(K,F) < 0 AND SS = 0 THEN GOTO 720
310 IF FWD(K,F) < 0 THEN FWD(K,F) = FWD(K,N)
320 SS = 0
330 SW = 0
340 NEXT H,F
350 open "fed.rat" for output as #2
360 FOR L = 1 TO 24
370 FOR M = 1 TO 84
380 FR = FWD(L,M)
390 write #2, fr, l, m
400 FOR T = 1 TO 3
410 IF M = T*L THEN GOTO 460
420 NEXT T
```

```
430 NEXT M,L
440 close #1,#2
450 END
460 PRINT "  FCRWARD RATE ("I","M") IS" FR
470 GOTO 430
480 IF SW = 1 AND J = 25 THEN GOTO 330
490 IF SW = 0 AND J = 25 THEN GOTO 120
500 IF YLD(I,A) = 0 THEN GCTC 140
510 TY1 = YLD(I,A)
520 AA = A
530 GOTO 140
540 PRY = TY2/TY1
550 DD = J
560 Z = DD - AA
570 MPRY = PRY**(1/Z)
580 Q = Z - 1
590 IF Q = 1 THEN GOTO 670
600 FOR G = 1 TO Q
610 W = AA + G
620 P = MPRY**G
630 YLD(I,W) = ((TY1 + 1)**P) - 1
640 MYL(I,W) = ((1 + YLD(I,W))**(W/12)) - 1
650 NEXT G
660 RETURN
670 W = AA + Q
680 P = MPRY
690 YLD(I,W) = ((TY + 1)**P) - 1
700 MYL(I,W) = ((1 + YLD(I,W))**(W/12)) - 1
710 GOTO 660
720 I = F
730 J = H
740 YLD(F,H) = 0
750 SW = 1
760 SS = 1
770 GOTO 150
```

APPENDIX A

Table A3

```
20 LAGGED CROSS-CORRELATIONS (AFTER HAUGH,1976)
30 OPTION BASE 1
40 DIM RHO(50)
50 DIM RHO2(50)
60 DIM CHI(50)
70 OPEN "CHISQ.L24" FOR INPUT AS #1
80 IF EOF(1) THEN CLOSE: GOTO 120
90 INPUT #1, X2,I,J
92 PRINT "  CHISQUARE ALREADY CALCULATED FOR  " I "," J " IS " X2
100 CHI(I) = X2
110 GOTO 80
120 INPUT "  NUMBER OF LAGS + 1  ", K
130 INPUT "  NUMBER OF CROSS CORRELATIONS  ", N
132 INPUT " SESSION START MONTH ", S
140 I = S
150 J = I + 1
152 T = 0
170 L = (2*K) - 1
172 PRINT "INPUT CCF FOR MONTHS " I "AND" J
180 FOR A = 1 TO L
190 INPUT "  RHO  ", RHO(A)
192 IF RHO(A) = 9999 THEN GOTO 272
194 RHO(A)=RHO(A)/1000
200 NEXT A
210 FOR B = 1 TO L
220 M = K-B
230 RHO2 = RHO(B)**2
240 T = T + (RHO2/(N - ABS(M)))
250 NEXT B
260 CHI(I) = T * N**2
270 I = J: GOTO 150
272 PRINT "      HAUGH TEST STATISTICS FIRST FIVE YEARS "
273 PRINT "                              24 LAGS
274 PRINT "
276 PRINT "
280 OPEN "CHISQ.L24" FOR OUTPUT AS #1
288 F = I - 1
290 FOR I = 1 TO F
300 J = I + 1
310 X2 = CHI(I)
320 WRITE #1, X2,I,J
330 PRINT " CHI SQUARE FOR TESTING (" I "," J ") IS " X2 " DF = " I
```

```
340 NEXT I
350 CLOSE #1
360 END
370 STOP
```

APPENDIX A

Table A4

```
C        A PROGRAM TO CALCULATE SIMULATED YIELD CURVES AND
C        THEIR VARIANCES. 100 YIELD CURVES WILL BE FORECASTED
C        FOR EACH STEP AHEAD FORECAST.
50      FORMAT(F10.8)
        DIMENSION F(552),VF(552),S(552),VS(552),FWD(100,24)
        DIMENSION SAVYLD(24),TYLD(100,24),VARNCE(24)
        DIMENSION ISN(50)
        INTEGER Z,A,B,Y,C,D
        C=24
        D=C+24
        DO 11 L=1,50
        READ(1,77)ISN(L)
11      CONTINUE
77      FORMAT(I12)
        ISEED=ISN(D)
        WRITE(6,39)ISEED,C,D
39      FORMAT('  ',I12,' IS THE SEED NUMBER ',I2,' FROM POS ',I2)
80      CALL INTGEN(ISEED)
        READ(4,30)F
        READ(5,40)VF
30      FORMAT(F6.4)
33      FORMAT(F10.6)
40      FORMAT(5X,F6.4)
        DO 20 I=1,100
        DO 25 Y = 1,23
        J=C+((Y-1)*24)
        CALL ANORM(REAL)
        FWD(I,Y)=((SQRT(VF(J))*REAL) + F(J))/1000
        IF(I.GT.1) GOTO 89
        WRITE(12,33)VF(J)
        WRITE(13,33)F(J)
89      WRITE(11,50)FWD(I,Y)
        IF (Y.EQ.1) GOTO 99
        SYLD = SYLD*(FWD(I,Y)+1)
88      T=12.0/Y
        TYLD(I,Y) = (SYLD**T)-1
        WRITE(8,50)TYLD(I,Y)
25      CONTINUE
20      CONTINUE
C        CALCULATE AVERAGE YIELD CURVES AND VARIANCES
        DO 70 A=1,24
        YLDSUM = 0.0
```

```
      DO 60 B=1,100
      YLDSUM = YLDSUM + TYLD(B,A)
60    CONTINUE
      SAVYLD(A) = YLDSUM/100.0
      WRITE(9,50)SAVYLD(A)
      VARSUM = 0.0
C       NOW CALCULATE VARIANCE
      DO 55 L=1,100
      VARSUM = VARSUM + ((TYLD(L,A) - SAVYLD(A))**2)
55    CONTINUE
      VARNCE(A) = VARSUM/100.0
      WRITE(10,50)VARNCE(A)
70    CONTINUE
      GOTO 200
99    SYLD = FWD(I,Y)+1
      GOTO 88
200   STOP
      END
```

APPENDIX A

Table A5

```
C       EXPECTATIONS HEDGE RATIO CALCULIATION
        DIMENSION FWD(24,2300),COUP(100),NP(100),MAT(100),PS(100)
        DIMENSION ACS(100),IT(5),MC(100),HR(8,8),P(24),HN(8,8),HD(8,8)
        DIMENSION C(8),PU(100),ACP(100),MF(8),CBA(100,8),CB(100,8),CD(8)
        DO 30 L = 1,100
        READ(1,15) NP(L),MAT(L),COUP(L),PS(L),ACS(L),MC(L),PU(L),ACP(L)
30      CONTINUE
        DO 10 I = 1,24
        DO 20 J = 1,2300
        READ(2,25) FWD(I,J)
20      CONTINUE
10      CONTINUE
        HRK = 0
        DO 40 M = 1,100
        II = (MAT(M) - NP(M))/6
        MCF = MC(M)
        IF(MCF .NE. 6) II = II + 1
        DO 60 JJ = 1,II
200     C(JJ) = COUP(M)*5
        IF(JJ .EQ. II) C(JJ) = C(JJ) + 1000.0
        MF(JJ) = MCF + (JJ - 1)*6
60      CONTINUE
        NR = NP(M)
        DO 64 KA = 1,100
        SM = 10000000.0/((PU(M) + ACP(M))*10)
104     DO 62 MG = 1,II
        D = 1
        IB = 1 + (KA - 1)*23
        IL = MF(MG) + (KA - 1)*23
        DO 66 KL = IB,IL
400     D = D*(1 + FWD(NR,KL))
66      CONTINUE
        DSQ = D**2
        CD(MG) = C(MG)/DSQ
C       NEXT WE CALCULATE THREE AT A TIME (MONTHS FORWARD) AND
C       CALCULATE APPROPRIATE HEDGE RATIO TO BE USED WITH EACH
C       FUTURES CONTRACT
        DO 90 KP = IB,IL
        KB = KP - IB + 1
600     P(KB) = D/(1 + FWD(NR,KP))
90      CONTINUE
        LC = MF(MG)/3
```

```
      IF(MCF .NE. 3) LC = IC + 1
      IF(MCF .EQ. 6) LC = LC - 1
      DO 92 JC = 1,LC
800   JD = 1 + (JC - 1)*3
      JE = JD + 2
      IF(JE .GT. MF(MG)) JE = MF(MG)
      HN(MG,JC) = 0
      PX = 1
      DO 12 JH = JD,JE
      PX = PX*P(JH)
12    CONTINUE
      HN(MG,JC) = CD(MG)*PX
92    CONTINUE
C     NEXT WE CALCULATE THE DENOMINATOR DPF/DR FOR EACH CONTRACT IN
C     SUCCESSICN
      DO 22 NN = 1,LC
202   NF = IB + (NN-1)*3
      NL = IB + 2 + (NN-1)*3
      DD = 1
      DO 24 MM = NF,NL
      DD = DD*(1 + FWD(NR,MM))
24    CONTINUE
      DDSQ = DD**2
      DDM = 1000000.0/DDSQ
      PF = 1
      DO 26 LL = NF,NL
      PF = PF*(1 + FWD(NR,LL))
26    CONTINUE
      HD(MG,NN) = DDM/DMM
      DMM = DD/PF
      HR(MG,NN) = HN(MG,NN)/HD(MG,NN)
      HRK = HRK + 1
      IF(HRK .EQ. 100) WRITE(4,35)M,KA,HR(MG,NN),MG,NN
      IF(HRK .EQ. 100) HRK = 0
402   CB(KA,NN) = CB(KA,NN) + HR(MG,NN)*SM
22    CONTINUE
62    CONTINUE
64    CONTINUE
      DO 72 MQ = 1,LC
103   CBA(M,MQ) = 0
      DO 74 NC = 1,100
      CBA(M,MQ) = CBA(M,MQ) + CB(NC,MQ)
74    CONTINUE
      CBA(M,MQ) = CBA(M,MQ)/100.0
      WRITE(3,45)CBA(M,MQ),M,MQ
72    CONTINUE
      DO 32 MX = 1,100
      DO 34 MY = 1,LC
      CB(MX,MY) = 0
34    CONTINUE
32    CONTINUE
```

```
40      CONTINUE
35      FORMAT(3X,I3,1X,I3,1X,F15.10,1X,I1,1X,I1)
45      FORMAT(3X,F12.4,1X,I3,1X,I2)
15      FORMAT(3X,2I2,F5.3,F8.5,F7.6,I1,F8.5,F7.6)
25      FORMAT(F9.8)
504     STOP
        END
```

APPENDIX A

Table A6

```
C     DURATION HEDGE CALCULIATION BASED ON FLAT YIELD CURVE ASSUMPTION
      DIMENSION YLC(24),COUP(100),NP(100),MAT(100),DUR(100)
      DIMENSION G(5),KK(100),MC(100),H(5),PS(100),ACS(100)
      DIMENSION CB(100),YLCM(100),YLC3(100),F(5),PU(100),ACP(100)
      DO 30 L=1,100
      READ(1,15)NP(L),MAT(L),COUP(L),PS(L),ACS(L),MC(L),PU(L),ACP(L)
15    FORMAT(3X,2I2,F5.3,F8.5,F7.6,I1,F8.5,F7.6)
30    CONTINUE
      DO 40 K = 1,24
      READ(2,25) YLC(K)
40    CONTINUE
25    FORMAT(3X,F8.6)
      DO 20 IH=1,100
      KK(IH) = 0
      XNUM=0
      XDEN=0
      MCF = MC(IH)
      I = (MAT(IH) - NP(IH))/6
      IF(MCF .NE. 6) I=I+1
      KK(IH) = I
      M = MAT(IH)
100   DO 10 J = 1,I
200   C = COUP(IH)*5
      IF(J .EQ. I) C=C+1000.0
      N = MCF + (J-1)*6
      CN = N * C
      D = (1 + YLC(M))**(N/12)
      YLCM(IH) = 1 + YLC(M)
      F(J) = CN/D
      XNUM = XNUM + F(J)
      G(J) = C/D
      XDEN = XDEN + G(J)
10    CONTINUE
      DUR(IH) = XNUM/(XDEN*12.0)
      HN = (-4.0)*((1 + YLC(3))**1.25)*DUR(IH)*XDEN
      YLC3(IH) = 1 + YLC(3)
      HD = (1 + YLC(M))*1000000.0
      H(IH) = HN/HD
      PP = (PU(IH) + ACP(IH))*10.0
      CB(IH) = (10000000/PP)*H(IH)
20    CONTINUE
      DO 50 JJ = 1,100
```

```
      WRITE(8,35)H(JJ)
      WRITE(9,45)CB(JJ)
50    CONTINUE
35    FORMAT(3X,F8.6)
45    FORMAT(3X,F9.4)
      STOP
      END
```

APPENDIX A

Table A7

```
C       DURATION HEDGE CALCUIATION BASED ON FLAT YIELD CURVE ASSUMPTION
C       ADJUSTED FOR DRI/DRJ BY REGRESSICN ON 3 MONTH YIELDS
        DIMENSION YLC(24),COUP(100),NP(100),MAT(100),DUR(100)
        DIMENSION G(5),BETA(24),KK(100),MC(100),H(5),PS(100),ACS(100)
        DIMENSION CB(100),YLCM(100),YLC3(100),F(5),PU(100),ACP(100)
        DO 30 L=1,100
        READ(1,15)NP(L),MAT(L),COUP(L),PS(L),ACS(L),MC(L),PU(L),ACP(L)
15      FORMAT(3X,2I2,F5.3,F8.5,F7.6,I1,F8.5,F7.6)
30      CONTINUE
        DO 40 K = 1,24
        READ(2,25) YLC(K)
        READ(3,55) BETA(K)
40      CONTINUE
25      FORMAT(3X,F8.6)
        DO 20 IH=1,100
        KK(IH) = 0
        XNUM=0
        XDEN=0
        MCF = MC(IH)
        I = (MAT(IH) - NP(IH))/6
        IF(MCF .NE. 6) I=I+1
        KK(IH) = I
        M = MAT(IH)
100     DO 10 J = 1,I
200     C = COUP(IH)*5
        IF(J .EQ. I) C=C+1000.0
        N = MCF + (J-1)*6
        CN = N * C
        D = (1 + YLC(M))**(N/12)
        YLCM(IH) = 1 + YLC(M)
        F(J) = CN/D
        XNUM = XNUM + F(J)
        G(J) = C/D
        XDEN = XDEN + G(J)
10      CONTINUE
        DUR(IH) = XNUM/(XDEN*12.0)
        HN = (-4.0)*((1 + YLC(3))**1.25)*DUR(IH)*XDEN
        YLC3(IH) = 1 + YLC(3)
        HD = (1 + YLC(M))*1000000.0
        H(IH) = HN/HD
        PP = (PU(IH) + ACP(IH))*10.0
        CB(IH) = ((10000000/PP)*H(IH))*BETA(M)
```

```
20      CONTINUE
        DO 50 JJ = 1,100
        WRITE(8,35)H(JJ)
        WRITE(9,45)CB(JJ)
50      CONTINUE
35      FORMAT(3X,F8.6)
55      FORMAT(F6.5)
45      FORMAT(3X,F9.4)
        STOP
        END
```

APPENDIX A

Table A8

```
C       A PROGRAM TO COMPUTE COMPARATIVE WEALTH CHANGES
        INTEGER X,Y,Z
        DIMENSION NP(100),MAT(100),PS(199),ACS(100),MC(100),PU(100)
        DIMENSION ACP(100),COUP(100),CBD(100),JC(100),CBE(100,8),PFB(100
        DIMENSION MYP(505),MYC(505),FP(505),N(505),PFD(100),WCD(100)
        DIMENSION WCDA(100),PFE(100),WCE(100),WCEA(100)
        DO 10 I = 1,100
        READ(1,15)NP(I),MAT(I),COUP(I),PS(I),ACS(I),MC(I),PU(I),ACP(I)
        READ(2,25)CBD(I)
        JX = MAT(I) - NP(I)
        JC(I) = (JX/3) + 1
        IF (JX .EQ. 3) GO TO 100
        IF(JX .EQ. 6) GO TO 100
        IF (JX .EQ. 9) GO TO 100
        IF (JX .EQ. 12) GO TO 100
        IF (JX .EQ. 15) GO TO 100
        IF (JX .EQ. 18) GO TO 100
        IF (JX .EQ. 21) GO TO 100
        GO TO 110
100     JC(I) = JC(I) - 1
110     JI = JC(I)
        DO 20 K = 1,JI
        READ(3,35)CBE(I,K)
20      CONTINUE
        SPT = PS(I) + ACS(I)
        PUT = PU(I) + ACP(I)
        PFB(I) = ((SPT - PUT)*10.0)*(1000000.0/PUT)
10      CONTINUE
        DO 30 NK = 1,505
        READ(4,45)MYP(NK),MYC(NK),FP(NK)
30      CONTINUE
        JMP = 611
        L = 0
        DO 40 J = 1,505
        IF (JMP .EQ. MYP(J)) GO TO 120
        JMP = MYP(J)
        L = L + 1
120     N(J) = L
        WRITE(10,17) J,N(J),JMP,MYP(J),MYC(J),FP(J)
40      CONTINUE
        DO 22 KZ = 1,100
        PFD(KZ) = 0
```

```
      PFE(KZ) = 0
22    CONTINUE
      DO 50 M = 1,100
      Z = 0
140   Z = Z + 1
      IF (NP(M) .EQ. N(Z)) GO TO 150
      GO TO 140
150   II = MYC(Z) - MYP(Z)
      IF (II .EQ. 2) GO TO 1000
      Y = 0
170   Y = Y + 1
      IF (MYC(Z) .EQ. MYC(Y)) GO TO 180
      GO TO 170
180   PFD(M) = (FP(Z) - FP(Y))*2500*CBD(M)
240   WRITE(12,25) CBD(M)
      WCD(M) = PFD(M) + PFB(M)
      WCDA(M) = ABS(WCD(M))
      JM = JC(M)
      DO 90 X = 1,JM
      IF(II .NE. 2) GO TO 310
      IF(X .NE. 1) GO TO 310
      PFE(M) = PFE(M) + (FP(MM) - FP(LL))*2500*CBE(M,X)
      WRITE(11,25) CBE(M,X),M,X,Y,Z,FP(MM),FP(LL)
      Y = 0
300   Y = Y + 1
      IF (MYC(Z) .EQ. MYC(Y)) GO TO 90
      GO TO 300
310   PFE(M) = PFE(M) + (FP(Y) - FP(Z))*2500*CBE(M,X)
      WRITE(11,25) CBE(M,X),M,X,Y,Z,FP(Y),FP(Z)
      Z = Z + 1
      Y = Y + 1
90    CONTINUE
      WCE(M) = PFE(M) + PFB(M)
900   WCEA(M) = ABS(WCE(M))
      GO TO 50
1000  LL = Z - 1
      JMT = MYP(LL)
1001  LL = LL - 1
      IF (MYP(LL) .EQ. JMT) GO TO 1001
      LL = LL + 1
      MM = 0
1070  MM = MM + 1
      IF (MYC(LL) .EQ. MYC(MM)) GO TO 1080
      GO TO 1070
1080  PFD(M) = (FP(LL) - FP(MM))*2500*CBD(M)
      GO TO 240
50    CONTINUE
      AWCD = 0
      AWCE = 0
      DO 80 MM = 1,100
      AWCD = AWCD + WCD(MM)
```

```
      AWCE = AWCE + WCE(MM)
      WRITE(7,75)WCD(MM),WCE(MM),WCDA(MM),WCEA(MM)
      WRITE(9,95)PFB(MM),PFD(MM),PFE(MM)
80    CONTINUE
      AWCD = AWCD/100.0
      AWCE = AWCE/100.0
      VARD = 0
      VARE = 0
      DO 88 KV = 1,100
      VARD = VARD + (WCD(KV) - AWCD)**2
      VARE = VARE + (WCE(KV) - AWCE)**2
88    CONTINUE
      VARD = (VARD/100)
      VARE = (VARE/100)
      WRITE(8,85)AWCD,AWCE,VARD,VARE
15    FORMAT(3X,2I2,F5.3,F8.5,F7.6,I1,F8.5,F7.6)
25    FORMAT(3X,F9.4,2X,I3,2X,I3,2X,I5,2X,I5,2X,F8.2,2X,F8.2)
35    FORMAT(3X,F12.4)
45    FORMAT(2I3,F4.2)
75    FORMAT(3X,F15.3,2X,F15.3,2X,F15.3,2X,F15.3)
85    FORMAT(3X,F15.3,2X,F15.3,2X,F15.3,2X,F15.3)
95    FORMAT(6X,F15.3,2X,F15.3,2X,F15.3)
17    FORMAT(3X,I3,2X,I2,2X,I3,2X,I3,2X,I3,2X,F5.2)
      STOP
      END
```

APPENDIX A

Table A9

```
C       FISHBURN TEST FOR DOWNSIDE RISK BASED ON RISK AVERSION
C       WHERE ALPHA GOES FROM 1 - 4
        DIMENSION PFD(100),PFE(100),DPF(4),EPF(4),FE(4),FD(4),FIS(4)
        DO 10 I = 1,100
        READ(1,35) PFD(I)
        READ(3,35)PFE(I)
 10     CONTINUE
        DO 30 K = 1,4
        DPF(K) = 0
        EPF(K) = 0
        KK = 0
        LL = 0
        DO 20 J = 1,100
        IF(PFD(J) .LT. 0) DPF(K) = DPF(K) + (PFD(J)**K)
        IF(PFE(J) .LT. 0) EPF(K) = EPF(K) + (PFE(J)**K)
        IF(PFD(J) .LT. 0) KK = KK + 1
        IF(PFE(J) .LT. 0) LL = LL + 1
 20     CONTINUE
        FD(K) = DPF(K)/KK
        FE(K) = EPF(K)/LL
        FIS(K) = FD(K)/FE(K)
        WRITE (2,25)FIS(K)
 30     CONTINUE
 25     FORMAT(3X,F10.5)
 35     FORMAT(3X,F15.3)
        STOP
        END
```

APPENDIX B

APPENDIX B

Table B1

FORWARD RATE FORECASTS
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
1 - 6 MONTHS FORWARD

MONTHS FORWARD

1	2	3	4	5	6
3.7845	3.4316	4.8671	4.4110	3.0788	5.8131
3.9831	3.4316	4.8671	4.4110	3.0788	5.5325
3.8724	3.4316	4.8671	4.4110	3.0788	5.4053
3.9596	3.4316	4.8671	4.4110	3.0788	5.3477
3.4409	3.4316	4.8671	4.4110	3.0788	5.3215
4.0500	3.4316	4.8671	4.4110	3.0788	5.3097
4.0500	3.4316	4.8671	4.4110	3.0788	5.3043
4.0500	3.4316	4.8671	4.4110	3.0788	5.3019
4.0500	3.4316	4.8671	4.4110	3.0788	5.3007
4.0500	3.4316	4.8671	4.4110	3.0788	5.3002
4.0500	3.4316	4.8671	4.4110	3.0788	5.3000
4.0500	3.4316	4.8671	4.4110	3.0788	5.2999
4.0500	3.4316	4.8671	4.4110	3.0788	5.2999
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998
4.0500	3.4316	4.8671	4.4110	3.0788	5.2998

FORWARD RATE FORECASTS
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
7 - 12 MONTHS FORWARD

MONTHS FORWARD

7	8	9	10	11	12
5.3348	5.1414	4.8912	5.5691	5.6395	5.3700
5.3348	5.1414	5.5006	5.5691	5.6395	4.7469
5.3348	5.1414	5.2668	5.5691	5.6395	5.3137
5.3348	5.1414	5.6275	5.5691	5.6395	5.2313
5.3348	5.1414	5.6275	5.5691	5.6395	5.4220
5.3348	5.1414	5.6275	5.5691	5.6395	4.7990
5.3348	5.1414	5.6275	5.5691	5.6395	5.2296
5.3348	5.1414	5.6275	5.5691	5.6395	5.0349
5.3348	5.1414	5.6275	5.5691	5.6395	5.2120
5.3348	5.1414	5.6275	5.5691	5.6395	5.1863
5.3348	5.1414	5.6275	5.5691	5.6395	5.2459
5.3348	5.1414	5.6275	5.5691	5.6395	5.0512
5.3348	5.1414	5.6275	5.5691	5.6395	5.1857
5.3348	5.1414	5.6275	5.5691	5.6395	5.1249
5.3348	5.1414	5.6275	5.5691	5.6395	5.1802
5.3348	5.1414	5.6275	5.5691	5.6395	5.1722
5.3348	5.1414	5.6275	5.5691	5.6395	5.1908
5.3348	5.1414	5.6275	5.5691	5.6395	5.1300
5.3348	5.1414	5.6275	5.5691	5.6395	5.1720
5.3348	5.1414	5.6275	5.5691	5.6395	5.1530
5.3348	5.1414	5.6275	5.5691	5.6395	5.1703
5.3348	5.1414	5.6275	5.5691	5.6395	5.1678
5.3348	5.1414	5.6275	5.5691	5.6395	5.1736
5.3348	5.1414	5.6275	5.5691	5.6395	5.1546

FORWARD RATE FORECASTS
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
13 - 18 MONTHS FORWARD

MONTHS FORWARD

13	14	15	16	17	18
5.4187	5.4187	5.6729	5.8855	5.4960	5.0401
5.4187	5.4187	5.7200	5.8855	5.4960	5.0401
5.4187	5.4187	5.7347	5.8855	5.4960	5.0401
5.4187	5.4187	5.7393	5.8855	5.4960	5.0401
5.4187	5.4187	5.7407	5.8855	5.4960	5.0401
5.4187	5.4187	5.7412	5.8855	5.4960	5.0401
5.4187	5.4187	5.7413	5.8855	5.4960	5.0401
5.4187	5.4187	5.7413	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401
5.4187	5.4187	5.7414	5.8855	5.4960	5.0401

FORWARD RATE FORECASTS
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
19 - 23 MONTHS FORWARD

MONTHS FORWARD

19	20	21	22	23
6.0108	6.0897	5.8875	6.0398	4.8234
6.4682	5.8424	5.8875	6.0398	4.8234
6.1607	5.8424	5.8875	6.0398	4.8234
5.5954	5.8424	5.8875	6.0398	4.8234
7.3248	5.8424	5.8875	6.0398	4.8234
5.4080	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234
5.9175	5.8424	5.8875	6.0398	4.8234

APPENDIX B

Table B2

FORWARD RATE FORECAST VARIANCES
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
1 - 6 MONTHS FORWARD

MONTHS FORWARD

1	2	3	4	5	6
0.4502	0.6141	0.4801	0.5161	1.2580	2.7744
0.9004	0.7667	0.6236	0.7052	1.6267	3.3447
1.3505	0.9193	0.7671	0.8944	1.9954	3.4619
1.8007	1.0718	0.9107	1.0836	2.3641	3.4860
2.2509	1.2244	1.0542	1.2727	2.7328	3.4909
2.7011	1.3770	1.1977	1.4618	3.1015	3.4920
2.8769	1.5296	1.3412	1.6510	3.4702	3.4922
3.0528	1.6821	1.4848	1.8401	3.8389	3.4922
3.2286	1.8347	1.6283	2.0293	4.2076	3.4922
3.4045	1.9873	1.7718	2.2184	4.5763	3.4922
3.5803	2.1398	1.9153	2.4076	4.9450	3.4922
3.7562	2.2924	2.0589	2.5967	5.3137	3.4922
3.9320	2.4450	2.2024	2.7859	5.6824	3.4922
4.1079	2.5976	2.3459	2.9750	6.0511	3.4922
4.2837	2.7501	2.4894	3.1642	6.4198	3.4922
4.4596	2.9027	2.6330	3.3533	6.7885	3.4922
4.6354	3.0553	2.7765	3.5425	7.1572	3.4922
4.8113	3.2079	2.9200	3.7316	7.5259	3.4922
4.9871	3.3604	3.0635	3.9208	7.8946	3.4922
5.1630	3.5130	3.2070	4.1099	8.2634	3.4922
5.3388	3.6656	3.3506	4.2991	8.6321	3.4922
5.5147	3.8181	3.4941	4.4882	9.0008	3.4922
5.6905	3.9707	3.6376	4.6774	9.3695	3.4922
5.8664	4.1233	3.7811	4.8665	9.7382	3.4922

FORWARD RATE FORECAST VARIANCES
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
7 - 12 MONTHS FORWARD

MONTHS FORWARD

7	8	9	10	11	12
2.3544	3.5000	2.0050	2.6400	4.7198	2.1739
2.3544	3.5000	2.2231	2.6400	4.7198	2.1739
2.3544	3.5000	2.2231	2.6400	4.7198	2.3555
2.3544	3.5000	2.5652	2.6400	4.7198	2.3555
2.3544	3.5000	2.5652	2.6400	4.7198	2.3555
2.3544	3.5000	2.5652	2.6400	4.7198	2.3555
2.3544	3.5000	2.5652	2.6400	4.7198	2.5678
2.3544	3.5000	2.5652	2.6400	4.7198	2.5678
2.3544	3.5000	2.5652	2.6400	4.7198	2.5855
2.3544	3.5000	2.5652	2.6400	4.7198	2.5855
2.3544	3.5000	2.5652	2.6400	4.7198	2.5855
2.3544	3.5000	2.5652	2.6400	4.7198	2.5855
2.3544	3.5000	2.5652	2.6400	4.7198	2.6063
2.3544	3.5000	2.5652	2.6400	4.7198	2.6063
2.3544	3.5000	2.5652	2.6400	4.7198	2.6080
2.3544	3.5000	2.5652	2.6400	4.7198	2.6080
2.3544	3.5000	2.5652	2.6400	4.7198	2.6080
2.3544	3.5000	2.5652	2.6400	4.7198	2.6080
2.3544	3.5000	2.5652	2.6400	4.7198	2.6100
2.3544	3.5000	2.5652	2.6400	4.7198	2.6100
2.3544	3.5000	2.5652	2.6400	4.7198	2.6102
2.3544	3.5000	2.5652	2.6400	4.7198	2.6102
2.3544	3.5000	2.5652	2.6400	4.7198	2.6102
2.3544	3.5000	2.5652	2.6400	4.7198	2.6102

FORWARD RATE FORECAST VARIANCES
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
13 - 18 MONTHS FORWARD

MONTHS FORWARD

13	14	15	16	17	18
2.8275	2.6857	1.6702	1.4970	0.9855	1.4569
2.8275	2.6857	1.8333	1.4970	1.0514	1.5429
2.8275	2.6857	1.8492	1.4970	1.1172	1.6289
2.8275	2.6857	1.8508	1.4970	1.1831	1.7150
2.8275	2.6857	1.8509	1.4970	1.2490	1.8010
2.8275	2.6857	1.8510	1.4970	1.3149	1.8870
2.8275	2.6857	1.8510	1.4970	1.3808	1.9730
2.8275	2.6857	1.8510	1.4970	1.4467	2.0590
2.8275	2.6857	1.8510	1.4970	1.5125	2.1450
2.8275	2.6857	1.8510	1.4970	1.5784	2.2310
2.8275	2.6857	1.8510	1.4970	1.6443	2.3171
2.8275	2.6857	1.8510	1.4970	1.7102	2.4031
2.8275	2.6857	1.8510	1.4970	1.7761	2.4891
2.8275	2.6857	1.8510	1.4970	1.8420	2.5751
2.8275	2.6857	1.8510	1.4970	1.9078	2.6611
2.8275	2.6857	1.8510	1.4970	1.9737	2.7471
2.8275	2.6857	1.8510	1.4970	2.0396	2.8331
2.8275	2.6857	1.8510	1.4970	2.1055	2.9192
2.8275	2.6857	1.8510	1.4970	2.1714	3.0052
2.8275	2.6857	1.8510	1.4970	2.2373	3.0912
2.8275	2.6857	1.8510	1.4970	2.3031	3.1772
2.8275	2.6857	1.8510	1.4970	2.3690	3.2632
2.8275	2.6857	1.8510	1.4970	2.4349	3.3492
2.8275	2.6857	1.8510	1.4970	2.5008	3.4352

FORWARD RATE FORECAST VARIANCES
FIRST TEST PERIOD
1 - 24 STEPS AHEAD
19 - 23 MONTHS FORWARD

MONTHS FORWARD

19	20	21	22	23
2.3362	1.9707	2.3567	2.1702	9.6059
2.3362	2.2874	2.3567	2.1702	9.6059
2.3362	2.2874	2.3567	2.1702	9.6059
2.3362	2.2874	2.3567	2.1702	9.6059
2.3362	2.2874	2.3567	2.1702	9.6059
2.3362	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059
2.6353	2.2874	2.3567	2.1702	9.6059

APPENDIX B

Table B3

FORWARD RATE FORECASTS
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
1 - 6 MONTHS FORWARD

MONTHS FORWARD

1	2	3	4	5	6
7.1961	7.4588	7.7067	8.2432	8.1402	8.0010
7.1949	7.4588	7.7067	8.2729	8.1402	8.0010
7.3214	7.4588	7.7067	8.3026	8.1402	8.0010
7.3219	7.4588	7.7067	8.3323	8.1402	8.0010
7.2640	7.4588	7.7067	8.3620	8.1402	8.0010
7.2638	7.4588	7.7067	8.3916	8.1402	8.0010
7.2903	7.4588	7.7067	8.4213	8.1402	8.0010
7.2904	7.4588	7.7067	8.4510	8.1402	8.0010
7.2783	7.4588	7.7067	8.4807	8.1402	8.0010
7.2782	7.4588	7.7067	8.5104	8.1402	8.0010
7.2838	7.4588	7.7067	8.5401	8.1402	8.0010
7.2838	7.4588	7.7067	8.5698	8.1402	8.0010
7.2812	7.4588	7.7067	8.5995	8.1402	8.0010
7.2812	7.4588	7.7067	8.6291	8.1402	8.0010
7.2824	7.4588	7.7067	8.6588	8.1402	8.0010
7.2824	7.4588	7.7067	8.6885	8.1402	8.0010
7.2819	7.4588	7.7067	8.7182	8.1402	8.0010
7.2819	7.4588	7.7067	8.7479	8.1402	8.0010
7.2821	7.4588	7.7067	8.7776	8.1402	8.0010
7.2821	7.4588	7.7067	8.8073	8.1402	8.0010
7.2820	7.4588	7.7067	8.8370	8.1402	8.0010
7.2820	7.4588	7.7067	8.8666	8.1402	8.0010
7.2820	7.4588	7.7067	8.8963	8.1402	8.0010
7.2820	7.4588	7.7067	8.9260	8.1402	8.0010

FORWARD RATE FORECASTS
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
7 - 12 MONTHS FORWARD

MONTHS FORWARD

7	8	9	10	11	12
5.6336	5.5086	5.9227	6.6532	5.9598	5.7918
5.6336	5.5086	5.9227	6.8758	5.9598	5.7918
5.6336	5.5086	5.9227	6.1487	5.9598	5.7918
5.6336	5.5086	5.9227	6.2256	5.9598	5.7918
5.6336	5.5086	5.9227	5.9744	5.9598	5.7918
5.6336	5.5086	5.9227	6.0009	5.9598	5.7918
5.6336	5.5086	5.9227	5.9141	5.9598	5.7918
5.6336	5.5086	5.9227	5.9233	5.9598	5.7918
5.6336	5.5086	5.9227	5.8933	5.9598	5.7918
5.6336	5.5086	5.9227	5.8965	5.9598	5.7918
5.6336	5.5086	5.9227	5.8861	5.9598	5.7918
5.6336	5.5086	5.9227	5.8872	5.9598	5.7918
5.6336	5.5086	5.9227	5.8837	5.9598	5.7918
5.6336	5.5086	5.9227	5.8840	5.9598	5.7918
5.6335	5.5086	5.9227	5.8828	5.9598	5.7918
5.6336	5.5086	5.9227	5.8829	5.9598	5.7918
5.6336	5.5086	5.9227	5.8825	5.9598	5.7918
5.6336	5.5086	5.9227	5.8826	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918
5.6336	5.5086	5.9227	5.8824	5.9598	5.7918

FORWARD RATE FORECASTS
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
13 - 18 MONTHS FORWARD

MONTHS FORWARD

13	14	15	16	17	18
6.0398	5.8836	6.1805	6.1707	7.5771	6.2711
6.0398	5.8836	6.1805	6.1707	4.3598	6.2711
6.0398	5.8836	6.1805	6.1707	7.2410	6.2711
6.0398	5.8836	6.1805	6.1707	4.8663	6.2711
6.0398	5.8836	6.1805	6.1707	6.5934	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711
6.0398	5.8836	6.1805	6.1707	5.8986	6.2711

FORWARD RATE FORECASTS
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
19 - 23 MONTHS FORWARD

MONTHS FORWARD

19	20	21	22	23
6.8715	5.9129	5.8113	6.0340	5.2879
5.4905	5.9129	5.8113	6.0340	5.2879
7.1631	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879
6.3914	5.9129	5.8113	6.0340	5.2879

APPENDIX B

Table B4

FORWARD RATE FORECAST VARIANCES
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
1 - 6 MONTHS FORWARD

MONTHS FORWARD

1	2	3	4	5	6
1.0884	0.8142	0.9079	0.6825	1.7044	2.4846
1.6254	0.9584	1.0959	1.3651	2.0067	3.0403
1.6905	1.1025	1.2838	2.0476	2.3090	3.5961
1.8484	1.2466	1.4717	2.7302	2.6112	4.1518
2.2278	1.3908	1.6596	3.4127	2.9135	4.7075
2.5313	1.5349	1.8475	4.0953	3.2157	5.2632
2.7346	1.6790	2.0354	4.7778	3.5180	5.8189
2.9656	1.8232	2.2234	5.4604	3.8203	6.3747
3.2427	1.9673	2.4113	6.1429	4.1225	6.9304
3.5056	2.1114	2.5992	6.8255	4.4248	7.4861
3.7475	2.2556	2.7871	7.5080	4.7271	8.0418
3.9956	2.3997	2.9750	8.1906	5.0293	8.5975
4.2533	2.5438	3.1629	8.8731	5.3316	9.1532
4.5081	2.6880	3.3509	9.5557	5.6339	9.7090
4.7585	2.8321	3.5388	10.2380	5.9361	10.2650
5.0102	2.9762	3.7267	10.9210	6.2384	10.8200
5.2640	3.1204	3.9146	11.6030	6.5407	11.3760
5.5171	3.2645	4.1025	12.2860	6.8429	11.9320
5.7693	3.4086	4.2904	12.9680	7.1452	12.4880
6.0218	3.5528	4.4784	13.6510	7.4475	13.0430
6.2747	3.6969	4.6663	14.3330	7.7497	13.5990
6.5275	3.8410	4.8542	15.0160	8.0520	14.1550
6.7801	3.9852	5.0421	15.6990	8.3543	14.7100
7.3171	4.1293	5.2300	16.3810	8.6565	15.2660

FORWARD RATE FORECAST VARIANCES
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
7 - 12 MONTHS FORWARD

MONTHS FORWARD

7	8	9	10	11	12
2.4032	3.1702	1.9091	1.7176	3.5478	1.6767
2.4032	3.1702	1.9091	1.7176	3.5478	1.6767
2.4032	3.1702	1.9091	1.9226	3.5478	1.6767
2.4032	3.1702	1.9091	1.9226	3.5478	1.6767
2.4032	3.1702	1.9091	1.9471	3.5478	1.6767
2.4032	3.1702	1.9091	1.9471	3.5478	1.6767
2.4032	3.1702	1.9091	1.9500	3.5478	1.6767
2.4032	3.1702	1.9091	1.9500	3.5478	1.6767
2.4032	3.1702	1.9091	1.9503	3.5478	1.6767
2.4032	3.1702	1.9091	1.9503	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767
2.4032	3.1702	1.9091	1.9504	3.5478	1.6767

FORWARD RATE FORECAST VARIANCES
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
13 - 18 MONTHS FORWARD

MONTHS FORWARD

13	14	15	16	17	18
2.0903	1.9510	3.6818	2.4903	1.8987	2.8173
2.0903	1.9510	3.6818	2.4903	1.8987	2.8173
2.0903	1.9510	3.6818	2.4903	1.8987	2.8173
2.0903	1.9510	3.6818	2.4903	1.8987	2.8173
2.0903	1.9510	3.6818	2.4903	1.8987	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173
2.0903	1.9510	3.6818	2.4903	2.2622	2.8173

FORWARD RATE FORECAST VARIANCES
SECOND TEST PERIOD
1 - 24 STEPS AHEAD
19 - 23 MONTHS FORWARD

MONTHS FORWARD

19	20	21	22	23
2.9818	1.7926	2.3908	2.6519	6.4163
2.9818	1.7926	2.3908	2.6519	6.4163
2.9818	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163
3.2937	1.7926	2.3908	2.6519	6.4163

APPENDIX C

APPENDIX C

Table C1

CONTRACTS SOLD DURATION HEDGE FIRST PERIOD

1	-26.7171	26	-26.9294	51	-38.6494	76	-37.1797
2	-10.2573	27	-10.2581	52	-33.2000	77	-52.4402
3	-26.4225	28	-26.4884	53	-23.5773	78	-29.6867
4	-30.5979	29	-6.7700	54	-13.5400	79	-34.0787
5	-36.5887	30	-6.5590	55	-17.1673	80	-26.3159
6	-30.4946	31	-6.6281	56	-42.2514	81	-57.6188
7	-3.3850	32	-10.2200	57	-31.1144	82	-29.8113
8	-16.3709	33	-38.2105	58	-10.2343	83	-3.4044
9	-27.2970	34	-16.7655	59	-19.9227	84	-42.0333
10	-30.1335	35	-13.4124	60	-13.6458	85	-13.4372
11	-16.7965	36	-6.8669	61	-47.7456	86	-30.7905
12	-3.2795	37	-23.0828	62	-44.9573	87	-30.3011
13	-3.3291	38	-3.4283	63	-29.8941	88	-10.2146
14	-27.1172	39	-38.4915	64	-6.6611	89	-10.1550
15	-13.1181	40	-33.5499	65	-10.1824	90	-19.6451
16	-19.9748	41	-19.9833	66	-33.9532	91	-36.9663
17	-13.3166	42	-6.6409	67	-45.4172	92	-26.8618
18	-6.6654	43	-6.6428	68	-20.3099	93	-23.6365
19	-10.1861	44	-10.2126	69	-16.9249	94	-19.9961
20	-19.9443	45	-10.0779	70	-49.0439	95	-34.2840
21	-9.9614	46	-9.9421	71	-41.8526	96	-23.3814
22	-16.6386	47	-16.5702	72	-3.4164	97	-33.2998
23	-13.6195	48	-3.3140	73	-6.7883	98	-6.8097
24	-9.9721	49	-23.8035	74	-20.5696	99	-23.8637
25	-16.6070	50	-13.2562	75	-26.8344	100	-9.9916

APPENDIX C

Table C2

CONTRACTS SOLD
ADJUSTED DURATION HEDGE
FIRST PERIOD

1	-21.1244	26	-23.2486	51	-27.7163	76	-26.6623
2	-9.6146	27	-9.6153	52	-26.6025	77	-34.8318
3	-21.1718	28	-19.5061	53	-15.6605	78	-20.9122
4	-26.0067	29	-5.5442	54	-11.0884	79	-28.9651
5	-29.3178	30	-4.8301	55	-16.1611	80	-18.5377
6	-20.2551	31	-5.2406	56	-28.0642	81	-40.5884
7	-2.7721	32	-9.5796	57	-26.8617	82	-23.8872
8	-13.1177	33	-28.1382	58	-8.8355	83	-3.4044
9	-22.3546	34	-11.1360	59	-17.5724	84	-30.1429
10	-23.8256	35	-8.9088	60	-11.7807	85	-11.4209
11	-14.2762	36	-6.4644	61	-33.6334	86	-25.2156
12	-2.4150	37	-16.9982	62	-33.1065	87	-21.7296
13	-3.0407	38	-3.1312	63	-22.0140	88	-9.5746
14	-23.0483	39	-30.4340	64	-4.7768	89	-8.3163
15	-9.6602	40	-26.5269	65	-9.9100	90	-15.7412
16	-18.2440	41	-14.3304	66	-22.5524	91	-29.2281
17	-12.1627	42	-5.8575	67	-32.5696	92	-19.2632
18	-5.7543	43	-5.8825	68	-16.6326	93	-20.0898
19	-9.9136	44	-9.5727	69	-13.8605	94	-17.2630
20	-13.2474	45	-8.5657	70	-32.5759	95	-28.0766
21	-8.7862	46	-7.8609	71	-33.0916	96	-20.7054
22	-11.0517	47	-13.1016	72	-3.4164	97	-24.5219
23	-12.7661	48	-2.6203	73	-6.6067	98	-6.3830
24	-6.6237	49	-19.4936	74	-18.7873	99	-21.0485
25	-14.7063	50	-10.4813	75	-23.7632	100	-7.1652

APPENDIX C

Table C3

CONTRACTS SOLD
EXPECTATIONS HEDGE
FIRST PERIOD

10.7446	1	1	10.9474	14	1	10.1299	31	1
10.4455	1	2	10.5837	14	2	10.4574	32	1
10.1633	1	3	10.2915	14	3	10.9622	33	1
10.4956	2	1	10.1465	15	1	10.9693	33	2
10.6538	3	1	9.8931	15	2	10.6061	33	3
10.3113	3	2	10.3546	16	1	10.6168	33	4
10.0417	3	3	10.3657	16	2	10.4372	34	1
11.0133	4	1	10.2679	17	1	10.2751	34	2
10.6446	4	2	10.0205	17	2	10.3888	35	1
10.6745	4	3	10.1706	18	1	10.1325	35	2
10.8426	5	1	10.4004	19	1	10.4112	36	1
10.8422	5	2	10.3887	20	1	10.6617	37	1
10.5216	5	3	10.3895	20	2	10.2798	37	2
10.0328	5	4	10.1923	21	1	9.8250	37	3
10.9678	6	1	10.3582	22	1	10.4420	38	1
10.6458	6	2	10.1973	22	2	10.9903	39	1
10.6818	6	3	10.4875	23	1	10.9953	39	2
10.3284	7	1	10.2479	23	2	10.7059	39	3
10.1433	8	1	10.2589	24	1	10.7058	39	4
10.0032	8	2	10.2780	25	1	10.8657	40	1
10.9810	9	1	10.1353	25	2	10.8625	40	2
10.6542	9	2	10.9041	26	1	10.5895	40	3
10.3825	9	3	10.4999	26	2	9.7658	40	4
10.8077	10	1	10.2131	26	3	10.3962	41	1
10.5001	10	2	10.4963	27	1	10.4032	41	2
10.5326	10	3	10.7313	28	1	10.1182	42	1
10.4063	11	1	10.3440	28	2	10.1383	43	1
10.2707	11	2	10.0706	28	3	10.4498	44	1
10.0197	12	1	10.3344	29	1	10.3284	45	1
10.1400	13	1	10.0218	30	1	10.2012	46	1

CONTRACTS SOLD
EXPECTATIONS HEDGE
FIRST PERIOD
(CONT.)

10.2870	47	1	10.8227	57	3	10.5488	68	1
10.1302	47	2	10.5005	58	1	10.5593	68	2
10.1139	48	1	10.3353	59	1	10.5056	69	1
10.9329	49	1	10.3399	59	2	10.3417	69	2
10.6089	49	2	10.5322	60	1	11.6738	70	1
10.1205	49	3	10.2892	60	2	11.3531	70	2
10.2405	50	1	11.3928	61	1	11.3851	70	3
9.9944	50	2	11.0628	61	2	11.0485	70	4
11.0779	51	1	11.1001	61	3	11.0626	70	5
11.0827	51	2	10.7621	61	4	11.3512	71	1
10.7794	51	3	10.7620	61	5	11.0504	71	2
10.7871	51	4	11.4551	62	1	11.0584	71	3
10.7760	52	1	11.0814	62	2	10.7638	71	4
10.7743	52	2	11.0932	62	3	9.6135	71	5
10.4591	52	3	10.7178	62	4	10.3242	72	1
9.6463	52	4	10.0772	62	5	10.3285	73	1
10.8562	53	1	10.7807	63	1	10.6629	74	1
10.5401	53	2	10.4027	63	2	10.6744	74	2
10.0558	53	3	10.4282	63	3	10.8456	75	1
10.4589	54	1	10.1933	64	1	10.4555	75	2
10.1973	54	2	10.3967	65	1	10.1612	75	3
10.5660	55	1	11.0460	66	1	11.0184	76	1
10.4291	55	2	11.0359	66	2	11.0212	76	2
11.5331	56	1	10.7518	66	3	10.7212	76	3
11.2149	56	2	9.8937	66	4	10.2426	76	4
11.2366	56	3	11.5431	67	1	11.7257	77	1
10.9278	56	4	11.2153	67	2	11.7304	77	2
9.7493	56	5	11.2365	67	3	11.4356	77	3
11.2092	57	1	10.9023	67	4	11.4212	77	4
10.7994	57	2	10.2491	67	5	11.1109	77	5

CONTRACTS SOLD
EXPECTATIONS HEDGE
FIRST PERIOD
(CONT.)

9.5814	77	6
10.7007	78	1
10.3730	78	2
10.4067	78	3
11.0581	79	1
11.0610	79	2
10.7178	79	3
9.8921	79	4
10.6412	80	1
10.3096	80	2
10.0377	80	3
11.5771	81	1
11.5894	81	2
11.2743	81	3
11.2861	81	4
10.9360	81	5
10.9371	81	6
10.7122	82	1
10.3644	82	2
10.3993	82	3
10.2879	83	1
11.4737	84	1
11.1484	84	2
11.1637	84	3
10.8489	84	4
9.6610	84	5
10.3888	85	1
10.1156	85	2
11.0492	86	1
10.7223	86	2
10.7533	86	3
10.9139	87	1
10.5769	87	2
10.6112	87	3
10.4519	88	1
10.4232	89	1
10.2123	90	1
10.2109	90	2
10.9331	91	1
10.9357	91	2
10.6533	91	3
10.1717	91	4
10.8331	92	1
10.5039	92	2
10.2377	92	3
10.8838	93	1
10.5179	93	2
10.0451	93	3
10.3729	94	1
10.3774	94	2
11.1196	95	1
11.1163	95	2
10.8193	95	3
9.9502	95	4
10.8022	96	1
10.4057	96	2
9.9107	96	3
10.8402	97	1
10.8383	97	2
10.4898	97	3
9.6683	97	4
10.3677	98	1
10.9756	99	1
10.6134	99	2
10.1109	99	3
10.2628	100	1

APPENDIX C

Table C4

CONTRACTS SOLD
DURATION HEDGE
SECOND PERIOD

1	-10.5329	26	-45.9357	51	-3.6711	76	-10.6961
2	-14.6128	27	-39.1218	52	-40.2809	77	-21.6025
3	-61.6973	28	-10.6781	53	-10.8012	78	-25.1562
4	-28.6937	29	-7.0251	54	-13.9777	79	-55.1048
5	-36.4543	30	-21.2967	55	-43.3260	80	-32.3852
6	-36.6288	31	-57.4182	56	-69.5234	81	-36.5414
7	-7.3064	32	-28.7363	57	-24.6976	82	-46.6985
8	-10.6483	33	-28.4739	58	-65.4377	83	-46.1565
9	-14.2882	34	-36.3277	59	-3.5125	84	-39.6911
10	-43.6320	35	-10.7162	60	-36.5325	85	-39.9618
11	-60.8635	36	-14.2041	61	-20.9617	86	-56.5456
12	-14.2480	37	-56.5811	62	-6.9889	87	-21.4477
13	-53.8951	38	-73.2150	63	-7.1492	88	-32.8067
14	-17.6268	39	-51.5242	64	-62.1403	89	-24.6611
15	-14.4527	40	-36.0812	65	-24.5873	90	-24.7008
16	-39.8064	41	-39.4306	66	-49.6196	91	-17.5413
17	-39.2245	42	-68.0879	67	-21.1089	92	-40.3662
18	-59.9070	43	-50.7044	68	-10.7693	93	-47.1786
19	-10.5633	44	-17.9673	69	-10.5700	94	-7.0467
20	-40.0959	45	-7.1307	70	-14.1978	95	-10.6531
21	-14.2374	46	-3.5233	71	-57.3946	96	-51.0761
22	-6.9858	47	-42.6296	72	-29.4685	97	-40.7797
23	-10.5545	48	-28.7025	73	-42.6618	98	-54.2638
24	-54.6716	49	-47.4806	74	-3.5254	99	-7.0165
25	-3.5594	50	-39.3059	75	-29.0075	100	-13.9333

APPENDIX C

Table C5

CONTRACTS SOLD
ADJUSTED DURATION HEDGE
SECOND PERIOD

1	-8.4398	26	-32.3585	51	-3.4559	76	-10.0692
2	-9.0753	27	-27.5586	52	-23.9901	77	-14.3488
3	-36.7450	28	-7.8633	53	-7.1744	78	-15.3833
4	-18.0578	29	-6.8372	54	-11.2001	79	-34.2228
5	-26.1421	30	-13.2263	55	-31.1622	80	-20.1128
6	-29.9968	31	-41.2980	56	-43.1775	81	-24.2715
7	-4.5377	32	-21.1614	57	-19.7897	82	-34.3888
8	-6.6131	33	-24.5821	58	-38.9727	83	-32.5140
9	-8.7374	34	-22.2147	59	-3.4186	84	-29.2285
10	-31.2894	35	-6.5530	60	-21.7577	85	-24.4370
11	-37.7993	36	-8.9391	61	-14.7661	86	-39.8324
12	-13.4130	37	-37.2258	62	-5.6000	87	-18.2294
13	-38.7641	38	-45.4702	63	-6.0765	88	-21.7908
14	-13.9370	39	-31.9991	64	-38.5922	89	-21.7518
15	-11.8359	40	-22.7070	65	-19.7013	90	-16.2511
16	-29.3134	41	-25.9421	66	-34.9535	91	-11.5407
17	-31.4298	42	-42.8497	67	-14.8697	92	-33.0575
18	-39.4140	43	-36.3611	68	-6.4139	93	-31.3370
19	-10.2807	44	-12.8847	69	-9.1253	94	-6.0835
20	-28.7536	45	-6.7128	70	-8.8175	95	-6.7043
21	-10.4844	46	-3.0418	71	-36.1201	96	-30.4194
22	-6.9858	47	-33.7059	72	-18.3014	97	-25.3262
23	-7.4349	48	-24.7795	73	-28.0680	98	-36.0431
24	-32.5608	49	-28.2780	74	-2.7874	99	-4.6163
25	-2.6211	50	-25.8601	75	-20.8019	100	-12.7260

APPENDIX C

Table C6

CONTRACTS SOLD
EXPECTATIONS HEDGE
SECOND PERIOD

10.8142	1	1	12.9118	11	2	11.7945	18	6
11.2928	2	1	12.3702	11	3	10.8043	19	1
10.7992	2	2	12.3719	11	4	12.1133	20	1
13.1238	3	1	11.8996	11	5	12.1359	20	2
13.1569	3	2	11.8885	11	6	11.6511	20	3
12.5603	3	3	11.0485	12	1	11.6528	20	4
12.5684	3	4	10.5857	12	2	11.0308	21	1
12.0659	3	5	12.8236	13	1	10.5358	21	2
12.0558	3	6	12.8343	13	2	10.5542	22	1
11.7285	4	1	12.3294	13	3	10.8226	23	1
11.3319	4	2	12.2867	13	4	12.9515	24	1
10.6523	4	3	11.9018	13	5	12.9588	24	2
11.9714	5	1	9.7717	13	6	12.3966	24	3
11.9836	5	2	11.0308	14	1	12.3385	24	4
11.5097	5	3	10.7320	14	2	11.9046	24	5
10.1228	5	4	11.2140	15	1	9.7533	24	6
12.0530	6	1	10.7430	15	2	10.7771	25	1
12.0630	6	2	11.9456	16	1	12.4004	26	1
11.6112	6	3	11.9557	16	2	11.9935	26	2
10.2133	6	4	11.4845	16	3	11.8875	26	3
11.0262	7	1	10.7053	16	4	11.4882	26	4
10.8672	8	1	11.7977	17	1	10.5162	26	5
11.0657	9	1	11.8128	17	2	11.7519	27	1
10.5819	9	2	11.3645	17	3	11.7651	27	2
12.5779	10	1	10.5990	17	4	11.2611	27	3
12.2134	10	2	12.7725	18	1	10.4998	27	4
12.0816	10	3	12.8012	18	2	10.9294	28	1
11.7197	10	4	12.2630	18	3	10.6157	29	1
9.9561	10	5	12.2789	18	4	11.1553	30	1
12.8876	11	1	11.8143	18	5	11.1713	30	2

CONTRACTS SOLD
EXPECTATIONS HEDGE
SECOND PERIOD
(CONT.)

12.9156	31	1	12.5329	38	6	10.6958	46	1
12.9227	31	2	12.5154	38	7	12.3909	47	1
12.4177	31	3	12.8671	39	1	12.0226	47	2
12.4082	31	4	12.5045	39	2	11.9029	47	3
11.9877	31	5	12.4201	39	3	11.5551	47	4
10.7917	31	6	12.0528	39	4	9.8057	47	5
11.7353	32	1	12.0490	39	5	11.7058	48	1
11.3480	32	2	11.8435	40	1	11.3848	48	2
10.6960	32	3	11.8351	40	2	10.7367	48	3
11.6806	33	1	11.3363	40	3	12.7801	49	1
11.2698	33	2	9.9993	40	4	12.3243	49	2
10.6282	33	3	11.9629	41	1	12.2161	49	3
11.9939	34	1	11.9897	41	2	11.7473	49	4
12.0098	34	2	11.4499	41	3	10.7746	49	5
11.4619	34	3	11.4523	41	4	11.8128	50	1
10.0574	34	4	13.5059	42	1	11.8296	50	2
10.9742	35	1	13.0953	42	2	11.3162	50	3
10.9744	36	1	12.9808	42	3	10.5619	50	4
10.5178	36	2	12.5736	42	4	11.1403	51	1
12.7220	37	1	12.5279	42	5	12.1158	52	1
12.7380	37	2	12.1229	42	6	12.1378	52	2
12.2078	37	3	10.6634	42	7	11.5589	52	3
12.1870	37	4	12.7291	43	1	10.7649	52	4
11.7598	37	5	12.3498	43	2	11.0797	53	1
10.5820	37	6	12.2631	43	3	10.8493	54	1
13.7238	38	1	11.8698	43	4	10.4016	54	2
13.3729	38	2	11.8636	43	5	12.6045	55	1
13.2791	38	3	11.2135	44	1	12.2003	55	2
12.9153	38	4	10.8760	44	2	12.0764	55	3
12.9085	38	5	10.8126	45	1	11.6986	55	4

CONTRACTS SOLD
EXPECTATIONS HEDGE
SECOND PERIOD
(CONT.)

9.9168	55	5	12.6378	64	4	9.7291	73	5
13.5903	56	1	12.2475	64	5	10.7166	74	1
13.2538	56	2	12.2361	64	6	11.8178	75	1
13.1411	56	3	11.4852	65	1	11.4505	75	2
12.7938	56	4	11.1297	65	2	10.7941	75	3
12.7607	56	5	10.0516	65	3	10.9743	76	1
12.3753	56	6	12.4903	66	1	11.3476	77	1
10.9431	56	7	12.1161	66	2	11.3589	77	2
11.5210	57	1	12.0378	66	3	11.7848	78	1
11.1824	57	2	11.6276	66	4	11.3410	78	2
10.0992	57	3	11.6301	66	5	10.2358	78	3
13.7048	58	1	11.0926	67	1	12.9378	79	1
13.2391	58	2	11.1267	67	2	12.9535	79	2
13.1021	58	3	10.9929	68	1	12.4980	79	3
12.6665	58	4	10.8737	69	1	12.4591	79	4
12.5847	58	5	10.9721	70	1	12.1290	79	5
12.1527	58	6	10.4925	70	2	9.9044	79	6
9.5494	58	7	12.7951	71	1	11.8005	80	1
10.6230	59	1	12.8058	71	2	11.3858	80	2
12.0171	60	1	12.2938	71	3	11.2903	80	3
12.0164	60	2	12.2643	71	4	12.0435	81	1
11.4624	60	3	11.8386	71	5	12.0339	81	2
10.0980	60	4	10.6554	71	6	11.5568	81	3
11.0152	61	1	11.9569	72	1	10.1902	81	4
11.0491	61	2	11.6115	72	2	12.5514	82	1
10.6138	62	1	10.9427	72	3	12.1748	82	2
10.8559	63	1	12.4154	73	1	12.0686	82	3
13.0752	64	1	11.9912	73	2	11.6951	82	4
13.0997	64	2	11.8698	73	3	10.7118	82	5
12.6359	64	3	11.4769	73	4	12.4242	83	1

CONTRACTS SOLD
EXPECTATIONS HEDGE
SECOND PERIOD
(CONT.)

12.0453	83	2	10.6473	91	2
11.9404	83	3	12.1222	92	1
11.5688	83	4	12.1293	92	2
10.5901	83	5	11.6692	92	3
12.0161	84	1	10.8979	92	4
12.0441	84	2	12.7349	93	1
11.5410	84	3	12.3563	93	2
11.5509	84	4	12.2384	93	3
12.1030	85	1	11.8443	93	4
12.1151	85	2	10.8542	93	5
11.5584	85	3	10.7020	94	1
11.5631	85	4	10.8854	95	1
12.6294	86	1	12.8681	96	1
12.6456	86	2	12.4139	96	2
12.1672	86	3	12.3277	96	3
12.1544	86	4	11.8305	96	4
11.7660	86	5	11.8306	96	5
10.5770	86	6	12.1625	97	1
11.3064	87	1	12.1629	97	2
11.3181	87	2	11.7165	97	3
11.9621	88	1	10.9426	97	4
11.5746	88	2	12.8930	98	1
11.4889	88	3	12.8946	98	2
11.4734	89	1	12.4082	98	3
11.1674	89	2	12.3769	98	4
10.1140	89	3	11.9906	98	5
11.5646	90	1	9.8592	98	6
11.1652	90	2	10.6291	99	1
10.0563	90	3	10.8224	100	1
10.9709	91	1	10.3624	100	2

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