ABSTRACT

HEAT AND MASS TRANSFER IN ONIONS

By

John R. Rosenau

The response of the product to its environment must be known if the processing of a biological product is to be optimized. The overall purpose of this study was to model the heat and moisture transfer response of onion bulbs to any given boundary condition.

The "Alternating Direction Explicit Procedure" for the solution of the transfer equations was adapted to a special finite difference grid system. This grid system was constructed orthogonal to the principle transfer directions within the onion bulb. The model was designed to be easily adapted to any axially symmetric body.

The model parameters required to model heat and moisture transfer in onions were obtained.

The heat transfer portion of the model simulated the actual process very well as indicated by small RMS and maximum differences between predicted and measured center temperatures. The model showed that for heat

transfer considerations, the bulb may be modeled as a sphere with thermal conductivity equal to the radial thermal conductivity of the bulb.

The mass transfer portion of the model was in only fair agreement with experimental weight loss measurements conducted as a test of the model. The differences are attributed mainly to the effect of respiration and the effect of the cracking and loosening of the outer scales during the drying process.

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A THESIS

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Agricultural Engineering

1970

G67101

ACKNOWLEDGMENTS

The interest, counsel, and time generously extended by the author's major professor Dr. F. W. Bakker-Arkema (Agricultural Engineering) is gratefully acknowledged. The author appreciates the help and guidance offered by Dr. J. V. Beck (Mechanical Engineering), Dr. W. G. Bickert (Agricultural Engineering), Dr. A. M. Dhanak (Mechanical Engineering), and Dr. R. Hamelink (Mathematics).

The author thanks Dr. G. E. Merva (Agricultural Engineering) and Dr. B. F. Cargill (Agricultural Engineering) for their interest and help in defining the research problem.

The financial assistance given the author by Michigan State University, the National Science Foundation, and the Agricultural Engineering Department is acknowledged.

Special thanks are extended by the author to his wife, Marion, for her help and encouragement during this study.

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SYMBOLS

B	Permeability ($lb_m ft/lb_f hr$)
C	Concentration ($moles/ft^3$)
C_p	Heat Capacity (Btu/F)
D	Diffusion Coefficient (ft^2/hr)
E	Internal Energy (Btu)
E_a	Activation Energy ($lb_f ft/mole$)
Gfap	Axial Geometric Factor (ft)
Gfrp	Radial Geometric Factor (ft)
H	Enthalpy (Btu)
$\overline{H}$	Molar Enthalpy (Btu/mole)
J	Molar Mass Flux ($moles/ft^2$)
Le	Lewis Number (dimensionless)
M_w	Molecular Weight of Water
Nu	Nusselt Number (dimensionless)
P	Pressure (lb_f/ft^2)
Pr	Prandtl Number (dimensionless)
R	Universal Gas Constant ($lb_f ft/mole°R$)
Re	Reynolds Number (dimensionless)
Sc	Schmidt Number (dimensionless)
Sh	Sherwood Number (dimensionless)

T	Temperature (°R)
V	Volume (ft^3)
$\bar{V}_i$	Partial Molar Volume (ft^3/mole)
$Vol_{i,j}$	Volume of the Node i,j (ft^3)
X	Mole Fraction (dimensionless)
c	Specific Heat (Btu/lb_mF)
$\bar{c}$	Molar Heat Capacity (Btu/mole F)
d	Diameter (ft)
h	Convective Heat Transfer Coefficient (Btu/hr ft^2F)
h_D	Convective Mass Transfer Coefficient (ft/hr)
j	Colburn j-factor (dimensionless)
k	Conductivity (But/hr ftF)
m	Moisture Content (d.b.) (dimensionless)
n	Moles (moles)
p	Partial Pressure (lb_f/ft^2)
q	Heat (Btu)
$\dot{q}''$	Heat flux (Btu/hr^2)
r	Radius (ft)
t	Time (hr)
x	Distance (ft)
y	Distance (ft)
z	Distance (ft)
v	Velocity (ft/hr)
Ψ_w	Water Potential (atm)
Ω	Mobility (moles ft/hr lb_f)

γ	Specific Gravity (dimensionless)
γ	Activity Coefficient (dimensionless)
μ	Chemical Potential (Btu/mole)
ρ	Density (lb_m/ft^3)
ρ_{dm}	Density of the Solids (lb_m/ft^3)
ω	Mass Fraction (dimensionless)

Subscripts

I	End Node Index
J	Surface Node Index
a	Axial Direction
a	Ambient
c	Based on Concentration
dm	Dry Matter
m	Based on Moisture Content
r	Radial Direction
i	Node Index
i	ith Species
j	Node Index
w	Water
s	Solids
s	Surface
wv	Water Vapor
x	Direction
y	Direction
z	Direction

μ	Based on Chemical Potential
c	Fixed Coordinate System
m	Coordinate System Moving with the "Medium"
n	Time Index
s	Coordinate System Moving with the "Solids"
v	Coordinate System Moving with the "Volume"
o	Saturated
o	Pure

I. INTRODUCTION

The remarkable efficiency of the agricultural production system in the United States has been achieved through advances in three basic areas: (i) the development of improved growing practices and genetic varieties, (ii) the mechanization of crop and animal production, and (iii) the evolution of new processing techniques to handle the resulting production increases. The development of these processing methods is extremely important as the benefits of increased production are wasted if the system for handling, processing, and distributing this production is inadequate.

Inherent to the methods incorporated for the processing of agricultural products is the response of the products to their total processing environment. If the effect of all of the mechanical, chemical, thermal, and biological forces acting on a product can be predicted, the development of optimal processing systems for that product is made possible.

This research project examines a part of the total problem outlined above--the development of a mathematical model simulating the response of yellow globe onions (var. Abbott and Cobb 192) to a convective heat and moisture transfer boundary condition.

There are two main reasons for choosing this thesis topic. First, the onion production industry in Michigan is in the process of rapid change. The harvesting and material handling operations have been totally mechanized as have the packaging and shipping operations. However, problems still remain. The first occurs just after harvest when the onions are "cured." Normally, this is done by leaving the covered pallets of freshly harvested and topped onions in the field. During the curing period, the onion necks dry, helping to prevent disease organisms from entering the bulb through the place where the green top was attached. In addition, the skins develop the traditional golden color, while the outermost skin drys, cracks, and falls off ("shucks"), taking with it any attached field dirt. With warm dry weather, curing takes about a week; with cool rainy weather, up to three weeks.

The author has found that the curing process can be shortened to about three days by passing air at 100 F and 90% relative humidity over the bulbs. More work is needed, however, to optimize this artificial curing process.

The processing step following curing is storage. While storage conditions of 32 F and 75% relative humidity have been recommended (Franklin _et al._, 1966), more work needs to be done to identify the various effects of respiration, moisture transfer, and spoilage on the weight loss of marketable onions in storage.

Another problem area in the processing sequence occurs when the bulbs are brought in from storage. If run through the packaging line at storage temperatures (about 35 F), water condenses on the bulbs. In the somewhat dusty atmosphere of the packing room, this moisture picks up dirt causing an inferior product. The onions thus need to be warmed before processing.

The optimal design of curing, storage, and heating equipment depends on the prediction of the response of the onion bulb to its environment. Thus, a mathematical model of the bulb is needed.

When the environment is significantly affected by interaction with the product (such as in the case of a deep bin), the model of the bulb may be included within larger models (Bakker-Arkema _et al._, 1969), to predict the response of the entire process. One problem associated with such nested models, however, is the large amount of computer storage and time required for their solution.

The second reason for the research was the hope that studying the heat and moisture transfer processes in onion bulbs would lead to a better understanding of these processes in other high moisture products. The onion has a relatively large size and a structure which allows sections to be removed with a minimum of cell damage. Thus, experiments may be performed on sections of the product to determine its transfer properties without changing these properties through the sectioning process.

II. LITERATURE REVIEW

2.1 Introduction

The investigation of the processes of heat and moisture transfer can be divided into two parts--the transfer processes occurring within the product, and the interaction of the product surface with the environment. Concerning moisture migration, Van Arsdel (1963) writes:

> The drying of a moist substance always involves the movement of a quantity of water away from a dry surface. The separation is usually regarded for purposes of analysis as the result of two successive phenomena: (1) migration of water within the moist body to its surface; and (2) conveyance of the vaporized water away from the body . . . the factors that determine the rate of movement of water within the body can be regarded as independent of the external conditions. A useful analysis of the process can be made on the basis of this simplified picture, even though in some cases it may become evident that vaporization is in fact occurring in an ill-defined zone within the moist body instead of only at its geometric surface.

The surface-environment interaction can be described by two convective transfer coefficients. The first, h, is defined by the equation

$$\dot{q}'' = h\ (T_s - T_a) \qquad (2.1\text{-}1)$$

and the second, h_D, by the equation

$$J_w^s = h_D \; (C_{wv_s} - C_{wv_a}). \tag{2.1-2}$$

In general, these coefficients are determined by the nature of the air flow pattern around the product. When radiation heat transfer is significant, h can be modified to include its effects.

Much research has been performed investigating these coefficients. Since onions are usually processed in deep bins and pallet boxes, the work of Barker (1965) in reviewing the subject of heat transfer in packed beds should be mentioned. By reference to Barker's article, a "Colburn j-factor" can be obtained for any given Reynolds number and product shape. The heat transfer coefficient h is then obtained by the relation

$$\frac{hd}{k_{air}} = Nu = j \; Re \; Pr^{1/3} \tag{2.1-3}$$

and the mass transfer coefficient h_D by

$$\frac{h_D d}{D_{air}} = Sh = j \; Re \; Sc^{1/3} \tag{2.1-4}$$

where the Reynolds number Re is given by the onion bulb diameter times the mass flow rate of air per square foot of bed area divided by the absolute viscosity. The Prandtl number Pr is given by the kinematic viscosity divided by the thermal diffusivity k_{air}, and the Schmidt number Sc by the kinematic viscosity divided by the mutual diffusivity of water in air D_{air}.

While the above formulas can only be considered approximate, they do characterize the interaction between the product and its environment. The transport processes within the product are outlined in the following sections.

2.2 Specific Heat

If heat is added to a simple closed constant pressure system in which no work other than that used to change the system volume is performed, the change in internal energy of that system is given by

$$\Delta E = q - P\Delta V. \tag{2.2-1}$$

The quantity H is defined by the equation

$$H = E + PV. \tag{2.2-2}$$

Thus, in this process,

$$\Delta H = q. \tag{2.2-3}$$

During the heat addition process the temperature of the system rises. This then is the basis for the definition of heat capacity, namely,

$$C_p = \left(\frac{\partial H}{\partial T}\right)_p . \tag{2.2-4}$$

If the system is homogeneous, the specific heat is given by

$$c = \frac{C_p}{\rho V} \tag{2.2-5}$$

and the molar heat capacity by

$$\bar{c} = \frac{C_p}{n}. \tag{2.2-6}$$

When the system can be considered a homogeneous mixture, the enthalpy of the system is the sum of the partial enthalpies, i.e.,

$$H = \sum_i n_i \bar{H}_i \tag{2.2-7}$$

where n_i is the number of moles of species i present and $\bar{H}_i$, the partial molal enthalpy, is defined by the equation

$$\bar{H}_i = \left(\frac{\partial H}{\partial n_i}\right)_{p,T} . \tag{2.2-8}$$

Likewise,

$$C_p = \sum_i n_i \bar{c}_i. \tag{2.2-9}$$

In general, $\bar{c}_i$ is not a constant for a particular species but varies with changes in temperature as well as in the relative mole fractions making up the mixture. Nevertheless, it is customary as a first (and usually quite accurate) approximation to consider the $\bar{c}_i$'s for a food system as being constant.

Siebel (1892) considered food as composed of solids and water for which he used the respective specific heats of 0.2 and 1.0 Btu/lb$_m$F.

Charm (1963) used this approach with the equation

$$c = 0.5\ \omega_f + 0.3\ \omega_s + 1.0\ \omega_m \tag{2.2-10}$$

wherein ω_f, ω_s, and ω_w refer to the mass fractions of fat, solids, and moisture. If this formula is applied to onions of 90% moisture content (w.b.), 9.8% solids non-fat, and 0.2% fat, c is calculated as 0.93 Btu/lb$_m$F. If the formula were applied to 80% moisture content onions (19.6% solids nonfat and 0.4% fat), c would be 0.86 Btu/lb$_m$F. Ordinanz (1946) gives identical values of c for onions at 80 and 90% moisture content (w.b.).

Reidel (1951) separated the solids portion of fruits and vegetables into X_o "soluble" and X_u "insoluble" fractions by examining the refractive index of the juice. He tested onions of 85.5% moisture content (w.b.) and used weight fractions of 13.% (of the total weight) as soluble solids, and 1.5% as insoluble solids in the equation

$$c = (1 - X_u)(1 - 0.57\ X_o) + 0.29\ X_u. \tag{2.2-11}$$

In this case c is 0.916 Btu/lb$_m$F. For 90% moisture content onions (w.b.), assuming the soluble and insoluble fractions are in the same proportion to each other, c would be 0.942 $Btu/lb_m F$.

In light of the above, it was concluded that further work on determining the specific heat of onions is not needed at the present time. Charm's equation (eqn. 2.2-10) was used to determine c in the calculations involving specific heat in this research.

2.3 Density

The importance of a product's specific heat has been described in the previous section. Density merits consideration in that it, when multiplied by the specific heat, expresses the system's heat capacity on a per unit volume basis.

It is often easier to determine a food product's specific gravity γ (by measuring its buoyancy in water) than its density. The density is then determined by multiplying the specific gravity by the density of water. While the density of water is slightly dependent upon its temperature, it does not vary from 62.27 $lb_m ft^3$ (at 70 F) by more than 0.4% in the range of 40 to 100 F (Holman, 1963). Thus, in this temperature range the following equation is sufficiently accurate.

$$\rho = 62.27\ \gamma \qquad (2.3\text{-}1)$$

The author was unable to find literature values for the density of onion flesh. It thus became necessary to determine this property experimentally.

2.4 Thermal Conductivity

In the one dimensional case, Fourier's equation of heat conduction

$$\dot{q}'' = - k \frac{\partial T}{\partial z} \tag{2.4-1}$$

defines a material's thermal conductivity k. Consideration of the law of the conservation of energy in conjunction with the above leads to the following familiar differential equation for the temperature history of a body in which there is no heat generation:

$$\rho c \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} (k \frac{\partial T}{\partial z}). \tag{2.4-2}$$

This simple definition of k, however, must be expanded when the multi-dimensional case is studied since thermal conductivity is a tensor and not a scalar property. Thus, in three dimensions (Arpaci, 1966),

$$\begin{vmatrix} \dot{q}''_x \\ \dot{q}''_y \\ \dot{q}''_z \end{vmatrix} = - \begin{vmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{vmatrix} \cdot \begin{vmatrix} \frac{\partial T}{\partial x} \\ \frac{\partial T}{\partial y} \\ \frac{\partial T}{\partial z} \end{vmatrix} . \tag{2.4-3}$$

If a material is orthorombic, and if its principle directions are coincident with the x, y, and z directions, the following equations are equivalent to equation (2.4-3):

$$\dot{q}_x'' = k_x \frac{\partial T}{\partial x} , \qquad (2.4\text{-}4)$$

$$\dot{q}_y'' = k_y \frac{\partial T}{\partial y} , \qquad (2.4\text{-}5)$$

and

$$\dot{q}_z'' = k_z \frac{\partial T}{\partial z} . \qquad (2.4\text{-}6)$$

The law of conservation of energy can be applied to these equations, along with pertinent initial and boundary conditions, to generate a model for the temperature-time history of a product. (See Section 2.7 of this chapter and all of Chapter IV.) Since this history is dependent upon the thermal conductivity of the product, these values must be determined and included in the mathematical model.

Van Arsdel (1963) gives a general description of the values of thermal conductivity which might be expected in biological products:

> In fresh fruits and vegetables, whose moisture content is very high, the conductivity is not far from that of pure water. As drying takes place, however, conductivity falls. If shrinkage is complete, so that the dry product is free from internal voids, the decrease in conductivity is only minor, but if the body becomes highly porous as it dries the low conductivity of the air in the open spaces reduces the overall conductivity markedly.

Since the onion is a high moisture content vegetable, one would expect its thermal conductivity to be close to that of pure water which is 0.349 Btu per hr ft F at 70 F (Holman, 1963). Since the thermal conductivity of water varies from the value at 70 F by a maximum of 4.9%, in the temperature range of 40-100 F, one would also expect the thermal conductivity of onion flesh to be quite constant within this range.

Reidy (1968) has conducted an extensive review of experimental techniques used to determine thermal properties. He noted that steady state techniques could not be expected to give accurate results with high moisture biological products due to the problem of moisture migration within the product during the test. He concluded that numerical methods should be used with transient type experiments involving these materials.

As the author has been unable to find values for thermal conductivity of onion flesh in the literature, one of the objectives of this research (see Chapter III and Chapter VI) was to determine values for the thermal conductivity of onion flesh.

2.5 Moisture Diffusion

Görling (1958) outlined the process of moisture movement within a biological product as a combination of five different mechanisms: (i) liquid movement caused by capillary forces, (ii) diffusion of liquids caused by concentration differences, (iii) surface diffusion, (iv) water vapor diffusion, and (v) vapor flow caused by differences in total pressure.

Before proceeding with the subject of diffusion, the distinction between the movement of moisture via diffusion and via bulk flow should be mentioned. Bulk flow is flow caused by the action of a pressure gradient or gravity on water existing within a relatively large channel such as a pipe. Poiseuille flow and Knudsen flow are familiar examples of bulk flow. While such flows exist in the xylem vessels of the larger plants during the growing process, the movement of water within the flesh of a biological product can be considered a diffusion process, i.e., the movement is caused by a chemical potential gradient (as discussed later in this section) and not by a total pressure gradient or gravity directly (Saravacos, 1962).

The traditional literature on the subject of drying used the theory and mathematics associated with heat transfer. Thus, the diffusion equation

$$J_w^s = - D_c' \frac{\partial C_w}{\partial z} \tag{2.5-1}$$

was written in obvious analogy to the heat conduction equation and all of the mass transfer mechanisms were grouped together under one overall diffusion coefficient D_c' and one driving force $\partial C_w/\partial z$.

Babbit (1940) pointed out that the true driving force for moisture movement is not the concentration gradient. He showed this by noting that water can be made to diffuse against a concentration gradient. He concluded that the driving force for diffusion was the gradient of the equilibrium water vapor pressure.

Kramer (1969) showed that the true driving force for diffusion of water through a medium is the gradient of the chemical potential of water $\partial\mu_w/\partial z$. Kramer, however, followed the soil physics literature convention and used the chemical potential in another form. By dividing the chemical potential by the molar volume of pure liquid water, the chemical potential is transformed into Ψ_w, the "water potential," which is dimensionally equivalent to pressure. Kramer mentioned that the transformation is employed simply for convenience.

By using the chemical potential gradient instead of the concentration gradient, a diffusion equation analogous to equation (2.5-1) can be developed. If J_w^m is the molar flux of water with respect to a reference frame moving at the same rate as the "medium," then

$$J_w^m = \frac{C_w \Omega_w \partial\mu_w}{\partial z} \qquad (2.5\text{-}2)$$

where Ω_w may be thought of as a mobility.* The solids

*Hartley and Crank (1949) presented a similar development but with emphasis on liquid diffusion. Their expression $1/\sigma_A \eta N$ is the same as Ω_w.

may also move with respect to this reference frame and the flux relative to the solids, J_w^s, is the flux needed to determine drying rates.

If v^{ms} is the velocity of the medium with respect to the solids,

$$J_w^s = J_w^m + v^{ms}\ C_w \tag{2.5-3}$$

and

$$J_s^s = 0 = J_s^m + v^{ms}\ C_s. \tag{2.5-4}$$

Therefore,

$$J_w^s = J_w^m - J_s^m \frac{C_w}{C_s}. \tag{2.5-5}$$

The rate at which the solids move with respect to the medium defines another mobility Ω_s by the equation

$$J_s^m = -\ C_s\ \Omega_s\ \frac{\partial \mu_s}{\partial z}. \tag{2.5-6}$$

From the Gibbs-Duhem equation (Moore, 1962),

$$d\mu_s = \frac{C_w}{C_s}\ d\mu_w. \tag{2.5-7}$$

Therefore,

$$J_s^m = C_w\ \Omega_s\ \frac{\partial \mu_w}{\partial z}, \tag{2.5-8}$$

$$J_w^s = - C_w \, \Omega_w \frac{\partial \mu_w}{\partial z} - \frac{C_w^{\;2}}{C_s} \; s \; \frac{\partial \mu_w}{\partial z} \, , \tag{2.5-9}$$

and

$$J_w^s = - (C_s \; \Omega_w + C_w \; \Omega_s) \frac{C_w}{C_s} \frac{\partial \mu_w}{\partial z} \, . \tag{2.5-10}$$

If D'_μ is defined by

$$J_w^s = \frac{C_w}{RT} D'_\mu \frac{\partial \mu_w}{\partial z} \, , \tag{2.5-11}$$

then

$$D'_\mu = \frac{RT}{C_s} (C_s \; \Omega_w + C_w \; \Omega_s) . \tag{2.5-12}$$

Since

$$\mu_w - \mu_w^\circ = RT \ln a_w = RT \ln (\gamma_w X_w) = RT \ln \frac{p_w}{p_w^\circ} \tag{2.5-13}$$

the equation for the molar flux of water may be stated in a number of equivalent ways. The most traditional involves the concentration of water. Thus,

$$\frac{\partial \mu_w}{\partial z} = RT \frac{\partial \ln a_w}{\partial z} = RT \frac{\partial \ln a_w}{\partial \ln X_w} \frac{\partial \ln X_w}{\partial z} . \tag{2.5-14}$$

But,

$$X_w = \frac{C_w}{C_w + C_s} \, . \tag{2.5-15}$$

Expanding dX_w yields

$$dX_w = \frac{\partial X_w}{\partial C_w} dC_w + \frac{\partial X_w}{\partial C_s} dC_s \tag{2.5-16}$$

or

$$dX_w = \frac{C_s}{C^2} dC_w - \frac{C_w}{C^2} dC_s \ . \tag{2.5-17}$$

From the Gibbs-Duhem relations,

$$\frac{dC_s}{dC_w} = - \frac{\overline{V}_w}{\overline{V}_s} \ . \tag{2.5-18}$$

Therefore,

$$dX_w = \frac{C_s\overline{V}_s + C_w\overline{V}_w}{C^2\overline{V}_s} dC_w = \frac{1}{C^2\overline{V}_s} dC_w \tag{2.5-19}$$

and

$$\frac{\partial \ln X_w}{\partial z} = \frac{1}{C_w \ C \ \overline{V}_s} \frac{\partial C_w}{\partial w} \tag{2.5-20}$$

yielding,

$$J_w^s = \frac{-(C_s \ \Omega_w + C_w \ \Omega_s)RT}{C \ C_s \ \overline{V}_s} \frac{\partial \ln a_w}{\partial \ln X_w} \frac{\partial C_w}{\partial z} \ . \tag{2.5-21}$$

This development shows that D_c' as used in equation (2.5-1) is given by

$$D'_c = (C_s\,\Omega_w + C_w\,\Omega_s)\,\frac{RT}{C\,C_s\,\overline{V}_s}\,\frac{\partial \ln a_w}{\partial \ln X_w}$$

$$= D'_\mu\,\frac{1}{C\overline{V}_s}\,\frac{\partial \ln a_w}{\partial \ln X_w}\,. \qquad (2.5\text{-}22)$$

At this point the distinction between D'_c and D_c should be explained. D'_c as defined in equation (2.5-1) relates the molar flux with respect to the solid to the concentration gradient. D_c as used in many texts on mass transfer (especially Bird, Steward, and Lightfoot, 1960) is the mutual diffusion coefficient which relates the molar flux with respect to the <u>volume</u> fixed reference plane to the concentration gradient. Thus,

$$J_w^v = -\,D_c\,\frac{\partial C_w}{\partial z} \qquad (2.5\text{-}23)$$

The relation of D'_c to D_c can now be determined. Since

$$J_w^v = J_w^m + v^{mv}\,C_w, \qquad (2.5\text{-}24)$$

$$v^{mv} = -\,v^{vm} = -\,\overline{V}_w\,J_w^m - \overline{V}_s\,J_s^m\,, \qquad (2.5\text{-}25)$$

and

$$J_w^v = J_w^m - C_w\,(\overline{V}_w\,J_w^m + \overline{V}_s\,J_s^m)\,, \qquad (2.5\text{-}26)$$

substituting for J_w^m and J_s^m into equation (2.5-26) yields

$$J_w^V = - (C_s \Omega_w + C_w \Omega_s) \bar{V}_s C_w \frac{\partial \mu_w}{\partial z} . \quad (2.5\text{-}27)$$

Substituting for $\partial \mu_w / \partial z$ gives finally

$$J_w^V = - (C_s \Omega_w + C_w \Omega_s) \frac{RT}{C} \frac{\partial \ln a_w}{\partial \ln X_w} \frac{\partial C_w}{\partial z} \quad (2.5\text{-}28)$$

which is identical with the result given by Hartley and Crank (1949). Thus,

$$\frac{D_c'}{D_c} = \frac{1}{C_s \bar{V}_s} . \quad (2.5\text{-}29)$$

When shrinkage upon drying is negligible, the partial molar volume of water $\bar{V}_w$ is zero and the term $1/C_s\bar{V}_s$ is unity. In this case, D_c' and D_c are identical.

Fish (1958) found that in potato starch gel, D_c' varied with temperature and the variation could be described by Arrhenius' equation as

$$D_c' = D_o' \exp \left(\frac{-E_a}{RT} \right) . \quad (2.5\text{-}30)$$

The constant D_o' varied little with moisture content but the activation energy E_a increased with decreasing moisture content.

Jason (1958) obtained good agreement between the experimental and theoretical results in the drying of fish muscle by using two values for D_c'--the first during the initial falling rate drying period and the second during the later stages of drying. He postulated that the transition from the first to the second was associated with the uncovering of the unimolecular water layer normally bound to the portein molecules.

Jason (like Fish) noted an Arrhenius type of relationship between the diffusion coefficient and temperature. His results agree with those of Fish in that D_o' was quite constant but E_a increased with decreasing moisture content.

Wang (1958) used a different equation to model moisture diffusion. While he solved a problem in simultaneous heat and mass transfer, the equations he used degenerate for a one dimensional isothermal case to

$$J_w^s = \frac{B'}{M_w} \frac{\partial p_w}{\partial z} \tag{2.5-31}$$

where p_w is the equilibrium vapor pressure. Since

$$\frac{\partial \mu_w}{\partial z} = RT \frac{\partial \ln p_w}{\partial z} = \frac{RT}{p_w} \frac{\partial p_w}{\partial z} , \tag{2.5-32}$$

equation (2.5-10) can be rewritten as

$$J_w^s = - (C_s \Omega_w + C_w \Omega_s) \frac{C_w}{C_s} \frac{RT}{p_w} \frac{\partial p_w}{\partial z} . \tag{2.5-33}$$

Thus,

$$B = \frac{M_w RT}{p_w} (C_s \ \Omega_w + C_w \ \Omega_s) \frac{C_w}{C_s} = \frac{D'_\mu \ M_w \ C_w}{\rho_{wv}} \ . \tag{2.5-34}$$

Since C_w includes all of the moisture existing per unit volume in the material, the gas law does not provide a simple relationship between p_w and C_w. One cannot say a priori that B' is a constant.

Young (1968) used a related approach to model simultaneous heat and mass transfer in spherical, porous, hygroscopic solids. He assumed that the gradient of the density of water vapor in the pore spaces ρ_{wv} is the driving force for mass transfer, thus making a distinction between ρ^m_{dm}, the overall moisture density and ρ_{wv}, the density of the water vapor in the void fractions of the material. Two further assumptions were that the overall moisture content (d.b.) was related to the vapor concentration in the void fraction and the temperature by

$$m = \alpha + \beta \ \rho_{wv} - \gamma \ T \tag{2.5-35}$$

where α, β, and γ are constants and that the diffusion coefficient associated with the vapor concentration gradient driving force could be modeled as

$$D'_{wv} = D_1 + D_2 m + D_3 T \tag{2.5-36}$$

where D_1, D_2, and D_3 are constants.

It can be shown that D'_{wv} is related to D'_{μ} by the equation

$$D'_{wv} = \frac{D'_{\mu} \; m \; \rho_{dm}}{\rho_{wv}} \; . \qquad (2.5\text{-}37)$$

While Young did not attempt to verify his model experimentally, he did solve the model numerically for many different input conditions. One important result was the illustration of the fact that the heat and mass transfer equations may be solved separately whenever temperature equilibrium is reached much faster than moisture equilibrium. Young developed a modified Lewis number as a criterion for this condition. (See Chapter VIII.)

In spite of the fact that D'_{wv} is not a constant, extremely good fits between experimental data and the corresponding mathematical models have been obtained in some cases by using constant values of D'_{wv}. For example, Roa and Bakker-Arkema (1969) found that in freeze-dried meat cubes, a model using a constant D'_{wv} and one using a second order polynomial D'_{wv} fitted the adsorption data almost equally well.

Another possibility for the driving force should be mentioned--the moisture content (d.b.), m. Since

$$\frac{\partial \mu_w}{\partial z} = RT \frac{\partial \ln a_w}{\partial m} \frac{\partial m}{\partial z} \; , \qquad (2.5\text{-}38)$$

equation (2.5-10) becomes

$$J_w^s = - (C_s\ \Omega_w + C_w\ \Omega_s)\ \frac{C_w RT}{C_s}\ \frac{\partial \ln a_w}{\partial m}\ \frac{\partial m}{\partial z}\ . \qquad (2.5\text{-}39)$$

The mass flux of water with respect to the solids J_w^s is given by

$$j_w^s = M_w\ J_w^s\ . \qquad (2.5\text{-}40)$$

Therefore,

$$j_w^s = - M_w\ RT\ (C_s\ \Omega_w + C_w\ \Omega_s)\ \frac{C_w}{C_s m}\ \frac{\partial \ln a_w}{\partial \ln m}\ \frac{\partial m}{\partial z} \qquad (2.5\text{-}41)$$

defining D'_m in the equation

$$j_w^s = - \rho_{dm}\ D'_m\ \frac{\partial m}{\partial z} \qquad (2.5\text{-}42)$$

as

$$D'_m = \frac{RT}{C_s}\ (C_s\ \Omega_w + C_w\ \Omega_s)\ \frac{\partial \ln a_w}{\partial \ln m}\ . \qquad (2.5\text{-}43)$$

It follows that

$$D'_m = D'_c\ C_s\ \overline{V}_s = D_c = D'_\mu\ \frac{\partial \ln a_w}{\partial \ln m}\ . \qquad (2.5\text{-}44)$$

Fish (1958) found that in potato starch gel the term $\partial \ln a_w/\partial \ln m$ varied from approximately 1.50 at moisture contents between 0.02 and 0.18 (d.b.) to approximately 0.02 near saturation. He also found that, at 35 C, D'_m was constant (2.4×10^{-7} cm^2/sec) for samples over 0.30 moisture content (d.b.). For samples below this moisture content, D'_m decreased rapidly (1×10^{-10} cm^2/sec. at 0.8% moisture content).

The mass of water included in an incremental volume of material dV is $\rho_{dm} m dV$. The law of the conservation of mass when combined with equation (2.5-42) yields in the one dimensional case

$$\frac{\partial m}{\partial t} = \frac{\partial}{\partial z} \left(D'_m \frac{\partial m}{\partial z}\right) . \qquad (2.5\text{-}45)$$

Thus, the use of the moisture content as the driving force for moisture transfer leads to an equation that is relatively simple to solve in that the driving force for moisture transfer and the measure of water existing within the material are the same. For this reason, the moisture content was used as the driving force in the mathematical model as described in Chapters IV and VI.

2.6 Mathematical Modeling

As shown in Sections 2.4 and 2.5, heat and moisture transfer within a material can be described by parabolic partial differential equations such as (2.4-2) and (2.5-45). This section outlines in general terms how these equations can be put into finite difference form, and describes a particular method--the "alternating direction explicit procedure" for solving the finite difference equations.

For the purpose of illustration, constant property, isotropic, two dimensional heat conduction shall be considered. The modifications needed to extend this to the heat and moisture transfer problem under consideration in this research are straightforward and are outlined in Chapter IV. The partial differential equation for the constant property, isotropic, two dimensional case is

$$\rho c \frac{\partial T}{\partial t} = k \left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) . \qquad (2.6\text{-}1)$$

In order to develop the finite difference approximations to this equation, a rectangular grid is superimposed on the material as shown in Figure 2.6.1. To make the notation easier, the node points are referred to by subscripts i, j, and n, with i and j referring to the increment in the x and y directions, and n to the increment in time.

By the Taylor series expansion theorem,

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j} - 2\, T_{i,j} + T_{i+1,j}}{\Delta x^2} + 0\ (\Delta x^2) . \qquad (2.6\text{-}2)$$

Similarly,

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i,j-1} - 2\, T_{i,j} + T_{i,j+1}}{\Delta y^2} + 0\ (\Delta y^2) . \qquad (2.6\text{-}3)$$

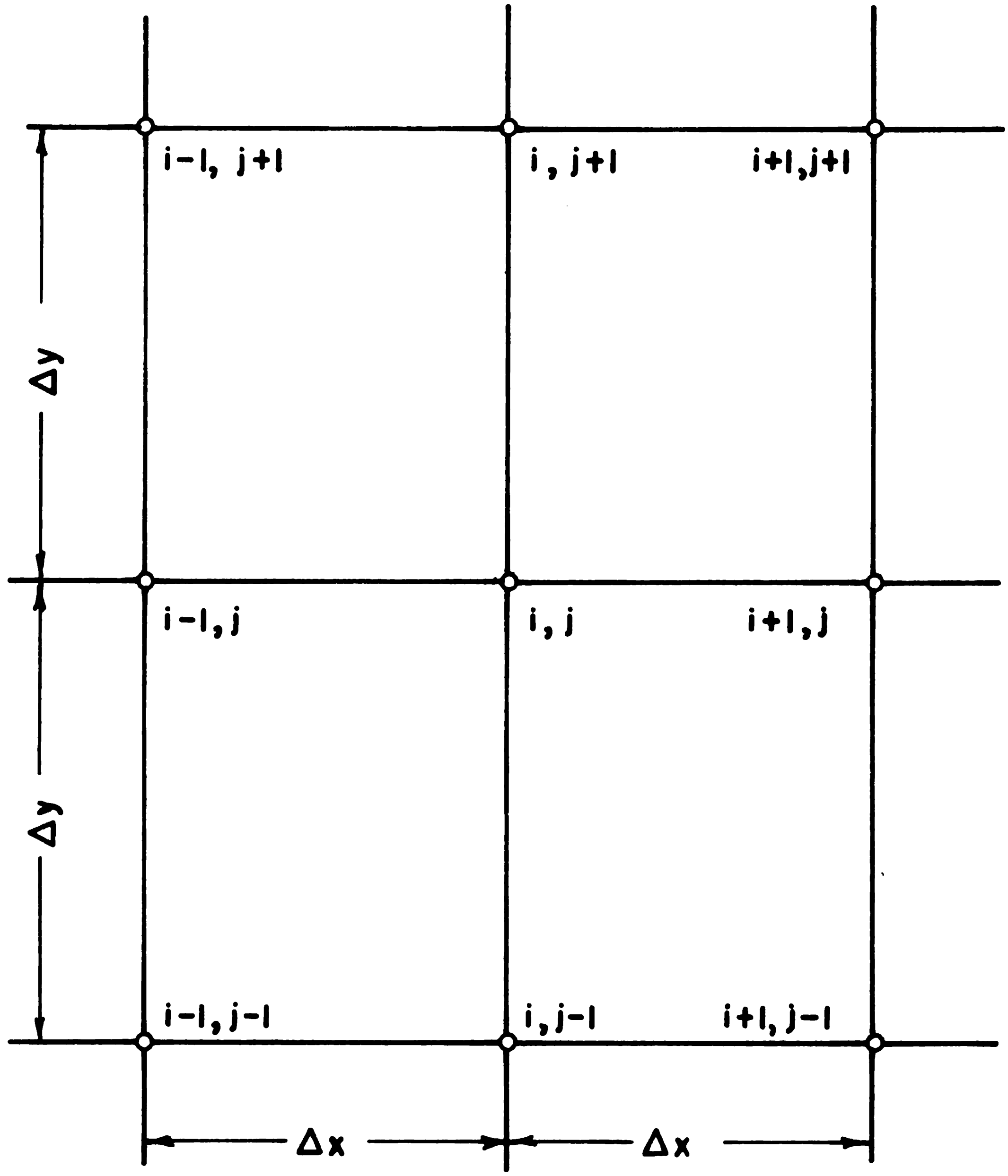

Figure 2.6.1. A Portion of a Two Dimensional Finite Difference Grid in Cartesian Coordinates.

Combining (2.6-2), (2.6-3), and (2.6-1),

$$\frac{dT_{i,j}}{dt} = \frac{k}{\rho c}\left[\frac{T_{i-1,j} - T_{i,j}}{\Delta x^2} + \frac{T_{i+1,j} - T_{i,j}}{\Delta x^2} + \frac{T_{i,j-1} - T_{i,j}}{\Delta y^2} + \frac{T_{i,j+1} - T_{i,j}}{\Delta y^2}\right] + 0(\Delta x^2) + 0(\Delta y^2) \quad . \tag{2.6-4}$$

Equation (2.6-4) is the basic equation upon which the various finite difference schemes are built. Given the initial temperatures at every node point, the numerical methods attempt to approximate the term $(dT_{i,j}/dt)_{ave}$ in the equation

$$T_{i,j}^{(n+1)} = T_{i,j}^{(n)} + \left(\frac{d\ T_{i,j}}{dt}\right)_{ave} \Delta t \tag{2.6-5}$$

in order to obtain the temperature-time history of the material. The evaluation of the right hand side of equation (2.6-4) by the inclusion of the node temperatures at different times gives rise to the various numerical methods for solution of the conduction problem.

The "forward difference explicit method" uses only node temperatures at time n in order to calculate the right hand side of equation (2.6-4). Thus,

$$\frac{T_{i,j}^{(n+1)} - T_{i,j}^{(n)}}{\Delta t} = \frac{k}{\rho c}\left[\frac{T_{(i-1,j}^{(n)} - T_{i,j}^{(n)}}{\Delta x^2} + \frac{T_{i+1,j}^{(n)} - T_{i,j}^{(n)}}{\Delta x^2} + \frac{T_{i,j-1}^{(n)} - T_{i,j}^{(n)}}{\Delta y^2} + \frac{T_{i,j+1}^{(n)} - T_{i,j}^{(n)}}{\Delta y^2}\right] \tag{2.6-6}$$

and rearranging,

$$T_{i,j}^{(n+1)} = T_{i,j}^{(n)} + \frac{\Delta tk}{\rho c}\left[\frac{T_{i-1,j}^{(n)} - T_{i,j}^{(n)}}{\Delta x^2} + \frac{T_{i+1,j}^{(n)} - T_{i,j}^{(n)}}{\Delta x^2} + \frac{T_{i,j-1}^{(n)} - T_{i,j}^{(n)}}{\Delta y^2} + \frac{T_{i,j+1}^{(n)} - T_{i,j}^{(n)}}{\Delta y^2}\right] . \tag{2.6-7}$$

Thus, the method is "explicit," meaning that the calculation of the temperature $T_{i,j}^{(n+1)}$ uses only one equation and not a set of simultaneous equations. Unfortunately, the method is unstable if either of the ratios $k\Delta t/\rho c\Delta x^2$ or $k\Delta t/\rho c\Delta y^2$ becomes larger than about 1/4. This requires that the time step Δt be kept so small that the method is impractical.

The "backward difference method" uses the node temperatures as evaluated at time n+1 for evaluation of the right hand side of equation (2.6-4). Since this yields a system of simultaneous algebraic equations giving the node temperatures at the time n+1, the method is an "implicit" one. While the method is stable for any Δt, the requirement of solving the simultaneous equations renders the method impractical.

The "Crank-Nicolson" method uses the average of the node temperatures at times n and n+1 for the right hand side of equation (2.6-4). The method is stable but has the same limitation as the backward difference method in that the requirement of solving the system of simultaneous algebraic equations created by using the node temperatures at time n+1 makes the method impractical.

A fourth method is known as the "alternating direction implicit procedure" (ADIP) or the "Peaceman-Rachford" method. This method uses node temperatures at time n in the first two terms within the brackets on the right hand side of equation (2.6-4) and temperatures at time n+1 in the second two terms. In calculating the temperatures at time n+2, temperatures at n+2 in the first two terms within the brackets and at n+1 in the second two are used. The systems of resulting simultaneous algebraic equations differ from those obtained in the backward difference or the Crank-Nicolson methods in that the unknown node temperatures appearing in any equation are all from the same nodal row or column. This makes the solution much easier since it is easier to solve I algebraic simultaneous equations J times than to solve I times J simultaneous equations once. This method is stable and practical but the "alternating direction explicit procedure" (ADEP), as described next, is faster and easier to program and was therefore used in this research (Allada and Quon, 1967).

The alternating direction explicit procedure (ADEP), like the ADIP, is a two pass system. On the forward pass, the first and third terms within the brackets on the right hand side of equation (2.6-4) are evaluated at time n+1, and the second and fourth terms are evaluated at time n. On the return pass the first and third terms of the right hand side of equation (2.6-4) are evaluated at time n+1 and the second and fourth at time n+2. The method thus progresses through two time steps during the full forward and return passes.

Barakat and Clark (1966) described a variation of the ADEP in which two separate solutions are generated. In one, the first and third terms within the brackets on the right hand side of equation (2.6-4) are evaluated at time n+1 while the second and fourth are evaluated at time n. In the other solution, the second and fourth terms are evaluated at time n+1 and the first and third at time n. The final solution is obtained by averaging the two solutions wherever desired. Thus, for a given Δt, this variation requires twice as much computer time and storage as the first ADEP method. Barakat and Clark, however, argued that the averaging ADEP is better than the non-averaging. They pointed out that the truncation error in the first pass includes the terms $- \frac{\Delta t}{\Delta x} \left. \frac{\partial^2 T}{\partial t \partial x} \right|_{(n+1)}$

and $- \frac{\Delta t}{\Delta y} \frac{\partial^2 T}{\partial t \partial y}\Big|_{(n+1)}$. This would indicate that, to obtain convergence, the ratios $\Delta t/\Delta x$ and $\Delta t/\Delta y$ must go to zero. The truncation errors on the return pass, however, have the opposite sign of the ones on the forward pass. This indicated to Barakat and Clark that these errors cancel in the averaging ADEP, relaxing the requirement that the ratios $\Delta t/\Delta x$ and $\Delta t/\Delta y$ go to zero. Barakat and Clark commented that this condition needs further examination. The topic is pursued further in Chapter IV.

III. OBJECTIVES

The objectives selected for this research are as follows:

1. To determine the density of onion bulbs.
2. To determine the thermal conductivity of onion flesh in the axial and radial directions.
3. To determine the moisture permeability of onion skins.
4. To develop a mathematical model that simulates the response of an onion bulb to its environment.

IV. MATHEMATICAL MODEL

4.1 Introduction

The partial differential equations governing the heat and moisture transfer in an orthorombic material were developed in Sections 2.4 and 2.6. It was also mentioned that, by consideration of initial and boundary conditions and of the laws of the conservation of mass and energy, a mathematical model suitable for computer solution could be developed to simulate the response of a biological product to its environment. Examination of the onion bulb, however, reveals that there are two considerations that should be mentioned before undertaking the development of such a model.

First, examination reveals that the bulb can be considered axially symmetric. This is the basis for the decision to use the cylindrical coordinate system. A reduction from a three dimensional problem to a two dimensional one is easily accomplished in this case by including only radial and axial dimensions.

The second consideration is that the principal directions of the flesh (i.e., "axial"--coplanar with the central axis and tangential to the onion rings, and "radial"--perpendicular to the rings) are dependent upon position. A grid system that is orthogonal to these principal directions is desirable because the node equations describing heat and mass transfer will then be simplified. Under such a system, transfer from one node to another is independent of the potential gradient existing at right angles to a line connecting the two nodal points. The following section describes the generation of such an orthogonal grid system.

The grid system is the basis for three computer programs. The first of these, HTRAN, was written to simulate the temperature response of a biological product under the assumption of no mass transfer. The second program, HMTRAN, simulates both the heat and moisture responses using the moisture content (d.b.) as the moisture transfer driving force. The third program, MTRAN, is identical with the second except that only moisture transfer is considered.

4.2 Finite Difference Grid

In the development of the finite grid system, two computer subroutines are required. The first, PERIM (z), associates with any point (z,r) the length of the radius passing through (z,r) to the surface. The second DPERIM (z), associates with (z,r) the slope of the tangent line to the surface with respect to the axis. (See Figure 4.2.1.)

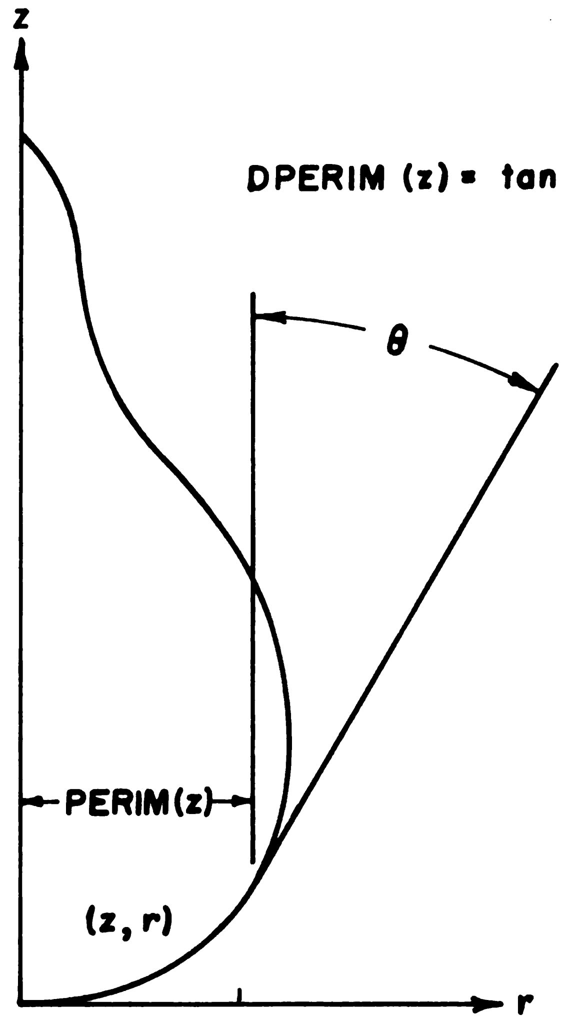

Figure 4.2.1. Relationship Between PERIM (z), DPERIM (z), and z.

Expressions for PERIM (z) and DPERIM (z) were determined for onion bulbs by fitting a ten term Fourier sine series to the onion shape. This was done by taking 35mm slides of ten individual onions. From the projected images, the length to maximum diameter ratio was found to be 3/2. The diameters corresponding to various points along the central axis of the bulbs were recorded. The ratios of the diameters to the maximum diameter of each bulb gave about 200 points outlining the shape of an average onion with unit diameter. The ten term series was fitted to these points by the least squares technique as performed by GAUSHAUS--a computer program supplied by the Michigan State University Computer Laboratory. Table 2 of the Appendix gives the resulting series. DEPERIM (z) is obtained by term by term differentiation of PERIM (z) with respect to z.

To develop the grid proper, node points were placed on the central axis. Since the base and the tip of the onion were thought to play a relatively important role in the mass transfer process, the node points were placed progressively closer together near the base and near the tip. When, for the purposes of model testing, the grid was developed for a sphere, eleven node points were placed on the central axis at points corresponding to 0.16, 1/16, 2/16, 4/16, 6/16, 8/16, 10/16, 12/16, 14/16, 15/16, and 16/16 of the diameter. When developed on the onion bulb, an additional point was placed at the length ratio of 9/16 so that the total grid would follow the bulb shape more closely (see Figures 4.2.2 and 4.2.3).

From each of the central axis node points, a line was projected outward along the principal material directions. This was done in the following manner: at each central axis node point, an incremental distance was formed as one hundredth of the distance between the two adjacent axial node points. Using this incremental distance, a grid line was projected outward from the node point adjusting itself to remain along the radial principal direction, which was always assumed at right angles to the axial principal direction. The axial principal direction was determined by assuming the tangent of the angle between the principal direction and the central axis is given by DPERIM (z) multiplied by the ratio r/PERIM (z). When the grid line so generated reached the surface of the bulb, it was sectioned to give nodes at points corresponding to 0/32, 8/32, 16/32, 20/32, 24/32, 26/32, 28/32, 30/32, 31/32, and 32/32 of the line length. The total grid is shown in Figures 4.2.2 and 4.2.3 for the two-inch sphere and the two-inch diameter onion bulb, respectively. When the grid was developed for HMTRAN and MTRAN, two additional rows of points were added near the surface at the ratios (as described above) of 61/64 and 63/64. This was done because HMTRAN and MTRAN had to be designed for cases in which the modeling time would be much less than the time constant for mass transfer.

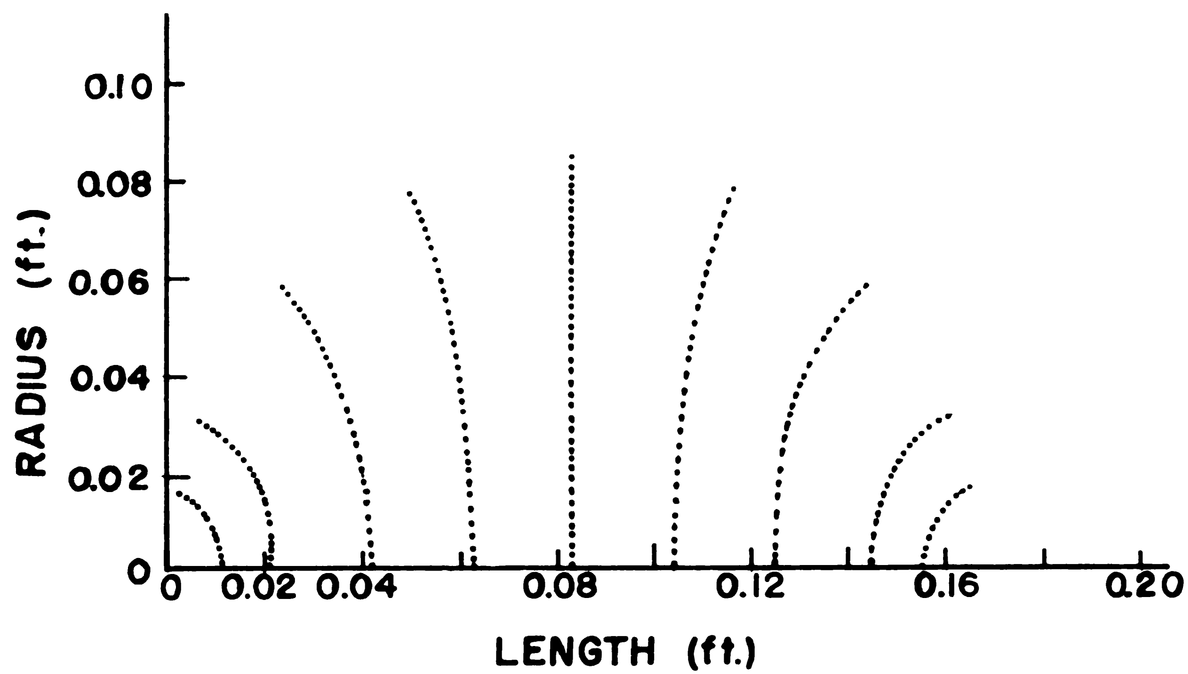

Figure 4.2.2. Node Points for a Two-Inch Diameter Sphere.

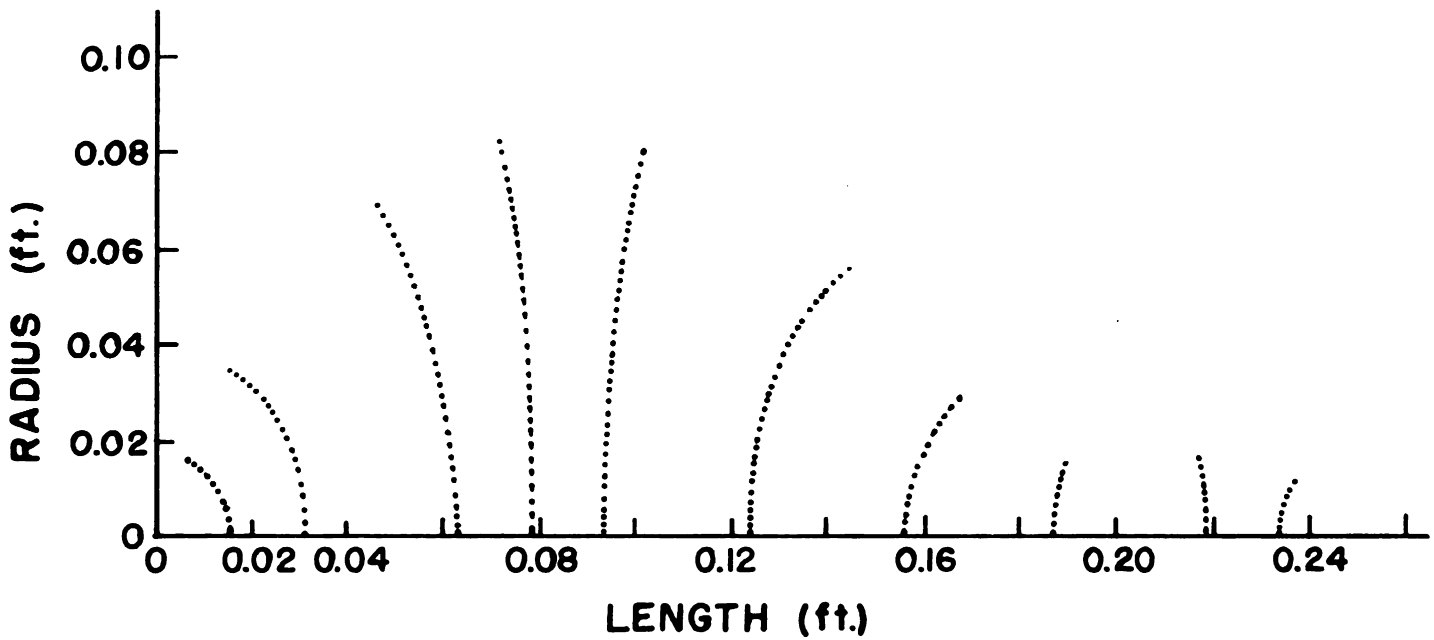

Figure 4.2.3. Node Points for a Two-Inch Diameter Onion.

After the grid was developed, the nodal volumes were determined. The volume of a general interior node (i,j) was calculated by the following equation (see Figure 4.2.4):

$$\mathrm{Vol}_{i,j} = \frac{\pi r_{i,j}}{8}\left[\frac{(B_1 + B_2)r_{i+1,j} + (A_1 + A_3)r_{i,j}}{r_{i+1,j} + r_{i,j}} + \frac{(B_5 + B_6)r_{i-1,j} + (A_1 + A_3)r_{i,j}}{r_{i-1,j} + r_{i,j}}\right]$$
$$\left[\frac{(B_7 + B_8)r_{i,j+1} + (A_2 + A_4)r_{i,j}}{r_{i,j+1} + r_{i,j}} + \frac{(B_3 + B_4)r_{i,j-1} + (A_2 + A_4)r_{i,j}}{r_{i,j-1} + r_{i,j}}\right] \qquad (4.2\text{-}1)$$

The volume of a surface node was calculated in the same manner. Referring to Figure 4.2.5,

$$\mathrm{Vol}_{i,J} = \frac{\pi r_{i,J}}{4}\left[\frac{B_2 r_{i+1,J} + A_3 r_{i,J}}{r_{i+1,J} + r_{i,J}} + \frac{B_5 r_{i-1,J} + A_3 r_{i,J}}{r_{i-1,J} + r_{i,J}}\right]$$
$$\left[\frac{B_3 r_{i,J-1} + A_2 r_{i,J}}{r_{i,J-1} + r_{i,J}} + \frac{B_4 r_{i,J+1} + A_4 r_{i,J}}{r_{i,J+1} + r_{i,J}}\right] . \qquad (4.2\text{-}2)$$

The volume of a general central axis node is given by (see Figure 4.2.6)

$$\mathrm{Vol}_{i,1} = \frac{\pi}{128}\,[A_2(3A_1 + B_1)^2 + A_4(3A_1 + B_6)^2] . \qquad (4.2\text{-}3)$$

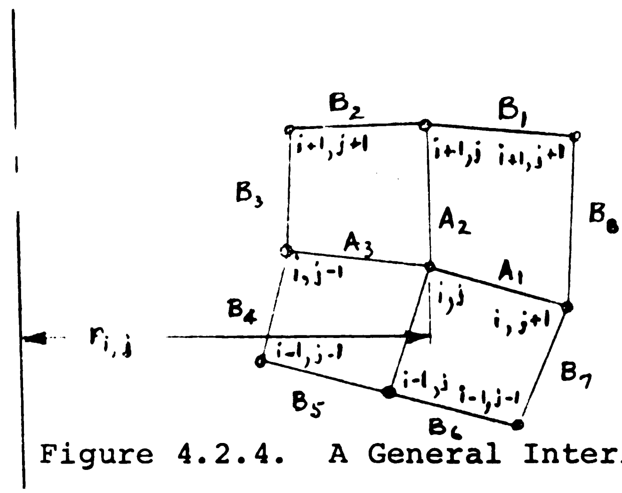

Figure 4.2.4. A General Interior Node.

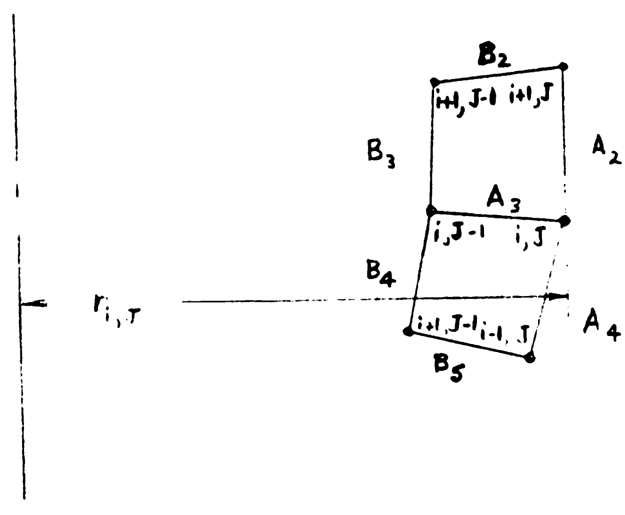

Figure 4.2.5. A General Surface Node.

The volumes of the two end nodes are simply (see Figure 4.2.7)

$$\mathrm{Vol}_{I,1} = \mathrm{Vol}_{1,1} = \frac{2\pi}{3} A_2^3 \; . \tag{4.2-4}$$

The transfer of heat or moisture from one nodal volume to another is directly proportional to what Dusinberre (1961) calls the "geometric factor." This geometric factor is defined as the effective transport area divided by the distance between the two node points.

Referring again to Figure 4.2.4 for a general interior node, the geometric factor between node (i,j) and node (i+1,j) is given by

$$\mathrm{Gfap}_{i,j} = 4\pi \frac{(B_1 + B_2) r_{i+1,j} + (A_1 + A_3) r_{i,j}}{6A_2 + B_3 + B_8} \tag{4.2-5}$$

The geometric factor between node (i,j) and (i,j+1) is given similarly as

$$\mathrm{Gfrp}_{i,j} = 4\pi \frac{(B_7 + B_8) r_{i,j+1} + (A_2 + A_4) r_{i,j}}{6A_1 + B_1 + B_6} \tag{4.2-6}$$

The geometric factor between two surface nodes (i,J) and (i+1,J) as shown in Figure 4.2.5, is

$$\mathrm{Gfap}_{i,J} = 2\pi \frac{B_2 \; r_{i+1,J} + A_3 \; r_{i,J}}{3A_2 + B_3} \tag{4.2-7}$$

The geometric factor between a surface node and the ambient surroundings is somewhat different from the above

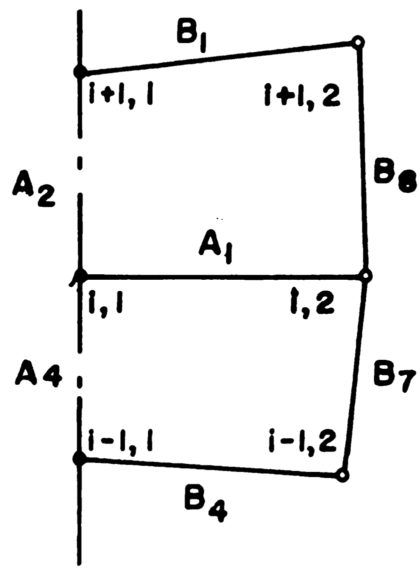

Figure 4.2.6. A General Central Axis Node.

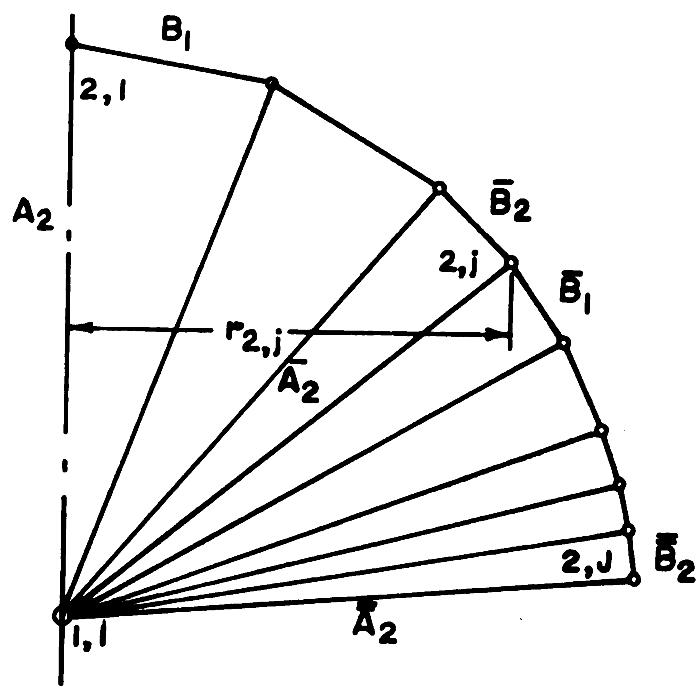

Figure 4.2.7. An End Node.

in that transfer between the two is proportional to a heat or mass transfer coefficient, instead of a conductivity or a mass diffusivity. Referring to Figure 4.2.5,

$$Gfrp_{i,J} = \pi\, r_{i,J}\, (A_2 + A_4)\ . \qquad (4.2\text{-}8)$$

With reference to Figure 4.2.6, the geometric factor between a central axis node and its axial neighbor is given by

$$Gfap_{i,1} = \frac{\pi (B_1 + A_1)^2}{4\ (3A_2 + B_8)} \qquad (4.2\text{-}9)$$

The geometric factor between a central axis node and its radial neighbor is given by

$$Gfrp_{i,1} = A_2 + A_4 + B_7 + B_8\ . \qquad (4.2\text{-}10)$$

The end nodes present a somewhat different case in that heat and moisture transfer takes place between them and all of the adjacent nodes as well as with the ambient. Thus, referring to Figure 4.2.7, the geometric factor between the end node (1,1) and the node (2,j) is given by

$$Gfap_{1,j} = \frac{2}{3}\,\frac{(\bar{B}_1 + \bar{B}_2)\ r_{2,j}}{\bar{A}_2} \qquad (4.2\text{-}11)$$

The geometric factor between (1,1) and the surface node (2,J) is

$$Gfap_{1,J} = \frac{2}{3}\,\frac{\bar{\bar{B}}_2\ r_{2,J}}{\bar{\bar{A}}_2} \ . \tag{4.2-12}$$

The geometric factor between (1,1) and (2,1) is given by

$$Gfap_{1,1} = \frac{2}{3}\,\frac{B_1^{\ 2}}{A_2} \ . \tag{4.2-13}$$

The geometric factor between (1,1) and the ambient is

$$Gfrp_{1,J} = \frac{\pi\bar{\bar{A}}_2^{\ 2}}{4} \ . \tag{4.2-14}$$

After having determined the geometric factors, the transfer parameters are assigned to each node. In HTRAN these consist of k_a, k_r, c, ρ, as well as h for the surface and end nodes. In addition to the heat transfer parameters, the initial values for D'_{ma}, D'_{mr}, and h_D have to be assigned in HMTRAN. In MTRAN, only D'_{ma}, D'_{mr}, and h_D are assigned.

This completes the development of the grid system. The next section describes the solution of the system of resulting node equations.

4.3 Solution of Node Equations

The Alternating Direction Explicit Procedure (ADEP) for the solution of the conduction equation was reviewed in Section 2.7. The present section describes how this method was utilized to solve the node equations resulting from the finite difference grid outlined in the previous section. The case of pure heat conduction is developed first, after which the modifications necessary to include moisture transfer are described.

As was mentioned in Section 2.7, the ADEP is a two pass method. However, when attempting to model a homogeneous, isotropic sphere for the purposes of model testing, it was found that, for the present grid system, the two pass system was not truly symmetrical. Instead, a four pass system was used which proved satisfactory. The first pass starts at the bottom center of the product and progresses row by row (moving outward upon each) until it reaches the top. If this progression is designated as northeast, the other three passes may be simply described by the directions southwest, southeast, and northwest.

The following paragraphs outline the equations used for the first pass. The equations used on the other passes are all similar.

For an interior node, as shown in Figure 4.2.4, the equations are the same as described in Section 2.7. Thus, for the first pass,

$$T_{i,j}^{(n+1)} = \frac{\rho_{i,j}\, c_{i,j}\, Vol_{i,j}\, T_{i,j}^{(n)}/\Delta t + k_a Gfap_{i-1,j} T_{i-1,j}^{(n+1)}}{\rho_{i,j}\, c_{i,j}\, Vol_{i,j}/\Delta t + k_a Gfap_{i-1,j} + k_r Gfrp_{i,j-1}}$$

$$+ \frac{k_r Gfrp_{i,j-1} T^{(n+1)} + k_a Gfap_{i,j} \left[T_{i+1,j}^{(n)} - T_{i,j}^{(n)} \right]}{}$$

$$+ \frac{k_r Gfrp_{i,j} \left[T_{i,j+1}^{(n)} - T_{i,j}^{(n)} \right]}{} \,. \qquad (4.3\text{-}1)$$

Heat transfer between the product surface and the ambient is proportional to the surface heat transfer coefficient. Thus, the above equations are modified for a surface node, as shown in Figure 4.2.5, to yield for the first pass,

$$T_{i,J}^{(n+1)} = \frac{\rho_{i,J}\, c_{i,J}\, Vol_{i,J}\, T_{i,J}^{(n)}/\Delta t + k_a Gfap_{i-1,J} T_{i-1,J}^{(n+1)}}{\rho_{i,J}\, c_{i,J}\, Vol_{i,J}/\Delta t + k_a Gfap_{i-1,J} + k_r Gfrp_{i,j-1}}$$

$$\frac{k_r Gfrp_{i,J-1}\, T_{i,J-1}^{(n+1)} + k_a Gfap_{i,J}\left(T_{i+1,J}^{(n)} - T_{i,J}^{(n)}\right)}{}$$

$$\frac{+\, h_i\, Gfrp_{i,J}\left(T_a - T_{i,J}^{(n)}\right)}{}\ . \qquad (4.3\text{-}2)$$

Since a central axis node, as shown in Figure 4.2.6, has only three faces, the first pass equation is

$$T_{i,1}^{(n+1)} = \frac{\rho_{i,1}\, c_{i,1}\, Vol_{i,1}\, T_{i,1}^{(n)}/\Delta t + k_a Gfap_{i-1,1} T_{i-1,1}^{(n+1)}}{\rho_{i,1}\, c_{i,1}\, Vol_{i,1}/\Delta t + k_a Gfap_{i-1,1}}$$

$$+ \frac{k_a Gfap_{i,1}\left(T_{i+1,1}^{(n)} - T_{i,1}^{(n)}\right)}{}$$

$$+ \frac{k_r Gfrp_{i,1}\left(T_{i,2}^{(n)} - T_{i,1}^{(n)}\right)}{}\ . \qquad (4.3\text{-}3)$$

The end nodes are unique in that each has more than four adjacent nodes as shown in Figure 4.2.7. The first pass equation for the node (1,1) is

$$T_{1,1}^{(n+1)} = \frac{\rho_{1,1}\, c_{1,1}\, Vol_{1,1}\, T_{1,1}^{(n)}/\Delta t + \sum_{j=1}^{J} k_a Gfap_{1,j}}{\rho_{1,1}\, c_{1,1}\, Vol_{1,1}/\Delta t + h_1\, Gfrp_{1,J}}$$

$$\underline{T_{2,j}^{(n)} - T_{1,j}^{(n)} \quad + h_1\, Gfrp_{1,J}\, T_a} \; . \tag{4.3-4}$$

The heat transfer model was tested by comparing the analytical and model temperature histories for the center of a two-inch diameter sphere initially at uniform temperature and subjected to a step change in ambient temperature. The surface heat transfer coefficient was set at 500 Btu/hr ft^2F, the initial temperature was uniform at 40 F, the ambient temperature was 100 F, the density was 58.8 lb_m/ft^3, the specific heat was 0.93 Btu/ lb_mF, and the axial and radial thermal conductivities were 0.3 Btu/hr ft F. The results, as shown in Figure 4.3.1, indicate that the model agrees reasonably well with the analytical solution which was calculated by the following equation (Grigull, 1964):

$$T = 100 - 60 \sum_{m=1}^{B} \frac{2 \sin v_n - v_n \cos v_n}{v_n - \sin v_n \cos v_n} \exp\left(\frac{-v_n^2\, kt}{\rho c\, r^2}\right) . \tag{4.3-5}$$

The eigenvalues v_n in the above equation are the solutions to the following transcendental equation:

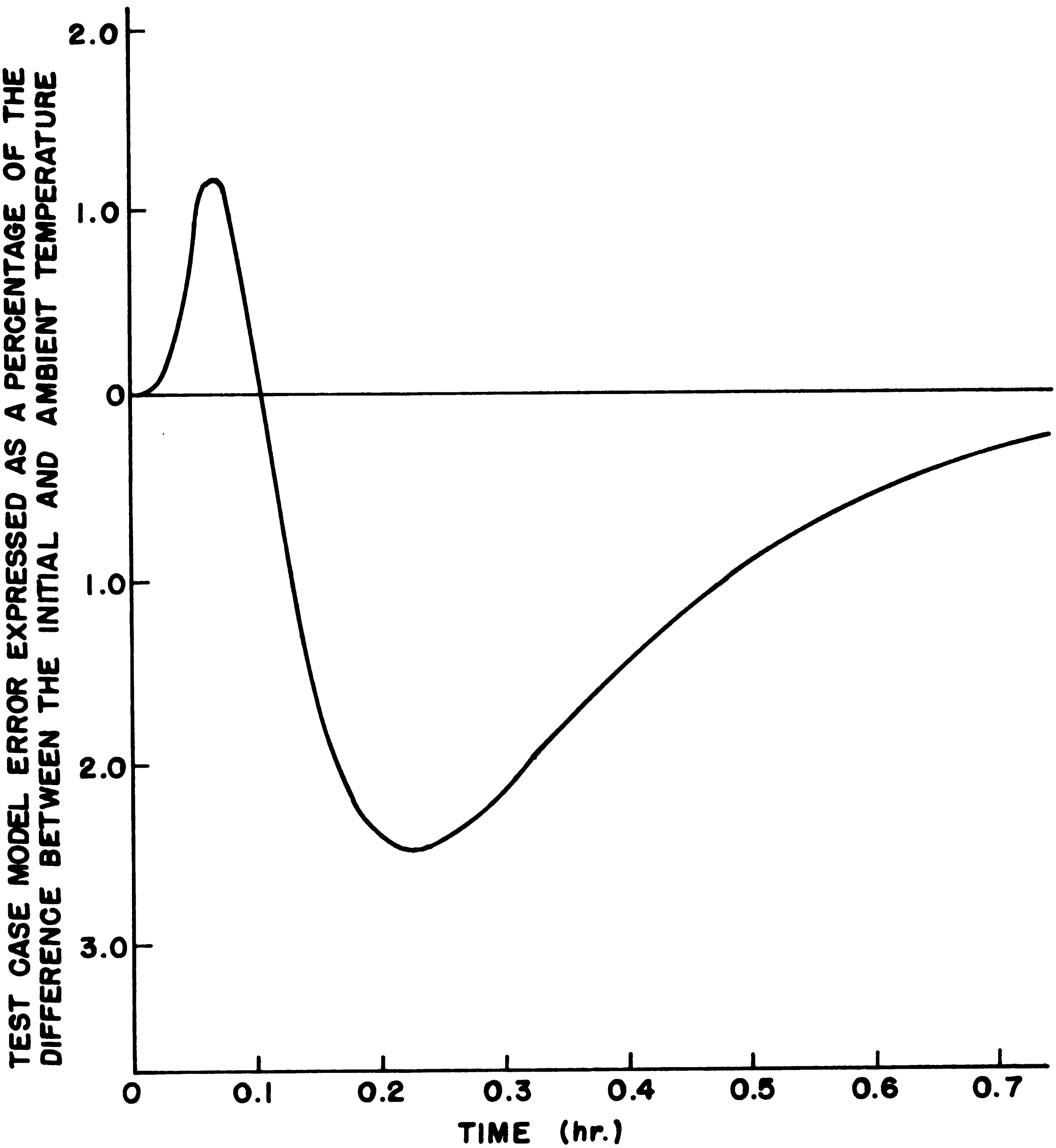

Figure 4.3.1. Test Case Model Error Using the ADEP on a Two-Inch Diameter Sphere.

$$v_n \cos v_n = (1 - \frac{hr}{k}) \sin v_n \; . \qquad (4.3\text{-}6$$

Only eight terms of the infinite series were used in determining the analytical solutions. The analytical results for times less than 0.04 hours were discarded due to the nonconvergence with only eight terms.

The author tried various modifications of the program in an attempt to minimize the error shown in Figure 4.3.1. As shown in the Appendix, HTRAN was written so that only a few statements would have to be changed to convert the ADEP to that of Barakat and Clark (see Section 2.6). When this was tried, however, it was found that the error was not affected. Since the Barakat-Clark scheme requires in this case four times more storage and four times more computing time, the Barakat-Clark modification was abandoned.

The second modification tried was to further subdivide the region by adding more nodes in the radial direction. This modification reduced the positive peak shown in Figure 4.3.1, but did not affect the negative one.

The third modification tried was to vary the axial spacing of the node points to one in which the incremental distances in z were more uniform. This did not significantly affect the error.

The fourth modification tried was to further subdivide the region by adding more nodes in the axial direction. This increased slightly the magnitude of both the positive and negative peaks shown in Figure 4.3.1.

The fifth modification tried was to reduce the time increment by a factor of 100. This did not affect the magnitude of the error.

The sixth modification tried was to replace the geometric factors based on the average area between the nodes by factors taking into account the fact that the area between nodes is not constant. For example, if this variation is assumed linear, the axial geometric factor between two general interior nodes becomes (see Figure 4.4.2)

$$\mathrm{Gfap}_{i,j} = \frac{\pi(B_1 + B_2)r_{i+1,j} - (A_1 + A_3)r_{i,j}}{A_2 \ln\left[\dfrac{(B_1 + B_2)r_{i+1,j}}{(A_1 + A_3)r_{i,j}}\right]} \tag{4.3-7}$$

If the variation is assumed quadratic, the radial geometric factor between two general interior nodes becomes (see Figure 4.2.4)

$$\mathrm{Gfrp}_{i,j} = \frac{\pi\sqrt{r_{i,j}(A_2 + A_4)\left((B_7 + B_8)r_{i,j+1} - (A_2 + A_4)r_{i,j}\right)}}{A_1 \tan^{-1}\sqrt{\dfrac{(B_7 + B_8 r_{i,j+1}}{(A_2 + A_4)r_{i,j}} - 1}}$$

(4.3-8)

When these equations were used in the model, the positive peak shown in Figure 4.3.1 remained the same but the magnitude of the negative peak was increased.

Barakat and Clark (1966) gave the following equation for the truncation error for a uniform grid in a cartesian coordinate system:

$$\xi_{i,j} = -\frac{\Delta t}{2}\frac{\partial T}{\partial t}\bigg|_{i,j,n+1} - \frac{\Delta t}{\Delta x}\frac{\partial^2 T}{\partial t \partial x}\bigg|_{i,j,n+1}$$

$$-\frac{\Delta t}{\Delta y}\frac{\partial^2 T}{\partial t \partial y}\bigg|_{i,j,n+1} - \frac{(\Delta x)^2}{12}\frac{\partial^4 T}{\partial x^4}\,{}_{i,j,n+1}$$

$$-\frac{(\Delta y)^2}{12}\frac{\partial^4 T}{\partial y^4}\bigg|_{i,j,n+1} + \ldots \quad (4.3\text{-}9)$$

The failure of the modifications to improve the model error appears to contradict equation (4.3-9). It would thus seem that a different expression for the truncation error would be needed for this grid system. The author, however, was unable to derive such an equation due to the complexities of the grid system.

As shown by the listings of HTRAN, HMTRAN, and MTRAN in the Appendix, the modifications necessary to model mass transfer are minor: first, since the diffusion coefficient D_m' is a function of the moisture content, it was re-evaluated for each node at the start of each group of four passes; and second, to prevent instability at the boundary, the mass flux at the surface was always evaluated at the time n+1 for each pass. This was done by approximating the boundary layer vapor concentration at time n+1 by

$$C_{wvs}^{(n+1)} = \frac{m_{i,j}^{(n+1)} \; C_{wvs}^{(n)}}{m_{i,j}^{(n)}} \; . \tag{4.3-10}$$

The boundary layer vapor concentration $C_{wvs}^{(n)}$ was evaluated as a function of $m_{i,j}^{(n)}$ by means of the straight lines approximation shown in Figure 7.2.7.

V. DENSITY

A very simple experiment was performed to determine the density of onion flesh. Eleven bulbs were weighed individually in air. A metal sinker was then sewed to each and each weighed in water. The temperature of the bulbs and of the water was 70 F. The moisture content of the bulbs was 9.00 (d.b.).

Table 1 of the Appendix shows the results. The statistical data shown was calculated with the aid of BASTAT, a basic statistical computer routine supplied by the Michigan State University Agricultural Experiment Station. As shown in the table, the average specific gravity was 0.944. This corresponds to a density of 58.8 lb/ft^3. The moisture content of the bulbs was 9.00 (d.b.). The assumption that shrinkage upon drying is negligible yields

$$\rho = \rho_{dm} (1 + m) \tag{5-1}$$

and

$$\rho_{dm} = 5.88 \ lb_m/ft^3. \tag{5-2}$$

Equations (5-1) and (5-2) were used in all calculations involving the density in this research.

VI. HEAT TRANSFER

6.1 Introduction

The thermal conductivity of wood is considerably greater parallel to the grain (0.23 Btu/hr ft F) than perpendicular to the grain (0.12 Btu/hr ft F) (Rohsenow and Choi, 1961). It was felt that this anisotropy might also be present in onion flesh. Thus, experiments were performed to determine the conductivity in both the "axial" and "radial" directions (see Section 4.1).

The mathematical model for heat and mass transfer has been described in Chapter IV. In most biological products, the time scale for heat transfer phenomena is much smaller than that for moisture transfer phenomena. This allows separation of the two for modeling purposes (Young, 1968).

When conducting an experiment to determine a property such as thermal conductivity, the experiment should be planned so that the measured quantity, such as temperature, is highly dependent upon the desired property. The "sensitivity coefficients," defined in this case by the ratio of the change in temperature at a particular location and time to an assumed change in thermal conductivity, give an indication of the accuracy obtainable in a particular experiment. More than one property may be obtained from a single experiment if the respective sensitivity coefficients are not closely correlated to each other and are of sufficient magnitude. The sensitivity coefficients $\Delta T/\Delta k_r$ and $\Delta T/\Delta k_a$ for the center (the point on the central axis at which the onion diameter is a maximum) of a two-inch diameter onion are shown in Figure 6.1.1. To determine these sensitivity coefficients, both k_r and k_a were assumed to be 0.3 Btu/hr ft F while the initial temperature was taken as 40 F with the boundary condition consisting of a step change in ambient temperature to 100 F with a heat transfer coefficient of 500 Btu/hr ft F. The changes in the center temperature versus time due to small (1%) changes in the assumed values of thermal conductivity were obtained by solving the model three times--once at the base values, once with an increased k_r, and once with an increased k_a. The sensitivity coefficients were calculated by dividing the resulting changes in center temperature by the causal change in thermal conductivity. As the figure shows, the center temperature is much more sensitive to k_r than to k_a. Thus, placing an onion initially at 40 F in a constant temperature bath at 100 F could be expected to give k_r (as described in Section 6.3) but not both k_r and k_a. An alternative method (described in the next section) was used to find k_a.

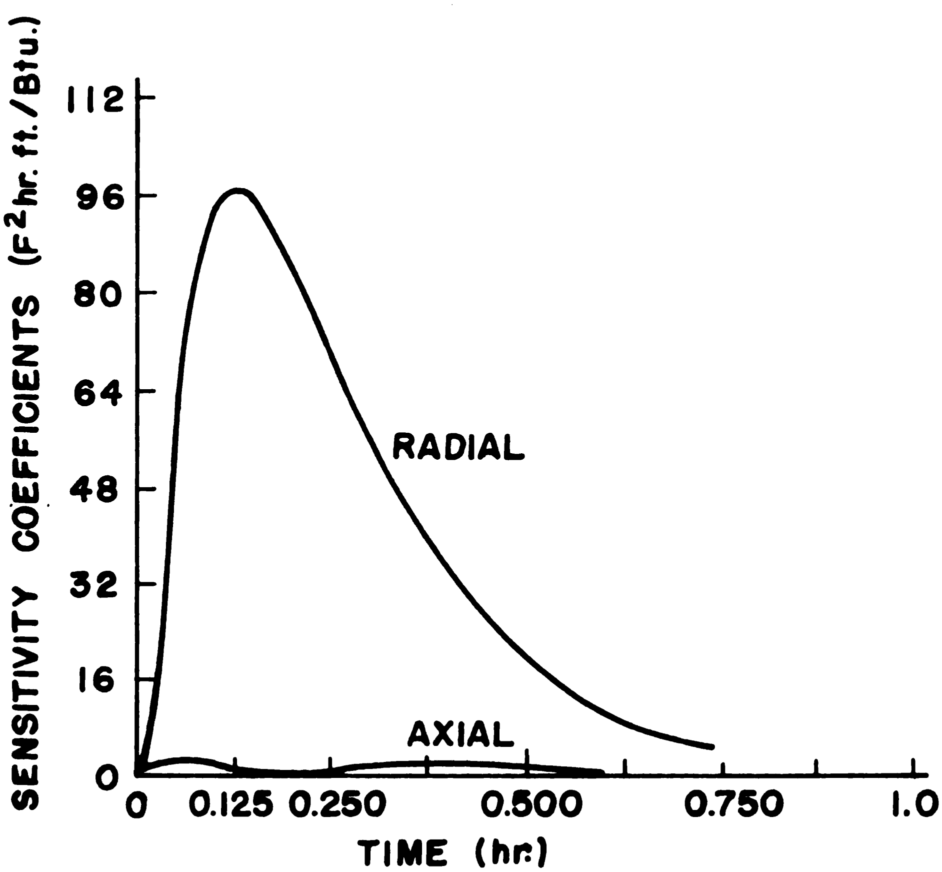

Figure 6.1.1. Temperature Sensitivity Coefficients at the Center of a Two-Inch Diameter Onion.

6.2 Axial Conductivity

As outlined in the previous section, the temperature history of an onion, when subjected to a step change in ambient temperature with a uniform heat transfer coefficient existing over its surface, is not highly dependent upon the axial conductivity of the onion flesh k_a. However, since the axial conductivity is a parameter in the model, and since it was desired to be able to model the case in which the surface heat transfer coefficient is a function of z (see Figure 4.2.1), determination of k_a was included among the objectives of the research.

In order to determine the axial conductivity, the sample holder shown in Figure 6.2.1 was constructed. The holder consists of two metal cups lined with insulation. A heat source consisting of a copper block one-inch in diameter by 0.66 inches long was placed in a cavity in the top cut. Using a cork borer, a one-inch diameter by one-inch long sample was cut from the center of an onion and placed in the bottom cup. After cooling the bottom cup and specimen to 40 F and heating the top cup and block to 100 F, the cups were brought together and the temperature histories of the block and sample were taken by means of three thermocouples attached to the block and five thermocouples imbedded in the sample. An existing system developed by Dr. J. V. Beck (Mechanical Engineering Department) was used to collect and record the data. The thermocouple voltages were amplified and sent to an IBM 1800

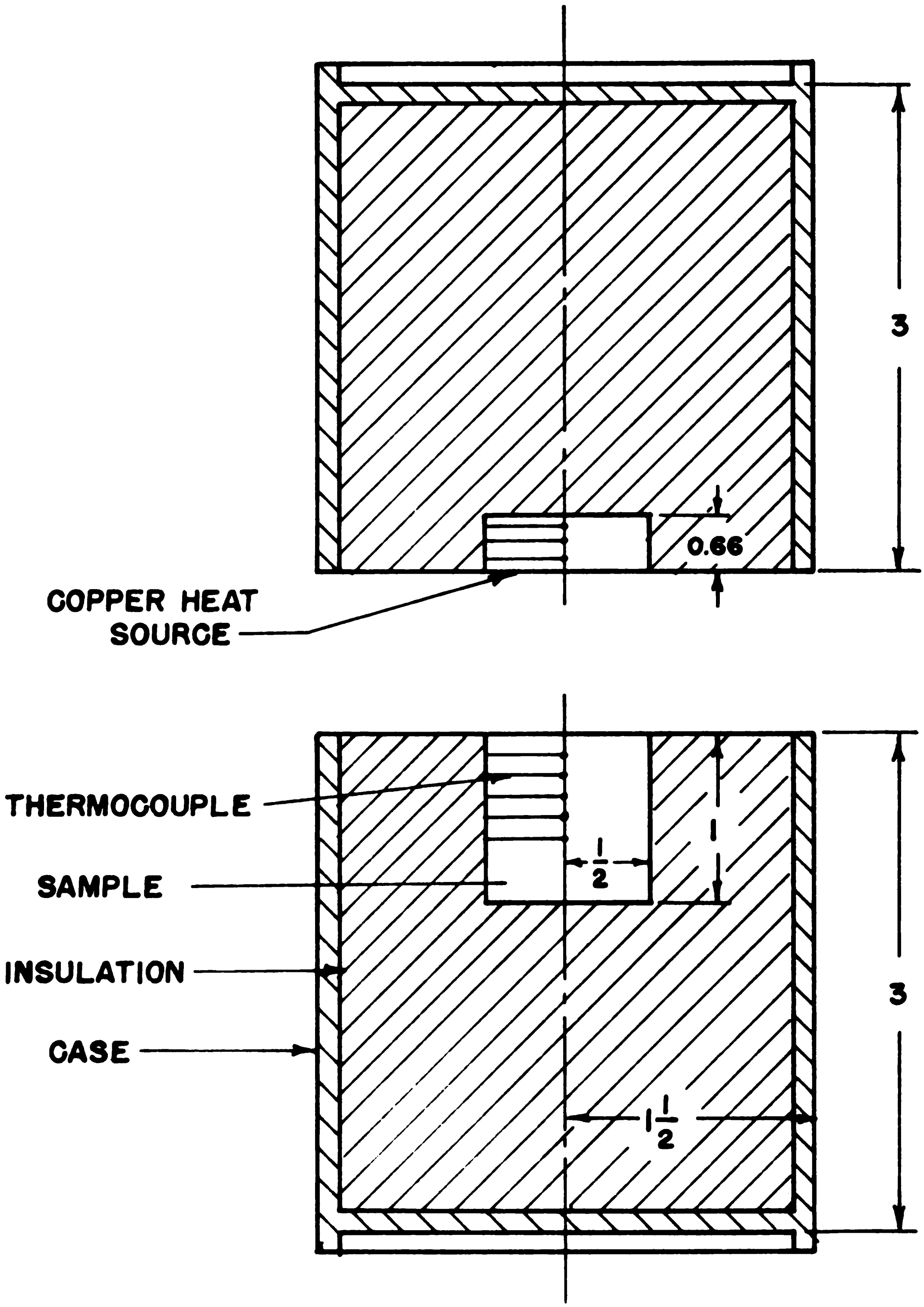

Figure 6.2.1. Cross Sectional View of the Sample Holder Used in the Axial Conductivity Tests.

hybrid computer which converted the voltages to temperatures, and punched the temperature-time histories onto cards. Each experimental run lasted 7.5 minutes during which 150 readings were taken at each thermocouple.

Two existing nonlinear estimation computer programs developed by Beck (1964, 1966, 1968) were used to analyze the data. The first used the temperature history of the copper block to estimate the heat flux from the block to the sample. The heat flux was assumed constant during each of the time intervals between the thermocouple readings. These discrete heat fluxes were adjusted by the program so that the sum of the square differences between the experimental block temperatures and the temperatures as generated by a one dimensional Crank-Nicolson finite difference model of the block, which included the discretized heat fluxes, was a minimum.

The second program used the temperature history of the sample and the heat fluxes determined by the first program to find the thermal conductivity of the sample. As in the first program, the sum of the squared differences between the experimental temperatures and those generated by a one dimensional Crank-Nicolson model of the sample was minimized--in this case by adjusting the thermal conductivity.

The results of five experimental runs are shown in Table 3 of the Appendix. The second column in the individual results section of the table shows the root mean square (RMS) difference between the calculated and the experimental sample temperatures for each run. From this column, it can be seen that the numerical model fits some runs better than others. It is believed that the higher RMS values were caused by inaccuracies in the measurement of the thermocouple position within the sample. Thus, it would seem justifiable to weight the results of each run by the inverse of the RMS value to obtain a better estimate of the axial conductivity. This weighted average is given in the results section of Table 3 as 0.30 Btu/hr ft F and was used in the calculations involving k_a in this research.

6.3 Radial Conductivity

As shown in Section 6.1, the temperature history at the center of an onion subjected to a step change in ambient temperature is highly dependent upon the radial thermal conductivity k_r. This is the basis for the experiments performed to determine k_r.

Thermocouples were placed at the center (the point on the central axis corresponding to the maximum diameter) of each of five onions. After cooling to a uniform temperature of 40 F, the onions were placed in an agitated water bath at 100 F. The thermocouple voltages were amplified and sent to the IBM 1800 hybrid computer which punched the temperature-time histories onto cards. Readings were taken every ten seconds over an interval of twenty-five minutes.

The convective heat transfer coefficient h was found to be 500 Btu/hr ft^2 F from the temperature history of a copper block placed in the bath. While the geometry of the onion is different from that of the block, it was felt that this manner of determining h was sufficiently accurate since the Biot number of a two-inch diameter sphere with a thermal conductivity of 0.3 Btu/hr ft F and a surface heat transfer coefficient of 500 Btu/hr ft F is 139. This would indicate that the surface heat transfer coefficient is sufficiently large so that the temperature response of the onion is governed by the transfer properties of the onion and not by the surface heat transfer coefficient.

To determine k_r, a non-linear estimation program, GAUSHAUS, as supplied by the Michigan State University Computer Laboratory, was used in conjunction with the heat transfer model described in Chapter IV to find a k_r that minimized the sum of the squared differences between the experimental and calculated temperatures for the center node of the onion. The results are shown in Table 4 of the Appendix. As in the case of axial conductivity, the weighted (by 1/RMS temperature difference between the calculated and experimental temperatures) mean is included (0.217 Btu/hr ft F).

6.4 Heat Transfer Simulation

The heat transfer model was solved for a set of convective heat transfer coefficients ranging from 0.5-500 Btu/hr ft^2 F and a set of bulb diameters ranging from 1-3 inches. The results are shown in Figure 6.4.1. The ratio of the heat transferred to that transferred at infinite time is plotted versus the dimensionless grouping $h^2t/\rho ck_r$ at various values of hd/k_r.

From this chart, it is possible to determine the time necessary to raise the average bulb temperature to any desired level if the initial temperature, the bulb diameter, the ambient temperature, and the convective heat transfer coefficient are known.

If Figure 6.4.1 is compared to the corresponding figure for spheres (Holman, 1963), it is seen that the two agree within the accuracy that these charts may be read. Thus, for process evaluation, such figures may be substituted for Figure 6.4.1.

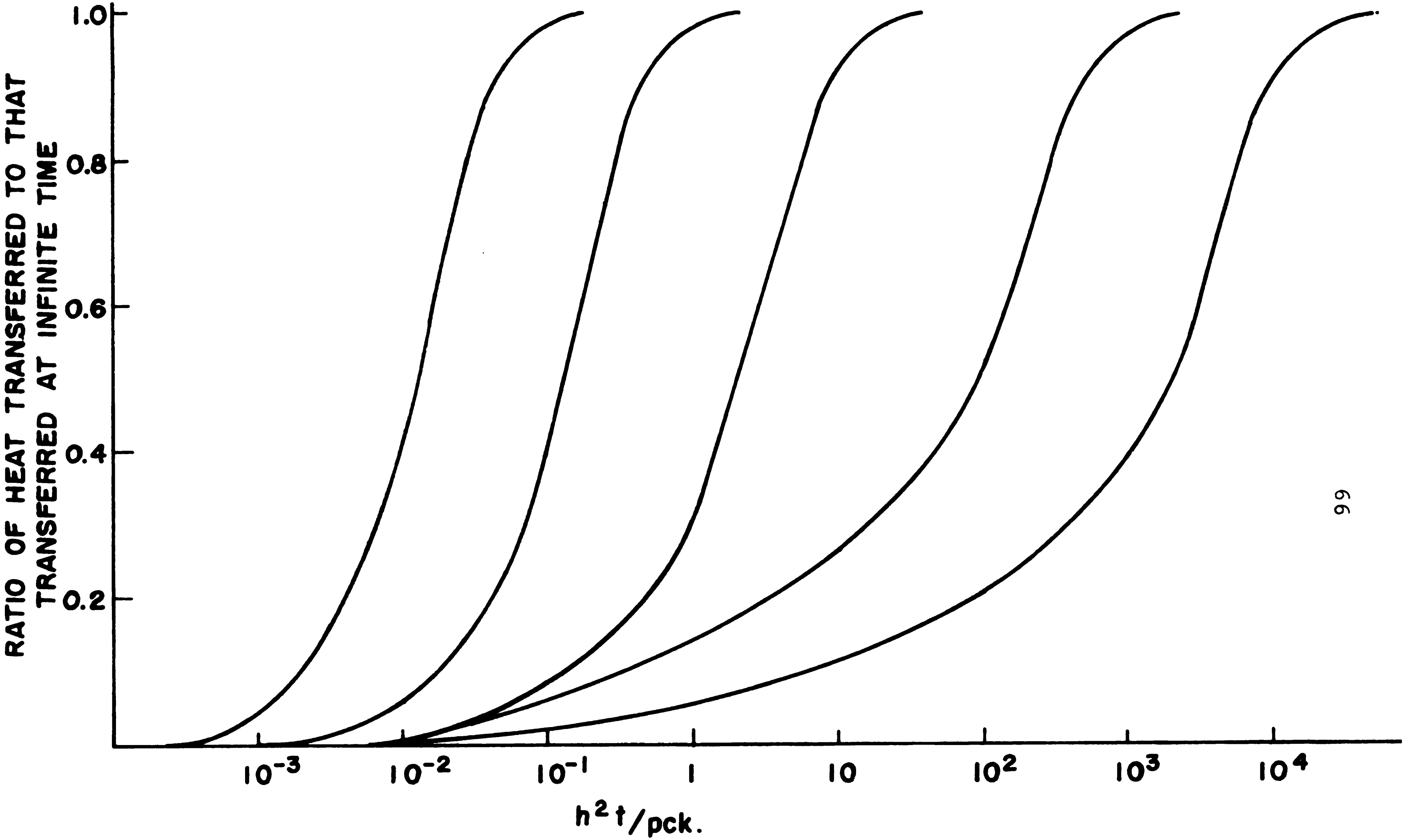

Figure 6.4.1. Heat Transfer to Onion Bulbs Subjected to a Step Change in Ambient Temperature.

VII. MOISTURE TRANSFER

7.1 Introduction

In Section 2.5 it was shown that D'_μ and D'_m are related by the equation

$$D'_m = D'_\mu \frac{\partial \ln a_w}{\partial \ln m} . \tag{7.1-1}$$

This means that the two equations modeling one dimensional moisture transfer

$$j_w^s = \frac{C_w M_w}{RT} D'_\mu \frac{\partial \mu_w}{\partial z} \tag{7.1-2}$$

and

$$j_w^s = - \rho_{dm} D'_m \frac{\partial m}{\partial z} \tag{7.1-3}$$

are equivalent if D'_μ and D'_m satisfy equation (7.1-1).

Section 7.2 of this chapter describes the experiments conducted to determine D'_μ and D'_m in onions. Section 7.3 describes the results of including D'_m in the mathematical model outlined in Chapter IV.

7.2 Determination of Moisture Diffusivity

A modified Denny osmometer as described by Dumbroff and Webb (1968) (Figure 7.2.1) was constructed to determine the moisture diffusivity of onion flesh. The osmometer consists basically of a distilled water chamber connected to a sample holder. The rate of water loss from the chamber can be monitored by observing the movement of the water meniscus in a capillary tube attached to the chamber. The unit was originally designed to be used with another chamber on the other side of the sample in which was placed a stirred sugar solution. Since sugar solutions become quite viscous at water activities below 0.95 and saturated near 0.9, this chamber was removed for water activities below 0.95. The entire osmometer was placed in an air stream with closely controlled temperature and relative humidity produced by an "Aminco-Aire"* unit. Close control of temperature was necessary since the position of the meniscus within the capillary tube is very temperature sensitive.

It was desired to model D'_{μ} over the respective ranges of water activity and temperature of 0.4 to 1.0 and 40 to 100 F. Inspection of equations (2.5-4), (2.5-8) and (2.5-12) shows that if the velocity of the medium with respect to the solids v^{ms} is small, Ω_s is small, and

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*A device producing a stream of constant temperature and humidity air manufactured by the American Instrument Co., Inc., Silver Spring, Maryland.

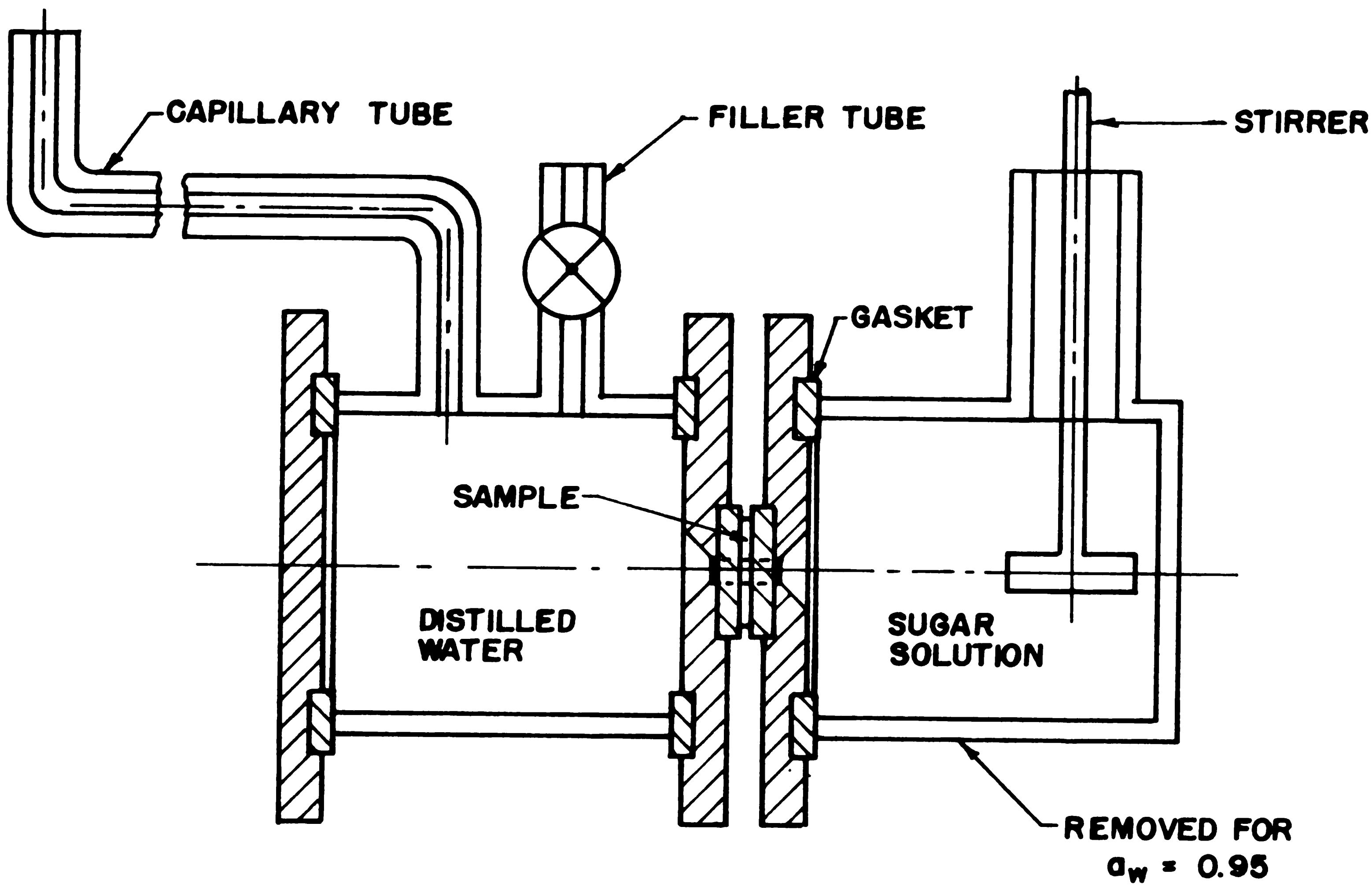

Figure 7.2.1. Cross Sectional View of the Denny Osmometer.

$$D'_{\mu} = R\ T\ \Omega_w \ . \tag{7.2-1}$$

This is the rationale behind attempting to approximate $D'_{\mu r}$ by

$$D'_{\mu r} = RT \left(\Omega_1\ m^{\Omega_2} + \Omega_3(T - 530)\right) \ . \tag{7.2-2}$$

Table 5 of the Appendix summarizes the results of the experiments conducted to determine Ω_1, Ω_2 and Ω_3 of equation (7.2-2). By equation (7.1-2),

$$j_w^s = - \frac{\rho dm\ m}{RT} D'_{\mu} \frac{\partial \mu_w}{\partial z} \ , \tag{7.2-3}$$

and

$$j_w^s \Delta L = \int_{\bar{a}_w}^{1} \frac{\rho_{dm}\ m\ RT\left(\Omega_1 m^{\Omega_2} + \Omega_3(T-530)\right) da_w}{a_w} \tag{7.2-4}$$

where $\bar{a}_w$ is the water activity of the fluid to the right of the sample (see Figure 7.2.1). Evaluation of this integral requires that m be expressed as an explicit function of a_w.

Saravacos (1960) determined the moisture isotherm for onion flesh at 80 F. His data are summarized in Table 6 of the Appendix and in Figure 7.2.2. In order to express m explicitly in terms of a_w, the isotherm was approximated as shown in Figure 7.2.2 by three connecting straight line segments passing through the points (0,0), (0.022, 0.22), (0.530, 0.863), and (9.00, 1.0). The first and last of these are not from Saravacos' data--the first

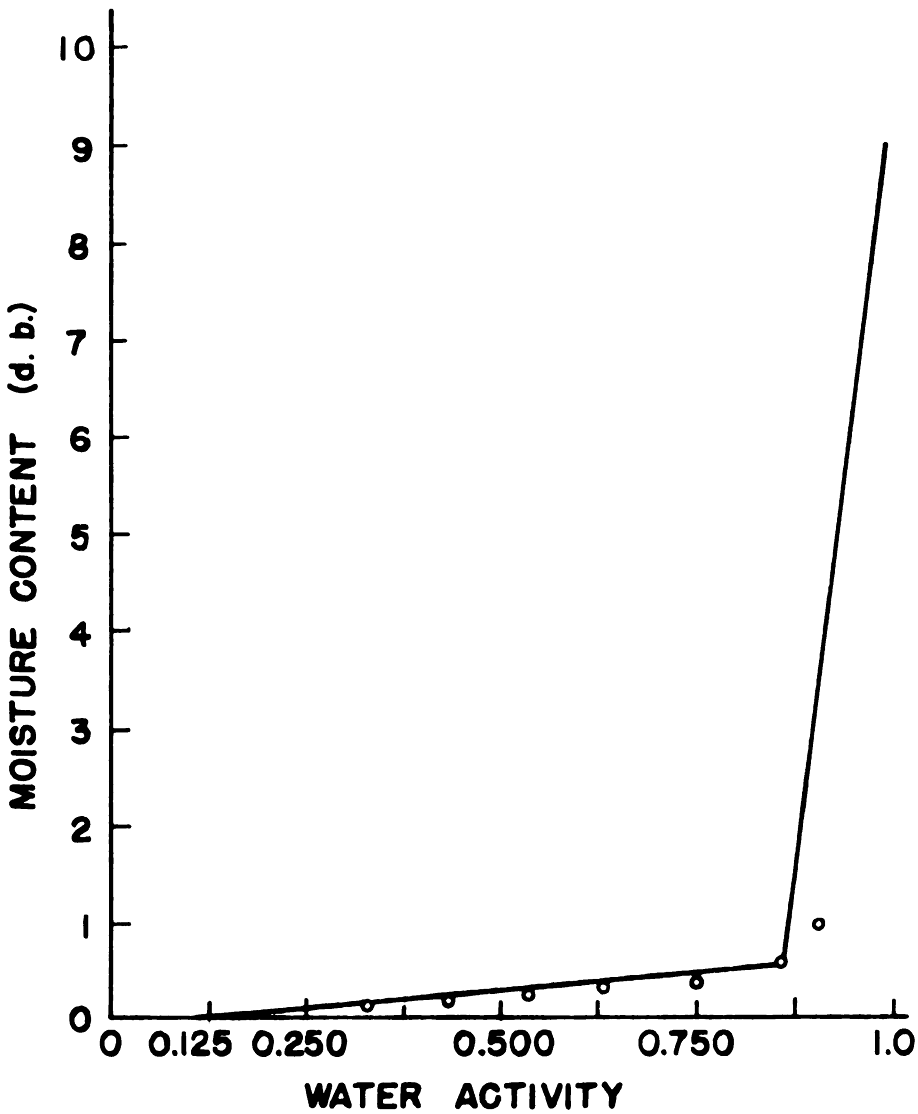

Figure 7.2.2. Onion Moisture Isotherm.

was included so that the straight lines approximation to the isotherm would pass through the origin; the second, so that the approximation would pass through the initial onion bulb moisture content at saturation. Thus, the simplified relationship between m and a_w is given by

$$m = .1\ a_w \text{ for } 0 \leq a_w \leq 0.22\ , \tag{7.2-5}$$

$$m = 0.15181026 + .79004666\ a_w \text{ for } 0.22 < a_w \leq 0.863\ , \tag{7.2-6}$$

and

$$m = -52.824817 + 61.8248175\ a_w \text{ for } 0.863 < a_w \leq 1.0\ . \tag{7.2-7}$$

The non-linear estimation routine, GAUSHAUS, was used again (see Section 6.3) to vary the parameters Ω_1, Ω_2, and Ω_3 so that the sum of the squared differences between the experimental mass fluxes and those predicted by equation (7.2-4) was a minimum. The integration of equation (7.2-4) was performed numerically by a Romberg integration routine. The resulting values are shown in the following equation:

$$D'_{\mu r} = RT\left(4.5535 \times 10^{-18}\ m^{6.023} + 1.779 \times 10^{-14}(T-530)\right). \tag{7.2-8}$$

Inspection of equation (7.2-9), which is shown in graphical form in Figure 7.2.3, reveals that negative values are predicted for $D'_{\mu r}$ when both the temperature and moisture content are low. This was prevented when using equation (7.2-8) in the mass transfer model by setting D'_m equal to zero whenever this occurred.

In Section 7.1, it was shown that D'_m is obtained by multiplying D'_μ by $\partial \ln a_w/\partial \ln m$. Differentiation of equations (7.2-5), (7.2-6), and (7.2-7) yields

$$\frac{\partial \ln a_w}{\partial \ln m} = 1 \text{ for } 0 \leq m \leq 0.022 \text{ ,} \tag{7.2-9}$$

$$\frac{\partial \ln a_w}{\partial \ln m} = \frac{m}{m + 0.15181026} \text{ for } 0.022 < m \leq 0.0.530, \tag{7.2-10}$$

and

$$\frac{\partial \ln a_w}{\partial \ln m} = \frac{m}{m + 52.824817} \text{ for } 0.530 < m \leq 1.0 \text{ .} \tag{7.2-11}$$

In Section 6.1, it was shown that the temperature history of the onion bulb is much more dependent upon k_r than k_a. Likewise, the mass transfer within the product is much more dependent upon D'_{mr} than upon D'_{ma}. With this in mind, it was decided to model D'_{ma} by simply assuming it to be a constant multiple of D'_{mr}. To determine the ratio between D'_{ma} and D'_{mr}, five experimental runs with 0.22-inch-thick slices cut from the center portion of the bulb were performed under the same conditions as run 6

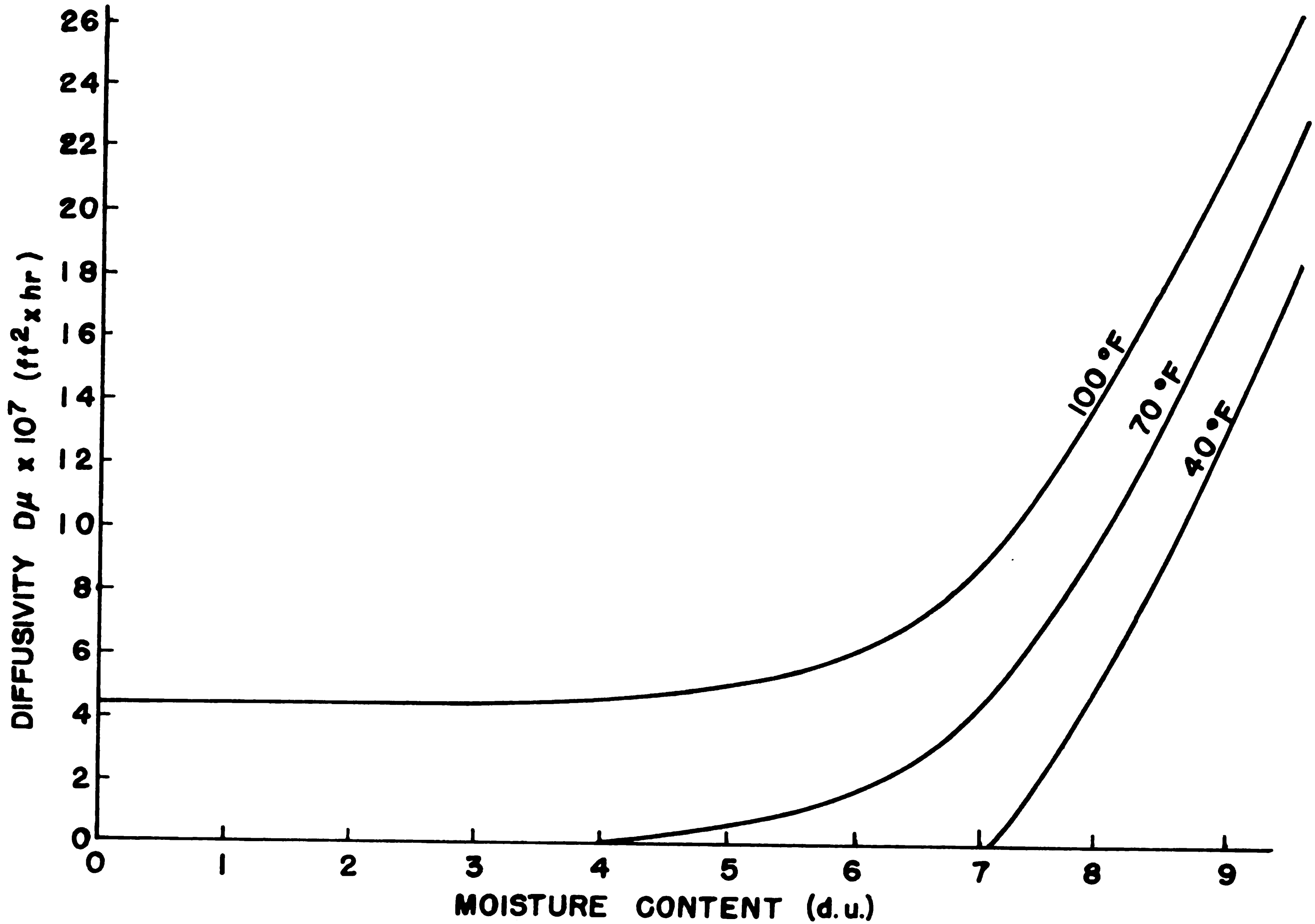

Figure 7.2.3. Predicted Moisture Diffusivity $D'_{\mu r}$.

of Table 5 of the Appendix. The average mass flux was 1.80×10^{-2} lb_m/ft^2hr. Comparing this result with 4.35×10^{-3} lb_m/ft^2hr, the mass flux obtained from equation (7.2-4) under the same conditions yields

$$D'_{\mu a} = 4.14 \times 10^2 \; D'_{\mu r} \; . \qquad (7.2\text{-}12)$$

Before concluding this section, the validity of a tacit assumption made earlier should be established. The above method rests upon the assumption that the resistance to mass transfer through the specimen lies wholly within the sample proper and the surface resistance to the right of the sample is negligible. (See Figure 7.2.1.) (The surface resistance to the left of the sample is obviously zero since the fluid is pure water.) Reynolds analogy states that the convective heat transfer coefficient and the convective mass transfer coefficient are related by

$$h_D = \frac{h}{\rho_{air} \; c_{air} \; L_e^{2/3}} \qquad (7.2\text{-}13)$$

where ρ_{air} is the air density, c_{air} the specific heat, and Le the ratio of the thermal diffusivity of air to the mutual diffusion coefficient of the air-water vapor mixture. If h is assumed to be 5 Btu/hr ft^2 F; ρ_{air}, 0.075 lb_m/ft^3; c_{air}, 0.24 Btu/lb_m F and Le, 0.845, then h_D is 312 ft/hr. Since

$$j_w^s = \frac{h_D M_w}{RT} (P_{wvs} - P_{wva}) , \qquad (7.2\text{-}14)$$

the maximum water vapor pressure difference from the surface to the ambient (using the mass flux of 1.4×10^{-3} lb_m/ft^2hr from run 4 of Table 5 of the Appendix) is 1.425×10^{-3} psi. Since the minimum water pressure drop from the pure water chamber to the ambient is 0.036 psi (run 5), the drop in pressure from the surface to the ambient is negligible.

7.3 Mass Transfer Model Testing

To obtain data with which to test the mathematical model outlined in the previous sections and Chapter IV, five two-inch diameter onion bulbs were placed in the air stream of an "Aminco-Aire" unit for twenty-four days. The temperature was maintained at 70 F and the relative humidity at 31%. One of the onions sprouted after a week; the average weight loss from the other four versus time is shown in Figure 7.3.1.

Predicted moisture losses for the same conditions were obtained by solution of the mathematical model and are included in Figure 7.3.1. The convective mass transfer coefficient h_D was set to 300 ft/hr. The initial moisture content was assumed to be 9.00.

As Figure 7.3.1 shows, there is only fair agreement between the predicted and experimental values. The difference between the two is believed due to three factors:

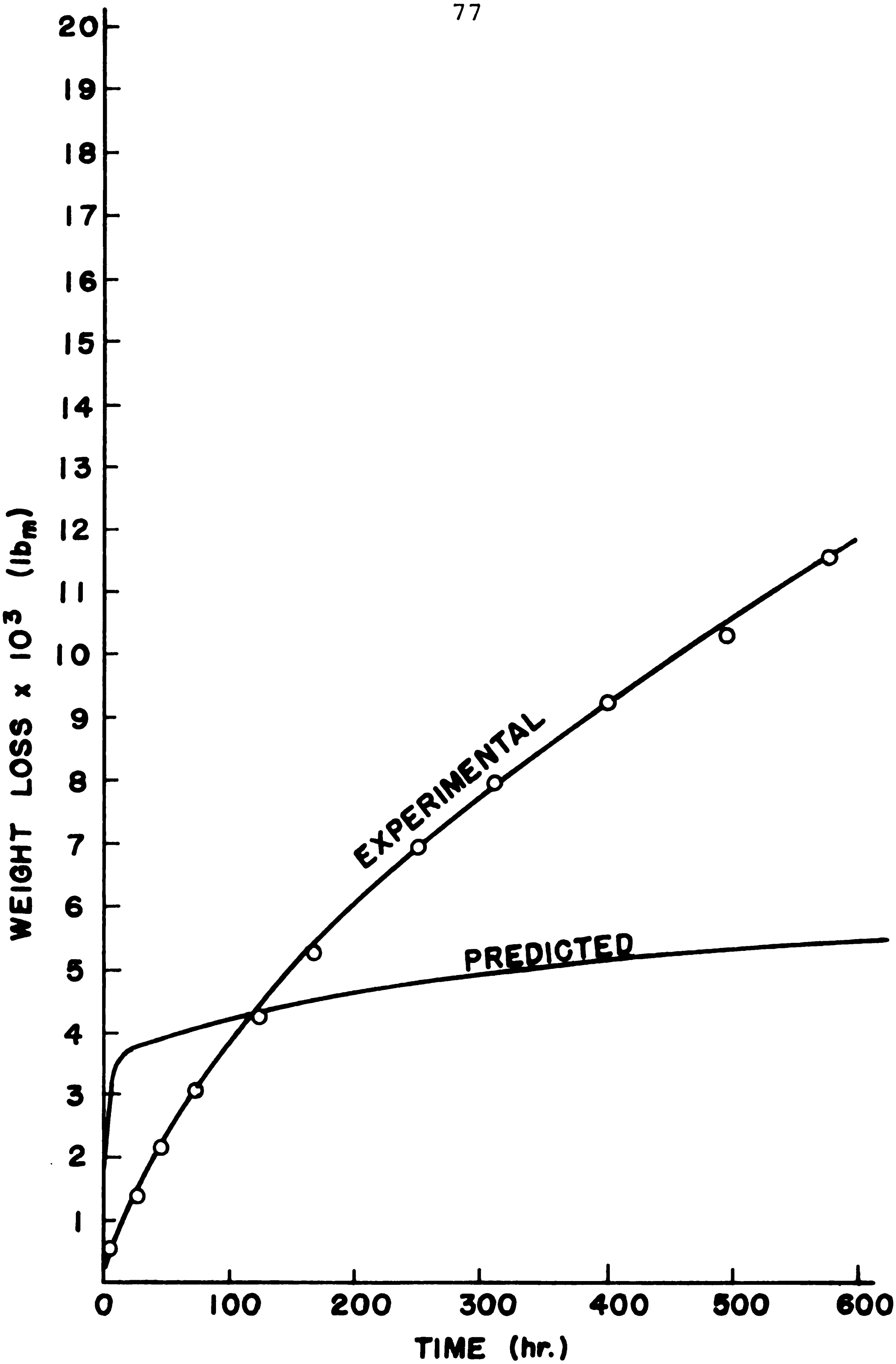

Figure 7.3.1. Moisture Loss From Onion Bulbs at 70 F and 31% Relative Humidity.

i) The predicted values are based on the assumption that the initial moisture content distribution is uniform. This accounts for the predicted moisture loss values being greater than the experimental values at small time values.

ii) Respiration losses are not accounted for in the predicted weight losses.

iii) As the onion dries, the outer scales crack and loosen uncovering additional surface area of higher moisture content. This accounts for the experimental moisture losses being higher at the longer times.

VIII. CONCLUSIONS

The first objective for this research as given in Chapter III was to determine the density of onion bulbs. The final value obtained was 58.8 lb_m/ft^3.

The second objective listed was to obtain values of thermal conductivity in the axial and radial directions. The final values obtained were 0.30 Btu/hr ft F and 0.217 Btu/hr ft F for the axial and radial thermal conductivity respectively.

The third objective was to obtain values for the moisture diffusivity. The predicted values for the radial diffusivity $D'_{\mu r}$ and the axial diffusivity $D'_{\mu a}$ are given by the equations

$$D'_{\mu r} = RT[4.553 \times 10^{-18}\ m^{6.023} + 1.779 \times 10^{-14}(T-530)] \tag{8-1}$$

and

$$D'_{\mu a} = 414\ D'_{\mu r}. \tag{8-2}$$

The final objective was to develop a mathematical model to simulate the response of the onion bulb to its environment. To achieve this objective, the Alternating Direction Explicit Procedure (ADEP) for solving the transfer equation in cartesian coordinates was adapted to a non-uniform grid system based on the onion shape.

The heat transfer portion of the model simulated the actual process very well as indicated by the small RMS and maximum differences between the predicted and experimental center temperatures observed during the tests for radial thermal conductivity (see Table 4).

The mass transfer portion of the model was in only fair agreement with the experimental weight losses observed as a test of the model.

Suggestions for Further Study

The author concludes that further work is needed:

i) to determine the magnitude of the weight losses due to respiration compared to those due to moisture loss,

ii) to measure the effect of the cracking and loosening of the outer layers of the bulb, and

iii) to include this last effect in the modeling of the mass transfer process.

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BIBLIOGRAPHY

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APPENDIX

TABLE 1.--Experimental values and results of the onion density determination data.

Sample	1	2	3	4	5	6	7	8	9	10	11
Weight in air (g)	69.620	85.346	74.902	68.792	70.540	69.052	75.912	70.263	84.188	67.287	66.963
Weight in water (g)	23.017	25.779	26.330	24.150	25.065	24.195	25.030	25.630	23.530	25.285	25.650
Weight of sinker in water (g)	29.158	29.158	29.158	29.158	29.158	29.158	29.158	29.158	29.158	29.158	29.158
Specific gravity	.9189	.9619	.9636	.9321	.9451	.9329	.9484	.9521	.9373	.9456	.9502

Results

Average specific gravity	0.94437
Standard deviation	0.01330
Standard error of the mean	0.0040
Density (by formula (2.3-1)	58.81 lb_m/ft^3

TABLE 2.--The Fourier series approximating the onion shape.

$$\text{PERIM (a)} = \frac{d}{2} \sum_{i-1}^{10} A_i \sin \left(\frac{i \pi z}{1.5 d}\right)$$

i	A_i
1	0.8322400
2	0.3689900
3	0.0462150
4	0.0074205
5	0.0822030
6	-0.0114820
7	0.0194130
8	0.0074970
9	0.0145270
10	-0.0046454

TABLE 3.--Results of the axial thermal conductivity experiments.

Run	k_a (Btu/hr ft F)	RMS Difference Between Calculated and Experimental Temperatures (F)
	Individual Experiments	
1	0.32518	1.38
2	0.251	1.44
3	0.322	0.56
4	0.324	1.36
5	0.241	2.55
	Results	
	Average Axial Conductivity	0.293 Btu/hr ft F
	Standard Deviation	0.042 Btu/hr ft F
	Standard Error of the Mean	0.019 Btu/hr ft F
	Weighted (by 1/RMS) Average Axial Conductivity	0.30 Btu/hr ft F

TABLE 4.--Results of the radial thermal conductivity experiments.

Run	k (Btu/hr ft F)	RMS Difference Between Calculated and Experimental Center Temperatures (F)	RMS Difference Between Calculated and Experimental Center Temperatures Using k = 0.217 Btu/hr ft F (F)	Maximum Difference Between Calculated and Experimental Center Temperatures Using k = 0.217 Btu/hr ft F (F)
		Individual Experiments		
1	0.224	1.14	1.34	2.42
2	0.209	0.81	1.21	2.34
3	0.215	0.35	0.41	1.07
4	0.224	0.64	0.97	1.65
5	0.216	0.78	0.78	1.72

Results

Average Radial Conductivity	0.217 Btu/hr ft F
Standard Deviation	0.0085 Btu/hr ft F
Standard Error of the Mean	0.0029 Btu/hr ft F
Weighted (by 1/RMS) Average Radial Conductivity	0.217 Btu/hr ft F

TABLE 5.--Results of the radial moisture diffusivity experiments.

Run	Fluid Left Side	Fluid Right Side	Right Side	Temperature (F)	Sample Thickness x 10^3 (in)	Mass Flux x10 (lb_m/hr ft^2)	Mass Flux x10 By Equation (7.2-4) (lb_m/hr ft^2)
1	Water	Air	0.40	70	3.7	5.58	6.60
2	Water	Air	0.65	70	3.3	2.02	7.40
3	Water	Air	0.15	70	3.5	6.48	6.98
4	Water	Air	0.65	70	3.0	14.1	7.40
5	Water	Air	0.90	70	6.5	6.63	3.76
6	Water	0.5m Mannitol sol'n	0.9911	70	5.0	3.24	1.92
7	Water	Air	0.78	40	3.8	0.00	1.18
8	Water	Air	0.65	100	3.7	13.3	14.1

TABLE 6.--Data points for the onion moisture isotherm at 86 F (Saravacos, 1960).

Water Activity	Moisture Condent (d.b.)
0.112	0.012
0.220	0.022
0.328	0.077
0.436	0.148
0.539	0.190
0.633	0.305
0.756	0.330
0.863	0.530
0.907	0.925

```
      PROGRAM HTRAN (INPUT,OUTPUT)
      DIMENSION PAX(35,21), PRA(35,21), T(35,21), RHO(35,21), CP(35,21),
     1 U(35,21), V(35,21), W(35,21), X(35,21), SU(35,21), SV(35,21), SW(
     235,21), SX(35,21), VFAP(35,21), VFRP(35,21), VOL(35,21), H(35), ST
     3(35,21), R(1000), Z(1000), C3(20)
      COMMON DIA,ZMAX,PI
      REAL KR,KA
      PI=3.14159265
      PRINT 63
C     SET UP GRID SYSTEM
      READ 56, DIA
      DIA=DIA/12.
      ZMAX=1.5*DIA
      I=12
      I1=I-1
      J=10
      J1=J-1
      DO 1 K=1,J
      PRA(1,K)=0.0
      PAX(1,K)=0.0
      PRA(I,K)=0.0
      PAX(I,K)=ZMAX
    1 CONTINUE
      A=1./16.
      PAX(2,1)=ZMAX*A
      PAX(3,1)=ZMAX*A*2.
      PAX(4,1)=ZMAX*A*4.
      PAX(5,1)=ZMAX*A*5.
      PAX(6,1)=ZMAX*A*6.
      PAX(7,1)=ZMAX*A*8.
      PAX(8,1)=ZMAX*A*10.
      PAX(9,1)=ZMAX*A*12.
      PAX(10,1)=ZMAX*A*14.
      PAX(11,1)=ZMAX*A*15.
      DO 2 K=2,I1
    2 PRA(K,1)=0.
```

```
      DO 7 K=2,I1
      DL=(PAX(K+1,1)-PAX(K-1,1))/100.
      R(1)=0.
      Z(1)=PAX(K,1)
      K1=2
      Z1=Z(K1-1)
    3 DRA=DL*COS(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      DZ=-DL*SIN(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      R(K1)=R(K1-1)+DRA
      Z(K1)=Z(K1-1)+DZ
      Z1=Z(K1)
      IF (Z(K1).GT.0..OR.Z(K1).LT.ZMAX) GO TO 4
      Z(K1)=Z(K1-1)
      R(K1)=R(K1-1)
      GO TO 5
    4 IF (R(K1).GE.PERIM(Z1)) GO TO 5
      K1=K1+1
      GO TO 3
    5 PRINT 59, DL,DRA,DZ,K,K1
      AK1=K1
      PRA(K,J)=R(K1)
      PAX(K,J)=Z(K1)
      C3(2)=.25
      C3(3)=.5
      C3(4)=.625
      C3(5)=.75
      C3(6)=.8125
      C3(7)=.875
      C3(8)=.9375
      C3(9)=.96875
      DO 6 K1=2,J1
      K2=AK1*C3(K1)
      PRA(K,K1)=R(K2)
    6 PAX(K,K1)=Z(K2)
    7 CONTINUE
      DO 8 K=1,I
```

```
    8 PRINT 57, (PAX(K,L1),PRA(K,L1),L1=1,J)
C     DETERMINE GEOMETRIC FACTORS AND VOLUMES
      DO 9 K=2,I1
      DO 9 K1=2,J1
      A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
     1**2)
      B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
     1**2)
      B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
     1**2)
      B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFRP(K,K1)=4.*PI*((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(6.*A1+B1
     1+B6)
      IF (K.EQ.I1) GO TO 9
      VFAP(K,K1)=4.*PI*((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(6.*A2+B3
     1+B8)
    9 VOL(K,K1)=PI*PRA(K,K1)*(((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(P
     1RA(K+1,K1)+PRA(K,K1))+((B5+B6)*PRA(K-1,K1)+(A1+A3)*PRA(K,K1))/(PRA
     2(K-1,K1)+PRA(K,K1)))*(((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(PRA
     3(K,K1+1)+PRA(K,K1))+((B3+B4)*PRA(K,K1-1)+(A2+A4)*PRA(K,K1))/(PRA(K
     4,K1-1)+PRA(K,K1)))/8.
      K1=J
      DO 10 K=2,I1
```

```
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
     1**2)
      B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
     1**2)
      VFRP(K,J)=PI*PRA(K,J)*(A2+A4)
      IF (K.EQ.I1) GO TO 10
      VFAP(K,K1)=2.*PI*(A3*PRA(K,K1)+B2*PRA(K+1,K1))/(3.*A2+B3)
   10 VOL(K,J)=PI*PRA(K,J)*(((B2*PRA(K+1,K1)+A3*PRA(K,K1))/(PRA(K+1,K1)+
     1PRA(K,K1))+(B5*PRA(K-1,K1)+A3*PRA(K,K1))/(PRA(K-1,K1)+PRA(K,K1))))
     2*(B3*PRA(K,K1-1)+A2*PRA(K,K1)+B4*PRA(K,K1-1)+A4*PRA(K,K1))/(PRA(K,
     3K1-1)+PRA(K,K1))/4.
      DO 11 K1=1,J
      VOL(1,K1)=2.*PI*PAX(2,1)**3/3.
   11 VOL(I,K1)=VOL(1,K1)
      PRINT 58
      K=1
      DO 12 K1=2,J1
      VFRP(1,K1)=0.
      VFRP(I,K1)=VFRP(1,K1)
      VFAP(I,K1)=0.
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
```

```
      VFAP(1,K1)=1.5*PRA(2,K1)*(B1+B2)/A2
   12 VFAP(I1,K1)=VFAP(1,K1)
      VFRP(1,J)=PI*B8**2/4.
      VFRP(I,J)=VFRP(1,J)
      VFAP(I,J)=0.
      K=1
      K1=J
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      VFAP(1,J)=1.5*PRA(2,J)*B2/A2
      VFAP(I1,J)=VFAP(1,J)
      VFRP(1,1)=0.
      K=1
      K1=1
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      VFAP(1,1)=1.5*B1*B1/A2
      VFAP(I1,1)=VFAP(1,1)
      DO 13 K=2,I1
      A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
     1**2)
      B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFRP(K,1)=A2+B8+A4+B7
      IF (K.EQ.I1) GO TO 13
      VFAP(K,1)=PI*(B1+A1)**2/(3.*A2+B8)/4.
   13 VOL(K,1)=A2*PI*(3.*A1+B1)**2/128.+A4*PI*(3.*A1+B6)**2/128.
```

```
      VFRP(I,1)=0.
      VFAP(I,1)=0.
      DO 14 K=1,I
      PRINT 60, K,(VFAP(K,L),L=1,J)
   14 PRINT 60, K,(VFRP(K,L),L=1,J)
      AREA=0.
      DO 15 K=1,I
   15 AREA=AREA+VFRP(K,J)
      PRINT 66, AREA
      PRINT 61
      DO 16 K=1,I
   16 PRINT 60, K,(VOL(K,L1),L1=1,J)
      VOLUME=0.
      DO 17 K=1,I
      DO 17 K1=1,J
   17 VOLUME=VOLUME+VOL(K,K1)
      PRINT 67, VOLUME
      KA=.3
      KR=.217
      DO 18 K=1,I
   18 H(K)=5.
      SET UP INITIAL CONDITIONS
      DO 19 K=1,I
      DO 19 K1=1,J
      T(K,K1)=40.
      RHO(K,K1)=58.8
      ST(K,K1)=T(K,K1)
      W(K,K1)=T(K,K1)
      V(K,K1)=W(K,K1)
      U(K,K1)=V(K,K1)
      SW(K,K1)=T(K,K1)
      SV(K,K1)=SW(K,K1)
      SU(K,K1)=SV(K,K1)
      SX(K,K1)=T(K,K1)
   19 CP(K,K1)=0.93
      SOLVE TEMP DISTRIBUTION BY ALLADA QUON
```

```
      HT=0.
      TIME=0.
      READ 56, DELT,TA
      PRINT 62, DELT
      DO 54 L=1,10000
      A1=RHO(1,1)*CP(1,1)*VOL(1,1)/DELT
      G=0.
      DO 20 K1=1,J
   20 G=G+KA*VFAP(1,K1)*(T(2,K1)-T(1,1))
      T(1,1)=(A1*T(1,1)+G+H(1)*VFRP(1,J)*TA)/(A1+H(1)*VFRP(1,J))
      DO 21 K1=2,J
      T(1,1)=(A1*T(1,1)+G+H(1)*VFRP(1,J)*TA)/(A1+H(1)*VFRP(1,J))
      DO 21 K1=2,J
   21 T(1,K1)=T(1,1)
      DO 23 K=2,I1
      A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
      B4=KA*VFAP(K-1,1)
      T(K,1)=(A1*T(K,1)+B4*T(K-1,1)+KR*VFRP(K,1)*(T(K,2)-T(K,1))+KA*VFAP
     1(K,1)*(T(K+1,1)-T(K,1)))/(A1+B4)
      DO 22 K1=2,J1
      A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
      B3=KR*VFRP(K,K1-1)
      B4=KA*VFAP(K-1,K1)
   22 T(K,K1)=(A1*T(K,K1)+B3*T(K,K1-1)+B4*T(K-1,K1)+KR*VFRP(K,K1)*(T(K,K
     11+1)-T(K,K1))+KA*VFAP(K,K1)*(T(K+1,K1)-T(K,K1)))/(A1+B3+B4)
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      B3=KR*VFRP(K,J1)
      B4=KA*VFAP(K-1,J)
   23 T(K,J)=(A1*T(K,J)+B3*T(K,J1)+B4*T(K-1,J)+KA*VFAP(K,J)*(T(K+1,J)-T(
     1K,J))+H(K)*VFRP(K,J)*(TA-T(K,J)))/(A1+B3+B4)
      A1=RHO(I,1)*CP(I,1)*VOL(I,1)/DELT
      G1=0.
      G=G1
      DO 24 K1=1,J
      G=G+KA*VFAP(I1,K1)*T(I1,K1)
   24 G1=G1+KA*VFAP(I1,K1)
```

```
      T(I,1)=(A1*T(I,1)+G+H(I)*VFRP(I,J)*(TA-T(9,1)))/(A1+G1)
      DO 25 K1=2,J
   25 T(I,K1)=T(I,1)
      DO 26 K=1,I
      DO 26 K1=1,J
   26 U(K,K1)=T(K,K1)
      G=0.
      DO 27 K1=1,J
   27 G=G+KA*VFAP(I1,K1)*(U(I1,K1)-U(I,J))
      U(I,J)=(A1*U(I,J)+G+H(I)*VFRP(I,J)*TA)/(A1+H(I)*VFRP(I,J))
      DO 28 K1=1,J1
   28 U(I,K1)=U(I,J)
      DO 30 KK=2,I1
      K=I+1-KK
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      B2=KA*VFAP(K,J)
      U(K,J)=(A1*U(K,J)+B2*U(K+1,J)+KR*VFRP(K,J1)*(U(K,J1)-U(K,J))+KA*VF
     1AP(K-1,J)*(U(K-1,J)-U(K,J))+H(K)*VFRP(K,J)*TA)/(A1+B2+H(K)*VFRP(K,
     2J))
      DO 29 KK1=2,J1
      K1=J+1-KK1
      A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
      B1=KR*VFRP(K,K1)
      B2=KA*VFAP(K,K1)
   29 U(K,K1)=(A1*U(K,K1)+B1*U(K,K1+1)+B2*U(K+1,K1)+KR*VFRP(K,K1-1)*(U(K
     1,K1-1)-U(K,K1))+KA*VFAP(K-1,K1)*(U(K-1,K1)-U(K,K1)))/(A1+B1+B2)
      A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
      B1=KR*VFRP(K,1)
      B2=KA*VFAP(K,1)
   30 U(K,1)=(A1*U(K,1)+B1*U(K,2)+B2*U(K+1,1)+KA*VFAP(K-1,1)*(U(K-1,1)-U
     1(K,1)))/(A1+B1+B2)
      A1=RHO(1,J)*CP(1,J)*VOL(1,J)/DELT
      G1=0.
      G=G1
      DO 31 K1=1,J
      G=G+KA*VFAP(1,K1)*U(2,K1)
```

```
31 G1=G1+KA*VFAP(1,K1)
   U(1,J)=(A1*U(1,J)+G+H(1)*VFRP(1,J)*(TA-U(1,J)))/(A1+G1)
   DO 32 K1=1,J1
32 U(1,K1)=U(1,J)
   DO 33 K=1,I
   DO 33 K1=1,J
33 V(K,K1)=U(K,K1)
   A1=RHO(I,1)*CP(I,1)*VOL(I,1)/DELT
   G=0.
   DO 34 K1=1,J
34 G=G+KA*VFAP(I1,K1)*(V(I1,K1)-V(I,1))
   V(I,1)=(A1*V(I,1)+G+H(I)*VFRP(I,J)*TA)/(A1+H(I)*VFRP(I,J))
   DO 35 K1=2,J
35 V(I,K1)=V(I,1)
   DO 37 KK=2,I1
   K=I+1-KK
   A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
   B2=KA*VFAP(K,1)
   V(K,1)=(A1*V(K,1)+B2*V(K+1,1)+KR*VFRP(K,1)*(V(K,2)-V(K,1))+KA*VFAP
  1(K-1,1)*(V(K-1,1)-V(K,1)))/(A1+B2)
   DO 36 K1=2,J1
   A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
   B2=KA*VFAP(K,K1)
   B3=KR*VFRP(K,K1-1)
36 V(K,K1)=(A1*V(K,K1)+B2*V(K+1,K1)+B3*V(K,K1-1)+KR*VFRP(K,K1)*(V(K,K
  11+1)-V(K,K1))+KA*VFAP(K-1,K1)*(V(K-1,K1)-V(K,K1)))/(A1+B2+B3)
   A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
   B2=KA*VFAP(K,J)
   B3=KR*VFRP(K,J1)
37 V(K,J)=(A1*V(K,J)+B2*V(K+1,J)+B3*V(K,J1)+KA*VFAP(K-1,J)*(V(K-1,J)-
  1V(K,J))+H(K)*VFRP(K,J)*(TA-V(K,J)))/(A1+B2+B3)
   A1=RHO(1,1)*CP(1,1)*VOL(1,1)/DELT
   G1=0.
   G=G1
   DO 38 K1=1,J
   G=G+KA*VFAP(1,K1)*V(2,K1)
```

```
38 G1=G1+KA*VFAP(1,K1)
   V(1,1)=(A1*V(1,1)+G+H(1)*VFRP(1,J)*(TA-V(1,1)))/(A1+G1)
   DO 39 K1=2,J
39 V(1,K1)=V(1,1)
   DO 40 K=1,I
   DO 40 K1=1,J
40 W(K,K1)=V(K,K1)
   G=0.
   DO 41 K1=1,J
41 G=G+KA*VFAP(1,K1)*(W(2,K1)-W(1,J))
   W(1,J)=(A1*W(1,J)+G+H(I)*VFRP(1,J)*TA)/(A1+H(1)*VFRP(1,J))
   DO 42 K1=1,J1
42 W(1,K1)=W(1,J)
   DO 44 K=2,I1
   A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
   B4=KA*VFAP(K-1,J)
   W(K,J)=(A1*W(K,J)+B4*W(K-1,J)+H(K)*VFRP(K,J)*TA+KA*VFAP(K,J)*(W(K+
  11,J)-W(K,J))+KR*VFRP(K,J1)*(W(K,J1)-W(K,J)))/(A1+B4+H(K)*VFRP(K,J)
  2)
   DO 43 KK1=2,J1
   K1=J+1-KK1
   A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
   B1=KR*VFRP(K,K1)
   B4=KA*VFAP(K-1,K1)
43 W(K,K1)=(A1*W(K,K1)+B1*W(K,K1+1)+B4*W(K-1,K1)+KA*VFAP(K,K1)*(W(K+1
  1,K1)-W(K,K1))+KR*VFRP(K,K1-1)*(W(K,K1-1)-W(K,K1)))/(A1+B1+B4)
   A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
   B1=KR*VFRP(K,1)
   B4=KA*VFAP(K-1,1)
44 W(K,1)=(A1*W(K,1)+B1*W(K,2)+B4*W(K-1,1)+KA*VFAP(K,1)*(W(K+1,1)-W(K
  1,1)))/(A1+B1+B4)
   A1=RHO(I,J)*CP(I,J)*VOL(I,J)/DELT
   G1=0.
   G=G1
   DO 45 K1=1,J
   G=G+KA*VFAP(I1,K1)*W(I1,K1)
```

```
45 G1=G1+KA*VFAP(I1,K1)
   W(I,J)=(A1*W(I,J)+G+H(I)*VFRP(I,J)*(TA-W(I,J)))/(A1+G1)
   DO 46 K1=1,J1
46 W(I,K1)=W(I,J)
   DO 47 K=1,I
   DO 47 K1=1,J
   T(K,K1)=W(K,K1)
47 X(K,K1)=T(K,K1)
   IF (TIME.GT.2.0) GO TO 55
C  CHECK TO MAKE SURE THAT THE STEPS JUST TAKEN ARE OK
C  IF NOT. BACKTRACK
   A=0.
   DO 48 K=1,I
   DO 48 K1=1,J
   B=ABS(T(K,K1)-ST(K,K1))
48 A=AMAX1(A,B)
   IF (A.LT.1.) GO TO 50
   DO 49 K=1,I
   DO 49 K1=1,J
49 T(K,K1)=ST(K,K1)
   DELT=DELT/2.
   GO TO 54
50 TIME=TIME+DELT*4.
   PRINT 65, TIME,DELT
   DO 52 K=1,I
   IF (L.GT.5) GO TO 51
   PRINT 60, K,(T(K,L1),L1=1,J)
51 CONTINUE
52 HT=HT+VFRP(K,J)*H(K)*(X(K,J)-TA)*DELT*4.
   PRINT 64, HT,DELT
   DO 53 K=1,I
   DO 53 K1=1,J
53 ST(K,K1)=T(K,K1)
C  DOUBLE STEP SIZE IF O.K.
   IF (A.GT..5) GO TO 54
   DELT=DELT*2.
```

```
      DELT=AMIN1(DELT,.0016)
   54 CONTINUE
   55 CONTINUE
C
C
   56 FORMAT (8F10.0)
   57 FORMAT (11(1X,F11.7))
   58 FORMAT (1H1,23H      VIEW FACTORS       )
   59 FORMAT (1X,F10.7,1X,F10.7,1X,F10.7,1X,I5,1X,I5)
   60 FORMAT (1X,I2,11(1X,F10.6),/,10(1X,F10.6),1X,I2)
   61 FORMAT (1H1,21H                  VOLUMES)
   62 FORMAT (1H1,35HRESULTS                 DELTA TIME =  ,F10.7)
   63 FORMAT (22H1HEAT TRANSFER PROGRAM,/,12H NODE POINTS)
   64 FORMAT (20H HEAT TRANSFERRED = ,F11.7,14H BTU,  DELT = ,F9.7)
   65 FORMAT (1X,22HTEMPERATURES,  TIME = ,F14.8,9H  DELT = ,F10.8)
   66 FORMAT (27H THE TOTAL SURFACE AREA IS ,F10.6,12H SQUARE FEET)
   67 FORMAT (21H THE TOTAL VOLUME IS ,F10.8,11H CUBIC FEET)
      END
```

```
      PROGRAM MTRAN (INPUT,OUTPUT)
      DIMENSION PAX(35,35), PRA(35,35), DMA(35,35), DMR(35,35), RHO(35,3
     15), RDM(35,35), MC(35,35), R(300), Z(300), C3(9), VFAP(35,35), VFR
     2P(35,35), VOL(35,35), HD(35), SMC(35,35), CW(35), AK(3), AN(3)
      COMMON DIA,ZMAX,PI
      REAL MC,MT
      PI=3.14159265
      RW=1545.33/18.
      READ 62, AK
      READ 62, AN
      PRINT 69
C     SET UP GRID SYSTEM
      READ 62, DIA
      DIA=DIA/12.
      ZMAX=DIA*1.5
      I=12
      I1=I-1
      J=12
      J1=J-1
      DO 1 K=1,J
      PRA(1,K)=0.0
      PAX(1,K)=0.0
      PRA(I,K)=0.0
      PAX(I,K)=ZMAX
    1 CONTINUE
      A=1./16.
      PAX(2,1)=ZMAX*A
      PAX(3,1)=ZMAX*A*2.
      PAX(4,1)=ZMAX*A*4.
      PAX(5,1)=ZMAX*A*5.
      PAX(6,1)=ZMAX*A*6.
      PAX(7,1)=ZMAX*A*8.
      PAX(8,1)=ZMAX*A*10.
      PAX(9,1)=ZMAX*A*12.
      PAX(10,1)=ZMAX*A*14.
      PAX(11,1)=ZMAX*A*15.
```

```
      DO 2 K=2,I1
    2 PRA(K,1)=0.
      DO 7 K=2,I1
      DL=(PAX(K+1,1)-PAX(K-1,1))/100.
      R(1)=0.
      Z(1)=PAX(K,1)
      K1=2
      Z1=Z(K1-1)
    3 DRA=DL*COS(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      DZ=-DL*SIN(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      R(K1)=R(K1-1)+DRA
      Z(K1)=Z(K1-1)+DZ
      Z1=Z(K1)
      IF (Z(K1).GT.0..OR.Z(K1).LT.ZMAX) GO TO 4
      Z(K1)=Z(K1-1)
      R(K1)=R(K1-1)
      GO TO 5
    4 IF (R(K1).GE.PERIM(Z1)) GO TO 5
      K1=K1+1
      GO TO 3
    5 PRINT 65, DL,DRA,DZ,K,K1
      AK1=K1
      PRA(K,J)=R(K1)
      PAX(K,J)=Z(K1)
      C3(2)=.25
      C3(3)=.5
      C3(4)=.625
      C3(5)=.75
      C3(6)=.8125
      C3(7)=.875
      C3(8)=.90625
      C3(9)=.9375
      C3(10)=.96875
      C3(11)=.984375
      DO 6 K1=2,J1
      K2=AK1*C3(K1)
```

```
      PRA(K,K1)=R(K2)
    6 PAX(K,K1)=Z(K2)
    7 CONTINUE
      DO 8 K=1,I
    8 PRINT 63, (PAX(K,L1),PRA(K,L1),L1=1,J)
C     DETERMINE GEOMETRIC FACTORS AND VOLUMES
      DO 9 K=2,I1
      DO 9 K1=2,J1
      A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
     1**2)
      B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
     1**2)
      B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
     1**2)
      B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFRP(K,K1)=4.*PI*((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(6.*A1+B1
     1+B6)
      IF (K.EQ.I1) GO TO 9
      VFAP(K,K1)=4.*PI*((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(6.*A2+B3
     1+B8)
    9 VOL(K,K1)=PI*PRA(K,K1)*(((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(P
     1RA(K+1,K1)+PRA(K,K1))+((B5+B6)*PRA(K-1,K1)+(A1+A3)*PRA(K,K1))/(PRA
     2(K-1,K1)+PRA(K,K1)))*(((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(PRA
```

```
     3(K,K1+1)+PRA(K,K1))+((B3+B4)*PRA(K,K1-1)+(A2+A4)*PRA(K,K1))/(PRA(K
     4,K1-1)+PRA(K,K1)))/8.
      K1=J
      DO 10 K=2,I1
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
     1**2)
      B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
     1**2)
      VFRP(K,J)=PI*PRA(K,J)*(A2+A4)
      IF (K.EQ.I1) GO TO 10
      VFAP(K,K1)=2.*PI*(A3*PRA(K,K1)+B2*PRA(K+1,K1))/(3.*A2+B3)
   10 VOL(K,J)=PI*PRA(K,J)*(((B2*PRA(K+1,K1)+A3*PRA(K,K1))/(PRA(K+1,K1)+
     1PRA(K,K1))+(B5*PRA(K-1,K1)+A3*PRA(K,K1))/(PRA(K-1,K1)+PRA(K,K1))))
     2*(B3*PRA(K,K1-1)+A2*PRA(K,K1)+B4*PRA(K,K1-1)+A4*PRA(K,K1))/(PRA(K,
     3K1-1)+PRA(K,K1))/4.
      DO 11 K1=1,J
      VOL(1,K1)=2.*PI*PAX(2,1)**3/3.
   11 VOL(I,K1)=VOL(1,K1)
      PRINT 64
      K=1
      DO 12 K1=2,J1
      VFRP(I,K1)=0.
      VFRP(1,K1)=VFRP(I,K1)
      VFAP(I,K1)=0.
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
```

```
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFAP(1,K1)=1.5*PRA(2,K1)*(B1+B2)/A2
   12 VFAP(I1,K1)=VFAP(1,K1)
      VFRP(1,J)=PI*B8**2/4.
      VFRP(1,J)=VFRP(1,J)
      VFAP(I,J)=0.
      K=1
      K1=J
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      VFAP(1,J)=1.5*PRA(2,J)*B2/A2
      VFAP(I1,J)=VFAP(1,J)
      VFRP(1,1)=0.
      K=1
      K1=1
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      VFAP(1,1)=1.5*B1*B1/A2
      VFAP(I1,1)=VFAP(1,1)
      DO 13 K=2,I1
      A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
     1**2)
      B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
```

```
      VFRP(K.1)=A2+B8+A4+B7
      IF (K.EQ.I1) GO TO 13
      VFAP(K,1)=PI*(B1+A1)**2/(3.*A2+B8)/4.
   13 VOL(K,1)=A2*PI*(3.*A1+B1)**2/128.+A4*PI*(3.*A1+B6)**2/128.
      VFRP(I.1)=0.
      VFAP(I.1)=0.
      DO 14 K=1.I
      PRINT 66. K.(VFAP(K.L).L=1.J)
   14 PRINT 66. K.(VFRP(K.L).L=1,J)
      AREA=0.
      DO 15 K=1.I
   15 AREA=AREA+VFRP(K,J)
      PRINT 72. AREA
      PRINT 67
      DO 16 K=1,I
   16 PRINT 66. K,(VOL(K.L1),L1=1,J)
      VOLUME=0.
      DO 17 K=1.I
      DO 17 K1=1,J
   17 VOLUME=VOLUME+VOL(K,K1)
      PRINT 73. VOLUME
C     SET UP INITIAL CONDITIONS
      READ 62. DELT,TA,RH
      T=TA
      A=ALOG(460.+TA)
      PWS=EXP(54.6329-12301.688/(460.+TA)-5.16923*A)*144.
      CAS=PWS/(RW*(460.+TA))
      CA=RH*CAS
      DO 18 K=1,I
      DO 18 K1=1,J
      RHO(K.K1)=58.8
      RDM(K,K1)=5.88
      MC(K.K1)=(RHO(K,K1)-RDM(K,K1))/RDM(K.K1)
   18 SMC(K.K1)=MC(K,K1)
      DO 19 K=1,I
      CW(K)=AW(MC(K.J),AK,AN)*CAS
```

```
   19 HD(K)=300.
C     SOLVE MOISTURE DISTRIBUTION BY ALLADA QUON
      TIME=0.
      MT=0.
      SMT=0.
      PRINT 68, DELT
      DO 60 L=1,5000
      DO 25 K=1,I
      DO 25 K1=1,J
      IF (MC(K,K1)-.022) 20,20,21
   20 L1=1
      GO TO 24
   21 IF (MC(K,K1)-.530) 22,22,23
   22 L1=2
      GO TO 24
   23 L1=3
   24 DMR(K,K1)=RW*(TA+460.)*(4.5535E-18*MC(K,K1)**6.023+1.779E-14*(TA-7
     10.))*MC(K,K1)/(MC(K,K1)-AK(L1))
      DMR(K,K1)=AMAX1(DMR(K,K1),0.)
   25 DMA(K,K1)=414.*DMR(K,K1)
      AM1=RDM(1,1)*VOL(1,1)/DELT
      GM=0.
      DO 26 K1=1,J
   26 GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*(MC(2,K1)-MC(1,1))/2.
      MC(1,1)=(AM1*MC(1,1)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+HD(1)*VFRP(1,J)*A
     1M2(MC(1,1),AK,AN,CAS))
      DO 27 K1=2,J
   27 MC(1,K1)=MC(1,1)
      DO 29 K=2,I1
      AM1=RDM(K,1)*VOL(K,1)/DELT
      BM1=(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,1)/2.
      MC(K,1)=(AM1*MC(K,1)+BM1*MC(K-1,1)+(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)
     1*(MC(K+1,1)-MC(K,1))/2.+(DMR(K,1)+DMR(K,2))*VFRP(K,1)*(MC(K,2)-MC(
     2K,1))/2.)/(AM1+BM1)
      DO 28 K1=2,J1
      AM1=RDM(K,K1)*VOL(K,K1)/DELT
```

```
      BM1=(DMA(K,K1)+DMA(K-1,K1))*VFAP(K-1,K1)/2.
      BM2=(DMR(K,K1)+DMR(K,K1-1))*VFRP(K,K1-1)/2.
   28 MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K-1,K1)+BM2*MC(K,K1-1)+(DMA(K,K1)+DM
     1A(K+1,K1))*VFAP(K,K1)*(MC(K+1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,K1
     2+1))*VFRP(K,K1)*(MC(K,K1+1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
      AM1=RDM(K,J)*VOL(K,J)/DELT
      BM1=(DMA(K,J)+DMA(K-1,J))*VFAP(K-1,J)/2.
      BM2=(DMR(K,J)+DMR(K,J1))*VFRP(K,J1)/2.
   29 MC(K,J)=(AM1*MC(K,J)+BM1*MC(K-1,J)+BM2*MC(K,J1)+(DMA(K,J)+DMA(K+1,
     1J))*VFAP(K,J)*(MC(K+1,J)-MC(K,J))/2.+HD(K)*VFRP(K,J)*CA)/(AM1+BM1+
     2BM2+HD(K)*VFRP(K,J)*AM2(MC(K,J),AK,AN,CAS))
      AM1=RDM(I,1)*VOL(I,1)/DELT
      GM1=0.
      GM=GM1
      DO 30 K1=1,J
      GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*MC(I1,K1)/2.
   30 GM1=GM1+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)/2.
      MC(I,1)=(AM1*MC(I,1)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+GM1+HD(I)*VFRP(I,
     1J)*AM2(MC(I,1),AK,AN,CAS))
      DO 31 K1=2,J
   31 MC(I,K1)=MC(1,1)
      DO 32 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CAS
   32 MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      GM=0.
      DO 33 K1=1,J
   33 GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*(MC(I1,K1)-MC(I,1))/2.
      MC(I,J)=(AM1*MC(I,J)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+HD(I)*VFRP(I,J)*A
     1M2(MC(I,J),AK,AN,CAS))
      DO 34 K1=1,J1
   34 MC(I,K1)=MC(I,J)
      DO 36 KK=2,I1
      K=I+1-KK
      AM1=RDM(K,J)*VOL(K,J)/DELT
      BM1=(DMA(K,J)+DMA(K+1,J))*VFAP(K,J)/2.
      MC(K,J)=(AM1*MC(K,J)+BM1*MC(K+1,J)+HD(K)*VFRP(K,J)*CA+(DMA(K,J)+DM
```

```
     1A(K-1,J))*VFAP(K-1,J)*(MC(K-1,J)-MC(K,J))/2.+(DMR(K,J)+DMR(K,J1))*
     2VFRP(K,J1)*(MC(K,J1)-MC(K,J))/2.)/(AM1+BM1+HD(K)*VFRP(K,J)*AM2(MC(
     3K,J),AK,AN,CAS))
      DO 35 KK1=2,J1
      K1=J+1-KK1
      AM1=RDM(K,K1)*VOL(K,K1)/DELT
      BM1=(DMA(K,K1)+DMA(K+1,K1))*VFAP(K,K1)/2.
      BM2=(DMR(K,K1)+DMR(K,K1+1))*VFRP(K,K1)/2.
   35 MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K+1,K1)+BM2*MC(K,K1+1)+(DMA(K,K1)+DM
     1A(K-1,K1))*VFAP(K-1,K1)*(MC(K-1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,
     2K1-1))*VFRP(K,K1-1)*(MC(K,K1-1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
      AM1=RDM(K,1)*VOL(K,1)/DELT
      BM1=(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)/2.
      BM2=(DMR(K,1)+DMR(K,2))*VFRP(K,1)/2.
   36 MC(K,1)=(AM1*MC(K,1)+BM1*MC(K+1,1)+BM2*MC(K,2)+(DMA(K,1)+DMA(K-1,1
     1))*VFAP(K-1,1)*(MC(K-1,1)-MC(K,1))/2.)/(AM1+BM1+BM2)
      AM1=RDM(1,1)*VOL(1,1)/DELT
      GM1=0.
      GM=GM1
      DO 37 K1=1,J
      GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*MC(2,K1)/2.
   37 GM1=GM1+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)/2.
      MC(1,J)=(AM1*MC(1,J)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+GM1+HD(1)*VFRP(1,
     1J)*AM2(MC(1,J),AK,AN,CAS))
      DO 38 K1=1,J1
   38 MC(1,K1)=MC(1,J)
      DO 39 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CAS
   39 MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      AM1=RDM(I,1)*VOL(I,1)/DELT
      GM=0.
      DO 40 K1=1,J
   40 GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*(MC(I1,K1)-MC(I,1))/2.
      MC(I,1)=(AM1*MC(I,1)+GM+HD(I)*VFRP(1,J)*CA)/(AM1+HD(I)*VFRP(I,J)*A
     1M2(MC(I,1),AK,AN,CAS))
      DO 41 K1=2,J
```

```
41 MC(1,K1)=MC(I,1)
   DO 43 KK=2,I1
   K=I+1-KK
   AM1=RDM(K,1)*VOL(K,1)/DELT
   BM1=(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)/2.
   MC(K,1)=(AM1*MC(K,1)+BM1*MC(K+1,1)+(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,
  11)*(MC(K-1,1)-MC(K,1))/2.+(DMR(K,1)+DMR(K,2))*VFRP(K,1)*(MC(K,2)-M
  2C(K,1))/2.)/(AM1+BM1)
   DO 42 K1=2,J1
   AM1=RDM(K,K1)*VOL(K,K1)/DELT
   BM1=(DMA(K,K1)+DMA(K+1,K1))*VFAP(K,K1)/2.
   BM2=(DMR(K,K1)+DMR(K,K1-1))*VFRP(K,K1-1)/2.
42 MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K+1,K1)+BM2*MC(K,K1-1)+(DMA(K,K1)+DM
  1A(K-1,K1))*VFAP(K-1,K1)*(MC(K-1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,
  2K1+1))*VFRP(K,K1)*(MC(K,K1+1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
   AM1=RDM(K,J)*VOL(K,J)/DELT
   BM1=(DMA(K,J)+DMA(K+1,J))*VFAP(K,J)/2.
   BM2=(DMR(K,J)+DMR(K,J1))*VFRP(K,J1)/2.
43 MC(K,J)=(AM1*MC(K,J)+BM1*MC(K+1,J)+BM2*MC(K,J1)+(DMA(K,J)+DMA(K-1,
  1J))*VFAP(K-1,J)*(MC(K-1,J)-MC(K,J))/2.+HD(K)*VFRP(K,J)*CA)/(AM1+BM
  21+BM2+HD(K)*VFRP(K,J)*AM2(MC(K,J),AK,AN,CAS))
   AM1=RDM(1,1)*VOL(1,1)/DELT
   GM1=0.
   GM=GM1
   DO 44 K1=1,J
   GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*MC(2,K1)/2.
44 GM1=GM1+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)/2.
   MC(1,1)=(AM1*MC(1,1)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+GM1+HD(1)*VFRP(1,
  1J)*AM2(MC(1,1),AK,AN,CAS))
   DO 45 K1=2,J
45 MC(1,K1)=MC(1,1)
   DO 46 K=1,I
   CW(K)=AW(MC(K,J),AK,AN)*CAS
46 MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
   GM=0.
   DO 47 K1=1,J
```

```
47 GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*(MC(2,K1)-MC(1,1))/2.
   MC(1,J)=(AM1*MC(1,J)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+HD(1)*VFRP(1,J)*A
  1M2(MC(1,J),AK,AN,CAS))
   DO 48 K1=1,J1
48 MC(1,K1)=MC(1,J)
   DO 50 K=2,I1
   AM1=RDM(K,J)*VOL(K,J)/DELT
   BM1=(DMA(K,J)+DMA(K-1,J))*VFAP(K-1,J)/2.
   MC(K,J)=(AM1*MC(K,J)+BM1*MC(K-1,J)+HD(K)*VFRP(K,J)*CA+(DMA(K,J)+DM
  1A(K+1,J))*VFAP(K,J)*(MC(K+1,J)-MC(K,J))/2.+(DMR(K,J)+DMR(K,J1))*VF
  2RP(K,J1)*(MC(K,J1)-MC(K,J))/2.)/(AM1+BM1+HD(K)*VFRP(K,J)*AM2(MC(K,
  3J),AK,AN,CAS))
   DO 49 KK1=2,J1
   K1=J+1-KK1
   AM1=RDM(K,K1)*VOL(K,K1)/DELT
   BM1=(DMA(K,K1)+DMA(K-1,K1))*VFAP(K-1,K1)/2.
   BM2=(DMR(K,K1)+DMR(K,K1+1))*VFRP(K,K1)/2.
49 MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K-1,K1)+BM2*MC(K,K1+1)+(DMA(K,K1)+DM
  1A(K+1,K1))*VFAP(K,K1)*(MC(K+1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,K1
  2-1))*VFRP(K,K1-1)*(MC(K,K1-1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
   AM1=RDM(K,1)*VOL(K,1)/DELT
   BM1=(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,1)/2.
   BM2=(DMR(K,1)+DMR(K,2))*VFRP(K,1)/2.
50 MC(K,1)=(AM1*MC(K,1)+BM1*MC(K-1,1)+BM2*MC(K,2)+(DMA(K,1)+DMA(K+1,1
  1))*VFAP(K,1)*(MC(K+1,1)-MC(K,1))/2.)/(AM1+BM1+BM2)
   AM1=RDM(I,1)*VOL(I,1)/DELT
   GM1=0.
   GM=GM1
   DO 51 K1=1,J
   GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*MC(I1,K1)/2.
51 GM1=GM1+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)/2.
   MC(I,J)=(AM1*MC(I,J)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+GM1+HD(I)*VFRP(I,
  1J)*AM2(MC(I,J),AK,AN,CAS))
   DO 52 K1=1,J1
52 MC(I,K1)=MC(I,J)
   DO 53 K=1,I
```

```
      CW(K)=AW(MC(K,J),AK,AN)*CAS
   53 MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      IF (TIME.GT.10000.) GO TO 61
C     CHECK TO MAKE SURE THAT THE STEPS JUST TAKEN ARE OK
C     IF NOT, BACKTRACK
      A=0.
      DO 54 K=1,I
      DO 54 K1=1,J
      B=ABS(MC(K,K1)-SMC(K,K1))
   54 A=AMAX1(A,B)
      IF (A.LT..5) GO TO 56
      MT=SMT
      DO 55 K=1,I
      DO 55 K1=1,J
   55 MC(K,K1)=SMC(K,K1)
      DELT=DELT/2.
      GO TO 60
   56 TIME=TIME+DELT*4.
      PRINT 71, TIME,DELT
      IF (L.GT.5) GO TO 58
      DO 57 K=1,I
   57 PRINT 66, K,(MC(K,L1),L1=1,J)
   58 CONTINUE
      SMT=MT
      PRINT 70, MT,DELT
      DO 59 K=1,I
      DO 59 K1=1,J
   59 SMC(K,K1)=MC(K,K1)
C     DOUBLE STEP SIZE IF O.K.
      IF (A.GT..10) GO TO 60
      DELT=DELT*2.
      DELT=AMIN1(DELT,10.)
   60 CONTINUE
   61 CONTINUE
C
C
```

```
62 FORMAT (8F10.0)
63 FORMAT (11(1X,F11.7))
64 FORMAT (1H1,23H     VIEW FACTORS       )
65 FORMAT (1X,F10.7,1X,F10.7,1X,F10.7,1X,I5,1X,I5)
66 FORMAT (1X,I3,12(1X,F9.6))
67 FORMAT (1H1,21H                  VOLUMES)
68 FORMAT (1H1,35HRESULTS                  DELTA TIME =  ,F10.7)
69 FORMAT (1H1,21HMASS TRANSFER PROGRAM,/,11HNODE POINTS)
70 FORMAT (19H MASS TRANSFERRED= ,F11.7,12H LB.  DELT= ,F15.8)
71 FORMAT (7H TIME= ,F15.4,8H  DELT= ,F15.8)
72 FORMAT (27H THE TOTAL SURFACE AREA IS ,F10.6,12H SQUARE FEET)
73 FORMAT (21H THE TOTAL VOLUME IS ,F10.8,11H CUBIC FEET)
   END
```

```
      PROGRAM HMTRAN (INPUT,OUTPUT)
      DIMENSION PAX(12,12), PRA(12,12), U(12,12), DMR(12,12), DMA(12,12)
     1, T(12,12), RHO(12,12), RDM(12,12), MC(12,12), CP(12,12), R(300),
     2Z(300), C3(12), VFRP(12,12), VFAP(12,12), VOL(12,12), H(12), HD(12
     3), ST(12,12), SMC(12,12), CW(12), AK(3), AN(3)
      COMMON DIA,ZMAX,PI
      REAL KR,KA,MC,MT
      PI=3.14159265
      RW=1545.33/16.
      PRINT 71
C     SET UP GRID SYSTEM
      READ 64, DIA
      DIA=DIA/12.
      ZMAX=1.5*DIA
      I=12
      I1=I-1
      J=12
      J1=J-1
      DO 1 K=1,J
      PRA(1,K)=0.0
      PAX(1,K)=0.0
      PRA(I,K)=0.0
    1 PAX(I,K)=ZMAX
      A=1./16.
      PAX(2,1)=ZMAX*A
      PAX(3,1)=ZMAX*A*2.
      PAX(4,1)=ZMAX*A*4.
      PAX(5,1)=ZMAX*A*5.
      PAX(6,1)=ZMAX*A*6.
      PAX(7,1)=ZMAX*A*8.
      PAX(8,1)=ZMAX*A*10.
      PAX(9,1)=ZMAX*A*12.
      PAX(10,1)=ZMAX*A*14.
      PAX(11,1)=ZMAX*A*15.
      DO 2 K=2,I1
    2 PRA(K,1)=0.
```

```
      DO 7 K=2,I1
      DL=(PAX(K+1,1)-PAX(K-1,1))/100.
      R(1)=0.
      Z(1)=PAX(K,1)
      K1=2
      Z1=Z(K1-1)
    3 DRA=DL*COS(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      DZ=-DL*SIN(ATAN(DPERIM(Z1)*R(K1-1)/PERIM(Z1)))
      R(K1)=R(K1-1)+DRA
      Z(K1)=Z(K1-1)+DZ
      Z1=Z(K1)
      IF (Z(K1).GT.0..OR.Z(K1).LT.ZMAX) GO TO 4
      Z(K1)=Z(K1-1)
      R(K1)=R(K1-1)
      GO TO 5
    4 IF (R(K1).GE.PERIM(Z1)) GO TO 5
      K1=K1+1
      GO TO 3
    5 PRINT 67, DL,DRA,DZ,K,K1
      AK1=K1
      PRA(K,J)=R(K1)
      PAX(K,J)=Z(K1)
      C3(2)=.25
      C3(3)=.5
      C3(4)=.625
      C3(5)=.75
      C3(6)=.8125
      C3(7)=.875
      C3(8)=.90625
      C3(9)=.9375
      C3(10)=.96875
      C3(11)=.984375
      DO 6 K1=2,J1
      K2=AK1*C3(K1)
      PRA(K,K1)=R(K2)
    6 PAX(K,K1)=Z(K2)
```

```
7 CONTINUE
  DO 8 K=1,I
8 PRINT 65, (PAX(K,L1),PRA(K,L1),L1=1,J)
C      DETERMINE GEOMETRIC FACTORS AND VOLUMES
  DO 9 K=2,I1
  DO 9 K1=2,J1
  A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
  A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
  A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
  A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
  B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
 1**2)
  B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
 1**2)
  B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
 1**2)
  B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
 1**2)
  B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
 1**2)
  B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
 1**2)
  B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
 1**2)
  B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
 1**2)
  VFRP(K,K1)=4.*PI*((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(6.*A1+B1
 1+B6)
  IF (K.EQ.I1) GO TO 9
  VFAP(K,K1)=4.*PI*((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(6.*A2+B3
 1+B8)
9 VOL(K,K1)=PI*PRA(K,K1)*(((B1+B2)*PRA(K+1,K1)+(A1+A3)*PRA(K,K1))/(P
 1RA(K+1,K1)+PRA(K,K1))+((B5+B6)*PRA(K-1,K1)+(A1+A3)*PRA(K,K1))/(PRA
 2(K-1,K1)+PRA(K,K1)))*(((B7+B8)*PRA(K,K1+1)+(A2+A4)*PRA(K,K1))/(PRA
 3(K,K1+1)+PRA(K,K1))+((B3+B4)*PRA(K,K1-1)+(A2+A4)*PRA(K,K1))/(PRA(K
 4,K1-1)+PRA(K,K1)))/8.
```

```
      K1=J
      DO 10 K=2,I1
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A3=SQRT((PAX(K,K1-1)-PAX(K,K1))**2+(PRA(K,K1-1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
      B4=SQRT((PAX(K-1,K1-1)-PAX(K,K1-1))**2+(PRA(K-1,K1-1)-PRA(K,K1-1))
     1**2)
      B5=SQRT((PAX(K-1,K1-1)-PAX(K-1,K1))**2+(PRA(K-1,K1-1)-PRA(K-1,K1))
     1**2)
      VFRP(K,J)=PI*PRA(K,J)*(A2+A4)
      IF (K.EQ.I1) GO TO 10
      VFAP(K,K1)=2.*PI*(A3*PRA(K,K1)+B2*PRA(K+1,K1))/(3.*A2+B3)
   10 VOL(K,J)=PI*PRA(K,J)*(((B2*PRA(K+1,K1)+A3*PRA(K,K1))/(PRA(K+1,K1)+
     1PRA(K,K1))+(B5*PRA(K-1,K1)+A3*PRA(K,K1))/(PRA(K-1,K1)+PRA(K,K1))))
     2*(B3*PRA(K,K1-1)+A2*PRA(K,K1)+B4*PRA(K,K1-1)+A4*PRA(K,K1))/(PRA(K,
     3K1-1)+PRA(K,K1)))/4.
      DO 11 K1=1,J
      VOL(1,K1)=2.*PI*PAX(2,1)**3/3.
   11 VOL(I,K1)=VOL(1,K1)
      PRINT 56
      K=1
      DO 12 K1=2,J1
      VFRP(I,K1)=0.
      VFRP(1,K1)=VFRP(I,K1)
      VFAP(I,K1)=0.
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      B3=SQRT((PAX(K+1,K1-1)-PAX(K,K1-1))**2+(PRA(K+1,K1-1)-PRA(K,K1-1))
     1**2)
```

```
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFAP(1,K1)=1.5*PRA(2,K1)*(B1+B2)/A2
   12 VFAP(I1,K1)=VFAP(1,K1)
      VFRP(1,J)=PI*B8**2/4.
      VFRP(I,J)=VFRP(1,J)
      VFAP(I,J)=0.
      K=1
      K1=J
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B2=SQRT((PAX(K+1,K1-1)-PAX(K+1,K1))**2+(PRA(K+1,K1-1)-PRA(K+1,K1))
     1**2)
      VFAP(1,J)=1.5*PRA(2,J)*B2/A2
      VFAP(I1,J)=VFAP(1,J)
      VFRP(1,1)=0.
      K=1
      K1=1
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      VFAP(1,1)=1.5*B1*B1/A2
      VFAP(I1,1)=VFAP(1,1)
      DO 13 K=2,I1
      A1=SQRT((PAX(K,K1+1)-PAX(K,K1))**2+(PRA(K,K1+1)-PRA(K,K1))**2)
      A2=SQRT((PAX(K+1,K1)-PAX(K,K1))**2+(PRA(K+1,K1)-PRA(K,K1))**2)
      A4=SQRT((PAX(K-1,K1)-PAX(K,K1))**2+(PRA(K-1,K1)-PRA(K,K1))**2)
      B1=SQRT((PAX(K+1,K1+1)-PAX(K+1,K1))**2+(PRA(K+1,K1+1)-PRA(K+1,K1))
     1**2)
      B6=SQRT((PAX(K-1,K1+1)-PAX(K-1,K1))**2+(PRA(K-1,K1+1)-PRA(K-1,K1))
     1**2)
      B7=SQRT((PAX(K-1,K1+1)-PAX(K,K1+1))**2+(PRA(K-1,K1+1)-PRA(K,K1+1))
     1**2)
      B8=SQRT((PAX(K+1,K1+1)-PAX(K,K1+1))**2+(PRA(K+1,K1+1)-PRA(K,K1+1))
     1**2)
      VFRP(K,1)=A2+B8+A4+B7
      IF (K.EQ.I1) GO TO 13
```

```
      VFAP(K,1)=PI*(B1+A1)**2/(3.*A2+B8)/4.
   13 VOL(K,1)=A2*PI*(3.*A1+B1)**2/128.+A4*PI*(3.*A1+B6)**2/128.
      VFRP(I,1)=0.
      VFAP(I,1)=0.
      DO 14 K=1,I
      PRINT 68, K,(VFAP(K,L),L=1,J)
   14 PRINT 68, K,(VFRP(K,L),L=1,J)
      AREA=0.
      DO 15 K=1,I
   15 AREA=AREA+VFRP(K,J)
      PRINT 74, AREA
      PRINT 69
      DO 16 K=1,I
   16 PRINT 68, K,(VOL(K,L1),L1=1,J)
      VOLUME=0.
      DO 17 K=1,I
      DO 17 K1=1,J
   17 VOLUME=VOLUME+VOL(K,K1)
      PRINT 75, VOLUME
      KR=0.217
      KA=.30
      DO 18 K=1,I
      H(K)=5.
   18 HD(K)=300.
C     SET UP INITIAL CONDITIONS
      DO 19 K=1,I
      DO 19 K1=1,J
      T(K,K1)=70.
      RHO(K,K1)=58.8
      ST(K,K1)=T(K,K1)
      RDM(K,K1)=5.88
      MC(K,K1)=(RHO(K,K1)-RDM(K,K1))/RDM(K,K1)
      SMC(K,K1)=MC(K,K1)
   19 CP(K,K1)=0.93
C     SOLVE TEMP AND MOISTURE DISTRIBUTION BY ALLADA ●UON
      TIME=0.
```

```
      HT=0.
      SHT=0.
      MT=0.
      SMT=0.
      READ 64, DELT,TA,RH
      READ 64, AK
      READ 64, AN
      CA=RH*CWS(TA)
      PRINT 70, DELT
      DO 62 L=1,5000
      DO 25 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CWS(T(K,J))
      DO 25 K1=1,J
      IF (MC(K,K1)-.022) 20,20,21
   20 L1=1
      GO TO 24
   21 IF (MC(K,K1)-.53) 22,22,23
   22 L1=2
   23 L1=3
   24 DMR(K,K1)=RW*(T(K,K1)+460.)*(4.5535E-18*MC(K,K1)**6.023+1.779E-14*
     1(T(K,K1)-70.))*MC(K,K1)/(MC(K,K1)-AK(L1))
      DMR(K,K1)=AMAX1(DMR(K,K1),0.)
   25 DMA(K,K1)=414.*DMR(K,K1)
      AM1=RDM(1,1)*VOL(1,1)/DELT
      A1=RHO(1,1)*CP(1,1)*VOL(1,1)/DELT
      G=0.
      GM=0.
      DO 26 K1=1,J
      GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*(MC(2,K1)-MC(1,1))/2.
   26 G=G+KA*VFAP(1,K1)*(T(2,K1)-T(1,1))
      MC(1,1)=(AM1*MC(1,1)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+HD(1)*VFRP(1,J)*A
     1M2(MC(1,1),AK,AN,CWS(T(1,1))))
      T(1,1)=(A1*T(1,1)+G+H(1)*VFRP(1,J)*TA)/(A1+H(1)*VFRP(1,J))
      DO 27 K1=2,J
      MC(1,K1)=MC(1,1)
   27 T(1,K1)=T(1,1)
```

```
      DO 29 K=2,I1
      AM1=RDM(K,1)*VOL(K,1)/DELT
      BM1=(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,1)/2.
      A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
      B4=KA*VFAP(K-1,1)
      MC(K,1)=(AM1*MC(K,1)+BM1*MC(K-1,1)+(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)
     1*(MC(K+1,1)-MC(K,1))/2.+(DMR(K,1)+DMR(K,2))*VFRP(K,1)*(MC(K,2)-MC(
     2K,1))/2.)/(AM1+BM1)
      T(K,1)=(A1*T(K,1)+B4*T(K-1,1)+KR*VFRP(K,1)*(T(K,2)-T(K,1))+KA*VFAP
     1(K,1)*(T(K+1,1)-T(K,1)))/(A1+B4)
      DO 28 K1=2,J1
      AM1=RDM(K,K1)*VOL(K,K1)/DELT
      A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
      BM1=(DMA(K,K1)+DMA(K-1,K1))*VFAP(K-1,K1)/2.
      BM2=(DMR(K,K1)+DMR(K,K1-1))*VFRP(K,K1-1)/2.
      B3=KR*VFRP(K,K1-1)
      B4=KA*VFAP(K-1,K1)
      MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K-1,K1)+BM2*MC(K,K1-1)+(DMA(K,K1)+DM
     1A(K+1,K1))*VFAP(K,K1)*(MC(K+1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,K1
     2+1))*VFRP(K,K1)*(MC(K,K1+1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
   28 T(K,K1)=(A1*T(K,K1)+B3*T(K,K1-1)+B4*T(K-1,K1)+KR*VFRP(K,K1)*(T(K,K
     11+1)-T(K,K1))+KA*VFAP(K,K1)*(T(K+1,K1)-T(K,K1)))/(A1+B3+B4)
      AM1=RDM(K,J)*VOL(K,J)/DELT
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      BM1=(DMA(K,J)+DMA(K-1,J))*VFAP(K-1,J)/2.
      BM2=(DMR(K,J)+DMR(K,J1))*VFRP(K,J1)/2.
      B3=KR*VFRP(K,J1)
      B4=KA*VFAP(K-1,J)
      MC(K,J)=(AM1*MC(K,J)+BM1*MC(K-1,J)+BM2*MC(K,J1)+(DMA(K,J)+DMA(K+1,
     1J))*VFAP(K,J)*(MC(K+1,J)-MC(K,J))/2.+HD(K)*VFRP(K,J)*CA)/(AM1+BM1+
     2BM2+HD(K)*VFRP(K,J)*AM2(MC(K,J),AK,AN,CWS(T(K,J))))
   29 T(K,J)=(A1*T(K,J)+B3*T(K,J1)+B4*T(K-1,J)+KA*VFAP(K,J)*(T(K+1,J)-T(
     1K,J))+H(K)*VFRP(K,J)*(TA-T(K,J)))/(A1+B3+B4)
      AM1=RDM(I,1)*VOL(I,1)/DELT
      A1=RHO(I,1)*CP(I,1)*VOL(I,1)/DELT
      GM1=0.
```

```
      GM=GM1
      G1=0.
      G=G1
      DO 30 K1=1,J
      GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*MC(I1,K1)/2.
      G=G+KA*VFAP(I1,K1)*T(I1,K1)
      GM1=GM1+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)/2.
   30 G1=G1+KA*VFAP(I1,K1)
      MC(I,1)=(AM1*MC(I,1)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+GM1+HD(I)*VFRP(I,
     1J)*AM2(MC(I,1),AK,AN,CWS(T(I,1))))
      T(I,1)=(A1*T(I,1)+G+H(I)*VFRP(I,J)*(TA-T(I,1)))/(A1+G1)
      DO 31 K1=2,J
      MC(I,K1)=MC(I,1)
   31 T(I,K1)=T(I,1)
      DO 32 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CWS(T(K,J))
      MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      T(K,J)=T(K,J)-HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT*1054./(RHO(K,J)*CP(K
     1,J)*VOL(K,J))
   32 HT=HT+VFRP(K,J)*H(K)*(T(K,J)-TA)*DELT
      G=0.
      GM=0.
      DO 33 K1=1,J
      GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*(MC(I1,K1)-MC(I,1))/2.
   33 G=G+KA*VFAP(I1,K1)*(T(I1,K1)-T(I,J))
      MC(I,J)=(AM1*MC(I,J)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+HD(I)*VFRP(I,J)*A
     1M2(MC(I,J),AK,AN,CWS(T(I,J))))
      T(I,J)=(A1*T(I,J)+G+H(I)*VFRP(I,J)*TA)/(A1+H(I)*VFRP(I,J))
      DO 34 K1=1,J1
      MC(I,K1)=MC(I,J)
   34 T(I,K1)=T(I,J)
      DO 36 KK=2,I1
      K=I+1-KK
      AM1=ROM(K,J)*VOL(K,J)/DELT
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      GM1=(DMA(K,J)+DMA(K+1,J))*VFAP(K,J)/2.
```

```
      B2=KA*VFAP(K,J)
      MC(K,J)=(AM1*MC(K,J)+BM1*MC(K+1,J)+HD(K)*VFRP(K,J)*CA+(DMA(K,J)+DM
     1A(K-1,J))*VFAP(K-1,J)*(MC(K-1,J)-MC(K,J))/2.+(DMR(K,J)+DMR(K,J1))*
     2VFRP(K,J1)*(MC(K,J1)-MC(K,J))/2.)/(AM1+BM1+HD(K)*VFRP(K,J)*AM2(MC(
     3K,J),AK,AN,CWS(T(K,J))))
      T(K,J)=(A1*T(K,J)+B2*T(K+1,J)+KR*VFRP(K,J1)*(T(K,J1)-T(K,J))+KA*VF
     1AP(K-1,J)*(T(K-1,J)-T(K,J))+H(K)*VFRP(K,J)*TA)/(A1+B2+H(K)*VFRP(K,
     2J))
      DO 35 KK1=2,J1
      K1=J+1-KK1
      AM1=RDM(K,K1)*VOL(K,K1)/DELT
      A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
      BM1=(DMA(K,K1)+DMA(K+1,K1))*VFAP(K,K1)/2.
      BM2=(DMR(K,K1)+DMR(K,K1+1))*VFRP(K,K1)/2.
      B1=KR*VFRP(K,K1)
      B2=KA*VFAP(K,K1)
      MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K+1,K1)+BM2*MC(K,K1+1)+(DMA(K,K1)+DM
     1A(K-1,K1))*VFAP(K-1,K1)*(MC(K-1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,
     2K1-1))*VFRP(K,K1-1)*(MC(K,K1-1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
   35 T(K,K1)=(A1*T(K,K1)+B1*T(K,K1+1)+B2*T(K+1,K1)+KR*VFRP(K,K1-1)*(T(K
     1,K1-1)-T(K,K1))+KA*VFAP(K-1,K1)*(T(K-1,K1)-T(K,K1)))/(A1+B1+B2)
      AM1=RDM(K,1)*VOL(K,1)/DELT
      A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
      BM1=(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)/2.
      BM2=(DMR(K,1)+DMR(K,2))*VFRP(K,1)/2.
      B1=KR*VFRP(K,1)
      B2=KA*VFAP(K,1)
      MC(K,1)=(AM1*MC(K,1)+BM1*MC(K+1,1)+BM2*MC(K,2)+(DMA(K,1)+DMA(K-1,1
     1))*VFAP(K-1,1)*(MC(K-1,1)-MC(K,1))/2.)/(AM1+BM1+BM2)
   36 T(K,1)=(A1*T(K,1)+B1*T(K,2)+B2*T(K+1,1)+KA*VFAP(K-1,1)*(T(K-1,1)-T
     1(K,1)))/(A1+B1+B2)
      AM1=RDM(1,1)*VOL(1,1)/DELT
      A1=RHO(1,J)*CP(1,J)*VOL(1,J)/DELT
      GM1=0.
      GM=GM1
      G1=0.
```

```
      G=G1
      DO 37 K1=1,J
      GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*MC(2,K1)/2.
      G=G+KA*VFAP(1,K1)*T(2,K1)
      GM1=GM1+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)/2.
   37 G1=G1+KA*VFAP(1,K1)
      MC(1,J)=(AM1*MC(1,J)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+GM1+HD(1)*VFRP(1,
     1J)*AM2(MC(1,J),AK,AN,CWS(T(1,J))))
      T(1,J)=(A1*T(1,J)+G+H(1)*VFRP(1,J)*(TA-T(1,J)))/(A1+G1)
      DO 38 K1=1,J1
      MC(1,K1)=MC(1,J)
   38 T(1,K1)=T(1,J)
      DO 39 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CWS(T(K,J))
      MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      T(K,J)=T(K,J)-HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT*1054./(RHO(K,J)*CP(K
     1,J)*VOL(K,J))
   39 HT=HT+VFRP(K,J)*H(K)*(T(K,J)-TA)*DELT
      AM1=RDM(I,1)*VOL(I,1)/DELT
      A1=RHO(I,1)*CP(I,1)*VOL(I,1)/DELT
      G=0.
      GM=0.
      DO 40 K1=1,J
      GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*(MC(I1,K1)-MC(I,1))/2.
   40 G=G+KA*VFAP(I1,K1)*(T(I1,K1)-T(I,1))
      MC(I,1)=(AM1*MC(I,1)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+HD(I)*VFRP(I,J)*A
     1M2(MC(I,1),AK,AN,CWS(T(I,1))))
      T(I,1)=(A1*T(I,1)+G+H(I)*VFRP(I,J)*TA)/(A1+H(I)*VFRP(I,J))
      DO 41 K1=2,J
      MC(I,K1)=MC(I,1)
   41 T(I,K1)=T(I,1)
      DO 43 KK=2,I1
      K=I+1-KK
      AM1=RDM(K,1)*VOL(K,1)/DELT
      BM1=(DMA(K,1)+DMA(K+1,1))*VFAP(K,1)/2.
      A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
```

```
      B2=KA*VFAP(K,1)
      MC(K,1)=(AM1*MC(K,1)+BM1*MC(K+1,1)+(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,
     11)*(MC(K-1,1)-MC(K,1))/2.+(DMR(K,1)+DMR(K,2))*VFRP(K,1)*(MC(K,2)-M
     2C(K,1))/2.)/(AM1+BM1)
      T(K,1)=(A1*T(K,1)+B2*T(K+1,1)+KR*VFRP(K,1)*(T(K,2)-T(K,1))+KA*VFAP
     1(K-1,1)*(T(K-1,1)-T(K,1)))/(A1+B2)
      DO 42 K1=2,J1
      AM1=RUM(K,K1)*VOL(K,K1)/DELT
      A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
      BM1=(DMA(K,K1)+DMA(K+1,K1))*VFAP(K,K1)/2.
      BM2=(DMR(K,K1)+DMR(K,K1-1))*VFRP(K,K1-1)/2.
      B2=KA*VFAP(K,K1)
      B3=KR*VFRP(K,K1-1)
      MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K+1,K1)+BM2*MC(K,K1-1)+(DMA(K,K1)+DM
     1A(K-1,K1))*VFAP(K-1,K1)*(MC(K-1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,
     2K1+1))*VFRP(K,K1)*(MC(K,K1+1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
   42 T(K,K1)=(A1*T(K,K1)+B2*T(K+1,K1)+B3*T(K,K1-1)+KR*VFRP(K,K1)*(T(K,K
     11+1)-T(K,K1))+KA*VFAP(K-1,K1)*(T(K-1,K1)-T(K,K1)))/(A1+B2+B3)
      AM1=RUM(K,J)*VOL(K,J)/DELT
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      BM1=(DMA(K,J)+DMA(K+1,J))*VFAP(K,J)/2.
      BM2=(DMR(K,J)+DMR(K,J1))*VFRP(K,J1)/2.
      B2=KA*VFAP(K,J)
      B3=KR*VFRP(K,J1)
      MC(K,J)=(AM1*MC(K,J)+BM1*MC(K+1,J)+BM2*MC(K,J1)+(DMA(K,J)+DMA(K-1,
     1J))*VFAP(K-1,J)*(MC(K-1,J)-MC(K,J))/2.+HD(K)*VFRP(K,J)*CA)/(AM1+BM
     21+BM2+HD(K)*VFRP(K,J)*AM2(MC(K,J),AK,AN,CWS(T(K,J))))
   43 T(K,J)=(A1*T(K,J)+B2*T(K+1,J)+B3*T(K,J1)+KA*VFAP(K-1,J)*(T(K-1,J)-
     1T(K,J))+H(K)*VFRP(K,J)*(TA-T(K,J)))/(A1+B2+B3)
      AM1=RUM(1,1)*VOL(1,1)/DELT
      A1=RHO(1,1)*CP(1,1)*VOL(1,1)/DELT
      GM1=0.
      GM=GM1
      G1=0.
      G=G1
      DO 44 K1=1,J
```

```
      GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*MC(2,K1)/2.
      G=G+KA*VFAP(1,K1)*T(2,K1)
      GM1=GM1+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)/2.
   44 G1=G1+KA*VFAP(1,K1)
      MC(1,1)=(AM1*MC(1,1)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+GM1+HD(1)*VFRP(1,
     1J)*AM2(MC(1,1),AK,AN,CWS(T(1,1))))
      T(1,1)=(A1*T(1,1)+G+H(1)*VFRP(1,J)*(TA-T(1,1)))/(A1+G1)
      DO 45 K1=2,J
      MC(1,K1)=MC(1,1)
   45 T(1,K1)=T(1,1)
      DO 46 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CWS(T(K,J))
      MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      T(K,J)=T(K,J)-HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT*1054./(RHO(K,J)*CP(K
     1,J)*VOL(K,J))
   46 HT=HT+VFRP(K,J)*H(K)*(T(K,J)-TA)*DELT
      DO 47 K=1,I
      DO 47 K1=1,J
   47 U(K,K1)=T(K,K1)
      G=0.
      GM=0.
      DO 48 K1=1,J
      GM=GM+(DMA(1,1)+DMA(2,K1))*VFAP(1,K1)*(MC(2,K1)-MC(1,1))/2.
   48 G=G+KA*VFAP(1,K1)*(U(2,K1)-U(1,J))
      MC(1,J)=(AM1*MC(1,J)+GM+HD(1)*VFRP(1,J)*CA)/(AM1+HD(1)*VFRP(1,J)*A
     1M2(MC(1,J),AK,AN,CWS(U(1,J))))
      DO 49 K1=1,J1
      MC(1,K1)=MC(1,J)
   49 U(1,K1)=U(1,J)
      DO 51 K=2,I1
      AM1=RDM(K,J)*VOL(K,J)/DELT
      A1=RHO(K,J)*CP(K,J)*VOL(K,J)/DELT
      BM1=(DMA(K,J)+DMA(K-1,J))*VFAP(K-1,J)/2.
      B4=KA*VFAP(K-1,J)
      MC(K,J)=(AM1*MC(K,J)+BM1*MC(K-1,J)+HD(K)*VFRP(K,J)*CA+(DMA(K,J)+DM
     1A(K+1,J))*VFAP(K,J)*(MC(K+1,J)-MC(K,J))/2.+(DMR(K,J)+DMR(K,J1))*VF
```

```
2RP(K,J1)*(MC(K,J1)-MC(K,J))/2.)/(AM1+BM1+HD(K)*VFRP(K,J)*AM2(MC(K,
3J),AK,AN,CWS(U(K,J))))
 U(K,J)=(A1*U(K,J)+B4*U(K-1,J)+H(K)*VFRP(K,J)*TA+KA*VFAP(K,J)*(U(K+
1I,J)-U(K,J))+KR*VFRP(K,J1)*(U(K,J1)-U(K,J)))/(A1+B4+H(K)*VFRP(K,J)
2)
 DO 50 KK1=2,J1
 K1=J+1-KK1
 AM1=RDM(K,K1)*VOL(K,K1)/DELT
 A1=RHO(K,K1)*CP(K,K1)*VOL(K,K1)/DELT
 BM1=(DMA(K,K1)+DMA(K-1,K1))*VFAP(K-1,K1)/2.
 BM2=(DMR(K,K1)+DMR(K,K1+1))*VFRP(K,K1)/2.
 B1=KR*VFRP(K,K1)
 B4=KA*VFAP(K-1,K1)
 MC(K,K1)=(AM1*MC(K,K1)+BM1*MC(K-1,K1)+BM2*MC(K,K1+1)+(DMA(K,K1)+DM
1A(K+1,K1))*VFAP(K,K1)*(MC(K+1,K1)-MC(K,K1))/2.+(DMR(K,K1)+DMR(K,K1
2-1))*VFRP(K,K1-1)*(MC(K,K1-1)-MC(K,K1))/2.)/(AM1+BM1+BM2)
50 U(K,K1)=(A1*U(K,K1)+B1*U(K,K1+1)+B4*U(K-1,K1)+KA*VFAP(K,K1)*(U(K+1
1,K1)-U(K,K1))+KR*VFRP(K,K1-1)*(U(K,K1-1)-U(K,K1)))/(A1+B1+B4)
 AM1=RDM(K,1)*VOL(K,1)/DELT
 A1=RHO(K,1)*CP(K,1)*VOL(K,1)/DELT
 BM1=(DMA(K,1)+DMA(K-1,1))*VFAP(K-1,1)/2.
 BM2=(DMR(K,1)+DMR(K,2))*VFRP(K,1)/2.
 B1=KR*VFRP(K,1)
 B4=KA*VFAP(K-1,1)
 MC(K,1)=(AM1*MC(K,1)+BM1*MC(K-1,1)+BM2*MC(K,2)+(DMA(K,1)+DMA(K+1,1
1))*VFAP(K,1)*(MC(K+1,1)-MC(K,1))/2.)/(AM1+BM1+BM2)
51 U(K,1)=(A1*U(K,1)+B1*U(K,2)+B4*U(K-1,1)+KA*VFAP(K,1)*(U(K+1,1)-U(K
1,1)))/(A1+B1+B4)
 AM1=RDM(I,1)*VOL(I,1)/DELT
 A1=RHO(I,J)*CP(I,J)*VOL(I,J)/DELT
 GM1=0.
 GM=GM1
 G1=0.
 G=G1
 DO 52 K1=1,J
 GM=GM+(DMA(I,1)+DMA(I1,K1))*VFAP(I1,K1)*MC(I1,K1)/2.
```

```
      G=G+KA*VFAP(I1,K1)*U(I1,K1)
      GM1=GM1+(DMA(I,I)+DMA(I1,K1))*VFAP(I1,K1)/2.
   52 G1=G1+KA*VFAP(I1,K1)
      MC(I,J)=(AM1*MC(I,J)+GM+HD(I)*VFRP(I,J)*CA)/(AM1+GM1+HD(I)*VFRP(I,
     1J)*AM2(MC(I,J),AK,AN,CWS(U(I,J))))
      U(I,J)=(A1*U(I,J)+G+H(I)*VFRP(I,J)*(TA-U(I,J)))/(A1+G1)
      DO 53 K1=1,J1
      MC(I,K1)=MC(I,J)
   53 U(I,K1)=U(I,J)
      DO 55 K=1,I
      CW(K)=AW(MC(K,J),AK,AN)*CWS(T(K,J))
      MT=MT+HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT
      IF (L.GT.5) GO TO 54
      PRINT 68, K,(MC(K,L1),L1=1,J)
   54 CONTINUE
      U(K,J)=U(K,J)-HD(K)*VFRP(K,J)*(CW(K)-CA)*DELT*1054./(RHO(K,J)*CP(K
     1,J)*VOL(K,J))
   55 HT=HT+VFRP(K,J)*H(K)*(U(K,J)-TA)*DELT
      DO 56 K=1,I
      DO 56 K1=1,J
   56 T(K,K1)=U(K,K1)
      IF (TIME.GT.6.) GO TO 63
C     CCCC CHECK TO MAKE SURE THAT THE STEPS JUST TAKEN ARE OK
C     CCCC IF NOT, BACKTRACK
      A=0.
      DO 57 K=1,I
      DO 57 K1=1,J
      B=ABS(T(K,K1)-ST(K,K1))
   57 A=AMAX1(A,B)
      IF (A.LT.4.) GO TO 59
      HT=SHT
      MT=SMT
      DO 58 K=1,I
      DO 58 K1=1,J
      MC(K,K1)=SMC(K,K1)
   58 T(K,K1)=ST(K,K1)
```

```
      DELT=DELT/2.
      GO TO 62
   59 TIME=TIME+4.*DELT
      PRINT 73, TIME,DELT
      SHT=HT
      SMT=MT
      DO 61 K=1,I
      IF (L.GT.5) GO TO 60
      PRINT 68, K,(T(K,L1),L1=1,J)
   60 CONTINUE
      DO 61 K1=1,J
   61 ST(K,K1)=U(K,K1)
      PRINT 72, HT,MT,DELT
C     CCCC DOUBLE STEP SIZE IF O.K.
      IF (A.GT.2.) GO TO 62
      DELT=DELT*2.
      DELT=AMIN1(DELT,.0016)
   62 CONTINUE
   63 CONTINUE
C
   64 FORMAT (8F10.0)
   65 FORMAT (11(1X,F11.7))
   66 FORMAT (1H1, 23H      VIEW FACTORS       )
   67 FORMAT (1X,F10.7,1X,F10.7,1X,F10.7,1X,I5,1X,I5)
   68 FORMAT (1X,I3,12(1X,F9.6))
   69 FORMAT (1H1, 21H                   VOLUMES)
   70 FORMAT (1H1, 35HRESULTS                 DELTA TIME =  ,F10.7)
   71 FORMAT (1H1, 30HHEAT AND MASS TRANSFER PROGRAM,/,1X, 11HNODE POINT
     1S)
   72 FORMAT (1X, 19HHEAT TRANSFERRED = ,F11.7, 25H BTU,  MASS TRANSFERR
     1ED= ,F11.7, 12H LB.  DELT= ,F9.7)
   73 FORMAT (1X, 22HTEMPERATURES,  TIME = ,F10.4,  9H  DELT = ,F10.8)
   74 FORMAT ( 27H THE TOTAL SURFACE AREA IS ,F10.6, 12H SQUARE FEET)
   75 FORMAT ( 21H THE TOTAL VOLUME IS ,F10.8, 11H CUBIC FEET)
      END
```

```
      FUNCTION PERIM(Z)
C     THIS FUNCTION IS USED WITH HTRAN, MTRAN,AND HMTRAN
      COMMON DIA,ZMAX,PI
      PERIM=DIA*(0.8322400*SIN(PI*Z/ZMAX)+0.3689900*SIN(02.*PI*Z/ZMAX)+0
     1.0462150*SIN(03.*PI*Z/ZMAX)+0.0074205*SIN(04.*PI*Z/ZMAX)+0.0822030
     2*SIN(05.*PI*Z/ZMAX)-0.0114820*SIN(06.*PI*Z/ZMAX)+0.0194130*SIN(07.
     3*PI*Z/ZMAX)+0.0074970*SIN(08.*PI*Z/ZMAX)+0.0145270*SIN(09.*PI*Z/ZM
     4AX)-0.0046454*SIN(10.*PI*Z/ZMAX))/2.
      END


      FUNCTION DPERIM(Z)
C     THIS FUNCTION IS USED WITH HTRAN, MTRAN,AND HMTRAN
      COMMON DIA,ZMAX,PI
      DPERIM=DIA*PI*(0.8322400*COS(PI*Z/ZMAX)+0.3689900*02.*COS(02.*PI*Z
     1/ZMAX)+0.0462150*03.*COS(03.*PI*Z/ZMAX)+0.0074205*04.*COS(04.*PI*Z
     2/ZMAX)+0.0822030*05.*COS(05.*PI*Z/ZMAX)-0.0114820*06.*COS(06.*PI*Z
     3/ZMAX)+0.0194130*07.*COS(07.*PI*Z/ZMAX)+0.0074970*08.*COS(08.*PI*Z
     4/ZMAX)+0.0145270*09.*COS(09.*PI*Z/ZMAX)-0.0046454*10.*COS(10.*PI*Z
     5/ZMAX))/(2.*ZMAX)
      END


      FUNCTION CWS(T)
C     THIS FUNCTION IS USED WITH HMTRAN
      A=ALOG(460.+T)
      CWS=EXP(54.6329-12301.688/(460.+T)-5.16923*A)*144./(1545.33/18.*(4
     160.+T))
      END
```

```
      FUNCTION AW(AM,AK,AN)
C     THIS FUNCTION IS USED WITH MTRAN AND HMTRAN
      DIMENSION AK(3), AN(3)
      IF (AM-0.022) 1,1,2
    1 L=1
      GO TO 5
    2 IF (AM-0.530) 3,3,4
    3 L=2
      GO TO 5
    4 L=3
    5 AW=(AM-AK(L))/AN(L)
      END
```

```
      FUNCTION AM2(AM,AK,AN,CAS)
      THIS FUNCTION IS USED WITH MTRAN AND HMTRAN
      DIMENSION AK(3), AN(3)
      IF (AM-0.022) 1,1,2
    1 L=1
      GO TO 5
    2 IF (AM-0.530) 3,3,4
    3 L=2
      GO TO 5
    4 L=3
    5 AM2=(AM-AK(L))*CAS/(AN(L)*AM)
      END
```