DYNAMIC ANALYSIS OF NONLINEAR SPACE FRAMES

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ABSTRACT

DYNAMIC ANALYSIS OF
NONLINEAR SPACE FRAMES

By

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In this thesis, a matrix formulation is presented for the analysis of dynamically loaded space frames. The effects of both material and geometric nonlinearities are taken into consideration. These effects are restricted, respectively, to the case of linearly elastic-perfectly plastic materials and the case of small rotations, i.e., the case in which the rotation angles are negligible with respect to unity.

The analysis begins with specifying two yield condition equations for a typical member cross-section. Then, the incremental force-displacement relations for a space-frame member are derived for several cases. Firstly, these relations are summarized for a linearly elastic member with the effects of geometric nonlinearities taken into account. Secondly, the relations for a member whose end cross-sections are yielding are derived again with the latter effects accounted for. Finally, the

incremental force-distortion relations are derived for an elastoplastic discrete model under geometrically linear conditions.

The analysis is further carried on by describing a mass lumping procedure that considers rotary inertia. Then, the equation of motion for a typical "free" joint is derived. Following that, the criterion by which a member cross-section is ruled to be yielding is described. It is shown next that the force vector acting on a yielding cross-section is prevented from proceeding beyond the yield surface. Finally, the steps of the numerical procedure employed to determine transient response are described.

A computer program is prepared for the implementation of the analysis. Three numerical problems are considered: a cantilever beam, a six-member space frame, and a two-bay two-story building frame. Dynamic loading is either provided by concentrated loads applied to free joints, or generated by ground motions due to earthquake.

Several comparative studies on the numerical problems mentioned above are presented. These studies show that good agreements exist between the results provided by the geometrically linear formulation and those given in a published report in which a different method was used. This may be construed as evidence for the validity of the present analysis. The method given here, however, requires significantly less computer time.

The comparative studies also show that plastic displacements as predicted by the geometrically nonlinear formulation are generally larger than those resulting from the geometrically linear version. But, on the whole, when axial loads are small (as compared to the corresponding Euler loads), the influence of geometric nonlinearities on numerical results does not seem significant. However, as axial loads increase, such influence rapidly grows.

DYNAMIC ANALYSIS OF
NONLINEAR SPACE FRAMES

By

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A	= Flexibility matrix;
A_x	= cross-sectionsl area;
A_y, A_z	= effective areas of shear;
B	= flow-constant matrix;
b	= flow constant;
D	= displacement transformation matrix defined by Equation (1-3);
E	= modulus of elasticity;
E	= matrix defined by Equation (2-31) or (2-51);
e	= axial strain;
e	= superscript denoting "elastic part";
F	= member force (vector);
F_G	= member force (vector) due to geometrically nonlinear terms;
F_x	= axial force;
F_{xp}	= fully plastic axial force;
F_y, F_z	= Shearing forces;
G	= modulus of shear;
G	= matrix defined by Equation (2-37);
G_N, G_P	= matrices defined by Equations (2-26);
H	= matrix defined by Equation (1-2);
I	= third order identity matrix;

I_b = branch inertia tensor expressed at branch mass center b;

I_j = branch inertia tensor expressed at joint mass center j;

J_j = joint inertia tensor expressed at joint mass center j;

K = generalized member stiffness matrix;

L = length of a member;

M_J = joint mass matrix expressed at joint J;

M_j = joint mass matrix expressed at joint mass center j;

M_x = torsional moment;

M_{xp} = fully plastic torsional moment;

M_y, M_z = bending moments;

M_{yp}, M_{zp} = fully plastic bending moments;

m_B = mass of branch B;

m_J = mass of joint J;

N = axial stress resultant;

N = subscript to denote negative end of a member;

$\overline{N}$ = position vector of member end N expressed in global coordinate system;

N_1 = subscript to denote section $x = (1-\beta)L$ of a member;

N_2 = subscript to denote section $x = (1-2\beta)L$ of a member;

P = subscript denoting positive end of a member;

$\overline{P}$ = position vector of member end P expressed in global coordinate system;

P_E = external joint force (vector);

P_I = internal joint force (vector);

P_J = force (vector) on joint J;

P_j = force (vector) on joint mass center j;

P_1 = subscript to denote section $x = \beta L$ of a member;

P_2 = subscript to denote section $x = 2\beta L$ of a member;

p = superscript denoting "plastic part";

R = rotation matrix (6x6);

r = rotation matrix (3x3);

S = elastic member stiffness matrix;

S_G = geometric member stiffness matrix;

s_G = submatrix of S_G;

T = force transformation matrix defined by Equation (1-1);

t = time;

t = superscript denoting matrix transposition;

U = member displacement (vector);

u_x, u_y, u_z = linear member displacements;

V = yield surface normal in dimensional force space

W = member distortion (vector);

$\overline{W}_b$ = position vector of branch mass center b expressed in global coordinate system;

$\overline{W}_j$ = position vector of joint mass center j expressed in global coordinate system;

$\overline{w}_j$ = position vector of joint mass center j expressed in local coordinate system;

X, $\dot{X}$, $\ddot{X}$ = joint displacement, velocity, and acceleration (vectors), respectively;

$\ddot{X}_J$ = acceleration (vector) of joint J;

$\ddot{X}_j$	= acceleration (vector) of joint mass center j;
$\ddot{X}_0$	= foundation acceleration (vector);
X, Y, Z	= joint (global) coordinates;
x, y, z	= member (local) coordinates;
Z_J	= responsiveness matrix for joint J;
α	= dimensionless parameter modifying moments and products of inertia of a joint rigid body;
β	= dimensionless parameter giving plastic-portion length of a yielding member if multiplied by 2L;
γ	= dimensionless parameter modifying direction of yield surface normal;
Δ	= prefix denoting "change" or "increment";
δ_y, δ_z	= axial shortenings of a member divided by L;
μ	= uniformly distributed mass per unit length of a member;
ρ	= density of material;
Φ	= yield function;
Φ_e	= elliptic yield function
Φ_p	= parabolic yield function
ϕ_y, ϕ_z	= axial force parameters;
ω_x, ω_y, ω_z	= rotational member displacements; and
∇	= prefix denoting "gradient."

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CHAPTER I

INTRODUCTION

This chapter presents the objective of the present work, previous related studies, and assumptions and limitations of the analysis developed. It also gives an outline of the investigation carried out and certain general definitions needed in the subsequent analysis.

1.1 Objective

Two types of nonlinearities occur in structural problems. The first type may be referred to as "geometric nonlinearities." They occur when deflections are large enough to cause significant changes in the geometry of the structure. In this case the equations of equilibrium must be formulated for the deformed configuration. The second type may be referred to as "material nonlinearities" which include any deviation from linear elasticity, such as nonlinearly elastic, or plastic, or viscoelastic behavior of the structural material.

The objective of this thesis is to develop a numerical method for the dynamic analysis of space frames, taking the effects of geometric and certain material

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nonlinearities into consideration. In order to accomplish the objective, a matrix formulation of the problem is derived. Furthermore, a computer program is prepared for the implementation of the analysis. Finally, numerical results of certain problems are obtained in order to demonstrate the validity and practicality of the method.

It seems hardly necessary to point out that the problem under consideration is a broad, and hence, difficult one. However, as it will become apparent later, the present work has had the advantage of using several earlier works as "building blocks." These earlier works will be described briefly in the following section.

1.2 Previous Studies

Recent advances in the field of computer technology have provided the necessary tools for the development of the analysis of geometrically nonlinear frames. Many papers concerned with such developments have appeared in recent years. Among these perhaps the pioneering works by Saafan (12),* Livesley (7), Argyris (1), and Johnson and Brotten (6) deserve special attention.

In a recent paper, Jenning (5) has incorporated the effects of change of geometry into certain displacement transformation matrices for members of plane frames.

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*Numbers in parentheses refer to entries in the list of references.

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He considered the axial shortening due to member inclination as being the only important nonlinear term in cases where displacements are not exceedingly large. However, he mentioned that accurate transformation matrices can be adopted in cases in which very large deformations are to be dealt with. Based on Jenning's formulation, Iverson (4) has derived stiffness coefficients for space-frame members. He applied his geometrically nonlinear formulation to elastic frames under dynamic loads.

Connor, Logcher, and Chan (2) have also derived a geometrically nonlinear formulation for the three dimensional case. Their derivation is restricted to the small rotation case, i.e., a case in which the squares of rotation angles are negligible with respect to unity. Zarghamee and Shaw (15) have given a similar formulation independently. They presented a more comprehensive expression for axial force than the one given by Connor, Logcher, and Chan.

The area of inelastic behavior of frames under static and/or dynamic loads has been of great interest in the past decade. Almost all published works on this subject have been partially based on the plastic potential theory. In a recent report, Morris and Fenves (9) have studied the inelastic behavior of space frames under static loads. They derived the incremental force-deformation relations for a member having any number of plastic hinges. In their report, they also derived the approximate yield surface equations for certain commonly used cross-sections, considering the interaction between bending moments, torsional moment, and axial force.

Nigam (11) has recently presented an elastoplastic formulation to study the response of dynamically loaded space frames. He derived the incremental force-displacement relations for a member whose one or two end cross-sections are yielding. His equations are basically the same as the ones derived by Morris and Fenves, although they apparently seem different. He then extended his formulation to the dynamic case. But, in this extension, he did not discuss such important questions as: how to handle the mass; how to prevent the force vector acting on a yielding cross-section from proceeding beyond the yield surface; and how to solve the equations of motion.

Wen (13) has also studied the elastoplastic behavior of space frames under dynamic loads. His formulation is based on dividing a yielding member into an elastic member with continuous flexibility and an elastoplastic member with lumped flexibility.

1.3 Assumptions and Limitations

The assumptions and limitations employed in this study are divided into the following five categories:

(1) Material

(a) The material is assumed to be linearly elastic--perfectly plastic.

(b) All stress-strain characteristics of the material are assumed to be time-independent.

(c) Strain-hardening effects of the material are neglected.

(2) Cross sections

(a) All cross-sections are assumed to have two axes of symmetry.

(b) Yielding is assumed to occur at individual cross-sections with no "spread length."

(c) A shape factor of 1.0 is assumed for all cross-sections. That is, cross-sections are assumed to make an abrupt transition from a completely elastic state to a state where unrestricted plastic flow can occur.

(d) It is assumed that cross-sections are free to warp under torsional loads, i.e., the case of "pure" torsion prevails.

(3) Members

(a) All members are assumed to be prismatic.

(b) The usual engineering theory of bending is assumed to be applicable.

(c) Torsion-flexure coupling effects are neglected.

(d) Members may have uniformly distributed gravity loads only. Concentrated loads at points other than the ends of members are not permitted.

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(4) Joints

(a) Joint loads, which are necessarily concentrated, may be statical and/or dynamical.

(5) Frame

(a) Changes in the geometry of the frame are restricted to the "small rotation" case, i.e., a case in which the squares of rotation angles are negligible in comparison with unity.

1.4 Outline of Present Study

In this study, two yield condition equations for a member cross-section, as used in Reference 13, are first specified. Then a series of incremental force-displacement relations for a member, taking the effects of geometric nonlinearities into account, are derived. Firstly, these relations are summarized for a linearly elastic member, as formulated by Connor, Logcher, and Chan (2). Secondly, the case of a member whose end cross-sections are yielding is studied. Finally, the incremental force-distortion relations for a discrete model, taken from Reference 13, are derived.

The dynamic analysis begins with a mass lumping procedure (see References 4 and 13) that accounts for rotary inertia. Then, the motion equation for a typical (ungrounded) joint is derived. In the dynamic solution, the criterion by which a member cross-section is rule

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ruled to be yielding is described first. It is shown next that the force vector acting on a yielding cross-section is prevented from proceeding beyond the yield surface. Finally, the transient response of the frame to dynamic loading is obtained by integrating the equations of motion numerically. Dynamic loading is either provided by concentrated loads applied to free joints, or generated by ground motion due to an earthquake.

Three numerical examples taken from Reference 13 are considered: a cantilever beam subjected to a pulse type of loading; a six-member space frame subjected to a step-function type of loading; and a two-story two-bay building frame subjected to the 1940 El Centro earthquake. These examples, on the one hand, illustrate the applications of the numerical method developed. On the other hand, they will provide a basis for comparing the numerical results of the present method with those reported in the above-mentioned reference.

1.5 General Definitions

Figure 1-1 illustrates a three-dimensional frame whose joints and members are all numbered. The joint count is separate from the member count. For convenience of computer programming, free joints are always numbered first.

Each member is arbitrarily assigned an orientation by specifying one of its ends as the positive end and the other as the negative end. Consider a member framed between two joints. It is then said that the member is positively incident to the joint at its positive end and negatively incident to the joint at its negative end. For example, in Figure 1-1, the member 2 is positively incident on the joint 1 and negatively incident on the joint 3.

Two classes of right-handed Cartesian coordinate systems are used. The origin of each coordinate system is either clear from context or specifically pointed out. The first class consists of a single joint (global) coordinate system arbitrarily chosen. The second class consists of all member (local) coordinate systems chosen in the following manner. The first axis of each member coordinate system coincides with the undeformed centroidal axis, and is directed from the positive end to the negative end of the member. The other two axes are coincident with the principal axes of inertia of a generic cross-section of the member.

In the next two chapters, force and displacement transformation matrices will be needed. These are given below for the sake of completeness. Referring to Figure 1-2, let $o_1x_1y_1z_1$ and $o_2x_2y_2z_2$ denote two coordinate systems whose axes are parallel and oriented in the same fashion. The force transformation matrix from the first system to the

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the second is then given by

$$T_{0_2 0_1} = \begin{bmatrix} I & 0 \\ H & I \end{bmatrix} \tag{1-1}$$

where I is the third order identity matrix, and

$$H = \begin{bmatrix} 0 & Z & -Y \\ -Z & 0 & X \\ Y & -X & 0 \end{bmatrix} \tag{1-2}$$

in which X, Y, and Z are the coordinates of the center 0_2 with respect to the system $0_1x_1y_1z_1$. The corresponding displacement tranformation matrix is given by

$$D_{0_1 0_2} = (T^{-1}_{0_2 0_1})^t \tag{1-3}$$

Referring again to Figure 1-2, let $0_2x_2'y_2'z_2'$ denote a third coordinate system rotated with respect to the system $0_2x_2y_2z_2$. The force transformation matrix from the second system to the third is given by

$$R = \begin{bmatrix} r & 0 \\ 0 & r \end{bmatrix} \tag{1-4}$$

where

$$r = \begin{bmatrix} \cos(x_2',x_2) & \cos(x_2',y_2) & \cos(x_2',z_2) \\ \cos(y_2',x_2) & \cos(y_2',y_2) & \cos(y_2',z_2) \\ \cos(z_2',x_2) & \cos(z_2',y_2) & \cos(z_2',z_2) \end{bmatrix} \tag{1-5}$$

in which the parenthesized quantities denote the angles between the axes indicated. The matrices R and r will be referred to as the "rotation" matrices. In this case, the displacement transformation matrix is also given by the matrix R. For example, the incremental member end displacement ΔU and the incremental displacement ΔX of the joint, connected to the member end being considered, are related by

$$\Delta U = R\Delta X \tag{1-6}$$

where ΔU and ΔX are expressed, respectively, in the member and joint coordinate systems, and R is the rotation matrix from the second system to the first.

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CHAPTER II

FORCE-DEFORMATION RELATIONS FOR A MEMBER

In this chapter, two yield condition equations for a member cross-section, to be employed for obtaining numerical solutions, are first specified. Then a series of incremental force-displacement relations for a member, considering the effects of geometric nonlinearities, are derived. Firstly, these relations are summarized for a linearly elastic member. Secondly, the case of a member whose end cross-sections are yielding is studied. Finally, the incremental force-distortion relations for a discrete model, as used in Reference 13, are derived.

2.1 Yield Condition Equation

In frame analysis, it is convenient (perhaps even necessary) that the yield condition equation for a member cross-section be formulated in terms of stress resultants. These resultants will be hereafter referred to as "force components" for short. Expressed in these terms, the yield condition equation defines the combination of the force components necessary to initiate yielding at a cross-section. Since a shape factor of 1.0 is assumed for all cross-sections, the initiation of yielding will coincide with the start of unrestricted plastic flow. Thus, the yield condition equation represents the relation among the force components for the initiation of unrestricted plastic flow. Finally, for an elastic-perfectly plastic material as assumed in this study, the yield condition equation remains the same as yielding progresses.

In the case of three-dimensional frames, the yield condition equation for a cross-section may be written symbolically as

$$\Phi(F_x, M_x, M_y, M_z) = 1 \tag{2-1}$$

where Φ is the so-called yield function, F_x is the axial force, M_x is the torsional moment, and M_y and M_z are the bending moments. Equation (2-1) represents a hypersurface in the four-dimensional force space spanned by the Cartesian coordinates F_x, M_x, M_y, and M_z. This hypersurface is called the yield surface.

The derivation of the exact yield condition (or yield surface) equation for a given cross-section is generally quite difficult. Numerical solutions are available only for a square cross-seciton. Morris and Fenves (10) have derived approximate lower bound yield surface equations for commonly used cross-sections, using a procedure suggested by Hodge (3) along with simplifying assumptions on the neutral axis position. Even those equations appear rather unwieldy for general application to frames with many members.

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In this study, for the sake of simplicity, two particular forms of yield functions taken from Reference 13 are employed. These functions do not correspond exactly to any commonly used cross-section. However, considering the assumptions made and uncertainties involved in applying the theory of plasticity to this problem, they may be regarded as reasonable approximations for obtaining numerical solutions.

The first function is given by

$$\Phi_p = \left|\frac{F_x}{F_{xp}}\right| + \left(\frac{M_x}{M_{xp}}\right)^2 + \left|\frac{M_y}{M_{yp}}\right| + \left|\frac{M_z}{M_{zp}}\right| = 1 \qquad (2\text{-}2)$$

in which F_{xp}, M_{xp}, M_{yp}, and M_{zp} are the fully plastic force components corresponding to F_x, M_x, M_y, and M_z, respectively. The function Φ_p will be referred to as the "parabolic" yield function. The second function is given by

$$\Phi_e = \left(\frac{F_x}{F_{xp}}\right)^2 + \left(\frac{M_x}{M_{xp}}\right)^2 + \left(\frac{M_y}{M_{yp}}\right)^2 + \left(\frac{M_z}{M_{zp}}\right)^2 = 1 \qquad (2\text{-}3)$$

the function Φ_e will be referred to as the "elliptic" yield function.

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It is apparent that the parabolic yield surface is enclosed by the elliptic one. Therefore, for engineering analysis, it is a more conservative one to use as it would normally indicate an occurrence of yielding before the elliptic yield function would do so.

2.2 Incremental Force-Displacement Relations for a Geometrically Nonlinear Member

The incremental force-displacement relations for a typical member can be derived by considering the statics, member geometry, and stress-strain characteristics of the material. Connor, Logcher, and Chan (2) derive these relations for an elastic member, considering the effects of geometric nonlinearities. This derivation will be outlined briefly in the next subsection for the sake of completeness.

When a member end cross-section yields, an unknown vector, namely, the plastic end displacement increment, is introduced in the incremental force-displacement relations. This unknown is determined, up to a scalar parameter, i.e., the "flow constant," by applying the plastic potential theory. (This theory is described in several textbooks among which the one by Malvern (8) is perhaps the most recent.) The flow constant, and consequently the force-displacement relations, can be determin

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determined by applying the condition that the force increment--to be theoretically correct, of infinitely small magnitude--acting on a yielding cross-section is tangent to the yield surface.

2.2.1 Elastic Member

Figure 2-1 shows the initial and deformed positions of a typical member PN. The x axis coincides with the undeformed centroidal direction and the y and z axes are coincident with the two principal axes of inertia. These axes constitute the local coordinate system for the member. The internal forces (F_x, F_y, F_z, M_x, M_y, M_z) and displacements (u_x, u_y, u_z, ω_x, ω_y, ω_z) are referred to in this coordinate system, and so are the end forces F_P and F_N with components F_{Px}, F_{Nx}, etc., and end displacements U_P, and U_N with components U_{Px}, U_{Nx}, etc.

Figure 2-2 shows the initial and deformed positions of a differential element AB. The centroidal point B is displaced to the point B'. Based on the assumption of small rotations ($\omega_y^2 << 1$, $\omega_z^2 << 1$), the longitudinal strain e at the point B', in the direction of the tangent to the deformed centroidal axis, is expressed by

$$e = \frac{du_x}{dx} + 0.5\ \omega_y^2 + 0.5\ \omega_z^2 \tag{2-4}$$

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$$\omega_y = -\frac{du_z}{dx} \tag{2-5a}$$

$$\omega_z = \frac{du_y}{dx} \tag{2-5b}$$

where u_x, u_y, and u_z are the translational displacements of the point B. The second and third terms in Equation (2-4) introduce the effects of geometric nonlinearities in the present formulation. If these terms are neglected the formulation returns to the geometrically linear case.

With reference to Figure 2-2, the force (stress resultant) tangent to the deformed centroidal axis is denoted by N. (Note that the same symbol N has also been used to denote the negative end of the member. But the meaning in every case where the symbol N appears should be obvious from context.) The force N and the moment M_z are related to the strain e and displacement u_y, respectively, by

$$N = EA_x\, e \tag{2-6}$$

$$M_z = EI_z \frac{d^2u_y}{dx^2} \tag{2-7}$$

in which E is the modulus of elasticity, A_x is the cross-sectional area, and I_z is the cross-sectional moment of inertia about the z axis. Note that Equation (2-7) is based o

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based on the usual engineering theory of bending which neglects the effects of shearing deformations.

The equilibrium equations consistent with the present geometric approximation ($\omega_y^2 << 1$, $\omega_z^2 << 1$) can be obtained by applying the principle of virtual work. The details are omitted since the procedure is quite straightforward. The final equations are then listed as

$$\frac{dN}{dx} = 0 \tag{2-8}$$

$$\frac{d^2M_z}{dx^2} - \frac{d}{dx}(N\omega_z) = 0 \tag{2-9}$$

Equation (2-8) states that the axial force N is constant along the entire length of the member. Consequently, Equation (2-6) can be integrated over the length of the member. This leads to

$$N = -\frac{EA_x}{L}(u_{Px}-u_{Nx}) + EA_x(\delta_y+\delta_z) \tag{2-10}$$

in which L is the length of the member and

$$\delta_y = \frac{1}{2L}\int_0^L \omega_z^2 dx$$
$$\delta_z = \frac{1}{2L}\int_0^L \omega_y^2 dx \tag{2-11}$$

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The governing equation for the displacement u_y is obtained by substituting Equations (2-5b) and (2-7) into Equation (2-9). Thus

$$\frac{d^4u_y}{dx^4} + \frac{\phi_y^2}{L^2}\frac{d^2u_y}{dx^2} = 0 \qquad (2\text{-}12)$$

where

$$\phi_y = \left[- \frac{NL^2}{EI_z}\right]^{0.5} \qquad (2\text{-}13)$$

= axial force parameter

Note that the parameter ϕ_y is a function of the axial force N which in turn depends on the displacement u_y. Rigorous solution of the differential equation (2-12) for the displacement u_y would require using iterative schemes. To avoid that, the latter equation should be solved under the assumption that the parameter ϕ_y (or axial force N) is known. The solution should then be introduced into Equations (2-7) and (2-10). The discrepancy between the assumed axial force N in solving Equation (2-12) and the one obtained in Equation (2-10) is of course a source of inaccuracy in the procedure. This point would be brought up again in later sections.

It is evident that Equations (2-7), (2-9), and (2-12) are associated with bending in the x-y plane. Similar equations can readily be written for bending in the x-z

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the x-z plane. The force-displacement relations are obtained by assembling the equations associated with flexure in the x-y plane, flexure in the x-z plane, and finally, twisting about the x axis. The end forces are then determined in terms of the end displacements from the latter equations by enforcing the proper boundary conditions. The lengthy details of this procedure are omitted. Only the relevant equations in matrix form will be given below.

The end force-end displacement relations are now expressed as

$$\begin{aligned} F_P &= S_{PP} U_P + S_{PN} U_N - F_G \\ F_N &= S_{PN}^t U_P + S_{NN} U_N + F_G \end{aligned} \tag{2-14}$$

in which S_{PP}, S_{PN}, and S_{NN} are the "elastic" stiffness matrices, and F_G is a column matrix containing the nonlinear terms due to the rotations ω_y and ω_z. These matrices are all expanded in Appendix I. Note that the effects of torsion-flexure coupling and warping restraint have been neglected in Equations (2-14). In addition, it has been assumed that the member does not have any releases. However, it will be shown later (see Appendix II) that end releases may be considered in the incremental form of the latter equations.

Later, the end force-end displacement relations in incremental form will be needed. To derive these, it is assu

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is assumed that the stiffness matrices S_{PP}, S_{PN}, and S_{NN} are constant within the increments (of forces and displacements). This assumption is a reasonable one since the elements of S_{PP}, S_{PN}, and S_{NN} vary slowly with ϕ_i ($i = y,z$) when ϕ_i is not close to 2π. ($\phi_i = 2\pi$ corresponds to N = the Euler load.) The incremental relations are thus expressed as

$$\begin{aligned} \Delta F_P &= S_{PP}\ \Delta U_P + S_{PN}\ \Delta U_N - \Delta F_G \\ \Delta F_N &= S_{PN}^t\ \Delta U_P + S_{NN}\ \Delta U_N + \Delta F_G \end{aligned} \qquad (2\text{-}15)$$

where the increment ΔF_G can be expanded as

$$\Delta F_G = S_G\ (\Delta U_P - \Delta U_N) \qquad (2\text{-}16)$$

The matrix S_G may be interpreted as the "geometric" stiffness matrix. The approximate form of this matrix used in the present analysis is developed in Appendix I.

Introducing Equation (2-16) into Equations (2-15), the approximate incremental relations are finally written as

$$\begin{aligned} \Delta F_P &= K_{PP}\ \Delta U_P + K_{PN}\ \Delta U_N \\ \Delta F_N &= K_{PN}^t\ \Delta U_P + K_{NN}\ \Delta U_N \end{aligned} \qquad (2\text{-}17)$$

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$$K_{PP} = S_{PP} + S_G$$

$$K_{PN} = S_{PN} - S_G \tag{2-18}$$

$$K_{NN} = S_{NN} + S_G$$

It follows from the symmetry of the stiffness matrices S_{PP}, S_{NN}, and S_G that the generalized stiffness matrices K_{PP} and K_{NN} are also symmetric.

If the member happens to have end releases, its incremental force-displacement relations must be modified accordingly to account for such releases. This modification can be carried out easily in a fashion quite similar to that presented in Reference 9. However, for the sake of completeness, an outline of the latter modification is given in Appendix II.

2.2.2 Elastoplastic Member

The assumption that the member is elastic is now dropped. It is instead assumed that both end cross-sections of the member are yielding. The end displacement increment ΔU can be decomposed into an elastic part and a plastic part. These two parts will be denoted by the superscripts e and p, respectively. Thus

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$$\Delta U_P = \Delta U_P^e + \Delta U_P^p$$
$$\Delta U_N = \Delta U_N^e + \Delta U_N^p \qquad (2\text{-}19)$$

The above decomposition is a basic assumption of the theory of plasticity.

Based on the plastic potential theory, the flow law adapted to the present problem is expressed by

$$\Delta U_P^p = b_P \; \nabla\Phi \; (F_P) = b_P V_P$$
$$\Delta U_N^p = b_N \; \nabla\Phi \; (F_N) = b_N V_N \qquad (2\text{-}20)$$

in which b is the flow constant (a positive scalar parameter), ∇ is the so-called "del" (gradient) operator, Φ is the yield function, and V is the outward normal to the yield surface at the point where the end force F meets the yield surface. A geometrical interpretation of the flow law is illustrated in Figure 2-3 for a simpler case in which the yielding condition depends on only two stress resultants. It is seen from this figure that the plastic end displacement increment ΔU^p is normal to the yield surface. Thus, the direction of the increment ΔU^p is determined by the plastic potential theory. However, the magnitude of that increment still remains to be determined.

During yielding, the end force vector F must stay on the yield surface represented by the equation $\Phi(F) = 1$. (The latter equation remains fixed since the effects of work ha

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work hardening are neglected.) This is specified analytically by

$$\Delta\Phi\ (F_P) = 0$$
$$\Delta\Phi\ (F_N) = 0 \tag{2-21}$$

These equations can be rewritten as

$$[\nabla\Phi(F_P)]^t\ \Delta F_P = 0$$
$$[\nabla\Phi(F_N)]^t\ \Delta F_N = 0 \tag{2-22}$$

or,

$$V_P^t\ \ \Delta F_P = 0$$
$$V_N^t\ \ \Delta F_N = 0 \tag{2-23}$$

Equations (2-23) will be used shortly to determine the magnitude of the incremental plastic end displacement ΔU^p. These equations state that the end force increment ΔF is tangent to the yield surface.

Since the material is assumed to be elastic--perfectly plastic, the end force increment ΔF is governed by the elastic end displacement increment ΔU^e only. Therefore, Equations (2-17) can be written as

$$\Delta F_P = K_{PP}\ \Delta U_P^e + K_{PN}\ \Delta U_N^e$$
$$\Delta F_N = K_{PN}^t\ \Delta U_P^e + K_{NN}\ \Delta U_N^e \tag{2-24}$$

Considering Equations (2-19) and (2-20), the above equations can be rewritten as

$$\Delta F_P = K_{PP}\ \Delta U_P + K_{PN}\ \Delta U_N - K_{PP}\ V_P\ b_P - K_{PN}\ V_N\ b_N$$

$$\Delta F_N = K_{PN}^t\ \Delta U_P + K_{NN}\ \Delta U_N - K_{PN}^t\ V_P\ b_P - K_{NN}\ V_N\ b_N \qquad (2\text{-}25)$$

At this point, it is convenient to define two 6x2 matrices G_P and G_N by

$$G_P = [K_{PP}\ V_P \mid K_{PN}\ V_N]$$

$$G_N = [K_{PN}^t\ V_P \mid K_{NN}\ V_N] \qquad (2\text{-}26)$$

If now the flow constants b_P and b_N are assembled into a column matrix B, Equations (2-25) can be expressed as

$$\Delta F_P = K_{PP}\ \Delta U_P + K_{PN}\ \Delta U_N - G_P\ B$$

$$\Delta F_N = K_{PN}\ \Delta U_P + K_{NN}\ \Delta U_N - G_N\ B \qquad (2\text{-}27)$$

Substitution of these equations into Equations (2-23) furnishes

$$V_P^t\ K_{PP}\ \Delta U_P + V_P^t\ K_{PN}\ \Delta U_N - V_P^t\ G_P\ B = 0$$

$$V_N^t\ K_{PN}^t\ \Delta U_P + V_N^t\ K_{NN}\ \Delta U_N - V_N^t\ G_N\ B = 0 \qquad (2\text{-}28)$$

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These equations are now combined into the following single equation:

$$\begin{bmatrix} V_P^t\ K_{PP} \\ \\ V_N^t\ K_{PN}^t \end{bmatrix} \Delta U_P + \begin{bmatrix} V_P^t\ K_{PN} \\ \\ V_N^t\ K_{NN} \end{bmatrix} \Delta U_N - \begin{bmatrix} V_P^t\ G_P \\ \\ V_N^t\ G_N \end{bmatrix} B = 0 \qquad (2\text{-}29)$$

Solution of Equation (2-29) for the (flow constant) matrix B yields

$$B = E\ G_P^t\ \Delta U_P + E\ G_N^t\ \Delta U_N \qquad (2\text{-}30)$$

in which

$$E = \begin{bmatrix} V_P^t\ G_P \\ \\ V_N^t\ G_N \end{bmatrix}^{-1} \qquad (2\text{-}31)$$

Equation (2-30) is finally substituted back into Equations (2-27) whereby the incremental force-displacement relations become

$$\begin{aligned} \Delta F_P &= (K_{PP} - G_P\ E\ G_P^t)\ \Delta U_P \\ &\quad + (K_{PN} - G_P\ E\ G_N^t)\ \Delta U_N \\ \Delta F_N &= (K_{PN} - G_P\ E\ G_N^t)^t \Delta U_P \\ &\quad + (K_{NN} - G_N\ E\ G_N^t)\ \Delta U_N \end{aligned} \qquad (2\text{-}32)$$

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The reduction to the case where only one end cross-section of the member is yielding is quite straightforward and thus omitted.

2.2.3 Elastic Return

When a cross-section reaches its yielding condition, a "plastic hinge" is said to form there (following the terminology in the simple plastic theory of structures). It frequently happens that a plastic hinge "unloads" and the corresponding cross-section becomes elastic again. This phenomenon is referred to as an "elastic return." It occurs whenever there is a reversal in the direction of the incremental plastic displacement at a yield hinge. It thus follows from Equations (2-20) that the elastic return at a plastic hinge is signalled by a negative flow constant.

To test for elastic returns, the column matrix B for each elastoplastic member is first calculated. The sign of each of the elements (one or two) of the matrix B is tested. Any yield hinge at which an elastic return is detected is then removed; in other words, the corresponding member end cross-section is considered to be elastic.

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2.3 Incremental Force-Distortion Relations for a Discrete Model of a Geometrically Linear Member

It is evident that for a "geometrically linear" member the force-deformation relations can be obtained directly from the previous results. These relations are, however, rederived in the present section for a geometrically linear discrete model taken from Reference 13. To this end, the derivation will have to be made in terms of distortions rather than displacements.

Figure 2-4 shows a model PN which is to replace a typical geometrically linear member PN whose P_1 and N_1 sections are yielding. It is assumed that each of the segments PP_1, P_1P_2, NN_1, and N_1N_2 (of the member) are infinitely rigid and of length βL where β is a nondimensional parameter to be chosen between zero and some fraction, say 1/8. Furthermore, it is assumed that the flexibility of the portion PP_2 is lumped at its midsection P_1. The same assumption is also made for the portion NN_2. The remainder of the model, namely, the portion P_2N_2 is assumed to be continuously elastic.

The distortion of the model referred to the end P is defined to be the displacement of the end P relative to the end N. In incremental form, it is given by

$$\Delta W_{PN} = \Delta U_P - D_{PN}\,\Delta U_N \tag{2-33}$$

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in which D_{PN} denotes the displacement transformation matrix from the local coordinate system through N to the same system through P.

Analogous to Equations (2-19), the incremental distortion ΔW_{PN} can be decomposed into

$$\Delta W_{PN} = \Delta W^{e}_{PN} + \Delta W^{p}_{PN} \tag{2-34}$$

The plastic distortion increment ΔW_{PN} can be expressed as

$$\Delta W^{p}_{PN} = D_{PP_1} V_{P_1} b_{P_1} + D_{PN_1} V_{N_1} b_{N_1} \tag{2-35}$$

If the flow constants b_{P_1} and b_{N_1} are assembled into a column matrix B, Equations (2-35) can be written as

$$\Delta W^{p}_{PN} = GB \tag{2-36}$$

in which

$$G = [D_{PP_1} V_{P_1} \,\vdots\, D_{PN_1} V_{N_1}] \tag{2-37}$$

The elastic distortion increment ΔW^{e}_{PN} can be given as a summation of

$$\Delta W^{e}_{PN} = D_{PP_1} \Delta W^{e}_{P_1P_1} + D_{PP_2} \Delta W^{e}_{P_2N_2} + D_{PN_1} \Delta W^{e}_{N_1N_1} \tag{2-38}$$

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The flexibility of the portion PP_2 is lumped at its mid-section P_1; therefore, the elastic distortion increment $\Delta W^e_{P_1P_1}$ is determined by

$$\Delta W^e_{P_1P_1} = A_{P_1P_1}\ \Delta F_{P_1} = A_{P_1P_1}\ T_{P_1P}\ \Delta F_P \tag{2-39}$$

in which $A_{P_1P_1}$ is the lumped flexibility matrix for the portion PP_2, and T_{P_1P} is the force transformation matrix from the local coordinate system through P to the same system through P_1. The main diagonal elements of the matrix $A_{P_1P_1}$ (which is diagonal) are given by

$$\begin{aligned} C_1 &= \frac{2\beta L}{EA_x} & C_2 &= \frac{2\beta L}{GA_y} \\ C_3 &= \frac{2\beta L}{GA_z} & C_4 &= \frac{2\beta L}{GJ} \\ C_5 &= \frac{2\beta L}{EI_y} & C_6 &= \frac{2\beta L}{EI_z} \end{aligned} \tag{2-40}$$

in which G is the modulus of shear, A_y and A_z are the effective areas of shear, and J is the constant of torsion. The increment $\Delta W^e_{N_1N_1}$ is similarly determined by

$$\Delta W^e_{N_1N_1} = A_{N_1N_1}\ \Delta F_{N_1} = A_{N_1N_1}\ T_{N_1P}\ \Delta F_P \tag{2-41}$$

where

$$A_{N_1N_1} = A_{P_1P_1} \tag{2-42}$$

Since the segment P_2N_2 is elastic, the superscript e can be omitted from the increment $\Delta W^e_{P_2N_2}$. This increment is readily given by

$$\begin{aligned}\Delta W^e_{P_2N_2} &= \Delta W_{P_2N_2} \\ &= A_{P_2P_2} \ \Delta F_{P_2} \\ &= A_{P_2P_2} \ T_{P_2P} \ \Delta F_P \qquad (2\text{-}43)\end{aligned}$$

where $A_{P_2P_2}$ is the ordinary flexibility matrix for the continuously elastic segment P_2N_2 referred to the section P_2. Note that the effects of shearing deformations may be included in the matrix $A_{P_2P_2}$.

Equations (2-39), (2-41), and (2-43) are substituted back into Equation (2-38). This leads to

$$\Delta W^e_{PN} = A_{PP} \ \Delta F_P \qquad (2\text{-}44)$$

or,

$$\Delta F_P = A^{-1}_{PP} \ \Delta W^e_{PN} \qquad (2\text{-}45)$$

in which

$$\begin{aligned}A_{PP} = {} & D_{PP_1} \ A_{P_1P_1} \ T_{P_1P} + D_{PP_2} \ A_{P_2P_2} \ T_{P_2P} \\ & + D_{PN_1} \ A_{N_1N_1} \ T_{N_1P} \qquad (2\text{-}46)\end{aligned}$$

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Considering Equations (2-34) and (2-36), Equation (2-45) is cast into

$$\Delta F_P = A_{PP}^{-1} (\Delta W_{PN} - GB) \tag{2-47}$$

Analogous to Equations (2-23), the condition that the force increments ΔF_{P_1} and ΔF_{N_1} are tangent to the yield surface is expressed by

$$\begin{aligned} V_{P_1}^t \; T_{P_1 P} \; A_{PP}^{-1} (\Delta W_{PN} - GB) &= 0 \\ V_{N_1}^t \; T_{N_1 P} \; A_{PP}^{-1} (\Delta W_{PN} - GB) &= 0 \end{aligned} \tag{2-48}$$

These two equations can be combined into

$$G^t \, A_{PP}^{-1} \; (\Delta W_{PN} - GB) = 0 \tag{2-49}$$

from which

$$B = EG^t \, A_{PP}^{-1} \, \Delta W_{PN} \tag{2-50}$$

where

$$E = (G^t \, A_{PP}^{-1} \, G)^{-1} \tag{2-51}$$

Equation (2-50) is substituted back into Equation (2-47) to furnish

$$\Delta F_P = (A_{PP}^{-1} - A_{PP}^{-1} \, G \, E \, G^t \, A_{PP}^{-1}) \, \Delta W_{PN} \tag{2-52a}$$

The incremental force-distortion relations are finally completed by having

$$\Delta F_N = -T_{NP} \, \Delta F_P \, . \tag{2-52b}$$

CHAPTER III

DYNAMIC ANALYSIS AND SOLUTION

In this chapter, the joint mass matrix is first constructed. The equation of motion for a given joint is derived next. Finally, the numerical procedure employed to determine the dynamic response is presented.

3.1 Dynamic Analysis

The dynamic analysis begins with the formulation of the equation of motion for a typical free (ungrounded) joint in the global coordinate system. This requires that a mass matrix be constructed for the joint. Such a construction may be carried out by lumping, at the joint, the "contributory" masses from all the incident members. The mass lumping could be done in a reasonable manner as long as there is no coupling between the various joint mass matrices. However, the more realistic the lumping procedure is, the more accurate the numerical results will be.

In this study, the joint mass matrix is constructed in a manner described by Iverson (4). The rotary inertia is taken into account in this construction. A nondimensional parameter, as introduced in Reference 13 to control the moments and products of inertia, is also included.

3.1.1 Joint Mass Matrix

To formulate the joint mass matrix, it is convenient to envisage a rigid body associated with the joint. To this end, each member with its length multiplied by α will be called a "branch," where α is a dimensionless parameter, to be chosen between zero and 1/2. The joint rigid body is then defined to be the collection of all the branches incident to the joint being considered. To keep the total mass of the rigid body independent of α, the density ρ of the material is divided by α. Furthermore, it is recalled that each member may have a uniformly distributed gravity load with a mass μ per unit length. Then each μ is also divided by α. It follows that α is introduced to modify the moments and products of inertia of the joint rigid body while leaving the rigid body mass the same.

The mass m_B of a typical branch B is presented readily by

$$m_B = 0.5\ L\ (A_x\ \rho + \mu) \tag{3-1}$$

The mass m_J of a generic joint J is obtained by summing the masses of all the incident branches. Thus

$$m_J = \Sigma\ m_B \tag{3-2}$$

It is seen that α does not appear in Equations (3-1) and (3-2).

Referring to Figure 3-1, the position vector $(X_j, Y_j, Z_j) = \overline{W}_j$ of the joint mass center j in the joint J coordinate system is determined by

$$X_j = \frac{\Sigma m_B X_b}{m_J}$$
$$Y_j = \frac{\Sigma m_B Y_b}{m_J} \tag{3-3}$$
$$Z_j = \frac{\Sigma m_B Z_b}{m_J}$$

in which $(X_b, Y_b, Z_b) = \overline{W}_b$ is the position vector of the branch mass center b in the same coordinate system, and each summation is to be taken for all the branches incident to the joint J. The vector $\overline{W}_b$ is given by

$$\overline{W}_b = \pm\ 0.25\ \alpha\ (\overline{P} - \overline{N}) \tag{3-4}$$

in which $\overline{P}$ and $\overline{N}$ are, respectively, the position vectors of the positive and negative ends of the member B in the global coordinate system. The positive or negative sign in Equation (3-4) is to be chosen according as the member B is positively or negatively incident to the joint J.

The branch inertia tensor I_b with respect to the local coordinate axes through b is determined next. Note that these axes are coincident with the principal axes of inertia for the branch B. Thus

$$I_b = \begin{bmatrix} C_1 & 0 & 0 \\ 0 & C_2 & 0 \\ 0 & 0 & C_3 \end{bmatrix} \tag{3-5}$$

where

$$C_1 = \iiint \rho \, (y^2 + z^2) \, dx \, dy \, dz$$

$$= \frac{\rho L}{2} (I_y + I_z) \qquad (3\text{-}6a)$$

$$C_2 = \iiint \rho \, (z^2 + \alpha^2 \, x^2) \, dx \, dy \, dz$$

$$+ \int \mu \, (\alpha^2 \, x^2) \, dx$$

$$= \frac{\alpha^2 L^3}{96} (\rho \, A_x + \mu) + \frac{\rho L}{2} \, I_z \qquad (3\text{-}6b)$$

$$C_3 = \iiint \rho \, (y^2 + \alpha^2 \, x^2) \, dx \, dy \, dz$$

$$+ \int \mu (\alpha^2 \, x^2) \, dx$$

$$= \frac{\alpha^2 L^3}{96} (\rho \, A_x + \mu) + \frac{\rho L}{2} \, I_y \qquad (3\text{-}6c)$$

The tensor I_b is now transferred to the joint mass center j by using the theorem of parallel axes. This leads to

$$I_j = I_b + m_B \begin{bmatrix} y_j^2 + z_j^2 & -x_j \, y_j & -x_j \, z_j \\ & z_j^2 + x_j^2 & -y_j \, z_j \\ \text{symmetric} & & x_j^2 + y_j^2 \end{bmatrix} \qquad (3\text{-}7)$$

where $(x_j, y_j, z_j) = \overline{w}_j$ is the position vector of the joint mass center j in the local coordinate system through b.

This vector is determined by

$$\bar{w}_j = r^t (\bar{W}_j - \bar{W}_b) \tag{3-8}$$

in which r is the rotation matrix (3x3) from the joint J coordinate system to the member B coordinate system. The definition of the matrix r is given by Equation (1-5).

Each branch inertia tensor I_j at the joint mass center j is now rotated to the global coordinate system through j. They are then summed to obtain the joint inertia tensor J_j at the joint mass center j. Thus

$$J_j = \Sigma \; r^t \; I_j \; r \tag{3-9}$$

Finally, the mass matrix for the joint J with respect to the global coordinate system through j is assembled as

$$M_j = \begin{bmatrix} m_J \; I & 0 \\ 0 & J_j \end{bmatrix} \tag{3-10}$$

where I is the third order identity matrix.

3.1.2 Equation of Motion for a Joint Rigid Body

Considering again the joint J and its mass center j in Figure 3-1, the equation of motion for the corresponding rigid body in the global coordinate system through j is expressed as

$$M_j \; \ddot{X}_j = P_j \tag{3-11}$$

in which $\ddot{X}_j$ is the acceleration (vector) of j and P_j is the resultant of all non-inertial forces acting on j. This equation can be rewritten as

$$M_j \, (T_{jJ}^{-1})^t \, \ddot{X}_J = T_{jJ} \, P_J \tag{3-12}$$

in which T_{jJ} is the force transformation matrix from the global coordinate system through J to the same coordinate system through j. Premultiplying Equation (3-12) by T_{jJ}^{-1}, it follows that

$$M_J \, \ddot{X}_J = P_J \tag{3-13}$$

in which

$$M_J = T_{jJ}^{-1} \, M_j \, (T_{jJ}^{-1})^t \tag{3-14}$$

The inverse of M_J may be referred to as the "responsiveness" matrix. This inverse is denoted by Z_J and given by

$$Z_J = M_J^{-1} = \begin{bmatrix} I/m_J - HJ_j^{-1}H & -HJ_j^{-1} \\ J_j^{-1}H & J_j^{-1} \end{bmatrix} \tag{3-15}$$

where H, as defined by Equation (1-2), contains the coordinates (X_j, Y_j, Z_j) of the joint mass center j in the global coordinate system through J (see Figure 3-1).

The force P_J acting on the joint J, in the global coordinate system through J, may be written as

$$P_J = P_I + P_E \tag{3-16}$$

where P_I is the internal force (vector) acting on J, and P_E is the external load (vector). The force P_I, in terms of the member end forces in their corresponding member coordinate systems, is given by

$$P_I = - \sum^{(P)} R\, F_P - \sum^{(N)} R\, F_N \tag{3-17}$$

in which the first summation is to be taken for all the members positively incident to the joint J, the second summation is to be taken for all the members with negative incidence, and R is the rotation matrix (6x6) from the appropriate member coordinate system to the joint J coordinate system.

Summarizing, the acceleration of the joint J is given by

$$\ddot{X}_J = Z_J\ (P_I + P_E) \tag{3-18}$$

in which all the quantities are in the global coordinate system through J. The responsiveness matrix Z_J and the force P_I are determined by Equations (3-15) and (3-17), respectively. The external load P_E is to be prescribed.

If the foundation is subjected to translational motion (e.g., due to earthquakes or blasts), it can readily be shown (see Reference 13) that the equation of motion becomes

$$\ddot{X}_J = Z_J\ (P_I + P_E) - \ddot{X}_0 \tag{3-19}$$

where $\ddot{X}_J$ is now the acceleration of the joint J relative to the foundation and $\ddot{X}_0$ is the acceleration of the foundation.

3.2 Dynamic Solution

The numerical method used to obtain the dynamic solution will be outlined in this section. First, the criterion by which a member cross-section is ruled to be yielding is described. It will be shown next that the force vector acting on a yield hinge is prevented from proceeding beyond the yield surface. Finally the steps of the solution procedure are described and the choice of time increment is discussed.

3.2.1 Insertion of a Plastic Hinge at a Cross-Section

It is presumed that a given cross-section is initially elastic. This presumption is expressed analytically by

$$\Phi(F) < 1 \tag{3-20}$$

in which Φ is the yield function and F is the force acting on the cross-section. As the frame is deformed, the force F may become so large that although Inequality (3-20) holds, yet

$$\Phi(F + \Delta F) > 1 \tag{3-21}$$

where ΔF is the force increment (say, corresponding to a small time increment) computed under the assumption that the cross-section is elastic.

If the above conditions hold, the cross-section is ruled to be yielding. A yield hinge is inserted at the cross-section and the force increment ΔF is recomputed accordingly, i.e., with the newly inserted plastic hinge taken into consideration.

3.2.2 Force Containment at a Plastic Hinge

Let it be assumed that for some cross-section, a situation as represented by Inequalities (3-20) and (3-21) prevails, i.e., the cross-section is yielding. With reference to Figure 3-2, the unit normal $V^{(1)}$ is determined by

$$V^{(1)} = \frac{\nabla\Phi(F)}{|\nabla\Phi(F)|} \tag{3-22}$$

where ∇ is the gradient operator. This unit normal is used to compute the force increment ΔF. If the force $F + \Delta F$ does not reach the exterior of the yield surface, i.e., $\Phi(F + \Delta F) \leq 1$, then the force increment ΔF is ruled to be acceptable and no further action is taken. On the other hand, if $\Phi(F + \Delta F) > 1$, then "force containment" becomes necessary since it is physically impossible for the force $F + \Delta F$ to proceed beyond the yield surface.

To this end, the unit normal $V^{(2)}$ is determined by

$$V^{(2)} = \frac{\nabla\Phi(F + \Delta F)}{|\nabla\Phi(F + \Delta F)|} \qquad (13\text{-}23)$$

The force increment ΔF is then recomputed by using an "average" normal V as given by

$$V = (1 - \gamma)\ V^{(1)} + \gamma\ V^{(2)} \qquad (3\text{-}24)$$

where γ is a scalar parameter to be chosen between zero and unity. The choice of a value for γ is discussed next.

Starting from zero, γ is advanced by a finite value up to the point where $\Phi(F + \Delta F)$ is no longer greatern than one. Limited numerical experience on the problems presented in the next chapter indicated that numerical results were quite insensitive to the value of γ. Thus, γ was taken to be either zero or one; in other words, if the unit normal $V^{(1)}$ was not acceptable, the unit normal $V^{(2)}$ was.

3.2.3 Numerical Procedure of Solution

Assume that the state of the frame is known at a given time t_1. This includes: the joint displacements, velocities, and accelerations; the member end forces; and the list of the existing plastic hinges. It is then desired to determine the state of the frame at the time $t_2 = t_1 + \Delta t$ where Δt is a finite time increment. To this end, the following procedure is employed:

(1) Compute the incremental displacement of each free joint from some numerical integration formula such as

$$\Delta X = [\dot{X}(t_1) + 0.5\ \ddot{X}(t_1)\ \Delta t]\ \Delta t \qquad (3\text{-}25)$$

(2) Compute the incremental displacement of each member end by rotating the proper joint displacement increment from the global coordinate system to the appropriate local coordinate system.

(3) If the solution is to be geometrically linear, compute the incremental distortion of each member from Equation (2-33).

(4) If the solution is to be geometrically nonlinear, compute the stiffness matrices for each member (elastic or elastoplastic) from Equations (A-1-8), (A-1-9), and (A-1-10). In this computation, the unknown axial force of a member at the time t_2 is approximated by the known axial force of that member at the time t_1. It will be shown, in the first example of the next chapter, that elimination of such an approximation would not affect the numerical results significantly.

(5) Compute the incremental member end forces from Equations (2-52) or (2-32) depending on whether the solution is to be geometrically linear or nonlinear. Then, increment the member end forces by the values just obtained.

(6) Compute the yield-function value for each member end force if the corresponding cross-section is elastic (not yielding), by using such equations as Equations (2-2) or (2-3). If this value happens to be greater than unity, insert a plastic hinge at the cross-cross being considered and return to Step (5). (See Subsection 3.2.1.)

(7) Remove every plastic hinge at which an elastic return is detected. At this point, it seems logical to return to Step (5) in order to recompute the member end forces if necessary. But, to avoid the possibility of removing and inserting a particular plastic hinge repeatedly within a single time-step, the return to Step (5) is neglected.

(8) Compute the internal force acting on each free joint from Equations (3-17).

(9) Compute the acceleration of each free joint from Equations (3-18) or (3-19) depending on the absense or presence of the foundation motion.

(10) Use such a numerical integration formula as

$$\dot{X}(t_2) = \dot{X}(t_1) + 0.5\ [\ddot{X}(t_1) + \ddot{X}(t_2)]\ \Delta t \qquad (3\text{-}26)$$

to determine the velocity of each free joint.

(11) Increment the free-joint displacements by the values obtained in Step (1).

The state of the frame is thus completely determined at the time t_2. The same procedure is repeated for advancing from the time t_2 to the time $t_2 + \Delta t$, and so on.

3.2.4 Choice of Time Increment

It is well known that a numerical procedure, as applied to a linearly elastic structure, is stable if the time increment used is smaller than a certain fraction (say, $1/\pi$) of the smallest period of natural vibration. This remark has to be ignored in the following discussion since the system being considered is not linearly elastic. However, knowledge of the smallest period is useful to the extent that it serves as a guide in choosing the time increment. In general, the problem of determining the smallest period is quite time-consuming. This problem becomes more complicated by the fact that the smallest period will change whenever a plastic hinge is inserted or removed. It would thus appear more appropriate that the smallest period be roughly estimated rather than rigorously computed. For this purpose, the following procedure suggested by Iverson (4) is employed.

The total mass of each free joint and the largest axial stiffness of all the members incident to this joint are used to compute

$$T = 2\pi \sqrt{\frac{\text{free-joint mass}}{\text{axial stiffness}}}$$

The smallest value of T for all free joints is taken as an estimate for the smallest period. This estimate is then used as an initial try for the time increment. In subsequent tries, increases or decreases are made if necessary. In general, the largest tolerable time increment (that yields a stable solution) should be used. Such a time increment is normally so small that the corresponding numerical results would not be significantly different from those using smaller time increments.

3.3 Computer Program

A general computer program is prepared to implement the formulation presented. The program is written in FORTRAN IV for the use on the CDC 3600 digital computer at Michigan State University. It is described and also listed in Appendix III.

CHAPTER IV

APPLICATIONS

This chapter presents three numerical examples taken from Reference 13. These examples, on one hand, illustrate the applications of the analysis developed in the preceding chapters. On the other hand, they provide a basis for comparing the present numerical results with those reported in the above-mentioned reference. The first example is a cantilever beam subjected to a pulse type of loading with a short duration. The second one is a three-dimensional rigid frame with a triangular plan-form subjected to a step-function type of loading. The final example is a two-story two-bay building frame subjected to the 1940 El Centro earthquake.

In the examples presented, forces are expressed in kips, and moments in kip-inches. The material is assumed to be structural steel having the following properties: density = 490 pounds per cubic inch, Young's modulus = 30,000 ksi, shear modulus = 12,000 ksi, normal yield stress = 33 ksi, and shear yield stress = 18 ksi. Linear displacements are expressed in inches, and rotational displacements in radians.

4.1 Cantilever Beam

Figure 4-1 shows a cantilever beam which has a uniform cross-section of 12WF53. The web of the beam is vertical (in the X-Y plane). The fully plastic stress resultants are 504, 275, 275 kips, and 73, 997, 2706 kip-inches, respectively. Each of these values represents the corresponding carrying capacity of the beam cross-section if the cross-section is subjected to that one stress resultant only. The external loads, in addition to the weight of the beam, are given in the figure. Note that the dynamic disturbance is supplied by a pulse type of loading with a duration of 0.01 seconds. The graphs to be presented in this example correspond to the elliptic yield function, a value of $\alpha = 0.5$, and a time increment of 0.0005 seconds.

To compare the results in Reference 13 with the present ones, the tip displacement of the beam in the Z direction is plotted versus time in Figure 4-2. Either graph in this figure corresponds to a value of $\beta = 0.0625$. It is clearly seen that a good agreement exists between the graph in the above-mentioned reference and the one furnished by the present formulation (geometrically linear).

It is now of interest to study the axial force effects on the response of the beam in both geometrically linear and nonlinear cases. To this end, the maximum absolute value of the tip displacements in the X and Z directions are plotted against the axial load (static, compressive) in Figure 4-3. Both graphs furnished by the geometrically linear formulation correspond to a value of $\beta = 0$. These two graphs are nearly straight while the graphs provided by the geometrically nonlinear formulation are highly nonlinear, particularly, where the axial load approaches 200 kips. If the axial load exceeds 200 kips (say, by 50 kips or more) the numerical results corresponding to the latter formulation indicate that the beam would collapse. It is to be noted that the axial load of 200 kips is equal to 40% of the axial carrying capacity (504 kips) of the beam cross-section and 26% of the Euler load (771 kips, for the cantilever case, of course) of the beam.

The numerical results based on the geometrically linear formulation (not presented here) indicated that the beam would collapse with an axial load between 400 and 500 kips. These loads are considerably larger than the 200 kip load indicated by the geometrically nonlinear formulation. Thus, it may be concluded that when members carry substantial axial loads, the geometrically nonlinear formulation, which is certainly more accurate, should be used.

To study the time-displacement response of the beam, the tip displacement in the axial (X) direction is plotted versus time in Figure 4-4. Both graphs plotted correspond to an axial load of -30 kips as shown in Figure 4-1. In addition, the graph provided by the geometrically linear formulation corresponds to a value of $\beta = 0$. This graph exhibits a permanent set of -0.002 inches which is entirely due to yielding. It also shows that there is no apparent physical vibration in the axial direction. This can be explained by noting that the axial shortening effects do not enter the geometrically linear formulation. (The high-frequency oscillations appearing in the above-mentioned graph are probably due to imperfections of the numerical integration technique employed in the solution procedure.) The graph furnished by the geometrically nonlinear formulation shows a much larger permanent set of -0.010 inches. This is partly due to the force interaction effects (-0.002 inches) and partly due to the axial shortening effects considered in the geometrically nonlinear formulation.

The period of axial vibration predicted by the geometrically nonlinear formulation is roughly 0.078 seconds. This period can be measured directly from the relevant graph presented in Figure 4-4. It is of some interest to compare this period with the one approximated by $2\pi\sqrt{m_B L/EA_x}$ where $2m_B$ is the mass of the beam. According to the data chosen for the beam, the latter period is calculated to be 0.087 seconds. This period agrees fairly well with the one mentioned earlier.

This comparison would thus mean that the beam appears to respond in the axial direction like a single-degree-of-freedom system if the geometrically nonlinear formulation is used.

It is recalled that in obtaining geometrically nonlinear solutions, the axial force in a given time-step, is approximated by the one obtained at the end of the previous time-step. The question may be raised that how much this approximation affects the accuracy of the resulting solutions.

To consider such effects, a geometrically nonlinear solution of the beam was obtained without using the above-mentioned approximation. This was accomplished by performing iteration on the axial force within each time-step. An axial load of 200 kips in compression was applied as a part of the static loading in order to create a more critical situation. The numerical results so obtained (not shown here) did not indicate any noticeable difference from those obtained with the approximation. For example, the resulting displacements differed by less than 3% in the axial direction and 2% in the transverse directions. It is thus concluded that the performance of iteration on the axial force would not significantly affect the numerical results. The advantage of using the approximation, of course, lies in a significant saving of computer time.

4.2 Triangular Frame

The frame to be considered as a second example is the one shown in Figure 4-5. The horizontal members (girders) and the vertical members (columns) are fabricated from 12WF53 and 12WF40 sections, respectively. The webs of the girders are vertical. The webs of the columns lie in the vertical planes containing the bisectors of the triangular planform. The external loads, in addition to the weight of the girders, are shown in the figure. A step-function type of dynamic loading is applied. The value of α employed is the same as the one used in the preceding example, that is, 0.5.

For the purpose of comparing the results in Reference 13 with the present ones, the displacement in the Z direction of the joint 1 is plotted versus time in Figures 4-6 and 4-7. These two figures correspond to the elliptic and parabolic yield functions, respectively. The value of β used in both cases is 0.125.

From Figure 4-6, it is seen that for the elliptic yield function, a reasonably good agreement exists between the two graphs. For the parabolic yield function, the two graphs shown in Figure 4-7 are also quite similar. As far as the displacement magnitudes are concerned, they differ by approximately 7%.

In order to compare the geometrically linear and nonlinear responses, the displacement of the joint 1 in the Z direction is plotted against time in Figure 4-8. The graphs presented in this figure correspond to the elliptic yield function. In addition, the graph furnished by the geometrically linear formulation corresponds to a value of $\beta = 0$ and a time increment of 0.001 seconds. For the time increment just mentioned, the geometrically nonlinear response turned out to be unstable. The largest time increment for which the geometrically nonlinear response became stable was 0.00005 seconds, i.e., 1/20 times 0.001 seconds.

It is seen that, the geometrically nonlinear solution indicated a larger response than that given by the geometrically linear solution. It should be remembered, however, that the axial loads in this case are rather moderate. As seen from the preceding example on the cantilever beam, if the axial loads were sufficiently large, the difference between the two solutions would be much more drastic.

4.3 Building Frame

As a final example, a two-story two-bay building frame, as shown in Figure 4-9, is considered. The member 9 (girder) of the frame has a 12WF53 cross-section. The other girders and the columns are made from 8WF40 and 8WF17 sections, respectively. The webs of the columns are parallel to the south-north direction while the webs of the girders are vertical.

The static loading, in addition to the weight of the girders, consists of a load of 1.50 kips per foot on the member 9 and a load of 0.75 kips per foot on the rest of the girders. The mass of these loads is lumped at the relevant joints in the same manner as the mass of the members themselves. The dynamic loading is supplied by subjecting the foundation of the frame to all three components of the 1940 El Centro earthquake. The sketches to be given in this example correspond to the elliptic yield function, a time increment of 0.0008 seconds, and a value of $\alpha = 0.5$.

Figure 4-10 shows a plan view of the distorted shape of the frame as predicted by the geometrically linear formulation. The solution corresponds to a value of $\beta = 0$. In Figure 4-11 are shown similar results obtained from a solution with the geometrically nonlinear formulation. The distortions in both figures are based on the member plastic displacements recorded at the end of two seconds of the ground motion. It is seen that the two distortions, suffered by the frame and computed by the two formulations, are comparable as far as the general appearance is concerned. However, as expected, the distortion magnitudes given by the geometrically nonlinear formulation are greater than those of the linear version.

4.4 Computation Time

In concluding the present chapter, it is instructive to consider the computation time for the method given in Reference 13 and that developed in the present work. To this end, for the three examples presented, the time increment, execution time, and real time interval of the several solutions are listed in Table 4-1. (In all cases, the computer execution time refers to the CDC 3600 digital computer at Michigan State University.) It is seen from this table that the execution time for the method presented in Reference 13 depends on the value of β, whereas for the present method (geometrically linear), they do not. It is also seen that the present method requires considerably less computation time. This is perhaps the most significant improvement of the present work over that reported in Reference 13.

CHAPTER V

SUMMARY AND CONCLUSIONS

In this study, a matrix formulation has been presented for the dynamic analysis of space frames. In the analysis, the effects of both geometric and material nonlinearities have been taken into account. A computer program has been prepared for the implementation of the analysis. Numerical results of three problems were obtained in order to demonstrate the validity and practicality of the formulation. These problems were: a cantilever beam subjected to a pulse type of loading; a six-member space frame subjected to a step-function type of loading; and a two-bay two-story building frame subjected to the 1940 El Centro earthquake.

Comparative data with and without the effects of geometric nonlinearities taken into consideration were shown in the form of graphs. Based on these graphs the following observations were found to be noteworthy:

(1) The geometrically nonlinear formulation developed in this work is obviously applicable to geometrically linear problems. Good agreements were found to exist between the numerical results obtained in

such applications and those reported in Reference 13 in which a different method was used. This may be construed as an evidence for the validity of the present analysis.

(2) Plastic displacements as predicted by the geometrically nonlinear formulation were generally larger than those resulting from the geometrically linear version. But, when axial loads were small, the influence of geometric nonlinearities on the numerical results presented herein did not seem significant. However, as axial loads increased, the influence rapidly grew. For the cantilever beam problem, the beam would collapse with an axial load equal to approximately 26% of the Euler load. This axial load is practically equal to only one half of the magnitude corresponding to a geometrically linear solution.

(3) The numerical results of the cantilever beam problem based on using the axial force of the previous time-step, for the calculation of the member stiffness matrices, differed only insignificantly from those resulting from the more accurate approach of iterating on the axial force. The advantage of not performing any iteration lies, of course, in a considerable saving of computer time.

Natural extensions of this work may include the incorporation of more accurate yield surface equations in the computer program, and an investigation into the possibility of reducing the number of degrees of freedom in order to facilitate applications to even larger structural systems such as high-rise building frames.

Table 4-1 Comparison of Computer Time between Reference 13 and Present Work

	Computer Execution Time in Seconds	Time Increment in Seconds	"Real Time" of Solution in Seconds
<u>Cantilever Beam</u>			
Reference 13 ($\beta=1/8$)	11	0.0004	0.06
Reference 13 ($\beta=3/32$)	14	0.0004	0.06
Reference 13 ($\beta=1/16$)	170	0.0004	0.06
Present Work	4	0.0005	0.08
<u>Triangular Frame</u>			
Reference 13 ($\beta=1/8$)	153	0.0005	0.4
Present Work	23	0.001	0.5
<u>Building Frame</u>			
Reference 13 ($\beta=1/8$)	998	0.0005	2.0
Present Work	315	0.0008	2.0

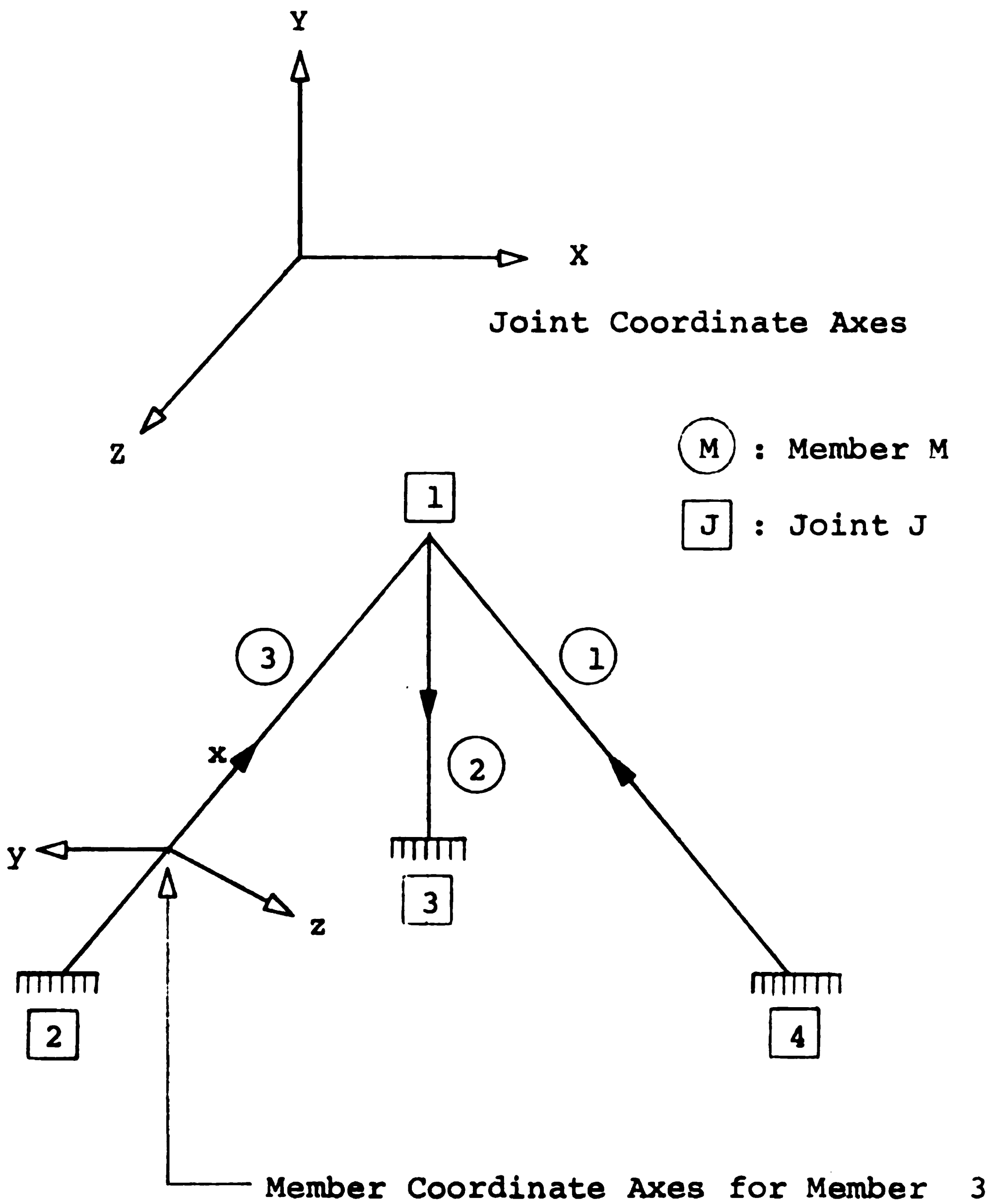


Figure 1-1 Typical Space Frame and Notation

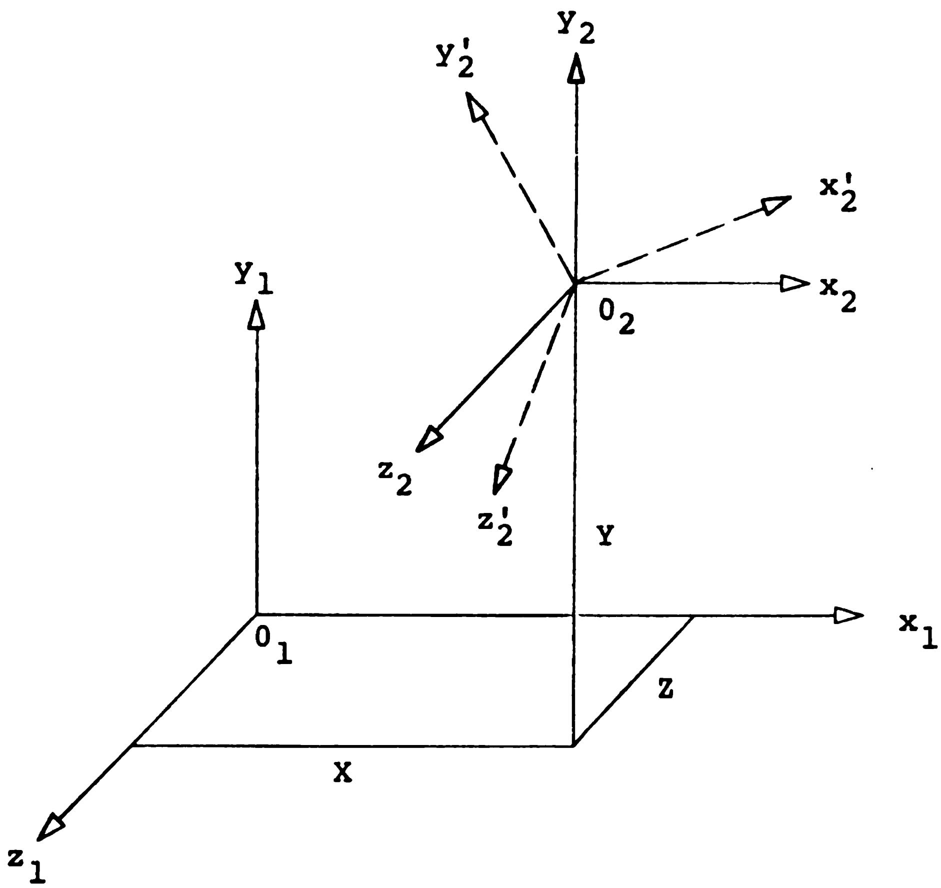


Figure 1-2 Different Coordinate Systems

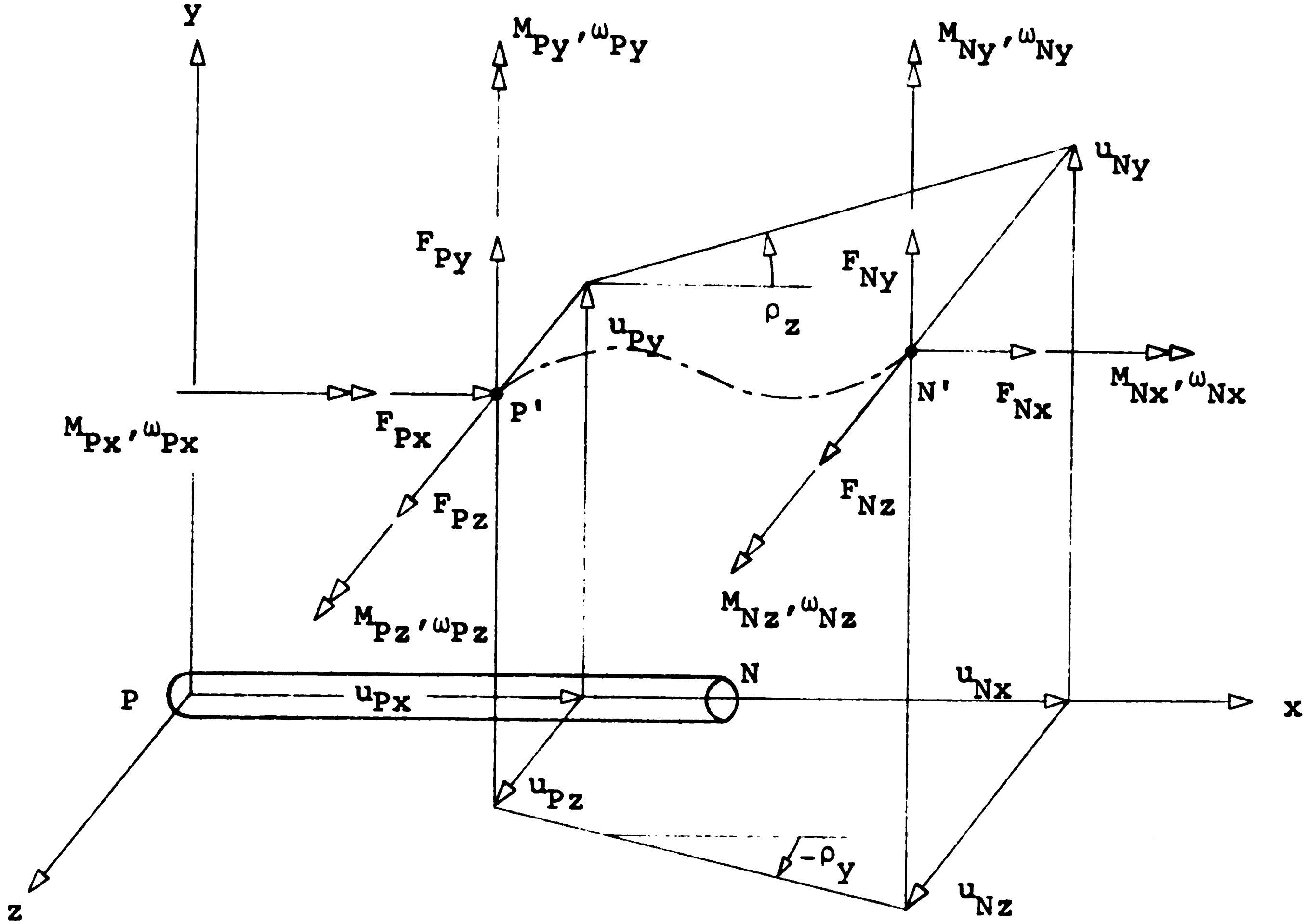


Figure 2-1 End Forces and Displacements of a Typical Member

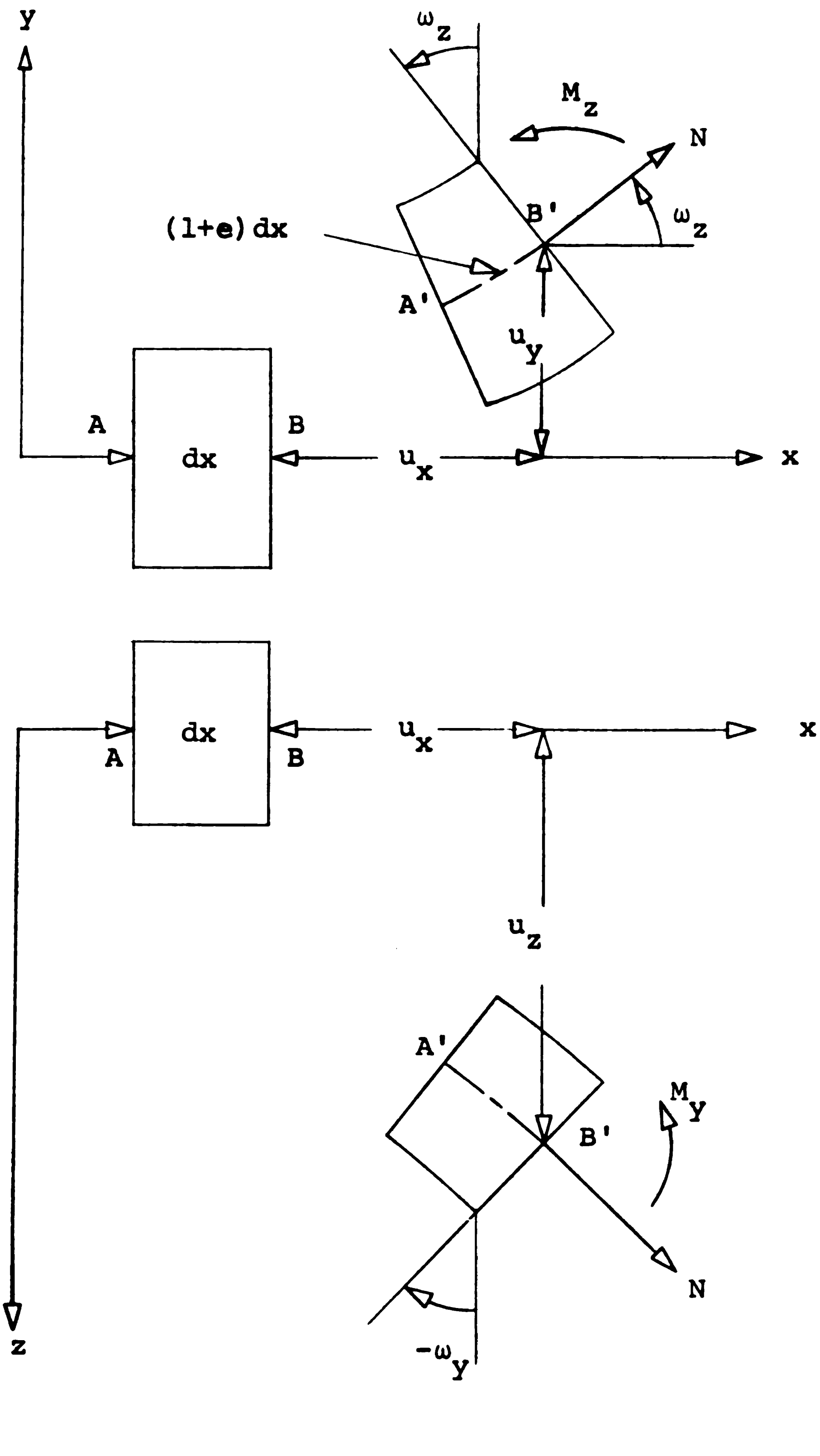


Figure 2-2 Free-Body of a Differential Element

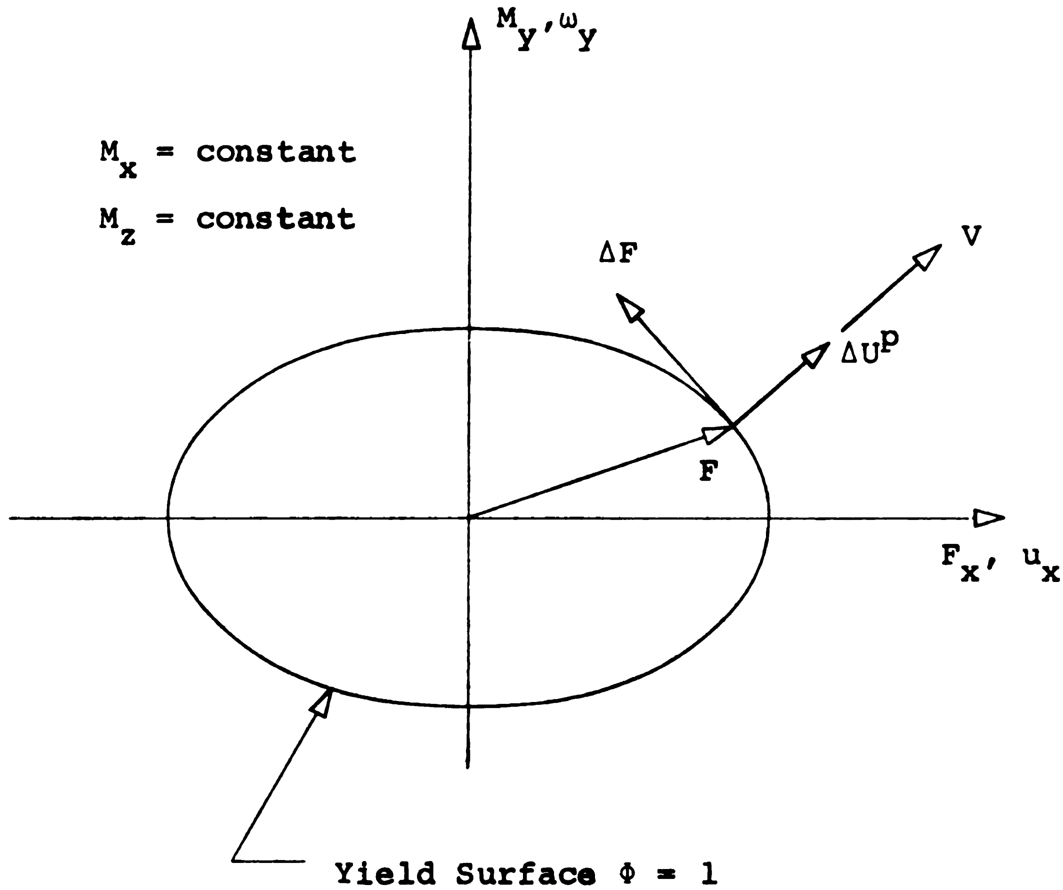


Figure 2-3 Geometrical Interpretation of Flow Law

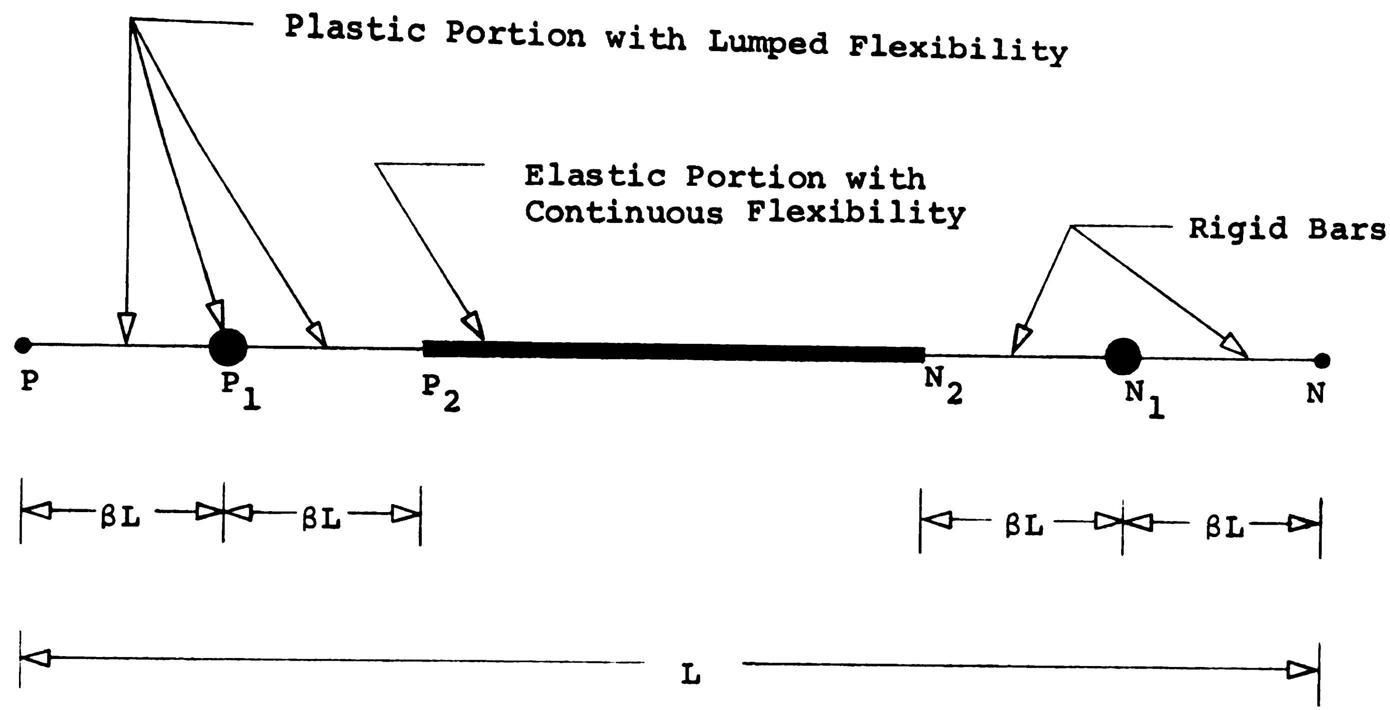


Figure 2-4 Discrete Model as Used in Reference 13

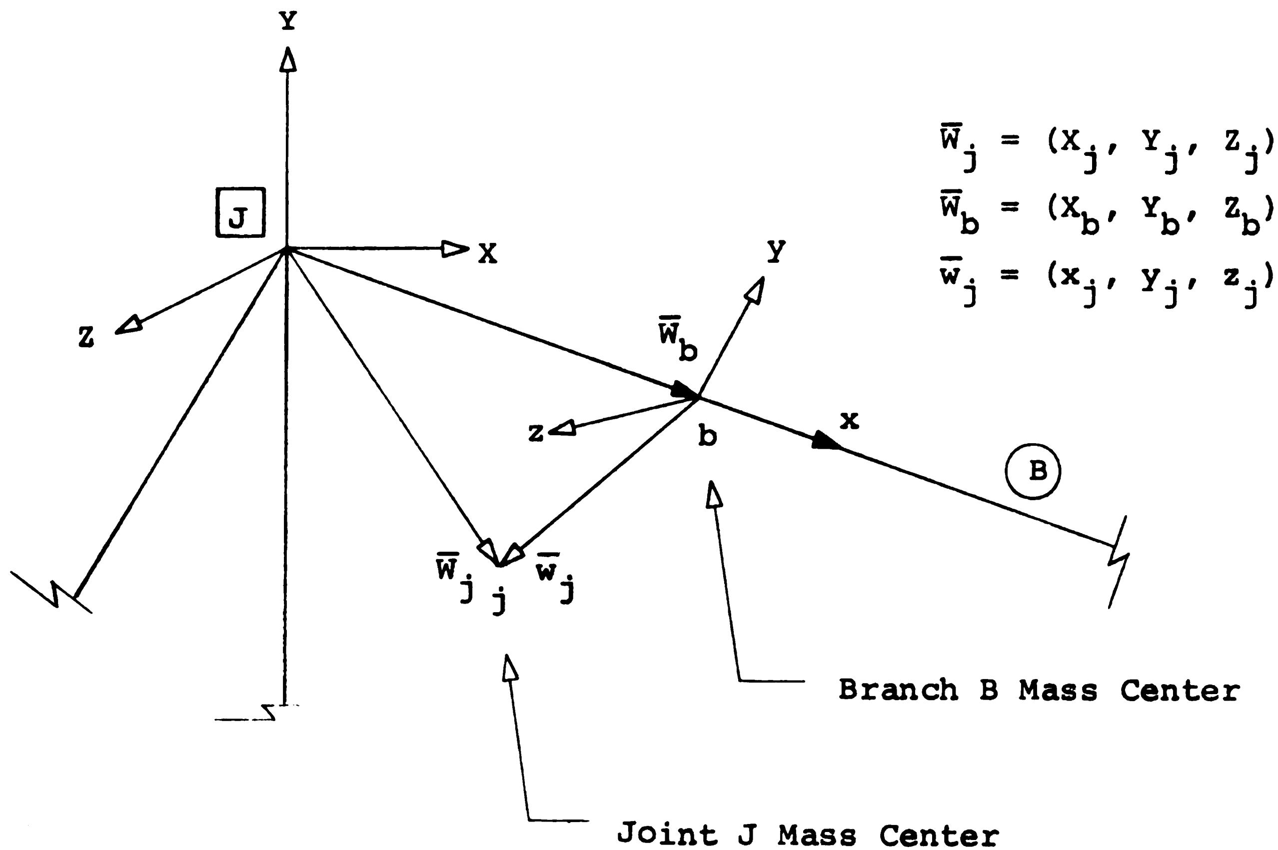
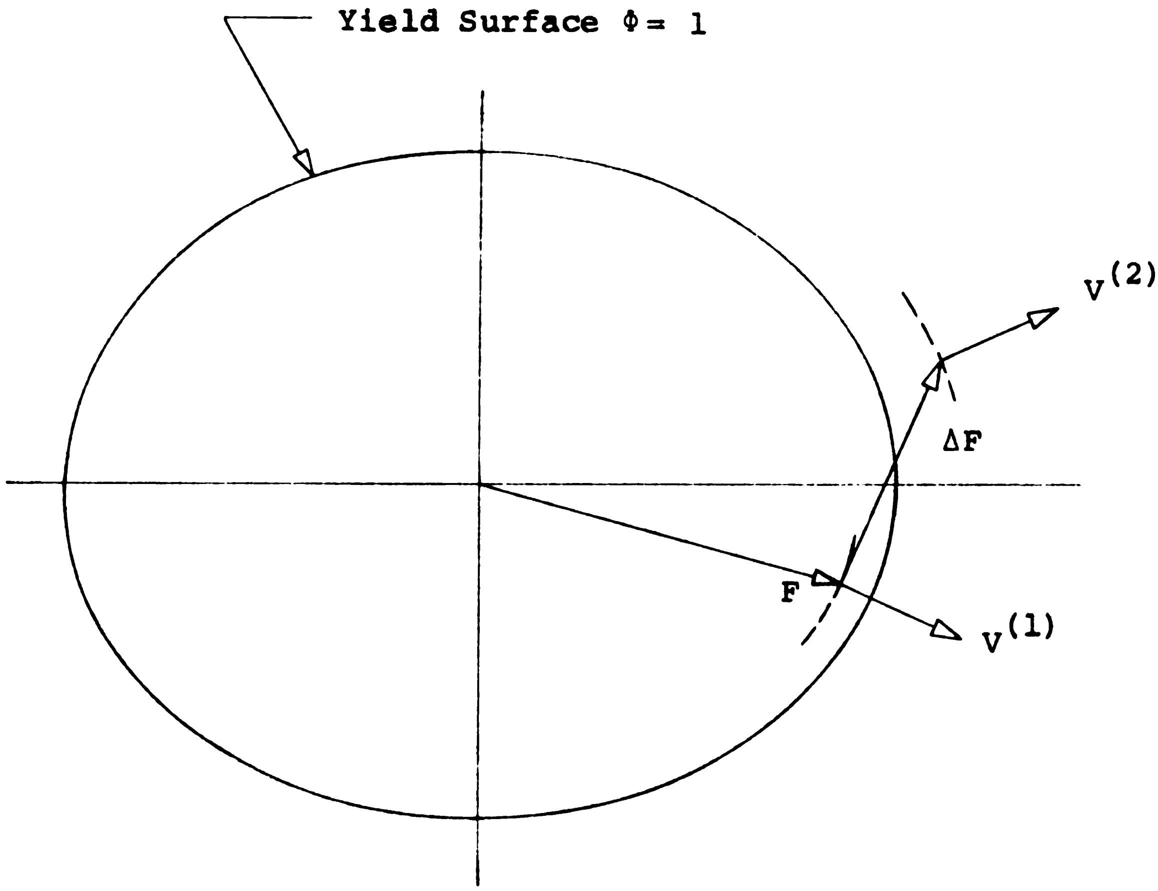
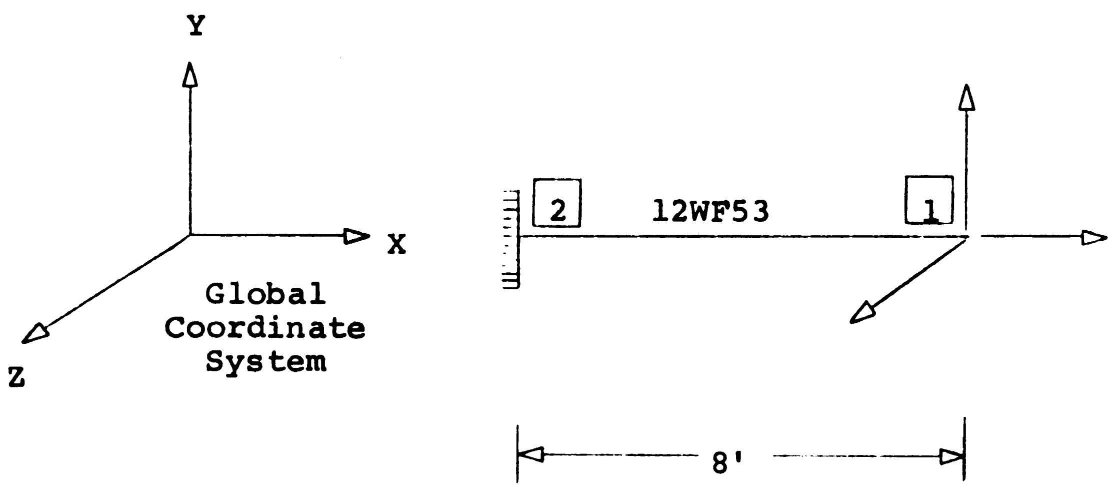


Figure 3-1 Rigid Body Associated with Joint J



igure 3-2 Treatment of Force Vector during Yielding



Joint	Axis	Static Load	Dynamic Load	Duration of Dynamic Load
1	X	-30 kips	--	--
1	Y	1 kip	-18 kips	0 to 0.01 sec.
1	Z	1 kip	8 kips	0 to 0.01 sec.

Figure 4-1 Cantilever Beam and Loading

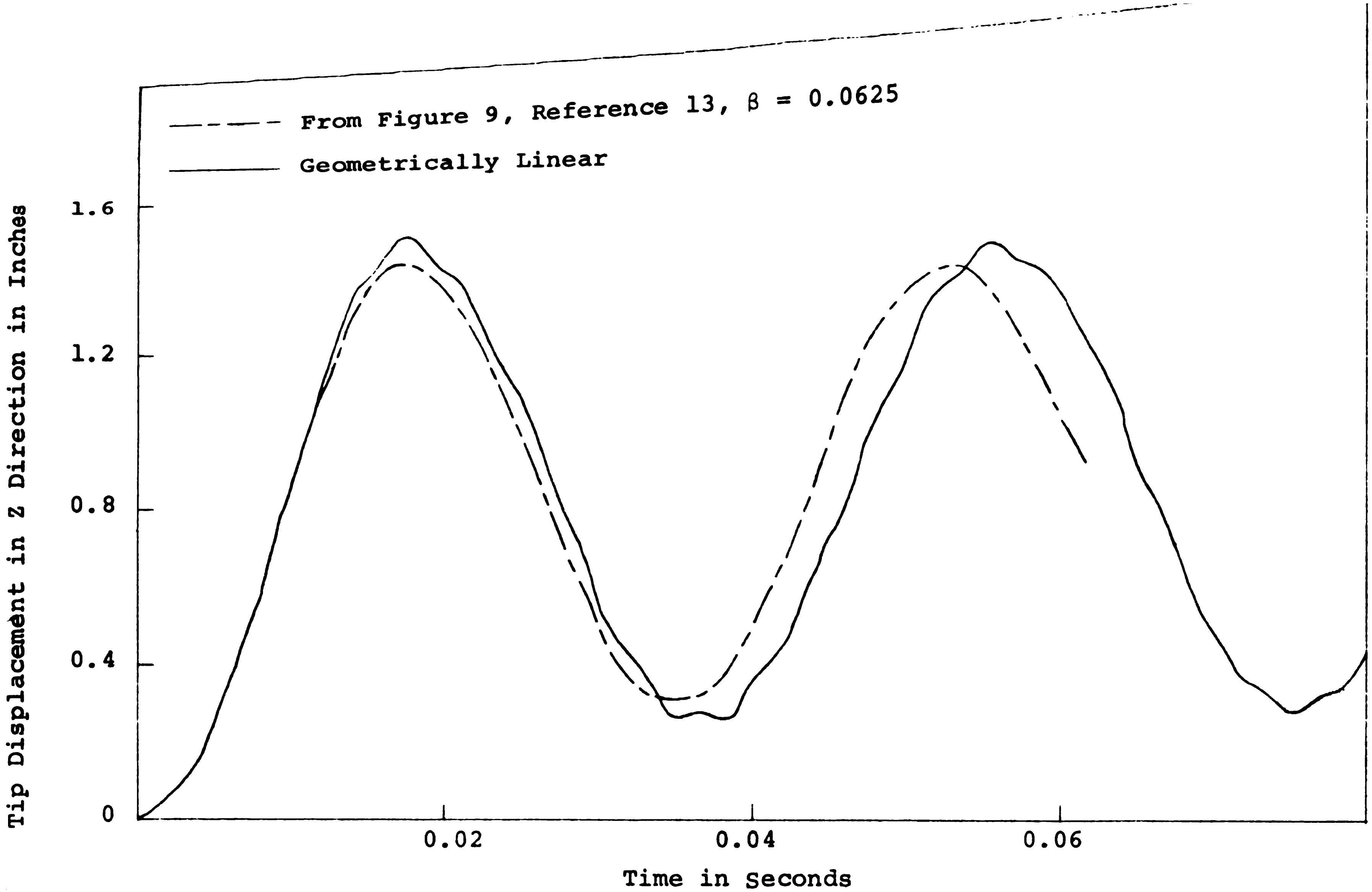


Figure 4-2 Transverse Displacement of Cantilever Beam versus Time

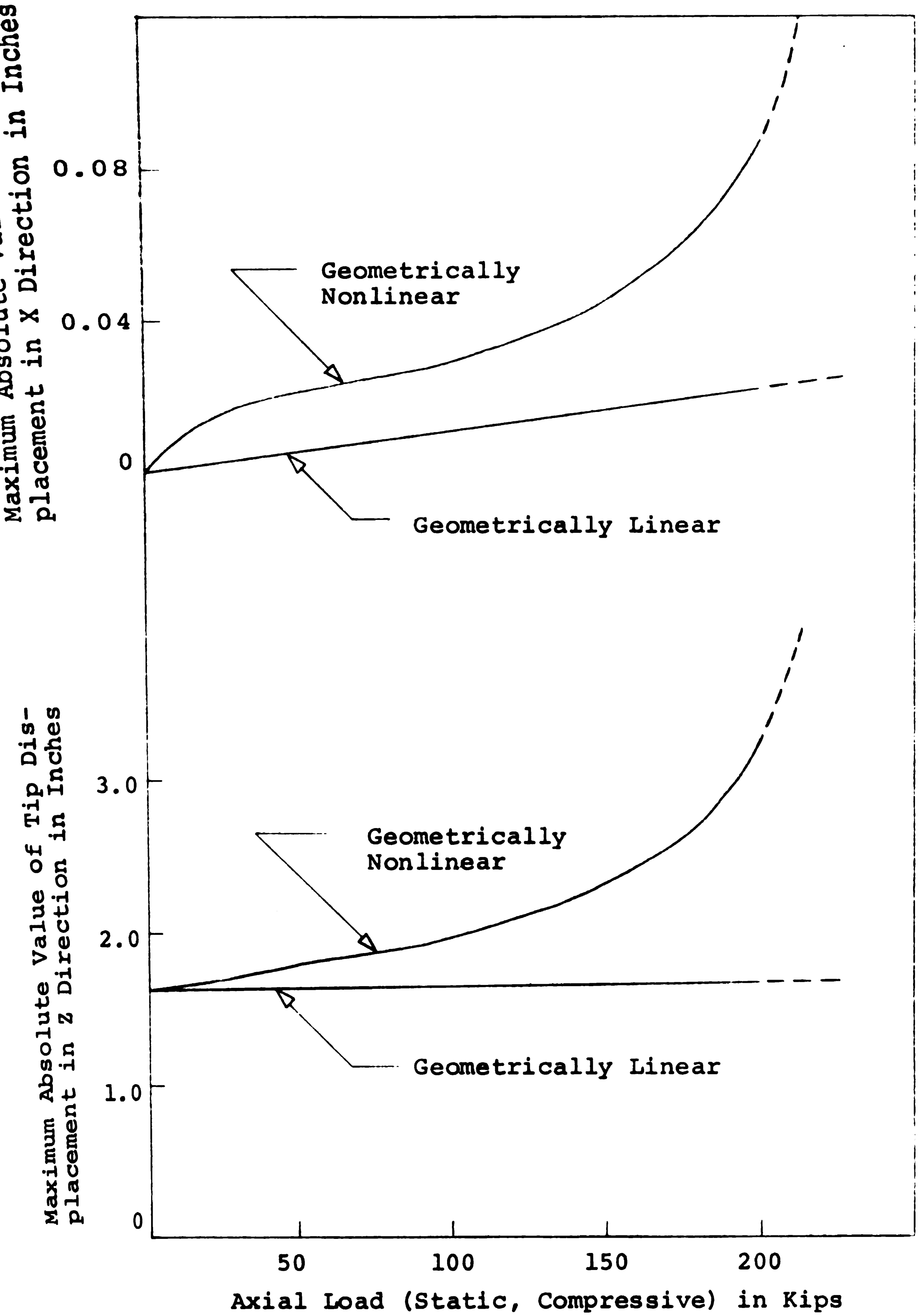


Figure 4-3 Effects of Axial Load

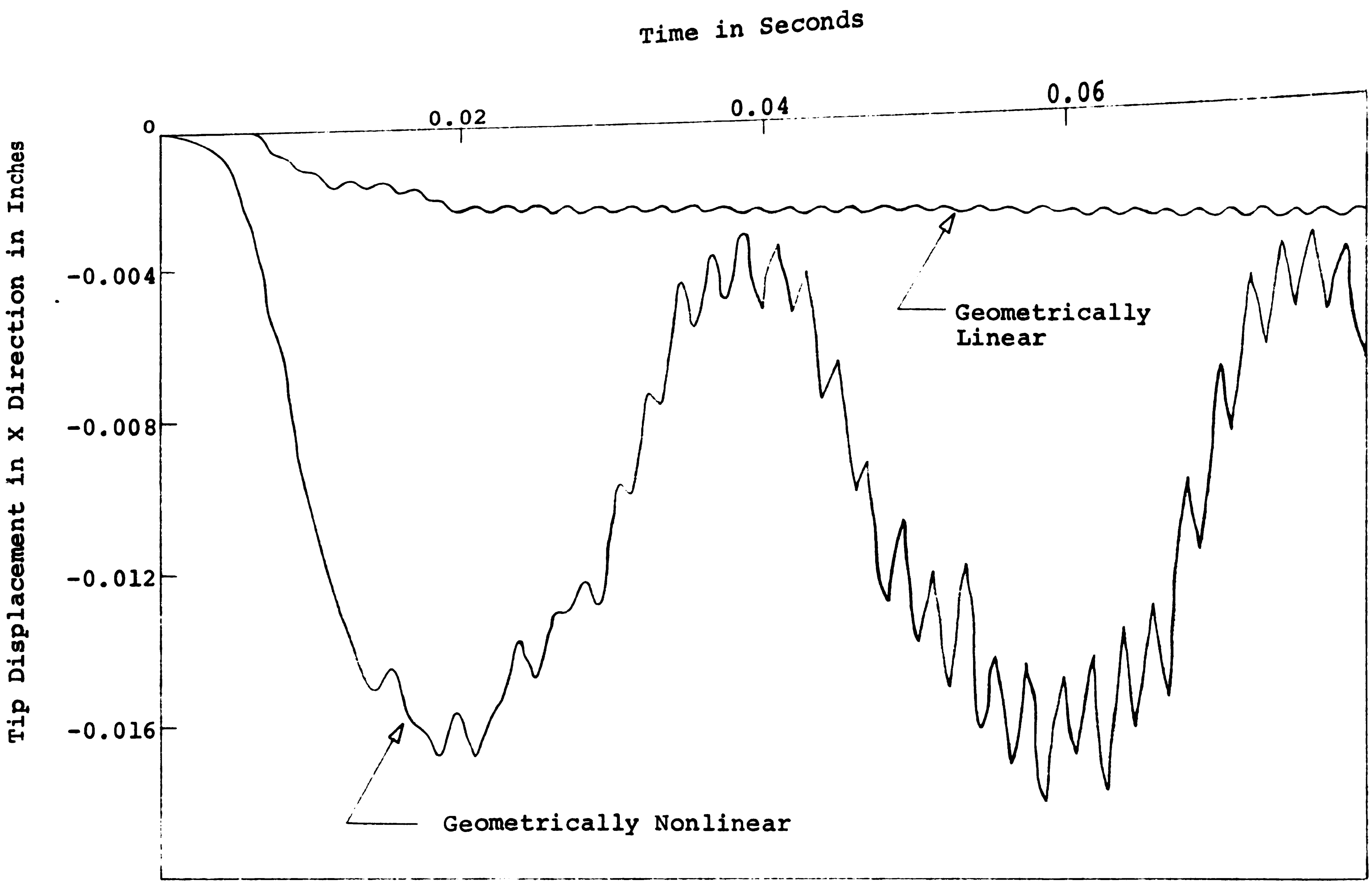
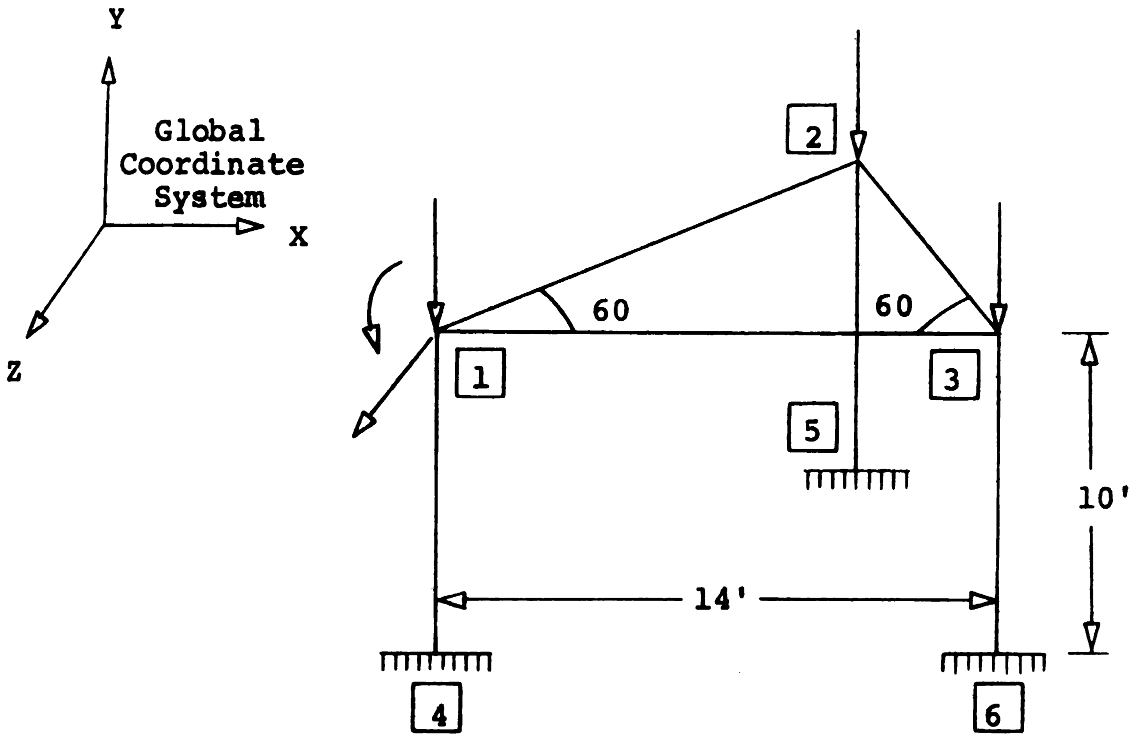


Figure 4-4 Axial Displacement of Cantilever Beam versus Time



Joint	Axis	Static Load	Dynamic Load	Duration of Dynamic Load
1	Y	-30 kips	--	--
1	Z	1 kip	20 kips	0 to ∞ sec.
1	Y	6 kip-in.	120 kip-in.	0 to ∞ sec.
2	Y	-30 kips	--	--
3	Y	-30 kips	--	--

Figure 4-5 Triangular Frame and Loading

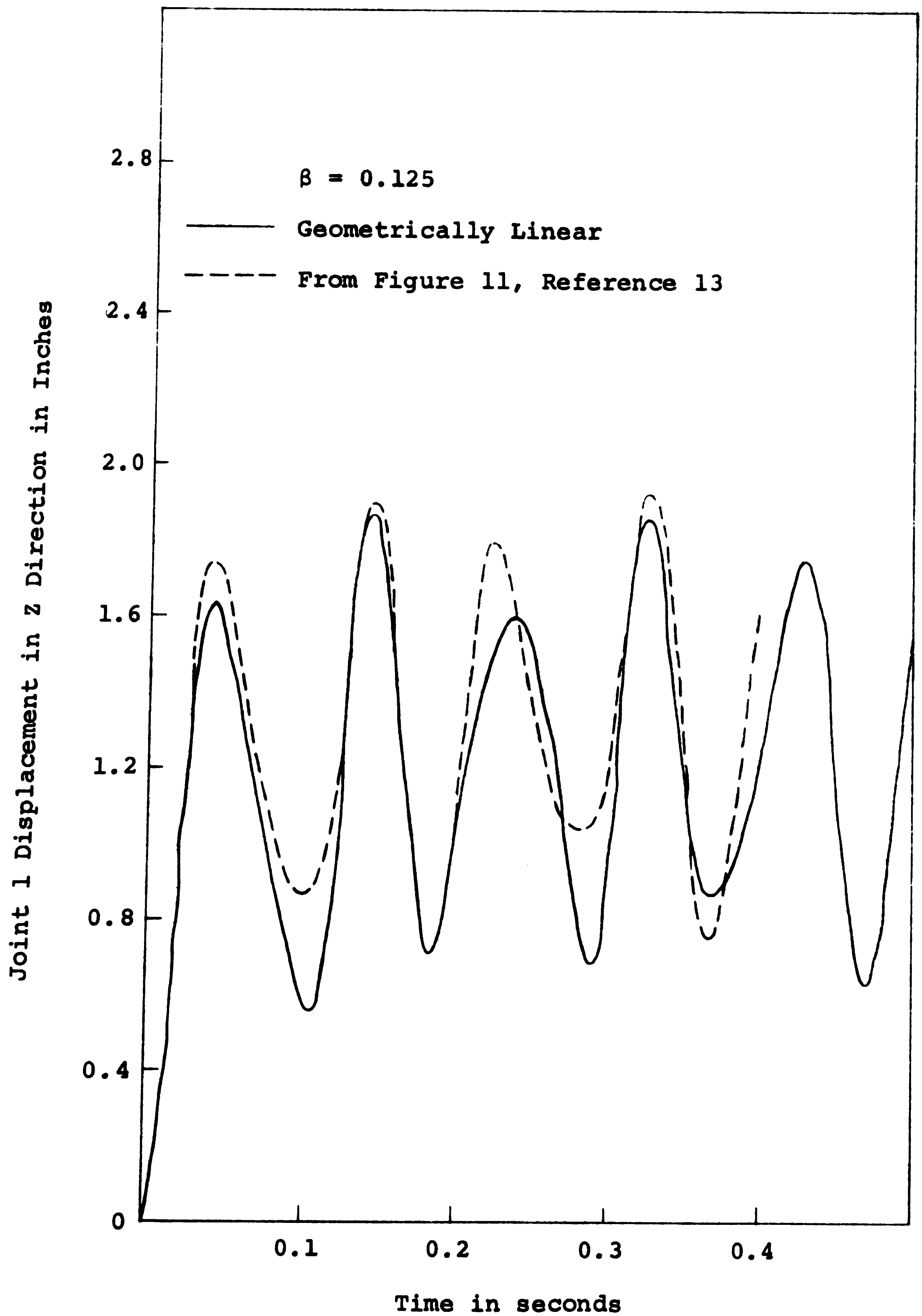


Figure 4-6 Response of Triangular Frame
(Elliptic Yield Function)

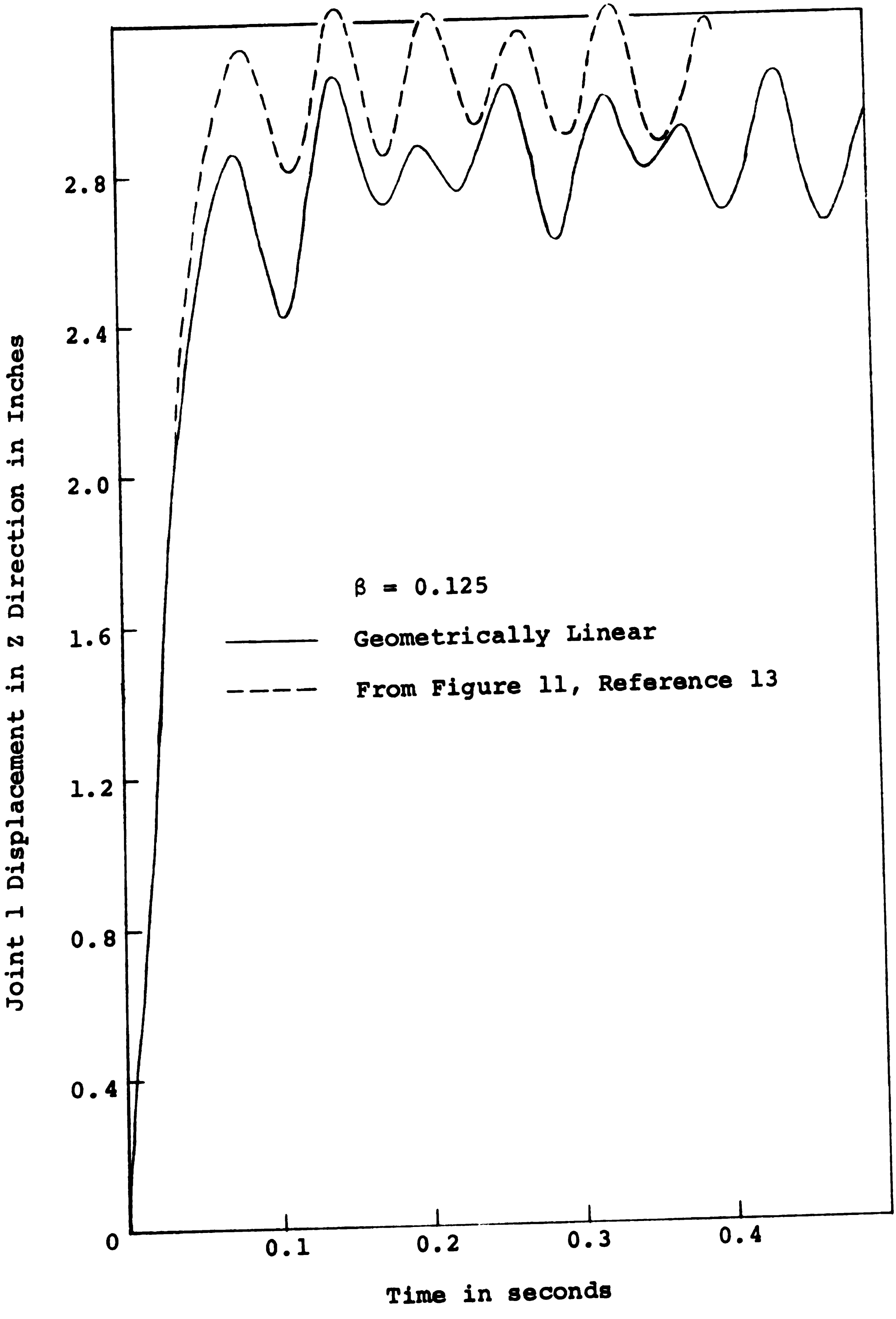


Figure 4-7 Response of Triangular Frame (Parabolic Yield Function)

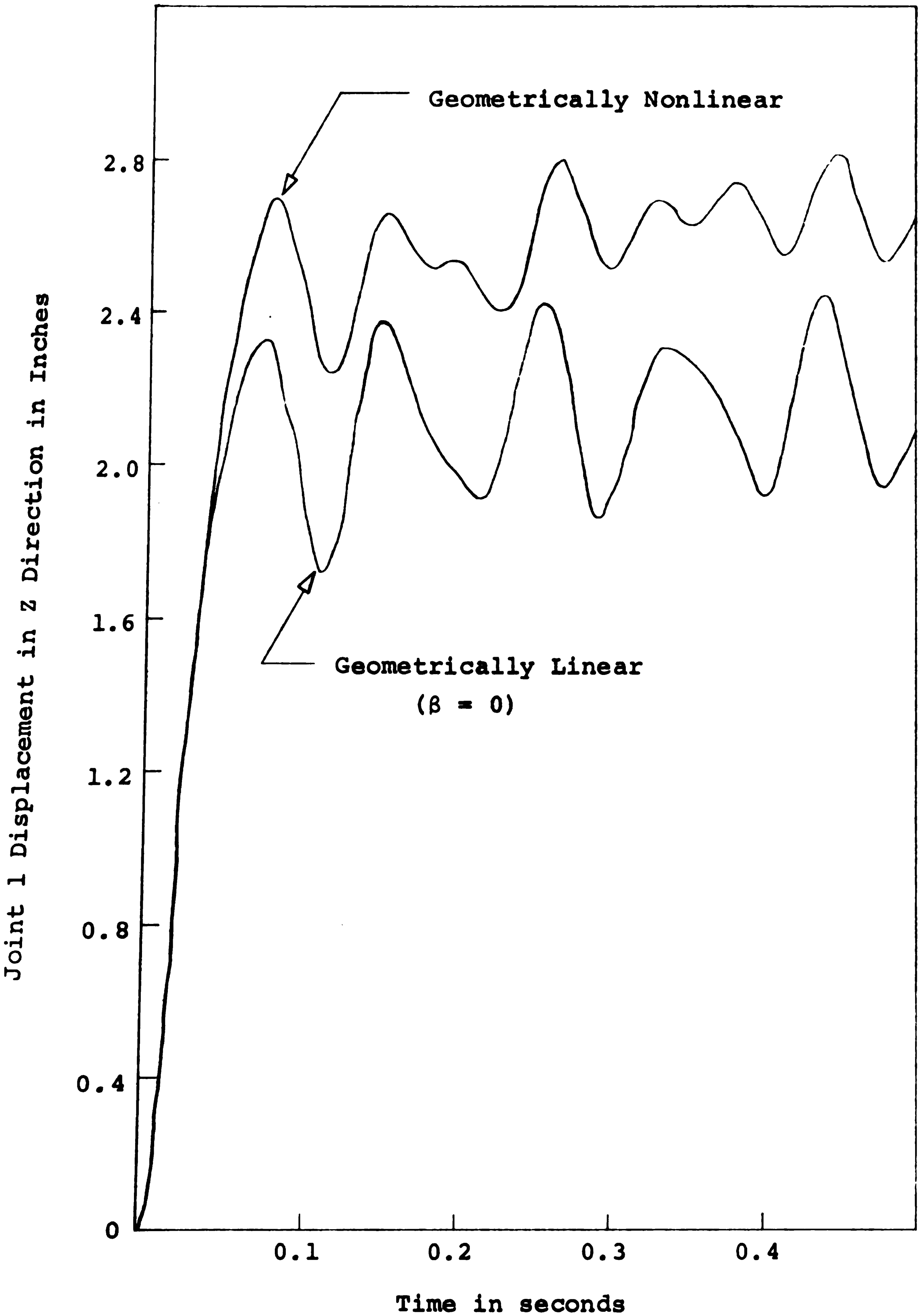


Figure 4-8 Comparison between Geometrically Linear and Nonlinear Responses of Triangular Frame

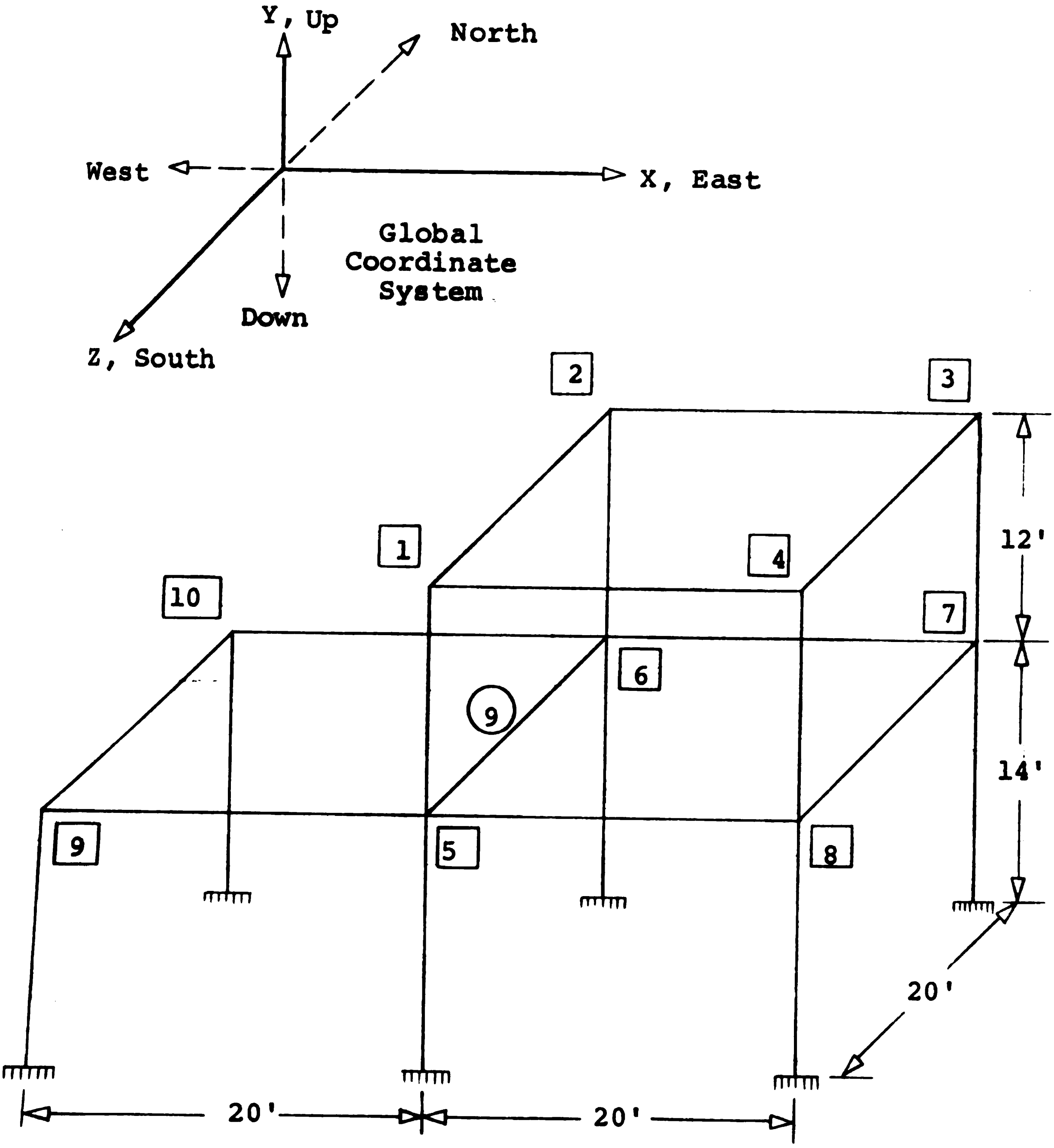


Figure 4-9 Building Frame

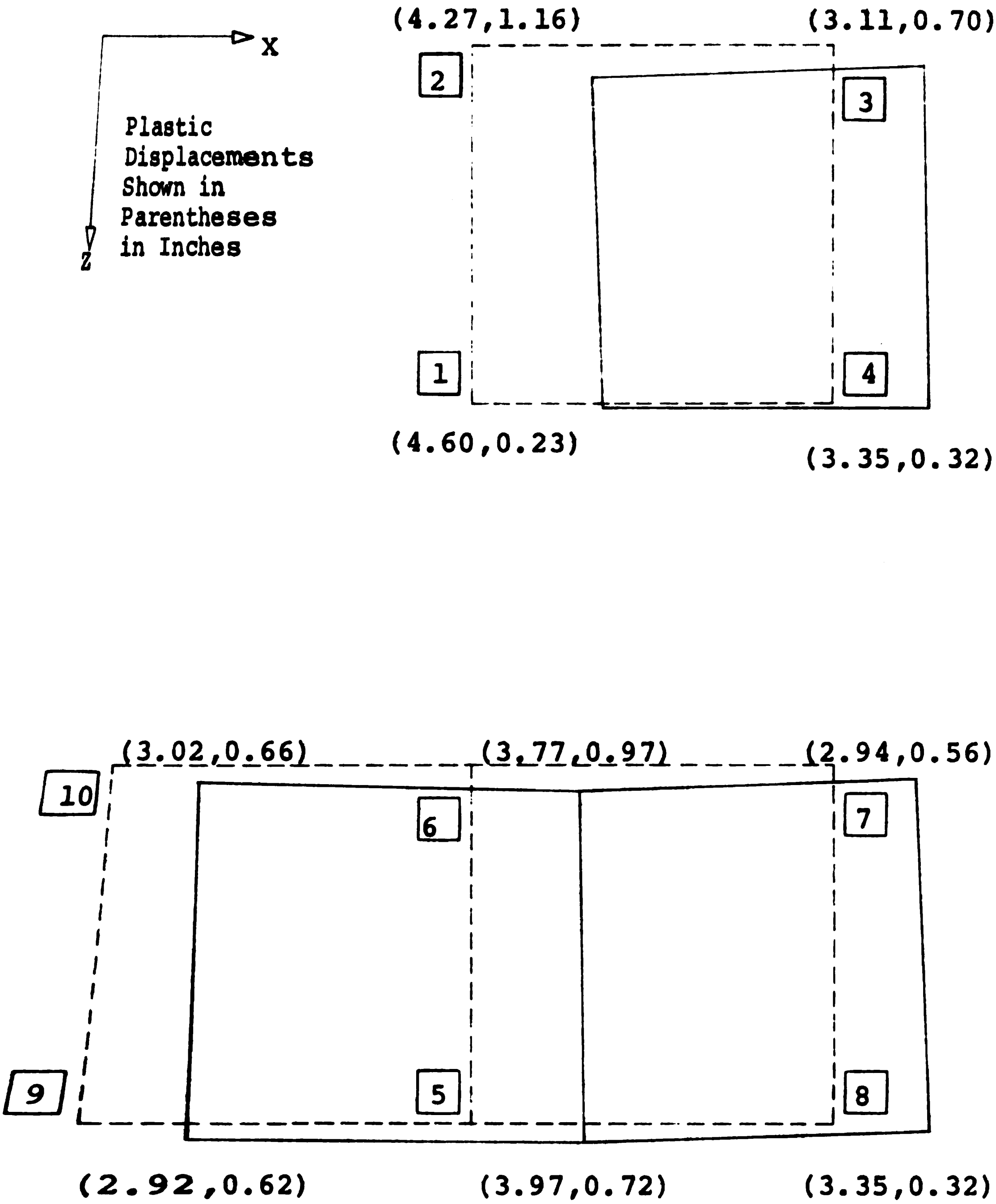


Figure 4-10 Plan View of Distorted Frame Predicted by Geometrically Linear Formulation

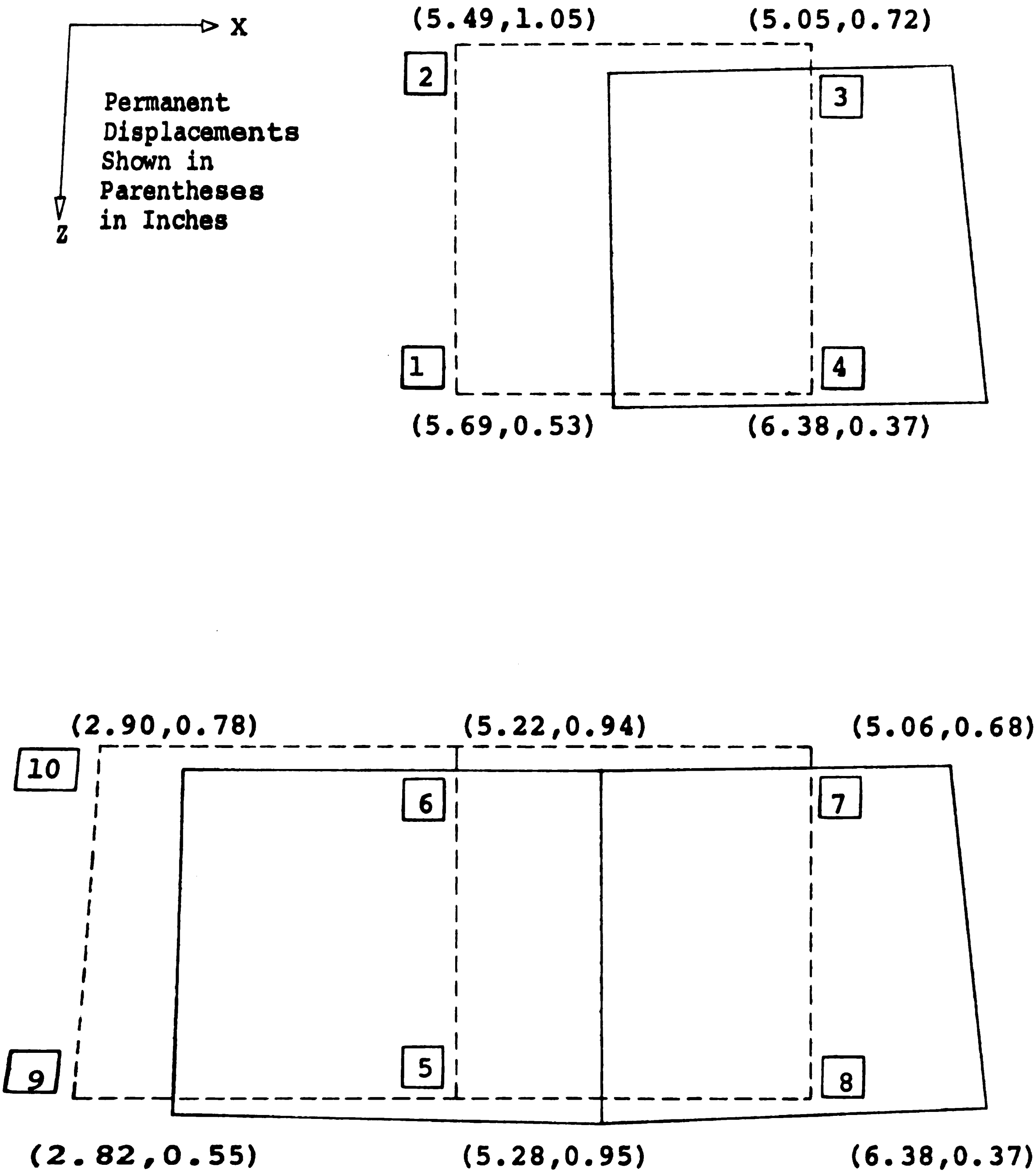


Figure 4-11 Plan View of Distorted Frame Predicted by Geometrically Nonlinear Formulation

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APPENDICES

APPENDIX I

STIFFNESS MATRICES

In this appendix, the elastic stiffness matrices are expanded, and the geometric stiffness matrix is derived.

A.1.1 Elastic Stiffness Matrices

The terms employed in the elastic stiffness matrices S_{PP}, etc., are first defined. The matrices themselves are expanded next. The expanded form of the matrix F_G is also given.

$$\begin{aligned} j &= y,\ z \\ k &= y,\ z \\ j &\neq k \end{aligned} \qquad \text{(A-1-1)}$$

$$\phi_j = \left[- \frac{F_{Nx} L^2}{EI_k} \right]^{0.5} \qquad \text{(A-1-2)}$$

$$\begin{aligned} C_j &= \cos \phi_j \\ S_j &= \sin \phi_j \end{aligned} \qquad \text{(A-1-3)}$$

$$h_{jx} = \frac{\phi_j(S_j - C_j\phi_j)}{2(1-C_j) - S_j\phi_j}$$

$$h_{jy} = \frac{\phi_j(\phi_j - S_j)}{2(1-C_j) - S_j\phi_j} \qquad \text{(A-1-4)}$$

$$h_{jz} = h_{jy} + h_{jx}$$

$$\rho_y = \frac{u_{Pz} - u_{Nz}}{L}$$

$$\rho_z = \frac{u_{Ny} - u_{Py}}{L} \qquad \text{(A-1-5)}$$

(ρ_y and ρ_z are the member chord rotations in the x-z and x-y planes, respectively.)

$$\delta_j = \frac{1}{2L}\int_0^L \omega_k^2\, dx \qquad \text{(A-1-6)}$$

$$F_G = [EA_x(\delta_y+\delta_z),\ \rho_z F_{Nx},\ \rho_y F_{Px},\ 0,\ 0,\ 0]^t \qquad \text{(A-1-7)}$$

$$S_{PP} = \left[\begin{array}{c|c|c|c|c|c}
\frac{EA_x}{L} & & & & & \\ \hline
 & 2h_{yz}\frac{EI_z}{L^3} & & & & h_{yz}\frac{EI_z}{L^2} \\ \hline
 & & 2h_{zz}\frac{EI_y}{L^3} & & -h_{zz}\frac{EI_y}{L^2} & \\ \hline
 & & & \frac{GJ}{L} & & \\ \hline
 & & & & h_{zx}\frac{EI_y}{L} & \\ \hline
\text{symmetric} & & & & & h_{yx}\frac{EI_z}{L}
\end{array}\right]$$

(A-1-8)

$$
S_{PN} = \begin{bmatrix}
-\frac{EA_x}{L} & & & & & \\
 & -2h_{yz}\frac{EI_z}{L^3} & & & & h_{yz}\frac{EI_z}{L^2} \\
 & & -2h_{zz}\frac{EI_y}{L^3} & & -h_{zz}\frac{EI_y}{L^2} & \\
 & & & -\frac{GJ}{L} & & \\
 & & h_{zz}\frac{EI_y}{L^2} & & h_{zy}\frac{EI_y}{L} & \\
 & -h_{yz}\frac{EI_z}{L^2} & & & & h_{yy}\frac{EI_z}{L}
\end{bmatrix} \tag{A-1-9}
$$

$$
S_{NN} = \begin{bmatrix}
\frac{EA_x}{L} & & & & & \\
 & 2h_{yz}\frac{EI_z}{L^3} & & & & -h_{yz}\frac{EI_z}{L^2} \\
 & & 2h_{zz}\frac{EI_y}{L^3} & & h_{zz}\frac{EI_y}{L^2} & \\
 & & & \frac{GJ}{L} & & \\
 & & & & h_{zx}\frac{EI_y}{L} & \\
 & \text{symmetric} & & & & h_{yx}\frac{EI_z}{L}
\end{bmatrix} \tag{A-1-10}
$$

A.1.2 Geometric Stiffness Matrix

To arrive at the geometric stiffness matrix S_G used in this study, the incremental rotations due to flexure are neglected, that is,

$$\Delta\omega_y = \Delta\rho_y$$
$$\Delta\omega_z = \Delta\rho_z \qquad \text{(A-1-11)}$$

The incremental chord rotations $\Delta\rho_y$ and $\Delta\rho_z$ are constant along their corresponding member. The approximation mentioned above is consistent with neglecting the change in the elastic stiffness matrices S_{PP}, etc., when forming the increments.

Consider now the matrix F_G produced in the previous section. Its increment is given by

$$\Delta F_G = [EA_x\,(\Delta\delta_y + \Delta\delta_z),$$
$$F_{Nx}\,\Delta\rho_z + \rho_z\,\Delta F_{Nx},$$
$$F_{Px}\,\Delta\rho_y + \rho_y\,\Delta F_{Px},$$
$$0,\ 0,\ 0]^t \qquad \text{(A-1-12)}$$

in which

$$\Delta\delta_y = \frac{1}{L}\int_0^L \omega_z\,\Delta\omega_z\,dx$$
$$= \frac{\Delta\rho_z}{L}\int_0^L \frac{du_y}{dx}\,dx$$
$$= \rho_z\,\Delta\rho_z$$
$$= -\frac{\rho_z}{L}\,(\Delta u_{Py} - \Delta u_{Ny}) \qquad \text{(A-1-13a)}$$

and similarly,

$$\Delta\delta_z = \frac{\rho_y}{L} (\Delta u_{Pz} - \Delta u_{Nz}) \tag{A-1-13b}$$

and also,

$$\begin{aligned} \Delta F_{Px} &= \frac{EA_x}{L} (\Delta u_{Px} - \Delta u_{Nx}) \\ &\quad - EA_x (\Delta\delta_y + \Delta\delta_z) \\ &= \frac{EA_x}{L} (\Delta u_{Px} - \Delta u_{Nx}) \\ &\quad + \frac{EA_x}{L} (\Delta u_{Py} - \Delta u_{Ny}) \rho_z \\ &\quad - \frac{EA_x}{L} (\Delta u_{Pz} - \Delta u_{Nz}) \rho_y \end{aligned} \tag{A-1-14a}$$

and

$$\Delta F_{Nx} = -\Delta F_{Px} \tag{A-1-14b}$$

It is now easy to show that the increment ΔF_G can be cast into

$$\Delta F_G = -S_G (\Delta U_P - \Delta U_N) \tag{A-1-15}$$

in which

$$S_G = \begin{bmatrix} s_G & 0 \\ 0 & 0 \end{bmatrix} \tag{A-1-16}$$

where

$$s_G = \frac{EA_x}{L} \begin{bmatrix} 0 & \rho_z & -\rho_y \\ & \rho_z^2 + \frac{F_{Nx}}{EA_x} & -\rho_y \rho_z \\ \text{symmetric} & & \rho_y^2 + \frac{F_{Nx}}{EA_x} \end{bmatrix} \quad \text{(A-1-17)}$$

APPENDIX II

INCREMENTAL FORCE-DISPLACEMENT RELATIONS FOR A GEOMETRICALLY MEMBER WITH END RELEASES

Assume that a geometrically nonlinear member PN, as shown in Figure 2-1, has a total of R releases at its negative end. Corresponding to the Ith release, assume a column matrix V_I defined by

$$V_I = [0, 0, 0, 1, 0, 0]^t \tag{A-2-1}$$

in which the only nonzero element is a "1" corresponding to the released force component (at the negative end of the member). For instance, the matrix V_I shown above corresponds to a released torsional moment.

The negative-end displacement increment ΔU_N can be decomposed into an elastic part ΔU_N^e and a part ΔU_N^r due to the releases being considered. Thus

$$\Delta U_N^e = \Delta U_N - \Delta U_N^r \tag{A-2-2}$$

The increment ΔU_N^r may be written as

$$\Delta U_N^r = \sum_{I=1}^{R} V_I \, b_I \tag{A-2-3}$$

in which b_I is a scalar factor which gives the magnitude of the displacement increment at the Ith release. It is now convenient to define a 6xR matrix G by

$$G = [V_1 \vdots V_2 \vdots \cdots \vdots V_R] \tag{A-2-4}$$

If the factors b_I (I=1, ..., R) are assembled into a column matrix B, Equation (A-2-3) reduces to

$$\Delta U_N^r = GB \tag{A-2-5}$$

Substitution of Equation (A-2-5) into Equation (A-2-2) furnishes

$$\Delta U_N^e = \Delta U_N - GB \tag{A-2-6}$$

The latter equation is now introduced into the incremental force-displacement relations given by Equations (2-17). This leads to

$$\Delta F_P = K_{PP} \; \Delta U_P + K_{PN} \; (\Delta U_N - GB) \tag{A-2-7a}$$

$$\Delta F_N = K_{PN}^t \; \Delta U_P + K_{NN} \; (\Delta U_N - GB) \tag{A-2-7b}$$

The condition that the negative-end force component increment in the direction of any release I is zero is expressed by

$$V_I^t \; \Delta F_N = 0 \tag{A-2-8}$$

Substituting Equation (A-2-7b) into Equation (A-2-8) and considering all the releases, it follows that

$$V_I^t \, K_{PN}^t \, \Delta U_P + V_I^t \, K_{NN} \, (\Delta U_N - GB) = 0$$

$$I = 1, \ldots, R \qquad \text{(A-2-9)}$$

These equations can be combined into the following single equation:

$$G^t \, K_{PN}^t \, \Delta U_P + G^t \, K_{NN} \, (\Delta U_N - GB) = 0 \qquad \text{(A-2-10)}$$

from which

$$B = E \, G^t \, K_{PN}^t \, \Delta U_P + E \, G^t \, K_{NN} \, \Delta U_N \qquad \text{(A-2-11)}$$

where

$$E = (G^t \, K_{NN} \, G)^{-1} \qquad \text{(A-2-12)}$$

Finally, substituting Equation (A-2-11) back into Equations (A-2-7), the following incremental force-displacement relations are obtained:

$$\begin{aligned} \Delta F_P &= (K_{PP} - K_{PN} \, G \, E \, G^t \, K_{PN}^t) \, \Delta U_P \\ &\quad + (K_{PN} - K_{PN} \, G \, E \, G^t \, K_{NN}) \, \Delta U_N \\ \Delta F_N &= (K_{PN}^t - K_{NN} \, G \, E \, G^t \, K_{PN}^t) \, \Delta U_P \\ &\quad + (K_{NN} - K_{NN} \, G \, E \, G^t \, K_{NN}) \, \Delta U_N \end{aligned} \qquad \text{(A-2-13)}$$

The extension to the case where the positive end of the member has also releases is straightforward and thus omitted.

APPENDIX III

COMPUTER PROGRAM

For the sake of completeness, the computer program written for the present study is outlined in this appendix. The routines which constitute the program are briefly described. The important identifiers used in the program are defined; and finally, a listing of the program is presented.

A.3.1 Description of Routines

The computer program developed consists of a main routine called DYNAMIC and four subroutines named INVERSE, RMATRIX, FMATRIX, and SMATRIX. The first subroutine INVERSE, taken from the M.S.U. computer laboratory, is used to invert square matrices as needed in the program. It is the only subroutine written in COMPASS (CDC 3600 machine language); therefore, its listing will be omitted. The second subroutine RMATRIX is used to compute the responsiveness matrix for a given joint.

The third subroutine FMATRIX has two versions. One of the two corresponds to the geometrically linear formulation (of a discrete model as described in Section 2.3) while the other is concerned with the geometrically nonlinear formulation. The linear version is used to compute the stiffness matrix for a given member. Note that the eight nonzero elements of the latter matrix are the only ones which are actually computed. The nonlinear version of the subroutine is used to compute certain factors (a total of eight) that appears in the elements of the elastic stiffness matrices for a given member. These factors are independent of the axial force. The effects of the latter force are considered in the subroutine SMATRIX.

Analogous to the subroutine FMATRIX, the fourth subroutine SMATRIX has also two versions. Both linear and nonlinear versions of this subroutine are used to perform the same task, namely, to compute the member end forces. They are also capable of inserting and/or removing plastic hinges when necessary. Each version of this subroutine has in turn two versions depending on the (type of) yield function to be used. In the listing, to be presented shortly, the linear version of the subroutine SMATRIX corresponds to the parabolic yield function while the nonlinear version corresponds to the elliptic yield function.

In the main routine DYNAMIC, the initial static solution is first read in as a part of the input. This solution is conveniently obtained by using the program STATIC written by Wen and Iverson (14). The rest of the input is read in next and immediately printed out. The identifiers are then initialized as necessary. In the following portion, the output of the program is monitored for printing and plotting. If the integration time exceeds the time limit specified, the program stops. The incremental joint displacements are computed next. The rest of the program is concerned with computing: the joint forces due to the member end forces, the ground acceleration if applicable, and the joint accelerations and velocities. The control is then sent back to the portion where the output is monitored.

A.3.2 Definition of Important Identifiers

The important identifiers used in each routine are defined below in alphabetical order. Any identifier which appears in more than one routine but maintains the same meaning is defined only once.

PROGRAM DYNAMIC

ACC(I,J)	=	Ith component of joint J acceleration
ALPHA	=	α
AMPLIFY	=	amplification factor times acceleration of gravity
AREAX(M)	=	cross-sectional area of member M
AREAY(M)	=	effective shear area of member M in y direction

AREAZ(M)	= effective shear area of member M in z direction
BETA	= β
COMP(I,M)	= projection of member M on Ith global axis
DATA A(I,J)	= Ith component of foundation acceleration in Jth reading
DATA T(I,J)	= time corresponding to DATA A(I,J)
DDIS(I,J)	= increment of DIS(I,J)
DEADM(M)	= uniformly distributed mass per unit length of member M
DENSITY	= density of material
DIS(I,J)	= Ith component of joint J displacement
DPLT	= time interval for plotting
DPRT	= time interval for printing
DT	= time increment
E	= modulus of elasticity
FNN(I,M)	= Ith component of force on negative end of member M
FORCE(I,J)	= Ith component of force on joint J exerted by incident members
FPP(I,M)	= Ith component of force on positive end of member M
G	= modulus of shear
GLINEAR	= true, if geometrically linear response desired; false, otherwise

IXX(M)	= torsional constant for member M cross-section
IYY(M)	= moment of inertia about y axis for member M cross-section
IZZ(M)	= moment of inertia about z axis for member M cross-section
JFREE	= number of free joints
JN(M)	= number, identifying negative joint of member M
JOINTS	= number of joints
JP(M)	= number, identifying positive joint of member M
LENGTH(M)	= length of member M
MEMBER(I,J)	= number, identifying Ith incident member on joint J; it is signed according to incidence
MEMBERS	= number of members
MLINEAR	= true, if material nonlinearity effects ignored; false, otherwise
NMEM(J)	= number of memebers incident on joint J
PEXT(I,J)	= Ith component of dynamic load on joint J
QUAKE	= true, if earthquake applied; false, otherwise
QTIME	= time lag
RESPON(I,J,K)	= element in Ith row and Jth column of joint K responsiveness matrix

RM(I,J,M) = element in Ith row and Jth column of rotation matrix (3x3) associated with member M

ROT(1) = cosθ, in which θ = angle measured clockwise from east-west axis to global X axis

ROT(2) = sinθ (see above for definition of θ)

SLIMIT(I,M) = Ith fully plastic stress resultant for member M cross-section

STLOAD(I,J) = Ith component of static load on joint J

T(K) = Kth component of foundation acceleration

TIME = time

TLIMIT = time limit

VEL(I,J) = Ith component of joint J velocity

XO ACC(I) = Ith component of foundation acceleration

XO DIS(I) = Ith component of foundation displacement

XO VEL(I) = Ith component of foundation velocity

SUBROUTINE SMATRIX
(linear version)

BB(1) = plastic-flow constant corresponding to: either section $x = \beta L$, or section $x = (1-\beta)L$ if section $x = \beta L$ not yielding

BB(2) = plastic-flow constant corresponding to section $x = (1-\beta)L$ if section $x = \beta L$ yielding also

CHI(1) = yield-function value for force on section $x = \beta L$

CHI(2) = yield-function value for force on section $x = (1 - \beta)L$

DV(I) = Ith component of member distortion

FP(I) = Ith component of positive-end force

GG(I,J) = element in Ith row and Jth column of matrix defined by Equation (2-37)

ITERATE = false, initially; true, if member end force not acceptable

LOOP = identifier to prevent endless looping

PDISP(I,M) = Ith component of plastic distortion of member M

PHI(1,M) = CHI(1), if section $x = \beta L$ of member M not yielding; constant, otherwise

PHI(2,M) = CHI(2), if section $x = (1 - \beta)L$ of member M not yielding; constant, otherwise

PLASTIC(1,M) = true, if section $x = \beta L$ of member M yielding; false, otherwise

PLASTIC(2,M) = true, if section $x = (1 - \beta)L$ of member M yielding; false, otherwise

S(I,M) = Ith stiffness element for member M

SS(I,J) = element in Ith row and Jth column of member stiffness matrix

V(I,1) = Ith component of yield surface normal corresponding to: either section $x = \beta L$, or section $x = (1 - \beta)L$ if section $x = \beta L$ not yielding

V(I,2) = Ith component of yield surface normal corresponding to section $x = (1 - \beta)L$ if section $x = \beta L$ yielding also

YIELDED(1,M) = false, before first yielding occurs in member M; true, afterwards

SUBROUTINE SMATRIX (nonlinear version)

BB(J) = plastic-flow constant; J = 1 corresponds to positive-end cross-section, J = 2 corresponds to negative-end cross-section

CHI(J) = yield-function value; J = 1 corresponds to positive-end force, J = 2 corresponds to negative-end force

DUN(I) = Ith component of negative-end displacement increment

DUP(I) = Ith component of positive-end displacement increment

ENDN = true, if only negative-end section yielding; false, otherwise

ENDP = true, if only positive-end section yielding; false, otherwise

ENDS = true, if both end sections yielding; false, otherwise

FN(I) = Ith component of negative-end force

GP(I,J) = element in Ith row and Jth column of matrix defined by Equations (2-26)

GN(I,J)	= element in Ith row and Jth column of matrix defined by Equations (2-26)
H21, ..., H33	= stiffness factors
P	= axial force
PDISN(I,M)	= Ith component of negative-end plastic displacement of member M
PDISP(I,M)	= Ith component of positive-end plastic displacement of member M
RHO2, RHO3	= ρ_y and ρ_z, respectively
S(I,M)	= Ith common factor of elements of elastic stiffness matrices for member M
SNN(I,J)	= element in Ith row and Jth column of direct stiffness matrix, referred to negative end
SPN(I,J)	= element in Ith row and Jth column of cross stiffness matrix,referred to positive end
SPP(I,J)	= element in Ith row and Jth column of direct stiffness matrix, referred to positive end
VN(I)	= Ith componenent of yield surface normal corresponding to negative-end section
VP(I)	= Ith component of yield surface normal corresponding to positive-end section
YIELDED(1,M)	= false, before first yielding occurs in positive-end section of member M; true, afterwards

YIELDED(2,M) = false, before first yielding occurs in negative-end section of member M; true, afterwards

SUBROUTINE RMATRIX

BIM(K,L) = element in Kth row and Lth column of branch inertia matrix

JIM(K,L) = element in Kth row and Lth column of joint inertia matrix

MMASS(I) = mass of branch I

UX, UY, UZ = coordinates of joint mass center with respect to local coordinate system through branch mass center

WX, WY, WZ = coordinates of joint mass center with respect to joint coordinate system

A.3.3 Listing of Program

```
      PROGRAM DYNAMIC
      LOGICAL QUAKE, GLINEAR, MLINEAR, GNONLIN
      DIMENSION ACC(6,50), VEL(6,50), PEXT(6,50), FORCE(6,50)
      DIMENSION DATA T(3,800), DATA A(3,800), ROT(2), T(3)
      DIMENSION XO ACC(3), XO VEL(3), XO DIS(3), INDEX(3)
      DATA (XO DIS = 3(0)), (XO VEL = 3(0))
      DATA (XO ACC = 3(0)), (INDEX  = 3(1))
      COMMON RESPON(6,6,50)
      COMMON/ TIME/ TIME, DT
      COMMON/ STIFF/ S(8,100), B(2,100)
      COMMON/ PLASTIC/ PLASTIC(2,100) $ LOGICAL PLASTIC
      COMMON/ YIELDED/ YIELDED(2,100) $ LOGICAL YIELDED
      COMMON/ DIS/ DIS(6,50), DDIS(6,50),
     1         PDISP(6,100), PDISN(6,100)
C
C  IDENTIFIERS WHOSE VALUES TO BE READ IN
C  FROM INITIAL STATIC SOLUTION
C
      COMMON/ TYPE/ DENSITY, E, G
      COMMON/ SIZE/ MEMBERS, JOINTS, JFREE
      COMMON/ AREA/ AREAX(100), AREAY(100), AREAZ(100),
     1         IXX(100), IYY(100), IZZ(100) $ REAL IXX, IYY, IZZ
      COMMON/ DEADM/ DEADM(100)
      COMMON/ COMP/ COMP(3,100)
      COMMON/ SLIMIT/ SLIMIT(6,100)
      COMMON/ LENGTH/ LENGTH(100) $ REAL LENGTH
      COMMON/ FORCE/ FPP(6,100), FNN(6,100)
      COMMON/ JPJN/ JP(100), JN(100)
      COMMON/ RM/ RM(3,3,100)
      COMMON/ MEM/ NMEM(50), MEMBER(10,50)
      COMMON/ STLOAD/ STLOAD(6,50)
C-----------------C
C  PROGRAM INPUT  C
C-----------------C
      LUN = 35
      CALL SKIPFILE( LUN )
      CALL SKIPFILE( LUN )
      READ(LUN)  MEMBERS, JOINTS, JFREE, DENSITY, E, G
      DO 505 M = 1,MEMBERS
 505  READ(LUN) AREAX(M), AREAY(M), AREAZ(M), IXX(M), IYY(M),
     1          IZZ(M), DEADM(M), (COMP(I,M), I=1,3),
     2          (SLIMIT(I,M), I=1,6), LENGTH(M),
     3          (FPP(I,M), I=1,6), (FNN(I,M), I=1,6),
     4          JP(M), JN(M), ((RM(I,J,M), I=1,3), J=1,3)
      DO 515 J = 1,JOINTS
 515  READ(LUN) NMEM(J), (MEMBER(I,J), I=1,10),
     1          (STLOAD(I,J), I=1,6)
```

```
      READ 531, QUAKE, QTIME, AMPLIFY
      READ 531, GLINEAR, BETA, DPRT, DPLT
      READ 531, MLINEAR, ALPHA, DTIME, TLIMIT
 531  FORMAT(L10,3E10.5,40X)
      IF( QUAKE )  READ(LUN) ROT, XO ACC, DATA T, DATA A
      PRINT 537, QUAKE, QTIME, AMPLIFY,
     1             GLINEAR, BETA, DPRT, DPLT,
     2             MLINEAR, ALPHA, DTIME, TLIMIT
 537  FORMAT(*1QUAKE=*L2*  QTIME=*F7.4*  AMPLIFY=*F7.2/
     1*0GLINEAR=*L2*  BETA=*F7.4*  DPRT=*F8.5*  DPLT=*F8.5/
     2*0MLINEAR=*L2*  ALPHA=*F5.2*  DTIME=*F8.5*  TLIMIT=*F8.5)
C-----------------C
C  INITIALIZATION  C
C-----------------C
      TIME = 0 $ DT = DTIME
      PRTIME = PLTIME =-0.5*DTIME
      GNONLIN = .NOT.GLINEAR
      DO 640 M = 1,MEMBERS
      B(1,M) = BETA*LENGTH(M)
      B(2,M) = LENGTH(M) - B(1,M)
      PLASTIC(1,M) = PLASTIC(2,M) = .FALSE.
      YIELDED(1,M) = YIELDED(2,M) = .FALSE.
      CALL FMATRIX( M )
      DO 640 I = 1,6
      PDISP(I,M) = PDISN(I,M) = 0
 640  SLIMIT(I,M) = 1.0/SLIMIT(I,M)
C
C  EXTERNAL DYNAMIC LOADS AT TIME = 0
C
      DO 670 J = 1,JFREE $ DO 670 I = 1,6
 670  PEXT(I,J) = 0
C
C  JOINT ACC., VEL., AND DIS. AT TIME = 0
C
      DO 680 J = 1,JOINTS $ DO 680 I = 1,6
 680  DIS(I,J) = 0
      DO 690 J = 1,JFREE
      CALL RMATRIX( J, ALPHA )
      DO 690 I = 1,6
      ACC(I,J) = VEL(I,J) = 0
      DO 685 K = 1,6
 685  ACC(I,J) = ACC(I,J) + RESPON(I,K,J)*PEXT(K,J)
      IF( I .LT. 4 )  ACC(I,J) = ACC(I,J) - XO ACC(I)
 690  CONTINUE
C-----------------C
C  PROGRAM OUTPUT  C
C-----------------C
      LUN = 25
      WRITE(LUN,2005) JFREE
2000  IF( TIME .LT. PRTIME )  GO TO 2030
```

```
      PRTIME = PRTIME + DPRT
      RTIME = TIMEF(4)
      PRINT 2001, TIME, RTIME
 2001 FORMAT(*1DYNAMIC VARIABLES AT TIME =*F8.5,75X
     1       *ELAPSED TIME =*-3PF7.2)
      IF( QUAKE )  PRINT 2002, XO DIS, XO VEL, XO ACC
 2002 FORMAT(*0GROUND MOTION*30X*X*15X*Y*15X*Z*//
     1       18X*DISPLACEMENT*8X3F16.8/
     2       18X*VELOCITY*12X3F16.8/
     3       18X*ACCELERATION*8X3F16.8)
      PRINT 2003
 2003 FORMAT(*-FORCE* 6X*S1*14X*S2*14X*S3*14X
     1       *S4*14X*S5*14X*S6*14X*S11*13X*S12*)
      PRINT 2004, (M, (FPP(I,M), I=1,6),
     1                FNN(5,M), FNN(6,M), M=1,MEMBERS)
 2004 FORMAT(/I4,8F16.8)
 2005 FORMAT( I4,6F16.8)
C
      PRINT 2006
 2006 FORMAT(*-DISPLACEMENT*)
      PRINT 2007, (J, (DIS(I,J), I=1,6), J=1,JFREE)
 2007 FORMAT(/I4,6F16.8)
C
      IF( GLINEAR )  PRINT 2012
      IF( GNONLIN )  PRINT 2013
 2012 FORMAT(*-PLASTIC DISTORTION*)
 2013 FORMAT(*-PLASTIC DISPLACEMENT*/)
      DO 2016 M = 1,MEMBERS
      L =-M
      IF( YIELDED(1,M) )  PRINT 2004, M, (PDISP(I,M), I=1,6)
      IF( YIELDED(2,M) )  PRINT 2005, L, (PDISN(I,M), I=1,6)
 2016 CONTINUE
C
 2030 IF( TIME .LT. PLTIME )  GO TO 2040
      PLTIME = PLTIME + DPLT
      WRITE(LUN,2032) TIME, ((DIS(I,J), I=1,6), J=1,JFREE)
 2032 FORMAT(F8.6, 6F12.8/(8X 6F12.8))
 2040 IF( TIME .GE. TLIMIT )  RETURN
C--------------------------------------C
C  INCREMENTAL FREE-JOINT DISPLACEMENTS  C
C--------------------------------------C
      DO 1001 J = 1,JFREE $ DO 1001 I = 1,6
 1001 DDIS(I,J) = ( VEL(I,J) + 0.5*ACC(I,J)*DT )*DT
      TIME = TIME + DT
C------------------------------------------C
C  FREE-JOINT FORCES DUE TO MEM. END FORCES  C
C------------------------------------------C
      CALL SMATRIX( BETA , MLINEAR, GLINEAR )
      DO 1506 J = 1,JFREE
      DO 1501 I = 1,6
```

```
 1501  FORCE(I,J) = 0
       N = NMEM(J)
       DO 1505 L = 1,N
       M = MEMBER(L,J)
       IF( M .LT. 0 )  GO TO 1503
C
C  POSITIVE INCIDENCE
C
       DO 1502 I = 1,3
       FORCE(I,J) = FORCE(I,J) - RM(1,I,M)*FPP(1,M)
      1                        - RM(2,I,M)*FPP(2,M)
      2                        - RM(3,I,M)*FPP(3,M)
 1502  FORCE(I+3,J) = FORCE(I+3,J) - RM(1,I,M)*FPP(4,M)
      1                            - RM(2,I,M)*FPP(5,M)
      2                            - RM(3,I,M)*FPP(6,M)
       GO TO 1505
C
C  NEGATIVE INCIDENCE
C
 1503  M =-M
       DO 1504 I = 1,3
       FORCE(I,J) = FORCE(I,J) - RM(1,I,M)*FNN(1,M)
      1                        - RM(2,I,M)*FNN(2,M)
      2                        - RM(3,I,M)*FNN(3,M)
 1504  FORCE(I+3,J) = FORCE(I+3,J) - RM(1,I,M)*FNN(4,M)
      1                            - RM(2,I,M)*FNN(5,M)
      2                            - RM(3,I,M)*FNN(6,M)
 1505  CONTINUE
 1506  CONTINUE
C----------------------C
C  GROUND ACCELERATION C
C----------------------C
       IF( QUAKE )  1710, 1790
 1710  XTIME = TIME + QTIME
       DO 1750 K = 1,3
C
C   INDEX(K) TO BE DETERMINED SUCH THAT
C   DATA T(K, INDEX(K)-1) .LT. XTIME .LE. DATA T(K, INDEX(K))
C
 1720  IF( XTIME .LE. DATA T(K, INDEX(K)) )  GO TO 1730
       INDEX(K) = INDEX(K) + 1
       GO TO 1720
C
C   GROUND ACC. IN EAST-UP-SOUTH COORD. SYSTEM INTERPOLATED
C   LINEARLY BY TWO SUITABLE CONSECUTIVE READINGS
C
 1730  T(K) = ( XTIME - DATA T(K, INDEX(K)-1) )*
      1       ( DATA A(K, INDEX(K)) - DATA A(K, INDEX(K)-1) )/
      2       ( DATA T(K, INDEX(K)) - DATA T(K, INDEX(K)-1) )
 1750  T(K) = ( T(K) + DATA A(K, INDEX(K)-1) )*AMPLIFY
```

```
C
C   GROUND ACC. IN GLOBAL COORD. SYSTEM
C
        GARB = ROT(1)*T(1) + ROT(2)*T(3)
        T(3) =-ROT(2)*T(1) + ROT(1)*T(3)
        T(1) = GARB
C
C   GROUND VEL., ACC., AND DIS.
C
        DO 1760 K = 1,3
        XO VEL(K) = XO VEL(K) + 0.5*( XO ACC(K) + T(K) )*DT
        XO ACC(K) = T(K)
1760    XO DIS(K) = XO DIS(K)
       1          + ( XO VEL(K) + 0.5*XO ACC(K)*DT )*DT
1790    CONTINUE
C--------------------------------- --------C
C   FREE-JOINT ACCELERATIONS AND VELOCITIES   C
C--------------------------------- --------C
        DO 1612 J = 1,JFREE $ DO 1612 I = 1,6 $ GARB = 0
        DO 1611 K = 1,6
1611    GARB = GARB + RESPON(I,K,J)*
       1         ( FORCE(K,J) + STLOAD(K,J) + PEXT(K,J) )
        IF( I .LT. 4 )  GARB = GARB - XO ACC(I)
        VEL(I,J)   = VEL(I,J) + 0.5*( ACC(I,J) + GARB )*DT
        ACC(I,J) = GARB
1612    DIS(I,J) = DIS(I,J) + DDIS(I,J)
        GO TO 2000
        END



        SUBROUTINE RMATRIX( J, ALPHA )
        DIMENSION MMASS(10), RIM(3,3), JIM(3,3), XXX(3,3)
        DIMENSION H(3,3), HJ(3,3), HJH(3,3)
        REAL MMASS, JMASS, JIM
        COMMON RESPON(6,6,50)
        COMMON/ TYPE/ DENSITY, E, G
        COMMON/ RM/ RM(3,3,100)
        COMMON/ DEADM/ DEADM(100)
        COMMON/ COMP/ COMP(3,100)
        COMMON/ LENGTH/ LENGTH(100) $ REAL LENGTH
        COMMON/ AREA/ AREAX(100), AREAY(100), AREAZ(100),
       1         IXX(100), IYY(100), IZZ(100) $ REAL IXX, IYY, IZZ
        COMMON/ MEM/ NMEM(50), MEMBER(10,50)
C
C   WX, WY, AND WZ = JOINT COORDINATES OF JOINT MASS CENTER
C
        JMASS = VXM = VYM = VZM = 0
        N = NMEM(J)
```

```
      DO 15 I = 1,N
      M = MEMBER(I,J)
      TEMP = SIGN( 0.25*ALPHA, M )
      M = IABS( M )
      MMASS(I) = 0.5*( DENSITY*AREAX(M)+DEADM(M) )*LENGTH(M)
      VXM = VXM + TEMP*COMP(1,M)*MMASS(I)
      VYM = VYM + TEMP*COMP(2,M)*MMASS(I)
      VZM = VZM + TEMP*COMP(3,M)*MMASS(I)
15    JMASS = JMASS + MMASS(I)
      JMASS = 1.0/JMASS
      WX = VXM*JMASS
      WY = VYM*JMASS
      WZ = VZM*JMASS
C
C  BIM = BRANCH INERTIA MATRIX, JIM = JOINT INERTIA MATRIX
C
      DO 20 L = 1,3 $ DO 20 K = 1,3
20    JIM(K,L) = 0
      DO 30 I = 1,N
      M = MEMBER(I,J)
      TEMP = SIGN( 0.25*ALPHA, M )
      M = IABS( M )
      VX = WX - TEMP*COMP(1,M)
      VY = WY - TEMP*COMP(2,M)
      VZ = WZ - TEMP*COMP(3,M)
      UX = RM(1,1,M)*VX + RM(1,2,M)*VY + RM(1,3,M)*VZ
      UY = RM(2,1,M)*VX + RM(2,2,M)*VY + RM(2,3,M)*VZ
      UZ = RM(3,1,M)*VX + RM(3,2,M)*VY + RM(3,3,M)*VZ
      BIM(1,2) = BIM(2,1) =-MMASS(I)*UX*UY
      BIM(2,3) = BIM(3,2) =-MMASS(I)*UY*UZ
      BIM(3,1) = BIM(1,3) =-MMASS(I)*UZ*UX
      TEMP = 0.5*DENSITY*LENGTH(M)
      BIM(3,3) = TEMP*IZZ(M)
      BIM(2,2) = TEMP*IYY(M)
      BIM(1,1) = BIM(3,3) + BIM(2,2)
      TEMP = ALPHA**2*LENGTH(M)**3*
     1        ( DENSITY*AREAX(M)+DEADM(M) )/96.0
      BIM(3,3) = BIM(3,3) + MMASS(I)*( UX**2+UY**2 ) + TEMP
      BIM(2,2) = BIM(2,2) + MMASS(I)*( UZ**2+UX**2 ) + TEMP
      BIM(1,1) = BIM(1,1) + MMASS(I)*( UY**2+UZ**2 )
      DO 30 K = 1,3
      DO 25 L = 1,3
25    XXX(K,L) = RM(1,K,M)*BIM(1,L)
     1         + RM(2,K,M)*BIM(2,L)
     2         + RM(3,K,M)*BIM(3,L)
      DO 30 L = 1,3
30    JIM(K,L) = JIM(K,L) + XXX(K,1)*RM(1,L,M)
     1                    + XXX(K,2)*RM(2,L,M)
     2                    + XXX(K,3)*RM(3,L,M)
C
```

```
C  JIM = INVERSE OF JOINT INERTIA MATRIX
C
      CALL INVERSE( JIM,3,3,H(1,1),1.0E-9,JR,H(1,2),H(1,3) )
C
C  RESPON = JOINT RESPONSIVENESS MATRIX
C
      H(1,1) = H(2,2) = H(3,3) = 0
      H(2,3) = WX   $  H(3,2) =-WX
      H(3,1) = WY   $  H(1,3) =-WY
      H(1,2) = WZ   $  H(2,1) =-WZ
      DO 50 K = 1,3
      DO 40 L = 1,3 $ HJ(K,L) = 0 $ DO 40 I = 1,3
40    HJ(K,L) = HJ(K,L) + H(K,I)*JIM(I,L)
      DO 50 L = 1,3 $ HJH(K,L) = 0 $ DO 50 I = 1,3
50    HJH(K,L) = HJH(K,L) + HJ(K,I)*H(I,L)
      DO 65 K = 1,3
      DO 60 L = 1,3
      RESPON(K,L,J) =-HJH(K,L)
      RESPON(K,L+3,J) = RESPON(L+3,K,J) =-HJ(K,L)
60    RESPON(K+3,L+3,J) = JIM(K,L)
65    RESPON(K,K,J) = RESPON(K,K,J) + JMASS
      PRINT 80, J, ALPHA, ((RESPON(K,L,J), K=1,6), L=1,6)
80    FORMAT(*0RESPONSIVENESS MATRIX FOR JOINT*I3
     1       *  AND  ALPHA =*F5.2, 6(/8X6E17.8))
      END



      SUBROUTINE FMATRIX( M )
C-------------------------------- C
C  GEOMETRICALLY LINEAR VERSION  C
C-------------------------------- C
      DIMENSION X(3), H(3), FLEX(8,3), FLEXIB(8)
      COMMON/ AREA/ AREAX(100), AREAY(100), AREAZ(100),
     1          IXX(100), IYY(100), IZZ(100) $ REAL IXX, IYY, IZZ
      COMMON/ PLASTIC/ PLASTIC(2,100) $ LOGICAL PLASTIC
      COMMON/ STIFF/ S(8,100), B(2,100)
      COMMON/ TYPE/ DENSITY, E, G
C
C  X(1) = LENGTH OF POSITIVE-END PLASTIC PORTION
C  X(2) = LENGTH OF NEGATIVE-END PLASTIC PORTION
C  X(3) = LENGTH OF ELASTIC PORTION
C
      X(1) = X(2) = 0
      IF( PLASTIC(1,M) )  X(1) = 2.0*B(1,M)
      IF( PLASTIC(2,M) )  X(2) = 2.0*B(1,M)
      X(3) = B(1,M) + B(2,M) - X(1) - X(2)
C
C  H(1) = DIS. BETWEEN POS.-END YIELD HINGE AND POS. END
```

```
C   H(2) = DIS. BETWEEN NEG.-END YIELD HINGE AND POS. END
C
      H(1) = 0.5*X(1)
      H(2) = B(1,M) + B(2,M) - 0.5*X(2)
      H(3) = X(1)
C
C   FLEX. MATRICES FOR PLASTIC PORTIONS
C
      DO 402 I = 1,2
      FLEX(7,I) = FLEX(8,I) = 0
      TEMP = X(I)/E
      FLEX(1,I) = TEMP/AREAX(M)
      FLEX(5,I) = TEMP/IYY(M)
      FLEX(6,I) = TEMP/IZZ(M)
      TEMP = X(I)/G
      FLEX(2,I) = TEMP/AREAY(M)
      FLEX(3,I) = TEMP/AREAZ(M)
402   FLEX(4,I) = TEMP/IXX(M)
C
C   FLEX. MATRIX FOR ELASTIC PORTION
C
      TEMP = X(3)/E
      FLEX(1,3) = TEMP/AREAX(M)
      FLEX(5,3) = TEMP/IYY(M)
      FLEX(6,3) = TEMP/IZZ(M)
      FLEX(4,3) = X(3)/( G*IXX(M) )
      FLEX(2,3) = X(3)**2*FLEX(6,3)/3.0
      FLEX(3,3) = X(3)**2*FLEX(5,3)/3.0
      FLEX(7,3) =-0.5*X(3)*FLEX(6,3)
      FLEX(8,3) = 0.5*X(3)*FLEX(5,3)
C
C   FLEX. MATRICES REFERRED TO POSITIVE END
C
      DO 404 J = 1,3
      FLEX(2,J) = FLEX(2,J)
     1            + H(J)*( H(J)*FLEX(6,J) - 2.0*FLEX(7,J) )
      FLEX(3,J) = FLEX(3,J)
     1            + H(J)*( H(J)*FLEX(5,J) + 2.0*FLEX(8,J) )
      FLEX(7,J) = FLEX(7,J) - H(J)*FLEX(6,J)
404   FLEX(8,J) = FLEX(8,J) + H(J)*FLEX(5,J)
C
C   FLEXIBILITY MATRIX
C
      DO 406 I = 1,8
406   FLEXIB(I) = FLEX(I,1) + FLEX(I,2) + FLEX(I,3)
C
C   STIFFNESS MATRIX
C
      S(1,M) = 1.0/FLEXIB(1)
      S(4,M) = 1.0/FLEXIB(4)
```

```
      TEMP = FLEXIB(2)*FLEXIB(6) - FLEXIB(7)**2
      S(2,M) = FLEXIB(6)/TEMP
      S(6,M) = FLEXIB(2)/TEMP
      S(7,M) =-FLEXIB(7)/TEMP
      TEMP = FLEXIB(3)*FLEXIB(5) - FLEXIB(8)**2
      S(3,M) = FLEXIB(5)/TEMP
      S(5,M) = FLEXIB(3)/TEMP
      S(8,M) =-FLEXIB(8)/TEMP
      END



      SUBROUTINE FMATRIX( M )
C-----------------------------------C
C  GEOMETRICALLY NONLINEAR VRESION  C
C-----------------------------------C
      COMMON/ TYPE/ DENSITY, E, G
      COMMON/ STIFF/ S(8,100), B(2,100)
      COMMON/ AREA/ AREAX(100), AREAY(100), AREAZ(100),
     1          IXX(100), IYY(100), IZZ(100) $ REAL IXX, IYY, IZZ
C
C  CONSTANT FACTORS OF ELEMENTS OF ELASTIC STIFFNESS MATRICES
C
      X = B(1,M) + B(2,M) $ TEMP = E/X
      S(5,M) = TEMP*IYY(M) $ S(8,M) = S(5,M)/X
      S(6,M) = TEMP*IZZ(M) $ S(7,M) = S(6,M)/X
      S(1,M) = TEMP*AREAX(M)
      S(2,M) = 2.0*S(7,M)/X
      S(3,M) = 2.0*S(8,M)/X
      S(4,M) = G*IXX(M)/X
      END



      SUBROUTINE SMATRIX( BETA, MLINEAR, GLINEAR )
C---------------------------------------------------------C
C  GEOMETRICALLY LINEAR VERSION, PARAB. YIELD FUNCTION   C
C---------------------------------------------------------C
      LOGICAL MLINEAR, GLINEAR, ITERATE
      DIMENSION DW(6), FP(6), V(6,2), H(2)
      DIMENSION PHI(2,100), CHI(2), BB(2), EE(2,2), XN(6)
      DIMENSION SS(6,6), GG(6,2), SG(6,2), XX(2,6), YY(6,6)
      DATA (SS = 36(0)), (GG = 12(0))
      COMMON/ TIME/ TIME, DT
      COMMON/ RM/ RM(3,3,100)
      COMMON/ SLIMIT/ SLIMIT(6,100)
      COMMON/ JPJN/ JP(100), JN(100)
      COMMON/ STIFF/ S(8,100), B(2,100)
```

```
      COMMON/ SIZE/ MEMBERS, JOINTS, JFREE
      COMMON/ FORCE/ FPP(6,100), FNN(6,100)
      COMMON/ LENGTH/ LENGTH(100) $ REAL LENGTH
      COMMON/ PLASTIC/ PLASTIC(2,100) $ LOGICAL PLASTIC
      COMMON/ YIELDED/ YIELDED(2,100) $ LOGICAL YIELDED
      COMMON/ DIS/ DIS(6,50), DDIS(6,50),
     1        PDISP(6,100), PDISN(6,100)
      DO 880 M= 1,MEMBERS
C--------------------------------C
C  INCREMENTAL MEMBER DISTORTION C
C--------------------------------C
      JPM = JP(M)
      JNM = JN(M)
      DO 305 I = 1,3
      DW(I) = DW(I+3) = XN(I+3) = 0
      DO 305 J = 1,3
      XN(I+3) = XN(I+3) + RM(I,J,M)*DDIS(J+3,JNM)
      DW(I+3) = DW(I+3)
     1        + RM(I,J,M)*( DDIS(J+3,JPM)-DDIS(J+3,JNM) )
305   DW(I) = DW(I) + RM(I,J,M)*( DDIS(J,JPM)-DDIS(J,JNM) )
      DW(2) = DW(2) + LENGTH(M)*XN(6)
      DW(3) = DW(3) - LENGTH(M)*XN(5)
C-----------------------------------------C
C  POS.-END FORCE IF MEMBER M NOT YIELDING C
C-----------------------------------------C
400   N = 0
      IF( PLASTIC(1,M) .OR.  PLASTIC(2,M) )  N = 1
      IF( PLASTIC(1,M) .AND. PLASTIC(2,M) )  N = 2
      IF( N .GT. 0 )  GO TO 440
      FP(1) = FPP(1,M) + S(1,M)*DW(1)
      FP(2) = FPP(2,M) + S(2,M)*DW(2) + S(7,M)*DW(6)
      FP(3) = FPP(3,M) + S(3,M)*DW(3) + S(8,M)*DW(5)
      FP(4) = FPP(4,M) + S(4,M)*DW(4)
      FP(5) = FPP(5,M) + S(5,M)*DW(5) + S(8,M)*DW(3)
      FP(6) = FPP(6,M) + S(6,M)*DW(6) + S(7,M)*DW(2)
      DO 420 J = 1,2
420   CHI(J) = ABS(FP(1))*SLIMIT(1,M) + (FP(4)*SLIMIT(4,M))**2
     1       + ABS(FP(5) + B(J,M)*FP(3))*SLIMIT(5,M)
     2       + ABS(FP(6) - B(J,M)*FP(2))*SLIMIT(6,M) - 1.0
      IF( MLINEAR )  800, 700
440   CONTINUE
C-------------------------------------C
C  POS.-END FORCE IF MEMBER M YIELDING C
C-------------------------------------C
      LOOP = 1
      H(1) = B(1,M) $ H(2) = B(2,M)
      IF( .NOT.PLASTIC(1,M) .AND. PLASTIC(2,M) )  H(1) = H(2)
C
C  YIELD SURFACE NORMALS
C
```

```
      DO 502 J = 1,N
      V(1,J) = SIGN( SLIMIT(1,M), FPP(1,M) )
      V(4,J) = 2.0*FPP(4,M)*SLIMIT(4,M)**2
      V(5,J) = SIGN( SLIMIT(5,M), FPP(5,M)+H(J)*FPP(3,M) )
502   V(6,J) = SIGN( SLIMIT(6,M), FPP(6,M)-H(J)*FPP(2,M) )
C
C  STIFFNESS MATRIX
C
      DO 503 I = 1,6
503   SS(I,I) = S(I,M)
      SS(6,2) = SS(2,6) = S(7,M)
      SS(5,3) = SS(3,5) = S(8,M)
C
C  GG MATRIX
C
505   ITERATE = .FALSE.
      DO 510 J = 1,N
      GG(1,J) = V(1,J)
      GG(2,J) =-V(6,J)*H(J)
      GG(3,J) = V(5,J)*H(J)
      DO 510 I = 4,6
510   GG(I,J) = V(I,J)
C
C  SG = SS*GG
C
      DO 511 I = 1,6 $ DO 511 J = 1,N $ SG(I,J) = 0
      DO 511 K = 1,6
511   SG(I,J) = SG(I,J) + SS(I,K)*GG(K,J)
C
C  EE = INVERSE OF T(GG)*SS*GG     T MEANS TRANSPOSE
C
      DO 512 I = 1,N $ DO 512 J = 1,N $ EE(I,J) = 0
      DO 512 K = 1,6
512   EE(I,J) = EE(I,J) + GG(K,I)*SG(K,J)
      CALL INVERSE( EE, N, N, XN(1), 1.0E-9, L, XN(3), XN(5) )
C
C  XX = EE*T(SG) = EE*T(GG)*T(SS)
C
      DO 514 I = 1,N $ DO 514 J = 1,6 $ XX(I,J) = 0
      DO 514 K = 1,N
514   XX(I,J) = XX(I,J) + EE(I,K)*SG(J,K)
C
C  YY = SG*XX = SS*GG*EE*T(GG)*T(SS)
C
      DO 518 I = 1,6 $ DO 518 J = 1,6 $ YY(I,J) = 0
      DO 518 K = 1,N
518   YY(I,J) = YY(I,J) + SG(I,K)*XX(K,J)
C
C  POSITIVE-END FORCE
C
```

```
      DO 552 I = 1,6
      FP(I) = 0
      DO 551 J = 1,6
551   FP(I) = FP(I) + ( SS(I,J)-YY(I,J) )*DW(J)
552   FP(I) = FP(I) + FPP(I,M)
      DO 553 J = 1,2
      CHI(J) = ABS(FP(1))*SLIMIT(1,M) + (FP(4)*SLIMIT(4,M))**2
     1         + ABS(FP(5) + B(J,M)*FP(3))*SLIMIT(5,M)
     2         + ABS(FP(6) - B(J,M)*FP(2))*SLIMIT(6,M) - 1.0
      IF( PLASTIC(J,M) .AND. ( CHI(J) .GT. 0 ) )  ITERATE = .TRUE.
553   CONTINUE
C
C  YIELD SURFACE NORMALS
C
      IF( ITERATE )  570, 590
570   LOOP = LOOP + 1
      DO 572 J = 1,N
      V(1,J) = SIGN( SLIMIT(1,M), FP(1) )
      V(4,J) = 2.0*FP(4)*SLIMIT(4,M)**2
      V(5,J) = SIGN( SLIMIT(5,M), FP(5)+H(J)*FP(3) )
572   V(6,J) = SIGN( SLIMIT(6,M), FP(6)-H(J)*FP(2) )
      IF( LOOP .LT. 4 )  GO TO 505
      PRINT 581, TIME $  CALL EXIT
581   FORMAT(*-LOOP IS TOO LARGE AT TIME =*F8.5)
590   CONTINUE
C-------------------------C
C  TEST FOR ELASTIC RETURN  C
C-------------------------C
      DO 640 J = 1,N
      BB(J) = 0
      DO 620 K = 1,6
620   BB(J) = BB(J) + XX(J,K)*DW(K)
      IF( BB(J) .GT. 0 )  GO TO 640
      IF( .NOT.PLASTIC(1,M) .AND. PLASTIC(2,M) )  J = 2
      PLASTIC(J,M) = .FALSE.
      IF( BETA .GT. 0 )  CALL FMATRIX( M )
      L = M $ IF( J .EQ. 2 ) L = -M
      PRINT 624, L, TIME, CHI(J)
624   FORMAT(*-MEMBER*I3* RETURNS  TIME=*F8.5*  CHI=*F12.8)
640   CONTINUE
C
C  PLASTIC MEMBER DISTORTION
C
      DO 660 I = 1,6
      DO 660 J = 1,N
660   PDISP(I,M) = PDISP(I,M) + GG(I,J)*BB(J)
C--------------------C
C  TEST FOR YIELDING  C
C--------------------C
700   DO 780 J = 1,2
```

```
      IF( PLASTIC(J,M)   )  GO TO 780
      IF( CHI(J) .LT. 0 )  GO TO 760
      PLASTIC(J,M) = YIFLDED(1,M) = .TRUE.
      IF( BETA .GT. 0 )  CALL FMATRIX( M )
      L = M $ IF( J .EQ. 2 ) L = -M
      PRINT 724, L, TIME, PHI(J,M)
724   FORMAT(*-MEMBER*I3* YIELDS    TIME=*F8.5*  PHI=*F12.8)
      GO TO 400
760   PHI(J,M) = CHI(J)
780   CONTINUE
C-------------------C
C  MEMBER END FORCES  C
C-------------------C
800   FPP(5,M) = FP(5)
      FPP(6,M) = FP(6)
      FNN(5,M) =-FP(5) - LENGTH(M)*FP(3)
      FNN(6,M) =-FP(6) + LENGTH(M)*FP(2)
      DO 830 I = 1,4
      FPP(I,M) = FP(I)
830   FNN(I,M) =-FP(I)
880   CONTINUE
      END



      SUBROUTINE SMATRIX( BETA, MLINEAR, GLINEAR )
C---------------------------------------------------------------C
C  GEOMETRICALLY NONLINEAR VRESION, ELLIP. YIELD FUNCTION  C
C---------------------------------------------------------------C
      LOGICAL ENDP, ENDN, ENDS, MLINEAR, GLINEAR, ITERATE
      DIMENSION DUP(6), DUN(6), FP(6), FN(6), VP(6), VN(6)
      DIMENSION PHI(2,100), CHI(2), BB(2), EE(2,2), XN(6)
      DIMENSION GP(6,2), GN(6,2), EGPT(2,6), EGNT(2,6)
      DIMENSION GPEGPT(6,6), GPEGNT(6,6), GNEGNT(6,6)
      DIMENSION SPP(6,6), SPN(6,6), SNN(6,6)
      DATA (SPP = 36(0)), (SPN = 36(0)), (SNN = 36(0))
      COMMON/ TIME/ TIME, DT
      COMMON/ RM/ RM(3,3,100)
      COMMON/ SLIMIT/ SLIMIT(6,100)
      COMMON/ JPJN/ JP(100), JN(100)
      COMMON/ STIFF/ S(8,100), B(2,100)
      COMMON/ SIZE/ MEMBERS, JOINTS, JFREE
      COMMON/ FORCE/ FPP(6,100), FNN(6,100)
      COMMON/ LENGTH/ LENGTH(100) $ REAL LENGTH
      COMMON/ PLASTIC/ PLASTIC(2,100) $ LOGICAL PLASTIC
      COMMON/ YIELDED/ YIELDED(2,100) $ LOGICAL YIELDED
      COMMON/ DIS/ DIS(6,50), DDIS(6,50),
     1        PDISP(6,100), PDISN(6,100)
      DO 880 M = 1,MEMBERS
```

```
C-----------------------------------------C
C   INCREMENTAL MEMBER END DISPLACEMENTS  C
C-----------------------------------------C
      JPM = JP(M)
      JNM = JN(M)
      DO 105 I = 1,3
      DUP(I) = DUP(I+3) = DUN(I) = DUN(I+3) = 0
      DO 105 J = 1,3
      DUP(I)   = DUP(I)   + RM(I,J,M)*DDIS(J,JPM)
      DUP(I+3) = DUP(I+3) + RM(I,J,M)*DDIS(J+3,JPM)
      DUN(I)   = DUN(I)   + RM(I,J,M)*DDIS(J,JNM)
105   DUN(I+3) = DUN(I+3) + RM(I,J,M)*DDIS(J+3,JNM)
C-----------------------------------------------C
C  FACTORS NEEDED IN MEMBER STIFFNESS MATRICES  C
C-----------------------------------------------C
      IF( GLINEAR )  GO TO 260
      P = FNN(1,M)
      ABSP = ABS( P )
C
C  TEMP = 1 PER CENT OF SMALLEST EULER LOAD
C
      TEMP = 0.025*S(8,M)
      IF( ABSP .LT. TEMP )  GO TO 260
      PHI2 = SQRT( ABSP/S(7,M) )
      PHI3 = SQRT( ABSP/S(8,M) )
      IF( P )  220, 260, 240
220   C2 = COS( PHI2 ) $ S2 = SIN( PHI2 )
      C3 = COS( PHI3 ) $ S3 = SIN( PHI3 )
      GO TO 250
240   C2 = 0.5*EXP( PHI2 ) $ S2 = EXP(-PHI2 )
      C3 = 0.5*EXP( PHI3 ) $ S3 = EXP(-PHI3 )
      C2 = C2 + 0.5*S2 $ S2 = C2 - S2
      C3 = C3 + 0.5*S3 $ S3 = C3 - S3
250   TEMP = -SIGN( 1.0, P )
      GARB = TEMP/( 2.0*( 1.0-C2 )/PHI2 - TEMP*S2 )
      H21 = GARB*( S2 - C2*PHI2 )
      H22 = GARB*( PHI2 - S2 )
      H23 = H21 + H22
      GARB = TEMP/( 2.0*( 1.0-C3 )/PHI3 - TEMP*S3 )
      H31 = GARB*( S3 - C3*PHI3 )
      H32 = GARB*( PHI3 - S3 )
      H33 = H31 + H32
      RHO2 = ( RM(3,1,M)*( DIS(1,JPM)-DIS(1,JNM) )
     1          + RM(3,2,M)*( DIS(2,JPM)-DIS(2,JNM) )
     2          + RM(3,3,M)*( DIS(3,JPM)-DIS(3,JNM) ) )/LENGTH(M)
      RHO3 = ( RM(2,1,M)*( DIS(1,JNM)-DIS(1,JPM) )
     1          + RM(2,2,M)*( DIS(2,JNM)-DIS(2,JPM) )
     2          + RM(2,3,M)*( DIS(3,JNM)-DIS(3,JPM) ) )/LENGTH(M)
      TEMP = P/LENGTH(M)
      GO TO 280
```

```
260    CONTINUE
       H31 = H21 = 4.0 $ H32 = H22 = 2.0 $ H33 = H23 = 6.0
       RHO3 = RHO2 = TEMP = 0
280    CONTINUE
C----------------------------C
C  MEMBER STIFFNESS MATRICES  C
C----------------------------C
       SPP(1,1) = SNN(1,1) = S(1,M)
       SPP(4,4) = SNN(4,4) = S(4,M)
       SPP(5,5) = SNN(5,5) = H31*S(5,M)
       SPP(6,6) = SNN(6,6) = H21*S(6,M)
       SPP(1,2) = SPP(2,1) = SNN(1,2) = SNN(2,1) = RHO3*SPP(1,1)
       SPP(1,3) = SPP(3,1) = SNN(1,3) = SNN(3,1) =-RHO2*SPP(1,1)
       SPP(2,3) = SPP(3,2) = SNN(2,3) = SNN(3,2) = RHO3*SPP(1,3)
       SPP(2,2) = SNN(2,2) = TEMP + RHO3*SPP(1,2) + H23*S(2,M)
       SPP(3,3) = SNN(3,3) = TEMP - RHO2*SPP(1,3) + H33*S(3,M)
       DO 310 J = 1,3 $ DO 310 I = 1,3
310    SPN(I,J) =-SPP(I,J)
       SPN(4,4) =-SPP(4,4)
       SPN(5,5) = H32*S(5,M) $ TEMP = H23*S(7,M)
       SPN(6,6) = H22*S(6,M) $ GARB = H33*S(8,M)
       SPP(2,6) = SPP(6,2) = SPN(2,6) = TEMP
       SNN(2,6) = SNN(6,2) = SPN(6,2) =-TEMP
       SPP(3,5) = SPP(5,3) = SPN(3,5) =-GARB
       SNN(3,5) = SNN(5,3) = SPN(5,3) = GARB
C---------------------------------------C
C  END FORCES IF MEMBER M NOT YIELDING  C
C---------------------------------------C
400    CONTINUE
       ENDP = ENDN = ENDS = .FALSE.
       IF( PLASTIC(1,M) .AND. PLASTIC(2,M) )  ENDS = .TRUE.
       IF( .NOT.ENDS .AND. PLASTIC(1,M) )  ENDP = .TRUE.
       IF( .NOT.ENDS .AND. PLASTIC(2,M) )  ENDN = .TRUE.
       IF( ENDS .OR. ENDP .OR. ENDN )  GO TO 480
       DO 440 I = 1,6
       FP(I) = FN(I) = 0
       DO 420 J = 1,6
       FP(I) = FP(I) + SPP(I,J)*DUP(J) + SPN(I,J)*DUN(J)
420    FN(I) = FN(I) + SPN(J,I)*DUP(J) + SNN(I,J)*DUN(J)
       FP(I) = FP(I) + FPP(I,M)
440    FN(I) = FN(I) + FNN(I,M)
       CHI(1) = (FP(1)*SLIMIT(1,M))**2 + (FP(4)*SLIMIT(4,M))**2
      1 - 1.0 + (FP(5)*SLIMIT(5,M))**2 + (FP(6)*SLIMIT(6,M))**2
       CHI(2) = (FN(1)*SLIMIT(1,M))**2 + (FN(4)*SLIMIT(4,M))**2
      1 - 1.0 + (FN(5)*SLIMIT(5,M))**2 + (FN(6)*SLIMIT(6,M))**2
       IF( MLINEAR )  800, 700
480    CONTINUE
C-----------------------------------C
C  END FORCES IF MEMBER M YIELDING  C
C-----------------------------------C
```

```
      LOOP = 1
C
C   YIELD SURFACE NORMALS
C
      IF( ENDN )   GO TO 501
      VP(1) = FPP(1,M)*SLIMIT(1,M)**2
      VP(4) = FPP(4,M)*SLIMIT(4,M)**2
      VP(5) = FPP(5,M)*SLIMIT(5,M)**2
      VP(6) = FPP(6,M)*SLIMIT(6,M)**2
      IF( ENDP )   GO TO 505
501   VN(1) = FNN(1,M)*SLIMIT(1,M)**2
      VN(4) = FNN(4,M)*SLIMIT(4,M)**2
      VN(5) = FNN(5,M)*SLIMIT(5,M)**2
      VN(6) = FNN(6,M)*SLIMIT(6,M)**2
C
C   GP, GN, AND EE MATRICES
C
505   ITERATE = .FALSE.
      IF( ENDN )   GO TO 513
      DO 510 J = 1,6
      GP(J,1) = SPP(J,1)*VP(1) + SPP(J,4)*VP(4)
     1         + SPP(J,5)*VP(5) + SPP(J,6)*VP(6)
510   GN(J,1) = SPN(1,J)*VP(1) + SPN(4,J)*VP(4)
     1         + SPN(5,J)*VP(5) + SPN(6,J)*VP(6)
      EE(1,1) =  GP(1,1)*VP(1) +  GP(4,1)*VP(4)
     1         +  GP(5,1)*VP(5) +  GP(6,1)*VP(6)
      IF( ENDP )   GO TO 523
513   DO 520 J = 1,6
      GP(J,2) = SPN(J,1)*VN(1) + SPN(J,4)*VN(4)
     1         + SPN(J,5)*VN(5) + SPN(J,6)*VN(6)
520   GN(J,2) = SNN(J,1)*VN(1) + SNN(J,4)*VN(4)
     1         + SNN(J,5)*VN(5) + SNN(J,6)*VN(6)
      EE(2,2) =  GN(1,2)*VN(1) +  GN(4,2)*VN(4)
     1         +  GN(5,2)*VN(5) +  GN(6,2)*VN(6)
      IF( ENDN )   GO TO 533
      EE(1,2) =  GP(1,2)*VP(1) +  GP(4,2)*VP(4)
     1         +  GP(5,2)*VP(5) +  GP(6,2)*VP(6)
      EE(2,1) = EE(1,2)
      CALL INVERSE( EE, 2, 2, XN(1), 1.0E-9, L, XN(3), XN(5) )
523   IF( ENDP )   EE(1,1) = 1.0/EE(1,1)
533   IF( ENDN )   EE(2,2) = 1.0/EE(2,2)
C
C   GPEGPT = GP*EE*T(GP)     T MEANS TRANSPOSE
C   GPEGNT = GP*EE*T(GN)   GNEGNT = GN*EE*T(GN)
C
      IF( ENDS )   GO TO 540
      J = 1 $ IF( ENDN ) J = 2
      DO 535 K = 1,6
      EGPT(J,K) = EE(J,J)*GP(K,J)
      EGNT(J,K) = EE(J,J)*GN(K,J)
```

```
      DO 535 I = 1,6
      GPEGPT(I,K) = GP(I,J)*EGPT(J,K)
      GPEGNT(I,K) = GP(I,J)*EGNT(J,K)
535   GNEGNT(I,K) = GN(I,J)*EGNT(J,K)
      GO TO 550
540   DO 545 K = 1,6
      EGPT(1,K) = EE(1,1)*GP(K,1) + EE(1,2)*GP(K,2)
      EGPT(2,K) = EE(2,1)*GP(K,1) + EE(2,2)*GP(K,2)
      EGNT(1,K) = EE(1,1)*GN(K,1) + EE(1,2)*GN(K,2)
      EGNT(2,K) = EE(2,1)*GN(K,1) + EE(2,2)*GN(K,2)
      DO 545 I = 1,6
      GPEGPT(I,K) = GP(I,1)*EGPT(1,K) + GP(I,2)*EGPT(2,K)
      GPEGNT(I,K) = GP(I,1)*EGNT(1,K) + GP(I,2)*EGNT(2,K)
545   GNEGNT(I,K) = GN(I,1)*EGNT(1,K) + GN(I,2)*EGNT(2,K)
C
C  END FORCES
C
550   DO 552 I = 1,6
      FP(I) = FN(I) = 0
      DO 551 J = 1,6
      FP(I) = FP(I) + ( SPP(I,J)-GPEGPT(I,J) )*DUP(J)
     1              + ( SPN(I,J)-GPEGNT(I,J) )*DUN(J)
551   FN(I) = FN(I) + ( SPN(J,I)-GPEGNT(J,I) )*DUP(J)
     1              + ( SNN(I,J)-GNEGNT(I,J) )*DUN(J)
      FP(I) = FP(I) + FPP(I,M)
552   FN(I) = FN(I) + FNN(I,M)
      CHI(1) = (FP(1)*SLIMIT(1,M))**2 + (FP(4)*SLIMIT(4,M))**2
     1 - 1.0 + (FP(5)*SLIMIT(5,M))**2 + (FP(6)*SLIMIT(6,M))**2
      CHI(2) = (FN(1)*SLIMIT(1,M))**2 + (FN(4)*SLIMIT(4,M))**2
     1 - 1.0 + (FN(5)*SLIMIT(5,M))**2 + (FN(6)*SLIMIT(6,M))**2
      IF( PLASTIC(1,M) .AND. (CHI(1) .GT. 0) ) ITERATE = .TRUE.
      IF( PLASTIC(2,M) .AND. (CHI(2) .GT. 0) ) ITERATE = .TRUE.
C
C  YIELD SURFACE NORMALS
C
      IF( ITERATE )  570, 590
570   LOOP = LOOP + 1
      IF( ENDN )  GO TO 571
      VP(1) = FP(1)*SLIMIT(1,M)**2
      VP(4) = FP(4)*SLIMIT(4,M)**2
      VP(5) = FP(5)*SLIMIT(5,M)**2
      VP(6) = FP(6)*SLIMIT(6,M)**2
      IF( ENDP )  GO TO 572
571   VN(1) = FN(1)*SLIMIT(1,M)**2
      VN(4) = FN(4)*SLIMIT(4,M)**2
      VN(5) = FN(5)*SLIMIT(5,M)**2
      VN(6) = FN(6)*SLIMIT(6,M)**2
572   IF( LOOP .LT. 4 )  GO TO 505
      PRINT 581, TIME $  CALL EXIT
581   FORMAT(*-LOOP IS TOO LARGE AT TIME =*F8.5)
```

```
590    CONTINUE
C---------------------------C
C  TEST FOR ELASTIC RETURN  C
C---------------------------C
       DO 640 J = 1,2
       IF( PLASTIC(J,M) )  610, 640
610    BB(J) = 0
       DO 620 K = 1,6
620    BB(J) = BB(J) + FGPT(J,K)*DUP(K) + FGNT(J,K)*DUN(K)
       IF( BB(J) .GT. 0 )  GO TO 640
       PLASTIC(J,M) = .FALSE.
       L = M $ IF( J .EQ. 2 ) L = -M
       PRINT 624, L, TIME, CHI(J)
624    FORMAT(*-MEMBER*I3* RETURNS  TIME=*F8.5*  CHI=*F12.8)
640    CONTINUE
C
C  PLASTIC MEMBER END DISPLACEMENTS IN GLOBAL COORD. SYSTEM
C
       IF( ENDN )  660, 650
650    DO 655 I = 1,3
       PDISP(I,M)   = PDISP(I,M)   + RM(1,I,M)*BB(1)*VP(1)
       DO 655 J = 1,3
655    PDISP(I+3,M) = PDISP(I+3,M) + RM(J,I,M)*BB(1)*VP(J+3)
       IF( ENDP )  670, 660
660    DO 665 I = 1,3
       PDISN(I,M)   = PDISN(I,M)   + RM(1,I,M)*BB(2)*VN(1)
       DO 665 J = 1,3
665    PDISN(I+3,M) = PDISN(I+3,M) + RM(J,I,M)*BB(2)*VN(J+3)
670    CONTINUE
C--------------------C
C  TEST FOR YIELDING  C
C--------------------C
700    DO 780 J = 1,2
       IF( PLASTIC(J,M)  )  GO TO 780
       IF( CHI(J) .LT. 0 )  GO TO 760
       YIELDED(J,M) = PLASTIC(J,M) = .TRUE.
       L = M $ IF( J .EQ. 2 ) L = -M
       PRINT 724, L, TIME, PHI(J,M)
724    FORMAT(*-MEMBER*I3* YIELDS   TIME=*F8.5*  PHI=*F12.8)
       GO TO 400
760    PHI(J,M) = CHI(J)
780    CONTINUE
C--------------------C
C  MEMBER END FORCES  C
C--------------------C
800    DO 830 I = 1,6
       FPP(I,M) = FP(I)
830    FNN(I,M) = FN(I)
880    CONTINUE
       END
```