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thesis entitled

MODAL-COHERENT EQUIVALENTS DERIVED FROM
AN RMS COHERENCY MEASURE

presented by

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has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Systems Science

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Date 8/6/79

MODAL-COHERENT EQUIVALENTS DERIVED FROM AN RMS COHERENCY MEASURE

By

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Electrical Engineering
and Systems Science

1979

ABSTRACT

MODAL-COHERENT EQUIVALENTS DERIVED FROM AN RMS COHERENCY MEASURE

By

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The modal and coherency analysis techniques have dominated the substantial research efforts which have been devoted to developing dynamic equivalents that may be used to reduce the complexity of power system transient stability studies. The equivalents derived from these two techniques have distinctly different structures and properties. Although both approaches have desirable features, each approach has been the subject of criticism and neither technique has gained complete acceptance. The objective of this research is to show that a dynamic equivalent which combines the best features of both modal and coherent equivalents can be derived based on a coherency analysis using the rms coherency measure.

An algebraic formula is derived which relates the expected value of the rms coherency measure, evaluated over an infinite observation interval, to the parameters of the power system state model and the statistics of the system disturbance. This expression is used to establish an important link between the modal and coherency analysis approaches to

power system dynamic equivalents by showing that the inertially weighted synchronizing torque coefficients, which determine the system modes, are also the basis of coherency aggregation when a particular probabilistic disturbance, called the modal disturbance is used to identify coherent groups for coherency based aggregation. This result allows a coherent equivalent to be derived which closely approximates a general purpose modal equivalent based on the same coherency measure and disturbance. An example system is used to show that the eigenvalues of the coherent equivalent derived from the modal disturbance closely approximate the system eigenvalues retained by the corresponding modal equivalent and that both of these equivalents are excellent general purpose equivalents, suitable for studying many different system contingencies.

The coherent equivalent based on the rms coherency measure and the modal disturbance is called a modal-coherent equivalent. This equivalent has the theoretical soundness and general purpose applicability of a modal equivalent and the power system component structure of a coherent equivalent.

An efficient computational algorithm, applicable to large scale systems is developed for constructing the modal-coherent equivalent. It is shown that the computational effort required to construct the general purpose modal-coherent equivalent is competitive with the total effort required to construct the set of coherent equivalents which would be needed for a transient stability study of a relatively small number of distinct system disturbances.

For

Pauline

ACKNOWLEDGMENTS

I would like to express my appreciation to the Chairman of the Department of Electrical Engineering and Systems Science, Dr. J. B. Kreer, for the financial support of the department during my first year of study. Drs. G. L. Park and R. A. Schlueter and the Division of Engineering Research are thanked for providing continuing financial support over the final two years of my doctoral program.

Special acknowledgment is made of the contribution of Dr. R. O. Barr, who had confidence in me and encouraged me to pursue advanced education.

The participation of Drs. P. McCleer and A. N. Andry on my guidance committee is also appreciated and acknowledged.

I am grateful to Dr. G. L. Park for the opportunity to participate on his wind energy research projects which contributed greatly to my overall professional development. Dr. Park is also thanked for his significant service on my guidance committee.

I would like to express my sincere appreciation to my thesis advisor, Dr. R. A. Schlueter, for his guidance throughout the course of my doctoral program. Dr. Schlueter served not only as advisor but also as a friend and his friendship will never be forgotten.

Most of all, I would like to thank my wife, Julie, for her love, patience and gentle encouragement.

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CHAPTER 1

INTRODUCTION

The extensive research efforts to develop dynamic equivalents suitable for use in simulations of power system response to disturbances have been dominated by the modal and coherency analysis approaches [1,2]. These two techniques have developed independently and appear to be distinctly different concepts, although both approaches have the common goal of reducing the complexity of power system analysis. Each approach has been the subject of criticism and neither technique has gained complete acceptance. This thesis explores the relationship between these two approaches and proposes a new approach to power system dynamic equivalents which combines the best features of modal and coherency analysis.

In order to define some relevant terminology, the transient stability problem for large scale systems and the need for dynamic equivalents are briefly reviewed. An overview of modal and coherency analysis is also given in order to provide a perspective for the contribution of this research. The specific objectives of the thesis are then discussed.

The Need for Dynamic Equivalents in Transient Stability Studies

Consider a large geographical area where the electric power needs are served by many interconnected utilities.

Let one particular utility be designated as the internal system (or study area) and let the remaining utilities in the interconnected network be designated as the external system. The transient stability problem is to determine how well-behaved the generators in the internal system will be when a disturbance occurs in the internal system. The types of disturbances considered in transient stability studies fall into four basic categories; generator dropping, load shedding, line switching or electrical faults.

Recognizing that the internal-external system model may contain several hundred interconnected generating units, each of which is characterized by nonlinear differential equations, it is apparent that the determination of the transient behavior of the internal system generators for any particular disturbance will entail the solution of hundreds of nonlinear, coupled, differential equations. Thus, the transient stability problem for modern power systems is characterized by a severe dimensionality problem. Historically, transient stability analysis was performed using the classical, second order, synchronous machine equations, which model only the rotor dynamics of the synchronous machine, to represent each generator in the system. This model was used in order to minimize the number of differential equations to be solved. More recently, it has become apparent that there is a need to increase the level of generator modelling to include the exciter dynamics and the effects of the turbine-governor system, in order to improve the accuracy of transient stability results. An increase in the level of generator modelling naturally aggravates the dimensionality problem.

Since the number of generators in the internal system model is generally a relatively small percentage of the total number of generators in the composite internal-external system, it is clear that the study area of interest contributes only fractionally to the over-all complexity of the transient stability problem. Put another way, the bulk of the computational effort expended by a particular utility in performing a transient stability study is consumed in computing the impact that the large external system has on the behavior of the much smaller internal system.

A crude approach to ease the dimensionality problem would be to simply neglect the external system and perform the transient stability study on the isolated internal system. In general, the coupling which exists between the internal and external systems, due to tie lines, is not sufficiently weak to permit such an approach. In fact, one of the reasons for installing the tie lines which couple neighboring utilities is to improve system performance under disturbed conditions and to neglect these lines in contingency analysis would ignore their important function.

A more reasonable solution to the dimensionality problem is to find some method to reduce the size of the external system model. Ideally, the reduction would be done in such a way that the impact that the external system has on the behavior of the internal system, would be preserved with respect to disturbances that occur inside the study area. A reduced order model of the external system which meets this objective is called a dynamic equivalent (or simply an equivalent).

Two approaches that have been developed for constructing dynamic equivalents for power systems are the modal and coherency analysis approaches. The characteristics of these two techniques are now discussed.

The Modal Analysis Approach to Dynamic Equivalents

The modal analysis approach [1] assumes that a reasonable simplification of the transient stability problem would be to use a linearized model of the external system. The linearization can be justified, since the response of the external system to remote internal system disturbances would be at most weakly nonlinear. The order of the linearized model is then reduced by performing a modal reduction based on controllability and observability considerations. The reduced order linear model of the external system is called the modal equivalent. The procedure for constructing the modal equivalent is

1. Establish the linearized model of the unreduced external system.
2. Compute the eigenvalues and eigenvectors of the linear model.
3. Put disturbances at the boundary between the internal and external systems and use rules of mode elimination [1] based on the eigenvectors and eigenvalues to eliminate any mode of the external system model which

i) is relatively unexcited by any boundary disturbance (uncontrollable modes).

ii) is relatively undetectable at the boundary (unobservable modes).

With respect to the coherency analysis approach which will be discussed later, the modal approach has two significant advantages. These are

1. The approach is theoretically sound.
2. The modal equivalent is general purpose and may be used to study any disturbance in the internal system.

The disadvantages of the modal approach (with respect to coherency analysis) are

1. The eigenvalues and eigenvectors required to perform mode reduction are expensive to compute for large systems and are virtually impossible to compute for system models larger than one hundredth order.
2. The mode reduction process destroys power system structure, that is, the reduced order linear model obtained with the modal procedure is no longer recognized as a linearized power system model. Because the modal equivalent is not represented in terms of power system components, present transient stability programs would have to be modified to accept the linear model representation.

To overcome the difficulty in computing the eigenvalues of a very large system model, it has been proposed that the external system be divided into several computationally manageable "multiple external system" models [3]. Such an approach is undesirable unless weakly coupled regions in the external system can be easily identified, otherwise there would be no guarantee that the eigenvalues of the multiple external system models would be the same as the eigenvalues of the undivided system model. A general procedure for producing modal equivalents which preserve a meaningful physical structure is presently being pursued [4]. At this time, it appears that any approach to power system dynamic equivalents which requires eigenvalue calculations is not practical.

The Coherency Analysis Approach to Dynamic Equivalents

The coherency analysis approach to power system dynamic equivalents is based on the coherency phenomena associated with the behavior of synchronous machines. Coherency is a disturbance-dependent phenomena and for a particular system disturbance, two generators are said to be coherent if the coherency measure between them, evaluated for that disturbance, is less than some prescribed threshold. There are various ways of defining the coherency measure between two generators all of which are a function of the difference between the voltage angles (swing curves) at the internal generator buses of the two generators. Two or more mutually coherent generators define a coherent group and the generators in a coherent group are said to "swing together" in response to the system disturbance for which the coherency measure was evaluated. More than one coherent group may be observed for a single disturbance.

It has been shown [5] that the coherent groups corresponding to a particular system disturbance may be accurately identified from a coherency analysis of the response to that disturbance of a simplified, linearized model of the internal-external system. Once coherent groups have been identified, a coherent equivalent is constructed using the coherency-based aggregation technique [2] to form one equivalent generator per coherent group observed in the external system. Although coherent groups are identified with a simplified, linear model, the aggregation technique is applied to the full nonlinear system model and the coherent equivalent obtained is a reduced order, nonlinear model. The transient stability study for the disturbance used to identify coherent groups is then performed using the coherent equivalent identified for that disturbance.

There is no theoretical justification for power system model aggregation based on a coherency measure dependent only on voltage angle. The foundation of the coherency analysis approach rests on the intuitive appeal of the concept that generators which swing together function as essentially a single generator and may therefore be aggregated.

Although the coherency analysis technique lacks the sound theoretical basis of modal analysis, it offers significant advantages.

1. Eigenvalues and eigenvectors are not required to construct the coherent equivalent, and the overall computational procedure has been shown to be efficient and applicable to very large systems [5].

2. The form of the equivalent obtained is a reduced set of equivalent generators and lines and may therefore be used with existing transient stability programs.

The coherency based aggregation technique presently uses a max-min coherency measure to identify coherent groups. The technique has been criticized because

1. The max-min coherency measure used to identify coherent groups has not been shown to be proportional to system parameters and thus the coherency measure must be determined by simulation [6].

2. The equivalent produced using the max-min measure is dependent on the particular disturbance used to identify coherent groups for aggregation such that a unique equivalent must generally be constructed for each disturbance to be studied [7].

Satisfactory responses to these criticisms have not yet been given.

One additional criticism is directed at both modal and coherency analysis. Quite often there are alternative solution approaches for solving a given engineering problem, but when this is the case, the various solution techniques are related and the solutions which are obtained are essentially identical. Dynamic equivalents derived from the present modal and coherency analysis techniques [1,2] have such different structures and characteristics that it is reasonable to question whether or not these approaches are mutually consistent.

The rules of mode elimination, based on controllability and observability considerations, used in modal analysis [1] are designed to preserve the states of the internal system model. Mode reduction based on preserving states, and not a coherency measure, does not guarantee that intermachine behavior is preserved and may also be sensitive to the choice of reference generator used to establish the state model. Thus, if present modal equivalents are also coherent equivalents, they are coherent equivalents by accident and not by design. Similarly, the present coherency approach [2] makes no effort to insure that system modes are preserved. The fact that coherent equivalents must be recomputed each time the location of the system disturbance is significantly changed is sufficient evidence to indicate that coherent equivalents do not preserve system modes. Thus, the present modal and coherency analysis approaches are not mutually consistent.

Neither modal nor coherency analysis has gained complete acceptance by the power industry. The criticisms of these approaches shed considerable light on the properties of a dynamic equivalent which the industry would find highly desirable. The characteristics of an "ideal" dynamic equivalent are now discussed.

The Ideal Dynamic Equivalent

An ideal equivalent should be a general purpose equivalent, that is, a single ideal dynamic equivalent would be suitable to study any disturbance which might occur within the study area for which the equivalent is derived. Therefore, the ideal equivalent should be based solely on system structure and not on the type or location of any particular system disturbance.

In addition to the general purpose property, an ideal equivalent should be both a modal and a coherent equivalent. Coherency is an important power system phenomena which is strongly related to transmission line power flows and peculiar to the behavior of synchronous machines. Any approach to power system model aggregation which produces an equivalent which significantly alters the coherency measure between generators would upset the power transfer characteristics of the system and neglect the essential nature of synchronous machine interaction. The characteristic modes of oscillation are similarly important. If an equivalent does not closely preserve system modes, then there is little chance that the equivalent can accurately predict the time response of the full system model for a large class of disturbances. Thus, an ideal dynamic equivalent should preserve system modes and the coherent behavior of generators.

Two final characteristics that an ideal equivalent should have are dictated by practical considerations. First, the equivalent should be represented in terms of normal power system components so that it can be used with existing transient stability computer programs. And second, the approach for deriving the equivalent must be computationally efficient and applicable to large scale systems.

The discussion of the characteristics of an ideal power system dynamic equivalent indicates that such an equivalent should be a general purpose equivalent which is both a modal and a coherent equivalent, which can be derived from an efficient procedure that preserves power system component structure. Neither modal analysis nor coherency analysis leads to an equivalent which has all of these properties. However, the characteristics of an ideal equivalent are a composite of the most desirable features of present modal and coherent equivalents. This suggests that a unified approach to dynamic equivalents which is consistent with the objectives of both modal and coherency analysis may result in a dynamic equivalent with ideal or nearly ideal properties. An early effort to unify modal and coherency analysis is now briefly reviewed.

A Coherent-Modal Approach to Dynamic Equivalents

Recognizing the need to insure that modal equivalents preserve the coherent behavior between generators, a recent paper [8] established rules of mode elimination for constructing modal equivalents based on a modal analysis of an rms coherency measure evaluated over an infinite interval. The rules of mode elimination are designed to eliminate modes which do not significantly change the rms coherency measure between any two generators by more than some arbitrary amount.

Modal equivalents which are derived to preserve a coherency measure may also be considered coherent equivalents and the approach suggested by [8] may be termed a coherent-modal approach. The coherent-modal approach would not lead directly to an ideal equivalent, since eigenvalues are required and since the equivalent does not retain power system component structure. However, the approach does provide some indication of how an ideal equivalent might be obtained.

The rules of mode elimination based on the rms coherency measure [8] were derived for both deterministic and probabilistic system disturbances. In a companion paper [9], those rules were applied to an example system, and modal equivalents were constructed for various step disturbances in mechanical input power. It was observed that when the step disturbance in the mechanical input powers on the system generators were zero mean, independent and identically distributed (ZMIID), that the process of mode reduction closely resembled a coherency-based aggregation since, the only modes eliminated by the rules of mode elimination were associated with the intermachine oscillations within a coherent group. It was further observed that the rules of mode elimination applied to deterministic disturbances did not resemble a coherency aggregation. Two hypotheses were proposed based on these observations. The first was that dynamic system structure can be identified through an rms coherency measure if probabilistic disturbances are used to identify coherent groups. The second hypothesis was that the eigenvalues of the coherent equivalent derived using the coherency-based aggregation technique to aggregate the coherent groups identified by the rms coherency measure and the ZMIID disturbance should closely approximate the system eigenvalues retained by a modal equivalent based on the same coherency measure and disturbance.

The above hypotheses, based on empirical observations of the behavior of coherent-modal equivalents, provides a major direction for this research. The specific objective of this thesis is now described.

Thesis Objective

The objective of this thesis is to develop the justification and the means for constructing a modal-coherent equivalent whose properties closely approach those of an ideal dynamic equivalent. It is emphasized that the term "ideal" is meant to be taken in the context of the preceding discussion.

The modal-coherent equivalent proposed in this research is constructed using the coherency-based aggregation technique [2] to aggregate the coherent groups identified by the infinite interval rms coherency measure and a particular probabilistic step disturbance in mechanical input powers called the "modal disturbance". Two properties of an ideal equivalent are realized by the modal-coherent approach as a natural consequence of the use of a coherency analysis and coherency-based aggregation to identify and construct the equivalent. These are

1. The coherent behavior of generators is preserved by the equivalent.
2. The equivalent is represented in terms of normal power system components.

It will be shown, that two additional properties of an ideal equivalent are achieved by the modal-coherent equivalent because the rms coherency measure and the modal disturbance can identify coherent groups which reflect the dynamic structure of the system. These properties are

3. System modes are closely preserved by the equivalent.
4. The equivalent is general purpose and may be used to study a large class of disturbances.

It will also be shown, that the final property of an ideal equivalent is realized by the modal coherent approach because

5. The infinite interval rms coherency measure can be evaluated for the probabilistic modal disturbance using an efficient technique which is applicable to large scale systems.

Thus, this thesis will develop a modal-coherent equivalent which is a general purpose coherent equivalent and an approximate modal equivalent that may be derived from a computationally efficient procedure which preserves power system component structure.

The justification for the modal-coherent equivalent is developed in Chapters 3 and 4. In Chapter 3, an algebraic relationship is derived relating the expected value of the rms coherency measure, evaluated over an infinite interval, to the parameters of the state model and the statistics of the system disturbance. This relationship is used to show that the rms coherency measure and a particular probabilistic system disturbance can reflect dynamic system structure. Thus, the first hypothesis proposed in [9] is verified theoretically. Using the same example system as [9], it is shown in Chapter 4, that the eigenvalues of the coherent equivalent constructed using the coherency-based aggregation technique to aggregate the coherent groups identified by the rms coherency measure and a particular probabilistic system disturbance closely approximate the system eigenvalues retained by the modal equivalent based on the same coherency measure and disturbance. It is further shown that both the modal and the coherent equivalent derived from this probabilistic disturbance are suitable for studying the effects of any disturbance which might occur outside the areas of the system aggregated to form these equivalents. Thus it is shown that a general purpose coherent equivalent which is an approximate modal equivalent can be derived from the rms coherency measure when an appropriate probabilistic disturbance is used to identify coherent groups. The appropriate probabilistic disturbance is shown to be the modal disturbance. The example system used is shown to be a special case where the above results apply to the ZMIID disturbance as well as the modal disturbance and thus the second hypothesis proposed in [9] is empirically verified.

Chapter 5 discusses the need for an approach to dynamic equivalents which is consistent with the objectives of both modal and coherency analysis and retains the structure of a coherent equivalent. The modal-coherent equivalent is proposed to meet this need.

The means for constructing the modal-coherent equivalent is developed in Chapter 6. An efficient method for evaluating the infinite interval rms coherency measure for the probabilistic modal disturbance is developed which is applicable to large scale systems. It is shown that the computational effort required to construct the general purpose modal-coherent equivalent is competitive with the total effort required to construct the set of coherent equivalents which would be needed to perform a transient stability study for a very modest number of distinct system disturbances.

The final chapter summarizes the contribution of this thesis and proposes topics for future investigation based on this research.

CHAPTER 2

POWER SYSTEM MODEL, GENERALIZED DISTURBANCE MODEL, AND THE RMS COHERENCY MEASURE

The objective of this chapter is to present the mathematical models used in the development of the modal-coherent equivalent. The linearized power system model, generalized disturbance model and rms coherency measure used for coherency analysis are defined, and the mechanism by which coherent groups are aggregated to form a coherent equivalent is briefly discussed.

2.1 Linearized System Model

Recent work on coherency-based dynamic equivalents at System Control Incorporated (SCI) has shown that a simplified model for coherency analysis can be derived with the following assumptions

1. The coherent groups of generators are independent of the size of the disturbance. Therefore, coherency can be determined by considering a linearized system model.
2. The coherent groups are independent of the amount of detail in the generating unit models. Therefore, a classical synchronous machine model is considered and the excitation and turbine-governor systems are ignored.

3. The effect of a fault may be reproduced by considering the unfaulted network and pulsing the mechanical powers to achieve the same accelerating powers which would have existed in the faulted network.

The first assumption may be confirmed by considering a fault on a certain bus, and observing that the coherency behavior of the generators is not significantly changed as the fault clearing time is increased. The second assumption is based upon the observation that although the amount of detail in the generating unit models has a significant effect upon the swing curves particularly the damping, it does not radically affect the more basic characteristics such as the natural frequencies and mode shapes. The third assumption recognizes that the generator accelerating powers are approximately constant during faults with typical clearing times. The above assumptions and their justifications are quoted from [11].

A linear model can be obtained by starting with the classical synchronous machine representation for each generator

$$M_i \frac{d}{dt} \omega_i(t) = PM_i(t) - PG_i(t) - D_i\omega_i(t) \qquad i=1,2,\ldots,N \tag{2-1a}$$

$$\frac{d}{dt} \delta_i(t) = \omega_i(t) \quad , \quad i=1,2,\ldots,N \tag{2-1b}$$

where,

- i a subscript for generator i
- N the number of generators in the system
- M_i inertia constant (p.u.)
- D_i damping constant (p.u.)
- ω_i generator speed (rad/sec)
- δ_i generator rotor angle (rad)

PM_i	mechanical input power (p.u.)
PG_i	electrical output power (p.u.)
ω_o	nominal synchronous generator speed (rad/sec)

Equations (2-1) are nonlinear due to the nonlinear relationship between PG_i and the bus angles in the interconnected network. For a lossless transmission network, the system network equations can be written as

$$\begin{aligned} PG_i &= \sum_{\substack{j=1 \\ j\neq i}}^{N} \frac{|V_i||V_j|}{X_{ij}} \sin(\delta_i-\delta_j) \\ &\quad + \sum_{\ell=1}^{K} \frac{|V_i||V_\ell|}{X_{i\ell}} \sin(\delta_i-\theta_\ell) \qquad i=1,2,\ldots,N \\ PL_k &= \sum_{j=1}^{N} \frac{|V_k||V_j|}{X_{kj}} \sin(\theta_k-\delta_j) \\ &\quad + \sum_{\substack{\ell=1 \\ \ell\neq k}}^{K} \frac{|V_k||V_\ell|}{X_{k\ell}} \sin(\theta_k-\theta_\ell) \qquad k=1,2,\ldots,K \end{aligned} \tag{2-2}$$

where,

$\lvert V_i\rvert,\lvert V_j\rvert$	magnitude of the complex voltages at generator buses i and j
$\lvert V_k\rvert,\lvert V_\ell\rvert$	magnitude of the complex voltages at load buses k and ℓ
θ_k,θ_ℓ	voltage angle at load buses k and ℓ
K	is the number of load buses
$X_{\cdot\cdot}$	is the impedance of the line directly connecting any two specified buses

The synchronous machine equations may be linearized using a first order Taylor series expansion, by introducing the deviations $\Delta\delta_i$, $\Delta\omega_i$, ΔPM_i, ΔPG_i about the nominal load flow conditions δ_i^o, ω_o, PM_i^o, PG_i^o. The resulting linear model has the form

$$M_i \frac{d}{dt} \Delta\omega_i(t) = \Delta PM_i(t) - \Delta PG_i(t) - D_i \Delta\omega_i(t) \qquad i=1,2,\ldots,N \tag{2-3a}$$

$$\frac{d}{dt} \Delta\delta_i(t) = \Delta\omega_i(t) \quad , \quad i=1,2,\ldots,N \tag{2-3b}$$

The network equations may also be linearized with real and reactive powers decoupled and written in polar form as

$$\begin{bmatrix} \Delta \underline{PG} \\ \Delta \underline{PL} \end{bmatrix} = \begin{bmatrix} \partial \underline{PG}/\partial \underline{\delta} & \partial \underline{PG}/\partial \underline{\theta} \\ \partial \underline{PL}/\partial \underline{\delta} & \partial \underline{PL}/\partial \underline{\theta} \end{bmatrix} \begin{bmatrix} \Delta \underline{\delta} \\ \Delta \underline{\theta} \end{bmatrix} \tag{2-4}$$

where,

$\underline{PG} = (PG_1, PG_2, \ldots, PG_N)^T$

$\underline{PL} = (PL_1, PL_2, \ldots, PL_K)^T$

$\underline{\delta} = (\delta_1, \delta_2, \ldots, \delta_N)^T$

$\underline{\theta} = (\theta_1, \theta_2, \ldots, \theta_K)^T$

K is the number of load buses

ΔPL_j deviation in power injection at load bus j

$\Delta\theta_j$ deviation in voltage angle at load bus j

The decoupling of real and reactive powers is justified for transmission systems which exhibit high X/R ratios (i.e. low loss networks). Line losses should have little impact on coherent behavior between generators and for the purpose of coherency analysis a lossless network will be assumed.

The power-angle Jacobian matrix in the network equations (2-4) is a sparse, symmetric and singular matrix. The network equations are not of full rank since the entries in any row, or column, sum to zero (the diagonal elements are the negative of the sum of the off-diagonal elements in any row or column). Thus a unique solution for $\Delta\underline{\delta}$ and $\Delta\underline{\theta}$, given $\Delta\underline{PG}$ and

<u>ΔPL</u>, cannot be obtained. This minor dilemma is solved by an angle referencing scheme which is discussed shortly.

Equations (2-3) and (2-4) are said to be a <u>synchronous frame</u> model since the deviations in bus angles and generator speeds are measured with respect to an external reference rotating at the nominal system speed, ω_o. If a step input in mechanical input powers, which is not balanced by an equivalent change in load, is applied to the synchronous frame model, the speed of the generators in the model will change. The deviations in generator angles in response to such a disturbance will appear as ramp functions when measured with respect to a reference that always rotates at the nominal synchronous speed, ω_o. Thus, the synchronous frame model has an eigenvalue at the origin (step input, ramp output). The analysis presented in this thesis requires a model which has all nonzero eigenvalues. Such a model may be obtained by referencing all of the bus angles in the system to the angle of an arbitrarily chosen reference generator. Referencing angles is a common practice in power system analysis and results in no loss of generality. Selecting generator N as the system reference, equations (2-3b) and (2-4) may be rewritten in the <u>machine N reference frame</u> as

$$\frac{d}{dt}\Delta\hat{\delta}_i(t) = \frac{d}{dt}\Delta\delta_i(t) - \frac{d}{dt}\Delta\delta_N(t) \qquad (2\text{-}5)$$

$$= \Delta\omega_i(t) - \Delta\omega_N(t) \quad , \quad i=1,2,.i.,N-1$$

and,

$$\begin{bmatrix} \underline{\Delta PG} \\ \underline{\Delta PL} \end{bmatrix} = \begin{bmatrix} \partial \underline{PG}/\partial \underline{\hat{\delta}} & \partial \underline{PG}/\partial \underline{\hat{\theta}} \\ \partial \underline{PL}/\partial \underline{\hat{\delta}} & \partial \underline{PL}/\partial \underline{\hat{\theta}} \end{bmatrix} \begin{bmatrix} \underline{\Delta \hat{\delta}} \\ \underline{\Delta \hat{\theta}} \end{bmatrix} \tag{2-6}$$

where,

$$\hat{\delta}_i = \delta_i - \delta_N \quad , \quad i=1,2,\ldots,N-1$$

$$\hat{\theta}_j = \theta_j - \theta_N \quad , \quad j=1,2,\ldots,K$$

In addition to eliminating the eigenvalue at the origin, the angle referencing scheme allows the network equations, in the generator N frame, to be uniquely solved for $\underline{\Delta\hat{\delta}}$ and $\underline{\Delta\hat{\theta}}$ given the values of $\underline{\Delta PG}$ and $\underline{\Delta PL}$.

When each of the generators in the system is characterized by the same damping to inertia ratio, that is

$$\frac{M_i}{D_i} = \sigma \text{ (a constant)} \quad , \quad i=1,2,\ldots,N \tag{2-7}$$

then the differential equations describing the generators may be written as

$$\frac{d}{dt}\Delta\hat{\omega}_i(t) = M_i^{-1}(\Delta PM_i(t) - \Delta PG_i(t)) - M_N^{-1}(\Delta PM_N(t) - \Delta PG_N(t)) - \sigma\Delta\hat{\omega}_i(t) \tag{2-8a}$$

$$i=1,2,\ldots,N-1$$

$$\frac{d}{dt}\Delta\hat{\delta}_i(t) = \Delta\omega_i(t) \quad , \quad i=1,2,\ldots,N-1 \tag{2-8b}$$

where,

$$\hat{\omega}_i = \omega_i - \omega_N \quad , \quad i=1,2,\ldots,N-1$$

Equations (2-8) are referred to as a uniform machine N frame model. The uniform damping assumption, (2-7), does result in some loss of generality. However, the assumption is partially justified by recognizing that damping to inertia ratios are typically small and in general the value of the damping constant is not accurately known. A thorough discussion of the synchronous frame, machine N frame and uniform machine N frame models may be found in [10].

A state variable model may be derived by writing the 2N-2 equations in (2-8) in vector form and using the network equations (2-6) to eliminate $\underline{\Delta PG}$ from the expression. The resulting model has the form

$$\dot{\underline{x}}(t) = \underline{A}\ \underline{x}(t) + \underline{B}\ \underline{u}(t) \tag{2-9}$$

where,

$$\underline{x} = \begin{bmatrix} \underline{\Delta\hat{\delta}} \\ \underline{\Delta\hat{\omega}} \end{bmatrix}, \quad \underline{u} = \begin{bmatrix} \underline{\Delta PM} \\ \underline{\Delta PL} \end{bmatrix} \tag{2-10}$$

$$\underline{A} = \begin{bmatrix} \underline{0} & \underline{I} \\ -\underline{M}\ \underline{T} & -\sigma\underline{I} \end{bmatrix}, \quad \underline{B} = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{M} & \underline{M}\ \underline{L} \end{bmatrix}$$

and,

$$\underline{M} = \left[\begin{array}{cccc|c} M_1^{-1} & & & & -M_N^{-1} \\ & M_2^{-1} & & & -M_N^{-1} \\ & & \cdot & & \cdot \\ & & & \cdot & \cdot \\ & & & M_{N-1}^{-1} & -M_N^{-1} \end{array}\right] \tag{2-11}$$

$$\underline{T} = \partial\underline{PG}/\partial\underline{\hat{\delta}} - (\partial\underline{PG}/\partial\underline{\hat{\theta}})\ [\partial\underline{PL}/\partial\underline{\hat{\theta}})]^{-1}\ (\partial\underline{PL}/\partial\underline{\hat{\delta}})$$

$$\underline{L} = -(\partial\underline{PG}/\partial\underline{\hat{\theta}})\ [\partial\underline{PL}/\partial\underline{\hat{\theta}})]^{-1}$$

The synchronizing torque coefficient matrix, $\underline{T}$, and the reflection matrix, $\underline{L}$, are found in the process of solving the network equations for $\underline{\Delta PG}$ in terms of $\underline{\Delta\hat{\delta}}$ and $\underline{\Delta PL}$ with the result

$$\underline{\Delta PG} = \underline{T}\ \underline{\Delta\hat{\delta}} + \underline{L}\ \underline{\Delta PL} \tag{2-12}$$

A linearized power system model has been developed and attention is now focused on a generalized disturbance model. The disturbance model will be shown to model deterministic as well as probabilistic system disturbances. The model was developed by Schlueter, in [8], and the presentation here follows that development.

2.2 Generalized Disturbance Model

The initial conditions of the linear model (2-9) are assumed random with

$$E\{\underline{x}(0)\} = \underline{0} \tag{2-13}$$

$$E\{\underline{x}(0)\ \underline{x}^T(0)\} = \underline{V}_x(0)$$

since the expected deviations from any operating state is zero but the variance of such deviations is nonzero. The rms coherency measure will be shown to depend on the covariance matrix $\underline{V}_x(0)$. The initial conditions are included not to reflect any specific type of disturbance but rather the effects on the state from some hypothetical disturbance whose statistics (2-13) may be inferred from internal and external operating conditions.

The input, composed of the deviations in the mechanical input power, $\underline{\Delta PM}$, and the deviations in load power, $\underline{\Delta PL}$, can be used to model

i) loss of generation due to generator dropping
ii) loss of load due to load shedding
iii) line switching
iv) electrical faults

These contingencies can be modelled by an input $\underline{u}(t)$ that has the following form

$$\underline{u}(t) = \underline{u}_1(t) + \underline{u}_2(t) \tag{2-14}$$

The vector function

$$\underline{u}_1(t) = \begin{cases} \underline{u}_1 & t \geq 0 \\ \underline{0} & t < 0 \end{cases} \tag{2-15}$$

represents

i) the loss of generation due to generator dropping
ii) the loss of load due to load shedding
iii) changes in load injections due to line switching

The modelling of these three disturbances requires determination of $\underline{u}_1$ and possible modification of the network before determination of matrices $\underline{A}$ and $\underline{B}$. The procedure used [11] for each disturbance type is discussed below:

<u>generator dropping</u> - the transient reactance of the generator dropped is omitted from the network and the deviation in the generator output PM_i of the generator dropped is set equal to the loss of generation.

<u>load shedding</u> - the load deviation PL_k for all buses k where load is shed should be set equal to the change in load caused by the load shedding operation.

<u>line switching</u> - the network is modified to represent the system after the line switching operation is performed. The load deviations, PL_k and PL_m, at buses to which this line is connected, are set equal to the changes at that bus which occur due to the particular line switching operation.

Note that in each case above all variables in $\underline{u}_1$ are zero unless otherwise specified and the operating point used to obtain matrices $\underline{A}$ and $\underline{B}$ is that obtained from the base case load flow even if network changes are made. The results obtained without finding the post disturbance load flow conditions is apparently satisfactory because the effects due to changes in the load flow are assumed to be confined to the study system and thus should not affect the coherency of the external system being equivalenced.

The vector function

$$\underline{u}_2(t) = \begin{cases} \underline{0} & t > T_1 \\ \underline{u}_2 & 0 \le t \le T_1 \\ \underline{0} & t < 0 \end{cases} \tag{2-16}$$

represents the effects of electrical faults where T_1 represents the fault clearing time and

$$\underline{u}_2 = \begin{bmatrix} \Delta PM \\ \hline \underline{0} \end{bmatrix} \tag{2-17}$$

represents the step change in generation output equivalent to the accelerating powers due to a particular fault. The change of mechanical powers, $\underline{\Delta PM}$, which corresponds to the accelerating powers on generators due to a particular fault is calculated by an ACCEL program [11], and has been shown to adequately model the effects of that fault when a linearized model based on pre-fault load flow conditions is used. Again the results obtained neglecting faulted and post fault load flow conditions is apparently satisfactory due to the fact that the effects due to changes in load flow conditions are assumed to be confined to the study system and should not effect the coherency of the external system being equivalenced.

The above model can be generalized to model the uncertainty of any particular disturbance and yet handle specific deterministic disturbance as a special case. If the size and location of an electrical fault is not known and if the clearing time T_1 for this fault is known, then a probabilistic description of this electrical fault is

$$E\{\underline{u}_2\} = \begin{bmatrix} \underline{m}_{21} \\ 0 \end{bmatrix} = \underline{m}_2 \tag{2-18}$$

$$E\{[\underline{u}_2 - \underline{m}_2]\ [\underline{u}_2 - \underline{m}_2]^T\} = \begin{bmatrix} \underline{R}_{21} & 0 \\ 0 & 0 \end{bmatrix} = \underline{R}_2$$

where $\underline{m}_{21}$ and $\underline{R}_{21}$ describe the uncertainty in accelerating power on all generators due to this electrical fault. This mean and variance should be determined based on observed historical records or hypothesized based on the present network and present internal and external conditions. If $\underline{R}_2 = \underline{0}$, and $\underline{m}_{21} = \underline{\Delta PM}$ for a specific fault, this generalized model then reverts to the deterministic model of a specific electrical fault.

The uncertainty due to a generator dropping, line switching, and load shedding disturbance can be modelled by

$$E\{\underline{u}_1\} = \begin{bmatrix} \underline{m}_{11} \\ \underline{m}_{12} \end{bmatrix} = \underline{m}_1 \tag{2-19}$$

$$E\{[\underline{u}_1 - \underline{m}_1]\ [\underline{u}_1 - \underline{m}_1]^T\} = \begin{bmatrix} \underline{R}_{11} & \underline{0} \\ \underline{0} & \underline{R}_{12} \end{bmatrix} = \underline{R}_1$$

where

1. $\underline{m}_{11}$ and $\underline{R}_{11}$ can describe the uncertainty in generation changes due to generator dropping when the particular station, the generator in the station, and the power produced on the generator are unknown.

2. $\underline{m}_{12}$ and $\underline{R}_{22}$ describe the uncertainty in the location and magnitude of the load being dropped by any manual or automatic load shedding operation.

3. $\underline{m}_{12}$ and $\underline{R}_{22}$ describe the uncertainty in the location and the change in injections on buses due to any line switching operation.

It should be noted that $\underline{\Delta PM}$ and $\underline{\Delta PL}$ are assumed uncorrelated because this model is to represent only one specific type of contingency at a time. For the same reason $\underline{u}_1$ and $\underline{u}_2$ are assumed uncorrelated with initial conditions and

$$E\{\underline{x}(0)\ \underline{u}_1^T\} = \underline{0} \tag{2-20}$$

$$E\{\underline{x}(0)\ \underline{u}_2^T\} = \underline{0}$$

The uncertain model of $\underline{u}_1$ can handle the case of a specific deterministic disturbance by setting $\underline{R}_1 = \underline{0}$ and $\underline{m}_1 = \underline{u}_1$ for the particular disturbance.

The probabilistic descriptions of generator dropping and line switching are made assuming the network changes associated with the deterministic disturbances of these types can be omitted. This assumption seems valid since the effects of retaining these elements in the network should be confined to the study system and should not seriously effect coherency of the external system being equivalenced.

2.3 RMS Coherency Measure

Early work by SCI [2] was aimed at identifying the particular coherency measure of voltage angle differences that would produce the best dynamic equivalent when the generator buses classified as coherent by each measure were equivalenced. Two particular measures compared at that time were the max-min and rms coherency measures

$$C^1_{k\ell} = \max_{t\varepsilon[0,T]}\{\delta_k(t) - \delta_\ell(t)\} - \min_{t\varepsilon[0,T]}\{\delta_k(t) - \delta_\ell(t)\} \qquad (2\text{-}21)$$

$$C^2_{k\ell} = \sqrt{\frac{1}{T}\int_0^T [\Delta\delta_k(t) - \Delta\delta_\ell(t)]^2 dt}$$

where

$$\Delta\delta_k(t) = \delta_k(t) - \delta_k(0)$$

The max-min coherency measure was shown to produce better dynamic equivalents than those constructed based on an rms measure of coherency when both measures are compared based on the same short observation interval. The result is not unexpected because the max-min measure should insure there are no large deviations between the equivalent and the unreduced model whereas the rms measure should only insure that average energy in the deviations is small over this short observation interval. Another way of viewing this difference in the two measures is that a max-min measure of coherency tends to measure the sum of the amplitudes of dominant modes of the system dynamics where rms coherency tends to measure the sum of the energy in dominant modes of angle differences $\delta_k(t) - \delta_\ell(t) \quad k, \ell = 1,2,\ldots,N$. The max-min measure of coherency is thus clearly a better measure over a short observation interval when a dynamic equivalent for one particular contingency occurring in one location is desired. However, if an equivalent is desired that best reflects the overall dynamics of the external system, the rms measure will be shown to be superior.

The rms measure of coherency between generator internal buses k and ℓ based on the uncertain description of disturbances is defined by

$$C_{k\ell}(t) = \sqrt{\frac{1}{t^n} \int_0^t E\{[\Delta\delta_k(\tau) - \Delta\delta_\ell(\tau)]^2\}d\tau} \qquad (2\text{-}22)$$

The integer n is chosen to be one if a load shedding, line switching, or generator dropping contingency occurs and zero if an electrical fault occurs. This integer is chosen as zero or one so that the above integral will be finite and non-zero for an infinite observation interval.

The computation of the rms measure is facilitated by constructing the intermediate quantity, $\underline{S}_x(t)$, which is a 2N-2 dimensional symmetric matrix which is defined in terms of the state vector of the linear model as

$$\underline{S}_x(t) = \frac{1}{t^n} \int_0^t E\{\underline{x}(\tau)\underline{x}^T(\tau)\}d\tau \qquad (2\text{-}23)$$

The coherency measure between any pair of generators k and ℓ, defined by (2-22), depends only on the generator angles. Therefore, the value of any $C_{k\ell}(t)$ will be determined by the upper left (N-1) x (N-1) submatrix of $\underline{S}_x(t)$. Defining the upper left (N-1) x (N-1) submatrix of $\underline{S}_x(t)$ to be $\hat{\underline{S}}_x(t)$, the coherency measure $C_{k\ell}(t)$ is related to the entries in $\hat{\underline{S}}_x(t)$ by

$$C_{k\ell}(t) = \begin{cases} \sqrt{\{\hat{\underline{S}}_x(t)\}_{kk} - \{\hat{\underline{S}}_x(t)\}_{k\ell} - \{\hat{\underline{S}}_x(t)\}_{\ell k} + \{\hat{\underline{S}}_x(t)\}_{\ell\ell}} & k,\ell \neq N \\ \sqrt{\{\hat{\underline{S}}_x(t)\}_{kk}} & k \neq N,\ \ell = N \\ \sqrt{\{\hat{\underline{S}}_x(t)\}_{\ell\ell}} & \ell \neq N,\ k = N \end{cases} \tag{2-24}$$

The matrix $\underline{S}_x(t)$ can be computed for the disturbance $\underline{u}(t)$, given by (2-14), by substituting the solution to the state equation

$$\underline{x}(\tau) = \begin{cases} \varepsilon^{\underline{A}\tau}\ \underline{x}(0) + \int_0^\tau \varepsilon^{\underline{A}v}\, dv\ \underline{B}\ (\underline{u}_1 + \underline{u}_2) & \tau < T_1 \\ \varepsilon^{\underline{A}\tau}\ \underline{x}(0) + \int_0^\tau \varepsilon^{\underline{A}v}\, dv\ \underline{B}\ \underline{u}_1 & \\ \qquad + \varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \underline{A}v\ dv\ \underline{B}\ \underline{u}_2 & \tau \geq T_1 \end{cases} \tag{2-25}$$

into equation (2-23), and taking the expectation term by term. The resulting form of $\underline{S}_x(t)$ was derived in [8] and shown to be

$$
\begin{aligned}
\underline{S}_x(T) = {} & \frac{1}{T^n} \int_0^T \varepsilon^{\underline{A}\tau} \, \underline{V}_x(0) \, \varepsilon^{\underline{A}^T\tau} \, d\tau \\
& + \frac{1}{T^n} \int_0^{T_1} \left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right] \left[\underline{R}_1 + \underline{R}_2 + \underline{m}_1\underline{m}_1^T + \underline{m}_2\underline{m}_2^T + \underline{m}_1\underline{m}_2^T + \underline{m}_2\underline{m}_1^T \right] \\
& \qquad \times \left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right]^T d\tau \\
& + \frac{1}{T^n} \int_{T_1}^{T} \left(\left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right] \left[\underline{m}_1\underline{m}_1^T + \underline{R}_1 \right] \left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right]^T \right) d\tau \\
& + \frac{1}{T^n} \int_{T_1}^{T} \left(\left[\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \, \underline{B} \right] \left[\underline{m}_2\underline{m}_2^T + \underline{R}_2 \right] \right. \\
& \qquad \left. \times \left[\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \, \underline{B} \right]^T \right) d\tau \\
& + \frac{1}{T^n} \int_{T_1}^{T} \left(\left[\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \, \underline{B} \right] \left[\underline{m}_2\underline{m}_1^T \right] \left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right]^T \right) d\tau \\
& + \frac{1}{T^n} \int_{T_1}^{T} \left(\left[\int_0^{\tau} \varepsilon^{\underline{A}v} dv \, \underline{B} \right] \left[\underline{m}_1\underline{m}_2^T \right] \left[\varepsilon^{\underline{A}(\tau - T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv \, \underline{B} \right]^T \right) d\tau
\end{aligned}
\qquad (2\text{-}26)
$$

Once the matrix $\underline{S}_x(t)$ is computed for a particular disturbance, the coherency measure between any pair of generators may be computed using (2-23). Based on the computed coherency measures, coherent groups of generators are identified (the identification procedure is discussed later in the thesis) and then aggregated using the coherency-based aggregation technique [2]. A short discussion of the aggregation technique is now presented.

2.4 Coherency-Based Aggregation Technique

A coherent equivalent is derived by aggregating coherent groups using the coherency-based aggregation technique [2]. Although coherent groups are identified using a simplified, linear power system model, the aggregation technique is generally applied to the nonlinear system description. Thus, the coherent equivalent derived for use in a particular transient stability study is a reduced order, nonlinear version of the complete system model. It is possible to use the coherency-based aggregation technique to aggregate the linearized model and obtain a simplified, linearized coherent equivalent. Comparisons between the system behavior observed with the unreduced, linear system model and the reduced order, linearized coherent equivalent for various disturbances are useful in inferring how well the nonlinear coherent equivalent will predict the behavior of the full nonlinear system model. The procedure for aggregating the linear system model using the coherency-based aggregation technique is now presented and the discussion will closely parallel the presentation in [12] for aggregating the nonlinear model.

A power system model consisting of N generators, N internal generator buses, N generator terminal buses, K load buses and L transmission lines is converted by the linearization process to a small signal model containing N generators, N internal generator buses and N(N-1)/2 equivalent lines. One equivalent line connects each possible pair of internal generator buses. The objective of coherency-based aggregation is to represent each group of coherent generators by a single equivalent generator and reduce the number of equivalent lines in the network to correspond to the number of generators retained in the aggregated model. The aggregation problem is to determine the parameters which characterize the equivalent generators and the reduced set of equivalent lines.

Consider the problem of aggregating a single coherent group containing m generators ($m < N$). The aggregation of multiple coherent groups can be accomplished by repeat applications of the procedure for aggregating a single coherent group. For convenience, let the generators be numbered such that the indices of the generators in the coherent group run from one to m. Each generator in the simplified linear model used to identify coherent groups is represented by the linearized classical synchronous machine equations (2-3). In order to determine the parameters which describe the equivalent generator, representing the coherent group in the reduced order linear equivalent, consider summing the synchronous machine equations of the members of the group,

$$\begin{aligned} \sum_{j=1}^{m} M_j \frac{d}{dt} \Delta\omega_j &= \sum_{j=1}^{m} \Delta PM_j - \Delta PG_j - D_j \Delta\omega_j \\ &= \sum_{j=1}^{m} \Delta PM_j - \sum_{j=1}^{m} \Delta PG_j - \sum_{j=1}^{m} D_j \Delta\omega_j \end{aligned} \tag{2-27}$$

By definition, coherent generators swing together, therefore each $\Delta\omega_j$ in (2-27) may be replaced by an equivalent speed deviation, $\Delta\omega_g$, which is characteristic of all the generators in the group. Replacing each $\Delta\omega_j$ by $\Delta\omega_g$, (2-21) may be rewritten as

$$\left(\sum_{j=1}^{m} M_j\right) \frac{d}{dt} \Delta\omega_g = \sum_{j=1}^{m} \Delta PM_j - \sum_{j=1}^{m} \Delta PG_j - \left(\sum_{j=1}^{m} D_j\right) \Delta\omega_g \tag{2-28}$$

It is immediately recognized that (2-28) is the synchronous machine representation of the desired equivalent generator since that equation may be written in the form

$$M_g \frac{d}{dt} \Delta\omega_g = \Delta PM_g - \Delta PG_g - D_g \Delta\omega_g \tag{2-29}$$

Comparing (2-29) and (2-28) it is clear that the parameters which describe the equivalent generator are

1. an inertia equal to the sum of the inertias of the members of the coherent group.
2. a damping constant equal to the sum of the damping constants of the members of the coherent group.
3. a deviation in mechanical input power equal to the sum of the deviations in mechanical input power on the individual generators in the coherent group.
4. a deviation in electrical output power equal to the sum of the deviations in electrical output power of the generators in the coherent group.

The parameters which describe the reduced set of equivalent lines can be determined from item four above. Each equivalent line in the unreduced linear model is characterized by a synchronizing torque coefficient which is determined from (2-11). The synchronizing torque coefficients may be used to compute the deviation in power flow from any internal generator bus j to another internal generator bus k, which is defined as

$$\Delta P_{jk} = T_{jk}(\Delta\delta_j - \Delta\delta_k) \tag{2-30}$$

The total deviation in the electrical output power produced by generator j is found by summing (2-30) over all possible choices of k,

$$\Delta PG_j = \sum_{\substack{k=1 \\ k\neq j}}^{N} \Delta P_{jk} = \sum_{\substack{k=1 \\ k\neq j}}^{N} T_{jk}(\Delta\delta_j - \Delta\delta_k) \tag{2-31}$$

Item four above may be expressed as

$$\Delta PG_g = \sum_{j=1}^{m} \Delta PG_j \tag{2-32}$$

Substituting (2-31) into (2-32)

$$\Delta PG_g = \sum_{j=1}^{m} \sum_{\substack{k=1 \\ k\neq j}}^{N} T_{jk}(\Delta\delta_j - \Delta\delta_k) \tag{2-33}$$

Recognizing that the coherent generators are assumed to have the same deviation in rotor angle, each $\Delta\delta_j$ in (2-33) may be replaced by an equivalent rotor angle deviation, $\Delta\delta_g$, and (2-33) may be rewritten as

$$\Delta PG_g = \sum_{j=1}^{m} \sum_{k=m+1}^{N} T_{jk} \ (\Delta\delta_g - \Delta\delta_k) \tag{2-34}$$

where the summation interval on the index k has been changed to drop terms which are zero. Reversing the order of summation, (2-34) may be written as

$$\Delta PG_g = \sum_{k=m+1}^{N} (\Delta\delta_g - \Delta\delta_k) \sum_{j=1}^{m} T_{jk} \tag{2-35}$$

Defining

$$T_{gk} = \sum_{j=1}^{m} T_{jk} \qquad (2\text{-}36)$$

and substituting T_{gk} into (2-35),

$$\Delta PG_g = \sum_{k=m+1}^{N} T_{gk} \; (\Delta\delta_g - \Delta\delta_k) \qquad (2\text{-}37)$$

Thus, T_{gk} is the synchronizing torque coefficient of the equivalent line which connects the equivalent generator representing the coherent group to the generator k which is external to the group. From (2-36), T_{gk} is the sum of the synchronizing torque coefficients in the unreduced linear model which connects external generator k to the individual generators of the coherent group.

Equations (2-33) through (2-37) provide considerable insight into the network aggregation mechanism of the coherency-based aggregation technique. The transition from (2-33) to (2-34) is accomplished by replacing the individual rotor angle deviations of the generators in the group by an equivalent deviation in rotor angle which is said to characterize all of the generators in the group. This step is equivalent to shorting together the internal generator buses of the members of the group. Once this is done, the m equivalent lines which used to connect a generator external to the group to the individual generators in the coherent group appear as m parallel connections between the external generator and the equivalent generator. Synchronizing torque coefficients of parallel lines add and thus equation (2-36) is obtained.

The coherency-based aggregation technique for aggregating a coherent group, as applied to the linear model (2-9), may be summarized as

1. Determine the parameters of the equivalent generator by comparing (2-28) and (2-29) to obtain

$$M_g = \sum_{j=1}^{m} M_j$$
$$D_g = \sum_{j=1}^{m} D_j \qquad (2\text{-}38)$$
$$\Delta PM_g = \sum_{j=1}^{m} \Delta PM_j$$

2. Compute the synchronizing torque coeff- cients of the lines which connect the equivalent generator to the other generators in the system using (2-36).

It should be pointed out that the aggregation technique has no impact on the parameters of generators external to the coherent group or the equivalent lines connecting such generators.

CHAPTER 3

THE RMS COHERENCY MEASURE AND DYNAMIC SYSTEM STRUCTURE

Two criticisms of the coherency analysis approach to power system dynamic equivalents based on the max-min coherency measure were referenced in Chapter 1. The first criticism is that the max-min coherency measure has not been expressed in terms of system parameters and as a result simulation is required to determine the coherency measure. The second criticism is that coherent equivalents derived from the max-min approach are disturbance-dependent and consequently a separate equivalent must be derived for each distinct system disturbance to be studied. The objective of this chapter is to show that these criticisms can be answered when the rms coherency measure, rather than the max-min measure, is used to identify coherent groups.

This chapter is divided into two sections. Section 3.1 will show that simulation is not required to evaluate the infinite interval rms coherency measure since the rms measure can be algebraically related to system structure and the statistics of the system disturbance. The second section discusses the relationship between the infinite interval rms coherency measure and dynamic system structure for the particular case of step disturbances in mechanical input power.

Two specific probabilistic step disturbances in mechanical input power are shown to leave the rms measure as a function of system structure alone, a result which suggests that disturbance-independent coherent equivalents may be derived from the rms measure.

3.1 Algebraic Relationship Between $\underline{S}_x(\infty)$, System Structure and Disturbance Statistics

In the following discussion, specific types of disturbances such as generator dropping, load shedding, and line switching will be referred to as step input disturbances while electrical faults will be called pulse disturbances. The dependence of the infinite interval rms coherency measure on system structure and disturbance statistics will be shown by evaluating the matrix $\underline{S}_x(t)$, defined by (2-23), over an infinite observation interval for step, initial condition and pulse disturbances. In the case of step disturbances, $\underline{S}_x(\infty)$ will be shown to be an explicit function of generator inertias, synchronizing torque coefficients and disturbance statistics, and a clear dependence on the same parameters will be shown for initial condition and pulse disturbances. The three types of system disturbances are now analyzed separately.

Random Initial Conditions

A simplified form for $\underline{S}_x(t)$ can be derived when only initial conditions are to be considered by substituting

$$n=0,\ \underline{m}_1=\underline{0},\ \underline{R}_1=\underline{0},\ \underline{m}_2=\underline{0},\ \underline{R}_2=\underline{0} \qquad (3\text{-}1)$$

into equation (2-26) with the result,

$$\underline{S}_{\underline{x}}(t) = \int_0^t \varepsilon^{\underline{A}\tau} \underline{V}_{\underline{x}}(0) \varepsilon^{\underline{A}^T\tau} d\tau \tag{3-2}$$

Differentiating the integrand of (3-2) with respect to τ, the right hand side (RHS) of (3-2) becomes

$$\int_0^t \frac{d}{d\tau} (\varepsilon^{\underline{A}\tau} \underline{V}_{\underline{x}}(0) \varepsilon^{\underline{A}^T\tau}) d\tau = \underline{A} \int_0^t \varepsilon^{\underline{A}\tau} \underline{V}_{\underline{x}}(0) \varepsilon^{\underline{A}^T\tau} d\tau + \int_0^t \varepsilon^{\underline{A}\tau} \underline{V}_{\underline{x}}(0) \varepsilon^{\underline{A}^T\tau} d\tau \underline{A}^T \tag{3-3}$$

Evaluating the left hand side (LHS) of (3-3), and substituting $\underline{S}_{\underline{x}}(t)$ as defined in (3-2) into the RHS of (3-3)

$$\varepsilon^{\underline{A}t} \underline{V}_{\underline{x}}(0) \varepsilon^{\underline{A}^T t} - \underline{V}_{\underline{x}}(0) = \underline{A}\ \underline{S}_{\underline{x}}(t) + \underline{S}_{\underline{x}}(t)\ \underline{A}^T \tag{3-4}$$

If all the eigenvalues of $\underline{A}$ have strictly negative real parts then as t approaches infinity, the first term on the LHS of (3-4) vanishes and $\underline{S}_{\underline{x}}(\infty)$ will be found as the solution to the Lyapunov equation

$$-\underline{V}_{\underline{x}}(0) = \underline{A}\ \underline{S}_{\underline{x}}(\infty) + \underline{S}_{\underline{x}}(\infty)\ \underline{A}^T \tag{3-5}$$

The solution to the Lyapunov equation, $\underline{S}_{\underline{x}}(\infty)$, and therefore the infinite interval rms coherency measure, will depend on system structure (through the dependence of the solution on the coefficient matrix $\underline{A}$) and on the statistics of the random initial conditions, $\underline{V}_{\underline{x}}(0)$.

<u>Pulse Input Disturbance</u>

The appropriate form of $\underline{S}_{\underline{x}}(t)$ for a pulse input can be obtained by substituting

$$n=0,\ \underline{V}_{\underline{x}}(0)=\underline{0},\ \underline{m}_1=\underline{0},\ \underline{R}_1=\underline{0} \tag{3-6}$$

into equation (2-26), in which case $\underline{S}_x(t)$ becomes

$$\underline{S}_x(t) = \int_0^{T_1} [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]\ [\underline{R}_2+\underline{m}_2\underline{m}_2^T]\ [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]^T\ d\tau$$

$$+ \int_{T_1}^{t} [\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv\ \underline{B}]\ [\underline{R}_2+\underline{m}_2\underline{m}_2^T]$$

$$[\varepsilon^{\underline{A}(\tau-T_1)} \int_0^{T_1} \varepsilon^{\underline{A}v} dv\ \underline{B}]^T\ d\tau \qquad (3\text{-}7)$$

Since the coefficient matrix $\underline{A}$ of the linear system model is nonsingular, the interior integrals may be evaluated as

$$\int_0^{\tau} \varepsilon^{\underline{A}v} dv = \underline{A}^{-1}(\varepsilon^{\underline{A}\tau} - \underline{I}) \qquad (3\text{-}8)$$

Defining

$$\underline{W}_2 = \underline{B}[\underline{R}_2+\underline{m}_2\underline{m}_2^T]\underline{B}^T \qquad (3\text{-}9)$$

and substituting (3-8) and (3-9) into (3-7)

$$\underline{S}_x(t) = \underline{A}^{-1} \left\{ \begin{aligned} &\int_0^{T_1} [\varepsilon^{\underline{A}\tau}\underline{W}_2\varepsilon^{\underline{A}^T\tau} - \varepsilon^{\underline{A}\tau}\underline{W}_2 - \underline{W}_2\varepsilon^{\underline{A}^T\tau} + \underline{W}_2]d\tau \\ &+ \int_{T_1}^{t} [\varepsilon^{\underline{A}\tau}\underline{W}_2\varepsilon^{\underline{A}^T\tau} - \varepsilon^{\underline{A}(\tau-T_1)}\underline{W}_2\varepsilon^{\underline{A}^T\tau} \\ &\qquad - \varepsilon^{\underline{A}\tau}\underline{W}_2\varepsilon^{\underline{A}^T(\tau-T_1)} \\ &\qquad + \varepsilon^{\underline{A}(\tau-T_1)}\underline{W}_2\varepsilon^{\underline{A}^T(\tau-T_1)}]d\tau \end{aligned} \right\} \underline{A}^{-1^T} \qquad (3\text{-}10)$$

Combining the first term in the first integral of (3-10) with the first term of the second integral, evaluating the remaining terms of the first integral, and making the change of variable $s=\tau-T_1$ in the remaining terms of the second integral (3-10) becomes

$$\underline{S}_x(t) = \underline{A}^{-1} \left\{ \begin{aligned} & \int_0^t \varepsilon^{\underline{A}\tau} \underline{W}_2 \varepsilon^{\underline{A}^T\tau} d\tau - \underline{A}^{-1}(\varepsilon^{\underline{A}T_1} - \underline{I})\underline{W}_2 \\ & - \underline{W}_2[\underline{A}^{-1}(\varepsilon^{\underline{A}T_1} - \underline{I})]^T + \underline{W}_2 T_1 \\ & - \int_0^{t-T_1} \varepsilon^{\underline{A}s} \underline{W}_2 \varepsilon^{\underline{A}^T s} ds \; \varepsilon^{\underline{A}^T T_1} \\ & - \varepsilon^{\underline{A}T_1} \int_0^{t-T_1} \varepsilon^{\underline{A}s} \underline{W}_2 \varepsilon^{\underline{A}^T s} ds \\ & + \int_0^{t-T_1} \varepsilon^{\underline{A}s} \underline{W}_2 \varepsilon^{\underline{A}^T s} ds \end{aligned} \right\} \underline{A}^{-1^T} \tag{3-11}$$

Defining,

$$\begin{aligned} \underline{V} &= \lim_{t\to\infty} \int_0^t \varepsilon^{\underline{A}v} \underline{W}_2 \varepsilon^{\underline{A}^T v} dv \\ &= \lim_{t\to\infty} \int_0^{t-T_1} \varepsilon^{\underline{A}v} \underline{W}_2 \varepsilon^{\underline{A}^T v} dv \end{aligned} \tag{3-12}$$

which satisfies the Lyapunov equation

$$-\underline{W}_2 = \underline{A}\,\underline{V} + \underline{V}\,\underline{A}^T \tag{3-13}$$

and substituting (3-12) and (3-13) into (3-11), $\underline{S}_x(\infty)$ becomes

$$\underline{S}_x(\infty) = \underline{A}^{-1} \left\{ \begin{aligned} & \underline{V} + \underline{A}^{-1}(\varepsilon^{\underline{A}T_1} - \underline{I})(\underline{A}\,\underline{V} + \underline{V}\,\underline{A}^T) \\ & + (\underline{A}\,\underline{V} + \underline{V}\,\underline{A}^T)[\underline{A}^{-1}(\varepsilon^{\underline{A}T_1} - \underline{I})]^T \\ & - (\underline{A}\,\underline{V} + \underline{V}\,\underline{A}^T)T_1 - \underline{V}\,\varepsilon^{\underline{A}^T T_1} \\ & - \varepsilon^{\underline{A}T_1}\,\underline{V} + \underline{V} \end{aligned} \right\} \underline{A}^{-1^T} \tag{3-14}$$

Using the series expansion

$$\varepsilon^{\underline{A}T_1} = \sum_{n=0}^{\infty} \frac{T_1^{\ n}}{n!} \underline{A}^n \tag{3-15}$$

equation (3-14) may be written as

$$\underline{S}_x(\infty) = \underline{A}^{-1} \left\{ \begin{array}{l} \underline{V} + [\sum_{n=1}^{\infty} \frac{T_1^{\ n}}{n!} \underline{A}^{n-1}] \; [\underline{A}\ \underline{V} + \underline{V}\ \underline{A}^T] \\ \\ + [\underline{A}\ \underline{V} + \underline{V}\ \underline{A}^T] \; [\sum_{n=1}^{\infty} \frac{T_1^{\ n}}{n!} \underline{A}^{n-1}]^T \\ \\ - (\underline{A}\ \underline{V} + \underline{V}\ \underline{A}^T) \; T_1 \\ \\ - \underline{V}[\sum_{n=0}^{\infty} \frac{T_1^{\ n}}{n!} \underline{A}^n]^T - [\sum_{n=0}^{\infty} \frac{T_1^{\ n}}{n!} \underline{A}^n]\underline{V} + \underline{V} \end{array} \right\} \underline{A}^{-1^T} \tag{3-16}$$

which, after cancellation of terms, may be written as

$$\underline{S}_x(\infty) = \sum_{n=2}^{\infty} \frac{T_1^{\ n}}{n!} (\underline{A}^{n-2}\underline{V} + \underline{V}\ \underline{A}^{n-2^T}) \tag{3-17}$$

If the fault clearing time, T_1, is sufficiently short, only the first term in the series will be required, and under this assumption

$$\underline{S}_x(\infty) = \underline{V}\ T_1^{\ 2} \tag{3-18}$$

Since $\underline{V}$ is the solution to the Lyapunov equation (3-13), $\underline{V}$ will depend on system structure due to the dependence of the solution on $\underline{A}$ and $\underline{B}$ and on the statistics of the pulse disturbance ($\underline{R}_2$ and $\underline{m}_2$) and the same conclusion will hold for $\underline{S}_x(\infty)$ and the rms coherency measure.

Step Input Disturbances

For step inputs, an explicit formula will be derived relating the rms coherency measure to system structure and the statistics of the step input. Substituting

$$n=1, \underline{V}_x(0)=\underline{0}, \underline{m}_2=\underline{0}, \ \underline{R}_2=\underline{0} \tag{3-19}$$

into (2-26) the desired expression for $\underline{S}_x(t)$ is

$$\underline{S}_x(t) = \frac{1}{t}\int_0^{T_1} [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]\ [\underline{R}_1+\underline{m}_1\underline{m}_1^T]\ [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]^T d\tau$$

$$+ \frac{1}{t}\int_{T_1}^{t} [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]\ [\underline{R}_1+\underline{m}_1\underline{m}_1^T]\ [\int_0^{\tau} \varepsilon^{\underline{A}v} dv\ \underline{B}]^T d\tau \tag{3-20}$$

Defining

$$\underline{W}_1 = \underline{B}[\underline{R}_1+\underline{m}_1\underline{m}_1^T]\underline{B}^T \tag{3-21}$$

and evaluating the interior integrals of (3-20) using (3-8)

$$\underline{S}_x(t) = \frac{1}{t}\underline{A}^{-1}\int_0^t [\varepsilon^{\underline{A}\tau}\underline{W}_1\varepsilon^{\underline{A}^T\tau} - \varepsilon^{\underline{A}\tau}\underline{W}_1 - \underline{W}_1\varepsilon^{\underline{A}^T\tau} + \underline{W}_1]d\tau\underline{A}^{-1^T} \tag{3-22}$$

As t approaches infinity, the first three terms in the time averaged integral (3-22) vanish, since the system model is asymptotically stable, leaving

$$\underline{S}_x(\infty) = \underline{A}^{-1}\underline{W}_1\underline{A}^{-1^T} = [\underline{A}^{-1}\underline{B}]\ [\underline{R}_1+m_1m_1^T]\ [\underline{A}^{-1}\underline{B}]^T \tag{3-23}$$

For the form of $\underline{A}$ given by (2-10), $\underline{A}^{-1}$ becomes

$$\underline{A}^{-1} = \begin{bmatrix} -\sigma(\underline{M}\ \underline{T})^{-1} & -(\underline{M}\ \underline{T})^{-1} \\ \underline{I} & \underline{0} \end{bmatrix} \tag{3-24}$$

and using the form for $\underline{B}$ also given in (2-10)

$$\underline{A}^{-1}\underline{B} = \begin{bmatrix} -(\underline{M}\ \underline{T})^{-1}\underline{M} & -(\underline{M}\ \underline{T})^{-1}\underline{M}\ \underline{L} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{3-25}$$

For simplicity, step disturbances in mechanical input power and load bus power injection will be considered separately. For step disturbances in mechanical input power, $\underline{m}_1$ and $\underline{R}_1$ as defined by (2-19) become

$$\underline{m}_1 = \begin{bmatrix} \underline{m}_{11} \\ \underline{0} \end{bmatrix}, \quad \underline{R}_1 = \begin{bmatrix} \underline{R}_{11} & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{3-26}$$

where $\underline{m}_{11}$ is the mean value of the step disturbance in mechanical input power and $\underline{R}_{11}$ is the covariance of the disturbance. Substituting (3-26) and (3-25) into (3-23), $\underline{S}_x(\infty)$ for step disturbances in mechanical input power is given by

$$\underline{S}_x(\infty) = \begin{bmatrix} [(\underline{M}\ \underline{T})^{-1}\underline{M}]\ [\underline{R}_{11}+\underline{m}_{11}\underline{m}_{11}^T]\ [(\underline{M}\ \underline{T})^{-1}\underline{M}]^T & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{3-27}$$

For step disturbances in load bus power injection, $\underline{m}_1$ and $\underline{R}_1$ become

$$\underline{m}_1 = \begin{bmatrix} \underline{0} \\ \underline{m}_{12} \end{bmatrix}, \quad \underline{R}_1 = \begin{bmatrix} \underline{0} & \underline{0} \\ \underline{0} & \underline{R}_{12} \end{bmatrix} \tag{3-28}$$

Substituting (3-28) and (3-25) into (3-23), $\underline{S}_x(\infty)$ for step disturbances in load bus power injection becomes

$$\underline{S}_x(\infty) = \begin{bmatrix} [(\underline{M}\ \underline{T})^{-1}\underline{M}\ \underline{L}]\ [\underline{R}_{12}+\underline{m}_{12}\underline{m}_{12}^T]\ [(\underline{M}\ \underline{T})^{-1}\underline{M}\ \underline{L}]^T & \underline{0} \\ \underline{0} & \underline{0} \end{bmatrix} \tag{3-29}$$

Thus, for step input disturbances, an explicit formula has been derived which relates the rms coherency measure, through its dependence on $\underline{S}_x$, to the parameters of the linear system model and the statistics of the disturbance, eliminating the need for simulation to determine the coherency measure as is required to determine the max-min coherency measure.

It has been shown that the matrix $\underline{S}_x(\infty)$, which defines the infinite interval rms coherency measure, can be related algebraically to system structure and disturbance statistics. For random initial conditions and the pulse type disturbance, the determination of $\underline{S}_x(\infty)$ was shown to entail the solution of Lyapunov equations. For step inputs, an explicit formula for $\underline{S}_x(\infty)$ was derived. In the next section, the significance of the relationship between the rms coherency measure and system structure for step disturbances in mechanical input power is discussed.

3.2 Disturbance-Independent Coherent Equivalents Derived from the RMS Coherency Measure

The relationship between the infinite interval rms coherency measure and dynamic system structure is now discussed for the special case of step disturbances in mechanical input power. It will be shown that if the step input is a ZMIID disturbance then the rms measure is determined strictly by the synchronizing torque coefficients of the linear system model. When the disturbance in mechanical input power at each generator bus is zero mean, independent of the disturbance at every other generator bus, and has a variance proportional to the square of the generator inertia, it is shown that the rms coherency measure is dependent solely on the inertially weighted synchronizing torque coefficients. These results indicate that coherent equivalents which depend on system structure, and not on the location of any particular system disturbance, may be derived by using one of the above disturbances and the rms coherency measure to identify the coherent groups for aggregation.

$\underline{S}_x(\infty)$ and Dynamic System Structure

The significance of $\underline{S}_x(\infty)$ for step disturbances in mechanical input power is now discussed. From equation (3-27), the upper left quadrant of $\underline{S}_x(\infty)$ which determines the coherency measure between any pair of generators is defined as $\hat{\underline{S}}_x(\infty)$ and is given by

$$\hat{\underline{S}}_x(\infty) = [(\underline{M}\ \underline{T})^{-1}\underline{M}]\ [\underline{m}_{11}\underline{m}_{11}^T + \underline{R}_{11}]\ [(\underline{M}\ \underline{T})^{-1}\underline{M}]^T \qquad (3\text{-}30)$$

This expression is valid for both probabilistic and deterministic disturbances. For probabilistic disturbances

$$\underline{m}_{11} = E[\underline{\Delta PM}]; \quad R_{11} = E[(\underline{\Delta PM} - \underline{m}_{11})(\underline{\Delta PM} - \underline{m}_{11})^T] \qquad (3\text{-}31)$$

and for deterministic disturbances

$$\underline{m}_{11} = \underline{\Delta PM}; \quad \underline{R}_{11} = \underline{0} \qquad (3\text{-}32)$$

As noted previously, equation (3-30) eliminates the need for simulation to evaluate the coherency measure as is required in the max-min approach.

The coherency aggregation procedure has been criticized because the equivalents derived using the max-min coherency measure are dependent on the disturbance used to identify coherent groups for aggregation. Inspection of equation (3-30) shows that the rms coherency measure will be a function of only system structure for any disturbance which satisfies

$$\underline{m}_{11}\underline{m}_{11}^T + \underline{R}_{11} = \underline{I} \qquad (3\text{-}33)$$

This condition is clearly met when the disturbance in $\underline{\Delta PM}$ is ZMIID, that is

$$\underline{m}_{11} = \underline{0}; \quad \underline{R}_{11} = \underline{I} \qquad (3\text{-}34)$$

Substituting (3-33) into (3-30)

$$\hat{\underline{S}}_x(\infty) = [(\underline{M}\ \underline{T})^{-1}\underline{M}]\ [(\underline{M}\ \underline{T})^{-1}\underline{M}]^T \qquad (3\text{-}35)$$

Since

$$(\underline{M}\ \underline{T})^{-1}\underline{M}\ \underline{T} = \underline{I} \qquad (3\text{-}36)$$

equation (3-35) shows that when the disturbance in mechanical input power is ZMIID that the rms coherency measure is a generalized inverse function of synchronizing torque coefficients, such that the coherent groups are determined by line stiffness.

Another disturbance of interest is the disturbance which causes

$$\underline{M}(\underline{m}_{11}\underline{m}_{11}^T + \underline{R}_{11})\underline{M}^T = \underline{I} \qquad (3\text{-}37)$$

in which case

$$\hat{\underline{S}}_x(\infty) = [(\underline{M}\ \underline{T})^{-1}]\ [(\underline{M}\ \underline{T})^{-1}]^T \qquad (3\text{-}38)$$

and thus the coherency measure is determined by generator inertias and synchronizing torque coefficients and the coherent groups identified for aggregation are determined by line stiffnesses weighted by the inertias of the generators at the ends of the lines. The disturbance which satisfies (3-37) is

$$\underline{m}_{11} = \underline{0}; \quad \underline{R}_{11} = \mathrm{DIAG}(M_2^2, M_2^2, \ldots, M_{n-1}^2, 0) \qquad (3\text{-}39)$$

This disturbance is clearly dependent on the choice of reference generator used to establish the state model. For applications with a preferred choice of reference, such as a system containing an extremely large generator, aggregation based on (3-39) may be satisfactory. A reference independent result can be obtained by allowing the covariance of the disturbance in $\underline{\Delta PM}$ to be

$$\underline{R}_{11} = \text{DIAG}(M_1^{\ 2}, M_2^{\ 2}, \ldots, M_{N-1}^{\ \ 2}, M_N^{\ 2}) \tag{3-40}$$

The use of (3-40) may be viewed as solving (3-37) for all of the N possible choices of reference generator.

Since the inertially weighted synchronizing torque coefficients also determine the modal structure of the system, the modal and coherent equivalents derived from the rms coherency measure and the disturbance (3-40) will be nearly identical. They will not be exactly the same unless the coherent generators are so tightly tied to each other that the coherency measure between them is zero. Thus, the rms coherency measure and the disturbance (3-40) can capture both modal and coherent structure of the system. This further justifies the use of an rms coherency measure, which is dependent only on voltage angle, as a basis for aggregation. The disturbance defined by (3-40) shall be called the <u>modal disturbance</u>.

Deterministic step disturbances in mechanical input power which will satisfy (3-33) or (3-37) do not exist. This is due to the fact that for any real vector, $\underline{v}$, the product $\underline{v}\,\underline{v}^T$ cannot be an identity matrix. However, it will be shown in Chapter 6 that the square root of the sum of the squares of the coherency measures observed for a sequence of N deterministic disturbances, where each of the N generators is, in turn, subjected to a step disturbance proportional to its inertia is mathematically equivalent to the expected coherency measure observed with the modal disturbance. A similar sequence of N deterministic disturbances where each generator, in turn, experiences a step disturbance of 1 p.u. would duplicate the effects of the ZMIID disturbance. Thus, to construct a coherent equivalent based solely on system structure using a single disturbance to identify coherent groups, a probabilistic disturbance is required. This result explains why the present coherency approach using a max-min coherency measure and a single deterministic disturbance to determine coherent generators has not been able to produce a coherent equivalent based on system structure.

In the next chapter it will be shown, using an example system, that when an appropriate probabilistic disturbance is used to identify the coherent groups for aggregation, that the resultant coherent equivalent has eigenvalues which closely approximate the system eigenvalues retained by a general purpose modal equivalent. It is also shown that this coherent equivalent is useful for studying many different system disturbances. The inability of a single deterministic disturbance to produce a good general purpose coherent equivalent is demonstrated by constructing the coherent equivalent for the location independent uniform deterministic (UD) disturbance where a 1 p.u. perturbation in mechanical input power occurs at each generator bus. It is shown that the eigenvalues of this coherent equivalent do not closely match the eigenvalues of the modal equivalent based on the same disturbance and that neither of these equivalents is well suited for studying other system disturbances.

CHAPTER 4

COMPARISON OF MODAL AND COHERENT EQUIVALENTS DERIVED FROM THE RMS COHERENCY MEASURE

In this chapter, the properties of modal and coherent equivalents derived from the rms coherency measure are investigated for an example system. Equivalents are constructed for two step disturbances in mechanical input powers which apply uniform excitation to the system generators. The first disturbance is the probabilistic ZMIID disturbance defined by (3-34) and the second is the uniform deterministic (UD) disturbance where a 1 p.u. disturbance occurs on each generator. The UD disturbance was suggested in [13] as a possible basis for constructing a general purpose coherent equivalent that would be useful for studying many different system contingencies. The ZMIID disturbance was proposed for the same purpose in [9].

Modal equivalents in this discussion are derived from the rules of mode elimination developed in [8,9] to preserve the rms coherency measure throughout the system model. These rules of mode elimination allow system modes to be eliminated which do not change the infinite interval rms coherency measure between any two generators by more than some arbitrarily established ε. Any coherent equivalent derived in this chapter is a linearized version of the nonlinear coherent equivalent which is obtained by aggregating coherent groups in the unreduced, nonlinear system model using the coherency-based aggregation technique [2]. The coherent groups to be aggregated for a particular disturbance are identified, using the unreduced linear system model, by evaluating the infinite interval rms coherency measure for that disturbance. A generator belongs to a coherent group whenever the rms coherency measure between that generator and each other member of the coherent group is less than some established threshold. The performance of an equivalent is judged by the ability of the equivalent to preserve the infinite interval coherency measure observed with the unreduced linear model of the example system.

The linearized, seven machine model of the Michigan Electric Coordinated System [14], used as the example system in [9] is the basis of discussion in this section. The linear model matrices, $\underline{M}$ and $\underline{M}\ \underline{T}$, for this system are

$$\underline{M}\ \underline{T} = \begin{bmatrix} 72.75 & 17.65 & 15.26 & 13.79 & 33.47 & 31.24 \\ 11.77 & 112.69 & 13.79 & 11.99 & 29.22 & 26.85 \\ 3.11 & 14.31 & 169.65 & -6.71 & 13.62 & 9.06 \\ 24.44 & 19.69 & 5.66 & 97.78 & 21.88 & 18.49 \\ 21.37 & 17.09 & 5.05 & .76 & 175.52 & -9.76 \\ 20.20 & 16.71 & 2.96 & -.81 & -7.95 & 177.13 \end{bmatrix}$$

$$\underline{M} = \begin{bmatrix} 1.28 & 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & -3.63 \\ 0.00 & 5.42 & 0.00 & 0.00 & 0.00 & 0.00 & -3.63 \\ 0.00 & 0.00 & 4.76 & 0.00 & 0.00 & 0.00 & -3.63 \\ 0.00 & 0.00 & 0.00 & 3.63 & 0.00 & 0.00 & -3.63 \\ 0.00 & 0.00 & 0.00 & 0.00 & 3.63 & 0.00 & -3.63 \\ 0.00 & 0.00 & 0.00 & 0.00 & 0.00 & 3.63 & -3.63 \end{bmatrix}$$

The eigenvalues of the system model are [9]

$\lambda_{1,2} = -.087 \pm j\ 7.415;$ $\lambda_{7,8} = -.087 \pm j\ 14.304$

$\lambda_{3,4} = -.087 \pm j\ 9.481;$ $\lambda_{9,10} = -.087 \pm j\ 12.756$

$\lambda_{5,6} = -.087 \pm j\ 10.389;$ $\lambda_{11,12} = -.087 \pm j\ 13.614$

Modal equivalents of the ZMIID and UD disturbances for this example system were previously constructed in [9]. The modal equivalent of the ZMIID disturbance was derived using the rule of mode elimination for probabilistic disturbances, equation (20) of [9]

$$\{\frac{1}{\lambda_j} \underline{e}_{k\ell}^T \underline{T}_j \underline{r}_j \underline{B}\ [\underline{R}_1 + \underline{m}_1 \underline{m}_1^T]^{\frac{1}{2}}\}_j < \varepsilon_4 \qquad \begin{matrix} j=1,2,\ldots,N+K \\ k,\ell=1,2,\ldots,N \end{matrix} \qquad (4\text{-}1)$$

where $\underline{T}_j$ and $\underline{r}_j$ are the right and left eigenvectors of the matrix $\underline{A}$ associated with the jth eigenvalue. It was shown in [9] that λ_7, λ_8, λ_9, λ_{10}, λ_{11} and λ_{12} could be eliminated since their effects on the rms coherency measure were small. The coherency measure between each pair of generators k and ℓ, $C_{k\ell}$, observed with the unreduced system model and with the modal equivalent obtained by eliminating λ_7 through λ_{12} is given in the first two columns of Table 4-1.

Using a coherency threshold of 0.0373, the data for the unreduced system in Table 4-1 shows that generators 3, 5, 6 and 7 for a single coherent group for the ZMIID disturbance. Although the coherency measure between generators 1 and 7 is 0.0360, generator 1 is not considered part of the coherent group since C_{13}, C_{15} and C_{16} are greater than the prescribed threshold. A coherent equivalent for this disturbance was constructed using the coherency-based aggregation technique

Table 4-1 Coherency Measure $C_{k\ell}$ for the Unreduced System and the Modal and Coherent Equivalents Obtained with a ZMIID Disturbance in Mechanical Input Power.

ZMIID DISTURBANCE

Coherency Measure $C_{k\ell}$

k-l	unreduced system	modal equivalent	coherent equivalent
1-2	.0633422	.0629925	.0659853
1-3	.0431537	.0328371	.0372406
1-4	.0552152	.0551852	.0583934
1-5	.0435705	.0406859	.0372406
1-6	.0430098	.0400304	.0372406
1-7	.0360426	.0330797	.0372406
2-3	.0648643	.0574771	.0596469
2-4	.0736876	.0734757	.0764275
2-5	.0637336	.0623392	.0596469
2-6	.0634330	.0618677	.0596469
2-7	.0556143	.0506651	.0596469
3-4	.0471674	.0364979	.0420465
3-5	.0373198	.0113935	0.0
3-6	.0365226	.0102275	0.0
3-7	.0339855	.0078583	0.0
4-5	.0457239	.0434552	.0420465
4-6	.0450240	.0423694	.0420465
4-7	.0448191	.0417165	.0420465
5-6	.0277408	.0012357	0.0
5-7	.0285475	.0135124	0.0
6-7	.0283460	.0127177	0.0

[2] to aggregate the coherent group. The reduced order system matrices of the resultant coherent equivalent are given in Table 4-2 along with a comparison of the eigenvalues of this coherent equivalent and the system eigenvalues retained by the modal equivalent.

Table 4-2 System Matrices and Eigenvalues of the Coherent Equivalent Derived from the ZMIID Disturbance.

$$\underline{M} = \begin{bmatrix} 1.288 & 0.000 & 0.000 & -0.965 \\ 0.000 & 5.422 & 0.000 & -0.965 \\ 0.000 & 0.000 & 3.630 & -0.965 \end{bmatrix}$$

$$\underline{M}\ \underline{T} = \begin{bmatrix} 59.14 & 5.91 & 16.99 \\ 1.10 & 96.91 & 15.72 \\ 16.74 & 8.77 & 98.15 \end{bmatrix}$$

Eigenvalues of the Coherent Equivalent	Retained Eigenvalues of the Modal Equivalent
$-0.087 \pm j\ 7.288$	$-0.087 \pm j\ 7.415$
$-0.087 \pm j\ 9.351$	$-0.087 \pm j\ 9.481$
$-0.087 \pm j\ 10.659$	$-0.087 \pm j\ 10.389$

In order to compare the coherency measure that would be observed with the coherent equivalent to the measure observed with the unreduced system model and the modal equivalent, an equivalent disturbance must be derived for the machine which represents the aggregation of generators 3, 5, 6 and 7 in the coherent equivalent. The disturbance

$$\underline{\tilde{m}}_1 = \underline{0}; \quad \underline{\tilde{R}}_{11} = \text{DIAG}(1,1,1,4); \quad \underline{R}_{12} = \underline{0} \tag{4-2}$$

will be analogous to the original ZMIID disturbance of the unreduced system because the variance of the disturbance on the equivalent generator should be the sum of the variances of the independent disturbances on the individual generators in the coherent group. The rms coherency measure determined with the coherent equivalent and the equivalent disturbance defined by (4-2), is given in the third column of Table 4-1.

Tables 4-1 and 4-2 show that

1. the eigenvalues of the coherent equivalent agree closely with the system eigenvalues retained by the modal equivalent.
2. the behavior of the generators external to the coherent group (generators 1, 2 and 4) is well preserved by both the modal and the coherent equivalent. This is indicated by the close agreement of the $C_{k\ell}$; $k=1,2,4$ and $\ell=1,2,...,7$; observed with the unreduced system model and the corresponding values observed with the two equivalents.
3. neither the modal nor the coherent equivalent preserves the coherency measure between members of the coherent group (generators 3, 5, 6 and 7). The modal equivalent has discarded modes which contribute primarily to the intermachine behavior in the coherent group and consequently cannot reproduce exactly this aspect of the system response. Coherent generators are aggregated to form a single equivalent generator in the coherent equivalent and thus the coherency measure between members of the group is constrained to be zero for the coherent equivalent.

4. the overall performance of the coherent equivalent in reproducing the effects of the ZMIID disturbance compares favorably with the performance of the modal equivalent.

A modal equivalent of the example system for the UD disturbance was constructed in [9] using the rule of mode elimination for deterministic step disturbances, equation (19) of [9]

$$\frac{1}{\lambda_j} \underline{e}_{k\ell}^T \underline{T}_j \underline{r}_j \underline{B} \ \underline{u}_j < \varepsilon \qquad k,\ell=1,2,\ldots,N \tag{4-3}$$

It was shown in [9] that λ_5 through λ_{12} could be eliminated since their effects on the rms coherency measure between any pair of generators were small for this disturbance. The coherency measure, $C_{k\ell}$, observed with the unreduced system model and the modal equivalent formed by eliminating λ_5 through λ_{12} are given in the first two columns of Table 4-3.

Using a coherency threshold of 0.0057 the $C_{k\ell}$ observed with the unreduced system show that generators 3, 4, 5, 6 and 7 form a single coherent group for this UD disturbance. These generators were aggregated to obtain a coherent equivalent using the coherency-based aggregation technique [2]. The reduced order system matrices of the resultant coherent equivalent are given in Table 4-4 along with a comparison of the eigenvalues of this coherent equivalent and the system eigenvalues retained by the modal equivalent of this disturbance.

Table 4-3 Coherency Measure $C_{k\ell}$ for the Unreduced System and the Modal and Coherent Equivalents Obtained with a UD Disturbance, $\underline{U}_1^T = (1,1,1,1,1,1,1)$, in Mechanical Input Power

UNIFORM DETERMINISTIC DISTURBANCE

Coherency Measure $C_{k\ell}$

k-l	unreduced system	modal equivalent	coherent equivalent
1-2	.0590053	.0571079	.0570579
1-3	.0474613	.0402162	.0560142
1-4	.0473871	.0458444	.0560142
1-5	.0450963	.0481081	.0560142
1-6	.0449203	.0475732	.0560142
1-7	.0417129	.0427413	.0560142
2-3	.0115440	.0168917	.0010436
2-4	.0116182	.0112636	.0010436
2-5	.0139091	.0089998	.0010436
2-6	.0140850	.0095347	.0010436
2-7	.0172924	.0143667	.0010436
3-4	.0000741	.0056281	0.0
3-5	.0023650	.0078918	0.0
3-6	.0025409	.0073570	0.0
3-7	.0057483	.0025250	0.0
4-5	.0022980	.0022636	0.0
4-6	.0024668	.0017288	0.0
4-7	.0056741	.0031031	0.0
5-6	.0001759	.0005348	0.0
5-7	.0033833	.0053667	0.0
6-7	.0032073	.0048319	0.0

Table 4-4 System Matrices and Eigenvalues of the Coherent Equivalent Derived from the UD Disturbance.

$$M = \begin{bmatrix} 1.288 & 0.000 & -0.763 \\ 0.000 & 5.422 & -0.763 \end{bmatrix}$$

$$\underline{M}\ \underline{T} = \begin{bmatrix} 45.08 & -1.77 \\ -26.77 & 102.74 \end{bmatrix}$$

Eigenvalues of the Coherent Equivalent	Retained Eigenvalues of the Modal Equivalent
$-0.087 \pm j\ 6.653$	$-0.087 \pm j\ 7.415$
$-0.087 \pm j\ 10.176$	$-0.087 \pm j\ 9.481$

The disturbance for the coherent equivalent which is analogous to the UD disturbance for the unreduced system model is

$$\tilde{\underline{u}}_1 = (1,1,5)^T \tag{4-4}$$

since the disturbance on the aggregated equivalent generator should equal the sum of the disturbances on the individual generators in the coherent goup. The coherency measure determined with the coherent equivalent and the equivalent disturbance, $\tilde{\underline{u}}_1$, is given in the third column of Table 4-3.

Tables 4-3 and 4-4 show that

1. there is much less agreement between the eigenvalues of the two equivalents than was demonstrated previously for the equivalents based on the ZMIID disturbance.
2. neither equivalent preserves the coherency measure between members of the coherent

group (generators 3, 4, 5, 6 and 7) for the same reasons as given in the case of the ZMIID disturbance.

3. the modal equivalent outperforms this coherent equivalent in preserving the behavior of the generators outside the coherent group (generators 1 and 2). This is shown by comparing the $C_{k\ell}$; $k=1,2$ and $\ell=1,2,\ldots 7$; observed with the equivalents to the corresponding values observed with the unreduced system model.

4. the overall performance of the coherent equivalent in reproducing the effects of the UD disturbance is not as good as that demonstrated by the modal equivalent.

One drawback in using coherent equivalents produced by the present coherency procedure has been that to accurately preserve the coherency measure between generators, a unique equivalent must generally be constructed for each particular contingency to be studied. Constructing an equivalent is usually expensive and it is desirable to find a single general purpose equivalent. Such an equivalent must be based on system structure and independent of any particular disturbance so that it can be used repeatedly to study many different contingencies. Thus, the disturbance used to identify a general purpose coherent equivalent must be able to excite all system modes such that the coherency measure identifies groups that are truly structurally coherent. If the disturbance cannot excite all system modes the coherency measure between generators may be artificially small such that an erroneous indication of coherency results and the coherent equivalent derived from the disturbance is over-aggregated and unsuitable for general purpose use.

As noted previously, it has been suggested that the coherent equivalents based on the UD and ZMIID disturbances might be good general purpose equivalents. To investigate the suitability of these equivalents for general purpose use, the modal and coherent equivalents derived for the example system for these two disturbances were used to determine the coherency measure between generators for several different deterministic disturbances. The results for two of these disturbances are presented in Table 4-5. The first disturbance is a 1 p.u. disturbance in generator one and the second is a 1 p.u. disturbance on both generators one and two.

Table 4-5 Comparison of the Coherency Measure $C_{k\ell}$ of the Unreduced System and the Modal and Coherent Equivalents Obtained with the UD and ZMIID Disturbances for Two Particular Deterministic Disturbances in the Mechanical Input Powers of Generators 1 and 2.

1 p.u. DISTURBANCE ON GENERATOR 1

Coherency Measure $C_{k\ell}$

k-l	unreduced system	ZMIID modal equivalent	ZMIID coherent equivalent	UD modal equivalent	UD coherent equivalent
1-2	.0212364	.0213576	.0224942	.0218728	.0213435
1-3	.0209738	.0199320	.0228720	.0197697	.0288660
1-4	.0250668	.0251325	.0268080	.0257062	.0288660
1-5	.0234050	.0238083	.0228720	.0233674	.0288660
1-6	.0232658	.0235698	.0228720	.0231615	.0288660
1-7	.0208011	.0201603	.0228720	.0199900	.0288660
2-3	.0002625	.0014256	.0003777	.0021031	.0075224
2-4	.0038304	.0037748	.0043137	.0038333	.0075224
2-5	.0021686	.0024506	.0003777	.0014945	.0075224
2-6	.0020293	.0022122	.0003777	.0012887	.0075224
2-7	.0004352	.0011973	.0003777	.0018828	.0075224
3-4	.0040929	.0052005	.0039359	.0059365	0.0
3-5	.0024311	.0038762	0.0	.0035977	0.0
3-6	.0022919	.0036378	0.0	.0033918	0.0
3-7	.0001726	.0002283	0.0	.0002203	0.0
4-5	.0016617	.0013242	.0039359	.0023388	0.0
4-6	.0018010	.0015626	.0039359	.0025446	0.0
4-7	.0042656	.0049722	.0039359	.0057162	0.0
5-6	.0001392	10002384	0.0	.0002058	0.0
5-7	.0026038	.0036479	0.0	.0033774	0.0
6-7	.0024646	.0034095	0.0	.0031715	0.0

Table 4-5 (con't.)

1 p.u. DISTURBANCES ON GENERATORS 1 AND 2

Coherency Measure $C_{k\ell}$

k-l	unreduced system	ZMIID modal equivalent	ZMIID coherent equivalent	UD modal equivalent	UD coherent equivalent
1-2	.0376040	.0368366	.0386318	.0260029	.0298829
1-3	.0182530	.0162779	.0184463	.0128650	.0309562
1-4	.0251251	.0252339	.0266943	.0372978	.0309562
1-5	.0210830	.0226476	.0184463	.0133778	.0309562
1-6	.0209007	.0221993	.0184463	.0136140	.0309562
1-7	.0141176	.0099643	.0184463	.0063831	.0309562
2-3	.0558570	.0531145	.0570781	.0388679	.0608392
2-4	.0627291	.0620705	.0653261	.0633008	.0608392
2-5	.0586869	.0594842	.0570781	.0393807	.0608392
2-6	.0585046	.0590360	.0570781	.0396169	.0608392
2-7	.0517215	.0468009	.0570781	.0323861	.0608392
3-4	.0068721	.0089559	.0082479	.0244329	0.0
3-5	.0028299	.0063696	0.0	.0005128	0.0
3-6	.0026476	.0059214	0.0	.0007490	0.0
3-7	.0041354	.0063135	0.0	.0064818	0.0
4-5	.0040421	.0025863	.0082479	.0239200	0.0
4-6	.0042244	.0030345	.0082479	.0236839	0.0
4-7	.0110076	.0152696	.0082479	.0309147	0.0
5-6	.0001822	.0004482	0.0	.0002361	0.0
5-7	.0069654	.0126832	0.0	.0069946	0.0
6-7	.0067831	.0122350	0.0	.0072308	0.0

The results in Table 4-5 show that both the modal and the coherent equivalent derived from the ZMIID disturbance consistently outperform the equivalents based on the UD disturbance in preserving the coherency measure observed with the unreduced system model. In one of the cases investigated but not presented here, the coherent equivalent based on the ZMIID disturbance was found to be superior to the coherent equivalent derived from the UD disturbance in reproducing the effects of the UD disturbance.

The UD disturbance does not identify a good general purpose coherent equivalent because it is not sufficiently robust to excite all system modes. Although this disturbance is not location dependent it correlates the disturbance between generators and apparently has difficulty exciting some intermachine modes. The problem is made clear by considering a similar deterministic case where the disturbance in mechanical power at each generator bus is proportional to the inertia of the generator, that is where the disturbance in mechanical input power is given by

$$\underline{\Delta PM} = (M_1, M_2, \ldots, M_N)^T \tag{4-5}$$

In this case, the input to the linearized state equation will be zero and no intermachine modes will be excited since

$$\underline{M}(M_1, M_2, \ldots, M_N)^T = \underline{0} \tag{4-6}$$

Thus, when the UD disturbance is used to identify coherent groups, generators that are of approximately the same inertia are likely to be found coherent regardless of the relative stiffness of the lines connecting them.

During the process of constructing the modal equivalent of the ZMIID disturbance for the MECS example system, it was noted in [9] that for the n=4 generator coherent group consisting of generators 3, 5, 6 and 7, that 2n-2=6 eigenvalues could be eliminated without seriously affecting the rms coherency measure. It was further observed that as each pair of eigenvalues identified for elimination was removed that the generators within the coherent group became more coherent until they practically oscillated as a single generator when all six eigenvalues were eliminated. Based on the observation that the process of mode elimination was essentially a coherency aggregation for this particular disturbance, it was concluded that the coherent equivalent would be nearly identical to the modal equivalent based on the same disturbance. The results in this section support this conclusion but a qualification must be mentioned. In general, the modal disturbance defined by equation (3-40) rather than the ZMIID disturbance would be required in order to obtain a coherent equivalent whose eigenvalues would approximate those of the corresponding modal equivalent. It was shown in Chapter 3 that the coherent groups identified by the modal disturbance are based on the inertially weighted synchronizing torque coefficients, which determine the modes of the system response, whereas coherent groups based on the ZMIID disturbance are identified solely by the synchronizing torque coefficients. When all generators in the system have the same inertia there is no real difference between the modal disturbance and the ZMIID disturbance. For the MECS example system, there is little variation in generator inertias and it will be shown in Chapter 6 that the coherent group consisting of generators 3, 5, 6 and 7 is also identified by the modal disturbance.

The example system has shown that the coherent equivalent derived by aggregating the coherent group identified by the rms coherency measure and a ZMIID disturbance is a good general purpose equivalent and closely approximates the modal equivalent based on the same disturbance. This is a significant result because it allows an approximate modal equivalent, which retains power system component structure, to be constructed without the need for computing eigenvalues as required by other approaches to generating modal equivalents.

CHAPTER 5

A MODAL-COHERENT EQUIVALENT

In Chapter 1 it was noted that the present modal and coherency approaches [1,2] have developed independently. Perhaps it was assumed that modal equivalents would preserve coherent system behavior and that coherent equivalents would likewise preserve system modes. Since this is not the case, it is reasonable to question whether or not the present modal and coherency approaches are mutually consistent. Good engineering judgment indicates that while there may be several solution techniques for solving a specific engineering problem, that all legitimate problem solutions should be related to each other and in essence, identical. In this chapter, a procedure which combines modal and coherency techniques in order to obtain a general purpose dynamic equivalent consistent with the objectives of both modal and coherency analysis is discussed and justified.

The value of stiff interconnections to improve system stability has long been recognized by power system planners and operators. Any approach to power system dynamic equivalents which might destroy coherent behavior would clearly be ignoring power system structure. The system modes are similarly important. If an equivalent is constructed which significantly alters the system eigenvalues, there is little possibility that the time response with the equivalent model will closely match the response of the unreduced system model for a large class of disturbances. Thus, it is important that an equivalent preserve both modal and coherent system properties.

Mode reduction with the present modal approach is based on preserving the magnitude of the internal system states while eliminating modes of the external system which are not controllable or observable with respect to the internal system. Since the system states are dependent on the reference generator chosen to establish the state model, equivalents that are derived to preserve state, and not a coherency measure, may not preserve coherent behavior and may also be reference dependent. In recognition of the need to eliminate any sensitivity to the system reference and to insure that coherent behavior is preserved,a modal approach based on a coherency measure was presented in [8]. The modal method presented there, derived rules of mode elimination based on preserving the infinite interval rms coherency measure between any two generator buses. Those rules may be viewed as analogous to the controllability and observability considerations developed in [1] since the coherency measure is used as the criterion for deciding which modes are controllable and observable. The modal approach based on preserving a coherency measure represents a conceptual rather than a computational advance over the modal method based on preserving system states. Both procedures require that system eigenvalues be calculated and neither method results in an equivalent which retains power system component structure. However, modal equivalents derived to preserve a coherency measure are also coherent equivalents and thus the modal analysis approach based on the rms coherency measure is more easily related to coherency analysis than the modal approach based on preserving system states.

The present coherency analysis approach uses the coherency aggregation technique [2] to aggregate the coherent groups identified by the max-min coherency measure for any single deterministic disturbance. Because a single deterministic disturbance cannot identify structurally coherent groups, the equivalents derived from this procedure are disturbance dependent, and might not closely preserve system modes. The inability to produce a general purpose, coherent equivalent based on dynamic system structure is a significant drawback for the coherency approach and is in contrast to the modal procedure which can generate a general purpose equivalent. The most important advantage of the coherency aggregation technique is that the form of the equivalent produced is a reduced set of equivalent generators and lines, that can be used directly in present transient stability programs.

The results in Chapters 3 and 4 suggest that a modal-coherent equivalent can be constructed which combines the best features of modal and coherent equivalents. This equivalent would be constructed using the coherency aggregation technique to aggregate the coherent groups identified by the infinite interval rms coherency measure for the modal disturbance defined by (3-40). The properties of the modal-coherent equivalent are

1. the system eigenvalues will not be required in order to construct the equivalent.
2. the eigenvalues of the equivalent will closely approximate the system eigenvalues retained by the modal equivalent based on the same disturbance and rms coherency measure.
3. the equivalent will be useful for studying any disturbance which might occur outside the coherent groups aggregated to form the equivalent.
4. power system component structure is retained and the equivalent can be used with existing transient stability programs.

Thus, the modal-coherent equivalent has properties which make it more desirable than pure modal or coherent equivalents.

An efficient computational procedure for constructing a modal-coherent equivalent for large scale systems is presented in the next chapter.

CHAPTER 6

COMPUTATIONAL ALGORITHM
FOR CONSTRUCTING THE MODAL-COHERENT EQUIVALENT

Computational difficulties associated with the construction of modal equivalents for large scale systems have prevented the modal analysis approach to dynamic equivalents from gaining widespread acceptance in the power industry, even though the modal technique is theoretically sound. The most critical difficulty is that the number of generators which can be accommodated ($\sim$50) is severely limited by the necessity of computing the eigenvalues and eigenvectors of the linear system model. In addition, when thousands of load buses are involved it is expensive to perform the network reduction essential to the construction of the coefficient matrices of the linearized model. On the other hand, the coherency analysis approach based on the max-min coherency measure has become popular in spite of its lack of a solid theoretical justification because the procedure is computationally efficient and capable of handling very large systems. The coherency approach avoids the computational difficulties associated with the modal approach since eigenvalues are not required, and since the integration scheme used to simulate the response of the linearized system model and compute the max-min coherency measure for any given disturbance does not require that the A and B matrices of the linear model be explicitly computed. In the previous chapter, it was shown that the modal-coherent equivalent has properties which make it more desirable than either a pure modal or a pure coherent equivalent in the study of power system response to disturbances. However, the ultimate acceptance and usefulness of the modal-coherent approach clearly depends on whether or not the construction of the modal-coherent equivalent is computationally competitive with the procedure for deriving a coherent equivalent from the max-min coherency measure. In this chapter, an efficient algorithm for constructing the modal-coherent equivalent is proposed. The algorithm does not require that the eigenvalues or the matrices of the linear system model be computed and it is well suited for large system applications.

The procedure for constructing the modal-coherent equivalent consists of the following three steps:

1. the evaluation of the rms coherency measure, over an infinite interval, between each pair of system generators for the modal disturbance.
2. the identification of coherent groups based on the computed coherency measures.
3. aggregation of the coherent groups using the coherency-based aggregation technique [2].

Three completely analogous steps are followed to construct a coherent equivalent from the max-min coherency measure.

This thesis does not propose any change in the coherency-based aggregation technique [2] which is used to aggregate coherent groups once they have been identified. Thus, step 3 is identical for both approaches and is not discussed further. The major point of competition between the two techniques, besides which one produces the most accurate and useful equivalent, is the manner in which the coherency measures are evaluated.

The remainder of this chapter is divided into three sections. The first section develops steps 1 and 2 of the algorithm for constructing the modal-coherent equivalent. In the second section, the procedure developed to evaluate the rms coherency measure is compared and contrasted with the procedure used to evaluate the max-min measure; and finally, the third section presents some computational examples to illustrate the algorithm.

6.1 Evaluation of the RMS Coherency Measure and Identification of Coherent Groups

The modal-coherent equivalent is constructed by aggregating the coherent groups identified from the expected value of the rms coherency measure evaluated over an infinite interval for the modal disturbance. The modal disturbance, defined by equation (3-40), is a probabilistic step disturbance in mechanical input powers which is zero mean with the covariance matrix

$$\underline{R}_{11} = \mathrm{DIAG}(M_1{}^2, M_2{}^2, \ldots M_{N-1}{}^2, M_N{}^2) \qquad (6\text{-}1)$$

The infinite interval rms coherency measure between any pair of system generators may be easily computed using equation (2-24) once the matrix $\hat{\underline{S}}_x(\infty)$ is known. As shown previously, the form of $\hat{\underline{S}}_x(\infty)$ for the modal disturbance is given by

$$\hat{\underline{S}}_x(\infty) = [(\underline{M}\ \underline{T})^{-1}\underline{M}]\ [\underline{R}_{11}]\ [(\underline{M}\ \underline{T})^{-1}\underline{M}]^T \tag{6-2}$$

Although equation (6-2) provides a useful insight into the coherency mechanism by relating the rms measure to the parameters of the linearized system model, it does not suggest an efficient approach for computing $\hat{\underline{S}}_x(\infty)$.

An efficient computational procedure for computing $\hat{\underline{S}}_x(\infty)$ for the modal disturbance will be developed by showing

1. that the expected value of the rms coherency measure observed with the modal disturbance is equal to the square root of the sum of the squares of the coherency measures computed for a particular sequence of N deterministic step disturbances.
2. for any deterministic step input disturbance the rms coherency measure when evaluated over an infinite interval, depends only on the steady state response of the generator angles to that disturbance.
3. that steady state generator angles may be efficiently computed using a well-known technique.

The Modal Disturbance Sequence

To demonstrate that the expected value of the rms coherency measure for the probabilistic modal disturbance can be expanded in terms of the coherency measures observed for a sequence of deterministic disturbances, let the covariance of the modal disturbance, $\underline{R}_{11}$, be written as

$$\underline{R}_{11} = \sum_{k=1}^{N} \underline{\Delta PM}^k \; \underline{\Delta PM}^{k^T} \tag{6-3}$$

where each $\underline{\Delta PM}^k$ is an N dimensional column vector whose ith entry is defined by

$$\{\underline{\Delta PM}^k\}_i = \Delta PM_i^k = \begin{cases} 0 & i \neq k \\ M_k & i = k \end{cases} \quad , \quad k,i=1,2,\ldots,N \tag{6-4}$$

Substituting (6-3) into (6-2) the matrix $\hat{\underline{S}}_x(\infty)$ can be rewritten as

$$\hat{\underline{S}}_x(\infty) = [(\underline{M}\;\underline{T})^{-1}\underline{M}] \; [\sum_{k=1}^{N} \underline{\Delta PM}^k \; \underline{\Delta PM}^{k^T}] \; [(\underline{M}\;\underline{T})^{-1}\underline{M}]^T \tag{6-5}$$

$$= \sum_{k=1}^{N} \hat{\underline{S}}_x^k(\infty)$$

where,

$$\hat{\underline{S}}_x^k(\infty) = [(\underline{M}\;\underline{T})^{-1}\underline{M}] \; [\underline{\Delta PM}^k \; \underline{\Delta PM}^{k^T}] \; [(\underline{M}\;\underline{T})^{-1}\underline{M}]^T \tag{6-6}$$

The significance of equations (6-5) and (6-6) is that the $\hat{\underline{S}}_x(\infty)$ matrix for the modal disturbance can be constructed by summing a sequence of $\hat{\underline{S}}_x$ matrices, $\{\hat{\underline{S}}_x^k(\infty) : k=1,2,\ldots,N\}$, which corresponds to the sequence of N deterministic step disturbances, $\{\underline{\Delta PM}^k : k=1,2,\ldots,N\}$, where each generator, in turn, is subjected to a disturbance in mechanical input power proportional to its inertia. Alternatively, it can be said that the expected value of the rms coherency measure for the modal disturbance is equal to the square root of the sum of the squares of the rms measures computed for the N deterministic disturbances, $\{\underline{\Delta PM}^k\}$. The sequence of disturbances defined by (6-4) is referred to as the "modal disturbance sequence".

Steady State Generator Angles and the Infinite Interval RMS Coherency Measure

It will now be shown that for any deterministic step input disturbance $\underline{u}^k(t)$ given by

$$\begin{aligned} \underline{u}^k(t) &= \underline{0} && \text{for } t < 0 \\ &= \underline{u}^k = \begin{bmatrix} \underline{\Delta PM}^k \\ \hline \underline{\Delta PL}^k \end{bmatrix} && \text{for } t \geq 0 \end{aligned} \qquad (6\text{-}7)$$

that the matrix $\hat{\underline{S}}_x^k(\infty)$ depends solely on the steady state generator angles exhibited by the linear system model in response to the step input. This result will allow the rms coherency measure for the modal disturbance to be computed from the steady state angle response of the system generators to each of the disturbances in the modal disturbance sequence.

For the deterministic step input disturbance $\underline{u}^k(t)$ given by (6-7) the matrix, $\underline{S}_x^k(\infty)$ is by definition

$$\underline{S}_x^k(\infty) = \lim_{T\to\infty} \frac{1}{T} \int_0^T \underline{x}^k(t)\ \underline{x}^{k^T}(t)\ dt \qquad (6\text{-}8)$$

where $\underline{x}^k(t)$ is the system state vector and the solution to the state equation for the step disturbance $\underline{u}^k(t)$. If the linear system model is asymptotically stable and the magnitude of each of the step inputs in the input vector, $\underline{u}^k$, is bounded, then the system states will be finite for all time and will eventually converge to some finite steady state values, $\underline{x}^k(\infty)$. Recognizing that the entries in the matrix integrand of equation (6-8) are well-behaved, finite functions of time, each of which converges in time to some constant value governed by $\underline{x}^k(\infty)$, $\underline{S}_x^k(\infty)$ may be written as

$$\underline{S}_x^k(\infty) = \underline{x}^k(\infty)\ \underline{x}^{k^T}(\infty) \tag{6-9}$$

By definition,

$$\underline{x} = \begin{bmatrix} \underline{\Delta\hat{\delta}} \\ \underline{\Delta\hat{\omega}} \end{bmatrix} \tag{6-10}$$

therefore,

$$\underline{S}_x^k(\infty) = \begin{bmatrix} \underline{\Delta\hat{\delta}}^k(\infty)\ \underline{\Delta\hat{\delta}}^{k^T}(\infty) & \underline{\Delta\hat{\delta}}^k(\infty)\ \underline{\Delta\hat{\omega}}^{k^T}(\infty) \\ \underline{\Delta\hat{\omega}}^k(\infty)\ \underline{\Delta\hat{\delta}}^{k^T}(\infty) & \underline{\Delta\hat{\omega}}^k(\infty)\ \underline{\Delta\hat{\omega}}^{k^T}(\infty) \end{bmatrix} \tag{6-11}$$

The rms coherency measure between any pair of generators is determined by the upper left (N-1) x (N-1) submatrix of $\underline{S}_x$, that is by $\hat{\underline{S}}_x$. From (6-11)

$$\hat{\underline{S}}_x^k(\infty) = \underline{\Delta\hat{\delta}}^k(\infty)\ \underline{\Delta\hat{\delta}}^{k^T}(\infty) \tag{6-12}$$

Thus, the rms coherency measure evaluated over an infinite interval for any deterministic step input disturbance is determined by the steady state angle response of the generators to that disturbance.

It can easily be shown that equations (6-6) and (6-12) are consistent expressions for $\hat{\underline{S}}^k(\infty)$ for the disturbances in the modal disturbance sequence. Letting $\underline{\Delta PL}^k$ equal zero in (6-7) the disturbance $\underline{u}^k$ becomes the k<u>th</u> disturbance in the modal disturbance sequence when $\underline{\Delta PM}^k$ is defined by (6-4). Setting the derivative of the state vector equal to zero as t approaches infinity in the state equation (2-9), $\underline{x}^k(\infty)$ becomes

$$\underline{x}^k(\infty) = -\underline{A}^{-1}\ \underline{B} \begin{bmatrix} \underline{\Delta PM}^k \\ \underline{0} \end{bmatrix} \tag{6-13}$$

Substituting (3-25) for $\underline{A}^{-1}\ \underline{B}$ into (6-13) and extracting $\underline{\Delta\hat{\delta}}^k(\infty)$ from $\underline{x}^k(\infty)$

$$\underline{\Delta\hat{\delta}}^k(\infty) = (\underline{M}\ \underline{T})^{-1}\underline{M}\ \underline{\Delta PM}^k \qquad (6\text{-}14)$$

Substituting (6-14) into (6-12) equation (6-6) is obtained confirming that $\underline{\hat{S}}_x^k(\infty)$ depends on $\underline{\Delta\hat{\delta}}^k(\infty)$. Equation (6-12) will be shown to be a convenient form for computing the infinite interval rms coherency measure for the modal disturbance.

Computation of Steady State Generator Angles for a Step Disturbance

The steady state generator angles required to calculate $\underline{\hat{S}}_x(\infty)$ for any step input, $\underline{u}^k(t)$, can be efficiently computed using a triangular factorization technique to solve the system network equations (2-6), at time equal to infinity for $\underline{\Delta\hat{\delta}}^k(\infty)$. The key to the approach is to show that the steady state deviation in the real power generations due to the step input, $\underline{\Delta PG}^k(\infty)$, which are required to set up the steady state network equations for solution, can be deduced from the entries in the disturbance vector and knowledge of the generator inertias and damping constants. The procedures for constructing $\underline{\Delta PG}^k(\infty)$ from $\underline{u}^k$ and solving the steady state network equations for $\underline{\Delta\hat{\delta}}^k(\infty)$ are now discussed.

Prior to the occurrence of the step input disturbance, it is assumed that the system is in a power balance. The sum of the real powers generated by the system generators exactly balances the sum of the powers demanded at the load buses (neglecting line losses). The total mechanical input power to the system is just sufficient to maintain the balance between generated and demanded powers and to allow the generators to operate at the synchronous speed, ω_o.

After the step disturbance $\underline{u}^k(t)$ is applied, the power balance between generation and demand may be temporarily upset. However, if the linear system model is stable then as t approaches infinity the balance between real power generation and real power demanded at the load buses must be restored. Thus, if there are no losses in the network

$$\sum_{i=1}^{N} \Delta PG_i^k(\infty) = - \sum_{j=1}^{K} \Delta PL_j^k \tag{6-15}$$

where $\Delta PG_i^k(\infty)$ is the steady state deviation in the electrical output power produced by generator i in response to the disturbance $\underline{u}^k(t)$, and ΔPL_j^k is the change in the power demanded at load bus j (a negative value implies an increase in load) which is specified by the disturbance $\underline{u}^k(t)$. If there is an excess (deficiency) of mechanical input power in the step input disturbance $\underline{u}^k$ which is not balanced by a change in load, then the system generators will accelerate (decelerate) to some new system speed, $\omega_o + \Delta\omega^k$, but remain synchronous.

Each generator in the system is modelled by the linear, classical synchronous machine representation

$$M_i \frac{d}{dt} \Delta\omega_i^k(t) = \Delta PM_i^k - \Delta PG_i^k(t) - D_i \Delta\omega_i^k(t) \ , \quad t \geq 0 \quad i=1,2,\ldots,N \tag{6-16}$$

As t goes to infinity, each generator will settle into some new steady $(\Delta\hat{\delta}_i^k(\infty), \Delta\omega_i^k(\infty), \Delta PG_i^k(\infty))$ in response to the disturbance $\underline{u}^k$ and

$$\frac{d}{dt} \Delta\omega_i^k(t) \rightarrow 0 \text{ as } t \rightarrow \infty \ , \quad i=1,2,\ldots,N \tag{6-17}$$

and since the generators maintain synchronism

$$\Delta\omega_i^k(t) \rightarrow \Delta\omega^k \text{ (a constant) as } t \rightarrow \infty \qquad i=1,2,\ldots,N \tag{6-18}$$

Thus, as t approaches infinity (6-16) becomes

$$0 = \Delta PM_i^k - \Delta PG_i^k(\infty) - D_i \Delta\omega^k \quad , \quad i=1,2,\ldots,N \tag{6-19}$$

Summing the N equations of (6-19)

$$0 = \sum_{i=1}^{N} \Delta PM_i^k - \sum_{i=1}^{N} \Delta PG^k(\infty) - \Delta\omega^k \sum_{i=1}^{N} D_i \tag{6-20}$$

The steady state change in system speed resulting from the step disturbance can be found by solving (6-20) for $\Delta\omega^k$ and using the substitution (6-15)

$$\Delta\omega^k = \frac{\sum_{i=1}^{N} \Delta PM_i^k + \sum_{j=1}^{K} \Delta PL_j^k}{\sum_{i=1}^{N} D_i} \tag{6-21}$$

Equation (6-21) may be substituted into (6-19) and solved for $\Delta PG_i^k(\infty)$ to obtain

$$\Delta PG_i^k(\infty) = \Delta PM_i^k - D_i \left[\frac{\sum_{i=1}^{N} \Delta PM_i^k + \sum_{j=1}^{K} \Delta PL_j^k}{\sum_{i=1}^{N} D_i} \right] \qquad i=1,2,\ldots,N \tag{6-22a}$$

Thus, the steady state deviations in real power generation are determined by the entries in the vector $\underline{u}^k$ and the damping constants of the system generators.

In general, the generator damping constants are not accurately known. When the uniform damping assumption is made, the damping constants can be eliminated from (6-22) in favor of the more accurately known generator inertias. Substituting σM_i for D_i, equation (6-22) becomes

$$\Delta PG_i^k(\infty) = \Delta PM_i^k - M_i \left[\frac{\sum_{i=1}^{N} \Delta PM_i^k + \sum_{j=1}^{K} \Delta PL_j^k}{\sum_{i=1}^{N} M_i} \right] \qquad i=1,2,\ldots,N \tag{6-22b}$$

Once the vector $\underline{\Delta PG}^k(\infty)$ has been computed using equation (6-22) the network equations may be solved for $\underline{\Delta\hat{\delta}}^k(\infty)$. The procedure for solving the steady state network equations is now addressed.

The synchronous frame network equations (2-4) are not of full rank and in order to obtain a solution it is necessary to reference all of the bus angles in the system to the angle of an arbitrarily chosen reference generator whose angle may be set equal to zero. Let the network equations in the generator N reference frame, given by equation (2-6), be written as

$$\underline{\Delta P}(t) = \underline{J}\ \underline{\hat{\Delta}}(t) \tag{6-23}$$

where

$$\underline{\Delta P} = \begin{bmatrix} \underline{\Delta PL} \\ \underline{\Delta PG} \end{bmatrix}, \quad \underline{J} = \begin{bmatrix} \partial \underline{PL}/\partial \underline{\hat{\theta}} & \partial \underline{PL}/\partial \underline{\hat{\delta}} \\ \partial \underline{PG}/\partial \underline{\hat{\theta}} & \partial \underline{PG}/\partial \underline{\hat{\delta}} \end{bmatrix}, \tag{6-24}$$

$$\underline{\hat{\Delta}} = \begin{bmatrix} \underline{\Delta\hat{\theta}} \\ \underline{\Delta\hat{\delta}} \end{bmatrix}$$

The matrix $\underline{J}$ is an (N+K) x (N+K-1) matrix constructed by evaluating the Jacobian matrix of the synchronous frame network equations at the system's nominal load flow conditions and dropping the column corresponding to the N$\underline{th}$ generator. As a consequence of the conversion from the synchronous reference frame to the generator N reference frame one of the rows in equation (6-23) is a redundant equation and may be deleted. By eliminating the last row of $\underline{J}$, equation (6-23) may be written as

$$\underline{\Delta \tilde{P}}(t) = \underline{\tilde{J}}\ \underline{\hat{\Delta}}(t) \tag{6-25}$$

where $\underline{\tilde{J}}$ is a symmetric, nonsingular matrix formed by deleting the last row of $\underline{J}$ and

$$\underline{\Delta \tilde{P}} = \begin{bmatrix} \underline{\Delta PL} \\ \underline{\Delta \tilde{P}G} \end{bmatrix} \tag{6-26}$$

with

$$\underline{\Delta \tilde{P}G} = (\Delta PG_1, \Delta PG_2, \ldots, \Delta PG_{N-2}, \Delta PG_{N-1})^T \tag{6-27}$$

Equation (6-23) is a constraint on the states of the differential equations representing the system generators and must be satisfied at any point in time. In particular as t approaches infinity the steady state network equations for a particular step input, $\underline{u}^k$, become

$$\underline{\Delta \tilde{P}}^k(\infty) = \underline{\tilde{J}}\ \underline{\hat{\Delta}}^k(\infty) \tag{6-28}$$

where,

$$\underline{\Delta \tilde{P}}^k(\infty) = \begin{bmatrix} \underline{\Delta PL}^k \\ \underline{\Delta \tilde{P}G}^k(\infty) \end{bmatrix} \tag{6-29}$$

Equation (6-28) may be solved by the triangular factorization of $\underline{J}$,

$$\underline{\tilde{J}} = \underline{\tilde{V}}\ \underline{\tilde{V}}^T \tag{6-30}$$

where $\underline{\tilde{V}}$ is a lower triangular matrix, and then solving the forward substitution

$$\Delta\underline{\tilde{P}}^k(\infty) = \underline{\tilde{V}}\ \underline{y}^k \tag{6-31}$$

for the vector $\underline{y}^k$ and finally solving the back substitution

$$\underline{y}^k = \underline{\tilde{V}}^T\ \underline{\hat{\Delta}}^k(\infty) \tag{6-32}$$

for the vector $\underline{\hat{\Delta}}^k(\infty)$. The desired steady state generator angles, $\underline{\Delta\hat{\delta}}^k(\infty)$, may be extracted from $\underline{\hat{\Delta}}^k(\infty)$ using the definition (6-24).

The steps of matrix factorization, forward, and backward substitution are essential steps used in the Newton-Raphson solution to the load flow problem and can be performed quite efficiently using numerical methods which employ optimal ordering techniques and exploit the sparsity of the matrix $\underline{\tilde{J}}$.

The procedure for evaluating $\underline{\hat{S}}_x^k(\infty)$ for a particular step input disturbance has been shown to entail

1. the conversion of the step input into an equivalent steady state deviation in real power generations, equation (6-22).
2. one solution of a set of linear equations, equation (6-28).

The bulk of the computational effort is expended in the solution of the linear equations which are generally of very large dimension (N+K-1).

The algorithm for evaluating the infinite interval rms coherency measure for the modal disturbance will involve the construction of N equivalent steady state disturbances in real power generations and N repeat solutions of the linear equations, one for each disturbance in the modal disturbance sequence. The algorithm is summarized as follows.

1. Factorize the Jacobian matrix, $\underline{\mathfrak{J}}$.
2. Establish the modal disturbance sequence $\{\underline{\Delta PM}^k : k=1,2,\ldots,N\}$, using equation (6-4).
3. For each $\underline{\Delta PM}^k$ compute the entries in the vector $\underline{\Delta PG}^k(\infty)$ using equation (6-22). (Note: $\underline{\Delta PL}^k = 0$ for every k.)
4. Construct the $\underline{\Delta \tilde{P}}^k(\infty)$ vector for each k using equation (6-29).
5. For each k
   a) Solve the forward substitution problem (6-31) for $\underline{y}^k$.
   b) Solve the back substitution problem (6-32) for $\underline{\hat{\Delta}}^k(\infty)$.
   c) Extract the steady state generator angles $\underline{\Delta\hat{\delta}}^k(\infty)$ from $\underline{\hat{\Delta}}^k(\infty)$ using the definition (6-24).
6. Compute the $\underline{\hat{S}}_x(\infty)$ matrix for the modal disturbance as

$$\underline{\hat{S}}_x(\infty) = \sum_{k=1}^{N} \underline{\Delta\hat{\delta}}^k(\infty)\ \underline{\Delta\hat{\delta}}^{k^T}(\infty) \qquad (6\text{-}33)$$

7. Use the relationship between $\underline{S}_x$ and $C_{k\ell}$ given in (2-24) to compute the rms coherency measure between each pair of generators in the system.

The major features of this algorithm are

1. Only limited system data is required; namely the generator inertias and damping

constants and the Jacobian matrix of the network equations evaluated at the nominal load flow condition. The Jacobian matrix is a normal output of a Newton-Raphson load flow program.

2. Network reduction to eliminate load buses is not necessary since the algorithm does not require knowledge of the synchronizing torque coefficient matrix, $\underline{T}$, or the reflection matrix, $\underline{L}$.

3. The triangular factorization technique, which is the main tool in the solution process, is a well-known technique which is frequently employed in power system analysis. Thus programming the algorithm for large scale system applications should not present any major problems.

Once the rms coherency measure has been computed between each pair of system generators for the modal disturbance (step 1) the next step in the construction of a modal coherent equivalent is to identify coherent groups from the computed coherency measures (step 2).

Identification of Coherent Groups for Aggregation

Once the value of the rms coherency measure between each of the N(N-1)/2 possible generator pairs has been computed, the coherent groups for aggregation may be identified. In Chapter 4, a simple rule for identifying coherent groups was discussed which was based on the specification of a coherency threshold, ε_c. The rule assigned generators to a coherent group whenever the coherency measure between each possible pair of generators in the group was less than ε_c. As the threshold increases, the number of generators retained in the dynamic equivalent decreases (coherent groups are generally fewer in number but contain more generators). Thus, a particular value of ε_c uniquely specifies an equivalent of a certain size. In general, it would be more desirable to have a rule which specified, a priori, the size of the desired dynamic equivalent rather than a coherency threshold. In this section, two rules are proposed which allow the coherency threshold to float while identifying groups which correspond to an equivalent of a predetermined size.

One method of identifying the groups from the coherency measures is to rank the observed values of $C_{k\ell}$ from the smallest to the largest using a ranking table. The ranking table would contain three columns, the first of which would be the rank, r, where $r=1,2,\ldots,N(N-1)/2$, the second would contain the pair k_r, ℓ_r corresponding to the indices which identify the generators which produce the rth smallest coherency measure, and the third column would contain the value of the rth smallest coherency measure, C_{k_r,ℓ_r}. A suggested format for the ranking table is shown in Table 6-1. Coherent groups are then identified by proceeding down the ranks of the ranking table (from the smallest to the largest) using some prescribed rule to separate generators into coherent groups until the system model has been reduced to the predetermined number of generators. Two possible "prescribed rules" are now described.

Table 6-1 Suggested Ranking Table Structure

rank r	k_r, ℓ_r		C_{k_r,ℓ_r}	
1	k_1, ℓ_1	←most coherent pair	C_{k_1,ℓ_1}	←smallest coherency measure
2	k_2, ℓ_2		C_{k_2,ℓ_2}	
.	.		.	
.	.		.	
.	.		.	
$\frac{N(N-1)}{2}$	$k_{\frac{N(N-1)}{2}}, \ell_{\frac{N(N-1)}{2}}$	←least coherent pair	$C_{k_{\frac{N(N-1)}{2}},\ell_{\frac{N(N-1)}{2}}}$	←largest coherency measure

The first rule is based on a transitive relationship. Let A, B and C stand for generators, then the transitive coherency rule may be stated as

"A coherent with B and B coherent with C implies A coherent with C"

A procedure for identifying coherent groups based on the ranking table and the transitive coherency rule is now described in a flowchart fashion

1. Establish the desired level of aggregation by letting N_c be the number of generators in the desired modal-coherent equivalent ($N_c < N$ = the number of generators in the unreduced system model.).

2. Set $r = 0$

   $NE = N - N_c$ = number of generators to be eliminated

   $ne = 0$ = an index which counts the number of generators eliminated so far

3. Set $r = r+1$

4. Decide which of the following four possibilities apply to generators k_r and ℓ_r,

   a) if neither k_r nor ℓ_r has been previously identified as belonging to a coherent group, then generators k_r and ℓ_r become the first two generators in a new group. Proceed to 5

   b) if generator $k_r(\ell_r)$ belongs to a coherent group but generator $\ell_r(k_r)$ does not then put generator $\ell_r(k_r)$ into the coherent group of which $k_r(\ell_r)$ is a member. Proceed to 5

c) if generators k_r and ℓ_r belong to different coherent groups, then merge the two coherent groups to form a single group containing all members of the two separate groups. Proceed to 5

d) if generators k_r and ℓ_r have already been placed in the same coherent group, return to 3 since the aggregation of generators k_r and ℓ_r has been decided previously.

5. Set ne = ne+1

6. If ne $<$ NE return to step 3 otherwise proceed to 7.

7. Terminate.

A second possible rule for identifying coherent groups using the ranking table is based on a commutative notion of coherency. A simple illustration is provided by again letting A, B and C stand for generators and letting G be a coherent group containing only generators A and B, then

"C belongs to the group G if and only if
C is coherent with A and C is coherent with B"

The procedure for identifying coherent groups based on the commutative rule differs from the iterative procedure described previously for the transitive rule only in step 4, which should be changed to

4. Decide which of the following possibilities apply to generators k_r and ℓ_r,
   a) if neither k_r nor ℓ_r has been previously identified as belonging to a coherent group, then generators k_r and ℓ_r become the first two generators in a new group. Proceed to 5.
   b) if generator $k_r(\ell_r)$ belongs to a coherent group but generator $\ell_r(k_r)$ does not then
      i) if $\ell_r(k_r)$ has been previously recognized as coherent with all members of the group to which $k_r(\ell_r)$ belongs except for $k_r(\ell_r)$, then add $\ell_r(k_r)$ to the coherent group containing $k_r(\ell_r)$. Proceed to 5.
      ii) if $\ell_r(k_r)$ has not been found previously to be coherent with all other members of the group to which $k_r(\ell_r)$ belongs, then recognize that k_r and ℓ_r are coherent but do not add $\ell_r(k_r)$ to the coherent group containing $k_r(\ell_r)$. Return to step 3.
   c) if generators k_r and ℓ_r belong to different coherent groups then
      i) if all possible generator pairs which can be selected from the members of the two groups except k_r and ℓ_r have been previously recognized as being coherent, then merge the two groups to

form a single coherent group containing all members of the separate groups. Proceed to 5.

ii) if at least one pair of generators which can be selected from the two groups other than k_r and ℓ_r has not yet been recognized as a coherent pair, then recognize k_r and ℓ_r as a coherent pair but do not merge the groups. Return to step 3.

Further research is needed to determine which of the two rules (or possibly other rules) performs best based on the performance of the equivalents derived from those rules in reproducing the behavior of the unreduced system model for various levels of aggregation and various system disturbances. Some conclusions regarding the relative merits of the two proposed rules based on experience with the MECS system model are presented with the computational examples in Section 6.3.

6.2 Comparison of the Algorithms for Constructing the Modal-Coherent Equivalent and Coherent Equivalents Based on the Max-Min Coherency Measure

In this section, the computational algorithm for constructing the modal-coherent equivalent is compared with the procedure used to construct coherent equivalents based on the max-min coherency measure. The discussion begins with a brief summary of the integration technique used to simulate the linearized swing equations and the generator clustering algorithm used to identify coherent groups based on the max-min coherency measure. The discussion will show that for any given deterministic step input disturbance that the evaluation of the infinite interval rms coherency measure requires significantly less computational effort than the evaluation of the max-min coherency measure for the same disturbance. In addition, it is shown that when a reasonable number of disturbances are included in the transient stability study case list for a particular internal system that the construction of the general purpose modal-coherent equivalent is likely to be computationally competitive with the construction of the set of coherent equivalents which would be required to complete the same transient stability study.

Trapezoidal Integration Technique for Processing the Linearized Swing Equations

The max-min coherency measure is evaluated for any given system disturbance by simulating the response of the linearized system model to that disturbance over a one to two second interval using a computational step size of about 0.1 second. Each generator is represented by the linearized, classical synchronous machine equations (2-3) and the system network equations are viewed as a set of algebraic constraints on the system states which must be satisfied at each discrete time point in the simulation interval. A trapezoidal integration technique is used to process the swing equations and it is assumed that $\Delta\delta_i(t)$, $\Delta\omega_i(t)$ and $\Delta PG_i(t)$ vary linearly over a computation interval while the model inputs $\underline{\Delta PM}(t)$ and $\underline{\Delta PL}(t)$ vary in a step-wise fashion. Reference [5] provides a complete discussion of the trapezoidal integration technique and includes the following summary of the steps in the procedure.

STEP 1 Initialize $\underline{\Delta\delta}(0)$, $\underline{\Delta\omega}(0)$ and $\underline{\Delta PG}(0)$

STEP 2 Increment time from t - Δt to t

STEP 3 Set $\underline{\Delta PM}(t)$ and $\underline{\Delta PL}(t)$ according to the disturbance being modelled.

STEP 4 Calculate the following variables for each generating unit

$$A_i(t-\Delta t) = (1-\frac{D_i\Delta t}{2M_i})\ \Delta\omega_i(t-\Delta t) - \frac{\Delta t}{2M_i}\ \Delta PG_i(t-\Delta t) + \frac{\Delta t}{M_i}\ \Delta PM_i(t)$$

$$B_i(t-\Delta t) = \pi f_o \Delta t \Delta\omega_i(t-\Delta t) + \Delta\delta_i(t-\Delta t)$$

$$C_i(t-\Delta t) = \frac{2M_i}{\Delta t}\ A_i(t-\Delta t) + \frac{2M_i}{t^2\pi f_o}\ (1+\frac{D_i\Delta t}{2M_i})\ B_i(t-\Delta t)$$

Note: f_o = nominal synchronous speed in Hz

$\Delta\delta_i$ is in degrees

STEP 5 Solve the following matrix equation for the new bus angles $\underline{\Delta\theta}(t)$ and $\underline{\Delta\delta}(t)$

$$\begin{bmatrix} \underline{C}(t-\Delta t) \\ \underline{\Delta PL}(t) \end{bmatrix} = \begin{bmatrix} HGG' & HGL \\ HLG & HLL \end{bmatrix} \begin{bmatrix} \underline{\Delta\delta}(t) \\ \underline{\Delta\theta}(t) \end{bmatrix}$$

Note: $HGG = \partial\underline{PG}/\partial\underline{\delta}$

$HGL = \partial\underline{PG}/\partial\underline{\theta}$

$HLG = \partial\underline{PL}/\partial\underline{\delta}$

$HLL = \partial\underline{PL}/\partial\underline{\theta}$

and HGG' is the matrix HGG with $\frac{2M_i}{\Delta t^2\pi f_o}\ (1+\frac{D_i\Delta t}{2M_i})$ added to the diagonal elements.

STEP 6 Calculate the new generator electric powers using

$$PG_i(t) = \frac{2M_i}{\Delta t} A_i(t-\Delta t)$$

$$- \frac{2M_i}{\Delta t^2 \pi f_o} \left(1+\frac{D_i \Delta t}{2M_i}\right)(\Delta\delta_i(t)-B_i(t-\Delta t))$$

STEP 7 Calculate the new generator speeds using

$$\Delta\omega_i(t) - (\Delta\delta_i(t) - B_i(t-\Delta t))/\pi f_o \Delta t$$

STEP 8 Stop if time t exceeds specified value; otherwise return to step 2.

The modified network matrix, in step 5 above, differs from the power-angle Jacobian matrix in equation (2-4) only in the diagonal terms corresponding to the internal generator buses. The modified terms are a direct result of the use of the trapezoidal integration technique. Since the modified network matrix is constant and retains the sparse, symmetric character of the power-angle Jacobian matrix, triangular factors may be precomputed and then used at each integration step to solve the modified network equations for $\underline{\Delta\delta}(t)$ and $\underline{\Delta\theta}(t)$.

The bulk of the computational effort in computing either the infinite interval rms coherency measure or the max-min measure for a particular deterministic step disturbance is expended in the triangular factorization technique used to solve the system network equations. For a single disturbance, the network equations are solved at least ten times in the max-min approach (based on a minimum simulation time of 1 second and an integration step size of 0.1 second). On the other hand, the infinite interval rms measure for a particular step disturbance can be computed by solving the steady state network equations just once. Thus, once the triangular factorization of the network Jacobian matrix has been performed, the rms coherency measure for a given deterministic step disturbance can be evaluated with about one tenth the effort required to process the swing equations.

Clustering Algorithm for Identifying Coherent Groups

Once the linearized swing equations for a given system disturbance have been processed, coherent groups based on the max-min coherency measure are identified using a generator clustering algorithm. The clustering algorithm minimizes the number of max-min coherency measures which must actually be computed by using a transitive coherency rule. One generator in each coherent group is designated as a reference generator for the group and any other generator is considered a member of the group if the max-min coherency measure between that generator and the group reference is less than some specified coherency threshold, ε_c. The algorithm begins by assigning the first generator as the reference for the first coherent group. The remaining generators in the area to be equivalenced are then examined, one by one, and a generator can either be combined with an existing group or the generator becomes the reference for a new coherent group.

The identification of coherent groups using the generator clustering algorithm would require significantly less computational effort than would the use of the ranking table method. The procedure based on the ranking table requires that all N(N-1)/2 possible coherency measures be computed and then ordered from smallest to largest, whereas the number of coherency measures actually computed using the clustering algorithm is dependent on ε_c and is bounded below by N (large value of ε_c) and bounded above by N(N-1)/2 (small value of ε_c).

The accuracy of the clustering algorithm in identifying coherent groups is suspect since the decision as to whether or not a particular generator belongs to a group may well depend on the arbitrary choice of group reference. Further research is needed to determine how accurate the clustering algorithm is and whether or not the ranking table technique is worth the additional computational effort.

No matter which rule for identifying coherent groups is ultimately shown to be best, the modal-coherent equivalent will have a computational advantage over the coherency approach. Structurally coherent groups are identified only once in order to construct the modal-coherent equivalent while the disturbance dependent coherent groups required to construct a coherent equivalent must be identified each time the system disturbance is changed.

A rough comparison is now drawn between the computational effort required to construct the general purpose modal-coherent equivalent and the effort required to generate a set of coherent equivalents. For purposes of the comparison it is assumed that both approaches will use the generator clustering algorithm to identify coherent groups.

Computational Comparison

For a particular internal system, a thorough transient stability study would require a large number of coherent equivalents; one for each distinct disturbance in the transient stability study case list. On the other hand, a single modal-coherent equivalent would suffice to study any disturbance which might occur within the study area for which the modal-coherent equivalent was constructed. Thus, a comparison between the modal-coherent approach to dynamic equivalents based on the rms coherency measure and the coherency analysis approach based on the max-min measure must recognize that a single modal-coherent equivalent replaces many coherent equivalents. An accurate comparison between the computational effort required to construct the modal-coherent equivalent and the effort required to construct a set of coherent equivalents cannot be made until the procedure for constructing the modal-coherent equivalent is programmed for large scale systems. At this time, the following general comments are offered based on a system containing N generators for which n distinct system disturbances are to be examined in a transient stability study.

In order to study n distinct system disturbances, n coherent equivalents would be needed. The construction of n coherent equivalents would require that the algorithm for constructing a coherent equivalent be executed n times. Thus there would be

1. n passes through the linear simulation algorithm in order to be able to compute the max-min coherency measure for each of the disturbances in the case list.

2. n passes through the generator clustering algorithm to identify the appropriate coherent groups for each disturbance.

3. n applications of the coherency-based aggregation technique to do the network reduction and generator aggregation for each disturbance.

Based on data from [5], approximately one quarter as much computer time is required to identify coherent groups using the clustering algorithm as is required to run the linear simulation; and the aggregation procedure requires about twice as much computer time as does the simulation. With the following normalizations

1 linear simulation	$\sim$ 4 units of computer time
1 identification of coherent groups (with the clustering algorithm)	$\sim$ 1 unit of computer time
1 coherency-based aggregation	$\sim$ 8 units of computer time

the total number of "units of computer time required to construct n coherent equivalents would be

$$(4 + 1 + 8)\, n = 13\, n \text{ units of computer time}$$

The construction of the general purpose modal-coherent equivalent would require

1. N solutions to the steady state network equations, one for each disturbance in the modal disturbance sequence.

2. 1 application of the clustering algorithm.

3. 1 application of the coherency-based aggregation technique.

Since virtually all of the effort in the linear simulation algorithm is consumed in repeat solutions of the network equations it may be assumed that one pass through the linear simulation algorithm corresponds to a minimum of 10 solutions of the network equations (with a minimum simulation time of 1 second and an integration step size of 0.1 second). The number of units of computer time expended in solving the steady state network equations N times would be

$$\text{N solutions} \times \left(\frac{\text{4 units of computer time}}{\text{10 solutions}}\right)$$

$$= 0.4\ \text{N units of computer time}$$

Thus, the total units of computer time to construct a modal coherent equivalent would be approximately

$$0.4\ N + 1 + 8 = 0.4\ N + 9 \text{ units} \quad .$$

The construction of the modal-coherent equivalent will be computationally competitive with the construction of n coherent equivalents if less computer time is required to generate the modal-coherent equivalent than the total time required to generate the n coherent equivalents or,

$$13\ n > 0.4\ N + 9$$

or,

$$n > \frac{N + 22.5}{32.5}$$

For a system containing 250 generators, the construction of the modal-coherent equivalent would be competitive with the construction of 9 coherent equivalents.

It may be possible to reduce the number of disturbances in the modal disturbance sequence while retaining the essential character of the modal-coherent equivalent. One approach would be to limit the disturbance sequence to include only those disturbances which correspond to generators in the internal system and a few external system generators which are first or second neighbors to the internal system. Such an approach would attempt to preserve the modal-coherent structure only in the internal system and in a boundary region surrounding the internal system. The underlying assumption is that the dynamics arising from the remote regions of the external system have little impact on the behavior of the internal system and therefore accurate aggregation of the remote external system is not necessary. Since this method would significantly reduce the computational effort required to construct a modal-coherent equivalent, further research is needed to determine the validity of the approach.

6.3 Computational Examples

The algorithm for evaluating the infinite interval rms coherency measure and identifying structurally coherent groups is now illustrated for two example systems. The first example illustrates the procedure described in Section 6.1 for the case when the load buses have not been eliminated from the network equations. Example number two demonstrates the procedure when the network reduction necessary to eliminate the load buses has been performed and the synchronizing torque coefficients between internal generator buses are known. The second example system was treated previously in Chapter 4 where the coherency measure for the modal disturbance was determined by constructing the linear model matrix ($\underline{M}\ \underline{T}$) and computing $(\underline{M}\ \underline{T})^{-1}$.

Example System 1

A schematic diagram of a 3 generator, 5 bus system is shown in Figure 6-1. The system data necessary to compute the rms coherency measure for the modal disturbance is given in the figure.

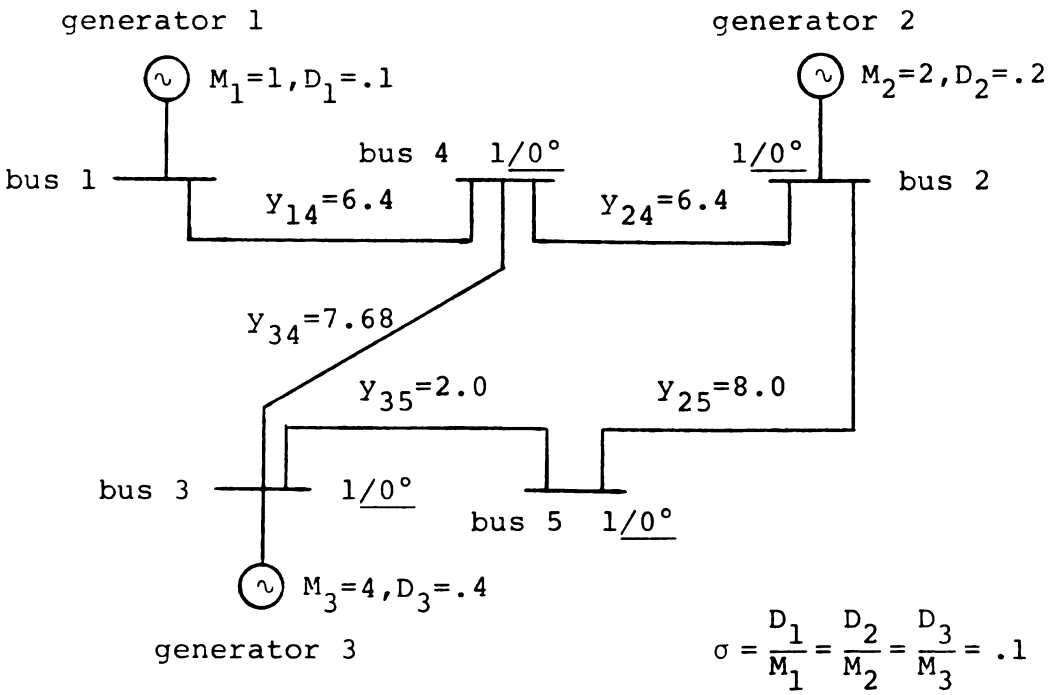

Figure 6-1 Three Generator Example System

The step by step procedure given in Section 6.1 to evaluate the rms coherency measure will be followed.

1. The synchronous frame network equations may be written as

$$\begin{bmatrix} \Delta PL_4 \\ \Delta PL_5 \\ \Delta PG_1 \\ \Delta PG_2 \\ \Delta PG_3 \end{bmatrix} = \begin{bmatrix} 20.48 & 0.00 & -6.40 & -6.40 & -7.68 \\ 0.00 & 10.00 & 0.00 & -8.00 & -2.00 \\ -6.40 & 0.00 & 6.40 & 0.00 & 0.00 \\ -6.40 & -8.00 & 0.00 & 14.40 & 0.00 \\ -7.68 & -2.00 & 0.00 & 0.00 & 9.68 \end{bmatrix} \begin{bmatrix} \Delta\theta_4 \\ \Delta\theta_5 \\ \Delta\delta_1 \\ \Delta\delta_2 \\ \Delta\delta_3 \end{bmatrix}$$

Using generator 3 as the reference generator, the network equations in the generator 3 reference frame may be written as

$$\begin{bmatrix} \Delta PL_4 \\ \Delta PL_5 \\ \Delta PG_1 \\ \Delta PG_2 \end{bmatrix} = \begin{bmatrix} 20.48 & 0.00 & -6.40 & -6.40 \\ 0.00 & 10.00 & 0.00 & -8.00 \\ -6.40 & 0.00 & 6.40 & 0.00 \\ -6.40 & -8.00 & 0.00 & 14.40 \end{bmatrix} \begin{bmatrix} \Delta\theta_4 - \Delta\delta_3 \\ \Delta\theta_5 - \Delta\delta_3 \\ \Delta\delta_1 - \Delta\delta_3 \\ \Delta\delta_2 - \Delta\delta_3 \end{bmatrix}$$

which is of the form

$$\underline{\Delta \tilde{P}} = \underline{\tilde{J}}\ \underline{\hat{\Delta}}$$

The triangular factorization of $\underline{J}$ may be accomplished by using the equations [15]

$$V_{i1} = \tilde{J}_{i1}/\sqrt{\tilde{J}_{11}} \qquad 1 \le i \le N + K - 1$$

$$V_{ij} = (\tilde{J}_{ij} - \sum_{k=1}^{j-1} V_{ik}V_{jk})/V_{jj} \qquad i < j < i \le N + K - 1$$

$$V_{ii} = (\tilde{J}_{ii} - \sum_{k=1}^{i=1} V_{ik}^{2})^{1/2} \qquad 1 < i \le N + K - 1$$

(6-34)

to obtain

$$\underline{\tilde{V}} = \begin{bmatrix} 4.525483 & 0.000000 & 0.000000 & 0.000000 \\ 0.000000 & 3.612278 & 0.000000 & 0.000000 \\ -1.414214 & 0.000000 & 2.097617 & 0.000000 \\ -1.414214 & -2.529822 & -0.953463 & 2.256304 \end{bmatrix}$$

2. The modal disturbance sequence for this system is

$\underline{\Delta PM}^1 = (1,0,0)^T, \quad \underline{\Delta PM}^2 = (0,2,0)^T, \quad \underline{\Delta PM}^3 = (0,0,4)^T$

3. Using equations (6-22b), the $\underline{\Delta PG}^k(\infty)$ vectors corresponding to the modal disturbance sequence may be written as

$$\underline{\Delta PG}^1(\infty) = (0.857143,-0.285714,-0.571429)^T$$
$$\underline{\Delta PG}^2(\infty) = (-0.285714,1.428571,-1.142857)^T$$
$$\underline{\Delta PG}^3(\infty) = (-0.571429,-1.142857,1.714286)^T$$

4. Recognizing that $\underline{\Delta PL}^k = \underline{0}$ for each disturbance in the modal disturbance sequence, the vectors $\underline{\Delta P}^k(\infty)$ needed to solve the steady state network equations may be written as

$$\underline{\Delta P}^1(\infty) = (0.000000,0.000000,0.857143,-0.285714)^T$$
$$\underline{\Delta P}^2(\infty) = (0.000000,0.000000,-0.285714,1.428571)^T$$
$$\underline{\Delta P}^3(\infty) - (0.000000,0.000000,-0.571429,-1.142857)^T$$

5. For each disturbance in the modal disturbance sequence

a) the solution to the forward substitution problem is

$$\underline{y}^1 = (0.000000,0.000000,0.408627,0.046047)^T$$
$$\underline{y}^2 = (0.000000,0.000000,-0.136209,0.575588)^T$$
$$\underline{y}^3 = (0.000000,0.000000,-0.272418,-0.621635)^T$$

b) the solution to the back sutstitution problem is

$$\hat{\underline{\Delta}}^1(\infty) = (0.057398,0.014293,0.204082,0.020408)^T$$
$$\hat{\underline{\Delta}}^2(\infty) = (-0.063776,0.516285,0.051020,0.255102)^T$$
$$\hat{\underline{\Delta}}^3(\infty) = (0.006378,-0.192951,-0.255102,-0.275510)^T$$

c) and the desired steady state angles are

$\underline{\hat{\Delta\delta}}^1(\infty) = (0.204082, 0.020408)^T$

$\underline{\hat{\Delta\delta}}^2(\infty) = (0.051020, 0.255102)^T$

$\underline{\hat{\Delta\delta}}^3(\infty) = (-0.255102, -0.275510)^T$

6. The $\underline{\hat{S}}_x(\infty)$ matrix for the modal disturbance is

$$\underline{\hat{S}}_x(\infty) = \sum_{k=1}^{3} \underline{\hat{\Delta\delta}}^k(\infty)\ \underline{\hat{\Delta\delta}}^{k^T}(\infty) = \begin{bmatrix} 0.109330 & 0.087463 \\ 0.087463 & 0.141399 \end{bmatrix}$$

7. The rms coherency measures between each pair of generators is

$$C_{12} = \sqrt{.109330 - 2(.087463) + .141399} = .275323$$

$$C_{13} = \sqrt{.109330} = .330651$$

$$C_{23} = \sqrt{.141399} = .376031$$

The above solution may be verified by constructing the synchronizing torque coefficient matrix $\underline{T}$, using the network Jacobian matrix in step 1 and applying equation (2-11) to obtain

$$\underline{T} = \begin{bmatrix} 4.4 & -2.0 \\ -2.0 & 6.0 \\ -2.4 & -4.0 \end{bmatrix}$$

then computing $\underline{M}\ \underline{T}$ with the result

$$\underline{M}\ \underline{T} = \begin{bmatrix} 5.0 & -1.0 \\ -0.4 & 4.0 \end{bmatrix}$$

and finally using equation (6-2) with the substitution

$$\underline{R}_{11} = \begin{bmatrix} 1.0 & 0.0 & 0.0 \\ 0.0 & 4.0 & 0.0 \\ 0.0 & 0.0 & 16.0 \end{bmatrix}$$

to compute $\underline{\hat{S}}_k$.

It can easily be shown that the computational procedure for evaluating the rms coherency measure based on computing the steady state response of the generator angles for each disturbance in the modal disturbance sequence is, in fact, a method of generating $(\underline{M}\ \underline{T})^{-1}$ directly from the network equations. For the first N-1 disturbances in the modal disturbance sequence the vector $\underline{M}\ \underline{\Delta PM}^k$ may be expressed as

$$\underline{M}\ \underline{\Delta PM}^k = \underline{1}_{k,0} \quad , \quad k=1,2,\ldots,N-1 \tag{6-35}$$

where the vector $\underline{1}_{k,0}$ is defined as an N-1 dimensional column vector whose ith entry is given by

$$\{\underline{1}_{k,0}\}_i = \begin{cases} 0 & i \neq k \\ 1 & i=k \end{cases} \qquad k,i=1,2,\ldots,N-1 \tag{6-36}$$

When a matrix of compatible dimension premultiplies $\underline{1}_{k,0}$ the resulting vector is the kth column of the matrix. Substituting (6-35) into (6-14)

$$\underline{\Delta\hat{\delta}}^k(\infty) = (\underline{M}\ \underline{T})^{-1}\ \underline{1}_{k,0} \tag{6-37}$$

and it is clear that the steady state generator angles for the kth disturbance in the modal disturbance sequence (k=1,2,...,N-1) is the kth column of $(\underline{M}\ \underline{T})^{-1}$.

For the example system it can be easily verified that $(\underline{M}\ \underline{T})^{-1}$ can be realized from the results in step 5 c)

$$(\underline{M}\ \underline{T})^{-1} = [\underline{\Delta\hat{\delta}}^1(\infty) \mid \underline{\Delta\hat{\delta}}^2(\infty)] = \begin{bmatrix} .204082 & .051020 \\ .020408 & .255102 \end{bmatrix}$$

Another interesting aspect of the information in the solution of the steady generator angles for the modal disturbance sequence is that it may be used to compute the synchronizing torque coefficients between internal generator buses. Since the vectors $\underline{\Delta\hat{\delta}}^k(\infty)$ corresponding to the first

N-1 disturbances in the modal disturbance sequence are the columns of the matrix $(\underline{M}\ \underline{T})^{-1}$ it is clear that these vectors are a linearly independent set for which weighting factors a_{ij} can be found such that

$$a_{i1}\underline{\Delta\hat{\delta}}^{1}(\infty) + a_{i2}\underline{\Delta\hat{\delta}}^{2}(\infty) + \ldots + a_{i,N-1}\underline{\Delta\hat{\delta}}^{N-1}(\infty) = \underline{1}_{i,0} \qquad i=1,2,\ldots,N-1 \tag{6-38}$$

For any vectors $\underline{\Delta PG}^{k}(\infty)$ and $\underline{\Delta\hat{\delta}}^{k}(\infty)$ corresponding to a disturbance where $\underline{\Delta PL}^{k} = \underline{0}$, equation (2-12) indicates that

$$\underline{\Delta PG}^{k}(\infty) = \underline{T}\ \underline{\Delta\hat{\delta}}^{k}(\infty) \tag{6-39}$$

Let the same set of a_{ij}'s which satisfy (6-38) be used to construct $\underline{\Delta P}_{i}$ which is defined as

$$\underline{\Delta P}_{i} = a_{i1}\underline{\Delta PG}^{1}(\infty) + a_{i2}\underline{\Delta PG}^{2}(\infty) + \ldots + a_{i,N-1}\underline{\Delta PG}^{N-1}(\infty) \tag{6-40}$$

Substituting (6-39) into (6-40)

$$\begin{aligned}\underline{\Delta P}_{i} &= a_{i1}\underline{T}\ \underline{\Delta\hat{\delta}}^{1}(\infty) + a_{i2}\underline{T}\ \underline{\Delta\hat{\delta}}^{2}(\infty) + \ldots + a_{i,N-1}\underline{T}\ \underline{\Delta\hat{\delta}}^{N-1}(\infty) \\ &= \underline{T}\ \{a_{i1}\underline{\Delta\hat{\delta}}^{1}(\infty) + a_{i2}\underline{\Delta\hat{\delta}}^{2}(\infty) + \ldots + a_{i,N-1}\underline{\Delta\hat{\delta}}^{N-1}(\infty)\} \\ &= \underline{T}\ \underline{1}_{i,0}\end{aligned} \tag{6-41}$$

and it is clear that $\underline{\Delta P}_{i}$ is the i<u>th</u> column of the synchronizing torque coefficient matrix.

Example System 2

When the network reduction necessary to eliminate the load buses from the linear system model has been performed, the procedure for evaluating the infinite interval rms coherency measure can be modified to take advantage of the reduction. In particular, there is a significant reduction in the order of the triangular factorization problem which is solved to obtain the steady generator angles for each disturbance in the modal disturbance sequence. The essential modifications for the case when the synchronizing torque coefficient matrix, $\underline{T}$, is known, is presented and then applied to the seven station MECS model.

If the synchronizing torque coefficient matrix, $\underline{T}$, and the reflection matrix, $\underline{L}$, are known, then the load bus angles, $\underline{\Delta\theta}(t)$, can be eliminated from the network equations. In this case the network equations are given by (2-12). For each step disturbance in the modal disturbance sequence, $\underline{\Delta PL}^k = \underline{0}$, and the steady state network equations for a particular disturbance k become

$$\underline{\Delta PG}^k(\infty) = \underline{T}\ \underline{\Delta\hat{\delta}}^k(\infty) \tag{6-42}$$

The matrix $\underline{T}$ is an N x (N-1) matrix and one row of (6-42) is a redundant equation. For the same reason described previously in the transition from the network matrix $\underline{J}$ to the matrix $\underline{\tilde{J}}$ the last row of $\underline{T}$ may be deleted to obtain

$$\underline{\Delta\tilde{P}G}^k(\infty) = \underline{\tilde{T}}\ \underline{\Delta\hat{\delta}}^k(\infty) \tag{6-43}$$

where $\underline{\tilde{T}}$ is an (N-1) x (N-1) symmetric matrix formed by omitting the last row of $\underline{T}$ and $\underline{\Delta\tilde{P}G}^k(\infty)$ is as defined in (6-27). Thus, when $\underline{T}$ is known the steady state generator angles can be computed by solving (6-43) using the triangular factorization technique.

The advantage in solving (6-43) rather than (6-28) for the steady state generator angles is that the dimensionality of the problem has been reduced from N+K-1 to N-1.

The procedure for constructing a modal-coherent equivalent when the matrix $\underline{T}$ is known, is now applied to the seven generator MECS model. Once again the step by step evaluation procedure given in Section 6-1 is followed.

1. The synchronizing torque coefficient matrix in the generator 7 reference frame for the MECS model is

$$\underline{T} = \begin{bmatrix} 29.89 & -4.14 & -6.53 & -2.42 & -3.53 & -3.86 \\ -4.14 & 16.54 & -1.82 & -0.91 & -1.62 & -1.73 \\ -6.53 & -1.82 & 30.64 & -4.96 & -5.12 & -5.70 \\ -2.42 & -0.91 & -4.96 & 22.26 & -4.44 & -4.88 \\ -3.53 & -1.62 & -5.12 & -4.44 & 37.81 & -12.65 \\ -3.86 & -1.73 & -5.70 & -4.88 & -12.65 & 38.86 \\ -9.41 & -6.32 & -6.51 & -4.65 & -10.45 & -10.04 \end{bmatrix}$$

Deleting the last row of $\underline{T}$ to obtain $\underline{\tilde{T}}$, the triangular factorization of $\underline{\tilde{T}}$ is obtained by replacing $\underline{\tilde{J}}$ with $\underline{\tilde{T}}$ in equation (6-34) with the result

$$\underline{\tilde{V}} = \begin{bmatrix} 5.4671 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -0.7572 & 3.9958 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ -1.1944 & -0.6818 & 5.3617 & 0.0000 & 0.0000 & 0.0000 \\ -0.4426 & -0.3116 & -1.0633 & 4.5646 & 0.0000 & 0.0000 \\ -0.6456 & -0.5277 & -1.1658 & -1.3429 & 5.8268 & 0.0000 \\ -0.7060 & -0.5667 & -1.2924 & -1.4772 & -2.8896 & 5.0773 \end{bmatrix}$$

2. Using (6-4) the modal disturbance sequence is

$$\underline{\Delta PM}^1 = (.7767,0,0,0,0,0,0)^T$$
$$\underline{\Delta PM}^2 = (0,.1844,0,0,0,0,0)^T$$
$$\underline{\Delta PM}^3 = (0,0,.2100,0,0,0,0)^T$$
$$\underline{\Delta PM}^4 = (0,0,0,.2752,0,0,0)^T$$
$$\underline{\Delta PM}^5 = (0,0,0,0,.2752,0,0)^T$$
$$\underline{\Delta PM}^6 = (0,0,0,0,0,.2752,0)^T$$
$$\underline{\Delta PM}^7 = (0,0,0,0,0,0,.2752)^T$$

3. The corresponding values of $\underline{\Delta PG}^k(\infty)$ can be obtained from (6-22b)

$$\underline{\Delta PG}^1(\infty) = (.5112,-.0630,-.0718,-.0941,-.0941,-.0941,-.0941)^T$$
$$\underline{\Delta PG}^2(\infty) = (-.0630,.1694,-.0170,-.0223,-.0223,-.0223,-.0223)^T$$
$$\underline{\Delta PG}^3(\infty) = (-.0718,-.0170,.1906,-.0254,-.0254,-.0254,-.0254)^T$$
$$\underline{\Delta PG}^4(\infty) = (-.0941,-.0223,-.0254,.2419,-.0333,-.0333,-.0333)^T$$
$$\underline{\Delta PG}^5(\infty) = (-.0941,-.0223,-.0254,-.0333,.2419,-.0333,-.0333)^T$$
$$\underline{\Delta PG}^6(\infty) = (-.0941,-.0223,-.0254,-.0333,-.0333,.2419,-.0333)^T$$
$$\underline{\Delta PG}^7(\infty) = (-.0941,-.0223,-.0254,-.0333,-.0333,-.0333,.2419)^T$$

4. The vectors $\underline{\tilde{\Delta}P}^k(\infty)$, defined by (6-27), may be obtained from the $\underline{\Delta PG}^k(\infty)$ vectors in step 3 by simply dropping the last entry in each vector.

5. The solution to the forward substitution problem, (6-31) for each k is

$$\underline{y}^1 = (.09350,.00194,.00768,-.00962,-.00629,-.00975)^T$$
$$\underline{y}^2 = (-.01153,.04022,-.00063,-.00341,-.00238,-.00403)^T$$
$$\underline{y}^3 = (-.01313,-.00675,.03176,.00009,-.00006,.00049)^T$$
$$\underline{y}^4 = (-.01721,-.00885,-.00970,.04845,.00080,.00214)^T$$
$$\underline{y}^5 = (-.01721,-.00885,-.00970,-.01184,.03413,.00363)^T$$
$$\underline{y}^6 = (-.01721,-.00885,-.00970,-.01184,-.01310,.03086)^T$$
$$\underline{y}^7 = (-.01721,-.00885,-.00970,-.01184,-.01310,-.02334)^T$$

and the solution to the back substitution problem (6-32) for each k is

$$\underline{\hat{\Delta}}^1 = (.01627,-.00034,-.00013,-.00333,-.00203,-.00192)^T$$
$$\underline{\hat{\Delta}}^2 = (-.00123,.00962,-.00073,-.00124,-.00080,-.00079)^T$$
$$\underline{\hat{\Delta}}^3 = (-.00117,-.00065,.00597,.00006,.00004,.00010)^T$$
$$\underline{\hat{\Delta}}^4 = (-.00222,-.00117,.00052,.01085,.00035,.00042)^T$$
$$\underline{\hat{\Delta}}^5 = (-.00264,-.00140,-.00039,-.00053,.00621,.00072)^T$$

$\underline{\hat{\Delta}}^6 = (-.00254,-.00132,-.00025,-.00040,.00078,.00608)^T$

$\underline{\hat{\Delta}}^7 = (-.00646,-.00474,-.00498,-.00542,-.00454,-.00460)^T$

The steady state generator angles are the $\underline{\hat{\Delta}}^k$ given above.

6. The matrix $\underline{\hat{S}}_x(\infty)$ can be computed as

$$\underline{\hat{S}}_x(\infty) = \sum_{n=1}^{7} \underline{\hat{\Delta}}^n \, \underline{\hat{\Delta}}^{n^T}$$

with the result

$$\underline{\hat{S}}_x(\infty) = 10^{-3} \begin{bmatrix} .32766 & .02369 & .02448 & -.03941 & -.02204 & -.01895 \\ .02369 & .02072 & .01301 & .00332 & .00427 & .00518 \\ .02448 & .01301 & .06143 & .03462 & .02121 & .02268 \\ -.03941 & .00332 & .03462 & .16016 & .03246 & .03404 \\ -.02204 & .00427 & .02121 & .03246 & .06469 & .03471 \\ -.01895 & .00518 & .02268 & .03404 & .03471 & .06309 \end{bmatrix}$$

7. The rms coherency measures can be found from $\underline{\hat{S}}_x$ using (2-24). The values obtained are given in the ranking table, Table 6-2.

In the following discussion it will be found convenient to represent the m generator coherent group consisting of generators $n_1, n_2, \ldots,$ and n_m as $(n_1, n_2, \ldots, n_m)$. The data obtained from the MECS model for the modal disturbance is now discussed.

It was stated in Section 6.2 that the coherent groups identified using the generator clustering algorithm, might be sensitive to the arbitrary choice of indices assigned to the system generators. This can be demonstrated using the data obtained with the MECS model. Let the threshold of coherency, ε_c, be .0795 and use the clustering algorithm as described in Section 6.2 to identify coherent groups based on the rms

Table 6-2 Ranking Table for the Modal Disturbance, MECS Example System

rank r	k_r, ℓ_r	C_{k_r, ℓ_r}
1	5-6	.007639
2	3-7	.007838
3	6-7	.007943
4	5-7	.008043
5	3-6	.008897
6	3-5	.009149
7	2-7	.010987
8	3-4	.012343
9	4-6	.012457
10	2-3	.012495
11	4-5	.012646
12	4-7	.012656
13	2-6	.013170
14	2-5	.013299
15	2-4	.016560
16	1-7	.018101
17	1-3	.018443
18	1-2	.020025
19	1-6	.020704
20	1-5	.020891
21	1-4	.023804

coherency measures given in Table 6-2. Let the generators be processed by the clustering algorithm according to the generator numbering system adopted in this thesis, that is in the order 1,2,3,4,5,6,7. The resulting coherent equivalent would be a 6-generator equivalent model consisting of generators 1,2,3,4,7 and an equivalent generator representing the coherent group (5,6). If the clustering algorithm processed the generators in reverse order, that is 7,6,5,4,3,2,1 corresponding to a simple reassignment of the generator indices, then the coherent equivalent defined by the same coherency threshold would be a 5-generator model consisting of generators 1,2,4,5 and an equivalent generator representing the coherent group (3,6,7). Thus, the coherent groups identified by the generator clustering algorithm are dependent not only on the selected coherency threshold but also on the arbitrary assignment of indices to the system generators.

Based on the results of the ranking table, Table 6-2, a modal-coherent equivalent for the 7-generator MECS model can be identified which retains any predetermined number of generators between 1 and 7. This can be accomplished using the commutative or the transitive rules described in Section 6.1. Table 6-3 shows the coherent groups identified as the commutative rule progresses through the ranking table. At each rank, r, the table indicates the coherent groups identified through that rank, the corresponding number of generators eliminated through that rank, ne(r), and the number of generators which remain in the modal-coherent equivalent if the

Table 6-3 Coherent Group Identification Based on the Ranking Table and the Commutative Rule, MECS Example System-Modal Disturbance

rank r	k_r, ℓ_r	coherent groups identified through rank r	number of generators eliminated through rank r ne(r)	number of generators retained in the equivalent identified at rank r 7-ne(r)
1	5-6	(5,6)	1	6
2	3-7	(5,6),(3-7)	2	5
3	6-7	(5,6),(3-7)	2	5
4	5-7	(5,6),(3-7)	2	5
5	3-6	(5,6),(3-7)	2	5
6	3-5	(3,5,6,7)	3	4
7	2-7	(3,5,6,7)	3	4
8	3-4	(3,5,6,7)	3	4
9	4-6	(3,5,6,7)	3	4
10	2-3	(3,5,6,7)	3	4
11	4-5	(3,5,6,7)	3	4
12	4-7	(3,4,5,6,7)	4	3
13	2-6	(3,4,5,6,7)	4	3
14	2-5	(3,4,5,6,7)	4	3
15	2-4	(2,3,4,5,6,7)	5	2
16	1-7	(2,3,4,5,6,7)	5	2
17	1-3	(2,3,4,5,6,7)	5	2
18	1-2	(2,3,4,5,6,7)	5	2
19	1-6	(2,3,4,5,6,7)	5	2
20	1-5	(2,3,4,5,6,7)	5	2
21	1-4			

Table 6-4 Coherent Group Identification Based on the Ranking Table and the Transitive Rule MECS Example System-Modal Disturbance

rank r	k_r, ℓ_r	coherent groups identified through rank r	number of generators eliminated through rank r ne(r)	number of generators retained in the equivalent identified at rank r 7-ne(r)
1	5-6	(5,6)	1	6
2	3-7	(5,6),(3-7)	2	5
3	6-7	(3,5,6,7)	3	4
4	5-7	(3,5,6,7)	3	4
5	3-6	(3,5,6,7)	3	4
6	3-5	(3,5,6,7)	3	4
7	2-7	(2,3,5,6,7)	4	3
8	3-4	(2,3,5,6,7)	4	3
9	4-6	(2,3,4,5,6,7)	5	2
10	2-3	(2,3,4,5,6,7)	5	2
11	4-5	(2,3,4,5,6,7)	5	2
12	4-7	(2,3,4,5,6,7)	5	2
13	2-6	(2,3,4,5,6,7)	5	2
14	2-5	(2,3,4,5,6,7)	5	2
15	2-4	(2,3,4,5,6,7)	5	2
16	1-7			
17	1-3			
18	1-2			
19	1-6			
20	1-5			
21	1-4			

group identification process were to terminate at rank r. Table 6-4 gives the same information for the transitive rule. The following observations are based on Tables 6-3 and 6-4.

1. The transitive rule tends to reduce the number of generators retained in the equivalent faster, that is at a lower rank, than the commutative rule. This result is not unexpected since more ranks must be examined to merge coherent groups under a commutative rule than a transitive rule.

2. The modal-coherent equivalents retaining 6,5,4 and 2 generators defined by the commutative and transitive rules are identical.

3. The two rules disagree on the coherent group which defines the 3-generator version of the modal-coherent equivalent. Corresponding to a 3-generator equivalent, the commutative rule identifies the coherent group (3,4,5,6,7) at rank 12 in Table 6-3 whereas the transitive rule identifies the group (2,3,5,6,7) at rank 7 in Table 6-4. Each rule previously identified a 4-generator equivalent containing the single coherent group (3,5,6,7). Since generators 2 and 7 are found to be a coherent pair at rank 7 in the ranking table, the transitive rule immediately merges generator 2 with the coherent group (3,5,6,7). The commutative rule also recognizes generators 2 and 7 as a coherent pair at rank 7, but must wait until generator 2 is also found to be coherent with generators 3,5 and 6 as well, before generator 2 can join the group (3,5,6,7). The commutative rule proceeds through the ranking table from rank 7, and generator 4 is found to be coherent with each member of (3,5,6,7) before generator 2 can meet that requirement. Thus, the commutative rule joins

generator 4 to the group (3,5,6,7) instead of generator 2 as observed with the transitive rule.

The disagreement over the coherent group which defines the 3-generator modal-coherent equivalent for the MECS model points out a fundamental difference in the character of coherent groups which are likely to be identified by the transitive and commutative rules. The synchronizing torque coefficient matrix for the MECS model given in step 1 shows that the synchronizing torque coefficients of the equivalent lines which connect generator 2 to the generators in the coherent group (3,5,6,7) are

$T_{23} = 1.82$
$T_{25} = 1.62$
$T_{26} = 1.73$
$T_{27} = 6.32$

Since generators 3,5,6 and 7 have essentially the same inertia, the torque coefficients determine the relative stiffness of the connection between generator 2 and each member of the group (3,5,6,7). The synchronizing torque coefficients clearly indicate that generator 2 is tightly coupled to generator 7 but not to the remaining generators in the group. Thus, the use of the transitive rule would be likely to identify coherent groups which contain "weak" members, that is a generator which is stiffly connected to just one member in the coherent group. The commutative rule avoids this problem by requiring that a generator be coherent with all members of a coherent group before that generator can join the group.

4. In Chapter 4, the ZMIID disturbance identified a 4-generator equivalent of the MECS model containing the single coherent group (3,5,6,7). Both the commutative and the transitive rules applied to the

ranking table of the modal disturbance identify the same 4-generator equivalent. Thus it is confirmed that there is no difference in the coherent groups identified by the ZMIID and modal disturbances for the 4-generator modal-coherent equivalent of the MECS model.

5. The 3-generator version of the modal-coherent equivalent defined by the commutative rule in Table 6-3 contains the single coherent group (3,4,5,6,7). The same coherent group was aggregated in Chapter 4 to construct the coherent equivalent based on the UD disturbance. Thus, the 3-generator modal-coherent equivalent of the MECS model is the same equivalent as the "UD coherent equivalent" in Chapter 4. The performance of the UD equivalent in preserving the rms coherency measure for various system disturbances was shown to be quite poor. It was also indicated that the eigenvalues of the UD coherent equivalent did not match the system eigenvalues retained by the modal equivalent based on the UD disturbance. The fact that the 3-generator coherent equivalents for the MECS model based on the modal and UD disturbances are identical is purely a coincidence, however, it does indicate that even if the modal-coherent approach is used to identify coherent groups there is the danger of over-aggregating and destroying the modal and coherent structure of the unreduced system.

Intuitively, the less model aggregation that is done to construct a dynamic equivalent, the more accurate the equivalent should be in predicting the behavior of the unreduced system model. An indication of the gradual degradation of the modal structure of the system that occurs as the number of generators retained by the modal-coherent equivalent decreases, is shown in Table 6-5. Table 6-5 compares the modes of intermachine oscillations in the unreduced linear system model with the modes of oscillation computed for the linearized 6,5,4,3 and 2-generator versions of the modal-coherent equivalent defined by the ranking table and the commutative rule as shown in Table 6-3. Except for the 4-generator and 3-generator versions, the modes corresponding to the equivalents in Table 6-5 were computed from linear models obtained by directly aggregating the unreduced linear MECS model using the procedure for linear model aggregation described in Section 2.4. As indicated earlier in the discussion, linear models of the 4-generator and 3-generator equivalents (and their modes) were previously derived in Chapter 4. The following comments are based on the results in Table 6-5.

1. The commutative rule identifies generators 5 and 6, at rank 1 in Table 6-3, as the most coherent pair of generators in the MECS model. The aggregation of these two generators to form the 6-generator modal-coherent equivalent removes the oscillation at 13.614 rad/sec observed in the unreduced system. Apparently, the oscillation at 13.614 rad/sec is associated exclusively with the intermachine behavior of generators 5 and 6 since the remaining modes of oscillation observed in the unreduced model are closely (and almost exactly) preserved in the 6-generator modal-coherent equivalent.

Table 6-5 Modes of Intermachine Oscillation for Modal-Coherent Equivalents Derived from the Commutative Rule

MODES OF INTERMACHINE OSCILLATIONS (rad/sec)

unreduced system	number of generators retained in the modal-coherent equivalent				
	6	5	4	3	2
7.415	7.414	7.226	7.288	6.653	8.680
9.481	9.481	9.432	9.351	10.176	
10.389	10.391	10.201	10.659		
12.756	12.761				
13.614		13.729			
14.304	14.309				

2. The 5-generator modal-coherent equivalent is constructed by aggregating two coherent groups, (5,6) and (3,7). The construction of the 5-generator equivalent may be viewed as a two step aggregation process in the unreduced system or a one step aggregation (of generators 3 and 7) in the 6-generator modal-coherent equivalent model. Taking the later view, the changes in the modes of oscillation from the 6-generator to the 5-generator equivalent indicate that the modes at 12.761 and 14.309 rad/sec in the 6-generator equivalent are associated with the intermachine oscillations of generators 3 and 7, and the group to group oscillations of (5,6) and (3,7). When generators 3 and 7 in the 6-generator model are aggregated to form the 5-generator modal-coherent equivalent, the modes at 12.761 and 14.309 rad/sec are replaced by a "new" mode at 13.729 rad/sec. It is emphasized that the new mode at 13.729 rad/sec in the 5-generator equivalent does not correspond to the oscillation at 13.614 rad/sec in the unreduced model. Rather, the new mode represents the effort by the coherency aggregation procedure to compensate in the 5-generator equivalent for the loss of one degree of freedom in representing the group to group behavior of (5,6) and (3,7), which was determined by two modes in the 6-generator equivalent and must be represented by a single "average" mode in the 5-generator equivalent. Notice also that the remaining modes in the 5-generator equivalent have shifted below the corresponding modes in the 6-generator equivalent.

3. The 4-generator modal-coherent equivalent can be constructed by aggregating the generators in the 5-generator equivalent model which represent (5,6) and (3,7). The transition of the modes of the 5-generator equivalent to the modes of the 4-generator equivalent indicates that the mode at 13.729 in the 5-generator equivalent, the new mode, is strongly associated with

the group to group oscillations of (5,6) and (3,7), since the aggregation of (5,6) and (3,7) to form the 4-generator equivalent effectively eliminates that mode. The modes which remain in the 4-generator equivalent correspond quite well with the low frequency (10.389 rad/sec and below) modes in the unreduced model. This also indicates that the high frequency modes 12.756, 13.614 and 14.304 of the unreduced model are all associated with the intermachine behavior of the coherent group (3,5,6,7).

4. In each of the transitions from the 6-generator to the 4-generator equivalent there is a slight but steady deterioration of the agreement between the low frequency modes of the modal-coherent equivalents and the corresponding modes of the unreduced system.

5. The modes of the 3- and 2-generator equivalents bear little resemblance to any of the modes of the unreduced system model.

The above observations indicate that the user of modal-coherent equivalents should not specify unnecessarily "small" equivalents. Unfortunately, there isn't an obvious way to tell whether or not a dynamic equivalent is over-aggregated. However, for the MECS model, the ranking table approach for identifying coherent groups does provide some indication that there might be a reasonably dramatic drop in performance between the 4-generator and 3-generator modal-coherent equivalents. Tables 6-3 and 6-4 indicate that for both the commutative and transitive rules that large jumps occur in the ranking table between the rank at which the 4-generator and 3-generator equivalents are identified. This suggests that the ranking table approach for identifying coherent groups may be able to provide information to suggest where the model aggregation process should be cut off in order to guarantee that modal and coherent system structure are adequately preserved. Further research is planned to investigate the uses of the ranking table in identifying coherent groups and in controlling the amount of model aggregation for large scale systems.

Summary of Chapter 6

In this chapter an efficient algorithm for constructing modal-coherent equivalents for large scale systems was developed. The feasibility of the algorithm was confirmed by applying it to two relatively small example systems. Programming of the algorithm for large scale systems is already underway.

CHAPTER 7

CONCLUSIONS AND FUTURE INVESTIGATIONS

The major results of this thesis are now summarized on a chapter by chapter basis and related topics for future research are proposed.

7.1 Thesis Review

In the first chapter, the properties of power system dynamic equivalents derived from the present modal and coherency analysis approaches are discussed. Both approaches are indicated to have considerable merit and at the same time significant drawbacks. The criticisms that have limited the acceptance of these two techniques are used to deduce the properties of an "ideal" dynamic equivalent which would be found highly desirable by the power industry. It is argued that an ideal equivalent should be

1. Suitable for studying any contingency that might occur inside the study area for which the equivalent is derived.
2. Simultaneously a modal and a coherent equivalent.
3. Derived from an efficient computational technique.
4. Expressed in terms of normal power system components.

The above properties are a composite of the most desirable features of present modal and coherent equivalents. This suggests that it may be possible to develop an ideal equivalent by properly linking the modal and coherency analysis techniques. A historical perspective for this research is provided by briefly reviewing the modal analysis approach based on the rms coherency measure [8,9], the so-called coherent-modal approach, which was an early effort to link modal and coherency analysis. It is indicated that the coherent-modal approach does not lead directly to an ideal dynamic equivalent but it does suggest that the rms coherency measure is the key to the development of a nearly ideal dynamic equivalent. The first chapter closes with a statement of the thesis objective which is to develop the justification and the means for constructing a modal-coherent equivalent whose properties closely approximate those of an ideal dynamic equivalent.

In the second chapter the mathematical models used in the development of the modal-coherent equivalent are defined.

The justification for the modal-coherent approach is developed in Chapters 3 and 4. In Chapter 3, the expected value of the rms coherency measure, evaluated over an infinite interval, is algebraically related to the parameters of the power system state model and the statistics of the system disturbance. For random initial conditions and pulse type disturbances (faults), an implicit relationship is developed which, in each case, takes the form of a Lyapunov equation. For step disturbances in mechanical input power or load bus power injection, an explicit formula is derived, relating the rms measure directly to system parameters and the statistics of the step disturbance. Two probabilistic step disturbances, the ZMIID disturbance and the modal disturbance, are shown to cause the rms coherency measure to depend solely on system parameters. Coherent groups based on the ZMIID disturbance are shown to depend strictly on line stiffnesses, while the coherent groups determined by the modal disturbance are determined by relative line stiffnesses.

Using an example system, it is shown in Chapter 4 that the eigenvalues of the coherent equivalent constructed by aggregating the coherent groups identified by the rms coherency measure and the ZMIID disturbance closely approximate the system eigenvalues retained by the modal equivalent based on the same coherency measure and disturbance. It is further shown that both the modal and the coherent equivalent based on the ZMIID disturbance are useful for studying any disturbance that might occur outside the areas of the example system aggregated to form these equivalents. These results indicate that a coherent equivalent which closely approximates a general purpose modal equivalent can be derived when the rms coherency measure and an appropriate probabilistic disturbance are used to identify coherent groups. The example system is a special case composed of many generating units with similar inertias and there is little difference between line stiffnesses and relative line stiffnesses. Consequently, the same set of coherent groups are identified using either the ZMIID or the modal disturbance. In general, this would not be the case and to identify a general purpose dynamic equivalent the modal disturbance would normally be preferred over the ZMIID disturbance since the coherent groups identified by the modal disturbance are determined by relative line stiffnesses which are a more complete description of dynamic system structure than line stiffnesses which are the basis for aggregation when the ZMIID disturbance is used to identify coherent groups.

Chapter 5 proposes that a modal-coherent equivalent can be derived by using the coherency-based aggregation technique to aggregate the coherent groups identified by the rms coherency measure and the modal disturbance. The justification for the approach rests on the analytical developments in Chapter 3 and the observations based on the example system in Chapter 4 which indicate that the rms coherency measure and the modal disturbance can identify coherent groups which properly reflect dynamic system structure.

The construction of the modal-coherent equivalent is a 3 step procedure which includes the evaluation of the expected value of the rms coherency measure for the probabilistic modal disturbance, the identification of coherent groups based on the computed coherency measure and finally, the aggregation of the coherent groups using the coherency-based aggregation technique [2]. Since the coherency-based aggregation technique is well established, the problem of developing the means for constructing the modal-coherent equivalent reduces to finding a procedure for computing the rms coherency measure and identifying coherent groups.

In Chapter 6, an efficient computational algorithm, applicable to large scale systems, is developed for computing the rms coherency measure for the modal disturbance. The algorithm expands the coherency measure for the probabilistic modal disturbance in terms of the coherency measures observed for a sequence of deterministic step disturbances known as the modal disturbance sequence. For each disturbance in the modal disturbance sequence the coherency measure is shown to depend strictly on the steady state response of the generator angles to that disturbance. It is shown that steady state generator angles for any deterministic step disturbance can be efficiently computed from the steady state network equations using a triangular factorization technique.

The coherent groups identified by the generator clustering algorithm, which is presently used to identify coherent groups based on the max-min coherency measure, were shown to be sensitive to the arbitrary order in which generators may be processed by the algorithm. To eliminate this sensitivity, a new approach for identifying coherent groups was proposed based on a ranking table in which the N(N-1)/2 possible coherency measures between the various pairs of generators are ordered from the smallest to the largest. Coherent groups are identified using the ranking table by proceeding down the ranks in the table and using either a transitive or a commutative coherency rule to assign generators to coherent groups. The ranking table approach has an additional advantage over the generator clustering algorithm since the size of the desired equivalent can be specified a priori and coherent groups may be identified to conform to the prespecified number of generators to be retained by the equivalent.

A rough comparison was drawn between the computational effort required to construct the general purpose modal-coherent equivalent and the effort required to construct a coherent equivalent based on the max-min coherency measure. It was shown that the computer time needed to construct the modal-coherent equivalent was approximately equal to the total time required to construct nine coherent equivalents, based on a system containing 250 generators. Thus, the modal-coherent approach will be computationally competitive with the coherency analysis approach based on the max-min coherency measure when a relatively modest number of disturbances are to be examined in a transient stability study.

7.2 Future Research

Based on the developments in the first six chapters it is concluded that the modal-coherent approach to power system dynamic equivalents represents a viable alternative to the present modal and coherency analysis techniques. However, further research is required to fully explore the relative merits of the approach. Some areas where further investigation is indicated are now discussed.

The procedure for evaluating the rms coherency measure for the probabilistic modal disturbance must be programmed for large scale systems. No major difficulties are expected in this task since the triangular factorization technique which is the heart of the computational procedure is an often used tool in power system analysis. This work is already underway.

There is also a need to investigate the suitability of the generator clustering algorithm and the ranking table methods for identifying coherent groups. Based on experience with the MECS example system it is believed that the ranking table procedure using the commutative rule will identify the most meaningful set of coherent groups for use with the modal-coherent approach. The use of the ranking table to limit the amount of aggregation in order to avoid constructing over-aggregated equivalents which do not preserve modal or coherent dynamic system structure is also a topic for future investigation.

Another item for future research which was briefly described in Chapter 6 is the possibility of reducing the number of disturbances in the modal disturbance sequence while retaining the essential character of the modal-coherent equivalent. The computational attractiveness of the modal-coherent approach would be significantly enhanced if it can be shown that only those disturbances corresponding to generators in the internal system and a few generators in the external system which are near neighbors to the internal system need be included in the modal disturbance sequence in order to preserve the integrity of the modal-coherent equivalent.

The performance of the dynamic equivalents derived in this research were judged on their ability to reproduce the coherency measure observed with the unreduced system model. Since these equivalents are intended for use in transient stability studies, there is a definite need to compare the time domain properties of the equivalents derived from the infinite interval rms coherency measure to the properties of equivalents derived from the max-min coherency measure which is evaluated over a short interval. Because the modal-coherent equivalent closely preserves system modes it is expected that the time response observed with the modal-coherent equivalent will closely match the time response of the unreduced system.

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