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DETERMINANTS OF FOOD CONSUMPTION IN RURAL SIERRA LEONE:
ESTIMATION OF A HOUSEHOLD-FIRM MODEL WITH APPLICATION
OF THE QUADRATIC EXPENDITURE SYSTEM

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John A. Strauss

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of the requirements for

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Economics

Carl K Eicher
Major professor

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By

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ABSTRACT

DETERMINANTS OF FOOD CONSUMPTION IN RURAL SIERRA LEONE: ESTIMATION OF A HOUSEHOLD-FIRM MODEL WITH APPLICATION OF THE QUADRATIC EXPENDITURE SYSTEM

By

John A. Strauss

This dissertation reports the derivation, specification and estimation of a household-firm model. The model is block recursive. First production decisions are made by maximizing short-run profits subject to a production function. These output and variable input values are then substituted into the budget constraint, which equates the sum of values of excess supply of goods and of labor to zero. The household then maximizes its utility subject to the budget constraint, and to a time constraint equating total time available to leisure plus labor time.

The data used are household level cross-section data from rural Sierra Leone. Price variation exists by region, permitting estimation of price effects on consumption and on output supply and labor supply and demand.

The household consumption-leisure choice component of the model (with profits held fixed) is estimated using a Quadratic Expenditure System with demographic variables. Seven commodities are used in the system: five foods, nonfood and household labor supply. This involves estimation of forty-two parameters by numerical maximum likelihood techniques.

Attention is paid to whether random disturbances on the expenditure system are distributed identically across households. They are found not to be, and this is incorporated into the estimation procedure. Engel

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curves are found to be significantly nonlinear; with marginal total expenditure on rice, the major staple, declining with higher total expenditure. Most foods are found to be reasonably price responsive with sizeable own price substitution effects, declining with higher expenditure. Aggregate labor supply is found to be price inelastic.

A system of output supply and labor demand functions is estimated. Six outputs are used, the same as used on the demand side. The production function used to derive these equations is a Constant Elasticity of Transformation - Cobb-Douglas function. The output data are censored; some households do not produce all outputs. The Tobit model is used to statistically account for this. Disturbances attached to different equations are assumed to be independent. This avoids the need to evaluate up to quintuple integrals, a very expensive procedure (possibly prohibitively so), allowing us to evaluate only single integrals, a manageable task.

Output elasticities with respect to own price are small, being under .5. The wage elasticity of labor demand is larger in absolute value, being less than minus one.

The results of the entire household-firm model are derived. The changes in consumption resulting from changes in total income when profits are allowed to vary in response to price changes are computed. In elasticity form these are important, being largest for lower expenditure households. These elasticities are then used in computing total elasticities of consumption with respect to price. The own price effects remain negative, except for root crops and other cereals for low expenditure households. The elasticities for low expenditure households are no longer higher in absolute value than for high expenditure households. Also, cross price elasticities are both positive and sizeable. Price elasticities c

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elasticities of marketed surplus are computed. Own price elasticities are all positive and sizeable, much higher than the output supply elasticities.

Effects of total expenditure and of prices on calorie availability are then computed using conversions from food composition tables. Elasticities of calorie availability with respect to total expenditure are found to be roughly .85, varying little by expenditure group. Price elasticities of calorie availability are generally positive, except with respect to rice and oils and fats prices for middle and high expenditure groups. For rice price the elasticity is around -.25 for the higher two expenditure groups, but .2 for the low expenditure group.

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ACKNOWLEDGEMENTS

This dissertation was written under the auspices of the project "Consumption Effects of Economic Policy," funded by the Agency for International Development (contract number AID/DSAN-C-0008). I am indebted to Professors Victor Smith (the project director) and Peter Schmidt for their patient assistance and their encouragement. Without their willingness to respond to ideas and to give a great deal of counsel this study would not have come to fruition. Thanks are due to Professor Carl Eicher for his interest in me these past five years and for his encouragement to work on this project. Professors Lindon Robison and Norman Obst have also benefited me greatly through their teaching and their general interest in my progress.

No study such as this can be conducted without an enormous amount of computer work. I received an enormous amount of programming assistance from Paul Wolberg and from George Sionakides. Also, Susan Chu provided programming help. Chris Wolf made available extra funds to use for computer work, as did Peter Schmidt.

An enormous amount of time went into data preparation. William Whelan and Victor Smith made particularly invaluable contributions. Will Whelan edited the data on market purchases among other work. Dr. Smith located information allowing us to express food consumption and production in terms of standard units of measurement. Both worked on obtaining conversions of quantities of foods into nutrients, using

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food composition tables. Derek Byerlee and Dunstan Spencer were generous with their time in answering questions concerning the raw data.

I give the largest share of credit for this study to my family. My parents have been a wonderful source of support. My daughter, Jennifer, has given to me much happiness and helped me to rediscover my humanness after having been a graduate student for so long. Finally, I thank my loving wife, Anna-Marie. She has sacrificed more than any other person these past seven years so that I might complete my formal education. This dissertation is dedicated to her.

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CHAPTER 1

SCOPE OF RESEARCH

Government policies affect the nutritional status of different population groups, sometimes intentionally but far more often without forethought. The nutritional well being of people, particularly persons with low income, has become an important consideration for governments of less developed countries. However, it is rare that policy planners have much indication how different policies will affect food consumption and thereby nutritional well being. This is especially so for people who operate their own firms and who can adjust outputs and inputs as well as labor supplied and consumption of goods and services in response to price and other socio-economic variables.

This dissertation is concerned with exploring the socio-economic determinants of food consumption of rural households in Sierra Leone, households that produce foods (and other goods) as well as consume them. Knowing these relationships it would be possible to trace the impact of such determinants on availability of nutrients to the household, especially of calories. This knowledge in turn may be of help in designing policies to increase the availability of such nutrients, which will be a crucial part of improving the nutritional status of individuals.

The importance of nutrition in the development process is well documented by Berg (1973), Reutlinger and Selowsky (1976), Dandekar and Rath (1971) and others. Reutlinger and Selowsky demonstrate the import

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importance of going beyond averages and looking at income distribution when examining calorie availability. As one example: per capita grain availability in Bangladesh was only one percent lower in 1974-75, a year in which widespread starvation was reported, than in the previous year. While emergency food aid flows show up in those figures, per capita grain production was down only 4.7 percent (IFPRI, 1977). Clearly, some people were much harder hit than others.

Of the economic variables, effects of prices and income on food intake come first to mind. Since calories come from all food sources, to trace the effects of prices and income on total caloric availability one needs to trace their effect on the consumption of all foods. This calls for a complete matrix of price and income elasticities, preferably different matrices for different income groups of households. Pinstrup-Anderson, de Londono and Hoover provide this for a set of urban households in Colombia using a method proposed by Frisch (1959) which uses only income elasticities, but at the expense of making extremely restrictive assumptions about household behavior. Others have derived such a matrix by estimating a complete system of demand equations. For rural households who produce goods as well as consume them, one needs to account for not only the direct effects of socio-economic variables on food consumption, but their indirect effects as well. The latter occur if the household is able to respond in its production patterns to changed socio-economic variables. That is, the rural household is both a producing and a consuming unit. This knowledge leads to use of so-called household-firm models in attempting to explain household food consumption behavior.

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Another concern of this research is to show that cross sectional data exhibiting geographic price variation can be successfully used in estimating both complete systems of demand equations and complete systems of output supply and input demand equations. Howe (1974) used cross section data in estimating systems of demand equations, but his data had no price variation so extraneous information had to be used to identify certain parameters statistically. Moreover, we show that systems allowing for a wide variety of behavior can be estimated when using a fair amount of commodity detail and including variables on demographic information.

The organization of the dissertation is as follows: Chapter 2 develops the household-firm model and makes it operational using a Quadratic Expenditure System (QES) and a multiple output Constant Elasticity of Transformation - Cobb-Douglas production function. How to incorporate household characteristic variables into the demand system is explored as is the effect of nonseparability of the utility function on the construction of aggregate prices.

Chapter 3 develops the general estimation procedures to be used and explores some possible econometric problems. Chapter 4 describes the data; both their preparation and sample characteristics. Chapter 5 reports results from estimating single equation demand regressions in share form as a vehicle for exploring which household characteristics to use in the demand system estimation. Chapter 6 reports the results of estimating the Quadratic Expenditure System and Chapter 7 does the same for the system of output supplies and input demands. For the latter, special econometric problems were encountered because many households specialized their production activities, producing none of several outputs. How this was handl

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was handled is discussed in detail. Chapter 8 uses parameter estimates from the demand and production sides of the household-firm model to trace the total effects of price and other variables on household consumption. It goes on to examine the effects of prices and total expenditure on caloric availability. Chapter 9 explores some implications of the model results for development in Sierra Leone and explores implications of the research for future modeling of household-firms.

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CHAPTER 2

DERIVATION AND SPECIFICATION OF HOUSEHOLD-FIRM MODEL

Introduction

In order to trace all the impacts of socio-economic variables on household food consumption it is necessary to account for those felt indirectly through influence on the production and labor supply activities of the household as well as directly on food consumption. This leads to modeling the household using so-called household-firm models. Economic models of household-firm behavior are not new. Seminal papers have been written by Nakajima (1969) and Jorgenson and Lau (1969). A further effort was provided by Lau and Yotopoulos (1974). All household firm models have a common structure of maximizing a utility function subject to three constraints: a production function and a time constraint and a budget constraint. Some models (e.g., Nakajima's subsistence model) hypothesize that markets do not exist and others (e.g., Jorgenson and Lau) explore intra-household distribution by using a social welfare function approach. These assumptions will be tailored to the problem at hand. For our purposes, we will assume households are semi-subsistence households. That is, they consume part of what they produce and sell the rest.

Derivation of the Household-Firm Model

Our unit of analysis is the household. We assume certainty and abstract from time. A household utility function is assumed with arguments being household consumption of various goods and of leisure.

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Goods may be bought or sold in the market and produced. Labor may be bought or sold in the market. Goods are produced using labor, land and fixed capital. Land is assumed fixed in total amount but must be distributed between uses. A time constraint exists equating household leisure plus labor time to total time available. Finally, a budget constraint exists equating the value of net product transactions plus exogenous income plus the value of net labor transactions to zero. Product prices and wage are taken exogenously by the household, markets are assumed to be perfectly competitive and family and hired labor are assumed perfect substitutes.

Formally, let the household maximize

$U = U(\bar{L}, X_i^c)$, where $\bar{L} \equiv$ leisure

$X_i^c \equiv$ good i consumed, $i=1, \ldots, n$

subject to: $G(X_i, L_T, D, \bar{K}) = 0$

$$X_i^c = X_i - S_i \qquad i=1, \ldots, n$$

$$S_L = L_H - L_T$$

$$\bar{L} = T - L_H$$

$$\sum_{i=1}^{n} p_i S_i + A + p_L S_L = 0$$

where $G(\cdot) \equiv$ implicit production function

$X_i \equiv$ production of good $i=1, \ldots, n$

$L_T \equiv$ total labor demanded

$D \equiv$ land

$\bar{K} \equiv$ fixed capital

$S_i \equiv$ net sales of good i (purchase if negative), $i=1, \ldots, n$

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S_L ≡ net sales of labor (purchase if negative)

A ≡ exogenous income

T ≡ total time available to household to allocate between labor and leisure

L_H ≡ total household labor time worked

p_i ≡ price of good i, i=1, . . ., n

p_L ≡ price of labor

Assume the utility function to be twice differentiable, increasing in its arguments and strictly quasi-concave. Assume the implicit production function to be twice differentiable, increasing in outputs, decreasing in inputs, and strictly quasi-convex. We will also assume interior solutions even though border solutions are easily handed algebraically (this is because estimation incroporating border conditions is very messy). We set up the Lagrangian function as

$$W = U(\bar{L}, X_i^c) + \lambda\left(\sum_{i=1}^{n} p_i(X_i - X_i^c) + A + p_L(T - \bar{L} - L_T)\right) + \mu(G(X_i, L_T, D, \bar{K})) \qquad (2.1)$$

Our first order conditions are:

$$\partial W/\partial X_i^c = \partial U/\partial X_i^c - \lambda p_i = 0 \qquad i=1, \ldots, n$$

$$\partial W/\partial \bar{L} = \partial U/\partial \bar{L} - \lambda p_L = 0$$

$$\partial W/\partial X_i = \lambda p_i + \mu\, \partial G/\partial X_i = 0 \qquad i=1, \ldots, n \qquad (2.2)$$

$$\partial W/\partial L_T = -\lambda p_L + \mu\, \partial G/\partial L_T = 0$$

$$\partial W/\partial \lambda = \sum_{i=1}^{n} p_i(X_i - X_i^c) + A + p_L(T - \bar{L} - L_T) = 0$$

$$\partial W/\partial \mu = G(X_i, L_T, D, K) = 0$$

These may be expressed in the more conventional way of equating marginal rates of substitution in consumption between goods to price ratios to marginal rates of transformation in production:

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$$(2.3)\quad \frac{\partial U/\partial X_i^c}{\partial U/\partial X_j^c} = \frac{p_i}{p_j} = \frac{\partial G/\partial X_i}{\partial G/\partial X_j} = \frac{-\partial X_j}{\partial X_i}\,, \quad i \neq j = 1, \ldots, n$$

$$\frac{\partial U/\partial \bar{L}}{\partial U/\partial X_i^c} = \frac{p_L}{p_i} = \frac{-\partial G/\partial L_T}{\partial G/\partial X_i} = \frac{\partial X_i}{\partial L_T}\,, \quad i = 1, \ldots, n$$

Graphically, for outputs, the household produces on its transformation function between two goods at the point at which the slope of the transformation curve equals relative market prices. Consumption is at the point of tangency between the same market possibilities line and the household indifference curves. Net marketed surpluses are measured by the usual trade triangles. In this case C-B of good j is sold and B-A of good i purchased. Between outputs and labor the same situation holds.

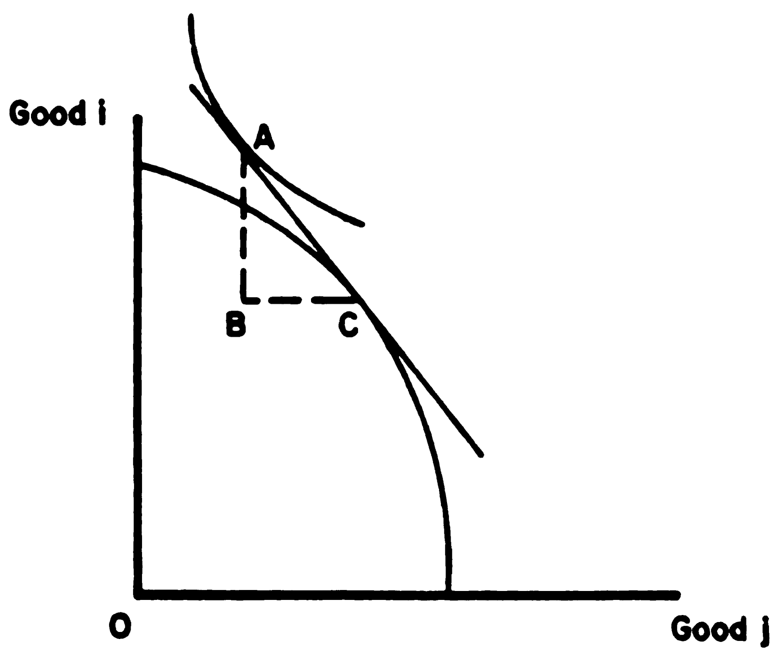

Figure 2.1

Household Equilibrium: Two Goods

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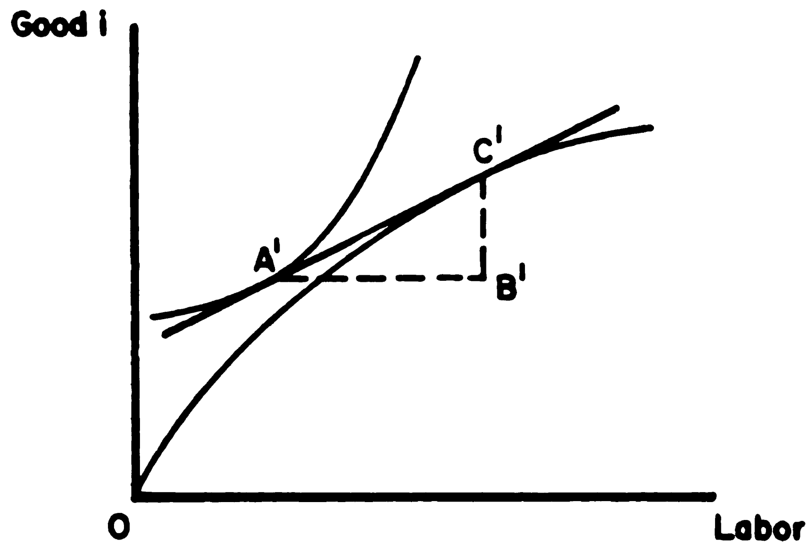

Figure 2.2

Household Equilibrium: Good and Labor

In the case pictured C'-B' of good i is sold and A'-B' of labor is hired.

An extremely important property of this model is that it is recursive. The household's production decisions are first made and subsequently used in allocating available "total income" between consumption of goods and leisure. This result is wholly dependent on the existence of markets for goods and labor. Intuitively this allows the family to separate its decisions on goods demanded and household goods supplied, the difference being hired (or sold out). This can be seen graphically in Figure 2.1 and 2.2. More formally, in the first order conditions, the partial derivatives with respect to outputs yield n equations in n+2 unknowns (n good outputs, total labor demanded and the ratio of two multipliers). Two more equations are added by the partial derivative with respect to total labor demanded and with respect to the multiplier of the implicit production function. This system of n+2 equations in n+2 unknowns can be solved in terms of all prices, the wage rate, fixed land and capital, the result of the quasi-

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the quasi-convexity of the implicit production function, first order conditions and the implicit function theorem. Such solutions may then be substituted into the budget constraint. That constraint plus the partial derivatives with respect to leisure and consumption of goods yields an additional n+2 equations in n+2 unknowns (n good consumptions, leisure and a multiplier), which may also be solved in terms of prices, the wage rate and nonearned income, since second order conditions are met.

Conditional on the production decisions this second set of n+2 equations is identical to the first order conditions of the labor-leisure choice problem. This, along with our assumptions about the utility function, implies that the usual constraints of economic theory apply: zero homogeneity of demand with respect to prices, wage rate and unearned income, and symmetry and negative semi-definiteness of the Slutsky substitution matrix. Likewise on the production side. The profit function (the profits equation after input demands and output supplies have been solved for in terms of prices of outputs and variable inputs and in terms of quantities of fixed inputs) is homogeneous of degree one in all prices and convex in prices.

When we later look at comparative static changes, from p_0-p_0 to p_1-p_1 in Figure 2.3, we can separate this movement into three parts. The total shift in consumption is from point A to point C. When we hold production fixed at point B, however, the household will be maximizing its utility by consuming at point E. The movement in consumption from point E to point C due to production moving from point B to point D we will later call the "profit effect." Rewriting the budget constraint, we have $A+\pi+p_L T-\Sigma p_i X_i^c - p_L \bar{L} = 0$, where $\pi = \Sigma p_i X_i - p_L L_T$ can be interpreted as short r

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as short run profits. When production changes in response to changing prices the effect on consumption will be caused by changing the X_js and L_T in the budget constraint, that is, by changing profits. The movement from point A to point E is the traditional labor-leisure choice model. It can be broken up into the traditional income and substitution effects (with real total income held constant).

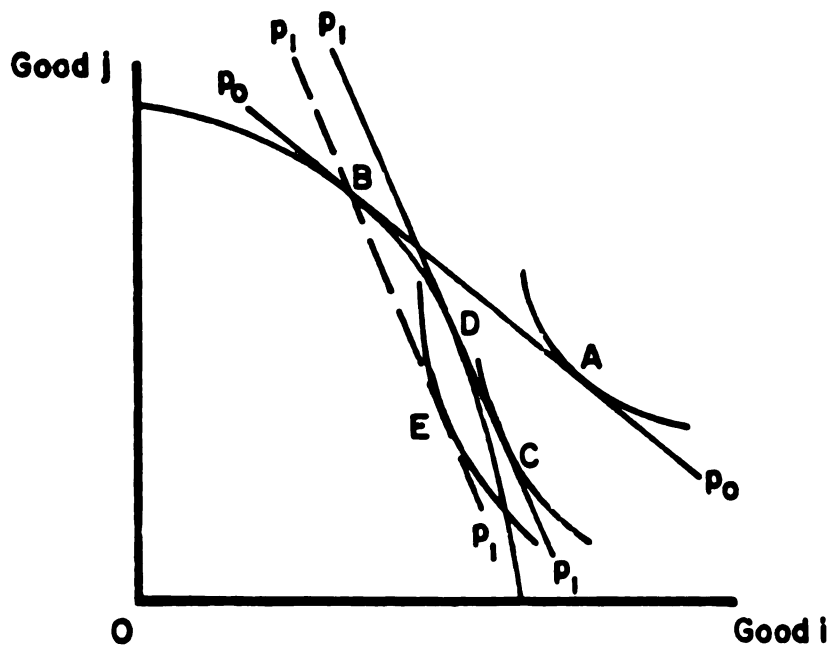

Figure 2.3

Effect of Price Change on Household Equilibrium

Specifying the Demand Side--The QES

When specifying the demand component of the household-firm model, we use systems of demand equations. Systems of demand equations relate an exhaustive set of expenditures to all prices and total expenditure (or income). Two broad approaches are used in specifying functional form. First, one can specify a particular functional form. This can be done either for the direct or indirect utility function, in which case one works fo

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works forward to derive the demand function; or for the demand functions, in which case one derives a class of direct or indirect utility functions giving rise that function. In doing so, three restrictions are generally imposed: an adding up of expenditure criterion, zero degree homogeneity in prices and expenditures, and symmetry of the Slutsky substitution matrix. Negative semi-definiteness of the substitution matrix is not imposed (though it could be) but is usually tested with the data upon estimation. These restrictions on parameters operate across demand equations as well as within each. This leads to one important advantage of systems estimation versus single equation estimation, that these cross equation restrictions may be incorporated into the estimation procedure. The adding up of individual expenditures to total expenditures (or total income in the household-firm model) results in the second advantage of systems estimation. Since both actual and predicted expenditures add to total expenditure a positive prediction error for one commodity must be offset by a negative error for another commodity. Hence, statistical errors are correlated between equations for a given household. Estimating a system can incorporate this fact leading to greater efficiency of the parameter estimates.

Alternatively to specifying a particular function, one can approximate an unknown direct or indirect utility function at a point to any desired degree of accuracy and derive the demand functions from the approximated utility function. Which approach one uses will depend on what relationships the research wants to highlight, number of observations available to use in estimation and so forth. As a general rule, approximating functions, when taken to the second degree of approximation as most have been thus far (e.g., trans-log or generalized Leontief), involve independent parameters to be estimated i

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estimated increasing as a multiple of the square of the number of commodities in the system. To decrease the number of parameters to be estimated additional constraints need to be placed on the system. Some specific functional forms have the number of parameters increasing as a multiple of the number of commodities included. This is achieved at the price of restrictions on the type of behavior admitted by that form. In general, the wider the range of behavior the functional form permits, the greater the number of parameters are.

One class of widely used expenditure equations is linear in income. Gorman (1961) has shown that this class of functions is generated by an indirect utility function of the form $V(p,y) = (y-f(p))/g(p)$, where $p \equiv$ vector of prices, $y \equiv$ expenditure and $f(p)$ and $g(p)$ are functions homogeneous of degree one, Pollak (1971a) derived the class of additive utility functions (of the form $U(x) = U(U_1(X_1)+U_2(X_2)+\ldots+U_n(X_n))$ giving rise to expenditure equations linear in income, one of which is the Klein-Rubin form $U(X) = \sum_{i=1}^{n} b_i \ln(x_i-c_i)$. This gives rise to the linear expenditure system:

$$(2.4) \quad p_iX_i = p_iC_i+b_i(y-\sum_{k=1}^{n} p_kC_k) \ , \qquad i=1, \ldots, n$$

$$\sum_{i=1}^{n} b_i = 1$$

The b_is are marginal budget shares. The C_is have traditionally been interpreted as "necessary quantities" of good i so that $y-\Sigma_k p_k C_k$ is the amount of expenditure available to be allocated after necessary consumption has been net (so called supernumary income). The trouble with this interpretation is that there exists no logical reason for the C_is to be positive; indeed when they are negative broader behavior is allowed by the function.

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For the purposes of this study the LES involves constraints on behavior which are unacceptably stringent. The major problem from our point of view with the LES, and with all other systems linear in total expenditure such as the S-branch utility system (Brown and Heien, 1972), is that it restricts Engel curves to be linear. We are interested in disaggregated food consumption for which there is more reason to believe Engel curves will not be linear. Indeed, some foods may be inferior goods. Less troublesome is the restriction that goods cannot be Hicks-Allen complements. Also, ordinary cross price elasticities are constrained to be negative, that is, income effects dominate substitution effects. Furthermore, if the C_is were constrained to be positive then own price elasticities would be constrained to be less than one in absolute value.

A generalization of the LES would allow for nonlinear Engel curves. One possibility is quadratic Engel curves. Howe, Pollak and Wales (1979) have shown that any quadratic expenditure system (QES) consistent with Engel aggregation (summing up of expenditures), zero homogeneity in prices and expenditures and symmetry of the substitution matrix is generated by an indirect utility function of the form $V(p,y) = -g(p)/(y-f(p))-a(p)/g(p)$, where $g(\cdot)$, $a(\cdot)$ and $f(\cdot)$ are all homogeneous of degree one. This function generates a class of quadratic expenditure systems of the form

$$(2.5)\quad p_i X_i^c = \frac{p_i}{g^2}\left(\frac{\partial a}{\partial p_i} - \frac{\partial g/\partial p_i}{g} a\right)(y-f)^2 + \frac{p_i \partial g/\partial p_i}{g}(y-f) + p_i \frac{\partial f}{\partial p_i}$$

While existence of an indirect utility function implies existence of a direct utility function, no closed form for the direct function associated with the QES has been derived. Thus, to extend the class of QES to the househ

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the household-firm model we must work with indirect utility functions. This presents no problem so long as we continue to assume interior solutions. As we have seen, one may solve for X_i^c and $\bar{L}$ as functions of the p_i, p_L, and $A+\pi+p_LT$, where the latter sum replaces income in the indirect utility function. Hence, to use the indirect utility function in deriving demand curves in the household-firm model we need an extension of Roy's identity. That Roy's identity extends itself is readily seen. Let $y=A+\pi+p_LT = \sum_i p_iX_i^c + p_L\bar{L}$. If we minimize y with respect to prices and wage rate subject to $U(X_i^c,\bar{L}) = U^*$ we obtain our optimum $y^*=y^*(p,U^*)$. Assuming $\partial y^*/\partial U^* \neq 0$ we can solve for $U^*=U^*(p,y^*)$. This is nothing but the indirect utility function $U^*=V(p,y^*(p,U^*))$. Differentiating with respect to p_i: $0 = \partial V/\partial p_i + \frac{\partial V}{\partial y^*}\frac{\partial y^*}{\partial p_i}$. As y^* is an expenditure function, by Shepard's lemma we have $\frac{\partial y^*}{\partial p_i} = X_i^c$. Hence, $X_i^c = \frac{-\partial V/\partial p_i}{\partial V/\partial y^*}$. Similarly, $\bar{L} = \frac{-\partial V/\partial p_L}{\partial V/dy^*}$

A formation of the indirect utility function used by Pollak and Wales (1978) is

$$(2.6)\quad V(p,y) = \frac{-\prod_k p_k^{a_k}}{y-\sum_k p_kC_k} + \lambda\prod_k p_k^{(a_k-d_k)},\ \sum_k a_k = \sum_k d_k = 1$$

This uses $g(p) = \prod_k p_k^{a_k}$, $f(p) = \sum_k p_kC_k$ and $a(p) = -\prod_k p_k^{(2a_k-d_k)}$, where the a_ks, C_ks and d_ks and λ are parameters to be estimated. There is no necessary reason for λ to appear. Dropping it in order to save a parameter we can extend 2.6 to the household-firm model in a natural way,

$$(2.7)\quad V = -\prod_{k=1}^{n+1} p_k^{a_k}/(A+p_LT+\pi-\sum_{k=1}^{n+1} p_kC_k) + \prod_{k=1}^{n+1} p_k^{(a_k-d_k)}$$

$\sum_{k=1}^{n+1} a_k = \sum_{k=1}^{n+1} d_k = 1$, where leisure is treated as the n+1 good.

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The resulting expenditure equation is

$$(2.8)\quad p_i X_i^c = p_i C_i + a_i\left(p_L T + \pi + A - \sum_{k=1}^{n+1} p_k C_k\right) - (a_i - d_i)\prod_{k=1}^{n+1} p_k^{-d_k}\left(p_L T + \pi + A - \sum_{k=1}^{n+1} p_k C_k\right)^2 \qquad i=1, \ldots, n+1$$

This has as a special case the linear expenditure system provided $a_i = d_i,\ \forall_i$.

As noted, the QES is a class of expenditure functions. In the foregoing example the function $a(p)$ was the multiplicative one $-\prod_k p_k^{2a_k - C_k}$. Had we chosen an additive function $a(p) = \sum_k p_k d_k$ (Howe, Pollak, and Wales, 1979) our indirect utility function would be

$$V(p,y) = \frac{-\prod_k p_k^{a_k}}{(A + \pi + p_L T - \sum_k p_k C_k)} - \frac{\sum_k p_k d_k}{\prod_k p_k^{a_k}}$$

and our expenditure system

$$(2.9)\quad p_i X_i^c = p_i C_i + a_i\left(A + \pi + p_L T - \sum_k p_k C_k\right) + \left(p_i d_i - a_i \sum p_k d_k\right)\prod_k p_k^{-2a_k}\left(A + \pi + p_L T - \sum_k p_k C_k\right)^2$$

It might be interesting, but costly in parameters, to find a more general specification of which these two are special cases. One possibility would be to let $a(P)$ be a CES type specification $a(P) = \left(\sum_k d_k p_k^{\rho}\right)^{1/\rho}$ $\sum_k d_k = 1$, which becomes an additive specification for $\rho=1$ and a multiplicative one for $\rho=0$.

Our main research interest is not to compare alternative specifications of the QES. We choose to use the specification of equations 2.7 and 2.8.

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Incorporating Demographic Variables into the Demand System

Since our unit of analysis is the household rather than the individual, we must decide how to incorporate household characteristics such as size and age distribution into our analysis. The discussion draws heavily upon Pollak and Wales (1978b, 1980). Two very general approaches are possible. We could assume that different household characteristics give rise to different utility functions. In this case the sample would need to be grouped by the appropriate characteristics and the system estimated separately for each group. This would drastically reduce the number of parameters one could estimate, necessitating a reduction in the size, and hence the interest, of the system. Alternatively, one can assume that different characteristics can be accounted for within a common utility function. This is the approach taken here.

One might ask why not simply replace expenditures and total expenditure by their per capita equivalents. Indeed, this is possible and implies that per capita consumption is what enters into the utility function. In the past this has been criticized for not allowing for different consumption requirements for different members of the household. Such reasoning has led to construction of consumer equivalents. Often this exercise is based on recommended caloric intake by age group and sex. Clearly, however, caloric "requirements" do not constitute the only relevant measure by which to weight different members of the household. Prais and Houthakker (1955) argue that each member ought to have a different weight for each consumption good. They hypothesize expenditure equations of the form $p_i X_i^c / s_i = f^i(p, y/s_o)$ $i=1, \ldots, n$ where $s_i \equiv$ the consumer equivalent for good i and $s_o \equiv$ the "income scale." They model s_i a

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model s_i as a linear combination of household characteristics and s_o they assume to be independent of expenditures. The trouble with the latter assumption is that the demand system may not satisfy the budget identity. Muellbauer (1980) corrects for this by defining s_o implicitly using the budget equation (i.e., $\Sigma s_i f^i(p, y/s_o) = y$) in which case s_o is a function of prices and total expenditure as well as of demographic characteristics. There is disagreement between Muellbauer (1980) and Pollak and Wales (1978b) over the question of the characteristics of any theoretically plausible demand system giving rise to the Muellbauer respecification of the Prais-Houthakker procedure. Muellbauer argues that preferences must correspond to a fixed coefficients utility function, that is no substitutability between goods consumption. Pollak and Wales try to establish that applying the Muellbauer modification to a demand system corresponding to an additive utility function results in a theoretically plausible system. They further try to show that applying this method to a system linear in expenditure will be plausible only if the underlying utility function is additive. Since we are interested primarily in systems which are neither linear in expenditure nor additive this way of incorporating demographic variables into our analysis will not be pursued further.

The idea of equivalence scales which vary by commodity can be implemented in other ways, which are generally applicable to all theoretically plausible demand systems. Moreover, using arbitrary assumptions in order to form such scales prior to estimation can be avoided by estimating them. One example, scaling, due to Barten (1964), hypothesizes arguments in the utility function to be consumption as a ratio to commodity equivalence scales, which are dependent only on demogra

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demographic variables: $U(X) = U(X_1^c/I_1, X_2^c/I_2, \ldots, X_n^c/I_n)$. The resulting indirect utility function is of the form $V(p,y) = V(p_1I_1, \ldots, p_nI_n, y)$. Maximizing with respect to the X_i^c s, assuming the I_is to be fixed in the short run, yields an expenditure system of the form $p_iX_i^c = p_iI_if^i(p_1I_1, \ldots, p_nI_n, y)$. Such a system retains consistency with all the usual theoretical constraints except for negative semi-definiteness of the substitution matrix. Under continuity assumptions on the utility function, however, the modified system will meet this criterion for I_i sufficiently close to one.

Under the scaling method of entering demographic variables the effect of changes in demographic variables operates analogously to price changes. We can write $\ln X_i^c = \ln I_i + \ln f^i(p_iI_i, y)$ so that

$$(2.10) \quad \frac{\partial \ln X_i^c}{\partial \ln \eta_t} = \frac{\partial \ln I_i}{\partial \ln \eta_t} + \sum_{j=1}^{n} \frac{\partial \ln f^i}{\partial \ln p_jI_j} \frac{\partial \ln I_j}{\partial \ln \eta_t}$$

where $\eta_t \equiv$ the t th demographic variable and $\frac{\partial \ln f^i}{\partial \ln p_jI_j} = \frac{\partial \ln f^i}{\partial \ln p_j} \equiv$ the cross elasticity of good i with respect to price j. Hence, the consumption elasticities with respect to demographic characteristics are an affine function of the price elasticities. It remains to specify the I_i. Two possibilities are polynomial and log linear. The polynomial specification is $I_i = 1 + (\sum_{r=1}^{K} \sigma_{ir}\eta_r)^{\theta_i}$, where the K η_rs are defined as above and the σ_{ir}s and θ_is are unknown parameters. There will be at most n(k+1) of these parameters which are in addition to other parameters in the model. Clearly then, the number, k, of demographic variables to be included will be limited by model size considerations. The log-linear specification is $I_i = \prod_{r=1}^{K} \eta_r^{\sigma_{ir}}$. A special case of the polynomial is the linear

$$I_i = 1 + \sum_{r=1}^{K} \sigma_{ir}\eta_r.$$

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Another method of entering demographic variables into demand analysis due to Pollak and Wales (1978b, 1980) is called translating. The direct utility function is of the form $U(X) = U(x_1 - v_1, \ldots, x_n - v_n)$ and the indirect utility function is $V(p,y) = V(p, y - \sum_{i=1}^{n} p_i v_i)$. As for the LES, the v_is may be interpreted as committed quantities of goods. However, there is no reason why these parameters should be positive. The expenditure system may be written $p_i X_i^c = p_i v_i + f^i(p, y - \Sigma p_i v_i)$. Again, negative semi-definiteness of the substitution matrix may hold only for v_i sufficiently close to zero. The effects of demographic variables, η_t, come through income in this modification. $\frac{\partial p_i X_i^c}{\partial \eta_t} = p_i \frac{\partial v_i}{\partial \eta_t} - \frac{\partial f^i}{\partial y} \sum_{j=1}^{n} p_j \frac{\partial v_j}{\partial \eta_t}$. Pollak and Wales dub the first expression the "specific" effect and the latter the "general" effect. The specification of the v_i has the same considerations as for the I_i in the scaling case. The linear specification would omit the one, however; $v_i = \sum_{r=1}^{K} \sigma_{ir} \eta_r$.

Other ways to enter demographic variables exist. Gorman (1976) has proposed to sequentially scale and then translate. The reverse would also be possible as Pollak and Wales note. The little experimenting which Pollak and Wales have done indicates that scaling may be slightly better, although most of their comparisons are not nested and non-nested statistical tests of the differences are not performed. Using the linear scaling specification and the QES, the demand side of the household-firm model would look like:

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$$(2.11)\quad p_iX_i^c = p_i(1+\sum_{r=1}^{K}\sigma_{ir}\eta_r)C_i+a_i(A+\pi+p_LT-\sum_k p_k(1+\sum_{r=1}^{K}\sigma_{kr}\eta_r)C_k)$$

$$-(a_i-d_i)\prod_k[p_k(1+\sum_r\sigma_{kr}\eta_r)]^{-d_k}(A+\pi+p_LT-\sum_k p_k(1+\sum_r\sigma_{kr}\eta_r)C_k)^2$$

$$-p_LL_H = -p_L(T-(1+\sum_r\sigma_{Lr}\eta_r)C_L)+a_L(A+\pi+p_LT-\sum_k p_k(1+\sum_r\sigma_{kr}\eta_r)C_k)$$

$$-(a_L-d_L)\prod_k[p_k(1+\sum_r\sigma_{kr}\eta_r)]^{-d_k}(A+\pi+p_LT-\sum_k p_k(1+\sum_r\sigma_{kr}\eta_r)C_k)^2$$

The first term of the second equation we can rewrite as

$$(2.12)\quad -p_L(T-C_L)+\sum_r\sigma_{Lr}C_L\eta_rp_L$$

Likewise, we can collect $T-C_L$ in the other expressions so as to avoid specifying T. Viewing only the above expression, only $\sigma_{Lr}C_L$ is identified. However, the σ_{Lr}s appear in the form $[p_L(1+\sum_r\sigma_{Lr}\eta_r)]^{-d_L}$, hence the σ_{Lr}s will be identified from that expression. Hence, C_L is over-identified and from the estimate $T-C_L$ so is T.

We can improve the realism of the model by noting that T, the "total" time available for household allocation will itself be a function of demographic variables. Moreover, this will not affect the budget identity. Writing $T = \sum_r\gamma_rm_r$ we have for the first expression

$$(2.13)\quad -p_L(\sum_r\gamma_rm_r-C_L)+\sum_r\sigma_{Lr}C_L\eta_rp_L$$

Now all the parameters are identified.

Alternatively, we can use translation. Modeling T as above we have for the expenditure system

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$$(2.14)\quad p_iX_i^c = p_iC_i + p_i\sum_{r=1}^{K}\sigma_{ir}\eta_r + a_i\left(p_L\sum_{r=1}^{q}\gamma_r m_r + \pi + A - \sum_{k=1}^{n+1}p_k\left(C_k + \sum_{r=1}^{K}\sigma_{kr}\eta_r\right)\right)$$

$$-(a_i-d_i)\prod_{k=1}^{n+1}p_k^{-d_k}\left(p_L\sum_{r=1}^{q}\gamma_r m_r + \pi + A - \sum_{k=1}^{n+1}p_k\left(C_k + \sum_{r=1}^{K}\sigma_{kr}\eta_r\right)\right)^2$$

Since leisure is not directly observed we subtract from both sides of the leisure expenditure equation the value of time available to the household. The left hand side becomes the negative of the value of household labor, which we do observe.

$$(2.15)\quad -p_L L_H = p_L C_L + p_L\sum_{r=1}^{K}\sigma_{ir}\eta_r - p_L\sum_{r=1}^{q}\gamma_r m_r + a_i\left(p_L\sum_{r=1}^{q}\gamma_r m_r + \pi + A\right.$$

$$\left. - \sum_{k=1}^{n+1}p_k\left(C_k + \sum_{r=1}^{K}\sigma_{kr}\eta_r\right)\right) - (a_i-d_i)\prod_{k=1}^{n+1}p_k^{-d_k}$$

$$\left(p_L\sum_{r=1}^{q}\gamma_r m_r + \pi + A - \sum_{k=1}^{n+1}p_k\left(C_k + \sum_{r=1}^{K}\sigma_{kr}\eta_r\right)\right)^2$$

This device avoids the need to impose values for T, such as a male having exactly sixteen hours per day available for work and leisure. With n+1 commodities, K translation demographic variables and q demographic variables for total time this system has at most (3+K) (n+1)-2+q parameters to estimate (fewer if some of the η_rs and m_rs are identical).

In the foregoing, we have made only the C_k parameters functions of demographic variables. In principle, the a_k and d_k parameters also might be functions of parameters. We might write $a_i = a_{i0} + \sum_r \xi_{ir}\eta_r$ subject to $\sum_i a_i = 1$. This latter constraint would imply that $\sum_i a_{i0} = 1$ and that $\sum_i \xi_{ir} = 0$, $\forall_r$. This might be one way to incorporate the hypothesis that different sources of income resulted in different expenditure patterns, a hypothesis that our formulation of the model does not permit exploration of.

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Both translation and scaling assume that household characteristics do not enter separately into the utility function. It is possible to enter demographic variables as separate arguments in the utility function. This is done for a linear logarithmic expenditure system by Lau, Lin and Yotopoulos (1978).

Comparison of alternative methods of entering demographic variables is not our purpose any more than comparing different forms for the QES. We ultimately use the translation specification, although use of the scaling specification was attempted and discarded for reasons outlined in Chapter 6.

Separability of Utility Function and Perfect Price Aggregation

One important issue of specification is the number of commodities to be included, hence the level of aggregation one uses. In a model of this size the number of commodities used will have to be limited, hence commodity groups will need to be formed. Since we are deriving our system using constraints implied by economic theory, the question naturally arises whether one can group commodities, in particular form aggregate price indices for the groups, and remain consistent with theory. Three ways exist to handle this question. One is to assume relative prices within each commodity group to be constant and form composite commodities as suggested by Hicks. The second approach is to use properties of separability on the utility function and derive the appropriate price indices accordingly. The third method is to ignore the question and form price indices in an ad hoc manner. Using the second method, Blackorby, Primont and Russell (1978) define strong price aggregation as the existence of linear homogeneous functions $\pi^i(p^i)$ such that y^r

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that $y^r = \theta^r(\pi^1(p^1), \ldots, \pi^n(p^n), y)$, where $p^i \equiv$ vector of prices in group i, $y \equiv$ total expenditures, $y^r \equiv$ expenditure on group r and θ^r is an expenditure equation homogeneous of degree one in prices and expenditure. They define the conditional indirect utility function as $H(y^1, y^2, \ldots, y^n, p) = \max_{x}(U(X)/\Sigma_r p^r x^r \leq y)$ and note it can be written as $H(h^1(y^1, p^1), h^2(y^2, p^2), \ldots, h^n(y^n, p^n))$ if and only if the direct utility function is weakly separable in the n commodity groups (that is, it can be written $U(X) = U(U^1(X^1), U^2(X^2), \ldots, U^n(X^n))$. In this case Pollak (1971b) has shown that one can derive a conditional demand system; expenditures within a group as a function of prices within the group and of group expenditure. The latter is a function of all prices and of total expenditure. Blackorby, et al. show that a sufficient condition for strong price aggregation is for H to have the form

$H(X) = U^*(\sum_{r=1}^{d} h^r(y^r, p^r) + U(h^{d+1}(y^{d+1}, p^{d+1}), \ldots, h^n(y^n, p^n)))$, where h^r is homogeneous of degree minus one in p^r for $r=d+1, \ldots, n$ and h^r is of the generalized Gorman polar form, $h^r = \psi^r(y^r/\pi^r(p^r)) + \Lambda^r(p^r)$, $\Lambda^r(p)$ being homogeneous of degree zero. It turns out that the generalized Gorman polar form yields expenditure equations linear in income. Hence, the class of QE systems does not meet this requirement. Indeed, the indirect utility functions as operationalized by Howe, et al. are not even separable (though this need not imply the same for the corresponding direct utility functions).

While Howe, et al. speculate the existence of a QES which is separable, we have been unable to derive such. The closest we have come is to derive systems quadratic in expenditure within groups but linear in total expenditure for group expenditures. One class of utility functions meeting Blackorby, et al.'s criterion for price aggregation is the S-b

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is the S-branch utility tree (Brown and Heien, 1972). Although this function is a generalization of the LES in that it allows for complementarities, it is also linear in expenditure, hence will not be pursued.

The LES is derived from an additive direct utility function and does give rise to price aggregates (Stone, 1970) but which depend on unknown parameters. To see this, add the LES expenditure equations for a commodity group:

$$(2.16)\quad \sum_{i\varepsilon I_r} p_i X_i^c = \sum_{i\varepsilon I_r} p_i C_i + \sum_{i\varepsilon I_r} a_i (y - \sum_i p_i C_i)$$

$$= \Big(\sum_{i\varepsilon I_r} \frac{(p_i C_i)}{\sum_{j\varepsilon I_r} C_j}\Big) \sum_{j\varepsilon I_r} C_j + \sum_{i\varepsilon I_r} a_i \Big(y - \sum_{r=1}^{n} \sum_{i\varepsilon I_r} \frac{(p_i C_i)}{\sum_{j\varepsilon I_r} C_j} \sum_{j\varepsilon I_r} C_i\Big)$$

$$= p^r C^r + a^r \Big(y - \sum_{r=1}^{n} p^r C^r\Big)$$

where $C^r = \sum_{j\varepsilon I_r} C_j$, $a^r = \sum_{i\varepsilon I_r} a_i$, $I_r \equiv$ group r and $p^r = \sum_{i\varepsilon I_r} p_i \frac{C_i}{\sum_{j\varepsilon I_r} C_j}$.

Price of group r is a weighted average of prices within group r with weights consisting of unknown parameters.

Given that the C_is are unknown two options exist. One is to estimate conditional expenditure equations within groups and to use the resulting estimators of p^r and C^r in the aggregate function (see Chapter 3). The second is to use proxies for p^r based on a price index. Ronald Anderson (1979) performed a Monte Carlo experiment using an additive perfect price aggregate model and found that multilevel estimation outperformed a variety of price index proxies using several criteria but that no type of index clearly outperformed any other. The better performance of the multilevel procedure was especially marked for cases in which some commodities entering into a commodity group were inferior.

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As noted, the QES we use is not separable, hence, perfect price aggregation is not of direct use to this research. However, as we explore in Chapter 3, use of a separable functional form such as the linear expenditure system allows, under certain statistical assumptions, estimation of conditional demand functions and composite demand functions in a multistage procedure. In principle, this extends the number of commodities one can realistically estimate, but again at the expense of linear Engel curves, at least of the group expenditures.

Specifying the Production Side

Specifying the production block of the household-firm model will involve a set of factor demand and output supply equations plus a short run profits function. We have initially specified an implicit production function of the form $G(X_i, L_T, D, \bar{K})$, where D and $\bar{K}$ are fixed. We could stop at this point, making operational this function (or its associated short run profit function which we have seen exists) using a flexible form such as the translog. However, we must be conscious of our parameter usage particularly since we are not primarily interested in the production side. The usual way to achieve parsimony in parameters is by using assumptions on the nature of the production function. Two general possibilities suggest themselves. At one extreme, we could assume non-jointness, that is the existence of individual production functions for each output. With fixed land and capital this would insure dependency of those outputs in whose production functions land and capital appeared on the corresponding output prices. However, assuming production functions to differ would entail at least nm para

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nm parameters, where n is the number of outputs and m the number of inputs. More importantly, there are inadequacies in our data for using this approach (see Chapter 7). Alternatively, we could assume some form of separability. One logical possibility would be to assume outputs as a group to be separable from inputs as a group. That is, $G(X_i, L_T, D, \bar{K}) = H(X_i) - F(L_T, D, \bar{K})$. We could further assume almost homogeneity of degree $\frac{1}{s}$, that is, $H(\lambda X_i) = F(\lambda^s L_T, \lambda^s D, \lambda^s \bar{K})$. That these assumptions are restrictive in the behavior they permit is true (for a survey see McFadden, 1978, and for an extension to multiple outputs see Lau, 1978). The question for this research is whether the answers to questions concerning food consumption which we are interested in are robust to assumptions on the production side.

Among the possible functional forms to use for inputs one appealing form is the Cobb-Douglas (CD). Its weaknesses are well known. Its strength for our purposes is its requiring only m+1 parameters. For outputs we might think of the counterpart to the constant elasticity of substitution function, the constant elasticity of transformation (CET) introduced by Powell and Gruen (1968). The function, of the form $H(X_i) = (\sum_i \delta_i X_i^c)^{1/c}$, where $\delta_i > 0$ and $c > 1$ to insure convexity, entails only m+1 parameters. Consequently, a CET-CD system would require n+m+2 parameters which must surely be pushing the lower bound of parameters in any reasonable system. Writing the CD function for inputs as $F(L_T, D, K) = A_o L_T^{B_L} D^{B_D} K^{B_K}$, we have

$$(2.17)\quad (\sum_i \delta_i X_i^c)^{1/c} = A_o L_T^{B_L} D^{B_D} K^{B_K}$$

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This production system requires one of two normalizations; either $A_o=1$ or $\sum_i \delta_i=1$. This can be seen since we can write the left hand side as $(\sum_i \delta_i)^{1/c}(\sum_i \delta_i^* X_i^c)^{1/c}$ where $\delta_i^* = \delta_i / \sum_i \delta_i$ and $\sum_i \delta_i^*=1$. In this case A_o and $(\sum_i \delta_i)^{1/c}$ are not distinguishable, so one would estimate $A_o^* = A_o / (\sum_i \delta_i)^{1/c}$ when using the normalization $\sum_i \delta_i^*=1$. Alternatively, we can leave the δ_is as they are and set $A_o=1$, which is what we have done in Chapter 7.

The parameter c can be transformed into $\frac{1}{c-1}$, the elasticity of transformation between outputs. That is $\frac{1}{c-1}$ is the elasticity of the ratio of two outputs with respect to the marginal rate of transformation, $-\partial X_i / \partial X_j$, between them. Since in a competitive equilibrium, which we assume, the marginal rate of transformation between outputs equals the relative price ratio, the elasticity of transformation between outputs is the elasticity of the ratio of two outputs with respect to their price ratio. For this production function the elasticity of transformation parameter is constant, hence the name CET. Moreover, it is the same for all pairs of outputs. Indeed, one generalization of this functional form would be to write it as a multilevel CET (Mundlak and Razin, 1971) to capture differing transformation elasticities between outputs from different groups.

The δ_i parameters have their meaning in the marginal rate of transformation. It is easily seen that $\frac{-\partial X_i}{\partial X_j} = \frac{\delta_j}{\delta_i}\left(\frac{X_j}{X_i}\right)^{c-1}$. On the input side, the B parameters have the usual meaning for a Cobb-Douglas specification, that is, the percent change in all outputs due to an infinitesimal change in the particular input. The sum of the B's is the degree of almost homogeneity.

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Maximizing profits subject to 2.17 (normalizing $A_o=1$) and to D and K being fixed, we arrive at the output supply and labor demand equations.

$$(2.18)\quad p_iX_i = B_L^{B_L/1-B_L}\,\delta_i^{-1/(c-1)}\,p_i^{c/(c-1)}$$

$$(\sum_k \delta_k^{-1/(c-1)}\,p_k^{c/(c-1)})^{(cB_L-1)/c(1-B_L)}$$

$$(D^{B_D}K^{B_K})^{1/(1-B_L)}\,p_L^{(-B_L/(1-B_L))}$$

$i=1, \ldots, n$

$$p_LL_T = (D^{B_D}K^{B_K})^{(\frac{1}{1-B_L})}\,B_L^{(\frac{1}{1-B_L})}\,p_L^{(-B_L/(1-B_L))}$$

$$(\sum_i \delta_i^{\frac{-1}{c-1}}\,p_i^{c/(c-1)})^{(\frac{c-1}{c(1-B_L)})}$$

These equations point out some of the simplifications made by selection of this functional form. Elasticities of value output with respect to fixed input are $\frac{B_i}{1-B_L}$, where i is either D or K. This means these elasticities are the same for all outputs. Also, the elasticities of value output with respect to wage $\frac{-B_L}{1-B_L}$ are identical for all outputs. Own price elasticities of value output and of value labor demand are not identical across commodities.

$$(2.19)\quad \frac{\ln p_iX_i}{\ln p_i} = \frac{1}{c-1} + p_i^{\frac{c}{c-1}}\,\delta_i^{\frac{1}{c-1}}\,(cB_L-1)/((1-B_L)(c-1)A)$$

where $A = \sum_i p_i^{c/(c-1)}\,\delta_i^{-1/(c-1)}$ and $\frac{\partial \ln p_LL_T}{\partial \ln p_L} = -B_L/(1-B_L)$.

Thus far we assume the implicit production function to be identical in all regions in Sierra Leone. One way to capture some differences is to allow for fixed regional effects, for instance, on the intercept term on the input function. Indeed, this is pursued in the estimation procedure. Of greater difficulty are possible differences in the remaining parameters.

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One could add slope dummy variables but at a large cost in parameters. Alternatively, one could assume that parameters vary, randomly around some mean with a disturbance which is identical for households within a region. This is essentially the Swamy (1974) specification for panel data adapted to a regional cross section. Of greater difficulty is the possibility that some outputs are not produced at all in some areas (which is true for our sample, see Chapters 4 and 7).

Estimating the household-firm model by agro-climatic region has appeal in principle, however, separating 138 households into eight regions will not leave sufficient data for estimation, and worse will leave no price variation as that is regional (see Chapter 4). Compromising may be possible but at the potential cost of having to reduce parameters to be estimated and reducing observed price and input differentials. Aggregation of outputs or inputs may help some but raises the same issues as on the demand side of the model. Hence, we assume that the production function is identical throughout rural Sierra Leone, but with certain parameters possibly varying with region.

As for the limited number of inputs, this specification is based on Byerlee and Spencer's (1977) extensive study of farm firms in Sierra Leone (also Byerlee, Spencer and Franzel, 1979). Fertilizer purchases are very limited and tractor services are hired by only a few mechanized farms in a particular area, Bolilands. This study is not concerned directly with changes in farming systems so these factors can probably be reasonably abstracted from (though they are included in our measure of capital flow--see Chapter 4).

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CHAPTER 3

ESTIMATION OF MODEL

Specifying the Error Structure

Specifying the error structure of the household-firm model can proceed in two ways. We can specify an error structure within the utility and production (or profit) functions and derive the appropriate error structure for the expenditure equations. The more common approach has been to append an error structure onto the demand and supply equations with, perhaps, some attention to properties of the error structure.

In the first approach we could add a stochastic component to the utility and production functions except that we are abstracting from uncertainty. Alternatively, we can assume randomness in parameters which reflects differences in household tastes. This has been pursued by Pollak and Wales (1969) and Wales and Woodland (1979). For this study randomness in demand parameters to account for differences in tastes makes sense only if we think important differences exist which are not due to demographic characteristics. Wales and Woodland append errors to first order conditions of utility maximization. Interpreting such errors as errors in allocation rather than deterministic components reflecting differences in tastes would lead to estimation of the structural first order conditions rather than the reduced form demand, expenditure or share equations. Deriving the likelihood function for the observed commodity and factor input demands

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demands and output supplies would be a straightforward (though messy) matter of taking the jacobian of the transformation from errors to observed variables and multiplying that by the likelihood function of the error terms, which we would assume.

If we are to be more conventional we can add errors to the reduced form. Here the question arises which form of the reduced form should errors be added to. The choices are threefold: for the demand (production) system they are quantity demand (supply) equations, expenditure (value supply) equations and share (share of profits) equations (there is no reason why the form for the demand and production sides ought to be the same). The choice will depend on in which form one expects the disturbances to have desirable properties. For household t let ε_t be an n vector error. Assume ε_ts to be iid $N(0,\Sigma)$ so that $\varepsilon=(\varepsilon_1', \varepsilon_2', \ldots, \varepsilon_T')'$ $\sim N(0, I_T \otimes \Sigma)$. On which form of the reduced form is this most likely to hold? In particular, on which form are the disturbances identically distributed? Pollak and Wales in most of their work on demand systems believe the share equations are the proper ones to which to add this error structure. Using experience from estimating Engel curves they feel the errors on expenditure equations have a heteroskedastic nature of the form $E(\varepsilon_{ti}\,\varepsilon_{tj}) = \sigma_{ij} y^2$, $y \equiv$ total expenditure. Hence, dividing each equation by y, resulting in share equations, is the appropriate solution. Alternatively, one might assume as did Pollak and Wales (1969) that errors on the demand equations have structure $E(\varepsilon_{ti}\,\varepsilon_{tj}) = \sigma_{ij} \hat{X}_i^c \hat{X}_j^c$ where the hats indicate non-stochastic portions. However the error structure is specified, residuals may be examined for the appropriateness of the specification, and if heteroskedasticity is suspected statistical tests may be performed.

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Without loss of generality assume error terms are added to the expenditure equations and value of factor demand and output supply equations. Subtracting the value of factor demand from the value of output supply equations yields the short run profit function, π. Assume profits have a stochastic component, also. Then $\pi = \hat{\pi} + \varepsilon$ and $\sum_i \varepsilon_{2i} - \varepsilon_{2L} = \varepsilon_\pi$, where ε_{2i} are disturbances added to the value of output supply and labor demand equations. Hence, for each household the sum of errors on the value of output supply equations less the disturbances on the value of labor demand and profits equations is zero.

One the demand side the disturbances also sum to zero. Formally we may write

$$(3.1)\quad p_i X_i^c = h^i(p, p_L T + \pi) + \varepsilon_{1i}, \quad -p_L L_H = h^L(p, p_L T + \pi) + \varepsilon_{1L}$$

provided $\sum_i h^i(p, p_L T + \pi) + h^L(p, p_L T + \pi) = \pi$, which is true for any theoretically plausible nonstochastic system, then $\sum_i \varepsilon_{1i} + \varepsilon_{1L} = 0$.

For any household t,

$$(3.2)\quad \begin{pmatrix} p_i X_{ti}^c \\ -p_L L_{tH} \\ -\pi_t \\ p_i X_{ti} \\ -p_L L_{tT} \end{pmatrix} = \begin{pmatrix} h_t^i(p, p_L T + \pi) \\ h_t^L(p, p_L T + \pi) \\ -g_t^\pi(p, K, D) \\ g_t^i(p, K, D) \\ g_t^L(p, K, D) \end{pmatrix} + \begin{pmatrix} \varepsilon_{t1i} \\ \varepsilon_{t1L} \\ -\varepsilon_{t\pi} \\ \varepsilon_{t2i} \\ \varepsilon_{t2L} \end{pmatrix}$$

and

$$(3.3)\quad \begin{pmatrix} i' & 0 & 0 \\ 0 & 1 & i' \end{pmatrix} \varepsilon_t^* = \begin{pmatrix} i' & 0 & 0 \\ 0 & 1 & i' \end{pmatrix} \begin{pmatrix} \varepsilon_{t1}^* \\ -\varepsilon_{t\pi} \\ \varepsilon_{t2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

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where i is a unit vector of appropriate dimension. Note that the equations have been stacked with the consumption block equations on top. Assume now that $\varepsilon_t^* = (\varepsilon_{t1}^{*\prime}, \varepsilon_{t2}^{*\prime})' \sim N(0, \Sigma^*)$. Then Σ^* is singular. This is easy to see because if $\sum_j \varepsilon_{t1j}^* = 0$ then $\sum_j \varepsilon_{t1j}^* \varepsilon_{t1k}^* = 0$, $\forall k$, and $\sum_j E(\varepsilon_{t1j}^* \varepsilon_{t1k}^*) = \sum_j \sigma_{1jk} = 0$, $\forall k$. this means that elements of each row (and column) of that part of Σ^* corresponding to ε_{t1}^* adds to zero. The same will be true for the section of Σ^* corresponding to $\varepsilon_{t\pi}$, ε_{t2}.

If we were to estimate this system using a maximum likelihood technique we would ignore one equation in the demand system and one equation in the production system (because we need to invert the covariance matrix). Which equations were dropped would not affect our results. Barten (1969) has proved that result for an error structure with one redundant equation. His result easily extends for a structure composed of two sub-structures each with one redundant equation. Assume that the labor supply and profits equations are dropped. Then we have n equations remaining in the consumption block and n+1 equations in the production block. We may rewrite the resulting system as

$$(3.4)\quad \begin{pmatrix} p_i X_{ti}^c \\ p_i X_{ti} \\ -p_L L_{tT} \end{pmatrix} = \begin{pmatrix} h_t^i\ (p, p_L T + \Sigma p_i X_{ti} - p_L L_T) \\ g_t^i\ (p, K, D) \\ g_t^L\ (p, K, D) \end{pmatrix} + \begin{pmatrix} \varepsilon_{t1} \\ \\ \varepsilon_{t2} \end{pmatrix}$$

Given our assumptions on ε_t^*, $\varepsilon_t = (\varepsilon_{t1}', \varepsilon_{t2}')' \sim N(0, \Sigma)$. Then the likelihood function for $\begin{pmatrix} p_i X_{ti}^c \\ p_i X_{ti} \\ -p_L L_{tT} \end{pmatrix}$ is

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$$(3.5)\quad L_t = (2\pi)^{\frac{-(2n+1)}{2}} |\Sigma|^{-\frac{1}{2}} ||J_{\varepsilon_t}|| \exp\{-\tfrac{1}{2}\varepsilon_t' \Sigma^{-1}\varepsilon_t\}$$

where J_{ε_t} is the Jacobian of the transformation of disturbances into dependent variables, and

$$(3.6)\quad |J_{\varepsilon_t}| = \begin{vmatrix} 1 & 0\ldots0 & -\partial h_t^1/\partial p_1 X_{1t} & -\partial h_t^1/\partial p_2 X_{2t} & \cdots & \partial h_t^1/\partial p_L L_T \\ 0 & 1\ 0.0 & -\partial h_t^2/\partial p_1 X_{1t} & \cdots & & \partial h_t^2/\partial p_L L_T \\ 0 & 0\ ..1 & -\partial h_t^n/\partial p_1 X_{1t} & \cdots & & \partial h_t^n/\partial p_L L_T \\ 0 & 0\ ..0 & 1 & 0 \quad \cdots & & 0 \\ & & & 1 \quad \cdots & & \\ & & & & & 0 \\ 0 & & & & 0 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} I_n & A \\ 0 & I_{n+1} \end{vmatrix} = 1, \text{ where A is nx(n+1) with}$$

$$A_{ij} = \begin{cases} -\partial h_t^i/\partial p_j X_{jt} & j=1,\ldots,n \\ \partial h_t^i/\partial p_L L_T & j=n+1 \end{cases}$$

If we assume the ε_t's to be independently, identically distributed for all t then the likelihood function for $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_T)'$ is

$$(3.7)\quad L = (2\pi)^{\frac{-T}{2}(2n+1)} |\Sigma|^{-T/2} \exp\{-\tfrac{1}{2}\sum_t \varepsilon_t' \Sigma^{-1} \varepsilon_t\}$$

$$= (2\pi)^{\frac{-T}{2}(2n+1)} |\Sigma|^{-T/2} \exp\{-\tfrac{1}{2} \text{ Trace } \varepsilon\Sigma^{-1}\varepsilon'\}$$

In our case this will be a nonlinear in parameters likelihood function. Barnett (1976) and Gallant and Holly (1980) have shown that under suitable regularity assumptions the consistent and asymptotically efficient properti

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properties of maximum likelihood estimators hold when the likelihood function is nonlinear in parameters. Moreover, the covariance of the asymptotic distribution of $\sqrt{T}(\hat{\beta} - \beta)$ continues to be $\lim_{T\to\infty} ((1/T)\ell)^{-1}$, where $\ell \equiv$ information matrix.

Effect of Non-Identically Distributed Errors

If the errors appended to the value equations are not identically distributed across households, this can easily be incorporated into the likelihood function. In the consumption block it may be that $E(\varepsilon_{t1i}\varepsilon_{t1j}) = \sigma_{1ij}\hat{X}^c_{ti}\hat{X}^c_{tj}$. Defining $F_{t1} = \begin{pmatrix} \hat{X}^c_{ti} & & 0 \\ & \ddots & \\ 0 & & \hat{X}^c_{tn} \end{pmatrix}$ we have

$\varepsilon_{t1} \sim N(0, F_{t1}\ \Sigma_{11}\ F_{t1})$, where Σ_{11} is the nxn upper left corner of Σ. On the production side it may be that errors appended to the quantity form are identically distributed. In this case $E(\varepsilon_{t2i}\varepsilon_{t2j}) = \sigma_{2ij}p_ip_j$ and $\varepsilon_{t2} \sim N(0, F_{t2}\Sigma_{22}F_{t2})$, where $F_{t2} = \begin{pmatrix} P_1 & & & 0 \\ & \ddots & & \\ & & p_n & \\ 0 & & & p_L \end{pmatrix}$ and Σ_{22} is the

(n+1) x (n+1) lower right corner of Σ. Of course, the F matrices need not be specified this way. Indeed, many different specifications can be used. Combining both sides we rewrite the likelihood function 3.7 as

$$(3.8)\quad L = (2\pi)^{\frac{-T}{2}(2n+1)} |\Sigma|^{-T/2} \prod_{t=1}^{T} ||F_t||^{-1} \exp\{-\tfrac{1}{2} \sum_{t=1}^{T} \varepsilon_t' F_t^{-1}\Sigma^{-1}F_t^{-1}\varepsilon_t\}$$

where $F_t = \begin{pmatrix} F_{t1} & 0 \\ 0 & F_{t2} \end{pmatrix}$.

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Block Recursivity of Model

Of interest for estimation purposes is how we specify Σ. If $\Sigma = \begin{pmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{pmatrix}$, that is, if disturbances on the demand side are independent of disturbances on the production side, then the likelihood function is the product of two such functions, one for the demand equations and one for the output supply and factor demand equations. This is due to the block diagonality of the covariance matrix of disturbances plus the block triangularity of parameters in the system (that is, the fact that commodity demand parameters do not enter into output supply and factor demand equations when decision making is recursive). Moreover, profits, π, will not be correlated with the consumption block disturbances. Hence, separate estimation will not result in inconsistent estimates. For any household t

$$(3.9)\quad L_t = (2\pi)^{-n/2} (2\pi)^{\frac{-(n+1)}{2}} |\Sigma_1|^{-\frac{1}{2}} |\Sigma_2|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}(h_t(i'y_2, Z_1, Z_2; \beta_1)', g_t(Z_2, Z_3, \beta_2)')\begin{pmatrix} \Sigma_1^{-1} & 0 \\ 0 & \Sigma_2^{-1} \end{pmatrix} \begin{pmatrix} h_t & (.) \\ g_t & (.) \end{pmatrix}\}$$

where $h_t \equiv$ demand side equations, $g_t \equiv$ production side equations, $y_2 \equiv$ value of output supplies and negative factor demands so $i'y_2$ $\equiv$ measured profits, $Z_i \equiv$ exogenous variables, and $\beta_i \equiv$ parameters. Then

$$(3.10)\quad L_t = (2\pi)^{-n/2} |\Sigma_1|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2} h_t' \Sigma_1^{-1} h_t\} (2\pi)^{\frac{-(n+1)}{2}} |\Sigma_2|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2} g_t' \Sigma_2^{-1} g_t\}$$

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If, however, the disturbance covariance matrix is not block diagonal then this property no longer holds. Parameters from the demand side are no longer separable from those of the production side. More importantly, profits are now correlated with consumption side disturbances, sc that separate estimation results in inconsistent estimates. In this case, the maximum likelihood estimator entails joint estimation of both the demand and production blocks of the system. In principle, the assumption of block diagonality is a testable one. We could estimate the system assuming block independence of the disturbances and use a Lagrange multiplier test (see Rao, 1973, pp. 418-20; or Breusch and Pagan, 1980), which requires only restricted parameter estimates.

Another reason to assume block diagonality is to increase computational tractability, thus allowing a larger problem to be examined. Separate estimation of the consumption and production sides of the models entails far fewer parameters being estimated for each separately. When using numerical maximum likelihood techniques the number of parameters being estimated greatly affects the cost and tractability of doing so. Hence, if we can estimate the subsystems separately we will be able to estimate many more parameters in total than if we did not. This means that we can include more commodity disaggregation and more demographic variables in our estimation, making the problem more interesting.

A further reduction in problem size to increase computer tractability can be accomplished by concentrating the likelihood function. If there exist no constraints on Σ_{11} and Σ_{22}, these would be obvious candidates, enabling reduction of $\frac{n(n+1)}{2}$ and $\frac{(n+1)(n+2)}{2}$ independent parameters respectively, a total of $(n+1)^2$. Maximizing the likelihood function with respec

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respect to elements of Σ^{-1} we obtain $\hat{\Sigma} = \frac{1}{T} \varepsilon ' \varepsilon$ and the concentrated likelihood function $L^* = K \left| \frac{1}{T} \varepsilon ' \varepsilon \right|^{-T/2}$, where $K \equiv$ constant =

$$(2\ )^{\frac{-T}{2}(2n+1)} \exp \{-\tfrac{1}{2}T(2n+1)\} \ .$$

Estimating Multilevel Demand Systems

We have seen how assuming some form of separability of the utility function can aid in forming price indices which in general depend on unknown parameters. A further property of weakly separable utility functions is that conditional demand functions may be derived which give quantity as a function of group expenditure and prices within the group, with group expenditures being a function of all prices and total expenditure, or income (see Pollak, 1971b). This raises the possibility of estimating our household-firm model using very aggregate commodity groupings and then estimate within group expenditure equations. By reversing the order of estimating one could possibly estimate the aggregate price indices from within group expenditure systems. To do this with theoretically plausible demand systems would require using in the household-firm model a function exhibiting the required separability attributes. This would rule out use of the QES. In addition, estimating within group expenditure systems would entail having to deal with estimation problems caused by some households not consuming any of certain goods (more on this below). With these qualifications in mind, we discuss some additional issues which would be involved in such multilevel estimation.

Multilevel systems of demand equations have been estimated by Braithwaite (1977, 1980), Deaton (1975) and R.W. Anderson (1979) among others. Fuss (1977) has estimated a multilevel system of input demand equations. One major econometric problem stands out. When intra-group deman

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demand systems are estimated separately by maximum likelihood techniques, there is an implicit assumption that disturbances on expenditure equations within a group are independent of the disturbances for aggregate group equations. Otherwise, the group expenditure variable in the conditional demand equation will be correlated with that equation's disturbance. This is completely analogous to our result on estimating the consumption subsystem separately from the production subsystem.

Even if the necessary independence of disturbances holds, there are still problems, but manageable ones. We would like disturbances on conditional demand equations for different groups to be independent if we estimate systems for these groups separately. If this is not true and we estimate the systems separately, our parameter estimates will be consistent, but efficiency will be sacrificed. If we estimate within group systems first and then use the resulting parameter estimates in the aggregate model there is the question of deriving the statistical properties of the resulting estimators given that we have estimated sequentially. We have in a sense two subsystems, an aggregate model and a collection of subaggregates. Assuming that disturbances of the two subsystems are independent, unconditional maximum likelihood estimation would still not be separate maximization of the two likelihood functions because parameters in the aggregate model are combinations of parameters in the within group systems. One could estimate the subsystems separately and obtain consistent parameter estimates, but efficiency would be lost because of cross equation parameter restrictions being ignored.

Theil (1974, 1975a,b) assumes that the covariance matrix of the error terms on the disaggregated expenditure system is proportional to the negative of the Slutsky substitution matrix. In his work he offers some sugg

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suggestions as to why this might be a plausible assumption. If we use an LES, the iith element of the Slutsky matrix is $-\frac{1}{K}a_i(1-a_i)$ (using our notation), and the ijth element is $\frac{1}{K}a_i a_j$. This follows from the additivity of the Klein-Rubin utility function (for instance, see Brown and Deaton, 1972). Suppose we have R groups. Using the LES we can form conditional demand functions of expenditures within each group as a function of prices within the group and of total group expenditure. Then we have a set of equations relating group expenditures to group price indices and to total expenditures, and separate sets of conditional expenditure equations. Using Theil's assumptions regarding the distribution of the error terms one can show that within group disturbances sum to zero for each group, that within group disturbances from different groups are independent, and that disturbances from every conditional demand equation are independent of disturbances from the across groups equations. The operational significance of these results is slightly limited by the fact that parameters of the across groups equations are combinations of the conditional expenditure equation parameters. Hence, as mentioned, asymptotical efficiency is sacrificed by maximization of separate likelihood functions.

To see the foregoing results we write

$$(3.11)\quad y^s = p^s C^s + a^s(y - \sum_s p^s C^s) + E^s$$

where (3.11) is identical to (2.16) with $E^s = \sum_{i\varepsilon s} \varepsilon_i^s$. Multiply 3.11 by a_i/a^s and subtract this from the expenditure equation for commodity i and one obtains:

$$(3.12)\quad p_i y_i = p_i \gamma_i + \frac{a_i}{a^s}(y^s - \sum_{j\varepsilon s} p_j C_j) + \varepsilon_i^s - \frac{a_i}{a^s}E^s,\ \forall_{i\,\varepsilon s},\ s=1,\dots,R$$

Let $V_i^s = \varepsilon_i^s - \frac{a_i}{a^s}E^s$,

since $a^s = \sum_{i\varepsilon s} a_i$, clearly $\sum_{i\varepsilon s} V_i^s = 0,\ \forall_s$.

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$$(3.13)\quad E(V_i^r V_j^s) = \sigma_{ij}^{rs} - \frac{a_i}{a^r}\sum_{i\varepsilon r}\sigma_{ij}^{rs} - \frac{a_j}{a^s}\sum_{j\varepsilon s}\sigma_{ji}^{rs} + \frac{a_i a_j}{a^r a^s}\sum_{i\varepsilon r}\sum_{j\varepsilon s}\sigma_{ij}^{rs}$$

$$= -\frac{1}{K}a_i a_j; + \frac{1}{K}a_i a_j + \frac{1}{K}a_i a_j - \frac{a_i a_j}{Ka^r a^s}\sum_{i\varepsilon r}a_i\sum_{j\varepsilon s}a_j = 0$$

Here we use the fact that $\sigma_{ij}^{rs} = -\frac{1}{K}a_i a_j$

$$(3.14)\quad E(V_i^r E^r) = \sum_{j\varepsilon r}\sigma_{ij}^{rr} + \frac{a_i}{a^r}\sum_{i\varepsilon r}\sum_{j\varepsilon r}\sigma_{ij}^{rr}$$

$$= \frac{1}{K}a_i(1-a_i) - \frac{1}{K}a_i\sum_{\substack{j\neq i\\ j\varepsilon r}}a_j - \frac{a_i}{a^r}\sum_{i\varepsilon r}\left(\frac{1}{K}a_i(1-a_i) - \frac{1}{K}a_i\sum_{\substack{j\neq i\\ j\varepsilon r}}a_j\right)$$

$$= \frac{1}{K}a_i - \frac{1}{K}a_i\sum_{j\varepsilon r}a_j - \frac{a_i}{a^r}\left(\frac{1}{K}\sum_{i\varepsilon r}a_i - \frac{1}{K}a^r\sum_{i\varepsilon r}a_i\right)$$

$$= \frac{1}{K}a_i - \frac{1}{K}a_i a^r - \frac{1}{K}a_i + \frac{1}{K}a_i a^r = 0$$

Similarly, $E(V_i^r E^s) = 0$, $r\neq s$. Consequently, group expenditure, y^s, can be treated as predetermined in the conditional demand equations just as profits are in the household-firm model with block independence.

Estimation with Censored Data

A potential statistical problem arises from the possibility that there will be zeroes for some households for some expenditures or output supplies. Clearly, the greater the level of aggregation the less likely this will occur. Still it may show up, especially for output supplies, for which households may specialize in more than for consumption. This is a problem mainly for estimation purposes and only if there are numerous zero observations. On the demand side if our utility function is $U(X_i^c)$ and our budget constraint $y = \sum_i p_i X_i^c$ then allowing for corner solutions we take Kuhn-Tucker conditions:

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$$(3.15)\quad \partial U/\partial X_i^c - \lambda p_i \le 0, \quad X_i^c(\partial U/\partial X_i^c - \lambda p_i) = 0, \qquad i=1, \ldots, n+1$$

$$y - \sum_i p_i X_i^c \ge 0, \qquad \lambda(y - \sum_i p_i X_i^c) = 0$$

$$X_i^c \ge 0$$

Hence, we do not consume X_i^c if the marginal utility of the money to purchase it is greater than the marginal utility of consuming it, when none is consumed. Obviously, we must constrain the utility function to allow for zero consumption, for instance, in the Klein-Rubin function zero consumption of good i implies $C_i<0$, or else the function will not exist. This raises a question if one derives a demand system from an indirect utility function, e.g., for the QES case, does the direct utility function which gives rise to it allow for zero consumption? Be that as it may for estimation purposes the problem is that of the censored distribution, or Tobit. If $p_iX_i^c \ge 0$ and if $p_iX_i^c = f^i(p,y) + \varepsilon_i$ then $\varepsilon_i \ge -f^i(p,y)$. In estimation, however, we assume $\varepsilon \sim N(O,\Sigma)$. Clearly, the dependent variable has its distribution piled up, or censored, at $-f^i(p,y)$. The expected value of the disturbances is no longer zero (giving rise to inconsistent estimators in the simple ols case). The usual solution would be to let $p_iX_i^{c*} = f^i(p,y)+\varepsilon_i$ and $p_iX_i^c = \max\ (0, p_iX_i^{c*})$ (assume no measurement error). Our case is a bit more complicated than this because of the budget constraint on the demand side. Assuming a theoretically plausible demand system, we have $\sum_i p_iX_i^{c*} = y$ or $\sum_i z_i^* = \sum_i p_iX_i^{c*}/y=1$. We observe $p_iX_i^c$, however, and denoting share of good i by z_i we must have $\sum_i z_i = 1$. If $z_i = \max\ (0, z_i^*)$ and if some z_i^*s are negative, this will not be so. Wales and Woodland (1978) normalize $z_i = z_i^*/\sum_{j\ \varepsilon\ J} z_j^*$, $J = \{j : z_j^* > 0\}$. They derive an extremely messy likelihood function for the z_i (with one share equation dropped). Basically, the function

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involves multiple integrals of probabilities under a multivariate normal distribution, one integral for each zero observation per household, so one household with three zeroes would involve one triple integral. With many households (300–400) Wales and Woodland find computation extremely expensive and so include only three expenditure categories. For the household-firm model expense may well be prohibitive. Alternatively (Wales and Woodland, 1979), one can append errors to the Kuhn-Tucker conditions and derive an appropriate likelihood function.

Of course, we have ignored measurement errors on our dependent variable. To indicate the problem we examine the simple Tobit case. Let $y^* = z\beta + \varepsilon$ and $y = \max(0, y^*)$, where y is the "true" variable. We observe $X = y + v$. Since X can now be negative (a few of the consumption observations are, see Chapter 4) there is no way to know which observations correspond to data at the point of censoring and which do not. Estimation is hopeless without bringing further information to bear.

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CHAPTER 4

DATA: PREPARATION AND SAMPLE CHARACTERISTICS

Sampling Procedure

The data were collected throughout rural Sierra Leone in 1974-75. This was done as part of a large project under the leadership of Drs. Dunstan Spencer and Derek Byerlee. That project was investigating the employment and output effects of alternative development strategies.

The sampling procedures are amply described elsewhere (e.g., Byerlee and Eicher, 1974; Spencer and Byerlee, 1977; King and Byerlee, 1977; Smith, Lynch, Whelan, Strauss and Baker, 1979). Very briefly, the rural area was divided into eight agro-climatic zones. Within each zone enumeration areas (EAs) were delineated and three were randomly selected. Within each enumeration area, 24 households were sampled. This set of households was visited twice weekly from March 1974 to June 1975 (with some households dropping out of the survey for various reasons). Data was collected on production and sales of commodities, on labor use by activity, on prices paid and received, and so forth. Roughly one-half of the sample was chosen randomly to participate in a consumption expenditure survey. These households were interviewed twice during one week in each month to record frequent purchases, and once a month to record large, infrequent purchases. This was designed to give purchase information for one week out of each month, as opposed to th

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to the production and labor use interaction which was collected weekly. The recall periods for the consumption survey were four days, with one of the days overlapping (See Lynch, 1980, for a detailed treatment of the method and of the different results resulting from different number of days recall). Of the 576 households in the production survey, 443 remained with reasonably complete data at the end. Households in three enumeration areas were dropped because of enumerator failure or dishonesty. Other households had to be dropped because of deaths, movement or other factors. For the consumption survey 203 households out of 250 initially in the survey remained at the end (King and Byerlee, 1977, p. 8).

Calculation of Quantity Data

Quantities of foods consumed annually were calculated for 128 foods. Since this was a much more disaggregated list than that used by Byerlee and Spencer, the calculations had to be computed from raw data. There were two components of consumption; quantities consumed out of own production and quantities purchased on the market. Quantities consumed out of own production were estimated as a residual. Estimates of production were taken as a starting point. From these quantities were subtracted quantities sold, wages paid out in kind and seed use for rice (the only commodity for which seed use data was available). Added were wages in kind received and rice seed purchased. Net gifts and loans were not accounted for. Change in storage from the beginning of the crop year to the end of the crop year was assumed to be zero. This was necessary because the beginning stocks data were not considered reliable by Byerlee and Spencer. After the above calculations had been made, commodities defined at different stages in production were grouped togeth

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together to avoid double counting. Disappearances of the more processed form of commodity were converted into units of the less processed form and then subtracted from quantities available for the less processed form. For instance, sales of rice flour were converted into cleaned rice equivalents and then subtracted from availability for household use of cleaned rice. Finally, having combined different stages of production, a "guesstimate" of the fraction of availability lost in storage was made for different crops and subtracted. Unfortunately, there are no very reliable data on this. Some very sketchy evidence is available from the National Academy of Sciences (1978).

For rice two different estimates were prepared. One used rice paddy production as measured by field cuttings. This was considered to be the most reliable production estimate by Byerlee and Spencer and is the one used (converted into clean rice equivalent) when estimating the system of output supplies and labor demand. However, reported sales of rice are considered by Byerlee and Spencer to be understated. If this is so, subtracting a low sales estimate from a good production estimate will leave a high availability estimate. An alternative measure was provided by measuring the production of cleaned rice, a later stage of processing, and subtracting disappearances from that. Most sales of rice are made before it is cleaned so beginning with this stage of production hopefully avoids much of the underreporting of sales. A possible problem with this measure is that the production of cleaned rice may be somewhat understated. Rice is cleaned fairly frequently and in small amounts so it may be easy for a respondant to forget some of what was cleaned. When both availability figur

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figures were made and compared to the few other estimates of rice consumption that exist, it was found that the measure using cleaned rice production corresponded much better (see Smith, Lynch, Whelan, Strauss, and Baker, 1979). Hence, the cleaned rice consumption figure was used in the demand part of this study.

In deriving the annual figures for production and net disappearances of foods, the same procedure was used for each component. This procedure was also used by Byerlee and Spencer in preparing their more aggregate estimates. Computation was carried out for 328 households.[1] First, the quantities were added for each month for each household. At this stage local units were converted into four standardized units using conversion factors supplied by Byerlee and Spencer. In general, these factors came from actual weighings made in local markets. For many households there was an incomplete accounting of the month. Perhaps an enumerator was sick, etc. If less than 16 days per month were accounted for the month was considered to be missing. If missing days numbered less than 16 the incomplete monthly totals were divided by the fraction of the month covered to arrive at a monthly total (the number of days missed were available for each month and for each household). Figures for missing months, in almost all cases two or less, were estimated by a procedure outlined in King and Byerlee (1977, pp. 73-75). The procedure assumes that the monthly distribution of a household's consumption is identical to that of other households in the

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[1]The remaining households in the set of 443 were considered by Byerlee and Spencer to be unfit for income analysis. Usually this was due to inadequate production data.

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same agro-climatic zone. Indices representing the proportion of annual consumption for the zone that occurred in each month were constructed from the non-missing data. These indices were calculated for 17 aggregated commodity groups. Using such a level of aggregation allowed the averaging to take place over a sufficient number of households to provide a meaningful average for the region. The indices for the missing month(s) were subtracted from unity and the result divided into the sum of the particular household's quantities for the good months. That is, the household's incomplete annual figure was divided by that proportion of annual consumption which takes place in the months for which the figures correspond, by an average household in the particular region. The resulting annual figures were then converted into kilograms.

These figures were then edited in a few instances for extremely large positive and large negative observations, taking into account household size and household income in the editing process.

Quantities of foods purchased were constructed in the same way. The day of overlap was removed, the figure coming from the shortest recall period being used. Monthly data were used only if data were available for at least three of the seven days for which data were collected. Households were dropped if they had less than six months of useable data. Monthly household totals were constructed by dividing the incomplete monthly data by the proportion of days in the month for which the household had reported. Missing months were filled in by using the same indexing procedure as was done for consumption out of home production.

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Quantities of foods purchased were only calculated for households which were in the good production sample and which met the criterion of having at least six good months of data. There were 140 such households.

Values of foods consumed were calculated by multiplying consumption out of home production by farm gate sales prices and adding that to the value of foods purchased, using purchase prices to make the latter calculation. This was done for each of the foods and these values were then added into the appropriate commodity groups. Valuing consumption out of own production at farm gate price implies this is the relevant opportunity cost; that is, the item could have been sold. This will not be strictly true for every household but it will be true for many. For some households, which are net purchasers, one could argue that they value consumption out of own production at purchase price providing that qualities of foods from own production and from the market are equal. In the limit this approach would value foods differently for each household and would run into serious problems of the resulting prices being endogenous to the household-firm model, as we shall see in the section on prices. Alternatively, we could argue that there are some quality differences between foods consumed out of home production and foods purchased. The latter after all have embodied in them certain services provided by persons in the market system. From this point of view the two sources of foods ought to have different prices and farm gate price and purchase price are the two best estimates available.

Value of nonfood consumption was taken as the sum of values purchased and values produced less values sold. Again, the former use purchase prices and the latter two sales prices. This had previously bee

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been computed for use in King and Byerlee (1977) and values were taken from that study. Of the 140 households having complete food consumption data two did not have nonfood expenditure information and so were dropped. This left 138 households, our final sample size.

Values of production were derived by multiplying quantities produced by farm gate sales price, and then added into the appropriate groups. Production of raw products was used; processed product production was not added in order to avoid double counting. For example for fish, only estimates of fresh production were used. Production of dried fish was not added to that.

Household labor supplied was measured in terms of male equivalents. Spencer and Byerlee (1977) found that wages for females over 15 were .75 of wages for males over 15, and children aged 11-15 had wages .5 of male adult wages. Under the assumption that relative wages reflect relative marginal productivities, hours of labor supplied were weighted by these factors and then summed. Labor supply includes work on all agricultural and nonagricultural activities in the household, plus labor sold out. It excludes such activities as food preparation, child care and so forth. The variable was derived by summing the weighted hours worked by all persons on these activities, and subtracting weighted hours worked by hired laborers.

Labor demanded by the household was estimated in the same way as labor supplied, but hired labor was included and labor sold out and labor used in processing agricultural products were excluded. The latter was excluded because processed agricultural products were not included in the production measures to avoid double counting.

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Calculation of Prices

Sales prices and purchase prices were calculated separately for each food commodity. The prices were calculated for each of the eight agro-climatic regions. Prices were available for each transaction a household made. In principle, we could have calculated prices for each of the 138 households. This would have created serious statistical problems. Assume that every household in a region faced the same set of sales and purchase prices. Still, different households have different demographic characteristics and different amounts of land and of capital. Hence, even with a common utility function, different households would buy and sell foods at different times of the year. Since prices will have a seasonal movement, calculating an average price for each household would result in those averages being different for each household, even though the households actually faced the same set of prices. The source of the different prices would be different household behavior. That is, prices would be endogenous to the household-firm model we use to explain household behavior. To then use these prices in estimating a system of demand equations would result in inconsistent parameter estimates since these "independent" variables would be correlated with the disturbances on the equations. It is in order to avoid this problem that we average prices of transactions across households. Region was chosen instead of enumeration area as the definition of market area because it was feared that the latter might be too small. Also, region is the area used by Byerlee and Spencer when they compute their prices.

Sales prices were calculated using the production sample of 328 households. Since the production and sales data for those households were considered useable by Byerlee and Spencer, it seemed to be unwise to throw out the information provided by those households not in our final sample of 138. The prices were calculated as total value of sales in a region divided by total quantity of sales in the region. All prices are in Leones per kilogram. Purchase prices were calculated in the same manner using only the smaller sample of 140 households. Sales and purchase prices were averaged to obtain a single average consumption price for each of the 128 foods. The weights used were the proportion of the value of total consumption (from purchases and from home production) in the region represented by the value of either consumption out of home production or of consumption from purchases. That is, the value of total consumption was added over households in a region; this was the denominator of the weight. Values of consumption from home production and from purchases were added separately across households in a region; these were the numerators of the weights. Hence, the weights were regional as were the prices. Prices of the 128 commodities were then aggregated into the appropriate groups, again using the proportion of value of group consumption represented by each component as the weight. Algebraically we have

$$(4.1)\quad P_i = \sum_{j \varepsilon i} \left(\frac{V_{ijH}}{V_i} P_{ijS} + \frac{V_{ijP}}{V_i} P_{ijP} \right)$$

where $P_i \equiv$ regional price of group i, $P_{ijS} \equiv$ sales price of food j in group i; $P_{ijP} \equiv$ purchase price of food j in group i, $V_i \equiv$ value of total consumption in the region for group i; $V_{ijH} \equiv$ value of consumption out of home production; $V_{ijP} \equiv$ value of consumption from market purchases. These are the prices used in estimating the quadratic expenditure system. The average was arithmetic not geometric. The latter is appropriate for estimating a translog system but the former is appropriate for estimating a linear expenditure system, which is a special case of the QES. As seen in Chapter 2, the QES is not separable so perfect price aggregators such as used by Anderson (1979) are not possible.

Farm sales prices for the 128 foods were aggregated into the same groups as the weighted sales and purchase prices were. In this case the weights were the proportion of value of regional sales for the group represented by each of its component foods. The weights were calculated using the large production sample of 328 households. These were the prices used in estimating the system of output supplies and labor demands. There is room for disagreement as to whether these weighted sales prices or the weighted "consumption" prices used in the QES estimation ought to have been used on the production side. On the one hand, the household-firm model does not distinguish between the two prices, indeed it assumes they are equal. From this point of view, we should use the same set of prices for each component of the model. However, looking at the dichotomous nature of the model, we first maximize short run profits subject to a production function. If this were done as a separate study sales prices are the appropriate ones to use.

Nonfood sales prices by region were available from the earlier work of Byerlee and Spencer. Nonfood purchase prices were not available. In deriving them we could not use the same procedures as were used for foods. The same item was often purchased in several different units. For foods a great deal of effort was expended by Victor Smith and William Whelan in obtaining conversions into a common unit, kilograms, but this was not done for nonfoods. However, we did have values of nonfood purchases. These had been used by King and Byerlee. We i

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took categories of nonfoods representing the bulk of expenditures on nonfoods. These were tobacco products, fuel and light, clothing, imported cloth and transport. Within each of these categories, we found one item which was the most important. These were cigarettes, kerosene, jongs (a local term used for clothing), imported cloth, and lorry rides. For these items it turned out that transactions were predominately in one unit, though different for different items. Average prices, by region, for these specific commodities in the specific unit were taken to represent prices for the particular group. These prices were combined into a nonfoods purchase price by weighting them by the proportion of value of regional nonfood purchases represented by purchases on all items in the group. The purchase and sales prices were then combined, again using proportion of value consumed as weight. Hence, the quantity unit of nonfood price is a hodgepodge of different units.

Wage was taken directly from Byerlee and Spencer's earlier work. It is expressed as Leones per hour worked for males over 15 years old.

Calculation of Production Inputs

Land is measured as total land area cropped, in acres. It includes land in perennial as well as annual crops. It is a simple sum of acres. No weighting to reflect different qualities (for example of swamp and of upland lands) was made because no such data were available.[2] For a very few households, data on this variable were missing. Since these households had useable data for all other variables, they were not

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[2]The rental markets are very thin and rental prices reflect a household's standing in the community as much as the economic value of the land (Spencer and Byerlee, 1977, pp. 21-24).

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not dropped. Byerlee and Spencer had classified households into many different farm types. From the production sample of 328 households we computed average land-labor use ratios for each farm type. Knowing the farm type and the labor use for these households we were able to estimate total land cropped.

Capital is measured as the value of its flow. For variable capital this represents no problem. However, variable capital for our sample is minuscule, mostly rice seed. Only a very little fertilizer is used and a little machinery hired, and these were added into the total. Since there are some values for variable capital, which is a flow, it was necessary to convert the stock of fixed capital into the equivalent flow in order to add the two. This raises many problems, but followed the lead of Spencer and Byerlee (1977, p. 46). In their work they used the formula

$$(4.2) \quad K = \frac{rV}{1-(1+r)^{-n}}$$

where $K \equiv$ annual service user cost, $V \equiv$ acquisition cost of capital, $n \equiv$ expected life of capital in years. In a perfect market the acquisition cost of the asset equals the discounted sum of its annual flows. Assuming the annual flows to be constant in real value, and assuming the flows start in year one, we obtain equation 4.2. Byerlee and Spencer use a discount rate of .1 and expected lives that were different for different types of capital (Spencer and Byerlee, 1977, pp. 47-48). The types of capital included are farm tools, animal equipment (includes fishing equipment), nonfarm equipment, livestock and tree crops. The coefficients $\frac{r}{1-(1+r)^{-n}}$ used are 1/5, 1/6, 1/13, 1/3.8, and 1/30 respectively.

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Ethnic Group

Household characteristic variables require little special comment on their preparation save the ethnic group of the household head. This variable was derived from two sources. For about half of the 138 households used in the analysis there was direct observation. From these it was apparent that within an enumeration area virtually all households were of the same ethnicity. As a check we had from the 1963 census (the 1974 census results were not available) the numbers of people by ethnic group living in each Chiefdom (an administrative unit that can be matched to our enumeration areas). The census was checked to see whether the dominant ethnic group within a Chiefdom was the same group shown by the data available from our sample. In all cases the groups matched. For those few enumeration areas for which there was no ethnic information from the sample, the dominant group as reported in the census was used. There was one Loko household, from enumeration area 53, in our sample. This was grouped with Mende households, the dominant group in the sample, because they were the second largest group within that enumeration area. The two other ethnic groups represented in the final sample of 138 households were Temne and Limba.

Commodity Definitions

The commodity definitions used in the study are given in Figure 4.1. The groupings represent a compromise between the number of commodities and the number of demographic variables to be used in the QES. With seven commodities and no demographic variables there would be 20 parameters to estimate. Adding a demographic variable adds seven

Commodity-Subgroup	No.	Components
Rice	1	
Root crops and other cereals	2	
Root crops		Cassava (including gari, foofoo and cassava bread), Yam, Water Yam, Chinese Yam, Cocoyam, Sweet potato, Ginger, Unspecified
Other cereals		Benniseed, Fundi, Millet, Maize (shelled), Sorghum, Agidi,[1] Biscuits (Natco)[1]
Oils and Fats	3	Palm oil, Palm kernel oil, Palm kernels,[2] Groundnut oil,[1] Coconut oil, Cocoa butter, Margarine,[1] Cooking oil,[1] Unspecified[1]
Fish and animal products	4	
Fish		Bonga (fresh), Bonga (dried),[1] Other saltwater (fresh), Other saltwater (dried),[1] Frozen fish,[1] Freshwater (fresh),[1] Tinned fish[1]
Animal products		Beef, Pork,[1] Goats and sheep (dressed), Poultry (dressed), Dear (dressed), Wild bird (dressed), Bush meat (dressed), Cow milk, Milk (tinned),[1] Eggs, Honey bee output, Unspecified[1]
Miscellaneous foods	5	
Legumes		Groundnuts (shelled), Blackeyed bean (shelled), Broadbean (shelled), Pigeon pea (shelled), Soybean (shelled), Green bean (in shell), Unspecified (shelled)
Vegetables		Onions, Okra, Peppers and Chillies, Cabbage, Eggplant, Greens, Jakato, Pumpkin, Tomato, Tomato paste,[1] Watermelon, Cucumber, Egusi, Other
Fruits		Orange, Lemon, Pineapple, Banana, Plantain, Avocado, Pawpaw, Mango, Guava, Breadfruit, Coconut, Unspecified
Salt and other condiments		Salt,[1] Sugar,[1] Maggicubes,[1] Unspecified[1]
Kolanut		
Nonalcoholic beverages		Coffee, Tea,[1] Soft drinks (bottled),[1] Ginger beer (local)[1]
Alcoholic beverages		Palm wine, Raffia wine, Beer (Star and Heineken),[1] Omole,[1] Gin (local), Liquor (Rum, etc.)[1]
Nonfoods	6	Clothing, Cloth, Fuel and light, Metal work, Woodwork, Other household and personal goods, Transport, Services and ceremonial, Education, Local saving, Tobacco products, Miscellaneous
Household labor	7	All farm and nonfarm production and marketing activities (for labor demand, work on processed agricultural products excluded), Labor sold out.[1] Excludes household activities such as food preparation, child care and ceremonies

[1]Commodity is not included in production figures for use in estimating system of output supplies and labor demand either because it is only purchased or because it is a more processed form of a commodity already counted.

[2]Not included in consumption data but included in production data.

Figure 4.1

Components of Commodities

parameters to be estimated and adding a variable to model the total time available adds another parameter. One demographic variable would probably mean using only household size but ignoring its composition. This does not seem to be a good strategy. Yet using more commodity groups would force some such compromise. On the other hand, the grouping we have used involves an extremely heterogeneous mix for miscellaneous foods. In principle, it would have been nice to separate legumes (mostly groundnuts) from fruits, vegetables and the other components of miscellaneous foods. Nutritionally, legumes are high in protein relative to the other components and also high in calories. Root crops (largely cassava) and other cereals (mostly sorghum) are also quite different nutritionally, especially in protein content. Yet if we use the economic criterion of grouping close substitutes and/or close complements root crops and other cereals probably meets that reasonably well. Rice is kept separate because it is the most important staple and because the government does have rice programs if not rice policies.

The other factor besides keeping the number of parameters to be estimated to a reasonable number was keeping the number of nonconsuming households for the groups to a very small number. In Chapter 3 we noted that zeroes in our dependent variable cause inconsistent parameter estimates, with the problem being small if the number of zeroes is small, and large if nonconsuming households are numerous. The methods for correcting for this were seen to be quite involved and extremely expensive. Hence we aggregated with this in mind. For example, this was a major consideration in grouping root crops with other cereals. Our final groupings have seven households not consuming root crops and other cereals and five not consuming oils and fats. All other groupings have no nonconsuming households. There are a few negative observations using our grouping, mostly in the groups for root crop and other cereals and oils and fats. These reflect errors in our data but are left in. As noted above, large positive and negative outliers were edited. Presumably there are also errors of overstating consumption left in our data. However, there is no basis for knowing which observations they are. As long as the average error is zero our statistical estimates will be consistent, since these are errors in dependent variables. To edit further by eliminating only the negative estimates would risk making the average error positive, leading to inconsistent estimates. Hence, this was not done.

Sample Characteristics

In viewing the characteristics of our sample and the results of estimating the household-firm model it will be useful to look at not only the sample means but also the means of households by total expenditure groups. Governments have begun to be interested in what happens to different income groups, particularly when they are concerned with nutritional issues. For our purposes we divide the sample into three groups: households spending under 350 Leones; those spending between 350 and 750 Leones; and those spending more than 750 Leones. To get an idea of how poor these households are note that the annual per capita expenditures in 1974-75 U.S. dollars are \$54, \$88 and \$136 respectively for the low, middle and high expenditure groups. For the capital city, Freetown (which was sampled for a migration component of the original study) when divided into three groups, the average income of the middle group is \$153. Hence, even our "high" expenditure households are quite poor both when compared to urban Sierra Leone and to other countries.

The sample characteristics of the variables appearing in the quadratic expenditure system are reported in Table 4.1 (for a more complete statistical description see Smith, Lynch, Whelan, Strauss and Baker, 1979). Expenditures on all commodities and the value of labor supplied increase with the expenditure group. As one can see from Table 4.2 rice comprises the largest average share of total expenditures for foods. The low share of expenditures on nonfoods, .33 at the sample mean, is a further indication of the poverty of these households. Household size rises with the expenditure group. Children under 10 as a proportion of total size is smallest for low expenditure households and largest for the middle expenditure group.

The production characteristics of these expenditure groups are reported in Table 4.3. Rice is the most important crop in value though its importance as a proportion of total value output diminishes for the high expenditure group. In general, value of production and of labor demanded increases with the expenditure group. Land area cropped does not change a great deal between expenditure groups, but value of capital flow jumps for the high expenditure group. The reason for this, and for the declining importance of rice for this group is the presence of nine households from Enumeration Area (EA) 13 in this group. These are commercial fishermen who also grow and sell a large amount of vegetables to the Freetown market. In their production characteristics they are quite different from the rest of the households,

Table 4.1

Mean Values of Consumption Related Data by Expenditure Group1

Variable	Expenditure Group: Low	Middle	High	Mean
Expenditures[2]				
Rice	58.2	125.2	262.9	146.7
Root crops & other cereals	10.7	32.4	147.4	61.3
Oils and fats	19.2	37.2	122.8	58.1
Fish and animal products	30.6	61.9	118.3	69.5
Miscellaneous foods	28.0	65.8	99.0	64.1
Nonfoods	90.0	190.1	324.0	199.9
Value of Household Labor	306.4	361.8	530.1	396.5
Prices[3]				
Rice	.25	.23	.27	.25
Root crops & other cereals	.36	.66	.63	.55
Oils and fats	.73	.62	.66	.67
Fish and animal products	.62	.60	.39	.54
Miscellaneous foods	.56	.58	.60	.58
Nonfoods	.62	.64	.75	.66
Household labor	.08	.08	.09	.08
Household characteristics[4]				
Total size	4.8	6.4	8.7	6.7
Members under 10 years	1.2	2.1	2.7	2.0
Members, 11-15 years	.5	.7	1.1	.8
Males over 15 years	1.7	1.8	2.6	2.1
Females over 15 years	1.4	1.8	2.3	1.8
Proportion Limba or Temne	.45	.29	.44	.39
Proportion northern	.43	.25	.40	.36
Number of households	44	51	43	138

[1]Households in the low expenditure group are those with total expenditure less than 350 Leones. Households in the middle expenditure group are those with total expenditure between 350 and 750 Leones. Households in the high expenditure group are those with total expenditure greater than 750 Leones.

[2]In Leones. One Leone = U.S. $1.1 in 1974/75.

[3]Weighted average of sales and purchase prices. In Leones per kilogram for foods and per hour of male equivalent for labor.

[4]In numbers.

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Table 4.2

Actual Average Total Expenditure Shares
By Expenditure Group

Commodity	Expenditure Group			
	Low	Middle	High	Mean
Rice	.25	.24	.24	.24
Root crops and other cereals	.05	.06	.14	.10
Oils and fats	.08	.07	.11	.10
Fish and animal products	.13	.12	.11	.12
Miscellaneous foods	.12	.13	.09	.11
Nonfoods	.38	.37	.30	.33

Table 4.3

Mean Values of Production Related
Data by Expenditure Group

Variable	Low	Middle	High	Mean
	Expenditure Group			
Value of Production[1]				
Rice	202.3	238.6	368.4	267.5
Root crops & other cereals	9.5	38.7	142.5	61.8
Oils and fats	39.7	93.9	162.9	98.1
Fish and animal products	9.6	26.2	198.1	74.5
Miscellaneous foods	25.5	54.5	145.3	73.5
Nonfoods	12.8	25.0	50.9	29.2
Value of Labor demand	293.8	373.5	572.4	410.0
Prices[2]				
Rice	.22	.20	.23	.22
Root crops & other cereals	.14	.12	.19	.15
Oils and fats	.46	.39	.36	.41
Fish and animal products	.53	.54	.39	.49
Miscellaneous foods	.27	.28	.28	.28
Nonfoods	1.18	1.29	1.50	1.32
Labor	.08	.08	.09	.08
Household Characteristics				
Cultivated land[3]	5.8	6.9	6.5	6.4
Capital[4]	34.5	34.0	78.7	48.1
Proportion in EA 13	0.00	.02	.21	.07

[1]In Leones. Valued by weighted sales prices.

[2]Weighted sales prices. In Leones per kilogram for foods and per hour of male equivalent for labor.

[3]In acres.

[4]Annual flow in Leones.

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as will be confirmed in Chapter 7 (this is not so true of their consumption characteristics). Indeed, it is useful to present in Table 4.4 the same material as in 4.3 only grouping households by the ten EA 13 households and the rest. The fishing households cultivate much less land than the other households (1.6 to 6.8 acres), but have considerably more capital in the form of boats and the like. Prices are also different with the price of fish and animal products being considerably lower.

Table 4.5 presents the quantities of production, total consumption and the difference, net marketed surplus, by expenditure group. Except for rice the high expenditure group tends to sell more or buy more than do lower expenditure groups. The only groups for which net purchases from the market are made are nonfoods, labor for middle and high expenditure groups and fish and animal products for low and middle expenditure groups. We have to remember, however, that these are net figures. A household may hire labor during peak season and sell labor in the offpeak season. The figures reported here combine these two transactions.

Finally, and not surprisingly, households specialize in production more than in consumption. Using our commodity definitions we have three households which do not produce rice, 19 which have no production of root crops and other cereals, 24 for oils and fats, 35 for fish and animal products, 12 for miscellaneous foods, and 59 for nonfoods. The relatively large number of zero outputs gives rise to statistical problems of the sort explored for the demand side in Chapter 3. These will be given much more detailed treatment in Chapter 7.

Table 4.4

Mean Values of Production Related Data by
EA 13-Non-EA 13 Households

Variable	EA 13	Non-EA 13
Value of Production[1]		
Rice	62.7	283.5
Root crops & other cereals	27.9	64.4
Oils and fats	20.6	104.2
Fish and animal products	733.5	23.0
Miscellaneous foods	331.8	53.3
Nonfoods	82.8	25.0
Value of Labor demand	954.7	367.5
Prices[2]		
Rice	.19	.22
Root crops & other cereals	.25	.14
Oils and fats	.37	.41
Fish and animal products	.17	.52
Miscellaneous foods	.15	.29
Nonfoods	2.23	1.25
Labor	.15	.08
Household Characteristics		
Cultivated land[3]	1.6	6.8
Capital[4]	214.3	35.1

[1]In Leones. Valued by weighted sales prices.

[2]Weighted sales prices. In Leones per kilogram for foods and per hour of male equivalent for labor.

[3]In acres.

[4]Annual flow in Leones.

Table 4.5

Quantities[1] Produced, Consumed, and Marketed by Expenditure Group

Commodity	Expenditure Group	Produced	Consumed	Marketed
Rice	Low	902.8	232.8	670.0
	Middle	1,164.3	544.3	620.0
	High	1,622.2	973.7	648.5
	Mean	1,227.5	586.8	640.7
Root crops and other cereals	Low	69.0	29.7	39.3
	Middle	335.8	49.1	286.7
	High	744.6	194.9	549.7
	Mean	422.1	111.5	310.6
Oils and fats	Low	85.5	26.3	59.2
	Middle	242.0	60.0	182.0
	High	447.2	186.1	261.1
	Mean	242.2	86.7	155.5
Fish and animal products	Low	18.0	49.4	-31.4
	Middle	48.3	103.2	-54.9
	High	508.7	303.3	205.4
	Mean	151.5	128.7	22.8
Miscellaneous foods	Low	93.0	50.0	43.0
	Middle	191.3	113.4	77.9
	High	515.3	165.0	350.3
	Mean	262.3	110.5	151.8
Nonfoods	Low	10.8	145.2	-134.4
	Middle	19.4	297.0	-277.6
	High	33.9	432.0	-398.1
	Mean	22.1	302.9	-280.8
Labor[2]	Low	3,963.8	3,800.3	163.5
	Middle	4,286.7	4,425.1	-138.4
	High	5,687.8	6,141.4	-453.6
	Mean	4,670.2	4,829.7	-159.5

[1]In kilograms for foods, hours for labor.

[2]Produced and Consumed correspond to supply and demand.

Caloric Availability

Having determined the quantities available for consumption from home production and from market purchases, nutrient availabilities may be calculated by using conversion rates available from food composition tables. This was done by William Whelan using FAO prepared food balance sheets specific to Africa (FAO, 1968). For this purpose, quantities purchased and available from home production were added without value weights for each of the 128 foods in our data. The nutritional composition of foods consumed from each source was thus assumed to be identical. The conversion into nutrients accounted for the inedible portion of each food (using figures available from the food composition tables). What was derived then was nutrients available for each food at the farm gate or retail level, taking out the inedible portion. Left in, however, is whatever part of the edible portion is wasted by the household before ingestion. This will vary vastly by household and by food. The FAO, in its calculations, assumes this to average ten percent (FAO, 1973, pp. 87-8).

Table 4.6 reports total caloric availability expressed per capita per day, and its sources by our five food groups for each of the expenditure groups. For this purpose caloric availability by food was summed into the five food groups and then totaled. Not surprisingly, caloric availability increases dramatically with expenditure group, particularly between the low and middle groups. The sample mean of 2109 cal/cap/day compares to an estimated availability of 2090 cal/cap/day computed by FAO from food balance sheets for the entire country (including urban areas) for a 1972-74 average and a 1975-77 average (FAO, 1980, pp. A41).

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Table 4.6

Calorie Availability and Its Components
by Food Group by Expenditure Group

Proportion of Calories from:	Expenditure Group Low	Middle	High	Mean
Rice	.44	.45	.43	.44
Root crops & other cereals	.17	.17	.15	.16
Oils and fats	.12	.12	.20	.16
Fish and animal products	.17	.10	.10	.11
Miscellaneous foods	.11	.15	.11	.12
Total calories per cap per day	1,188	2,132	2,608	2,109

The availability calculated from food balance sheets does cover urban as well as rural areas. It is formed by taking production, subtracting net exports, seed, feed, waste (storage and marketing), and net change of storage. The remaining figures are converted into units sold at retail level by further adjusting for processing. The FAO food balance sheet availability figures are comparable to ours, and so is their caloric availability figure (which also accounts for the inedible portion; FAO, 1972, p. 45).

The low availability figure for the low expenditure group is not unusual when compared to other budget studies. For example, a study conducted by the Vargas Foundation in Brazil, using 1960 data, found the lowest income decile having a caloric availability of some 1400 cal/cap/day (reported in Reutlinger and Selowsky, 1976, p. 11).

The availability figures can be compared to "requirements" per cap per day (the amount needed to maintain body weight with moderate activity) as computed for Sierra Leone by FAO (1980, p. A41). This "requirement" figure of 2300 calories per cap per day must not be taken too literally. It is computed using sex and age composition figures for Sierra Leone, and assuming an average weight for age. By further assuming that activity levels are "normal" (comparable to a reference adult in the United States), requirements figures by age group can be obtained and weighted to obtain a national requirements figure. This figure is directly comparable to the food balance sheets availability figures. It has built into it an allowance of ten percent for waste of edible portion between retail level and ingestion (FAO, 1972, p. 45). Hence, it is also comparable to the figures in Table 4.6. Three factors should be noted when comparing the figures. First, the availability figures are averages within the expenditure group. Hence, some in each group may have availability greater than 2300 calories per cap per day. Even for a household the figure is an average over a year. Secondly, the requirements may be interpreted as an average also. Sukhatme (1977) offers an estimate of 400 calories as the standard deviation, part of the variation being between persons and part being intra-individual (over time). We might subtract one standard deviation from the requirements and use that as an estimate of the "requirements" for the population, with average activity levels. However, as both Sukhatme (1977) and Srinivasan (1981) point out, even this procedure risks misclassifying household groups because of the usual type I and type II statistical errors. Thirdly, substitution is possible between food intake and activity levels. The FAO Ad Hoc Expert Committee recommended using 1.5 times the Basal Metabolic Rate (calories expended under resting, fasting conditions) as the energy cost of maintenance with minimal activity. For children who are growing it will need to be higher (FAO, 1973, pp. 36-7). Caloric availability below this amount would likely result in persons being underweight. Basal metabolic rates vary by individuals and over time. The energy cost of maintenance will vary even more due to its inclusion of even minimal activity levels. The only figures available on BMRs are from measurements at one laboratory in Boston over a 15 period (FAO, 1973, p. 107). Those figures are by weight and sex. If we take the "reference man" of 65 kilograms and the "reference woman" of 55 kilograms and average their BMRs this is 1588 calories/cap/day. Taking 1.5 times this and subtracting 20 percent of that to account for variation in BMRs, to arrive at a conservative estimate of daily maintenance requirements (see FAO, 1974, p. 49) we arrive at roughly 1900 calories per cay per day. However, the "reference" weights of 65 kilograms and 55 kilograms, derived from U.S. data, are probably high for our sample. Then the daily maintenance requirement figure should be even lower. Using 55 and 45 kilograms as reference weights for men and women and repeating the calculation we obtain 1735 calories per cap per day. Even without adjusting for lower weights, we need to average the 1900 with a requirements figure for children of different ages. For young children, even allowing for growth, these requirements are less than 1900 calories per day, so the population requirements figure corresponding to 1.5 x BMR will be lower than 1900. However, the mean availability for the low expenditure group is substantially below 1900 calories. In any case, we know that undernutrition is the major nutritional problem in Sierra Leone. UCLA (1978), in a national survey using anthropometric data, found that some 30 percent of children under five years are underweight (less than 80 percent of the standard weight for age). Information on pregnant and lactating women confirmed the undernutrition problem. Hence, it is not surprising to find indications of undernutrition for low expenditure households in our data, even if the extent of it may be overstated.

CHAPTER 5

CHOOSING DEMOGRAPHIC VARIABLES: SINGLE EQUATION SHARE REGRESSIONS

Introduction

The Quadratic Expenditure System with seven commodities and k household characteristic variables will have (3+k)7-2 parameters (excluding the total time parameters). Each demographic variable adds seven parameters to be estimated. The more the parameters the more iterations will be required for the computer to converge to a maximum likelihood solution and the greater the expense per iteration. Expense both in computer time and in research time will thus rise as the size of the problem increases. Having decided upon using the QES for estimation and believing that to use less than seven commodities, meaning five food groups, will result in too much aggregation for the research problem at hand, we must economize on the number of demographic variables we utilize. In principle, there are many such variables which might be included and for which we have data. The question arises, how should we choose between them?

$\bar{R}^2$ and C_p as Variable Selection Criteria

Many proposed solutions to this variable selection problem exist in the literature. For a review one can see Hocking (1976), Gaver and Geisel (1974), or Amemiya (1980). Of the non-Bayesian solutions, the only ones considered here, there is no one which dominates all others by any of the usual statistical criteria. Hence, some arbitrariness is involved in selecting the procedure to be used. We experimented with two criteria, Mallow's C_p and maximum $\bar{R}^2$. Both involve a trade-off between incurring bias in the predictions and reducing the variance of the predictions. It is easy to show (See Theil, 1971) that the expected variance of the error terms in an ols regression is lowest when the correct set of independent variables is used. It is also true that $\bar{R}^2 = 1 - \frac{\hat{\sigma}^2(n-1)}{SST}$, where $n \equiv$ number of observations and $SST = \sum_{i=1}^{n} (y_i - \bar{y})^2 \equiv$ total sum of squares. This implies that minimizing $\hat{\sigma}^2$, the computed variance of the regression disturbances, is equivalent to maximizing $\bar{R}^2$. It is also true that $\bar{R}^2$ will be increased only if the F-statistic for the variable(s) being removed is less than one. It turns out (See Hocking, p. 17) that this condition is a necessary one for the mean squared error of prediction to be lowered. That is, now assume we use only a subset of the "true" variables influencing our dependent variable. Then a necessary condition to lower the prediction mean square error (variance plus bias squared) is that the F-statistic of the variable(s) dropped be less than one.

Assume again that we know which variables are the true set. If we take the expected sum of squared prediction errors of a particular estimator of our dependent variable conditional on the values of our independent variables, and divide by the true regression variance we have the formula for Mallow's C_p statistic. Algebraically, we have $\frac{1}{\sigma^2} E\left(\sum_{i=1}^{T} (y_i - E(\hat{y}_i))^2\right) = \frac{E(RSS)}{\sigma^2} - T + 2p$, where $RSS \equiv$ residual sum of squares and $p \equiv$ number of regressors used in the estimate $\hat{y}_i$. Substituting the estimated RSS from the p-variable regression and $\hat{\sigma}^2$ from the regression using the complete set of independent variables, we have our statistic.

It turns out that C_p will be lowered if the F-statistic of the variable(s) dropped is less than two. This then is a more restrictive criterion than maximizing $\bar{R}^2$. It also is true (See Hocking, p. 18) that $C_p < p$ is a necessary condition for the mean squared error of prediction to be lowered compared to that of the full regression.

Specifications of Regressions

These criteria are for single equations. We want to choose the "best" demographic variables for our system of seven equations. To get at this we use these criteria for several single equations. If a set of demographic variables is included in the "best subset" for several commodities this will be an indication of its suitability for use in the systems estimation.

For functional form we try to mimic the QES. This means a squared term for our measure of total income and zero homogeneity of quantity demanded with respect to all prices and to total income. In share form the functional form is

$$(5.1) \quad \frac{p_i x_i^c}{y} = b_0 + b_1 \frac{y}{p_i} + \sum_{j=1}^{n} \gamma_j \frac{p_j}{y} + \sum_{k=1}^{K} \sigma_k \frac{p_i \eta_k}{y}$$

where $y \equiv$ total income, $p_i \equiv$ price of good i, $\eta_k \equiv$ household characteristic k. This equation meets the two aforementioned criteria.

The equations were run in share form because it was believed on the basis of other researchers' experience estimating demand equations that this form was least likely to exhibit heteroskedastic error terms (See for example Pollak and Wales, 1978a). The selection criteria used assume homoskedastic errors in derivation of their properties so it is preferable to correct for this before use of such criteria. As we shall see in Chapter 6, for our data the appropriate weights are predicted values of the dependent variable, not total income, hence the share equations are heteroskedastic also. How this affects our selection results is unclear.

Since total income is not directly observed, we need to use a proxy variable. Following our household-firm model total income can be defined as $TI=p_L T+\pi=p_L(\sum_{r=1}^{q}\gamma_r m_r)+\pi$. We have as estimate of π the value of goods expenditure less the value of household labor. We could substitute this formula into 5.1. The resulting equation would be nonlinear in parameters. By estimating the expenditure form (i.e., multiplying by total income) many of the nonlinearities disappear. We would still be left with some nonlinear parameter restrictions on the γ_r's; however, these could be ignored and ols run at the expense of efficiency, not bias of the parameters. This procedure would introduce many variables into the regressions. We would have terms $\frac{p_L}{p_i} m_r$ for each total time demographic variable r plus variable cross products $\frac{p_L^2}{p_i^2} m_r m_s$ and cross products of the total time variables and profits, π. In order to reduce the number of these variables, hence to test for more demographic variables, total expenditure was used as a proxy for total income. This creates some problems of bias because in the household-firm model total expenditure is endogenous, hence correlated with the error terms appended onto equation 5.1. We ignore this problem here and treat equation 5.1 as a traditional demand equation with total expenditure being a predetermined proxy for permanent income.

In running the share regressions we disaggregate foods into more than the five groups used in the rest of our analysis. Ideally, we should have used the same commodity definitions, but, the more disaggregated groups were useful for other parts of the project of which this dissertation is one part (see Smith, Strauss and Schmidt, 1981). The food groups used for the single equation estimation were rice, cassava, other cereals, palm oil, fresh fish, dried fish, groundnuts, fruits, vegetables, and salt and other condiments. The definitions of these groups follow Figure 4.1.

Prices of rice, cassava, palm oil, dried fish, groundnuts and nonfoods appeared in each equation in the full set of variables from which best subsets were chosen. Depending on the commodity, other prices also were included. Table 5.1 (pp. 81-83) indicates which prices were included for which equation. For the all important household characteristic variables, Figure 5.1 shows which were included. Except for the ethnic group dummy variables and the age of the head of household all have to do with household size and its age or sex composition. No variable is included for persons over 65 years because when added to the other components of household size they add to total size, perfect multicollinearity would result if all were included in an ols. The dependency ratio is the number of persons aged less than 15 or greater than 65 divided by those aged 16 to 65. If it were to prove to be a useful variable perhaps other size and composition variables could be dropped. Of variables not included in this exercise, four bear mentioning. Sex of household head was not used because less than five households in our sample had a female head. Religion of household head was not included because there were no direct observations on this variable. An indirect measure, years of Islamic education of the household head, was available but zero years for this variable need not imply

Variable	Definition
Size	Number of persons in household
Inf	Children aged five years and younger
Ych	Young children, aged six to 10 years
Ch	Children aged 11 to 15 years
Ad	Adults aged 16 to 65 years
Depr	Dependency ratio. Number of children under 10 plus adults over 65 divided by number of adults aged 16 to 65
Wiv	Number of wives of the head of household
Agehd	Age, in years, of the head of household
Limb	Dummy variable set to one if head of household is Limba
Temn	Dummy variable set to one if head of household is Temne

Figure 5.1

Definitions of Household Characteristics that the head of household is not Islamic. Moreover, preliminary analysis using this and years of English education for the household head indicated these were not useful variables.

Finally, whether the household lived in the northern, southern or eastern region was not included. Best subset selection using quantities rather than shares as dependent variable and including independent variables not suitable in our model was conducted by Smith, Strauss and Schmidt (1981). They found the region variables using these three regions were highly correlated with prices, which were calculated for the eight regions. In addition, they are correlated highly with the ethnic group variables. Indeed, in a few cases the correlation between the regional dummies and the set of other independent variables was virtually one (greater than .9999). The variables had perfect multicollinearity. In these cases the region variable(s) was usually dropped before best subset selection. In our case we felt that the price variables should be retained and the regional dummies dropped so as to avoid any perfect multicollinearity among the full set of independent variables to be considered. Nevertheless, these variables are a possible alternative to the ethnic group dummy variables and in the Smith, Strauss and Schmidt analysis they did well in the sense of being in the "best subset" equation for several commodities. Moreover, it was found in that analysis that coefficients of the eastern and southern region dummies were often of similar magnitudes suggesting they might be combined.

For cassava, other cereals, fresh fish, groundnuts and fruits the number of households not consuming was large enough to present a statistical problem (see Chapter 7 for detailed treatment). For these commodities only consuming households were used in the single equation analysis. This is a truncated data set, so the ols parameter estimates are biased (see Amemiya, 1973a; and Chapter 7). The variable selection is also affected by the truncation in an unknown way. For rice, palm oil, dried fish, vegetables, and salt and other condiments there were six or fewer nonconsuming households each, so all observations were used in the variable selection process, and truncation is not a problem.

Results

Equations were run and best subsets chosen on the basis both of minimum C_p and maximum $\bar{R}^2$. In general, use of the C_p criteria resulted in equations not including certain price and/or total expenditure variables which seemed a priori to warrant inclusion. The maximum $\bar{R}^2$ criteria did not present this problem, with one exception, so this is the criterion to which the results reported here correspond. The one exception was for the quadratic term in total expenditure. This was not always included in the chosen subset. When it was not the same equation was reestimated with this term. In these cases, the t-statistic for this variable will be less than one, so these cases are identifiable from Table 5.1. The fact that no quadratic term was chosen for some commodities is an important indicator of linear expenditure curves. These terms were put in here because linear expenditure curves will be explicitly tested for in the systems estimation. At this point we only want to mimic the QES form.

Results of the single equation regressions are reported in Table 5.1. Subsequent to the estimation a data error was discovered. Seven households were mistakenly classified as Mende rather than Temne. Rerunning

Table 5.1

Single Equation Share Regressions[1]

Commodity and Mean Share[2] of TEXP	$\bar{R}^2$	Constant Term	Independent Variables: Expenditure TEXP	Prices: PRB	PCA	PPO	PDF	PNF	PGN	POC	Other
(1) Rice .250	.317	.311 (5.96)	-1.71E-4 (-1.74)	-456.5 (-3.66)	743.1 (2.94)	-160.6 (-2.53)	196.8 (2.89)	427.2 (2.99)	-338.8 (-3.99)	-121.6 (-5.22)	3
(2) Cassava .027	.479	.031 (2.35)	-.487E-6 (-1.14)	64.7 (1.85)	-501.1 (-4.66)			-76.8 (-3.58)	109.4 (5.77)	28.9 (6.33)	3
(3) Palm Oil .075	.153	.123 (8.61)	-.839E-5 (-0.76)	200.9 (4.34)	211.7 (1.53)		84.7 (2.53)	-204.4 (-4.30)	117.7 (2.81)		3
(4) Groundnut .026	.203	.027 (2.27)		-168.9 (-2.19)	153.2 (1.45)	-106.2 (-4.55)	-184.9 (-3.40)	368.6 (3.73)	-751.2 (-3.47)	3	PFF 214.3 (3.45)

[1]Chosen on the basis of maximum $\bar{R}^2$ from all possible subsets of variables available to each equation. The dependent variable is the ratio of household expenditure on the ith food to total household expenditure $\left(\frac{q_i p_i}{TEXP}\right)$. Each independent variable has been deflated according to the the following schema:

Variable	TEXP	p_j	Household characteristic
Deflator	p_i	TEXP	$TEXP/p_i$.

where p_i is the price of the commodity represented by the dependent variable and p_j the price of any other commodity.

Price definitions are PRB≡rice, PCA≡cassava, PPO≡palm oil, PDF≡dried fish, PNF≡nonfoods, PGN≡groundnuts, POC≡other cereals, PFF≡fresh fish, PVG≡vegetables, PFT≡fruits, PCN≡salted other condiments. Definitions of household characteristics are given in Figure 5.1. The t-ratios are in parentheses.

[2]The mean expenditure share was calculated over all households.

[3]Not included in the available set.

Table 5.1--continued

of TEXP	$\bar{R}^2$	Constant Term	Expenditure TEXP	Independent Variables: Prices PRB	PCA	PPO	PDF	PNF	PGN	POC	Other
(5) Fish, fresh .041	.511	.066 (4.18)	-.286E-5 (-0.97)	330.9 (3.26)		-25.2 (-1.39)	419.6 (6.10)	-705.7 (-5.87)	1,468.8 (6.71)	3	PFF -286.2 (-4.75)
(6) Fish, dried .063	.346	.051 (5.05)	-.284E-5 (-0.91)		-421.0 (-5.06)	19.9 (1.60)		68.0 (4.01)	-86.5 (-4.09)	3	PFF
(7) Other cereals .045	.090	.136 (5.13)	-.123E-4 (-2.05)					-88.2 (-2.78)	88.2 (1.42)		3
(8) Vegetables .016	.230	.005 (0.68)	.370E-5 (2.15)	-22.1 (-2.00)	162.7 (3.18)		-28.0 (-3.91)	28.4 (3.36)	-11.0 (-1.23)	3	PVG, PFT -21.2 (-4.01)
(9) Fruits .001	.081	.13E-2 (0.97)			-84.7 (-3.10)	-15.6 (-2.68)			7.3 (1.63)	3	PVG, PFT 8.8 -29.7 (2.57) (-2.54)
(10) Salt and other condiments .023	.696	.014 (4.57)		-61.7 (-3.03)	107.6 (1.59)	-110.5 (-5.36)	109.8 (4.89)	-236.3 (-5.05)	702.5 (5.82)	3	PFF, PCN -201.0 63.7 (-6.04) (6.01)

Table 5.1--continued

Commodity[2]	Independent Variables: Household Characteristics										Number of Observations
	SIZE	INF	YCH	CH	AD	DEPR	WIV	AGEHD	LIMB	TEMN	
(1) Rice .250		43.5 (2.25)	-34.4 (-2.27)					1.5 (1.74)		-114.0 (-1.89)	138
(2) Cassava .027			-41.8 (-1.42)				37.9 (1.15)		364.5 (3.45)	331.0 (4.04)	114
(3) Palm oil .075						-6.1 (-1.77)				22.7 (2.04)	138
(4) Groundnut .026		8.4 (1.04)			7.5 (1.21)			0.9 (1.65)	249.7 (4.36)	154.7 (3.09)	103
(5) Fish, fresh .041	45.2 (2.80)	-43.2 (-2.81)	-30.6 (-1.83)	-54.6 (-4.06)	-58.4 (-3.05)	-17.8 (-1.56)					104
(6) Fish, dried .063	-3.01 (-1.34)								-59.1 (-3.59)		138
(7) Other cereals .045						10.5 (1.78)			294.3 (1.27)	29.0 (1.42)	113
(8) Vegetables .016											138
(9) Fruits .001							-4.9 (-1.13)				78
(10) Salt and other condiments .023	-1.2 (-1.86)	.92 (1.11)	1.2 (1.89)	1.7 (2.18)	1.0 (1.59)		-0.7 (-1.15)	0.1 (3.13)	4.2 (1.63)	3.3 (1.74)	138

several of the "best subset" regressions showed no major changes in coefficients except for the ethnic dummy coefficient. That is, the other coefficients were generally within one standard deviation of the estimates using the corrected data. The mistake was corrected before obtaining the system estimates.

At least one of the ethnic dummy variables (head of household being Limba or Temne) is selected in seven out of ten equations. For the cassava, groundnut and salt and other condiments equations the coefficients are similar in magnitude, suggesting that these variables could be combined into one. The infants and young children variables do moderately well, being selected in four out of ten equations. In two of these, for fresh fish and for salt and other condiments, their magnitudes are similar. Also, for the regressions using the full set of variables available for each equation (not shown here) the magnitudes are similar for several other regressions. Only for rice are they markedly different. The other variables each appear in three out of ten equations. Children aged 11-15 and adults between 16 and 65 years have similar coefficient magnitudes in the fresh fish and salt and other condiments equations. The household size coefficient has different interpretations depending upon which other composition variables appear with it. In the dried fish equation, in which it appears alone, the coefficient reflects the effects of changes in total household size on the predicted share. In the fresh fish and salt and other condiments equations all components of size except persons over 65 years appear. In these cases the household size variable's coefficients applies to a change in persons over 65. Dependency ratio as a single variable did not consistently capture the effects of household size and age composition. Number of wives of the household head and age of the household head do well in some equations.

In sum, the single equation results do not indicate clearcut choices for demographic variables except for the ethnic dummies, with some evidence that those two might be combined. As an alternative, we might try a regional dummy depending on whether the household lives in the north or south, with east combined with south. As noted the regional and ethnic group dummies are highly correlated. If infants and young children are combined into a second variable then it makes sense to include household size as a third. This would allow differing effects on consumption of persons under 10 and over 10. It would also allow demographic effects to come solely through size and not its composition (if the coefficients for the persons under 10 years variable were to prove to be insignificant). If number of parameters to be estimated in the QES were not a consideration we might add number of wives and/or age of the household head and/or split up the under 10 years and household size variables further. Our choice of household characteristic variables to include should not be viewed as the only one possible. However, problem size is a consideration and a choice has to be made. As mentioned, fewer commodities may sacrifice too much information as may a simpler demand system.

CHAPTER 6

QUADRATIC EXPENDITURE SYSTEM ESTIMATES

Specification

We now want to estimate the Quadratic Expenditure System equations 2.14 and 2.15 using the likelihood function given by equation 3.8. Our final specification is dictated by our commodity classification except for the translation parameters and for household total time. Chapter 5 explored single equation estimates for commodity shares, the major purpose of which was to discover which household characteristic variables were more powerful explanatory variables. From this exercise the set of chosen demographic variables comprised household size, children under 10 and either an ethnic dummy set to one if the household was Temne or Limba (Mende is the other group), or a regional dummy set to one if the household lived in the northern region. For total time available to the household the variables chosen were persons over 10, females over 15 and children aged 11-15. Children under 10 were found not to work by Byerlee and Spencer. Wage rates were found to differ for males over 15, females over 15 and children aged 11-15. Indeed, household labor supplied and labor demanded are in terms of male equivalents. Since these three components add to persons over 10 years old, one variable needs to be dropped to avoid perfect multicollinearity. Males over 15 was the variable dropped.

Since adding a child under 10 also increases household size by one the total effect of adding a child under 10 on the translation parameters will be the sum of the children under 10 and household size coefficients. The children under 10 coefficient may be interpreted as being the differential effect of children under 10 from persons over 10. Likewise, the coefficients on females over 15 years and on children aged 11-15 years show the differential effect on total time available to the household from males over 15 years.

From equation 2.14 or 2.15 we can see that the household characteristic variables are multiplied by prices when they enter the QES. An identification problem arises from our choice of demographic variables because wage times household size equals wage times persons over 10 plus wage times persons under 10. Hence, one of these variables must be dropped to avoid perfect multicollinearity. We drop the household size variable and rewrite equation 2.14.

$$(6.1)\quad p_i X_i^c = p_i C_i + p_i \sum_{r=1}^{3} \sigma_{ir}\eta_r + a_i\left(p_L m_1(\gamma_1 - \sigma_{71}) + p_L \sum_{r=2}^{3} \gamma_r m_r + \pi + A\right.$$
$$- \sum_{k=1}^{6} p_k\left(C_k + \sum_{r=1}^{3} \sigma_{kr}\eta_r\right) - p_L\left(C_L + (\sigma_{72} + \sigma_{71})\eta_2 + \sigma_{73}\eta_3\right)\Big)$$
$$-(a_i - d_i)\prod_{k=1}^{7} p_k^{-d_k}\Big(p_L m_1(\gamma_1 - \sigma_{71}) + p_L \sum_{r=2}^{3} \gamma_r m_r + \pi + A$$
$$- \sum_{k=1}^{6} p_k\left(C_k + \sum_{r=1}^{3} \sigma_{kr}\eta_r\right) - p_L\left(C_L + (\sigma_{72} + \sigma_{71})\eta_2 + \sigma_{73}\eta_3\right)\Big)^2$$

where we have used the fact that n+1=7, K=q=3. It is apparent from equation 6.1 that the coefficient of wage times persons over 10 $(\gamma_1 - \sigma_{71})$ is identified, but not its components. Likewise, for the coefficient of wage times children under 10 ($\sigma_{72}+\sigma_{71}$) (note that the effect of the ethnic dummy variable, η_3, is to add σ_{k3} to the price coefficient C_k). In consequence, total time, $T=\sum_{r=1}^{3}\gamma_r m_r$ is not identified. For the major questions in which we are interested this is not troublesome.

The final QES specifications which we estimate have seven commodities, three translation demographic variables and three total time demographic variables. The number of parameters is 42. That is, (3+3) 7-2+3 or 43 parameters, less one due to the identification problem. These systems in their expenditure form were estimated using the Davidon-Fletcher-Powell algorithm as available on the GQOPT package of numerical optimization routines. The DFP algorithm uses first derivatives of the likelihood function, but not second derivatives; an advantage. It is a variable metric algorithm. This means that when forming the direction to be searched in at iteration t, $-H_t \nabla L(\beta)$; H_t, which is a square matrix whose dimension is that of the parameter vector β, varies from iteration to iteration (∇ denotes a vector of first derivatives). The algorithm has many desirable features such as necessarily converging to the optimal point if the objective function is convex. For details, see a reference on nonlinear programming such as Avriel (1976).

Estimation

At first estimation was attempted of a QES with demographic variables entering through scaling. In the QES this involves raising the I_i scaling parameters to the $-d_i$ power (equation 2.11). As the d_i are not integers this requires the I_is to be positive for the function to exist. The I_is were specified as $I_i = \sum_{r=1}^{3}\sigma_{ir}\eta_r$, hence they had to be constrained to be positive. Unfortunately, the DFP algorithm kept getting "stuck" on an edge of the function where it was undefined (i.e., where I_i was almost zero for some i and some observation) and was unable to converge to a local optimum. Much effort was spent trying to obtain convergence, including use of several starting values for parameters and use of alternative algorithms. Finally, the translation specification was chosen because it has no undefined region. Alternatively, we might have specified the I_i as $I_i = \prod_{r=1}^{2} \eta_r^{\sigma_{ir}} e^{\sigma_{i3}\eta_3}$, which is necessarily positive and always defined since the η_rs are positive. Since we are not so interested in comparing the translation and scaling specifications this was not pursued.

Estimation using the translation specification was successful. Since there was question a priori whether the disturbances on the expenditure equations were identically distributed we took squared residuals from these equations and regressed them on variables which the variances were hypothesized to be proportionate to. In particular, they were regressed on a constant and the square of fitted value (i.e., $\mathrm{Var}(\varepsilon_{ti}) = \hat{X}_{ti}^2 \sigma_{ii}$), and a constant and the square of profits ($\mathrm{Var}(\varepsilon_{ti}) = \pi^2 \sigma_{ii}$). The results of the latter were mixed, in three out of six regressions the constant term being significant and not squared profits, and vice versa. As can be seen from Table 6.1 squared fitted values were very significant in five out of six regressions and significant at the .10 level in the sixth.[1] Moreover, regression standard errors for the regression using squared fitted values were uniformly lower than for the regressions using squared profits. The error specification giving rise to this result is

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[1]These results use residuals from estimation with the regional dummy. The qualitative results are the same when using residuals from the system using the ethnic group dummies.

Table 6.1

Regression Coefficients and Standard Errors for Regression of Squared Unweighted and Weighted QES Residuals on Squared Fitted Values[1]

Commodity	Equation	Constant	Squared Fitted Value	R^2 [2]
Rice	Unweighted	4,657.5	.78E-1	
		(2,130.8)	(.45E-1)	.02
	Weighted	.54	-.33E-5	
		(.11)	(.39E-5)	.01
Root crops and other cereals	Unweighted	7,032.8	.57	
		(4,478.3)	(.44E-1)	.55
	Weighted	2.0	.11E-4	
		(.96)	(.88E-4)	---
Oils and fats	Unweighted	1,928.3	.31	
		(875.2)	(.22E-1)	.58
	Weighted	9.3	-.22E-4	
		(2.51)	(.45E-4)	---
Fish and animal products	Unweighted	831.4	.24	
		(528.5)	(.59E-1)	.11
	Weighted	1.1	-.80E-4	
		(.29)	(.46E-4)	.02
Miscellaneous foods	Unweighted	1,428.4	.24	
		(594.2)	(.69E-1)	.08
	Weighted	1.9	-.12E-3	
		(.35)	(.61E-4)	.03
Nonfoods	Unweighted	5,107.1	.15	
		(2,580.8)	(.30E-1)	.15
	Weighted	.64	-.16E-5	
		(.21)	(.20E-5)	---

[1]Unweighted residuals are residuals from initial unweighted QES estimates, using regional dummy. Weighted residuals from the second stage QES estimates, which were weighted by fitted values from the initial estimates.

[2]-- indicates R^2 less than .005.

$\varepsilon_t \sim N(0, F_t \Sigma F_t)$ where F_t = diagonal $(| p_i\hat{X}^c_{ti}|)$. Alternatively, this amounts to weighting each equation for observation t and good i by $1/|p_i\hat{X}^c_{ti}|$. Clearly then the function is not defined for $|p_i\hat{X}^c_{ti}| = 0$.

The error specification using absolute fitted values was used and maximum likelihood estimation tried. Unfortunately, the algorithms kept stopping at a point at which $|p_i\hat{X}^c_{ti}|$ was nearly zero for some i and some t, but which were clearly not local optima.[2] Different starting values for parameters were tried, unsuccessfully. It was then decided to use for $p_i\hat{X}^c_{ti}$ the values from estimation of the expenditure form equations, and to treat these as constants (in an unrestricted maximum likelihood estimation these values will change every iteration as parameter values, and hence fitted values, change). This is an extention to regressions nonlinear in parameters of Amemiya's (1973b) suggested two step procedure for the linear regression case. He showed such two-step estimators to be consistent with a known distribution, but not asymptotically efficient. Halbert White (1980) has shown (theorem 2.4) that an unweighted, nonlinear least squares estimator is a strongly consistent estimator when error terms are not identically distributed, under some fairly weak regularity assumptions. What we have is a system of nonlinear seemingly unrelated regressions. Since estimating such equations jointly affects only efficiency, not consistency (assuming no misspecification), White's result is applicable to our first round estimators. In particular, our estimates of fitted values are consistent. That, in turn, means our second stage estimates are

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[2]Eigenvalues of the information matrix were used to check for optimality. At the function maximum these should all be positive.

consistent. These estimates are not unrestricted maximum likelihood and so are presumably not asymptotically efficient. Conditional on the first round estimates of fitted values they are mle and $\sqrt{T}\,(\hat{B}-B)$ should be asymptotically distributed as $N(O, \lim_{T\to\infty}(I/T)^{-1})$, with the information matrix calculated treating F_t as being fixed.

The second stage conditional maximum likelihood estimates were obtained with resulting parameters and their asymptotic standard errors shown in Table 6.2.

Regularity conditions were tested by computing eigenvalues of the Slutsky substitution matrix. The substitution matrix was computed as $\partial X_i^c/\partial p_j \mid_{du=0} = \partial X_i^c/\partial p_j + \hat{X}_j^c \partial X_i^c/\partial(p_L T + \pi + A)$ where $\hat{X}_j^c$ represents fitted value so that the matrix will be symmetric as imposed by the QES. For the system using the regional dummy regularity conditions held at 113 out of 138 sample points as against none when using the ethnic group dummy. The reason for the latter failure was a small negative (i.e., -.2) compensated own price elasticity for labor supply. The other compensated own elasticities were of the expected signs.

Using the regional dummy, twenty-two out of forty-two parameters have the absolute value of their coefficients greater than 1.96 times their standard errors, twenty-six have absolute values of coefficients more than 1.65 times their standard errors, and thirty have standard errors less than their coefficients' absolute value. The heteroskedasticity problem has nearly disappeared. Table 6.1 shows a significant constant term and insignificant coefficient for squared fitted values on four out of six regressions of squared weighted residuals on those variables. For one regression both constant and squared fitted value are significant and for the other the constant term is significant and the squared fitted value term borderline.

Table 6.2

Coefficients and Asymptotic Standard Errors
of Quadratic Expenditure Systems

Type of dummy variable	Regional		Ethnic Group	
Parameter	Coefficient[1]	Standard Error[2]	Coefficient[1]	Standard Error[2]
c_1	189.1	79.0	167.8	53.2
c_2	42.4	16.4	180.8	19.0
c_3	12.2	23.3	-128.4	41.3
c_4	9.3	21.9	10.9	15.5
c_5	6.8	13.9	10.7	29.1
c_6	.4	54.5	-1,907.4	698.7
c_7	-1,522.3	500.8	1,309.3	1,579.5
δ_{11}	7.3	15.0	8.7	10.7
δ_{12}	61.5	23.5	8.4	15.5
δ_{13}	214.0	73.1	102.1	52.2
δ_{21}	-9.8	5.6	40.2	3.8
δ_{22}	24.9	8.8	4.0	9.0
δ_{23}	-30.8	28.2	153.9	28.4
δ_{31}	-.6	5.0	-1.3	6.6
δ_{32}	11.4	8.4	6.9	7.5
δ_{33}	-47.1	19.9	19.6	14.7
δ_{41}	-3.7	2.9	-1.9	1.9
δ_{42}	11.0	4.3	1.5	2.9
δ_{43}	-4.2	19.9	-18.2	15.1
δ_{51}	-8.5	3.2	-5.1	2.8
δ_{52}	32.0	5.6	22.3	4.8
δ_{53}	20.8	20.2	-27.5	21.8
δ_{61}	-14.6	8.2	-27.2	22.6
δ_{62}	60.3	13.2	25.0	34.4
δ_{63}	37.7	37.9	97.1	115.4
$\delta_{72}+\delta_{71}$	-20.5	103.9	396.5	208.0
δ_{73}	-152.1	371.1	-2,129.3	993.4
$\gamma_1-\delta_{71}$	1,846.6	143.3	2,174.4	158.9
γ_2	-1,437.3	152.5	-1,461.6	229.7
γ_3	1,117.7	167.7	-1,628.5	251.8
a_1	.23162	.35E 1	.55362E-1	.20E-1
a_2	-.1405E 1	.11E 1	.13175	.42E-1
a_3	-.2803E-2	.36E-1	.420258E 1	.94E-2
a_4	.109989	.20E 1	.16796E 1	.90E-2
a_5	.7929E-1	.24E-1	-.2092E-2	.17E-1
a_6	.269242	.68E-1	1.0045	.58E-1
d_1	.23160	.35E-1	.55360E-1	.20E-1
d_2	-.1404E-1	.11E 1	.13170	.42E 1
d_3	-.2774E-2	.36E-1	.420263E 1	.94E-2
d_4	.109983	.20E-1	.16801E 1	.90E 2
d_5	.7928E-1	.24E-1	-.2086E-2	.17E-1
d_6	.269243	.68E-1	1.0044	.58E-1
Value of log-likelihood	3,487.7		-3,577.1	

[1] Single subscripts refer to commodity number as given in Table A.1 and double to commodity and demographic variable numbers. Demographic variable numbers for the δ's are 1-household size, 2-under 10 years, 3-regional or ethnic group dummy=1 if northern or Limba-Temne household. For the γ's the numbers are 1-over 10 years, 2-11 to 15 years, 3-females over 15.

[2] From information matrix calculated from second derivatives of log-likelihood function.

There were a few negative fitted values for all 138 observations. This is troublesome, but so are the solutions. We might have constrained fitted values to be positive in our estimation, however, judging from the experience of estimating the unconstrained maximum likelihood version weighting by fitted values (actually their absolute values), we would have gotten caught on an edge of the illegal negative space. Alternatively, we might have used a Tobit procedure (see Chapter 7), however, this involves numerically evaluating multiple integrals, a very expensive procedure which would have necessitated aggregating commodities a good deal more than we did. In the raw data there are a few zero values for expenditures, the most being seven for root crops and other cereals, and some small negative values reflecting either errors in the data or net withdrawal from storage over the year.

A series of Wald tests were run on different hypotheses and are reported in Table 6.3. First we test $H_o: a_i = c_i, \forall_i = 1, \ldots, 6$ (which implies $a_7 = C_7$ since $\sum_{i=1}^{7} a_i = \sum_{i=1}^{7} C_i = 1$). If this null hypothesis is true the QES simplifies into a linear expenditure system. The value of the statistic, which is asymptotically distributed as a chi-square variable with six degrees of freedom, is 19.0. This is significant at somewhat less than the .005 level; hence we can reject the hypothesis that we should have estimated a linear expenditure system. It may be that for individual commodities the hypothesis that $a_i = d_i$ is not rejected. In fact, this is true for miscellaneous foods and for nonfoods. The standardized normal statistics for testing $a_i = d_i$ are 1.2 and 0.1 respectively. The statistic for fish and animal products is 1.6 corresponding to a probability value of roughly .15. Miscellaneous foods and nonfoods are more highly

Table 6.3

Chi-Square Statistics from Wald Tests[1]

Test of	Statistic	Degrees of Freedom
1. LES as special case of QES	19.0	6
2. Household size coefficients	29.1	6
3. Children under 10 years coefficients	70.1	7
4. Equality with opposite signs of household size and children under 10 coefficients	100.1	6
5. Price coefficients	38.9	7
6. Ethnic group dummy coefficients	50.1	7
7. Equality with opposite signs of price and ethnic group dummy coefficients	18.1	7

[1]From QES with regional dummy.

aggregated commodities, hence, linear expenditure curves for them are not implausible. The coefficients on household size, which represent the effect of a unit change in persons over 10 on the commodity specific translation parameters, are jointly significant as are the coefficients for children under 10. Hence, children under 10 affect the translation parameters in a way different from household members over 10. Since the total effect of children under 10 on translation parameters is the sum of their coefficients plus household size coefficients, it is interesting to test whether the sum of these is jointly significantly different from zero. As can be seen, the statistic is 100.1 which with six degrees of freedom is highly significant. The price coefficients, the C_i's, are jointly significant as are the regional coefficients. This means that the price coefficients for southern households (for which the dummy is zero) are significant and significantly different from the price coefficients for northern households. Since the price coefficients for the latter are the sum of the southern price coefficients and the dummy coefficients we test whether this sum is jointly significantly different from zero, which it turns out to be at between the .025 and the .01 levels.

Expenditure Shares and Price Elasticities

Marginal total expenditure, marginal total income, price elasticities of demand and marginal effects of household characteristic variables are functions, using the QES, not only of parameters but also of data. Hence, one has to choose at which sample points to evaluate these. We have chosen to divide the sample into three groups based on total expenditure for this purpose. The dividing lines chosen are less than 350 Leones annual expenditure, between 350 and 750 Leones inclusive, and greater than 750 Leones (a Leone was worth U.S.$1.1 in 1974/75). The sample sizes for these groups are 44, 51 and 43 respectively. The main justification for such a division is that many observers are concerned with responses of people in different income groups, particularly the lower ones.

One can see from Table 4.1 that the lower expenditure group faces relatively lower prices for root crops and other cereals and nonfoods, but higher prices for oils and fats and fish and animal products.

Shares of marginal total expenditure are reported in Table 6.4. They are the extra shares of total expenditure spent on each commodity due to an infinitesimal change in total expenditure. As such, they add to one. They are derived from the marginal total income shares which are the same only due to a change in total income (remember total expenditure plus value of leisure equals total income). We can write $\partial p_i X_i^c / \partial(p_L T + \pi) = \partial p_i X_i^c / \partial(\sum_{i=1}^{n} p_i X_i^c) \cdot \partial(\sum_{i=1}^{n} p_i X_i^c) / \partial(p_L T + \pi)$ from which we solve for $\partial p_i X_i^c / \partial(\sum_{i=1}^{n} p_i X_i^c)$, the marginal total expenditure for good i. In general, they seem to be plausible. The marginal share for rice declines with higher total expenditure as one would expect although the .02 share for high expenditure households seems a little low. The low shares for root crops and other cereals is not surprising, though one would not have expected the marginal share to rise with expenditure. In particular, the share is not negative at our mean evaluation points. This is interesting because many observers have hypothesized that cassava may be an inferior good for higher income groups in West Africa. This may still be the case, however, since the group, root crops and other cereals, contains expenditures on sorghum

Table 6.4

Shares of Marginal Total Expenditure[1]
by Expenditure Group

Commodity	Expenditure Group Low	Middle	High	Mean
Rice	.22	.16	.02	.13
Root crops and other cereals	.03	.06	.12	.07
Oils and fats	.13	.20	.36	.23
Fish and animal products	.13	.11	.07	.11
Miscellaneous foods	.09	.07	.04	.07
Nonfood	.40	.40	.39	.39

[1]Partial derivative of commodity expenditure with respect to total income divided by partial derivative of total expenditure with respect to total income. Evaluated at expenditure group means using QES with regional dummy.

roughly equal to those on cassava, and sorghum may not be an inferior good. The marginal share for oils and fats rises sharply, perhaps too much so, for the high expenditure group. For nonfoods the marginal share is somewhat higher than the average share for all expenditure groups. It is not surprising that the average share of expenditures on foods should decrease as total expenditure increases (this is so since estimated average share is greater than marginal share). This is simply Engel's law.

Marginal total income shares are also reported, in Table 6.5. They will be needed when the entire household-firm model is examined in Chapter 8. For now we can note that the share of marginal expenditures on leisure out of an infinitesimal change in total income is .3 at the sample average, falling from .31 at the low expenditure group to .29 at the high expenditure group. Since total income is not identified, we cannot compute the average share of leisure out of total income, hence we cannot conclude how this average share is moving with rising total income.

Uncompensated price elasticities of demand (holding profits constant) are reported in Table 6.6. They correspond to a movement from point A to point E in Figure 2.3. For rice the own price elasticity declines in absolute value with expenditure group. Part, but not all, of this is due to an income effect declining with expenditure group. This is certainly not surprising. Root crops seem not to be price responsive. The higher expenditure group is slightly more responsive to price, partly due to an increasing income effect. The relative unresponsiveness of total household labor supplied to wage rate changes (-.06 to .28) is not really surprising since this is measuring total supply, not its

Table 6.5

Shares of Marginal Total Income[1]
by Expenditure Group

Commodity	Low	Middle	High	Mean
	Expenditure Group			
Rice	.15	.11	.01	.09
Root crops & other cereals	.02	.04	.09	.05
Oils and fats	.09	.14	.26	.16
Fish and animal products	.09	.08	.05	.07
Miscellaneous foods	.06	.05	.03	.05
Nonfoods	.27	.27	.28	.28
Leisure	.31	.31	.29	.30

[1]Partial derivative of commodity expenditure with respect to total income. Evaluated at expenditure group means using QES with regional dummy.

Table 6.6

Uncompensated Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-1.26	-.16	-.23	.02	.03	-.01	.01E-1
	Middle	-.78	-.13	-.31	.02	.02	-.02	.04E-1
	High	-.45	-.12	-.38	.05	.07	-.04	.01
	Mean	-.74	-.10	-.29	.03	.03	-.03	.01
Root crops and Other cereals	Low	-.02	-.15	-.02	-.02	-.02	-.02	.01
	Middle	-.02	-.26	.04	-.02	-.01	-.02	.01
	High	-.01	-.31	-.02	-.01	-.01	-.01	.01
	Mean	-.01	-.22	-.02	-.02	-.01	-.02	.01
Oils and Fats	Low	.04	.04	-.82	.05	.03	.05	-.02
	Middle	.01E-1	.04E-1	-1.10	.02E-1	.01E-1	.04E-1	-.02E-1
	High	-.01E-1	.05E-1	-1.25	.02E-1	.01E-1	.01	-.03E-1
	Mean	.04E-1	.01	-.97	.01	.01	.01	-.01
Fish and Animal Products	Low	.02	-.08	-.12	-1.29	.01	.01	.01E-1
	Middle	.03	-.06	-.15	-.92	.01	.01	.03E-1
	High	.06	-.05	-.15	-.81	.04	-.03E-1	-.04E-1
	Mean	.04	-.04	-.12	-.95	.02	-.01	.01E-3
Miscellaneous Foods	Low	.01	-.06	-.10	-.03E-1	-.99	-.01	.04E-1
	Middle	.01	-.06	-.14	-.03E-1	-.60	-.02	.01
	High	.04	-.04	-.14	.02	-.63	-.02	.01
	Mean	.02	-.04	-.11	.03E-1	-.71	-.02	.01
Nonfoods	Low	.10	-.16	-.21	.06	.06	-1.17	-.01
	Middle	.07	-.16	-.36	.02	.03	-.90	.01
	High	.14	-.12	-.38	.07	.08	-1.05	-.04E-1
	Mean	.09	-.11	-.30	.04	.05	-1.01	-.04E-1
Labor	Low	1.30	.72	1.81	1.38	1.03	1.39	-.06
	Middle	.56	.48	1.53	.71	.44	.74	.09
	High	.20	.31	1.16	.43	.31	.65	.28
	Mean	.47	.34	1.25	.67	.47	.78	.14

[1] Calculated at mean for each expenditure group. Uses QES with regional dummy.

allocation between uses. The negative sign for the low expenditure group is due to the income effect (see below) and gives some slight evidence for a backward bending supply curve. For other commodities, the own price elasticities are of sizeable magnitude and except for oils and fats they tend to decline in absolute value with higher expenditure groups. The oils and fats exception is partly due to the income effect increasing at higher total expenditure groups.

The cross price effects with respect to rice price are negative except for fish and miscellaneous foods. This is not surprising due to the large budget share of rice leading to a relatively large income effect. The fact that this is not as true for effects with respect to nonfood price is somewhat surprising since one would expect substitution effects of food commodities and rice to be larger than between rice and root crops. One can see that root crop demand is more responsive to changes in price of rice than rice demand is to changes in price of root crops. Since rice represents a larger budget share, its income effect is likely to be greater.

Income compensated price elasticities of demand are reported in Table 6.7. At the sample average and for all three expenditure group averages the substitution matrix was negative semi-definite.

As with the uncompensated elasticities there is a tendency for price responsiveness of rice to decline with total expenditure. All goods are Hicks-Allen substitutes except for root crops and rice at high expenditure levels. This is unlikely; however, the magnitude is small, -.01. Perhaps, then, it should be interpreted as suggesting independence. Also note that the substitution effects with respect to

Table 6.7

Income Compensated Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group	OF Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-1.05	.02E-1	.14	.26	.20	.23	-.08
	Middle	-.68	.01	.12	.17	.11	.15	-.09
	High	-.44	-.01	.01	.13	.11	.12	-.09
	Mean	-.65	.01	.09	.17	.13	.16	-.10
Root crops and Other cereals	Low	.03E-2	-.13	.02	.03E-1	.01E-1	.05E-1	-.01E-1
	Middle	.03E-2	-.23	.05	.01	.04E-1	.01	-.01
	High	-.01	-.27	.11	.02	.01	.04	-.02
	Mean	.03E-1	.20	.05	.01	.01	.18	-.01
Oils and Fats	Low	.04	.04	-.81	.05	.04	.05	-.02
	Middle	.04	.05	-.95	.05	.03	.06	-.03
	High	.01	.09	-.93	.07	.04	.13	-.08
	Mean	.04	.05	-.84	.06	.04	.08	-.04
Fish and Animal Products	Low	.13	.01	.08	-1.17	.10	.12	-.04
	Middle	.08	.02	.09	-.84	.06	.08	-.05
	High	.06	.01	.07	-.76	.06	.08	-.06
	Mean	.08	.01	.07	-.88	.07	.09	-.05
Miscellaneous Foods	Low	.09	.04E-1	.05	.09	-.92	.08	-.03
	Middle	.06	.01	.06	.06	-.56	.06	-.03
	High	.04	.05E-1	.03	.05	-.61	.05	-.04
	Mean	.06	.01	.04	.06	-.67	.06	-.04
Nonfoods	Low	.36	.04	.24	.35	.27	-.87	-.11
	Middle	.23	.07	.30	.25	.16	-.64	-.15
	High	.15	.08	.35	.23	.18	-.75	-.19
	Mean	.22	.06	.28	.26	.19	-.73	-.16
Labor	Low	.42	.04	.28	.42	.32	.38	.28
	Middle	.27	.07	.34	.30	.19	.28	.36
	High	.18	.08	.36	.26	.21	.32	.49
	Mean	.26	.06	.30	.31	.23	.31	.40

[1] Calculated at mean values for each group. Uses QES with regional dummy.

wage are small so that the compensated wage effects are largely income in nature, a result of changes in wage changing nominal total income as well as real income. Also, the response of household labor supply to wage rates, while small, does increase with expenditure group. Part of this fact may be due to wage rates increasing slightly with higher expenditure group.

The foregoing results were evaluated at expenditure group averages; in particular, the regional dummy variable was also averaged. Of course, no household head is reported as living part in the north and part in the south. Hence, marginal budget shares and price elasticities were calculated by expenditure level and region. The marginal budget shares for each expenditure group are nearly identical across regions. For own uncompensated price elasticities, the differences are small. In general, southern households tend to be a little less price responsive than northern households; however, the differences shrink with higher expenditure groups and for the high expenditure group are negligible. Since differences due to expenditure group are far greater than because of ethnic group the latter results are not reported, although they are available.

Changes in expenditure due to a marginal change in household composition variables are shown in Table 6.8. These changes are evaluated at the sample average except for the regional dummy variable which is set to one for northern households and to zero for southern households. One can see that the largest marginal expenditures are for rice, nonfoods, and oils and fats (except for changes in children under 10). For males over 15 the value of household labor supply is

Table 6.8

Change in Expenditure by Commodity Due to Marginal Change in Age-Group Variables by Region[1] (in Leones)

Commodity	Region	Age Group	Under 10	11-15	Males over 15	Females over 15
Rice	North		10.1	6.8	17.6	9.2
	South		9.7	7.0	18.4	9.5
Root crops and other cereals	North		4.3	-2.5	3.7	-1.2
	South		4.5	-2.7	3.4	-1.3
Oils and fats	North		-5.9	8.7	28.9	13.2
	South		-5.4	8.4	28.0	12.8
Fish and animal products	North		-1.8	2.0	10.9	4.0
	South		-1.9	2.1	11.1	4.1
Miscellaneous foods	North		10.1	-2.5	3.0	-1.2
	South		10.0	-2.4	3.2	-1.2
Nonfoods	North		8.7	5.6	39.2	13.0
	South		8.7	5.6	39.1	13.0
Household labor	North		25.5	18.1	103.3	37.0
	South		25.6	18.0	103.2	37.0

[1]Calculated at sample averages except for regional dummy variable.

also affected importantly. One can see that total expenditures increase for increases in each age, sex group. Also, region makes no real difference. Differences due to expenditure group are larger, which is not surprising since household characteristic variables affect expenditure through an income effect when entered into the demand system by translation. The differential effects at different expenditure levels are available, but not reported here.

For changes of all persons the marginal changes in goods expenditure less change in value of labor supplied equals zero since the sum of goods expenditure less the value of labor supplied equals the "profits" part of total income, which is constant. Persons under 10 do not affect household total time, therefore, the marginal change in leisure expenditure is equal to the negative of the change in value of household labor. This is not true, however, for changes in persons over 10.

Clearly, there are many interesting results in these tables. Of significance for development efforts is the general proposition that food demand is reasonably responsive to price (except for root crops and other cereals). Price as an important short run allocator of food consumption and hence caloric consumption has been stressed in recent years by such people as Mellor (1975) and Timmer (1978). Mellor has focused on the real income effect of price, which is supported here. However, we find own price substitution effects also to be important contrary to previous expectations. Partly this is due to the limited commodity disaggregation we have used (five food groups with two of staples). These results also supply information of some importance to the nutritional planner. For example, the negative uncompensated effects on root crops with respect to rice price means that decreases in rice consumption due to increases in rice price is not likely to be compensated by increases in cassava consumption, rather the opposite. Of course, in the longer run, people will shift their production and sales patterns when confronted by relative price changes, hence the need to estimate the production side of this household-firm model. With even more time investment in fixed production and human capital variables as well as changes in household size and composition will take place but these are outside the focus of this research.

CHAPTER 7

TOBIT ESTIMATES OF OUTPUT SUPPLY AND LABOR DEMAND EQUATIONS

Estimation with Censored Data

For estimating the system of output supply and input demand equations we begin with equation 2.18, derived from a Constant Elasticity of Transformation-Cobb-Douglas (CET-CD) multiple output production function. Following the discussion in Chapter 3, we add error terms which are distributed as $N(0,\Sigma)$ to these equations, which are in value form. If there were no other considerations, we could obtain our maximum likelihood estimates easily. However, we saw in Chapter 4 that for several of our six goods many households have no production. In particular, for production of nonfoods, oils and fats and fish and animal products this is so. If it is physically possible for households to produce these goods then the first order conditions from the maximization of profits subject to the production function are the Kuhn-Tucker conditions.

$$(7.1)\quad p_i - \mu\frac{\partial G}{\partial X_i} \le 0,\ X_i\left(p_i - \mu\frac{\partial G}{\partial X_i}\right) = 0 \qquad i=1, \ldots, n$$

$$-p_L - \mu\frac{\partial G}{\partial L_T} \le 0,\ L_T\left(-p_L - \mu\frac{\partial G}{\partial L_T}\right) = 0$$

$$G \le 0,\ \mu G = 0$$

Assume no technical inefficiencies, so that $G=0$, and assume that labor is always demanded, which is true for our sample, so that $p_L + \mu \frac{\partial G}{\partial L_T} = 0$. Then $\frac{p_i}{p_L} \leq \frac{-\partial G/\partial X_i}{\partial G/\partial L_T}$, $\forall_i$. The right hand side is the reciprocal of the marginal product of labor in producing good i. We have then that the value of marginal product of labor for good i is less than or equal to the price of labor. When this holds as an equality the good is produced and when it is an inequality the good is not produced.

This is the deterministic situation. Randomness can be accounted for in two ways. One can append error terms to the Kuhn-Tucker first order conditions. This was done for a system of demand equations by Wales and Woodland (1979). Doing this, and again assuming that labor is always demanded, we obtain

$$(7.2) \quad p_i \frac{\partial G}{\partial L_T} + p_L \frac{\partial G}{\partial X_i} - \varepsilon_L \frac{\partial G}{\partial X_i} + \varepsilon_i \frac{\partial G}{\partial L_T} \geq 0, \ \forall_i$$

$$G + \varepsilon_G = 0$$

The distribution of the transformed error terms will be normal if the original error terms were. The likelihood functions may then be derived. They will involve messy Jacobians of the transformation from the transformed error terms $\varepsilon_i \frac{\partial G}{\partial L_T} - \varepsilon_L \frac{\partial G}{\partial X_i}$, ε_G into the X_i's for ε's corresponding to goods produced by the household in question.

Alternatively, one can add error terms directly to the reduced form of output supply and input demand equations, as done for a demand system by Wales and Woodland (1978). This is akin to the Tobit model $y^*=g(x,\beta) + \varepsilon$, $y=\max(0,y^*)$, where y^* is not observed but y is. If $\varepsilon \sim N(0,\sigma^2)$ then $E(y) = E(y/y>0) \cdot P(y>0) + E(y/y=0) \cdot P(y=0)$, where $E(\cdot)$ is the expectations operator and $P(\cdot)$ is probability. Of course, $E(y/y=0)=0$ so $E(y) = E(y/y>0) \cdot P(y>0)$. $E(y/y>0) = g(x,\beta) + E(\varepsilon/y>0)$

and from Johnson and Kotz (1970) we have $E(\varepsilon/y>0) = E(\varepsilon/\varepsilon>-g(x,\beta))$ $= E(\varepsilon/\frac{\varepsilon}{\sigma}>\frac{-g(x,\beta)}{\sigma}) = \sigma f(g(x,\beta)/\sigma)/F(g(x,\beta)/\sigma)$, where $f(\cdot)$ is the standard normal density and $F(\cdot)$ is the standard normal distribution function. In particular, $E(\varepsilon/y>0) \neq 0$ so that regression using only observations with positive y's leads to inconsistent parameter estimates. This last implies that the mean of the disturbances using all observations on y, $E(\varepsilon/y>0) \cdot P(y>0)$ is also not zero, hence these OLS parameter estimates are inconsistent also. For the linear in parameters model Greene (1981) has shown $E(\hat{\beta}_{OLS}) = \beta F(\bar{X}\beta/\sigma)$, so that the lower the probability of a positive observation the greater is the bias. What is happening in this model is that the entire normal distribution of ε is not being observed. The lower tail in which $\varepsilon<-g(x,\beta)$, corresponding to y=0, is piled up at $-g(x,\beta)$, providing we observe y when it is equal to zero. This is so because we observe y, not y*. If y is not observed when it is zero, the distribution of ε is simply cut off or truncated at $\varepsilon = -g(x,\beta)$. The former situation (y observed), which we have in our data, is called censored data; the latter is called truncated data.

The foregoing applied to a single equation model. The output supply and input demand equations are a system but the same model is applicable. In this case ε is an n+1 vector with covariance matrix Σ. Also, there exist cross equation parameter restrictions, for instance that c is the same in all equations. The system can be estimated consistently using maximum likelihood techniques. The likelihood function is

$$(7.3)\quad L = \prod_0 l_t(\beta,\Sigma) \prod_1 l_t(\beta,\Sigma) \prod_2 l_t(\beta,\Sigma) \ldots \prod_K l_t(\beta,\Sigma)$$

where number subscripts correspond to the number of zero outputs and $l_t(\beta,\Sigma)$ is the appropriate density for household t. For households which produce all goods

$$(7.4)\quad l_t(\beta,\Sigma) = (2\pi)^{\frac{-(n+1)}{2}} |\Sigma|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}\varepsilon_t' \Omega^{-1} \varepsilon_t\}$$

For households producing all but one good

$$(7.5)\quad l_t(\beta,\Sigma) = \int_{-\infty}^{g_{ti}(X,\beta)} (2\pi)^{\frac{-(n+1)}{2}} |\Sigma|^{-\frac{1}{2}} \exp\{-\tfrac{1}{2}(\varepsilon_p', y)\Sigma^{-1}\binom{\varepsilon_p}{y}\}\, dy$$

where the ith good, put in the last position here, is not produced and ε_p are residuals for produced goods. For households producing all but two goods

$$(7.6)\quad l_t(\beta,\Sigma) = \int_{-\infty}^{g_{ti}(X,\beta)} \int_{-\infty}^{g_{tj}(X,\beta)} (2\pi)^{\frac{-(n+1)}{2}} |\Sigma|^{-\frac{1}{2}}$$

$$\exp\{-\tfrac{1}{2}(\varepsilon_p', y_1, y_2)\Sigma^{-1}\begin{pmatrix}\varepsilon_p\\ y_1\\ y_2\end{pmatrix}\}\, dy_1 dy_2$$

For households producing all but K goods the density $f_t(x,\beta)$ has the same form with the number of integrals equal to K, the number of goods not produced. In our data there are many households not producing one or two goods and a few households not producing as many as four goods. For these households the corresponding density involves evaluating a quadruple integral. This is not only extremely messy to program, but quite expensive to compute as well. Indeed, Wales and Woodland used only three commodities, one of which was always consumed, in their two papers.

One way around this difficulty would be to aggregate to, say, three outputs plus labor. Since one output is always produced and labor always demanded, this would involve at most double integrals, which would still be expensive, but perhaps manageable. An alternative not involving more aggregation is to assume Σ, the covariance matrix of ε, to have zeroes in certain places. If Σ were block diagonal then the multivariate density would be a product of densities of the outputs (and input) corresponding to each block. This would reduce the dimension of the multiple integrals to be evaluated. In the extreme case of assuming independence between each of the error terms, $l_t(\beta, \Sigma)$ would be the product of 7-K normal densities and K standard normal distribution functions. If K outputs were not produced, only a single integral would have to be evaluated, but one for each of the normal distribution functions corresponding to the K outputs not produced. However, evaluating a single integral K times is a much less costly and less difficult procedure than evaluating a K-dimension integral once. Although one need not go so far as assuming independence between all of the error terms, to choose which error terms are correlated in such a way as to result in block diagonality for Σ would seem to involve as much arbitrariness as assuming complete independence. Since the latter results in a considerably simpler estimation procedure, it was chosen.

It should be noted that one reason why this would be an unreasonable assumption for a demand system does not hold for output supplies and input demands. As we have seen for the demand side expenditures on goods plus value of household leisure equals total income, resulting in error terms summing to zero. Hence, the covariance matrix is singular, which it could not be if it were diagonal. However, this is not true for the values of output supply less value of input demand. On the other hand, one can argue that the probability of producing rice conditional on the household not producing any other commodity but demanding labor is not equal to the unconditional probability of producing rice. Clearly, in this case, the conditional probability is one, but the unconditional probability is not. Yet, independence of the error terms implies these probabilities are equal. Still, assuming independence does make the computation problem manageable. Moreover, ignoring cross-equation restrictions, maximum likelihood estimates assuming independence retain their consistency even if the assumption is violated. Hence, the assumption remains attractive statistically. All that would be sacrificed is asymptotic efficiency. The likelihood function to be maximized is thus

$$(7.7) \quad L = \prod_t [\prod_{i \varepsilon P} \frac{1}{\sigma_i} f(g_{ti}(\beta)/\sigma_i) \prod_{j \varepsilon NP} F(-g_{tj}(\beta)/\sigma_j)]$$

where $f(\cdot)$ is the standard normal density and $F(\cdot)$ the standard normal distribution function, P corresponds to goods produced, NP to goods not produced, and t to households. Taking the log-likelihood function, the first derivatives with respect to the jth element of β is

$$(7.8) \quad \frac{\partial \ln L}{\partial \beta_j} = \sum_t \sum_{i \varepsilon P} \varepsilon_{ti} \frac{\partial g_{ti}(\beta)}{\partial \beta_j} / \sigma_i^2 - \sum_t \sum_{k \varepsilon NP} f(g_{tk}/\sigma_k) \frac{\partial g_{tk}}{\partial \beta_j} / (\sigma_k F(-g_{tk}/\sigma_k))$$

The first derivative with respect to σ_j is

$$(7.9) \quad \frac{\partial \ln L}{\partial \sigma_j} = \sum_{\substack{t: \\ j \varepsilon P}} (\frac{\varepsilon_{tj}^2}{\sigma_j^3} - \frac{1}{\sigma_j}) + \sum_{\substack{t: \\ j \varepsilon NP}} f(g_{tj}/\sigma_j) g_{tj}(\beta) / (\sigma_j^2 F(-g_{tj}/\sigma_j))$$

These partial derivatives are used in the maximization procedure.

To justify use of the multivariate Tobit model one has to be convinced that there is positive probability of producing non-produced outputs. Looking at the data, many of the zero outputs are spread throughout all regions. That is, some households within an enumeration area will be producers and others not. In these cases, there is evidently no environmental reason why the particular good cannot be produced. There do exist some cases in which the zero observations are clustered geographically so that none of the particular output is produced by our sample of 138 in a particular enumeration area. This occurs for root crops and other cereals in EA 72, for oils and fats in EAs 52 and 53, for fish and animal products in EAs 32 and 72, and for nonfoods in EA 72. To get a better idea of whether there exist environmental constraints on production of those goods in these enumeration areas, the larger sample of 328 households for which production data were considered reliable by Byerlee and Spencer we examined. In all cases except for oils and fats in EAs 52 and 53, and fish and animal products in EA 72, there was some production of the good in question. For EAs 52 and 53, the 1970/71 Agricultural Survey of Sierra Leone showed that oils and fats were indeed produced in the Bombali areas in question. For EA 72 the Agricultural Survey indicated that game was captured. Since fish and animal products includes wild game, it was concluded that it was possible to produce this "good" in the area in question.

Another potential problem in using the Tobit model is misspecification of the production function. Instead of separability of all outputs and all inputs in the implicit production function, it can be argued that there are separate production functions for some outputs, perhaps for nonfoods, oils and fats and fish and animal products. As an example, one might hypothesize nonfood production as a function of nonfood labor and nonfood capital. With capital fixed either a Cobb-Douglas or a CES function implies zero supply of output if there is no capital. Hence, if households have no nonfood capital, the probability of producing nonfood output is zero. This approach runs into severe data problems with our sample. For example, there are households reporting no capital or labor use for fishing and animal product activities, yet reporting positive outputs.

Many households reporting zero production of nonfoods report positive labor use to produce nonfoods. When inputs are aggregated, as we have done, into total labor, total capital and total land, there is a greater chance than for using disaggregated inputs that such errors cancel each other out.

Another advantage in the CET-CD specification is that the supply of any output is a function of all output prices. A separate production function for nonfood, if it did not include land as an input would make nonfood supply a function of only nonfood price, wage and nonfood capital. This is a result of assuming labor can be freely sold and purchased, so that labor supply to the firm is not fixed.

For dependent variables, outputs and labor demanded, errors in data are not a serious statistical problem. For a single equation Tobit model suppose one observes $y_e = \max(0, y_e^*)$, where $y_e^* = y^* + v$, v being an error term uncorrelated with ε. This implies some reported zero production was really positive and vice versa. Then the likelihood function is $\prod_P \frac{1}{\sigma_u} f(g(\beta)/\sigma_u) \prod_{NP} F(-g(\beta)/\sigma_u)$, where $\sigma_u = \sqrt{\sigma_\varepsilon^2 + \sigma_v^2}$ and σ_ε^2 and σ_v^2 are not separately identified. However, β is identified.

Variable Selection

Variable selection is largely specified by choice of outputs, inputs and production function. It bears repeating here that land is not adjusted for quality as labor and capital flows are. The rental market for land is too small and influenced importantly by nonmarket factors such as whether the household is a member of the community or not (Spencer and Byerlee, 1977, pp. 21-23) to be used to adjust acreage for quality. No other data bearing on this question were available.

Acreage disaggregated by crop use was available but there may be different quality lands within each crop use. Moreover, the same data problems which exist for disaggregated capital exist for disaggregated land use. There is some room for variable selection after the outputs, inputs and production function have been specified providing one hypothesizes parameters of the production function to be a function of other variables. In production function analysis this has a time honored tradition when using cross-section, time series data (see Mundlak, 1961) as firm and time effects. This amounts to using shift dummies corresponding to firm or time when estimating the production function. More recently Mundlak (1980) has made slope parameters functions of certain variables. From their work studying production and labor use using the larger production sample of 328 households, Byerlee and Spencer concluded that one could group households by the two large regions, north and south, the same grouping which was used when estimating the quadratic expenditure system. Fitting completely different production functions for each region would reduce both sample size and price variation. If one could assume that the overall functions are the same but that certain parameters differ by region, then advantage may be taken of pooling the regions in estimation. Suppose one lets the shift parameter of the CET-CD production function vary by region. As we saw in Chapter 2, this function requires normalization by either the δ_i parameters summing to one or the shift parameter being unity. We have chosen the latter method. However, let $A_0=a_0+a_1D$, where $D \equiv$ dummy variable. Dividing both sides of equation 2 by A_0 gives the normalization which we use of the shift dummy equaling one. Now, however, the δ_i's are each divided by A_0 and the new coefficient will take on different values for each region. The coefficients thus derived $\delta_i/(a_0+a_1D)$ are a bit cumbersome. A simpler way to achieve this result and to maintain the normalization that $A_0=1$ is to make each δ_i depend linearly on the dummy variable $\delta_0=\delta_{i0}+\delta_{i1}D$. This introduces n new parameters rather than just one, where n is the number of outputs. However, it presumably allows somewhat more flexibility. In principle, all the coefficients might be allowed to vary with region. However, to keep matters simpler, only the equivalent to a shift dummy was permitted.

One other set of coefficients might in principle be allowed. These follow from the censored nature of the data. Notice from equation 2.18 that the deterministic output supplies and input demands are necessarily non-negative due to their multiplicative nature. Thus, $g_{ti}(\beta) \geq 0, \forall t, i$ resulting in $P(y_{ti}>0)=P(\varepsilon_{ti}> -g_{ti}(\beta)) \geq .5$. In principle, however, one would want the probability of a positive output to be allowed to vary between zero and one. One way to accomplish this would be to write $y^*_{ti} = g_{ti}(\beta) + \mu_i + \varepsilon_{ti}$, where μ_i is a constant to be estimated. This would add an additional seven parameters to be estimated and so was not done. However, in future work it might be tried. One reason excluding these parameters might not be detrimental to our results is that when evaluated at the sample average for independent variables $F(g_i(\hat{\beta})/\hat{\sigma}_i)$ is an estimator of the sample proportion with positive production of good i, which is always over half of our sample.

Estimates of Small CET-CD System in Value Form

With six outputs plus labor demand, the Constant Elasticity of Transformation-Cobb-Douglas production function has ten parameters, sixteen when the dummy variable is included, plus seven variances (which, because of the cross equation parameter restrictions and the Tobit estimation procedure, cannot be concentrated out of the likelihood function), a total of twenty-three parameters. Initial attempts at numerical maximum likelihood estimation ran into trouble. As a result, estimation of a smaller system was attempted. The smaller system had two fewer outputs, oils and fats and fish and animal products being aggregated with miscellaneous foods. The justification for this aggregation was that these were the two foods with the most zero outputs and the aggregation left the enlarged miscellaneous foods group with only two households having zero outputs. Maximum likelihood estimates of the seventeen parameters in the smaller system are shown in Table 7.1. Of the twelve parameters excluding the variances only four have asymptotic standard errors less than half the absolute values of their coefficients. In particular, all the δ_i parameters, the δ_{i0}s and the δ_{i1}s, have standard errors larger than their coefficients' absolute values. A Wald test of the joint significance of the four δ_{i1} parameters (associated with the regional dummy variable) gives a chi-square statistic of .08, abysmally low. Examining the residuals showed particularly high residuals for miscellaneous food output and labor demand for the ten households in enumeration area 13. Those households live in the coastal area near Freetown, the capital city. Their main production activities are fishing and growing vegetables. Indeed, most of the households are large commercial fishing households. Viewing production activities, they are possibly the most distinct set of households. In view of this, the regional dummy variable was redefined to be one for those ten households and zero for all others.

Table 7.1

Coefficients and Asymptotic Standard Errors of Aggregated CET-CD Systems[1]

Type of Dummy Variable: Parameter[2]	North-South Coefficient[2]	North-South Standard Error[3]	EA 13-Non-EA 13 Coefficient	EA 13-Non-EA 13 Standard Error
δ_{10}	.31E-5	.13E-4	.12E-1	.44E-2
δ_{11}	-.13E-5	.52E-5	-.19E-2	.61E-2
δ_{20}	.49E-5	.20E-4	.454	12.3
δ_{21}	1.53	13.2	-.438	12.3
δ_{30}	.95E-4	.31E-3	.282E-1	.95E-2
δ_{31}	.36E-4	.18E-3	-.264E-1	.87E-2
δ_{40}	195, 25.9	5.2E+5	2.05	1.0
δ_{41}	-29,612.5	1.5E+6	-1.49	.84
c	4.66	1.5	1.56	.14
β_D	.34	.8E-1	.15	.06
β_K	.16	.6E-1	.26	.05
β_L	.42	.3E-1	.44	.02
σ_1^2	45,655.4	6.0E+3	58,793.5	1.1E+4
σ_2^2	43,838.6	5.6E+3	62,948.6	1.0E+4
σ_3^2	186,273.0	2.1E+4	148,926.2	1.2E+5
σ_4^2	15,216.4	2.4E+3	15,194.9	2.5E+3
σ_5^2	106,942.8	1.3E+4	64,388.3	1.0E+4
Value of log-likelihood function	-3,741.9		-3,674.6	

[1]Estimated in value form.

[2]Numbered subscripts refer to commodity number and to type of variable, 0 for constant and 1 for dummy. Commodity numbers are 1-rice, 2-root crops and other cereals, 3-miscellaneous foods (including fish and animal products and oils and fats), 4-nonfoods, 5-labor demand.

[3]From information matrix calculated from second derivatives of log-likelihood function.

The smaller system was re-estimated with results also shown in Table 7.1. The log-likelihood value is roughly 66 higher than that for the system estimated with the north-south regional dummy. This is a very large difference. Eight of the twelve production function parameters have coefficients' absolute values more than twice their standard error and for nine this ratio is higher than 1.65 (corresponding to a .1 significance level for a two way test using standard normal tables). A Wald test of the joint significance of the δ_{i1} coefficients gives a statistic of 11.6 which corresponds to a probability level of roughly .02. For the δ_{i0} coefficients, the Wald test statistic is 12.9, a probability level of approximately .011. The residuals for the Enumeration Area 13 households are now much lower, which is reflected in the substantially higher log-likelihood value.

Estimates of Larger CET-CD System in Value Form

Having now seemingly good estimates from the system of four outputs and one variable input, we returned to the larger system of six outputs and labor demand. It was decided to use the dummy variable defined by the ten EA 13 households. In principle, this definition might not be preferable to the north-south definition when estimating the larger system. However, of the two outputs separated from miscellaneous foods, oils and fats and fish and animal products, these ten households distinguish themselves by their large production of fish; and of vegetables, left in the six output miscellaneous food category (see Table 4.4). Hence, it was felt that use of this dummy variable would continue to be preferable to using the north-south dummy. Use of both was felt not worth the extra expense and time involved in estimation.

Table 7.2 presents the results of estimation of the system of six outputs and labor demand using a dummy separating EA 13 from other households for the δ_i parameters. The standard deviations were estimated rather than the variances, because it was felt due to experience with the smaller system that convergence might be faster. Of the sixteen production function parameters, six have ratios of their coefficients' abslute values to their standard errors of more than two, seven have such ratios greater than 1.65 and eight have ratios greater than one. For the six δ_{i0} parameters, three (rice, oils and fats and miscellaneous foods) have their coefficients' absolute values greater than 1.29 times their standard errors, and for two it is greater than 1.65. For these parameters, a one-tailed test is appropriate since they are constrained to be positive, and 1.29 and 1.65 correspond to probability levels of .1 and .05 respectively. For the δ_{i1} parameters only one has its coefficient's absolute value more than 1.65 times its standard error (for miscellaneous foods). For the sum $\delta_{i0} + \delta_{i1}$, which corresponds to δ_{i0} for the ten EA 13 households, two (fish and animal products and miscellaneous foods) have absolute values of coefficients greater than 1.29 their standard error, and for one (miscellaneous foods) it is greater than 1.65 its standard error. So, for the δ_{i0}s, the δ_{i1}s and their sum, some coefficients are individually significant at the .10 level or better; however, as a group they are not. Wald test statistics of these parameters grouped are given in Table 7.3. With six degrees of freedom the probability value for the largest statistic, 6.0, is greater than .30. Given that the production function specification is Constant Elasticity of Transformation-Cobb-Douglas, it does not make sense to drop individual δ_{i0}s so long as the good in question is produced by the set of non-EA 13 households, which all are. It is felt, for reasons given above, that keeping the dummy variables is

Table 7.2

Coefficients and Asymptotic Standard Errors of CET-CD System in Value Form [1]

Parameter[2]	Coefficient	Standard Error[3]
δ_{10}	.17E-2	.97E-3
δ_{11}	.19E-1	.32E-1
δ_{20}	.80E-1	.91E-1
δ_{21}	-.12E-1	.17E-1
δ_{30}	.13	.84E-1
δ_{31}	2.2	26.4
δ_{40}	2.6298	4.7
δ_{41}	-2.6296	4.7
δ_{50}	.64E-1	.31E-1
δ_{51}	-.63E-1	.31E-1
δ_{60}	72.9	74.7
δ_{61}	-59.2	72.8
c	2.58	.24
β_D	.97E-1	.30E-1
β_K	.33	.30E-1
β_L	.45	.20E-1
σ_1	241.4	15.6
σ_2	226.8	14.8
σ_3	199.8	13.3
σ_4	183.9	15.8
σ_5	97.2	6.3
σ_6	121.8	9.7
σ_7	288.9	19.9
Value of log-likelihood function	-5,967.5	

[1]Uses EA 13-Non-EA 13 dummy variable.

[2]Single subscripts refer to commodity number as given in Table 4. . Double subscripts refer to commodity number and 1 for a dummy coefficient and 0 if not.

[3]From information matrix calculated from second derivatives of log-likelihood function.

Table 7.3

Chi-Square Statistics From Wald Tests
Using Estimates From CET-CD
System in Value Form [1]

Test of	Statistic	Degrees of Freedom
1. CET parameters for non-EA 13 households, δ_{i0}	5.9	6
2. CET dummy parameters, δ_{i1}	4.6	6
3. CET parameters for EA 13 households, $\delta_{i0} + \delta_{i1}$	4.1	6
4. Degree of almost homogeneity, $\beta_D+\beta_K+\beta_L$, different from one	10.8	1

[1]Using EA 13 - non-EA 13 dummy variable.

worthwhile. While non-significant dummies could be dropped, kept perhaps for fish and animal products and miscellaneous foods, re-estimation of the value system at this point was not considered worthwhile. In addition, there are six coefficients corresponding to the ten EA households so the fact that there is trouble in getting statistical significance for them may not be so surprising, and yet it may be that the true values of these δ_{i1} coefficients are different from zero.

The coefficient of c, 2.58, corresponds to an elasticity of transformation between outputs of .63. The Cobb-Douglas coefficients on capital flow, land and labor sum to .88 with a standard error of .04, hence the sum is significantly less than one. This would indicate that the production function is almost homogeneous of degree .88, using Lau's terminology (see Hasenkamp, 1976).

The coefficient on land, .1, is much lower than that for either capital, .33, or labor, .45. This is very different from the usual single agricultural output Cobb-Douglas results in which land's coefficient is the largest. Two reasons suggest themselves for this. First, some of our outputs such as fishing and animal products, oils and fats and nonfoods are not going to be much affected directly by land cultivated by the household. Capital and labor are far more important inputs for these activities. Perhaps, had the production function specification been to allow separate functions for these activities, the coefficient on land might have been higher for the remaining crop activities. Be that as it may, this was not possible as a result of the data inadequacies described earlier in this chapter. Given the output detail and function specification used, these coefficients may not be unreasonable. A second potential reason is the absence of any quality adjustments in defining the land variable. This misspecification affects all coefficients. Had the model been linear in parameters, however, and had increasing size of farm been associated with lower quality land, then the estimated coefficient for land would be lower than the true value. Whether this result applies here, given that the model is highly nonlinear in parameters, is not clear.

Effect of Censoring on Price Elasticities of Output Quantities

Elasticities of quantity outputs with respect to both prices and fixed inputs are derived as $\frac{Z_j}{E(X_i)} \frac{\partial E(X_i)}{\partial Z_j}$, where Z is either a price or fixed input. We have estimated value output and labor equations, but since price is nonstochastic we can divide expected value outputs by own price to derive expected quantity outputs. We have then $E(X_{ti}) = F(g_{ti}(\beta)/\sigma_i) g_{ti}(\beta)/p_i + \sigma_i f(g_{ti}(\beta)/\sigma_i)/p_i$. Taking the partial derivative with respect to own price, we have

$$(7.10) \quad \frac{\partial E(X_{ti})}{\partial p_i} = F \frac{\partial}{\partial p_i} \left(\frac{g_{ti}(\beta)}{p_i}\right) - \sigma_i f/p_i^2$$

The CET-CD production function is specified so that $\frac{\partial}{\partial p_i}\left(\frac{g_{ti}(\beta)}{p_i}\right) > 0$. This can be seen

$$(7.11) \quad \frac{\partial}{\partial p_i}\left(\frac{g_{ti}(\beta)}{p_i}\right) = \frac{X_i}{p_i}\left(\frac{1}{c-1} + \frac{c\beta_L - 1}{(1-\beta_L)(c-1)A} p_i^{\frac{c}{c-1}} \delta_i^{\frac{-1}{c-1}}\right)$$

$$\text{where } A = \sum_j \left(p_j^{\frac{c}{c-1}} \delta_j^{\frac{-1}{c-1}}\right)$$

The expression in parentheses simplifies to

$$(7.12) \quad p_i^{\frac{c}{c-1}} \delta_i^{\frac{-1}{c-1}} \beta_L(c-1) + (1-\beta_L) \sum_{j\neq i} p_j^{\frac{c}{c-1}} \delta_j^{\frac{-1}{c-1}}$$

which is positive given the convexity restrictions that $c > 1$ and $0 < \beta_L < 1$.

Thus, the first term in the expression for $\frac{\partial E(X_i)}{\partial p_i}$ is positive so that, ignoring the second term, expected quantity output responds positively to own price. However, the second term is negative and may be larger in absolute value than the first term.

This is a result of assuming that the disturbances attached to the value form of the system of output supplies and input demand are homoskedastic. In this case, when we divide each equation by own price to derive the equation for quantity, the error term also is divided by own price. Consequently, if the standard error on the value equation is σ_i the standard error on the quantity equation is $\frac{\sigma_i}{p_i}$. As price increases, this standard error drops. The expected value of the censored distribution of quantity outputs supplied and labor demanded $E(X_i)$, is a function of the expected value of the unobserved uncensored distribution and of that distribution's standard error (see page 125). Hence, increasing price increases the mean of the uncensored distribution, $g_i(\beta)$. However, the mean of the censored distribution may actually decrease if the decrease in the variance is sufficient. This situation is pictured in Figure 7.1.

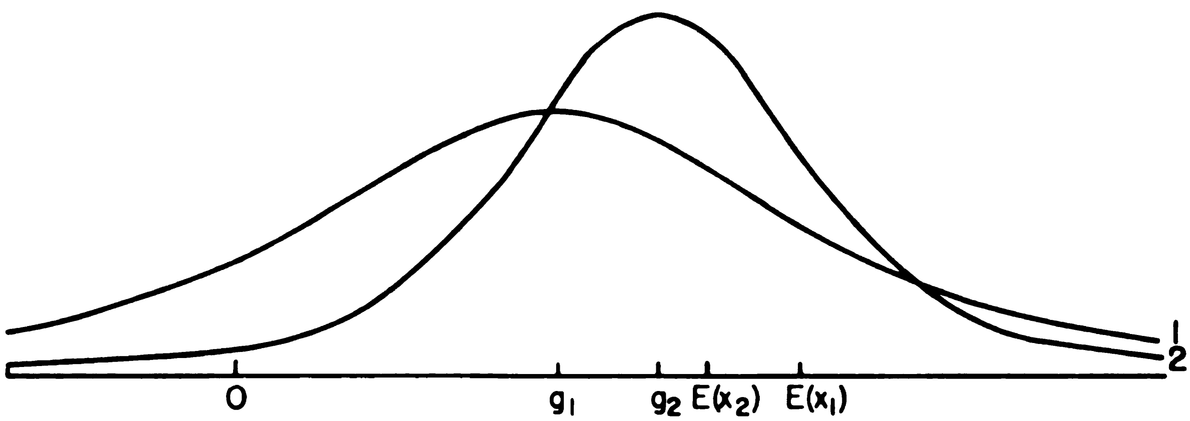

Figure 7.1

Effect of Price Change on
Mean of Censored Distribution

The own price elasticities for expected outputs for EA 13 and non-EA 13 households are given in Table 7.4. The only positive values are for rice in non-EA 13 households and for the sample mean, and fish and miscellaneous foods in EA 13 households. The own price elasticity for expected labor demand is negative in all cases. In this case, the effect of increasing wages decreasing the variance of the uncensored distribution associated with quantity labor demanded reinforces the effect of decreasing the mean of the uncensored distribution.

We can ask how believable these signs and magnitudes are. It is the author's opinion that they are not very believable, particularly in view of the fact that they are a consequence, though not a necessary one, of the way in which the system was estimated. Had the system been estimated in quantity form, we would have $\frac{\partial E(X_i)}{\partial p_i} = F(g_i(\beta)/ p_i\omega_i)\frac{\partial}{\partial p_i}(\frac{g_i(\beta)}{p_i}) > 0$, where ω_i is the constant standard error of the disturbance on the quantity equation i. Given that we have constrained the deterministic production function to allow only upward sloping supply curves (a well defined profit function would not exist if this were not true), it does not seem unduly restrictive to constrain the stochastic supply curves in the same way.

Testing Tobit Results for Heteroskedasticity

Besides this logical reason for re-estimating the system in quantity form, there is a potential statistical reason. When using Tobit estimation procedures, it turns out that if the error terms are heteroskedastic then maximum likelihood estimates which do not account for this are inconsistent, Hurd (1979), although perhaps not by much, Arabmazar and Schmidt (1980). Fortunately, it is possible to test for this, although with unknown power. Let our null hypothesis be that the error terms on the value form

Table 7.4

Own Price Elasticities of Quantity Supply and Labor Demand from CET-CD System in Value Form[1]

	Household Group		
Commodity	EA 13	Non-EA 13	Sample Mean
Rice	-.46	.48	.27
Root Crops and Other Cereals	-.58	-.90	-.87
Oils and Fats	-.90	-.54	-.69
Fish and Animal Products	.75	-.88	-.87
Miscellaneous Foods	.67	-.22	-.17
Nonfoods	-.16	-.91	-.88
Labor demand	-1.81	-1.48	-1.41

[1]Calculated as $\frac{p_i}{E(X_i)} \frac{\partial E(X_i)}{\partial p_i}$ at household group averages.

of the output system are homoskedastic with variance σ_i for the ith equation. From Amemiya (1973) we have for positive observations:

$$(7.13)\quad E(p_iX_i-g_i(\beta))^2 = \sigma_i^2 - \sigma_i g_i f/F$$

where f and F are standard normal densities and distribution functions evaluated at $g_i(\beta)/\sigma_i$. Also

$$(7.14)\quad E(p_iX_i-g_i(\beta))^4 = \sigma_i^2(3\sigma_i^2-3\sigma_i g_i \frac{f}{F} - g_i^3 \frac{f}{\sigma_i F})$$

Now clearly,

$$(7.15)\quad (p_iX_i-g_i(\beta))^2 = E(p_iX_i-g_i(\beta))^2 + \eta_i$$

where $E(\eta_i) = 0$ and

$$\begin{aligned}(7.16)\quad E(\eta_i^2) &= E((p_iX_i-g_i(\beta))^4-2(p_iX_i-g_i(\beta))^2E(p_iX_i-g_i(\beta))^2+(E(p_iX_i-g_i(\beta))^2)^2) \\ &= E(p_iX_i-g_i(\beta))^4-[E(p_iX_i-g_i(\beta))^2]^2 \\ &= 2\sigma_i^4-\sigma_i^3 g_i \frac{f}{F} - \sigma_i g_i^2 \frac{f}{F}(g_i+\sigma_i \frac{f}{F})\end{aligned}$$

Hence, if we take our estimates of $(p_iX_i-g_i)^2 - \sigma_i^2 + \sigma_i g_i \frac{f}{F}$, which are consistent under the null hypothesis, and divide by the square root of $E(\eta_i^2)$, that variable has mean zero and variance one. We can regress this variable (again note only for positive observations) on variables which we hypothesize the variance to be proportional to under the alternative hypothesis. Despite the dependent variable not being normal, in large samples the usual test statistics are asymptotically justified given that the dependent variable is independently, identically distributed with mean zero and finite variance, Schmidt (1976), pp. 56-60.

The question arises on what to regress our variable. Under the alternative hypothesis the error terms on the quantity form of the output system are homoskedastic. Hence, the variance for equation i in value form is $p_i^2 \omega_i^2$. The expected squared residual, under the null hypothesis, also has a term $\sigma_i g_i \frac{f}{F}$. Hence, a term $p_i g_i \frac{f}{F}$ may be an appropriate addition. If we add these terms as independent variables we have

$$(7.17) \quad (p_i X_i - g_i(\beta)) - \sigma_i^2 + \sigma_i g_i \frac{f}{F} = a_1 p_i^2 + a_2 p_i g_i \frac{f}{F} + \eta_i$$

where a_1 and a_2 are to be estimated. Then we should divide p_i^2 and $p_i g_i \frac{f}{F}$ by $\sqrt{E(\eta_i^2)}$ as we do the dependent variable.

This equation and an equation omitting the $p_i g_i \frac{f}{F}$ term were estimated and are reported in Table 7.5. The standard errors of the coefficients are computed using one as the regression standard error, because by construction $(E(\eta_i^2/E(\eta_i^2))^{\frac{1}{2}} = 1$. The lowest χ^2-statistic for testing the joint significance of $\hat{a}_1$ and $\hat{a}_2$ is 7.5, corresponding to a probability level of less than .024, with two degrees of freedom. Using only the price squared term, six out of seven coefficients are significant, the smallest probability value of those being less than .01. Hence, what statistical evidence can be gleaned supports the hypothesis that the error terms attached to the value output system are not homoskedastic. However, they do not suggest necessarily that the system in quantity form has homoskedastic errors.

Estimates of CET-CD System in Quantity Form

The system of output supplies and labor demand was re-estimated in quantity form. The commodity definitions and variables used were the same as for the larger system estimated in value form. Parameter estimates and their asymptotic standard errors are given in Table 7.6. Nine out of sixteen production function parameters have absolute values of coefficients

Table 7.5

Results of Regression Testing for Homoskedastic Errors on Positive Observations of CET-CD Systems[1]

Commodity	System Form	a_1	Standard Error	a_2	Standard Error	F-Statistic[2]
Rice	Value	2,172,230.	287,243.7	-11,457.8	1,604.4	59.51
		285,697.	112,792.5			6.4
	Quantity	10,207.7	9,459.1	309.6	281.4	1.2
		557.2	3,537.8			.2E-1
Root Crops and Other Cereals	Value	659,248.	315,977.0	5,377.7	4,368.4	7.5
		-300,609.	122,406.8			6.1
	Quantity	11,013.7	5,034.0	-595.7	522.0	6.2
		6,279.6	2,851.7			4.8
Oils and Fats	Value	56,504.6	69,471.4	77.4	1,430.7	8.7
		60,100.	20,375.4			8.7
	Quantity	-187.0	7,920.3	313.8	410.4	0.1
		5,203.4	3,610.7			1.4
Fish and Animal Products	Value	-71,675.5	24,924.0	1,460.3	2,357.0	23.1
		-58,219.5	12,228.7			22.7
	Quantity	40,317.	5,986.2	-5,758.9	811.2	55.8
		9,526.8	4,126.3			5.3
Miscellaneous Foods	Value	-142,488.	39,211.0	2,919.1	803.6	14.0
		-5,633.6	10,861.6			0.3
	Quantity	3,122.	1,846.2	389.7	170.2	14.7
		6,712.7	974.0			47.5
Nonfoods	Value	121.2	1,517.2	781.7	237.7	29.3
		4,040.6	939.0			18.5
	Quantity	-3,063.8	2,158.0	746.4	518.4	2.3
		-587.8	1,303.8			0.2
Labor	Value	6,365.1E+3	1,694.4E+3	-3,400.3		32.2
		5,050.3E+3	900.2E+3			31.5
	Quantity	12,225.4	3,559.3	-17.9	154.5	19.6
		11,957.2	2,702.5			19.6

[1]For equation form see equation 7.17. Weighted by $(E(\eta^2))^{\frac{1}{2}}$, see 7.16.

[2]Test of coefficient(s) equality with zero.

Table 7.6

Coefficients and Asymptotic Standard Errors of CET-CD System in Quantity Form[1]

Parameter[2]	Coefficient	Standard Error[3]
δ_{10}	.14E-5	.96E-6
δ_{11}	.26E-2	.13E-1
δ_{20}	.96E-5	.95E-5
δ_{21}	.29E-4	.92E-4
δ_{30}	.16E-2	.15E-2
δ_{31}	12.7	134.8
δ_{40}	.131223E-2	.15E-2
δ_{41}	-.131218E-2	.15E-2
δ_{50}	.7319E-3	.60E-3
δ_{51}	-.7307E-3	.60E-3
δ_{60}	90.8	107.7
δ_{61}	-78.8	108.5
c	4.25	.3
β_D	.69E-1	.3E-1
β_K	.36	.29E-1
β_L	.35	.17E-1
ω_1	1,008.4	63.1
ω_2	2,635.2	171.5
ω_3	512.7	34.7
ω_4	1,066.5	95.9
ω_5	504.0	32.4
ω_6	88.1	7.3
ω_7	2,924.2	184.4
Value of log-likelihood function	-6,071.0	

[1]Uses EA 13 - Non-EA 13 dummy variable.

[2]Single subscripts refer to commodity number listed in Figure 4.1. Double subscripts refer to commodity number and 1 for dummy coefficient, 0 if not.

[3]From information matrix calculated from second derivatives of log-likelihood function.

greater than their standard errors, with four having this ratio greater than two. For the δ_i parameters we again use the one-tailed test. One parameter (for rice) is significant at a probability level less than .1 (corresponding to a standard normal statistic of greater than 1.29) and two have probability levels of roughly .11. For the $\delta_{i0} + \delta_{i1}$ parameters, two have coefficient absolute values greater than 1.29 their standard errors. Wald test statistics of the joint significance of the δ_i parameters are low as is seen in Table 7.7. However, for the same reasons as for the estimates from the value system the quantity system is not re-estimated.

The coefficient c is now 4.25, corresponding to an elasticity of transformation between outputs of .31. The production function is almost homogeneous of degree .78, significantly less than one. The estimate of the coefficient for land is low, as it was for the system in value form.

Error terms corresponding to positive observations were tested for homoskedasticity in the same way as was done for the system in value form. Only now the alternative hypothesis is that error terms on the quantity equations have variance σ_i^2/p_i^2, where σ_i^2 is the constant variance on the value equations. Hence, the independent variables used were $1/p_i^2$ and $g_i \frac{f}{p_i F}$, both divided by $\sqrt{E(\eta_i^2)}$ as given by equation 7.16. The dependent variable was formed using g_i/p_i rather than g_i. The results, reported in Table 7.5, are mixed. For three outputs, rice, oils and fats and nonfoods, χ^2 statistics jointly testing the $\hat{a}_1$ and $\hat{a}_2$ coefficients are very low. For root crops and other cereals, the χ^2-statistic corresponds to a probability level of slightly

Table 7.7

Chi-Square Statistics From Wald Tests
Using Estimates From CET-CD System
in Quantity Form

Test of	Statistics	Degrees of Freedom
1. CET parameters for non-EA 13 households, δ_{i0}	3.6	6
2. CET dummy parameters, δ_{i1}	2.2	6
3. CET parameters for EA 13 households, $\delta_{i0}+\delta_{i1}$	2.4	6
4. Degree of almost homogeneity, $\beta_D+\beta_K+\beta_L$	37.6	1

under .05. For the other equations the joint test shows very high significance. Using only the $1/p_i^2$ term the same four equations show significant coefficients at .05 or better. For oils and fats the probability level of the coefficient is roughly .15 and for rice and for nonfoods it is much higher. Hence, the results of this test indicate heteroskedasticity in some, but not all, of the equations. This is a less than desirable result, but somewhat better than for the equations estimated in value form. Moreover, if neither of these forms has homoskedastic errors, the form which does is unclear.

Output Elasticities with Respect to Prices and Fixed Inputs-Quantity Form

Price elasticities of quantity of outputs supply and labor demand are given in Table 7.8 for EA 13 households, the remaining households and the sample average. The elasticities are evaluated at average values for these three groups. This is done rather than using only the sample mean values and setting the dummy to one for EA 13 households and to zero for the rest. The reason is that predicted quantities for EA 13 households using sample mean prices are wild. Prices faced by these ten households, particularly for fish and animal products, are very different (lower) than sample average prices, causing this aberrant behavior.

The formula used is again $\frac{p_j}{E(X_i)} \frac{\partial E(X_i)}{\partial p_j} = \frac{p_j}{E(X_i)} (F(\frac{g_i(\beta)}{p_i \omega_i}) \frac{\partial}{\partial p_j} (\frac{g_i(\beta)}{p_i}))$.
All the output elasticities are less than .5. In general, the more important the activity to the group of households, the more price responsive it is. For EA 13 households, fish and animal products and miscellaneous foods (remember vegetable production is important for these households),

Table 7.8

Elasticities of Expected Quantities of Outputs Supplied and Labor Demanded with Respect to Price From CET-CD System in Quantity Form[1]

With Respect to Price of	Household Group	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Mis-cel-laneous Foods	Nonfoods	Labor
Rice	EA 13	.08	.02E-1	.03E-2	.06E-1	.06E-1	.03E-1	.03
	Non-EA 13	.36	.03	.04	.02	.05	.01	.53
	Mean	.11	.01	.03E-1	.01	.02	.06E-1	.14
Root Crops and Other Cereals	EA 13	.08E-1	.12	.01E-1	.03	.03	.01	.18
	Non-EA 13	.03	.09	.01	.07E-1	.01	.04E-1	.16
	Mean	.02	.10	.04E-1	.01	.03	.08E-1	.20
Oils and Fats	EA 13	.02E-2	.04E-2	.02	.01E-1	.01E-1	.04E-2	.06E-1
	Non-EA 13	.02	.08E-1	.13	.06E-1	.01	.03E-1	.14
	Mean	.02E-1	.02E-1	.02	.02E-1	.03E-1	.09E-2	.02
Fish and Animal Products	EA 13	.04	.05	.07E-1	.45	.14	.06	.83
	Non-EA 13	.03	.01	.02	.08	.02	.05E-1	.20
	Mean	.02	.02	.05E-1	.09	.03	.09E-1	.23
Miscellaneous Foods	EA 13	.01	.02	.02E-1	.05	.35	.02	.27
	Non-EA 13	.02	.06E-1	.09E-1	.05E-1	.14	.03E-1	.11
	Mean	.01	.01	.03E-1	.01	.15	.05E-1	.13
Nonfoods	EA 13	.03E-1	.04E-1	.05E-2	.01	.01	.13	.06
	Non-EA 13	.04E-1	.01E-1	.02E-1	.09E-2	.02E-1	.04	.02
	Mean	.03E-1	.02E-1	.05E-2	.02E-1	.04E-1	.04	.03
Labor	EA 13	-.14	-.20	-.03	-.54	-.54	-.23	-1.37
	Non-EA 13	-.47	-.15	-.21	-.12	-.23	-.07	-1.17
	Mean	-.17	-.14	-.03	-.13	-.24	-.07	-.75

[1]Calculated at mean values for each household group using $\frac{p_j}{E(X_i)} \frac{\partial E(X_i)}{\partial p_j}$. Uses EA 13 - Non-EA 13 dummy.

have own price elasticities of .45 and .35 respectively. For non-EA 13 households rice is the most price responsive, having an elasticity of .35. Root crops and other cereals, oils and fats and miscellaneous foods have elasticities ranging from .09 to .14. Labor is much more elastic than is outputs for these households, being -1.37 and -1.17 for EA 13 and non-EA 13 households respectively.

For oils and fats (which includes palm kernels), a cash crop, the own price elasticity of .13 for non-EA 13 households is at first glance surprisingly low. However, it should be remembered that exogenous variables are averaged over households of which only some are major producers of oils and fats. This may bring price responsiveness down. More importantly, the stock of palm trees planted by the household is assumed fixed so the major response to price can come only by varying labor, that is, by varying the amount of fruit picked and processed (although command over more trees is possible by picking fruits off of trees growing wildly in the bush).

At the sample means price responsiveness tends to be low. The largest elasticities are for miscellaneous foods, .15, and for rice, .11. Except for rice and oils and fats, the elasticities are close to those for the non-EA 13 households, which is not surprising since they carry the larger weight in forming the sample means. The algebraic reason this is not so for rice and palm products is that the parameter $\delta_{i0}+.0725*\delta_{i1}$ is closer to $\delta_{i0} + \delta_{i1}$, the parameter for EA 13 households, than to δ_{i0}, that for non-EA 13 households. That is, δ_{i1} has a much larger value than δ_{i0} for rice and oils and fats. In the expression $\frac{\partial}{\partial p_i}\left(\frac{g_i(\beta)}{p_i}\right)$, $\delta_i = \delta_{i0}+\delta_{i1}D$ raised to the $-1/(c-1)$ power multiplies the remaining terms. Since the power is negative, the larger is δ_i the smaller this term tends to be.

Cross price elasticities of outputs tend to be low except with respect to wage rate. The latter is not surprising since labor demand is reasonably price responsive. The cross price elasticity with respect to wage can be written as the product of the own price elasticity of labor demand and the output elasticity of labor, where the latter is written $\frac{E(L_T)}{E(X_i)} \frac{\partial E(X_i)}{\partial E(L_T)}$. Cross price elasticities of labor demand are also not negligable. As with own price output elasticities, the more important the activity corresponding to the price changing, the more responsive labor demand is. The signs of the output cross elasticities are positive. That is increasing price of output i leads to increased production of output j. As output price changes, there is a substitution effect, that is movement along a production transformation frontier. This should be negative. There is also an output effect, a shift of the transformation frontier, due to changes in outputs other than i and j, and more importantly, due to changes in labor demand. An increase in price i should increase labor demand as well as output i, shifting the transformation frontier between goods i and j outward. Whether the outward shift of the transformation frontier is sufficient to outweigh the substitution effect is an empirical question. For the CET-CD production function, it turns out that sign $\left(\frac{\partial E(X_i)}{\partial p_j}\right)$ = sign $(c\beta_L - 1)$, which is positive for our estimates.

The price elasticities derived all assume that quantities, not prices, of land and of capital are fixed to the household. In the longer run, the reverse should be true, which should increase the price responsiveness of both outputs and labor. In the short run, a possibly interesting question is what are the expected output elasticities with respect to fixed inputs. If the data were not censored, the formula, given the production function used, would be $\beta_D/(1-\beta_L)$ for land and $\beta_K/(1-\beta_L)$

for capital. With our data, the formula is $F(g_i(\beta)/p_i\omega_i)\frac{g_i}{p_i}(\beta)\,\beta_D/((1-\beta_L)E(X_i))$ for land, and the same with β_K replacing β_D for capital. The former is the same for all outputs and labor. The latter is not, although the ratio of the land to capital elasticities is β_D/β_K for each output and for labor. These elasticities are presented in Table 7.9. The elasticities with respect to capital are roughly five times greater than those with respect to land. Again, the magnitudes are largest for those activities which are more important, for which $Fg_i/p_iE(X_i)$ is larger. These are fish and miscellaneous foods outputs for EA 13 households and rice for non-EA 13 households, and labor demand for both.

Table 7.9

Elasticities of Expected Quantities of Outputs Supplied and Labor Demand with Respect to Fixed Inputs[1]

Commodity	Household Group	WRT Land	Capital
Rice	EA 13	.03	.14
	Non-EA 13	.09	.49
	Mean	.04	.18
Root Crops and Other Cereals	EA 13	.04	.20
	Non-EA 13	.03	.15
	Mean	.03	.15
Oils and Fats	EA 13	.05E-1	.03
	Non-EA 13	.04	.21
	Mean	.07E-1	.04
Fish and Animal Products	EA 13	.11	.56
	Non-EA 13	.02	.13
	Mean	.02	.13
Miscellaneous Foods	EA 13	.11	.56
	Non-EA 13	.05	.24
	Mean	.05	.25
Nonfood	EA 13	.05	.24
	Non-EA 13	.01	.07
	Mean	.01	.07
Labor	EA 13	.10	.50
	Non-EA 13	.08	.42
	Mean	.05	.27

[1]Using CET-CD system with EA 13 - Non-EA 13 dummy in quantity form. Calculated at mean values for each household group using $\frac{Z_j}{E(X_i)} \frac{\partial E(X_i)}{\partial Z_j}$.

CHAPTER 8

HOUSEHOLD-FIRM MODEL RESULTS

Deriving Total Price Effects

Having estimated separately the demand system and production system components of the household-firm model we can now examine the model in its entirety. We have seen in Chapter 2 that consumption demand may be written $X_i^c = f(p, \eta, p_L T(m) + \pi(p, z))$, where $p \equiv$ prices, $\eta \equiv$ household characteristic variables, $T \equiv$ time available to the household, $m \equiv$ household characteristic variables, $z \equiv$ fixed inputs and $\pi \equiv$ profits. In Chapter 6 we examined the price elasticities holding profits constant. If we now allow profits to vary we can write $\frac{\partial X_i^c}{\partial p_j} = \frac{\partial X_i^c}{\partial p_j}\Big|_{d\pi=0} + \frac{\partial X_i^c}{\partial \pi}\frac{\partial \pi}{\partial p_j}$.

In elasticity form,

$$(8.1)\quad \frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial p_j} = \frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial p_j}\Big|_{d\pi=0} + \frac{p_j \partial X_i^c \partial \pi}{X_i^c\, \partial \pi\, \partial p_j}$$

The first term is simply the usual uncompensated elasticity of demand of good i with respect to price j. The second term is what we might call the "profit effect" in elasticity form. It can be simplified by noting that by Hotelling's Lemma $\frac{\partial \pi}{\partial p_j} = X_j$ (this derivative is taken allowing outputs and inputs to vary, see Varian, 1978, pp. 31-32). The term $\frac{\partial X_i^c}{\partial \pi}$ is easily gotten from the marginal total income expenditures in Table 6.5.

Two complications arise when implementing equation 8.1 with our data. First, $\pi = E(\pi) + u$, where u is an error term with mean zero, and is independent of price and fixed inputs. Then $\frac{\partial \pi}{\partial p_j} = \frac{\partial E(\pi)}{\partial p_j}$. However, due to the censoring in our data, Hotelling's lemma no longer holds.

We can write $\pi = \sum_{i=1}^{6} p_i X_i - p_L L_T$. From Chapter 7 we know that when using our parameter estimates from the quantity form of the production system $E(p_i X_i) = F(\frac{g_i(\beta)}{p_i \omega_i})\, g_i(\beta) + p_i \omega_i\, f(\frac{g_i}{p_i \omega_i})$, and likewise for $E(p_L L_T)$. Hotelling's lemma asserts that $\sum_{i=1}^{6} \frac{\partial g_i(\beta)}{\partial p_j} - \frac{\partial g_L(\beta)}{\partial p_j} = \frac{g_j(\beta)}{p_j}$, which is in fact true of the CET-CD production function. Then if the data were uncensored, so that the error terms had mean zero conditional on positive outputs, the lemma would apply. However, we have seen that $\frac{\partial E(p_i X_i)}{\partial p_j} = F(\frac{g_i}{p_i \omega_i}) \frac{\partial g_i}{\partial p_j}$ and $\frac{\partial E(p_i X_i)}{\partial p_i} = F(\frac{g_i}{p_i \omega_i}) \frac{\partial g_i}{\partial p_i} + \omega_i f(\frac{g_i}{p_i \omega_i})$.

Using this we have

$$(8.2)\quad \frac{\partial \pi}{\partial p_j} = \sum_{i=1}^{6} F(\frac{g_i}{p_i \omega_i}) \frac{\partial g_i}{\partial p_j} + \omega_j f(\frac{g_j}{p_j \omega_j}) - F(\frac{g_L}{p_L \omega_L}) \frac{\partial g_L}{\partial p_j} \qquad j=1,\ldots,6$$

and

$$\frac{\partial \pi}{\partial p_L} = \sum_{i=1}^{6} F(\frac{g_i}{p_i \omega_i}) \frac{\partial g_i}{\partial p_L} - F(\frac{g_L}{p_L \omega_L}) \frac{\partial g_L}{\partial p_L} - \omega_L f(\frac{g_L}{p_L \omega_L}) \qquad j=7$$

Clearly, if the estimated probabilities of production were identical for each output and for labor, we would have $\frac{\partial \pi}{\partial p_j} = F(\frac{g_j}{p_j \omega_j}) \frac{g_j}{p_j} + \omega_j f(\frac{g_j}{p_j \omega_j})$, which is almost Hotelling's lemma. However, equally as clear, it would be sheer coincidence if this were to occur. In any case, we have estimates for the necessary parameters so that $\frac{\partial \pi}{\partial p_j}$ can be constructed from our data.

Relation Between Sales Prices and Purchase Prices

A second complication arises because our study uses sales prices when estimating the production system, and a weighted average of sales and purchase prices when estimating the consumption system. Using superscripts of c for weighted consumption prices and s for sales prices, we have $\frac{\partial x_i^c}{\partial p_j^c} = \frac{\partial x_i^c}{\partial p_j^c}\bigg|_{d\pi=0} + \frac{\partial x_i^c}{\partial \pi}\ \frac{\partial \pi}{\partial p_j^s}\ \frac{\partial p_j^s}{\partial p_j^c}$. We need to make some assumption about $\frac{\partial p^s}{\partial p^c}$. Now $p^c = w^s p^s + w^p p^p$, where superscript p refers to purchase price and the w's are weights (see Chapter 4). It is certainly reasonable to suppose that purchase and sales prices move in the same direction (though this need not be so; for example, a better road system which reduces the costs of marketing may lead to a rising farm gate sales price and a falling retail purchase price). One possible assumption to make is that the marketing margin, the difference between sales and purchase price, will remain constant. If we think of the marketing margin as being determined by the demand and supply of marketing services and we assume a perfectly elastic market supply of marketing services, then providing that schedule does not shift, the marketing margin is constant. Then $dp^s = dp^p$, and since by construction $w^s + w^c = 1$, $dp^s = dp^c$ even if the weights are not constant. An alternative would be that $cp^s = p^p$, where c is a constant, presumably greater than one. Then $p^c = w^p cp^s + w^s p^s = (w^p c + w^s) p^s$ and $\frac{dp^c}{p^c} = \frac{dp^s}{p^s} + \frac{c-1}{w^p c + w^s} dw^p$, since $-dw^s = dw^p$. If the second term can be ignored we have the simple relationship that percent changes in weighted consumption prices and in sales prices are equal.

What relationship would hold over time is unclear because it depends partly on the source of the price changes, i.e., shifts in the supply schedule of marketing services versus autonomous increases in retail demand. What little evidence is in our data is inconclusive. A constant marketing margin implies that $p^c = a + p^s$, while the proportional assumption implies that $p^c = bp^s$. We have eight prices, corresponding to the eight regions, for six commodities (for wage, sales and purchase prices are assumed equal). Weighted consumption price was regressed first on a constant and sales price. Then sales price was subtracted from consumption price and that regressed on a constant. This amounts to testing the constant margin hypothesis using a restricted ols, the restriction being that the coefficient on sales price is unity. An F-test of the null hypothesis that b=1 can easily be computed. Likewise, the t-statistic on the constant coefficient in the unrestricted equation can be used to test whether a=0, that is whether proportionality exists between the two prices. The results of these tests, reported in Table 8.1, are inconclusive. Testing for a constant marketing margin the F-statistic is significant only for nonfoods and borderline, probability value between .1 and .05, for miscellaneous foods. Testing proportionality the t-statistics are significant only for oils and fats and nonfoods. Of course, only eight observations are involved so it is not surprising that conclusive evidence cannot be gleaned from this data. Moreover, equal constants and slopes between regions are assumed in this procedure, but this assumption is very likely not a good one. However, allowing different shift and/or slope parameters leaves us with even fewer degrees of freedom, and in the limit of one for each region, with none.

Since an assumption must be made it was assumed that sales prices and purchase prices are proportional. Further we assume weighted consumption price and sales price are proportional. Two reasons can be offered for making this assumption. First, our entire analysis assumes fixity of firm capital and total land. This is a short or medium run situation. In such a short time period it should be less likely that the marketing services supply schedule is horizontal than for the long run. That is, one would expect some upward slope of this supply

Table 8.1

Regression of Consumption Price on Sales Price and Tests of Constant Marketing Margin

Commodity	Dependent Variable[2]	Coefficients and Standard Errors[1] Constant	Sales Price	F-statistic[3]
Rice	p^c	.113 (.69)	.689 (.458)	
	$p^c - p^s$	.046 (.02)		0.5
Root crops and other cereals	p^c	.494 (.31)	-.244 (1.46)	
	$p^c - p^s$	.273 (.17)		0.7
Oils and fats	p^c	.433 (.14)	.665 (.33)	
	$p^c - p^s$	.301 (.04)		1.1
Fish and animal products	p^c	.313 (.20)	.493 (.41)	
	$p^c - p^s$	.105 (.11)		1.53
Miscellaneous foods	p^c	.021 (.12)	2.053 (.45)	5.5
	$p^c - p^s$	.284 (.06)		
Nonfoods	p^c	.443 (.14)	.180 (.09)	
	$p^c - p^s$	-.682 (.17)		76.9

[1]Standard errors are in parentheses.

[2]$p^c \equiv$ consumption price and $p^s \equiv$ sales price. There are eight observations.

[3]$F = \dfrac{SSE_R - SSE_u}{SSE_u/6}$, where $SSE_u \equiv$ sum of squared errors on unrestricted regression using p^c as dependent variable and $SSE_R \equiv$ sum of squared errors on restricted regression using $p^c - p^s$ as dependent variable. The statistic has one and six degrees of freedom.

schedule for the time horizon considered here. The second reason is that our elasticity calculations which follow will be much more understandable if both weighted consumption and sales prices move by the same infinitesimal percent. This would not be true if we assume a constant marketing margin. Table 8.2 shows mean ratios of consumption to sales prices. For rice and fish and animal products they are negligible. For root crops and other cereals they are large and quite variable when averaged over the three expenditure groups. The reason for this is that the sales price of cassava is much lower than prices for other components, particularly compared to sorghum, the other major component. The weight cassava's sales price receives in the group consumption price is the regional proportion of <u>value</u> of group consumption. This is low due to cassava's low price. It also varies substantially by region. The weight cassava's sales price receives in deriving the group sales price is the regional proportion of value of group sales. This is much higher than the weight it gets in the consumption price, because not much is sold of other group components. Hence, the low cassava sales price receives a low weight in deriving consumption prices and a higher weight in deriving group sales price. As noted these weights are highly variable by region. This, plus inclusion of the higher purchase prices in deriving consumption price explains the high and varying ratios for root crops and other cereals. The meaning of these ratios for our purposes is that if we assume a constant marketing margin, then a percent increase in consumption price will mean that sales price increases by more than one percent. For root crops and other cereals an increase of one percent in average consumption price for the middle expenditure group would imply a 5.5 percent increase in sales price for that group.

Table 8.2

Ratio of Consumption to Sales Prices

	Expenditure Group			
Commodity	Low	Middle	High	Mean
Rice	1.1	1.1	1.2	1.1
Root crops	2.6	5.7	3.3	3.8
Oils and fats	1.6	1.6	1.8	1.7
Fish and animal products	1.2	1.1	1.0	1.1
Miscellaneous foods	2.0	2.0	2.1	2.1
Nonfoods	0.5	0.5	0.5	0.5

This will result in a rather large profit effect. Worse yet, the percent increases in sales prices will be different for different groups so that reading a table of profit effects as elasticities will be quite misleading.

Profit Effects

The interested reader may find the tables derived under the constant marketing margin assumption in Appendix 8A. Table 8.3 reports the "profit effects" in elasticity form, the second term in equation 8.1, for low, middle, high and mean expenditure households assuming proportional prices. In most cases the effects are larger, often much larger, for the lowest expenditure households, declining with higher expenditure. Two reasons exist for this behavior. First, marginal expenditures out of total income for some goods decline with higher expenditure. Second, mean consumption of all goods and of labor supply increases with higher expenditure level. Then even for root crops and oils and fats, for which marginal expenditures out of total income rise with total expenditure level, the profit effect, which is in an elasticity form, falls. Goods having higher marginal expenditures, such as oils and fats and nonfoods, tend to have larger profit effects. This factor is also responsible for many of the cross profit effects being large. A change in total income generated by a changing price is distributed over all commodities according to the marginal expenditure out of total income.

The largest own profit effect, at the sample mean, is .27 for fish and animal prodicts. Oils and fats has an effect of .24. The other own effects at the mean household level are all lower than .17.

For the low expenditure group the largest own profit effect is .82 for rice, followed by .78 for fish and animal products and then .63 for oils and fats.

Table 8.3

Profit Effects in Elasticity Form by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.82	.63	1.44	.91	.66	.94	-.32
	Middle	.11	.15	.46	.16	.10	.18	-.11
	High	.05E-1	.08	.26	.05	.03	.11	-.07
	Mean	.08	.09	.35	.14	.09	.17	-.10
Root crops and other cereals	Low	.49	.38	.86	.54	.40	.56	-.19
	Middle	.14	.19	.56	.19	.12	.22	-.13
	High	.01	.16	.58	.12	.07	.24	-.15
	Mean	.12	.16	.55	.21	.14	.27	-.15
Oils and fats	Low	.36	.28	.63	.40	.29	.41	-.14
	Middle	.08	.11	.31	.11	.06	.13	-.07
	High	.03E-1	.04	.17	.03	.02	.07	-.04
	Mean	.05	.07	.24	.09	.06	.12	-.06
Fish and animal products	Low	.71	.54	1.24	.78	.58	.81	-.28
	Middle	.25	.35	1.05	.35	.22	.41	-.24
	High	.09E-1	.13	.45	.10	.06	.19	-.12
	Mean	.16	.21	.73	.27	.18	.36	-.20
Miscellaneous foods	Low	.23	.18	.40	.25	.19	.26	-.09
	Middle	.08	.11	.33	.11	.07	.13	-.08
	High	.05E-1	.06	.22	.05	.03	.09	-.06
	Mean	.06	.07	.25	.10	.06	.13	-.07
Nonfoods	Low	.12	.10	.22	.14	.10	.14	-.04
	Middle	.04	.06	.18	.06	.04	.08	-.04
	High	.02E-1	.04	.12	.02	.02	.06	-.04
	Mean	.04	.04	.14	.06	.04	.08	-.04
Labor	Low	-.56	-.43	-.99	-.62	-.46	-.64	.22
	Middle	-.13	-.19	-.56	-.19	-.11	-.22	.13
	High	-.08E-1	-.11	-.38	-.08	-.05	-.15	.10
	Mean	-.10	-.13	-.43	-.17	-.11	-.21	.12

[1]Calculated as $\frac{p_j}{X_i^c}\frac{\partial X_i^c}{\partial \pi}\frac{\partial E(\pi)}{\partial p_j}$ at expenditure group means, using parameter estimates from the CET-CD system in quantity form, and assuming proportional sales and purchase prices.

The reason for the large effect for rice is that the term $\frac{\partial E(\pi)}{\partial p}$ rises substantially when computed for the low expenditure group.

The signs of the profit effects with respect to goods prices are positive except for household labor supply. This is due to the marginal expenditures out of total income being positive for all goods. The sign in household labor is the opposite of the sign on household "leisure." Since "leisure" is a normal good for these households, labor supply is lowered as total income increases due to rising goods prices. With respect to wage rate the signs for effects on goods are negative, for the same reason. Profits are reduced as wage increases so expenditures fall. Household labor, however, increases in this case.

Total Price Elasticities of Consumption

Having derived the profit effects we can add these to the uncompensated elasticities with respect to price, which hold profit constant, to arrive at the total price elasticities of quantities of goods demanded and of labor supplied. These correspond to the movement from point A to point C in Figure 2.3 and are presented in Table 8.4. The own total price effects for commodities remain negative when profit effects are added except for root crops and other cereals at the low expenditure group. The fact that root crops and other cereals consumption responds positively to own price for low expenditure households is reflective of the lack of responsiveness of consumption to own price holding profits constant and of the higher profit effect for these households. In the other cases the short run responsiveness, holding profits constant, to own price is much greater and overwhelms the profit effect. However, the profit effect does have the interesting consequence that the total own price elasticities for several commodities such as rice, oils and fats,

Table 8.4

Total Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-.44	.47	1.21	.93	.69	.93	-.32
	Middle	-.67	.02	.15	.18	.12	.16	-.11
	High	-.44	-.04	-.12	.10	.10	.07	-.06
	Mean	-.66	-.01	.06	.17	.12	.14	-.09
Root crops and other cereals	Low	.47	.23	.84	.52	.38	.54	-.18
	Middle	.12	-.07	.52	.17	.11	.20	-.12
	High	.01E-1	-.15	.56	.11	.06	.23	-.14
	Mean	.11	-.06	.53	.19	.13	.25	-.14
Oils and fats	Low	.40	.32	-.19	.45	.32	.46	-.16
	Middle	.08	.11	-.79	.11	.06	.13	-.07
	High	.02E-1	.05	-1.08	.03	.02	.08	-.04
	Mean	.06	.08	-.73	.10	.07	.13	-.07
Fish and animal products	Low	.73	.46	1.12	-.51	.59	.80	-.28
	Middle	.28	.29	.90	-.57	.23	.40	-.24
	High	.07	.08	.30	-.71	.10	.19	-.12
	Mean	.20	.17	.61	-.68	.20	.35	-.20
Miscellaneous foods	Low	.24	.12	.30	.25	-.80	.25	-.09
	Middle	.09	.05	.19	.11	-.53	.11	-.07
	High	.05	.02	.08	.07	-.60	.07	-.05
	Mean	.08	.03	.14	.10	-.65	.11	-.06
Nonfoods	Low	.22	-.06	.01	.20	.16	-1.03	-.05
	Middle	.11	-.10	-.18	.08	.07	-.82	-.03
	High	.14	-.08	-.26	.09	.10	-.99	-.05
	Mean	.13	-.07	-.16	.10	.09	-.93	-.05
Labor	Low	.74	.29	.82	.76	.57	.75	.16
	Middle	.43	.29	.97	.52	.33	.52	.22
	High	.19	.20	.78	.35	.26	.50	.38
	Mean	.37	.21	.82	.50	.36	.57	.26

[1]Sum of uncompensated quantity elasticities and profit effects in elasticity form. Assumes proportional sales and purchase prices.

and fish and animal products no longer drop in absolute value with higher expenditure levels. Indeed, for rice the total own price elasticity is as low for low expenditure households as for high expenditure households. For root crops and other cereals, the negative response of consumption to own price is greater for high than for middle expenditure households. As seen in Table 6.6 this is mostly a result of the uncompensated (profits constant) price elasticities being higher in absolute value for the high expenditure group. Secondarily, the profit effects are slightly higher for the middle than for the high expenditure group. For household labor supply the response to wage is now positive at all expenditure levels, rising to almost .4 for high expenditure households and being roughly .25 at the sample mean. The fact that this is still rising with higher expenditure group is due to the classical demand substitution effects rising with expenditure as explained in Chapter 6.

In general, the total cross price effects are positive. Negative classical demand income effects are reversed in sign by the profit effects. The exceptions are for root crops and other cereals and oils and fats consumption with respect to nonfoods price, and for those two commodities with respect to rice price for the high expenditure group (and sample mean for root crops and other cereals). Some of the positive cross price elasticities are of large magnitude, for example, oils and fats consumption with respect to root crops and other cereals price. However, in general the cross price responsiveness declines with higher expenditure, as the profit effects do, and are not nearly so large when evaluated at the sample mean. For labor supply the cross price effects are negative, due to the profit effect. The cross effects with respect to wage rate are cut substantially from the effects when profits are held constant, but remain positive and non-negligible. Rises in the wage rate increase total income by increasing the value of time available to the household, but decrease total income by decreasing the profit component. Evidently, the former effect is the dominant one because the positive income effect, found by subtracting the income compensated from the uncompensated elasticities, is larger in absolute value than the negative profits effect.

Effects of Fixed Inputs

Prices are not the only exogenous variables in our household-firm model in which we are interested. The effect of changes in household characteristic variables on consumption was examined in Chapter 6, Table 6.8. Since these variables do not enter into the production side those are the total effects. On the production side, we can look at changes in consumption due to the profit effect of changes in fixed inputs. In elasticity form we have $\frac{Z_j}{X_i^c} \frac{\partial X_i^c}{\partial \pi} \frac{\partial E(\pi)}{\partial Z_j}$, where Z_j is either total land acreage or value of capital flow. These elasticities are reported in Table 8.5. The elasticities with respect to capital flow are larger than those with respect to land because the term $\frac{Z \partial E(\pi)}{\partial Z}$ is larger for capital than for land. This is a reflection of the higher expected quantity output elasticities with respect to capital as was reported in Chapter 7. As with the profit effects due to changes in prices, those profit effects are larger at lower expenditure levels, and for the same reasons. Also, they tend to be larger for commodities having larger marginal expenditures out of total income. The magnitudes of the elasticities are low, all being less than .05 at the sample mean with respect

Table 8.5

Quantity Elasticities with Respect to Fixed Inputs[1] by Expenditure Group

Commodity	Expenditure Group	With Respect To Total Land Cultivated	Value of Capital Flow
Rice	Low	.08	.43
	Middle	.01	.06
	High	.01E-1	.04E-1
	Mean	.01	.04
Root crops and other cereals	Low	.06	.33
	Middle	.02	.08
	High	.01	.05
	Mean	.01	.06
Oils and fats	Low	.15	.76
	Middle	.04	.23
	High	.04	.19
	Mean	.04	.20
Fish and animal products	Low	.09	.48
	Middle	.02	.08
	High	.08E-1	.04
	Mean	.01	.07
Miscellaneous foods	Low	.07	.35
	Middle	.01	.05
	High	.04E-1	.02
	Mean	.01	.05
Nonfoods	Low	.09	.50
	Middle	.02	.09
	High	.01	.08
	Mean	.02	.10
Household labor	Low	-.03	-.17
	Middle	-.01	-.05
	High	-.01	-.05
	Mean	-.01	-.05

[1]Calculated as $\frac{Z_j}{X_i^c} \frac{\partial X_i^c}{\partial \pi} \frac{\partial E(\pi)}{\partial Z_j}$, where Z_j is either acres of total land cultivated or Leones of capital flow.

to land, and .20 or less with respect to capital. It should be remembered that these elasticities reflect an autonomous change in these variables. In the longer run in which capital and total land can be varied, the elasticities of consumption with respect to price of capital and to price of land will not correspond to these short run figures.

Marketed Surplus Price Elasticities

We now have the total price elasticities of consumption of commodities and of labor supply. There are many questions which can be explored using these. One such is what happens to quantities sold or bought on the market when price changes and households have had a chance to adjust their production patterns as well as consumption. The response to price of marketed surplus, which can be either positive or negative, is an important question to governments interested in supplies to urban areas and to other rural areas. There is a very large literature on this both theoretical (for example, Krishna, 1962; and Dixit, 1969) and empirical (e.g., Behrman, 1966; and Medani, 1975, 1980). A review is provided by Newman (1977). Some empirical studies have not had data on consumption and production available separately. They used a reduced form and found the marketed surplus of subsistence crops negatively related to own price. In doing so many simplifications were made. For example, Behrman (1966) assumed zero expenditure and price elasticities of demand and Haessel (1975) assumed that production was fixed. Our data permit direct derivation of the elasticities of marketed surplus.

The only previous study to compute these elasticities from a structural household-firm model is Lau, Lin and Yotopoulos (1978), and they used only one aggregate agricultural commodity. Let $MS_i \equiv$ marketed surplus includes net sales plus in kind wages paid minus in kind wages received. Then $\frac{\partial MS_i}{\partial p_j} = \frac{\partial X_i}{\partial p_j} - \frac{\partial X_i^c}{\partial p_j}$ and in elasticity form

$$(8.3)\quad \frac{p_j}{|MS_i|} \frac{\partial MS_i}{\partial p_j} = \frac{X_i}{|MS_i|} \frac{p_j \, \partial X_i}{X_i \, \partial p_j} - \frac{X_i^c \; p_j \, \partial X_i^c}{|MS_i| \; X_i^c \partial p_j}$$

The elasticity of marketed surplus is then a weighted difference of output elasticities and of total price elasticities of quantities consumed. The weights are the ratio of quantity produced to surplus, for production, and quantity consumed to surplus, for consumption. Given our Tobit estimation of the production side, we use $\frac{\partial E(X_i)}{\partial p_j}$ in the first term. Also, the divisor is the absolute value of marketed surplus. This is used so that one can easily tell the sign of $\frac{\partial MS_i}{\partial p_j}$, that is whether production increases more or less than consumption.

If the sign of the elasticity is positive and the net surplus is positive, then an increase in price will result in more being sold on the market. If the elasticity is positive and the household is a net purchaser (a negative surplus), then an increase in price will lead to less being purchased on the market. A negative elasticity and a positive surplus will lead to less being sold to the market and a negative elasticity and a negative surplus means more will be purchased. We continue to assume proportional sales and purchase prices.

As Krishna pointed out, the magnitudes of the own price marketed surplus elasticities may be a good deal higher than the output elasticities if production is very much larger than surplus. Providing the total own price elasticities of consumption are negative, these will reinforce the effect of increasing production, further increasing the marketed surplus elasticity. Indeed, the only way in which this measure can be negative is for the total own price elasticity to be sufficiently positive and the ratio of consumption to marketed surplus be large enough that their product outweighs the effect of increasing production. Given our total price elasticities this will only be possible for root crops and other cereals for low expenditure households.

The matrix of marketed surplus price elasticities is shown in Table 8.6. All the own price elasticities are positive and reasonably high. There is a tendency for the price responsiveness of marketed surplus to decline at higher expenditure levels. In large part this is due to the absolute value of marketed surplus, part of the denominator, increasing with higher expenditure levels (see Table 4.5). The high magnitude of the own price elasticity for root crops and other cereals for low expenditure households occurs for this reason. If absolute changes in kilograms marketed due to a one percent increase in price were shown they would be roughly equal for the low and middle expenditure groups, rising for the high expenditure group. For household labor the large values of the marketed surplus elasticity with respect to wage rate are also caused by the small values of marketed surplus in the denominator.

The cross price elasticities of marketed surplus tend to be negative because of the strong profit effect in the cross total price elasticity of demand. The latter term is generally positive and often large. Since it is subtracted, after being weighted appropriately, from a generally small positive cross price effect on production, the difference will usually be negative. For example, an increasing price of root crops

Table 8.6

Price Elasticities of Marketed Surplus[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.89	.66	-.32	-1.05	-.47	-1.00	-18.45
	Middle	.73	.05	-.04	-.23	-.09	-.17	-5.74
	High	.75	.04	.09	-.12	-.03	-.08	-1.31
	Mean	.71	.06	-.03	-.72	-.05	-.15	-4.42
Root crops and other cereals	Low	-.11	3.10	-.31	-.70	-.34	-.58	-7.53
	Middle	-.09	.37	-.17	-.23	-.09	-.21	-5.54
	High	.02	.39	-.40	-.10	.02	-.25	-3.09
	Mean	-.08	.46	-.29	-.73	-.04	-.27	-6.61
Oils and fats	Low	-.08	.06	.79	-.58	-.27	-.50	-7.09
	Middle	-.07	-.01	.29	-.19	-.07	-.14	-2.56
	High	-.02E-1	-.01	.78	-.04	-.07E-1	-.09	-.58
	Mean	-.05	-.02	.44	-.44	-.04	-.14	-2.35
Fish and animal products	Low	-.18	.04	-.41	2.15	-.56	-.86	-10.84
	Middle	-.22	.03	-.29	1.81	-.22	-.43	-10.56
	High	-.09	.02	-.21	1.33	-.01	-.21	-2.56
	Mean	-.16	.02	-.33	5.94	-.08	-.38	-8.80
Miscellaneous foods	Low	-.05	.11	-.09	-.32	1.97	-.27	-4.22
	Middle	-.06	.03	-.06	-.13	1.29	-.12	-3.77
	High	-.06	.03	-.05	-.07	.49	-.08	-1.36
	Mean	-.06	.04	-.08	-.34	.81	-.12	-3.44
Nonfoods	Low	-.07	.08	.03E-1	-.30	-.17	1.12	-1.59
	Middle	-.09	.02	.06	-.14	-.09	.88	-1.24
	High	-.21	.04	.19	-.12	-.04	1.08	-.80
	Mean	-.12	.04	.09	-.52	-.06	1.01	-1.85
Labor	Low	-1.22	-5.45	-1.49	-3.30	-2.37	-.83	27.41
	Middle	-.58	-.60	-.37	-2.02	-1.29	-.56	16.41
	High	-.42	-.54	-.58	-.93	-.44	-.55	8.57
	Mean	-.49	-.72	-.51	-5.82	-.78	-.62	17.18

[1]Calculated as $\frac{X_i}{|MS_i|} \frac{p_j}{X_i} \frac{\partial E(X_i)}{\partial p_j} - \frac{X_i^c}{|MS_i|} \frac{p_j}{X_i^c} \frac{\partial X_i^c}{\partial p_j}$ and assuming proportional sales and purchase prices.

and other cereals will lead to a decrease in marketed surplus of oils and fats. That is, less oils and fats will be sold to the market. Also, a decrease in marketed surplus of nonfoods will take place. However, since nonfoods are purchased on the market (the surplus is negative) the decrease in marketed surplus means that more will be purchased on the market.

Some positive cross price elasticities exist. For example, the surplus for root crops and other cereals responds positively to all prices except for oils and fats and the wage rate. Also, the surplus for oils and fats responds positively to nonfoods price.

Some of the magnitudes of the cross price elasticities are fairly large. Again this is caused by the strong profit effect on consumption. The magnitudes do tend to fall with the higher expenditure groups, as they do for the own price elasticities. They are not negligible, however, so that ignoring them as most past studies have done would not seem to be a good idea.

Effects of Prices and Expenditure on Calorie Availability

This study is concerned ultimately with determinants of food consumption. This can be further translated into effects of prices and other variables in our model on availability to the household of different nutrients. Of greatest interest to development economists recently is caloric availability. Sukhatme's (1970) work indicating that sufficient caloric intake is usually accompanied by sufficient protein intake and caloric deficiencies with protein deficiencies is partly responsible for this attitude. More germaine to this study, Kolasa's (1979) summary of existing information based on anthropometric data concerning the nutritional situation in Sierra Leone found that chronic malnutrition (underweight for age) was the principal nutritional problem of children aged 0-5 years (the only population group for which a good deal of information was available). The little evidence which exists for other groups, principally pregnant and lactating women, also suggests that being underweight is the major problem. In view of these findings, only the impact on calories will be examined here, although one can in principle use our results to examine the impact of socio-economic variables on many nutrients.

We want to calculate $\frac{\partial cal}{\partial p_j} = \sum_{i=1}^{5} \frac{\partial cal}{\partial X_i^c} \frac{\partial X_i^c}{\partial p_j}$, where cal ≡ calories and 1-5 are our food groups. In elasticity form we want $\frac{p_j}{cal} \frac{\partial cal}{\partial p_j} = \frac{1}{cal} \sum_{i=1}^{5} \frac{\partial cal}{\partial X_i^c} \frac{p_j \partial X_i^c}{\partial p_j}$. The second term may be derived easily from Tables 6.6 and 8.4, the tables of price elasticities. We calculate effects on calories of price changes both when profits are constant and when they are variable. The difference will point out clearly the effect of allowing families to adjust their production patterns. In addition, the results from holding profits constant will be useful since they correspond to a short run situation which might be found at times.

Tables 8.7 and 8.8 report the effect on availability in kilograms of infinitesimal percentage change in prices, $\frac{p_j \partial X_i^c}{\partial p_j}$. They are of some interest in themselves because they show that the absolute magnitudes of changes in quantities of goods available caused by a change in the own price rises for higher expenditure groups. This result is expected, but different than for elasticities, which when profits were constant, declined with higher expenditure group. The absolute quantity changes due to cross price effects rise with expenditure group when profits are held constant, but profit effects result in many absolute changes

Table 8.7

Change in Quantities[1] of Foods Demanded Due to Infinitesimal Percentage Change in Price, With Profits Constant[2] by Expenditure Group

With Respect to Commodity	For Expenditure Group	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods
Rice	Low	-293.3	-4.8	-6.0	1.0	1.5
	Middle	-424.6	-6.4	-18.6	2.1	2.3
	High	-438.2	-28.1	-70.7	15.2	11.6
	Mean	-434.2	-11.1	-25.1	3.9	3.3
Root crops and other cereals	Low	-4.7	-4.5	-0.5	-1.0	-1.0
	Middle	-10.9	-12.8	-2.4	-2.1	-1.1
	High	-9.7	-72.5	-3.7	-3.0	-1.7
	Mean	-5.9	-24.5	-1.7	-2.6	-1.1
Oils and fats	Low	9.3	1.2	-21.6	2.5	1.5
	Middle	0.5	0.2	-66.0	0.2	0.1
	High	-1.0	1.2	-232.6	0.6	0.2
	Mean	2.3	1.1	-84.1	1.3	1.1
Fish and animal products	Low	4.7	-2.4	-3.2	-63.7	0.5
	Middle	16.3	-2.9	-9.0	-94.9	1.1
	High	58.4	-11.7	-27.9	-245.7	6.6
	Mean	23.5	-4.6	-10.4	-122.3	2.2
Miscellaneous foods	Low	2.3	-1.8	-2.6	-0.1	-49.5
	Middle	5.4	-2.9	-8.4	-0.3	-68.1
	High	38.9	-9.4	-26.0	6.1	-104.0
	Mean	11.7	-4.5	-9.5	0.4	-78.5
Nonfoods	Low	23.3	-4.8	-5.5	3.0	3.0
	Middle	38.1	-7.9	-21.6	2.1	3.4
	High	136.3	-28.1	-70.7	21.2	13.2
	Mean	52.8	-12.3	-26.0	5.1	5.5
Wage	Low	302.6	21.4	47.6	68.1	51.5
	Middle	304.8	23.6	91.8	73.2	49.9
	High	194.7	72.5	215.8	130.4	51.2
	Mean	275.8	37.9	108.4	86.2	51.9

[1] In kilograms.

[2] Calculated as $p_j \frac{\partial E(X_i^c)}{\partial p_j} \mid_{d\pi=0}$ at expenditure group means (see Table 6.6).

Table 8.8

Change in Quantities[1] of Foods Demanded Due to Infinitesimal Percentage Change in Price, with Profits Variable[2] by Expendituro Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods
Rice	Low	-102.4	14.0	31.8	45.9	34.5
	Middle	-364.7	1.0	9.0	18.6	13.6
	High	-428.4	-9.4	-22.3	30.3	16.5
	Mean	-387.3	-1.1	5.2	21.9	13.3
Root crops and other cereals	Low	109.4	6.8	22.1	25.7	19.0
	Middle	65.3	-3.4	31.2	17.5	12.5
	High	1.0	-35.1	104.2	33.4	9.9
	Mean	64.5	-6.7	46.0	24.5	14.4
Oils and fats	Low	93.1	9.5	-5.0	22.2	16.0
	Middle	43.5	5.4	-47.4	11.3	6.8
	High	1.9	11.7	-200.9	9.1	3.3
	Mean	35.2	8.9	-63.3	12.9	7.7
Fish and animal products	Low	169.9	13.7	29.5	-25.2	29.5
	Middle	152.4	14.2	54.0	-58.8	26.1
	High	68.2	18.7	53.8	-215.4	16.5
	Mean	117.4	18.9	52.9	-87.5	22.1
Miscellaneous foods	Low	55.9	3.6	7.9	12.3	-40.0
	Middle	49.0	2.5	11.4	11.3	-58.1
	High	48.7	4.7	14.9	21.2	-99.0
	Mean	46.9	3.3	12.1	12.9	-71.8
Nonfoods	Low	51.2	-1.8	0.3	9.9	8.0
	Middle	59.9	-4.9	-10.8	8.3	7.9
	High	136.3	-16.4	-48.4	27.3	16.5
	Mean	76.3	-7.8	-13.9	12.9	9.9
Labor	Low	172.3	8.6	21.6	37.5	28.5
	Middle	234.1	14.2	58.2	53.6	37.4
	High	185.0	46.8	145.1	106.2	42.9
	Mean	217.1	23.4	71.1	64.4	39.8

[1] In kilograms.

[2] Calculated as $p_j \frac{\partial E(X_i^c)}{\partial p_j}$ at expenditure group means using CET-CD estimates from system in quantity form and assuming proportional sales and purchase prices (see Table 8.4).

decreasing for higher expenditure groups. Since most of the cross effects are positive when profits are allowed to vary it is not clear a priori what the net effects of price changes on caloric availability will be. What is clear from Table 8.8 is that when profits vary the negative own price effects are larger in absolute magnitude for the high expenditure group but the positive cross effects are sometimes smaller for this group.

We now need the conversion from kilograms of our five food groups into calories, $\frac{\partial cal}{\partial X_i^c}$. In Chapter 4 we saw that these were available for each of our 128 foods from food composition tables. We now add up the calories available for each household from each of the foods into the five food groups, by first multiplying those conversion ratios by the sum of consumption out of home production and consumption from purchases. These are then summed over households. These numerators are then divided by the total quantity consumed of each of the five foods again summed over households; where quantity is defined as total value of consumption as defined in Chapter 4, divided by group price. These group quantities are then weighted sums of quantities in straight kilograms. The weights are the ratio of the sales or purchase price of an individual food (depending on whether it was purchased or not) to the consumption price of the group. This weight will, of course, vary by the eight agro-climatic regions to which prices correspond. The numerator, calorie availability, will also vary by household, because the components consumed within each food group vary. In other words, from a nutritional perspective, the aggregated commodity groups correspond to different commodities depending on the region and on the household. Heretofore, we have assumed that the commodities were identical for all households. For our previous economic analysis this last assumption makes sense. Now, however, it does not. Since we want to apply the caloric conversions to low, middle and high expenditure household groups separately, we calculate separate conversions for each group. The conversions may differ between groups for two reasons. First, the weights in calculating quantities for the denominator differ by region, particularly for root crops and other cereals (see Table 8.2). Second, the proportion of calories available for each food group from each of its components will differ by expenditure group. If we want to ask what would the effect be of price changes on caloric availability for a "typical" low expenditure household in our sample, it makes sense to use caloric conversions specific to that group.

Caloric conversion rates are reported in Table 8.9. The magnitudes for rice and for oils and fats do not require explanation, but the rest do. Comparing these rates to rates available for disaggregated foods in food composition tables shows large differences. For root crops and other cereals, cassava was assumed to have 1490 calories per kilogram and sorghum, 3420. These are the two major components of this group, yet both their calorie conversion rates are substantially below the sample mean group rate of 7506 calories per kilogram. The reason for this is as follows. The numerator in our calculation is the best estimate of actual calories available for our sample from the particular group. If we had divided this by the simple sum of kilograms consumed of the components of the root crops and other cereals group (e.g., kilograms of cassava plus kilograms of sorghum, etc.) the conversion rate would look reasonable. It would then be a weighted average of food composition conversion rates, with weights being the proportion of unweighted group

Table 8.9

Calorie Conversion Rates of Food Groups[1]
by Expenditure Group

Food	Expenditure Group			
	Low	Middle	High	Mean
Rice	3,759.1	3,848.6	3,664.6	3,743.3
Root crops and other cereals	8,679.4	10,270.6	5,956.1	7,505.6
Oils and fats	9,909.1	9,241.1	9,001.0	9,143.6
Fish and animal products	5,647.3	3,770.1	2,485.2	3,196.4
Miscellaneous foods	2,430.2	5,184.5	4,748.9	4,430.7

[1]In calories per kilogram of weighted quantity.

quantities for each component. For root crops and other cereals the dominant quantity weight is for cassava. Over 300 kilos per household of cassava is consumed by our sample while only about 50 kilos of sorghum are consumed. However, in deriving weighted quantities, the large quantity of cassava, most of which comes from home production, is multiplied by the ratio of cassava sales price to group consumption price. We saw earlier that this price ratio is very small in general. While the sorghum quantities are multiplied by ratios which are generally a little greater than one, those quantities are not large. The result is that weighted quantity of root crops and other cereals is much smaller than unweighted quantity. Hence, the large calorie conversion rate. Since the quantity units used in our model are weighted quantities, it makes sense to use calorie conversion rates which are in terms of the same weighted quantities.

Elasticities of caloric availability with respect to total expenditure are reported in Table 8.10. Total expenditure, as opposed to total income, is endogenous in our model, but those results should still be of interest. Elasticities with respect to total income cannot be computed from our model estimates since total income is not statistically identified, see Chapter 6, nor are actual estimates available without making further assumptions about the variable for total time available. The magnitudes are around .85 with little variation between expenditure groups. That the elasticity for the high expenditure group is slightly higher than for the low expenditure group is due to the marginal total expenditure share on oils and fats, an important contributor of calories, rising with higher expenditure group. This apparently offsets the declining total expenditure share on rice. The elasticity magnitudes we report compare to a

Table 8.10

Elasticities of Calorie Availability with Respect to Total Expenditure[1] by Expenditure Group

Expenditure Group			
Low	Middle	High	Mean
.85	.83	.93	.86

[1]Calculated as $\frac{TEXP}{Cal} \Sigma \frac{\partial Cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial TEXP}$ (see Table 6.4 for $\frac{\partial E(p_i X_i^c)}{\partial TEXP}$).

range of .15 to .30 used by Reutlinger and Selowsky (1976). They believed .15 and .3 to be the bounds on the calorie elasticity with respect to income. This belief was based largely on a set of cross-country regressions on per capita GNP of national calorie availability per capita (as computed from food balance sheets). The regressions were run separately for developing countries by region. Four functional specifications were used, three of which imposed a declining elasticity with higher income. When one calculates the calorie income elasticities using their equations for Africa and using a per capita GNP of U.S. $101, the per capita total expenditure in our sample, they range from .04 to .07 (Reutlinger and Selowsky, 1976, pp. 71-74). Possible sources of the different estimates are numerous. First, Reutlinger and Selowsky only had access to aggregate national data. For Africa these data are particularly weak. The variation in per capita GNP in their sample of 37 African countries is quite likely less than in total expenditure (or more properly the profits component of total income) for our sample of 138 households. Furthermore, our models are very different, to suit the different data available to each. In particular, we include price and demographic variables which they are unable to include. Finally, the marginal expenditure share on foods for our sample is very high at .61. Indeed, it may be higher than that for the average African country of U.S. $101 per capita, since the latter includes urban households which may have a lower marginal expenditure share on foods than a rural household of comparable income.

Our estimates of the total expenditure elasticity of calorie availability compare much better to those of Pinstrup-Anderson and Caicedo (1978). They estimate Engel curves from cross section household data in Colombia and find a calorie elasticity with respect to income of over .5 ranging to over .6 for low income households.

Tables 8.11 and 8.12 report calorie elasticities with respect to prices with profits held constant and allowed to vary. In the very short run, profits being constant, increases of commodity prices results in decreased caloric availability, except with respect to nonfoods price at the low expenditure group. There is no general pattern of elasticities across expenditure group, however, the absolute change in caloric availability often increases with higher expenditure group. For commodity prices the largest response of caloric availability is for changes in the price of rice, the major staple. These range from -.58 to -.28. This is a rather large impact suggesting the short run nutritional vulnerability of rural households to rice price increases.

When profits can vary the situation changes substantially. Now most of the commodity price elasticities of calories are positive. Increasing price may result in decreased consumption of that good, but the increase in total income is distributed on increases in consumption of other foods, enough so to increase total caloric availability. The exceptions to this are for rice and oils and fats prices at all but the low expenditure group, and for miscellaneous foods price at the high expenditure group. The magnitudes of the positive elasticities are not high for the sample mean, but some are sizable for the low expenditure group, and in general they tend to decline with higher expenditure group. Even absolute changes in calorie availability tend to decline with higher expenditure group except for changes in rice, oils and fats, and labor prices. For changes in rice and oils and fats prices, caloric availability increases for low expenditure households, but decreases for middle and high

Table 8.11

Elasticities of Calorie Availability with Respect to Price, Profits Constant[1] by Expenditure Group

With Respect to Price of:	Expenditure Group	Change in Kilocalories[2]	Elasticity
Rice	Low	-11.9	-.58
	Middle	-18.5	-.38
	High	-23.2	-.28
	Mean	-19.1	-.38
Root crops and other cereals	Low	-0.7	-.03
	Middle	-2.1	-.04
	High	-5.2	-.06
	Mean	-2.3	-.05
Oils and fats	Low	-1.5	-.07
	Middle	-6.0	-.12
	High	-20.9	-.25
	Mean	-7.4	-.15
Fish and animal products	Low	-3.9	-.19
	Middle	-4.0	-.08
	High	-6.9	-.08
	Mean	-4.2	-.08
Miscellaneous foods	Low	-1.5	-.07
	Middle	-4.4	-.09
	High	-6.3	-.08
	Mean	-4.2	-.08
Nonfoods	Low	0.2	.08E-1
	Middle	-1.1	-.02
	High	-1.9	-.02
	Mean	-0.9	-.02
Labor	Low	23.0	1.12
	Middle	28.0	.57
	High	36.5	.45
	Mean	28.1	.56

[1]Calculated as $\frac{p_j}{cal} \sum_i \frac{\partial cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j} \Big|_{d\pi=0}$ at expenditure group means.

[2]Change in kilocalorie availability due to infinitesimal percentage change in price, $\frac{p_j}{100} \sum_i \frac{\partial kcal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j} \Big|_{d\pi=0}$.

Table 8.12

Elasticities of Calorie Availability with Respect to Prices, Profits Variable[1] by Expenditure Group

With Respect to Price of:	Expenditure Group	Changes in Kilocalories[2]	Elasticity
Rice	Low	3.9	.19
	Middle	-11.7	-.24
	High	-16.7	-.20
	Mean	-12.8	-.26
Root crops and other cereals	Low	8.8	.43
	Middle	6.4	.13
	High	8.6	.11
	Mean	7.5	.15
Oils and fats	Low	5.5	.27
	Middle	-1.4	-.03
	High	-16.9	-.21
	Mean	-3.0	-.06
Fish and animal products	Low	9.8	.48
	Middle	11.5	.23
	High	3.9	.05
	Mean	8.8	.18
Miscellaneous foods	Low	2.9	.14
	Middle	0.6	.01
	High	-0.8	-.01
	Mean	0.3	.07E-1
Nonfoods	Low	2.6	.12
	Middle	1.5	.03
	High	1.1	.01
	Mean	1.9	.04
Labor	Low	12.2	.59
	Middle	19.8	.40
	High	27.3	.33
	Mean	20.3	.41

[1]Calculated as $\frac{p_j}{cal} \sum_i \frac{\partial cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j}$ assuming proportional sales and purchase prices.

[2]Change in kilocalorie availability due to one percent change in price, $\frac{p_j}{100} \sum_i \frac{\partial kcal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j}$.

expenditure households, and at the sample mean. For rice price the elasticities for the two higher expenditure groups are still sizably negative, between -.2 and -.25. Hence, when profit effects are accounted for, price increases would seem to lessen the discrepancy in calories available to the rural expenditure groups. For increases in rice price the mechanism behind this is increased availability for very low expenditure households and decreased availability for higher expenditure households. From Table 4.6 we see that the mean daily caloric availability per capita for high expenditure households is substantially above any reasonable level of "requirements." Although some households in this group will have calorie availability lower than the mean, it may be that lower availability will still allow these households to have available sufficient calories for weight maintenance under "normal" activity levels.

APPENDIX 8A

In Chapter 8 we assumed that sales and purchase prices were proportional in deriving the results of the full household-firm model. That assumption implied that a one percent change in one price was accompanied by the same percent change of the other price. In this appendix we present tables showing the full household-firm effects of price on consumption and on calorie availability when a constant marketing margin is assumed. As shown in Chapter 8 this will mean for goods other than nonfoods that a percent change in weighted consumption price is accompanied by a greater than one percent change in sales price (see Table 8.2). For nonfoods price the opposite will be true, and for wage the percent changes will be equal since only one wage figure is used. This means that the profit effects in "elasticities" shown in Table 8A.1 correspond to sales price changes of greater than one infinitesimal percent. Because of this the profit effects are generally larger, much larger with respect to root crops and other cereals price, than under the proportionality assumption. This mitigates even more the negative own price effects when profits are held constant. As one can see from Table 8A.5, however, the signs on the calorie elasticities are almost identical to the signs in Table 8.12, although some of the magnitudes are quite different.

Table 8A.1

Profit Effects in Elasticity Form[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.90	.69	1.58	1.00	.73	1.03	-.35
	Middle	.12	.17	.51	.18	.11	.20	-.12
	High	.06E-1	.09	.31	.06	.04	.13	-.08
	Mean	.09	.11	.39	.15	.10	.19	-.11
Root crops and other cereals	Low	1.27	.98	2.23	1.41	1.04	1.46	-.49
	Middle	.77	1.08	3.19	1.10	.66	1.24	-.73
	High	.04	.54	1.91	.40	.24	.78	-.49
	Mean	.47	.61	2.08	.79	.53	1.03	-.57
Oils and fats	Low	.57	.44	1.01	.64	.47	.66	-.23
	Middle	.12	.17	.50	.17	.10	.20	-.11
	High	.06E-1	.08	.30	.06	.04	.12	-.08
	Mean	.09	.12	.40	.15	.10	.20	-.11
Fish and animal products	Low	.85	.65	1.49	.94	.69	.97	-.33
	Middle	.28	.39	1.15	.39	.24	.45	-.26
	High	.09E-1	.13	.45	.10	.06	.19	-.12
	Mean	.18	.23	.80	.30	.20	.40	-.22
Miscellaneous foods	Low	.45	.35	.79	.50	.37	.52	-.17
	Middle	.16	.22	.65	.22	.13	.25	-.15
	High	.01	.13	.47	.10	.06	.19	-.12
	Mean	.12	.15	.52	.20	.13	.26	-.14
Nonfoods	Low	.06	.05	.11	.07	.05	.07	-.02
	Middle	.02	.03	.09	.03	.02	.04	-.02
	High	.01E-1	.02	.06	.01	.08E-1	.03	-.02
	Mean	.02	.02	.07	.03	.02	.04	-.02
Labor	Low	-.56	-.43	-.99	-.62	-.46	-.64	.22
	Middle	-.13	-.19	-.56	-.19	-.11	-.22	.13
	High	-.08E-1	-.11	-.38	-.08	-.05	-.15	.10
	Mean	-.10	-.13	-.43	-.17	-.11	-.21	.12

[1]Calculated as $\frac{P_j}{X_i^c} \frac{\partial X_i^c}{\partial \pi} \frac{\partial E(\pi)}{\partial P_j}$ at expenditure group means, using parameter estimates from the CET-CD system in quantity form, and assuming a constant marketing margin.

Table 8A.2

Total Quantity Elasticities with Respect to Price[1] by Expenditure Group

With Respect to Price of	For Expenditure' Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	-.36	.53	1.35	1.02	.76	1.02	-.35
	Middle	-.66	.04	.20	.20	.13	.18	-.11
	High	-.44	-.03	-.07	.11	.11	.09	-.07
	Mean	-.65	.01	.10	.18	.13	.16	-.10
Root crops and other cereals	Low	1.25	.83	2.21	1.39	1.02	1.44	-.48
	Middle	.75	.82	3.15	1.08	.65	1.22	-.72
	High	.03	.23	1.89	.39	.23	.77	-.48
	Mean	.46	.39	2.06	.77	.52	1.01	-.56
Oils and fats	Low	.61	.48	.19	.69	.50	.71	-.25
	Middle	.12	.17	-.61	.17	.10	.21	-.11
	High	.05E-1	.09	-.95	.06	.04	.13	-.08
	Mean	.09	.13	-.57	.16	.11	.21	-.12
Fish and animal products	Low	.87	.57	1.37	-.35	.70	.96	-.33
	Middle	.31	.33	1.00	-.53	.25	.44	-.26
	High	.07	.08	.30	-.71	.10	.19	-.13
	Mean	.22	.19	.68	-.65	.22	.39	-.22
Miscellaneous foods	Low	.46	.29	.69	.50	-.62	.51	-.17
	Middle	.17	.16	.51	.22	-.47	.23	-.14
	High	.05	.09	.33	.12	-.57	.17	-.11
	Mean	.14	.11	.41	.20	-.58	.24	-.13
Nonfoods	Low	.16	-.11	-.10	.13	.11	-1.10	-.03
	Middle	.09	-.13	-.27	.04	.05	-.86	-.01
	High	.14	-.10	-.32	.08	.09	-1.02	-.03
	Mean	.11	-.09	-.23	.07	.07	-.97	-.03
Labor	Low	.74	.29	.82	.76	.57	.75	.16
	Middle	.43	.29	.97	.52	.33	.52	.22
	High	.19	.20	.78	.35	.26	.50	.38
	Mean	.37	.21	.82	.50	.36	.57	.26

[1] Sum of uncompensated quantity elasticities and percent profit effects with respect to price. Assumes constant marketing margin.

Table 8A.3

Price Elasticities of Marketed Surplus[1] by Expenditure Group

With Respect to Price of	For Expenditure Group OF	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods	Nonfoods	Household Labor
Rice	Low	.94	.72	-.36	-1.15	-.52	-1.10	-20.25
	Middle	.73	.06	-.06	-.26	-.10	-.19	-5.97
	High	.77	.05	.05	-.13	-.03	-.10	-1.55
	Mean	.71	.05	-.05	-.84	-.05	-.17	-4.90
Root crops and other cereals	Low	-.29	7.89	-2.07	-1.87	-.93	-1.55	-19.86
	Middle	-.56	1.92	-1.01	-1.55	-.57	-1.31	-32.71
	High	.02	1.03	-1.33	-.37	.05	-.84	-10.42
	Mean	-.35	1.51	-1.13	-3.49	-.16	-1.09	-25.94
Oils and fats	Low	-.12	.13	1.05	-.89	-.42	-.77	-11.20
	Middle	-.10	-.01	.25	-.29	-.12	-.22	-4.04
	High	-.05E-1	-.02	.70	-.08	-.01	-.14	-1.14
	Mean	-.08	-.03	.37	-.84	-.07	-.23	-4.02
Fish and animal products	Low	-.22	.03	-.51	2.17	-.66	-1.03	-12.87
	Middle	-.24	.03	-.32	1.81	-.24	-.47	-11.49
	High	-.09	.02	-.21	1.33	-.01	-.21	-2.56
	Mean	-.18	.03	-.37	5.98	-.08	-.42	-9.68
Miscellaneous foods	Low	-.09	.17	-.22	-.62	2.79	-.55	-8.19
	Middle	-.12	.13	-.16	-.26	1.72	-.25	-7.55
	High	-.05	.05	-.23	-.10	.70	-.18	-2.91
	Mean	-.10	.06	-.22	-.65	1.13	-.26	-7.35
Nonfoods	Low	-.05	.11	.05	-.20	-.12	1.19	-.91
	Middle	-.08	.03	.09	-.09	-.07	.92	-.47
	High	-.21	.04	.23	-.11	-.04	1.11	-.46
	Mean	-.10	.04	.13	-.37	-.05	1.05	-1.05
Labor	Low	-1.22	-5.45	-1.49	-3.30	-2.37	-.83	27.41
	Middle	-.58	-.60	-.37	-2.02	-1.29	-.56	16.41
	High	-.42	-.54	-.58	-.93	-.44	-.55	8.57
	Mean	-.49	-.72	-.51	-5.82	-.78	-.62	17.18

[1] Calculated as $\frac{X_i}{|MS_i|} \frac{p_j}{X_i} \frac{\partial E(X_i)}{\partial p_j} - \frac{X_i^c}{|MS_i|} \frac{p_j}{X_i^c} \frac{\partial X_i^c}{\partial p_j}$ and assuming a constant marketing margin.

Table 8A.4

Change in Quantities[1] of Foods Demanded Due to Infinitesimal Percentage Change in Price,[2] with Profits Variable by Expenditure Group

With Respect to Commodity	For Expenditure Group	Rice	Root Crops and Other Cereals	Oils and Fats	Fish and Animal Products	Miscellaneous Foods
Rice	Low	-83.8	15.8	35.5	50.3	38.0
	Middle	-359.3	2.0	12.0	20.6	14.7
	High	-428.4	-7.0	-13.0	33.4	18.2
	Mean	-381.4	1.1	8.7	23.2	14.4
Root crops and other cereals	Low	291.0	24.7	58.1	68.6	51.0
	Middle	408.3	40.3	189.0	111.4	73.7
	High	29.2	53.8	368.4	118.3	38.0
	Mean	269.9	43.5	178.6	99.1	57.5
Oils and fats	Low	142.0	14.3	5.0	34.1	25.0
	Middle	65.3	8.3	-36.6	17.5	11.3
	High	4.9	21.1	-176.8	18.2	6.6
	Mean	52.8	14.5	-49.4	20.6	12.2
Fish and animal products	Low	200.5	16.9	36.0	-17.3	35.0
	Middle	168.7	16.2	60.0	-54.7	28.4
	High	68.2	18.7	55.8	-215.4	16.5
	Mean	129.1	21.2	59.0	-83.7	24.3
Miscellaneous foods	Low	107.1	8.6	18.1	24.7	-31.0
	Middle	92.5	7.9	30.6	22.7	-53.3
	High	48.7	21.1	61.4	36.4	-94.1
	Mean	82.2	12.3	35.6	25.7	-64.1
Nonfoods	Low	37.2	-3.2	-2.6	6.4	5.5
	Middle	49.0	-6.4	-16.2	5.2	5.7
	High	136.3	-23.4	-59.5	24.3	14.9
	Mean	64.5	-10.0	-19.9	9.0	7.7
Wage	Low	172.3	8.6	21.6	37.5	28.5
	Middle	234.1	14.2	58.2	53.6	37.4
	High	185.0	46.8	145.1	106.2	42.9
	Mean	217.1	23.4	71.1	64.4	39.8

[1] In kilograms.

[2] Calculated as $p_j \frac{\partial E(X_i^c)}{\partial p_j}$ at expenditure group means using CET-CD estimates from system in quantity form and assuming a constant marketing margin (see Table 8A.2).

Table 8A.5

Elasticities of Calorie Availability with Respect to Price, Profits Variable[1] by Expenditure Group

With Respect to Price of	Expenditure Group	Change in Kilocalories[2]	Elasticity
Rice	Low	5.5	.27
	Middle	-11.0	-.22
	High	-15.6	-.19
	Mean	-12.0	-.24
Root crops and other cereals	Low	24.0	1.17
	Middle	45.3	.92
	High	42.2	.52
	Mean	35.5	.71
Oils and fats	Low	9.6	.47
	Middle	1.2	.03
	High	-13.9	-.17
	Mean	-0.3	-.05E-1
Fish and animal products	Low	12.5	.61
	Middle	13.1	.27
	High	4.1	.05
	Mean	10.2	.20
Miscellaneous foods	Low	7.2	.35
	Middle	5.3	.11
	High	5.0	.06
	Mean	5.2	.10
Nonfoods	Low	1.4	.07
	Middle	0.2	.04E-1
	High	-0.4	-.05E-1
	Mean	0.5	.09E-1
Labor	Low	12.2	.59
	Middle	19.8	.40
	High	27.3	.33
	Mean	20.3	.41

[1]Calculated as $\frac{p_j}{cal} \sum_i \frac{\partial cal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j}$ assuming constant marketing margin.

[2]Change in kilocalorie availability due to infinitesimal percentage change in price, $\frac{p_j}{100} \sum_i \frac{\partial kcal}{\partial X_i^c} \frac{\partial E(X_i^c)}{\partial p_j}$.

CHAPTER 9

POLICY AND RESEARCH IMPLICATIONS

Introduction

These results have significant implications for the development process in Sierra Leone and for future modeling of this kind. First we state the obvious: prices and total income do affect household caloric availability, although the ability of the household being able to adapt its production pattern mitigates this effect. Response by the household in its role as a firm does make a difference. Secondly, for the representative low expenditure household to have caloric availability even at the level of 1900 calories per capita per day (see Chapter 4) would require increases in income of a magnitude not likely to occur anytime soon. With prices and household characteristics constant, an average low expenditure household would need an increase in annual total income of about 270 Leones to reach the availability level of 1900 calories per capita per day. This new level of total income (which we cannot compute since the original level is unknown, see Chapter 6) results in total expenditures being roughly 445 Leones. That figure is 88 percent higher than the total expenditure level of 237 Leones, which the representative low expenditure household spends (see Table 4.1). Assuming, optimistically, an annual growth rate in total expenditures of three percent, it would take nearly 22 years for an average low expenditure family to reach this point. Of course, if family size grew as total expenditure did, which is likely, then even longer would be needed.

Caution is needed here. Caloric availability at the household level says little about intake of individuals. For example, one of the variables in our model is household labor supplied, of which one part is labor supplied by lactating women. If, with increasing household total income, lactating women spend more time at home breastfeeding infants, the caloric intake of infants may increase more than suggested by total household availability. As another example, food waste may be influenced by variables such as total income.

Trade-Off Between Secular Growth and Short Run Nutritional Status

The price responsiveness, especially with respect to rice, of food availability and ultimately of calorie availability implies that there is a trade-off to be made between long run output growth and short run nutritional status. A secularly rising price of rice (remember this is total rice, swamp and upland rice are combined) may lead to increased output levels, and possibly to increased growth rates if technical change is endogenous, but will lower caloric availability for many rural households (assuming no other household variables change). Very low expenditure households may enjoy some nutritional benefits from such a rise. This implication will not change if we use the results when assuming a constant marketing margin (see Table 8A.5). Of course, in the long run households may invest in more capital (some embodying technical progress perhaps) and in more land. This would presumably be one result of a secular rise in rice price. As shown in Table 8.5 this will increase quantities of food availability, hence of calorie availability.

Whether this would offset the decreasing caloric availability due to increasing price will depend on how much capital and land increase, about which our results say nothing. At the sample mean the elasticity of caloric availability with respect to quantity of capital flow is .07. This elasticity is roughly four times lower than the calorie availability elasticity with respect to rice price. However, when both change there is an interaction effect and both elasticities will change also. Nevertheless, it seems that capital (or a combination of capital and technical change if the latter is capital augmenting) would have to increase more relatively than price for there not to be a net negative effect on caloric availability for a representative rural household.

In the longer run, rice price may be lower than otherwise if production growth has been stimulated. Distributional impacts of technical change have long been debated. Questions of access to technology cannot be addressed by these research results. However, differential price effects of technical change may be addressed. Most producers in rural areas would seem to be helped nutritionally by rice price being lower than it otherwise might be. However, those lowest expenditure households who are nutritionally worst off (see Table 4.6) may be hurt unless they participate in the technical change sufficiently. In that case the autonomous increase in total income due to the technical change would be enough to offset the lowered caloric availability due to a lower (than otherwise) rice price. These effects of price changes due to technical change are somewhat different from those generally postulated in the literature. Distributional impacts have been limited to examining the impact on pure consumers and on pure producers. Hayami and Herdt (1974) examine the impact on each with producers selling a portion of the crop (rice) to the market. However, consumption out of home production is assumed to be completely price inelastic and since purchases are ignored, total consumption of rice is assumed price inelastic. This enables them to examine the impact only on cash income. In their model a decline in rice price reduces cash income hence welfare, but differentially depending on the proportion marketed. In our model total income matters, not cash income, and consumption of rice is affected by price changes, though the decomposition of changes on consumption of home produced versus changes in consumption of purchased rice is not identified. Nevertheless, the price impact of technical change can now be positive on rural rice producing households, and is for representative households of all but the lowest expenditure group.

Rice Self-Sufficiency Impact on Calorie Availability

Another major policy thrust which may involve long run versus short run trade-offs is attempting to obtain self-sufficiency in rice. Whether this policy makes sense using static comparative advantage criteria is not at issue here. If, however, domestic rice prices are set above cif. Freetown plus transportation cost levels, there would seem to be an adverse short run impact on calorie availability for all but very low expenditure rural households (and a presumably adverse impact on urban households also). As before this implication is insensitive to the assumption made on the relationship between sales and purchase prices. If, in the longer run, a higher domestic rice price is only temporary and promotes an increasing level (and possibly growth rate) of rice production, then this adverse short run nutritional impact may lead to a positive long run impact. Exactly what the magnitudes might be will depend upon how much domestic prices are raised, and what effect that has on future supplies.

Export Promotion and Relation Between Market Orientation and Calorie Availability

A related trade policy question is to what extent to promote exports of cash crops such as palm oil, coffee and cocoa. Some people have argued in the past that increasing production of cash crops at the expense of subsistence crops will adversely affect nutritional status. These persons have argued that a reduced market orientation will result in better nutrition. In our household-firm model marketed surplus is endogenous, being simultaneously determined with production and consumption. As an endogenous variable it is affected by many exogenous variables. Hence, it stands to reason that one exogenous variable will affect marketed surplus and consumption differently than another, so that the relationship between marketed surplus and consumption should not be of only one kind. For example, if we examine oils and fats, of which palm oil is the lion's share in value of consumption (though palm kernels are included for production), an increase in price results in decreased calorie availability for high and middle expenditure groups but increased availability for the low expenditure group. Marketed surplus increases for all groups. Moreover, when we examine the sources of the change, they turn out to be the opposite of the sources which have heretofore been assumed. More, not less, is consumed of rice and root crops and other cereals when price of oils and fats increases (see Table 8.8). This is primarily because to the profit effect of increasing total income. As a result, less of these foods is marketed. Less, not more, oils and fats is consumed, and it is that reduction in consumption which is the source of lowered caloric availability. Moreover, even when we look at what happens to the production of rice and of root crops and other cereals, more is produced (see Table 7.8), not less, when price of oils and fats increases. Land area switched cannot be productive in the short run since it takes time to grow palm trees. Labor can be reallocated to picking from wild trees, but an increase in output prices raises demand for total labor, some of which is allocated to increasing rice and root crops and other cereals production. Even in the longer run when more land reallocation takes place, perhaps reducing subsistence crop production, total income increases even more and some of that will be allocated to increased consumption of foods, increasing caloric availability.

An increase in capital flow actually decreases the marketed surplus of oils and fats for the sample mean, and as seen from Table 8.5 it increases consumption of all foods. Alternatively, an increase in rice price decreases marketed surplus of oils and fats for the low and middle expenditure groups, Table 8.6, while increasing calorie availability for the low expenditure group and decreasing it for the middle expenditure group. Oils and fats consumption increases and rice consumption decreases when rice price increases. For the low expenditure group reduction in reliance on the market for oils and fats due to rice price changes results in the expected increase in caloric availability, but again for different reasons than commonly assumed. For the middle expenditure group the "expected" relationship does not hold.

Deriving Macro Predictions from Model Results

The above policy implications have been discussed from our estimation results derived from our sample. The sample, recall, was a multi-level random sample from most of the rural area. Our predictions of consumption and production can be added by households in each of the regions and converted to estimates for the population in each region, provided we know the sampling proportions. This work is being done by others as an extension of this dissertation. Converting our micro predictions into macro predictions will enable further policy analysis to be carried out. One example is the construction of food accounting matrices (see McCarthy and Taylor, 1980). These will enable easy viewing of the effects of discrete changes of variables in our model at the national level. Another possibility would be to estimate the caloric gap, calories necessary to raise all households above some minimal level, for rural Sierra Leone (see Reutlinger and Selowsky, 1976); as well as the increases in income necessary to eliminate it. If one had a general equilibrium model of the Sierra Leone economy one could integrate our model with the general equilibrium model and conduct policy analysis in that way (see Pinstrup-Anderson, de Londono, and Hoover, 1976).

Relationship of Research to Past Empirical Work

Our experience in formulating and estimating the household-firm model has implications for future research in this area. First though, it may be helpful to anchor this methodology more firmly in the existing literature, scant as it is. Lau, Lin and Yotopoulos (1976) estimated a profit function and input demand function using a Cobb-Douglas production function for an aggregate agricultural output. Their data were averages in each of two years of household data grouped by size of operation in Taiwan. They then used this data to estimate a Linear Logarithmic Expenditure System (1978) using aggregate agricultural (in kind) and nonagricultural (in cash) commodities, and leisure, as commodity definitions. This system assumes homogeneity of degree minus one in the indirect utility function resulting in expenditure elasticities with respect to total income being one for each group. They estimate the system using seemingly unrelated regressions with cross equation restrictions. In this case, which is not maximum likelihood estimation, parameter estimates are not invariant to the equation not estimated. Using both sets of estimates, they compute elasticities of marketed surplus as well as of quantities consumed.

Barnum and Squire (1979) use a Linear Expenditure System on the demand side with rice, a nonagricultural good and leisure as commodities (the households practiced monoculture). They use a Cobb-Douglas production function, which they estimate directly, for a single agricultural commodity, on the production side. Their data were from a cross section of households in Malaysia, exhibiting price variation only for labor. Their procedure in obtaining the LES parameter estimates is unusual and the statistical properties of their estimates, aside from consistency, are unclear. Their tests, however, are certainly inappropriate. First, they assume the error terms to be independent across demand equations, which is inconsistent with the sum of expenditure being total income. They then use ols instead of gls, in a strange way. They estimate the system unconstrained and obtain a partial set of parameters (partial because the others are in nonlinear form). They then construct new independent variables by using values obtained for those parameters.

This makes the model linear in parameters, hence, easier to estimate. These "variables" are then used to estimate the remaining parameters. However, the parameters which they are estimating include the same parameters which they assume values for when constructing their "independent variables." That is, they do not partition the variables into mutually exclusive sets as Stone did (1954), but into overlapping sets. They then iterate until convergence. Parks (1971) showed that the statistical properties of Stone's estimation procedure were unknown when the covariances between equations were unaccounted for. Moreover, the covariance matrix of parameter estimates derived from the procedure is not correct because the covariances between parameters held constant and parameters allowed to vary is not accounted for.

Singh and Squire (1978) pursue the results of Barnum and Squire. In addition, they propose using linear programming for the production side of the model, to extend it to multicrop households. Ahn, Singh and Squire (1980) do so using cross section household data from South Korea. They use six commodities including four foods: rice, barley, other farm produce and market purchased foods. They use an LES, using the same estimation procedure as did Barnum and Squire. Use of linear programming on the production side allowed more easily for commodity disaggregation on that side. Also, it easily handles the problem of specialization since it is a deterministic model. Further, risk can be easily incorporated into it. One disadvantage stems from its determinateness; statistical tests cannot be performed. In addition, one cannot get income group specific results without redoing the analysis for representative farms from each group. Nevertheless, it is an idea worth exploring further.

The empirical results from these studies are reported only at the sample mean. Lau, Lin and Yotopoulos report an own price elasticity of -.72 for their agricultural commodity, profits being held constant, and a total own price elasticity, profits being allowed to vary, of .22. They find that marketed surplus of the agricultural good responds positively to own price with an elasticity of about unity. The LES studies find a very small own price elasticity for rice in both Malaysia and Korea; -.04 and -.18 respectively. The total own price elasticities reported are .38 and .01 respectively. Hence, all these studies find that for the agricultural good profit effects outweigh negative own price effects holding profits constant. This is not generally confirmed for our data. The magnitudes of own price elasticities found in the Malaysia and Korean studies are much lower than we find, except for root crops and other cereals. The Malaysian figure seems particularly low.

For Korea and Taiwan, the difference in incomes between the farmers studied there and those studied in Sierra Leone is very large. That higher income farmers should have smaller own price elasticities for staples is not so surprising; indeed, it is confirmed in our results for rice.

The existing literature estimating Quadratic Expenditure Systems is small, because the system is relatively new. Howe, Pollak and Wales (1979) and Pollak and Wales (1980, 1978a) use only three or four commodities, none being labor supply. Data for households have been aggregated into groups raising the issue of whether certain constraints imposed at the household level hold. For example, symmetry of the Slutsky substitution matrix holds for groups only under certain restrictive conditions. Also, only time series or time series-cross section data have been used, except for Howe (1974). His cross section data had no price variation so he had to use extraneous information, on "subsistence requirements," to identify many of his parameters.

On the production side, this is the first work to apply the Tobit model to a multiple output production function. Heretofore, the only method used to account for specialization was mathematical programming. On the demand side Wales and Woodland (1978, 1979) have used the multivariate Tobit model without assuming independent error terms, but only for three commodities.

Future Research Possibilities

In sum, our research has shown that cross section household data can be successfully used to estimate price as well as income relationships of demand. This can be done using functional forms allowing for a wide variety of behavior, and it can be done for several commodities. The same holds true for the production side with the addition that zero outputs can be statistically handled in a proper way, provided certain simplifying assumptions are made. On the other hand, the numerical maximum likelihood procedures involved in estimation are costly in both computer and researcher time.

Much, however, remains to be explored. For the demand side of the household-firm model one particularly interesting possibility would be to define consumption from home production and consumption from market purchases as separate goods, for a major staple such as rice. Development economists sometimes hypothesize the former to be price inelastic and the latter more price responsive. In our model the two sources are not separable. Of course, a larger model might be tried or a different specification for the system or for entering demographic variables. Indeed, there are numerous small changes of this kind.

On the production side, two obvious possibilities exist. One is to estimate a system with more parameters, allowing for more flexible behavior. Alternatively, we might try Tobit estimation not assuming independence of errors across equations for the four outputs plus labor demand in the smaller system. This would involve at most triple integrals (see Chapter 7), but double and single integrals will be more numerous.

Other future research ought to include extending the household-firm model used here. In the first place, more can be done to make the model operational when the recursiveness assumption does not hold; perhaps the labor market does not exist. Specifying even simple utility functions such as the Stone-Geary (which gives rise to the LES), results in intractable algebra. However, one could approach the problem by specifying a flexible form for the reduced form equations. From the first order conditions we know which independent variables belong in each equation (if the model is not recursive all independent variables belong in all equations). Having specified a flexible form, one could constrain parameters so that certain restrictions were met. The question would be what restrictions to impose. Assuming no labor market, expenditures on goods would add to value of production less value of variable inputs other than labor. Zero homogeneity of consumption demand with respect to prices would be another restriction (this is implied by the first order conditions which would replace those in 2.2). Since flexible forms, even with these two restrictions involve many parameters, the number of commodities in such a system would probably have to be kept small.

In the longer run two other extensions of the model would seem to be worth exploring, provided data were available. First, the model might be made dynamic, either multiyear or multiseason. In this case, demand and supply out of storage would have to be accounted for. In a multiyear model investment in capital and land would need to fit in the model. Second, risk might be accounted for. On the production side, this is straightforward if one uses a programming model. On the demand side, it is not clear how to make it operational. The theory, using expected utility maximization, is probably not difficult to derive, but how that could be incorporated into a demand systems framework is unclear. Perhaps a flexible form might be a possible solution with an added risk parameter, for instance, the Pratt-Arrow absolute coefficient of risk aversion. However, to obtain household data for such a parameter would be quite expensive.

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