risk
economics

A NEW MEASURE OF ATTITUDE TOWARDS RISK

By

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In loving memory

of my father,

Oscar T. Fleisher, Jr.

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CHAPTER 1

INTRODUCTION: WHY DO WE NEED A NEW MEASURE OF ATTITUDE TOWARDS RISK?

The concept of an attitude towards risk which is unique to the individual is used both formally and informally by agricultural economists concerned with a wide variety of problem settings in which uncertainty is present. Attitudes towards risk are used formally in predictive models of agricultural sector participants' responses to changes in the economic, institutional or technological environments and in formulating extension recommendations. Perhaps more commonly, the truism that "all farmers are risk averse" is used by agricultural economists and policy makers in less formal analysis.

Young, et. al. (1979) state that "knowledge of risk preferences of individual agricultural producers is necessary for many useful private managerial and public policy analyses of decision making under risk." Agricultural economists have perceived the need to incorporate the effects of uncertainty into their analysis and have, over the past thirty years, engaged in numerous endeavors to quantify attitudes towards risk. Despite this accumulated body of research, the profession still has little consistent evidence supporting the assumption of risk aversion on the part of agricultural sector participants. Nevertheless, this assumption continues to be the cornerstone of much of the disciplinary and problem solving research done today.

Young, et. al. expressed their primary reservations with the elicitation and use of risk preferences as twofold: (1) the errors inherent in previously used measurement techniques, and (2) the possible temporal instability of preferences. It should also be noted that there is little evidence to support the common assumption that risk preferences elicited in one decision context can be generalized to other contexts, even at the same point in time.

This dissertation focuses on the problem of the instability of attitudes towards risk indicated by current measures. The research is motivated by the assumption that individuals' attitudes towards risk are stable, and that the perceived instability is due to the measures used. We begin with an examination of why currently used measures of attitudes towards risk result in the appearance of instability. A new measure of attitude towards risk is then developed which is hypothesized to be stable globally within one decision context, and also both across decision contexts and intertemporally.

Some Basic Concepts and Definitions

The study of attitudes towards risk and their formal use in analysis is part of a broader field of inquiry known as decision theory or decision analysis. Although used extensively by economists and agricultural economists, decision theory finds its home in no one discipline. Developments in decision theory are drawn from and used in many disciplines including economics, management science, mathematical psychology, finance, anthropology, sociology, statistics and systems science. While this conflux of numerous, seemingly unrelated, disciplines often creates definitional problems (Robison and Fleisher, 1983), it, at the same time, provides a varied and fertile set of conceptual frameworks from which to draw. In fact, the foundations for much of the development of the new measure of attitude towards risk presented in this dissertation come not only from economics, but also from recent theoretical work in other disciplines including mathematical psychology, management science and systems science.

An example of the definitional problems inherent in work in the field of decision theory is the term relative risk aversion; economists encountering this term immediately think of the Arrow-Pratt coefficient of relative risk aversion, described later in this chapter, while mathematical psychologists think of an entirely different concept, to be discussed in detail in Chapter 3. To avoid confusion, the mathematical psychologists concept of relative risk aversion is referred to as intrinsic risk aversion in this document.

While many terms will be defined as they are needed, several concepts used throughout the dissertation are defined here.

Risk and uncertainty, two of the most commonly used terms in decision theory, are probably the most difficult to define. In fact, Stiglitz has commented that, "Risk is like love; we have a good idea of what it is, but we can't define it precisely" (Roumasset, 1979, p. 4). Fleisher and Robison's definitions will be used here. They state (Fleisher and Robison, 1985, pp. 3-4):

> "Certain and uncertain are adjectives used to describe events. Events with only one possible outcome are defined as certain; the single outcome has a probability of one of occurring. Uncertain events are those with more than one possible outcome; each possible outcome has a probability between zero and one of occurring. Since events are either certain or uncertain, we cannot say that one event is more or less uncertain than another. Each event is either uncertain or certain.

> A class of uncertain events which alter the well-being of either a well defined class of decision makers or a single decision maker are called risky events. Riskiness, because it depends on the decision makers' attitudes, likes and dislikes, cannot be made more precise without first defining whose well-being is used to give meaning to the concept. Once we define the class of decision makers, we may be able to make comparative statements like action choice A's outcomes are more or less risky than B's. The important point to remember is that an event's riskiness depends on the preference of an individual or a class of individual decision makers. Riskiness cannot and should not be used interchangeably with the word uncertainty. Riskiness should also not be confused with the dictionary definition of the noun 'risk.' Risk is defined in the dictionary as the possibility of loss or injury. In the context of decision analysis, however, a risky event may result in favorable or unfavorable consequences for the decision maker.

Another distinction which needs to be made is between action choices, outcomes and attributes. Action choices are defined as the set of alternatives available to the decision maker, including the alternative of doing nothing. In a tractor purchase decision, a set of action choices might include buying brand A, buying brand B, or not purchasing a tractor. An outcome can be thought of as the result or consequence of an action choice. In the case of a tractor purchase, the outcome may be buying brand A tractor and having it live up to the purchasers' expectations. An attribute, on the other hand, is a specific characteristic of an action choice or an outcome. In the case of the tractor purchase decision, attributes of each tractor could include fuel efficiency, PTO and draw bar power.

Currently Used Measures of Attitude Towards Risk

Models of decision making and their application are as diverse as decision theory's bases. Some models of decision making are descriptive, attempting to capture and explain the cognitive processes used in making actual decisions. Others are prescriptive and are used to help decision makers make "better" decisions. The third group can be classified as predictive. These models make no attempt to formally model decision making processes. Instead, they treat the cognitive decision process as a black box; decision makers' preferred action choices are predicted <u>as if</u> they followed a certain set of procedures without assuming that these are the procedures actually used.

Although increasing use is being made of prescriptive models by agricultural economists working in farm management and related fields, economists, in general, tend to be primarily concerned with predictive models of decision making under uncertainty. The research presented here also falls into this category.

While there are numerous models which can be used to predict preferred action choices under uncertainty, Hey (1979) estimates that the expected utility hypothesis provides the basis for over 90% of the disciplinary work (in economics) on individual or firm behavior under uncertainty. The expected utility hypothesis was first formally deduced from a set of axioms by Ramsey (1931) and later developed more fully by von Neumann and Morgenstern (1944). The expected utility hypothesis asserts that if a decision maker's preferences satisfy certain conditions of "rationality", their utility function elicited over risky prospects can be used to predict their preferred action choices in other risky situations. Specifically, von Neumann and Morgenstern proved that their four basic axioms imply the existence of numerical utilities for outcomes such that the individuals' expectations for lotteries over these outcomes preserve their preference orderings, i.e. greater expected utility corresponds to higher preference. In von Neumann and Morgenstern's own words, they "practically defined numerical utility as being that thing for which a calculus of expectations is legitimate" (von Neumann and Morgenstern, from Schoemaker, 1982, p. 531). Because of their role in the formalization of the expected utility hypothesis, a utility function elicited over risky prospects is usually referred to as a von Neumann-Morgenstern utility function.

The four axioms of "rational behavior" which expected utility maximizers are assumed to follow are:

1. Ordering: If an individual confronts two risky prospective action choices, A and B each with more than one potential outcome or with a probability distribution of outcomes, he will prefer one of the two risky prospects or will be indifferent between them.

2. Transitivity: If the individual confronts three risky prospects, A, B and C and prefers A to B and B to C, then he will also prefer A to C.

3. Continuity: If an individual prefers A to B to C, then there exists a unique probability, p, such that he will be indifferent between B and a lottery of the form pA + (1 - p)C.

4. Independence: If action choice A is preferred to B and C is some other lottery, then the individual will prefer a lottery of pA + (1-p)C to the lottery pB + (1-p)C.

The four axioms of rational behavior provide both the mathematical foundations for expected utility theory and the behavioral conditions that an individual's preferences must meet to be an expected utility maximizer. Satisfaction of the behavioral requirements can be viewed as a precondition for expected utility maximization. If a decision maker's preferences conform to these axioms, a utility function u(x) can be formulated which reflects the preferences of the decision maker. A utility function u(x) derived for an expected utility maximizer has the following properties:

1. If x_1 is preferred to x_2 then $u(x_1) > u(x_2)$.

2. The utility of a risky prospect is equal to the expected utility of its possible outcomes.

3. The scale on which utility is measured is arbitrary; utility functions are unique up to a positive linear transformation.

In addition to the four formal conditions of "rational behavior" listed above, many applications of the expected utility hypothesis implicitly assume that several other conditions are met. These include:

1. That outcomes can be denominated in terms of a common unit.
2. That there is homogeneity of preferences for attributes of riskless outcomes[1].
3. That individuals can accurately educe their preferences.
4. That individuals subjective probabilities are the same as stated "objective" probabilities.[2]
5. That utility functions elicited using hypothetical decision contexts are applicable to actual decisions.

Different variants exist within the general expected utility framework depending on what type of probability transformations are allowed, how the outcomes are measured and how utility is measured.

While the axiomatization of the expected utility hypothesis was a major step in the development of decision theory, a suitable numerical measure of attitude towards risk was needed to arrive at quantifiable theories and predictions of decision makers' behavior. The application of the expected utility hypothesis to empirical problems was given impetus when Pratt (1964) and Arrow (1965) set forth a means of characterizing individuals on the basis of a numerical measure of their attitude towards risk. Pratt

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[1] Homogeneity of preferences implies that, at equilibrium, each individual values equally, or derives the same amount of utility from, the last unit of each commodity purchased and that this value or utility is identical with the value or utility received from an amount of money identical to the goods' price. This assumption ignores the existance of consumer surplus.

[2] Several variants of the expected utility hypothesis, such as the subjective expected utility and subjectively weighted utility models do not assume that this condition holds. Readers are referred to Schoemaker (1982) for an interesting exposition on different schools of probability theory related to assumption number four.

showed that a numerical measure of risk aversion, based on the rate of bending of an individual's von Neumann-Morgenstern utility function, is unique and invariant under positive linear transformations of the utility function. The measure he proposed, now known as the Arrow-Pratt coefficient of absolute risk aversion, is defined as:

$$(1.1)\ r(x) = -u''(x)/u'(x).$$

where x is income or wealth.

The Arrow-Pratt coefficient of absolute risk aversion can be taken at any arbitrary point on an individual's utility function. Two other measures of attitude towards risk can be shown to be related to the Arrow-Pratt measure. A risk premium measure of attitudes towards risk can be derived from the same individual's utility function by asking "for a small gamble with variance σ^2 and mean $x*$, what risk premium, π, would the individual be willing to pay to eliminate the uncertainty?" Pratt showed that the approximate relationship between these two measures, when the Arrow-Pratt coefficient is taken at the mean of the gamble, is:

$$(1.2)\ \pi = r(x*)\sigma^2/2$$

or that the risk premium, π, is equal to the value of the coefficient of absolute risk aversion at the mean of the gamble times the variance of the action choice divided by two. The certainty equivalent, CE, of the same gamble can be found by replacing π, the risk premium, with $x* - CE$. This can be expressed as:

$$(1.3)\ CE = x* - r(x*)\sigma^2/2 = x* - \pi$$

Pratt then inferred that the more risk averse the individual, the larger the risk premium he would require. Furthermore, he asserted that, at a point, or "in the small," individuals could be ordered according to their attitude towards risk measured in terms of a risk premium, a certainty equivalent, or the Arrow-Pratt coefficient of absolute risk aversion. A von Neumann-Morgenstern utility function and the three related measures of attitude towards risk are shown in Figure 1.

Note that these are all local measures of attitude towards risk. If the attitude towards risk is constant over the range of outcomes associated with one decision, Pratt showed that the utility function must take one of the forms:

$u(x)$ is a positive linear transform of x if $r(x) = 0$

$u(x)$ is a positive linear transform of $-e^{-cx}$ if $r(x) = c > 0$

$u(x)$ is a positive linear transform of e^{-cx} if $r(x) = c < 0$

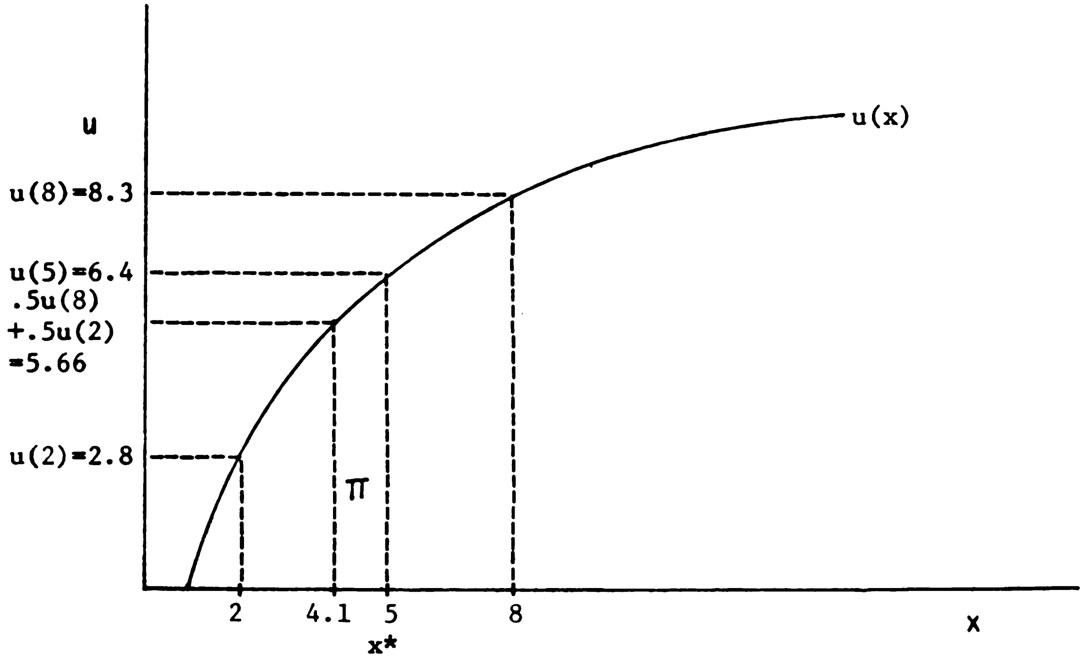
Pratt also defined a local measure of relative risk aversion defined as:

$$(1.4)\quad r_r(x) = -(x * u''(x))/u'(x)$$

This coefficient measures the elasticity of the marginal utility of wealth and is invariant not only with respect to changes in units of measurement of utility but also with respect to changes in units of wealth.

Numerous empirical studies have measured farmers' attitudes towards risk using one of the three related measures discussed above. The measured attitudes towards risk have often been used to order farmers according to their risk attitude, predict preferred action choices, or attempt to establish correlations between socio-economic attributes of individuals and their risk attitudes. Readers interested in a review and discussion of this literature are referred to Fleisher and Robison (1985).

Although the large body of literature on measurement of attitudes towards risk in agricultural settings will not be reviewed here, several of the points made by Fleisher and Robison must be noted because of their



$u(x) = 4 \ln x$ CE = 4.1
$r(x*) = 1/x*$ $\pi = .9$

When presented with and equal chance lottery between 2 and 8, or 5 for sure, the individual selects 5, since:

$$CE = x - r(x*)\,\sigma^2/2 = x* - \pi$$

which, in this case yields:

$$4.1 = 5 - .2 * 9/2 = 5 - .9$$

Figure 1: A von Neumann-Morgenstern Utility Function and the Relationship between Arrow-Pratt, Risk Premium and Certainty Equivalent Measures of Attitudes Towards Risk.

direct bearing on the present study. The first is the caveat that measures of attitudes towards risk taken at a point on the individuals' utility function are local measures and cannot justifiably be used to order individuals according to their attitudes towards risk "in the large". The second is that when utility functions are elicited using hypothetical money gambles, they have not proven very useful in predicting preferred action choices in "real world" situations. Third, when attitudes towards risk for the same individuals are taken at two different points in time, they are not stable (Halter and Mason, 1971; Whittaker and Winter, 1980; Love and Robison, 1984; Mosteller and Nogee, 1951). In addition, Hildreth and Knowles (1982) cite several empirical studies of individuals' risk preferences that produced apparently risk neutral or risk loving responses in cases where the decision makers' true preferences would be expected to be risk averse. These factors support the contention that something other than attitude towards risk influence preference rankings.

In addition, several studies have examined the conformance of decision makers' preferences with the behavioral conditions specified by the axioms of expected utility theory. Of particular concern is the independence axiom which assumes invariance of preference between certainty and uncertainty when other things are equal. According to Machina (1982, 1983), empirical/experimental research on the independence axiom has uncovered four types of systematic violations of the axiom: the common ratio effect, the common consequence effect, oversensitivity to changes in small probabilities and the utility evaluation effect. Only one type of the common consequence effect will be discussed here.[3]

Kahneman and Tversky (1979) observed a certainty effect by which decision makers viewed outcomes obtained with certainty as being disproportionately larger than those which are uncertain. A related experiment on the effect of certainty versus uncertainty was performed by Ellsberg (1961) showing systematic violation of the independence axiom. Other authors (e.g. Masson, 1974) have shown that violations of the independence axiom create discontinuities in the utility function at certainty. Weiss (1984) has demonstrated that the certainty effect can be modeled by allowing decision makers to have "two-rule" type measurable utility functions reflecting dichotomous behavior towards certainty and uncertainty. He also demonstrates (Weiss, unpublished) that predicted behavior under uncertainty when one allows for dichotomous behavior differs from the predictions made when this dichotomy is not allowed.

Despite these findings, most empirical applications of the expected utility hypothesis and other models of decision making under uncertainty which derive local measures of attitude towards risk treat them as true indicators of preferences and employ them in generalized conclusions about the risk attitudes of a population or the ordering of individuals within the population.

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[3] For a discussion of oversensitivity to changes in small probabilities, the reader is referred to Kuenreuther and Slovic (1978). A discussion of the utility evaluation effect may be found in McCord and de Neufville (1983) and a discussion of the common consequence effect can be found in Raiffa (1961) and Slovic and Tversky (1974).

Objectives of this Study

This goal of this undertaking is to contribute to the improvement of the application of decision theory in agricultural teaching, research and extension by developing a new measure of attitude towards risk which separates the influence of preferences for attributes of outcomes under certainty from like or dislike of uncertainty. The first step towards attainment of this goal is to show why the Arrow-Pratt coefficient of absolute risk aversion is not stable across decision contexts and therefore why agricultural economists can not continue to use coefficients derived in one decision context to predict preferred action choices in other decision contexts or at other points in time. The second step is to develop a new measure of attitude towards risk which is hypothesized to be stable and therefore can be used to generalize across decision contexts and compare individual's attitudes towards risk. The third step is to examine the new measure more closely through empirical application.

Overview of the Dissertation

This chapter has provided a brief introduction to the problem addressed in the dissertation. Chapter 2 examines the question of why risk attitudes appear to be unstable when measured using the Arrow-Pratt coefficient of absolute risk aversion. This includes a brief digression into the foundations of utility theory and some of the controversies, questions, an ambiguities in terminology which have arisen and often obscured our understanding of utility functions and what they actually represent. Chapter 3 introduces the concept of intrinsic risk aversion[4]. These chapters provide the foundation for Chapter 4, in which a new measure of attitude towards risk is developed and discussed. Chapter 5 discusses the procedures used to empirically examine the new measure and presents the limited conclusions that can be drawn from this exercise. Chapter 6 contains a summary of the research and observations regarding application of the results as well as a discussion of issues requiring further research.

The seasoned reader of dissertations will note that there is no chapter which is explicitly devoted to a review of the pertinent literature. Because the literature which provides the background for this study is so diverse, ranging from measurement theory to management science, individual sources are cited in the text where they are most applicable.

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[4] This concept is referred to as relative risk aversion in the mathematical psychology and much of the management science literature. Since this concept of relative risk aversion bears no relationship to the Arrow-Pratt coefficient of relative risk aversion, confusion will be reduced by referring to this concept by a new, if less descriptively accurate, term.

CHAPTER 2

WHY DOES THE ARROW-PRATT COEFFICIENT OF ABSOLUTE RISK AVERSION INDICATE UNSTABLE ATTITUDES TOWARDS RISK?

Various authors have suggested that the instability in empirically measured Arrow-Pratt coefficients of absolute risk aversion is due to a wide variety of problems including measurement error, subjectively held probability estimates which differ from those given in the elicitation procedure, discontinuities in the utility function and "other factors". While it is recognized that all of these arguments may have validity, they will not be examined closely in this chapter. Instead, we will focus on what the von Neumann-Morgenstern utility function actually represents and how this affects what the Arrow-Pratt coefficient of absolute risk aversion derived from the utility function actually characterizes.

We begin by using a measurement theoretic approach to examine what the utility function actually represents. This is followed by a brief digression into the development of utility theory and the expected utility hypothesis' place in this context. Special attention will be paid to two assumptions: constancy of the marginal utility of money, and homogeneity of preferences. This discussion is then used to examine what effect the violation of these assumptions has on the use of Arrow-Pratt coefficients as a numerical measure of attitudes towards risk.

What Does the Utility Function Actually Represent?

Measurement theory starts with an observed or empirical relational system (e.g. someones likes or dislikes for outcomes), called the domain. A mapping to a numerical relational system (e.g. a utility function), called the range, is sought. This mapping preserves all of the relations and operations (e.g. preferences) in the original observed or empirical system. Axioms of the expected utility hypothesis can be conceived of as the representation theorem which does the mapping. While many questions raised in examining the axioms as a representation theorem are of concern to those interested in decision theory, they are not the major focus of this chapter. Instead, our present concern is with what the representation theorem is actually mapping.

The expected utility hypothesis takes as its domain preferences over probability distributions over outcomes and maps this to a utility function whose range is the real line from which attitudes towards risk are derived. This mapping is shown in Figure 2.

The conceptualization of the domain and range for the utility function under uncertainty results from von Neumann and Morgenstern's use of risky prospects in developing utility functions. This utility function differs from the utility function in standard consumer theory in that the latter involves only certain outcomes. These utility functions map from the domain of preferences over riskless outcomes to a range on the real line. This standard consumer theory preference function will be referred to as a value function to distinguish it from a von Neumann-Morgenstern utility function over risky prospects.

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                                   -----
                                     A
---------------------------          X       -----------------------
   preferences over                  I        utility function whose
probability distributions  ---       O  ---> range is the real line
   over outcomes                     M       -----------------------
---------------------------          S
                                   -----
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[domain] [representation theorem] [range]

Figure 2: The von Neumann-Morgenstern Utility Function as a Homomorphic Mapping

Note that the expected utility hypothesis takes as its domain preferences over probability distributions over outcomes. If the representation theorem provides a mapping which preserves all relations and orderings in the domain (a homomorphic mapping), the resultant utility function incorporates both preferences over probability distributions and preferences over outcomes. The former may be viewed as containing information about attitudes towards risk while the latter contains information about preferences for riskless outcomes.

The obvious question, then, is if the utility function whose range is the real line incorporates information about both preference for riskless outcomes and attitude towards risk, can we use the rate of bending of the utility function [the Arrow-Pratt coefficient] as a measure of attitude towards risk? Before this question can be answered, we must have a better understanding of value and utility functions and the assumptions each embodies.

A Brief Digression into the Foundations of Utility

This digression into the foundations of utility is being made in the hope that it will clarify some of the confusion surrounding what the utility function actually represents. At the same time it will highlight the assumptions in the expected utility hypothesis which are a direct carryover from the earlier cardinalists and which give rise to the present concern over the use of the Arrow-Pratt coefficient of absolute risk aversion as a measure of pure attitude towards risk. Many attacks on the expected utility hypothesis are based on earlier criticisms of cardinality as used by the material welfare school [Baumol, 1951; Majumdar, 1958]. While some of these criticisms remain valid when examining the expected utility hypothesis, many others ignore the important distinctions which can be made between the early and new cardinalists.

Discussion of the historical foundations and concepts of utility are made more complex by the fact that the distinctions between what we shall call the early cardinalists, the ordinalists, and the new cardinalists cannot be made purely on the basis of debate over the "measurability" of utility. Cardinalists and ordinalists also disagree among themselves and with each other as to whether utility is an introspective or a behavioral concept, on its importance in welfare economics, and its use in determining preference orderings among risky prospects.

Early cardinalists viewed the fundamental natural operation in the measurement of utility as taking place within the mind of the subject; utility measurement was a process of introspectively weighing the amounts of satisfaction associated with different outcomes. Elicitation of utility was simply a matter of mapping these internal measures of satisfaction onto the real number line. It was argued that since individuals could not only order degrees of satisfaction but also judge their strength of preference for each outcome in a cardinal fashion (e.g. outcome A provides twice as much satisfaction as outcome B), the utility function derived from the mapping process was cardinal. In other words, not only was the mathematical representation viewed as cardinal, but the extra-mathematical interpretation was also cardinal in nature.

Three important assumptions about the nature of utility are implied by the early cardinalists. First, an increment of utility is not only quantifiable in principal, but also is measurable in terms of money. Secondly, utility is a measure of worth to the individual and does not necessarily imply any preference ordering; it is simply a measure of the amount of satisfaction an individual could obtain from a given outcome. Furthermore, this satisfaction refers directly to the meeting of physical needs, not the preference for more abstract desires. Third, utility is additive and interpersonally comparable.

It was argued that if we look upon money as a measure of general purchasing power, then there can be no objection to regarding it as an indicator of alternative welfare-yielding situations. Therefore, the amount of sacrifice, in terms of money, which a person would make rather than go without a particular thing is a measure of his welfare derived from it. The difficulty with this conclusion rests with a further assumption. If money provides the measuring rod of utility then, as with all measuring rods, its unit must be invariant; it must measure the same amount of utility in all circumstances. The first dollar must mean the same to the individual as the hundredth or millionth. This means that one has to assume constant marginal utility of money. The importance of this assumption will be discussed below.

Jevons was one of a group of economists who shifted the focus of utility from a purely individualistic approach to that of explaining behavior and conditions of exchange in a market economy. While Jevons still believed (albeit ambiguously) in the cardinality of utility, he did not view "total utility" for any good as being intrinsically measurable or interpersonally comparable. Instead, he hypothesized that interpersonal comparisons of welfare could be obtained through the use of the concept of marginal utility. He states in his second edition of _The Theory of Political Economy_ (1879, from Stigler, 1950, p. 317) that

> "It is from the quantitative effects of the feelings that we must estimate their comparative amounts. I never attempt to

> estimate the whole pleasure gained by purchasing a commodity; the theory merely expressed that, when a man has purchased enough, he would derive equal pleasure from the possession of a small quantity more as he would from the money price of it."

Jevons uses this to argue that, at equilibrium, welfare is maximized because each individual's marginal utility is equal, and thus, all individuals would obtain the same degree of satisfaction of needs from the last unit purchased. From this argument comes the assumption of homogeneity of preferences across individuals, implying that the last unit purchased by each individual is valued by each individual at the market price of that unit.

Marshall extended Jevons' concept of utility to formulate a hypothesis of diminishing marginal utility for goods. Marshall argued that an increment of utility (marginal utility) is quantifiable in principle and is actually measurable in terms of money. He also asserted the law of negatively sloping demand curves in all generality (Marshall, 1890, pp. 159-160, from Stigler,1950, p. 326):

> "There is then one law and only one law that is common to all demand schedules, viz. that the greater the amount to be sold, the lower will be the price at which it will find purchasers."

It is important to note that the Marshallian demand theorem cannot be derived from the hypothesis of diminishing marginal utility except in a one commodity world without contradicting the assumption of constant marginal utility of money with respect to changes in prices and income. If this difficulty is avoided by giving up the assumption of constant marginal utility of money, then money can no longer provide the measuring rod for utility and we can no longer express the marginal utility of a commodity in units of money.

With the advent of an ordinal concept of utility, utility was no longer viewed as a psychological entity measurable in its own right. Instead, utility was viewed as a convenient label for the explicit value of a function which described consumer behavior. Not all ordinalists argued that utility was not quantifiable in principle. Their fundamental argument centered on the fact that a basic understanding of the behavior of participants in an exchange economy did not depend on the assumption of cardinally measured utility. Assumptions regarding cardinality in extra-mathematical interpretation were viewed as extraneous to the issues at hand; since equally strong results for price theory could be obtained without this assumption, there was no reason to suppose that anything more than ordinal utility was required. The move to an ordinal concept of utility was not necessarily a total rejection of the mathematical concept of measurable utility; its extra-mathematical interpretation was simply unnecessary.

With the publication of von Neumann and Morgenstern's Theory of Games and Economic Behavior in 1944, the debate over the cardinality of utility was rekindled. From a measurement theoretic perspective, von Neumann-Morgenstern utility is cardinal because the utility scale has interval properties (it is unique up to a positive linear transformation) However, from a preference perspective, von Neumann-Morgenstern utility is ordinal in that it provides no more than ordinal rankings of lotteries. It differs from earlier cardinal utility in that utility is viewed not as "measurable" in the sense that it is correlated with any significant economic quantity such as the quantity of feeling or satisfaction or intensity. It is cardinally measurable only to the extent that the numerical operation of forming mathematical expectations on the basis of these numbers would be related to observable behavior, so as to be emprically meaningful.

Von Neumann and Morgenstern's utility also differed from early cardinalists in that they use risk choices to define utility differences, not to estimate introspectively ascertained differences. Therefore, von Neumann-Morgenstern utility should not be interpreted as cardinally measuring strength of preference for uncertain outcomes; as a preference measure it is strictly ordinal.

The distinction between this new cardinality in the purely measurement theoretic sense, and the earlier cardinalists is made more confusing by the strong similarities in notation and methods used to determine the outcome which maximizes utility. Ellsberg (1954) illustrates the source of this confusion with an example. Suppose an individual who prefers A to B, B to C and A to C must choose between having a lottery ticket offering A with probability p or C with probability 1-p. Both early cardinalists and von Neumann and Morgenstern would assign a triplet of numbers, U_A, U_B, and U_C, to the three outcomes with the property that $U_A > U_B > U_C$. This triplet is a utility index for the three outcomes since their order of magnitude reflects the order of preference.

Next, both would form the expression:

(2.1) U_B is greater than, less than, or equal to $pU_A + (1-p)U_C$

where p and 1-p are the respective probabilities of A and C.

In each case the individual would choose the prospect with the highest mathematical expectation of utility. Furthermore, for both the early cardinalists and von Neumann and Morgenstern, utility would be said to be measurable. The necessity of this assumption is seen if the expression is rewritten:

(2.2) $p(U_A-U_B) - (1-p)(U_B-U_C)$ is less than, greater than, or equal to 0

The rewritten expression shows clearly that the rules rely on comparing differences in utility. In order to retain consistency in results, it is necessary to find some operation that gives meaning to differences in utility numbers, and hence to the numerical operations implied by the rule. A utility index which is ordinal in the measurement theoretic sense (unique up to a monotone transformation) would not satisfy this requirement. Therefore, a more restricted set of indices, which summarizes the results of the additional operations as well as preferences, must be used. This requirement is satisfied by the set of indices which are invariant up to a positive linear transformation, or, in measurement theoretical terms, cardinal.

The similarity between von Neumann Morgenstern and early cardinalist indices is that both require a utility index determined up to a positive linear transformation. However, the differences between utility points could be different for the two indices. Moreover, the indices would, in general, but not always, be monotonic and not linear transformations of each other; preservation of orderings requires a positive linear transformation of utility but only a positive monotonic transformation of expected utility. Therefore, predictions based on the rule of maximizing expected utility could differ for the two approaches.

A more fundamental difference exists between the two approaches; early cardinalists would come to their conclusions regarding an individuals choice between the options, one of which is a risky prospect, without ever observing that individual's behavior in other risk situations. For von Neumann and Morgenstern, it would be necessary to observe the individual's behavior in other risk situations, involving different outcomes or the same outcomes with different probabilities. Utility is defined by an individual's choices among risky prospects. Where early cardinalists postulated a type of consistency between individual's risk choices and their feelings of relative differences in desirability of the outcomes, von Neumann and Morgenstern hypothesize simply a consistency between risk choices and other risk choices.

However, von Neumann and Morgenstern's break with the early cardinalists is not as complete as it may seem from the discussion above. They state (von Neumann and Morgenstern, 1944, p. 8):

> "..We shall therefore assume that the aim of all participants in the economic system, consumers as well as entrepreneurs, is money, or equivalently, a single monetary commodity. This is supposed to be unrestrictedly dividible (sic) and substitutable, freely transferrable and identical, even in the quantitative sense, with whatever "satisfaction" or "utility" is desired by each participant."

In addition, as Schoemaker (1982, p. 533) points out,

> "..(P)references among lotteries are determined by at least two separate factors; namely (1) strength of preference for the consequences under certainty, and (2) attitude towards risk. The NM utility function is a compound mixture of these two without direct resort to interval comparisons or strength of preference measures. As a preference theory, it is wholly ordinal. Nevertheless, it <u>implicitly</u> assumes that a neoclassical type of utility exists, otherwise it would not be possible psychologically to determine the certainty equivalence of a lottery."

The Importance of Heterogeneity of Preferences

In describing the environment in which the expected utility hypothesis may be applied, von Neumann and Morgenstern employ a very restrictive set of initial conditions and auxiliary assumptions.[1] For money to serve as the measuring rod of "satisfaction" or "utility" each individual must have constant marginal value[2] for money. For money as a measuring unit to be "...unrestrictedly dividable and substitutable, freely transferrable and identical for whatever satisfaction..." (von Neumann and Morgenstern, 1944, p. 8) preferences and the utility function must be homogeneous. In other words, indifference curves for any two goods, or one good and money, must be straight lines with slope of -1 and each individual must value goods at their market price.

Unfortunately, empirical evidence does not lend credence to the use of these initial conditions and auxiliary assumptions. The following examples will demonstrate the importance of violation of these conditions and assumptions when the von Neumann-Morgenstern utility function and Arrow-Pratt coefficients of attitudes towards risk are used to predict preferred action choices in the real world. As a base for comparison, Figure 3 shows the value functions for two individuals who conform to von Neumann-Morgenstern's set of initial conditions and auxiliary assumptions. The reader will recall that the independence axiom of the expected utility

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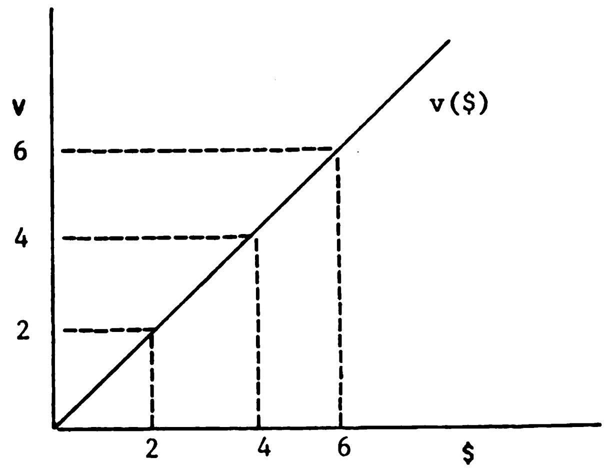
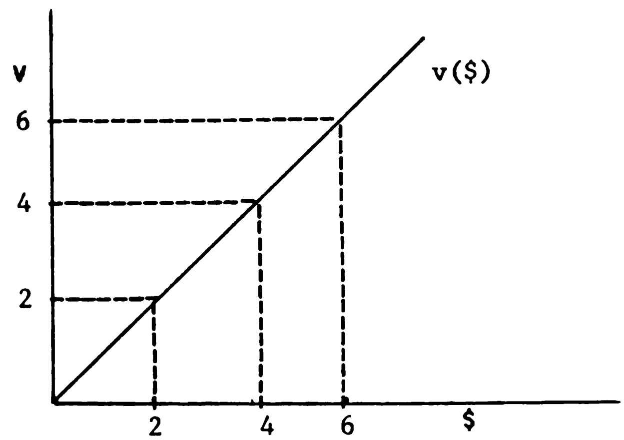
1 Giere (1979, pp.87-88) defines initial conditions as "...either the occurrence of a state of the system or a statement describing this occurrence." and auxiliary assumptions as "...additional assumptions about the system being studied and about its immediate environment."

2 In the literature, this phenomenon is usually referred to as constant marginal utility for money. Since we are distinguishing between utility as preference for risky outcomes and value as preference for riskless outcomes, the term constant marginal value for money is used here.



Individual A



Individual B

Figure 3: Value Functions of Two Individuals with Constant Marginal Value of Money

hypothesis assumes invariance of preference between certainty and uncertainty. Under this assumption, the individual's utility function is a linear transform of their value function.

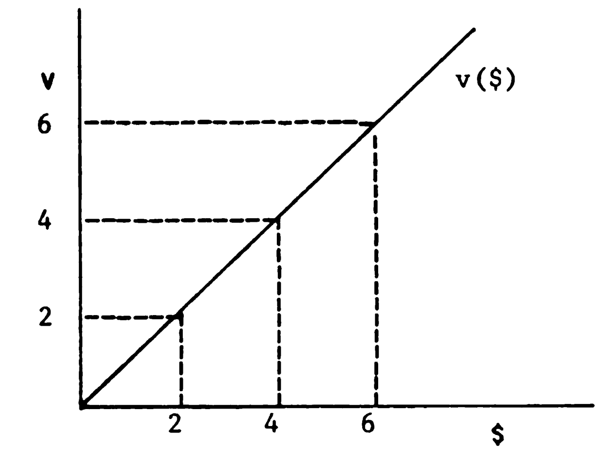
Both individual A and B exist in a world which consists of one monetary commodity. They have identical constant marginal value for money. We will assume for this case and the subsequent ones that individuals are actually risk neutral. This means that their risk premium will be zero, or in other words, their certainty equivalent will be equal to the mean of the lottery. If we present our two individuals with a choice of an equal probability gamble between \$2 and \$6, or \$4 with certainty (which is also the mean of the lottery), each will be indifferent between the lottery and the sure thing because their certainty equivalent is equal to the mean value of the lottery. For both individuals, the value of the lottery is:

(2.3) $.5*v(\$2) + .5*v(\$6) = .5*(2) + .5(6) = 4$

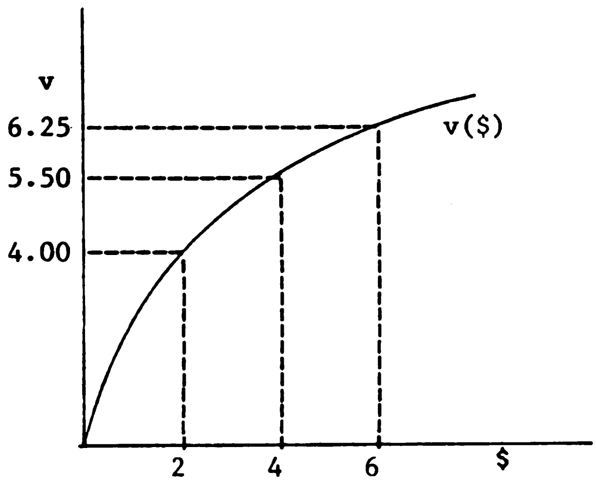
which is identical to the certain offer of \$4 which also has a value of 4. Any response other than indifference would be an accurate indicator of risk aversion or risk proneness.

Figure 4 presents the value functions for two individuals. In this case, individual B does not have constant marginal value for money. The two individuals are presented with the same choice between a gamble and a sure thing and individual A is indifferent between the two. But individual B is not; he would take the sure thing over the gamble. Using our current notions of attitude towards risk, we would say that this indicated that individual B was more risk averse than individual A with regard to lotteries with monetary outcomes. But in this case, that conclusion is not justified. The value of the lottery for individual B is:

(2.4) $.5*(v(\$2) + .5*v(\$6) = .5*(4) + .5*(6.25) = 5.125$



Individual A



Individual B

Figure 4: Value Functions of Two Individuals without Constant Marginal Value of Money

which is less than the value of the sure consequence to individual B since, for him, v($4) = 5.5. In this case the apparent risk averseness of individual B is due to his non-constant marginal value for money, not the requirement for a risk premium.

Figures 5 and 6 present cases where we further relax the assumptions made by von Neumann and Morgenstern. In these cases individuals are assumed to not live in a world which consists of just one monetary commodity; individuals A and B use the money to buy many different commodities. The individuals whose value functions and indifference curves are shown in Figures 5 and 6 are concerned with making a decision about tractor services which can be purchased for $1 per unit of service. In Figure 5, individuals A and B conform to von Neumann and Morgenstern's assumptions and have constant marginal value for money and homogeneous preferences and have indifference curves for money and tractor services of slope -1. We can present the two individuals with our standard choice between $4 for sure and an equal chance gamble of receiving $2 and $6. In this case both individuals are indifferent between the gamble and the sure thing, indicating risk neutrality.

This result does not hold in Figure 6 where individuals A and B do not have homogeneous preferences. Specifically, individual B does not have homogeneous preferences for money and tractor services; examination of the figure shows that individual B does not have a linear transformation between money and tractor services. The value the individual derives from $2 is 2 while the value the individual derives from $2 of tractor services is 3, the value he derives from $4 is 4 while the vlaue he derives from $4 of tractor services is 5 and the value he derives from $6 is 6 while the value he derives from $6 of tractor services is 8. If the individuals are

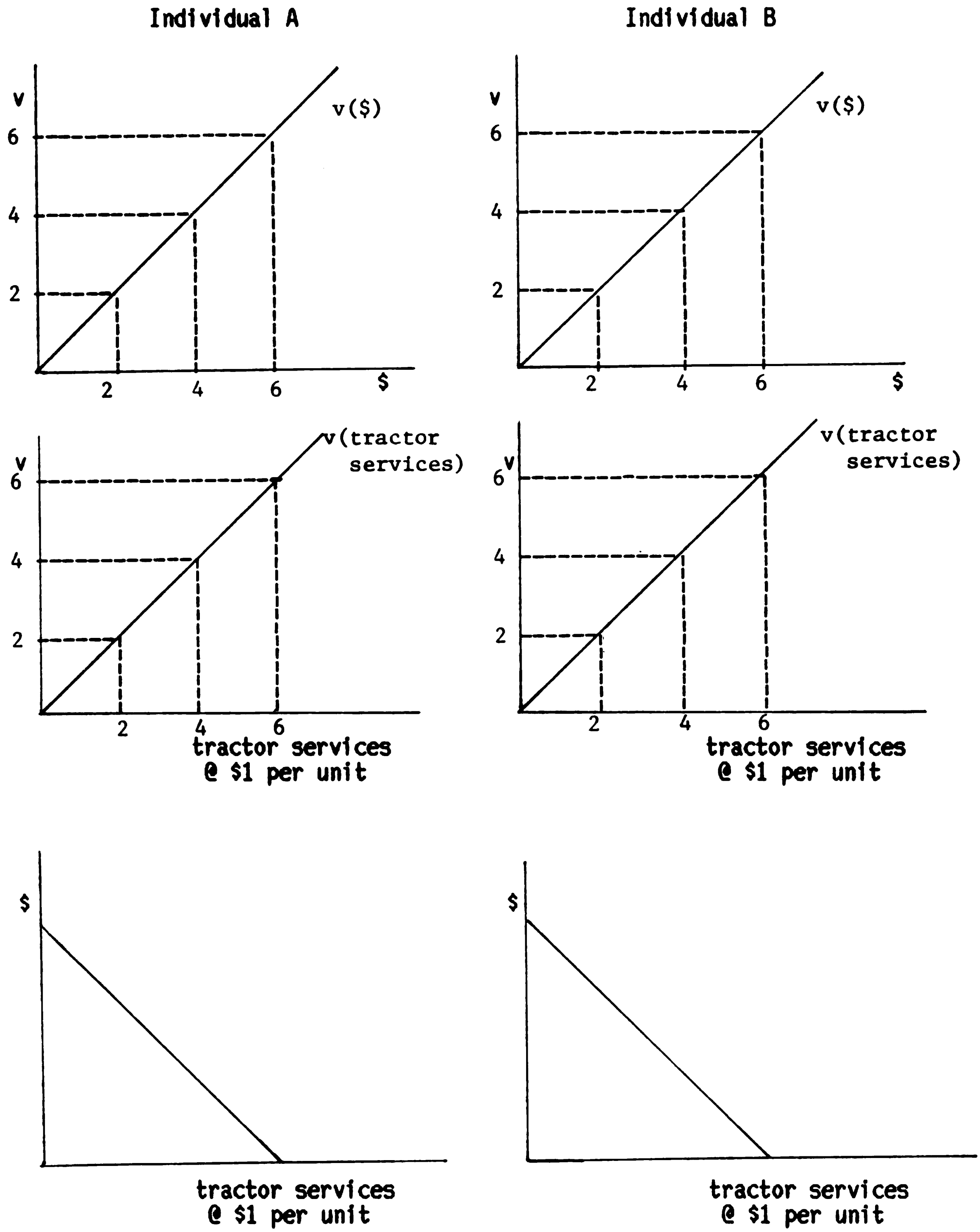


Figure 5: Value Functions and Indifference Curves of Two Individuals Who Have Constant Marginal Value of Money and Homogeneous Preferences

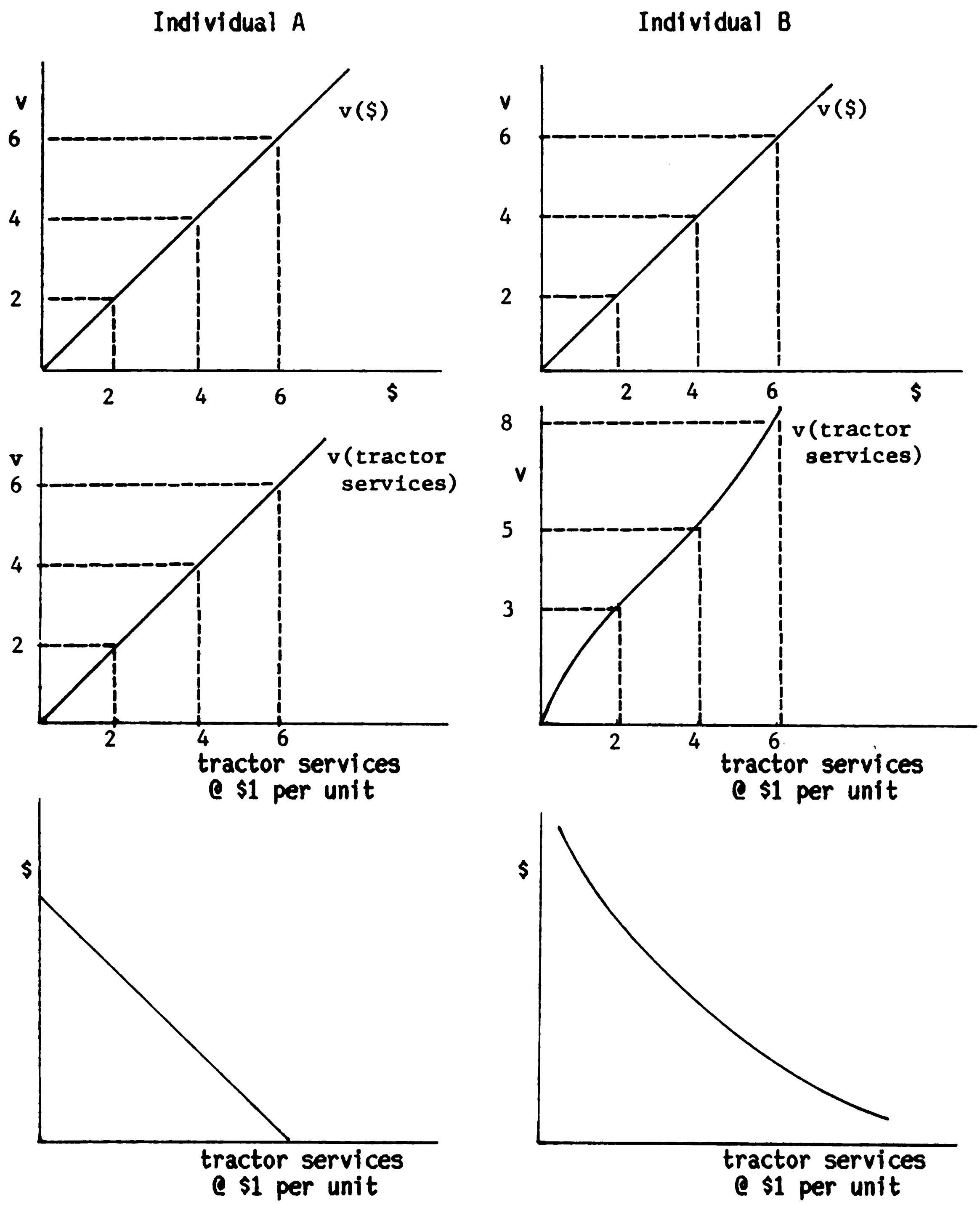


Figure 6: Value Functions and Indifference Curves of Two Individuals Who Have Constant Marginal Value of Money and Heterogeneous Preferences

primarily concerned with receiving tractor services and we present them with the standard choice between the gamble and the sure thing, individual A will be indifferent between the gamble and the sure thing while individual B will choose the gamble since:

(2.5) $.5*v(\$2) + .5*v(\$6)$

$= .5*v(\text{2 units tractor services}) + .5*v(\text{6 units tractor services})$

$= .5*(3) + .5*(8) = 5.5$

while the value of $4 of tractor services is 5. Therefore, the value of the lottery to the individual is greater than the value of the sure thing. Our current notions of attitude towards risk would indicate that individual B was risk prone relative to individual A. But again, if we examine individual B's preferences for riskless outcomes, we find that these explain apparently risk loving behavior when tractor services are the real concern of the individual. The value of the lottery to the individual is 5.5 while the value of the sure thing is only 5.

The examples used to this point all concern comparisons of risk attitudes of two individuals. Often, however, we use von Neumann-Morgenstern utility functions and coefficients of attitudes towards risk derived in one situation to predict the individuals preferred action choice in another situation. In other cases we determine attitudes towards risk in two different situations, compare them, and, if they are different, assert that the individual has an unstable attitude towards risk. Figure 7 shows the case where we have one individual faced with two different decisions. In the first case it is receiving tractor services, in the second case it is receiving services from a car. Note that the individual has heterogeneous preferences; the transformation between car services and money is nonlinear while the transformation between tractor services and money is

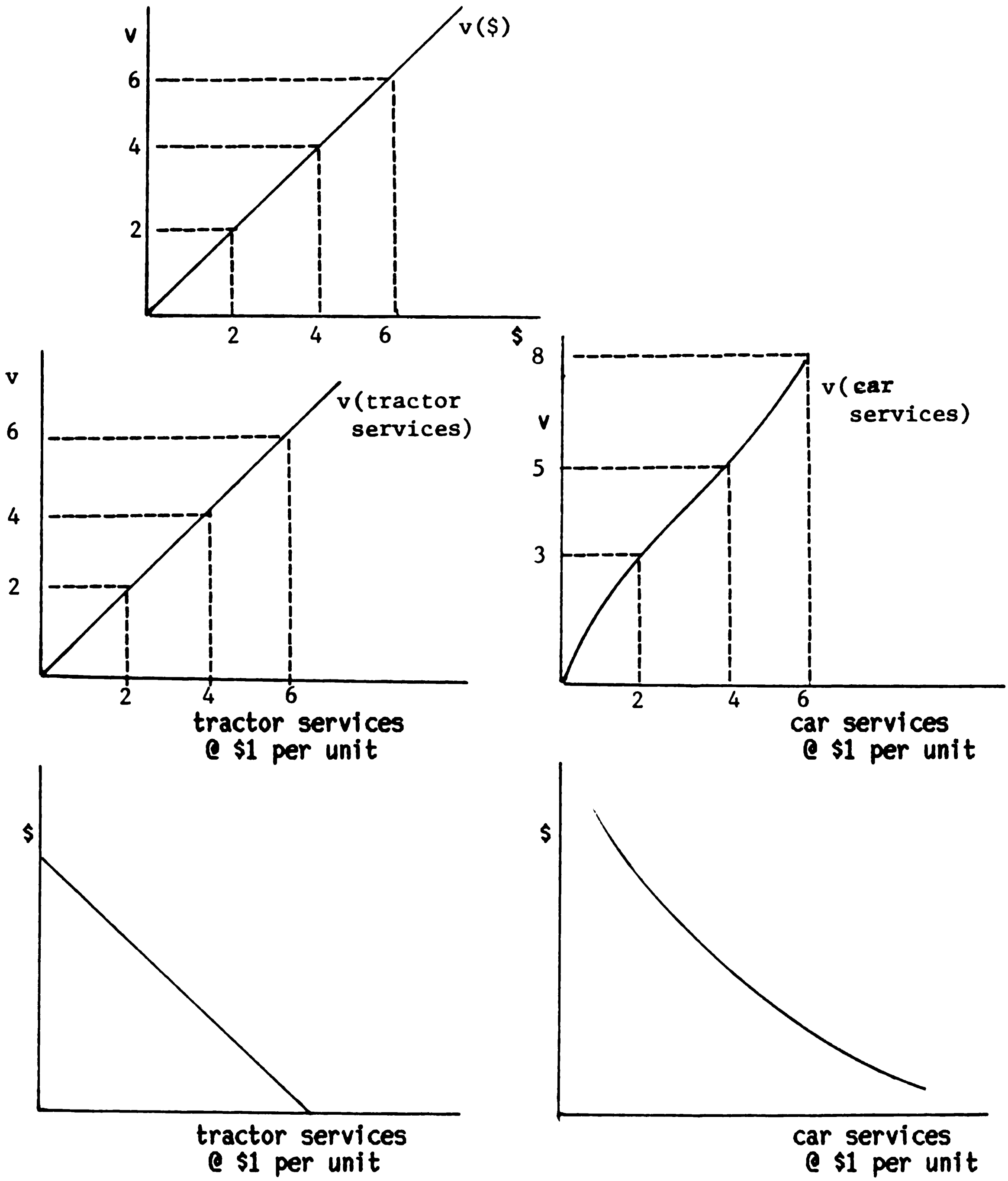


Figure 7: Value Functions and Indifference Curves of One Individual in Two Situations with Constant Marginal Value of Money and Heterogeneous Preferences

linear. When presenting the individual with the standard choice bwteen $4 for sure and an equal chance gamble between $2 and $6 when he is concerned with car services, we find that he prefers the gamble to the sure thing since the results of equation (2.5) hold; when presenting him with standard choice between $4 for sure and an equal chance gamble between $2 and $6 when he is concerned with tractor services we find that he is indifferent since the results of equation (2.3) hold. Does this mean that the individual's risk attitudes have changed? No, the "instability" in risk attitude only reflects heterogeneity of preferences for riskless outcomes.

More complex examples could be developed for combinations of the cases where the marginal value for money is not constant and individuals do not have heterogeneous preferences. However, these would serve only to make the same point as was made in the examples presented in Figures 3 through 7: for the Arrow-Pratt coefficient to measure only attitudes towards risk for outcomes having more than one attribute, the individual's preferences for riskless outcomes would have to conform to the initial conditions that (1) the marginal value of money is constant, (2) preferences for goods are homogeneous, and (3) there is a linear transformation between money and goods. In other words, we would have to be able to justify the assumption that the value functions were linear and of identical slope for each individual across commodities and that they were, in addition, of identical slope for all individuals.

Chapter 3

THE CONCEPT OF INTRINSIC RISK ATTITUDE: SEPARATING PREFERENCE FOR RISKLESS OUTCOMES AND ATTITUDE TOWARDS RISK

Recent theoretical work by mathematical psychologists suggests, as we do, that the perceived instability of Arrow-Pratt coefficient of risk aversion is due to the fact that the von Neumann-Morgenstern utility function is a confounding of attitudes towards risk and preferences under certainty. It has been hypothesized that if the two could be separated, we would find that individuals do have a stable pure or intrinsic attitude towards risk and that only preferences under certainty change across situations because of non-linear transformations between the actual attributes of outcomes and money. While a measure of pure attitude towards risk or a measure of strength of preference for certain outcomes cannot replace the expected utility hypothesis as a means of predicting preferred action choices under uncertainty, pure attitude towards risk, if stable, can provide a justifiable means of generalizing across situations or comparing individuals. Schoemaker (1982, pp. 534-535) cites five reasons to separate risk attitudes from strength of preference for certain outcomes, namely:

1. Emphasizing the point that there exist different types of cardinal utility, even within the categories of preferences for riskless and risky outcomes, that need only be related monotonically.

2. Providing a means for obtaining a measure of "intrinsic risk attitude".

3. If intrinsic risk attitude is stable, construction of u(x) can be simplified by building on v(x), strength of preference for riskless outcomes.

4. Group decision making can be simplified by aggregating individuals' v(x) function and transforming this to a von Neumann-Morgenstern utility function using a group measure of risk attitude.

5. Cardinal utility under uncertainty is useful for welfare economics.

This chapter will review the work that has been done on developing a measure of pure or intrinsic attitudes towards risk by mathematical psychologists and management scientists. Several studies which have measured individuals' utility and value functions and shown them to be non-identical will also be briefly reviewed. Several items are important to note at the outset. First, although various authors have expressed the belief that instability in Arrow-Pratt coefficients is due to the confounding of strength of preference for riskless outcomes and attitudes towards risk, no exposition of this problem analogous to the one presented in Chapter 2 has been published. Secondly, studies which have ascertained that the utility and value functions are not identical have not attempted to develop a relationship between the utility and value function. Thirdly, of the few studies which have developed a theoretical relationship between value and utility functions, only one has actually empirically tested this relationship.

How Can We Measure Pure or Intrinsic Attitude Towards Risk?

Unfortunately, pure or intrinsic attitude towards risk is not easy to measure. However, it can be obtained indirectly from knowledge of the decision makers' von Neumann-Morgenstern utility functions over risky prospects and their strength of preference or value functions over riskless outcomes. From a measurement theoretic standpoint, pure or intrinsic attitude towards risk can then be seen as the mapping from the value function over certain outcomes to the utility function for risky prospects as shown in Figure 8.

This is essentially a two stage process. First a von Neumann-Morgenstern utility function is obtained using the representation theorem contained in the expected utility hypothesis. This maps preferences over probability distribution functions over prospects to a utility function whose range is the real number line. A value function for the same certain outcomes is also mapped to the real number line using one of several available representation theorems. The second, and critical step, is to develop a transformation between the value function over certain outcomes to the von Neumann-Morgenstern utility function over risky prospects. This transformation encapsulates the individual's pure or intrinsic attitude towards risk, or the preference over probability distributions.

Intrinsic Risk Aversion [1]

Bell and Raiffa (1979) first introduced the concept of a pure measure of attitude towards risk by reference to lotteries whose outcomes had been specified in a particular manner. Specifically, the outcomes of the lotteries were first transformed to eliminate what, in our terms, are the problems of non-constant marginal value for money and heterogeneity of preference. To do this, they transformed both money and the pertinent

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[1] To avoid confusion with the concept represented by the Arrow-Pratt coefficient of relative risk aversion, the term intrinsic risk aversion is used to represent the concept often referred to by mathematical psychologists and management scientists as relative risk aversion.

STEP 1

preferences over probability distributions over outcomes ---- AXIOMS -----> utility function whose range is the real line

preferences over riskless outcomes -------- AXIOMS -----> value function whose range is the real line

[domains] [representation theorems] [ranges]

STEP 2

value function whose range is the real line -------------> utility function whose range is the real line

transformation encapsulating pure attitude towards risk

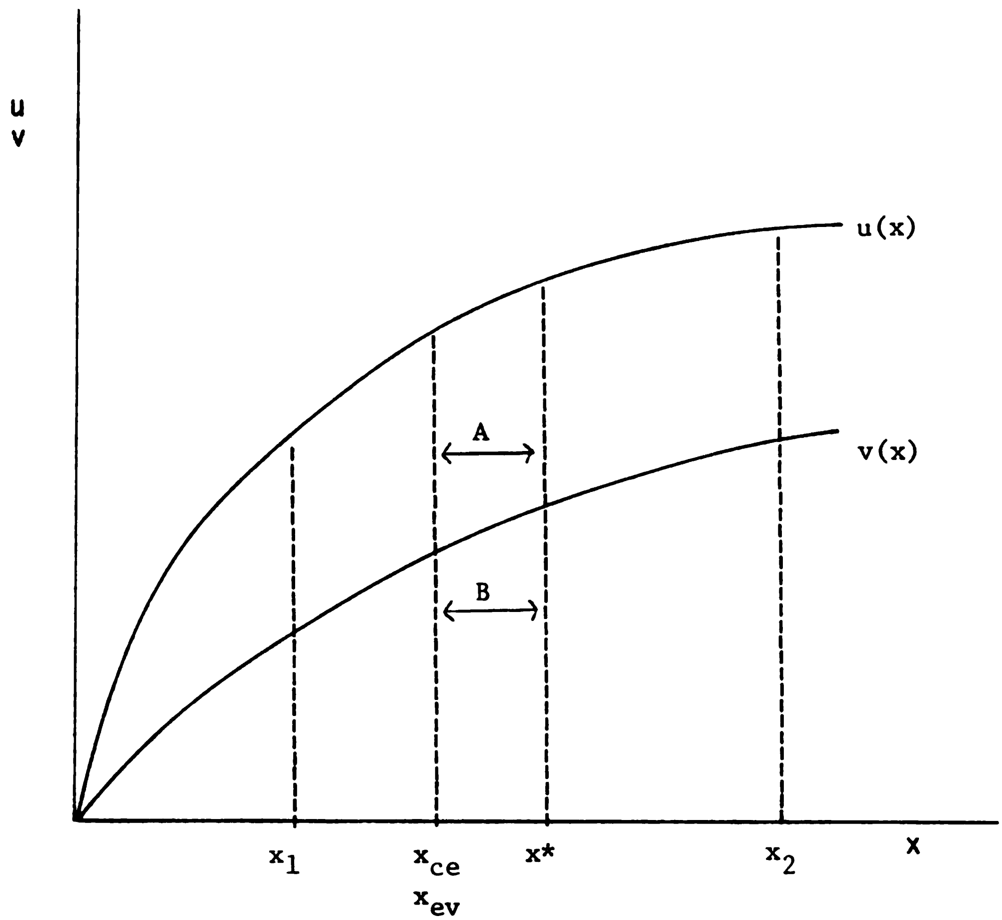
Figure 8: Pure Risk Attitude as a Function Transforming Value Functions to Utility Functions on the Real Line

attribute of the outcome to strength of preference scales. These scales were then further manipulated so that money and the attribute of concern have constant marginal values and a constant marginal rate of substitution. They hypothesized that the von Neumann-Morgenstern utility function measured over these transformed outcomes would always take the negative exponential form and that an individual's basic psychological attitude towards risk could be captured by the value of the negative exponent.

While Bell and Raiffa's work was an important step in the development of the concept of intrinsic risk aversion, the development of the theory must be attributed to Dyer and Sarin (1981). They propose a method by which preferences for risky alternatives can be determined relative to strength of preference for outcomes over the same range. The following is a summary of the most important aspects of their theory.

It is assumed that both the measurable value function $v(x)$ and the von Neumann-Morgenstern utility function $u(x)$ are monotonically increasing in x and are continuous and twice differentiable. Using the Arrow-Pratt measure of absolute risk aversion, defined as $r(x) = -u''(x)/u'(x)$, as a model, a local measure of the strength of preference of an individual for an asset at level x can be defined as $m(x) = -v''(x)/v'(x)$. Just as $r(x) >,=,< 0$ respectively indicate "risk aversion", "risk neutrality", or "risk preference", $m(x) >,=,< 0$ respectively indicate decreasing marginal value at x, constant marginal value at x, and increasing marginal value at x.

If the individual's value function and utility function exhibit the same rate of bending at one level of the attribute, x, as shown in Figure 9, it can be argued that the riskiness of the lottery has no effect on their preferences. Notice that the " risk premium", which represents the curvature of the utility function, and incorporates information about both



A = risk premium = $x^* - x_{ce}$
B = satiation sacrifice = $x^* - x_{ev}$
C = risk premium due to only attitude towards risk = $x_{ev} - x_{ce} = 0$

Figure 9: Value and Utility Functions for the Case of Intrinsic Risk Neutrality.

attitude towards risk and preferences for the outcomes under certainty, is identical to the satiation sacrifice, which represents the curvature of the value function sacrifice and incorporates information about preferences for the outcomes under certainty. Since the utility function incorporates information about preferences for outcomes under certainty and attitude towards risk, the curvature of the utility function which can be interpreted as the "risk premium" is actually attributable to the individual's preferences for outcomes under certainty, not an aversion or proneness to risk taking. At any point x, $m(x) = r(x)$ and the individual can be said to be intrinsically risk neutral. Note that at this point, x, both $m(x)$ and $r(x)$ could be $>, =, < 0$. The important point in the measurement of intrinsic risk aversion is that they are equal. When $v(x)$ is not equal to $u(x)$, the relationship between $m(x)$ and $r(x)$ can be used to define a local measure of intrinsic risk aversion.

An individual is said to be intrinsically risk averse if $m(x)<r(x)$, intrinsically risk neutral if $m(x)=r(x)$, and intrinsically risk prone if $m(x)>r(x)$. This indicates that the rate of bending of $r(x)$ is respectively greater than, equal to, or less than $m(x)$ at point x.

When a von Neumann-Morgenstern utility function is elicited, it incorporates both the individual's preferences for riskless outcomes and their attitude towards risk. A value function incorporates only preference for riskless outcomes. In principle, we can also define a utility function which maps only the individual's preference for the riskiness of outcomes independent of his preferenccs for the outcomes themselves. This is equivalent to eliciting $u(v)$ rather than $u(x)$. We can then define a measure of the rate of bending of this utility function as $b(x) = -h''[v(x)]/h'[v(x)]$, where the differentiation is taken with respect to v.[2]

Unfortunately b(x) cannot be measured directly. Instead, it must be derived from knowledge of the individual's value and utility functions. Using a method parallel to that used by Pratt (1964) in his proof of the coefficient of absolute risk aversion, it can be shown that an individual is intrinsically risk averse if and only if $b(x)>0$, intrinsically risk neutral if and only if $b(x)=0$ and intrinsically risk prone if and only if $b(x)<0$. Given:

(3.1) $u(x) = h[v(x)]$

Then:

(3.2) $\delta u(x)/\delta x = h'[v(x)]v'(x)$

(3.3) $\delta^2 u(x)/\delta x^2 = h'[v(x)]v''(x) + h''[v(x)](v'(x))^2$

Remembering that $r(x)=-u''(x)/u'(x)$, $m(x)=-v''(x)/v'(x)$, and $b(x) = -h''[v(x])/h'[v(x)]$, we can divide equation (3.3) by equation (3.2) and rearrange terms to obtain:

(3.4) $r(x) - m(x) = v'(x)*b(x)$

or

(3.5) $(r(x) - m(x))/v'(x) = b(x)$.

We know that $v'(x)>0$ from the assumptions of monotonic increases in v(x) and continuity. Therefore, we are primarily concerned with the relationship between r(x) and m(x) in the signing of b(x).

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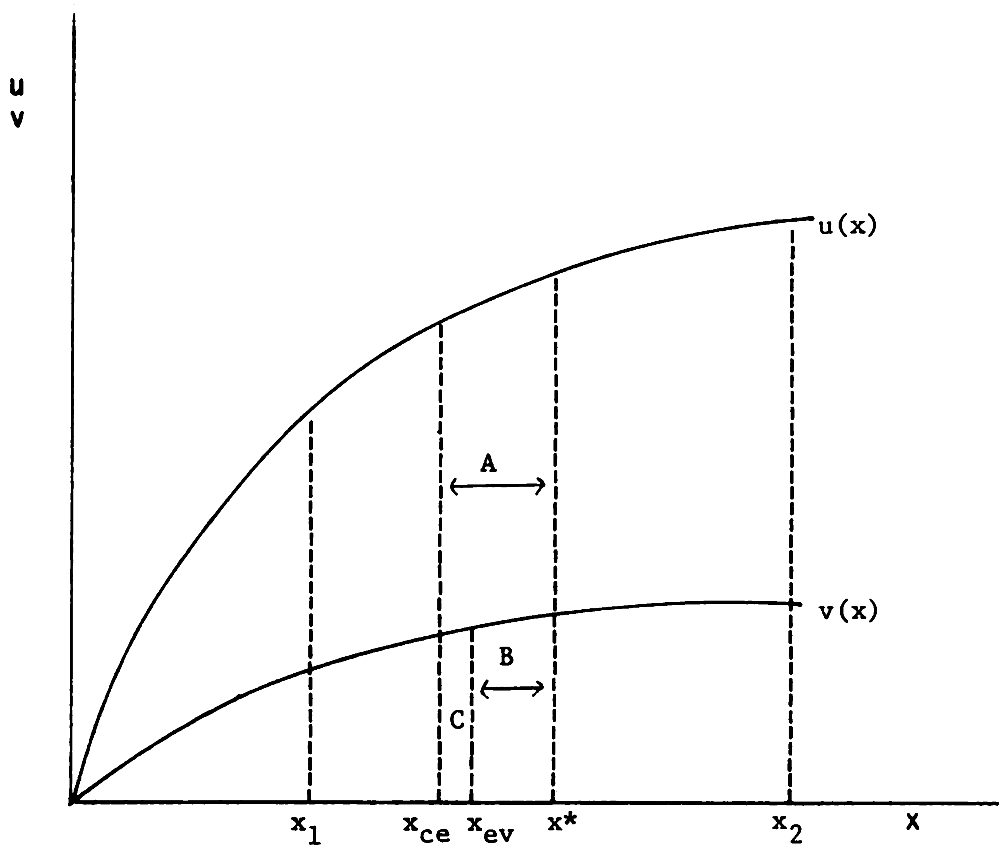
[2] It should be noted that the notation used in the literature to refer to the rate of bending of the function u(v) is $r_v(v)$ rather than b(x). It has been changed here to avoid confusion with the notation used for the Arrow-Pratt coefficient of absolute risk aversion, r(x).

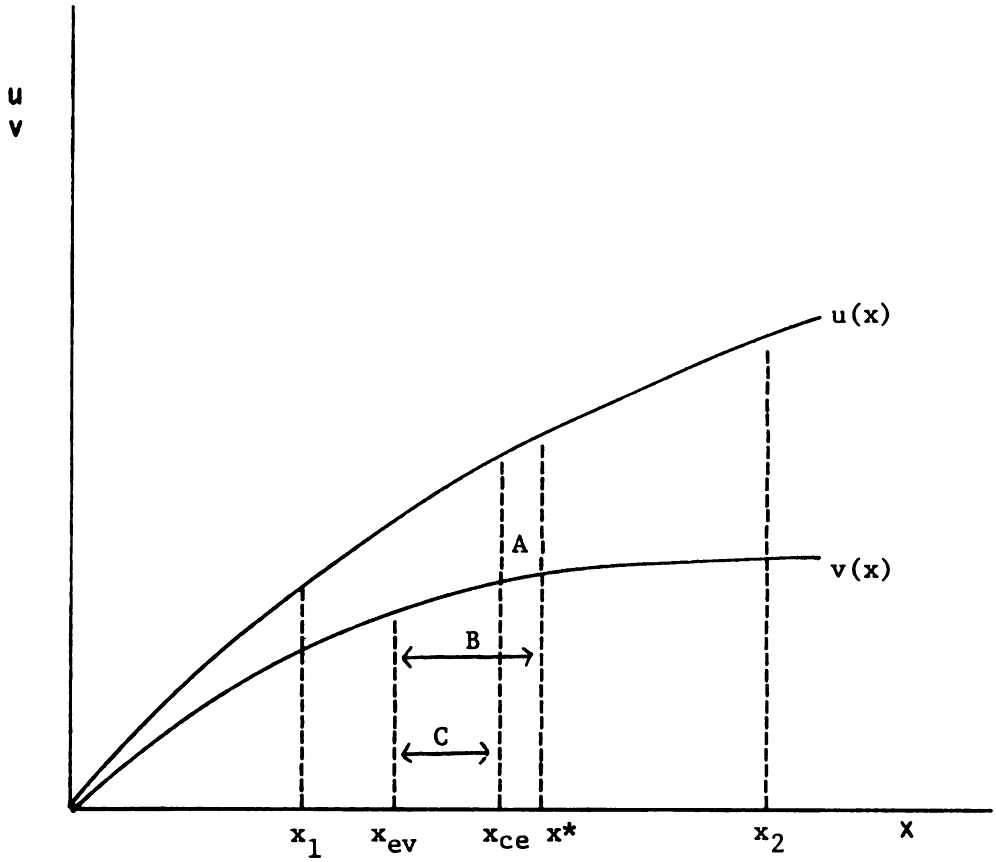
If $m(x)=r(x)$, indicating that both the value and utility functions bend at the same rate, $r(x)-m(x)=0$ and $b(x)=0$. This indicates that the rate of bending of the utility function over risky prospects is due solely to the non-linearity of the value function for the outcomes in riskless situations. Similarly, if $m(x)<r(x)$, indicating that the rate of bending of the utility function is greater than that of the value function, $r(x)-m(x)>0$, and $b(x)>0$. This implies that the rate of bending of the utility function is due, in part, to the non-linearity of the value function and, in part, to some other factor which is assumed to be attitude towards risk. Notice that we do not treat $r(x)$ as attitude towards risk; instead, attitude towards risk is only that part of $r(x)$ not attributable to $m(x)$. This is illustrated in Figure 10. The case of intrinsic risk proneness, when $r(x)<m(x)$ and $b(x)<0$ is illustrated in Figure 11. In both of these cases, in contrast to the case of intrinsic risk neutrality illustrated in Figure 9, the entire curvature of the utility function can not be explained by the individuals' preferences for outcomes under certainty measured via the satiation sacrifice. Since the utility function incorporates information about preferences for outcomes under certainty and attitude towards risk, the degree of curvature not attributable to preferences for outcomes under certainty can be attributed to intrinsic risk attitude.

Note that like the Arrow-Pratt coefficient of absolute risk aversion, Dyer and Sarin's measure of intrinsic risk aversion is a local measure. If Bell and Raiffa's hypothesis that rational individuals exhibit constant intrinsic risk attitude in risky situations is true, then, Dyer and Sarin argue, the hypothetical utility function for riskiness of outcomes should take one of the following forms:



A = risk premium = $x^* - x_{ce}$
B = satiation sacrifice = $x^* - x_{ev}$
C = risk premium due to only attitude towards risk = $x_{ev} - x_{ce} > 0$

Figure 10: Value and Utility Functions for the Case of Intrinsic Risk Aversion



A = risk premium = $x^* - x_{ce}$
B = satiation sacrifice = $x^* - x_{ev}$
C = risk premium due to only attitude towards risk = $x_{ev} - x_{ce} < 0$

Figure 11: Value and Utility Functions for the Case of Intrinsic Risk Pronenss.

u(x) is a linear transform of $-e^{-bv(x)}$ iff $b(x) = b > 0$

u(x) is a linear transform of v(x) iff $b(x) = 0$

u(x) is a linear transform of $e^{bv(x)}$ iff $b(x) = b < 0$.

These can be viewed as the function that transforms the value function into the utility function. These three functional forms are nearly identical to the forms of the utility function u(x) which Pratt proves are required in the cases of constant absolute risk aversion, constant absolute risk neutrality and constant absolute risk proneness. However, these transformations are from v(x) to u(x) rather than from x to u(x).

Comparing Value and Utility Functions

Numerous studies can be found in the marketing and management literature in which both utility and value functions are assessed over the same attribute and then compared. These studies have been concerned primarily with measurement techniques and have tested for differences between utility and value functions rather than trying to establish a relationship between the two. A very small sample of these studies is reviewed here to give a flavor of their results.

Tversky (1967) assessed value and utility functions for eleven subjects and reported that risky and riskless preference functions are not linearly related, even for fairly trivial outcomes. Fischer (1977) used ten subjects to test assessment procedures for value and utility functions. In the context of a job selection problem, he found that for five out of the ten subjects, the hypothesis that u(x) is a linear transformation of v(x) was not supported. Although Fischer hypothesizes that there exists some monotonic transformation between value and utility functions where they are not the same, he does not develop this concept further.

Allais (1979) reports utility and value functions for four subjects who took part in a large experiment on the assessment of subjective preferences conducted in 1952. $u(x)$ was not equal to $v(x)$ for any of the subjects. Currim and Sarin (1984) tested assessment procedures for riskless value and risky utility functions within the context of a job selection problem. They found that $u(x)$ was not equal to $v(x)$ for 30 of the 43 subjects involved in the study.

Testing the Hypothesis of Constant Intrinsic Risk Attitude

Krzysztofowicz (1983) presents the results of the only study which assesses u and v, tests the hypothesis of constant intrinsic risk attitude for all $x \in X$, and compares the intrinsic risk attitudes of decision makers for two decisions. The experiment he conducted supplies two utility functions and one value function for the same subject, attribute and decision context, but for two different decisions. Krzysztofowicz elicited responses from six water resource experts regarding announcement of the level of controlled releases from a multipurpose water reservoir given an uncertain supply of water. The individuals were presented with three scenarios. In the first scenario, the actual water supply was equal to the planned supply. Thus, no uncertainty was present. A value function was obtained to quantify the relative value of water through the strength of preference of the decision makers. The second scenario was termed the pessimistic decision environment and the decision makers were instructed to determine the utility of increments of water received during the season if, at the beginning of the season, the predicted water supply was at a very low level and thus, the announced planned release was low. The third scenario was termed the optimistic decision environment and the decision makers were instructed to determine the utility of increments of water received during the season if, at the beginning of the season, the predicted water supply was at a very high level and the announced planned release was high.

The hypothesis of constant intrinsic risk attitude was supported for both the "pessimistic" and "optimistic" decision scenarios. However, only three of the six subjects had the same intrinsic risk attitude for both decisions. The remaining three subjects changed their intrinsic risk attitudes from intrinsically risk averse to intrinsically risk seeking or vice versa. This result, although obtained from limited testing, points out the important distinction which must be made between constant intrinsic risk attitude for one decision (that the transformation between u(x) and v(x) is constant), and the idea of an intrinsic attitude towards risk which is constant across situations.

The reversal of intrinsic risk attitudes between decisions in Krzysztofowicz's study may be due, in part, to the construction of his experiment. Kahneman and Tversky (1979) have observed that individuals are generally risk seeking in the domain of losses and risk averse in the domain of gains. They argue that the utilities elicited will be influenced by how the questions are framed. The two scenarios analyzed in Krzysztofowicz's experiment can be viewed as two framings of the same outcome space; any outcome above the extremely pessimistic decision could be viewed by the decision maker as a gain while any outcome below the extremely optimistic decision could be viewed as a loss. Slovic, Fischhoff, and Lichtenstein (1982) report that if an outcome representing a monetary loss is framed in terms of an insurance premium rather than a sure loss, a reversal in risk attitude is often observed.

Because of the limited nature of the experiment and the possible influence of framing, Krzysztofowicz's results cannot be interpreted as conclusive evidence supporting or rejecting the hypothesis that risk attitude is independent of the decision context. Therefore, we continue with our hypotheses that: (1) there is some transformation between u(x) and v(x) for a given decision, which encapsulates risk attitude and (2) intrinsic risk attitude is constant across situations and may be termed the individual's intrinsic attitude towards risk. The first hypothesis addresses the question of whether the transformations between utility and value functions reflect actual preferences of decision makers. The second hypothesis addresses the question of whether individuals' intrinsic risk attitudes are independent of the attributes over which their preferences are assessed. This hypothesis seems intuitively plausible since any non-linear transformations between an actual attribute and the value of that attribute are incorporated in v(x) and, therefore, do not enter into the relationship between v(x) and h[v(x)].

The Adequacy of the Present Theory of Intrinsic Risk Attitude

Stiglitz (1969, pp. 664-666) has shown that for constant absolute risk attitudes to be exhibited, all indifference curves must be homothetic and all Engel curves must be straight lines through the origin. Dyer and Sarin, by explicitly separating the effect of non-linear value functions from attitude towards risk have obviated the need for homothetic indifference curves in the development of the concept of intrinsic risk attitude. However, Dyer and Sarin's development of constant intrinsic risk attitude implicitly contains another assumption which is equally troublesome. By using holistic preference judgements, Dyer and Sarin assume that one of the following two conditions hold: (1) that all decision problems are simple value problems, i.e. that outcomes refer to only one attribute, or (2) that preferences for attributes in multiattributed outcomes are independent and additive.

Extension agents, farm management specialists and others concerned with decision making in the food and fiber sector know that decision makers are often concerned with multiple attributes of any outcome. A farmer may be concerned not only with maximizing income, but also with his asset/debt ratio, quality of life, maintaining soil productivity, or implications of current decision outcomes for future flexibility in organization of the farm firm. Therefore, if our goal is to develop a measure of attitude towards risk which can be used in predicting preferred action choices for realistic decisions, we cannot assume that each possible outcome of a decision has only one attribute.

To do this, we must reconsider the assumption that preferences for attributes in multiattributed outcomes are independent and hence, additive. The assumptions of independence required for additivity are quite strong and easily violated.[3] Consider an outcome X with attributes (x_1, x_2). The assumption of strict independence of attributes required for additivity is, loosely speaking, that preferences for attribute x_1 are not influenced at all by the level of attribute x_2. The indifference map for this case is shown in Figure 12.

Keeney and Raiffa (1976, pp.330-332) show that if preferences over an outcome space for three attributes are compatible with an additive value

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[3] The independence conditions required for additive and multiplicative value and utility functions will be discussed in detail in Chapter 4.

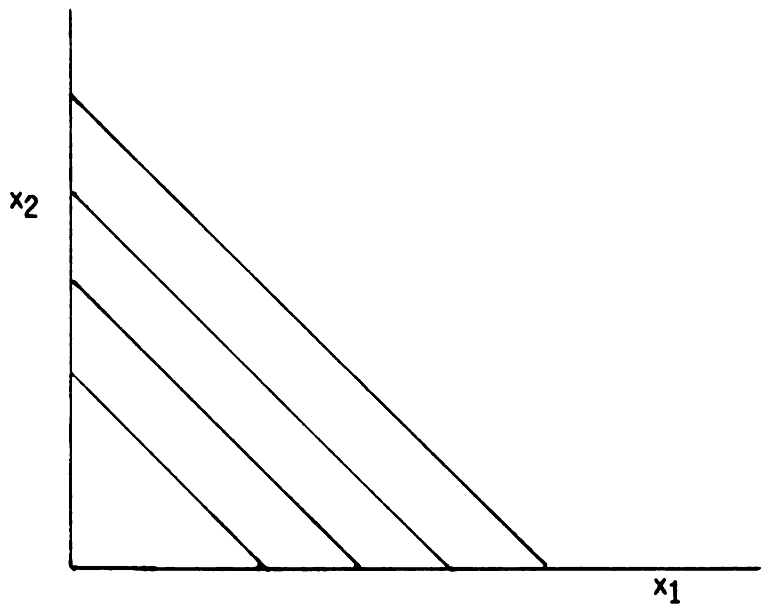


Figure 12: Indifference Curves for Two Attributes which Meet the Independence Conditions Required for Additivity.

function and conditions for a multiplicative utility function are met, then the utility function must have one of the following three forms:

u(x) is a linear transform of $-e^{-cv(x)}$, $c > 0$

u(x) is a linear transform of v(x)

u(x) is a linear transform of $e^{cv(x)}$, $c > 0$

The second case, analogous to Dyer and Sarin's constant intrinsic risk neutrality, implies that both the value and utility functions are additive. The first and third cases, analogous to Dyer and Sarin's constant intrinsic risk aversion and constant intrinsic risk proneness, imply that the value function is additive and the utility function is multiplicative. Again, the case of a multiplicative value function is not addressed.

Development of a more general set of transformations between value and utility functions which allow for cases of additivity or multiplicativity in both value and utility functions is the goal of Chapter 4. It will also be shown that the results obtained for the case when the value function is additive and the utility function is multiplicative are equivalent to those given by Dyer and Sarin and Keeney and Raiffa.

CHAPTER 4

A NEW MEASURE OF ATTITUDE TOWARDS RISK

Impetus for development of a new measure of attitude towards risk has come from the observations that marginal value of money is not constant, preferences are not homogeneous, and preferences for multiple attributes of outcomes do not necessarily meet the conditions required to assume additivity of the value or utility function. The first two observations are addressed by the concept of intrinsic risk attitude. The third must be addressed by the use of multi-attribute utility theory as a basis for a new measure of attitude towards risk.

The chapter begins with an examination of the independence conditions which are required to be met in order to assume additivity or multiplicativity of value and utility functions. This is followed by a brief exposition of the aspects of multiattribute utility theory which are pertinent to the development of the measure of attitude towards risk. This discussion includes some special problems which arise in the current application and which necessitate the weakening of some of the conditions of independence. The allowable transformations between value and utility functions which form the basis for measuring attitude towards risk are then developed. These results are used in a sample application and then compared to those obtained by Dyer and Sarin in their development of the concept of constant intrinsic risk attitude.

Independence Conditions for Additive and Multiplicative Value and Utility Functions

Using one of several representation theorems, value functions can be constructed which preserve orderings among multiattributed riskless outcomes. The form of the value function (additive or multiplicative) depends upon the degree to which preferences meet certain conditions of independence. Both additive and multiplicative value functions require the condition of mutual preferential independence.

> Preferential Independence: Attribute Y, where $Y \epsilon X$ is preferentially independent of its complement Y' if the preference order of consequences involving only changes in the levels in Y does not depend on the levels at which attributes in Y' are held fixed.

This definition implies that the conditional indifference curves over Y do not depend on the attributes Y'.

> Mutual Preferential Independence: Preferential independence must also hold for attributes in Y' with respect to Y.

For a value function to be multiplicative, it is required only that mutual preference independence and weak difference independence hold.

> Weak Difference Independence: Y is weak difference independent of the remaining attributes if the ordering of preferences on Y does not depend on the fixed level of the other attributes.

For a value function to be additive, a stronger condition must be met. In addition to mutual preference independence, strong difference independence must hold.

Strong Difference Independence: Attribute Y is strong difference independent of the remaining attributes if the preference difference between two levels of Y is not affected by fixed levels of the other attributes.

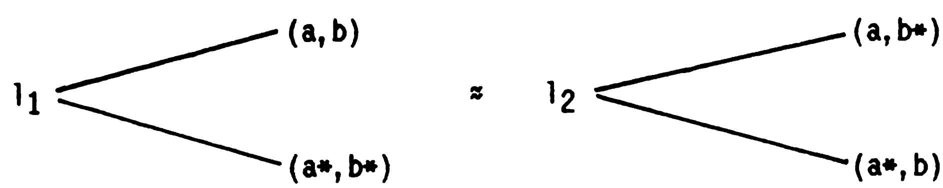
Similarly, additive and multiplicative utility functions can be distinguished on the basis of the restrictiveness of the independence conditions which preferences meet. Both additive and multiplicative utility functions require that the condition of utility independence is met.

Utility Independence: Attribute Y, where $Y \in X$, is utility independent of its complement Y' if the conditional preference order for lotteries involving only changes in the levels of attributes in Y does not depend on the levels at which the attributes Y' are fixed.

For a utility function to be multiplicative, only utility independence and mutual preference independence must hold. Additivity of a utility function requires that an additional condition, known as the marginality assumption, holds.

Marginality Assumption: Attributes Y, where $Y \in X$ are additive independent if preferences over lotteries on Y depend only on their marginal probability distributions and not on their joint probability distributions.

The marginality assumption is sometimes difficult to intuit, but can be made clearer using the following example. The marginality assumption requires that for any $a \in Y_1$ and $b \in Y_2$, the decision maker is indifferent between lotteries l_1 and l_2 shown below.



Multiattribute Value and Utility Functions

If preferences meet the independence conditions required for additivity discussed above, an additive multiattribute value function is written as:

$$(4.1)\quad v(x) = \Sigma w_i v_i(x_i)$$

where v_i is a single attribute value function and w_i is a scaling constant between zero and one.[1]

If preferences meet the independence conditions required for multiplicativity, a multiplicative multiattribute value function is written as:

$$(4.2)\quad 1 + wv(x) = \Pi[1 + ww_i v_i(x_i)]$$

where w is a constant reflecting the type and degree of non-additivity present.[2]

Similarly, if preferences meet the independence conditions required for additivity, an additive multiattribute utility function is written as:

$$(4.3)\quad u(x) = \Sigma k_i u_i(x_i)$$

where u_i is a single attribute utility function and k_i is a scaling constant between zero and one.

If preferences meet the independence conditions required for multiplicativity, a multiplicative multiattribute utility function is written as:

$$(4.4)\quad 1 + ku(x) = \Pi[1 + kk_i u_i(x_i)]$$

where k is a constant reflecting the type and degree of non-additivity present and must meet the conditions specified for w in footnote 2.

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1 Calculation of scaling constants is discussed in Chapter 5.

2 Calculation of w is also discussed in Chapter 5. w must be either $-1 \le w < 0$ or $w > 0$ and solve the condition $1 + w = \Pi(1 + ww_i)$.

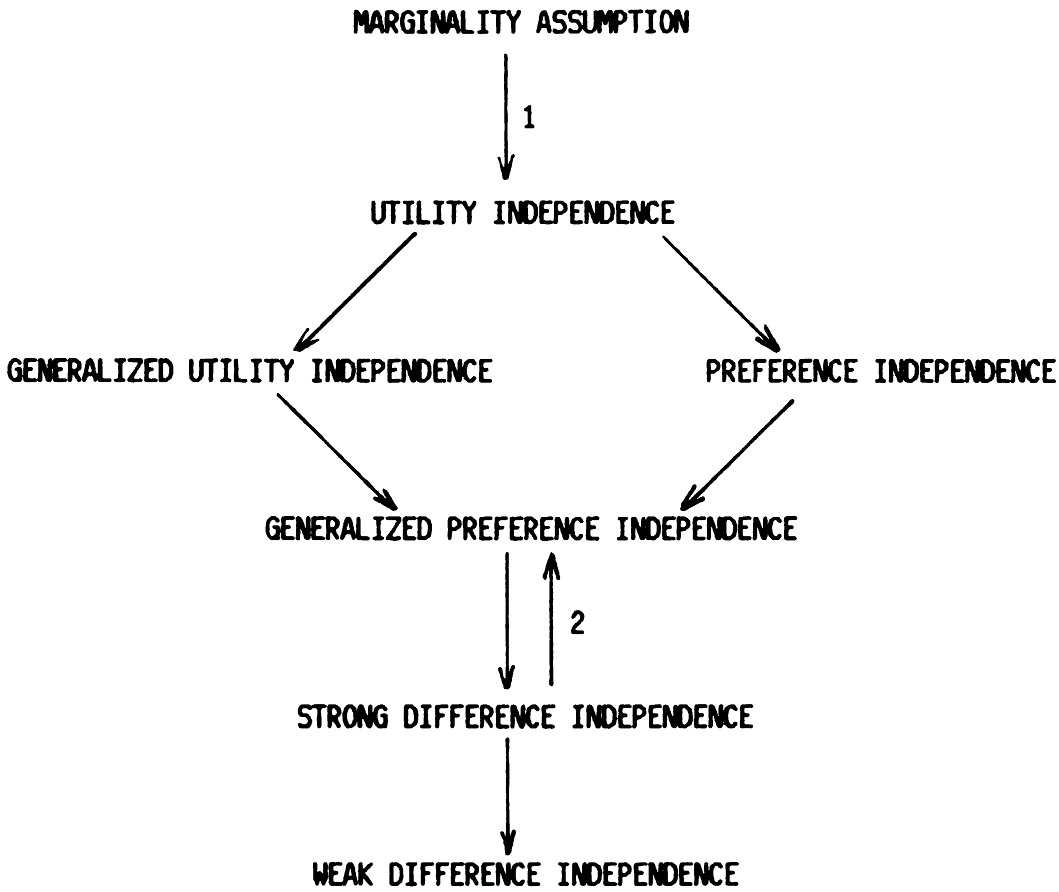
Reversals in Preferences and Sign Conditions

The multiplicative value and utility functions defined above do not allow for reversals in preferences over subsets of the attributes. In many situations, we would not expect reversals to occur in an individual's preferences. However, there are important cases where they do seem reasonable. For example, in an agricultural situation, one could expect that preferences for rain to increase or decrease depending on the level of sunshine available. If this occurs, the conditions of preference and utility independence required for multiplicative value and utility functions will be violated. It is impossible to have reversals in preference with the additive form of the value and utility functions. The violation of the independence conditions required for multiplicative functions can be avoided by weakening the independence conditions to those of generalized preference independence and generalized utility independence (Fishburn and Keeney, 1974, 1975).

> Generalized Preference Independence: Y_1 is generalized preferentially independent of Y_2 if, given any two levels of y_2, the two orderings of Y_1 are either identical, reversals of each other, or indifference exists among the Y_1.

> Generalized Utility Independence: Attribute Y_1, is generalized utility independent of attribute Y_2 if the conditional preference order for lotteries involving only changes in the levels of attributes in Y_1, given any two levels of y_2, are either identical, reversals of each other, or indifference exists among the Y_1.

Generalized preference independence is also known as sign dependence (Krantz, et al, 1971, pp. 329-332) or the sign axiom (Roskies, 1965). The relationships among independence concepts is shown in Figure 13.



1. Given $y, y', x, x' \in X$, x preferred to y and x' preferred to y' implies

2. Given $X = X_1 \times X_2 \times .. \times X_n$ for some positive integer n, each X_i is a convex subset of a finite dimensional Euclidean space and:
$\Sigma a(x)u(x) > \Sigma b(x)u(x)$ iff a is preferred to b for all $a, b \in A$ holds for some $u: X \to Re$ which is continuous in the relative usual product topology for X.

Figure 13: Relationships Among Independence Concepts[3]

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[3] Adapted from Fishburn and Keeney (1974, p. 300)

It has been noted that either $w>0$ or $-1\leq w<0$. If w is greater than zero and $1 + wv$ is a multiplicative value function with terms $1 + ww_i v_i(x_i)$ then $v' = a(1 + wv)$ where $a>0$ is also a multiplicative value function with terms $a_i(1 + ww_i v_i(x_i))$ where $\Pi a_i = a$. But if $-1\leq w<0$ and v is the original value function, $1 + wv$ produces an inverse ordering. However, $-(1 + wv) = -\Pi(1 + ww_i v_i(x_i))$, or:

(4.2a) $(\text{sgn } w)(1 + wv) = (\text{sgn } w)\Pi(1 + ww_i v_i(x_i))$

Analogous results apply the the multiplicative utility functions so that:

(4.4a) $(\text{sgn } k)(1 + ku) = (\text{sgn } k)\Pi(1 + kk_i u_i(x_i))$

These results are used in the proofs of the transformations between value and utility functions. Reference to (4.2) and(4.4) in the following sections will, unless otherwise stated, refer to the forms shown in (4.2a) and (4.4a).

Economic Interpretation of Parameters k and w [4]

Consider the two equal probability lotteries <A,C> and <B,D> shown in Figure 14 and assume that preferences are increasing in both Y and Z. It will be assumed that the individuals value functions are homogeneous. It can be shown that if <A,C> is preferred to <B,D>, $k>0$ and Y and Z can be viewed as complements, if <A,C> is indifferent to <B,D>, $k=0$, indicating an additive utility function and that preferences for Y and Z are independent, while if <B,D> is preferred to <A,C>, $k<0$ and Y and Z can be viewed as substitutes.

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[4] Parts of this section are adapted from Keeney and Raiffa (1976, pp. 240-241).

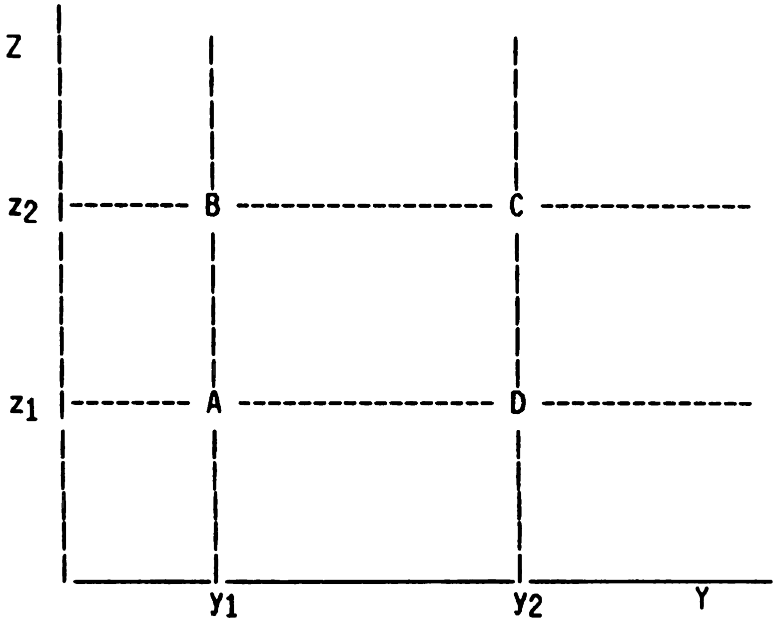

Figure 14: Using Lotteries to Interpret k in a Multiplicative Multiattribute Utility Function.

The lotteries are structured so that for <A,C>, the individual will either receive a high level of both Y and Z or a low level of both. For lottery <B,D>, the individual will receive either a high level of Y or Z, but not a high level of both. If <A,C> is preferred, it is as if the individual needs an increase in Y to complement and increase in Z in going from A to C. Otherwise, the full worth of the increase in Z could not be exploited. On the other hand, to prefer <B,D> implies that is important to receive a high level of at least one attribute and, given a high level of, say, Y, the increased preference due to an increase in Z is not as much. Thus Y and Z can be thought of as substitutes for each other.

To illustrate a complementary case, consider a farm manager who is thinking of investing in the use of a new, pest sensitive, high yielding variety of grain and a new, more expensive pest management system. Attribute Y can be viewed as the productivity of the new variety on his field, given existing soil types, drainage, etc. Attribute Z can be viewed as the successfulness of the new pest management system. If both the high yielding variety and the new pest management system are successful, the farm manager is very happy. However, if either fails (the HYV does not flourish on his soil type, or the pest management strategy does not keep away the targeted pests), the financial consequences may be as bad as if both had failed.

To illustrate a case of substitution, consider a farm manager who is thinking of using a new variety of corn on two separate fields which have very different soil and drainage characteristics. Attribute Y, in this case, is the yield from one field while attribute Z is the yield from the other. While the farm manager would like it if both fields flourished, achievement of attributes Y and Z would, most likely, be substitutes.

Fischer (1976) sets forth another interpretation for k. He argues that a $k>0$ could be indicative of complementary attributes, or the simple fact that the individual is willing to give up a sure thing in either attribute for the chance of receiving something which is better with regard to both attributes, i.e. risk proneness. Similarly, a $k<0$ could indicate attributes which are substitutes or simply an aversion to high variance outcomes.

These two interpretations of k are not necessarily contradictory; both can add insight into the formation of attitudes towards risk. If Keeney and Raiffa's interpretation of k is applied to the interpretation of w, then if $w>0$, the attributes can be said to be complements, and if $w<0$, the attributes can be said to be substitutes. Because a multiattribute utility function, like a von Neumann-Morgenstern utility function, incorporates both attitudes towards risk and information about the individual's value function, examination of k without information about w does not allow us to determine what portion of k is due to the complementarity or substitutability of the attributes and what portion is due to attitude towards risk.

Allowable Transformations Between Value and Utility Functions

Results which relate value and utility functions can be found scattered throughout the recent literature in measurement theory (e.g. Dyer and Sarin, 1979; Keeney and Raiffa, 1976; von Winterfeldt, 1979). Further work is needed to exactly define the class of allowable transformations between utility and value functions. In this section we will present proofs of the relationships between measurable value and utility functions. A common way to relate $v(x)$ and $u(x)$ is through the uniqueness theorems of their respective measurement theoretic representations. The proofs presented here differ from many earlier proofs in that we will rely on the use of Cauchy's four fundamental functional equations; this greatly simplifies the proofs and allows us to depart from the standard assumptions about the additivity of the value function.

Cauchy was not concerned with decision theory, utility functions, or value functions. Instead, he introduced the first systematic treatment of some basic functional equations to prove the powerful and practical nature of some rather abstract and original concepts he introduced such as the concept of limit and the concept of a continuous function. Cauchy's functional equations are applicable to the solution of many problems in economics (Gehrig, 1984). They are used here in a new application.

We begin with the general functional equation:

(4.5) $F[x_1, x_2, \ldots, x_n] = h\{G[x_1, x_2, \ldots, x_n]\}$

This general functional equation can be rewritten for the specific cases of additive and multiplicative value and utility functions.

For the case where the value and the utility function are additive, a general functional equation can be written as:

(4.6) $u(x) = \Sigma u_i(x_i) = h[\Sigma v_i(x_i)] = h[v(x)]$

This is of the same form as the general Cauchy equation:

(4.7) $h(x) + h(y) = h(x + y)$

Which, if h is continuous, has the general solution:

(4.8) $h(r) = ar$

Similarly, for the case where the value function is additive and the utility function is multiplicative, the functional equation, Cauchy equation, and general solution for the Cauchy equation can be written, respectively, as:

(4.9) $1 + ku(x) = \Pi(1 + kk_i u_i(x_i)) = h[\Sigma v_i(x_i)] = h[v(x)]$

(4.10) $h(x)h(y) = h(x + y)$

(4.11) $h(r) = e^{ar}$.

If the utility function is additive and the value function is multiplicative, the functional equation, Cauchy equation, and general solution to the Cauchy equation can be written, respectively, as:

(4.12) $u(x) = \Sigma u_i(x_i) = h[\Pi(1 + ww_i v_i(x_i))] = h[1 + wv(x)]$

(4.13) $h(x) + h(y) = h(xy)$

(4.14) $h(r) = a \ln r$

If both the utility and value functions are multiplicative, the functional equation, Cauchy equation, and general solution to the Cauchy equation can be written, respectively, as:

(4.15) $1 + ku(x) = \Pi(1 + kk_i u_i(x_i)) = h[\Pi(1 + ww_i v_i(x_i))] = h[1 + wv(x)]$

(4.16) $h(x)h(y) = h(xy)$

(4.17) $h(r) = r^a$.

The Cauchy equations and their general solutions can be used to solve the general functional equations for the cases of additive and multiplicative value and utility functions. The solutions, which can be viewed as the transformation between the utility and value functions, provide the basis for a new measure of attitude towards risk. Each possible case and its solution are given below. Proofs are contained in appendices 4A, 4B, 4C and 4D.

<u>Case</u> <u>1</u>: If the value and utility functions are additive, then:

(4A.1) $u = v$

<u>Case</u> <u>2</u>: If the value function is additive and the utility function is multiplicative, then:

(4B.1) $1 + ku = (1 + k)^v$

Case 3: If the value function is multiplicative and the utility function is additive, then:

(4C.1) $u = (\ln(1 + wv))/(\ln(1 + w))$

Case 4: If the value and utility functions are multiplicative, then:

(4D.1) $1 + ku = (1 + wv)^{(\ln(1 + k))/(\ln(1 + w))}$

Synthesizing and Applying the Results

It has been shown that there exists a specific set of allowable transformations between value functions and utility functions. To envision how these results can be applied in determining an individual's pure or intrinsic attitude towards risk, consider the individual whose value and utility functions are shown in Figure 15. Assume that the individual's value function is additive and that:

(4.18) $v_i(x_i) = \ln x_i$

and that the individual's utility function is multiplicative and that:

(4.19) $u_i(x_i) = (x_i - 1)(1/(e - 1))$

Note from panel A of Figure 15 that the utility function is linear. The Arrow-Pratt coefficient of absolute risk aversion, taken at any $x \epsilon X$ will be zero, indicating risk neutrality.

But our concern is not with the utility function alone; we are interested in separating pure attitude towards risk from the effects of non-constant marginal value. We may proceed in several directions, depending on what information is available about the individual's value and utility functions.

If we had actually elicited the individual's multiattribute utility function and determined k, which in this example is assumed to be e-1, we could test for the validity of the hypothesized transformation between

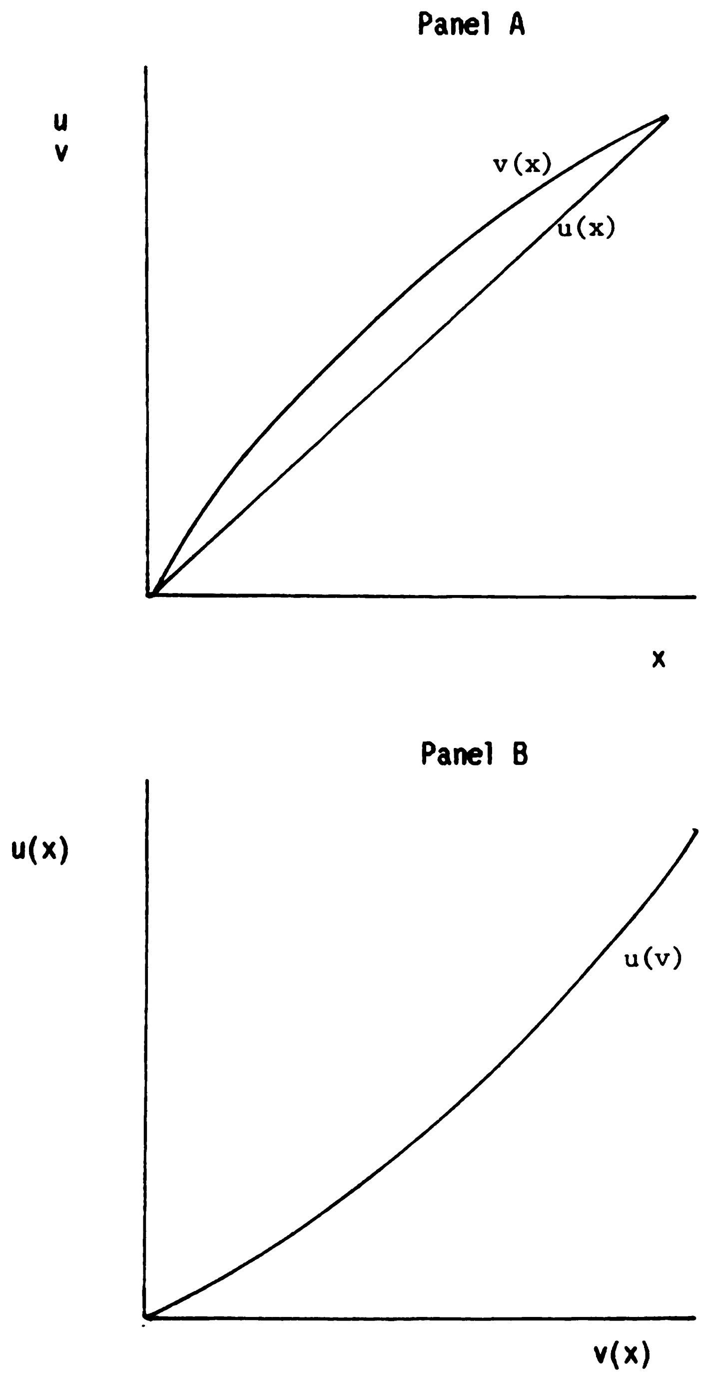

Figure 15: v(x), u(x) and u(v) for an Intrinsically Risk Prone Individual when $v(x) = \Sigma \ln x$.

additive value functions and multiplicative utility functions developed in the previous section. Taking the transformation:

$$(4A.1)\quad 1 + ku = (1 + k)^{v}$$

and substituting in (4.18), (4.19) and the value for k we obtain:

$$(4.20)\quad 1 + (e-1)[(x - 1)(1/(e-1))] = (1 + e - 1)^{\ln x}$$

which has the solution:

$$(4.21)\quad x = x$$

and thus, the transformation is valid.

Similarly, the value of k could be predicted from (4A.1), (4.18) and (4.19) and tested against the estimated k. In this case:

$$(4.22)\quad 1 + k[(x - 1)(1/(e-1))] = (1 + k)^{\ln x}$$

which has the solution:

$$(4.23)\quad k = e - 1$$

How do we interpret the k value? In this case, $k > 0$, which indicates risk prone behavior. This can be seen more intuitively by examining the function u(v) in panel B of Figure 15 which is of the form:

$$(4.24)\quad u(v) = (e^{v} - 1)/(e - 1)$$

The function is convex to the origin. A measure of the rate of bending of this function, analogous to the Arrow-Pratt coefficient, is:

$$(4.25)\quad -(\delta^2 u/\delta v^2)/(\delta u/\delta v) = -1 \text{ at all } v$$

which indicates constant risk proneness.

This information about the individual's pure attitude towards risk can be used in predicting preferred action choices in other situations. Assume that in a different decision situation, the decision maker's value function is again additive with:

$$(4.26)\quad v_i(x_i) = x_i^2$$

Using our knowledge of the transformation between additive value functions and multiplicative utility functions, and the value of k, the individuals von Neumann-Morgenstern utility function is easily obtained:

(4.27) $u(x) = (e^{x^2} - 1)/(e - 1)$

The Arrow-Pratt coefficient at a point x on this function is:

(4.28) $r(x) = [2e^{x^2}(2x^2-1))/(e-1)][(e^{(1-x^2)}/2x)-(1/(2xe^{x^2}))]$

which, when evaluated at x = 1.5, is -3.67, which indicates risk proneness as would be expected from the convex curvature of u(x) shown in panel A of Figure 16

But again, if we are concerned with separating the pure attitude towards risk from the effects of non-linearity in the value function, we want to know the rate of bending of u(v), not u(x). Because u(v) is:

(4.29) $u(v) = (e^v - 1)/(e - 1)$

the function, shown in panel B of Figure 16, is convex to the origin. Measuring the rate of bending of this function yields:

(4.30) $-(\delta^2u/\delta v^2)/(\delta u/\delta v) = -1$ at all v

which indicates constant risk proneness.

If only the Arrow-Pratt coefficient were to be examined, we would surmise that the individual's attitude towards risk had changed from risk neutrality to risk proneness between the two situations. But we can see from this example that the individual's pure attitude towards risk has not changed; the change in the rate of bending of u(x) was due to a change in the value function, not a change in pure attitude towards risk.

Now, let us look at another individual making decisions in the same context. This second individual is identical to the one above except that his k value is equal to -.8. Panel A of Figure 17 shows his value and

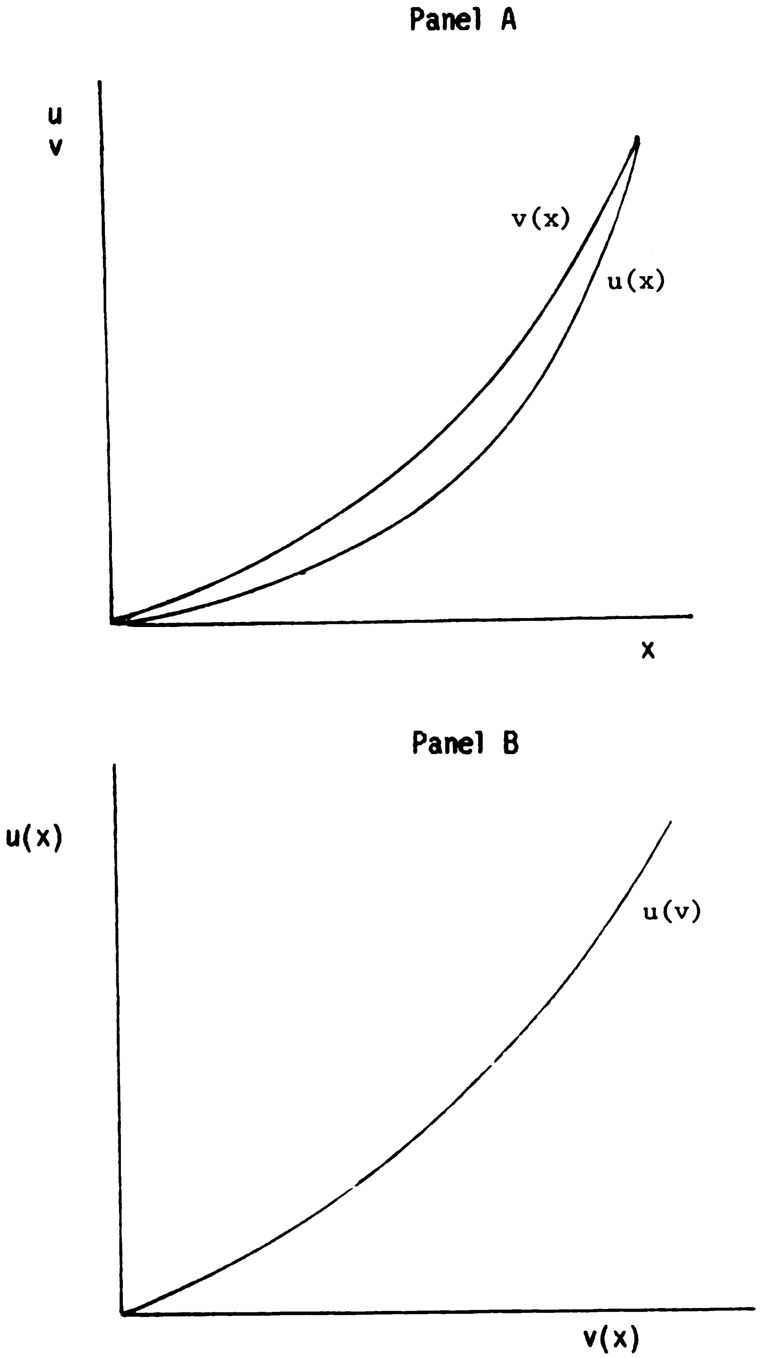

Figure 16: v(x), u(x) and u(v) for an Intrinsically Risk Prone Individual when $v(x) = \Sigma x^2$.

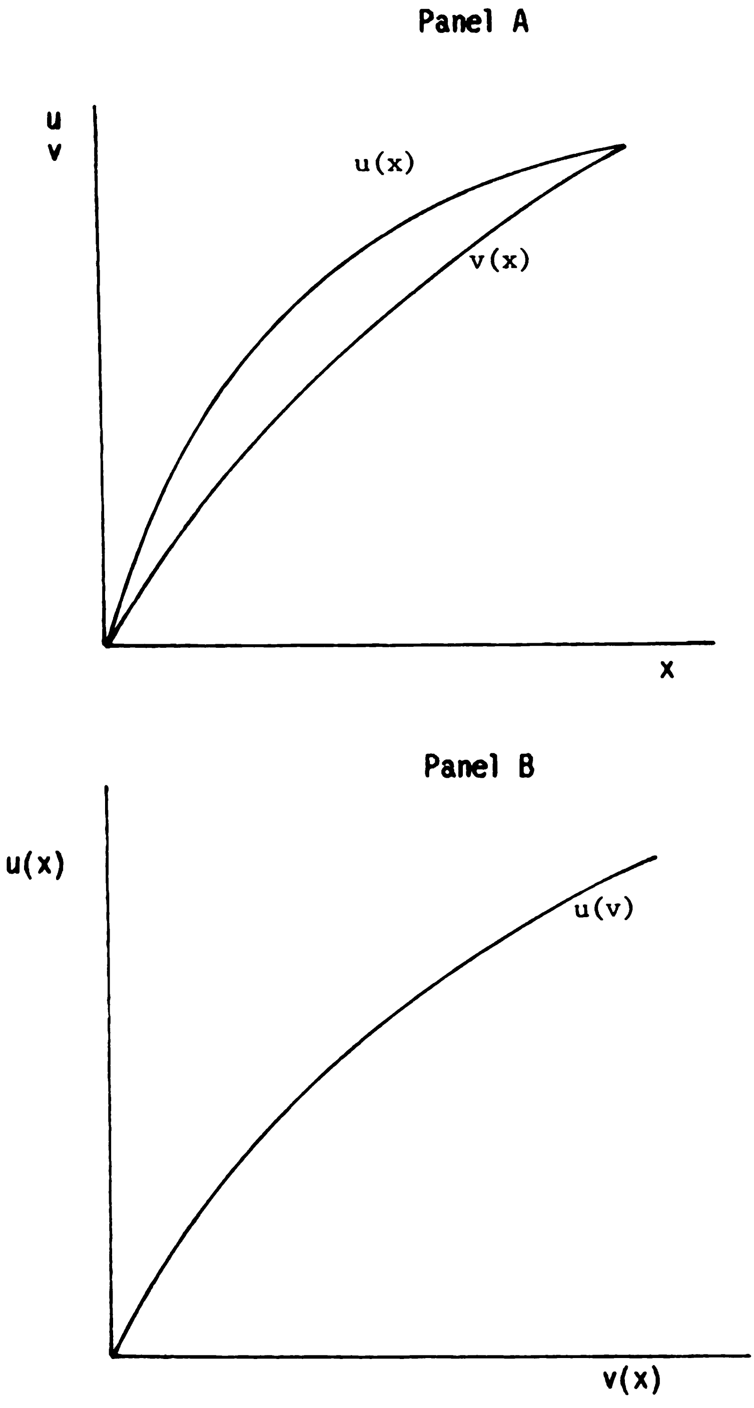

Figure 17: v(x), u(x) and u(v) for an Intrinsically Risk Averse Individual when $v(x) = \Sigma \ln x$.

utility functions when $v = \ln x$. This individual's Arrow-Pratt coefficient is:

(4.31) $r(x) = (\ln 5 + 1)/x$

which, when evaluated at $x = 1.5$ is 1.74, indicating aversion to risk. The function $u(v)$ for this individual is shown in panel B of Figure 17. Measuring the rate of bending of this function yields:

(4.32) $-(\delta^2 u/\delta v^2)/(\delta u/\delta v) = \ln 5 = 1.61$ at all v

which, like the Arrow-Pratt coefficient, indicates risk aversion, but a lower level than would have been indicated by examining only the utility function over x.

This individual is also faced with a second decision in which, like for the first individual, $v = x^2$. Panel A of Figure 18 shows this individual's value and utility functions for this situation. In this second situation, the individual's Arrow-Pratt coefficient is:

(4.33) $r(x) = 2x \ln 5 - (1/x)$

which, when evaluated at $x = 1.5$, is 4.16, indicating a very high level of aversion to risk. Is it possible that the individual's attitude towards risk has changed that much between the two situations? As before, if we examine the individual's utility function over v, shown in panel B of Figure 18, we see that it is concave to the origin, indicating some degree of risk aversion. Measuring the rate of bending of this function yields:

(4.34) $-(\delta^2 u/\delta v^2)/(\delta u/\delta v) = \ln 5 = 1.61$ at all v

which, like the Arrow-Pratt coefficient, indicates risk aversion, but at a much lower level than would have been indicated by examining only the utility function over x.

The examples discussed so far all involve cases where the value function is additive and the utility function is multiplicative. For these

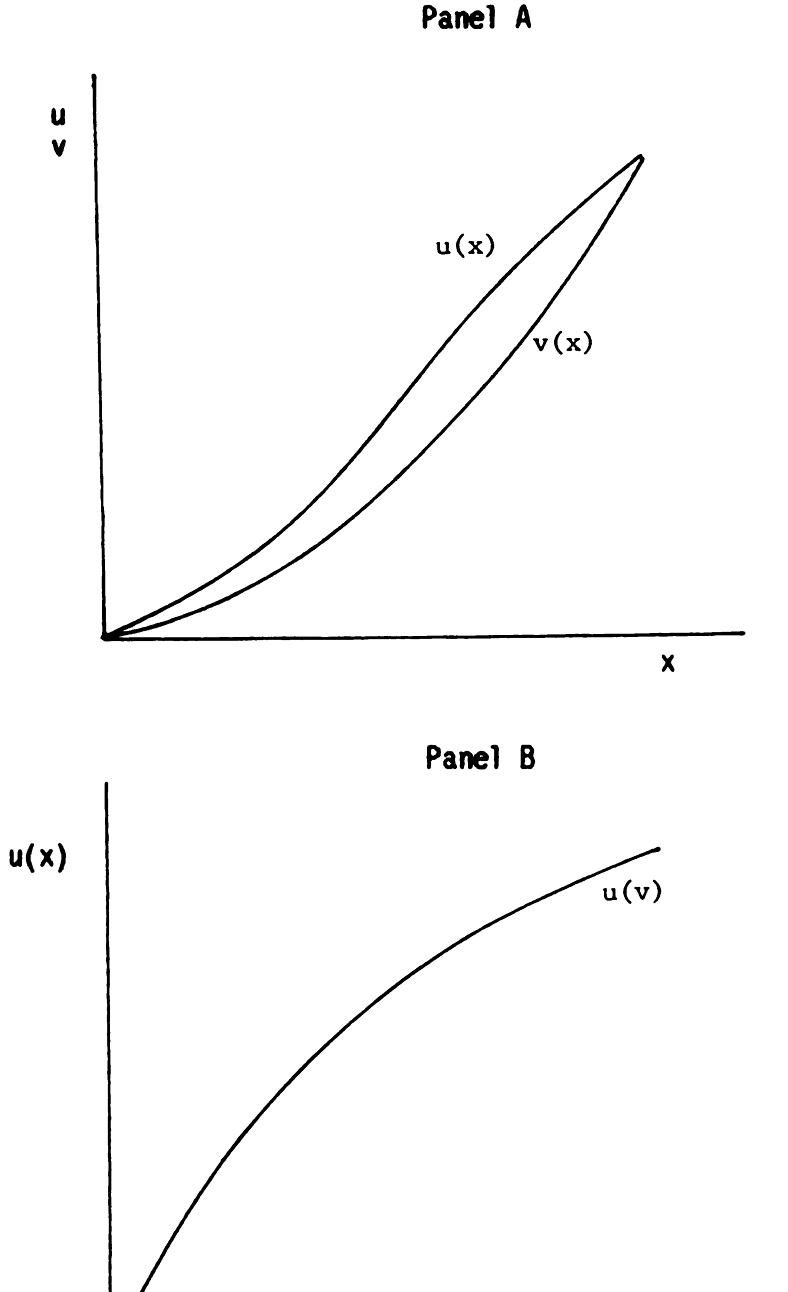

Figure 18: v(x), u(x), and u(v) for an Intrinsically Risk Averse Individual when $v(x) = \Sigma x^2$.

examples, the rate of bending of an individual's utility function over v, u(v), has been constant. Does the same hold for cases in which the value function is multiplicative and the utility function is either additive or multiplicative?[5] Preliminary results suggest that it does not.

Consider the case of an individual with a multiplicative value function where:

(4.35) $v_i(x_i) = \ln x_i$

and a multiplicative utility function. The transformation between the value and utility function takes the form:

(4D.1) $1 + ku = (1 + wv)^{(\ln(1 + k))/(\ln(1 + w))}$

Assume that k = -.5 and w = .9. Interpretation of these k and w values a la Keeney and Raiffa would suggest that for the multiattribute value function, the attributes were highly complementary while for the multiattribute utility function, the same attributes were substitutes. Fischer's interpretation of k would lead us to conclude that the individual is risk averse.

Given these values for k and w, the individual's von Neumann-Morgenstern utility function can be written:

(4.36) $1 - .5u(x) = (1 + .9 \ln x)^{(\ln (1 - .5))/(\ln (1 + .9))}$

or

(4.37) $u(x) = ((1 + .9 \ln x)^{-1.08} - 1)/-.5$

[5] Actual cases where the utility function is additive and the value function is multiplicative are rare. From a strictly measurement theoretic point of view, this condition is impossible since, as was shown in Figure 13, the independence conditions for additive utility imply the conditions for additive value. However, in some cases, economic theory or a priori knowledge about the attributes of outcomes involved may dictate that a multiplicative value function be used in conjunction with an additive utility function.

The value and utility functions for this individual are shown in panel A of Figure 19. The Arrow-Pratt coefficient, taken at x is:

$$(4.38)\quad (234/(125\ x + 112.5\ x\ \ln x)) + (1/x)$$

which, when evaluated at x = 2, is 1.076, indicating risk aversion.

The function u(v) for this same individual is shown in panel B of Figure 19. The rate of bending of this function is:

$$(4.39)\quad -(\delta^2 u/\delta v^2)/(\delta u/\delta v) = 234/(125 + 112.5\ x)$$

which, when evaluated at x = 2, is 1.153, also indicating risk aversion, but at a higher level than that indicated by the Arrow-Pratt coefficient.

Some interesting points can be made if we examine the same individual's Arrow-Pratt coefficient and rate of bending of the function u(v) when $v_i(x_i) = x_i^2$. In this case the Arrow-Pratt coefficient is:

$$(4.40)\quad r(x) = ((936\ x^2)/(250\ x + 225\ x^3)) - (1/x)$$

which, when evaluated at x = 2 is 1.128. The rate of bending of the function u(v) when x = 2 is .407. Note that at a given x, the rate of bending of u(v) when $v_i(x_i) = \ln x_i$ is not the same as the rate of bending of u(v) when $v_i(x_i) = x_i^2$. However, when the rate of bending of u(v) is evaluated at the same value of $v_i(x_i)$, it is the same for the two cases. When x = 2 and $v_i(x_i) = \ln x_i$, $v_i(x_i) = .693$. When x = .8326 and $v_i(x_i) = x_i^2$, $v_i(x_i) = .693$. The Arrow-Pratt coefficient, evaluated at this x is .718 and the rate of bending of the function u(v) is 1.153, identical to the case when $v_i(x_i) = \ln x_i$.

Therefore, we can conclude that when the value function is multiplicative, the rate of bending of the function u(v) will not be constant at all levels of v within one decision context. It will, however, be constant at a given level of v across decision contexts. This differs from the case where

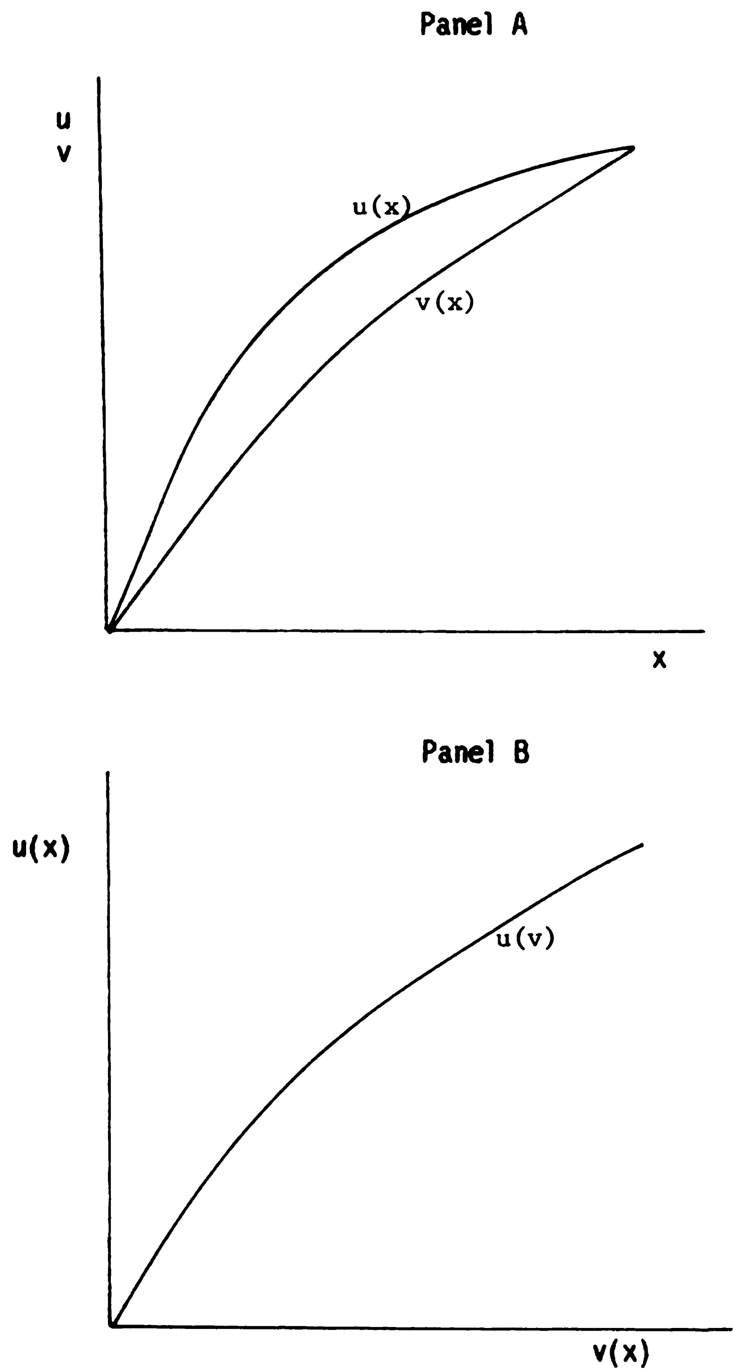

Figure 19: v(x), u(x) and u(v) for an Intrinsically Risk Averse Individual when $v(x) = \Pi \ln x$.

the value function is additive and the rate of bending of $u(v)$ is constant for all levels of v within one decision context as well as across decision contexts.

What relationship do these results bear to Dyer and Sarin's measure of intrinsic risk attitude? Recall from Chapter 3 that Dyer and Sarin establish a measure of intrinsic risk attitude, $b(x)$, such that:

(4.41) $b(x) = (r(x) - m(x))/v'(x)$

where $b(x)$ is the rate of bending of the function $u(v)$ and $m(x)$ is the rate of bending of the value function. This measure remains valid when the value function is additive and the utility function is either additive or multiplicative and when the value function is either additive or multiplicative.

Given this result, we can now re-examine Dyer and Sarin's hypothesis that the hypothetical utility function for riskiness of outcomes will take the forms:

u is a linear transform of $-e^{-bv(x)}$ iff $b(x) = b > 0$

u is a linear transform of v iff $b(x) = 0$

u is a linear transform of $e^{bv(x)}$ iff $b(x) = b < 0$

The second case, where $b(x) = 0$ follows directly from (4A.1). The first and third cases will be examined first for the situation where the value function is additive and the utility function is multiplicative. From Appendix 4B, this case can be written as:

(4B.1) $1 + ku = (1 + k)^v$

or, before standardization, as:

(4B.7) $(\text{sgn } k)(1 + ku) = e^{a(\text{sgn } k)v}$

When $k < 0$ and $b(x) > 0$, indicating risk aversion, (4B.7) can be rewritten as:

(4.42) $-(1 + ku) = e^{-av}$

or

(4.43) $u = (-e^{-av} - 1)/k$

which corresponds to Dyer and Sarin's hypothesis. Similarly, when $k > 0$ and $b(x) < 0$, indicating risk proneness, (4B.7) can be rewritten as:

(4.44) $u = (e^{av} - 1)/k$

which also conforms to Dyer and Sarin's hypothesis.

When the value function is multiplicative and the utility function is either additive or multiplicative, however, Dyer and Sarin's hypotheses regarding u are rejected. Therefore, we can conclude that Dyer and Sarin's hypothesis of constant intrinsic risk aversion hold for the cases when the value function is additive, but not when it is multiplicative.

APPENDICES TO CHAPTER 4

APPENDIX 4A

Case 1: If the value and utility functions are additive, then:

(4A.1) $u = v$

Proof:

Let $w_i v_i(x_i) = v_i'(x_i)$

$k_i u_i(x_i) = u_i'(x_i)$.

Assume that for the additive value function (4.1) and the additive utility function (4.3) there exists some continuous function h, such that, from (4.6):

(4A.2) $\Sigma u_i'(x_i) = h[\Sigma v_i'(x_i)]$

Let $x_j = x_j^0 = 0$ for all j not equal to i. (4A.2) is reduced to:

(4A.3) $u_i'(x_i) = h[v_i'(x_i)]$

Substituting the right hand side of (4A.3) into (4A.2) yields:

(4A.4) $\Sigma h[v_i'(x_i)] = h[\Sigma v_i'(x_i)]$

Now let $x_i = x_i^0 = 0$ for all $i = 3,4,\ldots,n$. (4A.4) can now be written:

(4A.5) $h[v_1'(x_1)] + h[v_2'(x_2)] = h[v_1'(x_1) + v_2'(x_2)]$

which is Cauchy's fundamental functional equation from (4.7) with general solution:

(4.8) $h(r) = ar$

Substituting (4.8) into (4A.2) yields:

(4A.6) $\Sigma u_i'(x_i) = a\Sigma v_i'(x_i)$

Since $\Sigma u_i'(x_i) = u$ from (4.3) and $\Sigma v_i'(x_i) = v$ from (4.1),

(4A.7) $u = av$

This solution of the functional equation is unique up to the specification of one parameter, a. To make the solution unique, the free parameter must be "used up". To do this, choose elements x^0, $x^1 \epsilon X$ such that $x^1 > x^0$ and defining $u(x^0) = v(x^0) = 0$ and $u(x^1) = v(x^1) = 1$. x^0 and x^1 can be viewed, respectively, as the worst and best alternatives available. Solving for a in (4A.6) yields:

(4A.8) $a = 1$

Therefore, the standardized equation takes the form:

(4A.1) $u = v$

APPENDIX 4B

Case 2: If the value function is additive and the utility function is multiplicative, then:

(4B.1) $1 + ku = (1+k)^v$

Proof:

Let $w_i v_i(x_i) = v_i'(x_i)$

$1 + kk_i u_i(x_i) = u_i'(x_i)$

Assume that for the additive value function (4.1) and the multiplicative utility function (4.4) there exists some continuous function h, such that, from (4.9):

(4B.2) $(\text{sgn } k)\Pi u_i'(x_i) = h[(\text{sgn } k)\ \Sigma v_i'(x_i)]$

(4B.2) is the compact form of the two functional equations $u' = h[v']$ for $k>0$ and $-u' = h[-v']$ for $k<0$.

Let $x_j = x_j^0 = 0$ for all j not equal to i. (4B.2) is reduced to:

(4B.3) $(\text{sgn } k)u_i'(x_i) = h[(\text{sgn } k)\ v_i'(x_i)]$

Substitution of the right hand side of (4B.3) into (4B.2) yields:

(4B.4) $\Pi h[(\text{sgn } k)\ v_i'(x_i)] = h[(\text{sgn } k)\ v_i'(x_i)]$

Now let $x_i = x_i^0 = 0$ for all $i = 3,4,\ldots n$. (4B.4) can now be written:

(4B.5) $h[(\text{sgn } k)\ v_1'(x_1)]h[(\text{sgn } k)\ v_2'(x_2)]$

$= h[(\text{sgn } k)\ v_1'(x_1) + (\text{sgn } k)\ v_2'(x_2)]$

which is Cauchy's fundamental functional equation from (4.10) with the general solution:

(4.11) $h(r) = e^{ar}$

Substituting (4.11) into (4B.2) yields:

(4B.6) $(\text{sgn } k)\Pi u_i'(x_i) = e^{(\text{sgn } k)\Sigma v'(x)}$

Since $v_i'(x_i) = v$ from (4.3) and $(\text{sgn } k)\Pi u_i'(x_i) = (\text{sgn } k)(1 + ku)$ from (4.4),

(4B.7) $(\text{sgn } k)(1 + ku) = e^{a\,(\text{sgn } k)\,v}$

Using the standardization conventions adopted in the proof of case 1, and solving for a yields:

(4B.8) $a = \ln(1 + k)$

Therefore, the standardized equation can be written:

(4B.1) $1 + ku = (1 + k)^v$

APPENDIX 4C

Case 3: If the value function is multiplicative and the utility function is additive, then:

(4C.1) $u = (\ln(1 + wv))/(\ln(1 + w))$

Proof:

Let $1 + ww_i v_i(x_i) = v_i'(x_i)$

$k_i u_i(x_i) = u_i'(x_i)$

Assume that for the multiplicative value function (4.2) and the additive utility function (4.3) there exists some continuous function h, such that, from (4.12):

(4C.2) $(\text{sgn } w) \Sigma u_i'(x_i) = h[(\text{sgn } w)\Pi v_i'(x_i)]$

(4C.2) is the compact form of the two functional equations $u' = h[v']$ for $w>0$ and $-u' = h[-v']$ for $w<0$.

Let $x_j = x_j^0 = 0$ for all j not equal to i. (4C.2) is reduced to:

(4C.3) $(\text{sgn } w)\ u_i'(x_i) = h[(\text{sgn } w)v_i'(x_i)]$

Substituting the right hand side of (4C.3) into (4C.2) yields:

(4C.4) $\Sigma h[(\text{sgn } w)v_i'(x_i)] = h[(\text{sgn } w)\Pi v_i'(x_i)]$

Now let $x_i = x_i^0 = 0$ for all $i = 3,4,\ldots,n$. (4C.4) can now be written:

(4C.5) $h[(\text{sgn } w)v_1'(x_1)] + h[(\text{sgn } w)v_2'(x_2)]$

$= h[(\text{sgn } w)(v_1'(x_1))(\text{sgn } w)(v_2'(x_2))]$

which is Cauchy's fundamental functional equation from (4.13) with the general solution:

(4.14) $h(r) = a \ln r$

Substituting (4.14) into (4C.2) yields:

(4C.6) $(\text{sgn } w) \Sigma u_i'(x_i) = a \ln((\text{sgn } w) \Pi v_i'(x_i))$

Since $(\text{sgn } w)\Pi v_i'(x_i) = (\text{sgn } w)(1 + wv)$ from (4.2) and $\Sigma u_i'(x_i) = u$ from (4.3),

(4C.7) $(\text{sgn } w)\ u = a \ln((\text{sgn } w)(1 + wv))$

Using the standardization conventions adopted in the proof of case 1, solving for a yields:

(4C.8) $a = 1/(\ln(1 + w))$

and the standardized equation takes the form:

(4C.1) $u = (\ln(1 + wv))/(\ln(1 + w))$

APPENDIX 4D

Case 4: If the value function and utility function are multiplicative, then:

$$(4D.1)\ 1 + ku = (1 + wv)^{(\ln(1 + k))/(\ln(1 + w))}$$

Proof:

Let $1 + ww_iv_i(x_i) = v_i'(x_i)$

$1 + kk_iu_i(x_i) = u_i'(x_i)$

Assume that for the multiplicative value function (4.2) and multiplicative utility function (4.4) there exists some continuous function h, such that, from (4.15):

$$(4D.2)(\text{sgn } w)(\text{sgn } k) \prod u_i'(x_i) = h[(\text{sgn } k)(\text{sgn } w) \prod v_i'(x_i)]$$

(4D.2) uses the same compact forms of the functional equations described in Appendices 4B and 4C.

Let $x_j = x_j^0 = 0$ for all j not equal to i. (4D.2) is reduced to:

$$(4D.3)\ (\text{sgn } w)(\text{sgn } k)\ u_i'(x_i) = h[(\text{sgn } k)(\text{sgn } w)\ v_i'(x_i)]$$

Substituting the right hand side of (4D.3) into (4D.2) yields:

$$(4D.4) \prod h[(\text{sgn } k)(\text{sgn } w)\ v_i'(x_i) = h[(\text{sgn } k)(\text{sgn } w) \prod v_i'(x_i)]$$

Now let $x_i = x_i^0 = 0$ for all $i = 3,4,\ldots,n$. (4D.4) can now be written:

$$(4D.5) \prod h[(\text{sgn } k)(\text{sgn } w)\ v_1'(x_1)]h[(\text{sgn } k)(\text{sgn } w)\ v_2'(x_2)]$$
$$= h[((\text{sgn } k)(\text{sgn } w)v_1'(x_1))\ ((\text{sgn } k)(\text{sgn } w)v_2'(x_2))]$$

which is Cauchy's fundamental functional equation from (4.16) with the general solution:

$$(4.17)\ h(r) = r^a$$

Substituting (4.17) into (4D.2) yields:

(4D.6) $(\text{sgn } w)(\text{sgn } k) \Pi u_i'(x_i) = (\text{sgn } k)(\text{sgn } w) \Pi v_i'(x_i)^a$

Since $(\text{sgn } w)v_i'(x_i) = (\text{sgn } w)(1 + wv)$ from (4.2) and $(\text{sgn } k)u_i'(x_i) = (\text{sgn } k)(1 + ku)$ from (4.4),

(4D.7) $(\text{sgn } w)(\text{sgn } k)(1 + ku) = (\text{sgn } k)(\text{sgn } w)(1 + wv(x))^a$

Using the standardization conventions adopted in the proof of case 1 and solving for a yields:

(4D.8) $a = (\ln (1 + k))/(\ln(1 + w)$

Therefore, the standardized equation is:

(4D.1) $1 + ku = (1 + wv)^{(\ln(1 + k))/(\ln(1 + w))}$

CHAPTER 5

EXAMINATION OF THE THEORY IN AN EMPIRICAL CONTEXT

This chapter introduces and explains the procedures used to collect data for use in an empirical examination of the model developed in Chapter 4 and draws some preliminary conclusions from the collected data. Before launching into a discussion of the specifics of the data collection and the conclusions which can be drawn from the data, it is important to examine some issues surrounding the assessment of multiattribute value and utility functions. It is to this topic that we now turn.

Holistic vs. Decomposed Assessment of Multiattribute Functions

During the 1970's the management science literature was filled with debate over the relative merits of various methods for assessing multiattribute value and utility functions. The two methods under consideration were that of holistic evaluation, also known as statistical estimation, and decomposed assessment, also referred to as algebraic composition.

Holistic evaluation of a multiattribute utility function determines the univariate utility functions and scaling constants from a utility function over holistic outcomes. In other words, the individual whose utility function is being determined is asked a series of questions about outcomes specified in terms of all of the relevant attributes. Assuming that the researcher knows the form of the individual's utility function, the multiattribute function can be fit using standard OLS procedures. The utility functions and scaling constants for individual attributes can be determined from the holistic utility function. These estimates are derived; they are inferences which are completely based on the appropriateness of the underlying preference structure.

A decomposed assessment procedure, while relying on the appropriateness of the same preference structure as the holistic evaluation procedure, differs from it primarily in that a univariate utility function is directly elicited for each each attribute and scaling constants are determined directly by questioning the respondent. These utility functions and scaling constants are then algebraically combined to determine the individual's multiattribute utility function.

Under the assumption of riskless choice, the over-all evaluation function used in the statistical approach need only satisfy the properties of an ordinal scale. However, if a decomposed value function is to be used, the value functions defined over each attribute must satisfy the properties of an interval scale, even though the over-all values generated must only be ordinal.

Numerous studies of holistic evaluation of multiattribute utility functions have generally indicated that people can assign values to multiattribute outcomes in a meaningful fashion. According to Fischer (1977), however, these studies have also revealed important shortcomings in this approach. These include:

1. Limited numbers of attributes can be used.

2. Significant random error is introduced, with the amount of error increasing with the number of attributes considered. This error has been shown to be an important source of suboptimality in both simulated and real world decision making.

3. Holistic evaluation is difficult to apply in situations where there are a large number of possible outcomes.

Several studies have been conducted to compare the efficacy of the holistic and decomposed approaches. They have almost exclusively been concerned with job selection problems where each outcome is comprised of attributes related to salary, location, and job task (Currim and Sarin, 1984; Huber, 1974; Huber, Daneshgar, and Ford, 1971; Barron and Person, 1979; Fischer, 1977). While some of the studies have been concerned with choices under certainty and some with choices under uncertainty, almost all have assumed, a priori, that preferences met the conditions required for additivity. The authors differ in their assessment of the predictive accuracy of the holistic and decomposed models.

Despite the inconclusive evidence in support of the superiority of either approach, there is one factor which is of overriding importance for this study. In statistical (holistic) approaches, the diagnosticity of the estimated function is influenced by many factors and represents a combination of the underlying reasons. The algebraic (decomposed) approaches, if carefully implemented, provide the direct means for interpretation of preferences for specific attributes, interactions between the attributes, and attitudes towards risk. Therefore, they provide better insight into the decision making process. For this reason, we will use the decomposed approach in developing the value and utility functions in our empirical test. However, there is one striking disadvantage of the algebraic approach which has been noted by several authors including Hauser and Urban (1979); the measurement techniques employed by the algebraic approach lack an error theory and thus prevent one from making statistical statements about parameter estimates. Therefore, consistency must be assumed or checked by repeated assessments.

Methods for Estimating Value and Utility Functions

There exists a plethora of methods for deriving value and utility functions, each with its attendant advantages and disadvantages. The breadth of methods will not be explored here, but a feel for the number of possibilities can be obtained by examining Table 1.

Numerous articles have been written on the presence of assessment errors in single and multiattribute utility functions (e.g. Hershey, Kunreuther and Schoemaker, 1982; Krzysztofowicz and Duckstein, 1980, Tversky and Kahneman, 1981; Keeney and Sicherman, 1976; Fishburn and Kochenberger, 1979). Sources of error cited by Hershey, et. al. include:

1. Response Mode Bias: Certainty equivalence methods generally yield greater risk-seeking that probability equivalence methods.

2. Dimensions of Lotteries: Probability and outcome levels used in reference lotteries induce systematic bias.

3. Domain of Lotteries: Combining gain and loss domains yields different utility measures than separate examinations of the two domains.

4. Inertia Bias: Responses to questions framed in terms of assumption of risk, e.g. the reservation price for entering into a speculative venture, may lead to an inertia or status quo bias.

5. Context Bias: Also known as framing, the way way information is presented to individuals regarding choices under uncertainty affects their final choice.

Krzysztofowicz and Duckstein focus on the errors which may result in a single attribute utility function and in the type of multivariate risk attitude obtained from the effects of range of outcomes. Their discussion

METHOD	CLASSIFICATION
1. Ranking	N/</1/D
2. Direct Rating	N/=/1/E
3. Standard Gamble 1	P/≥/1/E
4. Standard Gamble 2	P/≥/1/C
5. Direct Midpoint	N/=/1/C
6. Probabilistic Midpoint	P/≥/1/C
7. Direct Ordered Metric	N/</1/D
8. Probabilistic Ordered Metric	P/</1/D
9. Ranking	N/</2/B
10. Direct Ranking	N/=/2/B
11. Probabilistic Rating	P/≥/2/B
12. Sucessive Comparison	N/≈/2/B
13. Half-Value Sum	N/=/3/B
14. Direct-Ordered Metric	N/</3/B
15. Ordered Metric	N/</3/B
16. Probabilistic Ordered Metric	P/</3/B
17. Single Trade-Off	N/=/2/C
18. Double Trade-Off	N/=/3/B
19. Single Transformation	N/=/2/C
20. Double Transformation	N/=/2/C
21. Discrete Trade-Off	N/=/2/DC
22. Discrete Transformation	N/=/2/DC
23. Discrete Adjancency	N/=/2/DC
24. Saw-Tooth	N/=/2/CE

The 24 methods are classified by four aspects, a/b/c/d, defined as follows:

Aspect a
- N: the method does not use probabilities
- P: the method uses probabilities in scaling utilities

Aspect b
- ≥: the method is based on preference judgements
- <: the method is based on "direct" inequality judgements
- ≈: the method is based on indifference judgements
- =: the method is based on "direct" equality judgements

Aspect c
- The number denotes the number of factors involved in any one judgement

Aspect d
- B: the method is most applicable with binary factors or is applicable for estimating scale-transformation parameters
- C: the method is best used with continuous factors
- D: the method is best used with discrete factors
- E: the method is usable with discrete or continuous factors
- DE or CE: the method is applicable using one discrete and one continuous factor or one continuous and another factor.

Table 1: Methods for Estimating Value or Utility Functions[1]

[1] Adapted from Fishburn (1967, pp. 438-440)

is particularly important in the determination of scaling factors and will be discussed under that heading.

While these sources of error are often referred to as assessment error, implying that they are largely due to either the instrument used or the inability of respondents to accurately educe their preferences, they are all directly related to the behavioral content of the axioms of the expected utility hypothesis. Thus, many "errors in assessment" may actually reflect deviation of preference behavior from that assumed in the axioms.

The method used in this study to determine value functions for individual attributes is the mid-value splitting technique. To use this technique we first assign the worst possible amount of an attribute the designation x° and the best possible amount attribute the designation x^{*}. $v(x^{\circ}) = 0$ and $v(x^{*}) = 1$. The respondent is then asked to answer the following series of questions:

1. Give me an amount x' such that you would give up the same amount of another attribute, called Y, to go from x° to x' as you would to go from x' to x^{*}. $v(x') = .5$
2. Give me an amount x'' such that you would give up the same amount of another attribute, called Y, to go from x° to x'' as you would to go from x'' to x'. $v(x'') = .25$
3. Give me an amount x''' such that you would give up the same amount of another attribute, called Y, to go from x'' to x''' as you would to go from x''' to x^{*}. $v(x''') = .75$

The method used in this study to determine utility functions for the individual attributes is the certainty equivalent method. The worst possible amount of an attribute is designated x° and the best possible amount of an attribute is designated x^{*}. $u(x^{\circ}) = 0$ and $u(x^{*}) = 1$. All other attributes are assigned their worst possible amounts. The decision maker is asked to respond to the following series of questions:

1. Give me the certain amount of attribute x, called x', such that you would be indifferent between receiving $(x', y^{\circ}, z^{\circ})$ for certain or a lottery with a 50% chance of receiving $(x^{\circ}, y^{\circ}, z^{\circ})$ and a 50% chance of receiving $(x^{*}, y^{\circ}, z^{\circ})$. $u(x') = .5$

2. Give me the certain amount of attribute x, called x'', such that you would be indifferent between receiving $(x'', y^{\circ}, z^{\circ})$ for certain or a lottery with a 50% chance of receiving $(x^{\circ}, y^{\circ}, z^{\circ})$ and a 50% chance of receiving $(x', y^{\circ}, z^{\circ})$. $u(x'') = .25$

3. Give me the certain amount of attribute x, called x''', such that you would be indifferent between receiving $(x''', y^{\circ}, z^{\circ})$ for certain or a lottery with a 50% chance of receiving $(x', y^{\circ}, z^{\circ})$ and a 50% chance of receiving $(x^{*}, y^{\circ}, z^{\circ})$. $u(x''') = .75$

4. To check for consistency, we may ask the decision maker to specify the certain amount of attribute x, called x, such that they would be indifferent between receiving $(x, y^{\circ}, z^{\circ})$ for certain or a lottery with a 50% chance of receiving $(x'', y^{\circ}, z^{\circ})$ and a 50% chance of receiving $(x''', y^{\circ}, z^{\circ})$. x should be the same as x'.

The Number of Attributes Used

One of Fischer's criticisms of the holistic assessment procedure is that it is limited in the number of attributes which can be simultaneously assessed. While the approach taken here will be that of decomposed assessment, the question of the number of attributes of any outcome which should be included remains relevant.

Eliashberg (1980) argues that empirical studies on consumer preference judgements suggest that consumers are engaged in explicit tradeoffs between the two most important attributes of outcomes once the less important attributes pass their acceptable levels. He attributes this to the limited nature of human information processing capacity. Eliashberg's argument is supported by other researchers. Hansen (1969) chose the three most important attributes for each individual in a consumer choice problem and compared the model's performance against one which used all relevant attributes and noted that the predictive power of the model was not diminished with the restricted attribute set. Wilkie and Weinreich (1972) argue that, in accord with related results in choice theory and information processing, it appears that attitudes can be efficiently described (in predictive terms) with fewer attributes than are typically gathered in marketing research; the typical practice of using all attributes is likely to significantly understate the predictive power of the multiattribute model in marketing. In a study of consumer choice of cereal brands, Pekelman and Sen (1974) found that over 85% of their sample used just one or two attributes in making decisions. None used all five attributes listed. Shoemaker and Waid (1982) found that using the three most important attributes yielded better estimates than a model using all six attributes presented.

Determining and Interpreting Scaling Factors

Much of the discussion of attribute importance in the marketing literature is based on the assumption that the weighting or scaling factor w_i or k_i in the multiattribute value or utility function is an indicator of the attribute's importance to the decision maker. In fact, the studies discussed above defined the most important attributes to be those with the highest scaling factors. Keeney and Raiffa (1976, pp. 271-272) argue that this interpretation of the scaling factors is incorrect. The scaling factors are not easy to interpret since they depend on the choices of the maximum and minimum values of all of the attributes considered which, in turn, are dependent on the possible consequences of the decision problem at hand.

If, for example, we have assessed the scaling factors for a two attribute outcome decision problem and found one to be .75 and the other to be .25, we cannot say that the first attribute is three times as important as the second. In fact, it cannot be concluded that the first attribute is more important than the second. All we can say is that, starting from the minimum possible levels of both attributes, the decision maker would rather change the first factor from its lowest to its highest value than change the second factor from its lowest to its highest value. If, for example, the range of the second attribute is small, then the scaling factor for that attribute will be small, going to zero as the range shrinks to zero. The attribute may be very important, even though its range, and thus, its scaling factor, is very small.

Because of the misinterpretation of the scaling factors, several techniques for determining scaling factors have been proposed to circumvent the lengthy calculations required to determine them within the decomposed approach presented by Keeney and Raiffa (1976). The client-explicated technique is representative of these (Eliashberg, 1980). The client is asked to assign the most important attribute a value of one and then to indicate the relative importance of the remaining attributes, using as a reference the most important attribute. Variations of this technique give the individual 100 reference points and ask them to distribute them among the attributes on the basis of their importance.

The scaling factors basically serve to compress or stretch the scales of the value or utility functions over individual attributes so that they are consistent (Fishburn, 1967). They can be determined using one of several methods.

Keeney and Raiffa (1976) propose a method for determining the scaling constants for an additive value function. The method outlined below is a generalized form of the Keeney-Raiffa procedure which is applicable also to multiplicative value functions.

Given:

(5.1) $v(x_1^*, x_2^*, x_3^*) = 1$

(5.2) $v(x_1^\circ, x_2^\circ, x_3^\circ) = 0$

(5.3) $v_i(x_i^*) = 1$ and $v_i(x_i^\circ) = 0$ for all i

(5.4) $v(x_1, x_2, x_3) = \Sigma w_i v_i(x_i)$ + possible other terms[2]

the decision maker can be asked to specify an x_1' such that they are indifferent between receiving:

(5.5) $(x_1', x_2^\circ, x_3^\circ)$ and $(x_1^\circ, x_2^*, x_3^\circ)$

Using the conditions set forth in (5.3) and (5.4):

(5.6) $w_1 v_1(x_1') = w_2$

Analogous choices can be made to determine that:

(5.7) $w_2 v_2(x_2') = w_3$

(5.8) $w_3 v_3(x_3') = w_1$

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[2] This formulation becomes clearer if we recognize that both the additive and multiplicative value and utility functions are specific cases of a multilinear function when the conditions of mutual preference and utility preference, respectively, hold. The multilinear value function can be written:

$$v(x_1,x_2) = v(x_1,x_2^\circ) + v(x_1^\circ,x_2) + wv(x_1,x_2^\circ)v(x_1^\circ,x_2)$$

If $w = 0$, this becomes:

$$v(x_1,x_2)= v(x_1,x_2^\circ) + v(x_1^\circ,x_2)$$

If w is not equal to 0:

$$\begin{aligned} \text{let } v'(x_1,x_2) &= wv(x_1,x_2) + 1 \\ &= wv(x_1^\circ,x_2) + wv(x_1,x_2^\circ) + w^2 v(x_1^\circ,x_2)v(x_1,x_2^\circ) + 1 \\ &= [wv(x_1,x_2^\circ) + 1][wv(x_1^\circ,x_2) + 1] \\ &= v'(x_1,x_2^\circ)v'(x_1^\circ,x_2) \end{aligned}$$

Therefore, for an additive value function, there are no possible other terms while for a multiplicative value function, $1 + w$ is the remaining term.

$v_i(x_i')$ can be determined from the individual's value function over attribute i.

If the value function is additive with n attributes, only n-1 such indifference choices must be made because the scaling factors are constrained to meet the condition that:

(5.9) $w_1 + w_2 + \ldots + w_n = 1$

If the value function is multiplicative with n attributes, n such indifference choices must be made. In addition, w must be determined such that:

(5.10) $1 + w = \prod (1 + ww_i)$

For the case of three attributes described above, (5.10) becomes:

(5.11) $1 + w = (1 + ww_1)(1 + ww_2)(1 + ww_3)$

The resulting set of four equations, (5.6), (5.7), (5.8) and (5.11), can be solved as a simultaneous equation system to determine the four unknowns, w, w_1, w_2, and w_3.

This technique is known as certainty scaling. A similar technique, known as probabilistic scaling, is proposed by Keeney and Raiffa for determining the scaling constants for multiattribute utility functions.

Given:

(5.12) $u(x_1*, x_2*, x_3*) = 1$

(5.13) $u(x_1^\circ, x_2^\circ, x_3^\circ) = 0$

(5.14) $u_i(x_i*) = 1$ and $u_i(x_i^\circ) = 0$ for all i

(5.15) $u(x_1, x_2, x_3) = \Sigma k_i u_i(x_i)$ + possible other terms

then:

(5.16) $u(x_i*, x_j^\circ) = k_i$ for all j not equal to i.

The decision maker may then be asked, for what probability, p, are you indifferent between:

1. The lottery giving a p chance at (x_{1*}, x_{2*}, x_{3*}) and a (1-p) chance at (x_1°, x_2°, x_3°) and

2. The consequence (x_{1*}, x_2°, x_3°)

If the decision maker's answer is defined as p_1, then using (5.15), the expected utility of the lottery is p_1, and from (5.16), the utility of the consequence is k_1. Equating the expected utilities of the lottery and consequence, it is found that:

(5.17) $k_1 = p_1$

The values of each of the k_i's can be generated in an analogous fashion so that:

(5.18) $k_2 = p_2$

(5.19) $k_3 = p_3$

If the utility function is additive with n attributes, only n-1 such probabilities must be specified because the scaling factors are constrained to meet the condition that:

(5.20) $k_1 + k_2 + \ldots + k_n = 1$

If the utility function is multiplicative with n attributes, n such probabilities must be specified. In addition, k must be determined such that:

(5.21) $1 + k = \prod (1 + kk_i)$

For the case of three attributes described above, (5.21) becomes:

(5.22) $1 + k = (1 + kk_1)(1 + kk_2)(1 + kk_3)$

The resulting set of four equations, (5.17), (5.18), (5.19) and (5.22) can be solved as a simultaneous equation system to determine the four unknowns, k, k_1, k_2, and k_3.

Keeney and Raiffa also suggest that only one of the k_i's need be elicited using the probabilistic scaling procedure; the remainder can be determined using a variation of certainty scaling. The decision maker is asked to select a level of x_i, x_i', and x_j, x_j', such that, for any fixed levels of all of the other attributes, they are indifferent between:

1. A consequence yielding (x_i', x_j°) and
2. A consequence yielding (x_i°, x_j')

The utilities of these two indifferent consequences can be equated to yield:

(5.23) $k_i u_i(x_i') = k_j u_j(x_j')$

The major criticism of the procedures set forth by Keeney and Raiffa are their reliance on extreme levels of the attributes. While the range from x_i° to x_i^{*} must, by definition, cover the entire range of the attribute, x_i, the implications of, and preferences for, extreme levels are often difficult for the decision maker to assess. Although some researchers (e.g. Keeney and Sicherman, 1976; Krzysztofowicz and Duckstein, 1980) have realized that asking extreme value questions may be a shortcoming of the existing scaling techniques, the effects of the range of gamble outcomes on the individual's measured risk attitude has not received much attention in the literature. Studies of variance preferences in laboratory experiments where the range of gamble payoffs were very small have resulted in skepticism about range effects (Slovic and Lichtenstein, 1968). However, Myers and Sadler (1960) have shown a negative correlation between the size of the range of outcomes and the predictive power of the resulting function.

For the case of certainty scaling this problem is easily ameliorated by asking the decision maker a set of indifference questions at non-extreme values. For example, the decision maker may be asked to specify a level of x_1' such that they are indifferent between receiving:

(5.24) (x_1', x_2', x_3°) and (x_1'', x_2'', x_3°)

Since:

(5.4) $v(x_1, x_2, x_3) = \Sigma w_i v_i(x_i)$ + possible other terms

For the additive value function, there are no other terms. Equating the values of the two consequences yields:

(5.25) $w_1 v_1(x_1') + w_2 v_2(x_2') = w_1 v_1(x_1'') + w_2 v_2(x_2'')$

Rearranging the terms yields:

(5.26) $w_1 = w_2[(v_2(x_2'') - v_2(x_2'))/(v_1(x_1' - v_1(x_1'')))]$

Since the $v_i(x_i)$'s are easily obtainable from the individual's univariate value functions:

(5.27) $w_1 = \theta\ w_2$ where $\theta = [\cdot]$

For the multiplicative value function there is another term, and the solution can be written:

(5.28) $w_1[v_1(x_1')-v_1(x_1'')]=w_2[v_2(x_2')-v_2(x_2'')]$

$+ ww_1w_2[v_1(x_1'')v_2(x_2') - v_1(x_1')v_2(x_2'')]$

Similarly, for probabilistic scaling the decision maker may be asked, for what probability, p, are you indifferent between:

1. The lottery giving a p chance at $(x_1', x_2', x_3°)$ and a (1-p) chance at $(x_1'', x_2'', x_3°)$ and
2. The consequence $(x_1', x_2'', x_3°)$

For the additive utility function, the utility of the lottery is:

(5.29) $pk_1u_1(x_1') + pk_2u_2(x_2') + (1-p)\ k_1u_1(x_1'') + (1-p)k_2u_2(x_2'')$

and the utility of the sure consequence is:

(5.30) $k_1u_1(x_1') + k_2u_2(x_2'')$

Equating the utilities of the lottery and the sure consequence, and rearranging terms yields:

(5.31) $k_1[(p-1)u_1(x_1') - (1-p)u_1(x_1'')) = k_2p[u_2(x_2'') - u_2(x_2')]$

For the multiplicative utility function there is another term and the solution can be written:

$$(5.32)\ k_1[(p-1)u_1(x_1') - (1-p)u_1(x_1")) = k_2p[u_2(x_2") - u_2(x_2')]$$

$$+kk_1k_2[u_1(x_1')u_2(x_2")$$

$$-kk_1k_2[pu_1(x_1')u_2(x_2')+(1-p)u_1(x_1")u_2(x_2")]$$

where $u_1(x_1')$, $u_2(x_2')$=.75 and $u_1(x_1")$, $u_2(x_2")$=.25.

Before leaving the subject of scaling procedures it should also be noted that while the value and utility functions are elicited attribute by attribute, determination of scaling factors requires holistic judgements (or at least consideration of n>1 attributes at a time).

The Respondents

For this preliminary examination of the theory, subjects were recruited from the agricultural technology program at Michigan State University. The agricultural technology program offers a certificate to students after two years of study in technical agriculture. All of the individuals participating in the completion of the questionnaire had farm backgrounds and were enrolled in agricultural economics type courses within the agricultural technology program.

This group was selected because of their familiarity with decision making in agricultural settings and the relative ease with which the questionnaire could be administered. Fifty-nine students participated in the study.

The Questionnaire

Participants were given two hours to complete a questionnaire designed to elicit preferences within the context of different agricultural decisions. A total of three decision contexts were used; each participant responded to questions about two of the decisions. Each participant was presented with a booklet containing instructions, descriptions of each decision context and a questionnaire for each decision context.

A copy of the complete questionnaire for one decision context, that of selecting a variety of feed corn, can be found in Appendix 5A. The format of the questionnaire has been changed slightly due to page width restrictions.[3] The content is identical to that presented to the respondents. Descriptions of the other two decision contexts are contained in Appendix 5B. The questions for these decision contexts are functionally equivalent to those contained in Appendix 5A.

The questionnaire was designed to elicit several types of information including preference conformance with independence conditions, single attribute value and utility functions, and the scaling constants required to convert the single attribute functions into multiattribute functions.

After reading the instructions, each student was presented with a description of their first decision context.

The respondents were first asked to respond to a series of questions (#1-#4) designed to test whether preferences met the condition of generalized preference independence required for either an additive or a multiplicative value function. Preference for one variety over another in each case constitutes satisfaction of the condition of generalized preference independence.

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[3] Specifically, much of the text which was double spaced in the actual questionnaire is single spaced in the Appendix. In addition, scales used to elicit value functions in questions 12-14 have been compressed in the Appendix.

The next series of questions (#5-#7) checked for the presence of strong difference independence required for an additive value function. In each case, if the response for part (a) is identical to the response to part (c) and the response to part (b) is identical to the response to part (d), the individual is said to exhibit strong difference independence between the two attributes whose values change in the question.

Since the existence of an additive or multiplicative value function does not necessarily imply that the utility function meets the same conditions, generalized utility independence and the additional condition for an additive utility function, known as the marginality condition must also be checked.

Questions 8-10 check for the existence of generalized utility independence. If the condition holds, responses to parts (b) and (c) are "no". Question 11 tests for satisfaction of the marginality condition required for an additive utility function. Respondents should be indifferent for (a), (b), and (c) if this condition holds.

The next step is to elicit value functions for each of the attributes using a mid-value splitting technique. Because this technique may be difficult to understand, these questions are prefaced with an example. In the example and the representative question, the first response yields v(x)=.5, where x is the attribute in question, the second response yields v(x)=.25 and the third response yields v(x)=.75. Questions 12-14 elicit the value functions for the individual attributes.

Univariate value functions are only one of the two types of information needed to determine a multiattribute value function. The second type, information needed to determine scaling factors, is elicited using questions 15(a), (b) and (c). The first question in each part elicits information needed to determine the scaling constants using extreme values; the second question elicits the information needed to determine the scaling constants using non-extreme values.

Univariate utility functions are then elicited for each attribute in questions 16-18. In each question, the first response yields u(x)=.5, the second response yields u(x)=.25, the third response yields u(x)=.75 and the fourth response is a consistency check for u(x)=.5.

The final series of questions elicit the information needed to determine the scaling factors to convert the univariate utility functions into a multiattribute utility function. Because this technique also may be difficult to understand, the questions are prefaced with an example. As with the scaling constants for the value function, the first questions elicits probabilities using extreme values while the second question uses non-extreme values.

After responding to a functionally equivalent set of questions for the second decision context, respondents are asked to indicate their agricultural background and experience in making decisions in contexts similar to those presented in the questionnaire.

Most respondents took from one hour to one and a half hours to answer the set of questions for the first decision context and about one-half hour to answer the set of questions for the second decision context.

Problems With the Questionnaire

Decision theory has often be criticized by those interested in applied problem solving because of the cost, time and difficulties involved in obtaining reliable information with which to determine an individual's utility function. This study was not exempt from these difficulties.

Previous published studies which have obtained either an individual's multiattribute value or utility function for one decision have stressed the time involved to familiarize the respondent with the parameters of the decision and the techniques to be used in the elicitation of their utilities. Then the researcher or "decision analyst" works through the series of questions with the respondent, giving him or her constant feedback on the consistency and implications of their responses. If mathematical consistency is not obtained with their original responses, respondents are asked to reconsider and modify their responses until consistency is reached. For instance, when Keeney (1972) elicited the multiattribute utility function of a blood bank administrator for two attributes, shortage and outdating of supplies, he spent "several afternoons" completing these steps.

In contrast to these previous studies, the participants involved in this study were asked to complete a questionnaire which elicited information required to determine two multiattribute value functions and two multiattribute utility functions in only two hours. Although written and verbal explanations were given for the questions which were anticipated to present problems (the mid-value splitting technique for determining single attribute value functions and the probabilistic questions for determining scaling constants for multiattribute utility functions), they completed the questionnaires without feedback from the researcher. In addition, there were few formal or informal consistency checks.

Table 2 shows that many of the individuals had difficulty understanding some of the questions and hence, their questionnaires were not usable for further analysis.

Table 2: Number of Individuals Who Had Difficulty Answering Specific Types of Questions.

Type of Question	Number of Individuals
Mid-Value Splitting	25
Certainty Equivalent	11
Certainty Scaling[4]	1
Probabilistic Scaling[4]	23

Satisfaction of Independence Conditions

Although many of the responses were not usable for a complete analysis because of problems in responding to the questions noted in Table 2, almost all were usable to determine the degree to which independence conditions required for either additive or multiplicative multiattribute utility and value functions were met. The summary statistics for those individuals whose responses met the independence conditions are shown in Table 3. The statistics are compiled by decision context, not by individual. Since some individuals only completed the first half of the questionnaire, the same number of responses were not available for each decision context.

Table 3: Number of Responses Satisfying Independence Conditions

	Decision Context		
Independence Condition	Variety	Dryer	Loan
Generalized Preference Independence	39	39	34
Strong Difference Independence	4	12	5
Generalized Utility Independence	18	17	17
Marginality Condition[5]	9	10	8

[4] The first part of these questions do not use responses to earlier questions. If these were answered reasonably, the individual was regarded to have understood the question.

[5] Only those responses for which generalized preference independence was met are included in statistics for the marginality condition.

Those responses for which generalized utility independence was not met are not usable as this preference condition is required for either an additive or a multiplicative utility function.

Several interesting points can be made by observing the frequency of additive or multiplicative value and utility functions in those decision context responses in which the condition of generalized utility independence was met. Tables 4 and 5 contain the summary statistics for the usable responses where the condition of generalized utility independence was met. Again, these summary statistics are by decision context, not by numbers of individuals.

The reader will recall that the use of Dyer and Sarin's transformations between value and utility functions implicitly requires, in all cases, that the value function is additive. The utility function may be either additive or multiplicative. It can be noted from Table 4 that only nine out of the fifty value functions examined meet this requirement.

Table 4: Frequency of Additive and Multiplicative Value and Utility Functions by Decision Context.

Combination Rule	Decision Context Variety	 Dryer	 Loan
Value Function			
Additive	2	4	3
Multiplicative	15	12	14
Utility Function			
Additive	8	11	8
Multiplicative	9	5	9

For intrinsic risk neutrality to hold within Dyer and Sarin's framework, both the value and utility functions must be additive. Table 5 shows that only four of the cases examined meet this requirement. Five of the cases meet Dyer and Sarin's implicit condition for intrinsic risk proneness or aversion, that the value function is additive and the utility function is multiplicative. The remaining 41 cases can not be analyzed within Dyer and Sarin's framework. However, they should be capable of analysis using the transformations developed in Chapter 4.

Table 5: Frequency of Combinations of Different Types of Value and Utility Functions by Decision Context.

Combination	Decision Context Variety	Dryer	Loan
Additive Value and Additive Utility	1	1	2
Additive Value and Multiplicative Utility	1	3	1
Multiplicative Value and Additive Utility	7	10	6
Multiplicative Value and Multiplicative Utility	8	2	8

The Issue of Consistency of Responses

The only formal consistency test conducted in the questionnaire was an additional indifference question for each single attribute utility function where the respondent was asked to specify u(x)=.5 as a function of the extreme levels of the attribute and as a function of the levels of the attribute where u(x')=.75 and u(x'')=.25. If the responses to the two questions were identical, the respondent was assumed to be giving consistent responses to that utility function elicitation question.

An additional, informal consistency test can be made by examining the responses to the two sets of questions used to determine each scaling constant. The reader will recall from the earlier discussion of the questionnaire that each respondent was asked functionally equivalent sets of indifference questions using extreme values and then non-extreme values of the attributes. In none of the cases examined were the responses identical for the extreme and non-extreme questions. This may be due, in part, to the aforementioned difficulty that individuals have in responding to extreme value type questions.

However, the issue of consistency is not so easily dismissed. It is, in fact, quite important in reviewing the responses to the questionnaires. To determine the scaling constants needed to aggregate the single attribute value functions into an additive value function, an overidentified system of simultaneous equations was used. This system of equations can be written:

(5.6) $w_1 v_1(x_1') = w_2$

(5.7) $w_2 v_2(x_2') = w_3$

(5.8) $w_3 v_3(x_3') = w_1$

where x_i' is the individuals response to the indifference question and $v_i(x_i')$ can be determined from the individual's value function over attribute i.

The fourth equation for an additive value function constrains the values of the w_i's so that:

(5.9) $w_1 + w_2 + w_3 = 1$

For a solution to be found for the subsystem of equations (5.6), (5.7) and (5.8), the following must hold:

(5.33) $v_1(x_1') * v_2(x_2') * v_3(x_3') = 1$

If this condition does not hold, a convergent solution for the system of equations can not be found. Analogous arguments hold for the systems of equations needed to find the scaling factors for multiplicative value functions and both additive and multiplicative utility functions.

For the cases examined, the value of equation (5.33) ranged from .125 to 1.225. No set of responses met the condition set forth in (5.33), that $\prod v_i(x_i') = 1$.

The initial conclusion which can be reached from this finding is that the subjects responses are, in some way, 'inconsistent'. However, with the data available from the questionnaires it is not possible to determine the cause of this inconsistency. The central issue is whether the inconsistencies in responses are due to fuzzyness in human cognition, e.g. the inability of respondents to accurately educe their preferences, or whether the preference model used is not appropriate.

If the apparent inconsistency is due to fuzzyness in human cognition, the problem can be remedied by continuous feedback from the researcher to the respondent causing the respondent to modify their answers until mathematical consistency is obtained. An alternative approach would be to ask the same series of questions in numerous guises, determine the distribution of responses for each x_i', and then take the mean as the 'true' value. With the data available from the questionnaire, it is impossible to determine the type of distribution of possible 'fuzzy' responses or whether the responses are even near the mean or in either the right or the left tail of the distribution.

However, before this approach can be justified in another data collection effort, several other questions must be answered. Perhaps most disturbing is the question of how far the researcher can go in forcing mathematical consistency and still reflect the true preferences of the respondent. Although respondents apparently met the independence conditions for additivity or multiplicativity of their multiattribute value and utility functions (determined through responses to questions 1-11), their preferences may not, in fact, be accurately reflected by either the additive or multiplicative model.

The problem of 'inconsistency' of responses has been discussed by several researchers. Interesting work has been done in this area by Kahneman and Tversky (1979, 1982), who have demonstrated that apparent inconsistencies in responses are due to violations of the behavioral content of the axioms of the expected utility hypothesis stemming from human cognitive processes. Perhaps the most well known of these factors is framing, the effects arising when the same alternatives are evaluated in relation to different points of reference. Ellsberg (1961), in a test of the Savage axioms of expected utility, asked several eminent decision theorists to answer a series of questions to determine whether their preferences violated the axioms. He found that the respondents fell into three groups: those who were careful to calculate their responses in accordance with the axioms of the expected utility hypothesis and, thus, gave consistent responses; those who claim to be expected utility maximizers but whose responses were inconsistent, and; those who gleefully gave inconsistent responses. Ellsberg's report (1961, pp. 655-656) of their responses brings levity to this important issue:

> "...You might now pause to reconsider your replies. If you should repent of your violations -- if you should decide that your choices implying conflicts with the axioms were "mistakes" and that your "real" preferences, upon reflection, involve no such inconsistencies - you confirm that the Savage postulates are, if not descriptive rules for you, your normative criteria in these situations. But this is by no means a universal reaction; on the contrary, it would be exceptional.
>
> Responses do vary. There are those who do not violate the axioms, or say they won't, even in these situations (e.g. G. Debreu, R. Schlaiffer, P. Samuelson); such subjects tend to apply the axioms rather than their intuition, and when in doubt, to apply some form of the Principle of

Insufficient Reason. Some violate the axioms cheerfully, even with gusto (J. Marschak, N.Dalkey); others sadly but persistently, having looked into their hearts, found conflicts with the axioms and decided, in Samuelson's phrase, to satisfy their preferences and let the axioms satisfy themselves. Still others (H. Raiffa) tend, intuitively, to violate the axioms but feel guilty about it and go back into further analysis.

The important finding is that, after rethinking all their "offending" decisions in the light of the axioms, a number of people who are not only sophisticated but reasonable decide that they with to persist in their choices. This includes people who previously felt a "first-order commitment" to the axioms, many of them surprised and some dismayed to find that they wished, in these situations, to violate the Sure-thing principle. Since this group included L.J. Savage, when last tested by me (I have been reluctant to try him again), it seems to deserve respectful consideration."

Ellsberg goes on to state, in his conclusions (ibid., p. 669):

"...Are they foolish? It is not the object of this paper to judge that. I have been concerned rather to advance the testable propositions : (1) certain information states can be meaningfully identified as highly ambiguous; (2) in these states, many reasonable people tend to violate the Savage axioms with respect to certain choices; (3) their behavior is deliberate and not readily reversed upon reflection; (4) certain patterns of "violating" behavior can be distinguished and described in terms of a specified decision rule."

Without knowledge of the source of the apparent inconsistency in responses, a formal empirical test of the theory presented in Chapter 4 can not be completed. However, several points have been brought to light which can provide the basis for further research in this area. First is the finding that, although previous work on multiattribute value functions has assumed the existence of an additive value function, only nine of the fifty value functions examined meet this requirement. Second is the finding that, even though forty-nine of the fifty respondents have preferences which appear to satisfy the weaker independence conditions required for a multiplicative multiattribute value function, none of the responses were able to be analyzed fully because of inconsistent responses to questions designed to provide the information necessary to determine scaling constants. Because consistency checks were not incorporated throughout the questionnaire, it is not possible, at this juncture, to determine whether the inconsistencies are due to the inability of respondents to accurately educe their preferences or whether the preference model used is not appropriate. Resolution of these issues must precede an empirical test of the theory. They are discussed in greater detail in Chapter 6.

APPENDICES TO CHAPTER 5

APPENDIX 5A

INSTRUCTIONS

As you know, agricultural economists are often called upon to make extension recommendations, analyze the impact of farm policy and predict the impact of future events on the agricultural sector. One of the pieces of information which is often used in carrying out these tasks is some measure of peoples' attitudes towards risk.

Unfortunately, the methods we now use to gather this information are not very good. Therefore, our analyses are often not as good as we would like them to be. With better information, agricultural economists can do better work.

As part of the research project required for a Ph.D. in agricultural economics, I have developed a new measure of attitudes towards risk which I think is better than the ones that are now used. But before I can convince anyone else that the new measure is better, I have to test it. By consenting to participate in this study by completing a questionnaire, you will help me in doing a pilot test of the new measure.

In this questionnaire, you will be asked to answer the same series of questions about two different hypothetical decisions. In each case, you will first be given a description of the decision which you are being asked to make. This is followed by a series of questions relating to that decision. Many of the questions may appear to be repetitive, but each is included for a specific reason. You will be able to answer some of the questions without much thought, while others will be more difficult. This is not an exam, so take your time and think about the questions; there are no right or wrong answers. If there is something you do not understand, please ask me and I will try to explain.

The questionnaire packet will take about two hours to complete. You will not be penalized in any way if you decide during the two hours that you cannot complete the entire questionnaire. If, before you start, you feel that you will not be able to complete the entire questionnaire because of the length of time involved, I would appreciate it if you would let me know. To test the theory, complete questionnaires are needed.

To confirm that you freely consent to participate in the study, please sign you name on the line below. Your name will not be used in any part of the study. All of your answers will be treated with strictest confidence and you will remain anonymous.

If you are curious as to what your responses indicate about your attitude towards risk, I would be happy to send you the information which results from your questionnaire packet. If you would like to receive this information, please write your address under your name below. This is purely optional.

I freely consent to participate in this study

____

(please sign here)

Thank you very much for agreeing to participate in this study.

Beverly Fleisher

SELECTING A VARIETY OF CORN

For this decision situation, imagine that you have just been to talk with the local seed dealer about the variety of feed corn to plant this year. You have already decided that you want to use a 101-107 day long maturing variety. He has told you about several varieties which fit this requirement, but differ with regard to three other important characteristics. Seed for all of the varieties costs the same amount.

1. Potential Yield which, for the varieties you are considering, ranges from 100 to 120 bushels per acre, depending on the variety selected.

2. Field Losses which are due to pests (both insect and disease) and lodging. For the varieties you are considering, field losses range from 0 to 20% of potential yield, depending on the variety selected.

3. Nitrogen Requirements which, in the form of anhydrous ammonia, range from 150 to 200 pounds per acre, depending on the variety selected.

Suppose you must make choices between two varieties at a time. The five varieties the seed dealer told you about have the characteristics shown below.

Characteristics	Variety 1	Variety 2	Variety 3	Variety 4	Variety 5
Potential Yield (in bushels/acre)	100	110	110	110	100
Field Losses (as % of yield)	5%	5%	15%	15%	15%
Nitrogen Requirement (as lb./acre)	175	175	175	200	200

(1) In a choice between variety 1 and variety 2, would you (check one):

______ prefer variety 1 to variety 2?

______ prefer variety 2 to variety 1?

______ be indifferent between the two varieties?

(2) In a choice between variety 2 and variety 3, would you (check one):

______ prefer variety 2 to variety 3?

______ prefer variety 3 to variety 2?

______ be indifferent between the two varieties?

(3) In a choice between variety 3 and variety 4, would you (check one):

______ prefer variety 3 to variety 4?

______ prefer variety 4 to variety 3?

______ be indifferent between the two varieties?

(4) In a choice between variety 4 and variety 5, would you (check one):

______ prefer variety 4 to variety 5?

______ prefer variety 5 to variety 4?

______ be indifferent between the two varieties?

(5) Now suppose that you are aware of a variety with the characteristics shown under the heading variety 1 below and that there exists another group of varieties which have different potential yields and different percentage of field loss. In each of the four cases described below, please indicate the field loss you would accept to have the higher potential yield of variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	100	110
Field Losses	5%	?
Nitrogen Requirement	175	175

(a) I would accept a _______% field loss from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	100	110
Field Losses	10%	?
Nitrogen Requirement	175	175

(b) I would accept a _______% field loss from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	120
Field Losses	5%	?
Nitrogen Requirement	175	175

(c) I would accept a _______% field loss from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	120
Field Losses	10%	?
Nitrogen Requirement	175	175

(d) I would accept a _______% field loss from variety 2.

(6) Now suppose that you are aware of a variety with the characteristics shown under the heading variety 1 below and that there exists another group of varieties which have different percentage of field loss and different nitrogen requirements. In each of the four cases described below, please indicate the required nitrogen per acre you would accept to have the lower field loss of variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	110
Field Losses	15%	5%
Nitrogen Requirement	150	?

(a) I would accept a _______ lb/acre nitrogen requirement for variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	110
Field Losses	15%	5%
Nitrogen Requirement	175	?

(b) I would accept a _______ lb/acre nitrogen requirement for variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	110
Field Losses	10%	5%
Nitrogen Requirement	150	?

(c) I would accept a _______ lb/acre nitrogen requirement for variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	110
Field Losses	10%	5%
Nitrogen Requirement	175	?

(d) I would accept a _______ lb/acre nitrogen requirement for variety 2.

(7) Now suppose that you are aware of a variety with the characteristics shown under the heading variety 1 below and that there exists another group of varieties which have different potential yield and different nitrogen requirements. In each of the four cases described below, please indicate the potential yield you would require to to have the increased nitrogen requirement of variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	100	?
Field Losses	10%	10%
Nitrogen Requirement	175	200

(a) I would need a _______ bu/acre potential yield from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	?
Field Losses	10%	10%
Nitrogen Requirement	175	200

(b) I would need a _______ bu/acre potential yield from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	100	?
Field Losses	10%	10%
Nitrogen Requirement	150	175

(c) I would need a _______ bu/acre potential yield from variety 2.

Characteristic	Variety 1	Variety 2
Potential Yield	110	?
Field Losses	10%	10%
Nitrogen Requirement	150	175

(d) I would need a _______ bu/acre potential yield from variety 2.

(8) Now imagine that you are choosing between two varieties, but that you do not have complete information about them. You know the potential yield and field losses for variety 1, but you do not know its nitrogen requirements. The seed dealer does not know the exact nitrogen requirements of variety 2, but can tell you that that it has a 50% chance of having the characteristics shown in the first column under variety 2, and a 50% chance of having the characteristics shown in the second column under variety 2. What level of nitrogen requirement for variety 1 would make you indifferent between the two varieties?

Characteristics	Variety 1	Variety 2 50% chance	 50% chance
Potential Yield	110	110	110
Field Losses	10%	10%	10%
Nitrogen Requirement	?	200	150

(a) A _______ lb/acre nitrogen requirement for variety 1 would make me indifferent between the two varieties.

(b) Would your answer change if you knew that the potential yield for both varieties was 120 bu/acre?

_______ no

_______ yes

(c) Would your answer change if you knew that the field losses for both varieties was 20%?

_______ no

_______ yes

(9) Now imagine that you are choosing between two varieties, but that you do not have complete information about them. You know the potential yield and nitrogen requirements for variety 1, but you do not know its field losses. The seed dealer does not know the exact field losses for variety 2, but can tell you that that it has a 50% chance of having the characteristics shown in the first column under variety 2, and a 50% chance of having the characteristics shown in the second column under variety 2.
What level of field losses from variety 1 would make you indifferent between the two varieties?

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	110	110	110
Field Losses	?	10%	20%
Nitrogen Requirement	175	175	175

(a) A _______ % field loss from variety 1 would make me indifferent between the two varieties.

(b) Would your answer change if you knew that the potential yield for both varieties was 120 bu/acre?

_______ no

_______ yes

(c) Would your answer change if you knew that the nitrogen requirement for both varieties was 200 lb/acre?

_______ no

_______ yes

(10) Now imagine that you are choosing between two varieties, but that you do not have complete information about them. You know the nitrogen requirements and field losses for variety 1, but you do not know its potential yield. The seed dealer does not know the exact potential yield requirements of variety 2, but can tell you that that it has a 50% chance of having the characteristics shown in the first column under variety 2, and a 50% chance of having the characteristics shown in the second column under variety 2. What level of potential yield for variety 1 would make you indifferent between the two varieties?

Characteristics	Variety 1	Variety 2 50% chance	Variety 2 50% chance
Potential Yield	?	100	120
Field Losses	10%	10%	10%
Nitrogen Requirement	175	175	175

(a) A _______ bu/acre potential yield for variety 1 would make me indifferent between the two varieties.

(b) Would your answer change if you knew that the field losses for both varieties was 20%?

_______ no

_______ yes

(c) Would your answer change if you knew that the nitrogen requirement for both varieties was 200 lb/acre?

_______ no

_______ yes

(11) Now imagine that you are choosing between two varieties; you are not sure of the characteristics of each variety, but know that it has a 50% chance of having the characteristics listed in the first column under that variety, and a 50% chance of having the characteristics listed in the second column under that variety. For each of the three pairs of varieties below, you will be asked if you prefer one to the other, or if you are indifferent between them.

	Variety 1		Variety 2	
Characteristics	50% chance	50% chance	50% chance	50% chance
Potential Yield	120	100	120	100
Field Losses	20%	10%	10%	20%
Nitrogen Requirement	175	175	175	175

(a) Would you (check one):

_____ prefer variety 1 to variety 2?

_____ prefer variety 2 to variety 1?

_____ be indifferent between the two varieties?

	Variety 1		Variety 2	
Characteristics	50% chance	50% chance	50% chance	50% chance
Potential Yield	110	110	110	110
Field Losses	20%	10%	10%	20%
Nitrogen Requirement	200	150	150	200

(b) Would you (check one):

_____ prefer variety 1 to variety 2?

_____ prefer variety 2 to variety 1?

_____ be indifferent between the two varieties?

Characteristics	Variety 1 50% chance	50% chance	Variety 2 50% chance	50% chance
Potential Yield	120	100	120	100
Field Losses	10%	10%	10%	10%
Nitrogen Requirement	200	150	150	200

(c) Would you (check one):

_____ prefer variety 1 to variety 2?

_____ prefer variety 2 to variety 1?

_____ be indifferent between the two varieties?

An Example to Help in Answering Questions 12, 13, and 14

Suppose I am to be given a fruit basket with bananas, apples and oranges. I know that the number of bananas in the basket is constant, but that there is some flexibility in the number of oranges and the number of apples in the basket. I am asked to specify a number of apples, A, such that I would give up the same number or amount of oranges to move from 0 apples to A apples as from A apples to 10 apples.

I really like apples and am not too fond of oranges, although I occassionally like to eat them. I decide that I would give up alot of oranges, say 8 of them, to move from 0 apples to 4 apples and another 8 oranges to move from 4 apples to 10 apples.

For me, then, A would be 4. I would mark this on the line below as:

```
0   1   2   3   4   5   6   7   8   9   10
|___|___|___|___A___|___|___|___|___|___|
```

APPLES

I also put my answer, A, on the line below. Now, I am asked to specify another amount of apples, B, such that I would give up the same number of oranges to move from 0 apples to B apples as from B apples to A apples. I decide that I would give up 3 oranges to move from 0 apples to 3 apples and another 3 oranges to move from 3 apples to A, which is 4 apples. Note that the number of oranges I used in answering to the second question did not have to be the same I used in answering the first. I mark my answer on the line below as B.

```
0   1   2   3   4   5   6   7   8   9   10
|___|___|___B___A___|___|___|___|___|___|
```

APPLES

Now I am asked to specify another amount of apples, C, such that I would give up the same number of oranges to move from A apples to C apples as from C apples to 10 apples. I begin by marking my response, A, from the first question on the line below. I then decide that my response to this question is 8 apples. I also mark this on the line below.

```
0   1   2   3   4   5   6   7   8   9  10
|___|___|___|___A___|___|___|___C___|___|
```

APPLES

I then record my answers at the bottom of the page in the space provided.

(a) A = _______ apples

(b) B = _______ apples

(c) C = _______ apples

Questions 12, 13, and 14 ask you to answer questions about the corn varieties that are like this question. Feel free to refer back to this example if you have trouble understanding how to answer.

(12) Suppose the nitrogen requirement is the same for all of the varieties you are considering. Please specify the potential yield (in bu/acre), A, such that you would accept the same increases in field losses to move from 100 bu/acre to A bu/acre as from A bu/acre to 120 bu/acre. Please mark your response on all three of the lines below as A.

100 101 102 103 104 105 106....113 114 115 116 117 118 119 120

|___|___|___|___|___|___| .... |___|___|___|___|___|___|___|

POTENTIAL YIELD (in bu/acre)

Now please specify the potential yield, B, such that you would accept the same increases in field losses to move from 100 bu/acre to B as from B bu/acre to A bu/acre (which you have marked on the line below). Mark your response to this question on the line below as B.

100 101 102 103 104 105 106....113 114 115 116 117 118 119 120

|___|___|___|___|___|___| .... |___|___|___|___|___|___|___|

POTENTIAL YIELD (in bu/acre)

Check to make sure that your response, A, is also marked on the line below. Now, please specify the potential yield, C, such that you would accept the same increases in field losses to move from A bu/acre (which you have marked on the line below) to C bu/acre as from C bu/acre to 120 bu/acre. Mark your response to this question on the line below as C.

100 101 102 103 104 105 106....113 114 115 116 117 118 119 120

|___|___|___|___|___|___| .... |___|___|___|___|___|___|___|

POTENTIAL YIELD (in bu/acre)

Please write your responses below:

(a) A = _______ bu/acre potential yield.

(b) B = _______ bu/acre potential yield.

(c) C = _______ bu/acre potential yield.

(13) Now suppose that the field losses are the same for all of the varieties you are considering. Please specify the pounds of nitrogen per acre, D, such that you would require the same increase in potential yield to move from applying 150 lb/acre to applying D as you would to move from applying D lb/acre to 200 lb/acre.
Please mark your response on all three of the lines below as D.

```
150    155    160    165.....185    190    195    200
 |__|__|__|__|__|__| ..... |__|__|__|__|__|__|
```

NITROGEN REQUIREMENT (in lb/acre)

Now please specify the pounds of nitrogen per acre, E, such that you would require the same increase in potential yield to move from applying 150 lb/acre to applying E as you would to move from applying E lb/acre to D lb/acre (which you have marked on the line below). Mark your response to this question on the line below as E.

```
150    155    160    165.....185    190    195    200
 |__|__|__|__|__|__| ..... |__|__|__|__|__|__|
```

NITROGEN REQUIREMENT (in lb/acre)

Check to make sure that your response, D, is also marked on the line below. Now, please specify the pounds of nitrogen per acre, F, such that you would require the same increase in potential yield to move from applying D lb/acre (which you have marked on the line below) to F lb/acre as to move from applying F lb/acre to applying 200 lb/acre. Mark your response to this question on the line below as F.

```
150    155    160    165.....185    190    195    200
 |__|__|__|__|__|__| ..... |__|__|__|__|__|__|
```

NITROGEN REQUIREMENT (in lb/acre)

Please write your responses below:

(a) D = _______ lb/acre nitrogen requirement.

(b) E = _______ lb/acre nitrogen requirement.

(c) F = _______ lb/acre nitrogen requirement.

(14) Now suppose that all of the varieties you are considering require the same application of nitrogen per acre. Please specify the field losses, G, such that you would require the same increase in potential yield to move from 0% field losses to G% field losses as you would to move from G% field losses to 20% field losses.
Please mark your response on all three lines below as G.

0 1 2 3 4 5 6....13 14 15 16 17 18 19 20
|___|___|___|___|___|___|___|....|___|___|___|___|___|___|___|

FIELD LOSSES (in %)

Now please specify the field losses, H, such that you would require the same increase in potential yield to move from 0% field losses go H% field losses as you would to move from H% field losses to G% field losses (which you have marked on the line below). Mark your response to this question on the line below as H.

0 1 2 3 4 5 6....13 14 15 16 17 18 19 20
|___|___|___|___|___|___|___|....|___|___|___|___|___|___|___|

FIELD LOSSES (in %)

Check to make sure that your response, G, is also marked on the line below. Now, please specify the field losses, I, such that you would require the same increase in potential yield to move from G% field losses (which you have marked on the line below) to I% field losses as you would to move from I% field losses to 20% field losses. Mark your response to this question on the line below as I.

0 1 2 3 4 5 6....13 14 15 16 17 18 19 20
|___|___|___|___|___|___|___|....|___|___|___|___|___|___|___|

FIELD LOSSES (in %)

Please write your responses below:

(a) G = ______ % field losses.

(b) H = ______ % field losses.

(c) I = ______ % field losses.

Some of your responses from questions 12, 13, and 14 will be needed to answer question 15. Please transfer them from the previous pages to the spaces provided below.

B = _______ potential yield C = ________ potential yield

E = _______ nitrogen required F = ________ nitrogen required

H = _______ % field losses I = ________ % field losses

(15a) What level of potential yield for variety 1 would make you indifferent between variety 1 and variety 2 whose characteristics are shown below?

Characteristics	Variety 1	Variety 2
Potential Yield	?	100
Field Losses	20%	0%
Nitrogen Requirement	200	200

_______ bu/acre potential yield for variety 1 would make me indifferent between the two varieties.

Please write your answers for B, H, and I in the appropriate spaces below. Then answer the question: what level of potential yield for variety 1 would make you indifferent between the varieties shown below?

Characteristics	Variety 1	Variety 2
Potential Yield	?	B = _____
Field Losses	I = _____%	H = _____%
Nitrogen Requirement	200	200

_______ bu/acre potential yield for variety 1 would make me indifferent between the two varieties.

(15b) What level of nitrogen per acre required for variety 1 would make you indifferent between variety 1 and variety 2 whose characteristics are shown below?

Characteristics	Variety 1	Variety 2
Potential Yield	100	120
Field Losses	20%	20%
Nitrogen Requirement	?	200

_ ______ lb/acre nitrogen required for variety 1 would make me indifferent between the two varieties.

Please write your answers for B, C, and F in the appropriate spaces below. Then answer the question: what level of nitrogen per acre required for variety 1 would make me indifferent between the two varieties?

Characteristics	Variety 1	Variety 2
Potential Yield	B = ____	C = ____
Field Losses	20%	20%
Nitrogen Requirement	?	F = _____

_ _____ lb/acre nitrogen required for variety 1 would make me indifferent between the two varieties.

(15c) What level of field losses for variety 1 would make you indifferent between variety 1 and variety 2 whose characteristics are shown below?

Characteristics	Variety 1	Variety 2
Potential Yield	100	100
Field Losses	?	20%
Nitrogen Requirement	200	150

_______ % field losses for variety 1 would make me indifferent between the two varieties.

Please write your responses for E, F, and I in the appropriate spaces below. Then answer the question: what level of field losses for variety 1 would make me indifferent between the two varieties?

Characteristics	Variety 1	Variety 2
Potential Yield	100	100
Field Losses	?	I = ____%
Nitrogen Requirement	F = _____	E = ____

_______ % field losses for variety 1 would make me indifferent between the two varieties.

In questions 16, 17, and 18, notice that variety 2 has a 50% chance of having the characteristics shown in column 1 and a 50% chance of having the characteristics shown in column 2 under the heading variety 2.

(16a) Please specify the sure amount of field losses, J, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	? = J%	0%	20%
Nitrogen Requirement	200	200	200

J = _______ % field losses

(16b) Please also fill in your answer for J in the table below. Then specify the sure amount of field losses, T, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	? = T%	0%	J = _____%
Nitrogen Requirement	200	200	200

T = _______ % field losses

(16c) Please fill in your answer for J in the table below. Then specify the sure amount of field losses, L, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	? = L%	20%	J = _____%
Nitrogen Requirement	200	200	200

L = _______ % field losses

(16d) Please fill in your answers for T and L in the table below. Then specify the sure amount of field losses for variety 1 that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	?	T = ____%	L = _____%
Nitrogen Requirement	200	200	200

? = _______ % field losses

Please write your responses below.

J = ________ % field losses

T = ________ % field losses

L = ________ % field losses

? = ________ % field losses

(17a) Please specify the potential yield, M, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	 50% chance
Potential Yield	M = ?	100	120
Field Losses	20%	20%	20%
Nitrogen Requirement	200	200	200

M = _______ bu/acre potential yield

(17b) Please also write your answer for M in the table below. Then specify the potential yield, N, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	 50% chance
Potential Yield	N = ?	M = _____	100
Field Losses	20%	20%	20%
Nitrogen Requirement	200	200	200

N = _______ bu/acre potential yield

(17c) Please write your answer for M in the table below. Then specify the potential yield, O, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	 50% chance
Potential Yield	O = ?	M = ____	120
Field Losses	20%	20%	20%
Nitrogen Requirement	200	200	200

O = _______ bu/acre potential yield

(17d) Please write your answers for N and O in the table below. Then specify the potential yield for variety 1 that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	 50% chance
Potential Yield	?	N = ______	O = ______
Field Losses	20%	20%	20%
Nitrogen Requirement	200	200	200

? = _______ bu/acre potential yield

Please write your responses below.

M = _______ bu/acre potential yield

N = _______ bu/acre potential yield

O = _______ bu/acre potential yield

? = _______ bu/acre potential yield

(18a) Please specify the nitrogen requirement, P, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	Variety 2 50% chance
Potential Yield	100	100	100
Field Losses	20%	20%	20%
Nitrogen Requirement	? = P	150	200

P = _______ lb/acre nitrogen requirement

(18b) Please write your answer for P in the table below. Then specify the nitrogen requirement, Q, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	Variety 2 50% chance
Potential Yield	100	100	100
Field Losses	20%	20%	20%
Nitrogen Requirement	? = Q	P = _____	150

Q = _______ lb/acre nitrogen requirement

(18c) Please write your answer for P in the table below. Then specify the nitrogen requirement, R, for variety 1, that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	20%	20%	20%
Nitrogen Requirement	? = R	P = ______	200

R = _______ lb/acre nitrogen requirement

(18d) Please write your answers for Q and R in the table below. Then specify the nitrogen requirement for variety 1 that would make you indifferent between variety 1 and variety 2 with the characteristics shown below.

Characteristics	Variety 1	Variety 2 50% chance	50% chance
Potential Yield	100	100	100
Field Losses	20%	20%	20%
Nitrogen Requirement	?	Q = ______	R = _______

? = _______ lb/acre nitrogen requirement

Please write your responses below.

P = _______ lb/acre nitrogen requirement

Q = _______ lb/acre nitrogen requirement

R = _______ lb/acre nitrogen requirement

? = _______ lb/acre nitrogen requirement

An Example to Help in Answering Question 19

(19) Suppose you are considering two varieties. You know, for certain, that variety 1 has the the characteristics shown under variety 1. You also know that variety two has some probability of having the characteristics shown in the first column under variety 2 and some probability of having the characteristics shown in the second column under variety 2. What probability, p, of variety two having the characteristics in column 1 under variety two and probability (1-p) of variety two having the characteristics in column 2 under variety 2 would make you indifferent between the two varieties?

To answer these questions, it may be easier to think of variety 1 as a sure thing, and variety 2 as a gamble between the two columns. Since the probabilities on the two columns must add up to one, the probability for the second column occurring is just one minus the probability of the second column occurring. You do not need to specify the probability for the second column.

The following example might help to explain how to answer the questions: Let's say I am presented with a choice between two fruit baskets, the contents of which are shown below.

Characteristics	Fruit Basket 1	Fruit Basket 2 p chance	Fruit Basket 2 (1-p) chance
Apples	10	10	2
Oranges	5	15	5
Bananas	0	0	0

p = .7 would make me indifferent between the two fruit baskets. This says that I am indifferent between receiving 10 apples, 5 oranges and no bananas and playing a gamble in which I have a 7 out of 10 chance of getting 10 apples, 15 oranges and no bananas and a 3 out of 10 chance of getting only 2 apples, 5 oranges and no bananas.

Some of your responses from questions 16, 17, and 18 will be needed to answer question 19. Please transfer them from the previous pages to the spaces provided below.

T = ______ % field losses L = _____ % field losses

N = ______ potential yield O = _____ potential yield

Q = ______ nitrogen required R = _____ nitrogen required

(a) What p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	120	120	100
Field Losses	20%	0%	20%
Nitrogen Requirement	200	150	200

p = ______ would make me indifferent between variety 1 and variety 2.

(b) Please write your responses for L, N, O, and T in the appropriate spaces below. Then answer the question: what p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	O = _____	O = _____	N = _____
Field Losses	L = _____	T = _____	L = _____
Nitrogen Requirement	200	200	200

p = ______ would make me indifferent between variety 1 and variety 2.

(c) What p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	100	120	100
Field Losses	20%	0%	20%
Nitrogen Requirement	150	150	200

p = ______ would make me indifferent between variety 1 and variety 2.

(d) Please write your answers for N, O, Q, and R in the appropriate spaces below. Then answer the question: what p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	N = ____	O = ____	N = ____
Field Losses	20%	20%	20%
Nitrogen Requirement	Q = ____	Q = ____	R = ____

p = ______ would make me indifferent between variety 1 and variety 2.

(e) What p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	100	120	100
Field Losses	0%	0%	20%
Nitrogen Requirement	200	150	200

p = ______ would make me indifferent between variety 1 and variety 2.

(f) Please write your answers for L, R, Q, and T in the appropriate spaces below. Then answer the question: what p would make you indifferent between variety 1 and variety 2 shown below?

Characteristics	Variety 1	Variety 2 p chance	Variety 2 (1-p) chance
Potential Yield	100	100	100
Field Losses	T = ____	T = ____	L = ____
Nitrogen Requirement	R = ____	Q = ____	R = ____

p = ______ would make me indifferent between variety 1 and variety 2.

Thank you very much for participating in this study and completing the questionnaire. You have been a big help to me! If you are curious as to what your responses indicate about your attitudes towards risk, please be sure to write your address on the second page of the instructions. I anticipate having the results in about a month and would be happy to send them you.

By this time, you are probably tired of answering questions. But I have one last one for you. Would you take a moment and briefly tell me what your background in farming is. For example, do your parents farm or do you anticipate farming or working in an agricultural industry when you graduate? In addition, I would like to know if you have been involved in making decisions similar to the ones you answered questions about here.

THANKS AGAIN !!!

APPENDIX 5B

PURCHASING A CORN DRYER

For this decision situation, imagine that you own a 40,000 bushel corn operation and have decided to buy a grain dryer rather than have your grain dried at the elevator. You normally harvest corn at 25% moisture content and want to dry it down to 15% (grade 2). You have made a trip to the farm machinery dealer to discuss dryers, but have not yet decided which one to purchase. The dealer has told you about several dryers, some which are automatic batch dryers and some which are continuous flow. The dryers which you are considering vary with respect to:

1. Dryer Capacity which, for the dryers you are considering, ranges from 150 to 200 bushels per hour, depending on the dryer selected.

2. Cost of Drying Equipment which, for the dryers you are considering, ranges from $10,000 t0 $20,000.

3. Operating Cost which is comprised mainly of the cost of electricity or LP to run the dryer and, for the dryers your are considering, ranges from 9 to 13 cents per bushel.

FINANCING A COMBINE

For this decision situation, imagine that you have decided to purchase a new combine. The self propelled combine, with headers, is going to cost you $75,000. You have been shopping around for a loan to finance the combine. Several sources are willing to loan you the money, with at least the combine as collateral, but the loans differ with regard to:

1. Interest Rate which ranges from 11% to 15%

2. Years to Pay Back the Loan, also known as years to amortize the loan, which range from 7 to 10 years

3. Collateral Required which, in addition to the combine itself, ranges from none to an additional $20,000 worth of machinery and equipment.

CHAPTER 6

SUMMARY, CONCLUSIONS, AND ISSUES REQUIRING FURTHER RESEARCH

The research presented in this dissertation was motivated by the apparent instability of the Arrow-Pratt coefficient of absolute risk aversion across situations. Despite this perceived instability, the Arrow-Pratt coefficient has continued to be one of the major measures used to describe individuals' attitudes towards risk, order individuals according to their attitude towards risk, and generalize their responses to risky events across situations. Recent work by mathematical psychologists suggests the need to separate an individuals attitude towards risk from their strength of preference for riskless outcomes in order to examine the stability of each component across situations and over time. In essence, this approach calls for a re-examination of the behavioral assumptions underlying expected utility theory. In the dissertation, particular attention was given to three behavioral assumptions and the implications of their violation on the stability of the Arrow-Pratt coefficient of absolute risk aversion. The three assumptions are:

1. Individuals have constant marginal value for money.
2. Preferences are homogeneous.
3. Preferences remain invariant between certainty and uncertainty, other things being equal.

In response to evidence of consistent violation of these three assumptions, a new measure of attitudes towards risk was developed whose assumptions are believed to more accurately reflect individual's behavior.

This chapter briefly summarizes the research conducted, explores in further depth the methodological questions which have arisen, discusses issues requiring further research and examines what can be concluded at this juncture.

Summary

Chapter 2 begins with a brief review of the historical development of utility theory in order to discover the source of the three assumptions listed above. It was found that the assumption of constant marginal value for money stems from the early cardinalists' argument that money could be used as a measuring rod for utility and, as such, must have constant units of value just as each inch on a standard ruler must represent the same unit of measure. The assumption of homogeneity of preferences stems from Jevons and Marshallian demand theory which argues that the last unit of a good purchased by any individual gives him equal value or satisfaction as the monetary equivalent of that unit of good. Since, in a perfectly competitive market at equilibrium, each individual faces identical prices, it is argued that individuals' marginal utility is equal. Thus, all individuals would obtain the same degree of satisfaction of needs from the last unit purchased which is, in turn, the same degree of satisfaction they would obtain from having an amount of money equivalent to the price of that good. This assumption ignores the concept of consumer surplus.

If an individual's preferences meet the conditions implied by these two assumptions, their indifference curves for any two goods, or any one good and money, are straight lines with a slope of negative one. Indifference curves of this type can be obtained when an individual's value and utility functions over multiattributed outcomes are additive. If both the utility and value functions meet these conditions, the utility function must be a positive linear transformation of the value function. This positive linear transformation mathematically ensures that preferences are invariant under certainty and uncertainty.

The implications of violations of the assumptions of constant marginal value for money and homogeneous preferences for our ability to measure and compare attitudes towards risk using the Arrow-Pratt coefficient of absolute risk aversion were demonstrated graphically. Specifically, it was shown that, even if an individual actually has stable attitudes towards risk, if his preferences for riskless outcomes do not conform to these assumptions, his risk attitude, measured using the Arrow-Pratt coefficient, will appear unstable.

Chapter 3 begins with the conceptualization of a means whereby intrinsic attitudes towards risk can be determined by separating the effect of attitude towards risk on a von Neumann-Morgenstern utility function from the effect of preferences for riskless outcomes. Recent work on intrinsic risk attitude by Dyer and Sarin (1981) is discussed and illustrated graphically. (Note that what is referred to as intrinsic risk attitude is called relative risk attitude by Dyer and Sarin and others in the mathematical psychology and management science literature. The terminology was modified by this author to avoid confusion with an entirely different concept, the Arrow-Pratt coefficient of relative risk aversion.) It was shown that Dyer and Sarin's local measure of intrinsic risk attitude effectively compensates for violations of the assumptions of constant marginal value for money and homogeneous preferences. However, the global measures of intrinsic risk attitude developed by Dyer and Sarin place severe restrictions on the form which the individual's value function and utility function may take. Their measures require that one of the following two conditions hold:

1. All problems are simple value problems, i.e., that outcomes refer to only one attribute, or

2. Preferences for attributes in multiattributed outcomes are independent and additive.

The first condition does not allow for cases where, for example, a farmer is concerned with not only profits, but also his asset/debt ratio, quality of life, soil productivity, or implications of current decision outcomes for future flexibility in the organization of the farm firm. The second condition allows for consideration of several attributes, but requires that value of attainment of any level of one attribute is completely independent of the initial levels or changes in the levels of other attributes. In other words, the individual's multiattribute value function must be additive.

In Chapter 4, a new measure of attitude towards risk was developed which builds upon the work done by Dyer and Sarin. It begins with a closer examination of the independence conditions which preferences for attributes in a multiattributed outcome must meet in order to assume their additive or multiplicative combination. The new method of separating attitudes towards risk from preferences for riskless outcomes is more general than previously developed methods because it allows for consideration of a larger class of value functions. Specifically, while previous work requires that preferences meet strong independence conditions required for additive combination of preferences for different attributes of an outcome, the new theory only requires that preferences meet the weaker conditions of independence required for their multiplicative combination. The need for this generalization was confirmed by the finding that fifty out of fifty-nine respondents to a questionnaire administered by the researcher had preferences which met the conditions for multiplicativity while only nine respondents had preferences which also met the stricter conditions for additivity.

The new measure of intrinsic attitude towards risk was developed using Cauchy's four fundamental functional equations to ascertain the transformations between additive or multiplicative multiattribute value and utility functions. These transformations are then to determine intrinsic attitudes towards risk. These transformations allow one to do three things:

1. Given a utility function, u(x), and a value function, v(x), determine u(v), the utility for the riskiness of outcomes.
2. If u(v) is stable across situations, determine v(x) from u(x) or vice versa.
3. In modeling behavior, separate the influence of strength of preference for riskless outcomes from attitudes towards risk.

Use of the new measure is illustrated with hypothetical additive and multiplicative value and utility functions. It is shown that, while Dyer and Sarin's local measure of intrinsic attitude towards risk holds when the value function is either additive or multiplicative, the new, more general measure which allows for multiplicative value functions is needed for a global measure of intrinsic risk attitude.

The procedures used to collect data to examine the empirical validity of the new measure and the and the stability of intrinsic attitude towards risk across situations are discussed in the beginning of Chapter 5. This includes a description of the basic methods used in multiattribute utility theory as well as discussion of some of the current debate over certain aspects of these methods. The questionnaire used in this study is then described and discussed.

Attempts to use the data collected with the described instrument to examine the theory were beset by difficulties; the data could not be used to clearly support or refute the new measure's predictive power because of mathematical inconsistencies in the responses. The question which remains to be answered is whether these inconsistencies reflect 'fuzzyness' in human cognition which prevents individual's from accurately educing their preferences or whether these inconsistencies reflect the fact that individuals preferences do not conform to the behavioral assumptions underlying the methods used. This issue is discussed in the sections below.

Placing the Research in Context

This research has been primarily concerned with re-examining a subset of the behavioral assumptions of expected utility theory and their implications for measures of attitudes towards risk such as the Arrow-Pratt coefficient of absolute risk aversion. The reader will recall that the expected utility hypothesis assumes, among other things, invariance of preferences between certainty and uncertainty when other things are equal, homogeneity of preferences and constant marginal value of money. It has been argued that because these behavioral assumptions diverge from actual preferences, the Arrow-Pratt coefficient of absolute risk aversion does not reflect a decision makers' true attitude towards risk. Therefore, our methods of determining attitudes towards risk must be modified to more accurately reflect decision makers' behavior.

The test of a hypothesis is usually preceded by the formalization of an if, then statement such as, if the global measure of intrinsic risk attitude accurately reflects decision maker's preferences, then a utility function indirectly obtained using the individual's multiattribute value function and the appropriate transformation described by (4A.1), (4B.1), (4C.1) or (4D.1) will be a positive linear transformation of the individual's directly elicited von Neumann-Morgenstern utility function.

In a discussion of the basic elements of a good test of a hypothesis, Giere (1979, pp. 86-94) argues that this simple if, then statement is not sufficient; one needs to specify not only the hypothesis, H, and the prediction, P, but also the initial conditions, IC, and auxiliary assumptions, AA, which are expected to hold, such that the following two conditions are true.

Condition 1: If [H and IC and AA], then P

Condition 2: If [not H and IC and AA], then very probably, not P

Note that the if statement contains not only the hypothesis, but also the initial conditions and auxiliary assumptions.

It must be remembered that only a few behavioral assumptions of the axioms of expected utility theory were examined; the remaining content of the theory was assumed to reflect accurately decision maker behavior. (For summaries of recent research done on the behavioral and mathematical content of the independence and other axioms of expected utility theory, the reader is referred to Schoemaker (1982) and Machina (1983).) This strong assumption created somewhat of a methodological paradox in the collection and analysis of data in a study designed to test the new measure of attitudes towards risk: while the behavioral content of expected utility theory was being examined, the representation theorem underlying expected utility theory, including the independence axiom, was needed to collect the data. Several aspects of these assumptions are relevant in light of the inconsistencies pervading the collected data.

More formally stated, the initial conditions used in the study discussed in Chapter 5 were that all of the assumptions underlying the axioms of expected utility theory and multiattribute utility theory except the assumptions of constant marginal value of money, homogeneity of preferences and invariance of preferences between certainty and uncertainty are mathematically correct and accurately reflect decision maker behavior. The auxiliary assumptions employed relate to the methods used for measurement and include the assumptions that individuals can accurately educe their preferences and that individuals can understand and effectively work with probability concepts.

Assuming, for the purpose of argument, that the inability to complete the analysis implies an incorrect prediction, we might reason through a test of the hypothesis using the following logic:

First Premise:	If [H and IC and AA], then P
Second Premise:	Not P
Preliminary Conclusion:	Not [H and IC and AA]
Preliminary Conclusion:	Not H or not IC or not AA

At this point, we must make explicit our commitments to the truth of both the initial conditions and auxiliary assumptions. The emphasis above on the fact that only a few of the assumptions underlying the expected utility hypothesis were examined has alluded to the lack of commitment to the truth of the remaining assumptions. The remainder of this section and the ones following bring to light more issues which prevent us from continuing with the deduction that:

Additional Premise: IC and AA (are true)

Conclusion: Not H

Instead, we are left with the second preliminary conclusion:

Preliminary Conclusion: Not H or not IC or not AA

Most axiomatizations in measurement theory try to capture an important property of real numbers called the Archimedean property. Roberts (1979, p. 123) defines this property as follows:

> "If a and b are real numbers and a > 0, then there is a positive integer n such that na > b. That is, no matter how small a might be or how large b might be, if a is positive, then sufficiently many copies of a will turn out to be larger than b."

This property of the real number system is what makes measurement of any type, including utility, possible by making it possible to roughly compare the relative magnitudes of any two quantities by seeing how many copies of the first quantity are needed to exceed the second, larger quantity. This condition is necessary to achieve a homomorphic mapping from an abstract relational system, such as preferences, to the real line, yielding a utility function. The Archimedean property is embodied in the continuity axiom of expected utility theory which states that if an individual prefers A to B to C, then there exists an unique probability, p, such that he will be indifferent between B and a lottery of the form pA + (1-p)C. Note that this is the exact form of the questions used in the questionnaire to elicit information needed to determine the scaling factors for multiattribute utility functions.

It may be argued that the Archimedean axiom requires standards of accuracy which are unrealistic. For the interpolations involved in decision analysis to be valid, decision makers must not only have a good understanding of probability, but their responses must also be accurate and precise. However, the decision makers responses may often vary because of the inability to accurately educe preferences, changes in preferences over time, changes due to introspection about preferences evoked by closer examination, or in response to feedback about mathematical inconsistencies which may cause the decision maker to alter responses to the extent that they may or may not reflect true preferences.

Another set of assumptions which come into play in extending expected utility theory to multiattribute utility and value theory are mutual preference and mutual utility independence. These conditions require that the weight that the individual gives to each attribute will not vary with changes in the levels of other attributes. Mutual independence only holds if the weight given to one attribute when other attributes are at a specified level does not change when the attributes are at another level. Evidence of failure of preferences to meet this condition can be found if the individual has weights which are not constant. While generalized preference and utility independence allow for symmetric reversals in preferences to occur when moving from the domain of gains to the domain of losses, or vice versa, generalized independence does not allow for asymmetrical reversals in preferences. Violation of generalized independence can easily occur if the individual attaches a different weight to improving the level of an attribute than he does to maintaining or preventing a decrease in the level of the attribute. In the questionnaire, generalized independence was only checked over one domain (either gains or losses) rather than over both. Therefore, the apparent satisfaction of the conditions of generalized independence may be misleading.

Kahneman and Tversky (1979) describe this reversal of preferences about the status quo as the reflection effect and argue that preferences in the domain of losses are a mirror image of (symmetric with) preferences in the domain of gains. However, Friedman and Savage (1948) argue that preferences in the domain of losses are not symmetric with those in the domain of gains. Instead, they argue that the utility function in the domain of losses is much "steeper" than that in the domain of gains. They support this argument with the observation that individuals will, given the chance at significant gains, participate in unfair odds and, for the prevention of much smaller losses, accept equally unfair odds to preserve the resources which they hold. Although there is much disagreement with the specifics of the "everymans' utility function" Friedman and Savage developed in response to this observation, the observation has been reconfirmed by subsequent studies.

It is important to remember that the axioms of utility theory not only define "rational behavior" but go much further and define behavior which must be described both precisely and with mathematical consistency. This presents a dilemma to both the decision analyst and the decision maker: if the decision makers' preferences meet the conditions set forth in the axioms, the theory may be interpreted as a good descriptor of behavior and used to predict behavior while if the decision makers' preferences do not meet conditions set forth, the theory becomes prescriptive. In the latter case, the decision analyst may appeal to the apparent reasonableness of the axioms and the rationality of the decision maker in asking him to revise his responses. But a decision maker can be reasonable and have sound reasons for particular preference structures without conceding to the axioms; in this case, the decision analyst must reflect on how far they can push for mathematical consistency and still accurately reflect the preferences of the decision makers. Efstanthiou (1984, p. 310), in a discussion of this issue, argues that:

> "..in obtaining utility curves for single attributes and imposing the axioms of multiattribute utility theory, a dangerous mixture of descriptive and normative procedures will be produced."

Issues Requiring Further Research

The myriad issues which require further research can be classified into three major areas: mathematical content of the axioms, behavioral content of the axioms and the educement of preferences. The third area, although formally a subset of the behavioral content of the axioms, has enormous impact on the issue of mathematical consistency. Currently, research is being done in all three areas by researchers from many different disciplines. Three particularly promising areas of research directly related to the problems discussed above are described briefly below.

The issue of "fuzzyness" in human cognition has plagued decision analysis since its inception. In contrast to the uncertainty due to randomness of events external to the decision maker, this fuzzyness results from a form of uncertainty due to vagueness and imprecision internal to the decision maker. According to the proponents of fuzzy set theory, these distinct qualities must be modeled in different ways, the former using probability theory and the latter using fuzzy set theory. Fuzzy set theory provides a means of quantifying the degree of imprecision associated with any input into the decision process through the use of membership functions. Watson, Weiss and Donnell (1979) argue that probabilities and utilities can inherently only be represented by somewhat rough sets of numbers. Their "fuzzy decision analysis" method is motivated by the need to handle the imprecision accompanying the judgemental inputs into decision analysis in a systematic and consistent manner.

Zadeh (1965), one of the first to argue for a new fuzzy approach to systems analysis and decision making under uncertainty, holds that fuzzy set theory is not an alternative to probability theory and expected utility theory but is, instead, a parallel calculus to be used to handle the imprecision inherent in human cognitive processes. The central concept in fuzzy set theory is the membership function which numerically represents the degree to which an element belongs to a set. The function is valued between zero and one and is assessed subjectively with small values representing a low degree of membership in the set and high values representing a high degree of membership. For example, the statement "it will probably rain tomorrow" would have a higher degree of membership in a set regarding likelihood of rain than the statement "it might rain tomorrow". In applications of fuzzy set theory, the values used to represent degrees of membership are not elicited directly. Instead, they are taken from curves drawn by individuals to represent their degrees of belief that an event will occur.

The calculus of fuzzy sets is based on three propositions to which numbers indicating membership should conform. These propositions are analogous to those used in conventional set theory and include:

1. The degree to which X belongs to set A and set B is equal to the smaller of the individual degrees of membership.

2. The degree to which X belongs to either set A or set B is equal to the larger of the individual degrees of membership.

3. The degree to which X belongs to (not A) is one minus the degree to which X belongs to A.

The calculations involved in the decision analysis can be considered to be a functional relationship between the inputs regarding probabilities and utilities and the output of the analysis in the form of the expected utility of an action choice. The three relationships cited above are used to deduce the "fuzz" on the output given the fuzzyness of the inputs.[1] As with conventional utility analysis, probability distributions may be generated which characterize the range of possible outcomes for each action choice. Whereas the distributions obtained from conventional analysis are taken to be the true distributions, in fuzzy set theory the extent to which the distribution of inputs, probabilities and utilities implies a preferred action choice is only as large as the least level of implication for each set. Unless one distribution clearly dominates another, it cannot be said to indicate the preferred action choice. To determine the preferred action choice when two sets overlap, one must determine the extent to which one set is preferred over the other through the use of Zadeh's fuzzy calculus.

There remain questions regarding the axiomatization of a fuzzy set calculus which can be used to elicit membership functions. Experimental evidence does show, however, that individuals are able to draw curves or probability distributions to represent their perceived imprecision regarding degrees of belief such as "better than ever", "pretty likely", or "about X%". The precise shapes of these distributions are somewhat arbitrary, but this fact does not affect the inferences which can be drawn from fuzzy set analysis as it is the general shape of the distributions that matter.

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[1] For particulars of the mathematical methods used, see Watson, Weiss and Donnell (1979) and Zimmermann, Zadeh and Gaines (1984).

Machina (1982), in examining the mathematical content of the independence axiom of expected utility theory starts with the observation from elementary calculus that for a function to be differentiable, it must be locally linearly approximable. The independence axiom is like a linearity assumption for utility functions; it implies that the utility function may be represented as the expectation with respect to the given distribution of a fixed function defined over the set of possible outcomes. In other words, the utility function is constrained to be a linear function over the set of distributions of outcomes or, as commonly phrased, "linear in the probabilities". However, empirically determined utility functions do not always conform to the linearity condition because of the certainty effect or other phenomena.

Machina argues that, despite these violations, the basic concepts, tools and results of expected utility analysis are still applicable because they are not dependent upon the independence axiom. They can also be derived from a weaker assumption of local smoothness of preferences over alternative probability distributions. Machina shows that if you can assume that preference orderings underlying the utility function in a region are a differentiable functional, then the utility function is locally linear. Hence, the utility function is differentiable and the results of expected utility theory hold, at least locally since, at each point, a locally linear utility function can be obtained. With his generalized utility analysis, Machina elegantly proves that the assumption of smoothness of preferences and consistency in the shape of the utility function in a given region, in essence, rescues the independence axiom. Machina does not prove the local applicability of the other axioms of expected utility theory.

While Machina examines the mathematical content of the independence axiom, Weiss (1983, 1984), with equal mathematical rigor, examines the behavioral content of the axioms. Weiss begins with the observation that lotteries, which can be expressed as cumulative distribution functions, can take several forms including degenerate lotteries, where the probability of the outcome occurring is one, and continuous lotteries, where the probabilities of outcomes sums to one. Degenerate lotteries coincide with the case of certainty while continuous lotteries coincide with the case of uncertainty.

In Weiss' approach, decision makers compare lotteries according to their measurable utility and are allowed to use a "two-rule" measurable utility function which allows, but does not require, them to use one preference rule for choosing between degenerate lotteries and another for choosing between continuous lotteries. In a model of producer response to price uncertainty, Weiss finds that, when a decision maker is allowed to use a "two-rule" measurable utility function, he generally will choose a utility maximizing output level different from the one he would choose if he followed a more restrictive expected utility maximizing approach. In particular, if his marginal utility of money is greater under certainty than under uncertainty and both the expected utility and measurable utility models have interior solutions, then the latter model will generate a lower optimal production level than the former. Furthermore, it will prove possible to have a zero production level and a positive production level being simultaneously optimal (Weiss, 1983, p. 16). A particular pricing situation to which his analysis applies is that of random price that is truncated below the mean through the introduction of a support price.

These three examples illustrate the breadth of possibilities which can be explored by researchers in decision theory, all of which are useful in moving towards resolution of the common problems encountered in this dissertation. The first priority of this researcher is to obtain the mathematically consistent responses required to complete a test of the new measure of attitudes towards risk. This will require careful, lengthy procedures to elicit the multiattribute value and utility functions for actual decisions faced by a small group of decision makers. Particular attention needs to be paid to the issue of how far responses can be altered to meet the consistency required for analysis and still reflect decision makers' preferences. In the longer run, Weiss' suggestion (personal communication) that localized behavioral theorems based on preference orderings, analogous to the work done by Machina on the mathematical content of the axioms of expected utility theory, be developed seems promising and in keeping with the concerns which originally motivated this dissertation research.

<u>What Conclusions Can Be Drawn?</u>

There is strong evidence to refute many of the behavioral assumptions underlying the axioms of expected utility theory. It has been demonstrated that if decision makers' preferences violate any ohne of the assumptions of constant marginal utility of money, homogeneity, or invariance of preferences between certainty and uncertainty when other things are equal, the Arrow-Pratt coefficient of absolute risk aversion is a confounding of information about attitude towards risk and preferences for the attributes of outcomes under certainty. Conformance of decision makers' preferences with each of the three assumptions is necessary, but not sufficient, for the Arrow-Pratt coefficient to measure only attitude towards risk.

It has been shown that, when any one of these assumptions is violated, measurement of attitude towards risk requires consideration of the individual's value function as well as his von Neumann-Morgenstern utility function. Dyer and Sarin's measure of intrinsic risk attitude (known in the mathematical psychology and management science literature as relative risk attitude) introduces a useful approach to this problem. However, it was demonstrated that their method unduely constrains analysis to those cases where individuals' preferences for the attributes of outcomes under certainty are completely independent and additive.

A new measure of attitude towards risk that is more general than Dyer and Sarin's measure has been developed. This new measure is applicable to individuals whose preferences for attributes of outcomes under certainty or uncertainty are either additive or multiplicative.

It was found that determining individual's multiattribute value and utility functions without forcing mathematical consistency results in data that does not meet the level of consistency required to determine scaling factors for either the additive or multiplicative multiattribute models. At this juncture, it can not be determined whether the inconsistencies in responses to the survey instrument are due to problems with the new measure of intrinsic risk attitude, fuzzyness in human cognition, mis-specification of initial conditions and auxiliary assumptions, or poor design of the survey instrument.

The issue of mathematical consistency is particularly important for this new measure of attitudes towards risk. While many of the studies of multiattribute utility or value theory are concerned with either an individual's value function or their utility function, the new measure of attitudes towards risk is concerned with the relationship between the individual's value and utility functions. Therefore, both functions must be measured accurately and shown to reflect decision makers' preferences before the new measure of intrinsic attitude towards risk can be tested. Resolution of this question requires that other assumptions of expected utility theory and multiattribute utility theory undergo scrutiny similar to that received by the assumptions examined in this dissertation.

Despite the problems, caveats and unresolved issues discussed above, the new measure of intrinsic attitude towards risk may bring us one step closer to a means of accurately predicting preferred action choices and enable us to uniquely characterize individuals or classes of individuals by their attitude towards risk both across situations and over time.

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