ABSTRACT

MODIFIED KUBELKA-MUNK ANALYSIS FOR THE
APPROXIMATE THEORY OF MULTIPLE SCATTERING

by

Kaeo Songkhao

The transmission and reflection of light in light-scattering layers is discussed on the basis of a set of simultaneous differential equations originally due to Schuster. Formulas previously developed by Kubelka and Munk are briefly recapitulated and then extended to describe the exponential solutions, the semi-hyperbolic solutions, and hyperbolic solutions. The shortcomings of these formulas for practical applications is well known. To overcome these shortcomings, appropriate formulas have been developed. To test their validity, experiments were performed with pigmented lacquer films as specimens. Experimental and calculated results for lacquers containing individual pigments, and for lacquer containing much of the pigment have been compared in terms of infinite reflectance and CIE chromaticity coordinates. Tables of various parameters necessary for application of the formulas developed in this study have been constructed. Computer programs are given as well.

Comparison of experimental and calculated results shows that the method developed in the present study seem to give reliable estimates of reflectance and color both in the individual pigment and mixed-pigment

lacquer films. The method appears to be more accurate and convenient than the previous methods used.

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CHAPTER I

INTRODUCTION

1.1 Preliminary

The optical properties of a material comprise all of the modes whereby it interacts with light, or, more generally, with electromagnetic radiation. When radiant energy, in general, is transmitted through gases, liquids, or solids, radiation intensity, wavelength, direction of propagation, and plane of vibration may be affected by one or more of the following propagation phenomena [1]; reflection of radiation at the interface between two media which differ optically, refraction at the interface between the two media such that the direction of the beam undergoes a discontinuous change, absorption by which the energy of the beam is attenuated as it propagates through an absorbing medium, scattering of the radiation by particles of random distribution and polarization which changes or discriminates against the direction of the oscillating electromagnetic field. By far the most widespread mechanisms of the interaction, of light with matter are scattering and absorption. These processes are responsible for a wide variety of phenomena, including translucency, opacity and color.

Of the many methods in use for the characterization of physical-chemical properties of materials, light scattering is potentially one of the most useful methods, yet much under utilized. All matter scatters light, specular reflectance being merely a special case of backward scattering. This phenomenon is discussed in detail by Jenkins and White [2] in their chapter "Absorption and Scattering." If a scattering system is highly dense it may be called a turbid medium, where in the light scattered from one particle illuminate another particle at another point in the sample (secondary scattering), which particle in turn serves as the light source for still other particles (multiple scattering). Numerous authors have studied the propagation of light through such turbid media as a stellar atmosphere, translucent paper, opal glass, photographic emulsions, and so on. Although the optical phenomena of these are the same, the treatment may be different. The optics of turbid media have been widely studied, yet there is no satisfactory theory of their transmittance and reflectance properties. The exact theory of scattering from a turbid media is very complex. The mathematical treatment of multiple-scattering processes leads to a formidable mathematical situation, and solutions exist for only a limited number of situations. For this reason, in practice, it is of great importance to find simple analytical expressions which provide useful approximations to the exact solutions. The most widely accepted theory in the study of turbid media has been the Kubelka and Munk theory [3], which is, in fact, based on an earlier one put forward by the astronomer Schuster [4], who was studying the attenuation of light from stars by scattering and absorption before reaching an observer. The Kubelka and Munk theory contains quite a number of assumed simplifications. It is strictly applicable only to ideally diffuse light, i.e. light of unoriented radiation, both the incident light and that within the layer. In addition, the theory ignores optical phenomena at interfaces. Thus, other theories have been developed [5,6,7] which, in taking these deviations into account, are necessarily more complicated than the Kubelka and Munk theory. For several reasons, however, none of these theories have been accepted. A number of papers dealing with the analysis of the Kubelka and Munk theory have appeared from time to time in the literature of various fields. It is true, however, improvements in the Kubelka and Munk theory are still needed.

1.2 Purpose and Motivation

Although the success of the Kubelka-Munk theory in solving a number of optical problems in turbid media is rather impressive, it should not be overlooked that it is based on a fairly simple model which sometimes does not meet practical conditions. This theory assumes that the optical behavior of a layer which simultaneously scatters and absorbs can be described only by two terms, viz., the scattering and absorption coefficients. The theory results in solutions for the degrees of transmission and reflectance as a function of scattering coefficient, absorption coefficient and layer thickness, and including an optional substrate as well as a special case.

Often the question arises whether the Kubelka and Munk model is not too simple and should be improved. Instead of two fluxes, other authors [8,9,10] have studied models using four, six and more fluxes defined by different directions in space and associated with an increasing number of constants for every specimen or with assumptions about the spatial distribution of stray light from single scattering. The most complete theory is that of radiative transfer, leading to integral equations which can only be solved numerically with large computers. If the results of such calculations are compared with those using the Kubelka-Munk solutions, the empirical experience is confirmed that the simple Kubelka-Munk solution is very good approximation for various applications[11].

Recent literature on color in industry indicates a considerable need for an improvement in the Kubelka and Munk approach. Improvements of procedure and method in applying the Kubelka-Munk theory for the present-day practice is the purpose of this study. New formulas have been derived and we also have developed a procedure that will lead to more accurate and precise use of spectrophotometers and colorimeters. Experimentation had been performed to test the validity of these formulas and procedure in a variety of cases of single and multiple pigment systems. The limitations of these formulas and methods have also been specified. Although the method and formulas developed in this study were tested by applying them to a pigmented film it is expected nevertheless that many of the relations obtained will be helpful in other applications as well. The pigmented film was selected as a model for this study not only because it is the simplest model but also because of the importance to the field of surface coatings.

Since the solution of Kubelka-Munk analysis depends on the nature of the specimen including composition and preparation procedure, the specimen system including composition and preparation procedure, the specimen system used in this study is described fully.

Although the Kubelka-Munk theory treats both transmission and reflectance, the present study is restricted to the reflectance problem only. All surfaces reflect a certain amount of the incident energy. The nature of reflected energy depends not only on the type of incident beam, but also on the physical properties of the material under examination. Hence measurement of reflectance be used to study the optical properties of materials. The study of reflectance spectra has not yet become widespread because of the difficulties of interpreting them. Reflectance measurements are used widely in many applications, however, for example, in the fields of textiles, paints, paper, ceramics, metals, mining, and printing. In this study, our primary objective is to determine for several pigments the reflectance of an infinite-thickness film, the scattering and absorption coefficients of individual pigmented films, and then use them to predict the reflectance for any mixture of these pigments in an opaque film. This is possible because the scattering and absorption coefficients are mathematically related to the reflectance, and their combined effect is proportional to concentration of the pigment in a mixture.

1.3 Organization of Dissertation

The background of the problem and a review of turbid media theories are described in full detail in the beginning of Chapter II. The various derivations of the Kubelka-Munk theory and the method and formulas proposed in this study including their limitations are also described in that chapter. The related theories used in testing the validity of the method and formulas, color space, color mixture theories, and color difference formulas are also described briefly in Chapter II.

In testing the method and formulas proposed in this study, the pigmented lacquer films are used as the experimental system. The background knowledge of pigmentation and the specimen preparation are described in full detail in the first part of Chapter III. The description of the instruments, precautions, and procedures in measuring the specimen are also in Chapter III. The methods of calculation by using the method and formulas proposed in this study have been shown in the last part of Chapter III.

Results are given in Chapter IV.

The appendix includes Tables and Computer programs.

CHAPTER II

THEORY

2.1 Review of Theories of Turbid Media

The problem of calculating the optical properties of media which both scatter and absorb light is common to many disciplines [12] and it is related mathematically to the problem of describing the diffusion of neutrons [13] and the phenomenon of radiative transfer [14]. Consequently, there is a large volume of widely dispersed and loosely connected literature. The major theoretical developments in turbid medium theory have resulted from the work of astrophysicists and geophysicists interested in stars and planetary atmospheres. Important computation methods also have been developed by nuclear physicists, who face a similar problem when calculating the flux of neutrons in a reactor. Workers concerned with the optical properties of surface coatings, plastics, ceramics, photographic emulsions, papers, and textiles have developed a large literature which is poorly related to the broader material, but, nevertheless, has served the needs of these industries quite well [15].

Mathematical descriptions of the optics of turbid media may be divided into two categories [9]: (1) extremely general and broad treatments, often by astrophysicists, which, in most cases, lead to systems of integro-differential equations which cannot be solved analytically; (2) restricted special cases which may not describe the physical situation precisely, but do so adequately with respect to observable phenomena, and are mathematically tractable. Excellent and unusually broad reviews of these literatures have been written by Billmeyer et al. [16] and by Orchard [15].

Exact solutions of the multiple-scattering problem in turbid media have been discussed by Chandrasekhar [14], Grosjean [11], Case [18], Kourganoff [13], Sobolev [19], Van de Hulst [20], Chu and Churchill [21], Orchard [22] and Cheng [23]. Practical application of these integral solutions exist only for a limited number of situations, however, and the results so far are in a form too complex for ready application to cases of interest. Another, more recent, approach is the Monte Carlo method of random walk, using a digital computer [24]. This is not a good technique for thick, strongly scattering layers, where several hundred orders of scatter must be taken into account [25]. For this reason, in practice, it is convenient to use approximate theories since the latter can be used to obtain comparatively simple solutions, and correlations between the light scattering characteristics are discovered.

The theoretical treatments applied to turbid media can be classified roughly into those which deal with the problem in one dimension and those which consider it in two or three dimensions. The most widely adopted approach by far is the one-dimensional. In the one dimensional case, the medium is considered as bounded by two parallel plane and the propagation of the light is studied along a direction perpendicular to those planes. Thus the one-dimensional problem can be mathematically characterized by stating that everything depends on the thickness only, and on no other coordinate or angle. Hence planarity of the front and back surfaces is essential and any deviation from this condition would remove the one-dimensional nature of the problem. When the surfaces are irregular it is necessary to use the general three-dimensional theory of radiative transfer, which is well documented by Chandrasekhar [14]. Most theories for turbid media deal with plane parallel scattering layers and the two-flux method, partly because the theory for this geometry is simpler than for nearly all other cases, and also because most practical problems involve this geometry. Besides the advantages which they present, however, such approximations also may suffer from drawbacks with various degrees of importance, so that several authors [5,16,26,27,28,29] in recent years have tried to construct increasingly better formalisms. For simplicity and uniformity, in this study we also adopt the one-dimensional, two-flux model.

The earliest work related to (although not specifically concerned with) turbid media of one-dimensional theory appears to be a paper by Stokes in 1862 [30]. He was interested in evaluating the reflectance of a pile of glass plates each of which may or may not exhibit absorption. Beginning with the classic work of Stokes, a number of theories have been evolved during the past century to account for the observable optical properties of such light-scattering, light-absorbing media as piles of plates or films, translucent or partially opaque films, coatings and sheets.

The first paper of importance on the theory of multiple scattering of light in turbid media was written in 1905 by the astrophysicist Schuster [4], who suggested that the complicated situation of light traveling in all directions and being scattered and absorbed at all places in a turbid medium could be simplified by focusing attention on the rate of energy transfer, or flux, in the forward direction and in the reverse direction through the medium. Thus only two quantities are to be calculated at each point in the medium. It turns out that the differential equations which describe these two quantities are simple and easily solved. Several papers describing applications of these equations appeared before 1930, mostly published by astrophysicists and workers in the photographic industry [31,32,33,34].

In 1931, Kubelka and Munk [3] used essentially the same formalism in their treatment of light-scattering layers, suppressing the terms corresponding to the generation of light within the medium. Their work is very similar to that of Schuster, with the omission of the emission terms. Similar one-dimensional, two-quantities theories have been derived by several other authors [5,35,36,37,38]. The more inclusive historical review is summarized in the work of Beasley _et al_ [9].

Ingle [39] has shown that the work by Smith, Amy and Stokes are merely special cases of the Kubelka and Munk theory. Kubelka [40] has shown that the work by Gurevic is also a special case of the Kubelka and Munk theory. The K.M hyperbolic equations for reflectance and transmittance of a turbid, isotropically scattering medium have been found formally identical with the solution for reflected and transmitted fluxes of Chandrasekhar's radiative transfer equation for isotrophically highly scattering media [41]. The general equations of Kubelka and Munk are regarded as including all others of this type as special cases. This is why the most widely accepted theory in the study of multiple-scattering media is the Kubelka and Munk theory. Thus we will restrict our study to the Kubelka and Munk (hereafter K.M) theory.

2.2 Kubelka and Munk Theory

Excellent reviews of the principal theories have been given by Judd and Wyszecki [42], Kortum [43], Kubelka [40] and Ingle [34], and the latter two have demonstrated the equivalence of some of the theories. Additionally, Kubelka has presented a number of very useful equations stemming from the K.M theory.

The pioneering researches of Steele [44] and Judd [45], both of whom utilized the K.M theory, resulted in special solutions and various mathematical aids that have been responsible, at least in part, for the widespread use of this theory. Today, the K.M theory is employed for many scientific or industrial purposes, e.g., chromatographic systems [46], pharmaceutical products [47,48], paper and pulp material [44,49], textiles [50,51], foodstuff [54], radiant energy transfer [56], color measurements and color comparison [45,57,58,59,60,61,62,63,64,65].

2.2.1 Derivation of the K.M Equations

The K.M theory, the approximate two-parameter theory for multiple light scattering in a turbid medium has become popular for studying scattering media. This popularity is attributed to relative simplicity of its end formulas. The theory examines the transmission of a light beam through an infinite plane-parallel sheet of an isotropic scattering material illuminated on one side by a diffuse beam. It describes the light field by a system of two simple differential equations. The system of differential equations can be analysed under the assumption of an energy balance in the material from radiative transfer theory. The basic method is that of dividing the light flux into two parts: one flux, I, in the positive direction, and the other, J, in the negative direction

(Figure 2.1). The incident radiation is assuming for diffuse (i.e., the intensity is equal for all angles of incidence), and the radiation scattered sideways is assumed to be compensated for by an equal contribution from neighboring parts of the layer (i.e. the area investigated is either small in cross section compared with the total illuminated cross section of the sample or is large compared with the thickness of the sample). It also is assumed that the layer is illuminated with a monochromatic light source. For purposes of the calculus, the medium is taken homogeneous. The medium is assumed to be isotropic and not emit by radiation or fluoresce.

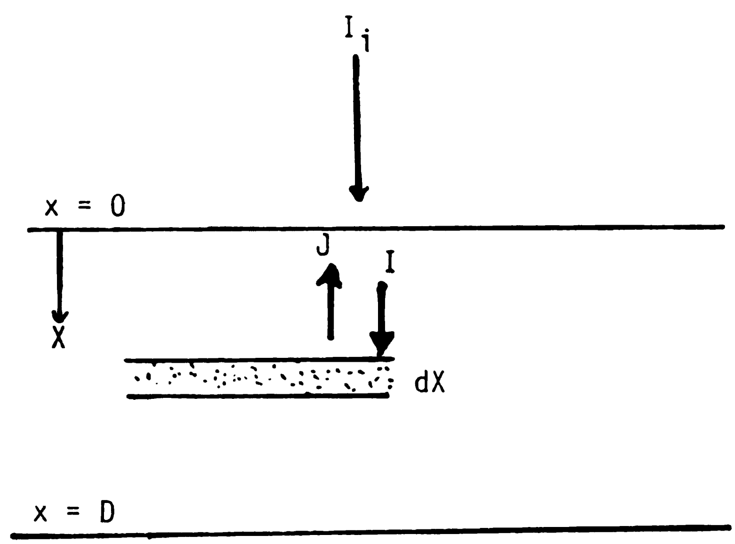

Fig. 2.1 Schematic representation of a layer of absorbing and light-scattering particles.

The following notation will be used:

X is the thickness of the layer. The positive X direction is taken downward, with X = 0 as the illuminated surface.

I_i is incident flux.

K is an absorption coefficient, defined by requiring that (KIdX) be the amount of the radiation absorbed from the flux I on passing through an infinitesimal layer dX.

S is a scattering coefficient, similarly defined by requiring that the flux scattered backward from I (and therefore added to J) in an infinitesimal layer dX is (SIdX).

On passing this layer, I will then be diminished by the amount absorbed and the amount scattered, but will be increased by the flux lost by scattering from J or:

$$dI = -(K + S)IdX + SJdX \tag{1}$$

$$dJ = +(K + S)JdX - SIdX \tag{2}$$

The signs have been chosen consistent with the fact that as X increases, I decreases, while J increases.

Equations (1) and (2) are the two fundamental simultaneous differentail equations which describe the absorption and scattering process for K.M theory.

It will be observed that only two directions of the incident and reflected beam are considered here, namely, those perpendicular to the surface of the layer. This is an approximation, but is considered to introduce a small error compared with that resulting from the approximation of ideal diffuse illumination within the sample [34].

These two fundamental simultaneous differential equations can be readily integrated, the general solutions being [56]:

$$I = A(1 - \beta)e^{\sigma X} + B(1 + \beta)e^{-\sigma X} \tag{3}$$

and

$$J = A(1 + \beta)e^{\sigma X} + B(1 - \beta)e^{-\sigma X} \tag{4}$$

where

$$\sigma = \sqrt{K(K + 2S)} \tag{5}$$

and

$$\beta = \sigma/(K + 2S) = \sqrt{K/(K + 2S)} \tag{6}$$

A and B are constants of integration to be determined by boundary conditions, σ is commonly referred to as the extinction coefficient, and β is an optical constant. If the integration is carried out over the entire thickness of the ideal diffusing sample layer, the appropriate boundary conditions are:

$$\begin{aligned} I &= I_0 \quad \text{for} \quad X = 0 \\ I &= I_d,\ J = 0 \quad \text{for} \quad X = X \end{aligned} \tag{7}$$

Solution for A and B under these conditions can be found, and it is now possible to obtain formulas of considerable practical importance from these solutions. For example, the transmittance of the layer is given by:

$$T = I_d/I_o = \frac{4\beta}{(1 + \beta)^2 e^{\sigma X} - (1 - \beta)^2 e^{-\sigma X}}$$

$$= \frac{2\beta}{(1 + \beta^2)\sinh\sigma X + 2\beta\cosh\sigma X} \qquad (8)$$

and the diffuse reflectance is

$$R = J_o/I_o = \frac{(1 - \beta^2)(e^{\sigma} - e^{-\sigma})}{(1 + \beta)^2 e^{\sigma X} - (1 - \beta)^2 e^{-\sigma X}}$$

$$= \frac{(1 - \beta^2)\sinh\sigma}{(1 + \beta^2)\sinh\sigma X + 2\beta\cosh\sigma X} \qquad (9)$$

As case of considerable interest is that in which the layer thickness of the sample is allowed to increase without bound, commonly referred to as infinite layer thickness. Setting $X = \infty$ in Equations (8) and (9), it is found that $T_\infty = 0$, as to be expected, and

$$R_\infty = \frac{1 - \beta}{1 + \beta} = \frac{1 - \sqrt{K/(K + 2S)}}{1 + \sqrt{K/(K + 2S)}} \qquad (10)$$

With some algebraic manipulation, Equation (10) can be written in the more familar form,

$$K/S = (1 - R_\infty)^2/2R_\infty \qquad (11)$$

The function K/S is commonly referred to as the Remission Function or K.M function.

The alternative method for solving the fundamental differential Equations (1) and (2), is by defining:

$$\frac{S + K}{S} = 1 + \frac{K}{S} = a \qquad (12)$$

Equations (1) and (2) may be written in the form

$$dI = - aSIdX + SJdX \qquad (13)$$

$$dJ = + aSJdX - SIdX \qquad (13')$$

If we divide Equation (13) by I, and Equation (13') by -J and add the two, we find, writing

$$J/I = r \quad , \qquad (14)$$

that $\quad -dr/SdX = r^2 - 2ar + 1$

or $$- S\int dX = \int \frac{dr}{r^2 - 2ar + 1} \qquad (15)$$

If the integration is carried out over the entire thickness of the sample layer, the appropriate limit of integration are

$r = (J/I) = R_g$ = reflectance of the background, for $X = X$

$r = (J/I) = R$ = reflectance of the layer, for $X = 0$

Equation (15) can be integrated by partial fractions to give:

$$2SX\sqrt{a^2 - 1} = \ln \frac{(R - a - \sqrt{a^2 - 1})(R_g - a + \sqrt{a^2 - 1})}{(R_g - a - \sqrt{a^2 - 1})(R - a + \sqrt{a^2 - 1})} \qquad (17)$$

When $X = \infty$, that is when the layer is of infinite thickness, $R_g = 0$ solve for R_∞ we have

$$R_\infty = 1/(a + \sqrt{a^2 - 1}) = a - \sqrt{a^2 - 1}$$

$$= 1 + K/S - \sqrt{K^2/S^2 + 2K/S} \qquad (18)$$

$$= \frac{S}{S + K + \sqrt{K(K + 2S)}}$$

With some algebraic manipulation, Eq. (18) can be written in the form of Eq. (11).

Writing $\sqrt{a^2 - 1} = b$ and rearranging Eq. (18), we can represent both a and b as functions of R_∞

$$a = 1/2(1/R_\infty + R_\infty) \text{ and } b = 1/2(1/R_\infty - R_\infty) \ , \qquad (19)$$

and a and b are mutually related by the expressions

$$b = \sqrt{a^2 - 1} \text{ and } a = \sqrt{b^2 + 1} \ , \qquad (20)$$

and are only used separately for convenient abbreviation. From Eq. (19) we have further

$$R_\infty = a - b = 1/(a + b) \tag{21}$$

If we insert Eq. (19) into Eq. (17), we obtain, for a layer of finite thickness X,

$$SX(1/R_\infty - R_\infty) = \ln\frac{(R - 1/R_\infty)(R_g - R_\infty)}{(R_g - 1/R_\infty)(R - R_\infty)} \tag{22}$$

or, resolving for R, we have

$$R = \frac{(1/R_\infty)(R_g - R_\infty) - R_\infty(R_g - 1/R_\infty)\exp[SX(1/R_\infty - R_\infty)]}{(R_g - R_\infty) - (R_g - 1/R_\infty)\exp[SX(1/R_\infty - R_\infty)]} \tag{23}$$

Equation (23) is a general expression for the exponential **solution** of K.M **theory.**

If we select an ideal black nonreflecting background, so that $R_g = 0$, and $R = R_o$, Eq. (23) becomes simplified to

$$R_o = \frac{\exp SX(1/R_\infty - R_\infty) - 1}{(1/R_\infty)\exp SX(1/R_\infty - R_\infty) - R_\infty} \tag{24}$$

Now, from Eq. (15) if the hyperbolic functions are used [34,38], the general solution is

$$SX = \int \frac{dr}{r^2 - 2\,ar + 1} = \frac{-2}{\sqrt{4a^2 - 4}} \begin{bmatrix} \text{Ar tanh} \\ \text{Arcoth} \end{bmatrix} \frac{2r - 2a}{\sqrt{4a^2 - 4}}$$

$$= \frac{1}{b} \begin{bmatrix} \text{Ar tanh} \\ \text{Arcoth} \end{bmatrix} \frac{a - r}{b}$$

Whether Artanh or Arcoth is to be used depends on the argument. Here $\frac{a - r}{b} \gg 1$, so that Arcoth must be used and we have

$$SX = \frac{1}{b} \left(\text{Arcoth}\, \frac{a - R}{b} - \text{Arcoth}\, \frac{a - R_g}{b}\right)$$

$$= \frac{1}{b}\, \text{Arcoth}\, \frac{b^2 - (a - R)(a - R_g)}{b(R_g - R)} \tag{25}$$

from which $$R = \frac{1 - R_g(a - b\coth bSX)}{a + b\coth bSX - R_g} \tag{26}$$

Equation (26) is the hyperbolic solution obtained from K.M theory which corresponds to Eq. (23), the exponential solution of K.M theory.

For an ideal black, non-reflecting background ($R_g = 0$, $R = R_o$), Eqs. (25) and (26) may be simplified to:

$$SX = \frac{1}{b}\, \text{Arcoth}\, \frac{1 - aR_o}{bR_o} \tag{27}$$

$$R_o = \frac{\sinh bSX}{a \sinh bSX + b \cosh bSX} \tag{28}$$

Using Eqs. (27) and (25), we obtain a, and hence R_∞ , as function of R, R_o, and R_g such that

$$a = 1/2(R + \frac{R_o - R + R_g}{R_o R_g}) \tag{29}$$

It may be noticed that in practical formulas, S always occurs together with the thickness X of the layer, that is as the product SX, called <u>Scattering Power</u> of the layer [45].

To find an analogous formula representing transmittance of such layer,

$$T = I_o/I_d \tag{30}$$

we multiply Eq. (28) by I; then by means of Eq. (14), we obtain for a layer of thickness X with a black background

$$J = R_o I = \frac{I}{a + b\coth bSX} \tag{31}$$

Inserting this into Eq. (13) we obtain

$$\frac{dI}{SI} = adX - \frac{dX}{a + b\coth bSX}$$

which now may be integrated. By replacing bSX = u, and $dX = \frac{du}{bs}$, we have

$$\frac{1}{S}\int_{I_o}^{I_d} \frac{dI}{I} = a\int_0^x dX + \frac{1}{bS}\int_0^{bSX} \frac{1}{a + b\coth u}\, du$$

and $\frac{1}{S} \quad \ln \frac{I_d}{I_o} = aX - \frac{1}{bS} \cdot \frac{1}{a^2 - b^2} (abSX - b \ln \frac{a\sinh bSX + b\cosh bSX}{b})$

Since according to Eq. (20), $a^2 - b^2 = 1$, we obtain finally

$$T = \frac{b}{a\sinh bSX + b\cosh bSX} \tag{32}$$

The inverse function of Eq. (32) is

$$SX = \frac{\operatorname{Arsinh} b/T - \operatorname{Arsinh} b}{b} \tag{33}$$

The exponential functions for the transmittance can be found by expressing a and b by Eq. (19), and inserting them in Eq. (32) we obtain by replacing the hyperbolic functions with exponential functions for the transmittance of the layer the expression:

$$T = \frac{(1 - R_\infty)^2 \exp(-\ bSX)}{1 - R_\infty^{\ 2} \exp(-\ 2bSX)} \tag{34}$$

which corresponds to Eq. (24) for the reflectance of the same layer using a black background. In these two equations T and R_o are represented as functions of R_∞ and SX.

From the relations so far derived, further equations can be obtained which relate the several variables. Thus, from Eqs. (28), (32) and (20), we arrive at

$$T = \sqrt{(a - R_o)^2 - b^2} \tag{35}$$

Many of the relations between the quantities R, R_g, R_0, R_∞, SX and T of K.M theory have been given in a composite list in a form suitable for practical use by Kubelka [40] and have been repeated and summarized by Judd and Wyszecki [42], and Kortum [43].

The usual current forms of the equations of K.M theory are in a semi-hyperbolic form. It is possible to rewrite these equations in fully hyperbolic forms which are both elegant and very convenient for calculation [60].

Define $Y = \ln(1/R_\infty)$ (36)

Eq. (20) then becomes:

$$a = \cosh Y \quad \text{and} \quad b = \sinh Y \tag{37}$$

Eq. (26), Eq. (27) and Eq. (32) become:

$$R = \frac{\sinh W - R_g \sinh(W - Y)}{\sinh(W + Y) - R_g \sinh W} \tag{38}$$

$$R_0 = \frac{\sinh W}{\sinh(W + Y)} \tag{39}$$

$$T = \frac{\sinh Y}{\sinh(W + Y)} \tag{40}$$

In these equations:

$$W = bSX = SX\sinh Y \tag{41}$$

Two useful subsidiary relations are:

$$F(R_\infty) = \frac{(1 - R_\infty)^2}{2R_\infty} = S\cosh Y - 1 \qquad (42)$$

$$R_\infty = \cosh Y - \sinh Y \qquad (43)$$

2.2.2 Use of the K.M Equations

The optical properties of turbid media can be calculated with fair success on the basis of K.M theory. This study is restricted to the consideration of turbid medium theory as applied to the problem of paint films. At the present time the K.M equations are almost universally used for multiple-scattering calculations in the paint industry. This wide acceptance is due to the fact that these calculations are easy and give rather good results in many situations. They have the drawback, however, that the scattering and absorption coefficients must be determined empirically, and then used only in similar systems under similar conditions of illumination and observation.

Judd and Wyszecki [42] provide an explanation with examples how K.M equations should be used to solve problems involving reflectance and opacity of colarant layers. Billmeyer and Abrams [67] have summarized the K.M equations for its practical application to all cases of interest plus those for other closely-related operations together with step-by-step procedures for their use.

There are many ways in which the solution to the basic K.M equations can be expressed depending upon the particular application under study. Two important cases for paints may be considered. The first one relates S and K to the reflectance of a film so thick that further increase in thickness does not change its reflectance, that is, optically an infinitely thick film. This is the use of Eq. (11). The second case applies to incompletely hiding films on a substrate. The equation of interest is Eq. (23) or Eq. (26). Note, however, that either case requires knowledge of the parameter a and R_∞. The application of these equations would be straightforward from the mathematical point of view except for the fact that, because of experimental errors it sometimes happens that a is slightly less than one, making b imaginary. This situation has no physical meaning, of course, but it is difficult to complete the computation for this case. Ross [68] has suggested a method to overcome this difficulty, which involves a series transformation for the computation of the hyperbolic functions. In such a case, however, the absorption coefficient K turns out to be negative, which means a medium must amplify light or it must fluoresce. The other problem is the reproducibility of R_∞. For dark paint, it usually is very simple to apply a sufficiently thick film to measure R_∞ directly, precisely, and accurately. Only as a paint becomes very white and low in hiding power does it become difficult to apply a film thick enough to satisfy the definition for R_∞.

2.2.3 Modification and Correction

2.2.3.1 Absolute and Relative Measurements

All the quantities arising in the K.M theory for a simultaneously scattering and absorbing layer are defined as the ratios of reflected or transmitted radiation to the incident radiation. In actual practice, however, only the transmittance of a layer may be determined absolutely, and diffuse reflectance is measurable only relative to a suitable white standard, such as MgO, $MgCO_3$, or $BaSO_4$. Under these conditions, one determines the ratio

$$R_{measured} = R_{sample}/R_{standard} \tag{44}$$

To determine the absolute R_{sample}, therefore, it is necessary to know the absolute reflectance R of the white standard. Many authors [69, 70,71,72] have measured and tabulated the absolute reflectance values of some white standards at various wave-lengths.

2.2.3.2 Surface Correction

Most of the practical problems associated with the use of K.M theory are due to neglect of the original postulates laid down for its application. The original K.M equations for reflectance, R, and transmittance, T, apply only to a planar scattering system of infinite extent, having no specularly reflecting boundaries and illuminated with diffuse light. In the derivation of the equations, Kubelka and Munk assumed that the refractive index of the layer is the same as that of the medium in which the measurements are made, which is not valid since usually the measurements are made in air. In practical use the K.M theory also must include the fact that light is reflected at the boundaries of the medium where the index of refraction changes abruptly.

It is convenient to consider first the energy balance just below the reflecting surface, and then to find the ratio of the incident and emergent fluxes (see Figure 2.2) [22]. Assume here that the upper boundary of the layer is a surface with specified directional reflection properties and that the lower boundary is a perfect absorber. If the upper surface is a specular reflector, its directional reflecting properties are described by Fresnel's equations. We now must take account of the interaction between the surface reflection and the true reflection of light by the diffuser.

Let R_m = apparent reflectance,

R = true reflectance,

T = true transmittance,

k_1 = external reflectance of surface

t_1 = external transmittance of surface = $1 - k_1$

k_2 = internal reflectance of surface

t_2 = internal transmittance of surface = $1 - k_2$

I_o = irradiance of surface.

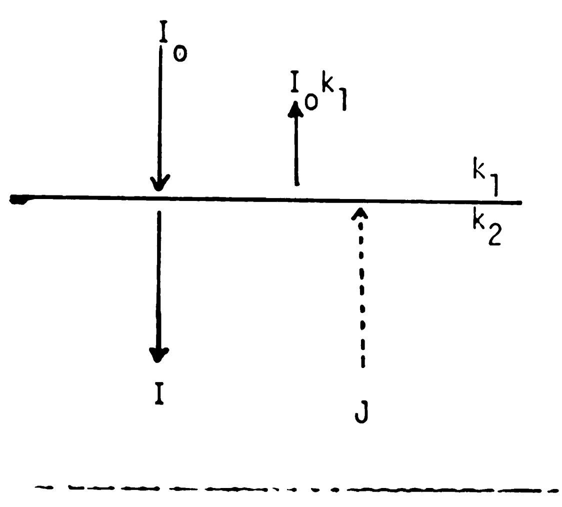

Fig. 2.2 Reflection from front surfaces of the layer.

In Figure 2.2 I is the reduced irradiance and J is the net reflected radiance just below the reflecting surface.
We have simply:

$$J = IR + Jk_2R \quad . \tag{45}$$

On writing $R^* = J/I$, we get

$$R^* = R + R^*k_2R \quad .$$

The apparent interior reflectance R^* is thus

$$R^* = J/I = R/(1 - k_2R) \quad . \tag{46}$$

Also we have

$$I = I_0t_1 \tag{47}$$

Now the emergent radiance (excluding the surface component) is, from Eqs. (46) and (47)

$$Jt_2 = I_0t_1t_2R^* \tag{48}$$

or

$$R_M = k_1 + \frac{t_1t_2R}{1 - k_2R} \tag{49}$$

This fact has been recognized by many earlier authors [5,28,57,58, 64,73,74,75,76,77] , but the Saunderson [57] correction seems to be the most widely used.

Saunderson used a General Electric recording spectrophotometer for the spectrophotometric measurements. In that instrument about half the specularly reflected light is lost through the entrance hole of the integrating sphere. His original surface correction was expressed as

$$R_m = \frac{k_1}{2} + (1 - k_1)\ (1 - k_2)\ \frac{R}{1 - K_2 R} \tag{50}$$

But the widely quoted equation that he used can be represented as Eq. (49) i.e.

$$R_m = k_1 + (1 - k_1)\ (1 - k_2)\ \frac{R}{1 - k_2 R} \tag{51}$$

Both k_1 and k_2 depend on, and may be calculated from, the refractive index of the sample for which the refractive index of the binder may normally be substituted [11]. This quantity depends on the wavelength of the light, but this small dependence is often neglected.

k_1 is given by the well-known Fresnel relationship

$$k_1 = \frac{(n - 1)^2}{(n + 1)^2} \tag{52}$$

where n is the ratio of the refractive index of the film to that of the surrounding medium. Corresponding values of the internal reflectance coefficient k_2 can be obtained from the relation [22]:

$$t_2 = t_1/n^2 \tag{53}$$

where t_1 = external transmittance of surface = $1 - k_1$

t_2 = internal transmittance of surface = $1 - k_2$

n = the refractive index of the film.

Eq. (53) follows directly from the definitions of t_1 and t_2 together with Snell's law of reflectance coefficients k_1 and k_2 with respect to the refractive index have been tabulated by Orchard [22].

For a glossy paint or other pigmented composition in a binder with a refractive index of about 1.5, k_1 is about 0.04. Saunderson found, by trial and error, that the value of 0.4 for k_2 produced the best color-matching prediction. A similar value has been obtained by Bridgeman [78]. Thus there is little doubt that $k_2 = 0.4$ is the optimum value for use in the application of Eq. (51) to practical color-matching problems. However, both theoretically and experimentally [74] k_2 has a value of 0.5 - 0.6. Many authors [78,79] use $k_2 = 0.6$.

Duncan [58] has developed from the correction equations of Saunderson [57] and Ryde [5], his formula for surface correction

$$R_m = \frac{(1 - k_1)(1 - k_2)R}{(1 - k_2R)} \tag{54}$$

He has used $k_1 = 0.04$ and $k_2 = 0.555$ for reflectance reading with the Beckman D-4 Spectrophotometer on glossy paint, and his formula then simplified to

$$R_m = \frac{0.427R}{1 - 0.555R} \tag{55}$$

This value of R_m is then used in the K.M function and the corresponding values of R_m and $F(R_M)$ have been tabulated [59] for convenient use.

Now if the lower boundary is not a perfect absorber, there will be a contribution from substrate reflectance. This modification has already been introduced by Kubelka and Munk, but, the complexity of the substrate reflectance is not generally appreciated [74]. It should be remembered that two types of reflectance can occur from the substrate of a film. The diffuse light transmitted through the film first encounters a boundary between the film and the bulk substrate. This boundary introduces a specular reflectance for the diffuse light, its magnitude depending on the relative refractive indexes of the film and the bulk substrate. Light that is transmitted across this boundary then can be reflected by scattering processes within the substrate. Generally, therefore, the reflectance of a substrate measured in air is not the value that applies when a film is laid on that substrate. This is due to the major changes in relative refractive index at the surface of the substrate and hence changes in the specular reflectance for diffuse light at the film-substrate boundary. An example of the magnitude of this effect has been given by Blevin [81]. To avoid this complicated phenomenon, it should be assumed that there is no incident flux on the substrate surface. This can be done in actual practice by using the ideal black substrate. In this study, therefore we will modify the technique of using K.M equations by deriving the required equations only for the case of the ideal black substrate.

2.3 Simplified Method for Computing K.M Coefficients

A knowledge of the absorption and scattering coefficients, K and S, respectively, of each optically significant component is required in using K.M equations to predict other optical properties of material. Unfortunately, these two coefficients are not available for use directly. In paint technology these two coefficients also depend on the paint system, and specific measurements which have only limited application. For example, the K.M absorption and scattering coefficients of a certain pigment from Manufacturer A might be measured and used for color formulation. If we wish to use instead another lot of pigment made by Manufacturer B, we would have to redetermine the absorption and scattering coefficients. It is, therefore, essential for any application of the K.M equations to determine these two coefficients.

When considering opaque materials all the various methods are easy to apply, and with few exceptions, produce result of acceptable accuracy. With translucent materials, however, many difficulties arise. Even with the simple K.M analysis the absorption and scattering coefficients of the material are quite difficult to determine, although methods have been described by Billmeyer and Abrams [67], and Caldwell [82]. Also the results obtained are frequently too inaccurate. General improvements in the theory used for the calculations would have only a minor bearing on the whole process. Often the question arises whether the K.M model is not too simple and should be improved. Instead of the two fluxes, other authors studied models using four, six and more fluxes defined by different directions in space and associated with an increasing number of constants. The most complete theory is that of radiative transfer, leading to integral equations that only can be solved numerically with large computers for limited situations. If the results of such calculations are compared with those using the K.M equation, the empirical experience is confirmed that the simple K.M solution is a very good approximation [11]. Some of the difficulties in using K.M equations should be by improved computational or experimental technique.

2.3.1 Developing of New Formulas

Many difficulties arise in the practical use of the K.M equations. When the parameter a is less than unity the result is an imaginary value for the parameter b, which has no physical meaning. The problem of reproducibility of R_∞ leads to variability and uncertainly in the values of S and K. In the latter case, it has been shown by Hoffmann *et al*. [65] that the assumption of constancy of the absorption and scattering coefficients cannot be maintained, and that these seem to be constant only for paint films which are so thick that the contrast ratio is greater than 0.96. The complicated phenomena of the substrate reflectance are the third major difficulty. To overcome all these problems, we have developed the approach described in this section.

In this technique the reflectance of two thickness of paint film should be measured over an ideal black substrate, and, by selecting the appropriate K.M equation for including the measured reflectance and thickness, two simultaneous equations will be obtained. These equations can be solved simultaneously for the scattering coefficient under the reasonable assumption that neither the S nor the R_∞ is a function of film thickness.

Caldwell [82] and Klien [53] have used a similar technique to find the K.M coefficients from transmittance measurements. In this study the technique will be used to find K.M coefficients from reflectance measurements. Eq. (9), Eq. (24) or Eq. (39) for the reflectance of the material must be solved for the constants.

Let us consider first Eq. (39). Cross-multiplying in Eq. (39) gives:

$$R_o \sinh(W + Y) = \sinh W$$

or

$$W + Y = \text{Arsinh}\left[\frac{\sinh W}{R_o}\right] \tag{56}$$

If two film thicknesses X_1 and X_2 on an ideal black substrate have reflectances R_{01} and R_{02} respectively, then from Eq. (41) we have:

$$W_1 = SX_1 \sinh Y \quad ,$$

and

$$W_2 = SX_2 \sinh Y \quad ,$$

and then from Eq. (56) we have:

$$W_1 + Y = \text{Arsinh}\left[\frac{\sinh W_1}{R_{01}}\right] \tag{57}$$

$$W_2 + Y = \text{Arsinh}\left[\frac{\sinh W_2}{R_{02}}\right] \tag{58}$$

Subtracting Eq. (57) from Eq. (58) yields

$$W_2 - W_1 = \text{Arsinh}\left[\frac{\sinh W_2}{R_{02}}\right] - \text{Arsinh}\left[\frac{\sinh W_1}{R_{01}}\right]$$

Selecting a film thickness such that $X_2 = 2X_1 = 2X$, which implies that $W_2 = 2W_1 = 2W$, we then have that:

$$W = \text{Arsinh}\left[\frac{\text{Sinh}2W}{R_{02}}\right] - \text{Arsinh}\left[\frac{\sinh W}{R_{01}}\right]$$

$$= \text{Arsinh}\left[\frac{\sinh 2W}{R_{02}}\sqrt{1+\left(\frac{\sinh W}{R_{01}}\right)^2} - \frac{\sinh W}{R_{01}}\sqrt{1+\left(\frac{\sinh 2W}{R_{02}}\right)^2}\right]$$

or $\quad \sinh W = \sqrt{\left(\frac{\sinh 2W}{R_{02}}\right)^2+\left(\frac{\sinh 2W \sinh W}{R_{01}R_{02}}\right)^2} - \sqrt{\left(\frac{\sinh W}{R_{01}}\right)^2 + \left(\frac{\sinh W \sinh 2W}{R_{01}R_{02}}\right)^2}$

$$= \sqrt{\left(\frac{2\sinh W\cosh W}{R_{02}}\right)^2 + \left(\frac{2\sinh W\cosh W\sinh W}{R_{01}R_{02}}\right)^2}$$

$$- \sqrt{\left(\frac{\sinh W}{R_{01}}\right)^2 + \left(\frac{2\sinh W\sinh W\cosh W}{R_{01}R_{02}}\right)^2} \; .$$

Since W_1 is not equal to W_2, this implies that sinhW is not equal to zero. Then the above equation can be divided through by sinhW from which we get:

$$1 = \sqrt{\left(\frac{2\cosh W}{R_{02}}\right)^2 + \left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2} - \sqrt{\left(\frac{1}{R_{01}}\right)^2 \quad \left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2} \; ,$$

$$1 + \sqrt{\left(\frac{1}{R_{01}}\right)^2 + \left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2} = \sqrt{\left(\frac{2\cosh W}{R_{02}}\right)^2 + \left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2} \; .$$

Squaring both sides, and regrouping the terms,

$$2\sqrt{\left(\frac{1}{R_{01}}\right)^2+\left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2}=\left(\frac{2\cosh W}{R_{02}}\right)^2-\left(\frac{1}{R_{01}}\right)^2-1\ .$$

Squaring again, we have:

$$4\left(\frac{1}{R_{01}}\right)^2+\left(\frac{2\sinh W\cosh W}{R_{01}R_{02}}\right)^2=\left(\frac{2\cosh W}{R_{02}}\right)^4-2\left(\frac{2\cosh W}{R_{02}}\right)^2\left(1+\left(\frac{1}{R_{01}}\right)^2+\left[1+\left(\frac{1}{R_{01}}\right)^2\right]^2\right.$$

or

$$4\left(\frac{1}{R_{01}}\right)^2+\left(\frac{4\sinh W\cosh W}{R_{01}R_{02}}\right)^2=\left(\frac{2\cosh W}{R_{02}}\right)^4-2\left(\frac{2\cosh W}{R_{02}}\right)^2-2\left(\frac{2\cosh W}{R_{01}R_{02}}\right)^2+\left[1+\left(\frac{1}{R_{01}}\right)^2\right]^2\ .$$

By using the trigonometric relation and regrouping the terms, we get:

$$\left(\frac{4\cosh W}{R_{01}R_{02}}\right)^2(\cosh^2W-1)-\left(\frac{2\cosh W}{R_{02}}\right)^4+2\left(\frac{2\cosh W}{R_{02}}\right)^2+2\left(\frac{2\cosh W}{R_{01}R_{02}}\right)^2=1-2\left(\frac{1}{R_{01}}\right)^2+\left(\frac{1}{R_{01}}\right)^4\ ,$$

or

$$16\frac{(\cosh w)^4}{(R_{01}R_{02})^2}-16\left(\frac{\cosh W}{R_{01}R_{02}}\right)^2-\frac{16}{R_{02}^4}(\cosh W)^4+\frac{8}{R_{02}^2}(\cosh W)^2+\frac{8}{R_{01}^2R_{02}^2}(\cosh W)^2=$$

$$\left(\left(\frac{1}{R_{01}}\right)^2-1\right)^2\ .$$

Multiplying through by $R_{01}{}^4R_{02}{}^4$ and again regrouping the terms,

$$16R_{01}{}^2(R_{02}{}^2-R_{01}{}^2)\cosh^4W-8R_{01}{}^2R_{02}{}^2(1-R_{01}{}^2)\cosh^2W-R_{02}{}^4(1-R_{01}{}^2)^2=0\ .$$

This equation is the quadratic equation in $\cosh^2 W$ in the form:

$$AX^2 + BX + C = 0 \quad ,$$

where

$$X = \cosh^2 W$$

$$A = 16R_{01}^2(R_{02}^2 - R_{01}^2)$$

$$B = -\ 8R_{01}^2 R_{02}^2(1 - R_{01}^2)$$

$$C = -\ R_{02}^4(1 - R_{01}^2)^2$$

and the solution:

$$X = \frac{-\ B + \sqrt{B^2 - 4AC}}{2A} \quad .$$

This will give us:

$$\cosh^2 W = \frac{8R_{01}^2 R_{02}^2(1-R_{01}^2) + \sqrt{64R_{01}^4 R_{02}^4(1-R_{01}^2)^2 + 64R_{01}^2 R_{02}^4(R_{02}^2 - R_{01}^2)(1-R_{01}^2)^2}}{32R_{01}^2(R_{02}^2 - R_{01}^2)}$$

$$= \frac{8R_{01}^2 R_{02}^2(1-R_{01}^2) + 8R_{01}\ R_{02}^2(1-R_{01}^2)\sqrt{R_{01}^2 + (R_{02}^2 - R_{01}^2)}}{32R_{01}^2(R_{02}^2 - R_{01}^2)}$$

$$= \frac{R_{02}^2(1-R_{01}^2)(R_{02} + R_{01})}{4R_{01}(R_{02}^2 - R_{01}^2)}$$

$$= \frac{R_{02}^{2}(1 - R_{01}^{2})}{4R_{01}(R_{02} - R_{01})} \quad , \tag{59}$$

or

$$\cosh W = \frac{R_{02}}{2}\sqrt{\frac{(1 - R_{01}^{2})}{R_{01}(R_{02} - R_{01})}} \tag{60}$$

From which we will have:

$$W = \text{Arccosh}\left[\frac{R_{02}}{2}\sqrt{\frac{(1 - R_{01}^{2})}{R_{01}(R_{02} - R_{01})}}\right] \tag{61}$$

Once W is known, R_∞ can be found by solving Eq. (57) for Y and then substitute the value of Y into Eq. (36) and solve for R_∞ or by using Eq. (43) directly. The other quantities in the K.M theory can then be found.

Now if we consider Eq. (24), cross-multiplying, rearranging and solving for SX we finally get:

$$SX = (\ln\frac{R_o R_\infty - 1}{R_o/R_\infty - 1})/(1/R_\infty - R_\infty)$$

By substituting the values of R_{01}, R_{02}, x_1 and x_2 as before we will have:

$$SX_1 = \frac{R_\infty}{1 - R_\infty^{2}} \ln \frac{R_\infty(R_{01}R_\infty - 1)}{R_{01} - R_\infty} \tag{62}$$

$$S X_2 = \frac{R_\infty}{1 - R_\infty^2} \ln \frac{R_\infty(R_{02}R_\infty - 1)}{R_{02} - R_\infty} \tag{63}$$

Since we set $X_2 = 2X_1 = 2X$, then when we divide Eq. (63) by Eq. (62) we have:

$$2 = \ln \frac{R_\infty(R_{02}R_\infty - 1)}{R_{02} - R_\infty} / \ln \frac{R_\infty(R_{01}R_\infty - 1)}{R_{01} - R_\infty} \quad .$$

Cross-multiplying and taking anti-log both sides yields:

$$\left[\frac{R_\infty(R_{01}R_\infty - 1)}{R_{01} - R_\infty}\right]^2 = \frac{R_\infty(R_{02}R_\infty - 1)}{R_{02} - R_\infty}$$

or

$$\frac{R_\infty^2(R_{01}^2R_\infty^2 - 2R_{01}R_\infty + 1)}{R_{01}^2 - 2R_{01}R_\infty + R_\infty^2} = \frac{R_{02}R_\infty^2 - R_\infty}{R_{02} - R_\infty}$$

Again cross-multiplying,

$$(R_{01}^2R_\infty^4 - 2R_{01}R_\infty^3 + R_\infty^2)(R_{02} - R_\infty) = R_{01}^2R_{02}R_\infty^2 - 2R_{01}R_{02}R_\infty^3 + R_{02}R_\infty^4 - R_{01}^2R_\infty + 2R_{01}R_\infty^2 - R_\infty^3$$

$$R_{01}^2R_{02}R_\infty^4 - 2R_{01}R_{02}R_\infty^3 + R_{02}R_\infty^2 - R_{01}^2R_\infty^5 + 2R_{01}R_\infty^4 - R_\infty^3 = R_{01}^2R_{02}R_\infty^2 - 2R_{01}R_{02}R_\infty^3$$

$$+ R_{02}R_\infty^4 - R_{01}^2R_\infty + 2R_{01}R_\infty^2 - R_\infty^3$$

Rearranging and regrouping the terms we get:

$$R_{01}^2R_\infty^4 + (R_{02}-2R_{01}-R_{01}^2R_{02})R_\infty^3 + (2R_{01}-R_{02}+R_{01}^2R_{02})R_\infty - R_{01}^2 = 0$$

and $(1-R_\infty^4)R_{01}^2 - (2R_{01}-R_{02}+R_{01}^2R_{02})R_\infty(1-R_\infty^2) = 0$

Since $1 - R_\infty^2$ can never equal to zero, we divide through the above equation, to give

$$(1+R_\infty^2)R_{01}^2 - (2R_{01}-R_{02}+R_{01}^2R_{02})R_\infty = 0$$

or

$$R_{01}^2R_\infty^2 - (2R_{01}-R_{02}+R_{01}^2R_{02})R_\infty+R_{01}^2 = 0$$

Again this equation is quadratic in R_∞ which can be solved to give

$$R_\infty = \frac{(2R_{01}-R_{02}+R_{01}^2R_{02}) - \sqrt{(2R_{01}-R_{02}+R_{01}^2R_{02})^2 - 4R_{01}^4}}{2R_{01}^2} \qquad (64)$$

Once R_∞ is known, the scattering coefficient S can be found by Eq. (62). The other quantities can be found by using the appropriate equations.

Now if we consider from Eq. (9)

$$R_0 = \frac{(1 - \beta^2)\sinh\sigma X}{(1+\beta^2)\sinh\sigma X + 2\beta\cosh\sigma X}$$

Cross-multiplying in this equation and by regrouping the terms and solving for β gives:

$$\beta = \frac{-R_0\cosh\sigma X + \sqrt{R_0^{2}\cosh^{2}\sigma X + (1-R_0^{2})\sinh^{2}\sigma X}}{(1 + R_0)\sinh\sigma X} \tag{65}$$

If two layers of thickness X_1 and X_2 and reflectances R_{01} and R_{02} respectively, are considered, such that $X_2 = 2X_1 = 2X$ and replaced $\cosh\sigma X$ by C_1, $\cosh 2\sigma X$ by C_2, $\sinh\sigma X$ by S_1 and $\sinh 2\sigma X$ by S_2 for convenience in writing, then

$$\beta = \frac{-R_{01}C_1 + \sqrt{R_{01}^{2}C_1^{2} + (1 - R_{01}^{2})S_1^{2}}}{(1 + R_{01})S_1} \tag{66}$$

and

$$\beta = \frac{-R_{02}C_2 + \sqrt{R_{02}^{2}C_2^{2}+(1 - R_{02}^{2})S_2^{2}}}{(1 + R_{02})S_2} \tag{67}$$

Dividing Eq. (66) by Eq. (67), and again cross-cultiplying and re-grouping the terms gives:

$$(1 + R_{01})R_{02}S_1C_2 - (1 + R_{02})R_{01}S_2C_1 = (1+R_{01})S_1\sqrt{R_{02}^{2}C_2^{2}+(1-R_{02}^{2})S_2^{2}}$$

$$- (1+R_{02})S_2\sqrt{R_{01}^{2}C_1^{2}+(1-R_{01}^{2})S_1^{2}}$$

Substituting $S_2 = 2S_1C_1$ and $C_2^{2} = 1 + S_2^{2}$, $C_1^{2} = 1 + S_1^{2}$ in the above equation, arranging the terms and dividing through by S_1, we have:

$$(1+R_{01})R_{02}C_2 - 2(1+R_{02})R_{01}C_1^2 = (1+R_{01})\sqrt{R_{02}^2+S_2^2} - 2(1+R_{02})C_1\sqrt{R_{01}^2+S_1^2}$$

Squaring both sides and arranging the terms yields:

$$(1+R_{01})^2(R_{02}^2C_2^2-R_{02}^2-S_2^2) + (1+R_{02})^2(4R_{01}^2C_1^4-4R_{01}^2C_1^2-4C_1^2S_1^2)$$

$$-4R_{01}R_{02}C_1^2C_2(1+R_{01})(1+R_{02})$$

$$= -4C_1(1+R_{01})(1+R_{02})\sqrt{R_{01}^2 + S_1^2}\sqrt{R_{02}^2 + S_2^2}$$

Substituting $C_2^2 = 1 + S_2^2$, $C_1^2-1 = S_1^2$, arranging the terms and dividing through by $-(1+R_{01})(1+R_{02})$ gives:

$$(1+R_{01})(1-R_{02})S_2^2 + 4(1-R_{01})(1+R_{02})C_1^2S_1^2+4R_{01}R_{02}C_1^2C_2 =$$

$$4C_1\sqrt{(R_{01}^2 + S_1^2)(R_{02}^2 + S_2^2)}$$

Again substituting $S_2^2 = 4C_1^2S_1^2$, multiplying out, arranging the terms and dividing through by $4C_1$ we have:

$$2(1-R_{01}R_{02})C_1S_1^2 + R_{01}R_{02}C_1C_2 = \sqrt{(R_{01}^2 + S_1^2)(R_{02}^2 + S_2^2)}$$

Substituting $C_2 = C_1^2 + S_1^2$, $C_1^2 - 1 = S_1^2$, multiplying out, and rearranging the terms, we have:

$$2C_1S_1^2 + R_{01}R_{02}C_1 = \sqrt{(R_{01}^2 + S_1^2)(R_{02}^2 + S_2^2)}$$

Squaring both sides gives:

$$4C_1^2S_1^4 + R_{01}^2R_{02}^2C_1^2 + 4R_{01}R_{02}C_1^2S_1^2 = R_{01}^2R_{02}^2 + R_{02}^2S_1^2 + R_{01}^2S_2^2 + S_1^2S_2^2$$

Substituting $C_1^2 = 1 + S_1^2$, $S_2^2 = 4C_1^2S_1^2$, arranging the terms and dividing through by S_1^2 we have:

$$4S_1^2+4R_{01}R_{02}+4S_1^4+R_{01}^2R_{02}^2+4R_{01}R_{02}S_1^2 = R_{02}^2+4R_{01}^2C_1^2+4C_1^2S_1^2 \quad ,$$

or $$4S_1^2(1+S_1^2)+4R_{01}R_{02}+R_{01}^2R_{02}^2+4R_{01}R_{02}S_1^2 = R_{02}^2+4R_{01}^2(1+S_1^2)+4S_1^2(1+S_1^2),$$

Arranging and regrouping the terms we have:

$$4(R_{01}R_{02}-R_{01}^2)S_1^2 = R_{02}^2+4R_{01}^2-4R_{01}R_{02}-R_{01}^2R_{02}^2$$

Again substituting $S_1^2 = C_1^2 - 1$, and solving for C_1 we have:

$$C_1 = \frac{R_{02}}{2}\sqrt{\frac{1 - R_{01}^2}{R_{01}(R_{02} - R_{01})}}$$

or

$$\cosh\sigma X = \frac{R_{02}}{2}\sqrt{\frac{1 - R_{01}^2}{R_{01}(R_{02} - R_{01}}} \tag{68}$$

and

$$\sigma = \frac{1}{X}\,\text{Arcosh}\left[\frac{R_{02}}{2}\sqrt{\frac{1 - R_{01}^2}{R_{01}(R_{02} - R_{01})}}\right] \tag{69}$$

From Eq. (65) we have β in the form:

$$\beta = \frac{-R_{01}\cosh\sigma X + \sqrt{R_{01}^2 + \sinh^2\sigma X}}{(1 + R_{01})\sinh\sigma X} \tag{70}$$

One σ and β are obtained, then the other parameters of K.M equations can be calculated. It is interesting to notice that σX is identical with W in Eq. (61).

Some of the relationships between the quantities obtained from this work are given in Table 2.2.

2.3.2 Limitations and Scope

Limitations of the technique described in this study depends on values of R_{01} and R_{02}. This can be considered from Eqs. (60), (64) and (68).

Let us consider Eq. (60) and Eq. (68). Since coshW and $\cosh\sigma X$ is greater than or equal to 1, from Eq. (60), or Eq. (68) we have the condition:

$$\frac{R_{02}^2(1 - R_{01}^2)}{4R_{01}(R_{02} - R_{01})} \geqslant 1$$

or $$R_{02}^2 - R_{01}^2R_{02}^2 \geqslant 4R_{01}R_{02} - 4R_{01}^2$$

$$4R_{01}^2 - 4R_{01}R_{02} + R_{02}^2 \geqslant R_{01}^2R_{02}^2$$

$$(2R_{01} - R_{02})^2 \geqslant R_{01}^2R_{02}^2$$

Taking the square root of both sides gives:

$$2R_{01} - R_{02} \geqslant R_{01}R_{02}$$

or $$(2 - R_{02})R_{01} \geqslant R_{02}$$

i.e. $$R_{01} \geqslant \frac{R_{02}}{2 - R_{02}} \tag{77}$$

This means that R_{01} and R_{02} must satisfy Eq.(77) otherwise Eq. (60) cannot be applied.

From Eq. (64), the value of R_∞ will be real if and only if the value within the square root sign is positive, or at least equal to zero. This will give us the condition:

$$2R_{01} - R_{02} + R_{01}^2 R_{02} \geqslant 2R_{01}^2$$

$$2R_{01}(1 - R_{01}) \geqslant R_{02}(1 - R_{01})(1 + R_{01})$$

or $$2R_{01} - R_{01}R_{02} \geqslant R_{02}$$

$$(2 - R_{02})R_{01} \geqslant R_{02}$$

Again this gives us the condition:

$$R_{01} \geqslant \frac{R_{02}}{2 - R_{02}}$$

which is the same as in Eq. (77).

We therefore have established the condition for the limitation of our technique, which we have tabulated in Table A.1 of Appendix A for use in inspecting R_{01} and R_{02} to determine whether they can be used.

There are several advantages in this method. Many difficulties that always arise in using K.M equations by other methods can be overcome. One of the most advantageous is that one can decide immediately from a qualitative inspection of spectrophotometric reflectances R_{01} and R_{02} of the sample whether they can be used.

Table 2.1 Definition of Symbols used in Table 2.2

Symbol	Definition
X	Thickness of layer
X_1, X_2	Thickness of layer 1 and layer 2 such that $2X_1 = X_2 = 2X$
R_{01}, R_{02}	Diffuse reflectance of layer 1 and of layer 2 with ideal black background
R_∞	Diffuse reflectance of layer so thick that further increase in thickness fails to change the reflectance, called reflectivity
T	Internal transmittance of layer
$S = (dR\ /dx)_{x \to 0}$	Coefficient of scatter, rate of increase of reflectance with thickness of a very thin layer for diffusely incident radiant energy
$K = (dT/dx)_{x \to 0}$	Coefficient of absorption, rate of decrease of transmittance with thickness of a very thin layer for diffusely incident energy
$a = (S+K)/S, = \cosh Y,$	Equal to $(1/R_\infty + R_\infty)/2$, whence: $R_\infty = a - (a^2 - 1)^{1/2}$
$b = (a^2-1)^{1/2} = \sinh Y,$	Equal to $(1/R_\infty - R_\infty)/2$, whence: $R_\infty = a - b$
$\beta = \sqrt{K/(K+2S)} = \sigma/(K+2S)$	Optical constant
$\sigma = \sqrt{K(K+2S)}$	Extinction coefficient
$\sinh U$	Abbreviation for $\frac{e^U - e^{-U}}{2}$, hyperbolic sine of U
$\cosh U$	Abbreviation for $\frac{e^U + e^{-U}}{2}$, hyperbolic cosine of U
ArsinhU	Inverse hyperbolic function of hyperbolic sine of U

Table 2.1 Definition of Symbols used in Table 2.2 (continued)

Symbol	Definition
ArcoshU	Inverse hyperbolic function of hyperbolic cosine of U
$Y = \ln(1/R_\infty)$	
$W = bSX = SX\sinh Y$	

Table 2.2 Formulas giving some of the relationships between R_∞, R_{01}, R_{02}, X, S, K, A, b, σ, β

Formula	Eq. Number
$W = \text{Arcosh}\left[\frac{R_{02}}{2}\sqrt{\frac{(1 - R_{01}^2)}{R_{01}(R_{02} - R_{01})}}\right]$	(61)
$Y = \text{Arsinh}\left[\frac{\sinh W}{R_{01}}\right] - W$	(56)
$\sigma = \frac{1}{X}\text{Arcosh}\left[\frac{R_{02}}{2}\sqrt{\frac{(1 - R_{01}^2)}{R_{01}(R_{02} - R_{01})}}\right]$	(69)
$\beta = \frac{-R_{01}\cosh\sigma X + \sqrt{R_{01}^2 + \sinh^2\sigma X}}{(1 + R_{01})\sinh\sigma X}$	(70)
$R_0 = \frac{(1 - \beta^2)\sinh\sigma X}{(1 + \beta^2)\sinh\sigma X + 2\beta\cosh\sigma X}$	(9)
$= \frac{\exp SX(1/R_\infty - R_\infty) - 1}{(1/R_\infty)\exp SX(1/R_\infty - R_\infty) - R_\infty}$	(24)
$= \frac{\sinh W}{\sinh(W + Y)}$	(39)
$R_\infty = S/[S + K + \sqrt{K(K + 2S)}]$	(18)
$= e^{-Y} = 1/\exp\{\text{Arsinh}[\frac{\sinh W}{R_{01}}] - W\}$	(71)
$= \cosh Y - \sinh Y$	(43)
$= \frac{1 - \beta}{1 + \beta}$	(10)

Table 2.2 Formulas giving some of the relationships between R_∞, R_{01}, R_{02}, X, S, K, a, b, σ, β (continued)

Formula	Eq. Number
$R_\infty = \dfrac{2R_{01}-R_{02}+R_{01}^2R_{02} - \sqrt{(2R_{01}-R_{02}+R_{01}^2R_{02})^2 - 4R_{01}^4}}{2R_{01}^2}$	(64)
$F(R_\infty) = (1-R_\infty)^2/2R_\infty = \cosh Y - 1 = a - 1$	(72)
$S = W/X\sinh Y$	(41)
$= [R_\infty/X(1 - R_\infty^2)] \ln \dfrac{R_\infty(R_{01}R_\infty - 1)}{R_{01} - R_\infty}$	(62)
$= \dfrac{\sigma - \beta K}{2\beta} = \dfrac{\sigma(1 - \beta^2)}{2\beta}$	(73)
$K = S(1 - R_\infty)^2/2R_\infty$	(74)
$= S(\cosh Y - 1)$	(75)
$= \sigma\beta$	(76)

2.4 Theory of Color Mixture

The validity of K and S values obtained in this study, will be tested according to the theory of color mixture and the color-difference formulas. Lower values of color difference between experimental and calculated are assumed to indicate greater validity of the values obtained.

The idea of being able to compute the color of a mixture of pigments in a paint extends back at least to 1940 [84]. Examples of calculating the color of pigmented plastics were given in 1942 by Saunderson [57]. By 1944 the whole theory of how to calculate color matches for the general case had been given by Park and Sterns [76]. Practical applications of the calculation of color matches for paints and textiles followed [58,59,83].

In K.M theory, K and S are additive, which implies that their values for mixtures can be calculated from the K and S of individual components in the mixture and the volume concentrations. An important step in this approach to the color of pigment dispersions is due to Duncan [84], who showed that if there is more than one kind of pigment dispersed in a film, the scattering and absorption coefficients of the film are additive functions of the scattering and absorption coefficients of the pigments.

2.4.1 The Single-Constant Theory

The basic assumption for calculating color mixture is that the light scattering and the absorption of colorant are proportional to the concentration, c. Almost all instrumental color matching at present is based on K.M analysis, but the two constants K and S are usually reduced to a single constant (K/S). Many people have pointed out the analogy between the equation in this form and Beer's Law. Davidson and

Hemmendinger [85] had proposed that if $F_i(R_\infty)$ are functions of K.M parameters, K and S, of the pigment i, and c_i are the concentrations of pigments i, such that $\sum_{i=1}^{n} c_i = 1$, then the K.M function of the mixture of these pigments should be:

$$F(R_\infty) = c_1F_1(R_\infty) + c_2F_2(R_\infty) + \ldots\ldots + c_nF_n(R_\infty) \tag{78}$$

For color prediction it is necessary, of course, to predict reflectance at wavelengths in the visible region.

2.4.2 Two-Constant Theory

For mixtures of pigments, Duncan [84] found that the effects of individual pigments are additive in proportion to their concentrations in the film:

$$K_m = c_1K_1 + c_2K_2 + \ldots\ldots + c_nK_n, \tag{79}$$

$$S_m = c_1S_1 + c_2S_2 + \ldots\ldots + c_nS_n, \tag{80}$$

where K and S are the values for the pigments specified by subscripts 1, 2, 3, ..., n, c; is the proportion of the specified pigment in the mixture, and K_m and S_m are the values for the film. The value of K/S for the mixture can be related in a simple way to values of K and S for the individual components:

$$(K/S)_m = (c_1K_1 + c_2K_2 + \ldots\ldots + c_nK_n)/$$

$$(c_1S_1 + c_2S_2 + \ldots\ldots + c_nS_n) \quad (81)$$

For a very wide gamut of colors in paints and plastics, the film is essentially opaque, and most of the scattering is supplied by the white pigment. In these cases, if we assume $S_1 = S_2 = \ldots\ldots = S_n$ and if we let w refer to the white pigment, the two-constant theory of Eq. (81) may be simplified as:

$$(K/S)_m = (c_1K_1 + c_2\,K_2 + \ldots\ldots + c_nK_n)/S_w$$

$$= c_1(K_1/S_w) + c_2(K_2/S_w) + \ldots\ldots + c_w(K_w/S_w), \quad (82)$$

or more simply;

$$(K/S)_m = c_1(K/S)_1 + c_2(K/S)_2 + \ldots\ldots + c_w(K/S)_w \quad (83)$$

Thus a single-constant (K/S), rather than two constants K and S separately, characterizes each pigment at a specified wavelength.

To predict the color of the known mixtures of pigments by the two-constant theory, the values of K and S for the pigments and the concentrations are inserted into Eq. (81). Calculations are made at a sufficient number of wavelengths for each of the mixtures so that a spectrophotometric reflectance spectrum can be drawn. Tristimulus values for the predicted spectrum then are computed and compared with the tristimulus values for the measured spectrum of the mixture to obtain a color difference.

2.5 Color Space

Numerous color systems exist. Fairly detailed accounts of the important systems are given by Judd and Wyszecki [42], and Wyszecki and Stiles [86].

2.5.1 The C.I.E. Colorimetric System

Of the various system of color measurement currently available, the C.I.E. method or trichromatic system is undoubtedly the most popular. In 1931 the Commission Internationale de l'Eclairage (C.I.E.) established a framework for color specification in which a given color may be expressed in terms of tristimulus values X, Y, Z based on three synthetic primaries. These tristimulus values are deduced from the spectrophotometric data.

The tristimulus values of a colored material are expressed by three integrals of the form [42]

$$X = k \int_0^\infty R_\lambda E_\lambda \, \bar{x}_\lambda d\lambda \ ,$$

$$Y = k \int_0^\infty R_\lambda E_\lambda \, \bar{y}_\lambda d\lambda \ , \qquad (84)$$

$$Z = k \int_0^\infty R_\lambda E_\lambda \, \bar{z}_\lambda d\lambda \ ,$$

where k is a normalizing factor, which for object colors is conventiently chosen as

$$k = \frac{100}{\int_0^\infty E_\lambda \bar{y}_\lambda \, d\lambda} \quad . \tag{85}$$

R_λ is a function of wavelength representing the reflection factor of the test sample, E_λ is a function of wavelength representing the spectoral energy distribution of illuminant, and $\bar{x}_\lambda$, $\bar{y}_\lambda$, and $\bar{z}_\lambda$ are the tristimulus values for the spectral response curves of the standard observer as specified by the International Commission on Illumination in 1931. Standard Tables of these values are published in numerous books [42,86,87,88,89, 90]. These functions are too complex to formulate analytically. Hence, the indicated integrations are usually approximated by the finite sums

$$X = k \sum_{400}^{700} R_\lambda E_\lambda \bar{x}_\lambda \Delta\lambda \ ,$$

$$Y = k \sum_{400}^{700} R_\lambda E_\lambda \bar{y}_\lambda \Delta\lambda \ , \tag{86}$$

and $$Z = k \sum_{400}^{700} R_\lambda E_\lambda \bar{z}_\lambda \Delta\lambda \ ,$$

with $\Delta\lambda$ usually taken uniformly as 10 or 20 millimicrons throughout the visible region 400-700 nm. The normalizing factor k is now defined as

$$k = \frac{100}{\sum_{400}^{700} E_\lambda \bar{y}_\lambda \Delta\lambda} \tag{87}$$

If the relative distribution of E_λ is unchanged, but its intensity is increased, the values of X, Y and Z are of course all increased proportionally. To obtain a measure of chromaticity, that is, to make the coordinates independent of intensity, they may be normalized so that their sum is equal to unity. Such chromaticity coordinates are given by

$$x = \frac{X}{X + Y + Z} \quad ,$$

$$y = \frac{Y}{X + Y + Z} \quad , \qquad (88)$$

and

$$z = \frac{Z}{X + Y + Z} \quad ,$$

Since the sum of the coefficients x, y and z is unity, any color may be represented graphically on a plane diagram with only x and y coordinates.

2.5.2 Color Difference Formulas

A number of color difference formulas are in use today, and a considerably larger number have been proposed over the years and then superseded. Detailed information and discussions of various color-difference formulas have been discussed by numerous authors [91,92,93,94]. The existence of so many color-difference equations has produced considerable confusion. No single formula can be considered as the best because no basis of judgment has been universally accepted. Most published discussion is concerned with the use of total or 'single-number' color-difference units. A total color-difference unit is not very revealing in many instances, for it gives no indication of the character of the difference. It does not show the relative size and direction of hue, saturation, and lightness differences.

The present study adopts the Hunter color-difference formula based on projective transformations of the C.I.E. chromaticity diagram in combination with a lightness scale. This formula was chosen in view of the wide use of the Hunter color-difference meter in industry [95].

In the Hunter color coordinate-space system [96] of L, a, and b, "L" measures lightness/darkness (100 = pure white, 0 = pure black), 'a' measures redness if positive and greeness if negative, and 'b' measures yellowness if positive and blueness if negative. These coordinate values can be transformed from the C.I.E. ones by:

$$L = 10(Y)^{1/2}$$

$$a = 17.5(1.02X - Y)/(Y)^{1/2} \tag{89}$$

and

$$b = 7.0(Y - 0.847Z)/(Y)^{1/2}$$

where X, Y, and Z are tristimulus for the C.I.E. standard observer and standard source C.

The total color difference, ΔE, is derived from the following equation:

$$\Delta E_{(Hunter)} = (\Delta L^2 + \Delta a^2 + \Delta b^2)^{1/2} \tag{90}$$

where ΔL, Δa, Δb are the difference between the Hunter coordinates of the two colors, i.e.:

$$\Delta L = L_1 - L_2 \quad ,$$

$$\Delta a = a_1 - a_2 \quad , \tag{91}$$

and

$$\Delta b = b_1 - b_2 \quad .$$

CHAPTER III

INSTRUMENTATION AND EXPERIMENTAL PROCEDURE

3.1 Sample Preparation

To apply the formulas developed in the previous chapter, a number of conditions must be satisfied in accordance with the assumptions in the development. The model takes the samples as a set of plane parallel layers. This choice has been made because the geometry is simple, and because most practical problems involve it. The model for the layers is a fairly uniform dispersion of particles in a turbid medium. A paint film is the most appropriate specimen for this study.

3.1.1 Vehicle

In the manufacture of a paint, pigments are dispersed in a fluid mixture consisting of binder, solvents, and other components. This mixture is known as the MEDIUM or VEHICLE. The binder is usually a resinous material solid at room temperature and serving as the medium to hold pigments in a dispersed state in the finished paint film. Sometimes the term "film former" is used instead of "binder." The solvent is a volatile liquid whose function is to dissolve the binder so that it will be a liquid state when the paint is formulated and when it is applied. The mode of drying, and the adhesive and mechanical properties of the films, are determined by the type of binders.

A vehicle must be selected carefully in order to satisfy the requirements for the intended application. For the present study a High Gloss Overprint Lacquer based on ELVACITE 2045/2046 (duPont ELVACITE Acrylic resins formulas 23) [97] was selected as the vehicle because of its clarity, gloss, adhesion, and non-yellowing characteristics. This specific vehicle, moreover, is colorless and almost completely transparent, thus minimizing optical interference, and it is quick-drying. Finally, it is poorly adherent to Mylar film and hence nonsupported lacquer films are easily prepared.

Compositions

The composition of the duPont ELVACITE Acrylic resins formula 23, High Gloss Overprint Lacquer based on ELVACITE 2045/2046 is as follows:

ELVACITE 2045	15% by weight	30% Solids	11.79% by volume	23.6% Solids
ELVACITE 2046	15% by weight		11.85% by volume	
Mineral Spirits	35% by weight		38.44% by volume	
Acetone	35% by weight		37.92% by volume	
	100%		100%	

Properties

ELVACITE 2045 is a bead polymer of isobutyl methacrylate with density 9.08 lb/gal, bulking value 0.1101, and specific gravity 1.09. It is the hardest of the weak-solvent-soluble butyl grades. ELVACITE 2046 is a bead polymer of n-butyl/isobutyl methacrylate 50/50 copolymer with density 9.04 lb/gal, bulking value 0.1106, and specific gravity 1.09. It is a medium-hardness butyl grade. Both ELVACITE 2045 and ELVACITE 2046 are soluble in mineral spirits, VM&P naphtha, and some alcohols. In lacquer technology two or more solvents are mixed so as (a) to control the flash point, price, etc., of the product, and (b) to ensure that the film deposited is free from defects such as the occurrence of undissolved resin in the film [98]. In this lacquer-type vehicle the solvents were acetone and mineral spirits.

Acetone, industrially the most important member of the acetone group, is dimethyl ketone, $CH_3.CO.CH_3$. It has a density of 0.791 at 20°C, and a boiling point of 56.2°C. It is extremely volatile and inflammable, and flashes at ordinary atmospheric temperature, with a flash point well below 0°C. The solvent power is extremely great, and extends to a very wide range of substances. It is completely miscible with water, and with many hydrocarbons and other organic liquids. It can therefore be used to convert many two-layer mixtures into one layer, i.e., single-phase, mixtures. As a rule, solutions in acetone have low viscosity.

Mineral spirits are petroleum fractions boiling in the 150°-190°C range, although a small percentage may boil up to 210°C. They are the most widely used solvents in the industry. They are also sometimes referred to as petroleum spirits, or mineral turpentine, or 'turps substitute', or VM & P naphtha and are known in British usage as White Spirits.

Preparation

ELVACITE bead resins dissolve at room temperature, but require constant agitation to prevent solvent-swollen granules of polymer from forming agglomerates and sticking to the walls of the container.

Acetone 350 gm and Mineral Spirits 350 gm were mixed in a 2000 ml beaker and ELVACITE 2045 and ELVACITE 2046 (150 gm each) were slowly added. Solvents and resins were mixed thoroughly with a glass rod. The beaker was put on a magnetic hot-plate stirrer, covered with a watch glass, and maintained with constant stirring until a clear solution formed. Time required for this was about 1-3 hours, depending on the conditions of the resins.

It is important that the polymer beads should be sifted directly into the vertex of the stirred solvent to speed wetting-out and dispersion. Solution time can be reduced by heating; but care is required to avoid over-heating and excessive solvent loss.

3.1.2 Pigments

Within the many basic types of pigment available to the modern paint formulator there are numerous subtypes. Pigments of the same nominal type and subtype as supplies by the various manufactures may be significantly different in characteristics [99]. In general, pigments may be classified into two main groups: white and colored. The pigment influences the working properties and the durability of the paint, as well as imparting color and hiding power to the paint. The reflection, refraction, and absorption of light by a pigment are the fundamental properties which determine hiding power; selective absorption is associated with the production of color. Knowledge of the optical pigment properties in physical terms is a basis for initial selection of a pigment. It must be stated, regrettably, that few useful data are known, even though they would be helpful in all calculations of optical efficiency [100].

In the present study the pigments were selected primarily for

for their optical characteristics, with the other physical, mechanical, and chemical properties being of secondary importance. Pigments for studies in the visible study were titanium dioxide (white), ferric oxide (red), and chromic oxide (green). Pigments for studies in the ultraviolet were V-8295 Low-infrared-reflectance and LG-3503-Virgo Blue Stain.

3.1.2.1 White Pigment

The characteristic "whiteness" of white pigment which the eye sees is the result of the diffuse scattering and reflectance of light by a myriad of fine particles which do not selectively absorb any of the wavelengths of visible light. Naturally, then, under red light they appear red; under blue light, blue; under yellow light, yellow. Only when the light by which they are illuminated is itself perceived as white can the pigments appear white. The opacity of white pigments when dispersed in a vehicle results from two factors: particle size, and the disparity between the refractive index and that of the paint vehicle, the greater the mismatch, the greater the opacity. If a colorless pigment has a refractive index the same as that of a paint vehicle, it becomes transparent when dispersed. When the difference in refractive index is relatively small, the pigment is referred to as extender rather than a white pigment.

WHITE: Titanium Dioxide (TiO_2)

Titanium dioxide has three inherent properties which make it suitable as a pigment: high refractive index, "whiteness", chemical inertness, and non-toxicity. The titanium dioxide in this study, Cyanamid Unitane OR-560 Titanium Dioxide (American Cyanamid Comapny), is an alumina-silica-treated rutile titanium dioxide. It is a chalk-resistant rutile grade designed to give greater hiding power and tinting strength in emulsion paints at higher levels of pigment volume concentration.

Special Properties of Cyanamid Unitane OR-560 Titanium Dioxide:

- -High brightness
- -Greater hiding power and tinting strength at elevated PVC levels
- -High oil absorption
- -Low sheen
- -Good dispersion in all types of equipment
- -Good chalk resistance and color retention.

Typical Physical and Chemical Data [101]:

TiO_2 content	90% minimum
Additives: Al_2O_3, SiO_2	
Specific Gravity	4.0
Solid Bulking (lb/gal)	33.3
Bulking value (gal/lb)	0.0300
Approximate pH	7.8
Oil absorption	26-28
Average Particle size	0.23 micron
Refractive index (Rutile TiO_2)	2.76

Note: Titanium dioxide pigments are manufactured products and are of two distinct types, anatase and rutile (a-titania and r-titania). Both are composed of very fine, colorless crystals of the same compound, TiO_2, but the structure of the crystals may assume either of two forms, which leads to two types of pigment with different properties. A third form, "brookite," also exists but as yet has no technical importance [99].

3.1.2.2 Colored Pigments

The field of colored pigments is vast with respect to both types of materials and shades of color. Hiding power of color pigments does not lend itself to a numerical system.. Several colored pigments from Pfizer Minerals Pigments & Metals Division have been selected for this study. Their brand names and their properties are as follows [102].

RED: Pure Red Iron Oxide R-2900

Pure REd Iron Oxides, Fe_2O_3, are produced by several methods which impart individual properties for specific applications. The R Type Reds, made by the thermal decomposition and oxidation of iron salts, are available in a range of colors from a light salmon red to a deep bluish maroon. These pigments are high in chemical purity, and contain a minimum of silica, alumina, calcium, and soluble salts.

Properties of Pure Red Iron Oxide R-2900

Iron Oxide	99.5%
Specific Gravity	5.15-5.20
Hiding	700-1100 sq.ft/lb
325 Mesh Retention	less than 0.10%
Oil Absorption	13-23
Particle Shape	Spheroidal

GREEN: Pure Chromium Oxide Green G-4099

Pure Chromium Oxide Green, Cr_2O_3, the most stable of all green pigments is made in a range of shades for use in paints, ceramics, rubber, plastics, concrete products, and other applications requiring superior durability.

Properties of Pure Chromium Oxide Green G-4099

Chromium Oxide	99.0%
Specific Gravity	5.20
Hiding	475-575 sq.ft/lb.
325 Mesh Rentention	less than 0.10%
Oil Absorption	13-19
Particle Shape	Rhombohedral.

V-8295 LOW-INFRARED-REFLECTANCE PIGMENT

V-8295 is a reddish-blue inorganic pigment manufactured without arsenic an important consideration in worker and consumer protection. It is the production of FERRO Corporation Color Division.

Typical Physical Properties

Composition	$CoLiPO_4$
Specific gravity	3.80
Average particle size (μ)	2.
Bulk density (gm/cu. in)	11.7
Weight/solid gal(lb)	31.7
Residue (+ 325 mesh screen)	0
Oil absorption (lb oil/100 lb pigment)	28.7

PEMCO LG-3503 Virgo Blue Stain

This stain is a dry blend of two of SCM Glidden-Durkee standard glaze stains [127].

Turq. GS-119	95.0%
B/Green GS-117	5.0%

The Turq. GS-119 is a calcined inorganic stain that is made from

ZrO_2, V_2O_5 and SiO_2.

The B/Green GS-117 is also a calcined, inorganic stain, but is manufactured using a combination of CoO and Cr_2O_3.

Both V-8295 and LG-3503 pigments were used to test our method in the ultraviolet region.

3.1.3 Pigment Dispersion

We have already seen that paint consists essentially of two components, the dry pigment and the liquid vehicle, which must be mechanically mixed. The basic operation is the dispersion of a pigment into the vehicle. For a stable dispersion the liquid must wet the solid particles.

In this study the pigments are powders. In the dry condition the particles are largely aggregated into clusters. Making paint is breaking-down the aggregates, and dispersing the individual particles into the vehicle. The dispersion process is to be distinguished from the grinding process, a reduction of the primary particle size. Modern treatments of pigment dispersion are given in references [103,104,105, 106,107,108]. In earlier times, and even well into the present century, the professional painter often mixed his own paints by hand, stirring the pigments into oil, varnish, or aqueous medium with a stick in a bucket. In such hand-mixed paints, the degree of dispersion of the pigment is far from complete. Larger or smaller amounts of the pigment are present as large, badly wetted agglomerates, and the resultant film usually has a sandy finish. To obtain a smooth, glossy surface the pigment must be properly dispersed, or ground, a process for which a wide variety of machines exists [109,110,111]. A machine-ground paint has the advantage over a hand-mixed paint that the pigment content may be very high and yet the paint will be easily brushable.

Starting with a dry pigment and a liquid vehicle, several separate stages in the dispersion process may be distinguished [105,112]. First is wetting, defined as the replacement of the pigment/air interface by the pigment/vehicle interface. Second is mechanical disaggregation. Third is stabilization of the dispersion, defined as the uniform distribution of dispersed particles throughout the entire workable mass. These three processes, though distinct, may occur almost simultaneously.

Dispersion is a generic term which does not fully describe the state of the material. A dispersion may be deflocculated, flocculated, or agglomerated, or a combination of the three. Breakdown of the aggregates takes place before dispersion of the individual particles. It is not always necessary, however, to break down the aggregates completely, and for each type of paint there is an acceptable <u>degree of dispersion</u>. This quantity, commonly known as <u>fineness of grind</u>, is measured usually by a <u>Fineness of Grind Gage</u> as described in ASTM D 1210-64 [113]. The gage consists of a hardened steel block (178 x 64 x 12.7 mm) whose top surface is ground flat and smooth. Centered in the top of this block is a shallow channel 12.7 mm wide by 128 mm long which is tapered uniformly in depth (lengthwise) from 100 μ at one end to zero depth at the other. The scraper is a wedge-shaped steel blade (89 x 38 x 6.4 mm) with the straight edges on the 89-mm side rounded to a radius of 0.25μ.

The test procedure is to place a puddle of the test paint in the deep end of the channel (gage resting on a flat horizontal surface). With the rounded edge of the scraper, the paint material is drawn down the length of the channel with uniform motion and with sufficient pressure to wi

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wipe clean the flat face of the gage. The thin wedge of paint remaining in the tapered groove is then immediately inspected. At some point along the channel, coarse particles or agglomerates will become visible. The finer the dispersion, the greater the distance from the deep end.

The degree of pigment dispersion as measured by the fineness-of-grind gage, is directly related to surface appearance; gloss, smoothness, texture, opacity, tinting strength, floating, flooding settling, sagging, film durability, gloss retentation, and other special properties. It should be remembered, however, that the fineness-of-grind reading is only indicative of the largest pigment agglomerates present in the pigment dispersion. It does not furnish any real index as to the particle distribution within the pigment dispersion.

The performance characteristics of paints in a given formulation are significantly controlled by the relationship of pigment and vehicle, specifically their respective volumes. The pigment-vehicle volume relationship will determine gloss, drying, stain removal, brushing, flow, holdout, viscosity, and other properties. This relationship is called Pigment Volume Concentration (PVC), and is expressed as the percentage of pigment volume to total volume. Thus

$$PVC = \frac{\text{Pigment Volume} \times 100}{\text{Pigment Volume + Vehicle-Solid Volume}}$$

When successive small increments of binder are added to a paint with extremely high pigment volume, a point is reached at which the paste or semi-paste product undergoes a sudden change in such properties as consistency, leveling, gloss, and permeability of the dry film [114]. This point, known as the Critical Pigment Volume Concentration (CPVC), is the specific

PVC at which the vehicle demand of the pigment is precisely satisfied; i.e., there are no voids among the pigment particles and there is no excess vehicle. The concept of CPVC has been most intimately associated with its effects on the performance properties of cured coatings. Such paint characteristics as gloss, blistering, rusting, hiding, scrubbability, stain removal, tensile strength, and many others are known to change radically with a transition through this point. Many excellent reviews of the subject of pigment concentration have been published [104,105, 114,115,116].

3.1.3.1 Pigment Volume Concentration

Practicable application and flow properties usually require that the pigment volume be reduced slightly below the critical value. In this study the highest PVC is 15% PVC, and the other PVC's are 1%, 5%, 10% PVC.

The proportion of pigments to 1 gm of vehicle dePont ELVACITE formula 23 for 1%, 5%, 10%, and 15% PVC calculated from the relation,

$$PVC = \frac{\text{Pigment Volume x 100}}{\text{Pigment Volume + Vehicle-Solid Volume}} ,$$

is as follows:

PVC	TiO_2 white	Fe_2O_3 red	Cr_2O_3 green
1%	0.00955 gm	0.01236 gm	0.01242 gm
5%	0.04977 gm	0.06439 gm	0.06470 gm
10%	0.10507 gm	0.13593 gm	0.13659 gm
15%	0.16687 gm	0.21589 gm	0.21693 gm

It is convenient to obtain the lower-PVC paint by adding the vehicle to the higher one. Consequently, we prepared paints of 15% PVC, and then obtained the lower PVC's by adding the vehicle (duPont ELVACITE formula 23) in the ratios calculated according to the above information. Starting from 1 gm of paint at 15% PVC, we added the following amounts of vehicle to obtain the required PVC of various paints.

For white TiO_2 ,

add	14.11751 gm	to obtain	1% PVC	
add	2.01635 gm	to obtain	5% PVC	
add	0.50407 gm	to obtain	10% PVC	.

For red Fe_2O_3 ,

add	13.54656 gm	to obtain	1% PVC	,
add	1.93516 gm	to obtain	5% PVC	,
add	0.48379 gm	to obtain	10% PVC	.

For green Cr_2O_3 ,

add	13.53449 gm	to obtain	1% PVC	,
add	1.93351 gm	to obtain	5% PVC	,
add	0.48338 gm	to obtain	10% PVC	.

3.1.3.2 Preparation of Paints

The required quantities of pigment and vehicle were mixed and dispersed in a Waring Commercial Blender - model 5011. This equipment is designed for high-speed blending, pulping, mixing, or reducing of materials. The container and jar lid were made of stainless steel. There are two switch settings for speed, LO and HI. Sharp stainless-steel blades whirling as fast as 23,000 rpm. almost instantaneously reduce ingredients to a homogeneous mass. The capacity of this blender is one quart mixing capacity. The procedure was the following:

(1) Measure the required quantity of vehicle into the blender container.

(2) Weigh the required quantity of pigment and add to the vehicle in the container.

(3) Mix the pigment-vehicle combination with a glass rod and observe for fluidity. If fluid, proceed to Step 4, otherwise continue stirring until fluid, then proceed to Step 4.

(4) Mix for 5 minutes in disperser. Start at LO speed for two minutes or so, follow at HI speed for about three minutes. A free-flowing paste composition is obtained, which is left to stand for a while to allow entrapped air to escape.

(5) Measure fineness of grind with Fineness-Of-Grind Gage. If the degree of dispersion is not in the required range, go back to Step 4. Repeat until the required degree of dispersion is obtained. Fill the paint in the tin can and cap tightly.

3.1.3.3 Preparation of Films

Many methods and devices for preparation of films have been developed. A fairly complete description of these methods and devices will be found in Gardner and Sward [117]. In the present study the films were cast with Microm Film Applicator Adjustable-Clearance Model (Paul N. Gardner Company) with 23 x 30 cm vacuum plate. In previous studies most workers prepared films on a Morest chart, black and white glass, ceramic tiles, or transparent plastic film, and then measured film reflectance without removing the substrate. Because the reflectance of the substrate is often changed by the applied paint there are unavoidable optical effects of such supported films -- even though there is a method for correcting for these effects [118]. Therefore in this study, the films were free, i.e., they are removed from the substrate after drying. Many methods have been suggested for obtaining free films [119]. Since the vehicle in this study adheres poorly to certain plastic films, a duPont Mylar Polyester Film Type S Gauge 100 with nominal thickness 0.001 in. was chosen as substrate for casting the paint films to obtain free films.

Mylar films were cut to the same size as the vacuum plate, i.e., 23 cm x 30 cm. The mylar film was placed on the vacuum plate lying on a level bench top. Drawdowns of each paint were made on the Mylar sheet with the Film Applicator. The cast films were allowed to air-dry at room temperature for about a day or more, depending on the thickness of the film. Without removing the substrate the dry films were then cut to obtain three 2 by 2 in. pieces for each film thickness. To obtain the double-thickness film, two pieces were patched together, paint side toward paint side, with mineral oil of refractive index 1.5 as a glue, and placed down on the base plate and carefully rolled over with the glass rod to ensure adhesion between those two films. The substrates were removed off the films before measuring the reflectance. Upon removal from the substrate showed a high gloss on a clean surface. The reflectance of this side of film was measured immediately after the substrate had been removed.

Various film thicknesses were cast for each paint to ensure obtaining satisfactory conditions, i.e., uniform thickness of single films, accurate double thickness, and reflectance to satisfy Eq. (77).

Paint film samples were all prepared from three single-pigment lacquers. Each pigmented lacquer was prepared at four different pigment

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volume concentrations, PVC, 1% PVC, 5% PVC, 10% PVC and 15% PVC. Samples of the mixed pigment lacquer films were prepared at 15% PVC only.

The experimental values of infinite reflectance are the averages reflectance of infinite film thickness, which were obtained by two methods: (a) patching thin films together one by one until the infinite film thickness was obtained; (b) casting the thin film and letting it dry and then repeatedly casting over it until the infinite thickness is obtained. It is quite difficult to cast a very thick film to obtain the infinite thickness with only one drawdown because of the formation of air bubbles. The first method seems likely to be the easiest way to obtain the infinite thickness film but considerable care must be exercised to avoid effects such as the rubbing effect which may give rise to incorrect measurements. The second method seems to be better, but it is quite tedious and time-consuming. We preferred the first method because of its similarity to the method of obtaining the double film thickness. To ensure the reproducibility of the infinite reflectance, at least four or five specimens had been used to find the average infinite reflectance for each sample.

3.2 Film Measurements

In the application of K.M theory, which relates the degree of reflectance to film thickness, the equations contain two parameters K and S which characterize the optical properties of the paint film. Conversely, these equations permit K and S to be determined from measurements of reflectance and film thickness. For the K.M single-constant theory we do not need to know the film thickness, but for the K.M two-constant theory, we must know it precisely. Without good film-thickness measurements, the results can be neither precise nor accurate. The second key to quantitative measurements is a good evaluation of reflectance of the apint film over the balck substrate. A small error here produces a large error in S because of the way that S is calculated from the reflectance values.

3.2.1 Measurement of Film Thickness

Many methods for measuring dry film thickness are known, but only a few are suitable for all types of specimens and circumstances [120, 121]. In this study, the thickness of a free chip of paint is measured directly with a commercial micrometer caliper. ASTM Method D 374, "Thickness of Solid Electrical Insulation," contains instructions for calibrating and using Machinist's Micrometers. A preferred procedure is set forth in ASTM Method D 1005, "Measurement of Dry Film Thickness of Organic Coatings." The method should give an accuracy of ±0.1 mil provided the film and panel surface are not distorted. Measurements of film thickness had been made after the measurement of reflectance for each film, so that the films will not be soiled inadvertently by the micrometer. Three or four measurements were made of each film, and the average was taken as the thickness of the film.

3.2.2 Reflectance Measurements

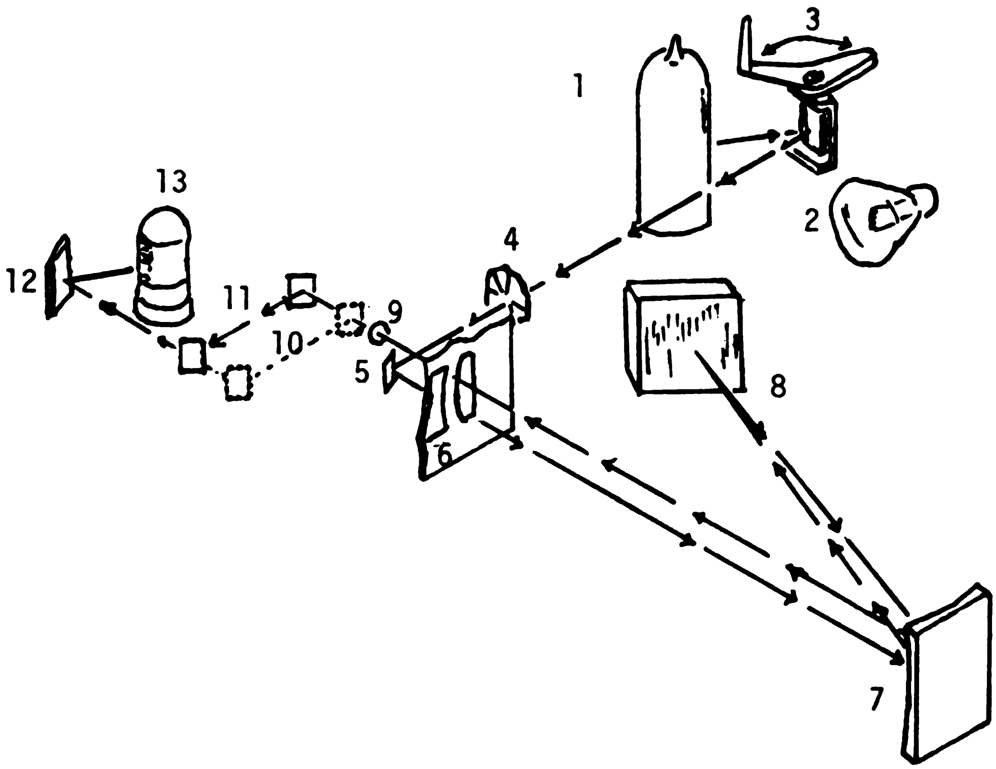
All surfaces reflect some fraction of the incident energy. The nature of the reflected energy depends not only on the type of incident beam, but also on the physical properties of the material under examination. Hence measurement of reflectance can serve to determine the optical properties of materials.

Measurement of reflectance spectra is not used as fully as it might be because such spectra are not easily interpreted. The technique is, however, hardly novel, particularly if one is speaking about diffuse-reflectance measurement, which deals with the measurement of light reflected in a diffuse manner (in contrast to specular or direct reflectance). Reflectance measurements are particularly suited for solid samples and thus provide an analytical tool in cases where samples are difficult to dissolve. Diffuse-reflectance measurement is rather widely applied in the following areas: dyestuffs, printing inks, paints and pigments, texttiles, paper, ceramics, building materials, mining, metallurgy, and others.

In this study the reflectance was measured with a Beckman DB-GT recording Spectrophotometer.

3.2.2.1 Spectrophotometric Components

The Beckman DB-GT Spectrophotometer in this study is a single-beam/double-beam, ratio-indicating or ratio-recording instrument which can make rapid transmittance and relfectance measurements in the near-ultraviolet/visible region. The spectrophotometer is housed in a compact case with front-panel controls and a direct-reading meter. Major components include two sources, source power supplies, monochromator, beam-switching device, detector, scale-expansion and zero-suppression controls, and amplifying, regulating, and log-converting circuitry. The reflectance attachment for the Model DB-GT Spectrophotometer consists of an integrating sphere equipped with a suitable end-on photomultiplier tube, of sample and reference holders, and of a lens system. The spectra were recorded on a Beckman Model 1005 Ten-Inch Laboratory Potentiometric Recorder. The optical diagram of the DB-GT Spectrophotometer is illustrated in Figure 3.1. Radiant energy from the tungsten or deuterium lamp is directed as a beam by the source-selector mirror to a seven-position filter wheel. In the filter-wheel position, 200 nm to 300 nm, the entrance beam passes unrestricted. Filters mounted in the other six positions allow transmission of specific spectral regions of the entrance beam.



1. Deuterium Lamp
2. Tungsten Lamp
3. Source Selector Mirror
4. Filter
5. Entrance Mirror
6. Slits
7. Collimating Mirror
8. Grating
9. Condensing Lens
10. Sample Beam
11. Reference Beam
12. Detector Focus Mirror
13. Photomultiplier Detector

Fig. 3.1 The Optical Diagram of the DB-GT Spectrophotometer.

3.2.2.2 Reflectance Measurements of Films

The measurement of reflectance is one of the most difficult problems of spectroscopy. Reflectance is not a simple specific property of a sample as is absorbance or polarization. Reflected light is influenced by many parameters such as illuminating light, illuminating aperture, surface texture of the sample, polarisation and absorbance of the sample. The light can be reflected diffusely, specularly, or as a mixture. In applying K.M equations to the calculation of K and S coefficients, various errors may arise because of deviations of the actual experimental conditions from the ideal conditions assumed in the equations. Many of the previous workers find K and S from K.M equations by measuring R_∞ , R_g, R and X using Eq. (11) and Eq. (22). This method has been used successfully to obtain K and S for very weakly absorbing (nearly white) powders. But it is not a satisfactory solution to the problem for a sample of the relatively high absorption coefficients, for even a thin layer appears infinitely thick, making it difficult to determine R, R_∞, and X to a satisfactory degree of accuracy. Many other authors find K and S from K.M equations by measuring R_o, R, R_g and X, first finding the K.M parameters a and b by using Eq. (29) and Eq. (20), then finding R_∞ from these parameters and the coefficients K and S are them obtained. There are some disadvantages in this method; the trouble in obtaining the K.M parameters a and b, the uncertain value of R_∞ and the complicated phenomena of the substrate reflectance, which have been mentioned before.

To test the applicability of the K.M theory as far as practical conditions are concerned, it is first necessary to investigate how far the assumptions involved in this theory can be fulfilled, and also whether the experimental measuring conditions limit or even exclude the applicability of the theory. For reasons discussed before, the equations of the K.M theory have been derived for diffuse incident irradiation. Therefore, we are limited practically to measuring diffuse reflectance only. The Beckman DB-GT Spectrophotometer is usable for these purposes.

To obtain the reflectance of the sample over the ideal black substrate, most of the previous authors used black glass or various designs printed in black and white on paper such as the Morest chart. These black substrates, however, do not give zero reflectance. In the present study the Ultraviolet Transmitting visible absorbing filter of Corning Glass Color Filter, Violet (Corning 7-54) served as the black substrate, which gives zero diffuse reflectance almost throughout the visible region.

In our technique, R_∞, together with the coefficients K and S, can be determined by reflectance measurements on the two films with a thickness ratio of 1:2 over the ideal black substrate. Diffuse reflectance spectra were recorded with a Beckman DB-GT Spectrophotometer equipped with a reflectance attachment having $BaSO_4$ as a reference. In all recording of the spectra, numerous repeated scans were made to insure the reproducibility and accuracy of the data.

The instruments had been carefully calibrated and always had been checked for accuracy by measuring Standard White $BaSO_4$, which should give 100% reflectance throughout the visible region, and by measuring the Violet filter (Corning 7-54) which should give 0% reflectance throughout the visible region.

The values of reflectance of two films, one as double the thickness of the other -- R_{01} and R_{02} -- as obtained from these measurements were then checked to determine whether they satisfied the condition in Eq. (77) at every wavelength. This check can be easily made with the aid of tabulated values for inspecting R_{01} and R_{02} in Table A.1 Appendix I. The data for the set of films (two films with a thickness ratio 1:2) that did not satisfy this condition should be deleted. And again when the films thickness had been measured, the data for the set of films that did not give a thickness close to the ratio of 1:2 should also be deleted. This descipancy could have arisen because of the nonuniform thickness of films.

The recorded reflectance spectra R_{01} and R_{02} of the valid set of films were then read out at 16 wavelengths from 400 to 700 nm at intervals of 20 nm. These reflectance values then were used along with film-thickness values to find the infinite reflectance R_{∞}, and the coefficients K and S, at each wavelength for further applications.

It is important to note that the reflectance values read out directly from the reflectance spectra are the values relative to $BaSO_4$. When the absolute reflectance values or the surface correction are to be considered, the inspection of R_{01} and R_{02} should be carefully made to insure that they still satisfy the condition in Eq. (77).

3.3 Calculation

All calculations of K, S, K/S, R_{∞} and other values from measured reflectance spectra were made on the CDC 6500 computer with programs written in FORTRAN language. These computer programs have been compiled in the Appendix II. Four different approaches dealt with the question of absolute reflection versus relative reflectance:

(a) With the relative reflectance values, i.e., with the measured reflectance values directly without any correction or interpolation.

(b) With the absolute reflectance values, i.e., through Eq. (44) for each measured reflectance value, employing the absolute reflectance of standard white $BaSO_4$ as reported by Gillespie et al. [72]. The values for converting reflectance relative to $BaSO_4$ to the absolute reflectance values are tabulated in Table A.3 APPENDIX I.

(c) With the relative reflectance values with surface correction, utilizing (1) the Saunderson correction, computing R from Eq. (50); and (2) the Duncan method, computing R from Eq. (55).

(d) With absolute reflectance values with surface correction, with both the Saunderson correction and the Duncan method. The measured reflectance values were first converted into absolute reflectance values, and then subjected to surface correction.

3.3.1 Determination of R_∞, $F(R_\infty)$, K and S for Individual Pigment

From the known values of R_{01} and R_{02}, the values of R_∞ can be determined directly either by using Eq. (64), or first finding W and Y from Eqs. (61) and (57) and then computing R_∞ using Eq. (71) or Eq. (43). The same result may be obtained by first finding σ and β from Eqs. (69) and (70) and then calculating R_∞ from Eq. (10). The values of R_∞ with the corresponding values of R_{01} and R_{02} have been tabulated in Table A.2 APPENDIX I, which may be used to find the values of R_∞ when R_{01} and R_{02} are known.

To check the applicability of our method, the calculated reflectance values R_∞ have been compared with the measured reflectance values R_∞. Various samples of infinite thickness films were prepared and repeatedly measured for R_∞, and the average of all experimental values were used for comparison. Note that if we started to find R_∞ by using absolute reflectance values or surface correction, it would be necessary to convert back to relative reflectance before comparing with the experimental reflectance values R_∞ .

When the values of R_∞ were known, $F(R_\infty)$ can be found by using Eq. (72) or can be read directly from Table A.6 in APPENDIX I. The values of K and S then can be determined using Eq. (62) and Eq. (74) or other appropriate equations when film thickness was known.

Computations were made at 16 wavelengths from 400 to 700 nm in 20 nm steps so that the results will be generally applicable to colored paint film.

3.3.2 Transforming Reflectance Spectra into CIE Coordinates

In order to simplify the comparison of computed and experimental results, the reflectance data were converted to CIE chromaticity coordinates. The comparison is then stated in terms of the Hunter color difference formula. It should be noted, however, that our computations are generally applicable to any region of the electromagnetic spectrum and our application here to colored (visually) films is only a special case.

The reflectance spectra are transformed into CIE coordinates by means of Eqs. (86) and (87) to find the tristimulus values X, Y, and Z, and Eq. (88) to find the chromaticity coordinates x and y. The 1931 CIE standard observer and the CIE standard source C were used.

Seven sets of the tristimulus values have to be determined: one from the measured values and six sets from the six cases of calculated values, relative values, absolute values, Saunderson correction with relative values, Saunderson correction with absolute values, Duncan method with relative values and Duncan method with absolute values.

The measured values and the calculated values of color coordinates were then compared by transforming the CIE tristimulus values into the Hunter color coordinate space system by Eq. (89). The Hunter color-coordinate differences between the experimental and calculated values can be found by Eq. (91); calculated values minus experimental values. The color difference between experimental and calculated values were then computed by the color-difference formula, Eq. (90). Total color difference ΔE, lightness difference ΔL, redness-greenness difference Δa, and yellowness-blueness difference Δb were then obtained and the results can readily be interpreted.

Values of R_∞ and K/S in all cases of the previous discussion may be obtained from the several Tables in APPENDIX I. The procedure for obtaining these values is illustrated in Figure 3.2 with each step in the diagram being described as follows:

Step I T.1 By Table A.1 in APPENDIX I, check the criterion

$$R_{01} \not> \frac{R_{02}}{2 - R_{02}} \quad .$$

If R_{01} and R_{02} satisfy this criterion, then they can be used with our method and one may proceed to the next step. If they do not satisfy the criterion, that sample can not be used in this case.

Step II T.2 Find R_∞ by Table A.2 in APPENDIX I.

Step III T.6 Find K/S by Table A.6 in APPENDIX I.

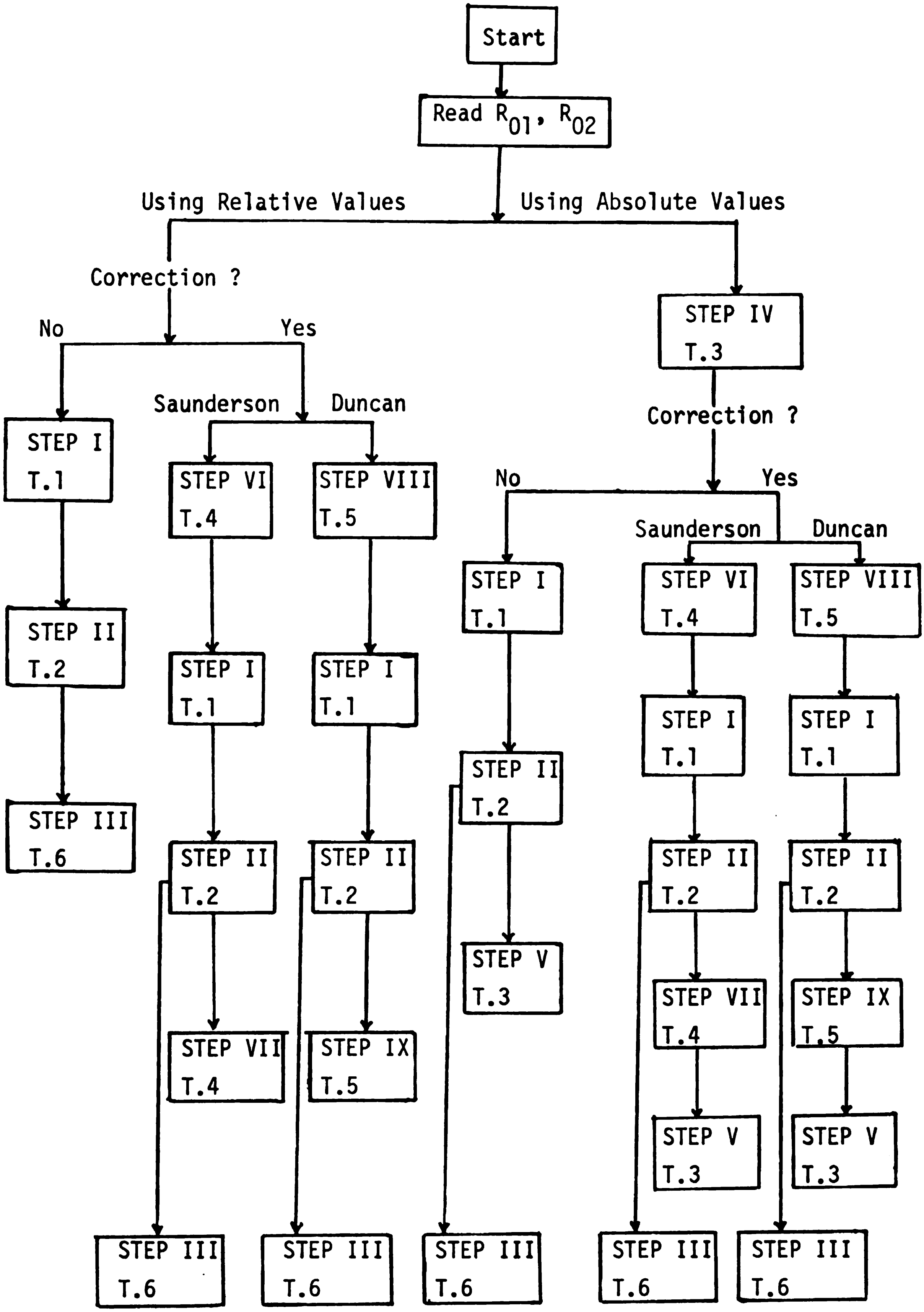


Fig. 3.2 Flow Chart for Determining K/S and R_∞ in Various Cases by using Tables in APPENDIX I.

Step IV T.3 Convert values R_{01} and R_{02} to absolute values by Table A.3 in APPENDIX I.

Step V T.3 Convert values R_∞ obtained in absolute values back to relative values by Table A.3 in APPENDIX I.

Step VI T.4 Correct R_{01} and R_{02} values by the Saunderson correction using Table A.4 in APPENDIX I.

Step VII T.4 Convert R_∞ obtained by the Saunderson correction back to the uncorrected values by using Table A.4 in APPENDIX I.

Step VIII T.5 Correct R_{01} and R_{02} values by the Duncan method using Table A.5 in APPENDIX I.

Step IX T.5 Convert R_∞ obtained by the Duncan method back to the uncorrected values by using Table A.5 in APPENDIX I.

Since it is tedious to calculate K/S in cases of the Saunderson correction and the Duncan method from the measured R_∞ each time it is needed, the values of K/S corresponding to different values of measured R_∞ for the Saunderson correction and the Duncan method are given in APPENDIX I, Table A.7 and Table A.8 respectively.

Duncan has developed his method of correction from the correction equations of Saunderson [57] and Ryde [29], but his formula is misprinted in the original paper [59], and the incorrect formula has been referenced and used in some texts [46]. Duncan tabulated values of K/S obtained by his method corresponding to measured values of R_∞ and indicated that K/S is zero when the measured values R_∞ are 96% and higher. This is not true; the value of K/S is zero when R_∞ is 96% but then increases as the value of R_∞ increases as shown in Table A.8 of APPENDIX I.

In most earlier work, infinite reflectance values were obtained by first determining the K.M parameters a and b. As we have noted earlier, that method leads to some difficulty. In this study infinite reflectance values are obtained directly without using those parameters. In many applications of K.M analysis, however, those parameters are required and for those applications we have constructed Table A.9 and Table A.10 in APPENDIX I for the Kubelka-Munk parameters a and b corresponding to infinite reflectance values respectively.

3.3.3 Computing the Reflectance of Paint Films Containing More Than One Pigment

In order to test the validity of the K and S coefficients and R_∞ values a number of film specimens containing two or more pigments were studied. If K and S or the Kubelka-Munk function, K/S, of the individual pigments are known, then the reflectance of the mixtures can be computed by the following methods:-

(a) A Single-constant theory. The values of the Kubelka-Munk function, K/S, of the multi-pigment system can be determined by Eq. (78) when the values of K/S of the individual pigments and their concentrations are known. The values of R_∞ then can be found directly from Table A.6 in APPENDIX I or by solving Eq. [11].

(b) Two-constant theory. From the known values of K and S of the individual pigments, the values K/S of the mixture can be determined by Eq. (81). The values of R_∞ are then determined as in the single constant theory.

All values so computed have been compared with the measured reflectance values. It is important to note as mentioned earlier that if the predicted reflectance values have been calculated from the absolute values and/or surface correction, they must be converted back to relative reflectance before comparing with the measured reflectance values.

3.3.4 Computing the Color of Paint Films Containing More Than One Pigment

When the predicted reflectance of the multi-pigment system is known at enough wavelengths in the visible region, the tristimulus values X, Y and Z of the system can be found by Eq. (86) or the CIE chromaticity x, y coordinates by Eq. (88). Computed values were compared with the experimental values by first transforming the CIE tristimulus both to the Hunter color coordinate system L, a, b by Eq. (89). Then the colors difference is found by Eq. (90).

CHAPTER IV

RESULTS AND DISCUSSION

Although several samples were prepared for this study, only a few of the crucial ones will be described in detail. A number of samples have been prepared in this study only to test the method. The results discussed here are intended to show how the method works.

4.1 Results of Calculated Values R_∞ of Single-pigment Lacquers

The most essential data in the K.M analysis are the values of R_∞ because the foundation of the theory is based on the reflectance of the infinite-thickness sample. Many methods have been described by earlier workers, but there are a number of disadvantages associated with them, all of which are avoided in the method developed for this study. Tables 4.2, 4.3, 4.4 and 4.5 show the comparison of calculated reflectance values with the average experimental reflectance values of infinite-thickness films at 15%, 10%, 5% and 1% PVC respectively, for TiO_2 pigment lacquer films. These experimental and calculated reflectance values have been transformed into color space and compared in terms of the Hunter color-difference units which are also shown in these figures. The same comparison for some samples of Fe_2O_3 pure red iron oxide, are shown in Tables 4.6 to 4.9: Cr_2O_3, pure chromium oxide green, are shown in Tables 4.10 through 4.12. The position of these colors

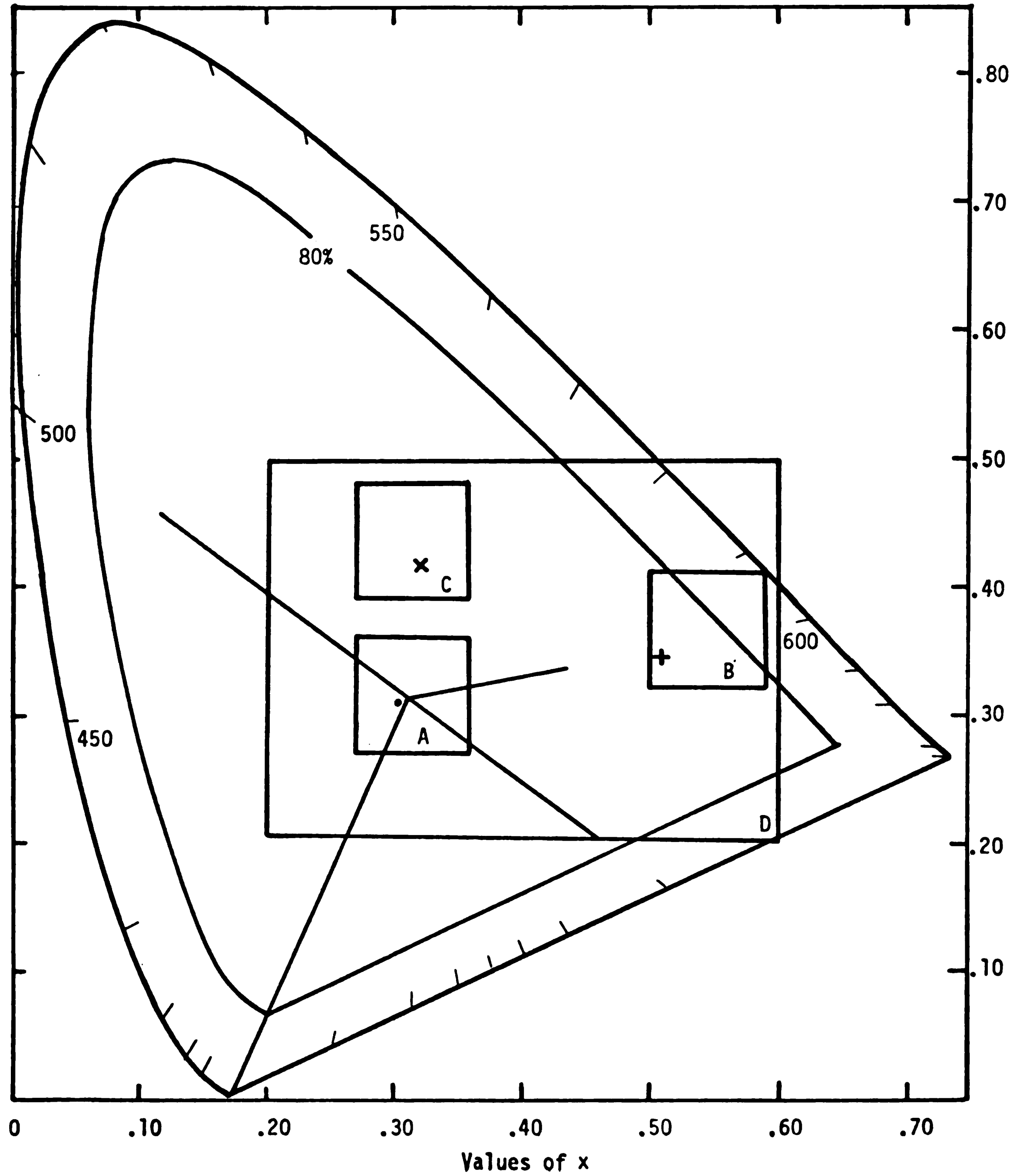


Fig. 4.1 Chromaticity coordinates by experiment of pigment lacquer films TiO_2 (. in area A), Fe_2O_3 (+ in area B) and Cr_2O_3 (x in area C). Areas A, B, C and D are reproduced at a larger scale in Figs. 4.2, 4.3, 4.4 and 4.5 respectively.

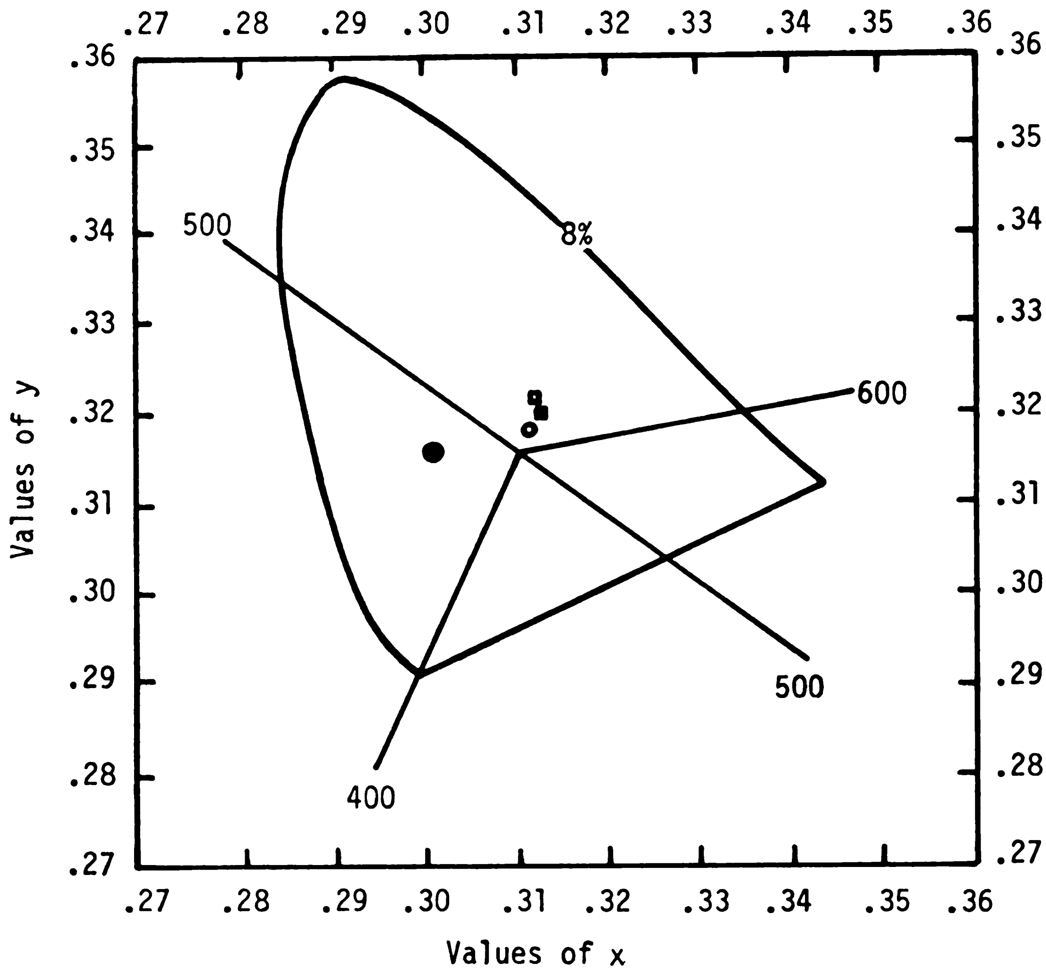


Fig. 4.2 Chromaticity coordinates of TiO_2 pigment lacquer films.

- ● experiment at 1 PVC
- ● calculated at 1 PVC
- ■ experiment at 15 PVC
- ◘ calculated at 15 PVC

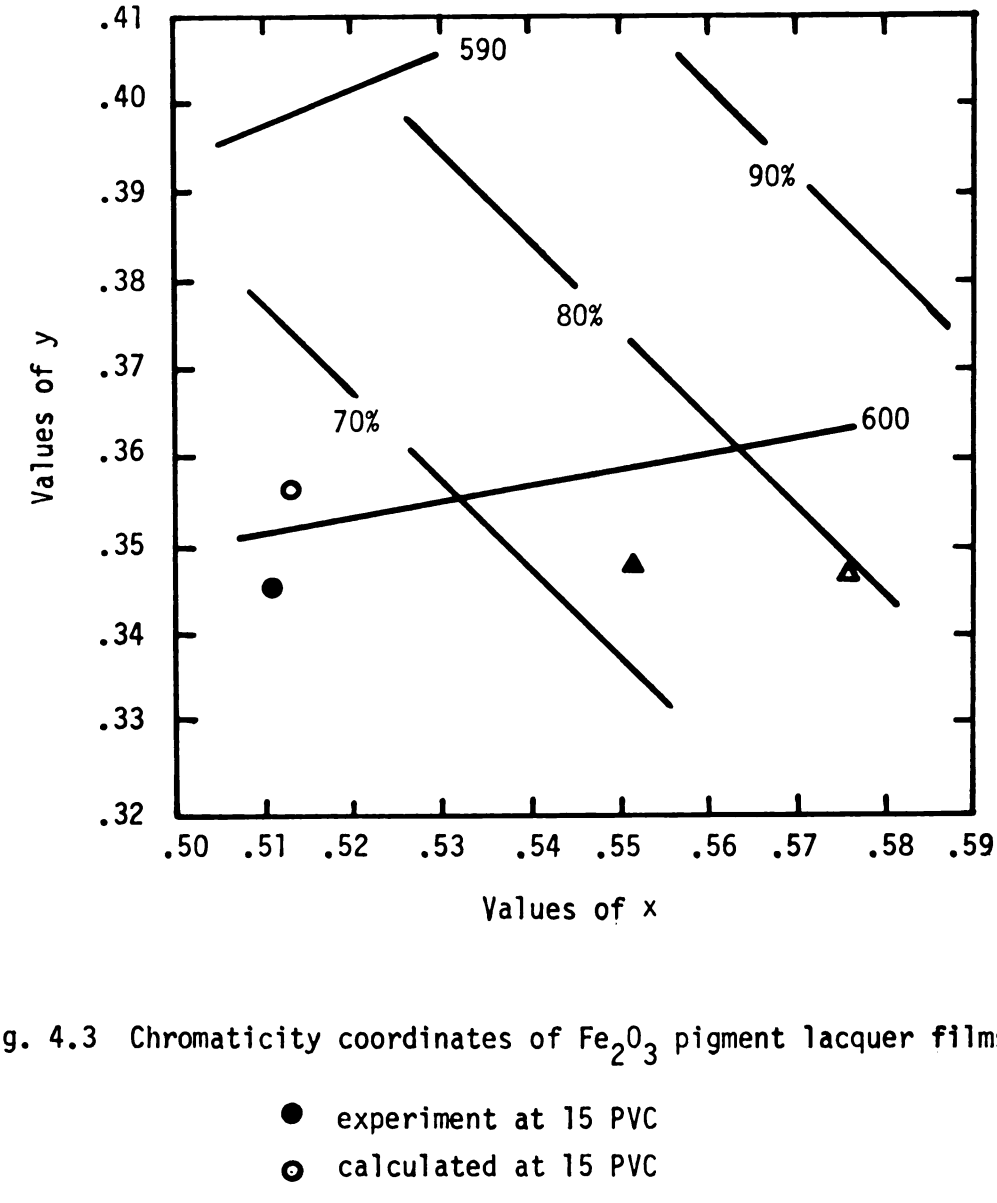


Fig. 4.3 Chromaticity coordinates of Fe_2O_3 pigment lacquer films.

● experiment at 15 PVC

○ calculated at 15 PVC

▲ experiment at 5 PVC

△ calculated at 5 PVC

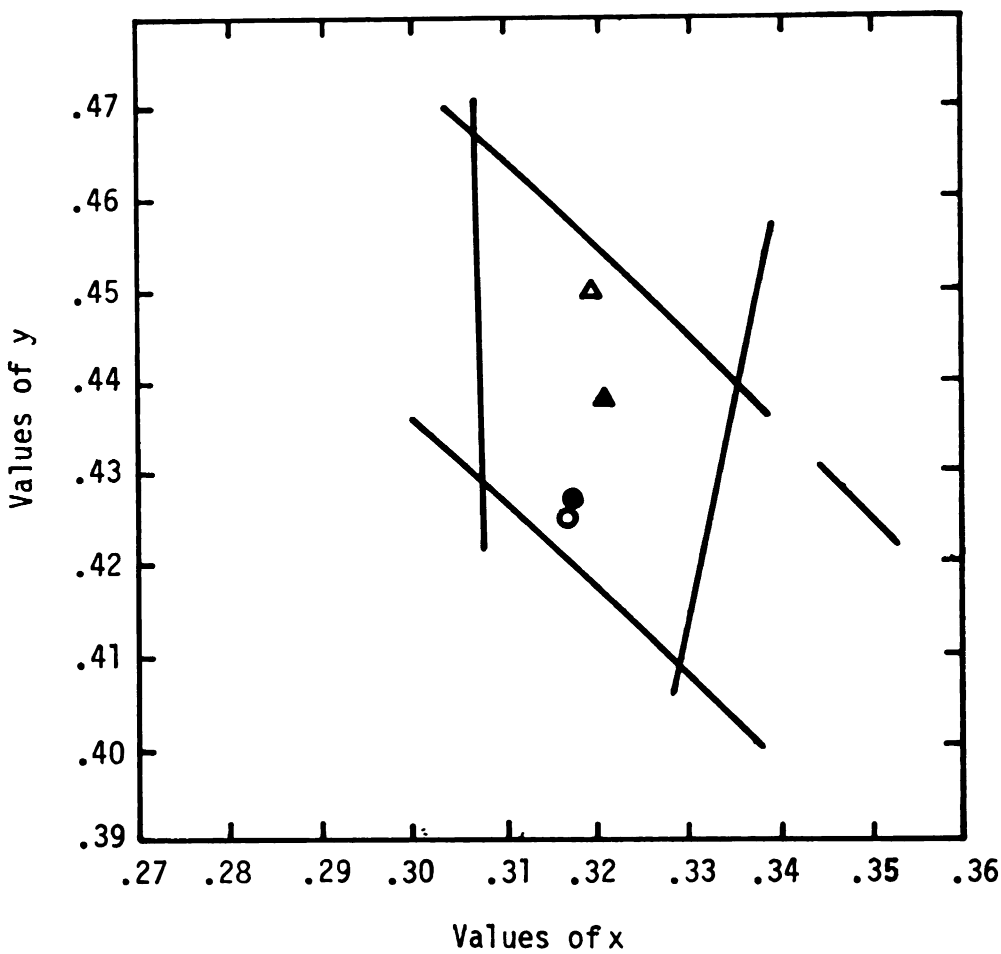


Fig. 4.4 Chromaticity coordinates of Cr_2O_3 pigment lacquer films.

● experiment at 15 PVC

○ calculated at 15 PVC

▲ experiment at 5 PVC

△ calculated at 5 PVC

Table 4.1 Lists of notations and abbreviations used in Tables 4.2-4.12.

Symbol	Definition
R	A value that cannot be calculated.
WL	Wavelength.
RO1, RO2	The measured reflectance of two film thickness, one double the thickness of the other, on black substrate.
RIM	The average reflectance of infinite-thickness film obtained by experiment.
RIN, RIAR	The calculated infinite reflectance obtained from the relative and absolute reflectance values respectively.
RISR, RIASR	The calculated infinite reflectance obtained through the Saunderson correction by using the relative and absolute reflectance values respectively.
RIDR, RIADR	The calculated infinite reflectance obtained through the Duncan method by using the relative and absolute reflectance values respectively.
DR, DRA	The difference between the calculated and experimental infinite reflectance using the relative and absolute reflectance values respectively.
DRS, DRAS	The difference between the calculated and experimental infinite reflectance through the Saunderson correction using the relative and absolute reflectance values respectively.
DRD, DRAD	The difference between the calculated and experimental infinite reflectance through the Duncan method using the relative and absolute reflectance values respectively.
X, Y, Z	The tristimulus values in C.I.E. color system.
SMLX, SMLY	The chromaticity coordinates x and y respectively.
DELTA L, DELTA A, DELTA B, DELTA E	The notations ΔL, Δa, Δb and ΔE used in the Hunter color difference formula respectively

Table 4.1 Lists of notations and abbreviations used in Tables 4.2-4.12 (continued).

Symbol	Definition
COR.	Correction
ABS.	Using the absolute reflectance values.
REL.	Using the relative reflectance values.

SAMPLE NO. 10015 TIO2 WHITE 15PVC
FILM THICKNESS = 136.0710 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.3000	.8830	.9430	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460	.9460
R02	.3090	.9000	.9530	.9620	.9670	.9700	.9700	.9700	.9700	.9700	.9700	.9700	.9700	.9700	.9700	.9700
RIM	.3250	.9080	.9620	.9710	.9740	.9800	.9830	.9840	.9850	.9860	.9880	.9890	.9900	.9910	.9910	.9870
RIN	.3093	.9018	.9542	.9659	.9755	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839
RIAR	.3093	.9018	.9542	.9658	.9752	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9819
RISR	.3093	.9017	.9542	.9659	.9755	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839	.9839
RIASR	.3093	.9017	.9541	.9658	.9751	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9824	.9818
RIDR	.3093	.9025	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RIADR	.3093	.9026	R	R	R	R	R	R	R	R	R	R	R	R	R	R

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0157	-.0062	-.0078	-.0051	.0015	.0039	.0009	-.0001	-.0011	-.0021	-.0041	-.0051	-.0061	-.0071	-.0071	-.0031
DRA	-.0157	-.0062	-.0078	-.0052	.0012	.0024	-.0006	-.0016	-.0026	-.0036	-.0056	-.0066	-.0076	-.0086	-.0086	-.0051
DRS	-.0157	-.0063	-.0078	-.0051	.0015	.0039	.0009	-.0001	-.0011	-.0021	-.0041	-.0051	-.0061	-.0071	-.0071	-.0031
DRAS	-.0157	-.0063	-.0079	-.0052	.0011	.0024	-.0006	-.0016	-.0026	-.0036	-.0056	-.0066	-.0076	-.0086	-.0086	-.0052
DRD	-.0157	-.0052	R	R	R	R	R	R	R	R	R	R	R	R	R	R
DRAD	-.0157	-.0052	R	R	R	R	R	R	R	R	R	R	R	R	R	R

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	96.04741	98.44129	112.7[illegible]254	.31259	.32038				
USE RELATIVE VALUES	95.67142	98.31055	112.22884	.31244	.32106	-.06591	-.44669	.23419	.50865
USE ABSOLUTE VALUES	95.55693	98.17304	112.20662	.31234	.32089	-.13527	-.41098	.15195	.45857
SAUNDERSON COR. REL.	95.67257	98.31244	112.22670	.31244	.32106	-.06496	-.44776	.23678	.51066
SAUNDERSON COR. ABS.	95.55657	98.17299	112.20426	.31234	.32090	-.13530	-.41153	.15332	.45953
DUNCAN COF. REL.	R	R	R	R	R	R	R	R	R
DUNCAN COF. ABS.	R	R	R	R	R	R	R	R	R

Table 4.2 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR TiO_2 AT 15 PVC

SAMPLE NO. 100102 TIO2 10PVC

FILM THICKNESS = 85.9536 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.2850	.8520	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750	.8750
R02	.3500	.9100	.9300	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320	.9320
RIM	.3700	.9160	.9580	.9680	.9680	.9700	.9720	.9730	.9740	.9770	.9810	.9820	.9820	.9830	.9830	.9830
RIN	.3723	.9402	.9686	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800	.9800
RIAR	.3722	.9397	.9674	.9779	.9779	.9761	.9761	.9761	.9761	.9761	.9761	.9761	.9761	.9761	.9761	.9746
RISR	.3729	.9401	.9689	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811	.9811
RIASR	.3729	.9397	.9678	.9787	.9787	.9768	.9768	.9768	.9768	.9768	.9768	.9768	.9768	.9768	.9768	.9751
RIDR	.3697	.9028	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RIACR	.3697	.9028	R	R	R	R	R	R	R	R	R	R	R	R	R	R

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
DS	.0023	.0242	.0106	.0120	.0120	.0100	.0080	.0070	.0060	.0030	-.0010	-.0020	-.0020	-.0030	-.0030	-.0030
DPA	.0022	.0237	.0094	.0099	.0099	.0061	.0041	.0031	.0021	-.0009	-.0049	-.0054	-.0059	-.0069	-.0069	-.0084
CHS	.0029	.0241	.0109	.0131	.0131	.0111	.0091	.0081	.0071	.0041	.0001	-.0009	-.0009	-.0019	-.0019	-.0014
DPAS	.0029	.0237	.0098	.0107	.0107	.0068	.0048	.0038	.0028	-.0002	-.0042	-.0052	-.0052	-.0062	-.0062	-.0079
CPD	-.0003	-.0052	R	R	R	R	R	R	R	R	R	R	R	R	R	R
CPAD	-.0003	-.0052	R	R	R	R	R	R	R	R	R	R	R	R	R	R

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	95.34252	97.49322	112.43699	.31232	.31936				
USE RELATIVE VALUES	95.69232	97.98818	113.90857	.31110	.31857	.25032	-.24316	-.53545	.63913
USE ABSOLUTE VALUES	95.35861	97.60952	113.70777	.31094	.31828	.05888	-.17668	-.68116	.70616
SAUNDERSON COR. REL.	95.78799	98.09384	113.99261	.31113	.31862	.30368	-.25708	-.51165	.64815
SAUNDERSON COR. ABS.	95.42183	97.67795	113.77337	.31095	.31830	.09350	-.18347	-.67237	.70319
DUNCAN COF. REL.	R	R	R	R	R	R	R	R	R
DUNCAN COF. ABS.	R	R	R	R	R	R	R	R	R

Table 4.3 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR TiO_2 AT 10 PVC

SAMPLE NO. 100052 TIO2 WHITE SPVC

FILM THICKNESS = 88.3412 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.3300	.7490	.7490	.7490	.7470	.7400	.7400	.7350	.7340	.7280	.7270	.7210	.7150	.7120	.7090	.7090
R02	.3350	.8350	.8400	.8400	.8400	.8390	.8380	.8370	.8350	.8310	.8280	.8250	.8230	.8220	.8210	.8190
RIM	.3520	.8920	.9470	.9510	.9550	.9580	.9620	.9640	.9650	.9650	.9690	.9690	.9690	.9690	.9690	.9690
RIN	.3351	.8785	.6927	.8927	.8962	.9069	.9031	.9108	.9052	.9036	.8946	.8967	.9033	.9071	.9113	.9022
RIAR	.3351	.8782	.8923	.8923	.8957	.9057	.9020	.9094	.9040	.9024	.8936	.8955	.9019	.9056	.9096	.9001
RISR	.3351	.8782	.8927	.8927	.8964	.9077	.9038	.9121	.9061	.9047	.8952	.8976	.9049	.9092	.9139	.9040
RIASR	.3351	.8779	.8923	.8923	.8959	.9064	.9026	.9105	.9048	.9033	.8941	.8963	.9034	.9075	.9120	.9016
RIDR	.3351	.8981	.9309	.9309	.9545	R	R	R	R	R	R	R	R	R	R	R
RIADR	.3351	.8972	.9280	.9280	.9442	R	R	R	R	R	R	R	R	R	R	R

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0169	-.0135	-.0543	-.0583	-.0588	-.0511	-.0589	-.0532	-.0598	-.0614	-.0744	-.0723	-.0657	-.0619	-.0577	-.0668
DRA	-.0169	-.0138	-.0547	-.0587	-.0593	-.0523	-.0600	-.0546	-.0610	-.0626	-.0754	-.0735	-.0671	-.0634	-.0594	-.0689
DRS	-.0169	-.0138	-.0543	-.0583	-.0586	-.0503	-.0582	-.0519	-.0589	-.0603	-.0738	-.0714	-.0641	-.0598	-.0551	-.0650
DRAS	-.0169	-.0141	-.0547	-.0587	-.0591	-.0516	-.0594	-.0535	-.0602	-.0617	-.0749	-.0727	-.0656	-.0615	-.0570	-.0674
DRD	-.0169	.0061	-.0161	-.0201	-.0005	R	R	R	R	R	R	R	R	R	R	R
DRAD	-.0169	.0052	-.0190	-.0230	-.0108	R	R	R	R	R	R	R	R	R	R	R

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	94.13409	96.41050	110.70963	.31247	.32003				
USE RELATIVE VALUES	87.98007	90.35597	104.60394	.31095	.31935	-3.13309	-.43291	-.58823	3.21709
USE ABSOLUTE VALUES	87.87645	90.23636	104.55053	.31089	.31924	-3.19602	-.40802	-.64218	3.28534
SAUNDERSON COR. REL.	88.06031	90.44816	104.61309	.31103	.31947	-3.08461	-.45137	-.52674	3.16165
SAUNDERSON COR. ABS.	87.94669	90.31710	104.55616	.31096	.31934	-3.15354	-.42426	-.58678	3.23560
DUNCAN COR. REL.	R	R	R	R	R	R	R	R	R
DUNCAN COR. ABS.	R	R	R	R	R	R	R	R	R

Table 4.4 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR TiO_2 AT 5 PVC

SAMPLE NO. 100012 TIO2 WHITE 1PVC
FILM THICKNESS = 72.3900 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.2350	.5590	.5590	.5530	.5530	.5510	.5510	.5420	.5350	.5350	.5300	.5280	.5210	.5180	.5150	.5100
R02	.2500	.6610	.6610	.6610	.6600	.6600	.6600	.6550	.6550	.6510	.6500	.6480	.6400	.6400	.6400	.6390
RIM	.3000	.8120	.8200	.8200	.8200	.8180	.8200	.8100	.8100	.8080	.8070	.8060	.8050	.8010	.8010	.8010
RIN	.4925	.7050	.7050	.7123	.7100	.7127	.7127	.7132	.7243	.7137	.7192	.7172	.7074	.7127	.7183	.7255
RIAR	.4921	.7049	.7049	.7122	.7099	.7124	.7124	.7129	.7240	.7134	.7189	.7169	.7072	.7124	.7180	.7250
RISR	.5496	.7039	.7039	.7114	.7091	.7117	.7117	.7124	.7241	.7131	.7190	.7170	.7071	.7127	.7187	.7266
RIASR	.5492	.7038	.7038	.7113	.7089	.7115	.7115	.7122	.7238	.7128	.7187	.7167	.7068	.7123	.7183	.7260
RIDR	.4792	.7058	.7058	.7137	.7112	.7140	.7140	.7147	.7269	.7153	.7215	.7193	.7089	.7146	.7208	.7289
RIADR	.4787	.7057	.7057	.7135	.7111	.7137	.7137	.7144	.7265	.7150	.7211	.7189	.7085	.7142	.7203	.7281

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	.1925	-.1070	-.1150	-.1077	-.1100	-.1053	-.1073	-.0968	-.0857	-.0943	-.0878	-.0888	-.0976	-.0883	-.0827	-.0755
DRA	.1921	-.1071	-.1151	-.1078	-.1101	-.1056	-.1076	-.0971	-.0860	-.0946	-.0881	-.0891	-.0978	-.0856	-.0830	-.0760
DRS	.2496	-.1081	-.1161	-.1086	-.1109	-.1063	-.1083	-.0976	-.0859	-.0949	-.0880	-.0890	-.0979	-.0883	-.0823	-.0744
DRAS	.2492	-.1062	-.1162	-.1087	-.1111	-.1065	-.1085	-.0978	-.0862	-.0952	-.0883	-.0893	-.0982	-.0887	-.0827	-.0750
DRD	.1792	-.1062	-.1142	-.1063	-.1088	-.1040	-.1060	-.0953	-.0831	-.0927	-.0855	-.0867	-.0961	-.0864	-.0802	-.0721
DRAD	.1787	-.1063	-.1143	-.1065	-.1089	-.1043	-.1063	-.0956	-.0835	-.0930	-.0859	-.0871	-.0965	-.0868	-.0807	-.0729

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	79.22075	81.11724	95.95781	.30909	.31649				
USE RELATIVE VALUES	69.98194	71.59232	83.23008	.31130	.31847	-5.45287	.17050	1.03727	5.55327
USE ABSOLUTE VALUES	69.95796	71.56687	83.21758	.31128	.31844	-5.46791	.17249	1.02514	5.56585
SAUNDERSON COR. REL.	69.94435	71.53692	83.15914	.31136	.31845	-5.48561	.20562	1.04151	5.58739
SAUNDERSON COR. ABS.	69.91862	71.50972	83.14607	.31134	.31842	-5.50169	.20755	1.02832	5.60082
DUNCAN COF. REL.	70.15614	71.77424	83.34602	.31142	.31861	-5.34543	.16231	1.10529	5.46092
DUNCAN COF. ABS.	70.12264	71.73872	83.32904	.31139	.31857	-5.36640	.16499	1.08807	5.47808

Table 4.5 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR TiO_2 AT 1 PVC

SAMPLE NO. 1 RED (FE2O3) 15 PVC
FILM THICKNESS = 97.0280 MICRONS

#L	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
RO1	.0200	.0200	.0200	.0200	.0200	.0200	.0200	.0210	.0330	.0780	.1630	.2100	.2250	.2420	.2730	.3160
RO2	.0250	.0250	.0250	.0250	.0250	.0250	.0290	.0300	.0440	.1010	.1840	.2200	.2380	.2580	.2900	.3360
RIM	.0300	.0300	.0300	.0300	.0300	.0300	.0310	.0350	.0480	.1000	.1890	.2300	.2460	.2660	.2990	.3460
RIN	.0267	.0267	.0267	.0267	.0267	.0267	.0364	.0368	.0495	.1107	.1872	.2205	.2388	.2592	.2912	.3375
RIAR	.0267	.0267	.0267	.0267	.0267	.0267	.0364	.0368	.0495	.1107	.1872	.2205	.2388	.2592	.2912	.3375
RISR	R	R	R	R	R	R	R	R	R	.1331	.1879	.2206	.2389	.2593	.2912	.3375
RIASR	R	R	R	R	R	R	R	R	R	.1333	.1879	.2206	.2389	.2593	.2912	.3375
RIDR	.0266	.0266	.0266	.0266	.0266	.0266	.0361	.0365	.0492	.1099	.1869	.2205	.2387	.2591	.2911	.3373
RIACR	.0266	.0266	.0266	.0266	.0266	.0266	.0361	.0365	.0492	.1099	.1869	.2205	.2387	.2591	.2911	.3373

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0033	-.0033	-.0033	-.0033	-.0033	-.0033	.0054	.0018	.0015	.0107	-.0018	-.0095	-.0072	-.0068	-.0078	-.0085
DRA	-.0033	-.0033	-.0033	-.0033	-.0033	-.0033	.0054	.0018	.0015	.0107	-.0018	-.0095	-.0072	-.0068	-.0078	-.0085
DRS	R	R	R	R	R	R	R	R	R	.0331	-.0011	-.0094	-.0071	-.0067	-.0078	-.0085
DRAS	R	R	R	R	R	R	R	R	R	.0333	-.0011	-.0094	-.0071	-.0067	-.0078	-.0085
DRD	-.0034	-.0034	-.0034	-.0034	-.0034	-.0034	.0051	.0015	.0012	.0099	-.0021	-.0095	-.0073	-.0069	-.0079	-.0087
DRAD	-.0034	-.0034	-.0034	-.0034	-.0034	-.0034	.0051	.0015	.0012	.0099	-.0021	-.0095	-.0073	-.0069	-.0079	-.0087

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	12.51278	8.47707	3.54020	.51010	.34558				
USE RELATIVE VALUES	12.41761	8.63117	3.16324	.51287	.35648	.26345	-1.72717	1.00980	2.01797
USE ABSOLUTE VALUES	12.41759	8.63116	3.16324	.51287	.35648	.26342	-1.72718	1.00978	2.01797
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COR. REL.	12.38794	8.59737	3.15674	.51313	.35612	.20587	-1.65890	.97011	1.93273
DUNCAN COR. ABS.	12.38795	8.59740	3.15675	.51313	.35612	.20592	-1.65904	.97014	1.93287

Table 4.6 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR : Fe_2O_3 AT 15 PVC

SAMFLE NO. 1 RED (FE2O3) 10 PVC

FILM THICKNESS = 51.0540 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.0220	.0220	.0220	.0220	.0220	.0220	.0220	.0230	.0320	.0710	.1560	.2060	.2250	.2400	.2750	.3200
R02	.0240	.0240	.0240	.0240	.0240	.0240	.0240	.0270	.0400	.0790	.1600	.2100	.2300	.2500	.2800	.3280
RIM	.0250	.0250	.0250	.0250	.0250	.0260	.0270	.0290	.0380	.0800	.1650	.2170	.2370	.2560	.2870	.3360
RIN	.0242	.0242	.0242	.0242	.0242	.0242	.0242	.0278	.0427	.0800	.1601	.2101	.2301	.2505	.2801	.3282
RIAR	.0242	.0242	.0242	.0242	.0242	.0242	.0242	.0278	.0427	.0800	.1601	.2101	.2301	.2505	.2801	.3282
RISR	R	R	R	R	R	R	R	R	R	.0816	.1601	.2101	.2301	.2505	.2801	.3282
RIASR	R	R	R	R	R	R	R	R	R	.0816	.1601	.2101	.2301	.2505	.2801	.3282
RIDR	.0242	.0242	.0242	.0242	.0242	.0242	.0242	.0278	.0425	.0799	.1601	.2101	.2301	.2504	.2801	.3282
RIADR	.0242	.0242	.0242	.0242	.0242	.0242	.0242	.0278	.0425	.0799	.1601	.2101	.2301	.2504	.2801	.3282

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0008	-.0008	-.0008	-.0008	-.0008	-.0018	-.0028	-.0012	.0047	.0000	-.0049	-.0069	-.0069	-.0055	-.0069	-.0078
DRA	-.0008	-.0008	-.0008	-.0008	-.0008	-.0018	-.0028	-.0012	.0047	.0000	-.0049	-.0069	-.0069	-.0055	-.0069	-.0078
DRS	R	R	R	R	R	R	R	R	R	.0016	-.0049	-.0069	-.0069	-.0055	-.0069	-.0078
DRAS	R	R	R	R	R	R	R	R	R	.0016	-.0049	-.0069	-.0069	-.0055	-.0069	-.0078
DRD	-.0008	-.0008	-.0008	-.0008	-.0008	-.0018	-.0028	-.0012	.0045	-.0001	-.0049	-.0069	-.0069	-.0056	-.0069	-.0078
DRAD	-.0008	-.0008	-.0008	-.0008	-.0008	-.0018	-.0028	-.0012	.0045	-.0001	-.0049	-.0069	-.0069	-.0056	-.0069	-.0078

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	11.20570	7.36957	2.95741	.52040	.34225				
USE RELATIVE VALUES	10.97805	7.26479	2.85494	.52034	.34434	-.19367	-.63924	.04342	.66935
USE ABSOLUTE VALUES	10.97805	7.26479	2.85494	.52034	.34434	-.19367	-.63925	.04342	.66935
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COR. REL.	10.97447	7.26027	2.85426	.52039	.34427	-.20205	-.62567	.03710	.65853
DUNCAN COR. ABS.	10.97448	7.26028	2.85426	.52039	.34427	-.20204	-.62569	.03711	.65855

Table 4.7 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Fe_2O_3 AT 10 PVC

SAMPLE NO. 5 RED (FE2O3) 5 PVC

FILM THICKNESS = 33.0835 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.0110	.0110	.0110	.0110	.0110	.0110	.0110	.0120	.0260	.0700	.1600	.2120	.2300	.2510	.2800	.3200
R02	.0120	.0120	.0120	.0120	.0120	.0120	.0120	.0130	.0270	.0750	.1630	.2180	.2400	.2630	.3000	.3550
RIM	.0160	.0160	.0160	.0160	.0160	.0160	.0180	.0200	.0300	.0740	.1600	.2030	.2230	.2420	.2750	.3230
RIN	.0121	.0121	.0121	.0121	.0121	.0121	.0121	.0131	.0270	.0754	.1631	.2182	.2405	.2636	.3017	.3599
RIAR	.0121	.0121	.0121	.0121	.0121	.0121	.0121	.0131	.0270	.0754	.1631	.2182	.2405	.2636	.3017	.3599
RISR	R	R	R	R	R	R	R	R	R	.0760	.1631	.2182	.2405	.2637	.3017	.3598
RIASR	R	R	R	R	R	R	R	R	R	.0760	.1631	.2182	.2405	.2637	.3017	.3598
RIDR	.0121	.0121	.0121	.0121	.0121	.0121	.0121	.0131	.0270	.0754	.1631	.2182	.2404	.2636	.3015	.3594
RIADR	.0121	.0121	.0121	.0121	.0121	.0121	.0121	.0131	.0270	.0754	.1631	.2182	.2404	.2636	.3015	.3594

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0039	-.0039	-.0039	-.0039	-.0039	-.0039	-.0059	-.0069	-.0030	.0014	.0031	.0152	.0175	.0216	.0267	.0369
DHA	-.0039	-.0039	-.0039	-.0039	-.0039	-.0039	-.0059	-.0069	-.0030	.0014	.0031	.0152	.0175	.0216	.0267	.0369
DHS	R	R	R	R	R	R	R	R	R	.0020	.0031	.0152	.0175	.0217	.0267	.0368
DHAS	R	R	R	R	R	R	R	R	R	.0020	.0031	.0152	.0175	.0217	.0267	.0368
DHD	-.0039	-.0039	-.0039	-.0039	-.0039	-.0039	-.0059	-.0069	-.0030	.0014	.0031	.0152	.0174	.0216	.0265	.0364
DHAD	-.0039	-.0039	-.0039	-.0039	-.0039	-.0039	-.0059	-.0069	-.0030	.0014	.0031	.0152	.0174	.0216	.0265	.0364

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	10.33383	6.52085	1.84276	.55121	.34783				
USE RELATIVE VALUES	10.68348	6.44510	1.43018	.57566	.34728	-.14877	3.14208	.95043	3.28605
USE ABSOLUTE VALUES	10.68348	6.44510	1.43018	.57566	.34728	-.14877	3.14207	.95043	3.28604
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COF. REL.	10.68166	6.44404	1.43000	.57565	.34728	-.15086	3.13915	.94910	3.28296
DUNCAN COF. ABS.	10.68166	6.44404	1.43000	.57565	.34728	-.15086	3.13914	.94910	3.28295

Table 4.8 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Fe_2O_3 AT 5 PVC

SAMPLE NO. 3 RED (FE2O3) 1 PVC
FILM THICKNESS = 70.6120 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.0170	.0170	.0170	.0170	.0170	.0170	.0200	.0200	.0300	.0760	.1650	.2170	.2320	.2580	.2850	.3300
R02	.0230	.0230	.0230	.0230	.0230	.0230	.0240	.0250	.0340	.0770	.1690	.2270	.2480	.2720	.3090	.3600
RIM	.0230	.0230	.0230	.0230	.0230	.0230	.0240	.0250	.0340	.0770	.1690	.2270	.2480	.2720	.3090	.3600
RIN	.0263	.0263	.0263	.0263	.0263	.0263	.0250	.0267	.0346	.0770	.1691	.2275	.2493	.2729	.3114	.3635
RIAR	.0263	.0263	.0263	.0263	.0263	.0263	.0250	.0267	.0346	.0770	.1691	.2275	.2493	.2729	.3114	.3635
RISR	R	R	R	R	R	R	R	R	R	.0770	.1691	.2275	.2493	.2729	.3114	.3633
RIASR	R	R	R	R	R	R	R	R	R	.0770	.1691	.2275	.2493	.2729	.3114	.3633
RIDR	.0262	.0262	.0262	.0262	.0262	.0262	.0250	.0266	.0346	.0770	.1691	.2274	.2491	.2728	.3112	.3631
RIACR	.0262	.0262	.0262	.0262	.0262	.0262	.0250	.0266	.0346	.0770	.1691	.2274	.2491	.2728	.3111	.3631

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	.0033	.0033	.0033	.0033	.0033	.0033	.0010	.0017	.0006	.0000	.0001	.0005	.0013	.0009	.0024	.0035
DRA	.0033	.0033	.0033	.0033	.0033	.0033	.0010	.0017	.0006	.0000	.0001	.0005	.0013	.0009	.0024	.0035
DRS	R	R	R	R	R	R	R	R	R	.0000	.0001	.0005	.0013	.0009	.0024	.0033
DRAS	R	R	R	R	R	R	R	R	R	.0000	.0001	.0005	.0013	.0009	.0024	.0033
DRD	.0032	.0032	.0032	.0032	.0032	.0032	.0010	.0016	.0006	.0000	.0001	.0004	.0011	.0008	.0022	.0031
DRAC	.0032	.0032	.0032	.0032	.0032	.0032	.0010	.0016	.0006	.0000	.0001	.0004	.0011	.0008	.0021	.0031

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	11.40086	7.26091	2.71406	.53335	.33968				
USE RELATIVE VALUES	11.50615	7.36570	3.09525	.52379	.33531	.19374	-.18567	-.65450	.70737
USE ABSOLUTE VALUES	11.50615	7.36569	3.09525	.52379	.33531	.19373	-.18568	-.65451	.70738
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COF. REL.	11.50084	7.36156	3.08371	.52405	.33544	.18613	-.18606	-.63652	.68878
DUNCAN COF. ABS.	11.50084	7.36157	3.08372	.52405	.33544	.18613	-.18609	-.63654	.68881

Table 4.9 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Fe_2O_3 AT 1 PVC

SAMPLE NO. 3 GREEN (CR2O3) 15 PVC
FILM THICKNESS = 87.1855 MICRONS

#L	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.0700	.0920	.0410	.0390	.0400	.0630	.1200	.1830	.1190	.0610	.0580	.0630	.0770	.1020	.1390	.1720
R02	.0770	.0930	.0480	.0460	.0470	.0710	.1300	.1840	.1210	.0690	.0630	.0700	.0840	.1110	.1430	.1750
RIM	.0750	.0940	.0490	.0460	.0490	.0720	.1300	.1850	.1220	.0690	.0650	.0700	.0850	.1110	.1450	.1760
RIN	.0778	.0930	.0494	.0475	.0485	.0722	.1309	.1840	.1210	.0702	.0635	.0709	.0847	.1119	.1431	.1751
RIAR	.0778	.0930	.0494	.0475	.0485	.0722	.1309	.1840	.1210	.0702	.0635	.0709	.0847	.1119	.1431	.1751
RISR	.0790	.0930	R	R	R	.0751	.1313	.1840	.1210	.0737	.0649	.0729	.0855	.1124	.1432	.1751
RIASR	.0790	.0930	R	R	R	.0751	.1313	.1840	.1210	.0737	.0649	.0729	.0855	.1124	.1432	.1751
RIDR	.0777	.0930	.0494	.0475	.0484	.0721	.1308	.1840	.1210	.0701	.0634	.0708	.0847	.1118	.1431	.1750
RIADR	.0777	.0930	.0494	.0475	.0484	.0721	.1308	.1840	.1210	.0701	.0634	.0708	.0847	.1118	.1431	.1750

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0012	-.0010	.0004	.0015	-.0005	.0002	.0009	-.0010	-.0010	.0012	-.0015	.0009	-.0003	.0009	-.0019	-.0009
DRA	-.0012	-.0010	.0004	.0015	-.0005	.0002	.0009	-.0010	-.0010	.0012	-.0015	.0009	-.0003	.0009	-.0019	-.0009
DRS	.0000	-.0010	R	R	R	.0031	.0013	-.0010	-.0010	.0047	-.0001	.0029	.0005	.0014	-.0018	-.0009
DRAS	.0000	-.0010	R	R	R	.0031	.0013	-.0010	-.0010	.0047	-.0001	.0029	.0005	.0014	-.0018	-.0009
DRD	-.0013	-.0010	.0004	.0015	-.0006	.0001	.0008	-.0010	-.0010	.0011	-.0016	.0008	-.0003	.0008	-.0019	-.0010
DRAD	-.0013	-.0010	.0004	.0015	-.0006	.0001	.0008	-.0010	-.0010	.0011	-.0016	.0008	-.0003	.0008	-.0019	-.0010

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	8.05131	10.84523	6.51620	.31682	.42676				
USE RELATIVE VALUES	8.04979	10.82997	6.57255	.31627	.42550	-.02317	.06302	-.12600	.14277
USE ABSOLUTE VALUES	8.04979	10.82997	6.57255	.31627	.42550	-.02317	.06302	-.12600	.14277
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COF. REL.	8.04499	10.82557	6.56456	.31629	.42561	-.02986	.05760	-.11870	.13527
DUNCAN COF. ABS.	8.04500	10.82557	6.56456	.31629	.42561	-.02986	.05761	-.11870	.13528

Table 4.10 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Cr_2O_3 AT 15 PVC

SAMPLE NO. 2 GREEN (CR2O3) 10 PVC
FILM THICKNESS = 76.2635 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
R01	.0650	.0830	.0390	.0380	.0410	.0610	.1130	.1720	.1100	.0590	.0560	.0600	.0750	.1000	.1320	.1670
R02	.0650	.0840	.0400	.0390	.0420	.0620	.1200	.1730	.1110	.0600	.0570	.0610	.0760	.1020	.1350	.1680
RIM	.0870	.1090	.0540	.0500	.0550	.0830	.1450	.2090	.1380	.0810	.0770	.0830	.0980	.1260	.1630	.1970
RIN	.0693	.0840	.0400	.0390	.0420	.0620	.1205	.1730	.1110	.0600	.0570	.0610	.0760	.1020	.1351	.1680
RIAR	.0693	.0840	.0400	.0390	.0420	.0620	.1205	.1730	.1110	.0600	.0570	.0610	.0760	.1020	.1351	.1680
RISR	.0697	.0840	R	R	R	.0620	.1207	.1730	.1110	.0601	.0571	.0611	.0760	.1021	.1351	.1680
RIASR	.0697	.0840	R	R	R	.0620	.1207	.1730	.1110	.0601	.0571	.0611	.0760	.1021	.1351	.1680
RIDR	.0692	.0840	.0400	.0390	.0420	.0620	.1204	.1730	.1110	.0600	.0570	.0610	.0760	.1020	.1351	.1680
RIACR	.0692	.0840	.0400	.0390	.0420	.0620	.1204	.1730	.1110	.0600	.0570	.0610	.0760	.1020	.1351	.1680

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	-.0177	-.0250	-.0140	-.0110	-.0130	-.0210	-.0245	-.0360	-.0270	-.0210	-.0200	-.0220	-.0220	-.0240	-.0279	-.0290
DRA	-.0177	-.0250	-.0140	-.0110	-.0130	-.0210	-.0245	-.0360	-.0270	-.0210	-.0200	-.0220	-.0220	-.0240	-.0279	-.0290
DRS	-.0173	-.0250	R	R	R	-.0210	-.0243	-.0360	-.0270	-.0209	-.0199	-.0219	-.0220	-.0239	-.0279	-.0290
DRAS	-.0173	-.0250	R	R	R	-.0210	-.0243	-.0360	-.0270	-.0209	-.0199	-.0219	-.0220	-.0239	-.0279	-.0290
DID	-.0178	-.0250	-.0140	-.0110	-.0130	-.0210	-.0246	-.0360	-.0270	-.0210	-.0200	-.0220	-.0220	-.0240	-.0279	-.0290
DRAC	-.0178	-.0250	-.0140	-.0110	-.0130	-.0210	-.0246	-.0360	-.0270	-.0210	-.0200	-.0220	-.0220	-.0240	-.0279	-.0290

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	9.26339	12.35563	7.26340	.32073	.42779				
USE RELATIVE VALUES	7.16324	9.85818	5.55456	.31729	.43667	-3.75284	.25047	-.86448	3.85926
USE ABSOLUTE VALUES	7.16324	9.85818	5.55456	.31729	.43657	-3.75284	.25047	-.86448	3.85926
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COR. REL.	7.16308	9.85753	5.55436	.31730	.43666	-3.75388	.25273	-.86517	3.86057
DUNCAN COR. ABS.	7.16308	9.85753	5.55436	.31730	.43666	-3.75388	.25272	-.86517	3.86057

Table 4.11 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Cr_2O_3 AT 10 PVC

SAMPLE NO. 3 GREEN (CR2O3) 5 PVC
FILM THICKNESS = 54.1020 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
RO1	.0700	.0700	.0320	.0300	.0370	.0570	.1150	.1700	.1000	.0510	.0510	.0580	.0720	.0970	.1320	.1690
RO2	.0720	.0790	.0330	.0310	.0410	.0580	.1160	.1740	.1060	.0540	.0530	.0590	.0730	.0980	.1330	.1700
RIM	.0630	.0800	.0390	.0360	.0410	.0590	.1120	.1690	.1110	.0600	.0560	.0610	.0760	.1020	.1320	.1690
RIN	.0721	.0803	.0330	.0310	.0415	.0580	.1160	.1741	.1064	.0542	.0531	.0590	.0730	.0980	.1330	.1700
RIAR	.0721	.0803	.0330	.0310	.0415	.0580	.1160	.1741	.1064	.0542	.0531	.0590	.0730	.0980	.1330	.1700
RISR	.0721	.0827	R	R	R	.0581	.1160	.1741	.1066	.0551	.0534	.0591	.0730	.0980	.1330	.1700
RIASR	.0721	.0827	R	R	R	.0581	.1160	.1741	.1066	.0551	.0534	.0591	.0730	.0980	.1330	.1700
RIDR	.0721	.0802	.0330	.0310	.0415	.0580	.1160	.1741	.1064	.0542	.0531	.0590	.0730	.0980	.1330	.1700
RIADR	.0721	.0802	.0330	.0310	.0415	.0580	.1160	.1741	.1064	.0542	.0531	.0590	.0730	.0980	.1330	.1700

COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FOR THE REFLECTANCE OF INFINITE THICKNESS

IN TERMS OF REFLECTANCE DIFFERENCE**

DR	.0091	.0003	-.0060	-.0050	.0005	-.0010	.0040	.0051	-.0046	-.0058	-.0029	-.0020	-.0030	-.0040	.0010	.0010
DRA	.0091	.0003	-.0060	-.0050	.0005	-.0010	.0040	.0051	-.0046	-.0058	-.0029	-.0020	-.0030	-.0040	.0010	.0010
DRS	.0091	.0027	R	R	R	-.0009	.0040	.0051	-.0044	-.0049	-.0026	-.0019	-.0030	-.0040	.0010	.0010
DRAS	.0091	.0027	R	R	R	-.0009	.0040	.0051	-.0044	-.0049	-.0026	-.0019	-.0030	-.0040	.0010	.0010
DRD	.0091	.0002	-.0060	-.0050	.0005	-.0010	.0040	.0051	-.0046	-.0058	-.0029	-.0020	-.0030	-.0040	.0010	.0010
DRAD	.0091	.0002	-.0060	-.0050	.0005	-.0010	.0040	.0051	-.0046	-.0058	-.0029	-.0020	-.0030	-.0040	.0010	.0010

IN TERMS OF COLOUR SPACE**

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY MEASURING	7.06795	9.63445	5.29389	.32132	.43800				
USE RELATIVE VALUES	6.75936	9.52507	4.88655	.31927	.44991	-.17670	-1.24282	.60095	1.39175
USE ABSOLUTE VALUES	6.75936	9.52507	4.88655	.31927	.44991	-.17670	-1.24282	.60095	1.39175
SAUNDERSON COR. REL.	R	R	R	R	R	R	R	R	R
SAUNDERSON COR. ABS.	R	R	R	R	R	R	R	R	R
DUNCAN COR. REL.	6.75831	9.52388	4.88485	.31928	.44994	-.17863	-1.24308	.60226	1.39280
DUNCAN COR. ABS.	6.75831	9.52388	4.88486	.31928	.44994	-.17863	-1.24308	.60226	1.39279

Table 4.12 COMPARISION OF CALCULATED VALUES IN TERMS OF REFLECTANCE AND COLOR FOR VARIOUS CASES WITH THE AVERAGE EXPERIMENTAL VALUES OF INFINITE FILM THICKNESS FOR Cr_2O_3 AT 5 PVC

in the chromaticity diagram are shown in Figure 4.1. The chromaticity coordinates of TiO_2 pigment lacquer films both experimental and calculated at 1 and 15 PVC are shown in Figure 4.2, and of Fe_2O_3 and Cr_2O_3 at 5 and 15 PVC are shown in Figures 4.4 and 4.5 respectively. The notations and abbreviations used in the tables throughout this chapter are indicated in Tables 4.1, 4.13, and 4.25.

Now let us consider the over-all calculated infinite reflectance values obtained using our modified theory and technique compared with the experimental values. In Table 4.2, we see that the reflectance difference at all wavelengths, except 400 nm, is less than 0.01. Such small differences have no real meaning since this is about the same magnitude as the experimental error. In terms of the color coordinates, a Hunter color difference ΔE less than 5.0 is almost imperceptible. To obtain the best results, the film thickness must be as thick as possible, but the double film thickness must not exceed the infinite film thickness, otherwise the K.M parameters, K and S cannot be found. The results will be worse if the film thickness is thinner as in Table 4.5. It is important to notice also that for lower PVC, a thicker film must be used in order to obtain the best results. If a film is too thin, the measured reflectance R_{o1} and R_{o2} will not satisfy the criterion in Eq. (77), and the infinite film thickness reflectance cannot be determined.

The results for the colored-pigment lacquers are worse than those for white ones. This is not a result of the analytical method, however, but is related to the experimental errors of measuring reflectance, a problem which will be discussed more fully in the section on error analysis. It is well to note that a specimen of constant reflectance leads to greater accuracy in the computation since the error of wavelength measurement is not a factor.

Consider the Saunderson correction and the Duncan method: the Saunderson correction cannot be applied at the lower reflectance (less than 0.04 for the single-thickness film) whereas the Duncan method cannot be applied at the higher reflectance (greater than 0.96 for the double-thickness film). This is because for the measured reflectance lower than 0.04 if the Saunderson correction is applied, the corrected reflectance values turn out to be negative, which has no physical meaning. In the case of the Duncan method, the K.M function approaches zero as the reflectance approaches 0.96, and the K.M function increases as the reflectance increases above 0.96, as has been pointed out in a previous chapter. This will cause R_{01} to be greater than R_{02} which would have no physical meaning.

It is, thus recommended that if the surface correction is to be applied, the Saunderson correction is appropriate in cases of higher reflectance and the Duncan method should be applied only in cases of lower reflectance.

The advantages and disadvantages of using absolute or relative reflectance values will be discussed in the section of prediction of reflectance and color of mixtures.

4.2 The Coefficients K, S and K.M Function K/S

In the K.M theory the scattering and absorption coefficients, S and K, are defined as the fraction of light scattered or absorbed per unit path length in an infinitesimally thin layer of material. A theoretical derivation of the absorption and scattering coefficients has been given Kubelka [40], Kortum [43], and an experimental verification of this resulted reported [122,123]. In that derivation, it is shown that the value of the absorption and scattering coefficient depends on the angular distribution of the light in the scattering material. If all the light passing through the differential layer is collimated and is perpendicular to the layer, then the Kubelka-Munk absorption coefficient will be equal to the linear absorption coefficient for the collimated light. However, there is usually some diffused light present. Therefore, some of the light takes a longer path through the layer and the Kubelka-munk absorption coefficient is larger than the linear absorption coefficient for collimated light. If the light is perfectly diffuse, the Kubelka-Munk absorption coefficient varies as twice the linear absorption coefficient for collimated light. Later in the derivation of the K.M theory, a constant factor of 2 arises [43]. This factor of two is sometimes eliminated from being carried through the equations by redefining the scattering and absorption coefficients as $S = 2s$ and $K = 2k$. This should always be kept in mind, otherwise it may lead to somewhat confusing results in applying these coefficients in K.M analysis. In this study we have chosen to use the coefficients K and S, and therefore the absorption coefficient, K in this work is the Kubelka-Munk coefficient which are approximately twice the actual physical absorption coefficient of the material.

Table 4.13 Lists of notations and abbreviations used in Table 4.14-4.24.

Symbol	Definition
WL	Wavelength.
R	A value that cannot be calculated.
K, KA	Absorption coefficient obtained by using the relative and absolute reflectance value respectively.
KS, KAS	Absorption coefficient obtained through the Saunderson correction using the relative and absolute reflectance values respectively.
KD, KAD	Absorption coefficient obtained through the Duncan method using the relative and absolute reflectance values respectively.
S, SA	Scattering coefficient obtained by using the relative and absolute reflectance value respectively.
SS, SAS	Scattering coefficient obtained through the Saunderson correction using relative and absolute reflectance values respectively.
SD, SAD	Scattering coefficient obtained through the Duncan method using the relative and absolute reflectance value respectively.
FR, FRA	K.M function obtained by using the relative and absolute reflectance value respectively.
FRS, FRAS	K.M function obtained through the Saunderson correction using the relative and absolute reflectance value respectively.
FRD, FRAD	K.M function obtained through the Duncan method using the relative and absolute reflectance value respectively.

N.B. unit of the absorption and scattering coefficients in tables are "per 10 micron".

SAMPLE NO. 10015 TIO2 WHITE 15PVC

FILM THICKNESS = 136.0710 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.0660	.0043	.0018	.0009	.0004	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
S	.0656	.8092	1.6807	1.5162	1.3880	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273	1.3273
FR	.7712	.0053	.0011	.0006	.0003	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
** USING ABSOLUTE VALUES																
KA	.0663	.0044	.0019	.0010	.0005	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
SA	.0851	.8043	1.6601	1.4972	1.3709	1.2952	1.2952	1.2952	1.2952	1.2952	1.2952	1.2952	1.2952	1.2952	1.2952	1.2798
FRA	.7785	.0055	.0011	.0006	.0003	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.0428	.0019	.0008	.0004	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
SS	.1648	1.8972	3.9911	3.6132	3.3147	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738	3.1738
FRS	.2598	.0010	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.0430	.0019	.0008	.0004	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
SAS	.1637	1.8853	3.9412	3.5670	3.2730	3.0955	3.0955	3.0955	3.0955	3.0955	3.0955	3.0955	3.0955	3.0955	3.0955	3.0580
FRAS	.2629	.0010	.0002	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0414	.0009	R	R	R	R	R	R	R	R	R	R	R	R	R	R
SD	.1830	2.4304	R	R	R	R	R	R	R	R	R	R	R	R	R	R
FRD	.2261	.0004	R	R	R	R	R	R	R	R	R	R	R	R	R	R
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0416	.0009	R	R	R	R	R	R	R	R	R	R	R	R	R	R
SAD	.1820	2.4090	R	R	R	R	R	R	R	R	R	R	R	R	R	R
FRAD	.2287	.0004	R	R	R	R	R	R	R	R	R	R	R	R	R	R

Table 4.14 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF TiO_2 AT 15PVC

SAMPLE NO. 100102 TIO2 10PVC
FILM THICKNESS = 85.9536 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.0356	.0013	.0004	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
S	.0673	.7088	.8315	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211	.8211
FR	.5293	.0019	.0005	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002	.0002
** USING ABSOLUTE VALUES																
KA	.0358	.0014	.0005	.0002	.0002	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003
SA	.0669	.7056	.8270	.8167	.8167	.8124	.8124	.8124	.8124	.8124	.8124	.8124	.8124	.8124	.8124	.8081
FRA	.5352	.0020	.0006	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0004
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.0215	.0006	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
SS	.1328	1.6795	1.9788	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557	1.9557
FRS	.1619	.0003	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.0216	.0006	.0002	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
SAS	.1319	1.6714	1.9678	1.9448	1.9448	1.9340	1.9340	1.9340	1.9340	1.9340	1.9340	1.9340	1.9340	1.9340	1.9340	1.9234
FRAS	.1641	.0004	.0001	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0221	.0009	R	R	R	R	R	R	R	R	R	R	R	R	R	R
SD	.1503	2.4304	R	R	R	R	R	R	R	R	R	R	R	R	R	R
FRD	.1474	.0004	R	R	R	R	R	R	R	R	R	R	R	R	R	R
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0223	.0009	R	R	R	R	R	R	R	R	R	R	R	R	R	R
SAD	.1494	2.4090	R	R	R	R	R	R	R	R	R	R	R	R	R	R
FRAD	.1493	.0004	R	R	R	R	R	R	R	R	R	R	R	R	R	R

Table 4.15 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF TiO_2 AT 10PVC

SAMPLE NO. 100052 TIO2 WHITE 5PVC

FILM THICKNESS = 88.3412 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.1147	.0031	.0023	.0023	.0021	.0016	.0018	.0014	.0016	.0016	.0020	.0018	.0015	.0014	.0012	.0015
S	.1739	.3667	.3593	.3593	.3535	.3358	.3370	.3254	.3253	.3153	.3164	.3057	.2944	.2890	.2838	.2857
FR	.6597	.0084	.0065	.0065	.0060	.0048	.0052	.0044	.0050	.0051	.0062	.0060	.0052	.0048	.0043	.0053
** USING ABSOLUTE VALUES																
KA	.1152	.0031	.0024	.0024	.0022	.0017	.0019	.0015	.0017	.0017	.0021	.0019	.0016	.0015	.0013	.0017
SA	.1729	.3657	.3583	.3583	.3525	.3340	.3353	.3238	.3237	.3138	.3148	.3042	.2930	.2876	.2824	.2837
FRA	.6664	.0086	.0066	.0066	.0062	.0051	.0055	.0047	.0053	.0055	.0066	.0063	.0055	.0051	.0047	.0059
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.0724	.0014	.0010	.0010	.0009	.0007	.0008	.0006	.0007	.0007	.0008	.0008	.0006	.0006	.0005	.0006
SS	.3387	.8573	.8418	.8418	.8286	.7878	.7904	.7636	.7629	.7390	.7406	.7154	.6893	.6768	.6646	.6685
FRS	.2136	.0016	.0012	.0012	.0011	.0009	.0010	.0008	.0009	.0009	.0011	.0011	.0009	.0008	.0007	.0009
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.0728	.0014	.0010	.0010	.0010	.0007	.0008	.0006	.0007	.0007	.0009	.0008	.0007	.0006	.0005	.0007
SAS	.3365	.8548	.8393	.8393	.8262	.7833	.7859	.7594	.7586	.7350	.7366	.7116	.6857	.6733	.6612	.6633
FRAS	.2163	.0016	.0012	.0012	.0012	.0009	.0010	.0008	.0010	.0010	.0012	.0012	.0010	.0009	.0008	.0011
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0696	.0004	.0001	.0001	.0000	R	R	R	R	R	R	R	R	R	R	R
SD	.3703	.9263	.9092	.9092	.8941	R	R	R	R	R	R	R	R	R	R	R
FRD	.1880	.0004	.0001	.0001	.0000	R	R	R	R	R	R	R	R	R	R	R
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0700	.0004	.0001	.0001	.0000	R	R	R	R	R	R	R	R	R	R	R
SAD	.3681	.9233	.9062	.9062	.8913	R	R	R	R	R	R	R	R	R	R	R
FRAD	.1902	.0005	.0001	.0001	.0000	R	R	R	R	R	R	R	R	R	R	R

Table 4.16 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF TiO_2 AT 5 PVC

SAMPLE NO. 100012 TIO2 WHITE 1PVC
FILM THICKNESS = 72.3900 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.0123	.0128	.0128	.0116	.0118	.0114	.0114	.0109	.0094	.0104	.0096	.0097	.0103	.0097	.0091	.0083
S	.0472	.2079	.2079	.1991	.1998	.1970	.1970	.1881	.1792	.1816	.1760	.1748	.1709	.1674	.1640	.1591
FR	.2615	.0617	.0617	.0581	.0592	.0579	.0579	.0577	.0525	.0574	.0548	.0557	.0605	.0579	.0552	.0519
** USING ABSOLUTE VALUES																
KA	.0125	.0129	.0129	.0116	.0119	.0115	.0115	.0110	.0095	.0105	.0098	.0099	.0105	.0098	.0092	.0084
SA	.0469	.2076	.2076	.1988	.1995	.1964	.1964	.1876	.1787	.1811	.1755	.1742	.1704	.1669	.1636	.1584
FRA	.2660	.0621	.0621	.0585	.0596	.0587	.0587	.0585	.0533	.0583	.0556	.0566	.0613	.0588	.0561	.0532
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.0042	.0063	.0063	.0056	.0058	.0056	.0056	.0053	.0045	.0051	.0046	.0047	.0050	.0047	.0043	.0039
SS	.0920	.4660	.4660	.4470	.4484	.4423	.4423	.4222	.4028	.4073	.3951	.3920	.3825	.3749	.3676	.3566
FRS	.0452	.0135	.0135	.0126	.0129	.0126	.0126	.0125	.0112	.0124	.0118	.0120	.0131	.0125	.0118	.0109
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.0043	.0063	.0063	.0057	.0058	.0056	.0056	.0053	.0046	.0051	.0047	.0048	.0051	.0047	.0044	.0040
SAS	.0913	.4652	.4652	.4462	.4476	.4407	.4407	.4207	.4014	.4059	.3937	.3907	.3812	.3736	.3663	.3549
FRAS	.0473	.0136	.0136	.0127	.0130	.0128	.0128	.0127	.0114	.0126	.0119	.0122	.0133	.0127	.0120	.0112
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0075	.0054	.0054	.0048	.0049	.0047	.0047	.0045	.0038	.0043	.0039	.0040	.0043	.0040	.0037	.0033
SD	.1091	.4897	.4897	.4697	.4712	.4647	.4647	.4437	.4234	.4282	.4154	.4122	.4025	.3946	.3869	.3755
FRD	.0688	.0110	.0110	.0102	.0105	.0102	.0102	.0101	.0089	.0100	.0094	.0096	.0107	.0101	.0095	.0087
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0076	.0054	.0054	.0048	.0050	.0048	.0048	.0045	.0038	.0044	.0040	.0040	.0044	.0040	.0037	.0034
SAD	.1085	.4888	.4888	.4689	.4703	.4631	.4631	.4422	.4220	.4268	.4140	.4109	.4011	.3932	.3856	.3737
FRAD	.0702	.0111	.0111	.0103	.0105	.0103	.0103	.0103	.0091	.0102	.0096	.0098	.0109	.0103	.0097	.0090

Table 4.17 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF TiO_2 AT 1 PVC

SAMPLE NO. 1 RED (FE2O3) 15 PVC
FILM THICKNESS = 97.0280 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.0677	.0677	.0677	.0677	.0677	.0677	.0382	.0405	.0512	.0499	.0711	.0986	.0884	.0802	.0760	.0674
S	.0038	.0038	.0038	.0038	.0038	.0038	.0030	.0032	.0056	.0140	.0403	.0715	.0729	.0758	.0882	.1036
FR	17.7625	17.7625	17.7625	17.7625	17.7625	17.7625	12.7645	12.6204	9.1230	3.5706	1.7643	1.3776	1.2128	1.0585	.8625	.6501
** USING ABSOLUTE VALUES																
KA	.0677	.0677	.0677	.0677	.0677	.0677	.0382	.0405	.0512	.0499	.0711	.0987	.0885	.0803	.0761	.0675
SA	.0038	.0038	.0038	.0038	.0038	.0038	.0030	.0032	.0056	.0140	.0402	.0714	.0727	.0756	.0880	.1033
FRA	17.8567	17.7813	17.7813	17.7813	17.7813	17.8001	12.7920	12.6477	9.1432	3.5795	1.7695	1.3819	1.2168	1.0621	.8656	.6541
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	R	R	R	R	R	R	R	R	R	.0188	.0504	.0701	.0616	.0547	.0502	.0425
SS	R	R	R	R	R	R	R	R	R	.0128	.0668	.1283	.1330	.1407	.1677	.2027
FRS	R	R	R	R	R	R	R	R	R	1.4673	.7547	.5467	.4629	.3883	.2991	.2099
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	R	R	R	R	R	R	R	R	R	.0188	.0505	.0702	.0617	.0548	.0503	.0427
SAS	R	R	R	R	R	R	R	R	R	.0128	.0666	.1279	.1326	.1404	.1673	.2018
FRAS	R	R	R	R	R	R	R	R	R	1.4693	.7576	.5490	.4649	.3900	.3005	.2114
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0646	.0646	.0646	.0646	.0646	.0646	.0364	.0385	.0474	.0423	.0527	.0687	.0605	.0537	.0491	.0415
SD	.0088	.0088	.0088	.0098	.0088	.0088	.0070	.0074	.0129	.0317	.0884	.1539	.1570	.1635	.1901	.2241
FRD	7.3289	7.3289	7.3289	7.3289	7.3289	7.3289	5.2356	5.1699	3.6685	1.3334	.5962	.4466	.3851	.3286	.2583	.1850
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0647	.0646	.0646	.0646	.0646	.0646	.0364	.0385	.0474	.0423	.0527	.0688	.0606	.0538	.0492	.0416
SAD	.0088	.0088	.0088	.0088	.0088	.0088	.0069	.0074	.0129	.0316	.0882	.1535	.1567	.1632	.1896	.2233
FRAD	7.3690	7.3369	7.3369	7.3369	7.3369	7.3449	5.2473	5.1815	3.6771	1.3371	.5982	.4482	.3866	.3299	.2594	.1863

Table 4.18 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Fe_2O_3 AT 15 PVC

SAMPLE NO. 1 RED (FE2O3) 10 PVC
FILM THICKNESS = 51.0540 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.2237	.2237	.2237	.2237	.2237	.2237	.2237	.1620	.1245	.1816	.2579	.2491	.2300	.1828	.2162	.1771
S	.0114	.0114	.0114	.0114	.0114	.0114	.0114	.0095	.0116	.0343	.1171	.1677	.1786	.1630	.2337	.2576
FR	19.6732	19.6732	19.6732	19.6732	19.6732	19.6732	19.6732	16.9719	10.7387	5.2883	2.2029	1.4851	1.2878	1.1215	.9251	.6874
** USING ABSOLUTE VALUES																
KA	.2237	.2237	.2237	.2237	.2237	.2237	.2237	.1620	.1245	.1817	.2581	.2493	.2302	.1830	.2165	.1775
SA	.0113	.0114	.0114	.0114	.0114	.0113	.0113	.0095	.0116	.0343	.1168	.1674	.1782	.1626	.2332	.2567
FRA	19.7769	19.6938	19.6938	19.6938	19.6938	19.7145	19.7145	17.0079	10.7622	5.3007	2.2090	1.4896	1.2920	1.1253	.9284	.6915
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	R	R	R	R	R	R	R	R	R	.1121	.1980	.1807	.1627	.1260	.1445	.1126
SS	R	R	R	R	R	R	R	R	R	.0283	.1922	.2992	.3250	.3011	.4420	.5005
FRS	R	R	R	R	R	R	R	R	R	3.9619	1.0300	.6038	.5007	.4184	.3270	.2249
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	R	R	R	R	R	R	R	R	R	.1119	.1982	.1809	.1630	.1262	.1448	.1130
SAS	R	R	R	R	R	R	R	R	R	.0281	.1917	.2984	.3242	.3004	.4409	.4985
FRAS	R	R	R	R	R	R	R	R	R	3.9788	1.0340	.6062	.5028	.4203	.3285	.2266
** USING DUNCAN METHOD WITH RELATIVE VALUES.																
KD	.2128	.2128	.2128	.2128	.2128	.2128	.2128	.1537	.1158	.1566	.1934	.1743	.1571	.1227	.1392	.1086
SD	.0252	.0262	.0262	.0262	.0262	.0262	.0262	.0220	.0267	.0770	.2523	.3578	.3807	.3493	.4968	.5504
FRD	8.1293	8.1293	8.1293	8.1293	8.1293	8.1293	8.1293	6.9844	4.3424	2.0328	.7662	.4870	.4127	.3513	.2802	.1974
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.2129	.2128	.2128	.2128	.2128	.2129	.2129	.1537	.1158	.1567	.1936	.1745	.1574	.1229	.1395	.1090
SAD	.0260	.0252	.0262	.0262	.0262	.0261	.0261	.0220	.0266	.0769	.2518	.3571	.3799	.3486	.4957	.5484
FRAD	8.1735	8.1381	8.1381	8.1381	8.1381	8.1469	8.1469	6.9997	4.3524	2.0380	.7687	.4888	.4143	.3527	.2814	.1988

Table 4.19 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Fe_2O_3 AT 10 PVC

SAMPLE NO. 5 HLD (PF203) 5 PVC
FILM THICKNESS = 33.0035 MICRONS

SAMPLE NO. 5 RED (FE2O3) 5 PVC
FILM THICKNESS = 33.0835 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.3537	.3537	.3537	.3537	.3537	.3537	.3537	.3658	.4664	.3422	.4295	.3410	.2846	.2614	.2063	.1477
S	.0088	.0088	.0088	.0088	.0088	.0088	.0088	.0098	.0266	.0603	.2000	.2435	.2373	.2543	.2552	.2595
FR	40.3283	40.3283	40.3283	40.3283	40.3283	40.3283	40.3283	37.2009	17.5046	5.6702	2.1479	1.4007	1.1994	1.0263	.8082	.5691
** USING ABSOLUTE VALUES																
KA	.3536	.3537	.3537	.3537	.3537	.3537	.3537	.3658	.4664	.3423	.4298	.3414	.2850	.2618	.2066	.1481
SA	.0087	.0088	.0088	.0088	.0088	.0088	.0088	.0098	.0266	.0602	.1996	.2430	.2368	.2537	.2547	.2586
FRA	40.5359	40.3697	40.3697	40.3697	40.3697	40.4111	40.4111	37.2775	17.5417	5.6834	2.1539	1.4051	1.2033	1.0318	.8112	.5728
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	R	R	R	R	R	R	R	R	R	.2325	.3301	.2444	.1983	.1776	.1347	.0914
SS	R	R	R	R	R	R	R	R	R	.0496	.3319	.4372	.4346	.4746	.4887	.5135
FRS	R	R	R	R	R	R	R	R	R	4.6844	.9945	.5590	.4564	.3743	.2756	.1780
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	R	R	R	R	R	R	R	R	R	.2323	.3304	.2448	.1987	.1779	.1350	.0918
SAS	R	R	R	R	R	R	R	R	R	.0493	.3310	.4361	.4335	.4734	.4874	.5114
FRAS	R	R	R	R	R	R	R	R	R	4.7064	.9983	.5613	.4583	.3759	.2769	.1794
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.3448	.3448	.3448	.3448	.3448	.3448	.3448	.3558	.4397	.2957	.3199	.2370	.1935	.1733	.1320	.0900
SD	.0204	.0204	.0204	.0204	.0204	.0204	.0204	.0228	.0610	.1350	.4297	.5207	.5092	.5458	.5517	.5679
FRD	16.9382	16.9382	16.9382	16.9382	16.9382	16.9382	16.9382	15.6034	7.2042	2.1910	.7444	.4552	.3800	.3174	.2393	.1584
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.3449	.3448	.3448	.3448	.3448	.3449	.3449	.3559	.4398	.2959	.3203	.2373	.1938	.1736	.1323	.0903
SAD	.0203	.0203	.0203	.0203	.0203	.0203	.0203	.0228	.0609	.1347	.4289	.5196	.5081	.5446	.5504	.5659
FRAD	17.0259	16.9559	16.9559	16.9559	16.9559	16.9736	16.9736	15.6360	7.2199	2.1965	.7468	.4568	.3815	.3187	.2403	.1596

Table 4.20 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Fe_2O_3 AT 5 PVC

SAMPLE NO. 3 RED (FE2O3) 1 PVC

FILM THICKNESS = 70.6120 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.0699	.0699	.0699	.0699	.0699	.0699	.1084	.0930	.1330	.2624	.1857	.1348	.1111	.1147	.0882	.0746
S	.0039	.0039	.0039	.0039	.0039	.0039	.0057	.0052	.0099	.0474	.0910	.1028	.0983	.1184	.1159	.1339
FR	18.0426	18.0426	18.0426	18.0426	18.0426	18.0426	19.0120	17.7625	13.4614	5.5309	2.0413	1.3115	1.1305	.9688	.7611	.5574
** USING ABSOLUTE VALUES																
KA	.0699	.0699	.0699	.0699	.0699	.0699	.1084	.0930	.1331	.2625	.1859	.1349	.1112	.1149	.0884	.0749
SA	.0039	.0039	.0039	.0039	.0039	.0039	.0057	.0052	.0099	.0474	.0908	.1025	.0980	.1182	.1157	.1334
FRA	18.1392	18.0617	18.0617	18.0617	18.0617	18.0808	19.0521	17.8001	13.4904	5.5438	2.0471	1.3156	1.1343	.9722	.7641	.5610
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	R	R	R	R	R	R	R	R	R	.2142	.1412	.0952	.0764	.0772	.0570	.0461
SS	R	R	R	R	R	R	R	R	R	.0473	.1524	.1857	.1807	.2225	.2232	.2654
FRS	R	R	R	R	R	R	R	R	R	4.5309	.9269	.5126	.4226	.3468	.2555	.1735
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	R	R	R	R	R	R	R	R	R	.2142	.1414	.0953	.0765	.0773	.0572	.0462
SAS	R	R	R	R	R	R	R	R	R	.0470	.1519	.1852	.1802	.2220	.2226	.2644
FRAS	R	R	R	R	R	R	R	R	R	4.5524	.9305	.5148	.4245	.3483	.2568	.1749
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0671	.0671	.0671	.0671	.0671	.0671	.1035	.0888	.1244	.2237	.1374	.0931	.0753	.0754	.0561	.0451
SD	.0090	.0090	.0090	.0090	.0090	.0090	.0132	.0121	.0227	.1050	.1956	.2208	.2121	.2547	.2514	.2920
FRD	7.4635	7.4635	7.4635	7.4635	7.4635	7.4635	7.8555	7.3289	5.4881	2.1318	.7025	.4217	.3549	.2960	.2231	.1544
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0672	.0672	.0672	.0672	.0672	.0672	.1035	.0888	.1244	.2239	.1376	.0933	.0754	.0755	.0562	.0453
SAD	.0090	.0090	.0090	.0090	.0090	.0090	.0131	.0121	.0226	.1048	.1953	.2203	.2116	.2542	.2508	.2910
FRAD	7.5042	7.4716	7.4716	7.4716	7.4716	7.4797	7.8726	7.3449	5.5004	2.1372	.7047	.4233	.3563	.2973	.2241	.1556

Table 4.21 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Fe_2O_3 AT 1 PVC

SAMPLE NO. 3 GREEN [illegible] 15 PVC
FILM THICKNESS = 87.1055 MICRONS

SAMPLE NO. 3 GREEN (CR2O3) 15 PVC

FILM THICKNESS = 87.1855 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.1127	.2148	.0917	.0895	.0906	.1022	.1088	.2045	.1831	.1010	.1236	.1091	.1157	.1106	.1516	.1618
S	.0206	.0486	.0100	.0094	.0097	.0171	.0377	.1130	.0574	.0164	.0179	.0179	.0234	.0314	.0591	.0832
FR	5.4671	4.4222	9.1370	9.5424	9.3360	5.9642	2.8844	1.8093	3.1916	6.1562	6.9090	6.0897	4.9452	3.5249	2.5651	1.9438
** USING ABSOLUTE VALUES																
KA	.1128	.2148	.0917	.0895	.0906	.1022	.1088	.2047	.1832	.1010	.1236	.1091	.1157	.1107	.1517	.1619
SA	.0205	.0485	.0100	.0094	.0097	.0171	.0376	.1128	.0572	.0164	.0179	.0179	.0233	.0313	.0590	.0830
FRA	5.4992	4.4275	9.1471	9.5529	9.3463	5.9780	2.8920	1.8146	3.1997	6.1704	6.9248	6.1037	4.9569	3.5338	2.5720	1.9521
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.0711	.1782	R	R	R	.0529	.0815	.1544	.1476	.0487	.0664	.0599	.0790	.0817	.1183	.1220
SS	.0167	.0597	R	R	R	.0110	.0541	.1963	.0837	.0097	.0094	.0116	.0221	.0401	.0926	.1414
FRS	4.2677	2.9851	R	R	R	4.8174	1.5068	.7863	1.7626	5.0376	7.0550	5.1710	3.5711	2.0361	1.2770	.8608
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.0709	.1782	R	R	R	.0527	.0815	.1546	.1477	.0485	.0663	.0597	.0789	.0818	.1184	.1228
SAS	.0164	.0596	R	R	R	.0109	.0539	.1958	.0835	.0096	.0093	.0115	.0220	.0400	.0923	.1409
FRAS	4.3151	2.9914	R	R	R	4.8372	1.5126	.7894	1.7695	5.0575	7.0916	5.1941	3.5864	2.0441	1.2819	.8719
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.0974	.1779	.0838	.0820	.0829	.0894	.0862	.1470	.1451	.0887	.1092	.0955	.0987	.0903	.1166	.1184
SD	.0462	.1065	.0229	.0214	.0222	.0386	.0829	.2403	.1246	.0370	.0403	.0403	.0522	.0694	.1281	.1783
FRD	2.1070	1.6701	3.6563	3.8285	3.7408	2.3163	1.0403	.6117	1.1640	2.3970	2.7113	2.3681	1.8889	1.3014	.9107	.6641
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.0975	.1779	.0838	.0821	.0829	.0895	.0863	.1471	.1452	.0887	.1093	.0955	.0988	.0904	.1167	.1186
SAD	.0460	.1064	.0229	.0214	.0221	.0385	.0827	.2398	.1244	.0369	.0402	.0402	.0521	.0693	.1278	.1778
FRAD	2.1234	1.6723	3.6506	3.8330	3.7452	2.3221	1.0433	.6137	1.1673	2.4030	2.7179	2.3740	1.8938	1.3051	.9134	.6674

Table 4.22 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Cr_2O_3 AT 15 PVC

SAMPLE NO. 2 GREEN (CR2O3) 10 PVC

FILM THICKNESS = 76.2635 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.1588	.2444	.2216	.2205	.2237	.2378	.1424	.2365	.2459	.2369	.2352	.2373	.2427	.2084	.1881	.2377
S	.0254	.0489	.0193	.0186	.0205	.0335	.0444	.1197	.0691	.0322	.0302	.0329	.0432	.0528	.0679	.1154
FR	6.2534	4.9935	11.5118	11.8311	10.9187	7.0933	3.2107	1.9766	3.5596	7.3609	7.7976	7.2249	5.6158	3.9510	2.7693	2.0601
** USING ABSOLUTE VALUES																
KA	.1590	.2445	.2216	.2205	.2237	.2379	.1425	.2367	.2461	.2369	.2353	.2374	.2428	.2085	.1882	.2379
SA	.0253	.0489	.0192	.0186	.0205	.0335	.0443	.1194	.0690	.0321	.0301	.0328	.0431	.0526	.0678	.1150
FRA	6.2855	4.9994	11.5243	11.8439	10.9306	7.1094	3.2189	1.9822	3.5685	7.3776	7.8151	7.2413	5.6289	3.9607	2.7766	2.0688
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.1058	.2017	R	R	R	.1804	.1091	.1812	.2027	.1761	.1682	.1783	.1976	.1692	.1487	.1833
SS	.0183	.0543	R	R	R	.0225	.0615	.2043	.0971	.0198	.0159	.0211	.0423	.0686	.1042	.1952
FRS	5.7947	3.7150	R	R	R	8.0358	1.7730	.8869	2.0869	8.8995	10.5727	8.4472	4.6736	2.4672	1.4269	.9388
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.1055	.2017	R	R	R	.1803	.1091	.1814	.2028	.1760	.1679	.1782	.1976	.1692	.1488	.1836
SAS	.0180	.0542	R	R	R	.0223	.0613	.2037	.0968	.0196	.0158	.0210	.0421	.0683	.1039	.1944
FRAS	5.8693	3.7233	R	R	R	8.0849	1.7798	.8904	2.0952	8.9569	10.6481	8.5002	4.6960	2.4772	1.4324	.9443
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.1385	.2057	.2032	.2026	.2043	.2086	.1142	.1726	.1974	.2086	.2084	.2086	.2073	.1705	.1462	.1747
SD	.0569	.1078	.0436	.0423	.0464	.0748	.0974	.2549	.1502	.0719	.0676	.0734	.0957	.1156	.1472	.2461
FRD	2.4351	1.9075	4.6562	4.7917	4.4046	2.7874	1.1723	.6770	1.3144	2.9001	3.0841	2.8428	2.1672	1.4754	.9928	.7098
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.1387	.2058	.2033	.2027	.2044	.2087	.1143	.1728	.1976	.2087	.2085	.2087	.2075	.1706	.1463	.1750
SAD	.0566	.1077	.0436	.0422	.0463	.0747	.0972	.2544	.1499	.0718	.0674	.0732	.0955	.1153	.1469	.2453
FRAD	2.4502	1.9100	4.6615	4.7971	4.4096	2.7942	1.1757	.6792	1.3180	2.9071	3.0915	2.8497	2.1727	1.4794	.9957	.7132

Table 4.23 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Cr_2O_3 AT 10 PVC

SAMPLE NO. 3 GREEN (CR2O3) 5 PVC

FILM THICKNESS = 54.1020 MICRONS

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
** USING RELATIVE VALUES																
K	.2840	.1609	.2997	.2953	.1891	.3324	.3464	.2418	.2092	.2347	.2689	.3331	.3410	.3466	.3440	.3344
S	.0475	.0306	.0212	.0195	.0171	.0435	.1028	.1234	.0557	.0284	.0318	.0444	.0580	.0835	.1218	.1650
FR	5.9748	5.2640	14.1532	15.1266	11.0731	7.6470	3.3680	1.9590	3.7530	8.2542	8.4460	7.5016	5.8845	4.1505	2.8257	2.0261
** USING ABSOLUTE VALUES																
KA	.2842	.1609	.2997	.2953	.1891	.3325	.3465	.2419	.2093	.2347	.2690	.3332	.3411	.3467	.3442	.3348
SA	.0473	.0305	.0212	.0195	.0171	.0434	.1026	.1232	.0556	.0284	.0318	.0443	.0578	.0833	.1215	.1645
FRA	6.0095	5.2702	14.1684	15.1427	11.0851	7.6642	3.3765	1.9646	3.7623	8.2727	8.4648	7.5185	5.8981	4.1606	2.8331	2.0347
** USING THE SAUNDERSON CORRECTION WITH RELATIVE VALUES																
KS	.2167	.0940	R	R	R	.2411	.2842	.1828	.1599	.1129	.1487	.2448	.2753	.2874	.2774	.2572
SS	.0408	.0244	R	R	R	.0242	.1485	.2087	.0708	.0094	.0109	.0260	.0534	.1073	.1887	.2803
FRS	5.3143	3.8533	R	R	R	9.9536	1.9144	.8760	2.2606	12.0339	13.6073	9.3990	5.1536	2.6787	1.4700	.9175
** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VALUES																
KAS	.2165	.0939	R	R	R	.2408	.2844	.1830	.1600	.1123	.1482	.2446	.2753	.2876	.2777	.2576
SAS	.0402	.0243	R	R	R	.0240	.1480	.2081	.0705	.0093	.0108	.0258	.0531	.1069	.1882	.2792
FRAS	5.3815	3.8612	R	R	R	10.0220	1.9219	.8794	2.2696	12.1176	13.7175	9.4615	5.1793	2.6897	1.4757	.9229
** USING DUNCAN METHOD WITH RELATIVE VALUES																
KD	.2452	.1389	.2790	.2761	.1742	.2939	.2758	.1776	.1714	.2103	.2408	.2940	.2929	.2845	.2665	.2450
SD	.1058	.0687	.0483	.0446	.0390	.0973	.2231	.2650	.1229	.0642	.0717	.0993	.1285	.1826	.2624	.3519
FRD	2.3177	2.0234	5.7782	6.1921	4.4726	3.0206	1.2359	.6701	1.3944	3.2775	3.3581	2.9593	2.2797	1.5577	1.0155	.6964
** USING DUNCAN METHOD WITH ABSOLUTE VALUES																
KAD	.2455	.1390	.2790	.2761	.1743	.2940	.2760	.1778	.1715	.2103	.2409	.2941	.2931	.2847	.2668	.2455
SAD	.1053	.0686	.0482	.0445	.0389	.0971	.2227	.2645	.1227	.0640	.0716	.0991	.1282	.1823	.2619	.3508
FRAD	2.3322	2.0260	5.7846	6.1990	4.4777	3.0279	1.2394	.6723	1.3982	3.2853	3.3660	2.9664	2.2854	1.5618	1.0185	.6998

Table 4.24 ABSORPTION AND SCATTERING COEFFICIENTS AND K.M FUNCTIONS FOR VARIOUS CASES OF Cr_2O_3 AT 5 PVC

The two phenomena of absorption and scattering in the K.M analysis are only considered in general terms. They depend on the system of pigments and method used. It may be well to bear in mind how these coefficients are obtained. In Table 4.14 to 4.17 the values of scattering and absorption coefficients, S and K, and the K.M function of TiO_2 white pigment lacquer at 15%, 10%, 5% and 1% PVC obtained by various methods are shown. These absorption and scattering coefficients and K.M functions obtained by various methods for Fe_2O_3 pure iron oxide red, Cr_2O_3 pure chromium oxide green, are shown in Tables 4.18 to 4.21, Tables 4.22 to 4.24 respectively. These coefficients cannot be obtained if the infinite reflectance cannot be determined. We thus see that at low reflectance values where the Saunderson correction is unapplicable, these coefficients cannot be determined by using the Saunderson correction. The same is true for the Duncan method at the high reflectance values.

The scattering and absorption coefficients ought to be constant at a specific wavelength for each pigmented lacquer. But it is found that the values calculated from K.M theory for scattering and absorption coefficients depend on the thickness of the paint film on which the reflectance measurements are carried out. The calculated values for K and S approach a constant value only at high film thickness. To obtain the best results, it should be remembered that the double film thickness has to be almost as thick as the thickness of the infinite thickness film.

Consider roughly the effect of changes in the pigment volume concentration, PVC. As the PVC increases the scattering coefficient increases whereas the absorption coefficient decreases for TiO_2 white pigment lacquer films. Both K and S are decreased as the PVC increases for

Cr_2O_3 pure chromium oxide green.

Now consider the results obtained by using the relative and absolute reflectance values with and without surface correction. The results obtained by using relative and absolute values are not significantly different at low reflectance but seem to differ considerably at the high reflectance. Since one of the basic assumptions of the K.M theory is that the reflectance are absolute values, neglecting to use the absolute reflectance values will result in more serious errors at high relfectivities than at low reflectivities.

The absorption coefficients K obtained by using surface corrections are almost about a half of those obtained by not applying the surface correction. It seems possible to say that the absorption coefficients obtained by applying surface correction are more physically meaningful, since it is about the same as the actual physical absorption coefficient of the material.

It is true, however, that these coefficients do not always give the same results because of differences in the tests and the pigment system. These include different PVC's, different vehicles and different degrees of dispersion.

4.3 Results of Predicted R_∞ and Color of Mixed Pigment Lacquers

The mixed-pigment lacquers were prepared containing two and three single-pigment lacquers in different proportions. These were all prepared at 15% PVC, using volume as the proportion of each pigment lacquer in the mixture. The infinite reflectance of these mixtures were obtained by averaging the experimental reflectance values. The chromaticity coordinates by experiment of each pigment lacquer films and their mixtures are

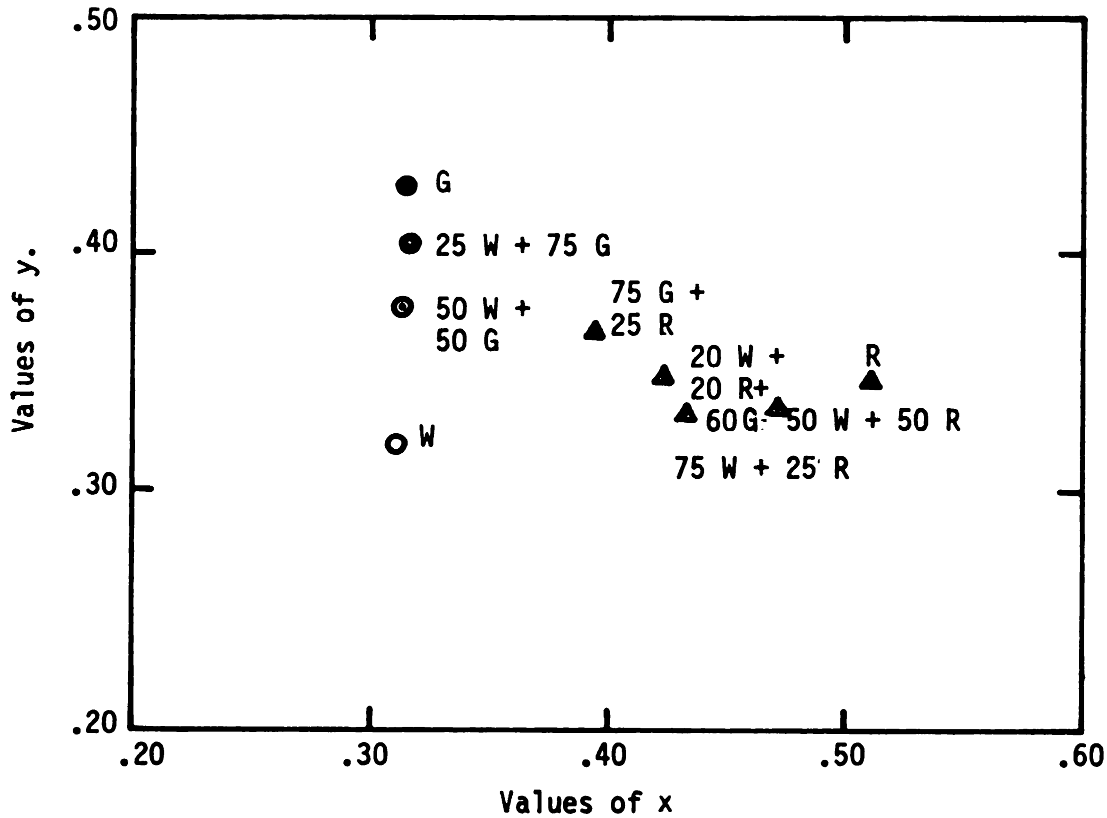


Fig. 4.5 Chromaticity coordinations by experiment of pigment lacquer films TiO_2, Fe_2O_3, Cr_2O_3 and their mixtures.

W for TiO_2

R for Fe_2O_3

G for Cr_2O_3

Antecedent numbers are percentage for each component of the mixtures.

shown in Figure 4.5. The calculated infinite reflectance and color coordinate space system compared with the experimental values are shown in Tables 4.26 to 4.32. Only the relative and absolute reflectance values without surface correction can be used for calculating infinite reflectance of every mixed-pigment lacquer films. This is because the single-pigment lacquer used in this study, TiO_2, gives a very high reflectance, hence the Duncan correction cannot be applied, and the colored-pigment lacquers, Fe_2O_3 red and Cr_2O_3 green, give a very low reflectance at some wavelengths, and hence the Saunderson correction cannot be used.

The results from the use of absolute reflectance values are not significantly different from those obtained using the relative values. This may result from the fact that the absolute reflectance values of the standard white $BaSO_4$ are too high. Using relative reflectance values seems to be adequate in practical use.

Consider the two color-mixture theories used. The single-constant theory give the better results than the two-constant theory. This is not a result of the theoretical advantage, however, but is related to the experimental errors of measuring and working procedure, a problem which will be discussed more fully in the error-analysis section.

It can be seen from Tables 4.26 to 4.32 that the two-constant theory cannot be used without surface correction. The results obtained by using the two-constant theory without surface correction are greatly different from the experimental values. It can be seen from the figures that the results obtained by using two-constant theory with surface correction, where applicable, give the better results.

It is sometimes argued that the single-constant theory is not

Table 4.25 Lists of notations and abbreviations used in Tables 4.26-4.32.

Symbol	Definition
WL	Wavelength.
R	The value that cannot be calculated.
REX	The infinite reflectance of the mixture obtained by experimentation.
RINM 1, RINM 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory with relative reflectance value respectively.
RMA 1, RMA 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory with the absolute reflectance value respectively.
RMS 1, RMS 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory with the relative reflectance value respectively.
RMAS 1, RMAS 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory through the Saunderson correction with the absolute reflectance value respectively.
RMD 1, RMD 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory through Duncan method with the relative reflectance value respectively.
RMAD 1, RMAD 2	The calculated infinite reflectance values of the mixture using single constant and two-constant theory through Duncan method with the absolute reflectance values respectively.
ONE CONST.	Single constant theory.
TWO CONST.	Two-constant theory.
REL., ABS.	Using relative and absolute reflectance value respectively.
DUN.	Duncan method.

Table 4.25 Lists of notations and abbreviations used in Tables 4.26-4.32 (continued).

Symbol	Definition
SAUND.	Saunderson correction.
W	TiO_2 pigment lacquer.
R	Fe_2O_3 pigment lacquer.
G	Cr_2O_3 pigment lacquer.

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.0900	.1420	.0850	.0750	.0800	.1030	.1600	.2360	.1800	.1110	.0990	.1040	.1200	.1500	.1860	.2200
RINM1	.0982	.1186	.0658	.0615	.0667	.0938	.1622	.2238	.1519	.0897	.0845	.0905	.1089	.1399	.1788	.2138
RINM2	.1431	.3546	.5827	.5586	.5686	.5292	.5163	.4548	.4517	.5210	.5128	.5088	.5106	.5112	.4873	.4761
RMS1	.0935	.1084	R	R	R	.0881	.1526	.2137	.1418	.0834	.0778	.0824	.0999	.1304	.1688	.2037
RMS2	.1596	.3337	R	R	R	.5571	.4849	.4297	.4201	.5512	.5516	.5228	.4975	.4827	.4555	.4480
RMC1	.0972	.1161	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMC2	.1377	.3517	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.0982	.1186	.0658	.0615	.0667	.0938	.1622	.2238	.1519	.0897	.0845	.0906	.1089	.1399	.1788	.2138
RMA2	.1431	.3540	.5814	.5572	.5672	.5263	.5134	.4520	.4488	.5181	.5098	.5058	.5077	.5083	.4844	.4720
RMAS1	.0935	.1084	R	R	R	.0881	.1526	.2137	.1418	.0834	.0779	.0824	.0999	.1304	.1688	.2037
RMAS2	.1599	.3332	R	R	R	.5552	.4824	.4273	.4175	.5494	.5499	.5207	.4950	.4801	.4529	.4443
RMAD1	.0973	.1161	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAD2	.1376	.3508	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	11.92185	15.07952	10.48123	.31806	.40231				
ONE CONST. REL.	10.27313	13.57278	8.61613	.31647	.41811	-1.99110	-1.54204	.74292	2.62570
TWO CONST. REL.	49.76102	49.53050	63.85002	.30502	.30360	31.54555	16.20359	-15.70574	38.78593
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. ABS.	10.27403	13.57430	8.61644	.31647	.41812	-1.98905	-1.54405	.74463	2.62581
TWO CONST. ABS.	49.50478	49.24829	63.68533	.30476	.30318	31.34477	16.26426	-15.86104	38.71167
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.26 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 25W•75G

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.1530	.2650	.1900	.1680	.1720	.2100	.2910	.3840	.3100	.2210	.1980	.2050	.2300	.2700	.3170	.3600
RINM1	.1264	.1611	.0930	.0873	.0942	.1297	.2141	.2849	.2018	.1245	.1177	.1256	.1490	.1873	.2336	.2737
RINM2	.2012	.5224	.7274	.7100	.7179	.6865	.6724	.6107	.6164	.6800	.6736	.6701	.6707	.6695	.6461	.6325
RMS1	.1118	.1336	R	R	R	.1073	.1880	.2591	.1751	.1010	.0936	.0997	.1227	.1611	.2073	.2478
RMS2	.2176	.4792	R	R	R	.6945	.6294	.5701	.5679	.6897	.6902	.6657	.6430	.6285	.6006	.5901
RMU1	.1240	.1529	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMU2	.1928	.4984	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.1265	.1611	.0930	.0873	.0942	.1297	.2142	.2850	.2019	.1245	.1177	.1256	.1490	.1874	.2337	.2738
RMA2	.2012	.5219	.7266	.7091	.7170	.6846	.6704	.6083	.6140	.6781	.6716	.6681	.6687	.6675	.6440	.6292
RMAS1	.1118	.1336	R	R	R	.1073	.1881	.2591	.1751	.1010	.0936	.0997	.1227	.1611	.2074	.2478
RMAS2	.2179	.4787	R	R	R	.6934	.6275	.5678	.5656	.6886	.6891	.6643	.6412	.6265	.5984	.5868
RMAU1	.1240	.1529	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAU2	.1928	.4977	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	22.71998	27.17870	22.37642	.31435	.37605				
CNE CONST. REL.	13.97268	18.03897	12.05144	.31711	.40939	-9.66089	-2.16136	1.86219	10.07333
TWO CONST. REL.	65.26769	65.42363	81.51868	.30756	.30830	28.75168	15.92851	-14.18018	35.79740
CNE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
CNE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
CNE CONST. ABS.	13.97583	18.04403	12.05262	.31711	.40942	-9.65493	-2.16680	1.86707	10.06969
TWO CONST. ABS.	65.09063	65.21832	81.41773	.30743	.30803	28.62466	15.98595	-14.28895	35.76445
CNE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
CNE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.27 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 50W+50G

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.1050	.1120	.1070	.1050	.1050	.1050	.1050	.1060	.1220	.1900	.3200	.3900	.4170	.4380	.4790	.5300
RINM1	.0837	.0926	.0927	.0927	.0927	.0927	.1210	.1221	.1561	.2857	.4034	.4458	.4675	.4904	.5244	.5696
RINM2	.2650	.7734	.8435	.8389	.8339	.8313	.8700	.8664	.8514	.8532	.8283	.8020	.8113	.8194	.8239	.8335
RMS1	R	R	R	R	R	R	R	R	R	.2661	.3519	.3965	.4200	.4449	.4820	.5319
RMS2	R	R	R	R	R	R	R	R	R	.8653	.7956	.7669	.7789	.7894	.7969	.8107
RMD1	.0809	.0867	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMD2	.2552	.7232	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.0837	.0926	.0927	.0927	.0927	.0927	.1211	.1222	.1562	.2859	.4037	.4461	.4678	.4908	.5248	.5703
RMA2	.2650	.7734	.8433	.8386	.8336	.8306	.8698	.8663	.8511	.8529	.8277	.8012	.8106	.8187	.8233	.8327
RMAS1	R	R	R	R	R	R	R	R	R	.2664	.3520	.3967	.4202	.4452	.4823	.5324
RMAS2	R	R	R	R	R	R	R	R	R	.8655	.7948	.7659	.7779	.7885	.7961	.8096
RMAD1	.0809	.0867	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAD2	.2553	.7230	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	23.37943	17.65211	12.52770	.43652	.32958				
ONE CONST. REL.	28.05735	21.65504	10.96700	.46239	.35688	4.52056	.38365	6.87026	8.23304
TWO CONST. REL.	81.52755	84.74741	97.58288	.30898	.32119	50.04394	-28.82445	-10.13843	58.63473
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. ABS.	28.07524	21.66692	10.96841	.46244	.35689	4.53331	.40042	6.88123	8.25000
TWO CONST. ABS.	81.48475	84.71399	97.55670	.30894	.32118	50.02578	-28.84451	-10.14667	58.63054
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.28 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 75W+25R

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.0610	.0610	.0610	.0610	.0610	.0610	.0610	.0610	.0790	.1510	.2550	.3060	.3240	.3460	.3810	.4310
RINM1	.0488	.0507	.0507	.0507	.0507	.0507	.0680	.0687	.0906	.1857	.2876	.3279	.3492	.3722	.4073	.4557
RINM2	.2091	.6584	.7510	.7413	.7323	.7276	.7869	.7814	.7583	.7615	.7252	.6883	.7020	.7139	.7213	.7363
RMS1	R	R	R	R	R	R	R	R	R	.1906	.2627	.3027	.3244	.3480	.3839	.4339
RMS2	R	R	R	R	R	R	R	R	R	.7856	.6877	.6509	.6668	.6811	.6918	.7114
RMD1	.0480	.0493	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMD2	.1954	.6055	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.0488	.0507	.0507	.0507	.0507	.0507	.0681	.0687	.0907	.1858	.2877	.3280	.3493	.3724	.4075	.4560
RMA2	.2091	.6582	.7503	.7406	.7315	.7261	.7860	.7804	.7571	.7603	.7236	.6865	.7002	.7123	.7198	.7342
RMAS1	R	R	R	R	R	R	R	R	R	.1908	.2627	.3028	.3245	.3481	.3840	.4341
RMAS2	R	R	R	R	R	R	R	R	R	.7851	.6861	.6490	.6650	.6794	.6902	.7091
RMAD1	.0480	.0493	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAD2	.1954	.6049	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	17.64505	12.64930	7.18622	.47078	.33749				
ONE CONST. REL.	19.38814	14.13323	6.01126	.49043	.35751	2.02834	-.05129	3.91923	4.41329
TWO CONST. REL.	71.76284	75.33085	86.04256	.30782	.32312	51.22750	-30.61804	-10.93810	60.67424
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. ABS.	19.39457	14.13714	6.01155	.49046	.35751	2.03355	-.04261	3.92372	4.41958
TWO CONST. ABS.	71.63738	75.20882	85.96014	.30771	.32305	51.15717	-30.63350	-10.97865	60.63001
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.29 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 50W+50R

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.0530	.0530	.0430	.0420	.0420	.0540	.0670	.0720	.0720	.0860	.1050	.1180	.1320	.1600	.1920	.2300
RINM1	.0534	.0576	.0415	.0398	.0418	.0511	.0792	.0917	.0892	.0773	.0786	.0852	.1021	.1301	.1656	.1993
RINM2	.0713	.0875	.0460	.0435	.0462	.0653	.1226	.1771	.1161	.0764	.0933	.1152	.1282	.1492	.1762	.2056
RMS1	R	R	R	R	R	R	R	R	R	.0800	.0765	.0812	.0978	.1263	.1622	.1959
RMS2	R	R	R	R	R	R	R	R	R	.0818	.1096	.1302	.1354	.1513	.1756	.2045
RMD1	.0532	.0573	.0414	.0397	.0417	.0509	.0782	.0899	.0886	.0770	.0782	.0847	.1014	.1293	.1647	.1981
RMD2	.0709	.0870	.0459	.0433	.0460	.0649	.1217	.1758	.1156	.0762	.0920	.1124	.1256	.1471	.1747	.2043
RMA1	.0534	.0576	.0415	.0398	.0418	.0511	.0792	.0917	.0892	.0773	.0786	.0852	.1021	.1301	.1656	.1993
RMA2	.0713	.0875	.0460	.0435	.0462	.0653	.1226	.1771	.1161	.0764	.0933	.1152	.1282	.1492	.1762	.2056
RMAS1	R	R	R	R	R	R	R	R	R	.0801	.0765	.0812	.0978	.1263	.1622	.1959
RMAS2	R	R	R	R	R	R	R	R	R	.0819	.1097	.1303	.1355	.1513	.1756	.2045
RMAD1	.0532	.0573	.0414	.0397	.0417	.0509	.0783	.0899	.0886	.0770	.0782	.0847	.1014	.1293	.1647	.1981
RMAD2	.0709	.0870	.0459	.0433	.0460	.0649	.1217	.1758	.1156	.0762	.0920	.1124	.1256	.1471	.1747	.2043

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	8.76783	8.02494	5.24284	.39789	.36418				
ONE CONST. REL.	7.66793	8.10418	5.15630	.36639	.38723	.13953	-7.41153	.33168	7.42026
TWO CONST. REL.	9.58549	11.30722	6.11108	.35497	.41873	5.29789	-13.63516	3.90647	15.14086
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	7.62200	8.03233	5.13993	.36654	.38628	.01305	-7.26498	.22948	7.26861
TWO CONST. DUN. REL.	9.47347	11.21278	6.08712	.35384	.41880	5.15717	-13.77218	3.80510	15.19039
ONE CONST. ABS.	7.66798	8.10424	5.15631	.36639	.38723	.13964	-7.41163	.33179	7.42037
TWO CONST. ABS.	9.58544	11.30719	6.11107	.35497	.41873	5.29785	-13.63532	3.90643	15.14099
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	7.62216	8.03259	5.13996	.36654	.38628	.01350	-7.26551	.22990	7.26916
TWO CONST. DUN. ABS.	9.47357	11.21290	6.08714	.35384	.41880	5.15735	-13.77227	3.80524	15.19057

Table 4.30 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 75G+25R

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.0470	.0470	.0450	.0440	.0430	.0500	.0550	.0580	.0620	.0880	.1380	.1500	.1630	.1890	.2200	.2600
RINM1	.0638	.0701	.0509	.0488	.0513	.0623	.0955	.1100	.1072	.0933	.0948	.1025	.1220	.1539	.1935	.2303
RINM2	.1283	.3357	.5462	.5229	.5280	.4948	.4983	.4425	.4355	.4974	.4840	.4732	.4791	.4838	.4677	.4634
RMS1	R	R	R	R	R	R	R	R	R	.0885	.0843	.0898	.1093	.1419	.1819	.2188
RMS2	R	R	R	R	R	R	R	R	R	.5350	.5131	.4819	.4678	.4600	.4408	.4390
RMD1	.0632	.0688	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMD2	.1227	.3332	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.0638	.0701	.0509	.0488	.0513	.0623	.0955	.1100	.1072	.0933	.0948	.1025	.1220	.1539	.1935	.2303
RMA2	.1283	.3351	.5447	.5214	.5265	.4918	.4953	.4398	.4325	.4944	.4810	.4702	.4762	.4809	.4649	.4595
RMAS1	R	R	R	R	R	R	R	R	R	.0885	.0843	.0898	.1092	.1419	.1819	.2188
RMAS2	R	R	R	R	R	R	R	R	R	.5331	.5111	.4796	.4653	.4575	.4384	.4355
RMAD1	.0632	.0688	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAD2	.1227	.3324	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	9.98316	8.10876	5.29684	.42684	.34669				
ONE CONST. REL.	9.23047	9.75611	6.30907	.36490	.38568	2.75888	-14.65696	.98392	14.94677
TWO CONST. REL.	47.06937	47.28897	59.78082	.30537	.30679	40.29109	-10.90946	-12.30986	43.51920
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. ABS.	9.23110	9.75681	6.30921	.36491	.38569	2.76001	-14.65723	.98488	14.94731
TWO CONST. ABS.	46.80938	47.00203	59.60519	.30511	.30637	40.08214	-10.84832	-12.46134	43.35376
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.31 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 20W+20R+60G

PREDICTED REFLECTANCE OF THE MIXTURE

WL	400	420	440	460	480	500	520	540	560	580	600	620	640	660	680	700
REX	.0480	.0490	.0490	.0490	.0490	.0490	.0500	.0530	.0570	.0880	.1500	.1800	.1980	.2160	.2420	.2800
RINM1	.0440	.0457	.0414	.0408	.0415	.0442	.0619	.0647	.0792	.1172	.1444	.1597	.1831	.2172	.2590	.3019
RINM2	.1383	.4317	.6031	.5850	.5805	.5613	.5979	.5535	.5433	.5857	.5586	.5320	.5439	.5539	.5504	.5566
RMS1	R	R	R	R	R	R	R	R	R	.1136	.1180	.1287	.1555	.1947	.2397	.2831
RMS2	R	R	R	R	R	R	R	R	R	.6213	.5529	.5175	.5216	.5262	.5214	.5294
RMC1	.0436	.0450	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMC2	.1295	.4112	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMA1	.0440	.0457	.0414	.0408	.0415	.0442	.0619	.0647	.0792	.1172	.1444	.1597	.1832	.2172	.2591	.3020
RMA2	.1383	.4311	.6016	.5836	.5792	.5586	.5954	.5508	.5405	.5831	.5559	.5293	.5413	.5514	.5479	.5530
RMAS1	R	R	R	R	R	R	R	R	R	.1137	.1180	.1287	.1554	.1947	.2397	.2832
RMAS2	R	R	R	R	R	R	R	R	R	.6198	.5508	.5152	.5192	.5238	.5191	.5261
RMAU1	.0436	.0450	R	R	R	R	R	R	R	R	R	R	R	R	R	R
RMAU2	.1295	.4103	R	R	R	R	R	R	R	R	R	R	R	R	R	R

****IN TERMS OF COLOR SPACE****

	C.I.E COORDINATE					COLOR DIFFERENCE IN HUNTER UNIT			
	X	Y	Z	SMLX	SMLY	DELTA L	DELTA A	DELTA B	DELTA E
BY EXPERIMENT	10.96440	8.32793	5.77221	.43745	.33226				
ONE CONST. REL.	11.17665	9.30380	4.96730	.43920	.36560	1.64397	-5.29013	3.35454	6.47619
TWO CONST. REL.	54.69472	56.32671	67.16313	.30696	.31611	46.19297	-18.57242	-8.86426	50.56975
ONE CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. REL.	R	R	R	R	R	R	R	R	R
ONE CONST. ABS.	11.17791	9.30467	4.96741	.43921	.36561	1.64540	-5.28831	3.35579	6.47572
TWO CONST. ABS.	54.46182	56.06670	67.00564	.30677	.31581	46.01955	-18.52287	-8.98383	50.41433
ONE CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. SAUND. ABS.	R	R	R	R	R	R	R	R	R
ONE CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R
TWO CONST. DUN. ABS.	R	R	R	R	R	R	R	R	R

Table 4.32 COMPARISION OF PREDICTED VALUES WITH THE EXPERIMENTAL VALUES

THE MIXTURE OF 50R+25G+25W

sufficiently accurate in certain types of colorant formulation, a two-constant theory involving the determination of separate K and S values leads to more accurate color predictions. This might be true, however, it seems likely that the single-constant theory is adequate for practical purposes, that is to say, that the errors in a predicted formulation that are due to the two-constant theory are about the same size or more as the errors in making up a formulation in practice. It may be well to bear in mind the following quotation in from the pioneering researcher whom utilized the K.M theory, resulted for the widespread use of this theory: Deane B. Judd [124]. "The complete truth is usally too complicated to be of industrial interest. You can go broke with the truth, if it comes in a complicated package. Each worker must seek a compromise between simplicity and accuracy."

It is not suggested here that a practical color-mixture formula should drop the two-constant theory whenever one wants to predict the color of a pigment dispersion. There will no doubt always be cases where the simpler single-constant theory is adequate. What is suggested is that in cases where the single-constant theory is found to be too inaccurate, the two-constant theory should be thoroughly examined.

4.3.1 Application of the Theory in the Ultraviolet Region

The K.M analysis is generally concerned with propagation and interactions of light with matter, i.e., scattering and absorption, not only in the visible spectrum, but also in any regions of the spectrum. The most important restriction on the K.M analysis is that it applies only to a single wavelength at a time. It is not specifically restricted to any region of the electromagnetic spectrum. Although applications of K.M analysis commonly are in the visible region, it is possible, for specific purposes, to extend the range into the ultraviolet or infrared regions. There is, however, a limitation concerning particle size effects. To reflect light of given wavelength efficiently, a particle must posses a diameter approximately one half of that specific wavelength or larger [126]. If a particle is small, compared with the wavelength, a ray of light can simply go around it without being refracted.

The Beckman DB-GT Spectrophotometer, which also is capable of recording reflectance spectra in the ultraviolet region, was used to obtain reflectance spectra in this region for testing the formulas and method developed in this study. Since the pigmented lacquer films of TiO_2, Fe_2O_3 and Cr_2O_3 which were used for our measurements in the visible region, have reflectances which are too low in the ultraviolet region to be of any use, it was necessary to find other pigments. PEMCO Virgo Blue Stain and FERRO Low Infrared reflectance red pigments were found to have sufficiently high reflectances. The experimental and calculated values for the infinite reflectance of both the individual pigments and their mixtures were obtained in the ultraviolet region at 300 nm and 350 nm. The calculated results are in good agreement with the experimental results as shown in Table 4.33. It is clear that for the case of appropriate particle sizes and sufficiently reflectances, the K.M theory can be applied in the ultraviolet regions giving agreement between experimental and calculated results as good as that obtained in the visible region.

Table 4.33 Comparison of computed and experimental infinite reflectance values in ultraviolet region of PEMCO Virgo Blue Stain and FERRO Low Infrared reflectance red lacquer pigment films.

Pigment lacquer	WL	R_{01}	R_{02}	R_∞ computed	R_∞ experiment
BLUE	300	.025	.026	.02604	.026
	350	.070	.071	.071015	.071
RED	300	.198	.201	.20105	.202
	350	.274	.305	.30937	.307
50 RED +	300			.045926	.053
50 BLUE	350			.11434	.105
80 RED +	300			.0852	.081
20 BLUE	350			.18179	.150
33 RED +	300			.036577	.047
67 BLUE	350			.094965	.098

4.4 Error Analysis

There are two types of error:systematic and random. A systematic error will cause the value of a measurement to be displaced a fixed amount from the value of the quantity measured, therefore, a systematic error is a constant error. Factors which cause systematic errors include the ambient environment, the calibration of the measurement equipment, the measurement technique employed, and the observer. Instrumental errors depend on the instrument used and the manner in which it is being used. A random error has random causes and will cause several independent measurements of a quantity to be distributed randomly about their average value. The cause of the random errors is the uncontrollable and unnoticed fluctuation of the many influencing factors of the measurement.

An important source for limitations in accuracy of the calculated values is the repeatability of the lacquer pigment films. There may be several reasons for this, and most are connected with the inaccuracy of working procedures and insufficient apparatus. The predicted result can, therefore, be valid only if the lacquer pigment films are obtained under the same conditions. The unavoidable errors for both experimental and calculated values will be now discussed in the following subsection.

4.4.1 Errors of the Measured Reflectance Spectra

The following are errors which could be caused in the reflectance spectra.

(a) Measurement errors due to the instrument and instrumental calibration. In this study the Beckman DB-GT Spectrophotometer with the Beckman Model 1005 Ten-Inch Laboratory Potentiometric Recorder was used. Under the best calibration; the error of the DB-GR Spectrophotometer by recorder output is ± 0.5% with reference to the reflectance. This means that the error is higher at the higher reflectance than the lower one. Besides the systematic error of the instrument, the error in setting the wavelength is the most serious cause especially in cases of the inclined spectra. Only a few nanometers error in wavelengths may cause much error in reflectance values at the steep portion of the spectra. This will have the result that calculated reflectance values at the region near 400 nm for TiO_2 and at various wavelengths of the colored lacquer pigment films will be less accurate than the other portion of the white one. The other cause of error is the readability in the graph paper recorded by the recorder. This error will cause a large percentage of error at low reflectance. (b) Errors caused by specimens. Besides the inaccuracy of working procedures for the pigmented lacquer that casues some difficulty with reproducibility of the results, the preparation of films is the other cause of inaccuracy. It is not possible to prepare films of reasonably uniform thickness for all the lacquer pigments used, the double film thickness is not really double. This will casue inaccuracy in R_{01}, R_{02} and of course the calculated R_{∞}. There are many other environmental factors that may be encountered as sources of errors in specimens. One of them is relative to the standard reference. One general statement is always true: the greater is the deviation between the spectral distribution of sample to standard, the more likely is the measured ratio of sample to standard to be in error.

In conclusion these errors may be classified into three types:
(1) the nature of pigmentation system and films prepared for measurement,
(2) the operational characteristics of the instrument being used and
(3) faulty analytical techniques. A number of specimens should be prepared

for measurements and the average values should be used to ensure the reproducibility. It is found that starting from the same lacquer pigment in preparing films for measurement is more reproducible than starting from the different lacquer pigment of the same formula. The error in reflectance measurement leads to the invalid results.

4.4.2 Errors in the Determinations of K/S, S and K

The value of K.M function, K/S, obtained with the technique modified in this study depends on the accuracy of R_∞ as can be seen from Eq. (11), where R_∞ depends on the accuracy of R_{01} and R_{02}. As pointed out earlier that the best results could be obtained when the thickness of double film thickness is almost as thick as of the infinite thickness film. The thinner the film the more inaccurate result will be obtained.

There are several possible sources of error in determinations of the absorption and scattering coefficients, K and S, S is dependent on the accuracy of R_{01}, R_{02} and film thickness, X. The error resulting from the thickness, X, of the sample was caused by the nonuniform thickness of the film, the working procedure in measuring film thickness and the instrument being used. In an attempt to offset these possible sources of error in the measurement of X, at least five measurements of each film thickness were made and averaged. However, an error of about 5-10% was still possible for the individual values of X. The error is even worse for the case of colored lacquer pigment films which have to be so very thin that the value of X is not large compared with variation in the accuracy of the instrument. The important thing to remember is that the absorption and scattering coefficients will tend to be constant only if the double film thickness approaches the infinite film thickness.

4.4.3 Sources of Error in Predicting Reflectance and Color of the Mixed Pigment Lacquer Films

Errors occurred both in the experimental and calculated reflectance values. For the experimental reflectance values, errors may be caused by the working procedure in mixing and the proportion of each individual pigment lacquer contained in the mixture. For the calculated reflectance values, there are several sources of error in each individual value as has been discussed previously. It is well to compare the results of the single theory and two-constant theory, since there are more sources of error for two-constant theory than the single one. This is especially true for the values of film thickness. The accuracy of the micrometer used in this study is about ± 0.1 mil with careful use. This is significantly large compared to the film thickness as which are of the order of 5 mils. This will cause quite a great error for the individual values of K and S, which explains why the single constant theory gives a more accurate result than two-constant theory, for the single-constant theory is not concerned with the thickness of films.

4.5 Conclusions

Several useful conclusions can be drawn from this study. First, the modified theory and method, as developed here are shown to give reliable estimates of infinite reflectance without the difficulties that arise in using the other methods. The second conclusion grows out of the observation that the coefficient K obtained directly from K.M theory without any corrections are almost twice the coefficient K obtained by applying the surface correction. As it has been mentioned in the section 4.2 that the K.M coefficients, K are approximately twice the true phenomena, this seems likely to be true that the actual physical absorption coefficient can be obtained by using surface correction. In other applications, such as radiative heat transfer, if the K.M analysis is to be used, it is strongly recommended that the surface correction must be made. The third conclusion, this study describes not only the use of the new developed formulas and method to predict the infinite reflectance of films containing a single pigment, wavelength by wavelength, but also the extension of these results to mixed-pigmented films and color prediction. The best results in formulation using K.M analysis in the system studied will be obtained for color prediciton of mixtures if the single constant theory is used; and if the two constant is used, the surface correction must be used. Two constant theory cannot be relied upon, however, in the formulation of a system containing the white pigment. The fact that the two constant theory failed in this case does not mean that the theory has no value. This form is used in industry, and it is very successful in the case of mixtures of colored pigments and also at low values of PVC (less than 5% PVC) [67,78,125]. Our choice of experimental PVC values for tne mixture was dictated by the requirements that many paints are formulated at levels of PVC about this value and upwards to or beyond the critical PVC. When the PVC is so high that the pigment particles are not separated by distances comparable with their own sizes, the mixing laws, Equations (79) and (80), no longer predict the proper K and S for use in the K.M theory [67]. The system used in this study provided a severe test for the K.M theories, firstly the mixtures containing white pigment TiO_2 which the single constant is preferred. Secondly the lacquer pigments of 15% PVC were used for the mixtures which are too high for the proper predicted K and S. It is the achievement of these conditions, plus a knowledge of the exact thickness of the films, which makes application of the two constant theory inapplicable for this study. The fourth conclusion, it has been demonstrated that the formulas and method developed here can be used to predict infinite reflectance in other regions of the spectra, specifically the ultraviolet. Finally, it is possible to say that the single constant theory-with or without surface correction-is adequate for practical use.

In summary, the modified theory and method, as developed in this study have been shown to give reliable estimates of reflectance and color both for the single and mixed pigment lacquer films. The calculated results were found to be in good agreement with the experimental determinations obtained in our laboratory.

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APPENDIX I

TABLES

This APPENDIX consists of 10 Tables:

Table A.1 Table for inspecting R_{01}, R_{02}. In this table the minimum values of R_{01} for a given R_{02} that is the limitation of the formulas and method developed in this study are listed.

Table A.2 From the known values of R_{01} and R_{02}, the values of R_{∞} can be found directly from this table.

Table A.3 The relative reflectance values which were measured relative to standard white $BaSO_4$ have been converted to the absolute reflectance values. The absolute reflectance values of standard white $BaSO_4$ used in this calculation at 400, 420, 440 460, 480, 500, 520, 540, 560, 580, 600, 620, 640, 660, 680, and 700 are 0.995, 0.999 0.999, 0.999, 0.999, 0.998, 0.998, 0.998, 0.998, 0.998, 0.998, 0.998, 0.998, 0.998, 0.998 and 0.997 respectively [72].

Table A.4 The reflectance values with the Saunderson correction are listed corresponding to the measured reflectance values. In this calculation the values of $k_1 = 0.04$ and $k_2 = 0.6$ were used.

Table A.5 The reflectance values corresponding to the Duncan method are listed. The values of $k_1 = 0.04$ and $k_2 = 0.555$ were used in computing for this table.

Table A.6 The Kubelka-Munk functions.

Table A.7 and Table A.8 are the tables of the Kubelka-Munk function corresponding to measured values R_∞ incorporating the Saunderson and Duncan correction respectively. These two tables can be used to find the Kubelka-Munk function with surface correction directly.

Table A.9 and Table A.10 are the values of the Kubelka-Munk parameters a and b corresponding to R_∞. These two parameters are necessary for some applications of K.M theory, such as in computing hiding power.

TABLE A.1

TABLE FOR INSPECTING R01, R02

(MINIMUM VALUES OF R01 CORRESPONDING TO GIVEN R02)

R02	(PERCENT R02) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	.000	.001	.001	.002	.002	.003	.003	.004	.004	.005
1.	.005	.006	.006	.007	.007	.008	.008	.009	.009	.010
2.	.010	.011	.011	.012	.012	.013	.013	.014	.014	.015
3.	.015	.016	.016	.017	.017	.018	.018	.019	.019	.020
4.	.020	.021	.021	.022	.022	.023	.024	.024	.025	.025
5.	.026	.026	.027	.027	.028	.028	.029	.029	.030	.030
6.	.031	.031	.032	.033	.033	.034	.034	.035	.035	.036
7.	.036	.037	.037	.038	.038	.039	.040	.040	.041	.041
8.	.042	.042	.043	.043	.044	.044	.045	.045	.046	.047
9.	.047	.048	.048	.048	.049	.050	.050	.051	.052	.052
10.	.053	.053	.054	.054	.055	.055	.056	.057	.057	.058
11.	.058	.059	.059	.060	.060	.061	.062	.062	.063	.063
12.	.064	.064	.065	.066	.066	.067	.067	.068	.068	.069
13.	.070	.070	.071	.071	.072	.072	.073	.074	.074	.075
14.	.075	.076	.076	.077	.078	.078	.079	.079	.080	.080
15.	.081	.082	.082	.083	.083	.084	.085	.085	.086	.086
16.	.087	.088	.088	.089	.089	.090	.091	.091	.092	.092
17.	.093	.093	.094	.095	.095	.096	.096	.097	.098	.098
18.	.099	.100	.100	.101	.101	.102	.103	.103	.104	.104
19.	.105	.105	.106	.107	.107	.108	.109	.109	.110	.110
20.	.111	.112	.112	.113	.114	.114	.115	.115	.116	.117
21.	.117	.118	.119	.119	.120	.120	.121	.122	.122	.123
22.	.124	.124	.125	.125	.126	.127	.127	.128	.129	.129
23.	.130	.131	.131	.132	.133	.133	.134	.134	.135	.136
24.	.136	.137	.138	.138	.139	.140	.140	.141	.142	.142
25.	.143	.144	.144	.145	.145	.146	.147	.147	.148	.149
26.	.149	.150	.151	.151	.152	.153	.153	.154	.155	.155
27.	.156	.157	.157	.158	.159	.159	.160	.161	.161	.162
28.	.163	.163	.164	.165	.166	.166	.167	.168	.168	.169
29.	.170	.170	.171	.172	.172	.173	.174	.174	.175	.176
30.	.176	.177	.178	.179	.179	.180	.181	.181	.182	.183
31.	.183	.184	.185	.186	.186	.187	.188	.188	.189	.190
32.	.190	.191	.192	.193	.193	.194	.195	.195	.196	.197
33.	.198	.198	.199	.200	.200	.201	.202	.203	.203	.204
34.	.205	.206	.206	.207	.208	.208	.209	.210	.211	.211
35.	.212	.213	.214	.214	.215	.216	.217	.217	.218	.219
36.	.220	.220	.221	.222	.222	.223	.224	.225	.225	.226
37.	.227	.228	.229	.229	.230	.231	.232	.232	.233	.234
38.	.235	.235	.236	.237	.238	.238	.239	.240	.241	.241
39.	.242	.243	.244	.245	.245	.246	.247	.248	.248	.249
40.	.250	.251	.252	.252	.253	.254	.255	.255	.256	.257
41.	.258	.259	.259	.260	.261	.262	.263	.263	.264	.265
42.	.266	.267	.267	.268	.269	.270	.271	.271	.272	.273
43.	.274	.275	.276	.276	.277	.278	.279	.280	.280	.281
44.	.282	.283	.284	.285	.285	.286	.287	.288	.289	.289
45.	.290	.291	.292	.293	.294	.294	.295	.296	.297	.298
46.	.299	.300	.300	.301	.302	.303	.304	.305	.305	.306
47.	.307	.308	.309	.310	.311	.311	.312	.313	.314	.315
48.	.316	.317	.318	.318	.319	.320	.321	.322	.323	.324
49.	.325	.325	.326	.327	.328	.329	.330	.331	.332	.332

R02	(PERCENT R02) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.333	.334	.335	.336	.337	.338	.339	.340	.340	.341
51.	.342	.343	.344	.345	.346	.347	.348	.349	.350	[illegible]
52.	.351	.352	.353	.354	.355	.356	.357	.358	.359	[illegible]
53.	.361	.361	.362	.363	.364	.365	.366	.367	.368	[illegible]
54.	.370	.371	.372	.373	.374	.375	.375	.376	.377	[illegible]
55.	.379	.380	.381	.382	.383	.384	.385	.386	.387	[illegible]
56.	.389	.390	.391	.392	.393	.394	.395	.396	.397	[illegible]
57.	.399	.400	.401	.402	.403	.404	.404	.405	.406	.407
58.	.408	.409	.410	.411	.412	.413	.414	.415	.416	.417
59.	.418	.419	.420	.421	.422	.423	.425	.426	.427	.428
60.	.429	.430	.431	.432	.433	.434	.435	.436	.437	.438
61.	.439	.440	.441	.442	.443	.444	.445	.446	.447	.448
62.	.449	.450	.451	.452	.453	.455	.456	.457	.458	.459
63.	.460	.461	.462	.463	.464	.465	.466	.467	.468	.470
64.	.471	.472	.473	.474	.475	.476	.477	.478	.479	.480
65.	.481	.483	.484	.485	.486	.487	.488	.489	.490	.491
66.	.493	.494	.495	.496	.497	.498	.499	.500	.502	.503
67.	.504	.505	.506	.507	.508	.509	.511	.512	.513	.514
68.	.515	.516	.517	.519	.520	.521	.522	.523	.524	.526
69.	.527	.528	.529	.530	.531	.533	.534	.535	.536	.537
70.	.538	.540	.541	.542	.543	.544	.546	.547	.548	.549
71.	.550	.552	.553	.554	.555	.556	.558	.559	.560	.561
72.	.562	.564	.565	.566	.567	.569	.570	.571	.572	.574
73.	.575	.576	.577	.579	.580	.581	.582	.584	.585	.586
74.	.587	.589	.590	.591	.592	.594	.595	.596	.597	.599
75.	.600	.601	.603	.604	.605	.606	.608	.609	.610	.612
76.	.613	.614	.616	.617	.618	.619	.621	.622	.623	.625
77.	.626	.627	.629	.630	.631	.633	.634	.635	.637	.638
78.	.639	.641	.642	.643	.645	.646	.647	.649	.650	.652
79.	.653	.654	.656	.657	.658	.660	.661	.663	.664	.665
80.	.667	.668	.669	.671	.672	.674	.675	.676	.678	.679
81.	.681	.682	.684	.685	.686	.688	.689	.691	.692	.693
82.	.695	.696	.698	.699	.701	.702	.704	.705	.705	.708
83.	.709	.711	.712	.714	.715	.717	.718	.720	.721	.723
84.	.724	.726	.727	.729	.730	.732	.733	.735	.736	.738
85.	.739	.741	.742	.744	.745	.747	.748	.750	.751	.753
86.	.754	.756	.757	.759	.761	.762	.764	.765	.767	.768
87.	.770	.771	.773	.775	.776	.778	.779	.781	.783	.784
88.	.786	.787	.789	.791	.792	.794	.795	.797	.799	.800
89.	.802	.803	.805	.807	.808	.810	.812	.813	.815	.817
90.	.818	.820	.821	.823	.825	.826	.828	.830	.832	.833
91.	.835	.837	.838	.840	.842	.843	.845	.847	.848	.850
92.	.852	.854	.855	.857	.859	.860	.862	.864	.866	.867
93.	.869	.871	.873	.874	.876	.878	.880	.881	.883	.885
94.	.887	.889	.890	.892	.894	.896	.898	.899	.901	.903
95.	.905	.907	.908	.910	.912	.914	.916	.918	.919	.921
96.	.923	.925	.927	.929	.931	.932	.934	.936	.938	.940
97.	.942	.944	.946	.947	.949	.951	.953	.955	.957	.959
98.	.961	.963	.965	.967	.969	.970	.972	.974	.976	.978
99.	.980	.982	.984	.986	.988	.990	.992	.994	.996	.998

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 1 PERCENT

R01	RINF
.00	R
.01	.0100
.02	R
.03	R
.04	R
.05	R

R02= 2 PERCENT

R01	RINF
.00	R
.01	R
.02	.0200
.03	R
.04	R
.05	R

R02= 3 PERCENT

R01	RINF
.00	R
.01	R
.02	.0400
.03	.0300
.04	R
.05	R

R02= 4 PERCENT

R01	RINF
.00	R
.01	R
.02	R
.03	.0450
.04	.0400
.05	R

R02= 5 PERCENT

R01	RINF
.00	R
.01	R
.02	R
.03	.0903
.04	.0533
.05	.0500

R02= 6 PERCENT

R01	RINF
.03	R
.04	.0301
.05	.0625
.06	.0600
.07	R
.08	R
.09	R
.10	R

R02= 7 PERCENT

R01	RINF
.03	R
.04	.1624
.05	.0834
.06	.0720
.07	.0700
.08	R
.09	R
.10	R

R02= 8 PERCENT

R01	RINF
.03	R
.04	R
.05	.1257
.06	.0901
.07	.0817
.08	.0800
.09	R
.10	R

R02= 9 PERCENT

R01	RINF
.03	R
.04	R
.05	.2012
.06	.1204
.07	.0981
.08	.0914
.09	.0900
.10	R

R02= 10 PERCENT

R01	RINF
.03	R
.04	R
.05	R
.06	.1827
.07	.1228
.08	.1067
.09	.1013
.10	.1000

R02= 11 PERCENT

R01	RINF
.04	R
.05	R
.06	.4023
.07	.1548
.08	.1283
.09	.1158
.10	.1111
.11	.1100
.12	R
.13	R
.14	R
.15	R

R02= 12 PERCENT

R01	RINF
.04	R
.05	R
.06	R
.07	.2533
.08	.1611
.09	.1353
.10	.1251
.11	.1210
.12	.1200
.13	R
.14	R
.15	R

R02= 13 PERCENT

R01	RINF
.04	R
.05	R
.06	R
.07	.6634
.08	.2174
.09	.1629
.10	.1431
.11	.1345
.12	.1309
.13	.1300
.14	R
.15	R

R02= 14 PERCENT

R01	RINF
.04	R
.05	R
.06	R
.07	R
.08	.3421
.09	.2052
.10	.1674
.11	.1515
.12	.1441
.13	.1409
.14	.1400
.15	R

R02= 15 PERCENT

R01	RINF
.04	R
.05	R
.06	R
.07	R
.08	R
.09	.2798
.10	.2021
.11	.1736
.12	.1603
.13	.1537
.14	.1508
.15	.1500

R02= 16 PERCENT

R01	RINF
.08	R
.09	.4613
.10	.2562
.11	.2034
.12	.1807
.13	.1693
.14	.1634
.15	.1607
.16	.1600
.17	R
.18	R
.19	R
.20	R

R02= 17 PERCENT

R01	RINF
.08	R
.09	R
.10	.3553
.11	.2406
.12	.2073
.13	.1854
.14	.1785
.15	.1732
.16	.1707
.17	.1700
.18	R
.19	R
.20	R

R02= 18 PERCENT

R01	RINF
.08	R
.09	R
.10	.6563
.11	.3154
.12	.2437
.13	.2127
.14	.1966
.15	.1878
.16	.1830
.17	.1806
.18	.1800
.19	R
.20	R

R02= 19 PERCENT

R01	RINF
.08	R
.09	R
.10	R
.11	.4507
.12	.2972
.13	.2447
.14	.2192
.15	.2052
.16	.1972
.17	.1928
.18	.1906
.19	.1900
.20	R

R02= 20 PERCENT

R01	RINF
.08	R
.09	R
.10	R
.11	R
.12	.3858
.13	.2859
.14	.2479
.15	.2263
.16	.2140
.17	.2067
.18	.2026
.19	.2005
.20	.2000

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 21 PERCENT

R01	RINF
.11	R
.12	.5856
.13	.3555
.14	.2851
.15	.2527
.16	.2340
.17	.2229
.18	.2163
.19	.2125
.20	.2106
.21	.2100
.22	R
.23	R
.24	R
.25	R

R02= 22 PERCENT

R01	RINF
.11	R
.12	R
.13	.4731
.14	.3400
.15	.2866
.16	.2586
.17	.2421
.18	.2321
.19	.2259
.20	.2223
.21	.2205
.22	.2200
.23	R
.24	R
.25	R

R02= 23 PERCENT

R01	RINF
.11	R
.12	R
.13	.9308
.14	.4243
.15	.3324
.16	.2893
.17	.2652
.18	.2505
.19	.2413
.20	.2356
.21	.2322
.22	.2305
.23	.2300
.24	R
.25	R

R02= 24 PERCENT

R01	RINF
.11	R
.12	R
.13	R
.14	.5922
.15	.3967
.16	.3294
.17	.2935
.18	.2724
.19	.2591
.20	.2507
.21	.2453
.22	.2421
.23	.2405
.24	.2400
.25	R

R02= 25 PERCENT

R01	RINF
.11	R
.12	R
.13	R
.14	R
.15	.5095
.16	.3847
.17	.3295
.18	.2988
.19	.2800
.20	.2679
.21	.2601
.22	.2551
.23	.2520
.24	.2505
.25	.2500

R02= 26 PERCENT

R01	RINF
.14	R
.15	.8236
.16	.4633
.17	.3772
.18	.3317
.19	.3050
.20	.2880
.21	.2769
.22	.2696
.23	.2648
.24	.2620
.25	.2604
.26	.2600
.27	R
.28	R
.29	R
.30	R

R02= 27 PERCENT

R01	RINF
.14	R
.15	R
.16	.6262
.17	.4450
.18	.3740
.19	.3354
.20	.3117
.21	.2963
.22	.2860
.23	.2791
.24	.2746
.25	.2719
.26	.2704
.27	.2700
.28	R
.29	R
.30	R

R02= 28 PERCENT

R01	RINF
.14	R
.15	R
.16	R
.17	.5553
.18	.4315
.19	.3736
.20	.3402
.21	.3189
.22	.3048
.23	.2952
.24	.2887
.25	.2845
.26	.2818
.27	.2804
.28	.2800
.29	R
.30	R

R02= 29 PERCENT

R01	RINF
.14	R
.15	R
.16	R
.17	.8679
.18	.5173
.19	.4239
.20	.3753
.21	.3458
.22	.3265
.23	.3135
.24	.3045
.25	.2984
.26	.2943
.27	.2917
.28	.2904
.29	.2900
.30	R

R02= 30 PERCENT

R01	RINF
.14	R
.15	R
.16	R
.17	R
.18	.6789
.19	.4947
.20	.4202
.21	.3784
.22	.3520
.23	.3344
.24	.3223
.25	.3139
.26	.3080
.27	.3041
.28	.3017
.29	.3004
.30	.3000

R02= 31 PERCENT

R01	RINF
.18	R
.19	.6102
.20	.4810
.21	.4193
.22	.3827
.23	.3568
.24	.3426
.25	.3313
.26	.3233
.27	.3177
.28	.3140
.29	.3116
.30	.3104
.31	.3100
.32	R
.33	R
.34	R
.35	R

R02= 32 PERCENT

R01	RINF
.18	R
.19	R
.20	.5721
.21	.4729
.22	.4203
.23	.3878
.24	.3661
.25	.3510
.26	.3403
.27	.3328
.28	.3275
.29	.3239
.30	.3216
.31	.3204
.32	.3200
.33	R
.34	R
.35	R

R02= 33 PERCENT

R01	RINF
.18	R
.19	R
.20	.7543
.21	.5489
.22	.4686
.23	.4229
.24	.3936
.25	.3736
.26	.3596
.27	.3495
.28	.3423
.29	.3372
.30	.3338
.31	.3315
.32	.3304
.33	.3300
.34	R
.35	R

R02= 34 PERCENT

R01	RINF
.18	R
.19	R
.20	R
.21	.6770
.22	.5342
.23	.4670
.24	.4266
.25	.4000
.26	.3815
.27	.3683
.28	.3588
.29	.3519
.30	.3470
.31	.3436
.32	.3415
.33	.3404
.34	.3400
.35	R

R02= 35 PERCENT

R01	RINF
.18	R
.19	R
.20	R
.21	R
.22	.6352
.23	.5251
.24	.4674
.25	.4313
.26	.4068
.27	.3896
.28	.3772
.29	.3681
.30	.3615
.31	.3564
.32	.3535
.33	.3515
.34	.3503
.35	.3500

RO1	RINF (RO2= 36 PERCENT)	RINF (RO2= 37 PERCENT)	RINF (RO2= 38 PERCENT)	RINF (RO2= 39 PERCENT)	RINF (RO2= 40 PERCENT)
.20	R	R	R	R	R
.21	R	R	R	R	R
.22	.8927	R	R	R	R
.23	.6094	.7602	R	R	R
.24	.5199	.5928	.7124	R	R
.25	.4593	.5175	.5821	.6805	1.0000
.26	.4366	.4725	.5171	.5755	.6600
.27	.4141	.4426	.4766	.5183	.5719
.28	.3979	.4216	.4491	.4815	.5208
.29	.3861	.4064	.4294	.4559	.4870
.30	.3775	.3952	.4150	.4375	.4632
.31	.3711	.3869	.4044	.4238	.4457
.32	.3656	.3808	.3964	.4136	.4327
.33	.3635	.3764	.3905	.4059	.4229
.34	.3614	.3734	.3863	.4003	.4155
.35	.3603	.3714	.3833	.3961	.4100
.36	.3600	.3703	.3814	.3932	.4060
.37	R	.3700	.3803	.3913	.4032
.38	R	R	.3800	.3903	.4013
.39	R	R	R	.3900	.4003
.40	R	R	R	R	.4000
.41	R	R	R	R	R
.42	R	R	R	R	R
.43	R	R	R	R	R
.44	R	R	R	R	R
.45	R	R	R	R	R
.46	R	R	R	R	R

RO1	RINF (RO2= 41 PERCENT)	RINF (RO2= 42 PERCENT)	RINF (RO2= 43 PERCENT)	RINF (RO2= 44 PERCENT)	RINF (RO2= 45 PERCENT)
.20	R	R	R	R	R
.21	R	R	R	R	R
.22	R	R	R	R	R
.23	R	R	R	R	R
.24	R	R	R	R	R
.25	R	R	R	R	R
.26	.8248	R	R	R	R
.27	.6465	.7736	R	R	R
.28	.5705	.6378	.7439	R	R
.29	.5244	.5709	.6325	.7246	R
.30	.4931	.5268	.5727	.6297	.7118
.31	.4707	.4996	.5338	.5755	.6289
.32	.4541	.4785	.5065	.5395	.5793
.33	.4417	.4627	.4865	.5138	.5456
.34	.4323	.4508	.4714	.4947	.5213
.35	.4251	.4417	.4600	.4803	.5031
.36	.4198	.4348	.4512	.4692	.4892
.37	.4159	.4296	.4444	.4607	.4785
.38	.4131	.4257	.4394	.4541	.4702
.39	.4113	.4230	.4356	.4492	.4639
.40	.4103	.4213	.4330	.4455	.4590
.41	.4100	.4203	.4313	.4429	.4555
.42	R	.4200	.4303	.4412	.4529
.43	R	R	.4300	.4403	.4512
.44	R	R	R	.4400	.4503
.45	R	R	R	R	.4500
.46	R	R	R	R	R

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 46 PERCENT

R01	RINF
.28	R
.29	R
.30	.8828
.31	.7035
.32	.6297
.33	.5839
.34	.5522
.35	.5291
.36	.5116
.37	.4982
.38	.4879
.39	.4798
.40	.4736
.41	.4689
.42	.4654
.43	.4629
.44	.4612
.45	.4603
.46	.4600
.47	R
.48	R
.49	R
.50	R
.51	R
.52	R

R02= 47 PERCENT

R01	RINF
.28	R
.29	R
.30	R
.31	.8390
.32	.6985
.33	.6317
.34	.5891
.35	.5592
.36	.5371
.37	.5203
.38	.5074
.39	.4973
.40	.4895
.41	.4834
.42	.4788
.43	.4753
.44	.4728
.45	.4712
.46	.4703
.47	.4700
.48	R
.49	R
.50	R
.51	R
.52	R

R02= 48 PERCENT

R01	RINF
.28	R
.29	R
.30	R
.31	R
.32	.8140
.33	.6959
.34	.6348
.35	.5949
.36	.5664
.37	.5453
.38	.5291
.39	.5166
.40	.5068
.41	.4991
.42	.4932
.43	.4886
.44	.4852
.45	.4828
.46	.4812
.47	.4803
.48	.4800
.49	R
.50	R
.51	R
.52	R

R02= 49 PERCENT

R01	RINF
.28	R
.29	R
.30	R
.31	R
.32	R
.33	.7981
.34	.6952
.35	.6385
.36	.6011
.37	.5740
.38	.5537
.39	.5380
.40	.5259
.41	.5163
.42	.5088
.43	.5030
.44	.4985
.45	.4952
.46	.4928
.47	.4912
.48	.4903
.49	.4900
.50	R
.51	R
.52	R

R02= 50 PERCENT

R01	RINF
.28	R
.29	R
.30	R
.31	R
.32	R
.33	R
.34	.7879
.35	.6961
.36	.6435
.37	.6078
.38	.5818
.39	.5622
.40	.5471
.41	.5352
.42	.5259
.43	.5186
.44	.5128
.45	.5084
.46	.5051
.47	.5027
.48	.5012
.49	.5003
.50	.5000
.51	R
.52	R

R02= 51 PERCENT

R01	RINF
.33	R
.34	R
.35	.7816
.36	.6943
.37	.6448
.38	.6148
.39	.5899
.40	.5709
.41	.5562
.42	.5446
.43	.5355
.44	.5283
.45	.5227
.46	.5183
.47	.5151
.48	.5127
.49	.5112
.50	.5103
.51	.5100
.52	R
.53	R
.54	R
.55	R
.56	R

R02= 52 PERCENT

R01	RINF
.33	R
.34	R
.35	R
.36	.7782
.37	.7015
.38	.6548
.39	.6222
.40	.5981
.41	.5797
.42	.5654
.43	.5541
.44	.5452
.45	.5381
.46	.5325
.47	.5282
.48	.5250
.49	.5227
.50	.5211
.51	.5203
.52	.5200
.53	R
.54	R
.55	R
.56	R

R02= 53 PERCENT

R01	RINF
.33	R
.34	R
.35	R
.36	R
.37	.7771
.38	.7056
.39	.6612
.40	.6299
.41	.6066
.42	.5887
.43	.5747
.44	.5636
.45	.5548
.46	.5479
.47	.5424
.48	.5382
.49	.5350
.50	.5327
.51	.5311
.52	.5303
.53	.5300
.54	R
.55	R
.56	R

R02= 54 PERCENT

R01	RINF
.33	R
.34	R
.35	R
.36	R
.37	.9701
.38	.7777
.39	.7106
.40	.6680
.41	.6378
.42	.6152
.43	.5978
.44	.5841
.45	.5732
.46	.5646
.47	.5577
.48	.5523
.49	.5481
.50	.5449
.51	.5427
.52	.5411
.53	.5403
.54	.5400
.55	R
.56	R

R02= 55 PERCENT

R01	RINF
.33	R
.34	R
.35	R
.36	R
.37	R
.38	.9366
.39	.7798
.40	.7162
.41	.6752
.42	.6460
.43	.6240
.44	.6069
.45	.5935
.46	.5828
.47	.5743
.48	.5675
.49	.5622
.50	.5581
.51	.5549
.52	.5526
.53	.5511
.54	.5503
.55	.5500
.56	R

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 56 PERCENT

R01	RINF
.37	R
.38	R
.39	.9230
.40	.7832
.41	.7224
.42	.6828
.43	.6544
.44	.6329
.45	.6172
.46	.6050
.47	.5925
.48	.5841
.49	.5774
.50	.5721
.51	.5680
.52	.5649
.53	.5626
.54	.5611
.55	.5603
.56	.5600
.57	R
.58	R
.59	R
.60	R
.61	R

R02= 57 PERCENT

R01	RINF
.37	R
.38	R
.39	R
.40	.9171
.41	.7877
.42	.7291
.43	.6908
.44	.6630
.45	.6420
.46	.6256
.47	.6126
.48	.6022
.49	.5939
.50	.5873
.51	.5821
.52	.5780
.53	.5749
.54	.5726
.55	.5711
.56	.5703
.57	.5700
.58	R
.59	R
.60	R
.61	R

R02= 58 PERCENT

R01	RINF
.37	R
.38	R
.39	R
.40	R
.41	.9159
.42	.7932
.43	.7364
.44	.6990
.45	.6719
.46	.6512
.47	.6351
.48	.6223
.49	.6120
.50	.6038
.51	.5972
.52	.5920
.53	.5879
.54	.5849
.55	.5826
.56	.5811
.57	.5803
.58	.5800
.59	R
.60	R
.61	R

R02= 59 PERCENT

R01	RINF
.37	R
.38	R
.39	R
.40	R
.41	R
.42	.9185
.43	.7996
.44	.7442
.45	.7076
.46	.6809
.47	.6606
.48	.6446
.49	.6320
.50	.6218
.51	.6137
.52	.6071
.53	.6020
.54	.5979
.55	.5948
.56	.5926
.57	.5911
.58	.5903
.59	.5900
.60	R
.61	R

R02= 60 PERCENT

R01	RINF
.37	R
.38	R
.39	R
.40	R
.41	R
.42	R
.43	.9245
.44	.8068
.45	.7524
.46	.7164
.47	.6901
.48	.6700
.49	.6543
.50	.6417
.51	.6317
.52	.6236
.53	.6171
.54	.6119
.55	.6079
.56	.6048
.57	.6026
.58	.6011
.59	.6003
.60	.6000
.61	R

R02= 61 PERCENT

R01	RINF
.42	R
.43	R
.44	.9342
.45	.8148
.46	.7610
.47	.7255
.48	.6995
.49	.6796
.50	.6640
.51	.6516
.52	.6416
.53	.6335
.54	.6271
.55	.6219
.56	.6179
.57	.6148
.58	.6126
.59	.6111
.60	.6103
.61	.6100
.62	R
.63	R
.64	R
.65	R
.66	R

R02= 62 PERCENT

R01	RINF
.42	R
.43	R
.44	R
.45	.9492
.46	.8236
.47	.7701
.48	.7348
.49	.7091
.50	.6893
.51	.6738
.52	.6615
.53	.6515
.54	.6435
.55	.6371
.56	.6319
.57	.6279
.58	.6249
.59	.6226
.60	.6211
.61	.6203
.62	.6200
.63	R
.64	R
.65	R
.66	R

R02= 63 PERCENT

R01	RINF
.42	R
.43	R
.44	R
.45	R
.46	.9777
.47	.8333
.48	.7796
.49	.7444
.50	.7188
.51	.6992
.52	.6838
.53	.6714
.54	.6615
.55	.6535
.56	.6471
.57	.6419
.58	.6379
.59	.6349
.60	.6326
.61	.6311
.62	.6303
.63	.6300
.64	R
.65	R
.66	R

R02= 64 PERCENT

R01	RINF
.42	R
.43	R
.44	R
.45	R
.46	R
.47	R
.48	.8439
.49	.7890
.50	.7543
.51	.7287
.52	.7091
.53	.6937
.54	.6814
.55	.6715
.56	.6635
.57	.6571
.58	.6520
.59	.6480
.60	.6449
.61	.6426
.62	.6411
.63	.6403
.64	.6400
.65	R
.66	R

R02= 65 PERCENT

R01	RINF
.42	R
.43	R
.44	R
.45	R
.46	R
.47	R
.48	R
.49	.8554
.50	.8000
.51	.7645
.52	.7388
.53	.7192
.54	.7039
.55	.6915
.56	.6816
.57	.6736
.58	.6672
.59	.6620
.60	.6580
.61	.6549
.62	.6527
.63	.6511
.64	.6503
.65	.6500
.66	R

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 66 PERCENT

R01	RINF
.49	R
.50	.8652
.51	.8109
.52	.7750
.53	.7492
.54	.7255
.55	.7140
.56	.7017
.57	.6917
.58	.6837
.59	.6772
.60	.6721
.61	.6680
.62	.6649
.63	.6627
.64	.6611
.65	.6603
.66	.6600
.67	R
.68	R
.69	R
.70	R
.71	R

R02= 67 PERCENT

R01	RINF
.49	R
.50	R
.51	.8825
.52	.8224
.53	.7858
.54	.7597
.55	.7398
.56	.7243
.57	.7119
.58	.7019
.59	.6938
.60	.6873
.61	.6822
.62	.6781
.63	.6750
.64	.6727
.65	.6712
.66	.6703
.67	.6700
.68	R
.69	R
.70	R
.71	R

R02= 68 PERCENT

R01	RINF
.49	R
.50	R
.51	R
.52	.8989
.53	.8346
.54	.7969
.55	.7704
.56	.7504
.57	.7347
.58	.7222
.59	.7121
.60	.7040
.61	.6975
.62	.6922
.63	.6881
.64	.6850
.65	.6827
.66	.6812
.67	.6803
.68	.6800
.69	R
.70	R
.71	R

R02= 69 PERCENT

R01	RINF
.49	R
.50	R
.51	R
.52	R
.53	.9187
.54	.8475
.55	.8085
.56	.7814
.57	.7610
.58	.7452
.59	.7326
.60	.7224
.61	.7142
.62	.7076
.63	.7024
.64	.6982
.65	.6951
.66	.6927
.67	.6912
.68	.6903
.69	.6900
.70	R
.71	R

R02= 70 PERCENT

R01	RINF
.49	R
.50	R
.51	R
.52	R
.53	R
.54	.9454
.55	.8614
.56	.8205
.57	.7926
.58	.7719
.59	.7558
.60	.7430
.61	.7327
.62	.7245
.63	.7178
.64	.7125
.65	.7083
.66	.7051
.67	.7028
.68	.7012
.69	.7003
.70	.7000
.71	R

R02= 71 PERCENT

R01	RINF
.54	R
.55	R
.56	.8765
.57	.8330
.58	.8042
.59	.7829
.60	.7665
.61	.7536
.62	.7432
.63	.7348
.64	.72[illegible]
.65	.7226
.66	.7184
.67	.7152
.68	.7128
.69	.7112
.70	.7103
.71	.7100
.72	R
.73	R
.74	R
.75	R
.76	R

R02= 72 PERCENT

R01	RINF
.54	R
.55	R
.56	R
.57	.8935
.58	.8402
.59	.8161
.60	.7942
.61	.7774
.62	.7642
.63	.7530
.64	.7451
.65	.7383
.66	.7328
.67	.7285
.68	.7253
.69	.7228
.70	.7212
.71	.7203
.72	.7200
.73	R
.74	R
.75	R
.76	R

R02= 73 PERCENT

R01	RINF
.54	R
.55	R
.56	R
.57	R
.58	.9133
.59	.8602
.60	.8284
.61	.8057
.62	.7885
.63	.7750
.64	.7642
.65	.7555
.66	.7486
.67	.7430
.68	.7387
.69	.7353
.70	.7329
.71	.7312
.72	.7303
.73	.7300
.74	R
.75	R
.76	R

R02= 74 PERCENT

R01	RINF
.54	R
.55	R
.56	R
.57	R
.58	R
.59	.9387
.60	.8753
.61	.8412
.62	.8175
.63	.7997
.64	.7858
.65	.7748
.66	.7660
.67	.7589
.68	.7533
.69	.7488
.70	.7454
.71	.7429
.72	.7413
.73	.7403
.74	.7400
.75	R
.76	R

R02= 75 PERCENT

R01	RINF
.54	R
.55	R
.56	R
.57	R
.58	R
.59	R
.60	1.0000
.61	.8919
.62	.8547
.63	.8297
.64	.8112
.65	.7969
.66	.7856
.67	.7765
.68	.7693
.69	.7635
.70	.7590
.71	.7555
.72	.7530
.73	.7513
.74	.7503
.75	.7500
.76	R

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO R01 AND R02

R02= 76 PERCENT

R01	RINF
.60	R
.61	R
.62	.9110
.63	.8670
.64	.8423
.65	.8229
.66	.8091
.67	.7964
.68	.7871
.69	.7797
.70	.7738
.71	.7692
.72	.7657
.73	.7631
.74	.7613
.75	.7603
.76	.7600
.77	R
.78	R
.79	R
.80	R
.81	R

R02= 77 PERCENT

R01	RINF
.60	R
.61	R
.62	R
.63	.9347
.64	.8844
.65	.8554
.66	.8350
.67	.8195
.68	.8074
.69	.7978
.70	.7902
.71	.7841
.72	.7794
.73	.7758
.74	.7731
.75	.7714
.76	.7703
.77	.7700
.78	R
.79	R
.80	R
.81	R

R02= 78 PERCENT

R01	RINF
.60	R
.61	R
.62	R
.63	R
.64	.9738
.65	.9015
.66	.8692
.67	.8474
.68	.8311
.69	.8185
.70	.8086
.71	.8007
.72	.7945
.73	.7896
.74	.7859
.75	.7832
.76	.7814
.77	.7803
.78	.7800
.79	R
.80	R
.81	R

R02= 79 PERCENT

R01	RINF
.60	R
.61	R
.62	R
.63	R
.64	R
.65	R
.66	.9214
.67	.8840
.68	.8603
.69	.8430
.70	.8298
.71	.8195
.72	.8114
.73	.8049
.74	.7999
.75	.7961
.76	.7933
.77	.7914
.78	.7903
.79	.7900
.80	R
.81	R

R02= 80 PERCENT

R01	RINF
.60	R
.61	R
.62	R
.63	R
.64	R
.65	R
.66	R
.67	.9472
.68	.9002
.69	.8738
.70	.8553
.71	.8414
.72	.8306
.73	.8221
.74	.8154
.75	.8102
.76	.8063
.77	.8034
.78	.8015
.79	.8004
.80	.8000
.81	R

R02= 81 PERCENT

R01	RINF
.67	R
.68	R
.69	.9145
.70	.8862
.71	.8690
.72	.8531
.73	.8418
.74	.8329
.75	.8260
.76	.8206
.77	.8165
.78	.8135
.79	.8115
.80	.8104
.81	.8100
.82	R
.83	R
.84	R
.85	R
.86	R

R02= 82 PERCENT

R01	RINF
.67	R
.68	R
.69	R
.70	.9412
.71	.9038
.72	.8813
.73	.8653
.74	.8532
.75	.8439
.76	.8366
.77	.8310
.78	.8268
.79	.8237
.80	.8216
.81	.8204
.82	.8200
.83	R
.84	R
.85	R
.86	R

R02= 83 PERCENT

R01	RINF
.67	R
.68	R
.69	R
.70	R
.71	.9801
.72	.9214
.73	.8954
.74	.8778
.75	.8649
.76	.8550
.77	.8473
.78	.8415
.79	.8370
.80	.8338
.81	.8316
.82	.8304
.83	.8300
.84	R
.85	R
.86	R

R02= 84 PERCENT

R01	RINF
.67	R
.68	R
.69	R
.70	R
.71	R
.72	R
.73	.9430
.74	.9108
.75	.8910
.76	.8769
.77	.8663
.78	.8582
.79	.8520
.80	.8473
.81	.8440
.82	.8417
.83	.8404
.84	.8400
.85	R
.86	R

R02= 85 PERCENT

R01	RINF
.67	R
.68	R
.69	R
.70	R
.71	R
.72	R
.73	R
.74	.9784
.75	.9282
.76	.9050
.77	.8893
.78	.8779
.79	.8692
.80	.8626
.81	.8577
.82	.8542
.83	.8518
.84	.8504
.85	.8500
.86	R

TABLE A. 2 VALUES OF INFINITE REFLECTANCE RELATIVE TO RO1 AND RO2

RO2= 86 PERCENT

RO1	RINF
.73	R
.74	R
.75	R
.76	.9457
.77	.9203
.78	.9024
.79	.8857
.80	.8804
.81	.8734
.82	.8652
.83	.8644
.84	.8619
.85	.8605
.86	.8600
.87	R
.88	R
.89	R
.90	R
.91	R

RO2= 87 PERCENT

RO1	RINF
.73	R
.74	R
.75	R
.76	R
.77	.9938
.78	.9378
.79	.9163
.80	.9021
.81	.8918
.82	.8843
.83	.8787
.84	.8747
.85	.8720
.86	.8705
.87	.8700
.88	R
.89	R
.90	R
.91	R

RO2= 88 PERCENT

RO1	RINF
.73	R
.74	R
.75	R
.76	R
.77	R
.78	R
.79	.9606
.80	.9317
.81	.9151
.82	.9036
.83	.8953
.84	.8893
.85	.8850
.86	.8821
.87	.8805
.88	.8800
.89	R
.90	R
.91	R

RO2= 89 PERCENT

RO1	RINF
.73	R
.74	R
.75	R
.76	R
.77	R
.78	R
.79	R
.80	R
.81	.9500
.82	.9291
.83	.9159
.84	.9066
.85	.9000
.86	.8954
.87	.8923
.88	.8905
.89	.8900
.90	R
.91	R

RO2= 90 PERCENT

RO1	RINF
.73	R
.74	R
.75	R
.76	R
.77	R
.78	R
.79	R
.80	R
.81	R
.82	.9771
.83	.9449
.84	.9289
.85	.9183
.86	.9109
.87	.9054
.88	.9025
.89	.9006
.90	.9000
.91	R

RO2= 91 PERCENT

RO1	RINF
.82	R
.83	R
.84	.9650
.85	.9432
.86	.9305
.87	.9221
.88	.9164
.89	.9127
.90	.9107
.91	.9100
.92	R
.93	R
.94	R
.95	R

RO2= 92 PERCENT

RO1	RINF
.82	R
.83	R
.84	R
.85	R
.86	.9600
.87	.9436
.88	.9337
.89	.9272
.90	.9230
.91	.9207
.92	.9200
.93	R
.94	R
.95	R

RO2= 93 PERCENT

RO1	RINF
.82	R
.83	R
.84	R
.85	R
.86	R
.87	.9876
.88	.9585
.89	.9459
.90	.9382
.91	.9334
.92	.9308
.93	.9300
.94	R
.95	R

RO2= 94 PERCENT

RO1	RINF
.82	R
.83	R
.84	R
.85	R
.86	R
.87	R
.88	R
.89	.9785
.90	.9593
.91	.9496
.92	.9440
.93	.9409
.94	.9400
.95	R

RO2= 95 PERCENT

RO1	RINF
.82	R
.83	R
.84	R
.85	R
.86	R
.87	R
.88	R
.89	R
.90	R
.91	.9758
.92	.9619
.93	.9547
.94	.9511
.95	.9500

RO2= 96 PERCENT

RO1	RINF
.91	R
.92	R
.93	.9761
.94	.9660
.95	.9614
.96	.9600
.97	R
.98	R
.99	R

RO2= 97 PERCENT

RO1	RINF
.91	R
.92	R
.93	R
.94	R
.95	.9765
.96	.9718
.97	.9700
.98	R
.99	R

RO2= 98 PERCENT

RO1	RINF
.91	R
.92	R
.93	R
.94	R
.95	R
.96	R
.97	.9828
.98	.9800
.99	R

RO2= 99 PERCENT

RO1	RINF
.91	R
.92	R
.93	R
.94	R
.95	R
.96	R
.97	R
.98	R
.99	.9900

RO2=100 PERCENT

RO1	RINF
.91	R
.92	R
.93	R
.94	R
.95	R
.96	R
.97	R
.98	R
.99	R

TABLE A. 3

ABSOLUTE REFLECTANCE VALUES CORRESPONDING TO MEASURED REFLECTANCE VALUES

AT 400 NM

RM	(PERCENT MEASURED REFLECTANCE) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
1.	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019
2.	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
3.	.030	.031	.032	.033	.034	.035	.036	.037	.038	.039
4.	.040	.041	.042	.043	.044	.045	.046	.047	.048	.049
5.	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059
6.	.060	.061	.062	.063	.064	.065	.066	.067	.068	.069
7.	.070	.071	.072	.073	.074	.075	.076	.077	.078	.079
8.	.080	.081	.082	.083	.084	.085	.086	.087	.088	.089
9.	.090	.091	.092	.093	.094	.095	.096	.097	.098	.099
10.	.101	.102	.103	.104	.105	.106	.107	.108	.109	.110
11.	.111	.112	.113	.114	.115	.116	.117	.118	.119	.120
12.	.121	.122	.123	.124	.125	.126	.127	.128	.129	.130
13.	.131	.132	.133	.134	.135	.136	.137	.138	.139	.140
14.	.141	.142	.143	.144	.145	.146	.147	.148	.149	.150
15.	.151	.152	.153	.154	.155	.156	.157	.158	.159	.160
16.	.161	.162	.163	.164	.165	.166	.167	.168	.169	.170
17.	.171	.172	.173	.174	.175	.176	.177	.178	.179	.180
18.	.181	.182	.183	.184	.185	.186	.187	.188	.189	.190
19.	.191	.192	.193	.194	.195	.196	.197	.198	.199	.200
20.	.201	.202	.203	.204	.205	.206	.207	.208	.209	.210
21.	.211	.212	.213	.214	.215	.216	.217	.218	.219	.220
22.	.221	.222	.223	.224	.225	.226	.227	.228	.229	.230
23.	.231	.232	.233	.234	.235	.236	.237	.238	.239	.240
24.	.241	.242	.243	.244	.245	.246	.247	.248	.249	.250
25.	.251	.252	.253	.254	.255	.256	.257	.258	.259	.260
26.	.261	.262	.263	.264	.265	.266	.267	.268	.269	.270
27.	.271	.272	.273	.274	.275	.276	.277	.278	.279	.280
28.	.281	.282	.283	.284	.285	.286	.287	.288	.289	.290
29.	.291	.292	.293	.294	.295	.296	.297	.298	.299	.301
30.	.302	.303	.304	.305	.306	.307	.308	.309	.310	.311
31.	.312	.313	.314	.315	.316	.317	.318	.319	.320	.321
32.	.322	.323	.324	.325	.326	.327	.328	.329	.330	.331
33.	.332	.333	.334	.335	.336	.337	.338	.339	.340	.341
34.	.342	.343	.344	.345	.346	.347	.348	.349	.350	.351
35.	.352	.353	.354	.355	.356	.357	.358	.359	.360	.361
36.	.362	.363	.364	.365	.366	.367	.368	.369	.370	.371
37.	.372	.373	.374	.375	.376	.377	.378	.379	.380	.381
38.	.382	.383	.384	.385	.386	.387	.388	.389	.390	.391
39.	.392	.393	.394	.395	.396	.397	.398	.399	.400	.401
40.	.402	.403	.404	.405	.406	.407	.408	.409	.410	.411
41.	.412	.413	.414	.415	.416	.417	.418	.419	.420	.421
42.	.422	.423	.424	.425	.426	.427	.428	.429	.430	.431
43.	.432	.433	.434	.435	.436	.437	.438	.439	.440	.441
44.	.442	.443	.444	.445	.446	.447	.448	.449	.450	.451
45.	.452	.453	.454	.455	.456	.457	.458	.459	.460	.461
46.	.462	.463	.464	.465	.466	.467	.468	.469	.470	.471
47.	.472	.473	.474	.475	.476	.477	.478	.479	.480	.481
48.	.482	.483	.484	.485	.486	.487	.488	.489	.490	.491
49.	.492	.493	.494	.495	.496	.497	.498	.499	.501	.502

RM	(PERCENT MEASURED REFLECTANCE) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.503	.504	.505	.506	.507	.508	.509	.510	.511	.512
51.	.513	.514	.515	.516	.517	.518	.519	.520	.521	.522
52.	.523	.524	.525	.526	.527	.528	.529	.530	.531	.532
53.	.533	.534	.535	.536	.537	.538	.539	.540	.541	.542
54.	.543	.544	.545	.546	.547	.548	.549	.550	.551	.552
55.	.553	.554	.555	.556	.557	.558	.559	.560	.561	.562
56.	.563	.564	.565	.566	.567	.568	.569	.570	.571	.572
57.	.573	.574	.575	.576	.577	.578	.579	.580	.581	.582
58.	.583	.584	.585	.586	.587	.588	.589	.590	.591	.592
59.	.593	.594	.595	.596	.597	.598	.599	.600	.601	.602
60.	.603	.604	.605	.606	.607	.608	.609	.610	.611	.612
61.	.613	.614	.615	.616	.617	.618	.619	.620	.621	.622
62.	.623	.624	.625	.626	.627	.628	.629	.630	.631	.632
63.	.633	.634	.635	.636	.637	.638	.639	.640	.641	.642
64.	.643	.644	.645	.646	.647	.648	.649	.650	.651	.652
65.	.653	.654	.655	.656	.657	.658	.659	.660	.661	.662
66.	.663	.664	.665	.666	.667	.668	.669	.670	.671	.672
67.	.673	.674	.675	.676	.677	.678	.679	.680	.681	.682
68.	.683	.684	.685	.686	.687	.688	.689	.690	.691	.692
69.	.693	.694	.695	.696	.697	.698	.699	.701	.702	.703
70.	.704	.705	.706	.707	.708	.709	.710	.711	.712	.713
71.	.714	.715	.716	.717	.718	.719	.720	.721	.722	.723
72.	.724	.725	.726	.727	.728	.729	.730	.731	.732	.733
73.	.734	.735	.736	.737	.738	.739	.740	.741	.742	.743
74.	.744	.745	.746	.747	.748	.749	.750	.751	.752	.753
75.	.754	.755	.756	.757	.758	.759	.760	.761	.762	.763
76.	.764	.765	.766	.767	.768	.769	.770	.771	.772	.773
77.	.774	.775	.776	.777	.778	.779	.780	.781	.782	.783
78.	.784	.785	.786	.787	.788	.789	.790	.791	.792	.793
79.	.794	.795	.796	.797	.798	.799	.800	.801	.802	.803
80.	.804	.805	.806	.807	.808	.809	.810	.811	.812	.813
81.	.814	.815	.816	.817	.818	.819	.820	.821	.822	.823
82.	.824	.825	.826	.827	.828	.829	.830	.831	.832	.833
83.	.834	.835	.836	.837	.838	.839	.840	.841	.842	.843
84.	.844	.845	.846	.847	.848	.849	.850	.851	.852	.853
85.	.854	.855	.856	.857	.858	.859	.860	.861	.862	.863
86.	.864	.865	.866	.867	.868	.869	.870	.871	.872	.873
87.	.874	.875	.876	.877	.878	.879	.880	.881	.882	.883
88.	.884	.885	.886	.887	.888	.889	.890	.891	.892	.893
89.	.894	.895	.896	.897	.898	.899	.901	.902	.903	.904
90.	.905	.906	.907	.908	.909	.910	.911	.912	.913	.914
91.	.915	.916	.917	.918	.919	.920	.921	.922	.923	.924
92.	.925	.926	.927	.928	.929	.930	.931	.932	.933	.934
93.	.935	.936	.937	.938	.939	.940	.941	.942	.943	.944
94.	.945	.946	.947	.948	.949	.950	.951	.952	.953	.954
95.	.955	.956	.957	.958	.959	.960	.961	.962	.963	.964
96.	.965	.966	.967	.968	.969	.970	.971	.972	.973	.974
97.	.975	.976	.977	.978	.979	.980	.981	.982	.983	.984
98.	.985	.986	.987	.988	.989	.990	.991	.992	.993	.994
99.	.995	.996	.997	.998	.999	1.000	1.001	1.002	1.003	1.004

TABLE A. 3 (CONT.)

ABSOLUTE REFLECTANCE VALUES CORRESPONDING TO MEASURED REFLECTANCE VALUES

AT 420,440,460,480 NM

	(PERCENT MEASURED REFLECTANCE)									
KM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
1.	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019
2.	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
3.	.030	.031	.032	.033	.034	.035	.036	.037	.038	.039
4.	.040	.041	.042	.043	.044	.045	.046	.047	.048	.049
5.	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059
6.	.060	.061	.062	.063	.064	.065	.066	.067	.068	.069
7.	.070	.071	.072	.073	.074	.075	.076	.077	.078	.079
8.	.080	.081	.082	.083	.084	.085	.086	.087	.088	.089
9.	.090	.091	.092	.093	.094	.095	.096	.097	.098	.099
10.	.100	.101	.102	.103	.104	.105	.106	.107	.108	.109
11.	.110	.111	.112	.113	.114	.115	.116	.117	.118	.119
12.	.120	.121	.122	.123	.124	.125	.126	.127	.128	.129
13.	.130	.131	.132	.133	.134	.135	.136	.137	.138	.139
14.	.140	.141	.142	.143	.144	.145	.146	.147	.148	.149
15.	.150	.151	.152	.153	.154	.155	.156	.157	.158	.159
16.	.160	.161	.162	.163	.164	.165	.166	.167	.168	.169
17.	.170	.171	.172	.173	.174	.175	.176	.177	.178	.179
18.	.180	.181	.182	.183	.184	.185	.186	.187	.188	.189
19.	.190	.191	.192	.193	.194	.195	.196	.197	.198	.199
20.	.200	.201	.202	.203	.204	.205	.206	.207	.208	.209
21.	.210	.211	.212	.213	.214	.215	.216	.217	.218	.219
22.	.220	.221	.222	.223	.224	.225	.226	.227	.228	.229
23.	.230	.231	.232	.233	.234	.235	.236	.237	.238	.239
24.	.240	.241	.242	.243	.244	.245	.246	.247	.248	.249
25.	.250	.251	.252	.253	.254	.255	.256	.257	.258	.259
26.	.260	.261	.262	.263	.264	.265	.266	.267	.268	.269
27.	.270	.271	.272	.273	.274	.275	.276	.277	.278	.279
28.	.280	.281	.282	.283	.284	.285	.286	.287	.288	.289
29.	.290	.291	.292	.293	.294	.295	.296	.297	.298	.299
30.	.300	.301	.302	.303	.304	.305	.306	.307	.308	.309
31.	.310	.311	.312	.313	.314	.315	.316	.317	.318	.319
32.	.320	.321	.322	.323	.324	.325	.326	.327	.328	.329
33.	.330	.331	.332	.333	.334	.335	.336	.337	.338	.339
34.	.340	.341	.342	.343	.344	.345	.346	.347	.348	.349
35.	.350	.351	.352	.353	.354	.355	.356	.357	.358	.359
36.	.360	.361	.362	.363	.364	.365	.366	.367	.368	.369
37.	.370	.371	.372	.373	.374	.375	.376	.377	.378	.379
38.	.380	.381	.382	.383	.384	.385	.386	.387	.388	.389
39.	.390	.391	.392	.393	.394	.395	.396	.397	.398	.399
40.	.400	.401	.402	.403	.404	.405	.406	.407	.408	.409
41.	.410	.411	.412	.413	.414	.415	.416	.417	.418	.419
42.	.420	.421	.422	.423	.424	.425	.426	.427	.428	.429
43.	.430	.431	.432	.433	.434	.435	.436	.437	.438	.439
44.	.440	.441	.442	.443	.444	.445	.446	.447	.448	.449
45.	.450	.451	.452	.453	.454	.455	.456	.457	.458	.459
46.	.460	.461	.462	.463	.464	.465	.466	.467	.468	.469
47.	.470	.471	.472	.473	.474	.475	.476	.477	.478	.479
48.	.480	.481	.482	.483	.484	.485	.486	.487	.488	.489
49.	.490	.491	.492	.493	.494	.495	.496	.497	.498	.499

	(PERCENT MEASURED REFLECTANCE)									
KM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.501	.502	.503	.504	.505	.506	.507	.508	.509	.510
51.	.511	.512	.513	.514	.515	.516	.517	.518	.519	.520
52.	.521	.522	.523	.524	.525	.526	.527	.528	.529	.530
53.	.531	.532	.533	.534	.535	.536	.537	.538	.539	.540
54.	.541	.542	.543	.544	.545	.546	.547	.548	.549	.550
55.	.551	.552	.553	.554	.555	.556	.557	.558	.559	.560
56.	.561	.562	.563	.564	.565	.566	.567	.568	.569	.570
57.	.571	.572	.573	.574	.575	.576	.577	.578	.579	.580
58.	.581	.582	.583	.584	.585	.586	.587	.588	.589	.590
59.	.591	.592	.593	.594	.595	.596	.597	.598	.599	.600
60.	.601	.602	.603	.604	.605	.606	.607	.608	.609	.610
61.	.611	.612	.613	.614	.615	.616	.617	.618	.619	.620
62.	.621	.622	.623	.624	.625	.626	.627	.628	.629	.630
63.	.631	.632	.633	.634	.635	.636	.637	.638	.639	.640
64.	.641	.642	.643	.644	.645	.646	.647	.648	.649	.650
65.	.651	.652	.653	.654	.655	.656	.657	.658	.659	.660
66.	.661	.662	.663	.664	.665	.666	.667	.668	.669	.670
67.	.671	.672	.673	.674	.675	.676	.677	.678	.679	.680
68.	.681	.682	.683	.684	.685	.686	.687	.688	.689	.690
69.	.691	.692	.693	.694	.695	.696	.697	.698	.699	.700
70.	.701	.702	.703	.704	.705	.706	.707	.708	.709	.710
71.	.711	.712	.713	.714	.715	.716	.717	.718	.719	.720
72.	.721	.722	.723	.724	.725	.726	.727	.728	.729	.730
73.	.731	.732	.733	.734	.735	.736	.737	.738	.739	.740
74.	.741	.742	.743	.744	.745	.746	.747	.748	.749	.750
75.	.751	.752	.753	.754	.755	.756	.757	.758	.759	.760
76.	.761	.762	.763	.764	.765	.766	.767	.768	.769	.770
77.	.771	.772	.773	.774	.775	.776	.777	.778	.779	.780
78.	.781	.782	.783	.784	.785	.786	.787	.788	.789	.790
79.	.791	.792	.793	.794	.795	.796	.797	.798	.799	.800
80.	.801	.802	.803	.804	.805	.806	.807	.808	.809	.810
81.	.811	.812	.813	.814	.815	.816	.817	.818	.819	.820
82.	.821	.822	.823	.824	.825	.826	.827	.828	.829	.830
83.	.831	.832	.833	.834	.835	.836	.837	.838	.839	.840
84.	.841	.842	.843	.844	.845	.846	.847	.848	.849	.850
85.	.851	.852	.853	.854	.855	.856	.857	.858	.859	.860
86.	.861	.862	.863	.864	.865	.866	.867	.868	.869	.870
87.	.871	.872	.873	.874	.875	.876	.877	.878	.879	.880
88.	.881	.882	.883	.884	.885	.886	.887	.888	.889	.890
89.	.891	.892	.893	.894	.895	.896	.897	.898	.899	.900
90.	.901	.902	.903	.904	.905	.906	.907	.908	.909	.910
91.	.911	.912	.913	.914	.915	.916	.917	.918	.919	.920
92.	.921	.922	.923	.924	.925	.926	.927	.928	.929	.930
93.	.931	.932	.933	.934	.935	.936	.937	.938	.939	.940
94.	.941	.942	.943	.944	.945	.946	.947	.948	.949	.950
95.	.951	.952	.953	.954	.955	.956	.957	.958	.959	.960
96.	.961	.962	.963	.964	.965	.966	.967	.968	.969	.970
97.	.971	.972	.973	.974	.975	.976	.977	.978	.979	.980
98.	.981	.982	.983	.984	.985	.986	.987	.988	.989	.990
99.	.991	.992	.993	.994	.995	.996	.997	.998	.999	1.000

TABLE A. 3 (CONT.)

ABSOLUTE REFLECTANCE VALUES CORRESPONDING TO MEASURED REFLECTANCE VALUES

AT 500,520,540,560,580,600,620,640,660,680 NM

(PERCENT MEASURED REFLECTANCE)

RM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
1.	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019
2.	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
3.	.030	.031	.032	.033	.034	.035	.036	.037	.038	.039
4.	.040	.041	.042	.043	.044	.045	.046	.047	.048	.049
5.	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059
6.	.060	.061	.062	.063	.064	.065	.066	.067	.068	.069
7.	.070	.071	.072	.073	.074	.075	.076	.077	.078	.079
8.	.080	.081	.082	.083	.084	.085	.086	.087	.088	.089
9.	.090	.091	.092	.093	.094	.095	.096	.097	.098	.099
10.	.100	.101	.102	.103	.104	.105	.106	.107	.108	.109
11.	.110	.111	.112	.113	.114	.115	.116	.117	.118	.119
12.	.120	.121	.122	.123	.124	.125	.126	.127	.128	.129
13.	.130	.131	.132	.133	.134	.135	.136	.137	.138	.139
14.	.140	.141	.142	.143	.144	.145	.146	.147	.148	.149
15.	.150	.151	.152	.153	.154	.155	.156	.157	.158	.159
16.	.160	.161	.162	.163	.164	.165	.166	.167	.168	.169
17.	.170	.171	.172	.173	.174	.175	.176	.177	.178	.179
18.	.180	.181	.182	.183	.184	.185	.186	.187	.188	.189
19.	.190	.191	.192	.193	.194	.195	.196	.197	.198	.199
20.	.200	.201	.202	.203	.204	.205	.206	.207	.208	.209
21.	.210	.211	.212	.213	.214	.215	.216	.217	.218	.219
22.	.220	.221	.222	.223	.224	.225	.226	.227	.228	.229
23.	.230	.231	.232	.233	.234	.235	.236	.237	.238	.239
24.	.240	.241	.242	.243	.244	.245	.246	.247	.248	.249
25.	.251	.252	.253	.254	.255	.256	.257	.258	.259	.260
26.	.261	.262	.263	.264	.265	.266	.267	.268	.269	.270
27.	.271	.272	.273	.274	.275	.276	.277	.278	.279	.280
28.	.281	.282	.283	.284	.285	.286	.287	.288	.289	.290
29.	.291	.292	.293	.294	.295	.296	.297	.298	.299	.300
30.	.301	.302	.303	.304	.305	.306	.307	.308	.309	.310
31.	.311	.312	.313	.314	.315	.316	.317	.318	.319	.320
32.	.321	.322	.323	.324	.325	.326	.327	.328	.329	.330
33.	.331	.332	.333	.334	.335	.336	.337	.338	.339	.340
34.	.341	.342	.343	.344	.345	.346	.347	.348	.349	.350
35.	.351	.352	.353	.354	.355	.356	.357	.358	.359	.360
36.	.361	.362	.363	.364	.365	.366	.367	.368	.369	.370
37.	.371	.372	.373	.374	.375	.376	.377	.378	.379	.380
38.	.381	.382	.383	.384	.385	.386	.387	.388	.389	.390
39.	.391	.392	.393	.394	.395	.396	.397	.398	.399	.400
40.	.401	.402	.403	.404	.405	.406	.407	.408	.409	.410
41.	.411	.412	.413	.414	.415	.416	.417	.418	.419	.420
42.	.421	.422	.423	.424	.425	.426	.427	.428	.429	.430
43.	.431	.432	.433	.434	.435	.436	.437	.438	.439	.440
44.	.441	.442	.443	.444	.445	.446	.447	.448	.449	.450
45.	.451	.452	.453	.454	.455	.456	.457	.458	.459	.460
46.	.461	.462	.463	.464	.465	.466	.467	.468	.469	.470
47.	.471	.472	.473	.474	.475	.476	.477	.478	.479	.480
48.	.481	.482	.483	.484	.485	.486	.487	.488	.489	.490
49.	.491	.492	.493	.494	.495	.496	.497	.498	.499	.500

(PERCENT MEASURED REFLECTANCE)

RM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.501	.502	.503	.504	.505	.506	.507	.508	.509	.510
51.	.511	.512	.513	.514	.515	.516	.517	.518	.519	.520
52.	.521	.522	.523	.524	.525	.526	.527	.528	.529	.530
53.	.531	.532	.533	.534	.535	.536	.537	.538	.539	.540
54.	.541	.542	.543	.544	.545	.546	.547	.548	.549	.550
55.	.551	.552	.553	.554	.555	.556	.557	.558	.559	.560
56.	.561	.562	.563	.564	.565	.566	.567	.568	.569	.570
57.	.571	.572	.573	.574	.575	.576	.577	.578	.579	.580
58.	.581	.582	.583	.584	.585	.586	.587	.588	.589	.590
59.	.591	.592	.593	.594	.595	.596	.597	.598	.599	.600
60.	.601	.602	.603	.604	.605	.606	.607	.608	.609	.610
61.	.611	.612	.613	.614	.615	.616	.617	.618	.619	.620
62.	.621	.622	.623	.624	.625	.626	.627	.628	.629	.630
63.	.631	.632	.633	.634	.635	.636	.637	.638	.639	.640
64.	.641	.642	.643	.644	.645	.646	.647	.648	.649	.650
65.	.651	.652	.653	.654	.655	.656	.657	.658	.659	.660
66.	.661	.662	.663	.664	.665	.666	.667	.668	.669	.670
67.	.671	.672	.673	.674	.675	.676	.677	.678	.679	.680
68.	.681	.682	.683	.684	.685	.686	.687	.688	.689	.690
69.	.691	.692	.693	.694	.695	.696	.697	.698	.699	.700
70.	.701	.702	.703	.704	.705	.706	.707	.708	.709	.710
71.	.711	.712	.713	.714	.715	.716	.717	.718	.719	.720
72.	.721	.722	.723	.724	.725	.726	.727	.728	.729	.730
73.	.731	.732	.733	.734	.735	.736	.737	.738	.739	.740
74.	.741	.742	.743	.744	.745	.746	.747	.748	.749	.751
75.	.752	.753	.754	.755	.756	.757	.758	.759	.760	.761
76.	.762	.763	.764	.765	.766	.767	.768	.769	.770	.771
77.	.772	.773	.774	.775	.776	.777	.778	.779	.780	.781
78.	.782	.783	.784	.785	.786	.787	.788	.789	.790	.791
79.	.792	.793	.794	.795	.796	.797	.798	.799	.800	.801
80.	.802	.803	.804	.805	.806	.807	.808	.809	.810	.811
81.	.812	.813	.814	.815	.816	.817	.818	.819	.820	.821
82.	.822	.823	.824	.825	.826	.827	.828	.829	.830	.831
83.	.832	.833	.834	.835	.836	.837	.838	.839	.840	.841
84.	.842	.843	.844	.845	.846	.847	.848	.849	.850	.851
85.	.852	.853	.854	.855	.856	.857	.858	.859	.860	.861
86.	.862	.863	.864	.865	.866	.867	.868	.869	.870	.871
87.	.872	.873	.874	.875	.876	.877	.878	.879	.880	.881
88.	.882	.883	.884	.885	.886	.887	.888	.889	.890	.891
89.	.892	.893	.894	.895	.896	.897	.898	.899	.900	.901
90.	.902	.903	.904	.905	.906	.907	.908	.909	.910	.911
91.	.912	.913	.914	.915	.916	.917	.918	.919	.920	.921
92.	.922	.923	.924	.925	.926	.927	.928	.929	.930	.931
93.	.932	.933	.934	.935	.936	.937	.938	.939	.940	.941
94.	.942	.943	.944	.945	.946	.947	.948	.949	.950	.951
95.	.952	.953	.954	.955	.956	.957	.958	.959	.960	.961
96.	.962	.963	.964	.965	.966	.967	.968	.969	.970	.971
97.	.972	.973	.974	.975	.976	.977	.978	.979	.980	.981
98.	.982	.983	.984	.985	.986	.987	.988	.989	.990	.991
99.	.992	.993	.994	.995	.996	.997	.998	.999	1.000	1.001

TABLE A. 3 (CONT.)

ABSOLUTE REFLECTANCE VALUES CORRESPONDING TO MEASURED REFLECTANCE VALUES

AT 700 NM

RM	(PERCENT MEASURED REFLECTANCE) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	.000	.001	.002	.003	.004	.005	.006	.007	.008	.009
1.	.010	.011	.012	.013	.014	.015	.016	.017	.018	.019
2.	.020	.021	.022	.023	.024	.025	.026	.027	.028	.029
3.	.030	.031	.032	.033	.034	.035	.036	.037	.038	.039
4.	.040	.041	.042	.043	.044	.045	.046	.047	.048	.049
5.	.050	.051	.052	.053	.054	.055	.056	.057	.058	.059
6.	.060	.061	.062	.063	.064	.065	.066	.067	.068	.069
7.	.070	.071	.072	.073	.074	.075	.076	.077	.078	.079
8.	.080	.081	.082	.083	.084	.085	.086	.087	.088	.089
9.	.090	.091	.092	.093	.094	.095	.096	.097	.098	.099
10.	.100	.101	.102	.103	.104	.105	.106	.107	.108	.109
11.	.110	.111	.112	.113	.114	.115	.116	.117	.118	.119
12.	.120	.121	.122	.123	.124	.125	.126	.127	.128	.129
13.	.130	.131	.132	.133	.134	.135	.136	.137	.138	.139
14.	.140	.141	.142	.143	.144	.145	.146	.147	.148	.149
15.	.150	.151	.152	.153	.154	.155	.156	.157	.158	.159
16.	.160	.161	.162	.163	.164	.165	.166	.168	.169	.170
17.	.171	.172	.173	.174	.175	.176	.177	.178	.179	.180
18.	.181	.182	.183	.184	.185	.186	.187	.188	.189	.190
19.	.191	.192	.193	.194	.195	.196	.197	.198	.199	.200
20.	.201	.202	.203	.204	.205	.206	.207	.208	.209	.210
21.	.211	.212	.213	.214	.215	.216	.217	.218	.219	.220
22.	.221	.222	.223	.224	.225	.226	.227	.228	.229	.230
23.	.231	.232	.233	.234	.235	.236	.237	.238	.239	.240
24.	.241	.242	.243	.244	.245	.246	.247	.248	.249	.250
25.	.251	.252	.253	.254	.255	.256	.257	.258	.259	.260
26.	.261	.262	.263	.264	.265	.266	.267	.268	.269	.270
27.	.271	.272	.273	.274	.275	.276	.277	.278	.279	.280
28.	.281	.282	.283	.284	.285	.286	.287	.288	.289	.290
29.	.291	.292	.293	.294	.295	.296	.297	.298	.299	.300
30.	.301	.302	.303	.304	.305	.306	.307	.308	.309	.310
31.	.311	.312	.313	.314	.315	.316	.317	.318	.319	.320
32.	.321	.322	.323	.324	.325	.326	.327	.328	.329	.330
33.	.331	.332	.333	.334	.335	.336	.337	.338	.339	.340
34.	.341	.342	.343	.344	.345	.346	.347	.348	.349	.350
35.	.351	.352	.353	.354	.355	.356	.357	.358	.359	.360
36.	.361	.362	.363	.364	.365	.366	.367	.368	.369	.370
37.	.371	.372	.373	.374	.375	.376	.377	.378	.379	.380
38.	.381	.382	.383	.384	.385	.386	.387	.388	.389	.390
39.	.391	.392	.393	.394	.395	.396	.397	.398	.399	.400
40.	.401	.402	.403	.404	.405	.406	.407	.408	.409	.410
41.	.411	.412	.413	.414	.415	.416	.417	.418	.419	.420
42.	.421	.422	.423	.424	.425	.426	.427	.428	.429	.430
43.	.431	.432	.433	.434	.435	.436	.437	.438	.439	.440
44.	.441	.442	.443	.444	.445	.446	.447	.448	.449	.450
45.	.451	.452	.453	.454	.455	.456	.457	.458	.459	.460
46.	.461	.462	.463	.464	.465	.466	.467	.468	.469	.470
47.	.471	.472	.473	.474	.475	.476	.477	.478	.479	.480
48.	.481	.482	.483	.484	.485	.486	.487	.488	.489	.490
49.	.491	.492	.493	.494	.495	.496	.497	.498	.499	.501

RM	(PERCENT MEASURED REFLECTANCE) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.502	.503	.504	.505	.506	.507	.508	.509	.510	.511
51.	.512	.513	.514	.515	.516	.517	.518	.519	.520	.521
52.	.522	.523	.524	.525	.526	.527	.528	.529	.530	.531
53.	.532	.533	.534	.535	.536	.537	.538	.539	.540	.541
54.	.542	.543	.544	.545	.546	.547	.548	.549	.550	.551
55.	.552	.553	.554	.555	.556	.557	.558	.559	.560	.561
56.	.562	.563	.564	.565	.566	.567	.568	.569	.570	.571
57.	.572	.573	.574	.575	.576	.577	.578	.579	.580	.581
58.	.582	.583	.584	.585	.586	.587	.588	.589	.590	.591
59.	.592	.593	.594	.595	.596	.597	.598	.599	.600	.601
60.	.602	.603	.604	.605	.606	.607	.608	.609	.610	.611
61.	.612	.613	.614	.615	.616	.617	.618	.619	.620	.621
62.	.622	.623	.624	.625	.626	.627	.628	.629	.630	.631
63.	.632	.633	.634	.635	.636	.637	.638	.639	.640	.641
64.	.642	.643	.644	.645	.646	.647	.648	.649	.650	.651
65.	.652	.653	.654	.655	.656	.657	.658	.659	.660	.661
66.	.662	.663	.664	.665	.666	.667	.668	.669	.670	.671
67.	.672	.673	.674	.675	.676	.677	.678	.679	.680	.681
68.	.682	.683	.684	.685	.686	.687	.688	.689	.690	.691
69.	.692	.693	.694	.695	.696	.697	.698	.699	.700	.701
70.	.702	.703	.704	.705	.706	.707	.708	.709	.710	.711
71.	.712	.713	.714	.715	.716	.717	.718	.719	.720	.721
72.	.722	.723	.724	.725	.726	.727	.728	.729	.730	.731
73.	.732	.733	.734	.735	.736	.737	.738	.739	.740	.741
74.	.742	.743	.744	.745	.746	.747	.748	.749	.750	.751
75.	.752	.753	.754	.755	.756	.757	.758	.759	.760	.761
76.	.762	.763	.764	.765	.766	.767	.768	.769	.770	.771
77.	.772	.773	.774	.775	.776	.777	.778	.779	.780	.781
78.	.782	.783	.784	.785	.786	.787	.788	.789	.790	.791
79.	.792	.793	.794	.795	.796	.797	.798	.799	.800	.801
80.	.802	.803	.804	.805	.806	.807	.808	.809	.810	.811
81.	.812	.813	.814	.815	.816	.817	.818	.819	.820	.821
82.	.822	.823	.824	.825	.826	.827	.828	.829	.830	.831
83.	.832	.834	.835	.836	.837	.838	.839	.840	.841	.842
84.	.843	.844	.845	.846	.847	.848	.849	.850	.851	.852
85.	.853	.854	.855	.856	.857	.858	.859	.860	.861	.862
86.	.863	.864	.865	.866	.867	.868	.869	.870	.871	.872
87.	.873	.874	.875	.876	.877	.878	.879	.880	.881	.882
88.	.883	.884	.885	.886	.887	.888	.889	.890	.891	.892
89.	.893	.894	.895	.896	.897	.898	.899	.900	.901	.902
90.	.903	.904	.905	.906	.907	.908	.909	.910	.911	.912
91.	.913	.914	.915	.916	.917	.918	.919	.920	.921	.922
92.	.923	.924	.925	.926	.927	.928	.929	.930	.931	.932
93.	.933	.934	.935	.936	.937	.938	.939	.940	.941	.942
94.	.943	.944	.945	.946	.947	.948	.949	.950	.951	.952
95.	.953	.954	.955	.956	.957	.958	.959	.960	.961	.962
96.	.963	.964	.965	.966	.967	.968	.969	.970	.971	.972
97.	.973	.974	.975	.976	.977	.978	.979	.980	.981	.982
98.	.983	.984	.985	.986	.987	.988	.989	.990	.991	.992
99.	.993	.994	.995	.996	.997	.998	.999	1.000	1.001	1.002

TABLE A. 4

REFLECTANCE VALUES WITH SURFACE CORRECTION CORRESPONDING TO MEASURED REFLECTANCE VALUES

THESE FIGURES INCORPORATE SAUNDERSON CORRECTION USING K1 = 0.04, K2 = 0.60

(PERCENT MEASURED REFLECTANCE)

KM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	-.111	-.108	-.105	-.102	-.099	-.096	-.094	-.091	-.088	-.085
1.	-.082	-.079	-.076	-.073	-.071	-.068	-.065	-.062	-.059	-.057
2.	-.054	-.051	-.048	-.045	-.043	-.040	-.037	-.035	-.032	-.029
3.	-.026	-.024	-.021	-.018	-.016	-.013	-.010	-.008	-.005	-.003
4.	.000	.003	.005	.008	.010	.013	.015	.018	.021	.023
5.	.026	.028	.031	.033	.036	.038	.041	.043	.046	.048
6.	.051	.053	.055	.058	.060	.063	.065	.067	.070	.072
7.	.075	.077	.079	.082	.084	.086	.089	.091	.093	.096
8.	.098	.100	.103	.105	.107	.109	.112	.114	.116	.119
9.	.121	.123	.125	.127	.130	.132	.134	.136	.139	.141
10.	.143	.145	.147	.149	.152	.154	.156	.158	.160	.162
11.	.164	.166	.169	.171	.173	.175	.177	.179	.181	.183
12.	.185	.187	.189	.191	.193	.195	.197	.199	.201	.203
13.	.205	.207	.209	.211	.213	.215	.217	.219	.221	.223
14.	.225	.227	.229	.231	.233	.235	.237	.239	.241	.243
15.	.244	.246	.248	.250	.252	.254	.256	.258	.259	.261
16.	.263	.265	.267	.269	.271	.272	.274	.276	.278	.280
17.	.281	.283	.285	.287	.289	.290	.292	.294	.296	.297
18.	.299	.301	.303	.304	.306	.308	.310	.311	.313	.315
19.	.316	.318	.320	.322	.323	.325	.327	.328	.330	.332
20.	.333	.335	.337	.338	.340	.342	.343	.345	.347	.348
21.	.350	.351	.353	.355	.356	.358	.359	.361	.363	.364
22.	.365	.367	.369	.371	.372	.374	.375	.377	.378	.380
23.	.382	.383	.385	.386	.388	.389	.391	.392	.394	.395
24.	.397	.398	.400	.401	.403	.404	.406	.407	.409	.410
25.	.412	.413	.415	.416	.418	.419	.421	.422	.423	.425
26.	.426	.428	.429	.431	.432	.434	.435	.436	.438	.439
27.	.441	.442	.443	.445	.446	.448	.449	.450	.452	.453
28.	.455	.456	.457	.459	.460	.461	.463	.464	.465	.467
29.	.468	.470	.471	.472	.474	.475	.476	.478	.479	.480
30.	.481	.483	.484	.485	.487	.488	.489	.491	.492	.493
31.	.495	.496	.497	.498	.500	.501	.502	.503	.505	.506
32.	.507	.509	.510	.511	.512	.514	.515	.516	.517	.518
33.	.520	.521	.522	.523	.525	.526	.527	.528	.529	.531
34.	.532	.533	.534	.536	.537	.538	.539	.540	.541	.543
35.	.544	.545	.546	.547	.549	.550	.551	.552	.553	.554
36.	.556	.557	.558	.559	.560	.561	.562	.564	.565	.566
37.	.567	.568	.569	.570	.572	.573	.574	.575	.576	.577
38.	.578	.579	.580	.582	.583	.584	.585	.586	.587	.588
39.	.589	.590	.591	.592	.594	.595	.596	.597	.598	.599
40.	.600	.601	.602	.603	.604	.605	.606	.607	.608	.610
41.	.611	.612	.613	.614	.615	.616	.617	.618	.619	.620
42.	.621	.622	.623	.624	.625	.626	.627	.628	.629	.630
43.	.631	.632	.633	.634	.635	.636	.637	.638	.639	.640
44.	.641	.642	.643	.644	.645	.646	.647	.648	.649	.650
45.	.651	.652	.653	.654	.655	.656	.657	.658	.658	.659
46.	.660	.661	.662	.663	.664	.665	.666	.667	.668	.669
47.	.670	.671	.672	.673	.673	.674	.675	.675	.677	.678
48.	.679	.680	.681	.682	.683	.684	.684	.685	.686	.687
49.	.688	.689	.690	.691	.692	.693	.693	.694	.695	.696

(PERCENT MEASURED REFLECTANCE)

KM	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
50.	.697	.698	.699	.700	.700	.701	.702	.703	.704	.705
51.	.706	.707	.707	.708	.709	.710	.711	.712	.713	.713
52.	.714	.715	.716	.717	.718	.719	.719	.720	.721	.722
53.	.723	.724	.724	.725	.726	.727	.728	.729	.729	.730
54.	.731	.732	.733	.733	.734	.735	.736	.737	.738	.738
55.	.739	.740	.741	.742	.742	.743	.744	.745	.746	.746
56.	.747	.748	.749	.749	.750	.751	.752	.753	.753	.754
57.	.755	.756	.757	.757	.758	.759	.760	.760	.761	.762
58.	.763	.763	.764	.765	.766	.767	.767	.768	.769	.770
59.	.770	.771	.772	.773	.773	.774	.775	.776	.776	.777
60.	.778	.779	.779	.780	.781	.781	.782	.783	.784	.784
61.	.785	.786	.787	.787	.788	.789	.789	.790	.791	.792
62.	.792	.793	.794	.794	.795	.796	.797	.797	.798	.799
63.	.799	.800	.801	.802	.802	.803	.804	.804	.805	.806
64.	.806	.807	.808	.809	.809	.810	.811	.811	.812	.813
65.	.813	.814	.815	.815	.816	.817	.817	.818	.819	.819
66.	.820	.821	.821	.822	.823	.823	.824	.825	.825	.826
67.	.827	.827	.828	.829	.829	.830	.831	.831	.832	.833
68.	.833	.834	.835	.835	.836	.837	.837	.838	.839	.839
69.	.840	.840	.841	.842	.842	.843	.844	.844	.845	.846
70.	.846	.847	.847	.848	.849	.849	.850	.851	.851	.852
71.	.852	.853	.854	.854	.855	.856	.856	.857	.857	.858
72.	.859	.859	.860	.860	.861	.862	.862	.863	.863	.864
73.	.865	.865	.866	.866	.867	.868	.868	.869	.869	.870
74.	.871	.871	.872	.872	.873	.874	.874	.875	.875	.876
75.	.877	.877	.878	.878	.879	.879	.880	.881	.881	.882
76.	.882	.883	.884	.884	.885	.885	.886	.886	.887	.888
77.	.888	.889	.889	.890	.890	.891	.891	.892	.893	.893
78.	.894	.894	.895	.895	.896	.897	.897	.898	.898	.899
79.	.899	.900	.900	.901	.901	.902	.903	.903	.904	.904
80.	.905	.905	.906	.906	.907	.907	.908	.909	.909	.910
81.	.910	.911	.911	.912	.912	.913	.913	.914	.914	.915
82.	.915	.916	.917	.917	.918	.918	.919	.919	.920	.920
83.	.921	.921	.922	.922	.923	.923	.924	.924	.925	.925
84.	.926	.926	.927	.927	.928	.928	.929	.930	.930	.931
85.	.931	.932	.932	.933	.933	.934	.934	.935	.935	.936
86.	.936	.937	.937	.938	.938	.939	.939	.940	.940	.941
87.	.941	.942	.942	.943	.943	.944	.944	.944	.945	.945
88.	.946	.946	.947	.947	.948	.948	.949	.949	.950	.950
89.	.951	.951	.952	.952	.953	.953	.954	.954	.955	.955
90.	.956	.956	.957	.957	.957	.958	.958	.959	.959	.960
91.	.960	.961	.961	.962	.962	.963	.963	.964	.964	.964
92.	.965	.965	.966	.966	.967	.967	.968	.968	.969	.969
93.	.969	.970	.970	.971	.971	.972	.972	.973	.973	.974
94.	.974	.974	.975	.975	.976	.976	.977	.977	.978	.978
95.	.978	.979	.979	.980	.980	.981	.981	.982	.982	.982
96.	.983	.983	.984	.984	.985	.985	.986	.986	.986	.987
97.	.987	.988	.988	.989	.989	.989	.990	.990	.991	.991
98.	.992	.992	.992	.993	.993	.994	.994	.995	.995	.995
99.	.996	.996	.997	.997	.997	.998	.998	.999	.999	1.000

TABLE A. 5

REFLECTANCE VALUES WITH SURFACE CORRECTION CORRESPONDING TO MEASURED REFLECTANCE VALUES

THESE FIGURES INCORPORATE DUNCAN CORRECTION USING K1 = 0.04, K2 = 0.555

		(PERCENT MEASURED REFLECTANCE)									
RM	*	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.	*	.000	.002	.005	.007	.009	.012	.014	.016	.019	.021
1.	*	.023	.025	.028	.030	.032	.034	.037	.039	.041	.043
2.	*	.046	.048	.050	.052	.054	.057	.059	.061	.063	.065
3.	*	.068	.070	.072	.074	.076	.078	.081	.083	.085	.087
4.	*	.089	.091	.093	.095	.097	.100	.102	.104	.106	.108
5.	*	.110	.112	.114	.116	.118	.120	.122	.124	.126	.128
6.	*	.130	.132	.134	.136	.138	.140	.142	.144	.146	.148
7.	*	.150	.152	.154	.156	.158	.160	.162	.164	.166	.168
8.	*	.170	.172	.173	.175	.177	.179	.181	.183	.185	.187
9.	*	.189	.190	.192	.194	.196	.198	.200	.202	.203	.205
10.	*	.207	.209	.211	.213	.214	.216	.218	.220	.222	.224
11.	*	.225	.227	.229	.231	.232	.234	.236	.238	.240	.241
12.	*	.243	.245	.247	.248	.250	.252	.253	.255	.257	.259
13.	*	.260	.262	.264	.265	.267	.269	.271	.272	.274	.276
14.	*	.277	.279	.281	.282	.284	.286	.287	.289	.291	.292
15.	*	.294	.295	.297	.299	.300	.302	.304	.305	.307	.308
16.	*	.310	.312	.313	.315	.316	.318	.320	.321	.323	.324
17.	*	.326	.328	.329	.331	.332	.334	.335	.337	.338	.340
18.	*	.341	.343	.345	.346	.348	.349	.351	.352	.354	.355
19.	*	.357	.358	.360	.361	.363	.364	.366	.367	.369	.370
20.	*	.372	.373	.375	.376	.377	.379	.380	.382	.383	.385
21.	*	.386	.388	.389	.391	.392	.393	.395	.396	.398	.399
22.	*	.401	.402	.403	.405	.406	.408	.409	.410	.412	.413
23.	*	.415	.416	.417	.419	.420	.421	.423	.424	.426	.427
24.	*	.428	.430	.431	.432	.434	.435	.436	.438	.439	.440
25.	*	.442	.443	.444	.446	.447	.448	.450	.451	.452	.454
26.	*	.455	.456	.458	.459	.460	.461	.463	.464	.465	.467
27.	*	.468	.469	.470	.472	.473	.474	.476	.477	.478	.479
28.	*	.481	.482	.483	.484	.486	.487	.488	.489	.491	.492
29.	*	.493	.494	.496	.497	.498	.499	.500	.502	.503	.504
30.	*	.505	.507	.508	.509	.510	.511	.513	.514	.515	.516
31.	*	.517	.519	.520	.521	.522	.523	.524	.526	.527	.528
32.	*	.529	.530	.531	.533	.534	.535	.536	.537	.538	.540
33.	*	.541	.542	.543	.544	.545	.546	.548	.549	.550	.551
34.	*	.552	.553	.554	.555	.557	.558	.559	.560	.561	.562
35.	*	.563	.564	.565	.567	.568	.569	.570	.571	.572	.573
36.	*	.574	.575	.576	.577	.578	.580	.581	.582	.583	.584
37.	*	.585	.586	.587	.588	.589	.590	.591	.592	.593	.594
38.	*	.596	.597	.598	.599	.600	.601	.602	.603	.604	.605
39.	*	.606	.607	.608	.609	.610	.611	.612	.613	.614	.615
40.	*	.616	.617	.618	.619	.620	.621	.622	.623	.624	.625
41.	*	.626	.627	.628	.629	.630	.631	.632	.633	.634	.635
42.	*	.636	.637	.638	.639	.640	.641	.642	.643	.644	.645
43.	*	.646	.647	.648	.649	.650	.651	.652	.652	.653	.654
44.	*	.655	.656	.657	.658	.659	.660	.661	.662	.663	.664
45.	*	.665	.665	.667	.668	.668	.669	.670	.671	.672	.673
46.	*	.674	.675	.676	.677	.678	.679	.679	.680	.681	.682
47.	*	.683	.684	.685	.686	.687	.688	.688	.689	.690	.691
48.	*	.692	.693	.694	.695	.696	.696	.697	.698	.699	.700
49.	*	.701	.702	.703	.703	.704	.705	.706	.707	.708	.709
50.	*	.710	.710	.711	.712	.713	.714	.715	.715	.716	.717
51.	*	.718	.719	.720	.721	.721	.722	.723	.724	.725	.726
52.	*	.726	.727	.728	.729	.730	.731	.731	.732	.733	.734
53.	*	.735	.736	.736	.737	.738	.739	.740	.740	.741	.742
54.	*	.743	.744	.744	.745	.746	.747	.748	.749	.749	.750
55.	*	.751	.752	.752	.753	.754	.755	.756	.756	.757	.758
56.	*	.759	.760	.760	.761	.762	.763	.763	.764	.765	.766
57.	*	.767	.767	.768	.769	.770	.770	.771	.772	.773	.774
58.	*	.774	.775	.776	.777	.777	.778	.779	.780	.780	.781
59.	*	.782	.783	.783	.784	.785	.786	.786	.787	.788	.789
60.	*	.789	.790	.791	.791	.792	.793	.794	.794	.795	.796
61.	*	.797	.797	.798	.799	.800	.800	.801	.802	.802	.803
62.	*	.804	.805	.805	.806	.807	.807	.808	.809	.810	.810
63.	*	.811	.812	.812	.813	.814	.814	.815	.816	.817	.817
64.	*	.818	.819	.819	.820	.821	.821	.822	.823	.824	.824
65.	*	.825	.826	.826	.827	.828	.828	.829	.830	.830	.831
66.	*	.832	.832	.833	.834	.834	.835	.836	.836	.837	.838
67.	*	.838	.839	.840	.840	.841	.842	.842	.843	.844	.844
68.	*	.845	.846	.846	.847	.848	.848	.849	.850	.850	.851
69.	*	.852	.852	.853	.854	.854	.855	.856	.856	.857	.858
70.	*	.858	.859	.859	.860	.861	.861	.862	.863	.863	.864
71.	*	.865	.865	.866	.866	.867	.868	.868	.869	.870	.870
72.	*	.871	.871	.872	.873	.873	.874	.875	.875	.876	.876
73.	*	.877	.878	.878	.879	.879	.880	.881	.881	.882	.883
74.	*	.883	.884	.884	.885	.886	.886	.887	.887	.888	.889
75.	*	.889	.890	.890	.891	.892	.892	.893	.893	.894	.895
76.	*	.895	.896	.896	.897	.898	.898	.899	.899	.900	.900
77.	*	.901	.902	.902	.903	.903	.904	.905	.905	.906	.906
78.	*	.907	.907	.908	.909	.909	.910	.910	.911	.911	.912
79.	*	.913	.913	.914	.914	.915	.915	.916	.917	.917	.918
80.	*	.918	.919	.919	.920	.921	.921	.922	.922	.923	.923
81.	*	.924	.924	.925	.926	.926	.927	.927	.928	.928	.929
82.	*	.929	.930	.930	.931	.932	.932	.933	.933	.934	.934
83.	*	.935	.935	.936	.936	.937	.938	.938	.939	.939	.940
84.	*	.940	.941	.941	.942	.942	.943	.943	.944	.944	.945
85.	*	.946	.946	.947	.947	.948	.948	.949	.949	.950	.950
86.	*	.951	.951	.952	.952	.953	.953	.954	.954	.955	.955
87.	*	.956	.957	.957	.958	.958	.959	.959	.960	.960	.961
88.	*	.961	.962	.962	.963	.963	.964	.964	.965	.965	.966
89.	*	.966	.967	.967	.968	.968	.969	.969	.970	.970	.971
90.	*	.971	.972	.972	.973	.973	.974	.974	.975	.975	.976
91.	*	.976	.977	.977	.978	.978	.979	.979	.980	.980	.981
92.	*	.981	.982	.982	.982	.983	.983	.984	.984	.985	.985
93.	*	.986	.986	.987	.987	.988	.988	.989	.989	.990	.990
94.	*	.991	.991	.992	.992	.993	.993	.993	.994	.994	.995
95.	*	.995	.996	.996	.997	.997	.998	.998	.999	.999	1.000
96.	*	1.000	1.000	1.001	1.001	1.002	1.002	1.003	1.003	1.004	1.004
97.	*	1.005	1.005	1.006	1.006	1.006	1.007	1.007	1.008	1.008	1.009
98.	*	1.009	1.010	1.010	1.011	1.011	1.011	1.012	1.012	1.013	1.013
99.	*	1.014	1.014	1.015	1.015	1.015	1.016	1.016	1.017	1.017	1.018

Table A.6

TABLE OF KUBELKA-MUNK FUNCTION, F(R)=K/S

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		R499.00050000	249.00100000	165.66616667	124.00200000	99.00250000	82.33633333	70.43207143	61.50400000	54.56005556
1	49.00500000	44.46004545	40.67266667	37.46803846	34.72128571	32.34083333	30.25800000	28.42020471	26.78677778	25.32526947
2	24.01000000	22.82002381	21.73827273	20.75063043	19.84533333	19.01250000	18.24376923	17.53201852	16.87114286	16.25587931
3	15.68166667	15.14453226	14.64100000	14.16801515	13.72288235	13.30321429	12.90688889	12.53201351	12.17689474	11.84001282
4	11.52000000	11.21562195	10.92576190	10.64940698	10.38563636	10.13361111	9.89256522	9.66179787	9.44066667	9.22858163
5	9.02500000	8.82942157	8.64138462	8.46046226	8.28625926	8.11840909	7.95657143	7.80042982	7.64968966	7.50407627
6	7.36333333	7.22722131	7.09551613	6.96800794	6.84450000	6.72480769	6.60875758	6.49618657	6.38694118	6.28087681
7	6.17785714	6.07775352	5.98044444	5.88581507	5.79375676	5.70416667	5.61694737	5.53200649	5.44925641	5.36861392
8	5.29000000	5.21333951	5.13856098	5.06559639	4.99438095	4.92485294	4.85695349	4.79062644	4.72581818	4.66247753
9	4.60055556	4.54000549	4.48078261	4.42284409	4.36614894	4.31065789	4.25633333	4.20313918	4.15104082	4.10000505
10	4.05000000	4.00099505	3.95296078	3.90586893	3.85969231	3.81440476	3.76998113	3.72639720	3.68362963	3.64165596
11	3.60045455	3.56000450	3.52028571	3.48127876	3.44296491	3.40532609	3.36834483	3.33200427	3.29628814	3.26118067
12	3.22666667	3.19273140	3.15936066	3.12654065	3.09425806	3.06250000	3.03125397	3.00050787	2.97025000	2.94046899
13	2.91115385	2.88229389	2.85387879	2.82589850	2.79834328	2.77120370	2.74447059	2.71813504	2.69218841	2.66662230
14	2.64142857	2.61659929	2.59212676	2.56800350	2.54422222	2.52077586	2.49765753	2.47486054	2.45237838	2.43020470
15	2.40833333	2.38675828	2.36547368	2.34447386	2.32375325	2.30330645	2.28312821	2.26321338	2.24355696	2.22415409
16	2.20500000	2.18609006	2.16741975	2.14898466	2.13078049	2.11280303	2.09504819	2.07751198	2.06019048	2.04307988
17	2.02617647	2.00947661	1.99297674	1.97667341	1.96056322	1.94464286	1.92890909	1.91335876	1.89798876	1.88279609
18	1.86777778	1.85293094	1.83825275	1.82374044	1.80939130	1.79520270	1.78117204	1.76729679	1.75357447	1.74000265
19	1.72657895	1.71330105	1.70016667	1.68717358	1.67431959	1.66160256	1.64902041	1.63657107	1.62425253	1.61206281
20	1.60000000	1.58806219	1.57624752	1.56455419	1.55298039	1.54152439	1.53018447	1.51895894	1.50784615	1.49684450
21	1.48595238	1.47516825	1.46449057	1.45391784	1.44344860	1.43308140	1.42281481	1.41264747	1.40257798	1.39260502
22	1.38272727	1.37294344	1.36325225	1.35365247	1.34414286	1.33472222	1.32538938	1.31614317	1.30698246	1.29790611
23	1.28891304	1.28000216	1.27117241	1.26242275	1.25375214	1.24515957	1.23664407	1.22820464	1.21984034	1.21155021
24	1.20333333	1.19518880	1.18711570	1.17911317	1.17118033	1.16331633	1.15552033	1.14779150	1.14012903	1.13253213
25	1.12500000	1.11753187	1.11012698	1.10278458	1.09550394	1.08828431	1.08112500	1.07402529	1.06698450	1.06000193
26	1.05307692	1.04620881	1.03939695	1.03264068	1.02593939	1.01929245	1.01269925	1.00615918	.99967164	.99323606
27	.98685185	.98051845	.97423529	.96800183	.96181752	.95568182	.94959420	.94355415	.93756115	.93161470
28	.92571429	.91985943	.91404965	.90828445	.90256338	.89688596	.89125175	.88566028	.88011111	.87460381
29	.86913793	.86371306	.85832877	.85298464	.84768027	.84241525	.83718919	.83200166	.82685235	.82174080

Table A.6 (cont'd)

R	(PERCENT OF REFLECTANCE) 0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
30	.81666667	.81162957	.80662914	.80166502	.79673684	.79184426	.78698693	.78216450	.77737662	.77262294
31	.76790323	.76321704	.75856410	.75394409	.74935669	.74480159	.74027848	.73578707	.73132704	.72689812
32	.72250000	.71813240	.71379503	.70948762	.70520988	.70096154	.69674233	.69255199	.68839024	.68425684
33	.68015152	.67607402	.67202410	.66800150	.66400599	.66003731	.65609524	.65217953	.64828994	.64442625
34	.64058824	.63677566	.63298830	.62922595	.62548837	.62177536	.61804671	.61442219	.61078161	.60716476
35	.60357143	.60000142	.59645455	.59293059	.58942938	.58595070	.58249438	.57906022	.57564804	.57225766
36	.56888889	.56554155	.56221547	.55891047	.55562637	.55236301	.54912022	.54589782	.54269565	.53951355
37	.53635135	.53320889	.53008602	.52698257	.52389840	.52083333	.51778723	.51475995	.51175132	.50876121
38	.50578947	.50283596	.49990052	.49698303	.49408333	.49120130	.48833679	.48548966	.48265979	.47984704
39	.47705128	.47427238	.47151020	.46876463	.46603553	.46332278	.46062626	.45794584	.45528141	.45263283
40	.45000000	.44738279	.44478109	.44219479	.43962376	.43706790	.43452709	.43200123	.42949020	.42699389
41	.42451220	.42204501	.41959223	.41715375	.41472947	.41231928	.40992308	.40754077	.40517225	.40281742
42	.40047619	.39814846	.39583412	.39353310	.39124528	.38897059	.38670892	.38446019	.38222430	.38000117
43	.37779070	.37559281	.37340741	.37123441	.36907373	.36692529	.36478899	.36265476	.36055251	.35845216
44	.35636364	.35428685	.35222172	.35016817	.34812613	.34609551	.34407623	.34206823	.34007143	.33808575
45	.33611111	.33414745	.33219469	.33025276	.32832159	.32640110	.32449123	.32259190	.32070306	.31882452
46	.31695652	.31509870	.31325108	.31141361	.30958621	.30776882	.30596137	.30416381	.30237607	.30059808
47	.29882979	.29707113	.29532203	.29358245	.29185232	.29013158	.28842017	.28671803	.28502510	.28334134
48	.28166667	.28000104	.27834440	.27669669	.27505785	.27342784	.27180658	.27019405	.26859016	.26699489
49	.26540816	.26382994	.26226016	.26069878	.25914575	.25760101	.25606452	.25453622	.25301606	.25150401
50	.25000000	.24850399	.24701594	.24553579	.24406349	.24259901	.24114229	.23969329	.23825197	.23681827
51	.23539216	.23397358	.23256250	.23115887	.22976265	.22837379	.22699225	.22561799	.22425097	.22289114
52	.22153846	.22019290	.21885441	.21752294	.21619847	.21488095	.21357034	.21226660	.21096970	.20967958
53	.20839623	.20711959	.20584962	.20458630	.20332959	.20207944	.20083582	.19959870	.19836803	.19714378
54	.19592593	.19471442	.19350923	.19231031	.19111765	.18993119	.18875092	.18757678	.18640876	.18524681
55	.18409091	.18294102	.18179710	.18065913	.17952708	.17840090	.17728058	.17616607	.17505735	.17395438
56	.17285714	.17176560	.17067972	.16959947	.16852482	.16745575	.16639223	.16533422	.16428169	.16323462
57	.16219298	.16115674	.16012587	.15910035	.15808014	.15706522	.15605556	.15505113	.15405190	.15305786
58	.15206897	.15108520	.15010653	.14913293	.14816438	.14720085	.14624232	.14528876	.14434014	.14339643
59	.14245763	.14152369	.14059459	.13967032	.13875084	.13783613	.13692617	.13602094	.13512040	.13422454
60	.13333333	.13244676	.13156478	.13068740	.12981457	.12894628	.12808251	.12722323	.12636842	.12551806
61	.12467213	.12383061	.12299346	.12216069	.12133225	.12050813	.11968831	.11887277	.11806149	.11725444
62	.11645161	.11565298	.11485852	.11406822	.11328205	.11250000	.11172204	.11094817	.11017834	.10941256
63	.10865079	.10789303	.10713924	.10638942	.10564353	.10490157	.10416352	.10342936	.10269906	.10197261
64	.10125000	.10053120	.09981620	.09910498	.09839752	.09769380	.09699381	.09629753	.09560494	.09491602

Table A.6 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
65	.09423077	.09354916	.09287117	.09219678	.09152599	.09085878	.09019512	.08953501	.08887842	.08822534
66	.08757576	.08692965	.08628701	.08564781	.08501205	.08437970	.08375075	.08312519	.08250299	.08188416
67	.08126866	.08065648	.08004762	.07944205	.07883976	.07824074	.07764497	.07705244	.07646313	.07587703
68	.07529412	.07471439	.07413783	.07356442	.07299415	.07242701	.07186297	.07130204	.07074419	.07018940
69	.06963768	.06908900	.06854335	.06800072	.06746110	.06692446	.06639080	.06586011	.06533238	.06480758
70	.06428571	.06376676	.06325071	.06273755	.06222727	.06171986	.06121530	.06071358	.06021469	.05971862
71	.05922535	.05873488	.05824719	.05776227	.05728011	.05680070	.05632402	.05585007	.05537883	.05491029
72	.05444444	.05398128	.05352078	.05306293	.05260773	.05215517	.05170523	.05125791	.05081319	.05037106
73	.04993151	.04949453	.04906011	.04862824	.04819891	.04777211	.04734783	.04692605	.04650676	.04608999
74	.04567568	.04526383	.04485445	.04444751	.04404301	.04364094	.04324129	.04284404	.04244920	.04205674
75	.04166667	.04127896	.04089362	.04051062	.04012997	.03975166	.03937566	.03900198	.03863061	.03826153
76	.03789474	.03753022	.03716798	.03680799	.03645026	.03609477	.03574151	.03539048	.03504167	.03469506
77	.03435065	.03400843	.03366839	.03333053	.03299483	.03266129	.03232990	.03200064	.03167352	.03134852
78	.03102564	.03070487	.03038619	.03006960	.02975510	.02944268	.02913232	.02882402	.02851777	.02821356
79	.02791139	.02761125	.02731313	.02701702	.02672292	.02643082	.02614070	.02585257	.02556642	.02528223
80	.02500000	.02471973	.02444140	.02416501	.02389055	.02361801	.02334739	.02307869	.02281188	.02254697
81	.02228395	.02202281	.02176355	.02150615	.02125061	.02099693	.02074510	.02049510	.02024694	.02000061
82	.01975610	.01951340	.01927251	.01903341	.01879612	.01856061	.01832688	.01809492	.01786473	.01763631
83	.01740964	.01718472	.01696154	.01674010	.01652038	.01630240	.01608612	.01587157	.01565871	.01544756
84	.01523810	.01503032	.01482423	.01461981	.01441706	.01421598	.01401655	.01381877	.01362264	.01342815
85	.01323529	.01304407	.01285446	.01266647	.01248009	.01229532	.01211215	.01193057	.01175058	.01157218
86	.01139535	.01122009	.01104640	.01087428	.01070370	.01053468	.01036721	.01020127	.01003687	.00987399
87	.00971264	.00955281	.00939450	.00923769	.00908238	.00892857	.00877626	.00862543	.00847608	.00832821
88	.00818182	.00803689	.00789342	.00775142	.00761086	.00747175	.00733409	.00719786	.00706306	.00692970
89	.00679775	.00666723	.00653812	.00641041	.00628412	.00615922	.00603571	.00591360	.00579287	.00567353
90	.00555556	.00543896	.00532373	.00520986	.00509735	.00498619	.00487638	.00476792	.00466079	.00455501
91	.00445055	.00434742	.00424561	.00414513	.00404595	.00394809	.00385153	.00375627	.00366231	.00356964
92	.00347826	.00338817	.00329935	.00321181	.00312554	.00304054	.00295680	.00287433	.00279310	.00271313
93	.00263441	.00255693	.00248069	.00240568	.00233191	.00225936	.00218803	.00211793	.00204904	.00198136
94	.00191489	.00184963	.00178556	.00172269	.00166102	.00160053	.00154123	.00148310	.00142616	.00137039
95	.00131579	.00126236	.00121008	.00115897	.00110901	.00106021	.00101255	.00096604	.00092067	.00087643
96	.00083333	.00079136	.00075052	.00071080	.00067220	.00063472	.00059834	.00056308	.00052893	.00049587
97	.00046392	.00043306	.00040329	.00037461	.00034702	.00032051	.00029508	.00027073	.00024744	.00022523
98	.00020408	.00018400	.00016497	.00014700	.00013008	.00011421	.00009939	.00008561	.00007287	.00006117
99	.00005051	.00004087	.00003226	.00002467	.00001811	.00001256	.00000803	.00000451	.00000200	.00000050

Table A.7

TABLE OF THE K.M FUNCTION F(R) CORRESPONDING TO MEASURED VALUES R
THESE FIGURES INCORPORATE SAUNDERSON CORRECTION USING K1 = 0.04, K2 = 0.400

(PERCENT REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0	R	R	R	R	R	R	R	R	R	R
1	R	R	R	R	R	R	R	R	R	R
2	R	R	R	R	R	R	R	R	R	R
3	R	R	R	R	R	R	R	R	R	R
4	R	191.30130005	95.30259605	63.30388802	47.30517598	37.70645995	31.30773994	26.73750740	23.31025807	20.64480957
5	18.51282051	16.76862636	15.31533742	14.08582087	13.03212465	12.11908397	11.32032520	10.61500031	9.99946302	9.42920946
6	8.92525253	8.46933218	8.05496658	7.67673509	7.33012048	7.01132832	6.71714792	6.44404424	6.19207300	5.95681322
7	5.73731343	5.53204814	5.33968254	5.15904363	4.98909641	4.82892416	4.67771203	4.53473326	4.39933758	4.27094143
8	4.14901961	4.03309815	3.92274822	3.81758050	3.71724260	3.62141119	3.52970254	3.44211754	3.35813953	3.27763198
9	3.20038647	3.12621095	3.05492812	2.98637407	2.92039705	2.85685633	2.79562124	2.73657026	2.67959029	2.62457587
10	2.57142857	2.52005644	2.47037343	2.42229899	2.37575758	2.33067831	2.28699459	2.24464382	2.20356700	2.16370804
11	2.12501677	2.08744148	2.05093633	2.01545723	1.98096248	1.94741259	1.91477017	1.88299975	1.85206771	1.82194210
12	1.79259259	1.76399034	1.73610792	1.70891922	1.68239937	1.65652408	1.63127250	1.60662145	1.58255078	1.55904091
13	1.53607306	1.51362928	1.49169241	1.47024601	1.44927435	1.42876238	1.40869565	1.38906032	1.36984311	1.35103129
14	1.33261261	1.31457534	1.29690820	1.27960033	1.26264130	1.24602109	1.22973005	1.21375888	1.19809863	1.18274070
15	1.16767677	1.15289884	1.13839919	1.12417038	1.11020522	1.09649678	1.08303837	1.06982353	1.05684600	1.04409975
16	1.03157895	1.01927795	1.00719131	.99531375	.98364015	.97216558	.96088524	.94979451	.93888809	.92816403
17	.91761572	.90723986	.89703250	.88699979	.87710800	.86738351	.85781282	.84839250	.83911926	.82998987
18	.82100122	.81215027	.80343407	.79484976	.78639456	.77806574	.76986069	.76177683	.75381166	.74596281
19	.73822785	.73060451	.72309055	.71569380	.70838213	.70118347	.69408581	.68709719	.68019570	.67337946
20	.66666667	.66004554	.65351436	.64707144	.64071512	.63444382	.62825595	.62215000	.61612447	.61017790
21	.60430888	.59851602	.59279795	.58715330	.58158095	.57607946	.57064765	.56528432	.55998828	.55475838
22	.54959350	.54449252	.53945438	.53447602	.52956240	.52470652	.51990940	.51517005	.51048755	.50586095
23	.50128937	.49677190	.49230769	.48789588	.48353565	.47922616	.47496664	.47075629	.46659434	.46248006
24	.45841270	.45439154	.45041587	.44648501	.44259827	.43875499	.43495452	.43119621	.42747944	.42380359
25	.42016807	.41657227	.41301562	.40949755	.40601750	.40257491	.39916926	.39580002	.39246605	.38916807
26	.38590557	.38267085	.37948203	.37632065	.37319224	.37009634	.36703250	.36400029	.36099927	.35802902
27	.35508912	.35217916	.34929875	.34644747	.34362495	.34083080	.33806465	.33532613	.33261467	.32993052
28	.32727273	.32464115	.32203544	.31945528	.31690033	.31437027	.31186478	.30938355	.30692628	.30449266
29	.30208240	.29969520	.29733077	.29498884	.29266912	.29037134	.28809524	.28584054	.28360698	.28139431

Table A.7 (cont'd)

(PERCENT REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
30	.27920228	.27703063	.27487912	.27274751	.27063556	.26854303	.26646971	.26441535	.26237973	.26036264
31	.25836386	.25638317	.25442036	.25247524	.25054758	.24863719	.24674388	.24486744	.24300769	.24116443
32	.23933747	.23752665	.23573176	.23395263	.23218909	.23044097	.22870808	.22699027	.22528736	.22359919
33	.22192560	.22026042	.21862151	.21699071	.21537386	.21377081	.21218141	.21060552	.20904299	.20749368
34	.20595745	.20443415	.20292366	.20142583	.19994053	.19846763	.19700701	.19555852	.19412205	.19269747
35	.19128466	.18988350	.18849386	.18711563	.18574869	.18439292	.18304821	.18171446	.18039154	.17907935
36	.17777778	.17648672	.17520607	.17393573	.17267558	.17142553	.17018549	.16895534	.16773500	.16652436
37	.16532334	.16413183	.16294974	.16177699	.16061348	.15945911	.15831382	.15717750	.15605006	.15493144
38	.15382153	.15272026	.15162754	.15054330	.14946745	.14839991	.14734061	.14628947	.14524641	.14421135
39	.14318422	.14216495	.14115347	.14014970	.13915357	.13816500	.13718394	.13621031	.13524404	.13428507
40	.13333333	.13238876	.13145128	.13052083	.12959736	.12868080	.12777108	.12686814	.12597193	.12508239
41	.12419945	.12332305	.12245314	.12158967	.12073256	.11988177	.11903725	.11819893	.11736676	.11654070
42	.11572067	.11490665	.11409856	.11329636	.11250000	.11170943	.11092460	.11014545	.10937195	.10860404
43	.10784167	.10708480	.10633339	.10558737	.10484672	.10411138	.10338130	.10265646	.10193679	.10122226
44	.10051282	.09980844	.09910906	.09841465	.09772517	.09704058	.09636083	.09568589	.09501572	.09435027
45	.09368951	.09303340	.09238190	.09173498	.09109259	.09045471	.08982129	.08919230	.08856770	.08794746
46	.08733154	.08671990	.08611252	.08550936	.08491039	.08431557	.08372486	.08313825	.08255568	.08197714
47	.08140259	.08083200	.08026534	.07970257	.07914368	.07858861	.07803736	.07748988	.07694615	.07640613
48	.07586981	.07533715	.07480812	.07428269	.07376085	.07324255	.07272777	.07221649	.07170865	.07120432
49	.07070336	.07020580	.06971161	.06922075	.06873321	.06824895	.06776796	.06729021	.06681567	.06634433
50	.06587615	.06541112	.06494921	.06449039	.06403465	.06358196	.06313230	.06268564	.06224197	.06180126
51	.06136349	.06092864	.06049669	.06006761	.05964139	.05921800	.05879743	.05837964	.05796464	.05755238
52	.05714286	.05673605	.05633193	.05593049	.05553170	.05513555	.05474201	.05435108	.05396272	.05357692
53	.05319367	.05281294	.05243472	.05205898	.05168572	.05131491	.05094654	.05058059	.05021704	.04985587
54	.04949708	.04914063	.04878652	.04843473	.04808525	.04773805	.04739312	.04705045	.04671001	.04637181
55	.04603581	.04570200	.04537037	.04504091	.04471359	.04438841	.04406534	.04374438	.04342551	.04310872
56	.04279399	.04248130	.04217065	.04186202	.04155540	.04125077	.04094811	.04064743	.04034869	.04005190
57	.03975703	.03946407	.03917302	.03888385	.03859656	.03831113	.03802755	.03774581	.03746589	.03718779
58	.03691149	.03663698	.03636424	.03609327	.03582406	.03555659	.03529085	.03502683	.03476451	.03450390
59	.03424497	.03398772	.03373213	.03347819	.03322590	.03297523	.03272619	.03247676	.03223293	.03198069
60	.03174603	.03150494	.03126541	.03102742	.03079098	.03055607	.03032267	.03009078	.02986040	.02963150
61	.02940409	.02917815	.02895366	.02873064	.02850905	.02828890	.02807018	.02785287	.02763696	.02742246
62	.02720935	.02699761	.02678725	.02657825	.02637061	.02616431	.02595935	.02575572	.02555341	.02535242
63	.02515273	.02495433	.02475722	.02456140	.02436685	.02417356	.02394152	.02379074	.02360120	.02341269
64	.02322581	.02303994	.02285529	.02267184	.02248958	.02230852	.02212863	.02194992	.02177238	.02159600

Table A.7 (cont'd)

(PERCENT REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
65	.02142077	.02124668	.02107373	.02090192	.02073123	.02056166	.02039319	.02022584	.02005950	.01989441
66	.01973033	.01956732	.01940539	.01924452	.01908471	.01892596	.01876824	.01861157	.01845593	.01830132
67	.01814773	.01799516	.01784359	.01769303	.01754346	.01739488	.01724729	.01710068	.01695505	.01681037
68	.01666667	.01652391	.01638211	.01624126	.01610134	.01596236	.01582430	.01568717	.01555095	.01541565
69	.01528126	.01514776	.01501516	.01488346	.01475263	.01462269	.01449363	.01436543	.01423810	.01411163
70	.01398601	.01386125	.01373733	.01361425	.01349200	.01337059	.01325000	.01313024	.01301129	.01289315
71	.01277582	.01265929	.01254355	.01242862	.01231446	.01220110	.01208851	.01197670	.01186566	.01175538
72	.01164587	.01153711	.01142911	.01132186	.01121535	.01110958	.01100455	.01090025	.01079668	.01069383
73	.01059170	.01049028	.01038958	.01028959	.01019029	.01009170	.00999381	.00989660	.00980008	.00970425
74	.00960910	.00951462	.00942082	.00932768	.00923521	.00914340	.00905225	.00896175	.00887190	.00878270
75	.00869414	.00860622	.00851894	.00843229	.00834626	.00826087	.00817609	.00809193	.00800839	.00792546
76	.00784314	.00776142	.00768030	.00759978	.00751986	.00744053	.00736179	.00728363	.00720605	.00712905
77	.00705263	.00697678	.00690150	.00682678	.00675263	.00667904	.00660600	.00653352	.00646159	.00639020
78	.00631936	.00624907	.00617931	.00611008	.00604139	.00597324	.00590560	.00583849	.00577191	.00570584
79	.00564029	.00557525	.00551072	.00544670	.00538318	.00532017	.00525765	.00519563	.00513411	.00507308
80	.00501253	.00495247	.00489290	.00483380	.00477519	.00471705	.00465938	.00460218	.00454545	.00448919
81	.00443339	.00437805	.00432317	.00426875	.00421477	.00416125	.00410818	.00405555	.00400337	.00395163
82	.00390033	.00384946	.00379903	.00374903	.00369946	.00365031	.00360160	.00355330	.00350543	.00345797
83	.00341094	.00336431	.00331810	.00327229	.00322690	.00318191	.00313732	.00309313	.00304935	.00300596
84	.00296296	.00292036	.00287815	.00283633	.00279489	.00275384	.00271317	.00267288	.00263297	.00259344
85	.00255428	.00251549	.00247708	.00243903	.00240135	.00236404	.00232708	.00229049	.00225426	.00221839
86	.00218287	.00214771	.00211289	.00207843	.00204432	.00201055	.00197713	.00194405	.00191131	.00187891
87	.00184684	.00181512	.00178372	.00175266	.00172193	.00169153	.00166146	.00163171	.00160229	.00157318
88	.00154440	.00151594	.00148779	.00145996	.00143245	.00140524	.00137835	.00135177	.00132549	.00129952
89	.00127385	.00124849	.00122343	.00119866	.00117420	.00115003	.00112616	.00110258	.00107929	.00105630
90	.00103359	.00101117	.00098904	.00096719	.00094563	.00092434	.00090334	.00088262	.00086217	.00084200
91	.00082211	.00080248	.00078313	.00076405	.00074524	.00072670	.00070843	.00069041	.00067267	.00065518
92	.00063796	.00062099	.00060429	.00058784	.00057165	.00055571	.00054003	.00052459	.00050941	.00049448
93	.00047979	.00046535	.00045116	.00043721	.00042351	.00041005	.00039683	.00038384	.00037110	.00035859
94	.00034632	.00033428	.00032248	.00031091	.00029957	.00028846	.00027758	.00026693	.00025650	.00024630
95	.00023632	.00022657	.00021704	.00020773	.00019864	.00018976	.00018111	.00017267	.00016445	.00015644
96	.00014864	.00014106	.00013369	.00012653	.00011957	.00011283	.00010629	.00009996	.00009383	.00008791
97	.00008219	.00007667	.00007135	.00006623	.00006131	.00005659	.00005206	.00004773	.00004360	.00003966
98	.00003591	.00003235	.00002899	.00002581	.00002283	.00002003	.00001742	.00001499	.00001275	.00001070
99	.00000883	.00000714	.00000563	.00000430	.00000316	.00000219	.00000140	.00000079	.00000035	.00000009

Table A. 8

TABLE OF THE K.M FUNCTION F(R) CORRESPONDING TO MEASURED VALUES R
THESE FIGURES INCORPORATE DUNCAN CORRECTION USING K1 = 0.04, K2 = 0.555

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		R212.87866889	106.07983476	70.48099760	52.68215744	42.00331429	34.88446816	29.79990476	25.94676695	23.02124530
1	20.64905402	18.70837496	17.09132935	15.72323188	14.55073592	13.53472060	12.64584526	11.86167294	11.16475268	10.54130740
2	9.98031542	9.47285447	9.01162448	8.59059496	8.20474053	7.84983985	7.52232102	7.21914132	6.93764270	6.67572684
3	6.43129520	6.20270068	5.98845829	5.78726306	5.59796355	5.41953990	5.25106558	5.09179206	4.94093591	4.79786792
4	4.66200378	4.53281621	4.40982812	4.29260683	4.18075897	4.07392616	3.97174117	3.87402461	3.78038207	3.69060152
5	3.60445109	3.52171712	3.44220235	3.36572444	3.29211455	3.22121611	3.15284373	3.08698225	3.02338501	2.96197709
6	2.90264658	2.84529197	2.78981755	2.73613367	2.68415629	2.63380656	2.58501039	2.53769612	2.49180419	2.44726683
7	2.40402781	2.36203219	2.32122806	2.28156637	2.24300073	2.20548720	2.16898418	2.13345220	2.09885384	2.06515358
8	2.03231764	2.00031395	1.96911199	1.93868269	1.90899840	1.88003274	1.85176059	1.82415795	1.79720193	1.77087068
9	1.74514330	1.71999981	1.69542112	1.67138895	1.64788578	1.62489485	1.60240010	1.58038612	1.55883814	1.53774197
10	1.51708401	1.49685118	1.47703093	1.45761118	1.43858034	1.41992723	1.40164111	1.38371164	1.36612887	1.34888320
11	1.33196539	1.31536653	1.29907804	1.28309161	1.26739927	1.25199328	1.23686621	1.22201085	1.20742025	1.19308769
12	1.17900668	1.16517095	1.15157441	1.13821119	1.12507561	1.11216216	1.09946551	1.08698051	1.07470215	1.06262559
13	1.05074614	1.03905925	1.02756049	1.01624559	1.00511041	.99415090	.98336316	.97274340	.96228792	.95199314
14	.94185560	.93187191	.92203878	.91235304	.90281150	.89341135	.88414945	.87502303	.86602930	.85716556
15	.84842918	.83981760	.83132833	.82295894	.81470707	.80657041	.79854672	.79063382	.78282955	.77513189
16	.76753876	.76004821	.75265831	.74536717	.73817298	.73107394	.72406830	.71715436	.71033046	.70359498
17	.69694633	.69038297	.68390338	.67750609	.67118965	.66495266	.65879373	.65271153	.64670474	.64077206
18	.63491226	.62912408	.62340634	.61775785	.61217747	.60666407	.60121655	.59583383	.59051485	.58525859
19	.58006404	.57493020	.56985612	.56484083	.55988341	.55498295	.55013856	.54534937	.54061453	.53593319
20	.53130453	.52672776	.52220208	.51772671	.51330091	.50892393	.50459504	.50031352	.49607869	.49188984
21	.48774631	.48364743	.47959256	.47558106	.47161231	.46768569	.46380060	.45995646	.45615268	.45238870
22	.44866396	.44497791	.44133001	.43771974	.43414657	.43060999	.42710952	.42364465	.42021490	.41681960
23	.41345888	.41013168	.40683776	.40357667	.40034798	.39715126	.39398609	.39085205	.38774875	.38467579
24	.38163276	.37861929	.37563500	.37267951	.36975247	.36685349	.36398224	.36113836	.35832152	.35553136
25	.35276757	.35002981	.34731776	.34463110	.34196953	.33933274	.33672042	.33413228	.33156802	.32902737
26	.32651003	.32401572	.32154418	.31909513	.31666830	.31426343	.31188027	.30951856	.30717805	.30485849
27	.30255964	.30028126	.29802312	.29578498	.29356661	.29136779	.28918829	.28702791	.28488641	.28276359
28	.28065924	.27857315	.27650512	.27445494	.27242243	.27040738	.26840960	.26642891	.26446511	.26251803
29	.26058747	.25867327	.25677524	.25489321	.25302700	.25117046	.24934141	.24752168	.24571712	.24392757

Table A. 8 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
30	.24215285	.24039283	.23864735	.23691624	.23519937	.23349659	.23180774	.23013270	.22847130	.22682342
31	.22518891	.22356764	.22195948	.22036428	.21878193	.21721278	.21565522	.21411060	.21257832	.21105825
32	.20955026	.20805424	.20657007	.20504762	.20363679	.20218746	.20074952	.19932285	.19790735	.19650291
33	.19510942	.19372677	.19235487	.19099360	.18964286	.18830256	.18697260	.18565287	.18434328	.18304373
34	.18175413	.18047438	.17920440	.17794409	.17669335	.17545211	.17422027	.17299774	.17178445	.17058029
35	.16938520	.16819908	.16702186	.16585345	.16469377	.16354274	.16240028	.16126633	.16014079	.15902360
36	.15791467	.15681394	.15572133	.15463677	.15356019	.15249151	.15143067	.15037760	.14933222	.14829446
37	.14726430	.14624161	.14522636	.14421847	.14321789	.14222454	.14123837	.14025932	.13928732	.13832231
38	.13736424	.13641303	.13546864	.13453101	.13360007	.13267578	.13175807	.13084690	.12994219	.12904391
39	.12815199	.12726639	.12638704	.12551390	.12464692	.12378604	.12293122	.12208240	.12123954	.12040258
40	.11957147	.11874618	.11792664	.11711282	.11630467	.11550214	.11470518	.11391370	.11312761	.11234731
41	.11157221	.11080246	.11003802	.10927885	.10852491	.10777615	.10703253	.10629402	.10556057	.10483213
42	.10410868	.10339018	.10267657	.10196784	.10126392	.10056480	.09987043	.09918078	.09849580	.09781546
43	.09713973	.09646857	.09580195	.09513982	.09448216	.09382893	.09318010	.09253563	.09189549	.09125964
44	.09062806	.09000072	.08937757	.08875859	.08814374	.08753300	.08692634	.08632372	.08572511	.08513048
45	.08453981	.08395306	.08337021	.08279122	.08221607	.08164473	.08107718	.08051335	.07995327	.07939688
46	.07884416	.07829509	.07774963	.07720776	.07666946	.07613469	.07560344	.07507567	.07455136	.07403049
47	.07351303	.07299896	.07248825	.07198088	.07147682	.07097605	.07047855	.06998430	.06949326	.06900542
48	.06852076	.06803925	.06756087	.06708560	.06661341	.06614429	.06567821	.06521515	.06475509	.06429801
49	.06384388	.06339270	.06294443	.06249906	.06205656	.06161693	.06118012	.06074614	.06031495	.05988654
50	.05946089	.05903798	.05861779	.05820031	.05778550	.05737337	.05696388	.05655702	.05615277	.05575112
51	.05535204	.05495552	.05456154	.05417009	.05378114	.05339469	.05301071	.05262918	.05225010	.05187344
52	.05149918	.05112732	.05075784	.05039071	.05002593	.04966348	.04930334	.04894550	.04858994	.04823665
53	.04788561	.04753681	.04719023	.04684586	.04650369	.04616369	.04582587	.04549019	.04515665	.04482523
54	.04449592	.04416871	.04384357	.04352051	.04319951	.04288054	.04256360	.04224868	.04193577	.04162484
55	.04131589	.04100890	.04070386	.04040077	.04009960	.03980034	.03950299	.03920753	.03891395	.03862223
56	.03833237	.03804435	.03775816	.03747379	.03719122	.03691046	.03663148	.03635427	.03607883	.03580514
57	.03553319	.03526297	.03499447	.03472768	.03446258	.03419918	.03393745	.03367738	.03341897	.03316221
58	.03290709	.03265359	.03240170	.03215142	.03190274	.03165564	.03141011	.03116616	.03092376	.03068291
59	.03044359	.03020581	.02996954	.02973478	.02950152	.02926970	.02903948	.02881067	.02858332	.02835743
60	.02813299	.02790999	.02768841	.02746826	.02724952	.02703218	.02681624	.02660168	.02638850	.02617670
61	.02596625	.02575716	.02554942	.02534301	.02513793	.02493416	.02473174	.02453061	.02433077	.02413223
62	.02393497	.02373898	.02354427	.02335081	.02315861	.02296765	.02277793	.02258945	.02240218	.02221614
63	.02203130	.02184767	.02166523	.02148398	.02130392	.02112503	.02094730	.02077074	.02059533	.02042107
64	.02024796	.02007597	.01990512	.01973536	.01956676	.01939925	.01923284	.01906753	.01890331	.01874017

Table A. 8 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
65	.01857810	.01841711	.01825719	.01809832	.01794050	.01778373	.01762801	.01747331	.01731965	.01716701
66	.01701538	.01686477	.01671516	.01656655	.01641894	.01627231	.01612667	.01598201	.01583832	.01569559
67	.01555383	.01541302	.01527316	.01513425	.01499627	.01485923	.01472312	.01458794	.01445367	.01432031
68	.01418787	.01405633	.01392568	.01379593	.01366707	.01353909	.01341199	.01328577	.01316041	.01303592
69	.01291228	.01278950	.01266757	.01254649	.01242625	.01230684	.01218826	.01207051	.01195358	.01183747
70	.01172218	.01160769	.01149400	.01138112	.01126903	.01115773	.01104722	.01093749	.01082854	.01072036
71	.01061295	.01050631	.01040043	.01029531	.01019094	.01008732	.00998444	.00988231	.00978091	.00968025
72	.00958031	.00948110	.00938261	.00928484	.00918778	.00909143	.00899578	.00890084	.00880660	.00871305
73	.00862019	.00852802	.00843653	.00834572	.00825559	.00816613	.00807734	.00798921	.00790175	.00781494
74	.00772879	.00764329	.00755844	.00747423	.00739066	.00730773	.00722543	.00714377	.00706273	.00698232
75	.00690253	.00682335	.00674479	.00666684	.00658950	.00651276	.00643662	.00636108	.00628614	.00621179
76	.00613803	.00606485	.00599226	.00592024	.00584880	.00577794	.00570764	.00563792	.00555876	.00550016
77	.00543212	.00536463	.00529770	.00523132	.00516548	.00510019	.00503544	.00497123	.00490755	.00484441
78	.00478180	.00471972	.00465816	.00459712	.00453660	.00447660	.00441712	.00435814	.00429967	.00424171
79	.00418426	.00412730	.00407084	.00401488	.00395941	.00390444	.00384995	.00379594	.00374242	.00368938
80	.00363682	.00358474	.00353312	.00348198	.00343131	.00338110	.00333136	.00328208	.00323326	.00318489
81	.00313698	.00308952	.00304251	.00299595	.00294983	.00290416	.00285892	.00281413	.00276977	.00272585
82	.00268235	.00263929	.00259665	.00255444	.00251266	.00247129	.00243034	.00238981	.00234969	.00230999
83	.00227070	.00223181	.00219333	.00215526	.00211759	.00208031	.00204344	.00200696	.00197088	.00193519
84	.00189988	.00186497	.00183045	.00179630	.00176254	.00172916	.00169616	.00166354	.00163129	.00159941
85	.00156791	.00153677	.00150601	.00147560	.00144556	.00141589	.00138657	.00135761	.00132901	.00130076
86	.00127287	.00124532	.00121813	.00119128	.00116478	.00113863	.00111282	.00108734	.00106221	.00103742
87	.00101296	.00098883	.00096504	.00094158	.00091844	.00089564	.00087316	.00085100	.00082917	.00080766
88	.00078647	.00076560	.00074504	.00072480	.00070487	.00068525	.00066594	.00064695	.00062825	.00060987
89	.00059179	.00057401	.00055653	.00053935	.00052247	.00050589	.00048960	.00047361	.00045791	.00044250
90	.00042738	.00041254	.00039800	.00038374	.00036976	.00035606	.00034265	.00032952	.00031666	.00030408
91	.00029178	.00027975	.00026800	.00025651	.00024530	.00023435	.00022368	.00021326	.00020312	.00019324
92	.00018362	.00017426	.00016516	.00015632	.00014774	.00013941	.00013134	.00012352	.00011595	.00010864
93	.00010157	.00009476	.00008819	.00008186	.00007579	.00006995	.00006436	.00005901	.00005390	.00004903
94	.00004440	.00004001	.00003585	.00003192	.00002823	.00002477	.00002154	.00001855	.00001578	.00001323
95	.00001092	.00000883	.00000697	.00000532	.00000391	.00000271	.00000173	.00000097	.00000043	.00000011
96	0.00000000	.00000011	.00000043	.00000096	.00000171	.00000266	.00000383	.00000521	.00000679	.00000858
97	.00001057	.00001277	.00001517	.00001778	.00002059	.00002360	.00002680	.00003021	.00003382	.00003762
98	.00004162	.00004581	.00005020	.00005478	.00005955	.00006451	.00006966	.00007501	.00008054	.00008626
99	.00009216	.00009825	.00010453	.00011099	.00011763	.00012446	.00013147	.00013865	.00014602	.00015357

Table A.9

TABLE OF KUBELKA-MUNK PARAMETER A V.S R INFINITY

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		R500.00050000	250.00100000	166.66816667	125.00200000	100.00250000	83.33633333	71.43207143	62.50400000	55.56005556
1	50.00500000	45.46004545	41.67266667	38.46803846	35.72128571	33.34083333	31.25800000	29.42926471	27.78577778	26.32526947
2	25.01000000	23.82002381	22.73827273	21.75063043	20.84533333	20.01250000	19.24376923	18.53201852	17.87114286	17.25557931
3	16.68166667	16.14453226	15.64100000	15.16801515	14.72268235	14.30321429	13.90686889	13.53201351	13.17664474	12.84001282
4	12.52000000	12.21562195	11.92576190	11.64940698	11.38563636	11.13361111	10.89256522	10.66179787	10.44066667	10.22850163
5	10.02500000	9.82942157	9.64138462	9.46046226	9.28625926	9.11840909	8.95657143	8.80042982	8.64966965	8.50407627
6	8.36333333	8.22722131	8.09551613	7.96800794	7.84450000	7.72480769	7.60875758	7.49610657	7.38694118	7.28087081
7	7.17785714	7.07775352	6.98044444	6.88581507	6.79375676	6.70416667	6.61694737	6.53200649	6.44925641	6.36861392
8	6.29000000	6.21333951	6.13856098	6.06559639	5.99438095	5.92485294	5.85695349	5.79062644	5.72581818	5.66247753
9	5.60055556	5.54000549	5.48078261	5.42284409	5.36614894	5.31065789	5.25633333	5.20313918	5.15104082	5.10000505
10	5.05000000	5.00099505	4.95296078	4.90586893	4.85969231	4.81440476	4.76998113	4.72639720	4.68362963	4.64165596
11	4.60045455	4.56000450	4.52028571	4.48127876	4.44296491	4.40532609	4.36834483	4.33200427	4.29628814	4.26118067
12	4.22666667	4.19273140	4.15936066	4.12654065	4.09425806	4.06250000	4.03125397	4.00050787	3.97025000	3.94046899
13	3.91115385	3.88229389	3.85387879	3.82589850	3.79834328	3.77120370	3.74447059	3.71813504	3.69218841	3.66662230
14	3.64142857	3.61659929	3.59212676	3.56800350	3.54422222	3.52077586	3.49765753	3.47486054	3.45237838	3.43020470
15	3.40833333	3.38675828	3.36547368	3.34447386	3.32375325	3.30330645	3.28312821	3.26321338	3.24355690	3.22415409
16	3.20500000	3.18609006	3.16741975	3.14898466	3.13078049	3.11280303	3.09504819	3.07751198	3.06019048	3.04307988
17	3.02617647	3.00947661	2.99297674	2.97667341	2.96056322	2.94464286	2.92890909	2.91335876	2.89798876	2.88279009
18	2.86777778	2.85293094	2.83825275	2.82374044	2.80939130	2.79520270	2.78117204	2.76729679	2.75357447	2.74000265
19	2.72657895	2.71330105	2.70016667	2.68717358	2.67431959	2.66160256	2.64902041	2.63657107	2.62425253	2.61206281
20	2.60000000	2.58806219	2.57624752	2.56455419	2.55298039	2.54152439	2.53018447	2.51895894	2.50784615	2.49684450
21	2.48595238	2.47516825	2.46449057	2.45391784	2.44344860	2.43308140	2.42281481	2.41264747	2.40257798	2.39260502
22	2.38272727	2.37294344	2.36325225	2.35365247	2.34414286	2.33472222	2.32538938	2.31614317	2.30698246	2.29790011
23	2.28891304	2.28000216	2.27117241	2.26242275	2.25375214	2.24515957	2.23664407	2.22820464	2.21984034	2.21155021
24	2.20333333	2.19518880	2.18711570	2.17911317	2.17118033	2.16331633	2.15552033	2.14779150	2.14012903	2.13253213
25	2.12500000	2.11753187	2.11012698	2.10278458	2.09550394	2.08828431	2.08112500	2.07402529	2.06698450	2.06000193
26	2.05307692	2.04620881	2.03939695	2.03264068	2.02593939	2.01929245	2.01269925	2.00615918	1.99967104	1.99323006
27	1.98685185	1.98051845	1.97423529	1.96800183	1.96181752	1.95568182	1.94959420	1.94355415	1.93756115	1.93161470
28	1.92571429	1.91985943	1.91404965	1.90828445	1.90256338	1.89688596	1.89125175	1.88566028	1.88011111	1.87460381
29	1.86913793	1.86371306	1.85832877	1.85298464	1.84768027	1.84241525	1.83719919	1.83200168	1.82685235	1.82174060

Table A.9 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
30	1.81666667	1.81162957	1.80662914	1.80165502	1.79673684	1.79184426	1.78698493	1.78216450	1.77737662	1.77262298
31	1.76790323	1.76321704	1.75856410	1.75394409	1.74935669	1.74480159	1.74027846	1.73578707	1.73132704	1.72689812
32	1.72250000	1.71813240	1.71379503	1.70948762	1.70520988	1.70096154	1.69674233	1.69255199	1.68839024	1.68425684
33	1.68015152	1.67607402	1.67202410	1.66800150	1.66400599	1.66003731	1.65609524	1.65217953	1.64828994	1.64442625
34	1.64058824	1.63677566	1.63298830	1.62922595	1.62548837	1.62177536	1.61808671	1.61442219	1.61078161	1.60716476
35	1.60357143	1.60000142	1.59645455	1.59293059	1.58942938	1.58595070	1.58249438	1.57906022	1.57564804	1.57225766
36	1.56888889	1.56554155	1.56221547	1.55891047	1.55562637	1.55236301	1.54912022	1.54589782	1.54269565	1.53951355
37	1.53635135	1.53320889	1.53008602	1.52698257	1.52389840	1.52083333	1.51778723	1.51475995	1.51175132	1.50876121
38	1.50578947	1.50283596	1.49990052	1.49698303	1.49408333	1.49120130	1.48833679	1.48548966	1.48265979	1.47984704
39	1.47705128	1.47427238	1.47151020	1.46876463	1.46603553	1.46332278	1.46062626	1.45794584	1.45528141	1.45263283
40	1.45000000	1.44738279	1.44478109	1.44219479	1.43962376	1.43706790	1.43452709	1.43200123	1.42949020	1.42699389
41	1.42451220	1.42204501	1.41959223	1.41715375	1.41472947	1.41231928	1.40992308	1.40754077	1.40517225	1.40281742
42	1.40047619	1.39814846	1.39583412	1.39353310	1.39124528	1.38897059	1.38670892	1.38446019	1.38222430	1.38000117
43	1.37779070	1.37559281	1.37340741	1.37123441	1.36907373	1.36692529	1.36478899	1.36266476	1.36055251	1.35845216
44	1.35636364	1.35428685	1.35222172	1.35016817	1.34812613	1.34609551	1.34407623	1.34206823	1.34007143	1.33808575
45	1.33611111	1.33414745	1.33219469	1.33025276	1.32832159	1.32640110	1.32449123	1.32259190	1.32070305	1.31882462
46	1.31695652	1.31509870	1.31325108	1.31141361	1.30958621	1.30776882	1.30596137	1.30416381	1.30237607	1.30059808
47	1.29882979	1.29707113	1.29532203	1.29358245	1.29185232	1.29013158	1.28842017	1.28671803	1.28502510	1.28334134
48	1.28166667	1.28000104	1.27834440	1.27669669	1.27505785	1.27342784	1.27180658	1.27019405	1.26859016	1.26699489
49	1.26540816	1.26382994	1.26226016	1.26069878	1.25914575	1.25760101	1.25606452	1.25453622	1.25301606	1.25150401
50	1.25000000	1.24850399	1.24701594	1.24553579	1.24406349	1.24259901	1.24114229	1.23969329	1.23825197	1.23681827
51	1.23539216	1.23397358	1.23256250	1.23115887	1.22976265	1.22837379	1.22699225	1.22561799	1.22425097	1.22289114
52	1.22153846	1.22019290	1.21885441	1.21752294	1.21619847	1.21488095	1.21357034	1.21226660	1.21096970	1.20967958
53	1.20839623	1.20711959	1.20584962	1.20458630	1.20332959	1.20207944	1.20083582	1.19959870	1.19836803	1.19714378
54	1.19592593	1.19471442	1.19350923	1.19231031	1.19111765	1.18993119	1.18875092	1.18757678	1.18640876	1.18524681
55	1.18409091	1.18294102	1.18179710	1.18065913	1.17952708	1.17840090	1.17728058	1.17616607	1.17505735	1.17395438
56	1.17285714	1.17176560	1.17067972	1.16959947	1.16852482	1.16745575	1.16639223	1.16533422	1.16428169	1.16323462
57	1.16219298	1.16115674	1.16012587	1.15910035	1.15808014	1.15706522	1.15605556	1.15505113	1.15405190	1.15305786
58	1.15206897	1.15108520	1.15010653	1.14913293	1.14816438	1.14720085	1.14624232	1.14528876	1.14434014	1.14339643
59	1.14245763	1.14152369	1.14059459	1.13967032	1.13875084	1.13783613	1.13692617	1.13602094	1.13512040	1.13422454
60	1.13333333	1.13244676	1.13156478	1.13068740	1.12981457	1.12894628	1.12808251	1.12722323	1.12636842	1.12551806
61	1.12467213	1.12383061	1.12299346	1.12216069	1.12133225	1.12050813	1.11968831	1.11887277	1.11806149	1.11725444
62	1.11645161	1.11565298	1.11485852	1.11406822	1.11328205	1.11250000	1.11172204	1.11094817	1.11017834	1.10941256
63	1.10865079	1.10789303	1.10713924	1.10638942	1.10564353	1.10490157	1.10416352	1.10342936	1.10269906	1.10197261
64	1.10125000	1.10053120	1.09981620	1.09910498	1.09839752	1.09769380	1.09699381	1.09629753	1.09560494	1.09491602

Table A.9 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
65	1.09423077	1.09354916	1.09287117	1.09219678	1.09152599	1.09085878	1.09019512	1.08953501	1.08887842	1.08822534
66	1.08757576	1.08692965	1.08628701	1.08564781	1.08501205	1.08437970	1.08375075	1.08312519	1.08250299	1.08188416
67	1.08126866	1.08065648	1.08004762	1.07944205	1.07883976	1.07824074	1.07764497	1.07705244	1.07646313	1.07587703
68	1.07529412	1.07471439	1.07413783	1.07356442	1.07299415	1.07242701	1.07186297	1.07130204	1.07074419	1.07018940
69	1.06963768	1.06908900	1.06854335	1.06800072	1.06746110	1.06692446	1.06639080	1.06586011	1.06533236	1.06480758
70	1.06428571	1.06376676	1.06325071	1.06273755	1.06222727	1.06171986	1.06121530	1.06071358	1.06021469	1.05971862
71	1.05922535	1.05873488	1.05824719	1.05776227	1.05728011	1.05680070	1.05632402	1.05585007	1.05537883	1.05491029
72	1.05444444	1.05398128	1.05352078	1.05306293	1.05260773	1.05215517	1.05170523	1.05125791	1.05081319	1.05037106
73	1.04993151	1.04949453	1.04906011	1.04862824	1.04819891	1.04777211	1.04734783	1.04692605	1.04650678	1.04608999
74	1.04567568	1.04526383	1.04485445	1.04444751	1.04404301	1.04364094	1.04324129	1.04284404	1.04244920	1.04205674
75	1.04166667	1.04127896	1.04089362	1.04051062	1.04012997	1.03975166	1.03937566	1.03900198	1.03863061	1.03826153
76	1.03789474	1.03753022	1.03716798	1.03680799	1.03645026	1.03609477	1.03574151	1.03539048	1.03504167	1.03469506
77	1.03435065	1.03400843	1.03366839	1.03333053	1.03299483	1.03266129	1.03232990	1.03200064	1.03167352	1.03134852
78	1.03102564	1.03070487	1.03038619	1.03006960	1.02975510	1.02944268	1.02913232	1.02882402	1.02851777	1.02821356
79	1.02791139	1.02761125	1.02731313	1.02701702	1.02672292	1.02643082	1.02614070	1.02585257	1.02556642	1.02528223
80	1.02500000	1.02471973	1.02444140	1.02416501	1.02389055	1.02361801	1.02334739	1.02307869	1.02281188	1.02254697
81	1.02228395	1.02202281	1.02176355	1.02150615	1.02125061	1.02099693	1.02074510	1.02049510	1.02024694	1.02000061
82	1.01975610	1.01951340	1.01927251	1.01903341	1.01879612	1.01856061	1.01832688	1.01809492	1.01786473	1.01763631
83	1.01740964	1.01718472	1.01696154	1.01674010	1.01652038	1.01630240	1.01608612	1.01587157	1.01565871	1.01544756
84	1.01523810	1.01503032	1.01482423	1.01461981	1.01441706	1.01421598	1.01401655	1.01381877	1.01362264	1.01342815
85	1.01323529	1.01304407	1.01285446	1.01266647	1.01248009	1.01229532	1.01211215	1.01193057	1.01175058	1.01157218
86	1.01139535	1.01122009	1.01104640	1.01087428	1.01070370	1.01053468	1.01036721	1.01020127	1.01003687	1.00987399
87	1.00971264	1.00955281	1.00939450	1.00923769	1.00908238	1.00892857	1.00877626	1.00862543	1.00847608	1.00832821
88	1.00818182	1.00803689	1.00789342	1.00775142	1.00761086	1.00747175	1.00733409	1.00719786	1.00706306	1.00692970
89	1.00679775	1.00666723	1.00653812	1.00641041	1.00628412	1.00615922	1.00603571	1.00591360	1.00579287	1.00567353
90	1.00555556	1.00543896	1.00532373	1.00520986	1.00509735	1.00498619	1.00487638	1.00476792	1.00466079	1.00455501
91	1.00445055	1.00434742	1.00424561	1.00414513	1.00404595	1.00394809	1.00385153	1.00375627	1.00366231	1.00356964
92	1.00347826	1.00338817	1.00329935	1.00321181	1.00312554	1.00304054	1.00295680	1.00287433	1.00279310	1.00271313
93	1.00263441	1.00255693	1.00248069	1.00240568	1.00233191	1.00225936	1.00218803	1.00211793	1.00204904	1.00198136
94	1.00191489	1.00184963	1.00178556	1.00172269	1.00166102	1.00160053	1.00154123	1.00148310	1.00142616	1.00137039
95	1.00131579	1.00126236	1.00121008	1.00115897	1.00110901	1.00106021	1.00101255	1.00096604	1.00092067	1.00087643
96	1.00083333	1.00079136	1.00075052	1.00071080	1.00067220	1.00063472	1.00059834	1.00056308	1.00052893	1.00049587
97	1.00046392	1.00043306	1.00040329	1.00037461	1.00034702	1.00032051	1.00029508	1.00027073	1.00024744	1.00022523
98	1.00020408	1.00018400	1.00016497	1.00014700	1.00013008	1.00011421	1.00009939	1.00008561	1.00007287	1.00006117
99	1.00005051	1.00004087	1.00003226	1.00002467	1.00001811	1.00001256	1.00000803	1.00000451	1.00000200	1.00000050

Table A.10

TABLE OF KUBELKA-MUNK PARAMETER B V.S R INFINITY

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		R499.9995000	249.9990000	166.6651667	124.99800000	99.99750000	83.33033333	71.42507143	62.49600000	55.55105556
1	49.99500000	45.44904545	41.66066667	38.45503846	35.70728571	33.32583333	31.24200000	29.40326471	27.76877778	26.30628947
2	24.99000000	23.79902381	22.71627273	21.72763043	20.82133333	19.98750000	19.21776923	18.50501852	17.84314286	17.22687931
3	16.65166667	16.11353226	15.60900000	15.13501515	14.68888235	14.26821429	13.87088889	13.49501351	13.13889474	12.80101282
4	12.48000000	12.17462195	11.88376190	11.60640698	11.34163636	11.08861111	10.84656522	10.61479787	10.39266667	10.17958163
5	9.97500000	9.77842157	9.58938462	9.40746226	9.23225926	9.06340909	8.90057143	8.74342982	8.59168966	8.44507627
6	8.30333333	8.16622131	8.03351613	7.90500794	7.78050000	7.65980769	7.54275758	7.42918657	7.31894118	7.21187681
7	7.10785714	7.00675352	6.90844444	6.81281507	6.71975676	6.62891667	6.54094737	6.45500649	6.37125641	6.28961392
8	6.21000000	6.13233951	6.05656098	5.98259639	5.91038095	5.83985294	5.77095349	5.70362644	5.63781818	5.57347753
9	5.51055556	5.44900549	5.38878261	5.32984409	5.27214894	5.21565789	5.16033333	5.10613918	5.05304082	5.00100505
10	4.95000000	4.89999505	4.85096078	4.80286893	4.75569231	4.70940476	4.66398113	4.61939720	4.57562963	4.53265596
11	4.49045455	4.44900450	4.40828571	4.36827876	4.32896491	4.29032609	4.25234483	4.21500427	4.17828814	4.14218067
12	4.10666667	4.07173140	4.03736066	4.00354065	3.97025806	3.93750000	3.90525397	3.87350787	3.84225000	3.81146899
13	3.78115385	3.75129389	3.72187879	3.69289850	3.66434328	3.63620370	3.60847059	3.58113504	3.55418841	3.52762230
14	3.50142857	3.47559929	3.45012676	3.42500350	3.40022222	3.37577586	3.35165753	3.32786054	3.30437838	3.28120470
15	3.25833333	3.23575828	3.21347368	3.19147386	3.16975325	3.14830645	3.12712821	3.10621338	3.08555696	3.06515409
16	3.04500000	3.02509006	3.00541975	2.98598466	2.96678049	2.94780303	2.92904819	2.91051198	2.89219048	2.87407988
17	2.85617647	2.83847661	2.82097674	2.80367341	2.78656322	2.76964286	2.75290909	2.73635876	2.71998876	2.70379609
18	2.68777778	2.67193094	2.65625275	2.64074044	2.62539130	2.61020270	2.59517204	2.58029679	2.56557447	2.55100265
19	2.53657895	2.52230105	2.50816667	2.49417358	2.48031959	2.46660256	2.45302041	2.43957107	2.42625253	2.41306281
20	2.40000000	2.38706219	2.37424752	2.36155419	2.34898039	2.33652439	2.32418447	2.31195894	2.29984615	2.28784450
21	2.27595238	2.26416825	2.25249057	2.24091784	2.22944860	2.21808140	2.20681481	2.19564747	2.18457798	2.17360502
22	2.16272727	2.15194344	2.14125225	2.13065247	2.12014286	2.10972222	2.09938938	2.08914317	2.07898246	2.06890611
23	2.05891304	2.04900216	2.03917241	2.02942275	2.01975214	2.01015957	2.00064407	1.99120464	1.98184034	1.97255021
24	1.96333333	1.95418880	1.94511570	1.93611317	1.92718033	1.91831633	1.90952033	1.90079150	1.89212903	1.88353213
25	1.87500000	1.86653187	1,85812698	1.84978458	1.84150394	1.83328431	1.82512500	1.81702529	1.80898450	1.80100193
26	1.79307692	1.78520881	1.77739695	1.76964068	1.76193939	1.75429245	1.74669925	1.73915918	1.73167164	1.72423606
27	1.71685185	1.70951845	1.70223529	1.69500183	1.68781752	1.68068182	1.67359420	1.66655415	1.65956115	1.65261470
28	1.64571429	1.63885943	1.63204965	1.62528445	1.61856338	1.61188596	1.60525175	1.59866028	1.59211111	1.58560381
29	1.57913793	1.57271306	1.56632877	1.55998464	1.55368027	1.54741525	1.54118919	1.53500168	1.52885235	1.52274080

Table A.10 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
30	1.51666667	1.51062957	1.50462914	1.49866502	1.49273684	1.48684426	1.48098693	1.47516450	1.46937662	1.46362298
31	1.45790323	1.45221704	1.44656410	1.44094409	1.43535669	1.42980159	1.42427848	1.41878707	1.41332704	1.40789012
32	1.40250000	1.39713240	1.39179503	1.38648762	1.38120988	1.37596154	1.37074233	1.36555199	1.36039024	1.35525684
33	1.35015152	1.34507402	1.34002410	1.33500150	1.33000599	1.32503731	1.32009524	1.31517953	1.31028994	1.30542525
34	1.30058824	1.29577566	1.29098830	1.28622595	1.28148837	1.27677536	1.27208671	1.26742219	1.26278161	1.25816476
35	1.25357143	1.24900142	1.24445455	1.23993059	1.23542938	1.23095070	1.22649438	1.22206022	1.21764804	1.21325766
36	1.20888889	1.20454155	1.20021547	1.19591047	1.19162637	1.18736301	1.18312022	1.17889782	1.17469565	1.17051355
37	1.16635135	1.16220889	1.15808602	1.15398257	1.14989840	1.14583333	1.14178723	1.13775995	1.13375132	1.12976121
38	1.12578947	1.12183596	1.11790052	1.11398303	1.11008333	1.10620130	1.10233679	1.09848966	1.09465979	1.09084704
39	1.08705128	1.08327238	1.07951020	1.07576463	1.07203553	1.06832278	1.06462626	1.06094584	1.05728141	1.05363283
40	1.05000000	1.04638279	1.04278109	1.03919479	1.03562376	1.03206790	1.02852709	1.02500123	1.02149020	1.01799389
41	1.01451220	1.01104501	1.00759223	1.00415375	1.00072947	.99731928	.99392308	.99054077	.98717225	.98381742
42	.98047619	.97714846	.97383412	.97053310	.96724528	.96397059	.96070892	.95746019	.95422430	.95100117
43	.94779070	.94459281	.94140741	.93823441	.93507373	.93192529	.92878899	.92566476	.92255251	.91945216
44	.91636364	.91328685	.91022172	.90716817	.90412613	.90109551	.89807623	.89506823	.89207143	.88908575
45	.88611111	.88314745	.88019469	.87725276	.87432159	.87140110	.86849123	.86559190	.86270306	.85982462
46	.85695652	.85409870	.85125108	.84841361	.84558621	.84276882	.83996137	.83716381	.83437607	.83159809
47	.82882979	.82607113	.82332203	.82058245	.81785232	.81513158	.81242017	.80971803	.80702510	.80434134
48	.80166667	.79900104	.79634440	.79369669	.79105785	.78842784	.78580658	.78319405	.78059016	.77799489
49	.77540816	.77282994	.77026016	.76769878	.76514575	.76260101	.76006452	.75753622	.75501609	.75250401
50	.75000000	.74750399	.74501594	.74253579	.74006349	.73759901	.73514229	.73269329	.73025197	.72751027
51	.72549216	.72297358	.72056250	.71815887	.71576265	.71337379	.71099225	.70861749	.70625097	.70389114
52	.70153846	.69919290	.69685441	.69452294	.69219847	.68988095	.68757034	.68526660	.68296970	.68067958
53	.67839623	.67611959	.67384962	.67158630	.66932959	.66707944	.66483582	.66259870	.66036803	.65814378
54	.65592593	.65371442	.65150923	.64931031	.64711764	.64493119	.64275092	.64057678	.63840870	.63624661
55	.63409091	.63194102	.62979710	.62765913	.62552708	.62340090	.62128058	.61916607	.61705735	.61495438
56	.61285714	.61076560	.60867972	.60659947	.60452482	.60245575	.60039223	.59833422	.59628169	.59423462
57	.59219298	.59015674	.58812587	.58610035	.58408014	.58206522	.58005556	.57805113	.57605190	.57405786
58	.57206897	.57008520	.56810653	.56613293	.56416438	.56220085	.56024232	.55828876	.55634014	.55439643
59	.55245763	.55052369	.54859459	.54667032	.54475084	.54283613	.54092617	.53902094	.53712040	.53522454
60	.53333333	.53144676	.52956478	.52768740	.52581457	.52394628	.52208251	.52022323	.51836842	.51651806
61	.51467213	.51283061	.51099346	.50916069	.50733225	.50550813	.50368831	.50187277	.50006149	.49825444
62	.49645161	.49465298	.49285852	.49106822	.48928205	.48750000	.48572204	.48394017	.48217834	.48041255
63	.47865079	.47689303	.47513924	.47338942	.47164353	.46990157	.46816352	.46642936	.46469900	.46297201
64	.46125000	.45953120	.45781620	.45610498	.45439752	.45269380	.45099381	.44929753	.44760494	.44591602

Table A.10 (cont'd)

(PERCENT OF REFLECTANCE)

R	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
65	.44423077	.44254916	.44087117	.43919678	.43752599	.43585878	.43419512	.43253501	.43087842	.42922534
66	.42757576	.42592965	.42428701	.42264781	.42101205	.41937970	.41775075	.41612519	.41450299	.41288416
67	.41126866	.40965648	.40804762	.40644205	.40483976	.40324074	.40164497	.40005244	.39846313	.39687703
68	.39529412	.39371439	.39213783	.39056442	.38899415	.38742701	.38586297	.38430204	.38274419	.38118940
69	.37963768	.37808900	.37654335	.37500072	.37346110	.37192446	.37039080	.36886011	.36733236	.36580756
70	.36428571	.36276676	.36125071	.35973755	.35822727	.35671986	.35521530	.35371358	.35221469	.35071862
71	.34922535	.34773488	.34624719	.34476227	.34328011	.34180070	.34032402	.33885007	.33737883	.33591029
72	.33444444	.33298128	.33152078	.33006293	.32860773	.32715517	.32570523	.32425791	.32281319	.32137106
73	.31993151	.31849453	.31706011	.31562824	.31419891	.31277211	.31134783	.30992605	.30850678	.30708999
74	.30567568	.30426383	.30285445	.30144751	.30004301	.29864094	.29724129	.29584404	.29444920	.29305674
75	.29166667	.29027896	.28889362	.28751062	.28612997	.28475166	.28337566	.28200198	.28063061	.27926153
76	.27789474	.27653022	.27516798	.27380799	.27245026	.27109477	.26974151	.26839048	.26704167	.26569506
77	.26435065	.26300843	.26166839	.26033053	.25899483	.25766129	.25632990	.25500064	.25367352	.25234852
78	.25102564	.24970487	.24838619	.24706960	.24575510	.24444268	.24313232	.24182402	.24051777	.23921356
79	.23791139	.23661125	.23531313	.23401702	.23272292	.23143082	.23014070	.22885257	.22756642	.22628223
80	.22500000	.22371973	.22244140	.22116501	.21989055	.21861801	.21734739	.21607869	.21481188	.21354697
81	.21228395	.21102281	.20976355	.20850615	.20725061	.20599693	.20474510	.20349510	.20224694	.20100061
82	.19975610	.19851340	.19727251	.19603341	.19479612	.19356061	.19232688	.19109492	.18986473	.18863631
83	.18740964	.18618472	.18496154	.18374010	.18252038	.18130240	.18008612	.17887157	.17765871	.17644756
84	.17523810	.17403032	.17282423	.17161981	.17041706	.16921598	.16801655	.16681877	.16562264	.16442815
85	.16323529	.16204407	.16085446	.15966647	.15848009	.15729532	.15611215	.15493057	.15375058	.15257218
86	.15139535	.15022009	.14904640	.14787428	.14670370	.14553468	.14436721	.14320127	.14203687	.14087399
87	.13971264	.13855281	.13739450	.13623769	.13508238	.13392857	.13277626	.13162543	.13047608	.12932821
88	.12818182	.12703689	.12589342	.12475142	.12361086	.12247175	.12133409	.12019786	.11906306	.11792970
89	.11679775	.11566723	.11453812	.11341041	.11228412	.11115922	.11003571	.10891360	.10779287	.10667353
90	.10555556	.10443896	.10332373	.10220986	.10109735	.09998619	.09887638	.09776792	.09666079	.09555501
91	.09445055	.09334742	.09224561	.09114513	.09004595	.08894809	.08785153	.08675627	.08566231	.08456964
92	.08347826	.08238817	.08129935	.08021181	.07912554	.07804054	.07695680	.07587433	.07479310	.07371313
93	.07263441	.07155693	.07048069	.06940568	.06833191	.06725936	.06618803	.06511793	.06404904	.06298136
94	.06191489	.06084963	.05978556	.05872269	.05766102	.05660053	.05554123	.05448310	.05342616	.05237039
95	.05131579	.05026236	.04921008	.04815897	.04710901	.04606021	.04501255	.04396604	.04292067	.04187643
96	.04083333	.03979136	.03875052	.03771080	.03667220	.03563472	.03459834	.03356308	.03252893	.03149587
97	.03046392	.02943306	.02840329	.02737461	.02634702	.02532051	.02429508	.02327073	.02224744	.02122523
98	.02020408	.01918400	.01816497	.01714700	.01613008	.01511421	.01409939	.01308561	.01207287	.01106117
99	.01005051	.00904087	.00803226	.00702467	.00601811	.00501256	.00400803	.00300451	.00200200	.00100050

APPENDIX II

COMPUTER PROGRAMS

The FORTAN PROGRAMS used in this study are compiled in this APPENDIX. They are:

1. PROGRAM TABLE, this program has been used to construct Tables A.1, A.2, A.3, A.4, A.5, A.6, A.7, A.8, and A.9 and A.10. Only formula statements and some descriptive statements have to be changed for each table.

2. PROGRAM PREDICT, this program has been used for calculating R_∞, K, S, K/S. The difference between experimental and calculated reflectance values and also in terms of color coordinates.

3. PROGRAM MIXTURE, this program has been used for K.M analysis in cases of predicting the reflectance and color of the mixture.

```
PROGRAM      TABLE                                     CDC 6500 FTN V3.0-P380 OPT=1

      PROGRAM TABLE(INPUT,OUTPUT)
      DIMENSION RM(100),RA(100,10),RR(100,11)
      INTEGER RM
      PRINT 11
   11 FORMAT(1H1,///////////,68X,*TABLE A. 2*//)
      PRINT 1
    1 FORMAT(59X,*TABLE FOR INSPECTING R01, R02*//,48X,*(MINIMUM VALUES
     XOF R01 CORRESPONDING TO GIVEN R02)*//)
      PRINT 14
   14 FORMAT(35X,*(PERCENT R02)*,48X,*(PERCENT R02)*/,14X,*R02*,2H *,2X,
     X*0.0*,2X,*0.1*,2X,*0.2*,2X,*0.3*,2X,*0.4*,2X,*0.5*,2X,*0.6*,2X,*0.
     X7*,2X,*0.8*,2X,*0.9*,7X,*R02*,2H *,2X,*0.0*,2X,*0.1*,2X,*0.2*,2X,*
     X0.3*,2X,*0.4*,2X,*0.5*,2X,*0.6*,2X,*0.7*,2X,*0.8*,2X,*0.9*//,14X,2
     X8(2H**),6X,28(2H**))
      DO 2 I=1,100
      RM(I)=I-1
    2 CONTINUE
      DO 3 I=1,100
      DO 3 J=1,10
      R=.01*(I-1)+.001*(J-1)
      RA(I,J)=P/(2-R)
    3 CONTINUE
      DO 7 I=1,100
      RR(I,1)=RM(I)      $ RR(I,2)=RA(I,1)    $ RR(I,3)=RA(I,2)
      RR(I,4)=RA(I,3)    $ RR(I,5)=RA(I,4)    $ RR(I,6)=RA(I,5)
      RR(I,7)=RA(I,6)    $ RR(I,8)=RA(I,7)    $ RR(I,9)=RA(I,8)
      RR(I,10)=RA(I,9)   $ RR(I,11)=RA(I,10)
    7 CONTINUE
      DO 5 I=1,50
    5 PRINT 6,((RR(I,J),J=1,11),(RR(I+50,J),J=1,11))
    6 FORMAT(7(2H  ),F3.0,3H * ,10(F5.3),3(2H  ),F3.0,3H * ,10(F5.3))
      END
MAP .02.12.57.  NORMAL                CONTROL
   ---TIME---LOAD MODE --L1--L2-----TYPE---------------USER---+---CALL-----------
   FWA LOADER  113741  FWA TABLES  112104
   -PROGRAM----ADDRESS-                        --LABELED---COMMON----ADDRESS-
   TABLE        000100
   SYSTEM$      010602                                EXECMSG       010602
                                                      CONEC10       010602

   KODER$       011727
   OUTPTC$      013351
   GETBA        013445
   SIO$         013464
   -----------END OF MAP----------
```

PROGRAM PREDICT CDC 6500 FTN V3.0-P380 OPT=1

```
      PROGRAM PREDICT(INPUT,OUTPUT)
      DIMENSION ICODE3(18),XBAR(32),YBAR(32),ZBAR(32),RIM(32),R1(32),R2(
     X32),SEC(32),RSTBA(32),RIN(32),FR(32),S(32),AK(32),R1A(32),R2A(32),
     XRC(32),RCA(32),RCS(32),RCD(32),RCAS(32),RCAD(32),
     XRIA(32),RIAR(32),FRA(32),SA(32),AKA(32),R1S(32),R2S(32),RIS(32),RI
     XSR(32),FRS(32),SS(32),AKS(32),R1D(32),R2D(32),RID(32),RIDR(32),FRD
     X(32),SD(32),AKD(32),R1AS(32),R2AS(32),RIAS(32),RIASR(32),FRAS(32),
     XSAS(32),AKAS(32),R1AD(32),R2AD(32),RIAD(32),RIADR(32),FRAD(32),SAD
     X(32),AKAD(32),DR(32),DRA(32),DRS(32),DRAS(32),DRD(32),DRAD(32),R(9
     X,32),WL(32),XD(9),YD(9),ZD(9),SMLX(9),SMLY(9),AL(9),BL(9),INDE(9),
     XX(9),Y(9),Z(9),VL(9),DL(9),DAL(9),DBL(9),DE(9),INDEX(9,32)
      INTEGER WL
      DATA N/16/
      DATA ISAM/4HSAMP/,IEND/4HEND /
      READ 100,(XBAR(I),I=1,N)
      READ 100,(YBAR(I),I=1,N)
      READ 100,(ZBAR(I),I=1,N)
      READ 100,(SEC(I),I=1,N)
  100 FORMAT(8F10.4)
      READ 200,(RSTBA(I),I=1,N)
  200 FORMAT(16F5.3)
      FK=1.E-30
      DO 60 I=1,N
   60 FK=FK+(SEC(I)*YBAR(I))
    1 READ 400,ICODE1,ICODE2,ICODE3
  400 FORMAT(A4,A4,18A4)
C     THE ICODES ARE USED TO SIGNIFY SAMPLE DESIGNATIONS AND END OF DATA
      IF(ICODE1.EQ.ISAM) GO TO 2
      IF(ICODE1.EQ.IEND) GO TO 999
    2 READ 500,D
  500 FORMAT(F10.5)
      PRINT 110,ICODE1,ICODE2,ICODE3
  110 FORMAT(1H1,A4,A4,18A4)
      PRINT 120,D
  120 FORMAT(/18H FILM THICKNESS = ,F10.5,8H MICRONS//)
      READ 200,(R1(I),I=1,N)
      READ 200,(R2(I),I=1,N)
      READ 200,(RIM(I),I=1,N)
      R1(0)=1.E-35$R1A(0)=1.E-35$R1S(0)=1.E-35$R1D(0)=1.E-35
      R1AS(0)=1.E-35$R1AD(0)=1.E-35$RIN(0)=1.E-25$RIS(0)=1.E-25
      RID(0)=1.E-25$RIAS(0)=1.E-25$RIAD(0)=1.E-25    $RSTBA(0)=1.E-25
      RIA(0)=1.E-25    $  RCAS(0)=1.E-35    $ RCAD(0)=1.E-35
      DO 11 I=1,N
C
      INDEX(1,I)=0  $  INDEX(2,I)=0  $  INDEX(3,I)=0
      INDEX(4,I)=0 $ INDEX(5,I)=0 $ INDEX(6,I)=0 $ INDEX(7,I)=0
C        USING THE RELATIVE REFLECTANCE VALUES
      RC(I)=R2(I)/(2.-R2(I))
C        CHECKING THE CRITERIA
      IF(RC(I).GT.R1(I)) GO TO 101
      RIN(I)=(2.*R1(I)-R2(I)+(R1(I)**2)*R2(I)-SQRT((2.*R1(I)-R2(I)+(R1(I
     X)**2)*R2(I))**2-4.*(R1(I)**4)))/(2.*(R1(I)**2))
      FR(I)=0.5*(1.-RIN(I))**2/RIN(I)
      S(I)=RIN(I)/(D*(1.-RIN(I)**2))*ALOG(RIN(I)*(R1(I)*RIN(I)-1.)/(R1(I
     X)-RIN(I)))
      AK(I)=S(I)*FR(I)
      GO TO 111
  101 INDEX(2,I)=I
C        USING THE ABSOLUTE REFLECTANCE VALUES
  111 R1A(I)=R1(I)*RSTBA(I)
      R2A(I)=R2(I)*RSTBA(I)
      RCA(I)=R2A(I)/(2.-R2A(I))
C        CHECKING THE CRITERIA
      IF(RCA(I).GT.R1A(I)) GO TO 221
      RIA(I)=(2.*R1A(I)-R2A(I)+(R1A(I)**2)*R2A(I)-SQRT((2.*R1A(I)-R2A(I)
     X+(R1A(I)**2)*R2A(I))**2-4.*(R1A(I)**4)))/(2.*(R1A(I)**2))
      RIAR(I)=RIA(I)/RSTBA(I)
      FRA(I)=0.5*(1.-RIA(I))**2/RIA(I)
      SA(I)=RIA(I)/(D*(1.-RIA(I)**2))*ALOG(RIA(I)*(R1A(I)*RIA(I)-1.)/(R1
     XA(I)-RIA(I)))
      AKA(I)=SA(I)*FRA(I)
      GO TO 222
  221 INDEX(3,I)=I
C       USING THE SAUNDERSON CORRECTION WITH RELATIVE REFLECTANCE VALUES
  222 IF(R1(I).LT.0.04) GO TO 331
      R1S(I)=(R1(I)-0.04)/(.36+.6*R1(I))
      R2S(I)=(R2(I)-0.04)/(.36+.6*R2(I))
      RCS(I)=R2S(I)/(2.-R2S(I))
```

```
C          CHECKING THE CRITERIA
      IF(RCS(I).GT.R1S(I)) GO TO 331
      R1S(I)=(2.*R1S(I)-R2S(I)+(R1S(I)**2)*R2S(I)-SQRT((2.*R1S(I)-R2S(I)
     X+(R1S(I)**2)*R2S(I))**2-4.*(R1S(I)**4)))/(2.*(R1S(I)**2))
      R1SR(I)=(.36*R1S(I)+.04)/(1.-.6*R1S(I))
      FRS(I)=0.5*(1.-R1S(I))**2/R1S(I)
      SS(I)=R1S(I)/(D*(1.-R1S(I)**2))*ALOG(R1S(I)*(R1S(I)*R1S(I)-1.)/(R1
     XS(I)-R1S(I)))
      AKS(I)=SS(I)*FRS(I)
      GO TO 333
  331 INDEX(4,I)=I
C       USING THE DUNCAN CORRECTION WITH THE RELATIVE REFLECTANCE VALUES
  333 R1D(I)=R1(I)/(.427+.555*R1(I))
      R2D(I)=R2(I)/(.427+.555*R2(I))
      RCD(I)=R2D(I)/(2.-R2D(I))
C          CHECKING THE CRITERIA
      IF(RCD(I).GT.R1D(I)) GO TO 441
      R1D(I)=(2.*R1D(I)-R2D(I)+(R1D(I)**2)*R2D(I)-SQRT((2.*R1D(I)-R2D(I)
     X+(R1D(I)**2)*R2D(I))**2-4.*(R1D(I)**4)))/(2.*(R1D(I)**2))
      R1DR(I)=.427*R1D(I)/(1.-.555*R1D(I))
      FRD(I)=0.5*(1.-R1D(I))**2/R1D(I)
      SD(I)=R1D(I)/(D*(1.-R1D(I)**2))*ALOG(R1D(I)*(R1D(I)*R1D(I)-1.)/(R1
     XD(I)-R1D(I)))
      AKD(I)=SD(I)*FRD(I)
      GO TO 444
  441 INDEX(6,I)=I
C     USING THE SAUNDERSON CORRECTION WITH THE ABSOLUTE REFLECTANCE VALUES
  444 IF(R1A(I).LT.0.04) GO TO 551
      R1AS(I)=(R1A(I)-.04)/(.36+.6*R1A(I))
      R2AS(I)=(R2A(I)-.04)/(.36+.6*R2A(I))
      RCAS(I)=R2AS(I)/(2.-R2AS(I))
C          CHECKING THE CRITERIA
      IF(RCAS(I).EQ.0) GO TO 551
      IF(RCAS(I).GT.R1AS(I)) GO TO 551
      R1AS(I)=(2.*R1AS(I)-R2AS(I)+(R1AS(I)**2)*R2AS(I)-SQRT((2.*R1AS(I)-
     XR2AS(I)+(R1AS(I)**2)*R2AS(I))**2-4.*(R1AS(I)**4)))/(2.*(R1AS(I)**2
     X))
      R1ASR(I)=(.36*R1AS(I)+.04)/((1.-.6*R1AS(I))*RSTBA(I))
      FRAS(I)=.5*(1.-R1AS(I))**2/R1AS(I)
      SAS(I)=R1AS(I)/(D*(1.-R1AS(I)**2))*ALOG(R1AS(I)*(R1AS(I)*R1AS(I)-1
     X.)/(R1AS(I)-R1AS(I)))
      AKAS(I)=SAS(I)*FRAS(I)
      GO TO 555
  551 INDEX(5,I)=I
C       USING THE DUNCAN CORRECTION WITH THE ABSOLUTE REFLECTANCE VALUES
  555 R1AD(I)=R1A(I)/(.427+.555*R1A(I))
      R2AD(I)=R2A(I)/(.427+.555*R2A(I))
      RCAD(I)=R2AD(I)/(2.-R2AD(I))
C          CHECKING THE CRITERIA
      IF(RCAD(I).EQ.0) GO TO 661
      IF(RCAD(I).GT.R1AD(I)) GO TO 661
      R1AD(I)=(2.*R1AD(I)-R2AD(I)+(R1AD(I)**2)*R2AD(I)-SQRT((2.*R1AD(I)-
     XR2AD(I)+(R1AD(I)**2)*R2AD(I))**2-4.*(R1AD(I)**4)))/(2.*(R1AD(I)**2
     X))
      R1ADR(I)=.427*R1AD(I)/((1.-.555*R1AD(I))*RSTBA(I))
      FRAD(I)=.5*(1.-R1AD(I))**2/R1AD(I)
      SAD(I)=R1AD(I)/(D*(1.-R1AD(I)**2))*ALOG(R1AD(I)*(R1AD(I)*R1AD(I)-1
     X.)/(R1AD(I)-R1AD(I)))
      AKAD(I)=SAD(I)*FRAD(I)
      GO TO 666
  661 INDEX(7,I)=I
  666 WL(I)=380+I*20
   11 CONTINUE
      PRINT 130,(WL(I),I=1,N)
  130 FORMAT(/1X,*WL*,2X,16I8//)
      PRINT 140,((R1(I),I=1,N),(R2(I),I=1,N),(RIM(I),I=1,N),(RIN(I),I=1,
     XN),(RIAR(I),I=1,N),(R1SR(I),I=1,N),(R1ASR(I),I=1,N),(R1DR(I),I=1,N
     X),(R1ADR(I),I=1,N))
  140 FORMAT(1X,*R01*,2X,16F8.4//,1X,*R02*,2X,16F8.4//,1X,*RIM*,2X,16F8.
     X4//,1X,*RIN*,2X,16F8.4//,1X,*RIAR*,1X,16F8.4//,1X,*R1SR*,1X,16F8.4
     X//,1X,*R1ASR*,16F8.4//,1X,*R1DR*,1X,16F8.4//,1X,*R1ADR*,16F8.4///)
C          COMPARING THE PREDICTED REFLECTANCE VALUES WITH THE MEASURED
C          REFLECTANCE VALUES
      DO 20 I=1,N
      IF(INDEX(2,I).EQ.I) GO TO 515
      DR(I)=RIN(I)-RIM(I)
  515 IF(INDEX(3,I).EQ.I) GO TO 616
      DRA(I)=RIAR(I)-RIM(I)
```

```
 616 IF(INDEX(4,I).EQ.I) GO TO 717
     DRS(I)=RISR(I)-RIM(I)
 717 IF(INDEX(5,I).EQ.I) GO TO 818
     DRAS(I)=RIASR(I)-RIM(I)
 818 IF(INDEX(6,I).EQ.I) GO TO 919
     DRD(I)=RIDR(I)-RIM(I)
 919 IF(INDEX(7,I).EQ.I) GO TO 414
     DRAD(I)=RIADR(I)-RIM(I)
C
 414 R(1,I)=RIM(I)
     R(2,I)=RIN(I)
     R(3,I)=RIAR(I)
     R(4,I)=RISR(I)
     R(5,I)=RIASR(I)
     R(6,I)=RIDR(I)
     R(7,I)=RIADR(I)
  20 CONTINUE
C
     ML=7 $ Y(0)=1.E-50
     XYZ=1.E-53
     DO 40 L=1,ML
     INDE(L)=0
     XD(L)=0    $    YD(L)=0    $    ZD(L)=0
     DO 30 I=1,N
     IF(INDEX(L,I).EQ.I) GO TO 201
     XD(L)=XD(L)+(R(L,I)*SEC(I)*XBAR(I))
     YD(L)=YD(L)+(R(L,I)*SEC(I)*YBAR(I))
     ZD(L)=ZD(L)+(R(L,I)*SEC(I)*ZBAR(I))
  30 CONTINUE
     GO TO 102
 201 INDE(L)=L     $     GO TO 40
 102 X(L)=100*XD(L)/FK
     Y(L)=100*YD(L)/FK
     Z(L)=100*ZD(L)/FK
C
     XYZ=X(L)+Y(L)+Z(L)
     SMLX(L)=X(L)/XYZ
     SMLY(L)=Y(L)/XYZ
C
     AL(L)=17.5*(1.02*X(L)-Y(L))/SQRT(Y(L))
     BL(L)=7.0*(Y(L)-.847*Z(L))/SQRT(Y(L))
     VL(L)=10.*SQRT(Y(L))
  40 CONTINUE
     DL(1)=0    $   DAL(1)=0    $   DBL(1)=0    $  DE(1)=0
     DO 50 L=2,ML
     IF(INDE(L).EQ.L) GO TO 50
     DL(L)=VL(L)-VL(1)
     DAL(L)=AL(L)-AL(1)
     DBL(L)=BL(L)-BL(1)
  39 DE(L)=SQRT(DL(L)**2+DAL(L)**2+DBL(L)**2)
  50 CONTINUE
     PRINT 150
 150 FORMAT(/1X,*COMPARING THE PREDICTED VALUES WITH MEASURED VALUES FO
    XR THE REFLECTANCE OF INFINITE THICKNESS*//)
     PRINT 160
 160 FORMAT(41H***IN TERMS OF REFLECTANCE DIFFERENCE****//)
     PRINT 170,(DR(I),I=1,N),(DRA(I),I=1,N),(DRS(I),I=1,N),(DRAS(I),I=1
    X,N),(DRD(I),I=1,N),(DRAD(I),I=1,N)
 170 FORMAT(1X,*DR*,3X,16F8.4/,1X,*DRA*,2X,16F8.4/,1X,*DRS*,2X,16F8.4/,
    X1X,*DRAS*,1X,16F8.4/,1X,*DRD*,2X,16F8.4/,1X,*DRAD*,1X,16F8.4//)
     PRINT 180
 180 FORMAT(31H***IN TERMS OF COLOUR SPACE****)
     PRINT 190
 190 FORMAT(//38X,*C.I.E COORDINATE*,27X,*COLOR DIFFERENCE IN HUNTER UN
    XIT*//,26X,*X*,9X,*Y*,9X,*Z*,7X,*SMLX*,6X,*SMLY*,10X,*DELTA L*,3X,*
    XDELTA A*,3X,*DELTA B*,3X,*DELTA E*//)
     PRINT 210,X(1),Y(1),Z(1),SMLX(1),SMLY(1)
 210 FORMAT(1X,*BY MEASURING *,7X,5F10.5//)
     PRINT 220,X(2),Y(2),Z(2),SMLX(2),SMLY(2),DL(2),DAL(2),DBL(2),DE(2)
 220 FORMAT(1X,*USE RELATIVE VALUES*,1X,5F10.5,6X,4F10.5//)
     PRINT 230,X(3),Y(3),Z(3),SMLX(3),SMLY(3),DL(3),DAL(3),DBL(3),DE(3)
 230 FORMAT(1X,*USE ABSOLUTE VALUES*,1X,5F10.5,6X,4F10.5//)
     PRINT 240,X(4),Y(4),Z(4),SMLX(4),SMLY(4),DL(4),DAL(4),DBL(4),DE(4)
 240 FORMAT(1X,*SAUNDERSON COR. REL.*,5F10.5,6X,4F10.5//)
     PRINT 250,X(5),Y(5),Z(5),SMLX(5),SMLY(5),DL(5),DAL(5),DBL(5),DE(5)
 250 FORMAT(1X,*SAUNDERSON COR. ABS.*,5F10.5,6X,4F10.5//)
     PRINT 260,X(6),Y(6),Z(6),SMLX(6),SMLY(6),DL(6),DAL(6),DBL(6),DE(6)
 260 FORMAT(1X,*DUNCAN COR. REL.*,4X,5F10.5,6X,4F10.5//)
```

```
PROGRAM        PREDICT                                   CDC 6500 FTN V3.0-P380 OPT=1
                   PRINT 270,X(7),Y(7),Z(7),SMLX(7),SMLY(7),DL(7),DAL(7),DBL(7),DE(7)
               270 FORMAT(1X,*DUNCAN COR. ABS.*,4X,5F10.5,6X,4F10.5//)
                   PRINT 280
               280 FORMAT(//16X,*FIG. 5.    COMPARISION OF CALCULATED VALUES IN TERMS
                  X OF REFLECTANCE AND COLOR FOR VARICUS CASES WITH*//,27X,*THE AVERA
                  XGE EXPERIMENTAL VALUES OF INFINITE FILM.THICKNESS FOR
                  XAT     PVC*)
                   PRINT 110,ICODE1,ICODE2,ICODE3
                   PRINT 120,D
                   PRINT 130,(WL(I),I=1,N)
                   PRINT 290
               290 FORMAT(1X,*** USING RELATIVE VALUES*//)
                   PRINT 310,((AK(I),I=1,N),(S(I),I=1,N),(FR(I),I=1,N))
               310 FORMAT(1X,*K*,4X,16F8.4//,1X,*S*,4X,16F8.4//,1X,*FR*,3X,16F8.4///)
                   PRINT 320
               320 FORMAT(1X,*** USING ABSOLUTE VALUES*//)
                   PRINT 330,((AKA(I),I=1,N),(SA(I),I=1,N),(FRA(I),I=1,N))
               330 FORMAT(1X,*KA*,3X,16F8.4//,1X,*SA*,3X,16F8.4//,1X,*FRA*,2X,16F8.4)
                   PRINT 340
               340 FORMAT(///1X,*** USING THE SAUNDERSON CORRECTION WITH RELATIVE VAL
                  XUES*//)
                   PRINT 350,((AKS(I),I=1,N),(SS(I),I=1,N),(FRS(I),I=1,N))
               350 FORMAT(1X,*KS*,3X,16F8.4//,1X,*SS*,3X,16F8.4//,1X,*FRS*,2X,16F8.4)
                   PRINT 360
               360 FORMAT(///1X,*** USING THE SAUNDERSON CORRECTION WITH ABSOLUTE VAL
                  XUES*//)
                   PRINT 370,((AKAS(I),I=1,N),(SAS(I),I=1,N),(FRAS(I),I=1,N))
               370 FORMAT(1X,*KAS*,2X,16F8.4//,1X,*SAS*,2X,16F8.4//,1X,*FRAS*,1X,16F8
                  X.4///)
                   PRINT 380
               380 FORMAT(1X,*** USING DUNCAN METHOD WITH RELATIVE VALUES*//)
               390 FORMAT(1X,*KD*,3X,16F8.4//,1X,*SD*,3X,16F8.4//,1X,*FRD*,2X,16F8.4)
                   PRINT 390,((AKD(I),I=1,N),(SD(I),I=1,N),(FRD(I),I=1,N))
                   PRINT 410
               410 FORMAT(///1X,*** USING DUNCAN METHOD WITH ABSOLUTE VALUES*//)
                   PRINT 420,((AKAD(I),I=1,N),(SAD(I),I=1,N),(FRAD(I),I=1,N))
               420 FORMAT(1X,*KAD*,2X,16F8.4//,1X,*SAD*,2X,16F8.4//,1X,*FRAD*,1X,16F8
                  X.4///)
                   PRINT 430
               430 FORMAT(///11X,*FIG. 5.    ABSORPTION AND SCATTERING COEFFICIENTS A
                  XND K.M FUNCTIONS FOR VARIOUS CASES OF                AT   PVC*)
                   GO TO 1
               999 STOP
                   END
MAP .14.38.28.  NORMAL               CONTROL
   ---TIME---LOAD MODE --L1--L2-----TYPE----------------USER---+----CALL-----------
   FWA LOADER  113742  FWA TABLES  111673
   -PROGRAM----ADDRESS-                        --LABELED---COMMON----ADDRESS-
   PREDICT     000100
   SYSTEM$     013341                                    EXECMSG      013341
                                                         CONECIO      013341
   INPUTC$     014466
   KODER$      014614
   KRAKER$     016236
   OUTPTC$     020013
   ALNLOGE     020107
   SORTE       020146
   GETBA       020170
   SIC$        020207
   ----------END OF MAP----------
```

PROGRAM MIXTURE CDC 6500 FTN V3.0-P380 OPT=1

```
      PROGRAM MIXTURE(INPUT,OUTPUT)
      DIMENSION ICODE6(18),ICODE3(18),XBAR(32),YBAR(32),ZBAR(32),R1(32),
     XR2(32),RSTBA(32),WL(32),REX(32),RN(7,32),INDEX(7,9,32),
     XRA(7,32),RS(7,32),RAS(7,32),RD(7,32),RAD(7,32),AB(7,32),ABA(7,32),
     XABS(7,32),ABAS(7,32),ABD(7,32),ABAD(7,32),SC(7,32),SCA(7,32),
     XSCS(7,32),SCAS(7,32),SCD(7,32),SCAD(7,32),F(7,32),FA(7,32),
     XFS(7,32),FAS(7,32),FD(7,32),FAD(7,32),SEC(32),C(7),XM(20),YM(20),
     XZM(20),SMLXM(20),SMLYM(20),DALM(20),DBLM(20),DLM(20),RR(20,32),
     XALM(20),BLM(20),VLM(20),DEM(20),XDM(20),YDM(20),ZDM(20),INDE(20),
     XINDEXX(20,32)
      INTEGER WL
      DATA N/16/
      DATA ITHE/4HTHE /,ISAM/4HSAMP/,IEND/4HEND /
      READ 100,(XBAR(I),I=1,N)
      READ 100,(YBAR(I),I=1,N)
      READ 100,(ZBAR(I),I=1,N)
      READ 100,(SEC(I),I=1,N)
  100 FORMAT(8F10.4)
C        XBAR,YBAR,AND ZBAR ARE THE C.I.E 1931 STANDARD OBSERVER
C        SEC CONTAINS WEIGHTING FACTORS FOR  C.I.E ILLUMINATING SOURCE C
      READ 200,(RSTBA(I),I=1,N)
C        RSTBA CONTAINS ABS. REFL. OF REFERENCE STANDARD WHITE BASO4
  200 FORMAT(16F5.3)
      FK=1.E-30
      DO 60 I=1,N
      WL(I)=380+20*I
   60 FK=FK+(SEC(I)*YBAR(I))
 1001 READ 400,ICODE4,ICODE5,ICODE6
  400 FORMAT(A4,A4,18A4)
C      ICODES ARE THE NAMES OF SAMPLES AND END OF DATA.
      IF(ICODE4.EQ.ITHE) GO TO 1002
      IF(ICODE4.EQ.IEND) GO TO 1000
 1002 READ 945,M
  945 FORMAT(I5)
C       M IS THE NUMBER OF PIGMENTS TO BE MIXED.
      READ 944,(C(L),L=1,M)
  944 FORMAT(7F10.5)
C       C IS THE CONCENTRATION OF EACH PIGMENT IN THE MIXTURE.
      READ 200,(REX(I),I=1,N)
C       REX IS THE EXPERIMENTAL INFINITE REFLECTANCE OF THE MIXTURE.
      KK=0
    1 READ 400,ICODE1,ICODE2,ICODE3
      READ 500,D
  500 FORMAT(F10.5)
C       D IS THE THICKNESS OF PIGMENTED FILM.
      READ 200,(R1(I),I=1,N)
      READ 200,(R2(I),I=1,N)
C      R1, R2 ARE THE REFLECTANCE OF SINGLE AND DOUBLE FILM THICKNESS ON
C      IDEAL BLACK BACKGROUND.
      RSTBA(0)=1.E-85
      KK=KK+1
C       COMPUTE R,K,S AND K/S FOR INDIVIDUAL PIGMENT.
      DO 11 I=1,N
      RN(KK,I)=1.E-75 $ RA(KK,I)=1.E-75 $ RS(KK,I)=1.E-75
      RD(KK,I)=1.E-75 $ RAS(KK,I)=1.E-83 $ RAD(KK,I)=1.E-85
      INDEX(KK,1,I)=0 $ INDEX(KK,2,I)=0 $ INDEX(KK,3,I)=0
      INDEX(KK,4,I)=0 $ INDEX(KK,5,I)=0 $ INDEX(KK,6,I)=0
      INDEX(KK,7,I)=0
C       USING RELATIVE REFLECTANCE VALUES.
      RC=R2(I)/(2.-R2(I))
      IF(RC.GT.R1(I)) GO TO 101
      RN(KK,I)=(2.*R1(I)-R2(I)+(R1(I)**2)*R2(I)-SQRT((2.*R1(I)-R2(I)+(R1
     X(I)**2)*R2(I))**2-4.*(R1(I)**4)))/(2.*(R1(I)**2))
      F(KK,I)=0.5*(1.-RN(KK,I))**2/RN(KK,I)
      SC(KK,I)=RN(KK,I)/(D*(1.-RN(KK,I)**2))*ALOG(RN(KK,I)*(R1(I)*RN(KK,
     XI)-1.)/(R1(I)-RN(KK,I)))
      AB(KK,I)=SC(KK,I)*F(KK,I)
      GO TO 222
  101 INDEX(KK,2,I)=1
  222 IF(R1(I).LT.0.04) GO TO 331
C       USING THE SAUNDERSON CORRECTION WITH RELATIVE REFLECTANCE VALUES.
      R1S=(R1(I)-0.04)/(.36+.6*R1(I))
      R2S=(R2(I)-0.04)/(.36+.6*R2(I))
      RCS=R2S/(2.-R2S)
      IF(RCS.GT.R1S) GO TO 331
      RS(KK,I)=(2.*R1S-R2S+(R1S**2)*R2S-SQRT((2.*R1S-R2S+(R1S**2)*R2S)**
     X2-4.*(R1S**4)))/(2.*(R1S**2))
      FS(KK,I)=0.5*(1.-RS(KK,I))**2/RS(KK,I)
```

```
      SCS(KK,I)=RS(KK,I)/(D*(1.-RS(KK,I)**2))*ALOG(RS(KK,I)*(R1S*RS(KK,I
     X)-1.)/(R1S-RS(KK,I)))
      ABS(KK,I)=SCS(KK,I)*FS(KK,I)
      GO TO 444
  331 INDEX(KK,3,I)=I
  444 IF(R2(I).GT..96) GO TO 551
C      USING DUNCAN METHOD WITH RELATIVE REFLECTANCE VALUES.
      R1D=R1(I)/(.427+.555*R1(I))
      R2D=R2(I)/(.427+.555*R2(I))
      RCD=R2D/(2.-R2D)
      IF(RCD.GT.R1D) GO TO 551
      RD(KK,I)=(2.*R1D-R2D+(R1D**2)*R2D-SQRT((2.*R1D-R2D+(R1D**2)*R2D)**
     X2-4.*(R1D**4)))/(2.*(R1D**2))
      FD(KK,I)=0.5*(1.-RD(KK,I))**2/RD(KK,I)
      SCD(KK,I)=RD(KK,I)/(D*(1.-RD(KK,I)**2))*ALOG(RD(KK,I)*(R1D*RD(KK,I
     X)-1.)/(R1D-RD(KK,I)))
      ABD(KK,I)=SCD(KK,I)*FD(KK,I)
      GO TO 111
  551 INDEX(KK,4,I)=I
C      USING ABSOLUTE REFLECTANCE VALUES.
  111 R1A=R1(I)*RSTBA(I)
      R2A=R2(I)*RSTBA(I)
      RCA=R2A/(2.-R2A)
      IF(RCA.GT.R1A) GO TO 221
      RA(KK,I)=(2.*R1A-R2A+(R1A**2)*R2A-SQRT((2.*R1A-R2A+(R1A**2)*R2A)**
     X2-4.*(R1A**4)))/(2.*(R1A**2))
      FA(KK,I)=0.5*(1.-RA(KK,I))**2/RA(KK,I)
      SCA(KK,I)=RA(KK,I)/(D*(1.-RA(KK,I)**2))*ALOG(RA(KK,I)*(R1A*RA(KK,I
     X)-1.)/(R1A-RA(KK,I)))
      ABA(KK,I)=SCA(KK,I)*FA(KK,I)
      GO TO 333
  221 INDEX(KK,5,I)=I
  333 IF(R1A.LT.0.04) GO TO 441
C      USING THE SAUNDERSON CORRECTION WITH ABSOLUTE REFLECTANCE VALUES.
      R1AS=(R1A-.04)/(.36+.6*R1A)
      R2AS=(R2A-.04)/(.36+.6*R2A)
      RCAS=R2AS/(2.-R2AS)
      IF(RCAS.GT.R1AS) GO TO 441
      RAS(KK,I)=(2.*R1AS-R2AS+(R1AS**2)*R2AS-SQRT((2.*R1AS-R2AS+(R1AS**2
     X)*R2AS)**2-4.*(R1AS**4)))/(2.*(R1AS**2))
      FAS(KK,I)=0.5*(1.-RAS(KK,I))**2/RAS(KK,I)
      SCAS(KK,I)=RAS(KK,I)/(D*(1.-RAS(KK,I)**2))*ALOG(RAS(KK,I)*(R1AS*RA
     XS(KK,I)-1.)/(R1AS-RAS(KK,I)))
      ABAS(KK,I)=SCAS(KK,I)*FAS(KK,I)
      GO TO 555
  441 INDEX(KK,6,I)=I
  555 IF(R1A.GT.0.96) GO TO 661
C      USING DUNCAN METHOD WITH ABSOLUTE REFLECTANCE VALUES.
      R1AD=R1A/(.427+.555*R1A)
      R2AD=R2A/(.427+.555*R2A)
      RCAD=R2AD/(2.-R2AD)
      IF(RCAD.GT.R1AD) GO TO 661
      RAD(KK,I)=(2.*R1AD-R2AD+(R1AD**2)*R2AD-SQRT((2.*R1AD-R2AD+(R1AD**2
     X)*R2AD)**2-4.*(R1AD**4)))/(2.*(R1AD**2))
      FAD(KK,I)=0.5*(1.-RAD(KK,I))**2/RAD(KK,I)
      SCAD(KK,I)=RAD(KK,I)/(D*(1.-RAD(KK,I)**2))*ALOG(RAD(KK,I)*(R1AD*RA
     XD(KK,I)-1.)/(R1AD-RAD(KK,I)))
      ABAD(KK,I)=SCAD(KK,I)*FAD(KK,I)
      GO TO 11
  661 INDEX(KK,7,I)=I
   11 CONTINUE
      IF(KK.LT.M) GO TO 1
      DO 321 I=1,N
      INDEXX(1,I)=0 $ INDEXX(2,I)=0 $ INDEXX(3,I)=0 $ INDEXX(4,I)=0
      INDEXX(5,I)=0 $ INDEXX(6,I)=0 $ INDEXX(7,I)=0
      INDEXX(8,I)=0 $ INDEXX(9,I)=0 $ INDEXX(10,I)=0 $ INDEXX(11,I)=0
      INDEXX(12,I)=0 $ INDEXX(13,I)=0
      FRMA1=0 $ FRMA2=0 $ FRMAS1=0 $ FRMAS2=0 $ FRMAD1=0 $ FRMAD2=0
      FRM1=0   $  FRM2=0   $  FRMS1=0   $  FRMS2=0   $  FRMD1=0
      FRMD2=0  $  AKM=0   $  AKMS=0   $  AKMD=0  $  SM=1.E-85
      SMS=1.E-85  $  SMD=1.E-85
      AKMA=0 $ AKMAS=0 $ AKMAD=0 $ SMA=1.E-85 $ SMAS=1.E-85 $SMAD=1.E-85
C      COMPUTE R, K, S AND K/S OF THE MIXTURE USING SINGLE CONSTANT AND
C         TWO CONSTANT THEORIES.
      DO 312 J=1,M
      IF(INDEX(J,2,I).EQ.I) GO TO 9021
      FRM1=FRM1+C(J)*F(J,I)
      SM=SM+C(J)*SC(J,I)
```

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      AKM=AKM+C(J)*AB(J,I)
      GO TO 902
 9021 INDEXX(2,I)=I
  902 IF(INDEX(J,3,I).EQ.I) GO TO 9061
      FRMS1=FRMS1+C(J)*FS(J,I)
      SMS=SMS+C(J)*SCS(J,I)
      AKMS=AKMS+C(J)*ABS(J,I)
      GO TO 906
 9061 INDEXX(4,I)=I
  906 IF(INDEX(J,4,I).EQ.I) GO TO 3121
      FRMD1=FRMD1+C(J)*FD(J,I)
      SMD=SMD+C(J)*SCD(J,I)
      AKMD=AKMD+C(J)*ABD(J,I)
      GO TO 907
 3121 INDEXX(6,I)=I
  907 IF(INDEX(J,5,I).EQ.I) GO TO 9071
      FRMA1=FRMA1+C(J)*FA(J,I)
      SMA=SMA+C(J)*SCA(J,I)
      AKMA=AKMA+C(J)*ABA(J,I)
      GO TO 908
 9071 INDEXX(8,I)=I
  908 IF(INDEX(J,6,I).EQ.I) GO TO 9081
      FRMAS1=FRMAS1+C(J)*FAS(J,I)
      SMAS=SMAS+C(J)*SCAS(J,I)
      AKMAS=AKMAS+C(J)*ABAS(J,I)
      GO TO 909
 9081 INDEXX(10,I)=I
  909 IF(INDEX(J,7,I).EQ.I) GO TO 9091
      FRMAD1=FRMAD1+C(J)*FAD(J,I)
      SMAD=SMAD+C(J)*SCAD(J,I)
      AKMAD=AKMAD+C(J)*ABAD(J,I)
      GO TO 312
 9091 INDEXX(12,I)=I
  312 CONTINUE
      IF(INDEXX(2,I).EQ.I) GO TO 912
      RR(2,I)=1.+FRM1-SQRT((1.+FRM1)**2-1.)
      FRM2=AKM/SM
      RR(3,I)=1.+FRM2-SQRT((1.+FRM2)**2-1.)
      GO TO 913
  912 INDEXX(2,I)=I  $  INDEXX(3,I)=I
  913 IF(INDEXX(4,I).EQ.I) GO TO 914
      RINMS1=1.+FRMS1-SQRT((1.+FRMS1)**2-1.)
      RR(4,I)=(.36*RINMS1+.04)/(1.-.6*RINMS1)
      FRMS2=AKMS/SMS
      RINMS2=1.+FRMS2-SQRT((1.+FRMS2)**2-1.)
      RR(5,I)=(.36*RINMS2+.04)/(1.-.6*RINMS2)
      GO TO 915
  914 INDEXX(4,I)=I  $  INDEXX(5,I)=I
  915 IF(INDEXX(6,I).EQ.I) GO TO 916
      RINMD1=1.+FRMD1-SQRT((1.+FRMD1)**2-1.)
      RR(6,I)=.427*RINMD1/(1.-.555*RINMD1)
      FRMD2=AKMD/SMD
      RINMD2=1.+FRMD2-SQRT((1.+FRMD2)**2-1.)
      RR(7,I)=.427*RINMD2/(1.-.555*RINMD2)
      GO TO 917
  916 INDEXX(6,I)=I  $  INDEXX(7,I)=I
  917 IF(INDEXX(8,I).EQ.I) GO TO 918
      RINMA1=1.+FRMA1-SQRT((1.+FRMA1)**2-1.)
      RR(8,I)=RINMA1/RSTBA(I)
      FRMA2=AKMA/SMA
      RINMA2=1.+FRMA2-SQRT((1.+FRMA2)**2-1.)
      RR(9,I)=RINMA2/RSTBA(I)
      GO TO 921
  918 INDEXX(8,I)=I  $  INDEXX(9,I)=I
  921 IF(INDEXX(10,I).EQ.I) GO TO 922
      RIMAS1=1.+FRMAS1-SQRT((1.+FRMAS1)**2-1.)
      RR(10,I)=(.36*RIMAS1+.04)/((1.-.6*RIMAS1)*RSTBA(I))
      FRMAS2=AKMAS/SMAS
      RIMAS2=1.+FRMAS2-SQRT((1.+FRMAS2)**2-1.)
      RR(11,I)=(.36*RIMAS2+.04)/((1.-.6*RIMAS2)*RSTBA(I))
      GO TO 923
  922 INDEXX(10,I)=I  $  INDEXX(11,I)=I
  923 IF(INDEXX(12,I).EQ.I) GO TO 924
      RIMAD1=1.+FRMAD1-SQRT((1.+FRMAD1)**2-1.)
      RR(12,I)=.427*RIMAD1/((1.-.555*RIMAD1)*RSTBA(I))
      FRMAD2=AKMAD/SMAD
      RIMAD2=1.+FRMAD2-SQRT((1.+FRMAD2)**2-1.)
      RR(13,I)=.427*RIMAD2/((1.-.555*RIMAD2)*RSTBA(I))
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              GO TO 925
          924 INDEXX(12,I)=I   $   INDEXX(13,I)=I
          925 RR(I,I)=REX(I)
          321 CONTINUE
C               TRANSFORM REFLECTANCE SPECTRA INTO THE CIE COLOR COORDINATE
              MM=13
              DO 80 K=1,MM
              XDM(K)=0   $   YDM(K)=0   $   ZDM(K)=0
              YM(0)=1.E-35
              INDE(K)=0
              DO 81 I=1,N
              IF(INDEXX(K,I).EQ.I) GO TO 801
              XDM(K)=XDM(K)+(RR(K,I)*SEC(I)*XBAR(I))
              YDM(K)=YDM(K)+(RR(K,I)*SEC(I)*YBAR(I))
              ZDM(K)=ZDM(K)+(RR(K,I)*SEC(I)*ZBAR(I))
           81 CONTINUE
              GO TO 802
          801 INDE(K)=K        $    GO TO 80
          802 XM(K)=100*XDM(K)/FK
              YM(K)=100*YDM(K)/FK
              ZM(K)=100*ZDM(K)/FK
              SMLXM(K)=XM(K)/(XM(K)+YM(K)+ZM(K))
              SMLYM(K)=YM(K)/(XM(K)+YM(K)+ZM(K))
C               TRANSFORM THE CIE COLOR COORDINATE SYSTEM INTO HUNTER COLOR UNITS
              ALM(K)=17.5*(1.02*XM(K)-YM(K))/SQRT(YM(K))
              BLM(K)=7.0*(YM(K)-.847*ZM(K))/SQRT(YM(K))
              VLM(K)=10.*SQRT(YM(K))
           80 CONTINUE
C               COMPUTE COLOR DIFFERENCE BETWEEN CALCULATED AND EXPERIMENTAL
C                 VALUES IN TERMS OF HUNTER COLOR DIFFERENCE UNITS.
              DLM(1)=0   $   DALM(1)=0   $   DBLM(1)=0   $   DEM(1)=0
              DO 82 K=2,MM
              IF(INDE(K).EQ.K) GO TO 82
              DLM(K)=VLM(K)-VLM(1)
              DALM(K)=ALM(K)-ALM(1)
              DBLM(K)=BLM(K)-BLM(1)
              DEM(K)=SQRT(DLM(K)**2+DALM(K)**2+DBLM(K)**2)
           82 CONTINUE
              PRINT 821
          821 FORMAT(1H1,///////////)
              PRINT 822
          822 FORMAT(10X,*PREDICTED REFLECTANCE OF THE MIXTURE*)
              PRINT 130,(WL(I),I=1,N)
          130 FORMAT(//1X,10X,*WL*,2X,16I7//)
              PRINT 823,((RR(1,I),I=1,N),(RR(2,I),I=1,N),(RR(3,I),I=1,N),(RR(4,I
             X),I=1,N),(RR(5,I),I=1,N),(RR(6,I),I=1,N),(RR(7,I),I=1,N),(RR(8,I),
             XI=1,N),(RR(9,I),I=1,N),(RR(10,I),I=1,N),(RR(11,I),I=1,N),(RR(12,I)
             X,I=1,N),(RR(13,I),I=1,N))
          823 FORMAT(10X,*REX*,2X,16F7.4/,10X,*RINM1*,16F7.4/,10X,*RINM2*,16F7.4
             X/,10X,*RMS1*,1X,16F7.4/,10X,*RMS2*,1X,16F7.4/,10X,*RMD1*,1X,16F7.4
             X/,10X,*RMD2*,1X,16F7.4/,10X,*RMA1*,1X,16F7.4/,10X,*RMA2*,1X,16F7.4
             X/,10X,*RMAS1*,16F7.4/,10X,*RMAS2*,16F7.4/,10X,*RMAD1*,16F7.4/,10X,
             X*RMAD2*,16F7.4//)
              PRINT 180
          180 FORMAT(41H                 ****IN TERMS OF COLOR SPACE****)
              PRINT 190
          190 FORMAT(//52X,*C.I.E COORDINATE*,25X,*COLOR DIFFERENCE IN HUNTER UN
             XIT*//,40X,*X*,9X,*Y*,9X,*Z*,7X,*SMLX*,6X,*SMLY*,8X,*DELTA L*,3X,*D
             XELTA A*,3X,*DELTA B*,3X,*DELTA E*//)
              PRINT 826,XM(1),YM(1),ZM(1),SMLXM(1),SMLYM(1)
          826 FORMAT(10X,*BY EXPERIMENT*,11X,5F10.5/)
              PRINT 827,XM(2),YM(2),ZM(2),SMLXM(2),SMLYM(2),DLM(2),DALM(2),DBLM(
             X2),DEM(2)
          827 FORMAT(10X,*ONE CONST. REL.*,9X,5F10.5,3X,4F10.5/)
              PRINT 829,XM(3),YM(3),ZM(3),SMLXM(3),SMLYM(3),DLM(3),DALM(3),DBLM(
             X3),DEM(3)
          829 FORMAT(10X,*TWO CONST. REL.*,9X,5F10.5,3X,4F10.5/)
              PRINT 830,XM(4),YM(4),ZM(4),SMLXM(4),SMLYM(4),DLM(4),DALM(4),DBLM(
             X4),DEM(4)
          830 FORMAT(10X,*ONE CONST. SAUND. REL.*,2X,5F10.5,3X,4F10.5/)
              PRINT 831,XM(5),YM(5),ZM(5),SMLXM(5),SMLYM(5),DLM(5),DALM(5),DBLM(
             X5),DEM(5)
          831 FORMAT(10X,*TWO CONST. SAUND. REL.*,2X,5F10.5,3X,4F10.5/)
              PRINT 832,XM(6),YM(6),ZM(6),SMLXM(6),SMLYM(6),DLM(6),DALM(6),DBLM(
             X6),DEM(6)
          832 FORMAT(10X,*ONE CONST. DUN. REL.*,4X,5F10.5,3X,4F10.5/)
              PRINT 833,XM(7),YM(7),ZM(7),SMLXM(7),SMLYM(7),DLM(7),DALM(7),DBLM(
             X7),DEM(7)
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PROGRAM       MIXTURE                                    CDC 6500 FTN V3.0-P380 OPT=1

          833 FORMAT(10X,*TWO CONST. DUN. REL.*,4X,5F10.5,3X,4F10.5/)
              PRINT 834,XM(8),YM(8),ZM(8),SMLXM(8),SMLYM(8),DLM(8),DALM(8),DBLM(
             X8),DEM(8)
          834 FORMAT(10X,*ONE CONST. ABS.*,9X,5F10.5,3X,4F10.5/)
              PRINT 835,XM(9),YM(9),ZM(9),SMLXM(9),SMLYM(9),DLM(9),DALM(9),DBLM(
             X9),DEM(9)
          835 FORMAT(10X,*TWO CONST. ABS.*,9X,5F10.5,3X,4F10.5/)
              PRINT 836,XM(10),YM(10),ZM(10),SMLXM(10),SMLYM(10),DLM(10),DALM(10
             X),DBLM(10),DEM(10)
          836 FORMAT(10X,*ONE CONST. SAUND. ABS.*,2X,5F10.5,3X,4F10.5/)
              PRINT 837,XM(11),YM(11),ZM(11),SMLXM(11),SMLYM(11),DLM(11),DALM(11
             X),DBLM(11),DEM(11)
          837 FORMAT(10X,*TWO CONST. SAUND. ABS.*,2X,5F10.5,3X,4F10.5/)
              PRINT 838,XM(12),YM(12),ZM(12),SMLXM(12),SMLYM(12),DLM(12),DALM(12
             X),DBLM(12),DEM(12)
          838 FORMAT(10X,*ONE CONST. DUN. ABS.*,4X,5F10.5,3X,4F10.5/)
              PRINT 839,XM(13),YM(13),ZM(13),SMLXM(13),SMLYM(13),DLM(13),DALM(13
             X),DBLM(13),DEM(13)
          839 FORMAT(10X,*TWO CONST. DUN. ABS.*,4X,5F10.5,3X,4F10.5/)
              PRINT 844
          844 FORMAT(////40X,*FIG. 4.    COMPARISION OF PREDICTED VALUES WITH THE
             X EXPERIMENTAL VALUES*//)
              PRINT 845,ICODE4,ICODE5,ICODE6
          845 FORMAT(50X,A4,A4,18A4)
              GO TO 1001
         1000 STOP
              END
```