EVALUATION OF EFFICIENT MARKET TESTS BASED ON DAILY STOCK RETURNS

by

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ABSTRACT

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Estimates of common stock betas (systematic risk) traditionally have been calculated using ordinary least squares (OLS) and monthly stock returns. With the availability of daily returns, betas computed using OLS and daily stock returns have recently appeared in the literature. However, Scholes and Williams [1977] have established that these betas are biased and inconsistent. The purpose of this paper is to investigate the effect of this bias and inconsistency on predicting security returns and the testing of the efficient market hypothesis.

Analytical, very large sample, results were derived establishing the effects of the inconsistency on the prediction of security returns. The effect of the inconsistency on the prediction error was very small. The increase of the mean squared prediction error was less than 0.10%. A computer simulation based on Merton's continuous time model was utilized to generate daily stock returns which allowed for the verification of the analytical result and the testing of small samples. The same tests conducted using simulated data were performed using daily stock returns from 283 firms listed on the New York Stock Exchange.

Each test was performed using OLS and Scholes and Williams estimated betas. Mean squared prediction errors were calculated. The summary statistics of the prediction errors were approximately

the same regardless of which estimator was used. A t-test and cumulations average residuals test was performed on several samples to establish whether or not an investigator would be able to find abnormal performance using the Scholes and Williams estimator more often than using the OLS estimator. Abnormal performance was found equally well employing either estimator. The implication of this study is that OLS estimators can be utilized in future research; furthermore, conclusions of past research, based on OLS estimators, are valid.

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I. INTRODUCTION AND OVERVIEW

Estimates of common stock betas (systematic risk) have traditionally been calculated using ordinary least squares (OLS) and monthly returns. These in turn have been used in testing the efficient market hypothesis. With the recent availability of daily returns through CRSP, several studies have employed betas computed using OLS and daily returns (Jaggi [1978] and Brown [1978]). However, Scholes and Williams [1977] and Dimson [1979] have shown that daily betas derived using OLS are both biased and inconsistent. An estimator is unbiased if its expected value is equal to its true value. An estimator is consistent if the estimator can be made to lie arbitrarily close to the true value with a probability arbitrarily close to one. In other words, as the sample size gets larger, the estimator approaches the true value (Maddala [1977, Chapter 4]). The purpose of this paper is to investigate the effect of this bias and inconsistency on predicting security returns and the testing of the efficient market hypothesis. The effect will be determined by comparing predictions and test results computed using OLS estimators, to those computed using consistent estimators derived by Scholes and Williams (S&W).

Scholes and Williams have established that OLS betas are biased and inconsistent. The effects of the bias and inconsistency on efficient market studies need to be determined. Efficient market studies usually divide a sample of stock market returns into two periods, one period is prior to an "event", and is used to estimate parameters (the alpha and beta for a firm). The second period is used to compute residuals or forecast errors. The errors are the difference between forecasts made using the parameters of the first period and the observations of the second period. The forecasts will be affected by beta and therefore, the effects of biased and inconsistent betas on prediction is the focus of this study.

Nonsynchronous Time Periods

The OLS bias is a result of nonsynchronous time periods occurring in daily returns. Nonsynchronous time periods arise from individual firm daily returns being computed from the last security transaction of the previous day to the last transaction of the present day. Consequently, the intervals over which daily returns are calculated are not of equal length. This causes serial correlation in daily return data which has been noted in several other research papers, Fisher [1966], Fama [1965], and Schwartz and Whitcomb [1977]. Serial correlation in daily return data is an indication of the common econometric problem of errors in variables, which produces biased and inconsistent OLS estimates of beta and alpha.[1]

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[1] The errors in variable problem is discussed in Johnston [1972], Chapter 9 and Maddala [1977], Chapter 13.

Eetas are usually computed using the market model as follows:

$$\tilde{R}_{nt} = \alpha_{nt} + \beta_{nt}\tilde{R}_{Mt} + \tilde{e}_{nt}, \tag{1.1}$$

where $\tilde{R}_{nt}$ = return on security n

α_{nt} = intercept for security n

β_{nt} = slope coefficient for security n

$\tilde{R}_{Mt}$ = market portfolio return

$\tilde{e}_{nt}$ = error term for that security n.

The error, $\tilde{e}_{nt}$, is assumed normal with mean zero and variance $\sigma^2(\tilde{e}_{nt})$. Merton [1973] has shown that equation (1.1) is consistent with continuous return theory. The main assumption is that all risky securities have prices distributed as infinitely divisible lognormal random variables. As a result of this assumption, continuously compounded returns R_{nt} on risky securities n = 1, 2, 3...N as calculated over any time period are joint normally distributed with constant mean μ_n, constant variance σ_n^2, and constant covariance.[2] Daily returns are those returns calculated over a time period of one day.

However, the data observed, returns calculated from prices quoted in the market (i.e. organized exchanges), are returns calculated from the last transaction of the

[2]See Scholes and Williams [1977, p. 310) for further discussion of this assumption. See Merton [1973] for other assumptions than the main one listed here.

previous day to the last transaction of the present day. The returns on the CRSP daily tapes are calculated in this manner. The observed return period may span more than a day or less than a day. In other words, observed daily returns are measured over nonsynchronous time periods. Since daily returns described by equation (1.1) cannot be observed, there are measurement errors in the data used.

Two daily returns have now been established, the daily return in equation (1.1) (the actual return), and the daily return that is observed (the observed return). The actual return is the continuously compounded return that is accumulated over a one day holding period. This return cannot be observed. The observed return is the return that is measured from last transaction to last transaction. Figure 1 shows the relationship between these two returns. S_{nt} represents the period during the day t where no transaction takes place[3]. The observed return period is over the period $(t-1-S_{nt-1}, t-S_{nt})$, and the return is designated R^s_{nt}. Equation (1.1) calculated on the observed series becomes:

$$R^s_{nt} = \alpha^s_{nt} + \beta^s_{nt} R^s_{Mt} + w_{nt} \qquad (1.2)$$

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[3]For consistency, the notation used in this paper is the same as that used by Scholes and Williams [1977].

As will be shown in chapter II, returns R^s_{nt} and R^s_{Mt} exhibit serial correlation.

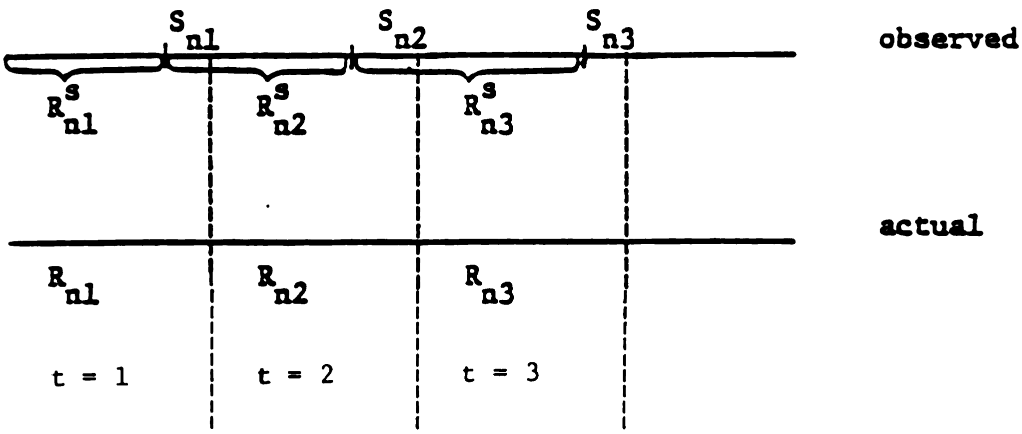

Fig. 1.1 The relationship between the actual and observed returns

Scholes and Williams [1977] and Dimson [1979] have shown that the problem described above will affect the estimation of beta. The fact that this problem exists is supported by Schwartz and Whitcomb [1977]. Therefore, the effects of nonsynchronous time periods on estimation have been established in the literature.

However, the effects of nonsynchronous time periods on the prediction of security returns have not been established. Prediction is important because prediction is an integral part of many efficient market studies. The market model (equation 1.2) is typically used to forecast or predict an expected return, and the residual, the difference between the expected return and actual return observed, is used as the basis of isolating abnormal performance.

Overview

The contribution of this study is to establish the effects of nonsynchronous time periods on prediction of security returns and, in turn, efficient market tests. The theoretical results are derived in chapter III. The market model is a model with stochastic regressors and therefore the results in chapter III are for infinitely large samples. Smaller samples may or may not result in similar conclusions. In order to establish the small sample results a computer simulation must be performed.

A computer simulation is used to obtain a large enough sample to get results congruous with the theoretical large sample results. Once the large sample size is established, smaller samples can be used to see how conclusions change as the sample size is reduced. The use of simulation requires a model, and once the model is obtained, parameter estimates are necessary.

The model used is based on the continuous time model put forth by Merton [1971]. This is the model assumed by S&W [1977]. The only difference is that the simulation model adds nonsynchronous time periods. Actual returns (those without errors) as well as observed returns can be simulated. The actual returns are used to determine the sample size at which the consistent beta estimators converge on the true beta. The derivation of the model is described further in chapter IV as well as the method of obtaining parameter estimates.

The effects of nonsynchronous time periods on efficient market tests will be investigated using a procedure similar to that used by Brown and Warner [1980]. An efficient market test using both the inconsistent OLS estimates and the consistent S&W estimates will be conducted using the simulated returns without abnormal performance and with abnormal performance added. The desire is to see if one estimator outperforms the other. Performance is measured by the ability to isolate abnormal performance. The results of this test are discussed in chapter V.

Results and conclusions based on simulation must be supported by real data results. This is necessary since simulation is based on a model that may or may not describe actual data. Therefore, the same tests performed using the simulated data were performed using the returns from 283 New York Stock Exchange firms. This was done to see if the simulation results were consistent with the results obtained using real firms. The results of these tests are discussed in chapter VI.

As mentioned earlier, Scholes and Williams and others have established that the use of daily returns in the market model results in biased and inconsistent estimates of alpha and beta. The basic uses of these estimates are:

1) the evaluation of risk,
2) prediction, and
3) the testing of the efficient market hypothesis.

This study addresses the last two uses. Does the fact that the estimates are biased and inconsistent affect prediction, and does the fact that the estimates are biased and inconsistent affect the conclusions of efficient market tests? The answers to these questions affect both the interpretation of past research and the design of future research. The conclusions of past research may be invalid. Efficient market tests conducted in the future may or may not have to be adjusted for the bias. This study will attempt to answer the above questions.

II. LITERATURE REVIEW

Scholes and Williams' [1977] (hereafter, S&W) analysis of beta leads directly to the subsequent analysis of this paper. Therefore, the major portion of the literature review chapter is devoted to the S&W analysis. The other articles included are either in support of S&W or an aid to further analysis.

Scholes and Williams

Scholes and Williams [1977] assume that at least one trade takes place each day and the periods of non-trading are independent and identically distributed across time.[4] These assumptions lead to relationships 2.1, 2.2, and 2.3:[5]

$$\text{Var}\ (R^s_{nt}) = \sigma^2_n + 2\ \text{Var}\ (S_n)\ \mu^2_n, \tag{2.1}$$

where R^s_{nt} = observed return for firm n at time t

σ^2_n = variance of the actual daily return = Var (R_{nt}),

S_n = period of non-trading for security n,

μ_n = the average return for security n, and

Var (S_n) = the variance of the period of non-trading.

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[4]This assumption appears reasonable since Hawawini [1980] shows the most significant cross correlation of returns occur at lags of ± 1 day.

[5]Derivation of these properties are shown in Scholes and Williams [1977, p. 325].

Equation (2.1) shows that the variance of the observed return series is greater than that of the actual return series. Consider that the time periods for which these returns are calculated are always changing. One return could be calculated for slightly more than one day (R^S_{n1}) while the next day's return may be of time period slightly less than one day (R^S_{n2}). R^S_{n1} could be greater in magnitude than the corresponding actual return (R_{n1}) because R^S_{n1} has a longer holding period. R^S_{n2} could be smaller in magnitude than the corresponding actual return (R_{n2}) because R^S_{n2} has a shorter holding period. The returns R^S_{n1} and R^S_{n2} would be more variable than R_{n1} and R_{n2} but have the same mean.

The covariance between the observed return of two securities is as follows:

$$\text{Cov}\ (R^S_{nt}, R^S_{mt}) = \sigma_{nm} - E[\max(S_n, S_m) - \min\ (S_n, S_m)]\ \sigma_{nm} + 2\ \text{Cov}\ (S_n, S_m)\ \mu_m \mu_n, \tag{2.2}$$

where σ_{nm} = the covariance between the actual returns of security n and security m,

$E[\max(S_n, S_m) - \min\ (S_n, S_m)]$ = the overlap in no-trade periods for which only one security was traded, and

μ_m, μ_n = the average return for securities m and n.

Two situations must be considered in discussing equation

(2.2). The first situation is when Cov (S_n, S_m) is positive (see Fig. 2.1), and the second situation is when Cov (S_n, S_m) is negative (see Fig. 2.2). Positive covariance indicates that when an increase in S_n occurs, an increase in S_m will more than likely occur. The corresponding period over which the returns are calculated for both securities will decrease and as a result, both returns will decrease. A decrease in S_n and S_m will cause an increase in the observed returns for securities n and m. The observed returns will exhibit a positive covariance due to the fluctuation of S_n and S_m since Cov $(S_n, S_m)\ \mu_n \mu_m$ is positive. Negative covariance implies a decrease in S_n which will probably be accompanied by an increase in S_m. The length of the time periods will vary in opposite directions and cause the observed returns to do likewise. Negative covariance (2 Cov $(S_n, S_m)\ \mu_n \mu_m$ negative) will reduce the observed covariance.

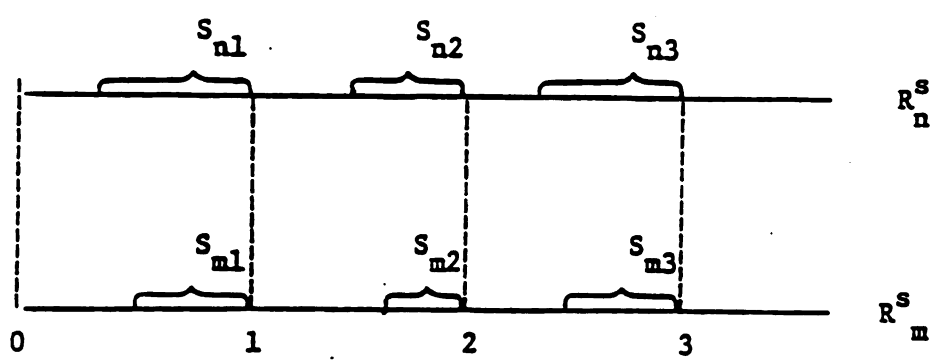


Fig. 2.1 Positive Covariance of Non-Trade Periods

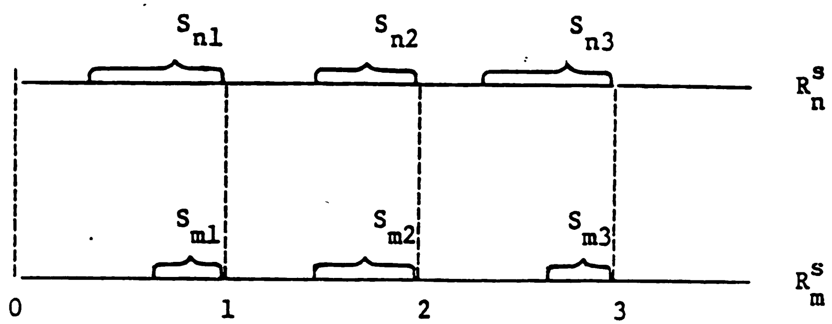


Fig. 2.2 Negative Covariance of Non-Trade Periods

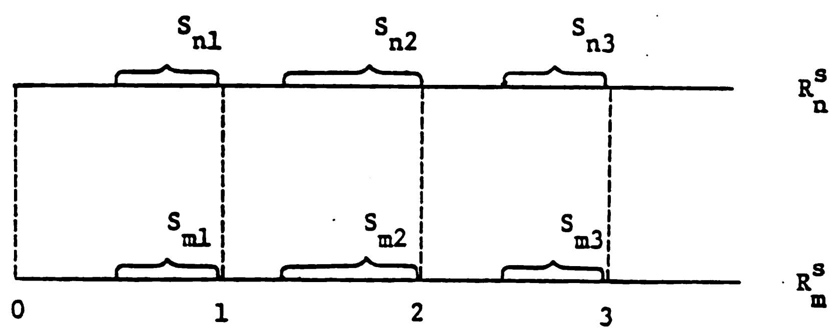


Fig. 2.3 Equal Non-Trade Periods

The other component of equation (2.2), $E[\max(S_n,S_m) - \min(S_n,S_m)]\sigma_{nm}$, is the expected length of time for which the time periods of R^s_{nt} and R^s_{mt} are not concurrent. During this period of time the two returns would not covary and the actual covariance is reduced. If $E[\max(S_n,S_m) - \min(S_n,S_m)] = 0$, then $S_n = S_m$ (see Fig. 2.3), but $\mathrm{Cov}\,(R^s_{nt},R^s_{mt}) \neq \sigma_{nm}$ because the covariance term $2\,\mathrm{Cov}\,(S_n,S_m)$ still has the same properties discussed in the previous paragraph.

The occurrence of Cov $(R^s_{nt}, R^s_{mt}) = \sigma_{nm}$ is when $S_n = S_m$ and $S_n = S_{nt}$ for all t.

The covariance between the observed return of security n and the prior day's observed return of security m is as follows:

$$\text{Cov}\ (R^s_{nt}, R^s_{mt-1}) = (E[\max(S_n - S_m, 0)]\sigma_{nm} - \text{Cov}\ (S_n, S_m)\ \mu_n\mu_m \qquad (2.3)$$

This equation is similar to equation (2.2) except $E[\max(S_n - S_m, 0)]$ is the expected overlap of the time periods for R^s_{nt} and R^s_{mt-1}. If $E[\max(S_n - S_m, 0)]$ is greater than zero then the two returns have a portion that is concurrent and as a result covary. The other component of the equation Cov $(S_n, S_m)\ \mu_n\mu_m$ results from the covariance of the length of time for returns R^s_{nt} and R^s_{mt-1}. If Cov (S_n, S_m) is positive and if the length of time for R^s_{nt} increases then R^s_{mt-1} decreases. Therefore, the sign of the term Cov $(S_n, S_m)\mu_n\mu_m$ must be negative.

The Scholes and Williams relationship is derived from equation (2.1), (2.2), and (2.3) using:

$$E[\max(S_n - S_m) - \min\ (S_n, S_m)] = E[\max(S_n - S_m, 0)] + E[\max(S_m - S_n, 0)] \qquad (2.4)$$

Substituting (2.4) into (2.2) leads to:

$$\mathrm{Cov}(R^s_{nt},R^s_{mt}) = \sigma_{nm}-(E[\max(S_n-S_m,0)]\sigma_{nm}-\mathrm{Cov}(S_n,S_m)\mu_n\mu_m)$$
$$-(E[\max(S_m-S_n,0)]\sigma_{nm}-\mathrm{Cov}(S_n,S_m)\mu_n\mu_m) \qquad (2.5)$$

Substituting (2.3) into (2.5) results in:

$$\mathrm{Cov}(R^s_{nt},R^s_{mt}) = \sigma_{nm}-\mathrm{Cov}(R^s_{nt},R^s_{mt-1})-\mathrm{Cov}(R^s_{nt-1},R^s_{mt}) \qquad (2.6)$$

Let X_{nM} represent the weight of security n in the market index M. Multiplying both sides of equation (2.6) by X_{nM} and summing over n - 1, ..., N results in:

$$\mathrm{Cov}(R^s_{nt},R^s_{Mt}) = \mathrm{Cov}(R_{nt},R_{Mt}) - \mathrm{Cov}(R^s_{nt},R^s_{Mt-1})$$
$$- \mathrm{Cov}(R^s_{nt-1},R^s_{Mt}) \qquad (2.7a)$$

where the market return is designated by M.

Multiplying both sides of equation (2.7a) by X_{nM} and summing over n = 1, ..., N results in:

$$\mathrm{Var}(R^s_{Mt}) = \mathrm{Var}(R_{Mt}) - 2\ \mathrm{Cov}(R^s_{Mt},R^s_{Mt-1}) \qquad (2.7)$$

Dividing (2.7a) by (2.7) leads to:

$$\frac{\mathrm{Cov}(R^s_{nt},R^s_{Mt})}{\mathrm{Var}(R^s_{Mt})} = \frac{\mathrm{Cov}(R_{nt},R_{Mt})}{\mathrm{Var}(R^s_{Mt})} - \frac{\mathrm{Cov}(R^s_{nt},R^s_{Mt-1})}{\mathrm{Var}(R^s_{Mt})}$$
$$- \frac{\mathrm{Cov}(R^s_{nt-1},R^s_{Mt})}{\mathrm{Var}(R^s_{Mt})} \qquad (2.8)$$

Substituting the following into (2.8):

$$\beta_n^s = \frac{Cov(R_{nt}^s, R_{Mt}^s)}{Var(R_{Mt}^s)} \quad \text{,the observed beta} \tag{2.9}$$

$$\beta_n^{s+} = \frac{Cov(R_{nt}^s, R_{Mt-1}^s)}{Var(R_{Mt}^s)}, \text{ one day lead observed beta} \tag{2.10}$$

$$\beta_n^{s-} = \frac{Cov(R_{nt}^s, R_{Mt+1}^s)}{Var(R_{Mt}^s)}, \text{ one day lag observed beta} \tag{2.11}$$

$$\beta_n = \frac{Cov(R_{nt}, R_{Mt})}{Var(R_{Mt})} \quad \text{, the actual beta} \tag{2.12}$$

results in the Scholes and Williams relationship:

$$\beta_n^s = \beta_n (1 + 2\rho^s) - \beta_n^{s+} - \beta_n^{s-} \tag{2.13}$$

where ρ^s equals the autocorrelation of the observed market. Equation 2.13 is estimated using the estimator

$$\hat{\beta}_{SW} = \frac{\hat{\beta}_{OLS} + \hat{\beta}_M^{S+} + \hat{\beta}_N^{S-}}{1 + 2\hat{\rho}^S}$$

where $\hat{\beta}_{SW}$ = the S&W estimator, and

$\hat{\beta}_{OLS}$, $\hat{\beta}_M^{S+}$, $\hat{\beta}_N^{S-}$ are OLS estimators.

The magnitude of Scholes & Williams relationship (equation 2.13) depends on the size of ρ^s, β_n^{s+} and β_n^{s-}. The market return is an average of individual security returns. An actively traded security would appear to lead the market because it is traded more often than the market average and, therefore, would appear to react to an event prior to a market reaction for the same event. As a result, in equation (2.16) β_n^{s+} would dominate. An inactive stock would appear to react more slowly than the market to an event, because it is traded less often than the market average and β_n^{s-} would dominate. A security which is traded approximately the same as the market should react to an event at approximately the same time as the market and β_n^{s+} and β_n^{s-} should be small. If β_n^{s+}, β_n^{s-} and ρ^s are insignificant the adjustment would also be insignificant. ρ^s has been computed on a daily basis by both Scholes and Williams [1976, p. 34] and Schwartz and Whitcomb [1977, p. 299]. Both Scholes and Williams, and Schwartz and Whitcomb found the autocorrelation of the value weighted market index 1971-1975 to be .291 while Scholes & Williams found the autocorrelation of the equally weighted market to be .458 (see table 2.1). The significance of $\beta_n^{s+}$and β_n^{s-}depends on the significance of the lead and lag covariances of the security with the market or, alternatively, the lead and lag cross correlations. The denominators of equations 2.9, 2.10, and 2.11 are all equal, therefore, differences can be attributed solely to differences in the numerators. The numerators are comprised of the concurrent, lead, and lag covariances respectively. Hawawini [1980] computed the lead and lag cross correlations for 50 firms with the S&P 500 index for 1 to 20 day leads and lags and found 72% of the 1 day lead cross correlations and 100% of the 1 day lag cross correlations to be statistically significant. As a result of these studies, it appears that these adjustments will make a difference.

Scholes and Williams empirically tested their relationship by applying the estimator to five portfolios formed from all stocks listed on the New York Stock Exchange and American Stock Exchange between January 1963 and December 1975. The portfolios were formed based on volume traded. Volume traded was used as a surrogate for the average number of transactions. S&W grouped, for example, the low volume securities together hoping to get mostly less actively traded securities. The results of their test are shown in tables 2.2, 2.3, and 2.4. As can be seen in the tables, the results agree with the theory. In table 2.2, all of the S&W betas are greater than the OLS betas, and in table 2.4, where the more actively traded securities are examined, the reverse is true. If equation 2.13 is rearranged, the difference between the OLS beta and the S&W beta can be computed as follows:

$$\beta^{s}_{n} - \beta_{n} = \beta_{n}\rho^{s}_{n} - \beta^{s+}_{n} - \beta^{s-}_{n} \tag{2.14}$$

Substituting the averages from tables 2.2, 2.3, and 2.4, the average differences between the OLS estimator of beta and the S&W estimator of beta are -.157, -.111, and .063 respectively.

Dimson [1979] relaxed the assumption that the security must be traded once in every period, and defined the market differently from S&W. Scholes and Williams' market is composed of securities traded every day while Dimson's market is the Market Portfolio, or in other words, a portfolio of all risky securities. As a result of this change, Dimson suggests the following beta adjustment (aggregated coefficients method):

$$\hat{\beta} = \sum_{k=-i}^{i} \hat{\beta}^{sk}_{n} ,$$

where i = the number of lags and leads.

Dimson states that for shares which trade in almost every period, the Scholes and Williams approach has results similar to the aggregated coefficients method.

Dimson states that for shares which trade in almost every period, the Scholes and Williams approach has results similar to the aggregated coefficients method.

The critical assumption between the two methods would appear to be the assumption of the security trading every day. Looking at Hawawini's study, the significance of the

TABLE 2.1

FIRST-ORDER AUTOCORRELATION COEFFICIENTS

FOR THE MARKET: 1971-1975.

Interval	Value-Weighted Index	Equally Weighted
1 day	.291	.458
2 days	.121	.335
1 week	.039	.283
2 weeks	.109	.289
1 month	.083	.134

SOURCE: Scholes and Williams [1976] p. 34.

TABLE 2.2

DAILY RETURNS ON LOW-VOLUME PORTFOLIOS REGRESSED ON VALUE-WEIGHTED MARKET RETURNS

Year	$\hat{\beta}^s_n = \hat{\beta}_{OLS}$	$\hat{\beta}_{SW}$	$\hat{\beta}^{s+} = \hat{\beta}^{s+}_{OLS}$	$\hat{\beta}^{s-} = \hat{\beta}^{s-}_{OLS}$	$\hat{\rho}^s$
1963	0.303	0.544	0.049	0.130	-0.058
1964	0.391	0.561	0.090	0.216	0.122
1965	0.524	0.647	0.045	0.352	0.212
1966	0.426	0.581	0.102	0.391	0.291
1967	0.556	0.651	-0.015	0.267	0.120
1968	0.600	0.775	0.125	0.462	0.266
1969	0.749	0.872	0.183	0.620	0.390
1970	0.679	0.809	0.185	0.565	0.383
1971	0,848	0.993	0.232	0.526	0.308
1972	0.596	0.661	0.121	0.364	0.317
1973	0.499	0.657	0.071	0.435	0.265
1974	0.346	0.431	0.024	0.307	0.284
1975	0.415	0.577	0.057	0.402	0.258
Average		0.674	0.098	0.387	0.243

SOURCE: Scholes and Williams [1977] p. 321.

TABLE 2.3

DAILY RETURNS ON INTERMEDIATE-VOLUME PORTFOLIOS

REGRESSED ON VALUE-WEIGHTED MARKET RETURNS

Year	$\hat{\beta}_{OLS}$	$\hat{\beta}_{SW}$	$\hat{\beta}^{s+}_{OLS}$	$\hat{\beta}^{s-}_{OLS}$	$\hat{\rho}^{s}$
1963	0.785	0.905	-0.039	0.005	-0.058
1964	0.754	0.851	0.071	0.233	0.122
1965	1.119	1.202	0.174	0.418	0.212
1966	1.005	1.149	0.248	0.565	0.291
1967	1.123	1.112	0.045	0.212	0.120
1968	1.187	1.274	0.264	0.501	0.266
1969	1.257	1.330	0.421	0.690	0.390
1970	1.248	1.305	0.418	0.638	0.383
1971	1.296	1.386	0.428	0.516	0.308
1972	0.989	1.021	0.299	0.381	0.317
1973	0.983	1.142	0.199	0.564	0.265
1974	0.724	0.830	0.116	0.462	0.284
1975	0.857	0.996	0.171	0.481	0.258
Average		1.115	0.217	0.436	0.243

SOURCE: Scholes and Williams [1977] p. 322.

TABLE 2.4

DAILY RETURNS ON HIGH-VOLUME PORTFOLIOS

REGRESSED ON VALUE-WEIGHTED MARKET RETURNS

Year	$\hat{\beta}_{OLS}$	$\hat{\beta}_{SW}$	$\hat{\beta}^{s+}_{OLS}$	$\hat{\beta}^{s-}_{OLS}$	$\hat{\rho}^{s}$
1963	1.495	1.336	-0.097	-0.217	-0.058
1964	1.355	1.290	0.199	0.049	0.122
1965	1.597	1.501	0.337	0.204	0.212
1966	1.725	1.564	0.452	0.297	0.291
1967	1.602	1.369	0.219	-0.122	0.120
1968	1.520	1.468	0.449	0.281	0.266
1969	1.531	1.501	0.682	0.458	0.390
1970	1.473	1.437	0.647	0.418	0.383
1971	1.445	1.441	0.535	0.349	0.308
1972	1.314	1.267	0.516	0.240	0.317
1973	1.318	1.314	0.375	0.316	0.265
1974	1.134	1.120	0.303	0.320	0.284
1975	1.174	1.172	0.312	0.290	0.258
Average		1.368	0.380	0.222	0.243

SOURCE: Scholes and Williams [1977] p. 323.

lead and lag cross correlations with the market index falls off dramatically after one day. Of the fifty securities tested, only 8% and 22% of the cross correlations were statistically significant for the two-day lead and lag respectively, compared to 72% and 100% for the one day lead and lag (see table 2.5). Therefore, it would appear that Scholes and Williams assumption is reasonable, and the Scholes and Williams adjustment will be used throughout the remainder of this study.

Scholes and Williams assume that the major cause of serial correlation is nonsynchronous time periods. Schwartz and Whitcomb [1977] investigate not only the serial correlation but also the causes of serial correlation. Schwartz and Whitcomb hypothesize that serial correlation is caused by the following possible reasons:

1) Measurement error
2) The 1/8 effect
3) Impact of Market Makers
4) The Fisher effect.

Measurement errors are defined as things such as recording and keypunch errors. The 1/8 effect refers to the rounding error resulting from reporting prices in 1/8 increments. This rounding transforms a smooth price series into a lumpy series, and is analogous to measurement error. Market Makers refer to NYSE specialists. The Fisher effect is attributed to the fact that infrequently traded securities have their last recorded price before the end of the day

TABLE 2.5

PERCENT OF 50 FIRMS IN WHICH

CROSS-CORRELATIONS ARE STATISTICALLY SIGNIFICANT

LAG IN DAYS	0	+1	-1	+2	-2	+3	-3	+4	-4
% of Significant Correlation	100	72	100	8	22	8	16	14	4

Source: Hawawini [1980]

(nonsynchronous time periods). Schwartz and Whitcomb conclude that there is little evidence in support of measurement errors, the 1/8 effect, and the impact of market makers, but that there is considerable evidence supporting the Fisher effect and that observed positive index autocorrelation could result from the Fisher effect impacting on thin issues. This conclusion supports S&W's assumption of nonsynchronous time periods.

Efficient Market Tests Using Daily Return Data

Betas have been shown to be biased, as documented earlier, when estimated using the market model and daily stock returns. However, many studies have used betas estimated with the market model and daily stock returns despite the bias. The incentive of conducting the current study is to investigate the potential impact of using a biased and inconsistent beta on such studies.

Brown [1978] tested for the abnormal performance using the market model to obtain expected returns. Daily returns were used in the estimation of beta and alpha. The purpose of his study was to see how fast the market reacted to reports of unusual earnings, and therefore, the cumulative average residuals were inspected. Based on the upward trend of the cumulative average residuals, Brown concluded market inefficiencies exist. However, Brown did not adjust for the bias in alpha and beta, and, as pointed out by Johnston [1972, Chapter 8], biased estimates may cause autocorrelation in the residuals of an OLS regression. The autocorrelation, and in turn, the bias, may account for the upward trend.

Jaggi [1978] conducted a study designed to test for market reaction to management forecasts. The expected returns were calculated using the market model, and the parameter estimates were computed using daily returns. Jaggi tested the residuals for significance and assumed a symmetric stable distribution. The symmetric stable distribution was used because residuals resulting from daily returns do not appear normal due to non-normal kurtosis [Fama and Roll, 1968, 1971]. This may also be due to the bias in the estimates, and Jaggi did not adjust for the bias.

Gheyara and Boatsman [1980] tested the market reaction to replacement cost disclosures. This study also used the market model to compute the expected returns, and daily returns were used to estimate beta and alpha. Gheyara and Boatsman computed both OLS estimates and S&W estimates, and found their results to be insensitive to which parameter estimation procedure was selected. They concluded the sample firms were drawn from the complete spectrum of trading intensity and the biases were diversified away. However, no tests were conducted to confirm this conclusion. It may also be that the bias does not affect prediction regardless of the spectrum of trading intensity.

Beaver, Christie and Griffin [1980] conducted a test similar to Gheyara and Boatsman. Beaver et. al. also recognized the bias problem and used monthly returns to estimate the market model parameters. Beaver states, "Monthly returns, rather than daily data, were used to assess beta because of the nonsynchronous nature of daily data and the attendant problems of beta estimation introduced." However, it is not certain that the bias affects prediction and daily returns could have been used.

The above four studies point out the need for information on the bias in OLS estimates and the effect of the bias and inconsistency on the prediction of returns. The intent of the current study is to provide this information.

Efficient Market Test Procedures

A method for evaluating different efficient market test procedures has been provided by Brown and Warner [1980]. Their desire was to see if one test procedure found abnormal performance better than another. In order to do this, a known abnormal performance was added to every firm's returns in the sample and different procedures were performed to see if the abnormal performance was found. The same test was performed on many samples to see if one procedure isolated abnormal returns more often than another. The current study investigates whether OLS estimators outperform S&W estimators in the same efficient market test.

Therefore, the only difference between the current study and Brown and Warner is that the same procedures with different estimators are being considered rather than different procedures.

Two of the tests Brown and Warner [1980] used will be used in this paper: a t-test and a cumulative average residual test (CAR). These two procedures are commonly used and were used in the four papers discussed in the previous section. The procedure is to divide a sample of firms' returns into 3 time periods (see figure 2.4). The first period (days -200 through -90) is used to estimate the parameters of the market model (equation 1.2) using OLS. These parameters are then used in the market model to forecast returns which correspond to the observed returns in periods 2 and 3. Residuals, the difference between forecast and observed returns, are calculated for each day of periods 2 and 3. The residuals from each day are averaged over all the firms in the sample so that there exists an average residual for every day. The average residuals in the second period are used to compute a standard deviation. The standard deviation is divided into the average residual on the event day (day 0) in the third period, which results in a t-statistic as follows (Brown and Warner [1980], equation A.11):

$$\frac{1/N \sum_{i=1}^{N} A_{i0}}{1/N\left(\sum_{i=1}^{N}\left[1/77 \sum_{t=-89}^{-11}\left(A_{it}-\left(\sum_{t=-89}^{-11} A_i/79\right)\right)^2\right]\right)^{1/2}}$$

where N = the number of firms in the sample, and

$A_{it} = R^{s}_{it} - \hat{\alpha}_i - \hat{\beta}_i R^{s}_{Mt}$, and

$\hat{\alpha}_i$, $\hat{\beta}_i$ = either the OLS estimates or the S&W estimates of α and β.

If the t-statistic is significant, then the conclusion is that abnormal performance exists. The entire set of third period average residuals is used in the CAR test as follows (Brown and Warner [1980], equation 1):

$$CAR_t = CAR_{t-1} + 1/N \sum_{i=1}^{N} A_{it}$$

where CAR_t = the cumulative average residual at time t.

These procedures are explained further when they are used.

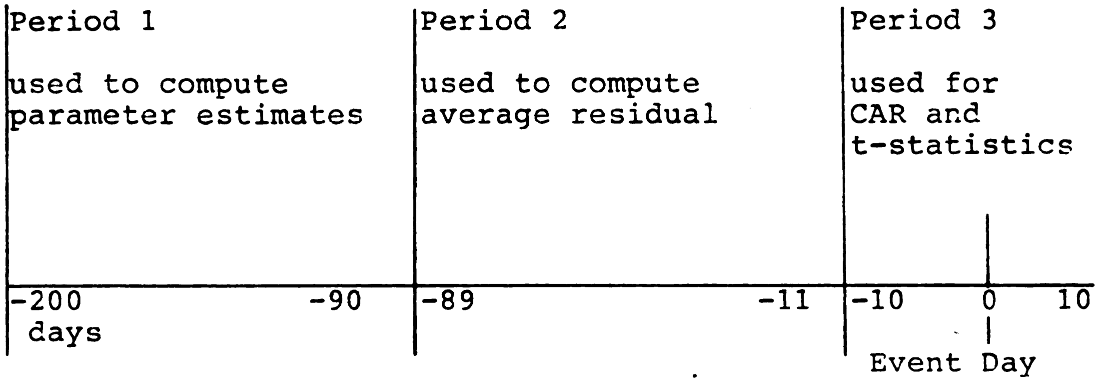


Fig. 2.4 Division of a Sample Used in a Brown and Warner Test

III. BETA BIAS AND MARKET MODEL PREDICTIONS

Scholes and Williams have established that OLS betas are biased and inconsistent. The effects of the bias and inconsistency on efficient market studies need to be determined. Efficient market studies usually divide a sample of stock market returns into two periods, one period is prior to an "event", and is used to estimate parameters (the alpha and beta for a firm). The second period is used to compute residuals or forecast errors. The errors are the difference between forecasts estimated using the parameters of the first period and the observations of the second period. The forecasts will be affected by beta and therefore, the effects of biased and inconsistent betas on prediction is the focus of this study.

Malinvaud [1970] provides a framework for studying the effects of inconsistency on the mean square prediction error. If the following model exists:

$$R^s_{mt} = R_{mt} + u_{mt} \tag{3.1}$$

$$R_{nt} = \beta R_{mt} + e_{nt} \tag{3.2}$$

where β = a parameter to be estimated,

u_{mt} = the measurement error of R^s_{mt}, and

e_{nt} = the error term for security n then the OLS estimate of beta is inconsistent (see also Maddala [1977, Chapter 13]). Prediction is defined as choosing a value R^P_{nt} for R_{nt}, and the general aim is to minimize $E(R^P_{nt} - R_{nt})^2$.

The prediction of R_{nt} given R^S_{mt} is:

$$R^P_{nt} = \tilde{\beta} R^S_{mt} \tag{3.3}$$

where $\tilde{\beta}$ = the estimator of β.

The resulting error is the following:

$$(R^P_{nt} - R_{nt}) = R^S_{mt} (\tilde{\beta} - \beta) + (\beta u_{mt} - e_{nt}). \tag{3.4}$$

The mean square prediction error conditional on R^S_{mt} is:

$$E[(R^p_{nt}-R_{nt})^2/R^s_{mt}] = (R^s_{mt})^2 E[(\tilde{\beta}-\beta)^2] + E[(\beta u_{mt} - e_{nt})^2]. \tag{3.5}$$

If $\tilde{\beta}$ is a consistent estimator then as the sample size gets larger $(\tilde{\beta} - \beta)$ approaches zero. If $\tilde{\beta}$ is inconsistent $(\tilde{\beta} - \beta)$ does not approach zero as the sample size gets larger, but approaches a value different from zero. As a result, the mean square prediction error for a large sample must be greater when an inconsistent estimator is used.

Market Model Predictions

The previous discussion assumed that the security return was observed without error and that the intercept term equaled zero. However, the security return is observed with error as follows:

$$R^{s}_{nt} = R_{nt} + v_{nt} \tag{3.6}$$

where v_{nt} = the measurement error of R^{s}_{nt}.

The underlying generating process (equation 3.2) with an intercept term added becomes:

$$R_{nt} = \alpha + \beta R_{Mt} + e_{nt} \tag{3.7}$$

where α= the intercept parameter to be estimated. Equation 3.7 is equivalent to the market model equation 1.1. Substituting equations 3.1 and 3.6 into equation 3.7 the process for generating observed returns becomes:

$$R^{s}_{nt} = \alpha + \beta R^{s}_{Mt} - \beta u_{Mt} + e_{nt} + v_{nt} \tag{3.8}$$

The prediction of R^{s}_{nt} given R^{s}_{mt} is:

$$R^{sp}_{nt} = \tilde{\alpha} + \tilde{\beta} R^{s}_{Mt} \tag{3.9}$$

where R^{sp}_{nt} = the predicted observed return and
$\breve{\alpha}$ = the estimator of α.

The resulting prediction error is:

$$R^{sp}_{nt} - R^{s}_{nt} = (\tilde{\alpha}-\alpha)+(\tilde{\beta}R^{s}_{Mt}-\beta R^{s}_{Mt})+(\beta u_{Mt}-e_{nt}-v_{nt}).$$

The mean square prediction error given R^{s}_{Mt} is:

$$E[(R^{sp}_{nt}-R^{s}_{nt})^{2}/R^{s}_{Mt}]=(R^{s}_{Mt})^{2}E[(\tilde{\beta}-\beta)^{2}]+E[(\beta u_{Mt}-e_{nt}-v_{nt})^{2}] \\ +E[(\tilde{\alpha}-\alpha)^{2}]. \tag{3.10}$$

If the OLS estimate of beta ($\hat{\beta}_{OLS}$) is substituted into equation 3.10, the mean square prediction error becomes:

$$MSPE_{OLS}=(R^{s}_{Mt})^{2}E[(\hat{\beta}_{OLS}-\beta)^{2}]+E[(\beta u_{Mt}-e_{nt}-v_{nt})^{2}] \\ +E[(\tilde{\alpha}-\alpha)^{2}]. \tag{3.11}$$

If the S&W estimate of beta ($\hat{\beta}_{SW}$) is substituted into equation 3.10 the mean square prediction error becomes:

$$MSPE_{SW}=(R^{s}_{Mt})^{2}E[(\hat{\beta}_{SW}-\beta)^{2}]+E[(\beta u_{Mt}-e_{nt}-v_{nt})^{2}] \\ +E[(\tilde{\alpha}-\alpha)^{2}]. \tag{3.12}$$

The S&W estimate of beta is consistent, and as the sample size gets large $(\hat{\beta}_{SW} - \beta)^2$ goes to zero and equation 3.12 becomes:

$$MSPE_{SW} = E[(\beta u_{Mt} - e_{nt} - v_{nt})^2] + E[(\tilde{\alpha} - \alpha)^2]. \tag{3.13}$$

Scholes and Williams [1977] found that the value for alpha, estimated using their relationship, was not statistically different from the OLS estimate of alpha. Therefore, the difference between equations 3.11 and 3.13 as the sample size gets large is:

$$MSPE_{OLS} - MSPE_{SW} = (R^s_{Mt})^2 [\hat{\beta}_{OLS} - \beta]^2, \tag{3.14}$$

or the difference is equal to the observed market return squared multiplied by the inconsistency squared.

An estimate of equation 3.14 was computed by selecting a sample of 100 observed market returns from the CRSP value weighted index. Each return was squared and multiplied by the differences between the OLS beta and the S&W beta squared. These differences were calculated using equation 2.14 and reported in chapter II. The results of this estimation are reported in table 3.1. The effect of these differences on total error depends on the size of the total error. Allen and Hagerman [1980] conducted a study investigating factors (indexes and return time periods) affecting forecast accuracy. The mean square prediction error was computed using the market model and monthly, weekly, and daily return data at various estimation periods. The firms used were those firms listed on the CRSP daily tapes for the period July 1, 1962 to December 31, 1976. The MSPE employing daily returns was approximately .0009 (Allen and Hagerman [1980], table IV). Assuming the worst case in table 3.1 (.00000081) the $MSPE_{OLS}$ would be .00090081. Compared to an $MSPE_{SW}$ of .0009, the ratio of $MSPE_{OLS}$ to $MSPE_{SW}$ would be 1.0009. The effect is very small.

A simulation will test the results of equations 3.11 and 3.13. Scholes and Williams [1976] tested their relationships 2.1, 2.2, and 2.3 using a simulation of only two firms, however, in order to test the above results a market must also be simulated. To simulate a market more than two firms are needed.

TABLE 3.1

ESTIMATES OF $MSPE_{OLS}$ - $MSPE_{SW}$

$\hat{\beta}_{OLS} - \hat{\beta}_{SW}$	average R^s_{mt}	average$(R^s_{mt})^2[\hat{\beta}_{OLS} - \beta)^2$
-.157	.000366	.00000081
-.111	.000366	.000000405
.065	.000366	.00000013

IV. SIMULATION MODEL STRUCTURE

The model used to generate stock market returns is the capital asset pricing model in continuous time. This is the model developed by Merton [1971] and assumed by Scholes and Williams [1976]. The major assumptions of the model are: all risky securities have prices distributed as infinitely divisible lognormal random variables, and the investment opportunity set is constant [Merton 1973]. Given these assumptions the model of returns (Cox and Miller [1966], chapter 5, and Merton [1971] equation 5) is:

$$\frac{dP}{P_0} = A\,dt + C\,dz(t) \tag{4.1}$$

where A is a vector of instantaneous expected returns for risky securities measured per unit of time and,

C is a nonsingular N X N matrix of constants such that,

$CC' = \Sigma = (\sigma_{nm})$ the variance - covariance matrix for the instantanious returns on risky securities, and

dz(t) is the increment of an N vector standardized Wiener process.

Equation (4.1) is a Wiener process. A Wiener process is a random process which has a normal distribution with an expected value of zero and a variance of t. The process is stochastic and not differentiable. In order to find a solution, Ito's lemma must be used. Ito's lemma is as follows (Haley and Schall [1979], chapter 10):

$$d\tilde{F} = \frac{\partial F}{\partial y}\, d\tilde{y} + \frac{\partial F}{\partial t}\, dt + 1/2\, \frac{d^2 F}{dy^2} (d\tilde{y})^2$$

The solution vector P(t) to equation (4.1) (Arnold [1974], chapter 8) is:

$$P(t) = P_0\, e^{[(A - 1/2 \Sigma X')t + CZ_n]} \tag{4.2}$$

where X' is the vector of market weights and,

Z_n is a vector of independent standard normal variates.

The partial derivates of 4.2 given $P = F(t,z)$ are:

$$\frac{\partial F}{\partial z} = P_0\, Cdz$$

$$\frac{\partial^2 F}{\partial z^2} = P_0\ (CC'X')dz^2 = P_0\ (\Sigma X')dt$$

$$\frac{\partial F}{\partial t} = P_0 \ (A - 1/2\ \Sigma X')dt$$

Substituting the partial derivates into Ito's lemma:

$$d\tilde{P} = P_0 \ A \ dt + P_0 \ C \ dz,$$

which is equavalent to equation (4.1).

The simulator generates prices in accordance with equation (4.2). The market is the sum of the prices multiplied by their market weights or:

$$P_{market} = P_0 \ e^{[(AX' - 1/2\ X\Sigma X')t + XCZ_n]} \tag{4.3}$$

Equation (4.3) is the matrix form of Merton's model for a portfolio (see Merton [1971] page 386, equation 35). Given the price vector, the return for firm n at time t will be:

$$R_{nt} = \ln P_{nt} - \ln P_{nt-1}$$

Prior to the price vector generation, however, estimates of A and Σ must be obtained, and t and Z_n must be generated.

Parameter estimates of the instantaneous variance of firms returns have been calculated in the pricing of options. The capital assets pricing model in continuous time was used by Black and Scholes [1972] to develop an options-pricing model. This model can be used to estimate a value for the instantaneous variance. The options-pricing model is:

$$C = SN(d_1) - e^{-r_f T} XN(d_2)$$

$$d_1 = \frac{\ln (S/X) + r_f T}{\sigma\sqrt{T}} + 1/2\ \sigma\sqrt{T}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

where C = the call price

S = the current stock price

X = the exercise price

r_f = the risk free rate

σ^2 = the instantaneous variance and,

T = the remaining time of the option.

The only parameter needed is the instantaneous variance (σ^2) (See Copeland & Weston [1980]). Studies in the options literature have shown that the implied standard deviation (ISD) is an adequate estimate of σ (see Latane and Rendleman [1976], Schmalensee and Trippi [1978], and Chiras and Manaster [1978]). For present purposes, eleven firms were chosen and their ISD's calculated to obtain an estimate of the market instantaneous variance, see table 4.1. The time at which the ISD's were calculated was October 4, 1977.

The same eleven firms were used to estimate the market average return. The expected return from 4.3 is:

$$(AX' - 1/2\ X\Sigma X')t \quad \text{or}$$

$$\bar{R}_n = at - 1/2\ \sigma_n^2 t$$

where a = the instantaneous expected return of the market.

The monthly average for each of the eleven firms was calculated for the four years ended December 31, 1977. The data were obtained from the CRSP monthly tapes. The instantaneous average was calculated for each firm and the average of the eleven firms was used for the market average. The resulting estimates for the market instantaneous average and variance are $a = .0003021$ and $\sigma^2 = .0001631$, see table 4.1.

The instantaneous variance - covariance matrix Σ is obtained by multiplying a symmetric positive definite matrix by a scalar such that

$$1/N^2 \sum_{i=1}^{N} \sum_{j=1}^{N} \sigma_{ij} = \sigma_M^2 \tag{4.4}$$

Any matrix can be used as long as it is N X N, symmetric and positive definite. For example,

$$D = \begin{bmatrix} 1 & 1/2 & 1/3 \\ 1/2 & 2 & 0 \\ 1/3 & 0 & 3 \end{bmatrix}$$

TABLE 4.1

THE INSTANTANEOUS EXPECTED MARKET RETURN

AND MARKET VARIANCE ESTIMATION

Firm	ISD/year	Monthly Variance	Monthly Average Return	Monthly Instantaneous Return
Avon Products Inc.	.10	.0008333	.005758	.0061745
Burroughs Corp.	.21	.0036750	-.003270	-.0014325
Digital Equip. Corp.	.28	.0065333	.011939	.0152057
Eastman Kodak Co.	.22	.0040333	-.011733	-.0097164
Homestake Mtg. Co.	.25	.0052083	.010315	.0129192
IBM	.10	.0008333	.006825	.0072417
Northwest Airls. Inc.	.28	.0065333	.011097	.0143637
Pennzoil Co.	.24	.0048000	.011993	.0143930
Tandy Corp.	.46	.0176333	.043400	.0522167
Upjohn Co.	.15	.0018750	-.006114	-.0023640
Xerox Corp.	.15	.0018750	-.013062	-.0093120
Total		.0538331		.0996896
Average		.0048939		.0090627
Daily Average*		.0001631		.0003021

* The daily average was computed assuming 30 days per month.

is a symmetric positive definite matrix. If matrix D is multiplied by .001914, the result is

$$\begin{bmatrix} .0001914 & .0000957 & .0000638 \\ .0000957 & .0003829 & 0 \\ .0000638 & 0 & .0005743 \end{bmatrix} = \Sigma$$

and $\frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N}\sigma_{ij}$ = .0001631 which is the instantaneous market variance from table 4.1.

Matrix C is obtained from matrix Σ using the square root method (see Naylor [1966]). The square root method is a set of recursive formulas for the computation of the elements of C as follows:

$$c_{i1} = \frac{\sigma_{i1}}{\sigma_{11}^{1/2}} \quad 1 \leq i \leq N$$

$$c_{ii} = (\sigma_{ii} - \sum_{k=1}^{i-1} c_{ik}^2)^{1/2} \quad 1 < i \leq N$$

$$c_{ij} = \frac{(\sigma_{ij} - \sum_{k=1}^{j-1} c_{ik} c_{jk})}{c_{jj}} \qquad 1<j<i\leq N$$

$$c_{ij} = 0 \qquad j> i.$$

For example, the C matrix for the Σ given in the previous paragraph is

$$C = \begin{bmatrix} .013847 & 0 & 0 \\ .0069174 & .0183044 & 0 \\ .0046116 & -.0017428 & .023452 \end{bmatrix}$$

and $C\,C' = \Sigma$.

Each firm has two time periods each day; the time until the last transaction and the time at the end of the day when no trade takes place. S&W used a poisson distribution for the number of transactions per day, and, as such, the time between transactions followed an exponential distribution. Oldfield and Rogalski [1980] indicate that time between transactions follow a gamma distribution. An exponential distribution is a special case of a gamma and appears to be adequate for the purpose of this paper. The use of the exponential distribution provided data consistent with the problem described by S&W, therefore, this distribution was considered adequate. Using a poisson generating function both the number of transactions and time between them can be generated. Once the times have been generated the observed returns and actual returns can be computed.

The vector Z_n, in equation (4.2), is a vector of independent standard normal variates and is generated using the method presented in Naylor [1966, pg. 97]. When the matrix C is multiplied by Z_n, a vector of multivariate normal variates is obtained. As a result, the returns are multivariate normal and the returns of all firms must be generated at the same time. Each observed return, however, has a different time period. This problem is handled by generating prices for all firms at each firm's ending transaction time. For example, if there are three firms in the market, each firm will have four prices generated for each day corresponding to the last transaction time of each firm and the end of the day (3 + 1). In figure 4.1, the last transaction times for firms 1, 2, and 3 are .65, .85, and .95 respectively. Therefore, prices must be generated for each firm at time .65 of the day as well as at .85 and .95 (see figure 4.2). A price must also be generated at the end of the day. The prices generated at the four time periods are based on the multivariate distribution CZ_n. From the prices in figure 4.2, the observed return for firm 1 in day 2 would be:

$$R^s_{12}=(\ln P_{11-.95}-\ln P_{1.65})+(\ln P_{1.90-.70}-\ln P_{11-.95})$$

The actual return would be (See figure 4.3):

$$R_{12}=(\ln P_{11-.93}-\ln P_{11-.95})$$

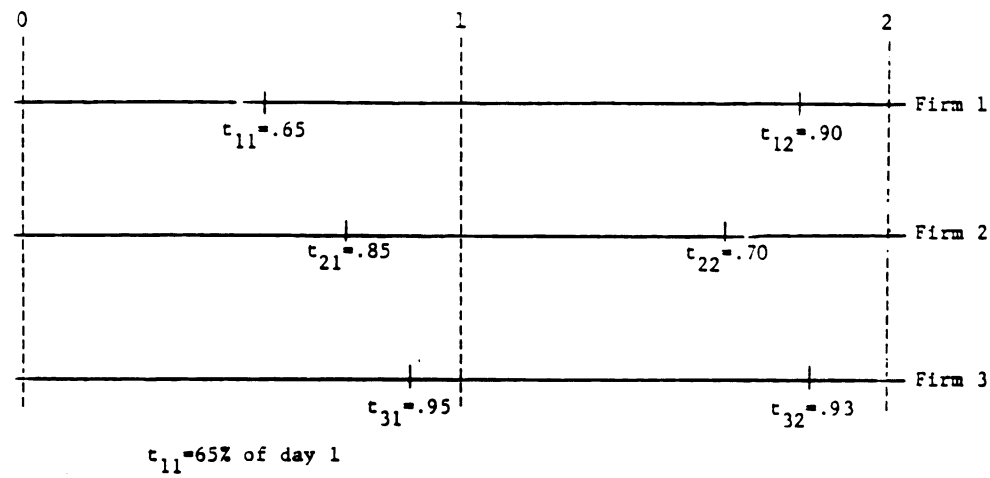


Fig. 4.1 Time Periods

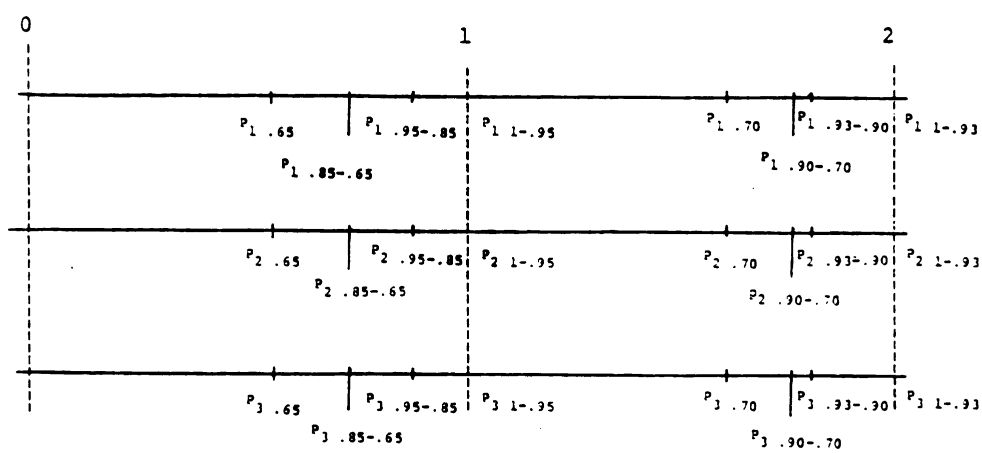
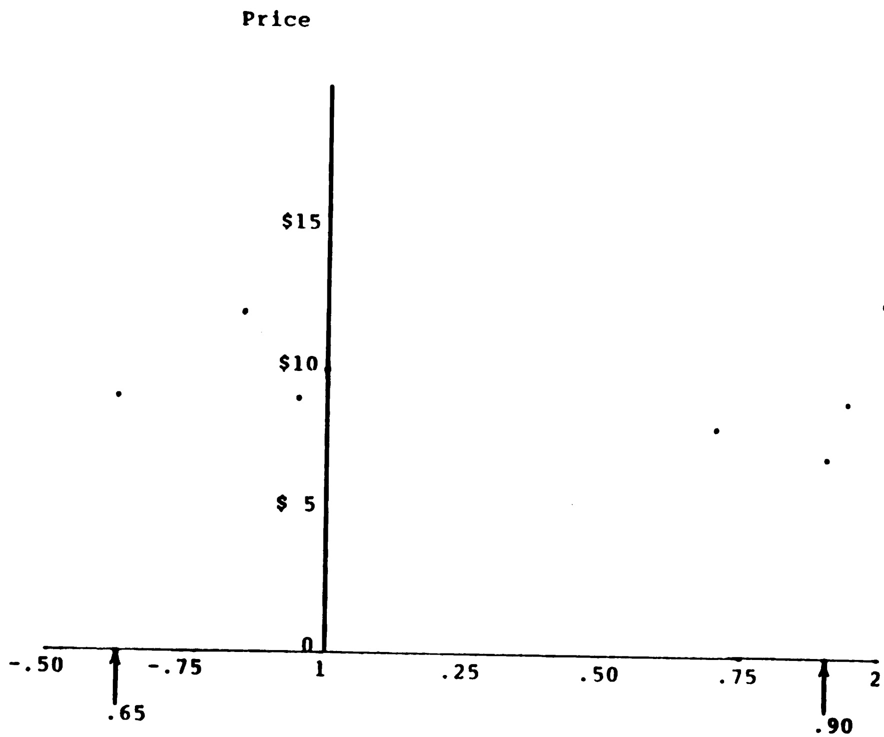


Fig. 4.2 Price Generation



$P_{1\ .65} = \$\ 9$
$P_{1\ 1-.95} = \$10$
$P_{1\ .90-.70} = \$\ 7$
$P_{1\ 1-.93} = \$12.50$

$R^{s}_{12} = (\ln P_{1\ 1-.95} - \ln P_{1\ .65}) + (\ln P_{1\ .90-.70} - \ln P_{1\ 1-.95})$

$R^{s}_{12} = (\ln 10 - \ln 9) + (\ln 7 - \ln 10) = -.25$

$R_{12} = (\ln P_{1\ 1-.93} - \ln P_{1\ 1-.95})$

$R_{12} = (\ln 12.50 - \ln 10) = .223$

Figure 4.3 The Four Prices Generated in Days 1 & 2

As a result, the actual returns are always generated over a time period of 1 and the observed returns over changing time periods. The observed time period in this case was 1.25.

The observed time periods depend on the average transactions chosen for each firm. The average number of transactions, a parameter for the poisson generator, affects the covariance between firms which in turn affects the variance and autocorrelation of the market. The time between transactions follows an exponential distribution and therefore the expected time between the last transaction and the end of the day is:

$$E(Sn) = 1/\lambda_n$$

where λ_n = the average number of transactions for firm n.

If firm n has a $\lambda_n = 2$ and firm m has a $\lambda_m = 100$, from equation (2.2) the observed covariance would be approximately 49% less than the actual covariance. Since the market variance is equal to the weighted sum of all the elements of the variance-covariance matrix (see equation 4.4), the observed market variance will be less than the actual market variance. S&W [1977] show that the first order autocorrelation will be:

$$\rho_M = 1/2 \left(\frac{\mathrm{Var}(R_M)}{\mathrm{Var}(R_M^s)} - 1 \right).$$

The greater the differences in the average number of transactions, the smaller the observed variance will be, and the smaller the observed variance, the greater the autocorrelation of the observed market will be. The average number of transactions will be based on a sample of 283 firms chosen from the New York Stock Exchange.[7] The number of transactions per day for the first three months of 1980 has been obtained (Woodruff [1981]) for each firm. A summary of the average transactions for those firms is listed in table 4.2.

The average number of transactions λ is used in a poisson generating function presented in Naylor [1966] as follows:

$$\sum_{i=0}^{x} t_i \leq \lambda < \sum_{i=0}^{x+1} t_i$$

where x = the Poisson variate

λ = the average number of transactions

t_i = $-\log r_i$, and r_i = a random variate with a uniform distribution

The time at which a transaction takes place in a day is the sum of the time between all previous transactions in the day.

---

[7]The 283 firms were chosen randomly by Woodruff [1981], but also conformed to the following two criteria:

1) Listed on the New York Stock Exchange
2) Listed in Value Line

TABLE 4.2

AVERAGE NUMBER OF TRANSACTIONS FOR 283 FIRMS

Class Midpoint	Percentage	Cumulative Percentage
2.5	4.3	4.3
7.5	12.6	16.9
12.5	12.6	29.6
17.5	11.0	40.5
22.5	7.6	48.2
27.5	6.0	54.2
32.5	6.0	60.1
37.5	5.0	65.1
42.5	4.7	69.8
47.5	3.3	73.1
52.5	2.3	75.4
57.5	3.0	78.4
62.5	3.3	81.7
67.5	2.3	84.1
72.5	2.0	86.0
77.5	1.7	87.7
82.5	0.3	88.0
87.5	1.3	89.4
92.5	0.7	90.0
97.5	1.0	91.0
102.5	0.7	91.7
107.5	1.0	92.7
112.5	0.3	93.0
117.5	0.7	93.7
122.5	0.3	94.0
127.5	0.7	94.7
132.5	0.0	94.7
137.5	1.3	96.0
142.5	1.0	97.0
147.5	0.3	97.3
152.5	0.0	97.3
157.5	0.0	97.3
162.5	0.0	97.3
167.5	0.0	97.3
172.5	2.7	100.0

minimum average transaction = 2.52/day
maximum average transaction = 286.08/day
mean = 41.4108
standard deviation = 44.0956

The goal in choosing the number of firms is to use the fewest that will adequately represent the market. The fewest number is desired because:

1) The larger the number of securities the more difficult it becomes to estimate a variance-covariance matrix. For example, with 20 firms there are 400 covariances and with 40 firms, 1600.
2) As the number of firms increases, the cost of the simulation increases dramatically. The cost corresponds to the number of calls to the random number generator. The number of calls equals the number of firms squared times the number of days to be simulated.

Most of the diversification effect is realized in portfolios of only twelve firms (Francis and Archer [1979], Tinic and West [1979]). The variance of portfolios of twenty or more is attributed mainly to the pairwise covariances (Fama [1976]). Therefore, the selected number of firms in this study is twenty since the pairwise covariances contribute more than the variances of the individual firms to the variance of the portfolio, and a portfolio of twenty will show substantial diversification. The number of firms may also affect the autocorrelation of the simulated market.

The autocorrelation of the observed market is affected by two factors, the average transaction time of the firms and their pairwise covariances. If a firm is added that has a small average number of transactions and its pairwise covariances with other firms in the portfolio are large, the autocorrelation of the portfolio will be increased. Figures 4.4 and 4.5 show the affects of adding more firms on the correlation of the portfolio. Figure 4.4 shows the results of adding approximately the same percentage of average transactions and approximately the same size covariances. After adding the first 5 or 6 firms, the changes in autocorrelation by adding more firms is slight. Figure 4.5 shows the results of adding firms that have a higher percentage of less than the average number of transactions than the firms in figure 4.4. Again, the largest increases to the autocorrelation occur with the first few firms and subsequent changes are smaller. Figure 4.5 does have a higher increase in autocorrelation than does figure 4.4, however, it is apparent that it would take a very large number of firms to approach the autocorrelation of the portfolio made up of the NYSE (the market), and also that the greatest change occurs with a relatively small number of firms. Therefore, adding more than twenty firms would not enhance the study unless a substantial number were added. The two figures (4.4 and 4.5) appear to have a tiered effect because the transaction times were always added in the same order. The first twenty were added from lowest number of transactions to highest, and the second twenty went from lowest to highest. If the order had been mixed up a more

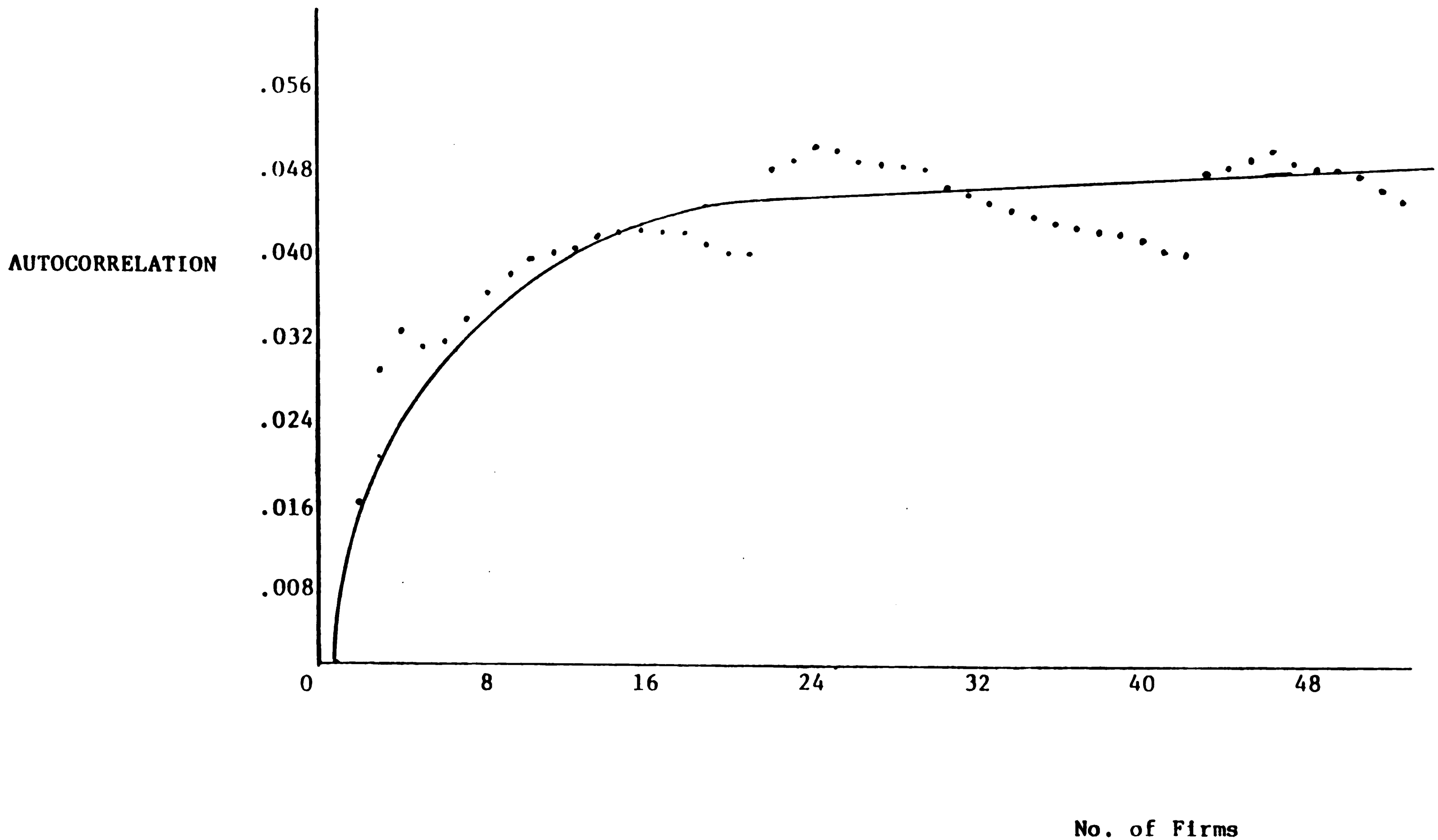


Figure 4.4 Autocorrelation of the Market as Firms are Added

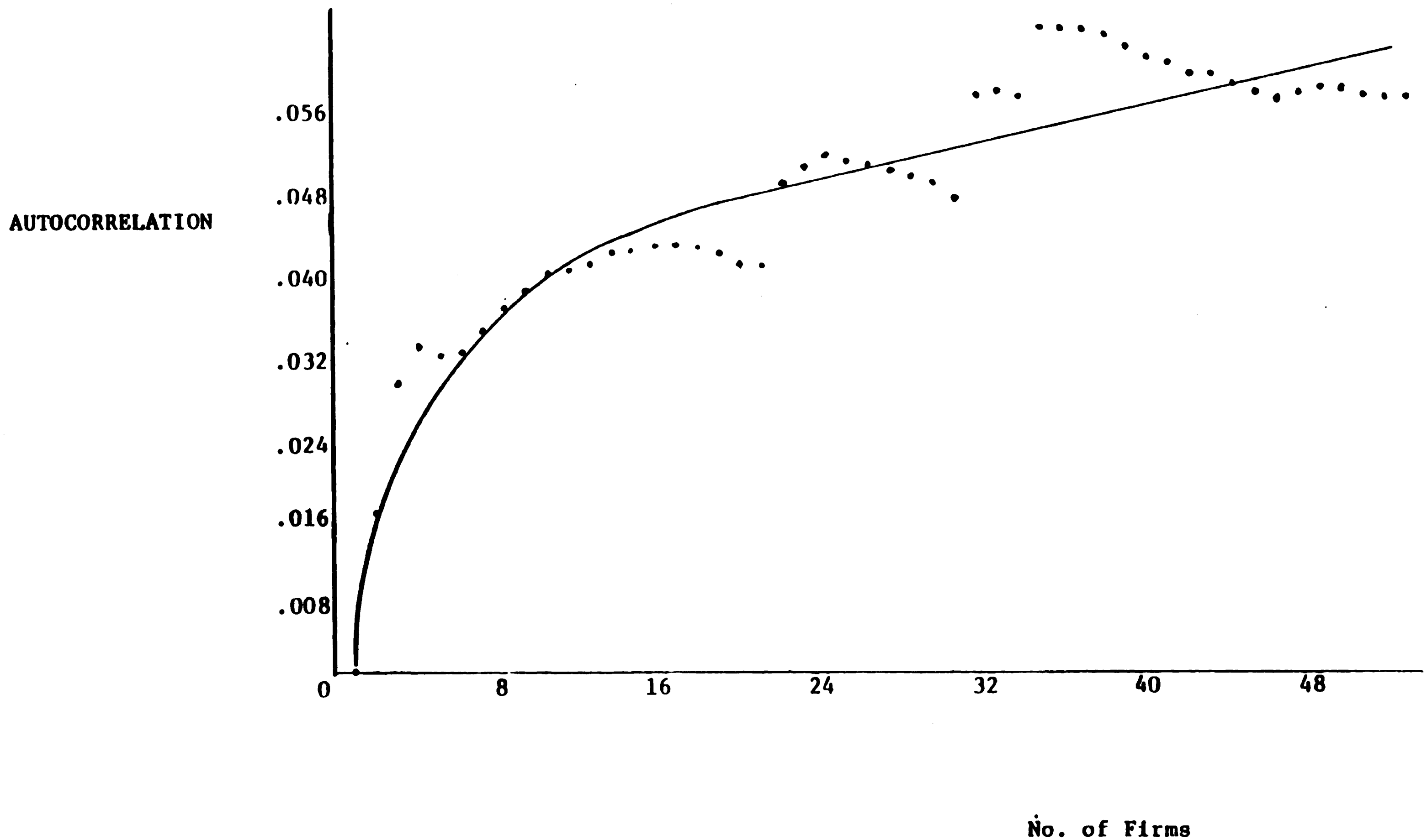


Figure 4.5 Autocorrelation of the Market Adding a Larger Proportion of Less Actively Traded Securities

random appearance would have occurred, and a relationship such as the solid line in figures 4.4 and 4.5 would be observed. One note of interest is that S&W found that the equally weighted market had a larger autocorrelation than the value weighted market. This is due to the fact that the equally weighted market has a higher proportion of firms with less than the average number of transactions. Figures 4.4 and 4.5 have the same relationship.

All that remains to be estimated in order for the simulation to be run is a twenty by twenty symmetric positive definite matrix. The matrix to be used is the matrix used by Jobson and Korkie [1980]. Jobson and Korkie used a twenty by twenty variance-covariance matrix in their study which they calculated from twenty firms randomly chosen. This matrix was used to eliminate the appearance of being able to manipulate the results by the matrix chosen. The matrix used was chosen independently from this study.

Verification of the Simulation

The above matrix was used with transaction times assigned to each firm and the simulation was run to see if the results would conform to what Scholes and Williams theorize (see table 4.3). Scholes and Williams indicate that the autocovariance of the firms should approximate zero. The average autocovariance of the twenty firms was .0047403. Measured variances for daily returns on large portfolios should typically understate the true variances. The variance of the observed portfolio was .0001376 and the true variance was .0001631. The theoretical value for the autocorrelation of the market (the portfolio of the twenty firms) is .102 the result of the simulation was .116. The theoretical variance is .0001361, the result was .0001376. The OLS beta of firms traded less than the market average will theoretically be understated. The average number of trades was 39. Ten firms had trades less than 39, and nine of the ten had understated betas. Ten of the ten firms with transaction times greater than 39 overstated beta. The results of this test indicate that the simulator gives results consistent with the theory.

TABLE 4.3

CONFORMANCE OF SIMULATED RESULTS TO

SCHOLES AND WILLIAMS' THEORY

Criteria	Theoretical Results	Simulated Results
autocovariance of the firm	approximately 0	.0047403
variance of the market	.0001361	.0001376
autocorrelation of the market	.102	.116
# of firms that understate Beta	10 out of 20	9 out of 20
# of firms that overstate Beta	10 out of 20	11 out of 20

V. SIMULATION RESULTS

The simulation described in chapter IV was run to establish the affects of nonsynchronous time periods on the bias in beta, the prediction ability of the market model using OLS and S&W estimates, and the results of efficient market tests. Different samples were obtained, using the same parameter estimates for the simulation, simply by changing the seed to the random number generator. The bias was examined using 10 different samples. Twelve samples were used to test the prediction ability of the market model, and forty-eight different samples were used for the efficient market tests.

Beta Bias

Scholes and Williams [1977] state that the beta estimated using OLS will be underestimated for firms with less than the average number of transactions, and over-estimated for firms with the average number of transactions. For firms with more transactions than the average, they were not certain if the beta would be over or underestimated. The simulator was run to see if these bias estimates could be substantiated. 1600 days were used to estimate the betas in eight of the samples. The average number of transactions for the twenty firms was held constant, and the random number seed was changed in the first five samples. A different set of random numbers was generated every time a different seed was used. The last three samples all had the same random number seed and the average number of transactions for each firm was changed.

The bias was calculated by subtracting the OLS estimate on the actual returns (returns without error) from the OLS estimate on the observed returns. A "bias" was calculated in the same manner using the S&W estimate.

The results of these 8 samples showed that the bias conformed to the S&W results no matter what set of transaction times was chosen. Therefore the remaining two samples used only one group of transaction times. These transaction times were chosen in proportions equivalent to table 4.2 in an attempt to represent the market. These times will be used in the remainder of this study.

Betas were computed for the above two samples at sample sizes starting at 100 days and ending with 5200 days. 5200 days corresponds roughly to 20.6 years. The betas and their corresponding biases are listed for one of the samples in table 5.1 for 100, 400, 1000, and 5200 days. The bias was calculated by subtracting the estimated beta from the true beta. The biases in table 5.1 are comparable to those reported by Scholes and Williams (tables 2.1, 2.2, and 2.3). In both cases, the less active securities have larger biases than the active securities. The magnitudes of the biases are also approximately the same. Figures 5.1 through 5.8 are plots of the biases in table 5.1. The average squared bias for the S&W betas is less than the average squared bias

TABLE 5.1

BETA ESTIMATES FROM SIMULATED RETURNS AND THE CORRESPONDING BIASES

Trans Time	True Beta	100 days Beta OLS	100 days Beta S&W	100 days Beta* Act	400 days Beta OLS	400 days Beta S&W	400 days Beta Act	1000 days Beta OLS	1000 days Beta S&W	1000 days Beta Act	5200 days Beta OLS	5200 days Beta S&W	5200 days Beta Act
2.5	.822	.720	.863	.868	.662	.786	.775	.706	.837	.855	.665	.807	.829
		(-.102)	(.041)	(.046)	(-.160)	(-.036)	(-.047)	(-.116)	(.015)	(.033)	(-.167)	(-.021)	(.007)
5.5	.644	.760	.900	.778	.582	.577	.668	.578	.625	.670	.589	.648	.652
		(.116)	(.256)	(.134)	(-.062)	(-.067)	(.024)	(-.066)	(-.019)	(.026)	(-.055)	(.004)	(.008)
6.0	1.275	.751	1.163	.937	1.089	1.190	1.140	1.237	1.276	1.253	1.218	1.301	1.280
		(-.524)	(-.112)	(-.338)	(-.186)	(-.085)	(-.135)	(-.038)	(.001)	(-.022)	(-.057)	(.026)	(.005)
7.5	1.895	1.417	1.402	1.741	1.552	1.626	1.763	1.752	1.788	1.808	1.849	1.878	1.894
		(-.478)	(-.493)	(-.154)	(-.343)	(-.269)	(-.132)	(-.143)	(-.107)	(-.087)	(-.046)	(-.017)	(-.001)
12.0	1.061	1.103	.857	.967	1.192	.984	1.151	1.112	1.162	1.091	1.111	1.098	1.096
		(.042)	(-.204)	(-.094)	(.131)	(-.077)	(.09)	(.051)	(.101)	(.030)	(.050)	(.037)	(.035)
13.0	.852	.522	.444	.472	.670	.796	.700	.760	.794	.768	.834	.866	.824
		(-.330)	(-.408)	(-.380)	(-.182)	(-.056)	(-.152)	(-.092)	(-.058)	(-.084)	(-.018)	(.014)	(-.028)
16.0	1.012	.939	1.208	.877	.986	.944	.950	1.031	1.020	1.011	1.028	1.038	1.016
		(-.073)	(.196)	(-.135)	(-.027)	(-.068)	(-.062)	(.019)	(.008)	(-.001)	(.017)	(.026)	(.004)
17.0	.803	.841	1.030	.949	.796	.850	.836	.782	.781	.779	.794	.799	.780
		(.038)	(.227)	(.146)	(-.007)	(.047)	(.033)	(-.021)	(-.022)	(-.024)	(0.009)	(-.004)	(-.023)
22.5	.674	.670	.687	.664	.672	.755	.659	.642	.664	.640	.672	.695	.670
		(-.004)	(.013)	(-.010)	(-.002)	(.081)	(-.015)	(-.032)	(-.010)	(-.034)	(-.002)	(.021)	(-.004)
27.5	.791	1.006	1.039	1.008	.914	.942	.901	.848	.874	.824	.796	.768	.782
		(.215)	(.248)	(.217)	(.123)	(.151)	(.110)	(.053)	(.083)	(.033)	(.005)	(-.023)	(-.009)
32.5	.852	.822	.520	.806	1.005	.841	.940	.912	.971	.889	.883	.879	.862
		(-.030)	(-.332)	(-.046)	(.153)	(-.011)	(.088)	(.060)	(.119)	(.037)	(.031)	(.027)	(.010)
37.5	1.073	1.074	1.050	1.009	1.186	1.112	1.128	1.089	1.031	1.064	1.096	1.068	1.063
		(.001)	(-.023)	(-.064)	(.113)	(.039)	(.055)	(.008)	(-.042)	(-.009)	(.023)	(-.005)	(-.010)
42.5	.540	.685	.744	.680	.637	.584	.629	.562	.527	.562	.566	.527	.559
		(.145)	(.204)	(.140)	(.097)	(.044)	(.089)	(.022)	(-.013)	(.022)	(.026)	(-.013)	(.019)
50.0	1.061	1.088	1.095	1.067	1.061	1.084	1.032	1.128	1.132	1.109	1.126	1.133	1.096
		(.027)	(.034)	(.006)	(0)	(.023)	(-.029)	(.067)	(.071)	(.048)	(.065)	(.072)	(.035)
57.5	.773	.864	.910	.815	.782	.815	.796	.800	.773	.805	.796	.779	.787
		(.091)	(.137)	(.042)	(.009)	(.042)	(.023)	(.027)	(0)	(.032)	(.023)	(.006)	(.014)
62.5	.895	1.150	1.069	1.137	.956	1.037	.945	.933	.946	.912	.911	.867	.886
		(.255)	(.174)	(.242)	(.061)	(.142)	(.050)	(.038)	(.051)	(.017)	(.016)	(-.028)	(-.009)
70.0	1.514	1.798	1.365	1.592	1.602	1.363	1.504	1.553	1.422	1.500	1.528	1.453	1.489
		(.284)	(-.149)	(.078)	(.088)	(-.151)	(-.010)	(.039)	(-.092)	(-.014)	(.014)	(-.061)	(-.025)
82.5	1.508	1.858	2.036	1.776	1.762	1.741	1.663	1.614	1.428	1.550	1.553	1.449	1.487
		(.350)	(.528)	(.268)	(.254)	(.233)	(.155)	(.106)	(-.080)	(.042	(.045)	(-.059)	(-.021)
122.0	.901	.867	.765	.845	.870	.857	.848	.875	.851	.857	.915	.878	.889
		(-.034)	(-.136)	(-.056)	(-.031)	(-.044)	(-.053)	(-.026)	(-.050)	(-.044)	(.014)	(-.023)	(-.012)
172.0	1.061	1.065	.839	1.008	1.021	.963	.972	1.078	1.042	1.053	1.078	1.070	1.059
		(.004)	(-.222)	(-.053)	(-.040)	(-.098)	(-.089)	(.017)	(-.019)	(-.008)	(.017)	(.009)	(-.002)
Average bias^2		.050	.063	.028	.017	.012	.007	.0039	.0036	.0015	.0025	.0010	.0003

* "act" is the OLS Beta estimate calculated from returns without error (R_{nt} as opposed to R^{*}_{nt}).

The bias is the number in parentheses.

TABLE 5.2

BETA ESTIMATES FROM SIMULATED RETURNS AND THE CORRESPONDING BIASES FOR 3 PORTFOLIOS FORMED BY TRANSACTION TIMES

Trans time	True beta	100 days			400 days			1000 days			5200 days		
		Beta OLS	Beta S&W	Beta Act	Beta OLS	Beta S&W	Beta Act	Beta OLS	Beta S&W	Beta Act	Beta OLS	Beta S&W	Beta Act
Less than aver.	1.092	.695 (-.397)	.779 (-.313)	.799 (-.293)	.759 (-.333)	.829 (-.263)	.841 (-.251)	.839 (-.253)	.887 (-.205)	.892 (-.200)	.858 (-.234)	.916 (-.176)	.913 (-.179)
Aver.	.851	.891 (.040)	.922 (.071)	.883 (.032)	.907 (.056)	.889 (.038)	.884 (.033)	.874 (.023)	.875 (.024)	.860 (.009)	.870 (.019)	.863 (.012)	.853 (.002)
More than aver.	1.109	1.267 (.158)	1.164 (.055)	1.196 (.087)	1.166 (.057)	1.130 (.021)	1.121 (.012)	1.142 (.033)	1.077 (-.032)	1.112 (.003)	1.130 (.021)	1.083 (-.260)	1.100 (-.009)
Average bias[2]		.061	.035	.031	.039	.024	.021	.022	.015	.013	.019	.011	.011

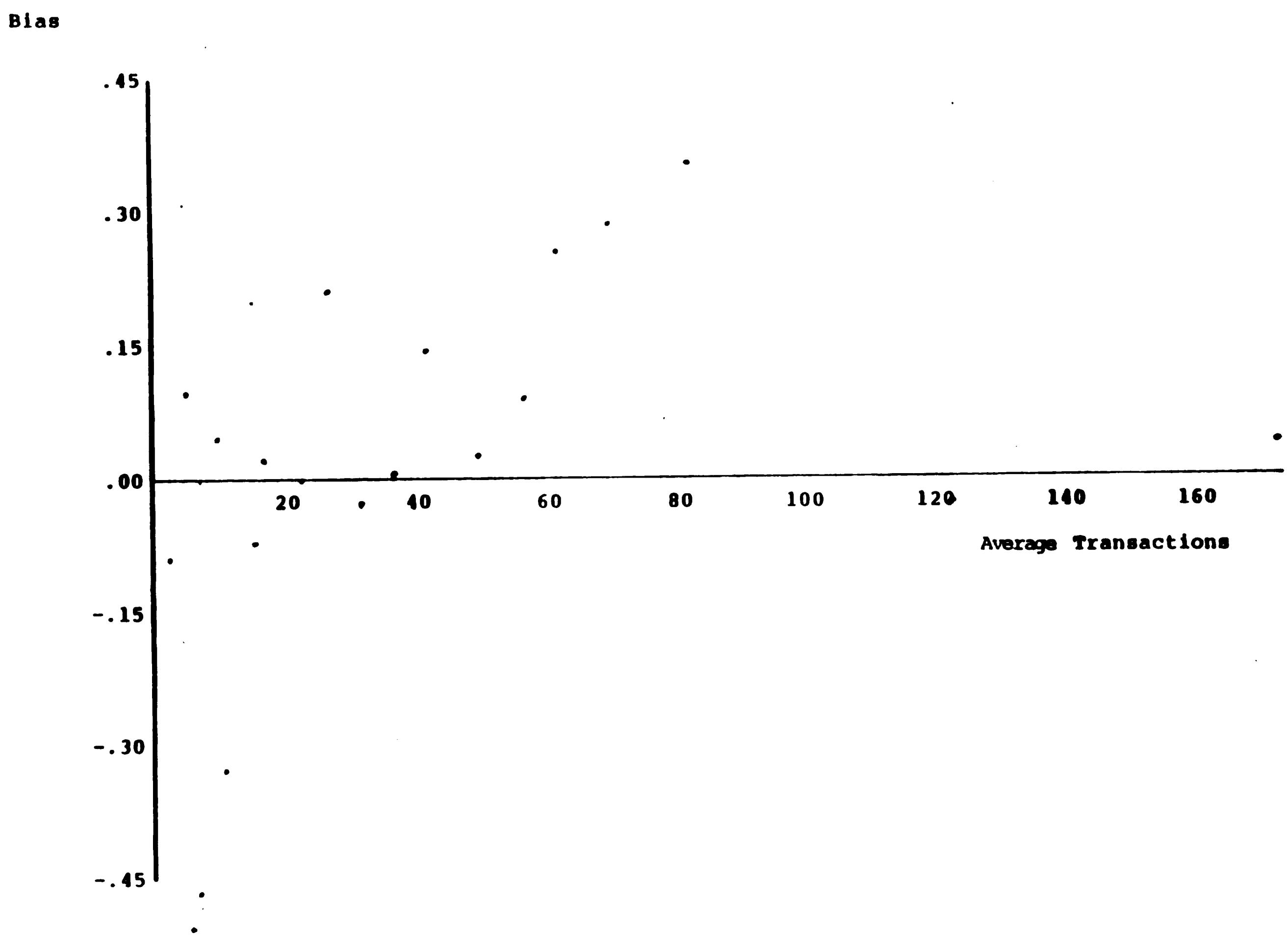


Figure 5.1 Bias in Beta Versus Average Transactions Per Day Estimation:
OLS and 100 days

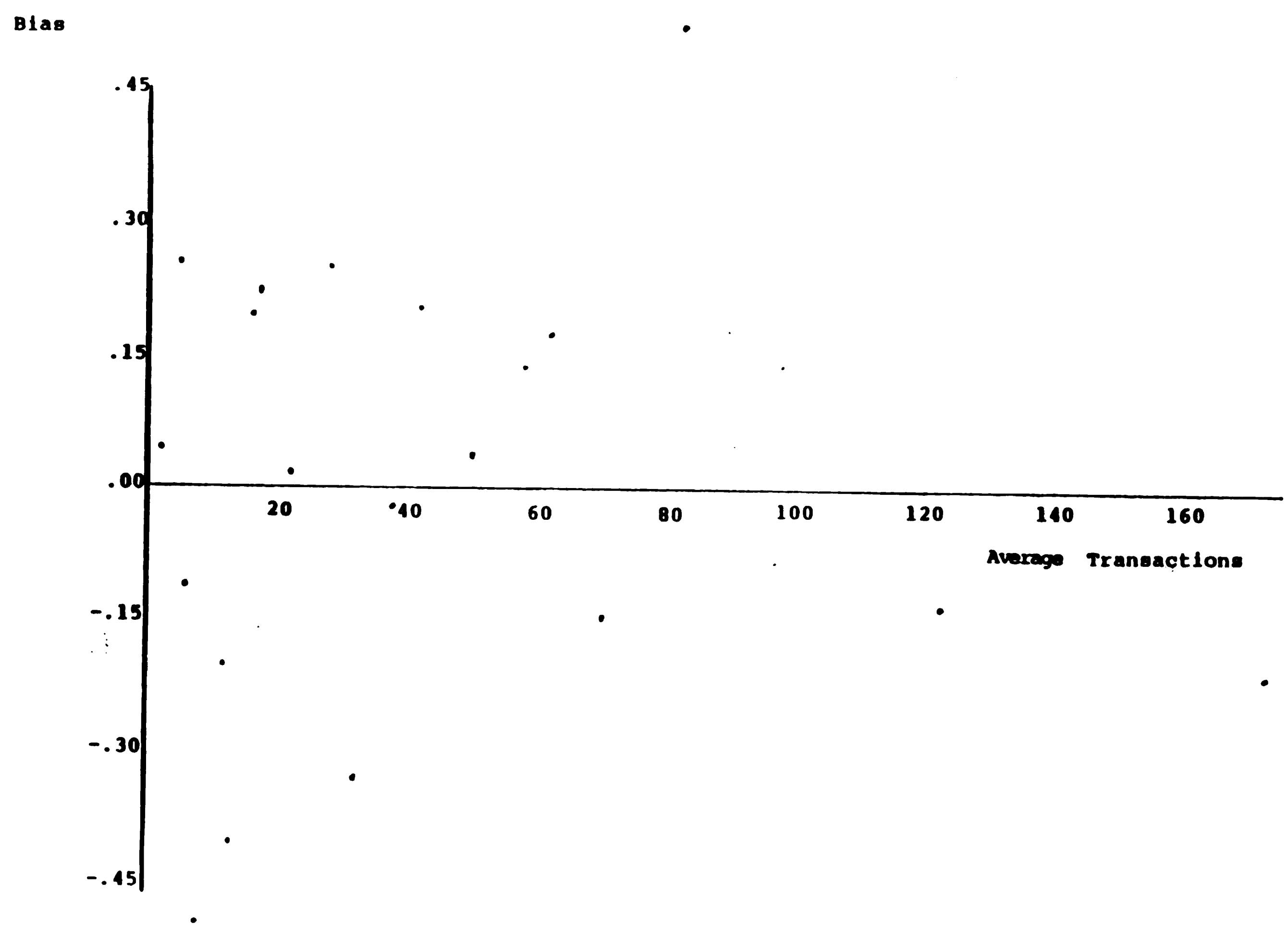


Figure 5.2 Bias in Beta Versus Average Transactions Per Day Estimation: S & W and 100 days

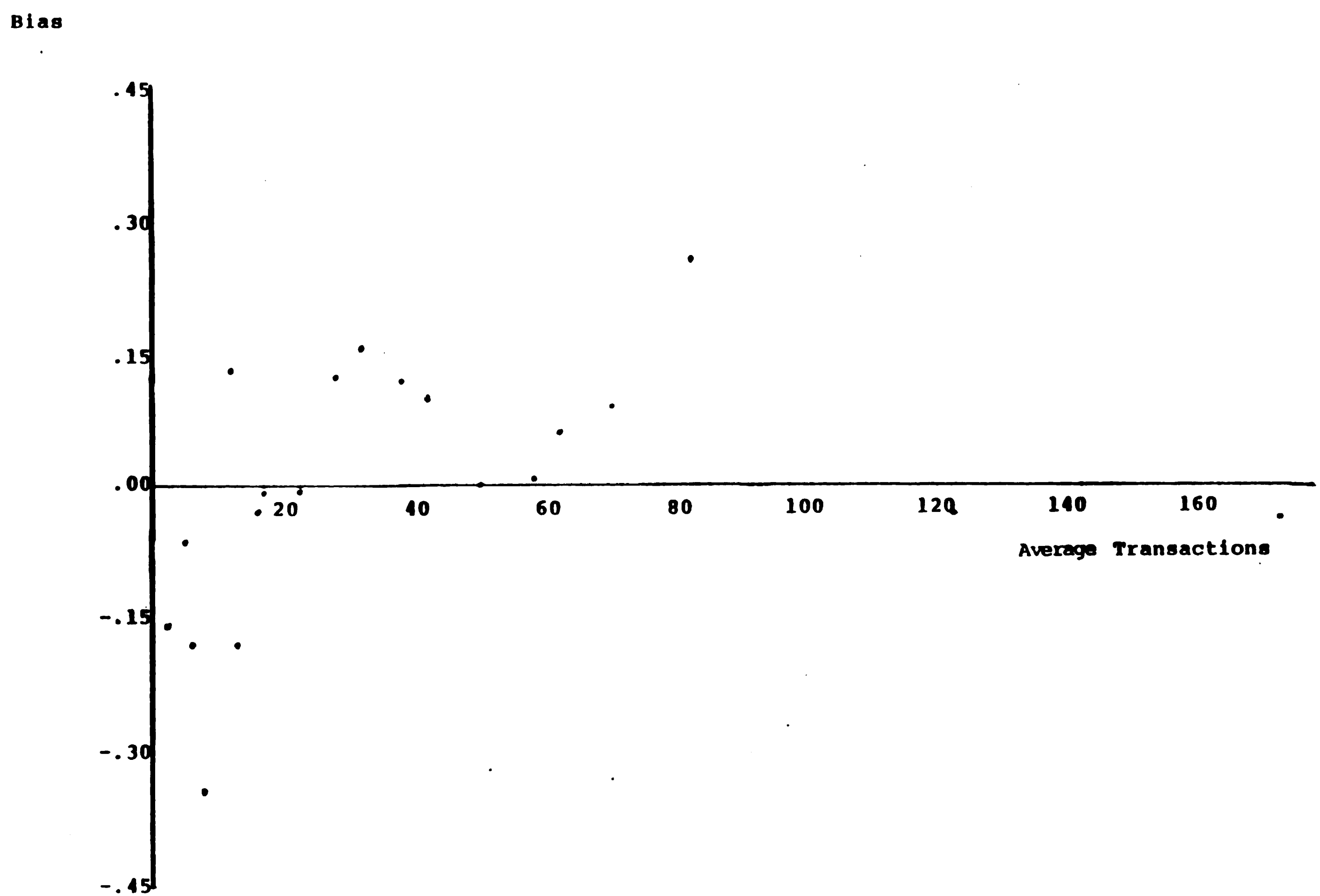


Figure 5.3 Bias in Beta Versus Average Transactions Per Day
Estimation: OLS and 400 Days

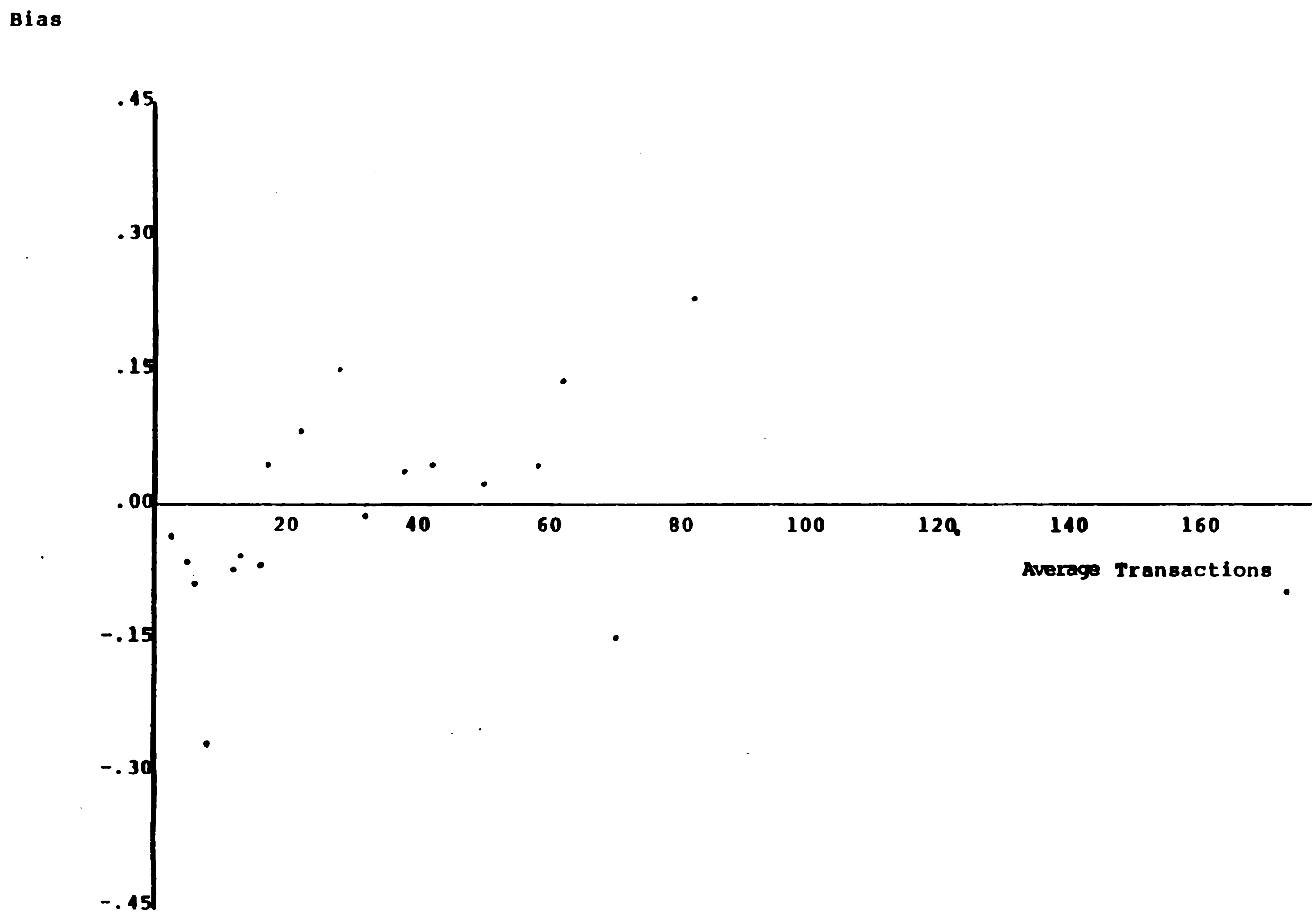


Figure 5.4 Bias in Beta Versus Average Transactions Per Day Estimation: S & W and 400 days

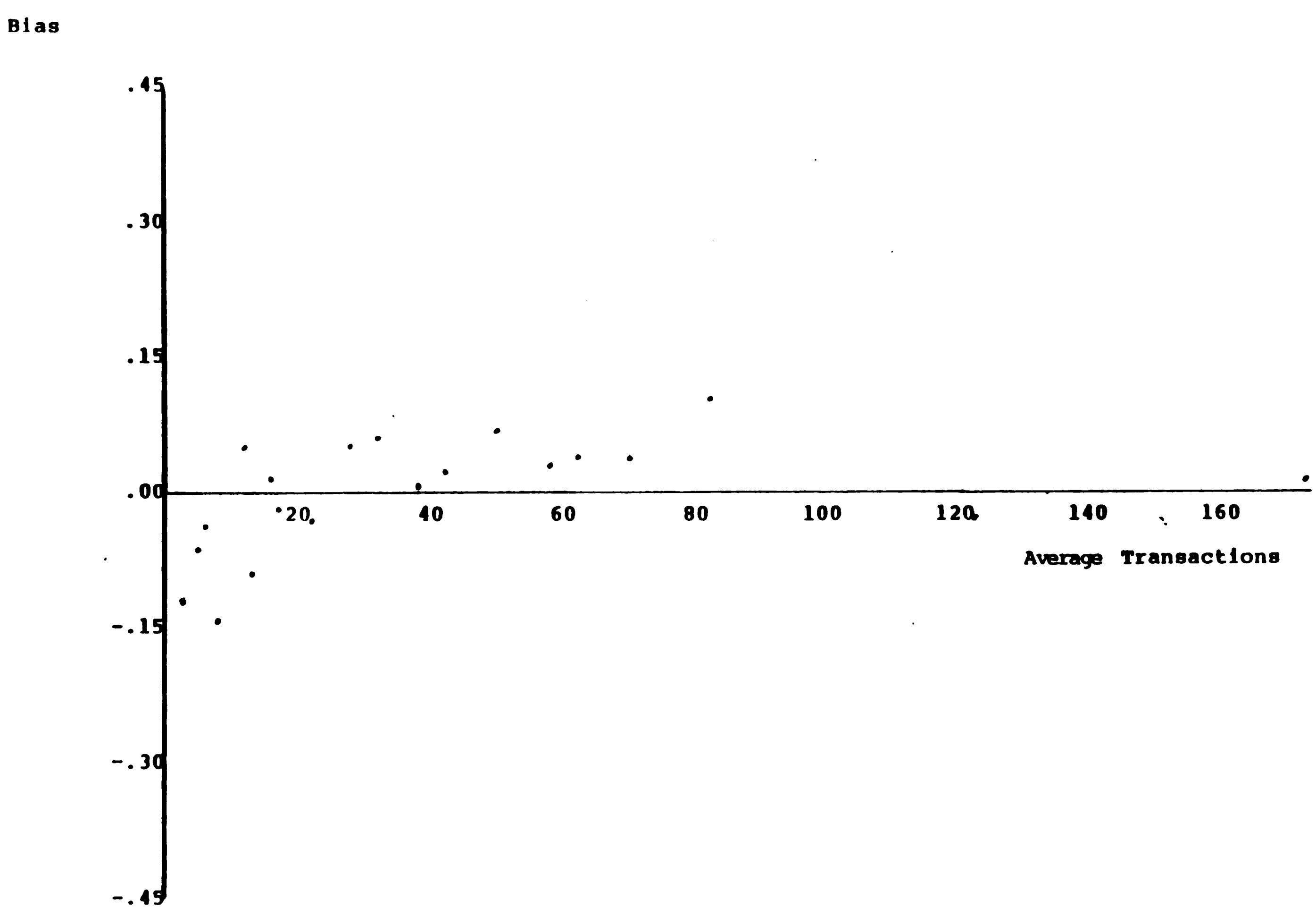


Figure 5.5 Bias in Beta Versus Average Transactions Per Day Estimation: OLS and 1000 Days

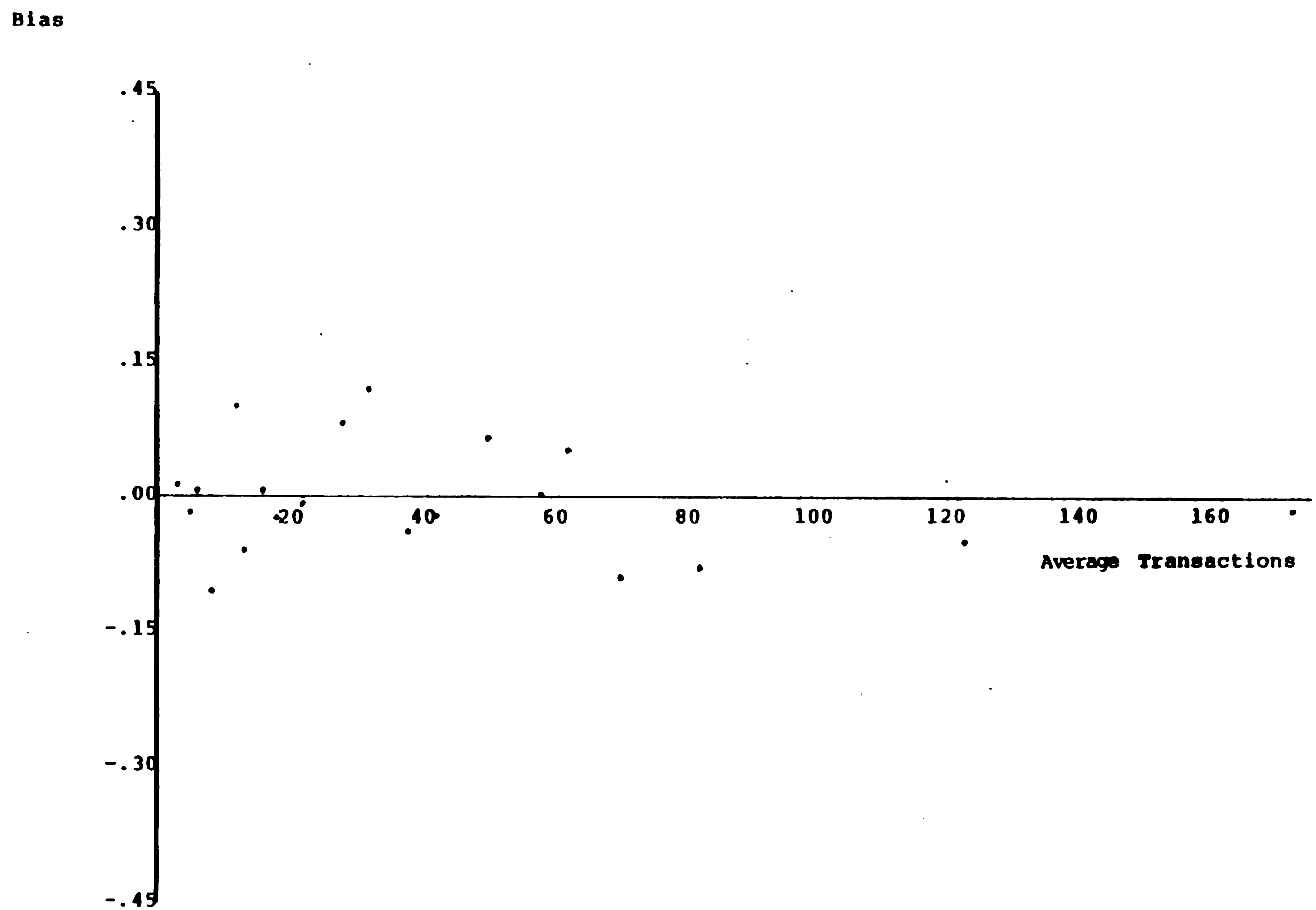


Figure 5.6 Bias in Beta Versus Average Transactions Per Day Estimation: S & W and 1000 Days

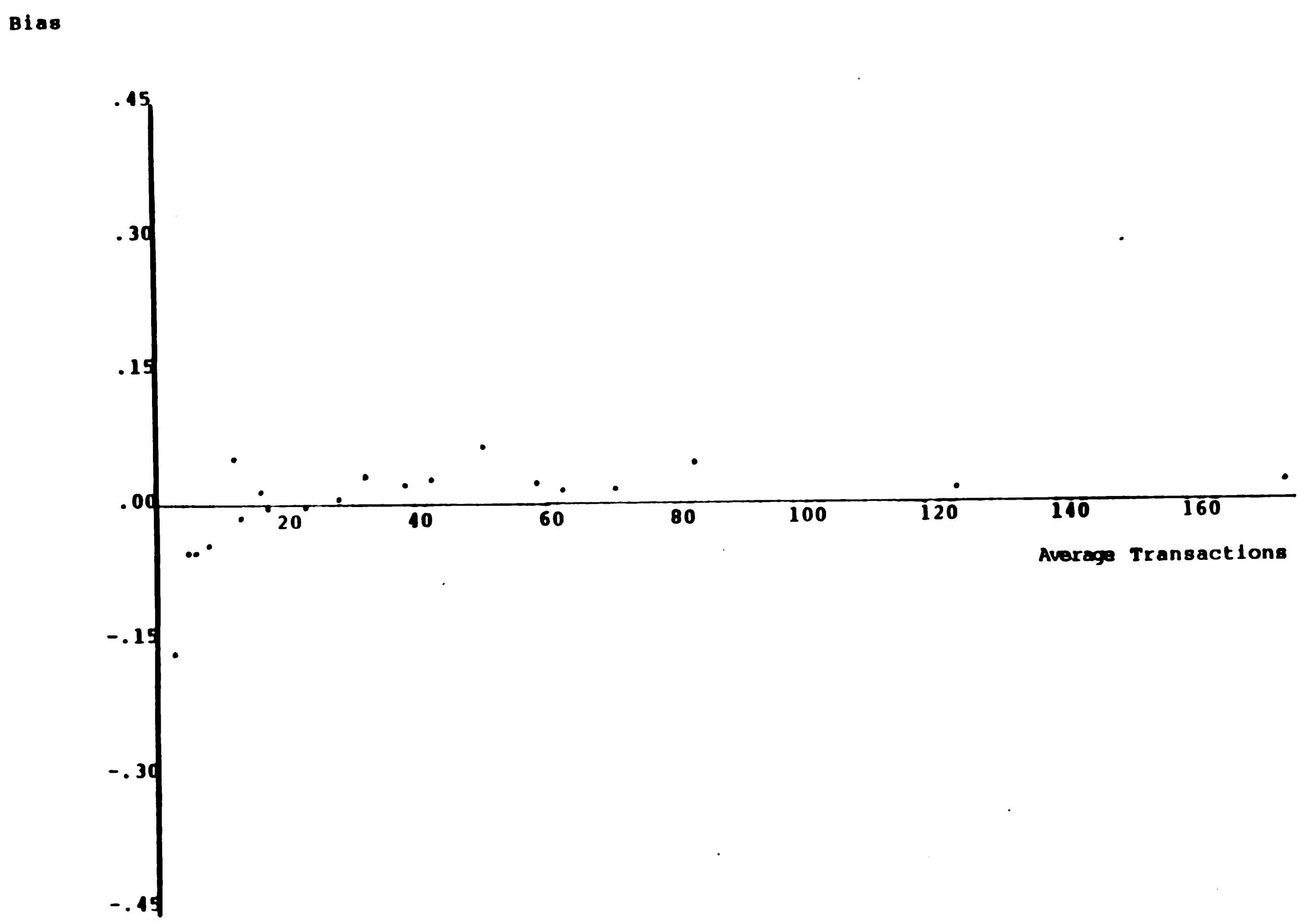


Figure 5.7 Bias in Beta Versus Average Transactions Per Day
Estimation: OLS and 5200 Days

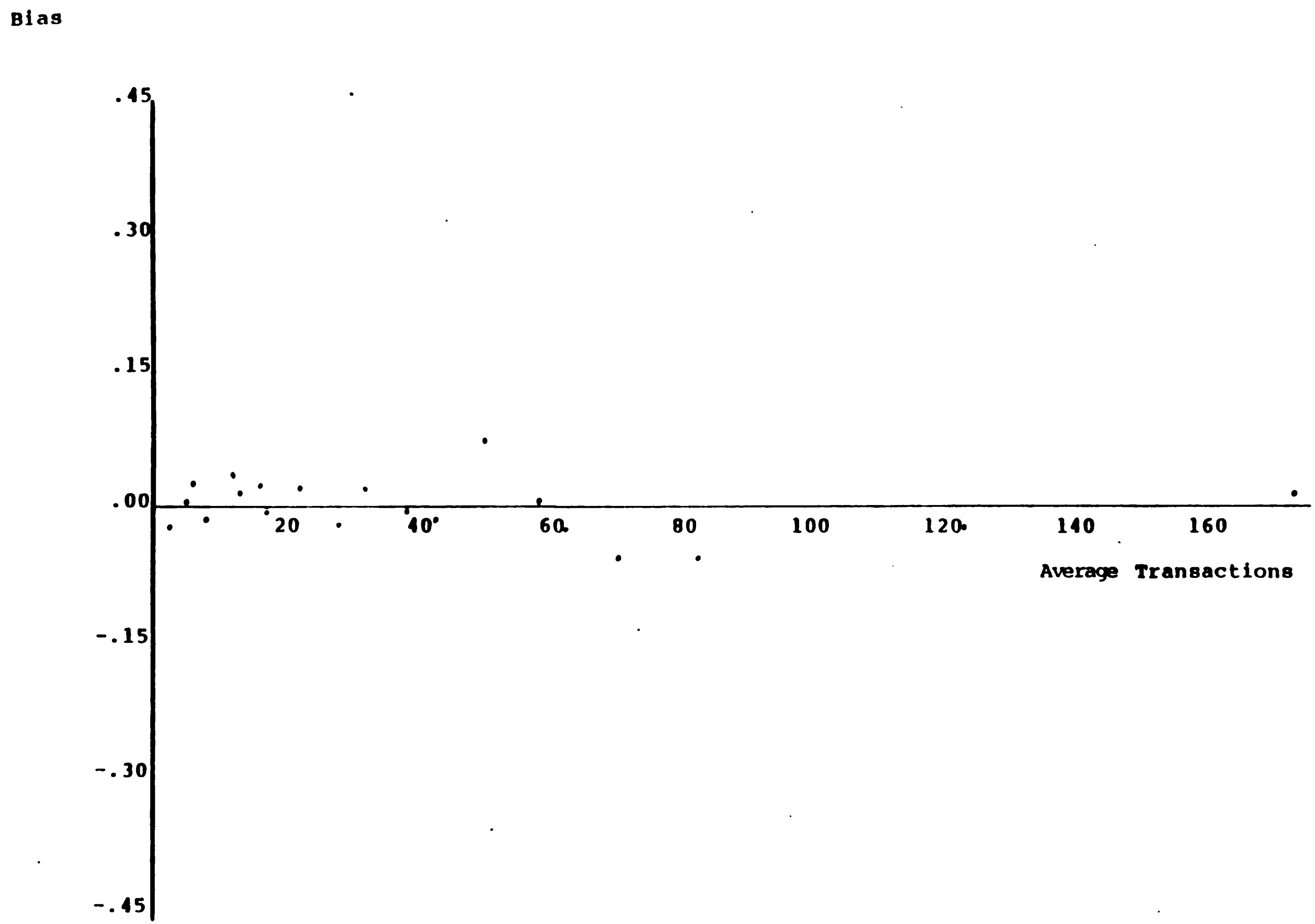


Figure 5.8 Bias in Beta Versus Average Transactions Per Day
Estimation: S & W and 5200 Days

for the OLS betas in all cases except at 100 days. Therefore, the S&W beta bias should appear closer to zero, and in fact they do. A definite pattern can be discerned in the plots of the OLS bias whereas a pattern can not be found in the S&W bias. Therefore, the bias has been eliminated using the S&W procedure.

Portfolios were made by grouping firms together by transaction times. Three portfolios were made. The first portfolio contained those firms with less than the average number of transactions. The second portfolio contained firms with close to the average number of transactions and the last portfolio contained firms with more than the average number of transactions. Betas and the respective biases were computed for these portfolios and the results are reported in Table 5.2. The results are consistent with the S&W conclusions and are similar to the individual results.

Prediction Results

The prediction test is designed to substantiate the results in Chapter III. In Chapter III it was shown that for infinitely large samples, the mean squared prediction error using the OLS estimate of beta will be larger than the mean squared prediction error (MSPE) using the S&W estimate of beta. Allen and Hagerman [1980] and Dutta [1975] use mean squared prediction errors as an indication of predictability, which is the method used in the subsequent test.

OLS estimates and S&W estimates were computed for alpha and beta starting at sample sizes of 100 and ending at sample sizes of 5200 days. A sample size of 5200 days corresponds roughly to 20.6 years. These estimates were then used to predict the next 100 days, using the market model (equation (1.2)), for each of the twenty firms. One hundred days were used for the prediction period because in subsequent tests a 100 day period is used. The subsequent tests are the efficient market tests fashioned after the Brown and Warner [1980] paper. A 100 day period is necessary to allow for enough observations to compute a variance for the residuals. Therefore, to be consistent, a 100 day period was used. The MSPE was then computed over the 100 day prediction period using the two estimates. The $MSPE_{OLS}$ was compared to the $MSPE_{SW}$ and the smaller MSPE was considered the better predictor. Twelve samples of twenty firms each were run by changing either the random generator seed for the estimation period or the prediction period. The results of these runs are shown in figures 5.9 and 5.10. The same runs were done forming portfolios out of the twenty firms. The portfolios were based on average transaction times. Each run consisted of three portfolios corresponding to less than average, average, and greater than average firms. The results of these runs are shown in figure 5.11.

The vertical lines above the zero horizontal line, in figures 5.9, 5.10, and 5.11, represent the number of times out of 12 that the $MSPE_{SW}$ was less than the $MSPE_{OLS}$ for each of the twenty firms or three portfolios. The vertical lines below the horizontal represent the number of times out of 12, the $MSPE_{OLS}$ was less than the $MSPE_{SW}$. As was shown earlier, the difference between these two measures is quite small and in some cases the two were virtually the same. Therefore, in some instances, the vertical line is shorter. For example figure 5.10, 5200 days, firm 16, the MSPE's for firm 16 were the same for 6 samples out of the 12 samples. The firms are listed from the lowest number of transactions to the highest, and the transaction times correspond to those in table 5.1.

It is not apparent from figures 5.9, 5.10, and 5.11 that either the S&W estimators or the OLS estimators result, on average, in better predictions of returns. However, there does appear to be a trend. The S&W estimators appear to result in better predictions for firms with a low number of transactions and possibly for those with a higher number of transactions. This is far more visible in figure 5.11 which shows the portfolio results. The portfolio results are better because at the individual firm level the major portion of the MSPE is due to the error term

(see equation 3.11 and 3.12). As portfolios are formed this error term is reduced. Therefore, based on the simulation, when firms are grouped together such that

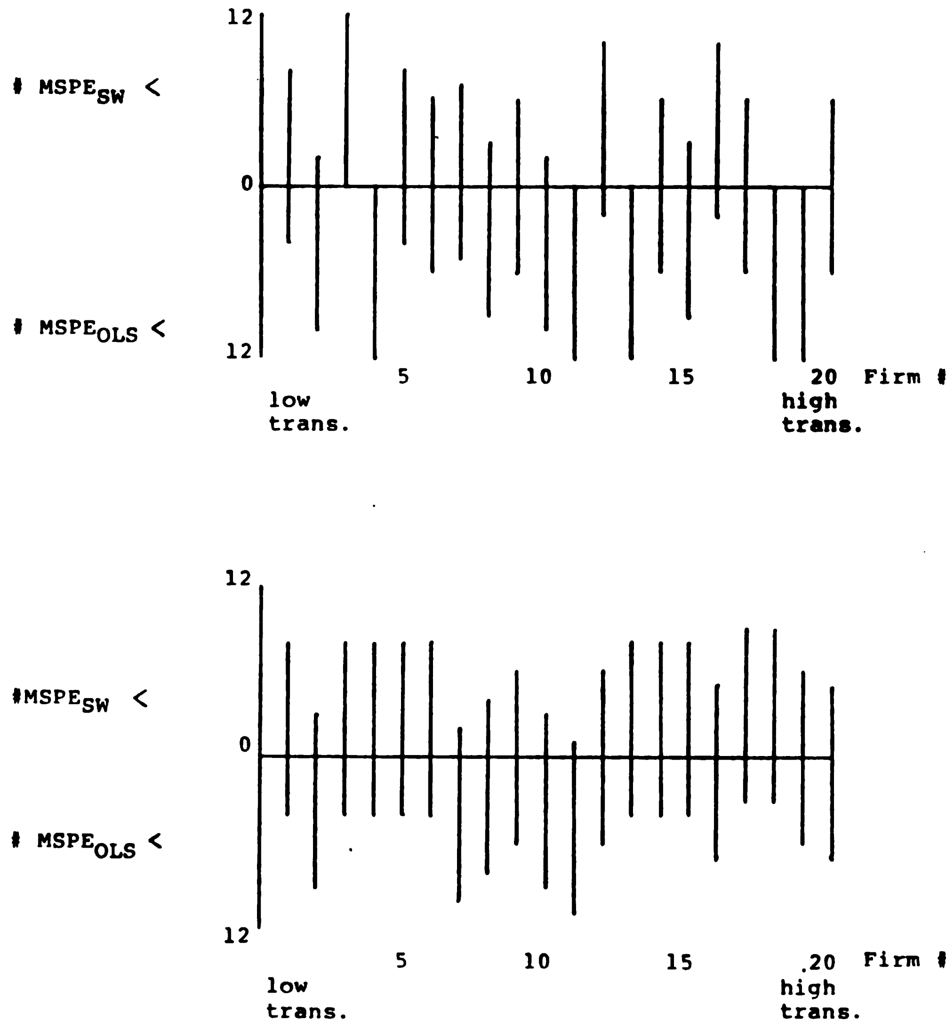


Figure 5.9 Number of Mean Squared Prediction Errors Out of 12 Replications Which are Smaller - Estimation Periods: 100 days and 400 days

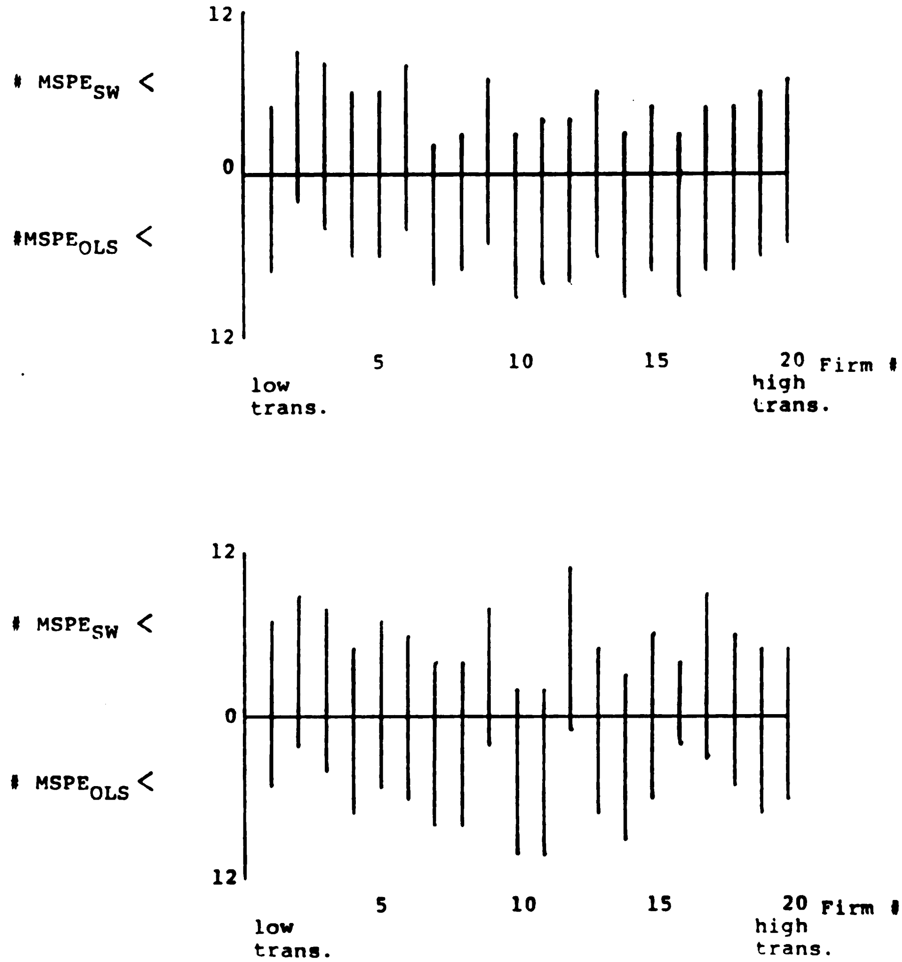


Figure 5.10 Number of Mean Squared Prediction Errors Out of 12 Replications Which are Smaller - Estimation Periods: 1000 days and 5200 days

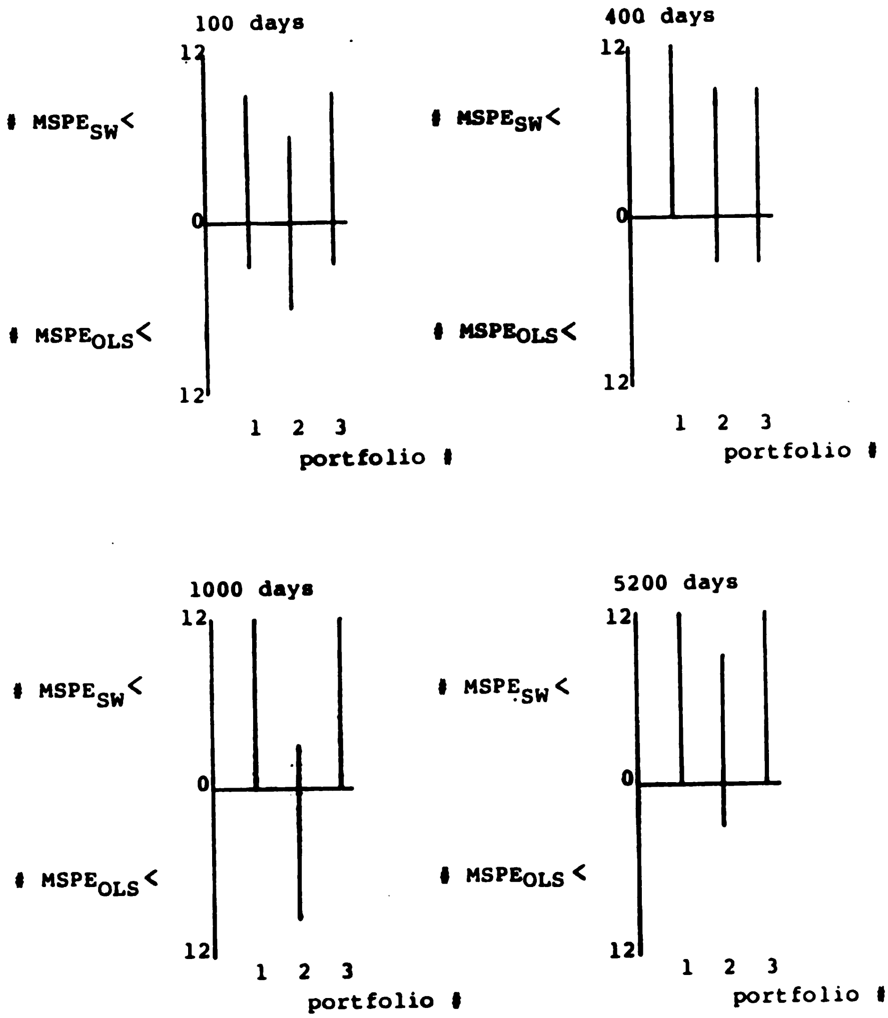


Figure 5.11 Mean Squared Prediction Errors Portfolio Results

the majority have a low number or high number of transactions, compared to the market average, then the S&W estimator should be used.

Efficient Market Test Results

The test of efficient market studies is similar to the technique used by Brown and Warner [1980]. The difference between this study and the Brown and Warner study is that Brown and Warner tested different procedures to see if abnormal performance was found using one procedure more often than another, where as this study uses the same procedure with different estimators. The desire is to find out if abnormal performances can be found more often using one method of estimation than another (OLS versus S&W). The market model will be used to predict expected returns. The expected returns will then be subtracted from the observed return to obtain a residual. The residuals are tested for abnormal performance.

The S&W beta and alpha and the OLS beta and alpha will be estimated over periods of 100 days to 5200 days. Abnormal performance will be tested over a 100 day period. A 100 day period was used, because 100 days allows for enough days, as will be seen later, to compute a variance for the residuals. Brown and Warner also utilized a 100 month period, therefore, the procedure employed can be the same as the Brown and Warner procedure. Ten firms from the twenty will be used in the abnormal performance test, and a total of 48 different samples will be selected. Only ten firms were used because if all twenty firms were used, the result would be to predict the market with the market, therefore less than twenty firms were desired and 10 were chosen. The abnormal performance will be tested using two different tests, the t-test and a cumulative average residual (CAR) test.

The t-test, as derived by Brown and Warner, tests for abnormal performance on an event day (t = 0). The 100 day test period is divided into the time prior to the event (t = -89 through t = -1) and 10 days after the event (t = 1 through t = 10) (see figure 2.4). An average market model residual is calculated on the event day (t = 0) as follows (Brown and Warner [1980], page 253, equation A.9):

$$1/N \sum_{i=1}^{N} A_{i0}$$

where N = the number of firms in the sample

$$A_{it} = R^{s}_{it} - \hat{\alpha}_i - \hat{\beta}_i R^{s}_{Mt}\ , \text{ and}$$

$\hat{\alpha}_i$, $\hat{\beta}_i$ = either the OLS estimates or the S&W estimates of α and β.

The standard deviation of the average market model residual is estimated on the basis of the standard deviation of the residual of each sample security over the period t = -89 to t = -11. At t = 0 the test statistic (Brown and Warner [1980], page 253, equation A.11) is:

$$\frac{1/N \sum_{i=1}^{N} A_{i0}}{1/N\left(\sum_{i=1}^{N}\left[1/77 \sum_{t=-89}^{-11}\left(A_{it}-\left(\sum_{t=-89}^{-11} A_i/79\right)\right)^2\right]\right)^{1/2}} \tag{5.1}$$

The above statistic is distributed Student - t with 78 degrees of freedom. Equation (5.1) is calculated for each of the 48 samples with abnormal performances added at t = 0 of .00, .01, .011, .013, .016, and .02. In other words, if an abnormal performance of .01 is to be added, .01 is added to each firm's return on the event day t = 0. These levels of abnormal performance were chosen in an attempt to obtain t-statistics that were close to the rejection boundary at the three test levels (.05, .025, and .01). For example, at the .025 test level, the boundary for rejection of the null hypothesis is a t-statistic of 2.00. The desire was to pick levels of abnormal performance such that many of the t-statistics were close to 2.00.

Tables 5.3 through 5.12 summarize the results of this test. The 48 replications of this test can be divided into 4 sets of 12. The first set of 12 replications was drawn using firms 1 through 10 or those firms with low to average number of transactions. The prediction results (figures 5.9 and 5.10) would indicate that for these 10 firms, on average, the S&W estimators would result in better predictions. Therefore, the expected result of the t-test would be that the S&W estimators would isolate abnormal performance sooner than the OLS estimators. Table 5.3 shows the percentage of replications where abnormal performance was found at each level of abnormal performance and at each test level. The percentage of abnormal performance found is greater for the S&W estimators in some instances but for the majority of the cases the percentages are the same. The average t-statistics, for the first set of twelve replications, are listed in table 5.8 and on average the S&W estimators result in a slightly higher t-statistic.

The second set of 12 replications was drawn using firms 11 through 20 or those firms with average to high number of transactions. The prediction results (figures 5.9 and 5.10) would indicate that for these 10 firms, on average, the OLS estimators would result in better predictions. The expected result of the t-test is that the OLS estimators will isolate abnormal performance more often than the S&W estimators. Table 5.4 shows the percentage of replications where abnormal performance was found, and in most cases the percentages are the same. Table 5.9 shows the average t-statistics and on average the OLS estimators result in a slightly higher t-statistic.

The third set of replications was drawn using firms 1 through 5, and 16 through 20, or those firms with a low or high number of transactions. The prediction results (figures 5.9 and 5.10) for these 10 firms indicate, on average, better predictions are obtained using the S&W estimators. The expected result of the t-test is that the S&W estimators will isolate abnormal performance more often than the OLS estimator. Table 5.5 shows the percentage of replications where abnormal performance was found. For the most part, the percentages are the same, but there are some instances where the S&W percentage is consistently higher. Table 5.10 shows the average t-statistics and on average the S&W estimators result in greater t-statistics.

The last set of replications was drawn using firms 6 through 15 or an average number of transactions. The prediction results (figures 5.9 and 5.10) for these 10 firms indicate, on average, that the OLS estimators result in better predictions. The expected result of the t-test is that the use of the OLS estimators will result in isolating abnormal performance more often than the S&W estimators. Table 5.6 shows some of the OLS percentages are greater than the S&W percentages, but most are equal. Table 5.11, however, shows that the average OLS t-statistic is slightly greater.

Tables 5.7 and 5.12 show the aggregated result of all 48 replications. As in the other tables, the percentages of

TABLE 5.3

A COMPARISON OF ALTERNATIVE PARAMETER ESTIMATES
SAMPLE: FIRMS 1 - 10
PERCENTAGE OF 12 REPLICATIONS WHERE THE NULL HYPOTHESIS IS REJECTED

Level of Abnormal Performance	Test Level	Number of days in estimation period: 100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	.050	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.025	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.010	.050	58.33%	66.67%	58.33%	66.67%	66.67%	66.67%	66.67%	66.67%
	.025	25.00%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
	.010	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
.011	.050	75.00%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	41.67%	41.67%	41.67%	41.67%	50.00%	41.67%	41.67%	41.67%
	.010	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
.013	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	75.00%	75.00%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.010	41.67%	41.67%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
.016	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
.020	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

TABLE 5.4

A COMPARISON OF ALTERNATIVE PARAMETER ESTIMATES
SAMPLE: FIRMS 11 - 20
PERCENTAGE OF 12 REPLICATIONS WHERE THE NULL HYPOTHESIS IS REJECTED

Level of Abnormal Performance	Test Level	Number of days in estimation period: 100 OLS	100 S & W	400 OLS	400 S & W	1000 OLS	1000 S & W	5200 OLS	5200 S & W
.000	.050	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.025	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.010	.050	41.67%	33.33%	41.67%	33.33%	41.67%	33.33%	41.67%	33.33%
	.025	8.33%	8.33%	16.67%	8.33%	16.67%	8.33%	16.67%	16.67%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.011	.050	58.33%	58.33%	66.67%	66.67%	66.67%	66.67%	66.67%	66.67%
	.025	41.67%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.013	.050	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	58.33%	58.33%	66.67%	66.67%	66.67%	66.67%	66.67%	66.67%
	.010	41.67%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
.016	.050	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.010	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
.020	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	91.67%	100.00%	91.67%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%

TABLE 5.5

A COMPARISON OF ALTERNATIVE PARAMETER ESTIMATES
SAMPLE: FIRMS 1 - 5 & 16 - 20
PERCENTAGE OF 12 REPLICATIONS WHERE THE NULL HYPOTHESIS IS REJECTED

Level of Abnormal Performance	Test Level	Number of days in estimation period: 100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	.050	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.025	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.010	.050	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	50.00%	50.00%	66.67%	75.00%	66.67%	66.67%	66.67%	66.67%
	.010	16.67%	8.33%	33.33%	25.00%	33.33%	33.33%	33.33%	33.33%
.011	.050	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	75.00%	75.00%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.010	25.00%	25.00%	33.33%	33.33%	33.33%	33.33%	33.33%	33.33%
.013	.050	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.025	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.010	58.33%	58.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
.016	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
	.010	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%	83.33%
.020	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	83.33%	91.67%	83.33%	91.67%	83.33%	91.67%	83.33%	83.33%

TABLE 5.6

A COMPARISON OF ALTERNATIVE PARAMETER ESTIMATES
SAMPLE: FIRMS 6 - 15
PERCENTAGE OF 12 REPLICATIONS WHERE THE NULL HYPOTHESIS IS REJECTED

Level of Abnormal Performance	Test Level	Number of days in estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	.050	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.025	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.010	.050	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
	.025	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
	.010	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
.011	.050	33.33%	33.33%	25.00%	25.00%	16.67%	16.67%	16.67%	16.67%
	.025	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
	.010	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
.013	.050	66.67%	66.67%	66.67%	58.33%	66.67%	58.33%	66.67%	66.67%
	.025	41.67%	41.67%	33.33%	25.00%	33.33%	33.33%	33.33%	33.33%
	.010	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%	16.67%
.016	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	66.67%	83.33%	75.00%	75.00%	66.67%	66.67%	83.33%	66.67%
	.010	50.00%	58.33%	58.33%	58.33%	50.00%	50.00%	50.00%	50.00%
.020	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%

TABLE 5.7

A COMPARISON OF ALTERNATIVE PARAMETER ESTIMATES

PERCENTAGE OF 48 REPLICATIONS WHERE THE NULL HYPOTHESIS IS REJECTED

Level of Abnormal Performance	Test Level	Number of days in estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	.050	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.025	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
	.010	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
.010	.050	50.00%	50.00%	50.00%	50.00%	52.10%	50.00%	52.10%	50.00%
	.025	25.00%	27.00%	32.00%	33.00%	33.00%	31.25%	33.00%	37.50%
	.010	12.50%	10.40%	16.70%	14.50%	16.70%	16.70%	16.70%	16.70%
.011	.050	62.50%	64.60%	64.60%	64.60%	62.50%	62.50%	62.50%	62.50%
	.025	39.60%	41.70%	43.75%	43.75%	43.75%	43.75%	43.75%	43.75%
	.010	14.50%	12.50%	16.70%	16.70%	16.70%	16.70%	16.70%	16.70%
.013	.050	83.33%	83.33%	83.33%	81.25%	83.33%	81.25%	83.33%	83.33%
	.025	64.60%	64.60%	66.70%	66.70%	66.70%	66.70%	66.70%	66.70%
	.010	39.60%	37.50%	41.70%	41.70%	41.70%	41.70%	41.70%	41.70%
.016	.050	95.80%	95.80%	95.80%	95.80%	95.80%	95.80%	95.80%	95.80%
	.025	87.50%	85.50%	85.40%	85.40%	83.30%	83.30%	87.50%	83.30%
	.010	77.10%	77.10%	77.10%	77.10%	75.00%	75.00%	75.00%	75.00%
.020	.050	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.025	98.00%	100.00%	98.00%	100.00%	100.00%	100.00%	100.00%	100.00%
	.010	91.70%	93.75%	91.70%	93.75%	91.70%	93.75%	91.70%	91.70%

TABLE 5.8

AVERAGE T - STATISTICS
SAMPLE: FIRMS 1 - 10

Level of Abnormal Performance		Number of days in the estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	average	.3434	.4552	.5295	.5334	.5289	.5431	.5277	.5336
	std dev	.5602	.4599	.4908	.5029	.4879	.5127	.4904	.5117
.010	average	1.9237	1.9113	2.0048	2.0104	2.0158	2.0206	2.0072	2.0135
	std dev	.4947	.4837	.5168	.5288	.5191	.5366	.5155	.5355
.011	average	2.0702	2.0569	2.1524	2.1581	2.1320	2.1683	2.1552	2.1614
	std dev	.4973	.4862	.5195	.5315	.5028	.5391	.5181	.5380
.013	average	2.3632	2.3487	2.4475	2.4535	2.4594	2.4638	2.4510	2.4574
	std dev	.5023	.4915	.5249	.5368	.5269	.5441	.5233	.5429
.016	average	2.8027	2.7850	2.8902	2.8966	2.9033	2.9070	2.8949	2.9017
	std dev	.5100	.4987	.5331	.5450	.5339	.5516	.5312	.5497
.020	average	3.4054	3.3684	3.4804	3.4873	3.4943	3.4941	3.4867	3.4933
	std dev	.5353	.5090	.5443	.5560	.5455	.5586	.5419	.5606

TABLE 5.9

AVERAGE T - STATISTICS
SAMPLE: FIRMS 11 - 20

Level of Abnormal Performance		Number of days in the estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	average	-.6183	-.6168	-.7295	-.7354	-.7399	-.7463	-.7271	-.7335
	std dev	.6593	.6319	.6960	.7280	.7001	.7293	.6959	.7242
.010	average	1.4364	1.4242	1.3597	1.3394	1.3416	1.3271	1.3619	1.3475
	std dev	.6833	.6434	.7042	.7464	.7148	.7462	.7101	.7399
.011	average	1.6433	1.5461	1.5595	1.5516	1.5498	1.5399	1.5653	1.5592
	std dev	.6851	.7421	.7128	.7466	.7168	.7461	.7119	.7406
.013	average	2.0636	2.0365	1.9745	1.9566	1.9644	1.9492	1.9821	1.9719
	std dev	.6824	.6488	.7179	.7576	.7224	.7528	.7159	.7462
.016	average	2.6781	2.7322	2.5999	2.5843	2.3406	2.5712	2.6072	2.3462
	std dev	.6955	.8541	.7237	.7612	.6842	.7603	.7225	.7138
.020	average	3.5059	3.4653	3.4323	3.4142	3.4323	3.4006	3.4409	3.4285
	std dev	.7053	.6651	.7338	.7727	.7378	.7712	.7325	.7635

TABLE 5.10

AVERAGE T - STATISTICS
SAMPLE: FIRMS 1 -5 & 16 - 20

Level of Abnormal Performance		Number of days in the estimation period: 100 OLS	100 S & W	400 OLS	400 S & W	1000 OLS	1000 S & W	5200 OLS	5200 S & W
.000	average	.4586	.5385	.5295	.5334	.5373	.5431	.5277	.5336
	std dev	.4707	.5698	.4908	.5029	.4940	.5127	.4904	.5117
.010	average	1.9237	1.9113	2.0048	2.0104	2.0162	2.0206	2.0072	2.0135
	std dev	.4948	.4837	.5168	.5288	.5190	.5366	.5155	.5355
.011	average	2.0702	2.0903	2.1524	2.1581	2.1636	2.1683	2.1552	2.1614
	std dev	.4972	.4986	.5195	.5315	.5216	.5391	.5181	.5380
.013	average	2.3632	2.3487	2.4475	2.4535	2.4598	2.4638	2.4510	2.4574
	std dev	.5023	.4915	.5249	.5368	.5268	.5441	.5233	.5429
.016	average	2.8027	2.7850	2.8902	2.8966	2.9029	2.9070	2.8949	2.9014
	std dev	.5100	.4987	.5331	.5450	.5348	.5516	.5312	.5504
.020	average	3.3887	3.3674	3.4804	3.5123	3.4943	3.4941	3.4867	3.4933
	std dev	.5205	.5090	.5443	.5625	.5455	.5585	.5419	.5606

TABLE 5.11

AVERAGE T - STATISTICS
SAMPLE: FIRMS 6 - 15

Level of Abnormal Performance		Number of days in the estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	average	-.5028	-.6168	-.7295	-.7354	-.7399	-.7463	-.7271	-.7335
	std dev	.7728	.6319	.6960	.7280	.7001	.7293	.6459	.7243
.010	average	1.4364	1.4242	1.3514	1.3394	1.3416	1.3271	1.3569	1.3475
	std dev	.6833	.6434	.7109	.7464	.7148	.7462	.7100	.7399
.011	average	1.6436	1.6295	1.5595	1.5516	1.5498	1.5400	1.5653	1.5592
	std dev	.6852	.6449	.7128	.7466	.7168	.7461	.7119	.7406
.013	average	2.0636	2.0365	1.9745	1.9618	1.9644	1.9492	1.9821	1.9719
	std dev	.6824	.6488	.7179	.7534	.7224	.7528	.7159	.7462
.016	average	2.6781	2.7322	2.5999	2.5843	2.5907	2.5712	2.6073	2.5962
	std dev	.6955	.8541	.7237	.7612	.7277	.7603	.7225	.7532
.020	average	3.5059	3.4653	3.4329	3.4142	3.4149	3.3931	3.4407	3.4286
	std dev	.7053	.6651	.7338	.7727	.7412	.7713	.7325	.7635

TABLE 5.12

AVERAGE T - STATISTICS

Level of Abnormal Performance		Number of days in the estimation period							
		100		400		1000		5200	
		OLS	S & W	OLS	S & W	OLS	S & W	OLS	S & W
.000	average	.0505	.0459	.0572	.0589	.0577	.0587	.0554	.0572
	std dev	.7060	.6712	.7540	.7588	.7515	.7571	.7352	.7485
.010	average	1.6817	1.6357	1.6516	1.5913	1.6876	1.6830	1.6905	1.6891
	std dev	.6094	.6001	.6801	.6207	.6389	.6446	.6331	.6371
.011	average	1.8549	1.8411	1.8652	1.8581	1.8241	1.8585	1.8652	1.8269
	std dev	.6040	.5642	.6454	.6413	.6779	.6355	.6257	.6634
.013	average	2.2047	2.1819	2.1635	2.1547	2.2110	2.2054	2.2147	2.2130
	std dev	.5914	.5547	.6138	.6064	.6189	.6234	.6133	.6168
.016	average	2.7210	2.7190	2.7112	2.7278	2.7349	2.7280	2.7388	2.7369
	std dev	.5881	.6107	.6176	.6170	.6047	.6094	.6010	.6038
.020	average	3.4147	3.3842	3.4294	3.4244	3.4331	3.4237	3.4397	3.4354
	std dev	.5924	.5486	.6066	.6166	.5988	.6015	.5952	.5986

the 48 replications where abnormal performance was found is about the same. The average S&W t-statistic is slightly less than the OLS t-statistic. However, when the results are not aggregated, the pattern follows that of the prediction tests, but the t-statistics of either the OLS estimators or the S&W estimators are not that different. The standard deviations are large in comparison to the t-statistics. Therefore, it would not matter which estimates were used.

Cumulative Average Residual Test

A cumulative average residual test is typically used when there exists only an approximate event date. The CAR's are computed for the days around this date and then plotted against time. The plots are examined to see if a pattern can be observed. If no abnormal performance exists the plot is expected to be flat or display virtually no slope. The CAR's are calculated as follows (Brown and Warner [1980], Equation 1):

$$CAR_t = CAR_{t-1} + 1/N \sum_{i=1}^{N} A_{it}$$

where CAR_t = the cumulative average residual at time t.

The time period covered in this test was a 21 day period from t = -10 to t = +10.

The cumulative average residuals were calculated for the same 48 replications that were run in the t-test (the previous section). Figures 5.12 through 5.19 are a few of the CAR plots. Figures 5.12 and 5.13 are two of the replications out of the 12 replications which were made using firms 1 through 10. Figures 5.14 and 5.15, 5.16 and 5.17, and 5.18 and 5.19 represent firms 11 through 20, 1 through 5 and 16 through 20, and 6 through 15, respectively. Parameters of the market model were estimated over periods of 100, 400, 1000 and 5200 days. The upper row of plots are the CAR's where no abnormal performance has been added. The bottom row of plots has an abnormal performance of .001 added to each day of the 21 day period. The x denotes the CAR using S&W parameters and the ° denotes the CAR using OLS parameters.

A CAR indicates abnormal performance when the drift of the CAR is greater, in either the positive or negative direction, than expected. The expected drift would be zero if no abnormal performance existed. Therefore, an important aspect is that when no abnormal performance is added, the drift should be small. The focus of this test is to discover which estimator (OLS or S&W) results in CAR's which are closer to zero. Table 5.13 shows the results of the CAR plots. The table is broken down into the 4 groups of 12 replications indicated in the preceeding paragraph and the total of all 48 replications. The data shown are the number of replications in which the CAR on

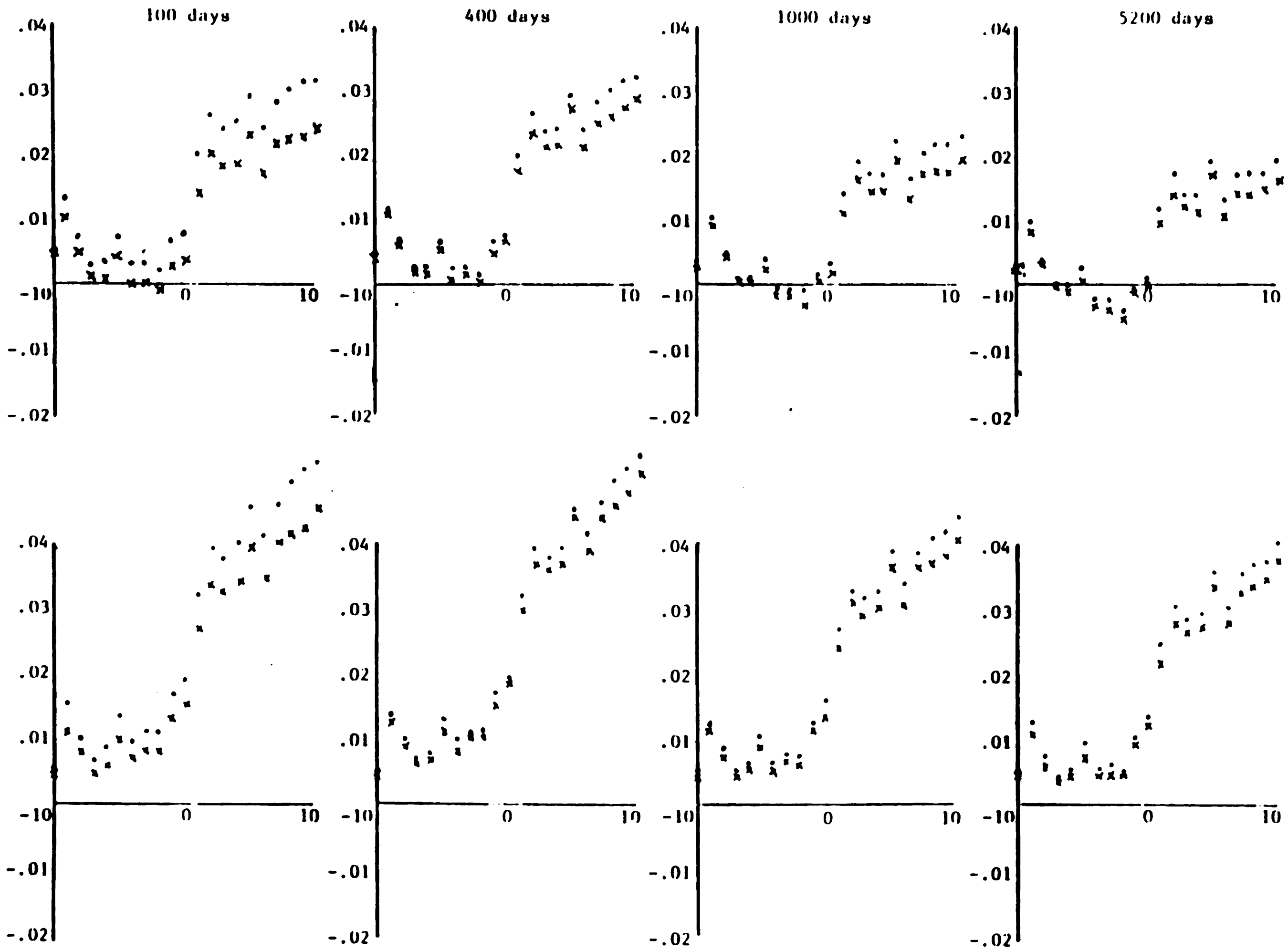


Figure 5.12 CAR Plots Firms: 1-10

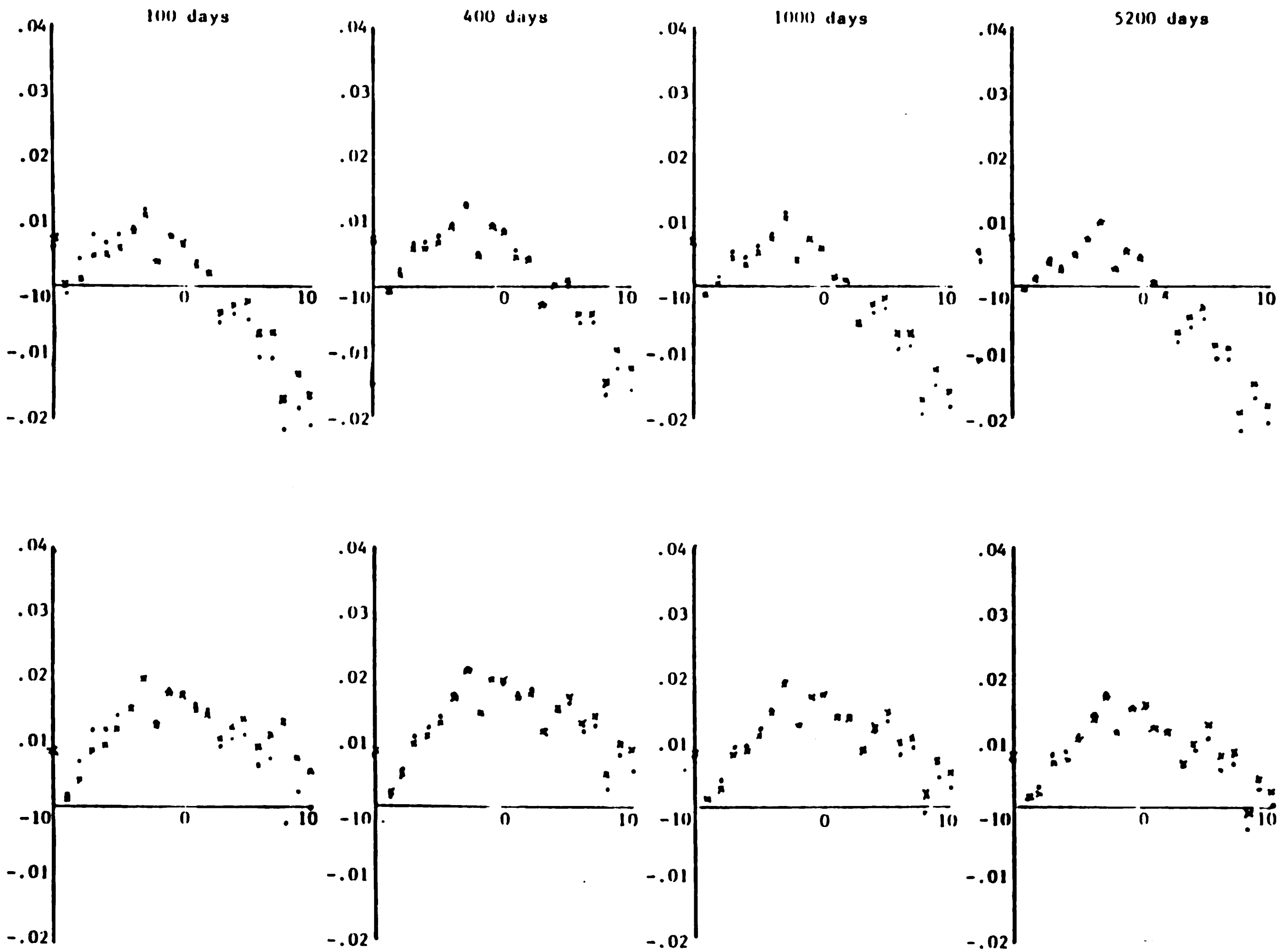


Figure 5.13 CAR Plots Firms: 1-10

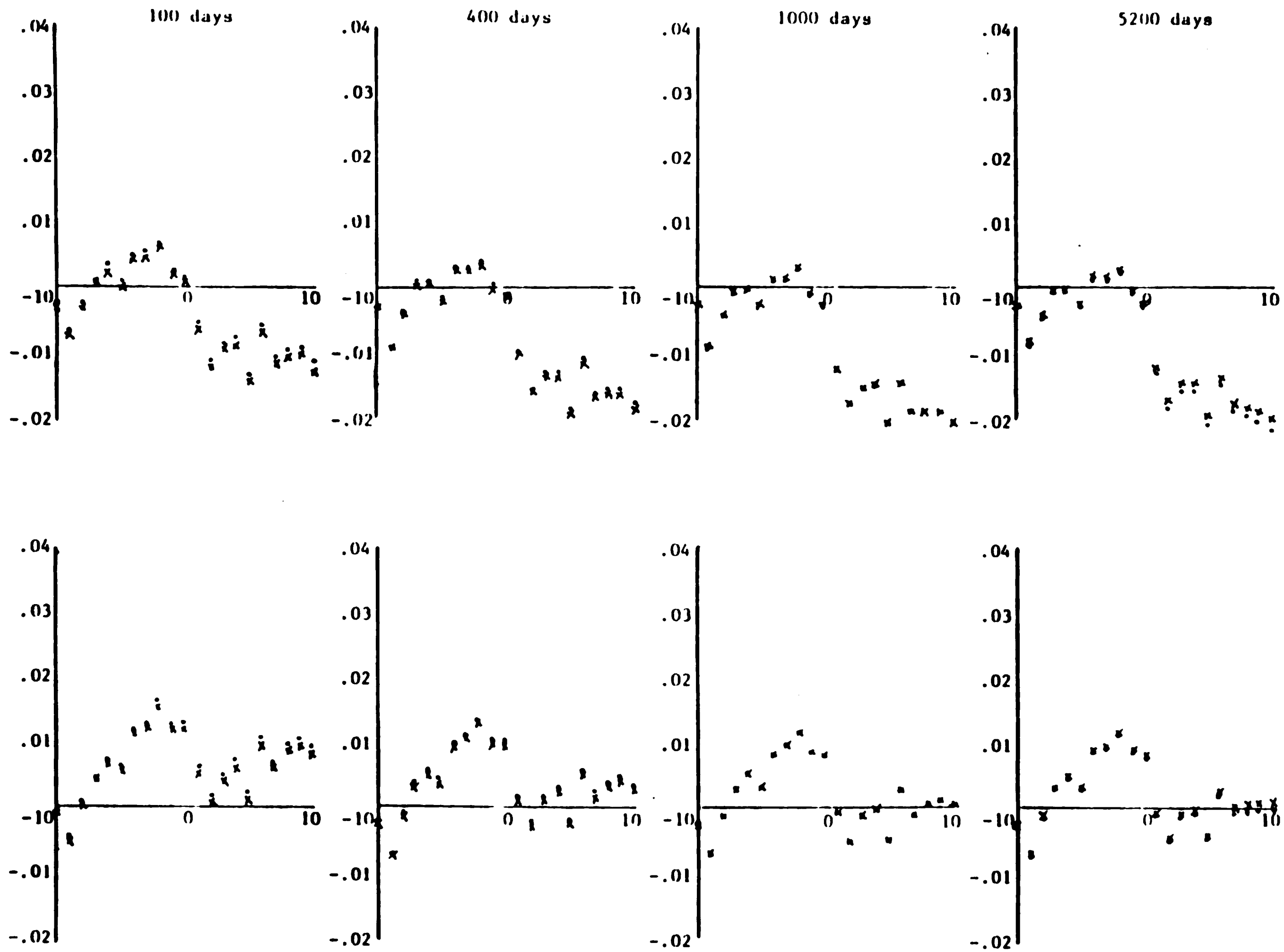


Figure 5.14 CAR Plots Firms: 11-20

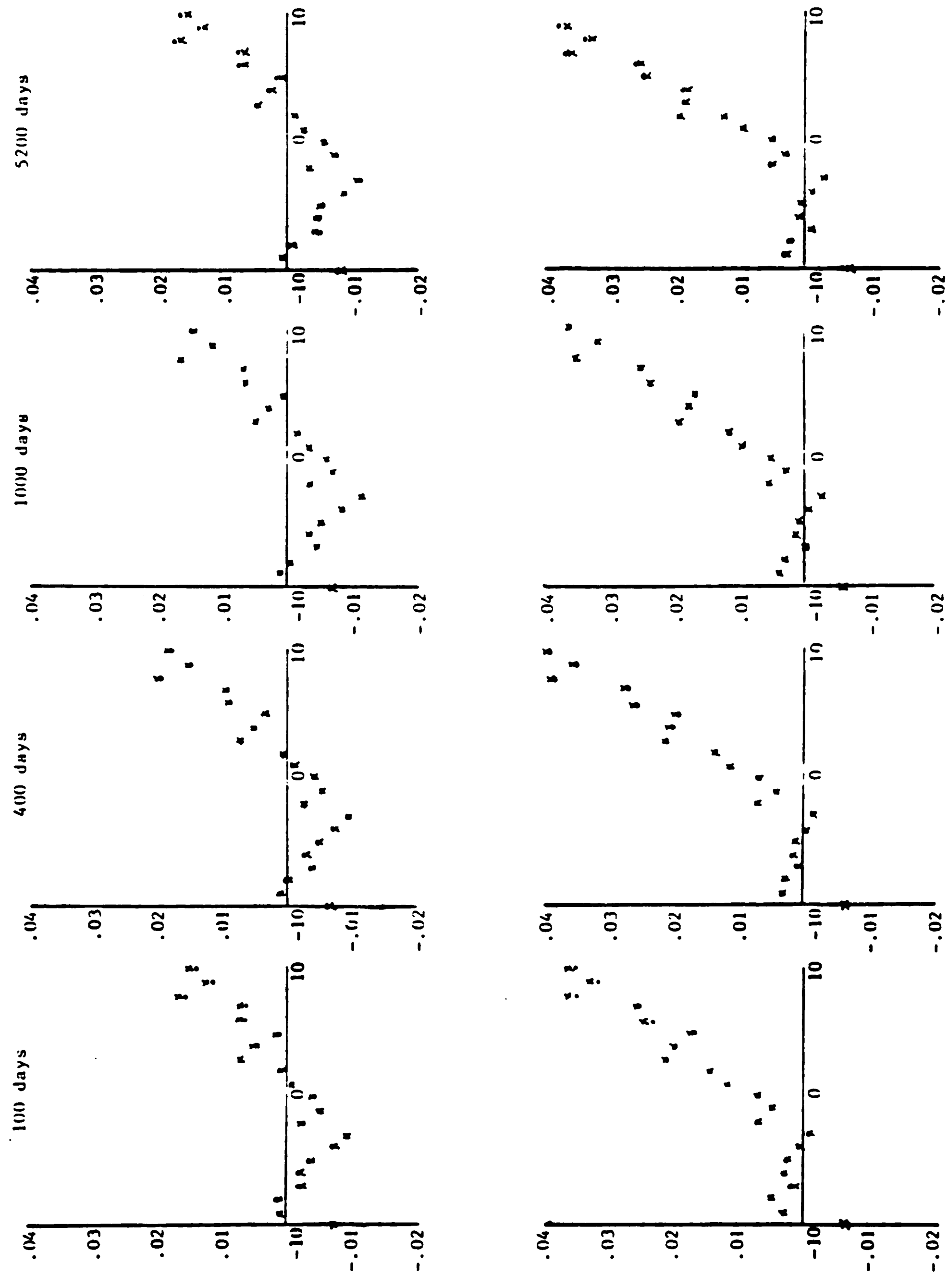


Figure 5.15 CAR Plots Firms: 11-20

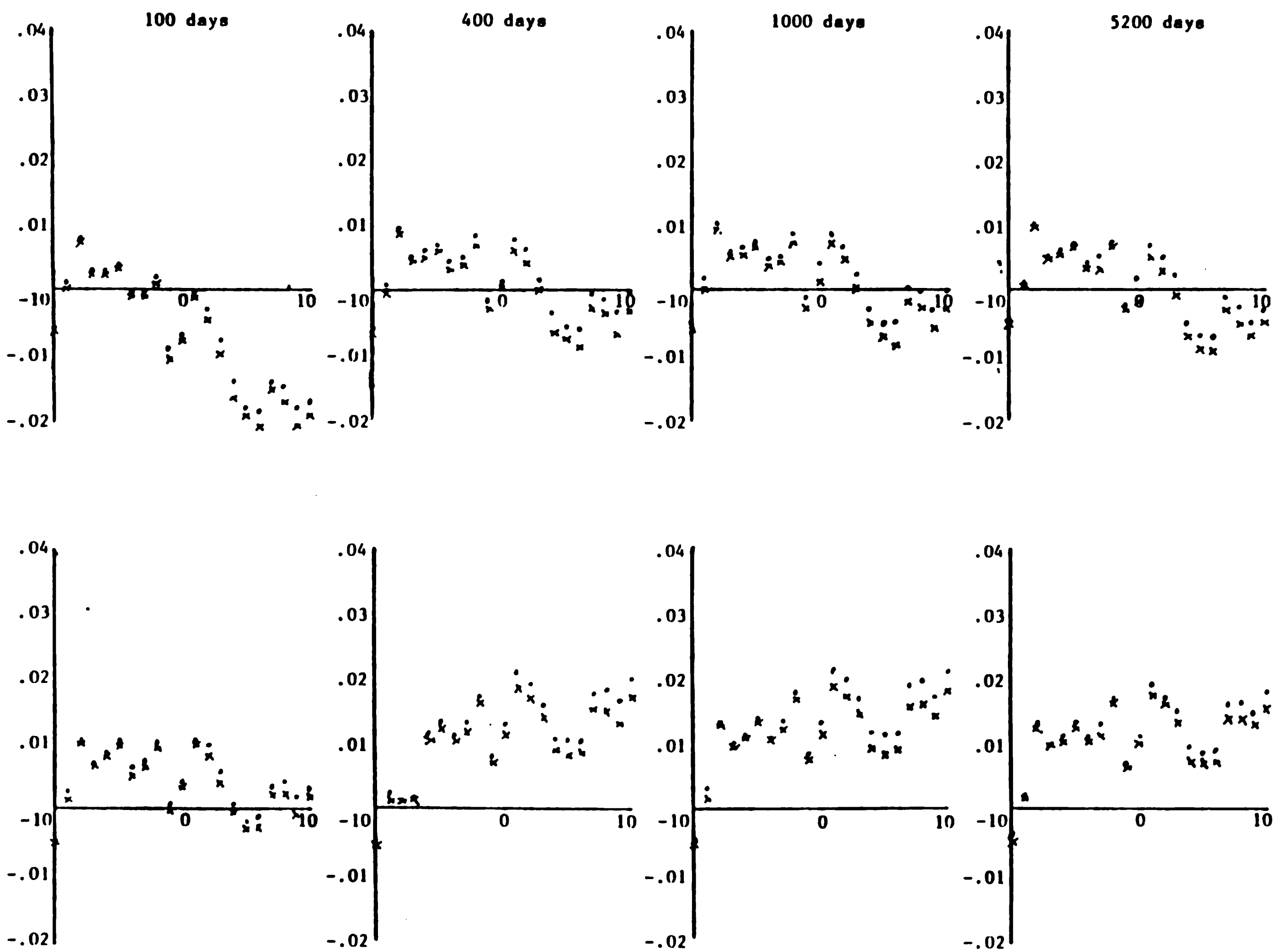


Figure 5.16 CAR Plots Firms: 1-5 and 16-20

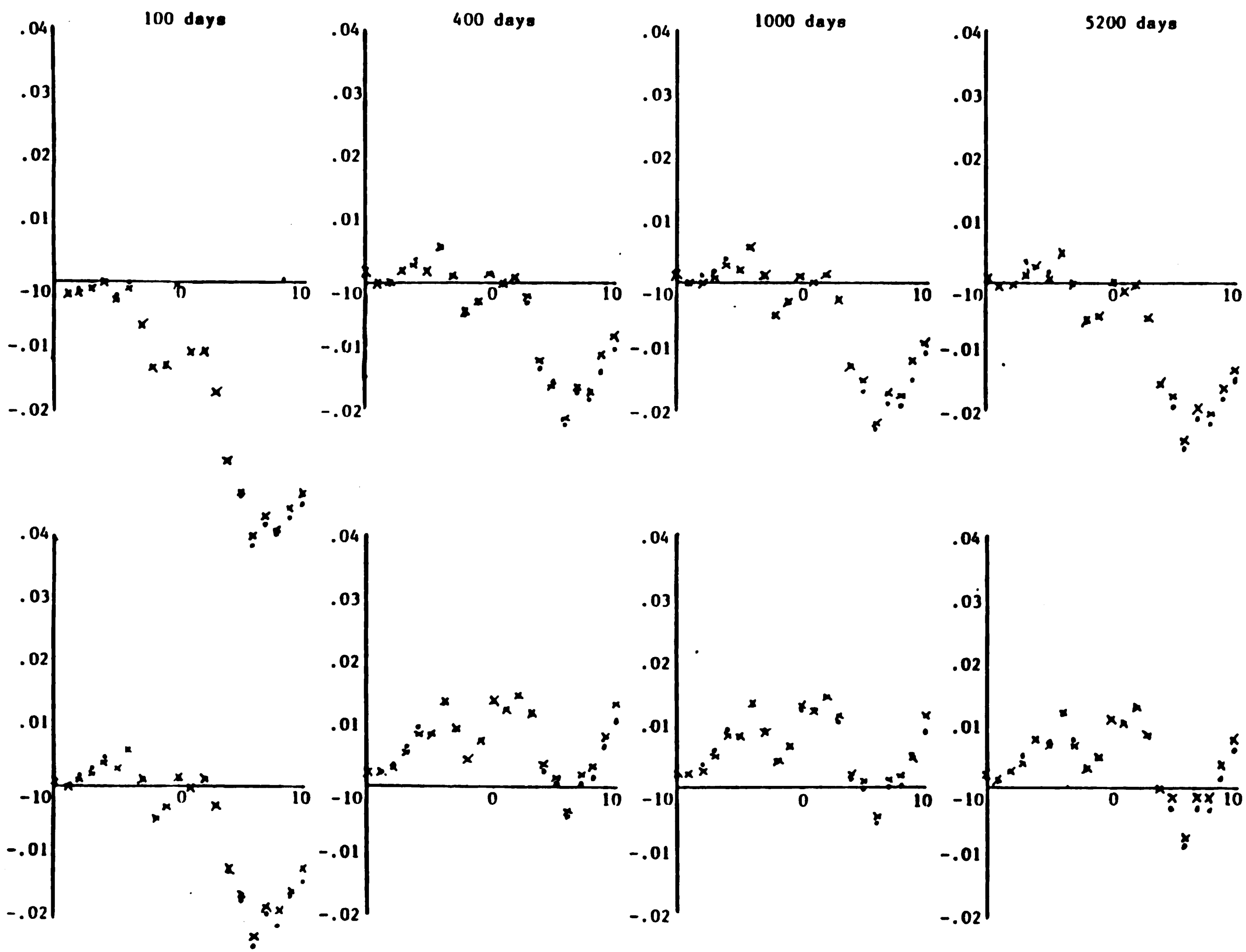


Figure 5.17 CAR Plots Firms: 1-5 and 16-20

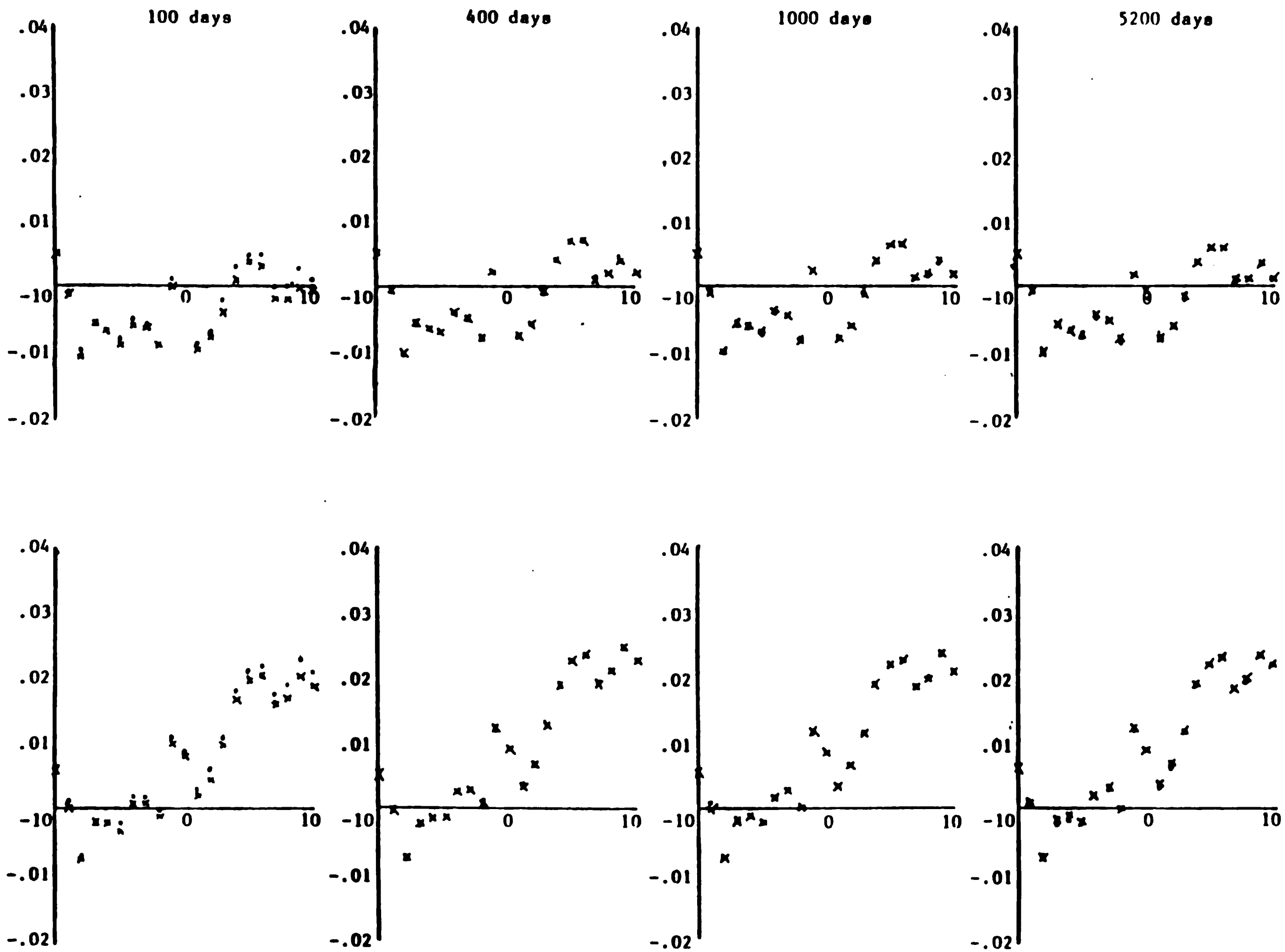


Figure 5.18 CAR Plots Firms: 6-15

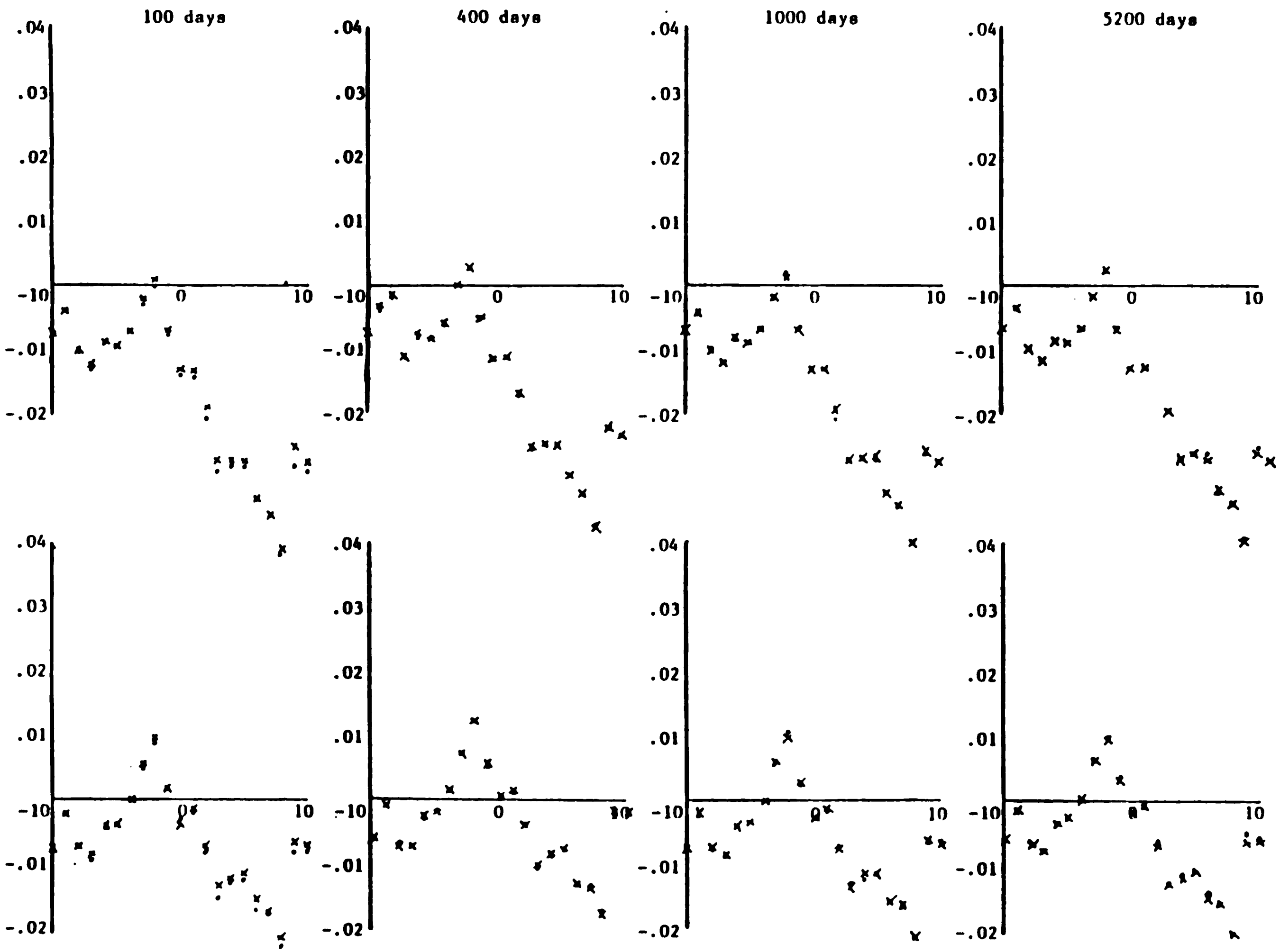


Figure 5.19 CAR Plots Firms: 6-15

the last day is closer to zero, as well as the average absolute difference between the CAR's computed using the OLS estimators and the S&W estimators at estimation periods of 100, 400, 1000 and 5200 days.

The results in Table 5.13 indicate that the S&W CAR is preferred in two cases, the case where the firms are comprised of less than average and average transactions, and the case of average and greater than average transactions. These results do not conform to the prediction results or the t-test results. However, the average difference between the two CAR's is small compared to the CAR itself. It is not apparent that the CAR test is significantly affected by the choice of estimator, nor is it apparent that a pattern exists as a result of the estimator used.

TABLE 5.13

NUMBER OF REPLICATIONS IN WHICH THE S&W CAR WAS CLOSER TO ZERO

Firms	Number of replications	Number of days used in estimation			
		100	400	1000	5200
1-10	12	10 (.00344)	8 (.00176)	8 (.00140)	8 (.00183)
11-20	12	8 (.00402)	8 (.00216)	8 (.00162)	8 (.00182)
1-5 & 16-20	12	6 (.00219)	6 (.00172)	6 (.00248)	4 (.00112)
6-15	12	6 (.00110)	6 (.00130)	6 (.00132)	4 (.00112)
Total	48	30 (.00269)	28 (.00174)	28 (.00171)	24 (.00147)

Note: The number in parentheses represents the average absolute difference between the CAR computed using OLS estimators and S&W estimators.

VI. TEST RESULTS AND CONCLUSIONS FROM TESTS USING NYSE FIRMS

The average number of transactions per day for the first three months of 1980 was obtained for each of 283 NYSE firms from Woodruff [1981]. The tests performed in Chapter V were performed using these firms. The above firms can be used because the average number of transactions are known (see table 4.2), and therefore, the results can be compared to the simulation results. The purpose of the test is to see if the results using real firms will be similar to those of the simulation in Chapter V.

The estimation period for the market model parameters was the four years from 1975 through 1978. A four year period was chosen because this would be a reasonable estimation period. A 100 day prediction period was chosen from 1979. The returns for these periods were obtained from the 1979 CRSP daily returns tape. The markets used were both the equally weighted and value weighted New York Stock Exchange index.

Conformance to Scholes and Williams

The conformance of the real return data to the conclusions of Scholes and Williams [1977] was reviewed in two ways, the autocorrelation of the market, and the direction of the S&W adjustment to beta. The autocorrelations of the market for the four years ending December 31, 1978, were .2210 and .4428 for the value weighted and equally weighted indices, respectively. The autocorrelation should be positive, and the above results are positive. Scholes and Williams [1976] found the autocorrelation for the value weighted index from 1971 - 1975 was .291 and for the equally weighted index it was .458. The autocorrelations of this study are similar to the S&W results.

Estimates of beta were computed using both OLS and the S&W method for the 283 NYSE firms. The beta estimates are listed in table 6.1. Table 6.2 shows the percentage of S&W betas whose relationship to the OLS betas was in accordance with the theory developed by Scholes and Williams. The 283 firms were divided into 5 groups based on their average number of transactions. Group 1 has the lowest average number of transactions and Group 5 the largest. Table 6.2 points out that the choice of a market will make a difference. The value weighted market will shift the market average towards the high transaction firms; therefore, the low-number-of-transactions firms are more extreme. The equally weighted market shifts the average down. As a result, opposite trends occur. The Scholes and Williams theory appears to be more accurate when the average number of transactions is much lower than the mean number of transactions, as is the case with the value weighted index, or much higher than the mean number of transactions, as is the case with the equally weighted index. Scholes and Williams indicate that this phenomenon will occur. The assumption has been made that the transaction times for

TABLE 6.1

BETAS FOR NYSE FIRMS USING THE VALUE WEIGHTED MARKET INDEX

Firm #	OLS	SW
1	.86799534	1.09691776
2	1.09307295	1.21086685
3	1.16552308	1.23716984
4	.79328076	1.04627835
5	1.61185499	1.72515542
6	.43693756	.77906905
7	.59301104	.80801571
8	1.00941150	.99845549
9	.50201425	.58386027
10	1.13163708	1.04336242
11	1.20979645	1.20109543
12	1.82941286	2.00592198
13	.4430333	.53999448
14	.72367386	.92254552
15	1.28292044	1.50517093
16	1.13431401	1.03791883
17	.50342528	.64369224
18	.45786744	.60843455
19	.52318220	.81828001
20	.53262888	.73833100
21	.81437602	.77811674
22	1.00994845	.82887003
23	2.32903613	2.52912823
24	.68101891	.71457889
25	1.37761525	1.51383506
26	1.12519175	1.18045399
27	.61829131	.64690958
28	.98547229	1.20208253
29	1.15579812	.88915184
30	1.47787378	1.30860783
31	1.38738912	1.38574642
32	1.13512105	1.21773316
33	.87367157	1.23400364
34	1.65022099	1.42499855
35	.86614330	.93911807
36	.62312693	.90843694
37	.65679657	.70528438
38	1.52198439	1.61588830
39	1.09988513	1.10332550
40	1.50525971	1.36119005
41	1.10208064	1.25623431
42	.97208796	1.19193901
43	1.53121692	1.53867365
44	.72740960	.85151821

TABLE 6.1 (cont.)

Firm #	OLS	SW
45	-.42686427	-.16663507
46	.80604794	.69593159
47	.44909390	.61920523
48	.89297932	.90366608
49	1.04931292	1.09753155
50	1.57427924	1.61217589
51	.69253258	.72185516
52	.55302855	.56243183
53	1.85627393	1.62915745
54	.79737892	.74762155
55	.68833701	.88079592
56	.86254290	.88963493
57	.55259868	.84270077
58	1.57227660	1.33709354
59	.46878293	.51346691
60	.93422054	1.13607476
61	1.22266131	1.10869330
62	1.32548572	1.22807535
63	1.31624059	1.27995983
64	1.11881463	1.13770204
65	1.33832181	1.22386464
66	.51580151	.58341939
67	1.03656434	1.25474322
68	1.18892523	1.20348127
69	.91701038	1.03858333
70	.40304795	.59687319
71	2.08065481	1.86146509
72	.76635454	.78620558
73	2.18324975	1.93664280
74	1.00002667	1.31608891
75	.58965257	.82316942
76	1.25264779	1.33625112
77	.52384970	.74900732
78	.76254744	1.00200475
79	.75434432	1.00169022
80	.75071080	.87090638
81	1.02993594	1.09900491
82	.84181294	.97692896
83	1.10168959	1.32499274
84	1.70189994	1.83387883
85	.37061292	.47504244
86	.81292932	.83773522
87	1.50129358	1.33609357
88	.56666924	.59393751
89	.93723655	1.12910472
90	1.43043519	1.23945972
91	1.60992371	1.27851196
92	.72661537	.83413539
93	.82177987	.95976595
94	.96365797	1.19166276

TABLE 6.1 (cont.)

Firm #	OLS	SW
95	.81746681	.79208001
96	1.86814809	1.67434978
97	2.08154702	2.07504472
98	.22433483	.33076316
99	.65618902	.69391690
100	1.33380052	1.57617427
101	.91930846	1.01234657
102	1.77132226	1.76747610
103	.91780638	1.02276226
104	1.11723072	1.11591803
105	1.03197372	1.30030258
106	1.06662798	1.31049297
107	1.67522632	1.90548998
108	.69336073	.80266855
109	.78829279	.77028904
110	1.21881972	1.36565774
111	.56889383	.74546714
112	1.14772767	1.04993038
113	.65223448	.82678769
114	.75062387	.86392535
115	1.94043964	1.96902349
116	.44675605	.69420928
117	1.10102999	.98992907
118	1.20832619	1.10396823
119	.94882733	1.07940014
120	1.72108321	1.82111874
121	.92668816	.96981492
122	.87408502	1.01616366
123	.28606179	.37233975
124	1.31725927	1.23629548
125	.93807816	.99644058
126	1.19709195	1.06734044
127	1.27098775	1.35365758
128	1.16461940	1.13610389
129	1.98369966	1.77704748
130	.95436045	.97977363
131	.76059220	.68462123
132	1.31781669	1.34024075
133	.60907226	1.15842612
134	1.44468559	1.49585089
135	.81319441	.83081004
136	1.16224497	1.37326309
137	1.43047434	1.39490625
138	1.25138263	1.62280369
139	1.85162716	1.60942348
140	1.53879614	1.90798689
141	1.39825630	1.74653994
142	.12454698	.26175358
143	.76166793	.79617506
144	1.23168741	1.24098316

TABLE 6.1 (cont.)

Firm #	OLS	SW
145	1.24490364	1.33221333
146	1.23069205	1.20306871
147	1.84289072	1.82470817
148	1.00419502	.96565300
149	.46406021	.63067464
150	1.31699139	1.56863943
151	.85922341	1.08406913
152	.81178315	.98746787
153	.77682008	1.09743109
154	.51486986	.69143481
155	.58106944	.64895196
156	1.13099450	1.17223293
157	.32772592	.56547183
158	1.06389896	1.36289998
159	.74027719	.79135565
160	.69339593	.89904881
161	.82112239	.99152289
162	1.14663773	1.36935798
163	1.26572109	1.39638999
164	.94948842	1.04515124
165	1.76022998	1.77103947
166	1.05106727	1.15630267
167	1.00039818	.97307397
168	1.05326349	1.19630916
169	.75784398	.85793136
170	1.39242427	1.21906675
171	1.42245564	1.23413961
172	1.20065374	1.27922200
173	.63832072	.75917158
174	1.08753977	.47619745
175	.80844300	1.10981090
176	.48967690	.61356708
177	1.28273829	1.38117565
178	.90291198	1.04179252
179	.92842374	.96965904
180	.58018379	.83486162
181	.44933924	.55764822
182	1.03838497	1.11407186
183	.89134908	1.05954133
184	.42497127	.57444782
185	1.83544393	1.61829760
186	1.06166856	1.16921747
187	1.03748806	1.13254389
188	.68234404	.77218015
189	1.33667594	1.60621411
190	1.65363864	1.87973293
191	.51646055	.51766883
192	.33901240	.40789151
193	1.97971738	1.83161165
194	1.27531214	1.28949434

TABLE 6.1 (cont.)

Firm #	OLS	SW
195	.49305114	.52605438
196	.41954297	.70376084
197	.41021150	.46763310
198	1.22528480	1.40735741
199	1.19775323	1.34154144
200	.90084176	1.00993934
201	1.30007934	1.23267702
202	2.27087929	2.45023079
203	1.50817661	1.41513992
204	1.57256434	1.62662591
205	.20113682	.33207239
206	1.33430270	1.40854854
207	1.79433441	1.67006753
208	.32186534	.34312532
209	.65538192	.86324113
210	.88468094	.80733517
211	.34224206	.45075682
212	.35341881	.50764868
213	1.09435089	1.13173328
214	.98620816	1.02985236
215	.84005030	1.03136545
216	1.68566235	1.55957321
217	.68581085	.64972187
218	1.75500333	1.67089820
219	.76996690	1.04312826
220	.42969757	.62104103
221	1.42567004	1.45636658
222	1.09247278	1.08109731
223	1.06027611	1.35583824
224	2.00364621	2.00054044
225	1.16517792	1.15370023
226	.86969576	.97241882
227	.52594963	.66307551
228	1.18311663	1.59307964
229	1.82521786	1.47334521
230	2.15454658	1.81820269
231	1.09964849	1.30959227
232	1.18914657	1.21784580
233	.68157984	1.04782326
234	.68010030	.72530328
235	1.19742477	1.21368480
236	.71869661	.85264956
237	1.69849989	1.63285819
238	1.06845504	1.27867515
239	.25989426	.31783766
240	.49137338	.54432194
241	.67787992	.57168695
242	.43624578	.55467057
243	.78045772	.94998724
244	1.48701271	1.43065897

TABLE 6.1 (cont.)

Firm #	OLS	SW
245	.76658832	.80187884
246	1.02736936	1.04985120
247	.90071247	.96746658
248	1.10896822	1.05062121
249	.49564721	.75198333
250	.59585694	.65348489
251	.71764131	.74288392
252	.92973282	.98151592
253	.87460414	.91378642
254	1.03600903	1.50499012
255	1.98006432	1.85515406
256	1.41333482	1.44574483
257	.74254611	.78956724
258	.69017353	.74260171
259	1.45837162	1.39449322
260	.31330771	.40817350
261	.84037429	1.04515182
262	.68616202	.76594395
263	2.18329953	2.06410916
264	.88065225	1.21917503
265	1.88861573	1.79187412
266	1.42085469	1.43528827
267	.96066871	1.09200958
268	1.15470234	1.00272855
269	.54450928	.60098237
270	.98877568	.86684846
271	.42241507	.60372486
272	1.14368701	1.27950760
273	.72746583	.92549787
274	1.50788843	1.46799440
275	1.22879529	1.29124999
276	1.34468358	1.71078495
277	1.67263805	1.53542976
278	.79051866	.93719123
279	1.29233912	1.38013572
280	1.08697485	1.09693994
281	1.77265334	1.73936387
282	1.68795559	1.46601402
283	1.67007996	1.77579190

TABLE 6.2

PERCENTAGE OF FIRMS IN WHICH THE BETA WAS ADJUSTED IN ACCORDANCE WITH S&W

	Transaction Group	Value Weighted Market	Equally Weighted Market
low	1	91%	58%
	2	87%	82%
	3	65%	84%
	4	48%	89%
high	5	44%	96%
	Total %	69.6%	79%

each firm in 1980 were approximately the same for 1975 - 1979, which may or may not be true. Therefore, 5 portfolios were made in an attempt to group firms by average number of transactions. The portfolio results were adjustments in the correct direction in all cases for both equally weighted and value weighted markets. Therefore, it appears these results are in conformance with S&W [1977].

Prediction Results

The prediction results are reported in table 6.3. The 283 NYSE firms were divided into 5 groups based on average number of transactions per day. The estimation period used was 1000 days and the prediction period was 100 days. One thousand days corresponds roughly to four years. The reason 1000 days was used is because 4 years would be a reasonable estimation period. In fact, most studies would use less than that. The results reported in table 6.3 are the percentage of firms in each group where the use of the S&W estimators resulted in lower mean squared prediction errors. The simulation results were taken from figure 5.10 for 1000 days in groups corresponding to approximately the same average transaction times as those listed in table 6.3. The simulated data have a definite pattern. The value weighted market results have a similar pattern, but not as distinct. The relationship to the average number of transactions is different for the value weighted than it is for the simulation. The average for the value weighted market is

TABLE 6.3

PREDICTION RESULTS FOR THE 283 NYSE FIRMS

ESTIMATION PERIOD 1000 DAYS, PREDICTION PERIOD 100 DAYS

Firm Group	Equally Weighted Market	Value Weighted Market	Simulation Market
1	47%	34%	60%
2	60%	23%	43%
3	65%	44%	38%
4	44%	44%	35%
5	49%	59%	48%

Note: This table indicates the percent of predictions where the S&W estimators resulted in smaller errors. It does not indicate how much smaller or larger the errors were. In fact, in most instances the differences were small.

probably greater than the average for the simulated market. Therefore, the high number of transactions have the highest percentage. The equally weighted results do not appear to have a pattern. However, in all three cases, equally weighted, value weighted, and simulation, the predictions using the S&W estimator were slightly worse more often than the OLS estimator, but the differences in prediction errors were quite small.

Efficient Market Test Results

The same tests as were conducted in the simulation (t-test and CAR), were conducted using the real firms. The CAR and t-test were performed using the 5 groups of firms chosen by average transaction times. In addition another 45 replications were performed using groups of firms chosen randomly. The tests were conducted twice for each of the above groups using parameters estimated from the value weighted market index and the equally weighted market index. The estimation period was 1975 - 1978, approximately 1000 days, and a 100 day test period was chosen from 1979. The test period was staggered for each firm so that the event day was not the same for all of the firms. This was done to eliminate any possible market effects, and follows the same procedure used by Brown and Warner [1980]. The results of these tests were also similar to the simulation results.

The results of the t-test are shown in table 6.4. The t-statistics in table 6.4 are higher than those in the simulation at the same level of abnormal performance added. For this reason, the levels of abnormal performance added were smaller in order to find t-statistics where significance is not indicated. The market variance used in the simulation could have been specified incorrectly, which would account for the smaller t-statistics resulting when simulated data were used. The market variance specified was an average of only 11 firms. This may be larger than the real market variance; therefore, the t-statistics may be smaller. The autocorrelation of the real observed market is larger than that of the simulation; therefore, the variance of the residuals may be smaller (Johnston [1972]) which again would account for larger t-statistics. The S&W parameters attempt to correct the autocorrelation and, in accordance with the above argument, the S&W average statistics should be smaller. The S&W average t-statistics for the 45 random groups in table 6.4 are indeed smaller than the OLS t-statistics. The same argument should apply to the value weighted and equally weighted markets. Since the value weighted market has a smaller autocorrelation than the equally weighted market, the t-statistics resulting from parameters estimated using the value weighted market index should be smaller, and indeed they are.

TABLE 6.4

T-STATISTICS FROM EFFICIENT MARKET TESTS

PERFORMED USING NYSE FIRMS

Firm Group	Level of abnormal performance added	Equally Weighted Market t-statistics		Value Weighted Market t-statistics	
		OLS	S&W	OLS	S&W
1	0	-.7279	-.7244	-.6696	-.6634
	.005	1.3551	1.3682	1.3974	1.3975
	.008	2.6049	2.6178	2.6376	2.6341
	.010	3.4381	3.4508	3.4644	3.4585
2	0	-.3907	-.4165	-.7437	-.6932
	.005	1.5475	1.5236	1.1847	1.2309
	.008	2.7105	2.6876	2.3418	2.3854
	.010	3.4858	3.4636	3.1132	3.1551
3	0	.9696	.9895	.9883	1.0110
	.005	2.5004	2.5222	2.5059	2.5334
	.008	3.4188	3.4419	3.4182	3.4467
	.010	4.0311	4.0550	4.0264	4.0557
4	0	-.8022	-.7969	-1.2055	-1.2095
	.005	1.0484	1.0487	.6599	.6496
	.008	2.1587	2.1560	1.7791	1.7650
	.010	2.8990	2.8942	2.5253	2.5087
5	0	.4658	.3121	-.2773	-.3471
	.005	2.7307	2.5734	2.0300	1.9610
	.008	4.0896	3.9302	3.4143	3.3458
	.010	4.9956	4.8347	4.3372	4.2691
average of random groups	0	-.0928	-.1128	-.3789	-.3613
	.005	1.7852	1.7715	1.5116	1.5114
	.008	2.9163	2.9021	2.6463	2.6443
	.010	3.6683	3.6558	3.4029	3.3995

The results shown in table 6.4 appear to be comparable with those in the simulation. The groups of firms with less than average numbers of transactions result in the t-statistics using the S&W parameters being slightly greater than the t-statistics using the OLS parameters. In addition, the average t-statistics from the 45 replications of randomly chosen groups result in the same relationship as that in table 5.12. The t-statistic using the S&W parameters is slightly less than that using the OLS parameter. However, in all instances the t-statistics are almost equal and the decision would not be affected by the estimator used.

The results of the CAR test, for which no abnormal performance was added, are in figures 6.1 and 6.2. The horizontal axis represents days and the vertical axis is the CAR. The x denotes the CAR's resulting from the use of S&W parameters and the ° denotes OLS CAR's. Figure 6.1 contains the CAR plots where the equally weighted market was used and figure 6.2 contains the value weighted plots. The plots represent the five groups of firms which were chosen based on their average numbers of transactions.

The plots in which the equally weighted index was used in the estimation of the parameters (figure 6.1) all resulted in the S&W CAR's being closer to zero. The use of the value weighted index (figure 6.2), however, resulted in two out of the three less-than-average-transaction-groups-S&W-CAR's being closer to zero, and the greater-than-average-transaction-groups-OLS-CAR's being closer to zero. Also, the two CAR's (OLS and S&W) were closer when the value weighted index was used as opposed to the equally weighted index. The 45 randomly chosen groups resulted in 28 and 25 out of 45 S&W CAR's being closer to zero for the equally weighted and the value weighted markets, respectively. It appears that the firms that were not chosen randomly had different results than those chosen randomly, and if groups contain firms of homogeneous average number of transactions, then the choice of estimator made a difference.

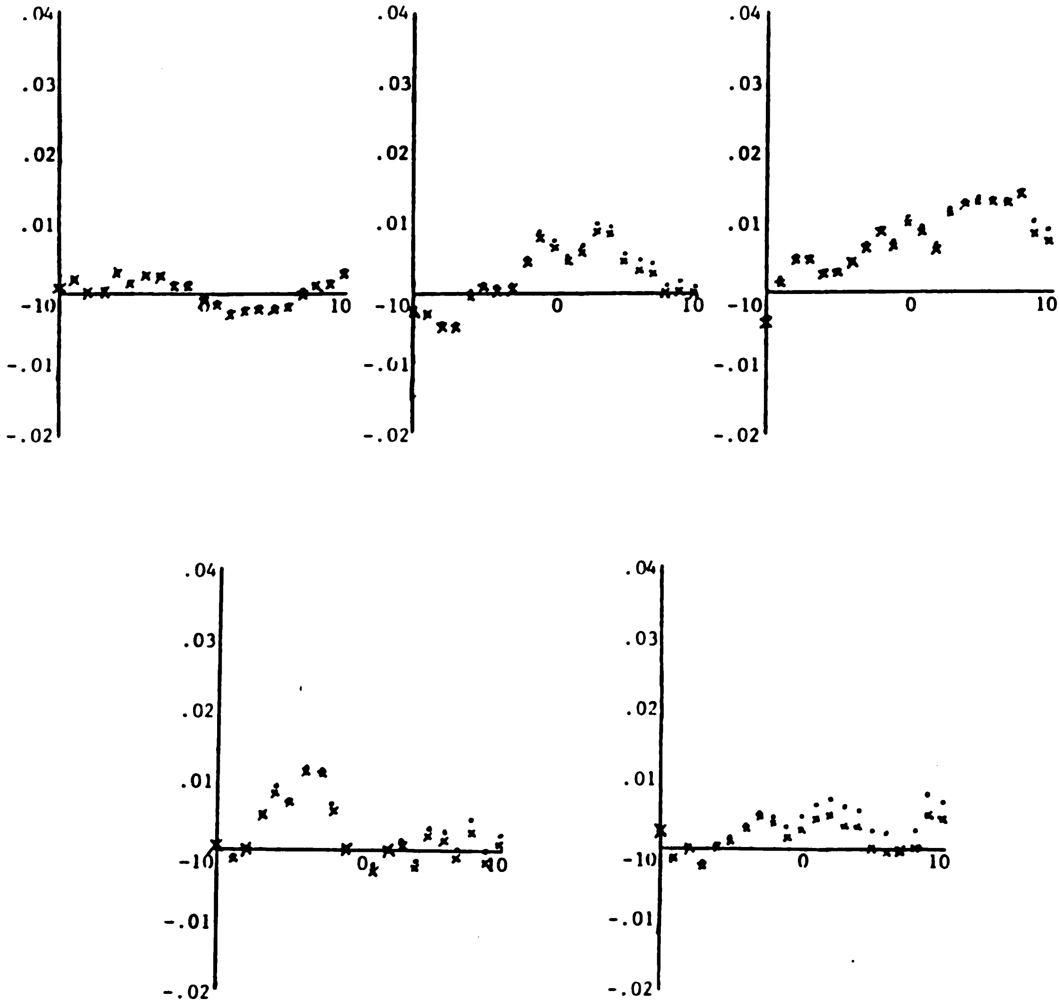


Figure 6.1 CAR Plots Equally Weighted Index

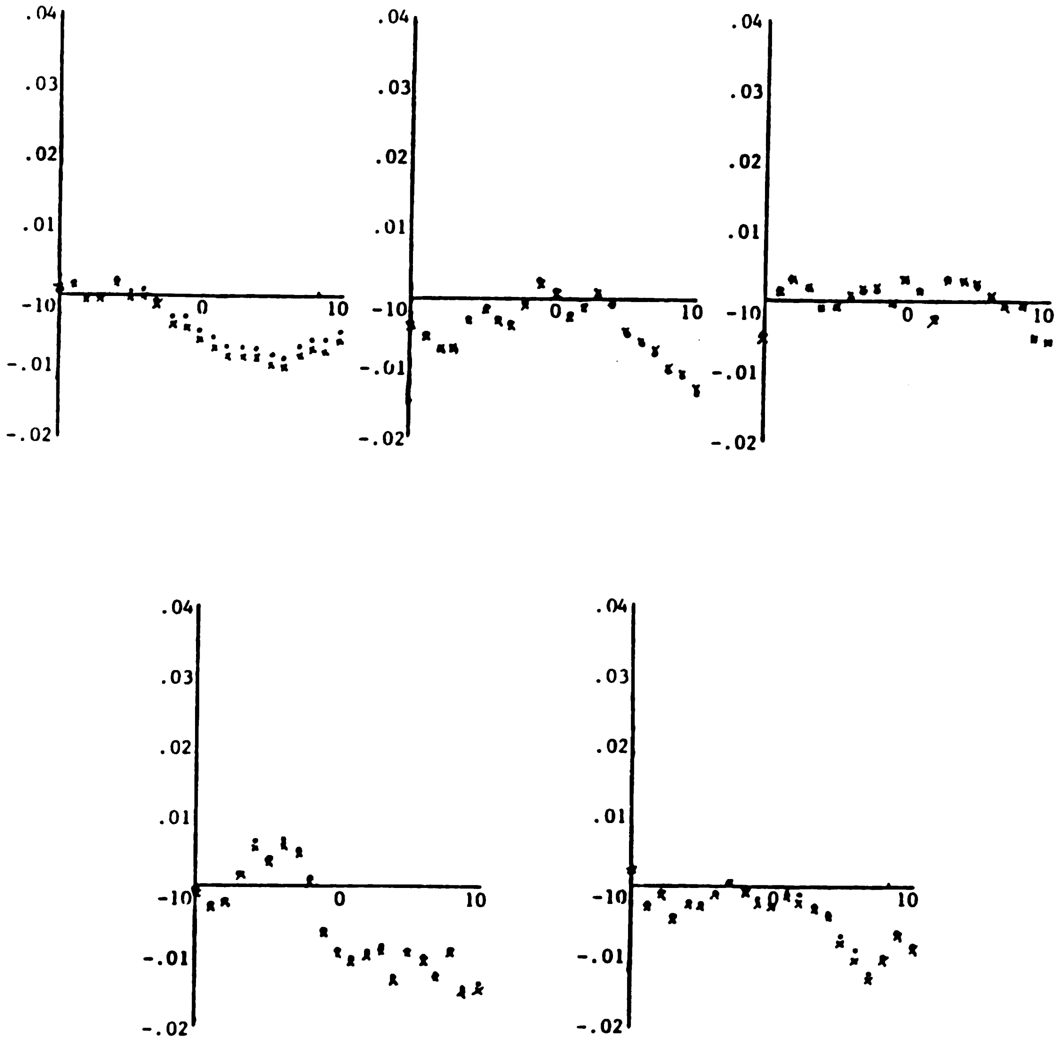


Figure 6.2 CAR Plots Value Weighted Index

VII. SUMMARY AND CONCLUSIONS

Studies conducted prior to this dissertation have established that OLS beta estimated using daily returns are both biased and inconsistent. This occurs because daily returns are computed over nonsynchronous time periods. A consistent estimator was established by Scholes and Williams [1977]. Scholes and Williams theorized that OLS betas would be underestimated for inactively traded securities and overestimated for all other securities. The effects of nonsynchronous time periods on estimation have been well established. The purpose of this study was to investigate the effects of nonsynchronous time periods on the prediction of security returns and, in turn efficient market tests.

The results of a theoretical analysis, for infinitely large samples, were that prediction errors were smaller when a consistent estimator was used. The reduction of the error, however, was very small. The theoretical results do not necessarily hold for small samples, and a simulation program was developed to generate return data which could be used to investigate the effects of nonsynchronous time periods when smaller samples were used.

The simulated return data were used to obtain estimates of beta and alpha employing both OLS and the S&W procedure (consistent estimators). These estimators were then used to predict returns, and the prediction errors were computed. The prediction errors resulting from the case of OLS estimators were generally slightly smaller than the prediction errors resulting from the S&W estimators. However, at sample sizes of approximately 1000 days, the prediction errors utilizing the S&W estimators were slightly smaller more often than the error resulting from the OLS estimators when the firm had a low average-number-of-transactions per day. The OLS estimators resulted in smaller prediction errors for firms with an average average-number-of-transactions or a high average-number-of-transactions more often than the S&W estimators.

Efficient market tests were also performed using the simulated return data. These tests consisted of a t-test and a cumulative average residual test (CAR). Known levels of abnormal performance were added to the returns for one day (an event day). The residuals were then calculated employing the two estimators, OLS and S&W. The t-tests were performed on both sets of residuals, the residuals resulting from the OLS estimators and the residuals resulting from the S&W estimators. If the group of firms used in the efficient market test contained a large number of firms with a low average-number-of-transactions per day, then the t-statistics resulting when the S&W estimators were utilized were slightly greater. If the group of firms did not contain the firms with the low average-number-of- transactions, then the t-statistics computed from average residuals resulting from the OLS estimators were slightly larger. However, the differences in t-statistics were very small, and the conclusions of the test would be the same regardless of which estimator was used. A CAR test was also conducted using the two sets of residuals, but this test was not affected by the choice of estimator.

The same tests that were performed using the simulated return data were performed using real return data for 283 NYSE firms. Each firms' average-number-of-transactions per day for the period January through March 1981 were known. Prediction errors resulting from the OLS estimators were smaller for a majority of the firms when the value weighted market index was used. The use of the S&W estimators resulted in the prediction errors being smaller when the firms contained approximately the average average-number-of-transactions and when the equally weighted market was used. However, overall, the prediction errors were smaller for the majority of firms when the OLS estimators were utilized regardless of what index was employed. The t-test and CAR test results were approximately the same as the simulation results. The t-statistics resulting from S&W estimators were slightly greater only when the group of firms used in the test contained predominately firms with a low average-number-of-transactions, but the difference in t-statistics was very small. The use of S&W estimators did not improve the CAR tests when the firms were chosen randomly, but if the equally weighted market index was used, the CAR's for groups containing a homogeneous average-number-of-transactions resulted in a slight improvement.

The evidence obtained in this study indicates that the prediction of security returns is not improved by S&W estimators. The conclusions of efficient market tests will not be altered unless the group of firms chosen contain mostly inactively traded securities; however, the differences in t-statistics were small and probably would not alter the conclusion. The implications for previous research is that the conclusions of those studies which did not account for nonsynchronous time periods still hold, and those that did account for the problem did not need to account for it. The implications for future research are that the OLS estimators are the only estimators necessary. The fact that the beta can be altered substantially by the S&W procedure has been previously established, but the impact of the difference in the OLS beta and the S&W beta has an extremely small affect on the prediction error, and, in turn, efficient market tests.

A need for further research in this area still exists. In current literature, the question of the effects of firm size on beta has been investigated (Banz [1981]). It may be that size is a surrogate for the average number of transactions and, in fact, there isn't a size effect, but only a nonsynchronous time effect. In other words, a relationship between size and the average number of trades may exist, and this may in part explain the size of effect.

The model in the current study is based on Merton's [1971] continuous time model. However, there has been discussion as to whether or not continuous time is an appropriate model (Oldfield and Rogalski [1980], French [1980]). The discussion centers on the weekend effect. The weekend effect refers to the fact that a high percentage of Mondays have negative returns. It may be possible to develop a consistent beta, based on a model other than the continuous time model, which results in better predictions than the S&W beta. Therefore, more work could be done in this area.

APPENDIX

APPENDIX 1

Programs Used in the Simulation

This appendix is composed of a brief description of each program used in the simulation followed by the fortran code for those programs.

Program GENPROS

Program GENPROS generates daily returns and nonsynchronous time periods from a random number generator. This program uses the returns generated to compute a market, OLS beta from both returns with nonsynchronous time periods and returns with equal time periods, and an S&W beta with nonsynchronous time periods. Subroutine POISSN generates time periods using a poissin distribution. Subroutine BETA computes the various betas given the returns generated in GENPROS. Subroutine MULVAR computes a vector of multivariate normal random deviates. Subroutine SIGMAI computes the square root of the variance-covariance matrix. Subroutine SORTT sorts the firms by transaction times.

Program RHO

Program RHO computes the theoretical autocorrelation of the market given a variance-covariance matrix and the average number of transactions per day for each firm.

Program MSPE

Program MSPE computes the mean squared prediction error for both the S&W and OLS estimators. This program also performs the Brown and Warner efficient market test procedures.

PROGRAM GENPROS

```
      PROGRAM GENPROS(INPUT,OUTPUT,TAPE1,TAPE2,TAPE5=INPUT,TAPE6=OUTPUT)
      COMMON/SUMS/SUMYYP(20),SUMXP,SUMYXM(20),SUMXM,SUMY(20),SUMXY(20),S
     *UMX,SUMXXP,SUMX2,SUMY2(20),SUMYYP(20),SUMYA(20),SUMXYA(20),SUMXA2,
     *SUMXXPA,SUMXA,SUMYYPA(20),SUMYA2(20),SUMYP(20),SUMYPA(20),SUMXPA
      COMMON/RMA/RMA(4)
      COMMON/RMO/RMO(4)
      COMMON /RA/RA(50,4),R01A(100,2)
      COMMON/RO/RO(50,4),R01(100,2)
      COMMON/BETA/BO(20),BA(20),BSW(20)
      COMMON/VAR/COVOXY,COVXYM,COVXYP,COVAXY,VAROX,VARAX,COVXXP,ROBAR,RM
     *ABAR,RMOBAR,RABAR,COVAAP
      COMMON/PR/PR(100)
      COMMON/T/T(100),ST(100),NFIRM(100),F(100)
      COMMON/Z/Z1(100),Z2(100)
      COMMON/SIGMA/RI(100),SIGMA(210),U(20),C(20,20)
      DIMENSION RVEC(1,20),WKVEC(3)
      DOUBLE DSEED
      DATA C/400*0/
      DATA U/20*0/
      DATA R01/80*0/
      DATA R01A/80*0/
      DATA Z1/100*0/
      DATA Z2/100*0/
      DATA BO/20*0/
      DATA BA/20*0/
      DATA BSW/20*0/
      DATA RA/80*0/
      DATA RO/80*0/
      DATA RMO/4*0/
      DATA RMA/4*0/
      DATA SUMYA2/20*0/
      DATA SUMYPA/20*0/
      DATA SUMYYPA/20*0/
      DATA SUMYA/20*0/
      DATA SUMXYA/20*0/
      DATA SUMYP/20*0/
      DATA SUMYYP/20*0/
      DATA SUMY2/20*0/
      DATA SUMXY/20*0/
      DATA SUMY/20*0/
      DATA SUMYXM/20*0/
      DATA SUMYXP/20*0/
      SUMXPA=0.0
      SUMXXPA=0.0
      SUMXA2=0.0
      SUMXA=0.0
      SUMXXP=0.0
      SUMX2=0.0
      SUMX=0.0
      SUMXM=0.0
      SUMXP=0.0
      DSEED=123.00
      NR=1
      KL=?
      IR=1
      WKVEC(1)=1.0
      KOUNT=0
      READ(5,10)P0,A,S,RAND
      DO 410 I=1,20
      PR(I)=P0
  410 CONTINUE
   10 FORMAT(4F11.6)
      CALL RANSET(RAND)
      DO 30 I=1,20
      READ(5,31)P(I)
   31 FORMAT(F10.5)
   30 CONTINUE
      CALL SIGMAI(S)
      WRITE(6,405)U
      KNT=101
   20 KOUNT=KOUNT+1
      IF(KOUNT.GT.5201)GO TO 999

      LK=3
      DO 622 LL=1,3
      RMO(LL)=RMO(LL+1)
      RMA(LL)=RMA(LL+1)
```

PROGRAM GENPROS

```
      LK=LK-1
  622 CONTINUE
      RMO(4)=RMA(4)=0.0
      DO 520 LL=1,20
      Z1(LL)=0.0
      Z2(LL)=0.0
      LK=3
      DO 527 LP=1,3
      RO(LL,LP)=RO(LL,LP+1)
      RA(LL,LP)=RA(LL,LP+1)
      LK=LK-1
  527 CONTINUE
  520 CONTINUE
      CALL POISSN(X,RAND)
      CALL SORTT
      TD=0.0
      DO 200 I=1,20
      CALL MULVAR(RAND,RVEC)
      TS=T(I)-TD
      DO 210 J=I,20
       Z1(NFIRM(J))=RVEC(1,NFIRM(J))*(TS**.5)+Z1(NFIRM(J))
  210 CONTINUE
      TD=T(I)
      IF(I.EQ.1)GO TO 203
      IP=I-1
      DO 220 J=1,IP
      Z2(NFIRM(J))=RVEC(1,NFIPM(J))*(TS**.5)+Z2(NFIRM(J))
  220 CONTINUE
  203 CONTINUE
  200 CONTINUE
      CALL MULVAR(RAND,RVEC)
      TS=1.0-T(20)
      DO 230 J=1,20
      Z2(NFIRM(J))=RVEC(1,NFIRM(J))*(TS**.5)+Z2(NFIRM(J))
  230 CONTINUE
      DO 250 N=1,20
      DO 260 NN=1,20
      IF(N.EQ.NFIRM(NN))MM=NN
  260 CONTINUE
      X=(((A*BI(N))-(U(N)/2.0))*T(MM))+Z1(N)
      XST=(((A*BI(N))-(U(N)/2.0))*ST(N))+Z2(N)
      PS=PB(N)*(EXP(X))
      PST=PS*(EXP(XST))
      RO1(N,2)=(ALOG(PS))-(ALOG(PB(N)))
      RO1A(N,2)=(ALOG(PST))-(ALOG(PS))
  405 FORMAT(' ',6F11.6)
      PB(N)=PST
      RMA(4)=RMA(4)+RO1(N,2)+RO1A(N,2)
      RMO(4)=RMO(4)+RO1(N,2)+RO1A(N,1)
      RA(N,4)=RO(N,4)=0.0
      RA(N,4)=RO1(N,2)+RO1A(N,2)
      RO(N,4)=RO1(N,2)+RO1A(N,1)
      RO1(N,1)=RO1(N,2)
      RO1A(N,1)=RO1A(N,2)
  250 CONTINUE
      RMO(4)=RMO(4)/20.0
      RMA(4)=RMA(4)/20.0
      IF(KOUNT.LT.3)GO TO 20
      CALL BETA(KOUNT,KNT)
      NN=0
      DO 50 K=1,20
      BET=BSW(K)-BI(K)
      B=ABS(BET)
      IF(B.LT..01)NN=NN+1
   50 CONTINUE
      IF(KOUNT.GT.5200)GO TO 999
      IF(NN.GT.19)GO TO 999
      GO TO 20
  999 CONTINUE
      END
```

SUBROUTINE POISSN

```
      SUBROUTINE POISSN(X,RAND)
      COMMON/T/T(100),ST(100),NFIRM(100),P(100)
      DO 20 I=1,20
      T(I)=0.0
      X=0.0
      A=-1.0*P(I)
      B=EXP(A)
      TR=1.0
    5 R=RANF(RAND)
      TR=TR*R
      IF(TR-B)10,8,8
    8 X=X+1.0
      T(I)=T(I)+(-1.0*(ALOG(R)))
      GO TO 5
   10 T(I)=T(I)/P(I)
      ST(I)=1.0-T(I)
   20 CONTINUE
      RETURN
      END
```

SUBROUTINE BETA

```
      SUBROUTINE BETA(KOUNT,KNT)
      COMMON/SUMS/SUMYXP(20),SUMXP,SUMYXM(20),SUMXM,SUMY(20),SUMXY(20),S
     *UMX,SUMXXP,SUMX2,SUMY2(20),SUMYYP(20),SUMYA(20),SUMXYA(20),SUMXA2,
     *SUMXXPA,SUMXA,SUMYYPA(20),SUMYA2(20),SUMYP(20),SUMYPA(20),SUMXPA
      COMMON/RMA/RMA(4)
      COMMON/RMO/RMO(4)
      COMMON /RA/RA(50,4),RC1A(100,2)
      COMMON/SIGMA/SI(100),SIGMA(210),U(20),C(20,20)
      COMMON/RO/RO(50,4),RO1(100,2)
      COMMON/BETA/BO(20),BA(20),BSW(20)
      COUNT=KOUNT
      SUMXP=SUMXP+RMO(4)
      SUMXM=SUMXM+RMO(2)
      SUMX=SUMX+RMO(3)
      SUMX2=SUMX2+(RMO(3)**2)
      SUMXXP=SUMXXP+(RMO(3)*RMO(4))
      SUMXA=SUMXA+RMA(3)
      SUMXA2=SUMXA2+(RMA(3)**2)
      SUMXXPA=SUMXXPA+(RMA(3)*RMA(4))
      SUMXPA=SUMXPA+RMA(4)
      DO 20 I=1,20
      SUMYXP(I)=SUMYXP(I)+(RO(I,3)*RMO(4))
      SUMYXM(I)=SUMYXM(I)+(RO(I,3)*RMO(2))
      SUMY(I)=SUMY(I)+RO(I,3)
      SUMXY(I)=SUMXY(I)+(RMO(3)*RO(I,3))
      SUMY2(I)=SUMY2(I)+(RO(I,3)**2)
      SUMYYP(I)=SUMYYP(I)+(RO(I,3)*RO(I,4))
      SUMYP(I)=SUMYP(I)+RO(I,4)
      SUMXYA(I)=SUMXYA(I)+(RA(I,3)*RMA(3))
      SUMYA(I)=SUMYA(I)+(RA(I,3))
      SUMYYPA(I)=SUMYYPA(I)+(RA(I,3)*RA(I,4))
      SUMYPA(I)=SUMYPA(I)+RA(I,4)
      SUMYA2(I)=SUMYA2(I)+(RA(I,3)**2)
   20 CONTINUE
      IF(KOUNT.NE.KNT)GO TO 999
      CNT=COUNT-2.0
      KNT=KNT+300
      VARX=SUMX2-((SUMX**2)/CNT)
      RHOM=(SUMXXP-((SUMXP*SUMX)/CNT))/VARX
      VARXA=SUMXA2-((SUMXA**2)/CNT)
      RHOMA=(SUMXXPA-((SUMXPA*SUMXA)/CNT))/VARXA
      DO 30 I=1,20
      BP=(SUMYXP(I)-((SUMXP*SUMY(I))/CNT))/VARX
      BM=(SUMYXM(I)-((SUMXM*SUMY(I))/CNT))/VARX
      B=(SUMXY(I)-((SUMX*SUMY(I))/CNT))/VARX
      VARY=SUMY2(I)-((SUMY(I)**2)/CNT)
      RHOP=(SUMYYP(I)-((SUMYP(I)*SUMY(I))/CNT))/VARY
      BA(I)=(SUMXYA(I)-((SUMYA(I)*SUMXA)/CNT))/VARXA
      BO(I)=B
      BSW(I)=(B+BM+BP)/(1+(2*RHOM))
      AACT=(SUMYA(I)-BA(I)*SUMXA)/CNT
      AONE=(SUMY(I)-SUMX)/CNT
      ASW=(SUMY(I)-BSW(I)*SUMX)/CNT
      AOLS=(SUMY(I)-BO(I)*SUMX)/CNT
      WRITE(6,40)I,KNT,BA(I),BO(I),BSW(I),RHOM,AACT,ASW,AOLS
   40 FORMAT(' ',2I6,'   BACT',F11.8,'   BOLS',F11.8,'   BSW',F11.8,'   RHO'
     *,3(F11.8,1X),F11.8)
      WRITE(1,50)BO(I),BSW(I),AOLS,ASW,KNT,AONE
   50 FORMAT(4F11.8,I5,F11.8)
   30 CONTINUE
  999 CONTINUE
      RETURN
      END
```

SUBROUTINE MULVAR

```
      SUBROUTINE MULVAR(RAND,RVEC)
      COMMON/SIGMA/RI(100),SIGMA(210),U(20),C(20,20)
      DIMENSION RVEC(1,20),WV(20)
      PI=22.0/7.0
      K=0
      DO 10 I=1,10
      R1=RANF(RAND)
      R2=RANF(RAND)
      Q=2.0*PI*R2
      K=K+1
      WV(K)=((-2.0*(ALOG(R1)))**.5)*(COS(Q))
      K=K+1
      WV(K)=((-2.0*(ALOG(R1)))**.5)*(SIN(Q))
   10 CONTINUE
      DO 20 I=1,20
      RVEC(1,I)=0.0
      DO 30 J=1,20
      RVEC(1,I)=(C(I,J)*WV(J))+RVEC(1,I)
   30 CONTINUE
   20 CONTINUE
      RETURN
      END
```

SUBROUTINE SIGMAI

```
      SUBROUTINE SIGMAI(S)
      COMMON/SIGMA/BI(100),SIGMA(210),U(20),C(20,20)
      DIMENSION KK(20)
      M=1
      K=0.0
      DO 20 I=1,20
      DO 30 J=1,I
      K=K+1
      READ(2,435)SIGMA(K)
  435 FORMAT(F10.8)
      SIGMA(K)=SIGMA(K)*.0000078712
      IF(J.EQ.1)GO TO 305
      IF(J.LT.I)GO TO 310
      IF(J.EQ.I)GO TO 315
  305 Q=SIGMA(1)**.5
      C(I,J)=SIGMA(K)/Q
      GO TO 320
  310 L=J-1
      Y=0.0
      DO 330 KM=1,L
      Y=(C(I,KM)*C(J,KM))+Y
  330 CONTINUE
      C(I,J)=(SIGMA(K)-Y)/C(J,J)
      GO TO 320
  315 L=I-1
      Y=0.0
      DO 335 KM=1,L
      Y=(C(I,KM)**2)+Y
  335 CONTINUE
      C(I,I)=(SIGMA(K)-Y)**.5
  320 CONTINUE
  321 FORMAT(' ',2I5,F13.10)
   30 CONTINUE
   20 CONTINUE
      K=210
      I=21
      DO 100 L=1,20
      KK(L)=K-1
      I=I-1
      DO 110 J=L,20
      U(I)=U(I)+SIGMA(K)
      K=K-1
  110 CONTINUE
      IF(L.LT.2)GO TO 100
      M=L-1
      DO 120 LL=1,M
      U(I)=U(I)+SIGMA(KK(LL))
      KK(LL)=KK(LL)-1
  120 CONTINUE
  100 CONTINUE
      DO 200 I=1,20
      U(I)=U(I)/20.0
      BI(I)=U(I)/S**2
  200 CONTINUE
      RETURN
      END
```

SUBROUTINE SORTT

```
      SUBROUTINE SORTT
      COMMON/T/T(100),ST(100),NFIRM(100),P(100)
      DO 10 I=1,20
      NFIRM(I)=I
   10 CONTINUE
      DO 350 L=1,19
      DO 310 M=1,19
      IF(T(M).LE.T(M+1))GO TO 310
      TG=T(M)
      KNT=NFIRM(M)
      T(M)=T(M+1)
      T(M+1)=TG
      NFIRM(M)=NFIRM(M+1)
      NFIRM(M+1)=KNT
  310 CONTINUE
  350 CONTINUE
      RETURN
      END
```

PROGRAM RHO

```
      PROGRAM RHO(INPUT,OUTPUT,TAPE1,TAPE2,TAPE5=INPUT,TAPE6=OUTPUT)
      DIMENSION T(100)
      VARS=VAR=0.0
      READ(5,10)IFIRM
   10 FORMAT(I5)
      DO 20 I=1,IFIRM
      READ(5,30)T(I)
   30 FORMAT(F10.5)
   20 CONTINUE
      DO 40 I=1,IFIRM
      DO 50 J=1,I
      READ(2,435)SIGMA
  435 FORMAT(F10.8)
      IF(I.EQ.J)GO TO 60
      G=(1.0/T(I))-(1.0/T(J))
      X=ABS(G)
      X=1.0-X
      VAR=VAR+(2.0*SIGMA)
      VARS=VARS+(2.0*SIGMA*X)
      GO TO 50
   60 VAR=VAR+SIGMA
      VARS=VARS+SIGMA
   50 CONTINUE
      RHOM=.5*((VAR/VARS)-1.0)
      WRITE(6,70)I,T(I),RHOM
   70 FORMAT(* FIRM*,I5,* TRANS.*,F7.2,* RHO*,F11.8)
   40 CONTINUE
      END
```

PROGRAM MPSE

```
      PROGRAM MPSE(INPUT,OUTPUT,TAPE1,TAPE2,TAPE5=INPUT,TAPE6=OUTPUT,
     *TAPE3)
      COMMON /FMT/ RMT(100)
      COMMON /RNT/ RNT(100)
      COMMON /ER/ EOLS(100),ESW(100)
      DIMENSION ROLS(100),RSW(100)
      DIMENSION BOLS(18,20),BSW(18,20),AOLS(18,20),ASW(18,20),KNT(18)
     *,AONE(18,20)
      DIMENSION VARASW(18),VAROLS(18),CARSW(18,21),POLS(18),PSW(18),
     *CAROLS(18,21)
      DATA VARASW/18*0/
      DATA VAROLS/18*0/
      DATA CARSW/378*0/
      DATA CAROLS/378*0/
      DATA POLS/18*0/
      DATA PSW/18*0/
      READ(5,30)IYEAR,IFIRM,NUM
   30 FORMAT(3I5)
      QNUM=NUM
      DO 901 MM=1,7
      DO 940 M=1,18
      VARASW(M)=VAROLS(M)=POLS(M)=PSW(M)=0.0
      DO 941 N=1,21
      CARSW(M,N)=CAROLS(M,N)=0.0
941   CONTINUE
940   CONTINUE
      READ(5,400)IFLG,AFF
  400 FORMAT(I5,F11.8)
      KOUNT=0
      BUFFER IN(2,1)(RMT(1),RMT(100))
      IF(UNIT(2))10,999,999
   10 CONTINUE
      DO 201 I=1,IYEAR
      DO 202 J=1,IFIRM
      READ(3,200)BOLS(I,J),BSW(I,J),AOLS(I,J),ASW(I,J),KNT(I),AONE(I,J)
  200 FORMAT(4F11.8,I5,F11.8)
  202 CONTINUE
  201 CONTINUE
      DO 20 I=1,IFIRM
      BUFFER IN(2,1)(RNT(1),RNT(100))
      IF(UNIT(2))40,999,999
   40 CONTINUE
      IF(I.GT.5.AND.I.LE.15)GO TO 20
      IF(IFLG.LT.2)GO TO 410
      DO 420 M=80,100
      RNT(M)=RNT(M)+AFF
  420 CONTINUE
      GO TO 430
  410 RNT(90)=RNT(90)+AFF
  430 CONTINUE
      WRITE(6,50)I,RNT(1),RNT(5)
   50 FORMAT(* *,I5,3X,2F15.12)
      DO 65 K=1,IYEAR
      SONE=0.0
      SWSE=0.0
      OLSSE=0.0
      DO 60 J=1,100
      RONE=RMT(J)+AONE(K,I)
      EONE=RONE-RNT(J)
      SONE=SONE+EONE**2.0
      ROLS(J)=RMT(J)*BOLS(K,I)+AOLS(K,I)
      EOLS(J)=RNT(J)-ROLS(J)
      RSW(J)=RMT(J)*BSW(K,I)+ASW(K,I)
      ESW(J)=RNT(J)-RSW(J)
      SWSE=ESW(J)**2+SWSE
      OLSSE=EOLS(J)**2+OLSSE
   60 CONTINUE
      SONE=SONE/99.0
      SWSE=SWSE/99.0
      OLSSE=OLSSE/99.0
      FONE=SONE/SWSE
      FSTAT=OLSSE/SWSE
      IF(MM.GT.1)GO TO 905
      WRITE(6,70)SWSE,OLSSE,FSTAT,KNT(K),SONE,FONE
   70 FORMAT(* *,*SWSE*,F15.10,* OLSSE*,F15.10,* FSTAT*,F15.10,2X,I5,10X
     *,*SONE*,F15.10,* FSTAT*,F15.10)
905   CONTINUE
```

PROGRAM MPSE

```
      AVSW=0.0
      AVOLS=0.0
      DO 300 N=1,79
      AVSW=AVSW+ESW(N)
      AVOLS=AVOLS+EOLS(N)
  300 CONTINUE
      AVSW=AVSW/79.0
      AVOLS=AVOLS/79.0
      VOLS=0.0
      VSW=0.0
      DO 310 N=1,79
      VOLS=VOLS+(EOLS(N)-AVOLS)**2
      VSW=VSW+(ESW(N)-AVSW)**2
  310 CONTINUE
      VOLS=VOLS/77.0
      VSW=VSW/77.0
      VARASH(K)=VARASH(K)+VSW
      VAROLS(K)=VAROLS(K)+VOLS
      POLS(K)=POLS(K)+EOLS(90)
      PSW(K)=PSW(K)+ESW(90)
      M=0
      DO 320 N=80,100
      M=M+1
      CARSW(K,M)=CARSW(K,M)+ESW(N)
      CAROLS(K,M)=CAROLS(K,M)+EOLS(N)
  320 CONTINUE
   65 CONTINUE
   20 CONTINUE
      DO 340 N=1,IYEAR
      POLS(N)=POLS(N)/QNUM
      PSW(N)=PSW(N)/QNUM
      VARASH(N)=(VARASH(N)**.5)/QNUM
      VAROLS(N)=(VAROLS(N)**.5)/QNUM
      TSOLS=POLS(N)/VAROLS(N)
      TSSW=PSW(N)/VARASH(N)
      CARSW(N,1)=CARSW(N,1)/QNUM
      CAROLS(N,1)=CAROLS(N,1)/QNUM
      DO 350 M=2,21
      CARSW(N,M)=CARSW(N,M-1)+CARSW(N,M)/QNUM
      CAROLS(N,M)=CAROLS(N,M-1)+CAROLS(N,M)/QNUM
  350 CONTINUE
      WRITE(6,351)N,TSOLS,TSSW,(CAROLS(N,M),M=1,21),(CARSW(N,M),M=1,21)
  351 FORMAT(' ',I5,2F11.8,3X,7F11.8,5(/,31X,7F13.10))
  340 CONTINUE
      REWIND 2
      REWIND 3
901   CONTINUE
  999 CONTINUE
      END
```

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