# THERMAL PROPERTIES OF GRAIN

Thesis for the Degree of Ph. D.

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Edward Arshak Kazarian

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This is to certify that the

thesis entitled

Thermal Properties of Grain

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Edward Arshak Kazarian

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# THERMAL PROPERTIES OF GRAIN

By

Edward Arshak Kazarian

AN ABSTRACT

Submitted to
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ABSTRACT

THERMAL PROPERTIES OF GRAIN

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The specific heat, thermal conductivity and thermal diffusivity of a few grains have been reported but the information is usually limited to a narrow range of moisture content. A knowledge of the thermal properties is necessary before heat-transfer analysis can be used for problems encountered in drying, storing, aeration and refrigeration of grain. The objective of this study was to determine the specific heat, thermal conductivity and thermal diffusivity of soft white wheat and yellow dent corn and to relate these properties on the basis of moisture content and temperature.

The specific heats of the wheat and corn were determined by standard calorimetric tests. A transient heat flow method with a line source of heat was used to determine the thermal conductivity of the grain. The thermal diffusivity was also determined by transient heat flow. Transient heat flow methods of determining thermal properties of hygroscopic materials minimize the problem of moisture migration or moisture changes in the material.

The thermal properties of soft white wheat were determined at moisture contents ranging from 0 to 20%. The thermal properties of yellow dent corn were determined at moisture contents ranging from 0 to 30%.

The specific heats of both wheat and corn were found to increase linearly with moisture content. The regression equations were determined. The specific heat of dry wheat was 0.334 Btu/lb-°F. The specific heat of dry corn was 0.350 Btu/lb-°F. There was no significant difference in the specific heat for a difference in mean temperature of 20°F.

The thermal conductivity also was found to increase linearly with moisture content for both grains. The thermal conductivity of dry wheat was 0.0676 Btu/hr-ft-°F. The thermal conductivity of dry corn was 0.0814 Btu/hr-ft-°F.

The thermal diffusivity of wheat decreased with increasing moisture content, but was not a linear function of the moisture content. The thermal diffusivity of dry wheat was 0.00359 sq-ft/hr. The thermal diffusivity of corn decreased with increasing moisture content up to 20%. For moisture contents above 20% the thermal diffusivity of corn increased with increasing moisture content. The thermal diffusivity of dry corn was 0.00395 sq-ft/hr.

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NOMENCLATURE

$\alpha$ = thermal diffusivity, sq-ft/hr

c = specific heat, Btu/lb-$^{o}$F

e = base of natural logarithm

k = thermal conductivity, Btu/hr-ft-$^{o}$F

M = moisture content, % w.b.

n = $\frac{1}{2}(\alpha\theta)^{-\frac{1}{2}}$, ft$^{-1}$

$\phi$ = temperature correction term, $^{o}$F

Q = quantity of heat input, Btu/hr-ft

r = radius of temperature measurement, ft

$\rho$ = density, lb/cu-ft

t = temperature, $^{o}$F

$t_c$ = center temperature of slab, $^{o}$F

$t_o$ = initial temperature, $^{o}$F

$\Delta t$ = temperature difference, $^{o}$F

$\theta$ = time, hr

w = weight, gm

w.b. = wet basis

x = distance, ft

z = $\alpha\theta/x^2$, dimensionless

# INTRODUCTION

Grain drying and storage are of prime importance to the agricultural industry. Each year large quantities of grain are dried and put into storage. Many of the problems encountered in drying and storing may be analyzed by using heat-transfer principles. For example, temperature changes in a grain bin due to external or internal temperature changes may be calculated by use of the basic heat-transfer equations. The use of these equations necessitates a knowledge of the thermal properties of the grains. The specific heat, thermal conductivity, and thermal diffusivity must be known before the equations of steady-state and transient heat flow can be used. Consideration of heat-transfer is not limited to problems of grain drying and storing but may be used for analysis of other processes, such as aeration and refrigeration.

Some information is available on the thermal properties of grain but it is scattered and often available only for a narrow range of moisture content. The thermal properties of hard wheat which is raised in Canada and the Dakotas have been investigated rather thoroughly, but very little work has been done with other grains. There is almost a complete lack of information on the thermal properties of yellow dent corn.

The thermal properties of hygroscopic materials are influenced by temperature and moisture content. The effect of temperature on the specific heat of wheat starch was reported by Rodewald and Kattein (1900). It was found that the specific heat of the wheat starch increased with temperature by 0.46% per degree centigrade. However, an increase of one % in the moisture content of the wheat starch resulted in a 2.50% increase in the specific heat. Therefore it was decided first to investigate the effect of moisture content on the thermal properties of grain. The secondary consideration would then be to determine the effect of temperature on the thermal properties of grain.

Soft white winter wheat and yellow dent corn are the grains most commonly grown in Michigan. Since no information is available on the thermal properties of soft white wheat, and only one value for the thermal conductivity of yellow dent corn has been reported, these two grains were used for the investigation.

Therefore, the objective of this investigation was to determine the specific heat, thermal conductivity, and thermal diffusivity of soft white wheat and yellow dent corn, and to relate these properties on the basis of moisture content and temperature.

The specific heat, thermal conductivity and thermal diffusivity were measured directly. The measured thermal diffusivity was compared to the thermal diffusivity

calculated from the measured values of the specific heat, thermal conductivity and density by the relationship:

$$\alpha = k/c\rho$$

Where $\alpha$ = thermal diffusivity, sq-ft/hr

$k$ = thermal conductivity, Btu/hr-ft-$^{\circ}$F

$c$ = specific heat, Btu/lb-$^{\circ}$F

$\rho$ = density, lb/cu-ft

Comparison of the measured thermal diffusivity with the calculated values has not been done previously.

## REVIEW OF LITERATURE

The thermal conductivity of oats was reported by Bakke and Stiles (1935). A steady state apparatus with oats placed between two concentric cylinders was used. The temperature difference was obtained by placing ice in the inner cylinder and then placing both cylinders into a constant temperature hot water bath. The heat flow was determined by measuring the amount of ice melted during the test.

Three series of tests were reported. For the first series, the thickness of the oats between the two cylinders was 0.94 cm. The samples were conditioned by addition of water to moisture contents ranging from 9.06 to 27.7% w.b.* The density of the oats was kept constant by using the same weight of grain to fill the area between the cylinders. For the first series of tests the thermal conductivity ranged from 0.0370 Btu/hr-ft-$^{\circ}$F for the oats at 9.06% moisture content to 0.0537 for the oats at 27.7% moisture content. The thermal conductivity was found to be a linear function of the moisture content.

$$* \text{ w.b. (wet basis)} = \frac{\text{moisture wt}}{\text{dry wt} + \text{moisture wt}} \times 100, \%$$

All reference to moisture content is given on a wet basis

For the second series of tests, the oats were uniformly filled into the area between the two cylinders with no attempt to keep the density constant. The reported values of the thermal conductivity ranged from 0.0370 Btu/hr-ft-$^{o}$F for oats at 8.70% to 0.0594 for the oats at 24.2%. The thermal conductivity data from the second series of tests were again presented as a linear function of moisture content.

The slope of the linear function obtained in the second series was greater than that obtained in the first series.

For the third series of tests, the sample thickness was 1.48 cm instead of 0.94 cm as used in the first two series. The thermal conductivity was again found to be a linear function of the moisture content and was in close agreement with the data obtained from the second series of tests. The authors concluded that the thermal conductivity of oats increases linearly with moisture content, but did not present an equation.

The thermal conductivity of wheat, maize and oats was reported by Oxley (1944). The grain samples were placed between two concentric spheres. The inner sphere was equipped with a heating element which provided the heat flow across the grain. After the steady-state was reached, the temperature difference across the grain was measured and used to calculate the thermal conductivity. The thermal conductivities of two samples of wheat were reported as 0.0872 Btu/hr-ft-$^{o}$F at a moisture content of 11.7% and 0.0920

for wheat at 17.8% moisture content. For the single sample of yellow maize at 13.2% moisture content, the thermal conductivity was reported as 0.102 Btu/hr-ft-$^{o}$F. Oats at 12.5% moisture content had a thermal conductivity of 0.075 Btu/hr-ft-$^{o}$F.

The thermal properties of hard wheat were reported by Babbitt (1945). The specific heat of a single sample was calculated from measured values of the thermal conductivity, thermal diffusivity and density. The wheat, at 9.2% moisture content with a bulk density of 53 lb/cu-ft, was enclosed in a metal cylinder. The sample was then heated with a wire stretched along the axis of the cylinder. The measured value of the thermal conductivity was reported as 0.0871 Btu/hr-ft-$^{o}$F, and the thermal diffusivity as 0.00446 sq-ft/hr. The specific heat as calculated from the relationship $c = k/\alpha\rho$ was found to be 0.370 Btu/lb-$^{o}$F. The temperature of the wheat varied from 79$^{o}$F to as high as 120$^{o}$F during the experiment.

The specific heat of hard wheat at different moisture contents was reported by Pfalzner (1951). The method of mixtures was used with 5 gram samples of wheat enclosed in a small copper capsule and dropped into water in a calorimeter. The water in the calorimeter had been cooled to about 34$^{o}$F and the wheat was held at room temperature before being dropped into the calorimeter. This would indicate a mean temperature of approximately 50$^{o}$F for the

wheat during the tests. The specific heats of 3 samples of wheat were determined at moisture contents ranging from 0 to 16%. Samples A and B were from the same year, and sample C from a different year. The specific heat was linearly dependent on moisture content and the results for the 3 samples were expressed as;

Sample A, $c = 0.283 + 0.00724\ M$

Sample B, $c = 0.301 + 0.00733\ M$

Sample C, $c = 0.288 + 0.00828\ M$

where c is the specific heat of the wheat and M is the moisture content, %, w.b. Pfalzner concluded that the apparent specific heat of bound water does not differ appreciably from that of free water over the range of moisture contents used.

Moote (1953), using the same method and procedure as Babbitt, studied the effect of moisture content on the thermal properties of hard wheat. The thermal conductivity, thermal diffusivity and density were measured for 2 samples of wheat. The moisture content of sample A was varied from 1.4% to 7.4% by intermittently adding water to the wheat. The moisture content of sample B was varied by intermittently drying from 13.6% to 5.3%. The specific heat was calculated from the measured values of the thermal conductivity, thermal diffusivity and the density.

The thermal conductivity was reported to vary linearly

with moisture content for both samples of wheat, with sample B having a slightly lower value than sample A. The conductivity values of sample A ranged from 0.0755 Btu/hr-ft-$^{o}$F at 1.4% moisture to 0.0874 at 7.4% moisture content. The thermal conductivity values for sample B varied from 0.0794 at 5.3% moisture content to 0.0967 at 13.6% moisture.

The calculated values of the specific heat were found to vary linearly with the moisture content. The specific heat of sample A ranged from 0.319 at 1.4% moisture to 0.407 at 7.4% moisture content. For sample B the specific heat varied from 0.357 at 5.3% moisture content to 0.460 at 13.6% moisture content. No attempt was made to relate the thermal diffusivity of the wheat to the moisture content.

Moote concluded that the thermal properties of dry wheat are as follows: thermal conductivity, 0.0726 Btu/hr-ft-$^{o}$F; thermal diffusivity, 0.00465 sq-ft/hr; and specific heat, 0.31 Btu/lb-$^{o}$F.

Disney (1954) reported the specific heat of Manitoba and Bersee hard wheat as measured by an ice calorimeter. The method used was to drop a known weight of the wheat at room temperature into the ice calorimeter and to measure the amount of ice melted.

The specific heat of the Manitoba wheat was measured

at seven different moisture contents ranging from 1.29% to 17.5%. The moisture content was changed by intermittently drying the wheat. Values of the specific heat reported ranged from 0.310 Btu/lb-$^{o}$F at 1.29% moisture content to 0.447 at 17.5% moisture content.

Specific heat measurements at twenty different moisture contents were reported for the Bersee wheat. Fifteen of the tests were run at moisture contents obtained by intermittently drying the wheat from 33.6 to 0.14%. Five tests were run after the moisture content of the sample was changed from 0.14 to 18.2% by the addition of water. The specific heat after conditioning the wheat to the various moisture levels by desorption was found to vary from 0.582 Btu/lb-$^{o}$F at 33.6% moisture content to 0.307 at 0.14% moisture. Conditioning the wheat by absorption gave specific heat values that ranged from 0.306 at 1.99% moisture content to 0.457 at 18.2% moisture content.

Disney did not express his data on specific heat as a linear relationship with moisture content.

Haswell (1954), using the same method and procedure as Disney, reported the specific heat of rice, oats and groats. For rough rice at moisture contents of 10.2, 13.5 and 17.0%, he expressed the specific heat by the regression equation: $c = 0.265 + 0.0107 M$. For shelled rice at 9.8, 14.5 and 17.6% moisture content, the data were expressed

by the equation: $c = 0.287 + 0.0091\ M$. Determining the specific heat of finished rice at moisture contents of 10.8, 14.6, and 17.4 gave the equation: $c = 0.282 + 0.009\ M$. Working with oats at 11.7, 14.8, and 17.8% moisture content, the specific heat was given by: $c = 0.305 + 0.0078\ M$. For groats at 11.8, and 17.6% moisture content, the linear relationship was reported as $c = 0.257 + 0.0119\ M$.

A transient heat flow method of determining thermal conductivities was reported by Hooper and Chang (1953). The method is based on using a line heat source of constant strength in a homogeneous, initially isothermal material. By measuring the amount of heat supplied, the temperature rise near the heat source and the time of heating, the thermal conductivity can be determined from the equation for heat flow from a line source of heat. Among the thermal conductivities of various materials that were reported was a single value of 0.077 Btu/hr-ft-$^{o}$F for wheat. The moisture content of the wheat was not specified.

The development of the transient heat flow method of determining the thermal conductivity of materials was reported in detail by Hooper and Lepper (1950). A thorough discussion of the apparatus and procedure and the solution to the basic equation was presented.

D'Eustachio and Schriener (1952) used the method of

heat flow from a line source to determine the thermal conductivity of cellular glass and silica gel. Their results were the same as results obtained using the guarded hot plate.

Lentz (1952) and DeVries (1952) individually reported on the thermal conductivities of soils as determined by using the line source of heat.

The reported thermal properties of grain are summarized in Table 1.

Table 1. Reported thermal properties of grain.

| Grain | Moisture content, % w.b. | Temperature, °F | Density, lb/cu-ft | Specific heat, Btu/lb-°F | Thermal conductivity, Btu/hr-ft-°F | Thermal diffusivity, sq-ft/hr |
|---|---|---|---|---|---|---|
| Corn (12)* | 13.2 | 80 to 88 | | | 0.102 | |
| Oats (2) | 8.7 | | | | 0.037 | |
| | 24.2 | | | | 0.0594 | |
| Oats (6) | 0 | | | 0.305 | | |
| Oats (12) | 12.5 | | | | 0.075 | |
| Rice (6) | 0 | | | 0.265 | | |
| Wheat (12) | 11.7 | 80 to 88 | | | 0.0872 | |
| | 17.8 | 80 to 88 | | | 0.0920 | |
| Wheat (1) | 9.2 | 79 to 120 | 53.0 | 0.370 | 0.0871 | 0.00446 |
| Wheat (13) | 0 | | | 0.283 | | |
| Wheat (11) | 1.4 | | 52.5 | 0.319 | 0.0755 | 0.00446 |
| | 7.4 | | 53.6 | 0.407 | 0.0874 | 0.00407 |
| Wheat (5) | 0.14 | | | 0.307 | | |
| (Bersee) | 4.2 | | | 0.322 | | |
| | 13.7 | | | 0.405 | | |
| | 19.9 | | | 0.476 | | |
| | 25.8 | | | 0.525 | | |
| | 33.6 | | | 0.582 | | |
| (Manitoba) | 1.29 | | | 0.310 | | |
| | 4.9 | | | 0.333 | | |
| | 10.1 | | | 0.367 | | |
| | 17.5 | | | 0.447 | | |
| Wheat (8) | | | | | 0.077 | |

* Numbers refer to the references.

# METHOD AND APPARATUS

## Specific Heat

The standard method of determining the specific heat of materials is by using a calorimeter. The material is heated to a constant temperature and then dropped into water at a known temperature in the calorimeter cup. This method is referred to as the method of mixtures. The temperature rise of the water is recorded until thermal equilibrium is reached. The weight of the sample, calorimeter cup and water are also measured. Then the specific heat of the material is calculated by equating the heat content of the material to that of the calorimeter cup and the water. Any factors that would affect the temperature measured in the calorimeter such as heat gain or loss, heat of stirring, etc., are combined into a temperature correction term. The equation for the method of mixtures is:

$$w_1c_1(\Delta t_1 + \phi) = w_2c_2(\Delta t_2 - \phi) + w_3c_3(\Delta t_2 - \phi)$$

where: $w_1$ = weight of the sample, gm

$c_1$ = specific heat of the sample, Btu/lb-$^{o}$F

$\Delta t_1$ = temperature change of the sample, $^{o}$F

$\phi$ = temperature correction term, $^{o}$F

$w_2$ = weight of the calorimeter cup, gm

$c_2$ = specific heat of the calorimeter cup, Btu/lb-$^{o}$F

$\Delta t_2$ = temperature change of calorimeter cup and water, $^{\circ}F$

$w_3$ = weight of water, gm

$c_3$ = specific heat of water, Btu/lb-$^{\circ}F$

The specific heat of the sample is calculated from the equation;

$$c_1 = \frac{(w_2c_2 + w_3c_3)\ (\Delta t_2 - \phi)}{w_1(\Delta t_1 + \phi)}$$

The specific heat determinations of grain were carried out with a Dewar-flask calorimeter. The calorimeter was further insulated with two inches of fiber-glass insulation to reduce the temperature correction for heat gain or loss from the surroundings.

The weights of the sample, calorimeter cup and the water were measured with a balance graduated to 0.01 gm. The temperatures of the sample and water were measured and recorded with a 30 gauge copper-Constantan thermocouple and a recording potentiometer. The recording potentiometer was set to record every 30 seconds. The apparatus for the specific heat measurements is shown in Fig. 1.

## Thermal Conductivity

The majority of the reported values for thermal conductivity of grain have been determined by steady-state heat flow across the grain. The objections to steady-state measurements are: (1) the long time required to attain

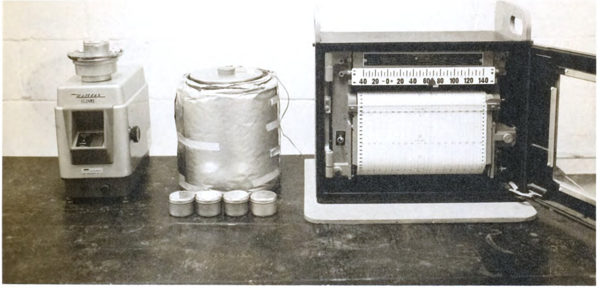


Fig. 1. Apparatus for determining the specific heat of grain.

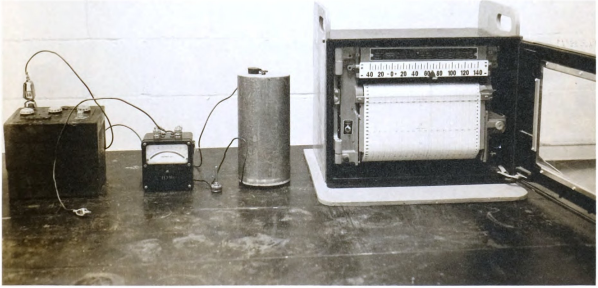


Fig. 2. Apparatus for determining the thermal conductivity of grain.

the steady-state conditions; and (2) the possibility of moisture migration due to maintaining the temperature difference across the grain for long periods of time. Both of these difficulties can be minimized by transient heat flow methods.

The method chosen for determining the thermal conductivity of grain was the transient heat flow in an infinite mass, initially at a uniform temperature, heated by a line heat source of constant strenght. The basic equation for the heat flow from a line source is:

$$\frac{\partial t}{\partial \theta} = \alpha \left( \frac{\partial^2 t}{\partial r^2} + \frac{1}{r} \frac{\partial t}{\partial r} \right)$$

where; $t$ = temperature at radius $r$

$\theta$ = time

$r$ = radius from the heat source

$\alpha$ = thermal diffusivity of the material

The solution for the temperature is given by Hooper and Lepper (1950) as;

$$t = \frac{Q}{2\pi k} I(rn)$$

where $t$ = temperature, $^{\circ}F$

$Q$ = heat input, Btu/hr-ft

$k$ = thermal conductivity of the material, Btu/hr-ft-$^{\circ}F$

$r$ = distance from heat source, ft

$n = \frac{1}{2}(\alpha \theta)^{-\frac{1}{2}}$, $ft^{-1}$

The function $I(rn) = A - \log_e (rn) + \frac{(rn)^2}{2} - \frac{(rn)^4}{8} + \ldots$

where A is a constant. If (rn) is sufficiently small, all the terms of the series except the first two may be dropped. Then $I(rn) = A - \log_e (rn)$, and the temperature is given by:

$$t = \frac{Q}{2\pi k}\left[A - \log_e (rn)\right]$$

The temperature rise between times $\theta_1$ and $\theta_2$ is given by:

$$t_2 - t_1 = \frac{Q}{4\pi k} \log_e (\theta_2/\theta_1)$$

where: $t_1$ = temperature at time $\theta_1$

$t_2$ = temperature at time $\theta_2$

and finally:

$$k = \frac{Q \log_e (\theta_2/\theta_1)}{4\pi(t_2 - t_1)}$$

The thermal conductivity is then calculated from the measured quantities of Q, $\theta_1$, $\theta_2$, $t_1$, and $t_2$.

The apparatus for the thermal conductivity tests consisted of a hollow brass cylinder 11 inches high and 5½ inches in diameter. The wall thickness of the cylinder was 1/32 inch. Both ends of the cylinder were plugged with 3/4 inch thick pieces of wood. A 6½ inch length of bare resistance heating wire was stretched between copper leads on the axis of the cylinder. The heater wire was surrounded by 2 3/4 inches of grain in the radial direction and 1 1/2

inches in the axial direction. A heater wire with a resistance of 5.45 ohm/ft was used.

Power for the heater wire was supplied by a 6 volt storage battery. A variable resistor was used to control the flow of current in the circuit. The current was measured by a 0-1 amp direct current ammeter graduated in 0.01 amp. A 30 gauge copper-Constantan thermocouple was secured to the middle of the heater wire. The thermocouple was insulated from the bare heater wire by a single layer of plastic electrical tape at the point of attachment. The thermocouple was placed at approximately 1/64 inch from the heater wire. Temperatures were again recorded with the recording potentiometer. The apparatus for the thermal conductivity tests is shown in Fig. 2.

A discussion of the possible sources of error using this method of determining the thermal conductivity of materials is given by Hooper and Lepper (1950). There are two sources of error that may affect the results. The first is the effect of dropping the terms in the I series and the second arises from the fact that the heat source is finite in diameter and length. For the apparatus constructed for these tests, the error caused by dropping the terms in the I series can be shown to be negligible. The radius of temperature measurement r, is 1/64 inch or 0.00130 ft. If the thermal diffusivity of the grain is assumed to be 0.004 sq-ft/hr, and the time is arbitrarily chosen as 1 minute,

n is found from the relation: $n = \frac{1}{2}(\alpha\theta)^{-\frac{1}{2}}$ to be 61.3 $ft^{-1}$. The quantity (rn) is then = 0.00130(61.3) = 0.0797. The quantity $(rn)^2/2$ = 0.00317. Dropping this term would cause an error of 0.125% which is within the limits of experimental error. The higher powers of (rn) would be smaller than the second power of (rn) and can also be dropped from the series.

The error caused by the finite heat source is determined from the test data and is discussed in the results of the thermal conductivity tests.

## Thermal Diffusivity

The method chosen for determining the thermal diffusivity of grain was transient heat flow in a slab, initially at a uniform temperature, with the faces suddenly lowered to and held at zero. The basic equation is;

$$\frac{\partial t}{\partial \theta} = \alpha \frac{\partial^2 t}{\partial x^2}$$

where: t = temperature

θ = time

α = thermal diffusivity

x = distance from face of the slab

The solution of the equation for the temperature at the center of the slab is given by Ingersoll, L.R., Zobel, O.J. and Ingersoll, A.C. (1954) as: $t_c = t_o\ S(z)$

where: $t_c$ = temperature at center of the slab, °F

$t_o$ = initial temperature of the slab, °F

$$S(z) = 4/\pi \left(e^{-\pi^2 z} - 1/3 e^{-9\pi^2 z} + 1/5 e^{-25\pi^2 z} - \ldots\right)$$

$z = \alpha\theta/x^2$, dimensionless

The series S(z) has been evaluated for values of z and is also given in Ingersoll, L.R., Zobel, O.J. and Ingersoll, A.C. Therefore, for selected values of $S(z) = t_c/t_o$, the values of z can be obtained, and with the measured values of $\theta$ and $x^2$, the diffusivity is calculated from the relation, $\alpha = zx^2/\theta$ .

The thermal diffusivity tests were conducted by enclosing the grain in a rectangular box. Three different size boxes were made to determine if the size of the sample had any effect on the measured thermal diffusivity. The inside dimensions of box #1 were 1.95 inches thick, 6 inches wide and 6 inches high. Box #2 had inside dimensions of 1.35 x 6 x 6 inches and box #3 was made with inside dimensions of 1.35 x 9 x 9 inches. The boxes were made by fastening two sheets of 24 gauge (0.025 inch) copper to a wood framework. A nylon thread was stretched at the center of the rectangular area and three 30 gauge copper-Constantan thermocouples were mounted on the thread. The thermocouples were located at the ¼ points along the thread. All the joints in the boxes were water-proofed with a mastic and the boxes were tested for leaks under water. The thermocouple wires leading from the boxes were wrapped with water-proof tape. A hole was provided in the top of the

boxes for filling and removing of the grain sample. The three boxes are shown in Fig. 3.

The temperatures at the center of the boxes were again recorded with the recording potentiometer. The apparatus consisting of one of the boxes and the potentiometer is shown in Fig. 4.

Fig. 3. The rectangular boxes used for the thermal diffusivity determinations.

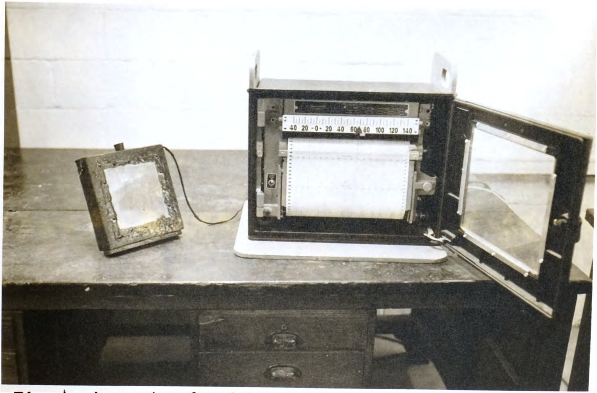


Fig. 4. Apparatus for determining the thermal diffusivity of grain.

# PROCEDURE

## Temperature Measurement

The accuracy of the thermocouple and recording potentiometer was checked against a second thermocouple and a manual potentiometer. Both thermocouples were made from the same spool of wire and were of equal length. The two thermocouples were immersed in ice water and the recording potentiometer was adjusted to read the same temperature as obtained from the manual potentiometer. Then temperatures were read at intervals from both potentiometers while the water warmed up to room temperature. The results of tests on four different thermocouples used with the recording potentiometer are shown in Table 2.

The temperatures measured over a 60 $^{o}$F range with the recording potentiometer were found to vary from 0.2 to 0.3 $^{o}$F from those measured with the manual potentiometer. Within a range of 15 to 20 $^{o}$F, the temperatures measured with the recording potentiometer varied by 0.1 $^{o}$F from the manual potentiometer. The thermal properties of grain were based on equations requiring only temperature differences so the recording potentiometer was sufficiently accurate .

Table 2. Comparison of temperatures measured with recording potentiometer and manual potentiometer.

| Temperature °F | | | | | | | |
|---|---|---|---|---|---|---|---|
| Thermocouple 1 | | Thermocouple 2 | | Thermocouple 3 | | Thermocouple 4 | |
| Man. | Record. | Man. | Record. | Man. | Record. | Man. | Record. |
| 32.1 | 32.1 | 32.1 | 32.1 | 32.1 | 32.1 | 32.1 | 32.1 |
| 32.1 | 32.1 | 32.6 | 32.6 | 32.1 | 32.1 | 32.1 | 32.1 |
| 32.1 | 32.1 | 34.7 | 34.8 | 33.6 | 33.6 | 34.2 | 34.3 |
| 35.1 | 35.1 | 37.2 | 37.2 | 39.8 | 39.8 | 44.9 | 44.9 |
| 37.8 | 37.8 | 46.9 | 46.8 | 44.3 | 44.3 | 49.0 | 49.0 |
| 42.2 | 42.3 | 52.1 | 52.3 | 48.1 | 48.1 | 50.1 | 50.1 |
| 45.3 | 45.3 | 55.2 | 55.2 | 51.2 | 51.3 | 53.8 | 53.8 |
| 48.0 | 48.0 | 57.5 | 57.5 | 52.9 | 52.9 | 56.1 | 56.2 |
| 49.9 | 50.0 | 59.3 | 59.4 | 54.5 | 54.5 | 58.5 | 58.6 |
| 51.4 | 51.4 | 60.7 | 60.8 | 55.6 | 55.7 | 59.8 | 59.9 |
| 52.7 | 52.8 | 62.1 | 62.3 | 57.0 | 57.1 | 60.6 | 60.6 |
| 53.7 | 53.7 | 63.0 | 63.2 | 58.6 | 58.7 | 61.2 | 61.2 |
| 54.8 | 54.8 | 64.1 | 64.2 | 60.1 | 60.3 | 61.8 | 61.9 |
| 55.5 | 55.7 | 64.5 | 64.7 | 62.4 | 62.6 | 62.3 | 62.4 |
| 56.2 | 56.3 | 65.1 | 65.3 | 63.4 | 63.7 | 62.8 | 63.0 |
| 57.0 | 57.0 | 65.7 | 65.9 | 64.2 | 64.6 | 63.6 | 63.7 |
| 57.5 | 57.7 | 65.9 | 66.2 | 65.6 | 65.8 | 64.0 | 64.2 |
| 58.0 | 58.1 | 66.2 | 66.4 | 66.1 | 66.4 | 64.4 | 64.5 |
| 58.5 | 58.7 | 66.6 | 66.8 | 66.8 | 67.1 | | |
| 58.9 | 59.1 | 66.7 | 67.0 | 67.3 | 67.6 | | |
| 59.3 | 59.5 | | | 67.7 | 68.0 | | |

## Grain Samples

Samples of soft white winter wheat and yellow dent corn were obtained and the moisture content of the grains was determined. Initial moisture content of the wheat was 12.7% and the moisture content of the corn was 13.2%. It was shown by previous investigators that significant

differences in the thermal properties of grain could be measured if the moisture contents of the grain samples differred by 4 to 5%. Therefore, it was planned to measure the thermal properties of the grains at moisture content increments of approximately 5%. The samples of wheat and corn were divided into two parts. One half of the grain was dried to approximately 10% moisture content. The other half of the sample was conditioned to about 15% moisture content by the addition of water. Disney (1954) had reported that wheat conditioned to higher moisture contents by the addition of water has the same thermal properties as wheat initially obtained at the higher moisture level. After determinations for the thermal properties were made on the grains at the 10% moisture level, the grains were dried to about 5% moisture content and eventually to 0% moisture content. Thus the data obtained at 0, 5 and 10% moisture levels for both grains were on the same sample of grain. The sample of wheat at 15% was tested and subsequently conditioned to 20% moisture content. The corn sample at 15% was conditioned to moisture levels of 20, 25 and 30% with determinations being made at each moisture level. Because the tests were conducted over a long period of time, the grains were kept in a 32°F temperature box when not being used.

The moisture content of the grain was determined by the oven drying method. After the grain had been conditioned

to approximately the desired moisture levels, 60 gram samples were obtained and dried in a 200°F oven for 3 to 4 days. The final weight was recorded and the moisture content calculated on the wet basis.

After running a series of tests to determine the thermal conductivity of the grain, a second 60 gram sample was obtained and another moisture determination was made. The moisture contents of the two samples did not vary over ± 2%, therefore no further moisture determinations were made. The values of the moisture content from the two samples were averaged and used as the moisture content of the grain. The moisture levels used for determining the thermal properties of the wheat and corn are shown in Table 3.

Table 3. Moisture content of wheat and corn samples used for determining the thermal properties.

| | Moisture content, % w.b. | | |
|---|---|---|---|
| Grain | Sample 1[a] | Sample 2 | Average |
| Wheat | 0.68 | 0.68 | 0.68 |
| | 5.40 | 5.50 | 5.45 |
| | 10.1 | 10.5 | 10.3 |
| | 14.4 | 14.4 | 14.4 |
| | 20.4 | 20.2 | 20.3 |
| Corn | 0.88 | 0.93 | 0.91 |
| | 5.10 | 5.07 | 5.08 |
| | 9.73 | 9.90 | 9.81 |
| | 14.7 | 14.8 | 14.7 |
| | 20.2 | 20.1 | 20.1 |
| | 24.8 | 24.6 | 24.7 |
| | 30.4 | 30.1 | 30.2 |

a. Moisture content determined before any tests were run.

## Density Measurements

It was not practical to measure the density of the grain samples for each of the tests conducted. Although the exact volume of the container holding the grain was known, the container could not be filled completely without vigorous and undue vibration. The density of the grain was therefore determined by filling a one liter graduate and weighing the amount of grain. An attempt was made to fill the graduate in the same manner as the test containers would be filled. Since the density could vary widely depending on how the container is filled, a total of ten determinations were made. The average of the ten tests was used as the density of the grain. The density measurements were conducted before any of the tests to determine thermal properties were conducted.

## Specific Heat

Several attempts were made to conduct the specific heat tests with the grain enclosed in a container. A small copper capsule, aluminum foil and polyethylene bags were used to contain the sample of grain dropped into the calorimeter. However many difficulties were encountered and consistent data could not be obtained. The main objection to this method was that unless a very small sample of grain was used the time to reach equilibrium

was 15 to 20 minutes. Consequently the temperature correction term was large compared to the measured temperature change in the calorimeter. When smaller samples were used, the temperature change in the calorimeter became too small to be accurately measured with the recording potentiometer.

In order to attain equilibrium between the grain sample and the water in a relatively short time, the grain was dropped directly into the calorimeter. When equilibrium is reached in a relatively short time the temperature change in the calorimeter would be greater and the temperature correction term would be smaller.

Since the specific heat of grain is one-third to one-half that of water, the maximum temperature change in the calorimeter for a given amount of grain was obtained when a minimum amount of water covering the grain was used. For 60 grams of grain, 40 grams of water would give the largest temperature change without an excess of grain in the calorimeter. The grain samples were held at room temperature and then dropped into cold water in the calorimeter. For each test, 40 grams of ice water were placed in the calorimeter cup and allowed to warm up to approximately 40°F. Then the grain sample was dropped into the calorimter and the calorimeter was shaken by hand to agitate the grain-water mixture. Equilibrium was reached in less than 30 seconds for the majority of the tests. Temperatures were recorded for 2 to 3 minutes after

equilibrium was reached.

Dropping the grain directly into the water in the calorimeter cup may have introduced two sources of error. First, the grain may undergo a change in moisture content and second, the heat of wetting may influence the measured temperature change.

It was not possible to measure directly if the moisture content of the grain was changed appreciably when dropped into the water for the period of 2 to 3 minutes required to run the test. The grain would be covered with surface water which could not be removed easily. Therefore an indirect method was used to determine if this was a source of error. Dry wheat was divided into ten 60 gram samples. Five of the samples were coated with paraffin by immersing the grain in melted paraffin. The grain was weighed after dipping to determine the amount of the paraffin coating. The other five samples of wheat were left uncoated. Tests were conducted on the ten samples to determine the specific heat. The data obtained for the uncoated and coated wheat are shown in Table 4.

Statistical analysis of variance of the data showed that there was no significant difference at the 5% level between the coated and uncoated wheat. Although the uncoated wheat may have undergone a slight change in moisture content, it would not affect the results of the specific heat determinations.

To determine if the heat of wetting would affect the measured temperatures in the calorimeter, wheat and water, both at room temperature were mixed together in the calorimeter. There was no measurable temperature change in the mixture after 10 minutes elapsed. This indicated that any heat evolved was too small to affect the temperature measurements.

Since the two expected sources of error were found to be negligible, the procedure described was used to determine the specific heat of wheat and corn at various moisture levels.

Table 4. Comparison of specific heat determined for paraffin coated and uncoated dry wheat.

Specific heat, Btu/lb-°F

| | Uncoated | Coated with Paraffin |
|---|---|---|
| | 0.381 | 0.370 |
| | 0.364 | 0.348 |
| | 0.348 | 0.345 |
| | 0.351 | 0.359 |
| | 0.294 | 0.387 |
| Average | 0.347 | 0.362 |

Temperature Correction

A typical time-temperature curve obtained during the calorimetric tests is shown in Fig. 5. The temperatures obtained before the sample of grain was dropped into the water are referred to as the initial rating period. Since the heating of the water in the calorimeter cup is

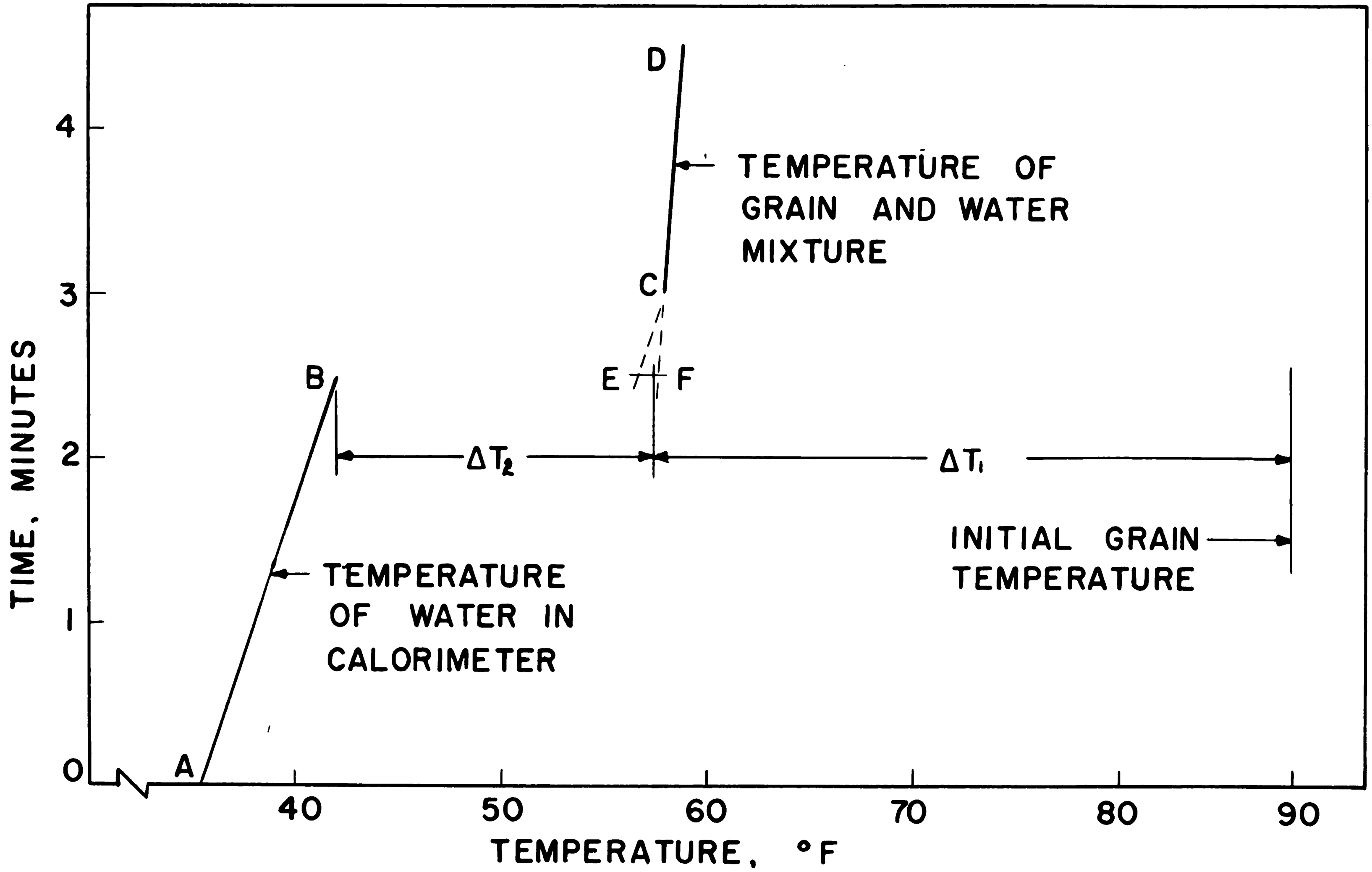


Fig. 5. Typical time-temperature curve obtained from calorimetric tests for determining the specific heat of grain.

power function, the curve may be approximated by a straight line if a small time interval is chosen. The initial rating period is shown by line A B in Fig. 5. The temperatures recorded after thermal equilibrium of the grain and water mixture was obtained are referred to as the final rating period and can also be approximated by a straight line. The final rating period is shown by line C D. The temperatures during the main period when the grain and water are exchanging heat is also a power function. Since temperatures were recorded every 30 seconds and equilibrium was obtained within that period of time, only the initial and final temperatures of the main period were recorded. These temperatures are shown by points B and C respectively in Fig. 5.

To obtain the temperature correction for heat gain into the calorimeter, lines C E and C F were drawn parallel to lines A B and C D respectively. The correct final temperature of the heat exchanging period is between points E and F. If the assumption were made that the heat gain into the calorimeter during the first half of the main period was given by the initial rating period, and for the second half by the final rating period, then the correct final temperature would be given by taking the distance half way between points E and F. This assumption is not valid because the curve from points B to C is a

power function and taking a simple average between the two rating periods would not give the proper correction. A more accurate method would be to determine the time required to reach the average temperature for the main period. This was done by taking the average initial and final temperatures of the main periods for all the tests and plotting a straight line between them on log-log graph paper. The relation between the temperature $t$ and time $\theta$ is given by: $t = c\theta^n$, where $c$ = constant at time $\theta = 1$, $n$ = slope of line on log-log paper. From the plot, c was found to be 41.7 and n was found to be 0.0906. Using this relationship, the time required to reach the average temperature during the main period was calculated and found to be 6.25 seconds. Since the total time of the main period was 30 seconds, the correct final temperature is found by taking (30-6.25)/30 or approximately 4/5 of the distance between points E and F in Fig. 5.

A second series of specific heat measurements were conducted on the wheat and corn by dropping the grain into hot water. This would give a higher mean temperature of the grain than used in the first series. The same procedure for temperature correction was used for the second series of tests.

## Thermal Conductivity

The first step in conducting the thermal conductivity

tests was to determine the current flow that would give a temperature rise large enough to be accurately measured. Using a sample of wheat at approximately 5% moisture content, it was found that a current of 0.490 amp would cause a temperature rise of 37.2°F after 10 minutes of heating. The temperature rise between 1 minute and 10 minutes of heating was 11.5°F. Using a sample of corn at about 5% moisture content and the same current of 0.490 amp the temperature rise was 9.7°F between 1 and 10 minutes of heating. Since a larger temperature rise was desirable the current was increased to 0.560 amp. This gave a temperature rise of 12.6°F between 1 and 10 minutes of heating.

To determine if using different currents had any effect on the thermal conductivity tests, additional tests were run on the corn sample. A total of 5 tests was conducted with a current of 0.490 amp and a total heating time of 10 minutes. Five additional tests were conducted with a current of 0.560 amp and a total heating time of 10 minutes. The results of the ten tests are shown in Table 5.

Although slightly higher values of thermal conductivity were obtained for the tests at the higher current, statistical analysis of the data showed no significant difference between them at the 5% level.

Table 5. Effect of current flow on the thermal conductivity determination of corn.

| | Thermal conductivity, Btu/hr-ft-°F | |
|---|---|---|
| Current | 0.490 amp | 0.560 amp |
| | 0.0842 | 0.0850 |
| | 0.0833 | 0.0850 |
| | 0.0841 | 0.0830 |
| | 0.0841 | 0.0835 |
| | 0.0816 | 0.0842 |
| Average | 0.0834 | 0.0841 |

Increasing the current is one way of obtaining a larger temperature rise. The other alternative is to lengthen the time of heating. Therefore 5 more tests were run on the corn sample using 0.490 amp current and a total heating time of 16 minutes. The comparison of the tests at different heating times is shown in Table 6.

Table 6. Effect of total heating time on the thermal conductivity determination of corn.

| | Thermal conductivity, Btu/hr-ft-°F | |
|---|---|---|
| | Heating time 10 minutes | Heating time 16 minutes |
| | 0.0842 | 0.0855 |
| | 0.0833 | 0.0848 |
| | 0.0841 | 0.0834 |
| | 0.0841 | 0.0843 |
| | 0.0816 | 0.0855 |
| Average | 0.0834 | 0.0847 |

A higher average value was found with the longer heating time, but statistical analysis again showed no significant difference between them at the 5% level.

A current of 0.490 amp. and a total heating time of 16 minutes was used for the corn samples. The same current but a total heating time of 10 minutes was chosen for wheat.

After determining the current and heating time to be used, the samples of grain for each test were placed in the cylinder and allowed to reach equilibrium with room temperature. Then the circuit was closed and the temperature rise was recorded. After each test the grain was removed from the cylinder and kept at room temperature for 10 to 15 minutes. Since the grain temperature was not increased greatly during the test, with the kernels next to the heater wire being the hottest, this length of time was sufficient to bring the temperature to a uniform level. The sample was again placed in the cylinder and another test was conducted. Care was taken to fill the cylinder in approximately the same way so the density would not vary greatly for the tests. Five determinations of the thermal conductivity were made for each sample of wheat and corn at the various moisture contents.

A typical time-temperature curve obtained during the thermal conductivity tests is shown in Fig. 6. Hooper and Lepper (1950) discussed a method to compensate for the finite diameter of the heater, which in effect replaces a small core of the grain. The difference in heat absorption between the heater and the displaced core can be considered as a heat production before the start of measured

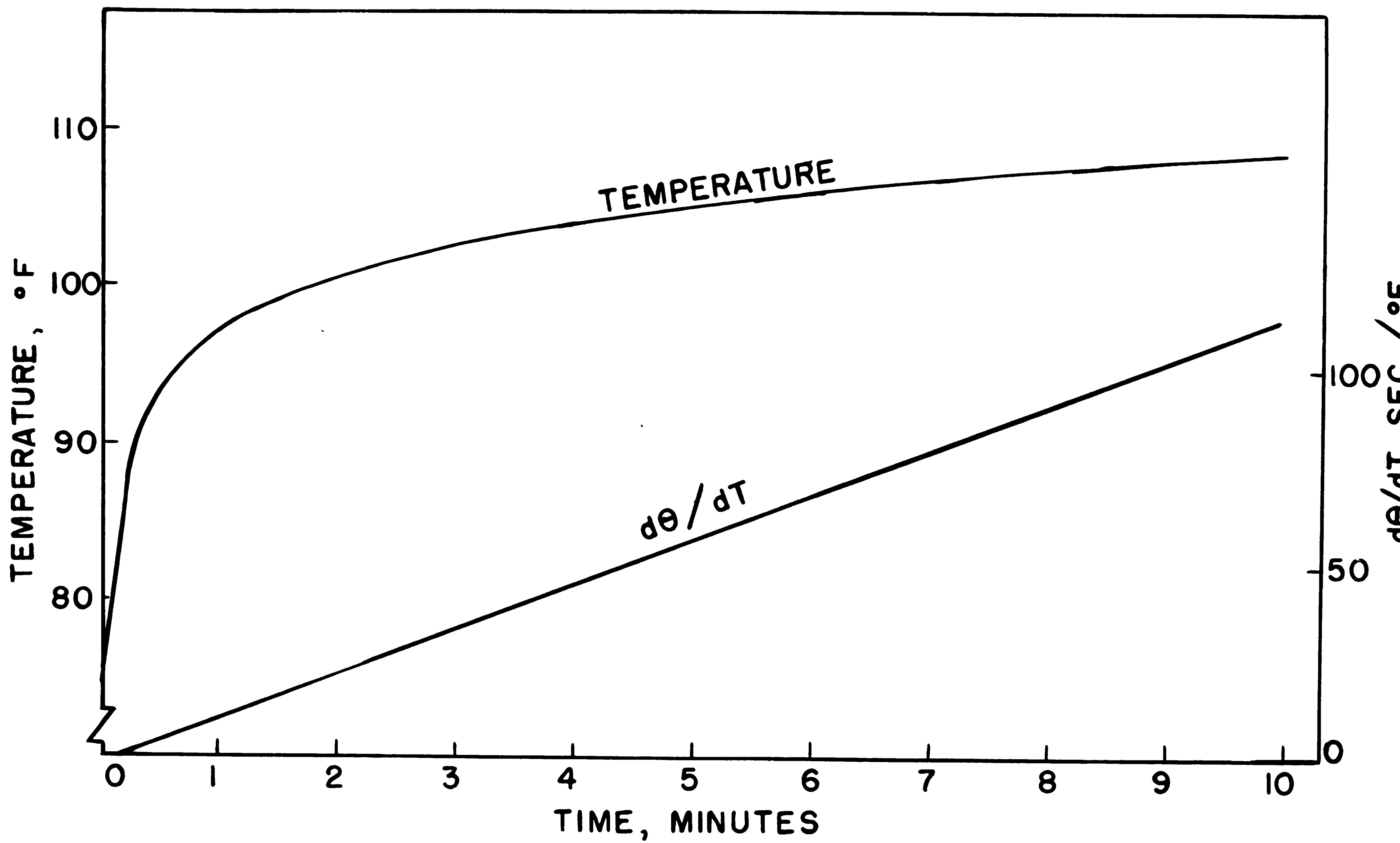


Fig. 6. Typical time-temperature curve obtained for the thermal conductivity determinations of grain.

time; that is, a time correction is subtracted from each observed time. The time correction can be obtained by plotting the values of dθ/dt against time and reading the value of time at dθ/dt = 0. For the data presented in Fig. 6, values of dθ/dt were obtained by drawing tangents to the time-temperature curve and determining the slope. Then the values of dθ/dt were plotted on the same graph to obtain the desired time correction. For the data given in Fig. 6, the time correction was 5 seconds. Hooper and Lepper (1950) have also shown that the time correction for most materials, and for a particular apparatus, is nearly constant. The time correction for one other test was determined and found to be 7 seconds. Therefore the corrections from the two tests were averaged and used as the time correction for all the conductivity tests.

## Thermal Diffusivity

The method chosen for determining the thermal diffusivity was transient flow in one direction across the material. The first step was to determine if the apparatus was closely approximating this condition. The three boxes constructed for the thermal diffusivity tests were filled with dry wheat and tests were conducted with each box. Using the three thermocouples mounted on the nylon string stretched at the center of the box, the temperatures were measured at the three locations. If the temperatures measured at the

two end locations fell on the cooling curve plotted by the center temperature, the heat flow would be unidirectional. For boxes 2 and 3, where the thickness was 1.35 inches, the temperatures measured at the 1/4 and 3/4 locations fell exactly on the curve plotted by the center temperature. For box 1, where the thickness was 1.95 inches, the temperature at the end locations fell on the center temperature cooling curve for the first 30 to 40 minutes of cooling. After this time, the temperature at the end locations varied slightly from the center temperature. If the tests were limited to 30 or 40 minutes, any of the three boxes could be used.

To determine if the size of the slab had any effect on the measured thermal diffusivity, tests were run on dry wheat using the three boxes. Five tests were conducted using boxes 1 and 2, and three tests were conducted with with box 3. The results of these tests are shown in Table 7.

Analysis of variance showed that there was no significant difference at the 5% level between the data obtained from the three boxes. It was decided to use Box 1 for the remaining tests because it would give a larger sample with which to work.

For each test the sample of grain was placed in the box and then held at room temperature until thermal equilibrium was assured. Then the box was placed in the ice bath and the temperature at the center was recorded

every 30 seconds. The sample was cooled until the temperature at the center reached ½ of the difference between the initial temperature and 32°F. Finely crushed ice was continually added to the ice bath to maintain a constant temperature. After each test, the grain was removed from the box until it reached room temperature before another test was conducted.

Table 7. Comparison of thermal diffusivity of wheat using different size slabs.

| | Thermal diffusivity, sq-ft/hr | | |
|---|---|---|---|
| | Box 1 (1.95"x6"x6") | Box 2 (1.35"x6"x6") | Box 3 (1.35"x9"x9") |
| | 0.00328 | 0.00359 | 0.00352 |
| | 0.00345 | 0.00361 | 0.00355 |
| | 0.00368 | 0.00362 | 0.00348 |
| | 0.00359 | 0.00354 | |
| | 0.00359 | 0.00360 | |
| Average | 0.00352 | 0.00359 | 0.00352 |

A typical cooling curve, obtained by plotting the temperature at the center of the slab against time is shown in Fig. 7. The thermal diffusivity was calculated for five points on the curve. The points were chosen at ratios of center temperatures to initial temperatures of 0.9, 0.8, 0.7, 0.6 and 0.5. The time required to reach the temperatures described by the ratios was determined and the diffusivity was found from the relation: $\alpha = zx^2/\theta$. The

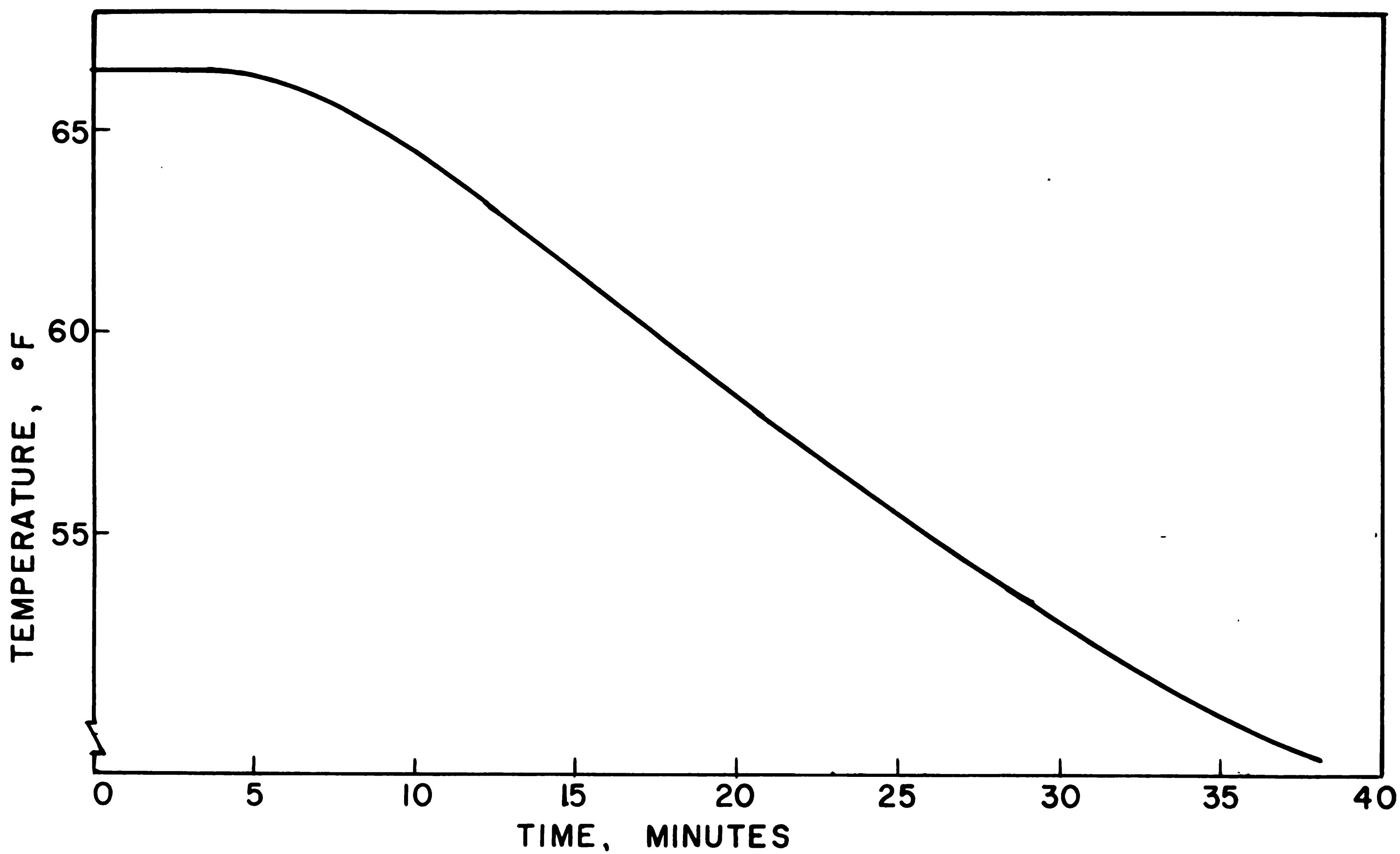


Fig. 7. Typical cooling curve obtained for the thermal diffusivity tests of grain.

average of the five calculations from each curve was used as the thermal diffusivity of the sample.

The thermal properties of dry quartz sand were determined. The measured values of the thermal properties were then compared to the published values to determine the accuracy of the methods and procedures used. The comparison of the measured values and the published values is given in Table 8. The published values were taken from Ingersoll, Zobel and Ingersoll (1954).

Table 8. Comparison of the measured and published thermal properties of quartz sand.

| | Density lb/cu-ft | Specific heat Btu/lb-°F | Thermal conductivity Btu/hr-ft-°F | Thermal diffusivity sq-ft/hr |
|---|---|---|---|---|
| Measured | 90.0 | 0.172 | 0.136 | 0.00838 |
| Published | 103 | 0.19 | 0.15 | 0.008 |
| % Variation | 12.6 | 10.5 | 6.66 | 4.75 |

A direct comparison could not be made because the quartz sand used in the tests had a density of 90 lb/cu-ft, and the published values were for sand of 103 lb/cu-ft. However the differences in percent between the measured and published thermal properties were less than the difference in density. Therefore, the apparatus was considered suitable for determining specific heat, thermal conductivity and thermal diffusivity of grain.

# RESULTS AND DISCUSSION

## Density

The results of the density measurements for the soft white wheat are shown in Fig. 8. For moisture contents of 0 to 10% the density of the wheat was fairly constant with an average value of 48.4 lb/cu-ft. Above 10% moisture content, the density decreased by 0.23 lb/cu-ft for a 1% increase in moisture content. This is expected since drying would shrink the kernels to a point beyond which very little shrinkage would occur by further removal of moisture.

The same results were obtained for the density measurements on the yellow dent corn. The density was approximately constant at 46.8 lb/cu-ft for moisture contents from 0 to 15%. The density decreased by 0.27 lb/cu-ft for an increase of 1% in the moisture content from 15 to 30%. The relation between density and moisture content for corn is shown in Fig. 9.

## Specific Heat

The specific heat values of soft white wheat obtained by dropping the grain into cold water are shown in Table 9. The mean temperature was taken as the average of the initial and final grain temperatures for all the tests.

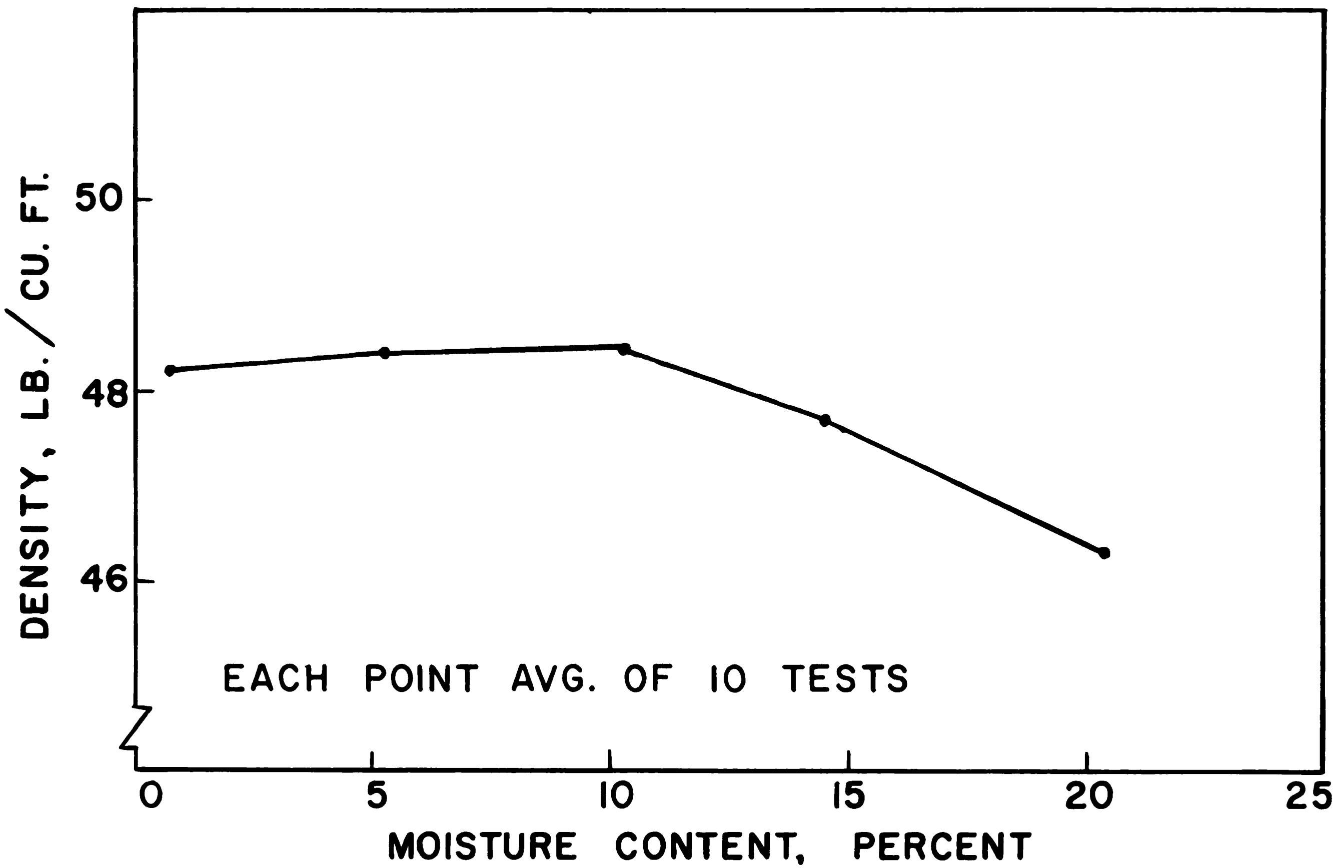


Fig. 8. Relation between density and moisture content for soft white wheat.

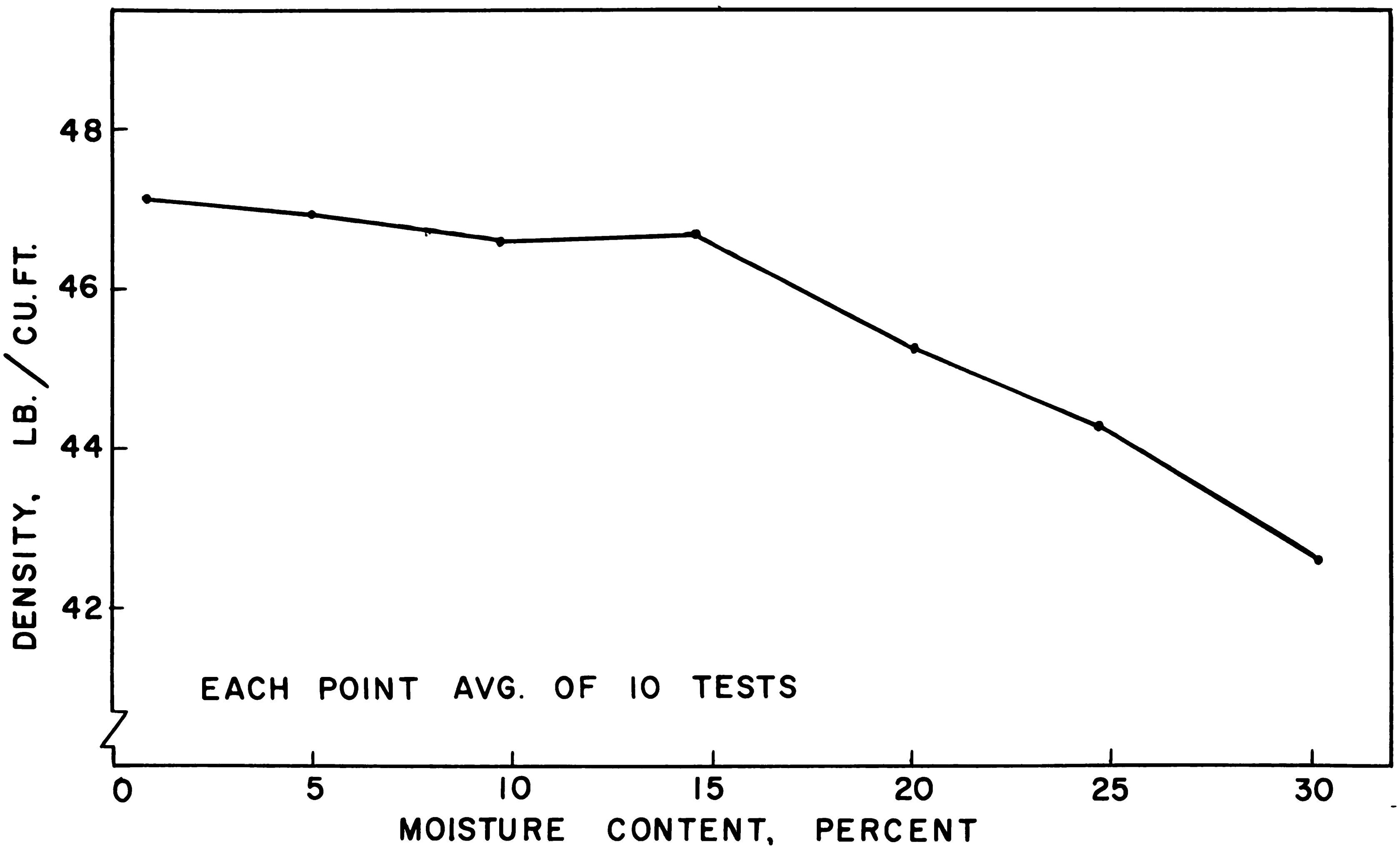


Fig. 9. Relation between density and moisture content for yellow dent corn.

For the wheat the mean temperature was 70.9 °F with a range of 51.2 °F to 89.0 °F.

Table 9. Specific heat values of soft white wheat at 70.9 °F (51.2 to 89.0 °F)

| | Specific heat, Btu/lb-°F | | | | |
|---|---|---|---|---|---|
| Moisture content, % | 0.68 | 5.45 | 10.3 | 14.4 | 20.3 |
| | 0.381 | 0.336 | 0.432 | 0.501 | 0.527 |
| | 0.364 | 0.315 | 0.432 | 0.490 | 0.517 |
| | 0.348 | 0.404 | 0.434 | 0.503 | 0.524 |
| | 0.351 | 0.402 | 0.416 | 0.496 | 0.527 |
| | 0.294 | 0.421 | 0.427 | 0.508 | 0.515 |
| Average | 0.347 | 0.375 | 0.428 | 0.500 | 0.522 |

The specific heat data appeared to be linearly dependent on the moisture content, therefore linear regression was applied. The regression equation for the wheat was: $c = 0.334 + 0.00977\ M$, where c = specific heat, Btu/lb-°F, M = moisture content, % w.b.

The standard error of estimate was 0.0180. The regression line and the standard error for soft wheat are shown in Fig. 10.

Pfalzner's (1951) work on the specific heat of hard wheat gave an average regression equation of: $c = 0.291 + 0.00761\ M$. In comparing the two equations, the constant for dry soft wheat is 14.1% higher than for the dry hard red wheat. The slope for the soft wheat is 28.3% greater than for the hard wheat. These differences in the regression

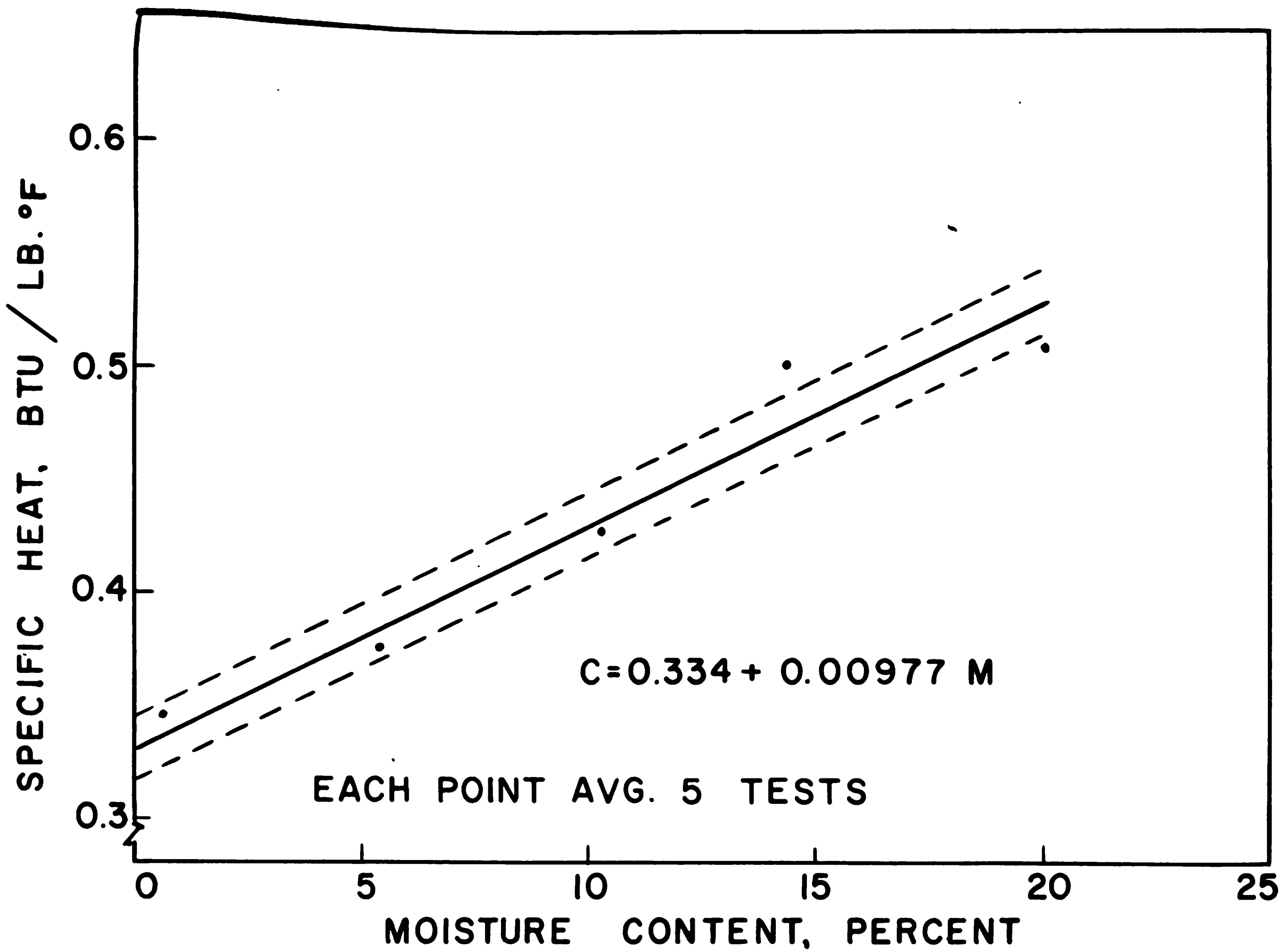


Fig. 10. Relation between specific heat and moisture content for soft white wheat at 70.9 °F.

equations are probably due to the physical and chemical differences in the two types of wheat and to the different methods used in obtaining the data. Pfalzner's data were obtained by enclosing the wheat in a container, therefore the specific heat reported is for bulk wheat. The data obtained by dropping the wheat directly into the water represents the specific heat of the individual kernels. Also the mean temperatures of Pfalzner's tests on hard wheat were approximately 50°F, while the data for soft wheat were obtained at a mean temperature of 70.9°F.

The specific heat determinations for the yellow dent corn dropped into cold water are given in Table 10. The mean temperature of the corn during the tests was 68.9°F with a range of 54.0 to 83.8°F.

Table 10. Specific heat values of yellow dent corn at 68.9°F (54.0 to 83.8°F)

Specific heat, Btu/lb-°F

| Moisture content, % | 0.91 | 50.8 | 9.81 | 14.7 | 20.1 | 24.7 | 30.2 |
|---|---|---|---|---|---|---|---|
| | 0.407 | 0.424 | 0.424 | 0.465 | 0.524 | 0.586 | 0.577 |
| | 0.348 | 0.393 | 0.447 | 0.483 | 0.504 | 0.572 | 0.593 |
| | 0.354 | 0.413 | 0.440 | 0.488 | 0.545 | 0.565 | 0.608 |
| | 0.375 | 0.397 | 0.437 | 0.497 | 0.542 | 0.552 | 0.561 |
| | 0.346 | 0.393 | 0.440 | 0.488 | 0.542 | 0.564 | 0.598 |
| Average | 0.366 | 0.404 | 0.438 | 0.484 | 0.531 | 0.567 | 0.588 |

The regression equation obtained for the specific

heat of corn was c = 0.350 + 0.00851 M. The standard error of estimate was 0.0155. Fig. 11 shows the data and regression line for corn at a mean temperature of 68.9°F.

The specific heat of the soft wheat obtained at the higher mean temperature of 93.1°F with a range of 77 to 115.1°F is given in Table 11. The higher mean temperature was obtained by dropping the grain into hot water in the calorimeter.

Table 11. Specific heat values for soft white wheat at 93.1°F. (77.0 to 115.1°F)

Specific heat, Btu/lb-°F

| Moisture content, % | 0.68 | 5.45 | 10.3 | 14.4 | 20.3 |
|---|---|---|---|---|---|
| | 0.304 | 0.368 | 0.443 | 0.477 | 0.555 |
| | 0.344 | 0.381 | 0.398 | 0.488 | 0.515 |
| | 0.322 | 0.350 | 0.438 | 0.488 | 0.526 |
| | 0.308 | 0.371 | 0.405 | 0.507 | |
| | 0.341 | 0.361 | 0.406 | 0.490 | |
| Average | 0.323 | 0.366 | 0.418 | 0.490 | 0.532 |

Analysis of variance of the specific heat of wheat at 70.9°F and 93.1°F showed no significant difference at the 5% level. However the data obtained at 93.1°F appeared to be consistently lower than the data obtained at 70.9°F. This indicated that possibly the different methods used, that is, dropping the grain into cold water compared to dropping it into hot water, may have influenced the data. To determine if the two methods did indeed give different

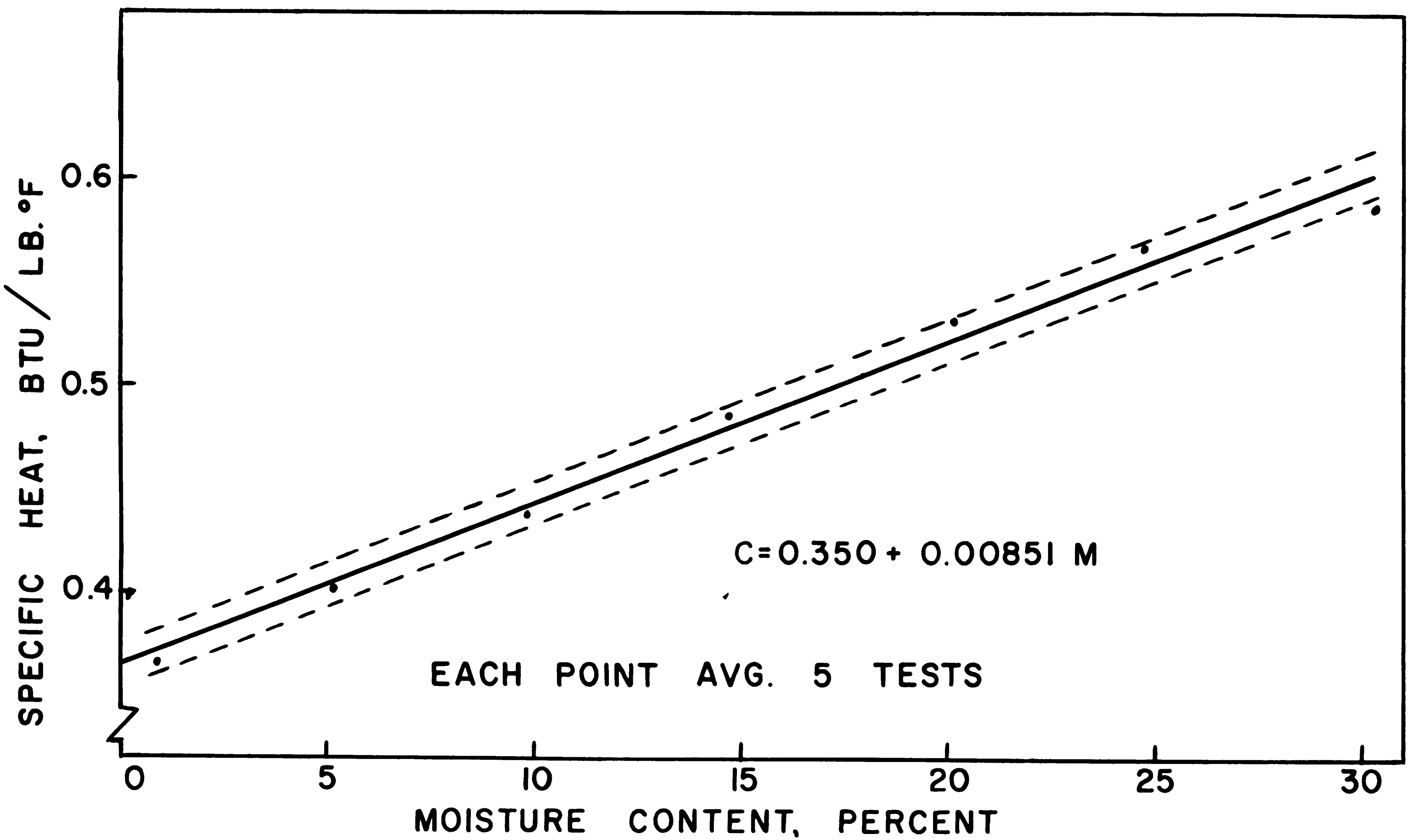


Fig. 11. Relation between specific heat and moisture content for yellow dent corn at 68.9°F.

results, data were obtained for aluminum pellets. The same method and procedure used for the grain were used to determine the specific heat of the aluminum pellets. The aluminum pellets had a known specific heat of 0.227 at a mean temperature of 85°F. The results of three tests where the aluminum pellets at room temperature were dropped into cold water gave a value of 0.229. The mean temperature of the aluminum was 61.8°F with a range of 49.0 to 74.8°F. For three tests where the pellets were dropped into hot water, the specific heat was 0.191. The mean temperature was 95.5 with a range of 75.7 to 116.5°F. It was apparent that the method of dropping the material into hot water did result in values that were low. No explaination could be found for this difference in the two methods.

The data for the specific heat of wheat and corn obtained at the higher mean temperature were not combined with the data obtained at the lower temperature but were analyzed separately.

For the soft white wheat at a mean temperature of 93.1°F (77.0 to 115.1°F), the regression equation was: $c = 0.311 + 0.0112\ M$. The standard error was 0.0121. The data and regression line for wheat are shown in Fig. 12.

The specific heat values for the yellow dent corn obtained at a mean temperature of 95.3°F (80.0 to 115.4) are shown in Table 12.

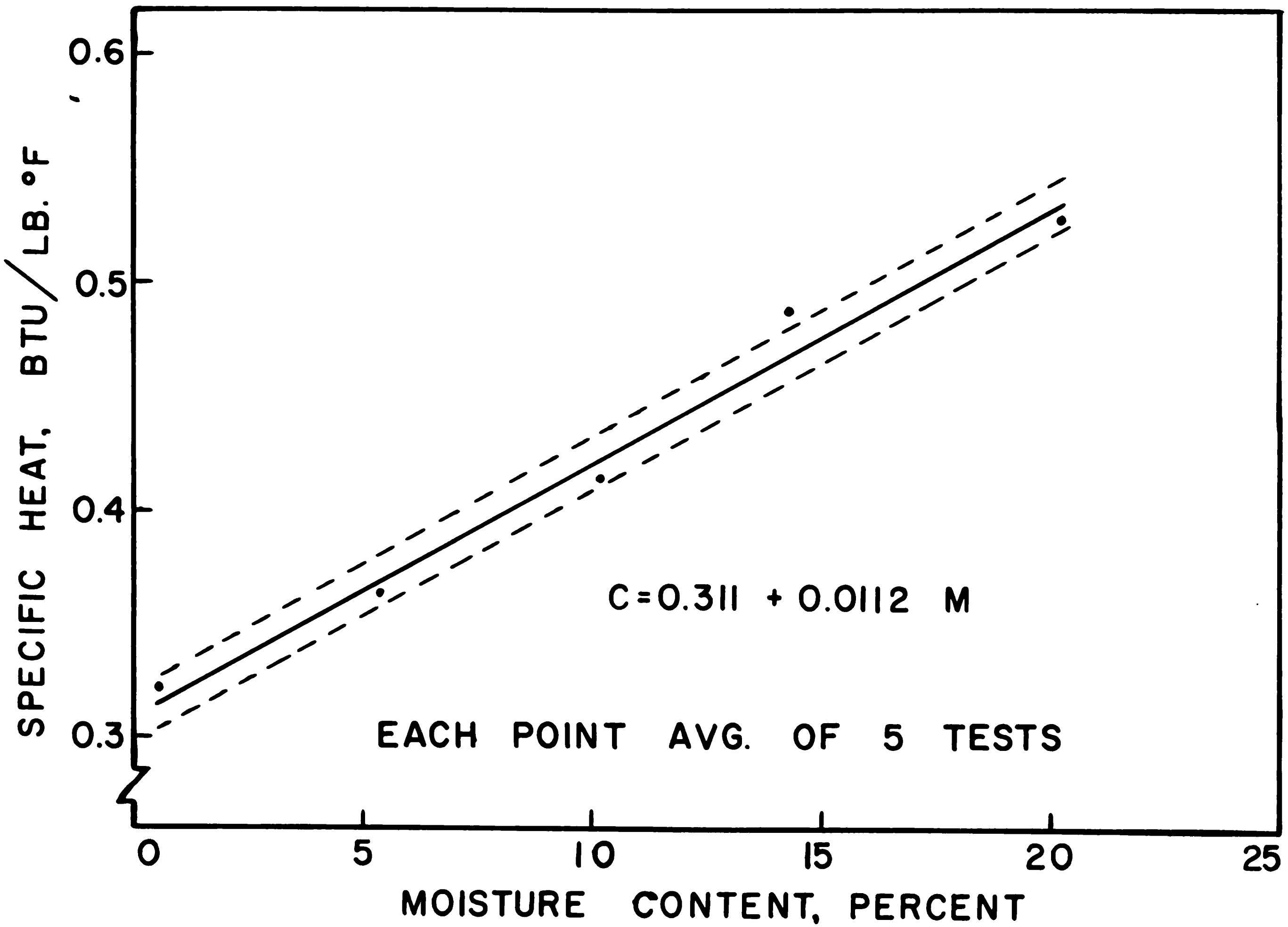


Fig. 12. Relation between specific heat and moisture content for soft white wheat at 93.1°F.

Table 12. Specific heat values of yellow dent corn at 95.3 °F. (80.0 to 115.4 °F)

Specific heat, Btu/lb-°F

| Moisture Content, % | 0.91 | 5.08 | 9.81 | 14.7 | 20.1 | 24.7 | 30.2 |
|---|---|---|---|---|---|---|---|
| | 0.330 | 0.367 | 0.435 | 0.468 | 0.523 | 0.575 | 0.592 |
| | 0.318 | 0.365 | 0.433 | 0.482 | 0.532 | 0.562 | 0.584 |
| | 0.326 | 0.370 | 0.437 | 0.482 | 0.536 | 0.563 | 0.590 |
| | 0.309 | 0.371 | 0.420 | 0.458 | 0.573 | 0.560 | 0.584 |
| | 0.309 | 0.361 | 0.428 | 0.453 | 0.554 | 0.568 | 0.570 |
| Average | 0.318 | 0.367 | 0.431 | 0.469 | 0.544 | 0.566 | 0.584 |

The regression equation for the yellow dent corn at 95.3°F was $c = 0.325 + 0.00949\ M$ with a standard error of 0.0204. The data are shown in Fig. 13.

## Thermal Conductivity

The results of the thermal conductivity determinations for soft white wheat are shown in Table 13.

Table 13. Thermal conductivity values for soft white wheat.

Thermal conductivity, Btu/hr-ft-°F

| Moisture content, % | 0.68 | 5.45 | 10.3 | 14.4 | 20.3 |
|---|---|---|---|---|---|
| | 0.0684 | 0.0692 | 0.0748 | 0.0755 | 0.0777 |
| | 0.0676 | 0.0704 | 0.0763 | 0.0770 | 0.0800 |
| | 0.0695 | 0.0710 | 0.0748 | 0.0816 | 0.0816 |
| | 0.0682 | 0.0715 | 0.0742 | 0.0816 | 0.0800 |
| | 0.0670 | 0.0715 | 0.0737 | 0.0777 | 0.0800 |
| **Average** | 0.0679 | 0.0706 | 0.0747 | 0.0786 | 0.0798 |

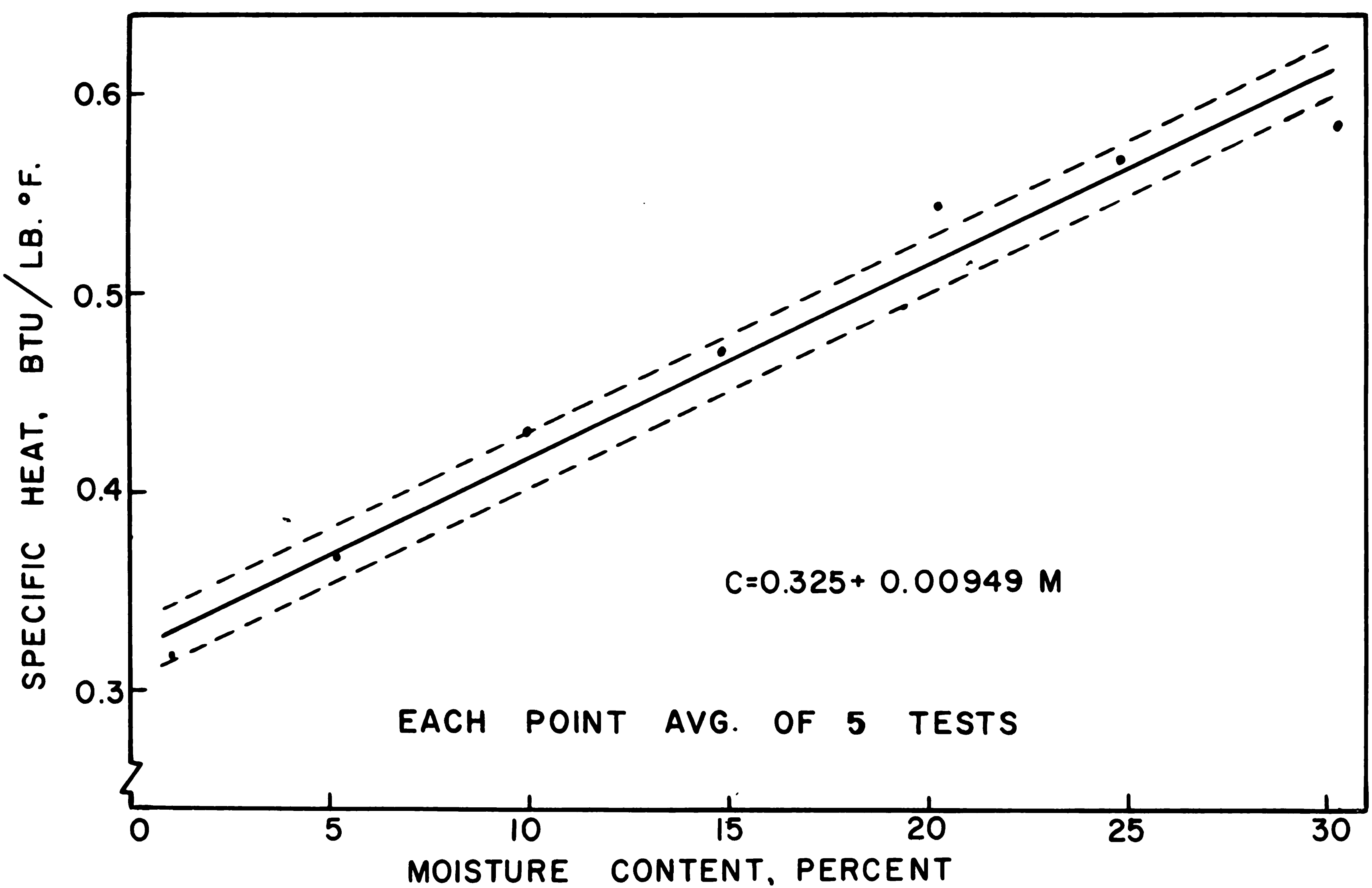


Fig. 13. Relation between specific heat and moisture content for yellow dent corn at 95.3°F.

The thermal conductivity values of the wheat appeared to be linearly dependent on the moisture content and therefore were analyzed by linear regression. The regression equation was $k = 0.0676 + 0.000654\ M$ and the standard error was 0.00118. The data and regression line for the thermal conductivity of wheat are shown in Fig. 14. The mean temperature for the wheat was determined in the following manner. A time-temperature curve was drawn using the average temperatures from all the tests on wheat. The curve drawn represents the temperature at a radius of 1/64 inch from the hot wire. For temperatures at increasing radii, the curves would be similiar but would fall below the curve for the 1/64 inch radius. The temperature curve for a radius of 2 1/4 inches would be nearly constant and equal to the initial grain temperature. Therefore the temperatures in the entire sample may be represented by the area under the time-temperature curve. This area was measured with a planimeter. The temperature where the area under the curve would be equal to one half of the total area was determined. This temperature was taken as the mean temperature for the grain. The mean temperature for the wheat was 87.7°F. The temperature range of the tests was 69.8 to 111.4 °F.

The thermal conductivity values on the yellow dent corn are given in Table 14.

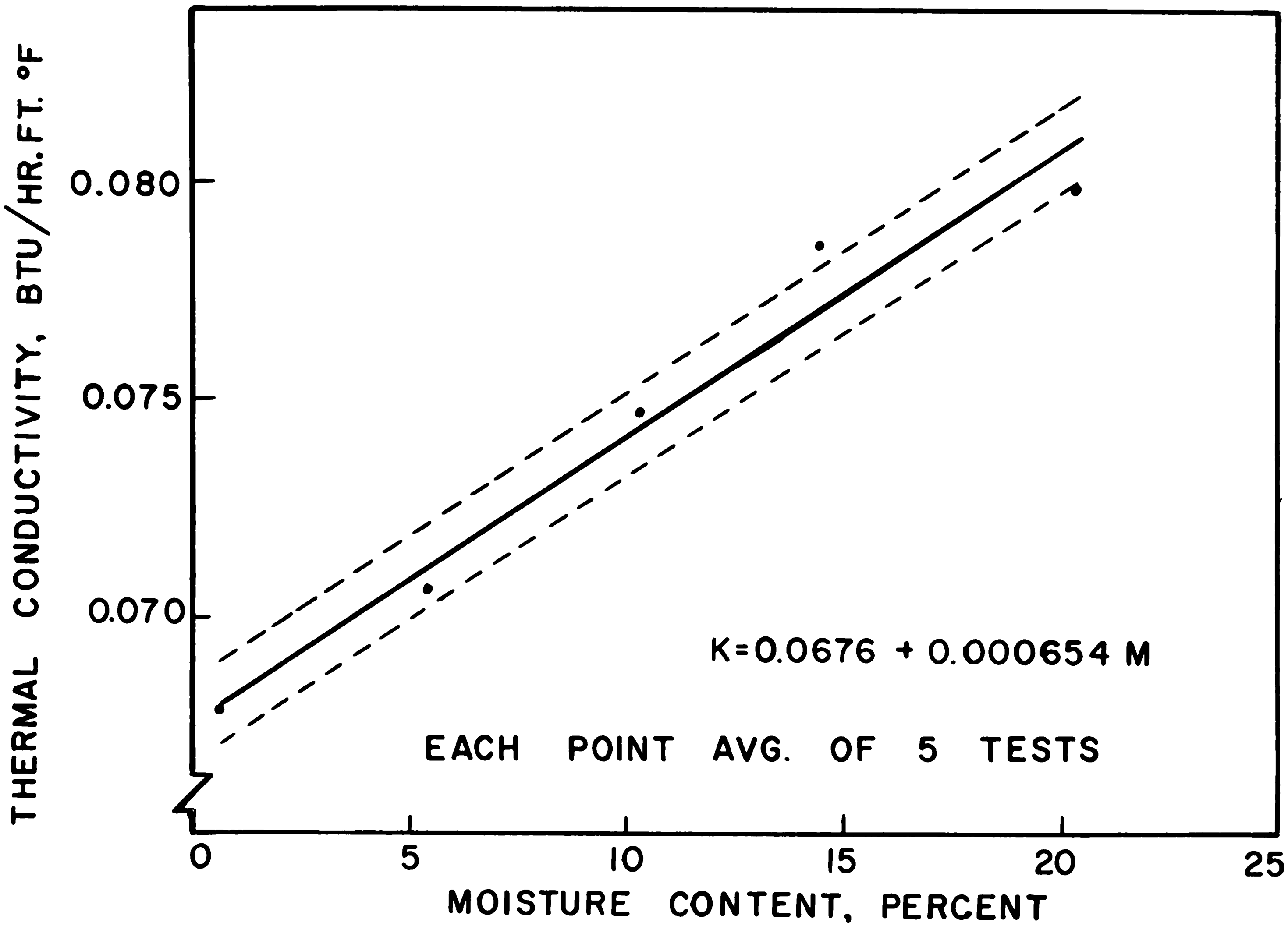


Fig. 14. Relation between thermal conductivity and moisture content for soft white wheat.

Table 14. Thermal conductivity values for yellow dent corn.

| Moisture content, % | Thermal conductivity, Btu/hr-ft-°F | | | | | | |
|---|---|---|---|---|---|---|---|
| | 0.91 | 5.08 | 9.81 | 14.7 | 20.1 | 24.7 | 30.2 |
| | 0.0820 | 0.0855 | 0.0886 | 0.0910 | 0.0907 | 0.0955 | 0.102 |
| | 0.0834 | 0.0848 | 0.0863 | 0.0919 | 0.0947 | 0.0990 | 0.107 |
| | 0.0794 | 0.0834 | 0.0894 | 0.0894 | 0.0964 | 0.0964 | 0.0964 |
| | 0.0834 | 0.0843 | 0.0855 | 0.0927 | 0.0964 | 0.101 | 0.0900 |
| | 0.0780 | 0.0855 | 0.0894 | 0.0945 | 0.0947 | 0.0990 | 0.103 |
| Avg. | 0.0812 | 0.0847 | 0.0878 | 0.0919 | 0.0945 | 0.0982 | 0.996 |

The data were again analyzed by linear regression resulting in the regression equation k = 0.0814 + 0.000646 M. The standard error of estimate was 0.00085. Fig. 15 shows the thermal conductivity data for the corn.

The mean temperature for the corn was determined in the same manner as for wheat. However since two different times were used for the conductivity tests on the corn, the mean temperatures were determined for each test separately. Sixteen minutes were used for the tests on the corn from 0 to 14.7% moisture content. The mean temperature for these tests was 89.5°F with a range of 70.0°F to 118.6°F.

The time for the test on 20.1, 24.7 and 30.2% corn was 10 minutes. The mean temperature for these tests was 95.4°F with a range of 69.4 to 126.6°F.

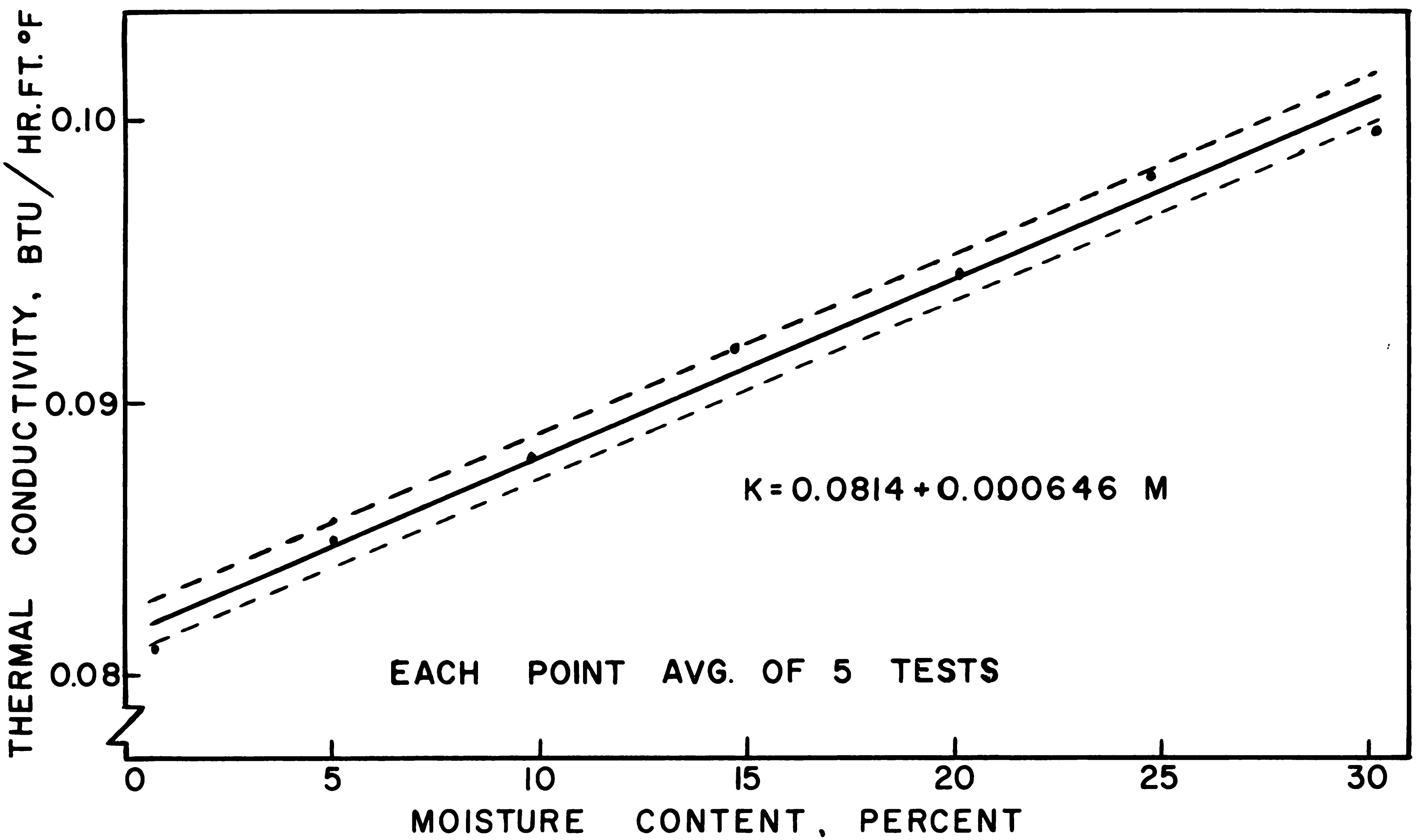


Fig. 15. Relation between thermal conductivity and moisture content for yellow dent corn.

Thermal Diffusivity

The results of the thermal diffusivity values for soft white wheat are shown in Table 15.

Table 15. Thermal diffusivity values for soft white wheat.

| | Thermal diffusivity, sq-ft/hr. | | | | |
|---|---|---|---|---|---|
| Moisture content, % | 0.68 | 5.45 | 10.3 | 14.4 | 20.3 |
| | 0.00359 | 0.00340 | 0.00327 | 0.00312 | 0.00301 |
| | 0.00361 | 0.00349 | 0.00333 | 0.00316 | 0.00315 |
| | 0.00362 | 0.00346 | 0.00339 | 0.00312 | 0.00304 |
| | 0.00354 | 0.00354 | 0.00326 | 0.00324 | 0.00314 |
| | 0.00360 | 0.00348 | 0.00331 | 0.00322 | 0.00316 |
| Average | 0.00359 | 0.00347 | 0.00331 | 0.00318 | 0.00310 |

The mean temperature for the thermal diffusivity tests was obtained in the same manner as described for the thermal conductivity tests. The mean temperature for the tests on the soft white wheat was 57.0°F with a range of 48.3 to 73.8°F.

Table 16 shows the thermal diffusivity values for the yellow dent corn. The mean temperature for the corn was 56.8°F with a range of 47.7 to 74.0°F.

Table 16. Thermal diffusivity values for yellow dent corn.

| | Thermal diffusivity, sq-ft/hr | | | | | | |
|---|---|---|---|---|---|---|---|
| M, % | 0.91 | 5.08 | 9.81 | 14.7 | 20.1 | 24.7 | 30.2 |
| | 0.00387 | 0.00389 | 0.00358 | 0.00342 | 0.00330 | 0.00344 | 0.00367 |
| | 0.00399 | 0.00376 | 0.00371 | 0.00360 | 0.00332 | 0.00333 | 0.00369 |
| | 0.00396 | 0.00389 | 0.00366 | 0.00352 | 0.00335 | 0.00348 | 0.00344 |
| | 0.00394 | 0.00378 | 0.00359 | 0.00350 | 0.00343 | 0.00345 | 0.00348 |
| | 0.00398 | 0.00373 | 0.00367 | 0.00352 | 0.00338 | 0.00350 | 0.00364 |
| Avg | 0.00395 | 0.00381 | 0.00364 | 0.00351 | 0.00336 | 0.00344 | 0.00358 |

The thermal diffusivity of the wheat decreased with increasing moisture content. The diffusivity of the corn decreased with increasing moisture content up to 20%, and for higher moisture contents the diffusivity was found to increase. This can be explained by the variation in the density with moisture content. As previously discussed the density was not a linear function of the moisture content for both wheat and corn. Since the thermal diffusivity can be obtained from the equation; $\alpha = k/c\rho$ , and with k and c varying linearly, $\alpha$ would vary inversely with the density, and would not be linear with moisture content. The measured values of the thermal diffusivity compared to the calculated values for soft wheat are shown in Fig. 16. The calculated values of the thermal diffusivity were 6.1 to 11.6% greater than the measured values for the wheat at the various moisture levels. The difference in the measured diffusivity and the calculated diffusivity can be shown to be acceptable.

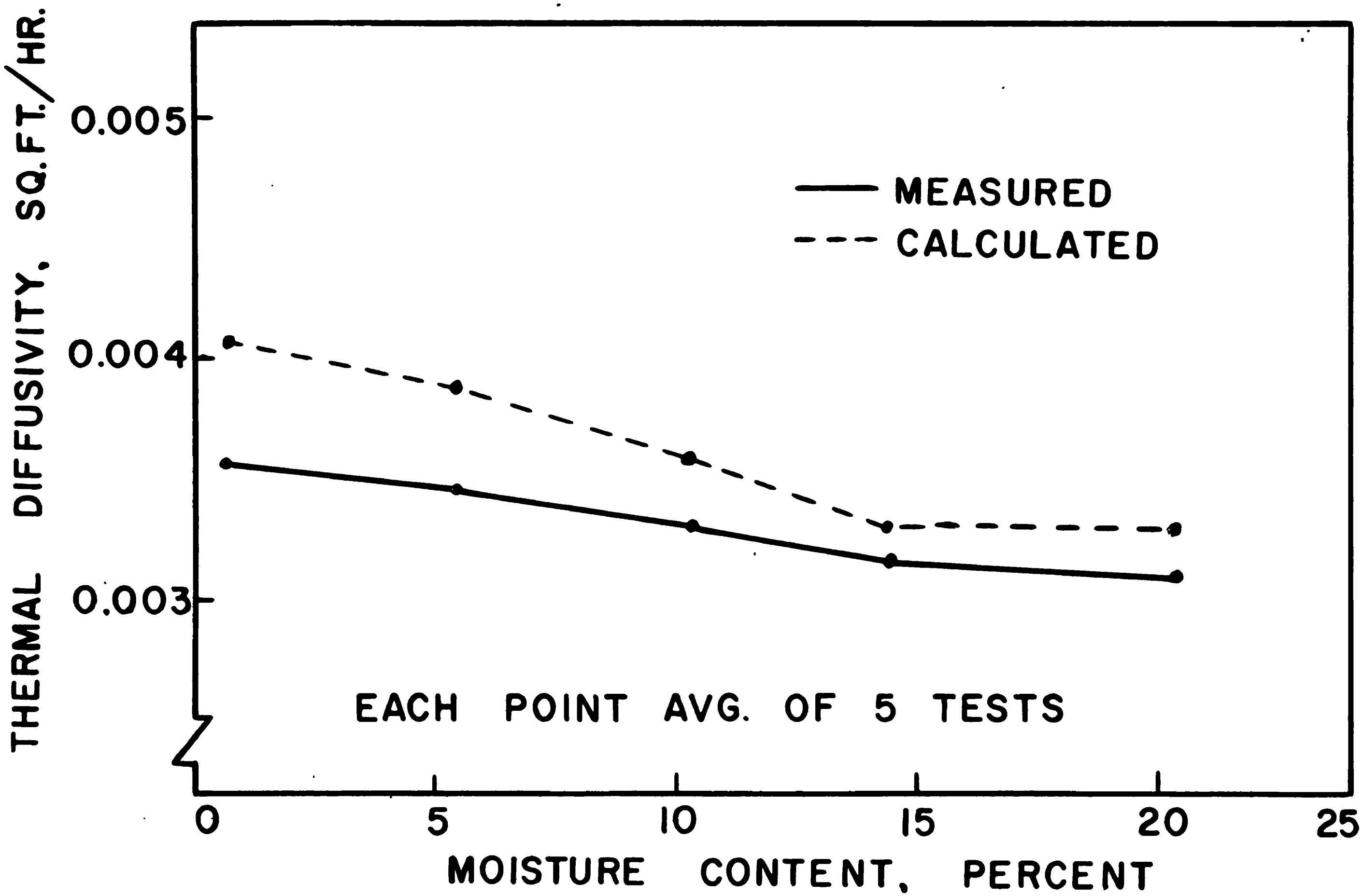


Fig. 16. Relation between thermal diffusivity and moisture content for soft white wheat.

For example, if the error in the temperature measurement was 1% it could have the following effect on the measured thermal properties. For specific heat, which was calculated by using the ratio of two temperature differences, the error could be 2%. The thermal conductivity was calculated using one temperature difference and could have an error of 1%. The thermal diffusivity calculated from the measured values of the specific heat and thermal conductivity could therefore have an error of 3%. The measured thermal diffusivity, which was found by using a ratio of two temperatures, could have a maximum error of 2%. Thus, an error of 1% in the temperature measurement alone, neglecting all other errors, may give a 5% difference between the measured and calculated thermal diffusivity. A similar error analysis assuming an error of 1% in all the measurements involved could result in a difference of 17% between the measured and calculated values of the thermal diffusivity. If all the possible sources of error including measurement error, personal errors, accidental errors, etc. were analyzed, the maximum error for such a comparison could be greater than 17%.

A comparison of the measured values and the calculated values of the thermal diffusivity for corn is shown in Fig. 17. The values differred by 9.8 to 16.1%, which again would be acceptable based on the previously discussed error analysis.

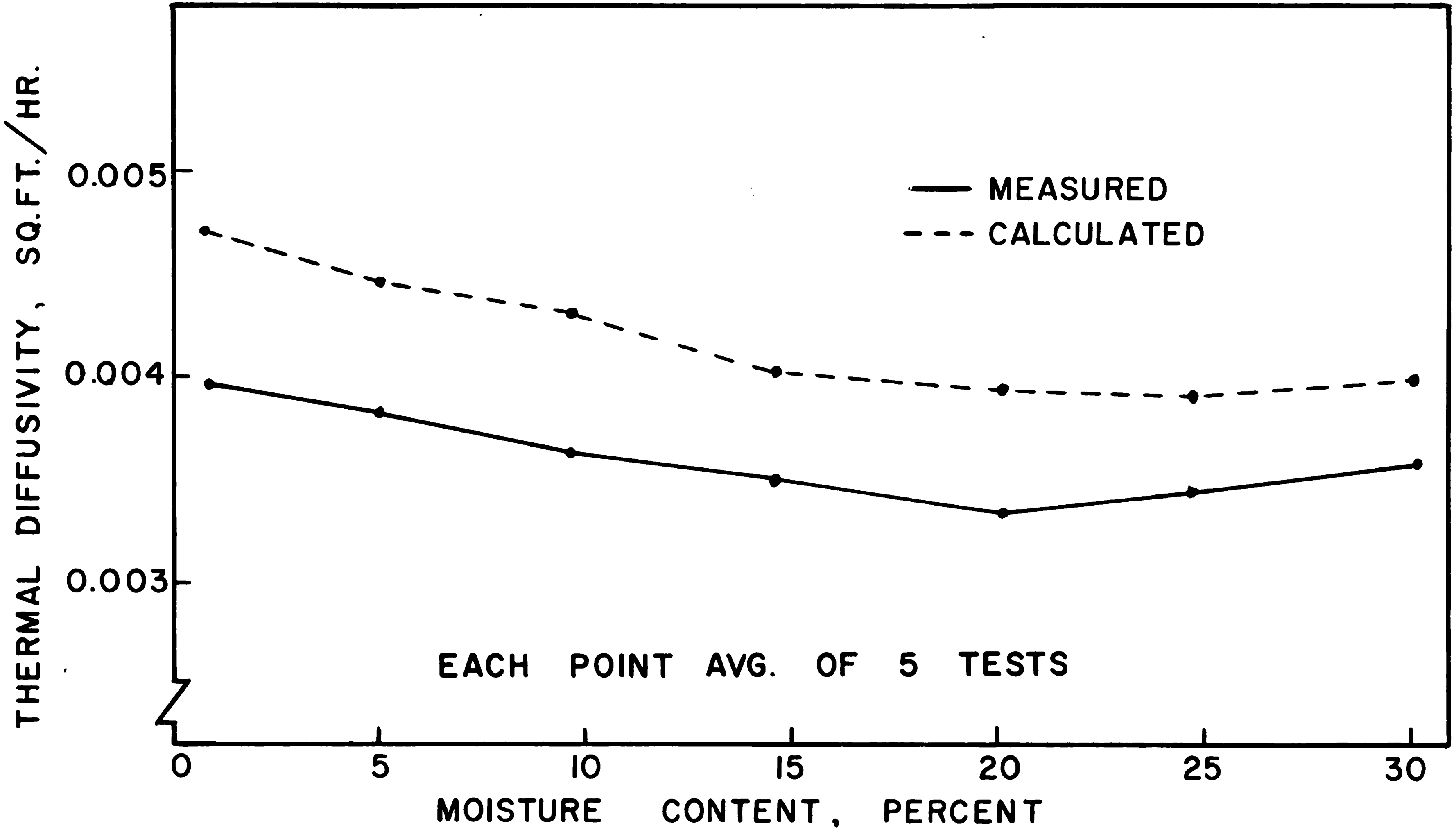


Fig. 17. Relation between thermal.diffusity and moisture content for yellow dent corn.

A summary of the thermal properties determined for soft white wheat is shown in Table 17. The thermal properties for yellow dent corn are summarized in Table 18.

Table 17. Summary of the thermal properties of soft wheat.

| Moisture content, % | Density lb/cu-ft | Specific heat, Btu/lb-°F | Thermal conductivity, Btu/hr-ft-°F | Thermal diffusivity, sq-ft/hr |
|---|---|---|---|---|
| 0.68 | 48.2 | 0.347 | 0.0679 | 0.00359 |
| 5.45 | 48.5 | 0.375 | 0.0706 | 0.00347 |
| 10.3 | 48.6 | 0.428 | 0.0747 | 0.00331 |
| 14.4 | 47.7 | 0.500 | 0.0786 | 0.00318 |
| 20.3 | 46.3 | 0.522 | 0.0798 | 0.00310 |

Table 18. Summary of the thermal properties of yellow dent corn.

| Moisture content, % | Density lb/cu-ft | Specific heat, Btu/lb-°F | Thermal conductivity, Btu/hr-ft-°F | Thermal diffusivity, sq-ft/hr |
|---|---|---|---|---|
| 0.91 | 47.1 | 0.366 | 0.0812 | 0.00395 |
| 5.08 | 46.9 | 0.404 | 0.0846 | 0.00381 |
| 9.81 | 46.6 | 0.438 | 0.0878 | 0.00364 |
| 14.7 | 46.7 | 0.484 | 0.0919 | 0.00351 |
| 20.1 | 45.2 | 0.531 | 0.0945 | 0.00336 |
| 24.7 | 44.3 | 0.567 | 0.0982 | 0.00344 |
| 30.2 | 42.6 | 0.588 | 0.0996 | 0.00358 |

The temperature range of all the various tests conducted was from 47.7 to 126.6 °F. Since the effect of temperature on the thermal properties is small, the data obtained may be used over this range. The conditions for many problems encountered fall within these temperatures.

## CONCLUSIONS

The results of this study and of other investigators show that the specific heat and thermal conductivity of grain is a linear function of the moisture content. The thermal diffusivity of grain is not a linear function of the moisture content.

The specific conclusions regarding the thermal properties of the grains used for this study are:

1. The specific heat of soft white wheat is a linear function of moisture content and is expressed by the equation, $c = 0.334 + 0.00977\ M$ for a temperature range of 51.2 to 89.0 °F.

2. The specific heat of yellow dent corn is a linear function of moisture content also. The relation for corn is $c = 0.350 + 0.00851\ M$ for a temperature range of 54.0 to 83.8 °F.

3. The thermal conductivity of soft white wheat is a linear function of moisture content expressed by the equation $k = 0.0676 + 0.000654\ M$ for a temperature range of 69.8 to 111.4 °F.

4. The thermal conductivity of yellow dent corn is a linear function of moisture content expressed by $k = 0.0814 + 0.000646\ M$ for a temperature range of 69.4 to 126.6 °F.

5. The thermal diffusivity of grain is not a linear function of moisture content. For soft wheat the thermal diffusivity varied from 0.00359 sq-ft/hr at 0.68% moisture content to 0.00310 sq-ft/hr at 20.3% moisture. The thermal diffusivity of yellow dent corn varied from 0.00395 sq-ft/hr at 0.91% moisture content to a minimum of 0.00336 sq-ft/hr at 20.1% moisture, then increased to 0.00358 sq-ft/hr at 30.2% moisture.

## SUGGESTIONS FOR FURTHER STUDY

1. The effect of temperature on the thermal properties of grain should be more thoroughly investigated.

2. The thermal properties of other grains and agricultural products should be determined.

3. Other properties such as absorptivity and emissivity should be investigated for analyzing radiant drying of grain.

4. Surface heat-transfer coefficients for grain and other products should be studied.

5. The linear and volumetric coefficients of thermal expansion should be determined.

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## APPENDIX

### Sample Calculations

**Specific Heat**

The equation for specific heat is;

$$c_1 = \frac{(w_2c_2 + w_3c_3)(\Delta t_2 - \phi)}{w_1(\Delta t_1 + \phi)}$$

For wheat at 0.68% moisture content;

$c_1$ = specific heat of sample

$w_1$ = weight of sample, 60.0 gm

$c_2$ = specific heat of calorimeter cup, 0.220 Btu/lb-°F

$w_2$ = weight of calorimeter cup, 49.1 gm

$c_3$ = specific heat of water, 1.00 Btu/lb-°F

$w_3$ = weight of water, 40.0 gm

$(\Delta t_2 - \phi)$ = corrected temperature change of water and calorimeter cup, 13.4 °F

$(\Delta t_1 + \phi)$ = corrected temperature change of sample, 29.8 °F

substituting into the equation;

$$c_1 = \frac{[(49.1)(0.220)+(40.0)(1.00)](13.4)}{(60.0)(29.8)} = 0.381 \text{ Btu/lb-}^\circ\text{F}$$

Thermal Conductivity

The equation for thermal conductivity is;

$$k = \frac{Q \log_e(\theta_2/\theta_1)}{4\pi(t_2 - t_1)}$$

For wheat at o.68% moisture content;

$k$ = thermal conductivity of sample, Btu/hr-ft-°F

$Q$ = heat input = $i^2R(3.42)$

$i$ = current, 0.490 amp

$R$ = resistance of heater wire, 5.45 ohm/ft

$\theta_2$ = time, 7.0 min

$\theta_1$ = time, 1.0 min

$t_2$ = temperature at time $\theta_2$, 106.4 °F

$t_1$ = temperature at time $\theta_1$, 96.3 °F

substituting into the equation;

$$k = \frac{(0.490)^2(5.45)(3.42) \log_e(7.0/1.0)}{4(3.14)(106.4 - 96.3)}$$

$k$ = 0.0684 Btu/hr-ft-°F

Thermal Diffusivity

The equation for thermal diffusivity is;

$$\alpha = zx^2/\theta$$

For wheat at 0.68% moisture content;

$\alpha$ = thermal diffusivity of sample, sq-ft/hr

$z$ = value obtained from $S(z)$, for $S(z) = t_c/t_o = 0.5$,

$z = 0.0947$

$x$ = distance from face of slab, for Box #2, $x = 0.113'$

$\theta$ = time, 0.338 hr

substituting into the equation;

$$\alpha = 0.0947(0.113)^2/0.338 = 0.00359 \text{ sq-ft/hr}$$