





THE 918

This is to certify that the

dissertation entitled
MATHEMATICS DEPARTMENT INSTRUCTORS'
CONCEPTUALIZATIONS OF THE ROLES
MATHEMATICS CONTENT COURSES PLAY
IN ELEMENTARY TEACHER EDUCATION PROGRAMS

presented by
Virginia L. Keen

has been accepted towards fulfillment
of the requirements for

Ph.D. degree in Education

Major professor

Date 8/2/94

**LIBRARY
Michigan State
University**

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.

DATE DUE	DATE DUE	DATE DUE
JAN 23 2011	_____	_____
SEP 18 2012 10 29 12	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____
_____	_____	_____

**MATHEMATICS DEPARTMENT INSTRUCTORS' CONCEPTUALIZATIONS
OF THE ROLES MATHEMATICS CONTENT COURSES PLAY
IN ELEMENTARY TEACHER EDUCATION PROGRAMS**

By

Virginia L. Keen

A DISSERTATION

**Submitted to
Michigan State University
in partial fulfillment of requirements
for the degree of**

DOCTOR OF PHILOSOPHY

Department of Teacher Education

1994

ABSTRACT

MATHEMATICS DEPARTMENT INSTRUCTORS' CONCEPTUALIZATIONS OF THE ROLES MATHEMATICS CONTENT COURSES PLAY IN ELEMENTARY TEACHER EDUCATION PROGRAMS

By

Virginia L. Keen

The subject of this study is the roles that mathematics department instructors see their mathematics content courses playing in elementary teacher education programs. These mathematics content courses are taught within one department (Mathematics) but are required as a part of a program within a different department (Teacher Education). While departmental policy from either department may suggest course content and design, instructors' decisions determine, in large part, the curricula of the courses. Instructors' commitments to particular roles for the courses and the grounding of their commitments are the focus of this study. This exploratory study describes six instructors' conceptualizations of the roles mathematics courses required in elementary teacher preparation should play. An analytic framework was developed and used to create portraits of each individual instructor's thinking. Analyses also examine three issues related to the critical role of students' learning of mathematical content across all of the instructors. The issues are: instructors' views of student preparation when entering the courses, the instructors'

choice of specific course content, and the type of student understanding sought by the instructors.

The roles identified by instructors were often idiosyncratic. The lack of input from outside of the Mathematics Education community is highlighted. Instructors' goals moved between two different expectations for student understanding: procedural understanding and conceptual understanding. The predominance of the goal of procedural understanding raises concern about the ability of the courses to serve programmatic roles aligned with the current reform movement in mathematics education.

Copyright by
VIRGINIA L. KEEN
1994

To
Ben and Shelly

ACKNOWLEDGMENTS

I mentally prepared an "Acknowledgments" section many times throughout the process of completing this work. Now that I am in a position to recognize all of those who have been instrumental in my ability to complete this dissertation, I feel overwhelmed, but I will try.

I wish to thank my Guidance Committee and Dissertation Committee members for their encouragement. Magdalene Lampert, Margret Buchmann, and Sharon Feiman-Nemser added very special dimensions to my Committees, and I thank them whole-heartedly for their encouragement. Two individuals were on both of my committees, Mary Kennedy and Glenda Lappan. Mary shared her wealth of knowledge about research on teacher education in ways that have shaped my work and serve to provide a model for the kinds of questions I will need to ask myself when undertaking future research. Glenda Lappan is a gem, precious and amazing. She took my writing on planes, buses, and who-knows-what-else, always responding to concerns I had and giving feedback with remarkable speed. She sat with me in times of desperation, providing support and, when necessary, a kick-in-the-pants. Her professionalism and personal warmth serve as witness to her knowledge, wisdom, and

compassion and as a model of the consummate mathematics teacher educator.

The road to the dissertation defense and the completion of the document has been much like a roller-coaster ride. The individual most responsible for guiding its course has been my Dissertation Director and the Chair of my Dissertation Committee, Daniel Chazan. Dan pushed without pushing me over the edge, though I did get close to going over a few times. This usually had very little to do with Dan's nudges, coming more from stresses outside the actual dissertation work. I hope that he feels a similar degree of satisfaction in its completion.

All of my Committee members have given so much to me, as professionals and as friends. I wish to publicly thank them here for all of their support. I look forward to continuing our relationships as I move into a new role.

This dissertation was totally dependent on the participation of a group of mathematics teacher educators. I commend them for their willingness to share their thoughts and for their openness in describing the courses they teach. I cannot thank them enough for their time and efforts.

To my family -- Jim, Ben, Shelly, my parents, Jim's parents, and all of the rest of our family: "Thank you" seems too simple, but, since words will never express the gratitude that I feel for all of your support and toleration of my often selfish actions, "Thank you" seems appropriate. To my friends: You know who you are and how much I have appreciated your smiles, encouragement, and questions. Some of you deserve special thanks for being willing to sit and read

sections and then talk with me about issues that were raised. I really enjoyed those conversations and hope that we will continue to get together to talk about other "important" things over coffee or a meal -- especially if chocolate is involved.

I have been richly blessed by those around me and hope that, in some way, I have touched your lives in a special way. I have made sacrifices in order to complete this work. In those sacrifices, others have been affected. I ask for forgiveness if the sacrifices have seemed too great, and I pray that the sacrifices are shown to be worthwhile, in the long run.

TABLE OF CONTENTS

LIST OF FIGURES	xii
CHAPTER 1: INTRODUCTION	1
Research Question and the Rationale for Studying This Question	6
CHAPTER 2: LITERATURE REVIEW	8
Studies of Mathematics Teacher Education	10
Studies of Mathematics Educators	14
CHAPTER 3: ANALYTIC FRAMEWORK	23
Possible Roles and Rationales	24
Learning Mathematical Content	26
Learning the Norms of Mathematics	30
Learning to Teach Mathematics	32
Gate-keeping	34
Feeling Good About Mathematics	36
Endnote	38
CHAPTER 4: RESEARCH METHODOLOGY	39
Site and Subjects	39
Primary Site Information and Background of the Instructors	39
Secondary Site Information and Background of the Instructor	44
General Overview of Sites and Subjects	45
Data Collection	47

CHAPTER 5: ANALYSIS	52
Instructors' Conceptualizations of the Roles	58
Kara: Multiple Roles and Mixed Messages	62
Cynthia: Developing Competence and Changing Perspectives	78
Jeff: Gate-keeping, Learning Content, and Developing Dispositions	97
Frank: Revising the Content	119
Gail: Reducing Anxiety	138
Craig: Content, Norms, Dispositions, and Attitudes ..	155
The Role of Learning Mathematical Content	176
What Should Students Know When They Enter the Course?	179
What Should Be the Mathematical Content of the Courses?	190
What Kind of Learning of Mathematics is Sought?	212
Summary of the Analyses	230
Characterizations of Mathematics and Teaching for the Learning of Mathematics	232
Final Thoughts	235
CHAPTER 6: CONCLUSIONS AND IMPLICATIONS	239
Instructors' Views of the Roles	240
Instructors' Views and the Analytic Framework	240
Instructors' Views of Learning Mathematical Content	242
Conflicting Views	245
Intra-instructor Conflicts and Dilemmas	245
Inter-instructor Variation and Isolation	246
Implications for Teacher Education	251
Need for Elementary School Connections	252
Need for On-going Conversations in Many Contexts	254
Need for the Goal of Conceptual Understanding	256
Future Research Directions	260
Summary	262
APPENDICES	265
Appendix A: Syllabi for Mathematics for the Elementary School Teacher Courses at Normal University	265

Appendix B: Interview Instrument One: Philosophical Interview	270
Appendix C: Interview Instrument Two: Content Interview	275
Appendix D: Interview Instrument Three: Observation Interview	279
Appendix E: Proposed Syllabus (Frank Adams) for Mathematics for the Elementary School Teacher First Course at Normal University	281
LIST OF REFERENCES	282

List of Figures

Figure 1.	Instructor Information	46
Figure 2.	Predominant Roles for Instructors	61
Figure 3.	Developing the Formula for the Area of a Circle	161

CHAPTER 1

INTRODUCTION

After having taught K-13 mathematics in various institutions in the United States, I spent several years teaching mathematics content courses for prospective elementary school teachers at a university. Conversations with others who taught these courses and with mathematics educators working in schools raised major questions and concerns in my mind about what the mission of the courses was: Was it to learn mathematics? Was it to learn how to teach? Was it to keep inept people out of the classroom? Was it to build positive attitudes, and, if so, toward what? Teaching these courses was hard; there were competing agendas and compelling problems that made the courses difficult to teach.

Struggles arose because many students came to mathematics courses for prospective elementary school teachers with weak backgrounds or only procedural knowledge of mathematics. If a goal was to be sure that the prospective elementary school teachers knew “enough” mathematics to successfully teach elementary school mathematics, how much was enough and how was I to teach it? Was covering a large amount of mathematics more or less important than the development of conceptual knowledge of a subset of the content? Courses were only fifteen weeks long and did not meet daily, magnifying the struggles. It was impossible to “cover” all of the mathematics taught in elementary education, let alone beyond that level. I wanted my students to understand mathematics beyond what they would be teaching in order to give them a sense of

the usefulness and connectedness of topics in the elementary curriculum to more sophisticated mathematics. Thus, my questions about the amount and depth of content coverage included concern for content that was not necessarily what students would teach.

The population of students in mathematics for elementary school teacher courses was often not only lacking in subject matter knowledge, but they also were not aware that they were lacking. They had generally been successful throughout their elementary school years and enjoyed being in school. They were comfortable in schools -- they wanted to spend their professional lives in schools. The notion that what they “knew” about mathematics was deficient was not part of their mentality. Part of my job, as I saw it, was to make them aware of this in a way that did not destroy their self-esteem or motivation to become a teacher. I paired this with a concern to improve their attitudes about mathematics. Many of the students saw mathematics as a subject to be taught, a necessary evil, much like a medicine that tastes bad but must be taken to get well. I wanted them to feel good about doing mathematics and to see mathematics as personally relevant in their world. The students did not seem to see these goals as reasonable or possible.

This is not to say that all students preparing to be elementary school teachers were not capable of doing or motivated to do mathematics. There were many who came to the courses with considerable knowledge and interest. This was a mixed blessing. This meant that there was a considerable spread in abilities, a situation that I attempted to use to my advantage. However, some capable students assumed, as did some of the less capable, that they

already knew all they needed to know and that taking the courses was simply fulfilling a requirement that they could not avoid. These students had taken calculus in high school or college in most cases. Often, they were looking for something different in the courses -- they wanted to learn how to teach mathematics to children. They felt confident and competent with mathematics as they had experienced it and were ready to get to the task of teaching it. At issue for them was how to teach it, not what mathematics is fundamentally or how they understood it. Actually, making sense of mathematics was threatening to many for whom the issue of understanding mathematics was difficult to appreciate because it was removed from their experience. They had been successful in traditionally taught and assessed mathematics classes. Their experience did not include really grappling with the concepts or understanding the rules and practices of the discipline.

There seemed to be a shared desire on the part of students, regardless of their mathematical preparation, for the courses to prepare them to teach mathematics. For most, the notion of learning mathematics with more depth and understanding seemed confusing and alien, even counter-productive to their aims. This undercurrent of skepticism complicated my task. I needed to convince them of the value of knowing and doing mathematics for its own sake as a way of preparing to become elementary school teachers.

With all of these concerns, what should I try to accomplish in these mathematics courses? What goals seemed especially important to accomplish in courses of this type? What would it mean to try to accomplish these goals? What would class look like?

How would desired roles influence the decisions and actions of the instructor? How did this type of course fit into the teacher education program for elementary certification? What responsibilities did I have and what roles were the courses to play in students' teacher preparation?

Their program required prospective elementary school teachers to take mathematics content courses and an elementary mathematics methods course. How were these to build on each other? How did the courses taught in the Mathematics Department relate to other courses in the general elementary program? I had rare contact with the Teacher Education Department, and what contact I had was not related to the role of the mathematics content courses in students' programs.

In this recounting of my struggles and dilemmas, I have noted the most pressing of my concerns. Teaching the mathematics content courses for prospective elementary school teachers held tremendous challenge. I knew I wanted students to have a different experience, in most cases, from what they had experienced in much of their K-8¹ education, plus I wanted them to go beyond that level of understanding. What roles did I see the courses playing in the preparation of prospective elementary school teachers? What was the mission of the courses -- what were students to learn, how were they to learn it, and why were they to learn it? What did I need to know and be able to do to make that possible for the students?

¹ Some institutions choose to focus their courses on K-6 teacher preparation. I chose to use K-8 throughout this study.

Concurrent with my questioning about the mathematics education of teachers, the National Council of Teachers of Mathematics (NCTM) was preparing a pair of documents, the Curriculum And Evaluation Standards for School Mathematics (NCTM, 1989) and the Professional Standards for Teaching Mathematics (NCTM, 1991), which set out a vision for K-12 mathematics education. A move toward a realization of this vision requires preparing teachers to think about, do, and teach mathematics in different ways. The question of what this means for mathematics teacher educators drew my attention.

Mathematics content courses that are required as a part of elementary teacher preparation are designed for a particular set of students. These students are preparing to teach mathematics as part of their professional careers. These careers will span time that should be, according to the vision described in the NCTM Standards documents (1989, 1991), a time of tremendous change in instructional practice. What are students learning in the required mathematics content courses that will assist them in becoming the kinds of teachers needed to support these changes? How are mathematics teacher educators responding to the multiple calls for change in university teaching -- both of mathematics courses and of mathematics education courses? What and how are they teaching mathematics education content courses, and why is it important to be doing this? It was with these and other questions I began my current examination of the mathematics content preparation of elementary school teachers through the eyes of mathematics teacher educators.

Research Question and the Rationale for Studying This Question

This exploratory study involves looking at mathematics education content courses through the eyes of the instructors of these courses. The research question is “How do mathematics department instructors of mathematics content courses for prospective elementary school teachers conceptualize the roles played by their courses?” These mathematics content courses are taught within one department (Mathematics) but are required as a part of a program within a different department (Teacher Education). While departmental policy from either department may suggest course content and design, instructors' decisions determine, in large part, the curricula of the courses. Instructors' commitments to particular roles for the courses and the grounding of their commitments are the focus of this study. Instructors' beliefs and assumptions, about students, mathematics, and mathematics teaching and learning ground the ways instructors conceptualize the courses. It is through consideration and analysis of these beliefs and assumptions that I hope to identify and describe the most critical roles that instructors see these courses playing in elementary teacher preparation programs.

By examining the meaning that a set of instructors make of these questions, we will learn more about the range of conceptualizations of the roles the courses play and learn more about the factors that instructors see as influencing the roles mathematics content courses play in teacher preparation. Additionally, this research will provide a basis upon which questions for discussion and further research can be built,

particularly with respect to current reform movements in mathematics education.

A new sense of the variety and strength of the beliefs that guide instructors can result from this examination of their views. Without this kind of groundwork research, we will remain unaware of or simply assume we know, based on our own pre-conceived notions, what the concerns and needs of mathematics teacher educators are at this time of reform in mathematics education.

At a time of great optimism for change in the way mathematics is experienced as a discipline in schools K-8, we need to consider what the current situation is in courses designed for prospective elementary teachers. What kinds of support and opportunities do instructors need to foster change? Ball (1992), in writing about hurdles to the implementation of the NCTM Standards (1989, 1991), acknowledged that the construction of ways to teach that support the vision of the Standards will require new learning, support and resources of new kinds by mathematics educators.

This study provides a picture of the ways instructors at one "extended"² institution, one that serves a large number of future teachers, see the courses they teach as a part of elementary teacher preparation. The roles they see the courses playing and their conceptualizations of the roles serve to inform recommendations made for improving the mathematical preparation of elementary school teachers the mathematics education of their future pupils.

² Extended institution is used to describe the university and community college (whose students often transfer to the university) in which the instructors taught.

CHAPTER 2

LITERATURE REVIEW

Mathematics education K-12 in the United States is currently the subject of much conversation inside and outside the education establishment. Society has declared the improvement of the mathematics education of children to be of paramount concern. Groups and individuals outside the educational sphere as prominent as the President of the United States are calling for improvement in the preparation of elementary school mathematics teachers to facilitate improvement in the mathematics education of children. Within the professional ranks, documents such as the NCTM Professional Teaching Standards (1991) and The Mathematical Association of America (MAA) A Call for Change (1991) describe in some detail what 'ideal' mathematics teachers in the classrooms of the 1990's and beyond must know and be able to do. They also make recommendations as to how elementary school mathematics teachers are to be prepared. Whether an outsider or insider to school mathematics education, all agree that the mathematics education of elementary school teachers is important to examine.

A common assumption or myth is that more subject matter knowledge is all that students needed (NCRTL, 1992; COMAP, 1992). The implication is that not only should prospective elementary school teachers take more secondary-level mathematics, but they should also take more mathematics after high school. Mathematics departments often create and design specific courses to be part of teacher preparation, and programs require them for teacher

certification. Elementary teacher education programs often involve one to three content courses. These courses for prospective elementary school teachers are usually the only mathematics courses students take in their college (teacher preparation) programs unless they work toward a mathematics major or minor.³ It is important to get a sense of how instructors intend for mathematics courses to fit into students' programs, what students are to learn, and how the courses are preparing them. Without this sense, recommendations for adding more courses only "add more courses" and do not necessarily prepare better elementary school mathematics teachers so that children get better mathematics instruction. Nor does adding courses address the challenges of varied abilities, poor attitudes or anxiety, and others with which I had struggled in my teaching.

Instructors, guided by departmental course descriptions, determine what and how to teach the content in their courses. The instructor's sense of the courses' functions shapes what they attempt and how they portray the content to students. This raises several questions. How do mathematics educators see their courses? What are their commitments and constraints? How are they thinking about mathematics reform and its effect on the courses they teach -- its effect on their own practice? To what extent do they feel much the same anxiety that K-12 mathematics teachers feel about being asked to rethink what it means to teach

³ One other possibility is that students take remedial courses prior to attempting the mathematics for the elementary school teacher courses.

mathematics? What models do they have for their own teaching? How are they seeking and giving support to their colleagues? Exposing the needs and concerns of mathematics teacher educators will provide us with a place to begin clarifying and addressing these needs and concerns. As Good (1990) suggested, "Any adequate improvement plan [in mathematics education] depends partly on the ability to define the problem clearly and accurately" (p. 24). Additionally, at a time when there is pressure to increase the mathematics course requirements for all prospective elementary school teachers, it seems important to document the conceptualizations of current courses by instructors responsible for designing and/or teaching them.

In this study, I examine how mathematics department instructors of mathematics content courses for preservice elementary teachers conceptualize the roles played by their courses. The extent to which the broader community of mathematics teacher educators responsible for these courses shares the concerns and challenges that I struggled with (embedded in sometimes conflicting but concurrently desired roles) particularly interests me.

Studies of Mathematics Teacher Education

The mathematical preparation of elementary school teachers at the University level has been the focus of numerous articles, conferences, and reports (Rising, 1969; Weissglass, 1977; Dossey, 1981; Carss, 1984; University of Chicago, 1992; COMAP, 1992). A consistent assessment is that elementary teacher preparation is deficient in many ways. The courses for prospective elementary

school teachers are under attack, yet we do not know much about their character or quality (Lampert, 1988).

A focus issue of the Journal of Research and Development in Education (JRDE, 1982) dealing exclusively with mathematics teacher education described the status of mathematics teacher education in the early 1980's. The editors had high-minded goals.

From the beginning it was our purpose to create or cause to be created a collection of ideas relative exclusively to mathematics teacher education that would provide new insights into our problems, provoke thinking about those problems and, perhaps with excessive enthusiasm on our part, generate a new dialogue among mathematics educators everywhere relative to attempts to achieve solutions or partial solutions. (Hooten, 1982, p. i)

Included in this collection were case studies of two elementary mathematics teacher education programs (Engelhardt, 1982; Hollis & Houston, 1982). These took the form of, in one case, a description of a program's history and a brief critique of the program, and, in the other, a description of a program's history, the principles of Competency-Based Teacher Education,⁴ various components of the program (e.g., technology, field experience), and a critique that hinted at struggles around ensuring children's learning and the lack of time and equipment to do adequate teacher preparation. In both case studies, the program critique implied that the task of preparing

⁴ Certain aspects of the program were identified: emphasis on using inquiry and discovery approaches when teaching, teaching mathematical structures, teaching students to consider learners' developmental levels when selecting instructional materials and tactics. These were stated baldly, with no explanation of how this was done or what the focus was. That is, were students being taught using inquiry or were they being taught how to teach using inquiry? This was left as implicit and, to me, unclear.

elementary teachers in the small mathematics components of their programs was impossible. This criticism that teacher preparation programs require insufficient mathematics course work is a common one.

In past criticism, the focus was chiefly on course requirements and topics covered in courses, with little note made of how or why content was dealt with in the courses (e.g., Burger et al, 1983; Dossey, 1984). Albers (1981) argued: "We now demand that elementary students know more mathematics. Should not their teachers also know more?" (p. 135). Z. P. Dienes (1972) took exception with the focus on the content when he suggested that what students are taught is less important than how they are taught because of the rapid changes in the body of mathematical knowledge and the use of technologies.

Today's world is increasingly based on wielding more and more complex structures. We shall need large numbers of people who are able to face unusual and unforeseen situations in the extremely complex world we are constructing. It is probably quite useless to make up a fixed curriculum of what children [prospective teachers] should learn, because what they are learning now is probably of little relevance to the problems they will have to meet twenty years from now. We can see this by looking back twenty years and seeing all the competencies required today that were not even thought of twenty years ago, such as the computer sciences. On the other hand, the development of curriculum should still receive serious attention, since we do have to teach children [prospective teachers] some mathematics even though it is not particularly important what mathematics we teach them. It is far more important to know how to teach what we wish to teach. This "how" should be determined by the need to train children to meet the unexpected. (p. 66)

He was addressing the teaching of children, but the implication is that teachers must be prepared to teach children to meet and challenge the unexpected.⁵

Professionals have in recent years come together to call for new ways of looking at subject-matter preparation in an effort to find ways to improve it (University of Chicago, 1992; COMAP, 1992; NCTM, 1991; Ball & McDiarmid, 1989). Concern for what content instructors covered in elementary teacher preparation is seen as necessary but not sufficient for improving preparation. Teacher candidates' and teachers' beliefs about mathematics and the teaching and learning of mathematics (Thompson, 1984; Kennedy & Schmidt, 1990; Cobb, 1983) as well as on teacher candidates' and teachers' subject matter knowledge (Ball, 1988) has been the focus of some research. The subjects have been, largely, prospective or inservice teachers, with mathematics teacher educators in the background.

Brown (1982) suggested another way of looking at subject-matter preparation. He called for "humanistic approaches" in the practice of teacher education, particularly mathematics teacher education. The humanistic perspective views mathematics as less clear and more ambiguous and controversy-laden than instructors portray it in most of school mathematics. Using problem solving as the content of concern, Brown argued the following:

It may very well be the case that what is needed is instruction in carrying on *genuine dialogue*, *encouraging controversy*, and *tempering inclinations at closure* [italics in original] with

⁵ Note that Dienes made these comments in 1972, over twenty years ago.

students if we wish to encourage a view among teachers of problem solving as problematic. (p. 7)

He admitted that he had no ready design for the kind of teacher education program this would require, noting that "even the most intellectually competent [graduate students] feel uncomfortable teaching anything as an unresolved or unresolvable problem" (p. 7). He did not argue that all of problem solving needs to be viewed as problematic, but he recommended that, "to the extent that education both of youngsters and of teachers is an effort to share personal, idiosyncratic insights and to understand fundamental unsolvable problems, [some attention should be paid to this] competing world view" (p.7-8).

Brown (1982) avowed that the most fundamental questions of teacher education are problematic. He identified a piece missing from conversations in teacher education.

What is missing then in the area of teacher education is not so much a collection of research findings upon which we might justify the organization and design of a particular program, but rather the courage and intelligence to allow all of us to become clearer on what we believe with regard to [problematic] questions. (p. 11)

There have been a few mathematics teacher educators and researchers who have asked some of these questions. Their work forms the foundation of this study and serves to launch it.

Studies of Mathematics Educators

Many recent reports on teacher preparation hint at the notion of studying mathematics teacher educators and their beliefs and actions (University of Chicago, 1992; COMAP, 1992; Thompson,

1992). Thompson (1992) raised several questions about the beliefs of mathematics teacher educators:

Research on teachers' beliefs and conceptions has stimulated reflection on our role as teacher educators. What do we believe about mathematics teaching and learning? To what extent are our methods of teaching and evaluating students' work consistent with our professed beliefs about mathematics and mathematics teaching and learning? What challenges do we face in changing our practices to better reflect our conceptions? (p. 141)

Implicit in this list of questions, raised in a review of the research in mathematics education related to teacher beliefs, is the recommendation that these questions drive research designed to investigate their answers.

Mathematics teacher educators have been primarily in the background in past research. Examinations of programmatic roles and instructors' goals and intentions for courses have been embedded within reports of research on student beliefs, student achievement, or student actions. One exception was Weissglass' report (1977) of his experience with and thinking about a mathematics content course for prospective elementary teachers. He had developed and used curricular materials that allowed him to teach the course in a mathematics laboratory using a small-group approach. In his report of this experience, he described his objectives for the course, the challenges to effective teaching it offered, the assumptions about learning on which he based the course design, and a description and evaluation of the project. This rare glimpse into a mathematics teacher educator's thought and action with respect to this kind of course indicates that some educators recognized many of the

concerns and hopes of more recent mathematics teacher educators and researchers some time ago.

Weissglass' (1977) objectives read much like the goals of the NCTM Standards (1989) on reasoning, communication, connectedness, problem solving, and motivation. The challenges to effectively teaching a course for prospective teachers that he identified were: 1) the high degree of math anxiety or math disinterest of students, 2) the wide range of mathematical backgrounds and abilities the students bring to the course, 3) the student dissatisfaction with concentration in the course on content -- not seeing the relevance to teaching of children, 4) the reliance on the lecture method, hindering meaningful student participation, and 5) methods of evaluation that interfere with student learning.

By adopting a small-group approach to instruction, Weissglass attempted to minimize these challenges, setting up a situation where students learned from each other, had some control over how they spent their time, used the experiences that they brought to the class, and taught each other. The particular approach used for the course is not the issue here. His examination of his objectives and assumptions and his decisions about how to deal with the challenges facing him is informative.

The "hows" and "whys" of his thinking and his students' learning concerned Weissglass. He began by defining "learning": "Learning consists of evaluating new information in relation to information that is already understood and storing it in a form that is available for use in new situations" (p. 379). From this, he determined his assumptions about what his students' learning

required of him. He assumed that students must be feeling good in order to learn and that students can evaluate new information only when they understand its relationship to information already assimilated. He based his decisions about his course design on these assumptions. It seems important to consider the extent to which the challenges to effective teaching of these courses remain the same, how assumptions about learning have changed, and how instructors are conceptualizing their courses today.

Recently, the research of a group of mathematics educators at Michigan State University (Wilcox, Schram, Lappan, Lanier, and Even) reported on the influence of an intervention program on beginning teachers' practice. The intervention was a sequence of three nontraditional mathematics courses, a methods course, and a curriculum seminar. The content courses emphasized the integration and interconnection of mathematical concepts. A major goal of the project was to challenge the beliefs and dispositions toward mathematics the students brought to the courses. The educators designed the courses to encourage students' reexamination of:

[Students'] fundamental beliefs about such questions as: What is mathematics? What does it mean to know mathematics? What mathematics do elementary school children need to study? How do we make decisions about what to include in the elementary curriculum? How do children learn mathematics? What is the role of the teacher in the mathematics classroom? (Lappan & Even, 1989, pp. 1-2)

The overall goal of the program was for prospective elementary teachers to experience "good mathematics -- taught well." The researcher-educators conceived of and enacted the intervention to "demonstrate the feasibility of developing in prospective elementary

teachers a more conceptual level of knowledge of mathematics and the teaching and learning of mathematics." (Wilcox et al, 1992, p. 2) They embedded the learning of mathematics in the context of learning to teach so that developing subject matter knowledge was linked with developing pedagogical content knowledge. They designed instruction to use problem situations, involve periodic reflections, and develop a community of learners.

Three facets of "good mathematics" concerned the instructor of the courses: the mathematics content, mathematical connections, and students' "doing" mathematics. She interpreted these facets from a perspective aligned with the NCTM Standards (1989, 1991). For example, she dealt with the thematically organized content in various ways allowing for integration of ideas and problem solving. The instructor of the mathematics courses attempted to create an environment in which students "could collectively and cooperatively learn ways to make sense of mathematics, invent procedures to solve new problems, and build models to understand mathematical situations" (Schram et al, 1988, p. 8).

The Michigan State University study included the conceptualizations of the mathematics content course by the instructor as a backdrop. Another project that included the mathematics teacher educator as an informant, but not as the primary focus, was the work of another group of educators: Borko, Eisenhart, Brown, Underhill, Behm, and Romagnano. The most important aspect of this work to the proposed study was the involvement of the mathematics methods instructor. The broader study examined the complete teacher education program of an

institution from the perspective of two different theories of learning (Eisenhart et al, 1991). Within the study was a consideration of the ways the mathematics methods instructor viewed the methods course, his theory of mathematics teaching and learning, the challenges he saw to effective teaching and learning, and the researchers' analyses of the messages the course conveyed to the students. While it involved the thoughts of the mathematics educator (Underhill, 1991), they were secondary to the study and represented the reflections of only one instructor.

Brown, Cooney, and Jones (1990) assessed that a golden opportunity to study mathematics teacher educators was, largely, ignored with the advent of "New Math" and the associated teacher enhancement programs. They bemoaned the fact that little research was done around "the most highly funded mathematics teacher education program in the history of the United States" (p. 641). As a result, we have very little sense of how the present call for change might be supported.

The New Math reform was top-down and curriculum driven; it involved the "what." The current reform is meant to involve all levels of participants (e.g., Standards authors include mathematics teacher educators, state mathematics supervisors, and classroom teachers) and includes curricular, instructional, and philosophical aspects -- the "what, how, and why." Unclear is what the "what, how, and why" of mathematics courses designed for prospective elementary school teachers are.

The studies of the past involving instructors' thinking behind the courses they taught are informative, but they involved single

instructors and did not focus central attention on the mathematics content courses for prospective teachers. What if the questions posed about the beliefs and future actions of prospective teachers within the courses in the Michigan State University study were posed to mathematics teacher educators?

What is mathematics? What does it mean to know mathematics? What mathematics do [prospective teachers] need to study? How do we make decisions about what to include in the courses? How do [students] learn mathematics? What is the role of the teacher in the mathematics classroom?

Addressing these questions could serve to orient discussions about and further research into problematic areas of mathematics teacher education.

An examination of the ways a set of instructors view the mathematics content courses (as the content portion of the mathematics content-methods sequence of courses) as a part of the preparation of elementary school teachers offers an opportunity to consider the different ways instructors define course content, the different expectations for students, and the different perspectives on teaching and learning mathematics that permeate the experience for students in the courses. How do instructors' assumptions about mathematics and the teaching and learning of mathematics shape course design? What roles do instructors see their courses playing in students' programs?

These questions lead back to the concerns raised in my own teaching. If instructors think that the mathematics content courses for prospective elementary school teachers are to play particular roles, what are the roles that instructors most value and support?

The roles that instructors believe to be most important and the meaning those roles hold for them will orient what content gets taught and how they attempt to teach the content in the courses.

With the atmosphere of reform surrounding mathematics education today, mathematics teacher education has a critical role to play (LeBlanc, 1977). LeBlanc asserted that, "Teacher education is a key factor, if not the most important factor in school reform" (p.223). If he is correct, the question of the similarity of the roles and goals of the mathematics courses for prospective elementary school teachers to the recommendations of the reform movement needs to be addressed.

More questions arise. To what extent are the goal of students' mathematical empowerment and a view of the discipline of mathematics as problematic a part of the experience of prospective teachers? What image of teaching is a part of this experience? The mathematics teachers and teacher educators are between the reform and the students, and they are seen as "key figures in changing the ways in which mathematics is taught and learned in schools" (NCTM, 1991, p. 2). The roles mathematics teacher educators see as most important for their courses to play guide their decision-making in the courses. The roles instructors expect courses to play have implications as to whether the needs of students in their teacher preparation are being well-served as set against the measure of the reform recommendations presented in documents such as the NCTM Standards (1989, 1991) and the MAA A Call for Change (1991).

My study intends to address several concerns raised in this examination of the literature. By focusing on the beliefs and actions

of mathematics teacher educators as they attempt to mathematically prepare prospective elementary school teachers, I hope to contribute to the dialogue about mathematics teacher education anticipated by the editors of the focus issue of JRDE cited earlier. The hopes of the editors are not unlike the aspirations I hold for this study: to provide new insights into the ways we view the mathematical preparation of elementary school teachers, to provoke thinking about the roles the courses can and should play, and generate dialogue among mathematics educators and teacher educators everywhere relative to improvement of mathematics teacher education and, ultimately, improvement of the mathematics education of all elementary school children.

CHAPTER 3

ANALYTIC FRAMEWORK

Mathematics content courses are a required part of elementary teacher preparation programs. These courses are taught as a service to the teacher preparation program within mathematics departments by departmental staff. As noted, programs generally require one to three mathematics courses along with a mathematics methods course. The programmatic purposes for the content courses and the preferred means for achieving those purposes were often veiled to me as an instructor. I held particular goals for the courses when I taught. These were instructional goals that I selectively developed. But, my question in this study asks about the programmatic roles that the courses are to play. How do instructors see the courses fitting into the teacher preparation of the students? What special mandates do these courses satisfy? How do these roles and the goals that individual instructors see as serving these roles mesh with those envisioned in the current reform movement for the classrooms in which their students will eventually be teaching?

As a way to begin thinking about the research question, I developed descriptions of five roles (elaborated upon below) that I anticipated would be major roles identified and stressed by the instructors in their reflections on the courses. A part of this study involves the extent to which the pre-described roles encompass the roles instructors see as important for the course to play. Additionally, I document the existence of major roles that were not consistent with these pre-described roles. The beliefs and

assumptions that were expressed by instructors are the evidence for the existence of the various roles, and those beliefs and assumptions expressed that did not fit with a particular role are of special interest, as evidence of other considerations and concerns with which instructors were dealing.

In order to clarify the instructors' beliefs and assumptions, I needed to ask several questions: What assumptions about mathematics, the learning of mathematics, and the teaching of mathematics served as the basis on which courses were designed? How did instructors see themselves supporting the roles that they valued in the text selection, in assignments, in use of class time, and in assessment? How were the roles seen as connected to teacher preparation, if at all? I investigate the answers to these questions with attention given to the attributes of the pre-described roles. It is to those roles and their descriptions we now turn.

Possible Roles and Rationales

I assert that mathematics education faculty have different ideas about the roles mathematics content courses required for elementary certification play in teacher preparation. Based on my experience, I see several possible roles and rationales mathematics educators might conceive of for the courses, each having a possible connectedness with teacher preparation. The five that I share here represent my best guess as to the most common roles accepted as valid by mathematics educators.

The roles of mathematics content courses in teacher preparation are:

- To provide students with the opportunity to learn the mathematics they need to know as K-8 classroom teachers (Content),
- To provide students with the opportunity to learn the history and conventions of mathematics and the preferred strategies of operation in mathematics they need to know as K-8 mathematics teachers (Norms),
- To provide students with the opportunity to learn how to teach mathematics K-8 (Teaching),
- To keep inept people out of the elementary school classroom (Gate-keeping), and
- To provide students with an experience that helps them feel good about themselves when dealing with mathematics (Attitude).

These are the most likely global roles that I foresaw. There are other roles, such as "to learn how to use technology to solve mathematics problems." They are important and were of interest to the study, but they were not seen as focal. I anticipated that there were roles that instructors would identify that I had not identified, but I set these five out as a place to start.

With each of these roles, there are differing interpretations possible, resulting in very different looking classrooms and having varied course requirements. Part of the study was to investigate what different instructors meant when emphasizing the same role or when using similar terminology. And, while many instructors may believe each of the roles identified above are valid, they could differ in their determination of the primary role and the reasons for

valuing each. These different interpretations include the mathematics educator's rationale supporting the role for the courses in teacher preparation. These rationales could involve an orientation toward serving the needs of the student, or they could reflect institutional concerns. It was also important to identify assumptions made about what students know and ought to know that play a part in various rationales for the roles courses play in teacher preparation. I used these to help distinguish the roles and rationales that support them.

Learning Mathematical Content

When the role of mathematics content courses in teacher preparation is seen as to teach for the learning of the mathematics students need to know to be able to teach K-8 mathematics, one issue is what content is necessary and sufficient to know. The content could be limited to the mathematics that is a part of the K-8 curriculum. It could be expanded to include other mathematical topics beyond the typical K-8 level. The role of learning mathematical content involves different expectations of sophistication, depending on how the content is defined.

Two assumptions about what content students know upon entering the courses result in different situations in classrooms. The first assumption is that the students enter the class not knowing the mathematics they need to know. The course is seen as a place where particular mathematical content is emphasized based on the needs identified by the instructor. These needs may be determined on the basis of the instructor's experience teaching the courses, assessment of needs at the beginning of each semester, or

the instructor's personal reading of the state-of-affairs in mathematics education, writ large.

When operating under this assumption, different situations can result. When instructors are concerned with helping students learn mathematics that they do not know, instruction can be oriented toward addressing areas of most common weakness, providing a valuable learning experience that results in strengthening teacher preparation. Instruction can deal with content that is generally seen to be a part of the elementary school curriculum that the instructor has assessed as missing from students' backgrounds. Alternatively, instruction may be oriented toward addressing areas that are personal "favorites" of the instructor or topics that the instructor has identified as critical but may be seen as peripheral to elementary teacher preparation by a majority of mathematics educators. Students enter not knowing this content because it has not been a part of their past, not because they have been unsuccessful in dealing with it.

A different assumption may play a part in a mathematics educators' belief that the role of the courses is to provide students with an opportunity to learn mathematics. Instructors may believe that students "know" K-8 mathematics, but not with the kind of understanding they value. This assumption can drive very different situations in classrooms than the first assumption. When the goal is understanding of mathematics that students may already have "mastered" or at least believe that they have mastered, instruction requires a different orientation. Many students realize that their

understanding is shaky, and they are anxious about this, adding complexity to the situation.

There are different conceptions of how understanding is developed, relying on different views of the nature of mathematics and psychology of learning mathematics. These conceptions suggest different use of class time. One conception has the underlying view that mathematics is a fixed body of knowledge that is transmitted to students. Showing students interconnectedness of topics, clarifying definitions, showing "why you do what you do when you do it," setting up problem solving situations and activities that lead students to discoveries desired by the instructor, all are part of this orientation. Errors are avoided by laying out correct paths to mathematical truths. The understanding of the content that is expected is of a procedural nature, involving sets of steps or reasons explaining the content.

An alternative conception of how understanding can be developed relies on a "problem-solving" view of mathematics and a social constructivist perspective on learning. Ernest (1989) describes a problem-solving view of mathematics as a dynamic view of mathematics as a continually expanding field of human inquiry, unfinished and open to revision. Challenging students' conceptions, using errors as paths to deeper understanding, strengthening and empowering students as mathematicians -- doers of mathematics, and giving them a sense of their own ability to make sense of mathematics, all are part of this orientation. The goal is understanding of concepts, involving interconnectedness of ideas and

many ways of dealing with and communicating about the mathematics.

These two different views of mathematics (as a fixed set of content to be taught and as an ever changing field of inquiry) and the differing curricular demands attached to each are important to keep in mind throughout the analyses. For the purposes of my analyses, I found it necessary to distinguish between these two views with respect to the understanding that is expected and developed: Understanding in the first situation I call "procedural understanding" and understanding in the second situation I call "conceptual understanding." Occasionally, instructors used the terms procedural and conceptual to describe the understanding they were expecting and developing. Instructors may not have used the terms in the way I have indicated here, but the context of their comments in the individual analyses makes clear how each was thinking about them. A more detailed description and comparison of the ways instructors talked about learning and understanding mathematical content is a part of the analysis when looking across instructors' responses.

In describing this role, I have assumed that instructors relate the learning of the content to the purposes of teacher preparation. However, the course could be seen as playing the role of developing mathematical content knowledge separate from any concern for future teaching. I document the extent to which instructors connect the learning of mathematical content to the students' status as prospective teachers.

A related issue is the extent to which instructors interact with instructors from other departments responsible for other

content or general education courses to make decisions about the content and design of the mathematics content courses. Instructors could be operating in isolation or they could be cooperatively developing the content course as a part of an integrated and coherent teacher education program. The programmatic role played by the course in learning the mathematical content could vary, depending on the level of inter-departmental collaboration.

Learning the Norms of Mathematics

The second role identified above is to offer students the opportunity to learn about doing the mathematics they need to know as a K-8 mathematics teacher. The focus is on the history and conventions of mathematics and the preferred strategies of operation in mathematics. This includes having a sense of what evidence is required for proof and knowing how truth is established. If learning mathematical content includes learning what and why, learning the norms of mathematics includes learning how: how to communicate, how to conjecture, how to justify and prove, how to present arguments, how to connect ideas.

The assumption of students' prior knowledge of this information is predominantly in the form of conventions, though students may have had experiences that introduced them to historical information and specific strategies. For example, a reasonable expectation is that most students in college would have completed high school algebra and learned a specific strategy for solving linear equations in one variable that involves solving for the unknown on the left-hand side of the equation following "standard procedures."

Different emphases on this role can result in very different situations in course content/emphasis. The history of mathematics could be used to move through course content, connecting history to content and content to content. The course could "tell the story" of mathematics, acknowledging that it is an unfinished story and encouraging students to continue investigating. Or, the history of mathematics as a focus could present mathematics as a story that was and is written by mathematicians "back then" and "out there," separate from the lives of the students. Connections between topics and with students could be minimal.

Learning the conventions of mathematics, whether in notation or procedures, is important in teacher preparation, but it is difficult to conceive of organizing a course around this. Strategies of operation could be emphasized as a vehicle to get at content, where the first and second roles are interwoven, acknowledging the importance of both.

A focus on the strategies of operation in mathematics is more problematic to consider. The question can be raised, "Are the strategies of operation in mathematics actually 'mathematics'?", making this a part of learning mathematics? An example of using strategies of operation as a focus different from mathematics content might help.

It is common, in this time of emphasis on problem solving in mathematics, to introduce students to Polya's four heuristics of problem solving. These heuristics may be a vehicle to get to mathematics or they may become the destination. Considerable class time can be spent learning to apply this four-step procedure to

problems. The focus could be on the procedure, not on the mathematical content of the task, and students would be expected to learn the procedure.

A course centered around learning the strategies of operating within mathematics, such as Polya's heuristics or two-column proof, may support beliefs of students that knowing how to do something is "the thing." Additionally, questions of when and why one chooses to use particular strategies might get less attention. Learning the norms of the discipline includes learning the value of conjecture and justification in mathematics. A course focusing on this would, by necessity, be tied to the content. The difference in emphasis on learning of mathematics and learning about the norms of mathematics could vary widely.

The history and norms of mathematics are important knowledge for all students. It is important to notice is how this plays out in the ways instructors emphasize the role of learning the norms of the discipline for content courses in teacher preparation.

Learning to Teach Mathematics

The third role that instructors might see mathematics content courses playing is that of offering students the opportunity to learn how to teach K-8 mathematics. The merging of content with methods in courses seems natural in the context of teacher preparation and, in fact, is recommended in the NCTM Standards (1991). However, there can be differences in the motivation for and emphasis in such a merging.

Instructors may see content and methods as a piece, not to be separated. They might approach the task of teaching content

explicitly relating decisions they make in methodology to students as a part of their instruction. Issues of how and why one presents content in particular ways are then on a par with the question of what content is to be emphasized.

In some situations, there is considerable conversation among different instructors -- those of the content courses and those of the methods courses. This is usually facilitated by a shared departmental assignment, either both in the Mathematics Department or both in the Department of Teacher Education. When colleagues cooperatively determine where, when, what, and how mathematics content and methods will be developed, the merging of content and methods may result in a coherent program for teacher preparation.

In other situations, there may be little conversation between content and methods instructors. There may be a sense of distrust or lack of respect between those responsible for teaching the content courses and those responsible for teaching the methods courses, especially when the courses are offered in different departments. In this situation, the role of the content course as a place to learn methods has a different motivation -- that of making sure that things get done "right," unsure that they will be done right if left to others to do.

The role of the mathematics content courses in teaching students how to teach mathematics is important to examine as the course fits into the whole of the students' teacher education program. Instructors may be very aware of the whole program and have a clear sense of the contributions of content specific courses

to students' teacher preparation. Or, instructors may be operating within the narrow confines of their direct responsibility for the teaching of the specific course or courses they teach within the mathematics department.

An instructor's decision to focus on how to teach mathematics in a mathematics content course might have a lot to do with student expectations. Many students view any course that is part of the requirements for the elementary education program as a "how-to-teach" course. Instructors who recognize this might choose to treat the course primarily as a methods course to avoid student dissatisfaction. It might be that the instructor prefers teaching methods content over mathematics content or feels that there is not enough time in the methods course to do all that is important, so the mathematics content course becomes a methods course.

Whatever the motivation, the students' level of content knowledge could suffer from a focus on other methodological issues, such as classroom management and lesson planning. Students might be happier for this, but their preparation as mathematics teachers might not include the depth of understanding of content that they need. This depends on how "methods" are defined and the extent to which content and methods are merged in both the content and methods courses.

Gate-keeping

The fourth possible role for a mathematics content course in teacher preparation is that of gate-keeping. This role has more than one motivation, as well, grounded in different assumptions. With a pro-active view of gate-keeping, students who come to content

courses underprepared are guided to courses that should be taken prior to the content courses for elementary teacher education or are supported within the courses to provide adequate preparation. The intention is to provide an environment in which students are appropriately prepared while reducing the possibility of allowing students to successfully exit the courses without having the mathematical expertise required for teaching the content. In this sense, gate-keeping keeps those individuals who have not exhibited fundamental understanding of mathematical ideas and content out of the K-8 classroom.

Another motivation for a gate-keeping role of a mathematics content course in teacher preparation stems from two perceptions. The first is that mathematics courses are traditionally perceived as difficult. Second, mathematics departments have a historic pattern of congratulating themselves on high failure rates in their departments as a sign of their high standards and keen insight into students: "If no one else will take the heat and flunk these students out, we will!" The mind-set is different from the pro-active and may involve situations having little to do with mathematical ability. This conception of the role of gate-keeping is very different, concerns filtering students out of the teacher preparation program, and results in a different set of expectations of instructors.

Either conception of the role of the course as a gate-keeper places the course in service to institutions. First, it suggests that the University can feel confident that its graduates are mathematically prepared to teach. Second, it suggests that school

systems hiring the graduates can feel confident that the new employees are mathematically prepared to teach.

Feeling Good About Mathematics

A mathematics content course could have a very different role for students. Rather than filtering them out and away from mathematics, it could serve the role of improving students' self-confidence with respect to knowing and doing mathematics. This, the fifth and final role to be considered here, is also a two-edged sword.

Providing students with an experience that helps them feel good about themselves when dealing with mathematics is a possible role for a content course. A mathematics instructor of courses required for elementary certification is often faced with students who have mathematics anxiety or, at the least, do not like mathematics. Changing students' affective perceptions of mathematics can be an important role for a mathematics content course, particularly since it may be the last mathematics course students ever take. How that role takes form and is emphasized is important to consider.

"Feeling good" may result from being empowered, gaining understanding and a sense of purpose for mathematical activity, and sensing the ability to know where to go for help when unsure. The course developed to achieve this will be different from a course whose goal is to have students do "fun" activities. The sense of "feeling good" is very different. One reflects a belief based on cognition, the other a belief based on affect.

Approaching the issue of helping students feel good about mathematics can be through the building up of the sense of relevance that mathematics has for students. The rationale is that if students see mathematics as a valuable and useful tool, they will have a better attitude toward mathematics. There are different ways to accomplish this. One could conceive of building relevance through activities that focus on using pizzas and beer, as one extreme. Another option would be to go on field trips to set up doing mathematical tasks -- figuring how much paint it would take to paint a water tower or how many steps it would take to get to the top of a hill. Instructors' conceptions of "relevance" and what it means to make mathematics relevant would orient their decisions about content and methods.

Assuming that the way a person feels about a subject affects the amount of time they want to spend on it (as a classroom teacher), improving prospective teachers' attitudes about mathematics can result in very different experiences for the elementary school children who become their students. A teacher who feels comfortable about mathematics will look forward to teaching it and spending time on it in class. But, the nature of the "feeling good" also affects the nature of the experiences of the elementary school children. If "fun" activities with limited content development captivate the prospective teacher in their courses, then this is likely the nature of the activities the prospective teacher will be comfortable using. If content rich activities are used that help build mathematical capabilities and confidence while improving

attitude, then the prospective teacher may be more likely to rely on content rich tasks that act to empower children in their teaching.

Endnote

In this study, the focus is on the programmatic roles instructors see as of particular importance for the courses to play. Within the courses, instructors work toward goals that serve the programmatic purposes. I see the roles as being roles of the course for students in their teacher preparation programs and goals as being goals for a particular course. Occasionally, I intermingle the word usage.

I have presented five roles that mathematics teacher educators might see as important for mathematics content courses in teacher preparation. I have indicated possible rationales for each and tried to indicate different motivations that I see attached to each, though sometimes implicitly, in my description. These roles and rationales act as the analytic framework to be applied to the data collected in the study. Data collection was designed to make full use of this framework. The sources and nature of the data and its collection follow.

CHAPTER 4

RESEARCH METHODOLOGY

Site and Subjects

This study examines courses in mathematics teacher education through the eyes of its instructors. The structure of the study is that of a case study of an “extended institution.” The case study involves six faculty members, five at a large, four-year institution and one at a near-by community college. The course work at the community college is specifically designed to satisfy the requirements of and receive the same credit as courses at the university, to which it feeds students.

Primary Site Information and Background of the Instructors

The primary site for my data collection was one of the first teachers' colleges west of the Allegheny Mountains, called Normal University⁶ (NU) in this study. It has a history as a normal school and now is a state-supported four-year university with graduate programs in many areas. This includes a Masters degree program in mathematics education. Normal University is a multipurpose university in the Midwest with an enrollment of approximately 25,000 students. Over 2000 students per year graduate from NU with teaching certification. With this huge output into the teaching market, the study of its courses has much to offer for the describing of the mathematics preparation of elementary school teachers.

⁶ All names of institutions and individuals are pseudonyms, and course numbers have been altered.

At the time of this study, the elementary teacher education program required two mathematics content courses, Math 115 and Math 116. The Teacher Education Department oversaw the elementary teacher education program, but the Mathematics Department was responsible for the teaching of the mathematics content courses. The same building housed the two departments on the NU campus. There were departmental syllabi developed within the Mathematics Department (see Appendix A) for instructors to follow that indicate the content to be covered, the sections of the text (all use Musser & Burger, 1991) and course supplements that pertain to the content, and the approximate number of days to be spent covering the content. The two syllabi were identical in terms of the objectives (taken from the NCTM Standards, 1989, p. 5 - New Goals for Students), recommendations about evaluation, and a recommendation for and description of the use of journals.

The outlines for the two courses' content, material, and number of days of instruction were somewhat different. The first course's outline was a list of topics. The content specified for the first course included: problem solving (5 days); sets, Venn diagrams, Cartesian product, applied problems, functions (3 days); number systems/properties (2 days); estimation (3 days); number theory (6 days); fraction concepts, estimation with fractions (4 days); decimals, ratio, percents (5 days); integers (3 days); and real numbers, introduction to algebra (2 days). In addition to the content included on the syllabus, topics could include optional ones, such as logic and work with different bases. Optional content was seen as

the content that individual instructors would teach that made the course more personally theirs.

The content outline for the second course was more elaborate than that of the first course. Rather than presenting the whole outline here, some parts are written out completely (in italics) and an paraphrased version of the rest of the list is given: *constructing and interpreting stem and leaf, bar, and line graphs, writing a narrative interpreting the graph* (2 days); measures of central tendency and attributes of distributions (4 days); *compare and contrast experimental probabilities with theoretical probabilities, sample space, events, students should be actively involved in finding experimental probabilities* (2 days); counting principles (3 days); simulations (5 days); sampling (3 days); *scatter plots, finding the line of best fit (graphically and algebraically), an excellent opportunity to further discuss the concept of function* (3 days); symmetry (2 days); angles (2 days); measurement (6 days); and constructions using ruler, compass, and Mira (4 days). The outline indicated tessellations as an optional topic.

The first course had no prerequisite requirement indicated in the university course catalog at the time of the study. However, the faculty syllabus indicated that the prerequisites for the course were two years of high school algebra and one year of geometry. The Mathematics Department had recently instituted prerequisite testing in the first course with recommendations made for course changes to developmental courses for those not performing satisfactorily on the test. Course change was voluntary. That is, students could test as unprepared for the course, but they could

remain in the course. In the future, this policy is expected to change so that students will be required to change to a lower level course if they test unsatisfactorily. The second course required that students achieve a "C" or better in the first course. Following successful completion of the two content courses, students must take the mathematics methods course, Math 374, in the department to complete elementary education requirements.

The mathematics education faculty held a set of meetings⁷ just prior to the study to discuss what the content courses should "be like" and whether or not they wanted to choose a different textbook. I call the set of instructors participating in these meetings the Mathematics Education Committee. It is a standing committee and meets as needed.

Five of the faculty members at NU participated in this study. The participating faculty included a lecturer and four faculty members. All were full-time employees.

The lecturer, Gail Laughlin, taught on a year-to-year basis with no tenure possibilities. She had worked in this way for over fifteen years. Her background was secondary mathematics education, but she had never held a full-time secondary teaching position. She had done substitute teaching for five years. Gail took a position at NU and had stayed with the University teaching a range of mathematics courses, usually either in mathematics education or

⁷ This involved all tenured and non-tenured, full-time mathematics department faculty teaching courses in the elementary education program who chose to attend. These were open meetings, but other possibly interested individuals from within or outside the mathematics department did not usually attend unless specifically invited.

business mathematics. She has actively participated in departmental conversations about the mathematics education courses and in the past has taken responsibility for developing the course packet along with other instructors.

Kara Sheffield had taught high school mathematics for five years before being hired at NU. She began her work at NU in the same way Gail did, as a temporary instructor. Though she does not have a terminal degree, Kara was given a tenure-track position three years ago. Prior to this, she took eighteen hours of mathematics course work beyond her Masters degree. Her teaching load was regular mathematics courses until the department found that they needed more instructors for the mathematics education courses in the late 1980's. Her personal interest in mathematics education supported her switch to teaching mathematics education courses. She has taken an active role in departmental meetings related to the mathematics education courses and anticipated working on revision of the course packet.

Craig Kennedy was a tenured faculty member with a mathematics education background. He taught high school mathematics for three years before working on his Masters and Doctoral degrees in mathematics education. He received an EdD from a large research-focused university before moving to the Midwest to teach mathematics education at a university in the same state as NU. He came to NU in the early 1970s. He taught mathematics education courses at all levels.

Jeff Sullivan was also a tenured faculty member at NU. His background included satisfying the undergraduate requirements for

secondary mathematics teacher certification, but he had never taught below the university level. He completed his doctorate in mathematics and had been teaching mathematics courses at NU for nine years. The semester immediately preceding the interviewing gave Jeff his first experience with teaching a course specifically designed as a part of elementary mathematics education.

The fifth instructor interviewed at NU was not actively teaching the courses, but Frank Adams was influential in the decision-making around the courses, including the assignment of the instructors. He taught the methods course that followed the two content courses as the key components of the mathematics teacher preparation for elementary certification. His career included teaching eleven years in public schools, two years in overseas teaching, and twenty-five years at the university level. Frank served on several committees over the years that had shaped the present sequence of content and methods courses at NU.

Secondary Site Information and Background of the Instructor

The secondary site was an urban community college with an enrollment of more than 16,000 students. Community College (CC) serves a very diverse population. Community College offers two content courses in elementary mathematics education. Many of the students in elementary education that transfer to NU receive credit for courses taken at CC. But, NU is only one of several institutions that students choose to transfer into after taking courses at CC. Of those that transfer, about one out of every four students in elementary education transfers into NU from CC. Since CC's imposition of a beginning algebra prerequisite the term before the

study began, there had been a significant drop in enrollment for the first course.

All instructors at CC use the same textbook (Devine, Olson, & Olson, 1991). A year earlier, a committee of mathematicians and mathematics education faculty⁸ at CC selected a textbook and set out the content to be covered in the two content courses. The catalog descriptions of the two beginning mathematics education courses offered at CC and NU are very similar in terms of content.

The instructor at CC, Cynthia Yoder, had taught full-time at CC for two years as a tenure-track faculty member. She had completed a Bachelor's degree in mathematics education and taught one year at a private high school before taking nine years off to raise a family. She attended night school and received a Masters degree in mathematics from NU. She had taught part-time at CC and a near-by university the three years prior to receiving her full-time appointment at CC.

General Overview of Sites and Subjects

The participants included both male and female subjects, younger and more mature educators, individuals with differing educational and experiential bases. Each subject was unique, but each shares characteristics with many other members of the mathematics education community having similar teaching responsibilities (see Figure 1). The orientation of NU toward teacher preparation as a primary focus and its position as a major contributor to the teacher job market recommends the study of these subjects.

⁸ All were members of the mathematics department faculty.

Name	Site	Status	Degree	Teaching Experience		
				Elem.	Sec.	College
Gail	NU	Non-tenure	MA - Math	0	5 yr (substitute)	10 yr part-time 6 yr full-time
Kara	NU	Tenure track	MA - Math	0	5 yr	5 yr part-time 15 yr full-time
Craig	NU	Tenured	EdD Math Ed	0	3 yr	20 yr
Jeff	NU	Tenured	PhD Math	0	0	9 yr
Frank	NU	Tenured	PhD Math Ed	0	11 yr 2 yr (overseas)	25 yr
Cynthia	OC	Tenure track	MA - Math	0	1 yr	3 yr part-time 2 yr full-time

Figure 1
Instructor Information

This study of instructors contributes to our understanding of the concerns, beliefs, and struggles of mathematics teacher educators as they attempt to prepare elementary school teachers. I determine the collectively-valued roles to be played by the course. Additionally, I consider the range of goals these subject-matter-specific courses are expected to address as a part of teacher preparation. In order to explore the conceptualizations of the courses by a set of instructors adequately, the number of participants was small and the methodology relied heavily on in-depth interviewing.

Data Collection

The ideas that instructors have about the roles their courses should play and how their beliefs and assumptions manifest themselves in their description of the courses are the focus of this study. In particular, I examine the roles that mathematics teacher educators see as of primary importance for mathematics content courses to play as a way of understanding how mathematics teacher educators see their courses fitting into students' teacher preparation programs.

I developed the interview instruments using several resources. Instruments previously developed for use in studies by the National Center for Research on Teacher Education (NC RTE, 1988a, 1988b) and the Survey of Mathematics and Science Opportunities (SMSO, 1992) were of particular value in developing card sort tasks that were a part of two interviews. After initial instrument development, I referred to the analytic framework to determine if I should include additional material in order to ensure a sense of all roles instructors were emphasizing.

The data collection took the form of a set of three, one to one-and-one-half hour interviews with associated observations in the participants' classrooms, when possible.⁹ The purpose of the observations was to better understand what the instructors said in the interviews. In some cases, the instructor was teaching or had taught both mathematics content courses in the elementary teacher

⁹ Kara, Craig, and Cynthia were observed during the interview cycle. Gail was observed prior to interviewing. Jeff and Frank were not observed.

preparation component in their department. Interview questions allowed instructors to respond with respect to either content course.

I structured the initial interview to identify the roles the instructor saw the courses playing in the students' programs and for the University. (see Appendix B) A major component of the interview was a card sort task. In this task, I indicated possible purposes of the course on cards that were to be placed in categories of Essential, Very Important, Somewhat Important, Not Very Important, and Not Important at All. I asked instructors to talk aloud as they placed the cards, indicating why they were putting the cards in the specific categories. The other component of the first interview involved getting the instructors' reactions to a set of strongly worded statements. I asked instructors to indicate if they agreed or disagreed with all or part of each statement. The set of statements and probes suggested possible roles a person might conceive a course to serve and rationales for the roles. My intent was not only to identify the roles the instructor valued and saw as part of the courses, but also to identify roles that the instructor valued but did not see the courses playing. With these two concerns went a third: the emphasis the instructor placed on each role. How strongly stated were the roles and how did the instructor prioritize the roles that he or she identified? Additionally, what constraints or barriers were there making roles difficult to enact? I also investigated the extent to which the instructor saw his or her task oriented toward service to the student or service to the institution.

The second interview (see Appendix C) was informed by material obtained prior to the interview, when possible. I asked for a copy of the course syllabus, the name of the textbook used and a list of assignments, a sample exam (possibly from a previous semester), and a copy of any handouts given -- particularly any description of grading procedures or how projects were to be completed. Throughout the interview, I attempted to clarify the roles the instructor saw the courses playing using questions that referred to the materials provided. For example, if an instructor said that a major role of the courses was to develop knowledge of mathematics, I sought to determine how this was interpreted in the materials used for the courses. In looking at exams, I selected tasks and asked the instructor to describe a good answer and clarify what counted as the "knowledge of mathematics" the instructor was attempting to develop. If there was a role encouraged in the materials that the instructor did not indicate as valued prior to this time, I asked about it, just as, if any role appeared to be missing in the materials (or at least veiled to me), I inquired about its absence (that is, I asked the instructor to help me find it in the materials). I designed some interview questions to elicit assumptions held by the instructor about the students taking the class: what students know, what they need to know, how they are characterized and how that affects the roles the course is expected to play for the students.

Between the second and third interviews, I observed a set of classes, when possible. The instructor and I mutually agreed on the number of observations. Using these observations, I built an interview (see Appendix D) for each participant that was directed at

having the instructor help me put the picture of the courses together and highlighting the ways the roles of the courses are brought to life. I asked the instructor to use specific examples from the observed classes to describe how he or she saw opportunities for the course to play a particular role for students. For example, if the instructor indicated in the first two interviews that building positive attitudes was a critical role for the course to play, I developed questions for him or her whose answers helped me enhance my sense of what "building a positive attitude" meant to the instructor. A question might be, "You mentioned that building positive attitude is an extremely important one for you in the course. Could you give me an example from what I observed in class that you see as an exemplar of how the course provides opportunities for doing this?" In this way, I was able to connect what the instructor said with what he or she did and enhance my understanding of what the instructor saw as enacting the roles.

If an instructor did not emphasize a particular role, but I observed actions in class that I related to the role, I asked questions such as, "What did you hope to accomplish in doing [example from observation]?" For example, if I thought an activity seemed focused on learning how to teach but the instructor had not emphasized this as a role for the course, the instructor's answer would help me understand how he or she saw the activity connecting with the roles of the course. There were many questions consistent across interviews, but some questions in this interview were specific to the individual.

These in-depth interviews provided access to the context of the instructors' behavior (Seidman, 1991, p. 4) thereby providing a way to understand the meaning of their behavior. By employing this set of in-depth interviews, I gained both a rich sense of the instructor's context and a clearer sense of the meaning of their words and actions. I gained a sense of the beliefs and assumptions undergirding instructors' intentions and actions in a way that set the roles they saw the course playing as focal.

CHAPTER 5

ANALYSIS

The analysis of the data takes two forms: individual and across the set of instructors. This introduction serves to describe the focus of each form of analysis and includes an explanatory note to help orient the reader to the more practical aspects of the analysis.

Instructors' Conceptualizations of the Roles

The first section is an analysis of the instructors' conceptualizations of the roles mathematics content courses are to play in prospective elementary school teachers' programs. This includes six individual analyses that intend to capture the particular emphases and special aspects of each instructor's characterization of the course or courses each taught. I use the pre-described roles to help describe the instructors' conceptualizations. The individual analyses are not exhaustive, but rather are intended to characterize what seem to be the most important things each instructor stressed. Additionally, the conceptualizations give a sense of the different perspectives brought to the teaching of the course by the instructors.

The order in which the instructors' conceptualizations appear is totally subjective. I placed them in such a way as to offer contrast, although almost any ordering of these analyses would offer contrast.

To develop the conceptualizations, I used the data in many different ways in an effort to immerse myself in each instructor's

sense of what the courses should accomplish and how instructors saw the courses offering opportunities for students to learn the content of the courses. My approaches ranged from a quantitative analysis of card sort responses to a look at how the observations of the classrooms of instructors informed my understanding of instructors' comments. I examined artifacts, such as course packets and sample exams, to see what kinds of material and activities instructors used to provide students with opportunities to learn what they intended and to see what expectations instructors had for student learning.

I crafted short "thumb-nail sketches" from which the present conceptualizations grew. I went through the interview transcripts carefully, pulling out important comments that I used as evidence for my interpretation of the roles and the instructors' valuing of the roles, building on the points I had used as the skeletons of the thumb-nail sketches. After doing this, I returned to my analytic framework to see if it proved to be sufficient. I saw its general adequacy, but I noted a deficiency. I acknowledge and explain this deficiency in the context of the instructor's conceptualization that most strongly highlights this weakness.

The Role of Learning Mathematical Content

I restricted the second analysis to a central role. Looking across the set of instructors, I focused on the role of learning mathematical content. The instructors were in mathematics departments teaching content courses that were a required part of a program outside their department. The ways instructors connected or did not connect the role of learning content to the status of the

courses as part of students' teacher preparation were important to consider in determining how instructors saw the course functioning within the students' program.

The analysis involves three issues related to the role of students' learning of mathematical content: student preparation for the courses, the specific content of the courses, and expectations for student understanding. These issues allow for a broad look across the instructors' conceptualizations, examining the range and variation of their interpretation of the role and a view into the ways instructors saw the courses serving a programmatic role.

The first issue focuses on the knowledge of mathematics instructors assumed students brought to the first course in the two-course sequence. Student preparation, in particular poor student preparation, was something about which all of the instructors acknowledged having some concern. I recount what instructors saw as poor preparation, how they perceived its impact on the content, and what their response was in the context of the course.

I examine the second issue, what the content of the courses should be, by looking first at the departmentally recommended content and design of the courses. Then, I look for variation in the instructors' choice of content and emphases on the content in the courses. I consider the distinctive aspects of the courses as portrayed by the instructors, and identify multiple influences on the courses' content and design.

Addressing the third issue involves looking within the role of learning mathematics to examine the instructors' primary goal:

understanding. The concept of understanding is one that is prominent in recent conversations about teaching. In particular, the phrase "teaching for understanding" has become near-cliché, with books and articles having been written about teaching mathematics for understanding (e.g., Burns, 1989; Hiebert & Carpenter, 1992; Prawat, 1989). The meanings of terms used in discussions of understanding often get blurred, so I carefully define the ways that I chose to use the terms needed to analyze the instructors' views. Using the language as I define it, I look at the ways that these instructors talked about understanding. At a time when the concept "understanding" is so widely used, the questions of how the instructors used it and what they saw as developing and supporting it seem important to investigate.

In the final piece of the analysis, I look for patterns in the ways instructors portrayed the roles. A pair of differing ways of viewing mathematics and teaching for the learning of mathematics emerge as critical influences on what instructors counted as mathematics and saw as appropriate instruction and assessment. I present these as two alternative characterizations that hold promise as food for thought and discussion.¹⁰

¹⁰ The two characterizations are not unlike, even similar to, the two alternative characterizations of mathematics teacher education research (analytic and humanistic) described by Brown, Cooney, and Jones (1990). I see both pairs of characterizations holding the promise of challenging mathematics educators and mathematics education researchers to consider alternative visions; examine issues, motivations, and goals; and make hard decisions about how to improve mathematics education.

Explanatory Note

The analyses used all sources of the data, from the departmental syllabus to sample exams to classroom observations to interviews. Again, these analyses do not represent the totality of all that could be learned from these data, but they set out what seemed to be of greatest importance and of greatest potential for acting as a catalyst for discussions involving the mathematics education community and the teacher education community.

Highlighting a few characteristics of the formatting used in the analyses may help the reader. In the interest of specifying where particular citations appear in the interview transcripts, each citation is followed by a set of parentheses indicating the page number and the interview type. The letters used to indicate the specific interview type are the following: P = Philosophical Interview (Interview One, Appendix B), C = Content Interview (Interview Two, Appendix C), and B = Observation Interview (Interview Three, Appendix D). If I took a quote from page 24 of Professor Sheffield's Content Interview, I identified it as (24 C) in the context of her conceptualization. When in the across-instructor analyses or in a different instructor's conceptualization, the instructor's initials are included, e.g., (KS, 24 C). I used three ellipsis points (. . .) to indicate a deleted portion of the transcript. I included descriptive information in parentheses, such as (short pause). When dialogue is included, I identified the interviewer using the initials, **GK**.

When communicating about work involving teacher education, it can be unclear to whom the word "students" refers. Except in

direct quotations from instructors, the word "students" refers to college-level learners. The words "pupils" or "children" refer to elementary-school-age learners.

Instructors at NU frequently used course numbers when talking about the courses. The courses numbered 115 and 116 are the two mathematics content courses of particular concern to this study, and 374 is the elementary mathematics methods course number. Instructors occasionally mentioned other course numbers. In those cases when the course being referred to seems unclear, course focus is indicated in brackets.

As background for reading the conceptualizations, it also may be helpful to remember that NU had just begun a prerequisite testing program. As noted in the information about the site, any action taken after the testing was purely voluntary at the time of the interviewing, so students could test as not ready for the Math 115 course, the first content course required in the elementary school teacher preparation program, but the students could choose to take the course, anyway. The department was making determinations about the testing and its voluntary/required status at the time of the interviewing.

INSTRUCTORS' CONCEPTUALIZATIONS OF THE ROLES

The individual analyses present each instructor's conceptualization of the courses as a picture of his or her commitment to the roles the courses play in students' teacher preparation. In addition, unique perspectives of individual instructors underline the breadth of concerns and influences on the roles the course is expected to play. The instructors' conceptualizations appear in the following order:

- Professor Kara Sheffield
- Professor Cynthia Yoder
- Professor Jeff Sullivan
- Professor Frank Adams
- Professor Gail Laughlin
- Professor Craig Kennedy

Many of the issues examined in the across-instructor analysis are fore-shadowed in the individual analyses. Some of the concerns are unique to specific individuals. Certain aspects are helpful to remember as most strongly associated with a particular instructor, though other instructors may share the same concern.

Professor Sheffield, Kara throughout the remainder of the study, held that the course had many roles to play in students' programs, but she found that conflicting goals and influences had an impact on what she was able to do in the course and how she was able to teach the course. These conflicting goals were a significant part of Kara's conceptualization.

Attempting to have the course play all of the pre-described roles and meet multiple goals was Professor Cynthia Yoder's considerable charge. In Cynthia's view, the needs brought to the course by the student population taking the courses at Community College (CC) complicated her situation. Her conceptualization reflected the high demands she placed on herself and her expectations for student learning.

As the only mathematician/non-mathematics educator¹¹ in the set of instructors, Professor Jeff Sullivan had a different background (and, it could be argued, mind-set) as he approached teaching the mathematics education content course. Jeff described how his lack of experience teaching the course had affected the roles the course played in his students' program and how he would do things differently another time. Jeff's conceptualization illuminated a deficiency in my analytic framework. He emphasized the development of particular dispositions that supported development of creative problem solvers.

Professor Frank Adams was unique in that his conceptualization emphasized the roles the course was not to play, rather than the roles it was to play. Within this, he emphasized the separation of content course content from methods course content.

¹¹ This assessment as a "mathematician/non-mathematics educator" is Jeff's own and that of the other mathematics educators in the department. However, his undergraduate preparation was as a secondary mathematics teacher. Other instructors, seen as mathematics educators, had minimal (five years or less) experience teaching K-12; for one, this was in a substitute role (5 years), for another, this was in a private school (1 year). (See Figure 1, p. 46.)

The instructor most oriented to a single, powerful role for the course to play was Professor Gail Laughlin. She was unique in her advocacy of reducing anxiety as the primary role of the courses in the elementary education mathematics program. Most instructors indicated believing that competence preceded confidence; Gail felt that positive attitude preceded the development of competence. Gail did not ignore other roles, adding complexity to her conceptualization, but the role of reducing anxiety was foremost in her mind.

The last instructor, Professor Craig Kennedy, held many of the same goals as other instructors. Unlike most instructors, Craig interpreted these goals most often through the lens of concern for students as individual learners as opposed to students as future teachers. In addition, Craig's awareness of his own learning in the context of teaching the courses set his conceptualization apart from others.

The following figure (Figure 2) indicates the central commitments to roles for the courses as indicated by the instructors. The italicized role or roles represent what I assessed as the instructor's primary intention for the course. Instructors rarely indicated no value for any of the pre-described roles, but the roles indicated here represent those that each instructor emphasized as most important throughout the interviews and were in evidence in their teaching, if observed.

<u>Instructor</u>	<u>Roles</u>
Kara Sheffield	<i>Content</i> and Attitude
Cynthia Yoder Teaching,	<i>Content</i> , Norms, Dispositions, and Attitude
Jeff Sullivan	<i>Content</i> , Norms, <i>Dispositions</i> , and <i>Gate-keeping</i>
Frank Adams	<i>Content</i> , <i>NOT Teaching</i> , Not Gate- keeping
Gail Laughlin	Content, Teaching, Gate-Keeping and <i>Attitude</i>
Craig Kennedy	<i>Content</i> , Norms, Dispositions, and Attitude

Figure 2
Predominant Roles for Instructors

Kara: Multiple Roles and Mixed Messages

Professor Kara Sheffield characterized the mathematics content courses in prospective elementary teachers' preparation as having multiple roles. She wove the roles together in her interview responses so that, content and method, attitude and gate-keeping, all held merit in her eyes. Kara often set the roles the courses were to play in the context of dilemmas posed by having conflicting goals or influences on attaining the goals. Students' choice of the pursuit of teaching as a career was the principal guide for Kara's emphases in the courses, but concerns beyond those of a departmental nature, related to the larger teacher preparation program, did not influence her.

Kara related her predominant concerns to both students' professional needs, having the necessary understanding of mathematics to teach competently, and students' personal situations, having the aptitude and attitude needed. The first two sections of this analysis address the roles that Kara emphasized, learning content and developing positive attitudes. The final section is an examination competing goals or influences that affected the roles Kara felt the courses should play for her students.

Content: Understanding for Teaching

Content for Understanding

Kara most strongly emphasized the goal of developing understanding of the mathematics that students studied in the courses. This was to be achieved through a particular strategy --

asking students to solve problems in many different ways. Kara assumed that students entered the class knowing how to operate computationally, and indicated that the purpose of this course work was to build conceptual understanding. This required more and different energy from other mathematics courses.

I think it takes more thinking, not at a high level, you know, mathematical thinking. But it takes more thinking and reasoning, because they've got to look at how it goes together. Yeah, there are two or three different approaches to this, for example, the greatest common factor. We do it two or three different ways, and I think there's even four or five ways that are shown. (13 C)

If students were willing and able to learn how to look at problems in many different ways, they would be more successful in the course; Kara saw this ability as critical for future teaching, and she connected success in the course with competence for future teaching.

Deeper understanding seemed to be synonymous with multiple representations¹² in the way Kara applied the phrase. The motivation for having the deeper understanding-different representations resided in the importance of this in the teaching of the content rather than learning the content as a student.

You can't just say "I know one[solution method]," you know, especially if you're going to teach. And I see it different, if they're going to teach and if they're not going to teach, that you need to know all the different ones. You can't just say "I'll find one that works for me." You have to find one that you can

¹² The word "representations" is used throughout the analyses to refer to concrete, pictorial, abstract, applied, and verbal depictions of mathematical concepts or ideas unless a restriction (e.g., concrete representations) is indicated.

make work for any kid that you might have. . . . There's different ways of learning. (5-6 B)

The different ways of learning that Kara mentioned were not restricted to use of different representations, though different representations might support different ways of learning.

Some people learn better visually, which I do not. People learn better by hearing it. Some people learn better by making themselves little notecards. Some people learn better by talking with another student. There's just so many things that. And I think tho-, I really think that those things need to be presented. (15 P)

So, different strategies for presenting content as well as different ways of representing the content were both seen as important parts of what students were to learn in the course.

For Kara, the mathematical concepts to be understood were typical K-8 content with some expectation of sophistication beyond that level, as alluded to above as "not at a high level": "They're learning a little bit more or a little bit deeper than they might want their students to know" (1 C). She placed emphasis on having many ways to look at concepts rather than having deeper insights: "We'd like them to have a little bit better background and know a little bit more than their students do and have a better feeling for the mathematics and a better feeling for how to problem solve to find out things" (14 P).

Kara described this "better feeling" for the mathematics developed in the courses both in terms of enjoying the study of mathematics and in terms of understanding the underlying reasonableness that permeates mathematical activity. She elaborated upon the latter goal as special to these courses: "There's

definitely a special role [for these courses] in explaining why and how some of the basic mathematics and different approaches and helping to understand them and not just do the process" (19 C).

For Kara, understanding the mathematics involved being competent with mathematical procedures and able to reason mathematically -- being able to "figure it out" on your own. Kara saw the content courses she teaches as providing places to gain conceptual knowledge of mathematics. Simply learning to compute or solve problems was not all that successful work in the courses required (institutionally defined success was obtaining a grade of "C" or better in the course). The skills were to be supported by knowledge of the underlying reasons for their appropriateness and use. Kara grounded her argument for learning this in a utilitarian philosophy: "What do you learn math for except to solve problems, to use it in everyday life, to use it in a job, to use it someplace. And you know, you're not learning math just to, so you can say, 'I know mathematics.' You're using it" (7 B). For Kara, students in the course needed to be competent with mathematical procedures and able to reason mathematically because they needed to be able to use this knowledge in their future teaching.

Having a variety of ways to present mathematical concepts was tied to the ability to communicate the content as well as to the ability to talk and write about concepts in meaningful ways. Kara identified these skills as essential to being good at mathematics and essential parts of the course.

Content for Teaching

Kara connected the goals of having students understand the content and being able to communicate this to others to the status of the students as prospective elementary school teachers. She rarely spoke of these goals as of value to students as individuals, but rather as of use in the practice of teaching.

If this person wants to be in an elementary classroom, it is important to get, to help them develop skills to, to learn to teach mathematics and to connect the concepts. I think it is important to keep 'em out if they don't have it, but I don't think, I think I see as my first goal, if you want to be a, a elementary school teacher, you do need to know these math concepts. You do have to learn them to do it. [vocal emphasis] (16 P)

Kara's consistent connection of goals with the status of the students as prospective teachers served to emphasize the distinctiveness of these courses from other courses in the Mathematics Department.

When talking about work on bases done in the course, Kara emphasized what students would learn about the teaching and learning of the content¹³ rather than what mathematical insights the students themselves would develop: "When I teach bases, I try to tell them, 'It's not that I think bases are real important and tomorrow we're going to be doing everything in base seven for the rest of our lives'" (9 P). She reasoned that working with base seven helped students see "how it feels" to work with an unfamiliar base to mimic the feelings of children and why children make the

¹³ The focus was on the students' learning the content as prospective teachers. This is not to imply that the focus was on learning how to teach the content.

mistakes they do through the experience of making those types of mistakes as students. The deeper insights encouraged for prospective teachers had more to do with having empathy for their future pupils than having a deeper insight into the mathematics itself.

As a requirement in elementary teacher preparation, Kara saw the course work as important beyond the individual to the profession: "I don't want someone teaching my child who doesn't understand basic concepts" (8 P). She implied that the courses for prospective elementary school teachers were to support, if not ensure, this understanding.

Aptitude and Attitude

Kara felt that not all people are cut out to be teachers; both aptitude and attitude are important. At a very basic level, students need an aptitude for teaching mathematics.

And if they [prospective teachers] really can not do this [the mathematics in the course], this is not their thing, it'd be like me trying to go into art. I could start from the bottom and probably stay at the bottom. If this is not their thing, I'd like them to find something else. (8 P)

The force intended in this comment is better understood knowing that Kara has poor vision, making the idea of her being an artist near absurd.

Kara saw aptitude as unchangeable, whereas attitude was malleable. She intended the courses provide an opportunity to develop and support the kind of attitude that a teacher of elementary school mathematics should have. She stressed a positive attitude toward mathematics, a "can do" attitude:

And, if you start to enjoy it, they [children] will. And it will make all the difference in the world. And without that confidence, and if you're not real confident, don't go in there and show 'em that (laughs), you know. Find some ways to find out more about it, or if you do a lesson that something didn't work out good, go find out a better way. Try to build your confidence. Cuz once you do it, once a problem comes out right, you're real happy and you're real confident, you know. And so I really like to stress that. (4 P)

Kara saw competence and success supporting confidence. The emphasis on confidence here was echoed in other comments, all related to the importance for teaching.

And I say, "You know, your students don't know everything you know and don't know. You've got to walk in there and you've got to be confident. You've got to make it fun. You've got to show them that you like it. If you don't, you're gonna be a crummy teacher, and if you can relate back to why you maybe don't like math or are having trouble now, I bet you can relate back to an elementary teacher." And I really emphasize, "You don't want to do to kids what somebody may have done to you." (4 P)

In this case, confidence was seen as going hand-in-hand with being able to make mathematics fun and avoiding the repetition of students' own possibly bad personal experience with mathematics.

The courses were to play the role of providing positive experiences that developed students' own positive attitudes and confidence, experiences much like those Kara hoped students' would design into their own teaching. The methods for such designing were not a part of the courses, however.

In this course, we really don't give them methods, and things like that. It's just content, but the idea of, like "Oh, math I'm going to teach isn't going to be that hard. This should be kind of fun. I hope I can learn how to do this." And that would be their goal for the next course, the content, the methods course is like "I hope I get some neat ideas on how to teach all this

stuff." And that would be, you know, I mean, that's the goal. But that "Oh, this stuff isn't that hard. And this stuff is kind of fun to do and I can make, I hope I can make this fun for kids to do." I think making it fun, building self-confidence and self-esteem, and having them want to do that with their students, is real important. (15 C)

Though the intention was to enhance students' personal attitudes toward mathematics through development of competence, Kara directly related the development of competence to its need for students as future teachers, not as mathematically literate human beings.

This "feeling good about being able to teach mathematics" was one of the two main goals Kara identified for the courses: "I guess I do see two main things, which is the math in the meaningful way and building confidence and empowerment and feeling good about being able to teach math" (1 B). The order in which these goals were mentioned was intentional. Her first goal was competency, with the personal realization of the competence as secondary: "My goals would also be that they would feel competent in problem solving and talking, working with mathematics. My main goal [would be] that students would be competent in talking about and working with and discussing mathematics"(1 C). Being competent was to be achieved through developing a deeper understanding of the mathematics.

Sending Mixed Messages

Mathematical Ideas as Interconnected vs. Staying One Day Ahead

Sometimes, Kara held conflicting goals concurrently. One tension Kara felt was in evidence in statements she made about what it means to understand mathematics in meaningful ways so as to be able to teach it. She identified developing students' ability to

interconnect mathematical ideas as an essential goal for the course and indicated that it was important that students see the curriculum as a whole.

I think that one of, some of the biggest problems that sta-, like how their math, and it's, like, this is the fractions, this is the decimals, this is the this stuff, and you never connect them all. They're this own little. And the more connections you make, (pause) and those connections, I think, are really what makes it, and so I really have liked them to look for connections. Sometimes I think I need to do a better job in helping them find them (laughs), but I think that's really an essential part of learning math. Otherwise, I think you're getting teachers who are up there doing page 2 and page 3 today, and page 4 tomorrow. Because it's on the page. (5 P)

The conflict came when talking about students who had not done well on tests and were concerned about their ability to perform the tasks of teaching:

I keep telling students who tell me, "Well, I don't do very good on tests. I panic." I say, "Can you do good in front of the classroom? You know, you're going to have your book, you're gonna have the night before to look over just that day's lesson. Can you do that?" you know, "If you've forgotten it, are you good at looking it up and being able to do it? That's what's going to matter in the long run. No one's going to be standing over there, grading you in that sense." (9 P)

Kara reiterated this in another interview, saying: "I said, 'When you're teaching it, you can go to a text book. You can look at things the night before. If you feel like you're understanding what's going on, then you should be pretty happy that you're learning something'" (3 B).

Kara saw both the essential nature of viewing interconnectedness of mathematical ideas in meaningful ways and the acceptability of preparing for instruction in the way described

above on a day-to-day basis. These two seemingly contradictory views represented a tension. Kara felt that the course should play the role of developing meaningful mathematical content knowledge, but it also needed to affirm students' abilities to deal with the tasks of teaching mathematics. Kara acknowledged that ideal understanding of concepts and the reality of student understanding did not always match. Furthermore, she acknowledged that there was no way to "cover" all the content that students might be called upon to teach.

Kara was expressing a common conflict in goals for teacher educators. On the one hand, the goal is developing in students a "big picture" of the domain being studied. On the other hand, the goal is setting students on the road to a career in teaching. Students do not always gain a big picture of the domain in the particular course we teach. We anticipate that they will develop it through the whole of the teacher preparation program in which they participate.

Teaching as Usual vs. Change

Mathematics teacher educators are currently faced with several recommendations for change in the mathematical preparation of elementary school teachers. These imply changes in the roles the content courses should play in students' programs. Kara sensed that she was sending mixed messages about what was important for students to learn and how they were to learn. She told them they should not lecture to students, but used lectures as the predominant method of instruction; she talked about developing students' abilities to communicate, but relied on written test

responses as the major opportunity for students to communicate mathematical ideas.

Kara was alone among instructors in airing conflicting feelings about the current reform and its impact on familiar practice. While some instructors noted the reality of change and the call for reform, no other instructor expressed the cautionary notes that Kara voiced. She verbalized cautions about a range of issues, from a specific concern about the selection of a new textbook to a broad concern about change in general. Kara's perspective shaped what she saw as appropriate roles for the courses in students' programs. Kara's cautions were both intellectually sound and personally grounded. They were calls for discussion and questioning.

Kara did not say she was unwilling to change. She saw the need for reasoned examination of the total situation, including the necessity to change lessons due to the use of a different textbook. Having had considerable experience using the current textbook, she was unwilling to "jump on the band wagon" to change texts. For her, agreeing to change textbooks would be based on careful analysis of textbooks, curricular content, departmental course design, and the ramifications of replacing the current textbook.

Kara cautioned against moving too quickly on complex issues. She reiterated this in her response to a question about how she saw changes in the content and design of the courses for prospective elementary mathematics teachers:

Oh, I, I really like what I, what I've been seeing as far as most of the shifts. I'm hoping that we're not going to go too fast and go too quickly. You know, when the new, the New Math came out, people like went, "Swee [throwing her arms into the air,

making this sound]," all extreme. And I think a lot, some of the stuff in the New Math is good and we still use, but they like went overboard. (9 C)

Notice Kara's language: "I really like what I've been seeing as far as most of the shifts." An important aspect is subtle; she has "seen" shifts. Her words do not infer personal action. In this distancing lies another layer to the conflict -- a very personal layer.

Seeing shifts and making shifts are very different. As noted in earlier sections, Kara has been making some gradual changes in her practice. But what is lost with a shift away from familiar practice? Consider what was implicit in the continuation of the previous comment:

But, I think it's [shifts are] gonna, it's gonna help a lot in, (short pause) being able to teach, you know, having some flexibility to talk about and reason through problems as opposed to, I really have been trying to do less and less lecture. I mean the way I was taught, the way I was taught to teach, was you get up, you lecture, you answer questions, you hopefully explain well. And, I mean usually, one of the things I get as good comments from students is that I explain things well, you know. So this is, I thought, "I'm doing really good. I can explain things well to [students]." At least 90% of the students say that, and that I'm slow and detailed, and all of this. (9 C)

A shift away from detailed explaining is a shift away from what she knows she does well and has ample encouragement for continuing to do. What the shift is to is not clear from her comments.

Unfortunately, this situation is a common one. If mathematics teacher education is to move toward the vision of the Standards, that vision needs to be exemplified and encouraged somehow. Kara supports reasoned action. She also acknowledges the constancy of

change. But, these do not always integrate smoothly. Questions arise.

I think things are changing constantly. And you always want change and you always want to listen to what's going on. [vocal emphasis] And sometimes it may not even sound real good. And I think you want to give it a chance. I don't know that you want to jump in, you know, unless, depending on how you feel about it, but there's just, you know with all the stuff with the Standards now, just recently even, and even some of that, I think, needs to be weighed and thought about. And then how do you implement them? And how do you assess them? And you know, all these things that are going on. I just think this, this is an exciting time to be in mathematics. And we've seen the, you know, back-to-basics and New Math and all these trends.
(18 P)

This time of reform may be an exciting time to be in mathematics, but it is also a complicated time. Kara raised some important questions.

Though Kara was alone in her expressions of caution about reform and change, she was probably not the only instructor thinking about this. Perhaps her own sense of confidence about what, how, and why she teaches the courses led her to be forward about her concerns. Alternatively, she may have simply felt more open about the interviewing process. Whatever the explanation, it seems reasonable to speculate that others share some of her concern. It is for this reason I see this conflict as important in the context of describing the roles that mathematics content courses play in teacher preparation. If mathematics teacher educators are not sure about the vision set forth in the NCTM Standards as compared with their own beliefs, how can they be expected to prepare prospective teachers to teach with "the vision of the Standards"? What roles

should the courses play and to what extent do the orientations of those roles depend on acceptance of the NCTM Standards as a guide to the enactment of those roles in students' programs? What would acceptance mean for the instructional practice of instructors like Kara? How can Kara's caution be used in productive ways to help her and her students consider the bases for decision-making in their teaching of mathematics? Caution is not a negative in and of itself. But, when caution serves as a wall around sacred practices, it can be destructive.

Kara gave no indication that her cautions were serving as walls to keep ideas out, but they did appear to be protective. She is attempting to make gradual changes in her practice, in this way making change less stressful and risky.

I'd like to see a lot more than just lecture. I've been gradually working in more things that I like to have them do in a group. (pause) I'm trying to work more and more of that in, and there are so many times, (pause) And I wish I had more, you know, I want to work more time in my class to do that kind of thing. And I see this movement as giving me a chance to do that. Cutting back on the lecture. Cutting back on the, you know, having to have these formal definitions of everything. (pause) And I think we're working away from some of that. And I like that. (9 C)

As Kara tried teaching as she interpreted the Standards, she was pleasantly surprised at what she saw, e.g., when talking about working in groups -- "And it is amazing what students can learn from each other and what they can tell each other." (8 P)

Summary

Of all the participants in this study, Kara's characterization of the roles mathematics content courses play in prospective

elementary teachers' preparation most closely reflected what I anticipated before beginning data collection. No single pre-defined role overshadowed any other to the extent that she discounted any of the others. Kara's conceptualization highlighted the complexity of issues of teaching and learning in the course in many and interesting ways. While the mathematics education community in the form of departmental colleagues and the NCTM Standards influenced her conceptualization, Kara did not mention collaboration or any active involvement in elementary classrooms¹⁴ or with faculty in other departments.

Kara had a self-proclaimed number one goal; she wanted students to understand the mathematics they were expected to teach as elementary school teachers. The simplicity of that statement concealed the question of what understanding is, the issue of what the nature and content of mathematics for elementary school teachers should be, and the acknowledgment of the students' status as prospective teachers as a critical influence on the content and instruction of the course. Additionally, several other statements made by Kara implied that gate-keeping was an important role for the course as a way of ensuring that students are properly placed and supported in their attempt to become teachers or guided away from teaching as a career. The role of developing and supporting a positive attitude toward mathematics and teaching mathematics permeated much of her description of the content and its focus. The pre-defined roles appeared to fit with Kara's characterization of the roles of the course. No other role seemed to apply.

¹⁴ Except as a parent.

I discussed the mixed messages that were sent to emphasize the complexity of the issues connected with determining the goals for the teaching and learning of mathematics in these courses and in teaching, in general. The dilemmas represent the many and varied struggles that all mathematics teacher educators must deal with as they attempt to prepare future elementary school teachers. In some cases, Kara explicitly identified the tension and acknowledged her struggle. In other cases, she did not acknowledge the conflicts embedded in her responses. This silence does not mean that she did not recognize them or that she was unaware of them. Acknowledged or not, the tensions had an impact on what roles she saw as attainable and reasonable.

Cynthia: Developing Competence and Changing Perspectives

Professor Cynthia Yoder explicitly and strongly held learning the mathematical content, learning the norms of mathematics, and developing confidence about doing mathematics as roles for the course. She also saw herself dealing with learning to teach mathematics implicitly, through modeling. Though Cynthia did not emphasize gate-keeping, it was there by design in the form of course requirements.

Cynthia's perceptions of what the students brought to the course she taught at CC guided her in establishing course requirements and in setting the classroom environment. Her beliefs about what the students should know and care about after taking the course also guided these decisions, more so than departmental policy. I use poor preparation and traditional views, the two perceptions of what students brought to the course that Cynthia stressed, to organize my analysis. I begin by considering how different aspects of students' poor preparation for the course had an impact on course content and design.

Content, Norms, and Teaching: Poor Preparation and Curricular Response

Cynthia perceived that students came into the course poorly prepared in many areas. She saw a role of the course to be to change this situation, and she responded by designing the course to address student needs. The three areas of weakness that had the greatest impact on the course content and her instructional practice were

J
a
co
re

cou
mo
skill
but
com
Arith

s
s
E
s
s

computational facility, conceptual understanding, and the ability to communicate mathematically. The first two areas involve learning mathematical content. The third involves the roles of both norms and teaching.

Poor Preparation in terms of Computational Skill

Cynthia acknowledged that the setting of the course in a large urban community college with minimal entrance requirements affected the diversity and caliber of students who were in the course. The content of the course in this setting had to include material that Cynthia assumed would not be necessary at the four-year institutions to which the students would transfer: "We are not only dealing with trying to teach them the concepts or where these concepts came from or problem solving, but we also have to remediate" (3 P).

To ensure basic computational facility, a requirement of the course was the completion, outside class, of Interactive Math modules or testing out of the Interactive Math program, a self-paced skills program using videotapes. This was part of Cynthia's syllabus but was not a Division requirement. Cynthia emphasized the value of computational ability in the context of two courses she teaches: Arithmetic and Mathematics for Elementary School Teachers.

I think it's just, after we learn to walk, we should be able to add, subtract, multiply and divide. (laughs) Okay, it's just an inherent thing that we all should have. Obviously one of the goals as written by the Division of this is that we have, for the Arithmetic class, one of the goals is that they can add, subtract, multiply and divide rational numbers. In the Math for Elementary School Teachers, that's really not one of the specific goals. It has become one of my goals because I have seen students who can't do it, and, certainly, we can't send

U
W

be
fe
ca
im
par

Cynth
in the
institut
backgr

them into the elementary school classroom if they can't do that. [vocal emphasis] (13 B)

By requiring the completion of the Interactive Math program, Cynthia was dealing with her concern about the students' ability levels while pursuing the goal of preparing students to move on in their program with adequate computational skills.

Poor Preparation in terms of Conceptual Understanding and its Value

Cynthia spoke of two aspects of poor preparation with respect to conceptual understanding; one was valuing it, and the other was developing it. She also emphasized the value of conceptual understanding to students as individuals and as future teachers.

Valuing Conceptual Understanding

Cynthia bemoaned the students' lack of concern for going beyond a procedural understanding of mathematical concepts. She felt their view that "We don't need to understand why, as long as we can do the algorithm," (2C) was shallow and a reflection of their immaturity. This satisfaction with procedural understanding particularly disturbed her in the context of teacher preparation:

When I have this attitude [just-give-me-the-rules] in a Math for Elementary School Teachers, it disturbs me tremendously because I just see this attitude being passed on to another generation of children, and that just really bothers me. And, maybe I haven't been in the profession long enough to distance myself, but it becomes a very emotional and very stressful thing. (4 B)

Cynthia discerned one potential basis for change in student attitudes in the new pre-requisite requirement that the department had instituted for the course. She felt that, with stronger mathematical backgrounds, students would come to the course appreciating the

value of understanding why mathematical properties and concepts are reasonable. As it stood, however, she felt that the course had to play the role of developing this appreciation in the students.

Developing Conceptual Understanding

Cynthia perceived that students were poorly prepared in terms of their actual conceptual understanding of the content. She described an exemplar of her definition of conceptual understanding following the interviewer's observation of an Arithmetic class¹⁵ lesson on perimeter and area that included a demonstration involving cutting out shapes.

Today, I could have just given them the formulas and given them some worksheets with some numbers on it and said, "Go to it." That's just not me. I can't get it out of my head that if you understand something, you will retain it better. You'll learn it better. And so, obviously, I went through the cutting out of the forms and trying to develop the concepts. (12 B)

When asked how this lesson would be different in a Mathematics for Elementary School Teachers class, Cynthia noted that the students would have done the activity in small groups, each group cutting out shapes and considering questions about perimeter and area. She emphasized the importance of the students developing a sense of what concepts mean and having a sense of the mathematical reasonableness of solutions.

Cynthia stated that "you're searching for whys" (11 C) throughout the course. When asked if the "you" was intended as singular or plural, she responded, "Most often, it's me," (11 C) and

¹⁵ Cynthia was not teaching the mathematics content course for prospective elementary school teachers at the time of the study. She was observed teaching the Arithmetic course.

laughed, before saying that she would want it to be plural. In other words, she wants the students to join her in the search and, ultimately, to make the search their own. She suggested that the difficulty arose because students are low ability: "They're not really wanting to know the whys and the wherefores and learn the concepts and understanding. It's just get-me-through-this-class kind of attitude" (4 B). She also blamed the diversity of her students for the difficulty in getting students to search for conceptual understanding, recounting a story of the uneasiness caused when a 50-year-old student was unable to control his enthusiasm about coming to really understand a concept. The younger students were "turned off" by his excitement.

Personal and Professional Value of Conceptual Understanding

Cynthia saw a role for the course in developing students' abilities to teach for conceptual understanding as well as in developing their own understanding. Her argument melded the need for individual insight into mathematical concepts with being able to teach well. Knowing recipes was not sufficient. Being able to examine concepts from different perspectives, a quality she saw as needed in good teaching of mathematics, required having deeper personal insight into concepts.

Cynthia indicated that she saw what she attempted to do in the Math for Elementary Teachers as different in nature from what students had experienced in the past. Notice in the following her use of the pronouns "I" and "we."

I'm assuming, from what I've been told, that they [students] have been told, they have been taught the mechanics. And I'm

getting away from the mechanics and going into concepts. I'm developing vocabulary, because we're often talking about "top numbers" and "bottom numbers" and "flipping" and we're "cross-multiplying" to divide fractions, . . . but I'm searching for the vocabulary. I'm searching for the mathematically sound whys, the wherefores, the understanding, okay? Because, what happens when fifth-grade Johnny or Suzi comes and says, "Why does this work?" Do you send them back to their seat, "Because it does?" To me, that's totally inadequate, especially, when fifth-grade Johnny or Suzi are potential mathematicians. You know, you don't want to turn off those whys. You know, you don't want them to go back to their seat and just do their worksheet and do the algorithm. You want to nurture. (12 C)

In this statement, Cynthia's use of pronouns pointed to her sense of opposite actions on the part of the students and herself: "I" am developing and searching; "we" are flipping and cross-multiplying. Cynthia's comments about moving from a singular to a plural "you're searching for whys" enhanced the sense that she was trying to motivate students to accept her vision of appropriate mathematical action.

Somebody said [in solving an algebraic equation] "We're going to move a number," and I just, "We don't 'move' in this class. We don't 'flip' in this class. We don't do any kind of magic in this class. We do it by mathematically correct operations." [vocal emphasis] (17 B)

She stressed the importance of reasoning and sound action for her students, both in her class and in the students' future classrooms.

Cynthia's rendering of students' poor preparation with respect to conceptual understanding was complex. They did not value or possess conceptual understanding as she intended and the sometimes nonsensical views students did possess were antithetical to

Cynthia's goals. She intended for her curricular responses to deal with these situations.

Poor Preparation in terms of Communication of Mathematical Ideas

When connecting conceptual understanding to teaching, Cynthia often promoted the importance of communication. For example, when describing the importance of being an effective problem solver, she connected the ability to communicate problem solutions to modeling of this that she did in the classroom. She noted that solutions needed reasons or justifications and that the ability to articulate reasoning was important to teaching.

Cynthia acknowledged the realities of teaching and the importance of accuracy in communication. She pointed out the need to write understandable tests, lesson plans, worksheets, and other materials to be used in teaching: "The vocabulary has to be correct and that the communication of ideas has to be correct" (11-12 B). In her relating of communication to teaching tasks, Cynthia gave a sense of what counts as communication to her. The communication involved in "writing a test" was not simply in the physical writing: "Just the instructions on a test, just the layout of the test. If they would be creating a word problem, to make sure that what's going into the word problem is correct" (12 B). She spoke of the need to be clear in the way vocabulary was used and solutions were written. Cynthia supported the development of some of these explicitly in the class, having students present work at the board or on the overhead projector and requiring the creation of problems related to course content as a part of students' weekly portfolio work.

C

portfolio

develop

of their

value,

to a q

for stu

already

E

c

f

c

n

t

c

s

a

a

f

e

c

Cynthia was the lone instructor interviewed who used portfolio work as any part of students' course work. Cynthia developed her own system for portfolio use in her course. Because of their unique structure and Cynthia's reasoned argument for their value, I present her description here as a piece. She was responding to a question about whether the course should provide an opportunity for students to think about how the mathematics that they had already learned was taught.

Every week, my students have a portfolio. And that portfolio consists of five questions. The first question is a reading from the Research Within Reach and then they have sub-questions on that reading. So, they are thinking about remediation. They're thinking about diagnostic tools. They're thinking about assessment. You know, we do fifteen readings out of that book. There, and also, I start them with Sheila Tobias' How to be Successful in Math, just to break some anxiety. They read the chapter "How to be Successful in Math" and "How to Read a Math Textbook," because they don't know how to read a math textbook. And, as teachers, they have to know, to be able to explain. So even though it isn't under our department's list of objectives, I feel that it is part, it is my objective for them to reflect on this, and to start to form ideas of how are they going to be a teacher. Also in their portfolios, they have writing exercises where, number two is a writing exercise, where some concept that we've developed, they have to explain as if they were tutoring a peer in this concept. So that I'm teaching them the writing skill. And then we have, generally, some word problems or problem solving that is out of the ordinary that applies to what they've, we've done that week. And then the fifth problem is generally write two problems and solve them and explain, and again as you would be teaching a peer, how you would come up with your solution. So, these portfolios are a lot of work for them. But, I think they really learn from them. [vocal emphasis] Even though, again, it's not a department objective, I want them to start thinking about how they learn. (4 P)

There is a staggering amount of information about Cynthia's hopes for the course embedded in this description of the portfolio assignments. She related many of her intentions for the portfolio work to developing communication of mathematical ideas. A partial list is: to learn about how research can be interpreted and used, to learn how to read a mathematics textbook, to use communication to reduce mathematics anxiety -- learning that students are not alone or helpless, to learn to give thorough mathematical explanations, to learn to write coherently, and to learn how to ask good questions.

As implied in the list of portfolio aims, teaching was not the only arena in which Cynthia saw the value of learning to communicate mathematically. Her assignment of the creation of problems as part of students' portfolio work hinted at her sense of the value of communication to understanding: "I think, when you write a problem, you're going beyond doing a problem" (19 C). Through talking and writing about problems, she felt that students gained a deeper insight into the mathematics used to solve the problems.

Cynthia designed the writing tasks to improve students' understanding and their ability to communicate that understanding and had only indirect connection to issues of grading. This disconnection from grading meshed with her stated lack of concern about a gate-keeping role for the course: "I'm not really concerned about being a barrier to anyone. I think that's going to happen. If the student's not going to be a teacher, they're not going to be a teacher" (4 P). Cynthia set standards that students were to meet in order to succeed in the course. If students finished the course able to

compute with facility, had demonstrated their understanding of concepts in their portfolios and on exams, communicating that understanding orally and in writing, the students would likely complete the course satisfactorily. If they did not achieve some part of this, students would need to retake the course or self-select out of the teacher preparation program.

Attitudes and Teaching: Traditional Views and Curricular Response

Dealing with poor student preparation was only part of what Cynthia wanted to accomplish in the context of the course. The way students viewed mathematics and the teaching of mathematics also concerned her. She saw a role of the course to be changing these perspectives. She perceived that students entered the class with a view of mathematics as irrelevant, mechanical, and boring. Their view of mathematics teaching was simplistic and based on their years of experience as students. This view was stereotypical, traditional, and not desirable in Cynthia's eyes. As with poor preparation, she responded by designing the course to address it.

Mathematics as Irrelevant

Students entered the course with a view that mathematics was lacking in meaning and purpose for them. Cynthia wanted students to see mathematics as a meaningful part of their personal and professional lives, a part that had underlying structure and pertinence, not mere utility. Her use of the word relevance set out four emphases: de-mystification, utility, understanding, and visualization.

A

mathem

workin

showin

that th

cubec

myste

prob

matl

imp

eve

rel

re

m

u

t

e

At a very basic level, Cynthia tried to de-mystify elements of mathematics that seemed "unreal" to students. For example, working on volume, she clarified what a cubic centimeter (cc) is by showing them a block, one centimeter on each side. She pointed out that the volume was found by taking one times one times one, one cubed. In this way, a symbol holding little significance and some mystery, "cc," gained meaning.

On a second level, Cynthia used everyday experiences in word problems or work on logic to help students see the usefulness of mathematics. One example she gave involved using logic and implications to determine if a student would get the car for the evening based on certain information. The focus was specifically on relevance for the students as learners in these first two cases.

For Cynthia's third emphasis, the content was to be made relevant to the students by helping them put the 'big picture' of mathematical ideas together. This connected relevance to understanding. The relevance of learning concepts in one context so that later work would be meaningful was explicitly a part of the course. The later work could be in further study of a concept as a student or in actual teaching of a concept as a teacher.

The "seeing" of reasonableness in the doing of mathematics was the fourth emphasis Cynthia highlighted within the aspect of making mathematics relevant in the course for prospective teachers. She used manipulatives and encouraged visualization and the use of pictures. If students could see, physically, that certain properties are true and can be useful in building understanding, Cynthia was

convinc

unders

visual

Feeling

With t

embed

utility,

role o

course

Mathe

dread

in the

could

throug

mathe

mathe

fabulc

convinced students were in a better position to build upon that understanding in valuable ways.

Cynthia had personal grounding for her emphasis on visualization.

To me, math is a very visual thing. . . . And one time [in a graduate class], we were doing something in second semester Real Analysis, and I just tried to draw a picture. And he [the professor] goes, "See it in your mind. I want you to see it in your mind." And that's what he would always say to us. "See it and close your eyes and see it and feel it." And that's kind of what I try to, see it, get a sense of it, feel it into your soul. And , it's just something that even with my Arithmetic students, I try to teach them that approach. (17 B)

Feeling the concept in the soul was what Cynthia was hoping for. With this feeling, the relevance of the concept would be deeply embedded in the individual.

Cynthia related all of these emphases, de-mystification, utility, understanding, and visualization, to her interpretation of the role of making mathematics relevant for the mathematics content course for elementary school teachers.

Mathematics as Forbidding

Students entered the course with a dread of mathematics and a dread of doing mathematics. They perceived themselves as stymied in their attempts to do mathematics, and they were unsure that this could ever change. Cynthia intended to change this perception through the nurturance of a different attitude toward doing mathematics to be accomplished through experiences in the course.

Cynthia saw supporting a positive attitude toward mathematics as a minor component in teacher preparation: "I have a fabulous attitude towards figure-skating, but, swoo, aside from

puddling

would e

Valuing

required

necessa

[future

enjoyment

F

about

positive

doing

Student

course

experie

student

paired

"I think

heard

Student

sense

feeling

classr

puddling along forward, I can't do anything, and there's no way that I would ever go on the pretense of being a skating coach" (9 P).

Valuing a positive attitude toward mathematics as a subject is not required for teaching, but it is beneficial: "Is it [positive attitude] necessary to get children to compute? No. [But] I don't think they're [future teachers] going to transmit a, an appreciation of math or enjoyment of math or will nurture students to go on in math" (10 P).

For Cynthia, the requirement was not for particular feelings about the discipline, but rather capability with the content. The positive attitudes that Cynthia wanted most to nurture were toward doing mathematics and feeling justifiably competent in doing it. Students were to gain these feelings through experiences in the course that built up content knowledge and that relied on experiences that were fundamentally different from that of most students up to this point in their mathematics learning. Cynthia paired "feeling good" with empowerment gained through competence: "I think they have to feel good, that, 'Yeah, I can do that!'" (4P). I heard this refrain throughout her responses.

You certainly don't want a student getting in front of a classroom thinking, "Can I do this? What am I gonna do? What happens if I make a mistake?" You know, just that whole lack of self-confidence is going to undermine their authority in the classroom. It's going to, I just see all sorts of problems coming up with not feeling good about their own knowledge of a subject. (14 B)

Students needed to feel that they could try problems and have a sense that they could find a solution. But, feeling capable and feeling that you have "the knowledge" to be the authority in the classroom are not the same meanings of empowerment. Cynthia sent

conflict

and stu

T

attitude

well by

mathem

how to

rest be

and su

C

own un

[

f

t

v

c

h

s

Cynthi

enhan

ideas

which

with t

attitu

Mathe

on the

classr

conflicting messages as to her intentions for students as learners and students as future teachers.

The students as learners brought a mixed bag of abilities and attitudes. Cynthia attempted to ensure that she prepared students well by offering them the opportunity to learn much more than pure mathematical content. She felt that she needed to help them learn how to learn, how to read a text, how to study for a test, and how to rest before a test. She supported this learning through assignments and suggestions.

Going beyond "content" to develop the ability to rely on their own understanding and justification was also a goal for the course.

[I want them to learn] that they can look at different examples for different things and draw conclusions or conjectures. That they don't have to be told all the time what's going on. That when they think about things, that they can come to conclusions and develop mathematical ideas on their own. . . . It's not a written objective of the course, or, but it's, it's something that helps build self-esteem. It helps build up their self-confidence. It relieves some math anxiety. (18 C)

Cynthia intended that experiences in the course either develop or enhance students' abilities to conjecture, to develop mathematical ideas on their own, and to feel confident in their ability, none of which was viewed as "mathematical content." These goals align with the roles of learning the norms and developing positive attitudes.

Mathematics Classes as Demonstration-and-Practice Sessions

Students entered the course with certain expectations based on their "traditional" experience. They had spent many years in classrooms for which mathematics instruction took the form of

teacher
problem
expecta
experie
challen
mathem

F
a
t
h
f
n

She c
comme
You ca
would
comm

expect
respec

Beca
that t

teacher lecture/demonstration followed by seatwork of practice problems. Cynthia perceived the need to change student expectations and attempted to accomplish this through the experiences she set up in the course. She also pointed to the need to challenge students' perceptions of what it means to teach mathematics.

Frequently I've had students verbalize to me that all you do in a math class is hand out math dittos, or "Why do we need to do this because we'll have, or know how to do it, because we'll have the Teacher's Answer Book?" And these attitudes frighten me. We need to change student attitudes about what a math classroom should be all about. (3 P)

She discouraged "survivalist responses," unlike Kara, who commented that "When you're teaching it, you can go to a text book. You can look at things the night before" (KS, 3 B). This comment would "frighten" Cynthia as much or more than the above student's comment.

Cynthia found that the teaching to support a change in student expectations was difficult. She spoke of this difficulty with respect to the use of discovery lessons.

I'm a very discovery-oriented, and, group work and conjecture kind of person. Sometimes the stuff is more difficult to motivate the discovery, the false discovery maybe, (short pause) of the things they already know. "We already know this. Why are we doing this?" "Well, you already know the algorithm, but do you understand why? Let's see if using these pattern pieces or these Cuisenaire rods we can understand why we're doing it." And so, like, the nurturing of answering the question why. (13 C)

Because Cynthia wanted students to understand why, she thought that the work to achieve this understanding was justified.

she ch
teache

When
need
and t

both
cours

Some of Cynthia's motivation for the instructional strategies she chose stemmed from the status of the course as a part of teacher preparation.

I see my classroom as being a model of what a math classroom should be. We do a lot of group work, we do a lot of interactive thinking, they do a lot of board work, they do their portfolios, so math isn't just putting a number under some kind of problem. And I think that's critical for us to change what's going on in mathematics classrooms. [Her Division Head] has said to me, "You teach as you have been taught." So, I feel I'm the beginning of changing that. (3 P)

When asked how she would respond to students who argued for no need to change since their experience was in traditional classrooms and they felt prepared, Cynthia replied:

"A lot of people have been studying the teaching mathematics, the development of mathematics, and we're finding that it is much better to nurture the understanding, rather than just to learn the rules. And so, you don't, are you going to drive the same Model-T that your great-grandfather drove, or are you going to try the new Probe that is coming soon?" We don't always do things the same way, even. . . . If you find out that something's better, then you try to do it the better way. (12 C)

Cynthia's "trying to do it a better way" has meant changes in both content and methods from the traditional mathematics content course for prospective teachers.

I'm sensing, that previously, teaching the content was an okay thing, and I don't think it's just okay anymore. I think bringing in the manipulatives, bringing in thinking, bringing in problem solving, you know. Problem solving, again, is a big thing, and, in doing problem solving, to have the teacher stand at the board and do several problems, (laughs) you're not teaching problem solving. Do you know what I mean? . . . So, I just get the feeling that group work is now essential. Maybe before, a teacher could lecture, I think group work is essential. I think the use of manipulatives is essential. (short pause) So much

m
to
th
(

"You h
develop
Cynthia
teacher
for usi

I
t
s
c
s
c

Cynthi
as a v
thems

design
profes
the c

more now is essential because we see that our teachers have to be think, teaching thinking skills. When you're teaching thinking skills, we first have to teach them how to think.
(12 P)

"You have to know it to teach it" was a frequent refrain, but it was developing personal knowledge of the content that overrode Cynthia's decision-making, not the status of the students as future teachers. Viewing students as learners was the primary motivator for using a variety of teaching strategies in the course.

I think that for themselves, they need variety. In general, they're not the strongest math students, and just to come and sit and listen to someone talk at them, just isn't a real enjoyable thing. Doing the different things (short pause) gives them as much, you know, makes this as pleasant and as enjoyable situation as it can be. I think it helps them relax a little bit. Relieves that anxiety. It just, sometimes, you learn better doing something a different way. (17-18 C)

Cynthia also saw her teaching as a way to model good practice and as a way to encourage students to learn the necessary content for themselves as learners and future teachers.

Cynthia found justification for the changes in the teaching and design of the course in policy documents and in conversations at professional conferences. When asked about the impact of these on the course, Cynthia replied:

They're critical to the way we are, to the way I design my course, okay. By the incorporation of more peer-learning and group work. Coopera-, er, the inclusion of manipulatives, okay, we use Cuisenaire rods and the pattern pieces for fractions. We use the base 10 blocks for decimals. And this is all part of the Standards. It's all part of, I went to the conference on Teacher Preparation, previous one at [another community college]. And so, obviously, you know, someone new, I'm seeing all of this, trying to incorporate all of this into our course. Not just so that we have a "with-it" course, but that our

n
s
le
no
ch
stu
cla
as
ther

students are exposed to this so that when they teach, when they go out to teach, this is part of them. (6 C)

So, Cynthia found additional support for her choice of content and instructional practice through outside informants in the shape of policy documents and conferences.

At CC, the college catalog included an "Education Studies Program" for students to use as a guide for course-taking. However, CC, while offering content courses that could be transferred to other university teacher education programs, had no Department of Education. Despite this, Cynthia contacted other instructors of the courses and mathematics educators off campus to discuss issues around the content and instruction of the mathematics content courses for prospective elementary school teachers.¹⁶

In these many and various ways, Cynthia justified and implemented changes in her own teaching as a vehicle for changing students' traditional views of what mathematics teaching and learning look like. Student expectations upon entering the course did not match what Cynthia wanted the students to anticipate. She chose content and methods for teaching the course intended to alter students' future assumptions about what goes on in mathematics classrooms. She made these choices to better prepare her students as she saw their needs, not because the students' program expected them to have an altered vision of mathematics teaching and learning.

¹⁶ Cynthia was not involved in elementary classrooms except as a parent.

Summary

Cynthia based her decisions about the content and instruction in the course on her perceptions of what students brought to the course, her beliefs about what students should know and care about when the course was complete, and the Division policy for the course. These all have been and are affected by many different influences, ranging from individual students to national professional organizations to other institutions.

The result was that Cynthia set near-impossible goals for herself and the course: "I have such high expectations of myself as an instructor and I try to do so much in the course" (26 B). Cynthia saw her focus on developing computational ability, conceptual knowledge, and communication skills as necessary for the good of the students. The students needed a strong content knowledge base from which to continue their work in teacher preparation. Cynthia saw, in her teaching of the content, "many wonderful by-products that are very important to these students can be attained" (4 P). Those "wonderful by-products" were just as important to Cynthia as the development of content knowledge. The by-product most valued by Cynthia involved challenging students' perceptions: perceptions of the relevance of mathematics, perceptions of their ability to do mathematics, and perceptions of what a mathematics class should look like. She hoped to change student perceptions of mathematics as an endeavor and to change student expectations for their own mathematics teaching.

Jeff: Gate-keeping, Learning Content, and Developing Dispositions

Professor Jeff Sullivan was new to teaching mathematics education courses. Though he had secondary mathematics teaching certification, he had never used the it, choosing to proceed through a doctoral program in mathematics and teach at the university level. His responses indicated the roles he thought the course work should play, but these roles were not necessarily the ones he felt the course played in the programs of the students who had been a part of section of the course he had taught.

The pre-described roles Jeff articulated were gate-keeping and learning content. Aspects of Jeff's responses went beyond the descriptions in the analytic framework to include developing particular dispositional attributes. While other instructors alluded to developing these attributes, Jeff did so more frequently and with greater ardor.

Gate-keeping: The Teachers' Responsibility in the Face of Poor Preparation

While many of Jeff's concerns echoed Cynthia's, there were differences. Both professors were concerned about poor student preparation for the course in terms of the "knowledge of the mechanics" -- algorithmic and computational facility. Cynthia had the Interactive Math component of her course to deal with this. Jeff

had not anticipated a need.¹⁷ He had little sense of students' capabilities going into the course: "One of the things that really would have been helpful is to have some feeling for where these people [students in course] are at, you know, mathematically. I didn't have the vaguest idea of what to expect out of them" (24 P).

Gate-keeping at the Beginning of the Course

Jeff thought having information from pre-tests would have been helpful, but he chose not to give his students the departmentally recommended voluntary placement test. Not giving the test was understandable since he had not anticipated poor preparation. However, even if he gave the placement test to a subsequent class, he indicated that would not trust the results. He harbored distrust for the departmental test on the basis of the experience of one of his students with the state certification test. A student that Jeff thought did not have the fundamental skills needed for teaching elementary mathematics passed the state certification test. The test represented an "institutional" level of monitoring. If the state test "doesn't catch them," how would a departmental test? As a result, Jeff saw monitoring as the course's responsibility. Jeff didn't see his concern for students' knowing "the mechanics" as a barrier. The monitoring role that he saw for himself and the course was meant to ensure that students were competent.

¹⁷ Recall that Jeff was given a syllabus that indicated students would enter the course with two years of high school algebra and one year of geometry in their backgrounds.

Jeff indicated that in the future he would develop and employ a self-designed test. His test-plan was not well-thought-out yet, but he had a sense of what to emphasize: "A large part would be fractions. . . . I have this, this vague feeling that it's when we bump into fractions in elementary school is where a lot of people begin to lose it" (8 P). He also sensed what might be involved in supporting the testing and the test's passage by students in some future class. He envisioned having a minimum passing level, multiple versions of the test, and support materials for those not passing but wanting to re-try. The procedure Jeff envisioned was essentially what Cynthia described as the Interactive Math component of CC. Students could also take preparatory courses before the mathematics education course in order to be more successful in the course and as a teacher.

Jeff saw the students' lack of preparation as a hindrance affecting what he was able to accomplish with the class: "It just makes it difficult to . . . attain goals" (6 C). Jeff felt that regular course work on problem solving, a major focus for the course, did not help with students' weakness. He found that students sometimes were not able to use the problem solving as an opportunity to improve their "basics" because the problems frustrated them. "Some people did not do well at problem solving. Some people thought it very frustrating because they didn't have enough manipulative skills to carry them through" (29 C). Having those fundamental manipulative skills was a critical pre-requisite to being able to accomplish the goals for the course that Jeff intended.

Student limitations disappointed Jeff because he saw much potential in the textbook and supplementary problems for developing

problem solving skills. There were many and varied problems to assign. He saw these problems setting a context for interesting mathematical discussions that would provide opportunities for students to talk about mathematical ideas. But, students were unable to approach some of the problems in meaningful ways due to their weak mathematical backgrounds.

Gate-keeping at the End of the Course

The need for a gate-keeping role for the course became important to Jeff only after what he perceived as a disastrous outcome from the course: Students successfully met his requirements without showing the ability to deal with the fundamentals of mathematics.

I don't want to ever have that happen again, where I have students that I know do not know what a fraction is . . . have the stamp of approval. I don't want that to happen. I don't want to sound like some mean old curmudgeon, hiding under the bridge. Some troll waiting to jump and grab the students. I mean that's,(pause) but on the other hand, we don't want to certify teachers that aren't ready. (26 P)

Giving his "stamp of approval" could have ramifications down the road, as he related in a story told to him by Frank Adams (another study participant). Frank had passed a weak student who claimed she would always be a Kindergarten teacher. A few years later, Frank had led an inservice for sixth-grade teachers in which this student (then a teacher) had participated. While not wanting to be a "curmudgeon," Jeff did not want to assign any sign of approval to a student not showing the desired content knowledge. Otherwise, Frank's experience could be repeated.

Jeff blamed his "well-defined, rigid grading scheme" for allowing students to complete the course successfully without having fundamental mathematical tools: "I was obligated to follow that grading scheme. Students did certain projects and certain homework assignments and so on and got a reasonable grade in the course, but I knew that they were really lacking in some very fundamental skills" (7 P). Students would never be able to do that again, if Jeff could help it: "I think it's our responsibility to make sure that unprepared people are not out there ruining a thousand more students" (3 P).

Jeff's sense of failure was based on a more general sense of the importance of the task he was undertaking -- preparing future teachers of mathematics.

JS: The whole thing made me nervous. I mean, I, I just felt tense about it. I'm not saying I went in there, petrified every day, but the whole idea made me nervous. And it was just because these people are going to be teachers!

GK: Did you feel a heavy weight on your shoulders?

JS: I did! [vocal emphasis] I did, I mean, I (pause) if a student doesn't do well in another course, like a business math course, well, you know, I feel that either I or the student themselves failed them. Somebody didn't make this work. I don't know if it's my fault or their fault. So what are the consequences? Well, if they don't know the math they need to do business, so what (laughs)? . . . As compared to a teacher out there that doesn't know mathematics, that's, there's a big difference. So, yeah, I, I was intimidated by it. (10 C)

Jeff consistently stressed the roles of the course as preparing future teachers with content knowledge and particular dispositions. But, the task of preparing the students as future teachers did not include teaching them how to teach in Jeff's view.

The bottom line was that much more rode on successful work by teacher and students in the Mathematics for Elementary School Teachers course: "Everything is more important in this course. I was terrified of this course. Everything was important because, if, if I failed and did a bad job then the consequences were dire" (6 P). As a result of this bottom line, Jeff saw the need for the course to play the role of monitor.

Content and Dispositions: Course Goals

Jeff emphasized two other goals for the course: students were to gain a conceptual understanding of mathematics and students were to become creative and effective problem solvers. The phrase "conceptual understanding" was not part of Jeff's working vocabulary, but his description of his primary goal for the course made it evident that conceptual understanding was what he was working toward. The second goal was very complex and went far beyond simple learning and application of heuristics. In fact, the goal held within it, implicitly, dispositional goals for the student. The analysis of Jeff's course goals is in two sections, related to the roles of developing content knowledge (both conceptual understanding and problem solving) and developing dispositional attributes.

Content Goals and Their Utility as a part of Teacher Preparation

Motivation for Understanding the Content

Jeff's primary content goal for the course was to help students "understand what's happening underneath" (12 P) the algorithms and procedures he assumed they already knew. As future teachers, the students needed this understanding.

If I could have what I wanted, I would want students to come out with the ability to answer their students' questions. You know, when their little kids say, "Look at this neat thing I have. Look! I got the right answer." And I would want the teacher to be able to say, "Wow! That's a wonderful method. Look what you've done." And be able to talk with them intelligently about it. As opposed to saying, "That's not what the book says. We don't do it that way." I would want them to know what's right and what's wrong. [vocal emphasis] (10 C)

Jeff did not stress the value of this knowledge for the students as individual learners. He also did not connect it to questions of how to teach, except as supporting appropriate responses to children.

When their student says, "Hey, can I do it this way?" that they have the understanding to answer that question, instead of just saying, "No! You have to do it my way." [spoken quickly, followed by laugh] We've all had teachers that said, were very rigid like that and sort of discouraged students from thinking and, and it's probably because they don't know themselves whether that process is correct. (9 P)

In the course, students as future teachers were to gain the content knowledge needed to be able to avoid responding with the quick "No!"

Developing in students an understanding of the algorithms used at the elementary school level was the "bare bones minimum" the course was to accomplish. Jeff identified "computational skills and the understanding that goes behind those computational algorithms" as the foundation, the basics: "So those, that I see as just absolutely a must for everybody. Now a person could go out and be a reasonable math teacher with that. I mean they at least would not destroy their students" (19 P). This understanding was necessary, and it required enough depth to allow flexibility.

I would want it to be clear in their minds that, simply understanding the algorithms that you expect the elementary

kids to learn is not sufficient. I would want them to be aware of that, that it takes a deeper understanding. So you have thirty kids in your classroom, and they all want to do this a different way. So, what are you going to, just say, "No. You have to do it my way," because you don't understand anything else? You have to understand the fundamentals underneath there. So I think, yeah, to teach mathematics requires a lot more than just knowing how to do an algorithm. . . . A lot of them say to me, "Well, why do we need to know this silly stuff for? We're not going to teach this." Well, yes and no. You're going to teach maybe elementary things, but you need to have a deeper understand of those things. (4 P)

The recurring message throughout these comments was the necessity for students as prospective teachers to understand the mathematical concepts and procedures they would be expected to teach well enough to deal with children's questions and understandings.

Content as Unifying Themes and First Principles

Jeff spoke of understanding the fundamentals as being "a road to better things," not an end in itself. He used the analogy of a writer that needs to know grammar. Grammar and mathematical fundamentals are necessary, but not sufficient for their respective end-products. Jeff spoke frequently about trying to develop unifying themes throughout the course work on the fundamentals of algorithms and properties. These themes included seeing properties of addition and multiplication as similar, especially in work with identities and inverses.

This business of sort of seeing addition and multiplication as sort of the same in the sense that, that there is this special element, the identity, that does nothing when you add it. There is a special element that does nothing when you multiply by it. And then, these numbers seem to come in pairs, these inverses. You can add two numbers to get back the identity or you can

multiply two numbers to get back the identity. I mean, that's a more, I guess, mature way of looking at our arithmetic operations than, than maybe they would have had before. (24 P)

Jeff intended work in the course to develop this more mature vision of computation in the students.

Jeff identified the need to expand the set of numbers used when a smaller set is shown to be deficient in some way. For example, when the set of whole numbers is shown to be deficient for subtraction -- there is no member of the set that satisfies the equation $4 - x = 7$, the set is expanded to whole numbers and their opposites, the set of integers. Jeff saw the work with unifying themes as critical for students in their preparation to become teachers: "As teachers, they need to see a bigger picture" (5 P).

Jeff saw the use of different representations as valuable for developing an intuitive sense of the meanings of concepts. He called this starting from first principles.

I would start from first principles and (short pause) try to develop it [a concept] in a natural way. I did that with fractions when I discovered that I was in trouble with that. We talked about what does it mean to take one half of three fifths. What's one half times three fifths? And then I'd draw pictures and di-, "Well, here's three fifths, now let's cut that in half," and, yeah, so when, when I discovered that, you know, that they really didn't know this stuff, then I went back to a sort of a very intuitive level. (7 C)

The impetus for this was his perception that students did not know first principles.

With fractions, (pause) I found out very early on that I was wrong, that they didn't know how to deal with fractions, that I really needed to just start with, you know, drawing the pie and cutting it in half. . . . I wound up going back and doing that. Drawing bars and slicing it into five equal pieces. (pause) And

I guess I don't, I don't recall that I had problem assignments where I asked them to draw the pie and slice it up. I, the problems that I had, eventually, then, have them work were, were more complicated. And they couldn't do 'em. (6 C)

In teaching the content for conceptual understanding, Jeff found that, in some cases, he needed to approach the concepts in very different ways than he had expected, and sometimes in ways that he felt ill-prepared.

Going beyond first principles

Implicitly, the return to first principles and development of unifying themes that Jeff desired was important for the students as learners. Jeff explicitly valued this kind of knowledge because it puts students in a better position to respond to questions that future pupils might raise. He saw the ability to respond to questions as essential to the future teaching that the students would be expected to do and an essential role of the course.

Understanding the fundamentals was not the extent of Jeff's content goals for the course, but it does represent what he saw as the most critical goal: "Ideally, we want our teachers to be more creative, to be able to do pattern guessing and be creative problem solvers. That also is very important, but I think it's secondary to understanding the fundamentals" (20 P). The primary goal for the course was that students in the course were to understand the algorithms and procedures. In addition, "our teachers" were to become creative and effective problem solvers. There was a clear order in Jeff's mind, and he felt some of his students had not gotten to the point of moving to the secondary goal.

And so they [some students] were never able to get to the point where they could do something fun. And (pause) unfortunately with almost all skills, sports, mathematics, cooking, you know, there are certain basics that you have to somehow get a grip on before you can do the really creative stuff. (29 C)

The fun part was being able to solve interesting problems, and solving interesting problems came only after gaining the undergirding fundamentals. There was little doubt that the first goal was preeminent: "You could have a teacher out there that might not be the greatest most creative problem solver, but if they, I mean when it comes to the fourth grade and they're supposed to be teaching fractions, they'd better know what a fraction is" (26 P).

When asked if discussions of problems could be the setting for developing the conceptual understanding that he saw as primary for this course, Jeff asserted the belief that it was possible, but difficult.

You probably could. It would take some work, but you probably could. Most of the problems we did, were not thought out well enough to do that. They didn't really, we did some things, we did some things. I've mentioned where you know, here's, here's a long computation, simple multiplication problem. Well, if we changed this one number, what's going to happen over here in the answer. So, it forces you to think about the algorithm a little bit. But, we did some stuff like that, but not enough. So, it probably could be done, it would take a lot of work to come up with the right types of problems. (21 P)

As with many questions about his pedagogical strategies, Jeff indicated positive feelings toward integrating development of conceptual understanding with the work on problem solving, but he acknowledged little attempt to do so. In many cases, he attributed his hesitance to a sense of his own lack of knowledge with respect to pedagogical decision-making. He would try strategies only if he

felt capable of doing them in reasoned ways. For example, he used manipulatives only as he knew and understood their use from his experience.

Jeff articulated developing conceptual understanding and problem solving abilities in students as goals for the learning of content in the course. But, he intended for work toward these goals to support the students' metamorphoses into creative and effective problem solvers, not only to be able to show the skills required for successful problem solving but to exhibit the dispositions of a successful problem solver.

Dispositional Goals and Their Role in Teacher Preparation

Teasing out the dispositional goals Jeff held for the course involved careful examination of the way he spoke about the "doing" of the mathematics with which he hoped students would gain facility. His comments implied that attributes of flexibility (described above), open-mindedness, tenacity and risk-taking, and confidence were to be developed through work in the course. He did not explicitly lay these mathematical habits of mind out as valued, but this was evident in the body of his comments.

Open-mindedness

Three aspects of open-mindedness seemed most significant to Jeff. Students were to be open to other people's ways of doing problems and an expanded view of mathematics -- its content. And, students were to be open to a view of mathematics as a tool.

The first aspect of open-mindedness involved tolerance of other people's ideas. Jeff wanted students to see the value of alternative explanations and be open to multiple ways of doing

problems. He described one particularly valuable discussion in which he felt students had not only shared in important ways, but they had also seen value in other people's explanations. He explicitly connected this with their future teaching.

JS: There was one exercise that really surprised me. It was on mental arithmetic. Just like little tricks that you can do for doing mental arithmetic. So, the way it was presented in the textbook was it was a chapter on how to do mental arithmetic. So, I thought, "Alright, we'll discuss these little tricks." So, I would discuss them, and what was really neat was I would say, "Well, okay. Now, let's try it this way." And they would say, "Well, that's not the way I would do it. I would do this." And the whole discussion was exactly what I was just talking about -- the students saying, "Well, I wouldn't do it that way. I'd like to do it this way." And then what do I say, "Well, that's not what the book said," or do you say, "Well, yeah, that's great! That's another way you could do it." And that, when we were having that discussion, I started talking to them about, how they ideally should respond to their kids.

GK: So that it became explicitly a part of the classroom experience?

JS: Yeah, it turned into, it turned into a model of, the discussion we were having turned into a model of how I would hope that they would respond to their kids. And we specifically talked about that. That you need to be tolerant of other people's ideas. (11 C)

Jeff remarked that students had "really listened" to the discussion. Not only that, it seemed that Jeff had listened to it. He was "surprised" by the discussion. It sounded as if this kind of discussion was what he wanted in the class but that he was not often able to orchestrate it.

Jeff struggled with the value of this discussion and others for learning content: "Those were fun discussions. I mean, they [students] were awake. They participated (laugh). They were very vocal, but I'm not sure that they came away really that much more

enlightened" (13-14 C). So, while he wanted to offer opportunities in the class that encouraged tolerance of other people's ideas, he did not want providing these opportunities to limit attainment of content goals.

Tolerance of alternative solutions was not to be a blanket affair. Jeff saw the value of developing a sense of the most elegant way to solve a problem, depending on the constraints.

I guess every situation might require different types of reasoning, so it might be finding out the right way to look at a situation, the right way to think about it, is what's really important. . . . The most enlightening way. . . . For whatever my purposes are at this time being, here are these different ways that we could view it, but this is the one that suits my purposes in this situation. (short pause) Yeah, yeah, being able to, I mean, whatever your situation is, pick out the right point of view that things suddenly become clearer. (22 C)

He connected his intention of making things clearer with the students as future teachers. The clearest way to explain was the one to be prized.

The second aspect of open-mindedness that Jeff wanted to encourage was an open-mindedness to what counts as mathematics or a mathematical problem. He found that students often did not see the problems he assigned as "math problems." The old definition of mathematics gave them something to "love to hate." They weren't sure how to respond to this new situation, and, in some cases, they chose not to respond.

There were people who went through that whole course with that, with that mind-set. You know, "Just tell me the way to do it. Don't, don't confuse me." And, and I didn't reach those people. They came out of the course with that same mind-set.

. . . My impression was that (short pause)there were probably some people who, who didn't open up. (11 C)

For the students, it was hard to believe differently, in part, because if mathematics is not the mechanical subject of their beliefs and does allow, even invite creativity, then all of those years of memorization were "wasted." Students had too much invested in maintaining the old view.

Along with seeing the content of mathematics in an expanded way, Jeff wanted students to develop a sense of mathematics as a tool.

You want people to feel like this is a great tool. "I can do something with this." And, that's certainly something that I would want to get everyone that teaches mathematics to have. . . . That's sort of an implicit objective. It's not something that I would write a lesson plan for. . . . It's important, but I'm not exactly sure how I would incorporate that into. . . . It's a very slippery objective. It's, I don't know how I would specifically plan for that other than try to be enthusiastic. Try to, try to come up with good problems in general, so that the students come away from the class with a good feeling of, "Hey this was interesting. I could do something with this." Yeah, I suppose that's sort of implicitly sort of the, built into everything that we do is to make them think that math is a good tool. (3 P)

As a tool, he saw mathematics as valuable beyond mere utility, similar to Cynthia.

I guess I would want students to know that mathematics is relevant in their daily lives but I would also want them to be able to go beyond that. To see the beauty and the power in being able to solve problems that may not be something they would do in their everyday life. I don't really see in our society today that we're. We're training people to be able to go to the grocery store, to be honest. It requires more than that, you have to be able to think beyond that level. And so yeah,

mathematics is relevant, I would want them to know that, but I would also want them to be able to go past that. (5 P)

Students were to be open to the notion that doing mathematics may involve using it as a powerful tool to understand or model situations beyond the routine of balancing checkbooks.

One form of open-mindedness toward doing mathematics leads to the third dispositional goal for the course, tenacity and risk-taking. Jeff saw openness to doing problems that did not look familiar as essential for students.

Openness to attempting problems. Now, there, there's something that's important! They'll say, "I haven't seen one like this before. You didn't work one like this in class, so therefore I shouldn't have to do it." . . . I feel pretty strongly about that. That people, I don't know, just, have, are open enough that they are willing to say, "Well, I, you know, I've never seen something like this before, but let me think about this for a minute," instead of just saying, "I can't do that." I think that's real important. "He didn't do one like that so why should I have to do one." (20 C)

This openness is not the same as open-mindedness, but being open-minded about what problems look like and what they require seemed a first step to Jeff's dispositional goal of tenacity and risk-taking.

Tenacity and risk-taking

Part of what Jeff hoped the course and its focus on problem solving would develop in students was the recognition that learning mathematics may require attempting problems that seem difficult or unusual, staying with problems that seem difficult, and accepting that learning can be painful but rewarding. Like Cynthia, he supported the development of tenacity in class discussions about appropriate study habits.

I talk to students not only in an elementary content course but all my classes about, what do you do to learn? How do you read a math book? What should you do to study? Here's a problem. You have no idea how to do. What should we do? Do we just throw up our hands and quit? No, we back up and we look at an example. So, yeah, I spend a lot of time in all my classes talking about, you know, the process of learning mathematics. I mean, how do you read a math book? Is this an Evelyn Wood speed-reading course? (6 P)

As hinted at in this comment, he suggested strategies for approaching problems as a way of encouraging students to attempt problems.

The problem sounds hard but, so what should you do? Well, you don't just quit. You do something. Try to look for a simpler problem. Work some examples. Look for patterns. So I guess, I mean that, that's part of the problem solving skills that we went through. I, I guess really was learning how to learn. How do you, when you're given a problem that's just completely different than anything you've seen before, what do you do? You don't just give up. And, so we talked a lot about, about how to do that. (25 P)

Attempting problems, taking risks, these were to be part of students' solution strategies: "I tried to get them to be more willing to take risks, like in problem solving. 'Boy, this is a hard problem.' Well, instead of just throwing our hands up, let's try something. Let's think about this. Let's look for patterns, work examples, and so on" (1-2 C).

When asked how important being able to solve problems was to being good at math, he shifted the focus to attempting problems and not giving up.

Being able to attempt many different kinds of problems, (short pause) gosh, there are so many problems out there that we could think about but we wouldn't be able to solve, so I don't know if it's all that important that we actually solve them

[vocal emphasis]. But maybe attempting to solve them is important. It's not like, if you don't solve the problem, then you're a failure. (20 C)

He emphasized this by referring to the centuries-long work mathematicians gave to Fermat's Last Theorem: "It's nice to be successful, but, you know, people worked on Fermat's Last Theorem for hundreds of years and failed. But, it (laugh) was still a good exercise" (20-21 C).

The reward was the exercise, itself, and it relied on staying with the task, even when it does not seem pleasant.

I guess I'm a little bit biased. Some people have this, we talked about this once before too, that mathematics has to be fun. Well, (short pause) it doesn't. And sometimes it's not. Sometimes it's hard work. And you have to be willing to accept the fact that sometimes it's hard work. (laugh) Anyway, (short pause) sometimes hard work is pleasant, and I mean, is rewarding in the end. But while you're doing it, it sometimes is difficult. (20 C)

Jeff noted that it could even be painful.

I think in almost any subject, not just mathematics, sooner or later, to change the way you think is physically and mentally painful sometimes. I mean, (short pause) yeah, you should have fun but not to the point where you say, "Oh, this isn't fun anymore, so I'm not doing it." It doesn't have to be fun. Sometimes it's hard and, and people have to be willing to accept that and deal with it. (6 P)

His basic premise was: "If it's fun, fine; if it's not, tighten your belt and deal with it" (6 P).

One of the aspects of being an effective problem solver that Jeff felt he had mixed success in achieving in the course was the willingness to risk and feeling comfortable with making

conjectures. He spoke of suggesting strategies to students as a way of encouraging this disposition.

We spend a lot of time doing that [conjecturing] with problem solving. "Well, I don't know, this is a hard problem. Could we look at a simpler problem? Let's look at a bunch of examples. Do we see anything, any pattern in these examples?" I think conjecture, pattern guessing, problem solving, those are real important skills. People are afraid to guess. But that, I mean that's what mathematics is. (6 P)

Here we see the issue of what mathematics is interwoven with the issue of risk-taking. At the same time, Jeff saw all of these tied to the fourth dispositional goal, confidence with doing mathematics and teaching mathematics.

Confidence

When asked what to give a one sentence description of the course goals, Jeff said:

The goals of 115, I guess, is to prepare people to be competent math teachers, so that they (short pause), they have the confidence, to teach mathematics to young kids. . . . You need to understand the content yourself before you have a chance of communicating it to someone else, so, the goal of this course then is to get them to be comfortable with the content. (1 C)

The goal is for student competence so that they will be confident teachers. Jeff saw a linear relationship: competence yielding confidence. "Developing empowerment and confidence with teaching math I think would come naturally with a deeper understanding of the material" (15 P). This relationship sets the dispositional goal of confidence up as both dependent upon competence and necessary to achieve the ultimate goal of being capable of sharing that competence with children in the ways that Jeff intended.

Achieving the goal of confidence was not seen as trivial. Jeff sensed that the students in the course were more "fragile" than students in other mathematics courses: "I just had the impression that these students were maybe a little more fragile. . . . Well, in terms of their confidence with mathematics. That might not be, that might not be true. But, I just sort of had that feeling" (10 C). He saw this weakness affecting the pool of elementary school teachers: "I might be wrong about this, but I would guess that there are some people that go into elementary ed., they make that career choice to avoid math and sciences. So, you have people in there that are really scared" (25 P).

Jeff assessed that students who came to the class prepared would leave the course with the content goals attained. This preparation included arriving with a certain level of confidence.

I suppose this is fairly obvious, but it was those stronger students that (pause) that came in with the mechanical skills that they needed and a certain amount of confidence that came away, getting out of the course what I had hoped that they would. That is, to see a bigger picture and to deal with tougher problems. (4 C)

In this course, students were to see the bigger picture and become creative and effective problem solvers. These capabilities represented the competence Jeff saw as necessary for feeling confident with mathematics and with teaching.

Jeff had the intention of reducing student anxiety toward mathematics and doing mathematics: "I went into this with the idea that I want to make it as natural and comfortable as possible" (11 P). But, he did not intend to work toward reducing the anxiety students might have toward teaching mathematics.

Reduce the anxiety students might have about teaching mathematics. (long pause) I don't know. Sometimes anxiety can have positive effects. If you're scared of something, sometimes you make the effort to, to struggle through this. You work harder sometimes. So, I don't know, reduce the anxiety students might have about teaching mathematics. I guess that, no. I would want them to be somewhat anxious about it. I mean not scared to death, but I'm anxious about it. That's part of the, I don't know, that's part of what makes it maybe a little bit of fun, part of what makes it, challenging. I mean, I still get anxious on the first day of class about my teaching (laughs) so. I don't know that's not something I would probably do. (11 P)

Jeff spoke of the anxiety as having positive effects. Since Jeff's goal was developing confidence to teach mathematics rather than simply confidence with doing mathematics, equating positive attributes to anxiety and not attempting to reduce anxiety about teaching was reasonable. Interestingly, he did not apply this logic to his treatment of anxiety toward the subject; that is, if students are scared of the subject, they may make the effort to struggle through it. He taught the subject with the intention of making it less "scary."

Summary

According to Jeff, the distinctive nature of this mathematics content course as a part of teacher preparation, particularly with respect to affecting certification, placed special responsibilities on the course and its instructor as Jeff experienced it. As a novice to the task of teaching mathematics education courses, he gained a sense of the roles the course should play in students' programs through the act of teaching the course. He had little interaction with others teaching the course, not being a part of the group of

mathematics educators who met occasionally to discuss the course. Like many other instructors, he alluded to occasional insights that came from his experiences connected with parenting elementary-school-aged children. Jeff suggested that having a sense of the whole teacher preparation program students were to complete for certification would be helpful in deciding what roles mathematics courses should play. In this, he was unique.

Jeff saw the task of preparing teachers as extremely important. He viewed the roles of learning mathematics and gate-keeping as important. In addition, he stressed the development of particular dispositional attributes: flexibility, open-mindedness, tenacity and risk-taking, and confidence.¹⁸ But, he limited his focus to achieving content goals and developing dispositional attributes. Pedagogical issues were only of minimal concern. This limitation was probably a reflection of Jeff's self-described status as "ill-equipped" to teach the course. He had given pedagogical issues some attention, but his own understanding of what it means to teach mathematics constrained his vision of what his instructional options were.

¹⁸ Throughout the remainder of this document, the "role of developing particular dispositions" will include these and other related aspects.

Frank: Revising the Content

As someone involved in mathematics education for many years, Professor Frank Adams brought a perspective that pre-dated that of the rest of the participants. His career spanned from the mid-fifties to the present. He was responsible for assigning instructors to teach the mathematics content courses for prospective teachers at NU and had input on the departmental syllabus for the courses, but he did not usually teach the courses himself. He taught the methods course and special content courses, such as Problem Solving. Frank did more outreach teaching than any other instructor interviewed, leading workshops for inservice teachers and parents. He often had children participate in the workshops so that he could model the instructional strategies he was suggesting. Most of his responses related to the first content course, Math 115.

What seemed most obvious about the way Frank saw the roles for the course to play was what Frank did not see as roles for the course.¹⁹ Frank thought the course should not involve learning how to teach mathematics and it should not be a gate-keeper. While he emphasized the learning of mathematical content, the content he described was not a part of the content prescribed in the departmental syllabus, except as optional material.

¹⁹ I struggled to determine what role Frank was committed to for the course. I was not as successful as I would have liked.

Content and NOT Gate-keeping

Syllabus Concerns

Though Frank had input on the syllabus, the content specified for the course did not reflect his preferences, in general. There were several reasons he saw it needing revision. He saw the recent imposition of pre-requisites as assuring stronger student preparation for the course, making some present content, such as work with fractions, unnecessary.

Within the last two or three years we've changed a lot in terms of requirements. We used to pretty much take anybody and try to adapt 115, at that time, to that particular student. Now we're requiring that they have at least a year and a half of algebra and a year of geometry before they get into 115, and we're assuming that background. (22-23 P)

With pre-requisites, he saw that better students would be in the courses, "therefore we ought to be looking at doing other things in that course than what we've done in the past" (5 C).

Another reason Frank thought the syllabus ought to be revised was that the content was no longer rigorous enough. He felt a return to work on groups and their properties and logic was needed to support the work on problem solving, the major focus of the first content course. These topics had, historically, been a part of the course.

FA: [Twenty years ago] we were more interested in (short pause) change in the student to understand some modern mathematics, some sets, some of the more modern languages and so on.

GK: Formal structures as opposed to?

FA: Right. As opposed to now, the emphasis is much more on problem solving, thinking skills, introducing them to some of the manipulatives and so on. (short pause) If I had any (short

pause) way, I would probably have them relook at that whole program and say the course that we're now teaching doesn't really meet the needs of the student and it should be beef it up a little bit. (3-4 C)

Frank saw beefing the course up (spending time on formal logic, truth tables, and formal sets) as a way to prepare students to teach in present-day classrooms, classrooms in which the curriculum is changing, requiring a different set of skills.²⁰

See my emphasis would be on, we would talk about what forms a group. Then, of course, we'd talk about closure and inverses and identities, commutative, and associative. And then we would talk about different systems that would satisfy that, and by, that way reviewing the whole numbers and fractions. Okay. And then we would go into a, a field properties, talk about field properties. Not necessarily doing the things that you do in abstract algebra, but at least looking at those properties and saying, okay which properties are relevant to an elementary school and so on, and what to emphasize. And then look at the different numeration systems that would apply for those. I'd like to see logic being there, along with problem solving throughout the whole course. Use of calculators throughout the whole course. I'd like to tie in logic with sets and, maybe even spend some time with going through some attributes with sets and electrical networks. Tie, show them that tie-in through the whole thing. Everybody doesn't agree with me. (5 C)

Frank saw great value in making these changes in content. He gave numerous responses related to these topics (not currently on the syllabus) to interview questions, even when I requested an example using content from the present version of the course.²¹

²⁰ The view of the importance of the study of logic is held by others (Chapman, 1972; Stone, 1972) and, historically, was more strongly held 20 years ago.

²¹ I point out instances of Frank's examples unrelated to syllabus content in this analysis.

Study of logic and group theory were foundational to versions of the course in the past, but it had been largely deleted over the years. During the set of Mathematics Education Committee meetings held just prior to the interviewing, instructors had discussed the course content and design at length. They discussed a new syllabus, supported by Frank. He saw the new syllabus as holding promise for achieving his recommendations for increased rigor and substance (see Appendix E). Logic and group theory were prominent on this syllabus.

From Frank's perspective as a mathematics educator familiar with the long history of mathematics education emphases and as a methods course instructor, there were other reasons for focusing attention on content different from the current focus on content similar to the elementary school curriculum. Frank wanted students to have a sense of the mathematics beyond the level at which they would be teaching. More importantly, he wanted students to learn mathematical content other than what was a part of the methods course. He frequently heard the complaint from students that they had seen and done the same activities he expected them to do in the methods course in a previous mathematics content course.

One of the biggest complaints, I think I've mentioned this before, is that a lot of students go through our 115 [content course] with certain instructors and feel that, then they get into the 374, the methods course, and they say, "Well, we've had all this." So, they end up not having the emphasis, in 115 and 116 should be in the content, whereas the emphasis in 374 should be to know materials to teach boys and girls, the idea is there. So, either we're doing the wrong thing, or giving them the wrong idea, if they think they already know that material. It should not be in the 115 and 116 part. (6 C)

This problem was of major concern to Frank, as indicated by the frequency with which he mentioned it. By shifting the content of the content courses to content such as group theory and logic, Frank could be more certain that students would not be complaining about redundancy.

It was not clear how Frank saw his content recommendations improving the way the course supported students' teacher preparation. But, he saw learning mathematical content as an important part of the course.

Understanding

While Frank spoke of students "gaining understanding" in the course, it was only somewhat important that they understand it in ways to help them teach it, not essential or even very important as all other instructors indicated. Also, what he meant by this phrase seemed to be very fuzzy. The following exchange centered on giving an example of work for the methods course that relied on student understanding gained in the content course.

FA: The way that I understand this [card sort - understanding content in meaningful ways] is that they understand the math. And so then I could then [in the methods course] take, "Okay, you understand the math. Now, how are you going to get that across to boys and girls?" So then I would build on that, I assume they already know the math and I don't have to work about it.

GK: When you say "the math", do you mean the?

FA: The basic content. . . . Let's say for example, if I gave them the three different percentage-type problems, 30% of 80 is what? [The three types he is referring to all involve "a% of b is c" where two of the three values, a, b, and c, are known] They would be able to do those problems, and they might do it basically from a rote learning standpoint, so my job would be, okay they now know how to do that. Are there ways that now

they can use to teach boys and girls those three percentage problems? Use the proportion method, use the triangular method, things like that.

GK: Would you think they would understand those methods when they came in?

FA: They would probably have, maybe, one of each one of those methods that they probably understand so they could do the problem. They're probably not looking at any other methods. So they understood it.

GK: Those different methods, or different ways of thinking about that, you wouldn't necessarily see as part of 115 or 116 [the content courses]?

FA: No. (10 P)

These comments suggest that, when students entered the methods class, he assumed nothing beyond a "rote learning standpoint." He saw his job in the methods class as one of showing them many ways to teach the concept to children. He did not expect that students would seek other ways of doing problems prior to the methods course. How this translated into understanding of concepts in the courses prior to the methods course was difficult to follow.

When asked about the value of developing adult-level insights into the mathematics that students would be expected to teach, Frank again expressed concern about developing students' understanding of group properties (not currently on the 115 syllabus).

FA: See one of the things that, and not everybody agrees with me, is that I feel that the students, once they get finished with 115 or 116, should understand the properties of a group. Okay? And how they are used in the elementary school. Okay? And the same with the properties of field. Okay? They do cover those properties now, in some respect, but a lot of times, it's just hodge-podge type thing, in the textbook. Not everybody agrees with me, but that would be, would so, to me I guess that would be a deeper understanding, insight into mathematics than we now are currently doing.

GK: Now would that be in knowing what those properties are, or would it be in being able to look at a set and determine if it was a group or if it was a field or

FA: I think there ought to be, we ought to, for example, look at, once we talk about the properties of the group, and then we can say okay, let's take the operation intersection, let's take sets, do they satisfy the properties of a group? If so, yes; if not, why not. We could use logic the same way. (16 P)

Though left unstated, it seemed that Frank was viewing work with group properties as a connecting theme in a way similar to the way Jeff Sullivan spoke of them. Avoiding "hodge-podge" and systematically looking at sets and their properties was seen as a way to make mathematics more meaningful. Making mathematics more meaningful would seem to support developing adult-level insights. However, when asked what insights he would want students to gain from this work, his response was confounding: "That math is not just addition, subtraction, multiplication, but that these concepts are throughout their everyday life" (16 P). Part of this final comment, that the insight gained from the work on the properties of a group would be that mathematics is not just computation, seemed reasonable. Insinuating that part of the insight would be that group properties are part of everyday life was puzzling, given his description of the work with groups.

Preparation

Frank indicated that having different ways of thinking about percent or other concepts was not an important part of the content course experience. Rather, "the math" was what students were to leave the content course knowing. He also indicated that "the math" was what was to be part of the pre-requisites. The juxtaposition of

these comments leaves one to wonder what content Frank thought students were to learn in the course.

Frank skirted the question of content coverage in a lengthy exchange in which I attempted to clarify what students were to learn in the course. I asked Frank how important it was for the course to act as a barrier for unprepared students.

FA: No, actually that [being unprepared] should, the way our set up is in here, it would be not very important, because that should be taken care of with the developmental area. . . . The developmental math is basically to take care of any deficiencies they have in math before they take 115, at least that would be my opinion. (pause) So in some respects, 115 shouldn't even have to deal with a student that's unprepared.

GK: So is there, what would you think they would be prepared for in 115. Is there something that they're prepared for in 115 that they need to have when they get out?

FA: Okay, you're talking about after they get out in terms of working with other, with students that they're teaching that are unprepared?

GK: Well, I guess if everything they would need to know mathematically they would know when they come into 115, then what is it they would learn in 115 that they would need to be prepared to do?

FA: I don't think, 115 basically would be to take care of any of the deficiencies they would have up until 115. 115 is a vast, different type of course in my estimation, that a lot of the topics they probably would not have had in a standard high school course in mathematics, or standard college, college algebra course or intermediate algebra. So (short pause) I guess my whole comment would be the act of, as a barrier for unprepared students, (short pause) I don't think 115 should have to deal with students, at least the way it's set up now, that are not ready for 115. I assume this means students are not ready for a normal 115 curriculum.

GK: Okay, what about students who wouldn't be ready for the normal methods class?

FA: In some respects, I am preparing them, they are preparing them to work with the content, to work with at least the content that they're working with in, in 374, which is a

methods course. . . . I don't think that's a problem. Once they go through 115 and 116, even if they don't enter, they don't understand all the content in both of those courses, I'm not sure it's that critical. (2 - 3 P)

He stressed that students who were unprepared for Math 115 and 116 were placed into a lower level course. When pressed to focus attention away from entry-level preparation to end-of-course preparation, Frank thought that the question referred to students' future pupils who were unprepared. He seemed unable to conceive that students in the course might be unprepared to go on in their teacher preparation.

Frank indicated both that developmental math took care of entering deficiencies and that 115 was to take care of deficiencies. Frank alluded to what he saw as the difference in the deficiencies dealt with in each context. The deficiencies dealt with in the developmental math course were computational; whereas, the deficiencies dealt with in 115 were in terms of students' lack of exposure to different topics, topics not normally a part of the standard K-12 curriculum, such as probability and statistics. But, Frank did not insinuate the need to attain any level of proficiency with the topics in 115. It appeared that the course was to give awareness of content, so that the methods instructor could focus on how to teach mathematics.

Frank did not emphasize the role of developing a deeper understanding of the content within the whole of the mathematics teacher preparation program. This is not to say deeper understanding was ignored, only not emphasized. This lack of emphasis was a reflection of his feeling that people with less

content knowledge sometimes do a better job of teaching than those with more content knowledge.

Ensure that only those students' who have a deep understanding of math are able to continue in the mathematics preparation program. (short pause) This is always a (laugh) a controversial thing. I feel that probably some of the people who are having the most difficult time in mathematics probably do as good a job, if not a better job, with their students in the elementary school than some of the ones who really have done a bang-up job. (5 P)

In fact, Frank went so far as to say that competence was not required: "I still think they could be, do a good job as an elementary teacher teaching mathematics without necessarily be that competent in that area" (4 P). The individual's attitude toward teaching and mathematics was of greater importance than their understanding of the content. Given a choice between a person with weak content knowledge and a positive attitude and a person with strong content knowledge and weak personal skills (apparently equating with non-positive attitude), Frank would rather have the person with a positive attitude teaching children. Thus, he was not prone to demand that students have a deep understanding of the content.

Frank also indicated that it was his feeling that school districts often overlooked students who had superior content knowledge, even paired with positive personal skills and attitudes, when they were hiring. His cynicism grew out of experiences with strong teacher candidates who were not hired by districts if other, less capable, candidates had personal contacts in the district. Frank

argued that school districts were not concerned about the subject matter preparation of the students in many cases.

Connecting Content to Everyday Life

Connection of the content to everyday use was a significant part of Frank's conceptualization of the course. He acknowledged that this reflected his current personal interest, yet, when asked for an example of the kind of interconnection of ideas he would like to see, he had difficulty.

FA: Interconnection of mathematics and, I happen to be pushing at this stage, and of course is my (laughs), you know, you get into different emphases. And now my big emphasis is that I feel we ought to be connecting math and science in the elementary school. [inaudible]

GK: Where do you see places in like 115 or 116 where that could be done? Could you give me an example of the kind of thing you'd like to see?

FA: (short pause) The big problem we have in a lot of math courses is that we do problems and they're not associated with anything out in everyday life (laugh). And so I would like to see us connecting math and science, let's say, in terms of a practical type thing. (short pause) I guess, you know, you could do things that would be, let's see, in science where (short pause) you're getting the science concept across and you're also getting the math. Let me see if I can come up with an example. My first example is M & Ms, but that's more statistics in math, or probability in math. (pause) Let's see what would be a? (pause) Well, let's take a typical discussion, let's say of a, of a parabolic curve. Now you could talk about it in science in terms of a parabolic telescope. You could talk about it in lenses. You could talk about it with just throwing an object. Okay? Now, you could then tie that into mathematics if you wanted to, depending on how far you wanted to go in terms of the algebra of it, or you could tie it in terms of, let's say (pause) paper folding with the parabola, or you could tie it in with string designs. Basically the whole type of thing, that could be a whole, or if you wanted to, you could eventually tie it in, in science, you could tie it in with, (short pause) wave, if you had a, a ripple tank, and wave

motion. They could make their own, if they wanted to, to show the parabola in there where the waves would eventually congregate. You could do the same thing with (short pause) the hyperbola and ellipse, if you wanted to do things like that, tying in both the, the orbit of the moon or the earth around the sun, or the moon around the earth with the, or the satellites with an ellipse in mathematics and so on. You could do some string designs, things like that, if you want to. (11-12 P)

When asked if this content was part of the course, Frank noted that it was not currently in the course, and that it was hard to come up with good examples. I felt that his inability to provide examples was odd, given that connections to everyday life were of particular interest to him. He stressed the importance of students seeing practical applications of mathematics.

I'd like to see a lot of practical, and the reason I mention science is that a lot of times you can use science ideas that would make it practical on that. I'd like to see more and more, too many times math is taught in a vacuum and all you're doing is manipulating either the letters or numbers, and then they feel that's the important thing, and it's not. Okay, you have to be able to use this in other fields. So, I don't think we do enough of that, and I don't think we do enough of that in probably 115 or 116. We probably don't do enough of that in all the basic level courses. (13-14 P)

Seeing mathematics as other than manipulating symbols was a reason for this emphasis on practical applications. Frank gave few examples of how this could be done with the concepts in the content courses.

NOT Teaching: Content and Methods Relationship

As alluded to above, Frank had some concerns about the content courses as their content and instruction affected the work to be done in the methods course. He wanted to avoid repetition of content coverage in the two settings. Furthermore, he did not want

students to focus on how to teach mathematics in the courses, but he stressed that students were to see models of good mathematics teaching in the courses.

Use of Manipulatives

In Frank's mind, the use of manipulatives in 115 was different from their use in the methods course. Frank saw 115 as a site for students to become aware of how materials can be used to learn mathematics. The learners were the students, themselves, in the 115 course.²²

I would assume that we would use [in 115] a lot of the materials that would be used in the [elementary] classroom. The only difference is that you would be using them on a more advanced math. For example, let's say, Cuisenaire rods, or colored rods could be used to show modular arithmetic. (7 P)

Frank wanted the content courses to provide students with enough experience using manipulatives so that students in the methods course would focus on how to teach using the materials rather than on how to use materials themselves.²³

²² Within the following statement we have another example of "examples" Frank gave involving content in the course that was content he felt should be a part of the course, but which was not agreed upon by all instructors as important to investigate in the course. Modular arithmetic was not consistently included.

²³ When asked if other courses in the Mathematics Department not related to mathematics education made use of manipulatives to teach related content, Frank initially indicated that they should not use manipulatives. Use of manipulatives in regular mathematics classes would "infringe" on the mathematics education work. After talking more about this in the context of the interview, he turned around 180°, saying that he would not anticipate that others would use them, but that instructors certainly could, even should, use manipulatives and modeling materials in regular mathematics

While Frank was quite vocal about the importance of the use of manipulatives in the content courses in the interviews, other instructors had the sense that the methods instructors did not want manipulatives used in the content courses. Gail Laughlin (another instructor in this study) made this comment in an interview.

We're supposed to stay away from manipulatives in there, save them for 374, I mean, we're supposed to. We still do some. I mean we still bring in base ten blocks so they can, they've seen 'em, and I still do the cup and chip model, chips for positive and negative integers. And then I bring in the cups for balancing with equations, but we just had, we met in May, four weeks in a row, a couple hours in the morning, talking about what's the role of the course. And there, you know, when I say "they", Donna Sterling [another methods instructor] and Frank Adams and, we're trying to keep manipulatives in 374, so that's why I didn't even think about it, you know. (GL, 16 P)

Gail insinuated that Frank had given her the impression in the Mathematics Education Committee meetings that he did not want manipulatives to be used in the course. As a result, she limited her use of manipulatives in the content course sections that she taught.

Gail's misunderstanding of Frank's perspective on the use of manipulatives in the content courses highlighted the importance of having a clear sense of what role the use of manipulatives was to play in each context, content and methods course. Frank was concerned about this and other issues that he saw blurring the lines between the two types of courses.

Separation of Content and Methods

Throughout his interviews, Frank stressed the separation of content and methods courses. As noted above, he indicated that the

courses. This appeared to be an instance of bringing up an issue in the interviewing that had not been raised elsewhere before.

use of manipulatives had a different purpose in the content and methods classes, resulting in different types of instruction and conversation around their use.

Occasionally, Frank pointed out specific examples of problems that arose when content and methods courses were not separated enough. As noted above, students were coming into the methods course saying that they have already done the activities that he wanted them to do in the methods course.

We have students who go through 115 and 116 and then take this methods course and say, "Oh, we've had all this." Okay? To me, 115 and 116, the emphasis should be on the content, using some of the materials that you would use in the classroom, but again, the final outcome is on the content, whereas in 374, I assume that the students know the content, they've used some of the materials, but the emphasis should be on how do you get that content across to the boys and girls, and different ways of approaching it using some of the manipulatives and other strategies. (22 P)

For Frank, the methods course, not the content course, included learning different ways of approaching content. His position was different from the way other instructors talked about the importance of learning multiple representations in the content course.

Frank voiced a different perspective, that of hoping that students learn that there are many different ways to look at problems, when I asked what students were to learn how to learn in the content course. This is a subtle difference: different ways of looking at problems as opposed to different ways of representing problems. Frank identified "learning to learn" as an essential

purpose of the course, so I asked him to give a sense of what students were to learn to learn in the course.

I would hope that they would learn that there are many different ways to look at problems and be able to, different ways of trying to solve problems. (short pause) That, that you can't teach math just as a hearing problem, where if they don't understand it, the instructor says the same thing again, but a little louder. Okay, that they would have to come up with different ideas to try to present that, to try to get the student to understand it. Not only does the instructor in 115 have to do that, but hopefully that would then mirror that for the student, that when they eventually teach that they would try to do the same thing. (18 P)

The different ways to look at problems, different ways to solve problems, different ideas to present, are part of the teaching and learning in the course. So, although Frank stated that he did not expect students to know different ways to solve problems as a result of their experience in the 115 course, he valued this knowledge for the students.

Role of "Modeling"

Frank did not want students to learn how to teach mathematics, as such, in the content courses, but he indicated that it was essential for the course to provide a model of good mathematics teaching. Students were to see appropriate teacher decision-making as a support for the courses' position in teacher preparation. Students were not to be taught explicitly how to teach mathematics in the courses, but they were to see modeled the kind of teaching that they should do in their future classrooms. Frank's view indicated an immense reliance on student mirroring of teacher actions.

I would hope that people who teach 115 would also mirror, since most teachers teach the way they were taught, mirror what's going on, what we hope that would be going on there in the elementary classroom, or in the class. So I would hope, for example, that one, manipulatives be used, emphasizing problem solving, thinking skill. There would be a, a course of a cooperative type situation where students might be sitting down working together. They would (pause) do, let's say, an activity together using concrete materials that would emphasize problem solving and work together on it. (9-10 C)

This model of teaching is what he hoped for, but he did not think it was the reality in the course: "Of the sections that we have, my guess would be that, probably during the semester, all have some of that in it, but probably most of them are, 80% of it is probably done by a lecture method" (10 C).

The "role of modeling" in this context was a role for the instructor to play, not for the course to play. The role of teaching for the learning of content overrode modeling when the two roles seemed to come into conflict. He stated the following:

Most people agree that they would like to do everything that we talked about, okay (laugh). In reality, I'm not sure that they really do it. Okay? (short pause) And there could be many different reasons for that. Obviously, doing that takes a great deal more time than just to lecture does. And also, even though our, I think our curriculum is set up to give them enough time and leeway for enrichment activities and so on, others feel they, pressed that they have to get this material across to be able to complete this whole material in a given length of time, by the end of this course for 115, so they have that background for 116. (10 C)

Frank assessed that instructors recognized that students were part of a "program," taking a sequence of courses that, together, formed students' mathematics teacher preparation. Frank saw that reality and the constraints embedded in the "real," whether only perceived

or actual, affected what instructors attempted. In order to cover the material, choices were made that moved away from the use of instructional strategies other than lecturing. In this way, the course's role for student learning of content took priority over the instructor's role of modeling good mathematics teaching methods.

Summary

Frank did not come out strongly for a particular role for the course to play, but he did come out against the course as a place to learn how to teach mathematics and as a gate-keeper. Learning how to teach came in the methods course he taught. Gate-keeping was not necessary since competence in mathematics was not essential for good teaching. But this description is too simple because, as indicated, students were to see how to teach in the course and Frank desired a more rigorous treatment of the subject.

The role of 115 that Frank emphasized the most was providing a background experience in the content using manipulatives so that students would be in a good position to learn how to teach mathematics in the methods course. As a methods course instructor, Frank wanted students to have an opportunity to learn about topics in the content course that they may not have had in their K-12 education. This opportunity was preparatory to learning to teach content that may or may not have been in the curriculum when the students were children.

Frank also felt that students were not being given a rigorous enough background in logic and group properties. These were two of several content topics that he wanted students to study in the courses, but which were not a part of the courses as they were

designed. Additionally, he wanted the concepts taught in the setting of everyday use. But, he found it difficult to give examples of what this kind of teaching would look like.

Frank said that the course was not to play a role in teaching students how to teach, but he emphasized that students should see a model of good mathematics teaching in their courses. In this additional way, the courses would be a supportive part of elementary teacher preparation.

Frank spoke frequently about what he wished was taught and how he wished it was taught. As a faculty member with an administrative role to play, it was reasonable for him have with this focus. Leadership often demands looking beyond what is to what should be. Frank did not indicate involvement in conversations with instructors or administrators outside the Mathematics Department, however. In this way, he focused his vision of "what should be" within the Mathematics Department.

Gail: Reducing Anxiety

Professor Gail Laughlin's conceptualization of the roles the mathematics education courses were to play in students' teacher preparation was at once very simple and very complex. Her primary goal was that of reducing anxiety, directly stated and re-iterated throughout the interviews. This sounds simple, but the complexity of this role manifested itself in the many different aspects of anxiety that were of concern.

Reducing Anxiety: Enabling Content Learning

Gail equated reducing anxiety with building students' confidence, helping students feel good and have fun in the courses, developing good attitudes, and providing a non-threatening environment. I examine toward what Gail wanted to reduce anxiety: mathematics, doing mathematics, being in a mathematics classroom, the teacher, teaching. I consider how Gail saw learning content fitting with reducing anxiety first in order to set reducing anxiety into the context of the content courses.

Reducing Anxiety and Learning Content

While most instructors intended the mathematics content courses for prospective teachers to be courses in which students learned content, Gail believed the central role the course should play was not content-specific. She felt that, if the classes were fun and students were turned on to the mathematics, the issue of learning content would be trivial.

See, I believe once you get a student past the anxiety, they see that mathematics is fun, you turn them on to mathematics, all these things that we talked about, the content is easy! Because they are turned on to it! I've seen it! They'll go research it. They'll look up something. They'll study something. The content is easy for them. I mean we're talking, you know, arithmetic and junior high and grade level stuff. That's not, to me, not that it's not important. It's essential, but they will get that once you get 'em past this other stuff. So, that's why I probably sound more like I'm into the philosophy and the math ed and all that. But, I, I, you know, I find you can teach them the content once they get over that fear, anxiety, hatred, all the bad things that they come into the course with. (21 P)

In Gail's mind, learning the content was trivial, in part, because the content itself was lower level, not requiring a great deal of sophistication. If students were able to focus on the content, getting past the question of difficulty or their feelings of frustration, they could easily learn it. With a positive attitude, learning would occur.

Developing positive attitudes and reducing anxiety toward mathematics was the fundamental goal of the course in Gail's mind.

You can always get the material, once you have a good attitude. If you really like it and you're turned on to it, you'll be morning, whether you're the teacher preparing for it or a student studying for an exam. So, I guess that's my number one thing. I want them to walk away with a good attitude. (11 C)

In this statement, as in several others, there was no confounding Gail's meaning. Gail saw positive attitudes preceding content learning. So, reducing anxiety in its many forms was the most critical role for the mathematics content courses for prospective teachers to play.

Building Confidence

When Gail equated reducing anxiety with building confidence, reducing anxiety had several objects, e.g., toward mathematics, toward teaching mathematics, toward being in a mathematics classroom. She handled reducing anxiety toward mathematics and building confidence in doing mathematics through strengthening aspects of students' mathematical content knowledge. On a superficial level, Gail saw her practice of giving extra credit for and encouraging students to use symbols for frequently used terms in mathematics such as "for all" ($\forall$) and "there exists" ($\exists$) supporting confidence-building. She noted that students got excited about using the symbols.

Gail learned, through experience, that she needed to start problem solving content with less difficult problems and move to the more difficult problems gradually in order to achieve the goal of reducing anxiety toward mathematics through confidence-building.

One of the biggest mistakes I made when we first brought in the problem solving a few years ago, I was so excited. I made up these problem solving cards and I couldn't wait to get in there and do that group work and I threw it at them the first day, and they were so lost. And, I mean, they were so scared. And it was (short pause) bad. I mean, I learned from it. And now, every time I teach it, more and more, I look for simpler problems that'll give them the confidence, you know, confidence-building that, that almost anybody could get. And they come, "Oh, I got that!" and they feel really excited and good and then, like, they are willing to try, they're ready to do something a little bit harder. (12 P)

Gail's choice of feeling words like "lost" and "scared" and her description of the situation as "bad" gave a strong sense of the negative effects she saw from this type of encounter in students'

experience. The problems were too hard. The situation itself was threatening. Inducing negative feelings went against her primary goal of reducing anxiety. She learned to start with simpler problems, and build confidence toward doing problem solving.

Reducing anxiety and building confidence about teaching mathematics concerned Gail because she saw reducing anxiety about teaching mathematics as intimately connected with, in fact synonymous with, reducing anxiety about mathematics, itself.

GL: Reduce anxiety students might have about teaching mathematics. (short pause) Isn't reduce the anxiety students have, oh, well, yeah. The two are one and the same, aren't they?

GK: Well, are they?

GL: Yeah. If you have anxiety about mathematics, how could you ever teach it?

GK: Is it possible to not have anxiety about teaching mathematics but to have anxiety about mathematics?

GL: Not if you're honest with yourself, I don't think. And that's what I tell them. You know, I mean, I've had people say, "Well, I can remember being in elementary school and we just had math once a week." Once a week? What?

GK: So they aren't anxious about teaching math in elementary school?

GL: Until they come into my course. (laughs) You mean, they're not anxious because they think they only got to do it once a week? So, my goal is to give them anxiety, right? (Laughs) . . . I look it as, if you're anxious about mathematics, you're going to steer away from it, and you're not going to do a good job teaching it. (9 P)

In this exchange, Gail noted that some students were not anxious about teaching mathematics because they thought they would not need to do it very often, as was their memory of elementary school. When asked to indicate how she responded to this student perception, she laughingly asked, "So, my goal is to give them

anxiety, right?" Increasing student anxiety about teaching mathematics, which would mean increasing anxiety about mathematics, did not fit with her goal of making students feel good about mathematics, teaching mathematics, or being in the class. Instead of challenging the notions that some students had about the time or content knowledge required for the teaching of mathematics and, consequently, introducing anxiety toward teaching into students' minds, Gail preferred reducing students' anxiety toward mathematics with the assumption that this also reduced anxiety toward teaching mathematics. Unlike Jeff, who saw having some anxiety about teaching as an important spur to better teaching, Gail attempted to avoid anxiety about teaching to minimize anxiety about mathematics. In this regard, Gail was consistent.

Another aspect of reducing anxiety and building confidence centered on the classroom environment and the teacher's role in facilitating both in the classroom. Providing a non-threatening environment was critical to Gail. Gail wanted students to have reduced anxiety toward being in a mathematics class, and her class, in particular. She saw the environment set by the teacher. A desirable environment and a good teacher were non-threatening. She attempted to provide such an environment for her students, much as her best teachers had done.

What your students remember ten years later isn't the material. It's how you taught. So, I am the mirror that they are seeing. I mean, I still remember one of my favorite teachers doing this when she stood and taught, and there are so many things that different teachers I've picked up that had such a impact on the way I teach. . . . I can still remember she was an excellent teacher, she was just excellent. And, I mean,

I, I like to think that I picked up a lot of her ways of teaching. . . . She was very personable, she wasn't threatening. . . . She was just so neat, and I'm trying to think of ways she taught. I guess it was mainly the non-threatening. I mean, she was always well-prepared, you know, presented the stuff. But the biggest thing is that she was non-threatening. (11-12 P)

Gail stressed the sense of good teaching as non-threatening and the good teacher as non-threatening as more noteworthy than being well-prepared. This ranking could reflect her belief that most of her teachers had had content knowledge, but that, in order to be a good teacher, you needed more. That "more" involved empathy.

Empathy. That's the biggest thing. I mean, a lot of your, you know, college professors think . (sigh) I don't know, they just don't have an understanding for the average, your typical student, like, whatever your typical student is. And they have no empathy for 'em and they just flunk 'em. They won't work with 'em, they don't care, they cut 'em down, they intimidate 'em. I mean, these kids are coming in with enough anxiety, they don't need somebody standing up there telling them how stupid they are. (21-22 P)

The instructor sets the environment through his or her own empathy or lack of empathy for the students.

The environment that Gail wanted to prevail in her classroom was one in which students could ask questions without fear of being treated as if they or their questions were "stupid." Her comments implied that she was so effective in this that she was working against her goal of confidence-building:

That's one of the things I talk about the first few weeks, you know, at first, when they ask a question, they're kind of shy. And it's like, "Well, I know this is a stupid question, but," you know, and I'm always saying, "No question is stupid, if you want to know why, you know, if you're not following on the board, stop. It might be because you don't see that two and three is five. Ask." And I never, I shouldn't say never, I'm sure

I've had my bad days, but I rarely, and I really try not to make them feel stupid. I still get things in evaluations, "She always thinks you're so smart, she makes you stressed. She thinks we're so smart, she thinks we know everything." And it makes them feel stupid because they don't. (7 B)

The credit that she gave to her students, when they felt it unwarranted, worked against her aim of having students feeling confident about their knowledge of mathematics. The students sensed that they were, in fact, stupid, because Gail implied that they met her high expectations of them, and they knew differently.

As Gail described it, a mathematics class should look and feel like a place where a student could comfortably ask questions and not feel intimidated by the teacher or the subject. She laughed with students about the way they had "learned" concepts before and how she expected them to understand concepts differently in the courses.

[In class, we] talk about why you do what you do when you do it- that's what they get so excited about. They come and say, "God, it's so neat. I never knew why I did that. I just did it. I never understood why I did that." You know, even something simple like inverting and multiplying with division, they didn't know. "It said invert and multiply, okay, I'll invert and multiply." We always get laughing in class, you know, or finding common denominators or whatever. They never understood why they had to do it. (17 P)

By laughing about past situations and pointing students toward the goal of understanding why they did things in solving mathematical problems, Gail felt that the students moved toward the understanding of the concepts as she intended.

Gail expressed conflicting feelings about whether the course should be portrayed as difficult. While wanting students to feel confident that they could do the work in the course, she explicitly

described the course as "not an easy course" in the course description she handed out at the beginning of class: "[Math 115] is not a course in arithmetic. It is not an easy course. It is a course that is designed to challenge you to think" (emphasis in original - GL Course Description). [Note that this conflicts with her stated view that students could easily learn the content because "The content is easy for them. I mean we're talking, you know, arithmetic and junior high and grade level stuff" (21 P).] When asked if she wanted to maintain the perception that the course was difficult, she said that she read this notion two ways. One reading left her feeling that she did not want to perpetuate that sentiment; she wanted students to feel confident about their ability to complete the course successfully. The second reading led her to say that the course was difficult, misunderstood by those outside the realm of education who would have the students believe that the course was void of anything of value.

Feeling Good

Gail also paired reducing anxiety with developing positive feelings. Gail noted that, with confidence, came feeling good. What students were to feel good about (e.g., about mathematics or about teaching mathematics) was not always clear.

Having students feel good occasionally conflicted with Gail's plans for group work as an instructional design. Sometimes, "feeling good" took priority; other times, having students work in particular groups took priority.

I could see that they [students] were always getting in the same groups, and so I came in with numbers and changed their

grouping, and they were really upset about it at first. But then they liked it because they got to meet new people. So, I'm kind of, that's a tough one. It's like if you force somebody in a group that they don't want to be in, they're not going to do well; they're not going to like it. But on the other hand, they get into these, where they're always in the same group, they don't get to know other people, they rely on the top student in that group to do all the thinking for 'em. (13 C)

Because Gail was sensitive to students' feeling good about being in the class, she was hesitant to form groups in ways that she felt might break up cliques of students, though she sensed that cliques were not a productive way for the groups to be formed. The issue of what students were to feel good about seemed important here. The focus was feeling good about being in the class or not being upset with the teacher.

Another instance of this focus on feeling good came when Gail talked about correcting student errors. She described an exam question which asked students to determine if a child had used a reasonable strategy to do a computation. The child's work included a gross error.

It's amazing. They'll [students] say, "Yeah, that's okay." I mean, they could check it on their calculator that it's wrong. They'll say this, "Oh, yeah. That student's fine." They'll tell me it's right. . . . It blows my mind.(laughs) And I point out to them, in a nice way, I say, "You guys, you're, you're, you're forgetting that you can always use your calculator. Multiply 37 times 56 and you can see that it's wrong. I mean, you should at least be able." That's what worries me. Are they gonna get out in the classroom and say to that student, "Oh, yeah. That's a nice short-cut. You can do that." I can't believe they would do that. (Laughs)(19 C)

In this situation, students' exam answers indicated that they lacked understanding of multi-digit multiplication and disregarded the

availability of calculators as a support. Gail pointed out to them, "in a nice way", that they had made a serious mistake. While the students' seeming lack of sense-making troubled her, having them feel good about the situation concerned her more.

To some extent, Gail's concern resided in having the students like her, as the teacher, and putting them at ease with her. She saw having pupils like the teacher and feel comfortable in the class as things that she wanted students to value for themselves as teachers, too. As a result, she actively attempted to support developing this relaxed feeling toward the teacher and the class.

When they do something wrong, give them a pat on the back, point out the good things. When I grade their quizzes and tests, I'm always writing personal notes on it. I put a smiley face if they do well. Talking with them, telling them a little bit about my personal life, just being personable with them. I think that's more than anything, is not being intimidated by the teacher, then they're not intimidated by the course as much. Getting to know me. [vocal emphasis] (4 B)

Again, she expressed concern for not intimidating or threatening students. In this case, the intention was to support students' positive feelings toward the teacher, which supported feeling good about being in the course. The issue of mathematics content learning was implicitly a part of the equation, as indicated above. That is, if students did not feel intimidated or threatened by the teacher, they would be inclined to work harder and learn the content. The closest thing to explicitly talking about feeling good about mathematics itself came when Gail spoke of using manipulatives.

Gail described using cups and chips in an activity that modeled balancing equations. In her recounting of the lesson, she expressed

surprise that students liked the activity and had gained some understanding of the process of solving equations.

I was amazed how much they liked it. It clicked for some of them, that, "Oh, whatever we do to the one side of the equation, we have to do to the other." That's my main reason for doing that is to show 'em that you got to do the same thing to both sides, whether you're adding, subtracting, multiplying or dividing. (4 B)

The fact that the students liked doing the activity was important to Gail. The reliance on activities and the desire for them to be enjoyable was a consistent part of Gail's instructional practice. This desire was especially clear when she talked about the importance of providing opportunities for students to have fun in the course.

Having Fun

Gail provided opportunities for students to have fun with the mathematics as a way to reduce anxiety. When asked how she planned for having fun in the course, Gail used the example of work with Miras. A Mira is a reflective piece of Plexiglas with a beveled edge and end braces to set the Mira upright on. A Mira can be oriented on or near a drawing and used to reflect the drawing in different positions, depending on the placement of the Mira. Gail had several sheets in the course packet that she felt added an element of enjoyment to the task of learning how to use the Mira.

Well, did you see the course packs and the stuff that we did? That was so cute. And see, we, you know, this is for elementary school, so it's pretty, some of the stuff is pretty simple. [looking through course packet and picking out sheets, . . .] We thought this was so cute. You move it around and you can put the wigs on her. It changes her hair style. This one you can put the car in the garage or you can bring the garage to

the car. Just things like that. I think, kids love this. And even the college kids got a kick out of it. I mean it really was to show them what they can do in elementary school. (8 B)

The cute activities were appropriate for elementary school children and gave the students a chance to learn how to use the Mira. The motivation for doing these activities was to get students comfortable with using the Mira. Because Gail did not assume that students had taken high school geometry, there was little discussion of the mathematical ideas at work in the activities. She felt that this would disadvantage students who had not studied formal geometry.

Providing activities that were fun and could be used in students' future teaching was an important part of Gail's planning and supported having students both enjoy the class and find the class interesting. For other instructors, selecting content that was interesting to the students or motivating interest on the part of students was important. Gail had a slightly different approach to the issue of interest. Her focus was less on the selection of interesting mathematical content and more on connecting the content to the students' future teaching to maintain interest.

My students in my 115 course are gonna lose interest if I get (short pause) far away from what they're gonna experience in the classroom. To me, you gotta bring it in, you know, "When you get out there, this is gonna happen, or that's gonna happen or when you're in the classroom," because then they can relate to it. It's like, that's, that's one of the things that they do positively comment on. You know, "It's neat because she, she brings in, we can really see where we're gonna use this. We can see where we really need this. We're gonna be using this in the classroom." (25 P)

The interest that she capitalized on was interest in teaching more so than mathematics. She capitalized on this interest in the course, in part, through her practice of having students do activities that they could use in their teaching.

Give students ideas for activities that they can do in their future classrooms. Oh, yeah. It's not an essential part of the course as far as the math department, but we're looking at essential as my standards, right? . . . Cuz I bring in extra stuff and tell them to start building a notebook and saving it for when they get out. So, it's essential in my course. (7 P)

Examples of the "extra stuff" that she brought in were poems, riddles, and articles from various sources. She saw these adding to the students' professional library of references as well as supporting her goal of building on their interest as future teachers.

Reducing Anxiety and Learning Content: Revisited

While Gail saw reducing anxiety as the primary role of the course, she did not want this emphasized at the expense of content.

Well, it's easy to say you want a good attitude. Ideally, you want to turn them on to math and have a good attitude, but your job also is to teach 'em content. When they go into 374 [methods course], they need to know this content. To me, this is arithmetic stuff, and so you can talk all you want about, "Oh, yeah, all I want to do is have good attitude." I mean, that's not all I want to do. [vocal emphasis] I want them to know this material, too. . . . You asked me to summarize (laughs) what was, if I had one goal, what would it be, and that would be to turn 'em on to math and have a good attitude. But there's a lot of goals. They need to know that content. They can't go out there and teach without knowing it. (19, 20 C)

The bottom line was that students had to know the content when they finished the course. The course provided a set of opportunities for learning the content intended to be non-threatening, supportive

of confidence-building, and enjoyable. She designed the activities to accomplish this while giving students activities that they might be able to refine and use in their own teaching.

In taking Gail's comments as a whole, there was no inconsistency between her avowed primary goal and the learning of content. She did not ignore content with her emphasis on attitudes.²⁴ Rather, Gail's premise was that, by developing positive attitude, content "would come" -- especially the relatively easy content of arithmetic that Gail saw the students needing to know.

While Gail felt strongly about reducing students' anxiety about mathematics, doing mathematics, and being in a mathematics classroom, she saw the learning of the content, as easy as it was, as a necessity. She did not feel compelled to pass students who did not satisfy her course requirements. She spoke of weeding out underprepared students the first few weeks of the semester. She also acknowledged that up to one third of the students in her classes were students retaking the course. Gate-keeping was a role the course was to play.

Without a pre-requisite requirement, students took the course without having an adequate base of content knowledge. When I asked

²⁴ At one point in the interviewing, Gail suggested that perhaps the particular content of the course was not important, and that she should give thought to selecting content that students could build positive attitudes around. Seeing the goal as reducing anxiety through the choice of content is a very different reason for not being concerned about the particular content to be learned than was suggested by Dienes (1972). Recall Dienes saw the particular content of less concern because the content is changing so fast. It is more important, in his view, to be concerned that students learn to "meet the unexpected" (p. 66).

Gail if this affected what content she taught in the course, she replied:

No. It just means they don't make it. (laughs) No, I don't lower my standards at all. They, you know, I explain to them and I, I will work with them on the side, because I feel bad that there wasn't a prereque [pre-requisite] in the book. It's not their fault, but, (pause) you know, they have to, you have certain goals and they got to meet 'em, if they don't, they're out of there, that's the way it is. (6 C)

The result was that many students did not pass and chose to retake the course rather than change their career goal. Gail indicated that the students often realized the value in retaking the course. In this way, she saw retaking the course enhancing students' attitudes. She also noted that, in the future, a developmental mathematics component taken prior to the course might reduce this problem.

The three strategies for reducing anxiety that Gail emphasized were three of many different ways that an instructor could organize instruction to maximize this role in the courses. Building confidence and providing opportunities for students to feel good and to have fun with mathematics and in mathematics class were important aspects of the courses for Gail. There were other goals for the course beyond reducing anxiety. In some cases, she actively pursued the goals (e.g., gate-keeping) in the courses; in other cases, she specified the goals (e.g., learning the norms) but made little attempt to meet them.

Summary

Gail had a singular focus on reducing anxiety to facilitate mathematics learning, and she did not look beyond this to consider what other courses outside the mathematics department contributed

to students' programs. She crafted her version of the course using her own preferences and experiences as informants, sometimes choosing to include content that others recommended against. She saw her experiences working with her son's teachers as important contributors to her teaching and decision-making.

Gail saw the mathematics content courses as essential to the preparation of elementary school teachers. In large part, this was because the courses offered an opportunity to reduce the anxiety students had toward mathematics, doing mathematics, and being in a mathematics classroom. Gail saw this as critical for their future performance of teaching duties with respect to mathematics. She made this point repeatedly throughout her interviews. And, she described several aspects of reducing anxiety that oriented her actions in the class. The three aspects of reducing anxiety elaborated upon in this analysis were building confidence, feeling good, and having fun.

Like Jeff, Gail felt a heavy responsibility for the preparation of the future teachers in her courses and believed that teaching the courses held greater demands than other mathematics teaching. Her reasoning took a different tack from Jeff's, however.

When you're offering a course to students that want to be teachers, you need a good teacher in that classroom teaching it. Because again, not only are you teaching them the content, you're teaching 'em how to teach, whether you want to or not. They're watching you and they're picking it up. And they're on your case all the time. I mean, if you breathe wrong, (laughs) they're onto your case. You can't get away with anything. Well, you can't get away with anything in there. I mean, it's much easier for me to teach the algebra and the business analysis and the business calculus courses. They kind of look up to you

and (short pause) they, they don't argue with you, you can, you pretty much have free rein to do what you want. But, in those math ed courses, they are studying to be teachers, and a lot of them are very opinionated, and I'm not saying that's bad. But, you better be on your toes. You'd better be ready, and you'd better know what you're doing, and you better know how to answer the whys and hows and, of everything you're doing. It's harder. It's just more difficult. (21 C)

Where Jeff's reasoning centered on the students' future pupils, Gail's perspective focused on her as the model of teaching. This perspective was consistent with the way Gail saw the demands on being good at mathematics versus the demands on being good at teaching mathematics. She saw the latter as holding greater demands -- for such things as flexibility of thinking, ability to model mathematical concepts, and being able to ask good questions.

Gail focused her comments powerfully on reducing anxiety and, as a result, seemed to be only secondarily concerned about issues of how to teach. This may be misleading, because she, more than many other instructors, voiced the need to deal with methods issues in the context of the course. The end-result was a sense that Gail saw all of the pre-described roles as being valid for the course. But, reducing anxiety had top priority.

Craig: Content, Norms, Dispositions, and Attitudes

For Professor Craig Kennedy, the roles the mathematics content courses play in students' teacher preparation complemented the further work in the department's methods course. He was perhaps in the best position to have this perspective so well-established in his mind, as an instructor of both content and methods courses at the time of the interviewing. He did not indicate that he works with elementary school teachers or faculty from other departments, but he took an active role in public education as a school board member in the rural community in which he lived.

Craig characterized the courses and their instruction as dynamic, in the process of change and improvement. He described the roles they were to play as evolving, as well. The current emphasis was on the learning of mathematical content with supporting roles in the learning of norms and the development of positive attitudes and particular dispositions.

Content Knowledge

Craig followed the departmental syllabus in terms of the content coverage and its problem solving focus. He monitored input from outside and inside the courses to determine the course orientation. These resources included the Mathematics Education Committee meetings, professional resources, and the students in the courses, themselves. Craig provided students with frames of reference that were to support their learning of mathematics. In the interviewing, Craig clarified his views on understanding and the

interconnectedness of mathematical topics. He also described the impact of reform on content issues facing the instructors of the courses.

The Changing Content

Craig saw the content of the courses going through a change, due in part to the initiation of placement testing and availability of courses at the remedial level. In the future, content coverage with whole numbers and fractions in the program would take the form of work in the methods course involving how to teach the content to children and would no longer be a focus of concentration in the content courses, Math 115 and 116.

We're constantly trying to upgrade. Now, at NU, we have a course 107 and 108 [remedial] where they really would be channeled if they don't have the math background like we'd like to take 115 or 116. So, we're working on that. And in recent curriculum revisions we've made for our 115 and for our 116, we're cutting out work with whole numbers and quite a bit of work with fractions and these kinds of things in these courses, hoping either they've had that before, or will get more with that in the methods course, later, where you get hands-on and go in-depth with how to teach this to kids. And we'd like to focus more on these courses, using the calculator, doing problems with other parts of the real number system, instead of focusing so much on the whole numbers and numbers like that. (3 C)

The shift would allow more concentration on problem solving and use of technology, two elements that Craig identified as essential for students to have experience with in the courses. Craig talked about this shift as being supported in the discussions of the Mathematics Education Committee that focused on the Math 115 and Math 116 courses. Unlike the other instructors, he thought the

Mathematics Education Committee meetings might result in some change in the content focus of the courses.

In the current departmental syllabus, problem solving was a major focus, but the use of technology was restricted to the use of four-function calculators. Craig wanted work with computers and graphing calculators to support the learning of mathematics within the courses, but the lack of equipment and knowledge (on the part of faculty) was holding him back. He hoped to spend time learning about using calculators.

That's one thing I would, thing I would like to do this summer is explore the graphics calculator more for its use in a course like this. Because I think, for example, when you're doing line of best fit and that kind of thing, it would be very nice to have that. And to have them or at least have one to show how, how you could do some of this with a graphics calculator and for certain functions. But we don't, I don't currently use a calculator, a graphics calculator in this course. (5 B)

Craig could see that there were places that the use of technology would make sense and add to the content coverage. He anticipated working toward using technology more as it became available to use.

Craig also stressed the impact of outside influences like the NCTM Standards, MEAP (Michigan Educational Assessment Program) testing, and professional reading on his version of the course. In this way, he was quite unique -- noting at several points the impact of these influences on the courses and his own thinking and acting.

GK: What's been the impetus for all the changes?

CK: Well, this is, the Standards, the national Standards, certainly, and what we see on the MEAP, you know, the new MEAP. (short pause) I think also just in general, where we are in our reading and thinking as professionals, probably we've had some of these beliefs for years, but (laugh - short pause) now, with the emphasis at the national and state level on some

change, it's easier in a sense to (short pause). . . . Of course, I think the national Standards and other things just came from people like us, anyway, that, so. (short pause) Didn't just start three years ago, out of the blue. It's been smoldering for many years. (4-5 C)

Craig found support for what he wanted to be doing in the classroom in these outside sources that reinforced things he had been thinking about for years. This support extended to both content concerns and concerns about how to teach the content.

Craig responded to what he saw as students' prior knowledge and suspected needs through his instructional decisions. How to teach the mathematics in the content courses for prospective elementary school teachers was connected intimately with the why.²⁵ Craig saw that students needed to understand the content personally at a level that was rich in interconnections in order to move toward learning how to teach it: "We don't focus on them teaching it, but for them understanding it, hopefully it will help them teach it" (6 P). The content and depth of understanding was geared to their chosen career path: "And, by deep, we mean, deep in the sense that we know they're going to be elementary teachers, for example. Not deep in the sense that a PhD would need" (6 P).

For concepts that students would have encountered prior to the courses, the goal was to build on the students' understanding. This building could take the form of setting the content in context.

²⁵ Notice that this "how to teach" does not involve the role of the students learning how to teach. Rather, it involves the decisions of the mathematics teacher educator for the teaching of the mathematics content courses.

Like on fractions, for example, [my design] is have it come up kind of incidentally, not as a topic to be, you know, go back to square one and go through unit fractions and so forth, so much as having them be able to use those ideas and concepts in problem solving situations. You know, measurement situations, that kind of thing. (7-8 C)

In this way, Craig anticipated that the students were reinforcing skills and gaining a sense of confidence in their ability to use them.

The building of understanding could also take the form of investigating concepts and relationships using manipulatives.

In 115, I've used base ten materials and Cuisenaire rods connected with fractions and decimal concepts. Grids, from the concrete to the, to the semi-concrete, or, pictorial. (pause) Three different kinds of representations of fractions, in terms of the area, length, and discrete object kind of thing. (short pause) Many times they've had that in s-, a lot of that, but they hadn't had a chance to sit back now, and, in a short period of time, take a glance back at all of it, you know, for a second time. And, "Oh, that's what fractions are all about!" (9 C)

The use of manipulatives was becoming an important aspect of the teaching and learning in the courses for Craig. He sensed that the students were able to consider things on a metacognitive level as a result of this approach to the content in the courses.

Frames of Reference for Learning Mathematics

Early each term, Craig presented a framework that he and the students used throughout the class to help talk about learning and doing mathematics.

I will do things like go through and have them think about the kinds of things you do in mathematics, like skills that you get developed, knowledge, understandings, generalizations, attitude, concepts. We'll discuss. (pause) You know the kinds of things that you try to, that you get involved with when you do mathematics or learn mathematics. And then, as we go

through the course, we'll, these words will become part of our vocabulary, you know. (22 C)

In addition to this set of six aspects involved in learning and doing mathematics (skills, knowledge, understandings, generalizations, attitude, concepts), Craig also explicitly drew students' attention to the different ways of representing concepts, extending Bruner's set of three -- inactive, iconic, and symbolic -- to include two categories added by Lesh -- applications and verbalizations. In terms used more frequently by Craig, the categories were concrete, pictorial, symbolic, applied, and verbal as ways of representing concepts.

We discuss both of these things [six things involved in doing mathematics and five ways of representing concepts], but we don't take a whole lot of time on it, fairly early in the course, and then it's, these are things we can refer back to as kind of a common, for many of the things that we do, see how this fits into these. (25 C)

Both sets of ideas were foundational in setting the tone for the courses. Students were to learn mathematics in various forms, and there were ways to learn mathematics that strengthened its meaning. Having this common foundation helped achieve the goal of greater student understanding.

If you feel good about yourself and you understand more of what this is all about by tying in the concrete to the pictures to the symbols then I, from what research has shown us, too, I mean you're going to remember more. The more you really understand what you're doing, not only are you going to be less afraid to try a new problem or whatever, but you're going to remember it better if you've helped develop it instead of just being lectured to it and all this stuff. (20 B)

This final statement gives a clue as to how Craig anticipated accomplishment of this goal. Lecturing was not his instructional strategy of choice. Rather, Craig tried to involve students in activities that required them to move from the concrete to the abstract, often in the context of applications requiring written or oral explanation.

The motivation for this development of concepts from concrete to abstract with applications and verbalization rested in the value of this to the students as learners. In talking about developing the formula for the area of a circle through a progression of activities, starting with a paper parallelogram and moving to a regular polygon with many sides all cut into triangles, drawing pictures (see Figure 2), and eventually writing the formula for the area of a circle, Craig stated the motivation: "The reason for doing this is to help them. Perhaps, give them some ideas of what they can do with kids. But, help them to understand better where the formula comes from and, and again to cut their anxiety" (9 B). He used this as an opportunity for students to investigate the concept of area, connecting concepts

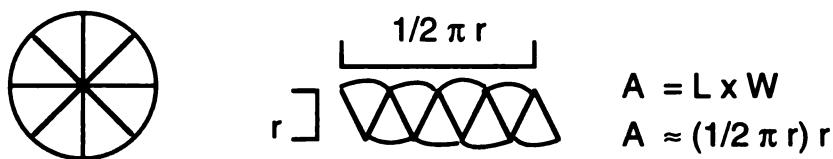


FIGURE 2
Developing the Formula for the Area of a Circle

to support learning, in particular seeing that the area formula for rectangles could be used to understand the area formula for a circle. This exemplifies how a task could involve interconnecting across concepts (area of rectangle/area of circle) and interconnecting meaning across representations (concrete, pictorial, and abstract).

Clarifying Ideas and Definitions in Thinking about Content

Craig worked to clarify terms as he spoke, often pointing out that a term was complex, e.g., "understanding." When asked if people not having a deep understanding of mathematical concepts should be kept out of teaching, he pointed this out.

It depends how much they don't understand, I suppose. If they don't understand a few concepts, understanding's a spectrum anyway, right? (laugh) What does it mean "to understand"? There are levels of that, as Davis, I think, had an article one time in Arithmetic Teacher Journal. Understand generalizations, understand (short pause) various things, there are levels of that. (13 P)

A bit later, he acknowledged that a certain level of understanding had to be a part of an individual's understanding or he or she did not belong in teaching, and he saw the courses as having the role of developing and enhancing that level of understanding.

The notion of competence was one of several that seemed to defy definition and specificity for Craig. Though he had difficulty specifying what competence was, Craig saw the whole mathematics teacher preparation program developing competent mathematical thinkers rather than relying on students gaining competence in a single course:

If you think of all the content courses, 115, 116, and whatever would go after that, when do they become competent? I'm not

sure they are competent like in 115, so I need to think of, eventually, at least? Not necessarily after 115, maybe 116, when does a person feel competent? (5 P)

Competence, though a nebulous notion, was something to be working toward, not necessarily something to be demanded in a single course.

Craig seemed to use the interviewing actively, as a site for professional growth, stimulating his thinking about ideas and terms that had occupied his attention previously. In answering questions, he noted, "I'm trying to think deeper on what the question means, maybe I'm not sure what all it means" (2 P). Not being sure what it all meant did not mean that he shied away from the questions, only that he recognized that the issues were not black and white. He struggled with multiple meanings of terms, trying to determine which meaning he was applying when responding to questions. The most extensive example of this was when talking about the importance on interconnectedness of mathematical ideas. He saw several meanings as viable, each being something he worked toward in the class.

Recall the emphasis Craig gave to laying the framework of six kinds of things involved in mathematics: skills, knowledge, understandings, generalizations, attitude, and concepts. He saw these as interconnected and wanted students to, as well. But, he saw interconnectedness of mathematical ideas as involving more.

GK: So, when you say interconnectedness, you're thinking of how all of these are interconnected? These six kinds of things?

CK: Yeah, and how together they give us, maybe, what we call mathematics. But also things like, (short pause) also, things like, concrete material, pictorial level, and (short pause) the abstract, say, applications, -- back to Lesh's model, and

verbalization, how that's all connected, as well as concepts and ideas are all connected. . . . This is more global[pointing to list of six kinds of things done in math], I guess, a lot of this is. And when you get down to the nitty-gritty of how regrouping, say, with decimals is similar to regrouping before you use decimals, is, and to help show that, I would use concrete materials, and the pictorial, and on to this symbolic. (22 - 23 C)

So, Craig viewed the six components of mathematics as interconnected as well as the five ways of representing mathematical concepts as interconnected, but on different levels and in different ways. He wanted students to be aware of and see these interconnections, too. For this reason, he explicitly used the terms in class and referred to them throughout the courses.

Craig also noted that he emphasized other connections. He saw connecting algebra to geometry and connections between the mathematics students were studying and the use of it in the real world as important. With all of these different meanings of connection and interconnection, Craig speculated that "maybe I don't have a well-defined, in my head, what I mean by connecting. I guess I use it in a lot of different contexts" (24 C). When asked if there was a specific sense of interconnectedness that he wanted students to leave the course with, his reply was, "I think I mean all of these things" (24 C).

Another word that raised the question of meaning was "unprepared." Were the courses to act as a barrier to those students who were unprepared?

It depends what unprepared means. . . . I try to help, if at all possible, to get through the course. I think it's always hazy whether they are prepared or not. What does that mean

(laugh)? I'd probably tend to think there might be a number of reasons and I can't exactly tell if they are unprepared anyway, at least in the current set-up that we have. . . . There's some testing we do now that, you know, that's, within the past year that's helping some. I think a little interview could, would be important, too, though. (3 P)

That is, on the front-end, it was hard to know what students brought to the course and just what being prepared would include. The issue of what unprepared on the out-end meant was less blurred: "If they can do the mathematics in a content course, that's, I guess, that's what I need to try to find out" (4 P).²⁶ The issue of being prepared to teach was not a part of this. Nor did Craig see the role of the courses to include sorting and screening the students as future teachers. That was the job of the methods course or courses in the School of Education.

While Craig did not ignore teaching issues in the courses, the students' future teaching was a reason for learning the mathematics in the courses. The students' chosen career path and the courses as designed specifically for prospective elementary school teachers affected the specific content and teaching of the mathematics, but the focus was on their mathematics learning and the resulting understanding: "Understand math in ways that help them teach it, sure, I would hope that's what we do. Well, we don't focus on them teaching it, but for them understanding it, hopefully it will help them teach it" (5-6 P). And, when thinking of challenging students' notions of how to teach, this was seen as accomplished "by having

²⁶ Recall the mathematics in the content course was specified in the syllabus for Craig.

them do the things I do, which are focused on them, not the kids, in these content courses" (5 P).

Vision of Reform as This Affected Decisions About the Content

Craig saw that reorientation to the vision of the reform movement was not yet occurring. As a result, his responsibility for offering opportunities for students to learn mathematics in ways that were more in line with the vision for future elementary classrooms was real.

I still have plenty of students that say, when we do work with say concrete materials and try to tie it in to show the meaning for different steps in an algorithm, that kind of thing, still a lot of them say they've never seen or done that before. I know it's true for me. I can remember manipulating sticks and stuff like that to make bundles of ten, years ago, but I think a lot of classrooms, least the kind of students I'm currently dealing with, they still, they're still coming through with not much of that in their background. So, in that sense, things haven't changed. There's still a need to do a lot of that with them before we get to thinking about how they can (laugh) use materials and things with kids. (15 P)

Craig's perceived need to do work with manipulatives and pictorial representations drove many of the changes in his own teaching, pushing him to re-think the wisdom of waiting to use manipulatives in the methods course, a recommendation supported by some in the department.

With the revisions he expected as a result of the recent Mathematics Education Committee meetings, he saw the opportunity to develop the use of manipulatives for learning mathematics.

I see as the years go by, more of a more of an emphasis on having students in that course, even, hands-on, instead of waiting to a methods course where we're looking at how to do hands-on stuff the way kids might do it. I feel more and more

that, we're spending more, we're cutting certain topics out, of courses like 115 and 116, so that we can bring in a few new topics and emphasize calculator more and so forth, but get more hands-on, as well. (4 C)

He saw the removal of work with whole numbers and the basics of fractions as offering an occasion for altering the way instructors taught the content. Increasing the amount of hands-on activities and developing ways different representations supporting the learning of the content were important to him.

Craig also seemed to look forward to the shifts, changes, and opportunities he anticipated as a result of departmental, state, and national initiatives. Part of his expectancy grew out of his sense that the professionals he worked with would come together to work in ways that supported these changes.

As we [faculty teaching the math ed courses] get to know each other better and talk about curriculum issues, and we are doing that more and more, I think, as, partly stimulated by the national and state initiatives, but, in sharing good ideas that work and things, you get more trust in each other, I think in a sense, and more sharing might find me becoming more like another person (laugh) in terms of my 374. Partly because I'm learning things they do that work well and that I hadn't even thought of or had been afraid to try, or something. And I think as our expertise in technology improves, too, each of us, you know, we share things that we're each trying that sound interesting to try. (10 C)

Craig sensed that building trust within the group of people responsible for teaching the courses, content and methods, would support risk-taking and openness in the teaching process. Such trust-building parallels the Standards recommendation for building communities of learners in classrooms. In this case, the learners are mathematics teacher educators.

Craig saw the content courses designed for prospective elementary teachers as unique and important courses, necessary for the preparation of teachers as much as the methods courses and playing a very distinct role in this preparation. His assessment was that the content and methods of the courses were different in some ways from any other mathematics course offered at NU. The differences arose from the added demands of the courses due to their very special student population.

I think many of the other courses are noticing the need to have like small group discussions, (short pause) use calculators, do estimating, cooperative learning, (short pause) talk about the concepts and ideas, discuss these, (pause) but, so I think that's a good sign. In general, in our math department at least, there are many more people than say five years ago that are doing, trying a lot of these kinds of things, say, in a Calc[ulus] 1 class even. Getting away from the pure lecture, assign problem-, give a few examples and assign problems strategy, but (short pause) in terms of concrete ma-, spending time with concrete materials and really trying to go (pause) to (laugh) at least to help people really (short pause) recover from math anxiety and get a good positive attitude toward math and the way it's used in the world, and (short pause) get a good grasp of various approaches for fundamental concepts and ideas that you need when you are teaching elementary school, that would be different in the 115/116. (short pause) I wouldn't care if the average person knew four different algorithms for multiplication or not. Maybe if they know one and do it well, that's fine. But if you're going to be a teacher, (laugh) [that isn't enough]. (13 C)

Craig saw these courses as critically important sites for student learning of the content necessary for teaching elementary school. The content was different from that of other courses and focused on understanding the concepts in a richer sense than students had when they entered the courses. Craig felt that the courses' designation as

a part of teacher preparation allowed him to approach the task of teaching the content more appropriately.

I think they need a course like this where they can, just geared for teachers. Where you are free as a, where I am free as a teacher to interject comments and ask questions related to teaching in general, and where I know I can focus on things that are going to help them get grounded in the kind of math they need to help teach elementary school. . . . It's a unique course. I think it's an important, unique course. (12-13 C)

The predominant goal was getting students grounded in the mathematics.

Norms, Attitudes, Dispositions: Interwoven to Support Content Learning

Like Jeff, Craig saw the need to develop dispositions in students. Craig also wanted to develop confidence and norms, particularly of communication. Unlike Jeff, these dispositions were to be developed specifically in support of student learning, with a sense of Craig's and Jeff's shared ultimate concern, having an understanding of the content that would support good teaching.

As noted earlier, Craig wanted students to be inclined to think about the components of mathematics and the different ways one could represent ideas, and to use these aspects to deepen their understanding of concepts. He also wanted them to learn how to learn mathematics through this process of thinking through concepts -- experiencing the learning of mathematics in a dynamic way. "What way of looking at the concept makes it more understandable to me?" was a question Craig hoped students would ask themselves when approaching a concept.

Craig's sense of the need for the students to construct the meaning of the mathematics for themselves came through when talking about his consideration of students' interests in selecting problems to assign.

I feel if you can't motivate and spark their interest somehow, kind of draw them into the topic, you're not going to accomplish much (laugh), cause they're the ones that have to learn it. We assume we know the material, (short pause) and so I try to do, what I could to, if it's a topic, a mathematical topic, or an application that they don't know much about, maybe it's something I'm interested in, but I would like them to be interested (laugh) in. I feel it is important (short pause) for them to tune in and connect and learn whatever math is within that problem or situation to be learned, they've got to have some, gotta know enough about it to have an interest, I guess. (10-11 P)

He tried to set up opportunities for students to learn what they needed to know to develop interest in the problems. If that meant explaining a bit about baseball in order to work problems on batting averages, then he did that.

Craig required students to complete a three-to-five page paper on some aspect of mathematics that they used in a job, sport, or hobby. He wanted to capitalize on student interest and build the students' awareness of the importance of mathematics. He saw this assignment, along with other activities in the courses, developing in students positive attitudes toward doing mathematics and a better sense of what mathematics is.

Through the papers that they write, whether they say it in here or not, and many of them do, at the beginning or the end of their paper, usually, how their attitude really has changed, through writing that paper or what we've done in the course,

toward what math is - what is math - or how oft-, how much it's important in their lives, kind of thing. (22 C)

Craig saw the changes in attitude fostered by preparing the papers as important not just for reducing students' anxiety, but also for increasing students' willingness to attempt problems and see mathematics as a meaningful and approachable endeavor. Having the disposition to attempt problems that appeared to be difficult was a quality he wanted to encourage.

With risk-taking, students learned to do more conjecturing and learned to verbalize their mathematical ideas in group work, in class discussions, and on paper.

I encourage them to, as I said earlier, come up with conjectures, verbalize what they see. Try to hold back, though a little bit, and not the first person that sees something yell it out. But, make observations and eventually verbalize them and talk to each other. Even in group problem solving, they're discussing the material, they're verbalizing, hopefully going deeper into the idea. (6 B)

Because a major goal was to go deeper into ideas, Craig encouraged conjecturing and verbalizing. Going deeper into the ideas was one of the motivations for the assignment of the major paper described above.

A lot of them have never been asked to kind of put down on paper and really think through how math is involved in something they've done maybe for years. . . . So they're writing about it and they're thinking about it and then we share these in small groups, too, when they bring these papers in. And so they each have to verbalize and, and talk about what's in the paper. And so they learn from each other too, not only what they just wrote but what, what other people wrote. They have to communicate this. (7 B)

Craig saw this act of communicating as supportive of his goals for the courses. As a result, he was giving considerable thought to increasing the amount of writing done in the courses for the future.

And I think there is more writing we, we could, and probably will start doing in even a course like this. Yeah, writing about what you thought about as you solved the problem. What were your biggest hurdles? I think that would be, it's a good idea that I'd like to head into, so that I do have writing from them besides the solutions to problems and attempts. (8 B)

By encouraging students to write, verbalize, communicate their ideas more frequently, Craig hoped that students would achieve greater understanding and be more comfortable sharing their ideas with others.

Craig felt that the kinds of discovery activities done in the class enhanced students' comfort in doing the mathematics and sharing ideas. The comfort came from feeling capable of approaching difficult problems.

From what looks like a very hard problem and nobody had solved the problem, instead of me putting the solution on the board, and then maybe or maybe not following it and perhaps not being able to repeat it on their own or solve another similar problem, I really let them, gave a couple hints then had them. And I saw in their, in their eyes and in their reactions, kind of like, "Hey, this is not bad." "Ah-ha!" and kind of finished it up. (3-4 B)

Craig intended that the activities build confidence and openness to attempting problems. A strategy Craig used involved giving hints and having students work on the tasks individually or in small groups. He indicated that, with more time available, he would not rely on hints as much, but that with the time constraints, he used hints as a way to reduce anxiety.

If I have to give a hint or suggestion of where we might start or ask some questions, "Where do you think makes a reasonable place to even start this problem?" But, give minimal you know guidance at first, but, and let them kind of cooperatively go from there and finish the problem. So, I think that helps reduce the anxiety because it's like somebody, you know, who doesn't tell you how to do everything but, you're learning how to solve a problem. (4 B)

By learning how to solve the problem, the students were gaining competence and confidence, a recurring theme throughout Craig's comments on the roles the courses were to play in the students' program of teacher preparation. He attempted to build confidence within the constraints he found himself placed.

But, without the content knowledge base, the program did not hold together, in Craig's eyes. Having a positive attitude without the content knowledge support as a preparation to teach was difficult for Craig to find sensible. When asked to react to the statement concerning a positive attitude overcoming much weakness in content knowledge, Craig struggled.

I think it's important to have a positive attitude, very important, but, can over-, that attitude can overcome, how is it going to overcome much weakness? (pause) I'm not sure if this means through having a positive attitude, you, it will help you (short pause) understand more? Or if your glowing, glowing personality or whatever will make up for your weakness, because you'll motivate the kids anyway, even if you don't know the content (laugh). (14 P)

Overcoming weakness in content knowledge with a positive attitude was, literally, laughable to Craig. Content knowledge was the base.

For Craig, development of this content knowledge was the focal role of the content courses with the methods course working in tandem for teacher preparation.

The way I see a content course is more helping the college students understand better, feel better about the content. And a lot techniques we use, the manipulatives and so forth, might be somewhat similar to what they would do with kids, but, more in the methods course, we'd focus on, hopefully, "Now you, yourself are pretty competent and how are we going to get this across to kids, now, at various grade levels?" (14 P)

So, while content and methods courses had different emphases, they shared the ultimate goal of preparing elementary school mathematics teachers. Craig was working through what trying to teach the content in ways that supported the development of teachers for changing mathematics teacher expectations meant for him as an instructor.

Summary

Getting students personally grounded in the mathematics of the course was the essential goal Craig stated over and over again. This grounding took a special form for the students as prospective teachers. It included understanding the concepts, being able to consider the concepts in various ways -- using different representations, having a sense of what mathematics is and how the concepts are interconnected, and gaining a positive attitude toward and particular dispositions toward doing mathematics. His view took in three of the pre-described roles (learning content, learning the norms of mathematics, and developing a positive attitude) as well as the role of developing particular dispositions. The focus was on the mathematical content in the courses, but the impetus was the adequate preparation of elementary school teachers through the whole program in which students took part.

After a series of Mathematics Education Committee meetings with his colleagues , Craig took away different expectations for the ways the courses would be changing in content and design. He anticipated being able to go deeper into concepts by reducing the breadth of content to be covered. He expected the deepening to involve greater use of technology, manipulatives, and alternative instructional and assessment designs.

While a teacher, Craig was also a learner -- a student of teaching and learning mathematics. As such, he used opportunities within the courses and the interviews to learn more about his students, his motivations, and his decision-making around the courses. In this way, Craig saw a personal professional role for the courses: the role of developing deeper insights into mathematics and mathematics teaching and learning.

THE ROLE OF LEARNING MATHEMATICAL CONTENT

The first section of my analysis introduced the individual instructors by indicating which of the roles mathematics content courses play in students' teacher education programs were most central to their thinking. In this second part of the analysis, I examine the ways all of the instructors viewed one of the roles, the role of learning mathematical content.²⁷

When the "purpose" cards in the first card sort task were clustered according to the roles with which they most connected and a numerical value was assigned to the ranking of the sort, the instructors as a group scored the role of learning mathematical content higher than all other roles. All but one instructor scored learning mathematics the highest of all the roles. The outlier was Gail Laughlin, for whom developing a positive attitude took a slight advantage.

Rather than attempt to examine all of the aspects that were a part of the instructors' concerns about teaching for the learning of mathematics in the courses, this analysis explores three questions related to learning mathematics in the courses. The questions are

²⁷ I struggled with how to identify the role as laid out on the list of roles (p. 25) -- to provide students with the opportunity to learn the mathematics they need to know as K-8 classroom teachers. People talked about this role differently; sometimes it was "to teach mathematics," other times it was "to provide opportunities for students to learn (construct personal meaning for) mathematics." The form I use is based on the earlier elaboration of roles sub-title -- learning mathematical content (p. 26).

phrased as if directed to the instructors, and I address them through my analysis of the data. They are:

- What mathematics should students know when they enter the course?
- What should be the mathematical content of the courses?
- What kind of learning of mathematics is sought?

Individual instructors and sets of instructors suggested alternative understandings of and responses to these questions. I noted the range and variation of interpretation of the role that instructors ascribed to the courses as embodied in these questions.

The analysis of the first question explores the influence of the students' prior knowledge of mathematics on the mathematics content of the courses. In particular, I focus on some instructors' concerns about the poor mathematical preparation with which many students entered the first course. I examine the extent to which instructors considered the prior knowledge or preparation students brought to the course when selecting mathematical content to be included and determining course design.

Second, I examine the specific mathematical content that instructors thought should be in these content courses. I indicate the curriculum intended by the department or division in which each instructor taught. Next, I describe the differences in content emphasis between instructors and examine influences on the instructors' choice of content, design, and emphases of instruction. I close the section by giving an example of how a similar influence had different effects on the ways instructors saw the role of learning content for the course.

With the third question, I look within the role of learning mathematics to its central goal. While instructors identified many goals, all of the instructors identified "understanding" as a critical goal for learning mathematics, i.e., students should understand the mathematics learned in the course. Instructors had differing interpretations and expectations of understanding that resulted in differing content and design. I needed to distinguish between the ways the individual instructors were using the term understanding in the context of the goal of understanding mathematics in the courses.

In the summary that concludes the analysis, I take a step back and look at the patterns of thought and action on the part of the instructors that seemed to orient their decisions, and, ultimately, the roles the courses could play in students' teacher preparation. I present two characterizations of mathematics and teaching for the learning of mathematics that emerged from my analyses.

What Should Students Know When They Enter the Course?²⁸

Most instructors felt that they should be able to assume that students entered the course with basic computational skills. Other aspects of preparation were occasionally noted, but the greatest concern resided in the lack of mathematical preparation²⁹ that many students exhibited upon entering the first course. The nature and degree of the instructors' concern and their responses to the concern in the context of the first course varied. As foreshadowed by the individual conceptualizations of the instructors, concern about students' preparation often impacted the content and design of the course. In this case, mathematical content that instructors thought students should have brought to the course, but did not, was of concern. My analysis begins with a description of each instructor's portrayal of student preparation for the course, its importance, and the instructor's response. It ends with a summary analysis of the issue.

²⁸ Because successful work in the first course (a grade of "C" or better) was the prerequisite for the second course, I focus on the first content course. This is an arguable choice but was made to enable a closer scrutiny of the issue rather than of the specific topics of content in courses. When explicitly a part of an instructor's response, poor preparation for the second course is also included. It should also be acknowledged that I use the terms lack of preparation and poor preparation interchangeably.

²⁹ Instructors had differing views of what counted as mathematical preparation. These are presented in the body of the analysis.

Kara

For Kara, poor mathematical preparation was an irritant for the students, making them more frustrated. When asked what being unprepared for the course meant, Kara said:

A lack of knowledge in mathematics is what I see as unprepared. They've either not had it or been very math anxious about it, avoided it, unable to do some things, probably in a lot of cases because they've just avoided it. . . . I think you find that once these people relax a little bit that they learn a lot and go, "This isn't as bad as I thought it was gonna be." (KS, 7 P)

Students were not "unprepared" because they could not do mathematics, it was because they had not taken mathematics courses that would have prepared them.³⁰ Kara viewed students as, generally, computationally prepared for the course, so the content of the course addressed the content they lacked. She thought students lacked underlying conceptual knowledge of arithmetic and general concepts of algebra and geometry. Her response in the context of the course was to provide opportunities for students to "learn a lot" in an environment that was intended to be non-threatening. In this way, students were to gain competence and confidence, thus becoming prepared to teach.

The phrase "poor preparation" was never used by Kara. It was a much different situation with Cynthia, who talked about poor preparation at length.

³⁰ Remember that the faculty syllabus that all instructors teaching the course at NU received indicated that the prerequisites for the course were two years of high school algebra and one year of geometry. The catalog did not indicate this. Rather, the only restriction indicated in the catalog was that the course was "open only to prospective elementary teachers."

Cynthia

For Cynthia, poor mathematical preparation was multidimensional. As mentioned earlier, the three dimensions she emphasized were: computational facility, conceptual understanding and its value, and communication of mathematical ideas. In the case of computational facility, Cynthia's perspective on its importance compelled her to act. Recall Cynthia's comment:

I think it's just, after we learn to walk, we should be able to add, subtract, multiply and divide. (laughs) Okay, it's just an inherent thing that we all should have. . . . In the Math for Elementary School Teachers, that's really not one of the specific goals. It has become one of my goals because I have seen students who can't do it, and, certainly, we can't send them into the elementary school classroom if they can't do that. (CY, 13 B)

Based on her experience, Cynthia did not assume that students had basic computational skills.³¹ Her response was to require that students successfully complete or test out of an individual mastery program called the Interactive Math Program, done outside of class. Students were tested and expected to complete certain video-supported modules. Entering computational deficiencies were to be corrected in this way.

Cynthia indicated that she was also distressed by the lack of concern for understanding mathematics on the part of her students:

³¹ The CC catalog did not indicate any prerequisite requirement for the courses. Students could enter and register for the mathematics education courses having had no mathematics beyond a high school General Mathematics course. This was changing at the time of the interviewing. A prerequisite of Beginning Algebra had been put in place in the division, resulting in a significant drop in enrollment for the first mathematics for the elementary school teacher course.

"They're not really wanting to know the whys and the wherefores and learn the concepts and understanding, it's just get-me-through-this-class kind of attitude" (CY, 4 B). She felt that, with stronger preparation, students would seek the answer to "why." She supported a Beginning Algebra-level prerequisite for the course, suggesting that this would increase students' appreciation of the whys. (This view seemed to be a result of her sense that students with algebra backgrounds were more versatile thinkers and were able to come up with different solution methods to problems.) In addition, she intended that classroom activities be opportunities for students to seek "the whys and wherefores" and that students would come to value this as a result of coming to know mathematics in a deeper way. She saw herself modeling the search for whys, and she hoped that students would join her in the search.

In response to students' inability to communicate mathematical ideas, Cynthia assigned significant written work, to which she responded. It required a great deal of time for both students and the teacher. She called this portfolio work. Goals included the development of students' ability to communicate, in general, and the ability to communicate about mathematics, in particular.

This understanding of students' poor mathematical preparation included the norms of the discipline -- how one communicates and supports conjectures and solutions -- as content. Cynthia emphasized correct use of language and notation as a preparation for further and deeper exploration of mathematical ideas and relationships. This strategy exemplified her sense of mathematics

content as a merging of what I have called learning the norms of mathematics with the learning of mathematical content.

The role of learning content that Cynthia expected the course to play included dealing with these three dimensions as entering deficiencies. Deficiencies were to become strengths, valued strengths. The course was to facilitate this shift from deficiencies to strengths for students as learners, first, and for students as future teachers, second.

Like Cynthia, Jeff saw poor preparation as of greater concern than Kara. Unlike Cynthia, who focused her attention on the students as learners, Jeff's concern was more focused on the problems for the future pupils of the prospective teachers in his charge.

Jeff

Jeff did not anticipate the poor mathematical preparation of his students, and, due to adherence to his pre-set, rigid grading scheme, he gave passing grades to students whom he felt did not know fundamentals, such as the meaning of fractions. Jeff felt the weight of this experience, stressing that he did not want it to happen again. Recall that he stated, "We don't want to certify teachers that aren't ready" (JS, 26 P).

Jeff's response was to plan on devising his own basic competency test to give to students upon beginning the course (even if the department had a required placement test). His intention was not to keep students out of teaching, but to be sure that they were competent to teach: "I'm not saying, you know, 'Too bad. You failed the test. You can never be a teacher.' . . . But, I think some time before they get there [elementary school classroom], they need to

become competent" (JS, 28 P). Jeff felt that either students should be identified and moved into a course that would prepare them mathematically for the mathematics education content course, or instructors should be prepared to give supplemental help to students.

Students were not seen as unprepared by Jeff in the range of ways to which Cynthia responded. He focused on computational and fundamental (e.g., what a fraction is) deficiencies of students. While he recognized students' inability to communicate mathematically, he did not see this as of particular concern except for explanation purposes.

Frank

Frank voiced strong confidence that, with the new prerequisite testing and subsequent placement of students, poor mathematical (computational) preparation would be a thing of the past. He saw curricular revision in the course paired with the prerequisite testing, to capitalize on the positive situation of well-prepared students.

One distinctive concern of Frank's was not mathematical preparation, but social and cultural preparation. He raised concerns about the narrow experience-base that many of the students brought to their teacher preparation program.

Most of them [students] are female. Many of them are from middle class backgrounds, (short pause) and some are probably not very realistic in terms of what they would expect when they go out and teach out there. Well, we get an awful lot of people who decided to become elementary teachers, who have graduated from, and spent most of their lives in, let's say Catholic schools, you know? And they're not (long pause)

prepared yet for what they're going to have to put up with and meet with in the, in the typical elementary school. And I think, so they're, in some cases, they're not ready for it. Hopefully, they, they will eventually have some pre-student teaching where they'll be able to see what a typical elementary class will look like without having a nun figure up in front, (laugh) or somebody. (FA, 1-2 C)

Frank's perspective, molded in part by his work teaching the methods course and overseeing students' practica, was oriented toward preparation to teach. He saw that many students had backgrounds that ill-equipped them for the challenges of teaching in urban public school settings. This aspect of preparation was at the fore-front of his thinking, knowing that many of the students would find few teaching positions available in more familiar settings.

When asked what, if anything, was done in the course to respond to this aspect of poor preparation, Frank indicated that he was not aware of anything being done. He also indicated his belief that it was more an issue for Teacher Education courses to address.

Gail

Like Kara, Jeff, and Frank, Gail assumed that students entered the course with computational skills, and she stated, "We don't cover that." Assessing the reality in the classroom, however, she acknowledged that she sometimes had students who were weak in some areas: "Even now, we're assuming they know how to work with fractions even, and a lot of them don't. So we end up helping them on the side, but it's not a part of the course" (GL, 11 P). She also planned to increase the emphasis on application problems involving fractions to provide an opportunity for students to work through any residual deficiencies in understanding fractions.

Gail felt that the level of mathematical preparation had improved in the recent past, supported by the prerequisite testing. She saw this improvement as allowing curricular changes, such as the deletion of computational work with fractions, to be viewed as reasonable. Prior to the prerequisite testing, when underprepared students took the course, they often failed it. They then had the choice of opting for a different career goal or of retaking the course. The number of students retaking the course was near one third, according to Gail. She expected this fraction to drop dramatically with the use of prerequisite testing.

This situation of high repeat levels corroborates two aspects of Gail's comments. First, if high numbers of students failed the course, it is likely that some were not adequately prepared to take the course. And, second, because students were not able to successfully complete the work on the mathematical content that was a part of the course, Gail's description of the course as not dealing with aspects of poor student preparation seems to be reasonable.

Assuming computational facility among her students, Gail's concern was more about the diversity in students' levels of preparation.

You got people that had pre-algebra or beginning algebra in ninth grade got a "D" in it, never took anything since. And then you got people that have been through calculus, are math majors, and decided to go into elementary teaching. And they're all in one classroom. (GL, 2 C)

With the wide range of backgrounds, Gail felt it was unfair to expect more sophisticated levels of mathematical thinking from all of the

students. Holding true to her concern for not intimidating or threatening students, she shied away from considering subjects, such as geometry, in depth. In this way, the mathematical content that students were expected to learn was restricted, based on Gail's perception of what all students could reasonably be expected to learn, given their collective mathematical backgrounds.

Craig

Craig saw a value in prerequisite testing, but he sensed that this did not tell the whole story of students' mathematical preparation. He suggested doing interviews with students as a way to determine if they were prepared for the courses. In this way, a sense of the ways in which students understood mathematics could help the instructor in responding to student needs.³²

In the current set-up, Craig dealt with entering computational deficiencies within the framework of the course. He responded to the needs of the students through the course work, but he sensed that the course would include different mathematical content if students entered well-prepared. Spending time reading and studying the NCTM Standards as a part of the course was one change he envisioned.

If they come into the course [now] a little, if you get, especially, a certain group that's a little more deficient than what you had hoped in certain areas, then you might have to emphasize that more. And it could affect some of the other goals, slightly. I hope not much. But, theoretically, if they really came in with good preparation from high school and, in terms of mathematics and attitude and so forth, then we could

³² Recall that Craig saw understanding as being a spectrum, ranging from recognition to conceptual understanding.

go right on. You know, look at the Standards and so forth. (CK, 3 C)

Poor preparation's impact on the content of the course involved emphasizing those topics in which students seemed to lack background. "Emphasizing" took the form of selecting instructional and assessment tasks that developed and used these topics. Craig spoke of assessing students' prior knowledge and building from that with the course content.

Student Preparation and its Effects

In the instructors' responses, we have six very different perceptions of what students' preparation is, how important an influence it is on the course content, and what the response needs to be in the context of the course. The range of emphases on poor mathematical preparation as an influence on the course content and design went from non-concern to a full-blown set of concerns that influenced content and design, particularly with respect to assessment, in dramatic ways.

Most instructors assumed that students had basic computational skills. Only Cynthia designed course requirements with an eye toward developing basic computational facility. Most instructors recognized that some remediation would need to take place within the course. Instructors relied on problem solving and application problems to accomplish multiple tasks, one of which was remediating computational skills. The context of the remediation was different across instructors. For one, it was something that she dealt with one-on-one with students. One instructor assumed that students had computational facility and, in

the context of the course, realized that he had problems as a result of his assumption. One instructor stated that there was no problem: Students were tested and placed in appropriate courses. As a stance held by someone not actively teaching the course at the time, perhaps this represents the response of the idealist.

Cynthia was the instructor with the broadest definition or sense of what students' poor preparation might involve. She felt that, if K-12 schools prepared students, students would be more able to communicate and more curious about the whys and the wherefores. She felt this represented preparation -- students should come to the class with these abilities and dispositions.

The diversity in the ways instructors perceived and responded to student preparation in the context of the course highlights the sometimes wide difference of opinion on the part of the instructors as to its influences on the course content and design. When instructors saw the need to assess and address poor mathematical preparation actively within the course, they adapted the content with the intention of dealing with the poor preparation while meeting the goals of additional mathematical content learning. When instructors assumed but did not confirm student preparation, instructors did not adapt the content, resulting in a situation where a significant number of students were unable to complete the course successfully in one semester. Overall, the instructors' goals did not mesh into a shared sense of the role of learning preparatory content.

What Should Be the Mathematical Content of the Courses?

The position of the courses in the context of teacher preparation strongly influenced how the individual instructors selected content for the courses. Instructors believed that students in the elementary education program needed to know the mathematics content of the elementary school curriculum,³³ but they also felt that students needed to know more.³⁴ The "more" was wrapped up in the reasons for knowing it. Instructors' beliefs influenced the choice of and emphasis on mathematical content and

³³ A question not addressed in this study but of great interest to me is on what evidence instructors base their view that the content topics on the syllabus reflect the content topics in the K-6 curriculum. As indicated in the instructor information (see Figure 1, p. 46), none of the interviewed instructors had elementary school teaching experience or preparation as an elementary school mathematics teacher prior to teaching these courses. Some instructors did outreach activities at the elementary school level, but these were usually described as times when the instructors would go to schools to do presentations, not as opportunities to examine the curricula being used.

³⁴ Generally, when instructors spoke of "more" in the sense of what content was needed "beyond the K-6 curriculum," they connected the more with gaining a deeper understanding of the K-6 curriculum than students were assumed to have when they entered the course. That is the focus of the next section of the analysis. However, an aspect that I see as troubling is that when describing the content that was beyond the K-6 curriculum that students should learn in the course, few mentioned the notion of functions. And, when talking of patterns, the patterns that students were to see seemed to be connected with particular problems as opposed to broad application. This may have simply been a case of lack of mention on the part of instructors rather than a reflection that developing deeper understanding of functions and patterns was not seen as important.

the ways they thought students should learn it. Departmental or division policy articulated an institutional view of what the content of the course should be. But, I found wide variation in content coverage and emphasis between instructors, raising questions about how the courses operate in the service of the teacher education program.

My analysis is structured with a description of the course content as recommended by the Mathematics Education Committees at each institution followed by an examination of the ways instructors emphasized content differently. The question of what counts as "mathematics" is raised by this examination, and I explore this by looking at the ways instructors talked about mathematics as content in the courses. I also examine the influences on instructors that either directly or indirectly had an impact on choice to teach for the learning of particular mathematical content.³⁵

Recommended Course Content

While instructors had the ultimate decision-making authority as to the specific content included in the two mathematics education content courses, departmental or division policy identified topics for minimal coverage. There were departmental (faculty) syllabi for the courses at NU and a shared textbook; there were no prepared syllabi at CC, but the textbook was shared.³⁶ The

³⁵ The content to be learned varied as well as the motivations for teaching it. As a result, I leave the specification of what and why to the body of the analysis.

³⁶ Recall that, at both institutions, a single textbook was used for the two course content sequence (Musser/Burger, 1991, at NU; Devine/Olson/Olson, 1991, at CC).

decision

were m

instituti

and no

part-tim

commi

mathe

the di

equiv

catalo

cours

elem

prere

user

reco

CC

ma

cou

37

te

an

M

3

m

u

decisions about the topics to be covered and the textbook to be used were made by Mathematics Education Committees at each institution. At NU, the Committee was made up of all of the tenured and non-tenured full-time mathematics education faculty,³⁷ though part-time faculty were encouraged to participate.³⁸ At CC, this committee included both mathematics education and regular mathematics faculty members. Essentially, any faculty member in the division who was interested in the courses participated.

There were no significant differences in the ways the equivalent courses at NU and CC were described in their respective catalogs. Neither catalog indicated a prerequisite for the first course, noting only that the course was intended for students in the elementary education program. Both included the first course as a prerequisite for the second course.

Instructors of each course at NU were given the syllabus and used it in whatever ways they chose, most taking the content recommendations seriously, though not religiously. In the case of CC, Cynthia indicated that the committee of mathematicians and mathematics educators had met the first year she had taught the course and had discussed a syllabus and selected a textbook to be

³⁷ "Mathematics education faculty" was defined loosely as anyone teaching courses designed specifically for prospective elementary and secondary teachers on a regular basis. They were all Mathematics Department faculty members.

³⁸ Participation by regular mathematics course instructors was not mentioned in conversations about the Committee and its work. It is unclear what encouragement they were given.

used in

rein³⁹

T

and Su

as info

were r

-

NCTM

Thes

desig

Ther

alter

oper

not

acti

it a

enc

39

app

rea

of

re

to

40

used in all sections. Prior to this time, instructors had a free rein³⁹ as to what the content and design of the course could be.

The syllabi for the courses at NU were described in the Site and Subjects section of the methodology.⁴⁰ They were referred to as informative by instructors, but recommendations on the syllabi were not always followed.

The course objectives were acknowledged as taken from the NCTM Standards (1989) and included the following:

Students will:

1. learn to value mathematics
2. become confident in their own ability
3. become mathematical problem solvers
4. learn to communicate mathematically
5. learn to reason mathematically

These were to be the instructors' guides in setting tasks and designing instruction and assessment.

Evaluation was to be determined by the individual instructor. There was a recommendation that evaluation include some form of alternative assessment, specifically encouraging more writing and open-ended questions. In the interviewing, several instructors were not sure what alternative assessment was. One instructor was actively using an alternative assessment form but did not recognize it as such. Some of those instructors using more writing and open-ended questions on quizzes and exams did so because they felt it

³⁹ The way Cynthia spoke of this, perhaps "free reign" is a more applicable phrase. Free rein implies freedom to choose within reasonable limits. Free reign implies omnipotent control. The lack of division recommendations or the ability to ignore recommendations without reprimand was the order of the day prior to Committee decisions one year earlier.

⁴⁰ The syllabi appear in Appendix A.

helped t

Comple

opportu

T

small s

ways jo

way th

into a

"you",

learne

tried c

faculty

This

was

and

in th

of to

supp

the

cov

41

vie

cla

42

43

us

as

of

Wa

helped them gauge how well students were learning the content. Completing assessment tasks was not spoken of as providing opportunities to learn the content by most instructors.⁴¹

The paragraph on each syllabus that recommended the use of small spiral notebooks as journals for class gave guidance on the ways journals might be used. The paragraph was written in such a way that it could be lifted from the faculty syllabus and put directly into a student syllabus for each course -- all the pronouns were "you", e.g., "Your journal entry should include a summary of what you learned in class." The journals were to be places where students tried out ideas and described their thinking process. None of the faculty required the use of journals in their sections, however.⁴² This observation matches the general perception that communication was to be used for assessment rather than instructional purposes.⁴³

The major difference between the course syllabi for Math 115 and Math 116, the two mathematics education content courses, was in the course outlines. The first course outline was a simple listing of topics matched with the sections from the text and supplementary materials to be used and the number of days to cover the topic. The second course outline not only indicated topics to cover, but also indicated aspects of the instruction to consider. For

⁴¹ Cynthia (not at NU, therefore not working with this syllabus) viewed assessment tasks as opportunities for learning and clarifying (student) understanding of the content.

⁴² None mentioned recommending their use in these courses.

⁴³ Again, Cynthia, not under the guidance of this syllabus, saw the use of writing as important. She talked of her use of the portfolio assignments as opportunities for students to think about the process of doing mathematics. She expected students to develop alternative ways of approaching the tasks and to write about them.

example:

theoretic

actively

*Scatter

algebra

concep

second

instruc

were

Instru

solvi

emp

Dif

ins

pr

p

a

c

example: "Compare and contrast experimental probabilities with theoretical probabilities, sample space, events, students should be actively involved in finding experimental probabilities (2 days)" and "Scatter plots, finding the line of best fit (graphically and algebraically), an excellent opportunity to further discuss the concept of function (3 days)." Notice how these portions from the second outline are more than a list of content. Qualities of instruction associated with the content are implied.⁴⁴

When instructors spoke of the course content, they generally were referring to the shared set of topics for courses.⁴⁵

Instructors consistently talked about the course as having a problem solving focus.⁴⁶ Within this focus, the content was differentially emphasized by instructors, however.

Different Emphases

Starting with very similar content descriptions, the instructors molded the courses with different emphases. I presented some aspects of this variation in the discussion of poor preparation. Some instructors saw the goals of the course little affected by what students brought to the course and others actively chose and emphasized content and selected instructional strategies based on their sense of what students lacked. The variation between

⁴⁴ The difference between the outlines is intriguing. My guess is that different instructors were responsible for preparing the outlines.

⁴⁵ These topics are identified in the Site and Subject section of the Methodology. The syllabi are found in Appendix A.

⁴⁶ I realize "problem solving focus" is vague, reflecting the way instructors spoke about this content. Elaboration on what this phrase implies resides in the comments cited from the instructors' interviews.

instruc

textbo

though

Impac

conter

appro

sense

work,

selec

differ

cours

instr

textb

textb

obje

does

prob

mid

to c

47

top

littl

lea

Ma

voi

instructors' emphases on content and the content of the required textbook and supplementary materials is one aspect that weaves through this discussion.

Impact on Textbook Selection

In Cynthia's department, an instructor who had taught the first content course for many years focused the course on logic, did not approve of the textbook used at NU (too "glitzy"), and, Cynthia sensed, did not approve of the way that she taught -- using group work, manipulatives, and portfolios. When a new textbook was selected recently at CC, its selection was shaped by the fundamental differences in the views of what content ought to be a part of the course held by the instructors, particularly Cynthia and this other instructor.

The committee that made the selection had reviewed several textbooks as part of its selection process. Cynthia described the textbook selection as being based on which text was "least objectionable."⁴⁷ The textbook selected was "a clean book. It doesn't, it's not offensive in any way" (14 C). She found it weak in problem solving and use of manipulatives. But, because it is such a middle-of-the-road text, she felt that it gave her a lot of freedom to do what she wanted.

It doesn't interfere. As I said, the book is so blasé that I do my own thing. The book kind of gives me an outline of what I'm

⁴⁷ Having a problem solving focus, including a large number of topics that implied the use of physical materials, and having too little material on logic would have resulted in objections from at least one instructor. Because this instructor had a PhD in Mathematics Education and had taught at CC for several years, his voice was strong.

covering. And I just go to other books, I use things that I've picked up at conferences, I just (short pause), I do my own thing. Okay. Because that's what I feel my responsibility is. (CY, 15 C)

Her "thing" included covering content very similar to that indicated in the NU syllabus for the first course; in Cynthia's course, this was problem solving, sets, number theory, estimation, and number systems through the real numbers. While another instructor used the same text for the course as Cynthia did, he spent a great deal more time on the sections related to logic and de-emphasized other content.

Deleting or De-emphasizing

Not all instructors at NU thought all of the topics on the syllabi were important or even appropriate to cover in the courses. In this case, the instructor might delete or de-emphasize certain content. If an instructor did not include content indicated on the syllabus, there was little chance students would exit the course with any greater understanding of the expected content than when they entered. This lack of coverage can be a concern in courses that are part of a sequence, where instructors of subsequent classes may hope to build on what was gained in previous courses.

Jeff did not follow the departmental syllabus closely, and students were not exposed to work in certain areas. Jeff felt that the departmental syllabus did not determine the requirements for the course. When asked how important it was for the course to satisfy the department's requirements, he responded: "Depends on what those requirements (laughs) are. I mean, I guess I have my own ideas. I'd rather follow my own ideas" (2 P). So, even though he was

a novice with this course, he felt free to follow his own ideas. He also solicited input from others responsible for teaching the course.

Yeah, there was a written syllabus, but I, (short pause) I talked to people as I kind of went through the course, and they would say, "Well, I don't really do this. I replaced this with something else. I find that that doesn't work well. I don't like that section of the book. So, here's what I do." So, there was a syllabus, but, (short pause) I sort of bent it a little bit to, to do things that I thought might be better. (4 C)

Bending the syllabus was seen as an acceptable practice. But, two issues were unclear: how much bending was reasonable when students were to learn mathematics within a sequence of courses and on what basis was content deemed "better."

As indicated above, the departmental syllabus was built around the coverage of particular sections in the textbook. Jeff adapted his coverage.

GK: Did you find the textbook itself, as a textbook, was helpful?

JS: Certain parts. There were other parts that I just thought, "I am not going to talk about this. I refuse." It was just,

GK: For example?

JS: Rounding numbers. You know, to my surprise, later on, I found out that probably I should have. That people somehow just don't know how to round numbers. (short pause) Stuff like how to add two decimal numbers, you know. I just, I am not going to stand (laughs) up there and describe how to add two decimal numbers. I did talk about (pause) you know, the place value of the numbers. That the digit in the third place is really how many hundreds you have, you know. It's not just a one's digit. So, I mean I did talk about place value and I would refer to that at different times in the course, but I, I just couldn't, couldn't bear to talk about how to add two numbers. (short pause) That was in the book. And I, you know, I just wouldn't do it. . . . I mean, there were parts of the book that were on the syllabus, and said, "Well, yeah, we're supposed to do this," and I found myself really fighting it. (pause) And, I

had a really hard time with certain sections, had a really hard time just keeping from boring the students to tears. (laugh) I (short pause) probably about half of it I thought was good, and probably the other half I really didn't enjoy too much. (JS, 4-5 C)

The emphasis in the text and the syllabus on learning "how to add two numbers" did not seem appropriate to Jeff. In my reading of the text, the text material implies not a focus on learning how to add two numbers, but on a variety of ways the operations and properties of whole numbers can be viewed.⁴⁸ This topic could be approached instructionally the way Jeff talked about learning to solve problems and gaining a sense of the structure of mathematics, but he saw the text as focused on learning how to add two numbers.

Some of the sections in the textbook seemed inappropriate to Jeff, so he "blew them off."

There were, as I mentioned on the syllabus, before on the syllabus, there were topics that I was supposed to cover, such as how to add two numbers, and I refused to do it, because I just assumed that these people know how to do this. I'm not going to teach that. So, that's how I responded to that. I mean, I just, either just said, "Well, you've all seen this before, but let me just refresh your memory. " and would do it really quickly. I mean, that's how I handled content that I assumed that they had seen before. I just kind of blew it off. (laugh) As opposed to things that they haven't seen before, you know, I would be very careful about it. (6 C)

Jeff chose to de-emphasize some topics that he assumed students already knew. Other content, he deleted: "I just said that I'm not doing this. I'm not going to teach 'em the long division (laugh)

⁴⁸ This approach includes non-standard algorithms and modeling of properties.

algorithm. So, no, I, I think it was, I mean, it was in the book, but I didn't do it" (12 C).

Two situations may have played key roles in how he perceived the syllabus and what he chose to de-emphasize or delete from the syllabus. One situation was that in Jeff's normal teaching responsibilities, there was no departmental syllabus to be followed, seriously or otherwise. Individual instructors made decisions, based on what they knew students would need to support their studies. And, consequently, the second situation was that Jeff's position as a newcomer to teaching this type of course may have disadvantaged him from his usual position of knowing what students knew and would need to know to support their studies.

There were other influences on what instructors chose as the content and design of mathematics education content courses, in general, and those at NU and CC, in particular. Some of the influences served to encourage particular choices; other influences served to constrain. These influences helped the instructors define the specific content to be covered in the courses as well as the role for the learning of that content for the courses.

General Professional Influences

The influences at the general professional level included national and state professional organizations and activities associated with them. At the national level, professional organizations have published documents that are intended to influence instructional decisions, e.g., NCTM Standards (1989; 1991) and MAA A Call For Change (1991).

The Standards were routinely identified as being either a part of the content or an important influence on instructors' thinking and acting. However, the instructors' limited use of or misuse of the vocabulary of the Standards raises a question as to the magnitude of their influence. The word "discourse", a term of major dimensions in the "Standards for Teaching Mathematics" (NCTM, 1991), was not used by any instructor at any time. One instructor thought that the word "empowerment" meant developing a class full of "people who want to show you how intelligent they are" rather than people who are feel justifiably competent and willing to trust their own ability to make sense of mathematical situations. The phrase "mathematical tasks" was not used with the richness implied in the Standards (1991). "Conjecture" was not always recognized.

Some of this discrepancy between what people described as being familiar with the Standards and their use of vocabulary may reside in the fact that some of the vocabulary is more prominent in the Teaching Standards (1991) than in the Curriculum and Evaluation Standards (1989), the document typically used in assignments. In Jeff's case, he had no direct experience with any recent documents, but he spoke in terms that could be easily translated as connected to these ideas. This was noted in his individual analysis with respect to "conceptual understanding". Another instance involves the phrase mathematical tasks. Jeff talked about how difficult the jobs of teaching this kind of course and making the decisions required as to content were:

I sort of immediately knew [when I got the teaching assignment] that this was, I'm going to have to think about

this. And as I thought, as I'm thinking about, "Well, what am I going to do with these ideas. Here's this chapter in the book. What am I going to do with this?" I mean, I found myself spending a lot more time and thinking a lot more carefully about what would be meaningful exercises, than, (short pause) I mean, I've been teaching calculus ever since I was a graduate student, and so that's, I mean, I kind of know how to teach calculus. (laugh) But I don't know how to teach Math for Elementary Ed Teachers. So, it was new. (26 C)

Knowing how to teach meant knowing how to "carefully select meaningful exercises". This is a reasonable way of talking about knowing how to "pose worthwhile mathematical tasks" when not used to that vocabulary. All this being noted, the Standards can not be said to have had a great influence on the content and design of these courses, yet.

On the state level, Craig mentioned that testing through the State Department of Education was influencing what he saw as important content for the courses. Because the content and design of the testing was changing and attempting to drive a change in the elementary curriculum, he saw a need to adapt the content and design of the courses. Other instructors were aware of state-level testing, but this was more in terms of the testing the students would take before being certified rather than testing in which their pupils would be involved.

Other general professional influences included conferences and professional reading. These influences were referred to by all but Jeff.⁴⁹ Instructors spoke of activities and ideas they brought back

⁴⁹ Jeff may attend conferences and do professional reading, but the conference sponsors and the journals he would have mentioned are probably not ones that he would have mentioned in the context of

and either tried out or shared in their classes. They valued attending conferences as opportunities to talk to other instructors, as well. Journals like The Arithmetic Teacher were seen as resources for reading assignments or inclusion in the course packet for students. Journals also provided a connection to the larger teaching and research community. Their value included serving as support along with other influences for recent changes in the course content and design.

The Standards, the national Standards. Certainly, and what we see on the [state test for children], . . . I think also just in general, where we are in our reading and thinking as professionals. Probably we've had some of these beliefs for years, but (laugh, short pause) now, with the emphasis at the national and state level on some change, it's easier in a sense to [change]. I think the national Standards and other things just came from people like us, anyway, that. So, (short pause) didn't just start three years ago, out of the blue. It's been smoldering for many years. (CK, 4 - 5 C)

These professional influences motivated instructors to varying degrees to define the content and the role of learning the content in different ways.

Local Influences

Local influences ranged from constraints on time and equipment to the influence of a colleague's curricular suggestion. While the pair of influences shared here is not exhaustive of those mentioned by the instructors, these two influences seemed to have the greatest impact on the instructors' ability or willingness to teach for the learning of particular mathematical content. They also

these interviews as having influence on his teaching of the mathematics education content course.

indicate the individualistic nature of the instructors' responses to influences on content coverage and emphasis.

Mathematics Education Committees

As members of committees that occasionally met to re-examine the courses' content and design, the instructors saw change in the course content as to be expected. The Mathematics Education Committee at NU met for a series of discussions just prior to the interviewing, and several instructors mentioned how they saw the discussions influencing the content and design of the courses. There were differing assessments of the extent to which the discussions would affect the courses.

Frank shared his recommendations for content coverage with the Committee at the meetings. Frank held strong beliefs about the need to include additional topics on the first course outline, such as logic and group properties. While some of this content was an informal part of the content, the lack of "necessary rigor" concerned him; he felt that the courses needed "beefing up." To say that the list of topics that he wanted the first course to include was staggering is, I believe, an understatement. His suggested course syllabus had seven topic areas, all to be associated with problem solving. He broke these topic areas down into sub-topics to be covered. For example, topic area one was "logic," and the sub-topics were arguments; validity; Eüler diagrams; compound statements -- disjunctions, conjunctions, implications; truth tables and tautologies; attributes; elementary networks; and story problems. (For a complete listing of his recommended syllabus for the first content course, see Appendix A.) It is reasonable to assume that

Frank saw breadth of content covered of greater value than depth.⁵⁰ His suggested syllabus implied considerable change in emphasis from the current syllabus.

Following the same meetings, Craig anticipated significant shift of work with whole numbers and fraction computation to the methods course, allowing for greater opportunities for developing the rest of the current content in meaningful ways that time did not allow, currently. This instructor did not anticipate the addition of new content, reflecting a value for depth rather than breadth of content coverage. Other instructors at the meetings assessed that there would be no change as a result of the meetings; possibly change would come after instructors selected a new textbook. At the time they chose a new text, instructors would determine which sections of the text to cover and develop new plans for instruction.

These instructors were all at the same set of meetings and heard and participated in the same discussions. They came away with very different impressions about how the discussions would affect course content and instruction and, ultimately, the role of learning content for the course. This perception serves to support one instructors assessment that the set of meetings only served to "muddy the waters."

Instructors sometimes wanted to see change in the mathematics content of the course, but they felt the constraint of working within the local system. As Cynthia's experience with

⁵⁰ The course has thirty-three days indicated for content-coverage on the syllabus and there are over forty sub-topics on Frank's list. It is unclear how he thought moving to this syllabus would develop "necessary rigor."

textbook selection exemplifies,⁵¹ changing the textbook did not equate with changing the mathematics content of the course. But, she had strong beliefs that this change needed to happen.

CY: In my mind, I've only been teaching this for two years, but I am aware of how it had been taught here, and it had been taught as a very, what the instructor thought was wonderful, that's what the instructor [taught], and I think that is the worst attitude to have. "Oh, I really like doing this, so we're going to do this." I think that's criminal. (short pause) Yeah, I think [change to teaching thinking skills] is shaping the textbooks. If it's shaping textbooks, it's certainly going to shape the way things are being taught.

GK: Did you say you are changing textbooks?

CY: No, I said this, I think the cry from the universities and from the workplace are saying we have to have change. And this cry for change is affecting how we, if you look at a book that was printed five years ago compared to a new edition book, there's just a tremendous difference. And when textbooks are going to start changing, you know, I mean, if you're not in, I mean, if you're not current in your field, if you're trying to sleep through all of this, and say, "No, I'm just going to do it the way I did it 20 years ago," the textbooks are gonna, you're gonna either be creating your own materials or (pause). (CY, 12 P)

Here, the change in content emphasis that she was advocating for the course, particularly as other instructors taught it, was an inclusion of more critical thinking and problem solving. She saw these as important aspects of the content. In the way that she spoke of them, she wedded the roles of learning mathematics, learning about doing mathematics, and learning particular dispositions toward doing mathematics.

⁵¹ Recall the new text had to be acceptable and usable by other instructors who wanted to maintain the content emphases of the past.

Differing Influence of Conversations with Colleagues: An Example

Instructors were aware of the ways different faculty members personalized the courses by emphasizing optional content. Some instructors wanted to see optional content as required. Others thought the optional content was not appropriate. The topic that received the most attention from the instructors within the interviews was that of having students learn to work within another base system, at least to the level of counting. Instructors' handling of the topic was very different.

Interactions with individuals as a part of or separate from the Mathematics Education Committee meetings⁵² influenced instructors decisions about what content to include and how to teach the content to varying degrees. One of the most obvious cases was related to the optional topic of the study of bases other than ten in the first course. One instructor, Donna Sterling, opposed the inclusion of the study of other bases, at least in part, because research that she had read discounted the instructional value of having students learn how to use other bases.⁵³ She shared what she had learned from the research with all of the instructors.

Individual instructors responded differently to her comments. For Jeff, teaching the course for the first time, Donna's argument influenced him to move away from having student work with other

⁵² Jeff was not a member of the committee, but he could attend the meetings. He had never done so.

⁵³ Because this instructor was not a part of the study, it is not known the extent to which her argument is based on these research results.

bases, even though his intuitive sense was that it held opportunities for learning mathematical content.

There's a chapter, a whole chapter in the book on other bases and I talked to one of the people here who teach 115 [the first course] regularly, Donna Sterling, and what she said was that she read research that said studying other bases has no carry over into base ten. In other words, you're wasting your time if you study other bases. It doesn't necessarily imply that they're gonna understand base ten any better, which sounds surprising to me. (short pause) So, based on her advice, I didn't study other bases. I mean, you know, other bases is a part of almost every one of those type of books, you know. (short pause) I can remember looking at these bo-, the textbook for this course when I was a graduate student, years ago. I mean, it had, part of it was to study other bases. Yeah, I was surprised about that. And I thought, and I thought, actually, that would have been an interesting thing to do. (short pause) But I didn't. (JS, 9 C)

He trusted that Donna, as a person more connected to mathematics education research, was right in her recommendation. His awareness that several textbook publishers included the study of other bases as mathematical content for prospective elementary school teachers did not hold the strength that his trust in Donna's good judgment and interpretation of the research did.

Kara indicated that she had chosen not to teach other bases the last time she taught the course. She based her decision, largely, on discussions with Donna and other instructors. Kara indicated that she felt that it had been valuable content to include.

Now one of the things that I'm not teaching this semester but I used to do for example with bases, and you know that's hard for a lot of those kids and I say, "I want you to just feel what a first grader would feel like and I'm not doing this just to make you confused or hard you know, try adding in that base. Well, didn't you know that four and five is really, say two in base

seven? And, and then you've got to carry a one," and they're like, you can see them stopping to think and then they're messing it up and putting the wrong, I said, "That's what your first graders are going to feel like. You have to think about" you know, and so like then they are like, "Yeah. I guess it would be hard for them." You know and so I'm trying to put them in the place of the kids and think how the kids are going to be thinking and, you know. I think that at that point, they're gonna, I want them anyway to be thinking how would a kid feel? How can I make the kid feel more comfortable? You know don't just, you can't just show them and expect that they're going to do it, or just it's the way it's done because that's the way you learned it, and maybe you did learn it by doing it over and over and over again. Eventually got drilled into your head. But was that the best way? (laughs) (KS, 3 B)

She mentioned several things she saw as resulting from learning this content, little of which had to do with the fundamental notion of place value. She cited students gaining a sense of how children feel when the students try to make sense of what does not, on the face of it, seem sensible. She wanted students to think about how they could relieve some of the anxiety around the doing of the mathematics. She voiced a philosophy of teaching: "You can't just show them and expect that they're going to do it, or just it's the way it's done because that's the way you learned it." She also shared a philosophy of learning: "Maybe you did learn it by doing it over and over and over again. Eventually got drilled into your head. But was that the best way?" With all of these aspects embedded in the task of learning about other bases, Kara was unsure of her decision to delete this from her version of the course. The argument of her colleague with the support of research findings persuaded her to make this choice, at least this semester. This response was an echo of one of the focal tensions, presented in her conceptualization of

the course, that Kara was dealing with -- the tension between familiarity and change. Her response was a cautious openness to consider an alternative.

Gail participated in many of the same discussions Kara did with Donna. She also had conversations with Frank, who also supported deleting work on other bases from the syllabus. For Gail, this was not sufficient.

I really like doing bases. It's just my thing. I like covering bases. I think it's good for the students to see. When they work on a base, say base five, base six, they make the same errors that their little first and second graders do in base ten. And they can understand why they are not carrying, borrowing, whatever. Why they're having trouble with long division, cuz they don't know their multiplication tables. All the things that they kind of take for granted, cuz they grew up with. So that's my bug. It's real important to me and I think it's neat. There's other teachers in the department, bases was taken out a long time ago. (GL, 5-6 C)

Gail knew that others had chosen to eliminate this content from the syllabus. Their opinion was not reason enough for her to remove it. She, like Kara, saw several purposes in retaining this content. Having students gain a sense of how children feel when the students try to make sense of it, themselves, was also important to her. She added different reasons that she connected to the learning of mathematics more generally: understanding how doing operations relies on place value and understanding the role of multiplication in division. For Gail, her decision to include this content came in spite of the recommendation from colleagues to remove it from the syllabus. Her own experience and gauging of the value of learning the content for students compelled her to act.

Summary

This final example shows how three instructors responded to a similar influence in different ways. Many of the instructors had other influences and constraints that were similar to those of their colleagues. Each instructor responded in his or her own way, making decisions about what mathematical content to include in the courses, what emphasis and kind of emphasis to place on it, and how to teach it.

Instructors each made their own choices. Mathematics Department policy had varying influence, and the Department of Teacher Education was not mentioned as having an effect on decisions. Instructors made decisions regarding content and instruction in isolation from any sort of programmatic discussion. And, instructors based their decisions on limited direct experience in elementary schools.

What Kind of Learning of Mathematics is Sought?

Instructors identified many goals for the courses. Among the two or three consistently seen as of greatest importance was that the mathematics learned in the course should be understood. In the context of the interviews, instructors indicated what understanding mathematics meant to them. The most prevalent phrase associated with their use of the word understanding was that of "finding the whys." What this phrase meant was rarely elaborated upon directly; it was left couched in allusions. While all of the instructors indicated that developing understanding of mathematics was an important, if not essential, part of the courses, they used the word differently. The different uses implied different expectations for the role of learning mathematical content in the courses. Consequently, the nature of the course as determined in instructor isolation is underlined, again.

Goals for understanding the content were tightly connected to the vision of mathematics the instructors held. For some instructors this meant that learning and understanding about the doing of mathematics and developing particular dispositions toward mathematics became woven into the learning of mathematics, as I defined it in the analytic framework. This difference highlights the question of what counts as "mathematics." Sometimes instructors focus on their students' understanding of mathematics as learners, while other times the focus is on understanding as it prepares them

to be future teachers. The ways instructors offered opportunities to develop the desired kind of understanding supported their focus.

I consider different meanings of "understanding" as a way to distinguish between the ways instructors used the word. Then, I examine procedural and conceptual understanding as goals for the learning of mathematics in the courses.

Defining Understanding

In order to clarify how instructors were using the term understanding in a way that helped identify what they intended with its use, I considered what aspects of knowing and doing mathematics they seemed to be focusing on when they spoke of understanding mathematics. I suspected that instructors did not all mean similar things when they spoke of understanding and described situations that exemplified understanding to them.

Defining Procedural and Conceptual Understanding

I needed a way to distinguish the ways of knowing and doing mathematics that instructors portrayed when they talked about understanding. I found this by distinguishing between two layers of understanding. One layer is based on having sets of explanations of algorithms or steps to justify actions. The other layer of understanding includes the first and extends beyond this to connect the procedures, their justifications, and other concepts and justifications in intricate webs of meaning.

The first layer involves procedural reasons. There are plans of action in mathematics that are procedures for explaining "why" but which, if not connected with any other mathematical relationship or

representation,⁵⁴ may lack depth of meaning. For example, explaining why one borrows 60 minutes from an hour when you are subtracting 1 hour 40 minutes from 3 hours may involve a straight-forward set of steps for renaming the 3 hours as 2 hours 60 minutes followed by the subtraction. This procedure does not require connections to renaming done in the subtraction of whole numbers or fractions or the subtraction of other kinds of units like feet and inches. Instructors may divorce procedures from other opportunities to connect them with other mathematical procedures, relationships, or representations. Because this form of understanding relies heavily on procedures, I call it procedural understanding.

A focus on procedural understanding includes developing an increasing number of fixed plans that tell students what to do at choice points. With procedural understanding, there is little awareness of the overall relationship between successive steps and the final goal. The learner is dependent on outside guidance for learning the procedures and the reasons justifying them. There are certain advantages to this type of understanding. It is usually easier for students to gain and requires less time and energy in instructor preparation. The rewards are more immediate and apparent, acting to build self-confidence. Students can often get right answers more quickly and reliably applying this understanding. However, if instructors encourage only procedural understanding, a significant part of the totality of mathematics is missing. This

⁵⁴ Recall that the word "representations" is used throughout the analyses to refer to concrete, pictorial, abstract, applied, or verbal depictions of mathematical concepts or ideas unless a restriction (e.g., concrete representations) is indicated.

partial view of mathematics is of particular concern when the preparation of teachers is an issue.

The second layer of understanding relies on the development of interconnections, student empowerment, and relevance. In the earlier example of subtracting 1 hour 40 minutes from 3 hours, students might be asked to investigate how they would find the difference, with the teacher monitoring how students approached the problem, looking to see what students were able to do and offering guidance when needed. The relationship of subtracting hours and minutes to subtracting whole numbers or fractions might be investigated as a resource for approaching the task. The task might have arisen in the context of planning for an afternoon party, knowing that there were three hours of class time and that there needed to be 1 hour 40 minutes used for academic work. Instructors might encourage students to draw pictures or use clock faces to move the minute hands through the act of finding the difference. Because this form of understanding relies heavily on the development of concepts and seeing their interconnections, I call it conceptual understanding.

A focus on conceptual understanding involves developing networks of interconnected representations and plans of action that are adaptable and generative. There is an awareness of the overall relationship between concepts, procedures, and goals. The learner is not dependent on an outside authority, though guidance in developing the understanding is valuable. There are certain advantages to this type of understanding. Conceptual understanding is more adaptable to new tasks, is easier to remember (though harder to learn), is

effective as a goal in itself, and encourages exploration beyond current knowledge.

As layers of understanding, procedural understanding and conceptual understanding are not disjoint. Conceptual understanding includes procedural understanding and takes full advantage of its benefits. In the context of this analysis, the question of the extent to which instructors extended their expectations for student understanding beyond the procedural to the conceptual became focal. None of the instructors fully articulated a reliance on the development of conceptual understanding, but all of the instructors gave glimpses into what they would like to do. The instructors differed in how many glimpses they gave and how fully developed their thinking about the development of conceptual understanding was.

In the following analysis, I mean to constrain the phrase "procedural understanding" to an emphasis on this type of understanding, only. I intend the phrase "conceptual understanding" to imply an emphasis on understanding including and beyond the procedural. The ways instructors talked about what they wanted students to understand and how they structured instruction for the purpose of developing understanding gave a sense of the type of understanding expected and encouraged. Working from the premise that all instructors saw understanding as "finding the whys,"⁵⁵ the

⁵⁵ There is ample evidence in the card sort tasks and the interviews that all instructors saw this as important, though certainly not the totality of what was meant by understanding. However, the additional aspects of understanding that instructors implied as also part of understanding are dealt with in the context of this discussion.

question becomes "finding whys for what goal" -- procedural or conceptual understanding?

Distinguishing Between Procedural and Conceptual Understanding:
Two Examples

All instructors spoke of developing understanding. Sometimes instructors implied that procedural understanding as defined above was the primary goal. At other times, it was conceptual understanding. I deliberately use words connected to occasions, not instructors, here. The same instructor might emphasize procedural understanding of one concept or procedure at one time and conceptual understanding of the same or a different concept or procedure at a different time. The following example highlights different ways of using the word understanding.

The examination across the semester of number systems and the properties that different number systems satisfied was important to the instructors. All instructors spoke of this as an integral part of the course. Within their descriptions of the emphases on and presentation of this content, instructors implied different pictures of what students were to learn and how they would accomplish student learning of the content. These exemplified how procedural and conceptual understanding were both a part of instructors' concern.

In one sense, instructors used the number system properties as categories to differentiate number systems, and used the language and notation to show whether sets of numbers had the properties. Instructors intended student understanding that was of a procedural nature, often involving categorization. The value in this context was

that students with this kind of understanding of sets and properties could respond knowledgeably and quickly about whether a studied set satisfied particular properties, such as closure or commutativity under a particular operation.⁵⁶ Instructors indicated that at some times this kind of understanding was their goal, and they gave assignments that reinforced it.

In another sense, this content, number systems and their properties, was a unifying theme that instructors used to help students build a rich sense of concepts like identity and inverse while giving meaning to the properties. Students were to develop understanding that included a "big picture" of the domain as a goal.⁵⁷ Instructors who spoke of developing conceptual understanding of number systems and other content did not de-value procedural understanding, only indicated that it was insufficient as the primary goal for the students in their courses.

A second example differentiating the ways instructors talked about understanding involves the use of an instructional practice and its motivation: use of manipulatives. One instructor saw work with manipulatives serving the purpose of giving students experience

⁵⁶ For example, are the whole numbers closed under subtraction? Students do a few subtraction examples and know to check to see if the answers are positive or negative. If negative, the set is not closed. This quick check may not include a sense of the fundamental notion of closure: The answer must be an element of the set under consideration.

⁵⁷ When you have a sense of the way concepts and properties build on each other and interrelate, you can use what you know about one situation to conjecture about patterns or generalize to a different situation. For example, by recognizing a deficiency of one number system and thinking about general properties, you can construct a new number system that does not have the deficiency.

with using materials prior to their work in the methods course, stressing that students should have considerable experience using manipulatives in the content courses but offering little sense of what kind of understanding students were to develop through the use of manipulatives. One physical representation of a concept or procedure was all that he felt was necessary for understanding in these courses. Students would develop other aspects of understanding in the methods course. A different instructor talked about work with concrete materials as motivated by the desire to help students understand mathematical concepts and procedures better by developing representations that students connected to their prior knowledge. These are very different perspectives on the use of manipulatives and the value of including work with physical materials for developing understanding.

It is too simplistic to say that the instructors fell neatly into two categories -- one set emphasizing procedural understanding, predominantly, and the other working beyond procedural toward conceptual understanding as goals for the courses. But, for the purposes of this analysis, differentiating in this way is helpful to push at the implications of these goals.

Procedural Understanding as a Goal for the Learning of Mathematics

When procedural understanding was the goal, the mathematical content to be learned often included a procedure for explaining the content or was exemplified in a representation that students were to connect with the content. The first example of renaming in subtraction involving hours and minutes is an example of this use of

a procedure explaining why. This kind of stand-alone model for understanding with connections and representations specified by the instructor develops a kind of mathematics that is easy to learn, can be used easily, and results in ways of doing tasks that reap quick and reliable answers.

Procedural understanding was more frequently the goal of instructors for whom affective aspects of learning were focal. With success built into procedural understanding, procedural understanding supported student confidence.⁵⁸ The advantages of procedural understanding made it a reasonable goal when working toward developing a positive attitude toward mathematics.

Instructors gave procedures and representations to students; explaining was generally the instructional method. This method is not the only means of teaching procedures and developing representations, but it does represent a common means for teaching for procedural understanding. Those who advocated procedural understanding as their goal indicated that the "telling of rules" that most students had experienced as pupils was not foundation enough for students to base their understanding of the concepts on if they were to be expected to teach. As teachers, they would be expected to explain why things were true to pupils and should not repeat their own experiences of memorizing rules to apply mindlessly.

⁵⁸ Confidence-building is a significant element. Albers (1981) observed, citing Leon Henkin: "The fact is that [prospective] teachers' needs are complex, that teachers' emotions and self-confidence play a great role in how they are going to present material" (p.135).

As noted, these instructors often saw understanding as embodied in representations of concepts or procedures. One instructor was comfortable as long as students had at least one representation. He saw the methods course as a place where students developed multiple representations of concepts. One instructor seemed to connect different representations with different ways of explaining. Lacking from these instructors' sense of understanding was the aspect of connections. Instructors working on procedural understanding made few explicit comments to indicate that they emphasized connecting representations, whether physical, pictorial, symbolic, applied, or verbal in meaningful ways. Rather, representations were stand-alone.

Instructors in this mode often presented the representations to students as representations of a particular concept. The expectation was that the students would directly connect the important aspects of the representation (and ignore those aspects that were distractive) with the appropriate parts of whatever the instructor represented, in the ways intended by the instructor. When students made inappropriate connections or did not see how a representation made sense, the instructor did not often become aware of this until exam time. When a misconception became evident, it was common for the instructor to correct it by explaining the correct understanding.⁵⁹

⁵⁹ Examples of this are Kara's comment that, "If they're misinterpreting, you want to know so that you can correct it" (KS, 10 P), and Cynthia's comment about "reverbalizing:"

Sometimes when they verbalize, and they verbalize incorrectly, I know to say, "Now wait a minute. Let's think about that." And then I'll reverbalize properly. Okay, somebody

One view held by an instructor was that interconnecting ideas was not particularly important in the context of the content courses. She felt that this was more a role for the methods course. As a part of the total teacher preparation program, she saw learning mathematics in the content courses for the purposes of providing background for teaching. Since the learning was for the students as future teachers, the connections could wait for the methods course. Had she viewed learning the content valuable for the students as learners, making connections might have been more important to her. For this instructor, connections were not unimportant, per se, only unimportant in the content courses.

For those instructors for whom the affective components of learning and motivation were highly prized (e.g., feeling good about mathematics, having fun in class), expectations that centered on developing procedural understanding, with its quicker right answers and ease to understand, were reasonable. Students who understood procedurally had reduced anxiety -- they knew what to do and why. But, instructors did not expect them to create alternative arguments or explanations for solutions or conjectures. These expectations were in the domain of conceptual understanding.

Conceptual Understanding as a Goal for the Learning of Mathematics

Some instructors spoke of listening to their students and building on the knowledge that students brought to the course in ways that connected the new knowledge with prior knowledge. From

said something, and I think It was the young lady next to you, said something and I restated it, and clarified it or put it into proper language. (CY, 8-9 C)

this base, they hoped to help students develop other interconnections, empowerment, and a sense of the relevance of mathematics, all important aspects of learning mathematics with conceptual understanding.

Like the other instructors, all of the instructors striving for conceptual understanding emphasized students having multiple representations. However, there was a belief that the representations were a necessary, but not sufficient part of understanding. Additionally, the ability to connect ideas was only important if there were representations to connect. The two actions, representing and connecting, worked together in these instructors' minds.

For those instructors for whom developing understanding involved building rich networks of interconnections, there was some variation as to the "look" of the representations that made up the networks. One instructor passionately described the importance of students' ability to see mathematical ideas in their souls and have rich visualizations that were components of the networks. "Hands-on discovery lessons" often supported connections between ideas and representations. For another, networks were made up of rich concepts that were connected by their multiple ways of representing them, often of an abstract nature. These connections arose from discussions, where ideas were "discovered." A third saw connections being on many different levels, ranging from connecting everyday uses with mathematical representations to connecting concrete, pictorial, abstract, applied, and verbal representations. In his description of understanding as a "spectrum," he highlighted his

sense that there were different ways of understanding. He also indicated that he saw developing conceptual understanding as the primary goal, but not often the attained goal.

One instructor admitted frustration that "discovery" activities she designed and used sometimes seemed contrived when attempting to get students to take a step back from procedures that they had memorized and look for ways of representing them and connecting to other representations.

Sometimes the stuff is more difficult to motivate the discovery, the false discovery maybe, of the things they already know. "We already know this. Why are we doing this?" "Well, you already know the algorithm, but do you understand why? Let's see if using these pattern pieces or these Cuisenaire rods we can understand why we're doing it." And so, like, the nurturing of answering the question why. (CY, 13 C)

This sentiment was fairly common among instructors, with varying levels of frustration around it.

This same instructor made moving between representations explicitly a part of what students were doing, connecting communicating about the concept with the physical representation.

First, they're doing the blocks [modeling equivalent fractions] and working with that. Okay, now, how do we take the blocks and transfer this to paper-pencil? How can I mathematically describe this trading three one-sixths for a half? Okay? And so, rather than assuming that they know it, I try to encourage them to use the manipulatives to understand why they're doing it. . . . And I encourage them to develop that alg-, their own algorithm, and, "Let's compare your algorithm and how you've stated it with what the book has or how I'm going to present it or what this handout has." So, rather than just saying, "Today, now, we're going to have fractions. . . . "We don't do that. We try to discover. We try to build concept and learn the why. (CY, 11 C)

Connecting the different representations of equivalent fraction was a part of this example. Manipulatives served as tools for alternative physical representations that were then connected to symbolic representations.

Instructors holding conceptual understanding as a goal also valued student empowerment. This stance contrasted dramatically with some of the other instructors who had little sense of what the word "empowerment" meant.⁶⁰ Instructors encouraged, even required (on assignments and exams), students to conjecture and show alternative solution methods to problems. Instructors emphasized helping students come to see their own ability to try problems and make decisions about the reasonableness of conjectures and solutions. In this way, instructors interrelated the roles of learning about doing mathematics and developing particular dispositional attributes with the role of learning mathematics.

While other instructors spoke of developing relevance for the mathematics either through connections to everyday use or to teaching, those hoping to develop conceptual understanding spoke of relevance as a much broader construct. Utility was only one part of relevance. Recall Cynthia's extensive set of aspects of relevance ranging from de-mystification (such as what a cubic centimeter looked like -- one centimeter on a side) to using understanding as the basis for further understanding, perhaps of another concept, process, or solution.

⁶⁰ One instructor thought that if he was developing students' empowerment, he was creating a class full of know-it-alls.

An important similarity of the instructors who most elaborately envisioned that students would develop conceptual understanding in the courses was their passion for the discipline. This passion was not evident in the ways that the instructors on the other end of the understanding spectrum spoke about mathematics. Passion for the discipline and a focus on the creativity embodied in the study of mathematics was not a shared aspect across instructors. This is not to say that the one set of instructors did not enjoy doing and teaching mathematics, only that the way they talked about what they expected students to learn and the way they portrayed the discipline was less organic than the set of instructors working toward conceptual understanding.

Instruction for conceptual understanding and instruction for procedural understanding were both reasonable ways for supporting development of mathematical content knowledge by students. The most important issue became what kind of understanding was the primary goal -- procedural or conceptual?

Analysis and Summary

As one instructor noted, there are different ways of understanding. This discussion presented one way of differentiating understanding into an admittedly simplistic pair. Throughout this discussion, I called these notions procedural and conceptual understanding.

The instructors all expressed the view that developing understanding of mathematics was a major goal of the course. Two layers of understanding were portrayed in the ways the instructors talked about instruction of the content for developing the kind of

understanding that each valued. Several instructors viewed their primary goal to be the development of procedural understanding. Their tasks involved making concepts less complicated by explaining the reasons why concepts, procedures, and solutions were reasonable or correct. While instructors valued representations, the instructors did not consistently interconnect representations in meaningful ways to ideas or each other.

Instructors in this mode seemed to replace the telling of the past with a new kind of telling. They were telling students why a procedure worked or how they could represent an idea with little input expected from the students. As future teachers, the students would, perhaps, be able to explain why things were true, but it was unclear that they would be in a position to explain why alternatives were true or untrue. Students were acquiring a "goody-bag of reasoning" much like they might acquire a "goody-bag of activities" to use in their future teaching when procedural understanding was the primary goal for content learning.

Some instructors viewed their primary goal to be development of conceptual understanding. Their tasks involved helping students approach the complexities of the content, feeling justifiably competent in approaching the content. Instructors emphasized aspects of interconnectedness of, empowerment with, and the relevance of concepts, procedures, and representations. Instructors saw this goal affecting the students' abilities to teach mathematics. In such an approach, students would be more able to think flexibly about the content and respond to pupils' questions in ways that referred to their own experiences of conjecturing and justifying,

multiple representations and multiple solution methods. The focus was on learning the content for individual empowerment, with influences on students' abilities to teach as by-products.

If you feel good about yourself and you understand more of what this is all about by tying in the concrete to the pictures to the symbols then I, from what research has shown us too, I mean you're going to remember more. The more you really understand what you're doing, not only are you going to be less afraid to try a new problem or whatever, but you're going to remember it better if you've helped develop it instead of just being lectured to it and all this stuff. (CK, 20 B)

With the content well-understood, attributes of self-confidence and risk-taking were supported along with cognitive abilities to connect and represent mathematical ideas.

Generally, those who worked toward procedural understanding focused considerable energy on affective goals. These instructors viewed the content students were to learn as easy, allowing them to focus their attention less on content and more on affective aspects. And, again, one of the advantages of procedural understanding is its connection with quick, reliable answers supporting the development of self-confidence. Prospective teachers needed the self-confidence to be able to present the content to pupils with surety.

Instructors who worked toward conceptual understanding held goals emphasizing cognition. Instructors saw the learning of the content as nontrivial, requiring significant mental energy. As one instructor commented: "Sometimes it's [math is] hard work. And you have to be willing to accept the fact that sometimes it's hard work. (laugh) Anyway, sometimes hard work is pleasant, and I mean, is rewarding in the end. But while you're doing it, it sometimes is

difficult" (JS, 20 C). Recognizing that learning the mathematics was a big challenge for the students, particularly when mathematics was understood to include the many aspects these instructors saw as a part of mathematics, itself, the instructors oriented their goals for the learning of mathematics toward the students as learners.

Thinking of the content as teachers would come later as a part of the methods course.⁶¹ This variation in cognitive and affective emphasis was another aspect that differentiated those whose primary goal was procedural understanding from those for whom conceptual understanding was the major goal.

⁶¹ When these instructors occasionally talked about explicit tasks within the courses that focused on teaching mathematics, these tasks were often conceptually-driven as opposed to procedure- or activity-driven. For example, one instructor asked students to think about content development throughout the elementary grades.

I kind of sneak these little things in, and I'll say, "Well, think about teaching a first grader, teach, think about teaching a fifth grader. And so you see how, over these five years, you have built, okay, from using dinosaur counters and teddy bear counters, all the way up to using pattern pieces or whatever, Cuisenaire rods, but do you see how you have built these number systems over this period of time?" And they're, "Oh, yeah." And then we do have textbook series and they do do projects out of the textbook series, and one project, for instance, will be discuss the teaching of fractions from grades da-da-da-da-da, and they have to go through the book and is there evidence of any fraction work in first grade, second grade, third grade, you know. And show the spiral, and show the development. (CY, 3-4 C)

SUMMARY OF THE ANALYSES

As I presented these analyses of the individual instructors and the role of learning mathematics content, I explicitly and implicitly suggested two ways instructors saw the programmatic roles of the course. Generally, I observed that instructors did not value the roles differently, but that they saw the roles, themselves, as different -- involving disparate motivations, enactments, and expectations. I found the study of the role that "learning mathematical content" played for the courses to be complex, filled with tensions and dilemmas. Instructors differed in what they assess students bring to the courses and how to respond. Determining the role that learning content is to play in students' programs was difficult due to variation in what instructors saw as the content of concern to the courses. This finding highlights the dearth of interaction between instructors. More significantly, I found a near-total lack of communication between the instructors, mathematics department staff, staff in other departments, and elementary school staff. Finally, I documented differing meanings of "understanding" and differing kinds of learning of mathematics sought by the instructors.

The object of this research was to study all roles and determine what roles the mathematics content courses should play in students' programs. The results of this examination of roles point to important differences in thought and action on the part of the instructors of the courses. These differences affect what individual instructors value overall as the roles the courses should play in

students' teacher preparation. In this summary analysis, I lay out two characterizations of mathematics and teaching for the learning of mathematics that emerged from my analyses of the instructors' individual and collective conceptualizations of the roles the course should play. Each instructor's goal for the understanding of the mathematics oriented his or her perspective.

In a few cases, instructors consistently (though never exclusively) spoke of the goal as either procedural or conceptual understanding; most instructors moved between these goals. For this reason, I do not specify particular instructors as espousing a particular goal. By laying out the two ways instructors talked about the mathematics and their goals in teaching for the learning of mathematics, the overarching goal for the course acts to interconnect the roles, summarize my findings, and point to areas for further study. Laying out these perspectives has particular value, as Brown, Cooney, and Jones (1990) suggest in their analysis of two research paradigms: "We focus on this dichotomy with the realization that the sharpening of distinctions can enable us to better unravel assumptions inherent in each of the perspectives" (p. 642).

I lay these two characterizations out, knowing that I risk portraying these goals as diametrically opposed and disjoint. They are not. The interrelationship of the goals and the complexities of practice show the dichotomy to be false, but I suggest here that these characterizations hold value for helping mathematics teacher educators clarify sometimes competing motivations and goals for

student learning of mathematics in their elementary teacher preparation.

Characterizations of Mathematics and Teaching for the Learning of Mathematics

The two characterizations represent patterns of thought and action that emerged from my study of the instructors' conceptualizations of the roles the mathematics content courses should play in students' preparation. These patterns suggested different goals for the learning of mathematics: procedural understanding and conceptual understanding.

Teaching for Procedural Understanding

Some instructors described mathematics in the courses as not difficult. Their focus was on teaching for procedural understanding.

Because instructors see the content as easy to learn, they see the role of developing positive attitudes holding great importance in the courses. Affective aspects around learning the content gain value, e.g., enjoying mathematics and feeling good about mathematics. Setting a non-threatening environment in the classroom is important. This non-threatening environment includes a pleasant instructor's demeanor, the sequential increase in difficulty of the problems to be worked on, and a passive role for the student (minimizing opportunities for embarrassment or discomfort). Setting this environment requires considerable emotional energy of the instructor. The rewards of learning are immediate and obvious -- students can do sets of problems quickly and accurately, building their self-confidence.

The instructors see the students predominantly as future teachers, rather than seeing the students as individual learners. They assume students already "know" the content at one level, so the goal is to provide the reasons for what they know. Instructors often see the content of the courses as restricted to K-8⁶² curriculum with minimal development beyond those topics. This curriculum determines and bounds the mathematics of study.

Procedural understanding has as its backbone sets of reasons that explain why certain things are true. The source of truth is the instructor or some other source of the reasoning. This source is usually separate from the students; it provides the reasons. Explaining "why" or seeing to it that students know why is the teacher's role. This means that in many situations the instructional mode is lecture/discussion.⁶³ The model of teaching that students observed is that of transmission.

The role that errors play in learning the content is that of letting the instructor know what explanations students do not understand. Instructors can then correct the errors by re-explaining or having another student re-explain.

Instructors teaching for procedural understanding expect students to know whats and whys -- "rules with reasons." Questions of "how" are missing -- how does this make sense, how does this connect with the other knowledge I hold, how might I approach this

⁶² One of the instructors in this study restricted this further to K-6.

⁶³ Different instructors used the words lecture and discussion differently, but generally they indicated that students did not often make contributions of mathematical substance in this mode.

differently if I started with different assumptions. Conceptual understanding includes this aspect.

Teaching for Conceptual Understanding

Sometimes, instructors talked about the mathematics in the courses as complex with a focus on conceptual understanding. Because instructors in this mode see the content to be harder to learn, the role of developing positive attitudes changes character. The goals are cognitive and dispositional, developing a positive attitude toward doing and persisting in the doing of mathematics. Setting a tone of investigation in the classroom is important. This tone includes expectations that students will conjecture, search for patterns, justify, and entertain alternative points of view or ways of operating. Setting this tone requires considerable mental energy of the instructor and the students.⁶⁴ The rewards of learning are slow in coming -- conceptual understanding of the mathematics takes longer to develop, but the mathematics is easier to remember, the mathematics is easier to adapt to new tasks, the mathematics often suggests new tasks and new ways of approaching tasks, and learning the mathematics can be intrinsically rewarding. The student's role is that of an investigator.

Instructors in this mode view students first as learners of the mathematics, though they take the long-term status of the students as future teachers into consideration when selecting tasks. Instructors consider and use students' prior knowledge to build

⁶⁴ This is not to say that considerable emotional energy is not required, as well, only that the cognitive demands are greater than when teaching for procedural understanding.

connections among multifaceted representations. They do not limit the content of the course by a focus on the K-8 curriculum, but this curriculum informs the choices the instructors make. The mathematics of study is organic, allowing and encouraging student contributions to new understandings.

Conceptual understanding has inquiry as its backbone. Students establish truth inside themselves through the process of conjecturing and justifying, alone and with others. The norms of the discipline are considered and used, but they are seen as guides and conventions that serve the learner and can be challenged. The instructional mode varies, depending on the content and task associated with learning the content. The model of teaching that students observe is that of facilitation.

The role that errors play in learning the content is that of providing opportunities for pushing thinking. Questions about the reasonableness of an error and the assumptions that lead to the error are important to investigate. Instructors are watchful to see how students understand the content in order to pose tasks that ask students to apply what they learn in ways that might unearth difficulties.

Instructors teaching for conceptual understanding expect students to know whats, hows, and whys. This learning of the content positions students to make productive use of that knowledge in whatever situation they might need it.

Final Thoughts

I realize that the dichotomy between teaching for the learning of procedural understanding and teaching for the learning of

conceptual understanding that I am suggesting is artificial, but I believe that it is important to lay out. It is artificial because instructors move between these goals for learning mathematics. It is artificial because these two goals are both valid and important, each in their own way -- they do not represent an either/or decision to be made. It is artificial because procedural understanding is embedded within conceptual understanding, so that setting their characterization out as a "dichotomy" moves attention away from their interrelated status.

On the other hand, laying teaching for procedural understanding and teaching for conceptual understanding out as dichotomous goals is important because it suggests specific patterns in instructors' ways of thinking and acting that affect the roles that the mathematics content courses can and do play for students. By examining these patterns in ways of thinking about mathematics and teaching for the learning of mathematics and ways of turning thoughts into action in the classroom, we are able to consider our alternatives.

Laying these goals out as dichotomous is also important because it helps focus attention on the question of expectations. Each goal holds different expectations for learning. And, generally, the level of competence and understanding that is expected is the maximum achieved.

Most instructors in this study were not explicitly aware of holding different goals for the learning of mathematics, though they did indicate some of the tensions they felt. Recall Kara's recognition: "Some of the things I think I want to do a lot, I'm not

doing a lot or that I think are important, I'm not stressing. (KS, 15 B)" This recognition came only after reflecting on her responses to interview questions eliciting her beliefs and then reflecting on her actions in the classroom.

I believe that instructors felt many tensions due to the pull between these two different goals, each holding different requirements and rewards for students and the instructor. For example, in the tension that Kara felt between portraying mathematics as a complex interconnected web or in having students feel that they knew enough to teach on a day-to-day basis, Kara was trying to balance her concern for learning of conceptual understanding, building interconnections and helping students see the bigger picture of the discipline, with her concern for her students' sense that they could use their knowledge of the rules and reasons why of procedural understanding to survive in the classroom. If students do not survive in the classroom, they will never come to thrive. If they survive, an important question to ask is what kind of understanding their pupils will be gaining. If prospective teachers have conceptual understanding, they will be able to make instructional decisions as to their goal for the learning of mathematics for their pupils. If the prospective teachers have procedural understanding only, how will they have a choice as to their goal for their pupils' learning? Are we assuming too much if we assume that, within the teaching, the prospective teachers will gain conceptual understanding?

It is also important to lay these goals out as a dichotomy because in some cases instructors' conceptualizations emphasized

the goal of teaching for procedural understanding as the predominant goal for students in the courses with teaching for conceptual understanding seen as an attractive extra. There were very few forays beyond procedural understanding, and instructors described them as exceptions from the routine of the course work. Instructors and program designers need to recognize the ramifications of opting for procedural understanding as the primary goal in these courses.

Instructors based their conceptualizations of the roles this course work is to play in students' programs on different goals -- one was teaching for procedural understanding, the other was teaching for conceptual understanding. There were trade-offs and tensions in the choosing of goals. Some instructors chose to focus their teaching on the development of procedural understanding with its quick, reliable rewards, rewards that supported instructors' affective goals, as well. The influences and constraints felt by these instructors often opposed teaching for development of conceptual understanding, even when the instructors themselves might have thought conceptual understanding a desirable goal. Other instructors chose to extend their expectations to the development of conceptual understanding, initiating students into the discipline in ways that supported instructors' cognitive and dispositional goals.



pt. 2-



LIBRARY
Michigan State
University

PLACE IN RETURN BOX to remove this checkout from your record.
TO AVOID FINES return on or before date due.

DATE DUE	DATE DUE	DATE DUE
JAN 18 2001		
SEP 18 2001		

MSU Is An Affirmative Action/Equal Opportunity Institution

c:\cbr\dtduea.pm3-p.1

CHAPTER 6

CONCLUSIONS AND IMPLICATIONS

I began this study by identifying questions raised by my experience as an instructor of mathematics courses designed for prospective elementary school teachers. I wondered what assumptions about teaching and learning organized instructors' thought and action and how instructors conceptualized the elementary mathematics education courses. I wondered about the extent to which instructors held shared values and views of what was to be accomplished in the courses and how their values and views meshed with those articulated in recent reform documents (NCTM, 1991; MAA, 1991). Most importantly, I wondered how instructors saw the courses fitting into the total teacher preparation program of their students. In this study, I focused on mathematics department instructors' views about the roles their mathematics content courses as service courses play in the teacher preparation program of their students.

The analyses that I presented represent a focused look at the thinking of a set of instructors responsible for teaching the mathematics content courses for prospective elementary teachers in an extended university setting, five instructors from a four-year university, NU, and one from a community college whose students transfer to NU. Their concerns and the dilemmas they described offer a fresh look into the issues surrounding the mathematical preparation of elementary school teachers. Teacher educators in both mathematics and education departments can use the analyses in

this study to orient discussions of the current and future roles of this type of course work.

This chapter includes a brief reprise of my findings, focusing on the instructors' conceptualizations of the roles and the discrepancies within and between instructors' conceptualizations. I highlight specific implications for teacher education that arise from the findings of this study and indicate possible directions my research can take in the future. I conclude with a return to the impetus for this study -- my own experience as an instructor of mathematics content courses for prospective elementary school teachers -- and suggest how this study has affected my own thinking.

Instructors' Views of the Roles

In preparing the analytic framework, I articulated my own preconceptions. I was making an educated guess based on the assumption that instructors would be operating under circumstances similar to those under which I had worked when teaching this kind of course. Overall, instructors confirmed my conjectures.

Instructors' Views and the Analytic Framework

When I described the role of developing positive attitudes, I tried to predict what the objects of the positive attitudes were. I anticipated and found within the set of instructors different objects, such as doing mathematics, being in a mathematics classroom, and teaching mathematics. Gail viewed developing positive attitudes as a precursor to learning mathematical content and oriented the course toward the goal of reducing mathematics

anxiety. All of the other instructors viewed the learning of content as a vehicle to reducing anxiety.

The majority of instructors held what I described in the analytic framework as a "proactive stance" on gate-keeping. Instructors felt that either the courses should include supports for students who lacked important preparation or that the department should identify and provide appropriate courses for students needing additional background before entering the mathematics education content courses. There was only a minimal sense that the course was to be used as a filter to keep students out of teaching.

The role of learning how to teach had a wide range of incarnations. This role's manifestations included brief discussions of how to teach a particular concept differently in the elementary classroom from the students' experience in their university course to course assignments requiring students to analyze elementary school textbook coverage of a concept across grade levels. The role of learning to teach mathematics was often implicit, to be accomplished by student observation of the instructor. Instructors tried to model good mathematics instruction. Many instructors were attempting to adapt their teaching methods to reflect the emphases in the NCTM Standards as they interpreted them.

Instructors said that learning mathematical content and learning the norms of mathematics were important. I had a hard time separating these into categories when I set out the framework, and I found some instructors put them back together. Yet, it was important for me to separate them in order to distinguish between the emphases that instructors felt needed to be given to each aspect.

Instructors' strong emphasis on the learning of mathematical content did not necessarily include a strong emphasis on learning the norms of mathematics, justifying my separation of these roles.

Through the analysis, I discovered one deficiency in my analytic framework. I had not separated the role of developing particular dispositions from the role of learning the norms of mathematics. The role "learning the norms of mathematics" was hard for me to describe in part because I left as implicit the dispositional aspects that I assumed to be part of the doing of mathematics. However, this study helped me clarify that the dispositions relate to the doer of the mathematics, not the doing. As such, the development of dispositions needs to be seen as a separate role. This role holds inclinations to think and act in particular ways.

For too long, we have not recognized that dispositions have been developed as a part of the study of mathematics. It is only now, as we consider developing different dispositions than have been developed in the past, that this role seems important to separate and study. If we do not recognize and value the development of different dispositions, it is unlikely that anything different will be in evidence in student learning.

Instructors' Views of Learning Mathematical Content

The instructors I studied made choices based on their beliefs and goals as to what was important for prospective elementary school teachers to know and care about. By making their beliefs and goals about the role of learning mathematics in the courses explicit and setting them side-by-side, I found patterns in the ways

instructors talked about and acted on the role. Two alternative characterizations of mathematics and the learning of mathematics emerged that seemed to orient the instructors' choices in content and design of the courses.

Instructors made choices based on their beliefs and assumptions about what kind of understanding their students needed to have as prospective elementary school teachers. Many chose to support developing procedural understanding in practice, even though they occasionally or regularly voiced beliefs that indicated that they valued conceptual understanding. Valuing conceptual understanding did not translate into seeing the development of conceptual understanding as a primary goal within the role of learning mathematical content.

The focal question became "What was to be developed -- procedural or conceptual understanding?" The instructors gave differing, often inconsistent answers, reflecting the complex set of understandings and influences on their beliefs and actions. The differences I found gave support to Ball's (1992) reading of the situation in mathematics education -- we hold differing views on such issues as what topics to cover, whether or how to use manipulatives, what makes a good problem, and what mathematics is and what it means to know, do, and use mathematics.

In their study of a prospective teacher, Borko et al (1992) asserted that the mathematics content courses that the student took as part of her teacher preparation program did not challenge her knowledge of fundamental concepts nor did it stress meaningful learning of mathematics. It appeared that "the university teacher

education program gave up an opportunity to foster her acquisition of the subject matter knowledge necessary for teaching middle school mathematics, and it reinforced her belief that her knowledge of mathematics was sufficient for teaching mathematics in elementary and middle schools" (p. 217-218). My study indicates that this student's experience was not an isolated instance. The beliefs and expectations of mathematics teacher educators justified their lack of demand for conceptual understanding or challenge of students' notions of the sufficiency of their understanding. In other words, instructors may be perpetuating poor subject matter knowledge and students' belief in the adequacy of their knowledge of mathematics by design and intent, though the instructors' intentions may be admirable. For example, the instructor may intend to improve students' attitudes toward mathematics by expecting procedural understanding, resulting in "more successful" work with the mathematics but limiting the students in terms of deeper insight into the concepts being used.

Eisenhart et al (1993) point to the need for mathematics teacher educators to support students in the examination of factors that create pressures for teachers to teach for procedural understanding as opposed to teaching for conceptual understanding. Implicit in this is the assumption that mathematics teacher educators have considered and reflected on the issues and dilemmas faced when teaching mathematics and that they have the goal of teaching for conceptual understanding. My study shows this assumption to be naive. Mathematics teacher educators must come to see dilemmas they are dealing with for what they are and respond

in ways that are more conducive to the development of instructional practices that will support development of students' conceptual understanding of mathematics.

Difficulties and tensions are a natural part of teaching these courses. It is essential to balance goals so that students have both a broad base of procedural understanding and a strong framework of conceptual understanding to work within. My study points to the fundamental tension between teaching for depth or breadth of content and the distinctive nature of these mathematics courses designed with a particular set of students in mind. Balancing the goals of teaching for procedural understanding and teaching for conceptual understanding takes on greater importance in this context. The challenge today is to bring a balance that more effectively prepares future teachers with mathematical knowledge, skills, and dispositions that will serve them in their future classrooms, knowing that the expectations placed on them as teachers may be different from yesterday and today.

Conflicting Views

The instructors expressed a wide range of interpretation of the roles and a wide variation in emphasis on the roles. The resulting image was one that highlights the multitude of dilemmas to which the individual instructors respond and the isolated nature of the instructors' work.

Intra-instructor Conflicts and Dilemmas

Instructors spoke of working toward many different goals, some of which were in conflict with each other. For example, an instructor desiring rigorous standards of mathematical preparation

for teaching (gate-keeping) also felt the need to consider the fragile mathematical status and mathematics anxiety of the students (attitude). This dilemma involved trying to establish high and rigorous standards, while acknowledging an audience that had a large number of mathematically damaged human beings, putting the needs of math-damaged students in tension with the demand for mathematical rigor.

Dilemmas did not arise only from valuing conflicting roles. As articulated in the analytic framework, there were also conflicting ways of viewing the same role. Instructors sometimes expressed and valued differing interpretations of the same role. For example, an instructor stressed the importance of "weeding out" students who came to class not knowing what they needed to know and also stressed the importance of supporting the development of knowledge that students should have had prior to beginning the course. Both are versions of gate-keeping.

I found that the same instructor could value two differing and conflicting interpretations of the same role at the same time or in different situations. He or she could also value two differing and conflicting roles at the same time or in different situations. Such conflicts set the instructor in a difficult position, resulting in decision-making that often reflected the instructor's response to the dilemma in the particular situation rather than decision-making that reflected a response to a programmatic orientation or design.

Inter-instructor Variation and Isolation

The responses of the instructors presented a cacophony of interpretations about the courses and what the courses were to

contribute to students' teacher preparation. Instructors at the same institution held widely varying views and were not compelled to support coherence within the mathematics departments' component, let alone the whole of the teacher preparation program. The program and departmental syllabi impose, theoretically, a consensus view from which all instructors are to operate. Imbedded in the program and syllabi are intentions for student learning, both the what and the how. Yet, the idiosyncratic nature of the course, determined by the individual instructors, led to differing expectations for the students. The variation in interpretation of and emphasis on the roles the courses were to play created the impression that teaching of these courses is an isolated activity.

Tensions

The instructors' visions of these courses as part of students' teacher preparation program included tension between the content and methods courses' content and design. Instructors spoke of "sneaking in" methods instruction and using the teaching of mathematics to elementary students as both a motivator and a backdrop for activities. Instructors felt comfortable saying, "You might do this with second graders," or, "How might you think about this with fifth graders?" or, "How do you see rational numbers being developed through K-6?" Instructors freely talked about these kinds of concerns in the context of the courses. But, when instructors sensed that they might be stepping into territory regarded as the province of the methods course, their reactions differed.

For example, the use of manipulatives raised questions in the instructors' minds. Instructors did not portray the use of

manipulatives and the rationale for their use similarly, and most questions related to whether their use in the content courses interfered with work done in the methods course.⁶⁵ One instructor's reaction was to include what she deemed as appropriate, even though she felt fairly sure that she was moving into the methods course territory. Another instructor's reaction was to include assignments that she saw as moving into methods course work as optional, but she provided strong encouragement for students to complete the assignments. Other instructors used manipulatives in very specific ways as tools for learning the content and did not relate their use to the students' future teaching of mathematics.

The Mathematics Education Committee meetings that instructors participated in hold promise as sites for discussing issues such as a consistent policy for the use of manipulatives in the courses, but only if instructors see the issues as important and if they discuss issues fully enough to move beyond the situation described by one instructor. He indicated that the meetings had "only muddied the waters." Muddying the waters is not all bad if it gets you to move toward clarifying and dealing with tensions in the long run. With the different agendas within this set of instructors, a move to a focused sense of purpose for the courses will doubtless

⁶⁵ When I asked Frank about using manipulatives in other courses in the department, he said that instructors should not use them. Then, he restated this: They would not use them. If someone was inclined to use the materials, Frank would say, "Fine." But, he had to work through the notion that anyone would use them because the idea was so novel to him. I see the thoughtful use of manipulatives as an area through which mathematics educators can reach out to others in the mathematics department.

involve significant discomfort and challenge to long-held notions of what it means to teach and do mathematics.

Isolation

Within any course, an instructor holds particular goals. These goals are often a reflection of the larger programmatic roles the designers and instructors of a particular program expect the courses to play. When talking with the instructors in this study, the individual instructor's goal and the instructor's vision of the roles for the mathematics courses to play were often one and the same. But, these goals and visions were not often informed by or connected to the vision of the roles of the courses articulated by program designers or anyone outside the immediate circle of mathematics teacher educators.

Instructors indicated that they had little, if any, interaction with faculty in the Teacher Education Department. Only Jeff indicated that having a sense of the "big picture" of the program was important for instructors in order to maximize the coherence, effectiveness, and efficiency in the elementary teacher preparation program. This situation implies that instructors determined the roles the courses should play with limited awareness of the larger teacher education program and the work done in other courses required as a part of its completion. Perhaps, with all of the concerns they were already dealing, expanding their set of informants to include others outside the mathematics department taxed their limits.

An alternative explanation for instructors' lack of mention of the larger teacher education program may be that they were already

well-aware of it. They gave no indication that this was the case. I had very little sense that the instructors knew anyone in other departments (except socially). At NU, the Teacher Education Department was in the same building as the Mathematics Department, a few floors away. But, interaction was not a norm.

I found instructors working in isolation or in small communities that did not consistently encourage in-depth consideration of the issues and dilemmas surrounding the tasks of teaching and learning of mathematics in the courses. The communities did not extend beyond the bounds of the mathematics education community to include faculty from other departments teaching either general education courses or other content area courses also required as a part of the elementary teacher education program. There was no on-going interaction with elementary school staff. In this way, instructors had limited input at the policy-making level for the course and its instruction, by design or default. Because the existing policy did not serve to encourage or require inter-instructor, inter-departmental, or inter-institutional collaboration, this is not a surprising finding.

Instructors spoke of working within the constraints they felt the environment imposed upon them, requiring the adjustment of their sense of what roles the courses could and should play. The lack of consistent interpretation of the roles and the wide range of the instructors' views present a picture of the course as disconnected from concerns for program coherence and as lacking departmental uniformity. The lack of communication between

individuals in different contexts served to support the individually determined nature of the course.

Implications for Teacher Education

As a member of the community of mathematics teacher educators, I am both distressed and optimistic about the future of mathematics education as a result of carrying out this study. My dismay is not based on a specific instructors' perspective or some dramatic exposition that resulted from completing this study, though the documenting of the prominence of the goal of procedural understanding of mathematics is significant, I believe. On a more general yet practical level, I am concerned about the different meanings instructors had for words and phrases, different from each other and different from the larger mathematics education community. For example, instructors often spoke of providing a non-threatening environment that encourages student learning. For some, this meant setting an affectively-pleasant environment, one in which instructors avoided student discomfort through the lack of expectation for student participation in discussions unless students felt sure that they were correct or through the selection of problems so that students would not be discouraged. Others saw providing a non-threatening environment as setting a cognitively-attractive environment, one in which instructors encouraged students to try out ideas without fear of quick "correction" and by selecting problems that challenged the bounds of students' knowledge in order to support student invention and sense of

investment.⁶⁶ If these instructors met to talk about the teaching of the courses and one instructor suggested as a shared goal "providing a non-threatening environment," all would nod their heads and smile in agreement. But, they would need to discuss in more detail what this meant in order to reveal underlying differences in view. My research highlights the importance of pursuing the question of what we mean when we describe aspects of our teaching and what we see as goals.

Several other issues are raised by this study. Three implications that seem particularly important to pursue are shared here: the need for elementary school connections, the need for on-going conversations in several contexts, and the need for the consistent goal of learning mathematics with conceptual understanding.

Need for Elementary School Connections

I see as noteworthy the fact that none of the instructors had any experience within elementary schools except as a parent or within short inservice projects. None of the instructors had pursued a long-term commitment or had developed relationships with elementary school teachers in which there was on-going dialogue about teaching and learning mathematics in the elementary school. I was not surprised that none had been a full-time elementary school teacher because it is unlikely that individuals who pursue masters

⁶⁶ This description feels much like the way I think of fear as being rooted in ignorance. With knowledge and understanding, whether of mathematical relationships or alternative cultural practices, fear is replaced by security.

degrees in mathematics⁶⁷ would have done their undergraduate work in elementary education.⁶⁸ But, I was surprised that no one was "connected" to elementary schools and the teaching of mathematics in the field, except as it was a part of student field work supervision.⁶⁹ I am now wondering to what extent this is the norm in mathematics teacher education.

Working within a department of teacher education, I see many colleagues who are working with elementary school teachers on an on-going basis and teaching mathematics in elementary school classrooms (for more about this kind of work, see the writings of Magdalene Lampert and Deborah Ball⁷⁰). What experiences in the field inform the practice of mathematics teacher educators who participate in the preparation of elementary school teachers? This question seems important to consider, as is the question that naturally follows: What difference do experiences in elementary mathematics classroom settings make in the beliefs and actions of mathematics teacher educators?

⁶⁷ To become a faculty member in a mathematics department, a masters degree in mathematics is usually the minimum acceptable credential.

⁶⁸ I do not mean to suggest that an undergraduate degree in elementary education is inherently poor preparation for a masters degree in mathematics, only that it is unlikely that there are many individuals who are interested in elementary education who would focus graduate work on mathematics separate from education.

⁶⁹ One instructor did, however, serve on a local school board.

⁷⁰ Drs. Lampert and Ball do not, however, teach mathematics content courses. Currently, instructors in departments of teacher education rarely are responsible for teaching content courses in elementary school teacher preparation programs. There is some indication that departments are beginning to work together on staffing this type of course.

At the very least, relationships in the elementary school environment inform mathematics teacher educators as to the curricula that are in place and actively used in classrooms. Few of the instructors interviewed indicated any question that the content of the syllabus might be deficient or misguided as a representation of the elementary school curriculum. The fact that content was in the university-level textbook appeared to be the basis for their confidence in the syllabus.

Need for On-going Conversations in Many Contexts

The environment of mathematics departments has not traditionally been supportive of inter-instructor communication about the teaching of courses. The Mathematics Education Committee meetings at both schools were significant in that they occurred at all. However, the conversations in the meetings and many of the other conversations instructors described were often instrumental⁷¹ in nature. Instructors discussed ideas for activities, problems, and assignments. They were not often at the in-depth, relational level: "How is this all going to fit together?" or, "How is our piece going to add something to the total teacher

⁷¹ My use of "instrumental" here extends Skemp's (1987) use of the words instrumental and relational. Instrumental implies the "what" and relational implies "what and why." Instrumental conversations around the courses would involve exchanging ideas for activities or describing what went on in class. Relational conversations would go beyond the exchange of the activity to include discussion of why the activity might be valuable to do or what issues might be raised for students, and go beyond description of what went on in class to a sharing of ideas about what students' responses and actions were indicating and how the lesson might be revised to offer greater opportunities for learning.

preparation program?" Additionally, there was no departmental encouragement (e.g., release time) for instructors to spend time and energy with colleagues from other departments or with elementary school teachers and students.

As a teacher educator who focuses attention on the mathematics portion of prospective teachers' preparation, this raises another question: How can conversations involving program designers and instructors in the same and different departments be encouraged and supported as a means to inform each other of concerns, goals, and dilemmas? This kind of collaboration will not be easy: "It is one thing to ask individuals what role they think a course plays. It is another thing to take the notion seriously enough so that we talk to colleagues who live in a university, which has as its norm academic freedom, about the fact that they are teaching a service course, serving a program" (S. Feiman-Nemser, personal communication, June 14, 1994). Instructors must come to see more clearly that what they are doing is not just serving the needs of students, but also provides a particular service for a curricular program. There is no room for an isolationist attitude: "I'm going to put my blinders on and do with this course what I want to do with this course." In order to clarify what roles each course in a program should focus on, there needs to be interaction among all parties and mutual respect and contribution of ideas. The findings of this study could serve as the catalyst for conversations within mathematics departments and among many departments. We must find ways to talk about service courses within teacher education programs that help define the roles more clearly.

The different ways of using vocabulary that are a part of discrepancies in interpretation between mathematics teacher educators underline the difficulty that accompanies serious dialogue. By recognizing up-front that language must be developed and used in similar ways in order to communicate ideas, members of differing educational communities can come together to work as a larger community to develop teacher education programs that are actively served by the service courses in other departments.

Need for the Goal of Conceptual Understanding

If the primary goal for mathematics learning in these courses does not change from procedural to conceptual understanding, the implication for students (and their future pupils) is that procedural understanding of mathematics will continue to be their primary tool for use unless they independently develop conceptual understanding. I do not believe that teaching for procedural understanding is wrong-headed. There are times when this is a valid and reasonable goal. But, it is not an appropriate primary goal in these mathematics courses. The expectation or goal of procedural understanding of mathematics for these courses represents a glass ceiling. While students may sense that there is more to understand, they may not have the tools available to do so.

If others believe as I do that the goal must change so that prospective elementary school teachers gain conceptual understanding, the implication for mathematics teacher education is that the content and design of the mathematics content courses for prospective elementary school teachers must focus on supporting the goal of developing conceptual understanding. Supporting this

goal involves a significant reorientation of beliefs and actions. If we take the instructors' responses as evidence of beliefs, there is good reason to believe that the reorientation of beliefs is underway. But, beliefs do not quickly or easily turn into actions when opposing influences and constraints are seen as difficult or impossible to overcome.

In their research, Borko et al (1992) suggested that: "For prospective teachers to take advantage of the opportunities we [mathematics teacher educators] provide, we must find ways to challenge fundamental beliefs about learning, teaching, and learning to teach" (p. 220). My research suggests that our own beliefs and expectations, as mathematics teacher educators, need to be examined. We must ask what "opportunities we provide" students and determine how best to proceed in order to offer the greatest opportunity to learn the content of these courses. As Simon (in press) suggested:

Mathematics teacher educators seem to operate in a similar [to classroom teachers] vacuum, devoid of explicit theory with respect to how teachers learn to teach mathematics. It is not enough to specify necessary courses (i.e., methods, content, practicum) nor content or competencies for teacher education programs. We must be guided by theories as to how mathematics teachers learn and how teacher education can effectively support these learning processes. . . . Key concepts in mathematics teacher education must be identified, unpacked in terms of their component understandings, and used to generate appropriate exploration, concept identification, and application activities. . . . Just as our goal in mathematics teaching is thorough and well-connected understandings, a parallel goal must be pursued in mathematics teacher education if mathematics teachers are going to apply and extend these learnings in their classrooms. (pp. 19-20)

With thoughtful consideration of what prospective mathematics teachers need to know and how this could be accomplished within the total teacher preparation program, mathematics teacher education can improve. Mathematics department instructors must be major players in any redesign.

As I noted in the Literature Review, there continues to be a call from some corners for more course requirements in mathematics for elementary teacher preparation (although none of the instructors in this study suggested this as an option). My earlier question of "more of what" stands as critical for consideration. My study indicates that the "yes" to more content courses must include an examination of the current imbalance in emphasis on procedural and conceptual understanding. We must address this imbalance in order to more fully prepare prospective teachers. In addition, we need to ask ourselves what the implications of changing our goal from teaching for procedural understanding to teaching for conceptual understanding are for our own practice.

There is a belief within the ranks of teacher educators that if we explain content to students in enough different ways and if we explain enough different concepts in enough different ways, students will be prepared to teach. This assumption is one we must examine thoroughly in mathematics education as teacher educators before adding course requirements. What counts as "preparation" to teach? Before we can say that we expect our elementary teachers to teach mathematics in creative ways and to share a discipline that is dynamic, interesting, and worth studying, students need to experience the discipline in this way in all courses in their teacher

education program.⁷² What does this mean for faculty in mathematics departments and in other departments?

Considerable attention is being given K-12 mathematics teaching and learning, and recommendations for improvement of K-12 mathematics education are visible at national, state, and local levels. At the same time, professional organizations have made recommendations about preservice elementary teacher preparation in mathematics. But, mathematics teacher educators, the crafters of many of the recommendations for improving K-12 and preservice mathematics education, must also to take a long, hard look at our own practice. I do not believe that the kinds of changes recommended for elementary mathematics instruction in the NCTM Standards (1989, 1991) proposed for the improvement of children's conceptual understanding, as I see them, will occur without serious examination, re-evaluation, and reorientation of elementary mathematics teacher preparation. It is inconceivable that children will develop conceptual understanding in an environment in which teachers are prepared to teach and expect pupils to develop procedural understanding.

It is unclear in what order things will happen for individual instructors, whether the belief in the primacy of developing conceptual understanding will precede changes in practice or

⁷² Experiencing the discipline as dynamic in their teacher education program is a minimum. Surely, we would prefer that their experience prior to their professional course sequence in mathematics would provide opportunities for students to experience mathematics learning as an interesting and exciting endeavor, as well.

whether changes in practice that result in greater student learning will convince the instructor of the value of focusing their teaching on the development of conceptual understanding. Both can and do happen. But, what can be done to support the reorientation, whatever way it happens for instructors? I see this question as of utmost importance to address if mathematics teacher education and, as an outgrowth, elementary school mathematics education is to improve.

Future Research Directions

Many issues could be illuminated by further study of the data analyzed here. I approached the data with a particular concern and narrowed my focus to close examination of the role of learning mathematical content. There are several other significant issues to study. I see the two characterizations of teaching and mathematics sketched in the Summary of the Analysis as worthy of considerably more thought, and I look forward to spending time working through numerous questions associated with these characterizations. I also want to examine in more detail the ways instructors structured opportunities to develop understanding, looking in particular at how instructors saw communication and the use of small groups and manipulatives as instructional strategies for developing understanding. I find expanding my concern to include students' beliefs and dispositions attractive. That is, I wonder what students think they are to learn in the mathematics content courses and what they think they need to know in order to teach mathematics in the elementary school. Each of these extensions of this study could contribute to the conversations initiated around this study.

There were many influences and constraints cited and/or noted as affecting the choices that instructors made in terms of the roles that the courses were to play. The influences that seem to hold the most promise for supporting instructors in reassessing their goals and moving toward teaching for conceptual understanding are important to determine. What are the influences and constraints on the beliefs and actions of instructors in terms of teaching with the goal of developing conceptual understanding? What mathematical and pedagogical preparation do instructors need; what physical and material needs must be met; how can a move to valuing and acting on that value of conceptual understanding be supported and modeled; why would an instructor who is satisfied, even encouraged, to teach for procedural understanding redefine his or her goal? The questions are not easy,⁷³ but we must work toward teacher empowerment.⁷⁴

An additional direction my research could take moves beyond the immediate concerns of the courses' participants to expand the circle of informants to other instructors in mathematics

⁷³ Much like difficult but interesting mathematics problems, questions of importance are complex. And, as attempting to solve mathematical problems is as much or more important than solving them, answering the questions is not the point. It is in the struggle to make sense and determine reasonable courses of action that progress and growth occur.

⁷⁴ Paralleling the call for student empowerment in doing mathematics, I suggest a call for the empowerment of mathematics teacher educators. Like some students of mathematics, some mathematics teacher educators may not want to be empowered. There is a lack of awareness that a problem exists. There is a hesitance to challenge the familiar. Empowerment calls for different expectations as to what it means to do the activity -- the teacher's role is altered significantly. This is risky business with pay-offs or losses that are incalculable on the front-end.

departments (not responsible for teaching mathematics education courses), instructors in teacher education departments, instructors of content-specific required courses in other departments, and elementary school teachers and administrators. Because the findings of this study indicate a situation of instructor isolation existing, further study including others' points of view could serve to lay the groundwork for the kinds of conversations that are currently missing from the elementary teacher education program.

Summary

In this study, I described the roles a set of mathematics department instructors see mathematics content courses playing in their students' teacher preparation program. I wondered if instructors were aware of, informed by, and supportive of the particular program that had been developed at their own institution. By examining this situation, one that is far less distanced from instructors than the reform initiatives, I hoped to consider what situation preceded attempts at reform. What I have described here could inform further steps toward achieving the kind of teacher preparation for prospective elementary school teachers that mathematics teacher educators have called for in reform documents.

As I return to the teaching of mathematics content courses for prospective elementary school teachers, I am both excited and nervous about teaching these courses. For the past five years, I have been studying teaching and learning, in general, and teaching and learning of mathematics, in particular, largely separate from personal teaching responsibilities. It is difficult to imagine how my

practice will have changed, but the one certainty is that it will continue to change. Listening to this set of instructors talk about their questions, decisions, and struggles has only served to convince me that I must keep my mind open and receptive while examining ideas and issues with colleagues, colleagues much like these instructors, who see their task as complex, important, and rewarding. I must also avoid treating my responsibilities as confined to the teaching of a particular course in a particular department. Isolationism keeps us from contributing to the bigger picture. I must acknowledge the value of making contributions beyond my individual classroom.

I am challenged to revisit my own teaching of these types of courses and consider how this study has affected my own thinking about what is important for students to learn in mathematics content courses that are a part of teacher preparation programs. I was struck by the fact that the instructors at NU had a departmental syllabus that was much more elaborate and pedagogically oriented than any I had seen. Yet, for the most part, instructors used only the information as to what sections of the book were to be covered as their guide for designing the courses. Syllabus suggestions for journal usage and alternative assessments are difficult to respond to if you do not have a sense of purpose or vision of what you are seeking. For example, in order to use alternative assessments, you need to know what represents an alternative assessment. Many of the instructors were not sure what the phrase "alternative assessment" implied. I am challenged to enhance my own and others' understanding and use of these pedagogical suggestions.

I come away from this study with a vivid picture of the way things are. What I described can help frame and inform productive conversations to lay out more concretely and vividly some of the choices, tensions, and dilemmas involved in designing these courses. This picture can help instructors feel less isolated because they will recognize that problems they experience are shared.

Discomfort and thinking about change were a part of the interviewing process for the instructors. They found the interviewing challenging. They came face-to-face with themselves, their beliefs and actions. When their beliefs and actions did not mesh, even if the instructors did not see an easy solution to the tensions, they became aware that they were making choices that they wanted to rethink. Facing these issues needs to be a part of my practice and that of my colleagues.

I am optimistic about the future of mathematics education as a result of my work in this study. The word "guardedly" should rightfully precede optimistic; that is, I am guardedly optimistic about the future of elementary school mathematics teacher preparation. The instructors in my study were willing to be asked and to ask themselves difficult questions. They were willing and anxious to be a part of a study that they saw as holding promise for contributing to conversations around elementary school mathematics teacher preparation. This willingness to explore teaching and learning of mathematics for the elementary school teacher provides inspiration for further study and hope for the improvement of preservice mathematics teacher education.

Appendix A

Syllabi for Mathematics for the Elementary School Teacher Courses at Normal University

Departmental Syllabus
Math 115
1993

Course Title:

Instructor:

Office:

Office Hours:

Required: Mathematics for Elementary Teachers: A
Contemporary Approach. 2nd edition by G. Musser
and W. Burger

Course Description: The first of a two course sequence which examines topics from the elementary school mathematics curriculum from an advanced standpoint. Topics include problem solving, number systems through the real numbers, number theory, algebraic concepts including functions, applications and mathematical modeling. The emphasis is on teaching for conceptual understanding. Required of all students in early elementary and later elementary curriculums. Open only to prospective elementary teachers.

Prerequisites: 2 years of high school algebra and 1 year of geometry

Course Objectives are taken from NCTM's Curriculum and Evaluation Standards for School Mathematics.

Students will:

1. learn to value mathematics
2. become confident in their own ability
3. become mathematical problem solvers
4. learn to communicate mathematically
5. learn to reason mathematically

Evaluation: Determined by the individual instructors.
Evaluation should include some form of alternative assessment including more emphasis on writing and open ended questions.

Departmental Syllabus, Math 115 (continued)

Journal: You should use a small spiral notebook for your journal. You should write a journal entry for each class period. Date each entry. Your journal entry should include a summary of what you learned in class. Be sure to include any questions that arise in your mind. Use your journal as a vehicle for recording how your beliefs and attitudes about mathematics and about yourself evolve throughout the semester.

What follows is an outline of the material to be covered:

Days	Allotted	Chapter	Topics
5 days		Chapter 1	problem solving
3 days		Chapter 2	sets, Venn diagrams, Cartesian product, applied problems, functions omit 2.2
2 days		Chapter 3	properties of number systems omit definitions of whole number operations, thinking strategies for learning the basic facts, ordering whole numbers, exponents
3 days		Chapter 4	estimation omit 4.2 and 4.3
6 days		Chapter 5	number theory
4 days		Chapter 6	fraction concepts, estimation with fractions
5 days		Chapter 7	decimals, ratio, percents
3 days		Chapter 8	integers
2 days		Chapter 9	real numbers, introduction to algebra omit 9.1

Departmental Syllabus
Math 116
1993

Course Title:

Instructor:

Office:

Office Hours:

Required: Mathematics for Elementary Teachers: A
Contemporary Approach. 2nd edition by G. Musser
and W. Burger

Course Description: The second of a two course sequence which examines topics from the elementary school mathematics curriculum from an advanced standpoint. Topics include elementary probability including both experimental and theoretical, elementary statistics including exploratory data analysis, descriptive statistics, and an introduction to inferential statistics, informal geometry including symmetry, similarity and congruence, and measurement. Required of all students in early elementary and later elementary curriculums. Open only to prospective elementary teachers.

Prerequisites: Math 115

Course Objectives are taken from NCTM's Curriculum and Evaluation Standards for School Mathematics.

Students will:

1. learn to value mathematics
2. become confident in their own ability
3. become mathematical problem solvers
4. learn to communicate mathematically
5. learn to reason mathematically

Evaluation: Determined by the individual instructors.
Evaluation should include some form of alternative assessment including more emphasis on writing and open ended questions.

Departmental Syllabus, Math 116 (continued)

Journal: You should use a small spiral notebook for your journal. You should write a journal entry for each class period. Date each entry. Your journal entry should include a summary of what you learned in class. Be sure to include any questions that arise in your mind. Use your journal as a vehicle for recording how your beliefs and attitudes about mathematics and about yourself evolve throughout the semester.

What follows is an outline of the material to be covered:

Days	Allotted	Chapter	Topics
2 days		Chapter 10	10.1 constructing and interpreting stem and leaf, bar, and line graphs, writing a narrative interpreting the graph
4 days		Chapter 10	10.2 mode, mean, median, lower quartile, upper quartile, range, box plots, measuring dispersion, variances, standard deviation, normal distribution, omit z scores
2 days		Chapter 11	11.1 compare and contrast experimental probabilities with theoretical probabilities, sample space, events, students should be actively involved in finding experimental probabilities
3 days		Chapter 11	11.2 Fundamental Counting Property, tree diagrams, probability tree diagrams, Pascal's triangle
5 days		Chapter 11	11.3 simulations using dice, spinners, random digit tables, coins omit conditional probability, odds and expected values
3 days		Supplement	sampling, using samples to make predictions about the population, biased samples, capture-recapture problems

Departmental Syllabus, Math 116 (continued)

3 days	Supplement Scatter plots, finding the line of best fit (graphically and algebraically), an excellent opportunity to further discuss the concept of function. You may use 15.1 as a reference for equations of lines, slope, linear functions, interpreting graphs of linear functions
2 days	Chapter 12 12.1 lines of symmetry, rotational symmetry, planes of symmetry
2 days	Chapter 12 12.2 angle measures optional tessellations omit 12.3
6 days	Chapter 13 omit 13.1 13.2 perimeter, area, Pythagorean theorem 13.3 surface area and volume
4 days	Supplement basic constructions using ruler and compass and MIRA , construct centroid, incenter, and circumcenter of a triangle, explore properties of quadrilaterals through constructions.

Appendix B

Interview Instrument One: Philosophical Interview

Interview One - Philosophical
Keen Research
1993

My goal is to learn about the ways instructors of mathematics content courses for prospective elementary school teachers see the courses they teach - what roles they see the courses playing in their students' programs. My interview questions have been chosen to help me learn about this.

1. On these cards are a number of possible purposes for student course work in mathematics. Please indicate how important these purposes are for mathematics courses by placing them into five stacks ranging from:

1	2	3	4	5
Essential	Very Important	Somewhat Importan	Not Very Important	Not Important At All

Please talk aloud as you place them to help me understand why you have placed them there. If you'd rather, wait until you've placed them and then describe your rationale for the sorting you did.

- A. Understand mathematics in meaningful ways.
- B. Challenge the typical student's perception of his/her understanding of mathematics.
- C. Satisfy the departmental requirement for a mathematics course.
- D. Understand the social and cultural context of teaching.
- E. Learn about and use technology in meaningful ways.
- F. Become an effective problem solver.
- G. Learn about and use research findings in their teaching.
- H. Develop a sense of empowerment with doing math.
- I. Learn the conventions of the discipline.
- J. Learn to plan lessons and units.
- K. Act as a barrier for unprepared students.
- L. Challenge students' notions of what it means to teach mathematics.
- M. Provide students with an experience that builds up a sense of the relevance of mathematics to daily life.
- N. Learn the value of conjecture in mathematics.
- O. Give students an opportunity to have fun with mathematics.

Interview One (cont'd.)

Please indicate how important these purposes are for mathematics courses by placing them into five stacks ranging from:

1	2	3	4	5
Essential	Very Important	Somewhat Importan	Not Very Important	Not Important At All

- P. Understand how they themselves learn mathematics.
- Q. Understand how children learn mathematics.
- R. Ensure that students are competent mathematical thinkers by requiring the attainment of certain standards.
- S. Develop students' ability to communicate mathematics orally and in writing.
- T. Give students ideas for activities that they can do in their future classrooms.
- U. Challenge students' notions of how to teach mathematics.
- V. Understand mathematics in ways that help them teach it.
- W. Develop a sense of how small groups can be used.
- X. Learn to appreciate mathematics as a problem solving domain.
- Y. Develop a sense of the interconnectedness of mathematical ideas.
- Z. Ensure that only those students who have a deep understanding of mathematics are able to continue in the teacher preparation program.
- AA. Give students an opportunity to learn math in an environment that is different from their earlier experiences.
- BB. Reduce the anxiety students might have about mathematics.
- CC. Reduce the anxiety students might have about teaching mathematics.
- DD. Bring the history of mathematics alive.
- EE. Develop conceptual understanding of mathematical concepts and procedures.
- FF. Understand the importance of collegial support.
- GG. Promote adequate facility with computational skills.
- HH. Provide students an experience in a math class that mirrors the kind of classroom experience I want students to value for themselves as teachers.
- II. Maintain the students' perception of the course as difficult.
- JJ. Learn problem-solving heuristics.
- KK. Develop a sense of empowerment and confidence with teaching math.

Interview One (cont'd.)

Please indicate how important these purposes are for mathematics courses by placing them into five stacks ranging from:

1	2	3	4	5
Essential	Very Important	Somewhat Importan	Not Very Important	Not Important At All

- LL. Provide students with an experience that helps them feel good about themselves when dealing with math.
- MM. Provide an opportunity for students to think about how the mathematics they have already learned was taught.
- NN. Provide students with an opportunity to learn mathematics in a way that is very different from their earlier experiences with learning math.
- OO. Provide students with a model for teaching math.
- PP. Provide students with an opportunity to discover mathematical ideas and concepts.
- QQ. Show students "why you do what you do when you do it" in mathematics.
- RR. Challenge the typical student's perception of what it means to understand mathematics.
- SS. Provide students with an opportunity to learn how to ask good questions.
- TT. Become familiar with alternative assessment methods.
- UU. Develop student confidence with doing math.
- VV. Learn the value of proof in mathematics.
- WW. Motivate students to pursue further study in mathematics.
- Other (please specify) _____

Interview One (cont'd.)

I'd like you to read the following statements and react to them.

(Statements in bold. Of importance: Extent to which agree/disagree and why.) The statements are comments made by different people, some read, some as part of conversation.

2. To maximize learning, the learner should be interested in the material and find pleasure in the activity of learning.

Probes

- 1) How does this affect what you select as course material?
- 2) Is interest to students considered?
- 3) How do you know it is interesting?
- 4) To what extent is the course material connected to issues around teaching? How?

3. The goal of a college curriculum for teachers should be to develop deeper, mature insights into the mathematics that they will be teaching to children.

Probes

- 1) Can you give me an example of an mature insight into a math topic that you hope your students will develop in the course?
- 2) To what extent should methods for teaching children be a part of this?

4. Content knowledge alone is not sufficient to guarantee successful teaching.

Probes

- 1) To what extent does this affect what you do in the mathematics content course work for prospective elementary school teachers?
- 2) Is content knowledge sufficient to teach prospective teachers? If so, do you think most people in the math department could teach the course work successfully? If not, what else is required?

5. Good teaching includes teaching students how to learn.

Probes

- 1) What are students learning to learn in the mathematics content course work?
- 2) How is this accomplished?

Interview One (cont'd.)

6. It is important to keep people who are lacking understanding of mathematical concepts out of the elementary school classroom.

Probes

- 1) Do you agree or disagree?
- 2) How do you know when they lack understanding?
- 3) When should those people be identified?
- 4) How should they be kept "out"?

7. A prospective teacher's positive attitude toward mathematics can overcome much weakness in content knowledge.

Probes

- 1) Do you agree or disagree?
- 2) What are the characteristics of an attitude toward mathematics that will support good teaching?

8. Mathematics content course work should be closely tied to the teaching of the content to children.

Probes

- 1) To what extent should methods be focal?
- 2) Are methods explicitly a part of the courses you teach? If so, how? If not, how do you think about the role of methods in a content course?

9. Little has changed in terms of the needs of prospective teachers in mathematics content course work for several years.

Probes

- 1) What needs remain the same? have changed?
- 2) What changes in demands on instructors have resulted from any changes?

10. The University is concerned about having capable graduates and school districts want capable teachers to hire. What is the role of this course work for supporting this situation? In what areas do students gain competence in this course work? Are there other areas that are supported?

11. Is there anything that I have missed? Have the questions seemed reasonable?

Appendix C

Interview Instrument Two: Content Interview

Interview Two - Content
Keen Research
1993

I'd like to ask you to walk me through the course. I am especially interested in what you are trying to teach students. I'll ask questions about things like the content of the class, its goals, and the nature of the students who you have. I will also ask questions about the ways in which you evaluate your students.

1. Describe the math content course for elementary teachers that you teach in a few sentences.
2. Summarize the goals of the course in a few sentences.
3. Can you characterize the student population (ability range, commitment, age, gender, ethnicity)?
4. How are the goals affected by the student population taking the course?
5. Who developed the course?
6. How has the course changed over the years? Why?
7. Have you made any major changes in the content or in way you teach the class in the last few years? Explain the changes and the basis on which the changes were made. (If more appropriate, set in context of the future - will you be making any major changes in the content or way you teach in the next few years and explain.)
8. Is there a common syllabus, text, assignments, tests? Why is the course set up this way?
9. How does the situation with prerequisites (either enforcement, lack of enforcement, or no prereq) affect what the content of the course is? How do you feel about this? Why?
10. How does the situation with prerequisites (either enforcement, lack of enforcement, or no prereq) affect how course is taught? How do you feel about this? Why?

Interview Two (cont'd.)

11. Oftentimes, you may want to students to learn more about something that they have had instruction on before. What are examples of content that you include in this course that are of this type? How do you structure instruction on a concept that you think all students have encountered before? How is this different from what they have done before? Choose a concept and explain.

12. Occasionally, you may want students to learn something that they have never had instruction on before. What are examples of content that you include in this course that might be of this type? How do you structure instruction on a concept that you think few students have encountered before? Choose a concept and explain. How would this instruction be different, if at all, from what you described earlier?

13. Is the mathematics content included in courses different from the mathematics included in other University mathematics courses?

If so, how? Why/why not?

14. Is the way mathematics is taught in courses different from the way mathematics is taught in other University mathematics courses?

If so, how? Why/why not?

15. In the best of all possible worlds, what would a student take away from the experience of participating in this course? That is, are there particular things that you expect them to know about or care about as a result of their participation in this class? Why are these important?

Now, I'd like to ask specific questions about the texts and readings that are associated with the course.

16. What text do you use? Who selected the text? How was the text selected? What are the advantages/disadvantages of this text relative to others? (What do you like most/least about the text?) How is the text used? Why is it used this way?

17. Are there other readings or supplementary materials that are particularly important for the course? What are they? Could you briefly describe them? Why are they important?

Interview Two (cont'd.)

Let's talk about the schedule of topics in the course.

18. Can you list for me the topics that are covered in the course? What trade-offs did you make in deciding on this content and sequence?

19. Can you walk me through a typical meeting of the class?

- a) How do things start?
- b) What do the students do?
- c) What do you do? Anything else?
- d) How does it end?
- e) Why is class organized in this way?

20. Are there any "special" lessons that you have developed for the course?

If so, can you describe them to me? Why are they important?

Now can you tell me about the course requirements and assignments?

21. What assignments are students required to do in the course?

- a) Do they have to write papers? Why? What do you hope they will learn?
- b) Do they have to do presentations? Why? What do you hope they will learn?
- c) Do they have to participate in discussions? Why? What do you hope they will learn?
- d) Are they required to meet with you independently or in groups? Why? What do you hope they will learn?
- e) Are they required to meet as groups outside class? Why? What do you hope they will learn?
- [f) Is attendance mandatory? Why?]

22. Are there different types or categories of assignments that students are required to complete? If so, can you describe each category of assignment for me? What is the value of each?

Interview Two (cont'd.)

23. Could you pick an assignment from among the different ones students are required to do that is a significant assignment and is representative of the kind of thing that you want students to learn or to think about in this class. Can you briefly describe it to me?

Why did you select this one?

What is the student required to do?

Take a minute to recall a good assignment that you received from a student. Can you describe it for me?

What made it good? Why?

How did you respond to it?

Take a minute to recall a poor assignment that you received from a student. Can you describe it for me?

What made it poor? Why?

How did you respond to it?

Why is this assignment part of the course? That is, what do you hope to accomplish with this assignment?

24. Are exams given in the course? If so, may I have a copy? What are the concerns that you have that influence the type of exam you give? Where are examples of this in the exam? What are the trade-offs you see in using exams to assess student ability?

25. By what process do you evaluate students?

What are the criteria you use?

Why do you use these?

How are different criteria/assignments weighted in the final evaluation?

26. Is there anything I have missed? Do these questions seem reasonable?

Appendix D

Interview Instrument Three: Observation Interview

Interview Three - Observation
Keen Research
1993

Ask before the observation: What is something you would want an observer to notice about your lessons? What would you want noticed about the students? What would you want noticed about the course?

After observing in your class, I want to ask a few questions to help me connect what you have told me about what you see the role of the course to be and what the students are involved in in class.

1. How typical were the class periods I observed? What was "typical"? What was not? Were there any surprises?
2. Can you pick an example of something that went the way you'd planned for it to go? What was important for the students to learn? How was it accomplished? Why was it important for them to learn it?
3. You mentioned that a major role the course plays is _____. Could you give me an example of how you see the course playing that role in the class periods I observed?
4. Identify an episode in the class period that seems to be an example of a role the course is to play according to the instructor and ask the instructor:
In class, the following situation occurred - (relate). What were you hoping students would learn? Do you think this was accomplished? Why(or why not) were you able to accomplish what you wanted to in this situation? What would you do differently another time? [If possible, have a few other examples from the classroom observation to ask the instructor about.]
5. *Place each card in the pile best indicating how important you think each of the following is to being good at mathematics.*

Essential	Supportive	Not Important
-----------	------------	---------------

 - a. Remembering formulas.
 - b. Knowing how to ask good questions about mathematical situations.
 - c. Thinking in a logical step-by-step manner.

Interview Three (cont'd.)

Place each card in the pile best indicating how important you think each of the following is to being good at mathematics.

Essential Supportive Not Important

- d. Having a strong understanding of concepts and procedures.
 - e. Having a basic understanding of how truth is established in mathematics.
 - f. Being able to think flexibly.
 - g. Being able to apply the standard algorithms.
 - h. Being able to give thorough mathematical explanations.
 - i. Having a good attitude toward mathematics.
 - j. Being able to model a concept in a variety of ways.
 - k. Openness to attempting problems.
 - l. Being able to solve many different kinds of problems.
 - m. Being able to perform mathematical tasks successfully.
 - n. Having the ability to interconnect concepts within mathematics and between mathematics and other domains.
 - o. Being able to communicate mathematical ideas.
 - p. Knowing how to use technology.
13. Is there anything I've missed or ignored? Did the questions seem reasonable?

Appendix E

**Proposed Syllabus (Frank Adams) for
Mathematics for the Elementary School Teacher First Course
at Normal University**

**Math 115 - Recommended Syllabus
Rough Draft
Spring, 1993**

- | | | | |
|---|--------------------------------|--------------|---------------|
| I. Logic | - Arguments, Validity | | |
| | - Euler Diagrams | | |
| | - Compound Statements | | |
| | - Disjunctions | Implication | |
| | - Conjunctions | | |
| (Problem Solving) | - Truth Tables and Tautologies | | |
| | - Attributes | | |
| | - El. Networks | | |
| | - Story Problem | | |
| II. Sets | - Venn Diagrams | | |
| | - Cartesian product | | |
| (Problem Solving) | - Functions - Relations | | |
| | - Story Problem | | |
| | - Equivalence Relation | | |
| III. Group Properties | - (Abelian) | | |
| | - Whole Nos. | Estimation | |
| | - Fractions | Calculators | |
| (Problem Solving) | - Rational Nos. | | |
| | - Story Problems | | |
| IV. Field Properties | - Whole Nos. | | |
| | - Fractions | | |
| (Problem Solving) | - Rational Nos. | Estimation | |
| | - Story Problems | Calculators | |
| V. Number Theory | - Factors | | |
| | - Primes | Estimation | |
| (Problem Solving) | - Number Patterns | Calculators | |
| VI. Decimals, Ratio, Percents with Story Problems | | | |
| (Problem Solving) | (Applied) | Estimation | |
| (Scientific Notation) | | Calculators | |
| VII. Three types of percents problem using | | | |
| Algebra approach | | | |
| Place Value using exponents | | | |
| (Problem Solving) | (Applied) | (Estimation) | (Calculators) |

LIST OF REFERENCES

LIST OF REFERENCES

- Albers, D. (1981). Teaching teachers, teaching students. In L. Steen & D. Albers (Eds.), Teaching Teachers, Teaching Students. Boston: Birkhäuser.
- Ball, D. (1988). Research on Teaching Mathematics: Making Subject Matter Knowledge Part of the Equation (Research Report 88-2). East Lansing, MI: National Center for Research on Teacher Education.
- Ball, D. (1992). Implementing the NCTM Standards: Hopes and Hurdles (Issue Paper 92-2). East Lansing, MI: National Center for Research on Teacher Learning.
- Ball, D. & McDiarmid, G. (1989). The Subject-Matter Preparation of Teachers (Issue Paper 89-4). East Lansing, MI: National Center for Research on Teacher Education.
- Ball, D. & Wilcox, S. (1989). Inservice Teacher Education in Mathematics: Examining the Interaction of Context and Content (Research Report 89-3). East Lansing, MI: National Center for Research on Teacher Education.
- Borko, H., Eisenhart, M., Brown, C., Underhill, R., Jones, D., & Agard, P. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? Journal of Research in Mathematics Education, 23, 3, 194-222.
- Brown, C. & Cooney, T. (1982). Research on teacher education: A philosophical orientation. Journal of Research and Development in Education, 15, 4, 13-18.
- Brown, S. (1982). On humanistic alternatives in the practice of teacher education. Journal of Research and Development in Education, 15, 4, 1-12.
- Brown, S., Cooney, T., & Jones, D. (1990). Mathematics teacher education. In W. Houston (Ed.), Handbook of Research on Teacher Education. New York: Macmillan.

Burns, M. (1989). Teaching for understanding: A focus on multiplication. In P. Trafton & A. Shulte (Eds.). New Directions for Elementary School Mathematics (1989 Yearbook). Reston, VA: The Council.

Carss, M. (Ed.). (1984). Proceedings of the Fifth International Congress on Mathematics Education. Boston: Birkhauser.

Chapman, L. R. (Ed.). (1972). The Process of Learning Mathematics. Oxford: Pergamon Press.

Clark, C. (1988). Asking the right questions about teacher preparation: Contributions of research on teacher thinking. Educational Researcher, 17(2), 5-12.

Cobb, P. (1986). Contexts, goals, and learning. For the Learning of Mathematics, 6, 2, 2-9.

Cobb, P. (1988). The tensions between theories of learning and instruction in mathematics education. Educational Psychology, 23(2), 87-103.

Commission on the Education of Teachers of Mathematics of the National Council of Teachers of Mathematics (1981). Guidelines for the Preparation of Teachers of Mathematics. USA: The Council.

Devine, Olson, & Olson (1991). Elementary Mathematics for Teachers (2nd ed.). Somerset, NJ: John Wiley and Sons.

Dienes, Z. P. (1972). Some reflections on learning mathematics. In W. Lamon (Ed.), Learning and the Nature of Mathematics. Chicago: Science Research Associates.

Dunkin, M. (1986). Research on teaching in higher education. In M. C. Wittrock (Ed.), Handbook of Research on Teaching. New York: Macmillan.

Eisenhart, M., Behm, L., & Romagnano, L. (1991). Learning to teach: Developing expertise or rite of passage? Journal on Education for Teaching, 17, 51-72.

Eisenhart, M., Borko, H., Underhill, R., Brown, C., Jones, D., & Agard, P. (1993). Conceptual knowledge falls through the cracks: Complexities of learning to teach mathematics for understanding. Journal of Research in Mathematics Education, 24, 1, 8-40.

Engelhardt, J. (1982). Elementary school mathematics teacher education at Arizona State University. Journal of Research and Development in Education, 15, 4, 65-68.

Ernest, P. (1989). The knowledge, beliefs, and attitudes of the mathematics teacher: A model. Journal of Education for Teaching, 15, 1, 13-33.

Gagne, R. (1983). Journal of Research in Mathematics Education. 14, 1, 7-18.

Garfunkel, S. & Young, G. (1992). In the Beginning: Mathematical Preparation for Elementary School Teachers. USA: COMAP, Inc.

Good, T. (1990). Building the knowledge base of teaching. In Dill, D. (Ed.), What Teachers Need to Know: The Knowledge, Skills, and Values Essential to Good Teaching. San Francisco: Jossey-Bass.

Good, T., Grouws, D., & Ebmeier, H. (1983). Active Mathematics Teaching. New York: Longman

Grouws, D. (Ed.) (1992). Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan.

Hart, K. (1983). I know what I believe: So I have to believe what I know? Journal of Research in Mathematics Education, 14, 2, 119-125.

Hiebert, J. & Carpenter, T. (1992). In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan.

Hollis, L. & Houston, R. (1982). Elementary school mathematics teacher education at the University of Houston. Journal of Research and Development in Education, 15, 4, 69-71.

Hooten, J. (1982). Preface. Journal of Research and Development in Education, 15, 4, i.

Kagan, D. (1992). Implications of research on teacher belief. Educational Psychologist, 27(1), 65-90.

Kline, M. (1977). Why the Professor Can't Teach: Mathematics and the dilemma of university education. New York: St. Martin's Press.

Lampert, M. (1988). What can research on teacher education tell us about improving quality in mathematics education? Teaching and Teacher Education, 4, 2, 157-170.

Lappan, G. & Even, R. (1989). Learning to Teach: Constructing Meaningful Understanding of Mathematical Content (Craft Paper 89-3). East Lansing, MI: National Center for Research on Teacher Education.

LeBlanc, J. (1977). Teacher education in elementary mathematics: A framework for needed research. In Proceedings of the Research-on-Teaching Mathematics Conference, May 1-4, 1977. East Lansing, MI: The Institute for Research on Teaching.

Mathematical Association of America, Committee on the Mathematical Preparation of Teachers (1991). A Call for Change: Recommendations for the Mathematical Preparation of Teachers. USA: Mathematical Association of America.

Michigan Council of Teachers of Mathematics (1991a). Proceedings of the Mathematics Education Conference on Mathematics for Elementary Teachers. USA: Michigan Council of Teachers of Mathematics.

Michigan Council of Teachers of Mathematics (1991b). Guidelines for the Initial Preparation of Teachers of Mathematics in Michigan. USA: Michigan Council of Teachers of Mathematics.

Musser, G. & Burger, W. (1991). Mathematics for Elementary Teachers: A Contemporary Approach (2nd ed.). New York: Macmillan Publishers.

National Center for Research on Teacher Education (1988a). Teacher Educator Interview. East Lansing, MI: NCRTE.

National Center for Research on Teacher Education (1988b). Teacher Educator Questionnaire. East Lansing, MI: NCRTE.

National Center for Research on Teacher Learning (1992). Findings on Learning to Teach. East Lansing, MI: NCRTL.

National Council of Teachers of Mathematics (1989). Curriculum and Evaluation Standards for School Mathematics. Reston, VA: The Council.

National Council of Teachers of Mathematics (1991). Professional Standards for Teaching Mathematics. Reston, VA: The Council.

National Research Council (1991). Moving Beyond Myths: Revitalizing Undergraduate Mathematics. Washington, DC: National Academy Press.

Pajares, M. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. Review of Educational Research, 62, 3, 307-332.

Polya, G. (1963). American Mathematical Monthly, 70, 605-619.

Prawat, R. (1989). Teaching for understanding: Three key attributes. Teaching & Teacher Education, 5, 4, 315-328.

Schmidt, W. & Kennedy, M. (1990). Teachers' and Teacher Candidates' Beliefs About Subject Matter and About Teaching Responsibilities (Research Report 90-4). East Lansing: National Center for Research on Teacher Education.

Schoenfeld, A. (Ed.). (1990). A Sourcebook for College Mathematics Teaching. USA: Mathematical Association of America.

Schram, P., Wilcox, S., Lanier, P., & Lappan, G. (1988). Changing Mathematical Conceptions of Preservice Teachers: A Content and Pedagogical Intervention (Research Report 88-4). East Lansing, MI: National Center for Research on Teacher Education.

Seidman, I. (1991). Interviewing as Qualitative Research: A Guide for Researchers in Education and the Social Sciences. New York: Teachers College Press.

Simon, M. (in press). Learning mathematics and learning to teach: Learning cycles in mathematics teacher education. Educational Studies in Mathematics.

Simon, M. (1993). Reconstructing Mathematics Pedagogy from a Constructivist Perspective. Working Draft. Pennsylvania State University.

Skemp, R. (1987). The Psychology of Learning Mathematics. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Stark, J., Lowther, M., Ryan, M., Bomotti, S., Genthon, M., Haven, C., & Martens, G. (1988). Reflections on Course Planning: Faculty and Students Consider Influences and Goals. Ann Arbor, MI: National Center to Improve Postsecondary Teaching and Learning.

Stone, M. H. (1972). Remarks on the teaching of logic. In W. Lamon (Ed.), Learning and the Nature of Mathematics. USA: Science Research Associates.

Survey of Mathematics and Science Opportunities (1992). Teacher Context Questionnaires and Background Notes (Research Report Series No. 55). East Lansing, MI: Third International Mathematics and Science Study.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. Educational Studies in Mathematics, 15, 105-127.

Thompson, A. (1992). In D. Grouws (Ed.), Handbook of Research on Mathematics Teaching and Learning. New York: Macmillan.

Underhill, R. (1991). Learning to teach: The program director's response. Journal for Education on Teaching, 17, 73-79.

University of Chicago (1992). On the Mathematical Preparation of Elementary School Teachers. Report of a conference held at the University of Chicago. Chicago, IL: The University of Chicago.

Weissglass, J. (1977). Mathematics for elementary teaching: A small-group laboratory approach. American Mathematical Monthly, 84(5), 377-382.

Wilcox, S., Lanier, P., Schram, P., & Lappan, G. (1992). Influencing Beginning Teachers' Practice in Mathematics Education: Confronting Constraints of Knowledge, Beliefs, and Context (Research Report 92-1). East Lansing, MI: National Center for Research on Teacher Learning.

Worth, J. (Ed.). (1988). Preparing Elementary School Mathematics Teachers. Reston, VA: National Council of Teachers of Mathematics.