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ABSTRACT

DYNAMIC ANALYSIS OF NONLINEAR ELASTIC FRAMES

by James K. Iverson

A method of solution for dynamically loaded elastic frame structures is presented. The nonlinear effects of geometry changes and axial thrust are considered in the solution. The method is developed to analyze three dimensional structures subject to joint loading or earthquake ground motion. A computer program written in FORTRAN for use on the Michigan State University CDC 3600 computer system was used to implement the solution.

The mass of the members is lumped at the joints. The lumping procedure accounts for rotatory inertia as well as translational inertia. Mass-proportional viscous damping is included in the analysis. The nonlinear effects are taken into account in the calculation of member forces. The analysis is formulated using the joint displacements as variables, and the transient response of the frame is obtained by a numerical integration of the equation of motion of the joint masses.

Several studies of a simple cubic frame are presented. The studies considered the order of magnitude of the nonlinear effects, and the influences of dissymmetry in the three-dimensional structure-load system. The results of the nonlinear studies show

that the consideration of geometry changes are significant only if relatively large displacements occur. Axial force effects on flexural stiffness may be significant for relatively common values of axial load. The investigations also indicate that dissymmetry in the distribution of mass, dissymmetry in stiffness, or a time lag in the motion of the supports may cause considerable variations in the response of the structure. Such variations in the response of the structure can be predicted only with a truly three-dimensional analysis.

The use of a damped dynamic solution to obtain a nonlinear static analysis is also demonstrated for a plane frame.

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by

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER I

INTRODUCTION

1.1 General

The advent of high speed digital computers has made possible considerable advances in the study of nonlinear behavior of structural frames. Generally speaking, there are two types of nonlinearities, material nonlinearities and geometrical nonlinearities. Much work has been done in the former class of problems, both in static analysis and dynamic analysis. However, practically all published works in this area are limited to plane frames. The influence of geometrical nonlinearities has been considered by a number of investigators. Their studies are, in general, related to the static behavior of structures.

The purpose of this thesis is to develop a method for the dynamic analysis of elastic space frames that takes into account the nonlinear effects of geometry changes and axial thrust. Because of the nonlinear nature of the problem, a linear static analysis is a necessary prelude to this study. In order to demonstrate the practicality of the method, a computer program is prepared to implement the analysis, and the results of certain numerical problems are examined.

1.2 Literature Review

In the area of nonlinear dynamic response of frame structures Wen and Janssen (23)* studied plane frames using a lumped mass and lumped flexibility model to investigate the effect of elastic-inelastic material properties. The method is applicable to frames with members having a general curvilinear moment-curvature relation. Clough, Benuska and Wilson (1) presented a procedure for computing the dynamic response of plane frames taking into account the nonlinear effects of inelastic deformations in yield hinges at the ends of the members. The procedure is more efficient in handling frames of larger size but is limited to members with bi-linear moment-curvature relation. It was used to study plane frames subjected to earthquake loadings and to evaluate the ductility requirements for various members. Damping was not included. A continuation of this work including damping effects was undertaken by Giberson (5).

Tezcan (19) presented a modal analysis solution for three dimensional elastic frames which included damping. By use of the acceleration response spectrum derived for the 1940 El Centro earthquake, he obtained certain "equivalent static loads" which were based on the natural periods of vibration of the structure. No nonlinear effects were considered in this study. In this and all of the above-mentioned papers masses were "lumped" at the joints or node points with no rotatory inertia effects considered.

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*Numbers refer to items listed in the List of References.

The area of static linear-elastic analysis of frames has been widely investigated. In particular, the following works have been referred to most frequently in the development of the linear static analysis employed for the present study. The network formulation of linear frames has been presented by Fenves and Branin (6). Their original work had the geometric properties of the members combined with the stiffness properties. Subsequently, Dimaggio and Spillers (3) and Livesley (13) independently presented versions of a network formulation wherein the geometric properties are combined with the incidence properties. This results in a much improved conditioning of the final simultaneous equations. The network formulation uses a single 6 x 6 member stiffness matrix instead of the more common 12 x 12 matrix of the direct stiffness formulation.

Gere and Weaver's book (4) contains a comprehensive treatment in the more common 12 x 12 stiffness matrix format. Weaver's book (21) is primarily a presentation of programming techniques and prepared programs for use in linear-elastic frame and truss analysis.

The nonlinear effects of large displacements and axial thrust in the stability of frames have recently been investigated by a number of researchers. Saafan and Brotton (17) presented a computer solution for plane frames which considers the effects of member chord rotations and axial thrust. Saafan (16) continued the study and included, in addition, the effects of member bowing on the apparent axial deformation. Jennings (11) derives stiffness coefficients for plane members, considering geometry changes, and studied a single bent plane frame dividing each member into several "submembers" to increase accuracy. Zarghamee and Shah (24) developed stiffness coefficients in three dimensions for space frames, that considered member chord rotations and bowing effects. They used an iterative solution method to solve for the displacements. Connor, Logcher and Chan (2) made a similar development in three dimensions and studied the load-deformation relations of a small three dimensional frame which had also been studied experimentally.

1.3 Scope and Outline of the Investigation

The investigation presented here will first develop a method of solution for the nonlinear analysis of dynamically loaded elastic space frames, and then incorporate it in a computer program written in Fortran for use on the C.D.C. 3600 Computer System at Michigan State University.

In the development of the analysis the following simplifying assumptions were made:

1) Frame members remain elastic throughout the solution and are of a uniform cross-section.
2) All joints are completely rigid.
3) All loads other than dead load are applied to the joints.
4) Torsional-flexural coupling may be neglected in the solution of the individual members.
5) The Bernoulli-Navier solution for a simple beam holds for relatively large end displacements.

6) The rotation angle of the member chord relative to the undisplaced position remains small enough that its sine may be approximated by the angle, its cosine may be approximated by unity, and the product of the sines of two such rotation angles is zero.

The development of the analysis is based on straightforward applications of principles of mechanics and, at places, a synthesis of known results. In the preparation of the computer program, emphasis was given to economy of computer time, ease of modification, and generality. The frame is first analyzed statically for the displacements and forces due to its dead loads, and these are used as initial conditions for the dynamic solution.

The dynamic solution is accomplished using the displacements of the joints as variables, and utilizing numerical integration to solve for the transient response of the joints to dynamic loading, either applied to the joints or through ground motion. A mass lumping procedure (22) that accounts for rotatory inertia is presented. Viscous damping is taken into account.

The nonlinear effects of geometry changes and axial thrust are accounted for in the determination of the frame reaction forces used in the equation of motion for each joint. The geometry effect enters into the analysis in two ways: a consideration of the rotation of the "chord" of the member (the straight line joining its two ends) and of the "bowing" of the member. The former affects the end rotations of the member and the axial strain, whereas the latter affects the axial strain. These considerations enter into the calculations of the member distortions. The effect of axial force (axial force is defined to be the end force acting in the direction of the chord) simply modifies certain stiffness coefficients of the member according to beam-column behavior.

The investigation also includes various studies made with the computer program developed. A simple cubic frame is studied under dynamic loading to investigate the nonlinear effects. The capability to treat three-dimensional systems is utilized to study the effects of dissymmetry in a similar cubic frame, and to investigate the response to three-dimensional ground motion from a recorded earthquake.

1.4 Definitions of Member Incidence and Coordinate Systems

For the purposes of the analysis, the members will be assumed to be directed (arrows on the individual members will be used in sketches) between the two joints at their ends. The direction or orientation may be arbitrarily assigned initially. However, once assigned, it must be fixed throughout the analysis. The member then is said to be directed from the joint at its "initial" end to the joint at its "final" end. The member is "positively" incident to the joint at its initial end and "negatively" incident to the joint at its final end. Thus, in Figure 1.1 the member K is positively incident to joint I and negatively incident to joint J.

Three coordinate systems are used in the analysis. (see Figure 1.1):

1) Structure Global Coordinate System:

This system consists of a single set of cartesian axes with its origin at any convenient fixed point. The system is oriented with the X and Z axes horizontal and the Y axis vertical. This system is used to specify the initial location (three dimensional vectors) of the joints.

2) Joint Global Coordinate System:

This system consists of one set of cartesian axes for each joint with the origin placed at the initial location of the joint with its axes parallel to the structure global axes. This system is used to describe the displacements of the joint and forces acting on the joint, both of which are represented by vectors with six elements. For displacements, the first three elements in each vector represent the translational displacements in the direction of the three coordinate axes taken in order, and the second three elements represent rotations about each of the ordered axes, respectively. The force vectors are made up of forces corresponding to these displacements.

3) Member Coordinate System:

This consists of two sets of rectangular cartesian axes per member, located with the origins at the location of each end of the unloaded member. Both sets of axes will be oriented with the first axis along the centroidal axis of the member directed from the initial end to the final end of the member. The second axis is directed along one of the principal axes of inertia and the third axis is directed along the other principal axis of inertia. These axes are used to describe the displacements and forces acting on the two ends of the member. As in the joint global system, the displacements and forces are described by six element vectors, with the first three elements being translations and corresponding forces in the directions of the three axes, and the second three elements being rotations and corresponding moments about the three axes.

1.5 Notation

The notation shown below has been used in this report:

A_x = the cross-sectional area of a beam;

A_y = the equivalent shear area for shear stiffness for shear force parallel to the member y axis;

A_z = the equivalent shear area for shear stiffness for shear force parallel to the member z axis;

$\{a\}$ = an arbitrary 6 element vector;

$[C]$ = the viscous damping matrix;

$\{d\}$ = member relative end displacements;

$\{DF\}$ = the vector of forces representing the damping force on each joint;

$\{e\}$ = vector of corrected relative member end distortions;

E = modulus of elasticity;

$\{F\}$ = vector of end forces;

G = shear modulus;

g = shear stiffness coefficient;

I_x = the member torsional constant;

I_y = the member moment of inertia about the member y axis;

I_z = the member moment of inertia about the member z axis;

[IRM] = intermediate member rotation matrix;

[IT] = inertia tensor;

J = torsional constant;

[K] = joint stiffness matrix of the entire structure;

$[K_m]$ = member stiffness matrix;

k = member stiffness coefficient;

L = member length;

M = bending moment;

M_i = bending moment on end i of member;

[M] = lumped mass matrix of the entire structure;

m = mass;

P = axial member load;

{P} = the vector of dynamic forces acting on the joints or on a particular joint if subscripted;

{RE} = the hyper vector of forces representing the frame reaction on the frame joints;

[RM] = member rotation matrix;

{ST} = the vector of static load forces acting on the joints;

S = shear force;

t = time;

Δt = time increment;

$[T]$ = force equilibrium matrix;

$\{u\}$ = the vector of displacements in member coordinates;

$\{x\}$ = the vector of displacements in joint global coordinates;

Δ_1 = angle defining relative rotation of member chord in the x-y plane (see Figure 2.3);

Δ_2 = same as above in x-z plane;

λ = damping constant;

$\{\tau\}$ = vector of end forces at one end of a member;

ρ = mass unit density;

$\Delta(\ )$ = a finite, but small increment of the variable preceded;

$_I\{\ \}$ = a vector quantity associated with the initial end of a member;

$_F\{\ \}$ = a vector quantity associated with the final end of a member;

$(\dot{\ })$ = $\frac{d\,(\ )}{dt}$;

$(\ddot{\ })$ = $\frac{d^2\,(\ )}{dt^2}$.

CHAPTER II

FRAME JOINT REACTIONS

2.1 General

The equation of motion for a space frame may be written as

$$[M]\{\ddot{x}\} + \{DF\} + \{RE\} = \{P\} \qquad .....(2-1)$$

in which all quantities are in global coordinates: $[M]$ is the mass matrix; $\{\ddot{x}\}$, the acceleration vector; $\{DF\}$, the damping force vector; $\{RE\}$, the "joint reactions"; and $\{P\}$ the external loads on the joints. The formulation of $[M]$ and $\{DF\}$, and the solution of the equation will be discussed in the next chapter. In this chapter the joint reactions $\{RE\}$ are derived.

In the linear static case $\{RE\} = \{P\} = [K]\{x\}$, where $[K]$ is the linear joint stiffness matrix. Since a linear static solution is a necessary prelude (to obtain the initial stresses and displacements) to the dynamic analysis and also to serve as background material for the derivation of $\{RE\}$, the linear static solution used in this analysis is outlined in the following section.

2.2 Linear Solution Used in Static Analysis

The linear static analysis is formulated using a stiffness solution in a manner outlined in the paper by Dimaggio and Spillers (3). Using this format each member's stiffness properties are represented by a 6×6 stiffness matrix containing the direct stiffness coefficients at the initial end of the member instead of the more common 12×12 matrix including the direct stiffness coefficients at both ends as well as cross coefficients. The overall joint stiffness matrix of the structure is assembled from these member stiffness matrices using what Dimaggio and Spillers call a "Modified Incidence Matrix" that includes the necessary geometry and member incidence information. To save computer storage space and computation time several modifications were made.

2.2.1 Basic Member Stiffness Matrix.--The "Basic Member Stiffness Matrix" $[K_m]$ is a 6×6 stiffness matrix containing the direct stiffness coefficients for the initial end of a member expressed in member coordinates. This matrix is symmetric and has 8 independent non-zero elements

$$[K_m] = \begin{bmatrix} k_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 & 0 & k_7 \\ 0 & 0 & k_3 & 0 & k_8 & 0 \\ 0 & 0 & 0 & k_4 & 0 & 0 \\ 0 & 0 & k_8 & 0 & k_5 & 0 \\ 0 & k_7 & 0 & 0 & 0 & k_6 \end{bmatrix} \qquad \text{.....(2-2)}$$

The non-zero elements are defined as:

$$k_1 = \frac{EA_x}{L}; \quad k_2 = \frac{12EI_z}{L^3}\,\frac{1}{1+2g_z};$$

$$k_3 = \frac{12EI_y}{L^3}\,\frac{1}{1+2g_y}; \quad k_4 = \frac{GJ}{L};$$

$$k_5 = \frac{4EI_y}{L}\,\frac{2+g_y}{2+4g_y}; \quad k_6 = \frac{4EI_z}{L}\,\frac{2+g_z}{2+4g_z}; \qquad \text{.....(2-3)}$$

$$k_7 = \frac{6EI_z}{L^2}\,\frac{1}{1+2g_z}; \quad k_8 = -\,\frac{6EI_y}{L^2}\,\frac{1}{1+2g_y};$$

where E and G are the modulus of elasticity and shear modulus, L is the member length, J is the member torsional constant, and I_y and I_z are the moments of inertia about the two principal axes of bending. The shear stiffness coefficients g_y and g_z are defined by

$$g_y = \frac{6EI_y}{GA_y L^2}$$

$$g_z = \frac{6EI_z}{GA_z L^2} \qquad \text{.....(2-4)}$$

where A_z and A_y are the appropriate shear stiffness areas for shear force along the z and y axes, respectively. For example, for I beams A_y will be taken as equal to the area of the web. For the purpose of this investigation the torsional constant J, of the I sections which were used, was computed using the approximation for thin rectangular sections

$$J = \sum_{i=1}^{3} \frac{1}{3}\, b\, h^3 \qquad \text{.....(2-5)}$$

where b and h are the long and short dimensions of the ith thin rectangular section.

2.2.2 Member Rotation Matrices.--The member rotation matrices are the transformation matrices that rotate a 6-element vector quantity (end force or end displacement) expressed in joint global coordinates to the member coordinates of each member.

Since the joint global coordinates for all joints have a common orientation, a single rotation matrix is sufficient for each member. In vector form if $\{a'\}$ is a vector expressed in member coordinates and $\{a\}$ is the same vector in global coordinates, $\{a'\} = [RM]\{a\}$.

Since this transformation is between two orthogonal coordinate systems it follows that the inverse transformation from global to member coordinates will be accomplished by the transpose of the transformation matrix above. That is $\{a\} = [RM]^t\{a'\}$.

The matrix [RM] is made up of the direction cosines between the axes of the two coordinate systems

$$[RM] = \left[\begin{array}{ccc|ccc} s_{11} & s_{12} & s_{13} & & & \\ s_{21} & s_{22} & s_{23} & & 0 & \\ s_{31} & s_{32} & s_{33} & & & \\ \hline & & & s_{11} & s_{12} & s_{13} \\ & 0 & & s_{21} & s_{22} & s_{23} \\ & & & s_{31} & s_{32} & s_{33} \end{array}\right] \qquad \ldots\ldots(2\text{-}6)$$

where s_{ij} denotes the cosine of the angle between the ith member coordinate axis and the jth joint global coordinate axis.

2.2.3 Force Equilibrium Matrix.--The member force equilibrium matrix is denoted by [T] and is defined by the relationship:

$$\{\tau\}_{\text{Final End}} = [T]\{\tau\}_{\text{Initial End}} \qquad \ldots\ldots(2\text{-}7)$$

where $\{\tau\}$ is the vector of member end forces. If the member end forces at both ends are expressed in member coordinates

$$[T] = \left[\begin{array}{ccc|ccc} -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & -L & 0 & -1 & 0 \\ 0 & L & 0 & 0 & 0 & -1 \end{array}\right] \qquad \text{.....(2-8)}$$

This follows from the requirements of equilibrium and under the assumption that no load is applied between the end points of the member.

2.2.4 Relative End Displacements.--The displacement of an end-loaded member may be described by its end displacements in member coordinates, ${}_I\{u\}$ and ${}_F\{u\}$, where the presubscript I indicates initial end and F indicates final end. These displacement vectors are related to the joint displacements:

$$\{u\} = [RM]^t\{x\} \qquad \text{.....(2-9)}$$

in which $\{x\}$ is, in global coordinates, the displacement of the joint corresponding to the member end.

Let ${}_I\{d\}$ be the displacement vector of the initial end of the member relative to the final end; i.e.,

$${}_I\{d\} = {}_I\{u\} + [T]^t \; {}_F\{u\} \qquad \text{.....(2-10)}$$

It follows that

$$\{\tau\}_{\text{Initial End}} = [K_m] \; {}_I\{d\} \qquad \text{.....(2.11)}$$

2.2.5 Joint Stiffness Matrix.--By use of the previously defined quantities the structural reactions at the joints may be expressed in terms of the displacements of the joints. The equilibrium equation for a statically loaded frame may therefore be expressed in matrix form as

$$[K]\ \{x\} = \{ST\} \qquad \text{.....(2-12)}$$

where $\{ST\}$ is the vector of joint static loads in joint global coordinates. The matrix $[K]$ so defined will be called the "joint stiffness matrix", or the "overall joint stiffness matrix".

This joint stiffness matrix has certain advantageous properties, namely, symmetry, sparseness and bandedness.

In this solution method the joint stiffness matrix will be considered as partitioned into 6 x 6 submatrices with each submatrix corresponding to a given joint and the 6 dimensions of the submatrix corresponding to the 6 coordinates of that joint. In expanded form $[K]\ \{x\} = \{ST\}$ then becomes:

$$\begin{bmatrix} \begin{bmatrix}6 \times 6\\ \text{Sub-}\\ \text{matrix}\end{bmatrix} & \begin{bmatrix}6 \times 6\\ \text{Sub-}\\ \text{matrix}\end{bmatrix} & \cdots \\ \begin{bmatrix}6 \times 6\\ \text{Sub-}\\ \text{matrix}\end{bmatrix} & \begin{bmatrix}6 \times 6\\ \text{Sub-}\\ \text{matrix}\end{bmatrix} & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} \text{Jt.1} \\ \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} \text{Jt.2} \\ \vdots \end{bmatrix} = \begin{bmatrix} \begin{bmatrix}ST_1\\ ST_2\\ ST_3\\ ST_4\\ ST_5\\ ST_6\end{bmatrix} \text{Jt.1} \\ \begin{bmatrix}ST_1\\ ST_2\\ ST_3\\ ST_4\\ ST_5\\ ST_6\end{bmatrix} \text{Jt.2} \\ \vdots \end{bmatrix}$$

.....(2-13)

where all quantities are expressed in joint global coordinates. Each row of the partitioned matrix equation expresses an equilibrium equation for a particular joint of the structure being considered.

Analyzing a row of the partitioned matrix [K] it can be seen that the diagonal term must reflect the direct action of all members incident to the joint corresponding to that row. Furthermore, there must be non-zero submatrices in each column corresponding to the "other" end for each member incident to that joint. All other submatrices in that row will be null.

To conserve computer storage the computer program developed for this study generates and stores only the non-zero 6 x 6 submatrices in the joint stiffness matrix, and only those that appear in the upper triangular portion of [K]. It is now necessary to compute the non-zero submatrices. Four possibilities exist:

1) A direct contribution from a member positively incident to the joint being considered, which is reflected in the 6 x 6 submatrix on the diagonal and is equal to $[RM]^t\ [K_m]\ [RM]$.
2) A direct contribution from a member negatively incident to the joint being considered which is reflected in the 6 x 6 submatrix on the diagonal and is equal to $[RM]^t\ [T]\ [K_m]\ [T]^t\ [RM]$.
3) An indirect contribution due to the displacements of the joint at the other end of a member positively incident to the joint being considered. This corresponds to an off-diagonal submatrix in the column

corresponding to the other joint, but in the row corresponding to the joint being considered, and is equal to $[RM]^t\ [K_m]\ [T]^t\ [RM]$.

4) The same contribution as in 3) from a member negatively incident is equal to $[RM]^t\ [T]\ [K_m]\ [RM]$.

2.2.6 Solution of the Equilibrium Equation.--The equilibrium equation $[K]\ \{x\} = \{ST\}$ is solved for the unknown displacements using the Gauss Elimination Method (18). The method was used in this investigation with the 6 x 6 submatrices as outlined in the previous section as elements, and considering the symmetry of the joint stiffness matrix. The basic method is well documented in most numerical analysis texts. The reduction equation is

$$[A]_{ij\ \text{reduced}} = [A]_{ij\ \text{present}} - [A]_{ik}[A]_{kk}^{-1}[A]_{kj} \quad .....(2\text{-}14)$$

where the subscript k corresponds to the diagonal pivot element (actually a 6 x 6 matrix) currently being used.

Because of symmetry $[A]_{ik} = [A]_{ki}^t$. The submatrix $[A]_{ki}$ is in the pivot row and is in the stored portion of [K]. Also, any of the reduced matrices will be symmetric since the joint stiffness matrix is symmetric. These two facts allow treating only the submatrices in the upper triagular portion of [K], which coincides with the treatment in the generation of [K]. Back substitution is accomplished in the usual fashion.

2.2.7 Member End Forces.--Member end forces are obtained from the computed displacements using the basic equations:

$$ {}_{I}\{\tau\} = [K_m]\ {}_{I}\{d\} = [K_m]\ {}_{I}\{u\} + [K_m]\ [T]^t\ {}_{F}\{u\} = $$

$$ [K_m]\ [RM]\ {}_{I}\{x\} + [K_m]\ [T]^t\ [RM]\ {}_{F}\{x\} \qquad .....(2\text{-}15) $$

and

$$ {}_{F}\{\tau\} = [T]\ {}_{I}\{\tau\}, \qquad .....(2\text{-}16) $$

where ${}_{I}\{x\}$ and ${}_{F}\{x\}$ are the displacements in joint coordinates at the initial end of the member and the final end, respectively.

2.3 Nonlinear Joint Reactions

The determination of the frame joint reactions $\{RE\}$, in the nonlinear dynamic analysis is accomplished by solving each member for its end forces corresponding to the current specified joint displacements and then, for each joint, summing the end forces for all the members incident to that joint. The basic problem then becomes the solution of a member with specified end displacements for its end forces. As in the linear solution the relative end displacements are first obtained and then the appropriate stiffness coefficients are applied to obtain the end forces. The nonlinear effects of geometry changes are accounted for in the determination of the relative end displacements, while the effects of axial force on flexural stiffness are reflected in the stiffness coefficients.

2.3.1 Determination of Relative End Displacements.--Figure 2.1 shows a member initially undistorted with the member axis coincident with the x axis. After distortion, the initial end will have undergone a displacement ${}_{I}\{u\}$ and the final end ${}_{F}\{u\}$. As in the linear case, these member end displacements are obtained from the appropriate joint displacements by a linear rotation as shown in Equation (2-9). The translations of the initial end of the distorted member relative to the final end are:

$$d_1 = {}_I u_1 - {}_F u_1$$

$$d_2 = {}_I u_2 - {}_F u_2 \qquad \text{.....(2-17)}$$

$$d_3 = {}_I u_3 - {}_F u_3$$

The primary nonlinear effect of geometry change is caused by the rotation of the "chord" of the distorted member relative to its initially undistorted configuration. The "chord" refers to the straight line jointing the end points of the member. Referring to Figure 2.1, chord rotations in the x-y and x-z planes are approximated by

$$\Delta_1 \cong \frac{d_2}{L}$$

$$\Delta_2 \cong \frac{d_3}{L} \qquad \text{.....(2-18)}$$

The relative displacements in the nonlinear solution are more conveniently expressed relative to the chord of the deformed member. The relative axial displacement, the axial twist, and the rotations at each end of the members, completely define the member distortions, e_i:

$$e_1 = d_1 - \frac{d_2^2}{2L} - \frac{d_3^2}{2L} - \frac{L}{30}(2e_3^2 - e_3 e_5 - 2e_5^2) - \frac{L}{30}$$
$$(2e_4^2 - e_4 e_6 - 2e_6^2) = \text{(axial shortening)}$$

$e_2 = {}_Iu_4 - {}_Fu_4$ = (relative twist about x axis)

$e_3 = {}_Iu_5 - \Delta_2$ = (rotation about y axis at initial end)

$e_4 = {}_Iu_6 + \Delta_1$ = (rotation about z axis at initial end) .....(2-19)

$e_5 = {}_Fu_5 - \Delta_2$ = (rotation about y axis at final end)

$e_6 = {}_Fu_6 + \Delta_1$ = (rotation about z axis at final end)

The second and third terms in the expression for axial shortening, e_1, represent the effects of chord rotation and the last two terms reflect the effect of apparent shortening due to "bowing". The terms due to chord rotation may be developed as follows. The true shortening of the chord distance after displacement can be represented by

$$\epsilon_{\text{chord rot.}} = L - [(L-d_1)^2 + d_2^2 + d_3^2]^{\frac{1}{2}} \qquad \text{.....(2-20)}$$

Expanding the second term in a binomial expansion and dropping terms over second order

$$\epsilon_{\text{chord rot.}} = d_1 - \frac{d_1^2}{2L} - \frac{d_2^2}{2L} - \frac{d_3^2}{2L} + - - \qquad \text{.....(2-21)}$$

For most cases $d_1^2 \ll d_2^2$ and $d_1^2 \ll d_3^2$, the term $\frac{d_1^2}{2L}$ is consequently ignored, thus

$$\epsilon_{\text{chord rot.}} = d_1 - \frac{d_2^2}{2L} - \frac{d_3^2}{2L} \qquad \text{.....(2-22)}$$

The correction terms for apparent member shortening due to bowing are shown in Appendix 1, for the case where the effects of axial force are considered and also where they are not considered.

The terms used in this investigation did not include the effects of axial force in the correction for axial shortening due to bowing. The special case shown in Figure 2.2 was computed to determine the magnitudes of the correction terms. The axial load used was equal to the Euler load for the member considered. The neglect of axial force in the bowing correction causes an error of approximately 12% in the total correction terms. The axial forces found in practical situations are normally considerably less than the Euler load to preclude buckling of the member. Consequently, the neglect of axial force effects in the bowing correction was considered justified.

2.3.2 Determination of Member End Forces.--In terms of the relative displacements e_i given above the member end forces at the initial end $_I\{F\}$ are computed using the following member stiffness relations:

$$
\begin{aligned}
{}_IF_1 &= \frac{EA}{L} e_1 = \text{(axial force)} \\
{}_IF_2 &= \frac{EI_z}{L^2} C_{3z} (e_4+e_6) = \text{(shear force)} \\
{}_IF_3 &= \frac{EI_y}{L^2} C_{3y} (e_3+e_5) = \text{(shear force)} \\
{}_IF_4 &= \frac{GJ}{L} e_2 = \text{(twisting moment)} \\
{}_IF_5 &= \frac{EI_y}{L} C_{1y}e_3 + \frac{EI_y}{L} C_{2y}e_5 = \text{(bending moment)} \\
{}_IF_6 &= \frac{EI_z}{L} C_{1z}e_4 + \frac{EI_z}{L} C_{2z}e_6 = \text{(bending moment)}
\end{aligned}
\qquad \text{.....(2-23)}
$$

The coefficients used in F_1 and F_4 are the usual axial and twisting stiffnesses. The derivation of the coefficients used in F_2, F_3, F_5 and F_6 is outlined in Appendix 1. Shear stiffness effects are not included.

In terms of the stiffness coefficients defined previously, these forces may be written as:

$$
\begin{aligned}
{}_IF_1 &= k_1 e_1 \\
{}_IF_2 &= k_7 \frac{C_{3z}}{6} (e_4 + e_6) \\
{}_IF_3 &= k_8 \frac{C_{3y}}{6} (e_3 + e_5) \\
{}_IF_4 &= k_4 e_2 \\
{}_IF_5 &= k_5 \frac{C_{1y}}{4} e_3 + k_9 \frac{C_{2y}}{2} e_5 \\
{}_IF_6 &= k_6 \frac{C_{1z}}{4} e_4 + k_{10} \frac{C_{2z}}{2} e_6
\end{aligned}
\qquad \text{.....(2-24)}
$$

where k_1, k_4, k_5, k_6, k_7, k_8 are as previously defined in Section 2.1.1 and

$$
\begin{aligned}
k_9 &= \frac{2EI_y}{L} \frac{1-g_y}{1-2g_y} \\
k_{10} &= \frac{2EI_z}{L} \frac{1-g_z}{1-2g_z}
\end{aligned}
\qquad \text{.....(2-25)}
$$

The coefficients C_{1y}, C_{2y}, C_{3y} and C_{1z}, C_{2z}, C_{3z} represent the "beam-column effects", and their derivation is outlined in Appendix 1.

The first subscript corresponds to the subscript used in the Appendix, the second indicates the appropriate plane.

Since these coefficients are functions of the axial force on the member, the member end forces cannot be computed until the axial force is known. Although an iterative procedure may be feasible, in the present investigation the axial force from the forces obtained in the previous time-step solution is used to obtain the C coefficients. That is, the axial force used is one time increment behind its true value. The type of numerical integration used requires extremely small time increments and the variation in the axial force was found to be quite small. A maximum variation of approximately 3% of the Euler Load was noted for one time increment in a member experiencing axial loads from 0 to 0.7 of the Euler load.

The forces at the final end of the member are now determined by equilibrium:

$$_{F}\{F\} = [T]\ _{I}\{F\} \qquad \text{.....(2-26)}$$

2.3.3 Intermediate Rotation Matrix.--Considering Figure 2.3, it can be seen that due to the chord rotation the orientation of the member end forces $\{F\}$, solved for as outlined in Section 2.3.2, is no longer the same as the original member coordinate axes which correspond to the undisplaced member orientation. For ease in the programmed computation, all member forces are given in the original member coordinate system. Therefore, the member forces $\{F\}$ must be transferred to that coordinate system. This is accomplished by the introduction of an intermediate rotation matrix [IRM] such that:

$$\{\tau\} = [IRM]\{F\} \qquad \text{.....(2-27)}$$

To develop [IRM] the effect of the twisting of the member will be neglected, the rotation angles will be assumed to be small enough so that $\cos \Delta \cong 1$, $\sin \Delta \cong \Delta$ and $\sin \Delta_1 \times \sin \Delta_2 \cong 0$. With these assumptions [IRM] appears as

$$\left[\begin{array}{ccc|ccc} 1 & \Delta_1 & \Delta_2 & & & \\ -\Delta_1 & 1 & 0 & & 0 & \\ -\Delta_2 & 0 & 1 & & & \\ \hline & & & 1 & \Delta_1 & \Delta_2 \\ & 0 & & -\Delta_1 & 1 & 0 \\ & & & -\Delta_2 & 0 & 1 \end{array}\right].$$

2.3.4 <u>Rotation to Joint Global Coordinates</u>.--The solution for the member forces is completed by rotating the member end forces in member coordinates, $\{\tau\}$, to joint global coordinates using the transpose of the appropriate rotation matrix as outlined in Section 2.2.2. After that, as mentioned previously, at a given joint, the forces of the incident members are summed to yield the corresponding joint reaction.

CHAPTER III

DYNAMIC ANALYSIS

3.1 General

The equation of motion (Equation (2-1)) was introduced in Section 2.1 Chapter II was devoted to the solution for the joint reactions {RE} in that equation. This chapter will discuss the formation of the mass matrix [M] and the solution for the damping force {DF}, and will present the procedure used to solve the equation of motion to obtain the response of the structure.

3.2 Formation of Mass Matrix

The procedure used to form the mass matrix was developed to permit a "lumped" mass system that accounts for rotatory inertia. The variables used in the dynamic solution are the displacements of the joints, hence the procedure lumps the mass of the frame at the joints. It is assumed that associated with each joint is a rigid body made up of one-half of the length of all members incident to the joint as shown in Figure 3.1. This typical joint mass has a "mass center" that is generally not located at the joint. The problem then is the determination of the properties of the rigid body, or joint mass. It is convenient to determine the properties about the joint mass center and to transform these to the coordinates of the joint, in which the equation of motion will be treated.

Initially, the mass matrix for each ½ member is computed. For descriptive purposes each ½ member will be called a "branch". The branch mass m_B for branch i is determined by

$$m_{Bi} = (\rho\ A_x + DM)\ \frac{L}{2} \qquad \text{.....(3-1)}$$

where ρ is the density and DM is the superimposed dead load mass per unit length. The branch inertia tensor about the branch mass center, in member coordinates, [ITB], is determined next. Since prismatic members are assumed, the member axes will also be the principal axes of inertia.

$$[ITB]_i = \begin{bmatrix} A & H & N \\ H & B & F \\ N & F & C \end{bmatrix} \qquad \text{.....(3-2)}$$

and

$$A = \int_{vol.} \rho(z^2+y^2)\,dV = \frac{\rho L}{2}\,(I_y+I_z)$$

$$B = \int_{vol.} \rho(z^2+x^2)\,dV + \int_{len.} DM\ x^2\ dL$$

$$= \frac{L^3}{96}\,(\rho A_x+DM) + \frac{\rho L}{2}\,I_y \qquad \text{.....(3-3)}$$

$$C = \int_{vol.} \rho(y^2+x^2)\,dV + \int_{len.} DM\ x^2\ dL$$

$$= \frac{L^3}{96}\,(\rho A_x+DM) + \frac{\rho L}{2}\,I_z$$

$$F = N = H = 0$$

where it has been assumed that the superimposed dead load is concentrated as a line along the longitudinal member axis, and all cross-sectional properties are for member i. The products of inertia F, N and H are zero since the member axes are principal axes.

The joint mass m, is computed next

$$m = \sum_{i=1}^{k} m_{Bi} \qquad \text{.....(3-4)}$$

where k is the number of branches incident to the joint being considered.

The global coordinates of the joint mass center are computed by taking moments about the joint global axes

$$X = \frac{\sum_{i=1}^{k} m_{Bi} X_i}{m}$$

$$Y = \frac{\sum_{i=1}^{k} m_{Bi} Y_i}{m} \qquad \text{.....(3-5)}$$

$$Z = \frac{\sum_{i=1}^{k} m_{Bi} Z_i}{m}$$

where $(X_i, Y_i, Z_i) = \overline{V}_i$ is the position vector of the ith branch mass center in joint global coordinates. $\overline{V}_i$ is determined by

$$\overline{V}_i = \tfrac{1}{2} (\overline{F}_i - \overline{I}_i) \qquad \text{.....(3-6)}$$

where $\overline{F}_i$ and $\overline{I}_i$ are the position vectors of the final and intial ends of member i in structure global coordinates.

The elements of the individual branch inertia tensors are transferred to the joint mass center using the parallel axis theorem

$$([ITB]_i)_{J.M.C.} = \begin{bmatrix} a & h & n \\ h & b & f \\ n & f & c \end{bmatrix} \quad .....(3\text{-}7)$$

and

$$\begin{aligned} a &= A + m(y^2+z^2) \\ b &= B + m(x^2+z^2) \\ c &= C + m(y^2+x^2) \\ f &= -myz \\ n &= -mxz \\ h &= -mxy \end{aligned} \quad .....(3\text{-}8)$$

where $(x,y,z) = \bar{U}_i$ is the position vector of the joint mass center relative to the ith branch mass center in member coordinates, and is determined by

$$\bar{U}_i = [RMT]_i \ (\bar{V}_i - \bar{W}) \quad .....(3\text{-}9)$$

where $[RMT]_i$ is the three dimensional rotation matrix for member i and $\bar{W} = (X,Y,Z)$ is the position vector of the joint mass center in joint global coordinates.

The branch inertia tensors expressed at the mass center are then rotated to joint global coordinates and summed to obtain the joint inertia tensor at the mass center [IT].

$$[IT] = \sum_{i=1}^{k} [RMT]_i^t \, ([ITB]_i)_{J.M.C.} \, [RMT]_i \qquad \text{.....(3-10)}$$

The joint mass matrix at the mass center, $[M]_{J.M.C.}$ can then be assembled

$$[M]_{J.M.C.} = \left[\begin{array}{ccc|c} m & o & o & \\ o & m & o & 0 \\ o & o & m & \\ \hline & 0 & & [IT] \end{array}\right] \qquad \text{.....(3-11)}$$

and is then inverted.

The inverted joint mass matrix at the joint mass center is now transferred to the joint,

$$[M]_J^{-1} = [H]^t \, [M]_{J.M.C.}^{-1} \, [H] \qquad \text{.....(3-12)}$$

where [H] is a force transformation which is explained as follows.

The transformation matrix [H] is defined as a matrix such that [H] transforms a force vector acting at the joint to an equivalant vector acting at the joint mass center in similarly oriented coordinates, that is,

$$\{F\}_{J.M.C.} = [H] \, \{F\}_J \qquad \text{.....(3-13)}$$

If the space coordinates of the joint mass center in joint global coordinates are X, Y, Z, then

$$[H] = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & & & \\ 0 & 1 & 0 & & 0 & \\ 0 & 0 & 1 & & & \\ \hline 0 & Z & -Y & 1 & 0 & 0 \\ -Z & 0 & X & 0 & 1 & 0 \\ Y & -X & 0 & 0 & 0 & 1 \end{array}\right] \qquad \text{.....(3-14)}$$

The relationship between displacements corresponding to these force sets then is

$$\{x\}_J = [H]^t \{x\}_{J.M.C.} \qquad \text{.....(3-15)}$$

It follows that

$$\{\ddot{x}\}_J = [H]^t \{\ddot{x}\}_{J.M.C.} \qquad \text{.....(3-16)}$$

The equation of motion of the joint about the joint mass center may be rearranged to form

$$[M]_{J.M.C.}\{\ddot{x}\}_{J.M.C.} = \{P\}_{J.M.C.} - \{RE\}_{J.M.C.} - \{DF\}_{J.M.C.}$$

$$= \{R\}_{J.M.C.} \qquad \text{.....(3-17)}$$

where $\{R\}$ is a resultant force vector. Then

$$\{\ddot{x}\}_{J.M.C.} = [M]^{-1}_{J.M.C.}\{R\}_{J.M.C.} \qquad \text{.....(3-18)}$$

Substituting the relationships in Equations (3-13) and (3-16)

$$\{\ddot{x}\}_J = [H]^t \, [M]^{-1}_{J.M.C.} \, [H] \, \{R\}_J \qquad \text{.....(3-19)}$$

From which it follows that

$$[M]_J^{-1} = [H]^t \, [M]_{J.M.C.}^{-1} \, [H] \qquad .....(3\text{-}20)$$

The mass matrix of the structure [M] is defined as a diagonal matrix containing the individual 6 x 6 joint mass matrices, computed as outlined above, as the diagonal elements.

3.3 Damping

The exact form of damping in most structures is practically unknown. However, damping forces are generally relatively small so that it is not essential that they be represented precisely in a mathematical analysis. For these reasons a simplified form of damping is assumed. A viscous (resistive force proportional to velocity) type of damping is assumed in this investigation, and the damping coefficient matrix, [C] is assumed to be proportional to the mass matrix. Writing $[C] = \lambda[M]$, the equation of motion becomes

$$[M]\{\ddot{x}\} + \lambda[M]\{\dot{x}\} + \{RE\} = \{P\} \qquad .....(3\text{-}21)$$

3.4 Numerical Solution Procedure Used In Dynamic Analysis

3.4.1 Solution Procedure.--The procedure used in the dynamic solution will be described in this section. Assume that the state of the frame is known at $t = t_1$. This includes the displacements, velocities and accelerations of the joints, and the internal forces of the members corresponding to the displacements. It is desired to determine the state of the frame for $t = t_1 + \Delta t$ where Δt is one time increment. Knowing this, the same procedure can be used to solve for the state of the frame at time $t = t_1 + 2\Delta t$, and so on.

The solution process for the typical step from t_1 to $t_1+\Delta t$ is outlined as follows:

1) Levy's numerical integration formula,

$$x_{t_1+\Delta t} = 2x_{t_1} - x_{t_1-\Delta t} + (\Delta t)^2 \ddot{x}_{t_1} \qquad .....(3\text{-}22)$$

is used to obtain the displacements at $t = t_1+\Delta t$ where the subscripts refer to the time at which the variables are to be evaluated.

2) The frame joint reactions $\{RE\}$ are determined for $t = t_1+\Delta t$ from the displacements computed in step 1, using the procedure outlined in Section 2.3.

3) The damping force is computed using the damping matrix given in Section 3.3. In order to save computation time, by avoiding iteration, which would otherwise be necessary, the velocities at $t = t_1$ are used to compute the damping force at $t_1 + \Delta t$; i.e.,

$$\{DF\}_{t_1+\Delta t} = \lambda[M]\,\{\dot{x}\}_{t_1} \qquad .....(3\text{-}23)$$

This was thought justifiable because of the approximate nature of the damping representation and the fact that velocities do not change a great deal in one time increment.

4) The equation of motion is solved for the accelerations at $t_1+\Delta t$:

$$\{\ddot{x}\}_{t_1+\Delta t} = [M]^{-1}\left[\{P\}_{t_1+\Delta t} - \{RE\}_{t_1+\Delta t} - \{DF\}_{t_1+\Delta t}\right] \qquad .....(3\text{-}24)$$

5) The velocities are computed using the Euler-Fox numerical integration formula

$$\dot{x}_{t_1+\Delta t} = \dot{x}_{t_1} + \frac{\Delta t}{2}[\ddot{x}_{t_1} + \ddot{x}_{t_1+\Delta t}] \qquad \text{.....(3-25)}$$

These 5 steps then complete the solution for all the variables at time = $t_1+\Delta t$, and a similar solution process may be repeated for the next advance into the time domain.

3.4.2 Time Increment Used in Numerical Integration.--It is well known that in the solution of problems of structural dynamics by numerical integration the time increment must be less than a certain fraction of the smallest period of vibration in order to assure stability of the integration procedure. For Levy's formula, this fraction is $1/\pi$ (20). Strictly speaking, this applies only to linear problems.

In order to obtain the smallest period, a solution of the eigenvalue problem would be necessary. This would be time consuming and a quick approximate estimate is desirable. To estimate the smallest period of a frame the following approximation is suggested. The translational mass is estimated for all free joints and the greatest axial stiffness for any member incident to each joint is used in the expression for the "period", T_a:

$$T_a = 2\pi\sqrt{\frac{\text{mass}}{\text{axial stiffness}}} \qquad \text{.....(3-26)}$$

The smallest T_a for all members is used as an estimation of the smallest natural period, T_s. This approximation was used for the stiffer cubic frame shown in Figure 4.1 to yield T_s = 0.0087 seconds versus 0.0052 seconds obtained from a modal analysis program. The approximation yields 0.0093 seconds for the flexible frame shown in Figure 4.1 versus 0.0041 seconds from the modal analysis. While in error by a factor of approximately 2, these approximations would still give a usable first estimate of the integration interval, if a generous factor of 0.1 or less is used in lieu of the coefficient $1/\pi$ mentioned earlier.

3.4.3 Change of Variables for Ground Motion Problems.--If the input of the problem is ground or support motion (assumed translational motion only), it is convenient to define the joint displacement $\{x\}$ as relative to that of the ground $\{x_o\}$. The absolute joint displacement is then $\{x\} + \{x_o\}$. Therefore, the equation of motion becomes

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + \{RE\} = \{P\} - [M]\{\ddot{x}_o\} \qquad .....(3\text{-}27)$$

in which $\{\ddot{x}_o\}$ is the ground acceleration. It is seen that $-[M]\{\ddot{x}_o\}$ plays the same role as an external loading. In this equation the damping force is assumed to be proportional to the relative velocity $\{\dot{x}\}$ and $[C] = \lambda[M]$ as before.

3.5 Computer Program

An outline of the program developed for this study is presented in this section, the program itself is given in Appendix 2. The major steps in the program are described in the order in which they are executed:

1) The basic physical information concerning the frame is input. This information includes the locations of all joints (in structure global coordinates), the incidence

relations and the geometrical and mechanical properties of each member. The input of member properties is presently limited to prismatic members with an I cross-section, although it is a very simple matter to modify the program to include any other cross-section so long as the members are prismatic. Since only I sections are considered, the depth, flange width, flange thickness, and web thickness are input for each member. Also, the program is presently developed to use a single modulus of elasticity, shear modulus and density for all members; however, the computational portion of the program is set up to use separate properties for each member if this is desired. Any superimposed dead loads in addition to the weight of the members and any static loads on the joints are also input at this time. Only concentrated joint static loads and uniform superimposed dead loads may be considered.

2) From the information input in 1, the member stiffness matrices and the member rotation matrices are computed. Then the joint stiffness matrix is assembled. As outlined in Section 2.2.2 only the nonzero upper-diagonal 6×6 submatrices of the joint stiffness matrix are generated. A "locator" matrix is used to indicate which of the 6×6 submatrices are nonzero and where it is stored.

3) The frame is next analyzed statically for displacements

and forces due to dead loads and any other static joint loads. The dead loads are treated by computing the end forces on each member due to its own weight and any additional superimposed uniform vertical dead load which may be acting on the beams. The negative of these end actions for all members incident to a given joint are then summed and taken as a fictitious joint load to be used in the static solution. The displacements and forces so determined are taken as the initial conditions for the dynamic solution.

4) The mass matrix is computed using the procedure outlined in Section 3.2.

5) The necessary data to perform the dynamic analysis is input. This includes the integration interval, how long the integration should continue, how often the "running" information should be output and what information should be monitored for maximum and minimum.

6) The dynamic loading information is input. Two types of dynamic loading are included in the system developed. One type is base motion (earthquake loading), the second is a dynamic forcing function applied to all specified free joints. Both types utilize a discrete input method to allow storage of a loading matrix in the core memory of the computer to decrease computation time. The loading is input for translational coordinates only and is input at every 0.01

second in three dimensions. If intermediate values are needed, a straight line interpolation procedure between the discrete points is used.

The base motion input is developed from the ground acceleration records of the El Centro, California, May 18, 1940, earthquake. The North-South, East-West and Up-Down directions were used. These records were converted to input data in a format such that the accelerations in the order of directions to be loaded for the particular investigation were assembled in sets of three with one set for each 0.01 of a second of elapsed time. If needed, straight line interpolation of the random accelograms is used.

To increase the capabilities of the program it is also possible to introduce a time lag in the base motion application to any of the support joints desired. The displacement of these lagging support joints must be computed relative to the displacements of the "base" support joints. If $\{x_o\}$ is taken as the displacements of the "base" support joints and $\{x_L\}$ is taken as the displacements of the lagging support joints the relative support joint displacement is the difference,

$$\{x_R\} = \{x_L\} - \{x_o\} \qquad \text{.....(3-28)}$$

The displacements of the support joints are computed by integrating the earthquake accelogram (using Levy's numerical integration method).

The forcing function input is specified so that, as in the base motion case, the three coordinates of the forcing function may be input at 0.01 second intervals. This allows a wide variety of forcing functions in all three dimensions to be used.

7) The dynamic analysis is accomplished using the procedure outlined in Section 3.4. Information is output at designated intervals during the solution to permit a running record of the displacements of one joint and the initial end forces of one member. The displacements of one coordinate is also plotted at this same interval. Designated displacements and forces are monitored for maximum and minimum.

8) The maximum and minimum values determined for the specified forces and displacements are output.

CHAPTER IV

APPLICATIONS

4.1 General

Several studies of structural behavior were made using the computer program which embodies the solution method developed in the previous chapter. Initially a comparison with a linear solution by a separate computer program using the normal modes method was made to verify the reliability of the method. The system is then used to demonstrate its applicability to the solution of nonlinear static problems by use of sufficiently large damping coefficients. Such a solution is compared with some published data on a nonlinear static problem.

The program is then used to solve a dynamically loaded cubic frame, first experiencing large geometry changes and then with large axial loads in the columns. Linear and nonlinear analyses are compared for both cases. Next an earthquake analysis is presented. To highlight the three-dimensional features of the behavior of a space structure, a cubic frame is studied for the effects of unsymmetrical mass distribution, and for variation of frame stiffness, both under earthquake loading. An analysis using a torsional type of loading is also presented. Finally, the variation of the response of a structure due to a time lag in the application of the earthquake shock wave is considered.

4.2 Comparison with Modal Analysis Solution

Although each part of the program was checked individually, and certain sections, for example, the statics part, were verified completely, it was deemed desirable to obtain a check on the dynamic part of the program. For this purpose another computer program was developed for the solution of dynamically loaded linearly elastic frames using the method of normal modes (see, for example, Hurty and Rubinstein (10)). The stiff cubic frame shown in Figure 4.1 was analyzed with both solution methods for a step function loading with a magnitude of 10 kips applied on joints 1, 2, 3 and 4 in the positive X-direction. As this loading caused sufficiently small displacements and internal forces, the nonlinear solution should agree with the linear normal modes solution.

The translation of joint 1 in the positive X-direction is shown in Figure 4.2 for approximately four fundamental periods of vibration. The maximum difference between the displacements from the two solutions was less than 1% of the maximum displacement, and the two solutions essentially coincide when plotted.

4.3 Static Displacements Using Damped Dynamic Solution

One of the interesting uses of the solution method presented here is the application to nonlinear static problems. A static solution may be accomplished by using a monotone-increasing dynamic load (a step function was used here) which attains a maximum magnitude equal to the corresponding static loading. The damping coefficient is made sufficiently large to cause the oscillation of the structure to decrease rapidly to a displacement configuration equal to the desired static one.

To illustrate the effect of the magnitude of the damping coefficients, the stiff cubic frame was studied with 10 kip loads on joints 1, 2, 3 and 4 for damping coefficients of λ = 10, 40 and 60 kip-in. / sec. (see Section 3.3). The results for the displacement of joint 1 in the X-direction are shown in Figure 4.3. Since the displacements are small, little nonlinear effect should be present. It is noted that a rapid approach to the linear static solution with little oscillation was obtained with the damping coefficient of 60 kip-in. / sec. which is approximately twice the eigenvalue (31.5) corresponding to sway displacements in the X-direction. This is essentially the critical viscous damping of the structure in the first mode.

To further illustrate the method, the nonlinear static behavior of the plane frame shown in Figure 4.4 was considered. This problem had been studied by Saafan (17). The results from the analyses with the method presented here are compared with Saafan's results in the same figure. The agreement was good, the maximum difference within the range of loads considered being approximately 10%. The difference in the two solutions is thought to be due to the fact that the method presented considers the intermediate rotations in the force orientation, which Saafan's analysis does not, and that Saafan considers the axial force effects in his bowing corrections, which this solution does not. This comparison also provides further verification of the reliability of the solution procedure developed.

4.4 Nonlinear Comparisons

The difference in the frame response as computed by the nonlinear analysis presented and by a linear analysis was studied to determine the areas where significant variations would appear and to determine the relative magnitudes of such variations. The linear analysis was obtained by deleting the nonlinear considerations used in the solution for frame joint reactions.

In the following two sections the nonlinear effects of geometry changes and (compressive) axial loads are considered. It is recognized that these effects cannot be really separated. However, the loading for each study was chosen so as to accentuate the effects under study.

4.4.1 Effect of Geometry Changes.--The study was made to consider the effects of geometry changes on the response of the structure. The flexible cubic frame was subjected to a step-function loading applied on joints 1, 2, 3 and 4 in the positive X-direction. All displacement components of joint 1 were monitored for maximum and minimum. The significant results for 15 kip and 20 kip loads are shown in Table 4.1. The greatest variation is seen to be in the vertical displacement of joint 1. This is attributed to the more accurate consideration in the nonlinear analysis of the axial displacement of column 6 (which is directly under joint 1).

Various internal forces throughout the frame were monitored and the results from the linear and nonlinear solutions are also shown in Table 4.1 for some of the significant forces (forces with relatively large magnitude) which showed differences. The variation in the more significant forces was less than 5% even for the 20 kip loading which produced a sway in the X-direction of approximately 0.3 of the column height. The moment about the Z-axis at the initial end of column 6 and the moment about the Y-axis at the initial end of beam 4 are shown as representative. The axial forces in most members showed considerable variation and again this is attributed to the more accurate solution for axial displacement that is used in the nonlinear solution. The bending moment about the X-axis at the initial end of member 6 exhibited substantial variation (19% maximum), but is of minor importance because of its relatively low magnitude.

It was found during this investigation that the nonlinear solution for axial force is quite sensitive to accuracy in the displacement solution. A shorter integration interval had to be used in order to obtain accurate axial forces.

4.4.2 Effect of Compressive Axial Forces.--The investigation of axial force effects was made using the flexible frame, subjected to a step function loading in the X-direction on all free joints. The magnitude of the step function was held at 10 kips. In addition, identical vertical static loads were applied to all four free joints and the magnitude of these static loads was increased to approximately the Euler load of the columns (42 kips). The same displacements and forces were monitored for absolute maximum as in the previous section. Representative results are shown in Figures 4.5, 4.6 and 4.7.

Linear dynamic solutions for all magnitudes of vertical static loads showed no significant variation from the solution with no static loads, for the response variables monitored (with the obvious exception of the maximum Y-displacement of joint 1 which increased as a result of the direct compression of the column by the statically applied load).

In general, the nonlinear analyses showed larger response than that of the linear analysis. The difference increases with an increase in the axial loads. Figure 4.5 shows the variation in displacements of joint 1 in the X and Y-directions. Considerable variation between the nonlinear and linear solutions was found with increasing axial loads as may be noted in the figure. Figure 4.6 shows two of the end forces of column 6. Again considerable increase (70%$\pm$) occurs. Figure 4.7 shows two initial end forces in beam 1.

4.5 Earthquake Loading

To illustrate the application of the method developed to an earthquake response problem, both the flexible and stiff cubic frames were subjected to the well known 1948 El Centro earthquake for a 5 second duration. The ground motions in all three dimensions were taken into account, with the X-axis corresponding to the North-South direction of the earthquake input.

To more closely simulate realistic conditions, a superimposed beam dead load of 0.03125 kips per inch was applied to members 1, 2, 3 and 4 to approximate a roof or floor dead load, and a damping coefficient of 1.8 kip-sec. / in., approximately 3% of the "critical" damping factor defined in Section 4.3, was used. The results are shown in Figure 4.8.

The maximum displacement and forces for certain joints and members which are considered significant are shown in the chart, at the bottom of the figure. It is seen that the same earthquake produces quite different responses in the two different structures which have similar mass distribution but considerably different stiffness properties.

4.6 Effects of Dissymmetry in Distribution of Mass, Stiffness, and Loading

The effects of dissymmetry in the frame were investigated for both mass distribution, stiffness variation, and loading pattern. The frame shown in Figure 4.9 was used with the flexible columns and beams shown in Figure 4.1. The superimposed dead load was set at 10 lbs. per inch. The loading used was one second of the El Centro earthquake from 1.5 seconds to 2.5 seconds. This portion was used since it produced the most violent motion during the long term response studies reported in Section 4.5.

Mass dissymmetry was introduced by varying the superimposed beam dead load on members 1 and 3, but keeping the sum of the dead loads on both beams at 20 lbs. per inch. The dead load is included in the mass lumping procedure, and consequently this produced unsymmetric mass distribution. The results are shown in Figure 4.10. The greatest variations in the forces studied occurred in the column bending moments. Columns 6 and 7 are shown as representative. As more mass is shifted from beam 3 (connected to column 7) to beam 1 (connected to column 6), considerable increase in the maximum bending moment in column 6 was experienced with a similar decrease in the maximum bending moment on column 7.

The maximum twisting moments on the columns also increased with the dissymmetry in mass distribution. The graph for column 5 is shown as a representative example in Figure 4.10. The maximum horizontal bending moment in the beams increased approximately three times, and the results for member 4 are shown in the same figure. The results illustrate that mass distribution is a very significant consideration in determining the stresses experienced by the members of a structural system under an earthquake loading.

Dissymmetry was introduced in the stiffness of the frame by replacing members 3 and 7 (in Figure 4.2) with members 3 and 8 in the stiff set of members shown in Figure 4.1. Again the maximum twisting and bending moments in the columns varied considerably. The values for column 5 are shown in Table 4.2 along with the maximum horizontal bending moments in beam 4 and the maximum displacement of joint 1 in the X-direction.

A point to be observed here is that if the bent composed of members 5, 1 and 6 is considered alone considerable difference in the forces it experiences results depending on which of the three dimemsional structures it is actually a part. For purposes of comparison this bent was analyzed as a plane problem subjected to the X-component of loading used above. The results are shown in the last column of Table 4.2. It is of interest to note that the results are closer to those of the unsymmetrical frame.

As a possible means of estimating the variation in structural dynamic response caused by unsymmetrical stiffness, static analyses of both the modified and original three dimensional frames were made with 10 kip loads on joints 1, 2, 3 and 4 in the positive X-direction. The ratio of the displacement of joint 1 in the X-direction for the unsymmetric frame versus the symmetric frame was found to be 1.10. The ratio of the same maximum displacements in the dynamic analysis was 1.34, an increase of approximately 20% over the static case.

To consider the effects of loading pattern, the same frame was loaded with 10 kip step loads on joints 1 and 4 in opposite directions in one case, and in the same direction in another, as shown in Figure 4.11. The maxima of the X-displacement of joint 1, the twisting moment and bending moment about the Z-axis for column 6 and the bending moment about Y-axis for beam 4 for the two solutions are shown in the table in the figure.

As would be expected, the twisting moment in column 6 and Y-bending moment in beam 4 are considerably higher for the reversed loading case. However, the relatively large values of the displacement and column bending moment which occurred in the reversed loading case as compared with the symmetrical case were unexpected as an essentially torsional mode response was anticipated. To explain this result the normal modes were obtained for the frame investigated. The first four modes and their periods are shown in Figure 4.12. These modes all primarily involve sway displacements of the columns.

It is of interest to note that the first torsional mode has a larger period than that of the translational mode in the X-direction. The large X-displacement of joint 1 and bending moment in column 6 would indicate that the response is mainly in the fourth mode.

A further examination of the time-history plot of the response (for its "periodicity") confirms this observation.

The foregoing would indicate that in the presence of marked dissymmetry in a three-dimensional structural-load system, the behavior of the system can be accurately predicted only through a three dimensional analysis.

4.7 Effect of Time Lag of Support Movements

In this section the method of solution was used to consider the effect of the finite speed of travel of an earthquake shockwave through the foundation material. The method of including this time lag in the analysis is discussed in Section 3.5. It is recognized that this is an approximation of the true situation since foundation damping, secondary wave fronts and similar effects are ignored, but the purpose here is to show a method of considering this phenomenom and to obtain a quantitative assessment regarding its effect on a simple three dimensional structure.

This study again used the cubic frame with the flexible members and with the cross member in the top level deleted as shown in Figure 4.9. Loading was one second of El Centro earthquake starting at 1.5 seconds and using all three dimensions. A superimposed dead load of 10 lbs. per inch on beams 1, 2, 3 and 4 was used to simulate a roof or floor loading. The time lag was applied to support joints 6 and 7 which would correspond to a shock wave moving in the positive X-direction. A speed of seismic wave of 1000 ft. per second is used assuming a sandy soil (8). The 20 foot span of the frame then yields a time lag of 0.01 seconds.

The maximum absolute values of the displacements of joint 1, the initial-end forces of members 1, 2 and 4 and moments at the ends of all columns were monitored. The variation of representative forces and displacements are shown in Table 4.3. Variation in the column end moments as illustrated by the moment about the X-axis at the initial end of column 8 was not large with a maximum of approximately 10%. Other variations of somewhat larger percentage, but for forces of lesser magnitude may be noted in the figure. Differences in displacements from those corresponding to zero lag time were generally small except for the case of 0.01 second lag time. In the latter case, a difference of 26% and 130% are noted for joint 1 in the X-direction and joint 2 in the Z-direction, respectively.

CHAPTER V

SUMMARY AND CONCLUSIONS

The report has presented a method of analysis for dymanically loaded elastic space frames considering the nonlinear effects of geometry changes and axial thrust. The solution method has been embodied in a computer program written in FORTRAN. The program has been used to study the behavior of a simple cubic frame under various conditions.

The analysis uses essentially a lumped parameter model. Numerical integration is utilized to obtain the transient response of the frame. The nonlinear geometry and axial thrust effects are accounted for in the determination of the frame reactions corresponding to the joint displacements. Consideration of geometry changes leads to corrections in the relative member displacements for rotations of the member chord and for apparent axial shortening due to bowing. The flexural stiffness coefficients are continuously revised to reflect the beam-column effects of the axial thrust.

The solution utilizes a mass lumping procedure which accounts for rotatory and translational inertia. Mass proportional damping may be included in the solution. The intial conditions for the dynamic analysis are determined by a linear static analysis of the frame under dead loads.

The computer program is verified by comparing a nonlinear solution under practically linear conditions with a solution obtained by the method of modal analysis. In addition, good agreement was found between a published nonlinear static analysis of a plane frame and the results obtained by the present program using a large damping coefficient in the dynamic analysis. This serves not only as a verification of the nonlinear aspects of the solution, but also demonstrates a very useful application of the solution method.

The applications of the computer program have also included studies of a simple cubic frame. These were made to demonstrate the magnitude of the nonlinear effects considered in the analysis; to investigate the effects of dissymmetry of mass, stiffness and loading and to study the effect of time lag in the motion of the supports. The last two effects can be studied only with a three-dimensional analysis.

The numerical results show that the geometric nonlinearities have relatively small effect on the major internal forces (those having larger magnitudes). For example, under a step loading which produced a maximum sway of 30% of the column length, the maximum column moments change only 5%. However, the axial thrust effect is quite significant. A variation of 30% in column moments was found for the case with static column loads equal to 50% of the Euler load.

The studies considering the effects of unsymmetrical mass and stiffness distributions demonstrated significant variations in maximum forces experienced throughout the frame as the frame properties were changed. Variations of 50% to 100% were noted in various maximum forces and displacements.

In conclusion, it has been demonstrated that a three-dimensional, dynamic, nonlinear analysis of structural frames is practicable with the use of a computer. Furthermore, the studies made with the computer program have shown that nonlinear geometry effects are not critical until quite large displacements occur. However, axial force seems quite significant. The studies of the various unsymmetrical frames indicate that three-dimensional analysis can provide significant increases in the accuracy of analysis of space structures.

Future extensions of this investigation could include: (i) a further study of the application of dynamic analysis using large damping coefficients to obtain nonlinear static solutions; (ii) the marriage of this solution with one that includes the effects of nonlinear material properties which is in progress at Michigan State University.

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Table 4.1.--Variation in Forces and Displacements in Geometry Change Comparision

All values maximum absolute experienced during 1 second of vibration. Displacements in inches and radians and forces in kips and kip-inches.

	15 kip loading		20 kip loading	
	Linear	Non-linear	Linear	Non-linear
Vertical disp. of joint 1	0.042	7.28	.0568	12.53
Disp. of joint 1 in X-direction	56.72	55.32	75.63	72.48
Rotation about X-axis of joint 1	0.0162	.0225	0.0218	0.0335
Rotation about Y-axis of joint 1	0.0775	0.0861	0.1034	0.1169
Rotation about Z-axis of joint 1	0.0852	0.0894	0.1132	.1186
Axial force in col. 6 (tension)	25.50	29.55	34.20	53.91
Axial force in beam 1 (tension)	5.51	9.71	7.33	28.97
Mom. about Z-axis init. end col. 6	3457	3540	4610	4825
Mom. about Y-axis init. end beam 4	565.0	583.9	754.6	766.9
Mom. about X-axis init. end col. 6	125.42	147.65	166.8	206.7

Table 4.2.--Effect of Dissymmetry in Stiffness

(Maximum Absolute Values)

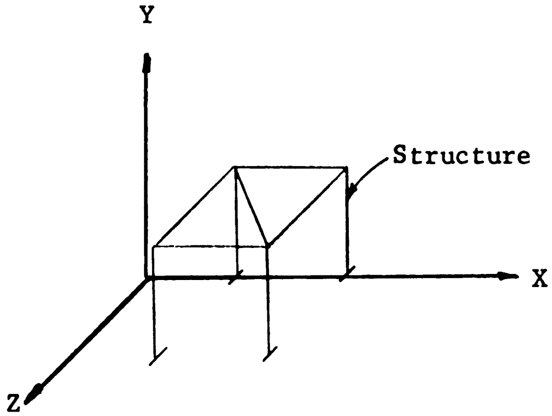
Item	Sym. Frame	Unsy. Frame	Plane Frame
Twisting mom. col. 5 (kip-in.)	17.85	38.84	- - -
Bending mom. Z-axis col. 5 (kip-in.)	131.7	180.	188.
Bending mom. Y-axis beam 4 (kip-in.)	14.38	59.7	- - -
Displ. joint 1 in X-dir. (inches)	2.41	2.23	2.61

Table 4.3.--Time Lag Studies

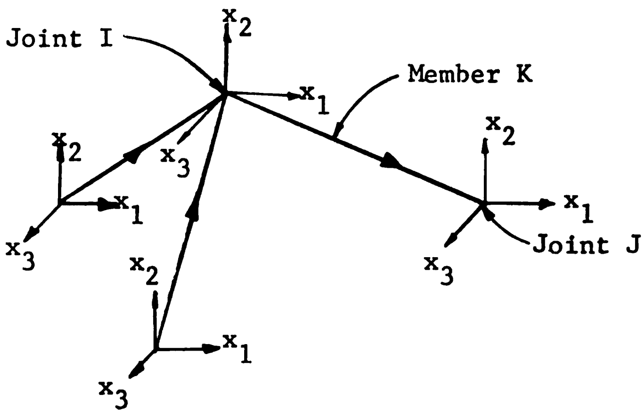
(Maximum Absolute Values)

Time Lag (sec.)	X mom. col. 8	Z mom. col. 8	Y mom. beam 2	X displ. of joint 1	Z displ. of joint 2
0.0	18.73	122.2	16.73	2.41	0.487
0.01	18.62	143.0	17.92	3.04	1.15
0.02	18.57	134.1	17.52	2.48	0.505
0.03	18.47	139.3	14.90	2.49	.513

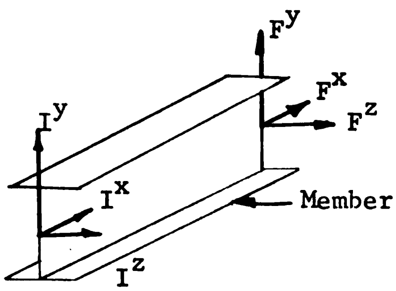
Time lag is the delay in application of earthquake load to joints 6 and 7.



Structure Global Coordinate Axes

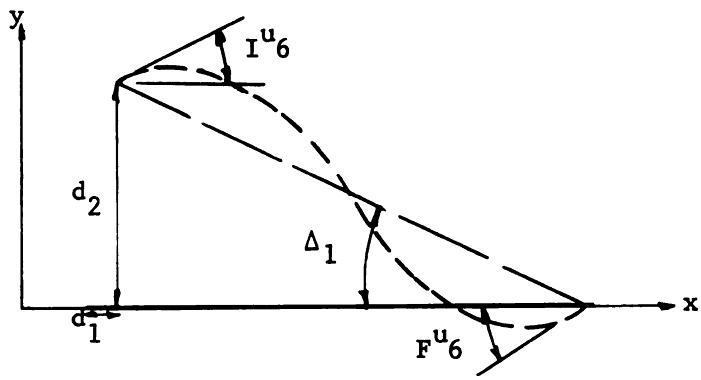


Joint Global Coordinate Axes

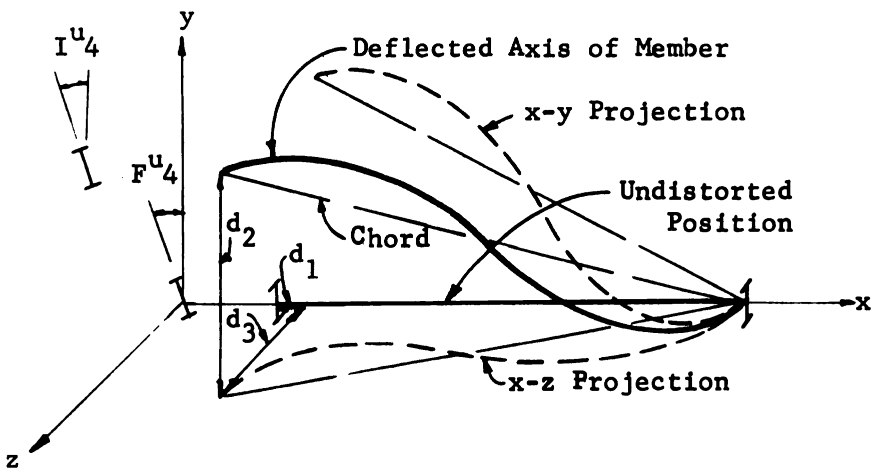


Member Coordinate Axes

Figure 1.1 Coordinate Systems

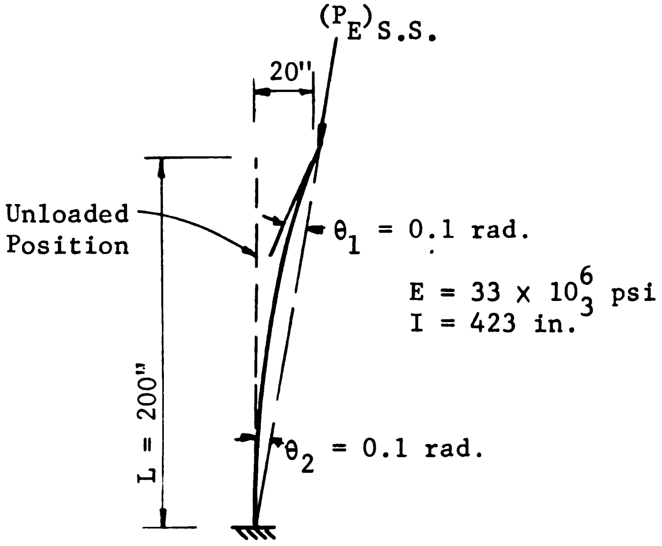


Intermediate Deflections in x-y Plane



Deflected Member in Three Dimensions

Figure 2.1 Member Relative End Displacements



Cantilever Column Investigated

$\frac{({}_Iu_2 - {}_Fu_2)^2}{2L}$	Bowing Ax. For. Not Considered	Bowing Axial Force Considered	Total No Ax. For.	Total With Ax. For.	% Diff.
1.000	0.333	0.501	1.333	1.501	12.5

Values in inches

Figure 2.2 Relative Magnitude of Geometry Correction Terms

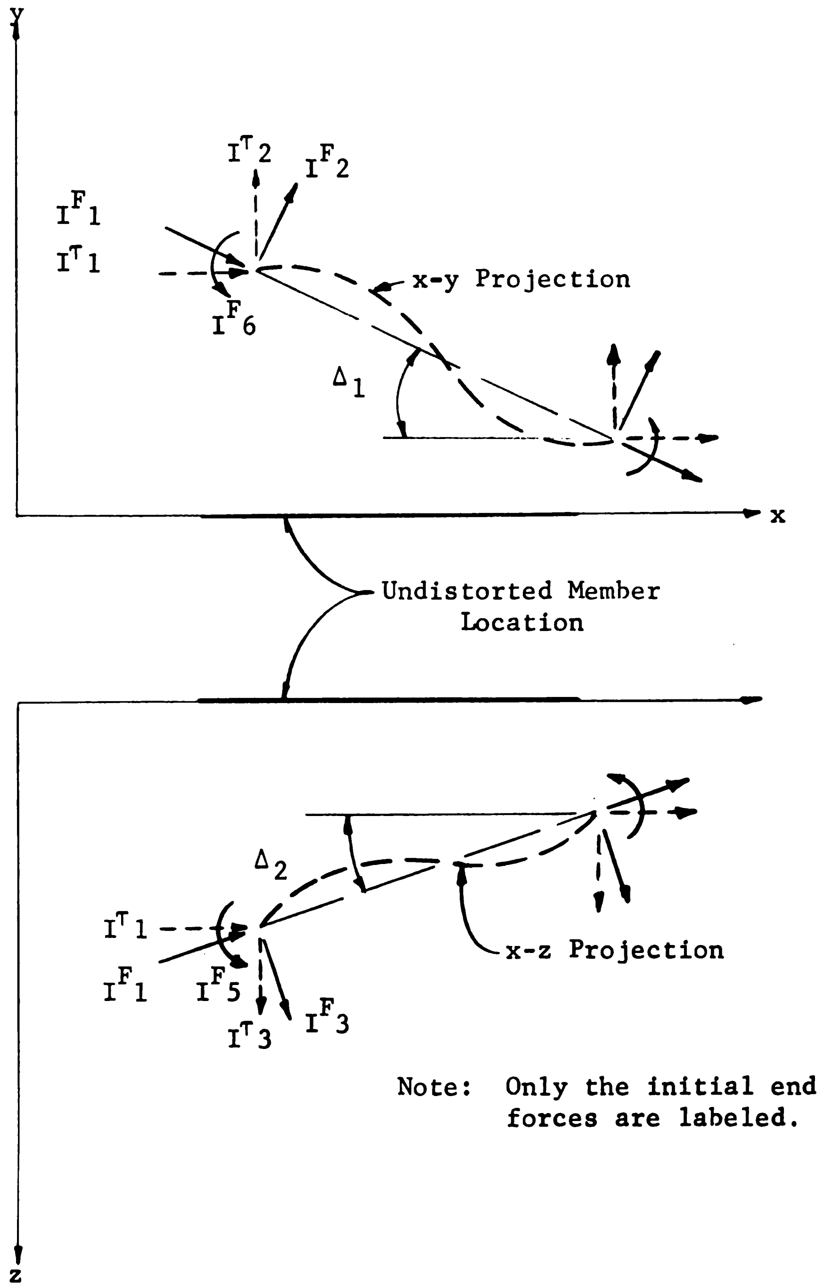
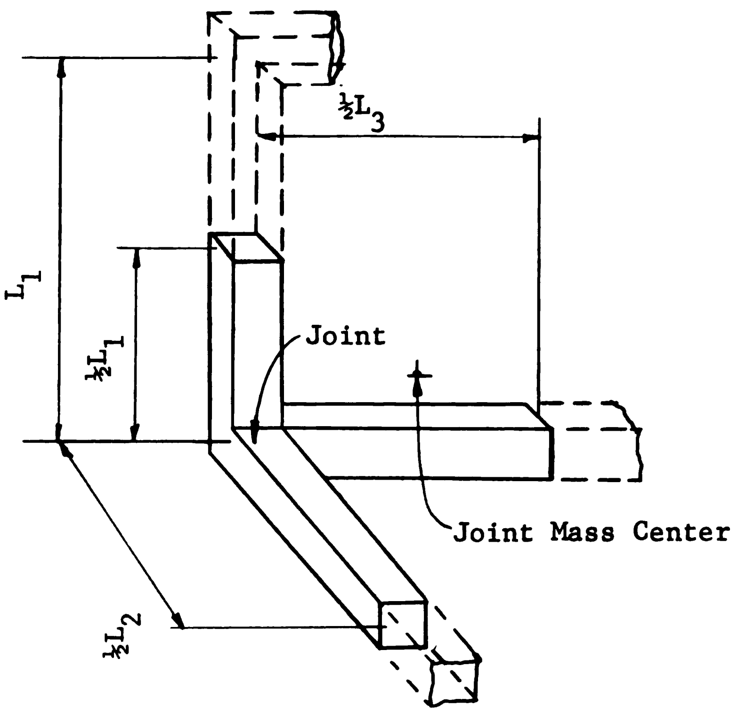
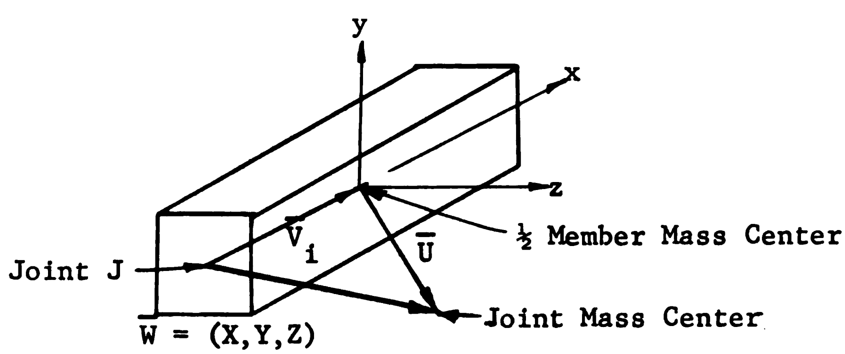


Figure 2.3 Rotation of Member End Forces Due to Chord Rotation

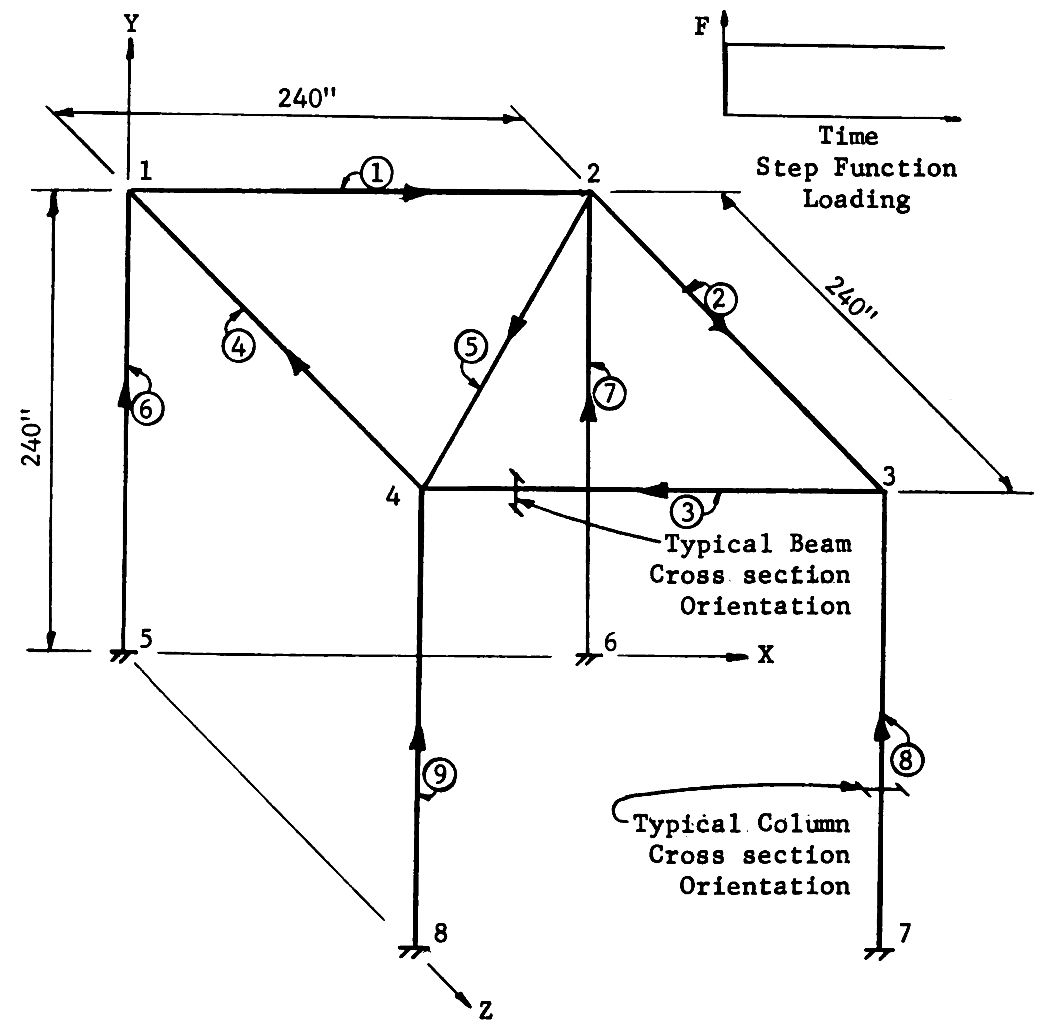


Typical Joint



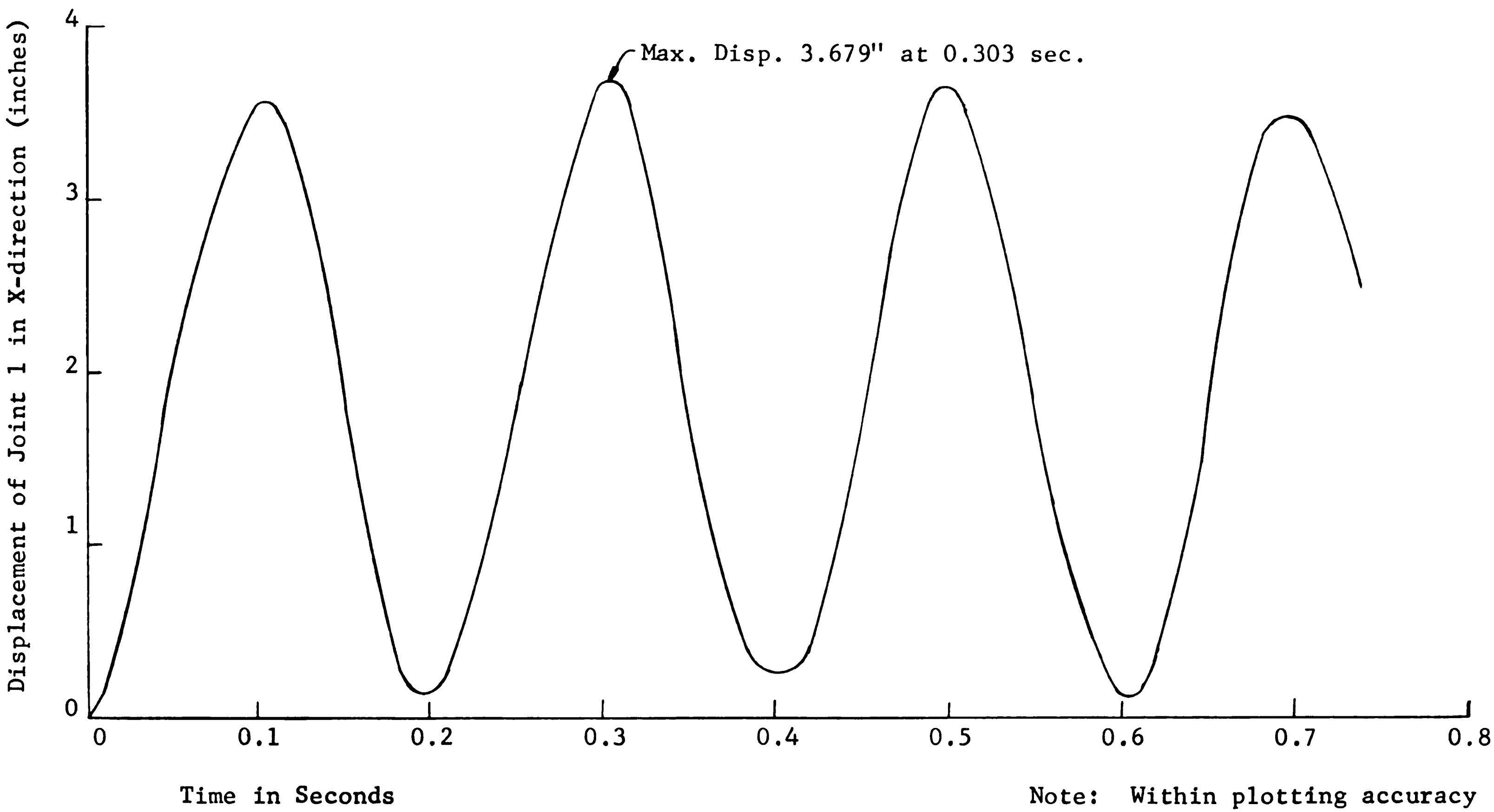
Typical Branch

Figure 3.1 Rigid Joint Mass



Flexible Members						Stiff Members			
Memb. No.	Flange Width	Flange Thick.	Total Depth	Web Thick.		Flange Width	Flange Thick.	Total Depth	Web Thick.
①	5.25	0.313	8.00	0.250		10.00	0.625	14.00	0.375
②	"	"	"	"		"	"	"	"
③	"	"	"	"		"	"	"	"
④	"	"	"	"		"	"	"	"
⑤	"	"	"	"		"	"	"	"
⑥	5.00	0.375	5.00	0.250		"	0.500	10.00	"
⑦	"	"	"	"		"	"	"	"
⑧	"	"	"	"		"	"	"	"
⑨	"	"	"	"		"	"	"	"

Figure 4.1 Cubic Frame



Load: 10 K step function on Joints 1,2,3,4
Frame: Cubic with stiff members.

Note: Within plotting accuracy there is no noticable difference in the two solutions.

Figure 4.2 Comparison of Solution Method with Modal Analysis

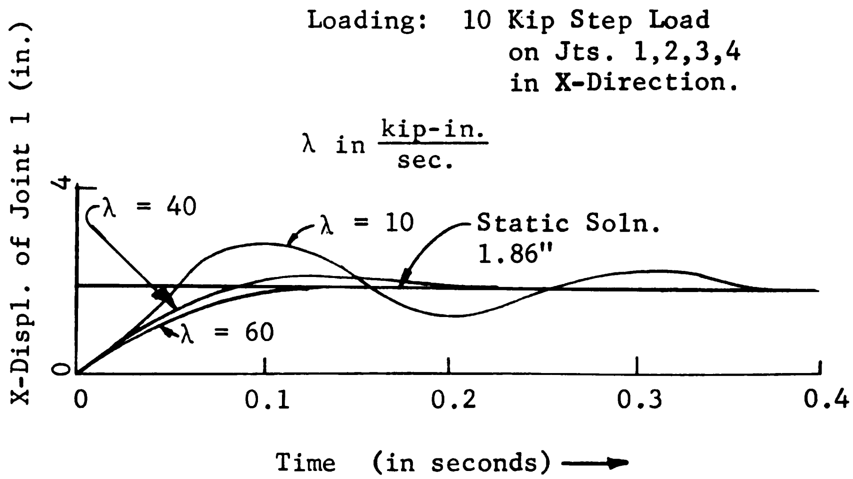
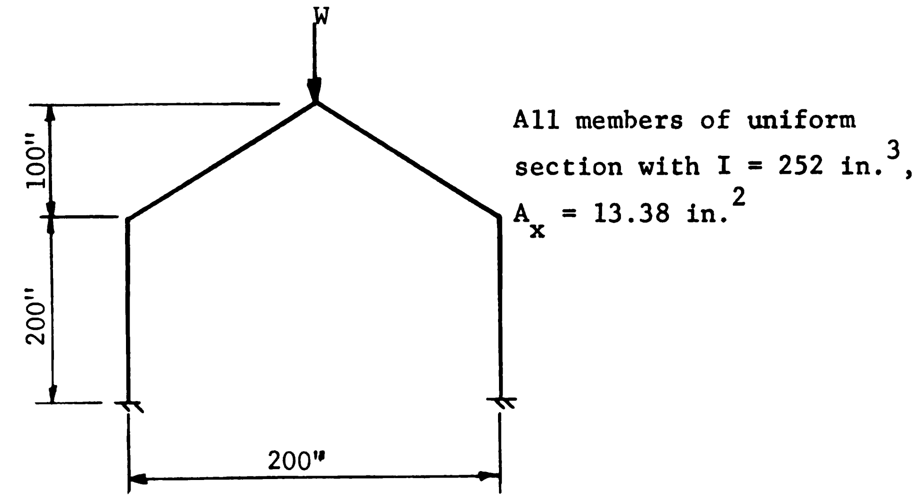


Figure 4.3 Damped Response of Stiff Cubic Frame



Plane Frame

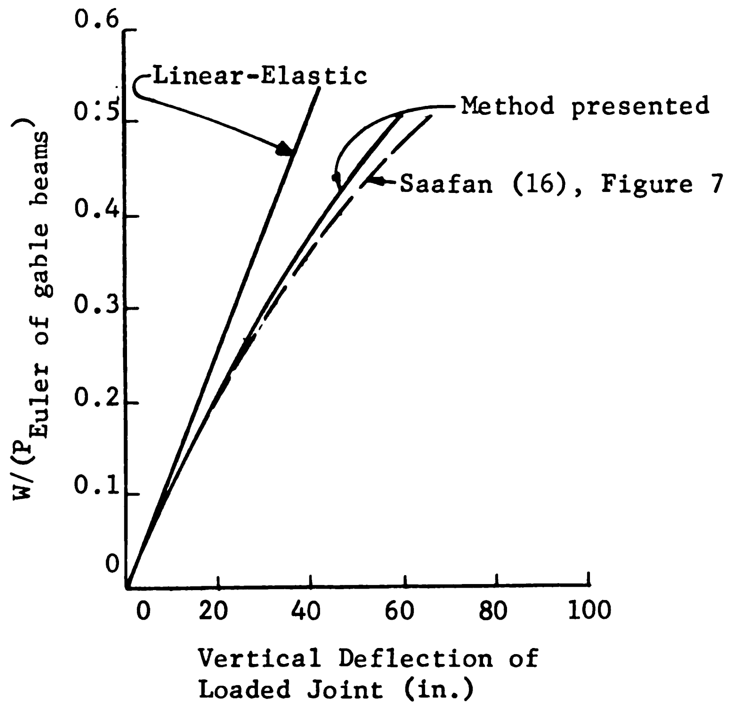
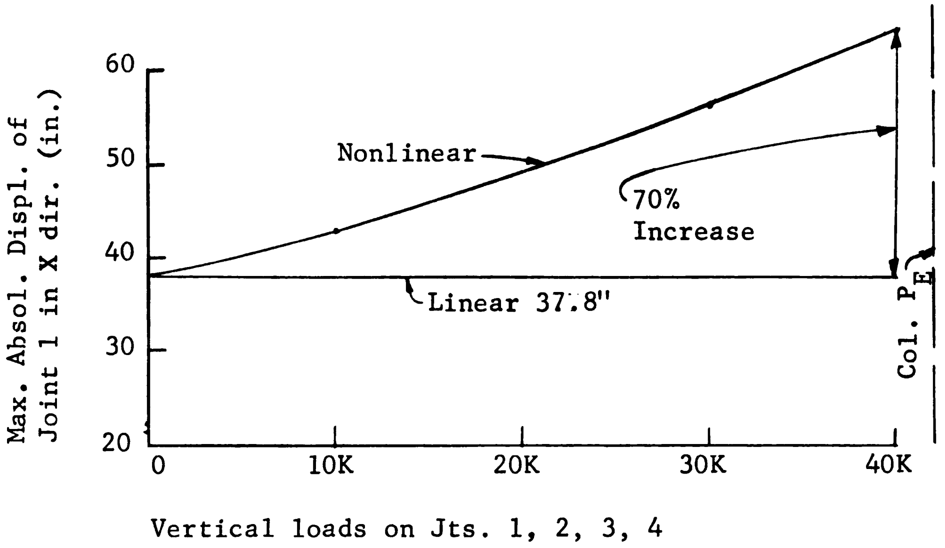


Figure 4.4 Static Solution of Plane Frame



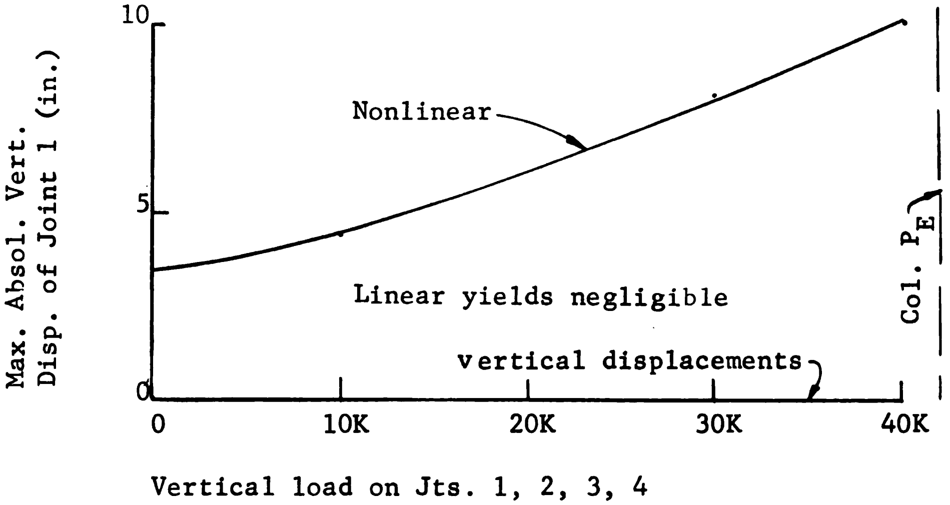
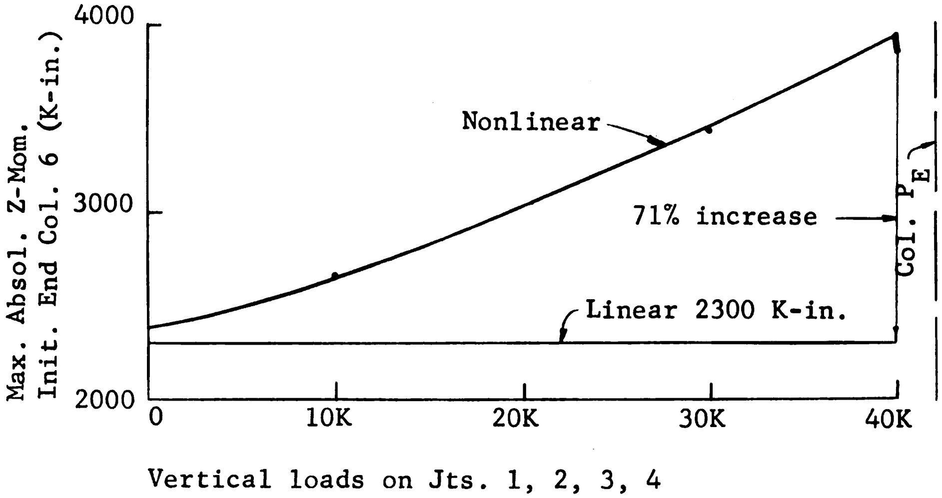


Figure 4.5 Axial Load Effect on Displacements



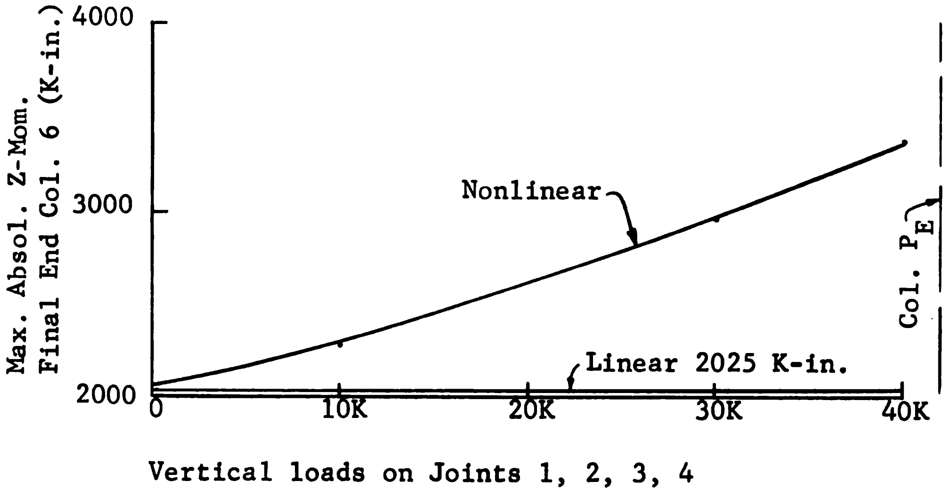


Figure 4.6 Axial Load Effects on Column Forces

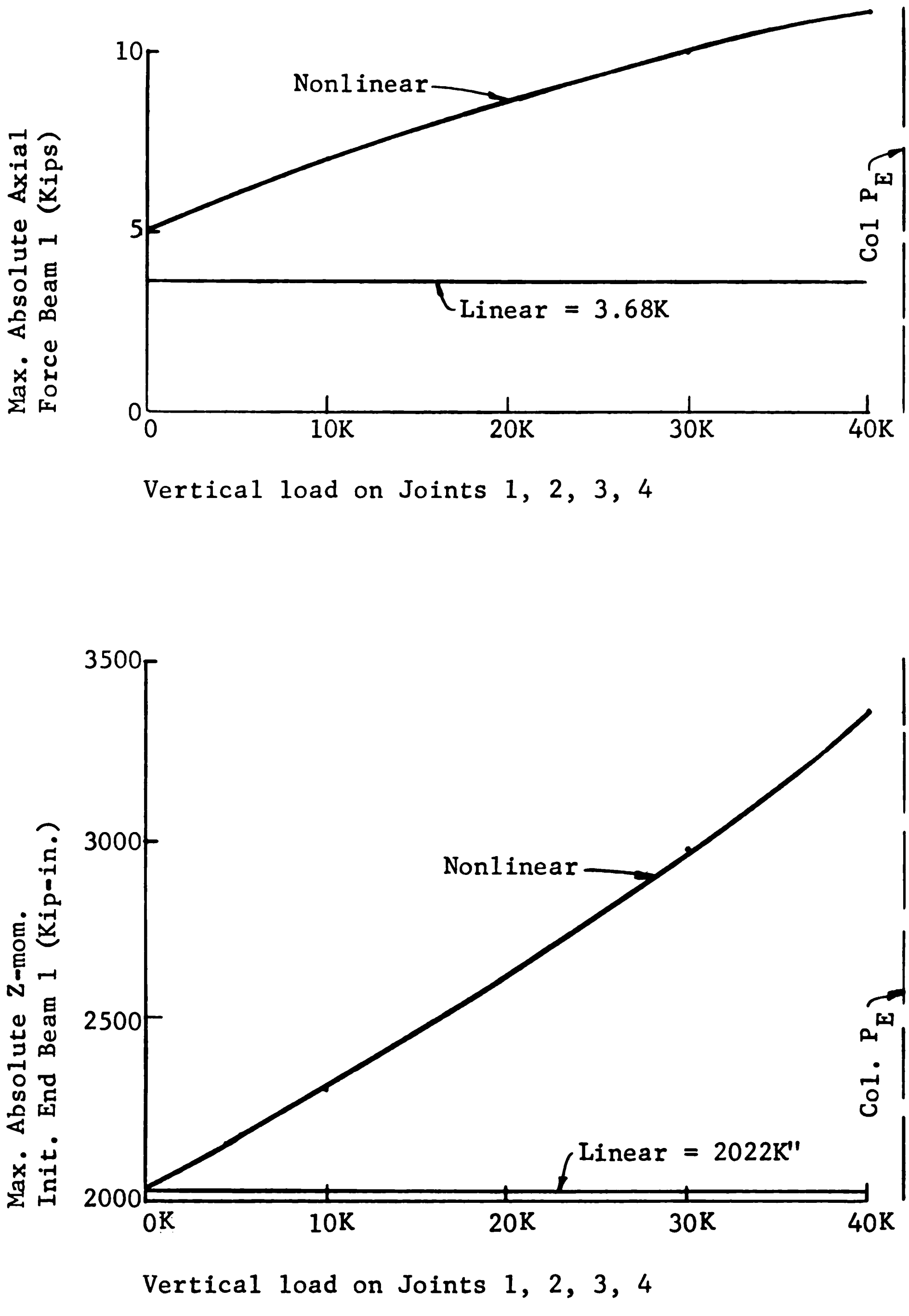
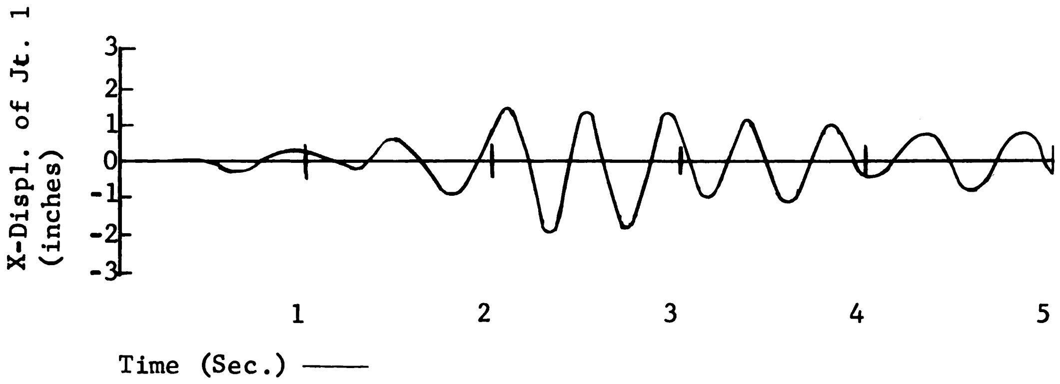
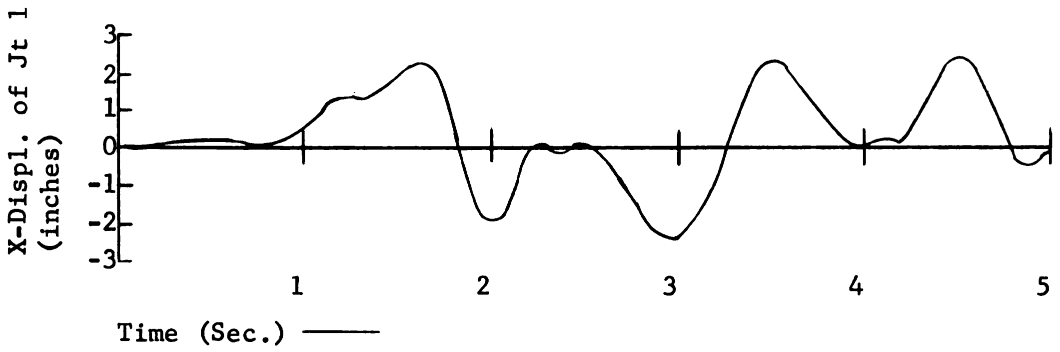


Figure 4.7 Axial Load Effects on Beam Forces



Stiff Cubic Frame Response

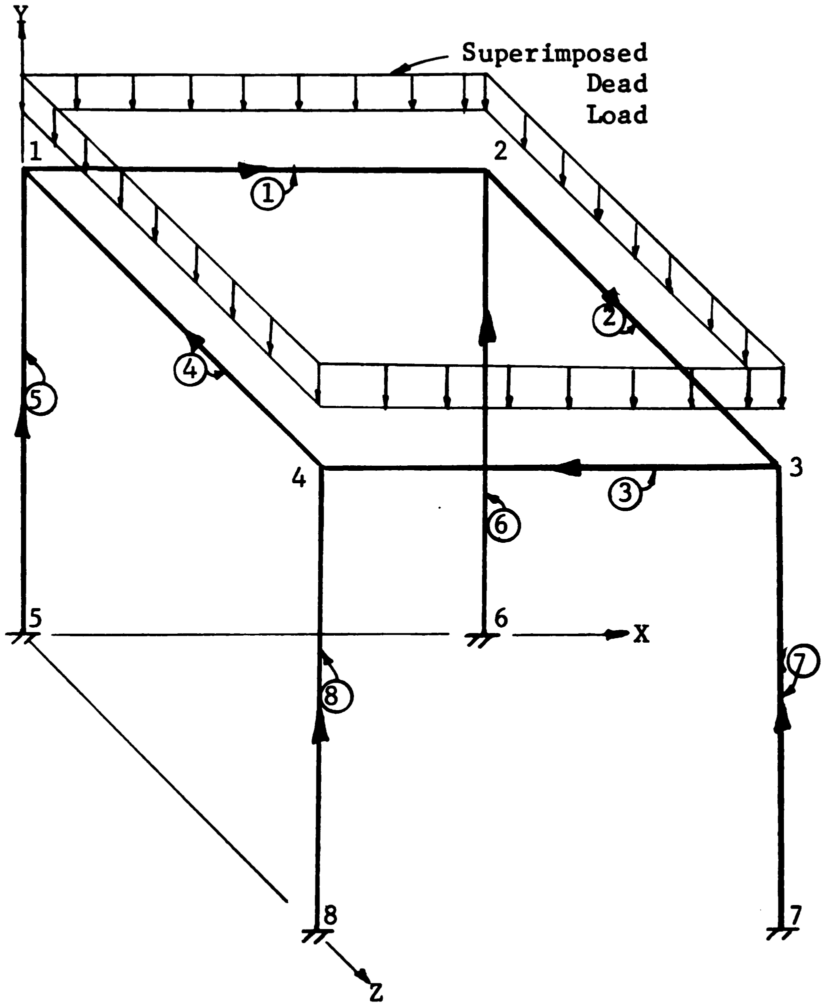


Flexible Cubic Frame Response

	Stiff	Flex.
Transl. Jt 1 in X-dir.	1.55"	1.96"
Transl. Jt 1 in Z-dir.	0.41"	1.34"
Z-Mom. Init. End Col. 7	1054K"	165K"
Z-Mom. Init. End Beam 1	852K"	261K"

Comparison of Max. Absolute
Values of Forces and Displacements

Figure 4.8 Response of Cubic Frames to 1948 El Centro Earthquake



Note: Column and beam cross-sections used in the Flexible Members in Figure 4.1 are used here.

Figure 4.9 Frame Used in Three-Dimensional Studies

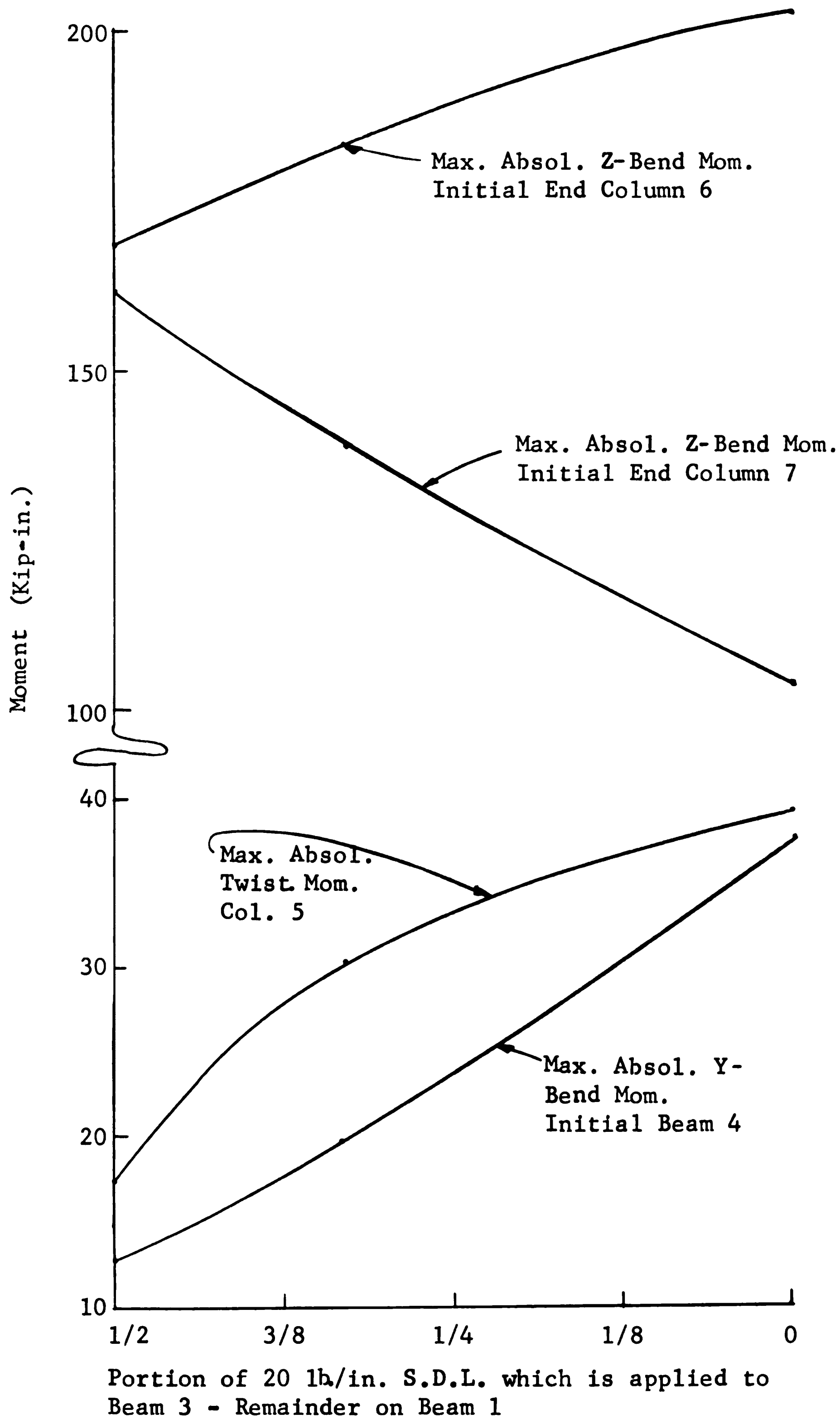
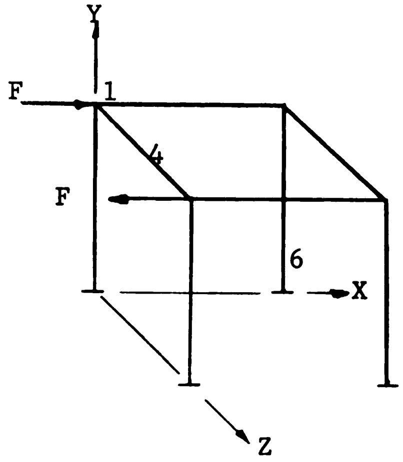
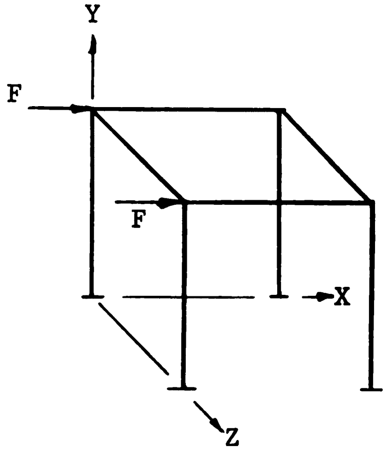


Figure 4.10 Effects of Unsymmetric Mass Distribution



Anti-symmetric Loading

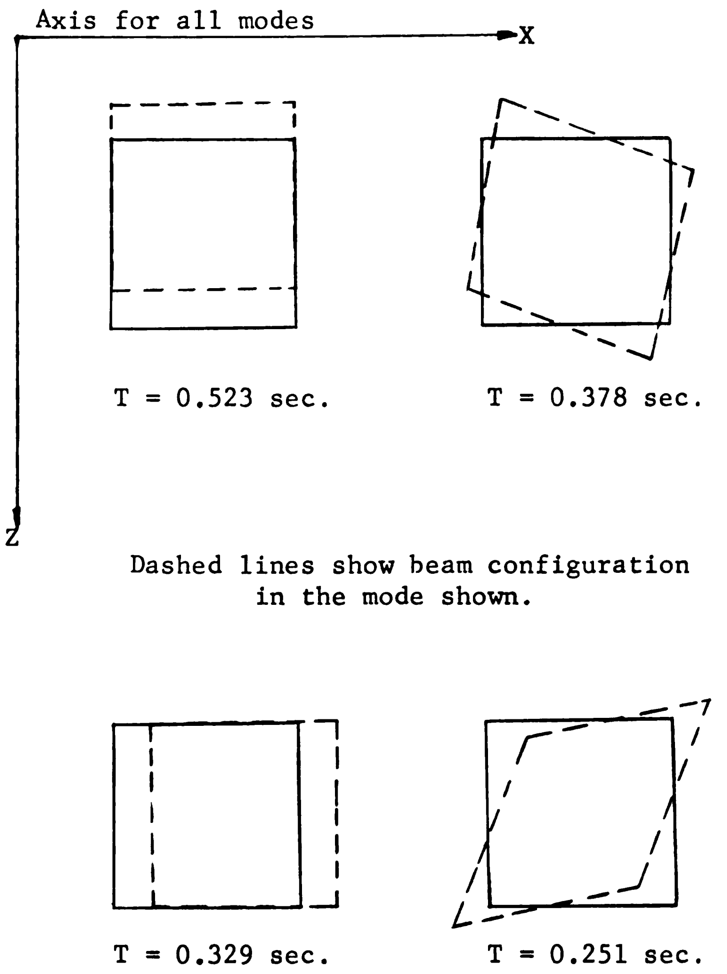


Symmetric Loading

Selected Maximum Displacements and Forces in 1 Second of Motion

	Symmetric Loading	Reversed Loading
Displ. Joint 1 in X-dir. (in.)	20.59	17.16
Twisting Mom. Col. 6 (Kip-in.)	-14.32	-284.6
Bend. Mom. Z-axis Col. 6 (Kip-in.)	1256	1021
Bend. Mom. Y-axis Beam 4 (Kip-in.)	55.3	416.4

Figure 4.11 Effect of Loading Pattern



Top view of frame is shown as modes primarily involve only column sway.

Figure 4.12 Normal Modes for Cubic Frame

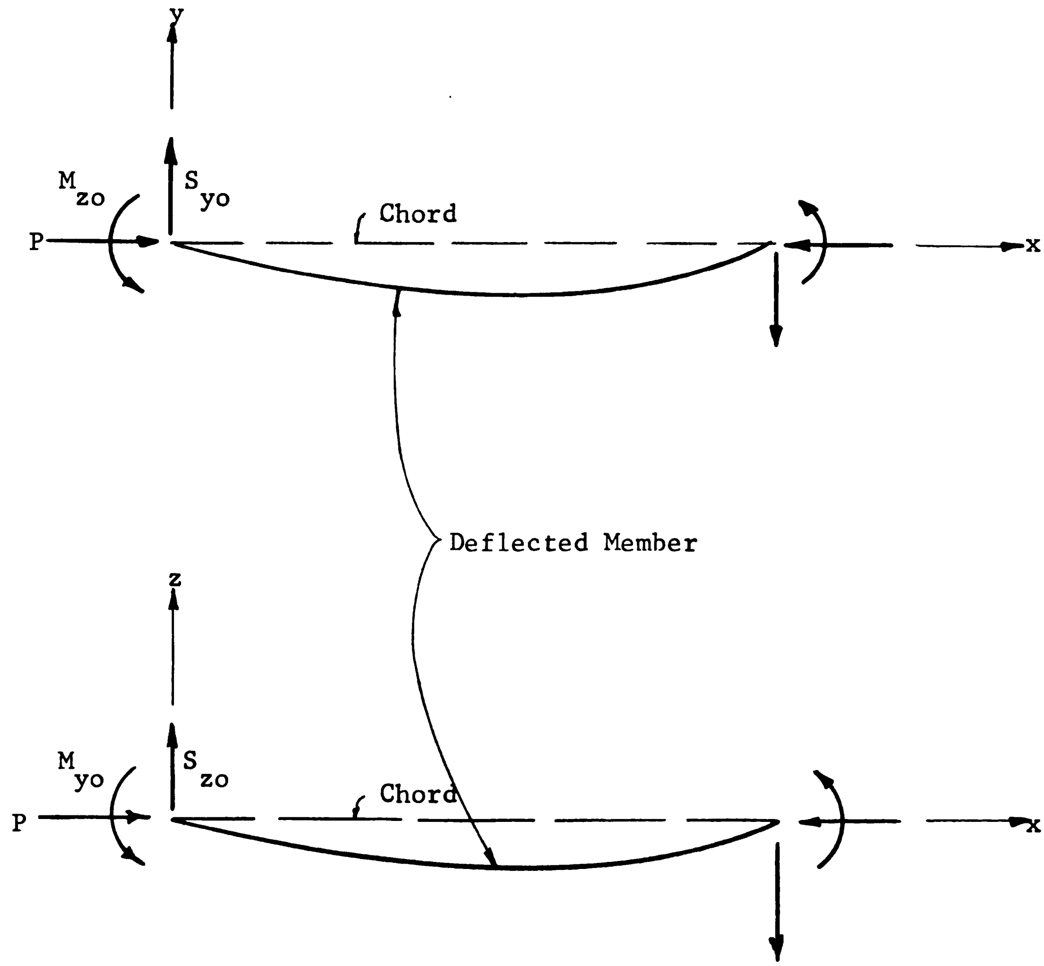


Figure A.1 Coordinates for Member Solution

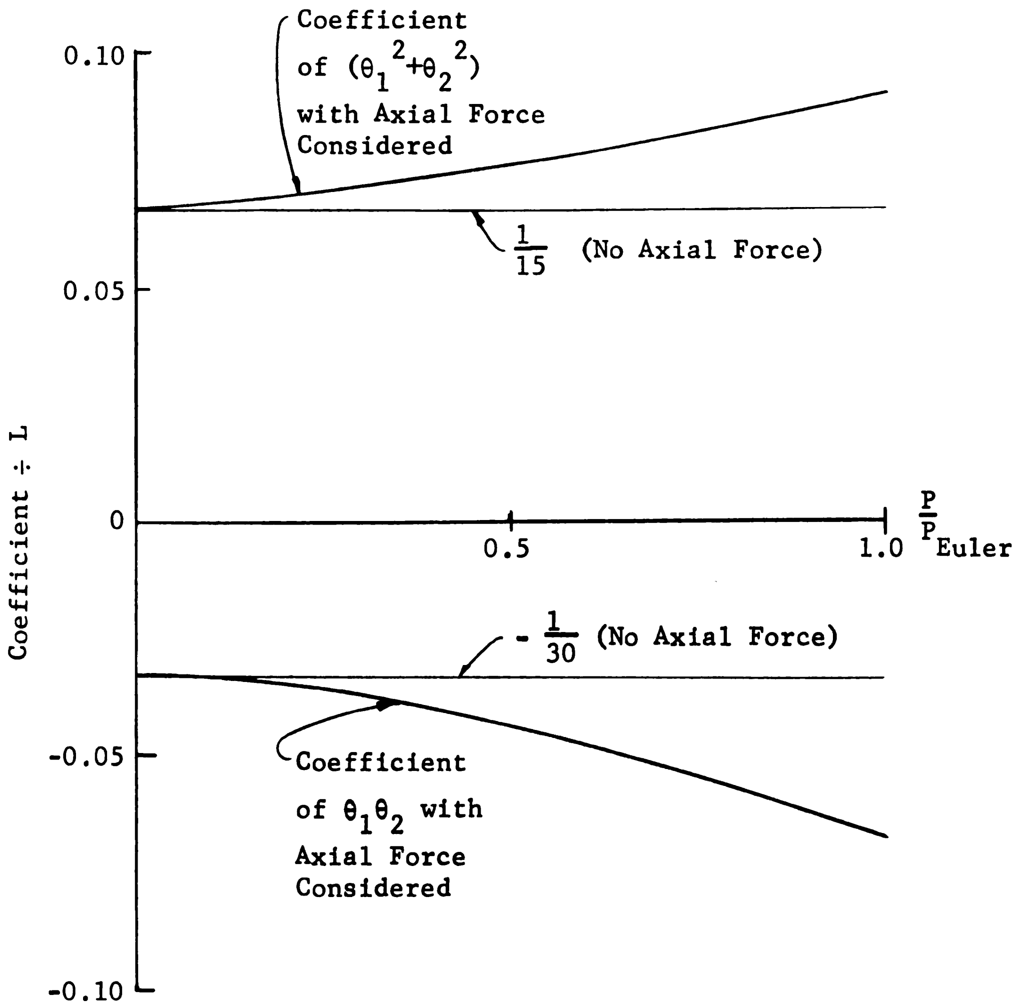


Figure A.2 Coefficients in Expression for Axial Shortening Due to Bowing

APPENDIX I

BEAM COLUMN SOLUTION

A.1.1 Stiffness Coefficients

In this section the stiffness coefficients of a member under the influence of an axial force are derived. These coefficients are available elsewhere, for example, in Reference 4. However, they are derived here for completeness and to outline the assumptions necessary to use these relationships in a three dimensional solution.

The assumptions that torsional-flexural coupling may be neglected and that the Bernoulli-Euler equations apply permit the displacement-force relationships for the individual member to be expressed by the three equations.

$$EI_z \frac{d^2y}{dx^2} = -M_{zo} + S_{yo}x - Py \qquad \text{.....(A-1)}$$

$$EI_y \frac{d^2z}{dx^2} = -M_{yo} + S_{zo}x - Pz \qquad \text{.....(A-2)}$$

$$\theta_{twist} = \frac{M_x}{GJ} \qquad \text{.....(A-3)}$$

where as shown in Figure A.1; P is the axial force, M_{zo} and S_{yo} are the moment and shear force components in the x-y plane at the initial end of the member, and M_{yo} and S_{zo} are similar components in the x-z plane. The differential Equations (A-1) and (A-2) are identical in form and consequently a single solution will be given and subscripts will be dropped. Rearranging and letting M_I be the end moment at the initial end and S the shear, yields the general differential equation:

$$\frac{d^2y}{dx^2} + \frac{P}{EI} y = \frac{1}{EI} (S x - M_I) \qquad \text{.....(A-4)}$$

which has the solution:

$$y = \frac{M_I}{P} \cos p x + \frac{\sin p x}{P \sin p L} [M_I (1 - \cos p L) - SL]$$

$$+ \frac{S}{P} x - \frac{M_I}{P} \qquad \text{.....(A-5)}$$

where $p^2 = \frac{P}{EI}$.

Differentiating for the slope yields:

$$\frac{dy}{dx} = \frac{M_I}{P} p \sin p x + \frac{p \cos p x}{P \sin p L} [M_I (1 - \cos p L) - SL]$$

$$+ \frac{S}{P} \qquad \text{.....(A-6)}$$

At x = 0, the slope $\frac{dy}{dx} = \theta_1$; and at $x \simeq L$, the slope $\frac{dy}{dx} = \theta_2$. Substituting these relationships yields two equations for M_I and S in terms of θ_1 and θ_2, the coefficients of which are the stiffness coefficients:

$$M_I = (C_1 \frac{EI}{L}) \theta_1 + (C_2 \frac{EI}{L}) \theta_2 \qquad \text{.....(A-7)}$$

$$S = (C_3 \frac{EI}{L^2}) (\theta_1 + \theta_2) \qquad \text{.....(A-8)}$$

where

$$C_1 = \frac{pL (\sin pL - pL \cos pL)}{\Phi} \qquad \text{.....(A-9)}$$

$$C_2 = \frac{pL\ (pL - \sin pL)}{\Phi} \qquad \text{.....(A-10)}$$

$$C_3 = \frac{(pL)^2\ (1 - \cos pL)}{\Phi} \qquad \text{.....(A-11)}$$

and

$$\Phi = 2 - 2 \cos pL - pL \sin pL \qquad \text{.....(A-12)}$$

The C coefficients then are the parameters reflecting the effects of the axial thrust. Identical coefficients apply for both Equations (A-1) and (A-2) with the appropriate moment of inertia used in computation of $p^2 = \frac{P}{EI}$.

Similar coefficients result for the case of negative axial force (axial tension):

$$C_1 = \frac{pL\ (pL \cosh pL - \sinh pL)}{\Phi} \qquad \text{.....(A-13)}$$

$$C_2 = \frac{pL\ (\sinh pL - pL)}{\Phi} \qquad \text{.....(A-14)}$$

$$C_3 = \frac{(pL)^2\ (\cosh pL - 1)}{\Phi} \qquad \text{.....(A-15)}$$

where now

$$\Phi = 2 - 2 \cosh pL + pL \sinh pL \qquad \text{.....(A-16)}$$

A.1.2 Solution for Bowing Effect on True Length

The apparent axial deformation, ϵ, caused by bowing may be approximated by the expression

$$\epsilon = \frac{1}{2} \int_o^L \left(\frac{dy}{dx}\right)^2 dx \qquad \text{.....(A-17)}$$

Substituting the expression for $\frac{dy}{dx}$ in Equation (A-6), integrating over the beam length and substituting for M_I and S in terms of θ_1 and θ_2 from the stiffness expressions in Equations (A-7) and (A-8) yields

$$\epsilon = \frac{1}{4p\Phi^2} [\alpha^3 - \alpha^2 (2 \sin\alpha + \sin\alpha \cos\alpha)$$

$$+ \alpha(2 + 2 \cos\alpha - 4 \cos^2\alpha) - 2 \sin\alpha + 2 \sin\alpha \cos\alpha] \qquad \text{.....(A-18)}$$

$$(\theta_1{}^2 + \theta_2{}^2) + \frac{1}{2p\Phi^2} [- \alpha^3 \cos\alpha + 3\alpha^2 \sin\alpha + 6\alpha$$

$$(\cos\alpha - 1) + 2\sin\alpha (1 - \cos\alpha)] \theta_1\theta_2$$

where $\alpha = pL$.

The above formula is quite lengthy and some simplification is possible if the effect of the axial force is neglected. The general differential Equation (A-4) is simplified to

$$\frac{d^2y}{dx^2} = \frac{1}{EI} (Sx - M_I) \qquad \text{.....(A-19)}$$

This equation may be solved by direct integration to yield,

$$\frac{dy}{dx} = \frac{M_I}{EI} (x - \frac{L}{2}) + \frac{S}{EI} (\frac{L^2}{6} - \frac{x^2}{2}) \qquad \text{......(A-20)}$$

The end moments with neglect of axial force become linear functions of the end displacements:

$$M_I = \frac{4EI}{L}\theta_1 + \frac{2EI}{L}\theta_2 \qquad \text{.....(A-21)}$$

$$M_F = \frac{4EI}{L}\theta_2 + \frac{2EI}{L}\theta_1 \qquad \text{.....(A-22)}$$

These relationships are substituted into Equation (A-20). The resulting expression for $\frac{dy}{dx}$ in terms of the end rotations is substituted into Equation (A-17) to yield

$$\epsilon = \frac{L}{15}(\theta_1^2 + \theta_2^2) - \frac{L}{30}\theta_1\theta_2 \qquad \text{.....(A-23)}$$

The coefficients of $(\theta_1^2 + \theta_2^2)$ and $\theta_1\theta_2$ from the more exact expression (A-18) and the simplified expression (A-23) were evaluated for various values of axial load, and the variation is shown in Figure A.2. It is seen that the differences of these coefficients are rather small for axial force less than one-half of the Euler load.

APPENDIX 2

COMPUTER PROGRAM

A.2.1 General

The computer program utilizes a main program called CONTROL and ten subroutines. Four of the subroutines STATIC, KSTORE, MAELIM and MAINVERT deal only with the initial static solution of the frame. The subroutine MEMPRO computes the member stiffness matrices, rotation matrices, the incidence relations, and the joint loads due to the dead load on the members. This data, necessary for the static solution, is also used in the dynamic solution.

A single subroutine called RMATRX computes the inverted mass matrix. The subroutine SETUP utilizes two M.S.U. library plotter subroutines PLOT and CHAR and is used to draw and title appropriate coordinate axes for use with the computer plotter, CALCOMP 563, which is used as one method of data output. The remaining three subroutines, INTEGN, MEMREA and ROTATE are used for the dynamic solution.

The program CONTROL is the controlling section for the entire program. It is also used to read in control data for the dymanic solution and to plot and print output data from this solution. The subroutine STATIC accomplishes a similar function in the static solution. All basic information concerning the frame is read by the subroutine MEMPRO. All other data is read either by STATIC or CONTROL and all data output occurs in these two routines.

The processes of the static and dynamic solutions are unrelated except as noted above, in using certain basic frame information from MEMPRO. Consequently, the basic outlines of the two solutions are given separately in the following sections along with the primary operations that are performed in the various subroutines.

A.2.2 Dynamic Solution

The main program CONTROL using the parameters DTIME and LOOP (see definitions of variables in Section A.2.4) calls the integration subroutine INTEGN which then integrates for LOOP cycles and returns to CONTROL which prints and plots the designated displacements and the current time. This process is repeated JS times. Subroutine INTEGN performs the numerical integration. It also calls the subroutine MEMREA which computes the frame reaction on each joint for the current displacement configuration, and monitors for maximum and minimum forces and displacements. MEMREA in turn calls ROTATE which is used simply to rotate forces or displacements from member coordinates to global joint coordinates, or vice-versa.

A.2.3 Static Solution

The main subroutine STATIC is called from CONTROL. STATIC performs no signicant computation but calls MEMPRO, KSTORE and MAELIM, in this order, and prints the required output data. MEMPRO, as outlined previously, reads the basic frame data, computes member stiffness and rotation matrices and incidence data, and computes dead load-joint loads. KSTORE forms and stores the joint stiffness matrices. MAELIM accomplishes the solution of the linear equations for the static displacements, and computes the end forces. MAINVERT simply inverts 6 X 6 matrices as needed in the solution of the linear equations.

A.2.4 Variables used in Computer Program

The variable names used in the program are listed below in the order encountered in the program:

Program CONTROL

MEMBS = number of frame members;

JINTS = number of frame joints;

NJFREE = number of free (non-support) joints;

U(i,j) = displacement of joint i corresponding to j coordinate;

MSTIFF(i,j) = jth stiffness coefficient for member i;

MSTIFO(i,j) = jth stiffness coefficient for member i with no axial effect;

SEC = total number of seconds of integration;

JS = number of times subroutine INTEGN will be called;

LOOP = number of numerical integration steps performed each time INTEGN is called;

DTIME = Δt for numerical integration;

JOPL, JCOPL = joint number and coordinate whose displacement is to be plotted at each return to CONTROL;

JOPR, MFORPR = joint number and member number of the joint-displacements and member-forces to be printed at each return to CONTROL.

JOINT = JOPC in R format;

ICOOR = JCOPL in R format;

NDIS, NFOR = number of displacements and forces to be monitored for maximum and minimum;

JDIS(i), JCORD(i) = ith joint number and its coordinate of the displacement which is to be monitored;

JFOR(i), JCORF(i) = ith member number (negative if negative end forces are desired) and the coordinate of the force which is to be monitored;

DAMP = damping coefficient λ;

NOLIN = parameter which directs nonlinear analysis if it is 1;

ILOD(i) = the dynamic loading incidence parameter; if it is 1 loading is applied as input, if it is -1 loading negative of that input and if 0 no loading is applied to joint i;

IBE = 100th of a second time lag in loading;

IE(i) = value is 1 if the ith joint has a time lag in loading;

IQAKE = parameter to direct earthquake;

FAC = scaling factor that multiplies the magnitude of loading input;

RLD(i,j) = the jth load value (of 3) at ix0.01 seconds;

MM = parameter used to control dynamic loading; MM = 1 when time is 0 and increases by 1 each 0.01 second;

TIME = time;

DISM(i), TMAXD(i) = minimum value and time of occurrence for the ith displacement monitored;

DISL(i), TMIND(i) = the minimum value and time of occurence for the ith displacement monitored;

FORM(i), TMAXF(i) = the maximum value and time of occurrence for the ith force monitored;

FORL(i), TMINF(i) = the minimum value and the time of occurrence for the ith force monitored;

Subroutine MEMPRO

COORD(i,j) = jth structure global coordinate of joint i;

JP(i), JN(i) = positive and negative joint of member i;

MODUL, SMODUL = modulus of elasticity and shear modulus of the material used in the frame;

WEIGHT = weight of the material in the frame in kips per cu. ft.;

DEADL(i) = uniform superimposed dead load on member i in kips per inch;

LE = number of members with superimposed dead load;

DENSTY(i) = density in kip-in. units;

E(i), G(i) = modulus of elasticity and shear modulus for member i;

AREAX(i), AREAY(i), AREAZ(i) = the cross-sectional area and the shear areas for shear load along the y and z axes respectively;

IXX(i), IYY(i), IZZ(i) = J, I_y and I_z respectively for member i;

COMP(i,j) = projection of member i on the jth structure global axis;

LENGTH(i) = length (in inches) of member i;

COSX(i), COSY(i), COSZ(i) = X, Y, and Z projections of member i each divided by the member length;

L1 = a logical variable used to protect against dividing by zero in the computation of the rotation matrices;

ALPHA(i) = rotation of the y axis of member i about the x axis measured in decimal degrees from the vertical plane containing the x-axis;

RM(i,j,k) = 6 X 6 (j and k) rotation matrix for member i;

ALFA = ALPHA (i) in radians;

SHG1, SHG2 = g_z and g_y;

NMEM(i) = number of members incident to joint i;

MEMBER(i,j) = member number of the jth member incident to joint i; it is negative, if negatively incident;

STLOAD(i,j) = static load on joint i corresponding to the jth global coordinate;

DLOAD = total member dead load per inch due to member weight and any superimposed dead load.

FF(i) = ith component of the member end force at the member end acting on the joint being considered.

Subroutine INTEGN

LOOP = number of steps of numerical integration accomplished on one call to this subroutine;

MMC, MM1 = parameters used for loading control;

MMB, MMB1 = lagging loading control parameters;

BLOD(i) = lagging loading load increment for one integration interval for the ith coordinate;

RLOD(i) = ordinary load increment for one integration interval for the ith coordinate;

U(i,j) = displacement of joint i corresponding to the jth joint global coordinate;

UM(i,j) = joint displacement as above at the previous integration step;

UMM(i,j) = joint displacement as above at two steps preceding the current time;

UDDT(i,j) = acceleration of the ith joint mass corresponding to the jth joint global coordinate.

UDTM(i,j) = acceleration as above at the previous integration step;

REA(i,j) = {RE} for the ith joint mass and the jth coordinate;

RLOAD(i,j) = current dynamic load on joint i corresponding to joint coordinate j;

SJA(i) = acceleration in the ith coordinate direction of ordinary support joints in earthquake loading;

SJAL(i) = acceleration in the ith coordinate direction of the lagging support joints in earthquake loading;

SJD(i), STDL(i) = ordinary and lagging support joint displacements for ith coordinate at most recent integration step;

SJDM(i), SJDLM(i) = ordinary and lagging support joint displacements for ith coordinate at previous integration step;

SJDMM(i), SJDLMM(i) = ordinary and lagging support joint displacements for ith coordinate at two previous integration steps;

RSJD(i) = relative displacement of the lagging support joints to the ordinary support joints for the ith coordinate;

RESPON(i,j,k) = inverse of the 6 x 6 joint mass matrix ([RM]) for joint i;

STLOAD(i,j) = static load on joint i corresponding to coordinate j;

UDT(i,j) = velocity of joint i corresponding to coordinate j;

Subroutine MEMREA

JP(i) = positive joint of member i;

JN(i) = negative joint of member i;

DELY = Δ_2 for the member being considered;

DELZ = Δ_1 for the member being considered;

DD(i) = e_i;

PP(i) = ith component of $_I\{F\}$;

PPN(i) = ith component of $_F\{F\}$;

PRM(i,j) = [IRM];

RFOR(i) = ith component of $_I\{\tau\}$;

RFORN(i) = ith component of $_F\{\tau\}$;

FOR(i) = ith component of the initial-end member end forces in joint global coordinates;

FORN(i) = ith component of the final-end member end forces in joint global coordinates;

P(i) = the axial force on member i, positive for member compression;

SKLI1 = $\sqrt{P/EI_y} \times L$;

SKLI2 = $\sqrt{P/EI_z} \times L$;

COSH1, SINH1, COSH2, SINH2 = Cosh (SKLI1), etc.;

PHI1, PHI2 = Φ for I_y and I_z respectively, (see Appendix 1);

S1I1, S2I1, --- = C_{1y}, C_{2y}, --- etc. (see Appendix 1);

Subroutine RMATRX

MMASS(i) = mass of $\frac{1}{2}$ of member i;

JMASS = joint mass;

XBAR, YBAR, ZBAR = coordinates relative to the joint of the joint mass center;

X1, Y1, Z1 = coordinates relative to the joint of the mass center of the member being considered.

RINERM(i,j) = the mass inertia tensor for the member being considered;

RINER(i,j) = the mass inertia tensor for the total joint expressed at the mass center;

RINERI(i,j) = the inverse of RINER;

RESPON(i,j,k) = inverse of the 6 x 6 lumped mass for joint i in joint global coordinates;

Subroutine ROTATE

L = a control variable indicating whether the rotation is for forces or displacements;

II = a variable indicating the member number of the appropriate rotation matrix;

B(i,j) = 6 element vector of displacements or forces in joint global coordinates, for joint i, if displacements, for member i, if forces;

JJ = joint number being considered in a displacement rotation;

C(i) = 6 element vector of displacements or forces in member coordinates;

Subroutine SETUP

PLOT = an M.S.U. Computer Lab. Library subroutine used to control the CALCOMP 563 plotter;

CHAR = a similar library subroutine used to print letters on the plotter;

KP = the coordinate number of the displacement to be plotted;

JOINT = the joint number to be plotted in R format;

ICOOR = the coordinate number to be plotted in R format;

SY = 100 times the inverse of the scale on the Y (vertical plot) axis;

Subroutine KSTORE

IAA(i,j) = the "layer" (first index number) of the location in KSTOR of the 6 × 6 submatrices; if the submatrix is null, the value is zero if the submatrix is below the diagonal, the value is the negative of the location of the transposed matrix in the upper diagonal portion of [K];

KSTOR(i,j,k) = the ith 6 × 6 (j and k) nonzero submatrix in the upper triangular portion of [K].

IND = the number of nonzero 6 × 6 submatrices stored in KSTOR:

Subroutine MAELIM

RLD(i,j) = the static load on the ith joint in the jth coordinate direction;

TEMP(i) = a 6 element temporary storage matrix used in the reduction of the loading vectors;

EDIS(i) = the 6 element vector of linear relative end deformations in global coordinates;

GK(i,j) = the 6 x 6 member stiffness matrix in global coordinates;

Subroutine MAINVERT

B(i,j) = the 6 x 6 submatrix of [K] which is to be inverted;

A(i,j,k) = KSTOR(i,j,k);

II = the ith index of KSTOR designating the 6 x 6 submatrix to be inverted.

A.2.5 Computer Program

```
 PROGRAM CONTROL
 COMMON
1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
2  LENGTH(100),NMEM(50),MEMBER (50,10),
3                RESPON (50,6,6), MEMBS, JINTS, NJFREE, IND
1  /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
2  ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
1  /3/
2  RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
3  ,GFOR(100,6),GFORN(100,6)
1   /4/
2  MSTIFO (100,8), COSX(100),COSY(100),COSZ(100),G(100),
3  FLANGW(100),FLANGT(100),DEPTH(100),WEBT(100),AREAY(100),
4  AREAZ(100), IXX(100)
1  /5/
2  JDIS(20),JCORD(20),JFOR(20),JCORF(20),DISM(20),TMAXD(20)
3  ,DISL(20),TMIND(20), TMAXF(20),FORM(20),TMINF(20),FORL(20)
4  ,NDIS,NFOR,NOLIN
1  /6/
2          KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
3  ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
4  REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
5  UDDT(50,6), RLD(500,3)
6              ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
1  /7/  ILOD(50),IE(50),IBE
 EQUIVALENCE
1                (MSTIFO(1),G(1)),(MSTIFO(101),FLANGW(1)),
2  (MSTIFO(201),FLANGT(1)),(MSTIFO(301),DEPTH(1)),
3  (MSTIFO (401),WEBT(1)),(MSTIFO(501),AREAY(1)),
4  (MSTIFO(601),AREAZ(1)),(MSTIFO(701),IXX(1))
5 ,(KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
6  (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
7  (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
8  RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
```

```
      TYPE REAL  MSTIFF,LENGTH,KSTOR,IXX,IYY,IZZ,MSTIFO
C  ZERO DISPLACEMENTS OF SUPPORT JOINTS *  *  *  *  *  *  *  *  *  *  *
C  THIS REQUIRES THAT THE SUPPORT JOINTS BE NUMBERED LAST  *  *  *  *  *  *  *
      NFSJ = NJFREE + 1
      DO 1001  I = NFSJ,JINTS
      DO 1001  J = 1,6
      U (I,J) = 0.0
1001  CONTINUE
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL STATIC
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL RMATRX
C  PRINT RESPON  *  *  *  *  *  *  *  *  *  *
      PRINT 107
107   FORMAT (1H1,////*RESPONSIVENESS MATRICES*////)
      DO 108  J = 1,NJFREE
      PRINT 109,J
109   FORMAT(1X///*JOINT NUMBER*I3,///)
      DO 108 K = 1,6
108   PRINT 110,(RESPON(J,K,L), L = 1,6)
110   FORMAT(1X/6E20.4)
C  COMPUTE MSTIFF IN COORDINATES FOR DYNAMIC ANALYSIS  *  *  *  *
      DO 18  I = 1,MEMBS
      SHG1 = 6. * E (I) * IZZ (I) / (G(I) * AREAY (I) * LENGTH (I) ** 2.
     1    )
      SHG2 = 6. * E(I) * IYY (I) / (G (I) * AREAZ (I) * LENGTH(I)**2.)
      COMY = 1. + 2. * SHG2
      COMZ = 1. + 2. * SHG1
      MSTIFF (I,1) = AREAX (I) * E (I) / LENGTH (I)
      MSTIFF (I,2) = 6.*E(I)*IZZ(I)/LENGTH(I)**2  /COMZ
      MSTIFF(I,3) = 6.*E(I)*IYY(I)/LENGTH(I)**2  /COMY
      MSTIFF (I,4) = G (I) * IXX (I) / LENGTH (I)
      MSTIFF(I,5) = 4.*E(I)*IYY(I)/LENGTH(I)*((1.+SHG2/2.)/COMY)
      MSTIFF(I,6) = 2.*E(I)*IYY(I)/LENGTH(I)*((1.-SHG2)/COMY)
      MSTIFF(I,7) = 4.*E(I)*IZZ(I)/LENGTH(I)*((1.+SHG1/2.)/COMZ)
```

```
      MSTIFF(I,8) = 2.*E(I)*IZZ(I)/LENGTH(I)*((1.-SHG1)/COMZ)
18    CONTINUE
C    PRINT MEMBER STIFFNESS MATRICES  *  *  *  *  *  *  *  *  *
      PRINT 55
55    FORMAT (1H1, *MEMBER STIFFNESS MATRICES FOR DYNAMIC ANALYSIS*////)
      DO 57  IM = 1,MEMBS
57    PRINT 56, IM,(MSTIFF     (IM,IL), IL = 1,8)
56    FORMAT  (1H0, I5, //, 5X, 8E16.8)
C   STORE INITIAL MSTIFF VALUES  *  *  *  *  *  *  *  *  *  *
      DO 7  I = 1,MEMBS
      DO 8  J = 1,8
8     MSTIFO (I,J) = MSTIFF (I,J)
7     CONTINUE
C   INTEGRATION CONTROL PARAMETERS  *  *  *  *  *  *  *  * *  *  *  *  *  *
      READ 121, SEC, DTIME, LOOP
121   FORMAT (/,2F10.5,I5)
      JS = SEC/(LOOP*DTIME)
C   PRINT INTEG CONTROL PAPAMETERS
      PRINT  122, SEC, DTIME, LOOP,JS
122   FORMAT (1H1, *TOTAL INTEG TIME =*, F7.4,5X,*INTEG INTERVAL =*,
     1  F7.5,5X,*LOOP =*, I5, *      JS =*,I5)
C   READ INPUT DATA TO DESIGNATE PLOT AND PRINT OF DISPLACEMENTS *  *  * *
      READ 80, JOPL,JCOPL, JOPR,MFORPR
C   BLANK OR TITLE CARD MUST PRECEDE DATA
80    FORMAT (/,2I5,/,2I5)
C   SET UP PLOT SHEET    *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C   ENCODE TO TITLE PLOT SHEET
      ENCODE  (2,408,JOINT) JOPL
408   FORMAT (R2)
      ENCODE (2,409,ICOOR) JCOPL
409   FORMAT (R2)
      CALL SETUP
     1  (JCOPL,JOINT,ICOOR)
C   READ INPUT TO TELL WHICH JOINTS-DISP AND MEMBS-FORCE TO MONITOR *  *  *  *  *
C   BLANK OR TITLE CARD PRECEDES DATA
```

```
      READ 60, NDIS, NFOR
C  READ NUMBER OF DISP AND FORCES
60    FORMAT (/,2I5)
C  READ DISP OT BE MONITORED
      READ 61,((JDIS (I), JCORD(I)),I=1,NDIS)
61    FORMAT (/3(16I5/))
C  READ FORCES TO BE MONITORED
      READ 62,  ((JFOR(I),JCORF(I)), I = 1,NFOR)
62    FORMAT (/3(16I5/))
C  INPUT DAMPING COEFFICIENT   *  *  *  *  *  *  *  *  *  *  *  *  *  *
      READ 153, DAMP
153   FORMAT (/F20.10 )
      PRINT 158, DAMP
158   FORMAT (1H4,10X,*DAMPING COEFFICIENT =*,F10.5)
C  INPUT PARAMETER TO DIRECT NONLINEAR ANALYSIS  *  *  *  *  *  *  *  *
      READ 159, NOLIN
159   FORMAT (/I5)
      IF (NOLIN .NE. 1) GO TO 160
      PRINT 161
161   FORMAT (1H0,10X,*THE ANALYSIS IS NONLINEAR*)
      GO TO 162
160   PRINT 163
163   FORMAT (1H0,10X,*THE ANALYSIS IS LINEAR*)
162   CONTINUE
C  LOAD APPLICATION DATA  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      READ 75, (ILOD(I), I=1,NJFREE)
75    FORMAT (/,3(16I5/))
      PRINT 76, (I,ILOD(I),I=1,NJFREE)
76    FORMAT (1H1,*LOADING APPLICATION DATA*,///,10X,*JOINT*,10X,
     1  *IS LOAD APPLIED*,///,50(10X,I5,15X,I5,/))
C  READ EQUAKE TIME LAG *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      READ 666, IBE,(IE(J),J=1,50)
666   FORMAT (/I5/50I1)
      PRINT 667,IBE,(IE(J),J=1,50)
667   FORMAT (1H1,*EQUAKE TIME LAG =*,I5,*100 THS OF A SECOND ON JOINTS
```

```
     1  WITH A 1*///,4(16I5/))
C  INPUT LOADING *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      READ 151,IQAKE,FAC
151   FORMAT (/,I5,F10.5)
      IF (IQAKE .NE. 1)  GO TO 260
C  EQUAKE LOADING
      READ 152, ((RLD(M,J),J=1,3),M=1,500)
152   FORMAT (/3F10.5/250(6F11.6/))
      GO TO 261
C  BLAST LOADING
260   CONTINUE
      DO 262  I = 1,500
      DO 262  J = 1,3
262   RLD (I,J) = 0.0
      READ 263,  ((RLD(M,J),J=1,3),M=1,300)
263   FORMAT (/3F10.5/150(6F10.3/))
261   CONTINUE
      PRINT 101,((RLD(M,J),J=1,3),M=1,500)
101   FORMAT (1H1,*    LOADING MATRIX*,250(10X,3F10.5,10X,3F10.5/))
      IF (IQAKE .NE. 1) GO TO 155
      PRINT 154, FAC
154   FORMAT (1H4,10X,*EARTHQUAKE LOADING*,10X,*SCALE FACTOR =*,F10.5)
      GO TO 156
155   PRINT 157, FAC
157   FORMAT (1H4,10X,*BLAST LOADING*,10X,*SCALE FACTOR =*,F10.5)
156   CONTINUE
C   ZERO LOADING CONTROL AND TIME  *  *  *  *  *  *  *  *  *  *  *  *  *
      MM = 1
      TIME = 0.0
C  ZERO DISM, TMAXD,TMINC,DISL,ETC   *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 72  I = 1,20
72    DISM(I) = TMAXD(I) = DISL (I) = TMIND(I) = FORM(I)=TMAXF(I)=
     1  FORL(I)=TMINF(I) = 0.0
C  PRINT DISPLACEMENTS AND FORCES TO BE PRINTED TO TITLE PRINT OUTPUT *  *  *
      PRINT 81, JOPL, JCOPL, JOPR, MFORPR
```

```
81    FORMAT (1H4,10X,*JOINT AND COORD TO BE PLOTTED*,2I5,///,10X,
     1  *JOINT WHOSE DISPL IS PRINTED =*,I5,10X, *MEMBER WHOSE END FORCE
     2S ARE PRINTED =*,I5,////)
C  START INTEGRATION   *  *  *  *  *  *  *  *  *  *  * *  *  *  *  *  *  *
      DO 205  I = 1,JS
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL INTEGN
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL PLOT  (U(JOPL,JCOPL), TIME, 1)
      IF (MFORPR .GT. 0)  GO TO 43
      MFOR = -MFORPR
      PRINT 44, TIME, (U(JOPR,J), J = 1,6), (FORN(MFOR  ,K), K = 1,6)
      GO TO 42
43    CONTINUE
      PRINT 44, TIME, (U(JOPR,J), J = 1,6), ( FOR(MFORPR,K), K = 1,6)
42    CONTINUE
44    FORMAT (1X,F7.4,5X,6F9.4,5X,6F9.2)
205   CONTINUE
C  FINALIZE PLOT STRING  *  *  *  *  *  *  *  *  * *  *  *  *  *  *  *  *
      CALL PLOT (U(JOPL,JCOPL), TIME, -1)
C  PRINT MONITORED DISP AND FORCES    *  *  *  *  *  *  *  *  *  *  *  *  *
      PRINT 181
181   FORMAT (1H1,10X,*JOINT*,5X,*COORD*,5X,*MAX DISP AND TIME*,10X,
     1  *MIN DISP AND TIME*,///)
      DO 180  I = 1,NDIS
180   PRINT 182,JDIS(I),JCORD(I),DISM(I),TMAXD(I),DISL(I),TMIND(I)
182   FORMAT (1H0,9X,I5,5X,I5,5X,F10.5,F7.4,10X,F10.5,F7.4)
      PRINT 82
82    FORMAT(1H0,10X,*MEMBER*,5X,*COORD*,5X,*MAX FORCE AND TIME*,10X,
     1  *MIN FORCE AND TIME*,//)
      DO 83  I= 1,NFOR
83    PRINT 84,JFOR(I),JCORF(I),FORM(I),TMAXF(I),FORL(I),TMINF(I)
84    FORMAT (1H0,9X,I6,5X,I5,5X,F10.2,F8.4,10X,F10.2,F8.4)
      END
```

```
      SUBROUTINE MEMPRO
C     THIS CALCULATES MEMBER PROPERTIES FOR PRIZ, MEMBERS, FIXED JOINTS,
      LOGICAL L1
      REAL   LENGTH, MSTIFF, MODUL, IXX, IYY, IZZ
      COMMON
     1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH (100), NMEM (50), MEMBER (50,10), RESPON (50,6,6)
     3  , MEMBS, JINTS, NJFREE, IND
     1  /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
     2  ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
     1   /4/
     2  MSTIFO (100,8), COSX(100),COSY(100),COSZ(100),G(100),
     3  FLANGW(100),FLANGT(100),DEPTH(100),WEBT(100),AREAY(100),
     4  AREAZ(100), IXX(100)
     1  /6/
     2          KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
     3  ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4  REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5  UDDT(50,6), RLD(500,3)
     6             ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
      EQUIVALENCE
     1                (MSTIFO(1),G(1)),(MSTIFO(101),FLANGW(1)),
     2   (MSTIFO(201),FLANGT(1)),(MSTIFO(301),DEPTH(1)),
     3   (MSTIFO (401),WEBT(1)),(MSTIFO(501),AREAY(1)),
     4   (MSTIFO(601),AREAZ(1)),(MSTIFO(701),IXX(1))
     5 ,(KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
     6   (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7   (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8  RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
      DIMENSION  FF(6)
      READ 10, MEMBS, JINTS, NJFREE
10    FORMAT  (3I5)
C  READ JOINT STRUCTURAL COORDINATES
      DO 88  J = 1,JINTS
```

```
88    READ 13, JINDX, (COORD (JINDX,K) , K = 1,3)
13    FORMAT (I5, 3F10.3)
C   READ MEMBER PROPERTIES  *  *  *  *  *  *  *  *  *  *  *  *  *  * *
      DO 66  K = 1,MEMBS
66    READ18,I,JP(I),JN(I),ALPHA(I),DEPTH(I),FLANGW(I),FLANGT(I),WEBT(I)
18    FORMAT(3I5,5XF10.4,4F10.3)
      READ 3,MODUL,SMODUL,WEIGHT,SIGMA,TAU
3     FORMAT(2E10.3,/F10.5,/2E10.3)
      DO 76  I = 1,MEMBS
76    DEADL(I) = 0.0
      READ 77, LE
77    FORMAT (/I5)
      DO 79  K = 1,LE
79    READ 78, I, DEADL(I)
78    FORMAT (I5,F10.5)
      DO 28 I = 1,MEMBS
      E(I) = MODUL
      G(I) = SMODUL
      DENSTY(I) = WEIGHT/(1728.*386.4)
C     COMPUTE GEOMETRIC PROPERTIES OF CROSS-SECTION
      AREAX(I) = (DEPTH(I)-2.0*FLANGT(I))*WEBT(I)+2.*FLANGW(I)*FLANGT(I)
      HT = DEPTH(I) - FLANGT(I)
      AREAY(I) = WEBT(I)*HT
      AREAZ(I) = 5./3.*FLANGW(I)*FLANGT(I)
      IXX(I) = 1./3.*(HT*WEBT(I)**3+2.*FLANGW(I)*FLANGT(I)**3)
      IYY(I) = FLANGT(I)*FLANGW(I)**3/6.
      IZZ(I) = HT**2*(HT*WEBT(I) + 6.*FLANGW(I)*FLANGT(I))/12.
C   COMPUTE FRAME GEOMETRIC PROPERTIES
      DO 19 K=1,3
19    COMP(I,K) = COORD(JN(I),K) - COORD(JP(I),K)
      LENGTH(I) = SQRT(COMP(I,1)**2 + COMP(I,2)**2 + COMP(I,3)**2)
      COSX(I) = COMP(I,1)/LENGTH(I)
      COSY(I) = COMP(I,2)/LENGTH(I)
      COSZ(I) = COMP(I,3)/LENGTH(I)
C   COMPUTE ROTATION MATRICES *  *  *  *  *  *  *  *  *  *  *  *  *  *
```

```
      L1 = ABS(COSX(I)) .LE. 0.001 .AND. ABS(COSZ(I)) .LE. 0.001
      IF(ALPHA(I).LE.0.001.AND.L1) GO TO 20
      IF(ALPHA(I).LE.0.001.AND..NOT.L1) GO TO 22
      IF(ALPHA(I).GT.0.001.AND..NOT.L1) GO TO 23
      IF(ALPHA(I).GT.0.001.AND.L1) GO TO 24
20    DO 21 J=1,3
      DO 21 K=1,3
21    RM(I,J,K) = 0.
      RM(I,1,2) = COSY(I)
      RM(I,2,1) = -COSY(I)
      RM(I,3,3) = 1.
      GO TO 26
22    RDEN = SQRT(COSX(I)**2 + COSZ(I)**2)
      RM(I,1,1) = COSX(I)
      RM(I,1,2) = COSY(I)
      RM(I,1,3) = COSZ(I)
      RM(I,2,1) = -COSX(I)*COSY(I)/RDEN
      RM(I,2,2) = RDEN
      RM(I,2,3) = -COSY(I)*COSZ(I)/RDEN
      RM(I,3,1) = -COSZ(I)/RDEN
      RM(I,3,2) = 0.
      RM(I,3,3) = COSX(I)/RDEN
      GO TO 26
23    ALFA = ALPHA(I)/360.*2.*3.1416
      RDEN = SQRT(COSX(I)**2 + COSZ(I)**2)
      RM(I,1,1) = COSX(I)
      RM(I,1,2) = COSY(I)
      RM(I,1,3) = COSZ(I)
      RM(I,2,1) = -(COSX(I)*COSY(I)*COSF(ALFA)+COSZ(I)*SINF(ALFA))/RDEN
      RM(I,2,2) = RDEN*COSF(ALFA)
      RM(I,2,3) = -(COSY(I)*COSZ(I)*COSF(ALFA)-COSX(I)*SINF(ALFA))/RDEN
      RM(I,3,1) =  (COSX(I)*COSY(I)*SINF(ALFA)-COSZ(I)*COSF(ALFA))/RDEN
      RM(I,3,2) = -RDEN*SINF(ALFA)
      RM(I,3,3) =  (COSY(I)*COSZ(I)*SINF(ALFA)+COSX(I)*COSF(ALFA))/RDEN
      GO TO 26
```

```
24    ALFA = ALPHA(I)/360.*2.*3.1416
      DO 25 J=1,3
      DO 25 K=1,3
25    RM(I,J,K) = 0.
      RM(I,1,2) = COSY(I)
      RM(I,2,1) = -COSY(I)*COSF(ALFA)
      RM(I,2,3) = SINF(ALFA)
      RM(I,3,1) = COSY(I)*SINF(ALFA)
      RM(I,3,3) = COSF(ALFA)
C   COMPUTE MEMBER STIFFNESS COEFFICIENTS FOR STATIC ANALYSIS  *  *  *
26    DO 27 J=1,8
27    MSTIFF(I,J) = 0.
      SHG1 = 6. * E (I) * IZZ (I) / (G(I) * AREAY (I) * LENGTH (I) ** 2.
     1    )
      SHG2 = 6. * E(I) * IYY (I) / (G (I) * AREAZ (I) * LENGTH(I)**2.)
      MSTIFF (I,1) = AREAX (I) * E (I) / LENGTH (I)
      MSTIFF (I,2) = 12.*E(I)*IZZ(I)/LENGTH(I)**3/(1.+2.*SHG1)
      MSTIFF (I,3) = 12.*E(I)*IYY(I)/LENGTH(I)**3/(1.+2.*SHG2)
      MSTIFF (I,4) = G (I) * IXX (I) / LENGTH (I)
      MSTIFF(I,8)    = -MSTIFF(I,3)   *LENGTH(I)/2.
      MSTIFF(I,5)    =-MSTIFF(I,8)   *LENGTH(I)/ 3.*(2.+SHG2)
      MSTIFF(I,7)    = MSTIFF(I,2)   *LENGTH(I)/2.
      MSTIFF(I,6)    = MSTIFF(I,7)   *LENGTH(I)/ 3.*(2.+SHG1)
28    CONTINUE
C     THE FOLLOWING SORTS OUT FOR EACH JOINT ITS ASSOCIATED MEMBERS
      DO 30 J = 1,JINTS
30    NMEM(J) = 0
      DO 31 I = 1,MEMBS
      J = JP(I)
      NMEM(J) = NMEM(J) + 1
      L = NMEM(J)
      MEMBER(J,L) = I
      J = JN(I)
      NMEM(J) = NMEM(J) + 1
      L = NMEM(J)
```

```
31     MEMBER(J,L) = -I
C   COMPUTE JOINT LOADS DUE TO DEAD LOAD  *  *  *  *  *  *  *  *  *  *
       DO 897   J = 1,JINTS
       DO 898  L=1,6
898    STLOAD(J,L) = 0.0
       K = NMEM(J)
       DO 897  M = 1,K
C   COMPUTE DEAD LOAD OF MEMBER (J,M)
       I = MEMBER (J,M)
       IP = IABS(I)
       DLOAD = WEIGHT*AREAX(IP) /1728
      1  + DEADL(IP)
       ALFA = ALPHA(IP)/360.*2.*3.14159
       CGAMMA = SQRT (COSX(IP)**2 + COSZ(IP)**2)
       SGAMMA = COSY(IP)
C   COMPUTE FIXED END MEMBER FORCES ON POST END IN MEMB COORS *  *  *
       TEMP = LENGTH(IP) * 0.5 * DLOAD
       FF(1) = TEMP*SGAMMA
       FF(2) = TEMP*CGAMMA*COSF(ALFA)
       FF(3) = -TEMP*CGAMMA*SINF(ALFA)
       FF(4) = 0.0
       FF(5) = TEMP*LENGTH(IP)*SINF(ALFA)*CGAMMA/6.
       FF(6) = TEMP*LENGTH(IP)*COSF(ALFA)*CGAMMA/6.
C   IF MEMB IS NEG COMPUTE NEG END FORCES  *  *  *  *  *  *  *  *  *
       IF (I .GT. 0)  GO TO 896
       FF(5) = -FF(5)
       FF(6) = -FF(6)
C   CONTRIBUTION TO JOINT LOAD = -RMT * FF   *  *  *  *  *  *  *  *  *
896    DO 897  JJ = 1,3
       SUM = SUB = 0.0
       DO 899  KK = 1,3
       SUM =SUM + RM(IP,KK,JJ) * FF(KK)
899    SUB = SUB + RM(IP,KK,JJ) * FF(KK+3)
       STLOAD(J,JJ) = STLOAD(J,JJ) - SUM
       STLOAD(J,JJ+3) = STLOAD(J,JJ+3) - SUB
```

```
897   CONTINUE
      END


      SUBROUTINE INTEGN
C   THIS SUBROUTINE PERFORMS THE NUMERICAL INTEGRATION
      COMMON
     1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH(100),NMEM(50),MEMBER (50,10),
     3                  RESPON (50,6,6), MEMBS, JINTS, NJFREE, IND
     1  /3/
     2  RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
     3  ,GFOR(100,6),GFORN(100,6)
     1  /6/
     2          KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
     3  ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4  REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5  UDDT(50,6), RLD(500,3)
     6             ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
     1  /7/  ILOD(50),IE(50),IBE
      EQUIVALENCE
     5  (KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
     6  (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7  (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8  RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
      DIMENSION  RLOD(6),BLOD(6)
     1  ,SJA(3),SJAL(3),SJDMM(3),SJDM(3),SJDLMM(3),SJDLM(3),RSJD(3)
     2  ,SJD(3),SJDL(3)
      TYPE REAL  KSTOR,KDU,MSTIFF,LENGTH,IYY,IZZ,MSTIFO
C   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DTSQ = DTIME**2
      DO 2000 IZ = 1,LOOP
```

```
C  LOADING CONTROL  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      MMC = TIME/0.01 + 1.
      IF (MMC-MM)  2019,2018,2018
2018  MM = MM+1
      MM1 = MM-1
      MMB = MM-IBE
      MMB1 = MMB - 1
      DO 2010  K = 1,3
      IF ((MMC-IBE) .GT. 0)  GO TO 200
      BLOD(K) = 0.0
      BLOD(4) = BLOD(5) = BLOD(6) = 0.0
      GO TO 2011
200   BLOD(K) = (RLD(MMB,K)-RLD(MMB1,K))/(0.01/DTIME)*FAC
2011  CONTINUE
2010  RLOD(K) = (RLD(MM,K)-RLD(MM1,K)) / (0.01/DTIME) * FAC
      RLOD(4) = RLOD(5) = RLOD(6) = 0.0
2019  CONTINUE
      IF (TIME .GT. 0.) GO TO 101
C  STARTING INTEGRATION PROCEDURE  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 100  I = 1,NJFREE
      DO 100  J = 1,6
C  TIME = 0
      UM(I,J) = U(I,J)
      UDT(I,J) = 0.0
      UDDT(I,J) = 0.0
C  U = U AT TIME 0
      REA(I,J) = STLOAD(I,J)
      IF (IE(I) .NE. 1) GO TO 205
      RLOAD (I,J) = 0.0
      GO TO 206
205   RLOAD(I,J) = RLD(1,J)*FAC+RLOD(J)
206   CONTINUE
      IF (ILOD(I))  301,302,303
301   RLOAD(I,J) = -RLOAD(I,J)
      GO TO 303
302   RLOAD (I,J) = 0.0
```

```
303   CONTINUE
100   CONTINUE
      TIME = DTIME
C  ZERO EQUAKE TIME LAG PARAMETERS
      DO 1005  K = 1,3
1005  SJDM(K) = SJD(K) =SJDLM(K)=SJDL(K) = RSJD(K) = SJA(K)=SJAL(K)=0.0
      GO TO 2000
101   CONTINUE
CNORMAL INTEGRATION PROCESS AFTER FIRST STEP *  *  *  *  *  *  *  *  *  *  *
C  RELATIVE SUPPORT JOINT DISPLACEMENT FOR EARTHQUAKE TIME LAG
      IF (IBE .EQ. 0)  GO TO 1010
      DO 1003  J =1,3
      SJA(J) = SJA(J) + RLOD(J)
      SJAL(J) = SJAL(J) + BLOD(J)
      SJDMM(J) = SJDM(J)
      SJDM(J) = SJD(J)
      SJDLMM(J) = SJDLM(J)
      SJDLM(J) = SJDL(J)
      SJD(J) = 2.*SJDM(J) - SJDMM(J) + DTSQ*SJA(J)
      SJDL(J) = 2.*SJDLM(J) - SJDLMM(J) + DTSQ*SJAL(J)
      RSJD(J) = SJDL(J)-SJD(J)
1003  CONTINUE
      NJ1 = NJFREE + 1
      DO 1002  I =NJ1,JINTS
      IF (IE(I) .NE. 1) GO TO 1002
      DO 1012  J = 1,3
1012  U(I,J) = RSJD(J)
1002  CONTINUE
1010  CONTINUE
C  ADVANCE U AND UDDT  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  * *  * *
      DO 105  I = 1,NJFREE
      DO 105  J = 1,6
      UDTM(I,J) = UDDT(I,J)
      UMM(I,J) = UM(I,J)
105   UM(I,J) = U(I,J)
C  SOLVE DYNAMIC EQUATION FOR UDDT AND INTEGRATE FOR U  *  *  *  *  *  *  *  *
      DO 107  I = 1,NJFREE
```

```
      DO 107  J = 1,6
C  EQUADE LOADING
      IF (IQAKE .NE. 1)  GO TO 108
      SUM = 0.0
      DO 109  K = 1,6
109   SUM = SUM + RESPON (I,J,K) * (STLOAD(I,K) - REA(I,K))
      UDDT(I,J) = SUM-RLOAD(I,J)-DAMP*UDT(I,J)
      GO TO 107
C  BLAST LOADING
108   SUM = 0.0
      DO 110  K = 1,6
110   SUM = SUM + RESPON(I,J,K) * (STLOAD(I,K) + RLOAD(I,K)-REA(I,K))
      UDDT(I,J) = SUM-DAMP*UDT(I,J)
C  INTEGRATION STATEMEMT
107   U(I,J) = 2.*UM(I,J)-UMM(I,J)+DTSQ*UDDT(I,J)
C  ADVANCE REA  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C  COMPUTE REA FOR PRESENT DISPL
      CALL MEMREA
C  ADVANCE RLOAD *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 115  I = 1,NJFREE
      DO 115  J = 1,6
      IF (ILOD(I))  305,306,307
305   CONTINUE
      IF (IE(I) .NE. 1) GO TO 207
      RLOAD(I,J) = RLOAD(I,J) - BLOD(J)
      GO TO 308
207   RLOAD (I,J) = RLOAD(I,J) - RLOD(J)
      GO TO 308
306   RLOAD(I,J) = 0.0
      GO TO 308
307   CONTINUE
      IF (IE(I) .NE. 1) GO TO 208
      RLOAD(I,J) = RLOAD(I,J) + BLOD(J)
      GO TO 308
208   RLOAD(I,J) = RLOAD(I,J) + RLOD(J)
```

```
308   CONTINUE
115   CONTINUE
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      IF (DAMP .EQ. 0.)  GO TO 2001
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
C  ADVANCE UDT   *  *  *  *  *  *  *  *  *  *  *  *  *  * *  * *  * *  * *  *
C    INTEGRATE FOR UDT
      DO 122  I = 1,NJFREE
      DO 122  J = 1,6
122   UDT(I,J) = UDT(I,J) + DTIME/2. * (UDTM(I,J) + UDDT (I,J))
2001  CONTINUE
C  INCREMENT TIME  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      TIME = TIME + DTIME
2000  CONTINUE
      END


      SUBROUTINE MEMREA
C  THIS SUBROUTINE COMPUTES THE MEMBER END FORCES CONSIDERING AXIAL THRUST
C AND GEOMETRY CHANGES
      COMMON
     1   /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH(100),NMEM(50),MEMBER (50,10),
     3                 RESPON (50,6,6), MEMBS, JINTS, NJFREE, IND
     1  /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
     2  ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
     1  /3/
     2  RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
     3  ,GFOR(100,6),GFORN(100,6)
     1  /4/
     2  MSTIFO (100,8), COSX(100),COSY(100),COSZ(100),G(100),
     3  FLANGW(100),FLANGT(100),DEPTH(100),WEBT(100),AREAY(100),
     4  AREAZ(100), IXX(100)
```

```
     1   /5/
     2   JDIS(20),JCORD(20),JFOR(20),JCORF(20),DISM(20),TMAXD(20)
     3   ,DISL(20),TMIND(20), TMAXF(20),FORM(20),TMINF(20),FORL(20)
     4   ,NDIS,NFOR,NOLIN
     1   /6/
     2           KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
     3   ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4   REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5   UDDT(50,6), RLD(500,3)
     6               ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
      EQUIVALENCE
     1                   (MSTIFO(1),G(1)),(MSTIFO(101),FLANGW(1)),
     2   (MSTIFO(201),FLANGT(1)),(MSTIFO(301),DEPTH(1)),
     3   (MSTIFO (401),WEBT(1)),(MSTIFO(501),AREAY(1)),
     4   (MSTIFO(601),AREAZ(1)),(MSTIFO(701),IXX(1))
     5  ,(KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
     6   (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7   (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8   RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9  (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
      DIMENSION
     1   RFOR(6),RFORN(6),RUI(6),RUF(6),DD(6),PP(6),PPN(6),P(100),
     2   PRM(3,3)
      TYPE REAL  KSTOR,KDU,MSTIFF,LENGTH,IYY,IZZ,MSTIFO
C  BEGIN
      DO 1  IC = 1,MEMBS
      JPI = JP(IC)
      JNI = JN(IC)
C  ROTATE U TO MEMBER COORDINATES * * * * * * * * * * * * * *
      CALL ROTATE (0,IC,JPI,RUI)
      CALL ROTATE (0,IC,JNI,RUF)
C  GENERATE RELATIVE END DEFORMATIONS  * * * * * * * * * * * * *
      DD(2) = RUI(4) - RUF(4)
      DELY = (RUI(3) - RUF(3)) / LENGTH(IC)
      DELZ = (RUI(2) - RUF(2)) / LENGTH(IC)
```

```
      DD(3) = RUI(5) - DELY
      DD(4) = RUI(6) + DELZ
      DD(5) = RUF(5) - DELY
      DD(6) = RUF(6) + DELZ
      IF (NOLIN .NE. 1) GO TO 100
      DD(1) = RUI(1) - RUF(1)
     1  - (0.5 * LENGTH(IC) * DELY**2 ) - (0.5 * LENGTH(IC) * DELZ**2 )
     2  - (LENGTH(IC)/30.) * (2.*DD(3)**2  - DD(3) * DD(5) +
     3  2. * DD(5)**2  + 2. * DD(4)**2  - DD(4) * DD(6) + 2. *DD(6)**2 )
      GO TO 101
100   DD(1) = RUI(1) - RUF(1)
101   CONTINUE
C  END FORCES IN ROTATED MEMBER COORD  *  *  *  *  *  *  *  *  *   *  *  *
      FOR(IC,1) = MSTIFF(IC,1) * DD(1)
      FOR(IC,2) = MSTIFF (IC,2) * (DD(4) + DD(6))
      FOR(IC,3) = -MSTIFF(IC,3) * (DD(3) + DD(5))
      FOR(IC,4) = MSTIFF(IC,4) * DD(2)
      FOR(IC,5) = MSTIFF(IC,5) * DD(3) + MSTIFF(IC,6) * DD(5)
      FOR(IC,6) = MSTIFF(IC,7) * DD(4) + MSTIFF(IC,8) * DD(6)
C  NEGATIVE END FORCES IN ROTATED MEMBER COORD  *  *  *  *  *  *  *  *  *
      DO 15  I = 1,4
15    FORN(IC,I) =-FOR(IC,I)
      FORN(IC,5) = -LENGTH(IC) *FOR(IC,3) -FOR(IC,5)
      FORN(IC,6) = LENGTH(IC) * FOR(IC,2) - FOR(IC,6)
      IF (NOLIN .NE. 1) GO TO 102
C  GENERATE PSEUDO ROTATION MATRIX  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 16  I = 1,3
16    PRM (I,I) = 1.
      PRM(1,2) = DELZ
      PRM(1,3) = DELY
      PRM(2,1) = -DELZ
      PRM(3,1) = -DELY
      PRM(2,3) = PRM(3,2) = 0.0
C  ROTATE TO MEMBER COORD  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 17  I = 1,3
```

```
      SUM = 0.0
      SUB = 0.0
      DO 18  J = 1,3
      SUM = SUM + PRM(I,J) * FOR (IC,J)
18    SUB = SUB + PRM(I,J) * FOR (IC,J+3)
      RFOR(I) = SUM
      RFOR(I+3) = SUB
      SUM = 0.0
      SUB = 0.0
      DO 19  J = 1,3
      SUM = SUM + PRM(I,J) * FORN(IC,J)
19    SUB = SUB + PRM(I,J) * FORN(IC,J+3)
      RFORN(I) = SUM
      RFORN(I+3) = SUB
17    CONTINUE
      GO TO 104
102   DO 103  I = 1,6
      RFOR(I) = FOR(IC,I)
      RFORN(I) = FORN(IC,I)
103   CONTINUE
104   CONTINUE
C  ROTATE RFOR AND RFORN TO GLOBAL COORD *  *  *  *  *  *  *  *  *  *  *
      CALL ROTATE (1,IC,1,RFOR)
      CALL ROTATE (-1,IC,1,RFORN)
      IF (NOLIN .NE. 1) GO TO 1205
C   CHANGE MSTIFF FOR AXIAL FORCE  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      IM = IC
      P(IM) = FOR(IM,1)
C  CHECK IF P GREATER THAN 0.001 OF EULER LOAD
      IF (ABSF (P(IM)) - 0.001  * 3.14159**2* E (IM) * IYY (IM)
     1  /LENGTH (IM)**2.)  1220,1220,1207
1220  DO 1221  I=1,8
1221  MSTIFF (IM,I) = MSTIFO (IM,I)
      GO TO 1205
C  COMPUTE S1I1 ETC THE CHANGE FACTORS
```

```
1207  SKLI1 = SQRT (ABS (P(IM)) / (E (IM) * IZZ (IM))) * LENGTH (IM)
      SKLI2 = SQRT (ABS (P(IM)) / ( E (IM) * IYY (IM))) * LENGTH (IM)
      IF (P (IM))   1212,1205,1210
1212  COSH1 = 0.5 * (EXP (SKLI1) + EXP (-SKLI1))
      COSH2 = 0.5 * (EXP (SKLI2) + EXP (-SKLI2))
      SINH1 = 0.5 * (EXP (SKLI1) - EXP (-SKLI1))
      SINH2 = 0.5 * (EXP (SKLI2) - EXP (-SKLI2))
      PHI1 = 2. - 2. * COSH1 + SKLI1 * SINH1
      PHI2 = 2. - 2. * COSH2 + SKLI2 * SINH2
CMODIFICATION FACTORS
      S1I1 = SKLI1 ** 2  * (COSH1 - 1.) / (6. * PHI1)
      S2I1 = SKLI1 * (SKLI1 * COSH1 - SINH1) / (4. * PHI1)
      S3I1 = SKLI1 * (SINH1 - SKLI1)/(2.*PHI1)
      S1I2 = SKLI2 ** 2  * (COSH2 - 1.) / (6. * PHI2)
      S2I2 = SKLI2 * (SKLI2 * COSH2 - SINH2) / (4. * PHI2)
      S3I2 = SKLI2 * (SINH2 - SKLI2)/(2.*PHI2)
      GO TO 1218
1210  PHI1 = 2. - 2. * COS (SKLI1) - SKLI1 * SIN (SKLI1)
      PHI2 = 2. - 2. * COS (SKLI2) - SKLI2 * SIN (SKLI2)
      S1I1 = SKLI1 ** 2  * (1. - COS (SKLI1)) / (6. * PHI1)
      S2I1 = SKLI1 * (SIN (SKLI1) - SKLI1 * COS (SKLI1)) / (4. * PHI1)
      S3I1 = SKLI1 * (SKLI1-SIN(SKLI1)) / (2. * PHI1)
      S1I2 = SKLI2 ** 2  * (1. - COS (SKLI2)) / (6. * PHI2)
      S2I2 = SKLI2 * (SIN (SKLI2) - SKLI2 * COS (SKLI2 )) / (4. * PHI2)
      S3I2 = SKLI2 * (SKLI2 - SIN(SKLI2)) / (2. * PHI2)
C    REVISE MSTIFF
1218  MSTIFF(IM,2) = MSTIFO (IM,2) * S1I1
      MSTIFF(IM,3) = MSTIFO (IM,3) * S1I2
      MSTIFF(IM,5) = MSTIFO(IM,5) * S2I2
      MSTIFF(IM,6) = MSTIFO(IM,6) * S3I2
      MSTIFF(IM,7) = MSTIFO(IM,7) * S2I1
      MSTIFF(IM,8) = MSTIFO(IM,8) * S3I1
1205  CONTINUE
1     CONTINUE
C  COMPUTE FRAME REACTION ON EACH JOINT   *  *  *  *  *  *  *  *  *  *  *
```

```
      DO 2 ID = 1,NJFREE
      DO 261  I = 1,6
261   REA(ID,I) = 0.0
      NM = NMEM (ID)
      DO 3  IJ = 1,NM
      IF (MEMBER (ID,IJ))  201,3,221
201   MEM = -MEMBER (ID,IJ)
      DO 211  I = 1,6
211   REA(ID,I) = REA(ID,I) +GFORN(MEM,I)
      GO TO 3
221   MEM = MEMBER (ID,IJ)
      DO 231  I = 1,6
231   REA(ID,I) = REA(ID,I) +GFOR(MEM,I)
3     CONTINUE
2     CONTINUE
C  MOMITOR DISP   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 72  I = 1,NDIS
      JOI = JDIS (I)
      JCOO = JCORD (I)
      IF (U(JOI,JCOO) .GT. DISM(I)) 73,74
73    DISM (I) = U(JOI,JCOO)
      TMAXD (I) = TIME
74    CONTINUE
      IF (U(JOI,JCOO) .LT. DISL(I)) 75,72
75    DISL (I) = U (JOI,JCOO)
      TMIND (I) = TIME
72    CONTINUE
C  MONITOR FORCES   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 76  I = 1,NFOR
      IF (JFOR(I) .GT. 0)  GO TO 85
      JOI = -JFOR(I)
      JCOO = JCORF (I)
      IF (FORN (JOI,JCOO) .GT. FORM(I))  82,83
82    FORM(I) = FORN(JOI,JCOO)
      TMAXF(I) = TIME
```

```
83    CONTINUE
      IF (FORN(JOI,JCOO) .LT. FORL(I))  84,86
84    FORL(I) = FORN(JOI,JCOO)
      TMINF(I) = TIME
86    GO TO 76
85    CONTINUE
      JOI = JFOR (I)
      JCOO = JCORF (I)
      IF (FOR(JOI,JCOO) .GT. FORM(I)) 77,78
77    FORM (I) = FOR (JOI,JCOO)
      TMAXF (I) = TIME
78    CONTINUE
      IF (FOR(JOI,JCOO) .LT. FORL(I)) 79,76
79    FORL(I) = FOR (JOI,JCOO)
      TMINF (I) = TIME
76    CONTINUE
2021  CONTINUE
      END


      SUBROUTINE RMATRX
C     THIS EVALUATES THE RESPONSIVENESS MATRIX OF THE SYSTEM
C   THIS SUBROUTINE REQUIRES THAT SUPPORT JOINTS BE NUMBERED LAST
      TYPE REAL    MSTIFF, LENGTH, KSTOR, IYY, IZZ
      TYPE REAL   JMASS, MMASS, MI3
      DOUBLE PRECISION RR,DET,RINER
      COMMON
     1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH(100),NMEM(50),MEMBER (50,10),
     3                RESPON (50,6,6), MEMBS, JINTS, NJFREE, IND
     1  /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
     2  ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
      DIMENSION
```

```
     1  XYZ (3,3), RINER (3,3), RINERJ (3,3), RINERM (3,3), RTJ (3,3),
     2  MMASS (100), RR (3,3), RINERI (3,3), XJINV (3,3),
     3  XJINVX (3,3), MI3 (3,3)
      EQUIVALENCE (RR(1),RINER(1))
      DO 11 J = 1,NJFREE
      NM = NMEM(J)
      JMASS = 0.
      XM = 0.
      YM = 0.
      ZM = 0.
      DO 12 N = 1,NM
      MK = MEMBER(J,N)
      I = IABS(MK)
      DO 10 K = 1,3
      DO 10 L = 1,3
10    RINER(K,L) = 0.
      MMASS(I) = DENSTY(I)*AREAX(I)*LENGTH(I)/2.
     1  + DEADL(I) * (LENGTH(I)/2.) / 386.4
      JMASS = JMASS + MMASS(I)
      IF(J.EQ.JP(I)) GO TO 13
      X1 = -COMP(I,1)/4.
      Y1 = -COMP(I,2)/4.
      Z1 = -COMP(I,3)/4.
      GO TO 14
13    X1 = COMP(I,1)/4.
      Y1 = COMP(I,2)/4.
      Z1 = COMP(I,3)/4.
14    XM = XM + MMASS(I)*X1
      YM = YM + MMASS(I)*Y1
12    ZM = ZM + MMASS(I)*Z1
      XBAR = XM/JMASS
      YBAR = YM/JMASS
      ZBAR = ZM/JMASS
      DO 15 K = 1,3
      DO 15 L = 1,3
15    XYZ(K,L) = 0.
```

```
      XYZ(1,2) = ZBAR
      XYZ(2,1) =-ZBAR
      XYZ(3,1) = YBAR
      XYZ(1,3) =-YBAR
      XYZ(2,3) = XBAR
      XYZ(3,2) =-XBAR
      DO 18 N = 1,NM
      MK = MEMBER(J,N)
      I = IABS(MK)
      IF(J.EQ.JP(I)) GO TO 16
      X1 = -COMP(I,1)/4.
      Y1 = -COMP(I,2)/4.
      Z1 = -COMP(I,3)/4.
      GO TO 17
16    X1 = COMP(I,1)/4.
      Y1 = COMP(I,2)/4.
      Z1 = COMP(I,3)/4.
17    XC1 = RM(I,1,1)*(X1-XBAR)+RM(I,1,2)*(Y1-YBAR)+RM(I,1,3)*(Z1-ZBAR)
      YC1 = RM(I,2,1)*(X1-XBAR)+RM(I,2,2)*(Y1-YBAR)+RM(I,2,3)*(Z1-ZBAR)
      ZC1 = RM(I,3,1)*(X1-XBAR)+RM(I,3,2)*(Y1-YBAR)+RM(I,3,3)*(Z1-ZBAR)
      RINERM(1,1) = 0.5*LENGTH(I)*DENSTY(I)*(IYY(I)+IZZ(I))
     1              + MMASS(I)*(YC1**2+ZC1**2)
      RINERM(1,2) = -MMASS(I)*YC1*XC1
      RINERM(2,1) = RINERM(1,2)
      RINERM(1,3) = -MMASS(I)*XC1*ZC1
      RINERM(3,1) = RINERM(1,3)
      RINERM(2,2) = 0.5*LENGTH(I)*DENSTY(I)*IYY(I)+2./3.*(LENGTH(I)/4.)
     1              **3*DENSTY(I)*AREAX(I)+MMASS(I)*(XC1**2+ZC1**2)
     2              + 2./3.*(LENGTH(I)/4.)** 3*DEADL(I)/386.4
      RINERM(2,3) = -MMASS(I)*YC1*ZC1
      RINERM(3,2) = RINERM(2,3)
      RINERM(3,3) = 2./3.*(LENGTH(I)/4.)**3*DENSTY(I)*AREAX(I) + 0.5*
     1        LENGTH(I) *DENSTY(I)*IZZ(I)+MMASS(I)*(XC1**2+YC1**2)
     2              + 2./3.*(LENGTH(I)/4.)** 3*DEADL(I)/386.4
      DO 18 K = 1,3
```

```
      DO 32 L = 1,3
      RTJ(K,L) = 0.
      DO 32 M = 1,3
32    RTJ(K,L) = RTJ(K,L) + RM(I,M,K)*RINERM(M,L)
      DO 18 L = 1,3
      RINERJ(K,L) = 0.
      DO 34 M = 1,3
34    RINERJ(K,L) = RINERJ(K,L) + RTJ(K,M)*RM(I,M,L)
18    RINER(K,L) = RINER(K,L) + RINERJ(K,L)
      DET=RR(1,1)*(RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))-RR(1,2)*(RR(2,1)*RR
     1(3,3)-RR(3,1)*RR(2,3))+RR(1,3)*(RR(2,1)*RR(3,2)-RR(3,1)*RR(2,2))
      RINERI(1,1) = (RR(2,2)*RR(3,3)-RR(2,3)*RR(3,2))/DET
      RINERI(2,2) = (RR(1,1)*RR(3,3)-RR(3,1)*RR(1,3))/DET
      RINERI(3,3) = (RR(1,1)*RR(2,2)-RR(2,1)*RR(1,2))/DET
      RINERI(2,1) = (RR(3,1)*RR(2,3)-RR(2,1)*RR(3,3))/DET
      RINERI(3,1) = (RR(2,1)*RR(3,2)-RR(3,1)*RR(2,2))/DET
      RINERI(3,2) = (RR(1,2)*RR(3,1)-RR(3,2)*RR(1,1))/DET
      RINERI(1,2) = RINERI(2,1)
      RINERI(1,3) = RINERI(3,1)
      RINERI(2,3) = RINERI(3,2)
C     THE RESPONSIVENESS MATRIX 'RESPON'
      DO 22 K = 1,3
      DO 35 L = 1,3
      XJINV(K,L) = 0.
      DO 35 M = 1,3
35    XJINV(K,L) = XJINV(K,L) + XYZ(K,M)*RINERI(M,L)
      DO 22 L = 1,3
      XJINVX(K,L) = 0.
      DO 22 M = 1,3
22    XJINVX(K,L) = XJINVX(K,L) + XJINV(K,M)*XYZ(M,L)
      DO 24 K = 1,3
      DO 24 L = 1,3
      MI3(K,L) = 0.
      IF(K.EQ.L) MI3(K,L) = 1./JMASS
24    CONTINUE
```

```
      DO 25 K = 1,3
      DO 25 L = 1,3
25    RESPON(J,K,L) = MI3(K,L) - XJINVX(K,L)
      DO 26 K = 1,3
      DO 26 L = 4,6
      LL = L-3
26    RESPON(J,K,L) = -XJINV(K,LL)
      DO 27 K = 4,6
      KK = K-3
      DO 27 L = 1,3
27    RESPON(J,K,L) = -XJINV(L,KK)
      DO 28 K = 4,6
      KK = K-3
      DO 28 L = 4,6
      LL = L-3
28    RESPON(J,K,L) = RINERI(KK,LL)
11    CONTINUE
      RETURN
      END


      SUBROUTINE ROTATE
     1  (L,II,JJ,C)
C   THIS ROTATES SPECIFIED DISP AND FORCE VECTORS
      DIMENSION  C(6)
      COMMON
     1     /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH(100),NMEM(50),MEMBER (50,10),
     3                 RESPON (50,6,6), MEMBS, JINTS, NJFREE, IND
     1  /3/
     2  RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
     3  ,GFOR(100,6),GFORN(100,6)
C
```

```
      IF (L .NE. 0) GO TO 2
C   ROTATE GLOBAL DISP TO MEMBER COORD
4     DO 1  I=1,3
      SUM = SUB = 0.0
      DO 1  J = 1,3
      SUM = SUM + RM(II,I,J) * U(JJ,J)
      SUB = SUB + RM(II,I,J) * U(JJ,J+3)
      C(I) = SUM
      C(I+3) = SUB
1     CONTINUE
      GO TO 5
C   ROTATES MEMBER FORCES TO GLOBAL COORD
2     DO 3  I = 1,3
      SUM = SUB = 0.0
      DO 6 J = 1,3
      SUM = SUM + RM(II,J,I) * C(J)
6     SUB = SUB + RM(II,J,I) * C(J+3)
      IF (L .EQ. -1)  GO TO 7
      GFOR(II,I) = SUM
      GFOR(II,I+3) = SUB
      GO TO 3
7     GFORN(II,I) = SUM
      GFORN(II,I+3) = SUB
3     CONTINUE
5     CONTINUE
      END


      SUBROUTINE SETUP
     1  (KP,JOINT,ICOOR)
C     SET UP THE PLOT SHEET
C   GET PEN CONTROL
      CALL PLOT (0.0, 0.0, 0, 100., 100.)
```

```
C    LEFT INDEX PEN
      CALL PLOT (0.0,-30.0,2)
C   ESTABLISH ZERO
      CALL PLOT (0.0,0.0,0)
C   TITLE SHEET
      CALL CHAR (1.30,1.30,6HJOINT ,6,0.0,0.15,0.15)
      CALL CHAR (1.30,2.425,JOINT,2)
      CALL CHAR (1.0,1.30,6HCOORD ,6)
      CALL CHAR (1.0,2.425,ICOOR, 2)
C   PLOT LOWER HALF Y AXIS
      CALL PLOT (0.5,0.8,2)
      CALL PLOT (4.0,0.8,1)
C   TITLE Y AXIS
      CALL CHAR (1.10,0.5,10H-    DISP   , 10,90.0)
      IF (KP - 3)  100,100,101
100   CALL CHAR (2.975,0.5,20H(4 IN D = 1 IN)    +,20)
      SY = 25.0
      GO TO 99
101   CALL CHAR (2.975,0.5,22H(0.05 RAD = 2 IN)    +,22)
      SY = 4000.
99    CONTINUE
C   PLOT UPPER HALF OF Y AXIS
      CALL PLOT (7.5,0.8,2)
      CALL PLOT (4.0,0.8,1)
C   PLOT X AXIS
      CALL PLOT (4.0,10.8,1)
C   TITLE X AXIS
      CALL CHAR (3.5,8.0,14H  1 SEC = 1 IN,14,0.0)
C   MARK X AXIS
      DO 102  J = 1,10
      R = J
      S = 11.8-R
      CALL PLOT (3.8,S,2)
102   CALL PLOT (4.2,S,1)
C   INITIALIZE FOR PLOTTING
```

```
      CALL PLOT (4.0,0.8,2)
      CALL PLOT (0.0,0.0,0.SY,100.)
      END


      SUBROUTINE STATIC
C   THIS SUBROUTINE CONTROLS THE STATIC SOLUTION
      DIMENSION  SLOAD(50,6)
      COMMON
     1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH (100), NMEM (50), MEMBER (50,10), RESPON (50,6,6)
     3  , MEMBS, JINTS, NJFREE, IND
     1  /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
     2  ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
     1  /3/
     2  RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
     3  ,GFOR(100,6),GFORN(100,6)
     1   /4/
     2  MSTIFO (100,8), COSX(100),COSY(100),COSZ(100),G(100),
     3  FLANGW(100),FLANGT(100),DEPTH(100),WEBT(100),AREAY(100),
     4  AREAZ(100), IXX(100)
     1  /6/
     2          KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
     3  ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4  REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5  UDDT(50,6), RLD(500,3)
     6             ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
      EQUIVALENCE
     1                 (MSTIFO(1),G(1)),(MSTIFO(101),FLANGW(1)),
     2  (MSTIFO(201),FLANGT(1)),(MSTIFO(301),DEPTH(1)),
     3  (MSTIFO (401),WEBT(1)),(MSTIFO(501),AREAY(1)),
     4  (MSTIFO(601),AREAZ(1)),(MSTIFO(701),IXX(1))
     5 ,(KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
```

```
     6  (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7  (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8  RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
      TYPE REAL    MSTIFF, LENGTH, KSTOR, IYY, IZZ
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL MEMPRO
      PRINT 11, MEMBS, JINTS, NJFREE
11    FORMAT (1H0, *NUMBER OF MEMBERS = *,I4, 10X, *NUMBER OF JOINTS = *
     1  , I4, 10X, *NUMBER OF FREE JOINTS = *, I4)
      PRINT 12
12    FORMAT (1H4, 10X, *JOINT COORDINATES*, //, 10X, *JOINT NUMBER*,
     1   10X, *X-COORD*, 10X, *Y-COORD*, 10X, *Z-COORD*, //)
      PRINT 15, (JINDX, (COORD (JINDX,K), K=1,3), JINDX = 1,JINTS)
15    FORMAT (17X, I5, 3F17.3,//)
      PRINT 16
16    FORMAT(///*MEMBER*5X*JP*5X*JN*7X*ALPHA*15X*DEPTH*11X*FLANGE  WIDTH
     1*7X*FLANGE THICKNESS*5X*WEB THICKNESS*/)
      DO 17 K = 1,MEMBS
17    PRINT2,K,JP(K),JN(K),ALPHA(K),DEPTH(K),FLANGW(K),FLANGT(K),WEBT(K)
2     FORMAT(I4,6XI3,4XI3,5XF8.4,12XF8.3,13XF8.3,2(12XF8.3))
      PRINT 5
5     FORMAT(///2X*I*1X*DENSITY*3X*LENGTH*3X*AREAX*4X*AREAY*4X*AREAZ*5X*
     1IXX*6X*IYY*6X*IZZ*)
      DO 198   I = 1,MEMBS
198   PRINT 6,I,DENSTY(I),LENGTH(I),AREAX(I),AREAY(I),AREAZ(I),IXX(I),
     1  IYY (I), IZZ (I)
6     FORMAT (I3,F8.5,3X,F6.2,2X,F6.2,4X,F5.2,3X,F6.2,4X,F5.2,3X,F7.2,
     1   1X,F7.2)
C  PRINT ROTATION MATIRCES
      PRINT 51
51    FORMAT (1H1, *ROTATION MATRICES* ////)
      DO 53    M = 1,MEMBS
53    PRINT 52, M, ((RM (M,J,K), K = 1,3), J = 1,3)
52    FORMAT  (5X, I5, // , 3(3E20.8  //))
```

```
C    PRINT MEMBER STIFFNESS MATRICES
      PRINT 55
55    FORMAT (1H1, *MEMBER STIFFNESS MATRICES*, ////)
      DO 57  IM = 1,MEMBS
57    PRINT 56, IM,(MSTIFF     (IM,IL), IL = 1,8)
56    FORMAT  (1H0, I5, //, 5X, 8E16.8)
C   PRINT MEMBER INCIDENCE INFORMATION
      DO 32 JY = 1,JINTS
      NM = NMEM(JY)
32    PRINT 33,JY,NMEM(JY),(MEMBER(JY,L),L=1,NM)
33    FORMAT(1X/////*JOINT NUMBER =*I3///*TOTAL NUMBER OF MEMBERS CONNEC
     1TED TO THIS JOINT =*I3///*THE MEMBER NUMBERS CONNECTED TO THIS JOI
     1NT ARE AS FOLLOWS WHERE + DENOTES POSITIVELY CONNECTED AND - DENOT
     1ES NEGATIVELY CONNECTED*//20I6)
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL KSTORE
C   PRINT NUMBER OF NONZERO 6X6 ELEMENTS IN KSTOR = IND
      PRINT 100, IND
100   FORMAT (1H1, 10X, *NUMBER OF 6X6 ELEMENTS IN KSTOR = *, I5)
C
C   PRINT IAA
      PRINT 102
      DO 99, I = 1,JINTS
99    PRINT 101, (IAA (I,J) , J = 1,JINTS)
101   FORMAT  (/, (20I6))
102   FORMAT  (1H4, 9X,*IAA MATRIX*////)
C   PRINT KSTOR
      PRINT 105
      DO 98, K = 1,IND
      PRINT 106 ,K
98    PRINT 104, ((KSTOR (K,L,M) , M = 1,6) L = 1,6)
104   FORMAT  (6 (5X, 6(3X, E16.8)//))
106   FORMAT  (//// 5X,  I5)
105   FORMAT  (1H1, 9X, *KSTOR ELEMENTS*////)
C   READ STATIC LOADS   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
```

```
      DO 788  I = 1,NJFREE
      DO 788  K = 1,6
788   SLOAD (I,K) = 0.0
C  READ NUMBER OF LOADED JOINTS
      READ 790, NLJ
790   FORMAT (/I5)
C  READ A) JOINT NUMBER  B) LOAD CORRESP TO CORD 1 THRU 6
C   ONLY LOADED JOINTS NEED BE LISTED
C
      DO 791  J = 1,NLJ
791   READ 789, K, ( SLOAD (K,L), L= 1,6)
789   FORMAT (I5, 6F10.3)
C  PRINT BEAM DEADLOADS  *  *  *  *  *  *   *  *  *  *  *  *  *  *  *  *  *
      PRINT 80,(I,DEADL(I),I=1,100)
80    FORMAT (1H1,////,10X,*DISTRIBUTED DEADLOAD ON BEAMS*,////,
     1  40(3(10X,I5,5X,F10.5)/))
C  PRINT JOINT STATIC LOADS  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      PRINT 794
794   FORMAT (1H1,////, 10X, *STATIC JOINT LOADS*////)
      DO 792  L = 1,NJFREE
792   PRINT 793, L, ( SLOAD (L,J), J = 1,6)
793   FORMAT (1H0, I5, 10X, 6F10.3)
C  PRINT JOINT DEADLOADS  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      PRINT 784
784   FORMAT (1H1,////,10X,*DEADLOAD ON JOINTS*////)
      DO 782  L=1,NJFREE
782   PRINT 783,L,(STLOAD(L,J),J=1,6)
783   FORMAT (1H0,I5,10X,6F10.3)
C  SUM STATIC AND DEAD LOADS   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 795  I = 1,NJFREE
      DO 795  J = 1,6
795   STLOAD (I,J) = STLOAD(I,J) + SLOAD(I,J)
C  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL MAELIM
C  PRINT DISPLACEMENTS   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
```

```
      PRINT 587
587   FORMAT (1H1,////,10X,*STATIC DISPLACEMENTS*,////)
      DO 588  I = 1,NJFREE
588   PRINT  589, I, (U(I,J), J = 1,6)
589   FORMAT (1H0,I5,10X,6E18.6)
C  PRINT END FORCES IN MEMBER COORDS   *  *  *  *  *  *  *  *  *  * *
      PRINT 24
24    FORMAT  (1H1, //*MEMBER END FORCES IN MEMBER COORD*////)
      DO 26 J = 1,MEMBS
26    PRINT 25, J, ( FOR (J,LLI), LLI = 1,6)
25    FORMAT (1H0, I5, 10X, 6E16.8)
      END


      SUBROUTINE KSTORE
C THIS FORMS THE 6X6 ELEMENTS OF THE JOINT STIFFNESS MATRIX AND RECORDS LOCAT
      COMMON
     1    /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2  LENGTH (100), NMEM (50), MEMBER (50,10), RESPON (50,6,6)
     3  , MEMBS, JINTS, NJFREE, IND
     1  /6/
     2          KSTOR (150,6,6), IAA (50,50), STLOAD (50,6)
     3  ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4  REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5  UDDT(50,6), RLD(500,3)
     6              ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
      EQUIVALENCE
     5  (KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
     6  (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7  (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8  RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),RLD(1))
      TYPE REAL   MSTIFF, LENGTH, KSTOR
```

```
      TYPE INTEGER   P, PP
C   IAA IS LOCATION MATRIX AND KSTOR IS THE STORAGE MATRIX
C   BEGIN
      DO 601   IA = 1,JINTS
      DO 601   JA = 1,JINTS
601   IAA (IA,JA) = 0
      PP = 0
      DO 901    JG = 1, JINTS
      PP = PP + 1
      P = PP
      DO 501   J = 1,6
      DO 501   K = 1,6
501   KSTOR (P,J,K) = 0.
      IAA (JG, JG) = P
      IGF = NMEM (JG)
 701  DO 901 IG = 1,IGF
      L = MEMBER (JG, IG)
      IF  (L)  801, 801, 803
 801  L = - L
C   CONTRIB TO DIAG TERM BY NEG MEMBER  *  *  *  *  *  *  *  *  *  *
      DO 1001  I = 1,3
      DO 1001,  J = 1, 3
      KSTOR (P,I,J) = KSTOR (P,I,J) + RM (L,1,I) * MSTIFF (L,1) * RM
     1  (L,1,J) + RM (L,2,I) * MSTIFF (L,2) * RM (L,2,J) + RM (L,3,I) *
     2 MSTIFF (L,3) * RM (L,3,J)
      KSTOR (P,I,J+3) = KSTOR (P,I,J+3) + ((-RM(L,2,I) * MSTIFF (L,2)
     1 * RM(L,3,J) + RM(L,3,I) * MSTIFF(L,3) * RM(L,2,J)) * LENGTH
     2  (L) + RM (L,2,I) * MSTIFF (L,7) * RM (L,3,J) + RM (L,3,I)
     3  * MSTIFF (L,8) * RM (L,2,J))
      KSTOR (P,I+3,J) = KSTOR (P,I+3,J) + (-RM (L,2,J) * MSTIFF (L,2)
     1  * RM (L,3,I) + RM (L,3,J) * MSTIFF (L,3) * RM (L,2,I)) * LENGTH
     2  (L) + RM(L,2,J) * MSTIFF (L,7) * RM (L,3,I)+RM(L,3,J) * MSTIFF
     3  (L,8) * RM (L,2,I)
 1001 KSTOR (P,I+3,J+3) = KSTOR (P,I+3,J+3) + (RM (L,3,I) * MSTIFF
     1  (L,2) * RM (L,3,J) + RM(L,2,I) * MSTIFF(L,3) * RM(L,2,J)) *
```

```
     2  LENGTH (L) ** 2. + (- RM (L,3,I) * MSTIFF (L,7) * RM (L,3,J)
     3  + RM (L,2,I) * MSTIFF (L,8) * RM (L,2,J)) * LENGTH (L) +(-RM
     4  (L,3,J) * MSTIFF (L,7) * RM (L,3,I) + RM (L,2,J) * MSTIFF (L,8)
     5  * RM (L,2,I)) * LENGTH (L) + RM (L,1,I) * MSTIFF(L,4) * RM
     6  (L,1,J) + RM (L,2,I) * MSTIFF (L,5) * RM (L,2,J) + RM (L,3,I)
     7  * MSTIFF (L,6) * RM (L,3,J)
C  FIND POSITIVE JOINT OF NEG MEMBER
C
      M = JP (L)
      IF (M - JG)  901,901,703
C  SET LOCATION OF OFF DIAG ELEMENT IN IAA   *   *   *   *   *   *   *   *   *
703   PP = PP + 1
      IAA (JG, M) = PP
      IAA (M, JG) = -PP
C  OFF DIAG TERM IN UPPER RH 1/2 FOR NEG MEMBER    *   *   *   *   *   *   *
      DO 1003 IC = 1,3
      DO 1003 JC = 1,3
      KSTOR (PP,IC,JC) = -(RM (L,1,IC) * MSTIFF (L,1) * RM (L,1,JC) +
     1  RM (L,2,IC) * MSTIFF (L,2) * RM (L,2,JC) + RM (L,3,IC) * MSTIFF
     2(L,3) * RM (L,3,JC))
      KSTOR (PP,IC,JC+3) = -(RM(L,2,IC) * MSTIFF (L,7) * RM(L,3,JC) +
     1  RM (L,3,IC) * MSTIFF (L,8) * RM(L,2,JC))
      KSTOR ( PP,IC+3,JC) = -(( - RM (L,2,JC) * MSTIFF (L,2) * RM
     1  (L,3,IC) + RM (L,3,JC) * MSTIFF (L,3) * RM (L,2,IC)) * LENGTH
     2  (L) + RM (L,2,JC) * MSTIFF (L,7) * RM (L,3,IC) + RM (L,3,JC) *
     3  MSTIFF (L,8) * RM (L,2,IC))
 1003 KSTOR (PP,IC+3,JC+3) = -((-RM(L,3,IC) * MSTIFF (L,7) * RM (L,3,JC)
     1  + RM (L,2,IC) * MSTIFF (L,8) * RM (L,2,JC)) * LENGTH (L) + RM
     2  (L,1,JC) * MSTIFF (L,4) * RM (L, 1,IC) + RM(L,2,JC) * MSTIFF
     3  (L,5) * RM (L,2,IC) + RM (L, 3,JC) * MSTIFF (L, 6) * RM (L,3,IC)
     4  )
      GO TO 901
C  THIS IS CONTRIBUTION TO DIAGONAL ELEMENT BY POSITIVE MENBER
 803  DO 1002 IB = 1,3
      DO 1002 JB = 1,3
```

```
      KSTOR (P,IB,JB) = KSTOR (P,IB,JB) + RM (L,1,IB) * MSTIFF (L,1)*RM
     1  (L,1,JB) + RM (L,2,IB) * MSTIFF (L,2) * RM (L,2,JB) + RM(L,3,IB)
     2  * MSTIFF (L,3) * RM (L,3,JB)
      KSTOR (P,IB,JB+3) = KSTOR (P,JB+3,IB) = KSTOR (P,IB,JB+3) + RM
     1  (L,2,IB) * MSTIFF (L,7) * RM (L,3,JB) + RM (L,3,IB) * MSTIFF
     2  (L,8) * RM (L,2,JB)
 1002 KSTOR (P,IB+3,JB+3) = KSTOR (P,IB+3,JB+3) + RM (L,1,IB) * MSTIFF
     1 (L,4) * RM (L,1,JB) + RM (L,2,IB) * MSTIFF (L,5) * RM (L,2,JB)
     2  + RM (L,3,IB) * MSTIFF (L,6) * RM (L,3,JB)
C  FIND NEG JOINT OF POST MEMBER *  *  *  *  *  *  *  *  *  *  *  *  *
      M = JN (L)
      IF (M-JG) 901,901,708
708   PP = PP + 1
      IAA (JG,M) = PP
      IAA (M,JG) = -PP
C  THIS IS OFF DIAG TERM IN UPPER RH 1/2 FOR POST MEMBER  *  *  *  *
      DO 1004   ID = 1,3
      DO 1004  JD = 1,3
      KSTOR (PP,ID,JD) = -(RM(L,1,ID) * MSTIFF (L,1) * RM (L,1,JD) + RM
     1  (L,2,ID) * MSTIFF (L,2) * RM (L,2,JD) + RM (L,3,ID) * MSTIFF
     2  (L,3) * RM (L,3,JD))
      KSTOR (PP,ID,JD+3) = - ((-RM (L,2,ID) * MSTIFF (L,2) * RM (L,3,JD)
     1  + RM (L,3,ID) * MSTIFF (L,3) * RM (L,2,JD)) *
     2  LENGTH (L) + RM (L,2,ID) * MSTIFF (L,7) * RM (L,3,JD) + RM
     3  (L,3,ID) * MSTIFF (L,8) * RM (L,2,JD))
      KSTOR (PP,ID+3,JD) = -(RM (L,2,JD) * MSTIFF (L,7) * RM (L,3,ID)
     1  + RM (L,3,JD) * MSTIFF (L,8) * RM (L,2,ID))
1004  KSTOR (PP,ID+3,JD+3) = - (( -RM(L,3,JD) * MSTIFF(L,7) * RM
     1  (L,3,ID) + RM (L,2,JD) * MSTIFF (L,8) * RM (L,2,ID)) * LENGTH
     2  (L) + RM (L,1,ID) * MSTIFF (L,4) * RM (L,1,JD) + RM (L,2,ID)
     3  * MSTIFF (L,5) * RM (L,2,JD) + RM (L,3,ID) * MSTIFF (L,6) *
     4  RM (L,3,JD))
 901  CONTINUE
      IND = PP
      END
```

```
      SUBROUTINE MAELIM
C  THIS SUBROUTINE PERFORMS GAUSSIAN ELIMATION AND BACK SUBSTITUTION
C  USING 6X6 SUBMATRICES AND SOLVES THE STATIC PROBLEM
      COMMON
     1   /1/ RM (100,3,3), MSTIFF (100,8), JP (100), JN (100),
     2 LENGTH (100), NMEM (50), MEMBER (50,10), RESPON (50,6,6)
     3 ,MEMBS,JINTS,NJFREE,IND
     1 /2/  ALPHA (100), E (100), COORD (100,3), IZZ (100), IYY (100)
     2 ,COMP(100,3),DENSTY(50),AREAX(100),DEADL(100)
     1 /3/
     2 RLOAD(50,6),U(100,6),UDT(50,6),REA (50,6),FOR(100,6),FORN(100,6)
     3 ,GFOR(100,6),GFORN(100,6)
     1 /6/
     2         KSTOR (150,6,6), IAA (50,50), RLD (50,6)
     3 ,UMM(50,6),UM(50,6),UDTMM(50,6),UDTM(50,6),REAMM(50,6),
     4 REAM(50,6),RLOAMM(50,6),RLOADM(50,6),PDT(50,6),RDT(50,6),
     5 UDDT(50,6), DLD(500,3)
     6            ,TIME,DTIME,LOOP,IQAKE,DAMP,MM,FAC
      EQUIVALENCE
     5 (KSTOR(1),UMM(1)),(KSTOR(301),UM(1)),(KSTOR(601),UDTMM(1)),
     6 (KSTOR(901),UDTM(1)),(KSTOR(1201),REAMM(1)),
     7 (KSTOR(1501),REAM(1)),(KSTOR(1801),RLOAMM(1)),(KSTOR(2101),
     8 RLOADM(1)),(KSTOR(2401),PDT(1)),(KSTOR(2701),RDT(1)),
     9 (KSTOR(3001),UDDT(1)),(KSTOR(3301),DLD(1))
      TYPE REAL  MSTIFF,KSTOR,LENGTH,IZZ,IYY
      DIMENSION
     1 B(6,6), IPC (6), TEMP(6)
     2 ,EDIS(6),GK(6,6)
      NOJ = NJFREE
      DO 100  K = 1,NOJ
CINVERT K,K ELEMENT   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      CALL MAINVERT (IAA(K,K),B)
C  ELIMINATION   *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      K1 = K+1
      DO 101  I= K1,NOJ
```

```
      DO 102  J = I,NOJ
      IF (IAA(K,I) .EQ. 0)  GO TO 101
      IF (IAA(K,J) .EQ. 0)  GO TO 102
      IF (IAA(I,J) .EQ. 0)  GO TO 110
      GO TO 111
110   IND = IND + 1
      IAA (I,J) = IND
      DO 112  L= 1,6
      DO 112  M= 1,6
112   KSTOR (IND,L,M) = 0.0
111   IAIJ = IAA (I,J)
      IAKI = IAA (K,I)
      IAKJ = IAA (K,J)
      DO 104  L= 1,6
      DO 104  M = 1,6
      SUM = 0.0
      DO 103  N= 1,6
      DO 103  NN = 1,6
103   SUM = SUM + KSTOR(IAKI,N,L)*B(N,NN)*KSTOR(IAKJ,NN,M)
      KSTOR (IAIJ,L,M) = KSTOR (IAIJ,L,M) - SUM
104   CONTINUE
102   CONTINUE
      DO 105 L = 1,6
      SUM = 0.0
      DO 106  N = 1,6
      DO 106  NN = 1,6
106   SUM = SUM + KSTOR (IAKI,N,L)*B(N,NN)*RLD(K,NN)
      RLD (I,L) = RLD (I,L) - SUM
105   CONTINUE
101   CONTINUE
100   CONTINUE
C    BACK  SUBSTITUTION   * * * * * * * * * * * * * * * * *
      DO 200  II = 1,NOJ
      JI = NOJ + 1 - II
      CALL MAINVERT (IAA(JI,JI),B)
```

```
      DO 208  K = 1,6
208   TEMP(K) = RLD (JI,K)
      JI1 = JI+1
      DO 210  J = JI1,NOJ
      IF (IAA(JI,J) .EQ. 0)  GO TO 210
      IAIJ = IAA (JI,J)
C   GENERATE  P - SUM OF A*U   *  *  *  *  *  *  *  *  *  * *  *  * *  * * * *
      DO 210  K = 1,6
      SUM = 0.0
      DO 209  L = 1,6
209   SUM = SUM + KSTOR(IAIJ,K,L)*U(J,L)
      TEMP(K) = TEMP(K) - SUM
210   CONTINUE
      DO 211 J=1,6
      SUM = 0.0
      DO 211 K = 1,6
      SUM = SUM + B(J,K) * TEMP(K)
211   U(JI,J) = SUM
200   CONTINUE
C  COMPUTE END FORCES   * *  *  *  *  *  *  *  *  *  *  *  *  *  *  *  *
      DO 69   I = 1,MEMBS
      JPI   = JP(I)
      JNI = JN(I)
      EDIS(1) =  U  (JPI,1) - ( U (JNI,1) - COMP(I,3) *  U  (JNI,5)
     1  + COMP (I,2) *  U  (JNI,6))
      EDIS (2) =  U  (JPI,2) - ( U (JNI,2) + COMP(I,3) *  U (JNI,4)
     1  - COMP(I,1) *  U (JNI,6))
      EDIS (3) =  U  (JPI,3) - ( U (JNI,3) - COMP(I,2) * U  (JNI,4)
     1  + COMP (I,1) *  U  (JNI,5))
      DO 70  J = 4,6
70    EDIS(J)    =  U  (JPI,J) -  U  (JNI,J)
      DO 65  J = 1,3
      DO 65  K = 1,3
      SUM = SUMM = SUMB = 0.0
      DO 66  L = 1,3
```

```
      SUM = SUM + RM (I,L,J) * MSTIFF(I,L) * RM (I,L,K)
      SUMM = SUMM + RM(I,L,J) * MSTIFF (I,L+3) * RM (I,L,K)
      GK (J,K) = SUM
66    GK (J+3,K+3) = SUMM
      DO 67  M = 2,3
      SUMB = SUMB + RM(I,M,J) * MSTIFF (I,5+M) * RM (I,5-M,K)
67    GK (J,K+3) = GK (K+3,J) = SUMB
65    CONTINUE
C   INITIAL END FORCES IN GLOBAL COORD   *  *  *  *  *  *  *  *  *  *
      DO 68  J = 1,6
      SUMF = 0.0
      DO 75  K = 1,6
75    SUMF = SUMF + GK (J,K) * EDIS (K)
68    GFOR(I,J) = SUMF
69    CONTINUE
C   INITIAL END FORCES IN MEMBER COORDS *  *  *  *  *  *  *  *  *  *  *
      DO 80  I = 1,MEMBS
      DO 80  J = 1,3
      SUM = SUB = 0.0
      DO 81  K = 1,3
      SUM = SUM + RM(I,J,K) * GFOR(I,K)
81    SUB = SUB + RM(I,J,K) * GFOR(I,K+3)
      FOR(I,J) = SUM
      FOR(I,J+3) = SUB
80    CONTINUE
C   NEGATIVE END FORCES IN MEMBER COORD
      DO 82  I = 1,MEMBS
      DO 83  J = 1,4
83    FORN(I,J) = -FOR(I,J)
      FORN(I,5) = -FOR(I,5) -LENGTH(I) * FOR(I,3)
82    FORN(I,6) = -FOR(I,6) + LENGTH(I) * FOR(I,2)
C   NEGATIVE END FORCES IN GLOBAL COORD *  *  *  *  *  *  *  *  *  *
      DO 84  I = 1,MEMBS
      DO 84  J = 1,3
      SUM = SUB = 0.0
```

```
      DO 85  K= 1,3
      SUM = SUM + RM (I,K,J) * FORN(I,K)
85    SUB = SUB + RM(I,K,J) * FORN(I,K+3)
      GFORN(J) = SUM
84    GFORN(J+3) = SUB
      END


      SUBROUTINE  MAINVERT (II,B)
C   THIS INVERTS SPECIFIED 6X6 MATRICES
      DIMENSION B(6,6)
      COMMON  /6/ A(150,6,6)
      DO 5 I = 1,6
      DO 5  J = 1,6
      B(I,J) = A(II,I,J)
5     CONTINUE
C
C    BEGIN
C
      DO 1 I = 1,6
      X=B(I,I)
      B(I,I)=1.0
      DO 2 J = 1,6
2     B(I,J) = B(I,J) / X
      DO 1 K = 1,6
      IF (K-I) 3,1,3
3     X=B(K,I)
      B(K,I) = 0.0
      DO 4  J = 1,6
4     B(K,J) = B(K,J)-X*B(I,J)
1     CONTINUE
      END
```

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