NATURAL CONVECTION HEATING OF LIQUIDS IN UNAGITATED FOOD CONTAINERS

Thesis for the Degree of Ph. D.
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John L. Blaisdell
1963

This is to certify that the

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NATURAL CONVECTION HEATING OF LIQUIDS
IN UNAGITATED FOOD CONTAINERS

presented by

John L. BLAISDELL

has been accepted towards fulfillment
of the requirements for

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I. J. Pflug
Major professor

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NATURAL CONVECTION HEATING OF LIQUIDS
IN UNAGITATED FOOD CONTAINERS

By

John Lewis Blaisdell

AN ABSTRACT OF A THESIS

Submitted to
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ABSTRACT

NATURAL CONVECTION HEATING OF LIQUIDS IN UNAGITATED FOOD CONTAINERS

by John L. Blaisdell

The natural convection phenomenon as it occurs in the thermal processing of canned foods seems to take many forms and degrees: the fluid may be confined only by the container, by large particles as well, or in small closed cells between layers of product; and in some instances there will be strong, perhaps even turbulent, convection, but in others the buoyant forces may not be sufficient to maintain or even establish convection. The natural convection phenomenon during this transient heating and cooling is closely related to the conduction mechanism, for not only does conduction prevail at the beginning and the end of transient heating (and in the solid portions of a product as well) but the mathematical solutions for solids with finite or infinite thermal diffusivity may be considered to be the lower and upper limits of convection transport rates.

The research reported in this dissertation is the result of an effort to understand more about convection heat transfer in unagitated food containers. Studies include a number of model systems consisting of several fluids, several sizes of glass containers and one size can. In the homogeneus Newtonian fluids of this study--water and 50

percent sucrose--several aspects of the phenomenon were observed. For large temperature differences, turbulent boundary layer but laminar core flow were observed. While the relatively fast moving (about 3 to 20 times as fast as the core) lateral eddies were being established, turbulent eddies arose from the bottom. These bottom eddies soon decreased in height to form temperature patterns not inconsistant with Benard cells or ring vortices. Soon after, the turbulent lateral eddies degenerated into a regular laminar boundary layer. These boundary layers appear to be equal to or thinner than the quasi-steady-state predictions due to the relatively large influence of exterior film coefficients in spite of only moderate container conductances. The horizontal profiles indicate an off-center cold point near the heel of the container which, from other considerations, appears to regress from near the heel to near the geometric center of the product. Conceivably, it leaves the region near the bottom of the container and of the lateral eddies as conduction becomes important and the lateral boundary layer no longer occupies the entire sidewall. Other 'cold points' exist at stagnation points and at vortex centers.

The generalizations drawn by Pflug and Blaisdell (1963) (on the basis of this research) as to the effects of viscosity, temperature difference and container geometry on the heat penetration parameters f, j and the lag time $t_a = f \log j$ appear to be subject to the restriction that

the exterior film coefficient remain constant, that the flow be in the same domain, e.g., turbulent or laminar, and that the effects of the container wall on the heat transfer and film properties at the wall be examined with particular care.

Quasi-steady-state film coefficients for vertical and horizontal flat plates when incorporated in a Newtonian heating model, i.e., infinite internal effective thermal diffusivity agree well with experimental values of f.

A procedure is described whereby the external film conductances of bodies heated under transient conditions can be determined using metal, can-sized cylinders. These procedures and cylinders were used to ascertain typical film conductances in processing equipment. These results compare favorably with predictions for finite plates and cylinders. The effects of these film and container conductances on the heat penetration parameters f and j for conduction and convection heating are illustrated.

The design and use of multipoint thermocouple rods, calorimetry, flow visualization, model systems, and dimensional analysis techniques are discussed as tools to assist interpretation of experimental heating curves for real systems. Errors in apparatus, temperature sensing systems, and in analysis are treated. The overall problem of transient conduction and convection in canned foods is examined in an analysis of the physical phenomenon and of the food engineering literature.

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SYMBOLS

A	Surface area of product; A_c, area of container covered by film of heating or cooling medium; A_f, area of product covered by film of heating or cooling medium.
A_n	All but the exponential portion of term n as defined in equation 2.5.
B	Empirical constant in Kaneko formulae, equations 6.18, 6.19, 6.20.
$\underline{b}$	Empirical constant, which is a function of Prandtl number, defined by equation 2.40.
b	Half-thickness of infinite slab.
C	Convection index, a measure of the amount of convection taking place, defined by equation 6.34.
C	Thermal capacitance, i.e., $C_p V$; C_f of fluid product; C_c of container; C'' of heated element.
C_d	Conversion factor for conduction heating.
C_r	Cooling rate or cooling coefficient.
C_v	Conversion factor for convection heating.
C_p	Specific heat at constant pressure.

D	Diameter of sphere or cylinder.
e	Napierian base (2.71828···)
F	Number of minutes required to reduce a given number of organisms at a given temperature; F_o denotes the time required at 250°F for an organism which has a z value of 18F, i.e. a 18°F temperature change is required for a 90 percent change in the thermal destruction rate constant; F_o values sometimes denote the equivalent process in u minutes at these conditions of 250°F and a z of 18.
f	Time required (in minutes) for the aysmptote of the heating or cooling curve to cross one log-cycle, that is the time required for a 90 percent change in temperature on the linear portion of the curve. Subscripts h, c, and numerals 2 and larger are used to denote heating, cooling, and second and successive f values if more than one linear portion is used to describe the heating and cooling curves. f_∞ is the f for a product when the overall heat transfer coefficient at the surface v is infinite; f_p is the estimate of f at p log cycles on the tangent to the heating curve from the start of heating to point p.
g	Gravitational constant.
G_n	Experimental constants; G_2 denotes the second such constant.

H_1, H_2	Relative errors in a heat penetration parameter at p_1 or p_2 value due to use of a tangent to heating or cooling curve rather than the aysmptote
h	Surface heat transfer coefficient; h_o outside film coefficient; h_i inside film coefficient; h_b at bottom of container; h_L on sides of container.
$J_n(\beta_n)$	Nth order Bessel function of first kind for the argument β_n.
j	Lag factor $(T_a-T_1)/(T_o-T_1)$; j_o lag at center of object; j_x lag at position x in object; $\hat{j}$ estimated lag at p lag cycles from the start of heating on the heating curve.
K	Coefficient used by Leggett and Sutton to denote the rate of cooling.
K_2	Exponential term in equation 6.13.
k	Thermal conductivity; k_f of film; k_c of wall of container; k_p of solid product.
L	Characteristic length of product in direction of flow; height of product in container, i.e. Za; L_c thickness of container wall.
M	Mass per unit area; M_f of product; M_c of container.
$\mathcal{M}$	Parameter in notation of Clark and coworkers which is equal to 2.303/f or τ.

$\underline{M}_o$	Prandtl number dependent derivative of the temperature distribution on investigations of Gebhart and co-workers.
N_{Bi}	Biot number $h/[k_p/a]$.
N_c	Capacitance ratio $(\rho C_p L)/(M_1 C_p)_f$.
N_{Gr}	Grashof number $L^3 \rho^2 g_c (T_s - T)/\mu^2$; subscripts denote length, temperature difference and fluid property temperature used.
N_{Gr}^*	Modified Grashof number based on height L of element and surface heat flux zt infinite time, i.e., steady state.
N_{Nu}	Nusselt number hL/k_f.
N_{Pr}	Prandtl number $C_p u/k$.
N_{Re}	Reynolds number $LV\rho/u$.
N	Exponent dimensionless.
P	Probability.
p	Number of log cycles on heating curve (p' on aysmptote) to point p (p').
Q	Heat transferred, not a rate.
$\underline{Q}$	Thermal capacity parameter, used by Gebhart and co-workers, defined in equation 2.41.
R	Weighting coefficient of Kaneko defined in equation 6.17.

R	Resistance 1/hA to transfer of heat.
r	Radius of sphere or infinite cylinder.
s	Laplace transform of time t.
T	Temperature; T_o initial temperature of product; T_1 temperature of coolant or heating media; T_a apparent initial temperature as defined by the linear portion of the heating or cooling curve, i.e., the ordinate value of the origin of the aysmplote of the curve; T_f mean temperature of the fluid, i.e., $(T_s+T_1)/2$ where T_s is the surface temperature; T_w temperature at 0.1 inch from wall, i.e. approximately T_s; T_x apparent temperature of heating or cooling medium; T_∞ the reference temperature in the bulk stream of convection fluid--presumeably at an infinite distance from the heat transfer surface:
$\underline{T}$	Generalized time variable, used by Gebhart and co-workers, defined by equation 2.40.
t	Time
t_a	Lag time defined as f·log j.
U	Overall coefficient of heat transfer; U_o initial value; U_∞ value at infinite time.
u	Dimensionless temperature $(T_1-T)/(T_1-T_o)$.

V	Volume of product.
V	Velocity of fluid in N_{Re}.
V	Temperature of solid in equation 2.16 and i v its Laplace transform; v temperature in equation 2.32.
v	Product of specific gravity and specific heat; a parameter used by Merrill.
W	Temperature of fluid in equation 2.17 and w its Laplace transform.
w	Mass flow rate.
x	Distance from center of product to point of measurement; ΔX thickness of wall segment.
Z	Half-cooling time.
Z	Thermometer temperature of solid in equation 2.16.
α	Thermal diffusivity
β_n	Nth root of the boundary equation, or the prescribed approximate boundary equation for a particular shape. The β_n for a slab, cylinder, or sphere are different.
ΔT	Temperature differential; T_s-T_∞ for idealized convection.
δ	Boundary layer thickness.

η_n	Nth root of equation 2.7.
λ_n	Nth root of equation 2.22.
μ	Viscosity of fluid.
ν	Kinematic viscosity of fluid μ/ρ ; ν_0 a reference value and ν_1 its thermal coefficient.
π	Pi (3.1416···).
ρ	Density.
σ	Specific speed as defined by Plank.
τ	Time constant.

SECTION I

INTRODUCTION

The phenomenon of natural convection heat transfer and fluid flow in enclosed spaces is of considerable importance in as diverse applications as the cooling of the earth, to the convection within rooms, but has not received much attention. The principal analytical and experimental investigations of this phenomenon have dealt with the special, steady-state cases of the cooling of homogeneous liquid reactors and of liquid cooling of turbine blades. These, however, are comparatively recent developments. Meanwhile, the food scientist and engineer, aware of the complexity of canned food processing, yet primarily concerned with the thermal destruction of microorganisms has relied on empiricisms and analogies to conduction heating.

Convection in Canned Food Processing

This investigation of transient natural convection heating of liquids in unagitated food containers was undertaken to broaden our understanding of the convection heat transfer phenomenon. Natural convection as it occurs in cans of foods takes many forms and degrees: the fluid movement may be restricted only by the container, or by large particles as well, or in small closed cells between layers of product; and in some instances there will be strong, perhaps even turbulent convection, but in others the buoyant forces may not be sufficient to maintain or even establish convection. Natural convection during transient heating and cooling is closely related to the conduction mechanism, for not only does conduction prevail at the beginning and the end of transient heating (and in the solid portions of a product as well), but in addition the mathematical solutions for solids with finite or infinite effective thermal diffusivity may be considered to the lower and upper limits of convection heat transfer rates.

Peterson (1961) suggested the following as an approach for studying both conduction and the natural convection heat transfer in cans of food:

> 1. Determination of the relative conductivity of the contents of the can.
> 2. Determination of the rate of travel of the heat through the material (transmission from molecule to molecule) (conduction) and transmission by currents (convection).
> 3. Determination of the patterns of distribution as affected by the contents of the can, the cylindrical shape of the container, etc.
> 4. Determination of the completeness of penetration of heat.
>
> These steps, naturally, are conducted in terms of the great body of the data on the thermal death rate of microorganisms, the variable of chief interest in sterilization by canning processes. (p. 79)

Most of the numerous investigations of the heat transfer aspects of canned foods have been from a conduction heating approach, although Jackson's (1940) classification of food products is evidence that convection is as important as

conduction in the thermal processing of foods. (See the recent reviews of Powers et al. (1962), Gallop (1960), Ball and Olson (1957), and Charlett (1955).) Therefore, an investigation of the natural convection phenomenon as it occurs in the transient heating and cooling of canned foods appears to be long over due.

Scope of the Present Investigation

This investigation is an experimental study and analysis of natural convection heating, without agitation, of low and moderate viscosity Newtonian fluids in several commercial sizes of glass jars and in one size of tin container. The processing temperature and initial temperature differences also were controlled variables. Transient local temperature histories at several axial positions and radial positions near the top and the bottom of the containers are the principle data examined. Dye movement, the development of the temperature patterns, and calorimetric measurements supplement these data.

This investigation was undertaken with the presumption that the rates, patterns, and temperature histories for transient natural convection could be correlated in terms of the dimensionless similarity groups which Okada (1940) had derived from an examination of the differential equations. Residual variation was expected, however, for Okada's development, like most studies of convection, neglected the inertial effects of the thin rising film and in the central descending core, and Lighthill's (1953) theoretical

considerations on free convection in tubes had predicted not only a similarity regime, but also two other flow regimes. For this study it was assumed that these dimensionless similarity groups could be incorporated in a model with a uniform temperature core and a controlling surface heat transfer resistance. This Newtonian heating model had been proposed by Jones (1931) and Schultz and Olson (1938), but has not received further consideration.

The experimental data are analyzed using the modified Newtonian heating model but with the nomenclature of Ball (1923) and are compared with the processing literature and the analytical approach of Hammitt and Chu (1962). This latter noteworthy approach to transient internal natural convection in filled vertical, cylindrical vessels, an outgrowth of earlier studies in which there was internal generation as in a nuclear reactor, came to the attention of the author during the writing of this dissertation but is included here for its significance for this problem.

SECTION II

THE PROBLEM

An appraisal of the literature has indicated that there are many problem areas in the natural convection heating of food products; these include: come-up (or come-down), variation in product characteristics from pack to pack, and surface resistance. Of these the surface resistance and container resistance and capacitance in themselves assume a much greater role in a study of the mechanism of convection heating, however. In the Jones (1931) or Schultz and Olson (1938) Newtonian heating models the surface resistance is presumed to be controlling. In the dimensional analysis approach of Okada (1940), the Nusselt, the Grashof, the Prandtl, and the Reynolds numbers--often associated with the idea of a boundary layer resistance--are introduced in addition to the Fourier modulus and the Biot numbers which are known to be important for conduction heating. Since come-up variations and variations in product characteristics may well have obscured the influence of the Grashof and Prandtl number factors, both the analytical and the experimental phases of this investigation will be limited to the response of Newtonian fluids to a step change in environmental temperature.

The phenomenon of natural convection heating of Newtonian liquids in unagitated food containers is anticipated to be much the same with particulate products, non-Newtonian fluids or superimposed agitation. In each instance the rates, temperature and flow patterns, and the temperature histories are affected by transient heat transfer through the container walls as opposed by internal forces. These above limitations were imposed in this dissertation in order to examine the homogeneous unagitated case, the most general problem which might be satisfactorily treated with available techniques. Ball and Olson (1957) have also concluded (p. 254) that solution of the problem of transient heating of the homogeneous fluid would be useful in correlating experimental data, but that natural convection in cooling canals, retorts, and racks (analogous to particulate products in containers) (p. 261) seemed too formidable.

Since an extension of this work to a solution of special cases like those referred to above may prove valuable, the possible implications of current studies in regard to the transient heating of packed beds, porous solids, and regenerators; the modification of non-dimensional heat transfer equations to approximate non-Newtonian viscosity; and the effects of body forces on forced and natural convection should be kept in mind.

Requirements

When this investigation was initiated there was no correlation for the effects of the factors represented by the Grashof, Prandtl, and Nusselt numbers and container geometry on the transient rates, patterns, and temperature histories during natural convection heating. Therefore, such a correlation was desirable.

There were several restrictions on the form of this correlation: (1) it must be a concise and simple way to express data; (2) the parameters chosen should have some physical significance; (3) it should retain as much as possible of the f and j approach which is widely accepted and for which several methods of process calculation (i.e., lethality) are available; (4) it should be compatible with most of the observed effects on f and j found with food products; and (5) the analysis upon which the correlation is based should suggest answers to some of the questions formulated at the beginning of this investigation.

The first two restrictions were imposed to assure that time-temperature histories could be reduced to a convenient form, a form which would both describe these curves and have some physical significance to the reader. It has been shown in the preceeding discussion of the literature, however, that the first of these criteria in some instances can be satisfied only by tabulations of temperature as a function of time.

The remaining restrictions were imposed in order to utilize as much as possible of the process calculation methods and the implications of the heat penetration literature.

It was hoped that the examination of the literature, further analysis of some specific problems, and subsequent experimental investigation would suggest answers to the following questions.

1. How does transient convective heat transfer develop? How is this development influenced by the Grashof, Prandtl, and geometric groups and the properties they represent?
2. May the temperature history and the motion of the fluid be readily correlated and predicted using dimensionless groups?
    a. Where, then, in terms of these groups and the temperature and flow patterns, is the cold point located and are there other locations which might receive a lower lethal value than their surroundings?
    b. What, then, can be used to estimate the temperature history of the element of the moving fluid which receives the least lethal value?
3. How does the amount of heat and rate of heat transfer vary with these dimensionless groups?
    a. What is the relationship between the rate of heat transmission and f?

b. Can estimates of the validity of the assumption of quasi-steady-state film properties readily be obtained? If so, what is a reasonable bound of this error?

4. When might the heat penetration curves for Newtonian fluids need to be approximated by more than one linear portion? If these lines are usually curved, how should the linear segment be chosen and how should they be reported? What is the physical signifigance of the several f values?

5. What is the effect of container resistance and capacitance, ratio of transfer area to volume of fluid, and container resistance relative to the interior fluid resistance on f and also on j?

6. What temperature and characteristic length should be used in the above relationships? How do these values compare to those characteristic of conduction heating?

7. Why is a large coefficient of variation encountered in convection heating? Is this related to Ball and Olson's (1957) convection index or to some index of temperature uniformity?

Method of Attack

Early in the investigation it became apparent that substitution of a functional representation of dimensionless group form for the several surface conductances, then

allowing for variation throughout the process was the most promising attack. Other approaches were not to be set aside, but were to be developed as far as is now practicable so that their possible merits and deficiencies could be examined.

Tentative answers to the above questions were to be determined through repeated analyses of the literature or suggested by analogies to other problems and supplementary experimentation. Certain of these problems were to be investigated experimentally, then contrasted with the working hypothesis.

In this section some considerations on alternate approaches will be presented. Also presented will be some considerations of the selection of characteristic values of properties, temperatures, and lengths. A derivation of the working hypothesis then will be presented. The appropriateness of the assumptions herein will be discussed in a later chapter. There they can be related to the experimental results. There, also, the relative value of high order approximation can be seen more clearly.

Correlation of Convection Heat Penetration Data

The starting point for these considerations of possible approaches to correlation of the heat penetration parameters f, j, and axial cold point position x/L was the Newtonian heating concept of a nearly uniform core with a controlling surface resistance.

The two simplest models from which the effect of shape on f could be estimated had several deficiencies: (1) neither the Jones (1931) nor the Schultz and Olson (1938) relations nor the approximations which followed took into consideration the large axial temperature gradient; (2) no means to correlate j or slow point position was forthcoming; and (3) only Jones (1931) had considered the low heat transfer rate through the headspace during convection. Nevertheless, the concept itself, with some dimensional analysis correlation of the film coefficients, might be employed. It was felt that simple modifications at first might be fruitful: they could be used to make simple comparisons, to examine in an approximate manner the possible effects of individual factors, and when applied to the literature (as in Section six) to form the basis for increased understanding of the natural convection phenomenon. It was hoped that ways might be found to account for heating medium film coefficient and container conduction natural convection transport properties, the variation of f and j due to property variation, and perhaps even the transport lag effects on j. Once schemes were developed some characteristic properties, temperature differences, and lengths could be determined. Before refinements can be made, however, it is essential to evaluate the assumption by Merill (1948) that the container wall capacitance can be neglected and that all the resistances at the walls can be lumped into a combined resistance to heat transfer, 1/U. If such an assumption is acceptable, the problem will be simplified and the same correlations may be applied to both glass and metal containers.

Effect of Resistance and Capacitance of f and j

The Effect of the Container on f and j

The effect of the heat capacitance and resistance of the container wall alone on the heat penetration parameters f and j may be estimated using a formula presented in Carslaw and Jaeger (1959) for a slab, initially at zero temperature, one face of which is suddenly maintained at a new temperature level and the other is cooled by direct contact with a well-mixed fluid which also was initially at zero temperature. (There are no film resistances on either face.)

The problem and its solution are restated below in terms of nomenclature used in this dissertation.

The differential equation is:

$$\frac{dv}{dt} = \frac{\partial^2 v}{\partial x^2} \qquad 2.1$$

where the boundary and initial conditions are:

$$v(0,t) = V \qquad t > 0 \qquad 2.2$$

$$k\,\frac{\partial v(L,t)}{\partial x} + M'C_p'\frac{\partial v}{\partial t}(L,T) = 0 \qquad t > 0 \qquad 2.3$$

and

$$v(x,0) = 0 \qquad 2.4$$

and v and V are chosen so $v = T - T_o$ and $V = T_1 - T_o$ to satisfy the initial conditions and $M' \cdot C_p'$ represents the (mass · specific heat)/(area heated) of the product.

The solution of this problem is:

$$\frac{v}{V} - 1 = -u - \sum_{n=1} \frac{2(\eta_n^2 + N_c^2)}{\left[\eta_n^2 + N_c^2) + N_c\right]} \left[\frac{\sin\left(\eta_n \frac{x}{L}\right)}{\eta_n}\right] \exp(-\alpha\eta_n^2 t/L^2) \quad 2.5$$

where

$$N_c = \frac{\Gamma C_p L}{M' C_p'} \quad 2.6$$

and η_n are the roots of the equation

$$\eta_n \tan \eta_n = N_c \quad 2.7$$

The f and j for the fluid may be found using the relation that:

$$\frac{d}{dx}\left[\log_a u\right] = \frac{\log_a e}{u} \frac{du}{dx} \quad 2.8$$

The equation for the tangent line and the definition of j and f follow therefrom. It can be shown that:

$$\log j_p = \log \frac{T_1 - T^o}{T_1 - T_o} = \log \sum_{n=1}^{\infty} A_n \exp(-\alpha\eta_n^2 t_p/L^2) - (-)\frac{t_p}{f_p} \quad 2.9$$

$$\frac{1}{f_p} = \frac{(\alpha\eta_n^2/L^2)\, A_n \exp(-\alpha\eta_n t_p/L^2)}{2.303 \sum A_n \exp(-\alpha\eta_n^2 t_p/L^2)} \quad 2.10$$

Hence:

$$1/f_\infty = \frac{\alpha \eta_1^2}{2.303L^2} \qquad 2.11$$

when A_n represents all but the exponential portion of term n in the summation equation 2.5.

<u>An illustration for glass walls</u>.--Consider the following system: water in a 16 oz. vegetable jar with the following physical characteristics:

Volume of product	454 ml.
Inside diameter	7.4 cm.
Fluid height	10.5 cm.
Weight of jar	211 grams
Measured jar thickness	0.27 cm.
Glass properties:	
Specific heat	0.2 cal/gm°C
Specific gravity	2.49 gm glass/gm water 4°C
Thermal diffusivity	0.005 cm^2/sec.

after the jar of Nicholas and Pflug (1961a), and the thermal properties of Shand (1948), Figure 2.4, Table 2.1, Table 2.6, respectively).

It will be fruitful to determine the f and j in this system and examine how they differ from those predicted by the Jones (1931) Newtonian heating model.

The dimensionless thermal capacitance ratio N_c defined by Equation 2.6 can be rewritten as:

$$N_c \approx \frac{MCp}{M'Cp'} \approx \frac{\rho' Cp' Ag(L)}{A_f(V_f/A_f)\rho' Cp'} \approx \frac{\rho' Cp' L}{(V/A_f)\rho' Cp'} \qquad 2.12\ a,b,c$$

The first relation yields 0.093 and the third relation yields

$N_c = 0.495\ (A_f/V_f)\ L = 0.085$. In the first of these relations (2.12a), it is presumed that the heat transmission is through the body only and that the fluid is distributed over the total heated glass surface including the glass in the headspace. The area of the glass A_g is somewhat larger than that of the heated fluid area A_f due to curvature and the glass in the headspace. The third relation (2.12c) therefore should yield a somewhat lower value and subsequently predict a somewhat longer time.

The root of Equation 2.7, obtained from Figure 2.1, is found to be 0.303 for $N_c = 0.093$. The value of f is:

$$f = \frac{(2.303)\ (0.268\ \text{cm})^2}{(5 \times 10^{-3}\ \text{cm}^2/\text{sec})(0.303)^2(60\ \text{sec/min})} = 6.02\ \text{min.}$$

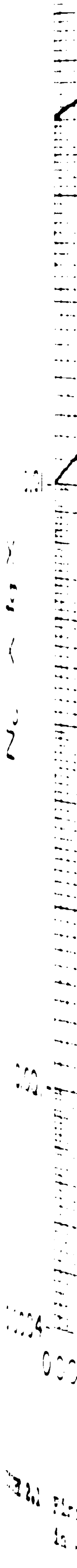
For this same system and $N_c = 0.085$, $\eta_1 = 0.287$

$$f = 6.7\ \text{min.}$$

When the Jones (1931) relation is used, and the V/A = 0.635 based on fluid dimensions is used, i.e., jar capacitance is neglected, f = 3.24, and when V/A = 0.576 based on external dimension is used, i.e., a jar capacitance equal that of water, f = 3.57. Kaneko's (1941) formula, incidentally, would yield f = 5.1 and 6.15, respectively.

For this case, since $\sin\left(\eta_n \frac{L}{L}\right) = 0.0930 \approx \tan\left(\eta_n \frac{L}{L}\right)$ or 0.0933, $N_c = 0.093$, and $\eta_1 - 0.303$ from Equations 2.5, 2.6, and the definition of j,

$$j = 1.02.$$



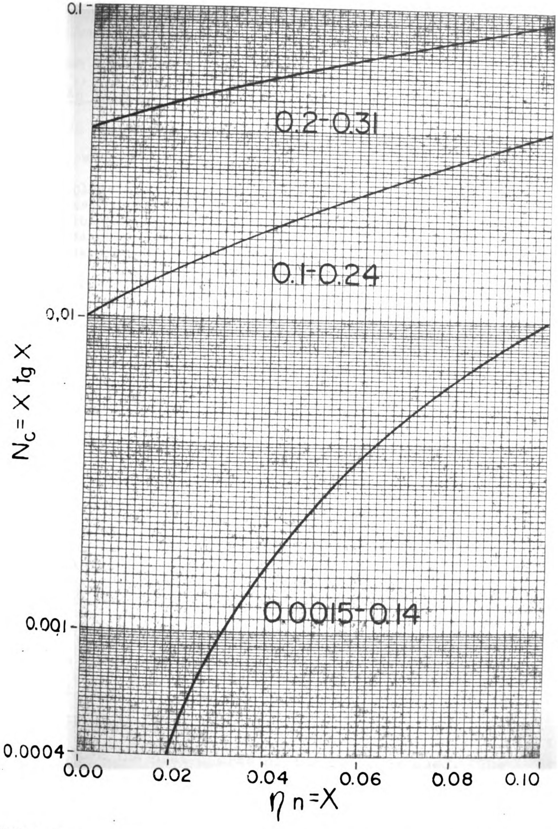


FIGURE 2.1 First root of $\eta_n \tan \eta_n = N_c$. Values of x tg x are from tables in Jahnke and Emde (1945).

This value is less than $j = 1.27$ for the infinite slab and more than $j = 1$ for the Newtonian heating model.

An illustration for thin metal walls.--Consider the following system: water in a 303 x 406 metal container with the following physical characteristics:

Volume of product	29.1 in^3, i.e., 1.0 lb.
Inside diameter	1.523 in.
Fluid height	3.800 in.
Measured weight of container	0.152 lb.
Measured wall thickness	0.010 in.
Wall properties:	
Specific heat	0.116 Btu/lb°F
Density	487 lb/ft^3
Thermal diffusivity (k = 26)	0.459 ft^2/hr.

after the container dimensions reported by Continental Can Company (1957) and the thermal properties of Eckert and Drake (1951, Table A-1 for a low carbon steel).

For this hypothetical problem $N_c = (0.152)(0.116)/(1)(1) = 0.0176$ and from Figure 2.1, $\eta_1 = .1322$ (the approximate N_c of Equation 2.12c = 0.0161), so that

$$f = 0.585 \times 10^{-3} \text{ min.}$$

and the lag factor:

$$j = 1.001.$$

The f value agrees very well with the values of $f = 1.32 \times 10^{-3}$ and 1.52×10^{-3} calculated for the same system by the Jones (1931) model when the area of the body and one end is used, and when the heated area of the fluid is used. The thermal capacitance of the system was incorporated in $C_p \rho V$. It seems that the heated fluid area should be used here, but not for glass containers.

The limiting j value.--The j values calculated in the above examples are very slightly larger than one would have anticipated from the physical considerations which had earlier led to the Newtonian conduction models and a predicted $j = 1$.

The lower limit of j is said to be 1 from the Newtonian heating model. To examine the limit it is convenient to express j in terms of trigonometric functions using Equation 2.7, consider

$$j_x = \frac{2(\eta_1^2+N_c^2)}{\eta_1^2+N_c^2+N_c} \cdot \frac{\sin \eta_1 \frac{x}{L}}{\eta_1} \qquad 2.13a$$

or

$$j_c = \frac{2 \sin \eta}{\eta + \sin \eta \cos \eta} \qquad 2.13b$$

From this and the properties of the $(\sin \eta)/\eta$ and $\cos \eta$ functions which both approach 1 as a lower limit it can be shown that when η is small:

$$\lim_{\eta_1 \to 0} j = 1 \qquad 2.14$$

It also can be shown that Equation 2.13b is similar in form to the trigonometic equation for the f value at the center of a slab (see Equation 7.32 of Ball and Olson, 1957) of thickness 2L. In this later case instead of:

$$\eta_1 \tan \eta_1 = \frac{M\,Cp}{M'Cp'} = N_c \qquad 2.7$$

the indicial equation is:

$$\beta_1 \tan \beta_1 = \frac{Lh}{2k} = N_{Bi} \qquad 2.15$$

It is for this reason that a thin, but not necessarily uniform temperature, body on the surface of a well-mixed fluid will cause j values less than or equal to those of a slab with negligible surface resistance. When N_c is very large there will be no heat transfer on the liquid surface, and since here a large exterior film coefficient was assumed the j value will be exactly the j = 1.273 for the slab with no resistances, i.e., N_{Bi} large. When N_c is small or N_{Bi} is small, the j value approaches 1, since the heat transmitted through the body is absorbed nearly uniformly.

The Effect of External and Container Resistances Combined on f and j

The above consideration of the effect of the capacitance and resistence of thin containers is also applicable to thin stationary boundary films. It was seen that there was little effect of such a layer on j for containers since the heat capacitance of the containers (or films) was small relative to the capacitance of the product. Since small film coefficients for slabs also cause small j values, the addition of film coefficients to the above problem is not anticipated to change the conclusions. However, the large errors observed when the container conductances k_c/L_c are lumped into the overall heat transfer coefficients may be

attributed to non-linear temperature gradients and hence might well be reduced by the presence of external and internal film conductances--film coefficients--each of which very likely would not be more than ten to twenty times larger, and in practice the internal film coefficient might be no more than three times larger than the conductance of a glass wall.

Wolfe (1959) has derived an equation for the temperature in an analogous problem: a slab, initially at zero temperature, which is heated at $x = o$ by radiation from a mass of well-mixed fluid whose initial temperature is also zero, and is cooled by radiation to another fluid maintained at temperature zero. Wolfe also included a fixed heat supply to the well-mixed fluid (room air) in his study of the dynamic response of air in a room to changes in external environmental temperatures. This heat source, not required in our problem, when neglected gives a trivial solution since if no heat were provided there would be no change from the zero initial temperature. Since the boundary conditions were not homogeneous, the problem was not of the Sturm-Liouville type, like those shown in Ball and Olson (1957) for infinite cylinders and slabs with and without boundary resistance, and other techniques than Fourier series alone were required. Wolfe determined a set of coefficients for these non-orthogonal functions by using boundary conditions to evaluate the integrals of the product of like or like and unlike index functions.

Pearce and Stephenson in the printed discussion of

Wolfe's paper claimed that the above problem can be readily solved by means of Laplace transforms and would yield identical results. It will be expedient to follow this suggestion in an attempt to solve the equations for a well-mixed fluid in a container with finite film coefficients at the container surfaces.

On the analytical solution.--The problem of a container wall heated by "radiation" from a fluid at surface x = L and cooled by "radiation" to a well-mixed otherwise insulated fluid may be expressed as follows:

The differential equations are:

$$\frac{\partial V}{\partial t} = \alpha \frac{\partial^2 V}{\partial x^2} \qquad \text{for the solid} \qquad 2.16$$

and

$$C_p M \frac{\partial W}{\partial t} = - h_o \left[W - V(0, t)\right] \qquad \text{for the liquid} \qquad 2.17$$

with boundary conditions

$$h_o \left[W - V(0,t)\right] = +k \frac{\partial V}{\partial x} \qquad 2.18$$

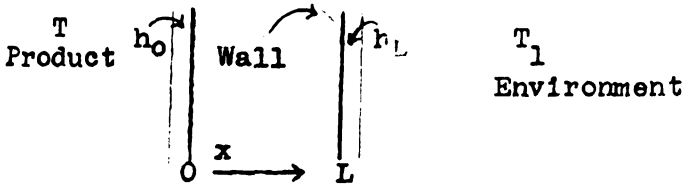
and

$$h_L = \left[F(t) - V(L,t)\right] = +k \frac{\partial V(L,t)}{\partial x} \qquad 2.19$$

and

$$W(0) = V(x,0) = T(0) - T_o = Z(x,0) - T_o = 0$$
$$= F(0) = G(0) - T_o \qquad 2.20$$

where T_o has been subtracted from the variable of interest T, the temperature of the well-mixed fluid, in order to obtain the desired initial conditions. This problem is shown schematically below:



Take the Laplace transform with respect to time t and let the transformed functions be represented by the corresponding lower case symbols and t be represented by s, to obtain the following transformed problem:

$$sv(x,s)-V(\cancel{x,}{}^{o}0) = \alpha \frac{\partial^2 v(x,s)}{\partial x} \qquad \text{for solid} \qquad 2.16a$$

and

$$sC_pMw(s)-w(\cancel{0}^{o})=-h_o[w(s)-v(C,s)] \qquad \text{for fluid} \qquad 2.17a$$

with boundary conditions

$$h_o[w(s)-v(0,s)] = +k\frac{\partial v(0,s)}{\partial x} \qquad 2.18a$$

and

$$h_L\,[f(s)-v(L,s)] = +k\frac{\partial v(L,s)}{\partial x} \qquad 2.19a$$

and

$$w(0) = v(x,0) = 0 \qquad 2.20a$$

but here F(o) was originally zero and at $t = o^+$ changed stepwise to a new value $T_1 - T_o$ hence

$$f\ (s) = \frac{T_1 - T_o}{s} \qquad 2.21$$

The equations which might become a Sturm-Liouville problem (See Mickley et al., 1957) and thence be solved by Fourier series are:

$$\frac{\partial^2 v(x,s)}{\partial^2 x} - \frac{\alpha}{s} v(x,s) = 0 \qquad 2.16b$$

$$k\frac{\partial v(0,s)}{\partial x} - \frac{h_o CpMs}{CpMS + h_o} v(0,s) \qquad 2.18b$$

$$k\frac{\partial v(L,s)}{\partial x} + h_L v\ (L,s) = + \frac{(T_1 - T_o)}{s} \qquad 2.19b$$

and

$$w(s) = \frac{h_o}{s(CpM) + h_o} \quad v(0,s) \qquad 2.20b$$

Even this problem is not a Sturm-Liouville problem, however, because the right hand term of Equation 2.19b is not equal to zero.

The usual strategem at this point is to define a new temperature variable equal to V(x,t) - F(t), when F(t) is a constant, to make the boundary equations homogeneous. Thus, for the initial constant temperature T_o has been subtracted from the real temperatures T, T_1, and Z in order to make the second terms (on the left) of Equations 2.16a and 2.17a vanish, another requirement of the Sturm-Liouville problem. If the suggested new variables (denoted by primes) were introduced, Equations 2.16a' and 2.16b' would include their initial values $V'(x,o) = Z_o - T_1 = T_o - T_1$ and $W'(o) = T_o - T_1$, Equation 2.19b' would lose the right hand term as desired, but Equation 2.18b' would gain a term $-h_o (T_1 - T_o)/CpMS + h_o)$ on the right hand side. Thus the usual stratagem does not simplify the problem.

Pearce and Stephenson did not encounter this problem because in their problem the initial conditions for the primed quality were zero, hence the transformed differential equation and its boundary conditions were homogeneous. For this reason they could construct a set of orthogonal functions of the form

$$A_n' \sin \sqrt{\frac{\alpha}{s}}\, x + B_n \sqrt{\frac{\alpha}{s}} \cos \sqrt{\frac{\alpha}{s}}\, x$$

satisfying the boundary condition and then could perform the inverse transform to obtain a solution for the temperature of the slab. Since this transformed equation is of the form

$V = C_n \exp - (D^2 n/L^2)t,$ W could be found, converted to the usual form log u: f and j could also be found.

None of three available ways to obtain such a solution will be employed at this time, however. Although finite Fourier (i.e., sin and cosine) transforms could be used then inverted much as in the inversion of the Laplace transformed equations; digital (or analogue) computers could be used as suggested by Pearce and Stephenson; or Wolfe's involved integral approach to the problem could be used, the complexity of this problem and simpler boundary condition problems, such as those treated by Hayakawa and Ball (1961) and by Jaeger (1940) in Crank (1957), p. 69-81, and the need for computer solution of an approximate model seemed to make continuation of the solution unprofitable at this time.

A continuation of this solution would be desirable when the need for more precise temperature predictions warrants a computer solution of the convection aspect of the overall phenomenon.

It is interesting to note that both investigators found that when the exterior film coefficient increased, the root of the indicial equation--which may be rewritten in terms of dimensionless groups such that:

$$\lambda \tan \lambda = \frac{\frac{h_2L}{k} \cdot \frac{h_1L}{k} \cdot \frac{Cp\rho L}{Cp'M'} - \lambda^2 \frac{h_1L}{k} + \frac{h_2L}{k}}{\frac{h_1L}{k} \cdot \frac{Cp\rho L}{Cp'M'} + \frac{h_2L}{k} \frac{h_1L}{k} - \lambda^2} \qquad 2.22$$

--becomes more sensitive to changes in diffusivity (length for our problem) and/or fluid heat capacity, and that increasing the magnitude of the fluid heat capacity lowers the value of the root. For a container these observations imply that lumping will be less satisfactory as the film coefficients increase relative to α, as the container wall thickness increases, as the relative heat capacity of the glass and product increases, or k decreases.

A second approximation.--The Newtonian heating models with container conductance K_c/L_c incorporated in the overall coefficient of heat transfer as suggested by Merrill (1948) seem not too satisfactory for containers like glass, even when the total heat capacitance is used. Although this approximation has been shown to give improved estimates when the internal and external surface film coefficients are of the same order as the container conductance, it would be useful to have a simple second approximation.

Kreith (1958) has suggested that lumped capacitance analysis methods, i.e., subdivision of the container wall into one or more uniform temperature increments, could be employed to account for the capacitance of the walls. When the wall temperature is assumed to be uniform, the problem becomes analogous to the response of a thermocouple in a well (see Eckert, 1958, Examples 5-7 and 5-8). The temperature change of the fluid is approximately:

$$\frac{T_1-T}{T_1-T_0} = W = \frac{\tau_1}{\tau_1-\tau_2} e^{-t/\tau_1} - \frac{\tau_2}{\tau_1-\tau_2} {}^{-t/\tau_2} \qquad 2.23$$

where the parameters τ_1, τ_2 are

$$\tau_1, \tau_2 = \frac{1}{2} (R_1C_1+R_2C_1+R_2C_2) \left[1 \pm \sqrt{1-\frac{4R_1C_1R_2C_2}{(R_1C_1+R_2C_1+R_2C_2)^2}} \right] \qquad 2.24$$

where:

C_1 = fluid capacitance $C_{p1} \rho_1 V_1$

C_2 = container capacitance $C_{p2}\rho_2 V_2$

C_p = specific heat

ρ = density

V = volume

R_1 = internal resistance $1/h_1 A_1$

R_2 = external resistance $1/h_2 A_2$

A = area

h = film coefficient h_1 of product, h_2 of heating medium

T = temperature, T of fluid, T_1 of heating medium, T_0 initial, of heating medium product and container wall

t = time

It is more convenient in the food industry to use the f parameters rather than the τ or time constants. From the form of Equation 2.23 it can be seen that the time constants are related to the f notation such that:

$$f = 2.303\,\tau \tag{2.25}$$

Thus, the time required for a 90 percent change in temperature on the linear portion of a heat penetration curve is 230.3 percent higher than the time required for a 63.2 percent change which the time constant τ represents. Also:

$$f_a = 2.303\left(\frac{C_p\rho V}{hA}\right)_a \tag{2.26}$$

Equation 2.24, rewritten in terms of the f of the independent components, i.e., the container and the product, becomes:

$$\tau_1, \tau_2 = \frac{1}{(2)(2.303)}\left(f_1+f_2\frac{h_1A_1}{h_2A_2}+f_2\right)\cdot \left[1\pm\sqrt{1-\frac{4f_1f_2}{\left(f_1+f_1\frac{h_1A_1}{h_2A_2}+f_2\right)^2}}\right] \qquad 2.27$$

Since $f = \frac{-1}{\frac{d\log_{10}u}{dt}} = \frac{-1}{\frac{\log_{10}}{u}\frac{du}{dx}} = \frac{2.303u}{du/dx}$

the apparent $\hat{f}$ for the system may be written:

$$\hat{f} = 2.303\,\frac{\tau_1 e^{-t/\tau_1} = \tau_2 e^{-t/\tau_2}}{e^{-t/\tau_1} - e^{-t/\tau_2}} \qquad 2.28$$

or

$$\hat{f} = 2.303\,\frac{\tau_1 - \tau_2 e^{-\left(\frac{\tau_1-\tau_2}{\tau_1\cdot\tau_2}\right)t}}{1 - e^{-\left(\frac{\tau_1-\tau_2}{\tau_1\cdot\tau_2}\right)t}} \qquad 2.28a$$

which becomes

$$\hat{f} = 2.303\,\tau_1 \qquad 2.29$$

for long times if for an approximation $\tau_1 >> \tau_2$

$$\log\hat{j} = \log\left[\frac{\tau_1 e^{-t_p/\tau_1}}{\tau_1-\tau_2} - \frac{\tau_2 e^{-t/\tau_2}}{\tau_1-\tau_2}\right] + \frac{t_p}{\hat{f}_p} \qquad 2.30$$

or

$$\hat{f}_p = \frac{\hat{f}_F\left[e^{-t/\tau_1} - e^{-t/\tau_2}\right]\left[10^{-t_p/f_p}\right]}{2.303\,(\tau_1-\tau_2)} \qquad 2.30a$$

or

$$\hat{j}_p = \frac{\hat{f}_p \left[1 - e^{-\frac{(\tau_1 - \tau_2)}{\tau_1 \cdot \tau_2} t_p} \right] e^{\frac{-t_p(2.303\,\tau_1 - f_p)}{f_p \cdot \tau_1}}}{2.303\ (\ _1 -\ _2)} \qquad 2.30b$$

which becomes

$$j_p \approx \frac{\tau_1}{\tau_1 - \tau_2} \qquad 2.31$$

for long times if $\tau_1 >> \tau_2$

If f_2 were negligible due to a negligible container capacitance τ_2 would vanish, hence j -->1, and the f would

become $\frac{2.303}{2} \left[\frac{Cp_1 \rho_1 V_1}{A_1} \right] \cdot \left[\frac{1}{h} + \frac{1}{h_2} \right] \left[\ 2\ \right]$

or equal to that predicted for Newtonian heating with $1/U =$ $(\frac{1}{h_1} + \frac{1}{h_2} + \frac{L_c}{k_c})$ if the decrease in capacitance were due to a decrease in thickness.

The predicted effect of container capacitance alone is an increase in f and an increase in j. For example, if f_1 = 6 min, $h_1 \approx h_2$, $A_1 = A_2$, $f_1/f_2 = 10$, the predicted f would be 12.3 min. or about 2-1/2 percent larger, and the predicted j would be 1.025 or once again about 2-1/2 percent larger than a container without this capacitance. Both of these predictions agree with the analytical prediction of an increased f and j due to capacitance alone and the predictions for the Wolfe (1959) and Pearce and Stephenson (1959) investigations. If the total capacitance of the product and the container were substituted into the equation for the Newtonian heating model, the predicted f would be 13.2 min. or 10 percent higher and the predicted j would be 1, or no increase.

This second order Newtonian heating model might prove to give better estimates for j. However, this approximation might not be as satisfactory for predicting f as combining film coefficients and either the total container capacitance, or the increased capacitance found if the container is assumed to have water-like capacitance into the Newtonian heating model.

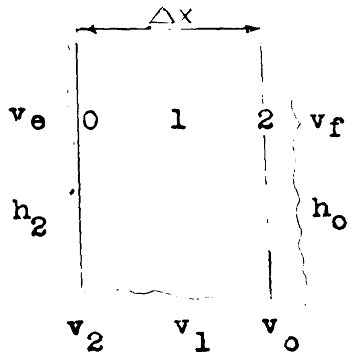
It should be noted that these formulas yield trivial answers, i.e., the container has no effect on f and $j = 1$, when either or both the film coefficients become very large. This artifact is due to the assumption that the container wall offered so little resistance to heat transfer that its temperature was uniform throughout. When the temperature in a glass wall is nearly uniform, the exterior and interior film coefficients will be equal to or less than the wall conductance, k_c/L_c, hence convection will be less ideal and the assumption of Newtonian heating is less satisfactory.

A third approximation.--Approximation of the effect of the container wall by a single uniform temperature solid is not satisfactory for most convection problems. The approximation seemed to fail because no allowance was made for the effect of the container conductance.

The approximation could be improved by dividing the wall into several equal thicknesses, ΔX, each of which is assumed to have a uniform temperature with a contact conductance equal to $k/\Delta X$. This analytical continuation, suggested by Kreith's (1958) approach is closely related to electrical analogues and to numerical methods for hand or digital computer solution. There are several of these approximations for treating the convective boundary condition (see Black, 1962) which may be used for numerical analyses. Some of these--Elrod's (1957) approximation for a semi-infinite solid and Black's (1962) approximation, for example--are not suitable for using in first order differential equations. Others, such as Dusinberre's (1961) lumped approximation (his Equation 3.1) for high Biot numbers N_{Bi}, ignore the heat capacity. Dusinberre has presented two approximations; the use of a one-half thickness, i.e., $\Delta X/2$, increment adjacent to the surface, or the use of a fictitious surface temperature to modify the lumped capacitance equations to include lumped resistance as well. The second approximation, used by Dusinberre (1961) as a first numerical approximation for a problem (his Example 6-3) like that investigated by Wolfe (1959), will be used here.

In the following examples lumping of both capacitance and resistance will be used in order to obtain a simple relation which may suggest a way to correlate heat penetration data for moderate convection in glass containers. The example should also illustrate how the container wall might be subdivided in order to obtain higher order approximation. If higher order approximations are desired, electrical analogues (Jakob, 1956, Chapter 20), higher order time approximation differential equations (Mickley <u>et al</u>., 1957 § 10.3 and 10.5), backward time step differential equations (Anderson <u>et al</u>., 1961a, b) and integral equations (Elrod, 1960; Goodman, 1960; and Yang, 1961, among others) should also be considered.

Consider the flow of heat through a container wall with film coefficients and nodes as sketched below:



The heat balances for these points assuming quasi-steady conductance are:

For the point 2

$$h_oA\,(v_e - v_2) = \frac{kA}{\Delta X/2}\,(v_2 - v_1) \tag{2.32}$$

For the point 1

$$\frac{kA}{\Delta X/2}(v_2 - v_1) - \frac{kA}{\Delta X/2}(v_1 - v_o) = A \Delta X C_p \rho \frac{dv_1}{dt} \qquad 2.33$$

For the point 0

$$\frac{kA}{\Delta X/2}(v_1 - v_o) = h_2 A(v_o - v_f) \qquad 2.34$$

For the fluid

$$h_2 \ A \ (v_o - v_f) = A \ M \ C_p \rho \frac{dv_f}{dt} \qquad 2.35$$

These equations may be reduced to:

$$(2 + N_{Bi_o}) \ v_2 = N_{Bi_2} \ v_e + 2 \ v_1 \qquad 2.32a$$

$$2 \ v_2 - 4 \ v_1 + 2 \ v_o = \frac{\Delta x^2}{2} \cdot \frac{dv_1}{dt} \qquad 2.33a$$

$$(2 + N_{Bi_o}) \ v_o = 2 \ v_1 + N_{Bi_o} \ v_f \qquad 2.34a$$

$$v_o - v_f = M \ C_p \rho \frac{dv_f}{dt} \qquad 2.35a$$

These heat balances may be rewritten to eliminate the surface temperatures (points 0 and 2). The resulting equations may be rewritten in the form:

$$\left[\frac{2 \ N_{Bi_2}}{2 + N_{Bi_2}}\right](v_e - v_1) - \left[\frac{2 \ N_{Bi_O}}{2 + N_{Bi_O}}\right](v_1 - v_f) = \frac{\Delta x^2 C_p \rho}{k} \frac{dv_1}{dt} \qquad 2.36$$

and

$$\left[\frac{2}{2+N_{Bi_0}}\right](v_1-v_f) = \frac{M\,C_p\,\rho}{h_o}\,\frac{dv_f}{dt} \qquad 2.37$$

These equations have the same form as those for the lumped capacitance problem except that the film coefficients h_1 and h_2 of equation 2.24 are replaced by their equivalent conductances: $1/\left[(1/h_1) + (\Delta X/2k)\right]$ and $1/\left[(1/h_2) + (\Delta X/2k)\right]$.

Conclusions

Simple approximations and analytical solutions, when available, have been used to show the great importance of container capacitance and the somewhat lesser importance of container conductance and film coefficients on the heat penetration parameters, f and j respectively. The container capacitance was shown to increase the j slightly, and increase which may prove not to be significant.

Several causes for variation of f and j are indicated: an increase in f and j due to the introduction of a container capacitance; an increase in f, but reduction in j with increased surface resistance; opposite effects on f and j due to convergence of the equations which accounted for container effects; and by analogy an increase in f and decrease in j due to thermocouple capacitance.

The lumped capacitance and resistance modification of the Newtonian heating model was shown to be subject to large errors in f when the container capacitance was large. These errors were indicated to be reduced when the temperature gradient was reduced. Thus, the modification gives better predictions when the internal and external film coefficients are relatively small. Approaches which might yield improved estimates of f and j have been pointed out.

Combination of Dimensional Analysis and the Newtonian Heating Model

Dimensional analysis of a physical problem combined with physical measurements is one of several ways to analyze convection heat transfer data. Other methods include exact or approximate solutions of the so-called boundary layer equations, analogy between heat, mass, and momentum transfer, and recently penetration theory (Calvert and Kapo, 1963). All but the last of these will be illustrated in the consideration of the processing literature in Section VI; for example: the dimensional analysis of Okada (1940a), the modification of Charm's (1963) similarity consideration, the dimensionless correlations for shape factors (Fagerson, 1950), and the many dimensional analysis correlations for thermo-syphon tubes, all illustrate the use of dimensional analysis; Eckert and Drake (1959) and Lighthill (1953) used approximate boundary layer solutions for the flat plate and thermo-syphon tube, respectively, and some of the aspects of the momentum and heat transfer analogies have been applied--the concept of eddy diffusivities as applied to local effective thermal diffusivities which was used to follow velocities in the core using temperature measurements, and the concept of equal effective momentum and mass diffusivities used to follow core velocities using dyes.

Dimensionless group correlations of experimental data have certain advantages which meet the requirements of this dissertation. Such correlations can be used to relate the effects of the many variables, which affect transient natural convection, in terms of a reduced number of variables. They are also useful when the exact or approximate boundary layer equations are impossible to solve without rather restrictive assumptions or differential equations have not or cannot be formulated because the physical mechanism and conditions are not known.

Dimensional group correlations do have limitations, however. These are the generally arbitrary form of the correlations and the possibility of regrouping the properties into unfamiliar or awkward new groups. These limitations are particularly serious when a relatively new field is being investigated. (Transient natural convection is such a new field.) For these reasons, it seems profitable to examine the form limitation further before proceeding into an examination of constant and then variable film coefficient correlations. Current practice will be examined as the need for additional groups arises in order to avoid awkward or unfamiliar groups.

On the Form of Correlations

When dimensionless ratios are determined from an analysis of the physical problem, its physical characteristics and the requirement that the product of the physical characteristics be nondimensional, equations of the form:

$$N_{Nu} = a\, N_{Gr}{}^{m}\, N_{Pr}{}^{n} \qquad 2.38$$

for example, are suggested. The physical significance of the equation often is not immediately evident even though this relation can be partially interpreted in terms of physical qualities. Such a relation for the above equation is:

$$\frac{\text{Total heat transfer}}{\text{Conductive heat transfer}} \approx \left[\frac{\text{Inertial force}}{\text{Viscous force}} \cdot \frac{\text{Buoyancy force}}{\text{Viscous force}}\right]^{m} \cdot \left[\frac{\text{Momentium diffusivity}}{\text{Thermal diffusivity}}\right]^{n} \qquad 2.38a$$

Equations of this form may be unsatisfactory if the mechanism changes or if a correlation over a moderate to wide range of parameters is desired.

Dimensionless ratios may also be determined from an examination of the differential equations for the problem or for an approximate model of the problem. An analysis of these equations may or may not suggest a form of an equation other than that illustrated in Equation 2.38 above. In some instances, such as the example of heat transfer to air flowing normal to a cylinder (Tao, 1960), an approximate model can be utilized to give both more accurate and more readily interpreted relations.

In this case heat transfer is assumed to be due to a laminar film and a buffer zone in series

$$\frac{1}{h_o \pi D} = \frac{\delta}{k \pi D} + \frac{1}{h_z} \qquad 2.39$$

or

$$\frac{k}{h_o D} = \frac{\delta}{D} + \frac{k}{h_z} \qquad 2.39a$$

where: h_o is the film coefficient

D is the diameter of the cylinder

k is the thermal conductivity of air

δ is the thickness of the laminar region

h_z is the conductance of the buffer zone

The fluid movement for natural convection when forced convection is negligible is entirely due to the Grashof number forces (Kreith, 1957, p. 304); hence the laminar boundary layer is inversely a function of the Grashof number. The heat transfer, however, is a function of both the Grashof and the Prandtl numbers. Therefore:

$$\frac{k}{h_o D} = \frac{K_1}{N_{Gr}^{m}} + \frac{K_2}{N_{Gr}^{n} N_{Pr}^{p}} \qquad 2.39b$$

or

$$N_{Nu} = \frac{N_{Gr}^{m+n} N_{Pr}^{p}}{K_2 N_{Gr}^{m} + K_1 N_{Gr}^{n} N_{Pr}^{p}} \qquad 2.39c$$

The last equation was said to be similar in form to published correlations for liquid-metal systems and is not greatly different from the Eckert and Drake (1959) correlation for laminar heat transfer from a flat plate

$$N_{Nu_x} = \frac{0.508\ N_{Pr}^{1/2}\ N_{Gr}^{1/2}}{(0.952\ N_{Gr} + N_{Pr}\ N_{Gr})^{1/4}} \qquad 6.41a$$

The above example illustrates how simple models, as well as physical considerations, may be used to advantage. The example also suggests that the film coefficients, as they appear in the Newtonian heating model for a container with negligible wall capacitance, for transient convection be treated in a similar manner. It is not immediately clear whether or not container capacitance or variable properties could be taken into consideration if this were done. This problem will be re-examined after a consideration of both current practice for analogous systems and of transient natural convection of confined fluids.

Constant Film Coefficient Models

Constant film coefficients for transient natural convection are a convenient artifice which may be useful in obtaining first order correlations. They are an artifice because they are assumed to be quasi-steady and to be characterized by some constant temperature difference and by constant transport properties as well. Nevertheless they will be useful to contrast with higher order approximations.

Previous Transient Studies with Appreciable Wall Capacitance

It will be useful to examine the relatively few studies in which there have been transient heat flow and appreciable wall capacitance, since the previous analysis emphasized the importance of this factor. These studies may also provide a basis for judging the possible implications of other transient or confined convection studies.

The dynamic response of heat exchangers having internal heat sources has been reported by Clark *et al*. (1957), Arpaci and Clark (1957, 1958), Yang *et al*. (1950), and Yang (1962). In this series of papers the experimental and analytically predicted behavior of the fluid and the electrically heated tube surrounding it when the walls are subjected to a step, a linear, and a sinusoidal (steady or transient periodic) change in power input are examined. The flow rate and the film coefficient are assumed constant. This system differs from the phenomenon found in confined natural convection in that there is a forced vertical flow without feedback of the exit stream (rising film) back into the heat exchanger. There is the simplifying knowledge of the energy input to the system. This system was analyzed using Laplace-transformed equations with the assistance of a digital computer. These investigations are of importance here due to their extensive literature review and their choice of parameters. The literature survey indicates that this system may be as analogous to the one

under investigation reported as any other then reported. In the analysis a number of parameters, which have appeared above, appear once again; these are the heat capacity ratio, the quantity $M = (hA)/(\rho C_p V)$ which is equal to 2.30/f for the fluid in Ball and Olson's (1957) notation and similar quantity for the wall. The nondimensional wall temperature response, fluid wall temperature difference, and the fluid temperature response are all correlated in terms of these quantities. It is interesting to note that the factor 1/f is multiplied by the ratio $(M + 1)/M$ or the total heat capacity of the system/the total heat capacity of the fluid. When the first element of the fluid subject to the step change reaches the end of the heat exchanger time also becomes a factor. During this first period the effects of relative heat capacity of the wall and the container are the same as that described above. This period is somewhat analogous to the first flow loop up the side wall after which time, time becomes important. (In canned foods, however, both the wall temperature and the bulk temperature change.) The relatively complex forms of their analytic equations do not suggest relatively simple dimensionless correlations beyond this initial period.

Transient natural convection from vertical elements with and without appreciable thermal capacity has been investigated by Gebhart (1961, 1962) and Gebhart and Adams (1962). In the first of these investigations integral

equations for the average surface temperature of a plate of finite thermal capacity subjected to a step change in heat flux were formulated. The negligible thermal capacitance case was solved for fluids having Prandtl numbers of 0.01 to 1,000 (the Prandtl number of water in the low acid processing range is about 3, while syrups and glycerols have Prandtl numbers one or two orders higher, respectively). Literature values for the quasi-steady-state velocity and temperature were used for the region in which similar laminar flows were found. This is the same similarity region as that treated by Lighthill (1953) for thermo-syphon tubes. In this analysis two parameters, a generalized time variable, and a thermal capacity parameter were introduced. These are:

$$T = \frac{\alpha \tau}{L_2} \cdot (\underline{b})^{2/5} (\bar{N}_{Gr_L}{}^{*} \bar{N}_{Pr})^{2/5} \qquad 2.40$$

and

$$Q = \frac{C''}{\rho C L \underline{M}_o} \cdot (\underline{b})^{2/5} (\bar{N}_{Gr_L}{}^{*} \bar{N}_{Pr})^{1/5} \qquad 2.41$$

where the variables are as previously defined except that $\underline{b}$ = Prandtl number dependent constant, $\underline{M}_o$ = a Prandtl number dependent derivative of the temperature distribution, C" = thermal capacitance of the element per unit surface area, and $N_{Gr_L}{}^{*}$ is the Grashof number based on total height L and the steady-state heat flux. The constants $\underline{b}$ and $\underline{M}_o$ were found to be relatively independent of the Prandtl number and the shape of the thermal boundary layer was relatively unaffected by the Prandtl number.

The parameter T appears to be a function of the Fourier modulus, an empirical power of a Rayleigh number and the Prandtl number sensitive "constant." The quantity which is raised to the 2/5 power is the square of the Nusselt number, however. It can be shown that the Fourier modulus • Nusselt number product is equal to the ratio of the time and the time constant for the fluid at the surface. Therefore, T is a restatement of the time and the resistances and capacitances of the lumped conductance model in terms of the quantities the Fourier modulus, the Prandtl, and the Rayleigh numbers which characterize the convection and conduction aspects of the phenomenon. Similarly the parameter Q is a restatement of the product of a capacitance ratio and the Nusselt number or of the ratio of storage fraction of the heat transferred by the wall to the total storage fraction.

In the second paper Gebhart (1962) investigated the effect of "appreciable thermal capacity." Large values of Q and/or slow variation of heat flux are said to cause one dimensional transient conduction, intermediate values to cause true convection transients, and further changes to cause quasi-steady convection. These true convection transients were said to exist only for a relatively small range of the Q variable. The analytic predictions of this paper were compared with experimental measurements of the third paper. Satisfactory agreement was obtained in the high Q range tested.

It is difficult to assess the implications of the relatively small convection range for step changes in plate temperature except at long times when the heat flux changes slowly. In this region the film coefficients in containers may very likely be quasi-steady.

In these investigations, in which wall capacitance was considered, two seemingly different approaches were taken, but in both instances the temperature responses were correlated in terms of the time constants of the system and their component resistances and capacitances. In the latter investigation in which the heat transfer coefficients were a variable the factors which influenced them were introduced into the parameters.

Some Other Studies in Transient Convection

There have been numerous investigations of forced unsteady or periodic convection (see recent reviews and investigations, such as Siegel, 1959; Goodman, 1962; and Siegel and Perlmutter, 1962) which undoubtedly have led to a clearer understanding of transient convection processes, but these and many of the free convection studies are beyond the scope of this investigation. All of the natural convection experiments and approximate solutions of the boundary layer equations have some implications in transient natural convection of totally confined liquids for a vertical container which can be variously divided into parallel horizontal plates and a vertical cylindrical surface which may be further approximated by a flat plate if the curvature is not excessive. The problem, here, differs in several respects, however. First, there is a moving stream at "a large distance" from the surface which may interfere somewhat with the boundary layer and may cause uncertainty as to the approximate reference temperature difference for the natural convection driving force; second, both the thermal and the velocity boundary layers are established during the transient period; third, the boundary layers must decrease in the upper regions of the container in order that the heated fluid may enter the core; and fourth, there is some uncertainty as to what influence heat transfer (i.e., through the ends of the container) in opposition to the boundary layer or core flow might have on the lateral boundary flows and on overall heat transfer.

Siegel (1958) examined transient free convection from a vertical flat plate utilizing the integral form of the boundary layer equations, and using the same approximations for the velocity and temperature profiles as Eckert (See Eckert and Drake, 1959, and modification by Lighthill, 1953, for thermo-syphon tubes). The method of characteristics was used to divide the phenomenon into three domains: the establishment of a boundary layer by conduction; the modification of the boundary layer as vertical flow develops; and a quasi-steady domain in which heat transfer coefficient is nearly independent of time. For a uniform wall temperature, the time required to reach a steady-state is said to be:

$$5.24\ (0.952 + N_{Pr})^{1/2}\ (N_{Ra_L} N_{Pr})^{-1/2}\ x^{1/2} < \frac{t}{L^2} <$$

$$7.10\ (0.377 + N_{Pr})^{1/2}\ (N_{Ra_L} N_{Pr})^{-1/2}\ x^{1/2} \qquad 2.42$$

or $$5.24\ (0.952 + N_{Pr})^{1/2}\ (g\beta \Delta T)^{-1/2}\ x^{1/2}$$

$$< \left[t^1 \text{ seconds}\right]\ 7.10\ (0.952 + N_{Pr})^{1/2} (g\beta \Delta T)^{-1/2}\ x^{1/2} \qquad 2.42a$$

with the uncertainty introduced to account for a range of arbitrary assumed velocity and temperature profiles.

The time required for the boundary layer flow to become two-dimensional was said to be:

$$1.80\ (1.5 + N_{Pr})^{1/2} (g\beta \Delta T)^{-1/2}\ x^{1/2} < \left[\frac{t\,\alpha}{L^2}\right] <$$

$$2.48\ (0.6 + N_{Pr})^{1/2} (g\beta \Delta T)^{-1/2}\ x^{1/2} \qquad 2.43$$

The results indicated that the time required to reach a steady-state was reduced as the Rayleigh-Prandtl number increased, that is, if g, β, or ΔT increased or u decreased. The results for a constant heat flux indicated that this time was also reduced for increased heat fluxes. This time required to establish a steady-state also decreased as the distance from the origin of the boundary layer decreased. The time required for the heating rate of the container to become established, perhaps proportional to $t_a = \hat{f} \cdot \log j$, then might also be proportional to the distance along the flow path from the origin of the lateral boundary layer. Since the predicted times are roughly inversely proportional to one-

third the square root of the Nusselt number, factors which increase heat transfer to confined fluids from flat plates (a decreased f) might also be expected to decrease the time required to attaon a steady-state (a decreased t_a perhaps). This studyalso emphasizes the need for retaining a Prandtl number in the "constant" in order to attain greater accuracy of prediction.

Goldstein and Eckert (1959) obtained excellent precision, a coefficient of variation of 1/2 percent, for $N_{Nu}/(N_{Ra})^{1/4}$ for a flat plate with uniform heat flux over a range of Rayleigh numbers from 10^1 to 10^8, but with a nearly constant Prandtl number. This study seemed to verify Siegel's predictions about the time required to establish steady boundary layers.

Sparrow and Gregg (1960) investigated the criterion for nearly quasi-steady free convection heat transfer in gasses. For a gas with properties ρu = constant, ρk = constant, c_p constant and buoyancy force $g(\rho_\infty - \rho)$ of $\frac{gc}{T}(T - T_\infty)$ with a solution expressed in terms of a stream function which is an infinite series in specified dimensionless parameters, the linear term only being retained, criterion for quasi-steady flow to 5 percent or 2 percent error are obtained.

For 5 percent error

$$\gamma = \frac{\dot{\Delta T}}{|\Delta T|}\left[\frac{\alpha T_\infty}{g|\Delta T|}\right]^{1/2} \leqslant 0.033 \qquad 2.44$$

For 2 percent error $\gamma \ll 0.013$

or for a liquid if T_∞ for gases can be replaced by its liquid equivalent $1/\beta$

$$\frac{d(T_s - T_\infty)}{dt} \frac{1}{(T_s - T_\infty)} \left[\frac{x}{2\beta(\Delta T)}\right]^{1/2} \qquad 2.45$$

where $(T_s - T_\infty)$ is the difference in temperature between the container wall and some unspecified reference temperature (at the same plane) in the core or bulk fluid. The effect of an axial temperature gradient in the bulk temperature upon this criterion should be explored. The appropriateness of these physical property variations to food fluids of Prandtl number 1 and higher should also be explored.

Upon examination of Equation 14 as expressed in Ball's terms, it is found that f may be incorporated into this criterion. Note that $[1/(T_s - T_\infty)]$ (d/dt) $[(T_s - T_\infty)]$ is (d/dt) $[\ln (T_s - T_\infty)]$ or $-2.303/f$ for Newtonian heating; hence, for 5 percent error:

$$\left[\frac{x}{g\beta(\Delta T)}\right]^{1/2} \leqslant 0.0143\, f_{(\text{seconds}} \leqslant 0.86\, f_{(\text{minutes})} \qquad 2.46$$

for 2 percent error

$$\left[\frac{x}{g\beta(\Delta T)}\right]^{1/2} \leqslant 0.0056\, f_{(\text{seconds})} \leqslant 0.33\, f_{(\text{minutes})} \qquad 2.47$$

for quasi-steady convection, where a fluid with property variation behaves like those of gases and $P_{Pr} = 0.72$. Since the buoyancy force β increases with temperature and the

viscosity increases with temperature for water, when temperature difference decreases heat transfer, somewhat higher f values might be required in order to equal quasi-steady convection than indicated above. For this reason quasi-steady convection might be achieved early in the transient steady-state process and unless f increased substantially the laminar heat transfer rates might be exceeded later in the transient process.

Hellums and Churchill (1962) investigated transient heat transfer from a vertical flat plate. This investigation confirmed the transition from conduction to convection, but seemed to indicate that the time required to attain steady-state convection is about one-half that predicted by Siegel (1958). A slight decrease in heat transfer before transition to steady-state convection, a decrease predicted by Siegel (1958) and Gebhart (1961), but not detectable by Goldstein and Eckert (1959) was calculated. This has been attributed to a too thick boundary layer due to its accelerating growth. This could be attributed to inertial forces, the same inertial forces which could cause the Prandtl number as well as the Rayleigh number to be important for predicting heat transfer rates.

Natural Convection within Vertical Boundaries

Some of the studies of natural convection have been discussed earlier with regard to their implications for the thermal processing problem. Here, however, the emphasis is

on the form of the correlation. A wide range of Rayleigh numbers could be encountered: in syrups and brines as the liquids of convection heating food products, e.g., 1 x 10^6 to 1 x 10^{10} for a 4-inch high container, with temperature differences from 100° down to 1°F; lower Rayleigh numbers would be anticipated with more viscous fluids or with partially confined fluids in which the characteristic length might well be of the same order as the equivalent diameter of the channel or cavity. In both of these ranges complicating changes in mechanism might be encountered, e.g., for beverages the N_{Ra} range includes transition from turbulence (or mixed turbulence) to laminar flow (at about $N_{Ra} = 10^9$ for flat plates, perhaps somewhat lower N_{Ra} for confined spaces); for lower N_{Ra} there is a gradual transition through several laminar flow regimes to conduction and some limiting Nusselt number ($N_{Nu} = 2$ for the exterior of spheres).

The form of some of the equations which have appeared in the literature and their range of application are summarized in Table 2.1.

The formulas of Table 2.1 are but a few of the many heat transfer correlations for various natural convection geometries. The apparent differences between them (even for the same flow region and geometry) and their similarities are both of interest here.

TABLE 2.1.--Some reported correlations for natural convection from vertical surfaces

---

I. Flat Plate
  A. Laminar Flow
    1. Eckert and Drake (1959)

$$(N_{Nu})_x = 0.508\ N_{Pr}^{1/2}(0.952+N_{Pr})^{-1/4}(N_{Gr})_x^{1/4}$$

where $\bar{h} = 4/3\ h_x$
for low N_{Pr}

    2. McAdams (1954)

$$(\bar{N}_{Nu_L})_f = 0.59(N_{Gr})_f^{1/4} \cdot (N_{Pr})_f^{1/4}$$

$$10^4 < N_{Ra} < 10^9$$

    3. Jakob (1949)
      a. Jacob and Linde (1933)

$$N_{Nu} = 0.555(N_{Gr} \cdot N_{Pr})^{1/4}$$

      b. Touloukian *et al*. (1948) (for large diameter cylinders)

$$\bar{N}_{Nu} = 0.726(N_{Gr} \cdot N_{Pr})^{1/4}$$

$$2.5 < N_{Pr} < 120$$

$$2 \times 10^8 < (N_{Pr} \cdot N_{Gr}) < 4 \times 10^{10}$$

  B. Turbulent Flow
    1. Eckert and Drake (1959)

$$(N_{Nu})_x = 0.0295(N_{Gr}\ )_x^{2/5}[1+0.494(N_{Pr})^{2/3}]^{-2/5}$$

    2. McAdams (1954)

$$(\bar{N}_{Nu_L})f = 0.13(N_{Gr})_{f,\Delta T}^{1/4} \cdot (N_{Pr})_f^{1/3}$$

$$10^9 > N_{Pr} > 10^2$$

    3. Jakob (1949)

      a. Jakob and Linde (1933)

$$\bar{N}_{Nu} = 0.129(N_{Gr} \cdot N_{Pr})^{1/3}$$

TABLE 2.1--Continued

b. Toulоukian *et al.* (1948)

1) $\bar{N}_{Nu}=0.0674(N_{Gr} \cdot N_{Pr}{}^{1.29})^{1/3}$

$2.5 < N_{Pr} < 120$

$4 \times 10^{10} < N_{Gr} < 9 \times 10^{11}$

2) $\bar{N}u = 0.086 (N_{Gr} \cdot N_{Pr})^{1/3}$

Same range but with detectable differences due to N_{Pr}

II. Vertical Confinement Only

A. Laminar Flow

1. Kraussold (1934) for k_e across vertical air layers r thick

a. $\frac{k_e}{k} = 0.11 \left[(N_{Gr})_r N_{Pr}\right]^{0.29}$

$6 \times 10^3 < (N_{Ra})_r < 10^6$

b. $\frac{k_e}{k} = 0.40 \left[(N_{Gr})_r N_{Pr}\right]^{0.20}$

$1 \times 10^6 < (N_{Ra})_r < 10^8$

2. McAdams (1954) for flow across vertical air layers $\not\propto$ thick.

$$(N_{Nu})_{f,b} = C \left[(N_{Gr})_{f,\ b,\Delta T} \cdot (N_{Pr})_f\right]^n \left(\frac{b}{L}\right)^{1/9}$$

for $2 \times 10^3 < N_{Gr} < 2 \times 10^4$; $b/L > 3, C=0.20,\ n=1/4$

for $2 \times 10^5 < N_{Gr} < 1.1 \times 10^7$; $C=0.071,\ n=1/3$

3. Jakob (1949) recommended $C = 0.18$ and 0.065 for the above ranges, respectively.

4. Nieman (1948) in Grober *et al.* (1954)

$$\frac{k_e}{k} = 1 + \frac{m(N_{Ra})_b}{(N_{Ra})_b + n}$$

TABLE 2.1--Continued

for Ra $< 10^8$

$m = 0.0236$
$n = 1.01 \times 10^4$
$r = 1.393$

5. Bodoia and Osterle (1962)

$$\left(\frac{h_L b}{L\,\Delta t k}\right)_{\text{average}} \equiv Nu = 0.630\left[N_{Pr}^{1/4}(N_{Gr})_b^{1/4}\frac{b}{L}^{1/4}\right]$$

10^3 to 10^4

$N_{Pr} \sim 0.70$

This compares favorably with Elenbaas (1948)

B. Turbulent flow

1. Siegel and Norris (1956)
Heat transfer for a two plate when 1×10^6 $\leq N_{Ra} < 1 \times 10^{10}$ was greater than that for laminar flow and uniform heat flux. Siegel (1954), i.e., $Nu_x = 0.453\ N_{Ra}^{1/4}$, was approximately correlated by Jakob's formula for turbulent flow but with an intermediate slope, little or no change for two heated plates when the bottom was closed, the same effects were observed when only one plate was heated, except that heat transfer was more seriously reduced by close plate spacings.

III. Spheres

Internal turbulent flow for alcohol, glycerol and water. (Schmidt, 1956)

$$N_{Nu_D} = 0.098\ (N_{Ra})_D^{0.345}$$

IV. Closed-End Tubes

A. Laminar Flow

1. Lighthill (1953) predictions

$$(\bar{N}_{Nu})_r = C_n\ \left(\frac{r}{L}\right)^n (N_{Ra})_r^{\ n}$$

TABLE 2.1--Continued

for $N_{Ra} < 4 \times 10^3$, n = 1

for $4 \times 10^3 < N_{Ra}$, but laminar, n = 1/4.

2. Martin and Cohen (1953)

   a. For Glycerine

      1) $(N_{Nu})_r = 0.0012\ (\frac{r}{L})(N_{Ra})_r$

      $10^3 < (N_{Ra})_r < 4.5 \times 10^6$

      decreasing stagnation in this region.

      2) $(N_{Nu})_r = 0.425\ (\frac{r}{L})^{0.28}(N_{Ra})_r^{\ 0.28}$

      $2.2 \times 10^4 < (N_{Ra})_r < 1.4 \times 10^6$

      Oscillation of temperatures occurred in this region.

   b. For Water and Air

      In the first region above the behavior is similar, but larger constant except for large brief drops in N_{Nu} at $(Ra)r$ at about 2×10^4.

3. Martin (1955)

   a. $(N_{Nu})_r = C(N_{Ra})_r(\frac{r}{L})$

      $C_1 = 0.00132$ for $800 < N_{Pr} < 1100$

      $C_1 = 0.00112$ for $1000 < N_{Pr} < 7600$

      for $(N_{Ra})_a < 1 \times 10^5$

      Decreasing stagnation in this region which vanishes at an N_{Ra} of about 1.5×10^4 to 2×10^4.

   b. $(N_{Nu})_r = C_1\ [(N_{Ra})\ \frac{r}{L}]^{0.28}$

      $C_1 = 0.42$ for $66 < N_{Pr} < 570$

      $C_2 = 0.44$ for $69 < N_{Pr} < 390$

      $2.1 \times 10^5 < (N_{Pr})_r < 1.8 \times 10^7$

      :an oscillatory region.

TABLE 2.1--Continued

B. Turbulent flow

1. Lighthill (1953)

   Predicted transition at N_{Ra} of about 1 x 10^4 (but somewhat later for small L/r ratios) with an immediate drop in heat transfer rate to a region having a slope of about 2/5 followed by a jump, but similar slope, to turbulent flow not filling the tube at successively larger N_{Ra} as the L/r ratio increased.

2. Martin (1955)

   a. Transition from laminar boundary layer flow (at high N_{Pr})(Transition at $(N_{Ra})r$ of about 3 x 10^5)

      $$(N_{Nu})r = 0.0325(N_{Ra})r^{.4}$$

      $$75 < N_{Pr} < 20c$$

      $$3 \times 10^5 < (N_{Ra})r < 3 \times 10^6$$

      $$L/r = 47.5$$

      with laminar core flows.

   b. Transition from laminar impeded-flow regimes. This transition was referred to in Martin and Cohen (1953) laminar flow study.

   c. Fully mixed turbulent flow. There is a general absence of the prolonged drop predicted by Lighthill (1953) and similarly no large jump.

3. Eckert and Jackson (1950) in Jakob (1957)

   The turbulent flow formula for the flat plates cited in Eckert and Drake (1959) (See above) was derived to solve a problem of thermo-syphon tubes with large superimposed body forces. It satisfactorily fitted experimental data for vertical flat plates and cylinders but not the data of Fuche and Diagula (1950) which was independent of $(N_{Ra})_L$ for the small a/L investigated.

In all instances there is appreciable conduction at low N_{Ra}. This is reflected in the lower limit of the formulas for laminar convection on flat plates, for heat transfer across channels, and the linear dependence of N_{Nu} for closed end tubes as the flow commences and begins to fill the entire tube. At higher N_{Ra}, the laminar convection is established such that N_{Nu} is proportional to about the fourth root of N_{Ra} in all geometries. Some of the differences within a geometric system may be attributed to a shift in the establishment or rate of heat transfer due to property effects. It would appear that a correlation of heat penetration data in this region should allow for this effect.

Transition to turbulence appears to be a complicated phenomenon for confined fluids--more complicated than for flat plates or large diameter vertical cylinders. In the latter instance turbulence with its thinner boundary layer (a larger power of N_{Ra}) seems to generally promote heat transfer as in the case of fluids in thermo-syphon tubes and perhaps in food containers as well if the Prandtl number is sufficiently large to permit a laminar boundary layer to be established before the onset of turbulence. Such a turbulent boundary layer for large diameter tubes might be approximated quite satisfactorily using correlations for convection outside flat plates. If a laminar boundary layer has not formed throughout the tube, there may be mixing and interference between the core and the boundary layer sufficient

to choke flow in the core and return warm fluid to the boundary layer earlier and hence reduce convective temperature differences, although the boundary layer itself may be thinner. Such a mechanism might be responsible for the decreased influence of the N_{Ra} at moderate values in what would have been a laminar region for flat plates, but appears to be a mixed-turbulent region for thermo-syphon tubes. In any event, in both of these latter confined geometries the effectiveness of N_{Ra} appears to be reduced from the 1/3 to 2/5 power dependence observed for flat plates. The N_{Pr} also appears to a higher power in turbulent flow. This is a reflection of the increased transfer of heat capacity by bulk transport, reduced conduction, and less influence of viscosity in this domain.

Natural Convection with Horizontal Boundaries

Some of the relations for natural convection will be cited later, but in relation to the flow eddies at the bottom of the container and the position of the slow point. Here the interest is principally in the magnitude of heat transfer in order to establish the relative contribution of the lateral and horizontal surfaces to overall heating. Such information would assist in the development and later discussion of a correlation for f, j, and here particularly for the slow point position, since experimental studies in thermo-syphon tubes indicate it to be very close to the bottom of the tube.

Some of the correlations for convection from and between horizontal surfaces are summarized in Table 2.2.

Once again there is a relatively large apparent difference among investigators, in spite of the great care taken by them. Despite these variations, it is evident for laminar flow that the Nusselt numbers, which are based on plate side length for single plates or on plate spacing for parallel plates are perhaps 40 or more percent lower than those for vertical plate enclosures but may or may not be reduced (but not as extensively) for turbulent flow. Reversal of the orientation from heating(cooling) of plates facing upward (downward) is seen to reduce the heat transfer still further by similar amounts. In the turbulent range the correlations for vertical and horizontal plates are relatively independent of the characteristic dimension. Since the appropriate ΔT may well be the cold point difference, the relative contribution of bottom and lateral surfaces might be the product of their relative areas, i.e., r/(2)(L), e.g., 1.5/(2)(4.5) and their relative constants--about 1/(1.4) or about 12 percent for a 300 x 408 container and perhaps 17 percent or more (based on plate and tube characteristic length of $\sqrt{\pi r^2}$ and 2a, respectively). Further, since the characteristic length for food containers might well be the slow point height, the relative heating contribution of the bottom and the top may well correspond to this position in the container, and hence be from 15 to 30 percent.

TABLE 2.2.--Some reported correlations for natural convection from horizontal surfaces

---

I. Single horizontal plates (McAdams, 1954, after Fishenden and Saunders, 1950)
  - A. Heated (cooled) plates facing upward (downward)
    - 1. Laminar flow

      $(\bar{N}_{Nu})_f = 0.54\ (N_{Ra})_f^{1/4},\quad T$

      $10^5 < N_{Ra} < 2 \times 10^7$
    - 2. Turbulent flow

      $(\bar{N}_{Nu})_f = 0.14\ (N_{Ra})_f^{1/3}$

      $2 \times 10^7 < N_{Ra} < 3 \times 10^{10}$
  - B. Heated (cooled) plates facing downward (upward)
    - 1. Laminar flow

      $(\bar{N}_{Nu})_f = 0.27\ (N_{Ra})_f^{1/3}$

      $3 \times 10^5 < N_{Ra} < 3 \times 10^{10}$
    - 2. Turbulent flow

      no correlation available

II. Vertically confined horizontal layers heated from below
  - A. Creeping flow
    - 1. Jakob and Gupta (1954)

      $(\bar{N}_{Nu})_f = 0.300\ (N_{Ra})^{1/4}$

      $2 \times 10^3 < N_{Ra} < 1.5 \times 10^5$
    - 2. O'Toole and Silveston (1960)

      $(\bar{N}_{Nu})_f = 0.0238\ (N_{Ra})_f^{0.816}$

      $\underline{1700} \leq N_{Ra} \leq \underline{3500}$
  - B. Laminar flow
    - 1. Jakob and Gupta (1954)

      as A(1) above or

      $(\bar{N}_{Nu})_f = 0.3615\ (N_{Ra})_f^{1/4}(L/D)^{0.058}$

      $L/D \leq 1.5$
    - 2. Poots (1958)

      $(\bar{N}_{Nu})_f = 0.17\ (N_{Ra})_f^{1/3}$

      $N_{Ra} > \underline{3500}$
    - 3. O'Toole and Silveston (1960)

      $(\bar{N}_{Nu})_f = 0.229\ (N_{Ra})^{0.252}$

      $\underline{3500} \leq N_{Ra} < 10^5$
  - C. Turbulent flow

TABLE 2.2--Continued

II. C. 1. Jakob and Gupta (1954)

$(\bar{N}_{Nu})_f = 0.126\ (N_{Ra})_f^{1/3}$

$3 \times 10^5 < N_{Ra} < 3 \times 10^9$

2. Globe and Dropkin (1959)

$(\bar{N}_{Nu})_f = 0.069\ (N_{Ra})^{1/3}(N_{Pr})^{0.74}$

$1.5 \times 10^5 < N_{Ra} < 6.8 \times 10^8$

$0.02 < N_{Pr} < 8750$

3. Malkus

$(\bar{N}_{Nu})_f = 0.085\ (N_{Ra})^{0.325}$

4. Nieman (1948) in Grober, Erk, Grigull (1957)

$$\frac{k_e}{k} = 1 + \frac{0.070\ N_{Ra}^{1.333}}{N_{Ra} + 3200}$$

$N_{R\varepsilon} > 10^8$

5. O'Toole and Silveston (1960)

$(N_{Nu})_f = 0.104\ (N_{Ra})_f^{0.305}(N_{Pr})_f^{0.084}$

$10^5 < N_{Ra} < 10^9$

On the Proposed Lumped Newtonian Convection Model

The temperature of the product can be shown to be a function of the Biot numbers N_{Bi} at the surfaces of the container, the Fourier modulus of the container (from Equation 2.35), a Fourier modulus of the product in which the characteristic length is the depth of the fluid and the Nusselt number of the product (from Equation 2.36). In these Fourier moduli the differential time dt is presumed to be the time.

The roots of the analytic solution of an analogous problem, heat loss from a room, seem to be expressed in terms of Biot numbers and capacitance ratios alone (see Equation 2.22), but these capacitance ratios may also be formed as the ratio of the product of the Fourier moduli and the Biot or Nusselt numbers, respectively. In the lumped capacitance and the lumped capacitance and resistance models the time constants (Equation 2.24) may be expressed in terms of resistances (or Biot numbers) and capacitances alone. Then it would seem that the Fourier moduli and the Nusselt number might eliminate the capacitances. Equations 2.28a and 2.30b show that the Fourier moduli appear in the relations of f and j only at relatively short times. For this reason Fourier moduli and Nusselt number effects upon the apparent $\hat{f}$ may be attributed to slow convergence (or perhaps a changing film coefficient) as the heat penetration data may be presumed to represent the asymptote having a slope f.

In the consideration of the engineering studies, it was noted that the relative contribution of the various surfaces and the N_{Pr} effects on each of these should be accounted for. The container could of course be partitioned into a truncated paraboloid and the residual part of the finite cylinder in order to proportion the capacitance for each surface and incorporate the film coefficient equations for these surfaces into the lumped capacitance and resistance model. For ease in computation, however, the equation for f is arbitrarily assumed to be of the form:

$$f = 2.303 \left(\frac{C_p \rho V}{hA}\right)\left(\frac{N_c}{N_c+1}\right)^n \tag{2.48}$$

where $$\frac{1}{hA} = \left[\frac{1}{\left(\frac{1}{h_o} + \frac{\Delta X}{R_w}\right)(2L+r)} + \frac{h_L}{2L} + \frac{h_b}{r}\right] r \tag{2.48a}$$

h_o = external coefficient determined from separate experiments

h_L = internal coefficient on lateral surfaces

h_b = internal coefficient on bottom surfaces

$C_p \rho V$ = fluid properties

N_c = heat capacitance ratio

n = an experimental exponent (less than unity)

It is further presumed that the film coefficient are in the form:

$$h_1 = \frac{k}{r} G_1 (N_{Pr})^m (N_{Ra})^P \left(\frac{r}{L}\right)^q \tag{2.48b}$$

$$h_b = \frac{k}{r} G_2 (N_{Pr})^s (N_{Ra})^v \tag{2.48c}$$

If it is further presumed that the convection from the top and the bottom will be in the same flow regime, i.e., N_{Ra} controlling, and hence s and p are equal, this can be seen from the summaries of the heat transfer correlations that these relations can be further simplified

$$f = 2303\,(C_p \rho)\,[N_c/(N_c+1)]^n \cdot 2 \cdot \left\{\left[\left(\frac{1}{h_o} + \frac{\Delta x}{k_w}\right)\left(2 + \frac{r}{L}\right) + \frac{k_f}{r} C_1 N_{Ra}^{\,p} N_{Pr}^{\,m} \frac{r}{L}^{q} \cdot \left(2 + \frac{C_2}{C_1} N_{Pr}^{\,s-p} \left(\frac{r}{L}\right)^{q+1}\right)\right]\right\} \qquad 2.49$$

This formula form seems to retain the lumping concept--simplified somewhat with capacitance lumping by use of $N_c/(N_c + 1)^n$ factor. The values C_p and the capacitance correction factor components could be determined fairly readily from water-like product and glass (or tin) properties. The heat flux has been partitioned into a container--heating medium and a product-size-shape component, each of which contains a shape correction of similar form, i.e., $G_5 t_b(\frac{r}{L})$.
In this proposed correlation there are seven experimentally determined constants; hence, many different experiments would need to be performed under a wide range of conditions in order to determine these values. Fortunately, some of these constants--perhaps all except n, G, and G_2, could be made identical to those for reported engineering studies for the same approximate flow domain in order to evaluate through G_1 and G_2 the appropriateness of these substitutions.

If the local (i.e., slow point) temperature is not used an x/L factor and perhaps a radial position factor would need to be incorporated into the correlation. The position of the line at t_p is fixed and the tangent slope $(-1/f)$ can be computed using Equation 2.48, but a correlation for j seems to require a knowledge of the initial transport lag, then stepwise calculation of u as a function of time. Because $\log j_p = u - \frac{t_p}{f_h}$, and $t_a = f \log j$, $t_a = fu - t_p$.

The slow point position might be correlated in the following form--a form which is suggested by solving for the vertical plane at which there is no overall net heat transfer for plug flows, i.e.,

$$\frac{x}{L} = G_3 \frac{G_1 (N_{Pr})^m (N_{Ra})^p \left(\frac{r}{L}\right)^q r^2}{G_2 (N_{Pr})^s (N_{Ra})^v \, 2rL} \qquad 2.50$$

or

$$\frac{x}{L} = G_4 \left(\frac{G_1}{G_2}\right) \left(N_{Pr}{}^{m-s}\right) (N_{Ra})^{p-v} \left(\frac{r}{L}\right)^{q+1} \qquad 2.50a$$

This is, of course, a simplified case, a case of no negligible wall resistance and large exterior film coefficient. Note that if x/L does prove to be a function of N_{Ra} then $N_{Ra}{}^{v-p}$ would be expected to appear in the second term of the internal film coefficient correction factor. If the natural convection domains (e.g., turbulent or laminar flow as conduction) for the bottom and the sides are similar and if the quantities m-s and p-v vanish as they might under these circumstances, then the slow point position is a function only of container shape, flow domain, and container orientation (i.e., tops up or tops down) where the ratio C_1/C_2 reflects these last two factors.

A more exact relation might be:

$$\frac{x}{L} = G_3 \frac{\left[\frac{1}{G_2 (N_{Pr})^s (N_{Ra})^v} + \frac{2 \Delta X}{k} + \left(\frac{1}{h_o}\right)_{bottom}\right] \pi r^2}{\left[\frac{1}{G_1 (N_{Pr})^m (N_{Ra})^p \frac{r}{L}^9} + \frac{\Delta X}{k} + \frac{1}{h_o} \quad side\right] 2 \pi r l} \qquad 2.50c$$

$$= G_3 \left[\frac{G_2 (N_{Pr})^s (N_{Ra})^n (2 \Delta X h_o + k_c) + (kh_o)_{bottom}}{G_1 (N_{Pr})^m (N_{Ra})^p \frac{r}{L}^9 (\Delta X_o + k_c) + (kh_o) \ side}\right]$$

$$\cdot \left[\frac{r}{L}\right] \left[\frac{G_1}{G_2} (N_{Pr})^{m-s} (N_{Gr})^{p-v} \left(\frac{r}{L}\right)^q\right] \left[\frac{h_o side}{h_o \ bottom}\right] \frac{1}{2} \qquad 2.50d$$

Variable Fluid Properties

The Sullivan and Holland (1961) approach.--These investigators have used some quite good assumptions as to temperature induced property variation--these include a linear specific heat, an exponential variation of viscosity, the somewhat less precise assumption that $(k/C_p)^{1-b}$ is a linear function of temperature and a linear change of k_{wall} and h_o a steam film coefficient--and an artful choice of transform variable to reduce the equation for the area (time) required to reduce the emperature difference to a given level to an implicit function of temperature. This function is quite involved as the exponential integral function of complicated arguments is involved. One iteration is used to correct for the temperature drop across the wall.

An unsuccessful search for a transform or property dependency assumption to reduce the natural convection equation was undertaken during this present investigation. The problem is essentially to reduce a function not simpler than

$$\int_a^b \frac{dT}{T^n}$$

but realistically $\displaystyle\int_a^b \frac{dT}{\left[(a_1+b_1T)(a_2+b_2T)a_3\mu^{-1/T}\right]^n}$ to an integrable form.

The term $\int_a^b T^{-n}dT$ for temperature could be reduced by the usual log differentiation strategem to produce the relation for a power dependent temperature difference. Such a strategem, but for a linear change in film coefficient, is illustrated below.

Linear change in film coefficient.--A linear change in film coefficient, a simple case, was examined in order to have a simple way to show the effect of such variation.

From a heat balance on the central core (neglecting container and film capacitances)

$$dQ/dt = UA(T_1 - T) = C\rho V dT/dt \tag{2.51}$$

$$\frac{dt}{U(T_1-T)} = \frac{A}{C\rho V}\ dt \tag{2.51a}$$

assume $U = a^{1}(1+b'T)$ 2.51b

$$\frac{dT}{a^1(1+b^1T)(T_1-T)} = \frac{A}{C\rho V}\ dt \tag{2.51c}$$

By separation by partial fractions the problem becomes

$$\frac{1}{a^1(1+b^1T_1)}\int_{T_o}^{T}\frac{dT}{(T_1-T)} + \int_{T_o}^{T}\frac{b^1dT}{(1+b^1T)} = \frac{A}{C\rho V}t \tag{2.52}$$

Hence: $\ln\left(\frac{T_1-T}{T_1-T_o}\right)\ln\frac{U}{U_o} - \left(\frac{A}{C\rho V}\right)V_1 t$ 2.53

and: $u = \frac{u}{u_o}\ t_o^{-t/fh_1}$

Now

$$\frac{\partial}{\partial t}\ln\left(\frac{T_1-T}{T_o-T_o}\right) = \frac{\partial \ln(U/U_o)}{\partial T} - \frac{U_1A}{C\ V} \tag{2.54}$$

Where $\frac{\partial \ln(U/U_o)}{\partial t} = \frac{1}{U/U_o}\ \frac{d(U/U_o)}{dT} = \frac{U_o}{U}\ \frac{1}{U_o}(0+a^1b^1)$ 2.55

$$= \frac{a_1b_1}{U}$$

and from (1)

$$\frac{\partial T}{\partial t} = \frac{UA(T_1-T)}{C\rho V} \tag{2.56}$$

or when the above have been substituted

$$\frac{\partial}{\partial t}\ln\frac{(T_1-T)}{(T_1-T_o)} = \frac{A(U_1-U-U_o)}{C\rho V} = \frac{-AU}{C\rho V} \tag{2.57}$$

therefore $\hat{f} = 2.303 \frac{C\rho V}{UA}$ 2.58a

$$f = 2.303 \frac{C V}{U_1 A}$$ 2.59a

Now the Ln j $(t_p) = \ln \frac{U(t_p)}{T(t_p)_o} - \frac{AU_1}{C\rho V} t_p + \frac{t_p(-)(-)AU(t_p)}{C\rho V}$ 2.60a

or

$$\text{Ln } j_{t_p} = \frac{\ln U(t_p)}{U_o} + \frac{A}{C_\rho V} (U-U_1)t_p$$ 2.60b

therefore the true $j = U_1/U_o$ 2.61

$$\hat{j} = \frac{(1+bT)}{(1+bT_o)} = \left(\frac{T_1-T}{T_1-T_o}\right) e^{\left(\frac{U_1 A}{C V}\right) t}$$ 2.62

$$= \frac{U}{U_o} 10^{-\frac{t}{b} + \frac{1}{\hat{f}}}$$ 2.63

This little exercise, completed earlier in the investigation, was incorporated with the geometric sensitive thermosyphon tube film coefficient correlations. It satisfies the requirements that can factors be temperature dependent, j being a function of changing flow domain and container r/L ratio (to a power) and that f and j be inversely interrelated by reason of variable film coefficients. The requirements are imposed by the conclusions from an analysis of the literature. The proposed correlation presented previously also meets these requirements and incorporates other factors such as relative contribution of surfaces and container capacitance as well.

Dimensionless property change factors.--The form of the proposed correlation might be further simplified by removal of the u factor from the N_{Ra} and the viscosity μ from N_{Ra} and N_{Pr} and introduction of reference viscosity, a reference temperature, and a $\nu_o/(\nu_1 \Delta T)$ factor when ν_o is the viscosity at a reference temperature and ν_1 is the temperature coefficient in order to use an initial reference fluid, initial temperature difference, and property change variables instead of computed N_{Ra} and N_{Pr} at a specified point in the transient process.

The Plug Flow Model

The concepts of a plug flow model have already been utilized in the proposed Newtonian model correlation for the slow point position and the proposal that f (approaching the slow point from above or below) be considered a function as the x/L ratio.

The heating of a container of a low viscosity liquid product, e.g., a beverage, is much like the heating of a composite slab from the top and the bottom if a uniform width plug is assumed. If this last assumption were true, the f would be expected to differ due to different effective diffusivities of the descending and ascending plugs, but to be uniform in each region. Although this uniformity is not the case, the model is not necessarily inexact. The variation might be attributed to either a space dependent property variation although actually the convection velocities might be decreasing--this would cause f to increase as the position lag away from the surface increased-- or due to a time dependent, effective film coefficient at the top or bottom of the container which causes convergence to make f appear to increase as the slow point is approached.

The j values for such a plug model would be anticipated to be equal to or below those of a composite slab heated at both surfaces by a transiently heated thick skin, a time dependent contact film coefficient. That is, j would be anticipated to be 1.27 or lower for such a model. Larger j values, therefore, are a sign that there is appreciable radial conduction across the boundary layer. Unfortunately, this hypothetical composite slab model is not orthogonal with the radial flow model (even for a negligible lateral contact resistance).

Other Approaches

Hammitt (1957, 1958a, 1958b) has recently formulated a computer program using an analysis modeled after that of Lighthill (1953) for transient convection with no initial transport lag in a finite cylinder with the heat absorption equated to an internal generation term of an earlier study. The possibilities of such an approach were seen from the earlier study, but its extension to the problem here was and is beyond the scope of this dissertation. An extensive analysis and experimental program was instituted instead, in order to clarify the problem. Now, however, if higher precision were desired or a program generalized enough to permit adequate treatment of the anomalies encountered with most food products could be formulated a computer could be used profitably.

Property Values and Lengths

Many of the property and geometric considerations have been discussed at length above and some of these have already been incorporated in the proper correlation form.

A substantial problem which remains, however, is the choice of a reference temperature suitable for estimating the film thermophysical properties.

For the moment, let us consider only the "local" film coefficient and assume that the quasi-steady state analysis is a good estimate. Sparrow and Gregg (1958) found a satisfactory reference temperature heat transfer by a gas with special property variations., $T_t = T_W - 0.38\ (T_W - T_\infty)$ and that this reference temperature was a good estimate for some other property variations as well. Moszynski (J.R., in discussion) illustrated that estimates made with the usual unweighted means are not too widely divergent from the above. This latter estimate, the mean of the wall, and the bulk temperature will be used here.

The overall local temperature differences will be used as a reference buoyancy force for its convenience rather than its appropriateness. It is difficult to formulate a concept of an improved temperature difference, moreover. The implications of the Sparrow and Gregg (1958) study of the effect of a non-isothermal free stream on boundary layer heat transfer will be introduced later in a discussion of the particular experimental systems.

Conclusions

The treatment of the problem of transient natural convection in food containers has been examined from the viewpoint of a modified Newtonian heating model. In light of the known departure from the Newtonian model, due to large axial temperature gradients, it can be seen that the approach should not be pursued too far. Exact answers for an inexact analogy could be attained.

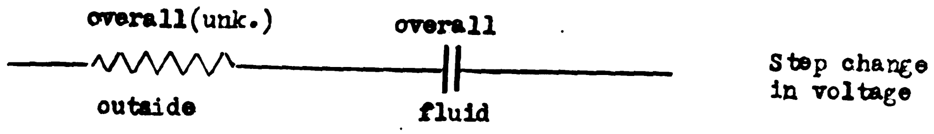
The above treatments can be restated in terms of electrical analogies as shown in Table 2.3 as suggested by Sunderland (1963). It can be seen how inexact are the models currently available. Even the model proposed here is electrically rather simple. Its principle contributions are the combination of parallel flow through walls which have capacitance and steady-state resistance, steady-state exterior resistance, and quasi-steady-state internal resistances to a fluid with capacitance that is a combination of moderately acceptable oversimplifications.

Since this model only gives insight into the behavior of a local temperature change--presumably the cold point--

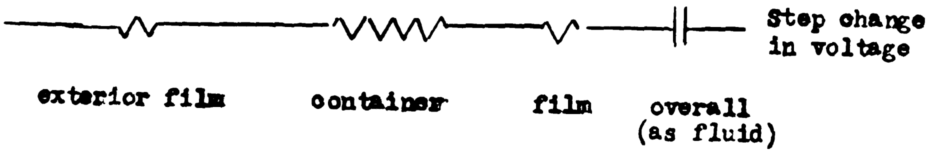
and its location, other approaches should be used to obtain meaningful estimates of j, or better, the temperature history. Also, quasi-steady film coefficients and residues may prove not to be satisfactory assumptions in practice. In addition, when in the future analogue or digital computers are applied, some provision should be made to account for the circulatory pattern and particularly the establishment of convection.

TABLE 2.3 Electrical analogy of modified Newtonian heating models.

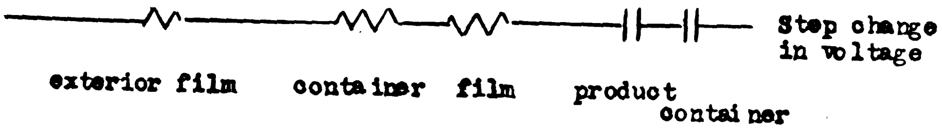
(1) Jones (1931), Schultz and Olson (1939), Ball and Olson (1957) Model.



(2) Merrill Model (1948)

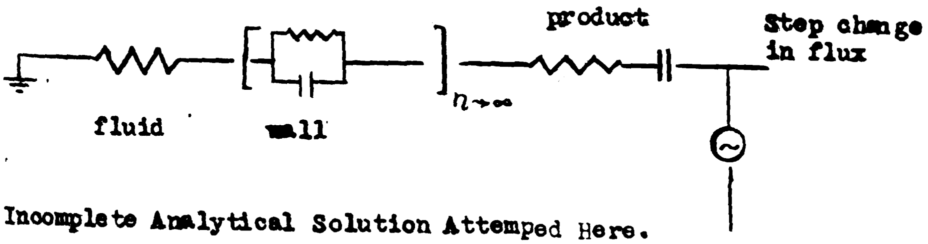


(3) Nicholas (1962)



(4) Analytical Problem of Wall Resistance Alone and Capacitance.

(5) Wolfe Problem (1949)



(6) Incomplete Analytical Solution Attemped Here.

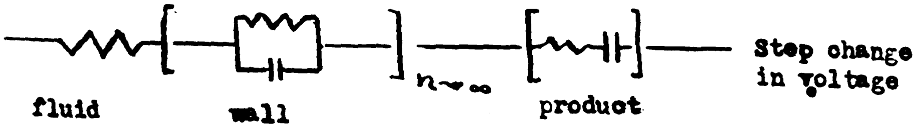
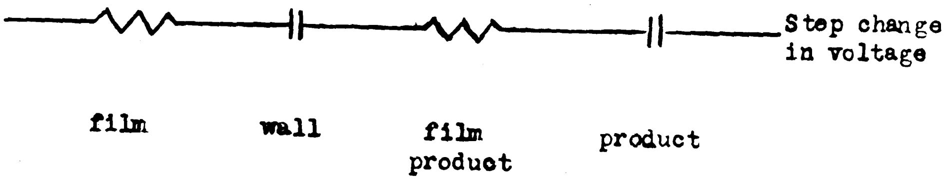
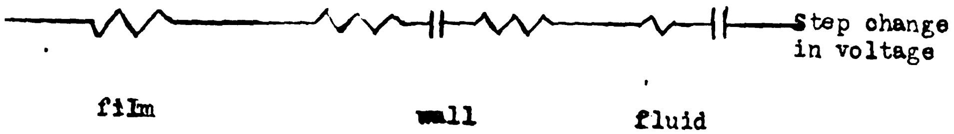


TABLE 2.3 Continued

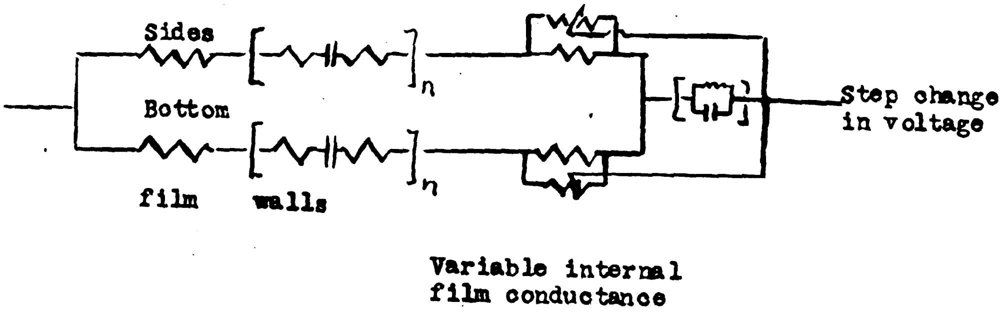
(7) **Thermocouple-Well, Without Resistance**



(8) **Thermocouple-Well With Resistance**



(9) **A Tentative Approach**



SECTION III

THE EXPERIMENT

The experimental phase of this investigation was undertaken to examine some of the relationships governing natural convection heating of liquids. The aspects of the phenomenon of particular interest were those whose effects appeared to be detectable in practice in spite of product and come-up variability. This phase was undertaken concurrently with the other phases (after a literature appraisal of the problem areas), in the absence of a proven correlation of thermal transport properties and characteristic dimension on the heat penetration parameters f and j. The aspects investigated, therefore, were not oriented towards the establishment of a correlation but towards the understanding of unanswered questions and an appreciation of the certainty (or lack of certainty) of the flow and temperature patterns for this system.

The first problem was the resolution of the effects of initial and processing temperature on heating rates into the effect of temperature difference on natural convection, and the effect of the most temperature dependent transport property, the viscosity. In earlier investigations for convection heating, differences in heat penetration corresponding to different final or different initial temperature had been observed and for conduction heating the f approached those corresponding to the thermal properties and the final temperature, and Joslyn (1928) had found an empirical correlation for viscosity effects alone.

The second problem was that of the effect of wall and exterior film resistances on the rate of heating. This problem will be shown to arise in the use of conversion factors (either convection or conduction) between glass and tin or between glass containers, in heating in steam-air mixtures and cooling canals, and may even arise in the processing of metal containers during come-up time in commercial retorts.

The third problem was the effect of container geometry. The finding of Nicholas and Pflug (196) that the surface-to-volume ratio, rather than the reciprocal relationship proposed by Schultz and Olson (1938), gave better predictions of the f parameter was disconcerting. The appropriateness of incorporating length to diameter ratios in a 'modified' Rayleigh number N_{Ra} or of using other shape factors as characteristic lengths also had not been confirmed.

Preliminary and substantiating experiments were performed to help answer some of the other preliminary questions. These tests ultimately included: (1) a study of temperature profiles during the inception of convection; (2) a comparison of the resultant temperature profiles with dye velocities using the techniques of Fagerson (1950), with Fagerson's films of dye motion and an experimental appraisal of the techniques of Tani (1940); (3) a check on the breadth of the central core, using horizontal temperature profiles; (4) a calorimetric study of the mixed-mean temperature; and (5) a careful but cursory check of temperature histories at long process times.

The Experimental Design

Temperature and Viscosity Effects

The separation of the temperature difference from the property effects can be accomplished by either (1) the use of a relatively constant viscosity fluid or (2) by choice of temperature levels so that the temperature difference effects were compared at the same viscosity level. A relatively constant viscosity fluid was sought among fluids for which transport processes were generally available. Although Bromotoluene was indicated to have a low temperature variation due to temperature (Perry, 1950), none of the commercial samples of the several isomers seemed to be substantially less temperature dependent than water. Glycerol, ethylene and propylene glycols as well as glycol ethers also were not substantially superior. The most promising fluid found, a commercial silicone fluid of the same viscosity range but approximately one-half the temperature coefficient of water, was not used, because the substantial size of the experimental program made preferable the use of only two fluids--water, and 50 percent sucrose by weight, fluids for which heat penetration data was available.

The choice of temperature level (fluid property values) at which to compare the effects of temperature difference, however, presumes that the experimentor knows what temperature difference and property level are controlling. There are at least three different levels at which to examine the effects of N_{Ra}, however, for it has been suggested (1) that the initial Rayleigh number N_{Ra} affects the onset of convection, (2) that some unspecified quasi-steady Rayleigh number during equalization affects the flow regime, and (3) that conduction, known to be controlled by final properties, rather than convection, is ultimately controlling. For the purposes of overall contrasts of the effects of initial temperature difference it was assumed that the properties at the mean temperature were controlling. This assumption permitted the choice of two successive equal temperature differences, namely heating from 60°F to 120°F, and 120°F to 180°F and the overall temperature difference 60°F to 180°F as treatments so as to provide orthogonal contrasts of the effect of initial temperature difference for equal mean temperature and of the effect of viscosity change corresponding to a change of temperature level of 60°F. Other meaningful contrasts, such as the effects of viscosity at the initial or final temperature can be made. These comparisons are not statistically independent of one another and are not independent of the temperature difference comparison (Walker and Lev, 1953).

The above contrasts of the effects of viscosity level provided a test of the practical importance of considering the change in viscosity when it is desired to process at a new

temperature level. The effect of viscosity level alone, and the viscosity-temperature difference interaction, can be tested at larger differences in treatments if two or more fluids of greatly different viscosities are used.

Containers and External Film Resistances

The reduced heat penetration which results when a container of large resistance such as glass is used instead of a metallic container has long been recognized. (A similar reduction due to exterior film resistance has also been detected occasionally.) The problem here is not so much an experimental survey of glass/tin factors but thepartitioning of overall heat transfer resistance into its various components. The problem then is an evaluation of the approach of Merrill(1948). If the heating lag of the container is small and the external and container resistances are known and a Newtonian-like heating model is assumed this approach of partitioning the overall heat transfer resistance permits an examination of the residual resistance due to transient natural convection of the product.

Since both forced and natural convection were anticipated to be important, the contrast between a low and a high container-exterior film resistance was to be obtained using glass and tinned-steel containers rather than a change in agitation rate. This contrast was limited to one container size since Townsend et al. (1949) had made detailed can/glass factor comparisons under commercial conditions.

The external film and the container resistances required for estimation of the internal film resistance were determined by an experimental investigation of quasi-steady-state exterior film coefficients on the surface of can shaped metal cylinders and from measured container thicknesses and their nominal physical properties.

An estimate of the forced and natural convection contribution to the external film coefficient was obtained by comparing film coefficients with and without agitation of the heating medium. This estimate was obtained only in one of the several baths used, the 180°F large water bath.

Container Geometry

There are two approaches used to describe the effects of changing the shape of a cylindrical vessel--the surface-to-volume ratio or its inverse, and the length to diameter or so-called "aspect" ratio.

Although control of either shape parameter was feasible the aspect ratio factors and in particular a nearly constant height was chosen for convenience. The nearly constant height of some relatively widely encountered sizes, the eight oz., sixteen oz. and the twenty-eight oz. containers permitted a large change in diameter (volume) while permitting nearly direct contrasts of heating rates at corresponding positions using only one special multi-point rod. This rod was also used for the standard quart (32 oz. size), an intermediate diameter but nearly double the height and for the 22 oz. cylinder jars. Heat penetration rate data for the upper portion of the container were sacrificed, of course. These additional containers permitted a contrast of the effects of container diameter versus container height. The diameter series was expected to reflect principally the rate versus capacitance concept inherent in the length to diameter ratio while the individual height comparison was anticipated to reflect principally the known effect of length of heat transfer surface on the rising fluid film.

Glass rather than metal containers were used for the geometry studies. The use of glass had several advantages. These included: (1) the availability of comparable data (for small come-up time) for acid fruit and vegetable products very frequently processed in glass; (2) the transparency of the container which permitted the use of the same container for visual studies of flow, observation of the position of the multi-point thermocouple rods, and visual checking by refraction (as in the case of convecting air) to assure that stratification of the syrup (and settling of the bentonite in a companion study to be reported elsewhere) had not occurred; and (3) the relative ease of affixing the Ecklund (1949) thermocouple packing glands to the closure and sealing of the container.

The Resulting Design

The nature of the natural convection phenomenon placed certain restrictions on the design of the experiment. Earlier in the analysis this investigation was limited to consideration of transient heating of homogeneous liquids when the come-up time was negligible. Here the anticipated natural variability

of the heat penetration data and the sensitivity of convection to variation in techniques and equipment performance suggested that the experiments be conducted during a limited period, that the experimental conditions be prescribed (or described) precisely, and that a moderate number of replications, such as six replicates, be used as control measures.

Local temperature histories supplemented by secondary flow experiments were to be the principal data obtained here. These temperature data were to be taken as gross indications of flow and temperature patterns at the central core. Since Nicholas and Pflug (1960) have indicated that the so-called cold point may vary throughout the process, small convenient increments (here 1/2 inch) were desirable in order to detect such movement. For the more common pint-sized containers this 1/2 inch increment requires nine different measurements.

The resulting experimental design is shown in Table 3.1. It represents a compromise between a desire to test the temperature response at each of two or more levels for each of the factors mentioned above and the overwhelming number (fifty-four) of heat penetration curves generated in each treatment: a completely factorial experiment of two fluids x two temperature differences x two temperature levels x two wall resistances x two heights x three diameters would have required ninety-six different treatment combinations. In spite of these compromises so much data was generated in this investigation that the corresponding cooling study--a complete replication of the above, the bentonite study and the mixing

TABLE 3.1 Experimental treatments. Six replications were made of each treatment combination

Lag	Height	Size	Viscosity	Temperature		
				60°ΔT_i		120°ΔT_i
				60 to 120°F	120 to 180°F	60 to 180°F
Metal	Av. Ht.	#503	W	X	X	X
			S	X	X	X
Glass	Average Height	8 oz	W	X	X	X
			S	X	X	X
		16 oz	W	X	X	X
			S	X	X	X
		28 oz	W	X	X	X
			S	X	X	X
	Mod.Tall	22 oz	W	X	X	X
			S			
	Tall	32 oz	W			X
			S			X

study must be reported at a later time.

The experimental conditions of the secondary experiments are also shown in Table 3.1. Most of these are conducted using the conditions of the same temperature treatment to water in 16 oz. glass.

The selection of the specific levels of these variables will be discussed in later portions of this section.

Materials

The Fluids

Distilled water and 50 percent sucrose solution were chosen as the fluids in these tests for their known thermal properties and their dissimilar viscosities and because they represented the class of Newtonian fluids found in commercial practice in juices and brined or syruped products.

The Rayleigh number, N_{Ra}, which will enter into later discussion, is shown in Figure 3.1 as a function of temperature. Certain other transport properties of these solutions are tabulated in the Appendix. The properties for sucrose were compiled from many sources. The specific heat and thermal coefficient and density, perhaps the most questionable of these data, are also shown in the Appendix.

The Containers

Specifications of the glass containers used in this investigation are given in Table 3.2 and 3.3. Standard 303 x 407, i.e., 3-3/16 in. seam O.D. x 4-7/16 in. overall height cans were also used.

TABLE 3.2 Container specifications.

Style name	Overflow capacity fl. oz	Weight oz, Av	Outside diameter in	Height to finish in	Height to shoulder in	Estimated liquid height, in	Wall thickness in	Bottom thickness in
8 oz cylinder	8-5/32	6-5/8	2-25/32	4-21/32	4.14	3.91	.133	.127
16 oz vegetable	16-1/4	7	3-9/64	4-25/32	4.23	4.06	.095	.127
22 oz cylinder	22-1/2	9-1/2	3-9/32	6-3/64	5.52	5.25	.102	.170
28 oz vegetable	28-3/8	11-1/2	4-1/16	5-1/64	4.46	4.14	.120	.147
32 oz rd quart	32-5/8	12-1/2	3-45/64	7-9/64	5.38 to spring line	5.73	.100	.208
303 can	16.2	5/8	3-3/16	4-6/16	———	3.80	.010	.010

Apparatus

The Waterbaths

The laboratory water baths used in the principal experiments are illustrated schematically in Figure 3.2. The 24 x 48 x 15 in. bath has been described by Pflug and Nicholas (1960). Prior to this study the bath was modified by the addition of an overflow at 11 inches above the bottom and a second line pressure reducing valve, which gave about 20 percent improvement in control. The earlier bath had had a point-to-point variation of less than ±1.25°F and a standard deviation at any one point of about ±.25°f. The agitation in this bath is provided by a Lightning Model L electric mixer with a 2 in. disk propeller placed about 11 in. from a rear corner of the bath at an 8 in. depth. The containers were placed in the center of the bath and were centered on the mixers.

An electrically heated 16 x 21 x 17 in. deep insulated 120°F bath and a water-cooled 21 in. I.D. x 18 in. 60°F tank were also used. The temperature variation within these baths was about 60 percent of that of the other bath. The Model L mixers, with the same propellers, again at an 8 in. depth, were centered and placed 4 in. to the left of center respectively on the rear of the baths.

In all of the baths the containers were placed about 5 in. apart in 3/8 x 1 in., 18 gauge, diamond-mesh galvanized-steel baskets (9 x 12 x 2-3/4 in.). These rested on other baskets at an eight inch depth.

The large 180°F water bath described above was used as the heating bath for the calorimetric studies, the incidence and the horizontal profile studies.

The flow studies were made in a transparent water bath which was similar to the one employed by Fagerson (1950a) for his dye studies and consisted of a 12 in. OD x 12 in. high Pyrex jar filled to a depth of 10 in. with distilled water. The bath was heated with a 1,000 watt intermittent heater (Fisher Cat. No. 11-463-10), and a 500 watt (Fisher Cat. No. 15-445-55) continuous heater. The temperature was regulated using a Fisher catalog No. 15-445-45 control unit and a Fisher "microset" Cat. No. 15-445-30, thermoregulator which had an accuracy of $\pm 0.02^{\circ}F$. The bath was agitated by a Fisher Fultork Lab Motor with slotted disk blades. The containers rested on a 9 in. OD steel disk (with 1/4 in. holes 3/8 in. on center, in a hexagonal pattern) which was placed 2 in. above the bottom of the bath. Because this bath had not more than a variation of $\pm 0.1^{\circ}F$, the least detectable by the present recording equipment even during the heating of a 16 oz. glass container, it was used for the runs at long process times.

The apparatus for testing the response of the cylinders--the devices used to determine the magnitude of surface conductances in the baths--in mixed and forced convection is shown in Figure 3.3. The apparatus is similar to that

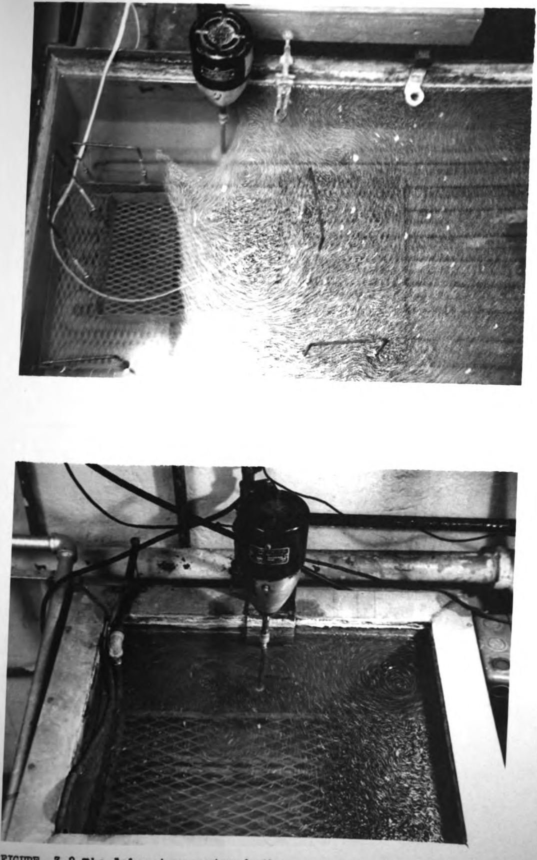
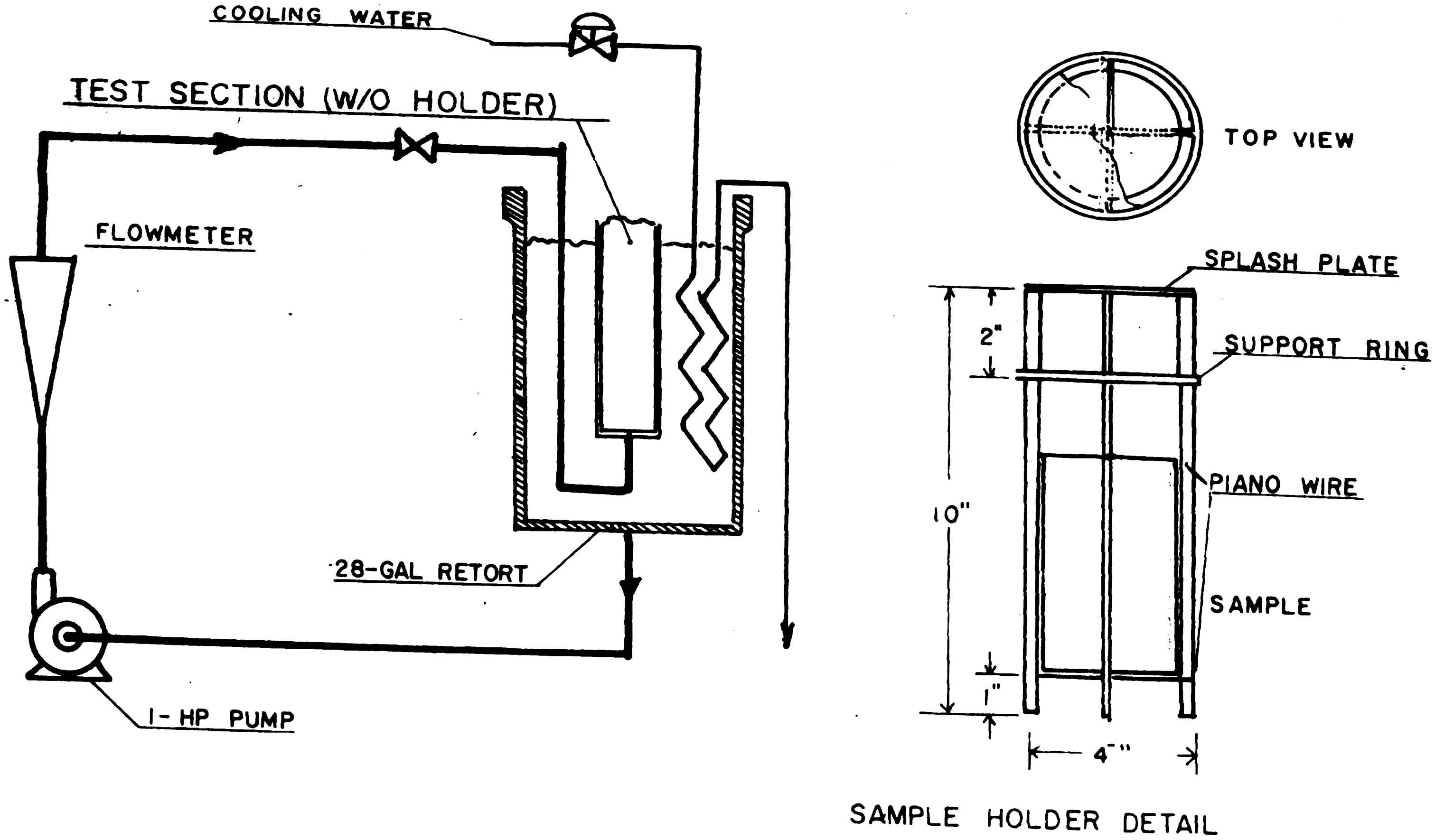


FIGURE 3.2 The laboratory water baths



SCHEMATIC DIAGRAM OF TEST APPARATUS

FIGURE 3.3 Schematic diagram of test apparatus for evaluation of film coefficient transducers developed by Pflug for the fruit cooling research of Motawi (1962). Flow rates from 10 to 110 GPM of water were used. The process water temperature was maintained at 70 $\pm 3/8^{\circ}$F.

The Thermocouples

Two types of multipoint thermocouple rods were used in this experiment. These are shown in Figures 3.4 and 3.5. These possessed the advantage of permitting comparison of heat penetration curves within the same containers without incorporation of the variances due to variations among jars and variations between immersions.

The first of these multipoint rods was a modification of the rod employed by Pflug (1960) for use in studies of the cold point position along the vertical axis of the container in cucumber products and model systems. This modified multipoint rod, and the companion rod which had horizontally placed thermocouple junctions instead of junctions along the length of the rod, will be described in some detail.

Both the modified vertical and horizontal multipoint rods were constructed with the following materials. The thermocouple wire used was No. 30 gauge enameled, cotton wrapped, fiberglass overwrapped copper-constantan thermocouple-wire. Smaller diameter (40 gauge was immediately available) would have reduced conduction errors and capacitance lags, but such fragile wire did not seem practical

FIGURE 3.4 Vertical profile multipoint rod

FIGURE 3.5 Horizontal profile multipoint rod

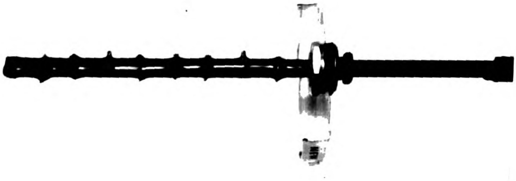


FIGURE 3.4

FIGURE 3.5

here. The multipoint rods were constructed using one-quarter inch O.D., one-eighth inch I.D. fibre tubing and one-eighth inch O.D. grooved fiber rods, as manufactured by National Vulcanized Fibre Company, Kennet Square, Pennsylvania. The resin used to seal the thermocouple assemblies was a thermosetting epoxy resin, Epocast 10-F, as manufactured by Furane Plastics Incorporated, 4516 Brazil Street, Los Angeles, California, and their No. 951 room temperature hardener. Some properties of these or similar materials are shown in Table 3.2.

The construction details for the vertical multipoint rod are as follows:

> An appropriate length (10 inches for standard glass containers up to 64 oz. capacity) of fiber rod was chucked in a lathe and then grooved to a depth equal to one-half the thermocouple wire diameter. Then the rod was drilled out to 5/32 in. inside diameter in order to provide room for the nine thermocouple wires. Then 7/16 in. holes for the thermocouples were drilled radially at an angle of 150° from the upper end of the rod. The thermocouples were introduced into the rod through these holes and were threaded through the rod progressively from the longest to the shortest lengths. The end wires had been sanded to remove the overwrap, and the portions which were to form the junctions had been bored. The

wires were snaked through the holes until the bare portion protruded through the holes. The bared thermocouple wires were led in opposite directions around the rod and then twisted to form a junction 180° from the hole, in such a manner that the wires fitted into their grooves. The junctions were then soldered with a resin-core tin-lead solder and clipped so that about 1-1/2 turns of wire were exposed. These junctions were varnished to reduce voltaic emf's. The rod was then given a light coat of the above resin, after separate sealing of the alternately placed holes. In order to reduce the internal shorting, due to internal voltaic emf, which had caused the first rods to fail after about 200 hours of operation, resin was drawn up into the rod. The transparent plastic vessel shown in Figure 3.6 was constructed for this purpose. The lead wires were sealed in the chamber using a rubber stopper, then the vessel was evacuated. The resin was drawn up until it filled the entire rod. Some failures were encountered at about 400 hours of operation, but they were not due to internal shorting. The failures were identified as due to flexing of the lead wires as they emerged from the rod. Subsequent rods were fabricated with a 2 in. length of rubber tubing, glued to the upper end of the rod. The rubber tube and the portion of the wires exposed to the water baths were then wrapped with plastic electrical insulating tape. No failures have been observed after about 800 hours of operation with these later rods.

The vertical rods used in these studies had more junctions, the first 1/8 in. above the tip and the remainder on alternate sides at 1/2 in. intervals. The large number of junctions were incorporated in one rod in hopes of detecting possible movement of the 'slow point' throughout the process.

The horizontal profile rods also had a large number of junctions in one assembly. Five couples were spaced horizontally every 1/2 in. apart at 1/8 in. above the tip and another row were placed 1/2 in. above the first. (These rods were constructed for use in confirming the apparent temperature uniformity within the descending core, the symmetry thereof, and for detecting, in a gross manner, the direction of heat transport, but the support technique described is adaptable to other arrangements.) The width of such a rod was limited by the closure of the container. In this instance, the rods were designed for the central container in the experiment, the 16 oz. vegetable jar. As Figure 3.5 shows, the thermocouple rod was constructed like a rake, with junctions located at each of the tines. This device, like the wrap-around exposed leads in the vertical rod, was used to increase the exposure of the leads to the ambient temperature in the vicinity of the junctions. Such

FIGURE 3.6 Vacuum vessel used to fill the core of multipoint rods with resin.

FIGURE 3.7 Jig used in the fabrication of the horizontal profile multipoint rods.

FIGURE 3.6

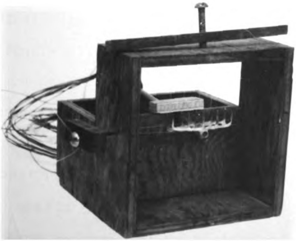


FIGURE 3.7

positioning also reduced the interference to flow in the vicinity of the measurements.

The special techniques used in fabricating the horizontal profile probe are described below.

The rod was drilled out to 3/16 in. ID instead of 5/32 in. ID in order to accommodate the increased number of leads. The size of the thermocouple holes was also increased by a thirty-second of an inch. After the wires had been snaked through (The author had better luck with threading the smooth lead wire up through the rod than threading the bared wires down through the rod.) the rod was placed in a jig.

This jig, shown in Figure 3.7, centered the rod and provided support for 0.012 in. OD nylon (leader) guy threads which were placed to coincide with the arms and the tines of the finished assembly. The lead wires were then tied to these guy threads and wrapped tightly with cotton thread. The wires and wrapping were next repeatedly varnished to render the cable impervious to moisture. The junctions were then made, trimmed and varnished as before. Considerable care in construction and later storage was required in order to obtain junctions which were not displaced from their common measured plane by 1/32 of an inch.

The holes were then sealed as before with epoxy resin. The cables were also coated with resin. This resin provided support for the arms and the tines of the probe. After several coats of the resin the guy wires were cut and the probe was removed from the jig. The evacuating vessel was again used to draw the resin into the rod. Care also was taken to seal around the exposed ends of the nylon guy threads during this evacuation. Finally, the wires were protected as above against failure due to flexing.

The multipoint thermocouple probes described above were compared with the single point rods used in this laboratory and with non-projecting type Ecklund (1949) thermocouples in a test of thermocouple responses. These single point rods were constructed using the above materials and techniques. Twenty-four gauge wire was used, however, and a one-half inch projection of the grooved fiber rod served as support for the thermocouple junction.

All of the above assemblies were secured and sealed in place using Ecklund (1949) rod-type thermocouple glands. An adapter was required in order to use the same thermocouple assemblies in tin cans as in the class containers. This adapter (Shown in Figure 3.8 and which resembled a 3/4 x 3/8 in. NPT brass bushing.) was soldered to the container lid.

These thermocouples were found to have individual f and f_2 of 6 and 36 sec. as contrasted to 6 sec. for a bare 24 gauge wire; 6 and 36 sec. for an extended type Ecklund rod; and 6 and 75 sec. for a chisel nose rod when heated from 60°

FIGURE 3.8 Adapters for vertical multipoint rods to facilitate their use in cans

FIGURE 3.9 The calorimeter and associated equipment

FIGURE 3.8

FIGURE 3.9

to 180° by natural convection.

The Potentiometers

Four different potentiometers were used in the several phases of this investigation. The first of these was a Model 8686 Portable Millivolt Potentimeter, as manufactured by Leeds and Northrup Company, with a one millivolt increment stepping switch and an error limit specified as ± (0.05% of reading $+3\mu v$). This instrument was used as a calibration and as a range input to the -0.10 to +1.00 mv (.01 LD) Least Division, abreviated hereafter as LD, potentiometer and as a calibrating input to the two other recording potentiometers. A + 50° to +350° span, (1°F LD) Model 153X72P12, 5 second print, 12 point strip chart recording potentiometer and a -100° to +250°F span (2°F LD) Model SY153X11, 6 second print, 12 point strip chart recording potentiometer were used in the principle experiments. These three recording potentiometers, manufactured by Minneapolis Honeywell, had design calibrated accuracies of about ±0.2°F, ± 0.8°F, and ±0.9°F, and sensitivities of ±0.1°F, ± 0.2°F and ±0.25°F respectively. The actual performance limits were reduced by frequent calibration and maintenance to approximately one-half of the above.

The recording millivolt potentiometer with its superior performance was used for the calorimeter studies. It was later modified to give a 4 second full scale pen speed and a chart speed of 2 in./min (LD 3 sec.), then was used with a Leeds and Northrup Model 1145714 Thermocouple Swith during the studies of inception, profiles at long process times and the surface conductances. The five-second print cycle recording potentiometer was later modified to a one-second print cycle and a 3 1/2 second full scale pen speed by an increase in balancing and chart motor speeds and then was used in the thermocouple response studies. This latter potentiometer was also used, but without modification, in the horizontal profile studies.

The Calorimeter

The simple calorimeter used to estimate the mixed mean fluid temperature is shown in Figure 3.9. It consisted of a 6 in. ID (6-7/8 in. OD) x 10-1/8 in. high thermos-brand dewar flask of 4300 ml capacity (with an integral aluminum stand) placed inside a 7 in. ID hole in a 10 x 10 x 13-1/4 in. styrofoam container. The top of this vessel was closed with a 1 in. thick cork lid. A slot was provided in the edge of this cork disk for the thermocouple assembly. The hand operated stirring device consisted of a 3-1/2 in. OD perforated disk fastened to an operating rod which projected through a 1/8 in. ID sleeve in the cork lid.

Test Cylinders

Several materials of known thermophysical properties were considered for use as test cylinders in the determination of overall film coefficients from heat penetration measurements. Their properties relative to water are shown in Table 3.3.

Cylinders 300 x 408, that is 3 and no sixteenth diameter by 4 and 8 sixteenths inches high, nearly the size of the 303 x 406 cans used in these investigations but of dimensions more convenient for calculation purposes, were turned from copper and aluminum billets. The materials were preferred for their machinability, their high thermal diffusivity (which assured that the surface conductance would be controlling even with high conductance), and their moderate and opposing change in thermal conductivity with temperature.

Thirty gauge copper-constantan thermocouple wire, with a guaranteed accuracy of ±1-1/2°F from -75° to 200°F but calibrated to within 0.2°F at 32° and 212°F were inserted in vertical holes at 0.00 ±0.02 and 1.35 ±0.02 in. from the axes of the cylinders. These holes had been drilled 3/32 in. diameter to a depth of 2 inches and then 1/16 in. in diameter to a 2.25 ±0.02 in. depth. The thermocouples had first been prepared by removal of the first two inches of the fiberglass overwrap, baring and soldering 1-1/2 turns of wire, and finally by varnishing of the junction, and the cotton wrap. The junctions were immersed in about 0.02. ml of an SAE No. 5 instrument oil which was placed at the bottom of the holes to assure wetted contact. Turnings from the billets were tamped around the leads. The top of the holes were sealed with a drop of electrical dope. No detectable reduction (i.e.,

TABLE 3.3.--Some thermophysical properties of selected materials relative to those of water at 68°F and their change in absolute value from 68° to 212°F. [Values from Eckert and Drake (1959), McAdams (1954), Stock Catalogue and Metals Handbook, Copper and Brass Sales, Inc.]

Material	Thermal Diffusivity	Thermal Capacitance	% Increase in Thermal Conductivity	% Increase in Thermal Capacitance
Copper, pure	572	0.82	-1.8	+1.5
Aluminum (1MG, .6Si, .25Cr, .25Cu)	372	0.58	+6.9*	+4.7
Brass (70Cu, 30Zr)	239	0.79	+15.6	+3.8
Lead	167	0.35	-3.5	+3.8
Carbon Steels (1C)	82	0.88	-4.0	+9.3
Constantan (60Co, 40Ni)	43		-2.3	+2.8
Ni-Cr Steel (20Ni, 15Cr)	27	0.87	+8.8	--
Hard Rubber	00.4	0.65	--	--
Water	1	1.0	+13.9	-0.3

*Of a similar alloy.

>0.1 ohm) in resistance (about 10 ohms) between the heads and the cylinders was noted due to abrasion of the insulation by the turnings.

Experimental Procedures

The Principal Experiment

The study of the effect of temperature and the effects of viscosity on heat penetration at several axial points within containers of several geometries was conducted using bath temperatures of 60°, 120°, and 180°F. These temperatures permitted 60° and 120°F initial temperature differences, representative of the differences encountered commercially, and low come-up times of 2 to 5 seconds.

The containers were filled at 130°F ±5 °F to a capacity corresponding to their nominal capacity (fluid ounces) of water in ounces avoirdupois. Due to the difference between the ounce units this stratagem produced a fill of about 96 percent before insertion of the thermocouple rods. These rods increased the apparent fill height another percentage unit. The resulting three percent headspace is below the 6 percent headspace at a 130°F sealing temperature, recommended by the National Canners Association for processes at 240° or 250°F, but the process here does not require this amount of headspace to prevent venting. Although the thermocouple rods need to be placed firmly against the bottom of the container, excessive pressure does permit undesirable venting.

The possibility of venting with resulting changes in volume of product and concentration made safeguards necessary. A gain (or loss) of one gram of fluid or a change in sucrose concentration of over 0.2 percent, the least scale division on the ABBE-3L Refractometer used, was selected as sufficient grounds for discard of the run affected. Normally the checks were made after the completion of three replicates of the temperature treatments. Any container which seemed to vent was checked immediately, and then the tests affected were discarded. Reruns were made immediately.

Two containers of each size were chosen at random from within 5 matched sets of 36 jars of one manufacturer. This scheme--the use of fixed jar sizes with jars matched to within ± 1 percent by weight--was used to reduce jar to jar induced variations since it was unreasonable to pick a sample which represented all of the jars of that design produced by all manufacturers. In the case of the 16 ounce vegetable jars, the sets had previously been used by Pflug and Blaisdell, (1962) in the steam-air velocity studies and the calorimetry studies.

The containers were equilibrated for a period approximately equal to 4 to 6 f transferred within two to five seconds, then processed in the first of the several baths. When the slowest point had reached within 3° to 5°F of the above processing temperature (when measurement errors were deemed excessive) the jars were shaken to reduce stratification and the magnitude of the opposing convection currents in the subsequent cooling process. The containers then were allowed to equilibrate until the process had continued for about six f values. The mean temperature of the containers throughout the fourth to sixth f period were taken as retort temperature and applied as a correction to the mean temperature of the bath for the preceeding period. These corrections to the mean compensated, in part, for the point-to-point temperature variations within the bath. The use of mean temperatures likewise reduced the error due to cycling of the bath temperatures.

The Inception of Convection

The study of the axial temperatures during the inception of convection was made using water in 16 oz. vegetable jars processed at 180°F with a 120°F rise. Temperatures were measured using the nine point vertical thermocouple rods and the single channel recording millivolt potentiometer and switch.

Preliminary studies of the time required for the temperature to rise one degree were made as a basis for an improved experimental design. Although repeated attempts were made with several switching schemes, no program was found which would permit recording of the initial temperature changes of even the five alternate points in the same test. In addition, the use of an incremental temperature change as an index that convection had begun was found to be unsatisfactory. This deficiency was due to the random slugging of the fluid at some points within the container--slugging which reduced as convection became fully established.

The studies, which will be reported subsequently, represent data for the selected points but separate immersions within the same container. These studies were limited to two tests at each position because an experiment which would permit more thorough examination of the interesting random slugging would have required an experiment of the size of the principal investigation itself.

The equilibrium, transfer, and correction procedures for these studies were the same as those in the principal experiment, described above.

The Flow Studies

The flow studies provided a check both on the conclusions of the above experiment and also on the reliability of the techniques of Fagerson (1950a) and Tani (1940). These investigators had used dye and an aluminum powder suspension, respectively. Other investigators had satisfactorily used techniques such as oil drops, discrete hollow plastic spheres with individual differences in shape or color which permitted later identification, or tellurium dye. Still others have used velometers and pilot tubes. The second group of techniques were adaptable to transient flows but were deemed unsuitable here either because it was considered difficult to match the range of densities encountered in transient convection or because they introduced a substantial-sized sensing element. The third group of techniques also introduced sizable elements but possessed initial and friction lags which would have required considerable calibration and compensating equipment. The dye and aluminum powder techniques, however, appeared to be simple. These latter techniques have their own disadvantages, however.

The experimental conditions investigated with the dyes and the aluminum powder were the same as those of the inception study above, except that the glass waterbath was used. Three contrasting dyes were selected for high color density at low concentrations after preliminary trials with other standard indicators. These were 0.5 percent Methylene Blue, 0.5 percent Safranin Orange and 0.1 percent by weight Fluorescein (Uranine) in aqueous solution. These were introduced nearly simultaneously through 20 gauge needles. The needles were located behind the central plane but discharged droplets at the central plane at the base of the heel, the top center and the bottom center of the container. The times required for the major dye element from the heel position to reach predetermined points in the container were estimated. The vertical positions were estimated with the aid of a translucent grid placed behind the container. The horizontal positions, which were subject to even more parallax and distortion, were likewise estimated using the grid. The actual lateral positions were estimated using the results of preliminary experiments in which apparent lateral positions were compared to positions when observed through the open top of the container (the upper portions of the finish were above the bath surface.)

A commercial aluminum powder (flakes) of approximately micron size was used in an appraisal of the techniques of Tani (1940) and to confirm with flow estimates the heat transfer results observed during the film conductance studies. The powder was clarified by sedimentation until the remaining fraction could be maintained in suspension in water for several hours. The container holding this fraction was agitated about one f prior to the test. The gross patterns of flakelet motion were then observed.

Aluminum powder was used in the water baths also. It provided a glimpse of the gross circulation in the baths. In this instance it was sprinkled on the top, rather than dispersed.

The Horizontal Profiles

The horizontal profiles at the bottom, and later at the top, were investigated in water in 16 oz. vegetable jars processed with a rise from 60° to 180°F. These studies were conducted to check the symmetry, breadth, and direction of the flow in the central core as it formed and as it diverged to rejoin the ascending streams.

The experimental procedures used were the same as those described in the principal experiment except that horizontal profile rods and the five-second print potentiometer were used throughout six replications in the same container. The horizontal profile rod was offset so that the plane of the junctions cut the container at the same diameter. When the probe was used at the bottom it rested on a small cork cone, inserted in the meniscus formed by the solidified resin in the core of the probe. The bottom central couple then was positioned 1/8 in. ±1/32 in. above the bottom of the container. The rod was positioned so that it was 1/8 in. ±1/32 in. below the surface of the liquid when employed at the top of the container. Since the headspace depth at room temperature with the probe in place was about 13/32 in. the top row of couples were above the shoulder of the jar.

The Calorimetry Study

A simple calorimetric study of the mixed-mean temperature was incorporated because it provides an estimate of the relationship between some means of the vertical temperature on a multipoint rod and the mean temperature in that container.

Several sets of 16 oz. vegetable jars were filled with 450 gms. of water then equilibrated at 100°F for over periods of about 5 f in length. Jars were processed at 180°F for 0, 3, 6, 9, 12, 15 or 61 minutes, then withdrawn quickly, wiped dry, then the contents were transferred to the equilibrated calorimeter (see Figure 3.9). The calorimeter had previously been filled with a charge of 900 gms of room temperature water about 2 f increments before. The transfer procedure required from 10 to 15 seconds. The temperature was followed for about 5 minutes, then the calorimeter was check weighed, wiped dry, reweighed, and recharged then for the next determination.

A schematic record of the temperature charts is shown in Figure 3.10. The temperature drop due to evaporation, equilibration of the calorimeter and heat loss to the surroundings was assumed to be linear. The intercept values at zero time were assumed to be the temperatures of the ballast and the equilibrated ballast and charge. The calorimetry constant, as determined in a similar manner with 450 gram charges at several temperatures, was incorporated in the heat balance to determine the temperature of the charge.

Temperature Profiles at Long Process Times.

Studies of temperature profiles at long process times require more precise control of bath temperatures, calibration to detect the small differences between the output of couples from the same spool of wire and recording than is commercial practice.

Although it had been anticipated that the heat transfer

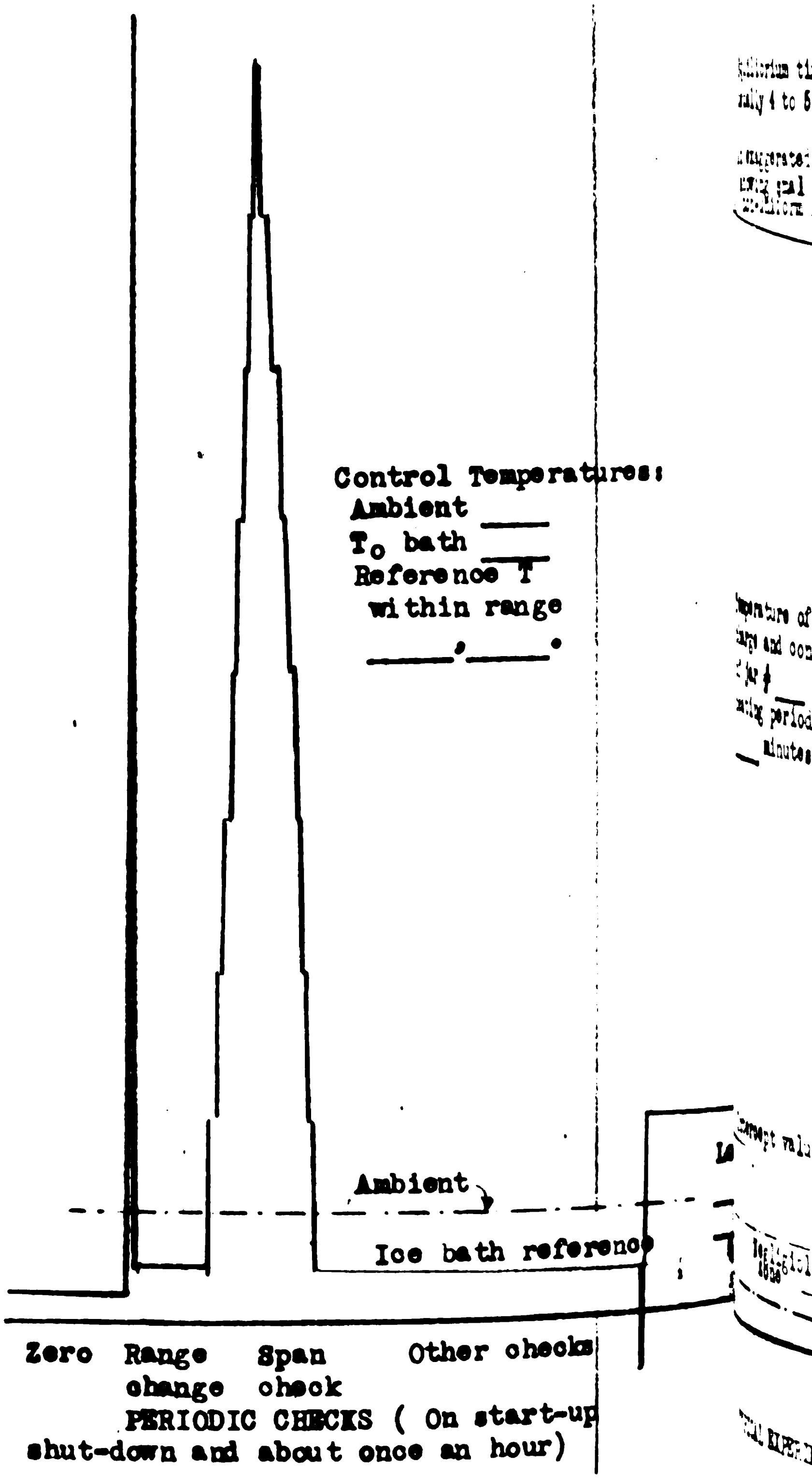
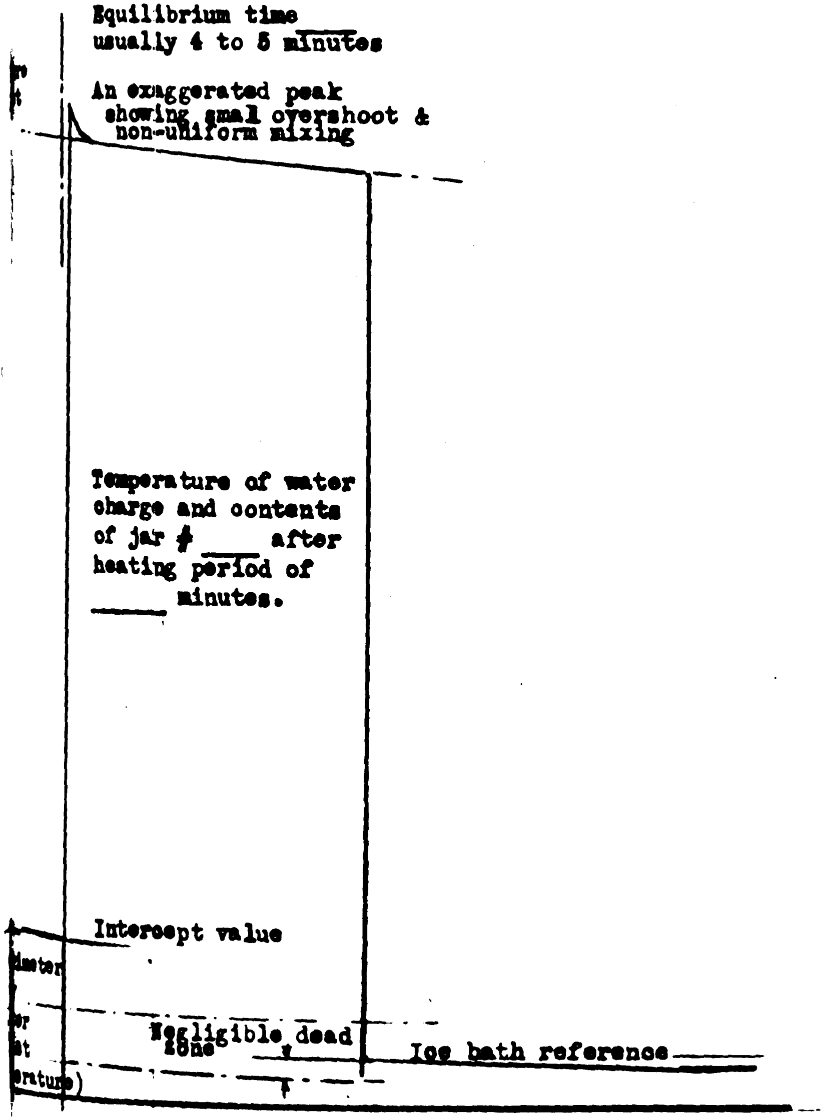


FIGURE 3.10 Schematic record of temperatures (emf's) dur



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rates would approach these conditions after long process times, information on this area had been sacrificed in the principal experiment in order to make a more intensive study of other factors using existing equipment. Nevertheless, two replications, which confirmed the suggested trends observed in the other studies, were made of water in 16 oz. glass processed through a 120° rise in the 180°F glass water bath. The recording millivolt potentiometer and the precision potentiometer range input which had a built-in 1.0 mv increment stepping switch were used in this study. The millivolts were converted to degrees Fahrenheit, then these data were treated as in the experiment above.

Film Conductances

The conductances in the waterbath were determined after the method of Merrill (1948) and Evans and Board (1954) in order to make better comparisons of heat penetration. The film conductance and the glass thicknesses as determined by destructive testing were to be used to estimate, by difference, the internally controlled heat transfer rates.

A vertical test section was used to provide comparisons between cylinders and thermocouple installations and as a basis in order to compare the magnitude of the measured film coefficients with some correlations for annuli, cylinders perpendicular to flows, and for plates perpendicular to flows, i.e., correlations for the various surfaces of the cylinders. Chilled cylinders at 32°F were inserted into a

4 in. vertical pipe. Water, maintained at 70 ±3/8°F, was pumped past the cylinder at flow rates from 10 to 110 GPM, mixed in the surrounding reservoir and then recirculated. Care was taken to assure that the cylinders and test frame were cooled in crushed ice for over four times their f, here about 15 min. and that they were transferred to the test section in not more than 3 seconds. The mean temperature of the cylinders throughout the third to fifth f'th portion of the heating period was compared to the mean process water temperature for that period. Differences were applied to correct the apparent processing temperature during the first two f periods.

The calibrated cylinders were then heated in the several baths used in the other experiments. Both left and right container positions in the baths were investigated since the jars attached to the 6 second print instrument were always processed in the left position and the jars attached to the 5 second print instrument were always processed in the right during the principal study. In order to evaluate the effect of position (of the two cylinders) the cylinders were (1) centered on a mixer (Position A), (2) placed in the corresponding position at the other end of the bath (Position B), and (3) centered on two mixers the second of which was 6 in. further away from the corner.

The natural convection contribution due to convection from the cylinders and also due to additional natural

convection from the heating (or cooling) elements were determined. For these tests the mixers (and heat sources) were turned off prior to the start of a run. The baths were partially insulated, covered and draped for evaluation of natural convection on the cylinders in order to reduce cooling convection patterns at the surfaces of the baths.

SECTION IV

RESULTS

The results will be presented in four parts: (1) The shape of heat penetration curves and the temperature and flow distribution; (2) Evaluation of the container and external film resistances; (3) A comparison of the f and j behavior as affected by position within the container, container, product, temperature difference and the temperature associated property variation; and finally (4) A comparison of slowpoint location, and the heating rate parameter f with those predicted from quasi-steady-state film coefficient correlations and Newtonian heating models.

The Shape of Heat Penetration Curves and Temperature and Flow Distribution

Typical heat penetration curves for a few of the many combinations are shown in Figures 4.1 to 4.4 and their three panels. (The mean values of parameters f and j for the treatment combinations are shown in Figures 4.17 and 4.18. The individual f and j values are tabulated in the Appendix.) Figures 4.1 to 4.4 present heat penetration curves for water or sucrose in 16-oz. No. 303 glass or tin containers when heated from 60° to 120° or 180°F, or from 120° to 180°F. The temperatures were measured at 1/8 in. above the bottom

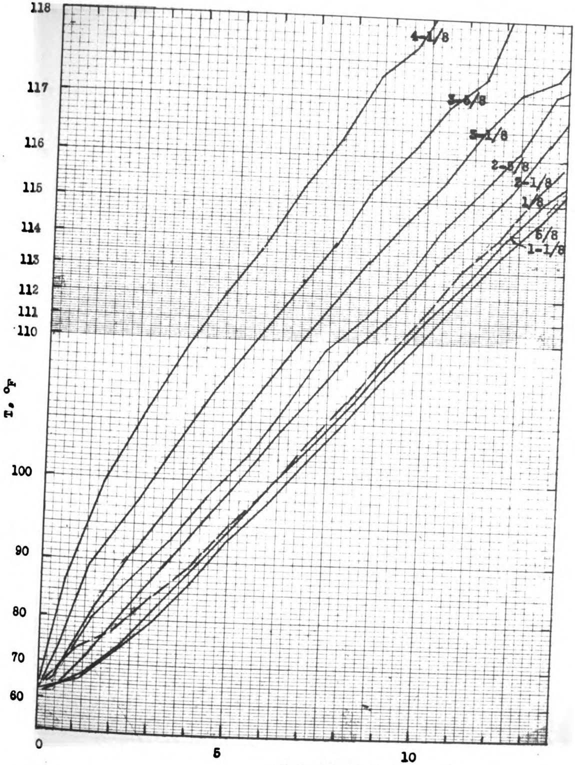

FIGURE 4.1a Typical heat penetration curves at various axial positions for water in 16 oz. vegetable jars. Run #13:60 to 120°F.

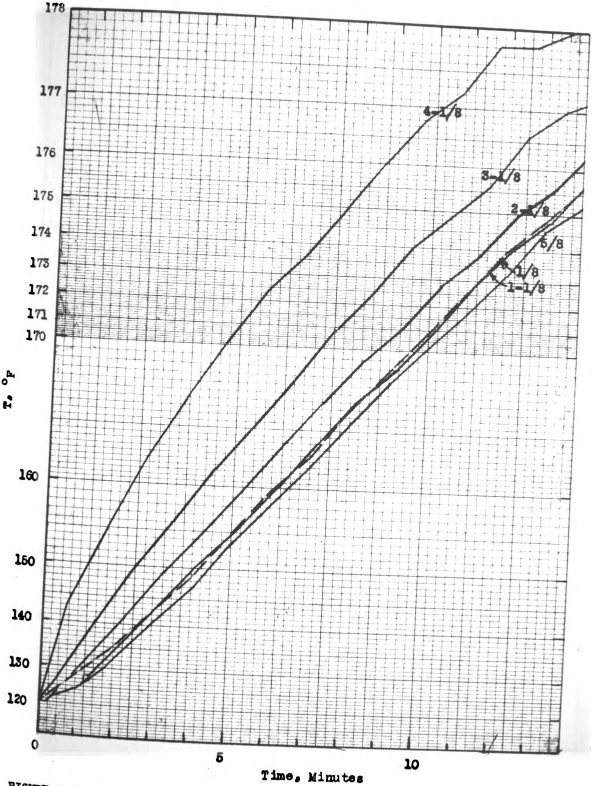

FIGURE 4.1b Typical heat penetration curves for water in 16 oz vegetable jars. Run #11: 120-180°F.

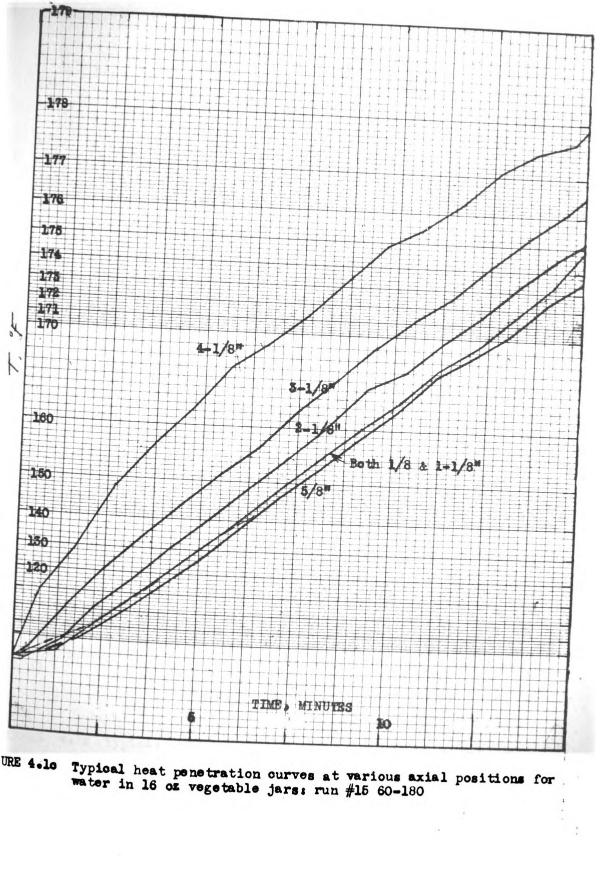

FIGURE 4.1c Typical heat penetration curves at various axial positions for water in 16 oz vegetable jars: run #15 60-180


TEMPERATURE, °F

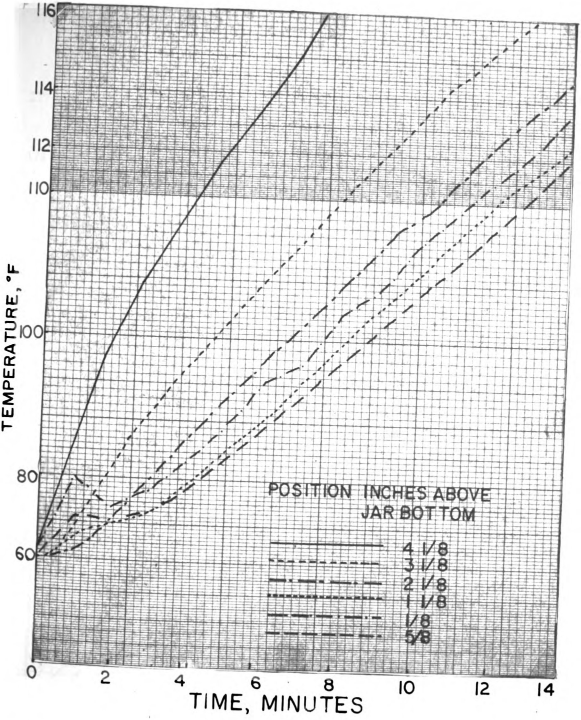

FIGURE 4.2a Typical heat penetration curves at various axial positions for 50 percent sucrose solution in 16 oz vegetable jars. Run#19.

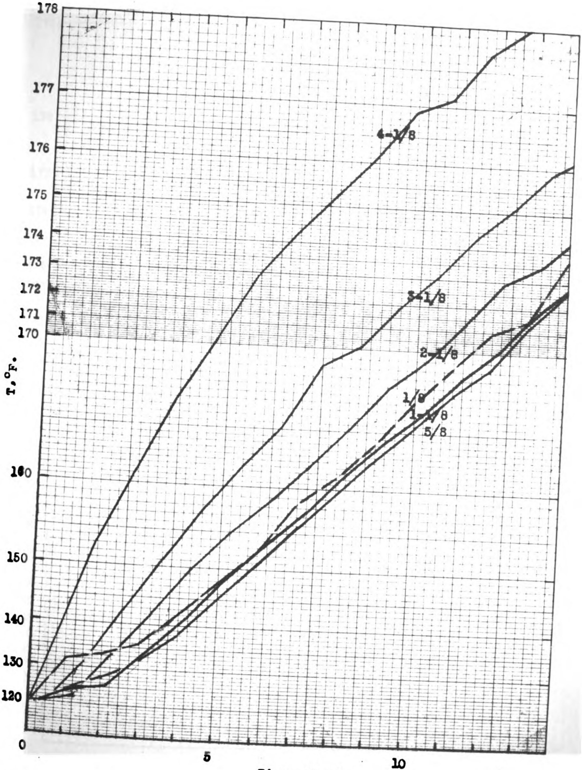

FIGURE 4.2b Typical heat penetration curves at various axial positions for 50 percent sucrose in 16 oz. vegetable jars. Run #97 120-180°F.

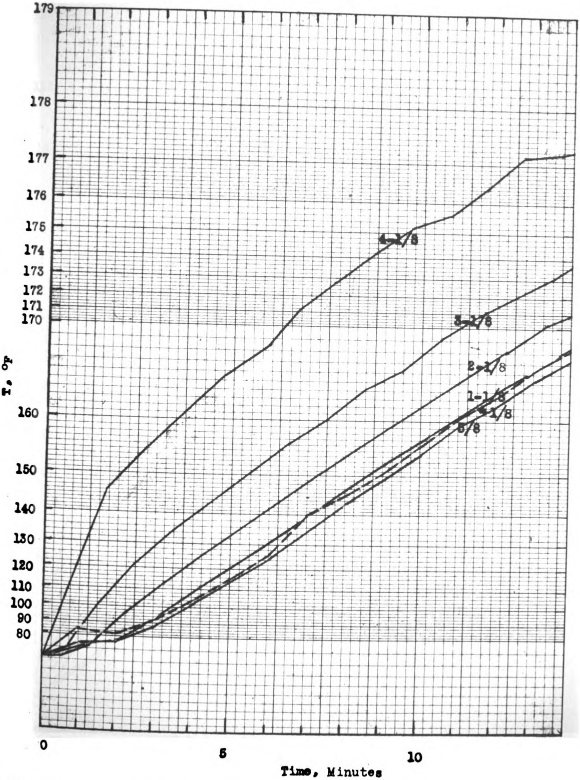

FIGURE 4.2c Typical heat penetration curves for 50 percent sucrose solution and various axial positions in 16 oz vegetable jars: run95 60-180

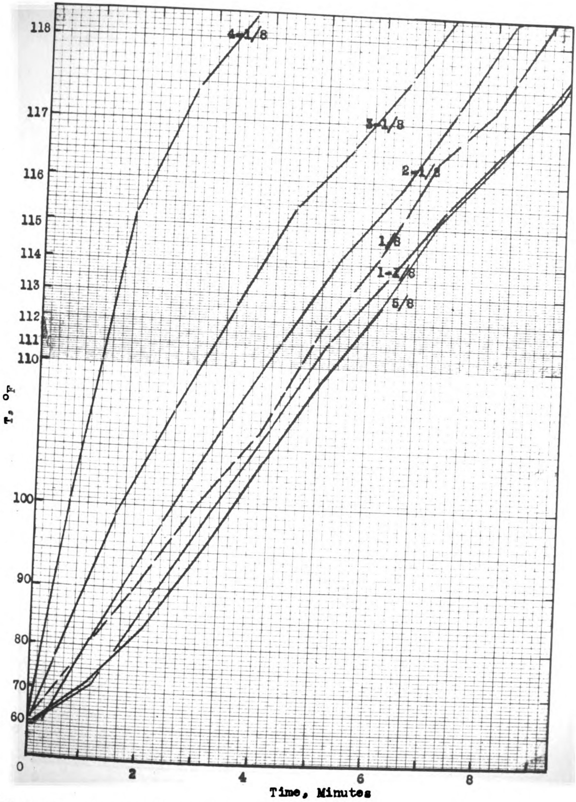

FIGURE 4.3a Typical heat penetration curves at various axial positions for water in 303 cans. Run #311,60-120°F.

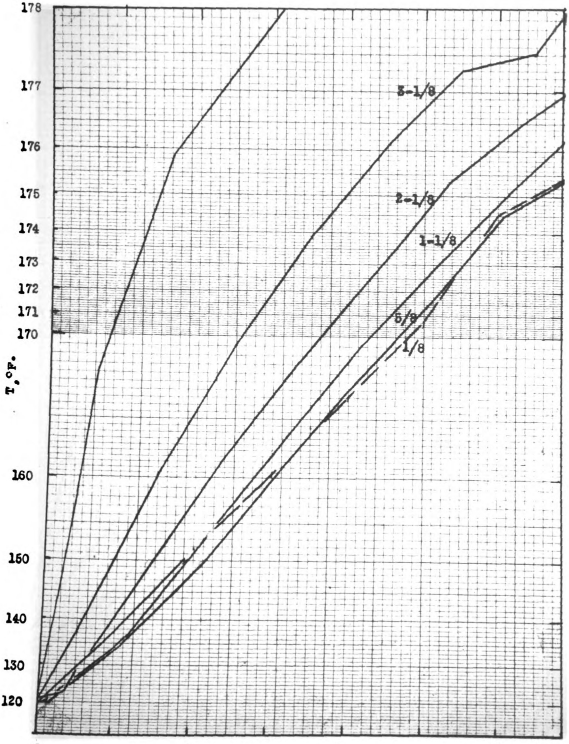

FIGURE 4.3b Typical heat penetration curves at various axial positions for water in 303 cans. Run # 315;120-180°F.

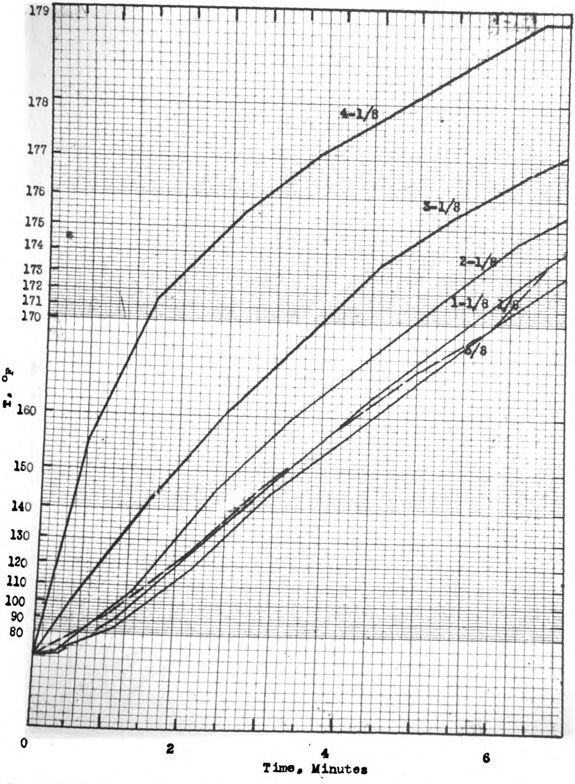

FIGURE 4.3c Typical heat penetration curves at various axial positions for water in 303 cans; run 307.

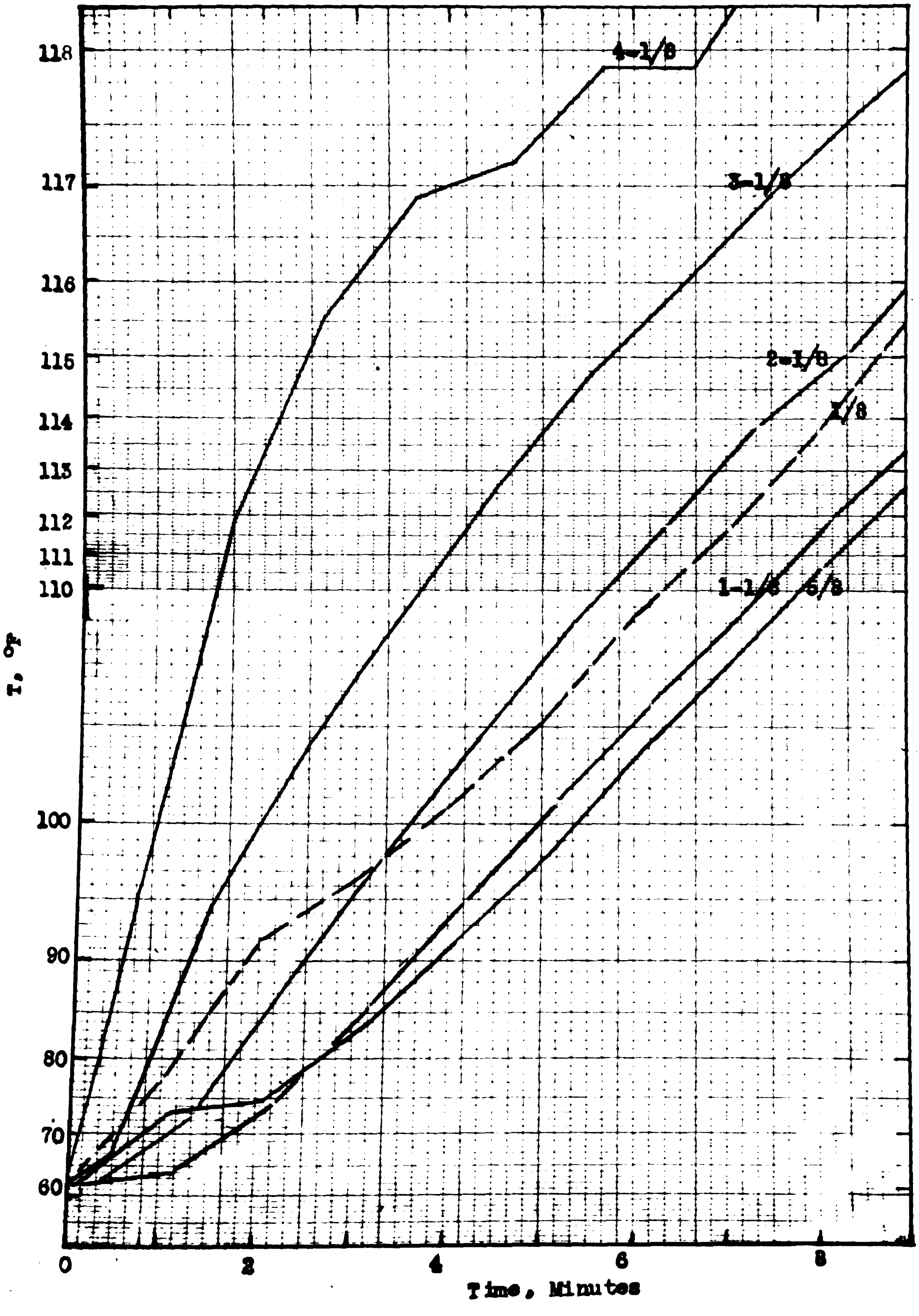

FIGURE 4.4a Typical heat penetration curves at various axial positions for 50 percent sucrose in 303 cans. Run #416: 60-120°F.

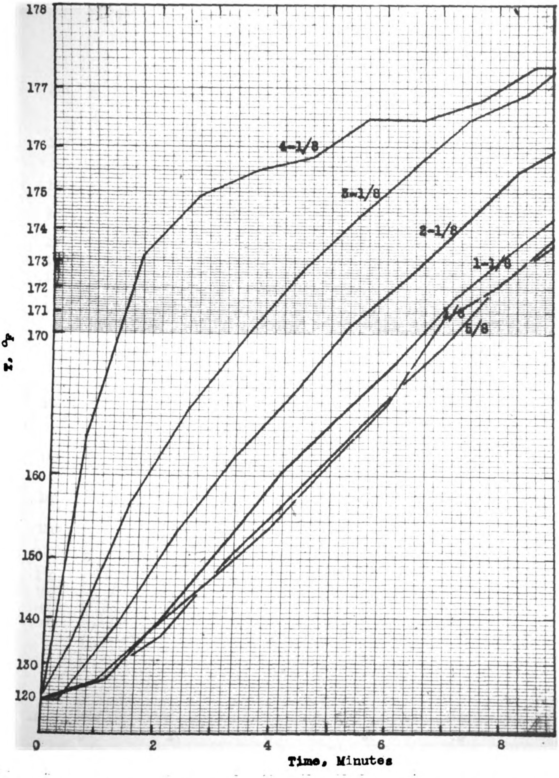

FIGURE 4.4b Typical heat penetration curves at various axial positions for 50 percent sucrose solution in 303 cans: run 429

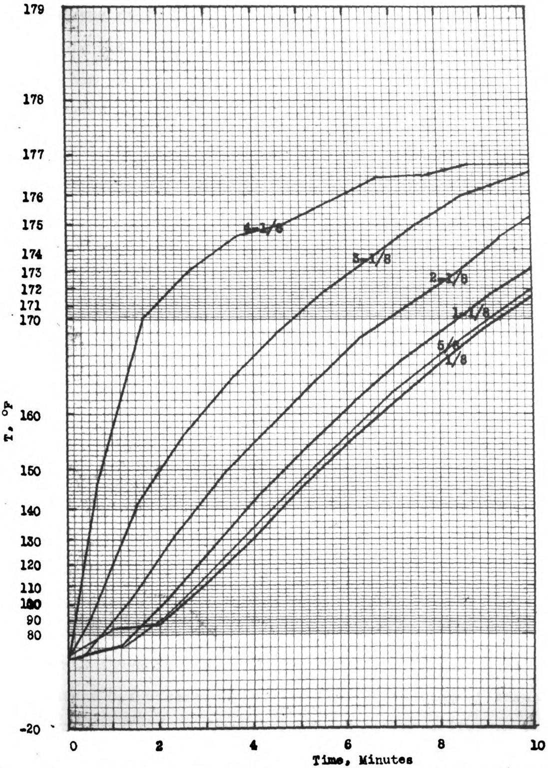

FIGURE 4.4c Typical heat penetration curves at various axial positions for 50 percent sucrose solution in 303 cans. Run #420:60-180°F.

surface of the fluid and on alternate sides of the multi-point thermocouple rod at successive 1/2 in. increments. Only data points on one side of the rod, except for point two, are shown. The temperature histories at the other side of the rod seem to correspond quite closely, considering variation in glass wall thickness. These curves illustrate aspects of the temperature histories and temperature distributions which are difficult to appraise when presented in tabular form. These aspects include the curvature of the heat penetration curves, that is, the validity of a linear approximation; the effect of the container wall on the shape of the heat penetration curve and the vertical temperature differences; and a rough check on similarity and the crossing of the heat penetration curves both initially and later in the transient process.

The Shape of the Heat Penetration Curves

The heat penetration curves for points in the vicinity of the cold point--for the combinations tested--appear to be fairly straight (once convection has been established) until relatively late in the process. Some curvature exists however, as is evident from the values of the heat penetration curve f parameter as shown in Table 4.1 (from point 2, Figure 4.11). The curvature (as assessed visually from overlaying of transparent heat penetration graphs) appeared to the writer to increase as the distance away from axial cold point increases. The curvature generally appeared to be increased by any of the following: a decreased initial temperature, decreased initial

temperature difference, a product with a moderately higher viscosity, a decreased wall thickness or an increase in exterior heat transfer coefficient. All of the above factors increase the relative importance of the interior film coefficient (and the influence of fluid properties, particularly viscosity, and of the temperature difference).

TABLE 4.1.--The heat penetration curve parameter f at 5/8 in. above the bottom of water in 16 oz. glass heated from 60° to 180°F and corresponding relative errors for several different thermocouple errors

Temperature (below 180°) at which f is measured, °F	f Minutes	Relative change in f (%)	Relative error in f for the following errors in (T_1-T) %			
			0.2°F	0.5°F	1°F	2°F
60 to 5 (range)	9.9	--	--	--	--	--
tangent at 60	9.3	-6.1	.33	.67	1.7	3.3
30	9.8	-1.0	.67	1.7	3.3	6.7
10	10.1	+2.0	2	5	10	20
5	10.8	+9.1	4	10	20	40
2	11.9	+20	10	25	50	100
1	17.6	+78	20	50	100	200

Similarity of Heat Penetration Curves Within a Container

It appears that the heat penetration curves at the various positions diverge sharply as convection and the axial temperature gradient are established, but appear to ultimately have very nearly the same slope (although significantly different j values) when the curves at long times--relative to f--could be observed without possible appreciable measurement errors. The time t/f required for the curves to become parallel seems to increase and the temperature difference between that of the fluid at the top and at the cold point seems to increase due to decreases of the initial temperature, initial temperature difference, or exterior film coefficient, or due to increases of the viscosity, or the container wall thickness. All of these trends correspond to slower heat transfer (that is, larger f values).

Regularity of Heat Penetration Curves

Heat penetration curves were generally smooth except when: (1) there was a change in controlling mechanism, as in the establishment of convection; (2) the measurements were taken near the origin of turbulent or laminar eddies; or (3) errors as in the limiting sensitivity as T approaches T_1 or incorrect instrument calibration transcription errors or plotting errors were made.

Abrupt changes in slope occurred when the controlling mechanism changed, as in the establishment of convection or as evident in the behavior of point 9 for sucrose in No. 303 cans. In this latter instance, the thermocouple was just beneath the surface of the fluid (less than 1/8 in.) and as the product heated the greater expansion of sucrose relative to water caused the lid to bulge as the headspace gases were compressed, hence the thermocouple junction moved from a sucrose convection region to an air conduction-convection region.

Erratic behavior near the source of eddies was sometimes evident in the behavior of points numbered 1 and 9 in particular, that is, about 1/8 in. above and about 1/8 in. below the horizontal fluid surfaces.

Errors, such as those mentioned above, were signalled by the appearance of pronounced curvature or "apparent" transition of mechanism. Reference to the multi-point records would invariably distinguish between these spurious results and real changes of mechanism.

Horizontal Temperature Profiles

Horizontal temperature profiles for water in 16 oz. vegetable jars, heated from 60° to 180°F are shown in Figures 4.5 and 4.6. These profiles are for the central 2 in. of the 2.95 in. diameter at 1/8 and 5/8 in. above the bottom and below the top fluid surface. The temperatures at five positions along the diameter at each of two levels were investigated simultaneously. The 6 to 7 replications were combined into half-profiles for the measurements at the top when observation revealed a slight off-center position of the horizontal multi-point rod as the probable cause for a slight asymmetry.

Quantitative measurements of fluid movement utilizing temperature measurements would require measurements of additional layers in order to obtain temperature histories on all sides of a given point to establish temperature and flow gradients and divergences at that point. Nevertheless the following comparisons can be used to indicate approximately the direction of heat flow: (1) higher temperatures near the outside of the

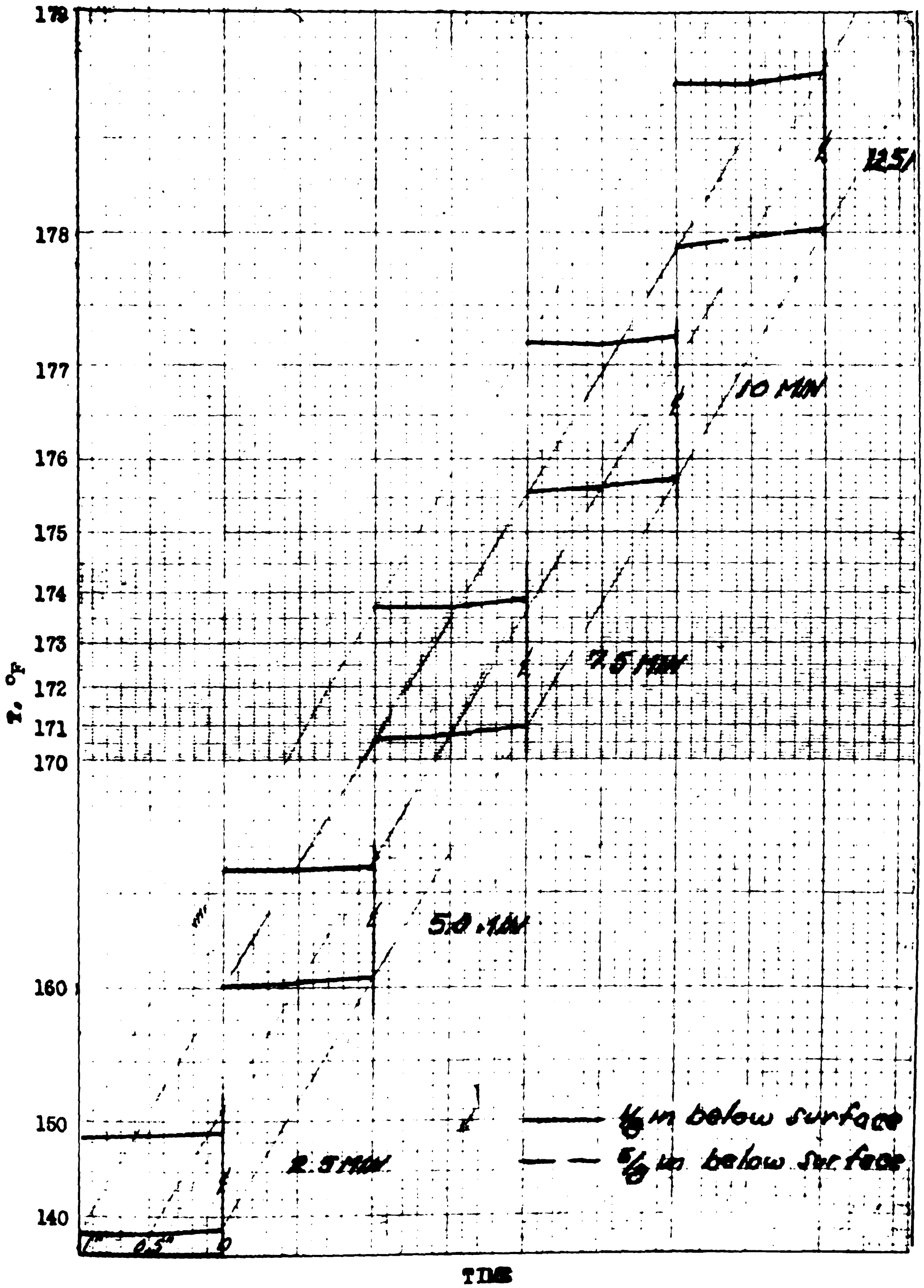

FIGURE 4.6 Horizontal profiles at 1/8 and 5/8in below the top surface of water in 16 oz vegetable jars heated from 60 to 180°f.

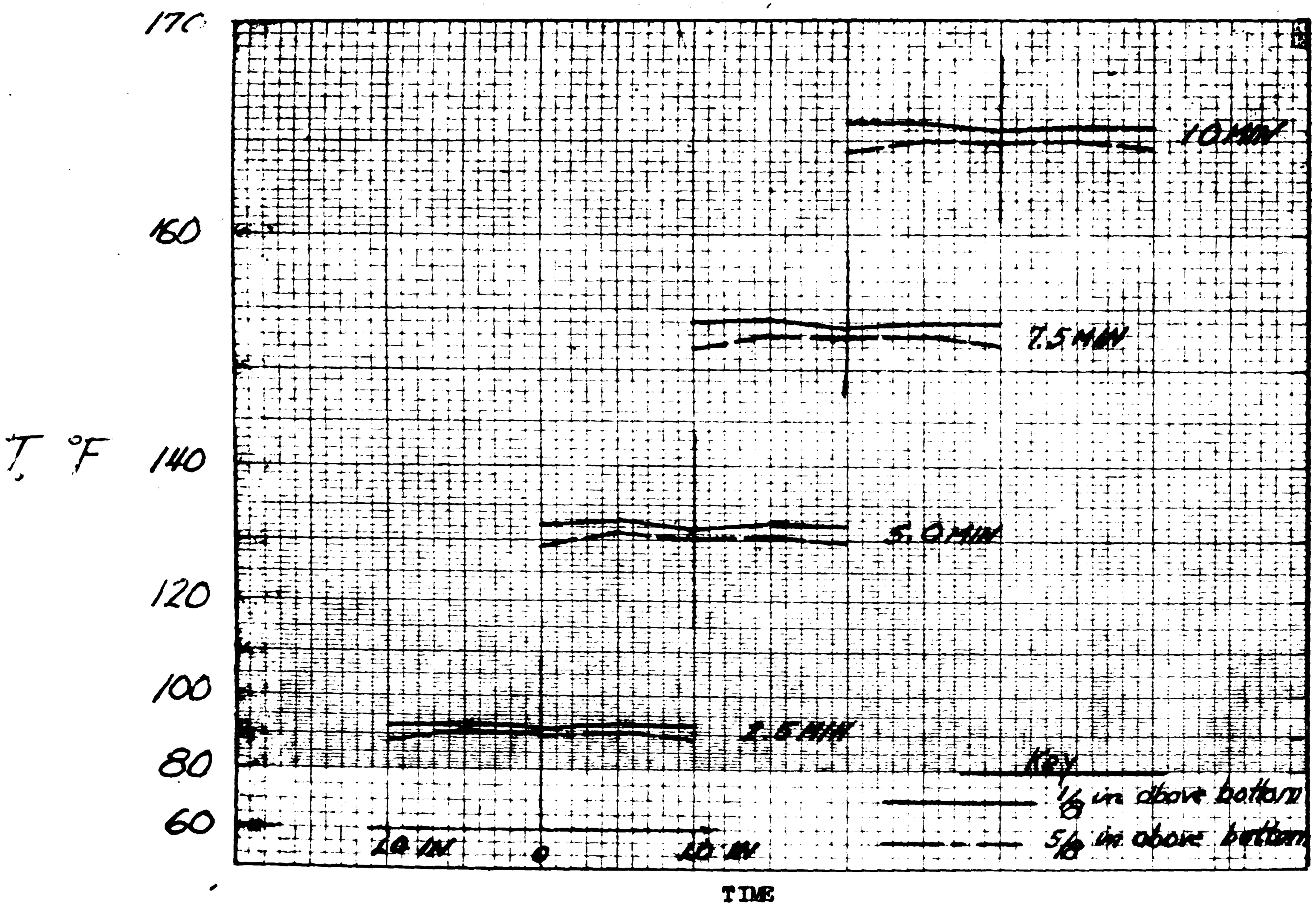

FIGURE 4.6 Horizontal profiles at 5/8 and 1/8 in above the bottom surface of water in 16 oz. vegetable jars heated from 60 to 180°F.

container indicate increased heat transfer due to conduction, and perhaps convection as well, but the converse is a sign of convective flow; (2) the temperature differences (on a logarithmic scale) between planes are an indication of the relative rate of transfer--heat transfer relative to the overall temperature potential available at that time; (3) the differences in the radial gradients (on a logarithmic scale) between positions can be interpreted as an indication of the relative difference in radial heat flow.

In both Figure 4.5 and Figure 4.6 the increasing relative gradient between layers as T approached T_1 may be interpreted as a sign of decreased heat transfer rates--decreased heat transfer which may be attributed to decreasing velocity of the convecting streams.

Within 5/8 in. of the top surface the fluid in the center of the container is consistently hotter than the surrounding fluid. This behavior may be attributed to increased convective flow at this position. The radial temperature gradient at 5/8 in. was greater than that at 1/8 in. below the surface. This suggests that although much of the fluid reached very near the top, the cooler outer portions of the boundary layer left the boundary layer and moved inward between 5/8 and 1/8 in. from the surface.

Near the bottom of the container, however, the fluid appeared to flow fastest at a position 5/8 in. or more from the center at 1/8 in above the bottom but somewhat nearer the center at 5/8 in. above the container bottom. Although the bottom of

the jar is curved away from the thermocouple junctions, the thickness of glass, which is more poorly conductive than water, simultaneously decreases, hence a nearly uniform conductance rather than increased heat transfer at the center might be presumed.

The Inception of Convection

The transition from conduction to convection and the corresponding ascension and decline of bottom convective flow during the establishment of lateral eddies are evident in the initial portions of the heat penetration curves shown in Figures 4.1 to 4.4. This phenomenon is shown in the individual time--emf(temperature curves) of Figure 4.7. These curves are for water in 16 oz. vegetable jars heated from 60° to 180°F. The temperature ordinate scale is slightly non-uniform as the emf-temperature conversion is slightly non-linear. The use of a linear temperature scale rather than the time-emf traces would have required the conversion of the emf to the corresponding temperatures at close intervals in order to retain the features evident in Figure 4.7. One such feature is random temperature variation which may be associated with random eddy movement. This random eddy movement is more pronounced initially and for points close to the origin of lateral and bottom convection, e.g., points number 9 and number 1.

The establishment of the quasi-steady turbulent flow of this particular system is also evident in Figure 4.8.

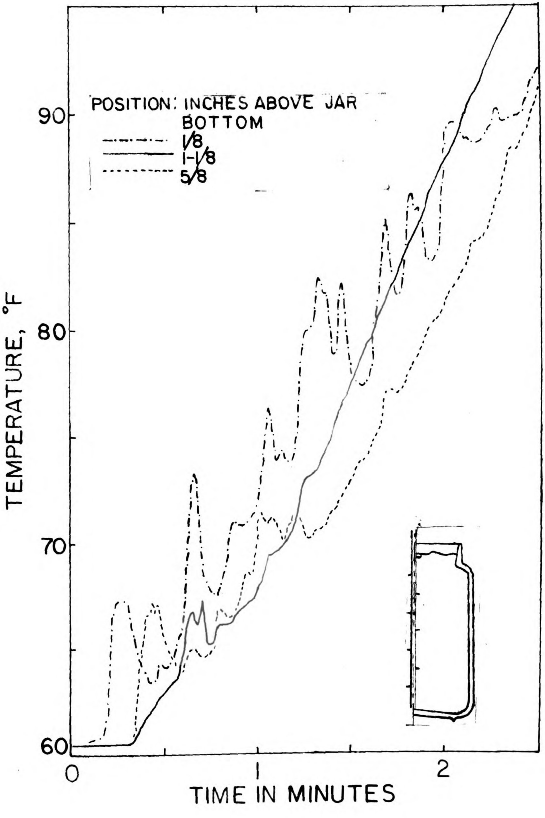

FIGURE 4.7a Temperature histories at various axial positions during the beginning of the heating period.

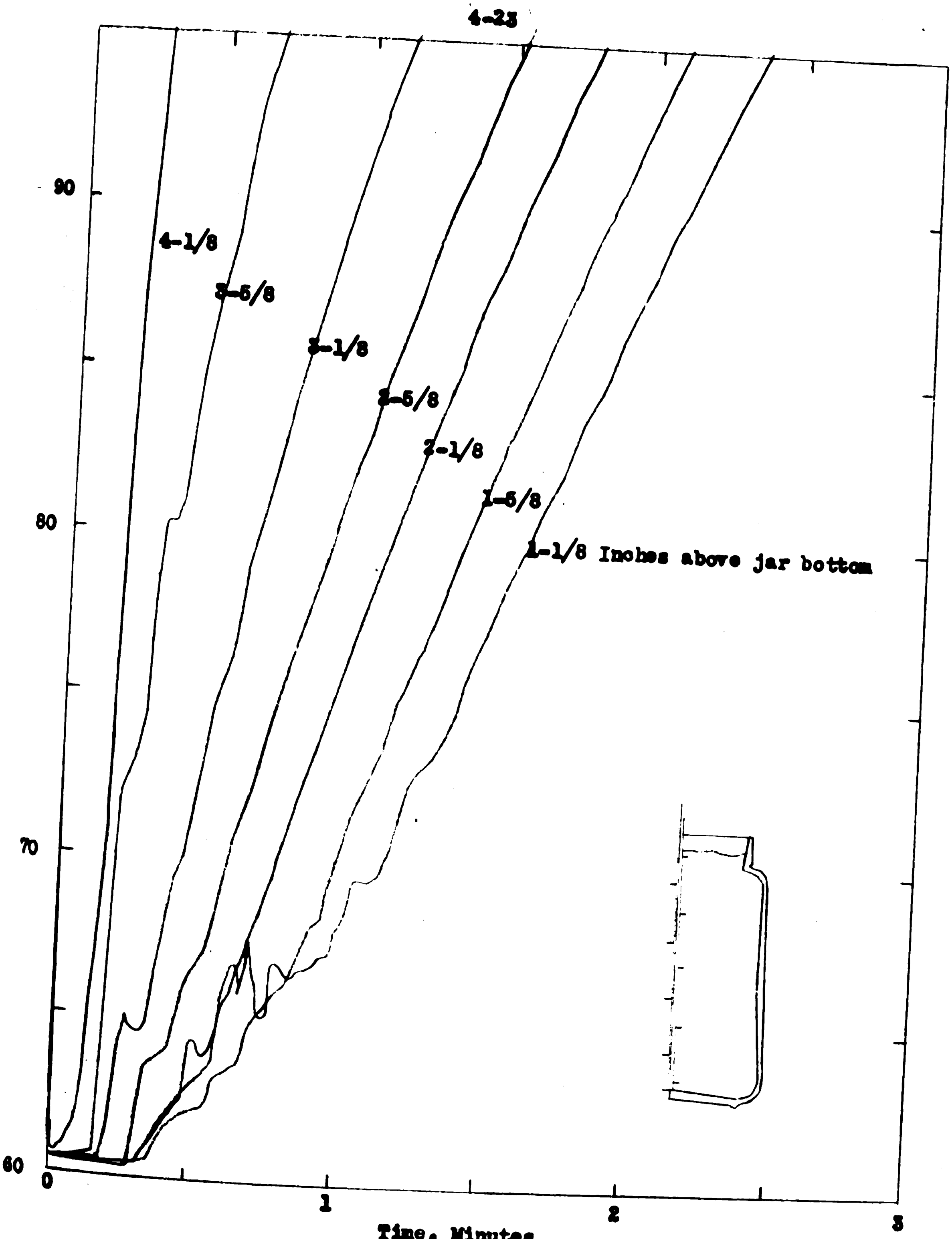

FIGURE 4.7b Time temperature histories at various axial positions during the beginning of the heating period.

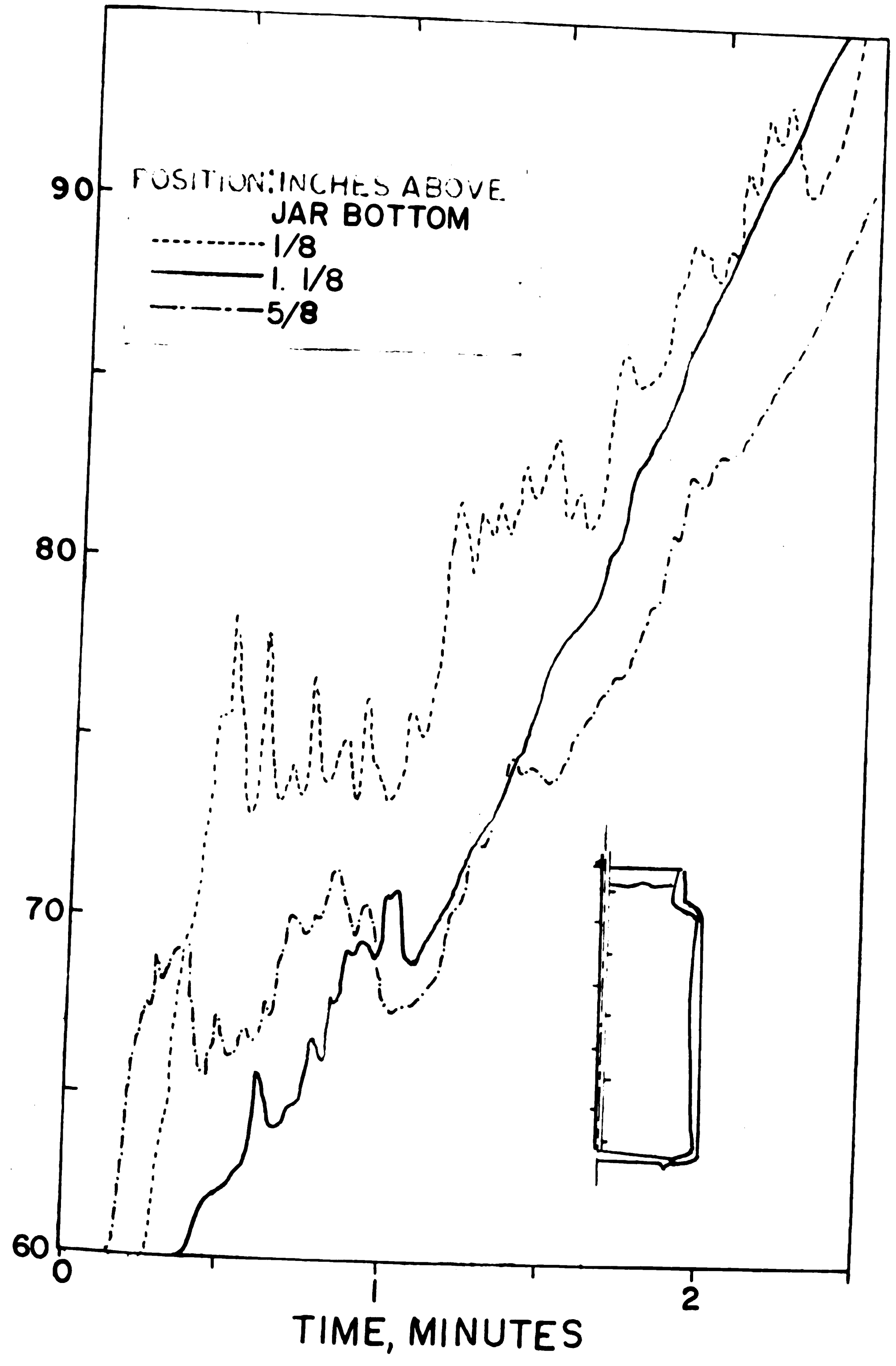

FIGURE 4-7e. Temperature histories at various axial positions during the beginning of the heating period.

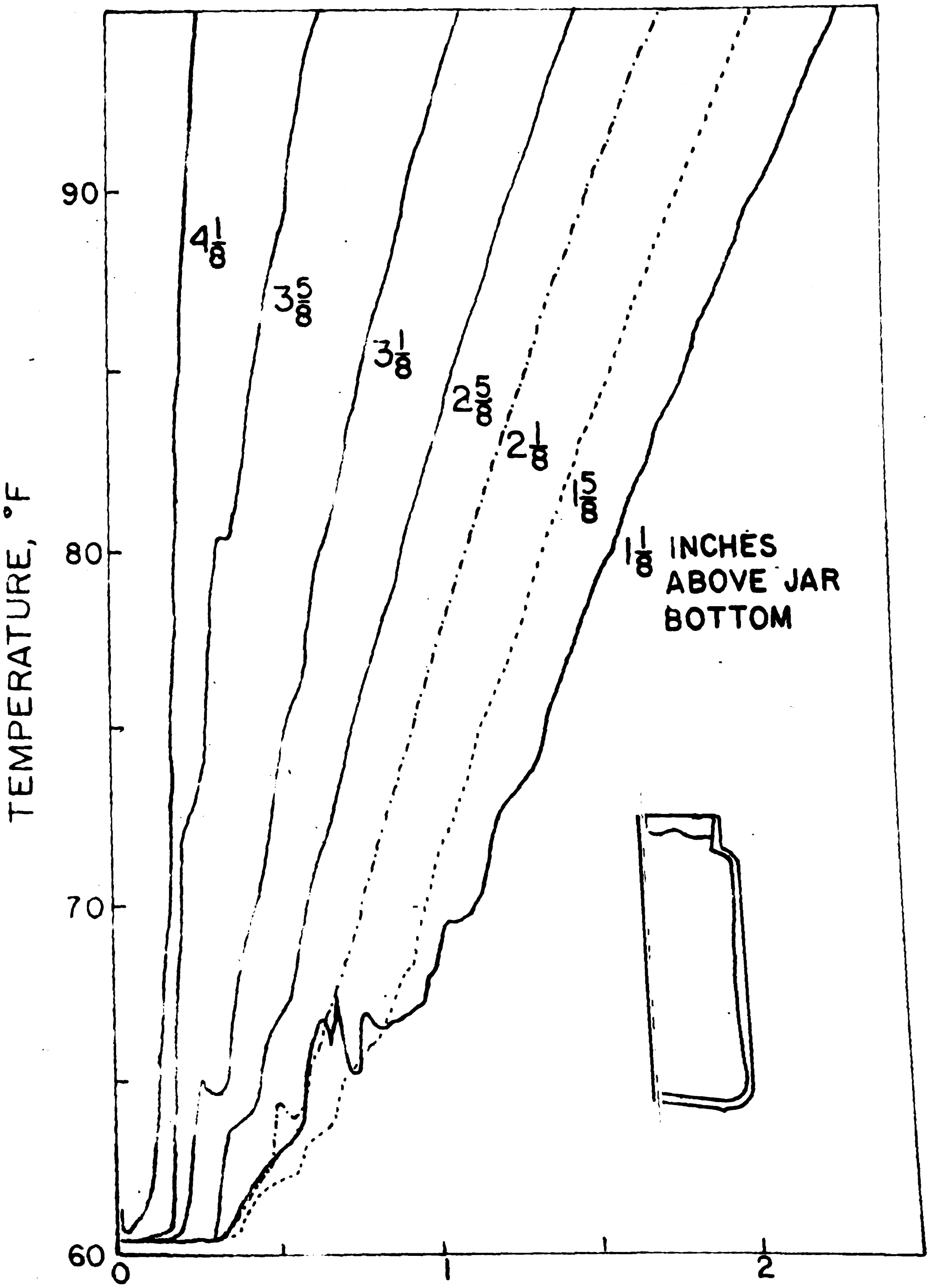

FIGURE 4.7d Temperature histories at various axial positions during the beginning of the heating period.

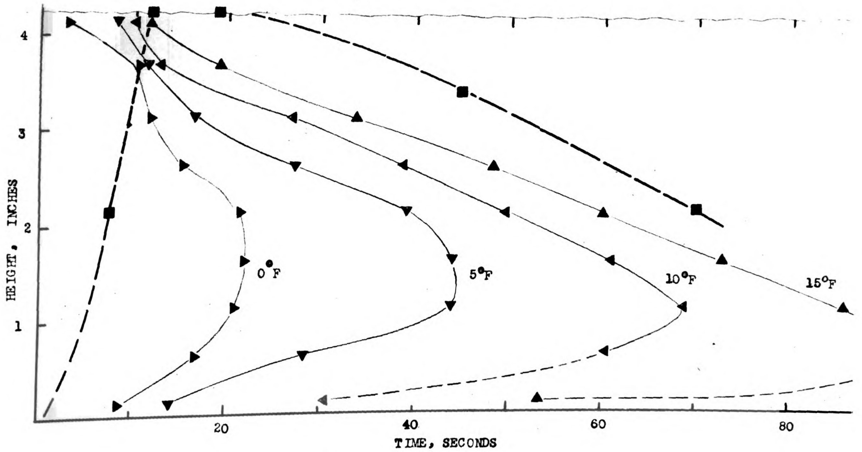

FIGURE 4.8 ISOTHERMALS AT VARIOUS HEIGHTS ABOVE JAR BOTTOM DURING HEATING AND POSITION OF DYE DROPLET RELEASED AT HEEL OF CONTAINER ■

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Here the times required for a given temperature level to be attained at a given position along the central axis are shown. These are measurements scaled from the previous figure. If the temperature fluctuated the several times at which an isothermal crossed the given position were averaged. In the case of the $0^{+\circ}$F isothermal the values are the intercepts of visually smoothed time-temperature histories.

Also shown in Figure 4.8 are the times required for a droplet of dye to move from the bottom heel to various vertical positions, first along the side wall and subsequently in the core.

The isothermals and qualitative dye measurements indicate that eddies from the bottom rise to about 40 percent of the container height initially. These eddies soon diminish in height as the nearly linear temperature gradient and nearly constant velocity lateral eddies form. By the time a droplet of dye, introduced at the heel at the start of heating, had reached the top--less than 8 seconds--its temperature had risen about 45°F. This same droplet descended with the core but at about 1/11th of this rate.

Qualitative Flow Studies

Individual dye droplets of different colors were released near the bottom center and heel and near the top center at about the same time. The dye droplets in the ascending portion of the lateral loops disintegrated in the turbulent flow near the top. Most of these smaller droplets moved laterally in the top 1/2 to 3/4 of an inch, i.e., within 12 to 18 percent of the top. Some of the lateral deflection may be attributed to the shoulder of the jar but this lateral movement was noticed even when the fluid surface was deliberately filled to below the shoulder. These droplets appeared to descend parallel to one another before they vanished. The dye droplets introduced into the core did not disintegrate but vanished at about 1/5th of the container height after some radial movement toward the ascending boundary layer. The dye droplets ascending from the bottom center stayed relatively intact but looped randomly and frequently these droplets did not stay in the same radial plane.

The films of dye movement used in the studies of Fagerson(1950) and Fagerson and Esselen (1950) were examined for details of flow structure. The technique itself was repeated experimentally. The dye streams which rose rapidly along the side wall did not ascend at the same rate. Some slight turbulence was evident at the top of these streams. It was not possible to distinguish droplet disintigration from overall diffusion but it was noticed that the shoulder of the container was responsible for an appreciable portion of the lateral deflection and some of the eddy streams appeared to bounce off the shoulder and descend immediately. Here also the fluid in the core had moved about halfway back to the wall by the time it had approached to within 1/5th of the container height. The flow from the bottom of the container was described by Fagerson and Esselen (1950) as ascending to about 1-1/2 in. (i.e., 38 percent) initially but only to 3/4 inch after five minutes.

The flow patterns were also indicated by the movement of aluminum flakelets in a rectangular prism formed by confining a ⊔-shaped aluminum channel in an open topped plastic box. (Two sides and bottom were heated by warm water which was suddenly introduced into one leg of the ⊔ and flowed out a weir in the other leg.) The gross flow patterns seen in the above two systems were evident here also. Turbulence, however, was indistinguishable from the normal flutter of the flakelets.

A composite sketch of the possible flow patterns which might exist in the temperature range investigated, i.e., from the establishment of the cold point to within 5°F of the heating temperature, is shown in Figure 4.9. This sketch is based on the horizontal profiles and the qualitative dye studies. The position of the horizontal profile measurement points are denoted by crosses. The position of the temperature measurement points in the Fagerson study are denoted by the open triangles. The behavior earlier--during the establishment of lateral and bottom convective flows--are shown in Figure 4.10.

Heat Penetration Curves at Long Times

Heat penetration curves at various positions for the later portion of the heating period are shown in Figure 4.11.

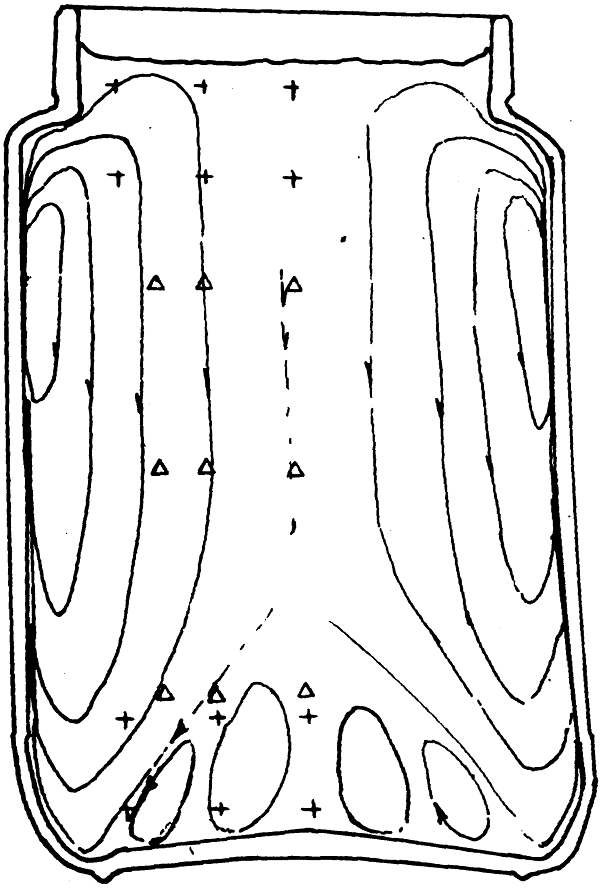


FIGURE 4.9 Flow patterns during heating for high N_{Ra}. The △ denote the location of Fagerson's (1950) temperature measurements; the + denote the location of the temperature measurements in this study.

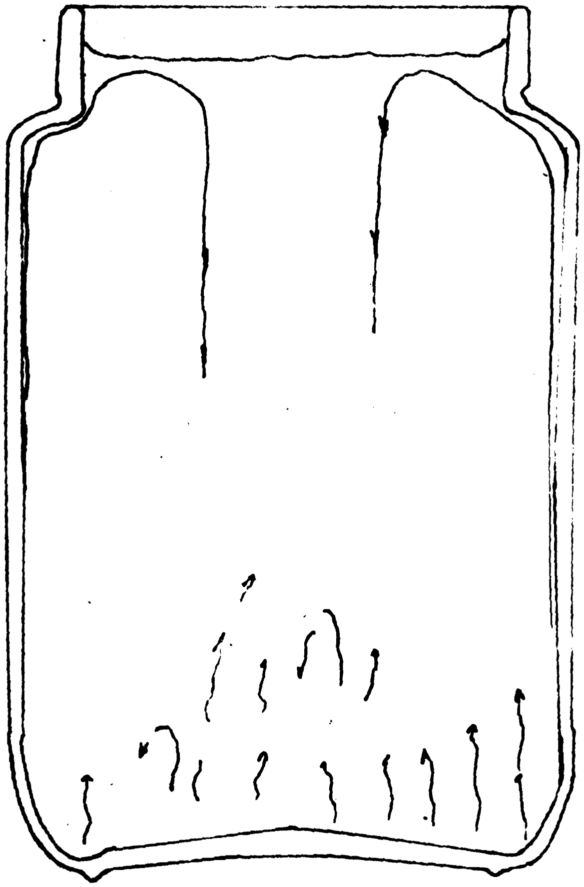


FIGURE 4.10 Flow pattern during the beginning of the heating period for high N_{Ra}

These curves are for water in 16 oz. vegetable jars heated from 60° to 180°F. The irregularities of Figure 4.8, except at 1/8 in. above the bottom surface of the fluid, appeared to vanish well before the slowpoint was within 20°F of the heating temperature. This detail was lost, however, in the conversion of the continuous emf-histories to temperature histories based on a few points as in this figure.

Calorimetry

The results of the calorimetry experiment to determine the mixed-mean temperature of water in 16° oz. vegetable jars are shown in Figure 4.12. These results are the means of seven observations at each point. The standard deviations are about 0.45°C except that at 61 minutes which was about twice as large.

Evaluation of Container and External Film Resistances

Evaluation of Test Cylinders

Estimated film conductances in the annular calibration apparatus are shown in Figure 4.13. These conductances were determined from Figure 4.14. The j measurements were not used here, due to the low sensitivity of j as a predictor of the Biot number at low Biot numbers region (see Figure 4.15+16) and unknown transfer associated errors, such as heat gain and non-uniform initial temperatures. (The range of lag values j determined at the beginning and end of the three to five second transfer period included the j's calculated for

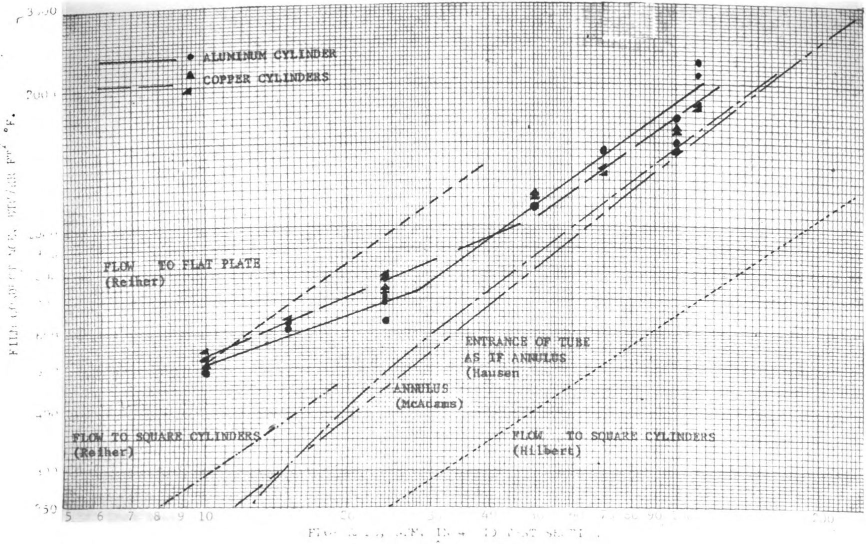


FIGURE 4.15 Experimental and predicted film conductances in the test annulus for various water flow rates.

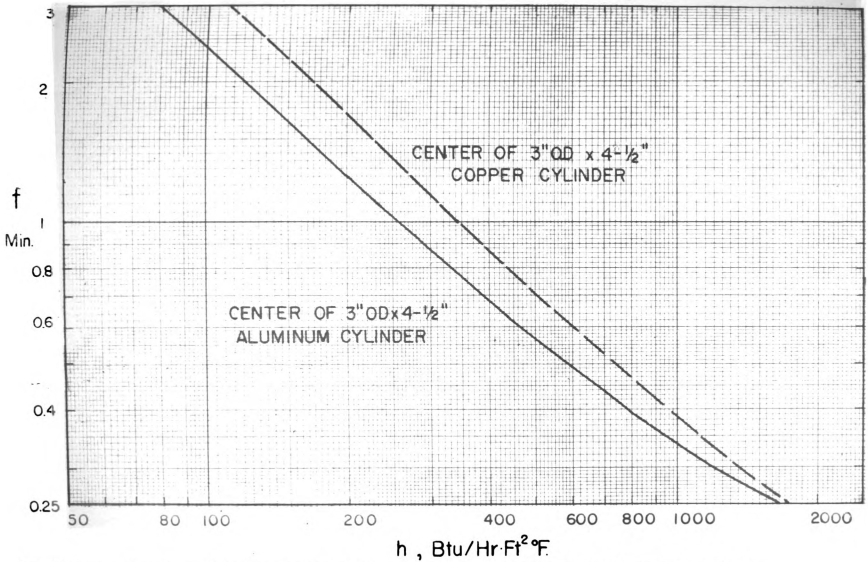


FIGURE 4.14 Heat transfer coefficient h as a function of f for the copper and aluminum cylinders

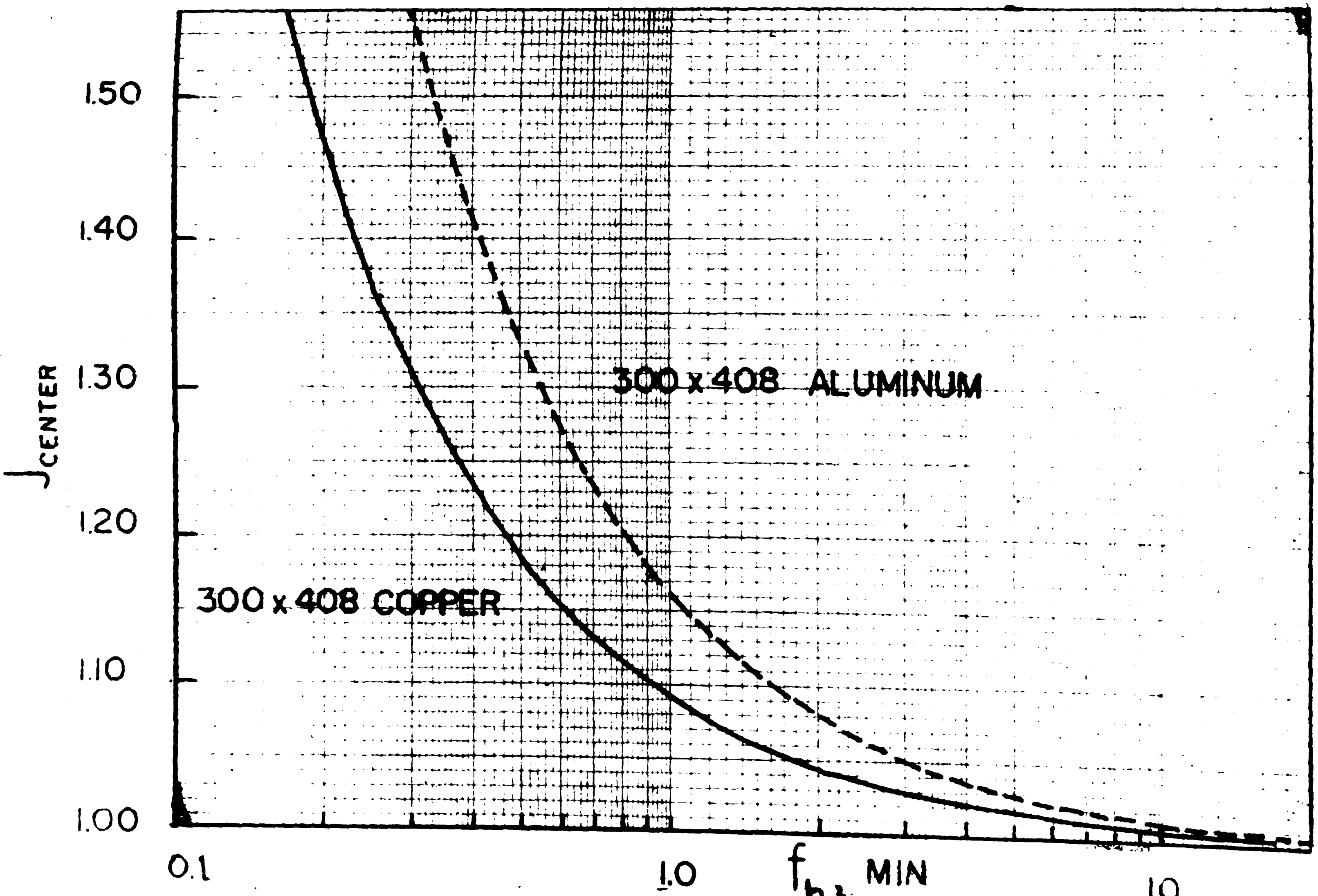


FIGURE 4.16 The relationship between the parameters f and J for the copper and the aluminum cylinders.

corresponding experimental f values.) Also shown are film conductances for inner or outer walls of concentric annuli in the turbulent region. The latter were calculated using the recommendation (Equation 9-32c) of McAdams (1954). Also shown for contrast are Hausen's (1940) equation (Equation p. 212, Eckert and Drake, 1959) using the hydraulic radius of the annulus formed by the cylinder, and the formulae of Reiher (1925) and Hilbert (1933) as reported by Jakob (1949), but modified by the usual correction for Prandtl number to convert data for air to other low Prandtl number fluids and the use of the equivalent square up-stream surface for flat plates and the equivalent equidimensional cylinder in the hydraulic radius. (The low flow region may correspond to the region in which natural convection would contribute more than 10 percent to the film conductance on a short tube (see Eckert et al., 1953 ; also Eckert and Drake, 1959).)

Data for the 0.9 and 0.0 r thermocouple positionswere not signigicantly different (P = 0.05) and so are not differentiated in Figure 4.13. There were no significant differences (P = 0.05) among the cylinders and the thermocouple installations.

Convection Heating and Cooling of Containers in Water

The effects of some processing variables--waterbath size and agitation, temperature difference, final fluid property level and heating vs. cooling--on the film conductances in several laboratory water-baths are shown in

Tables 4.2 and 4.3. The means and ranges are for two positions within the cylinders during two different immersions. The effects of the film coefficients shown in these tables are illustrated in the response of water in No. 303 cans as shown by Table 4.4.

Convection Heating of Containers in Steam-Air Mixtures

The results of a recent experiment (Koppleman, 1963) in which surface conductances in some steam-air mixtures were determined and the results of the f vs log velocity correlation of Pflug and Blaisdell (1961) are combined in Table 4.5 to illustrate the effects of large changes in film coefficient on the f and j.

The Container Wall

The results of the measurements of the thicknesses at several positions on the jars used in this investigation are shown in Table 4.6. Also shown are the results of estimation of glass thickness by the method of Merrill (1948); the method of Merrill but with a new empirical coefficient suggested by Alderson (1963); and by approximation of the glass thickness, suggested by Alderson (1963), which is determination of the area surface by division of the container into cones and cylinders of revolution and assuming that the glass is distributed uniformly over the sides but twice as thick over the container bottom.

TABLE 4.2.--Effect of initial temperature difference, and position in several laboratory water baths on apparent film conductance, expressed in Btu/hr ft^2 °F

Treatment			Individuals				Means for Process
			Aluminum Cyl.*		Copper Cyl.*		
Bath	Form	Cu Position	Means	Range (of 8)	Means	Range (of 8)	
60°	120	L	257	242-275	249	246-255	241
		R	250	235-275	217	195-237	
	180	L	248	242-256	246	242-250	245
		R	256	250-266	230	225-235	
120°	60	L	260	255-265	253	247-258	254
		R	256	230-282	252	243-264	
	180	L	283	275-293	279	273-285	282
		R	285	280-288	281	276-284	
180°	60	L	203	197-206	245	238-264	229
		R	251	220-280	220	201-241	
	120	L	215	206-220	231	226-235	223
		R	238	218-264	208	193-222	

*Cyl. = Cylinder.

TABLE 4.3.--Effect of agitation on the apparent film conductance, expressed in Btu/hr ft^2 ^{o}F. (4 measurements for each combination except as noted)

Treatment (60^o-180^o Process)	Individuals				Means for Process
	Aluminum Cyl.*		Copper Cyl.*		
Mixer Position**	Means	Range	Means	Range	
Natural Convection Cylinders Alone	163	157-167	188	185-190	175
Natural Convection Heat Source On	173	162-178	191	175-200	182
Mixed Natural and Forced Convection 1 Mixer on Left (at A)	214 (of 6)	202-225	240 (of 6)	230-245	227
Mixed Natural and Forced Convection 1 Mixer on Left (at A): a rerun	203	197-206	245	238-264	229
Mixed Natural and Forced Convection 1 Mixer on Right (at C)	221	208-232	220	217-231	220
Mixed Natural and Forced Convection 2 Mixers (1 each at A and B)	304	300-310	297	294-300	300

*Cyl. = Cylinder.

**Refers to mixer position indicated in Figure.

TABLE 4.4.--Effects of exterior film coefficients in water on f and j maxima (means of 6 replications) for water in No. 303 cans

Processing Temperature T_0 to T_1 °F	Mean h_o Btu/hr ft^2 °F	Parameters f min	 j	 $\bar{x}$ height in	Estimated h_i Btu/hr ft^2 °F
60-120	140*	8.7	1.26	.80	649
	254	5.5	1.47	.78	
60-180	180*	6.1	1.18	.48	276
	230	5.3	1.28	.51	646
	300	4.2	1.39	.39	1967

*Natural convection on cans but with the heat supply to processing equipment turned on.

TABLE 4.5.--Effects of exterior film coefficients on atmospheric pressure steam-air mixture on f and j (after Pflug and Blaisdell, 1961) for water in No. 303 vegetable jars

Processing Temperature T_0 to T °F	Velocity ft/sec	Mean[1] No. Btu/hr ft^2 °F	f min	j
100 to 165	.15	34	16.5	1.33
	.38	44	14.9	
	.58	45	14.3	
100 to 180	.15	73	13.0	1.34
	.38	75	12.0	
	.58	78	11.7	
100 to 195	.15	129	10.0	1.34
	.38	145	9.7	
	.58	154	9.5	

[1] Means of 4 replications for 60° initial temperature.

Factors Affecting the Parameters f and j

The parameters f and j for the various treatments and positions are shown in the panels of Figure 4.17 and Figure 4.18 as a function of dimensional position above the bottom surface of the fluid. The heat penetration parameters were arbitrarily determined over the same u range, i.e., from a u equal to 0.5 to a u equal to 0.0835 or in terms of temperatures are 90 to 115, 120 to 170, and 150 to 175°F for heating from 60 to 120, 60 to 180, and 120 to 180°f, respectively. The data for points 1, 2, 3, 5, and 7 which establish the slow point region and the temperature gradient above this region are reported.

The data for the maximum f and j, their dimensional position and the corresponding f or j are summarized in Tables 4.7 and 4.8. These data, being visually selected from curves drawn by visually weighing data with a coefficient of variation of approximately 10 percent, do not represent significant differences. The data do show trends summarized in Table 4.9 which have importance when discussed in light of other evidence.

FIGURE 4.17a,b,c,d. Heat penetration parameters f at various positions within the several containers for various processing conditions.

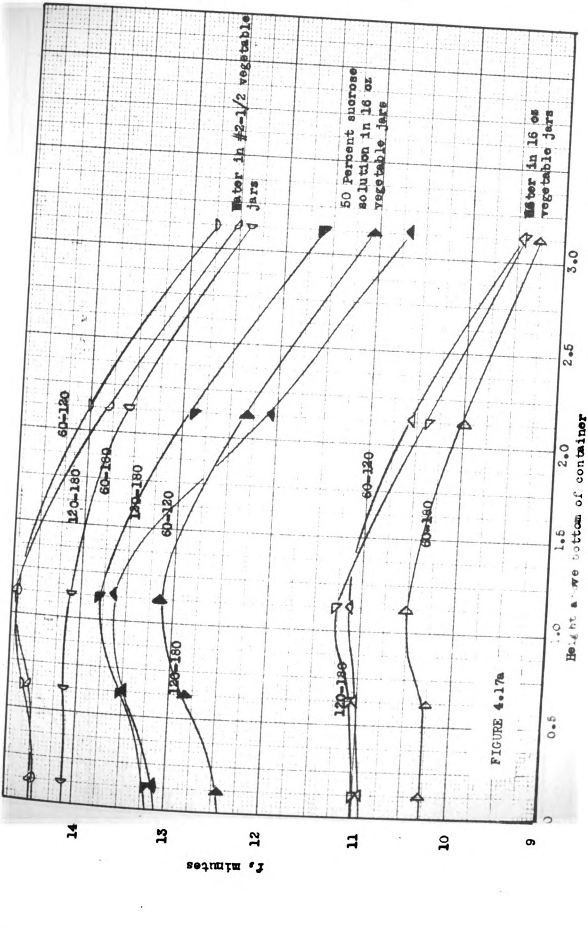

Water in #2-1/2 vegetable jars
50 Percent sucrose solution in 16 oz vegetable jars
Water in 16 oz vegetable jars
60-120
120-180
60-180
120-180
60-180
60-120
120-180
60-120
120-180
60-180
14
13
12
11
10
9
f, minutes
0
0.5
1.0
1.5
2.0
2.5
3.0
Height above bottom of container

FIGURE 4.17a

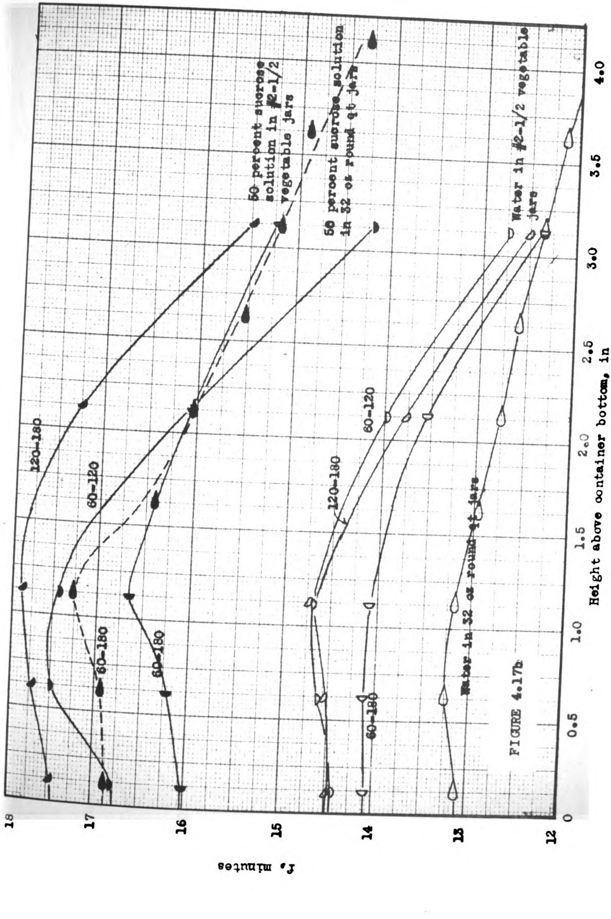


FIGURE 4.17b

50 percent sucrose solution in #2-1/2 vegetable jars
50 percent sucrose solution in 32 oz round qt jars
Water in #2-1/2 vegetable jars
Water in 32 oz round qt jars
120-180
60-120
60-180
60-180
120-180
60-120
60-180
Height above container bottom, in
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
f, minutes
12
13
14
15
16
17
18

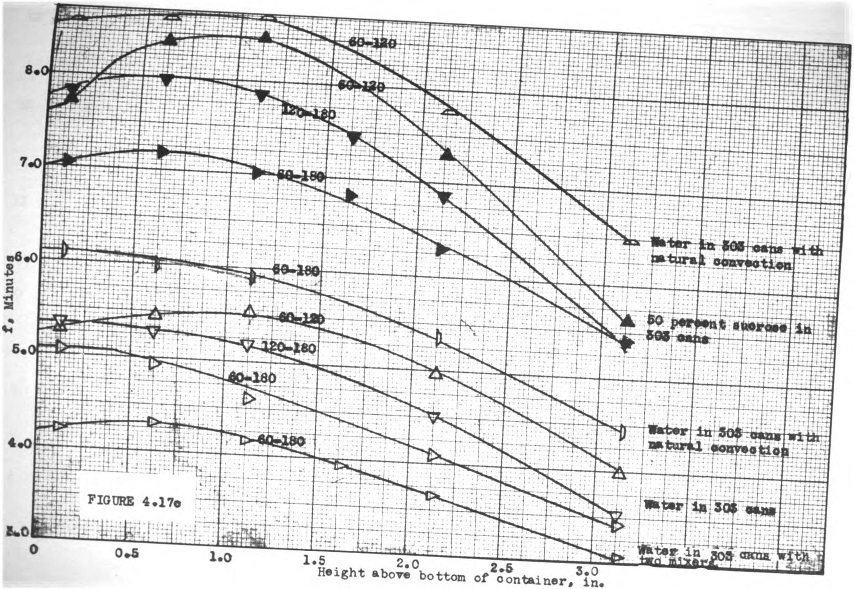

60-120
60-120
120-180
60-180
60-180
60-120
120-180
60-180
60-180
Water in 303 cans with natural convection
50 percent sucrose in 303 cans
Water in 303 cans with natural convection
Water in 303 cans
Water in 303 cans with two mixers
8.0
7.0
6.0
5.0
4.0
3.0
f, Minutes
0
0.5
1.0
1.5
2.0
2.5
3.0
Height above bottom of container, in.

FIGURE 4.17c

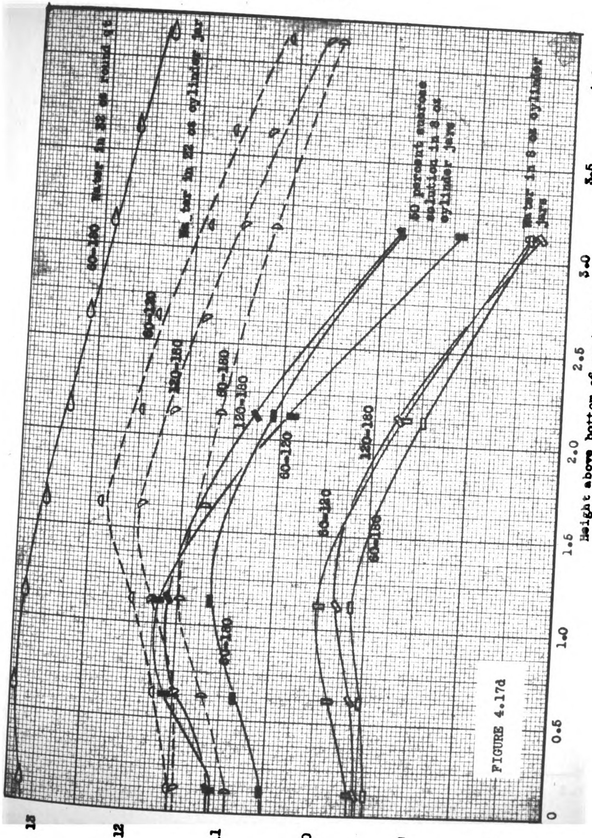

60-180 Water in 32 oz round qt
Water in 32 oz cylinder jar
60-120
120-180
60-180
120-180
60-120
50 percent sucrose solution in 8 oz cylinder jars
Water in 8 oz cylinder jars
120-180
60-120
60-180
60-120
Height above bottom of container, in
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
f, minutes
8
9
10
11
12
13

FIGURE 4.17d

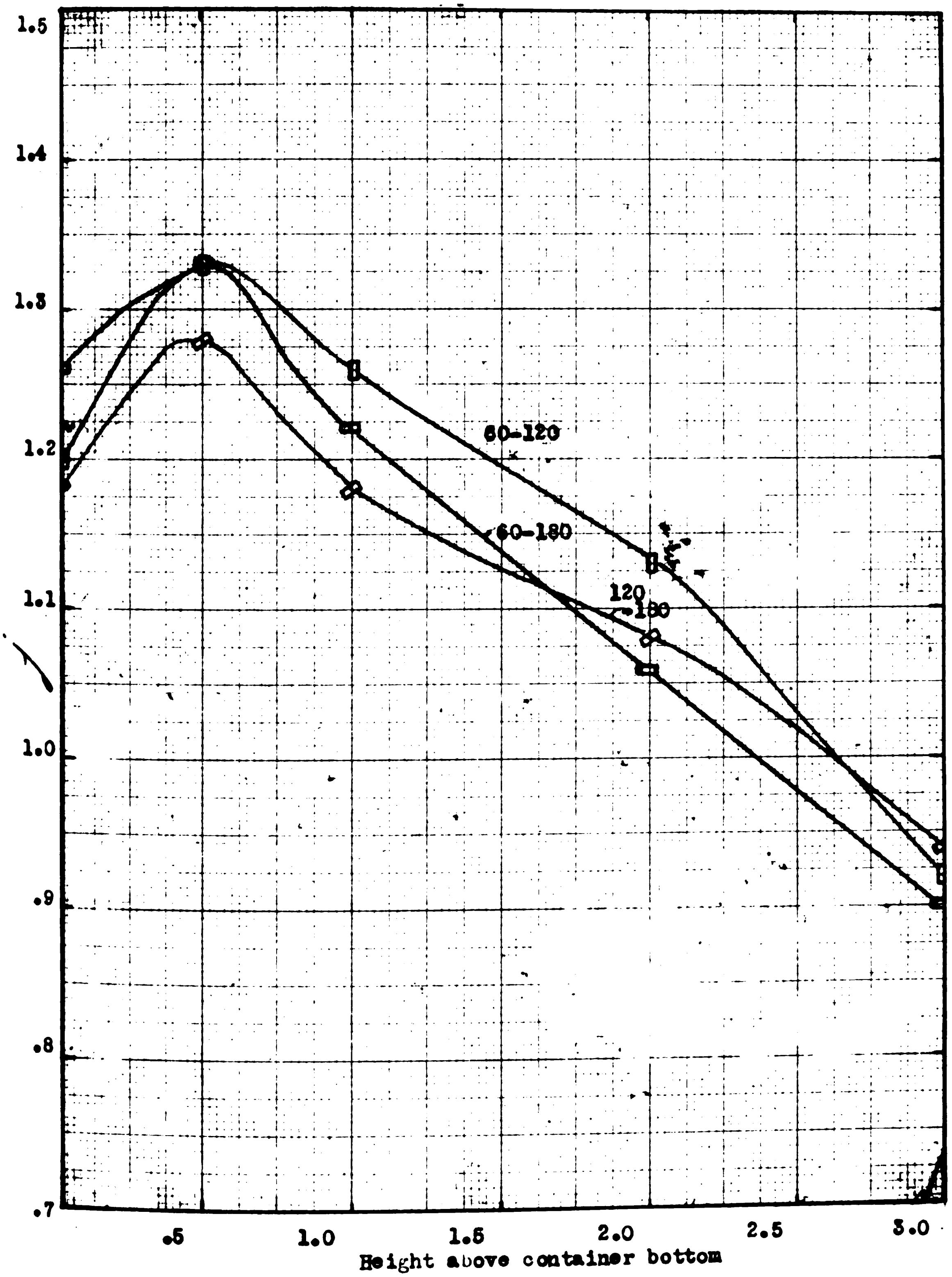
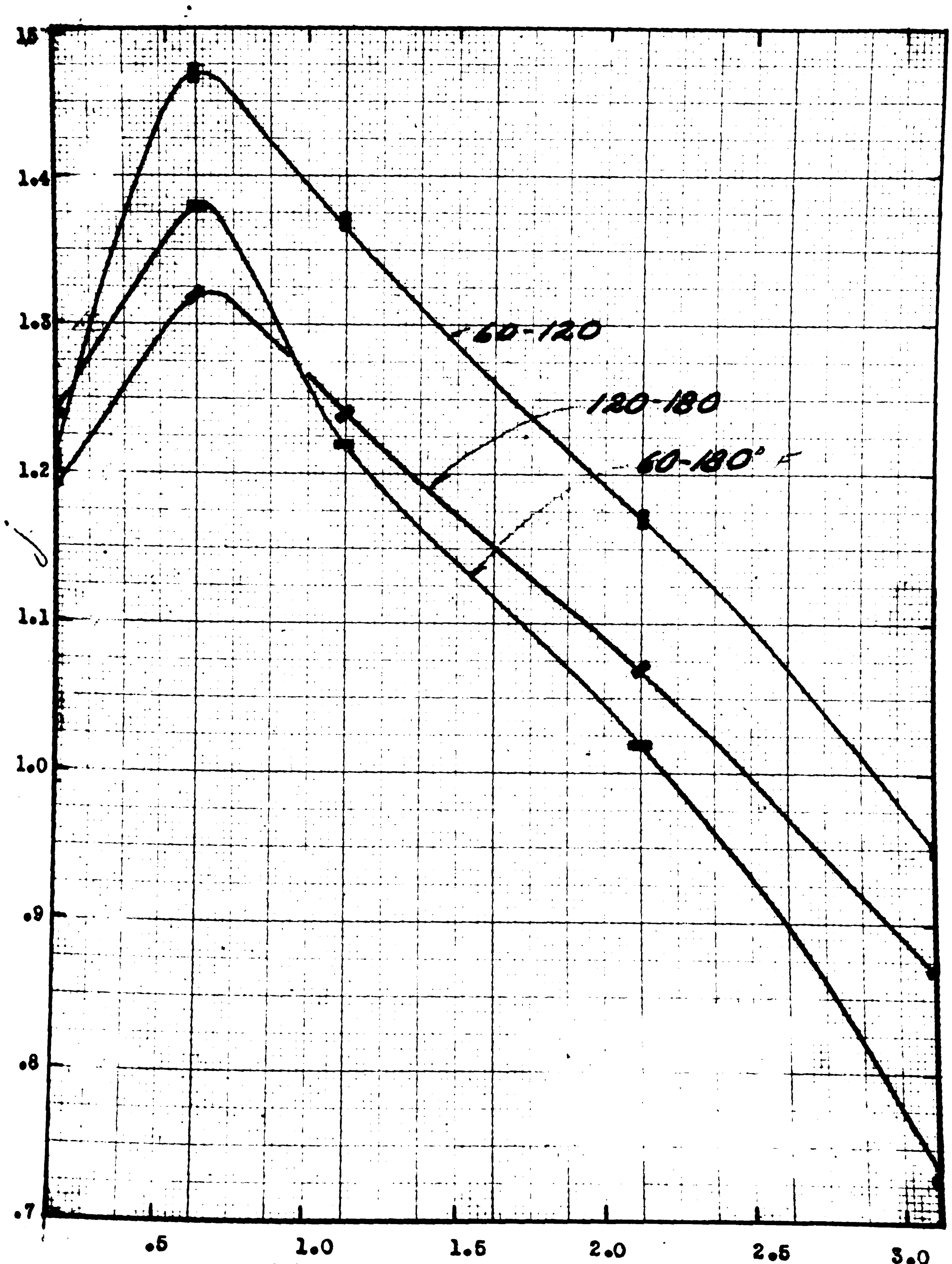


FIGURE 4.18a Heat penetration parameter j at various positions within 8oz cylinder jar containing water.



Height above the container bottom, in.

FIGURE 4.18b Heat penetration parameter j at various positions within 8 oz cylinder jars containing 50% sucrose solution

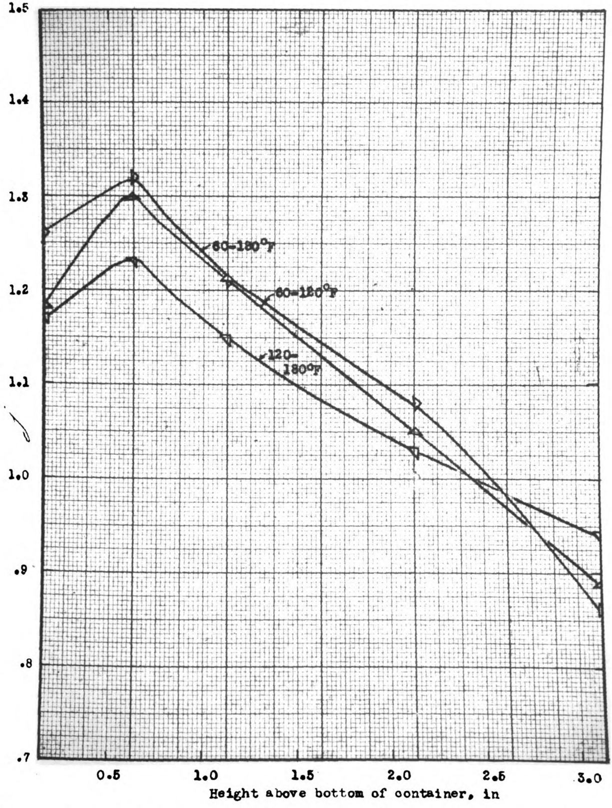


FIGURE 4.18c Heat penetration parameter j at various positions within 16 oz vegetable jars containing water

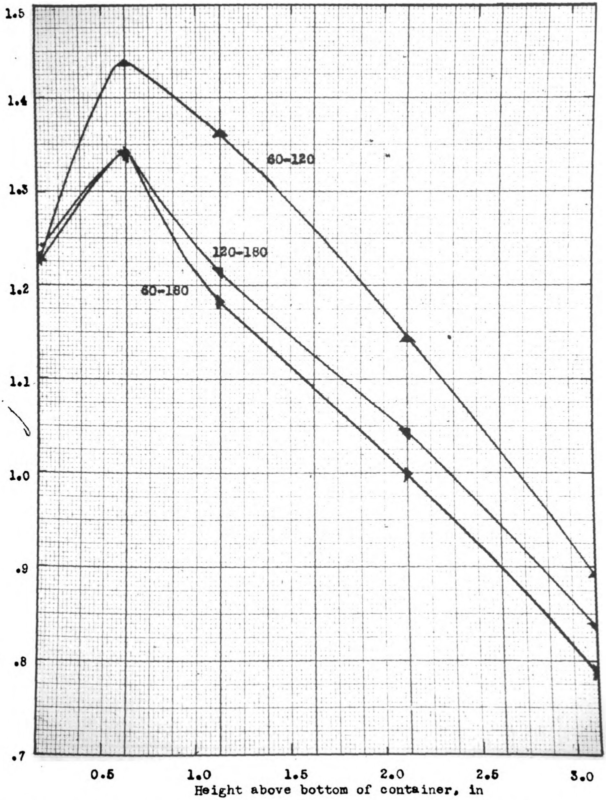


FIGURE 4.18d Heat penetration parameter j at various positions within 16 oz vegetable jars containing 50 percent sucrose solution

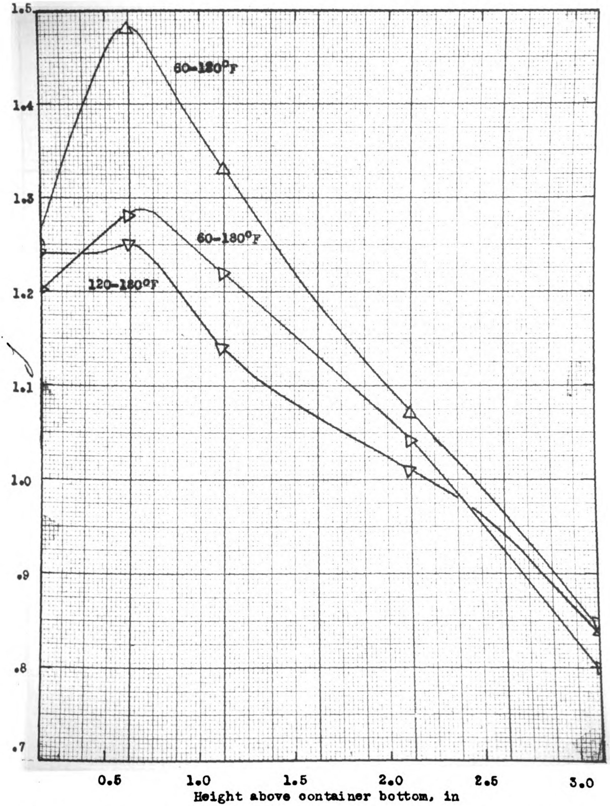


FIGURE 4.18e Heat penetration parameter j at various positions within 303 cans containing water

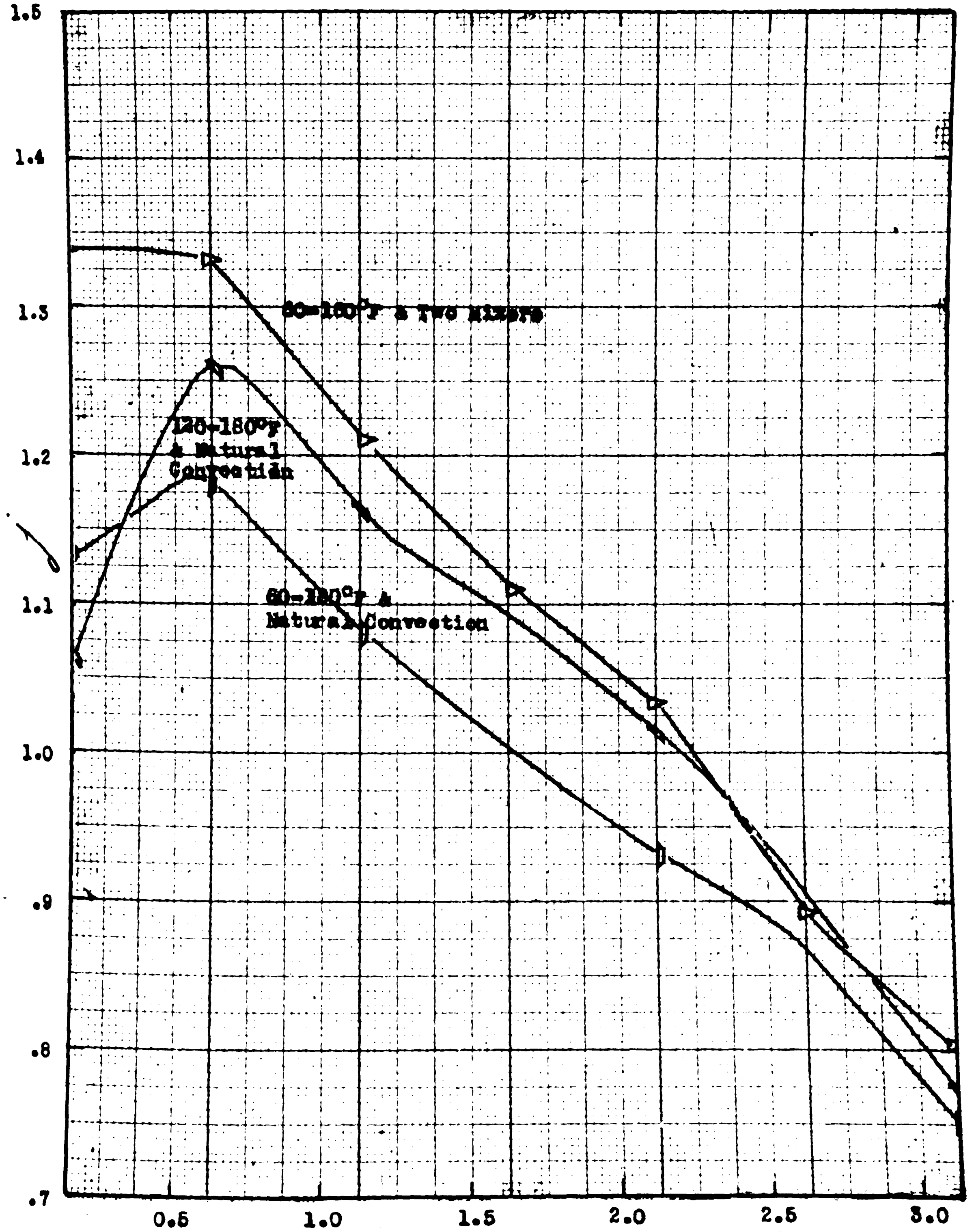


FIGURE 4.18f Heat penetration parameter j at various positions within the #303 cans containing water when the external agitation is other than a single mixer.

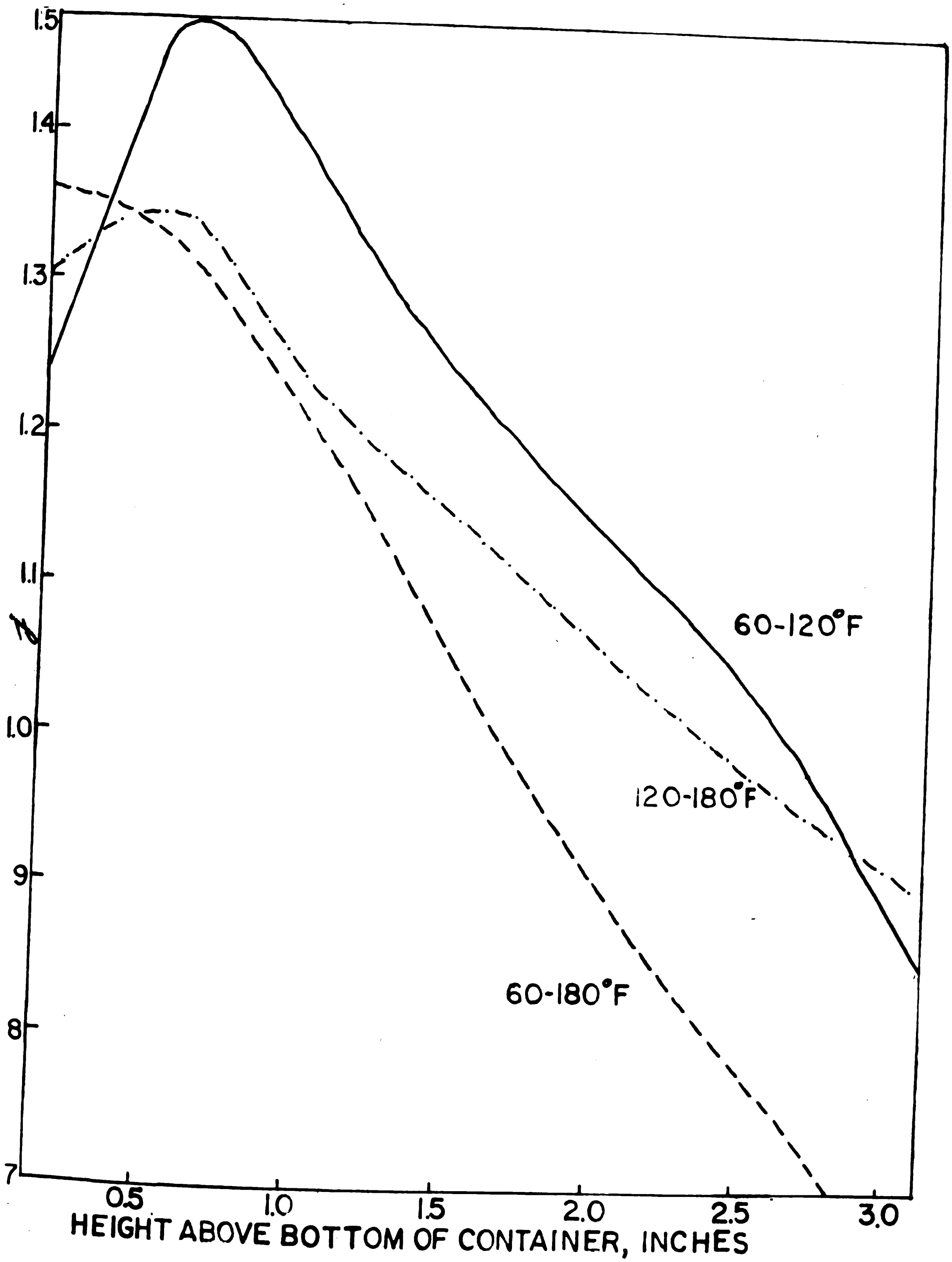


FIGURE 4.18g Heat penetration parameter j at various positions within 303 cans containing 50 percent sucrose solution

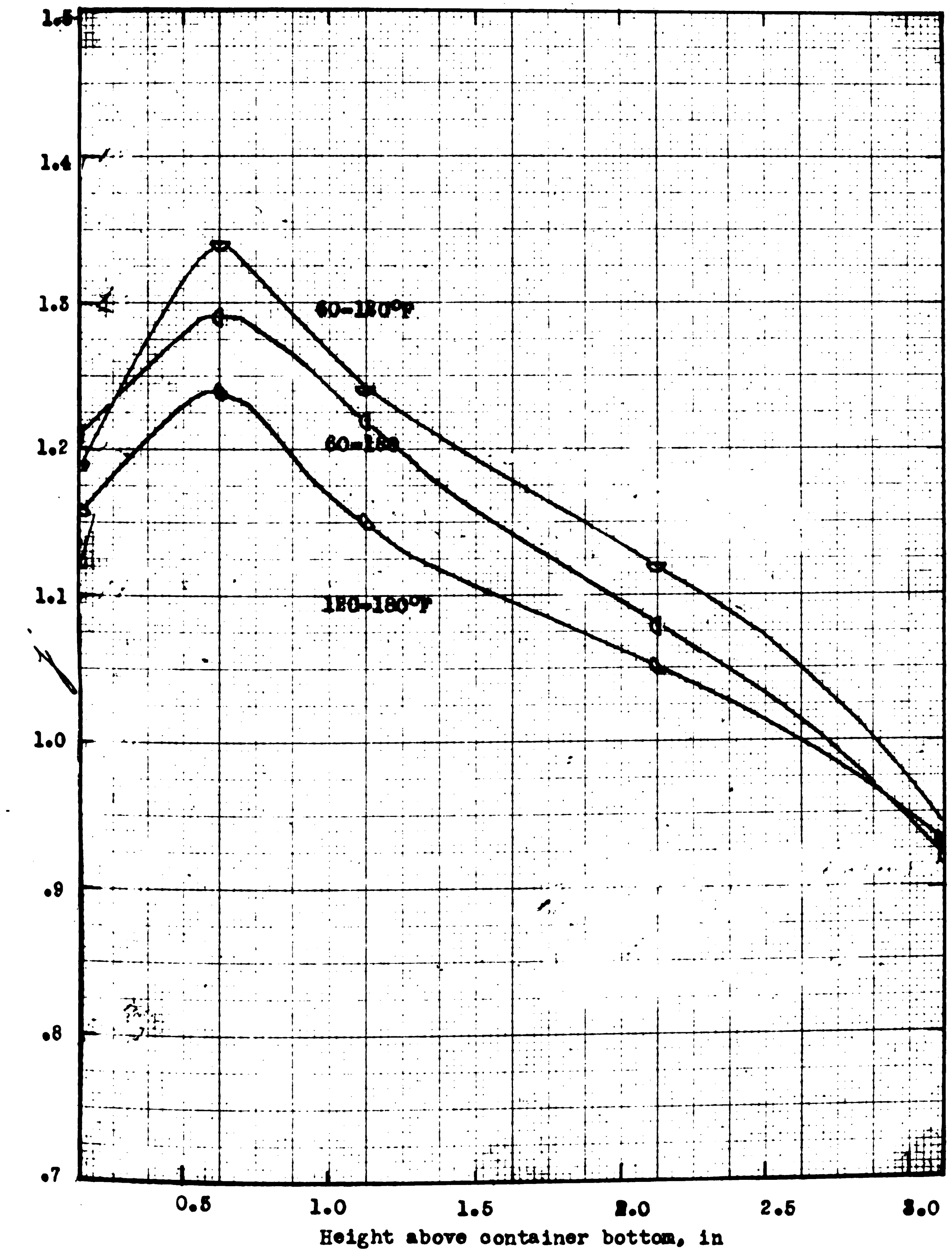


FIGURE 4.18h Heat penetration parameter j at various positions within #2-1/2 vegetable jars containing water.

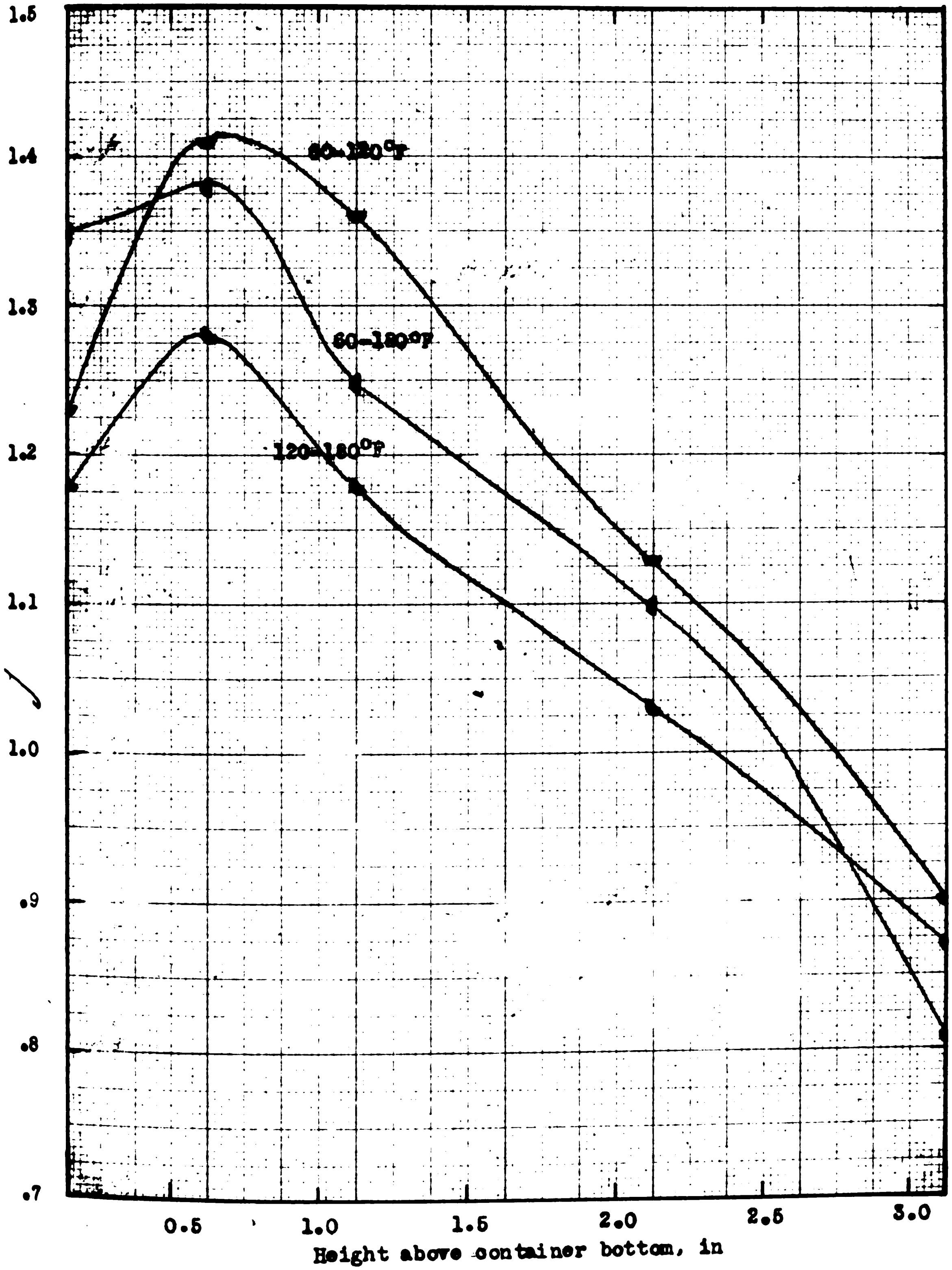


FIGURE 4.18i Heat penetration parameter j at various positions within #2-1/2 vegetable jars containing 50 percent sucrose solution

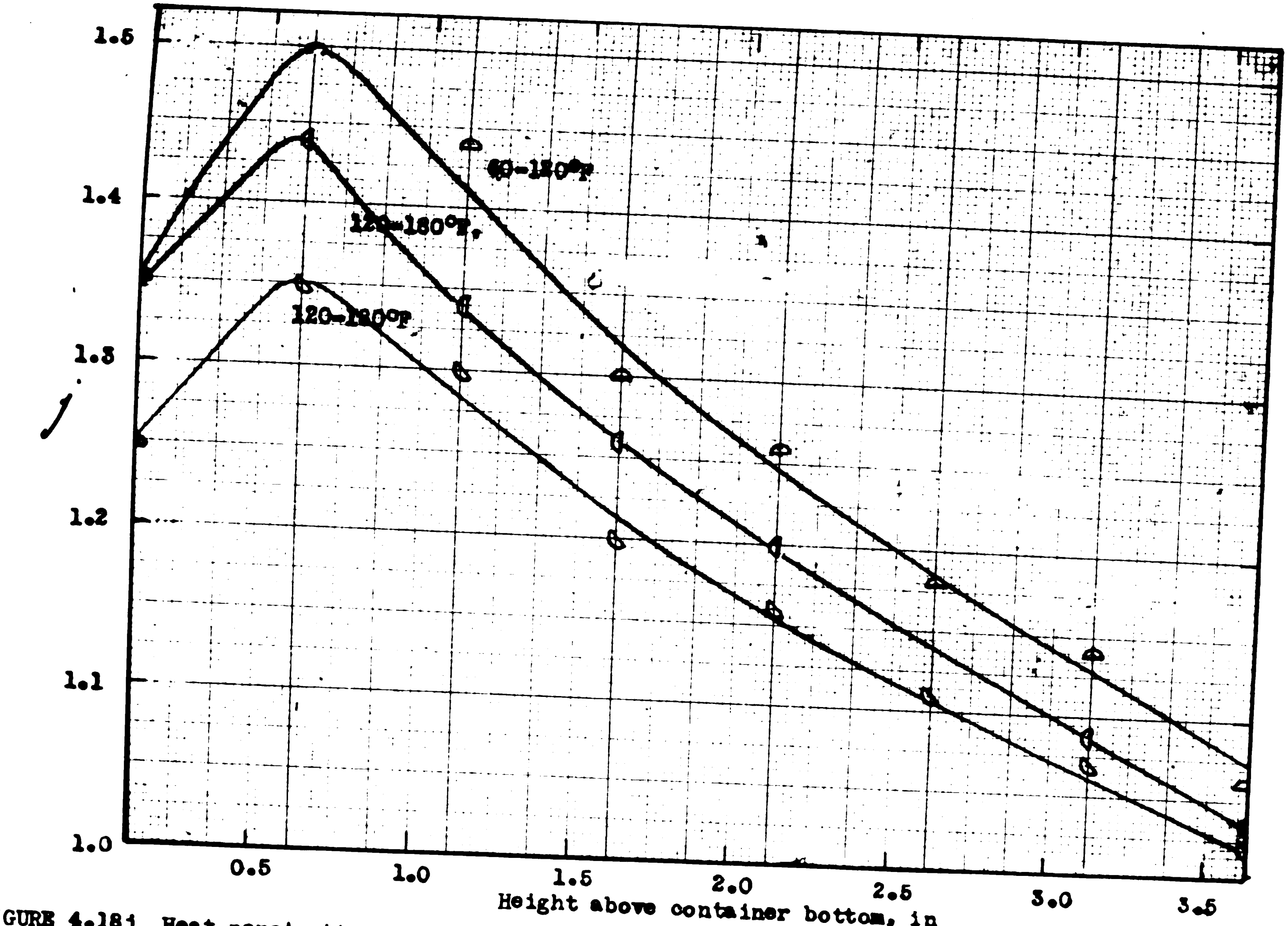


FIGURE 4.18j Heat penetration parameter j at various positions within 22 oz cylinder jars containing water.

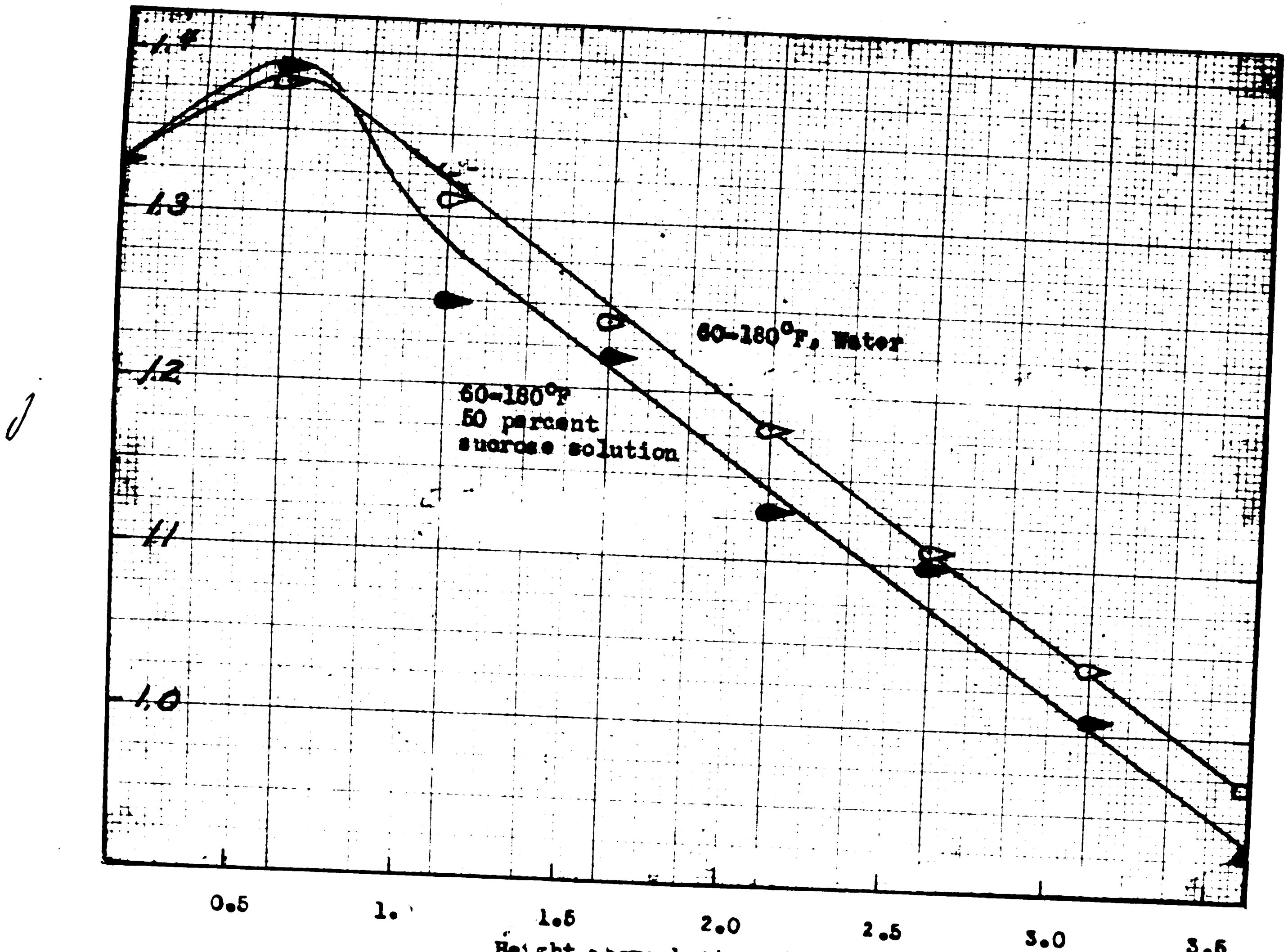


FIGURE 4.18k Heat penetration parameter j at various positions within 32 oz round quart jars

TABLE 4.6.--Estimated and measured container wall thickness, in inches

Jar	Merrill	Merrill-Alderson but (.104 Factor)	Calculated by Area	Measured			
				.8L	.5L	.2L	B
5 oz. Baby Food	.135	.097	.092	--	--	--	--
8 oz. Cylinder	.157	.133	.114	.115	.141	.143	.127
16 oz. Veg. Jar	.116	.084	.080	.095	.108	.083	.127
17 oz. Veg. Jar	.111	.079	.078	--	--	--	--
22 oz. Cylinder	--	--	--	.099	.100	.101	.208
28 oz. Veg. Jar (11 oz. wt.)	.135	.096	.095	--	--	--	--
28-3/8 ox. Veg Jar (11.5 oz. wt.)	.140	.101	.078	.098	.114	.095	.147
33 oz. Pickle Qt. (11.5 oz. wt.)	.109	.078	.084	--	--	--	--
32 oz. Round Qt.* (12.5 oz. wt.)	--	--	--	.099	.100	.101	.208

*Prints of this container are not available.

TABLE 4.7.--Location of maximum f and j and their corresponding j or f for water in various containers and various processing conditions

Container	Temperature °F	Maximum f				Maximum j			
		f,min.	j	height in	t_a,min.	f,min.	j	height in	t_a,min.
8 oz. Cylinder Jar	60-120	10.0	1.27	1.13	1.04	9.85	1.33	.63	1.22
	120-180	9.85	1.18	1.28	.71	9.65	1.33	.68	1.20
	60-180	9.65	1.22	1.13	.83	9.50	1.28	.58	1.02
303 Vegetable Jar	60-120	11.15	1.21	1.13	.92	11.00	1.30	.63	1.25
	120-180	11.3	1.15	1.13	.69	11.1	1.23	.63	1.00
	60-180	10.5	1.21	1.13	.87	10.3	1.32	.63	1.24
303 Can	60-120	5.5	1.34	1.08	.70	5.45	1.47	.63	.91
	120-180	5.35	1.24	.33	.50	5.30	1.25	.63	.51
	60-180	5.3	1.23	.33	.48	4.9	1.28	.68	.53
2-1/2 Jar	60-120	14.70	1.23	1.23	1.32	14.6	1.34	.63	1.86
	120-180	14.75	1.15	1.13	.89	14.55	1.24	.63	1.36
	60-180	14.15	1.29	.68	1.56	14.15	1.29	.63	1.56
22 oz. Cylinder Jar	60-120	12.3	1.34	1.63	1.56	11.7	1.50	.73	2.06
	120-180	11.9	1.28	1.53	1.27	11.5	1.35	.63	1.50
	60-180	11.5	1.33	.68	1.42	11.2	1.44	.68	1.77
32 oz. Quart Jar	60-180	13.3	1.38	.68	1.86	13.25	1.38	.63	1.85

TABLE 4.8.--Location of maximum f and j and their corresponding j or f for sucrose in various containers and various processing conditions

Container	Temperature °F	Maximum f				Maximum j			
		f,min.	j	height in	t_a,min.	f,min.	j	height in	t_a,min
8 oz. Cylinder Jar	60-120	11.7	1.41	.93	1.75	11.6	1.47	.63	1.94
	120-180	11.65	1.26	1.03	1.17	11.5	1.32	.68	1.39
	60-180	11.15	1.20	1.23	0.88	10.85	1.38	.63	1.52
303 Vegetable Jar	60-120	13.65	1.36	1.13	1.82	13.5	1.44	.63	2.14
	120-180	13.8	1.21	1.13	1.14	13.5	1.34	.63	1.72
	60-180	13.15	1.18	1.13	0.95	12.8	1.34	.63	1.63
303 Can	60-120	8.5	1.39	.98	1.22	8.45	1.47	.63	1.41
	120-180	8.0	1.33	.68	0.99	8.0	1.34	.58	1.02
	60-180	7.3	1.18	.63	0.52	7.1	1.34	.43	0.90
2-1/2 Jar	60-120	17.55	1.40	.88	2.56	17.5	1.41	.68	2.61
	120-180	17.90	1.18	1.13	1.29	17.7	1.28	.63	1.89
	60-180	16.65	1.22	1.28	1.43	16.6	1.38	.63	2.32
32 oz. Quart Jar	60-180	17.3	1.37	1.13	2.37	16.95	1.39	.63	2.42

TABLE 4.9.--Summary of effect of variables on convection heating as measured by the parameters f and j

Variable	Effect on f	Effect on f log j	Effect on j
Increasing diameter	Increase	Increase[1]	Increase or decrease
Increasing height	Slight increase[2]	Increase[1]	Increase[1]
Increasing viscosity	Increase	Increase	Increase or decrease
Increase T_1 constant ΔT	Decrease[2]	Decrease	Decrease
Increased ΔT constant T_o	Decrease	Decrease	Decrease
Constant T_1	Decrease[4]	Increase or decrease[5]	Increase or decrease[5]
Tin to Glass	Increase	Increase	Increase or decrease
Increased h_o	Decrease	Increase or decrease	Increase
Decrease x/L	Increase	Increase	Increase
particles Large	Decrease	Decrease	Decrease
Small	Increase	Increase	Increase

---

[1] Opposite effect--attributable to glass thickness-when comparing round or vegetable jars with cylinder jars.

[2] With some small exceptions.

[3] Except 16 oz. vegetable and sucrose in 28 oz. veg. jar.

[4] McAdams h estimates predict increase but Eckert h stimates predict decrease.

[5] Except 16 oz. containers for sucrose.

TABLE 4.10.--The effects of increase in level of treatment upon the heating rate parameter f

Part I. Increase in temperature level, i.e., $f_{120-180} - f_{60 \text{ to } 120°}$

Other Treatments		Height above container bottom, in.					
		1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	8 oz.	-	-	-	+	-	++
	16 oz.	+	+	+	-	0	++
	303 can	+	-	-	-	-	-
	22 oz.	+	-	-	-	-	-
	28 oz.	+	-	+	-	-	+
50% Sucrose	8 oz.	+	-	-	+	+	++
	16 oz.	-	+	+	+	+	++
	303 can	+	-	-	-	-	-
	28 oz.	+	+	+	+	+	+

Part II. Increase in product viscosity from water to sucrose.

8 oz.	60-120	+	+	+	+	+	++
	120-180	+	+	+	+	+	++
	160-180	+	+	+	+	-	+
16 oz.	60-120	+	+	+	+	+	-+
	120-180	+	+	+	+	+	-+
	60-180	+	+	+	+	+	--
303 can	60-120	+	+	+	+	+	+
	120-180	+	+	+	+	+	+
	60-180	+	+	+	+	+	+
28 oz.	60-120	+	+	+	+	+	++
	120-180	+	+	+	+	+	++
	60-180	+	+	+	+	+	++
32 oz.	60-180	+	+	+	+	+	+

Part III. Increase in temperature difference for constant T_o

Water	8 oz.	-	-	-	-	0	++
	16 oz.	-	-	-	-	-	+-
	303 can	-	-	-	-	-	-
	22 oz.	-	-	-	-	-	-
	28 oz.	-	-	-	-	-	++
50% Sucrose	8 oz.	-	-	-	+	+	++
	16 oz.	-	-	-	+	+	--
	303 can	-	-	-	-	-	-
	28 oz.	-	-	-	0	+	-+

TABLE 4.10--Continued

Part IV. Increase in temperature difference for constant T_1

Other Treatments			Height above container bottom, in.					
			1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
	Water	8 oz.	-	-	-	-	+	++
		16 oz.	-	-	-	-	-	-+
		303 can	-	-	-	-	-	+
		22 oz.	-	-	-	-	-	-
		28 oz.	-	-	-	-	-	-
	50% Sucrose	8 oz.	-	-	-	-	0	++
		16 oz.	-	-	-	-	-	-
		303 can	-	-	-	-	0	-
		28 oz.	-	-	-	-	-	-
Part V. Increase in container diameter for nearly constant height								
Water	16-8 oz.	60-120	+	+	+	+	+	++
		120-180	+	+	+	+	+	+-
		60-180	+	+	+	+	+	++
	28-16 oz.	60-120	+	+	+	+	+	++
		120-180	+	+	+	+	+	++
		60-180	+	+	+	+	+	++
50% Sucrose	16-8 oz.	60-120	+	+	+	+	+	++
		120-180	+	+	+	+	+	+-
		60-180	+	+	+	+	+	--
	28-16 oz.	60-120	+	+	+	+	+	++
		120-180	+	+	+	+	+	++
		60-180	+	+	+	+	+	++
Part VI. Increase in container height; diameter variable								
Water	22-8 oz.	60-120	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	22-16 oz.	60-180	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	28-22 oz.	60-120	+	+	+	+	+	-
		120-180	+	+	+	+	+	-
		60-180	+	+	+	+	+	-
Water	32-16 oz.	60-180	+	+	+	+	+	+
	32-22 oz.	60-180	+	+	+	+	+	+
	32-28 oz.	60-180	-	-	-	-	+	+
50% Sucrose	32-8 oz.	60-180	+	+	+	+	+	+
	32-16 oz.	60-180	+	+	+	+	+	+
	32-28 oz.	60-180	+	+	+	0	0	+

TABLE 4.11.--The effects of increase in level of treatment upon the lag factor j, i.e., $(T_1-T_a)(T_1-T_o)$

Part I. Increase in temperature level

Other Treatments		Height above container bottom, in.					
		1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	8 oz.	-	-	-	-	+	+
	16 oz.	-	-	-	-	+	-
	303 can	-	-	-	-	+	-
	22 oz.	-	-	-	-	-	-
	28 oz.	-	-	-	-	0	+
50% Sucrose	8 oz.	-	-	-	-	-	+
	16 oz.	0	-	-	-	-	-
	303 can	+	-	-	-	+	-
	28 oz.	-	-	-	-	-	-
Part II. Increase in product viscosity from water to sucrose.							
8 oz.	60-120	+	-	+	+	+	+-
	120-180	+	-	+	-	-	+-
	60-180	-	+	0	-	-	--
16 oz.	60-120	+	+	+	+	+	--
	120-180	+	+	+	+	-	+-
	60-180	-	+	-	-	-	0-
303 can	60-120	-	0	+	+	+	-
	120-180	+	+	+	+	+	-
	60-180	+	+	-	-	-	+
28 oz.	60-120	+	+	+	+	-	+-
	120-180	+	+	+	-	-	+-
	60-180	+	+	+	+	-	+-
32 oz.	60-180	0	+	-	-	-	-
Part III. Increase in temperature difference for constant T_o							
Water	8 oz.	+	0	-	-	-	-
	16 oz.	+	+	0	+	-	-
	303 can	-	-	-	-	-	-
	22 oz.	0	-	-	-	-	-
	28 oz.	+	-	-	-	-	-
50% Sucrose	8 oz.	+	-	-	-	-	-
	16 oz.	+	-	-	-	-	-
	303 can	+	-	-	-	-	-
	28 oz.	+	-	-	-	-	-
Part IV. Increase in temperature difference for constant T_1							
Water	8 oz.	+	+	+	-	-	--
	16 oz.	+	+	+	+	-	++
	303 can	-	+	+	+	-	-
	22 oz.	+	+	+	+	+	-
	28 oz.	+	+	+	+	+	-
50% Sucrose	8 oz.	+	+	-	-	-	-
	16 oz.	+	0	-	-	-	+-
	303 can	+	-	-	-	-	-
	18 oz.	+	+	+	+	-	+-

TABLE 4.11--Continued

Part V. Increase in container diameter for nearly constant height

Other Treatments			Height above container bottom, in					
			1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	16-8 oz.	60-120	-	-	-	-	-	++
		120-180	-	-	-	-	0	-+
		60-180	-	-	-	-	+	++
	28-16 oz.	60-120	+	+	+	+	+	-
		120-180	-	+	0	+	-	+
		60-180	-	-	+	0	+	-
50% Sucrose	16-8 oz.	60-120	-	+	+	+	-	-+
		120-180	+	+	-	-	-	++
		60-180	-	-	-	-	+	++
	28-16 oz.	60-120	+	-	+	+	+	+
		120-180	-	-	+	+	+	+
		60-180	+	+	+	+	+	+
Part VI. Increase in container height; diameter variable								
Water	22-8 oz.	60-120	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	22-16 oz	60-120	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	28-22 oz.	60-120	-	-	-	-	-	-
		120-180	-	-	-	-	-	-
		60-180	-	-	-	-	-	-
Water	32-16 oz.	60-180	+	+	+	+	+	+
	32-22 oz.	60-180	-	-	-	-	-	-
	32-28 oz.	60-180	+	+	+	+	+	+
50% Sucrose	32-8 oz.	60-180	+	0	+	+	+	+
	32-16 oz.	60-180	+	+	+	+	+	+
	32-28 oz.	60-180	-	+	0	+	+	-
Part VII. Increase in container resistance and capacitance								
Water	16-303	60-120	-	-	-	-	+	+
		120-180	-	-	+	+	+	+
		60-180	+	+	-	+	+	+
50% Sucrose	16-303	60-120	+	-	+	+	+	+
		120-180	-	0	-	0	+	+
		60-180	-	+	0	+	-	+

TABLE 4.12.--The effects of increase in level of treatment upon the lag time, $t_a = f \log j$

Part I. Increase in temperature level

Other Treatments		Height above container bottom, in.					
		1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	8 oz.	-	-	-	-	+	--
	16 oz.	-	-	-	-	+	--
	303 can	-	-	-	-	+	-
	22 oz.	-	-	-	-	-	-
	28 oz.	-	-	-	-	+	--
50% Sucrose	8 oz.	-	-	-	-	-	-
	16 oz.	-	-	-	-	-	-
	303 can	-	-	-	-	+	-
	28 oz.	-	-	-	-	-	--
Part II. Increase in product viscosity from water to sucrose							
8 oz.	60-120	+	+	+	+	+	--
	120-180	+	+	+	+	-	+-
	60-180	+	+	+	-	-	+-
16 oz.	60-120	+	+	+	+	-	--
	120-180	+	+	+	+	-	+-
	60-180	+	+	+	-	-	+-
303 can	60-120	-	+	+	+	-	-
	120-180	+	+	+	+	+	-
	60-180	+	+	+	-	-	+
28 oz.	60-120	+	+	+	+	-	+-
	120-180	+	+	-	-	-	+-
	60-180	+	+	+	+	-	+-
32 oz.	60-180	+	+	+	+		
Part III. Increase in temperature difference for constant T_0							
Water	8 oz.	+	-	-	-	-	--
	16 oz.	+	-	-	+	-	--
	303 can	-	-	-	-	-	--
	22 oz.	-	-	-	-	-	-
	28 oz.	+	-	-	-	-	--
50% Sucrose	8 oz.	+	-	-	-	-	--
	16 oz.	-	-	-	-	-	+-
	303 can	+	-	-	-	-	-
	28 oz.	+	-	-	-	-	+-
Part IV. Increase in temperature difference for constant T_1							
Water	8 oz.	+	+	+	-	-	--
	16 oz.	+	+	+	+	-	+-
	303 can	-	+	+	+	-	-
	22 oz.	+	+	+	+	+	-
	28 oz.	+	+	+	+	-	--
50% Sucrose	8 oz.	+	+	-	-	-	--
	16 oz.	-	-	-	-	-	++
	303 can	+	-	-	-	-	+
	28 oz.	+	+	+	+	-	+-

TABLE 4.12--Continued

Part V. Increase in container diameter for nearly constant height

Other Treatments			Height above container bottom, in.					
			1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	16-8 oz.	60-120	+	+	-	-	-	++
		120-180	+	-	-	-	-	--
		60-180	+	+	+	+	-	++
	28-16 oz.	60-120	+	+	+	+	+	--
		120-180	+	+	+	+	-	+-
		60-180	+	+	+	+	+	--
50% Sucrose	16-8 oz.	60-120	+	+	+	-	-	-+
		120-180	+	+	+	-	-	-+
		60-180	+	+	-	-	+	++
	28-16 oz.	60-120	+	+	+	+	-	0-
		120-180	+	+	+	+	-	+-
		60-180	+	+	+	+	-	+-

Part VI. Increase in container height; diameter variable

Water	22-8 oz.	60-120	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	22-16 oz.	60-120	+	+	+	+	+	+
		120-180	+	+	+	+	+	+
		60-180	+	+	+	+	+	+
	28-22 oz.	60-120	-	-	-	-	-	-
		120-180	-	-	-	-	-	-
		60-180	-	-	-	-	-	-
Water	32-16 oz.	60-180	+	+	+	+	+	-
	32-22 oz.	60-180	+	+	+	+	+	-
	32-28 oz.	60-180	+	+	+	+	+	+
50% Sucrose	32-8 oz.	60-180	+	+	+	+	+	+
	32-16 oz.	60-180	+	+	+	+	+	-
	32-28 oz.	60-180	+	+	+	+	+	-

Water	16-303	60-120	+	+	+	+	-	+
		120-180	+	+	+	+	-	-
		60-180	+	+	+	+	-	+
50% Sucrose	16-303	60-120	+	+	+	+	-	+
		120-180	+	+	+	+	-	+
		60-180	+	+	+	+	-	+

The experimental f values were increased by the use of products of higher viscosity, but differences in viscosity due to temperature level were sometimes obscured by relatively small increases in the exterior film coefficient, particularly in the 16 and 28 oz. vegetable jars. Increases in temperature differences with the same initial temperature (but with a 12 percent decrease in h) or with the same final temperature (and about 2-1/2 percent increase in h) decreased f. Decreases in the exterior film coefficient increased f. Increased diameter for a constant height increased f. The f for both the 22 oz. cylinder and 32 oz. increased over those of larger and smaller diameter but smaller height. However, the f of the 22 oz. container was larger than the f for the 32 oz. This may be due to thicker glass walls or constrained boundary layer flow for they are nearly the same height.

Increases in product viscosity and reduction in temperature level increased both j and f•log j. An increase in temperature difference at the same initial temperature (and about a 12 percent reduction in exterior film coefficient) reduced j and f•log j. An increased temperature difference but at the same final temperature (but with only about a 2-1/2 percent reduction in film coefficient) reduced j and f•log j only for sucrose in 303 vegetable jars and 303 cans.

Comparisons of Heat Penetration Data With Quasi-Steady-State Newtonian Model Estimates

The relations recommended by McAdams for natural convection film coefficients on vertical and horizontal flat plates (see Table 2.1 and Table 2.2) were incorporated in the proposed Newtonian heating model (Equation 2.48a). The measurements of container wall thickness from Table 4.6 and external film coefficients from Table 4.7 were used in order to estimate (1) the slow point position (using the heat balance hypothesized in Section II) and (2) the heat penetration rate parameter f. The driving force represented by initial temperature at the cold point, a constant uniform external resistance from Table 4.7, the internal area, the steady-state wall temperature gradient, properties evaluated at the arithmetic mean fluid temperature, i.e., $(T_S + T)/2$, the total heat capacitance of the container and the internal fluid area were used in these calculations. In addition the side of a square of equivalent area (and equivalent perimeter per unit area) was used in the calculation of the film coefficient on the bottom of the container. These overall U values were usually determined to within one percent after three oscillatory convergent iterations.

The slow point position estimates are contrasted with observed positions in Figure 4.19.

The estimates of f values are contrasted with experimental observations in Figure 4.20.

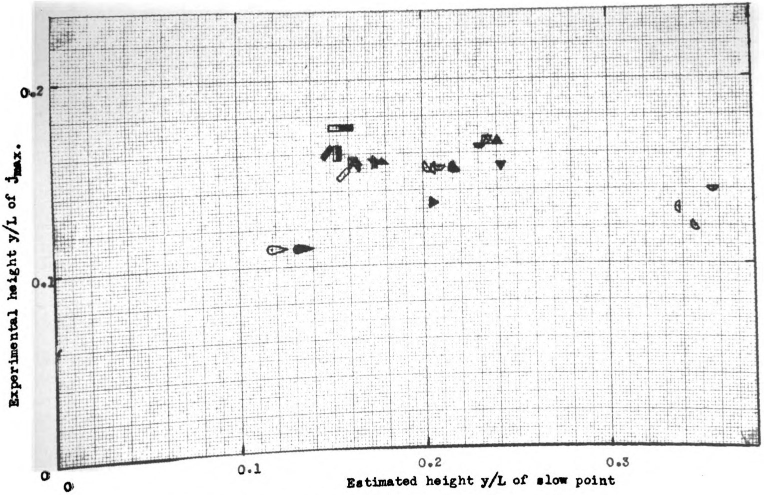


FIGURE 4.19 Scatter diagram of observed and predicted slow point position

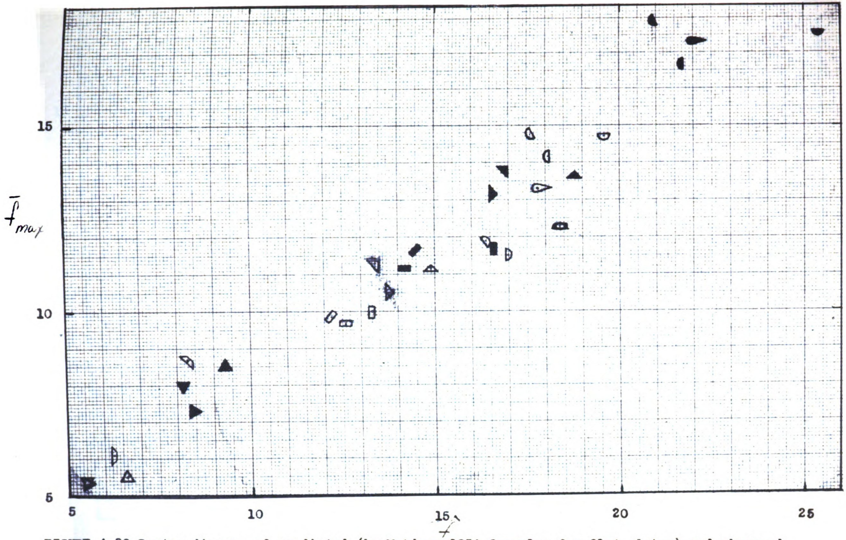


FIGURE 4.20 Scatter diagram of predicted (by McAdams,1954 formulae for flat plates) and observed f values.

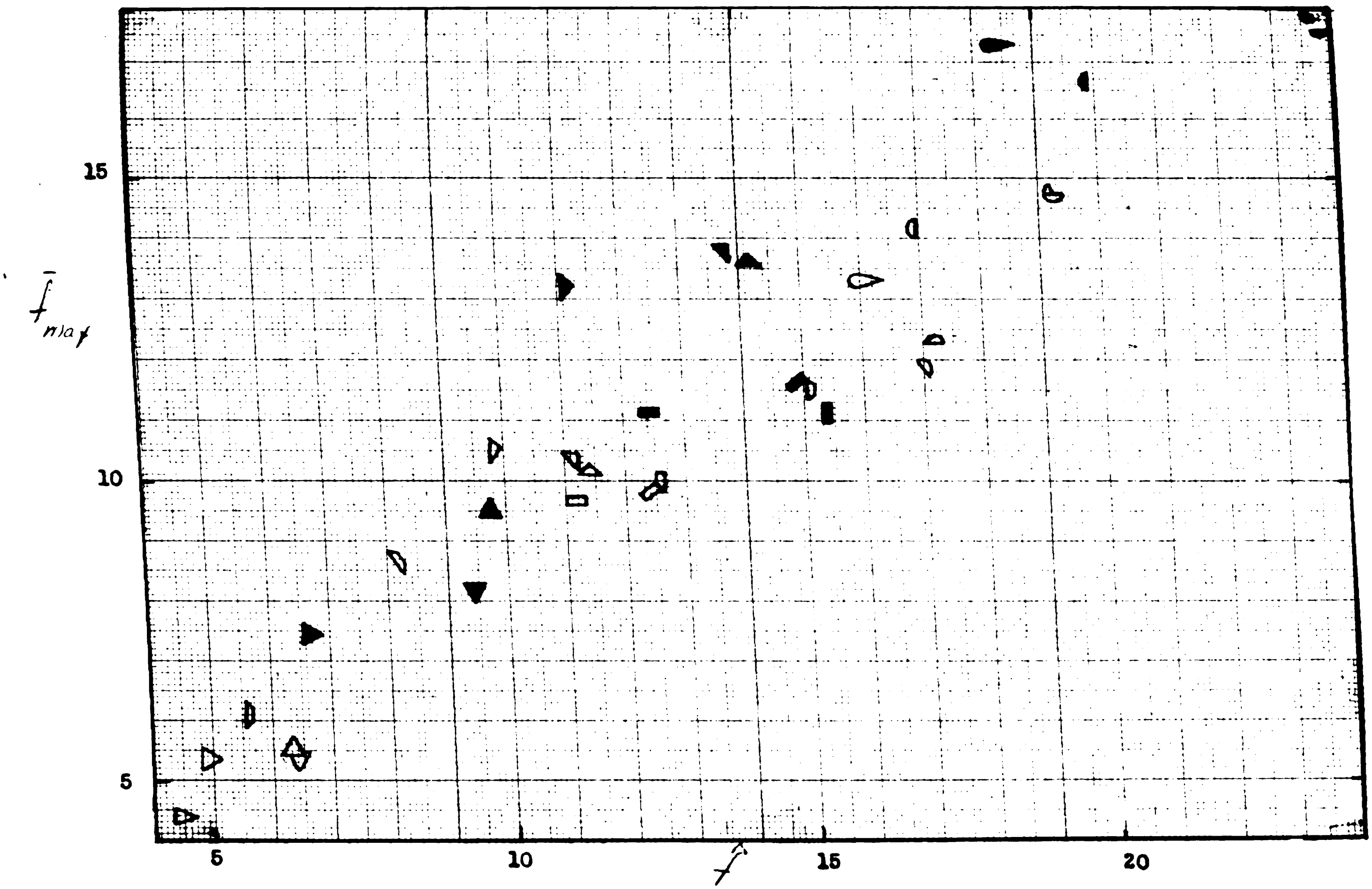


FIGURE 4.20b Scatter diagram of predicted (by McAdams,1954 formula for horizontal flat plates and Eckert & Drake,1959 formula for vertical plates) and observed f values.

SECTION V

DISCUSSION OF RESULTS

Part I. Possible Causes for Discrepancies Between Predicted and Experimental Results

There are many possible causes for discrepancies between the various models developed and discussed in Section II and the results as presented in Section IV. Foremost among these are the experimental errors themselves. There are other errors, however, due to the assumptions employed either in the experimentation or in the models. This section will be limited to a discussion of the errors of the particular system investigated. A discussion of the overall problem is left to the following section.

Experimental Errors

Errors in (T_1-T)

The recording potentiometer and thermocouples at various points in the container in a container and heating medium are the principal research tools available for the study of temperature histories (and corresponding heat transfer by convection of fluid). This system and the similar system which is used to control T_1 are responsible therefor for many of the errors in observation. If T_x is the assumed retort temperature, it can be shown that at long times:

$$\log \hat{j} = \log \frac{T-T_x}{T_0-T_1} + \frac{t}{f} \quad \text{and} \quad \frac{(\hat{f}-f)}{f} = \frac{T_1-T_x}{T-T_1}$$

where T is the temperature at time t, T_0 is the uniform initial temperature, and T_1 is the true retort temperature.

The drawing of heat penetration curves is a subjective skill, representing a compromise between the desire to use data at long times to assure the validity of the use of the first term approximation and to obtain a good estimate of the f near the process temperature T_1 for use in calculation of equivalent process times and the desire to avoid the measurement errors which increase in magnitude as T approaches T_1.

Townsend <u>et al</u>. (1956) have recommended that an investigator "draw a straight line through the points, ignoring the lag period and attempting to place the line not more than $1^{\circ}F$ from any point above 200-$210^{\circ}F$ for low acid products, and $140^{\circ}F$ for acid products." They "noted that a comparatively large variation in the position of the line when it nears retort temperature is much less significant in terms of temperature difference than a comparatively small variation at low temperatures." These appear to be workable criteria for drawing.

Townsend <u>et al.'s</u> (1956) criteria however, did not include a consideration of an upper limit beyond which errors in (T_1-T) might become excessive. The effect of these errors will be examined before the sources of these errors, and the control measures which were used in this investigation. The selection of the heating medium temperature is critical as is illustrated in Figures 5.1, 5.2 and 5.3 for the data given in Table 5.1. In Figure 5.1 the data were plotted assuming a retort temperature of $252^{\circ}F$. In Figure

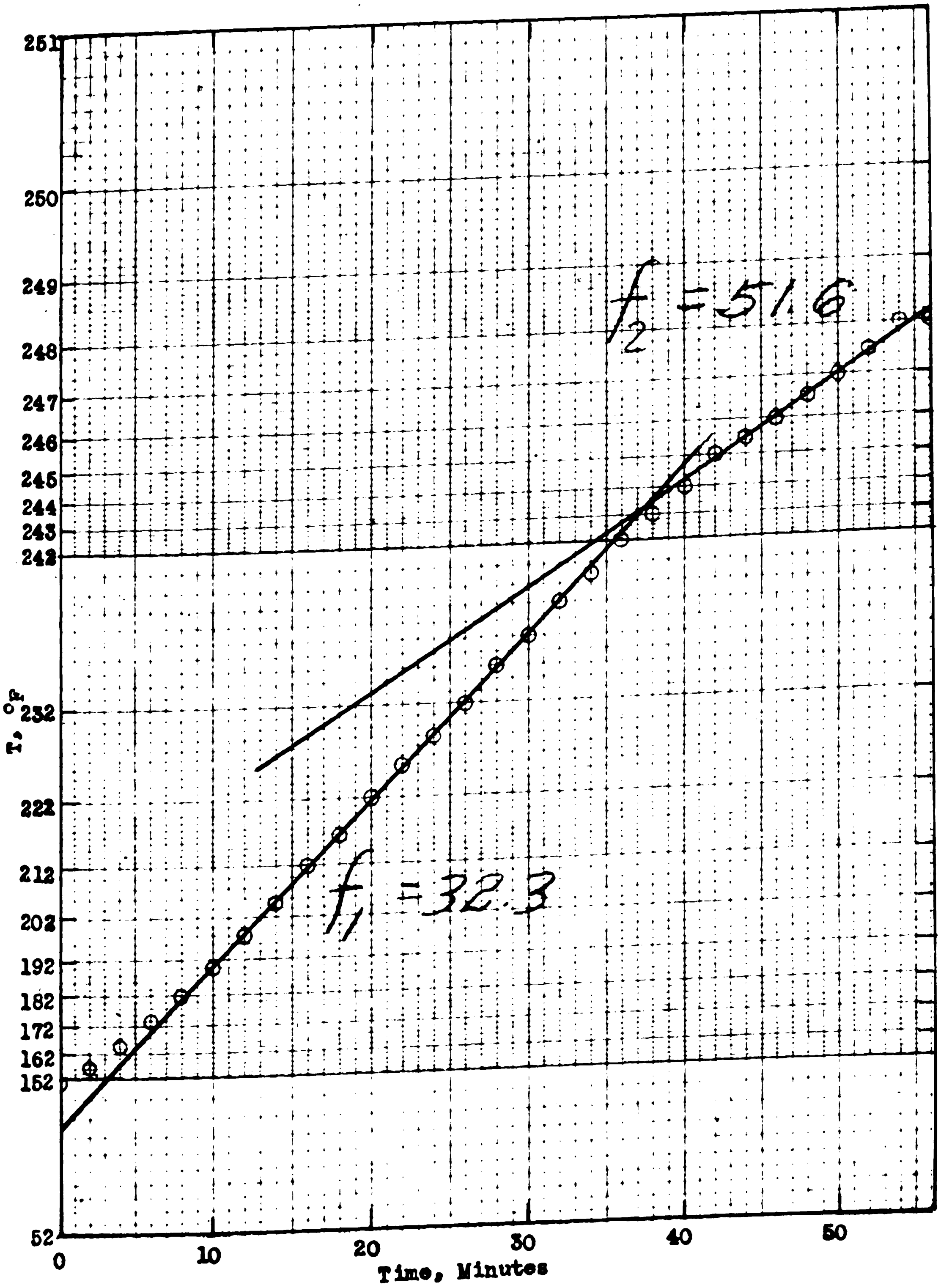


FIGURE 5.1 Heat penetration curve for data of Table 5.1 and T_1 of 252°F.

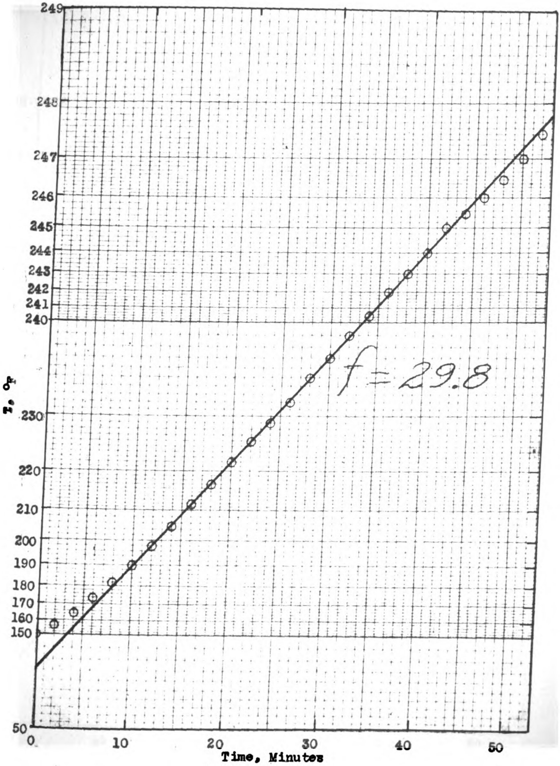


FIGURE 5.2 Heat penetration curve for data of Table 5.1 and T_1 of 250.

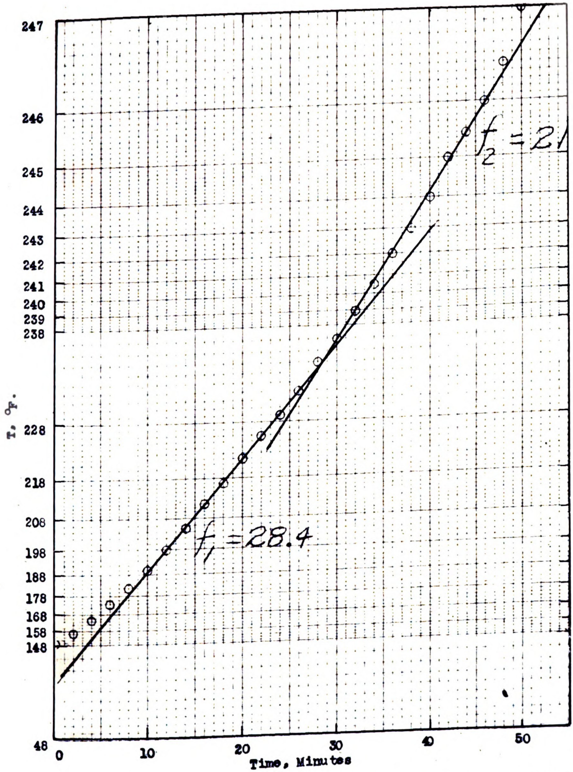


FIGURE 5.3 Heat penetration curve for data of Table 5.1 and T_1 of 248°F.

Table 5./ . Heating data

Time, min.	Temperature, °F	Time, min.	Temperature, °F
0	150	32	239
2	157	34	240.5
4	165	36	242
6	174	38	243
8	182	40	244
10	190	42	245
12	198	44	245.5
14	205	46	246
16	212	48	246.5
18	217	50	247
20	222	52	247.5
22	226	54	248
24	229	56	248
26	232	58	248.5
28	235	60	248.5
30	237	62	249

5.2, the data were plotted using a retort temperature of $250^{\circ}F$, and in Figure 5.3, the data were plotted using a retort temperature of $248^{\circ}F$. The variation in f and j and the presence of broken heating curves in some of these plots is an example of what can occur. With conduction heating products, this problem is much simpler than with convection heating products. With conduction heating products, it is approprate to make minor adjustments in the heating medium temperature until the curve becomes a straight line. Since convection heating curves are not straight lines, (see Table 4.1) the problem is much more difficult and requires a great deal more experience in properly treating this type of data.

Instrument-thermocouple errors.--Inadequate sensitivity of the instrument-thermocouple system produced by either a long, fine thermocouple of low electrical (and thermal) conductivity, or lack of instrument sensitivity, may result in obvious stair-like time-temperature data in the region close to processing temperature.

The errors in accuracy of the instrument-thermocouple system are more serious, however. These affect the heat penetration curve in much the same manner as an error in positioning the thermocouple in the heating fluid, since in both instances the effect is the same as an error in retort temperature. Copper-constantan thermocouples normally will be within plus or minus $0.75^{\circ}F$. However, measurement of temperatures inside a retort or other processing equipment is carried on under rather adverse conditions. Manual potentiometers are, in general, subject to fewer errors and problems than are automatic potentiometers such as those used in this investigation for the simple reason that a second emf system, the line voltage, is brought into use in the automatic potentiometer.

Errors in T_1.--The selection of the proper retort temperature to use in this data plotting procedure is not a simple matter. In general, there are 3 or 4 different temperatures that are available to use as the retort temperature. These are the temperature on the recording temperature control, the temperature of the indicating thermometer on the retort, the temperature of thermocouples in the retort and the temperature of thermocouples inside the containers that have been allowed to equilibrate with the heating media. It must be recognized, however, that the thermocouple inside the container is under a different set of physical conditions than the corresponding thermocouple outside the container. This situation can be severe in steam atmospheres; however, with water processing of glass containers it can be a nightmare. In the case of glass containers processed under water, the thermocouple in the water is in one electrolyte system whereas the thermocouple in the container is in a second electrolyte system with limited electrical paths between these two electrolytes. One path may be from failures in the insulation of the thermocouple wires when they pass through the heating medium to the heating media. Such failures in essence may create a situation not unlike that of a battery that will generate very small emf's that will either add or subtract from the final measured variable.

Perhaps the only way to be absolutely sure of the heating medium temperature to use with a given container is to heat the container sufficiently long to allow it to equilibrate and use this equilibration temperature as the heating medium temperature.

This is not free from errors because it is not uncommon for the retort temperature to drift upward during the course of the process so that the final temperature may be 1 or 2 degrees higher than the temperature during that portion of the heating period when most of the heat is transferred. (This may occur due to design of equipment with supply pressures and control valves adequate for the peak load which occurs during come-up. After this period the equipment capacity is excessive so that leakage through the control valves alone may be sufficient to cause this drift.)

It is desirable to control the processing temperature within ± 1/4°F as recommended by Townsend <u>et al</u>. (1956), however this is not always possible and it is increasingly more difficult to control the temperature in water baths, cooling canals, steam-air atmospheric pasteurizers, than in steam retorts. It is necessary to employ some method of correcting data when such changes in heating medium temperature occur. The analytical approaches of Thompson (1919, 1920), Riedel (1947), or Gillespy (1953) for a linear change in an otherwise steady environment temperature, or to Gillespy's solution for exponential come-up can be used for conduction heating products. For convection heating products a drift in retort temperature might be anticipated by using an equilibrium effective retort temperature. The equilibrium temperature can perhaps be corrected for this drift by observing the amount of drift that may have occurred in the temperatures in the heating media.

Control measures.--Accurate frequent calibration of the instrument thermocouple system and of thermocouples with one another, placement of the external fluid temperature thermocouple close to the container and precise control of retort temperature are control measures which may be used to reduce the magnitude of these errors.

In this experiment the equilibrium temperature of the thermocouple was compared to that indicated by the bath and the correction was applied to the mean apparent heating temperature during the period which was to be used for estimation of the parameters f and j. This stratagem was in addition to the use of a calibrated potentiometer-thermocouple system and thermocouple wire from the same lot.

These and other temperature measurement errors have been discussed intensively by Baker, Ryder and Baker (1953, 1961) and Dahl (1962) among others. One of these errors, spurious signals due to voltaic emf's at the thermocouple, in the container, or in the heating equipment or electrical leakage not compensated for, due to moderate resistance grounds, has appeared repeatedly in this investigation (particularly with the sensitive millivolt potentiometer) and in the field investigations of Pflug (1962). These errors appeared in spite of varnishing of thermocouples, junctions, using wire whose insulation was not frayed, and normal precautions of grounding of instruments, their cases, and process equipment. It is now the practice here to use common electrical circuits wherever possible and to take particular care to ground all equipment and the container to a common low resistance ground with low resistance straps and contacts. When working on wet floors it sometimes has been necessary to wear rubber footwear to avoid grounding the apparatus to other equipment or to floor drains. Another control measure is in-place calibration of the thermocouples; the use of equilibrium temperatures is in effect a calibration. Here we are following the practice of not using data closer than 5°F to the retort temperature. If Townsend's criteria of control, i.e., ± 0.25°F could be attained at five degrees below processing temperature in commercial equipment the relative error in f would be $(0.25^\circ)/(5^\circ F)$ or 5 percent, and if the control is ±1.0°F, which seems good for the data reported in the literature, the error is ± 20 percent. It is difficult to assess the magnitude of the errors as they occurred during this experiment. Consider the 50° to 350°F instrument used in this experiment: the sensitivity is ±0.2°F; the

estimated operational difference between thermocouples is ±0.4°F; and the control achieved within a test (using the 180°F waterbath) with the additional use of the equilibrium temperature is estimated to be about ±0.4°F. The estimated possible error due to the above for this system is therefore approximately ± 20 percent.

Thermocouple Induced Errors

The thermocouples introduce three errors in addition to those of calibration; these include:

1) A change in the velocity profile, and poorer convection heat transfer due to the increase in shear area, and reduction in core flow but an increased heat transfer area due to a slightly higher fluid height.
2) A lag in the apparent temperature due to the resistance, and small capacitance of the thermocouple.
3) A conduction error, generally causing a lead in apparent temperature, due to conduction of heat by the wire from the bath to the junction.

The velocity error on the overall pattern is presumed small since the additional vertical wall shear area is proportional to the ratio of the diameters of the thermocouple and the container, that is, 0.25/2.00 to 0.25/4 or six to twelve percent. This flow restriction of $(0.25/D)^2$ moreover is introduced in the center where the velocity is about one eleventh of that near the container wall.

The errors in heat penetration measurements in conduction heating products due to his lead wires and fittings have been estimated by Ecklund (1949, 1956). Cowell and co-workers (1959) investigated the effects of bare thermocouple wire of various diameters on the heating parameters f and j in comparison with a mathematical model proposed by Jaeger (1955). Corresponding errors for convective heating products do not appear to have been determined prior to this investigation. One approximation to the error is to treat the product as a conduction heating product with the observed "effective" thermal diffusivity. A second approximation used here is to assume that the fluid is a Jones-Olson + Schultz-Newtonian heating model and estimate the thermocouple lags using a procedure similar to that shown in Eckmann (1958) for a thermocouple in a well.

The f for various thermocouple assemblies installed in 16 oz. jar lids were determined by direct immersion in the 180° bath but without the usual mechanical agitation. The f's were on the order of 0.10; 0.10 and 0.60; 0.10 and 1.25; 0.10 and 0.60; min. for 24 gauge fiberglass thermocouple wire vertical multipoint rod, bakelite rod (conduction), and fiber rod (convection) thermocouple units. Only the second of the two constants f (the first corresponds to that for bare wire before heat is lost to the body of the rod) for the rods will be considered here.

The results of the effect of a 0.10 min., 0.60 min.

Table 5.2 . Errors due to resistance and capacitance of thermocouple elements.

True f of product in container, minutes	Percent relative error in f		
	f of thermocouple element alone, minutes		
	0.10	0.60	1.25
3	<<0.017	1.24	1.69
6	do	1.11	1.26
9	do	1.07	1.16
12	do	1.05	1.12
15	do	1.04	1.09
30	do	1.02	1.04

and 1.25 min. f couples on apparent $\hat{f}$ for containers having f of 3, 6, 9, 12, 15 and 30 minutes (but j-values of 1.0) are shown in Table 5.2. The corresponding j-values are of the order of 1.0.

Investigations of thermocouple errors suggest matching of thermal properties of the product and thermocouple probe to minimize error. Probes for convection heating should be highly conductive (about 24 times that of water--this is the effective thermal diffusivity as determined by Tani ,1940),be small in diameter--not only to minimize interference with product but minimum mass (lag) and cross sectioned area (heat gain from the environment), have a sufficiently long immersion length in an iso-j zone. and be inserted axially into the central core to reduce interference with flow. The multi-point rods used in this investigation are an attempt to meet these criteria but with compact but relatively rugged assemblies. The wrapping of the bare thermocouple wire which emerged from the rod halfway around the rod before making a junction achieved an immersion length of about 40 wire diameters. The immersion length of the horizontal multi-point rod was about 1/3 as great but the 0.10 in. diameter supports were in a region of temperature very close to that of the junction. Additional immersion length can be achieved with the sacrifice of flexibility or rigidity. A single fine bare wire inserted either through the top or through the sides of the container should be supported in order to assure a known position, but

fastening it to opposite sides (very difficult for glass containers) means that the thermocouples can only be used in that one container. The additional wires necessary for within-container comparisons substantially add to the complexity of the measurement system. A frame rather than the container itself might be used to support the thin wires (a scheme proposed by Sunderland, 1963). This frame could be used interchangeably in various sizes of glass containers by changing the closure size, but it is not readily apparent how the frame could be used in various cans. This later problem is solved for the vertical (but not the horizontal multipoint thermocouple rods) by inserting the probe into the opening of an Ecklund packing nut or other such adapter which is soldered onto the can lid after closing.

The Qualitative Nature of Flow Visualization

The qualitative nature of flow studies using chemical tracers or minute discreet particles, the two techniques which seemed most promising for study of the slow flows encountered in confined transient natural convection, was recognized in the design of the experiments. All three of the techniques employed, injection of dye droplets, layering of dye and suspensions of aluminum powder have their peculiar limitations however.

The injection of dye droplets had to be done with care in order not to impart an unwanted initial velocity to them. These droplets gradually faded due to diffusion. This is the reason that the core flow was not seen rejoining the boundary layer flow. This disadvantage could be overcome in future experimentation, however, by injection of dye droplets closer to the bottom and side of the container rather than only at the top center of the core as in this experiment.

This dye technique was deemed to be not suitable for study of flows at longer times since expansion forced the dye out of the needles continuously and the droplets from the top needles sank too rapidly and droplets from the bottom rose too rapidly due to their not being at quasi-equilibrium temperatures and injection velocities caused by this expansion. For these reasons no observations were made of the height of the lateral loops.

The layering technique used by Fagerson (1950) and in this experimentation was difficult to employ, for the diffusion process would begin immediately thus causing a dark background above the dye layer. The dye was all tooeasily mixed with the water during transfer to the heating bath. Since the dye rose on all sides and the core soon became colored it was difficult to assess radial extent of the somewhat darker boundary layer (darker because it was viewed on edge) even with a reference grid behind.

A satisfactory suspension of aluminum powder like that used by Tani (1940) was prepared, but here as in the layered dye technique the myriad of flashing flakelets obscured the motion of the particles in the boundary layer and a view from the top was necessary to determine the radial extent of the particular feature being viewed. It was not considered feasible to attempt two dimensional simultaneous photographs of droplet movement against a dark bottom layer or the movement of a particular particle among myriads of particles. (It was difficult to follow visually the motion of a particular particle even in the plastic side rectangular vessel in which there was little radial uncertainty.) Instead readily identifiable discreet colored droplets were injected then viewed radially (through a grid) and axially. In this way it was possible to assess radial and vertical position.

These techniques, however, are approximations of two different characteristics of the flow. Individual dye (here assumed not to diffuse) droplets and aluminum particles when photographed for brief intervals show the direction (the tangent to the recorded path) and the velocity (the length of the image relative to the exposure speed and the magnification of the camera system). The tangent to the path of a particle is the instantaneous direction of the stream lines. If the flow is steady, however, the stream lines no longer must be constructed from very short path lines but may be determined from long time exposures, or multiple short re-exposures, for the stream lines and paths of these particles are now coincident. Dye streams either from needles or from a layer at the bottom of the container are a measure not of the stream lines or of the path of particles but of the streak lines, that is the paths of fluid elements which have passed the point of the needle or from somewhere in the bottom layer. If the flow is steady the streak lines become coincident with the streamlines and the path lines.

The techniques employed here are therefore experimental approximations to the path and streak lines but have been used as approximate steady state stream lines in Figures 4.9 and 5.5 and in the diagrams of Tani (1940) and Ban and Kaziwara (1941). The photographs used by Fagerson (1950) and Fagerson and Esselen (1950) are more complex however: the assumption inherent in interpretation of these photographs is that the dye will be concentrated in the regions of greater fluid movement and that the approximate quasi-steady stream lines may be distinguished due to greater absorption by the boundary layer (when seen on edge) and in the core center as contrasted to the dark backgrounds.

Assumptions

The Assumption of Symmetry

A symmetrical flow and temperature distribution about the central axis has been assumed without question in previous investigations, due to geometric symmetry. There is some evidence within this experiment which indicates that although this is an excellent assumption, in practice it is not entirely correct. The irregular rate of rise of dye streams over the surface of the container when the layering technique of Fagerson (1950) was used, the observation that

glass thickness was not uniform either vertically or circumferentially, and the sensitivity of the metal cylinders used to evaluate the external film coefficients to position within the equipment all indicates an uneven flow pattern internally. The dye flow irregularity may be due in part to turbulence, as it might in the irregular flow pattern reported by Sparrow and Kaufman (1956) for free convection flow of water in a narrow vertical enclosure cooled at the top and open at the bottom to a heated reservoir. This latter investigation, the only one which the author has found which reports gross non-symmetric and irregular behavior, may be a special case of turbulent eddy flow (turbulent due to the tall height of the enclosure) in which the narrow apperture has restricted the formation of even turbulent eddy loops but in which the flow is not restrained laterally along the enclosure. Nevertheless the horizontal profiles were symmetric within the experimental time and temperature errors when the horizontal multipoint rod was placed vertically in the center of the container.

The Flow Regime

Film coefficients for the flow regime encountered in thermo-syphon tubes and between parallel horizontal plates have been summarized in a consideration of the engineering problem and the regimes themselves will be discussed in regard to the overall problem in Section VI. Since these flow regimes have been shown to have great influence on the phenomenon observed it is necessary to examine the presumed and apparent flow regimes in order to interpret other results.

The film coefficient computed by the McAdams (1954) formulae for vertical and horizontal plates using the assumptions outlined previously, are for Rayleigh numbers from 8×10^6 to 2.3×10^9, i.e., the laminar flow range for these formulae except for the lateral surface for water in 16 oz., i.e., 303 cans, in which the critical $N_{Ra} = 10^9$ is not exceeded within 20 to 30° of T_1 (with T_1 of 180°F) and very nearly this level, i.e., 0.7×10^9 in 16 oz. vegetable jars. (All the containers containing water and heated from 60 to 180°F had N_{Ra} which exceeded this level when the film coefficients were calculated using resistances corrected to the internal area and Eckert's relations for laminar flow on vertical flat plates.)

The investigations of Martin (1955) and Martin and Cohen (1954) of heat transfer in open thermo-syphon tubes with length to radius ratios ranging from 7.5 to 47.5 give little assistance in determining the probable flow regime in the jars used in this experiment, for from N_{Ra} based on container radius turbulent flow would be expected but on the basis of fluid height laminar flow would be expected. The transition between domains when based on radius have been indicated in Table 2.1. The transition on the height basis occurred at N_{Ra} from 2.4×10^{10} down to 8×10^9 as L/r decreased. Foster (1953) found transition occurring in thermo-syphon tubes at N_{Ra} ranging from 10^9 to

10^{10} as smaller diameter tubes or higher viscosity (and temperature coefficient of viscosity) fluids were used.

Calculations for the bottom film coefficient using the correlations of O'Toole and Silveston (1961) and the experimental coldpoint height as an estimate of the thickness of the confined horizontal layer (here insulated on the top) place the bottom flow in the turbulent region, i.e., $N_{Ra} > 10^5$ throughout the period in which observations were made. Calculations made assuming the effective height was lower--perhaps only half as high--might permit laminar eddy flow at the bottom late in the heating period for sucrose, however.

Turbulence--or at least random eddy motion--was detected in some instances. This behavior was detected during the flow visualization studies at the top of the side walls and to some extent in the bottom eddies during the initial phases of heating of water in 16 oz. vegetable jars with a 120° temperature difference. The Reynolds number for the dye droplet in the rising film was on the order of 1400. This is in excess of the N_{Ra} associated with turbulent flow in natural convection, (Eckert and Drake, 1959). Temperature fluctuations which correspond to irregular convective movement of fluid were observed initially at points near the bottom and near the top, i.e., near the discharge from the boundary layer. These fluctuations appeared to decrease in magnitude as the flow time away from these surfaces increased. These fluctuations also were noted to have disappeared before the cold point temperature had approached to within 20°F of the heating medium temperature. The shape of many of these fluctuations appear to be like those of Martin and Cohen (1954), i.e., a bell shaped curve skewed to the right, for turbulence in thermosyphon tubes (see Figure 4.7). Turbulence or lack of turbulence were not detectable in the tests which used the recording multipoint recording potentiometers since these instruments recorded the temperatures at discreet intervals (1 minute or 1.2 minutes). For this reason the temperature histories provide no information about the flow domain for other sizes or for sucrose solutions.

Transition to local turbulence at the top of the rising stream is enhanced by the inflection of the boundary (Schlichting, 1960, and Lin, 1955) due to discharge of fluid from the boundary layer into the core and local mixing of core-mixing such as reported by Hammitt (1958), Martin (1955), Martin and Cohen (1954), and Foster (1953). The transition all along this vertical boundary layer is retarded somewhat by the decrease in viscosity as the temperature increases. This effect does not appear to have been investigated experimentally however, (Schlichting, 1960, and Lin, 1955) but abundant evidence is available for the contrary effect observed in air, due to its increase in viscosity with increasing temperature. (See Gebhart, 1962, as well.) The viscosity of these fluids, among others, as a function of temperature is shown in Figure 5.4.

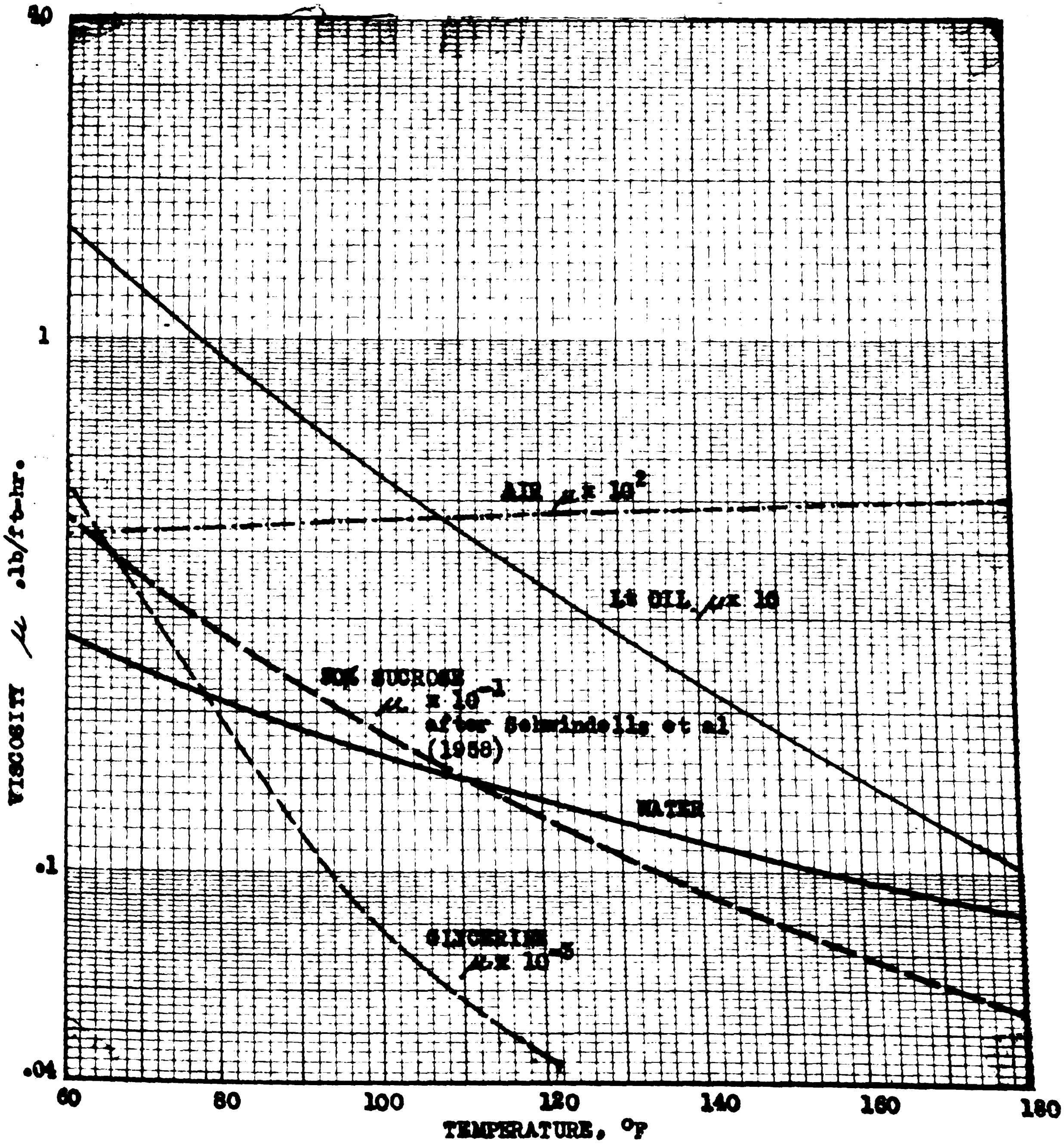


FIGURE 5.4 The temperature dependence of viscosity for some common fluids after Kreith (1958)

The cyclic behavior of the temperature at 1/8 in. above the bottom as point number 1 seen in Figures 4.1-4.4 and particularly in Figure 4.1 is attributed not to turbulence but to cycling of the bath temperature once the bath had recovered from the disturbances caused by adding cold containers. Variation between the apparent film coefficients (see Table 4.2 and Table 4.3) on a given cylinder within a treatment combination are nearly of the same magnitude as the difference between film coefficients with and without heat supply during a test, and may be confirmation of the influence of cycling of the bath temperature.

There was no indication from the motion of dye droplets that the central portion of the core was turbulent. Since turbulence deteriorated relatively soon in the heating period there is no major disagreement between these observations and those of Hammitt (1962), (p. 62) for steady state cooling of a larger vessel with internal generation but with slightly lower N_{Ra}, i.e. from 10^6 to 10^8. He had not observed turbulence in the boundary layer under conditions of laminar core flow. In addition Reynolds number calculated from the dye measurements is on the order of 10^2, well below the 2100 criteria near which turbulence in tubes may occur and hence on this basis might be presumed to be laminar. From this discussion it may be concluded that the boundary layer flow regime was generally laminar within 20 to 30°F of T_1 and the core flow was laminar throughout the heating periods investigated here. The flow at the bottom appeared

to be turbulent eddies up but not within 20 to 30°F of T_1. None of the systems previously investigated correspond well to the geometry of can and jars, however.

The Assumption of Quasi-Steady-State

Quasi-steady-state film coefficients have been computed and contrasted, with observed values through graphs of experimental versus predicted f values and in the evaluation of the film coefficient on the exterior of container-sized copper and aluminum cylinders. Is this a reasonable assumption however? The use of these steady-state coefficients for gases and for boundaries with negligible or appreciable capacitance has been reviewed briefly in an examination of the problem. Fluids in the food containers investigated here have opposite viscosity temperature coefficients to those for gases and are in contact with container surfaces which have moderate, not neglibible or appreciable thermal capacitance. These predictions can be used as a guide here however. Consider water in a 4 in. high container which has negligible capacitance or resistance. At about 65°F the $\beta = 1 \times 10^{-4}$ for water (for 50% sucrose it is about twice as large). According to the Sparrow and Gregg (1960) criteria as modified in Section II the heat transfer is steady-state within 5 percent when the f value is greater than 2.5, 3.1, 3.8, 5.1, 15.3, 30.6, and 306 for temperature differences of 120, 100, 80, 60, 20, 10, and 1°F respectively. The experimental f values for the most nearly comparable case, water in # 303 cans, were on the order of 5.4 and 8.0 for water and 50 percent sucrose respectively. If this criteria is

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appropriate the heat transfer would be quasi-steady up to within 22° and 75°F of T_1 for water and 50 percent sucrose respectively. According to the Siegel (1938) criteria the time required for the boundary layer to become two-dimensional for these same conditions but with a 100°F temperature difference are 5.3 and 5.5 seconds. The time required for the flow to cease being conduction is 1.9 seconds. These times would be increased by exterior resistances or capacitances however. These times will also be increased substantially by large Prandtl numbers (Goodman, 1962 and Sparrow and Gregg, 1958).

Other Assumptions

There are many other assumptions inherent in a quasi-steady-state Newtonian heating model; some of these have been introduced in an examination of the problem. These include (1) Negligible axial temperature gradient, (2) Negligible wall temperature gradient (both axial and longitudinal), (3) the use of constant exterior films coefficient and wall conductances, (4) the use of one temperature difference--the quasi-steady-state temperature difference between the wall and the fluid at the axial cold point--to estimate film coefficients and the arithmetic mean temperature to evaluate thermal property, (5) the assumption of no interaction between vertical and horizontal surfaces. Little or no information is available from the other investigation in order to evaluate the possible errors introduced by these assumptions however.

Part II. The General Flow Patterns

The flow patterns shown in Figures 4.10 and 4.11 confirm the gross flow patterns as reported by Bitting (1926-1927). Jackson and Olson (1940), Jackson (1940), Tani (1940), Fagerson (1950). The techniques of these several investigators were refined and used to supplement one another in order to examine the flow and corresponding temperature histories in greater detail.

From the calculations based on the Siegel (1958) formulae it can be seen that convection starts quickly. It also could be shown that a very small temperature difference between container wall and the bulk fluid temperature (see the lower limits for laminar flow in Tables 2.1 and 2.2) 0.01°F for water on a 4 in. vertical surface might be required in order to cause convection motion to start. The dye movement and the 0^+ °F isothermal shown in Figure 4.8 compare favorably for these estimates for negligible container-exterior film resistance and capacitance. The flow at the top had affected the temperature at 1/8 in. below the surface within 3 or 4 seconds. The dye droplet at the heel appeared to start moving upward about this same time. The flow at the bottom center (where the glass is thicker and near two heating surfaces as at the heel) appeared to start much later, but still within 6 to 8 seconds.

The flow patterns found in this investigation are not as simple as those described in most of the earlier investigations but neither are they as confused as in one widely reprinted artist's conception.

The rise and decline of the bottom eddies was comparable to the initial 38 percent x/L rise and the later 19 percent x/L relative height reported by Fagerson. Their random pattern had not been previously reported however. The decline of the height of these eddies as the lateral flow becomes established suggests that the position of this axial cold point is due to a balance of convective heat transfer from the sides and from the top of the container. This phenomenon could also be expected later but in the opposite direction as convection declined and conduction from the perpendicular to the direction of eddy movement became important.

The boundary layer formed on the side walls of the container resembles that found on flat plates (there is little curvature). The boundary layer appears to start to form near the top of the fluid as it does in thermosyphon tubes (Lighthill, 1953, Martin and Cohen, 1954, Martin, 1955) because the time required for the thermocouple at point # 9 appears to be equal to or less than the time required to induce a dye droplet at the heel to start moving upward. The mechanism for lateral movement of fluid from the boundary layer into the core before velocity pressures have been established has not been established. Pflug (1962) has suggested a probable mechanism, however. The fluid along the side wall expands upward as it is heated. This raises the fluid meniscus above the equilibrium level. The fluid then moves inward due to this slight gradient. The dye studies show however and the horizontal profiles at 1/8 and 5/8 in. below the fluid surface appears to confirm that only a small portion (the innermost portions of the boundary layer) rises to the top and spreads inward over the surface of the liquid as suggested by Jackson and Olson (1940). Instead a substantial portion appears to be deflected inward by container curvature and perhaps due to greater velocity pressures on the portion of the boundary layer nearer to the container wall. Bitting (1926-27) noted this discharge from the boundary layer as have Tani (1940) and Hammitt (1958).

Since sketches of fluid flow in a container are approximations of the quasi-steady stream lines the area between them represents flow tubes containing equal mass flow rates. The sketch published by Tani (1940) implies that there is a substantial established laminar profile in the control core as the flow area shown in the center is substantially smaller than that in the next flow tube (annulus). This type of velocity profile has also been proposed by Hamilton et al. (1954) and Hamilton and Lynch (1955). Most of the evidence including this investigation points to almost plug flow, i.e., a nearly flat velocity profile in the central core. (Jackson, 1940; Hammitt, 1958). There are small deviations from the flat velocity and temperature profile.

The variability of temperatures within the same container even when great care is taken (see Hammitt, 1958) have made it difficult to examine the core temperature profile in greater detail. In this investigation the upper and lower portion of the core (but not the center portion) were investigated with the multipoint horizontal profile thermocouple rod in order to compliment but not duplicate Fagerson's study (1950) of temperature profiles. Dye flows were observed however. Flow

visualization, Fagerson's movies and iso-t_a and iso-F_o contour maps (see Figure 6.9). Bitting's flow visualization and the radial temperature profiles of Hammitt all indicate that outside a thin rising boundary layer and a buffer or quiescent region there is a layer which is descending more quickly than the remainder of the core. In this layer the fluid is transported more rapidly and fixed particles in this area receive more equivalent minutes F_o than in the outer buffer zone or in the inner portion of the core. This layer appears to be nearer the surface, i.e., larger r/r_o for the No. 2-1/2 vegetable jars. The time required for a droplet of dye from the heel to reach a point along one of these stream lines of course increases as it becomes more distant along this path or if it were in a slower moving path. This is a refinement of the observation by Jackson that the lag factor increased from values of about 0.5 near the top to near unity near the bottom.

The sweep outward of fluid in the core before it reaches the bottom of the container has also been noted by Bitting (1926-27) and to a lesser extent by Tani (1940). This effect can be seen in the iso-F_o maps of Fagerson's data (see Figure 6.9). In these maps the iso-F_o lines are normal to the streamlines which form the boundaries for the flow tubes. This outward sweep as the fluid rejoins the ascending boundary layer and the by-passing of the area between the boundary layer and the more rapid streams within the core may be responsible for interaction which mask the effect of transport lay in a statistical of Fagerson's data.

This outward sweep of the core flow before it reaches the bottom makes it possible for the establishment (and detection) of the bottom flow patterns in food containers. The presence of bottom eddies in thermo-syphon tubes as hypothesized by Lighthill (1953) has not been confirmed experimentally in this system. The large L/r ratios and hence large h_sA_s/h_BA_B ratios investigated may be responsible for masking this phenomenon.

It can be seen from Table 2.2 that the flow regime at the bottom probably corresponds to either laminar flow (based on the McAdams formula for single flat plates) or to the lower portion of the turbulent region for flow confined between two hotizontal surfaces. Schmidt and Silveston (1959) have characterized this flow as a completely disorganized eddy pattern which is constantly shifting. The spacial arrangement of the bottom eddies is not clear: because if (1) the flow corresponded to Benard cells the fluid would rise in the center of the eddy loops, (2) Benard cells (which are known to appear above a N_{Ra} of 1,700 up to N_{Ra} of about 47,000) are usually thought to have a nearly hexagonal shape of torus whose diameter is about twice its height, and (3) it is not now clear what the effect of lateral confinement of the eddies is on such eddies. The effect of lateral confinement, however, is probably a function of the perimeter per unit area and hence may decrease as $\frac{1}{r^n}$ with increased r (as in the McAdams formulae for single horizontal plates).

The implication of the flow patterns discussed above are that the transport lag, $t_d = f \cdot \log j$, due to convection and supplemented by conduction increases along a stream line and as the flow area between stream lines increases, r regions, which receive less equivalent minute F_o, therefore will exist (1) at the center of buffer regions within (vortex centers) and between flow streams, (e.g., the buffer zone between the ascending boundary layer and the descending core) (2) within established bottom eddies or ring vortices (3) axially above the bottom flow and between the lateral eddies (4) at the heel of the container between the bottom eddies and the last fluid to return to the ascending boundary layer. When correlations of lag time t_a as a function of axial position and other physical and geometric factors are attained, the Food Engineer and Food Scientist may have adequate information--assuming plug flow--to calculate the lethal treatment imparted to particles traveling with the fluid to supplement estimates based on the temperature history of the 'coldest' spacial point.

The temperature histories near cold point (3) rather than near the cold point (4) (which was pointed out by Fagerson and Esselen, 1950 and Fagerson, Esselen and Licciardello, 1951) were investigated here. Movement of this cold point was implied by the analysis but neither confirmed (nor--the author feels--contradicted) in the experimental phase of this investigation for reasons which will be discussed shortly. The above discussion seems incomplete without a brief projection of the flow patterns for smaller N_{Ra} than investigated here. The boundary layers for the system investigated here are thin, less than 0.48 in. the measurement closest to the wall in this investigation, i.e., within the 0.75 in region not investigated by

Fagerson (1950), but between 0.1 and 0.6 in. the thermocouple locations used by Jackson and Olson (1940) and Jackson (1940). The thermal boundary layer thickness on a 4 in. high vertical plate for as a function of Rayleigh number and Prandtl number are shown in Table 5.2. These values were calculated using the formula for laminar flow on flat plates given in Eckert and Drake (1959) -

$$\frac{\delta}{x} = 3.93 \frac{(0.952+N_{Pr})^{1/4}}{N_{Pr}^{1/4} N_{Ra}^{1/4}}$$

TABLE 5.3.--Calculated thermal boundary layer thickness on a 4 in. vertical plate

N_{ra}	N_{Pr} 1	10	100
10^{10}	.059 in.	0.051 in.	0.050 in.
10^9	.105	.091	.089
10^8	.186	.161	.157
10^7	.354	.306	.299
10^6	.590	.511	.498
10^5	1.046	.906	.885
10^4	1.86	1.61	1.57

The velocity boundary layer thickness for natural convection based on an electronic computer solution of the boundary layer equations have been reported by Ostrach (1952). For this discussion however it is sufficient to use the approximation given by Schlichting (1960).

$$\frac{\delta_{\text{momentum}}}{\delta_{\text{thermal}}} = \sqrt{N_{Pr}}$$

This means that the velocity boundary layers which are not sensed by thermocouples are 1, 3.16 and 10 times thicker, respectively than those given in Table 5.2. These calculations are not appropriate for N_{Ra} or N_{Pr} such that there is appreciably curvature or core flow is restricted. The experimental observations of a thermal boundary layer greater than 0.1 in. (Jackson, 1940) but less than 0.475 in. and a velocity boundary layer of 0.1 in. or less (from flow visualizations studies) agree well with those estimated for $N_{Ra} = 10^9$, $N_{Pr} = 1$, approximately the condition of this and the several investigations cited.

This boundary layer thickens toward the top but grows most rapidly near the bottom. This is the reason that most of the core flow, particularly the central portion of the core returns to nearly the start of the boundary layer before rejoining it. The boundary layer would also be expected to thicken either late in the process or as more viscous fluids were used--both of which imply lower N_{Ra} values. When the N_{Ra} are such that the boundary layer occupies more than half of the flow area at the top (of thermo-syphon tubes) Lighthill (1953) has hypothesized that mixing of core and boundary [probably more here than for thermo-syphon tubes] will occur. The reduced velocities due to this mixing were hypothesized to cause the boundary layers to move upward causing a possibility of a stagnant fluid layer below. These domains have been repeatedly confirmed experimentally. Tani (1940), however, observed no decrease in height of the lateral eddy loops as T approached T_1 but Fagerson and Esselen (1950) reported that the eddy loops only rose to 1/2 the jar height after 27 minutes heating of water in No. 303 and No. 2-1/2 vegetable jars. The latter study used the same fluid but slower heating containers than the former. A sketch of a the possible flow pattern for these N_{Ra} is shown in Figure 5.5 . In this sketch some upward (downward) movement of the bottom (top) of the lateral eddies is indicated. Such displacement might be caused by the increased role played by conduction. As the conduction role increases less confidence can be placed in reliance on the stagnation points in the flow patterns as indicators of cold point position, e.g., the importance of the cold point at the bottom of the lateral eddy decreases and the importance of the axial cold point (now higher) increases as conduction through the bottom and normal to stream lines increases. The importance of the axial cold point (as determined using multipoint rods) for lower N_{Ra} and the irregular behavior of the bottom eddies as well as convenience suggest continued use of axial temperature histories as basis for spacial lethality calculations.

Part III. The Mixed Mean Temperature

The calorimetry experimental data were subject to moderate errors which generally increased as T approached T_1. Much of the variation found at 61 minutes may be attributed to heat storage error, an error which Short (1944) overcame by using 12-hour equilibrium periods. Other errors, such as spillage and evaporation, heat loss to surroundings, non-uniform agitation, and cycling of bath temperatures (and others which have been discussed by Sturtevant, 1959 and White, 1928 among others) are felt to be of lesser magnitude. Such equilibrium periods were not practical here.

It would seem that the most striking feature of this figure, the slope after the break, is due to such errors. It is interesting to note, however, that the slope corresponds to a coefficient of about 0.3 Btu/hr-ft^2-°F. This is near an experimental limiting natural convection film coefficient on vertical plates of 0.45 reported by McAdams (1954). Two other general observations can be made, however. First, at 10°F below the retort temperature an axial position of about 0.60 to 0.65 for the vertical profile studies (reported below) seems to correspond to the average temperature of the product; second, since heat loss experimental errors all would cause observations which were too high, any break seems to occur much less than 2°F from the retort temperature.

The first of these observations, the vertical displacement of the mixed mean temperature of the bottom and the prominant lateral eddies is to be expected for the bottom eddies would displace these latter flows upward. The axial position is not as high as would be expected from the mean of the slow point and container height because the thin fast moving portion of the eddies which is near the walls is quite warm relative to the core. It is evident that the axial cold point temperature is a conservative estimate of the mixed mean temperature and that it changes at nearly the same rate ($\frac{1}{f}$).

The second observation implies that the practice of heating a product at $1^{o}F$ above the nominal process temperature may satisfactorily prevent any under processing of a highly convective product due to transition to conduction heating.

Part IV. Overall Heat Transfer Coefficients

Exterior film coefficients and container conductances as well as the product film coefficient may play a significant role in transient convection heating and cooling of canned foods. Seemingly minor changes of processing conditions, or a change of heating medium therefore may cause detectable differences in heat penetration rates.

Differences among heating medium undoubtedly are detected only when the exterior film coefficients become important relative to other factors that affect the overall heat transfer coefficients. It will be seen in the succeeding section that these factors may include variations in retort operational practices such as retort temperature come-up time, venting, stacking and air agitation; and container-product variation such as fill, size, initial temperature, consistency and thermal diffusivity. The exterior film coefficients themselves, however, mau also contribute to the 7--but as high as 10 to 15 percent coefficient of variation of heating rates (Hicks, 1961) often encountered.

Estimation of film coefficients from present engineering correlations is difficult: (1) the shapes for which data presently are available are not those of the typical food container which is usually a finite cylinder, and (2) the fluid velocities within retorts, baths, tunnels and canals are difficult to measure and have not appeared in the literature. (Film coefficients for a few canned food systems have been reported and are shown in Table 6.7. These data can be used as guides.)

The purpose of this phase of the experiment was (1) the evaluation of heat transfer film coefficients, using metal objects of known geometry and thermal properties, (2) evaluation of the influence of low film conductances on the f and j_o of convection heating products, and (3) relation of the surface conductance to the parameters f and j of Ball (1923) using theoretical relationships.

The use of metal cylinders of known thermal properties in itself is not new to the thermal processing literature. They have been successfully used in the research of Merrill (1948), Evans and Board (1954), and Cowell et al. (1959). These investigators, however, did not describe the equipment or procedure for this promising technique.

Possible systems errors.--The effects of thermocouple conduction errors on heat penetration parameters f and j, of conduction heating foods have been discussed by Cowell et al. (1959). The effects of these errors were found to be neglibible here, due to the fortuitous use of thermocouples whose thermal properties closely match those of the product. More serious is the effect of the thermal property variation with temperature for the relative error in the f of a solid having a uniform internal temperature is approximately the same as the relative error in the thermal capacitance (see Table 5.1). Where film coefficients cause low N_{Bi}, the relative error in f will be substantially due to that of thermal capacitance while at high N_{Bi} both the thermal capacitance and the opposing thermal conductivity are important. Thus these cylinders would be anticipated to give apparent f values too high by 1.5 to 3.3 percent and 4.7 to -2.2 percent for copper and aluminum, respectively, and hence--since f is nearly proportional to h in the useful range from h = 1 to 1000, an average error of about 2 percent low would be anticipated when the aluminum and copper cylinder data are combined.

Discussion of the Film Coefficient Determination Method

The method and the equipment seem to give satisfactory results: (1) the differences between thermocouples appears not to be significant; (2) the variation within tests in forced flow seems acceptably small; (3) the difference between cylinders is substantially that anticipated from property differences (and the use of handbook values of thermal properties) and (4) the agreement between these and engineering correlations for other systems seems acceptable. The transient film coefficients would be anticipated to be slightly higher than steady-state coefficients. The lower values for most of the engineering correlations could also be due to the use of a reduced diameter in the equivalent diameter of the annulus (a diameter which had been reduced in order to account for shape of the cylinders). The correlations for flow perpendicular to flat plates would be anticipated to be somewhat higher than for the flat surfaces of the cylinder since the flow might not converge as fast behind a long cylinder as behind a flat plate.

Convection Heating and Cooling of Containers in Water

The effects of some processing variables on the film conductances in several laboratory water-baths are shown in Tables 4.2 and 4.3. The means and ranges are for two positions within the cylinders during two different immersions. The effects of the film coefficients shown in these tables are illustrated in the response of water in No. 303 cans as shown by Table 4.4.

There appear to be differences (but not significant) between water baths which might be attributed to different relative size (for the same mixer), increased temperature differences, and higher fluid temperature level--that is fluid properties between those of the initial and final temperature--but temperature difference seems to have more effect.

The data in Table 5.4 give insight into the components of the film coefficient. A substantial portion was due to natural convection (low turbulence to high laminar flow region on the side walls). Although the contribution of natural convection from the heating elements was small it is of the same order as that of much of the variation within tests and hence may have been responsible for variation in the 60 and 180°F baths in which the bath temperatures cycle--period was approximately equal to the f values. The agitation appears to increase the film coefficient by about one-third. This would indicate that the exterior film coefficients might drop to one-quarter or less of their initial value during the latter stages of the heating or cooling of canned foods and hence would contribute to a break in the heating rate very late in the process.

The natural convection coefficients for a 60-180°F treatment agree well with computed values of 226 and 103 Btu/hr ft^2 °F at 120° and 10°F below processing temperature using the recommended correlation of McAdams (1954) for natural convection.

The data in Table 5.5 show an inverse relationship between f and j, part of which might be attributed to the wall capacitance--exterior film coefficient effect (but its contribution would be anticipated to be greater for the data of Table 5.6 for glass with high exterior film coefficients). Much of the relation between f and j may very well be due to transport lag.

The high values of calculated convection film coefficients using the lumped capacitance-conductances model (the modified Merrill model described in Section II) is rather unexpected. If these high values are not due to the small differences in f and h relative to precision of the measurements they may be due to higher than steady-state internal film coefficients turbulence (suggested by Hammitt, 1958), higher surface temperatures on the surface of the transducer and hence to low film coefficients inertial effects of the core which might cause maintenance of faster boundary layer flows and possible error of about 12 percent due to weighting the contribution of heating through the top of the container in determining the overall film coefficient on the transducer. These factors are opposed by the nonlinear temperature gradient in relatively thick walled containers with moderate to higher internal and external surface film coefficients.

The application of the technique described above seemed to give satisfactory definition of the film coefficients on containers. It does not appear that the transducers or the technique in itself is responsible for the observed variation in f of either the cylinders or the canned foods, hence refinements such as those taken by Stolz (1959), i.e., measurement of temperature histories at several points within an object and computer calculation to determine the time rate of surface heat flux, are necessary at this time. The method of calculation of f from film coefficients, thermal properties and characteristic lengths is similar to its inverse, finding h from f as illustrated here. The problem of finding f for conduction products if head space resistance is to be accounted for is much more troublesome. It would appear that Equations 21, and 22, and 23 of Section 3:11 of the text by Carslaw and Jaeger (1959) should be used for the purpose rather than to construct graphs such as those shown by Cowell and Evans (1961) (but with the ratio of N_{Ra} on the two surfaces as additional parameters.)

Convection Heating of Containers in Steam-Air Mixtures

The results of a recent experiment in which surface conductances in some steam-air mixtures and the results of the f vs the logatithm of velocity correlation of Pflug and Blaisdell (1961), shown in Table 5.6, show the same behavior.

Glass Container Wall Thickness

The effects of container capacitance and resistance have been discussed in Section II. The problem here, however, is the determination of a wall thickness, either through measurement or estimation from the geometry of the container.

Merrill suggested a formula for estimation of glass height

$$L_c = \frac{0.145\ (\text{Wt of glass, oz. Av.})}{[(\text{Capacity, fl oz.}) \cdot (\text{Height, in.})]^{1/2}}$$

without derivation. The formula

$$Lc = \frac{L_c[2\pi rL+(0)\pi r^2]\ 0.145}{[\quad(\pi r^2L)(0.554\ \text{fl oz./in.}^3)(L)][0.69\ \text{in}^3/\text{oz.glass}]}$$

which neglects the bottom surface will give the same answer.

The empirical constant suggested by Alderson (1963), i.e., 0.104 instead of 0.145. Previously the formula seemed to be of the form

$t = \frac{t}{2} [2G_1 + G_2 \frac{r}{L}]$ where $G_1 = 2$ then $G_2 = 0$. Now the constant in the denominator is 2.79. Since r/L range from about 0.5 to 0.28 for baby jars to quarts with a mean of 0.34 the equation $t = \frac{t}{2.79} [2+2 \frac{r}{L}]$ would give approximately the same answer as Alderson's modification of Merrill's formula. For the containers contrasted in Table4.6 these modifications give equal or better answers than the basically superior methodd of Alderson. The Alderson method appears to have given poorer but satisfactory results because the container bottom was assumed to be twice as thick as the side walls although in these instances it was usually appreciably thinner and the horizontal projected area of the closure had already been taken into consideration in the frustums of right cones which were used to approximate the lateral surface.

The glass jar thickness, as determined by destructive testing, showed large variability with highly significant variation within wall positions in the containers which was ±25 percent and an interaction between wall thickness at various positions and the particular jar, which may be due to the method of manufacture. The nearly significant variation from jar to jar within a given size may have increased the range of f sufficient to have destroyed the sensitivity of a statistical analysis to real but small differences between T_o and T_1 treatments.

Part V. Discussion of f and j in Convection

The effect of container and product on the time-timperature heating pattern must be made in terms of f and j; however the time for the onset of heating, $t_a = f \log j$, i.e., the time for the fluid elements to move from the wall to the point in the container under consideration has more real meaning and the effect of variables on this quantity will be included in addition to their effect on f and j.

The Use of the Terms f and j for Convection Heating to Describe Convection Heating Data

The terms f and j are derived parameters for conduction heat transfer. Rigorous analysis of conduction problems permit correlation of f and j with the Biot number ha/k and other parameters of the conduction heating system. The significance of f and j in conduction heat transfer does not extend into convection heat transfer. However, it is possible to show that convection heat transfer can be described by the following equation if thermal properties and the overall heat transfer coefficients are constant

$$\log(T_1 - T) = \frac{-t}{f} + \log j\ (T_1 - T_o)$$

This equation also can be used to represent portions of the heating curve if these restrictions are not met. However, f can no longer be related to thermal diffusivity and distance but now is a function of the convection coefficient, viscosity, and container geometry among others. The physical significance of the lag factor j is even more remote in convection than is the f value. There is a thermal lag in convection heating as in conduction heating but this lag is now basically a time function, that is, a transfer lag that depends on the length of time required for the particle to move from the side wall stream to the point being measured (after the brief interval of conduction heating of the exterior film wall and the internal fluid until the critical N_{Ra} for the incidence of convection is attained). If the several lags in the system were due to non-interacting components it would be possible to partition the transfer lag into its several components using the techniques of system analysis as outlined by Caldwell, Coon and Zoss (1959) among others. The time when a given point begins to heat can be described in terms of the f and j nomenclature as f log j, (see Figure 5.6). Even though the basis of f and j is not as strong in convection as it was in conduction analysis, Pflug and Blaisdell (1963) believe these terms are as good as any terms that can be developed and their historical use and their general convenience is sufficient to warrant that they be used in convection analysis.

The status of methods for analysis of convection and combined convection and conduction data are tenuous. The available models all suggest heat penetration curves similar in shape to conduction but provide little or no information to predict j other than that tentatively proposed by Merrill (1948) and the treatment of container wall lags in Section II of this disertation.

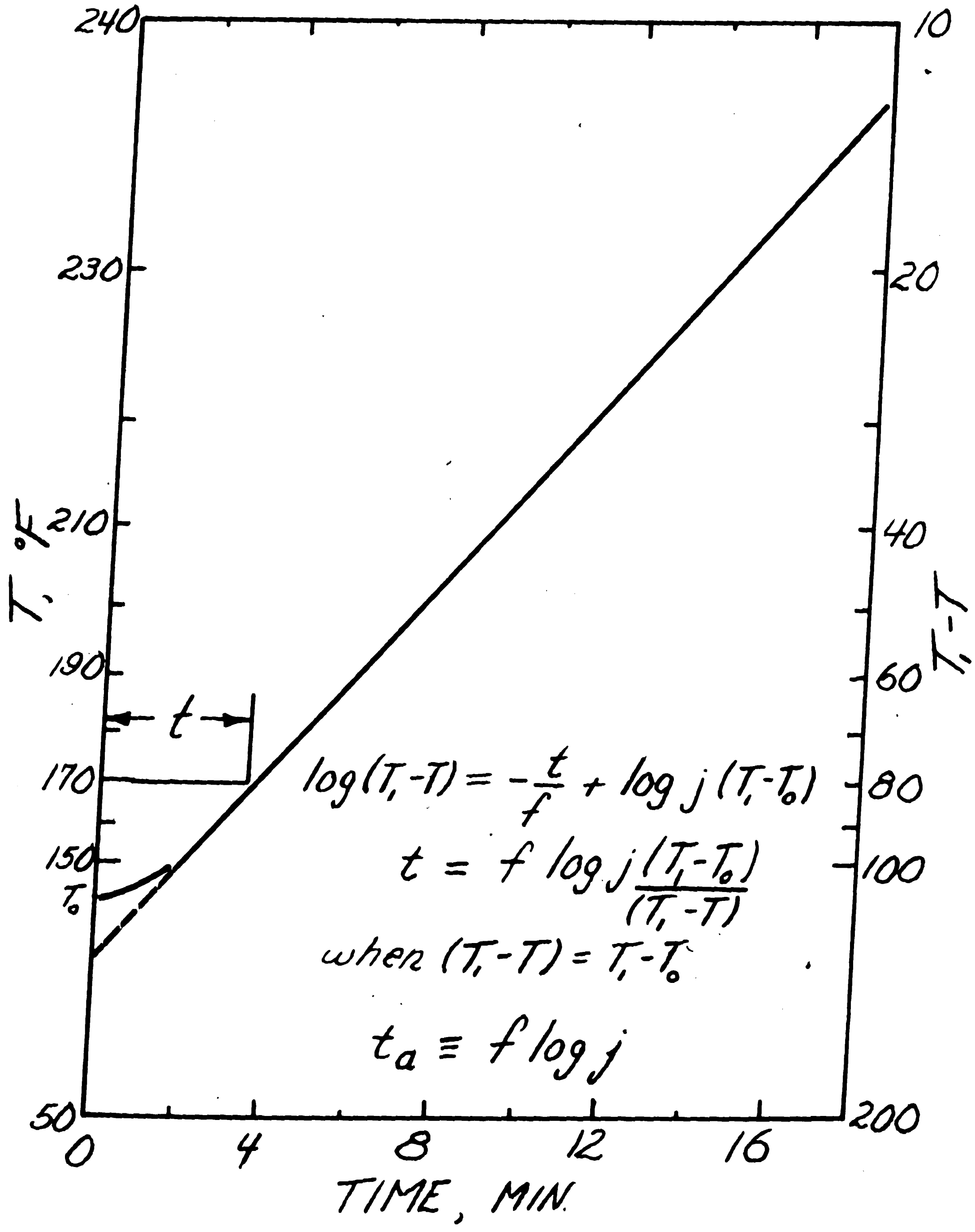

FIGURE 5.6 The time lag t_a for a heat penetration curve.

Comparison of a Small Diameter Container with a Large Diameter Container, Both of Which Are the Same Height

As the diameter of the container is increased, f increases and f log j increases; however, j may either increase or decrease depending on the relative magnitude of f and f log j. Consider a small diameter as opposed to a large diameter container. The ratio of the heat transfer surface to the volume of fluid decreases as diameter increases. The quantity of heat transferred to the container is a function of the surface area and therefore as the surface to volume ratio decreases, the heat transfer per unit volume decreases and hence f will increase; the side wall convection stream flow through the area $2\pi r\,\delta$ is a smaller percentage of the center slug area, $\pi(r-\delta)^2$, and hence the rate at which the slug in the center of the container moves downward will decrease causing an increase in f log j. The lag factor j would be expected to be nearly constant but be sensitive to changes in L_w.

Effect of Increasing Height, Diameter Held Constant

As the height is increased, holding the diameter constant, the heat transfer surface to fluid volume ratio $(r+2L)/rL$ or $(\frac{L}{L}+\frac{2}{r})$ will decrease slightly since the can is taller, the heat transfer through the bottom remains the same with height proportional to volume. The inside film coefficient for laminar flow of the side wall is inversely proportional to the fourth root of the height for laminar boundary layer flow;

therefore the inside film coefficient will decrease slightly with height. The net result is a small increase in f, an increase in f·log j and an increase in j. The cold point will move up very slightly as the height increases but the distance the cold point is from the bottom expressed as a ratio of this distance to the total height will decrease as the height of the container increases, in other words doubling the container height will not double the distance of the cold point from the bottom of the container.

Relative Values of f and j at Different Locations Along the Center Line of the Container

Moving from the top to the bottom of the container f, f·log j and j all increase until at the point of slowest heating, f, j and f·log j reach their maximum values. The f increases as we move from top to bottom of the container because the amount of fluid passing a given point along the center axis of the container decreases toward the bottom of the container. The wall film stream increases in thickness as we go from the heel of the container to the top; the fluid that is added to this rapidly rising stream comes from the central slug, and since the wall stream increases from bottom to top, the central slug velocity must decrease from top to bottom. The lag time, f·log j, increases because the flow path is greater and hence greater time is required for an element of fluid to move from the top of the wall stream to a measured point farther along the axis of the container. Since a greater mass of the core must be heated first before this position in the jar is effected, j will increase.

Considering a given location along the axis of the container in the vicinity of the cold point and moving from this point toward the wall stream, there will be a small decrease in f, f·log j will decrease slightly as will j increase slightly. The f will increase because we are moving into an area of higher fluid velocity. Considering the overall flow path and now moving away from an axial limitation, the slowest heating point, i.e., largest f and f·log j in the container will be near the heel where the downward flow pattern turns to become the side wall stream.

Effect of Viscosity

For any given system, increasing the viscosity as in sucrose versus water or a 60° to 120°F as contrasted to 120° to 180°F has the effect of reducing the boundary stream film coefficient and flow rate which in turn tends to increase f at any particular point. The lag time f log j, will increase but the lag value j may show very little effect or be smaller due to greater relative importance of thermal property variation of viscosity. Viscous materials such as high concentrations of sucrose or glycerol have a greater temperature coefficient of expansion than water and therefore have greater buoyancy for a given amount of temperature change; however, the viscosity may actually overshadow this buoyancy effect. It is possible for viscosity drag to be sufficient to prevent convection flow.

Increases in T

An increase in T at constant final temperature T_1 increase the driving force $g\rho\beta\Delta T$ but also increase the fluid viscosity if there is quasi-steady-state heating and no effect of momentum the heat transfer rate and internal temperature distribution (see Hammitt, 1958) will be equal at the same T_1-T. If Prandtl number effects on the coefficient G of ${N_{Ra}}^n$ in the equation $N_{Na} = G{N_{Ra}}^n$ are negligible as in the McAdams formulae, the initial temperature difference rather than the quasi-steady-state temperature alone is governing factor, the containers heated from 60°to 180°F instead of from 120° to 180°F are predicted to have smaller h and hence larger f and presumably transport lag, f · log j. If the equation of Eckert and Drake (1959) in which G is Prandtl-number dependent is used the converse is predicted; in addition sucrose is predicted to heat faster and water slower than predicted using the equation from McAdams (1954). The experimental data indicate that a relation like that of Eckert and Drake gives better predictions. If the Eckert and Drake formula had not improved the overall estimate for sucrose and water there would have been a greater possibility that the comparison of f over a constant u range (from u = 0.5 to u = 0.085) rather than at constant T was responsible for this effect. Comparisons at constant T have been made. An increased temperature difference but at the same final temperature (but with only about a 2-1/2 percent reduction in film coefficient) reduced j and f· log j only for sucrose in 303 vegetable jars and 303 cans. This seems to indicate that initial viscosity may have a detectable effect on the heating lag for products which heat slowly due to appreciable glass thickness or volume to surface ratio but this effect is overcome somewhat when internal resistance allows the wall to be heated to nearer the processing temperature earlier in the process.

Addition of Particles in the Fluid

When particles are introduced into a fluid that normally heats by convection, both j and f can be expected to decrease (see Table 5.4) because (1) the ratio of heated surface to actual fluid volume has now increased (the Volume of fluid has decreased),(2) the heat transfer coefficients between the fluid and the food product, which would remove heat from the fluid, will be much lower than the film coefficient between the fluid and the wall (the velocity in the center core will probably be less than 1/10 of the side wall film stream velocity) and (3) the thermal conductivity of the food products is comparatively low compared to container wall. As the size of the particles decreases, in other words, as we go from large particles to small particles, both j and f will increase because small particles will increase the fluid flow friction by reducing the size of the channels between them, and will have a greater amount of surface area and less internal resistance r/k and therefore will absorb heat from the fluid bulk at a higher rate.

TABLE 5.4.--Comparison of f and j for water and 50 percent sucrose solution in 303 jars with and without plastic spears (data of Nicholas *et al.*, 1960)

Fluid in Container	No Particles			Plastic Spears		
	f, min	j	t_a, min	f, min	j	t_a, min
Water	10.2	1.29	1.13	9.0	1.05	.19
50 percent Sucrose	12.3	1.32	1.48	12.0	1.13	.63

Changes of Movement in the Cold Point During a Heat Processing Cycle

As convection heating progresses, the flow patterns, bot side wall film stream and the slug moving down the center, decrease and the heat conduction through the bottom and sides will,toward the end of the logarithmic phase, move the cold point from the heel of the container back toward the geometric center of the container. The cold point along the central axis will, at the same time, be pushed toward the geometric center as the conduction of heat through the bottom of the container makes its presence felt through the bottom section of the container. To prevent the cold point from moving, heat flow from the top of the container and heat flow from the bottom must continue in about the same proportion. However, during the later stages of heating, heat flow from the bottom continues at an approximately constant rate whereas heat flow from the top decreases, which causes the cold point to move as shown in Figure 5.7.

Summary

In Table 4.9 are summarized the general conclusions that we have reached regarding convection heat transfer. In all generalizations there are exceptions and these will occur in convection heating due to variables such as container material, exterior film coefficient, and product behavior.

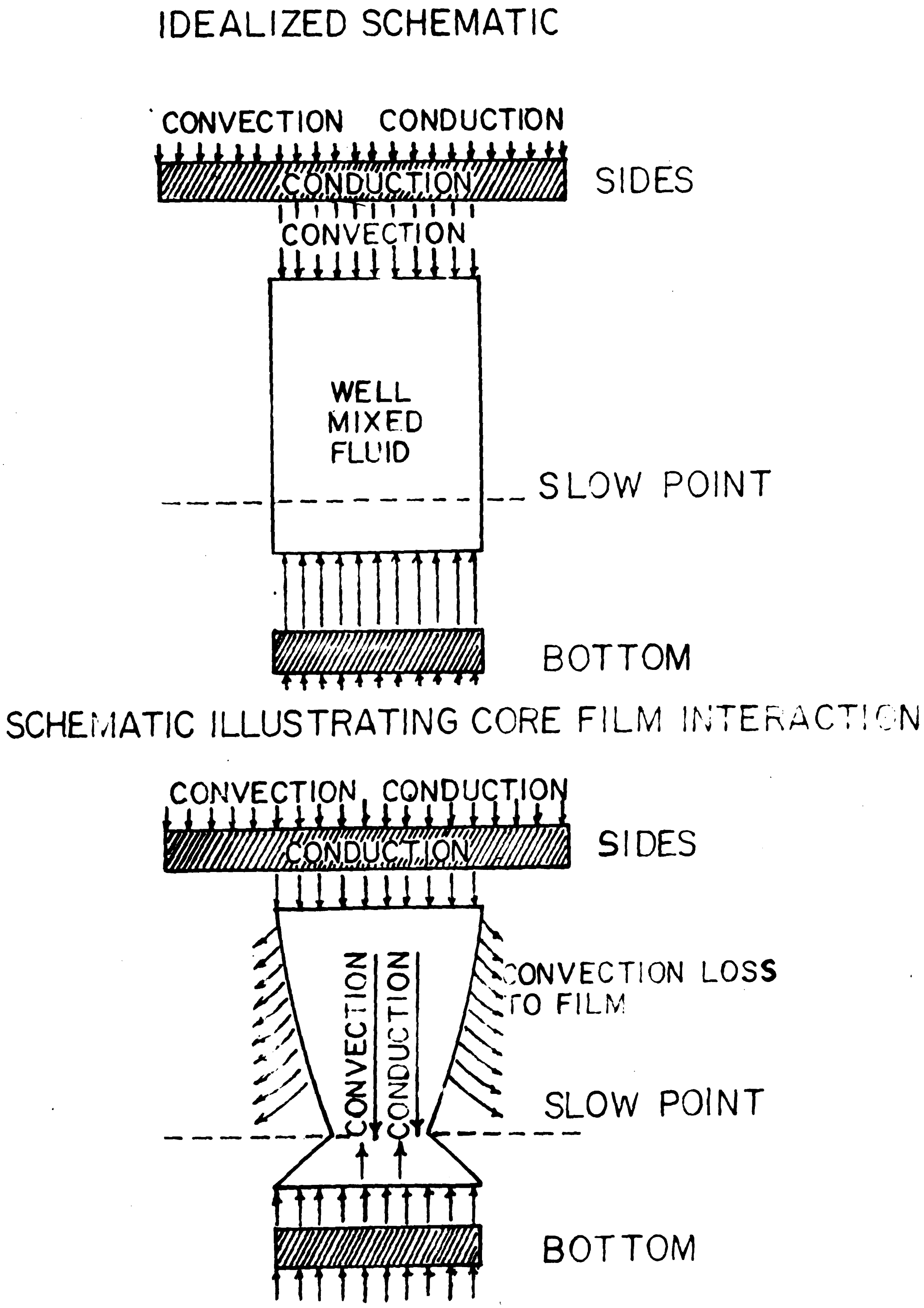

FIGURE 5.7 Schematic diagram of heat flow as it may effect the vertical position of the axial slow point.

Part VI. Predictions from Quasi-Steady State Film Coefficients

Slow Point Position

The prediction of axial cold position from a balance convection of heat flow from sides and top was unsatisfactory. This balance is shown schematically in Figure 6.14. In the region N_{Ra} investigated the McAdams and other predictors of heat transfer have higher N_{Ra} exponents than for heat transfer on vertical plates hence the slow point position was predicted to decrease rather than increase in this N_{Ra} range as N_{Ra} decreased. A subsequent correction for the parallel conduction components through the bottom and through the closure, a correction which would predict a conduction slow point above the geometric center due to greater steady state heat transfer through the bottom than through the headspace, predicted no detectible (a fraction of a percent) movement of the cold point. The estimator of cold point x/L was insensitive because there were not sufficient measurements taken below the cold point to adequately fix the position at which dT/dx = 0. The movement of cold point which is noticeable in Bentonite systems occurs for substantially lower N_{Ra} than those investigated here.

Heating Rate Parameter f

The McAdams (1954) and Eckert and Drake (1959) formulae for flat plates, rather than any of the correlations for thermosyphon tubes or fluids confined between horizontal plates were used because these correlations were for the N_{Ra} and N_{Pr} range of this investigation, the thermo-syphon tube investigations had been for substantially higher L/r, and the fluid was not confined by a rigid boundary at the cold point.

In the prediction of f using both the McAdams formulae, shown in Figure 4.20a, the correlation is due to container and sucrose-water differences rather than temperature treatments. In fact there is a persistant triangular pattern within these temperature treatments for a particular product and container. This observation suggested that fluid property might be responsible. It was also noted that the correlation could be brought to more nearly a 45^{o} slope by correcting the heat transfer for the difference between internal and external area. The new calculations which used the Eckert and Drake formula for the sides give noticeably improved predictions but appear to over-correct for property effects. Neither the two element or simple element lumped capacitance-conductance Newtonian heating models developed in the analysis were used here due to the experimental uncertainties in h_o, L_w and f, relative to differences between the f for the various treatments.

Change in f as T Approaches T_1

The non-linear nature of convection heat penetration curves has been shown both in the analysis of the problem and in a qualitative appraisal of the effect of container, viscosity, exterior film coefficient effects on the degree of curvature. Only the studies of heating at long times (see Table 4.1) appear to have sufficient precision to warrant a quantitative appraisal of curvature as T approaches T_1. In Figure 5.8

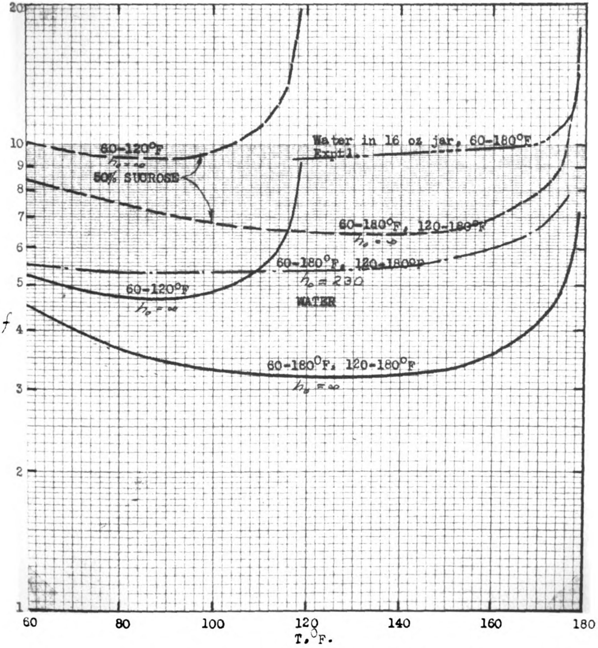

Figure 5.8 Relative change in heat penetration parameter f as a function of temperature of contents during processing. The calculated values are for #303 cans.

and 5.9 are shown the experimental f data for water in 16 oz. vegetable jars from Table 5.4 and f estimates for 16 oz. (303) cans. Figure 5.8a shows a pronounced relative change in f, i.e. (df/dT)/(f) = ln f with temperature. The higher order curvature is reduced by the exterior resistance on a can or by a glass container. The approximation of a linear change in overall heat transfer coefficient v, which is proportional to 1/f, used in Section II was an adequate approximation for intermediate T_1-T and it illustrated the nature of the interrelationship of f and j due to changes in heat transfer rates. The linear approximation does not appear to be satisfactory for smaller T_1-T values, however. The curves are aysmptotic to T_1 and since v is a function of $(T_1-T)^{1/4}$, and (T_1-T) is a function of time, these data were also compared as $\log_{10}$ f as a function of log (T_1-T). The logarithm parameter f appears to be a linear function of log T_1-T, i.e., log f = -m log (T_1-T) + log b, for values of (T_1-T) of 30 or less. The predicted curvature at larger values is more severe than the observed since the McAdams formula which estimates slightly higher h at high temperatures and low h at lower temperatures (N_{Pr}) for vertical plates was used. The temperature in the Newtonian model is now

$$\int - \frac{f(T_1-T)d[T_1-T]}{[T_1-T]} = dt$$

but here $f = b[T_1-T]^{-m}$, hence

$$\int_{T_1-T_o}^{T_1-T} -b[T_1-T]^{-m+1} \, d(T_1-T) = t$$

$$b[T_1-T]^{-m}\Big|_{T_1-T_0}^{T_1-T} = t$$

This expression is not explicit in log (T_1-T) and arithmetic t, and is not readily solved for an unknown m for given T_1-T and t data. A single f value no longer has physical significance, but two or more f values approximations might be preferable to determination of the F_o value using the experimental values of b and m in the ingregation of the equivalent lethality. It can be shown that such an integration would require tables of the incomplete gamma functions or more general tables of the confluent hypergeometric functions. The exponential integral function which appears in the development of the formula method (Chapter 12) in Ball and Olson (1957) and the error (probability) functions are special cases of the incomplete gamma functions. A treatment of convection like that of Ball (1923, 1928) and Ball and Olson (1957) for conduction would require tables for several values of m. Several values would be required because it has been shown that external resistance and property changes during heating reduce the m from its theoretical value (for laminar natural convection) of 0.25.

SECTION 6

DISCUSSION OF THE OVERALL THERMAL PROCESSING PROBLEM

> We cannot be impartial. We can only be intellectually honest; that is, aware of our own passions, on our guard against them and prepared to warn our readers of the dangers into which our partial views may lead them. Impartiality is a dream and honesty a duty.
> --Gaetano Salvemini, Historian

Transient heating and cooling with particular attention to the natural convection phenomena, as it occurs in food containers, will be treated in four parts: (1) a brief review of the methods of analysis of heat penetration data and their development; (2) a consideration of the temperature and flow patterns in convection heating and the validity of these methods, particularly the analogies to conduction models; (3) a brief examination of processing equipment factors influencing process heat transfer; and (4) consideration of the individual product, container, and heating medium factors influencing process heat transfer.

PART I METHODS OF ANALYSIS OF TRANSIENT HEAT TRANSFER

Conduction Heat Transfer

The Basic of the Parameters f and j (Ball, 1923)

Williamson and Adams (1919) developed equations for the transient temperature histories in some regular shapes using the Fourier series techniques. These solutions were limited to certain regular solids, e.g., the sphere, the cylinder, and the plate, whose properties were constant and isotropic, and whose surface transferred heat to the surroundings as a linear function of the temperature difference or was at some uniform temperature. Additional restrictions were that such bodies were initially at some uniform temperature and the surface temperature either changed suddenly to some new value or changed linearly with time.

About the same time Thompson (1919, 1920, 1922) reported series solutions for finite rather than infinite cylinders of the geometry of food containers when their surface temperature was changed suddenly to a new temperature that either remained constant or increased linearly to a new value and then remained constant. He also reported the solution for cooling in air, in which the heating medium temperature changes suddenly, but the large surface resistance to heat transfer permits only slow cooling of the container surface. These cases, which he also investigated experimentally, are the basis for most of the current analysis of the processing of conduction products.

Bigelow, Bohart, Richardson, and Ball (1920) presented a chart by Adams, based on Williamson and Adams (1919) which gave the logarithm of the fractional unaccomplished temperature rise, u, for several lengths to diameter ratios as a function of the non-dimensional time $\alpha t/r^2$ (now called the Fourier modulus) when the surface temperature of the container changed suddenly and then remained constant. These relationships were further illustrated with curves showing the temperature in various sizes of containers filled with a solid with water-like thermal properties and in one container size whose contents had various thermal properties. Although their own experimental time-temperature curves in rectilinear coordinates showed that the retort temperature did not change suddenly, they felt that the assumption of a sudden step-change in processing temperature was sufficiently accurate for most practical purposes. A portion of one of these curves was plotted as the logarithm of the unaccomplished temperature rise $T_{retort} - T_{in\ container}$ as a function of time.

Ball (1923) approximated the curve of the logarithm of the unaccomplished temperature rise as a function of time by a straight line representing a first term approximation, rather than the theoretical curve described by all the remaining terms of the slowly converging Fourier series solution of Thompson (1919). (See Figure 6-1 for this heat penetration curve and Figure 6-2 for an illustration of the mathematical problem.) This line was the asymptote to the curve and represented the heating curve best in the region in which thermal destruction was greatest. The parameters which describe the line on this heat penetration curve are f, the time in minutes for the curve to traverse one log cycle, and j, the ratio of the intercept of the asymptote and the intercept of the experimental curve. Ball related the f of the cans to thermal diffusivity, the diameter, and to the length to diameter

Transient state heat transfer differential equation where $u = \frac{(T-T_1)}{T_0-T_1}$

Differential Equation

$$\frac{\partial u}{\partial t} = \alpha \frac{\partial^2 u}{\partial x^2}$$

Solution: assuming surface film coefficient infinite solved for an infinite slab with coordinate zero at midpoint and uniform initial temperature throughout.

T_0 = initial temperature °F
T_1 = cooling medium temperature °F
t = time minutes
x = distance inches from center of slab
T = temperature (x, t)
$\alpha = K/\rho C_p$ (ft²/hr) x 144/60 = in²/min
K = thermal conductivity BTU, in/ft² hr F°
ρ = density lb/ft³
C = specific heat BTU/lb F°
a = half thickness of infinite slab

$$[illegible]\, e^{-\frac{\pi^2\alpha t}{4a^2}} \cos\frac{\pi x}{2a} - \frac{1}{3} e^{-\frac{9\pi^2\alpha t}{4a^2}} \cos\frac{3\pi x}{2a} + \frac{1}{5} e^{-\frac{25\pi^2\alpha t}{4a^2}} \cos\frac{5\pi x}{2a} \cdots$$

Figure 6.2 The basis for the heat penetration parameters is a mathematical treatment like that shown.

Figure 6.2a As a first term approximation this solution may be restated as shown below

$$\frac{(T-T_1)}{(T_0-T_1)} = \frac{4}{\pi} e^{-\frac{\pi^2\alpha t}{4a^2}} \cos\frac{\pi x}{2a}$$

$$\ln(T-T_1) = -\frac{\pi^2\alpha t}{4a^2} + \ln\frac{4}{\pi}\cos\frac{\pi x}{2a}(T_0-T_1)$$

Figure 6.2c The working equations are obtained from the above.

$$\log(T-T_1) = -\frac{\alpha t}{0.933a^2} + \log 1.27 \cos\frac{\pi x}{2a}(T_0-T_1)$$

$$f = \frac{1}{\frac{\alpha}{0.933a^2}} = \frac{0.933a^2}{\alpha}\ ;\ j = [illegible]$$

$$\log(T-T_1) = -\frac{t}{f} + \log j\,(T_0-T_1)$$

ratio of the containers. In this instance the surface conductance was assumed to be infinite as the film coefficients for condensing steam are large. Additional linear segments (see Figure 6-3) were to be drawn if the slope of the heating or cooling curve changed during a process.

There appear to be many excellent presentations of the mathematical solution for the sudden temperature change of a uniform finite cylinder to assist the reader in understanding the basis of the approach of Ball (1923, 1928): Hurwicz and Tischer (1952) present an exceptionally complete development of this case. Both Cooper (1937) and Okada (1940) cite an early analytical and experimental development of this case by Langstroth (1931). Hayakawa and Ball (1962) credit Kawaguchi and Okada (1940) with a presentation of the theoretical formulas comparable to that summarized by Ball and Olson (1957).

Other investigators have extended Ball's (1923) nomenclature to other shapes subject to the same conditions. Iwata (1940) treated oval cans in a somewhat similar manner. Okada (1940), following the work of Ball (1928), related the Bergman (1928) solution for brick-shaped rectangular cans to this nomenclature. Olson and Jackson (1942) presented equations for the relationship between f and the thermal diffusivity, and between j and position for brick, rectangular rod, infinite slab, finite cylinder, infinite cylinder, and sphere.

In view of the extensive treatment of heat conduction by separation of variables and Fourier (1878) series and by

Ball and Olson (1957) (who built upon the engineering texts of Ingersoll, Zobel, and Ingersoll (1948) and the classic mathematical texts of Carslaw (1921) and Carslaw and Jaeger (1948) and the approach of Ball (1923, 1928) in order to show the relationships between the mathematics of conduction heating and thermal processing problems), subsequent treatment of certain special cases as solved using Laplace, Finite Sine, and Hankel transforms by Hayakawa and Ball (1961), as well as the treatment of related problems and additional techniques in many engineering texts, the mathematics of the problem shall not be considered here.

Temperature Histories at Short Times

The non-linear or log portion of the heating curve, during which the series solution converges slowly since the exponential portion of the second and subsequent terms is still relatively large, assumrs great importance when changes in T_1 are made at high temperatures. The lag during the cooling portion of a thermal process has received considerable attention, as have the less important lags during the time the processing equipment is coming up to processing temperature.

Approximate cooling lag.--Ball (1923) approximated the lag portion of the cooling curve by an empirically determined hyperbola. The j of this cooling curve was assumed to be 1.41. He assumed that the temperature does not continue to rise sufficiently long so that the product was nearly uniform in temperature, such that the temperature does not rise after the heating medium is removed and cooling has begun. Hicks (1951), however, showed that in practice the product temperature at the center may continue to increase for a brief period before it is affected by the cooling of its exterior.

Gillespy (1951) estimated that the fictitious uniform temperature which would give the same j as that for an actual process would be that attained if the heating period was continued for 0.03 f minutes longer. (The tabulations for the integrals used in lethality calculations also incorporate factors to account for container shape and asymptotic convergence as well as for final temperature and heating vs. cooling temperature.)

Jakobsen (1954) suggested approximating the time-temperature history with the hyperbolic secant in order to attain an improved representation of the initial curved portions of the heating and curves. He related this approximation to the theoretical slope and to Ball's (1923) nomenclature.

Board, Cowell, and Hicks (1960) made an experimental investigation of the computation of the sterilizing values of commercial processes using the methods of Ball (1923) and of Gillespy (1951), but not that of Jakobsen (1954). Neither method was found to take into account the lethal value of the holding period in which the steam was turned off and the cooling water introduced into commercial retorts. Although Gillespy's method gave results quite

close to those obtained by graphical integration (Bigelow et al., 1920) when product movement such as that found by Hemler et al. (1952) and Powers et al. (1952) did not occur, in the latter cases Gillespy's method seriously overestimated the lethal value. Ball's method, on the other hand, generally underestimated the lethal value.

Approximate heating lag during come-up.--Ball (1923) approximated the come-up time lag that occurs in commercial retorts at the start of the process. During experimentation he found that the surface temperature averaged from 47 to 66 percent of the final process temperature and that this exposure period, as determined by graphical integration (Bigelow et al., 1920) varied from 82 to 91 percent of that which would have been attained at this average temperature. If 50 percent of the final process temperature is a good estimate of the average process temperature, as it would be during linear 'come-up' from the initial temperature of the product, and 85 percent is a good estimate of the lethal effectiveness of temperature, then the come-up time is 42 percent effective. This procedure was found to move the origin of the heating curve from the time the retort temperature attained its final constant value to a new point 42 percent of the come-up time earlier. The measured j of this curve is then calculated using the intercept at this new 'zero' time.

Alstrand and Benjamin (1949) made an experimental investigation which showed a percent effective time of 37.5

percent or very nearly that proposed by Ball. Logarithmic come-up increased the effectiveness to about 70 percent. (The erratic results for convection indicated an average percent effectiveness of 34 percent, but ranged from 18 to 65 percent.) Gillespy (1951) and Jakobsen (1954) have employed an effective time of 50 percent in accordance with European practice.

In 1953 Gillespy extended his treatment of the sterilizing value of a process to include complex processes and exponential come-up of the process temperature. The consideration of the exponential come-up was introduced in order to improve upon the estimate based on the assumption that the come-up is linear in those instances where the processing steam supply temperature is nearly that of the processing temperature. In these latter cases the come-up curves were said to be more nearly an exponential curve asymptotic to the steam supply temperature. The portion of the come-up time to be incorporated in the calculations, in order to achieve equivalent sterilization as calculated by a point-to-point integration, was determined for one length to diameter ratio as a function of the fraction excess of the temperature and the steam process temperature above the initial temperature, and as a function of the come-up time to f ratio. Corrections were given for other length to diameter ratios.

Charts for the special case in which the steam supply temperature is that of the processing temperature can be determined by combination of the solutions and tabulation given by Crank (1956) (see Equations 4.29 and 4.28 and Figures 4.3 and 5.4) for diffusion in the infinite slab and the infinite cylinder. Crank said that this exponential variation can be used to represent experimental attempts to create a sudden change in surface concentration (temperature here).

Tabulated experimental temperatures or lethalities for complex processes.--The general method described by Bigelow et al. (1920) or the Pasternik (1952) modification may be used to evaluate the lethal value of highly irregular processes. It is a graphical integration of the lethal values accumulated corresponding to the temperatures encountered during the thermal process. It, like the Simpson or trapezoidal rule integration of tabulated experimental temperatures, makes no assumption as to the form of the time-temperature history.

The capabilities of this method suggest that irregular temperature histories might be reported in tabular form or that some representative cumulated lethalities at one or more times might be tabulated as well. Some workers, as in the dissertations examined further by Powers et al. (1962), have reported their data in these forms. The precision gained by reporting data in these forms, which may well consume excessive journal space, is gained at the expense of readily interpreted heat penetration parameters, if the time-temperature histories are at all regular.

The experimentor may have little choice when studying a

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relatively regular heat penetration in some agitating processing equipment: he may remove the cans and measure the temperature of different cans at specified times or of the same can (which may cool during measurement) which is reinserted after each measurement (Gallop, 1960); or he may use an organism (Schmidt and Bock, 1956) or chemical (Pflug, 1952; Reed, 1963) with a known initial concentration and destruction rate. In the first instance the certainty of both the heating curve and the equivalent lethal process may be limited unless many more than the two minimum points required to fix a heating curve are known. In the second instance all knowledge of the heating (or cooling) phenomena are lost in order to attain more accurate information about the lethal value (for the test substance). For the moving but 'non-agitating' hydrostatic cookers, such as the "Hydron" unit (patented and manufactured by Mitchell Engineering Ltd., London, England), temperatures in process simulators have been successfully used (Bee et al., 1961) to obtain the customary heat penetration parameters f and j. Simulators are usually used for designing processes for Food Machinery Company reel-type agitating cookers.

Calculated temperatures for simple processes.--The counterpart of tabulation of experimental temperature time data at close time intervals is the calculation of these temperatures using the series solutions in spite of convergence problems. Olson and Schultz (1942) felt tabulation of the solutions of the cylinder and the slab was warranted for the following reasons: (1) the principle reported by Newman (1931, 1936) that certain solutions for different geometries could be combined to give solutions for other regular shapes could be used to apply this table to diverse geometries; (2) the heating equations were applicable to drying and other problems (see Newman 1931a, 1931b, 1931c); and (3) the accuracy of the then available Gurney and Lurie (1923), Schack (1930), and Newman (1936) charts particularly at the beginning of the heating period was limited. Their tabulations for the uniform cylinder and slab whose surface temperature is suddenly changed may well be still the most extensive tabulations. When these solutions are combined with one another or with tables of the probability integral solution for an infinite slab, as required, many solutions may be obtained. These include the infinite, semi-infinite, and finite cylinders; the semi-infinite, quarter-infinite, and eighth infinite solids; the infinite, the semi-infinite, and the quarter-infinite slabs; and infinite rectangular and the semi-infinite rectangular rods and finally the brick. To obtain the solution for a finite cylinder, for example, the solution for flow through the cylindrical surface (of an infinite cylinder) is multiplied by the solution for flow through the plane surface (infinite slab or plate). Only those product terms having the same index of summation need be retained. This is, of course, subject to certain restrictions.

The principle of Newman (1931, 1936) and Schultz and Olson (1942) has significance far beyond its use here, for:

(1) there are eleven separable orthogonal coordinate systems for the wave equation (see Morse and Feshbach (1953), Ch. 5) which include the familiar rectangular solids, the circular cylinders, and spherical coordinates; (2) the requirement that the boundary heat transfer coefficient (or surface temperature) be uniform (in a direction orthogonal to the direction of heat flow through a specified surface) although it may be a function of time alone is often a reasonable assumption; (3) the thermal conductivity is usually equal in all directions, although it may not be quite true in the case of meat (Hurwicz *et al*., 1956, and Lenz, 1961) or in 'woody' products (McLean, 1946) in which the conductivity along the grain may be greater than perpendicular to the fibers; and (4) the additional requirements that the initial temperature distribution also be expressed by a similar product solution can often be met by prescribing surface conditions and prior heat treatments from some time at which the temperature is uniform throughout until the actual zero time. (See Carslaw and Jaeger, 1959 , Sections 1.5, 6.6, and 8.4 for a partial proof.)

This device means that solutions for much more difficult problems or unidirectional heat flow for less familiar shapes might be extended to new three-dimensional solids, e.g., the solution by Iwata (1940) for oval cans perhaps might be expressed as a product of solutions for elliptical cylinders and the infinite slab, that solutions for potatoes could be expressed as products of the solutions for the surfaces which make up the prolate spheroidal coordinate system.

Although such solutions or convenient tabulations very likely would not be available, this principle suggests a means of expressing experimental results to account for changes in geometric factors.

Calculated temperatures for complex processes.--Riedel (1947) extended the treatment by Thompson (1919, 1920), for uniform cylinders whose surface temperature suddenly changes from ambient to the ambient in the processing equipment and, thence, rises linearly to the processing temperature and remains constant thereafter to include a processing temperature whose change could be described by any number of jumps and linear rises. Theoretically, any variation in the process temperature could be approximated by breaking the curve up into enough segments. There are practical limitations, however, since each term in Riedel's simplified formula represents an infinite series which converges only slowly and extensive tabulations like those of Olson and Schultz (1942) for the constant temperature portions are required. Even the conventional process at a constant temperature but with linear come-up and come-down times requires what to some would seem like an unreasonable amount of computation time. Riedel employed the formulas and tables that he constructed for a one-liter can (9.9 x 12.2 cm. or about a 314 x 413, i.e., a 3-14/16 in. diameter x 4-13/16 in. high can as designated in the United States canning literature) to illustrate various strategems, such as preheating or heating at a high temperature, then processing at a lower temperature (a process quite similar to the 'soaking' of billets in the metal industry) in order to achieve greater temperature uniformity, hence, less severely over-processed food at high temperatures.

Hicks (1951), following the techniques described in Carslaw and Jaeger (1959), developed an equation for the cooling portion of a simple process with instantaneous come-up and come-down and no surface resistance. This equation was then used to calculate the temperature histories (and subsequently the lethal process) and many positions within representative sized containers. The first term approximation of the series, such as used by Ball (1923) (see Figure 6.2), was employed except for the initial portion of the cooling process (where the terms converge slowly). This latter portion of the series was expanded in terms of error functions, and hence this series converged quickly at small values of time. The solution for the cooling portion was shown to be adequately represented by a straight line when the time was large as compared to the time at which cooling began.

Gillespy (1953) tabulated the necessary functions for shapes other than that calculated by Riedel (1947). Appended to this paper is a very complete restatement (in English) and development of the formulae used by these investigators.

This method leads to the use of one of the general methods described above. Some sense of the heat penetration phenomena is retained, however, if, as Gillespy (1953) requires, one obtains the thermal characteristics of the product from the f and j of a simple process.

Jakobsen (1954) has illustrated that Riedel's techniques can be adapted to his hyperbolic secant approximation to the heat penetration curve.

Hayakawa and Ball (1961) illustrated the Hankel, Laplace, and Finite Fourier transformations with some familiar and with some new assumed changes in surface temperature. The simple case of instantaneous come-up and come-down is illustrated. Also illustrated are the cases of linear come-up and come-down, variants of the cases presented by Riedel (1947) and Gillespy (1953) which might correspond to those encountered with a hydrostatic cook without holding in steam, and similar come-up and come-down but with a hold. A second type, the case of exponential come-up, then a hold, followed by a linear come-down is also a variant of the general problem as considered by Gillespy (1953). An analytic solution for the temperature with a quadratic come-up, a hold and a linear come-down also was presented. With an appropriate choice of constants this approximation should give results quite close to those for the exponential come-up case described above. They gave a more thorough consideration of the effect of time during the cooling process on apparent f and j than did Hicks (1951). They also considered the f and j for the linear come-up and come-down with hold case. The parameter f if measured at long times as compared to the come-up or come-down times was found to be unaffected by these regions. The parameter j, however, was affected.

Numerical approximations of cooling processes.--Charm (1960) has approached a related problem--cooling a container below that mass average temperature (equilibrated) which would readily support thermophiles, using finite difference equations. These equations which are applied to an infinite cylinder (since in a retort stack the heat transfer through the ends might be low) might also be extended to finite cylinders.

Numerical solutions by finite differences invariably lead to computer solutions. One of the practical problems with this method is stability and the small time increments often required for stability. New techniques are being developed (e.g., Brian, 1961), but others (e.g., Elrod, 1960; Goodman, 1960; Erdogan, 1962) are using computers to solve the integral equations for the same problem.

Numerical solution and subsequent integration of the corresponding lethal value are as flexible as tabulations of experimental temperature and are not restricted to constant or isotropic properties or simple environmental temperature histories. Like all predicting methods they require specification of property values which are often lacking. It is ironic that one way of determining interior or external properties is the analysis of the time-temperature histories of simple shapes like the sphere and the finite cylinder when subjected to a sudden change in environmental temperature (Ellerbrock and Ziemer, 1950; Chung and Jackson, 1954; Boberg and Starrett, 1958; Paschkis and Stolz, 1956a, 1956b; Stolz et al., 1959; Beck, 1962).

The Effect of Initial Temperature Distribution

In the derivation of the formulas which form the basis of Ball's (1923) heat penetration parameters, it was assumed that the initial temperature was uniform. Since containers may have just been hot filled or exhausted or may be waiting in a crate for processing, the assumption is infrequently met.

In the above consideration of temperature histories at short times, it was pointed out that the initial temperature might be taken into consideration by prescribing suitable previous temperatures T_1. The wide range of conditions which might be encountered following filling but prior to retorting make an analytical approach seem rather academic and impractical. Olson and Jackson (1942), however, have presented a simple formula for j in terms of T_w, the temperature at 0.1 in. from the can wall (approximately the surface temperature); T_1, the retort temperature; and T_o the initial temperature at the center.

$$j = 1.27 + 0.77 \, (T \;\; - T_w)/(T_1 - T_o)$$

No derivation of this linear relationship is given. It is interesting to note that at a uniform initial temperature j is that for a finite cylinder, while if the side wall were at retort temperature already the j becomes that for an infinite slab. Their experimental check showed this relationship gave reasonable results for values of the second term near one.

Alstrand and Benjamin (1949) made an experimental study which resulted in a recommendation that an average (mass average) temperature at the beginning of the process be used as an initial temperature instead of the above.

Conversion of Data to New Container Size

The desirability of being able to convert data for a product in one can size to estimates for another size was recognized by Bigelow *et al*. (1920), Ball (1923, 1928), and by Cheftel (1935), Olson and Stevens (1939), and Tabbert and Farrow (1941-1942), Townsend *et al*. (1948), Fagerson (1950), Townsend *et al*. (1956), Ball and Olson (1957), and undoubtedly by other investigators as well. All of these tables are for the simplest case, that of a cylinder at uniform initial temperature whose *surface temperature* suddenly was changed to a new level and no head-space resistance. Unfortunately, the corrections for seam heights and widths and head-space, and no head-space resistance appear not to be uniform and have only infrequently been reported.

Such conversions are of great potential importance and might be used as a tool to distinguish between conduction and convection (Jackson and Olson, 1940; Ball and Olson, 1957).

Temperature Dependent Thermal Properties

Evans (1958) made an experimental and analytical investigation on the effects of variability of product thermal properties on the apparent f and j. Since suitable transformations were not available to reduce the equation to an ordinary differential equation, the equation was integrated by the numerical procedure of Crank and Nicolson (1946). Such analytical results are, unfortunately, applicable only for the several conditions investigated although qualitatively the results are applicable to most processing conditions. The heat penetration curves were shown to take the same form as those without property variation. The f and j values were affected, however. These effects are tabulated in Table 6-1.

The effect of f and j seem to be opposing so that the resulting effects on the process value are decreased. The f approaches that which would be computed if the properties at the cooling or heating temperature are used. This was attributed both to the long portion of the process during which the product is nearly at the process temperature and to the increase in thermal diffusivity with temperature encountered here. It was also pointed out that temperature histories, due to the cylindrical and the plane surfaces, approach the asymptotic approximation at different rates so that for containers with one much larger dimension the linear portion is approached much more slowly than would be anticipated at first. It seems reasonable to believe that property value variation, film resistances, and the relative contribution of heat transfer from the lateral and end surfaces will assume much higher importance for convectively heating liquid and liquid-solid products. The failure of the thermal diffusivity to increase monotonically, i.e., always increasing, in the region above

250°F. and the differing temperature dependency of viscosity may well produce some interesting anomalies for high temperature, short-time processes in containers.

TABLE 6-1. Influence of some factors as they affect heat penetration in a No. 1 tall can (Evans, 1958).

	Heating		Cooling	
	f	j	f	j
Property variation	~RT (decrease)	+7%	~ at CT (increase)	-10%
Headspace resistance	+3.6%	-4.5%	+3.8%	-4.5%
Differing asymptotic approach of energy from lateral and end surfaces	+1.8%	-7.5%	+1.7%	-10%

Tsang (1960) has pointed out that variation in property values, i.e., $C_p \cdot \rho$, the product of product specific heat and density, from an analytical standpoint cannot be considered to be summed in a temperature dependent thermal diffusivity, but that the temperature dependence of the thermal conductivity and heat capacitance should be considered separately. Beulter and Knudensen (1953), however, have shown that the effect of density change is much smaller than that for the change in thermal conductivity.

Heisler (1946, 1947) has recommended that the properties of the surface film corresponding to the time-average temperature be used to calculate central temperatures. This procedure requires iteration because the surface

history is not known at first. The properties at two-thirds the estimated surface temperature (the environmental temperature at long times) are used in the initial calculation. In his problem the change in the surface heat transfer coefficient caused the major change in the Biot Number, N_{Bi}. This dimensionless group may be thought of as the ratio of the film coefficient h to the conductance of the solid, k/L, where L is the distance from the surface to the coldest point in the interior of the solid.

Finite Surface Film Coefficients

Okada (1940a, 1941) expressed Ball's heating rate parameter f in terms of the analytical expression of Thompson (1919), for finite cylinders, and Berger (1928), for rectangular solids, when, as in cooling, the surface heat transfer coefficient is important. The parameter f is a function of the length to diameter ratio, the container size, the thermal diffusivity and the Biot number, and if the surface heat transfer coefficient is very large, f is a function only of the first three of these factors. He was able to use the ratio of experimental data, f/f_∞, for two cases, a finite surface heat transfer and the very large heat transfer coefficient, respectively, to estimate the overall surface heat transfer coefficients. (An estimate, only, for one must know k, the surface heat transfer coefficient and the characteristic length to calculate the Biot number N_{Bi}.) Since the effect of film coefficient on the heating of canned fish products in boiling water was c

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presumed large, Okada was able to show that cooling in air was from 5.60 to 54 times as long and in water was 1.1 to 10 times as long as heating when the film coefficients ranged from 1.4 to 1.9 and 6.6 to 26 Btu/hr. ft.^{2} ^{o}F., respectively.

Studying conduction heating products, Evans and Board (1954) considered two factors: the effective conductance through the headspace and the effective conductance elsewhere on the container that are of great importance in convection heat transfer in containers. They established that in their equipment the film resistance on the outside of their container was, as is usually presumed for processes in steam or water, negligible for all practical purposes (h = 400 Btu/hr.ft.^{2} ^{o}F.). They found from experimental investigations--analyzed according to their development for a container with a finite top surface resistance--that headspace resistance was substantial when compared to the resistance on the other surfaces. It is understandable that Jackson and Olson (1940) found no significant effect of headspace for their conduction heating product since Evans and Board found no great difference in heating rate between the geometric center and the actual slowest heating point within the container.

Ball and Olson (1957) considered the effect of film coefficient on the heat penetration parameters f and j. They showed that for large Biot numbers, ha/k or hr/k, corresponding to the film coefficients of the order of several hundred for the agitated water in well mixed water baths or retorts or as large as the thousand or more Btu/hr.ft.^{2} ^{o}F. of film coefficients expected for processing in steam, that f and j were independent of the surface film coefficient. They anticipated, however, that h could become important in cooling cans. Relations were presented for the effect of Biot number on f and on j. These tables can be most readily interpreted as the effects of surface conductance as compared to product diffusivity (most product diffusivities would be anticipated to be near that for immobile water) for given size container of equal height and diameter. They also maintained that there were negligible effects for N_{Bi} as low as 10. These or similar tables are useful tools in determining surface conductances from heat penetration data. If the size of the conductance causes it to be important, these same tables could be used to correct for their effects when converting to new can sizes or new process equipment. These tables could also be used to determine the cooling characteristics of meat, fish, vegetable, or fruit products. The implications of this work on can factors, temperature distribution, and the technique for determining film coefficients as well as the aforementioned factors will be treated in more detail in later portions of this review, and have been illustrated in the experimental phases of this study.

Summary of Available Relations for f and j

Some of the available solutions which have been discussed are presented in Table 6.2. Particular attention is given to step changes in temperature for negligible and

Table 6.2 Summary of relations for f and g.

Table 6.2 Summary of relations for f and j.

Method	Geometry	Root equation	Intercept[a]	Slope (tan Ø)
Cooling rate	all	-	1	$\frac{-C_r}{2.303}$
Half-cooling time	all	-	1	$\frac{+\log(1/2)}{Z}$
Newtonian cooling	all	-	1	$\frac{-hA}{2.303C\ V}$
Leggett and Sutton	all	-	bump factor	$\frac{-K}{2.303}$
Backstrom	slab	$B_1^2 = \left(\frac{+8N_{Bi}}{3N_{Bi}+8}\right)$	$\left(\frac{2N_{Bi}}{3N_{Bi}+8}\right)e^{-0.114B_1^2}$	$\frac{-B_1^2}{2.303b^2}$
Rutov	slab	$B_1^2 = \left(\frac{2.5N_{Bi}}{2.4+N_{Bi}}\right)$	$10^{\left(\frac{0.11N_{Bi}}{1.3+N_{Bi}}\right)}$	$\frac{-B_1^2}{2.303b^2}$
Rutov	cylinder	$B_1^2 = \left(\frac{6N_{Bi}}{2.85+N_{Bi}}\right)$	$10^{\left(\frac{0.22N_{Bi}}{1.7+N_{Bi}}\right)}$	$\frac{-B_1^2}{2.303r^2}$
Rutov	Sphere	$B_1^2 = \left(\frac{10.3N_{Bi}}{3.2+N_{Bi}}\right)$	$10^{\left(\frac{0.33N_{Bi}}{2.1+N_{Bi}}\right)}$	$\frac{-B_1^2}{2.303r^2}$
Baehr	all	same as those of Ball and Olson below	$e^{0.10B_1^2}$	$\frac{-B_1^2}{2.303(b^2 \text{ or } r^2)}$

Table [illegible] (continued)

Table 6.2 (continued)

Method	Geometry	Root equation	Intercept	Slope (tan ∅)
Ball	all	-	j	$\frac{-1}{f}$
Ball and Olson	slab	$N_{Bi} = B_1 \tan B_1$	$\frac{2 \sin B_1}{(B_1 + \sin B_1 \cos B_1)}$	$\frac{-B_1^2}{2.303a^2}$
Ball and Olson	cylinder	$N_{Bi} \cdot J_o(B_1) = B_1 J_1(B_1)$	$(\frac{2}{B_1})(\frac{J_1(B_1)}{J_o^2(B_1) + J_1^2(B_1)})$	$\frac{-B_1^2}{2.303r^2}$
Ball and Olson	sphere	$N_{Bi} = 1 - B_1 \cot B_1$	$2(\frac{\sin B_1 - B_1 \cos B_1}{B_1 - \sin B_1 \cos B_1})$	$\frac{-B_1^2}{2.303r^2}$

surface resistance and to the generalized solutions of Riedel (1947) and Gillespy (1953). It will be shown that these solutions can be applied to products in which convection is suspected in order to confirm its presence by contrast, and may be applied to products having limited convection in small, confined space. The latter products correspond to group 2 of Jackson's (1940) classification.

Shown in Table 6.3 are the values for the slope, intercept, f, and the root of the boundary equations used by various investigators for calculation of the cooling times of fruit. Most of these are reasonable approximations only for the low Biot numbers, i.e., film coefficients of less than 1 to about 35 Btu/hr.ft.^{2} ^oF., encountered in that problem. In some instances researchers have assumed that there is no internal resistance, hence j = 1. These researchers' cooling rate coefficients in terms of f are also given.

Errors in the Determination of the 'Heating' Curve

The last assumption inherent in the use of the first term of asymptotic approximation (Ball, 1923) is that the tangent (or cord) is a good approximation of the asymptote and that this tangent can be readily attained.

Several writers have considered the effect of container shape and the value of u, the unaccomplished temperature rise, on the lag factor. Schultz and Olson (1938) presented a graph for the relationship between the j of the tangent to the heating curve at a specified point on the asymptote.

Cowell and Evans (1961) improved the accuracy of this graph and restated the problem in terms of the point on the heating curve. This graph is a family of bell-shaped curves which show a marked decrease in j as the square ratio of half-height to radius departs from values near 0.8 and which show an increasing reduction in j as the u value decreases.

Cowell and Evans (1961) also showed that the errors in f (with respect to the asymptote) are of similar shape (but of opposite direction and approximately several times less than those of j). The parameters f and j, therefore, vary in opposite directions due to these factors. These analytical relations are confirmed by the earlier experimental work of Evans (1958).

Since the point at which the tangent to the smoothed curve is drawn affects both the f and j, the weight of each datum of heat penetration curves will vary. This observation suggests that regression lines fitted to the "straight portion" of heat penetration curves (Hurwicz and Tischer, 1956a, 1956b) should include such weighting of the data. The curve might be fitted visually, weights for points as calculated from this first line would be found from tables and be used to calculate a new line. (Such a procedure, but for fitting probits to biological response data, has been described by Finney, 1952). If the necessary tables were to be calculated, they might well include the effect of the head-space resistance and the property variation, both of which cause nearly equivalent errors as shown in Table 6.1. Since Evans and Board (1954) have shown that the effect of can geometry (as it influences the Biot number) is appreciable, such tables of weights for j and f as a function of geometry and point on the heat penetration curve would also need to be tabulated for each of several Biot numbers. If the properties of the product are assumed to vary like those of water for a process from an arbitrary initial temperature, e.g., 130°F. to an arbitrary retort temperature, e.g., 240°F., no additional factors would need to be tabulated.

The influence of measurement errors.--The tables of weight proposed above to account the convergence, headspace, and property variation errors could readily grow to unmanageable size. There are measurement errors of nearly as large potential effects as these errors in assumptions. These measurement errors include the systematic conduction error induced by the presence of thermocouples (Board, 1953; Ecklund, 1956; Cowell et al., 1959; Baker, Ryder, and Baker, 1961), limited sensitivity (random) and accuracy (systematic) of the instrument-thermocouple system and the inaccuracy of positioning of the thermocouple junctions in the product, but more especially in the heating fluid. There is also the error due to temperature variation of the heating fluid (Ayrton and Perry, 1878; Townsend et al., 1956, pp. 13-10).

Cowell et al. (1959) have shown that both f and j are decreased by the presence of a thermocouple, since the diffusivity and conductivity of the wire are greater than those of the product. The error in f and the larger error in j were substantially reduced when 30 to 40 SWG wires were used. The error can be further reduced by using thermocouple materials other than highly conductive copper and if supports such as Ecklund (1949) thermocouple rods are used. The properties of the bulky rod and the product should be matched as closely as possible.

The conduction errors might be further reduced by using the maximum possible immersion length (Mohn, 1948; Rizika and Rohsenow, 1952; Beck, 1961; Baker, Ryder, and Baker, 1961) preferably parallel to the controlling heat transfer surface (Beck, 1961), although Cowell et al. (1959) found no advantage for such an arrangement.

In spite of these above precautions errors of several percent may remain. The problem is to calculate what the temperature history would be if the thermocouple were not present. Beck (1961) has illustrated for the semi-infinite solid how, with the aid of a computer, the actual temperature history could be calculated knowing the constant properties of the thermocouple and the product and the measured temperature history. One might also assume, as Evans (1958) did, that effects of relative errors on f and j were additive. Then one might modify Beck's approach to that for a finite cylinder; assume thermal properties like those of water and an arbitrary but generally acceptable thermocouple assembly; and a surface coefficient in the range of 275 to 400 Btu/hr.ft.^{2} ^{o}F. found by the C.I.S.R.O. workers; and tabulate the relative error in f and j at the center as a function of the square of the length to diameter ratio with the number of $\log_{10}$ cycles along the heating curve at which the measurement is made as a family of curves.

Inadequate sensitivity of the instrument thermocouple system, such as might occur if a long, fine thermocouple of low electrical (and thermal) conductivity is used, may cause the instrument to give stair-like heat penetration curves in the region close to processing temperature. The errors in accuracy of the instrument are more serious, however. These affect the heat penetration curve in much the same manner as an error in positioning the thermocouple in the heating fluid, since in both instances the effect is the same as an error in retort temperature. If T_x, the assumed retort temperature, is subtracted from both sides of an analytical expression for the temperature at the center of a can, a relation like that of Cowell and Evans (1961) for the temperature in the center of a can, it can be shown that at long times:

$$\log \hat{j} = \log \frac{T - T_x}{T_o - T_1} + \frac{t}{f} \qquad 6.1$$

and

$$\frac{(\hat{f} - f)}{f} = \frac{T_1 - T_x}{T - T_1} \qquad 6.2$$

where T is the temperature at time t, T_o is the uniform initial temperature, and T_1 is the true retort temperature. Frequent calibration of the instrument-thermocouple system and of thermocouples with one another, placement of the external fluid temperature thermocouple closed to the container and precise control of retort temperature (Townsend et al., 1956, recommends control to within ±0.25 F. for processing in steam and allows somewhat wider limits for processing of water) can be used to reduce this error. The author has illustrated a technique, that of using the equilibrated temperature at the end of a process which can be used to avoid some of these difficulties.

Ball and Olson (1957) have shown that the lag factor, at the center of a cylinder, is only slightly affected by position errors. An 8.3 percent radial or a 4.5 percent longitudinal displacement is required to produce a 1 percent error. Position errors are critical, however, when the difference in the lags at two or more points are to be used to obtain estimates of surface heat transfer rates (see Baker, Ryder, and Baker, 1961, pp. 223-5). Nicholas (1962) in Motawe (1962) has shown that this conclusion is also true for a spherical geometry.

The influence of drift of the environmental temperature.--Although it is desirable to achieve the control of the processing temperature within ± 0.25 °F. as recommended by Townsend et al. (1956), it is not always possible. Indeed, the author knows of no published data for heating in laboratory retorts which meet this criterion. It is increasingly more difficult to control the temperature in water baths, cooling canals, steam-air atmospheric pasteurizers, and refrigerated storages than it is in steam. It is, therefore, necessary to employ some method of correcting otherwise satisfactory data for such changes. If an analytical approach were desired, one could use the relations of Thompson (1919, 1920), Riedel (1947), or Gillespy (1953) for a linear change in an otherwise steady environment temperature, or to Gillespy's solution for exponential come-up. Approximate methods may be more satisfactory in practice, however.

Ayrton and Perry (1878) used two different methods to correct for a changing environmental temperature. The first was the use of the instantaneous environmental temperature. This strategem would seem to be most appropriate for a body with a nearly uniform internal temperature. Heisler (1946), has shown that the temperature at the center lags behind that of the surface by a time, which in Ball and Olson's (1957) nomenclature is equal to f·log (j surface/j center) e.g., f [log cos β_1 + log J_0 (γ_1)] where β_1 and γ_1 are the roots of the equations imposed by the boundary conditions for a finite cylinder. The second stratagem was the differentiation of the Fourier series equation for a sudden change in environmental temperature to obtain in our nomenclature, the equation:

$$\frac{1}{T-T_x}\frac{d\,(T - T_x)}{dt} = \frac{1}{T_o - T_x}\frac{dT_x}{dt} - \frac{2.303}{f} \qquad 6.3$$

This equation was solved for f and this value was substituted in the original equation to find the j value and from this the surface film coefficient as well.

A procedure for the analysis of response data, e.g., as a can in a retort crate or a fruit in a box, when the environmental temperature itself can be said to be described by a lag and a slope, is presented in Section 11.2, "Transfer functions from step response," of Caldwell, Coon and Zoss (1959). This method consists of a visual fit of the best asymptotic line to a plot of incomplete response vs. time, i.e., a heat penetration curve, followed by a plot of the difference between the experimental curve and the asymptote and then linear approximation. This process is continued until experimental and computational errors become excessive. Usually up to three terms can be obtained. A similar procedure has been suggested by Richards (1953) for cooling of fruit. This analysis procedure has one serious fault: the physical mechanisms causing these responses are not defined. One must rely on prior knowledge of the system or parallel analysis of the system components in order to determine individual responses and thereby an approximate mechanism. This means that one should measure the fluid temperature and if the surface coefficient on the solid product is not known, it should be measured also.

The variability of conduction heating data.--Hicks (1961) has made a thorough analysis of the uncertainties in canning process calculations, including the thermal heat transfer parameters f and j. He suggested that coefficients of variation of 7 percent for f are often encountered and sometimes reach as high as 15 percent. There appeared to be little information about the variability of j. It was pointed out that many factors affect both f and j, although j seemed to be affected more. Some of the factors, such as

property variation, headspace resistance, variation in drawing the curve, overall surface resistance, and retort temperature errors cause opposite effects in f and j and others, such as thermocouple conduction, error, cause, f , and j to change in the same direction. It would appear that the position in the object has little or no direct effect on f . The initial temperature distribution also seems to have little direct effect on f . However, these last two factors do affect j substantially. Disregarding these two instances of the independence of f and j, it would not be surprising if some factors did not appear to affect the sterilization value of a process, since f and j are oppositely affected by so many of the above factors.

Factors to be Considered in the Analysis of Conduction Heating Data

Drawing the straight line approximation to the heat penetration curve is a subjective skill and it represents a compromise between the desire to use data at long times (to assure the validity of the use of the first term approximation) and the desire to avoid errors, the measurements which increase in magnitude as T approaches T_1.

Townsend *et al.* (1956) recommend: "draw a straight line through the points, ignoring the lag period and attempting to place the line not more than 1° from any point above 200-210°F. for low acid products, and 140°F. for acid products." They noted that a comparatively large variation in the position of the line when it nears retort temperature is much less significant in terms of temperature difference than a comparatively small variation at low temperatures. These appear to be workable criteria for drawing. The suggestion corresponds to the recommendation that values of u of about 0.4 [e.g., $u = (240 - 200)/(240 - 140)$] be considered. This is nearly the criteria of a $u < 0.3$ that Baehr (1953, 1959) recommended for Biot numbers of approximating 35 and below. (Baehr, however, used the controlling geometry rather than all surfaces to estimate the time required, hence his estimates are too low.) In terms of the number of log cycles on the heating curve p' of Cowell and Evans (1961), these criteria are about .4 to .5, since $p' - -\log_{10} u$. The lowest p' value for which corrections for j and f are given is 0.6, however, corresponding to a u of .25 and hence a temperature of 225°, 15° below the retort temperature in this illustration.

Townsend et al.'s (1956) criteria did not include a consideration of an upper limit beyond which thermocouple errors might become excessive. This author's own arbitrary practice has been to use data not closer than 5°F. to the retort temperature. If Townsend's criteria of control, i.e., ±0.25°F is attained at 5° below processing temperature, the relative error in f is $(1/4^{\circ})/(5^{\circ})$ or 5 percent; however, if the control is ±1.0°F., which seems good for the data reported in the literature, the error is 20 percent. What, however, is the convergence error if data closer than 5°F. to heating or cooling temperature are not used? Returning to the above example, one finds that at 5° below retort temperature $u = 5/100$ or 0.05, hence $p' = 1.3$. At this value of p' the error in f is less than 0.8 percent and the error in j is less than 6.9 percent for the geometric ratios plotted by Cowell and Evans.

This example points out that the seemingly usable data lie in a rather narrow range, that is, from a u of 0.4 or 0.5 to $u \approx 0.05$. An extension of Cowell and Evans' (1961) charts for the usual sizes in which the can diameter is about equal to can height would permit extending the range of useful data. There appears little hope of reducing the 5°F. criteria suggested as an upper limit, however. To those interested in cooling at lower film coefficients this example may point out another need--a need for similar charts for film coefficients other than that for Biot number > 100.

When utilizing the Cowell-Evans charts, the author suggests that the log arithmic mean correction $(H_1 - H_2)/\ln(H_1/H_2)$, be used where H_1 and H_2 are the relative errors at the upper and lower criteria used.

There appears to be no great need for charts to correct for thermal property changes if one only wishes to fit the heating data with the best straight line, a line where slope and lag are to be used in lethality calculations for this product under these experimental conditions. If this were not the case, charts for one range, with a relative property change, h of about 400 as calculated by a Crank-Nicholson (1946) procedure might be quite useful. Some such approach is useful if diffusivity values are to be calculated from transient data.

It appears from the consideration of Cowell et al. (1959) and Ecklund (1956) that thermocouple corrections for j (and f as well) should be used. Since Cowell et al. have shown that the rod and the wire size used by Ecklund as reference thermocouples introduce errors, there appears to be a need for new experimentation using improved reference thermocouples. The new experiments might evaluate these so-called Ecklund reference couples and an internal fitting, rather than the 'chisel'-nose, plastic, rod-type thermocouple.

The exponential come-up time corrections of Gillespy (1953) appear preferable to those of Ball (1923).

The lethality of the cooling portion of a thermal process is said to usually lie between the estimate of Ball (Ball and Olson, 1957) and that of Gillespy (1951), and Board (Board et al., 1960). Ball's method, since it is conservative, would seem to be preferable. It is expected to fit best for small temperature differences at the end of heating time, and for immobile products. The general method should be used to evaluate a process if there is any reasonable doubt.

It appears that a computer would be required to examine the heat transfer phenomena and a general method to evaluate the lethality for complex processes. It would also appear that Gillespy's (1953) procedure is a workable method to predict lethality for a complex thermal process before experimental confirmation of the process is selected.

For slight changes in retort temperature, the use of the exterior temperature one lag time f log j earlier as the

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effective environmental temperature appears workable. More extensive changes might require the systems analysis approach.

Convection Heat Transfer

Transient natural convection in enclosed spaces is a complicated phenomenon as contrasted to conduction for there is flow as well as conduction. In its simplest sense, this phenomenon is the transient equalization of a nearly uniformly mixed fluid with a heat transfer resistance to the container wall. If one assumes that the temperature is uniform throughout, then the problem is analogous to so-called Newtonian heating of a solid of uniform temperature throughout. If, on the other hand, one assumes that the heat transfer by convection (bulk movement) is isotropic and constant throughout, then the analogous process is a solid whose effective thermal diffusivity is the sum of the conductive and the eddy diffusivities. When the convective or eddy diffusivity became small, as it might at small temperature differences, or in interstices of tightly-packed vegetable or fruit products, then the phenomenon would approach true conduction.

Before the advent of any proposed mechanisms or the development of methods of analysis of transient heat transfer data, investigators may have been intuitively applying these two different concepts. Early investigators of convection heating measured the temperature at the center of the containers the same as for conduction (see Ball and Olson, 1957). (In both of the idealized models this is the location of the least severe lethal treatment.) These investigators recognized the uniformity called for in the first model could be achieved in rapidly convecting or rapidly agitated products.

These two idealized approaches are essentially the only workable ones available today, although Okada (1940) has suggested that data might be correlated in terms of dimensionless groups which represented the conduction and convection contributions, and currently the assumption of a quasi-steady film coefficient is being used in engineering studies of transient convection on simple surfaces, such as flat plates. Since much of the experimental investigations are contrasts with the phenomena predicted by these approaches and an understanding of the approaches would enhance interpretation, two simple approaches will be reviewed below.

The Effective Diffusivity Model

Thompson (1919) used the second approach--the measurement of the apparent effective thermal diffusivity which water and/or brined vegetable products would have if the conduction theory applied. This diffusivity seemed to depend on the ratio of brine and solid product volumes. This diffusivity might also be expected to vary during the process, because fluid properties, such as viscosity, are much more temperature dependent than thermal conductivity. If the requirement that the experimentally determined effective diffusivity be isotropic is met or, in practical terms, the temperature distributions for convection heating products are like those for conduction heating, then all of the previous work on the effects of come-up, on complex processes, on can factors, and on convergence errors may be extended to convection heating.

Tani (1940), in his study of flow patterns and temperature distribution, relaxed the restriction of isotropism to permit the effective thermal diffusivity to depend on axial position in the container. He found that these local effective diffusivities are about 24 times those of still water. The time-temperature histories at a point can be analyzed using heat penetration curves, but conduction relations, discussed above, can no longer be used to predict the effects of come-up, etc.

The Uniform Temperature Model

Jones (1931) applied the convective approach of the well mixed fluid with surface heat transfer resistance. For this case the heat added by conduction is:

$$\frac{k}{\delta} = A\ (T_0 - T)$$

where:

A is the area of the film

δ is the constant film thickness (always small)

k is the constant thermal conductivity of the liquid

T is the mixed mean temperature of the fluid

T_0 is the initial temperature of the bath

T_1 is the temperature of the external bath

Heat is absorbed by the 'stagnant' layer of the fluid (neglecting the heat capacity of the film), hence:

$$\frac{k}{\delta} A \, (T_o - T) = V\rho \; C_p \frac{dT}{dt} \qquad 6.4$$

Where V is the fluid volume

C_p and ρ are the specific heat and density, respectively,

dT/dt is the instantaneous rate of rise of the fluid temperature.

The solution of this equation is:

$$\frac{T - T_1}{T_o - T_1} = e^{-\frac{A}{V} \frac{\alpha}{d} t} \qquad 6.5$$

where α is the thermal diffusivity. In this equation, A is taken as the area of the fluid in contact with the can neglecting the headspace area, and V is approximately the volume of the can. This formula was applied to the data of Magoon and Culpepper (1921) for distilled water in a No. 10 metal can whose initial temperature was 20° and whose final temperature was 100°C. At a temperature of 90°C. the appropriate film thickness was found to be 0.035 inch (a surface overall coefficient of heat transfer U of about 130 Btu/hr.ft.^{2} °F.). This value was applied to the Magoon and Culpepper (1921) data for No. 2 and No. 3 tin cans. Since these calculated data were said to correspond in position and in form (on a rectilinear plot of T vs t) to the experimental curves Jones suggests, "This (correspondence) suggests that the thickness of the film does not depend upon the size of the container though it will of course vary with the nature of the contents." (This assumption is true only when boundary layer flow is fully established.)

Schultz and Olson (1938) examined convection heating with the assumption of a uniform fluid temperature in order to obtain an estimate of factors to be used to convert convection heating data from one container size to another. They showed that in Ball's nomenclature:

$$f = 2.303\,(C_p \rho V)/VA \qquad 6.6$$

If the product is the same, h might be assumed to be independent of can size, then if the total surface area (including headspace) is effective and V is taken to be the container volume, the ratio of f 's for two sizes becomes:

$$\frac{f}{f'} = \frac{r\,L}{r + L} \cdot \frac{r' + L'}{r' L'} \qquad 6.7$$

where $\frac{r\,l}{r + L} = \frac{V}{A} = \frac{\text{harmonic mean of a and b}}{2}$

The Uniform Temperature Model with Variable Surface Heat Coefficient Transfer

In both of these treatments of a uniform temperature solid the lag factor was found to be unity. Ball and Olson (1957) have suggested that variation in the overall coefficient of heat transfer U (or k/δ) helps explain experimental values of j, values which were said to be less than unity.

The overall heat transfer coefficient, U, varies directly with the magnitude of the convection currents, currents which are induced by differences in buoyancy due to temperature differences. The overall heat transfer coefficient, U, is, therefore, anticipated to be greatest at the start of the equalization process. As these temperature differences decrease the value of U also decreases. First, conduction, as well as convection, becomes important. Ultimately the overall heat transfer coefficient U becomes that for conduction. These changes produce heating curves which are concave downward, hence j values less than unity are anticipated. It was suggested that "it should be possible to get a good correlation between U_0, U_∞, and j where U_0 is the initial and U_∞ the final coefficient of heat transfer." This is perhaps an oversimplification for although the initial and final values are for convection and conduction and hence initial and final values of f might be prescribed the position of these lines, hence, j depends on a knowledge of the temperature property value history, hence this approach would require that U be a prescribed function of temperature product properties and container factors.

Sullivan and Holland (1961) have treated an analogous problem, the determination of the area (time for our problem) for a fluid in a concentric pipe heat exchanger (can) to reach a given final temperature given specified property variation, the initial temperature, and a dimensional equation for forced convection (agitation). The applications of this for our cases are limited, however.

Since this reasoning seems sound, other factors must be responsible for the many convection investigations in which the j values are reported to be larger than unity.

Charm (1963) Variant of the Uniform Temperature Model

Charm (1963) has shown that if one assumes only the lateral surface to be effective for heat transfer then the ratio of f values for the same product is proportional to the ratio of the radii of the containers. (Under similar conditions for conduction, the f ratio would be proportional to the square of the ratio of the radii.)

He notes that "the Grashof modulus describes the conditions for similarity in the natural convection system," and from this concludes that one may calculate the temperature history for a new size (or product) if the temperature history in a container is known, and that the characteristic length in the Grashof number is the diameter of the container.

One must not succumb to the temptation of taking the logarithm of both sides of the relation:

$$N_{Gr} = \frac{D_1^{\,3} \rho_1^{\,2} g B_1 \Delta T_1}{\mu_1^{\,2}} = \frac{D_2^{\,3} \rho_2^{\,2} g B_2 \Delta T_2}{\mu_2^{\,2}} \qquad 6.8$$

solve for the temperature differences and differentiate with respect to time in order to use the known f for the first container, for one finds that the f's are identical since one does not know at what relative times the similarity is attained.

The choice of similarity criteria and characteristic length also seem to need further examination. It can be shown that the similarity of Prandtl number groups $(C_p\mu/k)$ is also required for natural convection heat transfer and if the core velocities are not small, the Reynolds number $\frac{Dv\rho}{u^2}$ is required as well. The diameter D is most frequently chosen as a characteristic length for horizontal cylinders and established flow in vertical cylinders while L is chosen as the length in the interior or exterior of vertical cylinders when the boundary layer is not restricted and $(r^4/L)^{1/3}$ is preferred for open thermo-syphon tubes, hence arbitrary choice of D for a characteristic length is suspect.

It might be preferable to substitute a dimensionless correlation for film coefficient (using $N_{Gr} \cdot N_{Pr} \cdot r/L$) into the Ball and Olson formula

$$\frac{f_1}{f_2} = \frac{C_1 \rho_1 V_1 U_2 A_2}{C_2 \rho V_2 U_1 A_1} \qquad 6.9$$

to obtain for Charm's case:

$$\frac{f_1}{f_2} = \left(\frac{C_1 \rho_1}{C_2 \rho_2}\right)\left(\frac{r_1}{r_2}\right)\left(\frac{k_2 r_1}{k_1 r_2}\right)\left(\frac{L_1 \mu_1 k_1}{r_1^4 \rho_1^2 g \beta_1 \Delta T_1 C_p} \cdot \frac{r_2^4 \rho_2^2 g \beta_2 \Delta T_2 C_{p2}}{\mu_2 k_2 L_2}\right)^n \qquad 6.10$$

or

$$\frac{f_1}{f_2} = \left(\frac{\alpha_2}{\alpha_1}\right)\left(\frac{\mu_1 k_1}{\rho_1^2 g \beta_1 C_{p1}} \cdot \frac{\rho^2 g B_2 C_{p2}}{\mu_2 k_2}\right)^n \left(\frac{r_1}{r_2}\right)^{2-4n} \left(\frac{L_1}{L_2}\right)^n \left(\frac{T_2}{T_1}\right)^n \qquad 6.11$$

where n is approximately 1/4 for laminar flow and approximately 1/3 for turbulent flow, and the second term is an inverse ratio of the respective Grashof-Prandtl numbers. This relation may be further simplified to:

$$\frac{f_1}{f_2} = \left(\frac{\alpha_2}{\alpha_1}\right)^{\frac{3}{4}} \left(\frac{\mu_1}{\mu_2}\right)^{\frac{1}{4}} \left(\frac{\rho_2}{\rho_1}\right)^{\frac{1}{4}} \left(\frac{r_1}{r_2}\right) \left(\frac{L_1}{L_2}\right)^{\frac{1}{4}} \left(\frac{\Delta T_2}{\Delta T_1}\right)^{\frac{1}{4}} \qquad 6.12$$

for laminar flow; hence, the f in convection is seen to be less sensitive to diffusivity changes than in conduction heating, proportional to the radius of the container, proportional to a power of the length of the side wall over which convection develops and directly proportional to the same power of the viscosity and inversely to the same power of the buoyant force.

The Plug Flow Model

In the above variants of the uniform temperature model there was no provision for the fluid film. Since the fluid film is assumed to be stationary, the heat transfer is across the film and into a stagnant core. If, on the other hand, one recognizes that (1) the heated fluid in the film becomes buoyant since it is hotter than the fluid in the core; (2) assumes that the fluid layer is of constant thickness, i.e., the fluid rises to the top of the container; (3) assumes that this fluid then sinks as hotter fluid covers it and descends to the bottom of the container before re-entering the rising film; and (4) assumes that the fluid rises so fast that the heat transfer between the rising film and the descending core is negligible, then the problem is analogous to the jacketed kettles with external heat exchangers, treated by Kern (1950). Inherent in this model is the assumption that heat transfer through the side walls is the principle mechanism, or alternately that there is some demarcation in the container between the regions heated principally by the lateral and bottom surface, respectively.

Kern presented two cases with implications for convection in cans. In the first the kettle contents of the jacketed kettle (can) are well mixed (conduction along the core is infinite), and in the second there is no mixing and axial conduction in the kettle (core of the can) is assumed negligible. For the case usually encountered in processing, the exterior temperature is constant, corresponding to a condensing or boiling vapor in heat exchanges, so only constant heat exchange fluids will be considered here.

In the first instance, with agitation, Kern found that:

$$2.303 \log \frac{T_1 - T}{T_1 - T_o} = \frac{-wC}{V\rho C} \left(\frac{K_2 - 1}{K_2}\right)t \qquad 6.13$$

where

$$K_2 = e^{UA/wC}$$

when w is the constant mass flow rate of the rising fluid stream in lb/hr. Hence, $j = 1$ and $f = 2.303 \frac{Ve}{Cw} \left[\frac{e^{UA/wC}}{e^{UA/wC}-1}\right]$.

This relation might be adapted to end-over-end agitation where the movement of the headspace through the container (see Clifcorn et al., 1950) is sufficiently rapid to cause uniform mixing of the core and additional heat transfer is obtained by inducing a velocity w due to the superimposed rotational axial body force. For still processes, although the natural convection is strongly temperature dependent, the above relation can be used to examine the heat penetration parameter f at some instant, assuming then that the heat transfer and fluid flow are quasi-steady. The overall coefficient of heat transfer is composed of the exterior coefficient (usually large), the container conductance (large only for thin metal or laminated film containers), and the interior film coefficient. This interior film coefficient, as it appears in the Nusselt number, hL/k, and the amount of liquid circulated due to natural convection are each functions of the Prandtl number $(C_p \mu)/k$, the Grashof number, $[(g\beta C\Delta T)\ L^3]/\mu^2$ and w is also a function of the heated area. Therefore, f is a function of the product mass, inversely a function of the product flow (which is a function of the cross-sectional area and the Grashof and Prandtl numbers) and directly a function of the bracketed terms.

When the flow in the central core, although slow, is so fast that axial conduction is negligible, Kern showed that the temperature at a point in the kettle remains constant until the interface of the hot fluid descends to that level, at which time the temperature increases suddenly and remains at this new level until all the fluid has once again passed through the heat exchanger.

Experimental temperature profiles, however, show a lag, depending upon position; an absence of step-wise response; and an axial temperature gradient. The "agitation," therefore, is not complete. Further, an assumption of negligible agitation or axial conduction can be partially justified only for the first circulation. Models of the behavior within the container ultimately should consider axial conduction.

Kern pointed out that agitation increases the film coefficient but for external heat exchangers, uniformity within the kettle means a lower temperature difference for heat exchange and hence slower heating. The analogous axial temperature gradient effects within containers therefore may need more study. These temperature gradient effects presumably are responsible for (1) the complex bracketed term in the above equation, (2) smoothing of the vertical profile, and (3) may influence the heating rate in the rising film.

Limitations of the Available Models

The available models suggest that the value of the heat penetration parameters f and j perhaps lie intermediate between those for an assumed uniform effective thermal diffusivity and those predicted on the basis of a uniform temperature distribution. The slope parameter f is seen to be a function of the product thermal diffusivity, buoyancy, the product viscosity, initial (or some other characteristic) temperature difference, and the container height and diameter, on the basis of an untested relationship. The lag factor j is perhaps subject to the time required for the core to circulate and is reduced by the tendency of natural convection to diminish during the process, but once again quantitative relationships are lacking. (Some qualitative relationships have been suggested as an outgrowth of the experimental phases of this investigation.)

There appears to be very little information even of an experimental nature on complex processes. The effect of initial temperature distribution appears not to have been investigated. Alstrand and Benjamin (1949) have shown the effect of come-up time on the equivalent lethal value of convection heating to be more variable than for conduction heating and that j was substantially increased while f was reported to be nearly invariant. Townsend et al. (1949) found similar behavior for convection in glass containers and considered convections for the lethal value of cooling. McConnell (1951) has considered the effect of drops in retort temperature upon the lethality of convection heating products. None of these investigations have been related to the models; hence, there is no indication as to what temperature difference and property values should be used in the formulae.

The heating medium, i.e., the exterior film coefficient, has sometimes been shown to influence the heating of fluid products. The container material (usually tinned steel or glass) has been shown to influence the heating of such products. Merrill (1948) combined these series resistances into an overall heat transfer coefficient U where

$$u = \frac{1}{\frac{1}{h_o} + \frac{L_c}{k_c} + \frac{1}{h_i}} \qquad 6.14$$

and h_o, h_i are the outside and inside film heat transfer coefficients, k_c is the thermal conductivity of the wall and L_c is its thickness. This is an accepted stratagem (see Carslow and Jaeger, 1959, Section 1.9) to simplify the relations for a composite body when the skin of the body is a poor conductor whose thermal capacitance may be neglected. When $1/h_o$ or L_c/k_c are large resistances compared to $1/h_i$, a dimensionless correlation for h_i can no longer be substituted directly into the Newtonian heating model relation for f . Merrill had moderate success in combining experimentally determined h_o, h_i, and L_c/k_c to predict f values. The combination of these two approaches, correlations for h_i and lumping of the heat transfer resistances seems to merit further consideration.

Since investigations of the thermocouple errors suggest matching of thermal properties of the product and thermocouple probe, it seems that the probes for convection should be highly conductive (at least 24 times that of water, i.e., at least as large as the effective convective thermal diffusivity of water as reported by Tani (1940)). They might also be small in diameter--not only to minimize interference with product flow. The need for long immersion length in a nearly iso-lag, i.e., an iso-j zone, as well as the need to reduce interference with flow suggests that thermocouples be inserted axially into the central core. Such techniques were used in the experimental phase of this investigation and have been discussed in the previous section. The effect of retort temperature errors would be expected to be approximately of the same magnitude as conduction. A drift in retort temperature might be anticipated by using an effective retort temperature somewhere between that at time t_a = f log (j surface/j center) earlier and the temperature at that instance, since there is some transport lag and inertial effects are presumed small.

There appears to be no published criteria on how to weigh the data when drawing a heat penetration curve for a convection heating product. It would appear that the line should represent the data in a region from about a u = 0.4 to u= 0.08 as in the conduction example, and if changes in slope above this region (u < 0.08), then the general method should be used.

It would now appear (after completion of the experimental work) that if temperature histories or f and j are to be reported for various positions they should be reported for a u at the cold point in this range rather than for the same u value but at different times. Some precision will be sacrificed for points warmer than the cold point but it now appears that the cold point temperature may be a useful tool in the estimation of f values. Since Ball and Olson's (1957) reasoning predict a strong curvature and thereby a relation between f and j, it would appear preferable to report either u or t as well as f and j. Then, when effects of the change in heat transfer coefficient U are known the data may be reinterpreted.

Combined Conduction and Convection Heat Transfer

The two principal models for ideal convection, the uniform temperature model and the uniform effective diffusivity model, seem to be bounds for real convection. It is, therefore, natural that investigators have attempted to combine these models. The simplest of these is the linearization of differential equations or the linear combination of solutions. A second approach is the correlation of results in terms of dimensionless groups. These dimensionless groups may be obtained from differential equations or from physical and dimensionless considerations alone. In both approaches the principal effort has been to obtain improved can factors.

Kaneko's Method (1941)

Kaneko (1941) compared the conduction relation for f from Okada (1940a), namely:

$$f = \frac{2.303}{[5.78(L/r)^2+9.87]}\,\frac{L^2}{\alpha} \qquad 6.15$$

and Schultz and Olson's (1938) relation for convection heating, i.e.,

$$f = \frac{2.303\ CpP}{U}\,\frac{L}{(1+L/r)} = \frac{2.303\ CpP}{V}\left(\frac{rL}{r+L}\right) \qquad 6.16$$

when applied to Lang's (1935) data for tuna fish in oil. This is essentially a solid product, but there is some convection of oil. He reported that the conduction estimate more clearly fitted the experimental data. (Olson (1947) suggested that the convection estimate was not appropriate in this instance, and that it might have been applied due to a translation misunderstanding.)

Kaneko made an interesting modification of the Schultz and Olson (1938) approximation as a result of the comparison of Equations 6.15 and 6.16. He noted the approximation represented by Equation 6.16 was strictly true [for conduction] only when the cans were geometrically similar, i.e., the length to radius ratio was constant. He therefore, weighted each heating surface in proportion to its distance from the surface, L/2 for the ends and r for the lateral surfaces. To obtain a weighting coefficient:

$$R = \frac{(L/2)2\pi r^2+r(2\pi rL)}{2\pi R^2 + 2\pi RL} = \frac{3}{2}\left(\frac{rL}{r+L}\right) \qquad 6.17a$$

$$= \frac{3}{2}\ \frac{L}{(1+L/r)} \qquad 6.17b$$

when this weighting value was introduced into the above formula, the following relation was obtained:

$$f \simeq \frac{2.303\ CpPVR}{rUA} = \left(\frac{3}{2}\right)\frac{2.303\ CpP}{BU}\left(\frac{rL}{r+L}\right)^2 \qquad 6.18a$$

$$=\left(\frac{3}{2}\right)\ \frac{2.303\ CpP}{rU}\left(\frac{L}{1+L/r}\right)^2 \qquad 6.18b$$

where B is an experimental constant. A plot of the squared term versus f_∞ , the parameter f for the product processed in steam as in the Okada approximation (1940), gave a linear relation. This curve was slightly concave upward, such that departures were experienced only at lower values of L, e.g., 10 cm, and generally only for low L/r ratios. Kaneko (1941) maintained that data for products in which there was stronger convection, such as that of Shiga Iwao for water in No. 2 and No. 5 cans could be compared using this relation. The experimental ratio of the f (a can factor) was 0.59 while the predicted ratio was 0.54.

The values for f for tuna fish in 211 x 109, 307 x 113, 401 x 201, and 603 x 408 cans were said to be nearly proportional to the $L^2/(1 + L/r)^2$ factor. Unlike the plot of f versus the Schultz and Olson factor, i.e., $L/(1 + L/r)$, mentioned above, the intercept appears to be zero, but a smooth curve connecting the points seem to the writer to be as concave downward as the Schultz and Olson curve had been concave upward. It would seem, therefore, that for such products in brine the can factor is some non-integer (between 1 and 2) power of the volume to surface ratio.

Ban and Kazewara (1941) have modified Kaneko's formula so as to apply to the cooling of the lateral surface alone and to the cooling of the top cover alone. For side cooling the weighting coefficient becomes r = R, hence:

$$f \simeq \frac{2.303\ Cp\rho r^2}{BU(2)} \qquad 6.19$$

or proportional to the square of the radius as it is in conduction. For lateral surface cooling the weighting coefficient becomes L/2, hence:

$$f = \frac{2.303\ Cp\rho\ L^3}{2\ BU} \qquad 6.20$$

and also is proportional to the square of the distance as for conduction. These formulae were evaluated using the f obtained from water-filled 4.5 x 14.5 cm and 6.0 x 6.8 cm cans <u>cooled</u> with water. The proportionality constants were reported to differ only by about 4 percent for total surface cooling, and about 10 percent for cooling of the lateral surfaces. The arithmetic means of the proportionality constants for each size were used as factors in the comparisons shown in Table 6.4.

Ban's Approximation (1941)

Ban assumed that the heat lost from a container is:

$$\frac{dQ}{dt} = - C_1 V \frac{dT}{dt} \qquad 6.21$$

where T is the temperature of the can contents [at center] and C_1 is the proportionality constant [which includes $C_p \rho$]. He further assumed that:

$$\frac{dQ}{dt} = C_2 \frac{k}{R} A(T - T_s) \qquad 6.22$$

where C_2 is a proportionality factor, R is Kaneko's weighting coefficient or equivalent length and T_s is the surface temperature. The heat lost through the surface is:

$$\frac{dQ}{dt} = UA\ (T_s - T_1) \qquad 6.23$$

Eliminating T_1 and integrating, he obtained:

$$(T-T_1) = (T-T_1)_o\ e^{-\left(\frac{C_2}{C_1}\right)\left(\frac{k}{R}\right)\left(\frac{A}{V}\right)\left(\frac{UR}{UR+C_2k}\right)t} \qquad 6.24$$

hence:

$$f = 2.303\ C_1 \left(\frac{V}{A}\right) r \left(\frac{1}{C_2k} + \frac{1}{UR}\right) \qquad 6.25$$

which for negligible surface resistance became:

$$f = 2.303\ \frac{C_1}{C_2} \left(\frac{V}{A}\right) \left(\frac{R}{k}\right) \qquad 6.26$$

This last formula is similar to the form of a Newtonian cooling body (hence Ban assumed that the surface resistance

TABLE 6.4. Some experimental and calculated can factors as reported by Kaneko (1941).

Can	Experimental Ratio	Calculated Ratio
Tall	f total/f side = 1.57	= 1.58
Flat	f total/f side = 0.79	= 0.77
Tall	f total/ f top = 0.19	= 0.18

was negligible and that the internal resistance could be concentrated at the surface of the body). He showed that:

$$f - f_{\infty} = 2.303 \frac{C_1 V}{AU} \qquad 6.27$$

and

$$\frac{f - f_{\infty}}{f_{\infty}} \simeq \frac{C_2 k}{vr} \qquad 6.28$$

if the above equations are valid.

These relations were evaluated using the data of Ito and Itani (1915) and Ito and Tawa (1916) (as had Okada (1940a, 1941) in the earlier mentioned analytical strategem to find exterior film coefficients). A plot of $\log [f_{\infty} /(f - f_{\infty})]$ from experimental values versus $\log (U/k)$ gave linear relationships with 45^{o} slope for each can size, which differed slightly with different intercepts as can geometry was varied and another different tight family of curves for rectangular cans. This seemed to substantiate the above approximation which predicted $f_{\infty} /(f - f_{\infty})$ was proportional to U/k. It was also shown that the proportionality constant C/R was inversely proportional to r, Kaneko's R value. Similarly $f - f_{\infty}$ was found to be nearly independent of can contents for air or for water cooling. A plot of log V calculated by Okada's method versus $\log V/A(f-f\ )$ the desired 45^{o} slope between the water and air data clusters although the data for cylindrical cans was about 30 percent of those predicted analytically and the data for rectangular cans were about 50 percent of those predicted analytically.

Merrill's Linear Combination (1948)

In contrast to Kaneko's lumping the internal transfer and the external heat transfer into one overall heat transfer coefficient linearization of the analytical conduction equation, Merrill (1948) attempted to combine Ball's (1923) relation for f for conduction heating and Schultz and Olson's (1938) relation for convection heating.

For a semi-infinite slab of half thickness b inches, v = specific gravity · specific heat, and all properties expressed in inches and minutes instead of in feet and hours as is customary in engineering work, this combination becomes:

$$f \simeq \frac{0.933a^2}{\alpha} + \frac{0.0831\ va}{V} \qquad 6.29$$

which was said to agree within 5 percent with values as determined from Gurney and Lurie charts (1923) and the values of Bachman (1938) when the latter were plotted on semilogarithmic coordinates. The range over which this comparison was made was not specified.

For an infinite cylinder:

$$f \simeq \frac{0.398r^2}{\alpha} + \frac{0.0416\ vr}{V} \qquad 6.30$$

This relation was said to have a maximum error of about 9 percent over the full range of Nusselt numbers.

For a finite cylinder:

$$f = \frac{0.0955\left(\frac{L}{D}\right)^2 D^2}{\alpha\left[(L/D)^2+0.427\right]} + \frac{0.0416\ (L/D)D}{2(L/D)+1}\ \frac{v}{V} \qquad 6.31$$

where once again the first term is the conduction controlling contribution and the second term is the surface controlling contribution.

The overall heat transfer coefficient was related to container thickness by the following equation:

$$\frac{1}{V} = \frac{1}{h'} + \frac{L_c}{0.0012} \qquad 6.32$$

where L_W is the average glass thickness in inches.

Equations 6.31 and 6.32 were evaluated using the results of heat penetration experiments in No. 2 and No. 3 cans and three weights of bottles with water, bentonite, and spaghetti as products, and experimentally determined h external film coefficients. The effective thermal diffusivities α used in the equations were determined experimentally in earlier experiments in which the external and container resistances were presumed negligible.

A brief consideration of the property values for which this method calls may serve to illumine the assumptions inherent in this method. The exterior film coefficient components of the overall film coefficients h were determined from the heat penetration curve of a lead cylinder of known diffusivity. The thermal diffusivity of steel and also a fireclay refractory material were determined using the same heating medium and temperature and the previously determined film coefficients. The comparisons of calculated and experimental f values for other shapes of the same materials, therefore, were a test of the validity of Equation 6.31 and also of the independence of h' with respect to geometry.

The modified effective thermal diffusivity values mentioned above, α values were determined rather circuitously. The experimental exterior film coefficient for water [it is not clear whether this was an agitated or unagitated water bath], and the exterior of can-shaped cylinders was said to be equal to the interior film coefficient for natural convection of water. Hence, one-half the film coefficient h' was used in Equation 6.31 to determine the effective thermal diffusivity α of <u>the core</u> from heat penetration curves. The value of the effective thermal diffusivity of the core, about 40 in^2/min. is about 2.5×10^3 times true diffusivity of still water. In contrast, overall effective diffusivities of 24 (Tani, 1940) and 16 (Thompson, 1919) times that of still water have been reported.

The effective core diffusivities were said to be inversely proportional to the square of the apparent viscosities. The basis for this functional form and the proportionality constant used are not given. It is interesting to note that if flow similarity in the core were attained, and the Reynolds number were the only similarity parameter then the viscosity influence would also be inversely proportional to the square of the viscosity. It might be inferred from this that the effective diffusivity contribution of the core to f is small; it is still largely determined by convection.

The comparison of actual and computed f values for the products and containers mentioned above in Merrill's Table IV, therefore realistically are a test of: (1) the effects of geometry changes, (2) secondary temperature induced viscosity changes in the vicinity of a region, and (3) k

p

t

s

t

t

r

d

t

p

*

T

c

a

f

i

o

f

[illegible]

i

j

t

/F

i

/[illegible]

partitioning of surface resistance and an unfortunate empirical relationship in which Equation 6.31 was made to fit the data. This table was said to show a satisfactory consistency throughout (perhaps due to an average error of +2.5 percent and a median and mode of +2.2 percent) although the errors for the 27 comparisons of three or more runs each ranged from +21.8 percent to -10.7 percent. Although the deviations are signs that further work should be undertaken, the relationships presented by Morrill give very good results, particularly since the range of Biot members for which this was applied was very wide, i.e., $1 \times 10^{-2} < N_{Nu} < 2.5 \times 10^3$. The controlling mechanism therefore ranged from nearly ideal convection to conduction.

The parameter j was shown to be generally intermediate between the j values for conduction (with a, the core effective diffusivity, as the controlling diffusivity) for an infinite slab and an infinite cylinder at a specified value of the Biot number. This finding is similar to that found for the effect of film coefficients and wall conductance K_c/L_c on the j of a fluid in contact with a slab as outlined in Section II of this investigation. The use of the latter j, it being larger and hence a conservative estimate for thermal destruction of microorganisms, was recommended.

Pagerson's Innovations (1950)

Pagerson (1950) tried two different methods to obtain improved can factors for the convection heating of Townsend et al. (1949) for 1 percent bentonite in four commercial glass jar sizes. The first approach was fitting a power^n of Schultz and Olson's $(rL/r + L)^n$ factor to the data. The overall heat transfer coefficients, U values, were assumed invarient, i.e., exterior film coefficient and jar wall thickness constant as well as an interior film coefficient independent of geometry. The resulting exponent was $n = 1.2$. This is a value intermediate between the $n = 1$ predicted by the Jones-Schultz and Olson model, and the $n = 2$ of Kaneko's model, as Kaneko's results seemed to indicate to us.

The second method was a dimensionless analysis, using r, L, [illegible], V, and $C_p \rho$ as factors. Since the properties and V for the Townsend <u>et al</u>. (1949) study were presumed constant, only exponents for r and L could be obtained. Hence:

$$\frac{f_1}{f_2} = \frac{r_1^{0.80} L_1^{0.42}}{r_2^{0.80} L_2^{0.42}} \qquad 6.33$$

It is interesting to note that these compare favorably with the 1.2 power of the length dimension found using the first method (r is approximately 50 to 80 percent of L) and the 1.25 power of length hypothesized in the modification of Charm's (1963) approach.

<u>Ball and Olson's Index of Convection (1957)</u>

Unlike Okada (1940a, 1941) and Ban (1941) who examined the convection (assumed external) contribution to heat transfer using a $(f - f_\infty)/f_\infty$ ratio, Ball and Olson (1957) examined the convection contribution utilizing the can factors.

A convection index C was defined as follows:

$$C = \frac{fC_d - f^1}{fC_d - fC_v} \qquad 6.34$$

where f and f' are known f values for two different sizes and C_d and C_v are the Olson and Stevens (1939) can factors for conduction heating and the Schultz and Olson (1938) can factors for convection heating, respectively. For pure conduction $fC_d = f'$ and, therefore, C = 0 and for pure convection $fC_v = f'$ and hence C = 1. The index, therefore, seems to take one value from 0 to 1. It would seem that this index should have equal or nearly equal value if estimates were to be made using f' instead of f. This new index would be:

$$C' = \frac{f'C_d' - f}{f' C_d - f'C_v} \qquad 6.35$$

but by definition $C_d' = 1/C_d$ and $C_v' = 1/C_v$, hence:

$$C' = \frac{\frac{f'}{C_d} - f}{\frac{f'}{Cd} - \frac{f'}{C_v}} = \frac{(f' - fC_d)\, C_v}{f' (Cv - Cd)} \qquad 6.36a,b$$

so that if C' is to equal C, i.e.,

$$C = \frac{(fCd - f')}{f (Cd - Cv)} = C' = \frac{(f_{Cd} - f)\, Cv}{f' (Cd - Cv)} \qquad 6.37a,b$$

or:

$$\frac{1}{f} = \frac{C_v}{f'} = \frac{1}{f' C_v'} \qquad 6.38a,b$$

This is a presumed equality which is true only if ideal convection occurs in both cans.

In the illustration of this method for real convection in 401 x 411 and 603 x 700 cans, Ball and Olson obtained an index value C = .92 with the above sizes being f and f', respectively. One can obtain C' = 0.87 for the same example by reversing the nomenclature. It would appear that these values would diverge as if less ideal convection were achieved. Since this index of convection appears to be ambiguous, the concept might be retained by defining f (or f') as that for the same product in a No. 10 tin can, 603 x 700, as for Ball's (1928) proposed index number for conduction or for a No. 2 tin can, 307 x 409, as has been informally used as a conduction index in the canning industry (Ball and Olson, 1957, p. 24).

The concept is a useful one: conduction and convection have been employed by Jackson and Olson (1940) to distinguish between conduction and convection of 3.25 percent bentonite in No. 10 cans using experimental data for the same product in No. 2 cans. The presumption is that the container resistance and the exterior and interior film coefficients are equal for the two sizes. Can factors for conduction heating products seem to be relatively insensitive to changes in h for this Biot number range, and very likely external dimensions could be used for glass containers, but it would be preferable to use the Merrill (1948) approach of lumped heat transfer resistances when comparing sizes for convection heating products.

Okada's Dimensionless Analysis (1940)

Okada analyzed the equations of fluid momentum, thermal energy, and continuity for an essentially incompressible constant viscosity fluid in order to resolve the dimensionless groups which could affect the temperature distribution in a container with a uniform external heat transfer coefficient. The ratio of u/u_o of two systems was found to be:

$$\frac{u}{u_o} = F^1\left(\frac{x}{L}, \frac{y}{L}, \frac{z}{L}, \frac{\alpha t}{L^2}, N_{Nu}, N_{Gr}, N_{Pr}, N_{Re}\right) \tag{6.39}$$

or a function of dimensionless position, the dimensionless time, $\left(h\ L/k, \frac{g\rho^2\beta\Delta t}{\mu^2}, \frac{Cp\mu}{k} \text{ and } \frac{L\rho V}{\mu}\right)$

where N_{Gr} becomes unimportant for forced convection and R_e becomes unimportant at small velocities.

To these numbers one may add $\nu_o/(\nu_1 \Delta t)$ where the ν is the reference value and ν_1 the temperature coefficient to account for the temperature sensitive kinematic viscosity and $\beta\Delta T$, but this second factor is presumably accounted for in the buoyancy terms in the Grashof number N_{Gr} (see Langhaar, 1951, pp. 126-7).

It also has been shown that if inertial forces are small then the Grashof number and the Prandtl number can be written as a product, sometimes called the Rayleigh number. (This Rayleigh number is not to be confused with the square root of the Weber number--the ratio of inertial and surface tension forces--which has also been called the Rayleigh number (see Boucher and Alves, 1959).) This $N_{Gr} \cdot N_{Pr}$ product has also been called the modified Grashof number (Ipsen, 1960, pp. 209-13). The Rayleigh number is sometimes modified further by multiplying it by the r/L or so-called aspect ratio to improve the correlations. Such a modified Rayleigh number was introduced in an extension of Charm's approach.

[The author would recommend the developments by Schlichting (1958), Ch. 14, Thermal Boundary Layers in Laminar Flow by Eckert and Drake (1959), Ch. 11, Free Convection; or by Kreith (1958), Ch. 7, Free Convection, to the reader who wishes to obtain a working knowledge of the development of the above dimensionless groups for free convection.]

Okada's conclusions, like all dimensionless group analyses, does not specify the form of the equation, but only possible effective groups. The temperature in a uniform temperature solid is known only to be some function of the Fourier modulus $\alpha t/L^2$, and the Nusselt number hL/k, where L is a characteristic length of the body. Langhaar (1951) has shown that for this particular example, the limit of the first term of a MacLauren series expansion for a small temperature difference corresponding to a small time change can be integrated to obtain the formula:

$$u = \frac{T_1-T}{T_1-T_0} = e^{-6(hL/k)\frac{\alpha t}{L^2}} \qquad 6.40$$

This device cannot be used to obtain the form for a correlation of f here. If the Rayleigh number appears to a fractional power, as is usual in practice, the temperature relation would be of the form $dT/(T_1 - T)^n$ where n is approximately equal to 1.25. Thus the integral would be of the form $\left(\frac{T}{T^n}\right)(T_1-T)^{n+1}$ rather than $\log_e (T_1-T)$ as in heat penetration curves. It would, therefore, appear that a heat penetration curve would be straight only if the influence of the temperature difference can be assigned to a particular fixed temperature difference for a particular system. This, contrary to previous assumptions, would require that inertial forces are controlling. Eckert and Drake (1959) have shown analytically that for a flat plate the Nusselt number is related to the Grashof and Prandtl numbers by an equation of the form:

$$N_{Nu} = C_3 Gr^{1/4} \frac{N_{Pr}{}^{1/2}}{(N_{Pr}+C_4)^{1/4}} \qquad 6.41$$

(where C_3 and C_4 are experimental constants) rather than a function of the Rayleigh number. This is analytical evidence that inertial effects may be important although not necessarily controlling.

Ball and Olson (1957) have pointed out that as the buoyancy forces decrease the heat transfer approaches that for conduction heating. It would seem, therefore, that an expression for the Nusselt number ultimately should include a constant term which is the conduction component. This constant would be 2 for a sphere. This corresponds to a sphere in an infinite conductive media at a different temperature. For flow external to spheres (Zenz and Othmer, 1960), data equations have been forced to have the theoretical limit of 2. The author knows of no application of such a theoretical limit for other geometrics or for the case in question, the internal region. An experimental limiting value of $N_{Nu} = 0.45$ has been reported for a slab, however (McAdams, 1954). It would seem that Kaneko's approximation for this constant value might prove to be a reasonable approximation for long times.

Merrill's (1948) consideration of the container as a skin resistance of negligible capacitance, when demonstrated to be a good assumption, might also be incorporated in the arbitrary form of the correlation.

It has been shown in the above discussion that Okada's dimensionless correlation, which has been said to have great promise, has many unresolved aspects, not the least of which is the functional form.

Although Okada (1940b) has cited Tani's (1940) observation that the temperature within various sizes of cans seemed to remain similar during heating, there is now some reason to doubt this conclusion in spite of no firm conclusions from the experimental phase of this investigation. Lighthill (1953) has hypothesized three domains in closed end tubes, only one of which is a domain in which the temperature profiles are everywhere similar. Yang (1960) has considered the necessary and sufficient conditions for laminar free convection external to vertical plates and cylinders. The only transient condition for cylinders for which similarity could be attained was for uniform but unsteady surface temperatures at large distances along the cylindrical surface. This corresponds to fully established boundary layer flow. The implications of the similarity domain and the restriction of wall temperature profile will be examined in a later consideration of the experimental and analytical temperature and flow patterns.

Limitations of the Combinations and Suggestions for Further Research

Merrill has shown that a linear combination of surface controlling and internal controlling *conduction* with *no headspace resistance* has an error of not more than 5 to 9 percent for predicting f. This is much better than the scatter about many engineering heat transfer correlations. This is also comparable to the precision of experimental heat penetration data (Hicks, 1961). Further departures of linear combination from experimental observations may be attributable in part to failure of the two terms to describe even the limiting cases, and the simplifications applied to determine property values.

All the conduction and convection representations except that of Evans and Board (1954) for conduction and Jones (1931), Kern (1950), and Charm (1963) for convection fail to consider that the headspace is relatively ineffective for heat transfer. None of these later models seem to have been applied by others. Also none of the models mentioned previously in a discussion of convection as mixed conduction-convection systems have allowed for finite axial conduction (but negligible radial conduction) for nearly ideal convection. Better components of a linear approximation for convection might be an isotropic core thermal diffusivity conduction model and a unidimensional conduction model with effective eddy and heat transfer through the top equal to that absorbed by "Newtonian" total surface heat transfer. For this proposed model the lag factor j due to convection would be 1 at the top and increased to those for an infinite slab (of twice the cold point depth, i.e., the container height minus the cold point height above the bottom) at the bottom. The Merrill (1948) hypothesis and the consideration of Section II of this investigation suggest similar relationships for j as a function of position. This trend will be discussed further in Part III of this section and has been discussed relative to one system in the previous section.

Okada (1940a, 1941) applied analytical equations to obtain a basis for a comparison of f and f_∞ where the latter is for negligible surface resistance. Hicks (1951) has pointed out that the results are not exact due to: (1) failure to consider the headspace resistance; (2) temperature dependence of the film coefficient; and (3) incorporation of internal film coefficient and eddy diffusivity into an effective diffusivity of the solid-liquid packs used in the experimental phase. It was also pointed out that it is not easy to find simple assumptions that are more exact.

A study of the components of f for constant properties for a particular case might entail: (1) the use of metal cylinder (Merrill, 1948; Evans and Board, 1954; Cowell et al., 1959) to determine the exterior coefficients; (2) heat penetration in a full can of product which had been immobilized by agar, a jel or bentonite, when exterior resistance was negligible in order to determine solid diffusivity; (3) a repeat of step (2) but with free fluid to determine by comparison with the results of part 2, and the results of an experiment (3') with a well-mixed product core, but constrained film flow to determine: (a) the internal film coefficient and the (b) vertical momentum diffusivity; (4) a repeat of step (3) and its components with headspace to determine the headspace contribution and its interactions; and finally (5) a repeat of steps (3) and (4) with finite surface resistance to explore the interactions due to a combined reduced driving force and non-uniform wall temperature distribution. The variability of heat penetration data encountered with present techniques as illustrated in the experimental phase of this investigation very likely would obscure the differences in the proposed comparisons. The comparisons suggested are also first approximations. Some portions of such a scheme might broaden our understanding of the mechanism, however. Steps (1), (2), and the conduction headspace contribution of (4) have been studied and analytical relations are available. Step (4) for a free liquid without a solid product could be examined with available techniques and analytical relations which may be forthcoming from investigations of homogeneous liquid reactors by Hammitt and coworkers, among others. There is also a developing literature for porous media, analogous to a product with slow convection. Analogies to fluidized beds might indicate how to correlate product particle shape and fill ratio (void fraction), the principle factors (Ball and Olson, 1957) for particular products.

Kaneko (1941) introduced a lumped resistance at the surface, hence incorporated isotropic conductivity into the surface resistance. Ban (1941) partitioned this relation into surface and conductive components and his application is subject to the errors of both Kaneko (1941) and Okada (1940, 1941). Merrill's approach gives improved estimates over the above for the region in which the internal film coefficient can be approximated. Ultimately, this method would require that internal film coefficients of homogeneous fluids be approximated by heating metal cylinders in a bath of product, and that film coefficients for non-homogeneous liquids be determined as in step (3') above. Figure 6.4 shows an apparatus which might be used. The effective thermal diffusivity of the core α required for Merrill's formula might then be determined by differences between experimental results and those predicted from the convection component.

Ball and Olson's (1957) convection index is somewhat ambiguous and is subject to the errors of Okada's (1940a, 1941) method, as well. Nevertheless, the Jackson and Olson (1940) comparison shows that can factors are a useful tool with which to distinguish conduction from convection.

Merrill's neglect of jar capacitance and his relation for j are relatively unevaluated: although improved models were developed in the analysis of Section II, the experimental data do not seem to warrant these refinements at present.

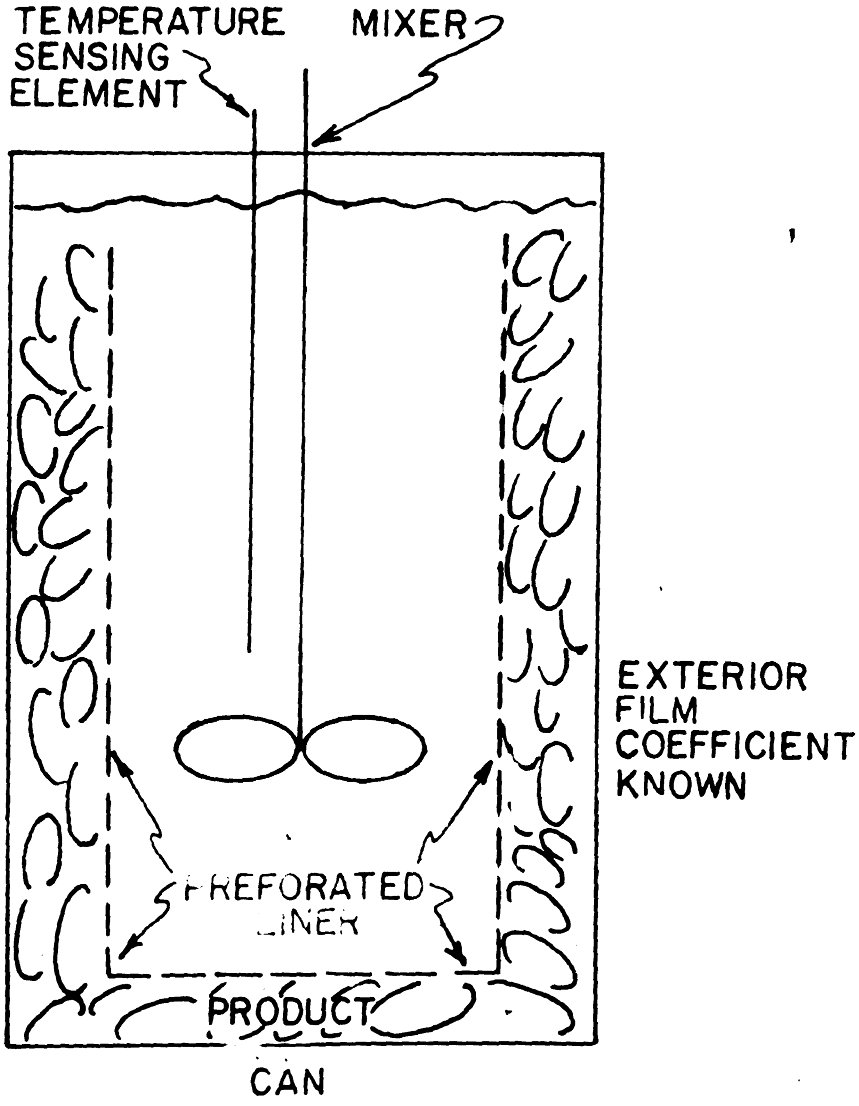


FIGURE 6.4 Device for determination of the internal film coefficient for liquids whose film is constrained by product.

Okada's method has not yet been formulated into working equations and has certain limitations which have been discussed above and in the treatment of Charm's (1963) proposal.

Status of the Analysis Methods

Ball's (1923) method of analysis of conduction heating is simple, readily related to an analytical basis, and to other methods, such as system engineering time constants or food cooling half cooling times, does not require approximation to obtain the roots of the boundary layer equations and unlike most other methods introduces the concept of a lag. The method has several deficiencies, however; the approximated series converges slowly so that the temperatures shortly after temperature changes are not represented, it is increasingly sensitive to measurement errors at long times, headspace effects may be appreciable and slow convergence may introduce errors even at long times, and thermal properties are often lacking. Only the first of these can be said to be a deficiency in the analysis method for an experimental curve which is to be used for most lethality determinations. It would seem that Gillespy's (1953) method could be used for prediction for complex processes.

The status of methods for convection and combined convection and conduction are tenuous. The available models all suggest heat penetration curves like the shape of those of conduction, but provide little or no information to predict the heat penetration parameter f and no information to predict the lag factor j other than that tentatively proposed by Merrill (1948).

PART II. TEMPERATURE AND FLOW PATTERNS

Experimental study of the temperature and flow patterns during transient heating or cooling of a food product are an important element in the examination of the mechanism of the process. Temperature measurements have been the principle quantity used in studies of convection. A knowledge of temperature histories and distributions is essential to the design of a thermal process. Since microorganisms are in suspension as well as on surfaces, both spacial or Eulerian temperatures and fluid or Lagrangian temperature histories should be known. It has been shown that present models do not permit the calculation of either for convection, and it will be seen that flow measurements of even a semi-quantitative nature are extremely difficult to obtain. For these reasons local temperature distributions have been studied.

Three models have been hypothesized: the uniform effective diffusivity, the uniform temperature, and the plug flow model. A preliminary examination of conduction profiles, before examination of experimental data for convection will be undertaken. It should provide a basis for later acceptance, rejection, or modification of the uniform effective diffusivity hypothesis. It had been the intent of the author to examine the engineering studies of transient natural convection on the various surfaces which make up a container. The rapid growth of this field will preclude such a treatment here. Instead, the author will refer to those studies which seem to have most immediate bearing on transient natural convection in enclosed spaces.

Conduction Patterns

Conduction Temperature Profiles

Jackson and Olson (1940) and Jackson (1940) studied the temperature distributions of 5 percent Bentonite, a thixotropic clay which appears to immobilize water at relatively low concentrations and hence approximates the thermal properties of high water content foodstuffs. In this study (in large, i.e., No. 10 cans) thermocouples at four radial positions and three elevations were used to determine the temperature at specified intervals. At the central plane the isochronals were initially strongly concave upward indicating large initial temperature gradients which are gradually equalized. The isochronals at the central axis of the container behaved similarly. Contrary to what one might expect from the Evans and Board (1954) study of headspace resistance the temperatures near the top of the containers are reported to rise faster than those at the bottom. This may have been due to thermocouple conduction error. It was noted in earlier discussion that the effect of headspace was small. Having this recent information, one might be tempted to accept an effect on f for the conduction and convection regions of 3.25 percent heating data at a .10 to .20 probability rather than rejecting at a .05 level.

The Influence of Container Position and Headspace on Slowpoint

Townsend et al. (1949) studied the heat penetration in several sizes of glass jars. Only the locations of the cold point or location of least lethality during the given heating process are reported. These are shown in Figure 6.5. They are cited here visually to illustrate that when the jars are on their sides (and all heating surfaces are glass) the slowest heating positions are above the geometric center of the product. When the caps are down the slow points also seem to be above the geometric center of the product. Since the headspace volume is not reported and the jar geometries are not known precisely, it is difficult to determine if the slow points are at or below the geometric center of the product. The relative heat transfer coefficients might indicate the probable direction: Evans and Board (1954) have indicated that the U through a tin enclosed headspace ranged from about 2 to 10 Btu/hr ft^2 ^{o}F; while U for a 0.2 in. thick glass wall is approximately 36 Btu/hr ft^2 ^{o}F which suggests that the slow points may be slightly above the geometric center.

The Iso-F_o Shells and the Influence of Jar Shape

Fagerson (1950) studied temperature profiles of 5 percent bentonite, but only in No. 2-1/2 and No. 303 glass jars. The isothermals at the central plane of the jar appeared to be similar in shape to those reported by Jackson and Olson (1940) and Jackson (1940). The isochronals for

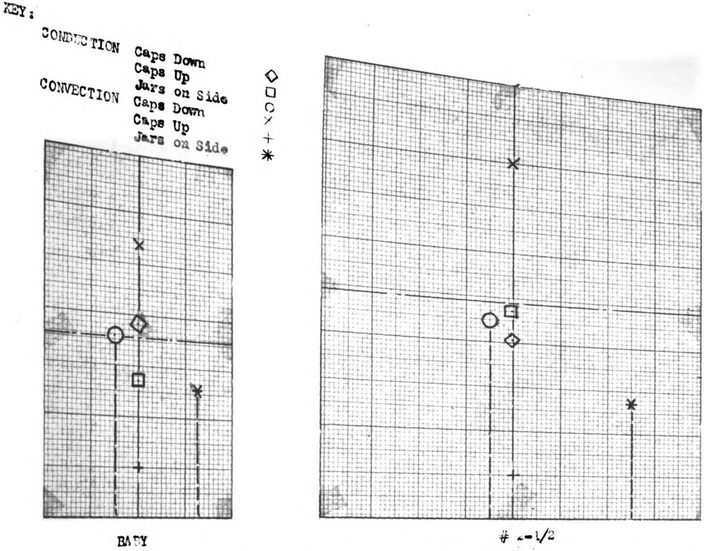
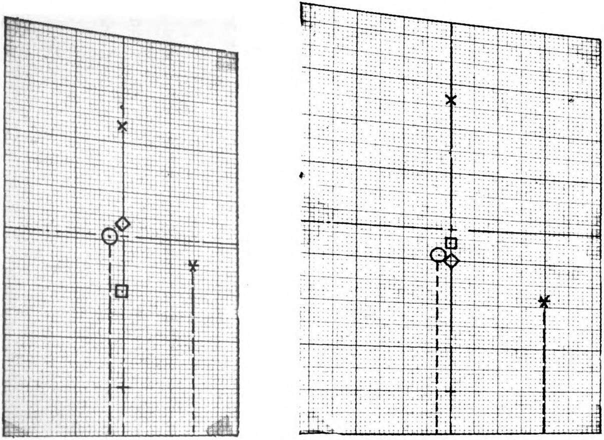


FIGURE 6.8 Location of slow heating region for baby, junior, #303 and No 2-1/2 jars containing 1 percent bentonite (convection) or five percent bentonite (conduction) when processed upright, inverted or on their sides, after the data of Townsend et al (1949)



JUNIOR

#303

FIGURE 6.5 Cont'd.

the central axis, however, showed faster heating near the bottom of the container during the initial stages, but the j's and reported F_o values indicate faster heating near the top of the container than at the bottom. Iso-F_o value curves--constructed from the nine reported values for each of the two sizes in the same manner as one might construct topographic profiles--are shown in Figure 6.6a, b. Radial symmetry was assumed. In both containers the lower F_o contours are egg-shaped with the small end up. This constriction at the top is greatest in the 16 oz. container. In this instance the curvature of side walls at the shoulder is relatively larger and the closure is relatively smaller than that for the 2-1/2 jar. Likewise the appearance of a hot zone near the bottom center of the 16 oz. jar may be due to the greater relative curvature of the bottom of this size jar. Although the F_o curves from all the F_o values taken together, suggest that the cold point is slightly above the geometric center of the product, the results of this investigation alone are inconclusive.

Effects Attributed to Anisotropic Conductivity

Hurwitz *et al*. (1956) studied temperature distribution in meat, which has a greater axial thermal conductivity than transfer to the fibers, in metal cans. They reported that the cold spot for heating and cooling combined may be a toroidal zone slightly above the geometric center of the product.

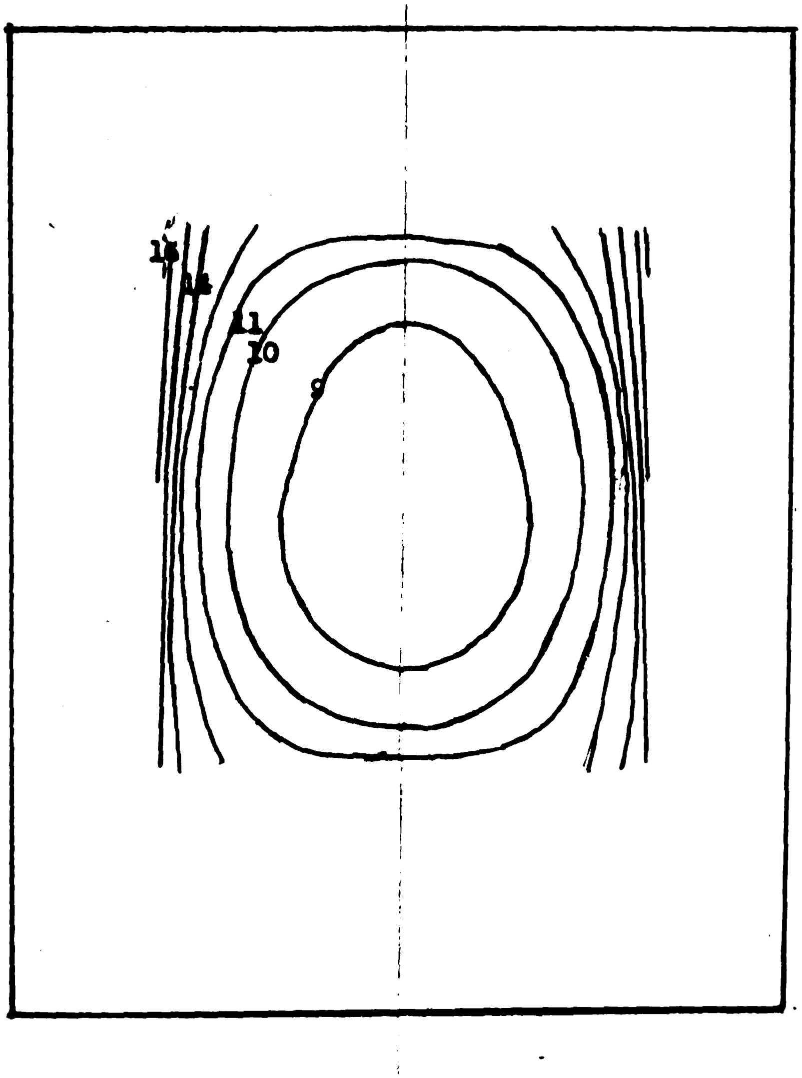


FIGURE 6-6a Iso-F_0 contours for 5 percent bentonite in 2-1/2 glass jars with 180°F initial temperature and 240°F water bath temperature for a minimum F_0 of 8, after the data of Fagerson and Esselen (1950). The contour interval is 1 minute.

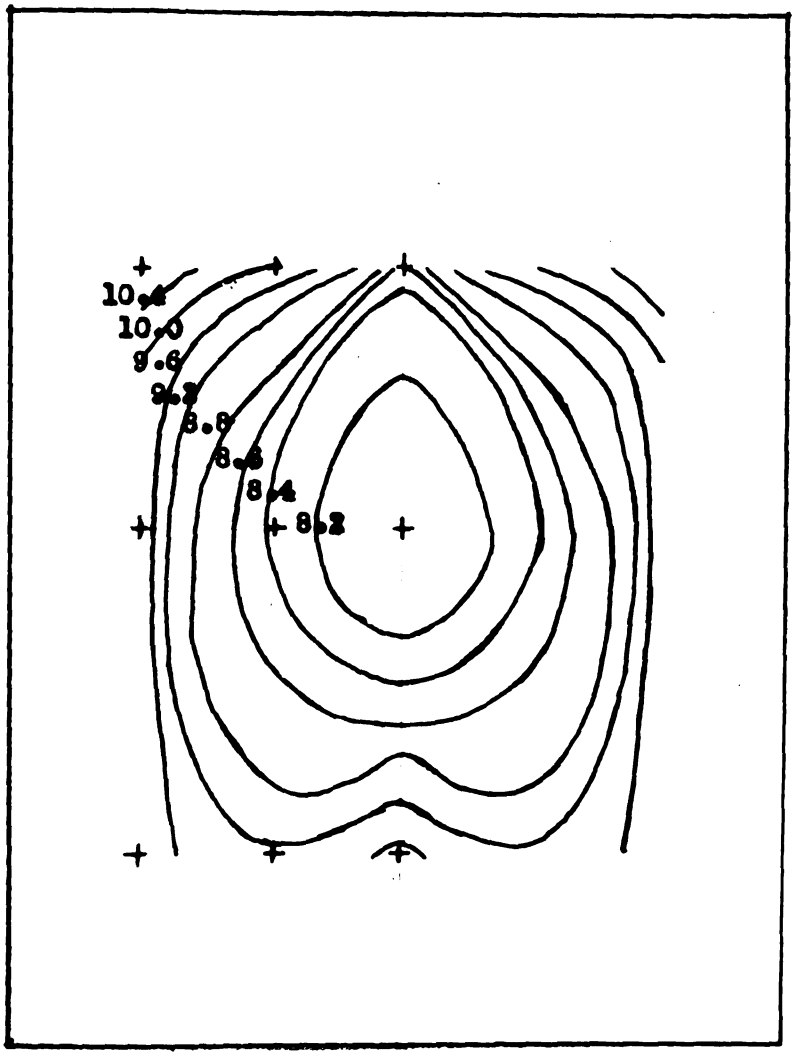


FIGURE 6-6b Iso-F_0 contours for 5 percent bentonite in 303 glass jars with 180°F initial temperature and 240°F water bath temperature for a minimum F_0 of 8, after the data of Fagerson and Esselen (1950)

This shape of cold region was attributed to the anisotropic thermal conductivity, to the initial temperature distribution caused by searing the meat, and by residual heating. Since computers are now more widely available and can readily handle McLean's (1946) formulae for transient heat transfer in anisotropic finite cylinders (round timbers), it might be feasible to obtain analytical results to determine if anisotropism is the cause for this toroidal-shaped cold zone.

Effects of Surface Resistance to Heat Transfer

The headspace resistance and isotropism both appear to distort the shape of the isotherms (or since f is constant the shape of the iso-j shells) from the analytically derived curves of Hicks (1951), Stumbo (1953), and Carslaw and Jaeger (1959), among others. Overall surface resistance or thermal property variation analytically would be anticipated to have greater effect on the position of these contours than on the general shape.

Surface heat transfer resistances reduce the level of j and increase f. Hence, a greater relative portion of the can volume would be anticipated to have nearly the same thermal destruction value as the center of the container. This effect is comparable to the greater relative portion of low lethal treatment, reported by Hicks (1956) to occur in centers of cans as the can size (particularly diameter) or the lethal value at the center were decreased. The greater uniformity of temperature obtained with appreciable surface heat transfer resistance is also analogous to the uniformity achieved in convection heating, uniformity which results in more severe thermal processes.

Effects of Variable Thermal Properties

Evans (1958) has shown that the f values approach those obtained at the final equilibrium temperature and the effects on j are in the opposite direction to the changes in thermal properties. In the region in which thermal diffusivity increases with temperature, the j values would be anticipated to be higher and closer spaced and hence the proportion of low lethal treatment is decreased slightly. The effects of property variation during the cooling portion of a process are in the opposite direction, however. It would appear that these opposing trends make it necessary to examine with great care the overall effect for a particular case.

Convection Patterns

An Early Flow Study

Bitting (1937) made a study of dye movement in 1926 and 1927 prior to the development of the uniform temperature or plug flow models. Quart and gallon glass jugs were coated with heavy agar. This agar was drawn to points at several heights and distances from the container walls. The path of dye flowing from these dye coated points was observed. It was concluded that "the convection currents in a jar or can in the standing position follow up the outside, become more rapid as they rise above the middle, are deflected inward near the top and turn downward but sweep outward before they reach the center of the jar" and that "a cold zone results below the center with the apex near the top and the base about 3/4 in. from the bottom of the jar."

These findings are substantially the same as those reported in later and more detailed investigations which are discussed below.

A different picture for horizontally-oriented containers was found. "The convection currents rose almost directly from the bottom with less following of the contour of the container," and "the time required in heating all parts was reduced from 15 to 30 percent in all fluid media, but not with equally good results when there was a combination of liquids and solids." These results were reported to have been confirmed later with thermal measurements.

Later investigations also confirmed these findings: heat transfer rates from heated horizontal are higher than from vertical plates, the Schleiren flow patterns reported by Schmidt (1956) or analytical patterns of Crawford (1961) or personal observation of flow in the heating of water in a spherical bottomed double boiler or steam kettle confirm the general flow pattern and Townsend et al. (1949) have investigated heat penetration in horizontal containers somewhat more intensively.

Temperature Patterns

Jackson and Olson (1940) and Jackson (1940) reported the temperature profiles of 1 percent Bentonite, a convection heating product, in No. 10 tin cans. The reported j values were substantially less than for conduction heating and the one heat penetration curve shown is substantially linear even with a nearly 180 degree temperature rise.

At the central plane the isochronals were essentially flat over the central 2-1/2 inches of the 3-7/8 inch radius, and the thermal boundary layer was between 0.1 and 0.6 inches thick. In the center of this thermal core the temperature rose somewhat more slowly than the rest of the core. This seems to be partial confirmation that the flow in the descending core sweeps outward before reaching the center. It is interesting to note that although this slower central heating is shown as a cusp, the condition of symmetry in the absence of a central heat or flow source suggest a zero radial temperature gradient at the center.

The isochronals at vertical axis were observed to concave upward for a plot of temperature versus position during the establishment of convection, and concave downward after convection had been established. This corresponds to faster convective than conductive transfer initially and the increasing importance of axial conduction thereafter.

Jackson and Olson have described the overall convection phenomenon as follows:

> Apparently the rising hot liquid forms a thin-walled tube adjacent to the can wall. As the hot liquid reaches the top of the can it spreads inward over the upper surface of the remaining liquid. The uniform temperatures shown across a horizontal plane indicate that the large cylinder of liquid within the rising tube is relatively quiescent. This cylinder must move downward slowly as the liquid in contact with the bottom is heated and enters the rising tube, and the liquid replaces that at the top of the cylinder.

They noted that the rise in temperature throughout most of the can is due to convective movement of liquid, although the mechanism was said to be complicated by some eddy currents and some transfer by conduction. Jackson (1940) commented that between the rising film and the descending core there would be a rather vague buffer region rather than a clear interface.

The reported j values seem to reflect the movement down the central core as suggested by the Kern (1950) model of plug flow. The j values were reported to be about 0.5 near the top of the can and to increase to a value near unity at a point about one inch from the bottom.

Some Details of the Patterns for Heating

Tani (1939) followed the convection heating currents in glass vessels 7.5 x 11.5 cm, similar to a 1-lb. tin can, and a vessel 1.34 times taller with a suspension of aluminum powder in water. Temperatures near the wall and also on the central axis, near the top, the geometric center, and near the bottom were measured concurrently with the flow, and also in a tin container.

An approximate shape of the eddy movement was suggested (See Figure 6.7). The initial disturbance was said to diminish rapidly and the eddy shape to remain approximately constant. A comparatively thin ascending current, which he surmised might be found even with particulate products, and a slowly descending central core was observed. Comparatively stable cellular eddies were found at the bottom when the temperature differences were small but they seemed to be combined with the general circulation for large temperature differences. Once established, however, the cellular eddies remain nearly constant in height in spite of the gradual reduction in local temperature and the axial temperature gradient.

Tani assumed that the thermal data followed a form similar to that of a conductively heating infinite cylinder. The relationship between cannonical temperature at each position and the Biot number was said to be independent of initial temperature difference and container diameter. The proportional gradient between vertical planes was said to be almost constant and independent of initial temperature and diameter. The radial temperature gradient was found to be small in comparison to the vertical gradient. The mean temperatures within the containers compared and found approximately equal to the theoretical conduction, have a diffusivity equal to the experimental effective diffusivity at a cannonical temperature of 0.5. This effective diffusivity

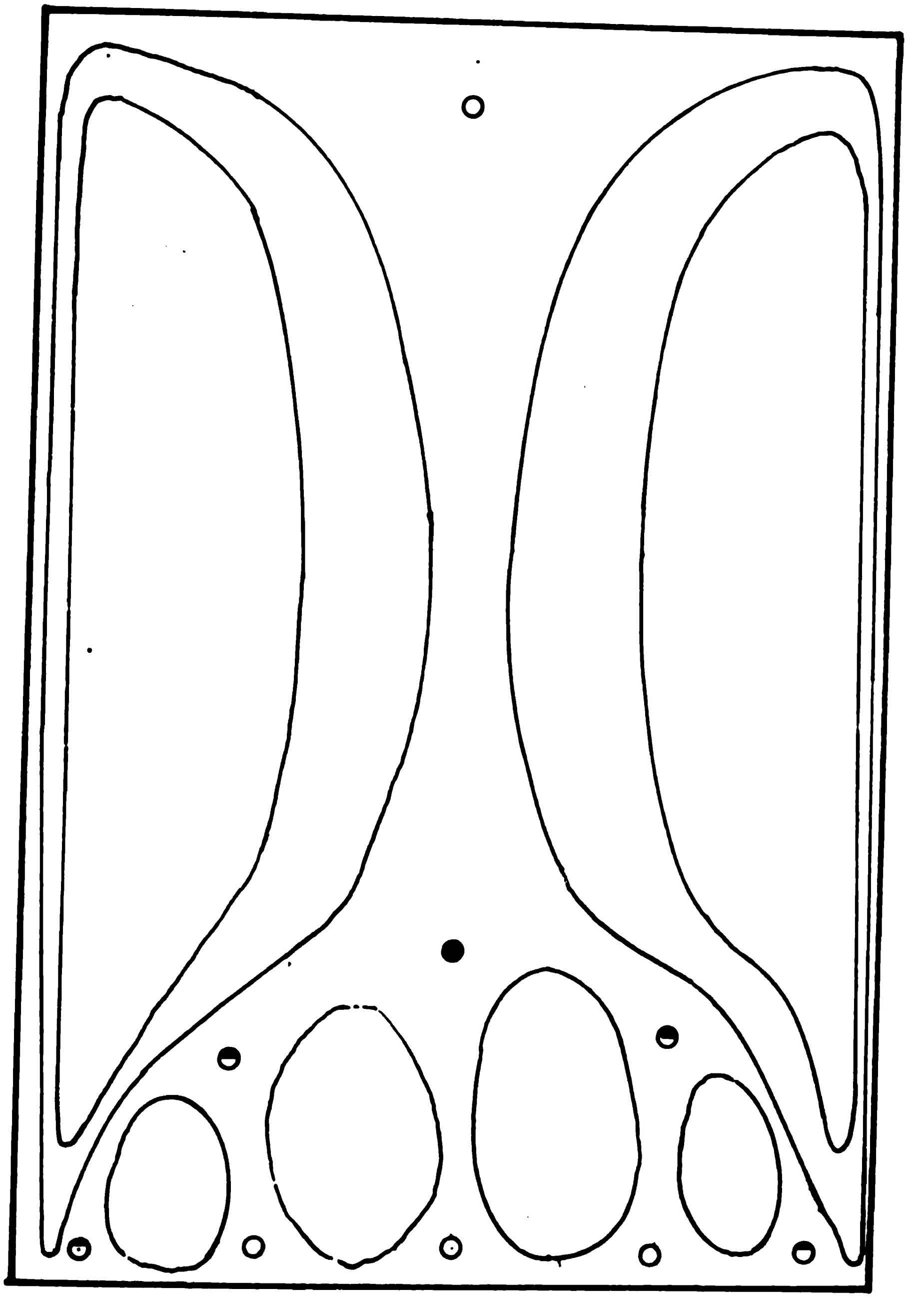


FIGURE 6.7 Flow patterns of water in heated 1 lb (7.5 cm x 11.5 cm) glass vessel after Tani (1940). To this diagram have been added circles to denote slow regions (for convection heat transfer) due to stagnation.

was 33.4 x 10^{-3} cgs. or 24 times that of still water.

These two studies seem to confirm the plug flow model. This model has not been expressed analytically, however. The observation that the temperature distributions were similar and that the temperature responses are expressible in the same way as for conduction, but anisotropic effective diffusivity suggests that dimensional analysis might be fruitfully employed.

Tani's flow lines seem to confirm a conical flow quiescent region in the central region, but here it does not reach to near the top. The sketch of the flow shows regions of flow separation (identified with circles on the figure) and of internal stagnation within the lateral and bottom eddies. Bitting (1937) reported a central cold point above the region for which Tani (1940) has reported several bottom eddies. Since minimum convection occurs at stagnation points, the separation points occur at balances between buoyancy forces, and the positions which receive the convection effects of lateral heating or bottom heating last are near the bottom of each eddy loop, one might expect the cold region to be somewhere between the two separation points indicated by the partially filled circles.

Some Details of the Patterns for Cooling

Ban and Kaziwara (1941) applied the experimental methods of Tani (1940) to the investigation of circulation and temperatures within 4.5 x 14.5 cm. and 6 x 6.8 cm. glass

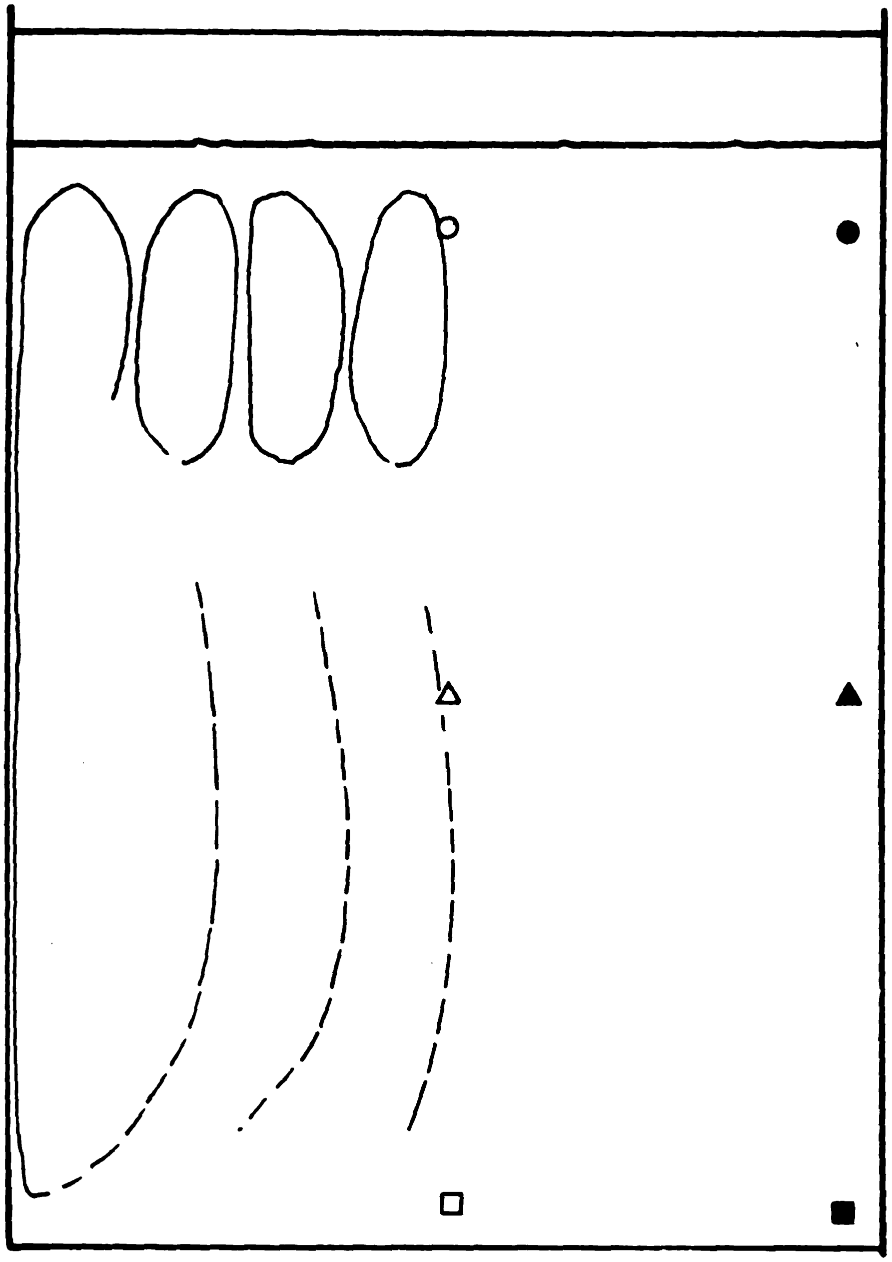


FIGURE 6.8 Possible flow patterns of water in cooled 1 pound (4.5 x 14.5 or 6.0 x 6.8 cm) glass vessels after Ban and Kasiwara (1941)

cylinders during cooling of the sides and also the total surface, and for the first container cooling of the top alone. The thin layer of descending fluid, slowly rising central core, with rapidly diminishing disturbance and approximately constant shape of eddy movement reported are moderately similar to, but opposite in direction to the heating behavior reported by Tani (1940). The cellular flow shown in Figure 6.6 was suggested as a possible pattern.

The logarithm of the temperature excess above the cooling temperature at the six points plotted against time was shown. These values were compared to the average temperature excess and then used to suggest isodeparture fractions (from the mean temperature) for the above cases.

The approximate formula for the effect of can dimensions upon f, derived by Kaneko (1940), was assumed to hold for the case and was modified to approximate side and top cooling. The mean values of the diffusivity so calculated using experimental f were 0.27, 0.51, and 0.60 cm^2/min. The ratios of experimental f for the several conditions were found to agree with calculated ratios which employed the mean experimentally determined diffusivities for the same cases. Surprisingly the f of the mean temperature for the tall can was smaller when cooled on lateral surfaces than when the total surface was cooled.

It is interesting to note that Ban and Kaziwara (1941) did not report the cooling flow patterns for total surface cooling to be the inverse of those reported by Tani (1940) for heating. Possible eddies were reported and in the description, vertical flows in the core were indicated to be even more tenuous.

Indications of the Effects of the Various Heat Transfer Surfaces

Two differences in the physical system may partially account for the suggested patterns. The bottom of the glass container used for the visualization studies is an admittedly poor conductor but any cooling of the bottom would be in opposition to vertical movement of the core and might cause the formation of a stagnant layer at the bottom. The temperature profiles reported seem to confirm the view and more uniformity is observed at the bottom of tall cans, as would be anticipated. The layering effect also seems to be less for side cooling alone, but not for a flat can. This last secondary effect may well be due to the decreased area and momentum.

It should also be noted that the decreased heat transfer through the headspace seems to barely sustain eddies, which were reported with greater certainty when heating from the more conductive bottom. The decreased heat transfer through the headspace during heating might interfere less with the height of the lateral eddies than cooling through the bottom.

Top cooling of the tall can gave slower cooling, but was reported to give higher effectiveness per unit of cooled area than had side cooling alone. This observation is not compatible with Evans and Board's (1954) values of 2 to 10 Btu/hr.ft.^{2} $^{\circ}$F. and anticipated side wall film coefficients of 60 and above. Their observation that the water from the top cover flowed down the side walls in a thin layer--a highly effective way to promote heat transfer--may be responsible for the apparent higher unit effectiveness of top cooling. If the containers were full, higher heat transfer through a cooled horizontal surface facing down than through a vertical plate of the same dimension may be responsible for this phenomenon.

Influence of Container Position and Headspace on the Slow Point

Townsend et al. (1949) studied the temperature distribution of 1 percent bentonite in jars as well as the conduction heating of 5 percent bentonite, only the "slow point" locations are reported however. These slow points are also shown in Figure 6.3. The slow points were higher for slightly faster heating which occurred when the jars were heated with lids down and were in higher absolute and relative position for the slightly faster heating which occurred when the jars were processed on their sides. This movement of the slow point would once again suggest that the so-called control slow point results from a balance of forces due to buoyancy from bottom and indirectly from lateral surface heating. If flows and temperature were similar, as Okada (1940) has suggested, movement of the slow point to the same relative axial (or for jars on their sides, radial) position. Movement of the slow point was reported only for horizontal cylinders, however.

Part of the answer to this apparent paradox may be due to a difference in overall flow mechanism in the container. The Schlieren photographs of a heated horizontal cylindrical container taken by Schmidt (1955) have shown that the heated fluid tends to rise vertically instead of following the walls. This is a confirmation of Bitting's (1937) findings. Since the overall influence of lateral surfaces on circulation is reduced, it seems reasonable to assume that an increase in jar diameter would increase the relative portion of the fluid rising immediately from the heated surface, and that this mechanism might explain the increase in relative position observed. At a nearly steady-state, that is when heat transfer is exceptionally slow, temperature gradients due to differing heat absorption might be presumed small. For such a condition the cooling of a horizontal container is analogous to the cooling of a filled horizontal annulus with a warmer inner tube (thermocouple). Crawford (1961) found that the thermal cold point was above the central axis and the stagnation region in the center of predicted flow loops for this slow convection was above the axis and to the side of the central plane. This point was nearer the axial height for low Grashof numbers than for Grashof numbers on the order of

30. It seems, therefore, that the flow patterns for horizontal cylinders are like those of vertical cylinders only for very low Grashof numbers. It also seems that once again the cold point was displaced. This displacement, however, was observed only when inertial forces were small.

The standard engineering heat transfer texts indicate that the average natural film coefficient h on a vertical plate is proportional to $\frac{(L^3)^{1/4}}{L}$ where L is the height of the plate. The strength of the side wall eddy loop decreases slightly with an increase in container height. This yields a decrease in effective buoyancy per unit volume.

The bottom eddies reported by Tani (1940) resemble a Benard cell structure (see Eckert and Drake, 1959) for convection between parallel plates, which forms only for Grashof • Prandtl numbers of about 1,700 or larger and deteriorates into irregular turbulence for higher values, i.e., 45,000. Here the depth may be proportional to the cold point height above the bottom surface. Thus, the lateral eddies are opposed by high reference temperatures due to conduction and some columnar flow (Ostrach, 1957) and buoyancy due to regular cellular movement or turbulent eddies.

In the first region the temperature decreases linearly with depth for a steady state but the transient heat flow which can be converted into buoyant flow decreases as the square of the depth; also the jar component is approximately a low 36 Btu/hr. $ft.^2$ ^oF.

When Benard cells, unconstrained by lateral boundaries, are formed, the Nusselt numbers have been reported by Jakob and Gupta (1954) to be for laminar flow.

$$N_u = 0.300 \, (N_{Gr} \cdot N_{Pr})^{1/4} \qquad 6.42$$

$$2 \times 10^3 < N_{Gr} \cdot N_{Pr} < 1.5 \times 10^5$$

for turbulent flow

$$N_u = 0.126 \, (N_{Gr} \cdot N_{Pr})^{1/3} \qquad 6.43$$

$$3 \times 10^5 < N_{Gr} \cdot N_{Pr} < 3 \times 10^9$$

McAdams (1954) recommends for laminar flow on vertical flat plates:

$$N_u = 0.59 \, (N_{Gr} \cdot N_{Pr})^{1/4} \qquad 6.44$$

Hence, an increase in height would cause a decrease in film coefficient proportional to .59 $L^{-1/4}$ for the lateral wall, but only (0.300 to 0.126) $\cdot$ (x/L or 0.2)$^{-1/4}L^{-1/4}$ or (.2 to .4) $L^{-1/4}$, when the cold point location is assumed to be about a x/L of 0.2. If the balance suggested in the experimental phase but not confirmed therein is correct this position is dependent on the L/D ratio of the container.

In the region in which both the bottom and lateral flows would be turbulent, little work seems to have been done to establish the relation for the Nusselt as a function of the Grashof•Prandtl number for horizontal layers with either a free or a confined upper surface. Tani's (1940) observations suggest that he observed the second and third of these regions, i.e., cellular and turbulent bottom flows.

Althought the heat flow and temperature differential briefly characterized above are responsible for the buoyancy (the Grashof number alone is responsible for this) and at an axial slow point there is no heat flow between the bottom and top flows, the shape of the container, as well as the effective heat transfer area for the top and bottom flows, greatly influences the ultimate balance. The momentum, thermal energy, and continuity conditions must be fulfilled as well. The geometric aspects are reported in the thermal processing literature. These aspects of the location of the cold point will be analyzed in a later section of this dissertation, following a review of later thermal processing studies and a review of the effects of geometry.

Fagerson and Esselen (1950) also studied the temperature profiles and slow points in glass jars, but supplemented these by visual and photographic observation of dye movement.

Since thermocouples were not placed closer than 3/4 of an inch from the wall, little can be said about the width of the thermal boundary layer other than it seemed to be less than this thickness. This does not conflict with an apparent width of between 0.1 and 0.6 inches noted for the Jackson (1940) and Jackson and Olson (1940) system.

Although there was general agreement with the Jackson and Olson description of the phenomenon including the general shape of the isochronals, there were some interesting differences. The first of these differences may be an artifact, but at all planes in both of Fagerson and Esselin's jars there was faster heating at the center of the core rather than the slower heating previously reported. In this investigation faster heating was observed in the center near the top but not in the possible bottom eddy region.

The parameter f generally increased from top to cold point of the container as in the present study. It was noted, however, that for the 16 oz. (303) vegetable jar that the f at 1 in. from the jar wall was larger than those above or below at the same radial position and larger than those at the jar center and also 3/4 in. from the wall at the same height. The effect of radial position is not as readily apparent, shown in Figure 6.9a, b.

The j values increased from top to bottom of the jar but most were greater than unity. This also was confirmed by this study although the j values were somewhat lower. Some of the overall increase in j value might be attributed to a thermal transport lag due to increased wall capacitance of the jars. There seem to be other factors as well, for heating to the same temperature with a lower initial difference (and higher initial viscosity) decreased the mean j values in every instance (not always significant at the 5 percent level, Fagerson, 1950) while generally increasing the f value. There seemed to be no simple relationship for j value with radial position for although the j's were smaller at about 1/3 r near the top and center planes the low j values near the bottom were normally closer to the container axis.

The F_o contour diagrams for convection, shown in

Figure 6.9a, b, are grossly different from those shown earlier for conduction heating. The curves, although symmetric radially, show no indication of the symmetry around a half plane, as for conduction heating. The slow points are well below the central axis and are so far displaced from the central axis that they are outside the region for which data were reported, although these are the F_0 distributions for a minimal observed F_0 of 8.0. The convection patterns differ between themselves. There appears to be less radial differences for the 2 No. 2-1/2 jars which might be said to have a wider proportion of uniform temperature core. The F_0 gradients for the No. 2-1/2 container, however, do seem to be steeper in the vicinity of the cold point. It is interesting to note that the F_0 contours for the 16 oz. jar are approximately normal to the flow lines reported by Tani (1940) for a 1 lb. jar and the F_0 contours for the No. 2-1/2 jar are approximately normal to the flow lines suggested by Figure 3-C of the Fagerson and Esselen paper.

It should be noted that unless the largest j and largest f are found at the same position within the container, both the "thermal" and "lethal" slow points will change during the process. Some evidence for a possible change was found in this experiment; further investigation is required however.

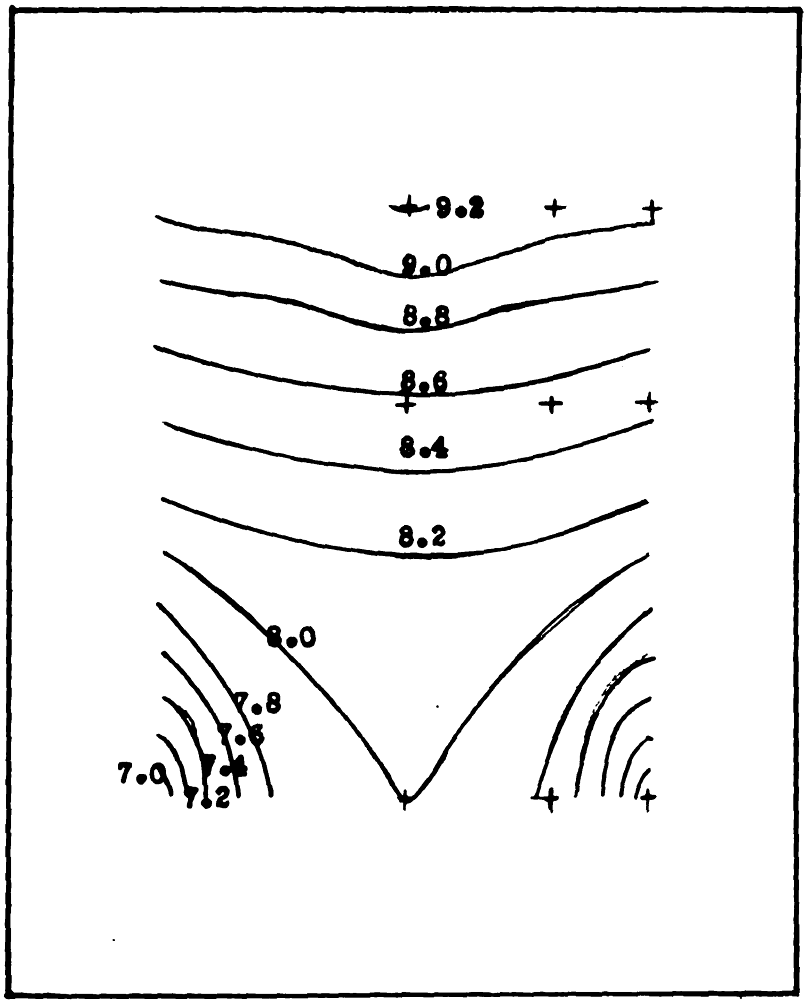


FIGURE 6-9a Iso-F_0 contours for 1 percent bentonite in No 2-1/2 glass jars with 180° initial temperature and 240° water bath temperature for a minimum F_0 of 8, after the data of Fagerson and Esselen (1950)

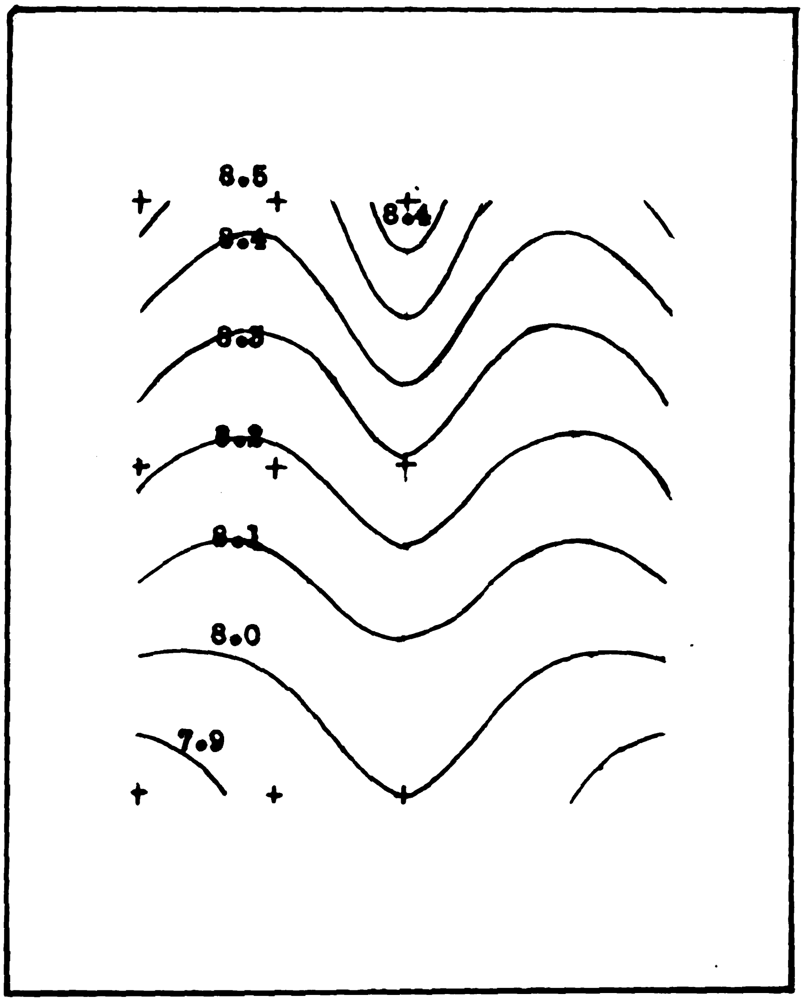


FIGURE 6.9b Iso F_0 countours for 1 percent bentonite in 303 glass jars with 180°F initial temperature and 240° water bath temperature for a minimum F_0 of 8, after the data of Fagerson and Esselen (1950)

Some Semi-Quantitative Flow Studies

Their results.--The dye studies in the No. 2-1/2 jars were reported by Fagerson (1950) to show a central column which formed within about 15 seconds and which reached the bottom about three to five minutes later. Hence, the core velocity is approximately 1/12th to 1/20th of that in the rising film. These are of the same order as the value of 1/11 found in this experiment. These ratios are of the same order, i.e., 1/3 to 1/20, as those reported by Hammitt (1958a) for a somewhat different system--a cylindrical vessel with internal heat generation (analogous to the heat sources and sinks in transient heating and cooling). This difference in results may be due to the difficulty in obtaining data of even a semi-quantitative nature. During this period bottom currents were observed to grow until they attained 1 and 1.5 inches in height (about an x/L of 0.18), but had decreased by the end of this period to about 3/4 of an inch. These currents may have been in the third domain, irregular turbulence. Tani's (1940) Benard cells (see Figure 6-7) were not reported, however.

Later in the heating process (by 20 minutes or about 2 f) the lateral loops were reported to be of similar shape, but only attained about an x/L of 0.8, by 27 minutes had decreased to an x/L of 0.5, and by the time the cold spot was within 8 ^{o}F. of processing temperature, a w of 0.084 reached only an x/L of 0.17. Tani (1940) had found no change in height of the lateral eddy loops.

Their limitations.--The geometry and the nature of the problem places severe limitations on the quality and type of measurements. Schlieren, or other photographs using the difference in refractive properties of fluids due to temperature changes, or photographs of dye or particle movements are of limited utility and validity. Only when jars are on their sides are non-refractive surfaces normal to the direction of observation and only in this position are the flows nearly uniform along the line of sight. This second limitation makes it necessary to have discrete readily identifiable particles or dye filaments and simultaneous observations in an axial direction when observing vertical cylinders. The vertical velocities are subject to only a little distortion as contrasted to the lens-like radial distortion by the jar and its contents. Streak photographs of readily suspended particles, such as micron-size alumina powder, are of limited value because identification of an individual particle is difficult and the supplemental axial photographs are also difficult to interpret (many particles are usually present). Tani (1940) has admitted to difficulty in interpretation of such motion, but unidirectional photographs. His observations seem generally very good in spite of his not detecting a possible change in lateral eddy height. Fagerson (1950) followed only unidirectionally the many filaments formed from dye which was initially layered at the bottom of the container.

Dye movements, like temperature measurements, are subject to some diffusion error since the temperature or concentration level progresses faster than the velocity level.

When such probes as for dye injection, thermocouple, thermal conductance, hot wire, or velocity pressures are introduced in order to fix the location at which the measurements are taken, the flow patterns may be disturbed. This seems to be preferable to uncertainty about position, however. Hammitt (1958) has discussed these techniques and found only the first two suitable for conditions similar to those found in convection in food containers. The dye techniques used in the investigation are suitable only for the initial stages of convection. Two dimensional photographs of rectangular cylinders would be a preferable technique.

Some Analogous Systems

Theoretical flow regimes in thermo-syphon tubes.—Lighthill's theoretical domains would seem to substantiate Fagerson's, rather than Tani's, observations on the change in height of the lateral eddy loops. Lighthill's system was a vertical tube closed at the bottom, but open to a reservoir at the top. When L/r is small (for a given $N_{Gr} \cdot N_{Pr}$) the flow is said to be boundary layer flow like that on a vertical plate. Since r is presumed large enough so curvature is negligible and the flat plate solutions usually do not take into consideration a velocity outside the boundary layer, for this solution, the boundary layer must be small compared to the whole cavity--a severe restriction. For this reason core flows are examined further.

In the first flow regime, up to the point at which the flow area in the core is no longer larger than the flow area of the film, the flow occupies all the height, and the flow and temperature profiles are those for a boundary layer. Boundary layer profiles were assumed which fill the requirements for the first derivatives and boundary conditions of the momentum and thermal energy equations. The temperature and boundary layers were assumed to be of equal extent, comparable to an $N_{Pr} = 1$ as for equal momentum and thermal diffusivities, even though inertial terms were neglected, i.e., comparable to an $N_{Pr} >> 1$. This apparent paradox has been shown to give generally valid results for $N_{Pr} > 1$. (N_{Pr} for water is >1.5 and other food syrups, brines, and sauces have appreciably higher N_{Pr} numbers.) This domain was hypothesized to be valid for a $(r/L) N_{Grashof} \cdot N_{Pr}$ above 3.4×10^3 and to approach the flat plate solution. The $N_{Gr} \cdot N_{Pr}$ values of the system investigated experimentally herein were about 10^4 to 10^6 higher.

Later in rapid convection processes the second domain might be encountered. In this region THE BOUNDARY LAYER DOES NOT OCCUPY ALL THE CONTAINER HEIGHT, and the profiles are not fully developed. Below this flow is a stagnant liquid layer. This region was predicted to persist down to a $(r/L)(N_{Gr})_a$ Top $\cdot$ N_{Pr} of 3.1×10^2 (or long into a transient process).

Below such a criterion a similarity region with fully developed flows is said to exist.

In the first region the $N_{Nu,a}$ was said to be proportional to the 1/4 power of the modified Rayleigh number and in the second and third regions to be proportional to the first power of this number.

Experimental evaluation of theory.--The second and third of these regions have been confirmed experimentally by Eckert and Diagula (1952); Foster (1953); Martin and Cohen (1954); and Martin (1955); among others. The last two of these publications noted reduced heat transfer above a modified Rayleigh number of $10^{4.4}$ to $10^{4.6}$, the eventual drop in performance upon transient heating predicted by Lighthill was not grossly evident but heat transfer was below that predicted for turbulent flow when no interference of turbulent boundary layer and cores would exist.

Although, as in Lighthill's thermo-syphon tubes, the boundary layer forms would be anticipated to decrease in height, rather than depth, for a transient heated closed tube, such a direction of recession with reduced modified Rayleigh number (temperature difference here) would seem to be necessary if lateral flow were to be retained. In Lighthill's analysis the critical modified Rayleigh number was taken at the top of the tube where the driving force between the cold entering core and the exterior surface is greatest. The greatest difference for transient heating is at the slow point. A decrease in height of lateral eddy loops may not be inconsistent with a vertical movement of the axial cold point as conduction heating is approached if the loops do not fill the tube, i.e., if they have less radial extent.

Slow Point Location

Hartnett and Welsh (1957) also have studied flow in thermo-syphon tubes and have illustrated both a cold fluid centerline temperature and a minimum film coefficient at an X/L of about 0.2. Later investigations with mercury do not show this minimum, but report a minimum at the bottom of the test section (Hartnett et al., 1958; and Larsen and Hartnett, 1961). Harnett (1963) was able to shed no light on this question or the projected (Hartnett and Welsh, 1957) study of bottom stagnation or possible bottom cellular flow as the research program was discontinued. Lighthill had hypothesized the possible existence of such body eddies for the first, or boundary layer regime.

Some Characteristics of Convection

In the above consideration of convection patterns it has been noted that the qualitative plug flow model, modified to include any or all of three regimes of conduction, Benard cells, irregular turbulent filaments, or bottom heating, and five possible regimes of lateral flow, and mixed turbulence boundary layer, briefly or not at all a laminar boundary layer, a non-similarity region of undeveloped flow with stagnant layers below, a fully developed similarity region filling the tube and ultimately conduction, describes the phenomenon.

When the temperature histories are characterized by Ball's (1923) heat penetration parameters, the f and j may vary with time of heating as a reflection of a change in the natural convection components of the overall surface coefficient of heat transfer. This variation was qualitatively observed to reduce j from the Newtonian model j = 1, but capacitance of the container and transport lag of the fluid, as lateral loops are established, may increase j's. Lag factor values below those predicted by effective diffusivities might be anticipated.

The relative position of the slow point has not been established, and the current apparatus appears not to be sensitive enough to detect the movement in rapidly convective systems such as those investigated here. It is thought that its position may be a complex relation depending on the modified Rayleigh number and container geometry. Conductive limiting cold regions may exist in the center of eddy loops, but convective limiting cold regions may exist at the bottom of eddy loops, particularly between the lateral and the nearest bottom eddy.

The temperature profiles may be distinguished from conduction by the speed of heat penetration, a wide, central region of nearly uniform temperatures in a radial direction, steep monotonic decreasing concave downward axial temperature gradients from top to bottom of the container. The temperature distributions may or may not be similar. Inertial forces are presumed small although it is apparent that they may be responsible for predicted f values which are substantially larger than predicted on the basis of quasi-steady-state film coefficients for flat surfaces, but if of even a moderate relative size will decrease curvature of the heat penetration curves.

Real Products

Many products in which convection may play a prominent role have either a non-Newtonian or non-homogeneous fluid and added particulate matter.

It is difficult to interpret heat penetration in such products with our present knowledge of the factors and mechanisms involved. Two aspects of studies of heat penetration in food products (and for comparison, in the frequently used model product--bentonite) are analogous to proposed behavior in model systems. These aspects are the possible change in position of the slow point due to container size and product, and some additional factors which may lead to changes in thermal and flow domains analogous to the changes in homogeneous Newtonian liquid models. The reader may refer to the cited publications for background material if he wishes to examine a particular system or mechanism.

Townsend et al. (1956) have reported that axial cold points are about 1/2 to 3/4 in. above the container bottom for convection heating, and near the geometric center for conduction heating products. When there is a gross change in the mechanism or regimes of heating, the axial slow point might be anticipated to be at an intermediate position depending on the time of heating of the lethal portion of the process.

Many causes which can be advanced for such 'breaks' in heat penetration curves are shown in Table 6.5.

The practices of many laboratories are reflected in the slow point locations tabulated in Table 6.5. These locations are reported to be invariant, some arbitrary height, or at a fixed axial ratio. Investigations at this laboratory indicate possible movement of the slow point during a process. Slow points may also be anticipated at constrictions between solid phases--e.g., between wings and legs of fowl (Bock,1961) as well as the interior or eddy loops and at stagnation points.

The particular toroidal zone suggested for black-eye peas by Powers et al. (1962) may be a result of effective anisotropic eddy difussivities which in convection are larger axially than radially, supplemental to radial conduction. In this instance the rising film will be constricted and the descending core retarded by the product, so that radial conduction, unimportant in rapid convection becomes important as well.

TABLE 6.5.--Some slow regions for convection as reported or recommended by various investigators

Material	Container	Position*			Investigators
		Caps Up	Caps Down	Side*	
A. Water	pint (302x411) jar	1.0 cm (0.4 in)			Pflug & Nicholas (Aug. 1961)
		0.8 cm (0.3 in)			Pflug & Nicholas (Aug. 1961)
	quart (309x702) jar	1.5 cm (0.6 in)			Pflug & Nicholas (Aug. 1961)
B. 1 percent Bentonite	baby (202x309) jar	1/2 in	7/8 in	2-1/4*; 3/8	Townsend *et al.* (1949)
	junior (208x401) jar	1/2 in	7/8 in	2-1/2; 1/2	Townsend *et al.* (1949)
	303 (303x411) jar	1/2 in	7/8 in	3-1/4; 5/8	Townsend *et al.* (1949)
		1/2; 3/4			Fagerson & Esselen (1950)
	2-1/2 (401x414) jar	1/2 in	7/8 in	3-1/2; 3/4	Townsend *et al.* (1949)
		3/4 in; 1-1/4			Fagerson & Esselen (1950)
	307 x 409 can	1/8; 1/2			McConnell (1952)
	603 x 700 can	1/8; 3/4			McConnell (1952)
C. 2 percent Bentonite		None Reported			several investigators
D. Sucrose Solutions	No. 2 cans	3/4 in			Jackson (1940)

TABLE 6.5--Continued

Material	Container	Position*			Investigators
		Caps Up	Caps Down	Side*	
E. Convection Food Products					
1. Clear soups, most brine packed products	No. 2 can	3/4			Alstrand & Ecklund (1952)
	No. 10 can	1-1/2			Alstrand & Ecklund (1952)
	All	1/3 h			Merrill (1948) Board (1953)
	All	1/2 to 3/4			Townsend *et al.* (1956)
2.a. Cucumber spears (brine or syrup)	303 (300x412) veg jar	0.90			Pflug & Nicholas (Nov. 1960)
	No. 2 can	0.88			Pflug & Nicholas (Nov. 1960)
	No. 2-1/2 veg. jar	0.90			Pflug & Nicholas (Nov. 1960)
	(See also slices)				
b. Cucumber slices	16 oz. cyl (300 x 412)	.2 h			Nicholas *et al.* (1957); Nicholas & Pflug (1961)
	16 oz. veg (214 x 508)	.2 h			Nicholas *et al.* (1957); Nicholas & Pflug (1961)
	22 oz. cyl (300 x 704)	.2 h			Nicholas *et al.* (1957); Nicholas & Pflug (1961)
	24 oz. veg (312 x 702)	.2 h			Nicholas & Pflug (May 1961)

TABLE 6.5--Continued

Material	Container	Position*			Investigators
		Caps Up	Caps Down	Side*	
	24 oz. veg (312 x 702)	1.4 in			Nicholas *et al.* (1957)
	28 oz. veg (400 x 500)	.2 h			Nicholas *et al.* (1957); Nicholas & Pflug (1961)
	32 oz. veg (310 x 702)	.2 h			Nicholas *et al.* (1957); Nicholas & Pflug (1961)
	All	between stacked slices			Bock (1961)
c. Cucumber wholes (syrups and brine)	16 oz. veg (300 x 412)	0.9 in			Nicholas & Pflug (1961)
	16 oz. cyl (213 x 508)	1.0 in			Nicholas & Pflug (1961)
	22 oz. cyl (300 x 704)	1.4 in			Nicholas & Pflug (1961)
	32 oz. reg. qt. (310 x 702)	1.3 in			Nicholas & Pflug (1961)
	32 oz.	1.0 and between pickles			Esselen *et al.*(1952)
	32 oz.	.75			Esselen *et al.*(1951)
	64 oz. reg 1/2 gal. (414 x 810)	1.3			Nicholas & Pflug (1961)
	128 oz. reg gal (605 x 1,000)	1.7			Nicholas & Pflug (1961)
3. Starchy-solids in liquid					
a. Hominy, whole potatoes, diced potatoes.	pint jar	1/2 in.			Derrosier & Esselen (1950)
	quart jar	1/2 in.			Derrosier & Esselen (1950)

TABLE 6.5--Continued

Material	Container	Position*			Investigators
		Caps Up	Caps Down	Side*	
	No. 2 can	1/2 in.			Derrosier & Esselen (1950)
	No. 2-1/2 can	1/2 in.			Derrosier & Esselen (1950)
b. Baked beans	pint jar	2 in.			Derrosier & Esselen (1950)
	No. 2-1/2 can	2 in.			
c. Pork and beans in molasses	pint jar	1/2 in.			Derrosier & Esselen (1950)
	quart jar	3/4 in.			Derrosier & Esselen (1950)
	No. 2 can	1/2 in.			Derrosier & Esselen (1950)
	No. 2-1/2 can	3/4 in.			Derrosier & Esselen (1950)
d. Starchy pastes		geometric center			Jackson (1940)
4. Others					
Diced carrots, cut green beans.	2-1/2 (401 x 414) jar	1-1/4; 3/4			Fagerson *et al.* (1950)
Apple juice	2-1/2 (401 x 414) jar	2; 3/4			Fagerson *et al.* (1950)
Carrots, green lima beans, whole kernal corn, Eng. peas, summer squash.	pint and quart jars	5/8 to 1 in.			Cover *et al.* (1955)

TABLE 6.5--Continued

Material	Container	Position* Caps Up	Caps Down	Side*	Investigator
Fruits in syrup	1, 2, 4, lb. jars	3/4 in.			Crang & Sturdy (1953)
Fruits in water	tin (401x411)	1 in.			McConnell (1949)
Fruits in syrup (18° Bricks and higher)		geometric center			McConnell (1949)
F. Broken Heating					
Bentonite 3.25 percent		3/4 in.			Jackson & Olson (1940)
Vacuum packed corn, brine packed spinach, syrup packed sweet potatoes, tomatoes.		3/4 in. and geometric center			Alstrand & Ecklund (1952)
Tomato juice	Wide range of can sizes from (300x 407) to (603x812)	1-1/2-2-1/2 in.			Frost (1940)
G. Stratified Products					
Model systems		slightly above interfaces			Nicholas, Pflug, & Mulvaney (1960)
Sterilmatic com		geometric center of sauce			Ball (1949)

TABLE 6.5--Continued

Material	Container	Position*			Investigators
		Caps Up	Caps Down	Side	
Fruits in wine		geometric center			Barnby & Voigt (1929)
H. In Product					
Tuberous vegetables.		1/4-1/2 in. in product			Alstrand & Ecklund (1952)
Potatoes, carrots, beets, etc.		1/4 in.			Townsend *et al.* (1956)

---

*Data reported as 2-1/4, 3/8 for containers on their side indicates position from cap, height or position from wall, height. When only one dimension is given it refers to a height on axis of jar.

TABLE 6.6.--Factors which may cause changes in local heat transfer mechanism or convection domain

---

1. Anomalous viscosity such as:
    - a. jel-sol transition (Jackson, 1940)
    - b. thixotropy
    - c. "real" plasticity
    - d. Bingham plasticity
    - e. phase change of fats
2. Stratification:
    - a. Layering of dissimilar fluids, one of which is eluted from the product by osmotic forces.
    - b. Layering of the same fluid upon release of water from the thermally degraded product.
    - c. Intentional stratification to cause improved overall processing; may not necessarily lead to breaks in the heating curve.
    - d. Stratification due to product movement.
3. Gross changes in the exterior coefficient of heat transfer.
    - a. Natural convection exterior coefficient (Olson)--a somewhat uniform change.
    - b. Change from a convective to a diffusion mechanism as in steam air mixtures (without agitation) near the saturation temperature.

PART III. EQUIPMENT FACTORS INFLUENCING PROCESS HEAT TRANSFER

In the foregoing considerations of methods of analysis of heat penetration data and of theoretical and experimental velocity and temperature distributions, it was assumed that the processing environment could be adequately controlled. There is a large and growing body of literature on the need for controlling certain aspects of the processing environment. These aspects include an adequate and calibrated temperature (and pressure) control system, adequate venting, proper stacking, and to a lesser extent controlled come-up time.

It has been shown that fluctuations of retort temperature are reflected in changes in cold point temperature at a somewhat later time. Some practical control measures recommended by Townsend et al. (1956) are close temperature (and pressure) control by calibrated controllers set to give a processing temperature about 1°F. high so that the temperature is always slightly above that for which the process was designed. Process time control is also important. The correction measures suggested in "Methods of Analysis of Heat Penetration Data" are somewhat tenuous, and hence are recommended only for obtaining reference values of thermophysical properties (α) with higher precision than allowed by the precision of available laboratory equipment. These system methods suggested for analysis of complex systmes seem to be a valuable research tool. Application of these to temperature distribution in properly stacked crates in retorts or cooling canals might clarify the phenomena of such cooling, but may well be impractical for more routine application in the canning industry.

Since Pflug and Blaisdell (1962) have shown that heating in steam-air mixtures is not retarded when the velocities are sufficiently high, the presence of air in a retort is a hazard only if the velocities are so low that diffusion or natural convection between points in a retort are major heat transfer mechanisms. Adequate venting is a must, however, if the retort fluid is not sufficiently agitated.

Come-up time enters into a consideration of heat transfer mechanisms because it may be responsible for causing gross errors in reporting values for f and j, and process lethality F_0. Experimentally based corrections for adequately specified come-up have been considered earlier. Examination of come-up for convection has analytical implications as well. First, it may have obscured other factors; second, it may give insight into the role of inertial forces in convection. If inertia forces are controlling, the convective velocities are established early in the process. This means that neither viscosity changes with temperature or temperature difference changes through a process will influence the internal natural convection; hence, the heat penetration curves will be essentially straight. Length and path of come-up during the establishment of the convection currents would be reflected in large changes in f and j. If inertial forces are negligible, f at a specified u below retort temperature should not be affected. Some intermediate region in which inertia can cause discernible effects but is not controlling seems now to be the case.

Alstrand and Benjamin (1949) have investigated the effect of come-up time and the manner (linear or exponential) of come-up on heat penetration in conduction, heating cream style corn, and 5 percent bentonite and also on convection heating 2 percent bentonite. The f values for conduction heating were made equal in accord with theory in order to examine the effects of come-up on experimental j. When come-up is not large with respect to f and the heat penetration curves are fitted to the data at a long enough time after the equipment is at processing temperature to assure convergence, this procedure is correct. Here the $t_{cu}/f<40$ percent convection heating curves were also reported to be essentially equal. The come-up times were about ±20 percent of f.

The data of Townsend <u>et al</u>. for 5 percent bentonite in tin and glass containers processed in water with different come-up time are shown in Figure 6-10. The decrease in f with increased come-up time (perhaps not significant statistically) might be explained on the basis of higher heat transfer rates due to more of the can being near the retort temperature. An increase in f would be introduced by slow convergence if the lines were not fitted sufficiently long

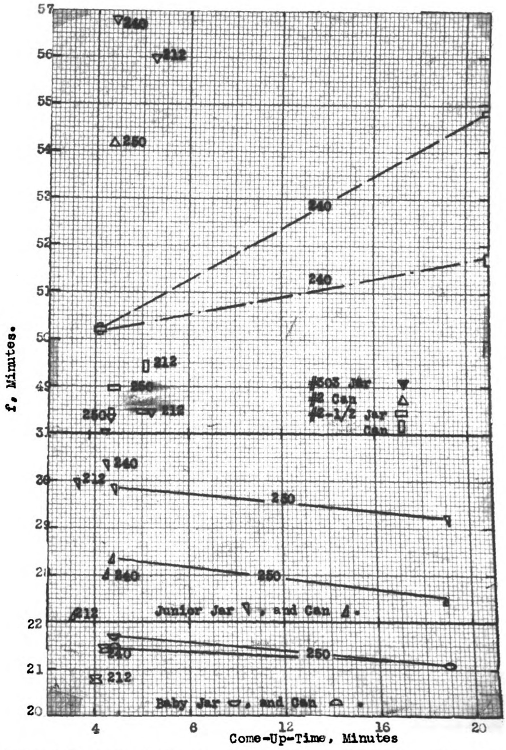


FIGURE 6.10 The effect of come-up time on the heat penetration parameter f (average) for five percent bentonite in various glass and metal containers precessed in water at various temperatures after the data of Townsend et al. (1949) Table 8.

after the end of the come-up period.

Townsend et al.'s (1949) data and approximate confidence limits for their studies of convection in tin and in glass, in water and in steam for various come-up times are shown in Figures 6-11 and 6-12. Two overall trends can be noted: a general slight increase in f as come-up time is increased and faster heating at higher processing temperatures (and initial temperature differences). The increase in f may be due to inertia although the slopes must have been determined at small temperature differences if the lines were fitted after the end of the longer come-up period. The process temperature effect may also be due to faster eddy and conductive transport as temperatures are increased. The Fagerson and Esselen data (1950) has been added to these figures.

A comparison of the Alstrand and Benjamin (1949) and the Townsend et al. (1949) data for 2 percent bentonite in No. 2 tin cans is made in Figure 6-13.

The principle variable (as shown) affecting the f value is the come-up time (Townsend et al. data).

The variable of next import is the heating medium--steam or water. The higher film coefficients in steam decrease f for Table 8 data steam, but not substantially for Table 9 data (Townsend et al.). Perhaps the degree of agitation of the water or the capacitance lag effect of the water on the shape of the retort come-up are responsible for the internal differences.

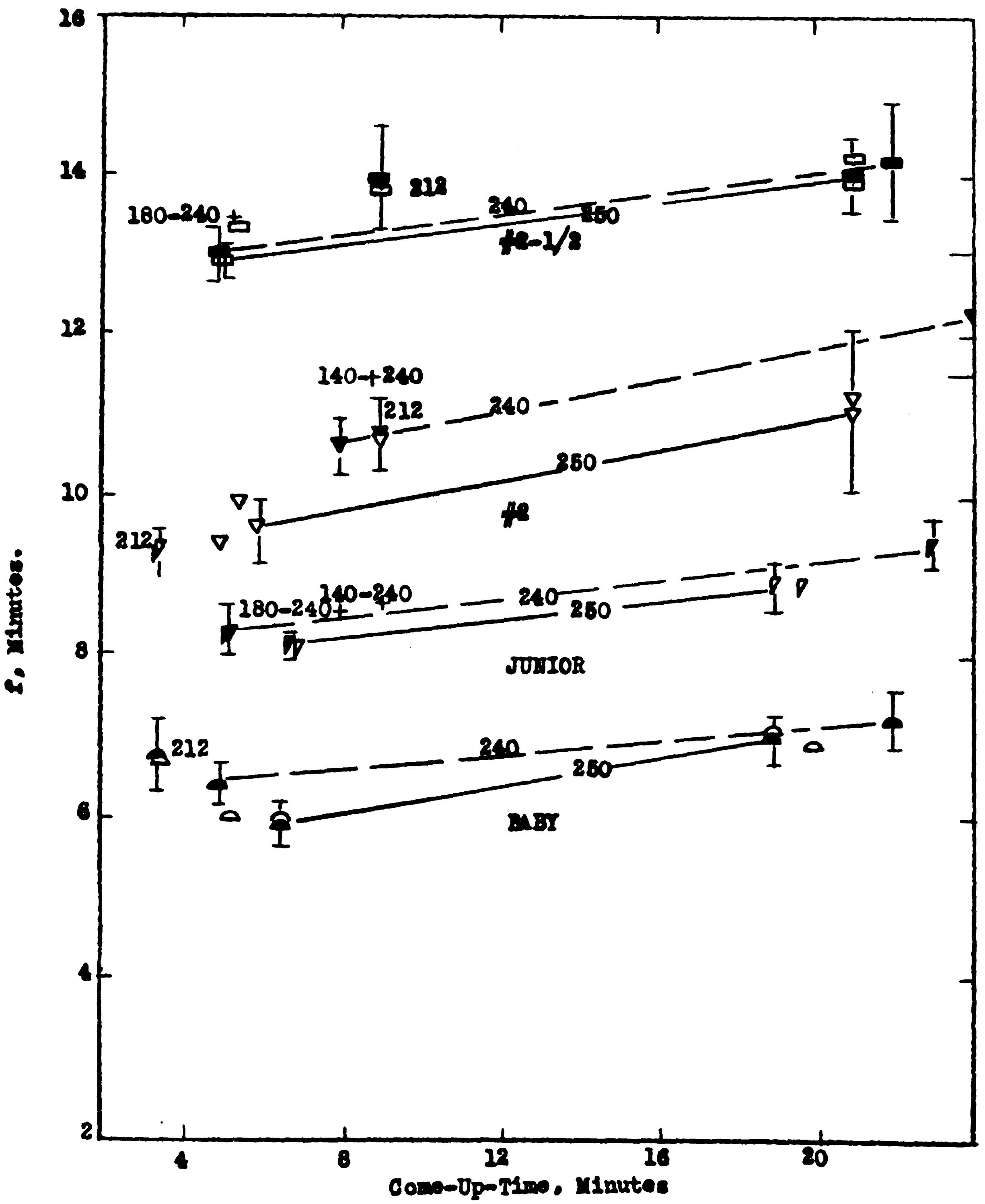


FIGURE 6.11 The effect of come-up-time on the heat penetration parameter f (average and 90 percent confidence interval based on range and mid-range) for one percent bentonite in glass jars processed in water at various temperatures after the data of Townsend et al (1949) Table 4 and means from Table 8. Also shown (as crosses) are the data of Fagerson and Esselen(1950).

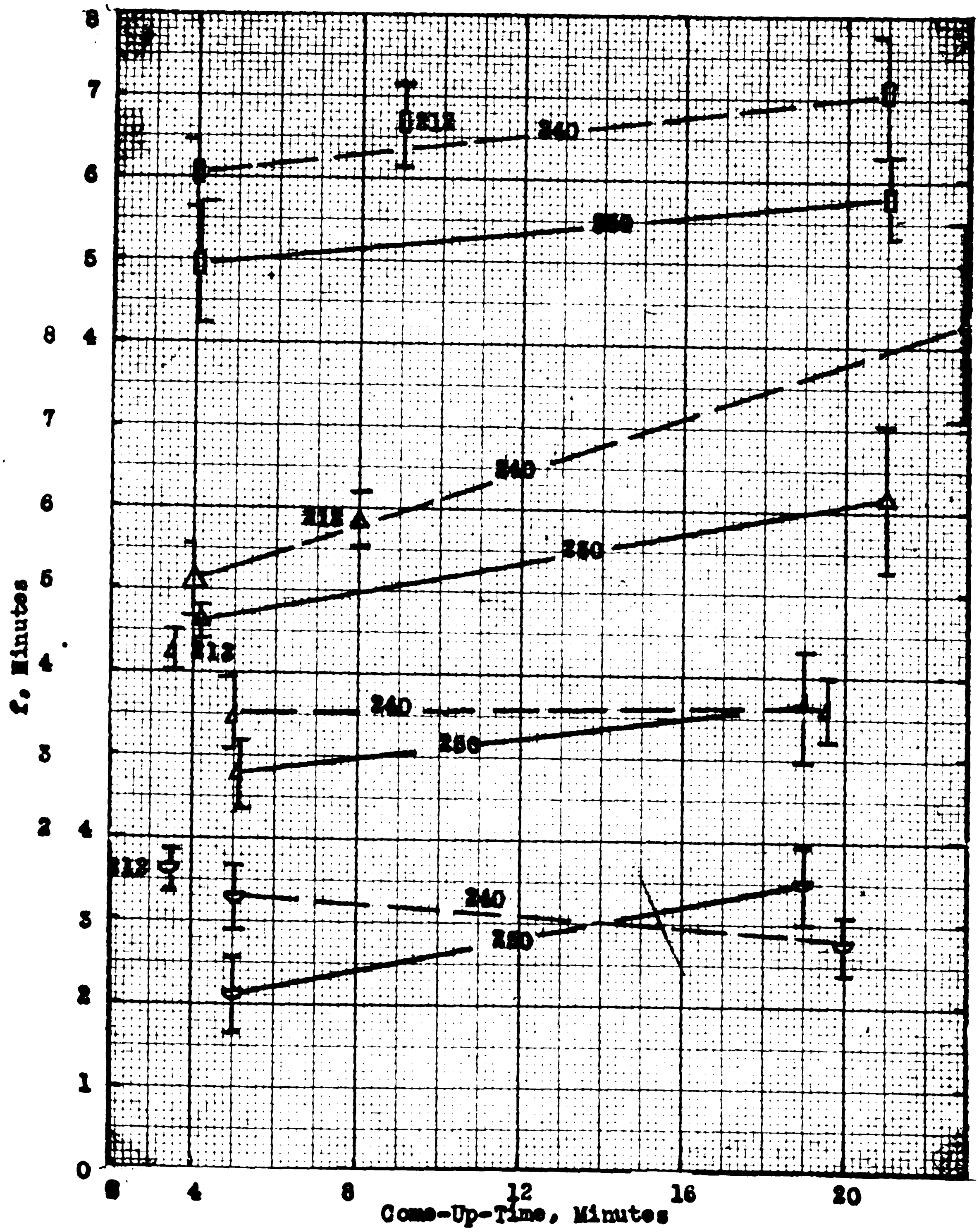


FIGURE 6.12 The effect of come-up-time one the heat penetration parameter f (average and 90 percent confidence interval based on range and mid-range) for one percent bentonite processed in cans at various temperatures after the data of Townsend et al (1949) Table 4.

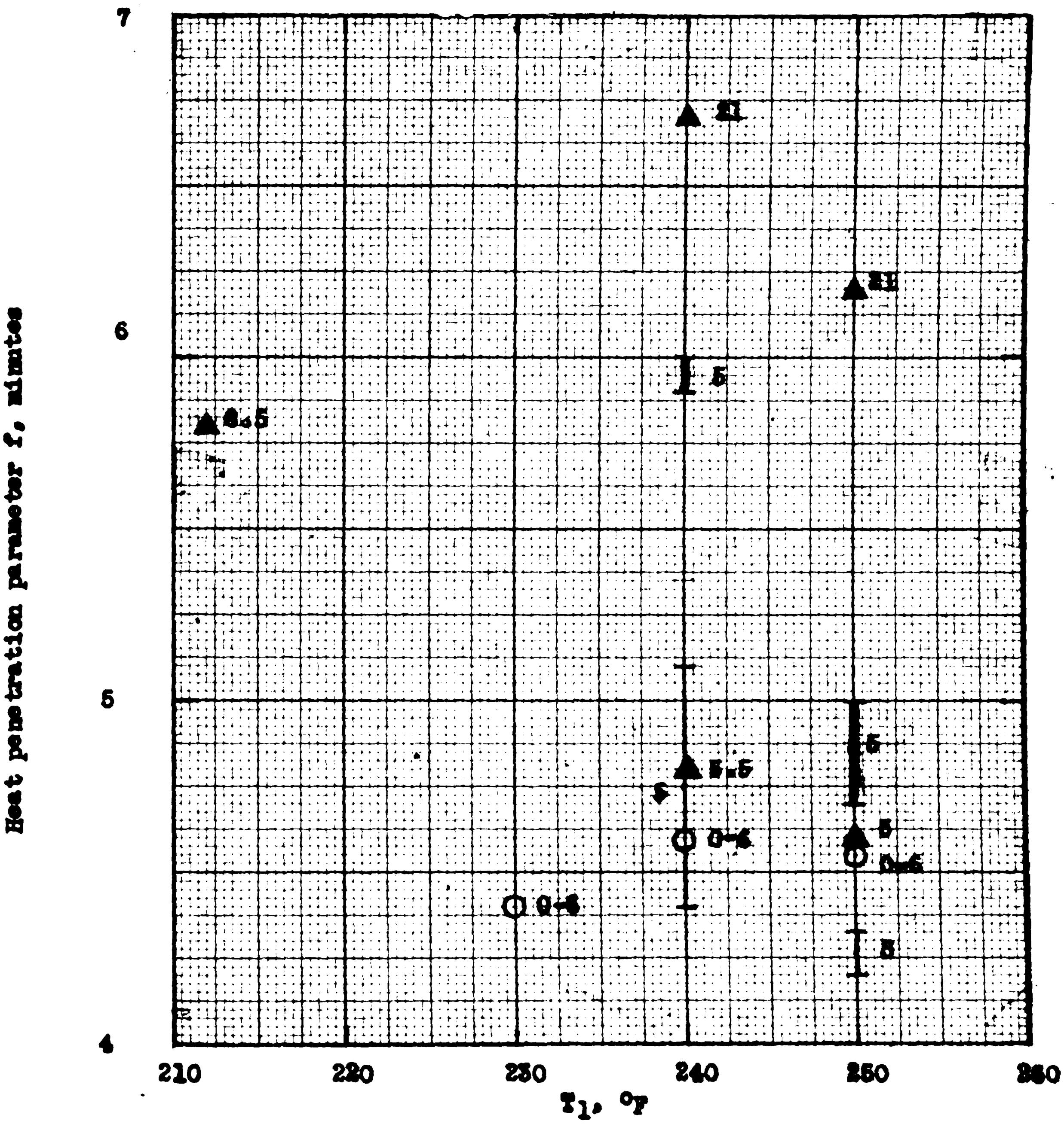


FIGURE 6-15 A comparison of published data for heating parameter f of 1 and 2 percent bentonite in No 2 tin cans, heated in water and steam for various come-up-times (noted on the graph) for a given T_1. The circles represent data from Alstrand and Benjamin (1949) for 2% bentonite with an approximate T_0 of 70°F. The remaining data are from Townsend et al (1949), Table 9 (bars) and Table 8 (triangles) with an approximate T_0 of 150°F. These data are for 1 percent bentonite.

Within the Townsend et al. experiment for similar conditions, an increase in retort temperature decreases the f in contrast to the Alstrand and Benjamin data. As the heating process may here be assumed to be principally a convective process and the temperature dependence of viscosity is large, the viscosity change in favor of a decreased f with increasing processing temperature may be assumed to be of more import than an increasing thermal diffusivity (for non-aqueous solutions the diffusivity decreases). Also of importance is the temperature difference which creates the flow due to density differences. If this last effect is considered, the Alstrand and Benjamin data and the Townsend et al. data can be reconciled. In addition, the known fractional power dependence of the convective film coefficient upon the temperature difference explains in part why this temperature difference effect does not predominate over other experimental factors within the Alstrand and Benjamin experiments. From the above considerations the inertia forces can be seen to have some effect.

It would seem desirable for further understanding of the role of inertial forces--and come-up path--to compare the temperature histories with those obtained for specified come-up and large or with negligible inertial.

For the exponential approach to retort temperature with the f of the retort equal to $\frac{2.303}{B}$ and not equal to $UA/C\rho V$) of the container when subjected to a stepwise response and subject to inertia, then:

$$\frac{(T_1 - T)}{T_1 - T_o} = \frac{B}{\left(B - \frac{UA}{C\rho V}\right)} e^{-\frac{UA}{CPV}t} - \frac{\left(\frac{UA}{C\rho V}\right)}{B - \frac{UA}{C\rho V}} e^{-Bt} \qquad 6.45$$

and for $B = UA/(C\rho V)$

$$\left(\frac{T_1 - T}{T_1 - T_o}\right) = \left(\frac{CPV - UAt}{CPV}\right) e^{-\frac{UA}{CPV}t} \qquad 6.46$$

Equations for other prescribed come-ups can be obtained by the Laplace transform or the method of undetermined coefficients because the ΔT_{Cr} appears in the Grashof terms of the overall film coefficient V. The value of such a critical ΔT_{Cr} is presently unknown; hence, the come-up path would have to be changed systematically in order to determine the possible existence of such a ΔT_{Cr}. This ΔT_{Cr} is possibly in the vicinity of the largest ΔT, between the surface temperature and the temperature of the fluid at the axial cold point, which is encountered during a process.

When inertial forces are negligible the differential equation

$$C\rho V \frac{dT}{dt} = U(T)A \left[T_1(t) - T\right] \qquad 6.47$$

could be integrated term-wise so as to account for the temperature dependency of U as well as the path of the environment temperature T_1.

The equipment factors mentioned above, temperature control, venting, and stacking, and come-up time, all can cause marked departures from design processing conditions. The temperature control effects may be comparable to the experimental errors in measurement of temperature differences mentioned in method of analysis; the venting and stacking may be associated with a change in exterior heating mechanism and so called "broken" heating curves, and the come-up path and time seemingly may cause more effects on heating rates than evidenced in conduction heating, due to the appearance of the temperature difference in the controlling overall surface coefficient of heat transfer.

PART IV. SOME PRODUCT, CONTAINER, AND HEATING MEDIUM FACTORS INFLUENCING PROCESS HEAT TRANSFER

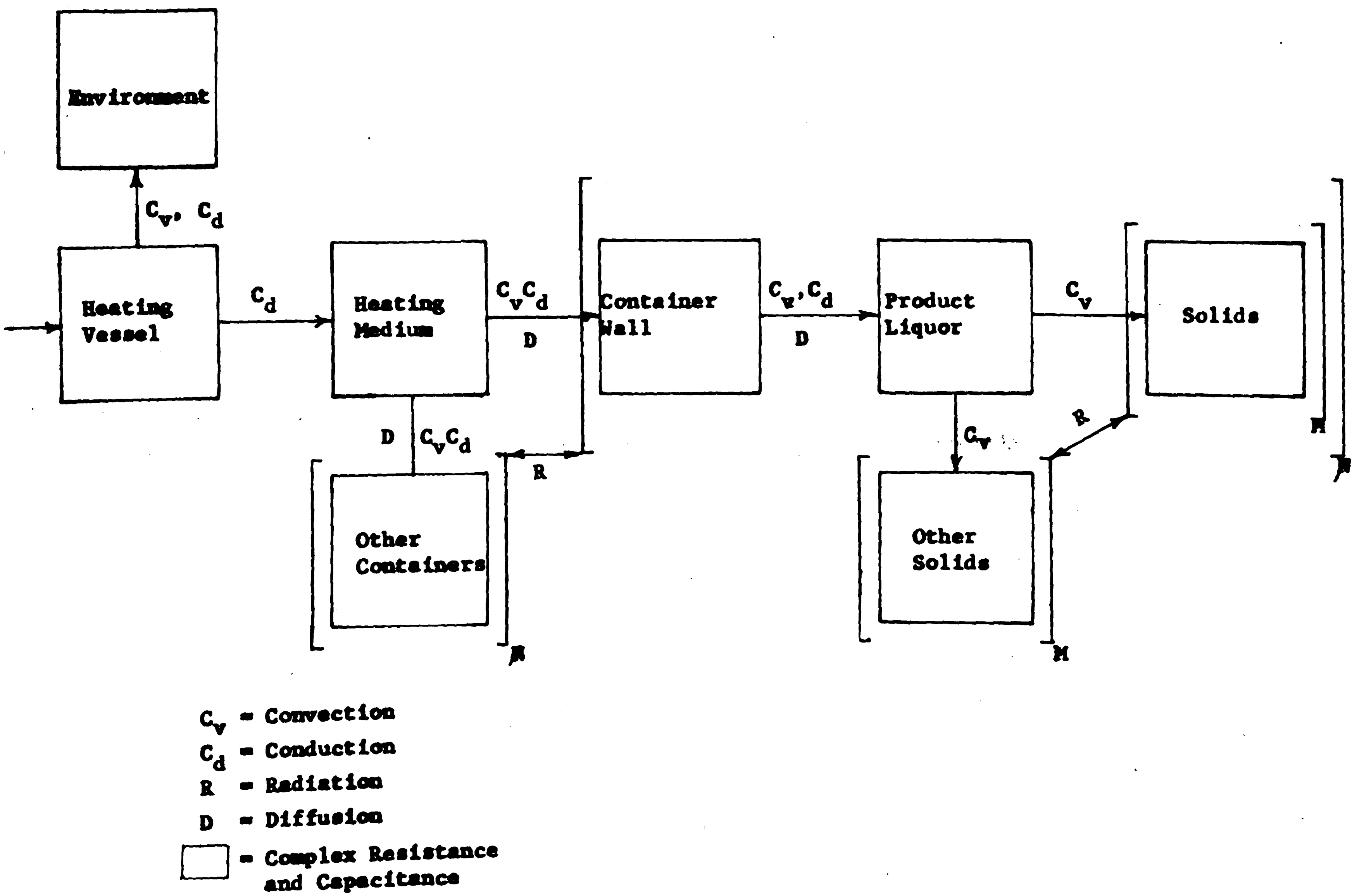
The phenomenon of transient natural convection of liquids in enclosed spaces is but one component of the transient system encountered in practice. The components of this system are the heating vessel, the heating medium, the container, all the product fluids (for vacuum process the head gas is important as well), and the product solids. These components are shown schematically in Figure 6-14. As the diagram illustrates, these components may be in series.

Each of the components may transfer energy or mass to the fluids by convection, conduction, radiation, or diffusion. These mechanisms are not all equally important nor are their transfer rates constant. The system capacitances may not be equally important. For this reason, only a few mechanisms, or system components may need to be considered.

In Part III above, it was pointed out that control of stacking, venting (and heating medium agitation) and of come-up may so reduce the effects of time and spacial temperature distribution within the heating vessel that they may be neglected in further considerations. If they cannot be so controlled, the overall phenomenon may become exceedingly complex.

The remaining components of the overall transient system, namely, the heating medium--the container--and the two closely interrelated product phases, will be considered briefly below. An examination of the magnitude of the transfer

FIGURE 6.14 Components and mechanisms of the transient natural convection system.



rates and the effects of the components may serve to clarify the phenomenon of transient natural convection.

Heating and Cooling Media

Exterior film conductances become important in the processing of canned foods when they produce recognizable differences in heating and cooling rates.

Differences between cooling rates of products cooled in air and those cooled in water were reported by Bigelow *et al*. (1920), Magoon and Culpepper (1921), and Jones (1931), and have been confirmed by subsequent investigations. Faster air cooling of glass containers compared to that of tin, attributed incidentally to a higher radiation emissivity, has been reported by Jones (1931b) and Esselin (1950), among others.

Differences between processing in water and in other media have not been consistently detected, however. Although Thompson (1919) reported a difference between convection heating products in cans in boiling water and steam at pressures above one atmosphere, he attributed this difference to the reduced product viscosity. Bigelow *et al*. (1920) were not able to determine a difference between steam and water, nor were Magoon and Culpepper (1921). Merrill (1938), however, reported faster heating in steam than in water at the same temperature. Roberts and Benjamin (1947) reported faster heating of tomatoes processed in steam, but McConnel (1949) found the converse for peach halves. Subsequently, Townsend *et al.* (1949) and Powers *et al.* (1952) detected no significant difference in the heating rates of bentonite suspensions between retort or atmospheric pressure processing in steam and in similar processes in water. Pflug and Nicholas (1961) found markedly faster convective heating in saturated steam and faster heating in water than in steam-air mixtures. They also confirmed the finding by Wehmiller and Wehmiller (1940) and Esselen *et al.* (1952) that heating in water sprays is not substantially faster than in water.

The factors which mask differences in heating and cooling rates caused by different heating media may include variations in operational practices, such as come-up time, venting, stacking, and air agitation; product fill, size, or consistency; and, to a lesser extent, initial temperature difference and processing temperature. It is interesting to note that Pflug and Nicholas (1961) found the above differences between media for model products in glass, which in itself is a large heat transfer resistance. In experiments the come-up time was negligible and the other factors were fixed. Later investigation by Pflug and Blaisdell (1961) of the effects of velocity showed that heating rates even in steam-air mixtures were greatly improved at moderate velocities.

Since improved product quality may be obtained by control of these factors during commercial processes, concern about exterior film conductances is not unreasonable.

TABLE 6.8.--Approximate Conductances of Containers and Product, Btu/hr ft^2 $^{\circ}$F

Material	Representative Thickness in.	Thermal Conductivity Btu/hr ft^2 $^{\circ}$F	Conductance Btu/hr ft^2 $^{\circ}$F
Glass Wall	0.125	0.6	58
Bottom	0.200	0.6	36
Tinned Steel	0.09	26	35,000
Water-like product	1.5	0.36	3

Film Conductances

Some surface and overall heat transfer coefficients for canned food products or analogous systems are shown in Table 6.7. These heat transfer coefficients become significant only when they are small in respect to the product interior film or equivalent conductance as to the container conductance, whichever is smaller. The findings of the investigations mentioned above illustrate this principle.

The film coefficients encountered during air cooling, on the order of 2 (largely due to radiation) are small with respect to the conductances of a liquid product film, 36 to 103, of glass, 58, or even a solid product. For air cooling the film coefficients, although about ten times as large, are still about 10 percent of those of a solid product. Since

Table 6 .7 . Some exterior surface and overall heat transfer coefficients, Btu/hr,ft^2,°F. (U values unless otherwise specified)

	U
A. COOLING IN STILL AIR	
1. Canned Fish (Okada, 1940, 1941)	U
Tall No. 3 (305 x 404)	1.8
Tall No. 4 (215 x 404)	1.4
7 oz. Tuna (305 x 109)	1.4
1/2 Kilo (303 x 207 x 100)	1.8
1/4 Kilo (215 x 201 x 104)	1.8
Large Rectangular (412 x 115 x 104)	1.8
Small Rectangular (401 x 112 x 104)	1.8
2. Water (Jones, 1931b)	
No. 2 (212-68°F)	1.7
3. Radiation from Surfaces (ASHRAE, 1960)	
($\epsilon = 0.9$, $\Delta T = 10$°F)	1.5-1.6
B. HEADSPACE (Evans and Board, 1954)	
1. At Low Temperature (32-104°F)	
No. 2-1/2 (401 x 411)	2.0
No. 1 Tall (301 x 411)	2.2-3.2
No. 1 Squat (401 x 211)	2.0-3.2
2. At Elevated Temperatures	
No. 2-1/2 (401 x 411) (75°F-240°F)	7.0-9.2
(83.5-241°F)	7.4-7.8
No. 1 Tall (301 x 411) (75-240°F)	6.7-10.5
(62.5-242°F)	8.0
1 lb. Squat (401 x 211)(75-240°F)	6.2-9.0
C. COOLING IN STILL WATER	
1. Canned Fish (Okada, 1940, 1941)	
Tall No. 3 (305 x 404)	24
Tall No. 4 (215 x 404)	10
7 oz. Tuna (305 x 109)	7
1/2 Kilo (303 x 207 x 100)	26
1/4 Kilo (215 x 201 x 104)	24
Large Rectangular (412 x 112 x 104)	21
Small Rectangular (401 x 112 x 104)	24

Table 6.7 (continued)

2. Unagitated Jacketed Kettle (Poste, 1924)	
Water to Water	43
3. Citrus Segments in Agitated No. 2 Can in Countercurrent Spray Cooler (Heid and Scott, 1937)	
86 rpm	36
4. Citrus Juice in Agitated No. 2 Can in Countercurrent Spray Cooler (Heid and Scott, 1937)	
0 rpm	38-45
70 rpm	88
100 rpm	285
261 rpm	145
555 rpm	295
D. HEATING IN WATER	
1. Agitated Water Baths	
No. 2, No. 10 can (68-212) (Jones, 1931a)	130
Unspecified (41-122) (Cowell *et al.*, 1959) $h_o = 270$	--
Unspecified (32-104) (Evans and Board, 1954) $h_o = 400$	--
312 x 708 (ambient to 150) (Merrill, 1948) (of lead-water) $h_o = 150$	70
(ambient to 190) (Merrill, 1948) $h_o = 175$	95
(ambient to 220) (Merrill, 1948) $h_o = 190$	105
(ambient to 250) (Merrill, 1948) $h_o = 210$	130
2. Citrus Juice in Agitated No. 2 Cans in Countercurrent Spray Cooker (Heid and Scott, 1937)	
70 rpm	180
3. Citrus Segments in Agitated No. 2 Cans in Countercurrent Spray Cooker (Heid and Scott, 1937)	
26 rpm	31
4. Unagitated Jacketed Kettles (Kern, 1950)	
Water-water	100
Water-water solutions	75-80

Table 6.7 . (continued)

Water-hydrocarbons	50
Water-medium organics	10-20

5. Natural Convection Flat Plate 4-1/2 in. high (McAdams, 1954)

180° Bath (120° T)	226
180° Bath (10° T)	103

E. HEATING IN STEAM

1. Can of lead-water, 312 x 708 (Jones, 1931a)

(ambient to 220) h = 620	175
(ambient to 240) h = 780	215
(ambient to 250) h = 880	240

2. Unagitated Kettles (Kern, 1950)

Steam-water	175-250

3. Condensation of Steam on Flat Plates (230°F steam, 10°F T) (Othmer, 1929)

0% air	2800
1.07 air	1400
1.96	920
2.89	730
4.53	480

the film coefficients for such cooling are relatively small, their effects have been repeatedly detected.

It can be seen that when water baths or retorts are used, the small conductance of glass, 58, becomes significant only for convection heating products, 36-103. For this reason the conversion factors between f for glass and f for tin as reported by Townsend et al. (1949) are near unity for 5 percent bentonite and are important for 1 percent bentonite.

The aforementioned factors for 1 percent bentonite also reflect a very small difference between processing in water and in steam as suggested by Figure 6.13. Since film coefficients for water and steam are reported to be 150 to 210 and above, and 620 to 880 and above, respectively, and the conductance of glass is about 58, such a small but detectable difference can be expected. The implication is that conductances of 150 to 210, as reported by Merrill (1948) or even lower are encountered in commercial equipment rather than 300 and above as is commonly inferred. The difference between water and steam becomes somewhat more important for convection in tin when $C = k_w/L_w = 36$ to 103, are encountered. This concept of controlling resistance can be used to estimate the approximate range of h of a given heating medium.

Since Pflug and Nicholas (1961) found no effect of the poorly conducting air film encountered with still steam-air mixtures even with 30 percent air for conduction products in glass and Pflug and Blaisdell (1961), found an effect for convection in glass ($k/L_w = 30$), one may infer that the film coefficient between 30 and 300, depending on velocity, is encountered with steam-air mixtures. These values have been examined in more detail in the previous section of this dissertation.

Determination of Film Conductances

Although heating rates of food products themselves have been used to obtain approximate values of the overall coefficients of heat transfer, e.g., references a, b, f, g of Table 6.7, estimation from heat penetration and models of known thermal properties are preferable, e.g., references, d, h, i, j of Table 6.7.

Okada's values which were determined from heat penetration in canned fish products cooled in air or water with thermal properties estimated from heat penetration heating in boiling water are uncertain. Neglect of the headspace resistance and the brine film coefficient both would tend to give a too low value of the apparent effective diffusivity, hence this reduced heat transfer would not be attributed to a low value of the surface conductance. The headspace would have more effect for these low film coefficients than for heating in boiling water and, therefore, might cause the apparent film coefficients to be too high.

Jones (1931a, 1931b) and Heid and Scott (1937) measured the overall coefficient of heat transfer using the Newtonian, i.e., uniform, temperature model, and the heat penetration in canned fluids. Jones neglected heat transfer through the headspace, but Heid and Scott did not. Nevertheless, the values reported by both investigators may be too small since Jones used the central temperature of unagitated cans and Heid and Scott may have used the central temperature of agitated cans. In both instances the use of too large temperature, i.e., fluid temperatures below the mixed mean temperature of the fluid, would cause the apparent overall coefficients to be too low. Schmidt (1955) avoided both of these errors, when investigating internal coefficients in spheres using transient histories, by filling the sphere, using the volumetric expansion of the fluid itself as a thermometer, mixing at the beginning and end of a small temperature change, and like Heid and Scott, the logarithmic mean temperature difference over the heating interval. This type of device, including that suggested in Figure 6-4, is a convenient way to obtain overall coefficients or internal coefficients for actual food products.

Metal cylinders of approximate size of the container in question are most suited for and may be preferable for measurement of external film coefficients. An investigator may extend the tables of Ball and Olson for the j of various shaped cylinders and for f for cylinders of equal semiheight and radius to the lower values of Biot numbers encountered with metal cylinders and extend the f tables to other shapes. From these tables curves for film coefficient (presumed constant) as a function of observed f or j for a particular metal cylinder may be constructed as shown in the previous section. Such measurements, of course, are subject to all the errors of heat penetration conduction heating food products except that of thé headspace.

Since the use of a cylindrical geometry requires construction of curves for that geometry or two-way interpolation in tables, engineering correlations for heat transfer to or from finite cylinders are not available for calibration purposes (indeed the finite length may cause unknown flow separation), some investigators have used spheres, which do not have these limitations, instead. This geometry is particularly advantageous if the time variation of surface conductance (see Stolz _et al_., 1959, Stolz, 1959) or to the free convection heat transfer properties of fluids (Boberg and Starrett, 1958) are to be determined. When time variation of surface conductances on a sphere or infinite slab (see Beck, 1962) is desired, temperatures at several positions in the body are required. Since high conductivity bodies of small dimension are used for sensitivity, position (j) and thermocouple errors become critical.

The desire for increased accuracy in the determination of the temperature dependency, introduces the additional thermocouples and thereby makes machine computation almost necessary. Both of the procedures cited use the integral equation approach, an approach which ironically is evaluated by comparison with the asymptotic solutions, i.e., Ball's (1923, 1928) approach, although not his nomenclature (Elrod, 1960). There is some question as to whether or not the film coefficients so determined satisfy the quasi-steady-state assumptions.

Precisely because engineering calculations for short cylinders are not available and there are no relations to assist interpretation of results for spheres in terms of finite cylinders, their use is suggested in the food industry. At this time it does not appear that the admitted temperature variation of film coefficients, natural convection in particular, warrants adoption of the above refinements. Some information about this variation may be obtained from analysis of short segments of the heating curve for the central temperature.

Comparison of Can and Jar Heating Data

Townsend *et al*. (1949) have examined heat penetration for conduction and convection heating products in tin and in glass in order to establish "rules of thumb" which would permit microbiologically safe conversion of data from tin to glass or from one glass container size to another. Since the objective here is to have increased understanding of the convection phenomenon, their data will be examined in new comparisons.

Conversion factors for tin to glass for 1 percent bentonite processed in water are shown in Table 6.9. The same systems, but with glass processed in water and tin processed in steam, as is commercial practice, are compared in Table 6.10. The difference between glass and tin is seen to decrease as processing temperatures (and initial temperature differences) decrease. There is also a general decrease in this difference at long come-up times in water when once again the film coefficients are anticipated to be lower.

TABLE 6.9.--Conversion ratios, f glass/f tin for 1 percent bentonite processed in water under various conditions, after data of Townsend et al. (1949), Table 8.

Temperature	Baby Baby	Junior Junior	303 #2	2-1/2 2-1/2
5 Min. Come-up-time				
250° F	3.33	3.24	2.05	2.43
240° F	2.14	2.68	2.06	2.44
212° F	1.86	2.22	1.85	2.12
20 Min. Come-up-time				
250° F	2.22	2.40	1.79	2.04
240° F	2.76	2.62	1.67	1.97
Recommended*	1.85	2.49	2.86	2.84

*Townsend et al.'s (1949) recommendation is of the slowest heating of 12 to 22 jars (Table 4) processed in water with 22 to 24 min CUT (but frequently with the largest temperature difference) and unspecified number of cans processed in steam with 5 min. CUT (Table 9), for processing temperature of 240° F.

The condition of comparison, including comparison of slowest heating containers is commercial practice however.

This is in agreement with the opposite trend shown in Table 6.10. In the latter instance temperature differences, i.e., natural convection, are important for water and so are affected by come-up, while steam condensing coefficients in the absence of air are not as sensitive. (The behavior of baby and junior containers, which were processed together, at 240° may be due to some secondary factor, such as the path of the come-up.) The individual ratios principally reflect the relative total weights of container and product (an estimate of the relative heat capacitance). In critical cases the difference would be anticipated to be less for tall containers, since the film coefficient decreases as the container heights increase.

The comparison shown in Table 6.10 must be interpreted with care, for a check comparison, Table 6.11, of the heating data of two tabulations shows poor agreement.

Tin containers heated faster in steam at 250°F. and 240°F. than in water except for the baby-size containers; hence, Townsend <u>et al</u>. have reported no significant difference. In this instance j is smaller rather than larger, also. This behavior may be due to errors in the experimental equipment, for some baby cans are reported to heat slower than junior cans even though the f ratios for constant film total area are 1.09; constant film top neglected, 1.14; top neglected with heating like a flat plate, 1.04; and top neglected with modified Rayleigh number controlled heating, 1.8.

TABLE 6.10.--Conversion ratios, f glass/f tin for 1 percent bentonite processed in water (Table 8) and in steam (Table 9 mid-ranges) under various conditions, after the data of Townsend *et al*. (1949)

	5 Min. Come-up-time			
Temperature	Baby / Baby	Junior / Junior	303 / #2	2-1/2 / #2-1/2
250° F	1.50	2.54	2.22	2.72
240° F	1.41	2.08	2.08	2.35
	20 Min. Come-up-time			
250° F	1.78	2.78	2.61	2.92
240° F	1.63	2.22	2.36	2.44*

*Note that Townsend *et al*. recommendations would tend to give a safe process. The reason for this is the generally larger number of containers tabulated in Table 4 produces greater variability and hence slower heating (larger f).

TABLE 6.11.--Conversion ratios, f tin/f tin for 1 percent bentonite in tin (means Table 8, mid-ranges Table 9 respectively) processed in water with 5 min. come-up time after the data of Townsend *et al*. (1949)

Temperature	Baby / Baby	Junior / Junior	303 / #2	2-1/2 / #2-1/2
250° F	0.58	0.74	0.95	0.92
240° F	0.66	0.76	1.01	0.97

TABLE 6.12.--Can factor comparison. Convection in tin processed in water** for (1 percent bentonite data Table 8, Townsend et al. (1949))

	5 Min. Come-up Time				20 Min. Come-up Time			
Temp.	Baby***	Junior	2	2-1/2	Baby	Junior	2	2-1/2
250°F	1.00	1.39	2.56	2.94	1.00	1.16	1.94	2.12
		1.00	1.84	2.12		1.00	1.68	1.84
			1.00	1.15			1.00	1.10
				1.00				1.00
240°F	1.00	1.11	1.71	1.96	1.00	1.36	2.68	2.88
		1.00	1.55	1.78		1.00	1.97	2.12
			1.00	1.15			1.00	1.07
				1.00				1.00
212°F	1.00	1.17	1.61	1.81				
		1.00	1.38	1.55				
			1.00	1.12				
				1.10				
Recommended* (1956)								
	1.00	1.15	1.65	1.88				
		1.00	1.44	1.64				
			1.00	1.14				
				1.00				

*Townsend et al. (1956), A Laboratory Manual for the Canning Industry, 2nd Ed., NCA.

**Processing tin in water is not usual canning practice.

***Elements across the diagonal of the matrix are the reciprocals of the elements shown.

TABLE 6.13.--Can factor comparison. Convection in glass processed in water for 1 percent bentonite data, Table 8, Townsend *et al*. (1949)

Temp.	5 Min. Come-up Time				20 Min. Come-up Time			
	Baby	Junior	2	2-1/2	Baby	Junior	2	2-1/2
250°F	1.00	1.33	1.57	2.15	1.00	1.25	1.56	2.23
		1.00	1.16	1.59		1.00	1.14	1.61
			1.00	1.37			1.00	1.35
				1.00				1.00
240°F	1.00	1.38	1.65	2.23	1.00	1.29	1.62	2.06
		1.00	1.19	1.61		1.00	1.26	1.60
			1.00	1.35			1.00	1.27
				1.00				1.00
212°F	1.00	1.39	1.60	2.06				
		1.00	1.15	1.48				
			1.00	1.29				
				1.00				
Recommended*								
240°F	1.00	1.23	1.76	1.99				
		1.00	1.43	1.62				
			1.00	1.13				
				1.00				

*Recommended by Townsend *et al*. (1949) Table 10, data are for approximately 22 min. come-up time at 240° processing temperature and were calculated by taking ratios of slowest heating upright jars as tabulated in Table 4. The number of replications was 15, 12, 19, 22 respectively compared to the 5 to 10 replications averaged and presented in Table 8.

TABLE 6.14.--Can factor comparison. Conduction in tin processed in water** for 5 percent bentonite data, Table 8, Townsend *et al*. (1949)

Temp.	5 Min. Come-up Time				20 Min. Come-up Time			
	202 x 214	211 x 210	307 x 409	401 x 411	202 x 214	211 x 210	307 x 409	401 x 411
250°F	1.00	1.31	2.50	3.33	1.00	1.31	----	----
		1.00	1.91	2.56		1.00	----	----
			1.00	1.34			1.00	----
				1.00				1.00
240°F	1.00	1.30	2.59	3.44				
		1.00	1.99	2.65				
			1.00	1.33				
				1.00				
212°F	1.00	1.30	2.64	3.53				
		1.00	2.02	2.70				
			1.00	1.34				
				1.00				
Recommended*								
	1.00	1.32	2.73	3.60				
		1.00	2.06	2.72				
			1.00	1.32				
				1.00				

*Recommended by Townsend *et al*. (1956) after Olson & Stevens (1939) for conduction heating products (for negligible film resistance).

**Processing tin in water is not usual canning practice.

TABLE 6.15.--Can factor comparison. Conduction in glass processed in water for 5 percent bentonite data, Table 8, Townsend *et al*. (1949)

	5 Min. Come-up Time				20 Min. Come-up Time			
Temp.	Baby	Junior	2	2-1/2	Baby	Junior	2	2-1/2
250°F	1.00	1.38	2.25	3.39	1.00	1.39	----	----
		1.00	1.62	2.45		1.00	----	----
			1.00	1.51			1.00	----
				1.00				1.00
240°F	1.00	1.42	2.24	3.46				
		1.00	1.58	2.44				
			1.00	1.54				
				1.00				
212°F	1.00	1.44	2.33	3.48				
		1.00	1.62	2.42				
			1.00	1.50				
				1.00				

TABLE 6.16.--Some characteristic lengths for external flows*

The Characteristic Lengths	1/L	L	
A. All Equal Contribution.			
According to 6.49:			
$\frac{1}{L_c} = \frac{1}{L} + \frac{1}{D}$	$\frac{D+L}{LD}$	$\frac{LD}{L+D}$	6.51a
or	$\frac{2r+L}{2rL}$	$\frac{2rL}{2r+L}$	6.51b
According to 6.50:			
$L_c = \frac{(2/4)\pi D^2 L}{\pi DL + 2\frac{\pi D^2}{4}}$	$\frac{2L+D}{LD}$	$\frac{LD}{2L+D}$	6.52a
or	$\frac{L+r}{Lr}$	$\frac{Lr}{L+r}$	6.52b
B. No Heating Through Top Surface.			
According to 6.48:			
$\frac{1}{L} = \frac{1}{L} + \frac{2}{D}$	$\frac{2L+D}{LD}$	$\frac{LD}{2L+D}$	6.53a
or	$\frac{L+r}{Lr}$	$\frac{Lr}{L+r}$	6.53b
According to 6.50:			
$L = \frac{(2/4)\pi D^2 L}{\pi DL + \frac{\pi D^2}{4}}$	$\frac{4L+D}{2LD}$	$\frac{2Ld}{4L+D}$	6.54a
or	$\frac{2L+r}{2rL}$	$\frac{2rL}{2L+r}$	6.54b

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*The investigation of Evans and Board (1954) for conduction heating products is also applicable here. They found conductances from 2 to 8 Btu/square ft($^{\circ}$F),hr--a negligible quantity if the product is well mixed by convection.

TABLE 6.17.--Some characteristic lengths for canned food products in cylindrical containers

System	Characteristic Length	Reference
1. Conduction		
a. Infinite slab		
1) Finite surface resistance	$L/\beta_1(N_{Bi})$	Ball & Olson (1957)
2) Infinite surface	$L/[2(1)+1]\pi$	Ball & Olson (1957)
b. Infinite cylinder		
1) Finite surface resistance	$r/\gamma\,(N_{Bi})$	Ball & Olson (1957)
2) Infinite surface resistance	$r/(1)\,(2.408)$	Ball & Olson (1957)
c. Finite cylinder		
1) Finite uniform overall surface resistance	$\left[\frac{B^2}{(L/2)} + \frac{\gamma^2}{r^2}\right]^{-1/2}$	Ball & Olson (1957)
2) Finite top surface resistance Infinite lateral resistance	$\left[\frac{\delta^2}{L^2} + \frac{(2.408)^2}{r^2}\right]^{-1/2}$	Evans & Board (1954)
3) Infinite overal resistance	$2\left[\frac{2.34 + (D/L)^2}{0.933D^2}\right]^{-1/2}$	Ball & Olson (1957)

Where β_1, γ_1, and δ_1 are roots of the indicial equation

$$\frac{hr}{k}\cot\beta = \beta \qquad \delta\cot\delta = \frac{-hl}{k}$$

$$\frac{hL/2}{k}\,J_1(\gamma) = J_0(\gamma)$$

2. Convection		
a-1. Newtonian Model--Total surface effective	$\left(\frac{1}{2}\right)\left[\frac{rl}{r+l}\right]^{1/2}$	Schultz & Olson (1938)
a-2. Newtonian Model--Top surface ineffective	$\left[\frac{rl}{r+2l}\right]^{1/2}$	Jones (1931)
b. Lumped resistance Model total surface effective	$\left[\frac{rl}{2r+2l}\right]^{3/2}$	Kaneko (1941) Ban (1941)
c. An Empirical Model total surface effective	$\left[\frac{rl}{2r+2l}\right]^{1.2}$	Fagerson (1950)
d. A Fitted Dimensional Analysis Model	$r^{0.8}\,L^{.42}$	Fagerson (1950)

Similar comparisons of the data for 5 percent bentonite were made for contrast. The trends mentioned above, except for differences in individual ratios due to geometry, were not observed.

Comparison of Heating Data Among Cans and Jars

Comparisons within the data of Townsend *et al.* (1949) were extended to size comparisons for glass and for cans containing 1 or 5 percent bentonite when processed in water. These are shown in Tables 6.12, 6.13, 6.14, and 6.15. For the shorter come-up times, lower processing temperatures (and initial temperature differences) generally reduced the differences between container sizes. This is in contrast to the larger differences observed in Table 6.14. In this latter instance the Biot numbers for the small cans are such that a further reduction in the magnitude of this ratio may affect the smaller cans more severely. In the former instance, since the properties and Δt are presumed equal at a particular temperature and hence may not affect the factors as the level changes, the above results may be due to a change in the flow mechanism from turbulent toward laminar flow and hence a reduced power of the geometric factors from $n = 1/3$ to $n = 1/4$ in equations 6.11, 6.42, and 6.43. It must be noted that the above comparisons also reflect changes in come-up time and come-up path in an undetermined manner.

The Container

In the previous discussion it has been pointed out that container shape at the shoulders and bottom of jars may affect the iso F_o profiles and that container heat transfer resistance must be considered in comparison with exterior and product resistances. There is another aspect of container shape and size which seems to deserve specific attention here. This is the problem of choosing an appropriate characteristic length both for use in film coefficient correlations and in the convection Fourier modulus. The right hand side of the familiar equation for the Newtonian model, $f = 2.303\ C\rho V/(UA)$ may be rewritten as $\frac{C\rho}{U(V/A)}\left(\frac{V}{A}\right)^2\frac{1}{t}$ in the form of the reciprocal of a Fourier modulus. In this instance, V/A becomes the characteristic length of the system. Several characteristic lengths, in addition to this one, have been introduced in the various models, hence they need to be examined further.

Natural convection heat transfer data have been traditionally correlated and with some success by equations in the form proposed by Lorenz (1881).

$$\frac{h_oL}{k_f} = C_P\ \frac{L^3\rho_f{}^2g\ \beta_f\Delta T}{\mu_f{}^2}\ \frac{C_pu}{k}_f \qquad 6.48$$

King (1932) correlated his own data of heating of 1/2 to 12 inch long vertical plates with those of other investigations of heating of vertical plates and also tubes. Included were the data of Colburn and Hougen (1930) for vertical tubes with small water flows. As confirmed by Jakob and Linde (1933), data for horizontal cylinders and spheres were well correlated with those of vertical tubes and cylinders when the characteristic length was defined as follows:

$$1/L = 1/L_{horizontal} + 1/L_{vertical} \qquad 6.49$$

where the characteristic circular dimension is the diameter and the characteristic spherical direction is the radius, and the characteristic dimension for blocks is as in (6.49) above.

McAdams (1954, pp. 172, 177) points out, however, that the data for the spheres and the blocks fell in a range where, due to the exponent, the geometric factor had substantially no influence. He also points out that if $L = D\pi/2$--the maximum distance traveled by the fluid--is chosen for horizontal cylinders, that the constant C in (6.48) above is the same as for the vertical plates and cylinders.

Tsuboushi and Sato (1960) have suggested an alternate representative linear dimension similar to the hydraulic mean depth:

$$L = 2V/A \qquad 6.50$$

They compared this definition to that of King using the data of Hermann (1936) and Yamagata (1943) for horizontal cylinders and spheres, and the data of Merk and Prins (1954) for a horizontal cylinder and sphere using the discrepancy--sphere-cylinder/sphere--and concluded that relation (6.50) is superior to (6.49) even for bodies having shapes as different as spheres and cylinders.

These characteristic lengths for a container with and without effective heating through the top surface are shown in Table 6.16. They are seen to be quite similar to those proposed in the food processing literature as summarised in Table 6.17.

It has been pointed out earlier that the characteristic length may well be some system length appearing to approximately a 1.25 or 4/3 power. This was evident in the intermediate fit of system 1c(1) and system 2b to data for canned fish products and the improved fit of both of the Fagerson formulas over the Schultz and Olson formulas for the data of Townsend et al. (1949).

It would appear from the consideration of relative resistances that for heating or cooling in media above film coefficients of about 50 that the contribution through the headspace should be neglected. It would also appear that since the radius is used in the thermo-syphon systems to correlate heat transfer data, it might well be used in both V and in the V/A characteristic. The characteristic length would also need to incorporate the L factor to insure forced similarity, hence the length would be in the form of Fagerson's second correlation. Unfortunately, this approach would mask the significance of an unheated top.

Comparison of experimental data seems to shed very little light on the appropriate characteristic length. Although Nicholas and Pflug (1961b) have shown that correlation is improved by neglect of the heating through the headspace they reported a plot of f versus the surface to volume ratio to give a better fit than the reciprocal relation produced by Jones (1931). The data of Nicholas and Pflug (1961b) for whole cucumber pickles, of Nicholas and Pflug (1961a) for cucumber slices, and of Hight (1963) for snap beans and pimentos were correlated with the Jones (1931), Schultz and Olson (1938), the two Fagerson formulas (1950), and the Nicholas and Pflug relation (1961b). No one formula consistently gave a higher correlation, but the Fagerson relations gave generally poorer results. It is interesting to note that the Nicholas and Pflug relation (1961b) gave nearly as consistent correlations as the others even when the bottom surface as well was subsequently neglected.

Product Factors

There are numerous factors which may influence heat penetration in canned food products. Some of these include fill, headspace, percent solids in liquid phase, proportion of solids in brine or syrup, syrup strength, viscosity, vacuum (in vacuum packed products), arrangement of pieces in the container, position of container in retort, tendency of product to mat, method of preparation, maturity, growing conditions, and formula changes (Townsend et al., 1956). To these Nicholas (1962) has added presoaking, jar size, point in jar, year, product shape, covering liquid composition (with respect to diffusion gradient as well as the aforementioned viscosity); covering liquid temperature, and bath temperature, which have appeared in cucumber processing studies.

Not all of the above factors are explicitly physical characteristics and not all the required physical properties are given here. For this reason there have been few attempts to characterize the effect of even the highly important fill ratio and the product viscosity. These factors, the orientation of the product and understandably the jar size and position within the jar often seem to be much more important than the thermal diffusivity of components, product buoyancy, size, shape (or shape factor) void fraction, headspace and vacuum.

It would seem desirable to study model product systems in order to establish the forms of dimensionless number

type correlations which could be used to examine a limited number of factors simultaneously, examinations which are presently quite difficult. Some techniques which might be used to partition the problem into velocities, film coefficients and conduction rates have already been mentioned previously.

Other techniques which may be used to examine are typified by the model stratified sucrose-water systems with and without model plastic spears (Nicholas et al., 1960). These studies showed grossly reduced heating rates when the sucrose layer was not deep enough to sustain its own lateral convection loops and indicated the slowest heating points just beneath the water-sucrose interface and slow points just above and below all system interfaces. These findings are compatible with the behavior of ideal systems. There are two anomalies reported that have significance for other systems.

The first of the anomalies is the faster heating of the product liquor upon the introduction of plastic spears. Higher fill ratio has been encountered with real cucumber spears (Nicholas, 1962). This is in direct contrast to the opposite behavior which has been observed in other products (e.g., cucumber slices, Nicholas, 1962; mustard and turnip greens, Alstrand and Ecklund, 1952; whole kernal corn, Thompson, 1919). These latter products have larger heat transfer area and small size and hence may quickly absorb

heat from the fluid as well as retard fluid flow. In the case of spears, vertical channels are provided for the ascending film and the descending core, the product absorbs heat from the liquid more slowly, and the amount of exterior heating area per unit volume of the liquor may even be increased.

The second anomaly is that the slower heating points for the layered systems with spears are much slower. This may be due to the restricted flow area for lateral flow at the top or bottom of the side eddy loops. Such a mechanism may also be responsible for the faster heating (Ball and Olson, 1957, p. 255) of asparagus when processed with tips down then with tips up. Since asparagus is brine covered, top flow is not appreciably restricted in either instance, but processing with butts down would restrict the formation and maintenance of the lateral loops by the descending core.

Viscosity

The product viscosity is the principle product property which is present in both liquid models and in real food products. This viscosity can be Newtonian as for water, brines, syrups, and glycerol-like fluids or any of a large number of so-called non-Newtonian fluids which may have a certain yield stress which must be exceeded to induce or maintain flow, a region in which molecules must be oriented to the flow direction and thixotropic--that is, work sensitive structure as well as gelatin or similar mechanisms which take place in a finite time and during a finite temperature range. Starch pastes, CMC agar, or alginate derivatives, juices with relatively high pectin contents and bentonite are among these materials (see Bigelow, 1922; Magoon and Culpepper, 1921, 1922; Joslyn, 1927; Irish et al., 1938; Jackson and Olson, 1940; Jackson, 1940; Fagerson, 1950; Oakes, 1956; Charm, 1963). The later group of products is large, but characterization of the viscosity is often difficult; hence, an examination of Newtonian fluids may more readily illustrate the effects of viscosity in the natural convection phenomenon.

Bigelow (1922), Magoon and Culpepper (1921, 1922), Joslyn (1927, 1928), Irish et al. (1928), and Jackson (1940) have shown that the effects of viscosity due to an increase in sucrose concentration only slightly reduces the heating rate when contrasted to fluids like pectin containing juice or starch. The largest relative change seems to occur between a concentration of 60 and 70 percent sucrose (Joslyn, 1928). More marked influence of sucrose concentration is noted during cooling (Magoon and Culpepper, 1922; Joslyn, 1928) which may be attributed to an increasing viscosity as well as the concurrent decreasing temperature difference. During heating, on the other hand, the simultaneous decrease of viscosity and temperature differences is said to be responsible for the relatively similar heat penetration in water and in sucrose solutions.

Joslyn (1928) has presented a correlation between the $\log_{10}$ of the apparent diffusivity and the $\log_{10}$ of the viscosity of sucrose solutions which has a slope of about -1/3. This is precisely the exponent which would be obtained if a controlling interior film characterized by a Rayleigh number for turbulent flow were substituted into the Newtonian model equation for 1/f as a function of film coefficient. Since these studies were conducted in No. 2-1/2 tin cans heated from 20° to 100°C. in a steam agitated boiling water bath, this substitution appears reasonable. Unfortunately, this apparent agreement does not confirm the Newtonian heating model and the neglect of inertial forces since the experimental viscosity values used in this experiment are much lower than those reported by Swindells et al. (1958). The corrected graph has a slope of about -1/5; hence the exponent on the Prandtl number correction for inertial forces seems to be about 0.0 to 0.11 when exterior resistances are accounted for.

Other Factors

In spite of the almost universal recognition of the effect of temperature difference on the degree of convection and the recognition that convection will be eventually followed by a conduction domain, there appears to have been little concern about the effect of temperature level or initial temperature differences on convection until fairly recently. These temperature differences are associated with the long recognized temperature dependent viscosity and also with the temperature thermal diffusivity and the processing medium, however. Those aspects and factors which may be more significant and hence have obscured these effects will be examined briefly below.

Thompson (1919) attributed the higher effective diffusivities he observed in processing in steam as contrasted to processing in water, to the higher processing temperatures employed, but this phenomenon was not examined further.

Bigelow et al. (1920) examined heat penetration in many products under many experimental conditions, but did not report a difference due to temperature difference. It is interesting to note, however, that a replot of the data of Figure 69 for water pack salmon (chosen for its known initial temperature, quick come-up, and smooth heat penetration curve in spite of poor convection) showed a slight increase in heat transfer with an increase in retort temperature.

Magoon and Culpepper (1921) reported that the temperatures in their containers "reached" the processing temperature at approximately the same time--an error which Ball and Olson (1951) have discussed at some length. Their data were not abstracted and plotted as heat penetration curves since the small scale of the figures did not seem to permit sufficient precision.

Magoon and Culpepper (1922) noted for water in quart jars processed at 116°C. that "the temperature of the retort is 'reached' [sic] six or seven minutes sooner when the initial temperature is 80° than when it is 20°C," a fact which makes higher initial temperatures important in order to secure more lethality for a given process time. These data, however, also show that the heating rate is increased when the initial temperature differences are larger (although high initial viscosity) and the product is processed at the same temperature. Since this conclusion was confirmed for water in No. 3 tin cans (which gave larger relative differences), Figure 6, string beans in quart glass jars, Figure 11, and string beans in No. 2 cans, the first such data examined, these comparisons were not continued. Replotting of the data of Figures 12 and 13 for string beans in No. 2 tin and No. 3 tin indicated slightly faster heating when the retort temperature was increased. The curves for the latter system appeared to be broken with a slope change within 7°C. of retort temperature.

Tani (1941) reported on the effect of different initial temperatures for heating of water in two different size containers. An evident crossing of the time temperature histories which would accompany smaller f and larger j's as the temperature differences increased may have obscured these effects. The curves seem too small to replot, however.

Jackson and Olson (1940) did not detect a difference in f for broken heating of 3.25 percent bentonite in No. 2 cans and unspecified initial temperatures, with retort temperatures of 200° and 270°F., but did detect a significant effect on f_2. This latter effect might be attributed to the higher thermal diffusivity. The former difference could be accepted if the null hypothesis were to be regulated with a probability P, such that $0.3 > P > 0.2$. The probabilities for the effect of initial temperature were even greater, hence no differences were detected. High closing temperatures and hence higher water vapor fraction in the headspace may have increased f_1 for $0.10 > P > 0.05$.

Townsend et al. (1949) did not report any effect of retort temperature or initial temperature on heating rates. Such differences are suggested in the previous examination of come-up time and container comparisons in this review, however.

In Alstrand and Benjamin's (1949) study of come-up path and time, increased retort temperatures appeared to increase the heating rates of cream style corn, but no consistent trends are to be observed for 2 or 5 percent bentonite.

A general but not always significant increase in heating rate, 1/f and increase in lag factor j for an increase in initial temperature difference but constant processing temperature is seen in the Fagerson and Esselen (1950) data for 1 percent bentonite in glass. There is some significant evidence that the converse is true for 5 percent bentonite. This latter phenomenon may be due to higher film temperatures hence faster heating.

The data of Powers et al. (1952) for bentonite in jars processed in boiling water and in steam and with different come-up times and initial temperature show no trend with temperature differences.

The one comparison which can be made for whole cucumber pickles, a highly variable product, as reported by Esselen _et al_. (1952) does not show an effect of initial temperature differences.

There seemed to be no consistent effects of temperatures for canned pimentos in glass as reported by Hight _et al._ (1954).

Hurwitz and Tisher (1952, 1956) and Evans (1958) have noted faster heating for meat and a water-like jel, respectively, due to increased thermal diffusivities.

Jaffe (1959) found a significant difference, a reduction, in f for fresh cucumber pickles in quart jars processed at 180°, 195°, and 211°F., but no significant difference in j value.

Pflug and Nicholas (1960) reported that the heating at the center of line pack fresh sweet cucumber spears in 16 oz. vegetable jars was significantly faster with 195° water bath than 180° water bath (135°-140°F. IT).

Nicholas and Pflug (1961a) maintained that the heating of whole No. 2 brine packed cucumbers, in 32 oz. vegetable jars, with about a 135°F. initial temperature was significantly faster at a water bath temperature of 200°, than at 160 or 180°F. if the two broken heating curves are eliminated from the analysis, yet report combined data.

Pflug and Nicholas (1961b), in their study of heating medium effects upon a convective, and also a conductive heating product (water and 5 percent bentonite suspension in 16 oz. and quart vegetable jars) reported that the heating rate increased with increases in temperature. Their bentonite data, however, do not in every case substantiate their conclusion. Both increases in thermal diffusivity and stronger convection currents accompanying the larger initial temperature differentials are suggested as possible explanations for the general phenomenon. They found evidence which suggested that f and j for convective products may be either interrelated or a function of a common parameter.

Pflug and Blaisdell (1961) found further evidence of a possible relation between f and j as is seen in the scatter diagram shown in Figure 6-15. They also showed differences in the heating rates at different processing temperatures.

There seem to be at least four factors which may contribute to the increase in heating rate (decrease in f) with higher retort temperatures: (1) an increase in effective thermal diffusivity of the product corresponding to the higher processing temperatures; (2) a corresponding increase in thermal diffusivity of the heating medium; (3) an increase in the diffusivity of water vapor through air and an increase in the saturation pressure of the steam, both of which may increase the surface heat transfer coefficient for steam-air mixtures at atmospheric pressure; and (4) a possible effect

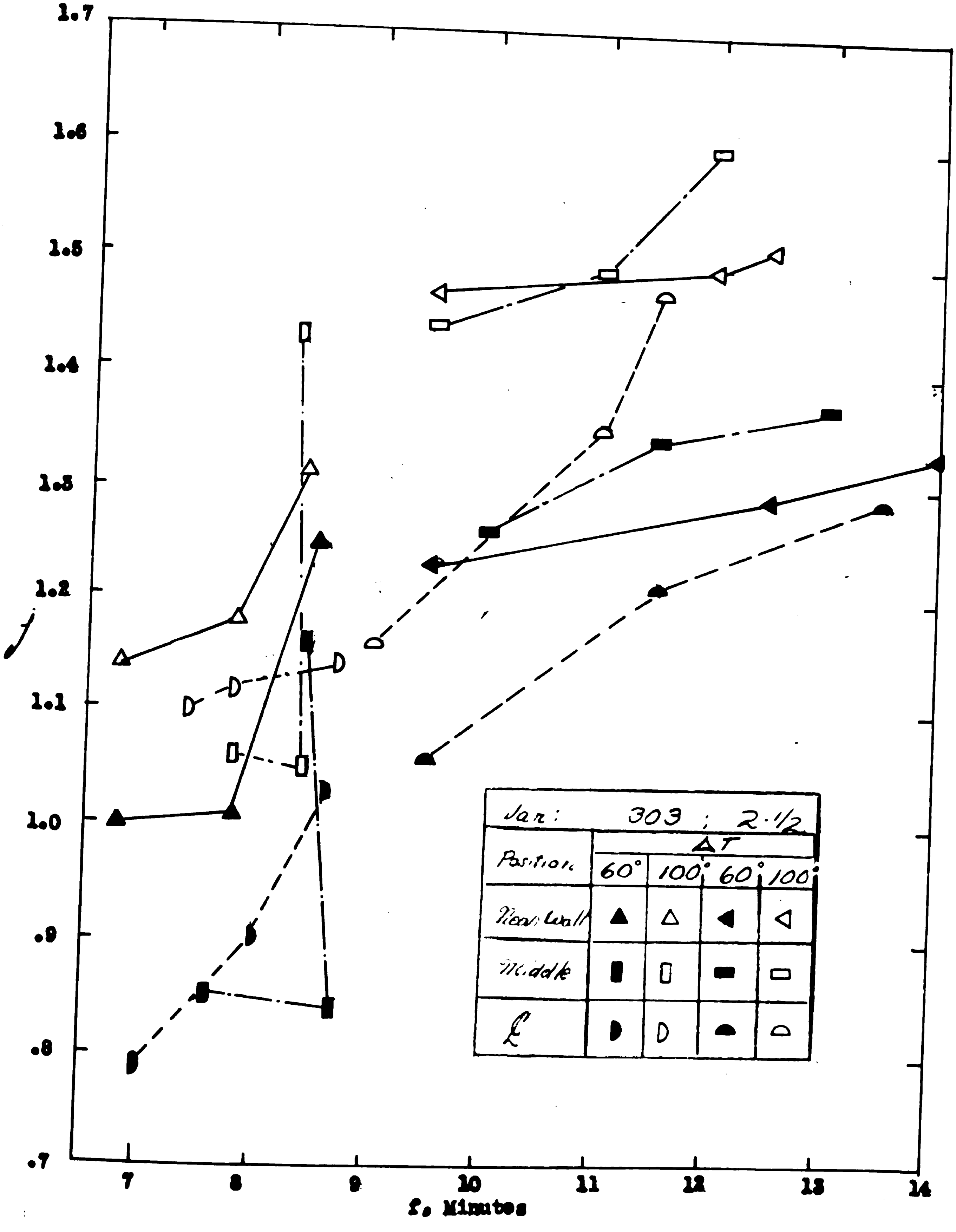

FIGURE 6.15b Scatter diagram of heat penetration parameters f and j after the data of Fagerson and Esselen (1950) Tables 1 & 2.

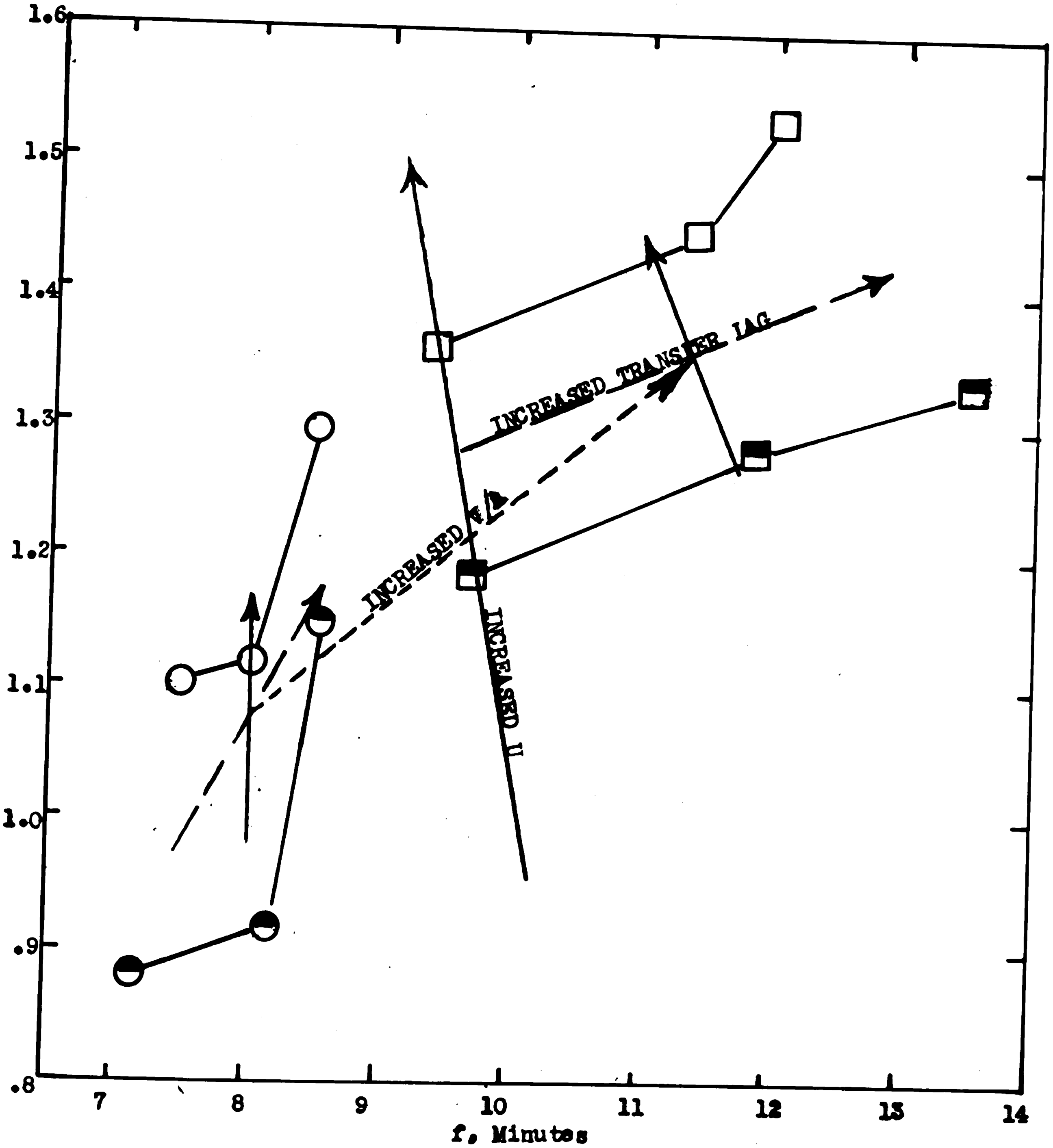

FIGURE 6.15b Abbreviated. Possible causes for the observed major effects are also shown. There are also unresolved interactions present.

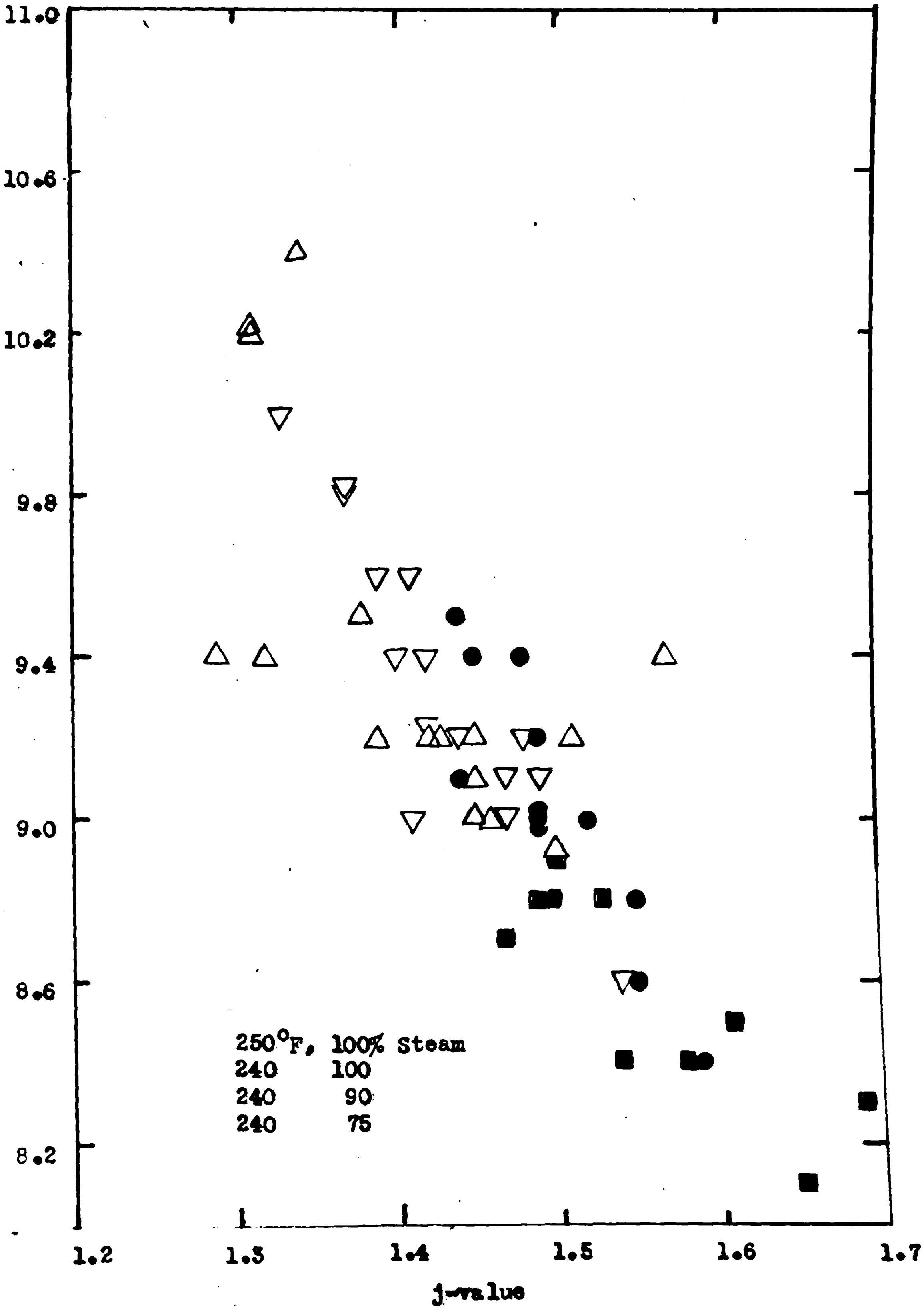

FIGURE 6.15 Scatter diagrams of the heat penetration parameters f and j after Figure 8. Pflug et al (1962). The data are for tomato puree heated in 0.750 inch wide vertical slottes and exposed to various steam-air mixtures.

of higher temperature rises on the degree of convection.

The effects of these factors are generally small, for example, perhaps less than $(\frac{120}{60})^{1/3}$ or $(\frac{120}{60})^{1/4}$, that is less than 141 to 119 percent for a severe case such as natural convection both inside and outside a container. These differences appear to be so small that experimental variability and change in product thermal properties may obscure significant differences, particularly with a viscous product or glass container.

PART V. STATUS OF CONVECTION HEATING STUDIES

Transient natural convection begins when temperature differences are sufficiently large. A rising film (heating) perhaps about 3/8 in. or less initially in water forms on the lateral surface of the container. The flow from this strean discharges near the top of the container. While the eflux from the boundary layers is settling, turbulent filaments may rise several inches into the container. These may decay into quasi-stable ring vortices like Benard cells. About this time the first elements of the lateral eddy are reaching the bottom and are being heated once again. (These side eddies descend at about 1/10 of the rate they rise.) The side eddies may decay from a mixed laminar-turbulent boundary layer to a laminar boundary layer with a stagnant layer below and thence to a fully established laminar boundary layer. Both bottom and side eddies ultimately decay into conduction controlled heating.

Cold points may exist above or below interfaces between phases and at the meeting of eddy loops. The cold point is thought to rise along the interface between these loops as heating continues and ultimately reach the center of a relatively stagnant core as the bottom loops decay completely.

The temperature profiles are noted for the nearly uniform radial temperature and nearly linear axial temperature gradients to the axial cold point.

It appears that correlations similar to those used for thermo-syphon heat transfer can be combined with surface resistance into a modified Newtonian heating model--i.e., negligible surface resistance, in order to correlate f, j, and cold point position. The model predicts that the heat penetration curves will not be straight. They may be sufficiently straight to be represented by no more than two straight lines in the processing region, however.

Film coefficients assume an importance not found in most conduction heating problems. The study of the interrelationships of f may reflect this concern.

The data of Fagerson and Esselen (1950) indicates that it takes less time to establish the faster rates encountered with increased surface conductances, slower heating rates, and a longer time to achieve them, both as the heat transfer surface per unit volume is reduced of fluid flow and as the fluid becomes either farther spacially or in terms from the discharge of the boundary layer. These transfer times may at times be reduced by the decreases of the overall surface conductance during the process to values less than zero time. It might be interesting to distinguish between the effects of finite film coefficient on high effective diffusivity products and those for convection in order to determine if a relationship between f and j can be used to distinguish convection from conduction heating.

SECTION VII

SUMMARY AND CONCLUSIONS

The physical problem of transient natural convection of liquids in unagitated food containers and the food engineering literature concerning the heating of canned foods have been examined in an attempt to broaden our understanding of the problem. Flow visualization, vertical and horizontal temperature profiles, calorimetric measurement of the mixed-mean fluid temperature, and film coefficients evaluated from the transient heating of metallic can-sized finite cylinders were used as experimental techniques to further explain the phenomenon.

Dye flow and vertical temperature profiles show that convection begins at the lateral surface and the boundary layer develops to fill the container. The time required for convection to begin in a 16 oz. vegetable jar compares favorably with the times predicted by Siegel (1958) for vertical plates with negligible thermal capacity. This time to initiate convection, is a function of the Prandtl number, the Rayleigh number, and the Fourier modulus of the fluid. Analysis shows that it is also a function of the Nusselt number, and the Fourier modulus of the container as well. Turbulent eddies rise from the bottom and penetrate well into ($x/L \approx .40$) the fluid above as the warmed central core descends. The flow starts later than the convection on the lateral surface, in part due to the thicker container thickness, but also due to the theoretical requirement that a critical N_{Ra} = 1708 be exceeded. The eddies decrease in height to about one-half their former level as the ring vortex pattern is established on the side wall. The laminar core descends at about 1/11 of the velocity of the ascending lateral boundary layer. This is in excellent agreement with the 1/3 to 1/20 range reported by Fagerson (1950) and Hammitt (1959). Turbulence was observed at high N_{Ra} from $10^{8.5}$ to $10^{9.2}$ but only in the bottom eddies and at the boundary layer inflection at the top of the lateral boundary layer. The boundary layer thickness and the heat transfer and the flow patterns agree well with those predicted from quasi-steady-state flows on vertical and horizontal flat plates.

The flow patterns and temperature profiles of this investigation, when combined with the investigations of the central portion (x/L) of the core by Fagerson confirm and refine the pattern suggested by Tani (1940). Cold points are shown to exist at stagnation points and vortex centers. The axial cold point is hypothesized to be positioned where heat transfer from the top and from the bottom is equal. This cold point would be predicted to move upward for smaller N_{Ra}, no movement was detected due to insufficient experimental precision and insufficient number of experimental temperature measurements below the cold point. The coldest of these cold points is near the heel of the container (as in the Fagerson study, 1950). The movement of the minimum cold point back to the central axis as convection declined was not investigated. The axial cold point was shown to be a conservative estimate of the mixed mean fluid temperature which corresponded to an axial position slightly above the geometric center of the fluid.

Quasi-steady-state film coefficients, a function of N_{Ra} and N_{Pr}, when based on the lateral and bottom fluid surfaces, and the temperature difference between the fluid surface and the cold point, were closely related to the observed heating rate parameters f. No correlation was developed for the lag factors j, but new quantity f.log j, which represents the transport lag due to conduction and convection, has been introduced in order to interpret the lags observed in convection. The effects of the several processing variables, including a hypothesized effect of particle size are summarized in Table 4.9.

The limited experimental data for small T_1-T and the predictions based on quasi-steady-state boundary layer theory and the Newtonian heating model indicate that heat penetration curves are curved but not greatly so for $T_1 - T > 10^oF$. The curvature is more pronounced as the internal film resistance becomes a larger proportion of the overall resistance to heat transfer, hence will be less pronounced for products in glass, and solids in brines or syrups unless a change in heat transfer mechanism occurs during heating or cooling. The fluid property changes are less important than the convective driving force T_w-T in determining the curvature of heat penetration curves. When the convection film coefficients are incorporated in this model the temperature

(T_1-T) raised to a non-integer power is a linear function of time. This expression is not convenient for direct graphical analysis of (T_1-T) versus time. The lethality integration leads to a complex expression in terms of the incomplete Gamma function. Representation of the heat penetration curves by two linear segments or the general method seem preferable to evaluation of these integrals.

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TABLE A.1 .--Some transport properties of 50% sucrose

T ^oF	ρ lb/ft^3	C_p Btu lb ^oF	μ lb. ft. hr.	k Btu hr. ft.^oF	d ft.^{2} hr 10^3	N_{Pr}	$\frac{1}{^oF}$ 10^3	N_{Gr}	$N_{Pr} \cdot N_{Gr}$ or N_{Ra}
	(1)	(2)	(3)	(4)					
32	77.27	0.745	108.27	0.238	4.14	338.9	0.164	3.48x10^4	11.80x10^6
50	77.02	0.741	61.01	0.246	4.31	183.8	0.183	12.16	22.32
68	76.76	0.739	37.58	0.252	4.14	110.2	0.202	35.1	38.7
86	76.47	0.739	24.45	0.259	4.26	69.8	0.220	89.6	62.4
104	76.16	0.739	16.92	0.264	4.37	47.4	0.237	200.	94.7
122	75.83	0.739	12.27	0.270	4.47	33.6	0.256	407.	136.8
140	75.47	0.739	9.22	0.275	4.59	24.8	0.274	765.	192.
158	75.07	0.740	7.11	0.279	4.67	18.9	0.293	1400.	264.
176	74.67	0.741	5.66	0.283	4.76	14.8	0.311	2255.	334

1. Bates et al. (1942).
2. Short (1944).
3. Schwendells (1958).
4. Reidel (1951).

TABLE A.2.--Some transport properties of water (from Kreith, 1958)

T °F	ρ lb/ft^3	Cp Btu lb °F	μ lb. ft. hr.	k Bty hr.ft.°F	d ft.^{2} 10^3 hr.	β N_{Pr}	10^4 °F	N_{Gr}	$N_{Pr} \cdot N_{Gr}$ or N_{Ra}
32	62.4	1.01	4.32	0.319	5.07	13.7	-0.37		
40	62.4	1.00	3.74	0.325	5.21	11.6	0.20	2.3×10^6	2.67×10^7
50	62.4	1.00	3.17	0.332	5.33	9.55	0.49	8.0	7.64x
60	62.3	.999	2.74	0.340	5.47	8.03	0.85	18.4	14.8
70	62.3	.998	2.37	0.347	5.57	6.82	1.2	34.6	23.6
80	62.2	.998	2.08	0.353	5.68	5.89	1.5	56.0	33.0
90	62.1	.997	1.85	0.359	5.79	5.13	1.8	85.0	43.6
100	62.0	.998	1.649	0.364	5.88	4.52	2.0	118.	53.3
150	61.2	1.00	1.051	0.384	6.27	2.74	3.1	440.	120.6
200	60.1	1.00	.738	0.394	6.55	1.88	4.0	1.11×10^9	209.
250	58.8	1.01	.569	0.396	6.69	1.45	4.8	2.14	310.
300	57.3	1.03	.454	0.395	6.70	1.18	6.0	4.00	472.
350	55.6	1.05	.378	0.391	6.69	1.02	6.9	6.24	636.

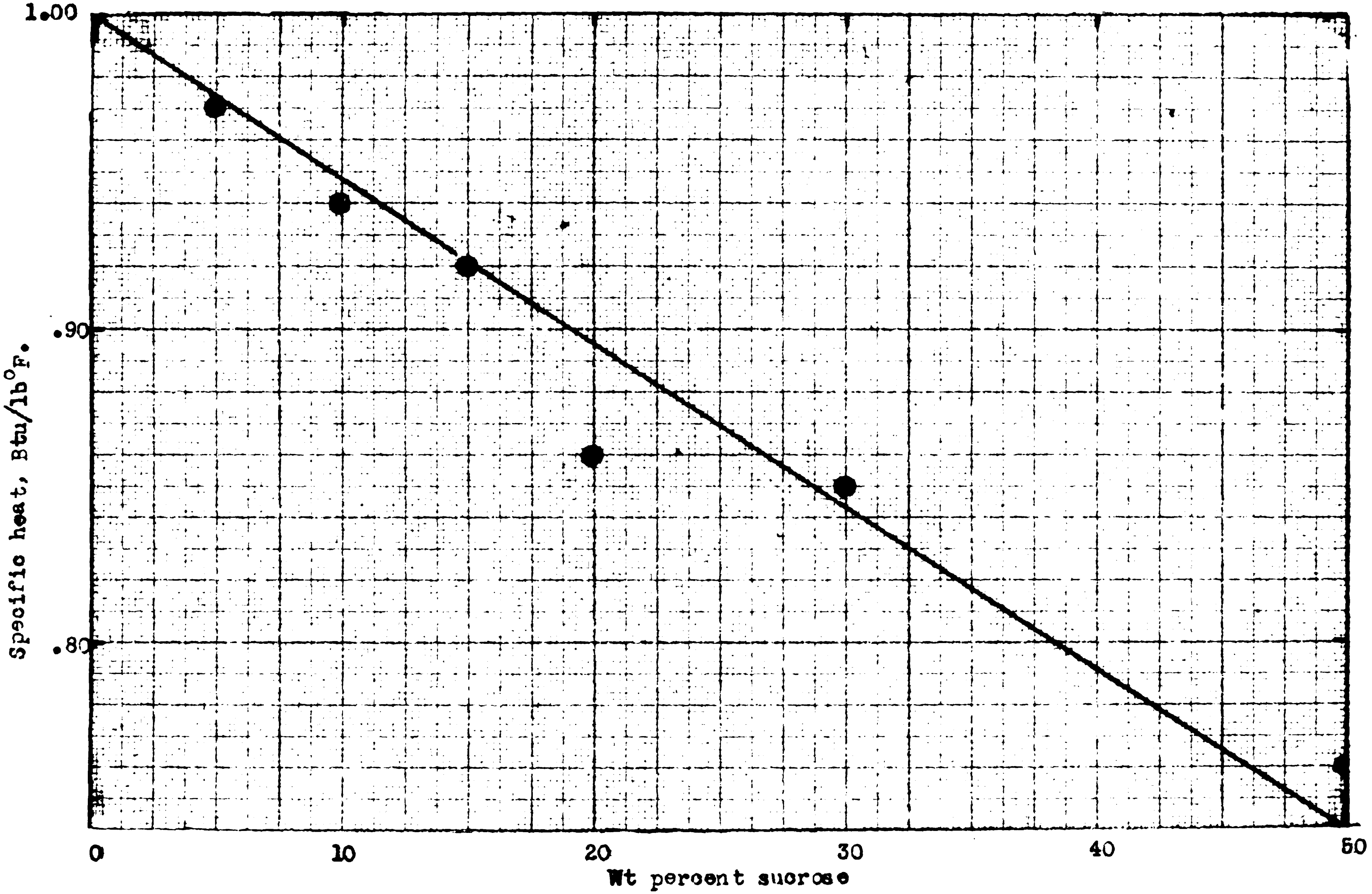

FIGURE A.1 Specific heat of sucrose after Short (1944).

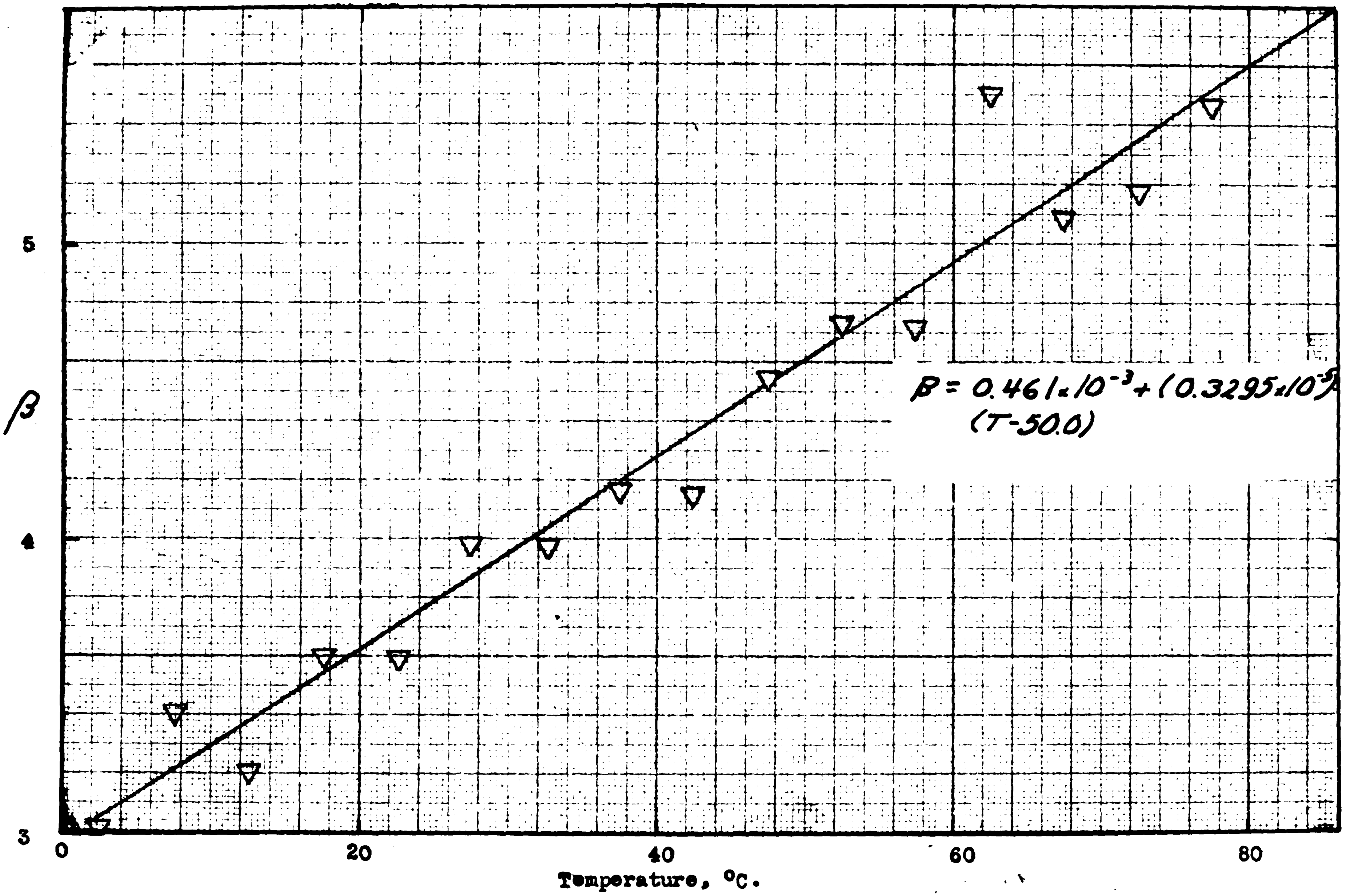

FIGURE A.2 The cubic thermal expansion coefficient (cgs system) after data from Swindells (1958)

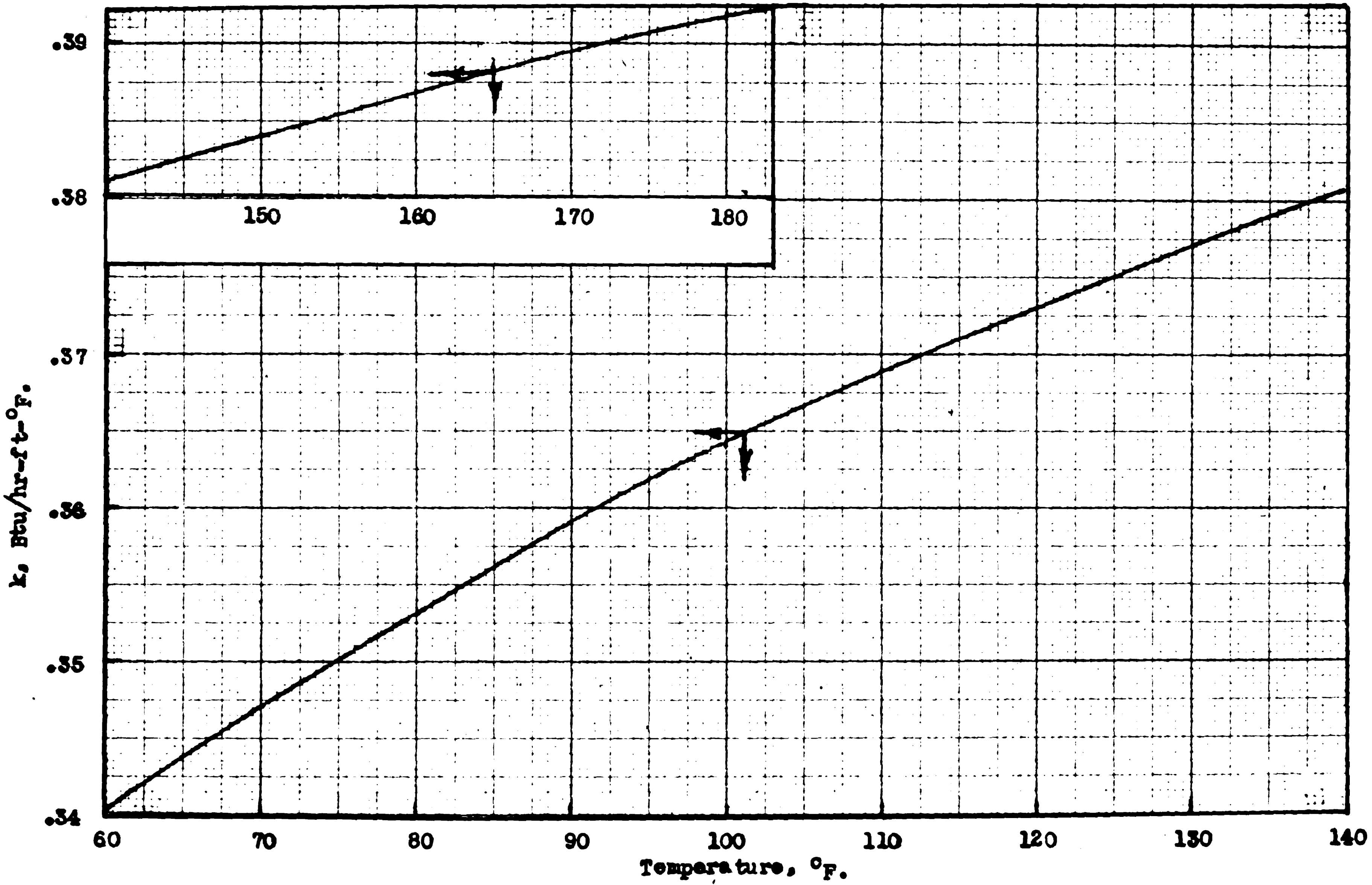

FIGURE A.3 Thermal conductivity of water after Kreith (1958).

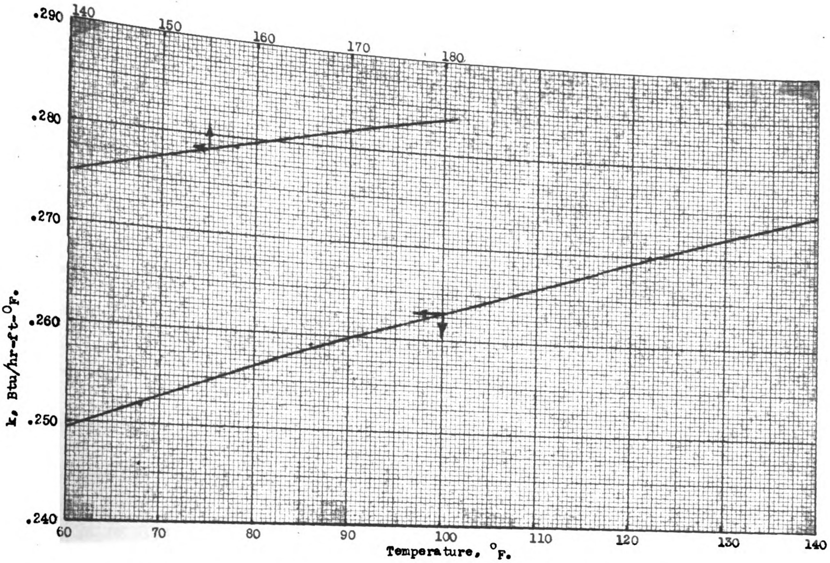

FIGURE A.4 Thermal conductivity of 50 percent sucrose after Reidel (1951), Kreith(1958).

Table A.3 . Summary of heat penetration parameters f and j data.
Where f is reported in minutes and $j = (T_1 - T_a)/T_1 - T_o$

Other Treatments			Position in container, in. measured from bottom of fluid in center of container												
			1/8		5/8		1-1/8		2-1/8		3-1/8		4-1/8		# of Tests
			f	j	f	j	f	j	f	j	f	j	f	j	
8 oz. jar	Water	60-120	9.59	1.20	9.36	1.33	10.02	1.26	9.18	1.13	7.92	.92	3.03	.88	6
		120-180	9.55	1.18	9.61	1.28	9.81	1.18	9.26	1.08	7.81	.94	3.13	.87	6
		60-180	9.42	1.26	9.53	1.33	9.66	1.22	8.98	1.06	7.92	.90	3.68	.72	6
	50% Suc.	60-120	11.08	1.21	11.58	1.47	11.71	1.37	10.38	1.17	8.70	.95	3.08	.92	6
		120-180	11.09	1.19	11.48	1.32	11.59	1.24	10.77	1.07	9.34	.87	3.30	.78	6
		60-180	10.54	1.24	10.86	1.38	11.16	1.22	10.60	1.02	9.34	.73	3.37	.63	6
16 oz. jar	Wat.	60-120	10.95	1.18	11.02	1.30	11.14	1.21	10.56	1.05	9.46	.88	4.47	.96	6
		120-180	11.01	1.17	11.11	1.23	11.28	1.13	10.39	1.03	9.46	.94	5.73	.80	6
		60-180	10.28	1.26	10.26	1.32	10.32	1.21	9.98	1.08	9.27	.86	5.15	.85	6
	50% Suc.	60-120	13.07	1.22	13.48	1.44	13.65	1.36	12.08	1.14	10.72	.89	4.02	.87	6
		120-180	13.14	1.22	13.52	1.34	13.82	1.21	12.90	1.04	11.67	.84	4.42	.82	6
		60-180	12.43	1.23	12.81	1.34	13.15	1.18	12.33	1.00	11.10	.79	3.32	.85	6
#303 can	Wat.	60-120	5.28	1.25	5.46	1.48	5.52	1.33	4.98	1.07	4.00	.84	1.50	.78	6
		120-180	5.35	1.24	5.28	1.25	5.17	1.14	4.50	1.01	3.55	.85	1.47	.61	6
		60-180	5.06	1.20	4.93	1.27	4.61	1.22	4.08	1.04	3.44	.80	1.58	.52	6
	50% Suc.	60-120	7.72	1.15	8.42	1.47	8.50	1.35	7.30	1.12	5.63	.85	2.33	.77	6
		120-180	7.82	1.31	7.98	1.34	7.85	1.22	6.85	1.04	5.38	.90	2.28	.48	6
		60-180	7.05	1.36	7.18	1.31	6.98	1.18	6.29	.87	5.39	.62	1.74	.62	6
	Wat.	60-120	8.62	1.06	8.67	1.26	8.72	1.16	7.75	1.01	6.46	.77	2.10	.78	12
		60-180	6.10	1.13	5.98	1.18	5.92	1.08	5.36	.93	4.43	.75	1.49	.64	14
		60-180	4.20	1.34	4.32	1.33	4.15	1.21	3.66	1.03	3.07	.80	1.25	.59	6
22 oz jar	Wat.	60-120	11.52	1.35	11.74	1.50	11.99	1.44	11.98	1.26	11.36	1.14	10.69	1.00	5
		120-180	11.46	1.25	11.48	1.35	11.78	1.30	11.65	1.16	11.00	1.07	10.19	.98	5
		60-180	10.89	1.35	11.20	1.44	11.47	1.34	11.13	1.20	10.66	1.09	10.06	.96	6
28 oz. jar	Wat.	60-120	14.44	1.19	14.62	1.34	14.72	1.24	13.98	1.12	12.73	.93	6.32	.84	6
		120-180	14.48	1.16	14.45	1.24	14.75	1.15	13.78	1.05	12.53	.93	6.97	.87	6
		60-180	14.12	1.21	14.14	1.29	14.10	1.22	13.58	1.08	12.42	.92	6.55	.84	6
	50% Suc.	60-120	16.79	1.23	17.50	1.41	17.44	1.36	16.06	1.13	14.20	.90	5.32	.90	5
		120-180	17.48	1.18	17.72	1.28	17.88	1.18	17.26	1.03	15.50	.87	6.70	.88	6
		60-180	16.03	1.35	16.25	1.38	16.65	1.25	16.07	1.10	15.23	.81	4.40	.96	6
32oz jar	W	60-180	13.16	1.33	13.28	1.38	13.20	1.31	12.78	1.18	12.39	1.04	11.85	.91	6
	S	60-180	16.88	1.33	16.95	1.39	17.30	1.25	16.07	1.13	15.23	1.01	14.33	.86	3

Table A.4 . Summary of lag time t_a (f·log j) data calculated from mean f and j for a treatment combination.

Other Treatments				Position in container, in. measured from bottom of fluid in center of container												# of Tests
				1/8		5/8		1-1/8		2-1/8		3-1/8		4-1/8		
				f	t_a	f	t_a	f	t_a	f	t_a	f	t_a	f	t_a	
8 oz jar	Wat.		60-120	9.59	.76	9.86	1.22	10.02	1.01	9.18	.49	7.92	-.29	3.03	-.17	6
			120-180	9.55	.69	9.61	1.03	9.81	.71	9.26	.31	7.81	-.21	3.13	-.19	6
			60-180	9.42	.95	9.53	1.18	9.66	.83	8.98	.23	7.92	-.36	3.68	-.53	6
	50% Suc.		60-120	11.08	.92	11.58	1.94	11.71	1.60	10.38	.71	8.70	-.19	3.08	-.11	6
			120-180	11.09	.84	11.48	1.38	11.59	1.08	11.77	.32	9.34	-.57	3.30	-.36	6
			60-180	10.54	.99	10.86	1.52	11.16	.96	10.60	.09	9.34	-1.27	3.37	-.56	6
16 oz jar	Wat.		60-120	10.95	.79	11.02	1.26	11.14	.92	10.56	.22	9.46	-.53	4.47	-.08	6
			120-180	11.01	.75	11.11	1.00	11.28	.69	10.39	.13	9.46	-.25	5.73	-.56	6
			60-180	10.28	1.03	10.26	1.24	10.52	.87	9.98	.33	8.27	-.61	5.15	-.36	6
	50% Suc.		60-120	13.17	1.14	13.48	2.14	13.65	1.82	12.08	.69	10.72	-.54	4.02	-.38	6
			120-180	13.14	1.14	13.52	1.72	13.82	1.14	12.90	.22	11.67	-.88	4.42	-.38	6
			60-180	12.43	1.12	12.81	1.63	13.15	.95	12.33	0	11.10	-1.14	3.32	-.23	6
305 can	Wat.		60-120	5.28	.51	5.46	.93	5.52	.68	4.98	.15	4.00	-.30	1.50	-.16	6
			120-180	5.35	.50	5.28	.51	5.17	.29	4.50	.02	3.55	-.25	1.48*	-.32*	6-4*
			60-180	5.06	.40	4.93	.53	4.61	.40	4.08	.07	3.44	-.33	2.58	-.45	6
	50% Suc.		60-120	7.72	.47	8.42	1.41	8.50	1.11	7.30	.36	5.63	-.40	2.38	-.26	6
			120-180	7.82	.92	7.98	1.01	7.85	.68	6.85	.12	5.38	-.25	2.28	-.72	6
			60-180	7.05	.94	7.18	.84	6.98	.50	6.29	-.38	5.39	-1.12	1.74	-.36	6
	Wat.	[illegible]	60-120	8.62	.22	8.67	.87	8.72	.56	7.75	.03	6.46	-.73	2.10	-.23	12
			60-180	6.10	.32	5.98	.43	5.92	.20	5.36	-.17	4.43	-.55	1.49	-.29	14
		[illegible]	60-180	4.20	.53	4.32	.54	4.15	.34	3.66	.05	3.07	-.30	1.25	-.29	6-4*
22oz jar	Wat.		60-120	11.52	1.50	11.74	2.07	11.99	1.90	11.98	1.20	11.36	.65	10.69	0	5
			120-180	11.46	1.11	11.48	1.50	11.78	1.34	11.65	.75	10.00	.32	19.19	-.09	5
			60-180	10.89	1.42	11.20	1.78	11.47	1.45	11.13	.88	10.66	.40	10.06	-.18	6
28 oz jar	Wat.		60-120	14.44	1.09	14.62	1.85	14.72	1.38	18.98	.69	12.73	-.40	6.32	-.48	6
			120-180	14.48	.93	14.45	1.35	14.75	.90	13.78	.29	12.53	-.40	6.97	-.42	6
			60-180	14.12	1.17	14.14	1.56	14.10	1.22	13.58	.45	12.42	-.45	6.55	-.47	6
	50% Suc.		60-120	16.79	1.51	17.50	2.61	17.44	2.33	16.06	.85	14.20	-.65	5.32	-.24	5
			120-180	17.48	1.26	17.72	1.90	17.88	1.29	17.26	.22	15.50	-.94	6.70	-.37	6
			60-180	16.03	2.09	16.25	2.27	16.75	1.61	16.07	.67	15.23	-1.39	4.40	-.08	6
32oz jar	S.W.		60-180	13.16	1.63	13.28	1.85	13.20	1.55	12.78	.92	12.39	2.21	11.85	.49	6
			60-180	16.88	2.09	16.95	2.42	17.30	1.68	16.07	.85	15.23	.07	14.33	-.94	3

Table A.5 . Heat penetration parameters f (in minutes) and at several axial positions in 8 oz. cylinder jars[1]

Part I. Values of f

Treatment			Position in container, inches above bottom						
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8 f_1	4-1/8 f_2
Water	60-120	99	9.6	9.9	9.9	9.2	8.0	3.3	7.4
		105	10.1	10.1	10.3	8.9	7.8	3.4	6.2
		111	10.0	9.9	10.0	9.9	8.1	3.2	7.7
		204	9.5	10.0	10.1	9.3	8.2	2.6	6.5
		210	9.0	9.5	9.8	8.8	7.7	2.7	6.0
		216	9.3	9.6	9.8	8.9	7.6	3.0	6.2
	120-180	103	9.3	9.6	9.6	9.1	7.6	3.3	7.2
		109	9.1	9.1	9.6	9.0	7.7	3.5	7.3
		115	9.5	9.6	9.6	9.1	7.9	3.6	8.3
		208	9.1	9.2	9.5	9.2	7.5	2.5	7.5
		214	10.1	9.9	10.5	9.4	8.2	2.7	8.2
		220	9.5	9.6	10.0	9.5	7.8	3.1	7.8
		101	9.0	9.1	8.9	8.5	7.7	3.6	7.2
		107	9.4	9.4	9.4	8.9	8.0	3.9	8.4
		113	9.2	9.2	9.2	8.9	7.7	4.0	7.8
		206	9.8	9.8	10.2	9.2	8.1	4.5	8.4
		212	9.6	9.8	10.3	9.2	7.9	2.9	9.7
		218	9.4	9.7	9.8	9.0	8.0	3.2	7.9
50% Sucrose	60-120	122	11.8	11.9	11.8	10.6	9.3	3.0	9.1
		128	11.0	11.7	12.1	10.3	8.9	2.5	8.1
		134	11.4	11.8	12.0	10.6	9.4	4.2	9.4
		227	10.9	11.3	11.3	10.4	8.4	3.4	6.6
		233	10.7	11.2	11.3	10.1	7.8	2.7	6.7
		239	10.5	11.4	11.6	10.1	8.4	2.7	6.6
	120-180	120	10.8	11.1	11.2	10.6	9.2	2.4	7.4
		126	11.2	11.4	11.4	10.8	9.4	3.4	8.7
		132	11.0	11.3	11.4	10.2	9.2	3.3	9.8
		225	11.5	11.6	11.7	10.8	9.2	3.5	11.8
		231	10.7	11.6	11.8	10.8	9.3	3.4	9.5
		237	11.3	11.7	12.0	11.2	9.6	3.8	9.6
		118	10.2	10.7	11.1	10.1	9.1	3.4	9.0
		124	10.4	10.9	10.6	10.1	8.7	3.5	8.7
		130	10.7	10.8	11.1	10.2	9.5	3.9	8.4
		223	10.8	11.0	11.4	11.0	9.9	3.7	8.9
		229	10.6	10.9	11.4	11.3	9.6	2.8	9.0
		235	10.4	10.7	11.1	10.8	9.2	2.9	9.2

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[1] *Data are from lines fitted visually to data over the U = 0.5 to U = 0.084 range.*

Table A.5 . (continued)

Part II. Values of log factor j

Treatment			Position in container, inches above bottom						
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8 f_1	f_2
Water	60-120	99	1.22	1.32	1.25	1.10	.90	.83	.39
		105	1.17	1.31	1.20	1.17	.93	.93	.51
		111	1.16	1.36	1.28	1.07	.92	1.04	.46
		204	1.15	1.25	1.28	1.12	.87	.81	.34
		210	1.28	1.42	1.30	1.18	.96	.89	.38
		216	1.19	1.31	1.28	1.12	.95	.80	.36
		103	1.15	1.27	1.19	1.07	.95	.78	.51
		109	1.16	1.31	1.21	1.11	.97	.92	.49
		115	1.19	1.26	1.24	1.16	.91	.88	.40
		208	1.18	1.28	1.17	1.07	.92	.88	.38
		214	1.15	1.27	1.13	1.07	.92	1.00	.42
		220	1.26	1.28	1.17	1.03	.96	.75	.36
	120-180	101	1.33	1.37	1.32	1.03	92	.76	.40
		107	1.25	1.29	1.23	1.07	.89	.72	.36
		113	1.25	1.35	1.25	1.10	.93	.72	.41
		206	1.32	1.36	1.12	1.04	.88	.53	.32
		212	1.25	1.29	1.17	1.06	.92	.85	.30
		218	1.19	1.32	1.21	1.06	.87	.71	.93
50% Sucrose	60-120	122	1.14	1.46	1.34	1.12	.95	1.00	.32
		128	1.23	1.45	1.38	1.18	.92	(1.16)	.34
		134	1.18	1.45	1.31	1.14	.94	.89	.33
		227	1.24	1.50	1.40	1.16	.94	.68	.34
		233	1.27	1.47	1.40	1.18	.96	.77	.32
		239	1.22	1.49	1.40	1.23	.99	1.01	.34
		120	1.23	1.37	1.28	1.08	.88	.83	.33
		126	1.17	1.32	1.25	1.03	.83	.83	.35
		132	1.18	1.32	1.22	1.10	.84	.84	.35
		225	1.18	1.33	1.32	1.15	1.00	.75	.33
		231	1.19	1.28	1.18	1.03	.83	.75	.35
		237	1.20	1.32	1.22	1.03	.83	.67	.38
	120-180	118	1.30	1.36	1.22	1.08	.76	.76	.35
		124	1.16	1.39	1.30	1.03	.80	.68	.35
		130	1.23	1.37	1.22	1.05	.76	.64	.35
		223	1.27	1.35	1.18	.94	.70	.58	.31
		229	1.17	1.39	1.17	1.00	.66	.74	.29
		235	1.28	1.44	1.24	1.02	.70	.69	.27

Table A.6 . Heat penetration parameters f (in minutes) and at several axial positions in 16 oz. vegetable jars[1]

Part I. Values of f

Treatment	$T_o - T_1$	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8 f_1	4-1/8 f_2
Water	60-120	1	11.3	11.2	11.4	10.6	9.5	2.8	8.4
		7	11.6	11.3	11.8	10.9	9.9	5.4	9.1
		13	11.0	11.5	11.1	10.4	9.6	4.8	8.2
		37	10.6	10.7	10.6	10.4	9.5	5.9	6.8
		43	10.5	10.6	10.8	10.6	9.2	4.5	6.6
		45	10.7	10.9	11.2	10.5	9.1	3.4	9.1
		5	11.5	11.8	11.8	10.9	9.8	5.6	9.6
		11	11.3	11.3	11.6	11.2	9.7	7.2	8.7
		17	11.2	11.2	10.8	10.5	9.5	5.8	9.0
		41	10.9	11.1	11.6	10.6	9.2	5.6	9.1
		47	10.3	10.4	10.4	9.0	9.4	4.6	9.5
		53	11.0	11.0	11.5	10.3	9.3	5.6	9.1
	120-180	3a	10.5	10.6	10.8	9.7	9.3	5.4	8.8
		9a	10.4	10.3	10.3	9.9	9.4	5.1	10.1
		15a	10.5	10.5	10.6	10.0	9.6	6.0	8.0
		35a	9.9	10.0	10.4	9.8	8.9	4.3	8.4
		45b	10.3	10.0	10.6	10.6	9.0	5.8	7.3
		51a	10.3	10.3	10.4	10.0	9.6	4.3	9.2
50% Sucrose	60-120	19	13.0	13.9	13.8	11.3	10.3	4.9	8.8
		93	13.2	12.9	13.1	12.2	10.7	3.1	10.8
		98	13.7	13.7	13.3	12.6	10.3	4.0	9.9
		183a	13.8	14.4	14.6	12.8	12.1	4.3	9.6
		189	12.5	13.0	13.5	11.6	10.9	5.1	5.7
		198	12.9	13.0	13.7	12.0	10.1	2.7	5.8
		97	12.9	13.5	13.7	13.0	10.6	6.1	9.5
		282	13.7	14.1	14.1	13.3	11.4	5.9	9.0
		288	14.1	14.1	13.7	12.1	11.0	3.6	9.9
		87	12.2	13.3	14.2	12.5	12.3	3.8	7.4
		193	13.4	13.6	14.1	13.9	12.7	3.4	9.9
		202	12.6	12.6	13.1	12.7	12.0	3.7	7.2
	120-180	95a	12.5	12.8	13.0	12.3	11.7	4.1	7.9
		280a	12.9	13.1	13.0	12.0	10.8	4.1	7.5
		286	12.6	13.5	13.5	12.4	11.1	3.6	11.1
		185	12.2	12.6	13.3	12.5	10.9	3.1	7.2
		191	12.1	12.4	13.0	12.4	11.4	3.1	6.4
		200	12.3	12.5	13.1	12.4	10.7	2.0	7.5

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[1] Data are from lines fitted visually to data over the U - 0.5 to U = 0.084 range.

Table A.6 (continued)

Part II. Values of log factor j

Treatment: Fluid	Treatment: T_o-T_1	Run #	Position in container, inches above bottom: 1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8: f_1	4-1/8: f_2
Water	60-120	1	.97	1.11	1.00	.88	.72		.41
		7	1.18	1.38	1.23	1.09	.89	.79	.47
		13	1.17	1.28	1.24	1.10	.89	.78	.48
		37	1.25	1.35	1.25	1.07	.94	.82	.71
		43	1.25	1.37	1.29	1.07	.93		.80
		49	1.23	1.33	1.23	1.09	.92		.38
	120-180	5	1.16	1.19	1.14	1.04	.91	.88	.51
		11	1.08	1.23	1.12	1.02	.91	.77	.47
		17	1.16	1.23	1.22	1.03	.97	.80	.52
		41	1.20	1.20	1.07	1.02	.90	.75	.50
		47	1.23	1.27	1.23	1.05	1.03	.83	.42
		53	1.17	1.24	1.13	1.03	.89	.74	.45
		3a	1.25	1.31	1.22	1.08	.75	.82	.51
		9a	1.26	1.33	1.23	1.08	.89	.81	.51
		15a	1.24	1.33	1.23	1.10	.90	.80	.57
		35a	1.31	1.34	1.19	1.10	.91		.53
		95b	1.23	1.32	1.15	1.08	.88	.83	.59
		51a	1.29	1.32	1.23	1.06	.85	.84	.43
50% Sucrose	60-120	19	1.19	1.40	1.40	1.11	.92	.85	.48
		93	1.20	1.48	1.32	1.09	.87		.37
		98	1.21	1.43	1.39	1.10	.91		.45
		183a	1.19	1.37	1.31	1.15	.85	.70	.35
		189	1.30	1.50	1.39	1.21	.87	.68	.43
		196	1.21	1.49	1.33	1.18	.92	.84	.39
	120-180	97	1.30	1.35	1.23	1.07	.98	.85	.49
		282	1.21	1.33	1.19	1.03	.89	.89	.52
		288	1.14	1.27	1.18	1.07	.82	.92	.38
		87	1.24	1.36	1.18	1.08	.80	.79	.39
		193	1.16	1.29	1.19	.94	.74	.75	.30
		202	1.27	1.41	1.29	1.08	.80	.75	.38
		95a	1.05	1.35	1.16	1.01	.73	.73	.42
		280a	1.25	1.31	1.25	1.04	.80	.84	.53
		286	1.33	1.26	1.16	1.03	.82		.29
		185	1.26	1.35	1.13	.88	.80	.67	.34
		191	1.27	1.39	1.19	1.01	.79	.83	.38
		200	1.24	1.41	1.22	1.01	.81	.86	.32

Table A.7 . Heat penetration parameters f (in minutes) and at several axial positions in 303 cans with one mixer[1]

Part I. Values of f

Treatment			Position in container, inches above bottom					
Fluid	To-T	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	60-120	311	5.6	5.6	5.6	5.2	4.5	1.5
		317	5.6	5.6	5.7	5.2	4.2	1.8
		247	5.2	5.4	5.4	4.8	3.9	1.5
		251	5.0	5.5	5.5	4.9	3.8	1.4
		256	5.1	5.3	5.5	4.9	3.7	1.4
	120-180	315	5.8	5.4	5.2	4.6	3.6	1.3
		323	5.6	5.7	5.4	4.6	4.2	1.6
		309	5.7	5.3	5.2	4.7	3.8	1.6
		243	5.2	5.3	5.1	4.5	3.5	--
		249	5.1	5.2	5.2	4.3	2.8	--
		257	4.9	5.0	5.1	4.5	3.5	--
		301	5.5	5.3	5.2	4.3	3.7	1.8
		313	5.2	5.0	4.3	4.0	3.5	2.1
		319	5.1	4.8	4.5	4.2	3.7	1.2
		241	5.2	5.1	4.6	4.1	3.7	2.0
		247	4.9	4.9	4.6	4.0	3.1	1.4
		253	4.6	4.7	4.6	4.0	3.1	1.1
50% Sucrose	60-120	416	8.3	8.3	8.1	6.9	5.3	2.0
		418	7.7	8.2	8.0	7.3	5.6	2.3
		428	8.1	8.5	8.7	7.8	6.2	2.7
		385	7.8	8.6	8.8	7.6	5.5	2.5
		387	7.5	8.6	8.8	7.2	6.0	2.5
		397	7.2	8.5	8.8	7.2	5.4	2.1
	120-180	422	8.3	8.5	8.1	7.0	5.7	1.9
		426	8.1	8.2	8.0	6.9	5.3	2.2
		429	8.0	8.3	8.0	7.0	5.7	2.1
		383	7.4	7.7	7.5	6.8	5.2	2.3
		391	7.6	7.7	7.7	6.7	5.1	2.4
		398	7.8	7.7	8.0	6.7	5.4	2.8
		420	7.7	7.4	7.3	6.8	5.8	1.5
		412	7.4	7.8	7.1	6.5	4.8	1.8
		424	7.0	7.1	7.0	6.1	5.4	1.7
		381	6.8	7.0	6.8	6.3	5.5	2.0
		389	6.6	6.8	6.7	6.0	5.0	1.9
		393	7.0	7.1	7.1	6.1	5.9	1.7

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[1] Data are from lines fitted visually to data over the U = 0.5 to U = 0.084 range.

Table A.7 (continued)

Part II. Values of log factor j

Treatment			Position in container, inches above bottom					
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Water	60-120	311	1.08	1.40	1.25	1.03	.76	.93
		317	1.23	1.50	1.33	1.04	.82	.86
		347	1.18	1.43	1.28	1.03	.84	.68
		245	1.37	1.59	1.44	1.16	.87	.77
		251	1.33	1.45	1.33	1.07	.87	.77
		256	1.29	1.51	1.33	1.09	.85	.68
	120-180	315	1.23	1.21	1.13	1.01	.86	.63
		323	1.07	1.18	1.07	1.03	.82	.63
		309	1.22	1.28	1.18	1.00	.81	.58
		243	1.31	1.26	1.16	1.01	.83	--
		249	1.27	1.24	1.17	1.04	.91	--
		257	1.33	1.35	1.15	.97	.85	--
		307	1.01	1.15	1.08	.88	.76	.46
		313	1.08	1.23	1.27	1.07	.80	.30
		319	1.22	1.33	1.30	1.03	.78	.58
		241	1.17	1.30	1.30	1.12	.76	.58
		247	1.36	1.32	1.18	1.05	.83	.50
		253	1.39	1.34	1.22	1.08	.87	.67
50% Sucrose	60-120	416	.98	1.51	1.39	1.11	.82	.94
		418	1.18	1.56	1.44	1.08	.84	.86
		428	1.09	1.49	1.31	1.03	.81	.76
		385	1.17	1.46	1.31	1.10	.90	.73
		387	1.21	1.40	1.33	1.15	.75	.67
		397	1.26	1.41	1.31	1.17	.98	.68
	120-180	422	1.32	1.28	1.21	1.06	.84	.52
		426	1.30	1.33	1.22	1.08	1.02	.48
		429	1.13	1.23	1.13	.93	.71	.56
		383	1.46	1.33	1.20	.90	.77	.50
		391	1.41	1.42	1.27	1.08	.93	.40
		398	1.26	1.45	1.30	1.21	1.15	.42
		420	1.27	1.33	1.10	.81	.54	.67
		412	1.27	1.15	1.11	.83	.67	.67
		424	1.39	1.33	1.16	.72	.61	.76
		381	1.47	1.33	1.28	.96	.67	.52
		389	1.38	1.41	1.29	.99	.67	.54
		393	1.35	1.31	1.14	.93	.56	.55

Table A.8 . Heat penetration parameters f (in minutes) and at several axial positions in 303 cans with no and double agitation for water only[1]

Part I. Values of f

Treatment			Position in container, inches above bottom					
Agitation	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Zero Mixing	60-120	34	9.0	8.9	8.9	7.9	6.6	2.2
		343	8.9	8.8	8.8	7.8	6.3	2.1
		345	8.3	8.8	8.6	7.6	6.4	2.2
		349	8.5	8.8	8.8	7.7	6.3	1.8
		361	8.6	8.5	8.6	7.8	6.6	2.2
		363	8.4	8.6	8.7	7.7	6.7	2.3
		365	8.5	8.5	8.7	7.7	6.8	2.1
		369	8.6	8.7	8.9	8.0	6.5	2.2
		351	8.7	8.5	8.5	7.6	5.9	2.0
		353	8.9	9.0	9.0	8.1	6.4	2.0
		371	8.6	8.7	8.9	7.8	6.4	2.2
		373	8.8	8.7	8.9	7.6	6.9	2.2
Double Mixing	60-180	325	5.2	5.9	5.9	5.2	4.3	1.5
		327	6.1	5.8	5.8	5.4	4.6	1.4
		331	7.5	6.3	6.1	5.6	4.8	1.7
		333	6.9	6.1	6.1	5.1	4.5	1.6
		259	5.9	6.1	5.8	5.4	4.5	1.5
		261	5.8	5.8	5.9	5.5	4.4	1.5
		265	6.0	6.1	5.8	5.6	4.4	1.6
		267	5.8	5.8	5.8	5.3	4.3	1.6
		335	6.4	6.1	6.1	5.7	4.5	1.6
		337	6.0	5.8	6.1	5.4	4.4	1.4
		339	5.9	5.8	5.9	5.4	4.6	1.3
		357	5.7	6.1	6.0	5.4	4.3	1.5
		359	5.7	5.9	5.8	5.3	4.5	1.5
		376	4.3	4.5	4.1	3.6	2.9	1.3
		378	4.3	4.3	3.9	3.6	3.0	1.1
		405	4.1	4.2	4.0	3.7	3.2	1.4
		407	4.1	4.4	4.4	3.9	3.3	1.2
		409	4.4	4.4	4.6	3.8	3.2	1.5
		374	4.2	4.3	3.9	3.5	2.9	1.2

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[1] Data are from lines fitted visually to data over the U = 0.5 to U = 0.084 range.

Table A.8 (continued)

Part II. Values of log factor j

Treatment			Position in container, inches above bottom					
Agitation	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8
Zero Mixing	60-120	34	1.06	1.24	1.14	.99	.75	.75
		343	1.01	1.25	1.16	.99	.77	.72
		345	1.08	1.24	1.17	1.02	.75	.68
		349	1.11	1.26	1.16	1.03	.81	1.03
		361	1.05	1.34	1.20	1.00	.76	.76
		363	1.09	1.26	1.21	1.04	.75	.72
		365	1.05	1.29	1.20	1.03	.76	.76
		369	1.04	1.23	1.13	.98	.73	.67
		351	1.06	1.29	1.17	1.00	.82	.82
		353	1.05	1.22	1.14	.98	.78	.93
		371	1.05	1.24	1.15	1.00	.80	.76
		373	1.02	1.25	1.15	1.02	.73	.76
Double Mixing	60-180	325	1.13	1.19	1.08	1.00	.75	.68
		327	1.08	1.25	1.14	.94	.71	.67
		331	.84	1.13	1.08	.90	.71	.67
		333	.94	1.08	.95	.94	.70	.67
		259	1.32	1.16	1.19	.94	.77	.67
		261	1.23	1.25	1.10	.92	.75	.67
		265	1.24	1.18	1.13	.91	.81	.64
		267	1.25	1.24	1.11	.90	.81	.59
		335	1.02	1.17	1.05	.92	.76	.68
		337	.99	1.18	.95	.85	.71	.55
		339	.94	1.15	1.04	.88	.67	.55
		329	1.16	1.12	1.07	1.02	.80	.69
		357	1.34	1.17	1.08	.96	.81	.59
		359	1.35	1.23	1.18	1.00	.74	.64
		376	1.41	1.31	1.28	1.02	.84	.69
		378	1.38	1.21	1.17	1.03	.84	.67
		405	1.43	1.41	1.23	1.01	.70	.67
		407	1.35	1.42	1.19	1.01	.81	.52
		409	1.07	1.36	1.11	1.07	.81	.44
		374	1.38	1.27	1.25	1.02	.82	.59

Table A.9 . Heat penetration parameters f (in minutes) and at several axial positions in No. 2-1/2 vegetable jars[1]

Part I. Values of f

Treatment			Position in container, inches above bottom						
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8 f_1	4-1/8 f_2
Water	60-120	135	14.7	14.7	14.8	13.7	12.8	6.6	10.5
		142	14.4	14.7	14.8	14.1	12.6	5.7	9.1
		148	14.7	14.7	14.7	14.0	12.8	5.6	11.1
		78	14.5	14.9	15.0	14.0	12.5	5.3	10.2
		79	14.5	18.6	14.7	14.5	13.3	7.6	11.2
		85	14.0	14.2	14.5	13.7	12.5	7.1	10.2
	120-180	140	14.6	14.2	14.3	13.9	12.2	5.6	9.3
		146	14.3	14.1	14.3	13.6	12.2	5.8	10.7
		152	14.7	14.8	15.0	14.0	12.7	7.1	11.0
		77	14.7	15.0	15.1	13.8	13.1	7.3	12.4
		83	14.4	14.7	15.0	13.8	12.2	7.7	11.5
		89	14.4	14.0	14.9	13.8	12.8	8.3	11.5
	60-180	138	14.4	14.6	14.4	13.9	11.8	6.1	11.8
		144	14.2	14.2	14.2	13.6	12.5	5.7	11.0
		150	14.5	14.1	13.9	13.4	12.3	6.1	10.5
		75	13.9	14.0	13.9	13.6	12.8	6.8	10.7
		81	13.8	14.0	14.2	13.3	12.4	7.4	10.3
		87	14.1	14.1	14.3	13.8	12.9	7.2	9.9
50% Sucrose	60-120	155	16.5	17.5	17.5	15.3	13.5	5.5	13.3
		161	16.8	17.6	17.2	15.7	14.2	5.1	14.1
		271	16.8	17.9	17.0	16.6	14.5	4.9	15.1
		165	17.2	17.4	17.6	16.3	14.5	5.8	15.0
		177	16.7	17.2	17.9	16.4	14.3	5.3	13.1
			--	--	a "tipped" jar		--	--	---
	120-180	159	17.3	17.5	17.6	17.0	14.7	9.3	16.4
		269	17.8	18.0	18.4	17.0	16.0	4.9	14.7
		275	18.4	18.3	18.9	17.2	15.4	6.4	16.1
		169	17.3	17.7	17.0	17.7	16.1	7.1	13.8
		175	16.7	17.1	17.5	17.9	15.5	6.6	12.2
		181	17.4	17.8	17.9	17.3	15.3	5.9	16.7
	60-180	163	16.6	16.6	16.6	16.2	14.4	3.8	14.9
		273	16.6	16.4	16.7	16.1	16.0	3.5	14.0
		157	15.7	15.9	15.7	14.7	13.9	3.6	14.1
		167	16.0	16.3	16.9	17.0	16.0	6.1	15.6
		173	15.3	15.8	16.6	16.1	14.9	4.9	13.7
		179	16.1	16.6	17.4	16.4	16.2	4.5	14.7

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[1] Data are from lines fitted visually to data over the U = 0.5 to U = 0.084 range.

Table A.9 (continued)

Part II. Values of log factor j

Treatment			Position in container, inches above bottom						
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	2-1/8	3-1/8	4-1/8	
Water	60-120	135	1.18	1.31	1.21	1.11	.91	.84	.57
		142	1.21	1.34	1.26	1.10	.92	.88	.61
		148	1.19	1.37	1.29	1.10	.93	.85	.48
		68	1.18	1.30	1.24	1.12	.95	.80	.52
		79	1.16	1.37	1.15	1.09	.93	.83	.56
		85	1.24	1.37	1.30	1.17	.95	.85	.59
	120-180	140	1.18	12.7	1.18	1.03	.94	.91	.56
		146	1.14	1.23	1.16	1.04	.91	.89	.47
		152	1.15	1.22	1.10	1.02	.90	.82	.53
		77	1.16	1.18	1.16	1.09	.93	.90	.51
		83	1.18	1.21	1.13	1.02	.94	.87	.54
		89	1.17	1.30	1.18	1.08	.95	.84	.59
	60-180	138	1.22	1.25	1.19	1.06	.84	.83	.45
		144	1.23	1.28	1.20	1.08	.93	.80	.44
		150	1.13	1.29	1.23	1.09	.94	.85	.44
		75	1.23	1.31	1.26	1.07	.91	.82	.41
		81	1.23	1.30	1.23	1.11	.96	.83	.60
		87	1.22	1.30	1.18	1.07	.91	.89	.64
50% Sucrose	60-120	155	1.27	1.41	1.31	1.17	.91	.86	.45
		161	1.22	1.40	1.34	1.13	.88	.86	.43
		271	1.23	1.40	1.38	1.09	.90	.95	.43
		165	1.23	1.42	1.39	1.13	.88	.93	.38
		177	1.21	1.43	1.38	1.15	.91	.91	.47
			--	--	a "tipped" jar		--	--	--
	120-180	159	1.22	1.24	1.15	1.05	.92	.97	.53
		269	1.19	1.27	1.17	1.02	.80	.93	.38
		275	1.13	1.23	1.12	1.00	.83	.77	.40
		169	1.22	1.31	1.17	1.09	.90	.88	.42
		175	1.13	1.35	1.27	1.02	.85	.85	.48
		181	1.22	1.29	1.22	1.00	.93	.91	.38
	60-180	163	1.26	1.30	1.26	1.01	.81	1.01	.35
		273	1.26	1.34	1.11	1.00	.69	1.00	.31
		157	1.36	1.42	1.32	1.15	.86	1.03	.34
		167	1.41	1.41	1.30	1.10	.81	.93	.50
		173	1.44	1.46	1.32	1.27	.93	.93	.47
		179	1.36	1.37	1.20	1.04	.76	.87	.33

Table A.10 . Heat penetration parameters f (in minutes) and at several other jar sizes[1]

Part I. Values of f

Treatment			Position in container, inches above bottom								
Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	1-5/8	2-1/8	2-5/8	3-1/8	3-5/8	4-1/8
					22 oz. glass (water)						
Water	60-120	296	11.9	11.5	11.9	12.6	12.2	12.1	11.4	11.3	10.8
		298	11.5	11.7	11.9	12.2	11.9	11.9	11.2	11.1	11.7
		300	12.0	11.8	12.1	12.2	11.9	11.7	11.4	11.1	11.0
		302	10.9	11.9	12.1	12.5	11.9	11.8	11.4	11.2	10.4
		304	11.4	11.8	12.0	12.5	12.1	11.9	11.4	11.1	10.6
	120-180	297	11.4	11.4	11.6	11.7	11.5	11.1	10.9	10.9	10.3
		299	11.5	11.5	11.8	11.7	11.5	11.4	11.0	10.6	10.2
		301	11.8	11.8	12.2	12.1	11.9	11.4	11.0	10.5	10.2
		303	11.4	11.5	11.7	12.3	11.7	11.2	11.0	10.8	10.3
		305	11.3	11.3	11.7	12.0	11.8	11.7	11.3	11.2	10.2
	60-180	280	10.7	11.0	11.5	11.3	11.0	10.9	10.6	10.4	9.8
		291	11.1	11.2	11.6	11.2	11.2	11.0	10.7	10.3	9.9
		292	10.8	11.6	12.0	11.1	11.1	--	10.7	--	10.2
		293	11.2	11.3	11.5	11.4	11.4	--	10.8	--	10.3
		294	11.0	11.5	11.4	11.3	11.3	--	10.7	--	10.2
		295	10.60	10.8	11.2	11.3	11.0	--	10.6	--	10.0
					32 oz. glass						
	60-180	431	13.5	13.6	13.5	13.5	13.3	13.1	13.0	12.8	12.6
		432	13.5	13.5	13.5	13.0	12.9	12.8	12.3	12.1	12.1
		433	13.5	13.6	13.3	13.1	12.9	12.6	12.5	12.4	11.8
		400	12.7	13.0	12.9	12.9	12.7	12.7	12.4	12.1	11.8
		401	12.9	13.0	13.0	12.7	12.5	12.3	12.0	17.5	11.5
		402	13.0	13.2	13.1	12.9	12.6	12.3	12.3	12.2	11.5
50% Sucrose		434	16.5	16.6	16.6	16.1	15.8	15.6	15.0	15.0	14.4
		403	16.6	16.7	17.1	16.2	15.5	15.1	14.8	14.5	13.6
		404	17.5	17.6	18.3	17.0	16.9	16.0	15.9	15.3	15.0

Table A.10 (continued)

Part II. Values of log factor j

Treatment Fluid	T_o-T_1	Run #	1/8	5/8	1-1/8	1-5/8	2-1/8	2-5/8	3-1/8	3-5/8	4-1/8
						22 oz. glass (water)					
Water	60-120	296	1.30	1.52	1.44	1.30	1.25	1.18	1.16	1.08	1.01
		298	1.39	1.52	1.46	1.32	1.27	1.17	1.15	1.05	.99
		300	1.28	1.46	1.42	1.28	1.25	1.17	1.10	1.05	.95
		302	1.70	1.43	1.38	1.27	1.25	1.17	1.10	1.04	1.00
		304	1.39	1.55	1.49	1.32	1.28	1.20	1.17	1.08	1.03
	120-180	297	1.24	1.32	1.27	1.21	1.16	1.13	1.08	1.01	.96
		299	1.28	1.33	1.27	1.18	1.17	1.12	1.07	1.02	.98
		301	1.25	1.35	1.29	1.20	1.15	1.13	1.09	1.04	.99
		303	1.22	1.33	1.32	1.18	1.15	1.05	1.04	1.00	.96
		305	1.28	1.43	1.35	1.21	1.18	1.10	1.06	1.01	1.00
	60-180	280	1.37	1.41	1.30	1.26	1.23	1.13	1.08	1.02	1.00
		291	1.32	1.43	1.35	1.27	1.17	1.11	1.07	1.01	.96
		292	1.32	1.45	1.34	1.27	1.19	--	1.09	--	.95
		293	1.33	1.44	1.33	1.25	1.19	--	1.10	--	.92
		294	1.37	1.43	1.34	1.26	1.19	--	1.10	--	.95
		295	1.40	1.48	1.36	1.26	1.23	--	1.10	--	.98
						32 oz. glass					
	60-180	431	1.28	1.34	1.27	1.19	1.14	1.07	.99	.93	.86
		432	1.36	1.36	1.28	1.24	1.18	1.11	1.07	1.01	.91
		433	1.32	1.36	1.32	1.24	1.18	1.12	1.04	.99	.94
		400	1.35	1.39	1.34	1.23	1.18	1.05	1.01	.95	.90
		401	1.32	1.42	1.34	1.27	1.20	1.14	1.07	1.00	.91
		402	1.36	1.40	1.33	1.25	1.19	1.15	1.03	.94	.92
50% Sucrose		434	1.39	1.42	1.27	1.24	1.11	1.09	1.02	.95	.84
		403	1.36	1.41	1.28	1.24	1.16	1.14	1.01	.94	.89
		404	1.23	1.35	1.20	1.19	1.11	1.08	1.00	.90	.84

Table A.11 . Summary of f and j data from study of horizontal temperature profiles in water in 16 oz. vegetable jars with $T_o = 60^o$, $T_1 = 180^oF$.

Part I. Profiles near top of container

Run	1		2		3		4		5	
#	f	j	f	j	f	j	f	j	f	j
A.	Top row (1/8 below free fluid surface) position in container									
447	7.2	.59	7.1	.61	7.1	.59	7.0	.61	7.2	.59
449	6.8	.60	6.9	.57	6.6	.63	6.8	.60	6.8	.64
451	7.1	.64	7.3	.60	7.4	.60	7.4	.60	7.3	.62
453	7.0	.62	7.4	.56	7.3	.57	7.4	.57	7.2	.59
455	7.1	.60	7.2	.55	7.2	.55	7.3	.55	7.4	.55
457	7.0	.61	7.0	.55	6.9	.59	6.9	.60	7.2	.58
459	7.2	.57	7.3	.53	7.3	.56	7 3	.54	7.3	.53
Mean	7.0	.60	7.2	.57	7.1	.58	7.1	.58	7.2	.58

B. Bottom row (3/8 in. below fluid surface) position in container

	6		7		8		9		10	
	f	j	f	j	f	j	f	j	f	j
447	7.4	.77	7.5	.76	7.2	.76	7.1	.76	7.2	.74
449	7.2	.82	7.0	.85	7.3	.81	7.3	.81	7.4	.79
451	7.9	.81	7.8	.79	7.7	.81	7.5	.84	7.7	.81
453	7.7	.76	7.5	.78	7.5	.76	7.4	.79	7.6	.78
457	7.8	.70	7.8	.71	7.9	.71	7.9	.72	8.1	.69
459	7.5	.79	7.5	.77	7.4	.69	7.8	.73	7.7	.75
Mean	7.6	.77	7.6	.77	7.5	.75	7.5	.77	7.7	.75

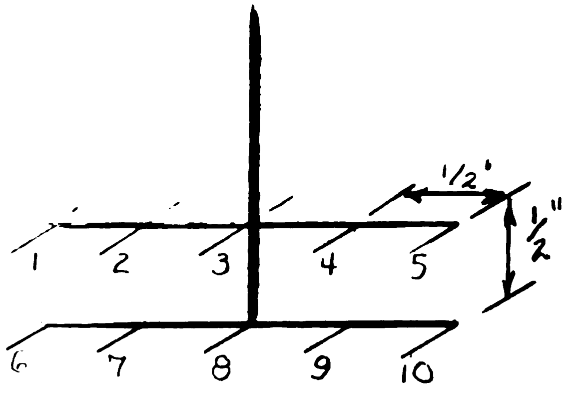

Table A.11 . (continued)

Part II. Profiles near bottom of container

Run	1		2		3		4		5	
#	f	j	f	j	f	j	f	j	f	j
A. Top row (5/8 inch above bottom)										
435	9.6	1.32	9.6	1.27	9.8	1.29	9.7	1.26	9.5	1.33
437	9.4	1.33	9.6	1.33	9.6	1.35	9.6	1.33	9.7	1.28
439	9.4	1.35	9.4	1.35	9.5	1.36	9.4	1.35	9.4	1.35
441	9.5	1.36	9.6	1.35	9.5	1.36	9.4	1.35	9.5	1.35
443	9.5	1.33	9.4	1.35	9.5	1.32	9.4	1.34	9.5	1.35
445	9.5	1.34	9.4	1.34	9.6	1.35	9.5	1.34	9.5	1.35
Mean	9.5	1.34	9.5	1.33	9.6	1.34	9.5	1.33	9.5	1.34

B. Bottom row (1/8 inch above bottom)

	6		7		8		9		10	
	f	j	f	j	f	j	f	j	f	j
435	10.0	1.35	9.7	1.34	9.6	1.34	9.9	1.26	9.7	1.34
437	9.7	1.37	9.7	1.33	9.6	1.39	9.5	1.39	9.6	1.38
439	9.5	1.43	9.7	1.36	9.6	1.36	9.7	1.38	9.7	1.40
441	9.7	1.42	9.6	1.36	9.6	1.36	9.6	1.37	9.7	1.40
443	9.6	1.37	9.6	1.36	9.6	1.37	9.6	1.39	9.7	1.37
445	9.7	1.40	9.7	1.34	9.7	1.36	9.7	1.35	9.7	1.39
Mean	9.7	1.39	9.6	1.35	9.6	1.36	9.7	1.36	9.6	1.38