

MICHIGAN STATE UNIVERSITY LIBRARIES
3 1293 01057 7165
LIBRARY
Michigan State
University

This is to certify that the

dissertation entitled

A NATURALISTIC STUDY OF ALGEBRA I VERBAL PROBLEM INSTRUCTION AND LEARNING

presented by

Gary Michael Klingler

has been accepted towards fulfillment
of the requirements for

PH.D degree in Education

William L Coker
Major professor

Date June 2, 1988

MSU is an Affirmative Action/Equal Opportunity Institution

O-12771



RETURNING MATERIALS:
Place in book drop to remove this checkout from your record. FINES will be charged if book is returned after the date stamped below.

DEC 5 4 1996

A NATURALISTIC STUDY OF ALGEBRA I VERBAL PROBLEM INSTRUCTION AND LEARNING

VOLUME I

By

Gary Michael Klingler

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Education

1988

ABSTRACT

A NATURALISTIC STUDY OF ALGEBRA I
VERBAL PROBLEM INSTRUCTION AND LEARNING

By

Gary Michael Klingler

The study followed three Algebra I teachers' unit of instruction of initial verbal problem content (linear equation structured), and then measured the effectiveness of the instruction on students' performance, both at the end of observed instruction (posttest) and ten weeks later (retention test). Nine students were interviewed after the intervening instruction.

Each teacher's instruction was largely lecture, expository oriented, and followed prescribed segments generally associated with effective teaching. Notably absent from all observed instruction was a daily segment devoted to mental computation, drill, or problem solving.

Teachers' decisions related to sequencing of content, pace of instruction, and student assignment patterns conformed to guidelines established by employed textbooks. Teachers' daily instructional routines were more varied due to differing perceptions of the major aims for the course, the priority of verbal problems within these aims, the preferred teaching role, and the students' capabilities and needs.

Two teachers' instruction proceeded from a procedural orientation to the content, and emphasized transmitting textbook skills and techniques of algebra. These teachers' verbal suggestions were primarily declarative statements, and their students reacted passively to the provided instruction. The third teacher's instruction proceeded from a problem solving orientation, which encouraged more active student participation, observation, and verbalization of thinking processes, and facilitated orderly learning from realistic problem settings, and resulted in an even balance of interrogative and declarative formats of verbal suggestions.

Students' verbal problem solving performance for both testing instruments was generally poor. Two classes of students' written solutions demonstrated heightened awareness and incorporation of the textbooks' general procedures. Student errors were mostly the result of incorrectly carrying out the actual processes called for by the general procedure. Students were generally most successful at solving equations, less successful at defining variables, and least successful at formulating equations for the verbal problems.

Student performance was lower on verbal problems which required recall and application of key relationships. Student processing was generally accomplished by mechanically applying direct translation techniques to

problem statements. The problem solving oriented classroom performed best on the posttest, while a different classroom, in which student practice was emphasized, performed best on the retention test.

ACKNOWLEDGMENTS

I thank the members of my dissertation committee, Professors William Cole (Director), Christopher Clark, Glenda Lappan, and Ben Bohnhorst for their able assistance in shaping this investigation from proposal stage to finished product. I am particularly indebted to Professor Cole for the time spent reviewing my preliminary drafts and for his insightful suggestions along the way.

I thank the study's three teachers and their students, and the district administrations for allowing me access to their classrooms, and enthusiastically cooperating as fellow researchers in the investigation.

I thank Jane Dumke and Jenny Schoch at the University of Wisconsin-Stevens Point for their assistance in preparing the final draft of the dissertation.

Lastly, I thank my wife, Colleen, my six children, and my father and mother for their continued confidence and support during my disappointments, and for their patience in my absence from the family. The dissertation is dedicated, with gratitude and affection, to all of them.

TABLE OF CONTENTS

LIST OF TABLES ix

LIST OF FIGURES xiii

CHAPTER I: STATEMENT OF THE PROBLEM 1

Purposes of the Study 7
Assumptions and Limitations 10
Hypotheses of the Study 19
Overview of the Study 24

CHAPTER II: REVIEW OF RELATED LITERATURE 26

Student Processing 27
Linguistic and Factual Knowledge 30
Schema Knowledge 44
Algorithmic Knowledge 56
Strategic Knowledge 62
Instructional Studies 86
Translation Training 91
Schema Training 98
Strategy Training 104
Algorithm Automaticity 117

CHAPTER III: METHODOLOGY 124

Pilot Study Phase 124
Teacher and Site Negotiation/Entry 133
Data Collection and Analysis 136
Textbook Analysis 137
Observations of Teachers' Instruction . 144
Student Performance Measures 153
Student Interviews 166

CHAPTER IV: INSTRUCTIONAL ENVIRONMENT PATTERNS 174

Textbook Analysis 175
Core Content 178
Verbal Problem Classifications 182

Analysis of Selected Textbooks 192
Textbook Comparisons 199
Author Input 203
Teacher Descriptions 204
Instructional Routines 229
Verbal Problem Instructional Patterns . . . 249
Summary 277

CHAPTER V: INSTRUCTIONAL SUGGESTIONS AND EXAMPLES 278

Instructional Suggestions 280
Homogenaity Tests 285
Independence Tests 287
Reaction 291
Instructional Examples 296
Quantity/Distribution of Examples . . . 297
Source of Examples 298
Content Focus of Examples 298
Sequential Coding Analysis 301
Sequential Coding Characteristics . . . 308
Summary 309

CHAPTER VI: STUDENT PERFORMANCE RESULTS 311

Pretest Results 312
Posttest Results 314
Summary Comments 335
Retention Test Results 337
Questionaire/Survey Section 338
Equation Solving Section 349
Verbal Problem Solving Section 355
Test Correlations 376
Student Interview Results 379
Survey Section 383
Verbal Problem Performance 388
Summary Comments 393

CHAPTER VII: CONCLUSIONS/RECOMMENDATIONS/
DIRECTIONS 398

General Problem Solving Model Instruction
and Incorporation 399
Conclusions 417
Recommendations 420
Influence of Textbook on Teachers'
Instructional Routines 423
Conclusions 435
Recommendations 437

Teachers' Characteristics' Influence on Instructional Routines 440
Conclusions 449
Recommendations 453
Teachers' Verbal Suggestions' Influence on Student Attention and Performance . . 458
Conclusions 464
Recommendations 466
General Student Performance Characteristics 469
Conclusions 483
Recommendations 486
Future Research Directions 494
Naturalistic Perspective 495
Problem Solving Processes 498
Instructional Programs 499
Role of Metacognition 500

APPENDIX A: Pilot Study Instrumentation 503

APPENDIX B: Study Coding Forms 505

APPENDIX C: Textbook Analysis Materials 508

APPENDIX D: Teacher Inventory Packet 512

APPENDIX E: Pretest and Student Information File 520

APPENDIX F: Posttest and Retention Test Instrumentation 523

APPENDIX G: Interview Related Materials 532

APPENDIX H: Contingency Tables: Suggestion Analysis 538

APPENDIX I: Process Coding of Teacher Examples 541

APPENDIX J: Individual Student Performance Results 544

APPENDIX K: Summary of Posttest and Retention Test Results 548

APPENDIX L: Posttest Item Analysis 556

APPENDIX M: Retention Test Item Analysis 561

APPENDIX N: Interview Results 565

LIST OF REFERENCES 574

LIST OF TABLES

Table		Page
1.1	Instructional Goals within Polya's Model . .	15
2.1	Mayer's Knowledge Type by Stage Classification	29
2.2	Mayer's Problem Solving Instructional Model	90
3.1	Summary of Teacher Observations Completed	148
4.1	Textbook Review Summary	175
4.2	Textbook Comparison: Space Allocation . . .	176
4.3	Textbook Comparison: Core Content of Verbal Problems	179
4.4	Textbook Comparison: Simple/Complex Exercises	183
4.5	Textbook Comparison: Exercise Format Classifications	185
4.6	Verbal Problem Difficulty: Format Types . .	187
4.7	Verbal Problem Difficulty: Textbook Format Classifications	188
4.8	Textbook Comparison: Summary Formats in Percentages	189
4.9	Textbook Comparison: Feature Evaluation . .	192
4.10	Teacher Comparison: Segment Time Means . . .	231
4.11	Teacher Comparison: Lesson Development Quality	255
5.1	Instructional Suggestions: Classification Examples	283

Table		Page
5.2	Instructional Suggestions: Summary Classification Types	284
5.3	Analysis of Variance for Teachers' Suggestion Quantity	285
5.4	Instructional Suggestions: Summary Homogenaity Tests	286
5.5	Instructional Suggestions: Summary of Independence Tests	288
5.6	Instructional Suggestions: Content/Placement Independence	290
5.7	Instructional Suggestions: Content/Format Independence	290
5.8	Instructional Suggestions: Placement/Format Independence	291
5.9	Coded Instructional Examples: Quantity/Distribution	298
5.10	Coded Instructional Examples: Content Focus	300
5.11	Coded Instructional Examples: Process Summary Stages 1 & 2	303
5.12	Coded Instructional Examples: Process Summary Stages 3 & 4	305
6.1	Pretest Results: Classroom Comparisons . . .	313
6.2	Classroom Pretest Differences: ANOVA Table	313
6.3	Problem Characteristics of Posttest Instrument	315
6.4	Scoring Techniques for Posttest Items . . .	317
6.5	Posttest Results: Summary of Classroom Performance	318
6.6	Posttest Results: Form Performance Comparison	319
6.7	Posttest Results: Classroom Performance by Test Item	321

Table		Page
6.8	Posttest Results: Class Comparisons by Problem Type	323
6.9	Posttest Results: Format Classification Comparisons	324
6.10	Verbal Problem Difficulty: Posttest Format Classifications	325
6.11	Posttest Results: Phase Success Proportions (Method I)	329
6.12	Posttest Results: Phase Success Proportions (Method II)	332
6.13	Retention Test Results: Survey - Item # 1	339
6.14	Retention Test Results: Survey - Item # 2	341
6.15	Retention Test Results: Survey - Item # 3	342
6.16	Retention Test Results: Survey - Item # 4	342
6.17	Retention Test Results: Survey - Relationships Items 2,3,4	344
6.18	Retention Test Results: Survey - Item # 5	346
6.19	Retention Test Results: Survey - Item # 6	347
6.20	Retention Test Results: Equation Solving Summary	350
6.21	Retention Test Results: Equation Solving Item Analysis	351
6.22	Retention Test Results: Equation Solving Error Analysis	353
6.23	Retention Test Results: Problem Solving Section - Item Classifications	357
6.24	Retention Test Results: Item Performance Summary	358
6.25	Problem Type Comparisons: Posttest/Retention Test	360
6.26	Retention Test Results: Class Performance - Format Classifications	362

Table		Page
6.27	Retention Test Results: Problem Difficulty Format Sequences	363
6.28	Retention Test Results: Problem Solving Section - Format Class Comparisons . . .	364
6.29	Retention Test: Summary - Conditional Proportions of Phase Success (Method I)	366
6.30	Retention Test: Summary - Proportions of Phase Success (Method II)	369
6.31	Correlation Measures between Test Instruments	376
6.32	Retention Test Results: Equation Solving Comparisons	378
6.33	Interview Results: Subject Test Measure Performances	381
6.34	Interview Instrument: Student Homework Patterns	386
6.35	Interview Instrument: Student Process Scores by Item	390

Table		Page
6.27	Retention Test Results: Problem Difficulty Format Sequences	363
6.28	Retention Test Results: Problem Solving Section - Format Class Comparisons . . .	364
6.29	Retention Test: Summary - Conditional Proportions of Phase Success (Method I)	366
6.30	Retention Test: Summary - Proportions of Phase Success (Method II)	369
6.31	Correlation Measures between Test Instruments	376
6.32	Retention Test Results: Equation Solving Comparisons	378
6.33	Interview Results: Subject Test Measure Performances	381
6.34	Interview Instrument: Student Homework Patterns	386
6.35	Interview Instrument: Student Process Scores by Item	390

LIST OF FIGURES

Figure		Page
2.1	Schematic Overview of the Problem Solving Process	68
2.2	Adapted Heuristic Problem Solving Model . .	70

CHAPTER I

Statement of the Problem

Mathematical problem solving has been identified as the major focus for curriculum and instructional development efforts of the 1980's (NCTM, 1980). The large number of recent books, articles, conference reports, research and curriculum development efforts also attests to the importance of problem solving within the school mathematics curriculum. There is substantial support for the notion that the ultimate aim of learning mathematics at every grade level is the ability to solve problems, including textbook verbal problems (Lester,1980). This importance of problem solving also explains the reason for inclusion of verbal problem content within every mathematical textbook, grade levels 1 - 12. Despite the recognized importance of mathematical problem solving within the curriculum, specific knowledge of how student abilities associated with problem solving develop, or how instruction assists such development remains less clear (Lester, 1980).

Partly responsible for the lack of clarity of current research efforts regarding the teaching and learning of mathematical problem solving has been the diversity of

problems, or task variables selected for research study (Lester, 1980). Most researchers operate under a broad definition of "problem" similar to that proposed by Lester (1978, p.54), where "a problem is a situation in which an individual or group is called upon to perform a task for which there is no readily accessible algorithm which determines completely the method of solution". It is assumed in this definition that the individual or group in question desires or is motivated to perform the task. Most mathematical problem solving research has used problem types classified as "puzzle problems", as exemplified by problems similar to the Tower of Hanoi, or simple translation story problem varieties (Lesh, 1979).

The selection of algebra verbal problems as the mathematical content for the investigation was in part determined by its relative position near the bottom of the "problem" spectrum, developmentally above only simple translation story problems. The investigator believes that present secondary mathematics instruction is characterized by few student opportunities to explore real world problems, or even to discuss challenging puzzle type problems. The closest schools come to providing opportunities for students to experience, receive instruction, and develop problem solving abilities resides in whatever the textbook provides in the verbal problem sections. Besides greater accessibility and opportunity

for research study, verbal problem solving skill development appears somewhat related to higher order mathematical problem solving skill development. Therefore, the verbal problem instruction which teachers deliver in algebra classrooms might also indicate guidelines for the direction that higher order problem solving instruction ought to pursue.

The particular choice of Algebra I verbal problem content for the investigation is also based on the investigator's belief that earlier grade level (K - 8) study of verbal problems emphasizes simple translation skills leading to the correct identification of an appropriate operation(s), whereas algebra verbal problems generally require students to utilize greater relational thinking within the planning and organization of solution strategies. Additionally, problem solving researchers believe contemporary high school algebra textbooks are primarily concerned with imparting a collection of algorithmic techniques for manipulating algebraic expressions and equations, resulting in the relative exclusion of any problem solving opportunities for algebra students (Lester, 1980). This pattern of algebra textbook construction somewhat explains results of the National Assessments which indicate that 17 year olds perform quite well on standard algebraic procedure items, and yet perform quite poorly when they are required to employ

these same procedures within verbal problem items (Carpenter et al, 1982). The lower verbal problem assessment results also indicate a need to address the issue of why algebra verbal problem related skills are either slower to develop, or are more quickly forgotten over time than are algebraic manipulation skills.

For algebra students, verbal problem content occupies a middle ground in the course experience, serving to connect the parallel developments of algebraic manipulation skills and real world problem solving or application skills. Verbal problem content is a form of vicarious problem solving, a substitute for real world problem solving, particularly useful within mathematical curriculums when more complex forms of problem solving opportunities are not present. Current trends in the placement of verbal problem content within algebra textbook designs similarly reflects this concern that students apply, almost immediately, the developing manipulative skills within verbal problem contexts. Based on investigator experience using algebra textbooks, it appears that present day textbook designs tend to disperse verbal problem exercises more equitably within chapters than was previously the case, and verbal problem exercises are placed more closely to the just completed manipulation skills which are to be applied in the verbal problems. Earlier textbook designs tended to clump verbal problem

exercises into one or two chapters in isolation from related manipulation skills. This placement trend indicates to the investigator that the inclusion of verbal problem exercises in the algebra course serves primarily an applicational function for further developing, reinforcing and practicing algebraic manipulation skills. However a primary applicational function for verbal problems also indicates that the development of verbal problem solving skills, separate or apart from manipulation skill development, is given less instructional treatment or is being downplayed in today's algebra classrooms. This trend also runs counter to the desires of mathematics educators who are calling for greater problem solving experiences for students within the mathematics curriculum. It should be noted however, that the types of verbal problems studied in the algebra curriculum has not changed considerably, at least not in the last 20 years, based on a review of the verbal problems employed in Paige and Simon's (1966) research study.

Mathematics education research has been criticized for its concentration on curriculum content and students' learning of that content, while excluding for the most part, the effect of teaching practices on the learning of such content (Brophy, 1986). Mathematical problem solving researchers have also noted the lack of

instruction-related studies, particularly studies of a "teaching experiment" type, where investigators observe student process development as it occurs in actual classrooms in which teacher instruction is permitted to progress as naturally as possible (Lester, 1980).

Most verbal problem solving research has instead attempted to control the teacher role variable either through the usage of self instructional materials or through observations of students' natural abilities within a laboratory setting. The teacher's instructional role in the development of problem solving abilities (of which verbal problem solving abilities are a subset) is generally assumed to be at least of equal importance to that of textbook or self instructional presentations, and yet there is neither characterizations of current teacher instructional practices within research, nor have there been attempts to measure the effectiveness that instructional practices have on the learning or development of problem solving related skills (Silver, 1985 and Grouws, 1985). Clinical studies, sometimes referred to as "teaching experiments" (Lester, 1980), such as those conducted by Kantowski (1977) and Putt (1978) more adequately address the shortage of classroom instruction-related descriptive studies, and make possible the collection of more qualitative data regarding the effectiveness of instructional practices on students'

problem solving behavior and performance.

In summary, ninth grade students are expected to develop and demonstrate proficiency in applying algebraic manipulation skills within verbal problem contexts during the Algebra I course, as a forerunner to developing more general problem solving skills. National Assessments indicate that such application proficiency does not occur for many secondary level students, due either to the nature of the received instruction within the Algebra I course, or due to quicker loss of learning in regards to verbal problem related skills than for algebraic manipulation skills (Carpenter et al, 1982). In either case, the characteristics of textbook and teacher presented instruction needs to be more fully documented, described, and compared as it is developed for students in Algebra I courses. Secondly there is a need to document, describe, and compare how algebra students translate the received instruction meaningfully (for them) into procedural sequences which lead to improved verbal problem performance.

Purposes of the Study

The two major purposes of this investigation are to describe and characterize current instructional patterns or presentations of Algebra I verbal problem content, and to analyze student verbal problem solving skill development, performance, and retention which results from

such provided instruction. The investigator believes these two purposes can be achieved through observations of typical, middle track, Algebra I classroom teachers, and through testing their respective classes of students. The choice of middle track Algebra I classes for the investigation, rather than accelerated Algebra I sections, is based on a belief that teacher provided instruction is more central to the former student groups' development of verbal problem solving skills. In accelerated Algebra I classes, students are generally held more personally accountable for learning such skills on their own through greater time spent practicing verbal problem exercises.

Some consideration has been given to the development of self instructional material packets related to verbal problems, and to the exclusive usage of student interviews and testing as measures of developing verbal problem solving proficiency. However both considerations minimize the impact of teacher and textbook roles, or fail to capture the more natural developmental process produced in school environments. Both considerations also hinder the development of descriptions of instructional patterns and student learning processes which represent the two major purposes of the investigation.

The first major purpose, observation and description of instructional patterns, requires the review of current techniques and practices employed by textbooks and

teachers within their instructional segments related to verbal problem content. Observations of several teachers' instruction of verbal problems provide a basis for comparing teacher actions in blending elements of textbook construction with their own conceptualizations into meaningful instructional segments. Primary emphasis in the resulting descriptions is given to the duration and qualitative characteristics of the development portion of the lesson, where teachers explain and reinforce new concepts in the verbal problem solving process. Secondary emphasis in the resulting descriptions is on comparing daily patterns and routines of teachers. Each of these emphases requires the further development of instruments, used after observations, that permit qualitative comparisons of textbook and teacher example presentations and quantify teachers' daily time usage. These become secondary purposes for this investigation.

Student performance and retention of verbal problem solving abilities relative to received textbook and teacher provided instruction represents the second major purpose of this investigation. A framework for evaluating a given teacher and textbook's instructional goals and presentations is developed by following student performance, through the analysis of student work samples, on two investigator designed tests (posttest and retention test). The relative effectiveness of each teacher's

instructional patterns related to verbal problems can then be compared in light of resulting student performance measures. The inclusion of a small sample of student interviews and oral reports of students solving verbal problems within the investigation's design add detail to investigator descriptions gained from the review of student performance work samples. Hence the design of three valid verbal problem solving instruments, posttest, retention test and interview instrument represents additional secondary purposes of this investigation.

A final major objective of the investigation is to suggest implications that current instructional practices have on the development of student verbal problem solving abilities, and speculate, from the verbal problem data related to instructional patterns and student performance, on the form of an appropriate model for the instruction of higher level problem solving. Within this objective there is also the possibility of formulating a sequential model of student processing development related to verbal problems, which in turn may suggest an appropriately sequenced model for the instruction of verbal problem content.

Assumptions and Limitations

Four assumptions have guided the development of this investigation into its present form. These assumptions relate to the investigator's view of classroom learning,

the "newness" of algebraic content for all students, Polya's four stage problem solving model as a connecting framework, and the need for naturalistic, observational research in this content domain.

This investigator's view of classroom learning is composed of two parallel elements. First student learning is viewed as a constructive process, in which the learner controls the pace of learning and directs the assimilation of content. The student in such a framework is assumed to be an active and motivated participant in the learning process. Knowledge is constructed in a way that makes sense to the student, and which builds on previous learnings in a relational way. A continual goal of learning is consistency and interiority of and between developed schemas (Skemp, 1979).

Second, the classroom exerts influence on learning when viewed as a social situation jointly constructed by the participants (teachers and students). Interpretation or understanding of the classroom setting is dependent upon the nature of shared transactions, goals, and activities of the participants. In mathematics classrooms, such as Algebra I classrooms, problems and exercises represent the shared transactions or common language spoken here.

The middle track Algebra I classroom is different from most other classrooms when viewed from this social

perspective. One can assume that the common language of algebra problems and exercises of this setting is "new" to all participants except one, the teacher. The students enter the setting with minimum experiences related to the content area, making for a more difficult situation for students than is true for other classroom settings. This aspect of the algebra class was demonstrated in a review of three textbook series in which it was found that the average percentage of new content was 30 percent for eighth grade textbooks, as opposed to 90 percent for Algebra I textbooks (Flanders, 1987). Therefore, the Algebra I setting requires students to give greater daily attention to new concept constructions, and to organize more immediate reconstructions in the development of a consistent knowledge base.

From a researcher or teacher perspective, the situation is somewhat advantageous. The researcher or teacher can assume a certain equality between entering students in terms of their algebraic knowledge. Admittedly, there are differences between entering students' computational abilities. However these differences are easier to separate than conceptual differences when analyzing student work samples. The researcher or teacher can also assume that most algebraic concept learning is a direct result of received classroom instruction. For the teacher, this last assumption places

added pressure and accountability for the planning and implementation of effective instructional lessons.

It is necessary to develop a framework which connects teacher and textbook instructional example presentations with students' processing actions employed in solving verbal problems. George Polya's problem solving model provides such a connecting framework (Polya, 1957).

Polya's four phase model is appropriate for several reasons besides connecting instructional examples with student processing performance. The model is incorporated into most algebra textbooks' teaching explanations and examples related to verbal problems. The model also appears in modified form in most contemporary elementary textbook series as a reminding guide in verbal problem sections. It has been argued by curriculum designers and mathematics educators that general models, such as Polya's model, are less effective and efficient for learning than instructional models which focus on specific problem solving strategy development (Ohio Department of Education, 1980). The few related research results comparing models have been inconclusive as to which problem solving instructional model is more effective for developing student abilities.

Polya wrote How to Solve It primarily for a college student audience, and to a lesser degree for secondary teachers. He does describe how he envisions his model to

be adapted by teachers to fit instruction. He believed problem solving required students to develop mental operations and self questioning techniques appropriate for each of the four phases. Instruction aids the development of such techniques through teacher modeling of the desired operations within instructional examples. These general, helpful problem solving suggestions or questions are referred to as heuristics (Polya, 1957).

Based on the investigator's experience, teachers tend to rely on a textbook's presentations and examples for verbal problem solving instruction. Algebra textbooks generally adopt a Polya model within their presentations. Therefore most teachers' instruction contains Polya's phase approach as a guiding framework for solving verbal problems.

Several researchers have used Polya's model to construct process-sequence coding forms for analysis of protocols gained from student problem solving interviews, with eighth graders solving verbal problems, and with calculus and geometry students solving problems (Goldin and McClintock, 1984). The investigator believes that Polya's model is therefore the best framework available because of its potential to connect all three participants of the classroom setting. Table 1.1 represents a preliminary attempt by the investigator to identify specific verbal problem instructional/student goals that

relate to each of the four phases of Polya's model.

Table 1.1
Instructional Goals Within Polya's Model

Polya's Model	Verbal Problem Instructional Goal
1. Understand the Problem	Able to use auxiliary elements to aid understanding/organization
	Able to define variables/unknowns
2. Devise a Plan	Able to write appropriate equation capturing relationships in problem statement
3. Carry out Plan	Able to solve constructed equation
4. Looking Back	Able to interpret/relate solution to problem question
	Able to check answer with problem statement

Some educational researchers believe that naturalistic variation research studies focusing on the effects of differences in quantity of instruction, teacher management skills, and degree of emphasis on active instruction should be replaced by experimental studies. Such experimental studies permit tighter control over naturalistic instructional variations, or studies in which teachers are trained to follow structured instructional guidelines (Brophy, 1986). Brophy's argument is based upon a separation of the issues of quantity and quality of instruction, and appears truthful for certain mathematical

content areas, such as for elementary school mathematical concepts where numerous naturalistic investigations have uncovered how students learn mathematical concepts from varied forms of instruction. However naturalistic investigations of instruction involving algebra verbal problem content, particularly the quality of such related instruction, has received less treatment. The investigator believes that the current research knowledge base related to algebra verbal problem content instruction is insufficient to conceive of the development of a related experimental study.

The investigator recognizes, and is willing to accept, the fact that most of the limitations related to this investigation are the direct result of the decision to implement a naturalistic, clinical study model. Instead of an experimental study where the quantity of instruction is carefully controlled and the quality of received instruction is confined to a prepared guideline, this investigation concentrates on description of both quantitive and qualitative characteristics of teacher and textbook instruction.

There are several possible pitfalls which might undermine the investigation's approach to data gathering and its comparative analysis of instructional forms. First, certain of the teachers selected for observation might lack equivalent classroom management skills, subject

matter knowledge, or efficient time management skills. Careful selection of teachers for the investigation controls these variables somewhat, as does the inclusion of instruments designed to measure the quantity of instructional time given to verbal problem content.

Second, the possibility exists that certain of the teachers selected for observation might be similar in their instructional approach to algebra verbal problems, and this hurts the development of comparative teacher and classroom models.

A third difficulty encountered in studying instructional quality in a coherent fashion requires the development of more sophisticated classroom observation instruments which capture qualitative differences between teachers' instruction apart from those differences that relate to quantity of instruction. To address this limitation, the investigator's field note taking during classtime is followed by a qualitative assessment of observed instruction with a separate instrument after review of the observational notes. Admittedly this sequence does not insure that the investigator's subjective evaluations of instructional quality is not effected by quantitative variations, in fact such separation is probably not realistically possible.

The selection of a local geographic unit and observations of only three classrooms within that

geographic unit limits the generalizability of the results to other Algebra I classrooms. The selection was based on its proximity to the investigator's place of employment and the expected ease of entry possible within the geographic unit's area schools.

There is also an awareness that student oral reports of their thinking, such as those anticipated within the student interview sessions, often do not reflect exact student processing or thinking (Nisbitt and Wilson, 1977), and are often influenced by aspects of the interview environment or student preconceived notions of the interview process (Schoenfeld, 1983). These limitations of the interview process make all collected oral reports more speculation than fact, and are to be analyzed accordingly.

There exists the possibility that the investigator's observational field notes of teacher instruction may reflect various biases maintained due to previous familiarity with the Algebra I setting and sensed frustrations related to the teaching of verbal problem content. Ideally fieldwork data collection should be undertaken by neutral observers, or someone unfamiliar with the observed setting. This unfamiliarity with the setting permits, at least initially, the observer to note more global diversity, and not be handicapped by the selective vision of one familiar with the setting. However the Algebra I setting does have a newness about it in the

mind of the investigator because it has been six years since his last secondary teaching experience. In some ways this previous teaching experience has some advantages, in that the investigator is more aware of teachers' frustration in their planning and delivery of instructional segments related to verbal problems.

This investigation, as are most clinical studies, is nonprescriptive in nature, but rather focuses on detailed descriptions and comparisons of instructional patterns. These developed descriptions need to be documented by observed classroom examples of the teacher behavior patterns described in order for the descriptions to be considered valid judgments and not just investigator opinions. The usage of several quantitative and qualitative instruments after classroom observations aids this data triangulation process of analysis. Similarly investigator evaluation of student performance through review of provided work samples is primarily a subjective process, and requires that the investigator determine beforehand an exact definition of scoring options and then maintain consistency in the evaluation of student work samples.

Hypotheses of the Study

Within a middle track Algebra I course setting, this investigation addresses the eight hypotheses listed below. The hypothese are listed under two general headings,

instructional environment patterns and student performance patterns. In order to indicate the purposes of the study more clearly, the hypotheses are stated here in a positive form. Additionally, for each listed hypothesis, the intended direction in which the study will proceed is indicated in a descriptive form.

I. Instructional Environment Patterns

A. Hypothesis 1: The inclusion and reinforcement of a general procedural model, such as Polya's or a textbook version, within a teacher's instruction improves student verbal problem solving performance.

Compare teaching techniques employed related to initial presentation, subsequent reinforcement, and evaluation in a context of meaning related to the textbook provided general procedural model for verbal problem solving.

B. Hypothesis 2: The employed textbook has great influence on teacher instructional decisions related to verbal problem content in regards to sequencing, planning, lesson development and presentation.

Compare teachers' instructional sequencing, presentations, and lesson developments with those provided within respective textbooks of each teacher.

C. Hypothesis 3: Teachers' instructional goals, expectations, and attitudes toward verbal problems have a substantial impact on the nature of lesson development segments and classroom routines.

req

pra

to

tea

sug

mod

stu

per

rel

perf

and

cont

Use data collected from teacher questionaires in regards to attitudes, expectations, and classroom practices in conjunction with instructional observations to formulate instructional pattern descriptions for each teacher of the study.

D. Hypothesis 4: Continual teacher reference to the general procedural model, via teacher suggestive comments and/or modeling within instruction of diverse verbal problem types is necessary for students to incorporate, generalize, employ, and retain the general model when solving verbal problems.

Relate and compare the quantity of teachers' oral suggestions and examples modeled related to the general model within the classroom setting with the degree of student incorporation shown within various student performance measures.

E. Hypothesis 5: Greater quantity of teacher provided suggestions for specific verbal problem types provides guidelines for student classification of verbal problems by type, which in turn improves student performance of familiar problem types and inhibits student performance of less familiar types.

Compare quantities of teacher oral suggestions related to specific verbal problem types with student performance results on these same verbal problem types, and also with performance on less familair problem contexts or situations.

stat

term

perf

a ge

resu

the

perf

inst

F. Hypothesis 6: The format in which a teacher's oral suggestion is framed (question or statement) does have an influence on the degree of student thinking or active processing. In particular a suggestion presented in a question format has greater impact on student attention and active processing.

Compare percentages of the formats (question or statement) in which teachers' suggestions are given in terms of immediate and retentive impact on student performance measures.

II. Student Performance Patterns

A. Hypothesis 7: In attacking and solving verbal problems, a high proportion of students will demonstrate an awareness of the general procedure within their completed work samples on varied verbal problem performance measures.

Design criteria indicative of the prevalent usage of a general procedure, and compare classroom performance results which indicate degree of student incorporation of the teacher developed procedural model. Relate classroom performance with quality and quantity of teacher provided instruction of the general model.

B. Hypothesis 8: Of the three central student processing components contained within the general problem solving model (those being variable definitions, equation formulations, and equation solutions), students will be most successful in solving equations, less successful in deriving variable definitions, and least successful in formulating equations.

succ

of c

whil

proc

deve

conn

sett

arra

this

trea

prob

part

Mathe

neede

of pr

segme

prepa

gain

which

gener

and t

relat

prese

and u

Compare overall student processing performance success for the three components, and also note patterns of classroom performance related to the three components while comparing teachers' provided instruction of each process skill.

The hypotheses of the study provide a framework for developing methods related to the investigation, and for connecting the three components of an instructional setting, teacher, textbook, and students, in a meaningful arrangement. The possible implications resulting from this investigation and its focus on instructional treatment and student performance related to verbal problem content might assist a diversity of interested parties within the mathematics education community. Mathematics problem solving researchers might receive needed information on how teachers adapt to the complexity of problem solving and formulate meaningful instructional segments within a classroom setting. Educators who prepare students in mathematics education courses might gain insights into the current instructional patterns which characterize the more effective teaching of this generally difficult content area. Curriculum designers and textbook authors might gain additional information related to the more effective sequencing and presentational characteristics which teachers appreciate and use within instruction from an employed textbook.

Lastly knowledge gained on the development of students' verbal problem solving abilities and constructions, strengths and weaknesses, holds promise for improving classroom teachers' instructional techniques and improved student learning and performance related to verbal problem content.

Overview of the Study

To address the eight hypotheses of the study, the investigator reviewed textbooks and observed portions of teachers' instructional segments related to verbal problem sets covered in three middle track Algebra I classrooms during the first four months of an academic school year (1986-87). Three student performance measures were developed and administered to the three classes by the investigator at the outset (pretest), after instruction was completed (posttest), and several months later (retention test). In addition nine students (three from each classroom) were interviewed after administration of the posttest. Students in the interview sessions orally reported their thinking as they worked through a set of verbal problems representing modifications of textbook provided problems.

Chapter II reviews research related to the hypotheses of the study. Chapter III focuses on the methods of data collection and analysis employed within this investigation. Chapter IV discusses results related to

the t

the f

Chapt

obser

withi

vario

recei

devel

Chapt

relat

stude

major

of th

effor

the textbook review and teacher observations relating to the formulation of instructional environment patterns. Chapter V identifies, describes, and reviews results of observed teacher provided suggestions and techniques used within instruction. Chapter VI reviews results from various student performance measures in the light of received instruction and its relationship to the development of student verbal problem solving abilities. Chapter VII includes a general discussion of the relationships found between instruction and resultant student learning, and includes a summary of the study's major findings and concluding comments on the implications of the study on instructional practice and future research efforts.

solvi

conte

revie

chara

verba

abili

which

aid t

stude

requi

effor

explic

proble

result

mathem

proble

cognit

Resnic

attemp

CHAPTER II

Review of Related Literature

This chapter reviews the literature concerning the solving of verbal problems and problem solving in contemporary mathematics and algebra classrooms. This review includes studies of both student performance characteristics and instructional studies related to verbal problem content.

Research efforts have improved mathematics educators' ability to analyze and assess the diversity of factors which effect school task difficulty, and have served to aid the identification of typical strategies employed by students when such research tasks can be related to required school tasks. However, problem solving research efforts have generally not begun their studies with an explicit framework or theory of how students process problems, nor does such a framework emerge from the results of these studies. The few theories of school mathematics learning in general, and school mathematics problem solving in particular have been developed by cognitive psychologists, Greeno (1978), Mayer (1982), and Resnick (Resnick and Ford, 1981). Greeno and Resnick both attempted to extend results from their studies of

eleme

highe

more

proce

discu

initi

the f

proce

compo

1985)

strat

proce

solvi

studi

offer

of ma

the s

acqui

cogni

variou

memor

betwe

inform

elementary age students with verbal problem tasks to higher level students involving verbal problem tasks in more complex content domains, such as algebra.

Mayer's (1982) theoretical model of student processing is the centerpiece for the following research discussion of student processing and is detailed initially. Then a review of research related to each of the four knowledge components of his model of student processing is undertaken. Two other recently introduced components, metacognition and student beliefs (Schoenfeld, 1985), are discussed within Mayer's fourth component, strategic knowledge, and conclude the section on student processing. Mayer's (1985) instructional model of problem solving is then introduced and analyzed and research studies related to its component parts are discussed.

Student Processing

Mayer believes that the field of cognitive psychology offers a new approach to the development of a psychology of mathematical problem solving. Cognitive psychology is the study of how humans process information, including the acquisition, storage and retrieval of knowledge. Within cognitive psychology it is assumed that humans possess various memory stores (short term, working, and long term memory) and memory processes for transferring knowledge between memory stores, and also for manipulating information in memory (Mayer, 1982).

Mayer's
commonly hel
cognitive ps
knowledge, r
behaviors, a
down into it
types of pro
are: lingui
sentences), s
relationship
procedures),
problems).

Secondl
as a series
representati
series of le
students pro
problem solv
solution sta
translate th
representati
includes the
operators. T
of a subject
solution sta
through the p

Mayer's theory of student processing is built on two commonly held views of cognitive psychology. First, cognitive psychology views learning as the acquisition of knowledge, rather than being solely composed of new behaviors, and that such acquired knowledge can be broken down into its component parts (Mayer, 1980). His four types of problem solving knowledge or its component parts are: linguistic and factual knowledge (how to encode sentences), schema knowledge (use of problem type relationships), algorithmic knowledge (how to perform procedures), and strategic knowledge (how to attack problems).

Secondly, cognitive psychology views problem solving as a series of mental operations that transform knowledge representations, rather than viewing problem solving as a series of learned behaviors. It is also assumed that students proceed through two distinct stages during problem solving activity, a representation stage and a solution stage. In the representation stage, subjects translate the problem statement into an internal mental representation or "problem space" (Hayes, 1981) which includes the given and goal states and allowable operators. The problem space is constructed on the basis of a subject's understanding of the problem. In the solution stage, subjects search for and devise a path through the problem space, continually applying operators

to newly dev

achieved. A

problems has

of two gener

(Jerman, 197

indicated th

more often t

representati

solution pro

emphasize co

representati

Mayer's

knowledge co

his belief a

primarily us

(taken from

Mayer'

Type of Knowl

Linguistic &
Schematic
Algorithmic
Strategic

Mayer s

required for

processing, a

to newly developed problem states until the goal state is achieved. A similar two stage process for solving verbal problems has been described in the literature, consisting of two general stages, translation and computation (Jerman, 1973 and Paige and Simon, 1966). Research has indicated that mathematical problem solving difficulties more often than not result from an inadequate representation of the problem rather than from faulty solution procedures, and yet instruction has tended to emphasize content related to procedures rather than representation (Simon, 1980).

Mayer's theoretical framework places the four knowledge components within the two stages on the basis of his belief as to how the four knowlege components are primarily used by subjects, and is indicated in Table 2.1 (taken from Mayer, 1982, p. 3)

Table 2.1

Mayer's Knowledge Type by Stage Classification

Type of Knowledge	Stage in Problem Solving Process	
	Representation (Understanding)	Solution (Searching)
Linguistic & Factual	X	
Schematic	X	
Algorithmic		X
Strategic		X

Mayer suggests that all four types of knowledge are required for successful student performance and processing, and indicates the complexity involved in

solving alge

weaknesses o

research has

routine prob

heuristic be

subject beli

believes tha

behaviors ar

problems, an

category to

convincing r

skills or su

performance

problems. I

attitudes an

"expert" act

student perf

metacognitio

strategic kn

Linguistic a

In order

from their re

must know ce

speech and pr

facts about t

the words in

solving algebra verbal problems. There are however three weaknesses or omissions within Mayer's framework which research has shown are important, at least for less routine problem solving tasks, those omissions being heuristic behaviors, metacognitive control skills, and subject beliefs (Schoenfeld, 1985). The investigator believes that both general and specific heuristic behaviors are relevant to the task of solving verbal problems, and has broadened Mayer's strategic knowledge category to include heuristic behaviors. There is less convincing research evidence that metacognitive control skills or subject beliefs have impact on student performance of routine tasks, such as algebra verbal problems. Instead it is speculated that positive attitudes and usage of metacognitive skills may separate "expert" activity from recognized successful or proficient student performance. These two areas of research, metacognition and beliefs are reviewed briefly within the strategic knowledge category.

Linguistic and Factual Knowledge

In order for students to formulate a representation from their reading of a verbal problem statement, they must know certain rules of language, such as parts of speech and pronoun referents, as well as knowing certain facts about the world related to the specific meanings of the words in a problem statement. It is obvious that such

knowledge ne
activation w
occur and a
for the exte
equation for

Beyond
problem if t
evidence reg
problem solv
(Lester, 198
readability
one study, L
of the verba
problem solu
Holtan (1976
readability,
were a large
solution per
studies in wh
mathematics
correlations
the reviewed
Vocabulary" a
variables),
"Spelling" al
computation,

knowledge needs to be accessible, capable of recall and activation within working memory, for understanding to occur and a representation to be developed, and in order for the external processes of variable definitions and equation formulation to be completed by students.

Beyond the obvious fact that students cannot solve a problem if they are unable to read it, the research evidence regarding the effect of reading ability on problem solving performance has been conflicting (Lester, 1980). The two most prominent studies of readability were done with elementary school subjects. In one study, Linville (1970) found that the difficulty level of the verbal problem syntax had a significant effect on problem solution, whereas in the second study, Knifong and Holtan (1976) found no such significant effect for readability, instead finding that computational errors were a larger factor in accounting for effects on problem solution performance. Aiken's (1972) review of research studies in which relationships between reading ability and mathematics achievement were given, noted that correlations between variables ranged from .40 to .86 in the reviewed studies. It was also noted that "Reading Vocabulary" and "Reading Comprehension" (both semantic variables), "Mechanics of English" (syntax variable), and "Spelling" all had sizable correlations with arithmetic computation, and even higher correlations with "Arithmetic

Reasoning" (
solving).

Within
knowledge re
secondarily
syntax varia
statements,
problem stat
grammatical
identified 1
related to g
variable stu
studies of h
algebra verb
least 2 synt
into the reg
studies, but
all studies.
syntax varia
in verbal pr
investigatio
that linguis
grade level.
predictors d
intermediate
for college s

Reasoning" (a construct believed related to problem solving).

Within task variable analysis, linguistic and factual knowledge relates primarily to syntax variables and secondarily to content variables. Classifications of syntax variables have focused on two components of problem statements, syntax variables describing length (words and problem statement), and syntax variables describing grammatical structure. For example, Barnett (1984) identified 18 variable constructs related to length and 19 related to grammatical structure in his review of syntax variable studies. In his review of linear regression studies of high school and college students solving algebra verbal problems, Barnett (1984) noted that at least 2 syntax variables related to length were entered into the regression equation on the first 12 steps in all studies, but the constructs were basically different in all studies. The only conclusion he reached was that syntax variables describing length play an important role in verbal problem difficulty and deserve continued investigation. Jerman's (1974) research studies indicated that linguistic predictor variables change with age and grade level. For instance "computational" variables as predictors decrease from being very important in the intermediate grades to having little value as predictors for college students.

The mos
related to v
category whi
phrases in a
technical vo
research, th
the mathemat
statement, a
setting of t
Somewhat dis
type" is con
variable. M
"problem typ
separate fro
investigator
(1984) and W
or setting,
type of cont
dichotomies,
concrete/abs
problem types
mathematic, e
statements.

Even wit
semantic cont
technical mat

The most important subdivision of content variables related to verbal problems is the "semantic content" category which refers to the meanings of critical words or phrases in a problem statement, particularly keywords or technical vocabulary (Webb, 1984). In task variable research, there is a subtle distinction between "content", the mathematical substance or meaning of a problem statement, and "context", the non-mathematical form or setting of the problem statement, variable types. Somewhat disturbing to the investigator is that "problem type" is considered a content rather than a context variable. Mayer (1982) and Silver (1982) both consider "problem type" a knowledge type (schema knowledge) separate from linguistic and factual knowledge. The investigator's position can also be justified by Kulm's (1984) and Webb's (1984) own inclusion of verbal context or setting, as contained in the problem statement, as one type of context task variable. Within this category dichotomies, such as familiar/unfamiliar, concrete/abstract are included, and it is claimed that problem types represent a similar partitioning of the non-mathematic, extraneous information within problem statements.

Even with this modification, the importance of semantic content variables, particularly key words and technical mathematical vocabulary, remains strong. By

"key words"
words into r
"less", "dec
statement in
subtraction
there seems
regards to t
in the teach
most mathema
who caution
approach (Ki
such as Scho
approach "de
position is
student proc
by key words
destroys the
relationship
the other ex
mainly, who
helping stud
1985 article
sixth grade
instructiona
also noted a
presessions

"key words" is meant the direct translation by students of words into respective operations. For example, words like "less", "decrease", "minus" and "below" within a problem statement indicate the likelihood for using the subtraction operation. It is interesting to note that there seems to be two opposite positions at work with regards to the usage of a key words instructional approach in the teaching of verbal problems. At one extreme are most mathematics educators (including the investigator) who caution against its use, considering it a nonviable approach (Kilpatrick,1969). Some educators are more vocal such as Schoenfeld (1982) who calls usage of such an approach "deceptive and fraudulent" teaching. This position is based on the belief that examples abound where student processing can just as easily be incorrectly cued by key words, and that such an instructional approach also destroys the necessity for students to understand the relationships contained in verbal problem statements. At the other extreme are practitioners, school teachers mainly, who believe key words have an enormous impact for helping students comprehend verbal problem statements. A 1985 article in Arithmetic Teacher demonstrated how one sixth grade teacher fully developed such a key words instructional approach (Havel, 1985). Schoenfeld (1982) also noted a disturbing finding given at the research presessions of the 1980 annual NCTM meeting which

indicated th
series, 97 p
approach wou

Most Al
units (2 - 4
using key wo
some cases a
forms of the
within an eq
generally im
student prac
exposure req
language pat
solve the ve
will gradual
approach whe
relational a
translation
transferabil
problem type
usage by stu
emphasis giv
provided ver

Webb's
studies with
verbal cue v

indicated that in a widely used elementary textbook series, 97 percent of the problems "solved" by a key word approach would yield correct answers.

Most Algebra I textbooks contain 1 or 2 instructional units (2 - 4 pages in length) in which students practice using key words to formulate algebraic expressions, and in some cases also provides practice in translating various forms of the verb, to be (is, was, are) into an equal sign within an equation formulation. These instructional units generally immediately precede initial instruction and student practice of algebra verbal problems. This initial exposure requires only that students translate English language patterns into correct mathematical equations to solve the verbal problems. It is assumed that students will gradually discard emphasis on a direct translation approach when verbal problem statements requiring a more relational analysis are introduced. Such direct translation techniques seem to have very limited transferability, even within the small sample of verbal problem types included within this investigation. Its usage by students would also depend on the degree of emphasis given by teachers and employed textbooks within provided verbal problem instruction.

Webb's (1984) review of semantic content variable studies with elementary school subjects indicated that verbal cue variables did not have a significant effect on

verbal probl

several elem

verbal probl

1972), along

Lochhead and

He concluded

verbal probl

mentally tha

Relational p

problem stat

subjects' fa

block" to su

statement to

other words,

to a key wor

instructiona

are largely

forms of ver

There h

subclass of

mathematical

such words a

integer, mea

they appear

to student a

represent pr

verbal problem difficulty. Mayer (1982) also reviewed several elementary level research studies related to verbal problems (Greeno, 1978 and Loftus and Suppes, 1972), along with the college level studies of Clement and Lochhead and his own studies with algebra verbal problems. He concluded that relational information presented within verbal problem statements is more difficult to represent mentally than other forms of presented information. Relational propositions, either explicitly given in problem statements or implicitly required to be drawn from subjects' factual knowledge banks tended to be a "road block" to subjects' attempts to move from a verbal problem statement to an internal representation (Mayer, 1982). In other words, it may be that students become so conditioned to a key words, direct translation approach because of instructional history and experienced success, that they are largely unable to adjust to more complex, relational forms of verbal problem statements.

There has been less research related to Webb's second subclass of semantic content variables, technical mathematical vocabulary. The mathematical meanings of such words as sum, product, function, rate or speed, integer, measures such as degrees, area and perimeter as they appear in verbal problem statements are often crucial to student attempts to understand, visualize, and represent problem information. There is general agreement

that reading
requires a sp
reading ordi
on problem d
suggestion th
reinforcemen
obvious.

Availab
type of cont
classificati
the investig
calculators,
verbal probl
and a calcul
all intervie
level genera
usage within
calculators
particularly
that calcula
generated ve
computations
numbers or i
integers. T
tends to ser
students wit

that reading mathematics textbooks or verbal problems requires a special ability, apart from that required of reading ordinary prose (Kane, 1968). The size of effect on problem difficulty is not known however, and the suggestion that instruction needs to include review and reinforcement of such mathematical vocabulary seems obvious.

Available equipment variations are also considered a type of content task variable within Webb's (1984) classification. This area needs a brief discussion since the investigator will permit students to use their own calculators, if they have them with them, on the designed verbal problem instruments (posttest and retention test), and a calculator will be provided by the investigator for all interviewed students. Teachers at this secondary level generally have similar guidelines for calculator usage within their classrooms, meaning if students have calculators with them during class they can be used, particularly on class assignments. Most students realize that calculators are not essential for the solving of generated verbal problem equations, since the related computations are generally simple operations with whole numbers or integers and resulting solutions tend to be integers. The usage of calculators in algebra classrooms tends to serve a confidence enhancing function for students with weaker arithmetic skills, and research has

shown that
students' c
(Reys, 1980
probably ne
students no
confidence.
calculators
problem set

An alt
analysis is
except the
problem stu
1977 and Ca
secondary sc
Caldwell and
primarily on
reference to
these studie
requiring al
constructed
convinced th
equivalent w
structure, a
in a set ther
of the proble
and being eit

shown that using a calculator does generally increase students' confidence in their ability to solve problems (Reys, 1980). Students regularly using calculators probably need the provided confidence boost, while students not using calculators already possess sufficient confidence. Other advantages gained from using calculators seem less relevant to the Algebra I verbal problem setting of this investigation.

An alternate methodology to linear regression analysis is to attempt to hold all task variables constant except the one variable under study as was done in verbal problem studies of elementary school subjects (Caldwell, 1977 and Caldwell and Goldin, 1979) and studies of secondary school subjects (Goldin and Caldwell, 1984 and Caldwell and Goldin, 1987). Discussion is focused primarily on the secondary school studies, with some reference to the elementary school studies. In all of these studies, sets of four verbal problems each, some requiring algebra techniques for solution, were constructed and tested until the researchers were convinced that the four verbal problems of a set were equivalent with regards to syntax, content, mathematical structure, and context setting. The four verbal problems in a set then were varied only on the basis of the context of the problem statement being either abstract or concrete and being either hypothetical or factual. The process of

the final

involved e

educators

consuming

The e

follows:

or symbolic

whereas a

situation

distance or

described a

hypothetica

initial sit

the initial

considered

computation

pairs, para

problems, w

one of the

be included

controlled

experimental

Another

inclusion at

student proc

explain stud

the final design of these sets was done carefully and involved extensive outside judgment of mathematics educators and teachers, and required a rather time consuming effort by the researchers.

The experimental classifications were defined as follows: An abstract problem statement described abstract or symbolic objects (such as numbers or plane figures), whereas a concrete problem statement described a real situation composed of real objects or people (such as distance or age). A factual problem statement merely described a situational relationship, whereas a hypothetical problem statement described not only an initial situation, but also described a possible change in the initial situation, and both situations needed to be considered in solving the verbal problem. Additionally computational skill tests were constructed in which item pairs, parallel to the skills needed to solve the verbal problems, were included. A student had to correctly solve one of the two computational items of a pair in order to be included in the verbal problem analysis, and hence controlled or removed computational differences as an experimental variable (Caldwell and Goldin, 1987).

Another positive aspect of these studies was the inclusion at the outset of three theoretical frameworks of student processing. These frameworks would later serve to explain student performance results and permit comparisons

between the

students).

predictions

First,

concrete pr

than abstra

respectivel

difficult s

concrete fa

abstract fa

Second

suggests AF

of intermed

difficult.

process (tra

computer pro

apply direct

solutions.

undergo addi

order to ref

described in

objects them

difficult th

Similarly hy

complex tran

this model.

between the two student samples (elementary and secondary students). Each theoretical framework and related predictions is therefore reviewed.

First, a strictly developmental model suggests that concrete problems and factual problems are less difficult than abstract problems and hypothetical problems respectively. In other words, a predicted easiest to most difficult sequence of problems for this model would be concrete factual (CF), concrete hypothetical (CH) - abstract factual (AF), and abstract hypothetical (AH).

Secondly, a direct translation or key words approach suggests AF problems as easiest, AH and CF problems to be of intermediate dificulty, and CH problems as most difficult. These predictions are based on the two stage process (translation and computation) incorporated within computer progams such as STUDENT (Bobrow, 1968), which apply direct translation techniques in their generated solutions. STUDENT requires that concrete problems undergo additional substitutions during processing in order to reference the number or quantity of objects described in a problem statement as distinct from the objects themselves, and hence concrete problems are more difficult than abstract problems for this reason. Similarly hypothetical problems require considerably more complex translation procedures than do factual problems in this model.

The th

indicates t

constructio

representat

representat

easiest, CH

AF and AH p

Goldin, 198

Subjec

taken from

and one hig

district.

include rep

enrollments

Results

concrete pro

least diffic

abstract fac

support for

results at t

student fami

textbook ana

- 80 percent

factual prob

for the seco

Data fr

The third framework, an information processing model indicates that a crucial stage in problem solving involves construction of a "nonverbal" or "imagistic" internal representation of the problem. This nonverbal internal representation model suggests that CF problems are the easiest, CH problems are of intermediate difficulty, and AF and AH problems are the most difficult (Caldwell and Goldin, 1987).

Subjects for the Caldwell and Goldin studies were taken from the seven elementary schools, one junior high, and one high school of a surburban Philadelphia school district. Care was taken in the secondary studies to include representative samples of mathematics course enrollments within the schools.

Results from the elementary study indicated that concrete problems (factual and hypothetical) were the least difficult, followed by abstract hypothetical and abstract factual problems. These results lended general support for the developmental model framework. The results at this level are at least partly due to greater student familiarity with concrete verbal problems, as a textbook analysis done by Caldwell (1977) revealed that 75 - 80 percent of textbook verbal problems are concrete factual problems. Such a textbook analysis was not done for the secondary school study however.

Data from the secondary school study best fit the

prediction
with some a
as well. A
difficulty
diminished
with the de
indication
involved.
interacted
hypothetica
concrete fa
problems we
factual pro
levels. Th
hypothetica
concrete co
concrete co
the distinct
described si
The research
for instruct
based on an

In seve
and Goldin a
data analysi
verbal probl

prediction made within the internal representations model, with some additional support for the developmental model as well. Abstract problems were consistently of greater difficulty than concrete problems, though differences diminished with increasing grade level which is consistent with the developmental model, and also gave some indication that formal operational thinking (Piaget) was involved. The factual-hypothetical variable strongly interacted with the concrete-abstract variable. Concrete hypothetical problems were much more difficult than concrete factual problems, and yet abstract hypothetical problems were actually less difficult than abstract factual problems at the elementary and junior high student levels. The greater difficulty associated with hypothetical problems occurred almost exclusively in concrete contexts. An offered explanation was that in concrete contexts, students find it difficult to maintain the distinction between internal representations of the described situation and the hypothetical change situation. The researchers conclude that their findings lend support for instructional models of problem solving competence based on an imagistic internal representation model.

In several respects, the above studies of Caldwell and Goldin are central to this study, particularly the data analysis employed. The primary classification of the verbal problems within this investigation is by "problem

type", whic
Caldwell ar
factual/hyp
textbooks t
to "problem
textbook's
easy for th
problem ins
themselves
therefore n
on the basi
Caldwell an
an addition
applied to
instruments
based on pr

A textl
investigati
certain prol
result in be
familiar cla
earlier elem
of problem s
theoretical
patterns, an
instruction.

type", which is a context setting variable similar to Caldwell and Goldin's abstract/concrete and factual/hypothetical variable classifications. Algebra textbooks tend to organize verbal problem sets according to "problem type" and teachers tend to follow the textbook's sequence within their instruction, making it easy for the investigator to identify and classify verbal problem instruction and the actual verbal problems themselves using a "problem type" classification. It is therefore natural to initially analyze student performance on the basis of the various problem type classifications. Caldwell and Goldin's classification system then provides an additional method of analyzing student performance, applied to the verbal problems which appear on developed instruments, after such instruments have been designed based on problem types.

A textbook analysis will be completed within this investigation to determine if greater familiarity with certain problem types and their classifications does result in better student performance for these more familiar classifications, as was done in Caldwell's (1977) earlier elementary study. Additionally their three models of problem solving competence provide predictive theoretical frameworks for analyzing classroom performance patterns, and how these patterns relate to provided instruction. Whereas their results can be said to

implicitly
of receive
includes s

The s
(Lester, 1
scheme for
is appropri
categorizat
of the prob
mathematica
solving the
structure/c
are fixed,
varied perm
student str

Schema Know

Mayer'
problems su
order to fu
representat
linguistic
to structure
whole which
statement to
Mayer, schem
a problem so

implicitly relate to provided instruction (no observations of received instruction was done), this investigation includes such observation.

The staff of the Mathematical Problem Solving Project (Lester, 1978) developed a theoretical categorization scheme for problems at an intermediate grade level which is appropriate in closing this section's discussion. The categorization scheme consisted of 4 dimensions: setting of the problem, structure and mathematical complexity, mathematical content, and applicable strategies for solving the problem. In this investigation, the structure/complexity dimension and the content dimension are fixed, while the setting of the problem dimension is varied permitting observation of the varied application of student strategies on the basis of the varied settings.

Schema Knowledge

Mayer's (1982) model of student processing of verbal problems suggests that research results indicate that in order to fully understand a problem and construct a representation or problem space requires more than just linguistic and factual knowledge. Students also need ways to structure the retrieved information into a meaningful whole which is accomplished by relating the given problem statement to a familiar problem type, or schema. To Mayer, schema and problem type mean the same thing, where a problem schema is a general representation for a class

of problem

Silver (19

which a sc

complex an

which is "

numerous e

schema rel

relatedness

relatedness

in attempti

Polya's (19

related pro

Schema

with interp

recalling o

term memory

often permi

based on di

probably ac

made in the

Silver belie

enormous exp

differing pe

within the s

From a

verbal probl

of problems (types), such as "motion" or "coin" problems. Silver (1982) uses a broader definition of schema, in which a schema "represents a prototypical abstraction of a complex and frequently encountered concept or phenomenon" which is "usually derived from past experience with numerous exemplars of this concept". Both notions of schema relate to the more general idea of problem relatedness. Research on student perceptions of problem relatedness within laboratory settings has been extensive in attempting to understand how students develop and apply Polya's (1957) most basic of heuristics, "think of a related problem".

Schemata have been shown to be associated not only with interpreting and encoding processes, but also the recalling of previously processed information from long term memory. It has also been demonstrated that schemata often permit multiple interpretations of text passages based on differing possessed schemata, and that schemata probably account for the differing nature of inferences made in the face of incomplete information (Silver,1982). Silver believes that a schema based theory contains enormous explanatory power with regards to students' differing performance on routine tasks, particularly within the solution of algebra verbal problems.

From a task variable analysis perspective, routine verbal problem typing and related student processing

analysis h
behavior c
which sche
plan devel
individual
terms, con
which augm
unstated g
problem st
algebraic
believes t
type inform
understandi
statement,
teachers' a
is an assum
during the
assumption
and less ca

The fi
theory and
studies of
subjects (1
to categori
into groups
of 12 algeb

analysis has lead to the conclusion that most such behavior can be called "schema driven inferencing", in which schema or problem type information drives student plan development and execution processes (Luger, 1984). An individual schema is a rather broad construction in these terms, consisting of its own unique semantic information which augments the problem statement with additional unstated givens such as implicit relationships, and also problem structure information relating to the underlying algebraic relationships (associated equations). Luger believes that the lack of explicit discussion of problem type information, which leads to inadequate and incomplete understanding of the semantic content of a verbal problem statement, is a major shortcoming of many secondary teachers' and textbooks' instruction. He notes that there is an assumption that semantic content will be "picked up" during the process of solving verbal problems, an assumption which is generally false, especially for slower and less capable students.

The first explicit research link between schema theory and mathematical problem solving was made in the studies of Hinsley, Hayes, and Simon (1977). Their subjects (14 high school and college students) were asked to categorize a series of standard algebra verbal problems into groups of related problems, and also to solve a set of 12 algebra verbal problems, some standard types (3) and

the rest n

students c

"river tra

problem ty

Secondly,

into "type

or read al

Thirdly, s

each proble

focusing at

statement a

instances t

such proble

researchers

schemata fo

the possess

of informat

Nevert

impact for

(14), and d

the subject

investigati

received ins

subjects rec

In a di

500 students

the rest nonstandard types. The results indicated that students categorize problems into "types", such as "age", "river travel" and "work" problem types, with 18 distinct problem types noted within student classifications. Secondly, students tended to classify the verbal problems into "types" even before they had completely assimilated or read all the information of problem statements. Thirdly, students possessed a body of information about each problem type which was potentially useful for focusing attention on important aspects of the problem statement and formulating problems' solutions. In many instances there was evidence of students totally using such problem type information in their solutions. The researchers concluded that their subjects did possess schemata for standard algebra verbal problems, and that the possessed schemata influenced encoding and retrieval of information during problem solving.

Nevertheless, these studies had limited practical impact for several reasons: the limited number of subjects (14), and diverse grade levels and higher ability level of the subjects. In addition as related to this investigation, the setting was laboratory, with neither received instruction or instructional history of the subjects recorded or noted in any way.

In a dissertation study, Chartoff (1977) investigated 500 students' perceptions of mathematical problem

similarity
graders to
algebra ve
which rang
similar.
recognized
a) how the
b) contexts
problem of
problem.
four dimens
dimension,
comparison
variance in
the four di
investigati
differences
differences
mathematica
indicated a
history of

Krutet
a seven yea
problem typ
mathematica
done to stu

similarity, with subjects ranging in age from seventh graders to college freshmen. Students rated pairs of algebra verbal problems on a continuous similarity scale which ranged from extremely dissimilar to extremely similar. He concluded that students at all grade levels recognized four fundamental problem similarity dimensions: a) how the problems are solved (mathematical structure), b) contextual setting, c) comparison with a generic problem of the same type, and d) the question posed by the problem. It is not clear where problem type fits into his four dimensions, most likely within the contextual setting dimension, or possibly within the generic problem type comparison dimension. His results did indicate a large variance in the degree to which individual students used the four dimensions within their judgments. His investigation shed little light on the issue of individual differences, except to note that such individual differences exist, and neither examined students' mathematical abilities to solve the verbal problems nor indicated any information regarding the instructional history of the study's subjects.

Krutetskii's (1976) series of studies, completed over a seven year span, developed problem sets that varied problem type within a set while maintaining common mathematical structure within a set. This variation was done to study the ability of students to generalize from

related ex
various pr
the conten
varied wit
"problem t
students t
problems w
of a grasp
structure
the type a
they had s
the proble
recalled o
statement
good and p
perception
structure f
implicatio
Krutetskii
due to the
analysis of
and his foc
often "gift

Buildi
examined st
mathematica

related examples or student solved problems across the various presentations within a problem set. Often times the content (arithmetic, algebra, geometry) was also varied within a problem set, which expands the notion of "problem type" considerably. He found that more capable students tended to rapidly generalize (and classify) the problems within a set, before solving them, on the basis of a grasp of the general features of the mathematical structure of the problems. More capable students recalled the type and general character of a problem's operations they had solved, but not the specific data or numbers in the problem statement. Less capable students generally recalled only the specific data or numbers in the problem statement or surface features. His research suggests that good and poor problem solvers differ considerably in their perceptions of important problem elements (surface or structure focused), and that this focus difference has implications for verbal problem solving ability. Krutetskii's studies have received criticism in the West due to the employed methodology being primarily the analysis of subject protocols within interview settings, and his focus on subjects who are highly capable, and often "gifted" mathematics students.

Building on Krutetskii's studies, Silver (1979) examined student perceptions of relatedness among mathematical verbal problems (some algebra types), and

also atten

verbal, ma

student pe

were 95 ei

accelerate

researcher

prealgebra

formal alg

In the fir

which cons

along two

contextual

groups of

then expla

found that

two additi

design of t

vaguely def

problems on

contained i

time. His

variations

second card

of verbal,

administere

Despit

also attempted to relate student performance on tests of verbal, mathematical, and problem solving ability to student perceptions of problem similarity. His subjects were 95 eighth graders in regular (as opposed to accelerated) eighth grade classrooms taught by the researcher himself. The curriculum consisted of standard prealgebra content, and students were not exposed to formal algebraic techniques for solving verbal problems. In the first study, subjects completed a card sorting task which consisted of 24 verbal problems, which were varied along two dimensions, mathematical structure and contextual details. Subjects were instructed to form groups of "mathematically related" verbal problems, and then explain the basis for their classifications. Silver found that students classified the verbal problems along two additional dimensions he had not considered in his design of the first study, those being question form (very vaguely defined) and pseudostructure, which related problems on the basis of common measurable quantities contained in problem statements such as age, weight or time. His second study expanded the verbal problem variations to include these two new dimensions and a second card sorting task was done. In addition a battery of verbal, mathematical, and problem solving tests was administered.

Despite the contrasting nature of subject

population
agree with
indicated
dominated
also found
performanc
classifica
structure
percentage
the contex
relationsh
scores for
relatednes
but not un
least used

Silve
pseudostru
pseudostru
problem so
mathematic
pseudostru
the solutio
Also, nearl
decreased t
(after prob
that some g

populations, the results of Silver's studies generally agree with Krutetskii's results. Silver's results indicated that the mathematical structure dimension dominated students' perceptions of problem relatedness. He also found a strong positive correlation between student performance on the various ability measures and student classification preferences along the mathematical structure dimension. A smaller, yet substantial percentage of students preferred to relate problems along the contextual detail dimension, and a negative relationship was found between the various ability measure scores for this group of students. Student problem relatedness along the pseudostructure dimension was small, but not uncommon, while sorting by question form was the least used student dimension.

Silver also conducted two studies related to the pseudostructure dimension. He found that presolution pseudostructure scores correlated significantly with the problem solving measure, but not with the general mathematical ability measures, suggesting that pseudostructural relationships are quite visible before the solution of problems, even to better problem solvers. Also, nearly equivalent numbers of subjects increased and decreased their pseudostructure scores on the second sort (after problem solutions completed). This data suggested that some good problem solvers abandoned the

pseudostru

while some

their numb

second sor

problem re

Silve

easily exp

equations,

not been t

solvers' s

of a succe

that more

mathematic

despite th

the common

similarity

Lastl

schema kno

(Mayer, 19

from 10 al

problem ty

template h

equation f

templates

templates

that studer

pseudostructure dimension after solving the problems, while some less capable problem solvers actually increased their number of pseudostructure classifications in the second sort, indicating a somewhat developmental nature of problem relatedness.

Silver also noted several of his verbal problems were easily expressible in terms of one or two linear equations, or required methods in which his subjects had not been trained. Examination of successful problem solvers' solutions of these problems indicated the usage of a successive approximation strategy. Thus it appeared that more capable problem solvers were able to perceive mathematical structural similarities among problems, despite the absence of the solution strategy that embodied the common structure, that being algebraic equation similarity.

Lastly Mayer's own work has focused on the issue of schema knowledge or problem types. In a first study (Mayer, 1981), he reviewed hundreds of verbal problems from 10 algebra textbooks. He identified 100 different problem types, or what he called templates, where each template has its own distinctive solution procedure or equation formulation. For example he identified 10 templates or versions of "current" problem type and 13 templates of "motion" problem type. It is not expected that students will organize and store verbal problem

content as

due to the

appears th

storage to

"problem t

umbrella o

related al

one unknow

In a

subjects r

recall the

noted freq

type and t

found that

textbooks

less commo

the correl

reached .8

systematic

statement

version of

their own

almost all

frequency

found in te

experiments

content as distinctive templates within long term memory due to the sheer number (100) of such templates. It appears that a more efficient and economical way for such storage to occur would be to group templates into isolated "problem types", which in turn are grouped under a larger umbrella or net based on common mathematical structure or related algebraic elements, such as linear equations in one unknown.

In a companion set of experiments (Mayer, 1982), subjects read algebra verbal problems and later tried to recall their content. In the earlier study, Mayer had noted frequency norms for textbook verbal problems both by type and template, and within his subsequent experiments found that problem types that occurred more frequently in textbooks were more likely to be recalled correctly than less common problem types. In several subject analyses the correlation between frequency and correct recall reached .85. Additionally errors in recall were systematic, in that a low frequency verbal problem statement was changed to a more familiar, higher frequency version of the verbal problem statement. Students made up their own verbal problems in another experiment, and almost all such "created" problems resembled high frequency verbal problem statements, similar to those found in textbooks. The combined results of Mayer's experiments suggest that if a verbal problem is unusual or

does not c

schemata,

student, t

statement

which in a

solution,

In a

investigat

impossible

approached

Some stude

inconsiste

translated

noted or d

third grou

the proble

and soluti

indicative

is similar

processing

remarkable

across the

second, equ

whereas the

approach wa

investigato

does not correspond to any of a student's learned schemata, or such a schema is not accessible to the student, then a student may change the verbal problem statement to fit a more familiar verbal problem statement, which in all likelihood will lead to an erroneous solution, of which the student may be totally unaware.

In a similar vein, Paige and Simon (1966) investigated students' "solutions" for a small set of impossible algebra verbal problems. Their subjects approached these verbal problems in three distinct ways. Some students, almost immediately, noted the inconsistencies in the problem statement, other students translated the problem into equations, and only later noted or did not note the inconsistency present, and a third group, a rather large proportion, actually changed the problem statement in order that a reasonable equation and solution (erroneous) resulted. The third approach, indicative of a lack of available and accessible schema, is similar to what Mayer found to be true of student processing in such cases. Paige and Simon, noting a remarkable degree of consistency of student behavior across the problems of the "impossible" set, labeled the second, equation translation approach, analytic thinking, whereas the first, immediate inconsistency recognition approach was labeled geometric or real world thinking. The investigator predicts that most Algebra I students of

this stud
thinking
of a funct
will reco
problem.
impossible
error was
point or p
effort.
included c
these pred

In su
schema kno
suggests t
central, c
problems.
naturally,
instructio
of the rev
fail to gi
received i
instructio
would appe
problem re

this study will generally use the second, analytic thinking and equation translation approach initially, kind of a functional fixedness (Luchins,1942), while only a few will recognize an inconsistency in an impossible verbal problem. When this approach results in a ridiculous or impossible answer, most of these students will believe the error was in their work, and most likely will quit at this point or possibly adapt the third approach in a retrial effort. As many as two such problems were planned to be included on the posttest form by the investigator to test these predictions.

In summary, this review of research pertaining to schema knowledge or problem typing of verbal problems suggests that such knowledge is an important, and possibly central, component in student processing of algebra verbal problems. Students generally make such classifications naturally, and seemingly independent of provided instruction, though this last aspect was not tested in any of the reviewed studies. All the reviewed studies also fail to give adequate attention to a description of the received instruction of their subjects, or include even an instructional history profile of subjects, yet instruction would appear to have an impact on students' perceptions of problem relatedness.

Algorithm

With

refers to

algebraic

equations

issue in

procedure

required

applicati

was locat

section i

Newe

mastery o

having st

applicatic

deliberate

after rela

an assumpt

concentrat

whole chap

equation s

practiced

practice l

to related

true that

assumption

Algorithmic Knowledge

Within this investigation, algorithmic knowledge refers to student abilities to solve equations or apply algebraic procedures to define variables or to formulate equations which lead to verbal problem answers. The major issue in this area is whether mastery of algebraic procedures applied within the equation solving process is required before students are exposed to verbal problem applications of such procedures. Since little research was located which dealt with this particular issue, this section is brief.

Newer algebra textbooks seem to assume that such mastery of algebraic procedures is not required before having students use these procedures in verbal problem applications. Within these designs, the spaced and deliberate placement of verbal problem sets immediately after related procedural practice exercises indicates such an assumption. Older algebra textbooks tended to concentrate the placement of verbal problem sets into whole chapters, after chapters of all required procedural equation skills were completed, and in which students practiced these skills extensively, believing such practice lead to mastery and was required before exposure to related verbal problem applications. It is probably true that neither position is totally correct in their assumptions. It is also an area where teacher decisions

with rega
deviate f
teacher b
textbook'
In either
this inve
come fort
textbooks
instructi
able to r
variation
problem s

It i
an importa
problems
such knowl
completely
has often
mechanical
relative e
1980). Si
not sustai
also know
apply them

In a
distinct p

with regards to content sequence is somewhat likely to deviate from that provided by the textbook, depending on teacher beliefs, and based on experience with the textbook's organization and placement of verbal problems. In either case the issue is not resolved, though within this investigation some directions and implications may come forth from the data. If differences between textbooks' presentational sequences and teachers' instruction is sufficient, then the investigator may be able to relate differing classroom performance to variations in the instructional organization of verbal problem sets.

It is generally agreed that algorithmic knowledge is an important component skill required to solve verbal problems (Mayer, 1982). As it relates to verbal problems, such knowledge is necessary to "carry out the plan" completely, Polya's third stage. Mathematics instruction has often overemphasized algorithmic knowledge of the mechanical procedures in algebra and arithmetic to the relative exclusion of other forms of knowledge (Simon, 1980). Simon argues that algorithmic knowledge alone will not sustain creative problem solving, and students must also know "when" to apply operators as well as "how" to apply them.

In a set of studies, it was determined that two distinct processes were required to solve linear equations

of the ty
Larkin, a
were call
process i
both side
variable,
One might
called "t
combining
the equat
"simplifi
distribut
these proc
(Mayer, La
shown that
algorithmi
expect thi
algebraic
character.

Lewis
experts an
operators,
more compl
into a sin
such exper
than the r

of the type formulated in this investigation (Mayer, Larkin, and Kadane, 1980). The two classes of processes were called "move" and "compute" processes. The "move" process involves applying one of the four operations to both sides of an equation using the same number or variable, such as adding 11 to both sides of an equation. One might recognize this process as what was formerly called "transposing". The "compute" process involves combining numbers or variables, both on the same side of the equation, and may be expanded to include other "simplification" processes, such as multiplying out distributive expressions. The researchers found that these processes were fairly automatic for college students (Mayer, Larkin, and Kadane, 1980). Since research has shown that it typically takes from 2 to 3 years for school algorithmic processes to become automatized, one would not expect this investigation's students first exposure to algebraic techniques to contain such an automatic character.

Lewis (1981) found several differences between experts and novices in their application of algebraic operators, most notably the tendency of experts to use more complex procedures, and to collapse several processes into a single step of written work. It is expected that such expert process tendencies will be more the exception than the rule for the subjects of this study. Most

students

process a

In g

National

17 year o

performed

performan

items (Ca

that thes

rather th

are from

course ex

reviewed

olds with

admittedl

mathemati

On tw

required t

problem st

and equati

respective

three rela

algebraic

equation s

percent co

involved a

students will tend to apply a single algebraic operator or process at a time in their solving of equations.

In general the algebra related results of the Second National Assessment of Educational Progress indicated that 17 year old students with one year of algebra background performed about 10 percentage points above the mean performance for all 17 year olds taking the test on all items (Carpenter et al, 1982). It must be kept in mind that these results more likely measure algebraic retention rather than acquisition since most subjects (17 year olds) are from one to two years removed from their Algebra I course experience. The focus of this discussion of the reviewed assessment results is on this group of 17 year olds with one year of algebra background, though admittedly this group contains some students with more mathematics background.

On two reviewed exercises where students were required to write simple algebraic expressions from given problem statements (a skill related to variable definition and equation formulation) 73 percent and 57 percent respectively of this group had correct responses. On three related exercises which involved simplifying algebraic expressions ("compute" process skill related to equation solving) results ranged from 46 percent to 82 percent correct responses, where the easier two exercises involved addition only, and the most difficult exercise

involved
(7 - 8(3
solving a
reviewers
the equat

The
statement
formulate
173 = x).
formulate
solved th
from two
For a sta
one was e
and the s
solutions.
papers we
formulatic
problem wa
the first

The a
one consid
had additi
many first
evident, a
about 20 p

involved subtraction within a distributive framework (7 - 8(3 - y)). On the one examined exercise related to solving a linear equation (considered "typical" by the reviewers) only 45 percent of this group correctly solved the equation.

The reviewers also noted a simple verbal problem statement of three sentences which asked students to formulate an appropriate equation (173 + x = 241 or 241 - 173 = x). Only 54 percent of this group correctly formulated the equation, whereas 86 percent correctly solved this same equation on a different exercise. Results from two standard verbal problem exercises were given. For a standard number problem, involving two numbers where one was explicitly given as three times the other number and the sum given, only 38 percent found correct solutions. It was noted for this exercise that many papers were left incomplete despite correct equation formulations being noted. Performance on a standard "age" problem was about half (20 percent correct) for that of the first verbal problem (Carpenter et al, 1982).

The assessment results are not very encouraging when one considers that as many as half of this group may have had additional mathematics experience. The mastery of many first year algebra topics and procedures was not evident, and verbal problem applications performance was about 20 percentage points lower yet. The authors

concluded
and stude
better to
until stu
abstract
Algebra I
course in
may sugge
opportuni
second po
in the hi
runs coun
Soviet Un
and 8 are
though the
mathematic
capability

In su
importance
solve equa
is believe
equation s
testing in
related co
be capable
minute cla

concluded that if present instructional practices continue and student performance remains stable, then it may be better to postpone algebra from the typical freshman level until students are more ready to deal with algebra's abstract nature. Another view expressed was that the Algebra I course might be considered a rite-of-passage course into mathematical abstraction where its foundation may suggest more rapid learning of algebra at the next opportunity (Algebra II) (Carpenter et al, 1982). The second position would suggest requiring 2 years of algebra in the high school curriculum, while the first position runs counter to Soviet curriculum practices. In the Soviet Union, it has been shown that students in levels 7 and 8 are capable of mastering algebra's abstraction, though the differing nature of both systems' elementary mathematics programs may account for this earlier capability.

In summary, the investigator is aware of the importance of algorithmic knowledge and student ability to solve equations in the total verbal problem process. It is believed that a short measuring instrument related to equation solving be included within one of the two major testing instruments (posttest or retention test). A related concern however is that both test designs have to be capable of completion by most students within a 40 minute class time frame. Both concerns are handled in a

unique wa

Strategic

This

investiga

within Ma

knowledge

plans for

since his

related t

technique

This note

analysis.

are strat

procedure

well defi

a general

algorithm

sequence

term, "heu

hints whic

problem sc

definition

heuristic

suggestion

(Goldin, 1

The p

unique way, and will be described in Chapter III.

Strategic Knowledge

This category of knowledge is broadened by the investigator as compared to its formulation and discussion within Mayer's (1982) model. Mayer defines strategic knowledge as varied techniques which help students develop plans for solution. There is an inconsistency however since his discussion focuses mainly on solution strategies related to equations, a rather minor part of the class of techniques which help students plan and process problems. This noted inconsistency is resolved within task variable analysis. The solution strategies of Mayer's discussion are strategies in a task variable sense, because they are procedures which narrow the set of possible moves to a well defined subset for consideration. A strategy then is a generalization of an algorithm, differing from an algorithm in that it does not single out a unique move or sequence of moves as an algorithm does (Goldin, 1984). The term, "heuristics" is reserved for general procedures or hints which help discovery and plan development during problem solution (Kulm, 1984), which is similar to Mayer's definition of this knowledge category. In many cases, a heuristic process often times precedes and initiates by suggestion a particular solution strategy or algorithm (Goldin, 1984).

The primary focus of this section is on research

which has
of heuris
statement
processes
related t
processes
related t
out of he
the end o

A he
"device",
"path" (K
methods"
and impro
verbal pr
suggestio
skill, st
variables
stages, p
"looking
problem s
affecting
various m
(Kulm, 19

The c
renewed re

which has identified and described the natural existence of heuristic processes inherent within verbal problem statements and which subjects use, and the effects such processes have on problem solving performance. Research related to the instructional development of heuristic processes is reviewed later in this chapter. Issues related to metacognition and subject beliefs, which grew out of heuristic process research is briefly reviewed at the end of this section.

A heuristic has been variously described as a "device", "technique", "rule of thumb" (Kilpatrick, 1967), "path" (Kantowski, 1975), and "efficient means and methods" (Polya, 1962) which activate mental processing and improves problem solving performance. With regards to verbal problems, inherent heuristics serve as hints or suggestions which initiate student activity, at either a skill, strategy, or algorithm level. Heuristic behavior variables are conjectured to affect all four of Polya's stages, particularly those of "devising a plan" and "looking back". Heuristic processes bridge the verbal problem statement and problem representation, as well as affecting transitions between problem statements and various mental and physical representations and solutions (Kulm, 1984).

The concept of heuristic reasoning has received renewed research interest since Polya's (1957)

reintrodu

"infancy"

of this d

classific

the inves

classific

content.

Lest

categorie

specific

general h

of tasks)

disjoint,

process r

An i

incorporat

heuristics

developed

three face

and task s

is its bro

In hi

Schoenfeld

processes,

Subject ap

reintroduction of the concept, yet it remains in an "infancy" stage of development (McClintock, 1984). Part of this difficulty relates to the description and classification of heuristic processes. An initial task of the investigator was to blend three located classifications which all relate to verbal problem content.

Lester (1980) classified heuristics into three categories, specific tool skills (physical actions), task specific heuristics (advice that is problem specific), and general heuristics (advice applicable to a broader range of tasks). The three classes are rather distinct and disjoint, which has advantages for the review of heuristic process research.

An information processing perspective which incorporated both general heuristics and task specific heuristics was the "General Problem Solver" (GPS) developed by Newell, Shaw, and Simon (1963) which had three facets; planning heuristics, means-ends analysis, and task specific heuristics. The advantage of this model is its broad coverage of the area.

In his discussion of heuristic process instruction, Schoenfeld (1984) identified two classes of heuristic processes, "cued" heuristics and "stage" heuristics. Subject approaches to problems or classes of problems in

which the
processes
such cuei
consisten
particula
and these
(Schoenfe
classific
solving m
investiga
student h
four stag

An a
the three
theory fo
verbal pr
analysis,
replaceme
and was i
movement
states, w
state. Pc
and recomb
and a simi
Kantowski
stages and

which the subject is cued to the patterned use of specific processes are called "cued" heuristics. In the absence of such cueing, heuristic processes often cluster consistently into groups which are usually applied at particular stages (in Polya's sense) of problem solutions, and these clusters are called "stage" heuristic processes (Schoenfeld, 1984). The primary advantage of Schoenfeld's classification is that it relates to Polya's problem solving model, which has been central to this investigation's development, and also serves to explain student heuristic behavior and performance within Polya's four stages.

An attempt is now made to integrate and interrelate the three classifications in order to formulate a coherent theory for how heuristic processes influence algebra verbal problem processing and performance. By means-ends analysis, Newell, Shaw, and Simon (1963) meant the replacement of a goal by a sequence of easier subgoals, and was indicated by a somewhat general and planned movement from the initial situation through intermediate states, which are gradually closer to the final goal state. Polya (1957) identifies the idea of decomposing and recombining as a general strategy in this approach, and a similar pattern of analysis/synthesis was noted by Kantowski (1977). In such a framework, Polya's four stages and their completion are analogous to subgoals

leading

for mean

Pla

are conc

allow fo

abstract

detailed

stages o

each requ

"stage" h

form and

general h

investiga

hints app

heuristic

to Schoen

isolated

activate

1980), su

defining

the exter

reasoning

To a

two figur

drawn fro

schematic

leading to a solution, and serve as a guiding framework for means-ends analysis of algebra verbal problems.

Planning heuristics (Newell, Shaw, and Simon, 1963) are conceptualized as systems of heuristic processes which allow for the construction of a potential solution in an abstract form, prior to carrying out the concrete, detailed solution actions. If one considers the four stages of Polya's model as subgoals, as was done above, each requiring resolution, then planning heuristics and "stage" heuristic processes bear close resemblance both in form and application. It is less clear where Lester's general heuristics fit in this formulation, though the investigator considers both Polya's model and general hints applicable for all verbal problem types as general heuristics. Lastly task specific heuristics are similar to Schoenfeld's (1984) "cued" heuristics, in that they are isolated units and verbal problem specific hints which activate various forms of specific tool skills (Lester, 1980), such as making tables, drawing diagrams, or defining variables. In other words, the tool skills are the external subject actions resulting from heuristic reasoning.

To additionally support this theoretical framework, two figures, Figure 2.1 and Figure 2.2 are included both drawn from Schoenfeld's work. Figure 2.1 is a copy of his schematic overview of the problem solving process

(Schoenf

indicate

processe

model.

indicate

product

student

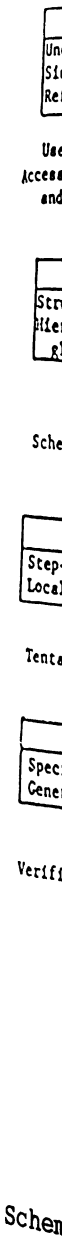
model in

Schoenfe

student

next sec

(Schoenfeld, 1984, p.447), which uses a flowchart to indicate the dynamic nature of the "stage" heuristic processes, and their relation within Polya's 4 stage model. The circles representing various solution forms indicate the subgoal nature of each stage, and also the product or external nature of each stage's completion. The student actions indicated within each stage of Polya's model in Table 1.1 relate to these product circles of Schoenfeld's model. His overview and discussion of student problem processing is discussed more fully in the next section related to instruction.


Un
Si
Re
Use
Access
and
Str
Sche
Step
Local
Tenta
Spec
Gene
Verifi

Schem

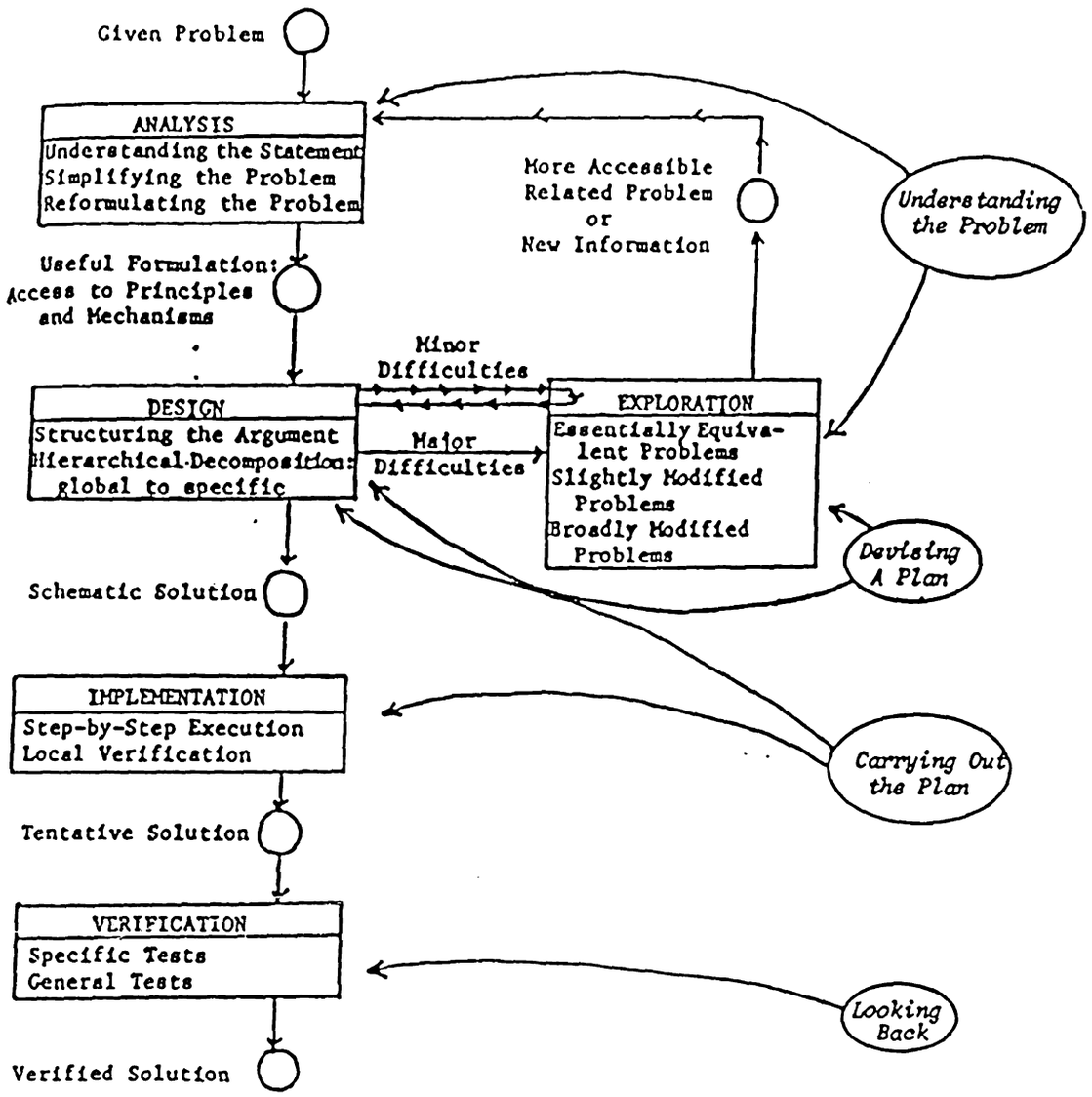


Figure 2.1

Schematic Overview of the Problem Solving Process

Fig

(1976) h

(1984, p

is expan

classifi

useful f

solution

at which

subject

related

Figure 2.2 represents a modification of Schoenfeld's (1976) heuristic problem solving model done by Lucas et al (1984, p.357). The number of processes within each stage is expanded and made more specific, while the stage classifications are nearly identical. The four ovals, useful formulation, schematic solution, tentative solution, and verified solution suggest the various points at which problem solving can be measured, particularly subject written work exemplifying specific tool skills related to verbal problem solution (Lucas et al, 1984).

ANALYS

DESIGN

IMPLEMENT
D
V
A

VERIFICATI
Ve
Ch
Ch
De
Ref
Inv

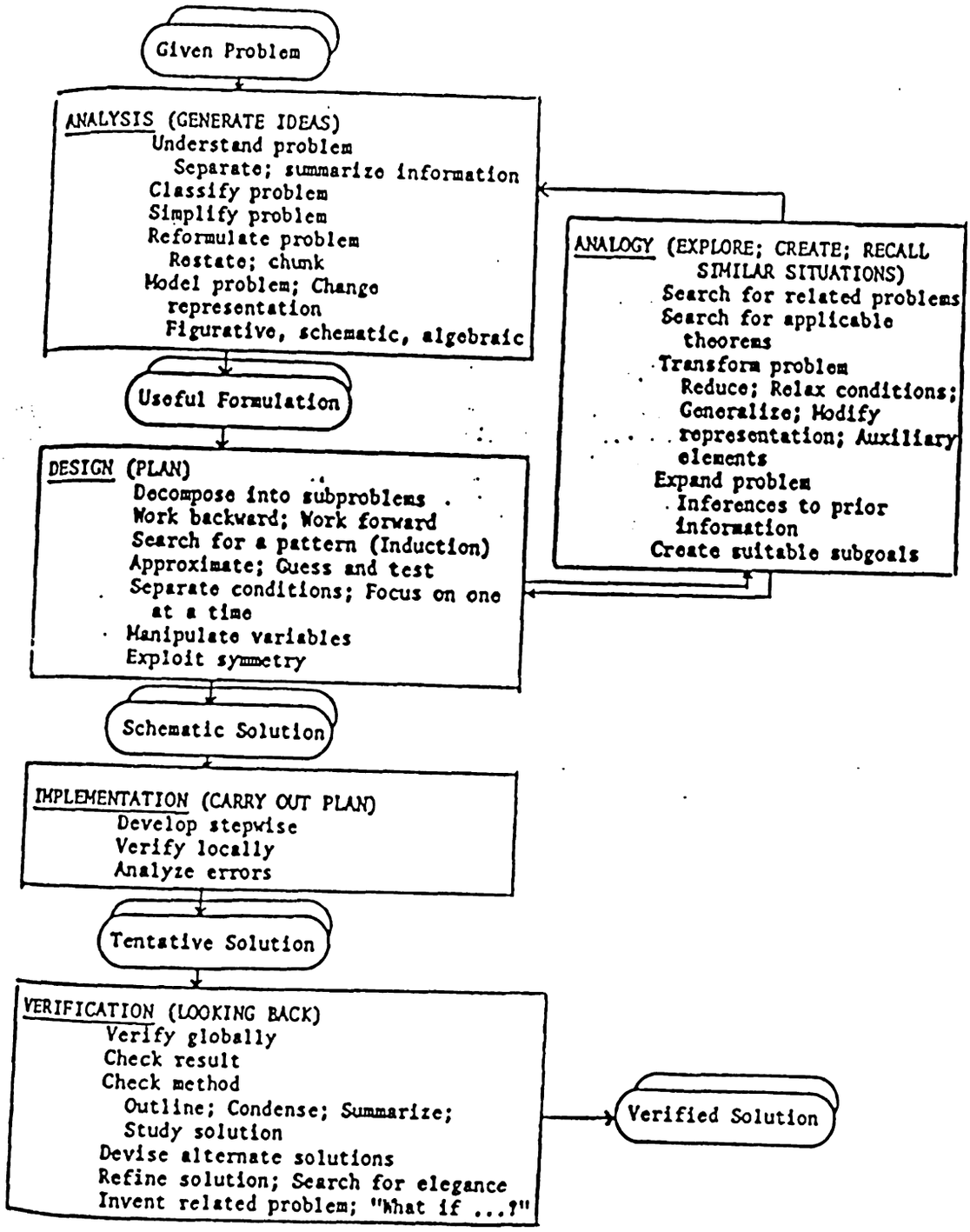


Figure 2.2

Adapted Heuristic Problem Solving Model

Res
identifi
(noninst
to the e
were con
all incl
task var
content
"thinkin
heuristi
forms or

Mucl
the sens
analyzin
classifi
heuristi
determine
heuristi
aptitude
and refi
Polya int
system, a
subjects
processes
codable.
figure, 2

Research studies in which the natural existence, identification and description of heuristic processes (noninstructional studies) can now be reviewed and related to the elements of the theoretical framework. All studies were considered exploratory, rather than experimental, and all include some verbal problem content as their primary task variable, though the subjects' ages and mathematical content domains varied considerably. All studies employed "thinking aloud" techniques for acquiring data on heuristic processing and developed process-sequence coding forms or checklists to collect the heuristic process data.

Much of Kilpatrick's (1967) work was developmental in the sense of creating mechanisms (methodology) for analyzing protocol material. His aim was to develop classifications of students according to their employed heuristic processes in solving verbal problems, and to determine how these classifications of students based on heuristic process patterns related to achievement and aptitude variables. Major effort was given to developing and refining the heuristic processes from the work of Polya into a process checklist and a process-coding system, applicable for the 56 above average eighth grade subjects of the study. He identified seven heuristic processes used by his subjects which were reliably codable. The seven heuristic processes were: 1) drawing a figure, 2) using successive approximation (guess and

check), 3

4) using

using tri

can envis

three maj

productio

represent

his revie

four of t

frequentl

unrelated

of other

trial and

success i

and erro

achieveme

or three

subjects

attempted

end, with

basis of

successiv

noted tha

achieveme

Webl

his explo

check), 3) questioning existence/uniqueness of solution, 4) using a deduction process, 5) using an equation, 6) using trial and error, and 7) checking the solution. One can envision where each of these processes fit into his three major divisions of processes: preparation, production, and evaluation, where preparation seems to represent a collapsing of Polya's first two stages. In his review of the problem solving protocols however, only four of the seven heuristic processes occurred more frequently, the drawing of figures, which was found to unrelated to success in problem solving or subject usage of other processes, the use of successive approximation trial and error (guess and check), which did relate to success in solving problems, the frequent usage of trial and error techniques, which did correlate with mathematics achievement, and checking work, which was only used on two or three specific problems. Finally he classified his subjects into four equal sized groups according to their attempted usage of an equation at least once at the upper end, with the three lower groups differentiated on the basis of decreasing usage of trial and error and successive approximation heuristic processes. He also noted that, in moving down the four groups, mathematics achievement and aptitude decreased at each level.

Webb (1975) modified Kilpatrick's coding system for his exploratory study involving 40 students in a second

year alg
Kilpatri
problem
categori
general,
From his
mathemat
to the p
of the v
larger p
mathemat
percenta
percent)
heuristi
more of
process.
problem
that bet
eight pr
frequent
Subjects
instrume
each use
few struc
intermed
equation

year algebra class. His aims were similar to those of Kilpatrick's study, being to determine relationships among problem solving ability and heuristic processes, categorize heuristic processes as problem specific or general, and cluster students according to process usage. From his regression analysis results, Webb found that mathematics achievement was, by comparison, highly related to the problem solving scores, accounting for 50 percent of the variance in total scores. He found a relatively larger percentage of variance solely attributable to mathematics achievement (20 percent), and a smaller percentage attributable to heuristic processes (13 percent), with a "common shared", 40 percent. The heuristic process, pictorial representation, accounted for more of the heuristic process variance than did any other process. Webb also found that most heuristics were problem specific (according to his classification), and that better problem solvers used a wider range of the eight problem specific heuristics he identified as used frequently by subjects and of high intercoder reliabilty. Subjects performed best on a verbal problem solving instrument if equations and trial and error processes were each used moderately to supplement one another, and where few structural solution errors were noted. More intermediate performance was noted for subjects who used equations more frequently and trial and error processes

less fre

error pr

frequent

Kilpatri

processi

and achi

subjects

instruct

techniqu

Bla

processe

of algeb

schools

were sim

the heur

associat

verbal p

heuristi

forms.

Polya's

defined,

Templati

their res

of schema

was defi

isomorphi

less frequently. The least adept subjects used trial and error processes more frequently and equations less frequently. This last result, which conflicts with Kilpatrick's finding that the frequency of trial and error processing was positively related to mathematics aptitude and achievement, is quite understandable given that Webb's subjects had received three additional years of instruction and practice in the usage of algebraic techniques, including equations.

Blake (1976) like Webb, examined the problem solving processes of 40 eleventh grade subjects in a second course of algebra. The subjects were selected randomly from 14 schools and had an I.Q. range of 115 to 125. His aims were similar to the earlier reviewed studies, identifying the heuristic processes used by algebra students and the associated patterns of such usage within the solution of verbal problems. His coding system included 12 basic heuristic processes, each containing several subdivided forms. Most of these heuristic processes were taken from Polya's heuristic list, with three "new" processes defined, templation, smoothing, and preservation. Templation included recalling of related problems for their results and methods' incorporation, actually a form of schema knowledge within this investigation. Smoothing was defined as altering a verbal problem to produce an isomorphic problem, and preservation dealt with extending

system p
form lai
in an or

Bla
and know
accounte
and 40 p
percent
and erro
cases",
to corre
contrast
and rand
processe
one four
decompos
All othe
infreque
of heuri
heuristi
verbal p
solution
and the
templati
"ineffect

system properties in a consistent fashion. Blake's coding form laid out these heuristic processes (and subdivisions) in an orderly way according to Polya's 4 stages.

Blake's findings indicated that heuristic processes and knowledge of mathematical content ("core" knowledge) accounted for approximately equal amounts of variance (41 and 40 percent respectively) with a "common -shared" 20 percent of variance. He found considerable usage of trial and error processes, particularly what he called "random cases", and found that such process usage related closely to correct solutions of the verbal problems, which contrasted with the results of Webb's study. Templation and random cases were the two most frequently noted processes, used by three fourths of the subjects. About one fourth of the subjects used analysis (problem decomposition), smoothing, and successive approximation. All other, included heuristic processes were observed infrequently. Blake concluded that both the total number of heuristic processes used and the number of different heuristic processes used related positively to successful verbal problem solving, and that subjects' attempts at solution relied heavily on "core" (mathematical content) and the most frequently used heuristic process, templation. He also believed students were largely "ineffective" in their usage of templation.

The
Kantowsk
teaching
instruct
was give
explorat
heuristi
included
Kilpatri
could be
analysis
first ye
processe
problem,
analysis
greater
No such
such as
and erro
(relatio

Kant
for her
ninth gr
heuristi
The 8 st
trained

The last two reviewed studies, Lucas (1972) and Kantowski (1975 and 1977), both involved a form of teaching, labeled heuristic teaching, in which explicit instruction of the maxims and components of Polya's model was given. However since both studies were clinical exploratory, rather than experimental studies, and had heuristic process identification as a major goal, they are included in this section's review. Lucas found that Kilpatrick's process-sequence coding form and checklist could be satisfactorily modified and extended for the analysis of verbal problem solving in the content area of first year calculus. He also found that five heuristic processes; kind of notation, applying method of related problem, applying result of related problem, reasoning by analysis, and organizing data were subject to change into greater usage through heuristic instruction or teaching. No such effect was found for other heuristic processes such as drawing/modifying diagrams, looking back, trial and error, reasoning by synthesis, or productivity (relationships, equations, or algorithmic processes).

Kantowski (1977) used geometry as a new content area for her "teaching experiment" with eight high ability ninth graders as subjects. The study had four phases and heuristic instruction was used for three units of content. The 8 students were tested frequently and were extensively trained to "think aloud" while solving problems during the

course c
subject
(modific
product
chapter)
the 44 p
individu
served a
solving
time.

Kan
heuristi
usage of
develope
processe
in solvi
indicate
regular
synthesi
heuristi
above th
of the s
interrel
goal ori
specific
heuristi

course of the study. The analysis of "thinking aloud" subject protocols included both a process sequence form (modification of Kilpatrick's scheme) and a process-product score (discussed more fully in methodology chapter). Process-product scores were assigned to each of the 44 problems solved by each of the eight subjects. Each individual subject's median process-product score served as a basis for judging relationships among problem solving process utility and ability level developed over time.

Kantowski's findings suggested that subjects given heuristic instruction were observed to increase their usage of heuristic processes as problem solving ability developed over time, and that increased usage of heuristic processes was directly and positively related to success in solving problems. She also found strong evidence to indicate that successful problem solvers exhibited more regular patterns of analysis (problem decomposition) and synthesis (problem recombination), both considered heuristic processes within her study. For those students above the median process-product score, 77 to 100 percent of the students' solutions showed such patterns. An interrelationship between this behavior pattern and other goal oriented heuristic processes (general or task specific) was also evident. Often a goal oriented heuristic process tended to initiate the regular pattern

of anal
occurre
This fi
theoret
heurist
verbal
regular
initiat
characte
use of
solutio
results
explicit
behavio
usage,
evidence

In
studies
generali
content
able to
process
process
the exce
modified
Kilpatri

of analysis and synthesis, and such combinations often occurred several times in a single solution to a problem. This finding relates closely with Schoenfeld's (1984) theoretical notions of "cued" heuristics and "stage" heuristics working in combination towards the solution of verbal problems, where the "stage" heuristics indicate regular patterns of student action (analysis/synthesis) initiated and aided by "cued" or task specific heuristic characteristics. Kantowski found some evidence for the use of related problems (schema knowledge) in problem solutions, where subjects were observed to use both results and methods of related problems. Finally, despite explicit instruction of "looking back" heuristic behaviors, she found little evidence of such process usage, nor was an increased usage of these processes evidenced during the course of the study.

In combining the results of these five reviewed studies with regards to heuristic processes, the following generalizations seem apparent. First, despite the varied content and subject ages of the studies, researchers were able to adapt process coding forms and in some cases process checklists capable of identifying heuristic process usage with primarily verbal problem content. With the exception of Blake's study, all reviewed studies modified Kilpatrick's (1967) model and maintained Kilpatrick's orientation of heuristic processes within

Polya's
include
design
analysi
1984).
the inv
heurist
problem
instruct
processi

Sec
small se
employe
investig
the patt
heuristi
Algebra
number c
these pr
specific

Thi
that pro
In both
further
"thinkin
scoring

Polya's 4 stage model. A task force meeting which included several of these researchers resulted in the design of a general process coding form and system for analysis of "thinking aloud" protocols (Lucas et al, 1984). This designed system provides a starting point for the investigator's design of an instrument to measure heuristic process characteristics of Algebra I verbal problem textbook instructional segments, teacher instructional segments, and student performance and processing within the interviews.

Secondly in each study, researchers identified a small set of heuristic processes which most students employed on a frequent basis. These results encourage the investigator to confine the search somewhat and focus on the patterns and combinations of frequent, student heuristic process usage for this sample of subjects in the Algebra I verbal problem domain. The expected small number of employed heuristic processes also indicates that these processes may be separable into general, task specific, and tool skill categories.

Thirdly the studies of Kantowski and Webb demonstrate that process-product scoring instrumentation is possible. In both studies, the scoring systems were used to make further quantitative comparisons between subjects' "thinking aloud" protocols. Such a process-product scoring system has also been used to analyze and evaluate

student
1980),
similar
the ver
instrum
perform
each ve

Fo
the heu
problem
sophist
heurist
a study
pairs o
mathema
grade s
(in a p
same he
They suc
of the s
found in
For both
processe
problems
noticeab
however,

student work samples on verbal problem tests (Zalewski, 1980), and the intention of the investigator is to similarly construct a process-product scoring system for the verbal problems used in this investigation's instruments, relating the varying degrees of student performance within five stages of written expression for each verbal problem test item.

Fourthly, most of the reviewed studies indicate that the heuristic processes employed by subjects on verbal problem tasks seldom approach the level of heuristic sophistication suggested by Polya's (1957) writings on heuristic techniques. This fact was also demonstrated in a study by Days, Wheatly, and Kulm (1979). They designed pairs of similar verbal problem tasks, which differed in mathematical structure (simple and complex) for eighth grade subjects. They found that both concrete and formal (in a Piagetian sense) subjects for the most part used the same heuristic processes on the simple structure problems. They suggested that the simple structure verbal problems of the study were typical of many of the verbal problems found in seventh and eighth grade mathematics textbooks. For both classes of subjects, the use of "higher level" processes was seldom noted for the simple structure verbal problems of the study. The formal subjects' advantage was noticeable for the complex structure verbal problems however, particularly their greater usage of more

sophist
process
assessm
capable
applica
which r
solving
structu

Fo

variabl
metacog
solving
variabl
experim
Silver,
metacog
expertis
primaril
mathemat

Met

one's ow
related
informat
believes
function
two comp

sophisticated production and evaluation heuristic processes. The study's results may also explain somewhat assessment results which indicate that students are more capable of solving routine, one-step verbal problem applications (typical of textbooks, simple structure) which require lower level processes, than they are at solving multi-step, nonroutine verbal problems (complex structure) which require "higher level" processes.

For the most part cognitive psychologists and task variable researchers shy away from discussion of metacognition and beliefs/values within their problem solving formulations since they are perceived as affective variables, and hence less capable of control within experimental studies. Three mathematics educators, Silver, Schoenfeld, and Lester however all believe that metacognition plays an important role in problem solving expertise. The discussion here will be brief and primarily theoretical, since little formal research on mathematical metacognition has been completed to date.

Metacognition "refers to one's knowledge concerning one's own cognitive processes and products or anything related to them" or "the learning-relevant properties of information or data" (Flavell, 1976). Lester (1982) believes that successful mathematical problem solving is a function of at least five component abilities, of which two component abilities are related to metacognition:

1) know
during
maintai
employe
with th
Adams (
ignored
believe
1982).

Si
researc
relatio
ability
student
conside
in whic
organiz
of thei
to them
argument
longer
differer
regulate
Lastly,
processe
attainin

1) knowledge about one's own cognition, before, after, and during a problem solving episode, and 2) the ability to maintain executive control (monitor and regulate) of the employed procedures during problem solving. He notes that with the exception of the work of Silver, Branca, and Adams (1980), problem solving researchers have largely ignored metacognitive behaviors, because they either believe them to be unimportant or unresearchable (Lester, 1982).

Silver, Branca, and Adams reviewed the available research related to metacognition, and generated several relationships between metacognition and mathematical ability. Their review suggests that high ability students' better apprehension of problem structure may be considered metacognitive planning and memory regulation, in which students prepare for elicited retrieval by better organizing incoming information into "chunks" on the basis of their experience of what is important and most useful to them in the future (problem structure and generalized arguments). They also note that high ability students' longer retention of mathematical content is likely due to differences in the way students organize information and regulate storage in memory, both metacognitive functions. Lastly, they note that it appears that awareness of mental processes appears to be a necessary condition for attaining expertise in a task domain. Once it appears

that a
solving
become
be larg
develop
signifi

Sc
with no
educato
effects
college
tasks w
he calle
similar
in "stag
processe
not expl
episodes
decisior
metacogn
ability
various
abandon
subject'
work wel

that a student has attained a high level of expertise in solving mathematical problems, metacognitive behaviors become so integrated into problem solving routines as to be largely unobservable, though their role in the development of problem solving ability is likely to be significant.

Schoenfeld's studies of college level problem solving with nonroutine tasks probably instigated mathematics educators to take a closer look at metacognition and its effects on mathematical problem solving. He found that college mathematics students and faculty attacked such tasks within a series of self-contained sequences, or what he called episodes (Schoenfeld, 1983). There is some similarity between his notions of episodes and stages (as in "stage" heuristics). He found that heuristic processes, such as cued or task specific heuristics, could not explain transitions or decision points between episodes, but rather that managerial and executive decisions were being made, which he equated with metacognition. These executive decisions required an ability to know when to apply and when not to apply various heuristic reasoning techniques, and also when to abandon such techniques. He also noted that when a subject's problem solving schemata (or productions) do not work well on a task, the necessary managerial skills take

over ar

solving

Sc

experti

(Schoen

possess

classes

specifi

nonrout

thought

has acc

demonst

experti

access

by the

manage

and heu

explici

distinc

minimal

to this

the cont

necessa

instruct

interest

expertis

over and serve to constitute what he means by problem solving expertise (Schoenfeld, 1983).

Schoenfeld differentiates between problem solving expertise as opposed to problem solving proficiency (Schoenfeld, 1982). He considers proficiency to mean the possession of a large number of schemata for generic classes of tasks in a domain, or what he calls domain specific proficiency. Most tasks, including even some nonroutine tasks capable of heuristic solution, can be thought of as schema driven types, in which the subject has access to the appropriate schema, and hence is demonstrating proficiency. On the other hand, defining expertise requires that the subject does not have ready access to a solution schema. Expertise is measured rather by the qualitative ability of the subject to utilize and manage available resources, including content knowledge and heuristic processes in deriving solutions. Though not explicitly stated by Schoenfeld, one can imply from these distinctions that metacognitive behavior probably has minimal impact on the verbal problem proficiency related to this investigation, due to the schema driven nature of the content and the assumed access and availability to necessary schemata provided by teacher and textbook instruction. This investigation is then primarily interested in measuring student proficiency, rather than expertise. The usage of metacognitive behaviors is not

expecte
student
profici
behavio
such me
aloud"
each cl

Sc

separat
profici
(1982)
conside
mathema
a broad
theoret
metacog
been de
outline
implica
and als
questio
prompts
Schoenfe
experime
methodo
hope of

expected for the majority of this investigation's students, but rather it is assumed that only the most proficient and successful students might engage in such behaviors. Therefore investigator attempts to identify such metacognitive behaviors is limited to the "thinking aloud" interviews of the three most capable students of each classroom.

Schoenfeld (1985) views beliefs/values as a totally separate variable effecting problem solving expertise and proficiency, whereas other researchers such as Silver (1982) and Callahan and Garofalo (1987) have tended to consider personally held knowledge and beliefs about mathematics and problem solving as being contained within a broader definition of metacognition. In fact, a theoretical student processing model which incorporates metacognitive behaviors at each stage of processing has been developed (Garofalo and Lester, 1985). Silver (1982) outlines four possible beliefs which might have important implications for how one approaches mathematical problems, and also suggests that many of Polya's heuristics and questioning routines might be considered metacognitive prompts. Yet student beliefs, such as Silver and Schoenfeld (1985) suggest, remain largely untestable in experimental research settings with present day methodology. It appears to the investigator that the only hope of identifying and analyzing beliefs and values

within
intervi
immedia
of such
solving

It
of obse
instruc
researc
instruc
the dev
even be
educato
of prob
instruc
general
be a be
which e
of most
on mathe
largely
and the
(Lester,
solving
Polya's

within this investigation would be in the "thinking aloud" interview settings, where student oral comments can be immediately probed by the investigator to reveal the depth of such beliefs and their impact on verbal problem solving.

Instructional Studies

It appears that this investigation by its inclusion of observations of teachers' verbal problem solving instruction is moving into rather virgin territory. Some researchers believe that a "well founded theory of instruction" for mathematical problem solving is still in the developmental stage (Webb, 1979). This conclusion may even be an overstatement, since several mathematics educators (Silver, Lester, and Grouws) have noted the lack of problem solving research studies in which teaching and instruction occupy the central focus of study. There is a general belief that a subject can be taught or trained to be a better problem solver, and identifying instruction which enhances student problem solving success is an aim of most problem solving research. However, the literature on mathematical problem solving instruction remains largely based on folklore, grounded in teacher intuitions, and the sage advice of master teachers, such as Polya (Lester, 1982). Besides influencing the study of problem solving processes, as was noted in the previous section, Polya's work has its most profound impact on the study of

instruc

are bel

solvers

has app

Th

present

behavio

automat

represe

(Schoen

problem

problem

represe

to what

behavio

accesse

behavio

behavio

process

for most

verbal p

not avai

that oth

student

resource

bring th

instruction in heuristic methods. His prescriptive ideas are believed to help students become better problem solvers by suggesting a simplified four stage model which has applicability for instruction (Lester, 1980).

The model of student verbal problem processing, presented in the last section, assumes that most students' behaviors and performance are attributable to an almost automatic and mechanical accessing of "situational representations and associated sets of responses" (Schoenfeld, 1985, p.365), or what Mayer terms schema or problem type knowledge. Certain features of the verbal problem statement seem to automatically activate a typical representation which brings along a set of expectations as to what are routine, ordinary, and productive heuristic behaviors. Such routine access has been called "schema accessed" (Schoenfeld, 1985), and such student mechanical behavior as "schema driven" (Silver, 1982). Such student behavior saves time and trouble, and allows for efficient processing, and is believed by the investigator to account for most students' behaviors in their attempts to solve verbal problems. It is only when such direct access is not available, such as with less familiar verbal problems, that other heuristic processes, metacognitive skills, and student beliefs assist students in gathering all the resources at their disposal in an efficient manner, and bring this knowledge to bear on a verbal problem. This

assumpt

(1985)

for rev

from hi

process

Fu

compari

success

poorly

benefic

in appl

as was

(AMPS)

possess

systems

Rather

heurist

construc

In

Mayer (

translat

and algo

instruct

listed i

mainly r

model of

assumption is the primary reason for selecting Mayer's (1985) instructional training model as a focusing point for reviewing instructional research, since it follows from his earlier (Mayer, 1982) model of student processing.

Furthermore, this investigation involves analyzing comparisons between average-ability novices who perform successfully and average-ability novices who perform poorly on given verbal problem tasks, while focusing on beneficial and nonbeneficial behaviors of the two groups in applying mathematical concepts to verbal problems, much as was done in the Applied Mathematical Problem Solving (AMPS) project (Lesh, 1985). As novices, they do not yet possess the well organized conceptual and procedural systems for producing a solution as experts might possess. Rather such systems of understandings, skills, and heuristics are "unstable", or in various stages of construction or development (Lesh, 1985).

In what might be classified as a position paper, Mayer (1985) outlined four areas of instruction: translation training, schema training, strategy training, and algorithm automaticity. Within each area of instruction, he briefly reviewed related research and listed instructional implications. The reviewed research mainly restated that given for his earlier theoretical model of student processing (Mayer, 1982), and included

few in

his fo

locate

model.

relati

implic

hypoth

instru

suggest

few instructional studies of any sort. This weakness of his formulation hampered investigator attempts to classify located instructional studies within his instructional model. Since his cited research had little instructional relationship to the proposed, listed implications, such implications would be better termed, intuitions or hypotheses. Table 2.2 is a reprint of Mayer's instructional model, and includes his implications as suggestions (Mayer, 1985, p. 126).

Issues

Transl
compi
solut

Schema
unde
exec

Strateg
proc
prod

Algorit
proc
resp

In

that May

Table 2.2

Mayer's Problem Solving Instructional Model

Issues	Suggestions/Impications
Translation Training: comprehension versus solution	Let students draw pictures or move blocks to represent propositions. Ask students to rephrase propositions. Ask students to reword propositions into another context. Ask students to derive equations or number sentences for problem sentences.
Schema Training: understanding versus executing	Mix problem types in exercises. Ask students to recognize problem types. Ask students to select relevant and irrelevant information. Ask students to draw or state problems.
Strategy Training: process versus product	Let students describe their solution strategies. Let students compare their solution process to that of "experts" such as in worked out examples. Provide direct instruction in strategies for specific problems.
Algorithm Automaticity: procedures versus response	Provide practice in basic algorithms before moving to complex algorithms. Evaluate and remediate performance in algorithms in terms of bugs in subprocesses.

In a follow-up reaction paper, Sowder (1985) notes that Mayer's focus on verbal problems and on studies in

In

that Ma

which s

severel

model f

solving

importa

problem

between

Sowder

process

major d

schema

search

problem

between

Transla

Th

reviewe

linguis

and syn

model,

shown i

field d

some me

and pri

In a follow-up reaction paper, Sowder (1985) notes that Mayer's focus on verbal problems and on studies in which subjects have had significant training in algebra severely limits the usefulness of Mayer's instructional model for the broader area of mathematical problem solving. Sowder does believe that verbal problems are an important class of tasks, and represent an "in-between" problem category or subset of routine problems, lying between exercises and what he calls "genuine" problems. Sowder developed two diagrams of simplified sequences of processing for routine and genuine problems, with the major difference being that genuine problems required a schema search, as opposed to routine problems where such a search is unnecessary due to the finding of a related problem schema. He also notes a similar distinction between proficiency and expertise much as Schoenfeld does.

Translation Training

This section is subdivided into three clusters of reviewed instructional research related to: prerequisite linguistic and factual knowledge (mathematical vocabulary and syntax), alternate approaches to Polya's 4 stage model, and specific tool skill instruction. It has been shown in expert-novice studies that problem solving in any field demands some minimal grounding in content, including some memorization of vocabulary, concepts, relationships, and principles, and that such memorized knowledge is a

necessa
solvin
geomet
explana
to solv
little
princi
both o
Kantows
content
and she
the use
problem
Webb's
problem
order f
concept

In
their b
forms.
committ
usage,
used de
sheets
sketche
teacher

necessary, but not sufficient condition for problem solving to occur (Larkin, 1981). Larkin noted that geometry and physics textbooks contained adequate explanations of principles in its instructional segments to solve associated problems, yet the textbooks presented little information about how to search for a useful principle or how to determine principle applicability, both of which can be considered heuristic behaviors. Kantowski (1979) noted that such minimal grounding in content would have to be specific to the content involved, and she believed that such grounding was necessary before the use of heuristic techniques served any value to the problem solver. This line of reasoning runs counter to Webb's (1979) contention that some sort of heuristic problem solving capabilities must exist before and in order for concepts to be acquired, or that acquisition of conceptual knowledge is dependent upon heuristics.

In Kantowski's study, students were allowed to use their books and reference sheets on the various test forms. She found however that on these tests, theorems committed to memory were easily activated for solution usage, whereas those theorems not recalled were often not used despite the fact that the textbooks and reference sheets were available for student use. Though only sketched briefly, the available research indicates that teachers need to review in advance the mathematical

also ne
explici
to dete
memoriz
Th
stateme
regress
verbal
structu
amounts
correct
review
found h
problem
it is g
problem
teacher
simples
general
1984).
then be
verbal
charact
easing
Ma
as char

also need to review and discuss this information explicitly with their students, thereby helping students to determine which aspects of this information should be memorized prior to heuristic and tool skill instruction.

The linguistic and syntactic nature of verbal problem statements was investigated by Jerman (1971). Using regression analysis techniques, he found that for algebra verbal problems possessing a standard underlying structure, syntax variables accounted for significant amounts of variance in the number of subjects achieving correct numerical solutions. Barnett (1984), in his review of two algebra verbal problem studies, similarly found high loadings for syntax variables related to problem statement length and word length. In this regard, it is generally agreed that textbook algebra verbal problem statements, which make up the primary content for teacher instruction of verbal problems, use the shortest, simplest, and most direct language possible, and are generally devoid of irrelevant information (Caldwell, 1984). The only implication on teacher instruction would then be that teachers exhibit some care that test and quiz verbal problem designs exhibit the same syntax characteristics of studied verbal problems, therefore easing student understanding of verbal problem statements.

Many years of exposure to verbal problem statements as characterized above often leads to students believing

M

as cha

that a

statem

A prog

belief

stress

of ver

direct

algebr

of the

formul

or sys

are pr

a deve

Dahmus

percei

used i

instru

verbal

student

Ba

effecti

to Poly

process

individ

Many years of exposure to verbal problem statements as characterized above often leads to students believing that all given information or data of a verbal problem statement must be used in formulating a correct solution. A program which attempted to capitalize on this student belief was developed by Dahmus (1970). His approach stresses a "direct-pure-piecemeal-complete " (DPPC) attack of verbal problem statements. A student is encouraged to directly translate problem statement information into algebraic expressions by concentrating on only a few words of the problem statement at a time. Eventually these formulated expressions are put together to form equations, or systems of equations. The direct translation processes are primarily driven by instruction of "key words". From a developmental and instructional history perspective, Dahmus's approach has appeal, since it builds on what is perceived to be the key words approach to verbal problems used in earlier grades. Dahmus (1970) claimed that his instructional approach was successful in improving algebra verbal problem solving performance with a small sample of students.

Bassler, Beers, and Richardson (1975) tested the effectiveness of Dahmus's instructional model as opposed to Polya's 4 stage instructional model and related processes with 48 ninth grade students as subjects. Seven individualized, instructional units for each approach were

devise
measur
equati
instru
criter
on the
variab
less s
relate
conclu
eighth
of Pol
that i
repres
these
seems
from a
perspe
approa
the sa
T
approa
algebr
relati
explic
One al

devised and implemented. For the verbal problem posttest measure analysis, two criterion scores were developed, an equation score and a solution score. The Polya instructional group scored notably higher on the equation criterion, while no differences were noted between groups on the solution criterion. Both groups capably defined variables, yet the Dahmus instructional approach group was less successful in combining data and variables into related equations. The researchers tempered their conclusion somewhat after examination of seventh and eighth grade textbooks, which revealed explicit treatment of Polya's model within these textbooks. They concluded that instruction using Dahmus's approach may have represented a new, and possibly conflicting technique for these students based on their instructional histories. It seems that both instructional approaches have some appeal from a developmental and instructional history perspective, and explains somewhat why aspects of both approaches are generally included and can co-exist within the same Algebra I textbook.

The biggest difficulty for students within Dahmus's approach is the transition or transfer of his methods to algebra verbal problem statements in which the equation relationships and required content knowledge is not given explicitly, but is rather implicitly assumed to be known. One algebra textbook attempted to smooth this transition

senten

their

equati

explic

additi

proble

This i

instru

and co

or dir

V

three

proble

five b

approx

data,

tool s

than h

defini

consid

experi

contro

instru

exampl

subjec

proble

sentences as to their "condition" (1 or 2) related to their impact on variable definition formulations (1) or equation formulation (2). It is possible that such explicit and external written student work may additionally help and improve student reading of verbal problem statements, though its merit has not been tested. This investigation's design incorporates both instructional approaches, giving primacy to Polya's model, and considering both approaches to be general heuristics, or directors of means-ends analysis.

Vos (1976) investigated the relative effectiveness of three instructional treatments on the acquisition of five problem solving behaviors across grades 9, 10, and 11. The five behaviors were: a) drawing diagrams, b) approximating, c) constructing equations, d) classifying data, and e) constructing charts, which represent mostly tool skills or students' written constructions, rather than heuristics or the processes involved in variable definition, equation formulation and solution, which are considered more algorithmic in nature. The two experimental instructional treatments (third treatment a control group) involved varying the presence of written instructions which included a relevant, worked out example, and a checklist of possible behaviors given to subjects as they attempted to solve a series of verbal problem tasks. The two algebra classes of the study

subjec
proble
exhibi
skills
Treatm
instru
T
design
test s
skill.
grades
of the
studen
contro
least
(Webb
skills
entiti
V
tool s
with e
instru
studen
explic
is not
specif

subjects as they attempted to solve a series of verbal problem tasks. The two algebra classes of the study exhibited the greatest usage of the five behaviors (tool skills), and performance was best for the Behavior Treatment groups, which received only the written instructions/example presentation and no checklist.

The Mathematical Problem Solving Project (MPSP) designed 3 instructional skills' booklets, for a guess and test skill, a table making skill, and a organized listing skill. After implementation with over 700 students in grades 4, 5, and 6 participating, a summative evaluation of the booklets was completed. The results indicated that students using the skills' booklets performed better than control students in solving "process" problems, and at least as well in solving standard textbook verbal problems (Webb and Charles, 1977). One might recognize the three skills developed in the booklets as representative of entities generally labeled problem solving "strategies".

Vos's study indicated that students can apply these tool skills when they are prompted by written instruction with examples, which is assumed to be true of classroom instruction. However it is less clear as to whether students consider using such tool skills when such explicit presentation of written instruction and examples is not immediately provided. Also left unclear is whether specific instruction in how, when, and why such tool

skills

improv

improv

skills

Schema

M

one to

having

types,

and re

as phy

langua

the ce

which

impact

issue

knowle

proble

verbal

locate

this i

S

found

(1983)

studen

develo

skills are appropriate for solving verbal problems improves student performance, or even if such instruction improves student awareness of the purposes which tool skills serve in the problem solving process.

Schema Training

Mayer's (1985) discussion of schema training leads one to believe that such training consists solely of having students practice recognition of verbal problem types, naming and categorizing verbal problems by type, and representing verbal problems in alternate forms such as physical, pictorial, symbolic, or in alternate language. This instructional formulation severely limits the central position occupied by problem type schemata, which is claimed by the investigator to have the greatest impact on student verbal problem performance. A greater issue for the investigator is the contained depth of knowledge and related action heuristics attached to single problem type schemata, due to the belief that most algebra verbal problem processing is largely schema driven. No located research studies has been found that addressed this issue.

Some support for the investigator's position was found in two studies, Greeno (1980) and Riley et al (1983), which investigated early elementary school students' verbal problem solutions. Both studies found a developmental pattern occurred in which children begin

with t

and on

gradua

schema

contex

grader

instru

mathem

M

proble

instru

suppor

the th

grade

proble

at a t

O

signif

proble

within

teache

instru

Broder

of pro

and pr

colleg

with the idea that all verbal problems are of one type, and only later, possibly due to instruction, do children gradually begin to differentiate among different types of schemata. Additionally, Zimmerman (1979) found that contextual cues carried an inordinate influence on sixth graders verbal problem solutions, even when the instructional problems involved a variety of underlying mathematics.

Mayer's suggestion of training students to recognize problem types, using a mixture of problem types within instruction, something textbooks seldom do, has received support. Sumagaysay (1972) found that a gradual mix of the three types of percentage problems within seventh grade instruction resulted in better performance on such problems than either instruction which focused on one type at a time or instruction of all three types at once.

One aspect of instruction which might have significant impact on student development of enriched problem type schemata is the use of worked out examples within instruction (both textbook and teacher), and teacher modeling of samples during verbal problem instruction. The classic instructional study of Bloom and Broder (1950) showed that like other "habits", the habits of problem solving can be altered by appropriate training and practice. Bloom and Broder recruited a group of college freshmen, called the "remedial group", who

posses

for su

failed

proble

imitat

studen

on exa

remedi

their

proble

transc

result

retaki

remedi

retaki

Howeve

more t

the in

receiv

and be

solvin

their

the tr

and op

learne

conclu

possessed the ability, knowledge, and motivation needed for success on comprehensive examinations, and yet somehow failed to correctly solve many of the examinations' problems. This remedial group of students were taught to imitate and make use of the processes used by "model" students, a group similar in aptitude, but more successful on examination problems. In 10 - 12 training sessions, remedial students were asked to "think aloud" and describe their problem solving processes as they worked through problems, and then compared their employed processes with transcripts of the processes used by the model group. The results showed improved performance in additional retakings of the comprehensive examinations by the remedial group as compared with a matched group's retakings who did not participate in the training. However, the positive effects of the training were due to more than just the incorporation of worked examples within the instructional training. The remedial group also received additional practice with examination problems, and became more skillful at analyzing their own problem solving behaviors (which relates to metacognition) than their matched group counterparts. It was also noted that the trained students expressed high levels of confidence and optimism (metacognitive aspects) concerning their learned problem solving abilities. Bloom and Broder also concluded that a certain amount of specific background

informa

solving

basic k

A

of info

(equiva

related

solutio

problem

volunte

mathema

Data wa

subject

problem

comple

proble

For th

verbal

statem

The fi

meanin

togeth

solvin

solvin

solvin

the ta

information was essential, and that methods of problem solving, by themselves, cannot serve as a substitute for basic knowledge of subject matter.

A study by Kulm and Days (1979) explored the transfer of information acquired in solving each of four categories (equivalent, special case, similar, and generalization) of related problems, and one unrelated problem on the solution of two target problems, one algebraic verbal problem and one puzzle problem. The subjects were 35 volunteer prospective elementary school teachers from four mathematics classes, with all but two subjects female. Data was collected in seven separate sessions, five subjects per session, in which subjects solved an assigned problem from one of the five categories, and then completed a "guessing" booklet containing the target problems, in what appeared to be a "cloze" type procedure. For the analysis of subject performance, the algebraic verbal problem was divided into three parts: the statement, the solution set-up, and the solution itself. The findings showed that the solution set-up (possibly meaning variable definition and equation formulation together) was the most difficult of the three problem solving stages. More learning or transfer was gained by solving a more general or similar problem form than by solving a simpler case or an equivalent problem form for the target algebraic problem, and this superiority was

especi

stage.

verbal

comple

more e

were s

produc

played

that t

proble

algebr

necess

extrac

S

using

effect

acquis

while

equati

experi

exampl

the ta

proble

own ur

The ex

a coll

especially noteworthy in the crucial solution set-up stage. The results indicated that solving algebraic verbal problems is different from rule learning in that a complex-simple sequence of problem presentation appeared more effective. Both types, algebraic and puzzle problems were similar in that the general-specific sequence produced significant transfer of information, and context played an important role in this transfer. They concluded that the performance differences between the two types of problems indicated that the strategies used for solving algebraic verbal problems may be more specific, necessitating that students solve a problem where extraction of these specific algebraic methods can occur.

Sweller and Cooper (1985) tested the applicability of using worked examples as an instructional tool, and its effects on schema acquisition and heuristic process acquisition. The subjects were level 8 and 9 students, while the content was algebraic techniques (solving linear equations) rather than verbal problems. In each experiment, the experimental group received a worked example or examples to study prior to the presentation of the target task. The control groups were given a similar problem or problems to solve from which to develop their own understanding prior to the target task presentation. The experimental groups outperformed the control groups on a collection of target task problems, both in terms of

dissim

acquis

method

instru

acquis

that w

the re

moves,

They a

domain

indica

proble

throug

T

quanti

of tea

proble

studen

retent

worked

from c

segmen

dissimilar character. They concluded that schema acquisition was enhanced by such an instructional example method, and that heavier use of worked examples within instruction reduces the time period for schema acquisition. The reason for this advantage appears to be that worked examples tend to focus student attention on the relations between relevant problem states and relevant moves, and away from the immediate goal of the problem. They also conjectured that if problem solving skill is domain specific, as the research literature seems to indicate, then learning how to solve particular types of problems may be the only skill that can be acquired through schooling.

The above reviewed studies indicate that the quantity, sequential nature of presentation, and quality of teacher modeling of worked examples of various verbal problem types within instructional segments each effect students' problem type schemata acquisition, and possibly retention. All three aspects of teacher presentations of worked examples appear capable of investigator description from observations of each teacher's instructional segments.

Strate

T

most c

the br

resear

knowle

solvir

studie

compor

and wh

heuris

teache

T

exempl

comput

For ex

solvir

of cor

recogn

which

formul

to app

instru

that

are no

60).

Strategy Training

This area of instructional research is clearly the most difficult of the four areas to review, due both to the broad nature of the category and the extensive research efforts spawned by a belief that strategic knowledge is of central importance in mathematical problem solving. Hence the investigator has selected those studies for review which contain an instructional component (either self-instructional materials or teacher) and which relate directly to the three strategy areas: heuristic processing behaviors, metacognition, or teacher/student beliefs.

The importance of strategic knowledge has been best exemplified by cognitive psychologists' attempts to build computer program models of mathematical problem solving. For example, Greeno (1978) developed a computer model for solving geometric proofs. Besides incorporating aspects of conceptual and linguistic knowledge and concept recognition within his model, he included a component, which he called strategic knowledge, which set goals and formulated solution plans. He noted that training in how to approach problems is not generally a part of formal instruction. In his review of textbooks, Greeno noted that "principles used in setting goals and forming plans are not mentioned in any text that I have examined" (p. 60). Yet he goes on to present evidence that such

strat

learn

refer

they

to "t

gener

1980)

compa

task

solvi

hypot

in ta

train

trans

the u

hypot

plann

perfo

speci

resul

instr

provi

techn

resul

strategic knowledge is required for problem solving to be learned.

Greeno's conception of strategic knowledge primarily refers to what are known as "general" heuristics, since they involve planning and subgoal generation, as opposed to "task specific" heuristics, which are less generalizable and hence more problem specific (Lester, 1980). Early instructional research studies attempted to compare which of the two kinds of heuristics, general or task specific, had greater impact on student problem solving performance. For example, Wilson (1967) hypothesized that high school students given instruction in task specific heuristics would perform better on his training tasks (near transfer), and more poorly on his far transfer tasks than similar students given instruction in the use of planning and means-ends heuristics. The hypothesis was not confirmed, and Wilson found that the planning heuristics group actually demonstrated better performance on the training tasks, than did the task specific heuristics group. A possible explanation for the results was the short duration of the instruction (self-instructional materials) being only two, 3 hour sessions.

Smith (1973) expanded upon Wilson's instruction to provide greater student practice of the general heuristic techniques within more diverse task environments. His results however were similar to those of Wilson's study.

Both
often
withi

the v
stude
of a
subje
consi
stage
instr
quest
speci
instr
expec
dicta
study
instr
instr
signi

study
exper
class
taugh
probl

Both studies indicated that the strong transfer power often attributed to general heuristics was not evident within their subjects' performance.

Post and Brennan (1976), in an instructional study of the verbal problem solving abilities of tenth grade students found similar results with regards to instruction of a general heuristic procedure. The experimental subjects were taught a general heuristic procedure consisting of four phases, which were similar to Polya's 4 stages. Within each phase, subjects were given specific instructions or suggestions, which related to Polya's questions for each stage, or might be referred to as task specific heuristics. It should be noted that their instructional model closely resembles the investigator's expectations as to the nature of teacher instruction dictated by Algebra I textbooks' formulation, though this study used self-instructional booklets as the mode of instruction. Hence there is some indication that instruction in a general heuristic procedure had no significant effect on problem solving performance.

More encouraging results were found in a dissertation study done by Ashton (1962), in which an instructional experiment was performed involving ten ninth grade algebra classes. One of two classes in each of five schools was taught to use heuristic methods for solving algebra verbal problems, while the other class of the pair was taught the

"text
taugh
heuri
disco
sugge
incor
provi
and p
teach
heuri
outpe
with
one (
the s
trans
was s
identi
to pr
result
conclu
substa
conten
In add
ideas
solver

"textbook" method. In each school, the same teacher taught both contrasting methods for a ten week period. The heuristic method consisted of aiding students in the discovery of algebra verbal problem solutions through the suggestive questions from Polya's list, and explicitly incorporated Polya's 4 stage model. The "textbook" method provided model solutions or problem types for imitation and practice, with model solutions presented by both the teacher and textbook. The results indicated that the heuristic method class in each of the five schools outperformed their "textbook" method counterpart class, with differences significant ($p = .05$) in all schools but one ($p = .07$). A limitation of this study was that all the subjects were female.

Since initial results in terms of performance and transfer with regards to instruction of general heuristics was somewhat discouraging, research attention turned to identifying and relating usage of task specific heuristics to problem solving performance. For instance, from the results of the AMPS project evaluation, Lesh (1983) concluded that good problem solvers in general bring both substantive content ideas in combination with powerful content related processes to bear on a problem solution. In addition, it was found that students without relevant ideas in a particular domain were in general poor problem solvers in that domain, even when extensive training in

the u

strat

perio

Polya

recei

heuri

that

strat

the u

stude

appro

the h

solve

heuri

conte

instr

the e

the l

study

devel

techn

progr

unit

instr

the use of general, content-independent heuristics and strategies was provided.

Lucas's (1972) calculus study involved an eight week period of instruction using a heuristic style based on Polya's writings with one class, while a second class receiving "normal" instruction served as a control. The heuristics class also received, and assumedly read, papers that defined and demonstrated task specific heuristic strategies, as well as problem assignments that encouraged the use of these heuristics. Based on his review of student performance in the three scored phases of approach, plan, and result, he concluded that subjects in the heuristic treatment were superior in their ability to solve calculus problems. Lucas further indicated that heuristics can be taught without infringing on normal content, although he noted that the reading of written instructions in heuristics required extra class time for the experimental group.

The importance of Kantowski's (1975) study rests with the longitudinal nature of observations which allowed the study of variations in student processing as they developed, and also permitted heuristic instructional techniques to be varied as student learning was in progress. Three geometry units were taught, where each unit contained two instructional phases, a readiness instruction phase and an instructional phase using

heuri

instr

vague

task

of go

the p

instr

of fi

stude

of st

With

heuri

contr

varia

heuri

instr

gains

obser

devel

must

and a

were

remai

realj

appro

heuristic techniques. The nature of these heuristic instructional techniques used by Kantowski is somewhat vague, but seemingly included general (goal-oriented) and task specific heuristics. The marked increase in the use of goal-oriented heuristics for the eight subjects from the pretest to the posttest was attributed to the two instructional phases of the study.

Schoenfeld (1979) studied the impact of instruction of five heuristic behaviors on upper division college students' (N = 4) performance in comparison with a group of students (N = 3) who did not receive such instruction. With the exception of "drawing a diagram", the other four heuristic behaviors of the study, induction, proof by contradiction, simpler cases (reduce the number of variables), and establishing subgoals represent general heuristics related to mathematical content. The instructional group's performance was superior in terms of gains from pretest to posttest. More significantly, he observed that students cannot be expected to naturally develop useful heuristics on their own, but rather they must receive explicit instruction both in characterizing and applying heuristics. He found that even when students were able to recognize such heuristics, they often remained unable to apply them, due to their failure to realize situations when using such heuristics was appropriate.

study
varia
teach
had c
instr
solvi
instr
actir
teach
strat
Polya
teach
stude
simpl
clari
feedb
group

stude
relat
and t
quest
strat
diffe
instr

An important feature of Putt's (1978) dissertation study was that he made no attempt to control the teacher variable by using either self-instructional materials or teacher scripts, as most of the already reviewed studies had done. Putt compared two methods of problem solving instruction on the behavior of fifth grade students solving "process" (nonalgorithmic) problems. Both instructional methods were based on Polya's 4 stage model acting as a general heuristic. One treatment required teachers to make students aware of Polya's heuristic strategies or questions within each of the four stages of Polya's model. The second treatment involved no direct teaching of heuristic strategies, nor did teachers help students realize generalizations. Instead students were simply given problems to solve and teachers primarily clarified words and terms, and provided students with feedback on how they were doing in the second treatment group.

Putt's major conclusions were that fifth grade students can learn to solve process problems in a relatively short period of instructional time (4 weeks), and that students can be taught to ask themselves questions to aid problem understanding and to use specific strategies for solving problems. Several qualitative differences were also noted between students in the two instructional groups. Students in the heuristic

instr

stepw

strat

quest

probl

group

stepw

than

(Lest

revie

inver

probl

targe

notal

probl

capab

task

regul

envir

learn

use

situa

verba

resul

instructional group named Polya's 4 stages as helpful steps in solving problems, used more of the highlighted strategies, and were generally more active (asking questions) in their attempts to attack and understand the problems. Students in the non-heuristic instructional group named the four arithmetic operations as helpful steps, and actually used a wider variety of strategies than the heuristic group in their problem solutions (Lester, 1980).

In attempting to summarize the results of these reviewed studies of heuristic instruction, the investigator notes that most of the studies used verbal problems, varied according to content and grade level, as target tasks in measuring student performance. The notable exception being Putt's usage of "process" problems. The studies indicate that students are indeed capable of learning heuristic strategies, both general and task specific, through provided teacher instruction with regular domain content and in regular classroom environments. It also appears clear that once students learn how to identify, apply, and when to appropriately use these heuristic processes in verbal problem situations, there is every reason to expect improved verbal problem solving performance or proficiency as a result of such training.

stage

to ai

heuri

model

centr

and f

instr

imagi

desig

for e

teach

notic

belie

solvi

isola

calle

stage

inclu

Schoe

leas

frame

requi

stude

helpf

sugge

The reviewed studies also tended to use Polya's 4 stage model as a framework or umbrella (general heuristic) to aid the organized study and placement of task specific heuristics within the stages. This notion of Polya's model as a framework or organizer for strategy study is central to this investigation's design. It also explains and fits well with Schoenfeld's (1984) heuristic instructional model detailed earlier. The investigator imagines that Schoenfeld struggled with the problem of designing a practical, down-to-earth instructional model for elementary and secondary school teachers to use in teaching heuristics. His instructional discussion noticeably departs from any mention of metacognition and beliefs/values, both central components of his problem solving model. Rather he focuses his discussion on isolated "cued" heuristics, and clusters of heuristics called "stage" heuristics and their instruction, where stage connotates his names for Polya's stages, and includes one additional stage, analogy exploration. Schoenfeld views students and teachers as operating, at least in verbal problem situations, within a general framework (textbook version of Polya's model), which requires constant teacher explanation and review within student practice settings, and that teachers are most helpful when they provide students with hints and suggestions (task specific heuristics) which can be

const

stude

belie

of "

stude

mathe

same

perce

abil

conce

prese

expe

for

under

One

incor

and

inclu

and

heavy

desig

of th

sever

constructed into stage heuristic procedures by the students themselves.

Schoenfeld's conceptions of metacognition and student beliefs grew out of his 1979 study and comparison reviews of "thinking aloud" protocols of high school and college students (considered novices) with efforts of college mathematics professors (considered experts) solving the same set of nonroutine problems. In these comparisons, he perceived great differences between groups in their ability to monitor and use the available resources and conceptual knowledge at their disposal. However, at the present time school instructional studies, particularly experimental studies, of metacognition seem to be waiting for a more precise definition and formulation of the underlying components of metacognition to be developed. One exception to this lack of instructional studies incorporating metacognition is the work of Silver, Branca, and Adams (1980) who suggest that instruction needs to include considerable class time being spent in discussing and thinking about processes students employ, and place a heavy emphasis on planning or "thinking before doing", designed to improve both student awareness and monitoring of their own processes and thinking.

In regards to beliefs, Schoenfeld (1985) identified several student beliefs and corollaries which together

made

stude

to wh

only

belie

geome

that

argu

solv

surr

and

on b

samp

exam

conf

math

situ

abou

wher

teac

deve

gath

teac

made up what he called a "make sense" epistemology. These student beliefs were defined in very practical terms, and to which most secondary teachers can relate and agree. His only explanation for the development of these student beliefs was based on his own observations of high school geometry students' classroom experience, where he noted that received instruction tended to exclude mathematical argumentation and deduction as useful techniques for solving problems.

Recently there has been a growing body of research surrounding teachers' beliefs and conceptions of teaching and mathematics, and the influence these conceptions have on both instruction and teaching performance. A small sample of that research is reviewed here. Shroyer (1978) examined elementary teachers' decision making when confronted with "critical moments" in teaching mathematics, and found that teachers responded to such situations in expected ways that reflected their beliefs about teaching. She was able to isolate several examples where instructional decision making was driven by teachers' previous classroom experiences, which had developed into teacher beliefs.

Shirk (1972) developed conceptual frameworks, gathered from a variety of sources, for four preservice teachers enrolled in a mathematics methods course, and

then
his c
group
teach
teach
ways
mathe
also
concr
from

in d
thei
for
prac
beli
teac
teac
beha
aspe
math
nota
mann
cont

teac

then attempted to relate these conceptual frameworks to his observations of each teachers' instruction of small groups of junior high students. He observed that teachers' conceptions appeared to be "activated" in teaching situations, "causing" the teachers to behave in ways that were consistent with their conceptions of mathematics teaching and their roles as teachers. Shirk also found no discernible change in the teachers' conceptions as they went through the methods course, or from their participation in the study.

Thompson (1982) conducted case studies (each 4 weeks in duration) of three experienced junior high teachers and their conceptions of mathematics and mathematics teaching for possible relationships to teachers' instructional practices. Her findings indicated that teachers' views, beliefs, and preferences about mathematics and its teaching had a significant, though subtle, role in shaping teachers' characteristic patterns of instructional behaviors. She found beliefs that related to both general aspects of teaching as well as to specifics of teaching mathematics. Thompson also observed that there was a notable consistency between teachers' conceptions and the manner in which they typically presented mathematical content.

Lastly, a study by Cooney (1985) demonstrated that a teacher's conceptions of mathematics and mathematics

teaching can change dramatically upon initial entry into the reality of the secondary mathematics classroom. The case study followed a single teacher, Fred, from his preservice student days through his first year of teaching secondary mathematics. During his preservice days, Fred participated in six extensive interviews, some involving statement sorting tasks, and from which Cooney compiled a picture of Fred's conceptions of mathematics. After ten weeks of teaching in his first secondary position in a rural school, Fred was observed for nine consecutive days teaching a diversity of mathematics courses.

Fred demonstrated a particularly strong belief in a problem solving orientation during the preservice interviews. Fred believed solving problems was the principal activity and essence of mathematics. He enjoyed working recreational mathematics problems himself, and believed such problems had motivating potential for his future students. Applications and textbook verbal problems were important to Fred only to the extent that they could be used to develop heuristic techniques, which were to him the central focus of teaching problem solving.

The observations of Fred's actual first year teaching revealed an entirely different picture of Fred's conceptions of problem solving. The textbook became for Fred the primary determiner of curriculum, and for the most part, his method of presentation also, which was

considered "typical" (homework review, new material explained, assigned seatwork) by Cooney. Fred's daily lesson plans consisted of a brief list of content to be covered, appropriate examples, and the day's assignment. Fred had quickly become consciously aware of the time demands required for a problem solving orientation, confessing that "it is easier to teach by the book and leave heuristics out completely". From his observations of Fred, Cooney concluded that Fred came to believe that recreational and nonroutine mathematical problems served only a motivating function in the classroom, to set off a "spark" of student attention and interest. When in some of his classes, there was no student reaction to these "spark" problems, Fred came to believe that students lacked internal motivation, which resulted in his adoption of a typical method of content presentation.

In summary, teachers' conceptions of content and teacher role appear to effect many aspects of instructional practice. In addition, teachers' conceptions and instructional practices are shaped by their students and experience in the classroom environment.

Algorithm Automaticity

In solving algebra verbal problems, students generally pass through three phases which involve application of previously studied manipulation skills or

procedures. The three phases which all result in student written work, and hence can be considered products of the problem solving process are: variable definition, equation formulation, and equation solution. It is clear that equation solving requires application of associated manipulation skills which together constitute a procedure or algorithm. It is less clear that the other two phases, particularly equation formulation, constitute procedures or algorithms. Their inclusion as algorithms is based on their product nature, and the belief that they represent algebraic subskills necessarily mastered before the introduction of verbal problem content. In most research, variable definition and equation formulation are considered from a process perspective, and together constitute a "setting up" phase. The primary advantage of separating these two student actions or products involves the analysis of student work samples by the investigator, and the fact that based on their training in the textbook procedure for solving verbal problems, students come to conceptualize these two processes as totally distinct phases. Investigator analysis of student work samples can uncover error patterns, and also locate in which of the three phases student errors are most likely to occur. It is assumed that students receive extensive practice in solving linear equations, some limited practice in

defining variables, and almost no practice in formulating equations within classroom instruction.

There appears to be an absence of research efforts that analyze Algebra I students' performance of algebraic manipulation skills, such as simplifying algebraic expressions or solving linear equations, other than the analysis done of National Assessment results, reviewed earlier.

Matz (1982) focused on students' algebraic error patterns related to algebraic expression and equation manipulation in her development of an explanatory theory of algebraic competence. Her framework is helpful for the investigator's analysis of student error patterns, but is otherwise not very practical for this investigation. She proposes that student knowledge in addressing unfamiliar algebraic manipulation tasks has two components, base rules which are extracted from prototype examples (textbook, teacher, or student generated), and extrapolation techniques which bridge the gap between base rules and unfamiliar tasks. Matz claims that most commonly observed manipulation errors are due to either incorrect generation of an extrapolation technique, or correspond to unmade developmental, conceptual changes required in moving from arithmetic to algebra content. A third class of errors, called processing errors, include

computational mistakes which are not consistent in nature, and often times are corrected by students upon reflection.

The two major extrapolation techniques identified by Matz as often misused in algebra are revising rules using a linearity argument and overgeneralized accomodation of rules. Examples of the first type of error include $(a + b)^2 = a^2 + b^2$. Examples of the second type of error include not realizing that the zero is a crucial element in solving equations like $(x - 3)(x - 4) = 0$ as opposed to equations like $(x - 3)(x - 4) = k$, where k does not equal 0.

Matz calls the the second class of errors, caused by unmade developmental changes required of algebra content and its associated base rules, planning errors. She notes that working arithmetic problems means executing algorithms, whereas adjusting to algebra involves the composition of a plan, as well as carrying out the plan. Furthermore, such plans are often pieced together on the fly, where the next step in a plan is at times not determined until the results of the previous step are completed. This fact is particularly noteworthy in the solving of linear equations, where there is a constant need for students to assess progress towards the goal state, to keep the goal state in mind, and also to judge when to discontinue the procedure. In Matz's model this class of planning errors normally leads to the production

of "buggy" algorithms of equation solving so often witnessed on student work samples of equation solving.

One Soviet study by Doblaev (1969) provides information related to the methods students use in their constructions of variable definitions and equation formulations. His subjects were 160 level 7 and level 9 algebra students. The task was a set of algebra verbal problems, some of which were routine textbook problem types, which was completed in "thinking aloud" interviews. In his analysis of the protocols, Doblaev identified two distinct methods, called A and B, which subjects used to set up equations for the verbal problems. Method A subjects began defining variables almost immediately after a complete reading of the verbal problem statement, and subjects' variable definitions used the statement's question as the beginning point for defining all verbal problem variables. Equation formulation for this group involved a "piecing together" or blending process of the defined variable expressions and a rereading of the problem statement to pull out necessary relationships. Method B subjects' behavior exhibited greater reflectivity in their initial reading of the verbal problem statement, and subjects often pondered quietly during this time. From his probing of students during this quiet time, Doblaev found that most students were developing a greater understanding of the problem's elements, and beginning

construction of an imagined form or representation that the required equation would take. Only when such a representation was accomplished internally did these subjects formally define variables and formulate the equation. He found that the external, written equations were generally consistent with the subjects' descriptions of imagined forms of problem representation.

The following aspects of Doblaev's (1969) analysis of method (A or B) differences are considered noteworthy for this investigation. He found that about 2/5 of his subjects consistently preferred method A, and the remaining 3/5 of his subjects consistently used a method B approach. Method selection for 5 of the 9 verbal problems of the task was highly dependent on characteristics of the verbal problems, three favoring a method B approach and two favoring method A. Doblaev also found a developmental trend in his subjects where in level 7 almost equal proportions preferred both methods, while level 9 subjects preferred method B over method A by a 2 to 1 margin. The two methods also related to level of achievement, defined subjectively in the study, where weaker students preferred method A (2 : 1 ratio) and better students preferred method B (2.5 : 1 ratio), and average students had equal proportions of preferential methods. Lastly Doblaev was able to demonstrate a clear-cut relationship between preferred student method and the two teachers and their

instruction, from whose classes the subjects were drawn. For one teacher, subjects preferred method A (2 : 1 ratio), while for the other teacher the ratio was reversed in favor of method B.

This concludes the review of the literature. Chapter III describes the employed methodology for the study, both its development and final form.

CHAPTER III

Methodology

An important aspect of this investigation is the development and design of instruments for analyzing characteristics of textbook verbal problem presentations and examples, teacher instruction and conceptions of verbal problem content and related examples, and student verbal problem solving performance and processing at various points in time. Therefore this methodology chapter is composed of three distinct sections. These sections are the Pilot Study Phase, Teacher and Site Negotiation/Entry, and Data Collection and Analysis Techniques. The Data Collection and Analysis Techniques section includes discussion of the four dimensions of the study, textbook review, teacher observations, student written performance measures, and student interviews.

Pilot Study Phase

Three main purposes were served by conducting a Pilot Study Phase to this investigation: to refamiliarize the investigator with the Algebra I instructional setting, to test related observational and interview techniques within the setting, and to become aware of additional needs

related to data collection instrumentation for the actual study. These purposes were accomplished within the pilot study through observations of one Algebra I teacher, Alice and her class during a three month period, February 1 - May 1, 1986. Alice was also one of the three teachers participating in the actual study.

Alice was observed by the investigator on twelve occasions during this period teaching verbal problem content. Field notes were taken and later rewritten and reviewed by the investigator of each of these observations of Alice's instruction. Additionally an hour per week was given to informal dialogue between Alice and the investigator regarding questions arising from the investigator's observations of Alice's instruction, and also regarding Alice's conceptions of algebra verbal problems, algebra teaching, and students' verbal problem performance patterns. Alice was always cooperative in these discussions, and exhibited a keen ability to verbalize her views in these matters in clear and concise terms based on her extensive experience (over 20 years) teaching algebra at the ninth grade level. In most cases, the investigator found that the field notes and weekly dialogue with Alice adequately captured the regularity of Alice's instructional patterns, and the underlying basis or teacher beliefs upon which her instructional patterns and practices were constructed.

The second and third purposes of the pilot study, testing, design, and refinement of data collection instrumentation for the actual study presented a greater challenge. An initial concern involved the development and testing of a process coding instrument capable of characterizing employed processing of verbal problem content presented in textbook examples, teacher instructional examples, and interviewed students' "think aloud" protocols of problem solutions. It was assumed by the investigator that the development of such a process coding instrument with broad capabilities was feasible, given that teachers' instructional examples generally conform to textbook example presentations, and students' solution paths generally conform to received teacher and textbook instructional example presentations. Additionally, reviewed research indicated that several process coding forms for reliably characterizing student process activity had been developed and were capable of modification within an algebra verbal problem content focus.

The process coding form developed for the pilot study, labeled Suggestion/Heuristic Coding Form, can be found in Appendix A. Blake's (1976) Process-Sequence Coding System, taken from Goldin and McClintock (1984, Table 5.9, p. 208), was used as a framework by the investigator to develop this instrument. Blake's system of 39 processes was selected as a framework because of its relationship to algebra verbal

problem content, and its sequential listing of processes according to Polya's 4 stage model. The pilot study's coding form deleted several of Blake's processes, due to their nonapplicability in Algebra I, subdivided other of Blake's processes into additional, more detailed components, and explicitly organized the resulting 47 processes into Polya's 4 stages according to the primary function each process served in the verbal problem solving process. The investigator assumed and expected the testing of this instrument in the pilot study to result in a refined form composed of fewer process categories (less than 30) based on the observations of process usage by textbooks, teachers, and students. It was also expected that other modifications to this process coding form might be necessary to facilitate its diverse applications in the instructional setting. This initial process coding form was tested during the pilot study to analyze ten textbook instructional examples, ten teacher instructional examples, and for each of the fifteen verbal problem solutions generated within the three student interviews conducted.

The only other instrument designed for the pilot study was an algebra verbal problem test, which was administered to the whole class of students near the end of the pilot study. The verbal problem test was composed of five problems which related to verbal problem content covered by Alice during the pilot study period, and not the content

related to the actual investigation's time period. The investigator's major purposes for constructing this instrument was to evaluate the degree of student learning and transfer related to verbal problem content one could expect of this level of student, and also for the investigator to practice the designing of verbal problem sets that were within students' abilities to solve.

The five verbal problems of the test instrument were printed on a single page, and presented to the class of students with explicit investigator instructions to show all their solution work (and also to not erase any written work) on separate paper provided by the investigator. The test was administered during one of Alice's regular class periods, and all students completed (within their capabilities) work on the test in less than 30 minutes. The actual test form can be found in Appendix A.

One week after the classroom test administration, three volunteer students from Alice's class completed the same exact set of verbal problems in separate interviews with the investigator. Interviewed students did not have access to their previous written work on the same test form. In the interviews, each of the five verbal problems was presented individually by the investigator in the same order of appearance as the class test form. The interviews then involved a mix of student retrospection and introspection, where students were directed by the

investigator to verbalize their thoughts and written actions as they were solving the same verbal problems a second time. Each interview was audiotaped for later review by the investigator. During the interview sessions, the investigator remained mostly passive, except to provide minimal feedback to direct questions asked by the student or to provide encouragement to the student to continually verbalize when lengthy periods of student quiet was evident. While the students were working on the verbal problems, the investigator was also completing a process coding form for that problem, noting the sequence of observed processes being employed by the students. Each interview session closed with a series of questions requesting students to verbalize their attitudes towards mathematics and algebra, verbal problems, their Algebra I teacher, and homework patterns. The duration of each interview was 45 minutes.

The pilot study indicated that the investigator's presence in the classroom, actively compiling field notes during teacher instruction, did not significantly disturb either the teacher's instructional patterns or students' behavior or classroom participation. Student performance on the investigator designed verbal problem test was poor however, indicating that the investigator had overestimated student ability to transfer skills to verbal problems consisting of new and varied contexts different from

familiar textbook contexts. The test was too difficult for most students, and led the investigator to conclude that any verbal problem constructions for student tests in the actual study would need to conform closely to those verbal problem contexts experienced in the respective classrooms.

The pilot study demonstrated the feasibility of using one process coding form for analyzing teacher example presentations, textbook example presentations, and student processing of verbal problems. The pilot study coding form consistently captured process features of the three elements' presentations. Additionally, the coding form results indicated that all three elements' presentations were procedurally similar both in terms of the choice of processes used and sequence of usage. The investigator decreased the large number of processes on the coding form from 47 to 27 processes for the actual study's coding form. Each of the resulting 27 processes had been observed to be used at least once in the various analyses where the coding form had been employed in the pilot study. The investigator also felt it was necessary to add a space for comments next to each listed process to detail either a textbook's explanation of a process, or a teacher's verbal explanation of a specific process, or a student's verbalization related to a specific process. The modified process coding form to be used in the actual study was labeled Coding Form 1, and can be found in Appendix B.

The pilot study exposed several weaknesses with regards to the planned design for the actual study's data collection and analysis. It became evident to the investigator that many of the proposed methods of data collection were too simplistic to capture any more than surface features of the instructional setting and student performance, and of minimal service in answering the proposed questions of the study. In most cases, the investigator attended to these perceived weaknesses by developing added instrumentation, borrowed or modified from other sources to provide a greater degree of depth to the study's data. This added instrumentation and related analysis techniques are discussed in detail within the data collection and analysis section.

The pilot study revealed five major weaknesses of the proposed methodology for the actual study, and these weaknesses can be summarized as follows:

1. The usage of investigator field notes of teachers' instruction to fully characterize the verbal problem instructional environment was inadequate. Other instruments which provide both quantitative and qualitative information, and which can be completed by the investigator quickly after teacher observations are needed.

2. The textbook analysis undertaken by the investigator during the pilot study did not contain adequate detail for making textbook comparisons (as is required in the actual study), nor did it contain author input of any kind on the nature of their textbook designs related to verbal problems. More structured and comparative instrumentation is needed, and textbook authors need to be queried for the rationales of their designs.

3. The pilot study clearly indicated that the investigator had not given adequate consideration in the proposal of how student performance, in the form of students' written solutions, was to be evaluated and analyzed. A scoring system is needed for the investigator's review of student work samples.

4. The small sample (3) of students interviewed indicated that less capable Algebra I students cannot be expected to verbalize their thought processes very well in the interview setting. The investigator should focus on identifying more capable and more verbal oriented students for the interviews during the classroom observations. The student attitude/ background section of the interview also needs greater specific structuring.

5. The actual study of 3 teachers, as opposed to the pilot study of one teacher, will not permit the one hour weekly discussions between investigator and teachers as was done with the one teacher of the pilot study. Instead the development of an alternate form of instrumentation (survey) is needed which relates to teacher instructional practices and conceptions of the setting and verbal problem content.

In summary, the pilot study's major purposes, investigator refamiliarization with the Algebra I setting and the capabilities of its students and testing and refinement of proposed instrumentation proceeded about as expected by the investigator. The pilot study however was more effective in its ability to identify specific weaknesses of the investigator's proposed methodology for the actual study, which was not expected by the investigator, but definitely indicated the value of the pilot study phase for the final formulation of the methodology used in the actual study.

Teacher and Site Negotiation/Entry

The study required the participation and cooperation of three Algebra I teachers and one of their respective Algebra I classes. The investigator initially conceived the three teachers of the study to be composed of two teachers from the same public junior high site as the pilot study site, with the third teacher selected from the local parochial high school. This design would permit investigator comparisons between two of the teachers' instructional patterns related to verbal problem content, where a common textbook orientation was used. This design proved unworkable however, when the only other teacher of Algebra I at the public junior high school site, besides Alice, refused to participate in the actual study. Alice, the teacher of the pilot study, had already agreed to participate.

Therefore the other public junior high school of the district was approached, and all three Algebra I teachers at this school volunteered to participate in the actual study. One teacher was selected by the investigator from this site on the basis of the time of his Algebra I class during the day, which best fit the investigator's available time commitments. The only teacher of Algebra I in the local parochial school agreed to the participation of one of her two classes for the study. At each site, the investigator completed necessary forms specified by the

respective school systems for proposed school related research, and followed designed channels which moved from central administration, to local school administration (principals), and finally to the teachers themselves. This negotiation process was begun in early August, 1986 and was completed in late August, 1986.

The selection of three teachers in three different school sites, and who employed three different textbooks had several advantages. First, it could be assumed that the three teachers would probably not communicate about the study with each other during the data collection phase of the study. Secondly, the addition of a third textbook series for the study increased the diversity of organization and presentational characteristics, improving investigator comparison capabilities undertaken in the textbook review. The third Algebra I textbook contained a very different organizational structure for verbal problem content in comparison to the study's other two textbooks, and such diversity also related well to the investigator's hypotheses related to textbook constructions. The addition of a third textbook did necessitate a rather quick review and analysis prior to the beginning of the school year. The other two textbooks used in the study had been thoroughly reviewed during the summer (1986), and their review at least provided the guidelines and instrumentation to make such an analysis possible.

The investigator met individually with each of the three teachers of the study at their respective schools just prior to the beginning of the school year (September, 1986). The purposes and methodology of the investigation were explained by the investigator, as well as clarification and reassurance of its observational and non-interventionist nature. Each teacher agreed to the minimal demands on their class preparation time, to notify the investigator when instruction related to verbal problems was planned, and to relinquish three class periods for investigator testing purposes. Each teacher expressed their enthusiasm for the investigation's purposes and the potential help the results of the investigation might provide for their own teaching methods related to verbal problem content. None of the three teachers expressed any concern at these meetings about having their teaching observed by the investigator, or believed that the investigator's presence might effect classroom behavior. At the close of these meetings, each teacher was requested by the investigator to complete the Teacher Inventory Packet (survey instrument related to teacher conceptions) prior to the investigator's initial contact with each teacher's class.

The three teachers were assigned code names, Alice, Betty, and Carl. Alice and Carl taught in the two different, public junior high school (grades 7 - 9)

settings of the same school district, and Betty taught in the same district's parochial high school (grades 9 - 12). The school district serves an area population of approximately 40,000 people, living in both residential and rural settings, and includes several small, bordering cities. The major city within the district has a population of 25,000 people, and is also home for a state university campus (enrollment: 9000 students) and headquarters for a major nationwide insurance operation, which are the area's two major employers. Farming, particularly potato farming, is the major industry of the surrounding rural areas. All ninth graders of the school district attend school at one of the three research sites. The student body and staffs are predominantly white Caucasian, with minority representation in the schools at less than 5 percent.

Data Collection and Analysis

This section is divided according to the four major phases of data collection: textbook analysis, observations of teachers' instruction, student performance measures, and the student interviews. With the exception of the retention test (student performance measure) given in early March, 1987, all data for the actual study was collected between August and December, 1986.

Textbook Analysis

Unable to locate within the research literature any evaluation schemes for analysis of secondary mathematics textbooks, the investigator designed several instruments which provided increasing degrees of specificity related to quantitative comparisons of the three Algebra I textbooks of the study. A qualitative evaluation instrument was also developed by the investigator, though admittedly this instrument reflects investigator bias for what are considered important qualities of textbooks' constructions, and also involved investigator subjective evaluations, based on textbook comparisons, for each textbook in each included category.

Initially the investigator reviewed the three textbooks, which were coded A, B, C as related to the teachers of the study (Alice, Betty, and Carl), to determine how many pages in each textbook were necessary to reach a point where common linear equation verbal problem types was completed, and also to where it was anticipated that teachers' instruction would finish prior to the Christmas break, 1986. There was a noted similarity between each textbook's construction and organization of the first four or five chapters, which aided this process. There was greater organizational differences among the three textbooks in the chapters beyond this point. The rest of the investigator's textbook analysis however was

confined to only these first four or five chapters of each textbook where the organizational differences were less apparent.

For each textbook, a page by page review was completed of these initial four or five chapters, with the investigator noting the amount of space (pages) given to four purposes: explanation, oral (classroom) exercises, written exercises, and "other". The "other" category included chapter reviews and tests, skill sharpener (review) sections, and optional application sections. The first three categories of purposes were also further delineated as to whether the algebra content referred to manipulation skills, verbal expression skills, or verbal problems. A closer review of verbal problem explanation sections and related exercises was then undertaken by the investigator.

The investigator identified five distinct problem types which the three textbooks covered to some degree within the reviewed pages. The five verbal problem types were geometric formula types (such as perimeter or sum of angles in a triangle), consecutive integer problem types, age related problem types, distance/rate/time related problem types, and coin/money related problem types. A sixth, broad category was added, called simple translation problem types, which included all verbal problems potentially solvable by direct translation techniques of

the verbal problem statements into variable definitions and the requisite linear equation. The six verbal problem type categories together composed what the investigator classified as the core content of the study. For each textbook, the investigator recorded the number of pages of textbook explanation and the number of written exercises related to the six verbal problem types of the core content.

Within the core content of the six verbal problem types, the investigator performed two other quantitative textbook comparisons according to two other research classifications of verbal problems. Charles and Lester (1982) classified problems into one of six types: drill exercise, simple translation problem, complex translation problem, process problem, applied problem, or puzzle problem. Textbook verbal problems are either simple translation problem types or complex translation problem types within their classification system. Within their classification system, the distinction between simple and complex problem types is based on a complex translation problem requiring at least two steps (multi-step) of translation into mathematical sentences (equations). Since all verbal problems of the identified core content required the formulation of only one linear equation, the investigator's analysis instead considered complex translation verbal problems to be ones which required

student processing of inferred relationships contained within verbal problem statements. The simple translation problem type then related to those verbal problems where the investigator believed that an appropriate equation could be accomplished by direct translation of the verbal problem statement.

The second classification scheme of verbal problems located within the research literature and employed in the textbook analysis was that developed by Caldwell and Goldin (1987). Their classification scheme is based upon categorizing verbal problem statements along two dimensions, concrete as opposed to abstract, and factual as opposed to hypothetical. Definitions for these dimensions has already been discussed in the research review (Chapter II). Within their scheme, every verbal problem is one of four types: abstract factual (AF), abstract hypothetical (AH), concrete factual (CF), or concrete hypothetical (CH). Counts were noted by the investigator of each of the four classifications within the verbal problems (exercises) of the core content according to the six problem types.

The investigator also reviewed all textbook examples related to the core content, and analyzed each example's presentational characteristics and suggested sequential processing using Coding Form 1. No direct analysis was done of this investigator coding of textbook examples, but rather it was completed for comparison purposes as related

to teacher instructional example presentation of the same content material, where commonalities of suggested processes and sequence could be noted during teacher observations.

The investigator developed a Feature Evaluation Form for the subjective evaluation of the textbooks' verbal problem core content. The developed form can be found in Appendix C. The form has four major subdivisions: A. General Model, B. Textbook Explanations, C. Textbook Examples, and D. Exercise Sets. Each major subdivision has at least three listed features which received investigator evaluation, and the textbook explanation subdivision (B.) has the most such features evaluated, six. The investigator evaluated each of the resulting 16 features for each textbook using a scale of 1 to 5, where 1 = lowest rating and 5 = highest rating. Rating subtotals within the major subdivisions, as well as the grand total of ratings were computed by the investigator.

Lastly an Author Questionaire was designed by the investigator and sent individually to each of the listed authors of the three reviewed Algebra I textbooks in August, 1986. The three page survey can be found in Appendix C. The first page of the survey was identical to the last page of the Teacher Inventory Packet, and requested authors to rate the importance (according to one of 5 levels) of eight given purposes for the study of

Algebra I, and also to rank order the eight purposes from their perspective in regards to importance. Authors were also asked to answer seven other questions related to their organizational preferences for verbal problem exercises and explanations, the ability level for whom the text was designed, the intended general problem solving model employed, and their thoughts on future design modifications related to verbal problem content presentation. Since only one Author Questionaire was returned as of December 1, 1986, and no questionaires were returned due to incorrect addressing, a second mailing to the authors was done in early December, 1986.

The analysis of the data collected from the textbook review was done on the basis of comparisons between the three textbooks (A, B, C) of the study in all cases. The three textbooks are never mentioned by name or publisher. An initial comparison relates the number of pages and percent of reviewed pages given to each of the four purposes: explanation, oral exercises, written exercises, and "other" category. Secondly, the core content of six categories of verbal problem types is developed, and the number of related explanatory pages and written exercises for each category is compared among the three textbooks.

The verbal problem exercises which composed the core content are then analyzed according to the two alternate classification systems introduced: simple versus complex

classifications, and the four way classifications resulting from the abstract/concrete and factual/hypothetical distinctions. Numerical counts of each of these classifications is compared by textbook within the confines of the six identified verbal problem type categories. There is also more extensive discussion of the three predictive models of problem solving competence introduced by Caldwell and Goldin (1987), and in the analysis the investigator compares aspects of student familiarity and practice with the four verbal problem classifications within the three models of problem solving competence. Within each quantitative comparison of the textbooks' verbal problem content, the investigator highlights both the common organizational elements shared by the textbooks and the most obvious differences between textbooks' design and organization of verbal problem content within the discussion.

The investigator's analysis of the subjective evaluations of the three textbooks, generated by the Feature Evaluation Form, receives greater discussion related particularly to the investigator's rationale for the assigned ratings. In the closing summary section, each of the three textbooks is discussed separately, and the investigator's perceptions of each textbooks' comparative strengths and weaknesses related to verbal problem content presentation are revealed. Most of the textbook review and

analysis was completed prior to the beginning of the teacher and classroom observations.

The analysis of the survey information from the textbook authors' completion of the Author Questionaire is reviewed along comparative lines also, both distinctions between the several authors of a single textbook, and also between the groups of authors for each textbook. Most of the investigator's attention is on the authors' perceptions of importance (ratings and rankings) related to the eight possible purposes of Algebra I instruction (page 1 of the survey). The investigator can also compare these ratings and rankings of purposes with those of the three teachers of the study, completed in the Teacher Inventory Packet.

Observations of Teacher Instruction

The three teachers of the study were to complete the seven page investigator designed, Teacher Inventory Packet prior to the investigator's introduction to the respective classes. The Teacher Inventory Packet was designed to aid the investigator's characterizations of teacher conceptions and instructional practices through the survey information provided by each teacher. The packet had four distinct sections, and its contents can be found in Appendix D. The first three parts (six pages) involved modifying many of the questions and formats of a teaching survey used by Good, Grouws, and Ebmeier (1983) so that they related to algebra content or algebra verbal problem content

instructional practices. Part I requested teachers to rate their classroom practices between two given opposite, paired descriptions, and there were 17 of these to complete. Part II related to teacher opinion of confidence, challenge, and satisfaction related to the teaching of algebra content, and asked teachers to express their degree of agreement with seven statements. Part III was composed of 19 open ended questions requiring short teacher responses and opinions, or estimation of time and percent allotments related to classroom practices. Part IV (last page) requested teachers to rate and rank order the eight possible purposes of Algebra I instruction, as was included in the Author Questionaire. The data collected from the Teacher Inventory Packet was to be used in conjunction with the teacher observations in the development of comparative descriptions of each teachers' instructional practices and conceptions of the Algebra I classroom setting.

Initial investigator contact with each of the study's three classes was done either during the second or third week of the school year (September, 1986). The respective teacher of each class introduced the investigator as a mathematics instructor at the local university, and explained that the investigator was interested in their class as a research site. The investigator in turn explained the general purposes of the study and its verbal

problem content focus, the joint roles of the investigator as researcher and classroom aide, the observational note-taking posture to be employed during the teacher's instruction, and also a willingness to help students on their assignments during seatwork time. Students were then directed by the investigator to complete both a general information sheet (one page) and an investigator designed verbal problem pretest. Both materials can be found in Appendix E. The Student Information File asked students to provide some background information (name, grade level, last year's mathematics class and teacher), their present class schedule, and also to write brief statements related to two questions. One question asked students to comment on their feelings and concerns about taking the Algebra I class, while the second question asked students to describe their ability and past experience in doing textbook verbal problems. Investigator review of these completed information sheets was brief, and certain characteristics of common response to the two questions were noted by the investigator.

The verbal problem pretest contained six verbal problems, three per page, with space provided for student work and answers. Five of the six problems related to the five distinct problem types of the core content (simple translation type excluded), while the sixth verbal problem required students to manipulate a formula (trapezoid area)

for a missing piece of information (shorter base). Since no algebraic procedures for solving verbal problems had been taught yet in the classrooms, the pretest instrument primarily measured student and classroom abilities to use guess and check strategies. Its intended purpose was to ascertain whether the three classrooms of students were of approximately equal natural intelligence and ability. The investigator scored each item of the pretest by examining the student's provided answer, and assigning either two points (totally correct answer), one point (partially correct answer) or zero points otherwise. Hence a perfect score of 12 was possible. Means of each class's performance were then compared as to whether classroom differences were significant.

After this initial session with each class, the investigator attempted to attend each class meeting where verbal problem content instruction was the focus of the day's lesson. There were occasions when such classroom observations were not possible, due either to the investigator's own teaching commitments, or when both Betty and Carl's verbal problem lessons landed on the same day, since their class meeting times overlapped somewhat. On these few latter occasions, the investigator selected which school site's instruction and observation was more critical to the needs of the study. The investigator believes that the daily observations of each teacher's verbal problem

instruction probably represented between 65 and 80 percent of each teacher's class days devoted to the identified core content of verbal problems.

On five occasions, unknown to the investigator prior to arrival at the school site, teachers had revised their daily lesson plans in order to provide additional review of nonverbal problem related content, rather than the scheduled verbal problem content. Field notes were still completed by the investigator during these instructional review sessions, and used for comparison purposes in the analysis of teacher instructional characteristics. Table 3.1 presents a summary of the number and type of investigator observations of each teachers' instruction.

Table 3.1

Summary of Completed Teacher Observations

Teacher	Verbal Problem Content Focus	Other Content Focus	Total Observations
Alice	11	1	12
Betty	8	2	10
Carl	7	2	9
Totals	26	5	31

The investigator used prepared forms for the taking of field notes during the classroom observations. The prepared forms contained space for placement of five minute time intervals along the left hand margin, and generally

two to three pages of field notes were gathered during each classroom observation. The investigator additionally noted the time explicitly in the field notes where it was felt that a change in a teacher's instructional focus with regards to various segments of the lesson was observed, and also when a teacher's instructional example presentations began and ended. Each day's field notes were rewritten and reviewed the same day as the observations. Teacher example presentations for a given day were reviewed from the field notes along with investigator recollections of such presentations, and the revised process coding form (Coding Form 1) was completed for each presented instructional example. The origin of the example, the sequence of teacher processes employed in the example, and specific comments/hints/suggestions provided by the teacher during example presentation were all noted by the investigator on Coding Form 1. Generally three instructional examples were reviewed on a single coding form using different pen colors for each example.

The pilot study had indicated that the usage of only field notes for the teacher observations likely would be insufficient to serve investigator purposes related to comparing teachers' usage of instructional time, comparing the quality of delivered teacher instruction, and also the resulting descriptions of each teacher's instructional routine related to verbal problem content. An additional

coding form, Coding Form 2 (see Appendix B), was developed by the investigator and completed after each teacher observation, to provide additional quantitative and qualitative information to aid this process of instructional routine description. One section of Coding Form 2 permitted the investigator to note the amount and percent of classroom time a teacher's attention was given to eight different purposes or segments of instruction. It is assumed that teachers' instructional actions define a regular routine or pattern, and that this pattern can be decomposed into distinguishable segments or purposes. The eight segments: organization/administrative, drill/mental math, homework review, concept review, lesson development, controlled practice, seatwork/homework, and transition/other adequately cover the range of teacher actions related to purposes, though admittedly there may be some overlapping of time commitments during a teacher's lesson.

A second section of Coding Form 2 permitted the investigator to subjectively evaluate the quality of the lesson development segment (introduction of new material) of the daily lessons within a common structure for all teachers. Teachers' verbal problem lesson development was evaluated on the basis of nine attributes considered characteristic of effective explicit instruction, where the investigator evaluated teacher performance of each

attribute on a five point scale (1 = low, and 5 = high). Four of the attributes, task orientation, clarity, enthusiasm, and frequency of teacher talk were taken from Good and Brophy's (1984) summary of effective characteristics of classroom instruction. The other five attributes represented the degree of teacher attention to prerequisites, relationships, concrete or alternate representations, discrimination perceptions, and concept generality, which are all considered characteristic of active and more successful mathematics teachers (Good, Grouws, and Ebmeier, 1983). A third section of the same form permitted investigator categorizations of teacher verbalizations and behavior when introducing new activity or content or when making assignments. Its purpose was to signal to the investigator teacher differences with regards to enthusiasm, enjoyment, and attitudes related to verbal problem sections. This third section was taken in nearly exact form from Good and Brophy's Form 4.1 (1984, pg. 124).

The analysis of the data collected from the investigator's teacher observations and completion of the related, developed instrumentation resulted in investigator formulations of descriptive characterizations of each teacher's instruction at three different levels. At the most general level, each teacher's classroom and students were described. At a second level, each teacher's pattern of daily instruction or instructional routine is

characterized fully and a teacher's actions and practices within the routine are related to their conceptions of the Algebra I content and verbal problems, their students, and their experience teaching this course to this level of student. At the third level, both quantitative and qualitative characteristics of each teacher's lesson development segments related specifically to verbal problem content is focused upon and described in greater detail.

The investigator also daily reviewed the field notes of the teacher observations specifically searching for instances when the teacher provided students with verbal suggestions and associated modeling of a behavior within their instruction designed to somehow aid student performance in solving verbal problems. Each such identified teacher suggestion was classified according to format (question or statement), in what segment of the lesson it appeared (homework review, lesson development, or controlled practice/seatwork), and whether it related to the general problem solving model, or was generally applicable advice (general heuristic), or was specifically applicable to one verbal problem type (task specific heuristic), or was related to specific tool skill development or usage. Quantitative comparisons between the three teachers were then compiled according to the investigator's schemes of suggestion categorizations. The investigator admits that there is limited usefulness to

this analysis of teacher instructional suggestions. There is little available research to indicate if differences in the format in which a suggestion is posed, or the suggestion's segment placement in the lesson, or the suggestion's intended purpose have any substantial or differential impact on student performance or its improvement related to verbal problems.

Student Performance Measures

The investigator designed three instruments, pretest, posttest, and retention test intended to measure student ability and performance characteristics related to the core content of algebra verbal problems. The contents of the pretest and the scoring method used by the investigator for its evaluation were described in the preceding section. Its intended purpose was to insure that each school's methods of student selection for Algebra I or Algebra I-A (accelerated track) were somewhat equivalent with regards to students' entering capabilities and characteristics.

The study's posttest development reflected five investigator concerns. First, the posttest had to include problem examples of all six verbal problem types identified as core content for the study, and which had received instructional treatment in all three classrooms. Secondly, the investigator was interested in student performance and processing of unsolvable verbal problems, and in the ability of students to identify contradictions within

problem statements. Thirdly, there was interest in student performance differences due to variations in numerical data types presented in verbal problem statements and to variations in the sequence of sentence presentation in problem statements. Fourthly, it was believed that one nonroutine, multi-step verbal problem be included to observe if any students were capable of transferring and extending their verbal problem skills. Fifthly, the posttest's length had to be carefully controlled in order that a large percentage of a class's students could capably complete the test within the 40 minutes of a class period. These diverse concerns lead the investigator to develop two posttest forms, A and B. The usage of two test forms also discouraged student cheating or copying, though this was not a major concern of the investigator. The two posttest forms, labeled A and B, can be found in Appendix F.

The investigator believed the two posttest forms of 10 verbal problems each represented equal difficulty for the students of the study. In terms of the core content's six verbal problem types, both posttest forms contained one simple translation type (identical on both forms), two geometric oriented types, one consecutive integer type, two age related types, two distance/rate/time types, and two coin/money related types. The sequence of problem types was the same for both test forms. The first two verbal problems, a simple translation problem type and a geometric

oriented problem type were identical on both forms and presented orally by the investigator to the classes at the beginning of test administration, while the remainder of problems were presented in a written format. The third identical problem (# 5) of both forms was an age related problem which had no solution.

The pair of problems (# 3 and # 4) were respectively a coin/money problem type involving wage computation and hence less familiar, and a standard distance/rate/time problem type. The wording for this pair of problems was identical on both forms, and variation involved the inclusion of one decimal number, rather than all whole numbers, in the data of the problem statement for one of the pair of problems on each form. The pair of problems (# 6 and # 7) were similarly worded, but used variation in wording (# 6) or numerical data (# 7) which resulted in one of the pair of problems on each form to have no solution. On both forms, problem six was a consecutive integer problem type and problem seven a geometric oriented problem type involving the finding of the dimensions of a rectangle given its perimeter and a relationship between its length and width dimensions.

Problem eight used a more familiar coin/money context (find the number of dimes and quarters in a collection), and the variation between forms related to the sequential order of presented sentences in the verbal problem

statement. Problem nine employed an age related context, was identical on both forms with the exception of the first names used in the problem statement, though the problem statement wording used was more difficult to comprehend or analyze than typical textbook age related problem types. Problem ten required two applications (nonroutine) of the distance/rate/time relationship, and each form's problem shared a common solution path from beginning to end. The problem information (data and names) were varied on each form and resulted in different solutions, though both problems were presented in a similarly realistic problem situation. The two test forms were typed into a two page format, five verbal problems per page, with space provided between problems for student work and blanks provided for student entry of their solutions for each problem.

The posttests were administered to each class during one regularly scheduled class meeting by the investigator within two weeks after the respective teachers' instruction of the core content had been completed. The posttest administrations were done during three different weeks of December due to differences in teacher pacing and textbook sequencing of the core verbal problem content. Adjacent students received alternate forms of the posttest, and the investigator made no comment regarding there being two different forms of the posttest. Calculator usage was permitted, though calculators were not provided by the

investigator or respective teachers.

The first two problems were read aloud by the investigator as often as students requested, usually four or five times per problem. The students then worked individually on the eight remaining problems of the posttest during the class period. No cases were noted by the investigator of time being a constraint on student completion of the posttest forms. Most students finished what they were capable of completing within thirty-five minutes, and all test forms were collected during the class periods. With the exception of Alice's class, all students showed their solution work right on the test forms. Alice's students used their own paper in solving the ten problems since Alice felt the provided workspace on the test forms was insufficient, a fact she expressed to the investigator prior to the posttest administration. In her class, student papers were collected with the test forms, and involved only minor inconvenience in the investigator's analysis.

Investigator scoring of student work samples for the individual test items and for the test form as a whole was a problem revealed by the pilot study. Two research studies (Webb, 1979 and Kantowski, 1977) had demonstrated the feasibility of employing a scoring system based on levels of satisfactory completion of several phases for a single verbal problem. Webb's (1979) scoring system had

three phases: Approach, Plan, and Result, with zero or one points awarded in the Approach phase, and zero, one, or two points awarded for the student's Plan and Result. Kantowski's (1977) process product scores were based on awards of zero or one point for each of the following: suggesting a plan of solution, persistence, looking back, absence of structural errors, absence of executive errors, absence of superfluous deductions, and correctness of result. The investigator's scoring scheme for analysis of student performance involved separating a verbal problem's solution path into its component parts according to the general problem solving model advocated by the textbooks, and assumed incorporated and followed by the study's students. Five component parts were identified: variable definitions, equation formulation, equation solving, interpretation/checking, and correctness of result. Scores of zero, one, or two were assigned by the investigator for each of the five phases based on the review of student work for a particular verbal problem. A correctly completed solution path would then result in a student score of 10 for an item. A more detailed description of the characteristics considered in partial scoring for a given phase is provided in Chapter VI, with the actual student performance results. Additionally, the investigator noted in the review of student work samples any indication of a student attempting to define variables for a verbal

problem, which suggested that the student was aware of the general problem solving model, where construction of variable definitions represents a student's first written output.

The investigator's developed scoring scheme provided a wealth of information for analysis and discussion. All analyses of student performance were done at a classroom level. Comparisons of student performance by mean scores, for individual test items, for the two forms, for the six problem types, and for other introduced classification systems were completed. In all cases, these comparisons served to initiate investigator thought as to what aspects of the observed teacher instruction within the classrooms may have accounted for these noticeable differences or patterns of classroom student performance. The investigator also analyzed student error patterns with particular attention given to alternate (and incorrect) variable definitions and equation formulations.

The investigator was also interested in which of the five component phases students experienced the greatest difficulty, and what percent of a class's students were able to navigate their way from the correct completion of one phase and move into the next phase. Therefore an analysis was developed around the notion of conditional probability, such that at each of the five phases a class's success rate (proportion) was calculated for those who had

demonstrated successful completion of the immediately preceding phase. An example is the proportion of a class of students who correctly formulated an equation for a problem given that this same subset of students had all constructed correct variable definitions. Again these proportional comparisons were carried out at a classroom level, both for individual items and for the posttest as a whole.

The last classroom measure of student performance developed by the investigator was a retention test instrument. Besides serving as a retention measure related to verbal problem solving skills and abilities, the retention test contained two other sections, an attitude survey and an equation solving section considered important to the investigator's overall purposes. The retention test form is found in Appendix F. The attitude survey was composed of seven items, six requiring structured responses and one open ended item. The one open ended item appeared last in the section, and requested students to indicate what they might say to an eighth grade student who questioned them about the Algebra I class and their respective teacher.

The initial item of this section asked students to rank order five concerns (1 = most important to 5 = least important) according to students' perceptions of the relative importance of each concern. In this item, the

investigator attempted to get at the motivational structure for students' participation in the class and related learning orientations. The first two listed concerns related to immediate student concerns for passing the course and achieving a good grade. The third concern, understanding the course content on a day to day basis was less immediate and represented student awareness of a learning orientation. The fourth and fifth concerns tended to address more future oriented purposes for class participation, one cognitive (solid knowledge base) and one affective (increased confidence). The analysis of this item focused on the immediate/future distinction of orientation and associated patterns of students' concern rankings.

For the other five items of the attitude survey, students were to circle one of the listed statements which best reflected their perceptions with regard to the posed area of questioning. Four items related to student perceptions of ability and attitude towards the algebra class as a whole and towards verbal problems specifically. The fifth item requested students to estimate how useful they believed the Algebra I class would be for their future high school program. The three items related to algebra class ability, verbal problem ability, and algebra class attitude were similar in that each item's five possible response selections ranged from very positive to very

negative aspects in equivalent steps. This parallelism of response selection was done deliberately in order that classroom correlations among pairs of the three items might be computed within the data analysis. These correlations served to indicate relationships between students' perceptions of ability in the two areas, and also relationships between ability in the two domains and general classroom attitude.

The verbal problem attitude item had six possible choices for student response, in which four of the choices involved variations in the degree of difficulty (easy or challenging) and the degree of satisfaction (enjoyable or not enjoyable) in solving verbal problems. The other two choices were a neutral, lacking opinion choice and a total dislike choice. The item related to future course usefulness had four choices of response, varying from very useful to no usefulness whatsoever. Class counts of student responses for these two items were used to compare the three classes' perceptions related to verbal problem attitudes and course usefulness. The investigator additionally analyzed separately those students who expressed total dislike for verbal problem exercises in each class by reviewing their responses for the verbal problem ability item and the algebra class attitude item. In this way the investigator searched for whether the reasons associated with this total dislike of verbal

problems was more due to poor success or ability with verbal problems, and also whether this negative attitude was more specific to verbal problem content.

The equation solving section of the retention test form was composed of seven linear equations for the students to solve. Each of the seven equations were modifications of the equations the investigator expected students to formulate for the seven verbal problems of the following verbal problem section. Modifications to the expected equations involved transposing certain terms of each equation, while maintaining the same numerical constants and parenthetical constructions. The seven equations were not in the same order as they would be formulated in the verbal problems, and appeared on pages separate from the verbal problems. Not a single student noted the similarity between sections to the investigator either during or after retention test administration.

The investigator analyzed each student's work samples for the seven equations, and awarded one point for each correctly derived solution, so that student scores for this section ranged from zero to seven. Secondly, student equation solving errors were reviewed, with the investigator categorizing students' initial errors as either computation (real numbers), computation (variable terms), distributive property related, or procedural (transposition) error. The few uncodable errors not

fitting within this categorization scheme were omitted from the analysis, as were occasions where students made no written attempt to solve a given equation. Classroom comparisons were then made for the means of the three classes for this section, as well as for student performance on the individual equations and quantities of errors within the various error categories.

The unique design of the equation solving section, as modifications of the expected equations of the verbal problem section, permitted the investigator to compare student abilities to solve similar linear equations as they appear in the two formats. The investigator noted the occasions where students had correctly formulated equations for the verbal problems and then compared student success rates in solving both the formulated verbal problem equation and the related equation of the earlier equation solving section.

Based on the relatively poor student performance on the investigator designed verbal problems of the posttest, the seven verbal problems for the retention test were taken directly from the three textbooks of the study, and reflected an intermediate level of difficulty. There were two verbal problems of a geometric formula problem type included, and one problem each of the other five verbal problem types. The scoring of student performance for the verbal problem section, on the basis of student provided

work samples, was identical to that method used for the posttest, with zero, one, or two points awarded for each of the five phases' degree of completion and correctness, making 10 the maximum possible score for one verbal problem, and 70 the total possible score for this section. Classroom comparative analyses of these results followed similar lines developed for the posttest analysis. Additionally correlations were computed relating classroom performances on the equation solving section and the verbal problem section of the retention test and for the verbal problem sections of the posttest and retention test.

All retention test forms were six pages in length, identical in content, and sequenced with the attitude survey first, equation solving section second, and the verbal problem set last. The retention test form provided adequate workspace for the students to write out their solution attempts on the test form itself for both sections, the equations and the verbal problems. Students were encouraged to show all their written work and not to erase any work, both within the oral instructions given by the investigator at the beginning of test administration and also within the written instructions contained on the test form.

After the students had received the retention test form and listened to the investigator's instructions regarding the test's three sections and the time available

for completion, students completed the entire test form at their own pace without any further investigator interruptions. The retention test was administered to both of Betty's Algebra I classes (hence 4 total test administrations), since students in her two classes had been intermixed at the beginning of the second semester (January, 1987). The two administrations for her classes did not create any problem for the retention test analysis since both classes had taken the earlier posttest (at Betty's request) and results were easily separable by student name.

The retention test was administered in all four classrooms by the investigator during the first two weeks of March, 1987. Approximately ten weeks of instruction had taken place since the posttest administrations. All three teachers of the study indicated to the investigator during the retention test administration that little instruction or review of verbal problem content had taken place since the posttest administration. The investigator did note in all four classes some student discomfort and frustration during their completion of the retention test, particularly within the verbal problem section.

Student Interviews

The nine individual student interviews (three students from each class) were all conducted during the same school week, just prior to the December, 1986 Christmas vacation.

The elapsed time between class posttest administrations and the interviews ranged from two days to two weeks. Each student interview lasted approximately 45 minutes (one class period), and was audiotaped and later replayed while the investigator completed Coding Form 1 related to student process usage for each verbal problem. Eight of the nine interviews were conducted in isolated small rooms within the respective school buildings during regular school hours. The only exception to this format was the interviewing of one of Betty's students after school in an empty university classroom.

The process of identifying students for the interviews was focused on selecting more capable students with regards to verbal problem solving ability. Because it was impossible to complete the analysis of the posttest results prior to this selection process, the investigator relied on perceptions of student ability based on the completed observations of student participation in each classroom. For each classroom, the investigator identified five students who were believed to be more capable verbal problem solvers and whose schedules allowed opportunity for in-school interviewing. Each proposed list was double checked with the respective teachers to insure that each student on the lists was performing at an above average level (B grade or higher) within the Algebra I course. The five students in each class were then given a two page form

to deliver to their parents which described the elements of the study and the interview format, and requested parental permission to conduct the interview with their son or daughter. The consent form's content, purpose, and format had received the earlier approval of the University's Commission on Research with Human Subjects, and had also received necessary administration (principal) approval at each school site. The Consent Form can be found in Appendix G.

Within two days of consent form distribution to the students, 12 of the 15 forms had been returned to the investigator. One student's parent denied permission for her daughter to be interviewed, while the other 11 returned forms indicated parental consent for the investigator to interview their child. Investigator questioning of the three students who failed to return the consent form all revealed student disinterest in being interviewed. At all three school sites, the three students eventually interviewed were among the investigator's initial lists of potential interview canidates.

The interview instrument (Appendix G) had three components, a background information survey, a perceived confidence level measure, and a verbal problem section. The background information survey included questions related to previous math course takings, Algebra I grades to that point, student perceptions of their teacher's instructional

strengths and weaknesses and the algebra content in a general way, and students' analyses of their strengths and weaknesses related to solving verbal problems. The survey concluded with a hypothetically posed verbal problem assignment for which students were to conjecture on the time and place of completion, and describe how they typically completed such homework assignments. The investigator recorded student responses on a prepared form as they were given by the student. This first component took approximately five minutes per interview to complete. The investigator's analysis of this component focused primarily on comparisons of students' responses to the hypothetical homework assignment and students' patterns of homework completion, with some discussion and description related to the students' background information.

The second component of the interview instrument was a one page form on which the investigator filled in percentage estimates provided by the student of their perceived ability to correctly set up a given verbal problem type's variable definitions, to correctly formulate a related equation (assuming correct variable definitions), and to correctly solve the formulated equation. This process was completed six times, for five of the six verbal problem types under study (simple translation type excluded) and a last classification described as a nonroutine, multi-step verbal problem type. The resulting

analysis focused on differences for each student with regards to confidence levels associated with the three phases (variable definition, equation formulation, and equation solving) of verbal problem solution, and also how students ordered the verbal problem types in terms of perceived difficulty. Comparisons between the nine students' mean confidence levels for the three phases were developed, and later compared for perceived accuracy with the actual performance of students in the three phases on the verbal problem set. This second component of the interview was generally completed in ten minutes, leaving 30 minutes for the interviewed students to consider and complete the third component, the verbal problem set.

The investigator designed three different verbal problem sets for the interview instrument, one set for each of the three classrooms of the study. The investigator considered the three designed sets to be of equal difficulty, with five of the six verbal problems of each set coming from the respective class's employed textbook. It was assumed that each set's design then better reflected problem statement wording to which the students were more familiar. The sixth verbal problem of each set was identical, being nonroutine in nature, age related in content, and required students to more carefully analyze and separate relationships and given information within the problem statement before proceeding to define variables and

formulate equations. More information than necessary to solve the problem was given, and the problem required the finding of three ages for three members of a family. The six verbal problems were presented to the interviewed students, one at a time, with each problem statement on a separate sheet of paper.

In each interview, the investigator presented the sixverbal problems in the same order according to problem type, with the one common nonroutine verbal problem always presented last. At the beginning of the verbal problem section, the investigator instructed the student to read each verbal problem silently, and then to verbalize their thoughts and written actions as they worked towards completing a written solution for the problem. While a student was working a particular verbal problem, the investigator remained quietly and attentively listening to the student's verbalizations, occasionally nodding affirmatively to a student's direct questions or gently reminding students to resume verbalizing when lapses of 30 seconds without student verbalization were noted by the investigator. Two of the nine interviewed students did require constant reminders to continue verbalizing.

Students determined when to stop work on a particular verbal problem, either when they believed the problem solution was finished or where a student's progress was stalled. Students were then presented with the next verbal

problem of the set. After students had at least read and attempted solutions for each of the six verbal problems and if sufficient time remained in the interview, students were permitted to reselect problems for further study and attempts at solution.

Interviewed students' work samples for the verbal problem sets were reviewed by the investigator with the audiotape of the interview playing, and the scoring method employed was identical to that used for scoring the student work samples of the posttest and the retention test. Points (0, 1, or 2) were awarded for each of the five phases' completion and degree of correctness, resulting in 10 points possible per problem and 60 points possible for the instrument. For each verbal problem attempted, students' process usage was noted on Coding Form 1, though this information was not included in the investigator's analysis since it added no new or relevant information to the study's direction. Comparisons of the nine students' overall performance and individual item performance were completed, and patterns of relative success noted for the various problem types. The audiotapes were reviewed a second time in which the investigator noted the amount of time students were actively attempting to solve each problem and also recorded specific student verbal comments made during the interview which related to features of student thinking and processing patterns. On the basis of

these verbal comments and students' demonstrated written performance, tentative characterizations of various developmental levels of student verbal problem solving competence were constructed.

The results of the various investigator analyses are each done separately, with Chapter IV focusing on the textbook analysis and the descriptions of each teacher's instructional routine gained from the investigator's observation of each teacher's instruction and the teacher responses on the Teacher Inventory Packet, with Chapter V focusing on the results of the investigator's analysis of teachers' verbal suggestions, and with Chapter VI focusing on the results of the investigator's analysis of student performance on the designed instruments of the study, including the interview results. In Chapter VII, the results compiled in the three preceding chapters are discussed and interrelated according to the eight hypotheses for the study, and related conclusions and recommendations are formulated.

CHAPTER IV

Instructional Environment Patterns

This chapter focuses on the characteristics and environmental patterns of the three Algebra I classrooms which formed the basis for this study. The investigator has analyzed the data to describe the regularities which characterize the two major instructional components' functioning within the setting. The first component, the three textbooks, will receive initial treatment. The second component consists of three sections focusing on the three teachers in the study. These three sections will proceed from general teacher descriptions and conceptions, to general instructional routines, and finally to specific glimpses of verbal problem related instruction. Formulated conceptions were derived from classroom observations and teacher responses on the Teacher Inventory Packet discussed in Chapter III. Teachers' instructional routines were formulated from investigator observations. Lastly the developmental segment of each observed lesson was reviewed for instructional styles which characterized each respective teacher's lessons on verbal problems.

Textbook Analysis

The investigator assumed that the three teachers of the study would follow the textbook sequence in their instructional planning. Each algebra textbook in the study was examined to identify verbal problem sets which would receive instructional treatment during the first four months of the 1986-87 school year. The identification of a common core of verbal problem types representative of a student's initial experiences with verbal problems was necessary. An exercise was considered a verbal problem if three criteria were met. First, the problem statement was presented in a written or pictorial format. Second, the exercise suggested the defining of a variable or unknown. Third, an equation was the suggested means for finding the problem's solution. This last requirement permitted differentiation between verbal expression exercises and verbal problems. Table 4.1 summarizes the investigator's review of each textbook in the analysis.

Table 4.1

Textbook Review Summary

Textbook	Teacher	Sections	Reviewed pages
A	Alice	1.1- 5.2	1- 168
B	Betty	1.1- 4.10	1- 153
C	Carl	1.1- 5.11	1- 189

The page by page review of each textbook initially separated page content into four categories: explanations, oral exercises, written exercises, and other. The "other" category included chapter separators, chapter reviews and tests, skill sharpeners, and other optional application sections. Fractions of a page to twelfths was used to categorize each page. Each textbook section was further coded to indicate its skill or content focus among three contents: manipulation, verbal expression, or verbal problem. The number of exercises and verbal problem examples in each section were counted and coded accordingly. Table 4.2 displays the number of pages and percent of reviewed pages in each of the four space allocation categories.

Table 4.2

Textbook Comparison: Space Allocation

Space Purpose	Textbook pages		
	A	B	C
Explanations	49 3/4	51 1/12	91 1/12
	(29.6%)	(33.4%)	(48.2%)
Oral exercises	11 11/12	13 1/4	13 3/4
	(7.1%)	(8.7%)	(7.3%)
Written exercises	48 1/6	47 5/12	38 1/6
	(28.7%)	(31.0%)	(20.2%)
Other	58 1/6	41 1/2	46
	(34.6%)	(26.9%)	(24.3%)
Totals	168	153	189
	(100%)	(100%)	(100%)

The percent figures are more representative in making textbook comparisons of space allocation. Space devoted to oral or classroom exercises is similar for the three textbooks. Space allocated for written exercises is somewhat misleading. Despite the highest percentage of space allocation and the second highest number of pages allocated for written exercises, textbook B has the fewest written exercises. Textbook A has the most written exercises, followed by textbook C. Written exercise differences will be discussed more fully later in this section.

The space allocation breakdown does demonstrate differences between textbooks in their explanatory, instructional treatment of algebra content. The fifteen to twenty percent difference between textbook C and the other two textbooks is indicative of differences in textbook author's instructional intentions. It appears that the authors of textbooks A and B include greater percentages of pages devoted to the "other" content and written exercise categories, and include less instructional treatment and example presentations than textbook C. Textbook C contains no skill sharpeners or review sections, other than chapter reviews. It contains no historical notes, career or consumer applications, or challenges. While clearly the least modern in its style, format, and attractiveness, textbook C clearly emphasizes

its instructional purposes and intentions.

A striking similarity existed between the three textbooks in regards to the three content categories noted earlier. Each textbook allocated approximately 60 percent of its instructional space to manipulation skills, 20 percent to verbal expression skills, and 20 percent to verbal problems. For each content category, textbook variance was less than three percent from these figures.

Core Content

Following space allocations, the next focus was the identification of the core content of verbal problems and the comparative treatment of this content. Seven distinct problem types were initially noted for the core content. Because textbook B contained no percent related problem types in the reviewed pages (appears later in this textbook), six problem types remained for analysis. The six problem types are noted in Table 4.3, with the number of pages of explanation, and the number of practice (written) exercises for each problem type by textbook.

Table 4.3

Textbook Comparison: Core Content of Verbal Problems

Problem Type	Textbooks					
	A		B		C	
	Pages	Exer.	Pages	Exer.	Pages	Exer.
1. Simple translation type	4 1/8	52	4 1/4	135	3 1/4	39
2. Geometric formula types	2 1/8	47	1	12	6 1/4	43
3. Integer problems	1 1/2	22	0	16	1 1/2	19
4. Age related	2	28*	1 1/2	13	1	16
5. Distance/rate/ time related	1 1/2	18	2	16	2 1/2	15
6. Coin/money related	2	19**	3/4	15	3 1/2	24
Totals	13 1/4	186	9 1/2	207	18	156

* includes 10 exercises from supplementary text (Alice)

** includes 15 exercises from supplementary text (Alice)

It appears from Table 4.3 that students using the three textbooks receive approximately equal exposure to the six problem types which make up the core content. This is particularly true for exercise counts related to the last four problem types. Students do not receive equal exposure to the first two problem types however.

There are two reasons, besides differences in quantities of problem types, why students in the three classrooms did not receive equal exposure to the six problem types. One reason relates to the difficulty in analyzing textbook B by problem type, and the second reason relates to the instructional assignment patterns of

the three study teachers. Textbook B has greater context diversity and multiple problem types in some of its exercise sets. Also its verbal problem exercise sets are more evenly distributed and in greater numbers within its design. Its placement of verbal problem exercises after relevant manipulation skill explanations, not done in the other two textbooks, accounts for the large number of exercises in the simple translation category. In conclusion, several design features of this textbook limited the investigator's ability to classify written exercises into the six problem types.

The second, more important reason for unequal classroom familiarity to the six problem types relates to the instructional assignment patterns employed by each classroom teacher. Based on classroom observations, it can be concluded that Alice's students were assigned 90 percent of the verbal problem exercises, or her students attempted 167 problems of the core content. Betty assigned her class 50 percent of textbook B's verbal problem exercises, or students attempted 104 problems of the core content. Carl assigned his class 60 percent of textbook C's verbal problem exercises, or students attempted 94 problems of the core content. This exposure or familiarity component to the six problem types is assumed accurate for Alice's and Carl's classes of students. All students in these respective classes did

attempt to solve the assigned textbook verbal problems daily. The investigator believed that a majority of Betty's students however did not attempt to solve assigned sets of verbal problems. Reasons for this belief will be discussed more fully later in this chapter. In general, the investigator was able to identify six verbal problem types to serve as a common core of verbal problems for the three classes of the study. The three textbooks do provide approximately equal coverage of the six problem types. Classes of students in the study however, did not receive equal exposure to the six types. It appears that Alice's students will have the greatest familiarity with the core content of verbal problems, and Betty's students the least familiarity.

The explanatory or instructional treatment each textbook provides of the core content verbal problems is also indicated in Table 4.3. Textbook B has the fewest pages of instructional treatment of the core content, and also the fewest instructional examples, 11. Textbook A is midrange on both measures with 19 instructional examples. Textbook C has the most pages of instructional treatment of the core content, and the most instructional examples, 40. The effect of these differing instructional treatments is less clear from research than the familiarity or exposure factor. While research has shown that the ability of students to solve various problem

types is dependent on student familiarity with different problem types (Stigler et al, 1986), the impact of differing quantities of textbook explanations and examples is possibly improved or diminished based on mediation within individual teachers' presentations of textbook material.

Verbal Problem Classifications

A detailed analysis was performed on the core verbal problem exercises of each textbook. The classification technique utilized categorized a problem into one of six types: drill exercise, simple translation problem, complex translation problem, process problem, applied problem, or puzzle problem (Charles and Lester, 1982). A textbook verbal problem is either a simple translation type or a complex translation type within this classification system. Verbal problems for which equation formulation could be accomplished by direct translation of the problem statement were classified simple translation problems. Verbal problems for which equation formulation required student processing of inferred relationships within the problem statement were classified complex translation types. Complex translation types were defined differently in this analysis from that proposed by Charles and Lester. Problems which involved at least two steps of translation into mathematical sentences were defined complex translation types in their classifications. Since no

exercises of the reviewed core of textbook verbal problems involved more than one equation for solution, the investigator adjusted the two types accordingly. Table 4.4 shows the results of this classification applied by problem type to the core verbal problem exercises of each textbook.

Table 4.4

Textbook Comparison: Simple/Complex Exercises

	A		B		C	
Problem type	Simp.	Comp.	Simp.	Comp.	Simp.	Comp.
1. Simple translation	48	4	100	35	37	2
2. Geometric formula	30	17	2	10	32	11
3. Integer related	10	12	10	6	13	6
4. Age related	18	10	5	8	10	6
5. Distance/ Rate/Time	5	13	10	6	9	6
6. Coin/ Money	10	9	7	8	5	19
Totals	121	65	134	73	106	50
Percent	(65%)	(35%)	(65%)	(35%)	(68%)	(32%)

From Table 4.4, the total percentages of each classification, simple or complex, for each textbook is nearly identical. There are about two simple translation types for each complex translation type exercise across textbooks. Differences are greatest for problem types related to geometric formulas, age related, and

Distance/Rate/Time related exercises. Simple translation type exercises generally dominate the first half of an exercise set, while complex types appear in the last portion of an exercise set. It can be assumed that students are more familiar with simple translation types for two reasons. First, the sheer number of exercises favors simple types, and secondly the order of presentation suggests students complete more of the simple types.

A second classification scheme for verbal problems developed by the investigator closely parallels the work of Caldwell and Goldin (1987). A verbal problem exercise was labeled one of four formats of problem dependent upon the problem statement of the verbal problem. The four formats are: abstract factual (AF), abtract hypothetical (AH), concrete factual (CF), and concrete hypothetical (CH). An abstract problem statement describes symbolic objects, such as numbers or figures, while a concrete problem statement describes a real situation. A factual problem statement describes declaratively a situation, while a hypothetical problem statement describes additionally a possible change in the situation. A hypothetical problem requires the problem solver to consider both the described situation, and also the change element relationship. The investigator classified each core content exercise of each textbook according to this

classification scheme. The summary results of this analysis appear in Table 4.5.

Table 4.5

Textbook Comparison: Exercise Format Classification

	Textbooks											
	A				B				C			
Prob. Type	AF	AH	CF	CH	AF	AH	CF	CH	AF	AH	CF	CH
1.Sim. Tr.	10	6	36	0	47	3	76	9	37	0	2	0
2.Geom.Fm.	31	9	7	0	2	0	6	4	34	2	5	2
3.Int. Pr.	10	12	0	0	9	4	2	1	16	3	0	0
4.Age Rel.	0	0	0	28	0	0	9	4	0	0	2	14
5.D/R/T	0	0	6	12	0	0	11	5	0	0	15	0
6.Coin Re.	0	0	12	7	0	0	12	3	0	0	14	10
Totals	51	27	61	47	58	7	116	26	87	5	38	26
Percent	27	14.5	33	25.5	28	3	56	13	56	3	24	17

The most notable distinction is that abstract/concrete classifications are heavily dependent upon the six problem types. Age related, D/R/T related, and coin/money related types are presented exclusively with concrete problem statement formats in all three textbooks. Greater diferences between textbooks are apparent for the simple translation and geometric formula exercise types. Textbook C particularly emphasizes abstract problem statements over concrete problem statements for these two problem types.

Caldwell (1977) found 75-80 percent of elementary textbook verbal problems to have concrete factual problem statements. The three algebra textbooks do exhibit a greater diversity of the four verbal problem formats than found in Caldwell's study. This result is not surprising since from a developmental perspective it is assumed that students are moving from a concrete operational level to a formal operational level during this age transition. Yet textbook B still maintains a high percentage (56 percent) of concrete factual formats in its exercise sets.

Caldwell and Goldin (1987) hypothesized that dependent upon one's perspective for the development of problem solving competence, differing sequences of problem difficulty involving the four format types result. These problem difficulty sequences are set forth for the three perspectives of problem solving competence along with Caldwell and Goldin's results for the elementary study (grades 1-6) and secondary study (grades 7-12) in Table 4.6. The three perspectives are described in greater detail in Chapter III.

Table 4.6

Verbal Problem Difficulty: Format Types

Models/Results	Least Difficult	Intermediate Difficulty	Most Difficult
Developmental	CF	CH - AF	AH
Direct Translation	AF	AH - CF	CH
Internal Representation	CF	CH	AF -AH
Elementary Study	CF -CH	AH	AF
Secondary Study	CF	CH	AH -AF

In both of Caldwell and Goldin's studies of verbal problem solutions, concrete problem statements were less difficult than abstract problem statements. However this difference in student performance on the two kinds diminished with increasing student age. For elementary students, the difference was 14 percent more correct, for junior high students 13 percent more correct, and for senior high students 3 percent more correct. The authors concluded that the elementary results best fit the predictions of the developmental perspective, and are partially explained by greater student exposure to concrete problem statements. The data of the secondary study fit the predictions made by the internal representation perspective, with some support for the developmental perspective as well.

The investigator considered the quantity or percentage of each of the four formats to indicate

textbook authors' conceptions of problem difficulty. Therefore it was assumed that student familiarity or exposure to the four formats was directly related to problem difficulty. The resulting verbal problem difficulty sequences for the three textbooks based on percentage exposure to the four formats is displayed in Table 4.7.

Table 4.7

Verbal Problem Difficulty: Textbook Format Classification

Textbook	Least Difficult	Intermediate Difficulty	Most Difficult
A	CF	AF - CH	AH
B	CF	AF	CH - AH
C	AF	CF - CH	AH

Textbooks A and B are similar to one another in regards to format classifications of verbal problem difficulty. Both textbooks resemble the developmental model sequence of problem difficulty (see Table 4.6). Textbook C more closely resembles the direct translation model sequence of problem difficulty. The secondary study results (Caldwell and Goldin, 1987) indicate textbooks A and B more closely conform in formats with generated student performance on verbal problems.

The relative treatments each textbook provides of the four formats also permits predictions of differing

classroom performance on verbal problems of the four formats. Again it was assumed student exposure or familiarity with the four formats would relate to student performance levels. The summary percentage of each format kind for each textbook are displayed in Table 4.8.

Table 4.8

Textbook Comparison: Summary Formats in Percentages

Format Classifications	Textbooks A	B	C
AF	27	28	56
AH	14.5	3	3
CF	33	56	24
CH	25.5	13	17
Total	100	100	100

It is predicted that Carl's class (textbook C) will perform better on AF format verbal problems than the other classes based on greater AF format familiarity. Similarly Alice's class (textbook A) will perform better on CH and AH format verbal problems, or in general on all hypothetical formats based on greater student familiarity. Betty's class (textbook B) will perform better on CF format verbal problems than the other classes. These predictions will be checked using item analysis of the posttest and retention test results.

The core of six verbal problems types for the study share a common underlying equation structure. Each verbal

problem type requires the construction and solving of a single linear equation in one unknown, or variable. Junior high students' perceptions of verbal problem similarity or relatedness was studied extensively using problem sorts by Silver (1979). Results indicated high ability students categorize problems based on mathematical structure characteristics, particularly underlying equation structure. Lower ability students generally categorize problems based on shared conceptual problem information, clustering verbal problems into various content types, similar to the investigator's problem types. The authors of textbooks A and C explicitly label all exercise sets and related explanations by problem type. This explicit "type" labeling appears intentional by these authors. According to Silver's results, explicitness would aid natural categorization processes of middle and low ability students, possibly its intended audiences. Textbook B provides explicit labels for three problem types: integer problems, age related problems, and D/R/T problems. Context and conceptual problem information within its exercises is more varied. The authors of textbook B attempt to limit student categorization of verbal problems into specific problem types, possibly intending their textbook for a higher ability audience. The study's focus is on middle ability students, and research results indicate these students are

more likely to categorize problems into problem types. Therefore the design of textbooks A and C appear more directed at Algebra I students, and possibly more beneficial to these students' performance on verbal problems.

Quantitative comparisons of verbal problem explanations and exercises fail to distinguish qualitative textbook differences related to organization and content. An evaluation form was designed by the investigator which incorporated both quantitative and qualitative characteristics of textbook design related to verbal problems. The designed instrument served to clarify investigator preferences of textbook construction. These preferences include: clear and consistent usage of a general model, clear, consistent and connected explanations and examples, and adequate quantities of explanations, examples, and exercises. The instrument also permitted formulation of descriptions of each textbook's design characteristics, along with respective strengths and weaknesses. Table 4.9 lists the subjective ratings assigned to each of the sixteen features selected for evaluation under four headings. The four headings were general model, explanations, examples, and exercise sets. A scale of one to five was used to rate each listed feature (one lowest, five highest).

Table 4.9

Textbook Comparison: Feature Evaluation

Feature	A	B	C
A. General Model			
1. Quality of model	3	4	5
2. Maintained consist.	1	5	4
3. Student usage clar.	3	5	3
subtotal	7	14	12
B. Textbook Explanation			
1. Quantity	4	1	5
2. Clarity objectives	3	3	5
3. Consistency/connect.	2	3	5
4. Layout appeal	4	5	2
5. Student readability	5	3	4
6. Suggested sequence	4	3	4
subtotal	22	18	25
C. Textbook Examples			
1. Quantity	2	1	5
2. Clarity/consistency	5	5	5
3. Diversity/complete.	2	3	5
subtotal	9	9	15
D. Exercise Sets			
1. Quantity	4	5	3
2. Clarity/readabilty	3	5	4
3. Diversity/complete.	2	5	4
4. Degree of difficulty	2	5	4
subtotal	11	20	15
Grand Total :	49	61	67

Analysis of Selected Textbooks

The following descriptions will focus upon a single textbook at a time, and the rationale for the ratings designated in Table 4.9. Textbook B was rated highest under the General Model heading. Its five step procedure for solving verbal problems is closest to Polya's model, and is introduced early in the textbook (Chapter 1). The five steps are labeled explicitly and consistently in

every example and sample reviewed. It presents to the student a clear model for processing verbal problems, and reinforces the importance of a general model for attacking all verbal problems types. General model development implicitly assumes verbal problem solving demands a combination of translation skills and relational thinking skills.

Another strength of this textbook is the exercise sets. The quality, clarity, diversity, and degree of difficulty within its exercises is superior to the other textbooks' exercises. Quantity of exercises is sufficient. Particularly noteworthy is the presence of multiple contexts within a single exercise set, and the greater challenge of its B and C level exercises.

Textbook B's major weaknesses are rooted in its instructional treatment of verbal problems. Verbal problem exercise sets are organized in three different ways. Most often, they appear immediately after manipulation skill exercise sets, and are designated as "Problems" with a single sample illustration. Secondly, some verbal problem sets appear as separate entities, given a problem type label, with no worked sample provided. Thirdly, some verbal problem sets receive greater instructional treatment in separate textbook sections. The first two locations are problematic for a teacher because the authors' intention for instructional

treatment is not clearly specified. A teacher must decide whether these problem sets are to be assigned with the preceding manipulation exercise sets, or are to receive separate instructional treatment and assignment. This locational diversity coupled with an inadequate quantity of verbal problem examples and explanations damages this textbook's readability, connectedness, and completeness ratings.

Textbook A's design strengths emanate from its concern for superficial characteristics which its authors believe are appealing to teachers and students. Its instructional explanations are clear, colorful, and generally appealing to the eye of the investigator. Boxed formats are used frequently in each section for emphasis of major points. The large print and simple vocabulary improve student readability of the textbook. Alice mentioned to the investigator that the inclusion of skill sharpeners within chapters as mini-reviews of earlier concepts, and the large number of practice exercises included in this textbook were primary reasons for its selection as a textbook for middle ability algebra students. Based on her years of teaching experience, Alice believed these features were essential for this level of students. She believed these students required greater practice experiences to gain skill mastery, and that student retention of the material could be

strengthened through more frequent review within chapters. This textbook also contained adequate clarity and consistency within its explanations and instructional examples.

The major weakness of textbook A is its inability to develop a consistent general model/procedure for processing verbal problems. Each time a new verbal problem type is encountered, a new or modified procedure is identified for usage on that particular problem type. Five such procedures were noted within the reviewed verbal problem content. Each identified problem solving section contains such a procedure which is boxed for emphasis. The commonalities between these procedures suggested to the investigator that the primary skills required for verbal problem solving are translation oriented skills. In three of these boxed procedures, instruction focuses on finding the two conditions within the problem statement which lead directly to the defining of variables, and to the construction of the problem's equation respectively. The relational nature of equation formulation receives minimal instructional consideration. The verbal problem examples and exercise sets reflect a similar low expectation of students' abilities to relationally think and transfer developed skills. There is minimal context diversity within an exercise set, and the increasing degree of difficulty within a set is accomplished in small

increments. Initial problems in an exercise set contain an identical structure to the instructional examples provided.

In analyzing the exercise sets of this textbook (A), the investigator noted a definite pattern. Pairs of consecutive problems (one odd, one even) contained almost identical contexts. It could not be determined whether this pattern was deliberate or not. The pattern, if recognized by teachers, would work well for classroom assignment practices. A teacher might assign all odd or even problems of a set one day, and the remaining problems the following day. This practice would insure context diversity within each assignment. The investigator believes that the combination of many diverse procedures organized around problem type categories and problem sets of limited context diversity somewhat inhibits verbal problem solving ability development. This combination could also nurture a common student belief that algebra learning is piece-meal, and unconnected to previous learning. The organization of textbook A facilitates student practice and teacher testing techniques on a narrow band of verbal problem solving skills. However its organizational design fails to encourage the broader power algebraic techniques possess for solving mathematical problems. Furthermore, in some cases verbal problem sets were located within manipulation exercise sets under the

label, "Application". Usually a new relationship or formula is introduced with an example, and several verbal problems which directly employed the relationship followed. At this point in the analysis, it became apparent to the investigator that textbook A possessed all the important design features generally associated with individualized learning programs. However the textbook is intended for usage in whole class instructional settings.

Textbook C is organized and designed markedly different from the two previously reviewed textbooks. Its design received more favorable investigator ratings. Investigator preference for its organization and design was based upon the authors' inclusion of several unique features. These features make explicit a conceptual model of verbal problem solving competence most similar to the investigator's conceptualization. This conceptual model hypothesizes that verbal problem solving ability involves primarily relational thinking mixed with translation skills.

The first unique inclusion is an entire chapter on formulas, and equation formulations. The chapter is located appropriately between a chapter which contains verbal problems requiring only simple translation skills, and a chapter composed of twelve verbal problem sections requiring relational thinking skills. The two other textbooks each contain only one section related to

equation formulation.

The second unique inclusion appears in their general model for " using equations to solve problems" procedure. The authors include a step, "find a word sentence to suggest an equation" before any student writing is to begin, acknowledging the importance of an early relational perspective. Translations skills are suggested later only after representation of the unknowns. The general model also includes a separate interpretation step, "write the conclusion that settles the problem", before the final checking step. Due to the model's length and wordiness, the procedure is clumsy, and not likely to be recalled by students in its entirety. Yet within its example constructions, the authors exhibit extreme care and consistency to include each step, without explicit listing, of the general model in their solution discussions.

The third unique design feature is the separation of most verbal problem types into a single chapter. This separate placement possibly suggests an author intention that verbal problem solving development requires separate and different instructional treatment from that provided for other algebra content. Presently, it is not clear what impact this textbook's unique design has on student learning, retention, or attitude.

Verbal problem exercises of textbook C are adequate

in terms of quantity and context diversity. Its explanation sections are drab, colorless, and needlessly lengthy. Its layout characteristics and student readability features received lower investigator ratings.

Textbook Comparison

The three textbooks of the study did contain an identifiable core of six verbal problem types within initial instructional chapters. Five problem types relate to the problem statement content or contextual problem representation. The sixth problem type, simple translation problems, contain varied content and problem statements, but share a translation skill orientation for equation formulation. The six problem types receive equal treatment in textbooks A and C. Four problem types also receive equal treatment in textbook B, while it contains fewer geometric formula type problems and considerably more simple translation types.

Space allocation comparisons revealed that verbal problems represented a twenty percent space commitment in each textbook's design. Verbal problems reprsented one third the space devoted to algebraic manipulation skills. Substantial difference in instructional treatment of verbal problem content was noted. Textbook C has nearly twice as many pages of instructional treatment, and nearly four times as many examples as does textbook B. Textbook A is between these two textbooks on both measures, but

contains less than half the instructional examples that textbook C contains. Student exposure or practice of verbal problem exercises was also not equal for the three classes. Observations of teachers' assignment patterns led to investigator estimates of student practice of verbal problem exercises for each class. These estimates indicated Alice's class had more practice on the exercises than Betty's or Carl's classes which had about equal practice of related verbal problem exercises.

Several classification systems were employed to compare the verbal problem exercises of the three textbooks. Within the simple/complex translation system, the three textbooks exhibited similarity on the core content exercises. Overall each textbook contains two simple translation types for every one complex translation type. Within three problem types, some differences were noted between textbooks.

Within the four format type classification, core content exercises of textbooks A and B are fairly similar. Textbook C has a much higher percentage of abstract factual formats, and also a much lower percentage of concrete factual formats than the other two textbooks. The investigator related these results to Caldwell and Goldin's (1987) hypothetical models of problem difficulty, and the results of their elementary and secondary studies. From the differing patterns of textbook format

classifications, several predictions of differing classroom performance were generated by the investigator for later testing and discussion related to student performance patterns.

The three textbooks were compared for explicit labeling characteristics of their respective exercise sets. Explicit labels for all but one problem type are provided in textbooks A and C, whereas only three problem types of the core content are explicitly labeled in textbook B.

The subjective feature evaluation of the three textbooks indicated investigator preference of textbook C. This preference was based upon its explicit expectation that students will need to use a combination of relational constructions of conceptual knowledge and some translation skills to solve verbal problems. Textbook C makes this expectation clear within its unique organization and its incorporated relational thinking suggestions. This relational perspective of algebra verbal problem solving agrees with current research results which indicate students do construct problem relationships based on available conceptual knowledge (Riley et al,1983). Textbook B maintains a similar perspective but presents its perspective less explicitly in its design and organization. Textbook A maintains an opposite perspective in which total dependence for equation

formulation is based on student ability to translate written problem information into appropriate symbolic and algebraic representations.

The other difference among textbooks was the treatment accorded and the development of a general model for verbal problem solving. The investigator preferred the development of a single general model for students to solve all verbal problem types, and that the general model incorporate Polya's four step model. All three textbooks' general models do incorporate the four steps of Polya's model. Two textbooks, B and C, develop a single general model while textbook A continually modifies its general model for each problem type encountered. The investigator's preference is based on a belief that the model itself is more likely to be incorporated and retained within student constructions when the same model is continually practiced and reinforced during instruction. However it is possible that the development of separate models for each problem type could lead to more effective learning within this particular student population. Silver (1979) demonstrated that middle and low ability students (such as those in this study) cluster verbal problems on the basis of context or problem type similarities. The instructional inclusion of a separate procedure for each problem type then provides the student a more complete package of conceptual and procedural

knowledge related to a particular problem type. Each problem type schemata is then stored in memory separately, and can be recalled once the problem type of a verbal problem is recognized. Paige and Simon (1966) found that most students can identify specific algebra verbal problem types from the reading of a single sentence of a problem statement. However they also found that such an immediate identification ability did not translate into any significant performance advantage. Several weaknesses of this instructional approach may help to explain Paige and Simon's results. First, memory requirements are greater in this approach since problem type procedures must be highly automatized and the quantity of these procedures is rather large. Secondly, there is no central control framework in this approach to aid student monitoring of progress. An incomplete or incorrect recall of a procedure is fatal for student processing. Thirdly, such problem type classifications have limited transfer capability to problems where context similarity is less obvious or possibly even inappropriate.

Author Input

A final phase of the textbook analysis was a survey of textbook authors' views on the purposes of the Algebra I course, and the relative importance of verbal problems within their textbook designs. A three page questionaire was designed and mailed to the ten living authors with a

return, stamped envelope. The major question of the survey required the ranking of eight potential purposes of the Algebra I course. This same question was included in the Teacher Inventory Packet for comparison purposes. The original mailing in August, 1986 resulted in one return. A second mailing in late November, 1986 resulted in one additional completed questionaire. Both respondents were authors of the same textbook series. Given the low rate of return of the Author Questionaire, no further analysis of the textbook authors' views was completed.

Teacher Descriptions

This first, teacher related section is very general, and characterizes each teacher's classroom setting, daily routine, attitudes, and perceptions related to algebra and its teaching. The form used is narrative description. General descriptions of the students of each classroom are also included in this section. Characterizations were developed from the Teacher Inventory Packet results, classroom observations, and occasions of informal dialogue between teachers and the investigator. Each teacher receives individual treatment, followed by investigator comparisons of the three teachers of the study.

Alice

Alice's class was composed of 18 ninth graders in a public junior high setting. This class was designated

Algebra I, as opposed to Algebra I-A (accelerated track). This class was the only Algebra I class Alice taught during the 1986-87 school year. In addition, Alice taught two Algebra I-A sections, two eighth grade mathematics classes (level 1), and supervised a study hall. She was the elder statesman of an eight member mathematics and computer department. The mathematics office and the classroom where she taught all her mathematics classes were both located on the third floor. Alice's main extracurricular role, a role she enjoyed immensely, was to serve as coordinator and mentor for a small group of selected students and their training for Math Counts, a national mathematics competition.

Alice had taught algebra for 20 years, primarily at the junior high level. Within and around her teaching schedule she had raised a family within this same community, and was seriously considering retirement at the end of the current school year.

The Algebra I class met five days a week in the morning for forty-five minutes. Alice had a preparation period before this class, and monitored the study hall immediately after this class. A class size of 18 was small in comparison to her other classes. There were no changes in this class's composition during the entire school year.

The classroom had a single entry door which led to

the front section of the classroom where a teacher desk was located. There was an eight foot walkway between the front chalkboard and the student desks. Student desks were neatly arranged in five rows of six desks facing the front chalkboard. Two other walls contained additional chalkboards running almost their entire lengths. The outside wall was dominated by eight foot windows and shades. The view below from the classroom was of a busy residential street and private housing. In the distance, buildings of a local university campus and hospital were visible. Twenty large, colorful posters were placed around the three interior walls above the chalkboards. Each poster contained a cute saying or cartoon related to learning, and used various animals to make their points. These posters remained up the entire school year. The rear wall bulletin board contained mathematical related materials which did not directly relate to the content of any of Alice's classes (enrichment ideas). The small, front bulletin board contained five envelopes (one for each of Alice's classes) into which make-up work or special assignments were to be placed by students for grading.

Alice always taught from the front of the classroom, and placed her teacher's manual on a three foot high, flat topped, wooden pedestal at the beginning of class. Her instructional materials consisted simply of chalk, the

front chalkboard, and a wooden dowel rod stored on the chalkboard tray. Alice's supplies, records, and other instructional materials were kept in her mathematics office desk, rather than in her classroom desk since other teachers used her classroom during the school day.

During observations, the investigator sat in an unoccupied desk in the middle of the row of student desks nearest the window. Alice allowed her students to seat themselves at the beginning of the school year, and that became their fixed seating arrangement for ten weeks. At the end of ten week marking periods, students were allowed to seat themselves for another ten week period. Early in the school year most students sat in the front half of the classroom. The investigator did note in his last visit (March, 1987) that a majority of the students were seated in the rear half of the classroom.

Alice expressed a high level of confidence in her ability to teach algebra and verbal problems, even to the point of believing that this area is a personal strength. As such, she found algebra teaching to be challenging, personally rewarding, and enjoyable. She preferred teaching a class composed of mixed ability level students, which was true of Algebra I students. Alice had high and consistent expectations of her students. She expected 75 percent of her students to be successful and earn B level grades or higher on all algebra coursework, including

verbal problems.

Alice was business like in her management of the classroom, and in her interactions with both the entire class and with individual students. Teacher initiated questions focused exclusively on algebra related content. Student initiated interactions were mainly confined to specific questions of the day's homework review, or individual queries for assistance during seatwork time. Alice was very concerned that each student made daily progress in the learning of algebra skills. At all times, Alice was comfortable, confident, and relaxed in her homework review and individual tutoring roles. Yet, often Alice appeared less comfortable and confident during lesson development instructional activity. The investigator inferred from this behavior pattern and from the shortness of her lesson developments, that Alice preferred certain teaching functions, homework review and seatwork helping, over other functions, such as whole class, new material presentations. In discussions with students, they expressed that their respect for Alice was based on their perception of her demonstrated concern for their learning, and on the equitable and noncritical treatment they received in her class.

Alice seldom criticized individual students, though on occasion she would scold the entire class for poor quiz or test performance, or for their inability to complete

what she considered easy assignments. Praise was given orally to individual students for demonstrated progress in learning content, or in the form of positive comments written on tests or graded assignments. Alice seldom had to remind students to pay attention during her explanations and instructional segments. However the investigator was not convinced that such facial attention was synonomous with an active learning posture. Students seldom initiated questions during the lesson development segment, and the few such initiated questions related to specific, surface features of the solving procedures used in her examples. During these lesson development segments, there was also a sense of tension and quiet, possibly symptoms of anxiety within Alice's students which disappeared when lesson development ended and the seatwork segment began. The investigator sensed the nature of an implicit agreement between parties, teacher and students, in which students gave attention and effort during instructional segments with minimal student input, in exchange for receiving adequate time to complete assignments and receive any necessary teacher assistance required.

The investigator observed a minimal level of student energy and enthusiasm in this class's students. Like Alice, they too were business like in their approach to learning algebra. When the whole class was in session few

student initiated questions were observed. Yet during provided seatwork time, Alice and the investigator were kept active answering student questions related specifically to the assigned homework. A majority of this class's students required constant teacher or investigator presence, reinforcement, and feedback on their progress with the individual problems of an assignment. These students were neither secure, nor confident in their learning of algebra. However, these students did put forth excellent effort on all their assignments, and did demonstrate a willingness to persevere in their efforts. Despite a lack of observed student enthusiasm in Alice's class, the investigator inferred from student conversations that students enjoyed Alice's class, due to the supportive and nurturing environment they experienced there.

Betty

Betty's class was composed of 27 students in a parochial high school (grades 9-12). Of the 27 students, 21 were ninth graders and six were tenth graders. One student was demoted to a prealgebra class at the beginning of the second semester (January, 1987). This demotion prevented the student from taking the retention test.

Betty taught both sections of regular Algebra I offered by the school. A different teacher taught the one section of accelerated Algebra I offered. Betty was the

junior member of a three person mathematics and computer department. Besides the two sections of Algebra I, Betty taught all three semester computing courses (two introductory courses and one advanced course), and supervised a study hall. Betty was involved in many extracurricular activities at the school. These activities included being the coordinator of the computing club, a forensics aide, and a class sponsor. She enjoyed the diversity these activities entailed, and she attended most school sponsored functions.

This class met in a first floor classroom where business related courses were mostly taught during the school day. The room was adjacent to the main entry area of the school and the hallway included entry doors for the gym and school office. With the exception of this class and the study hall, Betty's classes met in a second floor classroom which housed the microcomputer laboratory. Both Algebra I classes met in this second floor classroom during the second semester.

Betty was beginning her third year of teaching, and all her teaching experience was in this same school. Her father also taught mathematics at a high school level in a suburban school district 125 miles away. Betty had received her teaching degree from the university located in the same city in which she was presently teaching. Her student teaching had been done within the public system

there. She is single, and shares living arrangements in the community with another single teacher from the same high school.

The observed Algebra I class met five days a week for forty-five minutes in the early afternoon. The school had three alternate schedules to accomodate interruptions to the regular school day. Alternate scheduling effected one observation, limiting class time in this instance to forty minutes. Despite this classroom not being her normal one, Betty was always ready to begin class on time due to the previous time period being her lunch period. The following hour she taught the advanced computing class. Her other Algebra I class met in the morning, and both classes contained the same number of students. Class scheduling procedures of the school dictated that students have different schedules for the first and second semesters. As a result, Betty's two Algebra I classes were intermixed in equal proportions at the beginning of the second semester (January, 1987), and both sections were administered the retention test during the second semester.

As one entered the classroom, the teacher desk was on the right and the student desk area was to the left. The front of the classroom was the longer wall, behind which was the main hallway. This front wall contained two chalkboards and a business oriented bulletin board.

Thirty-five desks (seven rows of five desks each) were crammed into an area facing the front wall. Opposite the front wall was the outer wall composed of windows and ledge only. A large lawn space, a busy two lane street, and the grounds of a convent dominated the view outside. The short wall to the students' right was glassed in and contained a business equipment lab work area. A door connected the classroom to this work area. The wall to the students' left had two additional chalkboards. This area was more open and contained a teacher desk, a file cabinet, and the pencil sharpener.

Betty formulated a seating chart based on initial student self-seating patterns at the beginning of the school year. Students sat in the same seats for a ten week marking period. In the initial seating chart, males dominated to Betty's left as she faced the class and females dominated to her right. For the subsequent three marking periods, Betty designed the classroom seating arrangements, and her designs reflected a greater intermixing of sexes. During observations, the investigator sat near the back of the row of student desks nearest to the door and to Betty's right as she faced the class.

Betty taught from the front of the classroom, and placed her teacher's manual, class seating chart, and record book on a podium positioned against the center row

of student desks. Only five feet separated the front chalkboards and the student desks. Additional materials were placed on a small table under the front bulletin board to Betty's right. Except for two observations when Betty used the side chalkboards in her explanations, all written instruction was done on the front chalkboards. No other instructional aids, besides chalk and chalkboards, were observed to be used in Betty's class.

Betty expressed a high level of confidence in her ability to teach algebra concepts, but a lower level of confidence in her ability to teach verbal problem content. She believed her background and knowledge base related to verbal problems was weaker. For Betty, this lower confidence level related to verbal problem instruction resulted in a differing expectation for student verbal problem performance. Betty expected 70 percent of her students to be successful with manipulation skills, yet she expected only 50 percent of her students to be successful with verbal problem content.

Betty preferred teaching high ability mathematics students. Her justification for this preference involved the decreased instructional time these students required to reach automatized and mastery level performance. This conceptualization of differing student ability levels was reflected in Betty's teaching of the Algebra I class. She believed that these average ability students required

considerably more structured and explicit procedures for every concept within the content. She also operated on an assumption that average ability students' learning was more dependent on her instruction and related explanations. By increasing the length of teacher directed instructional time, Betty believed she could overcome perceived weaknesses in these students' mathematical abilities. She maintained tight control of the class during instructional segments related to concept review, homework review, and new content development. Students were in control of their learning only during seatwork time. In summary, Betty perceived her instructional role to be director, disseminator, and controller of learning in all its aspects.

A majority of Betty's students were not attentive during her explanations, nor did they participate in class discussions or board work. Based on the classroom observations, only nine students (one third of the class) gave consistent attention to Betty's instruction and participated fully in classroom interactions. Betty noticed that student effort was low based on assignment completion rates early in the school year. She adjusted her homework evaluation procedures in November of the school year. At this point, she began taking ten minutes daily to check students' homework completions, going around the class and recording scores for degree of

completeness in her record book. Often this process was followed by Betty's criticism of the class for its lack of effort. A further adjustment was made at the beginning of the second semester. Students were then required to complete their homework in notebooks which Betty periodically collected and evaluated. This second adjustment improved students' daily preparation and homework completion rates. It was not clear to the investigator whether the class's lack of effort resulted in Betty's greater teacher controlled instruction, or whether Betty's teacher-centered instruction led to these students' general lack of effort. This class of students responded mostly to grades, and the passing of this required course. Betty sensed this student concern for grades, and attempted to improve effort and learning attitudes of these students by incorporation of grades related to homework completion. She expressed several times to the investigator her concern as to whether students were learning the content. In the light of such comments, the creation of a teacher-centered environment represented to the investigator a rational reaction to this particular class of students.

Interaction patterns were limited in Betty's class, and most interactions were teacher initiated. Betty asked questions of students whom she believed were attentive, prepared daily, and most likely to respond correctly.

Occasionally when she selected a nonvolunteer (where one of the above criteria was absent), calling the student by name, she usually received an incorrect or evasive response. These situations confirmed and reinforced her perceptions of these "other" students. Betty appeared frustrated by her students' general inattentiveness, uncaring attitudes, and lack of effort.

Betty's students expressed to the investigator varying degrees of respect for Betty as a person and as a teacher. All questioned students respected her as a person because of her young age, her extensive school activity participation, and associated characteristics of with-it-ness. Only her better students sensed her concern for their learning and respected her teaching methods.

When Betty's daily lecturing was completed and an assignment given for the next day, a clamor of activity followed which seldom related to the assigned mathematical task. Students requested leave for the lavatory, or for a drink of water, or requested specific grade information of Betty. Most students did not begin the assignment in class, instead leaving it for homework, and using the available time for conversation with classmates. Betty, on occasion, freed herself in order to circulate around the room and help individual students when requested to by working students. Betty relied mainly on infrequent test and quiz evaluations to provide information on student

learning. This practice limited her understanding of students' daily learning progress.

In conclusion, Betty's class of students was energetic and enthusiastic. Yet this energy and enthusiasm did not carry over into their motivation to learn the algebra content. They appeared confident of their mathematical abilities, yet the investigator believed they overestimated their capabilities. Based on classroom observations and limited student conversations, the investigator believed attitudes related to their Algebra I experience were generally negative. Students accepted the fact that this class was required in their high school program, tolerated Betty's instructional attempts, but withheld active participation and effort. Related to verbal problem sets, this negativism was more pronounced as students orally expressed their dislike every time Betty began her instruction of verbal problem content.

Carl

Carl's class was composed of 22 ninth graders of a public junior high school. One student switched district schools early in the study, and was not included in the count of 22 students. The class was designated Algebra I, and had the same connotation as that used in Alice's school. Carl taught one other Algebra I class, two sections of Geometry I-A, one general math class, and

monitored a study hall. All his mathematics classes met in the same end room on the second floor. Carl had an additional desk and workspace area in the mathematics department office on the same floor. The seven person mathematics department was respected and stable in its composition. The building principal emphasized to the investigator the high status he accorded the mathematics department based on his perceptions of its teaching expertise. Carl's main extracurricular role was as coach of the school's ninth grade football and basketball programs. He enjoyed being a part of a sports program where district emphasis was on student participation and physical ability development, rather than solely on winning. Several of his students addressed him as "coach", though Carl discouraged this practice.

Carl had ten years experience teaching algebra, primarily in the same junior high school. He also has a family (two elementary aged children), and lived within the school district.

The Algebra I class under consideration met five days a week in the early afternoon for forty-five minutes. Carl had a preparation period before this class, and monitored the study hall after this class. The class size was smaller than his morning Algebra I class of 26 students. With the exception of three students who transferred into the school at the beginning of ninth

grade, students shared two years of mathematics at the same school at a level 2 designation.

The classroom had one entry door and a short walkway which led to the rear section of the classroom. A bulletin board covered the entire length of the rear wall. Carl initially placed optical illusion cards and explanations on this bulletin board. Later cartoons and interesting problems were added. The classroom had a contemporary design with five small window ports along the outer wall which overlooked the lawn area and front entry of the school building. The front section had a teacher desk and a file cabinet against the outer wall. A seven foot walkway existed between the front wall and the student desks. Two chalkboards dominated the front wall and an additional chalkboard ran along the interior wall. The rear section of the interior wall contained a small bulletin board on which Carl placed a daily classroom time schedule and school related announcements. There were 30 student desks arranged in five rows of six desks facing the front of the classroom.

Carl taught from the front of the classroom from a centered four foot high wooden table. The table held an overhead projector, teacher manual, record book, and seating chart. He focused the overhead projector on a screen hung above and covering the front chalkboard for most of his instruction. He taught mostly from a standing

or leaning position around the overhead. The front and side chalkboards were used only occasionally within his instruction.

Student seating arrangements were determined by the students, and maintained for ten week periods. During observations, the investigator sat in the rear of the class in the row of student desks nearest the door. Students generally sat in the desks closest to the front of the room and along the outer wall.

Carl expressed a high level of confidence in his abilities to teach algebra and verbal problems. His feelings and attitudes related to the enjoyment and challenge of algebra teaching were slightly lower. Carl preferred teaching middle ability students, as this class supposedly contained. He expected 80 percent of his students to be successful and earn B level grades or higher on both algebra manipulation skills and verbal problem content.

Carl's classroom presentations were well organized and reflected his belief (expressed to the investigator) that learning was a combined responsibility of the teacher and the student. He expected students' active attention and involvement during his presentations. Most students conformed to these expectations. He often requested the whole class to respond to his questions. Carl permitted students to blurt out answers to his questions when a

single student was not the focus of his question, or to even ask questions during his presentations. His behavior encouraged student-initiated questions, particularly questions related to incomplete student understanding or procedural justifications. Carl also encouraged student examples of creativity, method diversity and efficiency. He listened carefully to student suggested alternate methods, and examined why or why not such methods were generalizable or justifiable.

Students often challenged Carl with their curiosity and their desire for clarity and consistency. Carl demonstrated patience while listening to a student's question or explanation before attempting to interpret for the class its nature. Occasionally such exchanges between Carl and his students lead to chaotic and boisterous episodes where some students drifted away and talked with each other. Carl exhibited a keen sense for when the class was getting away from him on these occasions, and immediately called for the class to return their attentions to the present task.

Each individual student's learning preference was treated respectfully by Carl in his teaching. Five or six students preferred less active participatory roles, and were not forced by Carl to become like their more active classmates. Instead Carl provided these students greater individual assistance during the seatwork segment, showing

respect for their preferred learning orientation. He provided sufficient time daily for students to begin their assignments, and with the investigator circulated about the class encouraging students of the correctness of their methods, or explaining mistakes in their methods. As a result, Carl acquired sufficient information to evaluate the day's lesson development, his related teaching, and his students' ability to complete the assignment.

All students expressed openly to the investigator their enjoyment of Carl's class. The more active students appreciated the receptivity and the noncritical analysis their ideas received from Carl. The less active students appreciated the respect Carl gave their learning posture. All students were generally enthusiastic and confident of their abilities. Particularly, the more active students pushed Carl for clarity, consistency, and justification within their understandings. Students were motivated to learn, and more than willing to put forth sufficient, daily effort to complete assignments.

Comparisons

The classroom environments of the study were structurally similar, and represent typical secondary classroom settings. The structure of Betty's classroom is slightly different in that the front of its classroom is along the longer dimension. Also Betty's class met in a business course classroom, not her regular classroom,

which required teacher movement of materials every day. Time alotted daily for Algebra I was nearly identical for each class, either 44 or 45 minutes. Two classes (Betty and Carl) met in the afternoon, while Alice's class met in the morning.

The teaching schedules of Alice and Carl were similar, requiring three mathematically related preparations daily. On the other hand, Betty's primary responsibilities were computer science related with her two classes of Algebra I. Betty and Carl taught two sections of Algebra I daily, while Alice taught one section. Algebra teaching experience varied greatly, Betty with three years experience, Carl with ten years experience, and Alice with 20 years experience. Most of their teaching experience was gained in the same schools in which they were presently teaching. All three teachers were active in their respective school's activities.

Each teacher expressed the highest confidence level possible in rating their ability to teach algebra concepts. Alice and Carl also rated their ability to teach verbal problem content at the same high level, while Betty rated her ability to teach verbal problem content slightly lower, though still above average. Each teacher expected a high percentage of their students to be successful with manipulation skills, the actual percentages being Alice 75 percent, Betty 70 percent, and

Carl 80 percent. Betty expected a lower success rate, 50 percent, related to verbal problem content, while Alice and Carl expected the same success rate for verbal problem content as for manipulation skills. Together these results indicate that Alice and Carl believe their knowledge base and teaching experience is sufficient to reflect a confident attitude towards the teaching of verbal problem content. Possibly due to less experience, Betty is less confident of her ability to teach verbal problem content, and also for its successful learning by her students.

Each teacher expressed enjoyment of and challenge with the teaching of algebra. Interestingly, each teacher's indicated preference of student ability level was different. Alice preferred mixed ability groupings of students, Betty preferred high ability groupings, and Carl preferred middle ability groupings. For Alice's and Carl's preferences, ramifications on their teaching assignments is minimal, while Betty's preference for high ability groupings may hinder her ability to teach lower and middle ability algebra students.

Each teacher used a minimum number of available instructional resources in their teaching. All explanations, homework review, and constructed examples were done on the chalkboard by Alice and Betty. Carl used an overhead projector 80 percent of the time and the

chalkboard 20 percent of the time in his instructional functions. All three teachers used a whole class instructional format, directing the class from the front of the room. Each teacher, to some extent, incorporated most textbook examples into their instructional lessons, and student practice of verbal problems was mainly confined to the textbook exercise sets. Alice differed from the other teachers in her usage of supplementary materials. She used an old textbook for further review of two problem types, and on three occasions used additional worksheets of verbal problems drawn from supplementary materials as a quiz or homework. Carl designed and assigned his students a two page worksheet of mixed problem types as a chapter review. Betty used no supplementary materials.

Based on investigator observations of the teachers and informal conversations with them, differences were noted between teachers' degree of exerted instructional control and permitted student interaction patterns. Attempts were made to draw inferences from these differences, and relate them to differing teacher perceptions of the learning process. For Betty, the major responsibility for learning resides with the teacher. She maintained the tightest teacher control over her students during allocated class time. Little class time was provided for students to work verbal problem exercises or

for controlled practice. During the school year, she developed different procedures to coerce students into completing homework assignments. Teacher-student exchanges were primarily teacher initiated, and included only a small portion (1/3) of her students on a regular basis. Alice, on the other hand, believes the major responsibility for learning resides in the student. Alice's verbal problem instruction was short, and involved direct modeling of the desired procedural steps in a couple of examples that students were expected to follow. During Alice's presentations students listened passively and copied the examples. The vast majority of student initiated questions arose during seatwork/practice time when responsibility for understanding resides with the student. Students in Alice's class were practicing verbal problem exercises either at the chalkboard during homework review segments, or at their seats individually over 75 percent of allocated class time. Carl occupies a position, between Alice and Betty, which holds that learning is a shared responsibility between parties, teacher and students. Students are expected to put forth sufficient effort, actively contribute during teacher presentations, and push for understanding. Carl both directs the daily plan and objectives, and facilitates learning by appropriate usages of examples and questions. Hence, the inclusion of a daily controlled practice

segment serves as a transition from teacher responsibility to student responsibility.

Students' mathematical backgrounds within Alice's and Carl's classes (both public junior high schools) were nearly identical. All students in their classes were ninth graders, and 85 percent of their students had, at the same school, two years of previous equivalent mathematical instruction, of which Alice and Carl were familiar. Alice and Carl were less familiar with the backgrounds of the two or three exceptions per class who transferred form the local parochial school system at the beginning of the ninth grade. Betty's students had more diverse backgrounds and included two grade levels. Of Betty's 27 students, 21 were ninth graders and six were tenth graders. Her students came to the high school at grade 9 from the four area parochial grade school mathematical programs, and 2 students came from the public system's program. Student placement in Algebra I, Accelerated Algebra I, or General Math was not regulated by Betty or the high school staff. Placement was determined at the end of the eighth grade by the respective grade school teachers' recommendations.

All Carl's students expressed to the investigator a belief in their ability to learn algebra skills and to solve verbal problems. About one third of Alice's and Betty's students expressed uncertainty in regards to their

ability to learn how to solve verbal problems. Only students of Betty's class openly expressed displeasure with verbal problem content. Students of Alice's and Carl's classes were more enthusiastic towards and challenged by mathematical tasks, and demonstrated consistent effort and perseverence. Betty's students were active and enthusiastic, but not in regards to their algebraic lessons and assignments.

Instructional Routines

The focus of this section is on characterizing each teacher's daily instructional routine. Teacher descriptions were formulated from investigator observations of each teacher, and quantitative data from Coding Form 2 (found in Appendix B), completed after each observation. The top section of investigator designed Coding Form 2 contains a list of eight activity segments, with spaces provided to indicate minutes and percentage of class time devoted to each listed segment. The segments represent different teacher purposes which occupy teacher time during a class period. A similar form was used in observations of effective and less effective mathematics teachers in studies done by Good, Grouws, and Ebmeier (1983). The eight segment categories are identical to those used in their studies, and are: administrative, drill/mental math, homework review, concept review, lesson development, controlled practice, seatwork, and

transition/other.

The relative time compositions of these lesson segments form what the investigator means by a teacher's instructional routine. Specific teacher characteristics and beliefs also influence a teacher's time management, and these are included, where appropriate, in the individual teacher descriptions. Teacher responses related to instructional practices on the Teacher Inventory Packet form the basis for teacher beliefs, along with the teacher observations. A comparison summary of each teacher's mean times per class period for the eight lesson segments was completed, and is displayed in Table 4.10. The summary includes only observations where verbal problem solving was the content focus for the day, and new content was developed during the class period. The entries in parentheses indicate the range of time for each segment by teacher.

Table 4.10

Teacher Comparison: Segment Time Means (minutes)

	Alice	Betty	Carl
Number of Observations	10	8	7
Class Period Length	44	43.5	45
Administrative/Management	2.9 (1-6)	4.6 (2-11)	2.6 (2-5)
Drill/Mental Math	0	0	0
Homework Review	7.4 (0-15)	14.9 (0-25)	8.7 (0-20)
Concept Review	4.3 (0-18)	3.6 (0-10)	1.1 (0-4)
Lesson Development	8.3 (3-16)	11 (5-17)	19.9 (13-25)
Controlled Practice	5.4 (0-13)	1.1 (0-9)	5.9 (3-13)
Seatwork/Homework	15.3 (0-25)	7.3 (1-16)	6 (0-14)
Transition/Other	.4 (0-3)	1 (0-8)	.9 (0-4)

There are several commonalities within the three teachers' instructional routines which will be examined before addressing differences. From Table 4.10 one notes that each teacher required less than five minutes per class period for administrative and management functions. Classes for the most part were well behaved and required minimum teacher time or action to maintain student attention. Teachers of the study managed time efficiently.

There was not a single occurrence of a teacher providing time for mental math or additional drill in numeric related skills. All three teachers expressed to the investigator their belief that the time used for this segment could be better used in focusing on the daily lesson content.

While each teacher listed, on the teacher survey, twenty minutes as the optimum time devoted daily to lesson development of new content, only Carl came close to this optimum time frame for lesson development. The controlled practice segment means fail to distinguish the number of occasions these segments were observed in each classroom. Carl used a controlled practice segment within each observed day. Controlled practice was used in Alice's class nine out of the ten observed days. On the other hand, Betty was observed including a controlled practice segment in her instruction only once (for nine minutes). The remainder of this section focuses on the investigator descriptions of each teacher's instructional routine.

Alice

Alice's instructional routine was stable and easy for the investigator to follow and separate into segments during observations. Her routine for teaching manipulation skills content, observed on one occasion, was nearly identical to her routine for teaching verbal problem content.

Alice took attendance daily at the beginning of class, sometimes even before the bell rang. She expected students to have yesterday's assignment out on their desks ready for correction when she returned after posting the attendance in the hallway. She then read answers to the even problems assigned (odd problem answers were in the back of the book). Students corrected their own assignments. Alice collected student assignments once during observed days. For several observations, Alice gained a sense of how students had done on an assignment by requesting a show of hands indicating the number of problems missed.

Alice then asked students which problems of the assignment deserved further review. She wrote a list of problem numbers that students requested on the chalkboard. She implicitly required that a student requesting a problem solution had attempted to solve the problem. For two observations, Alice did all requested problems one by one on the front chalkboard, providing clear justifications and modeling of the required steps. For the seven other observations (no homework review for one observation), individual students did the requested problems on the front and side chalkboards.

Alice had an efficient procedure for involving students in the homework review segments. As the problems were requested, Alice wrote the problem numbers at the top

of the chalkboards, with adequate workspace provided. She then went down her class roster asking students by name, if they could do one of the remaining numbered problems. If a student indicated affirmatively, the student copied the problem solution from their work onto the chalkboard, and received five extra credit points for their effort. If a student replied negatively, Alice moved on down the class roster. If no students had taken a problem or problems when she had completed a roster sequence, Alice asked for volunteers from the class. On two observed days, no student had a problem solution for a requested problem, resulting in Alice doing the problems. On subsequent days, Alice started the procedure at the next name lacking the five point entry in her gradebook, and only asked students without the five point entry first. This process insured that most students received an opportunity to display homework problem solutions on the chalkboard. Alice believed these homework review segments were important, and that students appreciated the active involvement they entailed. She expected students to copy problem solutions from the board which they had been unable to solve correctly. On several occasions Alice explicitly reminded the class of the importance of this daily practice.

Alice then reviewed each student's boardwork, one problem at a time, noting the importance of various steps,

and her expectations of included student writing for each problem. The investigator coded this explicit review by Alice as concept review. Alice moved briskly through this segment usually taking less than four minutes to complete. On one occasion Alice had individual students explain their problem solutions. She was not happy with the students' explanations on this occasion, and appeared anxious due to the longer time required to complete this particular homework review segment.

The lesson development segment, related to introducing new content, was a relatively short segment of Alice's instructional routine. For six of the ten observations, new content received five or less minutes of instructional treatment. Alice deliberately kept her instruction of new content brief. Several times she mentioned to the investigator her concern that students not become frustrated completing assignments, particularly verbal problem assignments. She believed the provision of class time for completion of half an assignment, and also where student reception of immediate teacher assistance was possible, lowered student frustration. This pattern of Alice's routine is reflected in the higher mean time given to seatwork, over twice that of the other teachers. The mean seatwork time of 15.3 minutes daily is somewhat misrepresentative of average seatwork time, since for one observation no assignment was made and hence zero minutes

of seatwork time was coded. The adjusted mean seatwork time (minus this one observation) was 17 minutes.

However, on one particular observation, Alice deviated from her normal instructional routine. On this day Alice sensed correctly, based on the homework review, that students had not understood the previous day's assignment. She changed her daily plan, and spent 18 minutes reteaching the involved concepts. She still carried out her planned lesson development of new content, though she shortened it to 4 minutes. Alice realized only 13 minutes of class time remained. All 13 minutes remaining was devoted to a controlled practice segment, and as noted above, no homework assignment was made. Interestingly on the following day with no homework assignment to review, Alice's lesson development was 16 minutes (her longest). These two days demonstrated to the investigator the importance Alice placed on providing sufficient seatwork time daily, which appeared to be 15 minutes. For Alice, this belief became the central framework which dictated the rest of her instructional routine. Time for lesson development of new content was dependent on the class time remaining after homework and concept review less the 15 minute required seatwork time at the end of the class period.

The pattern of low lesson development time and a fixed, high seatwork time daily suggested to the

investigator that Alice preferred interacting with students during seatwork time on a one-to-one basis. This preference could be just be a practical matter, in that the short lesson development segments, dominated by teacher talk and modeling, failed to permit sufficient teacher-student interaction, or possibly indicated that Alice did not enjoy the center stage, lecturing role that new lesson development segments generally demand of teachers.

Alice concluded her lesson development on nine occasions (of ten observations) with a controlled practice segment. Students practiced one or two problems from the written exercises from which the daily assignment would come. Students were to do these problems (told explicitly) in their notebooks, and use them as completed model problems. Time was provided for students to attempt to work the problem(s) individually, and Alice then gave a complete demonstration of the written solution. This segment was usually finished in less than five minutes.

Betty

Betty's instructional routine was the most variable of the three teachers. The investigator found it difficult to separate the homework review and concept review segments of Betty's daily lessons. Betty's administrative /management time is slightly higher due to more occasions which required teacher effort to regain

attention and correct student misbehavior. This segment category also included thirty seconds daily given to class prayer. Betty took attendance at the beginning of the class period, and posted the form on the hallway wall. Each day several students arrived just before the bell signaling the beginning of class. Students were generally boisterous and conversational during the first minutes of class. There was not an expressed expectation on Betty's part that students be ready to begin.

Betty daily told students to take out their homework for correction, and she read the answers to the assigned even exercises. This sequence changed at mid-semester when Betty began her instruction by checking for assignment completion by circulating throughout the classroom. Betty then listed student requests for problem solutions by number on the chalkboard. She did not place any requirements on students' requests of problems. It was fairly obvious to the investigator that some students requesting problems had not attempted them, or in some cases only a single student had difficulty on a requested problem. The resulting list was usually large, about 75 percent of the assigned problems. On most observed days, Betty did the requested problems one by one on the front chalkboard. On two observed days, volunteers did three or four problems (one each) at a time on the chalkboard. Since there was nothing to gain from a student perspective

(extra credit or otherwise), it was not surprising that the same six or seven students did all the requested boardwork. On one day, Betty asked the students to vote their preference for having students or teacher (her) do the requested problems. The investigator sensed that Betty already knew what the response would be before she asked the question, yet asked anyway out of obligation. On most occasions, Betty preferred not to relinquish control to students, but rather to maintain control of the homework review segment.

Betty viewed the homework review and the intermixing of concept review as central to her instruction. Possibly too much time was given to these segments, over one third of her class time. Betty mentioned to the investigator on two occasions that most of her homework and concept review was actually lesson development, though obviously no new content was developed. The investigator inferred that Betty's perspective on the centrality of the homework review segment related to the low expectation Betty had for the previous day's lesson development segment. It was as if Betty was thinking, "I expect my lesson of yesterday to have failed, but the homework review today will permit me to pick up the pieces". Theoretically, it appears possible to accomplish. However such thoughts might result in student perspectives, such as "I don't need to pay attention today, for tomorrow Betty will clarify this

mess for me". The net effect of such a student perspective is that students are always one day behind the content pace.

Student attitudes and attention during the homework review segments changed. Most students were enthusiastic during the formation of the list of requested problems. Yet once explanations of the requested problems was begun by Betty or students, student attention was away from the front chalkboard. In fact, students who requested a specific problem often failed to attend to the solution of that very problem. Betty seldom mentioned to students to copy boardwork, and only a few students copied such solutions. Therefore, the net effect of the homework and concept review segments had minimal impact on aiding student learning.

Betty's lesson development segments depended on the number of provided samples or examples contained in the textbook related to the upcoming assignment. Despite a sufficient quantity of time for this segment (mean: 11 minutes), Betty's instruction was just a reworking on the chalkboard of the textbook provided solutions of verbal problem examples. Only on three occasions did Betty include an unsolved problem, each one taken from an exercise set, in her lesson development.

A controlled practice segment was observed only once in eight observations. Lesson development led directly

into the assignment for the day on the seven other observations. The most logical reason for this lack of controlled practice segments was related to time constraints, in that normally lesson development segments concluded within seven minutes of the class's end. The posting of the assignment signaled to most students that class was over. A small group of students (6 - 9) utilized the few minutes of available time to begin their assignments.

In conclusion, despite her control and careful direction of the homework and concept review segments, and sufficient time devoted to new content development, Betty lacked daily information of student progress and learning. Grades were not taken on the daily assignments, homework was not collected or reviewed by Betty, and class results were not requested by Betty. Most of Betty's efforts during the first semester were directed at getting students to complete assignments, with less concern directed towards the correctness of such learning.

Carl

Carl had a stable instructional routine he followed daily. Students entered his classroom boisterously, and took their assigned seats. He allowed students to remain conversational as he recorded and posted the attendance. When he returned to the front table, a simple comment, such as "Let's get down to work" instructed students to

quiet down and take out the previous day's assignment.

Students corrected their own assignments or exchanged assignments with a neighboring student for correction as Carl directed. All answers of an assignment were given by Carl, as the textbook did not provide any answers. On randomly selected days, about twice per week, Carl recorded student scores (percentages), given orally by the students, into his gradebook. He also collected and reviewed students' assignments about once per week. These methods provided Carl with explicit data related to students' daily performance. On days when no recording of scores or collection was done, Carl requested a show of hands for various performance levels.

Carl used the overhead projector to go over requested student problems from the assignment. Student involvement was limited to student verbalizations of their ideas made from their seats. In an assignment of ten problems, three or four problems received additional review by Carl. One problem at a time would be student requested and treated by Carl. When a problem was suggested, Carl asked for a show of hands to indicate how many students had difficulty on the requested problem. Three or four hands were necessary before Carl perceived a need to review a problem for the whole class.

Carl's treatment of a requested problem was interesting, in that he became a passive listener to a

student's explanation of progress, and then recorded the student's thoughts on the overhead. Problem discussion began with the student requesting the problem, insuring that the student had indeed attempted the problem. When a student reasoning or processing error occurred, Carl first attempted to get the focused student to identify the error. If such attempts failed, Carl requested volunteers to find the error, and to justify its nature. Students were highly attentive and participatory during this segment. Eventually Carl completed all steps of the problem in one frame with all errors removed. At this time Carl encouraged other students to share alternate solution paths they had used. As a student explained a differing procedure, Carl evaluated its correctness and justifications, or noted the reasons for its failure. For two problems, on two different days, Carl varied this process of homework review. Students who had correct answers for a requested problem were permitted to display their solution on the overhead and explain their processes, while Carl passively observed.

Specific content review was practically nonexistent, or consisted of a brief summary given by Carl after the homework review. Lesson development began quickly after the homework review was completed, and was the longest segment of Carl's daily instruction. The shortest time given to lesson development was 13 minutes, while on two

observations it spanned 25 minutes. The day of the 13 minute lesson development had the longest homework review segment (20 minutes). The two, 25 minute days of lesson development had two or less minutes given to homework review. These figures indicated a pattern, whereby homework review and lesson development segment times together approached 30 minutes daily, or two thirds of available class time.

All seven observations of Carl's instruction included a controlled practice segment at the end of lesson development. A minimum of two examples per day were used in these segments. Usually one example was made up by Carl, and the second example came from the problem set of the upcoming assignment, thereby aiding transition to the assigned problem type. Students were told to do the problems on scrap paper, while Carl circulated around the class and observed their efforts. Then a student was chosen among volunteers to give their answers. If a student was correct (as was usually the case) that student's explanation was recorded by Carl on the overhead, as the student verbalized their solution process. A show of hands was requested, so Carl knew how many students were able to derive a correct answer. More time and examples were done during this segment if Carl perceived incomplete student understanding.

Carl then made the daily assignment, and most of his

students started the assignment in class. Though mean seatwork time was short (six minutes), students realized the importance of this time for receiving specific help from either Carl or the investigator. If they completed two problems correctly during this time, they were quite confident they could complete the whole assignemnt. Carl's daily assignments were relatively shorter (sometimes as few as 5 or 6 problems were assigned), but Carl showed great care in his selection of problems for an assignment. He made sure the assigned problems contained a wide diversity of contexts. If students began work on an assignment in class, they could be assured only 15-20 minutes of homework time would be necessary to complete the assignment.

Comparisons

This section will focus on clarifying and summarizing teacher differences with respect to the daily instructional routine each teacher follows. The lesson segment designations serve as a framework for the discussion which follows.

The instructional routines of Alice and Carl were relatively stable and easy for the investigator to code into the various segments. Betty's routine was stable yet presented difficulties to the investigator's coding by lesson segments, particularly the separation of homework and concept review segments. Betty also had a misguided

belief that content review of previously introduced materials represented new lesson development.

Time given for homework review was similar for Alice and Carl, while Betty spent almost twice as much time reviewing homework than the other teachers. Procedures for homework review were similar for Alice and Betty, though Alice required greater student involvement during this segment. Carl's homework review procedures required less active, physical involvement and more student verbal interaction.

Carl's major instructional focus during a class period was on lesson development of new content. Alice spent the least amount of time daily for lesson development. For Alice, lesson development time was dependent on the available time between the homework and concept review segments and the 15-20 minutes of time deemed necessary for student seatwork at the end of a class period. Betty's lesson development was determined by the quantity of pages and examples her textbook provided of new content, and was slightly more than Alice's time for this segment.

Alice and Carl's instruction regularly included controlled practice segments, while Betty's routine included one such segment in eight observations. The apparent reason for the exclusion of controlled practice segments in Betty's routine was the limited time available

for such practice after lesson development.

The investigator inferred, from observations and teacher comments, different teacher perspectives on who bears responsibility for student learning. Alice placed primary responsibility for learning on her students, believing that student practice of verbal problems is more valuable to learning than teacher delivered instruction. Betty placed primary responsibility on the teacher and textbook for learning, believing her control and explicit delivery of instruction facilitated learning. Carl took a more balanced position in which he accepted his instructional responsibilities seriously, but also saw the need for daily student discussion and practice to improve student learning.

Lastly in this section, several teacher differences related to specific items of the Teacher Inventory Packet (see Appendix D) will be reviewed. It appeared more appropriate to discuss these specific differences where direct comparisons of teachers was addressed. For instance, Betty and Carl expressed confidence they could predict at the start of the school year where in the textbook their classes would be in January. Betty's confidence was based on her practice of covering each textbook section sequentially as laid out, and showed little regard for students' learning progress. Carl's confidence was based on his four years of teaching

experience with the same textbook. Alice's lower confidence might reflect her realization, based on experience, of the wide diversity of student competencies these classes generally exhibited.

The previous descriptions relate closely to teacher expectations for the amount of time daily students should practice skills on written assignments. Carl indicated 15 minutes was sufficient, while Betty indicated 30 minutes was necessary, and Alice 40 minutes. Of these amounts Carl believed half an assignment should be completed in class, Alice about five sevenths, and Betty about three eighths of an assignment. Each teacher's seatwork segments conformed closely to their indicated expectations.

Alice and Betty believed evaluation standards and procedures were the same for all students, while Carl indicated there were occasions when it was necessary to modify standards somewhat for particular students. A similar difference was noted in regards to standards each teacher uses in making a decision to reteach a whole unit on the basis of unit exam results. Teachers were asked what percent of their class doing poorly on an exam would result in the reteaching of the unit. For Carl this percent was lowest at 40 percent, for Betty 50 percent, amd for Alice 60 percent. Carl, in particular, was more sensitive to individual learning differences.

Teacher beliefs about students helping one another in the class were different. Carl expected students to help one another with their assignments. Alice expected little such student tutoring to occur. By choosing a neutral response to this item, Betty revealed a lack of awareness of the situation in her class. Each teacher believed help was initiated by both students and themselves, though in differing degrees. Each teacher encouraged their students to solve verbal problems along lines demonstrated in class. Each teacher listed themselves as the primary person a student should approach in seeking help. Students recognized this hierarchy of helping persons. In all nine student interviews, students acknowledged their respective teachers as the primary person they approach for help. It is only when such help is not possible or available, that students approach a fellow student or family member with their problem.

Verbal Problem Instructional Patterns

The previous section focused on characterizing each teacher's instructional routine in terms of segment differences. This section focuses on one segment, lesson development, related to verbal problem content. Initially certain questions of the Teacher Inventory Packet (see Appendix D for a complete listing of questions) are analyzed which related to teacher perceptions of algebra and verbal problem content. The purpose of this analysis

is to uncover differences in teacher perceptions which relate to differences in verbal problem instructional patterns. Secondly, the subjective evaluation of each teacher's lesson development is discussed briefly. The remainder of this section contains characterizations of each teacher's verbal problem content development.

Questions of Part I of the Teacher Inventory Packet required teachers to mark a position which most accurately described their instruction between two given statements. Items numbered 11, 14, and 17 are given below, and together attempted to identify a teacher's position on whether instruction focused more on manipulation/procedural skills or more on theoretical foundations, understanding, and justification.

	1	2	3	4	5	
11. Algebra is taught strictly as a manipulation oriented subject.	_	_	_	_	_	Algebra is taught with emphasis on theoretical foundations.
14. In my algebra class, I emphasize basic algebraic manipulation procedures.	_	_	_	_	_	In my algebra class I emphasize the underlying reasons for various procedures.
17. Understanding the methodology used is not critical.	_	_	_	_	_	Understanding the methodology of why a method gives correct answers is important.

The three teachers chose no extreme (1 or 5) positions for these three items, indicating teacher

differences to be of minor degree. Betty favored the left side consistently (manipulation/procedure oriented), while Carl consistently favored the right side (theoretical/understanding oriented). Alice's responses were not consistent in either direction.

A similar purpose was intended for the last page of the packet which requested teachers to rank eight given purposes of algebra instruction (same purposes and format as author survey contained). The ranking of the eight purposes indicates indirectly a teacher's perspective and priorities in the teaching of algebra and verbal problem content. Differences between teachers were more pronounced on this ranking item. Alice and Betty ranked verbal problems high (2 and 3) and all manipulation oriented objectives in their top five. Both teachers ranked "solutions of first and second degree equations" of highest importance (1). Carl, on the other hand, ranked verbal problems low (7), and ranked relationships between algebraic rules and arithmetic, and understanding/ justification of algebraic procedures, one and two respectively. All three teachers gave the purpose, algebraic techniques as powerful tools for problem solving, a middle ranking (3,4, or 5). Betty and Alice similarly prioritized the purposes according to specific skill emphasis, while Carl placed greater emphasis on understanding/justification purposes, or what might be

considered higher order objectives.

Part III of the Teacher Inventory Packet contained items which were more open ended, requiring teacher writing of responses or percentage estimates in most cases. Several items were selected for their relevence in characterizing different teacher perspectives. One item requested three reasons for the inclusion of verbal problems in the Algebra I course. Carl listed only one reason, stating that verbal problems "get students to think logically and abstractly in using deductive and inductive reasoning". Carl did not mention the applicational nature of verbal problems. Betty stated that first, verbal problems "foster a problem-solving-thinking through ability". Secondly, she noted the practical application and practice nature of verbal problems, and lastly noted the practice of specific translational skills required in verbal problem solving. Alice suggested verbal problems first, "improve reading comprehension", and secondly aid development of "ordered and structured processing and thinking". All three teachers referred to verbal problems as aiding organized thinking. Carl best expressed an understanding of the complexity involved in solving verbal problems, and the associated relational requirements on student thinking.

Carl also had the best understanding of the importance that vocabulary and implicit relationships

within verbal problem statements have on student performance. He stated that he reviewed problem sets for vocabulary and implicit relationships prior to assignment, and explicitly covered these areas within lesson development. Alice stated she gave little attention to such things, and Betty stated she did about half the time. Observations verified the accuracy of these responses from the Teacher Inventory Packet.

Teachers were also requested in Part III to list specific causes of student difficulty with verbal problems. Each teacher listed reading comprehension and analysis (attack) skills as the primary cause of student difficulty. Alice and Betty mentioned affective traits of students as a secondary cause. Both teachers mentioned an unwillingness on the part of some students to think, while Betty also stated that there was an "inbred anxiety and fear about word problems". Carl believed student inability to translate words or phrases into algebraic expressions was a secondary cause.

In the remediation of student difficulty with verbal problems, each teacher suggested student rereading of the problem more carefully towards understanding of the problem's key elements. Betty added "tell them to relax" before rereading. Alice noted on a different item that middle ability students' biggest barrier to success with verbal problems was "fear of failure". As was noted

earlier, this belief dominated the structure of Alice's daily routine. Each teacher then suggested a return to the steps of the general procedural model presented in their respective textbook. Betty stated "refer them back to the five steps in the book". Only Carl mentioned helping students "break down" sentence phrases and translating phrases. Carl exhibited a keen sense of specific student weaknesses related to verbal problems, particularly within the equation formulation phase.

Subjective Evaluation

After each teacher observation, the investigator evaluated the quality of the teacher's delivered lesson development segment using the lower right section of Coding Form 2. This form is described in detail in Chapter III, and can be found in Appendix B. Each of the nine categories was evaluated on a scale of 1 (poor) through 5 (excellent). The mean ratings of lesson development quality are given in Table 4.11 for the nine categories and each teacher. The few nonverbal problem solving lesson developments observed were included in these results.

Table 4.11
Teacher Comparison: Lesson Development Quality

	Alice	Betty	Carl
Number of observations	11	9	8
Category			
1. Task Orientation	4.2	2.4	4.6
2. Clarity	4.3	3.0	4.9
3. Enthusiasm	3.3	2.3	3.8
4. Frequent Teacher Talk	4.2	4.1	4.4
5. Attending Prerequisites	3.1	2.4	4.5
6. Attending Relationships	3.3	3.0	4.0
7. Attending Representations	2.5	2.4	3.1
8. Attending Discrimination	2.2	2.6	4.0
9. Generality of Concepts	3.0	2.2	3.5
Total Rating	30.1	24.4	36.8

The results of the lesson development evaluations indicate similarities and differences between teachers. For instance, the high ratings for frequent teacher talk indicate all three teachers lectured, and dominated interactions during lesson development segments. The last five categories were more effected by differences in the quantity of time given to lesson development by each teacher. For these categories the investigator judged sufficiency of examples and explanations according to what the investigator believed was possible for the teacher to accomplish.

Teacher differences are developed in the following

descriptions of each teacher's planning, organization, and delivery of verbal problem content. For each teacher, the textbook was the primary determinant for planning and lesson delivery. Yet each teacher approached this task from a different perspective which resulted in differing presentations of the same content.

Alice

In Alice's textbook, core verbal problems are neatly organized into nine chapter sections of this textbook, each section designated: Problem Solving and Applications. For two such sections, additional relationships are included within section exercise sets. In one exercise set, the formula: A+B+C+D = 360 is introduced, and in the other exercise set, the student is asked to recall (introduced earlier) that "the sum of the measures of angles of a triangle is 180 ". Three of the nine chapter sections correspond with three investigator designated problem types: integer problems, age related problems, and D/R/T problems. Two other sections together correspond with the geometric formula problem type. The four other sections (including the percent related problems) were classified simple translation problem types. Four exercises of one section and fifteen exercises from a supplementary textbook were classified coin/money related problems type. To some extent, all six identified problem types received coverage in Alice's instruction.

Verbal problem sections are located within chapters with the intention that such sections provide student practice of immediately preceding manipulation skills. Alice ignored the authors' sequencing intentions, and instead she first clumped together and taught a chapter's manipulation skill sections. Then the remaining three or four verbal problem sections were taught separately in order of textbook appearance. Exercise sets of verbal problem sections received almost total coverage by Alice in her explanations and assignments in two successive class periods. She assigned exercise sets in one of two patterns. Either the odd exercises of a set were assigned, followed by the even exercises the next day, or the first half of an exercise set was assigned with the remainder of the set the following day.

Alice's daily and weekly planning for this class required minimal effort. She planned and wrote the topic sequence for a week's lessons at a time, including the assigned exercises she would make each day. Quizzes and tests were taken directly from supplementary materials provided by the textbook publishers. Quizzes and tests lacked the same diversity of contexts noted earlier in the textbook analysis. On two occasions Alice designed supplementary practice worksheets of verbal problems taken from other sources. These materials exhibited greater context diversity, and encouraged the combined review of

several problem types simultaneously.

Alice's instruction did avoid one weakness of this textbook noted earlier by the investigator. The textbook identifies a new general procedure for each verbal problem type encountered within each chapter section. Alice avoided this weakness by not calling student attention to the boxed summaries found in each section. Instead she drew student attention to the fact that additional features could be incorporated into the four step model she required students to follow in their written solutions.

A chapter section of verbal problems received two days of instructional treatment by Alice. On the first day, Alice initially focused on the inherent relationship(s) contained in the verbal problems under study. She used concrete examples in her discussion to aid recall of the relationship(s). Alice assumed all students had been exposed to the studied relationships in their previous mathematical experiences. One or two example problems were selected from the exercise set of a section for classroom discussion and completion. Alice included explanation and purposes related to textbook suggestions for the inclusion of auxiliary devices (charts, tables, and diagrams). She expected students to write these example solutions in their notebooks, as she wrote the solution steps on the chalkboard. Steps were

clearly and explicitly defined in her presentations, and Alice consistently followed these steps in all verbal problem instruction. The well ordered pattern of steps began with the defining of variables, then finding the related equation, then solving the equation, and finally, posting the results next to the variable definitions. Observed student board work, during homework review segments, indicated general conformance with Alice's expectations for written solutions. Alice evaluated student written work based on the correct completion of the four steps, and awarded partial credit for the degree of student progress on a problem. In her instruction of verbal problems, Alice's primary objective was student development of an orderly sequence of processes to follow in solving verbal problems. She believed the development of organized mathematical thinking was the greatest need of these students.

Alice's lesson development segment included many student-teacher interactions, and involved students in every phase of an example's solution. As many as five or six students contributed input to the solution of a single example. Alice provided students time to read the example (exercise) silently, and then chose a student to read the problem aloud. That student had to identify what the problem was asking for, and possibly attempt to define variables for the problem's elements. A second student,

assigned or selected from volunteers by Alice, was requested to locate the sentence of the problem statement which related to the problem's equation, and read it orally. This student then suggested a possible equation. Alice wrote the suggested equation on the chalkboard, paused a moment to study it, and then asked the class their opinion of the proposed equation. Student positive nods or quiet indicated to Alice classroom agreement with the formulated equation, while student hand raising indicated disagreement or suggestions for alternative or modified equations. Alice seldom criticized students' suggestions, and students displayed an openness to conjecture guesses. Normally it required two or three student attempts before a correct equation was formulated. Once agreement on an equation was reached, a different student was chosen to solve the equation, verbally stating the procedure followed as Alice recorded the work on the chalkboard. Other students aided in this process, speaking out when in disagreement or as called upon by Alice when progress stalled. As a last step a student was identified to interpret the equation's solution. For this process Alice wrote the numerical answers indicated by the student next to the definitions of the variables at the beginning of the problem. About half the time a check of the solutions was carried out by Alice, and involved a quick review as to whether the solutions satisfied the

problem statement.

For the first example, students were told to record all written work from the chalkboard into their notebooks after a solution was completed. For a second example students were told to record solution steps while class discussion was proceeding. Alice then selected one or two problems from the exercise set to be done as controlled practice by the class. Students seldom made errors during these practice sessions, and when errors occurred they were individually remediated by Alice. Correct solutions for these problems were quickly sketched on the chalkboard with minimal comment by Alice.

The second day of instruction for a chapter section generally included one additional example taken from the remaining problems of the exercise set, and one controlled practice exercise. Lesson development time was greater for the first day of a chapter section, about twice as long, as for the second day of instruction.

Summary/ Reaction

Alice's lesson development segments lacked creativity, yet related to specific objectives and purposes Alice desired to be acomplished. Two objectives were accomplished in Alice's presentations. First, students were provided a well structured and sequential pattern for the processing of verbal problems. Alice's four step model resembles Polya's model, and received

consistent and explicit treatment. Secondly, Alice's short presentations allowed students to complete assignments efficiently in a short amount of time, and with a minimum amount of frustration.

The investigator believed Alice could have made greater use of concrete and practical examples, and spent more time developing encountered relationships. Her treatment of relationships was sketchy and concerned with related formulas, rather than with relational understanding. Alice made little attempt to connect new concepts with previously learned concepts in her instruction. Her textbook stresses that translational processes are to be used in formulating variable definitions and equations. Alice accepted this approach, and her instruction reflected this approach to verbal problem solving. In this approach the first goal is to separate the sentences of the problem statement as to whether they address the variable definition or the equation formulation. The individual sentences are then analyzed, and translation processes are applied to each sentence. The investigator noted that many student errors in equation formulation remained, despite Alice's greater attention and time given to the teaching of translation processes. However, reliance on translational processes to solve verbal problems appeared sufficient for all verbal problem types encountered during the study.

Betty

Verbal problem sets within Betty's textbook appeared in a diversity of locations. Since Betty followed the textbook sequence exactly in her instructional planning, such location diversity might not appear problematic for Betty. However, Betty's sole reliance on textbook examples for her instructional presentations increased the difficulties associated with diverse verbal problem locations. As the textbook analysis noted, textbook B contains a small number of instructional examples (11). This paucity of examples limited students' exposure to verbal problem types and associated methods of solution in Betty's class. The combination of textbook characteristics and Betty's instructional patterns relegated verbal problem content to a secondary status position in the total content package.

Betty planned for this class a week at a time, noting the section number, the content focus or skill, and the related assignment of exercises for each day in her plan book. Written detail of each day's lesson development was not recorded, nor was it necessary since each day's development reviewed the textbook's explanations completely point by point.

Verbal problem exercises appeared in three forms, at the end of a manipulation skill set, as an isolated problem typed set, or as an entire section within a

chapter. Each locational form received diferent instructional treatment by Betty. As a chapter section, verbal problem sets received one day of instructional treatment. At the beginning of the lesson development segment, students were told to open their textbooks to the appropriate chapter section. Betty then read the sentences which explained the focus and central relationship(s) of the section. Each textbook example was discussed (with students' books remaining open) with the class answering questions that Betty brought forth. Mostly, Betty's questions were directed to students by name who Betty believed were easily distracted during these discussions. Such directed questions allowed Betty to maintain a limited degree of class attention. During these discussions, students were not very attentive or intellectually challenged, since a quick glance at the open textbook generally revealed to the student the answer that Betty expected. Betty's total reliance on the textbook for her instructional presentations and examples was true for all content in chapter sections, not just verbal problem sections.

When verbal problem exercises appeared in an isolated, problem typed set, no textbook explanations or examples were provided. When these sets appeared in the textbook, Betty used the first problem of the exercise set as the only example within her instruction. The exercise

set was added to the assignment of the previous chapter section. Students paid greater attention to Betty's demonstration in these instances, and Betty was more willing to call on student volunteers in these discussions. If unable to find a student volunteer, Betty called on her more attentive and better students.

When verbal problems appeared at the end of a manipulation skill set, a textbook sample problem was provided. In her lesson development, Betty covered the section's manipulation examples, and then jumped to the sample in the verbal problem exercise set. Classroom discussion of the sample problem closely resembled that which characterized examples of a chapter section. These sets of verbal problem exercises were assigned with the associated manipulation exercises of the day's lesson. For all three locations of verbal problem sets, generally half (odd or even numbers) of an exercise set was assigned for homework. It was observed that a majority of students usually failed to complete assignments. Students generally attempted to solve the verbal problems of a chapter section, since that represented the entirety of an assignment. However, because verbal problems in the other two locations were at the end of usually long assignments, student attempts to solve these verbal problems was minimal.

One pattern of Betty's lesson development deserves

further discussion. Betty liked to construct well structured, explicit, and sequential steps for algebraic procedures. For instance, solving a linear equation had a seven step sequence. Betty believed these structured, explicit procedures served a summary function purpose for students, and aided student recall. Students obligingly copied them from the chalkboard into their notes. However the investigator did not observe students using their notes in completing assignments, although Betty referred explicitly to the various step numbers in both her review of homework and example presentations.

The same pattern of explicit step procedures was evident in Betty's development of a general problem solving model. The textbook's five step model incorporates the four steps of Polya's model. Betty explicitly introduced the five steps, including the last checking step, early in the school year because of its location in Chapter 1 of textbook B. Students then continuously reviewed examples using the general procedure, though it was not reformalized by Betty in her instruction for a two month period. At this time Betty reviewed the model's five steps and told students to show steps numbered 2-5 on their paper for every verbal problem (step 1 involved reading the problem). Initially she required students to record the number for each step also. Limited observed chalkboard work of students indicated

these students completed all steps of the procedure in their solutions. However, since the same ten students did all the homework review boardwork, it was less clear whether the other seventeen students had adopted the general procedural model. Betty's evaluation of student verbal problem work on tests or quizzes did not focus on the general model employed, but rather awarded full credit for a correct answer, or half credit where a computational or procedural error was apparent.

Summary/ Reaction

Betty's lessons followed a regular routine each day. Sequence and presentation were totally dependent upon the employed textbook. The clarity of her explanations was dependent upon the clarity of the textbook presentations, which the investigator did not rate highly or consider of sufficient quantity to be effective learning aids. Betty was consistent in employing a single, general procedural model for verbal problem solving. She attempted to have students internalize this model through teacher modeling and by explicitly requiring students to use the model on their assignments.

Two weaknesses of Betty's verbal problem instruction were apparent. Verbal problems received insufficient instructional treatment in comparison to other content, and Betty's students lacked necessary experience processing unsolved verbal problems. Partial blame for

insufficient instructional treatment of verbal problems lies with the textbook authors, in the sense that there are not enough examples presented, and was discussed earlier. Yet Betty might have altered her fixed instructional routine for verbal problem content if she had realized the limitations of textbook presentations, primarily the quantity of such presentations. Her insecurities with regards to verbal problem content and/or her inability to evaluate her teaching or possible alternatives prevented her from modifying or embellishing textbook presentations. Betty's verbal problem instruction demonstrates a case of teacher-textbook mismatch. If Betty persists in maintaining her present instructional routine, her students might be better served if Betty adopted a textbook similar to textbook A. Its more extensive number of examples and greater chapter section treatment of verbal problems provide students greater content exposure and opportunities for student learning from textbook explanations. One might argue that Betty's more motivated students learn primarily from textbook explanations presently.

The second weakness, a lack of student experience in processing unsolved problems, results directly from Betty's instructional routine. Except for three problems used as examples from the three isolated verbal problem sets, all of Betty's instructional examples were solved

completely for students within textbook explanations. Students continually exposed to worked out examples do not receive an opportunity to develop or practice the necessary thinking processes, or even to observe such processes in teacher modeling in the solving of verbal problems. The homework review segments conducted by Betty also failed to provide opportunities to practice these processes for the less motivated students. The lack of controlled practice segments further inhibited student opportunity. Only the more motivated students, who actively attempted verbal problem exercises on their own daily, gained sufficient experience and practice of necessary thinking processes.

Carl

In Carl's textbook, verbal problems appeared only in separate chapter sections. Two sections of verbal problems for which translation skills lead directly to equation formulation are in the third chapter of this textbook. The fourth chapter contains one verbal problem section for which geometric formulas/relationships lead to equation formulations. With one exception, the fifth chapter contains all verbal problem sections of more relational equation formulations. The one exception involves equation solving with parentheses (distributive property), which Carl covered earlier in the school year with Chapter 3 material related to equation solving. Carl

told the investigator during the study that he had spent an additional two weeks of class time this year assuring student mastery of algebraic manipulation and equation solving skills. He believed the additional time spent on these content areas would lead to improved performance on equation formulation problems and on verbal problems. Observations of student work samples of verbal problems and student input during Carl's lesson developments confirmed the correctness of Carl's beliefs. Fewer careless or procedural errors related to equation solving were noted in this class's work. Classroom discussions indicated that for most students the linear equation solving process was automatic, efficient and accurate.

Carl's lesson planning related to verbal problem content was detailed and polished, although not written out explicitly. He had obviously studied the explanatory pages, examples, and the exercise set of a textbook section carefully. On most days one verbal problem section received instructional treatment and assignment. The last four sections of verbal problems (Chapter 5) were covered in two days, two sections per day. For each day's planning, Carl constructed a realistic problem situation to which he felt students could relate and take on a hypothetical role posed within the situation. The constructed situation also demonstrated the essential relationships of the textbook section. For example, such

realistic situations included defining student roles as a store manager (percent markup, sales), a production manager (money/earning related), a family member (age related), and a scheduling supervisor (D/R/T types).

Carl began lesson development by defining the role he wanted each student to imagine presently. He then placed the students in a realistic situation by identifying a familiar, local company or industry in which the students were employed. Hypothetical situations were then discussed related to the students' position in the company. Carl then added numbers to the situation, and posed a related problem. Carl constructed the numbers on the spot, during lesson delivery, since solutions seldom came out evenly (unless this too was deliberately planned). Students were allowed to think aloud, and share their ideas and answers with classmates for several minutes. When convinced that several students had determined hypothetical answers to the posed problem, Carl selected one student who indicated they believed they had a correct solution. He listened to the student's answer, and requested the student to explain (justify) the solution path. Once the explanation was completed, Carl reviewed the student's arguments step by step for the class, and discussion centered on its merits or imperfections. This questioning process might continue with several students modifying the solution path until

Carl sensed the class agreed upon the posed problem's solution and employed reasoning.

Carl then moved to the overhead and asked students to suggest algebraic ways to solve the posed problem, if these methods had not come forth in the preceding discussion. Variables were defined and the equation was formulated with student assistance, and shown on the overhead. Carl then solved the equation, posted the results by the variable definitions, and explicitly checked the solutions with the verbal problem statement. Equation solving and checking were done quickly, and with less student interaction. Usually he posed one or two additional problems related to the initially constructed situation and devised role. Each new posed problem required greater understanding of the underlying relationship(s), and commanded greater attention to using algebraic techniques to achieve solution. Carl gave careful attention to the sequencing of these posed problems.

Carl then sought to formalize the relationship(s) by reviewing the textbook explanation of the focused verbal problem section with the students. He selectively reviewed some or all of the provided textbook examples, depending on whether an example had a varied context or required a completely different equation formulation. Textbook explanations and examples were covered quickly

(5-7 minutes) with Carl talking and writing on the overhead the steps he expected students to include in their assignments.

Lesson development ended and the controlled practice segment began when Carl requested students to take out a sheet of scrap paper to work some problems themselves. Usually two problems were assigned for controlled practice, and were selected from the written exercise section of the upcoming assignment. On two occasions, Carl orally posed a practice problem he had designed before class. Once students had attempted both problems, Carl did each problem on the overhead with student assistance. Student transition to the day's assignment of exercises was relatively easy. The few student questions generated during seatwork related to specific concepts contained within problem statements of the exercises.

The most active and participative students sat directly in front of Carl as he faced the class, and formed a "V" with himself as the vertex. The more quiet five or six students sat near the front and window areas, or to Carl's left as he faced the class. This arrangement was noticeable, but may have just been coincidental.

Summary/ Reaction

Carl's lesson development segments were considerably different from the other teachers qualitatively, and in terms of quantity of time and examples. Besides being

longer, these segments contained more realistic examples beyond those contained in the textbook. Examples were the primary means for the development of each section's central relationship(s). Students were encouraged to be active participants in Carl's lesson developments, and student-teacher interactions were lively and relevant exchanges.

Carl's purposes and those of his employed textbook were harmonious. Some verbal problem sets stressed translational skills, while most verbal problem sets suggested a more relational formulation. These two levels of verbal problems were clearly in separate locations of the textbook. Carl's instruction similarly separated verbal problem levels and the associated skills for each level.

Carl developed a general procedural model of written expectations for his students more by modeling the desired steps than by explicitly defining the procdure. His four step model closely matched Polya's model, and was a simplification of the textbook's seven step procedure. Three "thinking" steps were dropped from the textbook procedure, yet these steps received continued modeling by Carl in his instruction. Carl assumed students would internalize these steps from his modeling of them, and use them in their analysis of problem statements. He also stressed the checking (looking back) step in his

instruction.

Carl's assignments were shorter than for the other teachers, generally representing half of a textbook exercise set. The lack of a pattern in Carl's assignments indicated Carl reviewed exercise sets carefully, and selected exercises which represented a diversity of problem contexts.

Comparisons

Lester (1980) has described two categorizations of problem solving instruction which are relevant for comparing the study's three teachers' verbal problem instruction. The first categorization distinguishes three types of instruction: teaching for problem solving, teaching about problem solving, and teaching via problem solving. The first type, often emphasized in current algebra textbooks, focuses on student acquisition of specific tool skills useful for problem solving, such as translation or equation solving skills. The second type typically involves a teacher modeling appropriate behavior and/or directing student attention to salient procedures and strategies. The third type presents algebraic content/relationships in the context of problems to be solved. Alice's instruction was mostly of the first type (teaching for problem solving), though she also modeled problem solving behavior and directed student attention to important points. Betty's instruction closely followed

textbook lines, and was of the first type exclusively. Carl's instruction was mostly of the second type (teaching about problem solving). However, through the use of constructed situations and related problems, Carl often taught mathematical relationships via problem solving (type three).

If this first categorization scheme is considered a continuum (not Lester's intention), the three teachers are located in quite different positions. Polya (1957) and Wickelgren (1974) encourage teachers to move towards the third type of instruction, teaching via problem solving. Teaching via problem solving, even verbal problem solving, presents students greater opportunities to access and evaluate knowledge, to monitor progress, to justify procedures, and to develop control strategies (Schoenfeld, 1983). The three teachers' instruction differed markedly in presenting opportunities for students to develop these cognitive and metacognitive skills.

The second categorization focuses on the purposes of the problem solving instruction. Problem solving instruction can be designed to: develop specific tool skills, develop master thinking processes, teach task-specific heuristics (not readily generalizable advice), teach general heuristics (widely applicable advice), and involve students extensively in solving problems without direct teacher intervention. Unless one considers Polya's

model as a master thinking process or general heuristic, little of the teachers' instruction is designed to accomplish these two purposes. Betty's instruction is focused on developing specific tool skills only. Alice's instruction is intended to develop task-specific heuristics and specific tool skills in an environment of heavy student involvement without direct teacher intervention. Carl's instructional purposes resemble Alice's instructional purposes, but are accomplished in an environment where teacher modeling of tool skills and heuristics is of greater importance than student practice.

Summary

This chapter has reviewed the two components, textbook and teacher, which represent the instructional variables of the study. The textbook review demonstrated large differences in organization, location, and instructional treatment of verbal problems. Minor differences were found in each teacher's daily instructional routine. Greater differences were noted in teachers' lesson development segments related to verbal problems. The implications of teacher and textbook differences on student performance and learning will be done in Chapter VII. Chapter V will look at the nature of specific teacher suggestions given to improve verbal problem solving performance.

MICHIGAN STATE UNIV. LIBRARIES
31293010577165

MICHIGAN STATE UNIVERSITY LIBRARIES
3 1293 01569 8396

V. 2

LIBRARY
Michigan State
University



RETURNING MATERIALS:
Place in book drop to remove this checkout from your record. FINES will be charged if book is returned after the date stamped below.

A NATURALISTIC STUDY OF ALGEBRA I VERBAL PROBLEM INSTRUCTION AND LEARNING

VOLUME II

By

Gary Michael Klingler

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Education

1988

CHAPTER V

Instructional Suggestions and Examples

The previous chapter focused on descriptions of the instructional components, employed textbooks, and teachers of the study. These descriptions detailed characteristics of each classroom's expository content presentation along organizational and sequential dimensions. These descriptions primarily characterize instruction of knowledge and comprehension level objectives. Yet research has shown that instruction of higher level objectives, such as application and analysis, which involve cognitive skills, such as formulating and solving mathematical problems, requires supplemental instruction designed to address students' metacognition (Brophy, 1986). Brophy called such supplemental instruction "strategy training". Strategy training adds to the knowledge about skills with instruction in operative knowledge (how to use the skills) and conditional knowledge (when and why to use the skills). However, the observed content of teacher dialogue and suggestions in verbal problem solving did not lend itself to the three fold (skill, operative, and conditional) knowledge classification scheme Brophy proposed.

A model specific to verbal problem solving was instead selected for the analysis of observed teacher initiated suggestions (Riley et al, 1983). The model was developed and found appropriate for analyzing and explaining elementary students' difficulties related to verbal problems. The authors believe the model is also applicable for explaining student algebra verbal problem performance. The model conjectures that verbal problem solving requires three kinds of knowledge: problem schemata, action schemata, and strategic knowledge. Problem schemata organize, expand, and develop a coherent structure for representation of the problem statement. Action schemata mediate and carry out the choice of an action, operation, or procedure required to solve the problem. Strategic knowledge is composed of production rules organized to serve a planning function for the student. The production rules guide and clarify student planning and proceed from the general to the specific in a top-down manner (Riley et al, 1983).

One way that teachers develop these three kinds of knowledge is through the use of verbal suggestions within their instruction. One method of classifying teachers' verbal suggestions is according to their function or content purpose within the model's three components of knowledge, and this classification method is developed initially. Two other methods of categorizing teachers'

verbal suggestions are developed, by format (question or statement) and placement within instructional segments. The key issue addressed is how teachers' verbal suggestions and example presentations possibly influence student verbal problem solving performance in a positive way.

The chapter has two sections, the first section analyzes teacher suggestions and the second section analyzes teacher example presentations. In each section, the investigator searches for ways teachers' verbal suggestions or example presentations possibly impact on students' development of problem schemata, action schemata, and strategic knowledge. The analyses compare teachers' instructional efforts to promote and assist this developmental process related to training students in processing and solving verbal problems.

Instructional Suggestions

A major instructional role of teachers is to provide students with explicit suggestions or hints which may assist students' solving of problems or exercises. Investigator review of observation field notes revealed 182 instances of teacher oral expression intended to suggest help for the students, or an average of about six suggestions per class period. Analysis focused on identification and classification of each noted teacher suggestion. Each suggestion was classified according to

its content focus, lesson segment placement, and oral format. Oral format referred to a suggestion's presentation as a statement or a question. Lesson segment placement referred to the suggestion's location in the daily lesson. Three lesson segment categories were used: homework review segment, lesson development segment, and seatwork/ controlled practice segment.

The content focus classification involved combining elements of the earlier discussed verbal problem solving model of Riley et al (1983) and the problem solving instructional variable classifications of Lester (1980). The verbal problem solving model conjectures the need for three kinds of knowledge: strategic, action schemata, and problem schemata. The general procedural model (similar to Polya's model) developed in each class served as a framework for student planning, monitoring, and assessment. The steps of Polya's model (understand the problem, devise a plan, carry out the plan, and look back) indicate subgoals of the planning process. Hence one content category included teacher references to a general model. Instructional references to the steps of Polya's model usually were given in the form of actions such as: read and understand the problem, define the variable(s), formulate the equation, solve the equation, or check the results. Teacher suggestions which focused on particular actions (steps) or related subskills necessary to

accomplish these actions were categorized specific tool skills (Lester, 1980), and are related to the action schemata classification of Riley et al (1983). Teacher suggestions intended to improve problem perception and representation (problem schemata) were divided into two categories, either task-specific heuristics or general heuristics. Task-specific heuristics were intended to address a specific class of problems, whereas general heuristics had applicability across classes of problems. Each teacher suggestion then was classified for a single content focus of the four types: general model, specific tool skill, task-specific heuristic, or general heuristic.

Prior to the analysis of teacher suggestions according to the above described categorization scheme, six examples (two for each teacher) are displayed with investigator classification in Table 5.1. Each classification is ordered first by content focus, secondly by lesson segment placement, and lastly by oral format.

Table 5.1

Instructional Suggestions: Classification Examples

Teacher/Date	Teacher Dialogue	Classification
Alice 10/9/86	Will,read sentence right out of the book. Pick it up just like you read it. (translate)	tool skill seatwork/cp statement
Alice 10/27/86	Make a picture or chart, then an equation, solve it, then answer the question.	general model lesson devel. statement
Betty 10/8/86	What variable shall we use? What shall it stand for? Any other variables defined?	tool skill lesson devel. question
Betty 9/9/86	Do 2 things, choose variable and represent facts, and know P=2l+2w so write equation, need equation.	general heur. homework rev. statement
Carl 12/3/86	Apart after 2 1/2 hours.(uses chart). Basically all you do is fill in chart and find equation.	task-specific lesson devel. statement
Carl 11/10/86	What are we to find? S:a numb. How many? What kind? S:3 consecutive integers.	tool skill homework rev. question

Each teacher dialogue considered a suggestion falls into one of 24 classifications (4 content foci x 3 lesson segment placements x 2 oral formats). Initially totals for each of the nine major categories are displayed in Table 5.2. Subsequently several contingency tables were constructed to test pairs of independent variables (teachers, content focus, lesson segment, and oral format) for independence or homogenaity.

Table 5.2

Instructional Suggestions: Summary Classification Types

	Alice	Betty	Carl
Number of observations	12	10	9
Mean suggestions per class period	5.9	5.3	6.4
Median suggestions per class	6.5	5.5	6
A. Content focus classification			
1. General model	7	9	3
2. Tool skills	30	22	33
3. Task-specific heuristic	18	7	10
4. General heuristic	16	15	12
B. Segment placement classification			
1. Homework review	23	26	19
2. Lesson development	34	26	38
3. Seatwork/controlled practice	14	1	1
C. Oral format classification			
1. Question	10	12	29
2. Statement	61	41	29
Totals each classification	71	53	58

A review of teacher suggestions by classification types from Table 5.2 indicates the following: for content focus, differences appeared relatively minor, for segment placement, Alice was noticeably different in making more suggestions during the seatwork/controlled practice segments, and for oral format, Carl was noticeably different showing a greater percentage usage of question formatted suggestions than his counterparts.

Table 5.2 also indicates that the mean number of coded suggestions per class period for the three teachers is similar, ranging from Betty's 5.3 per class period to Carl's 6.4 per class period. The number of observed

suggestions in a class period ranged from a low of two suggestions which occurred once (Betty) to eleven suggestions which also occurred once (Carl). A one way ANOVA was performed for comparing the teachers' mean number of suggestions per class period. The results are displayed in Table 5.3.

Table 5.3

Analysis of Variance for Teachers' Suggestion Quantity

	df	MS	F
Treatments (teachers)	2	3.12	0.593
Error	28	5.26	

The low F ratio of 0.593 indicated a close similarity between the three teachers' daily distribution of suggestions.

Homogenaity Tests

In addressing the question of whether the three independent variables: content focus, lesson segment placement, and oral format occurred in roughly equal proportions in the three teacher populations, three hypotheses were tested. The three hypotheses tested for homogenaity the three teachers' verbal suggestions' content focus, segment placement, and oral format, and involved contingency tables, and the X^2 statistic. The individual contingency tables appear in Appendix H, where expected cell values are given in parentheses. A level of

significance of .05 was arbitrarily chosen for each test statistic. Table 5.4 provides a summary of the results related to the tests for homogenaity across the teacher populations.

Table 5.4

Instructional Suggestions: Summary Homogenaity Tests

Independent Variable	Computed	Table	Conclusion
1. Content focus	8.93	12.59	Fail to reject
2. Segment placement	20.94	9.49	Reject
3. Segment placement (excluding SW/CP segment)	3.14	5.99	Fail to reject
4. Oral formats	21.13	5.99	Reject

The test of homogenaity for lesson segment placements resulted in rejection of the null hypothesis (row 2, Table 5.4). The obvious reason for the rejection was the disproportion in the number of teacher suggestions in the seatwork/controlled practice segment (Alice 14, Betty and Carl 1, Table 5.2). A secondary hypothesis was that the proportion of teacher suggestions within the homework review and lesson development segments was the same for all three teachers. Row 3 of Table 5.4 reflects the results of this secondary hypothesis test.

The results of the tests of homogenaity for the three independent variables indicate that the proportion of suggestions by content focus types and by placement in the homework review and lesson development segments are

similar for the three teacher populations. Oral formats (question or statement) of suggestions is not homogeneous across the three populations, however.

Independence Tests

Investigator attention then turned to testing independence of pairs of variables: content focus, lesson segment placement, and oral format for each fixed teacher population. In several tests, variable pair analyses required modification of included variable categories and/or test abandonment due to expected cell frequencies being less than five in more than one cell of the contingency tables (Goldman and Weinberg, 1985). The results of the variable independence tests for individual teachers is summarized in Table 5.5. For each test, α was arbitrarily set at .05 and degrees of freedom was found as (the number of rows -1) times (the number of columns -1) for the contingency tables constructed. The individual contingency tables for the five tests capable of completion are given in Appendix H.

Table 5.5

Instructional Suggestions: Summary of Independence Tests

Test of Variables	Teacher	Exclusion or not possible	Computed X^2	Table X^2	Conclusion
A. Content focus & les. seg. place.	A	gen.mod.&SWCP	7.62	5.99	Reject indep.
	B	not possible	-	-	-
	C	not possible	-	-	-
B. Content Focus and oral form	A	not possible	-	-	-
	B	not possible	-	-	-
	C	general mod.	2.63	5.99	Fail to rej.
C. Les.Seg. and oral format	A	SW/CP	3.17	3.84	Fail to rej.
	B	SW/CP	.44	3.84	Fail to rej.
	C	SW/CP	1.67	3.84	Fail to rej.

Table 5.5 indicates the difficulty encountered in trying to test for independence of variables in the individual teacher populations. In four cases, the test was not possible, and in the other five tests at least one classification had to be excluded. Alice's placement of suggestions in homework review segments or lesson development segments was not independent of her content focus being tool skills, task-specific heuristics, or general heuristics. The oral format of Carl's suggestions was not influenced by his content focus (with the general model classification excluded). For all three teachers, the oral format employed (question or statement) was not dependent upon its location in the homework review segment or lesson development segment.

The inability to test for variable independence led the investigator to consider the suggestion data as a

sample from a single population, Algebra I classrooms, and to perform tests of variable independence on the combined teacher data. Three tables were constructed to reflect this alternate perspective. Table 5.6 displays the contingency table for variables: content focus and two lesson segment placements (seatwork/controlled practice segment excluded). The requirement on expected cell count size was violated otherwise. Table 5.7 displays the contingency table for variables: content focus and oral format. Table 5.8 displays the contingency table for variables: lesson segment placement (seatwork/controlled practice segment excluded) and oral format. The inclusion of the seatwork/ controlled practice segment results in rejection of independence for these last two variables. For all three tables numbers in parentheses represent expected cell counts.

Table 5.6

Instructional Suggestions: Content/Placement Independence

Content Focus	Segment Placement		Totals
	Homework Rev.	Lesson Devel.	
General Model	5 (7.0)	12 (10.0)	17
Tool Skills	38 (31.5)	39 (45.5)	77
Task-specific Heur.	10 (13.9)	24 (20.1)	34
General Heuristics	15 (15.6)	23 (22.4)	38
Totals	68	98	166

H_0: Content Focus and Segment Placement independent
H_A: Content Focus and Segment Placement not independent
Computed X^2 = 5.13 Table $X^2_{3,.05}$ = 7.81
Conclusion: Fail to reject H_0.

Table 5.7

Instructional Suggestions: Content/Format Independence

Content Focus	Oral Format		Totals
	Question	Statement	
General Model	2 (5.3)	17 (3.7)	19
Tool Skills	32 (23.8)	53 (61.2)	85
Task-specific Heur.	8 (9.8)	27 (25.2)	35
General Heuristics	9 (12.1)	34 (30.9)	43
Totals	51	131	182

H_0: Content Focus and Oral Format are independent.
H_A: Content focus and Oral Format are not independent.
Computed X^2 = 8.33 Table $X^2_{3,.05}$ = 7.81
Conclusion: Reject H_0

Table 5.8

Instructional Suggestions: Placement/Format Independence

Lesson Segment	Oral Format		Totals
	Question	Statement	
Homework Review	25 (20.5)	43 (47.5)	68
Lesson Development	25 (29.5)	73 (68.5)	98
Totals	50	116	166

H_0: Segment Placement and Oral Format are independent.
H_A: Segment Placement and Oral Format are not independent.
Computed X^2 = 2.41 Table $X^2_{1,.05}$ = 3.84
Conclusion: Fail to reject H_0

The following brief comments can be made regarding the independence of various pairs of variables related to classifications of suggestions. A suggestion's intent or content focus and its oral format was not dependent upon whether the teacher was in the homework review segment or the lesson development segment of the day's lesson. However, a suggestion's oral format was dependent upon the suggestion's content focus.

Reaction

The above analysis of teacher suggestions or helpful dialogue is limited in its ability to address the major questions related to instructional variables. Teacher suggestions are intended to aid students organize concepts, facts, and procedures in meaningful ways. However, the specific dimensions and characteristics of effective instruction which aid students in structuring

their knowledge remained elusive. Several researchers, most notably Cooney and Henderson (1972) and Henderson (1967, 1969) have developed descriptions of various instructional strategies that teachers employ in classrooms intended to aid student organization of knowledge. Cooney and Henderson (1972) identified nine organizing relations which characterized secondary (grades 7-12) teaching dialogue. The nine organizing relations were developed through an interaction of logical considerations and analyses of video tape transcripts. A limitation of the present study was an inability to develop such detailed classifications of teacher suggestions, since fieldnotes often failed to capture sufficient context of teacher dialogue which videotaping did for them. However, Cooney and Henderson (1972) were unable to determine the most effective teacher means to aid student knowledge organization in their study.

Based on reading comprehension instructional research, Brophy (1986) believed "strategy training" instruction in mathematics needed to be unusually explicit and detailed, and include considerable cognitive modeling (where teachers overtly verbalize thought processes) in addition to more typical expository presentations. That such instruction must be detailed and explicit was shown in studies by Schoenfeld (1979) and Putt (1978).

Putt's study is particularly noteworthy (despite its

fifth grade subjects) since it is one of the few instructional variable studies done in actual classrooms with students learning from teachers rather than self instructional materials. Lester (1980) called such research "teaching experiments", and saw a growing trend in their employment for instructional variable research. Putt designed two instructional methods, both based on Polya's four stage model for instruction of process problems (verbal problems not lending themselves to direct algorithmic procedures). In the heuristic treatment groups, teachers made students aware of heuristic questioning strategies suggested by Polya in each of the four stages (cognitive modeling). The control groups received no explicit heuristic training, though students were given identical sets of problems to solve. Teachers in these groups answered student questions related to tool skills, such as clarifying terms, words, and procedures. Results indicated that students can, in a short time (four weeks), learn to ask themselves questions which aid understanding of the problem, and to use specific strategies to solve problems. Students of the heuristic treatment groups also had a greater awareness of Polya's four stage model, and indicated the model was helpful for solving verbal problems.

Several comments regarding teacher suggestions of the present study appear relevant in light of the above

framework. Each teacher's instruction was sufficiently explicit and detailed, and focused adequately on the four content areas (general model, tool skills, task-specific heuristics, and general heuristics) which all relate to effective instruction of higher level cognitive objectives. Carl's fewer suggestions (3) related to the general model, all given on the day of its introduction, were compensated for by his consistent and implicit modeling of the general model in all instructional presentations. Within cognitive modeling all teachers seldom addressed monitoring strategies (asking questions of oneself) or repair strategies (knowing what to do when one is confused or an error is made), both critical components of students' metacognitive awareness as suggested by Brophy (1986). Teachers' instruction generally trained students to process problems thoughtfully, to identify relevant domain specific knowledge, skills, and procedures within a general model framework, and to develop schemata for particular problem types.

The other two classifications of teacher suggestions, lesson segment placement and oral format, lend themselves to common sensical notions of teaching practices. A teacher suggestion has greatest impact if given during the lesson development segment, lower impact if given during the homework review segment, and the least impact if given

during the seatwork segment. It is assumed that students are paying greater attention to the teacher's dialogue during lesson development, and less attention during the homework review segment since some of the students' attention is directed to correcting their work. The practice of teacher dialogue during the seatwork segment tends to interrupt and disturb student work progress, and is explicitly discouraged within Good, Grouws, and Ebmeier's (1983) instructional model. The high number (14) of such seatwork suggestions for Alice can be considered of minimal help for students. Betty gave an equal number of suggestions in the homework review and lesson development segments, Alice 50 percent more in the lesson development segment , and Carl twice as many in the lesson development segment.

While not able to validate with research, the investigator believes that a question format for a suggestion is a more valuable aid to student processing than a statement format. Research has shown that more effective teachers tend to use greater quantities of interrogative (question) formats than do less effective teachers (Evertson et al., 1980 and Friedman and Stomper, 1988). Presented as a statement, a suggestion does not encourage student response, or call forth any further processing or recall. A question, on the other hand, generally demands a response, reflection and processing,

and more actively involves the learner. Carl had the highest percentage (50 percent) of question formats as compared with Betty, 22.6 percent question formats, and Alice, 14.1 percent question formats. On both of these latter measures, Carl emerges as a "better" teacher or similar to more effective teachers due to his greater placement of suggestions in the lesson development segment and greater usage of question formats for his suggestions.

Instructional Examples

After exiting the classroom on days of teacher observations, the investigator reviewed the day's field notes for verbal problem related examples presented during the lesson development or controlled practice segments. For each such example presented, the teacher's process sequence was coded by numbers sequentially according to the process category labels established on Coding Form 1 (see Appendix B). To be categorized, a process had to be explicitly modeled or verbalized by the teacher. The related verbalizations or teacher actions were also noted next to the related process category in the description column provided. Examples were excluded from investigator coding which lacked detailed teacher treatment or when they were presented during homework review segments. Such exclusions were warranted since in the first instance, there was insufficient teacher input for investigator coding, and in the second instance, examples served more

of a reinforcement purpose as opposed to an instructional purpose.

This section's results related to instructional examples proceeds from general issues such as daily quantity and distribution of coded teacher examples to specific patterns of teacher process usage within example presentations. Analyses included quantity/distribution characteristics, teacher sources of examples, content focus of coded teacher examples, process content summaries, and patterns of teacher processing. For each example presented, the investigator noted the date, source, verbal problem content focus, and its sequential process coding. A hypothetical sequential process coding might be: 1A,H,I.2H.3B.4A (no check) which meant (refer to Appendix B): reads problem, identifies wanted/given/needed, defines variables, equation constructed relationally, algebraic procedures carried out, correctly interprets solution, and no explicit check. Source referred to the example's text location or whether it was teacher developed, while content noted to which one of the six verbal problem types (content core) or an "other" category (for percent types) an example was related.

Quantity/Distribution of Examples

A summary of the quantity and daily distribution of coded examples for each teacher is displayed in Table 5.9.

A summary of the quantity and daily distribution of coded examples for each teacher is displayed in Table 5.9. Its purpose is to provide a comparison of teachers' example presentation patterns.

Table 5.9

Coded Instructional Examples: Quantity/Distribution

Category	Alice	Betty	Carl
Total number coded examples	19	11	22
Number of days when:			
1 example coded	1	7	0
2 examples coded	4	2	3
3 examples coded	2	0	2
4 examples coded	1	0	0
5 examples coded	0	0	2
Total number of days	8	9	7
Number of coded examples per day (mean)	2.4	1.2	3.1

Table 5.9 indicates that, except for one day, Alice and Carl presented more than one verbal problem example per day in their observed instruction, whereas Betty generally presented a single example daily (7 of 9 observed days). On the average, Alice presented twice as many and Carl two and a half times as many examples per day as compared to Betty. The relatively low number (11) of coded examples for Betty was due partially to the low number of textbook provided instructional examples, and partially to her instructional pattern of treating verbal problem content and manipulative skill development within

minutes, with Alice averaging about 3 minutes per example, Betty 4 minutes per example, and Carl 5 minutes per example.

Source of Examples

The source of coded teacher examples reinforced the investigator's claim, alluded to in Chapter IV, that Carl was the more creative or original of the three teachers in his planned instruction of verbal problem content. Of the 22 coded examples observed in Carl's classroom, 12 were teacher developed and verbally presented to the class, and 10 were selected from the employed textbook. All of Alice's 19 examples and Betty's 11 examples were selected from textbook provided examples, samples, or exercises.

Content Focus of Examples

A summary of coded examples by content focus is displayed in Table 5.10. The "other" category was necessitated by investigator observations of Carl on two days in which percent type verbal problems (which did not fit the six defined core problem types) was the day's instructional focus.

Table 5.10

Coded Instructional Examples: Content Focus

Verbal Problem Type	Alice	Betty	Carl
1. Simple Translation	5	8	7
2. Geometric Type	0	1	0
3. Integer Type	4	0	0
4. Age Related	7	2	2
5. D/R/T Related	2	0	2
6. Coin/Money Related	1	0	3
"other"	-	-	8
Totals	19	11	22

A zero entry in Table 5.10 does not indicate that a teacher did not provide examples related to these problem types, but rather indicated that the investigator was not able to observe that teacher when such examples were presented. The investigator determined through teacher questioning that Alice presented two examples in one day of geometric type verbal problems and an additional two examples of coin/money related types. Betty presented one example on each of two days respectively for integer and coin/money related type verbal problems, and two examples in one day of D/R/T related verbal problem types. Carl presented three examples related to geometric type verbal problems and two examples of integer types during two separate days' instruction. In summary, students in all three classes attended to at least a minimum of one example of each problem type during class time. Student exposure to simple translation type examples, both in terms of quantity and instructional days, was generally

greater than for the other five example types, with the exception being Alice's extensive three day treatment of age related problem types. Betty presented students one example for three problem types and two examples for two verbal problem types. Alice and Carl presented a minimum of two examples for each of the five other core content verbal problem types besides simple translation types.

Sequential Coding Analysis

The remainder of this section focuses on the results of the investigator's sequential coding of instructional examples generated using Coding Form 1 and follow-up pattern analyses of these generated sequence codes. Initially summary counts were generated for each teacher's employed processes, either modeled actions or verbalizations, observed during example presentations. All observed teacher actions and verbalizations were codable using the list of processes developed for Coding Form 1. The investigator eliminated several processes which were listed on Coding Form 1 from the summary analysis to provide a better overall picture of actual teacher actions or processes. Processes labeled 2A, 2B, 2C, and 2E were not observed in a single teacher's example presentations, and hence were eliminated from further analysis. Certain processes in parts 3 and 4 of Coding Form 1 (Carrying out the Plan and Looking Back/ Interpretation stages) were not processes or actions at all, but rather represented a lack

of action or a terminal state. Processes labeled 3D, 3E, 3F, 4B, 4C were eliminated from further analysis for this reason. Refer to Appendix B for specific explication of each of these eliminated process categories.

The analysis of employed teacher processes within instructional examples is divided into two parts. First, processes within the first and second stages of Polya's model (Understanding the Problem and Devising a Plan) are summarized in Table 5.11, and secondly processes within the third and fourth stages of Polya's model (Carrying out the Plan and Looking Back) are summarized in Table 5.12. This separation permits better interpretation and teacher comparisons within the separate stages of Polya's model. A process category could be coded more than once in a single example, if a teacher returned to a particular process after an intervening process was observed and coded. Such multiple coding of the same process did occur within several examples, particularly for Alice.

Table 5.11

Coded Instructional Examples: Process Summary Stages 1 @ 2

	Alice	Betty	Carl
Number of Coded Examples	19	11	22
1. Stage: Understanding the Problem			
A. Reads Problem	18	11	14
B. Rereads for essentials	0	2	2
C. Restates prob. statement	1	0	6
D. Requests/states clarific.	6	0	1
E. Recalls related need	0	0	4
F. Exploratory manipulation	0	0	2
G. Diagram/chart constructed	8	2	9
H. Identifies want./giv./nd.	16	7	8
I. Defines variables	25	13	12
subtotal	74	35	58
2. Stage: Devising a Plan			
D. Chart construction	2	0	3
G. Equation constr. (trans.)	12	8	17
H. Equation constr. (relat.)	9	3	3
subtotal	23	11	23

Within the Undertanding the Problem Stage, Alice generally modeled or verbalized more processes per example than Betty or Carl did. The mean number of processes per example within this stage was Alice 3.9 processes, Betty 3.2 processes, and Carl 2.6 processes. All of Betty's examples and all but one of Alice's examples began with teacher or student reading of the problem to the class. Since 12 of Carl's examples were developed and presented orally to the class by Carl, explicit reading of the problem was less evident. Carl showed the greatest diversity of process usage within this stage, using all 9 listed processes at least once. Betty was the least

diversified teacher, and followed the most consistent process sequence in this stage, 1A,H,I. Alice and Betty were very conscious of closing this stage with a defining the variable process, and included this process in every coded example.

All teacher examples coded by the investigator reached a point where an equation was constructed either by use of a chart, translation processes, or relational thinking, or some combination of these processes as indicated within the Devising a Plan stage of Table 5.11. Both Alice and Carl tended to construct and use charts and diagrams to aid equation formulation in some instances, whereas Betty did not. All three teachers favored using translational processes for equation formulation over relational processes, though the proportions varied. In particular, the coding of Carl's examples as to equation construction, translational (2G) or relational (2H), appeared to contradict the patterns related to Carl's instructional routine and lesson development noted in Chapter IV. Carl's instructional routine and lesson development tended towards a relational emphasis rather than a translational focus. A closer review of the fieldnotes revealed that both positions coexisted in Carl's instruction. Initial teaching of verbal problem content was relationally oriented to develop the central relationships of the content section. These relationships

were then developed into formula formats, such as value coin type1 + value coin type2 = total value, or original price - discount = sale price. However, when examples were done for the class, Carl focused more on translating the problem statement's content into numeric values and variables, filling the slots of the developed formulas, and hence were coded translation oriented by the investigator.

Table 5.12

Coded Instructional Examples: Process Summary Stages 3 @ 4

	Alice	Betty	Carl
Number of coded examples	19	11	22
3. Stage: Carrying Out Plan			
A. Plan in 2 carried forth	4	0	4
B. Algo./arith. procedure use	4	7	16
C. Logical justific. provided	0	0	2
subtotal	8	7	22
4. Stage: Looking Back/Interp.			
A. Correctly interprets solu.	6	4	14
D(b). Check by: problem state.	3	2	9
D(c). Check by: equations	0	4	1
subtotal	9	10	24

The results of Table 5.12 reveal distinctly different teacher patterns of example treatment in the third and fourth stages of Polya's model. Carl completed the Carrying Out the Plan stage in all 22 of his coded examples. In other words, all Carl's 22 formulated equations were solved by some means during example presentations. Betty solved 7 of the 11 formulated

example equations. For the remaining four examples, Betty terminated discussion after the equation was formulated, and students were expected to complete or follow the textbook's completion of the example through the third and fourth stages. More than half (11 of 19) of Alice's example presentations were terminated after the examples' equation formulations were presented to the class. Once an equation was solved (end of Stage 3), all teachers explicitly interpreted and posted problem solutions for over half their equation-derived solutions.

Carl checked 10 of the 14 problem solutions, mainly by inserting the numerical solutions in the problem statement and validating the truth or falseness of the resulting statements. Betty checked all four problem solutions, but preferred to replace variables of the equation with solution values, and then verify the correctnss of the resulting equation statements. Alice checked problem solutions half the time (3 of 6), and used the problem statement to validate the solution in each case.

Certain emphases and sequential patterns were evident for each teacher's example presentations. Alice emphasized the Understanding the Problem stage the most in her example presentations. Her sequence through this stage was predictable. Generally, reading the problem was followed by identification of given quantities and needed

information, and then moved to the explicit defining of variables. When the textbook suggested the use of charts or diagrams to aid information organization, Alice included these processes in example presentations. Alice assumed students had mastered the equation solving, interpretation and checking processes, and gave less attention to these processes in example presentations.

Betty's example presentations conformed closely with the textbook's suggested steps for solving verbal problems. These steps were to read the problem, define the unknowns, set up the equation, solve the equation, and check the results. The only major process addition to this sequence was Betty's verbalizing and modeling the helpfulness of identifying wanted and given information of a problem statement, and that all problem information was usually used in the finding of the problem's solution. Betty terminated several examples once the equation was formulated, and neglected checking equations' solutions when time constraints dictated such action.

Carl's example presentations showed the greatest diversity of process usage in all four stages. Students were provided a broader exposure to the variety of possible solution paths and potential processes which might aid understanding the problem. He was also more likely to work an example to completion and model processes in all four stages. This consistent modeling of

the whole process sequence through the four stages, and for more examples might also explain why Carl's students received and required fewer explict suggestions related to the general model (see Table 5.3), since the modeling provided sufficient reinforcement of the general model used.

Sequential Coding Characteristics

The sequential coding of all 52 teacher presented examples is given in Appendix I. A small number of teacher process patterns are noted in this analysis. Alice and Betty followed the four stages sequentially (1 to 2 to 3 to 4), with no reversions back to earlier stages in all their example presentations. In four of Carl's 22 examples, there was a reversion back to an earlier stage process within an example's presentation.

Because stages labeled 2, 3, 4 generally included one or at most two distinct processes within each stage, patterns beyond those already discussed were limited. However, teachers exhibited sequential differences in their usage of the nine processes for Stage 1: Understanding the Problem. Only four different process sequences were observed for Betty in this stage, with 1A,H,I. the most common sequence (5 times in 11 examples). Alice followed eight different process sequence variations within this stage (greater diversity), yet the 1A,H,I. sequence was still the most common sequence (9 times in 19

examples). Carl had the greatest diversity of process sequences in this stage (15 different sequences) with 1A,H,I. and 1C. occurring most frequently (3 times each in 22 examples). Examination of sequential process patterns revealed the same patterns of used process diversity as did the analysis of the quantification characteristics of processes.

Summary

The purpose of this chapter was to describe and comparatively analyze instructional suggestions and example presentations for their ability to aid student metacognition and strategy training, which are believed important for developing cognitive skills, such as verbal problem solving. Results should be interpreted in light of the fact that this area of research still remains rather virgin territory for research, and the study's nature in this area exploratory. A weakness of the study was a lack of inter-coder reliability measures related to suggestion classifications and process coding, since the investigator alone completed all classifications and process coding.

Based on common sensical notions of "good" teaching practices, Carl was rated a somewhat more effective teacher in his delivery of oral suggestions. His greater usage of question formats and greater placement of suggestions in the lesson development segment led to this

conclusion.

The coding of teacher processes employed in the presentation of examples was shown to be a feasible objective for research study, and the process list (Coding Form 1) devised for the study worked well in assisting the process coding of teacher examples. Within example presentations, several patterns were revealed in regards to teacher process diversity, teacher attention to the four stage general model, and teacher termination decisions.

The results of the various student performance and attitude measures are discussed next in Chapter VI.

CHAPTER VI

Student Performance Results

This chapter focuses on student performance results and related data analysis on the four investigator designed instruments for the study. Each student participating in the study completed a pretest, a posttest, and a retention test. In addition nine students, three from each class, participated in an individual interview with the investigator, and completed an instrument designed for the interviews. The analysis focused on the main questions of the study listed in Chapter I, and the questions which emerged in Chapter IV which related to the textbook and instructional analyses.

The chapter has four major sections related to: pretest results, posttest results, retention test results, and interview results. Primary emphasis within the chapter is given to the display and discussion of summary results of each classroom's performance. Individual student results and details of employed statistical techniques can generally be found in various appendices.

Pretest Results

A pretest form (Appendix E) composed of six verbal problems was administered in the three classrooms during the first two weeks of school (September, 1986). There were three problems on each of 2 pages. The problem types of the verbal problems were: geometric oriented (#1 and #2), consecutive integer (#3), money oriented (#4), age related (#5) and distance/rate/time oriented (#6). Students were instructed to use any method, including guessing, to find the solution to each problem. Since no algebraic techniques had yet been taught, it was assumed most students would use guess and check strategies and intuitive logic to solve the problems. Generally this was the case based on investigator review of student work samples. Zero, one or two points were awarded per problem dependent upon student provided answers to each verbal problem. Two points were awarded for a completely correct answer to all parts of a question. One point was awarded if one part of a multiple part answer was correct, and zero was awarded if no response was given or answer was totally incorrect. In general student scores on the pretest were low as expected, and are displayed in Table 6.1.

The main purpose for using a pretest was to check the assumption that the three classrooms of students in the three different schools (and three different algebra

programs) had similar previous experience and ability related to verbal problem content at the outset of the study. Numbers in parentheses indicate the number of students attaining particular scores with twelve being the maximum possible score.

Table 6.1

Pretest Results: Classroom Comparisons

	Classroom			
	Alice	Betty	Carl	Total Group
Number of Students	18	27	22	67
High Score	11(1)	11(2)	11(1)	11
Low Score	0(2)	0(4)	0(1)	0
Median Score	3(4)	4(2)	6(2)	4
Mean Score	3.67	4.48	5.59	4.63
Variance	6.94	12.18	7.97	9.69

A one way analysis of variance (ANOVA) was done to determine if classroom differences were statistically significant (null hypothesis: $u_a = u_b = u_c$, $\alpha = .05$). The summary ANOVA table is shown in Table 6.2.

Table 6.2

Classroom Pretest Differences: ANOVA Table

Source of Variability	SS	df	MS	F
Between Classrooms	37.62	2	18.81	2.00
Within Classrooms	602.05	64	9.41	
Total Variation	639.67	66		

Classroom performance differences were not statistically significant on the pretest instrument ($F_{2,60,.05} = 3.15$), indicating that prior to instruction students' backgrounds and verbal problem solving abilities were not significantly dissimilar. No item analysis was performed on the pretest results since the scores were quite low, and such an analysis was not related to the study's questions. Individual student results for the pretest are listed in Appendix J by classroom teacher.

Posttest Results

The posttest was administered to the three classes of students within two weeks of each teacher's completion of the identified core verbal problem content. The three groups completed the posttest during three different weeks of December, 1986, in the order of Alice's class, Betty's class, and lastly Carl's class.

Two posttest forms, A and B, were developed, and each class of students completed equal proportions of each form. Entire copies of the two posttest forms can be found in Appendix F. The forty-five minutes allocated for test administration was sufficient for all students to complete the posttest. Table 6.3 displays a summary of problem characteristics related to the posttest items.

Table 6.3

Problem Characteristics of Posttest Instrument

Problem Number	Content Type	Four-way Classification	Forms A/B variations	Format presented
1.	Simple Translation	Abstract/Factual	Same	Oral
2.	Geometric Type	Abstract/Factual	Same	Oral
3.	Money/Coin Related	Concrete/Factual	Different (Decimal Data)	Written
4.	D/R/T Related	Concrete/ Hypothetical	Different (Decimal Data)	Written
5.	Age Related	Concrete/ Hypothetical	Same	Written
6.	Integer Type	Abstract/ Hypothetical	Different (No Solution)	Written
7.	Geometric Type	Abstract/Factual	Different (No Solution)	Written
8.	Money/Coin Related	Concrete/Factual	Different (Statement Sequence)	Written
9.	Age Related	Concrete/ Hypothetical	Different (names used)	Written
10.	D/R/T Related	Multi-step Concrete/ Hypothetical	Different (role reversal of 2 names)	Written

Posttest items numbered 1, 2, and 5 were identical on both forms of the posttest, with numbers 1 and 2 read orally four times by the investigator in each classroom. For items numbered 3 and 4, the provided data contained one entry on each form which was not a whole number in its written form ($7.50 in problem 3, Form A and 1.8 kph in

problem 4, Form B). The content was otherwise similar for these two items. For items numbered 6 and 7, the content was nearly identical, but the inclusion of different numbers as data for the verbal problems resulted in one problem of each form (#7, Form A and #6, Form B) which had no solution. Problem #8 contained identical data on both forms, but the ordering of the first two sentences of the problem statement was reversed on the two forms. Verbal problems #9 and #10 were identically worded, except the first names used on the two forms were different.

The first nine problems were designed by the investigator to closely resemble the verbal problem types students had encountered in their classroom experiences, and were typical of textbook verbal problems. Problem number 10, being multi-step, represented an investigator attempt to evaluate whether students could go beyond textbook exercises in their problem solving abilities. Since only two or three students per class showed any measurable progress on this item, results related to this problem were not included in the posttest analysis.

The scoring of student work on the posttest instrument was based on investigator awarding of zero, one, or two points in each of five distinct phases of problem progress and completion. Each problem then had a possible score of 10 points. The employed scoring system is detailed for each of the five phases in Table 6.4.

Table 6.4

Scoring Technique for Posttest Items

Phase 1: Variable Definitions
- Score 0 if no defining attempted or more than one error in variable definitions.
- Score 1 if exactly one error made in variable definition.
- Score 2 if explicit, written variable definitions correct.

Phase 2: Equation Formulation
- Score 0 if no equation formulated or error in connected relationships.
- Score 1 if correctly formulated relationship for incorrect variable definition.
- Score 2 if written equation formulation correct.

Phase 3: Equation Solving
- Score 0 if error with any formulated equation noted or processing discontinued.
- Score 1 if correctly solved an erroneously formulated equation.
- Score 2 if correctly solved a correctly formulated equation.

Phase 4: Interpretation/ Checking Process
- Score 0 if error in interpreting equation result or termination with no answer given.
- Score 1 if interpretation of result done correctly with variable definitions, but answer not correct and check not performed, which might indicate error.
- Score 2 if interpretation of result done correctly with variable definitions, and correct answers result (no check required).

Phase 5: Correct Answers
- Score 0 if incorrect answers to any of questioned parts of the problem.
- Score 1 if correct answers given, yet lack of work to indicate nature of derivation.
- Score 2 if all requested answers correct and work present to indicate derivation.

For each student, counts were also noted of student attempts at variable definitions for each verbal problem.

While the above scoring system might appear complex, it was found to be a workable system for analyzing student posttest work samples. This scoring system was also used to score the problem solving section of the retention test and for scoring the interview instrument. With item number 10 excluded from the scoring, the highest attainable score was 90 on the posttest instrument.

Individual student posttest scores, listed by classroom teacher, appear in Appendix J. A summary of the posttest results for each classroom and for the total group of all 67 students in the study is displayed in Table 6.5. Numbers in parentheses refer to student counts of those particular scores.

Table 6.5

Posttest Results: Summary of Classroom Performance

	Alice	Betty	Carl	Total Group
Number of students	18	27	22	67
High Score	42(1)	51(1)	67(1)	67
Low Score	14(1)	0(3)	9(1)	0
Median Score	27.5	15	38	25
Mean Score (Percent Equivalent)	27.72 (30.80)	16.85 (18.72)	37.32 (41.46)	26.49 (29.43)
Variance	46.21	193.44	227.08	237.86

A one way analysis of variance was done to determine if classroom differences were statistically significant.

Between classroom differences were significant ($F = 15.92$, as opposed to $F_{2,60,.01} = 4.98$) at an α level = .01. A two tail t-test between Alice and Carl's classes showed significant differences at a .05 level, but not at a .01 level. The variance for Alice's class was considerably lower, indicating her students' posttest scores were more closely grouped together. In fact 14 of her 18 students scored between 20 and 31 on the posttest. Carl's class performed noticeably better than the other classes, while Alice's class outperformed Betty's class.

Differences in student performance related to which form (A or B) was completed are shown in Table 6.6. Mean results are given for each form by teacher and for the total group. Analysis indicated very little difference between the two forms.

Table 6.6

Posttest Results: Form Performance Comparison

Form	Alice	Betty	Carl	Total Group
A	27.78	16.86	37.91	26.56
B	27.67	16.85	36.73	26.42

A closer look at the performance for the individual items which more greatly differentiated the two forms (items #3,4,6,7) for each classroom resulted in only one instance where a significant difference (α = .01) was noted, that being Carl's class performance related to item

number 6 (consecutive integer problem).

Differing investigator design of items 6 and 7 on the two forms was done to determine if verbal problems not having a solution had a negative influence on student verbal problem solving performance. By pairing student performance on the two, no solution problems (6B and 7A) of the posttest, and comparing that performance with that on the two, one solution problems (6A and 7B) of the posttest forms, an indication of this influence was tested. While student performance on the two, one solution verbal problems was better (mean = 3.34) as compared to performance on the two, no solution verbal problems (mean = 2.52), the difference was not statistically significant (α = .05).

Similarly, data variations in the two forms related to questions 3 and 4 were designed to determine if the inclusion of decimal form data (one entry) hindered student performance in solving verbal problems. Again combining results for paired items 3A and 4B (decimal form data) and items 3B and 4A (whole number data) was completed and the results compared. Although student results favored whole number data (mean = 2.66) over decimal form data (mean = 2.09), the difference was not statistically significant (α = .05).

Consideration was given to comparing oral and written format presentations on student performance, which was not

Consideration was given to comparing oral and written format presentations on student performance, which was not part of the posttest design, and was not possible since none of the seven written format problems was similar enough in problem type or level to perform such a comparison.

An item analysis was completed comparing classroom performance related to specific problem types and classifications of verbal problems. Table 6.7 displays the comparison of student performance in the three classrooms on the nine scored items of the posttest.

Table 6.7

Posttest Results: Class Performance by Test Item

Problem Number	Mean Group	Mean Alice	Mean Betty	Mean Carl	ANOVA Level
1.	6.57	7.50	4.11	8.82	$p < .01$
2.	3.04	4.39	1.67	3.64	$p < .01$
3.	0.87	0.22	1.15	1.05	NS
4.	3.88	2.28	3.48	5.68	$p < .05$
5.	1.82	1.61	0.85	3.18	$p < .01$
6.	3.16	3.61	0.26	6.36	$p < .01$
7.	2.70	3.00	1.30	4.18	$p < .01$
8.	2.94	2.28	3.30	3.05	NS
9.	1.45	2.61	0.74	1.36	$p < .01$

Carl's class outperformed the other two classes on five items (numbers 1,4,5,6,7), and did not have the

and lowest on three items (numbers 3,4,8). Betty's class performed highest on two items (numbers 3 and 8), and lowest on six items (numbers 1,2,5,6,7,9). Betty's highest class performance on items 3 and 8 is noteworthy since both verbal problems were coin/money related, and classroom differences were not significant only for these two items. Most of the appreciable differences between classrooms can be attributed to the exceedingly lower scores of Betty's class on most items. Differences in performance between Alice and Carl's classes were only significant for two items (numbers 5 and 6) at a $\alpha = 0.01$ level of significance.

The individual posttest items were classified into the six verbal problem types which composed the core curriculum for the study. Total group performance on the six verbal problem types was used to order the information for Table 6.8, which displays the classroom performance for each of the six verbal problem types.

Table 6.8

Posttest Results: Class Comparison by Problem Types

Content Type	Item Number	Mean Group	Mean Alice	Mean Betty	Mean Carl	ANOVA Level
Simple Translation	1	6.57	7.50	4.11	8.82	$p < .01$
D/R/T Related	4	3.88	2.28	3.48	5.68	$p < .05$
Integer Type	6	3.16	3.61	0.26	6.36	$p < .01$
Geometric Types	2,7	2.87	3.70	1.49	3.91	$p < .01$
Coin/Money	3,8	1.91	1.25	2.23	2.05	NS
Age related	5,9	1.64	2.11	0.80	2.27	$p < .01$

Classroom performance differences based on problem type were significant for five of the six problem types. Only for coin/money related problem types were differences not significant. While in Alice and Carl's classes coin/money related problem types represented the lowest performance of any problem type, Betty's class showed this problem type as a relative strength. The simple translation problem type was the least difficult verbal problem type for all three classes, and the age related problem types had lower results in all three classes. Relative secondary strengths differed appreciably for the three classes of students. Carl's students solved integer types and D/R/T related types at a higher level of success. Alice's students had greater success on

geometric and integer types. Betty's students did more favorably on the D/R/T related problem type.

The other verbal problem classification system developed for the study in Chapter IV involved formats of verbal problem statements into abstract/concrete and factual/hypothetical distinctions. Student posttest results were analyzed within this resulting four way classification to compare classroom performances for significant differences, and to compare student results with the speculations resulting from the textbook analysis developed in Chapter IV. The summary of classroom posttest results for the four format classifications is displayed in Table 6.9.

Table 6.9

Posttest Results: Format Classifications Comparison

Format	Related Items	Mean Group	Mean Alice	Mean Betty	Mean Carl	ANOVA Level
Abstract/ Factual	1,2,7	4.10	4.96	2.36	5.55	$p < .01$
Abstract/ Hypothetical	6	3.16	3.61	0.26	6.36	$p < .01$
Concrete/ Factual	3,8	1.91	1.25	2.23	2.05	NS
Concrete/ Hypothetical	4,5,9	2.38	2.17	1.69	3.41	$p < .01$

A one way analysis of variance for the group means of the four format types revealed no significant difference ($F = 2.53$). The individual class results and the total group mean scores of the four format types were used to

construct tables for analyzing the degree of problem difficulty for the four formats by classroom, similar to that done in Chapter IV (Tables 4.6 and 4.7). The results are shown in Table 6.10, together with the hypothesized results generated in Chapter IV (Table 4.7) for comparison purposes. The four format types are listed in abbreviated form: AF = Abstract/Factual, AH = Abstract/Hypothetical, CF = Concrete/Factual, and CH = Concrete/Hypothetical.

Table 6.10

Verbal Problem Difficulty: Posttest Format Comparisons

	Least Difficult	Intermediate Difficulty	Most Difficult
Posttest - Group	AF	AH	CH - CF
Alice - Hypothesized	CF	AF - CH	AH
Alice - Posttest	AF	AH	CH - CF
Betty - Hypothesized	CF	AF	CH - AH
Betty - Posttest	AF - CF	CH	AH
Carl - Hypothesized	AF	CF - CH	AH
Carl - Posttest	AH - AF	CH	CF

Of the three perspectives of problem solving competence outlined by Caldwell and Goldin (1987), the total group's performance and Alice and Carl's classes' performance most closely resembled the direct translation perspective. The sequence of problem difficulty for this perspective (direct translation) from least to most difficult was AF, AH, CF, CH (see Table 4.6). Betty's sequence of problem difficulty, based on the posttest

results, more closely resembled the developmental perspective, which had a sequence of CF, CH, AF, and AH.

The investigator also developed sequences of hypothetical performance based on textbook exercise exposure counts for the four format classifications (see Table 4.7). These hypothesized sequences were reasonably accurate for Betty, not as accurate for Carl, and almost completely reversed for Alice. In comparing the total group's performance results with those of Caldwell and Goldin's (1987) elementary and secondary studies, one finds a complete reversal of problem difficulty, with abstract types preferred over concrete types, opposing their results. Performance of the total group of students on an abstract item had a mean of 3.87 as compared to a mean performance of 2.19 for a concrete item. This difference between abstract and concrete item performance was statistically significant at an α level of .01 ($t = 4.79$).

In analyzing student performance on the posttest items, the investigator was particularly interested in which phases of the general procedural model student processing errors were most likely to occur, and the relative success probabilities associated with each phase. By phases the investigator is referring to the substeps of the general problem solving model: variable definitions, equation formulation, equation solving,

interpreting/checking results, and denoting the correct problem solution. It should be noted that these five phases were the central features around which the scoring system for the posttest was derived.

This part of the posttest analysis of student results was done in two different ways. For one method, counts were done where students were awarded two points in a phase (indicating complete correctness) for each item, and then these counts were used to generate proportions (conditional probabilities) for the degree of progress for each phase. The second method involved the totaling of process scores (by class) awarded in each phase category, determining means for each class and phase, and formulating proportions of the total possible earned score for each phase category. Both methods, in different ways, are indications of the apparent strengths and weaknesses demonstrated in student processing of verbal problems.

The employed methods hereafter will be referred to as Method I (conditional proportions) and Method II (process score proportions). In Method I the two phase categories, interpreting/checking and correct solution, were collapsed into one phase, since correct interpretation of the solution for an equation usually leads to a correct answer(s). In this method, the investigator also judged whether a student's work sample indicated an attempt to define variables. This addition permitted a more accurate

appraisal of student abilities to correctly define the variables for a problem, and also a measure of student incorporation of the advocated general, procedural model of processing, since the first written step of the model is variable definitions.

The summary tables which follow were derived from six individual classroom tables (3 for each method) displayed in Appendix K. The first three tables of Appendix K (organized by classroom) indicate counts for each of the nine items of cases where students earned two points in a phase. The second three tables in Appendix K indicate process scores for each item in the five phases, by classroom.

In Method I, proportions are computed based on the number of students who successfully completed a phase, given that they had already successfully completed the immediately preceding phase. For instance, the proportion of students who had correctly formulated the equation for a problem, given that a correct variable definition had been made. Table 6.11 displays these proportions for the various phases for each class, and for the total group of students in the study.

Table 6.11

Posttest Results: Phase Success Proportions (Method I)

Phase Description	Total Group	Alice	Betty	Carl
Variable Definition Attempted	.73	.79	.56	.87
Correct Variable Definition	.54	.58	.50	.54
Correct Equation Formulation	.54	.41	.49	.69
Correct Equation Solving	.83	.77	.70	.94
Correct Interpretation and Solution	.84	.74	.91	.85
Probability of Correct Completion Sequence	.148	.107	.087	.259

The attempt at variable definition category indicated how well the general procedural model (Polya's model) had been incorporated into student processing. For Alice and Carl's students, the model appeared well ingrained in student thinking, as attempts at variable definition for a problem occurred almost 80 percent of the time. A further indication of the degree of this ingrainedness was that all 18 of Alice's students and 21 of Carl's 22 students attempted variable definitions for five or more of the nine verbal problems. The one student exception in Carl's class was noticed by the investigator to quit working on the posttest after the fifth problem, at which time four of the five problems tried indicated an attempt at

variable definition. On the other hand, ten of Betty's 27 students failed to meet this criteria of five attempts at variable definitions for the nine problems, indicating less incorporation of the suggested general, procedural model.

Student abilities to correctly define the variables for a problem and correctly formulate the problem's equation were equivalent when the total group was considered, indicating these two phases posed the greatest difficulty in student processing. While the difference between classes is small for the variable definition process, Carl's class showed a noticeable advantage in their ability to formulate the correct equations. Alice's class showed a precipitous drop in this same ability, while Betty's class remained at a 50 percent level of ability. In all classes, there is marked improvement in student abilities to correctly solve equations, and in interpreting the results into correct solutions. Particularly notable is Carl's classroom obtaining a 94 percent success rate in solving equations.

The results indicate that remedial instructional efforts in Alice's class should focus on improving student abilities to formulate equations, and in Carl's class on improving student abilities to correctly define the problems' variables. In Betty's class initial instructional efforts need to refocus on the centrality of

following a general procedural model, followed by instructional efforts to improve specific variable definition and equation formulation skills.

The last row of entries (probability of completing sequence) of Table 6.11 was computed by multiplying the five entries above in each column, as these proportions were considered conditional probabilities for each phase. These computed sequence probabilities indicate that a student selected from Alice or Betty's class had about a 1 in 10 chance to complete the entire sequence correctly, while a student in Carl's class had about a 1 in 4 chance of successful completion of the sequence.

The second method of analysis (Method II) of student performance within the various phases used the process scores awarded by the investigator for each students' work sample. Proportions for each phase and class were determined by dividing the classes' mean process score for a phase category by the possible score 1(18 in all 5 phases, 9 problems x 2 points each) for a phase category. The proportion of attempted variable definitions is found by dividing a class's mean number of attempts by nine (the possible number of attempts). The resulting proportions are displayed in Table 6.12 for each class and the total group, and represent class success rates for each phase of student work.

Table 6.12

Posttest Results: Phase Success Proportions (Method II)

Phase Description	Total Group	Alice	Betty	Carl
Attempt Variable Definition	.72	.78	.57	.87
Variable Definition	.45	.53	.34	.52
Equation Formulation	.29	.25	.18	.44
Interpretation/Check	.25	.27	.15	.37
Correct Answer(s)	.18	.16	.11	.28
Overall Performance	.29	.31	.19	.42

With the exception of the variable definition phase, in which Carl and Alice's students were equally successful, Carl's students maintain a performance superiority over Alice's students for all subsequent process phases. The proportion rates for the equation formulation phase fall more dramatically for Alice and Betty's students than for Carl's students. About 40 percent of Carl's students, 30 percent of Alice's students, and less than 20 percent of Betty's students were able to successfully navigate through the procedural phases.

A detailed item analysis was performed on the nine items of the posttest. A summary of related information for each item can be found in Appendix L. The investigator listed separate comments for each of the nine

items of the posttest regarding patterns of student performance.

Item #1: Verbal problem was presented orally by the investigator. Highest student performance in all 3 classes for this simple translation type. Even here on this simplest of verbal problems, 11 of Betty's 27 students did not attempt variable definitions. Notable also is Carl's students better performance once the variables were correctly defined.

Item #2: Problem was presented orally by the investigator. All 3 classes of students were unsuccessful in variable definitions (overall success rate: 35%). Since 28 of the 39 related errors used n,2n,2n-4 rather than n,2n,n-4, this may be due to the oral presentation. All 9 of Betty's students equation formulation errors were due to the lack of awareness of the conceptual relationship needed (180 degrees = sum of angles).

Item #3: Lowest student performance item. Students' inability to even attempt variable definitions indicated total unfamiliarity with this problem type. There appeared a lack of conceptual understanding of wage computation, and that 24 hours total implies variable definitions of n and 24-n for number of hours.

Item #4: Familiarity with this D/R/T related problem obvious in number of attempts at variable definitions (80% used a chart). Large number of errors in variable definitions (18) showed just poor translation skills, particularly for Betty and Carl's students. Errors related to equation formulation (20) showed an inability to conceptualize the real situation and/or the distance, rate, time relationship.

Item #5: This age related problem had no solution. Only 4 students realized there was no solution (all from Carl's class). Students, particularly in Alice and Betty's classes tried to use the "twice" information in the variable definition, rather than for equation formulation. For 8 of the 16 equation formulation errors, students reversed the provided data information, so that a positive numerical age would result from equation solving, a finding in common with Paige and Simon's (1966) work on nonsolvable algebra verbal problems.

Item #6: This consecutive integer problem was totally unfamiliar to Betty's students. This was primarily due to the minimum instructional time given to this problem type by Betty and the textbook. Equation formulation errors for Alice and Carl's students indicated poor translation skills in writing the equation as stated in the problem statement, from left to right, or plain oversight of certain problem information (2 times).

Item #7: This geometric problem type involved the concept of perimeter of a rectangle. A large number of students in all 3 classes (20) failed to read the problem statement carefully enough to correctly translate variable definitions. About half the students had an incorrect concept of perimeter here, though it is possible that the no solution version of Form A caused some students to use the incorrect relation: length + width = perimeter, which at least resulted in a positive length and width.

Item #8: Betty's students did comparatively well on this coin related item, while Alice's class did relatively poor. Again there is evidence that direct translation skills needed for variable definition are relatively weak in all 3 classes. Alice's students were particularly weak in formulating the equation, in most cases forgetting to include values for the coins. This shows a conceptual misunderstanding of the basic notion of equation.

Item #9: This age related, B level problem proved extremely difficult for all 3 classes, particularly the variable definitions. Most students (44 of the 46) who attempted variable definitions used the first sentence of the problem statement to define variables which more normally would be expected to be used for equation formulation. Of the 44 attempts this way, 39 resulted in errors. Only 2 students correctly defined the variables as n, 54-n as was expected by the investigator.

Summary Comments

With the exception of item number 3 (the wage related verbal problem), students generally were able to relate to the verbal problem types of the posttest problems, recognizing them as similar to previous verbal problem work in class. This problem recognition was evidenced by the number of students able to attempt variable definitions for the given problem statements, and was notably higher for Alice and Carl's students. Only about 60 percent of Betty's students regularly attempted variable definitions for the problems, as compared to about 85 percent of Alice and Carl's students.

Particularly for the integer type problem (item #6), Betty's students lacked almost total recognition of its content type, as only four of 27 students even attempted variable definitions.

Student recognition of problem types generally lead to student processing and attempted definition of variables for the problems. Two types of knowledge appear requisite for variable definition and equation formulation to operate correctly. One type of knowledge is a translating ability from written English words and phrases into algebraic symbolization, while the other type of knowledge demands recall of stored concepts and relating these into applied relationships. Generally translation skills serve as useful tools in variable definitions, and conceptual applications are more useful for the equation formulations. Exceptions to this general pattern for the posttest items include the age related problems (items #5 and 9), where conceptual application skills are not required in either phase, and the integer type problem (item #6) where the pattern of usage of these two types of knowledge is reversed. Momentarily excluding item number 1 from consideration, due to its minimal necessity of translation skills (one number 14 more than another) and the simple concept (sum) required, the investigator reviewed student errors made on the rest of the posttest items more carefully. For problems where translation

skills were requisite in either phase, 70.6 percent (168 of 238) of the noted errors were due to incorrect translation of the problem statement. For problems where conceptual applications were necessary in either phase, 74.6% (50 of 67) of the errors made were due to either not knowing the key relationship or to an improper application of the key relationship. It appears that neither type of knowledge is adequately developed in student's processing to enable students to navigate their way through the variable definition and equation formulation phases.

Retention Test Results

The investigator designed a more comprehensive instrument for the retention test which included a questionnaire/survey section, and an equation solving section, besides a verbal problem solving section. A copy of the entire retention test form can be found in Appendix F. The same retention test was administered to all three classes of students during the same week (March 9 - 13, 1987), and during normal class time. It was necessary to administer the test for both of Betty's Algebra I sections since students of the original class had been intermixed with students of her other section at the beginning of the second semester (January, 1987). Of the 67 students who completed the posttest, 66 students also took the retention test. The one exception was one of Betty's students who was demoted to a prealgebra class at the

beginning of the second semester. The elapsed time between posttest and retention test administrations varied from 10 weeks (Carl's class) to 12 weeks for Alice's class.

All three teachers felt compelled to inform the investigator at the time of the retention test administration that very little verbal problem content had been covered during the intervening time after the posttest (which the investigator was already aware of from the textbook analysis), and though such information was not requested by the investigator.

Questionnaire/Survey Section

This section was composed of seven items. The first item required students to rank order a list of five provided concerns that the investigator believed students could understand and rate for their personal importance. The next five items used a multiple choice format, and the last item was open-ended, asking students to respond with a sentence or paragraph to a question. Since a majority of students left the last survey question blank, and hence little information was gained from this item, results for this item were excluded from the analysis. Two of the multiple choice items (numbers 2 and 4) related to students' perceptions of their ability related to algebra content and verbal problems, respectively, and two of these items (numbers 3 and 5) related to attitude

perceptions towards the algebra class and verbal problems respectively. The last multiple choice item (number 6) asked students to evaluate the perceived usefulness of their algebra class for their future high school program.

For the first item, the rank ordering of the five concerns, the investigator initially computed the mean class rankings for each listed concern and tallied counts of extreme positions (1 or 5) for each listed concern by class and for the total group of 66 students. Results are posted in Table 6.13. It should be noted that a lower mean ranking for a concern indicates greater relative importance for that concern.

Table 6.13

Retention Test Results: Survey - Item #1

	Alice			Betty			Carl			Group		
Concern	Mean	1	5	Mean	1	5	Mean	1	5	Mean	1	5
Pass Course	2.83	7	6	2.50	9	5	3.05	6	7	2.77	22	18
Good Grade	2.56	4	2	2.35	7	0	2.00	8	0	2.29	19	2
Daily Work	2.83	3	1	2.27	8	0	2.68	3	2	2.56	14	3
Solid Base	3.61	3	8	3.69	1	8	3.50	3	5	3.61	7	21
Gain Confidence	3.17	1	1	4.19	1	13	3.77	2	8	3.77	4	22

The present concern of greatest importance to Alice and Carl's students, and for the total group was getting a good grade in the course. For Betty's students almost equal importance was given to the first three concerns.

The large number of extreme rankings for the first concern, pass the course, was expected in that students can most readily evaluate their present position for this concern, and hence responded based on their perceptions of present classroom performance.

One might consider a dichotomy where the first three concerns relate to a student who is present/concrete oriented, and the last two concerns relate to a student who is future/depth oriented. For the total group of students, and in each class, about 80 percent of these students would be classified as present/concrete oriented and the other 20 percent future/depth oriented, or in a ratio of 4 : 1.

Patterns of complete rankings were limited in nature. Eight patterns appeared more than twice and accounted for 45 percent of student rankings (30 of 66). The most any patterned ranking occurred was five times, which happened for two patterns. The most any patterned ranking occurred in one class was three times.

Items numbered 2, 3, 4 were similar in nature, each having five selections from which to choose, and the five choices (a,b,c,d,e) represented a continuum from high ability or very positive attitude to very poor ability or very negative attitude. For all three items, choice c represented average ability or a neutral attitude. Due to these similarities, a numerical scale was constructed

(a = 5, b = 4, c = 3, d = 2, e = 1) which permitted more direct comparison of item responses, and allowed computations of correlations among the items. Table 6.14 displays the results for item number 2 of the survey for the three classes and the total group of students. The item asked the students to circle the choice which best described their ability in the algebra class.

Table 6.14

Retention Test Results: Survey - Item #2

Number of Student Responses	Alice	Betty	Carl	Group
a) very confident (5)	3	1	8	12
b) slight confident (4)	3	6	2	11
c) about average (3)	9	12	6	27
d) below average (2)	3	6	5	14
e) very weak skills (1)	0	1	1	2
mean item score	3.33	3.00	3.50	3.26

Table 6.14 indicates that in Alice and Betty's classes, about half the students perceived themselves to be average in algebra ability, and few students in either class marked an extreme position (5 or 1). In marked contrast, eight of Carl's students expressed a high level of confidence in their abilities. For Betty's class the frequency counts were symmetric about an average ability rating, and the mean is also at the average (3.00). Students in Alice and Carl's classes both rated themselves somewhat above average.

Similar summation tables were constructed for items numbered 3 and 4, and the results are displayed as Table 6.15 and Table 6.16 respectively. Item number 3 asked students to circle the choice which best described their attitude towards the algebra class, and item number 4 asked the same question as item number 2 for their ability with verbal problems.

Table 6.15

Retention Test Results: Survey - Item #3

Number of Student Responses	Alice	Betty	Carl	Group
a) high enjoyment (5)	4	1	4	9
b) positive experience (4)	4	7	8	19
c) neutral feeling (3)	7	11	8	26
d) dislike class (2)	3	5	2	10
e) hate attending (1)	0	2	0	2
mean item score	3.50	3.00	3.63	3.35

Table 6.16

Retention Test Results: Survey - Item #4

Number of Student Responses	Alice	Betty	Carl	Group
a) excellent ability (5)	0	0	0	0
b) above average ability (4)	3	2	2	7
c) average ability (3)	11	13	15	39
d) below average ability (2)	2	6	3	11
e) very poor ability (1)	2	5	2	9
mean item score	2.83	2.46	2.77	2.67

There is a drop in student perception of verbal problem ability as compared to general algebra ability, to

a slightly below average rating. This drop is rather consistent for all three classes, from .50 for Alice's class to .73 for Carl's class. There were also no students in the study who considered themselves "excellent ability" verbal problem solvers, despite 12 students who indicated they considered themselves very confident of their algebra abilities. Only 10 percent of the students even considered themselves as "above average ability" with regards to verbal problems.

Students' attitudes towards the algebra classes showed minimum differences with the ratings of algebra ability, being only slightly higher ratings, or the same rating (Betty's class). This last point dictated further examination of the relationships between student responses to these three items (numbers 2, 3, and 4), particularly the degree of consistency of response shown. Correlation coefficients were computed between items for the group and each class separately, and variations, and the direction of variation was noted for each student. The results are shown in Table 6.17.

Table 6.17

Retention Test Results: Survey - Items 2,3, and 4

	Alice	Betty	Carl	Group
Items 2 and 3				
Number of students which: same ability and attitude	8	17	8	33
algebra ability higher	5	5	6	16
algebra ability lower	5	4	8	17
correlation between items	.41	.73	.51	.57
Items 2 and 4				
Number of students which: same ability ratings	10	11	8	37
algebra ability higher	8	14	11	33
algebra ability lower	0	1	3	4
correlation between items	.78	.74	.40	.61
Items 3 and 4				
Number of students which: ability and attitude same	7	11	8	23
verbal problem ability higher	1	2	1	4
verbal problem ability lower	10	13	16	39
correlation between items	.56	.59	.57	.59

The computed correlation for each pair of items showed a consistency with regards to the total group of students for the study, with each correlation approximately equal to .60. Yet despite this consistency of the total group, only for items 3 and 4 (general class attitude and verbal problem solving ability) did this consistency maintain itself across the three classes.

Hence general attitudes and verbal problem solving ability ratings are more stable across classes. For Alice's class the greatest linear relationship (.78) existed between the two ability level ratings (algebra and verbal problem solving). Both correlations, related to attitude, were lower in her class than in the other two classes, and this indicated that general class attitude had less influence on student ability ratings, and vice versa. It also indicated that Alice's class is more homogeneous in regards to verbal problem solving abilities, a fact noted in the posttest results where Alice's class had a lower variance (46.21, as compared to 193.44 and 227.08).

For Betty's class, there was a stronger linear relationship between class attitude and general algebra abilities than in the other two classes. A similar moderate correlation was found "between" the ratings for the two abilities (general algebra and verbal problems). Her students appeared aware that the two abilities complement each other or were related. Yet student attitude was more influenced by perceived general algebra ability than by verbal problem solving ability, which contrasted her group from the other two classes.

The relatively lower correlation between the two ability level ratings for Carl's class, in marked contrast to the other two classes, indicated that his students perceived these two skill areas as more isolated, or less

related. Some of this perceived skill isolation may be due to the employed textbook's organizational practice which treated about 75 percent of the verbal problem sections in a single unit or chapter. In the other two classes, verbal problem content was more evenly distributed throughout the employed textbooks.

Item number 5 of the survey asked students to select a response from the six choices which best described their attitude towards doing algebra verbal problems. The actual choices are given in the following table. Two dimensions were included within the selections, degree of difficulty (easy or challenging) and degree of enjoyment. Table 6.18 indicates the total number of responses for each selection by class and for the total group.

Table 6.18
Retention Test Results: Survey - Item #5

Student Responses	Alice	Betty	Carl	Group
a) easy/enjoyable	0	0	1	1
b) challenging/enjoyable	6	3	4	13
c) neutral/no opinion	5	7	9	21
d) challenging/not enjoy	5	9	7	21
e) easy/not enjoyable	0	0	0	0
f) total dislike	2	7	1	10

A rather large percentage of students (21 of 66) selected the neutral or no opinion response, particularly in Carl's class (9 of 22). In assessing the degree of difficulty dimension, only one student selected an "easy" related response (a or e). About 20 percent of the

group's students found verbal problems enjoyable. Betty's class particularly expressed the least enjoyment of (3 of 26) and the greatest dislike (7 of 26) of verbal problems.

It appears that Alice's class had the most positive attitude towards verbal problems, and Betty's class the least positive attitudes, while Carl's class had a neutral (tolerant) attitude towards verbal problems. Of the ten students who expressed total dislike for verbal problems, nine students indicated below average ability with verbal problems (item 4, response d or e), and six of those nine rated their ability very poor (response e).

Item number 6 of the survey asked students to rate on one of four levels how useful they believed the algebra course was for their high school program and beyond. Table 6.19 indicates the total number of student responses for each selection by class and for the total group.

Table 6.19

Retention Test Results: Survey - Item #6

Student Responses	Alice	Betty	Carl	Group
a) very useful	3	8	11	22
b) some usefulness	13	11	8	32
c) little usefulness	2	6	3	11
d) no usefulness	0	1	0	1

Over 80 percent of the total group of students perceived some usefulness for the algebra course, and the percentages were higher for Alice and Carl's students, approaching a 90 percent level. Seven of the ten students

who expressed total dislike for verbal problems (item 5) still perceived some usefulness for the course. Exactly one third of the students perceived the algebra course as very useful to their future plans.

As mentioned earlier, the last item of the survey (item #7), which required a written response, was dropped from the data analysis for the following reasons. A majority of the students skipped the item altogether, and those that did respond in writing to the question generally focused on their teacher for the course rather than the algebra content, as was intended by the investigator.

Summary Comments

The survey section of the retention test indicated that students in the three classes share similar perceptions with regards to concerns, ability ratings, course attitudes, and course usefulness. About 80 percent of the students can be categorized as present/concrete oriented in their concerns related to the course. This implies that passing the course, grades, and day to day learning are their primary motive for course participation.

Their perceptions of their general algebra ability and course attitudes are slightly above average. However there was a notable drop in all three classes for their ratings of perceived abilities related to verbal problem

content. Only one student perceived verbal problems as easy, while about half of the students perceived verbal problems in a negative light, as not enjoyable. Over 80 percent of the students did perceive the algebra course to be useful within their high school program.

The few differences between class perceptions which did occur related to the two ability ratings and general attitude towards the algebra course. It appeared that Alice and Betty's classes had more greatly integrated their perceptions of verbal problem solving ability with general algebraic ability, while Carl's class rated these same abilities in greater isolation to one another. Course attitude corresponded more closely to student perceptions of verbal problem solving ability for Alice and Carl's classes, while general algebraic ability was a better predictor of course attitude for Betty's class.

Equation Solving Section

The equation solving section of the retention test was composed of seven equations. The seven equations were rewritten versions of the same seven equations that students were expected to employ in the verbal problem solving section. The investigator was evidently successful in this "disguising" effort, since not a single student noticed any relationship between the two sections. The exact nature of the seven equations appears in Appendix F, as part of the retention test form. Each

student completed the same seven equations presented in the same order, though this order did not correspond with that used in the verbal problem solving section.

The derived solution to the equations was the only criterion used for evaluating students' equation solving ability. One point was awarded for a correct solution, and zero points otherwise. A score of 7 is then a perfect student score. A summary of the results of the equation solving section for each class and the total group is given in Table 6.20. Individual student results for this section are posted in Appendix J.

Table 6.20

Retention Test Results: Equation Solving Summary

Statistic	Alice	Betty	Carl	Group
Mean Score	4.94	4.42	4.59	4.62
Variance	2.29	4.01	3.30	3.25

As can be noted, the three classes performed about equally well on this section, and differences between classes, using analysis of variance (ANOVA) were not statistically significant ($F = .44$). The initial scoring evaluation was followed by an item analysis, whose results are displayed in Table 6.21. Proportions of correct solutions were used to enable comparisons between the three classes' performance.

Table 6.21

Retention Test Results: Equation Solving - Item Analysis

Item number/Equation	Alice	Betty	Carl	Group
1. x + x - 180 = -3x - 80	.89	.50	.59	.64
2. 5(x -2) + 10(x +1) = 90	.72	.81	.73	.76
3. 64x = 48(7 -x)	.72	.85	.68	.76
4. x + 25 - 4x = 10	.83	.50	.73	.67
5. -3(x + 3) = 7 - 4x	.61	.65	.73	.67
6. 3x + 60 = 8x	.89	.85	.86	.86
7. x + (x + 2) - 26 = 4x - (x + 4)	.27	.23	.23	.24
Proportion correct - 7 items	.71	.63	.65	.66
Success Proportion - Solving Equation Phase of Posttest	.77	.70	.94	.83

A comparison of the total group's success rate on these equations with the conditional success rate on the posttest's phase scoring indicated that there was a drop in student equation solving performance during the elapsed time between tests in solving these types of linear equations. The drops in performance were similar for Alice and Betty's students, and more pronounced for Carl's students.

Item number 6 was clearly the easiest of the seven equations, requiring only two steps to solution. Item number 7 was clearly the most difficult item, with only one in four students able to correctly derive the solution. The most common error pattern for this item (21 of 50 student errors) was an inability to remove the parentheses for the term -(x + 4) correctly, a distributive property related error. There is a marked

similarity in student performance for the other five items, with performance proportions ranging from .64 to .76 for the total group.

Greatest classroom performance differences appeared with respect to items numbered 1 and 4. Betty's students scored appreciably lower on these two items, as only 50 percent of her students correctly solved these two equations. At the same time, her students' performance on items numbered 2 and 3 was better than the other classes. Items numbered 1 and 4 require only combining like terms and transposition of terms. Part of this differing performance on these items relates to Betty's instruction which maintained an approach of changing all equation subtractions into related additions, despite the fact that her textbook later removed this approach from its instruction. Since her approach adds more multiple signed numbers into the equation, resulting in greater complexity and student confusion applying the transposition process, the equation solving results for her class are not surprising. For items numbered 2 and 3, which both required applications of the distributive property, Betty's students outperformed their counterparts, demonstrating solid student familiarity with this property's application within equations.

An error analysis was conducted by reviewing student work samples for each equation item. Of the 159 student

errors, 24 of the errors reviewed (22 in Betty's class) showed no student attempt to solve, and 2 errors were considered uncodable within the investigator's classifications. The remaining 133 student errors were classified into one of five categories: 1) computation (real numbered values), 2) computation (variable terms), 3) distributive property errors, 4) procedural errors (transposition), and 5) quadratic form errors. This last category of errors were cases where students made the given linear equations into quadratic equations, and was undoubtedly due to more recent learnings related to quadratic equations. In cases where more than one student error was evident, the first appearance of an error was the category to which the error was assigned. Table 6.22 displays the proportion of errors classified in each error category for the classes and for the total group.

Table 6.22

Retention Test Results: Equation Solving Error Analysis

Error Classification	Alice	Betty	Carl	Group
Computation (Real Numbers)	.29	.24	.35	.29
Computation (Variable Terms)	.20	.22	.13	.18
Distributive Property Types	.29	.20	.19	.22
Procedural/Transposition	.22	.24	.31	.26
Quadratic Formed Types	.00	.10	.02	.05
Total Number of Errors Classified	35	46	52	133

Almost equal proportions (47 percent and 48 percent) of codable errors were due to computational related errors and to conceptual misunderstandings (distributive property and procedural types). Combining the two computational error categories for the three teachers indicated nearly equal proportions for each class. Alice's class made relatively more errors related to the application of the distributive property than students in the other two classes. On the other hand, Carl's students made relatively more procedural errors, a result which runs contrary to the greater instructional time he devoted to linear equation solving in the first part of the school year.

Summary Comments

The student performance results for the equation solving section indicated that students in the three classes demonstrated equal ability to solve linear equations. Differences between classes were confined to specific equation types encountered. While students in Alice and Carl's classes performed quite similarly on all equation items, Betty's students floundered somewhat on equations where only transpositions and combining like terms were required. However her class did relatively better on linear equations which required application of the distributive property. The error analysis showed a high degree of consistency for the three classes, and

indicated that nearly equal proportions of errors were attributable to computation difficulties as to conceptual misunderstandings.

Verbal Problem Solving Section

This section of the retention test is composed of seven verbal problems taken directly from the textbooks used by the three classes. It was felt that the inclusion of textbook formulated verbal problems might improve student performance over that of the investigator designed verbal problems contained in the posttest form. Two of the selected problems were B level difficulty and five were A level difficulty. All test forms were identical, containing two verbal problems selected from two of the employed textbooks and three from the remaining textbook. The number of problems in this section was decreased to seven, from the ten of the posttest, in an effort to permit completion of the entire retention test within a forty-five minute time frame. In hindsight, it probably would have been better if the retention test form was restructured into an ordered sequence of verbal problems, equation solving, and survey questions, rather than the employed sequence. While time was not a factor in any student being able to complete the entire retention form, it was obvious to the investigator that student motivation and effort related to the verbal problem section was

diminished by their previous completion of the other two sections first.

The analysis used for this section's verbal problem performance was similar to that developed for the posttest form. Item scoring was based on investigator awards of zero, one, or two points for each of five phases, or a total possible score of 10 points per problem. Seventy then represented a perfect score for this section. The individual test items for this section are listed with the entire retention test form in Appendix F, and individual student scores for this section are listed with other performance results, by classroom, in Appendix J.

Verbal problems for the retention test were selected carefully in order that each of the six problem types were represented and that each of the four format classifications were represented at least once in the verbal problem set. The resulting structure of these classifications is shown for each of the seven verbal problem items in Table 6.23.

Table 6.23

Retention Test: Problem Solving Section - Item Classifications

Item Number	Problem Type	Format Classification
1.	Geometric Type	Abstract/Factual
2.	Simple Translation	Abstract/Hypothetical
3.	D/R/T Related	Concrete/Hypothetical
4.	Geometric Type	Abstract/Factual
5.	Integer Type	Abstract/Factual
6.	Age Related	Concrete/Factual
7.	Coin/Money Related	Concrete/Hypothetical

Results were tabulated for each individual test item and for the total test performance by class. Analyses of variance tests were conducted to indicate any differences in student performance between classes on individual items. Table 6.24 provides a summary of item mean scores and related significant results for each class and the total group.

Table 6.24

Retention Test Results: Problem Solving Section - Item Performance Summary

Item Number	Mean Group	Mean Alice	Mean Betty	Mean Carl	F -Value	Significant Differences
1.	1.38	1.72	1.46	1.00	0.75	NS
2.	4.02	5.33	2.92	4.23	1.96	NS
3.	0.30	0.78	0.19	0.05	2.20	NS
4.	1.39	2.28	0.96	1.18	1.97	NS
5.	3.39	6.06	0.88	4.18	16.53	$p < .01$
6.	1.82	3.61	1.38	0.86	4.97	$p < .05$
7.	1.73	2.33	1.88	1.05	2.14	NS
all items	14.03	22.11	9.67	12.55	6.74	$p < .01$
% of total	20.04	31.58	13.81	17.93		

One immediate conclusion is the overall low student performance on these verbal problems. The mean score for the whole group was 14.03, or at a 20 percent correct level. Considering these problems were taken directly from the employed textbooks, the results demonstrate that verbal problem solving abilities erode fairly quickly if not supplemented by regular review or practice in subsequent instruction.

Another major development which occurred was the sequential ordering of the three classes' performance from that found for the posttest results. On both verbal problem tests, Betty's class scored lowest. However Alice and Carl's classes flip-flopped positions on the two tests, with Carl's class significantly outperforming Alice's class on the posttest and Alice's class performing substantially better on the retention test. One possible

explanation for this reversal might be the differing instructional centrality placed on student practice of verbal problems exhibited in these two classrooms. This issue will receive further treatment in the following chapter.

There were only two items (numbers 5 and 6) for which classroom performance differences were significant. Item number 5 was a consecutive integer type problem, and Betty's class scored appreciably lower on this problem type in comparison with the other two classes, as was the case for the posttest results. The relatively better performance of Alice's students for item number 6, the age related problem, is less explainable in terms of the posttest results. It may be that the non-solution nature of the age related problem of the posttest somewhat obscured her students' abilities to solve this type of problem. It was also interesting that the entire group of students performed almost identically on the two geometry type items (numbers 1 and 4). An ordering of student performance scores was done for the six problem types, and compared with performance results for the same six problem types used for the posttest. The results are shown in Table 6.25.

Table 6.25

Problem Type Comparisons: Posttest/Retention Test

Problem Content Type	Ranking Retention Test	Mean Score Retention Test	Mean Score Posttest	Ranking Posttest
Simple Translation	1	4.02	6.57	1
Integer Type	2	3.39	3.16	3
Age Related	3	1.82	1.64	6
Coin/Money Related	4	1.73	1.91	5
Geometric Type	5	1.39	2.87	4
D/R/T Related	6	0.30	3.88	2

Student performance actually improved slightly for two problem types, integer types and age related problems. For both problem types, the improved performance was most likely due to the non-solution nature of these problems on the posttest form. There was also not much difference between tests for the coin/money related problem type, most likely due to the similar context of one related verbal problem on each test form.

The student performance differences related to the other three problem types were more pronounced. The selected verbal problems for the retention test related to the geometric and D/R/T types were at a higher level of difficulty (B level problems) than those used for the posttest, and this partly explains the discrepancy in performance. One might also argue that these two problem

types require greater conceptual understanding and retrieval of necessary relationships than other problem types. Hence lower performance may indicate a diminishing retention of these concepts/relationships over time. There does not appear to be any reason for the lower student performance on the simple translation problem of the retention test since the problems on each test were nearly identical in context and degree of difficulty.

A closer look at classroom performance showed that Alice's students improved performance from the posttest for three problem types: integer types, age related types, and coin/money related types. Betty's students similarly showed improvement for the first two listed problem types of Alice's students, integer types and age related types. Carl's students' performance decreased substantially for all six problem types. The overall decrease in student mean scores for each teacher were a two percent decrease for Alice's class, a 32 percent decrease for Betty's class, and a 61 percent decrease for Carl's class.

Student performance in the problem solving section with regards to the alternate, four way format classification (abstract/factual, abstract/hypothetical, concrete/factual, concrete/hypothetical) was done similarly to that conducted for the problem type classification. The results are displayed in Table 6.26.

Table 6.26

Retention Test: Class Performance - Format Classification

Classification	Test items	Mean Group	Mean Alice	Mean Betty	Mean Carl	F Value	p Value
Abstract/ Factual	1,4,5	2.05	3.35	1.10	2.12	10.71	p<.01
Abstract/ Hypothetical	2	4.02	5.33	2.92	4.23	1.96	NS
Concrete/ Factual	6	1.82	3.61	1.38	0.86	4.97	p<.05
Concrete/ Hypothetical	3,7	1.02	1.56	1.04	0.55	3.18	p<.05

The primary purpose for the inclusion of this classification was to compare the study's student performance results with the hypothetical problem solving models developed by Caldwell and Goldin (1987). The composite posttest results most closely resembled the hypothesized direct translation model (AF, AH - CF, CH) sequence of problem difficulty. The sequence of problem difficulty for the retention test results is shown in Table 6.27, with the results of the posttest shown in parentheses.

Table 6.27

Retention Test: Problem Difficulty Format Sequence

Teacher	Lowest Level	Intermediate Level	Highest Level
Alice	AH (AF)	CF - AF (AH)	CH (CH - CF)
Betty	AH (AF - CF)	CF - AF - CH (CH)	-- (AH)
Carl	AH (AH -AF)	AF (CH)	CF - CH (CF)
Group	AH (AF)	AF - CF (AH)	CH (CH - CF)

On the retention test, the sequence of problem difficulty for Alice's students most closely resembled the performance of the entire group. The sequence of problem difficulty for Betty's students for the retention test was almost totally reversed from that developed from the posttest results, while for Carl's students the order was nearly identical for both test results.

The inclusion of only one exercise of an abstract hypothetical classification on each test form severely limited the usefulness of the test results in making comparisons with Caldwell and Goldin's (1987) models of problem solving performance, and in general the validity of classifying verbal problems according to these four formats. Whereas the posttest results indicated some similarity with the proposed direct translation model, the retention test results indicated no similarity with any of the hypothesized problem solving models, or with actual results from their studies. Abstract/Hypothetical problem

types do not appear easiest in any of their models or results. The position of the abstract/hypothetical format as easiest was also not explainable in terms of textbook exposure or treatment since this format type received the least exposure or practice of the four types in all three textbooks (Chapter IV). Another possible explanation for the relatively lower student performance on concrete format types may have been their location near the end of the retention test instrument when student effort and motivation was diminished.

The four individual format classifications: abstract, concrete, factual, and hypothetical were tested for significant statistical differences in student performance for the three classes of students, and for paired opposites. The results are indicated in Table 6.28.

Table 6.28

Retention Test Results: Problem Solving Section - Format Class Comparisons

Individual Format	Related test items	Mean Group	Mean Alice	Mean Betty	Mean Carl	F-Value	Difference
Abstract	1,2,4,5	2.55	3.85	1.56	2.65	10.91	$p < .01$
Concrete	3,6,7	1.28	2.24	1.15	0.65	7.59	$p < .01$
Factual	1,4,5,6	2.00	3.42	1.17	1.81	14.09	$p < .01$
Hypothetical	2,3,7	2.02	2.81	1.67	1.77	2.51	NS

Performance differences between classes were statistically significant for three of the formats, with

the hypothetical format being the only exception. Performance comparisons of opposite formats (abstract vs. concrete, and factual vs. hypothetical) revealed no significant differences between factual and hypothetical formats ($t = -.07$), and significant differences between abstract and concrete formats ($t = 4.59$). Greater problem inclusion of the four formats and a more equitable distribution of the four individual types might have provided greater stability of results. The analysis indicated that abstract format verbal problems, regardless of factual or hypothetical format were easier for the students to solve. The only theoretic model of Caldwell and Goldin (1987) for which abstract verbal problem types emerge as easiest is the direct translation model.

A complete item breakdown, similar to that done for the posttest verbal problems, was completed for the seven verbal problems of the retention test. Counts of the number of students who successfully completed each of the five phases were tallied by teacher, in anticipating development of student proportions accomplishing each subsequent phase successfully, given successful completion of the previous problem solving phase. The item breakdown is displayed in Appendix K, and also includes notes related to the error analysis for each item in two of the phases, variable definition and equation formulation. Table 6.29 displays the summary of conditional proportions

for each phase's successful completion for each teacher and the total group. The proportions in parentheses represent the same values for the posttest results.

Table 6.29

Retention Test: Summary - Conditional Proportions of Phase Success (Method I)

Proportion Who:	Group	Alice	Betty	Carl
Attempted Variable Definition	.60(.73)	.84(.79)	.47(.56)	.56(.87)
Correct Variable Definition	.60(.54)	.63(.58)	.58(.50)	.59(.54)
Correct Equation Formulation	.45(.54)	.49(.41)	.40(.49)	.43(.69)
Correct Equation Solving	.77(.83)	.82(.77)	.75(.70)	.73(.94)
Interpret/Answer Correctly	.71(.84)	.70(.74)	.67(.91)	.75(.85)
Probability Completing Entire Phase Sequence Correctly	.09(.15)	.15(.11)	.06(.09)	.08(.26)

For the total group of students, there were drops in performance from the posttest results for each phase except the variable definition phase. Since each classroom showed a similar improvement for this phase between tests, one can conclude that variable definition is a student skill which does improve with longer participation in the algebra course.

Alice's students' skills improved in every phase category, with the exception of interpreting/answer

correctly. It appears that an increased emphasis on student practice of verbal problems during initial instruction results in improved retention of verbal problem solving skills, and improved student performance during the algebra course of necessary subskills. Betty's students improved in their ability to correctly define variables, and to correctly solve formulated equations. Both gains relate to skills which probably received instructional treatment and student practice, outside of a verbal problem context, during the intervening timespan between test administrations.

The lower student performance of Carl's group in all phase areas, with the exception of variable definitions, leads one to conjecture that initial student learning related to verbal problems was somewhat superficial, isolated, and temporary in nature. Their learning seems to have lacked interiority, or reached a relational comprehension level. The employed textbook's organization of verbal problems into almost a single chapter of the textbook somewhat accounts for this happening in Carl's class only. A drop in student performance for the two tests was expected, based on the investigator's experience in similar classrooms. This expected drop was greater in the case of Carl's students, and the improved performance of Alice's students was not expected. The improved performance of Alice's students appears explainable in the

context of a differing initial instructional focus, that being greater emphasis on student practice of verbal problems, used in conjunction with a textbook which used a distributed verbal problem organization.

Correctly formulating a verbal problem's related equation remained the most difficult student skill (it was of equal difficulty with the variable definition skill on the posttest). Student ability to solve equations remained stable between tests, and emerged as the highest level performance skill on the retention test.

Considering these proportions as conditional probabilities for each phase of completion, and combining them to form a probability for the successful completion of the entire process resulted in Alice's students being twice as likely as Carl's students, and three times as likely as Betty's students, to complete the entire process correctly.

As was done for the posttest, an alternate method (Method II) involved the process scores awarded by the investigator for each students' work sample. Proportions of the total possible score, 14 (7 problems at 2 points) for each phase were developed for each classroom and the whole group. The resulting proportions are displayed for each classroom and the whole group in Table 6.30. The proportions in parentheses relate the same measures for the posttest results (see Table 6.12).

Table 6.30

Retention Test: Problem Solving Section -
Summary Proportions of Phase Success (Method II)

Earned Proportion Process Score	Group	Alice	Betty	Carl
1. Variable Definition	.39(.45)	.57(.53)	.29(.34)	.36(.52)
2. Equation Formulation	.18(.29)	.28(.25)	.13(.18)	.15(.44)
3. Equation Solving	.18(.30)	.31(.32)	.12(.17)	.15(.45)
4. Interpretation Checking	.14(.25)	.25(.27)	.08(.15)	.11(.37)
5. Correct Answer	.12(.18)	.17(.16)	.08(.11)	.13(.28)
Overall Performance	.20(.29)	.32(.31)	.14(.19)	.18(.42)

Investigator scoring by phases showed that Alice's class success rate on the retention test was similar to that for the posttest results, with losses in any one phase being minimal, and some improvement noted in several phase categories. The decrease in student performance for Betty's class was about what the investigator expected for all three classes. Carl's decrease in performance was more than expected, though student ability to formulate variable definitions remained relatively high, which indicated his students were able to begin solution of the verbal problems.

As was done for the posttest verbal problems, the investigator studied student error patterns made on the seven items, particularly focusing on the variable definition and equation formulation phases. These results

are listed in Appendix M. From this error analysis, several summary comments related to the individual test items follows.

Item 1: This geometric type problem involved a rather simple variable definition (x,2x), while equation formulation required relating the perimeters of 2 polygons (square, equilateral triangle). Fewer than one third of Carl's students even attempted a variable definition, indicating a lack of understanding of the problem statement. Variable definition errors were not patterned (6 diverse error types noted), while 17 of the 22 equation formulation errors were due to an inability to apply the concept of polygon perimeter. Only 3 (1 in each class) of 25 were able to formulate the equation correctly.

Item 2: This simple translation, number type problem was straight forward in its sentence structure, where the first sentence is used for variable definitions, and the second sentence used for equation formulation. Yet almost half of Betty's students failed to attempt variable definitions. The intervening study of two variable systems interfered somewhat in Alice and Betty's classes, as 5 students were unable to derive the two necessary relationships then required in a two variable system. Equation formulation errors were mainly due to an inability to include or apply the distributive property correctly within their equation constructions.

Item 3: Only about 30% of the students were able to even begin work on this distance/rate/time (B) problem, probably because distance was asked for and yet time was the variable needing to be defined. Only 15% of those beginning the verbal problem were able to correctly define the variable (x,7-x), and errors in this phase reflected an incorrect focus on the distance as variable. The item's construction did not permit evaluation of student conceptual understanding of the D/R/T relationship since so few students made sufficient progress on this verbal problem.

Item 4: This geometric type problem appeared less familiar to Betty and Carl's students as less than half of their combined groups even attempted variable definitions. The variable definitions for the 3 angles of the triangle required separation of language into relationships, a weak area for these students. Again the usage of two variable letters was apparent (as in item 2), yet progress was not aided within such a construction as necessary relationships were not addressed. Nine of the equation formulation errors centered on inadequate comprehension or recall of the angle sum relationship for a triangle being 180 degrees.

Item 5: Betty's class in particular was unable to recognize or recall this consecutive integer type problem, or even begin work (as was true of the similar posttest item). Variable definition errors were mainly due to defining consecutive integers, rather than consecutive even integers, and students in each class made this error. It is less clear whether this error pattern is due to careless reading of the problem statement or is conceptually based. Equation formulation errors were too diverse to find patterns.

Item 6: This age related problem proved extremely difficult for Carl's students to begin, as only 5 of 22 students attempted to define variables for the problem, which may have been due to the implicit nature of the problem statement in regards to variable definition, something of which students were less familiar. This inability was reflected in the variable definition errors, where 7 of 11 errors included the 4 times (explicitly given) in variable definitions. Equation formulation errors generally showed a lack of logical thought, and appeared to be meaningless combinations of pieces of problem information.

Item 7: All 3 groups showed a greater response to the real, concrete nature of this coin related verbal problem. The sentence structure of this problem was simple to follow, first sentence variable definition and second sentence equation formulation. Yet over one third of the variable definition errors used n and 4n, rather than n and n + 4. To receive full credit for variable definition, the adjusted coin amounts had to be indicated somewhere, and in 3 cases this adjustment was not specified. Many students again employed two variable letters, and in 3 cases, no relationship was developed between the two variable letters employed. Only 4 of 30 students were able to formulate the correct equation for this item, with 12 errors due to the exclusion of values for the coins (as was similar to the posttest results), while 9 other students made no attempt at equation formulation. It also appears that trial and error techniques are more successful for deriving correct answers for this type of verbal problem than expected techniques (10 correct solutions as opposed to 2 correct solutions).

Summary Comments

It can be assumed that little instruction related to verbal problem content was undertaken during the time interval between test administrations, a period ranging

from 10 to 12 weeks for the three classes. This assumption is based on the investigator review of employed textbooks for the classes, and partly on teacher reports to the investigator at the time of retention test administration. All three teachers indicated there was no review of the six verbal problem types constituting the core content of the study.

One would then expect students' verbal problem related skills to become "rusty" over time due to lack of usage or review. Indeed the overall mean score percent for the total group of students dropped from 29 percent to 20 percent of the total possible points for the two tests, which represented a 31 percent decrease in performance. Yet within the separate classes, student performance changes were quite different. Carl's students' performance was highest for the posttest (41.5 percent of total possible points), and yet on the retention test they earned only about 18 percent of the total possible points which represented a 57 percent decrease in performance. Admittedly his students had the most to lose because of their better posttest performance. Betty's students' performance was lowest on the posttest (18.7 percent of total possible points), and on the retention test also (13.8 percent), which represented a 26 percent decrease in performance. However Alice's students' performance actually improved on the retention test, though only

slightly, under a one percent increase in performance. One can conjecture that Alice's unique form of verbal problem instruction, which emphasized extensive student practice of verbal problems and provided greater class time for such practice, paid dividends for students' retention of such content. Other explanations for the improved performance of Alice's class appear less plausible. For instance, the textbook organization of verbal problems into a single chapter somewhat explains the significant drop in student performance for Carl's class, where a student focus of day to day preparation would lead to better immediate results, but have less retentive power. Yet Alice's scores improved and Betty's dropped despite the fact that both employed textbooks contained similarly spaced placements of verbal problems in their content organizations.

The poor retention results for Carl's class also resulted in the three classes' performance becoming more equivalent. On the posttest, significant differences were noted for five of the six verbal problem types (coin/ money related type being the only exception). On the retention test significant differences were noted for only two of the six verbal problem types, age related and integer type problems. For the integer type problems, this difference was mainly due to Betty's class, which had

extremely poor scores on both items of this problem type on both tests.

For the total group, student performance improved slightly for two verbal problem types, integer problem types and age related problem types, and decreased appreciably for geometric problem types and the D/R/T related problem type. This decrease in student performance for these two problem types was most likely due to the greater conceptual and relational knowledge required for these problem types. However, it can be argued that the more complex nature of the problem statements used on the retention test, particularly that used for the D/R/T type problem, account for some of this decrease in performance.

It was interesting to note that all three classes improved in their ability to correctly complete variable definitions on the retention test, as compared to their results for the posttest. This result indicated that all three classes probably had more experiences in algebraic expression manipulation during the time interval between test administrations which resulted in improved variable definition capabilities. For all other phase processes, student performance in Betty and Carl's classes decreased from that of the posttest results. However, based on both analysis methods employed, Alice's students improved their process performance for each phase between tests, with the

exception of correctly interpreting equation solutions into problem answers.

Test Correlations

Two Pearson correlation coefficients were computed for the student performance results of the two tests, posttest and retention test. One coefficient related student performance on the equation solving and verbal problem solving sections of the retention test, and the other coefficient related student performance on the verbal problem solving section of the retention test with the results of the posttest. These correlations were computed for each class and the total group. The results are displayed in Table 6.31.

Table 6.31

Correlation Measures Between Test Instruments

Correlation between:	Alice	Betty	Carl	Group
Equation Solving and Verbal Problem Solving Sections of Retention Test	.422	.436	.323	.404
Verbal Problem Solving Section of Retention Test and Posttest	.194	.806	.623	.488

The wide divergences for the classroom correlations relating the verbal problem solving section and the posttest results was indicative of the totally different student performance patterns of the three classes on the

retention test's verbal problem solving section. The correlations between the two sections of the retention test were consistent across classes, and indicated that equation solving ability is related to verbal problem solving ability, though not to as great an extent as one might expect. The lower correlation for Carl's class on this measure may indicate further the compartmentalized nature of student algebra learnings which appeared to have taken place for verbal problems, and which was also noted earlier for the retention test and survey results. In this case, student focus on day to day learning and achievement tend to block out connected relationships which exist between various algebra contents studied.

It was noted earlier that the equations given for the equation solving section were rewritten versions of the expected equations students would use for the verbal problem solving section. This design permitted the investigator to analyze student equation solving ability under two different formats, within verbal problems and outside verbal problems for the same basic equation. In cases where students correctly formulated an equation in the verbal problem section, one would expect a high degree of conformity in their ability to solve each of the two nearly identical linear equations. There were 76 cases where students of the study correctly formulated equations for the verbal problems of the retention test. For these

cases, the investigator checked student ability to solve the equation for the verbal problem and the related equation from the equation solving section of the test. The summary results for each class and the total group, given in proportions, is displayed in Table 6.32.

Table 6.32

Retention Test Results: Equation Solving Comparisons

	Alice	Betty	Carl	Group
Number Correctly Formulated Equations - Verbal Problems	33	21	22	76
Proportion Both Equations Solved Correctly	.49	.62	.50	.53
Proportion Only Solving Verbal Problem Equation Correctly	.33	.10	.23	.24
Proportion Only Solving Equation Section Correctly	.06	.14	.23	.13
Proportion Solving Neither Equation Correctly	.12	.14	.04	.10
Proportion Solving Verbal Problem Equations Correctly	.82	.71	.73	.76
Proportion Solving Related Equation Section Correctly	.55	.76	.73	.66

The results of the analysis indicated several patterns of student linear equation solving ability. Students actually had greater equation solving success when solving their own formulated equations derived from verbal problems, than when the equation was given in an altered form in isolation from a verbal problem context. This pattern was true only for Alice's class, and was

substantially different. The proportion of students solving both equation forms correctly was somewhat smaller than expected, only slightly better than 50 percent (.53 of the total group), and was fairly consistent across classes.

Students had the greatest success formulating equations for verbal problem items numbered 2 and 5 (29 and 26 correctly formulated equations, respectively, or 72 percent of all correctly formulated equations). For item number 2, almost equal proportions of students solved the two equation forms correctly (20/29, verbal problem and 22/29, equation section), but less than half (48 percent) solved both equation forms correctly. For item number 5, a greater proportion of students correctly solved the verbal problem equation form (19/26) than the equation section form (8/26), and only 31 percent correctly solved both equation forms. Table 6.32 also indicated that students in the study can successfully solve these linear equation types in at least one of their two given forms 90 percent of the time, yet were only able to correctly solve both equation forms on the same test 53 percent of the time.

Student Interview Results

Nine student interviews, three students from each class, were conducted during the same school week, December 15 - 19, 1986. Elapsed time from the posttest

administration ranged from several days for Carl's students to about 2 1/2 weeks for Alice's students. The posttest results were not completed prior to the necessity of the selection process for the interviews, and hence required the investigator to develop an alternate process for the selection of students for the interviews. On the basis of verbal problem classroom observations of each class, the investigator developed a list of five possible subjects from each class for the interviews. Each possible subject was believed by the investigator to be of above average ability with regards to verbal problem content.

Each of the fifteen listed students received a two page form consisting of a description of the study and its purposes, and a request for parental permission to conduct the interview with their son or daughter. One student in each class failed to return the signed permission slip within the three days given, and one student's parent (Alice's class) denied permission to interview their child. For Alice's class then, the three students returning affirmative parental permission slips were used, and for Betty and Carl's classes, the investigator selected three of the four students in the pool, based on ease of scheduling and which students were felt to be the best candidates for the interviews.

The nine students interviewed were labeled A1, A2, A3, B1, B2, B3, C1, C2, and C3, where the A, B, C designated their classroom or teacher (Alice, Betty, Carl) and the 1, 2, 3 ordering designated the sequence in which they were interviewed. Table 6.33 indicates each students' performance and class ranking for the posttest and for the verbal problem solving section of the retention test.

Table 6.33

Interview Results: Subject Test Measure Performances

Student	Posttest Score(90)	Class Rank	Retention Test Score(70)	Class Rank
A1	35	3	48	2
A2	25	11(tie)	16	11(tie)
A3	23	15	19	8
B1	51	1	42	1
B2	24	8	14	8
B3	7	19(tie)	3	16(tie)
C1	67	1	39	1
C2	55	3	8	14
C3	57	2	26	2
Mean Score and Related Rank	38.22	16 of 67	23.89	14 of 66

The investigator's ability to select above average performing students based on classroom observations was questionable for Alice and Betty's classes, though the

three students with the best performance on the posttest were selected in Carl's class. The mean scores of the interview group for the posttest and the retention test (verbal problem section) ranked in the top quarter (75th percentile) in each case. The correlation between posttest and retention measures for this group of students was .53, or slightly higher than was found for all students of the study. As it turned out, the investigator selected a high, a middle, and a low performing student in each of Alice and Betty's classes.

With one exception (B1), each student was interviewed for 45 minutes during school hours, in an isolated room at the school site. Subject B1 was interviewed after school for 60 minutes in a university classroom. Each interview was audiotaped in its entirety, though for two of the interviews (A3 and B2) the audio recording was garbled and not interpretable. The investigator took extensive notes on prepared forms during each interview. The audio recordings served three functions: to validate student work samples and investigator notes taken during the interview, to aid investigator recall of student comments made during the interview which the investigator failed to note, and to develop time frames for each verbal problem's completion. Obviously the two garbled recordings did not permit such validation, missed comments, or time frame developments to be completed for these two students.

Each interview followed the same sequential format. This format included a series of seven entry related questions (background, 10 minutes), a development of percent confidence assessments for three process skills for each of six hypothetical verbal problem types (10 minutes), and the completion of six verbal problems, presented one at a time (20 to 25 minutes). A complete copy of the interview instrument for each class of students, listed by teacher, can be found in Appendix G. The results for the first sections (background) are treated briefly in the following analysis, followed by a more detailed review of students' performance for the verbal problems. Discussion of students' confidence assessments (Appendix N) is limited, since the results did not relate to the purposes and questions of the study.

Survey Section

The seven questions of the background section dealt with students' mathematical backgrounds, earned and perceived grades, algebra strengths and weaknesses, verbal problem strengths and weaknesses, perceived purposes of verbal problems, and student homework patterns.

Students' mathematical backgrounds were more varied for the parochial school students, than for the public school students. Five of the six public school students had been tracked upon entry into the seventh grade at the junior high school they were presently attending. The one

exception had transferred at the ninth grade level from an area parochial grade school (K - 8), and as standard school procedure had not been tested upon entry, but rather had been placed into Algebra I (rather than Algebra I-A).

First quarter grades for the nine students ranged from A (four students) to C+ (two students), with the two students earning C+ grades both coming from Carl's class (C2 and C3).

Students were asked which content areas of algebra content already studied were easiest for them and which areas posed greater difficulty for them. Six students mentioned equation solving or simplifying expressions as easier areas of algebra content. Two students mentioned graphing (the current topic of study) and one student surprisingly mentioned verbal problems as easier areas. Two students could think of no particular content area as more difficult, while four students believed verbal problems were the most difficult content area for them. The other three students each listed a different content area as most difficult ("tough" equations (multi-step types), graphing, and polynomials).

One question of this section asked students to evaluate their perceptions of their one greatest strength and one greatest weakness related to verbal problem content. In regards to perceived strengths, four students



responded equation solving, four students responded variable definitions, and one student was unable to identify a single strength. In regards to perceived weaknesses, six students mentioned equation formulation as the most difficult aspect, and three students gave various forms of expression related to the reading or "imagining" of a problem situation, and the understanding or "intent" of a problem statement.

One question asked students to describe their perceptions of the reasons for the presence of verbal problems within the Algebra I content. Evidently four of the interviewed students had never considered the question before, since they did not express a single reason, or responded weakly, "same as the reason the rest is there". The other five students all mentioned the real life, applicational nature of verbal problems, giving purpose for the study of other algebraic skills.

The last question of this section asked students to answer five questions related to a hypothetical assignment of eight verbal problems (problem type unspecified) given by their respective teacher. The questions were designed to investigate student homework patterns related to verbal problems. The five questions were as follows:

a) How many of the 8 problems would you complete in class?
b) Where would you most likely complete the remainder of the assignment?
c) Estimate the total time it would take you to complete the eight verbal problems.

d) Would you likely need assistance to complete the assignment?
e) To whom or where would you go if you needed assistance with a problem of the assignment?

Using these letter designations for the questions, Table 6.34 indicates the nine interviewed students' responses for the five questions.

Table 6.34

Interview Instrument: Student Homework Patterns

Student	Question a)	Question b)	Question c)	Question d)	Question e)
A1	6	study hall	1 hour	possibly	teacher
A2	all 8	study hall	20 min.	not likely	teacher
A3	4	at home	1 hour	not likely	mom
B1	4 or 5	at home	30 min.	maybe	sister
B2	2 or 3	at home	1-1.5 hours	not likely	teacher
B3	4	at home	45 min.	possibly	dad/another student
C1	3 or 4	study hall	45 min.	occasionally	text/ notes
C2	3	study hall/ at home	20-30 min.	possibly	dad
C3	3 or 4	study hall	30 min.	yes	teacher/dad notes

The estimates for classroom completion (question a) of the assignment were accurate based on investigator observations of classroom time given for assignments, and indicated a sharp student awareness of instructional time characteristics of their respective classes. About half the students complete their assignments in school provided



study halls and half complete assignments at home. Since the public schools of the study generally provide ninth grade students at least one study hall per day in student schedules and there is greater variability for the parochial school in its provision of student study halls, one can easily guess which is the parochial school from the data provided.

Most students believed that from 30 minutes to one hour was sufficient time to complete the assignment. A majority of the students (6 of 9) believed there was a likelihood that they would need assistance on at least one verbal problem of the assignment. In four cases students would initially seek assistance from their respective teacher (2 of 3, for Alice), and four other students would first seek assistance from a family member (dad, mom, or sister). Only one student (C1) thought about reviewing classroom notes or the textbook as a first source of assistance. This last finding indicates that these students are overly dependent on other people in their environments as sources of assistance, rather than demonstrating a more independent and persistent style of learning.

In general, the data indicated a wide diversity of homework options and patterns available and employed by students of the study. Students' quick responses to the interviewer's questions of this entire section

demonstrated a well developed student awareness of ability and time management (a part of student metacognition) for this level of formal schooling.

Verbal Problem Performance

The completion of the first two sections of the interview generally required twenty minutes. The verbal problem section began with the investigator directing each student to show all work and verbalize their thoughts as much as possible while completing the six verbal problems of this section. Each verbal problem was written at the top of a separate sheet of paper and presented one at a time by the investigator, and in the same sequence for all nine students. Students determined when to discontinue work on a problem and move to the next presented verbal problem. Once students had read and analyzed each of the six verbal problems once, they were allowed to select one or more problems for further thought and work, if sufficient time remained at the end of an interview.

The verbal problems used for this section were different for each classroom's set of three interviewed students. In other words, the three students interviewed from Alice's class completed identical sets of six verbal problems, but the interviewed students from Betty and Carl's classes completed different sets of six verbal problems. The investigator carefully selected the three sets of verbal problems to be of equal difficulty, with

five problems of each set taken almost verbatim from the employed textbooks of each class, and the sixth verbal problem, common to all three interview test forms was an investigator designed nonroutine, age related verbal problem type. The purpose of this design was to lessen the influence of problem statement variations used in the three textbooks, and to use language which was more familiar to the students' previous exposure and practice.

The scoring for each verbal problem followed the format developed for the posttest and the problem solving section of the retention test. Investigator awards of zero, one, or two points in each of the five phase categories made 10 points the possible score for each verbal problem, and 60 points the possible score for this section. Process scores of the nine interviewed students for the six verbal problems are given in Table 6.35.

Table 6.35

Interview Instrument: Student Process Scores by Item

Student	Item #1	Item #2	Item #3	Item #4	Item #5	Item #6	Total Process Score
A1	10	3	10	4	2	1	30
A2	4	2	3	2	1	3	15
A3	2	1	2	2	4	9	20
B1	10	10	10	10	10	10	60
B2	2	3	10	4	4	1	24
B3	0	0	2	2	0	0	4
C1	10	10	4	10	10	3	47
C2	10	4	4	3	10	3	34
C3	10	0	4	10	10	10	44
Mean Process Score	6.4	3.7	5.4	5.2	5.7	4.4	30.8

Correlations between student process scores of the interview instrument, posttest results and the verbal problem solving scores of the retention test for this subgroup of nine students were computed. The correlation between the process scores of the interview instrument and the posttest was .876, and the correlation between the interview instrument and retention test was .694. These higher correlations indicated that the three investigator designed instruments did indeed measure the same set of verbal problem solving skills, and that investigator process scoring exhibited consistency for all three tests.

There was some similarity between student estimated confidence of the six verbal problem types and student performance for the six problem types. The notable exception to this similar arrangement was the coin/money related type (item #2) for which student performance was lower than that for the nonroutine verbal problem type (item #6). A possible explanation for this perception difference might be that the real and concrete (monetary) nature of this problem type obscures from student sight the relational knowledge requisite for solving this verbal problem type. It was also noteworthy that eight of the nine students interviewed attempted variable definitions for all six problems of the set. One student (B3) only attempted variable definitions for 2 of the 6 verbal problems.

Average time per problem for the seven students for whom the audiotape functioned properly did not indicate any relationship to students' earned process scores. Student B1, who earned a perfect score (60), averaged six minutes per problem, and yet student C1, who earned the second highest score, averaged less than two minutes per problem. The other five students all averaged between three and four minutes per problem. Item number 1 required the least average amount of time per student (2:30), and items numbered 5 and 6 the greatest amount of time per student (3:51 and 4:18 respectively). The other

three items all averaged about 3 1/2 minutes per student to complete.

The nine students estimated their abilities to form correct variable definitions and correct equation formulations almost equally, and their actual performance closely matches their estimates for the variable definition phase, with the exception of student B3. Yet this group of nine students as a whole considerably overestimated their ability to formulate verbal problem equations. In fact the nine students' performance related to equation formulation was no better than that for the entire group of students (67) on the posttest form (.54). Four of the students (B1, C1, C2, and C3) were more accurate in their estimates related to equation formulation, and these four students attained the four highest scores on the interview's verbal problem section.

It was also evident that once students were able to formulate a correct equation, there was almost complete assurance that students would derive correct answers for the verbal problems (20 of 21 cases). This result demonstrated that students' equation and interpretive skills were highly developed and automatized. While all Algebra I students interviewed appeared able to project realistic appraisals of their abilities to define variables and to find correct answers from correctly formulated equations, only higher ability students

demonstrated a realistic awareness of their ability to formulate equations. Average or below average students tended to considerably overestimate this same ability.

Summary Comments

On the basis of the above analysis, one might be discouraged by the lack of new insights or deeper understanding which the interviews appeared to provide. This appearance is deceiving, and is partly due to the way the interview results were developed in the analysis, more along group tendencies than along individual student tendencies. Individual student strengths and weaknesses related to verbal problems came clearly into focus for each and every student interviewed. At the end of each 45 minute observation of a student working on verbal problems, and listening to their related comments, the investigator was able to construct a detailed and composite representation of each student's verbal problem solving ability. These investigator perceptions were written down and noted on the audiotape after each interview session for later review and thought. During available time at the end of three or four of the interview sessions, the investigator shared some of these perceptions with the interviewed students, and requested feedback from the students. In each case there was general affirmation that the perceptions were correct. In a way it was fortunate that selected students for the

interview were at different developmental levels related to verbal problem solving skills, because they appeared to be a good representative sample of the entire group of Algebra I students, based on their performance levels for the posttest and retention test.

These investigator perceptions were not discussed in the preceding section since the investigator believed them to be highly speculative in nature, and based on too small a sample to merit discussion there. For students of average or below average ability, the interview sessions brought forth excellent diagnostic profiles of student verbal problem solving skills. It was easy to perceive, for example, that student B3 did not have an operative understanding of a general problem solving model or procedure to begin working on the verbal problems. He seldom was able to begin work on a problem, and his actions were mostly random and misguided, indicating no thought of a predesigned plan.

Students A1 and C2 both began written work on the verbal problems without adequate reflection upon the verbal problem statement or the development of an understandable image of the problems' relationships. From an observer's standpoint, it appeared that each student immediately realized a related problem type, recalled a previously done model of that problem type, and began matching problem information with the recalled model.

Other students, such as A3 and B2 considered verbal problem solving as a kind of trial and error manipulative game. They usually followed the general procedure in attempting to write out variable definitions and equation formulations, but lacked practical skills for how these two processes were to be accomplished, as evidenced by their lower scores for these two phases. They generally resorted to trying various random and illogical combinations until they seemed to "look" correct or worked to give an answer, any answer. Student A2 was the analyzer of this group of students, able to translate isolated phrases of the problem statement into variable definitions quite successfully. However the translation techniques he had developed were less useful for equation formulations where a relational synthesis was more necessary.

The interview sessions also served to highlight small details of the differences between better students. Students C1 and C3 demonstrated an excellent ability to perform all related processes required within a general problem solving procedure, with the possible exception of explicitly checking or looking back. They were also able to recognize when derived answers were probably in error based on appearances and tests of reasonableness. They were however less flexible in their ability to repair errors. They generally failed to locate the source of

their errors despite extensive backtracking and review of their work samples. In most cases they would give up the searching process after a short time, reread the problem and begin a whole new solution process. For C1 particularly, if the derived answer to a problem was not deemed correct on his first attempt, he preferred to just move onto the next verbal problem anyway.

Student B1, on the other hand, knew that time would not be a constraint for her interview session, and was determined at the outset to correctly solve all six verbal problems. This perseverance was necessary since four of her six initial solution attempts ended in error. Two of these errors would have escaped notice had she not checked her solutions with the problem statements. In any case, where solutions were derived, she methodically checked her solutions with the problem statements. In repairing recognized errors, she would reread and ponder the problem statement for 15 or more seconds, and then review each process step from the top of the previously completed work, checking its logical correctness. Obviously these repair strategies eventually resulted in correct answers for all verbal problems. While her process skills may not have been as well developed as some of the other students interviewed, her perseverance, confidence and checking/repair skills compensated for the weaknesses in ability of which she appeared completely aware.

The interview sessions then were of practical value for diagnosing specific verbal problem solving weaknesses of average or below average performing students, and also in highlighting certain metacognitive aspects of better student performance. In Chapter VII (Summary and Conclusions), the investigator will detail a conjectured model of five levels of student verbal problem solving performance, and related student characteristics for each level of performance. The interview data provides the primary basis for the construction of this model.

CHAPTER VII

Conclusions/Recommendations/Directions

The three previous chapters discussed the individual results related to each of the three aspects of this investigation: instructional patterns of textbooks and teachers, teacher verbal suggestion patterns, and student performance patterns. This chapter relates these three aspects and provides answers to the eight hypotheses generated for the study in Chapter I. The eight hypotheses are grouped according to similarity of hypothesis focus into five major subdivisions for this discussion. The five major subdivisions are: General Problem Solving Model Instruction and Incorporation (hypotheses 1, 4, and 7), Textbook Influence on Teachers' Instructional Routines (hypothesis 2), Teacher Characteristics' Influence on Instructional Routines (hypothesis 3), Teacher Verbal Suggestions' Influence on Student Attention and Performance (hypotheses 5 and 6), and General Student Performance Characteristics (hypothesis 8).

In each of the five subdivisions, the investigator restates the relevant hypothesis or hypotheses, summarizes the related findings of the study, formulates conclusions

based on the findings, and notes recommendations based on the formulated conclusions. The last section of the chapter discusses directions for future research efforts.

General Problem Solving Model Instruction and Incorporation

At the outset of the study, the investigator assumed that a major objective of textbook and teacher instruction of verbal problem content was student learning and incorporation of a general procedural model, designed to assist students in solving any algebra verbal problem. Hence three of the eight hypotheses of the study related to such instruction and student incorporation/usage of a general procedural model. The three hypotheses related to a general procedural model were:

Hypothesis 1: The inclusion and reinforcement of a general procedural model, such as Polya's or a textbook version, within a teacher's instruction improves student verbal problem solving performance.

Hypothesis 4: Continual teacher reference to the general procedural model, via teacher suggestive comments and/or modeling within instruction of diverse verbal problem types is necessary for students to incorporate, generalize, employ and retain the general model when solving verbal problems.

Hypothesis 7: In attacking and solving verbal problems, a high proportion of students will demonstrate an awareness of the general procedure within their completed work samples on varied verbal problem performance measures.

Findings

The three textbooks of the study each contained explicit instructional treatment of a general procedural model composed of specific steps to follow when solving verbal problems. These steps in the textbooks' models closely resembled the steps of Polya's model and investigator expectations regarding construction and sequence, in that they all contained steps related to reading and understanding problem elements (assumed in textbook A), to devising a plan which included formulating variable definitions and an equation, to carrying out the plan by solving the formulated equation, and to looking back by interpreting and checking the equation's solution.

Despite the presence of these commonalities within each textbook's treatment of a general procedural model, differences between the textbooks' models were noted along three dimensions, degree of detail, consistency of portrayal, and introductory placement1. Opposed to textbook A and B's skeletal four and five step general procedures, textbook C's general procedure included two additional steps, one related to locating a word sentence suggestive of an equation prior to any written formulations, and a separate interpretive step in which conclusions are drawn from the equation's solution before the formal checking step. In both textbooks A and B, the interpretive step is subsumed under the checking step.

The locating of a problem sentence suggestive of an equation was not explicitly treated in textbook B, whereas textbook A had several verbal problem sections in which students were trained to locate and identify conditions within the sentences of a problem statement which related to variable definitions and equation formulation respectively. Such explicit instructional treatment appeared to assume that such an ability to identify conditions is a separate tool skill apart from the steps of a general model, and further assumed that students will incorporate this tool skill during their initial reading and understanding of a verbal problem.

Possibly because their general procedures were shorter, textbooks A and B consistently labeled numerically each step or action within solved example problems corresponding to the steps of the general procedure. In its solved example presentations, textbook C refrained from making such numerical correspondences, but rather concentrated its explanation on the steps of its general procedure which required students' written actions.

Both textbooks B and C were highly consistent in their portrayal and reinforcement of a single, general procedural model in all example presentations, whereas textbook A introduced modified and specific procedures (boxed and highlighted in red) within every new verbal

problem chapter section. Alice's delivered instruction avoided any reference to these highlighted textbook procedures, but rather focused students' attention on the appropriateness and applicability of a single, four step general procedure, since she believed such highlighted procedures lead to greater student confusion.

Textbooks A and C introduced their respective general procedures for solving verbal problems once it was clear that students were about to begin more formal and regular instruction of verbal problem content. On the other hand, textbook B introduced its general procedure early in the school year (first chapter) and students also immediately practiced the model with one set of simple translation verbal problems. The general procedure was then regularly reinforced in subsequent verbal problem sections and sets as they occurred in the textbook's organization.

Two of the three hypotheses (4 and 7) required the investigator to establish criteria for what constituted student awareness and incorporation of a general procedural model on the basis of the review of student work samples. Since the first written step in all three textbooks' general procedures was defining variables, the investigator concluded that a student had demonstrated awareness of the general procedure if a student's reviewed work sample for a verbal problem included an attempt to define variables (correctly or incorrectly) for that

verbal problem. Using this criteria for student awareness of the general procedure, it was a simple matter to establish that a student demonstrated incorporation of the general procedure if the student demonstrated awareness for a majority of the verbal problems of an instrument's set (5 of 9 for the posttest instrument and 4 of 7 for the retention instrument).

Using these established criteria, the student performance results (posttest and retention test) support the hypothesis that a high proportion of students demonstrated awareness and incorporation of the general procedure within their work sample solutions. For the posttest, 83.5 percent of the students (56 of 67 students) demonstrated incorporation of the general procedure, and 73 percent of all reviewed work samples indicated student attempts to define variables. The corresponding figures for the retention test were somewhat lower, 68 percent of the students (45 of 66 students) demonstrated incorporation and 60 percent of reviewed work samples indicated variable definition attempts. The results indicate that about 15 percent of the students had forgotten the general procedure during the intervening ten week time span, during which the general procedure had not been practiced within their classes.

There were however differences in the degree of student incorporation and awareness between the three

classes of the study. For the posttest, all 18 of Alice's students and 21 of Carl's 22 students demonstrated incorporation of the general procedure, whereas only 17 of Betty's 27 students demonstrated such incorporation. Proportions of student work sample solutions indicating student awareness of the general procedure were 87 percent for Carl's students, 79 percent for Alice's students, and 56 percent for Betty's students.

For the retention test, all 18 of Alice's students again demonstrated incorporation of a general procedure in their work samples, and somewhat surprisingly demonstrated increased awareness, as 84 percent of their work sample solutions included attempted variable definitions. On the other hand, 15 of Carl's 22 students and 12 of Betty's 26 students demonstrated general procedure incorporation on the retention test, and 56 percent and 47 percent respectively of student work samples included attempted variable definitions. An apparent lack of student effort and perseverance, as compared with posttest effort, in completing the verbal problem section of the retention test on the day of test administration appeared partly responsible for the rather large percentage drops for the retention test in both categories (incorporation and awareness) for Carl's class. Other aspects of Carl's instruction and the design of his employed textbook (discussed later in this section) may have also resulted

in his students' poorer retention test performance.

The study's method of scoring student work samples of verbal problem solutions, based on the degree of completeness and correctness of the five phases of a general procedural model, favored students who demonstrated general procedure incorporation and penalized students who failed to demonstrate such incorporation. Students who used alternate procedures and yet derived correct solutions without supporting work were only awarded one point for each correct solution. One of Betty's students correctly derived solutions for four of the nine verbal problems of the posttest using a guess and check strategy, though this was rather exceptional for this group of students. Excluding this one student, no other student who failed to demonstrate general procedure incorporation, and hence tended to rely on alternate procedures, correctly solved more than two verbal problems on either test instrument. One might predict that because more of Betty's students used alternate procedures than did the students of the other two classes that most correctly derived solutions resulting from alternate procedures were accomplished by Betty's students. However the actual results contradicted this expectation in that Carl's class had the greatest number of such alternate procedure, correct solutions, 20, followed by Alice's class with 15 such solutions, and Betty's class with 13

such solutions. This finding seemed to indicate that students who have been more successful at learning to incorporate the general procedure into their verbal problem solution plans were also more likely to be successful in their attempts to derive solutions using alternate procedures, than were students who had failed to accomplish such general procedure incorporation.

There were also certain verbal problem types for which students using alternate procedures tended to have better success rates in deriving solutions. For instance from the posttest results, eight such students derived a correct solution for the consecutive integer problem type, and for the retention test results, seven students and ten students derived correct solutions for the age related problem and the coin/money problem types respectively, using alternate procedures.

One cannot however conclude from the study's findings that students' demonstrated incorporation of a general procedure alone insured improved verbal problem solving performance over those students for whom such incorporation did not occur. This was most apparent when one examined the bottom half (lower 50 percent) of student scores for the posttest and the retention test results for Betty's class and the retention test results for Carl's class. In all three cases, almost equal numbers of students' scores were noted who demonstrated general

procedure incorporation as those who did not demonstrate incorporation, though the mean scores for the incorporation subgroup was higher in all three cases. Additionally in Alice's class where all 18 students demonstrated general procedure incorporation on both testing instruments, the range of student scores was less than that of the other two classes indicating that such incorporation tended to equalize student ability to solve verbal problems. It is also recognized however that Alice's class had the fewest number of students of the study's three classes, and this may also account somewhat for the lower range of scores on both testing instruments.

At the outset of the study, the investigator believed two factors, teachers' verbal suggestions and teachers' modeling (see Hypothesis 4) were the two primary determining factors which effected the degree of student awareness and incorporation of a general procedure in solving verbal problems. The study's findings indicated that at least two other factors, extent of student practice of the general procedure and level of student attentiveness during teacher homework review and lesson development segments need inclusion. Both added factors are more student controlled than teacher controlled, though teachers, through their expectations and classroom management practices can exert influence on these factors.

Based on the classroom observations of each teacher,

the investigator's findings related to the relative influence of these four factors are discussed in comparing the three classrooms of the study. In terms of verbal suggestions directed towards encouraging the use of a general procedure, both Betty and Alice's instruction included a greater quantity (9 and 7 verbal suggestions respectively), and also a more extended reinforcing treatment than did Carl's three verbal suggestions all given the same day in which Carl introduced the general procedure to his students.

Investigator rating of each teacher's instructional modeling of the general procedure was done along two dimensions, quantity of such modeling and the qualitative consistency demonstrated within teacher presentations. Carl's instructional modeling of the general procedure was best along both dimensions. The quantity of examples reviewed daily in his homework review and lesson development segments far exceeded (about 3 times as many) that of the other two teachers. His presentations clearly and consistently emphasized the step nature of the general procedure, and his modeling often included discussion of related thinking processes students should practice in analyzing and understanding relational propositions contained in verbal problem statements. Carl's instruction generally included some non-textbook examples along with all textbook provided examples, and this tended

to broaden student exposure to more varied verbal problem contexts of the central relationships and resulted in greater student interest and participation within his instructional segments.

Alice's instructional modeling of the general procedure was both highly consistent and explicit in its reinforcement of the step nature of the general procedure, though her modeling was seldom removed from the textbook provided verbal problem examples or exercises. The quantity of examples modeled daily generally included the review of two or three problems of the previous day's assignment, and the modeling of one or two example problems during the lesson development segment.

Generally Betty daily modeled two verbal problem solutions during the homework review segment and the one provided textbook example or sample of a verbal problem set during the lesson development segment. She also made explicit reference to the steps of the general procedure as she modeled each verbal problem solution. Betty's instructional modeling was the least, both in terms of quantity and quality, though still it appeared to be potentially adequate.

Teachers exert some control over the extent of student practice of the general procedure, both by the quantity of verbal problems they assign and by their expectations regarding student effort towards completing

these assignments. The extent of student practice of the general procedure in Alice's class exceeded that of the other two teachers. In addition to four assignments of verbal problem exercises taken from supplementary course materials, Alice assigned over 90 percent of all verbal problems contained in the textbook. She expected students to daily attempt completion of these assignments and daily provided class time for students to begin these assignments, thereby insuring that all assignments were capable of completion within reasonable time frames. It appeared to the investigator that Alice's expectations were generally accomplished by her students, in that all students generally derived solutions for at least 75 percent of all verbal problems in every daily assignment.

Carl's daily verbal problem assignments generally represented 50 percent of all textbook verbal problem exercises. He selected the assigned problems from a verbal problem set carefully however, so that they represented a diversity of contexts and variable demands on student thinking and processing. Homework assignments were of manageable length, composed of less than ten problems, and dependent on students' abilities were generally capable of attempted completion in less than one hour's time. All Carl's students appeared to have attempted solutions for all verbal problems assigned daily, and Carl kept his students accountable for this

expected daily effort in completing homework assignments by recording grades for homework scores at unannounced points during the semester and collecting students' homework periodically.

Betty's daily verbal problem assignments similarly represented 50 percent of all textbook verbal problem exercises. Yet based upon the investigator's classroom observations of students' assignments and Betty's comments to her students and the investigator, the composition and length of her daily assignments generally resulted in a majority of students not even attempting solution of the assigned verbal problems. Most of the core content's verbal problem sets within Betty's textbook followed instructional chapter sections which were focused on equation solving manipulation skills. In such cases, the following verbal problem set's purpose was to have students practice and apply the newly introduced equation solving skills in a verbal problem setting. Since Betty appeared unwilling to deviate from her instructional pacing pattern of covering approximately one chapter section per day, her daily homework assignments generally included both practice of the relevant manipulation skills of a chapter section and also a sampling of the verbal problem set which followed within a chapter section. The resulting length of such assignments appeared to be beyond the time limits which most students were willing to commit

daily to algebra, and appeared to result in a patterned habit, in which a majority of students neglected to attempt the verbal problem portion of the assignment, and yet did attempt to complete the manipulation portion of the assignment. This student habit was somewhat reinforced by Betty's own dislike of verbal problems and low expectations for her students' ability to successfully comprehend verbal problem content, and also by a lack of any teaching practices, at least initially, designed to hold students accountable for attempting completion of verbal problem assignments. Not until well into the semester when she began daily checking for homework completion did she appear aware that her students had developed this habit of neglecting completion of verbal problem assignments. Additionally, her students seemed to realize that Betty's test and quiz designs contained only a small number of verbal problems, and hence their inability to solve verbal problems would not have any substantial impact on their grade for the course. Betty's awareness of the problem and her initiating attempts at remediating this student habit of neglecting verbal problem assignments may have come too late in the course to improve some students' awareness and incorporation of the general procedure in solving verbal problems.

The posttest results for Betty's class support the investigator's observations described above. Four verbal

problem types, simple translation, integer, geometric and age related problem types were treated as applications within textbook manipulation sections. For all four of these problem types, Betty's class performance was significantly lower than was the performance for the other two classes. On the other hand, distance/rate/time and coin/money related problem types received separate chapter section treatment in Betty's textbook, and the results for Betty's class for these problem types were more similar to the results for the other two classes.

Both Alice and Carl expected and received rather complete student attention during their daily instruction, modeling and review of examples and exercises. For Alice's class, this student attention was primarily a passive attentiveness, with the exception of students active in the writing up on the chalkboard of their verbal problem solutions during the homework review segment. A majority of Carl's students remained more actively attentive, and often highly participatory during all instructional lesson segments. On the other hand, Betty's instruction, modeling and review of verbal problem exercises and examples failed to motivate a majority of students to even listen attentively, and sadly but not surprisingly, the nonattentive students tended to be the same ones who had not attempted completion of the previous day's verbal problem assignment. In effect the

nonattentiveness of this group of students prevented them from making any progress or improvement related to verbal problem content or for the learning and incorporation of a general procedure.

The fact that 15 of Betty's 27 students demonstrated general procedure incorporation on the posttest attested to the fact that Betty's usage of verbal suggestions and her instructional modeling related to the general procedure was adequate for these students. These 15 students were also more likely to be attentive during Betty's instruction and had adequately practiced the general procedure in completing the assigned verbal problems. The fact that the 12 students who demonstrated general procedure incorporation on the retention test were a subset of the 15 students demonstrating incorporation on the posttest adds further credence to the argument that Betty's adjustments to improve student accountability had minimal impact, or came too late in the course to effect student learning and incorporation of the general procedure. The retention rate, 12 of 15 students or 80 percent, was quite high and provides additional evidence that Betty's initial instruction and modeling of the general procedure was sufficient for this group of students who demonstrated personal effort and daily attentiveness to Betty's instruction.

Attempting to identify which of the four factors

accounted for the differences between Alice and Carl's students' incorporation of the general procedure for the posttest and retention test results is more difficult. In both classes, all students were motivated to be attentive during classroom instruction and to attempt completion of verbal problem assignments. Students in both classes generally accepted these responsibilities without much teacher encouragement or threat. The fact that Carl's students demonstrated greater awareness and usage of the general procedure than Alice's students within their solutions on the posttest suggested that more extensive teacher modeling of the general procedure combined with a greater quantity and variety of contextual examples lead to greater student incorporation of the general procedure, at least for this acquisition phase of learning. In effect Carl's better instructional modeling appeared to compensate for the fewer explicit verbal suggestions he employed and the less extensive practice of verbal problems his students experienced.

Alice's greater usage of verbal reminders related to the general procedure and the greater extent of student practice of verbal problems during the acquisition phase in her class appeared to explain the better retention of the general procedure by her students, though the separation of each factor's effect was not possible from the student performance results. It would seem more

logical however, that the better retention of the general procedure was due to greater student practice in solving verbal problems. An alternate, yet possible, explanation for the differing degree of general procedure usage for Alice and Carl's classes on the two instruments, related to the actual verbal problems used in each instrument's design. The nine verbal problems of the posttest were designed by the investigator and the context situation for two of these verbal problems was posed differently from those encountered within textbook exercises. There were only small differences in student performance for these two items, slightly favoring Carl's students over Alice's students, and Carl's inclusion of examples demonstrating greater context diversity within his instruction may have accounted for these minor differences in students' posttest performance. The seven verbal problems of the retention test however were drawn from the study's three textbooks. Alice's students performed better and demonstrated greater retention of the general procedure on this instrument, possibly because Alice's instruction and modeling of the general procedure tried to develop clearly separable schema for each textbook problem type based on the contextual settings of the various problem types, and students practiced such textbook problem types more extensively.

Conclusions

1. All three teachers' instruction contained sufficient treatment and reinforcement of a textbook's general procedure for solving verbal problems to lead the investigator to conclude that each teacher considered student incorporation of a general procedure an important objective of their verbal problem instruction. Additionally each teacher consistently utilized a general procedure as a guiding framework for their discussions of solutions to verbal problem examples and exercises.
2. It is probable that a general procedure will become a part of students' working repertoire for solving verbal problems, if students demonstrate reasonable attentiveness to teachers' verbal problem instruction and follow the reception of this instruction with a reasonable effort aimed at completing teacher assigned verbal problem sets. General procedure retention however is more dependent on both the extent of student practice and the degree of student effort and attention put forth during the initial learning of the general procedure.
3. Of the three teachers of the study, Alice's instruction was the one most specifically

targeted toward this objective, and all her students demonstrated general procedure incorporation at both testing points. Teaching factors which influenced Alice's success included her constant and explicit reminders to individual students to always show all steps of the general procedure within their written verbal problem solutions, adequate quantity and excellent consistency of teacher modeling of the general procedure, and provision for extensive student practice of the general procedure.

4. Carl's daily instructional pace and assignment patterns for verbal problem sets were probably more typical of most Algebra I teachers' instruction than was Alice's pattern of assigning nearly all textbook verbal problems of a set over a two day period. Carl's careful selection of verbal problems for homework assignments and his more extensive modeling of a general procedure in solving both textbook and nontextbook examples somewhat compensated for less student practice in his class, and still resulted in a high degree of student demonstrated incorporation of a general procedure.
5. A large number of Betty's students failed to accomplish general procedure incorporation, and

reasons for this failure were in large part due to these students operating at low levels of attentiveness and effort during Betty's initial verbal problem instruction when the general procedure was introduced and reinforced with practice exercises. Betty appeared unaware that these students were failing to practice the procedure within their assignments and were not comprehending the concept of a general procedure during these initial phases of instruction. Once she became aware of these students' predicament, her teaching adjustments focused only on making students accountable for completing her verbal problem assignments, and did not attempt to remediate student comprehension difficulties related to a general procedure. Students adjusted to these new teacher demands by continuing to leave such exercises largely incomplete, or copying other students' solutions (giving up personal responsibility in a sense), or resorting to nonalgebraic strategies which were less likely to be successful as the level of problem difficulty increased during the course.

Recommendations

1. Algebra I teachers' initial verbal problem instruction is crucial in its effects for student learning and incorporation of a general procedure. Initial instruction should stress a general procedure's importance as a planning structure or guiding framework to follow in solving most verbal problems. Additionally, teachers need to simplify the textbook provided procedure into its central, written algebraic components to facilitate student encoding and later its recall with regards to its form and sequence, and also student understanding of its inclusion and relationship to other learned algebra skills.
2. During this initial phase of verbal problem instruction, teachers are primarily shaping student behavior to conform to the step by step processing of a general procedure, and should realize that this may be the first such occasion for some students where a mathematics teacher has expressed interest and concern for the entire solution process rather than just the answer to a verbal problem. Teachers should gently encourage students and carefully monitor students' progress to insure that students show all necessary

written work for each step of the general procedure, even for the simplest of verbal problems, and are experiencing some success in applying and following the steps of a general procedure. Betty was the least effective teacher in carrying out these two teaching functions during this initial phase, and the results for her class clearly demonstrated their respective importance.

3. Once this initial phase of instruction has successfully laid the groundwork for future verbal problem instruction and general procedure incorporation, teachers have several available options for solidifying and reinforcing learning related to a general procedure. A teacher, such as Alice, who strongly believes in the power of student practice for student learning and incorporation, and who appears to prefer one to one diagnostic-remediation teaching rather than whole class instruction should provide greater student opportunities for practice, and additionally set aside blocks of class time daily for students to begin these assignments where the teacher or class aide can assist students individually in their processing attempts. Some teacher explanation and consistent modeling of

the general procedure is still required for each new verbal problem type encountered, so that students understand the central relationships employed in each verbal problem set, and also are aware of teacher expectations for the contents of their written solution formulations.

More teachers probably share Carl's belief that student verbal problem learning is enhanced in a whole class instructional format. In this case, teachers' instruction should highlight (much as Carl's instruction did) a general procedure by providing explicit and extensive demonstration of its usefulness in solving verbal problem examples and exercises, in which students come to know teacher expectations, and which also encourages greater student interaction and participation in the process.

4. Teachers should exhibit care in their selection of textbook verbal problem exercises for student homework assignments, and also control the length of these assignments so that students can complete them in a reasonable period of time. Betty's practice of combining both manipulation and verbal problem exercises of a chapter section into a single and often lengthy homework assignment resulted in generally diminished

student effort and even some indifference to completing the verbal problem portion of the assignments. Hence it is suggested that teachers emulate the practices of the two other teachers of the study who structured the length and variability of their assignments more reasonably, possibly based on their greater experience teaching this level of student, and who also avoided mixing manipulation and verbal problem exercises in the same assignment.

Influences of Textbook on Teachers' Instructional Routines

Textbooks serve distinct functions for the two participants (teacher and student) of an Algebra I classroom. For students, textbooks are potentially a source for written explanations, demonstrations of sample solutions, and practice exercises. For the teacher, textbooks serve a central role, in that they provide a complete course catalog of content objectives in an organized framework which teachers use to determine the sequence, planned design, and delivered form of their instruction. The second hypothesis for the study addressed the question of what aspects of teachers' instructional routine are most influenced by employed textbooks.

Hypothesis 2: The employed textbook has great influence on teacher instructional decisions related to verbal problem content in regards to sequencing, planning, lesson development, and presentation.

Findings

Betty's sequencing of content never once deviated from the employed textbook's sequencing, as she daily worked her way through the textbook, section by section, in her instruction. She believed her textbook contained adequate coverage of all content, and hence no supplementary materials were ever used in her classroom. She also indicated on the Teacher Inventory Packet that it was important to her and her students that they know exactly where in the textbook they would be at any point in time during the school year. Her rigid adherence to the textbook's content sequencing was one way to insure students knew such information. Such consistent sequencing also had advantages for the management of both daily and weekly student absences which were more likely to occur in Betty's larger class. It was also possible that less familiarity with the course's textbook, less experience teaching this course, or the ease of planning which resulted from such sequencing were other reasons for Betty's decision to sequence her instruction exactly according to that suggested by the textbook.

Alice and Carl did deviate more from the textbook's

sequencing than did Betty, though both teachers' instructional sequencing primarily followed the suggested textbook sequence. Carl occasionally brought together sections from different chapters to form units of instruction which had a common focus, such as all equation solving technique sections. Alice's few reorganizations involved reordering the sections of a single chapter, so that initially all manipulation related sections were treated and then were followed by treatment of the chapter's verbal problem sections.

Both Alice and Carl appeared more aware of their respective textbooks' instructional and exercise limitations related to verbal problem content. Alice used a different set of older textbooks to provide students with additional practice on two verbal problem types, and also graded her students' performance on three instruments of ten verbal problems each taken from additional supplementary materials. Besides Carl's usage of nontextbook examples in lesson development segments which he created, he also developed a review sheet of 12 verbal problems, taken from old tests, composed of mixed practice of all six verbal problem types since his textbook failed to provide such mixed practice.

The organization of a textbook also exerted influence on the pace or speed of instruction each teacher decided to employ in their classes. The pace of Alice's

instruction was the most stable and consistent of the three teachers. Most verbal problem exercises in Alice's textbook were organized into chapter sections. These verbal problem chapter sections were covered by Alice over a two day period, and included two short explanatory lectures, one each day, two verbal problem assignments, odd numbered exercises the first day and even numbered exercises the second day, and two homework review segments, one per assignment. Alice's instructional pace then guaranteed a minimum of two day's worth of class time for five of the six verbal problem types of the study. The one exception, the coin/money problem type came later in Alice's textbook, with systems of linear equations. Alice's students practiced this one verbal problem type only one day, in which Alice reviewed the two examples and assigned ten problems from another textbook. Alice generally covered nonverbal problem sections of her textbook in one day, and her ability to accommodate this double dosage for verbal problem sections was accounted for by her rapid coverage of the first two chapters of the textbook, based on her assumption that this initial course content represented a review of that covered in the eighth grade program of these students.

Betty's instructional pacing was highly dependent on her textbook's sectional organization, and exhibited no differentiation between verbal problem sections and

manipulation oriented sections. Betty attempted to cover one textbook section of content per day, regardless of its size or content. Generally four textbook sections per week were covered, since usually one day per week the length of her homework review segment did not permit sufficient time for lesson development of the next section. However students' lack of comprehension of certain verbal problem content never appeared to effect or change Betty's pace of instruction. Three of the six verbal problem types of the study, age related, distance/rate/time related, and coin/money problem types, were contained in separate sections of the textbook, and hence received the equivalent of one day's instructional treatment. As part of a manipulation textbook section, the consecutive integer problem type received less than 15 minutes instructional treatment. While student practice of the other two verbal problem types was greater than one day, the placement of these exercises was diverse and received less explicit and structured instructional treatment by either Betty or her textbook.

Carl generally paced his instruction to treat one content section of his textbook each day. His textbook had the greatest number of distinct sections devoted to verbal problems and never combined verbal problem applications with related manipulation skills in the same textbook section. In two observed cases Carl did combine

two verbal problem sections into one day's instruction, where consecutive textbook sections involved only minor variation of applied central relationships. On one observed day he did not begin a new section of content when it became apparent during the homework review segment that a majority of students had failed to comprehend the previous day's assignment and additional review was necessary. Carl exhibited the most flexibility in his daily pacing of instruction, based in large part on how students were progressing in correctly completing assignments. Alice also demonstrated such awareness of student progress and flexible pacing, but to a lesser extent.

The textbook served as each teacher's major resource for their written formulation of lesson plans. These weekly written plans were brief and noted the relevant textbook section, the major concepts or relationships developed, a brief note on the number and type of textbook examples present, and the intended assignment from the section's exercise set. Assignment patterns for Betty and Alice demonstrated amazing consistency from day to day, leading the investigator to conclude that neither teacher closely reviewed exercise sets for potential difficulties related to vocabulary or differing problem context, as Carl's more irregular selection of assigned problems and his instruction of difficulties appeared to indicate.

Carl's lesson plans related to verbal problems also included reference to a real world situation he intended to introduce in the day's lesson development segment which he believed had greater potential than the textbook examples to motivate and interest his students in the content.

Textbooks A and B's occasional placement of both manipulation skill and verbal problem exercises together in the same chapter section of a textbook had the potential to present planning difficulties for these teachers. Alice split such chapter sections into two days of instructional treatment, one day for each skill focus, and in a sense always maintained separation of verbal problem content from algebraic manipulation content. On the other hand, Betty attempted to cover the entire section in one day, including coverage of examples for both focuses, and made a single and generally longer assignment for these sections composed of both manipulation and verbal problem exercises.

Each teacher's lesson development for verbal problem sections began with one or two minutes devoted to discussing the central relationship used within the associated verbal problem type solutions (for formulating equations). Betty simply restated the textbook's provided explanation of the relationship as the class read along with their textbooks open to the requested page. Carl and

Alice demonstrated and briefly explained the relationship with students having their textbooks closed, though generally Alice's explanation more closely resembled that presented by the textbook. In both these classes, student attention was directed to the teacher, and not divided between textbook and teacher as in Betty's class.

Once this introduction of central relationships was completed, Alice then directed students to open their textbooks to the relevant section, and to read the first provided solved example silently. With minimal student participation, Alice then modeled the logical thinking and processing steps she wanted her students to follow in a second reading of the example. If more than one solved example was provided in a section, they were covered in a similar manner. Once this process was completed, Alice selected an unsolved verbal problem from the exercise set for class discussion and solution. Students silently read the exercise, and then Alice guided the discussion of several selected students, seeking class consensus for what she wrote of the solution process on the chalkboard. For controlled practice, Alice assigned an additional exercise from the set for which each student attempted the solution process individually. After adequate wait time, Alice demonstrated how she expected students to write their solutions, and reinforced the step nature of processing and thinking she required of her students.

Carl's lesson development segments bore some resemblance to Alice's, but were more extensive and ordered differently. Carl moved from class discussion of the central relationship, to class discussion and multiple student participation in the suggested solution processes related to nontextbook examples created from the proposed, simulated setting he developed. Three or four such examples were treated at this time with active class participation in the derived solutions. Carl then directed students to open their textbooks to the related section, and he explicitly reviewed the written steps and the justification for each written textbook action for the three to five textbook provided examples in each section. For controlled practice, one exercise was selected from the problem set for students to do individually. Students then explained their solution process for the exercise, and Carl recorded the associated written actions on the overhead as the student directed him. Once a correct solution was derived in this way, Carl then encouraged students who may have used alternate solution paths (correct or incorrect) to share their processes with the class. Carl and the class then analyzed these alternate methods for their relative efficiency if the method led to correct solutions, or for logical weaknesses if the method led to an incorrect result. The investigator sensed that these discussions concluded with most students sharing a

common perspective for how to efficiently and correctly proceed with the day's verbal problem assignment.

Betty's lesson development segments followed exactly the textbook provided explanation of a section. She or students read all textbook material and the problem statements for provided examples. Betty encouraged, through questions directed to individual students, assistance in the solution process for examples as she demonstrated the written steps of the process on the chalkboard. Students appeared mostly apathetic to her desire for their participation, since the questions were mainly aimed at nonattentive students and only required an ability to restate the steps of the solution process which the open textbook provided. Her textbook's relative small quantity of verbal problem examples generally meant only one example or sample was covered prior to assignment. On two occasions, no textbook example was provided for a verbal problem exercise set. Betty proceeded to select the first exercise from the set for class discussion, and student attention and participation was notably better within these two discussions than that exhibited for the solved textbook examples' discussion. Only one instance of a controlled practice segment was noted in Betty's class.

Except for a few additions by Alice, all verbal problem assignments for each teacher came from their

respective textbooks. This pattern indicated that the teachers were generally pleased with the quality and quantity of each textbook's verbal problem exercises. However this also implied that the quality of textbook provided exercises largely determined the upper bound of teachers' expectations of students with regards to problem solving content for the course.

A priority for the state in which the study was conducted is that teachers incorporate in their instruction the principles of an "effective teaching" model, and all three teachers' instruction indicated some awareness of effective teaching's major elements. However not a single occasion was observed where a teacher's instruction included a separate drill review, mental mathematics exercise, or problem solving segment as is recommended by the model. A probable explanation for this omission from the teachers' instruction was the lack of such included materials within the respective textbooks or teacher manuals, though Alice did include the manipulation skill sharpeners her textbook provided within her instruction. Since all three teachers generally believed that such segments represented an inefficient usage of class time and tended to detract from the focus of the daily lesson plan, it is somewhat questionable whether these teachers would have included this segment even if the textbook provided such aids in its development.

Lastly no teacher's instruction or assignments ever required all students to read textbook explanation sections or review solved textbook examples on their own. Rather each teacher's lesson development segments included explicit treatment of all textbook provided explanations and review of all examples of a chapter section, essentially making student reading of algebra content explanations and text unnecessary. Only students who missed a teacher's instruction for a given day were ever encouraged to read textbook explanations prior to beginning related assignments, and it can be assumed that most such students probably only read textbook explanations after they had become stuck on a verbal problem of an assignment. In effect, the three teachers were not training their students in how to read and comprehend mathematical text, as is required in other disciplines such as science or history. Such a teaching practice precludes any possibility that these students will construct and incorporate mathematical knowledge on their own, or become independently responsible for their learning. Rather in all three classes, students daily appeared to become more dependent on the teacher's provided instruction, more mechanized and conforming to teachers' expectations in their verbal problem solutions, and largely failing to gain an appreciation for the logical, interconnected structure of algebra material and

Lastly no teacher's instruction or assignments ever required all students to read textbook explanation sections or review solved textbook examples on their own. Rather each teacher's lesson development segments included explicit treatment of all textbook provided explanations and review of all examples of a chapter section, essentially making student reading of algebra content explanations and text unnecessary. Only students who missed a teacher's instruction for a given day were ever encouraged to read textbook explanations prior to beginning related assignments, and it can be assumed that most such students probably only read textbook explanations after they had become stuck on a verbal problem of an assignment. In effect, the three teachers were not training their students in how to read and comprehend mathematical text, as is required in other disciplines such as science or history. Such a teaching practice precludes any possibility that these students will construct and incorporate mathematical knowledge on their own, or become independently responsible for their learning. Rather in all three classes, students daily appeared to become more dependent on the teacher's provided instruction, more mechanized and conforming to teachers' expectations in their verbal problem solutions, and largely failing to gain an appreciation for the logical, interconnected structure of algebra material and

its application in verbal problems. Only Carl's instruction indicated any teacher attempt to broaden students' perceptions of their own capability to construct and logically interrelate their algebra knowledge. His probing discussions of realistic, nontextbook examples required students to explain justifications for their suggested actions and encouraged divergent thinking of procedural alternatives. Both Alice and Betty appeared to operate under an assumption that students at this level of mathematical competence were largely incapable of reading and comprehending mathematical text, and hence structured their instruction more closely to that provided by their respective textbooks to overcome this believed deficiency. Not surprisingly, the nine interviewed students all indicated that their first choice for assistance in completing an assignment would be an authority person, most likely their teacher, but also a parent or older sibling. Only one of the nine students mentioned the textbook as a possible source for assistance, and it was that student's third listed source.

Conclusions

1. The generally held belief that secondary mathematics teachers rely heavily upon textbooks for the development of their instruction was substantiated for the three Algebra I teachers of the study. Specifically, the content sequencing,

the pace of instruction, the content and length of teachers' lesson development presentations, and the assignment patterns related to verbal problem content were all largely determined by the organization and contents of the employed textbooks. Whether such influence extends to other algebra content besides verbal problems was less clear, though the limited observations of such other content instruction gave every indication of a similar pattern of influence.

2. Teacher adjustments to textbook provided explanations and examples, including provisions for supplementary materials, was nonexistent for Betty's instruction, at a minimal level for Alice's instruction, and somewhat greater for Carl's instruction, particularly his daily lesson development segment. The limited nature of teachers' modifications to textbook verbal problem content meant that these students' exposure to mathematical problem solving was mostly confined to the learning of how to process solutions for typical textbook verbal problems.
3. Since the three teachers' instruction included explicit treatment of all textbook provided explanations and examples, students were never required to read or comprehend mathematical text

explanations or solved example presentations on their own. As a result, students tended to become more dependent as the year progressed on their teacher or other authority figures in their families for constructing the nature of their learning for them, rather than becoming more independent learners, responsible for their own construction of knowledge. Particularly in Alice and Betty's classes, students also became more mechanical in their thinking related to verbal problems, and in their attempts to conform to teacher expectations for written formulations of solutions.

Recommendations

1. Algebra I teachers should provide some opportunities for this level of student to read mathematical text and follow the mathematical logic and algebraic techniques employed in solved textbook examples, thereby enabling their students to become more independent learners of content and more flexible and diversified in their solution processes within mathematical problem situations. Teachers in their instruction can and should broaden student perceptions of the real world applicability of verbal problem techniques with a minimal amount

of effort and creativity required, much as Carl's instruction indicated.

2. Mathematics educators responsible for methods courses for preservice secondary teachers should include additional training in how to properly analyze and critique textbooks' organizational patterns, and practice designing daily lessons and assignments which build upon textbook provided instruction, rather than merely restate such instruction. Without such specific training directed at forming teachers to be reflective critics and evaluators of their own and textbooks' instruction, there is a greater tendency for beginning teachers to develop habits of total dependency on textbooks for the content and presentational form of instruction.
3. Since the curriculum for the Algebra I course appeared largely determined by the selected textbook for the course, responsibility for what constitutes the course content, its sequencing and pacing, the form of its instructional delivery, and the degree of challenge presented largely resides with textbook authors and publishers. Admittedly research has provided little direct assistance to authors in their attempts to carry out these curriculum

responsibilities with regards to placement of verbal problems, how they should be designated, or even the intended purposes they should serve for the course material. Present Algebra I textbook constructions severely limit students' opportunity to become any more than mechanical thinkers and processors of verbal problems, rather than the independent and flexible real world problem solvers desired for all citizens of our culture. At a minimum, textbook authors should include practical, real world situations (as Carl did), and samples of drill review, mental mathematics, or problem solving segments in their teacher editions, so that teachers can become more aware of available alternatives they might add or develop in their instruction. Some teachers, such as Alice and Betty, would likely ignore such alternative suggestions since they believe provided textbook explanations and examples are sufficient for their purposes and fit well with their instructional routines and methods. Other teachers, particularly beginning teachers would more likely appreciate the flexibility provided by alternative suggestions in their planning activities, and would be more likely to experiment with differing instructional

formats and methods geared towards improving both student performance and teaching effectiveness.

Teacher Characteristics' Influence on Instructional Routines

An assumption made at the outset of the study was that teachers' daily instructional routines and teaching practices become highly stable through their experience teaching the Algebra I course over a number of years. Based on this experience, teachers develop personal goals, attitudes, and beliefs about both verbal problem content and the capabilities of this level of student which influence teacher decisions with regards to the form, duration, and contents of instructional routines generally, and lesson development segment characteristics specifically. The primary source for the findings regarding teachers' beliefs about verbal problem content, teaching practices, and student capabilities came from teacher responses on the Teacher Inventory Packet, while investigator observations of each teacher's instruction provided data for the formulated composition of each teacher's daily instructional routine. The only hypothesis of the study which reflected the influence of teacher characteristics on instructional routine was the following:

Hypothesis 3: Teachers' instructional goals, expectations, and attitudes toward verbal problems have a substantial impact on the nature of lesson development segments and classroom routines.

Findings

All three teachers believed that a teacher centered, traditional expository style of content presentation and demonstration was the most effective format for verbal problem instruction. Greater variations occurred between teachers with regards to their perceptions for the purposes of algebra and verbal problems, capabilities and needs of their students, and their role in the learning process. Despite a common belief in the effectiveness of whole class instruction done in an expository mode, the three teachers' instructional routines were quite different, and teachers' conceptualizations in these three areas largely accounted for most of these differences. Within this section, the findings related to teachers' conceptualizations in these three areas are reviewed initially, and then the investigator speculates on how these teacher conceptualizations translated into each teacher's instructional practices and routines.

Alice and Betty perceived the primary purpose of Algebra I to be equipping their students with the necessary algebraic techniques deemed important for students to function in the abstract world of mathematical symbolization. Both teachers rated solutions of first and

second degree equations and simplifications of algebraic expressions/abstract symbolizations as high priority objectives, along with the solution of verbal problems/applications. Additionally, both teachers listed student application of algebraic techniques and improvement of students' reading comprehension and translation skills as primary purposes for the inclusion of verbal problems in the course. Secondarily, both teachers noted that verbal problems foster ordered and structured thinking abilities.

Carl, on the other hand, rated all specific algebraic technique objectives, including the solution of verbal problems, lower and believed rather that algebra served primarily to connect and expand student understanding of rules established in arithmetic, and assist student understanding and justification capabilities for mathematical concepts. For Carl, verbal problems were included in the course to "get students to think logically, abstractly, and deductively".

Alice's perceptions of entering students' capabilities and needs were the most clear and apparent of the three teachers, possibly because she was the only one of the three teachers to regularly teach this level of student in eighth grade content courses. She believed that this level of student entered Algebra I operating with generally weaker and highly mechanical arithmetic

skills, or at a rote level of understanding. Secondly, she sensed that a major obstacle to these students' algebra learning was their inability to recall previous knowledge and relate it to new content in a meaningful way. Thirdly, she perceived these students to be less confident, and more aware of their limitations with regard to their mathematical abilities, and hence less participatory in whole class settings for fear of teacher and peer ridicule. Rather she perceived these students as becoming more accustomed and dependent on one to one teacher attention as a preferred learning style. Fourthly, she believed and expected all these students to become successful at solving verbal problems, provided they received sufficient, structured guidance and reinforcement from the teacher. As their teacher, her responsibilities included providing such structured guidance through modeling and verbalizations, providing extensive practice to combat deficiencies in student recall abilities, and providing adequate seatwork time daily for students to receive personal teacher assistance in their learning. Her teaching roles during class time were to organize and structure student learning, and be a helper or mentor in the learning process.

Betty appeared the least confident of the three teachers with regards to all students having the potential to become successful at solving verbal problems. As

compared with other algebra content, she rated her ability to teach verbal problem content somewhat lower. She also commented that students' difficulties with verbal problems were often due to "an inbred anxiety and fear about word problems". The investigator's observations of her instruction gave every indication that Betty appeared to expect only her better students to be capable of successfully understanding and completing verbal problem assignments. Her better students' daily verbal problem performance largely determined her instructional decisions to move on to new content, a pattern not evident for other algebraic manipulation content where she expected a higher success rate of all her students. Additionally her course evaluation instruments (primarily tests) included only minor assessment of verbal problem solving abilities, less than 15 percent of content and score for all instruments.

Betty appeared well aware of these students' generally lower mathematical abilities and the wider range of these abilities, as compared with accelerated students, but she appeared largely unaware and unconcerned about the specific nature of her students' learning weaknesses, needs, or preferred learning styles. She expressed a willingness to help students with their difficulties, but requesting such help was the students' responsibility, and was done generally outside of class time. Students were successful in her Algebra I course based on the degree of

personal responsibility they took for learning the course content. Her major role as teacher was to serve as transmitter and mediator for the course content between textbook and students, insuring that all content was covered and presented in a manageable form. Her secondary role was to grade students and evaluate their learning and effort in the course.

Although his perceptions were less apparent to the investigator, Carl appeared to operate under assumptions similar to those of Alice with regards to these students' mathematical and learning limitations. He also realized that his students had developed a variety of ways and styles of learning, and he showed respect for these individual differences, rather than attempting to bring about student conformance to his expectations and ways. Whereas Alice largely adapted her instructional routine to her perceptions of students' capabilities and needs, Carl designed his instructional routine to address and bring about improvement in students' capabilities. If his students were largely mechanical in their mathematical processes, then his instruction should encourage student discussion not only of what they did, but also the reasons and rationale for their actions. If these students were largely passive and dependent on a teacher as the authority for mathematical correctness, then his instruction should encourage students to take a more

active and participatory role in the process of acquiring knowledge, verbalizing and reflecting daily upon their learning, and becoming more independent directors of their own thinking and rate of learning.

Carl's perceived role for a teacher was as facilitator of meaningful learning, learning which contained both depth and applicability. In this regard, verbal problem content took on added meaning in Carl's instruction since it contained greater potential, than did other algebra content, for class discussion of alternate forms of reasoning and processing and greater immediate applicability. Carl's lesson development segments for verbal problem content were both longer and involved greater student interaction than for other algebra content, and indicated Carl's awareness for such added functions for verbal problem content. He did not however force his teaching methods and beliefs on those students whose behavior indicated a preference for a more passive posture to learning the course content. He was never observed by the investigator to call on a student who did not volunteer to participate in class discussions, and he generally used daily seatwork time to assist these more passive students individually, while also insuring that each of these students had comprehended the main points of the day's lesson and could successfully begin their assignments.

In summary, the structure of each teacher's instructional routine was largely determined by their conceptualizations of the purposes for verbal problem content, their students' capabilities and needs, and their perceived role in the learning process. Alice adapted her instructional routine to fit her perceptions of these students' learning limitations and dependence, while placing a high priority on verbal problem content. Her students were most active during the homework review segment, which represented a nonthreatening situation for these students. Lesson development segments were deliberately kept brief to permit every student to receive some one to one personal teacher assistance during the longer seatwork segments, which were never less than 15 minutes. Lastly the time students spent practicing verbal problem solutions was twice that for other sections of content.

Betty placed a lower priority on developing students' verbal problem abilities in her instruction than she did for other algebra content, and in a sense relegated verbal problems to an accomplishable objective for only her better students. Her instruction indicated less awareness and concern for students' capabilities, difficulties, and needs. Within her instructional routine, she carried out her role as mediator of textbook provided content, and rather obligingly and somewhat unenthusiastically carried

out other teaching functions such as homework review and checking, evaluation of student learning, and classroom management. In Betty's class more than the other two classes, responsibility for learning verbal problem content rested in large part on students' shoulders, and their willingness to seek out additional help, pay attention to Betty's lectures, restudy textbook examples, and put forth effort on their verbal problem assignments.

Compared to Alice and Betty, Carl had higher purposes, beyond the textbook content, for the Algebra I course. Even if Carl disliked verbal problem content and its textbook presentation, he did recognize and use verbal problems' potential for developing the kind of meaningful and applied learning situations he wished to encourage throughout the course. His skillfully directed class discussions and posed questions during the homework review and lesson development segments demonstrated that teachers can improve this level of students' capabilities to learn from active and reflective teacher-student interactions. The investigator sensed that once these daily discussions were concluded, most students had comprehended the verbal problem material adequately to confidently and even enthusiastically complete their homework assignments individually. In Carl's class, there was also a sense that students were taking greater responsibility for their own learning as the school year progressed, towards an

acceptance of a mutual respect for each participants' (teacher and student) role in the learning process.

Conclusions

1. The hypothesis that teachers' instructional goals, expectations, and attitudes toward verbal problems have substantial impact on the nature of instructional routines, received support from the findings. Teacher's perceptions for the relative importance of verbal problems in relationship to other objectives for the entire Algebra I course resulted in different levels of instructional treatment and coverage of verbal problem content. Alice ranked verbal problems among other manipulation skill objectives, and her instructional time for verbal problem sections was twice that for other content sections. Betty ranked verbal problems right below the two manipulation skill objectives, and in a sense verbal problems served primarily an application or finished product function for manipulation skill content. Her instruction downplayed the importance of verbal problem skill development as a course objective, based largely on her belief that not all students were capable of success with verbal problems. Though Carl ranked verbal problems low as a course objective, his

instruction was both longer and more interactive for verbal problem content, because the verbal problem content provided a better environment for accomplishing those objectives he ranked highest.

2. The study identified two other classes of teacher perceptions which had implications for developed instructional routines, namely teachers' perceptions of student capabilities and needs, and the teachers' role in the learning process. Betty's expectations for students were low for verbal problem content, and combined with her perceived role as mediator of textbook content resulted in predictable instructional routines built around textbook presentations. Alice and Carl's perceptions of students' capabilities were similar, though more apparent and operable for Alice. However because of differing perceptions of their roles in the learning process, their instructional routines were notably different. Alice molded her instructional routine to fit her role as helper and mentor and to accommodate perceived student weaknesses and needs. On the other hand, Carl designed his instructional routine to encourage change and improvement in perceived areas of student weakness, and facilitate greater student awareness of their

thinking processes, and therefore become more responsible for their learning.

3. Of the three teachers, only Betty expressed some uncertainty and insecurity in her ability to teach verbal problem content, and also mentioned an effective reason (inbred anxiety) for students' difficulty in solving verbal problems. Generally her instruction reflected a neutral attitude towards verbal problem content, something she was obligated to teach because it was in the textbook. However there were several observed occasions, when in beginning new verbal problem content, her subtle comments indicated a more negative attitude, and she never scolded or corrected students who indicated their displeasure for verbal problems by howling or otherwise commenting out loud. Her class's attitude towards verbal problems was more negative than the other two classes, and suggested that teachers' attempts to mask their attitudes and beliefs from students, and avoid transmitting them to their students, even when they are aware of them (which in Betty's case was somewhat questionable), is not likely to be successful.

4. The study's findings suggested a possible relationship between the length of teaching experience and the dominance among various conceptualizations teachers use in designing the specific nature of their instructional routines. With the least teaching experience, Betty's instructional routine was largely determined by her perceptions of the content purposes for the course, and showed minimal awareness or concern for students' capabilities and needs. With greater teaching experience, Carl was more aware of students' capabilities and needs, though these beliefs were still prone to fluctuations gained from new experience, and remained at an unconscious level of thought. His instructional routine represented a blending of his perceptions of course objectives and purposes with his beliefs about students' capabilities. With the most teaching experience, Alice's instructional routine was mostly shaped by her perceptions of students' capabilities and needs, which were highly conscious and solidified to the point that newly encountered experience was not likely to change or modify them. The suggestion is that teachers' perceptions change with gained teaching experience in predictable ways, though due to

the small sample of teachers in the study, it is equally plausible that these differing conceptual frameworks of teacher operation developed independently of experience.

Recommendations

1. The purpose and related instructional objectives for the inclusion of verbal problems in the Algebra I course remains a troublesome issue for mathematics educators, textbook authors, and practitioners (teachers), and needs further clarification and resolution. Educators believe verbal problems occupy a broad middle ground between exercises and "genuine" problem solving. Textbook designs based on placement and organization of verbal problem sets indicated differing degrees of primacy given for application and problem solving functions. The Algebra I teachers believed that verbal problems improved students' reading comprehension and mathematical translation abilities, as well as their ability to think logically and sequentially, though specifying how these objectives are accomplished by verbal problems remains less clear. If improving students' reading comprehension was an important objective for the teachers, one wonders why teachers

provided no opportunities for students to read mathematical text other than verbal problems.

2. The study's findings supported other research results indicating minimal transfer effects from verbal problem solving situations to "genuine" problem solving situations. The investigator believes that Carl's verbal problem instruction provides a possible guideline for the direction teachers' instruction should attempt to move. Teachers should connect the contexts and central relationships contained in verbal problem sets to more realistic situations in their instruction, and involve their students more actively in the solution process, and in which teachers can then reinforce the need for and importance of "thinking algebraically" in problem situations.
3. Since only one teacher appeared consciously aware of their perceptions and associated influence on their instructional routines, teachers should periodically be provided opportunities to participate in training activities which aid their abilities to identify and reflect upon their developed conceptualizations of the instructional goals for the courses they teach, of the capabilities and needs of students they teach, and of their role and preferred teaching

style. In some of these areas, such identification and awareness instruments are presently available, while for other areas they are not. These suggested training activities should be carried out in a confidential and nonthreatening manner, and be directed by either designated "master teachers" of a school district or by consulting teams from outside the school district at regular two or three year intervals, with voluntary teacher participation, and also should be done in preservice methods courses. The training should increase teachers' awareness of their operational frameworks, and teaching strengths and weaknesses, while to a lesser extent, encouraging teachers to constantly reflect upon and evaluate their instructional routine, and consider changes they might institute due to their increased awareness. Inservice programs designed to bring about change in teachers' usage of instructional methods have been only marginally successful, and there is some question whether established teacher conceptions can be modified at all (Thompson, 1985), yet the suggested training activities appear a logical first step in bringing about change.

4. As an extension of the above recommendation, teacher evaluators (primarily building principals) should be additionally trained in how to characterize teachers' instructional routines, rather than just focusing on teacher actions and performance of duties. Follow up discussions could then be broadened beyond identification of teachers' strengths and weaknesses to include why teachers rationally select the routines they use, and how such routines could be made more efficient to improve student learning, and even modified if both parties felt such action was necessary. Less experienced teachers, such as Betty, are likely to maintain stable and largely ineffective routines unless interventions make them aware of their conceptions' impact on their instruction and provide them realistic alternatives to consider.
5. The role that teaching experience exerts on teachers' conceptions and the resulting impact on instructional routine design needs further study. The study's findings indicated a minor trend where inservice teachers were dominated by one conception for a period of years, were then somewhat more flexible for a period of years, and

then became more inflexible as they neared retirement. Alice may have passed through such stages, and though her instruction appeared effective for student learning (particularly retention), the investigator could not help but sense pessimistic overtones of resignation within Alice's instructional routine, where her teaching was directed at accomplishing simplistic goals. Therefore inservice interventions should be differentially structured for teachers with differing amounts of teaching experience. Early interventions should focus on broadening teachers' conception awareness, and train teachers to be more reflective evaluators of their lesson designs. More experienced teachers should gain more from a discussion of and rationales for alternative instructional methods, and yet remain flexible enough to experiment with innovative instructional routines and teaching methods. The investigator has witnessed some dramatic changes in experienced elementary teachers' instruction gained from their participation in a summer problem solving workshop, and a similarly timed intervention for secondary teachers might have similar effects. Interventions for the most experienced teachers

should focus on renewing the excitement for teaching they once possibly exhibited by demonstrating how their developed conceptions and instructional routines may be inhibiting students' potential to grow and learn independently of a teacher.

Teacher Verbal Suggestions' Influence on Student Attention and Performance

The investigator believed that teachers' verbal suggestions, given during instructional episodes, influenced student attention and resulting performance related to verbal problem content. Polya (1957) believed problem solving instruction should assist students in internalizing the questions within the phases of his model and result in students operating under their influence in problem solving settings. Vygotsky (1978) similarly emphasized external to internal dimensions of conceptual evolution, and sought ways that teachers could enhance students' problem solving capabilities by modeling heuristic suggestions that would gradually be internalized by the students. Hence it appears that teachers do enhance this transition from external to internal dimension through some combination of explicit verbal suggestions and teacher modeling. The role of teachers' verbal suggestions in this transition process was one focus for this investigation and two related hypotheses

were suggested.

Hypothesis 5: Greater quantity of teacher provided suggestions for specific verbal problem types provides guidelines for student classification of verbal problems by type, which in turn improves student performance of familiar problem types, and inhibits student performance of less familiar types.

Hypothesis 6: The format in which a teacher's oral suggestion is framed (question or statement) does have an influence on the degree of student thinking or active processing. In particular, a suggestion presented in a question format has greater impact on student attention and active processing.

The two hypotheses focus on two aspects of teachers' oral suggestions, the quantity and preferred format (question or statement) employed. It was believed that the quantity of a teacher's suggestions positively influenced the quality of students' developed schemata for the various verbal problem types, while a greater quantity of question formats was believed to enhance student attention and participation and hence result in better student performance and learning. Research studies by Evertson et al (1980) and Friedman and Stomper (1988) indicated that more effective junior high school and college teachers asked more process questions and elicited more correct responses to all types of questions in their instruction, and that more effective teachers used a greater percentage of interrogative formats (questions) as their instructional mode of choice during all phases of a

lesson. The related percentage figures of interrogative formats used by more effective teachers was 46 percent in the junior high setting (Evertson et al, 1980, p. 172) and 71 percent in the college setting (Friedman and Stomper, 1988, p. 172).

Findings

The analysis of teachers' oral suggestions is not a well defined domain at present, and the investigator's work in this area can best be described as exploratory. Teachers' oral suggestions were taken from the investigator's fieldnotes and classified along three dimensions, content focus (general model, tool skills, task-specific heuristic or general heuristic), lesson segment placement (homework review, lesson development, or controlled practice/seatwork), and oral format (question or statement).

The study's results provided little support for the hypothesis that relative quantities of teachers' oral suggestions related to specific verbal problem types influenced student performance on these same verbal problem types, primarily because so few of teachers' oral suggestions were classified as task specific heuristics. The greatest quantity of each teachers' suggestions (over 40 percent for all three teachers) related to tool skills such as how to use a diagram, chart, or table or towards improving specific processes of the solution procedure.

The quantities of task specific heuristics were much lower for the three teachers, 18 for Alice, 7 for Betty, and 10 for Carl, and so separating them according to verbal problem types was rather meaningless. Alice's greater quantity of task specific heuristic suggestions might have accounted for some of the improved performance noted for her class on the retention test. However the extent of this effect cannot be separated from that resulting from the additional student practice of all verbal problem types provided only within Alice's instruction.

The findings offered greater support for the notion that greater teacher modeling and explicit instructional treatment of specific verbal problem types of the core content, resulted in better student performance on the posttest for those same verbal problem types. All three classes did poorly on the age related problem type, due to the inclusion of an "impossible" age related problem type on all posttest forms, and hence this problem type is not considered in these classroom comparisons. The coin/money related and distance/rate/time related problem types presented the most difficulty for Alice's students on the posttest. The coin/money related problem type received the least instructional treatment and modeling (one day) of the six problem types in Alice's instruction. Another possible explanation for students' lowered performance on these two problem types related to Alice's instructional

emphasis on mechanical translation means for formulating equations, whereas these two verbal problem types required greater student understanding of the central relationships to facilitate equation formulation.

The integer related and geometric oriented problem types presented the most difficulty for Betty's students on both the posttest and retention test. The integer related problem type received minimal instructional treatment (one example demonstrated), and Betty's instructional treatment of the geometric oriented problem type was mostly disorganized, much in the same way her employed textbook treated this verbal problem type. Student performance for coin/money related, distance/rate/time related and age related problem types was somewhat better, and interestingly all three of these problem types were organized into separate textbook sections, and hence received greater and more organized treatment by Betty.

With the exception of simple translation problem types, Carl's instruction gave equal treatment for the other five verbal problem types, and hence it was somewhat surprising that his students scored lower on the coin/money related problem type. Lower student performance for this verbal problem type was most likely due to the different context used in one of the two designed problems of the posttest related to this problem

type, which comparatively lowered student scores more than for the other problem types.

There was greater variation between teachers' oral suggestions with regards to the relative frequency of question or statement formats for their suggestions. Oral suggestions were phrased as questions 14 percent of the time by Alice, 29 percent of the time by Betty, and 50 percent of the time by Carl. In evaluating student attention patterns for the three classes, the investigator sensed that most of Carl's students were actively attentive to his instruction, most of Alice's students were passively attentive to her instruction, and most of Betty's students were inattentive, and those students who were attentive to her instruction were primarily passive. Hence the findings for Carl's class indicated that a greater usage of question formats within his instruction encouraged more active student attention and involvement in the learning process. On the other hand, teachers' suggestions composed mainly of declarative formats resulted in more passive student attention (Alice's class) or even a high degree of student inattention (Betty's class), possibly because declarative formats fail to stimulate student attention, and actually discourage further student interaction and discussion.

Regardless of whether one uses the subjective evaluations of each teacher's instructional segments or

the students' posttest results as measures of teaching effectiveness, Carl emerges as the most effective of the three teachers in the study. Therefore the findings support other research results indicating that more effective teachers use a higher percentage of interrogative formats in their instruction. It is also noteworthy that Carl used a slightly higher percentage of interrogative formats (50 percent of the time) in his instructional suggestions than did the more effective teachers in the Evertson study of junior high students (46 percent of the time, Evertson et al, 1980).

Conclusions

1. There were only minor variations between both teachers' daily quantity of oral suggestions and the quantity of suggestions directed specifically towards improving schema development (task specific heuristics) for the six verbal problem types of the core content. Therefore, classroom performance differences for the six verbal problem types were more likely due to respective teachers and textbooks providing less instructional treatment of certain verbal problem types, and hence less student exposure and practice. Other minor influences such as specific characteristics of certain of the posttest items and one teacher's emphasis on

mechanical means for formulating equations additionally explained classroom performance differences for the six verbal problem types. Since Alice's class outperformed the other two classes for every verbal problem type on the retention test, and Alice provided the greatest number of task specific suggestions of the three teachers, a minor relationship between quantity and student retention of verbal problem type seems to be warranted.

2. The study's findings indicated that the way a teacher frames their oral suggestions, either as a question or statement, strongly influenced student attention and interaction during the instructional learning process. Based on subjective evaluations of each teachers' instruction and better class posttest performance, Carl was considered the most effective of the three teachers. His instruction contained the highest relative percentage of interrogative formats for his suggestions, and his class was highly interactive and involved. Greater usage of mostly declarative formats for suggestions resulted in students being mostly passively attentive during the other two teachers' instruction, and required

teachers to more carefully monitor their classes to maintain student attention.

3. The student performance results for the retention test stood in opposition to the generally held assumption that greater student involvement and interaction during the introductory and encoding phases of concept acquisition results in better retention and easier recall of the relevant concepts. For this study of verbal problem content, the assumption was false. Carl's class had the highest level of student interaction, yet performed lower on the retention test than did Alice's class of passive learners. Therefore, it appears that for algebra verbal problem content, greater instructional treatment and student practice of verbal problems results in better student retention of this content.

Recommendations

1. The limited usage of task specific suggestions was not likely to assist students in constructing well structured and detailed schemata for the six verbal problem types. Therefore student problem type schemata probably were based mostly on common surface and contextual features of verbal problem statements and associated tool skills, such as using a table for the D/R/T problem type.

Due to the nature of received instruction, students neither realized the bigger picture of their efforts, namely the common algebraic structure underlying all studied verbal problem types, nor developed detailed problem type schemata. Instead they tended to rely primarily on their ability to mechanically translate problem statements into variable definitions and equations, which for certain of the verbal problem types was more difficult. Teachers should broaden their instruction to include some reference to the common equation structure found in classes of verbal problems, and also include discussion of the purposes for introducing various tool skills, such as making a diagram or constructing a table. Their instruction should better assist students in developing more detailed and complete schemata for specific verbal problem types. One way teachers could accomplish these objectives would be to have students construct variable definitions and equations for all verbal problems of a set, and actually derive solutions for a limited number of items in their verbal problem assignments.

2. Research, including this study's findings support the need for teachers to more greatly

balance their pedagogical moves between declarative and interrogative formats. Such balance increased student attention and interactive involvement, lead to more effective and productive instructional episodes, and resulted in better student performance. Teachers should be trained to pose both more questions in their instruction and more process oriented questions that require further interactive discussion and elaboration. Such questions seem particularly appropriate for any kind of problem solving content, in which teachers allow students to direct the teacher's modeling of problem solutions as the students verbalize their thinking and actions. Carl's instruction and class demonstrated that this level of student was capable of such interaction and discussion of students' thinking processes.

3. It appears unrealistic to expect students to practice verbal problems to the extent Alice was able to accomplish in her instruction, and still be attentively receptive to the quality of lessons recommended above. Teachers' daily practice assignments should be brief (30 minutes to one hour), focused on a limited number of concepts and skills, and be capable of high rates

of successful completion. Verbal problems or other applications should not be mixed with other algebra content in practice assignments. One way for teachers to keep verbal problem assignments manageable would be to adopt the first recommendation of having students construct partial solutions for all verbal problems of a set and complete solutions for some verbal problems. Teachers can then still maintain their instructional pace for covering the course content, and also provide sufficient student practice of associated processes within verbal problems.

General Student Performance Characteristics

Despite the fact that only one hypothesis related to student performance, student performance characteristics were an important aspect of the study. Certain aspects of student performance have already been noted in previous sections of this chapter to demonstrate points related to teachers' and textbooks' provided instruction. The one hypothesis also lends itself to a broader discussion of other specific abilities demonstrated by students of the study. The following hypothesis related to student performance characteristics.

Hypothesis 8: Of the three central student processing components contained within the general problem solving model (those being variable definitions, equation formulations, and equation solutions), students will be most successful in solving equations, less successful in deriving variable definitions, and least successful in formulating equations.

The primary data sources for constructing the findings related to student performance characteristics were the students' written work on the two major instruments of the study, the posttest composed of ten verbal problems and the retention test composed of seven equations and seven verbal problems. Analysis proceeded along classroom dimensions and for the total group of students. Results from the nine student interviews are also used to supplement the results from the other student performance measures, and are also reviewed separately since they provided greater detail into student thinking and processing because of the combined verbal and written features of the sittings.

Findings

The process scoring system developed for analyzing students' written solution work for verbal problems was based on the assumption that most students were aware of and followed the phases of the general procedure. This assumption was most valid for Alice and Carl's class, while less valid for Betty's class, and was discussed in a

previous section of this chapter. For the posttest, 73 percent of students' written solutions indicated student following of the general procedure, while the figure was only 60 percent for the retention test's verbal problems. The investigator compensated for these general procedure differences between classes by considering phase success rates, phases being variable definition, equation formulation, equation solving, and interpretation/answer, as conditional probabilities dependent on successful completion of each previous phase in the data analysis. This methodology allowed better comparisons between classes with regards to students' capabilities associated with the four student processing components.

The total group of students and each class of students were more successful at correctly solving equations and correctly interpreting their results than they were for the other two processes, variable definition and equation formulation, both for the posttest and retention test results. In fact for the retention test, the proportion of students correctly solving their own formulated equations for the verbal problem set exceeded the proportion of students correctly solving the equivalent equation presented in isolation in the equation solving section (.76 as opposed to .66), though this effect mainly resulted from Alice's class performance, in which students were 1 1/2 times more likely to solve their

own formulated equations (.82 as opposed to .55). Betty's class was more successful solving the equations presented in isolation, whereas Carl's students were equally successful for both formats.

For students' equation solving skills, only one error analysis was completed by the investigator, and that was done for the equation solving section of the retention test. For this section, classroom performance differences were not significant, and students' equation solving skills were notably consistent between classes, both in terms of which equations posed the greatest difficulty and in the error patterns students committed. All three classes scored lowest on the one equation which required application of the distributive property for a quantity preceded only by a subtraction sign (no constant coefficient present), and scored best on the one equation which required only one transposition and one computation. Betty's students were somewhat more successful than other classes on equations requiring the distributive property application, and somewhat less successful on more direct transposition items. Both patterns were most likely the result of Betty's presentation of differing methods for solving linear equations, which in one case is advantageous and for the other case is more likely to result in more computational type errors.

Students' equation solving errors were partitioned

into five categories, two related to computation (real number and variable types), two related to conceptualization (transposition and distributive property types) and one related to alternate quadratic formulation. The proportion of student errors related to conceptualization and computation were nearly equal, 47 and 48 percent respectively, with differences again small between classes, and indicated that student errors were equally likely to be due to computational carelessness as to conceptual misunderstanding. Considering that the retention test was administered after three quarters of the school year had been completed, the large percentage of errors related to computation was not expected.

The student performance results were less conclusive in determining which of the two other central processes, variable definition and equation formulation, posed the greatest difficulty for students. Also differences between classes and differences in student performance for the study's two instruments were more substantial. For the total group of students, the proportion correctly completing both phases for the posttest was identical (.54), whereas for the retention test, students improved in their ability to correctly define variables (from .54 to .60) while their ability to correctly formulate equations decreased (from .54 to .45). All three classes improved between the posttest and retention test in their

ability to define variables, suggesting that students' ability to translate written text (the primary skill related to variable definition) into algebraic formats developed incrementally, and possibly apart from provided instruction.

Carl's students were notably more successful in correctly formulating equations for the posttest items (.69 phase success rate), yet this class advantage failed to carry through on the retention test, where his students performed only slightly better than the lowest class (Betty). The .69 success rate for the posttest appeared due to the inclusion of a chapter in Carl's textbook, just before the major verbal problem chapter, which aimed its instruction at improving students' equation formulation skills through the use of geometric formulas. Neither of the other two employed textbooks provided such additional development and student practice of this process. Yet it appeared that due to lack of student usage in the intervening time period between posttest and retention test administration, the enhancement of this process by the textbook was largely washed away from students' memory and learning.

Alice's students improved in their ability to formulate equations for the verbal problems of the retention test, and this was not anticipated by the investigator, primarily due to the nature of Alice's

observed instruction which emphasized the mechanical and translational aspects of how to formulate equations from verbal problem statements. Because the posttest results indicated that 75 percent of student errors in formulating equations were due to incorrect comprehension or application of conceptual relationships, and Alice had not emphasized these conceptual relationships in her instruction, it was assumed by the investigator that most of her students would forget or fail to retain these key relationships, and would result in some drop in student retention performance. A possible explanation for this improved performance appeared to be the greater extent of student practice of these key relationships apart from received instruction. It may have been that the time required for these students to consolidate their learning of the key relationships from this more extensive practice was greater than that allowed for by the posttest.

Therefore, one can conclude that students are more likely to retain and even improve their success at defining variables, whereas student ability to successfully formulate equations diminishes over time. However, student results for Alice's class contradicted this second trend. Additionally the high correlations (.81 for Betty's class and .62 for Carl's class) between the verbal problem section scores for the two instruments indicated that this decreasing retention in all areas was

fairly uniform across all students in these two classes, whereas this was not the case for Alice's students.

The error analysis for the posttest results identified certain patterns of student performance which prevented these students from more successful completion of these two processes. Over 70 percent of students' variable definition errors were the result of poorly developed or inappropriately applied translation skills. Particularly troublesome in all three classes were verbal problem statements where a whole amount of a measure was given, and students were to construct variable definitions for two partial amounts which together totaled the whole amount. Two verbal problems of the posttest suggested such a variable definition construction. For one verbal problem, only 4 of 23 student attempts to define variables were correct, with eight errors due to students inferring equal values for the two variables to be defined. For the second verbal problem, 2 of 46 student attempts to define variables paid any attention to the whole amount given as a means for defining their variables. The other 44 students attempted to define variables using the first sentence of the problem statement, which appeared to be designed for formulating the equation for the verbal problem. Similar results were noted for the one problem of the retention test where such variable definitions were required (only 3 of 20 correct definitions). Admittedly,

this type of variable definition requires students to infer the relationship between variables rather than directly translate the given relationship between variables normally provided in most verbal problem statements, and for which student performance was appreciably better.

Student errors and deficiencies related to equation formulation processing were primarily the result of students' inability to correctly recall and apply conceptual relationships they supposedly had learned and practiced. A commonly observed pattern in students' equation formulation errors was where students appeared to gather any leftover data in the verbal problem statement (after variable definition was complete), and put these numbers and variables into rather meaningless combinations, but which had an acceptable appearance to the student. This pattern was most apparent in two of the student interviews, where probing questions revealed that neither student had any understanding for the concept of an equation, or even what they meant when they replied two equal quantities being an equation. Students' inability to recall key relationships was most noticeable for the two geometric problem types included on both instruments, which required recall of two relationships, one for perimeter of a square or rectangle, and one for the sum of the angles of a triangle. Additionally about half the

students forgot to include values for the coins in their formulated equations for the two coin/money problem types (one on each instrument).

Three other aspects of student performance characteristics, students' transfer capabilities, students' solution strategies for "impossible" verbal problems, and student performance results for the alternate format classification system, are reviewed briefly. The investigator had considered inclusion of an instrument specifically designed to measure students' transfer capabilities either in addition to or in place of the retention test. However because the pilot study results had indicated only minor transfer effects from the received verbal problem instruction, the investigator decided instead to include two verbal problems on the posttest, one with a less familiar contextual setting (one's wage earnings) and one which required two differential applications of the D/R/T relationship. Both these verbal problems were framed in more realistic settings than typical textbook verbal problems. Since student performance for these two items was poor, no further consideration was given to measuring transfer effects on the retention test, though one verbal problem of the interview instrument contained superfluous data for the finding of three persons' ages who belonged to the same family.

Three students, all using an alternate guess and check technique, correctly solved the less familiar contextual setting, earnings problem of the posttest, with only one other student even reaching the point of correct equation formulation. Three students also correctly solved the multi-application problem of the posttest, one using standard algebraic techniques and the other two using guess and check techniques with a drawing as a model. Four other students completed the first application correctly, but were unable to proceed with the second application. This multi-application item was removed from consideration in the scoring of the posttest results, due to so few students even attempting to solve it.

All nine interviewed students expressed confusion in their initial reading of the superfluous data problem, as if they sensed something was not quite right or familiar about the problem. Eight of the nine students then focused on specific aspects of the problem statement which best fit their schema for age related problems while temporarily ignoring the other superfluous and interfering data of the problem statement, and proceeded to derive a solution for the verbal problem. Three students, one from each class, correctly solved the problem using standard algebraic techniques, but only one student verbally noted the presence of extra and unneeded information and saw how

this other information could be developed as an alternative way to derive the solution.

The investigator expected Alice and Betty's students to perform poorly on these few transfer items since their instruction was confined entirely to textbook verbal problem types, whereas more was expected of Carl's students because of greater instructional exposure to more realistic and less familiar contextual settings for verbal problems. While more of Carl's students attempted solutions for these transfer items, their efforts failed to result in any difference in performance as compared to students in the other two classes.

Since the students had never encountered verbal problems which had no solution in their practice of textbook verbal problems, one such age related verbal problem was included on all posttest forms. The investigator was interested in how students would react or change their equation formulations when their correct algebraic solution techniques resulted in an impossible answer, such as the age of a person being negative one. This inclusion also permitted the investigator to compare results with those found in Paige and Simon's (1966) study of students' verbal problem solving techniques for such "impossible" problems.

Two students, both from Carl's class, recognized the contradictory nature of the problem statement information

in their initial reading of the verbal problem and wrote "no solution" for their answer and explained why. Of the remaining 65 students, 59 attempted to solve the problem using algebraic methods, with 20 students correctly defining variables for the requested ages at both time periods, and only four students correctly formulating equations. Three of these four students correctly solved the formulated equation, but only two students recognized that the negative result implied that the problem had no solution. More remarkable was the fact that eight of the 16 incorrectly formulated equations involved reversing roles within the problem information to make the formulated equation result in a positive and more realistic solution. The investigator's later review of these eight students' work samples noted four cases where students had erased part of their work for this item, possibly indicating that the reversal came about after a "correct" negative answer was derived.

A similar pattern of student performance was found for a second "impossible" verbal problem related to the perimeter of a rectangle on one form of the posttest, and hence completed by half the students. No student realized the impossible nature of the situation during their initial reading of the verbal problem, which was somewhat expected, since the inconsistency was less apparent. Four of the 34 students completing this form reached the point

of correctly solving a correctly formulated equation for the problem (solution = -1 = rectangle width), and yet none of the four students realized that this meant that the proposed situation was impossible. Hence none of the 34 students achieved a "correct" answer of no solution. Again interestingly eight of the nine student equation formulation errors used one length + one width = perimeter, which does result in a more sensible positive answer, whereas only two of the five student equation formulation errors on the opposite form showed a similar error pattern. In summary, students of the study exhibited some of the same behavior patterns identified by Paige and Simon (1966) in their attempts to solve "impossible" verbal problems where appropriate schema was apparently lacking.

The student performance results related to the four format classifications: abstract hypothetical, abstract factual, concrete hypothetical, and concrete factual for verbal problem statements identified by Caldwell and Goldin (1979) mostly contradicted the results of their secondary study. On both instruments, the students performed significantly better for verbal problem statements with abstract formats over concrete formats, with no significant differences between factual and hypothetical formats of problem statements. The sequencing of the four formats according to problem

difficulty levels for the students did not directly fit any of the researchers' three hypothetical perspectives, though student performance most closely resembled the direct translation perspective of the three perspectives. This stood in opposition to the researchers' findings that a predominance of an information processing perspective was found for their secondary school subjects.

Conclusions

1. The student performance results support the hypothesis for this section, in that students were most successful at solving formulated equations for verbal problems at both testing points, and demonstrated equal ability to define variables and formulate equations for the posttest's verbal problems. However these students were more successful at defining variables than formulating equations for the retention test's verbal problems. Additionally these results conformed with the amount of separate and specific instruction provided by textbooks and teachers, and student practice of the three processing components.
2. The findings for the three classes indicated that students' proficiency in solving linear equations was maintained and relatively stable throughout the students' experience in the course.

Students' abilities to define variables actually increased through greater experience in the course, though it was left unexplainable why this effect occurred in all three classes. Students' equation formulation abilities deteriorated somewhat over time for students in two of the classes, mainly due to inabilities to recall the key relationships required, since they were not used by the students during the long period of time between test administrations. The third class more extensively practiced the key relationships during acquisition, which probably strengthened the learning of these key relationships, and facilitated better recall when needed during the retention test.

3. There was little direct testing for transfer effects of the verbal problem instruction. For the two items of the posttest requiring transfer however, the results were not encouraging. Expected differences in student performance on these two items due to differences in received instruction did not occur, as there were no significant class differences on these items. The results for the two "impossible" verbal problems, one problem common to both posttest forms, suggested that most students were highly

mechanical in their orientation for solving verbal problems. Even in the few cases where students correctly solved correct equations, they were unable to interpret what their unusual result implied, and a good number of students actually reversed or changed relationships used in verbal problem statements so that more realistic and possible solutions were derived. Interviewed students' performance on the superfluous data verbal problem was somewhat better, as most students just ignored the extra data and operated with the part of the data that fit their schema for the problem type.

4. The results for the four format classifications of verbal problem statements suggested that students' learning and operation most closely resembled the direct translation perspective of the three hypothesized perspectives of Caldwell and Goldin (1987). This conclusion fits well with other observed characteristics of students' performance, in suggesting highly mechanical behavior when solving verbal problems. However it may also have resulted from an insufficient quantity of the four formats on testing instruments, limiting the effect and usefulness of this classification scheme, or was possibly

due to an unequal degree of difficulty for the verbal problems used of the four formats. The results stood in opposition to Caldwell and Goldin's findings for their secondary study, though it is likely that the study's results are specific for this level of student's mathematical competence.

Recommendations

1. Earlier it was shown that textbooks largely determined teachers' selection of content, and textbooks provided much less practice, with one exception, of the two processes (variable definition and equation formulation) shown to be in greatest need of improvement. Therefore it is recommended that textbook authors consider ways to increase practice of these two processes in their constructions. It was shown that the one textbook which contained such additional practice of the equation formulation process did result in better process performance for this class on the posttest, but not the retention test. Since present textbook designs mainly confine student development and practice of these two processes primarily to verbal problem solving situations, the earlier made recommendation that teachers should have students

define variables and formulate equations for all verbal problems of an assignment, but actually solve equations for only a sample of these formulations appears applicable here also. That Alice's students practiced these two processes more extensively with more verbal problems, which resulted in improved performance for both processes on the retention test was noteworthy, and lends further support to this recommendation.

2. Students' ability to formulate equations for verbal problems decreased over time, possibly because students failed to recall key relationships, since these relationships were never used or practiced after initial verbal problem instruction. Therefore teachers should include more regular review during the school year of verbal problems in their instruction, primarily targeting such review to reinforce the required key relationships, and thereby improve student recall capabilities of the key relationships.
3. This level of student exhibited highly mechanical behavior in their algebra verbal problem solutions, and probably for other algebra content as well, though not an issue addressed by this study. Part of this mechanical behavior of students was attributable to teachers'

instruction, which for two classes emphasized student learning of associated procedures for every verbal problem type. Such a procedural orientation to instruction had certain positive benefits for students, but it also denied students any possibility to discover and construct their own techniques and procedures, and therefore make them more meaningful and capable of greater transfer potential to more realistic problem solving situations. Both researchers and mathematics educators believe and desire that verbal problem solving skills lead to improved thinking and reasoning abilities, and possibly other problem solving abilities.
For such to occur in the Algebra I classroom, major modifications in teachers' instruction and pedagogical orientation, as well as in textbook presentational characteristics would be required. Most teachers would have to be retrained to plan and deliver instruction which contains more elements of a problem solving orientation, and textbook authors would have to rethink their content presentations, in order to challenge students' perceptions of algebra and mathematics as largely procedurally oriented, and initiate greater student thinking and problem solving. At

the present time, both tasks appear insurmountable given the cost involved and the slow rate at which changes in the secondary school setting become accepted practice. Additionally, it is recommended that further research studies of instruction be carried out in Accelerated Algebra I settings and other secondary mathematical settings to ascertain whether this procedural orientation of instruction is specifically directed to this level of student and/or this particular course. Ideally this research would include matched case studies of the same teacher using the same textbook, and hence teaching highly similar content to two classes of different ability level designations.

4. Compared to Betty, Alice and Carl possessed greater daily awareness of how their students were progressing in their learning of verbal problem content. Though one cannot dismiss the effect of greater teaching experience, it also appeared that both teachers' instructional practices were designed to more actively interact with their students during classtime, and in a sense interview and question their students, which in turn aided their awareness. The

investigator noted a similar greater awareness of students' strengths, weaknesses, processing, and thinking from the nine student interviews of the study than was gained from the entire review of 66 students' work samples. This awareness of student progress and assessment of learning are obviously important for designing future instructional episodes, and for keeping the class somewhat progressing together in their learning, rather than increasing individual differences among students as the school year progresses. This situation was noted in Betty's class where she relied almost solely on students' written test performance for her assessment of learning progress. Therefore teachers should be encouraged to incorporate opportunities for student interaction and interviewing within their instructional routines and designs. Alice and Carl's instructional routines demonstrated that there are a variety of times during class when such opportunities present themselves, and can be developed. Greater opportunity for interactions with students should improve both teachers' awareness and assessment of students' capabilities and learning progress, as well as students' own awareness of their capabilities and

learning, thereby additionally serving a metacognitive function for students.

5. The wide variety of student performance exhibited in the nine student interviews led the investigator to speculate that students' verbal problem solving performance might best be described by a series of five levels that relate to movement from a "novice" position to that of a relative "expert" in the field. It is left quite unclear whether the proposed sequence of levels matches students' development, or even whether students must progress through all levels. Slightly exaggerated descriptions of students' characteristic behaviors and mode of operation at each level are given, and the investigator found at least one interviewed student who fit each level of student performance. Since three students in the second level displayed two rather distinctive orientations which resulted in highly similar performance, the second level was subdivided into A and B components.

Level 1: "Novice" (B3)	General procedure for solving verbal problems only occasionally operative, all three phase subskills in development and not automatized, largely unable to verbalize thinking or rationale for processing actions.

Level 2A: "Random Operator" (A3 and B2)	Well aware of general procedure's importance as a guiding structure, and follows rigidly, strong equation solver, weakest at defining variables and formulating equations, often guilty of gathering all problem statement information illogically and without meaning in a trial and error fashion. Judgments of correctness based mostly on perceptual appearance characteristics. Lacks confidence in abilities, due to awareness of perceived weaknesses.
Level 2B: "Analyzer/ Translator" (A2)	Awareness of general procedure less clear since seldom is able to proceed any farther than defining variables. Well developed analysis and translation skills for variable definition phase, yet largely unable to think relationally when such synthesis is required for formulating equations. Surprisingly confident of ability despite apparent weaknesses.
Level 3: "Mechanical Operator" (A1 and C2)	Well aware of general procedure and follows its sequence mechanically, behavior appears schema driven (written action immediately follows reading), with no time given to reflection or problem understanding, phase subskills generally automatized, unable to judge reasonableness of intermediate or final results in realistic terms. Confidence level fluctuates considerably depending on verbal problem type.
Level 4: "Proficient Operator" (C1 and C3)	Quite similar to "mechanical operator" in terms of mechanically following general procedure and highly developed phase skills yet different in ability to reflect upon and monitor operation within phases, and has established "critics" for reasonableness and realistic expectations. Despite a high and stable level of confidence in abilities, fails to demonstrate flexibility in repairing errors and not very persevering in overcoming realized errors.

Level 5: "Expert" (B1) Seldom makes procedural errors of any kind, very reflective at every phase and highly capable of verbalizing thinking and rationale for all actions. Formally checks every derived solution with problem elements, and anticipates conceptual errors. Demonstrates well developed sense of strengths and weaknesses. Believes capable of successfully solving any verbal problem if given sufficient time, and leads to high levels of confident operation and perseverance. Able to flexibly review and repair work, and locate conceptual errors in reasoning.

Though based on only a small sample of nine interviewed students, the investigator is sufficiently confident in these formulated levels of student performance to conjecture that if two teachers each carefully reviewed ten verbal problem work samples for the same student, and then studied the characteristics for each level description, there would be common agreement between the two teachers regarding where this student was located within the five levels. At the time of the posttest administration, the investigator believes that 90 percent of Carl's students were operating at either level 3 or 4, with the other ten percent somewhere in level 2. At the same time about equal proportions of Alice's class were operating at levels 2 and 3. About half of Betty's students were operating at level 1 with about four students each in levels 2, 3, and 4. Admittedly, there may be students

who shift between the operational modes described in the five levels, but there does appear to be remarkable consistency in students' capabilities, behavior patterns, and operational orientation, when it comes to solving verbal problems of any type.

Future Research Directions

As might be expected from the exploratory nature of this investigation and the small sample of teachers' observed instruction, the results point mathematics education researchers in certain directions, without providing much concrete, directional signposts or definitive answers. Lester (1985) raised four issues in need of special attention for future research on problem solving instruction which provide a model for this section's discussion. These four issues are the following:

1. Shift away from experimental and toward naturalistic methodologies.
2. Increased interest in problem-solving processes in addition to various product variables.
3. Increased interest in the development of problem-solving instructional programs that are "classroom ready".
4. Growing interest in the role of metacognition in problem solving.

Naturalistic Perspective

Over the past several years, there has been a notable increase in the quantity of problem solving instructional research proceeding from a naturalistic perspective, much as Lester predicted. A naturalistic perspective imposes minimum constraints on independent, mediating, and dependent variables, and studies the factors influencing teaching and learning behavior holistically, as opposed to atomistically. This research paradigm, by its very assumptions, presents four predictable difficulties for mathematics education researchers, due to its reliance on qualitative methods, its attempts to balance rigor with relevance, its expansionist stance, and its purpose to discover, rather than verify theory.

The investigator was somewhat surprised by the lack of general availability to qualitative instrumentation from the research community for such needs of the study as evaluating the quality of textbooks' constructions, or evaluating teachers' instruction, or classifying teachers' verbal suggestions. In many of these instances, lack of availability meant the investigator had to design this instrumentation, which obviously does not build on the research results of others, or permit much replication of results within this paradigm. The reporting of research results generally gives only scant reference to employed methodology and instrumentation, and provides little

assistance to other naturalistic researchers for whom such information is crucial in order to compare results from the generally small sample of case studies which each researcher is carrying out. There appears a need for greater collegiality and sharing of instrumentation among researchers who work from this perspective, possibly even including a forum (conference, book, or journal) in which qualitative methodology developments is the focused topic. The investigator would appreciate other researchers using and testing his designed instrumentation in other areas, in order to ascertain whether these instruments are valid and more generally applicable.

The investigator also recommends that further research be completed to determine whether the developed descriptions of teachers' instructional routines and student performance patterns related to verbal problem content remain valid for different locations and settings, for different mathematical content areas, and for differing levels of students. An initial recommendation for future research would be to study two Algebra I teachers' instruction of verbal problem content, preferably in two divergent school settings, using the same textbook for their instruction of two classes each, where the students are of different designated ability levels. This recommended research might uncover the different ways teachers modify and shape their instruction

to fit their perceptions of students' capabilities and needs within their aims for the course, for which teacher differences were quite profound in this investigation. Additionally, such research increases control of content coverage (same textbook) and instructional orientation (same teacher), while permitting closer comparisons of teachers' designed instructional routine for each level of student, and resulting differences in student performance.

Naturalistic research relies heavily upon the investigator's ability to infer relevant relationships and "discover" patterns of behavior from observations and collected data, or in a sense is more concerned with describing relevant features of the setting in an understandable way. This leaves to future researchers the task of sorting out these relationships and behaviors into more carefully controlled and rigorously designed experiments. The instructional classroom setting contains a diversity of variables capable of influencing student performance, and controlling some of these variables in different combinations might provide a continual flow of new insights about problem solving instruction.

Two related concerns experienced by the investigator in adopting a naturalistic perspective involved balancing expansionist tendencies with reductionist tendencies, and a tension of whether to focus on discovering or initiating theory or relating results to existing theory and research

results. In order to relate this investigation to other more experimental oriented problem solving and instructional research results often meant expanding aspects of the study's instruments, such as inclusion of "impossible" problems or analysis of student format classification performance, as the study was developing and in progress. In most cases, such expansion due to concern for research relatedness had minimal success, tended to detract from the investigation's main purposes and questions, and did result in some management problems. Therefore future researchers, operating from this perspective would be well advised to focus more of their efforts on initiating issues and discovering theory for further research, rather than verifying and relating their results to existing research. The investigator did set out a hypothetical model of student behavior and performance at the outset of the study, and kept this perspective during the study. Future researchers might look at alternate ways students process verbal problems despite provided instruction in standard techniques, or focus their efforts on students' natural performance and solutions to verbal problems where no instruction was or is provided.

Problem Solving Processes

The investigator's paper and pencil instruments and methodology was more successful at identifying global

aspects of teacher and textbook example processing and student processing, than it was in identifying the specific thinking processes behind such student and teacher behavior. The investigator was able to analyze what went wrong, without knowing why or what led to either student error or success. Similar global aspects of teachers' conceptions were possible from the written response format of the Teacher Inventory Packet. The nine student interviews, limited as they were, provided a greater degree of depth to the specific thinking processes students were using during their solution of verbal problems. Therefore, it is recommended that future research efforts, aimed at identifying problem solving processes, include greater interviewing of both students and teachers in their designs. Otherwise specific thinking processes will remain mostly missed by the observing researcher, and much behavior and processing left unexplainable. Paper and pencil instrumentation is still required and important in that it provides adequate measurement for the products of student learning.

Instructional Programs

The generally assumed positive relationship between the ability to solve verbal problems and the ability to solve more "genuine" or process problems was not supported by this investigation's results, though it may be specific to this level of student who exhibited predominantly

mechanical behavior and processing in their solving of verbal problems. Future research efforts need to explore this assumed relationship more carefully with differing levels of students. The development of "classroom ready" instructional programs for the teaching of mathematical problem solving will require rather radical reorientation of both teachers' and students' behavior patterns. When one considers that Carl's instruction was highly creative, motivating, and interactive, and yet resulted in minimal changes in students' problem solving performance and thinking capabilities, the problems facing instructional program designers remains large. Additionally some of the earlier made recommendations for textbook authors, teachers, and mathematics educators might possibly improve verbal problem instruction and performance, without having any significant impact on desired improvement of students' general mathematical problem solving abilities.

Role of Metacognition

Even if one expands metacognition to include students' beliefs and attitudes, study effects due to metacognition were rather minimal in comparison to the cognitive capabilities required for successful verbal problem solving performance. Again, this minimal effect for metacognition may be due to the mechanical behavior observed for both the teachers and the students of this level of course, and needs further research, both within

other courses and with different levels of students. The study's students were never required to evaluate alternatives as in more real world problem solving, but rather were only required to monitor progress and control movement through the general procedure. Students' ability to monitor their progress only had minor influence on student behavior, in comparison to student ability to perform the three major processes.

The nine student interviews revealed greater metacognitive differences between better performing students, in terms of greater awareness of ability and limitations, related compensation and repair strategy development, and perseverance traits. The limited instructional suggestions related to metacognitive actions largely fell on deaf student ears, who for the most part were just struggling to master more basic cognitive processes. Future research efforts will need to clarify which specific aspects of metacognition are most desirable and influential for successful mathematical problem solving (including verbal problem solving), and also when the optimum time for introduction of metacognitive processing may be, along with cognitive instruction, or after cognitive instruction has resulted in a certain level of content mastery.

The study identified and detailed relationships between a diversity of variables within the Algebra I

setting. It was shown that both the organization of textbooks and teachers' conceptual frameworks influence teachers' decisions related to the verbal problem content studied, the teaching methodology employed, the instructional routines designed, and the orientation used in lesson development segments. It was also demonstrated that these instructional decisions of teachers led to predictable patterns of student processing and performance related to verbal problem content. Future research is needed to validate and replicate the nature of these relationships, and may require studying these relationships in greater isolation and detail. The study also described both positive and negative characteristics of Algebra I textbooks, teachers' instruction, and students' performance related to verbal problem content, and therefore it is suggested that researchers consider implementation of certain of the study's recommendations as interventionist strategies within their future designs.

APPENDICES

APPENDIX A

Pilot Study Instrumentation

Suggestion/Heuristic Coding Form

Date: _____ Type: Text/Teacher/Student

Description: _____

Processes:

1. A.	a)	Reads problem	
	b)	Rereads for essentials	
	c)	Restates problem statement	
	d)	Separates, analysis components	
	e)	Requests definitions of terms	
B.	a)	Recalls related problem type	
	b)	Recalls related relationship	
	c)	Recalls related method/results	
C.	a)	Exploratory manipulations	
	b)	Draws diagram	
	c)	Modifies diagram	
	d)	Identifies unknowns	
	e)	Defines variables	
2. A.	a)	Deductive algorithmic approach	
	b)	Systematic trial and error	
	c)	Random trial and error	
	d)	Equation construction (piecemeal)	
	e)	Equation construction (relational)	
B.	a)	Estimates	
	b)	Templation/recall match	
	c)	Smoothing/modify problem statement	
	d)	Case studies	
	e)	Deduction	
	f)	Inverse deduction	
	g)	Analogy	
	h)	Organizational chart construction	
	i)	Elemental analysis	
	j)	Relational links noted	
	k)	Sets up equation(s)	

Suggestion/Heuristic Coding Form - Continued

3.	a) Guess and check carried forth		
	b) Algorithms - algebraic		
	c) Algorithms - arithmetic		
	d) Obtains solution		
	e) Work discontinued due to error		
	f) Work discontinued due lack progress		
	g) Logical justification		
4. A.	a) Checks solution in equation		
	b) Checks solution retracing steps		
	c) Checks with algorithmic processes		
	d) Checks using reasonableness stands.		
	e) Does not check		
	f) Checks with logical review of work		
B.	a) Refers to question of prob. state.		
	b) Correct interpretation/results		
	c) Incorrect interpretation/result		
	d) No solution determined		
	e) Expresses concern/comments		

Pilot Study Verbal Problem Instrument
(Class and Interviews)

1. The sum of two integers is seven. The sum of their respective squares is 85. Find the two integers.

2. The length of a rectangle is 2 cm longer than its width. The area of the rectangle is numerically 20 cm^2 more than its perimeter in cm. Determine the dimensions (length and width) of the rectangle.

3. Jerry has three times as many dimes as nickels in his collection. The total value of the coins is $4.55. How many of each coin (nickels and dimes) does he have?

4. Two fertilizer brands, A and B, contain respectively 8% and 12% nitrogen by weight. How much of each brand should be mixed to obtain 30 pounds of a 9% nitrogen mixture?

5. Two trains leave the same station in opposite directions, the passenger train at 1:00 PM and the freight train at 2:00 PM. The passenger train's average rate is 8 miles per hour faster than the freight train's rate. At 5:00 PM the two trains are 221 miles apart. Find each train's average rate.

APPENDIX B

Study Coding Forms

Coding Form 1: Process Coding of Examples

Date:_____

Type: Text: _____ Page: _____ Teacher: Self/borrowed Text: _____ Page: ____ Student: ___ Reference: _____

Description: _____

1. Understanding the Problem	Sequence(#)	Comments/Hint Suggestions
A. Reads Problem	_____	_____
B. Rereads, essentials separated	_____	_____
C. Restates problem statement	_____	_____
D. Requests or states clarification	_____	_____
E. Recalls related need (specify)	_____	_____
F. Exploratory manipulation	_____	_____
G. Diagram/chart constructed	_____	_____
H. Identifies wanted/given/ needed	_____	_____
I. Defines variables/unkown	_____	_____
2. Devising a Plan		
A. Estimates solution	_____	_____
B. Templation/match recall	_____	_____
C. Smoothing/analogy	_____	_____
D. Chart construction	_____	_____

Coding Form 1 - Continued

E. Random trial and error _____ _____

F. Systematic trial & error _____ _____

G. Equation construction (elemental translation) _____ _____

H. Equation construction (relational whole) _____ _____

3. Carrying out the Plan
   A. Plan in 2 carried forth _____ _____

   B. Algebra/arithmetic procedures used correctly _____ _____

   C. Logical justification provided _____ _____

   D. Correct solution obtained _____ _____

   E. Work discontinued/error _____ _____

   F. Work discontinued/other _____ _____

4. Looking Back/Interpreting Results
   A. Correctly interprets solution _____ _____

   B. Incorrectly interprets solution _____ _____

   C. No solution identified _____ _____

   D. Checks solution by:
      a. reasonableness standards only _____
      b. with problem statement _____
      c. in equation(s) _____
      d. algebraic procedures review _____
      e. entire logical development _____

Coding Form 2: Teaching Evaluation Instrument

Use: To be completed after every teacher observation

Purpose: To judge time usage/evaluate quality lesson development

Date: _____ Teacher: _____

Referring to observational notes: Determine amount of time in minutes for each classroom activity. Period = ___ min.

Minutes	% period	Activity
_____	_____	Organization/Administration
_____	_____	Drill/Mental Math
_____	_____	Homework Review
_____	_____	Concept Review (non-development)
_____	_____	Lesson Development
_____	_____	Controlled Practice
_____	_____	Seatwork/Homework Assignment
_____	_____	Transition/Other
_____	_____	Totals

When teacher introduced new activity or makes assignment

Behavior Categories	Code
1. Gushes, dramatic build-up	_____
2. Predicts group enjoyment	_____
3. Mentions info or skills to be learn	___
4. No attempt to motivate into act.	___
5. Apologizes or Sympathesizes	_____
6. Bribes, promises for good work	_____
7. Warns/reminds group of test	_____
8. Threatens punishment	_____
9. Presents activity as punishment	_____
10. Other (specify)	_____

Evaluator ratings of lesson development, scale 1-5 (1 = poor, 5 = excellent) Quality

	Day Rating
1. Task orientation	_____
2. Clarity	_____
3. Enthusiasm	_____
4. Frequent teacher talk	_____
5. Attending to prerequisites	_____
6. Attending to relationships	_____
7. Attending representations, models	_____
8. Attending to perception discrimination	_____
9. Generality of concepts	_____

APPENDIX C

Textbook Analysis Materials

Textbook Feature Evaluation Form

Feature	Ratings (1 - 5) Text A	Text B	Text C
A. General Model			
1. Quality of model	____	____	____
2. Maintained consistency	____	____	____
3. Student usage clarity	____	____	____
category subtotal	____	____	____
B. Textbook Explanation			
1. Quantity	____	____	____
2. Clarity: objectives	____	____	____
3. Consistency/ connectedness	____	____	____
4. Layout appeal	____	____	____
5. Student readability	____	____	____
6. Suggested sequence	____	____	____
category subtotal	____	____	____
C. Textbook Examples			
1. Quantity	____	____	____
2. Clarity/consistency	____	____	____
3. Diversity/ completeness	____	____	____
category subtotal	____	____	____
D. Exercise Sets			
1. Quantity	____	____	____
2. Clarity/readability	____	____	____
3. Diversity/ Completeness	____	____	____
4. Degree of difficulty	____	____	____
category subtotal	____	____	____
Grand total ratings	____	____	____

Textbook Author Survey

Name: _____ Publisher: _____

Algebra Text: _____ Date: _____

Consider for a moment the planning and design of your respective textbook for use with typical ninth grade algebra students. Below are listed several possible purposes, expectations, or abilities one might expect students using your textbook to accomplish in varying degrees during a one year Algebra I course. For each listing, describe in the scale at the right your position on its relative importance to you in the design of this textbook.

	Very Impor.	Some Impor.	Neutr.	Less Impor.	Little Impor.
___ Simplification of algebraic expressions	___	___	___	___	___
___ Solution 1st and 2nd degree equations	___	___	___	___	___
___ Understanding/justification procedures	___	___	___	___	___
___ Solution textbook word problems/applications	___	___	___	___	___
___ Relationships algebra geometric interprets.	___	___	___	___	___
___ Relationships algebra arithmetic rules	___	___	___	___	___
___ Relationships of algebraic symbol forms equivalences (i.e. equations & functions)	___	___	___	___	___
___ Algebraic techniques as tools for general problem solving use.	___	___	___	___	___
___ Other (specify) _____	___	___	___	___	___

In the left hand column above, rank order the 8 (or 9) items given as to their relative importance in the design of this textbook. (1 = most important consideration, 8 = least important consideration)

Textbook Author Survey - Continued

When you designed this particular textbook, what math ability level did you target for its use? (check all that apply)

_____ high ability level classes
_____ mixed ability level classes
_____ low ability level classes
_____ mixed ability level classes
_____ other (specify) _____
_____ no particular target class

Describe briefly your preference for the organization of textbook word problem exercises. (Examples: by category type, separate sections presentation or as applications for immediately preceding manipulative skills)

I assume you believe your stated preference above is also the most effective organization for students to gain skills in solving word problems. Upon what knowledge is this belief based? (Research, personal experience, testing, other).

Does your textbook contain and maintain an identifiable, sequential general problem solving pattern or orientation for solving its word problems or exercises? _____
If yes, describe it.

From whose ideas did this orientation originate?

Assume that today you were asked to design a textbook for Algebra I, would there be any changes (additions or deletions) of content, placement, or order of presentation? Describe.

Textbook Author Survey - Continued

Would there be any rethinking or reorientation on the inclusion of word problems or in their instructional significance or presentation?

Describe.

Other comments related to this survey:

Thank you for your help and cooperation in completing this survey for my study. Please return the completed survey in the preaddressed stamped envelope provided.

APPENDIX D

Teacher Inventory Packet

Name _____
School _____

Part I: Classroom Procedures

Please check the point within each of the following scales which most accurately describes your Algebra I classes. Please respond according to what actually happens, not what you think should happen or what you would like to happen. Please answer all questions.

	1	2	3	4	5	
1. I give an Algebra test once every 3 weeks.	__	__	__	__	__	I give a test once every week.
2. Algebra learning objectives are the same for all students in the class.	__	__	__	__	__	Algebra learning obectives are set for each student separately.
3. Students are encouraged to get a lot of help with their algebra problems.	__	__	__	__	__	Students are encouraged to solve algebra problems without a lot of teacher help.
4. Almost all help is initiated by students asking for it.	__	__	__	__	__	Almost all help is initiated by my seeing to it.
5. Daily lesson plans are stable, not usually subject to change.	__	__	__	__	__	Daily lesson plans are changed very frequently.
6. Evaluation standards/procedures are the same for all students.	__	__	__	__	__	Evaluation standards/procedures are different for each student.

Teacher Inventory Packet - Continued

	1	2	3	4	5	
7. On a typical day, I give a lecture presentation during 3/4 of class time.	__	__	__	__	__	I almost never give a lecture presentation.
8. Students frequently help one another during AND after algebra class.	__	__	__	__	__	Students seldom help one another during AND after algebra class.
9. I encourage students to solve word or story problems the way that I have demonstrated.	__	__	__	__	__	I encourage students to solve word or story problems any way that they desire.
10. I present an an algebra concept, then illustrate the concept by working several problems (deductive).	__	__	__	__	__	I present the class with a series of problems, then together we develop concepts and method of soln (inductive)
11. Algebra is taught strictly as a manipulation oriented subject.	__	__	__	__	__	Algebra is taught with emphasis on theoretical foundations.
12. Word problems or stories are taught as applications of algebraic procedures.	__	__	__	__	__	Word problems or stories are taught as problem solving opportunities or challenges.
13. I can usually predict where my students will be in the textbook in January.	__	__	__	__	__	I can't usually predict where my students will be in the textbook in January.
14. In my algebra class, I emphasize basic algebraic manipulation procedures.	__	__	__	__	__	In my algebra class, I emphasize understanding the underlying reasons for various algebraic procedures.

Teacher Inventory Packet - Continued

	1	2	3	4	5	
15.	Appreciation of algebra's power for solving problems is of high importance.	__	__	__	__	__ Appreciation of algebra's power for solving problems is not vital.
16.	Heavy emphasis is placed on textbook word problems or practical applications.	__	__	__	__	__ Little emphasis is placed on textbook word problems or practical applica.
17.	Understanding the methodology of why a given method gives the correct answer is important to me.	__	__	__	__	__ Understanding the methodology used is not critical to me.

Part II: Teacher Opinion

Select the appropriate choice for each statement.

A = Agree
B = Somewhat agree
C = Undecided
D = Somewhat disagree
E = Disagree

___ 1. I feel confident teaching algebra.

___ 2. I feel confident teaching algebra word problem techniques.

___ 3. I find teaching algebra challenging and stimulating

___ 4. I find teaching algebra rewarding and enjoyable.

___ 5. In terms of teaching skill, algebra in comparison to other math subjects I have taught is a personal strength.

___ 6. My basic function as an algebra teacher is to convey my knowledge of algebra to the students in a direct manner.

___ 7. I feel I have a good sound background and knowledge base about algebra word problems.

Teacher Inventory Packet - Continued

Part III: Completion

For each blank complete with mostly numerical estimations.

1. Most of my students complete ___% or more of all the word problems in their textbook associated with each lesson that is taught.

2. When you use practice exercises to reinforce algebra skills, approximately what percentage are:

   ____% oral work or chalkboard work.
   ____% written work to be done in class.
   ____% written work to be done outside of class.
   ____% other (specify) _____

   100% total

3. Before you would reteach a unit or chapter, what percent of your class would have had to have done poorly on an exam? _____%

4. When some students do poorly on tests or otherwise indicate that they have not understood a unit in algebra, what are three (3) things you do to improve the situation.

   1. _____

   2. _____

   3. _____

5. On the average, I spend about ____ minutes a day developing algebra concepts and skills, and have students practice these skills through homework and problem assignments _____ minutes a day.

6. Sometimes students have difficulty solving algebra word problems or stories. Briefly describe or relate suggestions you give to help students solve word problems.

   1. _____

   2. _____

   3. _____

Teacher Inventory Packet - Continued

7. When I introduce and assign students algebra word problems, I go over the vocabulary and relationships within a problem and point out what various words mean about ____% of the time.

8. Before I start presenting the algebra lesson for the day, I spend about ____ minutes per day going over the previous lesson.

9. When algebra homework assignments are checked, what percentage would fall into the following categories?

____% I check the students' papers.
____% An aide checks the students' papers.
____% Students check their own work.
____% Students check each others' work.

100% total

10. If you had your choice, what type ability level in algebra would you prefer to teach? (check one)

_____ Mostly high ability
_____ Mostly average ability
_____ Mostly low ability
_____ A mixture of ability levels

11. I expect about ____% of my students to be successful on algebra manipulation skills and procedures (successful = 80% = B).

12. I expect about ____% of my students to be successful on algebra word problem solving (same success level).

13. How many years (including this year) have you taught Algebra I at a middle ability level? ____ years
At a higher or accelerated ability level? ____ years

14. What differences are you aware of in your teaching of algebra word problem solving to Algebra I (middle track) vs. Algebra I (high ability track) ?

_____
_____
_____

Teaching Inventory Packet - Continued

15. What purposes or reasons do you see for the inclusion of extensive word problems or applications in the Algebra I course? (Rank 1, 2, 3)

_____
_____
_____

16. What reasons would you put forth to explain the apparent inability of some students to successfully solve word problems despite these same students' ability to successfully employ necessary manipulation skills and procedures? (Rank 1, 2, 3)

_____
_____
_____

17. How much emphasis (importance) do you think your textbook authors put on verbal problem solving skills in Algebra I. (place check)

_____	_____	_____	_____	_____
very important		equal import. with manipulative skills		little importance

Give any reasons you have for this judgment?

_____
_____

18. How much in agreement are you with the authors' emphasis or importance placed on verbal problem solving. (place check)

_____ sufficient emphasis
_____ too much emphasis on verbal problems
_____ insufficient emphasis (curriculum requires modifications/additions)

Teacher Inventory Packet - Continued

19. How much in agreement or disagreement are you with the textbook's handling of verbal problems with regards to (for each category, place check)

	Total agreement		Undecided		Total disagreement
a) sequencing/organization/ location	____	____	____	____	____
b) example problems used	____	____	____	____	____
c) presentations/explanations	____	____	____	____	____
d) associated problem sets (amount)	____	____	____	____	____
e) associated problem sets (degree of difficulty)	____	____	____	____	____
f) extensiveness/sufficiency	____	____	____	____	____

Teacher Inventory Packet - Continued

Below are listed 8 possible purposes, expectations, or objectives one might expect Algebra I students to accomplish in varying degrees during a one year course. Reflecting upon your experience in teaching this level course and others, mark the scale at right for each item with your personal opinion of the relative importance you give in your teaching to each item listed. (place check)

	Very Impor.	Some Impor.	Neutr.	Less Impor.	Little Impor.
___ Simplification of algebraic expressions	___	___	___	___	___
___ Solution 1st and 2nd degree equations	___	___	___	___	___
___ Understanding/justification procedures	___	___	___	___	___
___ Solution textbook word problems/applications	___	___	___	___	___
___ Relationships algebra geometric interprets.	___	___	___	___	___
___ Relationships algebra arithmetic rules	___	___	___	___	___
___ Relationships of algebraic symbol forms equivalences (i.e. equations & functions)	___	___	___	___	___
___ Algebraic techniques as tools for general problem solving use.	___	___	___	___	___
___ Other (specify) _____	___	___	___	___	___

In the left hand column above, rank order the 8 (or 9) items given as to their relative importance and incorporation into your teaching and instruction of Algebra I. (1 = most important consideration, 8 = least important consideration)

APPENDIX E

Pretest and Student Information File

Student Information File

Class: Algebra I (Teacher: _____)

Grade Level: _____

Name: _____

Present Complete Schedule:

Hour	Class/Teacher
1	_____
2	_____
3	_____
4	_____
5	_____
6	_____
7	_____
8	_____

Last Year's Math Class:

_____|_____|_____
school teacher grade

Anticipated grade for this class: _____

What are your feelings/concerns about taking this algebra class?

Describe your ability and experience in doing textbook word problems?

Verbal Problem Solving Pretest Instrument

Name: _____
School: _____
Teacher/Hour: _____

For each of the 6 problems given, try to solve the problems using any methods including guessing, and show all work done in the spaces provided. Place answer in space provided. If unable to solve, move on to the next problem.

1. The figure below is a trapezoid. The formula for the area of a trapezoid is $A = 1/2h\ (b_1 + b_2)$ or $A = .5 \times h \times (b_1 + b_2)$ where h = height, b_1 and b_2 are the two bases of the trapezoid. Find the length of the ? base, given the area is 195 cm^2 .

   Drawing: Trapezoid FIGURE
   h = 10 cm and 1 base = 24 cm labeled.

   ? = _____ cm

2. A rectangular bulletin board has a perimeter measure of 60 inches. The length of the board is 4 inches more than the width. Find the length and width of the rectangular board.

   length = _____inches
   width = _____inches

3. Find 3 consecutive odd numbers whose sum is 93.

   3 numbers are ____, ____, ____

Pretest Instrument - Continued

4. At a refreshment stand, hot dogs cost $0.80 each and soft drinks cost $0.50 each. If a person bought twice as many hot dogs as soft drinks and the total bill (without sales tax) was $12.60, how many of each were purchased?

hot dogs = _____
soft drinks = ____

5. Sue is ten years older than her brother Larry. Six years from now, Sue will be twice as old as Larry. How old is each person now?

Sue = _____ years old
Larry = ____years old

6. At 1:00 p.m., a freight train averaging 30 miles per hour leaves the station in one direction. An hour later (2:00 p.m.) a passenger train leaves the same station in the opposite direction. At 5:00 p.m. the trains' engines are 249 miles apart. Find the average speed or rate of the passenger train?

Passenger train = ____mph

APPENDIX F

Posttest and Retention Test Instrumentation

Posttest - Form A

Name _____

School _____ Date _____

1) (Read out loud). "One integer is 26 more than a second integer. The sum of the 2 integers is 14. Find the 2 integers."

2 integers are ___ & __

2) (Read out loud). " One angle of a triangle is twice the measure of a second angle. The third angle is 4 degrees less than the second angle. Find the degree measure of all 3 angles of the triangle."

1st angle = ____degrees
2nd angle = ____degrees
3rd angle = ____degrees

3) As a stockboy, John earns $4 per hour when he works during the week and $7.50 per hour on weekends. One week's check was for $124.00 for 24 hours of work. Find the number of hours John worked on the weekend?

_____ hours

4) Two joggers, Mary and Tom eat lunch and then begin jogging in opposite directions, starting at the same time. Tom's rate or speed is 2 km/hour faster than Mary's rate. After 5 hours, Tom and Mary are 60 km apart. Find the speed or rate of each jogger.

Mary = ____km/hr. Tom = ___km/hr.

Posttest Form A - Continued

5) Jean is seven years older than her sister, Helen. Five years ago, Helen was twice as old as Jean was then. Find Jean's and Helen's present ages.

Jean = ____years old
Helen = ____years old

6) Find 3 consecutive odd integers such that twice the sum of the first two integers is 12 less than four times the third integer.

integers are ___,___,___

7) The length of a rectangle is 12 inches more than twice its width. If the perimeter is 18 inches, find the length and width of the rectangle.

length = ____inches
width = ____inches

8) A collection of dimes and quarters has a total value of $5.05. The number of dimes in the collection is one more than twice the number of quarters. How many dimes and quarters does the collection contain?

dimes = _____
quarters = _____

9) Jeremy's age next year will be twice Paul's age of last year. Their present ages total 54 years. How old is each boy now?

Jeremy = ___ years, Paul = ____years

10) John's sister, Mary, leaves Stevens Point heading north on US51 at 9:00 AM averaging 40 miles per hour. At 10:00 AM John realizes Mary has his keys with her, and sets out at 60 mph to catch her. Once he catches her, John realizes he must be back in Point by 2:30 PM At what speed would he need to travel to be back at exactly 2:30 PM? John's speed = ____mph

Posttest - Form B

Name _____

School _____ Date _____

1) (Read out loud). "One integer is 26 more than a second integer. The sum of the 2 integers is 14. Find the 2 integers."

2 integers are ___ & __

2) (Read out loud). " One angle of a triangle is twice the measure of a second angle. The third angle is 4 degrees less than the second angle. Find the degree measure of all 3 angles of the triangle."

1st angle = ____degrees
2nd angle = ____degrees
3rd angle = ____degrees

3) As a stockboy, John earns $4 per hour when he works during the week and $6 per hour on weekends. One week's check was for $124.00 for 24 hours of work. Find the number of hours John worked on the weekend?

_____ hours

4) Two joggers leave the same point in opposite directions, and at the same time. Jogger A runs at a rate of 1.8 km per hour faster than jogger B. After each jogger has run for 5 hours, A and B are 51 km apart. Find the speed or rate of each jogger.

Mary = ____km/hr. Tom = ___km/hr.

Posttest Form B - Continued

5) Jean is seven years older than her sister, Helen. Five years ago, Helen was twice as old as Jean was then. Find Jean's and Helen's present ages.

Jean = ____years old
Helen = ____years old

6) Find 3 consecutive even integers such that twice the sum of the first two integers is 12 less than four times the third integer.

integers are ___,___,___

7) The length of a rectangle is 12 inches more than twice its width. If the perimeter is 66 inches, find the length and width of the rectangle.

length = ____inches
width = ____inches

8) A coin collection is made up of dimes and quarters. The number of dimes in the collection is one more than twice the number of quarters. The collection has a total value of $5.05. How many dimes and quarters does the collection contain?

dimes = _____
quarters = _____

9) Martha's age next year will be twice Susan's age of last year. Their present ages total 54 years. How old is each girl now?

Martha = ___ years, Susan = ____years

10) Mary's brother, John, leaves Stevens Point heading north on US51 at 9:00 AM averaging 45 miles per hour. At 10:00 AM Mary realizes John has her keys with him, and sets out at 60 mph to catch him. Once she catches him, Mary realizes she must be back in Point by 3:30 PM. At what speed would she need to travel to be back at exactly 2:30 PM? Mary's speed = ____mph

Retention Test Form

Name: _____
School/Teacher: _____

Questionnaire Section: Answer each of the seven questions honestly and reflecting your own personal beliefs.

1. Order the following list of concerns from 1 (most important), 2,3,4,5 (least important) for you to attain in this algebra course. (Read all 5 before placing #'s 1-5 next to each).

____ My main concern is to pass this required course.

____ My main concern is to get a good grade in this class.

____ My main concern is to understand the material on a day to day basis.
____ My main concern is to acquire a solid knowledge base for my future math courses.
____ My main concern is to gain confidence in my math abilities.

2. Circle <u>one</u> which best describes your <u>ability</u> and success in this algebra class.

   a) I am very confident of my algebra skills.
   b) I am slightly confident of my algebra skills.
   c) My algebra skills are about average.
   d) My algebra skills are below average.
   e) My algebra skills are very weak.

3. Circle <u>one</u> which <u>best</u> describes your <u>attitude</u> about this algebra class.

   a) I have really enjoyed this class.
   b) The class has been a somewhat positive experience for me.
   c) I have rather neutral feelings about the class experience.
   d) In general, I have disliked this class experience.
   e) I really hate attending this class.

4. Circle <u>one</u> which <u>best</u> describes your <u>ability</u> and success on word problems from this course.
   a) Very good, excellent ability to do.
   b) Above average ability to do.
   c) About average ability to do.
   d) Below average ability to do.
   e) Very poor, weak ability to do.

Retention Test Form - Continued

5. Circle <u>one</u> which <u>best</u> describes your <u>attitude</u> towards doing algebra word problems.

   a) Easy, yet enjoyable
   b) Challenging, yet enjoyable
   c) Neutral - lacking opinion
   d) Challenging, yet not enjoyable
   e) Easy, but not enjoyable
   f) Total dislike for such exercises

6. Circle <u>one</u> as to how <u>useful</u> you feel this course will be in your future high school program and beyond.

   a) Very useful
   b) Some usefulness
   c) Little real usefulness
   d) No usefulness whatsoever

7. Suppose an eighth grader asked you about taking an algebra class next fall with your present teacher, what would you tell them about the class and your teacher.

Equation Solving Section: Solve each equation for x, show all work and steps in space provided.

1. $x + x - 180 = -3x - 80$

   $x =$

2. $5(x - 2) + 10(x + 1) = 90$

   $x =$

3. $64x = 48\ (7 - x)$

   $x =$

4. $x + 25 - 4x = 10$

   $x =$

Retention Test Form - Continued

5. $-3(x + 3) = 7 - 4x$

$x =$

6. $3x + 60 = 8x$

$x =$

7. $x + (x + 2) - 26 = 4x - (x + 4)$

$x =$

Retention Test Form - Continued

Verbal Problem Section: Read and solve each word problem for what is called for. Write your numerical answers in spaces provided. Show all work on this sheet, and do not erase any work.

1. The side of a square is twice as long as the side of an equilateral triangle (a triangle with all sides equal). The perimeter of the square is 60 cm more than the perimeter of the triangle. What are the lengths of the sides of the square and triangle?

each side square = ____cm
each side triangle = ____cm

2. Two numbers differ by 3. Four times the smaller number decreased by three times the larger number is 7. Find the two numbers.

numbers are ___ & ___

3. How far in kilometers can a person drive into the country at an average speed of 64 km/hr and return over the same roads at an average speed of 48 km/hr if the total trip takes 7 hours?

distance = ____km

4. In triangle ABC, the measure of angle C is three times the measure of angle A, and the measure of angle A is 80 degrees less than the measure of angle B. Find the measure of each angle.

angle A = ____degrees
angle B = ____degrees
angle C = ____degrees

Retention Test Form - Continued

5. Find 3 consecutive even integers whose sum is 26 more than 4 times the smallest integer.

Numbers are ___, ___, ___

6. A woman was 25 years old when her daughter was born. The mother's present age is 10 years more than 4 times her daughter's present age. Find the present age of mother and daughter.

Mother = ____years old
Daughter = ____years old

7. John has 4 more dimes than nickels. After spending 2 nickels and 3 dimes, he has coins worth 90 cents left. How many of each coin does he have left?

Nickels left = _____
Dimes left = _____

APPENDIX G

Interview Related Materials

Parental Consent Form

December 10, 1986

Dear Parents:

This semester as part of my dissertation research through Michigan State University, E. Lansing, Michigan, I have been observing your son's or daughter's mathematics (Algebra I) class at (respective school). I have received the necessary approval from the (repective school district) Administration and (teacher's name), the teacher involved. My purpose is to analyze the instruction students receive in regards to verbal or word problems (stories), and their ability to make sense out of such instruction and practice, and use this knowledge effectively to solve verbal problems. I have sat in on about 10 such classes since September, observing and recording teaching sequences, and serving as an aide when students are working on verbal problems.

I am presently at the point where I need to assess and analyze what abilities the students have developed. To accomplish this goal, I would like to interview 3 students for a period of 45 minutes each, one at a time. Students will be chosen in consultation with (teacher name) and done at (respective school) during school hours. Preferably they will be done during a student's study hall period, or during regular class time.

The interviews will be conducted in a small tutoring room, the student and I alone, and will be audiotaped. I will first explore student perceptions and attitudes about algebra, verbal problems, and homework, asking students to comment on each area with open-ended questions. Then I will ask the student to attempt to solve 5 verbal problems similar to those he or she has seen during the semester's work. As the student is working, I will ask them to verbalize aloud as much as possible of their thinking as they proceed. Minimal help will be provided to the student.

Parental Consent Form - Continued

At any time during the interview, the student has the right to stop work on a problem, discontinue the interview process for whatever reason without interviewer questioning of the reasons, or to remove themselves from the research study completely.

It should be pointed out that I did seven similar interviews last year with Algebra I students, and found them very cooperative to the described procedures. The researcher will be the only person allowed access to the produced audiotapes and the written work done by the students during the interview. Any reference to the interview results will use a coding system to preserve total anonymity of the subjects involved. All data collected will remain in the possession of the researcher. Parents have the right to request that their son or daughter's interview results be excluded from the study, and the researcher will abide by their decision at any time in this matter. If you desire more details of the study or wish to clarify something, feel free to contact me at UWSP (346-4151) or at home (344-2533).

Sincerely,

Gary Klingler
Math and Computing

My son or daughter, _____ and I have carefully read the description provided above, and have agreed that he or she will be willing to participate in the interview described, if he or she is chosen to be.

_____ _____ _____
student signature date parent/guardian sign.

Student Schedule

Hour	1	2	3	4	5	6	7	8
Class								

I do not wish my son or daughter to participate in the proposed interview at this time.

_____
parent/guardian signature

Interview Instrument Design

Section A. Entry Questions - all 9 students

1. Describe your mathematical background, schools attended and teachers you have had since sixth grade.

2. What was the grade you received for the algebra course for the first quarter? Estimate your grade for the present quarter.

3. What types of algebra exercises or content do you find easiest? What types do you find most difficult?

4. Think of your present algebra teacher. How does he/she compare with other math teachers you have had? Is he/she best, better, about the same, worse, or worst?

5. Focus on word problems or story problems in algebra. In doing word problems, what do you think are your strengths?
   What are your weaknesses?

6. What purposes do you think there are for doing word problems in algebra, or why do you think they are included in the algebra content?

7. Suppose your teacher explains a word problem section in class, and then assigns 8 word problems for you to complete for class tomorrow.
   a) How many of the 8 problems would you likely finish in class?

   b) Where would you finish the rest of the assignment? (study hall, at home, library, others)

   c) How long would you estimate it would take you to complete all 8 word problems?

   d) Would it be likely that you would need any help to finish all 8 problems of the assignment?

   e) If one problem proved troublesome, where would you likely seek additional help? (textbook, notes, brother/sister, mom/dad, teacher, school friend, or just not do)

Interview Instrument Design - Continued

Section B. Skill Confidence Assessment

Suppose a story or word problem came from your textbook from a particular section: A (easiest), B or C. For each content type described of a problem, tell me the percent of the time or the probability that you could set-up the problem, find the correct equation (assuming a correct set-up), and solve the equation correctly and derive a correct answer (assuming correct equation formulation).

Content Type	Section	% Probability
1. Number, consecutive integer type	B	Set-up _____% Equation _____% Correct Answer ____%
2. Coin, money related	A	Set-up _____% Equation _____% Correct Answer ____%
3. Measurement/geometric	A	Set-up _____% Equation _____% Correct Answer ____%
4. Age related	A	Set-up _____% Equation _____% Correct Answer ____%
5. Distance/rate/time	A	Set-up _____% Equation _____% Correct Answer ____%
6. Non-text, nonroutine multi-step problem	A	Set-up _____% Equation _____% Correct Answer ____%

Interview Instrument Design - Continued

Section C. Verbal Problem Sets: By Teacher and Textbook

1. Alice - completed by students, A1, A2, A3

1. Find 3 consecutive odd integers such that their sum decreased by the second integer equals 50.

2. Joe sets aside dimes and quarters to pay tolls (requires 35 cents). He has four more quarters than dimes collected so far. The total value of his collection is $7.30. How many dimes and quarters has he collected so far?

3. The length of a rectangle is 8 inches more than twice its width. The perimeter of the rectangle is 112 inches. Find the length and width of the rectangle.

4. Mrs. Montez is 20 years older than Mrs. Cooke. Sixteen years ago, Mrs. Montez was 3 times as old as Mrs. Cooke was then. How old are Mrs. Montez and Mrs. Cooke presently?

5. Two bicyclists, Jack and Harry started at the same time from opposite ends of a bike trail, and rode towards each other. Harry rides at a speed of 4 miles per hour faster than Jack rides. The trail is 112 miles long and they meet after 4 hours of riding. Find the speeds or rates of Jack and Harry.

6. Nancy has 2 male children, Seth and Ben. Seth was 6 years old when Ben was born. Presently Nancy is twice as old as Seth and three times as old as Ben. The sum of their ages presently is 66 years. How old are Nancy, Seth and Ben today?

2. Betty - completed by students, B1, B2, B3

1. Find four consecutive integers such that the sum of the two greatest integers subtracted from twice the sum of the two smallest integers is 15.

2. Susan makes $3 an hour working after school and $4 an hour working on Saturday. Last week she worked a total of 12 hours and made $42.50. How many hours did she work on Saturday?

3. The length of a rectangle is 8 inches more than twice its width. The perimeter of the rectangle is 112 inches. Find the length and width of the rectangle.

Interview Instrument Design - Continued

4. Len is 10 years older than Marie. In six years, Len will be twice as old as Marie. How old is each now?

5. Barry and Juan live 72 km apart. At noon each leaves his house and bicycles toward the other. Juan rides at a rate 2 km per hour faster than Barry. They meet at exactly 3:00 PM. Find the rate or speed of each boy.

6. Nancy has 2 male children, Seth and Ben. Seth was 6 years old when Ben was born. Presently Nancy is twice as old as Seth and three times as old as Ben. The sum of their ages presently is 66 years. How old are Nancy, Seth and Ben today?

3. Carl - completed by students, C1, C2, C3

1. Three numbers have the sum of 119. The second number is 8 greater than the first number, and the third number is 3 greater than the first number. Find the three numbers.

2. Twenty-one nickels and dimes are worth $1.65. Find the number (how many) of each type of coin.

3. The length of a rectangle is twice its width. If the length measure is increased by 6 meters, and the width is decreased by 2 meters, the perimeter is 44 meters. Find the dimensions of the original rectangle.

4. Otto is 10 years older than Yusef. In 8 years, twice Otto's age will equal 3 times Yusef's age. How old is each child presently?

5. Two freight trains start at the same time from two points 480 km apart, and travel toward each other. One train travels at a rate or speed 10 km per hour faster than the second train. They meet in 6 hours (not a collision). Find the rate or speed of each train.

6. Nancy has 2 male children, Seth and Ben. Seth was 6 years old when Ben was born. Presently Nancy is twice as old as Seth and three times as old as Ben. The sum of their ages presently is 66 years. How old are Nancy, Seth and Ben today?

APPENDIX H

Contingency Tables: Suggestion Analysis
Independence Tests

Teacher: Alice
Variables: Content Focus and Segment Placement
Exclusions: General Model (Content Focus)
Seatwork/Controlled Practice (Segment Place.)

		Segment Placement: Homework Review	Segment Placement: Lesson Development	Total
Content Focus	Tool Skill	14 (9.47)	9 (13.53)	23
	Task Specific Heuristic	3 (7)	14 (10)	17
	General Heuristic	4 (4.53)	7 (6.47)	11
	Total	21	30	51

H_0: Content Focus and Segment Placement are independent.
H_A: Content Focus and Segment Placement not independent.
Computed X^2 = 7.62 Table $X^2_{(.05, 2df)}$ = 5.99
Conclusion: Reject Null Hypothesis.

Teacher: Carl
Variables: Content Focus and Oral Format
Exclusions: General Model (Content Focus)

		Oral Format: Question	Oral Format: Statement	Total
Content Focus	Tool Skill	20 (17.4)	13 (15.6)	33
	Task Specific Heuristic	5 (5.27)	5 (4.73)	10
	General Heur.	4 (6.32)	8 (5.68)	12
	Total	29	26	55

H_0: Content Focus and Oral Format are independent.
H_A: Content Focus and Oral Format are not independent.
Computed X^2 = 2.63 Table $X^2_{(.05, 2df)}$ = 5.99
Conclusion: Fail to reject Null Hypothesis

Independence Contingency Tables - Continued

Teacher: Alice
Variables: Segment Placement and Oral Format
Exclusion: Seatwork/Controlled Practice (Segment Place.)

		Segment Placement		
		Homework Review	Lesson Development	Total
Oral Format	Question	6 (3.63)	3 (5.37)	9
	Statement	17 (19.37)	31 (28.63)	48
	Total	23	34	57

H_0: Segment Placement and Oral Format are independent.
H_A: Segment Placement and Oral Format are not independent.
Computed X^2 = 3.17 Table $X^2_{(.05,\ 1df)}$ = 3.84
Conclusion: Fail to reject Null Hypothesis

Teacher: Betty
Variables: Segment Placement and Oral Format
Exclusion: Seatwork/Controlled Practice (Segment Place.)

		Segment Placement		
		Homework Review	Lesson Development	Total
Oral Format	Question	7 (6)	5 (5)	12
	Statement	19 (20)	21 (20)	40
	Total	26	26	52

H_0: Segment Placement and Oral Format are independent.
H_A: Segment Placement and Oral Format are not independent.
Computed X^2 = 0.44 Table $X^2_{(.05,\ 1df)}$ = 3.84
Conclusion: Fail to reject Null Hypothesis

Independence Contingency Tables - Continued

Teacher: Carl
Variables: Segment Placement and Oral Format
Exclusion: Seatwork/Controlled Practice (Segment Place.)

		Segment Placement		
		Homework Review	Lesson Development	Total
Oral Format	Question	12 (9.67)	17 (19.33)	29
	Statement	7 (9.33)	21 (18.67)	28
	Total	19	38	57

H_0: Segment Placement and Oral Format are independent.
H_A: Segment Placement and Oral Format are not independent.
Computed X^2 = 1.67 Table X^2 (.05, 1df) = 3.84
Conclusion: Fail to reject Null Hypothesis

APPENDIX I

Process Coding of Teacher Examples

Part 1: Distinct Process Sequences

Teacher: Alice
Number of Coded Examples: 19

Process Sequences (see Coding Form 1)	Quantity
1A,H,I.2H.3A.4A	3
1A,H,I.2H.3A.4A,D(b)	1
1A,H,I.2G.3F	2
1A,H,I.2G.3B	1
1A,I,H,I.2G	1
1A,D,H,I.2G	1
1G,A,D,G,G,I,I.2D,G	2
1A,G,H,I.2G,H	1
1A,H,I.2G.3B.4A,D(b)	1
1A,G,I.2H	1
1H,I.2G.3B.4A,D(b)	1
1A,H,D,I,I.2G	2
1A,H,D,I (bell)	1
1A,C,H,I,I,I.2H	1
Total	19

Teacher: Betty
Number of Coded Examples: 11

Process Sequences (see Coding Form 1)	Quantity
1A,G,I.2H	1
1A,G,I.2G	1
1A,H,I.2H.3B.4A	1
1A,H,I.2H.3B.4D(b)	1
1A,H,I.2G	1
1A,H,I.2G.3B	2
1A,I.2G.3B.4D(b and c)	1
1A,I.2G.3B.4A,D(c)	1
1A.2A.1H,B,I,I.2G.3D.4A,D(c)	1
1A,H,I,B,I.2G.3B,D.4A,D(c)	1
Total	11

Distinct Process Sequences - Continued

Teacher: Carl
Number of Coded Examples: 22

Process Sequences (see Coding Form 1)	Quantity
1A,I.2G.3B.4A,D(b)	1
1A,B,I.2G.3B.4A,D(b)	1
1A,G,G,H.2D.3C,D	1
1A,G,G,H,I.2D,H.3B,D.4A	1
1A,E,H,F.3D.4A	1
1A,D,E,F,I.2H.3C.4A,D(b and c)	1
1A,H,G.2D,H.3B,D.4A,D(b)	1
1A,E.2G.3B.4A	1
1A,E,I.2G.3B,D.4A	1
1C,E.3D.2G	1
1C.3B,B.4A.2G.4A	1
1B.2B,G.3A.2G.3A	1
1G.2G.1H.3B	1
1G.2B.3B	1
1C,G,I.2G.3B	1
1C,C.2G.3A	1
1C.3A	1
1A,H,I.2G.3B,D.4A,D(b)	3
1A,G,H,I.2G.3B,D.4A,D(b)	1
1A,I.3D.4D(b).1I.2G.3B.4A	1
Total	22

Summary Example Process Quantities
(see Coding Form 1)

	Alice	Betty	Carl	Total
Number of examples	19	11	22	52
Process Category				
1. Understanding the Problem				
A	18	11	14	43
B	0	2	2	4
C	1	0	6	7
D	6	0	1	7
E	0	0	4	4
F	0	0	2	2
G	8	2	9	19
H	16	7	8	31
I	25	13	12	50
Subtotal: Phase 1	74	35	58	167

Summary Example Process Quantities - Continued

	Alice	Betty	Carl	Total
2. Devising a Plan				
D	2	0	3	5
G	12	8	17	37
H	9	3	3	15
Subtotal: Phase 2	23	11	23	57
3. Carrying Out the Plan				
A	4	0	4	8
B	3	7	16	26
C	0	0	2	2
D	0	2	11	13
F	11	3	0	14
Subtotal: Phase 3	18	12	33	63
4. Looking Back				
A	6	4	14	24
D (b)	3	2	9	14
D (c)	0	4	1	5
Subtotal: Phase 4	9	10	24	43
Total Processes used (all examples)	124	68	138	330
Mean Number of Processes per example	6.52	6.18	6.27	6.50
Percent of processes per phase				
Phase 1	59.7%	51.5%	42.0%	50.6%
Phase 2	18.5%	16.2%	16.7%	17.2%
Phase 3	14.5%	17.6%	23.9%	19.1%
Phase 4	7.3%	14.7%	17.4%	13.1%

APPENDIX J

Individual Student Performance Results
Teacher: Alice

Student	Pretest Score (12)	Posttest Form	Posttest Score (90)	Retention Test Eq. Score (7)	Retention Test P.S.Score (70)
1.	5	B	28	3	31
2.	3	B	14	4	12
3.	3	A	25	6	33
4.	11	A	40	6	16
5.	0	B	28	5	9
6.	4	A	35	6	48
7.	3	B	23	4	19
8.	2	A	30	5	16
9.	6	B	31	7	22
10.	2	A	20	4	13
11.	3	B	31	7	9
12.	5	B	25	6	16
13.	1	A	22	5	17
14.	2	A	30	6	37
15.	4	B	27	5	21
16.	6	A	24	2	17
17.	0	B	42	6	50
18.	6	A	24	2	12
Totals	66		499	89	398
Mean	3.67		27.72	4.94	2.11
Variance	6.94		46.21	2.29	157.28

Individual Student Performance Results - Continued

Teacher: Betty

Student	Pretest Score (12)	Posttest Form	Posttest Score (90)	Retention Test Eq. Score (7)	Retention Test P.S.Score (70)
1.	0	B	7	4	3
2.	3	B	11	4	2
3.	1	A	35	5	7
4.	8	A	27	7	37
5.	6	B	39	6	26
6.	4	B	34	5	16
7.	2	B	3	2	0
8.	1	B	24	4	14
9.	10	B	0	0	1
10.	2	A	0	5	0
11.	6	A	16	6	1
12.	7	B	20	6	8
13.	5	B	9	4	4
14.	0	A	3	5	0
15.	0	A	2	7	6
16.	2	A	2	0	1
17.	0	A	20	5	11
18.	8	B	36	4	22
19.	11	A	30	6	17
20.	1	A	15	4	7
21.	6	B	7	4	3
22.	9	A	51	6	42

Individual Student Performance Results - Continued

Teacher: Betty (continued)

Student	Pretest Score	Posttest Form	Posttest Score	Retention Test Eq. Score	Retention Test P.S.Score
23.	4	A	22	7	7
24.	5	B	12	0	2
25.	11	B	17	5	0
26.	6	A	13	4	16
27.	3	A	0	not taken/demoted	
Totals	121		455	115	253
Mean	4.48		16.85	4.42	9.73
Variance	12.18		193.44	4.01	129.64

Teacher: Carl

Student	Pretest Score (12)	Posttest Form	Posttest Score (90)	Retention Test Eq. Score (7)	Retention Test P.S.Score (70)
1.	8	B	51	6	2
2.	4	B	40	3	15
3.	2	B	37	4	22
4.	7	A	34	6	15
5.	2	B	21	3	4
6.	7	A	24	5	4
7.	8	A	57	6	26
8.	4	B	44	4	13
9.	8	A	16	2	6
10.	4	A	9	5	0

Individual Student Performance Results - Continued

Teacher: Carl (continued)

Student	Pretest Score	Posttest Form	Posttest Score	Retention Test Eq. Score	Retention Test P.S.Score
11.	7	A	32	4	3
12.	4	A	49	6	10
13.	6	B	34	4	0
14.	10	B	15	6	8
15.	5	B	39	4	13
16.	7	A	50	6	12
17.	0	B	47	7	24
18.	2	B	30	5	13
19.	8	B	46	6	26
20.	3	A	24	3	13
21.	11	A	67	6	39
22.	6	A	55	0	8
Totals	123		821	101	276
Mean	5.59		37.32	4.59	12.55
Variance	7.97		227.08	3.30	97.59

APPENDIX K

Summary of Posttest and Retention Test Results

Posttest Summary: Conditional Counts

Teacher: Alice

Item Number	Attempts	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
		Number Counts			
1. 18	16	16	16	13	13
2. 18	17	7	5	3	3
3. 18	2	0	0	0	0
4. 18	15	12	1	1	1
5. 18	17	5	0	0	0
6. 18	18	14	2	1	0
7. 18	14	8	4	3	1
8. 18	15	8	1	1	1
9. 18	14	4	1	1	1
Totals 162	128	74	30	23	20

Teacher: Betty

Item Number	Attempts	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
		Number Counts			
1. 27	16	15	11	8	8
2. 27	21	10	1	1	1
3. 27	7	3	0	0	0
4. 27	20	12	8	7	5
5. 27	21	4	1	0	0
6. 27	4	3	0	0	0

Posttest Summary: Conditional Counts - Continued

Teacher: Betty (continued)

Item Number	Attempts Definition	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
		Number Counts			
7. 27	14	7	2	1	1
8. 27	19	13	9	6	6
9. 27	15	1	1	0	0
Totals 243	137	68	33	23	21

Teacher: Carl

Item Number	Attempts Definition	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
		Number Counts			
1. 22	21	19	19	19	19
2. 22	22	4	4	4	3
3. 22	14	1	1	0	0
4. 22	21	14	10	9	9
5. 22	21	11	3	3	2
6. 22	22	22	14	13	9
7. 22	19	12	7	7	4
8. 22	16	9	6	5	5
9. 22	17	1	0	0	0
Totals 198	173	93	64	60	51

Posttest Summary: Student Process Scores

Teacher: Alice

Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	30	32	27	24	22	135
2.	24	21	14	14	6	79
3.	3	0	1	0	0	4
4.	14	1	3	1	0	19
5.	17	1	4	7	0	29
6.	28	5	9	10	13	65
7.	17	9	16	10	2	54
8.	17	2	13	5	4	41
9.	11	8	13	11	4	47
Totals	161	79	100	82	51	473
Possible	324	324	324	324	324	1620

Teacher: Betty

Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	31	22	21	21	16	111
2.	28	5	5	5	2	45
3.	10	5	7	5	4	31
4.	27	21	18	15	13	94
5.	13	2	4	4	0	23
6.	6	0	0	0	1	7
7.	17	5	6	5	2	35
8.	27	21	15	14	12	89

Posttest Summary: Student Process Scores - Continued

Teacher: Betty (continued)
Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
9.	4	7	6	2	1	20
Totals	163	88	82	71	51	455
Possible	486	486	486	486	486	2430

Teacher: Carl
Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	38	40	38	39	39	194
2.	26	19	15	14	6	80
3.	2	11	6	3	1	23
4.	28	27	25	26	19	125
5.	25	7	18	13	7	70
6.	44	28	31	19	18	140
7.	24	19	23	18	8	92
8.	18	14	13	11	11	67
9.	2	11	11	4	2	30
Totals	207	176	180	147	111	821
Possible	396	396	396	396	396	1980

Retention Test Summary: Conditional Counts

Teacher: Alice
Number Counts

Item Number	Attempts Definition	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
1. 18	14	8	1	1	0
2. 18	18	14	9	8	7
3. 18	5	2	2	2	0
4. 18	18	5	2	2	2
5. 18	18	16	14	9	5
6. 18	15	10	4	4	4
7. 18	18	12	1	1	1
Totals 126	106	67	33	27	19

Teacher: Betty
Number Counts

Item Number	Attempts Definition	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
1. 26	16	11	1	1	0
2. 26	14	9	9	6	6
3. 26	9	1	0	0	0
4. 26	10	5	2	1	1
5. 26	6	4	3	2	0
6. 26	13	9	2	2	2
7. 26	18	11	3	3	1
Totals 182	86	50	20	15	10

Retention Test Summary: Conditional Counts - Continued

Teacher: Carl
Number Counts

Item Number	Attempts	Variable Definition	Equation Formulation	Equation Solving	Inter-pretation Answer
1. 22	7	6	1	1	1
2. 22	19	15	10	6	5
3. 22	6	0	0	0	0
4. 22	13	6	1	0	0
5. 22	20	14	9	8	6
6. 22	5	3	1	1	1
7. 22	16	7	0	0	0
Totals 154	86	51	22	16	12

Retention Test Summary: Student Process Scores

Teacher: Alice

Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	17	3	6	5	0	31
2.	28	19	19	16	14	96
3.	4	4	4	1	1	14
4.	14	4	10	9	4	41
5.	34	27	21	17	10	109
6.	20	11	12	11	11	65
7.	27	3	5	4	3	42
Totals	144	71	77	63	43	398
Possible	252	252	252	252	252	1260

Retention Test Summary: Student Process Scores - Continued

Teacher: Betty
Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	23	3	8	3	1	38
2.	19	20	12	13	12	76
3.	2	0	2	0	1	5
4.	13	4	2	3	3	25
5.	8	7	6	2	0	23
6.	18	4	6	4	5	37
7.	22	8	9	3	7	49
Totals	105	46	45	28	29	253
Possible	364	364	364	364	364	1820

Teacher: Carl
Total Process Scores

Item	Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	13	2	3	2	2	22
2.	30	21	16	16	10	93
3.	0	0	0	0	0	1
4.	18	3	2	0	3	26
5.	28	19	18	14	13	92
6.	6	2	3	2	6	19
7.	15	0	3	1	4	23
Totals	110	47	45	35	39	276
Possible	308	308	308	308	308	1540

APPENDIX L

Posttest Item Analysis

Item 1
Content Classification: Simple Translation Type
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	16	16	21
Number correctly defining variables	16	15	19
Number correctly formulating equations	16	11	19
Number correctly solving equations	13	8	19
Number correctly interpreting and correct ans.	13	8	19
Number other correct ans. nonstandard derivation	0	0	0
Process Mean Score	7.50	4.11	8.82
Item Variance	10.74	17.56	9.39

3 errors in defining variables Patterns: n, 26-n (2)
4 errors in equation formulation Patterns: n + 26 = 14 (2)

Item 2
Content Classification: Geometric Type
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	17	21	22
Number correctly defining variables	7	10	4
Number correctly formulating equations	5	1	4
Number correctly solving equations	3	1	4
Number correctly interpreting and correct ans.	3	1	3
Number other correct ans. nonstandard derivation	0	0	0
Process Mean Score	4.39	1.67	3.64
Item Variance	7.90	4.31	8.72

39 errors in defining variables Patterns: n,2n,2n-4 (28)
11 errors in equation formulation Patterns: unaware sum = 180 deg. (10)

Posttest Item Analysis - Continued

Item 3
Content Classification: Money/coin related
Format Classificiation: Concrete/Factual

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	2	7	14
Number correctly defining variables	0	3	1
Number correctly formulating equations	0	0	1
Number correctly solving equations	0	0	0
Number correctly interpreting and correct ans.	0	0	0
Number other correct ans. nonstandard derivation	0	2	1
Process Mean Score	0.22	1.15	1.05
Item Variance	0.30	7.48	1.85

19 errors in defining variables
Patterns: assumed equal times n and n (8)
one variable only used in problem (4)
4 errors in equation formulation
Patterns: careless computational error (2)

Item 4
Content Classification: Distance/Rate/Time related
Format Classificiation: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	15	20	21
Number correctly defining variables	12	12	14
Number correctly formulating equations	1	8	10
Number correctly solving equations	1	7	9
Number correctly interpreting and correct ans.	1	5	9
Number other correct ans. nonstandard derivation	0	2	1
Process Mean Score	2.28	3.48	5.68
Item Variance	4.80	15.41	14.79

18 errors in defining variables
Patterns: x constant by variable, rather than + (15)
20 errors in equation formulation
Patterns: incomplete(8), adds rates only = distance (6)

Posttest Item Analysis - Continued

Item 5
Content Classification: Age related
Format Classificiation: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	17	21	21
Number correctly defining variables	5	4	11
Number correctly formulating equations	0	1	3
Number correctly solving equations	0	0	3
Number correctly interpreting and correct ans.	0	0	2
Number other correct ans. nonstandard derivation	0	0	2
Process Mean Score	1.61	0.85	3.18
Item Variance	1.90	1.75	7.58

39 errors in defining variables
Patterns: using "twice" in defing variables (20)
incompletion of ages 5 years ago (8)
16 errors in equation formulation
Patterns: relationship reversed to be possible (8)
Helen = 2 x Jean's original age (2)

Item 6
Content Classification: Integer Type
Format Classificiation: Abstract/Hypothetical

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	18	4	22
Number correctly defining variables	14	3	22
Number correctly formulating equations	2	0	14
Number correctly solving equations	1	0	13
Number correctly interpreting and correct ans.	0	0	9
Number other correct ans. nonstandard derivation	7	1	0
Process Mean Score	3.61	0.26	6.36
Item Variance	6.84	0.43	11.48

5 errors in defining variables
Patterns: used consecutive integers n,n+1, n+2 (2)
23 errors in equation formulation
Patterns: reversal placement of 49(11),forgot 2 x left (5)

Posttest Item Analysis - Continued

Item 7
Content Classification: Geometric type (perimeter)
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	14	14	19
Number correctly defining variables	8	7	12
Number correctly formulating equations	4	2	7
Number correctly solving equations	3	1	7
Number correctly interpreting and correct ans.	1	1	4
Number other correct ans. nonstandard derivation	0	0	0
Process Mean Score	3.00	1.30	4.18
Item Variance	7.65	4.99	11.20

20 errors in defining variables
Patterns: used n and n + 12 (9)
used 2n and n + 12 (4)
14 errors in equation formulation
Patterns: used length + width = perimeter (10)

Item 8
Content Classification: Money/coin related
Format Classificiation: Concrete/Factual

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	15	19	16
Number correctly defining variables	8	13	9
Number correctly formulating equations	1	9	6
Number correctly solving equations	1	6	5
Number correctly interpreting and correct ans.	1	6	5
Number other correct ans. nonstandard derivation	2	0	1
Process Mean Score	2.28	3.30	3.05
Item Variance	5.86	15.45	16.62

20 errors in defining variables
Patterns: n and 2n used (6), 2n and 2n+1 used (4)
14 errors in equation formulation
Patterns: forgot coin values (7),errors in distributing(3)

Posttest Item Analysis - Continued

Item 9
Content Classification: Age related
Format Classificiation: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	18	27	22
Number attempting to define variables	14	15	17
Number correctly defining variables	4	1	1
Number correctly formulating equations	1	1	0
Number correctly solving equations	1	0	0
Number correctly interpreting and correct ans.	1	0	0
Number other correct ans. nonstandard derivation	1	0	2
Process Mean Score	2.61	0.74	1.36
Item Variance	6.60	1.82	1.67

40 errors in defining variables
Patterns: used n and 2n for present ages (9)
used n - 1 and 2n + 1 for present ages (6)
used n + 1 and 2n - 1 for present ages (6)

4 errors in equation formulation
Patterns: none, 4 different error types

APPENDIX M

Retention Test Item Analysis

Item 1
Content Classification: Geometric Type
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	14	16	7
Number correctly defining variables	8	11	6
Number correctly formulating equations	1	1	1
Number correctly solving equations	1	1	1
Number correctly interpreting and correct ans.	0	0	1
Number other correct ans. nonstandard derivation	0	1	0
Process Mean Score	1.72	1.46	1.00
Item Variance	3.15	2.90	4.95

12 errors in defining variables
Patterns: n, 2n not as sides in figure (3)
22 errors in equation formulation
Patterns: failure to relate perimeters of 2 figures (13)

Item 2
Content Classification: Simple Translation Type
Format Classificiation: Abstract/Hypothetical

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	18	14	19
Number correctly defining variables	14	9	15
Number correctly formulating equations	9	9	10
Number correctly solving equations	8	6	6
Number correctly interpreting and correct ans.	7	6	5
Number other correct ans. nonstandard derivation	0	0	0
Process Mean Score	5.33	2.92	4.23
Item Variance	16.94	17.75	13.61

13 errors in defining variables
Patterns: include 4n (5), x and y used not related (5)
10 errors in equation formulation
Patterns: distributive property errrors (7)

Retention Test Item Analysis - Continued

Item 3
Content Classification: Distance/Rate/Time related
Format Classificiation: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	5	9	6
Number correctly defining variables	2	1	0
Number correctly formulating equations	2	0	0
Number correctly solving equations	2	0	0
Number correctly interpreting and correct ans.	0	0	0
Number other correct ans. nonstandard derivation	1	1	1
Process Mean Score	0.78	0.19	0.05
Item Variance	4.42	0.32	0.05

17 errors in defining variables
Patterns: time variable not defined (6),
equal times used (assumed) (5)
1 errors in equation formulation
Patterns: distances not equated (1)

Item 4
Content Classification: Geometric Type
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	18	10	13
Number correctly defining variables	5	5	6
Number correctly formulating equations	2	2	1
Number correctly solving equations	2	1	0
Number correctly interpreting and correct ans.	2	1	0
Number other correct ans. nonstandard derivation	0	1	3
Process Mean Score	2.28	0.96	1.18
Item Variance	10.09	4.52	1.58

25 errors in defining variables
Patterns: failure to relate 3 angles to 1 variable (13)
11 errors in equation formulation
Patterns: unaware sum of 3 angles of triangle = 180 (6)

Retention Test Item Analysis - Continued

Item 5
Content Classification: Integer Problem Type
Format Classificiation: Abstract/Factual

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	18	6	20
Number correctly defining variables	16	4	14
Number correctly formulating equations	14	3	9
Number correctly solving equations	9	2	8
Number correctly interpreting and correct ans.	5	0	6
Number other correct ans. nonstandard derivation	0	0	1
Process Mean Score	6.06	0.88	4.18
Item Variance	8.17	3.79	16.54

10 errors in defining variables
Patterns: x, x + 1, x + 2 used (7)
8 errors in equation formulation
Patterns: none, 6 different error types noted

Item 6
Content Classification: Age related
Format Classificiation: Concrete/Factual

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	15	13	5
Number correctly defining variables	10	9	3
Number correctly formulating equations	4	2	1
Number correctly solving equations	4	2	1
Number correctly interpreting and correct ans.	4	2	1
Number other correct ans. nonstandard derivation	3	0	4
Process Mean Score	3.61	1.42	0.86
Item Variance	13.90	7.45	4.79

11 errors in defining variables
Patterns: x, 4x used (4), 4x and 4x +10 used (3)
15 errors in equation formulation
Patterns: incomplete or no attempt (6)

Retention Test Item Analysis - Continued

Item 7
Content Classification: Money/coin related
Format Classificiation: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	18	26	22
Number attempting to define variables	18	18	16
Number correctly defining variables	12	11	7
Number correctly formulating equations	1	3	0
Number correctly solving equations	1	3	0
Number correctly interpreting and correct ans.	1	1	0
Number other correct ans. nonstandard derivation	1	5	4
Process Mean Score	2.33	1.88	1.05
Item Variance	5.18	5.71	1.28

22 errors in defining variables
Patterns: n, 4n used (7)

26 errors in equation formulation
Patterns: no values used (12),
no equation attempt (9)

APPENDIX N

Interview Results

Student Phase Confidence Estimates
(all data entries in percents)

Student: A1

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	75	75	80
2. Coin/Money Related - A	75	75	80
3. Geometric Type - A	85	85	90
4. Age Related - A	75	75	80
5. D/R/T Related - A	80	80	85
6. Nonroutine Multi-step - A	70	70	75
Mean phase estimate	76.7	76.7	81.7

Student: A2

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	60	85	90
2. Coin/Money Related - A	90	100	95
3. Geometric Type - A	75	75	75
4. Age Related - A	60	70	90
5. D/R/T Related - A	80	85	95
6. Nonroutine Multi-step - A	80	90	95
Mean phase estimate	74.2	84.2	90

Student Phase Confidence Estimates - Continued

Student: A3

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	90	85	100
2. Coin/Money Related - A	60	50	70
3. Geometric Type - A	20	30	75
4. Age Related - A	10	10	20
5. D/R/T Related - A	50	60	80
6. Nonroutine Multi-step - A	25	30	55
Mean phase estimate	42.5	44.2	66.7

Student: B1

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	95	95	100
2. Coin/Money Related - A	95	95	100
3. Geometric Type - A	90	90	95
4. Age Related - A	95	95	100
5. D/R/T Related - A	95	95	100
6. Nonroutine Multi-step - A	90	90	95
Mean phase estimate	93.3	93.3	98.3

Student Phase Confidence Estimates - Continued

Student: B2

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	85	70	90
2. Coin/Money Related - A	90	87	90
3. Geometric Type - A	87	87	90
4. Age Related - A	90	90	90
5. D/R/T Related - A	85	85	90
6. Nonroutine Multi-step - A	75	75	90
Mean phase estimate	85.3	82.3	90

Student: B3

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	60	60	80
2. Coin/Money Related - A	85	90	100
3. Geometric Type - A	40	35	60
4. Age Related - A	70	70	90
5. D/R/T Related - A	80	80	90
6. Nonroutine Multi-step - A	50	40	50
Mean phase estimate	64.2	62.5	78.3

Student Phase Confidence Estimates - Continued

Student: C1

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	90	85	90
2. Coin/Money Related - A	60	40	65
3. Geometric Type - A	75	60	75
4. Age Related - A	80	85	85
5. D/R/T Related - A	90	85	85
6. Nonroutine Multi-step - A	50	45	50
Mean phase estimate	74.2	66.7	75

Student: C2

Content Type	Variable Definition	Equation Formulation	Correct Answer
1. Consecutive Integer - B	80	75	70
2. Coin/Money Related - A	85	85	90
3. Geometric Type - A	65	60	70
4. Age Related - A	95	90	95
5. D/R/T Related - A	60	60	65
6. Nonroutine Multi-step - A	50	50	50
Mean phase estimate	72.5	70	73.3

Student Phase Confidence Estimates - Continued

Student: C3

	Variable Definition	Equation Formulation	Correct Answer
Content Type			
1. Consecutive Integer - B	95	95	100
2. Coin/Money Related - A	90	90	100
3. Geometric Type - A	95	95	100
4. Age Related - A	90	90	100
5. D/R/T Related - A	80	80	100
6. Nonroutine Multi-step - A	60	60	100
Mean phase estimate	85	85	100

Interview Instrument: Conditional Counts
Nine Interviewed Students

Item Number	Attempts	Number Counts Definition	Variable Definition	Equation Formulation	Equation Solving	Interpretation Answer
1.	9	8	5	5	5	5
2.	9	8	6	3	2	2
3.	9	8	7	4	3	3
4.	9	9	7	3	3	3
5.	9	9	7	4	4	4
6.	9	8	4	3	3	3
Totals	54	50	36	22	20	20

Interview Instrument: Student Process Scores
Nine Interviewed Students

Item	Total Process Scores: Variable Definition	Equation Formulation	Equation Solving	Interpretation & Check	Correct Answer	Item Total
1.	16	10	11	11	10	58
2.	12	6	7	4	4	33
3.	14	6	11	12	6	49
4.	16	6	10	9	6	47
5.	14	8	10	11	8	51
6.	11	7	8	8	6	40
Totals	83	43	57	55	40	278
Possible	108	108	108	108	108	540

Interview Instrument: Problem Solving Item Analysis

Item 1
Content Classification: Consecutive Integer Type
Format Classification: Abstract/Hypothetical

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	2	3
Number correctly defining variables	3	2	3
Number correctly formulating equations	1	1	3
Number correctly solving equations	1	1	3
Number correctly interpret. and correct answer	1	1	3
Process Mean score	5.33	4.00	10.00

0 errors in defining variables

3 errors in equation formulation

Interview Problem Solving Item Analysis - Continued

Item 2
Content Classification: Coin/Money Related
Format Classification: Concrete/Factual

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	2	3
Number correctly defining variables	2	2	2
Number correctly formulating equations	0	1	2
Number correctly solving equations	0	1	1
Number correctly interpret. and correct answer	0	1	1
Process Mean score	2.00	4.33	4.67

2 errors in defining variables

3 errors in equation formulation

Item 3
Content Classification: Geometric Type (Perimeter)
Format Classification: Abstract/Factual

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	2	3
Number correctly defining variables	2	2	3
Number correctly formulating equations	1	2	0
Number correctly solving equations	1	2	0
Number correctly interpret. and correct answer	1	2	0
Process Mean score	5.00	7.33	4.00

1 errors in defining variables

4 errors in equation formulation

Interview Problem Solving Item Analysis - Continued

Item 4
Content Classification: Age Related
Format Classification: Concrete/Hypothetical

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	3	3
Number correctly defining variables	2	3	2
Number correctly formulating equations	0	1	2
Number correctly solving equations	0	1	2
Number correctly interpret. and correct answer	0	1	2
Process Mean score	2.67	5.33	7.67

2 errors in defining variables

4 errors in equation formulation

Item 5
Content Classification: D/R/T Related
Format Classification: Concrete/Factual

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	3	3
Number correctly defining variables	2	2	3
Number correctly formulating equations	0	1	3
Number correctly solving equations	0	1	3
Number correctly interpret. and correct answer	0	1	3
Process Mean score	2.33	4.67	10.00

2 errors in defining variables

3 errors in equation formulation

Interview Problem Solving Item Analysis - Continued

Item 6
Content Classification: Nonroutine, Age Related
Format Classification: Concrete/Factual

	Alice	Betty	Carl
Number of students	3	3	3
Number attempting to define variables	3	2	3
Number correctly defining variables	1	1	2
Number correctly formulating equations	1	1	1
Number correctly solving equations	1	1	1
Number correctly interpret. and correct answer	1	1	1
Process Mean score	4.33	3.67	5.33

4 errors in defining variables

1 errors in equation formulation

LIST OF REFERENCES

LIST OF REFERENCES

Aiken, L.R. Language factors in learning mathematics. Mathematics Education Reports. Columbus, Ohio, 1972. (ERIC Document Reproduction Service No. ED 068 340)

Ashton, M.R. Heuristic methods in problem solving in ninth grade algebra. Unpublished doctoral dissertation, Stanford University, 1962.

Barnett, J. The study of syntax variables. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Bassler, O.C., Beers, M.I. and Richardson, L.I. Comparison of two instructional strategies for teaching the solution of verbal problems. Journal for Research in Mathematics Education, 1975, 6, 170-177.

Blake, R.N. The effect of problem context upon the problem solving processes used by field dependent and independent students: a clinical study. Unpublished doctoral dissertation, University of British Columbia, 1976.

Bloom, B.S. and Broder, L.J. Problem-solving processes of college students. Chicago: University of Chicago Press, 1950.

Bobrow, D. Natural language input for a computer problem-solving system. In M. Minsky (Ed.), Semantic information processing. Cambridge, Mass.: MIT Press, 1968.

Brophy, J. Teaching and learning mathematics: Where research should be going. Journal for Research in Mathematics Education, 1986, 17(5), 323-346.

Caldwell, J. The effects of abstract and hypothetical factors on word problem difficulty in school mathematics. Unpublished doctoral dissertation, University of Pennsylvania, 1977.

Caldwell, J. Syntax, content, and context variables in instruction. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Caldwell, J.H. and Goldin, G.A. Variables affecting word problem difficulty in elementary school mathematics. Journal for Research in Mathematics Education, 1979, 10, 323-336.

Caldwell, J.H. and Goldin, G.A. Variables affecting word problem difficulty in secondary school mathematics. Journal for Research in Mathematics Education, 1987, 18(3), 187-196.

Callahan, L.G. and Garofalo, J. Metacognition and school mathematics. Arithmetic Teacher, May, 1987, 22-23.

Carpenter, T.P., Corbitt, M.K., Kepner, H.S., Lindquist, M.K., and Reys, R.E. Student performance in algebra: Results from the national assessment. School Science and Mathematics, 1982, 82(6), 514-572.

Charles, R. and Lester, F. Teaching problem solving: What why and how. Palo Alto, CA: Dale Seymour Publications, 1982.

Chartoff, B.T. An exploratory investigation utilizing a multidimensional scaling procedure to discover classification criteria for algebra word problems used by students in grades 7-13. (Doctoral dissertation, Northwestern University, 1976). Dissertation Abstracts International, 1977, 37, 7006A.

Cooney, T.J. A beginning teacher's view of problem solving. Journal for Research in Mathematics Education, 1985, 16(5), 324-336.

Cooney, T.J. and Henderson, K.B. Ways mathematics teachers help students organize knowledge. Journal for Research in Mathematics Education, 1972, 3(1), 21-31.

Dahmus, M.E. How to teach verbal problems. School Science and Mathematics, 1970, 70, 121-138.

Days, H.C., Wheatley, G.H., and Kulm, G. Problem structure, cognitive level, and problem-solving performance. Journal for Research in Mathematics Education, 1979, 10(2), 135-145.

Doblaev, L.P. Thought processes involved in setting up equations. In J. Kilpatrick and I. Wirzup (Eds.), Soviet studies in the psychology of learning and teaching mathematics, (Volume III). Chicago: The University of Chicago Press, 1969.

Evertson, C.M., Emmer, E.T., and Brophy, J.E. Predictors of effective teaching in junior high mathematics classrooms. Journal for Research in Mathematics Education, 1980, 11, 167-178.

Flanders, J.R. How much of the content in mathematics textbooks is new? Arithmetic Teacher, 1987, 35(1), 18-23.

Friedman, M. and Stomper, C. Effective college instruction in basic mathematics. Journal for Research in Mathematics Education, 1988, 19(2), 169-174.

Garofalo, J. and Lester, F.K. Metacognition, cognitive monitoring, and mathematical performance. Journal for Research in Mathematics Education, 1985, 16(5), 163-176.

Goldin, G.A. Structure variables in problem solving. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Goldin, G.A. and Caldwell,J. Syntax, content, and context variables examined in a research study. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press.

Goldin, G.A. and McClintock, C.E. Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Goldman, R.N. and Weinberg, J.S. Statistics, an introduction. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1985.

Good, T.L. and Brophy, J.E. Looking in classrooms (Third Edition). New York: Harper & Row, Publishers, 1984.

Good, T.L., Grouws, D., and Ebmeier, H. Active mathematics teaching: Empirical research in elementary and secondary classrooms. New York: Longman, 1983.

Greeno, J.G. A study of problem solving. In R. Glaser (Ed.), Advances in instructional psychology. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1978.

_____. Some examples of cognitive task analysis with instructional implications. In R.E. Snow, P. Frederico, and W.E. Montague (Eds.), Aptitude, learning and instruction. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1980.

Grouws, D.A. The teacher and classroom instruction: Neglected themes in problem-solving research. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Havel, P. Students categorize, then solve problems. Arithmetic Teacher, 1985, 33(3), 19-23.

Hayes, J.R. The complete problem solver. Philadelphia: The Franklin Institute Press, 1981.

Henderson, K.B. A model for teaching mathematical concepts. Mathematics Teacher, 1967, 60, 573-577.

_____. Abstracting, generalizing and explaining - processes or relations? Mathematics Teacher, 1969, 54, 600-605.

Hinsley, D., Hayes, J.R., and Simon, H.A. From words in equations. In P. Carpenter and M. Just (Eds.), Cognitive processes in comprehension. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1977.

Jerman, M. Instruction in problem solving and an analysis of structural variables that contribute to problem solving difficulty (Technical Report No. 180). Stanford University: Institute for Mathematical Studies in the Social Sciences, November, 1971.

_____. Individualized instruction in problem solving in elementary school mathematics. Journal for Research in Mathematics Education, 1973, 4, 6-19.

Jerman, M. Problem length as a structural variable in verbal arithmetic problems. Educational Studies in Mathematics, 1974, 5, 109-123.

Kane, R.B. The readability of mathematical English. Journal of Research in Science Teaching, 1968, 5, 296-298.

Kantowski, M.G. Processes involved in mathematical problem solving (Doctoral dissertation, University of Georgia, 1974). Dissertation Abstracts International, 1975, 36, 2734A. (University Microfilms No. 75-23764).

_____. Processes involved in mathematical problem solving. Journal for Research in Mathematics Education, 1977, 8(3), 163-180.

_____. The teaching experiment and soviet studies of problem solving. In L.L. Hatfield (Ed.), Mathematical problem solving: Papers from a research workshop, ERIC, 1978.

Kilpatrick, J. Analyzing the solution of word problems in mathematics: an exploratory study. Unpublished doctoral dissertation, Stanford University, 1967.

_____. Problem solving in mathematics. Review of Educational Research, 1969, 39, 523-534.

Knifong, J.D. and Holton, B. An analysis of children's written solutions to word problems. Journal for Research in Mathematics Education, 1976, 7, 106-112.

Krutetskii, V.A. The psychology of mathematical abilities in school-children. Chicago: University of Chicago Press, 1976.

Kulm, G. The classification of problem-solving research variables. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Kulm, G. and Days, H. Information transfer in solving problems. Journal for Research in Mathematics Education, 1979, 10(2), 94-102.

Larkin, J. Enriching formal knowledge: a model for learning to solve textbook physics problems. In J.R. Anderson (Ed.), Cognitive skills and their acquisition. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1981.

Lesh, R. Mathematical learning disabilities: Considerations for identification, diagnosis, and remediation. In R. Lesh, D. Mierkiewicz, and M.G. Kantowski (Eds.), Applied mathematical problem solving. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education, 1979.

_____. Modeling students' modeling behaviors. Paper presented at the annual meeting of the American Educational Research Association, Montreal, April 1983.

_____. Conceptual analyses of problem-solving performance. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Lester, F.K. Mathematical problem solving in the elementary school: Some educational and psychological considerations. In L.L. Hatfield and D.A. Bradford (Eds.), Mathematical problem solving: Papers from a research workshop. Columbus, OH: ERIC/SMEAC, 1978.

_____. Research in mathematical problem solving. In R. Shumway (Ed.), Research in mathematics education. Reston, VA: National Council of Teachers of Mathematics, 1980.

_____. Building bridges between psychological and mathematics education research on problem solving. In F.K. Lester and J. Garofalo (Eds.), Mathematical problem solving: Issues in research. Philadelphia: The Franklin Institute Press, 1982.

Lewis, C. Skill in algebra. In J.R. Anderson (Ed.), Cognitive skills and their acquisition. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1981.

Linville, W.J. The effects of syntax and vocabulary upon the difficulty of verbal arithmetic problems with fourth grade students (Doctoral dissertation, Indiana University, 1970). Dissertation Abstracts International, 1970, 29, 4310A. (University Microfilms No. 70-7957).

Loftus, E.F. and Suppes, P. Structural variables that determine problem-solving difficulty in computer-assisted instruction. Journal of Educational Psychology, 1972, 63, 531-542.

Lucas, J.F. An exploratory study in the diagnostic teaching of elementary calculus (Doctoral dissertation, University of Wisconsin-Madison, 1972). Dissertation Abstracts International, 1972, 32, 6825A. (University Microfilms No. 72-15,368).

Lucas, J.F., Branca, N., Goldberg, D., Kantowski, M.G., Kellogg, H. and Smith, J.P. A process-sequence coding system for behavioral analysis of mathematical problem solving. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Luchins, A.S. Mechanization in problem solving. Psychological Monographs, 1942, 54(6), Whole No. 248.

Luger, G.F. Applications of problem structure. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Matz, M. Towards a computational theory of algebraic competence. Journal of Mathematical Behavior, 1982, 3(1), 93-167.

Mayer, R.E. Frequency norms and structural analysis of algebra story problems into families, categories, and templates. Instructional Science, 1981, 10, 135-175.

_____. The psychology of mathematical problem solving. In F.K. Lester and J. Garofalo (Eds.), Mathematical problem solving: Issues in research. Philadelphia: The Franklin Institute Press, 1982.

Mayer, R.E. Implications of cognitive psychology for instruction in mathematical problem solving. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Mayer, R.E., Larkin, J.H., and Kadane, J. Analysis of the skill of solving equations. Santa Barbara: Department of Psychology, Series in Learning and Cognition, Report No. 80-2, 1980.

McClintock, C.E. Heuristic processes as task variables. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

National Council of Teachers of Mathematics. An agenda for action: Recommendations for school mathematics of the 1980's. Reston, VA: The National Council of Teachers of Mathematics, Inc., 1980.

Newell, A., Shaw, J.C., and Simon, H.A. Empirical explorations with the logic theory machine: a case study in heuristics. In E.A. Feigenbaum and J. Feldman (Eds.), Computers and thought. New York: McGraw-Hill, 1963.

Nisbett, R.E. and Wilson, T.D. Telling more than we can know: Verbal reports on mental processes. Psychological Review, 1977, 84, 231-279.

Ohio Department of Education. Problem solving: a basic mathematics goal (Book 2). Columbus, OH: Ohio Department of Education, 1980.

Paige, J.M. and Simon, H.A. Cognitive processes in solving algebra word problems. In B. Kleinmuntz (Ed.), Problem solving: Research, method and theory. New York: John Wiley and Sons Inc., 1966.

Polya, G. How to solve it (2nd Edition). Garden City, NY: Doubleday and Company, Inc., 1957.

_____. Mathematical discovery: On understanding, learning and teaching problem solving (2 volumes), New York: John Wiley and Sons Inc., 1962.

Post, T.R. and Brennan, M.L. An experimental study of the effectiveness of a formal versus an informal presentation of a general heuristic process on problem solving in tenth-grade geometry. Journal for Research in Mathematics Education, 1976, 7(1), 59-64.

Putt, L.J. An exploratory investigation of two methods of instruction in mathematical problem solving at the fifth grade level. Unpublished doctoral dissertation, Indiana University, 1978.

Resnick, L.B. and Ford, W.W. The psychology of mathematics for instruction. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1981.

Reys, R.E. Calculators in the elementary classroom: How can we go wrong? Arithmetic Teacher, 1980, 27(3), 38-40.

Riley, M.S., Greeno, J.G. and Heller, J.I. Development of children's problem-solving ability in arithmetic. In H.P. Ginsberg (Ed.), The development of mathematical thinking. New York: Academic Press, 1983.

Schoenfeld, A. Can heuristics be taught? Unpublished manuscript (SESAME Project), University of California, Berkeley, 1976.

_____. Teaching problem solving in college mathematics: the elements of a theory and a report on the teaching og general problem solving skills. In R. Lesh, D. Mierkiewicz, and M. Kantowski (Eds.), Applied mathematical problem solving. Columbus, OH: ERIC/SMEAC, 1979.

_____. Some thoughts on problem-solving research and mathematics education. In F.K. Lester and J. Garofalo (Eds.), Mathematical problem solving: Issues in research. Philadelphia: The Franklin Institute Press, 1982.

_____. Episodes and executive decisions in mathematical problem solving. In R. Lesh and M. Landau (Eds.), Acquisition of mathematical concepts and processes. New York: Academic Press, 1983.

_____. Heuristic behavior variables in instruction. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Schoenfeld, A. Metacognitive and epistemological issues in mathematical understanding. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates INc., 1985.

Shirk, G.B. An examination of conceptual frameworks of beginning mathematics teachers. Unpublished doctoral dissertation, University of Illinois, Urbana-Champaign, 1972.

Shroyer, J.C. Critical moments in the teaching of mathematics. Paper presented at the annual meeting of the American Educational Research Association, Toronto, March 1978.

Silver, E.A. Student perceptions of relatedness among mathematical verbal problems. Journal for Research in Mathematics Education, 1979, 10, 195-210.

_____. Knowledge organization and mathematical problem solving. In F.K. Lester and J. Garofalo (Eds.), Mathematical problem solving: Issues in research. Philadelphia: The Franklin Institute Press, 1982.

_____. Research on teaching mathematical problem solving: Some underrepresented themes and needed directions. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Silver, E.A., Branca, N.A., and Adams, V.M. Metacognition: the missing link in problem solving? In R. Karplus (Ed.), Proceedings of the fourth international conference for the psychology of mathematics education. Berkeley, CA, 1980.

Simon, H.A. Problem solving and education. In D.T. Tuma and F. Reif (Eds.), Problem solving and education: Issues in teaching and research. Hillsdale, NJ: Lawrence Erlbaum Associates Inc., 1980.

Skemp, R.R. Intelligence, learning and action. New York: John Wiley and Sons, 1979.

Smith, J.P. The effect of general versus specific heuristics in mathematical problem-solving tasks. Unpublished doctoral dissertation, Columbia University, 1973.

Sowder, L.K. Cognitive psychology and mathematical problem solving: a discussion of Mayer's paper. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Sumagaysay, L.A. The effects of varying practice exercises and relating methods of solution in mathematics problem solving. Dissertation Abstracts International, 32, 6751A, 1972.

Sweller, J. and Cooper, G.A. The use of worked examples as a substitute for problem solving in learning algebra. Cognition and Instruction, 1985, 2(1), 59-89.

Thompson, A.G. Teachers' conceptions of mathematics and mathematics teaching: Three case studies. Unpublished doctoral dissertation, University of Georgia, 1982.

_____. Teachers' conceptions of mathematics and the teaching of problem solving. In E.A. Silver (Ed.), Teaching and learning mathematical problem solving: Multiple research perspectives. Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers, 1985.

Vygotsky, L.S. Mind in society: the development of higher psychological processes. Cambridge, MA: Harvard University Press, 1978.

Vos, K.E. The effects of three instructional strategies on problem-solving behavior in secondary school mathematics. Journal for Research in Mathematics Education, 1976, 7, 264-275.

Webb, N.L. An exploration of mathematical problem-solving processes. Unpublished doctoral dissertation, Stanford University, 1975.

_____. Processes, conceptual knowledge, and mathematical problem-solving ability. Journal for Research in Mathematics Education, 1979, 10(2), 83-93.

_____. Content and context variables in problem tasks. In G.A. Goldin and C.E. McClintock (Eds.), Task variables in mathematical problem solving. Philadelphia: The Franklin Institute Press, 1984.

Webb, N.L. and Charles, R.I. Module development and formative evaluation (Tech. Rep. 3) Mathematical Problem Solving Project. Bloomington, IN: Mathematics Education Development Center, 1977.

Wickelgren, W.A. How to solve problems. San Francisco: W.H. Freeman and Company, 1974.

Wilson, J.W. Generality of heuristics as an instructional variable. Unpublished doctoral dissertation, Stanford University, 1967.

Zalewski, D.L. A study of problem-solving performance measures. In J.G. Harvey and T.A. Romberg (Eds.), Problem-solving studies in mathematics. Madison, WI: Wisconsin Research and Development Center for Individualized Schooling, 1980.

Zimmerman, C.W. Contextual details and mathematical problem difficulty. Dissertation Abstracts International, 1979, 40, 143-44A.

MICHIGAN STATE UNIV. LIBRARIES
31293015698396