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HOW DISCOURSE STRUCTURES NORMS:
A TALE OF TWO MIDDLE SCHOOL MATHEMATICS CLASSROOMS

presented by

BETH A. HERBEL-EISENMANN

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Cheryl Rosaen
Major professor

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HOW
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HOW DISCOURSE STRUCTURES NORMS:
A TALE OF TWO MIDDLE SCHOOL MATHEMATICS CLASSROOMS

VOLUME I

By

Beth A. Herbel-Eisenmann

A DISSERTATION

Submitted to
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in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

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2000

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ABSTRACT

HOW DISCOURSE STRUCTURES NORMS: A TALE OF TWO MIDDLE SCHOOL MATHEMATICS CLASSROOMS

By

Beth A. Herbel-Eisenmann

My experiences as a student and a teacher of mathematics have led me to pursue the topic of this dissertation—discourse patterns and norms in two "reform-oriented" mathematics classrooms. The two 8th grade classrooms that form the focus of this dissertation were using the Connected Mathematics Project, an NSF-funded curriculum project. I was intrigued by the teachers and their teaching because I noticed the students seemed to have similar understandings, but each classroom felt different to me as a participant-observer.

These classrooms offered a context that allowed me to study differences in the context of similarity. The teachers had many attributes in common (detailed in Chapter 5): similar academic backgrounds and professional development activities, same certification, same school, same curriculum and similar enthusiasm for it, same heterogeneous group of students, similar student-understandings, etc. However, the teaching in the two classrooms was different. Drawing from the sociolinguistics and mathematics education literatures, I describe the social and sociomathematical norms of the two classrooms in terms of the classroom discourse which they were embedded in and carried by. I also interpret student understandings whenever possible throughout the thesis, taking a social constructivist perspective.

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In the year prior to commencing my dissertation study (1997-1998), I completed classroom observations and student interviews as part of my practicum work and research assistantship. These were used to form preliminary hypothesis about student understandings and the classroom environment. The data used for this dissertation was collected over the next two years (1998-2000). During the first, I observed and audio- and video-taped students on a weekly basis. In addition, students were interviewed about their algebraic understandings and their classroom experience. The second year, one of the classrooms was observed to trace the formation of the norms in the classroom. The teachers took part in four extensive interviews, in which they were asked about influencing experiences related to their teaching and the norms in their classroom (in terms of the expectations, rights and roles of themselves and their students). They were also asked to react to transcripts and video from their classroom, offering their perspective about their intention in their language patterns and teaching.

The ideas I investigate in this dissertation include how social and sociomathematical norms are embedded in and carried by the classroom discourse in each classroom (Chapters 6 and 7). I also discuss aspects in the teachers' professional lives that influenced the ways they think about and work to establish and maintain the norms in their classrooms (Chapter 5). In Chapter 8, I look across the two classrooms to offer what I see as being similar and different, which has allowed me to locate differences in: the overall structure of teacher talk, the positioning of the teacher with respect to the locus of authority, the way each teacher draws from potential other knowledge sources in the classroom (i.e. students and the textbook), and the way each teacher draws attention to the common knowledge constructed in the classroom.

Dedicated to my husband, Joe, and our son, Kaleb,

for all their support, love and patience during this process.

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Many people have enabled and assisted my work for this dissertation. I am indebted to them all and thank them for all their time, encouragement, words of wisdom, etc.

- This dissertation was supported in part by a Spencer Research Training Grant and the Connected Mathematics Project.
- I would like to thank my dissertation committee, David Pimm, Cheryl Rosaen, Jenny Denyer, Glenda Lappan, and Jack Smith for their encouragement, guidance, expertise and time. Each offered insight and advice in his/her area of expertise during the data collection, analysis, and writing process. I would like to thank each of them for working through drafts of various chapters. As my advisor and chair of the thesis, David Pimm allowed me to work through multiple interpretations of my data and offered direction through a sociolinguistic perspective. He provided detailed guidance throughout the analysis, writing and editing process. His knowledge and insights have been truly helpful and inspiring! Cheryl Rosaen provided guidance in developing thoughtful and rich teacher interview protocols and offered never-ending support in my thinking and writing process. She and Jenny Denyer asked thought-provoking questions that pushed my thinking about both the interpretation and the writing process. I thank them both for the detailed feedback they gave on the case studies. Jenny is also the person who introduced me to and encouraged my interest in discourse analysis, giving me a lens with which to view classrooms that I had not had prior to taking her course. As an author of the CMP curriculum and someone who

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knew the teachers personally. Glenda Lappan offered insightful feedback on the teacher interview protocols, the case studies (related to my interpretations of the teaching) and the details I gave about the mathematical content. Jack Smith has supported my work since I have arrived at MSU, taking the time to discuss my ideas and offering feedback whenever I have requested it. He has especially pushed my thinking with respect to the content and analysis of my data. I greatly appreciate the time my committee members have spent to read, meet and offer feedback on this thesis.

- Discussions with colleagues were essential in my writing of this thesis. Conversations with Jo Lesser, Brian Yusko, Joan Hughes, Shari Levine-Rose, Cindy Carver, Ruth Berry and others involved in the Spencer RTG program helped me to articulate my ideas and make sense of my data. They also offered suggestions about readings that they thought might be helpful and took time to discuss my work whenever I needed. David Labaree's insights into what my research could contribute to the field of education were especially helpful. I am indebted to all of these people for their time, support and assurance.
- I am deeply thankful to the teachers and students who welcomed me into their classrooms and allowed me to video- and audio- tape them at any time. They were flexible and open to talking with me whenever I requested and took part in open and candid discussions about mathematics and what it was like to be a teacher and student in these classrooms. I would also like to thank Elizabeth Phillips and Jack Smith for introducing me to the teachers who were the focus of this dissertation.

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- I thank my husband, Joe, for enduring my sleepless nights and stressed out days with endless support, encouragement, love and guidance.
- I thank my son, Kaleb, for making me smile and giving me lighthearted reprieve from my work. His never-ending curiosity inspires me!
- I thank my parents and family for the many ways they have offered their support and for always telling me that I could do anything I wanted to do.
- Kathy Lessard has offered much support throughout my program and writing. She has been a great resource, friend and source of advice about a myriad of things. I thank her for the many things she has done for me!!

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF TRANSCRIPTION SYMBOLS

K:	Karla's turns
J:	Josh's turns
Mark:	the code-name of the speaker
[inaudible]	indecipherable speech
[the slope]	transcriber's situational elucidations or comment
[writing]	description of non-linguistic communication
so/	/ indicates a one-second pause
so//	// indicates a two-second pause
[3-second pause]	indicates the length of pauses over two seconds
So	indicate word or words stressed by the speaker
So	(bold) highlighted by the transcriber for the attention of the reader
So-	(-) indicates utterance interrupted or not completed

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CHAPTER 1: INTRODUCTION

In this dissertation, I explore the mathematical culture of two 8^{th} grade classrooms both using the Connected Mathematics Project (CMP). I examine the social and sociomathematical norms that each teacher established in his/her classroom. More specifically, I describe discourse patterns in both teacher talk and textbook writing (i.e. talk and representation in general and as associated with the mathematical register) and the norms that have been established between the teachers and students. I chose teacher talk and textbook writing because they are the main language forms of the expert "voices" students hear and see in a classroom. I undertook this study in order to examine my sense of difference between two "reform-oriented" classrooms which on the surface were rather similar (similar academic backgrounds, same certification, same school, same curriculum and similar enthusiasm for it, same heterogeneous group of students, similar professional development activities, similar student understandings).

In many discussions that have ensued since the release of the NCTM *Standards* and curricula that resulted from this push for new forms of teaching and learning, the debate has focused on traditional ideas vs. reform-oriented ones. However, little attention has been given to the diversity among and within just one of these paradigms (i.e. reform-oriented ones). Since the reforms are still fairly new, many teachers who have developed a reform-oriented stance are still working to implement such practice in their classrooms. Also, as with any text, interpretations of what it means to implement such teaching is ambiguous because the reforms are underdetermined (Ball, 1997). Therefore, taking a closer look at the differences and similarities in two such "reform-oriented" classrooms

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could prove useful, adding some shades of gray to the larger spectrum referred to as "reform-oriented."

My study offers to informed teachers, teacher educators, and curriculum developers an account of different teacher routes to a possibly similar end, enriching perspectives on the actual diversity of what monolithically might be called an "NCTM-reform classroom." In particular, my study explores the effects of variation in discourse patterns (e.g. pronoun usage, voice, encoding of authority, mix of technical and non-technical mathematical language) on classroom interactions and student understanding.

Background

I grew up in a family where mathematics was valued—my father taught algebra and geometry in the high school I attended and I had him for both of those classes. The clearest memory I have of my 9th grade algebra class consists of me watching my father write a series of definitions and properties on the board while I hurried to copy them into my notebook. He then explained what we were to do with those properties and how we were supposed to solve the problems. I had learned how to "do school" well and I did exactly as I was told to do--I manipulated the symbols in the manner he had described and I got the right answer most of the time.

It was that year that I was first invited to participate on the "math team." As a participant on this math team, I attended competitions in which we took individual exams and then worked in small groups on application problems. I always did fairly well on the individual exams—all I had to do was manipulate symbols. However, I rarely contributed anything to the small group portion of the exam because I really had no idea how to solve that type of problem. No one seemed to notice, anyway.

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The rest of my high school experience was really more of what I just described—the teacher told me what process to follow and I did. I rarely asked questions and never questioned the process I was being shown to solve the problems. I was considered one of the top five mathematics students in my school and attended the math competition every year. However, the group exams still eluded me. Even though I got the correct answer on the problems that focused on symbol manipulation, I was perplexed by those word problems!

My senior year, I met with the school counselor to discuss where I wanted to continue my education and in what I was going to major. He knew that I was good at math and since I was very people-oriented, we talked about my going into teaching. One of the best teacher preparation programs in the state had a recruiter visit my high school the next week. I spoke with him and he told me about a scholarship for which I could apply because I had a rather high ACT score. I applied for and received it. The decision was made—I would attend this small university and become a high school math teacher.

I took the math placement exam and found that I could begin calculus my first quarter. I knew that I wanted to teach high school and decided to take trigonometry because I had not had an entire course that focused on this. In addition, my older sister (whom I thought was much smarter than I) had started out in calculus her first year and found it to be quite a struggle. I did very well in trigonometry—again, it was substituting and manipulating things to find an answer and I could definitely do that!

Calculus went well until the last quarter. I had gotten through the others by cramming for over a dozen hours prior to an exam. I could recognize a "type" of problem and recall the process I was supposed to use to solve it. In my last quarter of calculus, the

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manipulation got more complex and I did not know exactly what to do. I still earned a B in the class, but really did not see the big picture. I "did school" well, though, and graduated Summa Cum Laude.

One could say my "apprenticeship" in teaching had been an effective one for I began my career teaching the way I had been taught. I spent the majority of my time talking while students listened and took notes. I thought breaking the problems into enough "steps" would appease students and help them succeed. I even recall handing out sheets of paper with all of the "steps" for each type of problem listed for students to refer to when studying for an exam. My students really were not very successful or motivated to copy the methods I was prescribing. However, I did not know what else to do.

Spring of that year I was encouraged by my undergraduate advisor to attend the national NCTM conference in New Orleans. I did and was more than impressed with the activities and experiences about which I heard other teachers talk. *This* was the kind of activity that I knew, deep down, my students needed to experience—challenging mathematical tasks that students could work on and discuss in small groups and then we could talk about as a class. I returned to my classroom with some great ideas and when I implemented them, I found I now had an additional complexity to attend to in my classroom—my students were actually talking to me!

However, I was not sure what to do to direct that discussion in a mathematically powerful way. Once the students began to talk, it was hard to keep them on task and I found myself falling back into a more didactic format—it was easier and I was in control. I implemented some alternative activities, but kept the conversation fairly controlled.

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When I left teaching middle school to get my master's degree at Northern Arizona University, I began teaching content courses to pre-service teachers. I found that these budding professionals had many problems talking about the mathematics: explaining their thinking, articulating their understanding, making connections and identifying the "big" ideas in the content. I began to incorporate more discussions about "why" and "how" and to talk about my own struggles with these as a teacher. I also gave writing assignments and essay exams, planned more group work, etc. to get students to articulate their thinking. I really enjoyed working with pre-service teachers and decided that this was what I wanted to do with my life.

This decision took me to Michigan State University. My first semester, I co-taught the methods course for math and language arts for pre-service elementary teachers. One of the projects in which we had our students engage consisted of a hypermedia investigation of Deborah Ball's 3rd grade mathematics class. The hypermedia lab consisted of a "virtual classroom," i.e. videotape of all the math lessons taught, access to student and teacher journals, transcripts, and student work. This virtual classroom was set up to engage pre-service teachers in thinking about alternative ways to teach mathematics. One of the purposes of our course project was to show students how important the language arts were for getting students to articulate their mathematical ideas. We discussed the NCTM discourse standards and had students pose questions about the discourse in Ball's classroom. They then worked in small groups to find evidence for answering their question in the classroom videotape, teacher's journal, student journals, etc. This classroom activity became the budding seed for my current dissertation work, because it was my first glimpse of a teacher orchestrating powerful

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mathematical discussion. It also gave me a taste of how empowering this type of discussion could be for students. I had never imagined third grade students could explore and talk about mathematical ideas in the ways that the children in this classroom were. It was thrilling to me as a teacher and as a researcher.

During the next two and a half years, I worked in two 8th grade mathematics classrooms as a graduate student researcher for the Connected Mathematics Project (CMP). The teachers with whom I worked were experienced CMP teachers who had piloted the curriculum. I have to admit I was quite taken aback by what I observed in their classrooms. Students were engaged in the types of problems with which I had struggled while participating in the math team in junior high and high school *and* this made up the majority of their experience.

Another unusual emphasis that appeared in these classrooms was that they actually spent time talking about the graphs and tables, something that had been virtually ignored in my experience as a student. Students were also talking to the teacher and to one another. I found myself wishing that I could have had their experience when I was a student as I made some mathematical connections I had not been aware of before my observations. For example, I finally saw the connection between the slope of a linear equation and the change in the y-values (when the x-values increase by one) in the table. I had only been taught to look across the table (from the x to the y column) and notice the relationship between the two variables. I had not noticed that *down* the table there was an additive rate that was constant which happened to be the slope of the line! I remember thinking, "This is really cool! I wish I had learned mathematics like this before I had begun to teach!"

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The work I originally did in these classrooms was associated with student-understandings of algebraic functions (for an instance of this work see Herbel-Eisenmann, Smith, & Star, 1999). In fact, all of the work I did in the first year was solely about the students. However, being a teacher myself, I could not help but notice the teaching in the two classrooms. Intuitively, I felt there was a difference in the teachers and the teaching in the two classrooms, but I could not identify exactly what might be contributing to that sense of difference. While I started out investigating student understanding, I found myself thinking more and more about the teaching. At one point in my data collection, it became clear that I needed to choose between these two foci because only one could be pursued for my dissertation. I had come to a fork in the road.

A choice was made—my true interest lay in understanding more about the teaching in the two classrooms. My questions evolved to focus on the norms and how they were related to the discourse patterns in the classroom. While I originally intended to focus both on the students and the teachers in the classroom, this became impossible because one of the teachers took an administrative position the year I would have collected the additional data I needed. This change required me to make significant changes in my intended study. I would have liked to have looked for evidence of effects of being in the different classrooms. Unfortunately, this was not possible.

My emerging analysis shaped the questions that I intended to pursue further. While highlighting the mathematics that took place in the discussions, I am not able to make claims about student understandings as I had originally intended. Instead, my dissertation focuses solely on the norms and how those were embedded and carried in the classroom discourse. This, in turn, led me to pursue issues of teacher positioning and

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authority. By focusing on the forms and functions of talk in the two focus classrooms, I offer a closer look at two "reform-oriented" classrooms.

Some Layers of Context

The National Context: Reforming Mathematics Teaching and Learning

The National Research Council (NRC) (1989) and the National Council of Teachers of Mathematics (NCTM) (1989, 1991) have recently defined new goals for mathematics education[1]. The authors of the *Standards* (1989, 1991) proposed a vision of students studying much of the same mathematics as what was generally being taught, but with a quite different emphasis. They defined mathematical literacy as students doing the following: learning to value mathematics, becoming confident in one's own ability, becoming a mathematical problem solver, learning to communicate mathematically, and learning to reason mathematically. The *Standards* define "knowing" mathematics as "doing" mathematics. They also envision mathematics as problem solving, communicating, reasoning, and connecting.

With this backdrop regarding the NCTM reform, it is important to highlight the stance regarding communication this document takes because it is quite different from what is currently the norm in mathematics classrooms. The NCTM reform movement advocated sense making and conceptual understanding by using the tools of language, representation, etc. Discourse was seen as a way for students to make sense of mathematics; communication was viewed as being a large part of mathematics.

More specifically, discourse was defined by NCTM (1991) as "ways of representing, thinking, talking, agreeing, and disagreeing" (p. 34). The authors of this document also pointed out that inherent in the discourse of a classroom are related issues of what it means to "do" and "know" mathematics, as well as issues about power and

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authority. In many mathematics classrooms, the text and teacher are typically viewed as the authority. However, this document encouraged reasoning and evidence as the basis for what counted as legitimate mathematical activity, drawing the locus of authority away from the teacher and the text. The teacher's role was described as that of a facilitator, one who initiated and orchestrated student discourse to foster student learning. In doing so, the teacher should:

- pose questions and tasks that elicit, engage, and challenge each student's thinking;
- listen carefully to students' ideas;
- ask students to clarify and justify their ideas orally and in writing;
- decide what to pursue in depth from among the ideas that students bring up during a discussion;
- decide when to provide information, when to clarify an issue, when to model, when to lead, and when to let a student struggle with a difficulty;
- monitor students' participation in discussions and decide when and how to encourage each student to participate (p. 35).

The Curricular Context: The Connected Mathematics Project

In an attempt to make the *Standards* more concrete, the National Science Foundation (NSF) announced funding for the development of reform-oriented curriculum. NSF wanted curriculum to be developed that embodied the ideas explicated in the *Standards* document. The Connected Mathematics Project (CMP) (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1998b) was one such curriculum to receive funding and was the one used in the two classrooms that were the focus of this dissertation.

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Broadly speaking, the CMP curriculum is a middle school problem-centered curriculum where almost every problem occurs in a "real life[2]" context. The mathematical goals of CMP can be summarized in the following statement:

> All students should be able to reason and communicate proficiently in mathematics. This includes knowledge and skill in the use of vocabulary, forms of representation, materials, tools, techniques, and intellectual methods of the discipline of mathematics including the ability to define and solve problems with reason, insight, inventiveness and technical proficiency (philosophy statement, CMP, revised 1997).

CMP is organized into units centering on big mathematical ideas. Students develop understanding and reasoning by exploring a set of problems that embody these ideas. Extensive problem sets are included throughout each unit that help students practice, apply, and extend their understanding and reasoning. Periodic reflections help students make connections among a set of "big" mathematical ideas and applications, contained within a given unit. For example, in the algebra strand students are introduced to linear, quadratic and exponential relationships in three representations: tabular, graphical, and symbolic.

In the spirit of the reform (and in addition to the multi-representational approach), a characteristic feature of this curriculum is that it takes a functions approach to the teaching and learning of algebra instead of a more traditional symbolic-manipulation approach. Students are asked to define, observe, model, analyze, etc. variables and make predictions about the data in terms of input/output and in relationship to how one variable depends on another.

The CMP curriculum places a notable emphasis on fostering discourse in its training sessions and in its pedagogical model. In the "Introduction to CMP" workshop, one of the main pedagogical strategies that is advocated is fostering discourse to pull the mathematics out of the

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mathematics out of the classroom discussion. The pedagogical model introduced is a "launch," "explore," "summarize" format that utilizes many prompts for teachers to encourage their students to talk in the classroom. For example, teachers are cued to spur student discussion about mathematical topics (e.g. "Why are there differences among your predictions?" (Lappan et al., 1997a, p. 14b) "How did you determine the length of the race? Why is that length reasonable?" (Lappan et al., 1997a, p. 34i).

Reform-Oriented Algebra Context: Why It Warrants Study

To illustrate how CMP is different from a more traditional version of algebra, I would like to discuss briefly the goals defined in the teacher's guide for one CMP unit, *Say it with Symbols*[3] (Lappan et al., 1998a) with respect to the discourse it may foster. The goals for students are listed as such:

1. Review and strengthen their understanding of the conventional *order of operation* rules **in the context of practical problems**;
2. Evaluate expressions by applying the rules of order of operations;
3. Write symbolic sentences that **communicate their reasoning**;
4. Develop tools for manipulating symbolic expressions **in ways that are both connected to and independent from tabular, graphical, and contextualized reasoning**;
5. Recognize applications of the distributive and commutative properties;
6. Recognize and **interpret** equivalent expressions;
7. **Explain the reasoning underlying** the solution of linear equations;
8. To **make sense of symbolic expressions** involving addition, subtraction, multiplication, division, and exponents;
9. To judge the equivalency of two or more expressions **by examining the underlying reasoning and the related tables and graphs**;
10. Apply the properties for manipulating expressions to solving linear equations;

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11. Solve simple quadratic equations with some sense of basic factoring and "undoing" techniques (p. 1g, boldface added).

Some of the expectations and the nature of the discourse associated with this curriculum are highlighted when focusing on the boldfaced words and phrases. These expectations could lead to a very different discourse in comparison with a discourse associated with a more conventional curriculum. For example, increased attention to student explication and justification encourages students to discuss, write, read and listen to one another in the same manner as traditional communication encourages students to listen to the teacher. There is also a decreased emphasis on memorization and manipulation and more emphasis on identifying, interpreting, exploring, reasoning, etc.

More generally, there are at least six dimensions that are different in the two types of classrooms (CMP vs. a more traditional curriculum) (Star, Herbel-Eisenmann & Smith, 1999), as outlined in Table 1. It is obvious from the table that the discourse in the two types of classrooms could be very different from one another.

More specifically (and related to my previous work), one difference in the discourse of these 8th grade CMP classrooms appeared in the ways students talk about mathematical ideas. In my traditional experience, I was taught that slope was defined as the ratio of the horizontal change with the vertical change, i.e. rise/run. However, in these classrooms students knew and articulated the concept of slope in multiple ways: (1) constant rates, (2) "the number attached to the x" in an equation, (3) "the slantiness" of the line in a graph, (4) "what it goes up by" ("it" refers to the change in the y-values in a table), etc. The ways "of talking, acting, interacting, valuing, and believing, as well as the spaces and material "props" the group uses to carry out its social practices" (Gee, 1992, p. 107) are different from more conventional classrooms. I argue that the discourses associated with the tr

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associated with the two classrooms that are the focus of this dissertation are different from a more conventional classroom and warrant closer investigation. And, as I show, these two "reform-oriented" classrooms were also different from one another.

Table 1
Some Key Dimensions of Differences between traditional Algebra I and 8th grade CMP

Algebra I	8th grade CMP
The *fundamental objects* in the curriculum	
Equations & symbolic expressions	Functional relationships represented in tables, graphs, and equations
Typical problems in the curriculum	
"Solve," "factor," "multiply," symbolic expressions or verbal statements with request to find a numerical value (word problems)	Verbal statements with tables, graphs, or symbolic expressions with request to find values and describe, explain, predict, etc.
Typical solution methods	
Complete the correct steps in symbolic procedures in the correct order	Relate verbal statements to tables, graphs, or equations; Compute or manipulate that representation; Interpret the results verbally
The role of *practice*	
Significant practice on particular problem types (in class and homework)	Similarities between problems are less salient; extended work on fewer, more open problems (in class and homework)
The role for *technology* for representing and calculating	
Used in balance with pencil & paper computation, which is more highly valued	Supports students' work on most all problems
Elements in a typical *lesson*	
Review homework, present new content, provide time for work on next assignment	More variation across lessons; Some mix of teacher presentation, small group work, and whole group discussion

When I began my observations in these two "reform-oriented" classrooms, I did not have a language to articulate the differences I felt between them. After my first year and a half in the classrooms, I enrolled in an introductory discourse analysis class. After engaging in many of the readings for this class, I felt that this was the lens through which would view these two

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I could view these two classrooms and actually account for the differences I felt. In a sense, that body of literature gave me a "voice" with which to describe what I was feeling. In the next chapter (and in some of the subsequent ones), I discuss many of the terms and concepts that I have acquired through the literature.

Upcoming Chapters

I begin this exploration of mathematical norms and discourse patterns by sharing the overarching framework that guided my thinking throughout my analysis (Chapter 2). I then describe the study by articulating the questions that guided it, the data I collected, and my process of analysis (Chapter 3). In Chapter 4, I offer a more detailed analysis of the CMP unit that was the focus of the classroom discourse in the two classrooms, *Thinking with Mathematical Models*, because it appeared to interact with the each teacher's talk in different ways. In Chapter 5, I introduce the teachers who are the subject of the two case studies included in Chapter 6 and 7. I lay out their backgrounds, philosophy of teaching in general and about teaching mathematics, highlighting the similarities and differences between the two teachers.

Each case study (Chapter 6 and 7) focuses on describing and illustrating the three main discourse patterns in the talk each teacher used. For instance, Karla's predominant talk patterns were "metacomments"; Josh mainly utilized series of questions. Because each pattern was pervasive in the teacher talk, I claim there were certain norms associated with them. These norms are discussed in the second section of each chapter. The two case studies are then compared and contrasted in Chapter 8, showing how the talk patterns (while using different surface forms) served the same broad purpose. However, I contend that these different surface forms may have implications for the way each teacher positioned him herself

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Chapter 9 revisits the research questions and summarizes the main findings of the study. I also offer limitations and implications of this work. The value of this study is explored, focusing on professional development of experienced teachers and then outlining a master's level course that focuses on discourse in mathematics classrooms.

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[1] I want to acknowledge that there is a newer version of the NCTM *Standards* that were released in the year 2000. However, since the teachers/teaching that form(s) the focus of this dissertation were operating under the earlier *Standards*, these are the ones to which I will refer and draw on throughout my dissertation.
[2] "Real Life" is being used for problems based on real experiences that may not be directly related to the students every day experience. See Boaler (1997) for a discussion of some of the difficulties with this notion.
[3] I chose this unit because it is one that would be considered as close to traditional algebra as any unit in the CMP curriculum. If differences appeared in this unit, one could assume the differences to be even greater in the units that were less symbol-dependent. Also, this unit was one of the two units that were video- and audio-taped in the two classrooms.

CHAPTER 2: OVERARCHING FRAMEWORK

My initial introduction to discourse analysis gave me a way to account for differences I felt in two "reform-oriented" mathematics classrooms. I began to understand the role of language in the classroom and how it differed from language in society in general. For example, in Cazden's (1988) *Classroom Discourse*, I first read about the pervasive Initiate-Respond-Evaluate (I-R-E) structure that exists in many classrooms. This book also introduced me to the relationship between language and authority structures and led me to read other literature related to this issue. Cazden cited many significant pieces of literature that became important to my articulation of the differences I felt in the two classrooms. Many of these (and others) will appear throughout this dissertation[1]. In this chapter, I introduce some of the readings that I have adopted from sociolinguistics and others from mathematics education.

Key Concepts and Terms Defined

Discourse Analysis

Discourse analysis is an ambiguous term. Typically it involves:

> [...] naturally occurring connected spoken or written discourse [....] refers to attempts to study the organization of language above the sentence or above the clause, and therefore to study larger linguistic units, such as conversational exchanges or written texts. It follows that discourse analysis is also concerned with language in use in social contexts, and in particular with interaction or dialogue between speakers (Stubbs, 1983).

Schiffrin (1994) recently compared many of the different approaches to discourse analysis, including speech act theory, interactional sociolinguistics, ethnography of communication, pragmatics, conversational analysis and variation analysis. The roots of these traditions come from disciplines such as philosophy, sociology, and anthropology. Although Schiffrin's main goal was to differentiate these approaches, she also argued that

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> [Although] these approaches originated in different disciplines (and are relevant to broader topics within each discipline), they all attempt to answer some of the same questions: How do we organize language into units that are larger than sentences? How do we use language to convey information about the world, ourselves and our social relationships? (p. viii)

And she concludes that "all the approaches to discourse view language as social interaction, and all are compatible with a functionalist rather than a formalist paradigm" (p. 415). The functionalist paradigm: (1) views language as primarily a societal phenomenon, (2) explains linguistic universals as deriving from the universality of the uses to which language is put in human society, (3) explains children's acquisition of language in terms of the development of the child's communicative needs and abilities in society, and (4) studies language in relation to its social function (pp. 21-22).

I have drawn on a number of different accounts. For example, in Chapter 7, I use the Theory of Politeness to interpret Josh's use of tag-forms. In Chapter 6, I use the idea of indirect speech acts[2] to address the illocutionary force of hedged statements that Karla and her students make.

In making inferences about the norms that are carried and embedded in the discourse, I draw from ethnography of communication. Similar to Mishler[3] (1972), I "show how different [...] norms are carried in the language used, primarily in the structure of the teachers' statements and in the type of interchange developed between them and the children" (p. 267). Ethnography of communication focuses on "the social functions of particular forms of language used in specific contexts" (p. 268). This form/function emphasis is carried throughout the case studies and the cross-case account. By attending to these, I have argued that different forms serving similar purposes may are attributed to diff

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Social and Sociomathematical Norms

Two types of norms are the focus of this dissertation—social norms and sociomathematical norms (Yackel & Cobb, 1996). Social norms are general and not subject specific: for example, they may be related to how students are expected to use equipment in the classroom (in this case, maybe how they are expected to use graphing calculators), how students are expected to work in groups, or how students are supposed to explain or justify their thinking. Sociomathematical norms are more content specific; they are related to ways of functioning mathematically that are recognized and accepted by the larger mathematical community. For example, what counts as "mathematically different, mathematically sophisticated, mathematically efficient, and mathematically elegant" (Yackel & Cobb, 1996, p. 461) would be considered to fall under this definition. One may construct an argument in any domain, but understanding what counts as an acceptable mathematical argument is a sociomathematical norm.

In some sense, the norms in the classroom are invisible. They do not surface explicitly and tend to show up when there is a breach of the norm that has been established either on the part of the teacher or the students (Cobb, Yackel, & Wood, 1993). These norms can be identified by analyzing the evolving patterns and regularities that occur in classroom social interactions (Cobb et al., 1993).

My study has pursued many of the same ideas as Cobb, Wood, and Yackel's work; however, there are also differences. Some of these include: age level of the students (their work focuses on primary grades, mine on upper middle school), content (their focus is on operations and place value, mine is on algebra), level of intervention on of the research

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the part of the researcher (their setting was an intentional teaching experiment, my role was that of participant-observer), and existence of a printed curriculum (they helped in developing lessons suitable for the classroom's context, the classrooms in my study have adopted a pre-existing reform-oriented curriculum). My study extended their work to a very different context from the one they studied.

Another difference between Wood et al.'s work and my study included the range of focus for the analysis. They pursued making sense of six aspects of the classroom (E. Yackel, personal communication, August 28, 1999) as shown in Table 2. My study used similar themes, but focused only on the portions of the table that are in bold print: the social norms, sociomathematical norms, and student's individual mathematical conceptions.

Table 2
Six Aspects on Which One May Focus Classroom Inquiry

Sociological	Psychological
Social norms	Individual beliefs
Sociomathematical norms	Individual beliefs about mathematics
Taken-as-shared classroom mathematical practices	**Student's individual mathematical conceptions**

In attending to these three aspects, my main focus has been on the social and sociomathematical norms as they are embedded in and carried by the classroom discourse. Student's individual mathematical conceptions are addressed, but are secondary in nature to the norms in the classroom.

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Common Knowledge

In the process of writing this dissertation, it became clear to me that some of the ideas presented in Edwards and Mercer's (1987) *Common Knowledge* were key to my analysis and interpretation. In this book, Edwards and Mercer investigated discourse in elementary classrooms and focused primarily on science. In the upcoming case studies, this idea of "common knowledge" is very important because many of the discourse patterns each teacher used were related to the way such common knowledge was established, defined and then continually woven through the class sessions (which I show in the two case studies presented). More importantly (and similar to Edwards and Mercer's findings) many of these discourse patterns were also related to the ways in which the common knowledge was being controlled in each classroom, although the teacher's choices[4] differed in the way in which they controlled that knowledge. I explicate, in detail, Edwards and Mercer's main ideas that are related to my analyses and interpretation. These are "common knowledge" and "control of common knowledge."

Common Knowledge, Context and Continuity

Edwards and Mercer are concerned with the ways in which knowledge is "presented, received, shared, controlled, negotiated, understood and misunderstood [...] and what that knowledge means to people and in how and to what extent it becomes part of their common knowledge, their joint understanding" (p. 1). An overarching belief that they have is that all education is fundamentally about the evolvement of some mutual understanding, some interdependence of positions.

They discuss how this is related to shared knowledge, something that differentiates humans from other animals. We are the only species that can share knowledge by representing it outside of the context in which it was generated. People

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tend to discuss, exchange views, negotiate beliefs and understandings. Discourse is part of the process of establishing shared understanding which influences future discourse. Some of the elements involved in this include: "offering new information, reference to existing past experience, requests for information, and texts or 'checks' on the validity of interpretations of information offered" (p. 6). Edwards and Mercer claim that using these elements allow people to construct (through discourse) a continuous experience greater than any individual occurrence.

The authors claim to be concerned more with content than form—in "what people say to each other, what they talk about, what words they use, what understandings they convey, and with the problematics of how these understandings are established and built upon as the discourse proceeds" (p. 10). In this way, they interpret people's meanings rather than just coding the forms of their language[5]. In fact, they are quite critical of attending only to form because of its limitations (e.g. "it was devised to reveal linguistic structures, not educational or cognitive processes" (p. 10)). In my analysis, I am using some of the tools of discourse analysis of which they are critical. However, in going back toward the form and function of the language, I am not ignoring meaning. I am working in the same spirit as Edwards and Mercer, but am extending their analysis to attend closely to form and function and through these (along with my knowledge of these teachers, students and classrooms[6]), I interpret the meaning of the classroom talk.

They claim that attention to content, meaning and context is important to examining how common knowledge is established. Also important is the notion of "*given* and *new information*" (p. 11)-the ways in which the information offered by the speaker is taken as known by the hearer ("given"), or else is presumed to be unknown to the hearer

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A related idea (to "given" and "new" information) is that of context. They define context as "everything that the participants in a conversation know and understand, over and above that which is explicit in what they say, that contributes to how they make sense of what is said" (p. 63). The unfolding of context through time they call "continuity." Kindred to Mead's 'emergent present', it is summarized by Griffin and Mehan (1981) as "that which is going on in the present inexorably becomes the past, informing and reforming the present, while future events inform the sense of the present" (p. 190).

According to Edwards and Mercer, two types of context are pertinent to the form and content of any discourse: linguistic and non-linguistic contexts. "The linguistic context is the speech or text that precedes and follows any given utterance, while the non-linguistic context includes the time and place, the social occasion, the persons involved, their behaviour and gestures, and so on" (pp. 63-64). All dialogue is dependent upon its context for its meaning, and context is not physical but mental.

Essential to the link between discourse and knowledge is the idea that context is not physical, but is in the mind of the participants. From an outsider's view, context is usually actual and definable; for the participants, it is more a matter of interpretation and recollection—"what they think has been said, what they think was meant, what they perceive to be relevant" (p. 66). What matters is what the participants in the communication understand and see as relevant. They describe education to be the

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establishment of shared mental contexts (joint understandings) between teacher and children which allow them to interact in educational discourse together successfully.

> One of the purposes of classroom communication [...] is to further the knowledge and understanding of children about topics which, however implicitly stated and ill defined, constitute the curriculum. For this to occur at all, a child and teacher must mutually establish a universe of discourse. For children, this amounts to more than learning how to take part in linguistic rituals (p. 49).

They are making a distinction between students merely saying what is required of them (learning what should be said when) and actually understanding what they are saying. For example, when a teacher asks the same question twice, the respondent may give a different answer the second time not because she has changed her mind, but because she has learned that being asked the same question twice indicates the answer she gave was an incorrect one (Cazden, 1988; Edwards & Mercer, 1987).

Not only did Edwards and Mercer find that the teachers used lessons to create the contexts of talk and experience (which predominated over any talk of events from the "real" world), but they also found that students appeared to operate with this ground rule-- "if the teacher asks a question related to work done in class, then the answer is to be found in what the lesson has actually covered" (p. 75). References made to out-of-school activity were typically dismissed or not taken up by the teacher, unless they happened to be something that she believed to be part of all of her student's out-of-school experience.

What are some ways that teachers establish and define common knowledge? One way is by talking through the process: that is, by using words to define the meaning in the activity. When a teacher demonstrates something, she often talks her way through the activity. This talk seems to have at least three functions: (1) to direct and hold the pupils' attention to activity and to align the joint attention of the group to particular pieces of the ways, Dr. Wright [illegible]

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activity; (2) to highlight the important aspects of the activity—to point to what pupils ought to be noticing and remembering; (3) to provide a common vocabulary for the actions needed to communicate the joint understandings to each other (pp. 79-80). What begins as a concrete physical context of joint activity later serves as a "shared mental context of experience and understanding" (p. 80). Since the teacher and students worked through the activity together, and have conceived how to talk about that activity, they can now begin to exchange understandings with words alone. "The joint activity and discourse of the past became a shared mental context for the present" (p. 80).

Continuity (i.e. the growth of shared contexts through time) was a feature to which the teachers themselves made explicit mention in interviews. However, these references actually played out in implicit ways in the classroom. They found that lessons typically began in such a way that teachers introduced students to the work to be done, and made continuity links with what had previously taken place. For example, we have all heard teachers ask, "Do you remember when…?" In asking such questions, the teacher is referring-back to bring ideas forward and connect them to up-coming activity.

Forming continuity is not unproblematic; difficulties sometimes arise with regard to the understanding established in lessons. Metacognitive and metadiscursive comments tend to occur at moments when students "seemed not to have grasped some significant principle, procedure, or instruction that had been dealt with previously" (p. 84). In the asymmetry of teacher-pupil conversations, these mismatches are important to the learning process. Edwards and Mercer contend that they occur in Vygotsky's 'zone of proximal development': "at precisely the points at which common knowledge is being created.

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And it is the adult who takes the leading role in drawing attention to them, talking about them, establishing knowledge which is both common and communicable" (p. 84).

The notions of "zone of proximal development" and "scaffolding" have two ideas which are important to the analysis of context and continuity.

> First, they embody the important principle that much of the acquisition of culture, including both formal and informal education, takes place in the context of guidance by some person, whether parent, teacher or more competent peer. It is a process of guided discovery, in which an individual's competence begins as his or her part in a social transaction. Joint activity and shared conceptions carried by language are major constituents. Second, there is the notion of internalization, in which the natural end-product of the learning process is a competent individual who has become able to perform alone, or in new contexts, activities and conceptualizations which could earlier be achieved only with the teacher's help (p. 86).

This latter idea is connected to Bruner's (1983) idea of a 'handover' process: that is, the handing over of knowledge and proficiency of an adult ultimately to a child. This idea of handover deserves more comment. In Bruner's inquiry into toddlers' learning of games (including "language games"), he described the process as one in which an adult set up a game and scaffolded it to ensure the child's inappropriate actions/words could be corrected by intervention. The adult removed the scaffold piece-by-piece as the child's reciprocal actions began to stand on their own. Bruner related this process to handing over both knowledge and actions through modeling, scaffolding and monitoring the child.

This term that they have adopted is filled with unhelpful resonances in suggesting a simple transmission model of learning. In Jean Lave's words, "Handover is yet another euphemism for transmission -- same old, same old in my book" (personal communication, August, 24, 2000). This metaphor frames knowledge as a physical object and as if it can be given from one to another. Even though I do not agree with a transmission model of learning, the fact of the matter is that teachers want students to an something ar

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learn something and in doing so, they have to give up control in terms of who is responsible as the active agent in classroom activity. I do not like the connotations of this term and will not use it in making claims about what is happening in my classrooms.

Although Edwards and Mercer claim all of the teachers in their study appeared to operate with something like handover in their teaching, none of them talked about it in their interviews. The teachers did sometimes tell students in their class that they would be expected to do some of the things that they themselves were doing. However, the authors claim that none of the lessons showed the handover process as being a complete success. They cite the teacher's continuing to perform the activities themselves or offering students easier versions of the defined task as evidence that handover did not really take place—that in the end, the students were not performing the activities without the help of the teacher. The students "remained dependent on the teacher to set the agenda, define the tasks and criteria for success, furnish the significant concepts, and generally control the learning process" (p. 91). It is in this asymmetry of power and knowledge that Edwards and Mercer find the problems in achieving handover of competence in school.

In summary, common knowledge is constructed through joint activity and discourse and becomes the contextual basis for future communication. Context is the common knowledge of the speakers called up by the discourse. It is problematic because ideas of each other's mental context may be only partially right, or may be completely incorrect. Similarly, continuity may be problematic since it is characteristic of context; it is context as it develops through time in the process of joint talk and action. Teachers have the task of 'scaffolding' children into the universe of educational discourse. This is accomplished by creating, through joint action and talk with the child, a contextual

backdrop for educational activities. One of the main purposes of education, according to Edwards and Mercer, is thereby to develop a common knowledge.

Control of Common Knowledge

The creation of joint understandings "takes place in the context of a power relationship between teacher and pupils, with the teacher representing and providing for the pupils an accepted wider culture of educational ideology, knowledge and practice" (p. 17). The teacher is in a role which allows her to control the discourse, to determine what is relevant to talk about, and to act as mediator of valid knowledge. Edwards and Mercer contend that students seem to accept this to be the teacher's role, also. The fact that classrooms operate on the assumption of common knowledge and values provides a "powerful means of encouraging pupils to adopt them without question" (p. 75).

They delineate, more specifically, classroom communication based on the extent to which the teacher controls the "nature, content and coding of knowledge" (p. 130). The features of classroom discourse they include[7]:

1. Elicitation (and non-elicitation) of pupils' contributions.

Even though students sometimes offered impromptu contributions, they were still not devoid of teacher control—"the teacher set the agenda, defined the topic of discussion, and established in advance the criteria of relevance and appropriateness of any contributions the pupils might offer" (p. 131). 'Spontaneous' contributions were ones in which students offered information that had not been previously taught without being called on by the teacher. Most contributions, however, fell into the 'elicited' category and followed the familiar initiate-respond-feedback (I-R-F) format[8]. In this structure, student contributions were constrained by the teacher's questions which:

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function[ed] as discursive devices through which the teacher is able to keep a continual check on pupils' understandings, to ensure that various concepts, information or terms of reference are jointly understood, so that subsequent discourse may be predicated on a developing continuity and context of intersubjectivity [...they] also function[ed] in defining and controlling what that knowledge and understanding will be (p. 132).

2. Significant markers, e.g. special enunciation, formulaic phrases, ignoring pupils' contributions,

3. Joint-knowledge markers, e.g. simultaneous speech, 'royal' plurals, repeated discourse formats[9],

Teachers sometimes use discursive devices to note the importance of what is being said. These devices include such things as special enunciation, the use of formulaic phrases, shifts in intonation, etc. Some of these seemed to function as highlighting important information (marking less important information as 'asides'), marking questions (as in use of rising intonation) or confirmation of answers (falling intonation), and denoting boundaries of pedagogic significance (changes in rate and loudness of speech). In one example given, the important curriculum-oriented content was stressed with careful, clear enunciation, while 'asides' were marked by a drop in volume and a sudden increase in the rate of speech (p. 137).

Formulaic phrases acted as noteworthy guidelines through which certain observations and conclusions were given distinction, were repeated, and were instituted as expressions of understanding. Typically, they arose during teacher-pupil dialogue (sometimes offered by students) and were taken up and encouraged by the teacher as correct and appropriate. The pedagogic function these seemed to serve was as "jointly understood encapsulations of the significant empirical findings that the pupils have been guided to discover" (p. 141). These phrases embody the essential equivalence of the results derived from classroom activity and marked the status of certain understandings as common knowled

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"common knowledge." These were marked often by their repeated use together with other devices, e.g. simultaneous speech, the teacher's use of "we" (implying joint understanding or no difference between the teacher's version and those of the students).

Another overt marker of common knowledge was speech in unison: a rehearsal of common understanding was performed through simultaneous enunciation of common language. People gave voice to the same words or meanings (not just speaking at the same time, as in overlapping speech). This typically occurred "at exactly those points where important issues of joint understanding were at stake; where the teacher was at pains to ensure[...] that her own interpretations [...] should prevail (pp. 140-141).

4. Cued elicitation of pupils' contributions.

This was characterized by I-R-E types of discourse in which the teacher offered heavy clues to the students about the information that was required. It may be accomplished by the wording of a question, but was often achieved in some other way—"intonation, pausing, gestures or physical demonstrations" (p. 142). The "best interpretation" Edwards and Mercer offer for the pedagogical function of cued elicitation is that it "embodies an educational process in which the pupils are neither being drawn out of themselves [....] nor being taught directly in the 'transmission' sense" (p. 143). Instead, it appeared that students were inculcated into what became a shared discourse with the teacher. Students were actively cued to "create" the shared knowledge, instead of sitting and listening to the teacher talk. In this way, it seemed to fit nicely into the process defined by Vygotsky's zone of proximal development, "in which pupils' knowledge is aided and scaffolded by the teacher's questions, clues and prompts to achieve additional insight that pupils by themselves seemed incapable of" (p. 143). This seemed to constitute a solution to the 'teacher's dilemma[10]'. Edwards and Mercer point out that the danger of cued elicitation is that "it can give a false impression of the extent to which pupils understand, and are ultimately responsible for, what they are saying and doing" (p. 143), which in turn can lead to procedural, ritual understandings.

5. Paraphrastic interpretations of pupils' contributions,
6. Reconstructive recaps, and
7. Implicit and presupposed knowledge (p. 131).

These three discursive devices were described as ways the teacher maintained tight control over the content of the common knowledge. When paraphrasing what students said and later recapping events, the teacher was able to redefine things as "altogether, neater, nicer, and closer to the lesson plan" (p. 146). Similarly, assuming certain things as "known" or "understood" allowed the teacher to hinder disagreement and direct the discourse and interpretation in the way most productive to the goals of the lesson. These reconstructive paraphrases played another function in the 'feedback' stage of the I-R-E sequence; "they provide an opportunity for the teacher not only to confirm what the pupils say, but to recast it in a more acceptable form, more explicit perhaps, or simply couched in a preferred terminology " (pp. 146-147).

What really matters during activity is the interpretation of what is being done; the words that define and declare the experience and the principles that are summarized in the words are clearly important to interpreting events. Typically teachers do this through reconstructive recapping. When providing those words, the teacher sometimes eliminates some and chooses others from the common vocabulary, managing the discursive process in which certain descriptions and versions of events are given as the foundation of joint understanding.

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By imposing her own way of interpreting activity, the teacher may use implication and presupposition in powerfully persuasive ways. By presuming a particular version as correct, by not offering it as open to scrutiny, she was able to direct student thinking. The use of presuppositional implication in educational contexts:

> [...] serves to introduce certain items of knowledge and assumption as things to be accepted without question, as understood but not on the agenda for discussion or disagreement, and, in a more general sense, is therefore available to the teacher as an instrument of control over what is known and understood (p. 152).

A general finding that Edwards and Mercer were surprised by the extent to which the teachers controlled the common knowledge in the classroom. These teachers appeared to be progressive and facilitative on the surface. However, on closer examination, the extent of teacher control became more apparent—"the freedom of pupils to introduce their own ideas was largely illusory; the teacher retained strict control over what was said and done, what decisions were reached, and what interpretations were put upon experience" (p. 156).

These categories are used to argue that the teacher's control over how knowledge is expressed often created as 'procedural' rather than 'principled[11]' student understandings--

> [students] frequently remain embedded in rituals and procedures, having failed to grasp the overall purpose of what they have done [...] saying and doing what seems to be required, rather than working out a principled understanding of how and why certain actions, expressions and procedures are appropriate or correct (p. 130).

They believe that this contributes to the ineffectiveness of education; that there is no final handover of knowledge and control to the students.

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Link to My Study

The literature I have presented here will appear throughout many of the chapters. "Common knowledge" is a notion that is prevalent in the upcoming textual analysis and in the case studies. In addition, each teacher controlled the common knowledge in the two classrooms. These methods of control are identified throughout the examples in the two case studies, using Edwards and Mercer's terminology. In the case studies, I also make claims about the norms that are carried by and embedded in the teacher-talk. I use social and sociomathematical norms to distinguish with whom or what each norm is associated: the teacher/teaching, the students or the mathematical content. I present these ideas at the outset because they are so pervasive. Other literature and tools/concepts of discourse analysis will be included at relevant points in the chapters that follow.

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[1] I only make reference to a few main ideas in this chapter. Many other relevant concepts and references will be introduced and defined throughout my dissertation in an attempt to make the text more continuous and coherent for the reader.

[2] This is defined as "cases in which one illocutionary act is performed by way of performing another (Searle, 1975, p. 60).

[3] Mishler focused on first grade classrooms and not only made claims about norms carried in the language, but also about cognitive strategies that were valued.

[4] Although the word "choice" implies an explicit level of consciousness, I am not suggesting that the teachers were always aware of the decisions they were making. In fact, in the interviews it was quite apparent that neither teacher was aware of the repeated language patterns I brought to their attention when I asked them about particular examples from the classroom transcripts.

[5] Edwards and Mercer recognize that some people consider this a less rigorous and objective way of attending to discourse: however, they contend that it is imperative to answering the questions they are pursuing.

[6] I would like to remind the reader that I have spent 2.5 years working with the teachers in these classrooms and prior to this analysis spent 3-5 days every week in the classrooms with the students (for the entire school year).

[7] The magnitude of teacher control increases as one descends the list, although Edwards and Mercer state that the list is not comprehensive and precludes an exact notion of hierarchy or order.

[8] This is essentially the same as the initiate-respond-evaluate (I-R-E) sequence to which Cazden (1987) refers. I will use I-R-E throughout this document to refer to this sequence and will not distinguish between the feedback as being an evaluation or not.

[9] Following Edwards and Mercer's format, I will treat the above two categories together in my explanation; this will also be the case with the last three (paraphrastic interpretations, reconstructive recaps, and implicit and presupposed knowledge).

[10] Edwards and Mercer define this as the students needing to generate their own understandings of things that have been predetermined by the teacher or curriculum—specific activities that were planned at the outset.

[11] Edwards and Mercer point out that common knowledge can be problematic if students are engaging in ritual knowledge. They define ritual knowledge as "a particular sort of procedural knowledge, knowing how to do something" (p. 97). They make a distinction between ritual and principled knowledge, while claiming that each has its place. For example, it would be grossly inefficient to always return to first principles in solving problems. Ritual knowledge produces fast answers, lending itself as a great aid to memory. Principled knowledge is "essentially explanatory, oriented towards an understanding of how procedures and processes work, of why certain conclusions are valid, rather than being arbitrary things to say because they seem to please the teacher" (p. 97). Performance becomes ritual when the conditions of knowing what is appropriate or when one method might be appropriate over another aren't met and there is no principled understanding to fall back on. I have thought about the common knowledge in these classrooms in terms of it being ritual vs. principled. It seems to me that both of these forms of knowing are valued and are part of the classroom discourse, but the data that I have collected does not illuminate which is prominent in terms of the teacher's goals for each lesson (as Edwards and Mercer used teacher interviews to determine some of the goals of each lesson and used this to clarify which form of knowledge was being focused on). They also used the classroom discussion to generate further examples of goals and student interviews to make claims about student understandings. I do have much of this data and will argue in Chapter 8 that there are differences in how students in each classroom engage with mathematical content; however, a more detailed analysis of student understandings are part of my research agenda and will not be addressed in this dissertation.

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CHAPTER 3: CARRYING OUT MY STUDY

The Inquiry

In this chapter, I describe my study by attending not only to the context of the study, but also to the data collection and analysis procedures. The process I detail allowed me to investigate differences I felt in two "reform-oriented" classrooms. My interest in this study has developed as a result of my own experiences as a student and teacher of mathematics. I am fascinated by the new genre of curriculum that focuses on mathematics in context because these problems eluded me as a student of mathematics. I am also intrigued by the ways in which this type of curriculum may be taught.

Through detailed discourse analysis, I have explicitly identified the norms that were embedded in and carried by the discourse in two classrooms by locating the three most prominent discourse patterns and using these to define some of the classroom norms in terms of the rights, roles, responsibilities and expectations that seemed to be attached to these reoccurring patterns. By describing them, I have been able to locate similarities in functions for different surface forms in the two classrooms. These differences point to variance in locus of authority, teacher positioning (with respect to the students in the classroom), and ways in which each teacher draws from knowledge sources (i.e. the textbook and students) in each classroom.

Guiding Questions

The questions that guided this study include:

A. What are the most prominent discourse patterns in each classroom?

- What are some of the social and sociomathematical norms embedded in and carried by them?

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- What are some of the things that influence the way the teachers think about and the way they work to establish and maintain their social norms and sociomathematical norms in their classrooms?

B. What are some of the similarities and differences across the two classrooms?

In addition to these questions, I constantly attend to the sense the students seem to make of the mathematics in the interpretation of the classroom transcripts. While I had intended to make that a focal point of the dissertation, a change occurred that directed my attention more toward the teachers and the teaching. This change included one of the teachers taking an administrative position beginning the year I had intended to investigate some preliminary findings from my early work in these classrooms. These changes will be described in more detail later in this chapter.

Site Description

The Local Context: The Town, District, and School

This study takes place in a rural town, Payson, that is located near a large university in the midwest. The population of the town is approximately 4000 people. The majority of the population is middle class (but not affluent) and white.

The school district consists of three elementary schools that feed into one middle school in Payson. The students at the middle school all attend the same high school. The district enrollment is 2,516 students, 400 of which attend this school. The ethnic breakdown of the school district includes 97.79% White, 1.63% Hispanic, 0.17 % each Black, Asian/Pacific Islander, and Other, and 0.07% American Indian/Eskimo/Aleutian. The district is also listed at 9.91% students in poverty.

In 1991 this school was chosen as one of the first sites (out of 55 districts/schools) to pilot the CMP materials as they were being authored. Throughout the process of during the mate

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editing the materials, the teachers used the units in the classroom and offered feedback to the authors about changes they suggested. Because of this involvement, the teachers who teach in this building are not only very experienced with the curriculum, but they are also very supportive of it and the approach to learning mathematics it embodies.

Inside the School: Classrooms, Teachers[1], and Students

Two of the teachers involved in piloting the CMP materials were involved in this study, Josh and Karla. Both taught 8th grade mathematics for the past two years; Josh was part of piloting the 8th grade units and taught them for the past six years. Between the two teachers, they encompassed the entire 8th grade student population, one for which there was no tracked mathematics classes. In addition, each teacher had a partial teaching assignment at the 7th grade level—in math for Josh and in science for Karla.

One of each teacher's classes was the focus of this study: Josh's first hour class and Karla's second hour. These classrooms were chosen because Josh and Karla felt that they were fairly representative of the 8th grade class as a whole. Also, Josh and Karla shared a preparation period during their third hour, which sometimes allowed the three of us to discuss scheduling (i.e. when observations, interviews, etc. would take place) and to carry on casual conversations about classroom episodes and the students in the class.

The scheduling of classes was set up on a quarter system. Because there was a semester career course that was required, the class schedule for students changed in January. Due to this, there were different populations of students in each class during the course of the school year. The number of students[2] in each class across the year are given in Table 3 below.

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Table 3
Number of Student Participants in Each Classroom by Semester

Classroom	Fall semester	Spring Semester
Josh	15 boys 12 girls	14 boys (11 from Fall[3]) 10 girls (5 from Fall)
Karla	12 boys 6 girls	9 boys (4 from Fall) 11 girls (4 from Fall)

Mathematical Context: What Students Have Had and What They Will Be Doing

The 8th grade CMP curriculum is made up of eight units which mainly focus on algebraic functions. The school year begins with a 7th grade unit, *Moving Straight Ahead*. The classes then use many of the 8th grade units and study linear, exponential and quadratic functions. In order to help the reader make sense of what the classes have done (mathematically) and what they were doing during the time that is the focus of this dissertation, I offer a brief description of the mathematical context of the two classrooms.

The Previous (and First) Unit: *Moving Straight Ahead.*

At this point in the school year, students have completed the CMP unit *Moving Straight Ahead.* (Lappan et al., 1997a). In that unit, the teachers and students worked on the concepts of slope and y-intercept in linear functions. They have used tables, graphs, and equations in contexts that were given in the text. These ideas build on a unit students completed in the 7th grade entitled *Variables and Patterns* (Lappan et al., 1997b). Some of the main points given in the Overview in *Moving Straight Ahead* are as follows:

- Students make conjectures about what relationship is characteristic of straight lines.
- Students discover that the rate at which y is changing relative to x is the coefficient of x.

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- Students learn to recognize linear situations from contexts, tables, graphs or equations. They also strengthen their understanding of the connections among these representations.

- Students explore the concept of *y*-intercept in several contexts (such as head start given to a participant in a race or the initial cost for renting a number of items).

- Students investigate the general form of a linear equation, $y = mx + b$. The *y*-intercept, b, and the slope, m, are key ideas in determining the equation of a line.

- Students learn how to determine the slope of a line given any two points on the line. The *y*-intercept is found by reasoning from the rate in a table or from a graph.

- More sophisticated methods for finding the *y*-intercept and an equation of a line are developed in the *Thinking with Mathematical Models* unit (pp. 1a-1b, Teacher's Edition).

This overview is given to describe the mathematical context of students up to this point. Students have been introduced to slope, y-intercept and linearity. They have explored these ideas using four representations: tables, graphs, equations and re-occurring contexts that the curriculum offers.

The Current Unit: *Thinking with Mathematical Models*.

In moving to *Thinking with Mathematical Models* (Lappan et al., 1998c) students have performed experiments to model various linear functions. From the perspective of the authors, the focus of this unit is:

> [...] mathematical models:what they are, how they are constructed, and what they enable us to do. [...] By choosing appropriate variables, collecting and graphing data, and manipulating the variables to see how relationships are affected, mathematicians can predict outcomes.
>
> The investigations in this unit introduce students to several kinds of algebraic relationships—including linear relationships, inverse relationships, and exponential relationships—used to model real-life situations. Students also explore other interesting relationships with graphing calculators [...] In this unit, they will review and deepen their understanding of both kinds of linear

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relationships and their representation in all three forms: tabular, graphic, and symbolic (p. 1a, Teacher's Edition).

This unit explores models in linear and non-linear situations. When doing so, students spend time both with data that is given in the textbook and with collecting their own data. The data is often recorded in a table and students are asked to generate a "graph model" for the data: that is, they are asked to generate a "line of best fit" that can then be used to make predictions for points that are not included in the data.

For example, in the first and second[4] Investigations, students collected data to examine the relationship between the strength of a paper bridge and two other variables that may affect the strength--the thickness and the length, respectively. Students construct paper bridges out of strips of paper and place an empty cup in the center of the bridge. They then fill the cup with pennies (to measure the breaking weight) and record the data. After the data has been collected, students are asked to decide if the data is linear or not by examining tables and graphs of the data. They are also asked to draw a graph model and to predict what might happen at points beyond what they had collected.

The four Investigations that make up *Thinking with Mathematical Models* explore data that is given and collected that result in linear, close to linear, and non-linear relationships. In between Investigation 1 and Investigation 2, students are given a Check-Up to assess what they have learned. This Check-Up is included in Appendix A.

Data Collected

1998-1999 School Year

I collected data primarily through participant observation and interviewing. I sometimes took part, as each teacher did, by moving around the classroom, asking students questions about their work, reminding them of directions that had been given and helping s

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and helping students with questions about problems they were assigned. At other times, I remained at my computer and video camera, taking field notes and collecting data. I also engaged in conversations with the teachers about student's questions, the content being studied and the "level" students seemed to be at in their understanding of the content. Many of these interactions were summarized and described in my field notes.

Observations

Part of the data collected consisted of two years of observations where detailed field notes were taken in which much of the classroom discussion was captured on a laptop computer as it took place. The first year of observations was in conjunction with another study and was used to gain insight into the classroom interactions and students' algebraic understandings. Questions about the classroom environment were formed during these observations and were used to guide the dissertation research. During the second year of observations, the actual dissertation data was collected. Usually within 24 hours, these notes were re-read and questions, comments, summaries, etc. were typed into the notes. Also, the language used which pertained to functions was color-coded based on the categories I developed in an earlier analytic memo (see Appendix B).

In addition to the field notes, two units were audio-taped and videotaped during the second year of observation. The first was *Thinking with Mathematical Models*, the second unit the classes used. It consisted of situations that were mostly linear (the latter ones were non-linear) in which students would perform an experiment, graph the data and find a line (or curve) of "best fit." This unit was audio-taped and videotaped in Karla's classroom; Josh's classroom episodes were audio-taped[5].

In the other unit[6], *Say it with Symbols*, contextual representations were fewer in number and less in focus. Most of the unit centered on understanding and representing situations wit

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situations with symbols. Students not only learned to do this, but also learned to solve and compare equations and expressions using tables, graphs and symbols. This was also a unit where one would find properties that are typically associated with algebra, e.g. distributive and commutative properties. During this unit, the last three weeks were audio- and videotaped in both classrooms[7].

The audio-tape was used to capture more of the actual classroom discussion than field notes allowed. When studying the language used pertaining to mathematical concepts, for example, I felt it important to have exactly what language was used and in what order. In both classrooms, the videotapes were used to observe any nonverbal actions associated with some of the language the class was generating for mathematical concepts (e.g. the term "swoopy down" referred to a decreasing exponential which was sometimes accompanied by a downward curving motion with the hand).

Student Interviews

Student interviews were held four times throughout the year. In the first interview, students were presented with an open-ended question (see Appendix C) which allowed them to talk generally about what they knew about linear functions and to describe what they considered important in defining linear functions.

The second task (see Appendix D) was given to a few students to see how students dealt with multiple relationships (i.e. linear, exponential, and quadratic) when they were presented all at the same time. Only two pairs were interviewed in each classroom due to time constraints and the change in my data collection schedule[8].

A third task was given at the request of Elizabeth Phillips, one of the co-authors of CMP. In this task, students worked in groups of three and were given a set of six problems (see Appendix E). Students were asked to generate and define categories in which they co

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which they could place the six problems. Once they had their categories established and defined, an additional six problems (see Appendix E) were added and students had to either place these problems into one of their categories or change/add categories to include the new problems. After they did this, each small group met with me and discussed their categories. The brief interviews were fairly structured in that all of the groups were asked a similar set of questions (see Appendix F). The purpose of this task was to help me uncover the big ideas behind algebra, from the perspective of the student. Students also had to answer questions that required them to articulate and refine their definitions, create more or less categories, justify the placement of their problems, etc.

The last task students engaged in consisted of two parts: two partnered written problems and an interview that focused on a problem different from the two they had already completed. The two problems that students worked on together in class consisted of NAEP Table Problem and Generating Patterns (see Appendix E). A range of students were chosen (based on gender and ability level) to be interviewed. The interview problem was the Phone Plans Problem (see Appendix E). This problem focused on a set of piecewise-linear functions. It allowed students to talk about familiar linear ideas, but the problem was atypical because each plan consisted of two parts, each of which could be represented by a linear function with defined domains.

Additional Data Sources

In addition to the observations and interviews, many written artifacts were collected. Both Karla and Josh photocopied partner quizzes and check ups from most of the linear units students covered. Because Karla had a student worker who did photocopying for her, I accumulated more written work for her students.

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1999-2000 School Year

Observations

I observed daily in Karla's classroom during the first unit, *Moving Straight Ahead*, to trace the genesis of the language patterns and observe how norms and discourse patterns were established in this classroom. Again, I audio- and videotaped each class period to capture more of the verbal and nonverbal discourse in the classroom. The classroom that I worked in was again a "typical" 8^{th} grade class, as determined by Karla. It consisted of 26 students: 17 girls and 9 boys. Since there was not a comparative class (because Josh was no longer teaching), I did not interview the students from this cohort. My intentions were to observe how Karla established the norms and discourse patterns in her classroom. To accomplish this, no interviews with students were needed.

Teacher Interviews

In addition to this work in the classroom and with students, I conducted a series of four interviews with Karla and Josh[9]. In the first interview, I had them talk about the norms in their classrooms in terms of the roles and responsibilities they expect of their students and themselves. I also had them react to a vignette from my fieldnotes in which three boys were having a small-group discussion: each boy was using a different representation (i.e. a graph, a table, and an equation) and were not really collaborating. I probed with questions that required Josh and Karla to talk both about the mathematical understandings of the students and the way in which they were interacting. Hoping to understand more about their teaching philosophy, I also asked them to talk about how their thinking or practice had changed in the past ten years.

The second interview allowed me to ask follow-up questions from the first interview and to get Josh and Karla to talk more about their expectations for students.

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Two ideas that I pursued from the first interview were related to: what each considered an "ideal" classroom to be; and what the "big ideas" of algebra were and how they used these in their classroom. To get them to be more explicit about their expectations for students and themselves, I asked them to list what these would be if they were to make a poster of them for their classrooms. In this way, I hoped to get them to talk more about what they saw their (and their students) roles, rights and responsibilities to be.

I began the third interview by asking the teachers to talk about the things that had influenced the way they taught. Also, by the time this interview took place, my analysis of the classroom discourse allowed me to use specific examples of emerging patterns and I asked each teacher to interpret some of these patterns for me. Then, to bring these two parts of the interview together, I asked them if they thought any of the influencing factors had actually encouraged those particular discourse patterns.

The fourth interviews were the least alike. I drew on particular statements each teacher made in the previous interview and asked clarifying questions. After we concluded with that portion of the interview, I showed each teacher a short video clip from the classroom observations made in their classrooms.

The clip that I chose took place during the last unit, *Say it with Symbols*. The portion of the lesson I focused on was an activity in which each teacher used a balance to model solving equations. I selected this because both Karla and Josh chose to do this activity, even though the authors did not include it in the final draft of the text (i.e. it was part of the pilot textbook but not in the published textbook). Also, I had noticed a difference in the manner in which Josh and Karla each enacted this activity and was curious to know more about it. Following Josh's modeling of the problem on the balance, he had a stud

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he had a student record the symbols on the board. In contrast, Karla worked back and forth—sometimes she modeled the problem first and at other times began with the symbols. Also, both activities were very teacher-directed and I wanted to see what each teacher had to say about that. During the interview I found Karla to be quite distressed by the example I had chosen because it was one that she had been thinking about for quite some time. Therefore, we did not view the entire thing. She asked me to turn it off and we spent time talking about why she was dismayed by the activity.

Since part of my dissertation focuses on differences in the two classrooms, I completed the last interview by asking Josh and Karla if they had ever observed in the other's classroom. Both had, but a very limited number of times. I asked them how they saw their classrooms as being similar and different.

Data Analysis Procedures

Overall, the process I used to explore the classroom data was discourse analysis. I first listened to the entire set of audio-tapes a total of three times within a week's time period. While listening to the data, I recorded (independently each time) repeated words or phrases that the teacher spoke on hard copies of the field notes that I had taken during the observations.

After recording these words/phrases, I compiled them and began to look for points of convergence in their form and function. In doing so, I located those that were most dominant in terms of sheer number in the classroom talk. I also grouped words/phrases into larger categories based on their form. I then found literature that would be useful for interpreting each form. For example, after compiling Karla's words/phrases I noticed that she often used meta-talk about classroom activities. I used

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Stubb's (1975) and Pimm's (1987) work to define the category "metacommenting" more precisely and to interpret the functions of the metacommenting in Karla's classroom.

After identifying the most prominent discourse patterns in each classroom, I located classroom interactions for my focus class periods. This process began by outlining a larger timeline of the data which consisted of describing the larger activities that took place and narrowing this down to more specific episodes based on interactions in the classroom and events that seemed significant to the study proposed. It also allowed me to differentiate between activities that included the entire class and those that focused on small-group or individual work. My analysis focused only on the whole-group discussions because I did not have the equipment to capture the discussion that took place in small groups.

The focus class periods that were transcribed were chosen based on common activities across the two classrooms and the concepts that were being taught. For example, both classroom transcripts include the teacher handing back and going over a Check-up, introducing and discussing an experiment in which students had to collect data, and going over homework that had been assigned. The process of discourse analysis also helped to separate description from interpretation, which was particularly useful in characterizing the two classrooms.

Once I located particular classroom interactions to be analyzed more thoroughly, I transcribed the lessons and used these to detail the functions of the forms I had identified. The functions were determined by finding every example of every form in the transcripts. These forms were first pulled from the transcript and examined as separate from the context and then were examined as part of the classroom context to identify the function associated w

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associated with them. The context was very important to determining the function of the talk. For example, upon reading subsequent interactions, I was able to determine that many of Josh's questions that appeared "open" (i.e. as having many correct answers) soon became "closed" (i.e. had one right answer) through his subsequent reformulations of the question.

Throughout the interpretation process, I returned to the third and fourth teacher interviews in which I asked the teachers to talk about some of the emerging patterns of discourse. In these interviews I asked each teacher: (1) if s/he noticed that they repeatedly said this; (2) if they could name this, what they would call it; (3) what they thought they were doing with that type of statement; and (4) what they thought that did for the students in the class. The teacher's interpretation is offered alongside mine whenever possible, one way that I triangulated the data.

In this dissertation, I used two class periods from each classroom to do a fine-grained analysis of the forms and functions of the talk. In order to check the representativeness of my claims for the larger database, I returned to the audio-tapes from *Thinking with Mathematical Models*. After writing the two case studies, I listened to these tapes again to make sure the forms that I described were apparent in each lesson. I found that all of the three most prominent discourse patterns that I described in each case did indeed exist in *every* class session to which I listened. In addition, the extended transcripts (which took place during March of the same school year) included in Appendix O and P further evidenced the reoccurring patterns I described in each case study.

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Another purpose for my reviewing the tapes was to help trace some of the patterns and to look for discrepant cases. For example, I needed to further investigate one of the forms of questioning Josh used (i.e. What's the rise?...the run?...the slope?...the y-intercept?...the equation for the line?) to see if he was using this line of questioning as a form of scaffolding. In addition, I checked to see if this particular form of questioning appeared often or only after Josh handed back quizzes (as it had in the focus class period). The results of these are addressed in Josh's case study.

Summary

My experiences as a student and a teacher of mathematics have led me to pursue the topic of this dissertation—discourse patterns and norms in two "reform-oriented" mathematics classrooms. The two 8^{th} grade classrooms that form the focus of this dissertation were using the Connected Mathematics Project, an NSF-funded curriculum project. I was intrigued by the teachers and their teaching because I noticed the students seemed to have similar understandings, but each classroom felt different to me as a participant-observer.

These classrooms offered a context that allowed me to study difference in the context of similarity. The teachers had many attributes in common (detailed in Chapter 5). However, the teaching in the two classrooms was different. Drawing from the sociolinguistics and mathematics education literatures, I describe the social and sociomathematical norms of the two classrooms in terms of the classroom discourse which they were embedded in and carried by. I also interpret student understandings whenever possible throughout the thesis, taking a social constructivist perspective.

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In the year prior to commencing my dissertation study (1997-1998). I completed classroom observations and student interviews as part of my practicum work and research assistantship. These were used to form preliminary hypothesis about student understandings and the classroom environment. The data used for this dissertation was collected over the subsequent two years (1998-2000). During the first. I observed and audio- and video-taped students on a weekly basis. In addition, students were interviewed about their algebraic understandings and their classroom experience. The second year, one of the classrooms was observed to trace the formation of the norms in the classroom. The teachers took part in four extensive interviews, in which they were asked about influencing experiences related to their teaching and the norms in their classroom (in terms of the expectations, rights and roles of themselves and their students). They were also asked to react to transcripts and video from their classroom, offering their perspective about their intention in their language patterns and teaching.

The ideas I investigate in this dissertation include how social and sociomathematical norms are embedded in and carried by the classroom discourse in each classroom (Chapters 6 and 7). I also discuss aspects in the teachers' professional lives that influenced the ways they think about and work to establish and maintain the norms in their classrooms (Chapter 5). In Chapter 8, I look across the two classrooms to offer what I see as being similar and different, which has allowed me to locate differences in: the overall structure of teacher talk, the positioning of the teacher with respect to the locus of authority, the way each teacher draws from potential other knowledge sources in the classroom (i.e. students and the textbook), and the way each teacher draws attention to the common knowledge constructed in the classroom.

[1] I have chosen to give a detailed account of the teachers in a separate chapter so will not include that information here.

[2] One striking difference in these numbers is that Karla had a smaller number of students in her classes, scheduled as such to allow her to prepare for teaching 7th grade science for the first time.

[3] The numbers in the parenthesis represent the number of students who carried over from Fall semester.

[4] This investigation is included in Chapter 4, pp. 52-53. It was one activity that took place during the October 15th class period.

[5] This difference in data collected is due to the changing nature of this study—when I originally proposed this work I had intended to mainly study Karla's classroom. I was also working in Josh's classroom as part of my assistantship with CMP. As differences/similarities emerged, I became more interested in looking closely at both classrooms and video- and audio-taped in both classrooms later in the school year.

[6] This unit is the last algebra unit of the school year.

[7] The last three weeks were captured because that is when Josh informed me that he had taken a new position for the Fall of 1999 as Assistant Principal/Junior High Athletic Director. I had his (and the students') permission to videotape and began the day after he informed me of this change.

[8] At this time, Josh informed me that he had taken a new position that would begin Fall, 1999. To collect the remainder of the data I needed in his classroom, I had to abandon the student interviews to audio- and videotape in his classroom while they finished their last algebraic unit, *Say It With Symbols*.

[9] For copies of these four interviews, see Appendix G.

CHAPTER 4: ANALYSIS OF *THINKING WITH MATHEMATICAL MODELS*[1]

In the upcoming description and analysis of the teaching and discourse, I realize that there is another force at play that is influential. In a sense, another "teacher" that exists in the classroom is the textbook that is being used, for "mathematics texts are there to be worked on, and when such texts are present, readers have to situate themselves in relation to them" (Love & Pimm, 1996, p. 372).

Following the analysis of the classroom transcripts, I noticed that the teachers used the textbooks differently. The manner in which each teacher used the textbook is addressed in Josh's case study (Chapter 7) and the cross case (Chapter 8). I became intrigued by how the textbook may be interacting with the teacher-talk and decided to look closely at the repeating patterns in the textbook that was used during the focus class periods. This analysis uncovered that particular aspects of the teacher-talk were similar to ones that appeared in the textbook. And, the patterns that each teacher used were different.

In this chapter, I concentrate on the style of the writing (authoritative structures), within which is embedded: (1) the construction of the reader and the acknowledgement of the teacher, (2) the way in which the text constructs reasoning and (3) the use of cohesive devices. In doing so, I view the text material as an objectively-given structure, rather than as a subjective scheme (Otte, 1983). I emphasize these structures in particular because they surfaced as prominent in the classroom discourse analysis. By introducing the textbook in this manner, I hope to provide the reader with some insight into ways in which the textbook may be influencing or interacting with the classroom discourse. More specific d

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specific details about the linguistic constructs (e.g. hedging, modal expressions and modal verbs) used in passing here are discussed in the upcoming case studies[2].

Style of Writing

In all, *Thinking with Mathematical Models*[3] consists of 12, 766 words (not including the Table of Contents)[4]. When analyzing these words, there are ways to locate the relationship and roles of author and reader in the style of writing. Three ways to address these are to focus on the use of imperatives, pronouns and modal verbs and expressions (Morgan, 1996). Each of these will be addressed here.

Imperatives

Two of the most commonly occurring forms that exist in the textbook are questions[5] and imperatives (commands). In fact, within the 747 sentences/questions[6] that exist in the textbook, there are 165 questions and 335 embedded imperatives[7], which are broken down by type of text sub-structure in Table 4.

The questions in the text most commonly began with how (64 times), what (57 times) and which (20 times) and appeared in all sections of the text, although were most prominent in the ACE[8] problems. Many of the questions in the ACE problems were followed by instructions to "explain your reasoning" or "give evidence/reasons."

Table 4
Frequency of Questions and Imperatives by Section of the Text

	Questions	**Imperatives**
Introduction	3	7
Investigation 1	40	47
ACE problems	22	46
Mathematical Reflections[9]	6	8
Investigation 2	25	36
ACE problems	20	36
Mathematical Reflections	4	3

Investigation 3	12	30
ACE problems	14	35
Mathematical Reflections	3	4
Investigation 4	7	34
ACE problems	9	35
Mathematical Reflections	0	14
Totals	165	335

Often there was a string of imperatives given. For example, when directions were given for an experiment[10] a string of imperatives would occur (my underlining of imperative verb forms):

2.1 Testing Bridge Lengths

In this problem, you will experiment with paper bridges of various lengths. What relationship do you expect to find between the length of a bridge and its breaking weight? Do you think longer bridges will be stronger or weaker than shorter bridges?

Equipment: eight 4-inch-wide strips of paper with lengths 4, 5, 6, 7, 8, 9, 10, and 11 inches, two books of the same thickness, a small paper cup, and about 50 pennies

Directions:

- Make paper bridges by folding up 1 inch on each long side of the paper strips.
- Suspend one of the bridges between the two books. The bridge should overlap each book by about 1 inch. Place the paper cup in the center of the bridge.
- Put pennies in the cup, one at a time, until the bridge crumples. Record the length and breaking weight of the bridge.
- Repeat this experiment to find the breaking weights of the remaining bridges (p. 26).

Other places repeated imperatives occurred were in the investigation problems, e.g.--

Problem 2.1

A. Do the experiment described to find the breaking weights of paper bridges of lengths 4, 5, 6, 7, 8, 9, 10, and 11 inches. Organize your data in a table. Study the

table, and look for a pattern. Do you think the relationship between bridge length and breaking weight is linear?

B. Make a graph of the (length, breaking weight) data from you experiment, and describe the pattern you see. Do the data appear to be linear?

C. Draw a straight line that seems to model the trend in the data. Do you think your graph fits the data satisfactorily? Explain.

D. Use your graph model to predict breaking weights for bridges of lengths 4.5, 5.5, and 6.5 inches. Make bridges of these lengths, and test your predictions.

E. How is the relationship between bridge length and breaking weight in this problem similar to and different from the linear relationships you studied in the last investigation (p. 27)?

--and in the ACE problems:

3. Since Betty raised her prices, cookie sales have fallen. Betty calls in a business consultant to help. The consultant suggests that Betty conduct a customer survey. Betty's customers are asked which of several amounts they would be willing to pay for a cookie. Here are the results:

Price	$1.75	$1.50	$1.25	$1.00
Customers willing to pay this price	100	117	140	175

a. Make a graph of these data, and draw a straight line or a curve the models the trend.

b. Use your graph model to predict the number of customers who would be willing to pay $1.35 and the number who would be willing to pay $2.00.

c. Do you think predictions based on your graph model are accurate? Explain.

d. The shape of this graph resembles the shape of another graph you have drawn. Look back at your work in this unit. Which situation has a graph similar to this one (pp. 42-43)?

Some of the most common verbs include: *explain* (46 times), *make* (e.g. make a table, make a graph; 35 times), *use* (e.g. use the graph to, use the table to; 29 times), *write* (e.g. write an equation; 29 times), and *describe* (28 times). Many of these imperatives invite students to be active participants in the experiments in which they are taking part: others require them to articulate and explain their thinking.

Morgan (1996) claims that imperatives influence the roles and relationships between the author and reader of textbooks.

> The use of imperatives and of other conventional and specialist vocabulary and constructions characteristic of academic mathematics marks an author's claim to be a member of the mathematical community which uses such specialist language and hence enables her to speak with an authoritative voice about mathematical subject matter. At the same time it constructs a reader who is also a member of the same community and is thus in some sense a colleague (p. 6).

However, given the unequal relationship between the reader and authors in school, it is probably more likely the case that this type of language is seen as a way to inculcate students into the mathematical community. The roles that are constructed for the reader may be different depending on the type of imperative that is used. Rotman (1988) distinguishes between inclusive imperatives (e.g. "let's go") and exclusive imperatives (e.g. "go") in mathematical writing. Inclusive imperatives are addressed to what he calls a "thinker" and "demand that speaker and hearer institute and inhabit a common world or that they share some specific argued conviction about an item in such a world" (p. 9). This form is also used to make suggestions and is considered less direct than exclusive imperatives. Exclusive imperatives construct the reader as a "scribbler" who performs actions (e.g. write, draw). Most of the imperatives in this unit are exclusive, constructing the reader to be a "scribbler."

<u>Pronouns</u>

First person pronouns (i.e. I and we) indicate the "author's personal involvement with the activity portrayed in the text" (Morgan, 1996, p. 5). For example, "We will show..." suggests that the author is speaking with the authority of the mathematical community. Another way in which "we" may be used is to indicate assumed mathematical activity performed by the reader, e.g. "In the last section, we...."

First person (singular and plural) pronouns were entirely absent from this unit. This may obscure the presence of human beings in the text, affecting "not only [...] the picture of the nature of mathematical activity but also distanc[ing] the author from the reader, setting up a formal relationship between them" (p. 6).

The second person pronoun, "you," must also be examined to understand this construction of roles and relationships between reader and author of the text.

> Addressing the reader as *you* may indicate a claim to a relatively close relationship between author and reader or between reader and subject matter [....] On the other hand, some uses of *you* appear to be attempts to provide expressions of general processes rather than being addressed to individual readers [...] particularly where children are struggling both to formulate generalisations and to communicate them (Morgan, 1996, p. 6).

In this unit, the word "you" occurs 263 times (397 when "your" is also included). There seem to be about four categories that "you" falls into, which are given in Table 5.

Table 5
Forms of "You" in the Text

Form	Examples	Number of Instances
You + verb	You find, you know, you think	165 times
You + modal verb[11]	You will, you can find, you would	56 times
Inanimate object (as subject) + animate verb + you (as direct object)	The graph shows you, the equation tells you	37 times
You + hedged[12] verb	You might have found	5 times

The first pattern (you + verb) is most apparent in the textbook. It seems that the authors are defining what they think the reader is doing: that is, they are telling "you" about yourself. When doing so, they are sometimes controlling the common knowledge by telling the reader what they think or know. At other times, they are defining and drawing attention to the common knowledge by telling the reader what they "find."

The third form (inanimate object (as subject) + animate verb + you (as direct object) strikes me as peculiar. Inanimate objects perform activities that are typically associated with people. Having an animate verb with an inanimate object is almost contradictory. When would a graph actually "tell" you something?

Morgan points out that agency can be obscured through the "use of processes without any indication that these are actually performed by anyone" (p. 4), indicating an absolutist image of mathematics as a system that can act independent of humans. This appeared in this third form. Often in this unit, representational objects were actors in verbal processes. For example, "Testing paper bridges to find out how thickness affects strength and then fitting a straight line to your experimental data *introduces you* to the idea of a linear graph model" (p. 4, italics added). In this case, a long, abstract process (i.e. "Testing paper bridges to find out how thickness affects strength and then fitting a straight line to your experimental data") "introduces"—an act that this cannot, in reality, perform. In addition, the text creates an image of the author as being absent. There is no "voice of the author" present, only someone omniscient who dictates what is being done.

This is not to say that the processes are the only actors in this textbook. In fact, many of the contexts have human actors in them. The actors are named by occupation (e.g. civil engineers), by group (e.g. riders on a bike tour or the student government association), or by name (e.g. Chantal and Charlie). The age of the actors also varies, but many are school-aged children. These people typically performed activities—collected data, analyzed it, represented it, drew conclusions, made predictions, etc.

Modality[13]

Another way in which relations may be located is in the modality of the text, which may appear in modal auxiliary verbs (e.g. can, will, could), adverbs (e.g. certainly, probably), or adjectives (e.g. I am sure that…). Modality allows one to see relations between author, reader and subject matter through "indications of the degree of likelihood, probability, weight or authority the speaker attaches to the utterance" (Hodge and Kress, 1993, p. 9). As I stated above, there are only five instances where "you" is combined with a hedged verb. This is one way in which the authors may acknowledge that the audience is unknown to them. For example:

> 2.2 Keeping Things Balanced
>
> The pattern you saw in the length, breaking weight) data show up in many other important and familiar problems. For example, you may have thought about this problem when you were a child:
>
> *How do people of different weights balance on a teeter-totter?*
>
> You probably discovered that the lighter person has to sit farther from the balance point (called a *fulcrum*) than the heavier person does.
>
> [cartoon drawings of two pairs of people sitting on teeter-totters]
>
> You might have found the balance point by trial and error, but there is a mathematical relationship between weight and distance. The following experiment will help you discover this relationship (p. 28, underline added, italics in original).

Within this, three of the five "you + hedged verb" in the textbook take place. The problem begins with the authors asserting that this experience is "important and familiar," assuming this to be true. Then, because they cannot be certain whether this is the case, they continue by hedging the verbs in the next three sentences: "*may* have thought," "*probably* discovered," and "*might* have found." These are things the authors cannot know about their reader and one way to acknowledge this is to make these statements less baldly by hedging the verbs.

Other hedges that occur in the textbook include: *about* (12 instances of this as being used to approximate), *might* (7 times), and *may* (5 times). These can indicate certainty about what is being expressed. The most prominent modal verbs that occur in the text include: *would* (55 times), *can* and *will* (40 times each), which are followed by *could* (13 times) and *should* (11 times). As indicated by the most prominent forms (and those of which there are few: "probably," "might," "may"), the text represents a fairly certain viewpoint ("must," "would," "can," "will") of mathematics.

A more explicit way authority is embedded in the text is included in the examples of when, where and how mathematics is used in the world and knowledge about those occupations. For instance, in the first two investigations, there are boxes that ask, "Did you know?" Within each box, the authors describe various things engineers have to take into consideration when they build a bridge and introduce engineering language for these things, e.g. dead load and live load. So, not only does the author know the mathematics, but they know much more than that—they know detailed information about occupations that use mathematics, too!

Construction of Reasoning

There were few instances of "because" and "so" (only 3 and 12 respectively) being used to express connective reasoning. The word "so" usually came prior to a main conclusion or a restatement of information that had been given. It seemed that the focus for reasoning came more often after students had answered a question; they typically were asked to provide an explanation[14] or to give reasons for their answer.

How, then, was reasoning constructed in this text? It seems to occur more in a narrative telling of how someone came to a particular conclusion. For example:

> 1.2 Drawing Graph Models
>
> A class in Maryland did the bridge-thickness experiment. They combined the results from all the groups and found the average breaking weight for each bridge. They organized their data in a table.

Thickness (layers)	1	2	3	4	5
Breaking weight (pennies)	10	14	23	37	42

> The class then made a graph of the data. They thought the pattern looked somewhat linear, so they drew a line to show this trend. This line is a good *model* for the relationship because for the thicknesses the class tested, the points on the line are close to points from the experiment.
>
> [graph is given]
>
> The line that the Maryland class drew is a graph model for their data. A **graph model** is a straight line or a curve that shows a trend in a set of data. Once you fit a graph model to a set of data, you can use it to make predictions about values between and beyond the values in your data (p. 7, boldface in original).

In this problem, there is some use of terms that carry connected reasoning, e.g. "so" and "because." However, there is a particular order of events that are modeled in this problem, also. This order is: (1) did the experiment, (2) combined the results, (3) found the average, (4) organized data in a table, (5) made a graph, (6) thought about the pattern (which looked linear), (7) drew in a line to show the trend. These exact events (and the order in which they happen) are those in which the classes themselves just engaged. In fact, they again engage in these exact activities in the next investigation when they do non-linear modeling. Also, this was used as a model for what followed: students then made the same kind of predictions as the students in this problem.

Following this story-type problem was a series of commands in which the students were supposed to follow. Morgan (1996) claims that the combination of temporal themes and imperatives would construct an algorithm. In this unit, the word "then" is used temporally 20 times; it is used to show cause only once. These story-form problems are always followed by a series of questions and/or imperatives.

The use of a re-occurrence of events was one way the text seemed to preserve continuity. Another way this was achieved was through referring-back to past events (either in this unit or prior ones). For example, following the above problem:

> 1.3 Finding Equation Models
>
> In your earlier work, you saw that linear relationships can be described by equations of the form $y = mx + b$, where m is the slope and b is the y-intercept. The line drawn to model the data in the Maryland students' bridge-thickness experiment is shown below. By looking at the graph and comparing vertical change to horizontal change, the students found that the slope of the line is about 8.7. Since the line passes through the origin, its y-intercept is 0. They wrote the equation $y = 8.7x$ to represent the line (p. 9, italics in original).

In a sense, the authors are giving meaning to the activity and reminding students what they "saw" at a previous time (Edwards and Mercer, 1987). Similarly, the text would state what "you found" at the beginning of each new Investigation and then proceed to tell students what "you will" do:

> Investigation 2: Nonlinear Models
>
> In the last investigation, you tested paper bridges of various thicknesses. You found that thicker bridges are stronger than thinner bridges, and you discovered that the relationship between thickness and breaking weight is approximately linear. In this investigation, you will explore relationships with different types of models (p. 26).

By telling students what "you found," the authors seem to be controlling student sense-making and defining what the class should have taken away from the activity, possibly in an attempt to define and control the common knowledge in the classroom. In fact, in some ways the authors need to do this because they are counting on a common readership that has done and found certain things in order to proceed. They also preserve continuity by connecting the past to what students will be doing: that is, they will be exploring relationships with "different types of models."

Another way continuity is preserved is in the repeated use of mathematical terminology. In the first investigation, students are introduced to "model" and "graph model." It is in that investigation in which these terms are highlighted (with italics first and then boldface type) and defined. After that, it is assumed everyone now knows the word as it appears throughout the rest of the textbook.

Summary

The analysis of this unit has led me to conclude that the style of writing of the text is pretty authoritative. The repeated imperatives in the experiments, problems and ACE construct the author of the text as having an unequal relationship with the reader whose role is to inculcate students into the mathematical community. Because many of these imperatives are "exclusive," the role of the student is constructed as a "scribbler."

The textbook is devoid of first-person (singular and plural) pronouns, indicating that the presence of human beings is concealed. Morgan claims that this affects how mathematical activity is presented and sets up a distance relationship between the author and reader. In the second-person pronouns, the authors: (a) seem to define what the readers should be doing, sometimes controlling and drawing attention to the common knowledge of the classroom, and (b) obscure agency by having inanimate objects perform animate activities. Morgan contends the latter indicates an absolutist image of mathematics as a system that can act independent of humans.

The modal verbs in the textbook are more apparent in the problem in which shared information is assumed: that is, where the authors draw on an experience that they cannot know if the students have been a part of or not (i.e. balancing on a teeter-totter). To soften their assertions, they hedge the verbs in the problem. The most prominent forms of asserting in the text (i.e. *would, can, will*) indicated that it represents a fairly certain viewpoint of mathematics. This is further apparent in the forms of which there are few, e.g. probably, might, may.

Reasoning was modeled in a narrative form: that is, it appeared in what other people did in the textbook. Model classes were introduced as participating in certain activities which were discussed and followed as the way certain events would occur. Usually this model example was followed by a series of imperatives. Morgan claims that the combination of temporal themes and imperatives would construct an algorithm to be followed. A series of re-occurring events seemed to be one way in which continuity was preserved in the textbook. Other ways this was achieved include: referring-back to past events, stating what "you found," pointing to what "you will" do and introducing and continuing to use introduced mathematical terminology. All of these could be seen as ways that the textbook attempted to define and control the common knowledge of the classroom. This makes sense because the authors can only imagine and draw on a common readership that has done and found certain things. Without this, the activity would not be able to proceed.

Some of the discourse patterns I discuss in the case studies (Chapters 6 and 7) are similar to those that have appeared in the textbook. For example, Karla preserved continuity by referring back to activities, discussions and contexts in the classroom. This overlapped with the textbook's use of "you found" which referred back to activities and contexts. In addition, Karla used narrative structures that were similar to those the textbook used. In Josh's classroom, the most prominent discourse pattern was his use of strings of questions. This also happened to be one of the most noticeable aspects of the textbook. I have preceded the case studies by this textual analysis with the intent of inviting the reader to continue to attend to the aspects of the textbook that are similar/different from the forms of address each teacher incorporates in his/her classroom. These will be taken up further in some of the sections in following chapters.

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[1] I want to remind the reader that this analysis is a portrayal of this particular unit within the larger CMP curriculum. Because this unit focuses on mathematical modeling, it has some aspects to it that are probably not as apparent in some of the other units. Therefore, one should not conclude that this analysis is indicative of what appears in other CMP units, with the exception that there is still an emphasis on functional relationships in problem-solving contexts using multiple representations (i.e. graphs, tables and equations).

[2] These are to be found in Chapters 6 and 7.

[3] Before offering the detailed analysis of this unit, I would like to point out that the type of texts that the NSF curriculum introduced may be a new genre in that they were the first to be written "in context." Because of this, there are many more words in the textbooks than more traditional textbooks which more often consist of strings of equations and symbols. The literature on textual analysis from which I am drawing was based on the analyses of student texts and more traditional versions of mathematical textbooks.

[4] I have used both an electronic copy of the textbook and a published copy of the textbook in doing this analysis. Some of the counts were done by the computer, others I did by hand. For instance, the number of words in the textbook was counted by the computer and the number of imperatives were counted by hand because there was often more than one in a given sentence.

[5] A detailed account of the literature pertaining to teacher-questioning will be given in Chapter 7.

[6] In counting these, I discounted lines in which there were headings (e.g. Problem 2.1), single-word entries (e.g. Directions) and have equations given without text describing the equation (e.g. in the ACE problems). Sentences were used in counting.

[7] By using the term "embedded imperatives," I am acknowledging that I included all imperatives in a sentence, not just the one with which the sentence began. For example, "Make a graph of the (length, breaking weight) data from your experiment, and describe the pattern you see" (p. 27) was counted as being two imperatives because the reader was being directed to "make a graph" and "describe the pattern."

[8] ACE problems are those that appear after the Investigations and is short for Applications, Connections and Extensions.

[9] An interesting point about the Mathematical Reflections is that even when the book states, "these questions will help you..." about half of the reflections (or more) are actually imperatives.

[10] The majority of the activity in this unit is having students engage in experiments. I would like to recognize that giving instructions for experiments lends itself to more imperatives than the units that have few or no experiments. *Models* is an exception to the other texts because of its experimental emphasis.

[11] I will comment more on the modal verbs in the next section, so this form will not be addressed right now.

[12] Hedges are "linguistic pointers to moments of uncertainty" (Rowland, 1995, p. 328).

[13] Details about modality and hedging are given in Chapter 6.

[14] Explicit explanation for how to do things was also absent from the text. However, I did not address that here because it is possible that this has more to do with the fact that the focus in this unit is on mathematical modeling. It may be possible that explanations are not as pertinent to mathematical modeling than other mathematical content, so I do not want to make inferences about this due to the fact that it might be attributable to the content in this particular unit.

CHAPTER 5: INTRODUCTION TO THE TEACHERS

Before proceeding, I would like to introduce the teachers that are the focus of the two case studies by describing their backgrounds: that is, I share details about their education, experience and professional activity. I continue by drawing on their interviews to offer some of their philosophy of teaching in general and about teaching mathematics specifically, highlighting the many similarities that exist between them, as well as some differences.

Background

Both Karla and Josh grew up in the same state and described the counties in which they grew up as fairly conservative. They both attended universities within the state when they graduated from high school. Although their ages differ by about seven years, they both emerged from their undergraduate programs with a Bachelor of Science in Elementary Education. Karla minored in mathematics and science: Josh in science. Josh later returned to school and received his Master's degree in Elementary Mathematics Education from a university in the Southwest. Both earned certification to teach grades K-8 in all subject areas and K-9 in science and mathematics.

Although Josh began his teaching career outside of the state, he came to Payson in 1986 to teach 6th grade at one of the three feeder elementary schools; Karla began her teaching career at the same school in the same year and taught 4th, 5th, and 6th grades during her eight years there. Josh spent five years at the elementary level and moved to Payson Middle School in 1991 when all sections of sixth-grade students in the district were relocated there. Karla made the switch to the middle school in 1993 when she decided to teach 7^{th} and 8^{th} grade mathematics and science. Josh and Karla have been colleagues and friends outside of school for many years.

While teaching 6^{th} grade at the elementary school, Josh became involved in the Middle Grades Mathematics Project[1] (MGMP). Josh indicated that this was the first time that he had considered that there were multiple ways for solving a given problem:

> [...] when you teach traditionally and when your texts are traditional in nature, there usually is *a* way or a *correct* way of doing something and probably when I started teaching MGMP is when I first really started looking at, wow, that's kind of a cool way of thinking about it, or maybe that's not a real efficient way of doing it, but, you know, it makes sense mathematically and I'll be darned, we got the right answer, so [...] I think it's probably been more in the last ten years of my teaching where I really saw the validity of looking at alternative strategies (Interview J-1, 11/23/99[2]).

With the switch to the middle school, Josh was asked to pilot early versions of the Connected Mathematics Project (CMP) beginning with the 6^{th} grade units and continuing by moving into the 7^{th} and then 8^{th} grade. Shortly thereafter, Karla joined him at the school and also became involved in piloting CMP at the 7^{th} and 8^{th} grade levels.

While piloting CMP, both Josh and Karla had many people working in their classrooms to get feedback and help with implementation. In Karla's words:

> [...] we had MSU here all the time and had constant support. In fact, we even had people running in five minutes before our class started going, "Here's the new lesson. Oh, that didn't work out? Try this one out. Here's another one." There was constant feedback, which was nice. I've grown up with it (Interview K-1, 11/23/99).

Josh and Karla were also part of a group of piloting teachers who met and discussed the units at least two or three times per month. In those meetings, they talked about "what didn't work [...], the order of the problems [...] and you couldn't help compare the discussions that happened in your classroom" (Interview J-4, 1/12/00).

In conjunction with piloting this curriculum, the county school district received a large Eisenhower grant to offer summer professional development activity related to the NCTM *Standards* and implementation of reform-oriented mathematics teaching. These workshops took place for one week during each of the summers from 1991 through 1995 and both Karla and Josh participated in all of them. Some of the presenters and organizers of these workshops were colleagues from Josh and Karla's school. The activities that they engaged in ranged from observing teaching of CMP lessons by model teachers to discussing the meaning of "discourse" as it was presented in the *Standards*.

Both Josh and Karla became very enthusiastic about CMP and showed this enthusiasm by becoming involved in the professional development activities CMP offered for its teachers. Josh not only helped with the introduction workshops for CMP, but also traveled out of state to help other schools who were implementing the program. Karla attended both the introduction workshop and the Users Conference[3] and began to lead some of the professional development activities while I was collecting the data for my dissertation. In addition to CMP development, both Karla and Josh began to attend workshops that offered instruction in the use of technology, i.e. graphing calculators and computer based data collection devices. Both were avid users of graphing calculators and encouraged their students to use them whenever they felt they wanted or needed.

During the 1997-1998 school year, Josh and Karla joined an area algebra project[4]. In their activities with this project, they were involved in discussions with other professionals about the nature of algebra, as well as its teaching and learning. In addition, they were involved in discussing and editing a series of problems that were developed to assess student algebra learning at the end of the 8th grade. Their classrooms became sites for this assessment. These assessment materials and corresponding student solutions were then used to develop instructional materials for pre-service teachers. Josh and Karla were also involved in the development of these instructional materials, which intended to support teachers' thinking and understanding of: (1) the nature of algebra as mathematical content, and (2) their own understanding of algebra, as well as (3) how to examine and evaluate students' responses to various algebra problems.

Josh and Karla are strong proponents of the CMP curriculum. They have shown their enthusiasm and support for the curriculum in at least two ways. The first was illustrated above—they represent themselves as teachers of CMP at broader levels than just within their building. In the second way, they have shown their support at local and regional levels in defending the curriculum when it has come under fire at the school, as well as beyond the school's four walls (e.g. at the district and regional level).

Since the CMP units are quite different from traditional mathematics textbooks, they also carry with them the controversy and adjoining criticism that often accompanies any reform-oriented curriculum (for examples of such criticisms and controversy see Askey, 1992; Dillon, 1993; Jackson, 1997a; Jackson, 1997b). In order to prepare for such backlash, Josh and Karla spent one summer (with other teachers in their building) mapping the CMP curriculum onto the state standards. That way, if they (the teachers and textbooks) were ever accused of not preparing their students sufficiently, they could point to all of the connections between CMP and the state standards[5].

General Beliefs about Teaching and Teaching Mathematics

Both teachers have described their involvement in the activities associated with CMP as highly significant to changes in their teaching. When they were asked if their teaching had changed at all in the past ten years, both teachers said that it had. They each described what their beginning teaching style had been like:

> I was a very traditional teacher for the first six to seven years of my teaching career where I was the person who had the math knowledge and it was my job to take what I knew and present it to the kids and kinda ram it down their throats (Interview J-1, 11/23/99).

> When I first started teaching, I'd show how you multiply two-digit numbers and I'd do a few examples and then give them a worksheet and tell them, now do this and I'd go sit down and that was like my down time to get everything caught up when they were doing their practice. I could just correct them really quick, they were right or wrong answers and that was it [....] I set all the expectations before in that today was page 7, tomorrow page 8, next day page 9 and that's all there was to it. If you didn't get it, too bad, we'll go on to page 8 [....] It was just the next page in the book, the next chapter and that's all it was (Interview K-1, 11/23/99).

While both teachers described their previous teaching in terms of a traditional paradigm, Josh actually used the word "traditional" many times in each of the four interviews to describe his early teaching. After their involvement in the CMP piloting and professional development, however, they both said that they had changed: (1.) how they thought about the mathematical content, (2.) how they defined their role as teacher in the class, (3.) how they thought students should be engaged to learn[6].

Mathematical Content

There were at least two ideas related to mathematical content that appeared important in the interviews with the two teachers: (1.) "big ideas" and how these influenced their teaching, and (2.) the role of solution methods in the classroom.

Big Ideas

Both Josh and Karla talked often about "big ideas" in mathematics and in algebra in particular. Karla said those big ideas changed her definition of algebra: "I see algebra now more as a study of relationships and different ways of looking at things and how they change" (Interview K-1, 11/23/99). The "big ideas" repeatedly appeared when the teachers talked about how they made their teaching decisions.

Karla indicated that the big ideas helped her and her students from getting lost, as well as helped connect one day's lesson to the next:

> I try to look every day at what's this problem supposed to be about because the kids and I can get lost in the text of the problem if it's about bugs or frog jumping. I can get lost in that versus it's supposed to be about looking at slope this time or looking at something going down and coming back up again [....] I know when I lose the big idea of what that problem is supposed to be about or even what the whole investigation is supposed to be about, we can go off on tangents and then everybody's confused [....] We're focused on what this math idea was then the next day's big idea could connect with the one before it (Interview K-2, 12/17/00).

Karla also talked about how the big ideas kept her questions focused on the goals that she wanted to achieve mathematically. Josh seemed to use these in a similar manner; however, his focus was on keeping his own purpose defined:

> [...] before you start a lesson, before you start a particular problem you really want to say, "Okay, what now, what do I really want to get out here?" I mean, what do I really want the kids to take away from this? What do they really have to have a firm understanding? And that sounds real simple, but with CMP there's sometimes so many other ideas that are, that come up with that particular problem, sometimes you get lost and you kind of forget what the real purpose was (Interview J-2, 12/17//99).

Before being introduced to mathematics in terms of the "big ideas," both Karla and Josh talked about the textbook setting their pace. Now Karla liked the flexibility that focusing on the big ideas offered to her teaching. Prior to thinking about mathematics in this manner, she had scheduled her teaching to move page by page, day by day.

> [...] it's flexible enough that if there are certain parts of a unit that we need to slow down on, we can or certain things can speed up [...] how we progress on the big ideas changes from day to day almost and I don't have that expectation anymore because I said we're going to be on page 7, we're going to be on page 7 (Interview K-1, 11/23/99).

Solution Methods

As I mentioned earlier, Josh's biggest reported change was located in the fact that he now saw that there were multiple approaches to solving mathematics problems and believed that these were important to student learning of the material. He did state that his teaching methods were still "fairly traditional"; however, the big change for him came in the way he thought about the content and that directly affected the way that he taught.

> [Students] kinda know that there's more than one way of solving a problem and that we're looking for multiple strategies, multiple ways of writing solutions and that we want them to talk and explain how we got those solutions. So, I think them knowing that coming in made my job a whole lot easier and I just try to facilitate that idea a little bit [....] I try to ask some of the right questions, although, many times I found my questions were sometimes leading, maybe too leading, but to me that was the way (Interview J-1, 11/23/99).

Karla originally chose to teach math because "it would be easy because there is a right and a wrong answer and that's all there is to it" (Interview K-1, 11/23/99). However, she also now believed that there was more than one way to tackle a problem—"it's not just cut and dry anymore" (Interview K-1, 11/23/99). She stated that she still believed that there was a "right answer, [but] the way kids go about the answer or the way they explain the answer can be so much different" (Interview K-1, 11/23/99). This change made her job as a teacher much harder because she had to keep track of everything students were saying.

In addition to using the "big ideas" in their teaching and addressing multiple ways for solving problems, both Josh and Karla indicated that they, themselves, had learned a lot of mathematics while using the CMP textbooks[7]. They both said that the textbooks helped them to understand some of the why's and to see some of the connections. For example, Josh said that he had just taken for granted that division by zero was undefined. After teaching the CMP curriculum, he now knew why it was undefined.

Role as Teacher

Although both Josh and Karla recognized that many of the changes in their teaching resulted from the more innovative curriculum, they were also aware that this alone was not enough to change what was happening in their classrooms. They recognized that they needed more to change their 'traditional' teaching:

> It was probably two or three years after we had started to look at curriculum differently when we started thinking, "Oh, my gosh, we can't just do a curriculum differently without the teacher being different!" (Interview K-2, 12/17/99).

> MGMP was such a gigantic step and huge change for me, the teacher talk at that time I thought was crucial. If you would've given me [...] just the student materials, I probably would've butchered that. I would've taken that and taught it as traditionally as possible (Interview J-4, 1/07/00).

Josh described this "teacher-talk" as being most influential in changes in his teaching. When I asked him what he thought this had done, he replied:

> [...] the teacher talk was a great benefit because that was really the first time when I really started pushing the 'why's' and getting at alternative strategies and really developing a true understanding and asking the right kinds of questions [....] [MGMP] gave big ideas, just like in CMP. So you kind of knew going into it where you want to eventually go, but it would literally give you questions, sample questions to ask. And, they would list possible student responses of which there were many and it would ask-if the student was looking at it from this point, maybe ask this question (Interview J-4, 1/07/00).

Although Josh recognized that questions might help him get students thinking about alternative strategies and reasoning, this aspect of his teaching was difficult because he "many times found my questions were sometimes leading, maybe too leading, but to me that was the way" (Interview J-1, 11/23/99).

Karla described her biggest change as resulting from the collegial interactions the piloting process had offered her. In particular, Karla was influenced by two of her colleagues, Marcie and Tammy. She described her interactions with Marcie as:

> infiltrat[ing] my life and making me think about things and reading things and working with other colleagues [...] and starting to think "Oh my gosh, I don't know what these kids know!" Then I started to worry about the fact that there was more than just the answer. Then I started worrying all the time about do they really know the answer or are they just copying what I'm doing? (Interview K-1, 11/23/99).

She also said knowing this had changed the way that she taught because she now tried to listen to students more and base her lesson on their ideas:

> [...] before I felt like I had to be the all knowing, all powerful, I know what's best for you [...] just watch me and do this again, you'll get it [.... Now] I listen to them say things like, "But I traced the graph and got that y-intercept," that makes me go, "We need to talk about y-intercept some more" (Interview K-1, 11/23/99).

Following observations, Marcie and Karla would sit down and talk about the class session. Marcie would ask Karla questions: "What was your mathematical goal today? What do you think the kids learned today?" Karla said she would respond, "I don't know what they learned today. We were supposed to do blah, blah, blah." When Marcie pointed out that Karla was really talking about what they were "supposed" to do, Karla replied, "Wow, I never really thought about it that way" (Interview K-2, 12/02/99).

Not only did Karla begin to think more about what students were understanding and what she might do with that information, but she also began to realize that she needed to let students have more control:

> I knew I had to change. I couldn't just answer all their questions. I still do, but I try not to say, "You're right, you're wrong—the answer's 36, now let's go onto the next one." It's, "Why do you think so?," "What do other people think?," I try not to be the overall judge all the time. I try to create an atmosphere where they'll do that. I don't always, but that's my goal [....] If I'm the all-knowing and I give

> all the answers, they don't feel like they have to do anything (Interview K-4, 1/07/00).

Josh had a similar "eye opener" while discussing student work at some of the district project meetings. He said he recognized that his students were not doing some of the things that other teacher's students were doing made him realize he may have been limiting them. When I asked him how this influenced what he did in his classroom, he mentioned that he began to listen more carefully to what students were saying:

> I think I was a little more willing to listen. Sometimes if a kid starts explaining how he or she is looking at a problem, when they first start talking and showing what they're doing, it makes absolutely no sense. But, if you give them a chance to really go on about it [...] down the road you get a little glimpse of how the person is thinking. And often times, it'll make sense and if it doesn't at least down the road if you really listen you'll see [...] where the kid really is on track and then you maybe can take what they know and kinda either develop it or refine it some way (Interview J-3, 12/17/99).

Engaging Students to Learn

Karla said that her increased attention to listening to students had created a different view about student learning for her. It made her believe that students needed to struggle through some of the mathematics to understand it. This realization seemed to be more prominent in the context of pacing: that is, when Karla talked about being rushed and having to present information more, she noticed that students did not seem to understand the mathematics as well because they were not engaged in it in the same way:

> [...] when I fall back [into] "Do this and do this and copy what I have on the board," and then I give a quiz and no one knows what they are doing anymore [....] I actually end up losing in the long run. I know that there's been times when [...] I felt like I did too much of it. I wrote it all on the board and, "Okay, write it in your notes." [...] if I ask them tomorrow, I don't know that anyone would be able to do something with a new situation [...] because I did it and they just watched me do it and I know that it's not as powerful as when they struggle through it themselves and become part of it (Interview K-1, 11/23/99).

One of the ways that Karla encouraged her students to talk more by not requiring them to raise their hands because she did not want to suppress their ideas:

> I don't have a, "You must raise your hand and I decide who talks" thing. I'm afraid of stifling an idea when it's coming out. I try to set it up that when they are doing something and an idea comes to you, you can get it out on the table. I try to make it so they can disagree. I'm always asking, "What do other people think about that?" without myself saying, "I don't agree with this because." I don't want them to wait for me to say, "Yes, it's right" or that there is *the* correct answer [....] I have an expectation of, "Okay, three's the answer, why is it the answer? Where did you get three? Can you explain to somebody else how it's three?" (Interview K-1, 11/23/99).

Because Josh now believed that there were multiple solution methods for mathematics problems, he recognized that this directly affected how he engaged students in learning. For example, he now had students working in small groups, focusing on generating multiple solution strategies:

> Twelve years ago I never would have dreamed of having kids work in groups of three or four for an entire classroom period on a set of problems. And, I look back on it and I can't imagine why I wouldn't have done that (Interview J-1, 11/23/99).

While Josh thought working in small groups was "wonderful," he also stated that he "still tried to keep a pretty good handle on the direction [...because] I've always felt that I had to have control over the classroom" (Interview J-1, 11/23/99).

Small-group interactions led Josh to see how powerful co-operative learning could be. Early in his involvement with CMP, he first observed a "truly gifted student" and students who struggled with the mathematical content both benefit from working together—"[it] opened my eyes that kids with multiple abilities could do well in this type of curriculum. That not only could they do well, but that the strength of the curriculum was having heterogeneous grouping" (Interview J-3, 12/17/99).

Karla had a similar experience that changed her beliefs about tracking students.

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Prior to CMP, she had been a proponent of tracking:

> I used to be a huge tracking person. In my second hour[8], I would have never said that they could attack algebra a few years ago. I would have said that they still need to work on their multiplication facts and drill those fractions and I would really be pushing for those low level kids—don't even let them attempt algebra [...] It is the most incredible class, listening to the way they talk to each other and things some of these special ed[ucation] kids say about algebra-it just blows my mind. I think a lot differently about what kids can do (Interview K-1, 11/23/99).

Not only did Josh and Karla change how they thought about mathematical content, teaching and learning, but they also now believed that all students could benefit from working together on mathematics and that all students could gain access to higher-level mathematics without being "fast" with basic facts (e.g. adding, subtracting, multiplying and dividing).

Summary

Josh and Karla grew up in the same state in which they both received an undergraduate degree in elementary education. While Josh began his teaching career outside of the state and worked on his Master's degree in elementary mathematics education, they both ended up with the same certification that allowed them to teach K-8 in any subject matter and K-9 in mathematics and science. They both came to the Payson School district in the same year and taught in the same two schools there. At the middle school, they both were heavily involved in piloting and adopting the Connected Mathematics Project in similar ways. While teaching this curriculum, they attended many of the same professional development activities that helped them think about changing their teaching practices to become more aligned with the NCTM *Standards*.

These activities and experiences changed the way Josh and Karla thought about mathematical content, the way that mathematics should be taught and how students shoul

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should be engaged in the classroom. Both now saw the mathematical content as being made up of "big ideas." While these big ideas guided teaching decisions and helped focus classroom activities for both teachers, Karla also described them as allowing her more flexibility in her pacing. She could now base her pace on what she heard students say instead of moving page-by-page. The biggest change for Josh related to content was that his perception about what a mathematics solution could look like had changed. He firmly believed that multiple solutions were important and this influenced much of what he did in his classroom. Karla also believed this to be true and no longer saw mathematics as being "cut and dry."

Even though the curriculum had an impact on their teaching, both Josh and Karla recognized the fact that they also had to think differently about their role as teachers in the classroom. For Josh, thinking about teacher-talk had led him to think harder about the types of questions he asked in the classroom and how this might be related to student understanding. He recognized that this was hard to do and that many of his questions were maybe "too leading." However, he also believed that this type of control was important to keeping the conversation focused on the mathematics.

The collegial interactions with which Karla was involved made her think differently about her teaching. She began to listen to students more and use that to guide her teaching. She also recognized that she needed to change her role in the classroom and not always be the "all-knowing." Instead of telling students they were correct, she began to use questions, hoping students would begin to judge solutions and ideas.

This listening to students led Karla to believe that students needed to struggle with the mathematics to "truly own it." She recognized that whenever she moved back into presenting information, students did not understand the mathematics in the same way. Josh's belief in multiple solution methods led him to incorporating co-operative learning in which students would engage in finding and discussing multiple solutions to problems. Josh found this powerful for students, especially when students of multiple abilities were assigned to work together.

The idea of having heterogeneous groups was advocated by the NCTM *Standards* as well as the CMP curriculum. Both Josh and Karla believed that one of the strengths of the curriculum was having students of varying abilities work together. In fact, it had changed their view about tracking and about what students needed to know to engage in more complex mathematical ideas. They no longer believed that student had to be "fast" with mathematical facts to move on to more abstract ideas like algebra.

One may think that two such similar teachers who have had so many similar experiences and were teaching such similar populations of students would teach very much alike. However, as a participant-observer in the classroom, I felt that there were differences in the classrooms. In the next two chapters (6 and 7), I describe the most prominent discourse patterns that occurred in each classroom. While doing so, I claim that certain norms were embedded in and carried with these patterns because they were so pervasive. In Chapter 8, I use these analyses to discuss some of the differences that existed, including issues of authority, teacher positioning and the ways in which each teacher drew from and used other sources of knowledge (i.e. the textbook and students) in the classroom.

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[1] MGMP is the predecessor to the CMP program.

[2] I would like to remind the reader that the teacher-interview data was gathered the year following the classroom-observation data.

[3] This workshop is directed at teachers who are experienced users of CMP.

[4] This was a group of teachers, administrators, university professors and graduate students who received Eisenhower funds to meet and discuss change in the teaching and learning of Algebra.

[5] The principal at Payson told me this in a casual conversation during one of my visits to the school.

[6] Although I recognize that these are intimately related, I will address each separately in the following sections. There will be some overlapping ideas across the three.

[7] I do not have a similar example for Karla. This claim comes from casual conversations I've had with her.

[8] The class to which Karla is referring is her second hour 8th grade math class during the 1999-2000 school year and was not part of the classroom observation data.

CHAPTER 6: KARLA'S CASE

In this chapter, I offer a case study of Karla's classroom, discussing the three main teacher discourse patterns apparent in Karla's talk[1]. These include: (1) classroom talk and mathematical contexts used to create and define "common knowledge," (2) metacommenting, and (3) expressions of uncertainty (including modal verbs and expressions used for politeness and hedging). In each segment that follows, I describe the tool or concept of discourse analysis that I have employed and offer examples and interpretation of Karla's use of them. I show how Karla creates an environment where continuity and context are discussed and preserved, talk about talk and actions are prevalent, and discussions are often fuzzy and uncertain.

As I elaborate on the discourse patterns, I also claim that certain social and sociomathemathematical norms are embedded and established in the classroom through these discourse patterns. Before proceeding with the discourse patterns, however, I present a larger context of the classroom sessions by describing both the activities and discussion that took place[2] in each of the two focus class periods.

Focus Class Periods

October 15, 1998

During this class period, Karla first returned the Check-Up given after Investigation 1, which students completed a day earlier. She proceeded by going through the correct answers with students. In some cases, Karla gave the correct answer or solution; in others, she asked students to talk about answers/solutions they generated.

Karla introduced Investigation 2 as a similar bridge activity to the one students had done in Investigation 1. However, in this Investigation, students looked at the relationship between the length of the bridge and the breaking point. In this experiment, the relationship that resulted was non-linear[3]. This was the first time students were given a non-linear situation to think about in class. Karla reminded students of the last bridge experiment they did in Investigation 1 and then discussed the differences in this second investigation (e.g. that they were going to be changing the length of the bridges, not the thickness). Students got into small groups and collected and recorded the data in tables.

When students finished collecting their data, they were asked to enter their tabular values into a large table on the white board in the front of the classroom. Once everyone's data was entered into this table, Karla discussed the variation in the data that was written on the board and pointed out some of the differences in data collection strategies that she, herself, had observed. She then posed a question about which average would allow students to even out the errors in their data—the mean, the median, or the mode: that is, which type of average would take the extreme high and low values and level them out. After they decided that they should find the mean, Karla assigned this task as homework. During the next class period, this mean data was discussed.

October 20, 1998

The previous day, students were asked to complete problem 1 and 3 in the ACE[4]. In these two problems, students were asked to think about the price of cookies at Betty's Bakery. In problem 1, students were given three possible plans for increasing the price of cookies to help the bakery create a profit, but that would not shock the customers. They were asked to generate and compare a table and graph for each plan, decide if any of the plans might be linear, and make a recommendation as to which plan Betty should incorporate. In problem 3, students were told that cookie sales had fallen and the business consultant who was called suggested that they conduct a customer survey to decide how much to raise the price. The survey data was given: students were asked to make a graph model and to predict the number of customers who would be willing to pay various prices for the cookies. Students were asked to find another graph from this unit that was similar to the graph model they drew. Karla began this class period by going through these previously assigned problems.

After this, Karla introduced Investigation 3.2 by modeling the problem in front of the class, setting up eight glass beakers, the first of which was filled with water. Karla first modeled the experiment without telling students how many ounces of water there were in the first glass. She described pouring half of the water into the next glass and then taking that glass and pouring half of its contents into the next glass and so on. However, she did not actually pour any water.

Next, she made the experiment more specific by telling students that there were 32 ounces of water in the first glass and proceeded to actually pour the water from one glass to the next. Students orally described the contents by telling her that she had 16, then 8, then 4, etc. ounces in each glass. After Karla poured the water into the last glass, she told her students that they should talk about what was happening. A class discussion followed about whether this data was linear or not and why students thought it may be one or the other. Students graphed the data and discussed whether they thought the points on the graph should be connected or not. Karla asked students how they would describe the data to someone. In concluding the class discussion, Karla reminded her students about what they had done so far, i.e. that they had looked at linear, "kinda" linear, and models that represented the data. She assigned them more ACE problems (#4-7) to work on in class. Students worked on these problems in small groups until the class session ended and their homework was to finish any problems they had not completed.

The Case

The case study that follows is broken into two main parts. First, I offer an analysis of the three discourse patterns that were most prominent in Karla's classroom talk. These three patterns include Karla's use of: 1.) classroom talk and mathematical contexts to create and define "common knowledge," 2.) metacommenting, and 3.) expressions of uncertainty (including modal verbs and expressions used for politeness and hedging). In each segment, I describe the tool or concept of discourse analysis that I have employed and then offer examples and interpretation of Karla's use of them. I show how Karla creates an environment where continuity and context are discussed and preserved, talk about talk and actions are prevalent, and discussions are often fuzzy and uncertain.

Within the summary of each section, I highlight how these patterns work to control the common knowledge in the classroom. As I elaborate on the discourse patterns, I also claim that certain norms are embedded and established in the classroom by these discourse patterns. I contend that the following norms are embedded within the discourse patterns I describe:

- Through Defining Common Knowledge:
    - This classroom is a community with a common history; and
    - There are connections in both the content (especially in terms of the multiple representations that are discussed) and the contexts that must be recognized.
- Through Metacommenting:

- Plurality should be endorsed and acclaimed;
- Contributions will be valued and credited;
- Mathematics is in the mind of the beholder;
- Uncertainty is something that should be acknowledged;
- The teacher has the right to summarize and contribute as she sees fit; and
- Everyone should hypothesize the possibilities with respect to the mathematics.

- Through Modal Verbs /Expressions and Hedging
    - The big ideas are stated and people must take responsibility for their own learning;
    - Mistakes are okay to make (or I'll protect you); and
    - There are relationships beyond what has already been discussed and explored, but the ideas for thinking about them are similar (e.g. you still need to ask the same questions of yourself when you are trying to decide what is happening).

In the second part of the chapter, I begin with the norms. I draw on the patterns of discourse to make inferences about the social and sociomathematical norms Karla established for her students in terms of expectations, rights, roles, and responsibilities. In addition, I draw from the teacher interviews to offer Karla's perspective.

Discourse Forms and Functions

Context and Continuity

As I stated in Chapter 2, Edwards and Mercer (1987) consider what it means to establish "common knowledge" in the classroom. In their discussion, they highlight the importance of context and continuity in developing common knowledge. They argue that context is best thought of as mental rather than either linguistic or situational: that is, a characteristic of general understandings that exist between people who communicate. *Context* is referred to as "everything that the participants in a conversation know and understand, over and above that which is explicit in what they say, that contributes to how they make sense of what is said" (p. 63). The context includes not only spoken language but also meanings and interpretations of what is said. *Continuity* is defined as "the development of such context through time" (p. 63). For teachers and students to be able to understand each other and develop a shared conception of their undertaking, according to Edwards and Mercer, it is acutely important that they can relate discourse to context, and, over time can build a joint frame of reference. This shared conception is the "common knowledge" of the classroom.

The Teller of Stories

One common thread that ran through Karla's classroom discourse was her reference to occurrences located in past classroom activity. When doing so, she often used general temporal back-references such as "yesterday" or "last year" or prompted memories of common classroom occurrences by saying "remember." A second strategy she used to replay a common memory was to refer to past problems from the text specifically by the name or activity of the problem. For example, she would refer to "Chantelle's birthday money" or "Betty's bakery." All of these strategies played a role in establishing and formulating what could be considered "common knowledge" in this classroom and played a role in developing the context and continuity of the activity.

"Yesterday," "Remember": This classroom is a community with a common history. In the more general use of Karla's language, especially when she referred to what happened the day before, there was at least one striking similarity. In each example, Karla summarized what "we found." For example:

> ***Example K-1***
>
> K: So it looks like it's going up six cents this time. **Yesterday** we did that gigantic chart with Chantelle's birthday money and **we found** out that 8% was not the same as 8 dollars, so last night when you were looking at this, it was the same idea, that 5% is not going to be the same as adding 5 cents every time. Is she going to reach a dollar eighty sooner or later?

Following this statement (and others like it), students never responded, "No we didn't"—indicating that they were willing to go along with the memory that Karla replayed as she portrayed it. In making this statement, Karla was doing a number of interesting things. First, she was making temporal reference in a narrative manner, thus posing this as a story to be told rather than facts to be given. She used ways of talking that are valued often outside of schooling, rather than "empirical verification and logical requiredness" (Bruner, 1991, p. 4), which is more often used in mathematics classrooms (and in schools in general). Second, her use of the royal plural "we" tacitly does not allow students to disagree with what she was saying—they must accept the findings as she stated them (Rowland, 1992). The use of 'we' suggests a "joint understanding in which there was no difference between the teacher's intentions and interpretations and those of the pupils" (Edwards & Mercer, 1987, p. 141).

Third, Karla reminded her students that, mathematically, determining an 8% increase was not the same thing as constantly adding eight every time. Karla was referring back to the fact that these two ideas were not mathematically equivalent and that students must remember this when they moved on to a similar problem—5% vs. 5 cents.

In addition, she made a connection from the past (8% increase was not the same as an $8 increase) to a more current problem (5% increase is not the same as a $5 increase).

Another striking feature of all of the examples similar to the one just given were that, similar to Edwards and Mercer's findings, almost all occurred in the context of difficulty. By "context of difficulty," I mean that they appeared at a time when students seemed not to remember something that the teacher possibly considered "common knowledge." In this case, students did not remember that taking a percent increase of a number was not the same as increasing at a constant rate. These back-references appeared at "moments when the pupils seemed not to have grasped some significant principle, procedure or instruction that had been dealt with previously" (Edwards & Mercer, 1987, p. 85). However, Edwards and Mercer described these mismatches in communication as an important part of the learning process. The mismatches were described as occurring in Vygotsky's 'zone of proximal development', "at precisely the point at which common knowledge is being created" (p. 84). The adult's role in this process was described as taking a leading role in "drawing attention to them, talking about them, establishing knowledge which is both common and communicable" (p. 84).

This type of difficulty also occurred when Karla used the word "remember." For example, in one class session, the class discussed whether they should connect the points (which they called "dots") on a graph that was curvilinear (i.e. increasing exponential in this particular example). In doing so, a student asked if he should connect the points using straight-line segments rather than the curves that she had drawn on the board. Karla might have realized that the student did not understand that the curve was not only representative of the entire relationship, but that it also occurred between each of the points they had graphed as she responded:

Example K-2

> K: Not necessarily. We always assume that, um, even on our constant rate problems. We always assumed, like in between this dot and this dot, you would connect them like that. But **remember** in *Variables and Patterns*[5], we looked at, we could connect dots this way [drew a straight-line segment] or this way [drew a curved segment]. We looked at times like when you were riding from this point to this point and this is showing that they are riding at a constant rate all the time [drew a straight-line segment between the points]. This is showing where they kinda started out slow and then took off [drew a curve between two points--an increasing exponential]. This is showing where they kinda started out fast and then slowed down [drew a curve between two points on a decreasing exponential]. We've always connected with straight-lines 'cuz we've assumed that constant rate is going to stay there. But, in this case we know there isn't a constant rate, so really that curvy makes more sense.

In this turn, Karla referred to two different types of relationships that they have explored: linear and exponential. She reminded students of these two types by referring back to an experience they had the year before in *Variables and Patterns* and continually used "we" to portray the experience as something that they developed and found jointly. In *Variables and Patterns*, there were extended opportunities for discussion about whether students should connect the points on a graph or not. This was typically determined by whether the data was discrete or continuous based on the context (e.g. if they were graphing time as the independent variable, it would be continuous because the measurement was continuous, so students would be expected to connect the points they graphed). However, since *Variables and Patterns* only discussed linear relationships, students had not grappled with how to connect the points when they appeared to be curves instead of lines. Karla seemed to be reminding them that the reason they connected linear functions with straight-line segments was because the rate was constant, so the slope would remain the same throughout the entire function. When connecting points in a curvilinear function, students needed to attend to how the rate was changing—in the first example Karla gave, the rate "started out slow and then took off" and in the second, the rate "started out fast and then slowed down." Even though she was not making the explicit connection between the change in rate with the physical appearance of the graph, she seemed to be connecting these two ideas in her examples.

These explicit references to past experiences seemed to function as an assurance that students had developed a joint understanding with the teacher of particular ideas that the teacher thought were significant and applicable to what was currently being discussed. In some ways, these were understandings that were foundational for students to be able to do what the teacher was asking them to do: that is, many were implicit and presupposed knowledge that Karla may have been using to take students to the next step mathematically (e.g. moving from drawing lines to drawing curves).

"Betty's Bakery," "Chantelle's birthday money": There are connections in both the content and the contexts that must be recognized. In a similar manner, Karla often referred to past problems from the text that the students had engaged in by the name (e.g. Betty's Bakery problem) or by the activity (e.g. the stacking bridge problem) in which they had engaged. In these more specific references, there were at least two things Karla would do. In the first, she would make the past the present in such a way that students could bring the mathematical ideas from the past conversations to the present discussion. For example:

Example K-3

K: Let's back up a minute. I'm hearing something that we tried to discuss yesterday, but I think we need to discuss it again. Um, we did **Chantelle's birthday money** yesterday and it was increasing by 8% each time. And it was found that we'd forgotten what to do to find a percent of a number. So, if I want to find five percent of this and then add it on, like we were doing with sales tax and stuff, how do I find five percent of a dollar? Samantha?

This reference to "Chantelle's birthday money" came after students repeated a difficulty that Karla had thought they had tended to the day before. In the previous day's discussion, students struggled to remember how to find the percent of a given number. By reminding students exactly which problem and of the exact numbers they had discussed, Karla was increasing the likelihood that students would recollect the previous discussion and the content involved in that discussion. Similarly, she more generally referred to "sales tax and stuff" that they had done previously, helping students connect to other mathematical ideas in which similar processes were used.

When students continued to display difficulties with the idea that increasing by a percent was not the same as an additive increase, Karla again repeated this "common knowledge" that she thought students should have acquired. She even ended her turn with a reference to what "we thought," tacitly capturing students in agreement with what she was saying. Furthermore, by posing these as thoughts, she modeled what they should be thinking about by this point in time.

Example K-4

K: You have to do it this way because it's not the same pattern as this one. This one was increasing five cents, this one is increasing five percent. And remember, with **Chantelle's birthday money**, we first wanted it to go up eight dollars every time. Then we thought, "Gee it's much better to go up 8% every time, she's going to have a lot more money in her account."

Another function this referencing of problems seemed to have was to allow students to make connections across the content, signaling them that not every day in class was in isolation of the others.

Example K-5

K: So it looks like it's going up six cents this time. Yesterday we did that gigantic chart with **Chantelle's birthday money** and we found out that 8% was not the same as 8 dollars, so last night when you were looking at this, **it was the same idea**, that 5% is not going to be the same as adding 5 cents every time. Is she going to reach a dollar eighty sooner or later?

In fact, some of the classroom activities engaged students in actively finding problems that they thought were the same and writing about the similarities they observed. Here, Karla described what they were expected to do:

Example K-6

K: After you've gone through all your graph models, then you want to refer back to the problems we've done and if you've drawn a graph model that you thought did **something like this**, you have to look through all the problems that we've done and write down a problem that might be **similar to that**. Did we, have we done any that have swooshed down like this?
Ss: Yeah
K: Like, which one?
Sam: Like the one we just did
K: So, if I had one that had that kind of a trend to it, I'd write down the problem 3.2 did that, also. Or the water problem did that type of thing. So, you want to go back through all of your problems and **find similar ones** if there are any for the models that you draw in ACE number four. And then ACE 5, 6, and 7, give you situations, Betty's bakery comes back [...]

In referring students to common past experiences (and often naming them), Karla may have been attempting to form a continuous experience for her students. She was also establishing that these connections were things that students should be thinking about and addressing. This "habit of mind[6]" appeared to be grasped by some students because there was evidence on tape and in my field notes that students began to engage in this type of referring-back talk. For example, in one class period they were discussing a problem and Xavier offered, "This is like the Pizza Pirate problem"—referring back to a problem they had engaged in the year before. In short, Xavier adopted Karla's "habit of mind." This type of connected, continuous learning experience can be very powerful. In this case, it appeared that students have internalized this habit of mind. Some students *have* begun to ask themselves where else they had seen various forms of these problems—making connections for themselves and voicing it to the class.

Summary

In this section, I have shown how Karla used general and specific references back to past classroom problems and activities. When she referred back in a general manner, she used temporal references such as "yesterday" or "last year," often employing what Edwards and Mercer called "reconstructive recaps." More specific back-references occurred when she referred to specific problems or activities from the texts the students were using. In these references, Karla brought the past into the present and created and controlled the "common knowledge" of the classroom. By doing so, she developed the context and continuity of the classroom activity through "repeated discourse formats." This was important to student understanding because the common knowledge was made an object to talk about and was made less implicit to students.

This aspect of Karla's talk, as I mentioned in Chapter 4, was similar to the textbook in that it also stated what was "found." However, the textbook usually stated "you found" and Karla usually said, "we found." This difference in pronoun usage may have pointed student attention to the joint construction of the ideas. The textbook, like Karla, also referred back to past problems and reminded students what they had done in previous investigations, possibly in an attempt to form a continuous experience for students.

As in the next section on meta-commenting, Karla was weaving together certain aspects of the classroom that she saw important for students to attend to. Here she seemed to be shaping the common knowledge in terms of the text and the specific context of the classroom in which she was working based on her knowledge and memory of activities and conversations that occurred in the classroom. This was another way to create a continuous experience for students and to draw on multiple texts to do so—the text of classroom discussion and activity as well as the problem situations CMP units offered throughout the three years of the curriculum.

Meta-commenting

Meta-commenting has been described as a characteristic aspect of classroom discourse (Stubbs, 1975). In meta-commenting, a teacher takes student utterances and treats them as appropriate items for comment themselves. Stubbs connects this idea to that of "formulating" (Garfinkel & Sacks, 1970) in which:

> A member may treat some part of the conversation as an occasion to describe that conversation, to explain it, or characterise it, or explicate, or translate, or summarise, or furnish the gist of it, or take note of its accordance with rules, or remark on its departure from the rules (p. 23).

Pimm (1994) points out that this strategy is appropriate for teaching. In one example, he discusses how the teacher monitors what is going on and attends to whether students in the class can hear what others are saying through her meta-comments. The meta-comments that the teacher makes include rebroadcasting student contributions. Other functions meta-comments can have include avoiding feedback (so as not to get trapped in the I-R-E sequence) while still taking a turn, putting student's contributions into opposition without the teacher having to decide among them, and drawing the class' attention to the form of a student's utterance (pp. 140-141).

Many of these functions are similar to those served by "revoicing." O'Connor and Michaels (1996) define revoicing as "a particular kind of reuttering (oral or written) of a student's contribution by another participant in the discussion" (p. 71). The functions they explicate include: (1) revoicing for reformulation and (2) revoicing to create alignments and oppositions within an argument. In this particular article, O'Connor and Michaels look at teacher revoicing. This gives rise to two related participation frameworks that they briefly address at the end of their article. These included both student revoicing of one another's comments in classroom discussions in Japan, and the teacher asking a second student to evaluate another student's utterance. In this latter instance, students are challenged to engage with one another's ideas.

When teachers reformulate, they may do so for a variety of reasons including "clarification of content or relevance or the introduction of new terminology for familiar ideas" or "in order to advance her own agenda, changing the contribution slightly so as to drive the discussion in another direction" or "to reach a wider audience than the student reached" (pp. 74-75). Part of teaching students to explain their reasoning includes helping them to articulate their understanding. Teachers may use reformulating to clarify, make connections, or fill in missing elements of a conversation. By taking a student's contribution, the teacher is taking on the role of "animator" (p. 76). Even though the teacher is doing the talking, the student is still getting credit for his/her ideas. Also, even if the teacher reutters or rephrases the student's contribution, the student is allowed to agree or disagree and is given credit for the reformulated version.

When a teacher adds or deletes material from a student's utterance, she may refine, emphasize, or restate aspects of the student's words to position the student in terms of the content and goals she is trying to teach. In Edwards and Mercer's terms, these "paraphrastic interpretations of student contributions" may serve to control the common knowledge in the classroom because they match the classroom discourse more to the goals of the lesson than what was actually spoken. The positioning that revoicing allows can be used to set up oppositions or create alignments with regard to the content and/or regarding other student's ideas.

In summary, revoicing must have two participants, the "inference maker/animator", the "inference source/principal" and the "proposition" (p. 82) which is the reformulated utterance that is spoken or written by the animator. O'Connor and Michaels also include some larger purposes for revoicing based on some classroom excerpts they give: to guide students to consider different bases on which they can refine a random guess into one supported by some sort of evidence; and to rebroadcast the contribution of a student who is being overlooked (which may serve goals for the social organization of the classroom).

Even though the functions for metacommenting and revoicing are similar and overlapping, the forms are where the distinction between them may be made. Metacommenting is a broader linguistic strategy than revoicing. When a teacher revoices, s/he typically restates or reformulates something a student said prior to the teacher's turn. In metacommenting, the teacher's turn does not necessarily need to be associated to a particular individual. For example, a teacher may relay something that a group of people were inquiring about instead of a single person. In doing so, he/she is telling the story of a collective group, rather than one person and might not say, "___ said."

Metacommenting appears to be a larger set of which revoicing is a subset. All of the forms and functions delineated in O'Connor and Michaels' article are encompassed by the ones described in Pimm's (following Stubbs) metacommenting. However, as I will show, the notion of metacommenting needs to be extended to include talk about actions or processes, and not just talk about words themselves.

Pimm (1994) raises the question of how a teacher may go about returning to a normal conversation once the meta-comments are no longer the center of the conversation. He indicates that these transitions "need to be handled smoothly if they are not to be too disruptive of attention and focus, yet not so smoothly as to pass unnoticed, as they are important sites for teaching and, hence, potential learning" (p. 143).

The Narrator of Activity and Talk

One of Karla's most prominent discourse patterns was that she often used meta-commenting in her talk to students. These meta-comments took on various forms, which included saying "some people…other people," "Robby said…," "in our minds…," "you lost me…" and "so, …" (this was typically marked by an elongated sssooo or a punctuated SO, each followed by a pause). In some of these cases, she employed revoicing strategies to make a meta-comment; in others she did not. Some of the functions these meta-comments played include: referring back to past conversations and actions, naming particular students when revoicing what the student said, creating conversations, signaling that there was a break in communication, summarizing a conversation, etc.

"Some people…other people…": Plurality should be endorsed and acclaimed. Karla often referred to past conversations or actions of people. However, she did not state who exactly the "people" were. The various versions of this type of statement include: "many people," "some people," "other people," "a couple of people," "a few people" or similar versions where she replaced "people" with "of you" (e.g. many of you). There seemed to be three functions of these types of statements: to make the ideas/strategies of a smaller group of students available to the entire class, to tacitly let students know that what they were doing was what she wanted them to do, and to tacitly disapprove of something someone said/did in class.

When Karla used these statements to make ideas/strategies available to the whole class, she sometimes replayed what a student(s) did/said. For example, in one class period, Karla handed back a quiz and went through the answers for students. In doing so, she came to a problem where some students had approached her during the quiz to ask her a question. She replayed this story for the whole class:

Example K-7

> K: And then **a couple of people** came up to me and they said, "This doesn't feel right." So then I suggested, "Graph it." So then when they looked at their graph they said, "Oh that has to be right because here's that point two-five. Of course it's going to cross higher than that on the y-axis." And then they said, "Oh yeah it has to cross up there around nine" that it makes sense. A **few people** even stapled a piece of graph paper to their test where they actually put this on a larger sheet of graph paper and they could see their y-intercept.

In this replaying of the conversation, Karla was making a discussion that she had with a smaller group available to the entire class. As teacher, she was in a position of privilege that allowed her to eavesdrop on the groups in the classroom. In a sense, she was acting as a narrator replaying a story. In doing so, she was casting the story as "constructed dialogue" (Tannen, 1989, p. 25), creating complex participation structures that invite higher degrees of involvement on the part of the listener(s).

Tannen believes that this "creation of voices occasions the imagination of a scene in which characters speak in those voices, and that these scenes occasion the imagination of alternative, distant, or familiar worlds, much as does artistic creation," making the conversation more vivid and effective. In using a more narrative form, Karla was creating a story in which the students themselves were actors and had a dialogue with her. By telling it in this manner, students could almost imagine the event as it happened (or create the image because they know the players) and could thereby remember it in a more vivid way.

Another tacit point Karla was making included that students can and should use their graphing calculators to check their work. Graphing calculators have been used in the classroom since the beginning of the school year and here she was subtly pointing to one way that they should be used—as a tool for checking their work[7]. She was also pointing toward the fact that students should think about whether their answer made sense or not. In short, she was using this as an example of what students should do and how they should think when working through a problem.

Karla used replaying also to make students aware of the array of strategies and conversations that were going on in the classroom during small group work. Many of the problems in the curriculum (especially the ACE problems) were done in small groups. Because students did not always discuss the solutions with the entire class, Karla often made explicit what other groups had done or said to make the ideas and strategies available to those who did not have her "birds-eye" view of the classroom. Often when she did this, she used the words "some people" followed by "other people." For example, students had worked in small groups to build bridges as part of Investigation 2.1, described in the beginning of this chapter[8]. After students performed the experiment and recorded the data, Karla led a whole-class discussion about the data they had collected. When doing so, she began by talking about the accuracy of the data collection.

Example K-8

K: You made your bridges a lot more accurate. You did place them on your books a lot more accurately. You were trying to drop your pennies in all the same, but a couple of things that we hadn't talked about but I started noticing as I went to look at your groups, um, what you consider a broken bridge was different from group to group. **Some groups** considered it a broken bridge when it hit your table. **Other groups** considered a broken bridge when it just started to bend. So, that makes a huge difference in how many pennies you could put in there. How you folded your sides up-- **some people** folded sides up so tight it held the cup, **others** barely folded it up, and that might have made a difference. Anything else?

Karla was pointing out the different ways that students defined a "broken bridge" to help students see why there was so much variation in the numbers recorded in the table. There was also a subtle suggestion that in experimentation taking care to define the process was part of collecting good data. The subtext of this type of contribution included the fact that there was no one right approach to the problem. This was one way that Karla allowed plurality into the public discourse. As is the case in this example, some of these were inherently contrastive.

Karla also used these types of statements to tacitly point to acceptable and unacceptable activities. In other words, she discussed things people did or said that were what she was looking for and also for what she thought was unacceptable or not quite what she had intended them to do. An example of the "correct" strategies included:

Example K-9

K: **Most people** I could see had drawn a table off to the side and were trying to use your slope and work it backwards, because you did this [drew a table on the board—x's given are 2, 1, 0 and y was given as 5]. **You** knew that the slope was –2 and we talked about this several times and--in fact yesterday right before the Checkup you gave me some points and it just happened to work out that we had a negative slope, so we had worked through one in the Reflection just before this, too. If the slope is –2, as the x's get bigger, what are the y's doing?

Jonah: Getting smaller

In this segment, Karla indicated what "most" of the students that she could see had done. She reminded them that this was something they had done prior to taking the quiz (so it should be "common knowledge") and then proceeded to go through the problem by modeling the method that she had previously shown them—work backwards in the table (using the slope) to find the y-intercept.

In the next example, Karla made this type of meta-comment, but tacitly disapproved of what students had done.

Example K-10

> K: [Problem] 2A. I recommended using your graphing calculator. I could tell some of you did and some of you did not. They[9] purposely set the equations up that way. The only two that were linear were the first and the last one. The middle two, you had division [meaning the equations were of the form y = c/x where c was some real number]. **Many people** instead in their explanation were saying, "Well because it's division and division and multiplication are related, it must be linear also." However, when you threw those in your GC and hit graph, what did it look like?

In the problem, students were given four equations and were asked to identify which were linear and explain why those were linear. However, in the two middle problems, the equations were of the form y = c/x + b instead of y = mx +b. Karla pointed out that just because students knew that multiplication and division were related did not necessarily mean that the equations were all linear. The "incorrectness" of the thinking was further indicated by her reminding them of what she had "recommended" and told them what they did "instead...," (pointing to the fact that they had done something *instead* of doing what was correct). Furthermore, when she began the last sentence with "however," she marked the fact that if they had checked the equations on the graphing calculator, it would have been clear that the other two equations were not linear.

In each of these examples, Karla brought language forward as an object of consideration, transforming something that was stated into an object to be thought about. Also, in choosing to use the word "people," Karla may have been trying to protect the students who were incorrect by "shielding" (Rowland, 1995) them. In other words, she was protecting students by not explicitly telling them they are wrong in what they did and by not naming who it was who did the things she described. She pluralized the people to whom she was referring either because she did not want to or could not name them all.

"____ said...": Contributions will be valued and credited. Another way Karla would meta-comment included naming who said or did something. This was different from the last section in that she often said "So-and-so said," therefore the person who contributed (or the person who acted) was specifically named. Another way that this type of contribution was different from the last was in its pointing toward ideas that were correct or in the direction that Karla wanted the conversation to go. In the last segment, I described "people" as possibly being a shield to point to things that were not quite what Karla wanted them to be (either in terms of how they were talked about or in terms of the process being used). In naming who said (or did) what, Karla typically pointed only to statements and processes that pushed further the big mathematical idea that was the focus on the conversation. This type of naming frequently occurred in Karla's reformulations of student contributions. She reported in an interview that she often did this to give the students a sense of ownership for their ideas/statements[10].

Example K-11

> Robby: Maybe because they seen [sic] [inaudible] goes up 4, and then 8 [inaudible]

K: Sort of a pattern there, isn't there? **Like Robby said** there is almost a multiplication or division pattern there. Since it isn't a constant rate pattern, it

> makes it not linear. So, it does do something kinda neat that we will look at later in here. Um, your explanation for how you recognize things as linear from a table were very good. Most of you talked about looking for that constant rate or finding what it's going up or down by.

Karla took what Robby said and reformulated it to show that the pattern that he indicated was not the constant rate patterns that they had explored so far. She named the pattern in a more precise and articulate way—it was not just going up by 4, then 8; it was a multiplication or division pattern. In reformulating Robby's contribution in this manner, she "foregrounds a key aspect of the academic content" (O'Connor and Michaels, 1996, p. 79): a multiplication or division pattern did not result in a linear situation because it was not a "constant rate pattern" (i.e. one in which the y-values of a table increased in a constant additive manner as the x-values increased by one). She was also controlling the common knowledge by using paraphrastic interpretations of student contributions that were more detailed and more specific in terms of the content in the classroom.

At other times, Karla repeated something from a group of responses when there was a lot of overlapping talk. She chose a student and gave him/her "voice." This may serve the purpose of "rebroadcasting the contribution of a child who is in danger of being overlooked" (O'Connor and Michaels, 1996, p. 86), as well as taking the conversation in the direction Karla wanted it to go. Many times in the transcripts there was a sea of voices talking after Karla asked a question. In fact, sometimes it was hard to make out all of the contributions by various students. In many of these instances, Karla followed up by pulling out statements from students and saying them aloud to the class, ignoring some student contributions. Often, these statements were ones that were correct and that Karla thought to be most relevant to her goals of emphasizing the "big mathematical idea[11]."

Example K-12

K: Is Samantha right or wrong?

Ss: Wrong/Right [overlapping speech]

K: Wrong? And **Robby said** it was going to be non-linear because it wasn't going down at the same rate every time. **He said** it wasn't going to be decreasing by the same amount every time because every time you cut it in half, you had a smaller amount to take it from. So, I think that, that's showing up on my table here because I can see here there's a huge jump, 16. Here there's only eight. [points to the next value] Four. So, it does appear like it's not going to be linear.

Example K-13

K: So, what's this graph going to do?

Ss: Keep going [overlapping speech].

Becky: Get closer and closer to zero, but never cross it.

K: It's gonna [laughs] keep getting closer and closer and closer to zero, but it's never going to touch it, theoretically. Because, as **Mark said**, I keep cutting it in half, cutting it in half, cutting it in half, cutting it in half.

In each of these cases, the student contribution was not distinguishable on the tape due to the many voices present after Karla asked a question. However, Karla heard many of these students and chose to restate (or reformulate—since the contribution was not captured, it could be either) what one student had said and gave him credit for what he said. In doing so she was shaping the conversation in the way that she thought to be most profitable toward the goals that she had set—to distinguish linear from non-linear situations. She also ignored some student responses possibly in an attempt to maneuver the conversation toward the big ideas in the classroom.

"I think...", "In my mind...": Mathematics is in the mind of the beholder. In some of Karla's meta-comments, she focused on the mind and thinking—i.e. metarepresentational talk (Olson & Ashington, 1993, p. 9). She may have indicated to students that thinking was a focus of what they were doing—that mathematics was mental and thinking was involved.

Example K-14

K: Yeah, so we knew that you had done something to cheat. So we're going to look at Mark's way of doing it. Instead of making thicker bridges, we're going to make shorter bridges and we **already think** if we make a shorter bridge, we're going to need more pennies.

Example K-15

K: Exactly. I didn't **think** about it until I started listening to the different groups. You're right. I had **an image in my mind** of what I thought a broken bridge was, but it was obvious that every group had a different **idea** of what a broken bridge was.

Mark: What's your **image**?

K: My **image** was that just when it started to buckle.

Example K-16

K: At this point, if I **think** about all the things we've been doing in this Investigation, actually in this whole book. Um, we started with the **idea** of a graph model and an equation model where we had data, especially the bridge thickness one **comes to my mind**. It wasn't perfect.

Example K-17

K: I wanted to talk about what you guys were saying about, was there a constant rate or something else going on. And we're going to look at another problem and **decide first in our minds**, what type of graph it might be and then, um, look a little more carefully at it. And here's the situation. I have eight cups over here. And I'll just kinda **think it through for a minute.**

In Example K-14, Karla was telling students that they "already think" something, based on past experience in the classroom. In doing so, she was pointing to something that she believed should be common knowledge by this point: that is, she was making presupposed knowledge explicit. She used this opportunity to tell students what they already think and to point toward the fact that now they were actually going to do this activity (making the bridges shorter).

Example K-15 showed Karla separating her ideas from the small group's ideas and that she had a different definition for what indicated a "broken bridge." In doing so, she pointed out that she had her image and every group seemed to have a different image from hers. Because of this, the data that students collected was not as accurate as it could have been. One student, Mark, took up her idea of "image," asking her to be more specific and explicit with her definition of "broken bridge."

In Example K-16, Karla went back to summarize all of the activities that they had done thus far in *Thinking with Mathematical Models*. However, she began by saying "if I think…" and then continued by talking about all of the activities as things "we" did. By switching pronouns, she may have indicated that these were things that she was thinking about, but activities in which they all took part (e.g. they were all involved in building the bridges and filling the cups with pennies to see when the bridges would break). In other words, she may have been modeling what students should be thinking about, but the activity as something that had jointly taken place.

Karla was moving students forward in Example K-17, beginning with something they were saying that might have been confusing for students, i.e. what type of pattern was in the data. The process that she was advocating was that they "decide first in our minds…and then look more carefully at it." In pointing out that this was the order that they should do it in, she was subtly telling them that when they merely think about something definite conclusions cannot be drawn. In order to draw valid conclusions, students must then look more carefully at the situation. Conversely, she may have wanted students to be aware of the critical need to think before doing something.

Olson and Ashington (1993) assert that this type of metarepresentational talk used by the teacher will lead to student reflection and articulation of their thinking and its expression. In addition, they hypothesize that:

> teachers who make more use of metacognitive and metalinguistic language will have students who do the same, and these students will better be able to understand their own and other's beliefs and intentions: that is, how their own beliefs are held and how others' statements are to be taken (p. 20).

"You Lost Me...": Uncertainty is something that should be acknowledged. Another pattern that appeared in the transcripts included segments when Karla would voice her own confusion. When she did this, she either did it as a pedagogic move or else she was genuinely confused, which triggered this type of pedagogic move. For example, she may have said, "You lost me...." When making statements like this, Karla superficially lessened her role as authority and signaled that there was a break in communication—that students needed to back up and go over something again. This typically appeared when someone introduced a process for doing something that hadn't been explored in class yet or that was different from what other students were doing: that is, it may not be considered "common knowledge" in the classroom. This was indicated by the fact that Karla continued to ask clarifying questions and other students in the room indicated that they were confused through their facial expressions (which Karla picked up on—see "I could tell by your face" at the end of Example K-18).

In the following example, students were given a point (5, 9.5) and the slope (1.5) of a line. They were asked to find the y-intercept so they could write the equation of the line. During this extended transcript[12], Karla made this type of statement[13] many times.

Example K-18

K: The piece that we're trying to figure out is this b. And you said this b was your up-front money or your up-front stuff, or your starting point or y-intercept. Is this [the given point] my y-intercept?

Fs: No

K: No. How do I know this is not my y-intercept? If it was my y-intercept-

Keith: Oh, cuz it'd be the first, the 9.5

Jonah: It'd be at zero

K: Yeah, because if it were, x is zero

[Ms talking under her]

K: I don't know this [pointing to the y-intercept]. I've got to find it. How do I find it if this [the slope and one given point] is all I know?

[...] [Karla addresses a student who is off task]

Becky: You know what you can do? Ms. Delmont, but you can put an equation in the graph and just calculate it out.

K: How?

Becky: If you put y = 1.5x and then go to the table and find where five is.

Ms: Yeah, but then the starting point would be zero.

Becky: No, when five is x, you find whatever y is and then whatever the difference is between y, that y and your other y-

Ms: 7.5

Becky: -is the [inaudible]

K: **Help**. [laughter]

Sam: Come again?

Becky: You put in y = 1.5x in the graphing calculator

K: Okay

Becky: And you go to table

K: Let's do that because **she lost me after that**. But if you're putting 1.5x into your calculator and you know it's crossing at zero, zero

Keith: Ms. Delmont, it's pretty bad when the students have to teach you

K: I know

[Ss laugh]

Ms: Ms. Delmont, you're just playing dumb right now.

Ms: Yeah.

[...] [Students get graphing calculators if they didn't already have one]

K: So, you said put 1.5x in even though you know that's not the right equation?

Becky: Yeah.

K: Okay, then you wanna do what?

Becky: Go to table and look for where five is, it's 7.5, right?

K: Yeah.

Becky: And then whatever the difference is between that one and 9.5 is your [inaudible] or your [inaudible]

K: So, you're saying the y-intercept is two?

Becky: Yep. And then you can [inaudible] 5x + 2 and where x is five

Keith: Ms. Delmont, can you say that in English so we can write that down?

K: **I'm not sure that I understand her.** I'm going to ask if I'm correct. On her calculator--and you can look at it on yours--um, she said she put in 1.5x, even though she knew that wasn't right. That's assuming it crossed at zero, zero. She went down to five and at that equation, it was at 7.5. So, then you took the difference between 9.5 and 7.5? And said the new y-intercept should be 2.

Karla began by asking students how to find the y-intercept if they were given the slope (1.5) and a point (5, 9.5) that was on the line. Becky first offered her strategy—one of using an estimated equation and the graphing calculator to come up with the y-intercept. From my classroom observations, there was no indication that this strategy has been taught in prior class sessions—that is, this would not be recognized as a common way of finding the y-intercept. Karla also indicated this when she said, "Help." This was a sign to Becky that she needed to back up and explain what she was doing.

After repeating her method a second time, Keith requested that Karla "say that in English." In asking Karla to do this in this manner, Keith may have indicated that he perceived Karla's interpretation of Becky's method as important to his ability to understand it. Furthermore, his request that Karla say it in "English," may have indicated his awareness of the difference between his own understandings and Becky's. Also, he may have believed that part of Karla's role in the classroom is to act as interpreter of mathematical processes, that her interpretation was important to his understanding of the topic at hand. Karla played this role through re-describing what Becky said and asked students to follow along on their own graphing calculator. In doing so, she reformulated what Becky said and tried to add more detail to her method. She then called on another student to help translate Becky's method.

Example K-18 (continued)

K: Xavier, **help us out** here.

> Xavier: You do the same thing, except take 1.5 and times it by five and then divide that by [inaudible]

K: Why are you doing that?

> Xavier: Cuz it's the same thing. What she did was she found what 1.5 was or 5, wait now. [inaudible] Then, she divided, then she took away [inaudible]

K: Oh, that's what she was actually doing?

Xavier: Yeah, but from the graph and the table instead of [inaudible] not the equation.

K: Why is she multiplying it by five?

Ss: Because [overlapping talk]

Becky: [inaudible]

Xavier: That's what it would be originally.

Becky: And we were trying to find when y, or when x is five, whatever y is.

K: Okay. Why are you multiplying it by five, though?

Becky: You're figuring out if you have an up-front, what it would be when x is five.

Xavier: The five would be by itself and then take away what's the x. The 7.5 by itself and 9.5 and it would be 2, so it'd start at two.

K: So, this is like you took away five groups of 1.5?

Xavier: Yeah, [inaudible]

K: Right. **I don't know, what do you think** [asking other students in class]?

When Karla still did not understand what Becky was doing, instead of using revoicing further, she called on Xavier (one of the more knowledgeable students in class) to act as interpreter for Becky, relinquishing her role as "interpreter." When doing this, Karla called on Xavier to explicate Becky's method and reasoning, drawing him to interpret the thoughts of another. Xavier described what he thought Becky was doing, which seemed to make more sense to Karla because she then began to ask more specific questions (e.g. why they took away five groups of 1.5 rather than some other number).

In Goffman's terms (1974, 1981), Karla was inviting Xavier to "animate" Becky. When explaining Becky's reasoning, Xavier was taking on the role of Inference Maker (i.e. making inferences about why an interlocutor said or did something). This was the "converse of externalizing one's own reasoning" (O'Connor and Michaels, 1996, p. 91). Both learning to elaborate one's own reasoning and that of others were "necessary activity within collaborative intellectual work" (p. 91). As in the excerpt of Victoria Bills (O'Connor and Michaels, 1996, p. 90), Karla was "skillfully bringing students' thinking in contact with each other" and into contact with her own thinking and ideas. She extended an invitation to the class to judge the validity of the strategy by asking them what they thought about Becky's method.

After receiving little response to this request, Karla did not seem convinced that the method made sense to everyone. She indicated this by calling on Samantha, whom she thought looked confused. Samantha confirmed her perception by shrugging her shoulders. Karla then redirected, drawing on "common knowledge"--what students already knew about a y-intercept—possibly because this presupposed knowledge made things "nicer and neater" than the alternative possibilities Becky had suggested.

Example K-18 (continued)

K: Samantha, what do you think?
[Samantha shrugs]
K: I could tell by your face. [laughs] Let me ask this question--what do we know about a y-intercept?

These types of "confused" statements followed by conversations directed to clarify, slow down or elaborate student's ideas also appeared after an activity had taken place and Karla was trying to summarize what had occurred. Students would give her the answers that they had generated and she would use this to signal that they needed to slow down and talk through what they had done.

Example K-19

K: In the first glass, we started out with 32 oz and then you guys all started talking all the way down and **I kinda got lost** on all the numbers and that was why I wanted to write 'em, so I'd get a better idea of what you were saying.
Sam: 32, 16-
K: Hold on, **if you go too fast, I get lost.**
Sam: 16, 8.
K: So, on the second one, you're saying there would be 16?
Sam: Then 8, then 4, then 2, then 1, then .5, then .25.

Because Karla had a privileged role as teacher in the classroom, she can say, "I'm lost," without appearing that she did not know anything. In fact, in a couple of class sessions, students responded to this by saying that she did, in fact, know what was going on (as in the extended example given earlier when one student said, "You're just playing dumb right now"). This was a powerful way to get students to think about their language and articulation without actually telling them to do so because she was not explicitly saying to them, "you need to explain yourself further." By indicating her confusion, she signaled that students should back up and start again or should slow down and give more detail. In other words, the students in Karla's classroom had learned the cue that this type of talk indicated a command to explicate their thinking further. In this way, it seemed that learning what to "do" (in terms of discourse patterns) was intricately intertwined with articulating processes and understanding.

<u>"So, ...": The teacher has the right to summarize and contribute as she sees fit.</u> One word that appeared frequently in Karla's talk in the classroom was the word "so." Typically, when Karla said it, it was drawn out (sssooo) or punctuated (SO) and was followed by a pause. In some occasions, this made it a significant linguistic marker to indicate that a meta-comment was going to follow. The meta-comments that followed this word were often statements that summarized or concluded a conversation that was ongoing in the classroom. Other times, they indicated a change of direction (e.g. a student may have been off task and Karla wanted to direct the attention back to what they had been doing) in the discussion, were used to move on to a new topic or they marked a clarifying comment or question after a student made a contribution.

Karla often took long turns (some of her longest turns ran almost 250 words long as compared to about 50 words in some of the student's longest turns). In most of these turns, she used the word "so" often. Unlike a "so" that is used to mark a revoicing statement (O'Connor and Michaels, 1996, p. 81), this use of "so" did not seem to lend itself as a way to create a slot for students to have a turn. In this context, it was spoken after a series of statements related to the ongoing classroom conversation, many of which indicated some kind of concluding or summarizing statement related to the classroom activity or conversation. In the following example, a student finished explaining a solution method. Karla ended this episode by restating student contributions, using the word "so" to let students know that she was now describing what they were doing.

Example K-20

K: If I take 1.5 away every time, how many times am I taking 1.5 away?

Ss: Five.

K: Five. It takes me five moves to get down to that zero. **So**, really when they did it this way, that's what they were doing. They wanted to see how many times are they goin' to be taking this away to get her back to zero. They're going to do that five times for a total of seven and a half that they're going to take away. We were doing it this way—where we took a point we knew and then used our slope and kinda worked it backwards. If I'm at 9 and a half and worked backwards, where am I now?

Ss: Six.

K: If I take away one and a half? And if I keep using that same slope. And then [Ss give numbers] and then [Students continue to give numbers] **So**, I am going to end up in the same place that they did. We just took them away one at a time to get back here. They were saying, "I know I'm going to be taking it away and I'm going to have five groups of it," **so** they did it in that many steps. But most of us would do it that way [...]

Example K-21

K: We just have to be careful to remember this is how I'm rising. I'm rising from one up to 11. **So**, I'm rising a total of ten. I'm running from two to seven, **so** I'm running five. Now if I take rise divided by run, Samantha, what's the slope?

Samantha: Two

Example K-22

> K: Yeah, it looks kinda funky. It had these two little curvy things. **So**, it definitely was not linear. **So**, when you divide by x, it's not the same thing as multiplying by x. That's what those two were getting you to look at.

In Example K-20, the last two uses of "so" were used to make the point that both ways would get students to the same place—whether you subtract one 1.5 at a time until you have subtracted five of them or if you take away five 1.5's at once. These statements summarized the main points of the talk and "so" drew student attention to what her point was in summarizing: that is, it served as a significant marker that highlighted the importance of the statement that followed. "So" served the same function in Examples K-21 and K-22. After some talk about the mathematics, Karla made a point about what they were talking about—she was drawing their attention to what the concluding point was that they should walk away from in this conversation. In Example K-21, it was the result of subtracting the change in the y and x values. In Example K-22, since the graph is "curvy," she let students know that they can conclude that the equation was not linear, which further allowed them to determine that dividing by x was not the same as multiplying by x in an equation.

In these instances, "so" was acting as a gist marker: that is, the next thing to be said was the main point of her turn. By doing this, she made a shift in generality by pointing to a truth that was larger than the other facts she had stated so far. In other words, she was formulating the common knowledge to be taken away from the conversation. Although many of the ideas that she was reiterating came from students, Karla often used her own words to summarize, rather than words of students. By doing this, she may have indicated to students that this was part of her role in the classroom—to summarize and clarify the activity of the classroom. In fact, in a conversation with Karla, she told me that she often tried to do this. She attributed this to the fact that her own mathematics teachers had never done this. Often, she felt like she left the class not knowing what they were doing—never knowing the purpose of what was going on in the classroom. She told me that she never wanted her students to feel that way.

It was not the case that Karla always used "so" in this manner. In other instances, Karla sometimes used "so" to direct students' attention back to the conversation when their talk became off task from the topic at hand. Another way Karla used "so" was to indicate a change of direction. These happened more often in an I-R-E sequence when Karla would repeat what a student said and then move on to the next question.

Example K-23

K: If the slope is –2, as the x's get bigger, what are y's doing?
 Jonah: Getting smaller
K: Getting smaller, **so** if I go the other way my x's are getting smaller, what should my y's do?
 Robby: Get bigger
K: Get bigger, **so** this should be increasing by 2 this way [...]

In this example, it was clear that Karla was changing the direction from what they were talking about to the next, related, idea: that is, she was moving the conversation along. She tacitly approved what the student said by repeating it and then used "so" to indicate that she was moving forward.

Similar to O'Connor and Michaels (1996), the last function this linguistic marker served was to precede a clarifying question, through a revoicing of something a student said. For example

Example K-24

Robby: Like, it starts out at 2

K: **So**. you're picking this point to work it from?
Robby: Yeah
K: Okay
Robby: [inaudible] y. no [inaudible]
K: And the rate, we know is two?
Robby: Okay. then the equation [inaudible]
K: **So**. is it 2x – 3? And you would've taken two groups of negative two. or two groups of two away. right? You would've taken four away from here?

Example K-25

K: You didn't do your homework. Okay. Keith. what's it going to do?
Keith: It's going to go up by fives until like sixteen weeks.
K: **So** I just keep adding five cents?
Keith: Yeah.

In each of these examples. Karla's use of "so" marked the beginning of her turn. further indicating that she was revoicing the student's contribution. Also. this marker provided "both opportunity to respond and a responsibility to ratify or reject the correctness of the revoiced utterance" (O'Connors and Michaels. 1996. p. 81). In revoicing. the student and teacher. for a moment. were on an equal conversational footing. This footing was quite different from the positioning accomplished with an I-R-E sequence. In using revoicing. Karla was "accepting students' response from the start. using it as a base for the warranted inference. and then allowing the student the right to evaluate the correctness of the teacher's inference" (p. 82). The acceptance of the student response appeared in the fact that Karla used part of what the student said. However. she modified the contribution slightly and ended her turn with rising intonation. signaling the student that they were to let her know if her interpretation was correct or not.

Summary

In this section. I have described how Karla used metacommenting to: refer to past conversations or actions of people. revoice student contributions. refer to the importance of thinking in mathematics. signal a breakdown in communication. and summarize/conclude the main points of the classroom conversation. These metacomments were marked by particular linguistic cues (e.g. "some people…other people…," "so-and-so said…," "you lost me…" and "so.…").

When referring to past conversations or actions of people, Karla recognized talk and processes that were both what she would consider correct and those that were not quite what she was looking for. In Edwards and Mercer's terms, she was making "reconstructive recaps" of things students said or did. She would also broadcast strategies that small groups were using to make these strategies known to the larger classroom. When using this strategy, Karla often acted as narrator of a story to be told. She even used what Tannen would call "constructed dialogue," which invited higher degrees of involvement on the part of the listener. She offered her "birds-eye" view of the classroom as stories to be told. Karla also used this strategy as a way of allowing plurality into the classroom discourse—in some cases these approaches were inherently contrastive.

In naming student contributions, Karla often revoiced student offerings. However, she also reminded students of strategies that individuals had offered during past classroom conversations. Often, Karla used this as an occasion to reformulate in more mathematically correct and precise ways—sometimes using language that would be considered more official. Other times, Karla used this strategy as a way to credit students for their ideas, giving them ownership of their contributions. According to Edwards and Mercer, these "paraphrastic interpretations of pupil contributions" control the common knowledge by making student contributions adhere more closely to the big ideas the teacher may be trying to veer the conversation toward.

The meta-representational talk Karla used indicated that thinking was a focus of mathematics. In some cases, she referred back to ideas associated with common knowledge—reminding students of what "we already think," making presupposed knowledge more explicit. She also focused on thinking through strategies and processes before even applying them to the problems that were being discussed. However, she pushed this further by reminding students that they needed to actually do the processes before deciding whether their conclusions were valid or not. In a sense, she encouraged them to be reflective both before and after they worked through a problem because she emphasized that they think through it first and then continually asked them to check their work to make sure the solution made sense after they had completed the problem (most often by using a graphing calculator).

Karla also used meta-comments to tacitly signal a break in communication. Often, this strategy appeared when students introduced ways of solving problems that were not like those that Karla (and possibly the other students in class) considered common ways of doing something. This pedagogic move was either genuine or was triggered when Karla sensed some students may have been confused. In this type of move, Karla would sometimes bring other students in to act as interpreter. In using this strategy, Karla got students to articulate themselves more fully by signaling them to do so.

Possibly, the most explicit marker that Karla employed was that of using the word "so" to signal that a meta-comment was to follow. In using this device, Karla was letting students know that a summative point was going to be made that transcended others stated prior to that one. These types of "so" were marked further by their intonation—they were either punctuated or drawn out and were followed by a pause. By using this "significant marker" in this manner, Karla was implicitly pointing to and defining the common knowledge as it was being developed in the classroom.

In describing Karla's use of these forms, I have broadened the notion of metacommenting to include comments on actions, not just words. When Karla commented on people's actions or processes, the talk had a similar function as metacommenting in that it was still commenting on something at a meta-level. However, in Pimm's discussion about metacommenting, all of the teacher's metacomments focused on words that were spoken, not actions that were taken. I have shown that Karla also used metacomments to bring actions and processes to student's attention.

Expressions of Uncertainty, Possibility and Hypothetical

Language that indicates vagueness or uncertainty can be identified in many ways. One common way is through examining modality. Stubbs (1996) defines modality as:

> ways in which language is used to encode meanings such as degrees of certainty and commitment, or alternatively vagueness and lack of commitments, personal beliefs versus generally accepted or taken for granted knowledge. Such language functions to express group membership, as speakers adopt positions, express agreement and disagreement with others, make personal and social allegiances and contracts (p. 202).

When we speak or write, we are often vague, indirect and unclear about to what we are committed. This has many uses, two of which are politeness and hedging.

Politeness

Brown and Levinson (1978) define politeness in terms of mutual social motivation where interlocutors address each other's need to preserve face. The term "face" comes from Goffman's (1967) notion which ties it to:

> [...] notions of being embarrassed or humiliated, or 'losing face'. Thus face is something that is emotionally invested and that can be lost, maintained, or enhanced, and must be constantly attended to in interaction (Brown & Levinson, 1987, p. 61).

It is advantageous to attend to one another's face because it works in everyone's best interest. Brown and Levinson contend that recognition of face and the necessity to pay attention to it is universal.

"Face" consists of a public self-image with two wants:

-positive face: aspiration to be appreciated and valued by others or the need for social approval, and

-negative face: concern for personal rights and freedoms, such as freedom to choose one's own actions and to avoid imposition.

Some acts (i.e. "face threatening acts" or FTA's) intrinsically threaten face. For example, requesting that someone share his/her solution to a mathematical problem because it is unique, interesting, concise, etc. would threaten his/her negative face. Criticizing or disagreeing with someone's solution would threaten their positive face.

Cazden (1988) translates this model to school classrooms more specifically. She claims that:

> 1.) Teachers inevitably engage in FTAs. They constrain students' freedom and criticize their behavior and their work, often in public.
>
> 2.) Teachers can soften the effects of such acts by various politeness strategies. Two important strategies express intimacy ("positive" politeness) or deference and respect ("negative" politeness). Note that, in this model, "positive" and "negative" are terms of description, not evaluation.
>
> 3.) The seriousness of any act, to teacher or student, depends on their perceptions of social distance (D), relative power (P), and a ranking (R) of the imposition of the teacher's act at a particular moment (p. 162).

While any single teacher will use a combination of negative and positive politeness forms, how they are used will depend on the D, P, and R. Teachers by virtue of their institutional position have their role defined as an authority in the classroom. A mathematics teacher, in particular, deals with claims made regularly. Naturally, a mathematics teacher may be expected to say "right" or "wrong," based on their role as a more knowledgeable other and representative of the discipline of mathematics.

More specific to mathematics education, Bills (2000) recently looked at the strategies for redressive action to FTA's in one-on-one interactions with two secondary mathematics students. The hierarchy for these strategies include:

1.) do FTA without redressive action;

2.) do FTA on-record with positive politeness;

3.) do FTA on-record with negative politeness;

4.) do FTA off-record—the safest strategy identified by Brown and Levinson, includes giving hints and clues, under- and overstating, being ironic and using rhetorical questions, to which Bills adds asking a question rather than making a statement;

5.) don't do the FTA (p. 41).

She found that the vast majority of the FTAs were occasions where the teacher wanted to 'correct' something that the student had written or said or seemed to be thinking. She also proposes that "a major influence on the teacher in her widespread use of politeness strategies was her wish to downgrade the apparent relationship between teacher and student" (p. 46). Although Bills suggests that the teacher's formal power (i.e. the power endowed by the culture) is not under threat, she posits that her sapiential power (i.e. power due to the teacher's greater knowledge of the mathematical culture) appears reduced through her politeness strategies by "giving weight to the students' opinions, wishes and cognitive concerns" (p. 46).

Hedging

Another function of modal verbs and expressions is to hedge. Hedges are "words or phrases whose job it is to make things fuzzier" (Lakoff, 1972). In addition to using modals to hedge, there are many other forms that can allow a speaker to make their assertions and ideas fuzzier. More particular to this study, Rowland (1995) has looked at how hedging is used in mathematical conversations in interviews with students. In doing so, he delineates two main categories of hedges: Shields and Approximators. Shields are described as "identifiable fuzzy preludes" (p. 334) that lie outside the proposition. Their effect is one that distances the speaker from the proposition, but does not modify the proposition. An example of a Shield would include "I think that…." An Approximator, in contrast, lies inside the proposition and alters it by making the proposition more vague. Examples of Approximators include "sort of," "about" and "a little bit." Each of these two categories is broken down further into two subcategories.

Shields can be subdivided into Plausibility Shields (PS) and Attribution Shields (AS). PS (e.g. "I think," "maybe" "probably") involve a position held or a belief considered and designates doubt that it will be fulfilled by events or will stand up to inspection. AS (e.g. "According to…") implicate some degree or quality of knowledge to a third party. This type of Shield was described as being used most often by the interviewer (i.e. Rowland) as a "teacher-like device for meta-comment[ing] on the activity" (p. 335). In one example, Rowland described his use of this type of Shield as a "ploy for being non-committal about the contribution of one child, in order to obscure evaluation of her answer and to encourage participation of the other child" (p. 335). In doing this, he restated what one student said with the intention of not evaluating the comment and putting it out for inspection to the other child in the interview.

The two types of Approximators that are delineated are Adaptors and Rounders. Adaptors attach vagueness to nouns, verbs or adjectives through words or phrases like "a little bit," "somewhat," "kind of," and "fairly." They exemplify the hedges where the issue is class membership and suggest, but do not define, the extension of the category, example, etc. Rowland describes how this type of Approximator tended to be something he used as the interviewer to comment on children's contributions to make indirect comments on their predictions, generalizations, and explanations. Rounders are more common in the domain of measurements and quantitative data. They act as a qualifier to the number being given (e.g. "it was about/around/approximately 20").

Defining words and phrases that fit into these categories are not easily done. In fact, one argument Rowland made in this article was that the students he interviewed were using some of these hedges to create time to think and to "shield" themselves from accusation of error. He also argues that the hedge "about," which is classified as an Approximator, was used by students to serve Shield-like ends. So, the context is important and must be considered when defining the function of various hedges.

<u>The Fuzzy Talker</u>

The third discourse pattern that was prominent in Karla's classroom was her use of this language of uncertainty, possibility, and the hypothetical. Many of her expressions used modality (Stubbs, 1986, 1996) for these two purposes: to hedge and to be polite.

Language that expresses uncertainty and vagueness have become of interest to the mathematics education community recently (e.g. Rowland, 1995; 1999). However, in many of the articles that have appeared in journals, the focus has been on student's use of this vague language. In this section, I will focus on how Karla uses modal verbs and expressions and hedging to create expressions of uncertainty, possibility and hypothetical. In the classroom transcripts, the use of modal verbs (e.g. could, would, should, might) and hedges allowed Karla to offer advice, make content expectations explicit, and make a link from what students already know to what they will be coming to know. In offering advice, Karla told students what she "would" do. In setting expectations, she expressed what students "will" and "can" do in terms of the mathematical content. When moving past the relationships about which students were already knowledgeable (i.e. linearity), Karla used fuzzy language to create a contrast to new ideas and relationships (i.e. modeling and non-linear relationships).

"I would," "I could": Everyone should hypothesize the possibilities with respect to the mathematics. One modal verb that Karla used frequently in combination with "I" was "would." In using this combination, she often said what "I would" do—giving indirect advice to students. She did not offer this as something that they (i.e. the students) had to do, but as a possible way of doing or thinking about something—based on what she would do, given the choice. The advice appeared to be a suggestion, not a mandate. However, given the privileged role Karla had as teacher in the classroom, this advice may have been interpreted as a mandate by students in her class. In this way, it appeared as if Karla was tacitly controlling the common knowledge through using the modal verb "would" to suggest what students might do. For example, after a discussion about the type of average (i.e. mean, median or mode) to use to even out errors in the data students had collected, Karla summarized the discussion by saying:

Example K-26

K: And that's what it's [i.e. the mean average] going to do. It's going to take some of the high pieces and put them with the low-- try and make everybody equal. So, **I would** agree, the mean makes the most sense to me because we want to try to equal out everybody and when I find the mean, that's really what I'm doing. I'm trying to make all the data pieces equal, if they **could** be.

Mark: If they **could** be equal
K: So, **I would** find the mean for each one.

In this example, Karla summarized the discussion by telling her students that she agreed with them, that the "mean makes the most sense." She continued by saying that what they were "trying" to do was make all the data pieces equal and qualified this by saying "if they could be." This type of statement was ambiguous. Karla may have been tacitly claiming that "making all the data pieces equal" cannot be done in all instances. "Could" might be used in the sense that these data pieces were trying to be equal; or in the sense that it was a logical possibility. In this last interpretation, Karla may have been saying, "If, in my imagination, I make all of my data pieces equal, what would they be equal to?" In this case, the answer would be the mean. After Mark reformulated Karla's "could" statement, Karla concluded by telling her students that she "would" find the mean for each row of data collected by the various groups in class. The shift from "could" to "would" moved from less to more commitment (Stubbs, 1986) to the ideas that were being expressed. This indirectly tells students that this was what they should have been doing; however, it left it open as to whether there were other defensible strategies for dealing with the error in the data collected.

This shift from "could" to "would" deserves more comment. Mark picked up on this uncertain language and cast it back to Karla, drawing her attention to the vagueness of her claim. In other classroom sessions, students appropriated "could." For example, the day after the above discussion, students recorded their mean data on the board (given below) and Karla asked them whether the data appeared linear or not.

Length	Pennies
4	76
5	53

6	40
7	18
8	12
9	9
10	7
11	6

In the discussion that followed, at least three[14] arguments were posed by students for why this data "could" be linear. In the first argument, Becky told Karla that it could be linear if only one group had collected the data, instead of many groups collecting the data. Her reasoning included that there would be fewer errors and more consistency in the data collection, hence, making the data "more linear" than it appeared.

When Karla pointed out that they eliminated some of the errors by taking the mean of the data collected, Becky and Sam then said, "You **could** make this linear" and suggested that they could "find the average of the differences" and then use the average as the constant rate for the entire table. They recommended that they find the first differences in the table (which would be 23, 13, 22, 6, 3, 2, 1) and then determine the average of these differences. This average could be used as the constant rate for the entire table. Karla pointed out that this would not make sense because there was a huge discrepancy between the first difference, 23, and the average difference of ten. This discrepancy would not allow anyone to use the model to predict anything because it was too far from the actual difference. Karla then asked, "Why are you trying to make this linear when it's obviously not?"

Even after two instances of "could" were explicitly dismissed, Sam proceeded to push for one more way that they "could" make the data linear. In this argument, Sam discussed the idea of dividing the number of pennies by the number of inches, i.e. find "how many (pennies) per inch." This "pennies per inch" would have resulted in a different rate for each data point. Given the context of the classroom, this suggestion could have made sense to the students in at least two ways. In the first, students were using language that they had come to associate with slope as a rate, i.e. something "per" something else (e.g. miles per hour and dollars per month[15]). Also, finding pennies per inch allowed a "standardization" throughout the entire table. Instead of each line showing a relationship between a different number of inches with a measured number of pennies, they had found a pattern for the entire table in terms of pennies per inch. This exemplified their understanding of slope (in a linear relationship) as a rate. In this case, they were calling that rate "pennies per inch." However, this idea of having a rate for each data point was not mathematically correct, as Karla pointed out when she again asked, "Why are you trying to make this linear when it's obviously not?"

This was a clear example of students appropriating the hypothetical use of "could" for their own means. They may have been using it to stall for time or to resist Karla's authority in the classroom or to signal her that they were not ready to move on to another mathematical relationship. It appeared that "could" had become a way of trying to make ideas fit into what students already knew, instead of thinking about the range of possibilities that might exist. The type of language being used was hypothetical, creating an environment where students could put their ideas out there for discussion. One way for them to do this was to offer what "could" be.

In the next example, Karla just finished having students graph data from an experiment she had demonstrated. In the demonstration, she poured water from one glass container into another, into another, etc. Each time, she poured half of the water from the container into the next container, beginning with 32 ounces, then 16, then 8, etc. After students graphed the data, they discussed whether the graph appeared linear or non-linear and then Karla asked students if they should connect the dots (i.e. points) or not.

Example K-27

K: It does do a swoopy. Should I connect the dots?
Sam: Yeah, it seems like it, yeah.
K: In *Variables & Patterns* we decided whether to connect the dots or not, we asked ourself [sic] if there was data in between--instead of this idea of just showing a trend. Is there data in between? Is there a glass 1.5?
Ss: No.
K: There **probably could be**, but that **would** be **kinda weird**. I don't think I **would** connect the dots on this one. If you did, with that idea that you want to show the trend of the data, I **guess** that's okay. But, there really isn't data in between on these.

This example began with a common occurrence in Karla's classroom: Karla used a formulaic word that the class associated with an exponential graph, "swoopy." In discussions I have had with Karla, she indicated that students introduced this word almost every year when they did exponential functions. She used this language to refer to the graphs because students used this word. She did this partially because she believed that it was important to value student ways of talking. She also did this because she realized that students enjoyed having their own ways to talk about the mathematical ideas because they felt "ownership" for them. In interviews with students, many students described Karla's class as "fun." One student in particular told me that he thought one of the reasons the class was fun was because Karla used words like "swoopy," indicating that Karla's intuition about this was correct (at least for this student).

After Karla asked if they should connect the dots, Sam responded affirmatively, although appeared tentative in her decision (i.e. "it *seems* like it"). Karla responded, using a "reconstructive recap," pointing out that they had decided about this in a unit they completed the previous year—*Variables and Patterns*. By doing so, she indicated that

Sam's response had not been correct. She reminded them that they were supposed to ask themselves if there was data in between the graphed points or not (hence distinguishing between discrete and continuous data). She applied this strategy specifically to the data they just collected when she asked, "Is there a glass 1.5?"

Karla hedged her answer twice with "probably could be" and removed herself from the possibility even further by stating (in a more definite manner) that "that **would** be kinda weird." The pervasive use of hedging in her response may be attributed to the fact that she was in the midst of a face-threatening act with Sam, who had given an incorrect answer. She offered students indirect advice by hedging the proposition "I would connect the dots" with "I don't think."

"Probably," "I don't think," and "I guess" are all Plausibility Shields. By using these, Karla was casting doubt on the idea that the points should be connected in this situation—given the fact that it would not make sense to have half glasses (or some size glass that would be in between the glasses that she used) in the experiment. This use of hedging seemed to indicate a tension in Karla's teaching--referred to as the "didactic tension" (Mason, 1988). She appeared to want to leave the option to offer ideas open as long as students justified their choice, but also wanted them to know about how she was thinking about it. In doing this, she was subtly telling students that she was open to different answers. However, she might be closing the door to their interpretations by telling them what she would do—as the authority in the classroom.

Example K-28

> K: You're first going to decide, "Does it look like this might be trying to go linear?"; "Does it look like it's trying to be one of those swoopy things?," etc. After you've drawn the graph models in that you can draw, then go back to any

that you might've done that were linear. If you drew something that was linear, find the slope of the line that you drew. Can you find slope of a line?

Ss: Yeah, sure.

K: How?

Adam: Divide the rise by the run.

K: The rise and the run. Draw that stair step in, that's probably how I **would** do it. But, then it also says this, so all of you with highly organized notebooks, this **would** be very simple. And if you don't have a highly organized notebook, you might have to panic and go through your um, problems a little bit.

In this example, Karla was going through the directions for part of the homework for the next day. She was ordering the things that students were going to do—first, decide if the graph was linear or not (again using the student word "swoopy"), then draw in graph models, then go back to any graph models that are linear and find the slope of the line. However, in describing the task, she phrased it as questions they might ask themselves, "Does it look like this might be trying to go linear?" or "Does it look like it's trying to be one of those swoopy things?" In posing it in this manner, she was designating the decision as tentative (e.g. "might" and "trying to") because they were only looking at a graph and the data was based on experimental data, so it was less reliable. In using "might," she may have indicated that there could be a range of possibilities. In the last part of her instructions, she asked students if they could find the slope of a line (i.e. do they know how to find the slope). After Adam told her they can find it by dividing rise by the run, Karla repeated part of what he said and then told them that drawing in a stair step was "probably how I would do it." Again, she was specifying a possible way to find the slope—draw in the "stair steps" (i.e. the horizontal and vertical changes look like stairs when they are drawn in—subsequent observations showed this to become a formulaic phrase for determining the slope of a line from the graphical representation). By using "probably," again, she was stating the position that she held for how this should be done. However, she was also pointing out that there were other possibilities that students have used in this classroom, including subtracting the x-values and the y-values in two pairs of points that were on the line. By using "probably," she indicated that there were other ways to do it, but the way that she, herself, would do it was to use the graph and draw in the stair step.

"You can..."; "You will...": The big ideas are stated and people must take responsibility for their own learning. In various portions of the classroom conversation, Karla would be more explicit about what her expectations were of students. When putting forth these expectations, she more often used "can" and "will." Stubbs (1996) asserted that "can" is associated with "universal truth" (p. 220). He also argued that "will" expresses inference; it is hypothetical and predictive (p. 222). This was different from her use of "could" and "would" in at least two ways. When using "can" and "will," Karla tended to emphasize specific concepts students were supposed to understand; that is, rather than focus on the process that students could have used to find an answer, this type of talk focused on the actual mathematical ideas that she expected students to understand. In this way, it made more explicit what knowledge Karla was now going to consider "implicit and presupposed." Also, it was different on its level of definitiveness.

After grading the first unit test (called a "Check-up"), Karla described what her expectations for students were at this point:

Example K-29

K: There's still some very low grades, though. If your grade is continuing to fall you've got to do something about it now because at this point I'm expecting that you **can** find slope, you **can** find y-intercept and I'm moving on. I'm not going back there again. I spent eight weeks on slope and y-intercept and that's plenty. If your grade is still below a "C" then it's up to you to do something. There's tons of opportunities here at school for you to do that, it's just, when are you going to take that responsibility? Math lab is here. You **can** talk to Mrs. Gregson. There are tutors available. The National Honor Society students at the high school get credit for tutoring you for free. You **can** even get yourself a tutor by speaking to Mrs. Gibson, so you've got to decide, "How am I going to fix this for myself because the class is moving on without me." Let's see [...] [calls out each student's name and hands back Check-up]

In Karla's interviews, I asked her about this type of talk. She laughed as she read this quote and said that she sounded "bitchy." In stating that she "spent eight weeks on slope and y-intercept and that's plenty," she was pointing to a tension she felt—that of time and pacing. She had expressed concern for always being at least two days (and sometimes up to a week) behind Josh. She felt that she needed to continue teaching at the same or a faster pace so that she could cover what she was expected to cover. In some sense the modal verbs that she has chosen breach politeness—she told them what she expected them to be able to do at this point and if they cannot do these things, they needed to take responsibility for what they did not know. After discussing the answers to the check-up, she reiterated her expectations, again using "can" and "will":

Example K-30

K: We're going to move on now. And it's going to be assumed from this point on, if something's linear, that **you will** recognize that it's linear in a table, graph or equation. You **can** tell me about the slope or the rate and then tell me about the starting point or the y-intercept. If you're still struggling with those main ideas, now it's up to you to catch up because I'm going to assume from this point that everyone **can** talk about those three things. And, if you still are worried that you're not quite sure how to do some pieces of that, **you can** come to me and I **can** give you some suggestions about what **you can** do that **will** help you catch up. [...]

In defining the expectations in this manner, she was explicitly pointing out what she required students to know and do at this point of the school year. She used "will" which indicated her future inferences about what they know about linear relationships in terms of tables, graphs, and equations.

Another way she used "will" was to project what she wanted students to come to class with the next day. This was different in that it did not focus on the content, per se, but what she expected them to physically produce or do in terms of their homework. This was typically used after an assignment had been given.

Example K-31

> K: Yep, you're going to need to get that data copied down before you leave because you're running out of time so **you'll** need to take it home to do your averages… the mean average. If you haven't had time to get all of this copied, **I'll** leave it up and you **can** run in here before you go to lunch and copy it down
>
> Ms: Thanks
>
> K: When you come to class tomorrow, **you'll** have computed all of these mean averages. **You'll** have a graph where you haven't connected the dots because c, d, e are going to ask you to do something with your graph. So, don't connect your dots until you read through c, d, and e. Length is here and pennies is going up and down. **I'll** leave the table up until after lunch so you **can** copy it down.

In this example, Karla told her students that they needed the class averages to do their homework that night—they "will" need to copy them down so they could take it home. She left this as an option by saying they "can" run in before lunch. In her last turn, she told them what they need to have done before class tomorrow—compute all the mean averages and have a graph that does not have the dots connected.

<u>"Probably" "Seemed": Mistakes are okay to make (or I'll protect you)</u>. One of the ways Karla made her talk more vague was through hedging. In the above examples, Karla used words like "probably" and "seemed." Often, this was used in the context of pointing out mistakes students had made—possibly to soften the assertion. For example, after handing back the Check-up described earlier, Karla went through some of the mistakes students made and tried to account for why she thought they got some of the problems incorrect.

Example K-32

K: You said **things like**: "I knew it was an equation that was linear because it had the number multiplied times x and the something added on"—really you were talking about y = mx + b, you just described it in **a little more** detail. Those were the **kinds of things** I looked for in 2B. 3A had four points. You first had to decide which two were linear and then write the equations for them. Some people **seemed** to slide right through and missed where it said to write the equations. Graph A and B are linear.

Example K-33

K: Some people just **seemed** to skip it. You talked about how it had to be a straight-line and not a curvy thing—on the back, #4, find the equation. Some people drew the line in and that was really helpful for people to see that it had a negative slope. They saw this decreasing so they knew they had to end up with a –2 instead of a positive 2. And then using the slope to work it backwards, you found the y-intercept. If you missed it, half a point on four, that's **probably** where it was, **probably** on the y-intercept. Because I could see that many people were **trying to** find their y-intercept they just used their slope backwards.

In Example K-32, Karla was doing a number of interesting things. First, she was replaying a common answer. However, mathematically speaking, this common answer was not standard. Students described a linear equation in terms of it being a "number multiplied times x and the something added on." She related this to the more official way of talking about linear equations—it was an equation of the form y = mx + b. However, she seemed to be valuing their response more than this official form when she said "you just described it *in a little more detail*." This may be related to Karla's use of words like "swoopy": that is, she was valuing the way in which students were describing the equation. By doing so, she may have indicated to students that their ways of communicating were just as (or more) important than more mathematical ways of talking.

Karla was not very specific about what she was looking for when she ended this part with "those were the kinds of things I looked for," leaving the interpretation up to students as to what did not fall into the category of "those kinds of things." By using these forms of Adaptors. Karla was leaving the range of possible answers open. Students were to interpret their answers in terms of these broad categories.

Karla then moved into talking about why students got the next problem wrong—some just "seemed" to slide through it. This use of past tense indicated politeness (Stubbs, 1996, p. 222) because it signaled remoteness. In using past tense, the speaker shifted back in time, distancing the speaker from the hearer and putting a hedge on the illocutionary force. In hedging this, Karla may have indicated that she did not think students would have skipped these problems intentionally—thus, reducing the face threatening act. In addition, there was a deliberate vagueness regarding to whom she was referring—"some people" rather than telling the class exactly to whom she was referring, thus shielding the people who skipped the problem.

In Example K-33, Karla did something similar to this last point. She let students know that she thought they "seemed" to skip the problem because if they had seen it, they would have completed it. She also pointed out, that on the next problem, if people missed half a point it was "probably" because they did not have the y-intercept. In saying it in this manner, she subtly pointed out the most common reason for getting points off on this problem—but, doing so without pointing fingers at any specific student. She also softened her interpretation by using hedges like "seemed" and "probably." Also, in her last statement, she made her point even less harsh by saying that she could see that many people were "trying to" find the y-intercept, they just did not succeed (although she left this last part off completely). By making all of these statements less harsh through hedging, Karla may have been redressing the FTAs to student's positive face.

+"Kinda," "Sorta": There are relationships beyond what has already been discussed and explored, but the ideas for thinking about them are similar. Karla used the hedges "seemed" and "probably" frequently was during discussions about non-linear situations and discussions about data that had been collected (or was given) from "real" situations. The forms of these hedges were the same, but the function here was more like an Approximator (e.g. the function "seemed" to be perfectly linear). These hedges were directly related to the mathematical relationship rather than the written or oral answers students offered. Many times additional Approximators like "kind of" and "sort of" appeared alongside these.

In the previous unit, students spent ample time working with and discussing linear relationships. Most students (in class and in interviews) could identify a linear table, graph, and equation based on finding the rate of change (i.e. the slope) and the starting point (i.e. the y-intercept). In *Thinking with Mathematical Models*, students were introduced to one related idea—linear modeling. This was different from linear in that the rate of change was not exactly the same every single time because of error in data collection or because the event was happening in the "real" world. Students were also introduced to non-linear relationships in terms of modeling. In this case, students were asked to think about a non-exact rate of change that was not additively constant (i.e. the y-values in the table do not increase additively for each change of one in the x-values).

In dealing with these new ideas, Karla used an extensive number of hedges to soften the transition from "perfectly" linear situations to modeling linear.

Example K-34

K: The potato problem, they gave more information than we needed to know. I've heard this--to try to get you to sort through the information that was necessary. All they wanted to know was that if I plant an old potato, about how many new potatoes are going to grow from it? All that stuff about cutting it up and dividing some from each piece and all that, that was just some background information about how if you wanted to go home and do this, you **could** do this.

Ms: It was easy.

Sam: I was confused.

K: I know [this was directed to Sam].
[Ss comment—overlapping speech]
K: 20 potatoes. So then 2 potatoes and we **could** get forty and three, **hopefully** sixty. So, your table is going to increase **that way**. Why did they use the word graph model when the data **seemed** perfectly linear?
[...] [dealing with student behavior]
K: They used the word graph model when it **seemed** to be a perfect fit. Why did it say that?

In this example, Karla was referring to the problem 7 in Appendix A, the "potato problem." The information that was given showed a constant increase of 20, but students were asked to make a table and a "graph model." This seemed contradictory because other experiments that were given did not give an exact constant increase when students either collected their own data or the data was given from an experiment. It was obvious, from Sam's contribution, that this was confusing to some students. Karla tried to soften the transition by saying they "could" get forty and that by three it was "hopefully" sixty. However, the problem itself contradicted Karla's interpretation—it was stating that each potato segment *will* yield 20 potatoes, not "about" 20 potatoes. However, students may have come to understand "graph models" as not being completely regular or precise.

In comparing the language Karla used in the transcript to the language the text used, an interesting contrast was taking place. The language that Karla used was very hypothetical ("if," "could") and vague ("seemed") when she referred to the potato problem. However, the language that the text used was very absolute ("will produce," "will yield..."), not allowing students to think about the fact that the data may deviate from the increase of 20 every time. This was contradictory to the idea of "model" that had been established in the classroom as one of being tentative and not "perfect." In fact, "perfect" had become Karla's way of distinguishing from data that was exactly linear (especially in hypothetical and presented data, rather than modeling data or data that was collected by students).

Another interesting thing Karla was doing here was using the word "they" to refer to the text. In doing so, she seemed to be using this as an Attribution Shield, distancing herself from the language of the text. When she did this, she was identifying the authors of the text as a third party that was separate from her and the classroom. This allowed Karla to play the role of interpreter of the text—distancing herself from the authority. The language of the text was absolute, which was not the type of language that students had come to associate with the mathematical models with which they had been dealing. To interpret this difference, Karla used "they" as an Attribution Shield.

This same type of fuzzy language was used when students were beginning to be exposed to non-linear modeling problems. Situations that were non-linear were just being introduced and their defining characteristics did not fit with what students already knew about linear (e.g. the fact that tables went up in a constant additive pattern and had a starting point which may or may not be zero). In order to cushion this new type of relationship, Karla used a lot of uncertain language about what was "kinda" happening.

Example K-35

> K: And then if you're not quite sure whether it's linear or not because it **might not** turn out **perfect**, remember the thickness **didn't quite** turn out **perfect**. The second thing you're going to do is graph it.

Example K-36

> K: This is showing where they **kinda started out slow** and then took off. This is showing where they **kinda started out fast** and then slowed down. We've always connected with straight-lines cuz we've assumed that constant rate is going to stay there. But, in this case we know there isn't a constant rate, so really that curvy makes more sense.

Example K-37

K: Your x is increasing and your y is decreasing and not at a constant rate. Is there **some sort of a** pattern going on though?

Example K-38

K: So, if I had one that had **that kind of a trend** to it, I'd write down the problem 3.2 did that, also. Or the water problem did **that type of thing**.

Example K-39

K: **Sort of a pattern** there, isn't there? Like Robby said there is **almost** a multiplication or division pattern there. Since it **isn't a constant rate pattern**, it makes it not linear. So, it **does do something kinda neat** that we will look at later in here [...]

Example K-40

K: Yeah, it **looks kinda funky**. It had these **two little curvy things**. So, it definitely was not linear. So, when you divide by x, it's not the same thing as multiplying by x. That's what those two were getting you to look at [...]

In each of these examples, Karla was indicating a different rate or trend than those to which students had become accustomed. The ones that they had been dealing with were linear patterns—ones that increased at a constant additive rate in the y values for each single increase in the x values. However, these constant increases had become ones that were "perfect" and now they were moving on to ones that were not perfect. To differentiate this new type of pattern further, Karla softened the differences by hedging them—they "aren't quite perfect" or they now "kinda start out slow and then took off" or they were "some sort of pattern." These Adaptors allow Karla to suggest some type of class membership different from linear without actually defining the difference. In doing so, she may have been hoping that students would come to a point where they themselves can discover the difference and articulate it to her, instead of having to vocalize this difference herself. Karla also stated that they "look kinda funky" because they are "little curvy things." These more informal ways of talking about non-linear functions were not only used by Karla, but were often offered, repeated and used by students in the class. According to Stubbs (1996), forms that end in –y encode informality and vagueness, therefore exhibiting less than full commitment to the proposition.

Summary

In this section, I have shown how Karla used modal verbs and expressions both to be polite and to hedge. When she used modal verbs, she shifted back and forth from "would" to "could"—offering advice in the first case, and allowing the classroom talk to be hypothetical in the latter. When offering advice, she did so politely so as not to force students into a particular way of doing something. Karla may have been more polite because she wanted students to do more and share more. By not putting her method out there as the only way to do something, she allowed students space to share their ideas and explore the content through their talk. She used politeness as one way of getting students to contribute their ideas and to show respect to the ideas that students were sharing. However, in doing so she was not always explicit about the mathematical processes she wanted students to use.

When defining the actual big ideas students needed to know, Karla was much more explicit and used more definitive modal verbs—"can" and "will." She outlined the knowledge that was going to be presupposed and implicit in future conversations through telling students what she was going to "assume" they "can" do. These were typically defined by what her expectations were in terms of the mathematical content. By the fifth week in her class they "can" talk about both the slope and the y-intercept in linear relationships. They "can" work with these concepts in all three representations (i.e. the graph, table and equation). I believe that this connected with Karla's push toward the big mathematical ideas that guide her teaching. Whereas with the above, she wanted students to define how they get to the mathematical ideas (i.e. the processes), she actually defined the mathematical content to be learned. By explicitly telling students what her expectations were, she shared her goals with her students and told them what she required at certain points of the school year.

In the last two sub-sections, I described similar surface forms (i.e. "probably," "seemed," "sort of" and "kind of") that served different functions. The first set of examples showed Karla using these forms to protect students as she discussed incorrect solutions they had given on a quiz she handed back. In the other set, these words were used to bridge to the new content that students were exploring. The same surface forms were used to make the ideas of linear and non-linear mathematical modeling appear different from "perfectly" linear. It was difficult to say exactly when these modal verbs and expressions were being used to hedge and when they were being used to be polite. In fact, these different functions may be undetectable to students. When students heard, "some just **seemed** to skip it" or "these **seemed** to be linear," they may not have been able to distinguish between the two differing functions. In the first, Karla was being polite and in the other she was hedging the mathematical content. Since politeness and hedging were different--yet overlapping--functions, this same surface form might have been confusing to students. In fact, if students were interpreting the function of this language as the way that Karla was being polite, they may not attend to it in the same manner. They may have ignored the importance of the content-referring function: that is, when Karla was saying something "seemed" linear, students may have only heard "linear" because they thought that she was just being polite. What Karla may have been trying to do was to differentiate between theoretical mathematics in which "perfectly" linear representations exist and real life (and not so perfect) experience.

It appeared that at least two norms were apparent in this fuzzy talk—a politeness norm and a hedging norm. The politeness norm created a more open environment that allowed students to contribute their ideas. The hedging norm established talk from Karla that may have appeared less authoritative in nature. At this time, I believe that I am not in a position to say that one form links with a certain function or another. However, if I have had difficulty making this distinction after careful deliberation of the transcripts, this may say something about student's interpretation of this fuzzy talk; it may be a confusing environment to be in. As part of my research agenda, I intend to use student interviews to try to distinguish how students make sense of these by the ways that they describe what is happening in the mathematical situations. Classroom observations indicated that students do use the fuzzy language to differentiate "perfectly" linear situations from "kinda" linear situations. However, I would like to gather more evidence about student-talk before inferring how they seem to make sense of the function of these language forms.

From Forms to Norms

In the introduction to this case, I put forth that each of the forms that I was about to describe carried with it a norm related to what it meant to "be" in Karla's classroom: that is, the repeated discourse forms carry with them certain expectations, rights, roles and responsibilities. I now return to the list of the norms that I described, but discuss them as they were related to the mathematical content, the teacher and the students in the classroom.

Those norms that were associated with mathematical content included:

- There are connections in both the content (especially in terms of the multiple representations that are discussed) and the contexts that must be recognized;
- Plurality should be endorsed and acclaimed;
- Everyone should hypothesize the possibilities with respect to the mathematics;
- Mathematics is in the mind of the beholder; and
- There are relationships beyond what has already been discussed and explored, but the ideas for thinking about them are similar (e.g. you still need to think about the same things when you're trying to decide what is happening).

Each of these norms is related to the ways the mathematical content was to be viewed and explored. In addition, the first four on the list were discourse patterns that students seemed to recognize and incorporate into their own talk. Students verbally made connections between like problems that they had already encountered and ones to which they were being introduced. Not only did they make connections between problems based on the process that they used to solve the problem, but they also related the contexts across investigations and past units. This "habit of mind" was also requested by some of the problems in the textbook. Students also offered multiple solution methods and took part in discussions about these processes, tacitly acknowledging that part of what it meant to "do" mathematics in the classroom consisted of generating more than one way to solve a problem.

Often, students hypothesized their ideas about the mathematical problems being discussed. This was particularly apparent in their use of "could"; students often hypothesized what "could" be the case, following Karla's modeling of such talk.

However, as I showed in that section, they sometimes appropriated the use of this word for their own means. Recognizing that thinking processes were important when doing mathematics appeared in students contributions about "thinking" and "images." Students offered what they thought but also picked up Karla's language for these notions.

The last norm seemed less apparent for students and there were conflicts in the ways the textbook addressed these relationships and the way the class talked about them. The textbook portrayed the modeling relationships in which data was collected as not being an exact constant additive relationship. However, the language the textbook used was absolute and exact. Karla attempted to show students that they were supposed to think about the content in the same manner, but used much "fuzzier" language for talking about the situations. Mathematically linear situations were "perfectly" linear; those that were based on an experiment in a modeling situation were "kinda" linear. However, Karla seemed to want students to know that they needed to think about the situations in the same way, even though they were slightly different. In a sense, she was again trying to instill a "habit of mind" that encouraged students to inspect and explore the relationships in the same way, e.g. they were supposed to look carefully at the rate of increase in the table to determine if the relationship was linear or not.

Only one norm seemed associated with the teacher alone:

- The teacher has the right to summarize and contribute as she sees fit.

Karla's role in the classroom seemed to be keeping students apprised of the activity. She summarized the classroom discussion and contributed as she saw fit. This was especially apparent in her use of the word "so," which indicated she was about to metacomment on the talk or activity in the classroom. In fact, Karla told me that during her experience as a student, she often felt lost or as if she did not know what the big point was. In an attempt to help her students not feel this way, she often summarized what was occurring in the classroom and believed that this was part of her role as teacher in the classroom.

The norms that seemed associated with student's rights, roles, responsibilities and expectations (although these were modeled by and seemed to be associated with Karla herself, too):

- Mistakes are okay to make (or I'll protect you);
- The big ideas are stated and people must take responsibility for their own learning;
- Contributions will be valued and credited;
- Uncertainty is something that should be acknowledged; and
- This classroom is a community with a common history.

Karla used hedging to protect students from blame when they provided incorrect answers. For instance, when she discussed incorrect answers, she told her students what they "seemed" to be "trying to" do. She stated in one of her interviews that she still "coddled" students and that she was continually aware that it was difficult for students this age to share answers. She wanted students to be able to contribute and one way that she might have encouraged this was to not single out particular students by avoiding explicit corrections and by hedging the correct solution method.

In terms of the "big ideas," Karla said that she sometimes told students what she expected they "can" do to "hold them accountable and again try to let them know that there are big ideas that they have to know to move on" (K-Interview 4, 1/07/00). Students needed to be responsible for their own learning, too, and she told them what they needed to know and what their options were for getting additional help if they needed. She also recognized that students seemed to be more responsible for their own learning than when they were in 7th grade: "By 8th grade they seem more responsible for their own learning, 'I've got to do something or I'm not going to be able to use a [inaudible] or know what's going on. I've gotta somehow be involved, whether that means just sitting here and listening and taking notes or speaking out in class'" (Interview K-2, 12/02/99).

Karla often revoiced student contributions, stating the name of the person who contributed the idea. In doing so, she let students know that their ideas were valued. She also wanted students to have ownership for their ideas. She also wanted students to know "that math is something that each person can do and come up with these things. It isn't just, 'the next page has an example and a definition to write down; I can come up with these things, too'" (Interview K-3, 12/17/99). Karla wanted students to know that they had the "power" to generate mathematical ideas themselves: it was not a process in which only mathematicians and textbooks could partake:

> I guess I just try to personalize it and give them ownership. It isn't just some mathematician out there that decided that multiplication and division patterns were non-linear. Robby, you found that and we all agree with Robby. Or, Mark's way of doing it, if it's making it shorter, what did that do to the data, instead of, "the book says now we're going to make it shorter." So, I guess by personalizing it, it's giving them more ownership in what's going on (Interview K-3, 12/17/99).

Students seemed to recognize this as they often shared their ideas and sometimes credited each other for ideas.

Uncertainty was acknowledged by Karla whenever students introduced novel ways of solving problems or went faster than she thought they should. She wanted students to know that it was okay to not be sure of something. In fact, this was another attribute that appeared to be adopted by students as they sometimes stated that they were "confused" or that they needed Karla to "say that in English."

Classroom community building was important to Karla. In fact, the main role she identified for her students was to "be a member of the class" (Interview K-2, 12/02/99). The way that she defined this included "be the ones making decisions, the ones that are going to give an idea, or respond to an idea." Karla also attributed her not wanting to be the "all-knowing" because she believed that when students made decisions about the content, it helped in the "community building." However, she also acknowledged that students didn't always buy into the role she wanted to play in the classroom: "But I don't know if they feel that their role is to say something, disagree, to be the judge. They still see me as doing that" (Interview K-2, 12/02/99).

Much of Karla's emphasis in the interviews focused on students and how they interacted. It appeared that many of these were also carried by and embedded in her talk, as many of the norms that I have uncovered are associated with her students and what she expected of them. During the teacher interviews, I found many of the answers Karla gave focused on the students first and the mathematics second. For example, I gave both she and Josh the same vignette of three students working together to solve a problem (see Appendix H for this scenario). Karla's initial reaction was, "I wish student three would say something more to student one and two. Personalities are probably a part of this..." (Interview K-1, 11/23/99). Much of what she said and did focused on students and the ways that they were expected to interact and participate in class.

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[1] For a more extended transcript from Karla's classroom, see Appendix N.

[2] See Appendix I for a timeline of events and where the examples took place within those events.

[3] More specifically, this relationship ends up being hyperbolic. However, this functional relationship is not really explored in depth because the focus at this point is getting students to differentiate between relationships that are linear and those that are not.

[4] These two problems have been included in Appendix J.

[5] This is the name of a unit that students completed in the beginning of their 7th grade school year.

[6] I am using this term in a Deweyan sense (Dewey, 1938)—it goes beyond a habit and becomes part of a person's disposition: that is, Xavier is seeking a continuous experience for himself by actively engaging in looking for connections between the problems in which he has engaged.

[7] This inference was not based on explicit statements by Karla about how the GC should be used in class. Instead, it was based on repeatedly observing Karla requesting that students use a GC to check their work in almost every class session I attended. It was almost habitual in that every time a solution was worked out, Karla requested that students take out their GC to check their solutions.

[8] This activity is given in Chapter 4, pp. 52-53.

[9] Karla used the word "they" to refer to the authors of the textbook. Her use here seemed to indicate that she was interpreting what they had intended in writing the problem, indicating that part of her role as teacher was to offer or interpret the author's intentions.

[10] Karla told me in her interviews that she named the person who talked because she believed that it was important for her students to have ownership of their ideas.

[11] This interpretation is inferred based on the fact that, in an interview, Karla told me that the big mathematical idea was very important for her in developing and pushing the classroom conversation. She said that she often re-read the teacher notes in the Teacher's Edition to remind herself of the "big mathematical idea" so that she could keep the conversation focused on this and so that it wouldn't diverge too much away from this goal.

[12] The extended transcript took place in about a 15 minute segment. It has been broken up into parts to include my description and interpretation of what was happening in each part of the transcript. Short portions were excluded that weren't related to the mathematical conversation (i.e. ones related to classroom management and about getting graphing calculators were deleted).

[13] See boldfaced text.

[14] I recognized these three arguments but they were not made an explicit part of the conversation.

[15] This is language I've heard them use in class to refer to other situations and that pertained to slope.

CHAPTER 7: JOSH'S CASE

In this chapter, I offer a case study of Josh's classroom, discussing the three main teacher discourse patterns apparent in Josh's talk[1]. These include: (1) questioning, (2) expressions establishing authority, and (3) statements about knowing. In each segment that follows, I give a brief account of the tool or concept of discourse analysis that I have employed and then offer examples and interpretation of Josh's use of them. I show how Josh creates an environment where questions are abundant, the mathematical authority resides with Josh and the text, and knowing is attributed to the teacher, the textbook and as characteristic of a "model" student.

As I elaborate on the discourse patterns, I also claim that certain norms are embedded and established in the classroom by these discourse patterns. In each summary section, I use Edwards and Mercer's categories to point out how the discourse patterns are associated with Josh's control of the common knowledge in the classroom. Before proceeding, however, I present a larger context of the classroom sessions by describing both the activities and discussion that took place in each of the two focus class periods[2].

Focus Class Periods

<u>October 6, 1998</u>

Prior to this class period, students had completed collecting data for Investigation 1. In that investigation, students measured the breaking weight of a bridge. This variable was compared with the thickness of the bridge—i.e. students increased the thickness of the bridge and then measured its breaking weight to determine how the strength of the bridge changed as the bridge got thicker. At the beginning of the class period, Josh had each group report orally the data as he recorded it on the white board.

After the data was recorded, there was a whole-group discussion about the data. Josh asked students if they noticed any consistency in the data and students talked about why each group's data was not exactly the same. Students then gave the averages for the data (which had been assigned as homework the day before) and there was a brief discussion about why he had asked them to do that. At the end of this discussion, Josh asked the class if they thought the data appeared to be linear or not.

To "find out," students were asked to open their books to Problem 1.2[3], "Drawing Graph Models" and Josh gave them graph paper to graph the class average data points. Students did this with their seat partner. Josh drew a co-ordinate system on the board and they discussed the scale they had chosen. After Josh drew a graph of the averages on the board, there was a discussion about whether students thought the graph was linear or not.

This discussion ended when Josh asked, "How are we going to show the linear pattern?" and called on Cory to read Problem 1.2 from the textbook. Problem 1.2 introduced a group of students from Maryland who were claimed to have done the same bridge problem and gave their data. In this problem, students are introduced to the terms "graph model" and "line of best fit." After Josh reiterated the meaning of these terms, he called on a student to come to the board to draw in what she thought was a "line of best fit" for the data.

The class then turned to discussing whether there were other options for the "line of best fit" and what those might look like. They also talked about using this line to predict values that were not included on the graph. Josh handed out a worksheet on which there were nine graphs. Students were asked to draw in the line or curve of best fit for each. Individual students were called up to the overhead and drew each. There was a class discussion about whether each is the line/curve of "best" fit. Josh reminded the class that the graph needed to be drawn so that predictions could be made.

Josh called on a student to read about "equation models" in Problem 1.3 and asked students to go back to the class average data from the beginning of the class period. Students worked in small groups to come up with an equation that went with the line of best fit they drew. The class discussed these equations as a group and Josh brought the discussion to a close by going through his own approximation and telling students which equations seem reasonable to him. No homework was assigned.

October 13, 1998

The previous day, students had taken the Check-up given between Investigation 1 and 2. Josh began this class period by returning this and going through the answers with the class. Students were then asked to open their books to Investigation 2, focusing on non-linear models. A student was chosen to read this bridge problem in which the class was asked to explore how the bridge length affected the strength of the bridge. Students then got into small groups and worked together to collect data and record it in tables.

A large table was put on the white board in which each group recorded its data. After each group put their data in this large table, Josh asked them to open their books to Problem 2.1 and do parts A, B, and C as homework. Students continued to work in small groups until they either finished the problem or the class period ended.

The Case

The case study that follows is broken into two main parts. In Part I, I offer an analysis of the three discourse patterns that were most prominent in Josh's classroom talk. These three patterns include Josh's use of: (1) questioning, (2) expressions that establish authority, and (3) references to "knowing." In each segment, I give a brief recount of the tool or concept of discourse analysis that I have employed and then offer examples and interpretation of Josh's use of them. I show how Josh creates an environment where questions are asked frequently, the authority exists with Josh and the textbook, and the sources of knowledge are "model" students, Josh and the textbook.

As I elaborate on the discourse patterns, I also claim that certain norms are embedded and established in the classroom by these discourse patterns. I contend that the following norms are embedded within the discourse patterns I describe.

- Through Questioning:
    - Pay attention to and agree with the common knowledge;
    - There are sub-questions that allow you to focus in on the content;
    - Know the reasons for the common knowledge;
    - Strongly established common knowledge is a decision to be made; and
    - Plurality is important in determining what is mathematically reasonable and what is not.
- Through Reference to Reasonableness and Agreement:
    - As *an* authority of mathematical knowledge, one of the teacher's roles is to clarify the content;
    - The textbook is part of the mathematical authority and should be drawn from in many ways; and
    - The teacher has the right to judge the reasonableness of a solution.
- Through talk about "knowing"
    - Model students want to know; and
    - "Knowing" sometimes resides in the textbook.

In the second part of this chapter, I summarize the discourse patterns in terms of the norms that are embedded within them. I draw on the patterns of discourse to make inferences about the social and sociomathematical norms Josh established for his students in terms of teacher and student expectations, rights, roles, and responsibilities. I also offer Josh's interpretations based on the ideas he offered in his interviews.

Discourse Forms and Functions

Questioning

Ordinarily, teaching involves the asking of prolonged sequences of questions (Bellack, Kliebard, Hyman, & Smith, 1966; Hoetker & Ahlbrand, 1969) that permit a teacher to control the direction and continuation of a discussion while assuming a necessary degree of attention and maintaining order (Westbury, 1973). A large range of literature has focused on classifying questions based on the type of information they elicit (for a review of some of this work, see Hargie, 1983). Others (e.g. Goody, 1978) have focused on classifying questions based on relationships: that is, there are certain types of questions that are appropriate to ask (or not) based on the relationship that you have with the person to whom you are addressing the question. This is especially true where there are issues of power and authority at play, which is definitely the case in relationships established between teachers and students in school.

Although much of the literature does not address the difficulties associated with classifying teacher questions[4], Churchill (1978) devotes a section in her book to discussing problems with determining whether a question has been asked or not. She associates this difficulty with the interplay between the forms and functions of questions. For example, if I say, "I wonder if this is a square," students may treat this as a question and "answer" it. I may actually be wondering aloud, demonstrating wondering about the situation at hand. This form can serve as a question, even if it does not end with rising intonation. If I say to a student, "Would you please go to the board and put up your solution?" the form denotes a question, but the intended function is a directive or command: that is, a student does not respond by answering, s/he responds by going to the board. Coulthard (1985) indicates that distinguishing forms from functions is important when analyzing discourse for similar reasons.

Both the forms and functions of teacher questions have been highlighted in educational literature. Since there is a plethora of literature related to questions in the teaching/learning environment, I shall explicate those forms and functions pertinent to the case study I will present. After I discuss some of the forms and functions of questions, I will delineate some teacher strategies identified in the literature that are used to help students answer the questions in an "appropriate" manner. Often when a teacher asks a question that is open-ended, students could produce many different answers. However, typically the teacher has in mind exactly which answer s/he will accept. I will discuss some of the ways teachers try to focus the questions they ask to help students "successfully" answer their questions.

Forms of Questions

Barnes (1969) analyzed teacher questions as falling into the following four categories based on the form of the question.

1. *Factual* ("What?" questions)
    - i. naming
    - ii. information
2. *Reasoning* ("How?" and "Why?" questions)
    - i. 'closed' reasoning—recalled sequences
    - ii. 'closed' reasoning—not recalled
    - iii. 'open' reasoning
    - iv. observation

3. *'Open' questions not calling for reasoning*[5] (e.g. What can you tell me about these two functions?)

4. *Social*

i. control ('Won't you...?" questions)

ii. appeal ("Aren't we...?" questions)

iii. other (p. 17, italics in original).

During the 1960s and 1970s, the vast majority of teacher questions were shown to be "*factual*" or "recall" in nature (Bartolome, 1969; Davis & Tinsley, 1967; Gall, 1970; Gallagher, 1965). More recent studies have not been carried out to see if this is still the case. Teachers asking this type of question will find out whether their students know specific mathematical facts, but they will gain little, if any, information about whether the students actually understood a given concept.

Reasoning questions require pupils to "think aloud" (p. 17): that is, they require students to construct, or reconstruct, (from memory) logically organized information. These questions may be "open" or "closed[6]" in nature: that is, they may have a number of different acceptable answers, or they may have only one. In order to determine whether a question is "open" or not, we need to examine the teacher's reaction to student responses (Edwards & Furlong, 1978). Often a teacher asks a seemingly open question, but will only accept one answer. For example, "What could be an equation for that line?" may be an 'open' question, inviting exploration of possibilities. However, if the teacher will only welcome the simplest form of the equation, the question is no longer "open"—there is only one acceptable answer. Barnes calls questions that appear open "pseudo-questions" (p. 17). Reasoning questions may also pertain to what students observe, i.e. observation questions. These questions are associated with phenomena that are immediately present to students and focus on their insight related to the phenomena.

Social questions are related to imposition. If a control question is asked, a teacher is directly imposing his/her wishes on a student. The less directive form, appeal questions, ask pupils to "agree, or share an attitude, or remember an experience" (p. 18). Barnes considers an appeal question to be less directive because it is possible for students to reject them without necessarily offending the teacher. However, the wording of a question sometimes makes it difficult for students to disagree. One commonly used form that this particular type of question may take is that of a "tag-question."

A tag, in its "usage as well as its syntactic shape is midway between an outright statement and a yes-no question: it is less assertive than the former, but more confident than the latter" (Lakoff, 1975, p. 15). Therefore, this form is often used in a situation in which neither a statement nor a question is appropriate. A tag question is used "when the speaker is stating a claim, but lacks full confidence in the truth of the claim" (p. 15). The way that a tag question is worded favors positive or confirming responses. For example, if I ask, "Is this a linear function?" I am less likely to receive an affirmative answer than if I say, "This is a linear function, isn't it?."

In some situations, tagging a statement or question is more legitimate than making an outright statement. For example, if I have heard/seen something only indistinctly, and have reason to believe that someone else has had a better view, I may ask him/her about it in tag form. An example of this would include: "The teacher said the quiz is Tuesday, didn't he?" We also find tags when the speaker knows as well as the addressee what the answer must be, and doesn't need confirmation. For example, when making small talk: "It's a nice day, isn't it?" In some cases, tags are used to seek corroboration with the speaker's opinions. There are many possible interpretations of tags like these. One includes the possibility that the speaker has a particular answer in mind—yes or no—but is hesitant to state it boldly.

When a tag form is present it does not necessarily mean that the information that proceeds it is a question. It may, in fact, be a tagged proposition: that is, a proposition followed by a tag. If there is not a raised intonation with the tag, the hearer may respond with something like "Mmm," marking that they are merely taking their turn and not really engaging in answering a question. Judging whether a tag is a question or not is interpretative and partially depends on the presence (or in the case of a proposition, absence) of a raised intonation. Interpretation of tag forms also relies on the larger context of the situation. In schools there are issues of power and imposition that may not be as firmly present in other contexts and this must be considered when explaining tags.

Lakoff claims that tags are much more likely to be used by women than men because they provide "a means whereby a speaker can avoid committing himself, and thereby avoid coming into conflict with the addressee" (pp. 16-17). The problem she associated with this is "a speaker may also give the impression of not being really sure of himself, of looking to the addressee for confirmation, even of having no views of his own" (p. 17). This last criticism, Lakoff argues, is the one most often leveled at women.

Britton (1969) relates tag questions and similar expressions (e.g. 'sort of', 'you know') to what Bernstein (1971) called 'sympathetic circularity'. "They constitute an appeal to the opinions and sentiments held in common by the group" (p. 108). Tag questions are appeals for confirmation. One may think that these expressions of sympathetic circularity appear only in mutually supportive speech (p. 108). Bernstein, however, suggests that when speakers of 'a restricted code[7]' are put in the context of explaining differences, they will tend to use the question tags to minimize 'sociological strain'—the stress, perhaps, they feel when group solidarity was reduced. Britton points out that "as the argument warms up, 'like', 'sort of', 'I mean', 'you know', 'don't you', and so on tend to be replaced by the more emphatic form of appeal, 'right [?]'" (p. 109).

This 'sociological strain' could be interpreted by politeness phenomena as face-threatening acts. Strategies for positive politeness are those which express approval of the hearer or approval of or concern for the hearer's wants, or convey in-group membership (which are similar to sympathetic circularity). Bills (2000) indicates that inclusion is a prevalent strategy that teachers employ to help students save positive face. This may be done by using 'we' or 'us' (what Edwards and Mercer call "royal plurals") to include the speaker and hearer or by using tag questions to seek the hearer's agreement or approval.

Functions of Questions

Ainley (1988) focuses on the purposes of teacher questions, naming tag questions "pseudo-questions." She claims that these questions are used to "establish acceptable behavior, and social contact" (p. 93). The other purposes she delineates for teacher questions include: to get information ("genuine" questions), to find out if the subject knows the answer ("testing" questions), and to provoke the subject to think further about a problem ("directing" questions) (p. 93). One main difference between genuine questions and testing questions is that in the first the teacher does not know the answer; in the latter, the teacher does know the answer and students are aware of this fact. Directing questions are broken into the following three sub-categories:

a.) structuring questions, typically a sequence of questions which 'activate' the subject's existing knowledge in such a way that new connections become clear,

b.) opening-up questions which suggest new areas of exploration, such as 'What would happen if…', 'Why do you think…?'

c.) checking questions which encourage pupils to think again about a statement, such as 'Are you sure?', 'Is that right?', 'Do you agree?' (p. 93).

Difficulty may arise when interpreting a question as "genuine" or "testing." One way to help differentiate these is by using Mishler's (1972) "structure of alternatives" (i.e. the numbers and types of alternative answers a question might allow and the structure that these might have). Mishler delineates three ways that teachers may distinguish the options they allow:

1.) a set of alternatives organized as a "tree" with earlier answers branching into later possibilities,

2.) an array of points scattered about a topological "region," and

3.) a "matrix" within which the answer is to be found (pp. 282-283).

In the first, the questions are more divergent and the conversation is like a tree in that each new question depends on a previous answer. This branching process does not appear to have a predetermined end. The second focuses students on a region of possibilities: that is, the points in the region are not connected or divided into clusters, but the correct answer lies somewhere in the region. The last arranges a fixed set of alternatives, among which lies the correct answer. The first two categories seem to be enveloped in what Ainley calls "genuine"; the last seems to be similar to the "testing" questions because there is one right answer.

Bills (2000), drawing on Ainley and using the framework of politeness phenomena, indicates that what is intended when asking a question may not be interpreted in the same manner. For example, a teacher may be asking a question to assess what a student knows or to find out where a student is in terms of the content, but the student may interpret this strategy as a way that a teacher is being polite. Students may "see the teacher's questions as an attempt to save their face, whilst pointing out their mistake, rather than as any sort of pedagogic strategy" (p. 44). Bills contends that this may mean that students see questions predominantly as intended to convey that a student has made a mistake. The student, in turn, may respond by assuming a mistake has been made and then second guessing him/herself instead of attending to the literal content of the question. This may be problematic because "the student will 'politely' correct it [the mistake] without engaging with the reason why the correction was needed" (p. 44).

Ainley (1987) discusses the relationship between questioning and telling. She points out that:

> *any* type of questioning conveys information. By asking questions you indicate what is of interest to you. [...] this is emphasized by the fact that the student believes that the teacher already knows the answer(s). One reason for asking questions is precisely to draw attention to something (p. 26, italics in original).

Furthermore, these types of "focusing" questions (i.e. ones that draw attention to something) (Love & Mason, 1995) are common to teaching situations. In fact, in some ways, by pointing out certain aspects and not others, the teacher may be controlling the direction of a lesson along a pre-determined route.

This "predetermined route" may be similar to what Edwards and Mercer refer to when they claim that most questions do not serve the purpose of getting information. Instead, they see teacher questions as "part of the discursive weaponry available to teachers for controlling topics of discussion, directing pupils' thought and action, and establishing the extent of shared attention, joint activity and common knowledge" (p. 46). By asking questions, teachers are letting students know what is important in the activity in which they are engaging, controlling what students attend to and establishing ways of talking and noticing. They help to define and control the common knowledge in the classroom because they show students what is important (from the teacher's viewpoint).

Teacher-Strategies for Helping Students Answer Questions

As we have seen, teachers ask many types of questions for many purposes. In most cases, the teacher already has an answer to his/her question in mind, even if the question is of the "open" form (i.e. a question for which there are many possible answers). However, the success that students have in answering these questions varies. In many cases, the students do not produce an "acceptable" answer and in others they produce no answer at all. Analyzing discussion in preschool classrooms, French and MacLure (1983) have outlined two interactive strategies that teachers may use to help students produce the answers they are seeking: preformulating (PF) and reformulating (RF). Preformulating "is a strategy which teachers use, in advance of asking some question, which orients children to, or provides them with, material prerequisite to the provision of an appropriate answer" (p. 201). There are multiple types of PF questions that relate to the focus questions (called nuclear utterances or NU). In some, the material that the teacher establishes as shared knowledge is physically present and an answer is not expected because the PF is directing student attention. In others, the teacher does require the children to give an answer to the PF, thus enabling her to check that they have oriented to the shared material (which is a past event or something the teacher supposed to be "common knowledge" and not perceptually present), before she asks the question. When teachers introduce new interpretative recipes and bodies of knowledge, a PF then refers to information to be known only to the teacher, as the basis for answering her nuclear question (pp. 198-201).

What do teachers do when their initial attempt to help students "appropriately" answer their questions has failed? One strategy they employ is that of reformulating (RF) the question, making the range of possible answers smaller. The three-part organization of a reformulating sequence is: 1.) Nuclear Utterance (NU) from the teacher (Wh- (or sometimes how-) question), 2.) No/Inappropriate answer from the students, 3.) Reformulator (which makes the teacher's question more specific). There are five types of reformulators (pp. 204-207) given in Table 6. These are given in terms of how much they

Table 6
Five Types of Reformulators

Type of RF	Definition	Example
Type 1	Teacher indicates that the appropriate value for the variable lies within a range of a subset of values; e.g. "quadrilateral" is a subset of "polygon"	(NU) What polygon is that? (no response) (RF) What quadrilateral is that?
Type 2	A yes/no question is used to make the original question more specific. The teacher selects one particular value for the variable that is *inappropriate* and expresses it as a yes/no question. The preferred answer is a negative one and is typically a member of the same semantic set as the desired 'right' answer, and often is its polar opposite or antonym.	(NU) What type of relationship is that? (RF) Was it a LINEAR relationship?
Type 3	Superficially like Type 2 in that they have yes/no form, also. However, here the preferred answer is the positive choice—the teacher presents an *appropriate* value.	(NU) What else did you see? (no response) (RF) Did you see a linear pattern?
Type 4	The teacher presents a pair of values for the variable. However, one of them is preferred and appropriate and one is	(NU) How did they decide? (no response)

	inappropriate.	(RF) Did they look at the graph or the table?
Type 5	The teacher actually states the correct answer and merely requires the child to confirm it. The teacher has done so much in specifying her original question that there is little left for child to do—s/he only has to confirm what the teacher said.	(NU) What's the reason for doing graph models? (no response) (RF) Is it just to see the pattern? (no response) (RF) It's to help make predictions, isn't it?

narrow the range of appropriate answers (type 1 narrow it the least...type 5 the most) students have to choose from when answering the teacher.

Whether such simplifications do more in the end to help a child answer a particular question is an important empirical question. However, one paradox of the teacher-question-student-answer structure is that pupil's answers are important to the lesson advancing, and yet the answer anticipated by the teacher is often not apparent (Cazden, 1988). Thus, even though a teacher may ask questions to move a lesson along it may actually prolong the lesson because students may not be giving the answer that the teacher is expecting. These "guessing games" that teachers play with students, Edwards and Furlong (1978) contend, are just one way of controlling knowledge and children—"one can ask more direct questions, for example, or insist on silent attentiveness and recitation of rituals" (Edwards and Mercer, 1987, p. 30).

The Asker of Questions

The predominant discourse pattern in Josh's classroom was that he asked many questions. In fact, some of his turns had as many as five questions in a row. These questions had many of the forms described and served many different functions. About two or three times per class period, he even explicitly said that he was going to ask a question, drawing student attention to the question through a metacomment[8]

Example J-1

> Okay—**let me ask this question again**. How many of you guys think this is a linear relationship between bridge thickness and the breaking weight? How many think it's not? How many aren't sure?

Often these questions were "assessing" in nature; they checked to see how many students did/thought particular things, possibly allowing Josh to stay in touch with a wider range of students than just those who often contributed to class discussion.

Table 7 summarizes the forms and functions of many of Josh's questions in class. In this section, I shall describe more fully the functions of these recurring forms in Josh's classroom.

Table 7
Josh's Questions: Forms and Functions

Form	Function	Example	Other Attributes
Tag questions	To establish common knowledge and to check to see if students were listening	When it goes up by a constant rate it's linear, isn't it?	May have used this form to be polite when students answers were not precise enough
(Proposition) + Bare "Why" questions	To draw attention to the common knowledge and reasons for it	When we see a line that slants down, the slope is negative. Why?	Many questions began "open" and became "closed" through follow-up questions
Focusing questions	Important sub-questions that get to the common knowledge	What's the rise?...What's the run? ... What's the slope?...What's the y-intercept? ...What's the equation for the line?	Appeared mainly when assessments were returned to students—Scaffolding did not appear to take place
Dichotomous questions	To make a mathematical	Is it linear or not?	Part of firmly established common

	decision		knowledge
Questions that allowed various solution methods	A limited invitation to diversity	Does anyone have something else?	"Limited" because Josh was judge and jury of ideas

<u>"(proposition), isn't it?," "(proposition), right?": Pay attention to and agree with the common knowledge</u>. Many of the turns Josh took began with tag questions (Lakoff, 1975) that often functioned as pseudo-questions (Ainley, 1988). In fact, these questions seemed, more particularly, to serve two important functions in Josh's classroom: 1.) as a way that Josh politely added to student contributions, and 2.) as a way for Josh to highlight and demarcate the formation of common knowledge in his classroom.

Most of Josh's tags began in a similar way to a reformulator (RF): that is, they were prefaced by a wh- or how question. However, the form deviated from the structure of a reformulator in that students actually answered the question. In response, Josh then revoiced the answer, ending with a tag. For example:

Example J-2

J: Well, how many think the classroom average is gonna be more accurate than just looking at one of the groups? Why? Laurie, why do you think this is gonna be probably a more accurate picture than looking at any one of these?
Laurie: 'Cuz you got it over a series of data
J: It's a lot more data to look at, **isn't it**?
Ss: Mm-hmm

Example J-3

J: Why does that give you the run? (4 second pause) Why? That's where one of you guys-
Cory: That's what's on the x-axis
J: That's what's on the x-axis, **isn't it**? You're going from zero on the x-axis/ to four on the x-axis. That's the run. That's the horizontal change. So if the run is four, what's the rise?/ You're going from negative three on the y up to/ one/which is?

In Example J-2, Josh began by asking "how many think" the classroom average will be more accurate than just looking at an individual group's data. In framing the question in this manner, he assessed how many people agreed with something that had been discussed the previous day. However, he moved beyond the assessment to pushing students to justify by asking "Why?" (the wh- question). After no response, Josh reformulated (Type 1) the question, directing it to one student and making it more precise (i.e. he included the proposition the "why" question was actually about—Why is it more accurate?). Laurie responded with a somewhat vague answer and Josh revoiced, making the contribution more exact.

When he revoiced, he tagged the proposition. In doing so, he appeared to be doing at least two things. First, he may have been checking to see if students were listening to what Laurie said. They showed that they were, indeed, listening when they responded with "mm-hmm." Another possibility was that they have learned what was expected of them following this type of question and they "cued" the expected response. In other words, they may have tuned into their slot in the I-R-E sequence so often established in classrooms. This format is very evident in this exchange (and many others) in Josh's classroom—he asks a question, a student answers it, and (in this exchange) Josh offers a positive evaluation by revoicing what the student said. By choosing a Type 5 reformulator, Josh merely required students to confirm the ideas that were put forth.

Another function this tagged proposition may have had was to indicate and focus student attention on part of the common knowledge that had been established in this classroom. This is similar to Ainley's interpretation of pseudo-questions in that something "acceptable" is the focus. However, instead of focusing on "acceptable *behavior*." Josh focused on an "acceptable *answer*." In using a tag, he softened the proposition rather than stating it baldly.

In Example J-3, Josh asked why the horizontal change (referred to as "that") gave the run. After a four second pause and no response, he shortened his question to just, "Why?" He appeared to begin to talk about the run specifically in terms of an answer given on the quiz when Cory answered his question. Josh revoiced what Cory said and tagged this with, "isn't it?" Again, this exchange followed an I-R-E format.

By tagging Cory's response, Josh appeared to be checking to see if students agreed with something that he assumed to be part of the common knowledge of the classroom at this point in time as well as affirming Cory's correct response. The "run" of a line was introduced in the previous unit (*Moving Straight Ahead*) and had been the topic of discussion numerous times in the past month. After Josh repeated Cory's contribution, he continued by detailing the process for thinking about the "run"—you "go from" zero on the x-axis to four. He then asked what the rise was and almost immediately broke this idea down in terms of moving from the negative three to one on the y-axis.

Related to this use, Josh also reformulated a student contribution, and then added information and tagged this addition. In doing so, he may have been attempting to lessen the impact of changing the student's response. In tagging the additional information, Josh may be trying to be polite to the student—allowing their ideas to come forth without his contribution appearing to "correct" what the student has said.

Example J-4

J: [...] Half of this class/ I bet at least half of this class circled/ the third table [6 second pause-writing on the board] Is that linear? [higher tone]
Ss: No [overlapping one another]
J: There's a pattern/ What's happening?/

Abram: **It's doubling**

J: **It's doubling each time**/ It goes up by two, then it goes up by four, then it goes up by eight, sure there's a pattern it's doubling, but that's not linear [3 sec] If you graph that, you're not going to get a line, you're gonna get some kind of a curve that's getting steeper as the x increases [3 sec] **aren't you**?

Ss: Mm-hmm

Example J-5

J: Not bad? 17.4, 17?

[Ss comment, overlapping so talk is inaudible]

J: 23.2, 23.

Ss: **Not bad**

J: **Not bad at all**? Last point gets a little ways away from our data, but when we graphed it on our line, that point was away from our data, too, **right**?

Abram: Yep

Example J-6

J: What's the slope of that line? First of all, positive or negative slope?

Ss. Negative

J: It's going down, **right**?

Ms: Yep

J: So, take a look at this, what is the rise? You're at five/ on the y/ and you end up at/ one at the y? What's the rise?

Ss: Four; **Negative four** [overlapping each other]

J: It's **negative four**. You're going down, **aren't you**?

Ss: Yeah

Example J-4 began with Josh stating that half of the class made the mistake that he was going to address. The table that students were given had a pattern, but it was not a linear pattern[9]; it was doubling. Josh seemed to be trying to make the point that the pattern had to be a particular type to be linear. In this case, he asked students what this table was doing—it was doubling, not going up additively. Josh expanded Abram's contribution by adding "each time," indicating positive feedback to Abram's response. He then continued his turn by detailing the pattern in terms of addition. The additive pattern +2, +4, +8, which is not constant (although he does not state this latter fact). The point was made again when Josh said, "sure there's a pattern, it's doubling, but that's not linear." An additional representation (i.e. the graph) was then brought into the discussion to further substantiate his point: "if you graph that, you're not going to get a line, you're gonna get some kind of a curve." When students failed to respond (denoted by the three second pause), Josh checked to see if they agreed by tagging his proposition. He appeared to be checking to see if students agreed with what he was putting forth as part of the "common knowledge"—that a doubling pattern was a pattern, but not a linear one. He also seemed to be focusing student attention on the common knowledge. However, by choosing a tag form, Josh cued a positive response, which may not have allowed him the insight that he sought: Do students think ___?

The connection between the table and graph again surfaced in Example J-5. In this example, Josh compared data from the table of values that the students collected and the table associated with the equation model students approximated; that is, in class they got 17.4 for one y value and their equation model they wrote generated a value of 17 (similarly, the class got 23.2 and the equation model generated a value of 23). He asked students if they thought the approximation was a good one or not, indicated by "Not bad?." When Josh read off the values 23.2 and 23, students verified that this was a good estimate ("Not bad"). Josh expanded this by adding "at all" and then connected the table they observed to the graph they had explored previously. When they drew in the graph model, the last point on the model deviated slightly away from the line of best fit. Josh pointed out that it makes sense that the last point on the table would be further away than the others because it had been on the graph. At this point in the discussion, there were multiple references to the fact that the graph, table and equation models are connected. In ending his turn with "right?" Josh appeared to check to see if this idea was common knowledge and to assess whether students were listening. Abram suggested he was listening by responding "Yep."

In Example J-6, Josh tagged additional information twice. Actually, the additional information that he tagged was exactly the same (but takes place in two different places in the discussion). The proposition that Josh asserted each time was that if the slope of the line was negative, then the line would be "going down": that is, if students noticed that the line was slanting downward (from the left to the right in the graph), the slope would be a negative real number. The first time that Josh tagged this, he asked students if the slope was positive or negative. When students replied that it was negative, he checked to make sure that students thought this because the line was going down. At least one male student agreed with this. However, it appeared that some students did not grasp this part of the "common knowledge" because in the next turn Josh asked the question again and some students replied with a positive response, even though the line was going down. Josh picked up the correct answer that he heard (ignoring the one that was incorrect) and then followed this with, "You're going down, aren't you?" Again, he appeared to be checking to see if students were paying attention and was also defining part of the common knowledge by asserting the proposition prior to the tag. By tagging the proposition, he was attempting to be polite to those who got the answer incorrect.

In each of these examples, after Josh initiated a response through asking a question, he restated or reformulated something a student said and then added further detail to the contribution. Each time he tagged this additional information. This may be one way that Josh was polite to students. Instead of asking them to be more exact or for more detail, he restated/reformulated the student-contribution and then tagged the pertinent detail that he added. In doing so, Josh still proclaimed his ideas, but was doing so less baldly. In this way he may have added credibility to the student response, but made it more detailed and exact. In Edwards and Mercer's language, he may have been using this form as a way of "interpreting student contributions" to make them more mathematically correct and related to the mathematical goals of the lesson.

In light of Lakoff's claim that tags are more often used by women, this seems a peculiar language pattern for Josh who is a mathematically knowledgeable, confident, experienced male teacher. I contend that he used this type of language to help students save face: that is, to be polite and not embarrass students when they gave an answer that was not as precise or detailed as Josh might want it to be.

The larger context of "school" is critical to this issue. Within school there are many issues related to power and imposition. The teacher, by the very nature of his role in the classroom, is continuously threatening students' face—" constraining their freedom of action; evaluating, often negatively, a high proportion of student acts and utterances; and often interrupting student work and student talk" (Cazden, 1988, p. 163). Teachers will mitigate their face threatening acts with some form of redressive action: in this case, Josh relied on the softening effect of a tag form. One aspect that determines the participants' assessment of face threatening acts is the issue of power[10]. The perception of power is dependent upon "teacher's confidence in her authority, and on family and peer-group respect for that authority" (p. 164). In Josh's case, he appeared to have the utmost confidence in his role as authority in the classroom.

Not only did tagging play a role in Josh's being polite, but it may also aid in establishing common knowledge in the classroom because what proceeded the tags were important propositions that pointed to connections (between representations in these examples) and details associated with bigger mathematical ideas (e.g. linear functions). Josh may have been using this form as "joint knowledge marker"—drawing student attention to the common knowledge of the classroom.

"Proposition" + "Why?": Know the reasons for the common knowledge. Another question type Josh used was similar to the tag form. In this form, Josh would also begin by stating an important mathematical proposition. However, instead of ending the proposition with a tag, Josh would often follow up with a bare why-question. This form seemed to have a similar function to the tag-form: it served to draw student attention to important common knowledge and to point out that knowing the reasons for that common knowledge was important. The proposition was sometimes stated by Josh and at other times was stated by a student when s/he answered a question Josh had asked.

In the following example, Josh discussed the inconsistencies in the data students had collected in the bridge activity. He then turned to talking about the averages that they took. In the extended example, a proposition is stated (by Josh) and is accompanied by "Why?" Later, a proposition is stated by a student and Josh followed up with "Why?"

Example J-7

J: These [i.e. the y-values that each group had collected] are pretty consistent, again this group for some reason had some pretty low numbers until they got to the breaking part of [inaudible] these I think were actually very consistent, um, I **wouldn't expect it to be all exactly the same. Why not? Why** did we get all, **why** didn't all the groups get the same data for each and every layer? Abram?

Abram: Cuz, like, maybe, like, they didn't have an exact inch [overlap] or something.

J: Yeah, there's okay--

Abram: They might have [inaudible]

J: That might have affected, now I tried to make sure you guys were only having a one inch overlap, but, as I was walking around, some of you maybe didn't control that very well. Anything else that might effect the data?

PJ: **People may have used the same paper over and over again and the paper got weaker**
J: You don't think so? Did you hear what he said? Say that again. PJ
PJ: That group probably [inaudible]
J: **Why**? What makes you think that?
PJ: Well, maybe they got the paper [inaudible]
J: So, once it snapped, the paper was weakened, so it doesn't hold as many pennies when you use it?
Abram: Looks like that group dropped it from higher or something, put like more pressure on it
J: I don't know. I don't know. Well, let me ask this question before we look at the averages. Um, or, let's go ahead and **look at the averages first**. What'd you guys get? What was the average breaking weight for a bridge with thickness of one layer? Carl, what'd you find?

Referring back to this example shows how the I-R-E structure appeared in Josh's classroom talk. In the example, Josh began by initiating with a question: "Why did all the groups get the same data for each and every layer?" He then called on Abram to answer his question and Abram responded. Josh proceeded by evaluating Abram's response—he said that "might" have affected the data and then indicated that this probably was not the reason because he had attended to that as he had walked around. Josh then initiated the next response by asking, "Anything else that might effect the data?" PJ responded and Josh asked him to repeat what he said so that everyone could hear it. This would be considered a deviation from the I-R-E structure because instead of evaluating the answer right away, Josh asked PJ to repeat it so everyone else in the class could hear it. Instead of evaluating PJ's repeated response, Josh asked him why he thought that might be a reason. PJ responded and Josh indicated that this was a valid reason by repeating what he said. Abram then offered an additional reason without Josh initiating it and Josh did not recognize the contribution. Instead, he controlled the pace and direction of the conversation by saying, "I don't know. I don't know. Well, ..." The phrase "I don't know" seemed to function as a way for Josh to disengage. That Josh was ready to move on to another topic was further indicated by, "Well, let me ask..."

In the first part of this extended example, Josh began by stating that he did not expect the data each group had collected to come out the same. This assertion was followed by "Why?" He rephrased the question by putting the "why" in front of the previous assertion, possibly because students had not immediately responded. Abram began to explain this difference in terms of how much overlap each group's bridge might have had. In the directions, students were asked to have the bridges overlap the books by one inch on each end. If students had not been careful, they would have had variation in the length of the bridge, which in turn would affect the strength of the bridge. Josh may have indicated that this answer was not really the reason for which he was searching when he pointed out that he "tried to make sure you guys were only having one inch overlap." In other words, he had attended to that, so it probably was not the reason they had so much variation in their data collection.

When Josh asked for additional reasons, PJ offered the possibility that some groups may have used the same paper over and over, asserting that this might have made the paper weaker. Josh asked PJ to state that proposition again and then requested that PJ explain his thinking. In doing so, Josh is "metacommenting," pointing to the fact that PJ has said something to which others should attend. Also, by cueing PJ to repeat what he said, Josh controlled who said what when, acting as the medium through which all classroom conversation should pass (Edwards and Mercer, 1987; Cazden, 1988); that is, he "retrospectively elicited" (Edwards and Mercer, p. 132) PJ's response, inviting a student response after it had already been made. The fact that Josh did not require Abram to explain why he thought the overlap would matter, but did ask PJ to justify his thinking may insinuate that PJ's answer was novel or that his reason was not apparent or well articulated. This was further apparent in Josh's amplification of PJ's response. Josh deemed his response important enough to repeat it aloud. Abram added another possible explanation to the original question and Josh replied, "I don't know. I don't know. Well, let me ask you this question…." In responding in this manner, it appeared that Josh has had enough reasons and was ready to move on. "I don't know. I don't know" seemed to signal Josh's disengagement from the conversation. These statements really do not mean that Josh "doesn't know" and they are said at a much lower volume than the other statements that Josh has made. The word "Well" indicated that Josh was channeling the direction of the conversation to talk about the class average. In doing so, he controlled the pace of the classroom conversation and marked the fact that he was ready to move on to the next topic of discussion.

Example J-7 (continued)

[…] [a few minutes later—after going through the table of average values that the class calculated]

J: 33? Christy, that's what you got for the average? You rounded to the nearest penny? Okay, now, **why** did we take a classroom average? [Ss are quiet] Cuz I just wanted to make you guys work? **Why** the classroom average? PJ?

PJ: Cuz those are-- not all of them are correct so we took all of them and average them

J: Well, how many **think the classroom average is gonna be more accurate than just looking at one of the groups**? **Why**? Laurie, **why** do you think this [the average value] is gonna be probably a more accurate picture than looking at any one of these?

Laurie: 'Cuz you got it over a series of data

J: It's a lot more data to look at, isn't it?

Ss: Mm-hmm

During this part of the conversation, Josh wanted students to think about why they were finding averages (rather than using the actual values that each group had collected).

Josh appeared impatient with the lack of student response when he answered his own question with a nonviable reason : "Cuz I just wanted to make you guys work?" In doing so, Josh was not asking a rhetorical question, but clearly the opposite; he responded to his own question with a somewhat ridiculous answer (in question form) and then repeated his original question.

PJ's response appeared to be not what Josh was looking for because he reformulated his question and in the process did not incorporate any of PJ's words (as he often does when he evaluates a student contribution). In a sense, this reformulation was a negative sanction for PJ's contribution. Josh reworded the question to make it more assessment-like ("how many think...") and more focused—he no longer wanted to know why they took the average, but now wanted to know how many thought the average was "more accurate" than any individual group's data. By reformulating the question in this way, the possible range of answers appeared to be similar to those a Type 3 reformulator allowed. Students now knew part of why the average was taken—it was more accurate.

However, Josh did not end the discussion here. Instead, he pushed further by asking why students thought the average was more accurate. Laurie answered this question with a somewhat vague answer[11] and Josh reformulated and tagged her contribution, checking to see that students were listening and drawing their attention to an important mathematical idea: averaging a large set of data will give you a more accurate representation than just taking data from one collection.

Josh did not always reformulate his "why" questions to help students answer them in the manner that he was seeking. In the following example, Josh discussed the values that students were going to use on the x- and y-axis when graphing the data in the previous example. During the discussion, he asked the same question twice. This example was complicated and unusual in that there was a tentativeness in Josh's voice that was quite uncommon and that contrasts with Josh's often strong "I" voice. It appeared that Josh was trying to make a small point about something that was not part of the common knowledge. However, since Josh's "I" voice was normally very strong, he needed to soften his assertions a lot, adding many hedges to his normal pattern of talk.

Example J-8

J: Five is—the way you want to go. Okay, um, I don't necessarily agree with the scales. If I was gonna do this, I might, **I think a scale of 2 might have shown a little better on the y-axis and I possibly would have gone by a scale of .5 on the x axis** too, **why**?

PJ: Why? It goes up by ones

J: It does go up by ones, so **why would I even think about using a scale of .5** on the x?

Ms: [quietly] To spread it out?

J: [no response right away] You guys don't have to agree with me, I'm just saying that that's what I would have done. **Why** would I have chosen a scale of point 5 instead of--

Lisa: To spread it out

J: Right, to spread the data out, to get a little better picture instead of cramming all your data into one little corner of your graph, but, um, anyhow so this is what you guys wanted to do, so, I'll do it that way. [graphs points] So, one layer's going to hold five pennies? Two is gonna hold 11. Just about to 10. Three is going to hold 17. Let's see, 15, some, right around there?

Fs: Yeah

In this example, Josh asked students what values they used when they graphed the data from their table. Students indicated that they used a scale of one on the x-axis and a scale of 5 on the y-axis. Josh stated that he did not "necessarily agree" with what they had done and shared the numbers that he "thinks" he "might" have "possibly" used, "if" he were going to do this. In hedging in this manner, Josh may have suggested to students that he was not very certain about the numbers he had chosen. These hedges may have been an attempt to soften Josh's proposition because he was making a point that was not central to the mathematical goals of the lesson[12]. In fact, the values that the students had offered made more sense to the given context and were actually more similar to values they had used for graphs they had made in the past.

PJ seemed to pick up on Josh's uncertainty because he questioned Josh explicitly. In doing so, PJ crossed into the realm of something that has been designated as part of the teacher's role in this classroom: he asked a direct why-question. Rather than address PJ's question, Josh restated part of PJ's point, implying agreement. He then re-seized the initiative and suggested that his role was to ask the questions; he again asked the question with which he began this conversation. This may have hinted that PJ's response was not acceptable. It was pretty clear that Josh's role was to ask the questions; he wanted the students to do the thinking. He may have indicated that other students in class needed to think about the question he had posed and wanted them to answer PJ's question.

After Josh's attempt to get a response failed, he began his next turn with, "You guys don't have to agree with me." By making this explicit statement, it was evident that typically students do need to agree with him. This declaration connoted a break down in the way things are normally done in this classroom. Students may been thinking, "Yeah, but that means we're *supposed* to use those values." There was further evidence that this interaction was not typical in the unusual amount of hedging Josh was doing: "*If* I was gonna do this, I *might*, I *think*... *might* have shown a *little* better...I *possibly would* have...." "I'm *just* saying...." "that's what I *would* have done" (if I had done it).

Laurie interrupted Josh's restated question with the correct answer. Josh finished the I-R-E sequence with an explicit evaluation of her response. He then reformulated her contribution to include the fact that spreading out the data will allow a better picture of the graph than the one that the students had recommended. In doing so, he offered his interpretation of Laurie's contribution, making the reasoning more explicit and controlling the joint understandings that were being formed.

In some instances, it became apparent to me as an observer that there was a specific answer that Josh was searching for through his repeated "why" questions. In the following example, the class discussed graph models and read a section in the text about this. Josh moved the conversation to why they were doing graph models.

Example J-9

J: [...] **Remember what a graph model is—it's a line or a curve that shows the trend in the data, it fits the data**. So that all the points are pretty close. Um, I don't know. **Why** do we do this? What's the purpose of a graph model? Abram, what's the purpose?

> [Abram's hand goes up right away]
>
> Abram: To show the linear relationship

J: Yeah. **I could maybe see** that it's linear just from looking at the table or if just looking at the way the plots are pointed. **Why** did I draw the line in? Just to show the pattern? Christy?

> Christy: To get a better look at what the data is trying to tell you

J: Well, **maybe that's part of it**. Look back at your definition for a graph model. Look at your definition of a graph model. What does it say? Read that last paragraph to yourself on page 7. Lance, what's the purpose here? **Why** do we even bother doing this?

> Lance: So you can make predictions [reading this directly from his notebook] values between values in the data

J: **Oh**, so if I want to predict how many pennies of thickness nine is gonna hold? How many? You tell me. Use your graph model, use your graph, to predict how many pennies a bridge thickness nine is going to hold. How many? Raise your hand, how many pennies a bridge of thickness nine layers is gonna hold? Cory, what'd you say?

Josh began by preformulating his upcoming question, reading the definition of graph model from the textbook. He then turns the discussion to address why they do graph models—which he rephrased three times in a row. He moved from "why do we do..." to "what's the purpose of..." to directing this question to a student that raised his hand. This shift from a why-question to a what-question was common in Josh's reformulators. In Barnes' terms, this shift represents movement from a "reasoning" question to a "factual" question, possibly lowering the cognitive strain on students. However, it also seemed to suggest a shift to a Type 1 reformulator, which reduced the range of answers by indicating an appropriate subset from which students may draw.

Josh hedged his reformulation of Abram's contribution to help indicate that this was not what he was looking for ("I *could maybe* see...") and told him that he could have gotten that same answer from things other than the graph model. He then used something similar to a combination of a Type 1 and a Type 2 reformulator. First, he narrowed the range of possible answers to a subset; it was not just about a graph model, but about why they drew in the line. Second, his Type 2 reformulator that immediately followed hinted at an answer that was not correct; the line was not drawn in "just to show a pattern." In the excerpt from the textbook that he had just read, he had stated that the graph model "shows the trend in the data." How students interpret "trend" may be problematic—to them, it may be just a pattern and it might not be considered that a trend usually continued in a similar manner. This would be important to understanding a "graph model" and to understanding exactly what Josh was looking for.

Josh posed this "why" question a third time—reformulating it as, "Why do we even bother doing this?" By asking the question in this manner, Josh seemed to be saying that we *do* bother with this, so there must be a reason. In this classroom, the focus was not just on how to do something, but also the reasons for doing them. What was valued were actions with reasons.

Additionally, Josh directed student attention to the reason given in the textbook, indicating that specific reasons can sometimes be found in the book (i.e. the book is a source of "reasons"). Lance read the purpose directly from the notes that he had taken: "So you can make predictions for values between values in the data." This was definitely the answer Josh was looking for, announced by his drawn out "Oh," followed by, "so if I want to predict...." Josh was now ready to move on to a more specific question that required students to predict a value based on the graph model they had just drawn.

In using the form "proposition" + "why," Josh seemed to push student thinking. However, many of the questions that appeared "open" in nature soon became "closed" in that they were asked repeatedly reformulated until students came up with what Josh had in mind. In this way, Josh may have "cued [the] elicitation of student contributions." This was characterized not only by the I-R-E format that took place, but also by the continued reformulating of questions until students produced the answer for which Josh was searching, offering "heavy clues about the information that was required."

By pushing on the "why" Josh established the fact that reasons for actions were important. In fact, in his interviews he said that following up with "why" questions gave him "a pretty good idea about whether the kid really had a grasp of what we were doing" (Interview J-3, 12/17/99). In many cases this norm appeared transparent to students because they did not verify their reasoning without Josh explicitly asking for it. Another possible interpretation for this may be that it was so much part of Josh's role to ask "why" that students did not see a need to give their reasoning right away. They may have known that Josh would ask that soon, so they waited to respond until after the justification had been pursued.

"What's the rise? ...the run? ...the slope? ...the y-intercept? ...the equation?: There are sub-questions that allow you to focus in on the content. In some of Josh's questioning, the same form functioned as a prompt to show students that there were sub-questions they needed to address to engage in a larger task. These questions seemed to serve as a way for Josh to model key questions that students should ask themselves when engaging in the mathematical activity. In this particular example, the larger task was to find a linear equation. Josh seemed to indicate that in order to find the equation of a line, students needed to know particular information: the rise, the run, the slope and the y-intercept.

Example J-10

J: [...] Give me two points that are on the line in graph B. Jaylen Hartz, give me two points on the line for graph B.
    Jaylen: Uh// 0, 0/ and uh// 4, 1
J: Okay// 0,0 and 4,1/ Great/ What's the **slope**?/ What's the **rise**? You're going from zero on the y up to one? What's the **rise**?
    Ss: One
J: One. What's the **run**? You're going from zero to four on the x?
    Ss: Four
J: So the **slope** is? .25?//
    Ss: .25 [in unison]
J: And the **y-intercept** i::s?/
    Ss: Zero
J: So, y = 1/4 x? Or y = .25x would be your **equation** [3 sec] Any questions on the first page?// [...]

This excerpt occurred after Josh had handed back a quiz. In fact, this sequence of questions was repeated three times (with slight variation each time) in the 15 minutes that Josh went through the problems on the quiz. The example that I gave was the second time this sequence occurred. During the first one, Josh began by asking what the slope between two given points was and a student replied by subtracting the x-values in each of the two ordered pairs. The student who replied had difficulty telling Josh what those values gave him: that is, he had difficulty naming the process that he was doing as "finding the run." Josh proceeded by asking about the rise, the slope and the y-intercept in an I-R-E format similar to those in Example J-10.

In Example J-10, Josh began by asking Jaylen for two points on the line. Jaylen told him two ordered pairs and Josh used these two ordered pairs as a launch into finding the rise, run and slope. There was very little hesitation between the time Josh asked what the slope was and the time he broke this idea down by asking about the components of slope: the rise and then the run. Furthermore, Josh marked the values that were used from each point and the movement that took place on the respective axis to get from one point to the other (i.e. "You're going from zero on the y up to one?"). In asking these, Josh was modeling the type of questions students should ask themselves about the situation. He was structuring the interaction as a guide for how students should think. At the end, he showed students what they then do with these components: they write them in their respective position in the equation (in this case y = 1/4x or y = .25x).

It was important to trace this "repeated discourse format" across time. Josh may have been using "discourse as scaffold" (Cazden, 1988, p. 101). For these questions to be scaffolded instruction, the assistance would need to be gradually withdrawn until students had learned to ask themselves these questions independently. Josh may have felt that he needed to continue to ask these guiding questions throughout the lesson because students were not answering the questions easily (or quickly), suggesting that they were not ready to take on this task by themselves. In addition, prior to the class period, Josh had told me that he was quite disappointed by the grades students had received on the quiz. By repeatedly asking this sequence, Josh may have provided a model, thereby suggesting the kind of information that should be included when trying to write a linear equation.

Three important features of scaffold include: making it possible for the novice to participate in the mature task from the very beginning; and doing this by providing support that is both adjustable and temporary (Cazden, 1988, p. 107). This type of scaffolding can play an important role for providing support in guiding students in their zone of proximal development.

> If, in fact, the novice takes over more and more responsibility for the task at hand, [...], then we can infer, retrospectively, that our help was well timed and well tuned, and that the novice was functioning in his or her zone of proximal development, doing at first with help what he or she could very soon do alone (p. 107).

An interesting question remains: does Josh fade out or not? After retracing Josh's questioning strategies through the *Models* unit, it seemed that this exact form of questioning only appeared when Josh returned assessments. On October 22, Josh handed back a partner quiz and scaffolded in the same manner: that is, he did not fade out and talked students through the steps as he did in the transcript from October 6th. This form did not seem to be a scaffold in the sense that Josh continued to use it and had students eventually take over the process. Instead, it indicated that Josh was annoyed because students did not know what they "should" know (i.e. the "common knowledge").

Josh said that he thought he was "holding their hands a little too much" (Interview J-3, 12/17/99) when he read these examples. However, he said that this was "something that they really should know" and that he "might [...] have been pissed off [...] 'cuz I'm really racing through this [referring to the pace he asked the questions]" (Interview J-3, 12/17/99). He also said that he thought he did this more often when he was:

> really trying to hammer something[....] this is a very traditional way of teaching. This is this, this is this, and you know once in a while I will turn around and snap [snaps his fingers...] I hope I didn't do this all the time. But, although, maybe there's some value in doing it this way as well. I think, when you're really trying

to hammer something in to kids. When you really fire it off at them it really kinda keeps them on task as to what you're trying to do. I would certainly hope I didn't do this that much, but I'm sure I did [laughs] (Interview J-3, 12/17/99).

When reviewing the tapes, I noticed that there was at least one other form that may have allowed Josh to use discourse as scaffolding. Whenever a graph was drawn, Josh used the same flow of questions to get students to set up and label the graph. This sequence of questions included: How did you label your x-axis? [...] How did you label your y-axis? [...] What scale did you use? [...] For your x-axis? [...] For your y-axis? Typically, this was followed by Josh leading the class through the data for the table. I also traced this succession of questions and found that Josh asked them in at least half of the class sessions. As with the previous sequence of questions, Josh's role did not change and he posed the same series of questions across time.

In interviews with students, it was quite apparent that they thought about the equation of a line in terms of these components. Also, when they drew their graphs it was obvious that they knew that they had to label the axes and define the scales for each. In addition, I heard students have conversations during my observations that indicated that students also used these smaller steps to complete these tasks in class sessions.
So, even though Josh's questioning did not act as a scaffold across this time period, students seemed to have recognized that these smaller steps made up the larger process or result. In other words, Josh's modeling of questions to ask oneself (when completing these two tasks) seemed to have been taken up by students.

"Is it linear or not?": Strongly established common knowledge is a decision to be made. During the first unit, *Moving Straight Ahead*, students explored linear relationships for seven weeks. It is possible that Josh believed that knowing whether something was linear or not was so much part of the common knowledge, that students should not even have to think about it any more: that is, he considered this knowledge to be "presupposed and implicit." In fact, he explicitly stated such expectations after he returned the Check-up to students:

Example J-10

J: Okay/ guys, let's// let's/ take a look at this Check-up. Um/ I'm not going to, I don't want to/ stand up here and lecture you guys again, but/ like I mentioned yesterday for seven weeks we've done nothing but linear patterns, linear relationships, linear equations for seven weeks. And there are some things/ quite honestly/ that **you shouldn't even have to think about right now**/ If you are given two points on any line, you should be able to find an equation, you should be able to find slope, you should be able to either/ work backwards in a table or use a symbolic method to find y-intercept/ You should be able to do all that stuff.

Often, Josh began the discussion about a new problem by asking if the relationship was linear or not. In doing so, he seemed to imply that this was a mathematical decision to be made (i.e. that it's something that they "shouldn't even have to think about" by now), rather than a mathematical activity to be explored. Also, he followed up many of these with "why," again indicating that reasons were important.

Example J-11

J: So, if one group makes a, kinda a, screws up as far as collecting their data, it's not gonna affect things as much since we're averaging six groups. Um, **linear or not linear**?

PJ: It's linear until the last one [last point on the graph model], I don't know, on the average

Abram: It's pretty linear

J: Pretty linear? Let's find out. Uh, what I'd like you guys to do is this: If you look in your books on page 7. (Ss open books) Problem 1.2 is called drawing graph models. And what it has you look at, is it has you look at a classroom set of data from Maryland. And, they plotted the data and they tried to **decide** if it's **linear or not**. Now, we're gonna look at that in a couple of minutes, but I'd rather look at our data. So, what I'm gonna do is I'm gonna give you a sheet of graph paper and I'd like you and your seat partner to talk this over. And I'd like you to graph the classroom average.

In this example, Josh concluded the discussion about the data collected by asking students to predict whether they thought the relationship in the graph appeared linear or not. PJ thought it was linear up until the last point they had graphed on their model. Abram thought it was "pretty" linear. As with Karla's classroom. "pretty linear" became a way to indicate linear models—they were not "perfectly" linear because the perfect relationship only exists in the textbook-generated linear relationships, not in the modeling ones. Josh responded, telling them they were going to look at this soon, but referred to a group of students from Maryland who had engaged in the same bridge activity. In a sense, they were being referred to as a "model" group who had done the same thing and who also had to decide if their relationship was linear. Since this group of students had had to decide this, Josh was going to have them engage in the same activity. He turned student attention to the data they collected first.

In other examples where Josh pointed out that they needed to decide if the relationship was linear or not, he often followed up with a question that pushed students to articulate *why* it was/was not linear.

Example J-12

J: Okay. Um, is that linear?
    Ss: Yeah/No/Sure [all three answers are coming from Ss]
[J laughs and shakes his head back and forth]
J: I hear a yeah, I hear a no, I hear a sure.
    Fs: No
    [Ss laugh]
J: **Linear or not**?
    Lisa: Yeah.
    Ms: Yes
J: Lisa, you say it's linear?
    Lisa: Yeah
J: What makes that linear?

Example J-13

J: Zero/ Okay, that's linear. Circle it/ Look at the second one [3 sec] y equals four divided by x. We haven't looked at any of those kinds of equations, but you should still be able to tell **if it's linear or not**/ How could you tell **if that's linear**?

Cory: It's not in standard form/

In both of these examples, Josh asked students to justify how they knew it was linear. In Example J-12, there appeared to be disagreement from students as to whether the model was linear—Josh verified this by telling them that he was hearing "yeah, no and sure" from students. He asked, again, whether they thought the relationship was linear. Lisa believed that it was and Josh called on her to justify her reasoning.

In Example J-13, students had been given a form of an equation with which they were unfamiliar, $y = 4/x$. Josh recognized that they had not discussed this type of equation, but told them that they should "still be able to tell if it's linear or not." Josh may have been saying that the definition of "linear" was so much part of the common knowledge in the classroom that students should be able to use this knowledge to distinguish whether an unfamiliar equation fit that criteria. Not only should students be able to judge this, but they should also be able to justify their decision.

I also found at least two other forms of this type of "decision" questioning. Students were expected to be able to determine whether something was the rise or run and if a slope was positive or negative. In Example J-14, after a student had told Josh to subtract two x-values as part of finding the slope between two given points, Josh asked him whether that gave him the rise or the run. In doing so, Josh seemed to advocate the need to know the name of the process that the student described.

Example J-14 (see also Example J-3[13])

J: What does that give you, the **rise or run**?
Abram: Um/ the rise
J: Bad guess
Abram: The run
J: Why does that give you the run? [4 sec] Why? That's where one of you guys-
Cory: That's what's on the x-axis.

Josh indicated that Abram did not think about his response when he said, "Bad guess." Josh then posed a question to the class that forced them not to guess, but to justify *why* that gave them the run. After a four second pause, Josh asked the question again and began to make a point about something he had observed in a student's answer. Cory interrupted with the correct answer, helping Josh move on to the next phase—finding the rise for the equation.

In the next example, Josh began the discussion about a graph by asking what the slope of the line was. He then backed up to something that was observable from the picture—deciding whether the slope was positive or negative. This decision could be made by looking at the slant of the line. If the line slanted upward, the slope was positive; if it slanted downward, it was negative. Josh wanted students to recognize this and make this decision before they even thought about finding the slope. Often in the discussions, students forgot to put a negative in front of the slope. Here, he is pointing out that they need to decide this "first of all."

Example J-15

J: What's the slope of that line? First of all, **positive or negative** slope?
Ss. Negative.
J: It's going down, right?
Ms: Yep.

Furthermore, after Josh asked them whether it was positive or negative, he added why it was negative: "it's doing down." This point was tagged, requiring a positive response.

When Josh read these examples, he said that he had not been aware that he sometimes posed questions in this manner. His interpretation of these questions was "that could be kinda limiting. I'm not sure if I do that intentionally as much as maybe I actually do it too much sometimes. I kinda limit them into their paths that they actually take" (Interview J-3, 12/17/99). This type of questioning, from Josh's perspective, may have been a way to direct the conversation because the form only allowed certain possibilities. This may have allowed Josh to maintain control of the direction of the conversation.

"Does anyone have something else?," "How many have...": Plurality is important in determining what is mathematically reasonable and what is not (Limited Invitation to Diversity). There seemed to be two major functions for questions that focused on allowing students to present multiple ways for solving problems. One of these included getting processes or answers out for discussion. The other was used more as an informal assessment of the answers people were getting.

To get students to offer their processes for solving problems, Josh often asked questions like, "What do you think?" or "What did you say?" For example, after students had filled in the average class value[14] for the bridge-thickness problem in Investigation 1, Josh turned the discussion to determining whether students thought the data was linear.

Example J-16

J: Laurie, **you say it's linear**?
    Laurie: Yeah
J: What makes that linear?
    Laurie: [inaudible]
J: Say that so we can all hear
    Laurie: It's pretty much going up like straight—[stops]
J: Okay. It's pretty much going up at the same steepness. PJ, **what do you think**?

PJ: I think no because the last point is not like, following with the pattern. To be linear it has to go up like in a consistent pattern.

J: Laurie, **what'd you say**?

Laurie: But it's experimental data.

PJ: Yeah, but [inaudible]

J: So, uh, I think we have two schools of thought here. One is that it's experimental data so it may not be a perfect line. And the other school of thought is that it's not linear because one of the pieces of data is kinda away from the rest of the pattern.

Abram: For me, to be linear it doesn't have to be exact, it can be off a little bit.

J: Okay—let me ask this question again. How many of you guys think this is a linear relationship between bridge thickness and the breaking weight? How many think it's not? How many aren't sure?

[about 2/3 of the Ss raise their hands to show that they think it is linear]

J: Well, I tell you what, I kinda agree with what Laurie just said. That this is experimental data and in experimental data you're not always gonna get this perfect pattern. If I looked at this graph, I would say there looks to be a linear relationship. It looks to be pretty close to being in a line. I agree, PJ, that last point kinda screws things up a little bit because it's away from the rest of the data a little bit. But, I think that there is a linear pattern here. So, how are we going to show the linear pattern? Let's go ahead and take a look and read problem 1.2 where it says, "Drawing graph models." Cory, would you read that please on p. 7.

In this example, Josh asked the class if an equation was linear or not. Laurie answered in the affirmative and Josh asked her how she knew that it was linear. She began to answer his question and then quit in mid-sentence. Josh paraphrased her contribution and called on PJ to see what he thought. PJ's idea contradicted Laurie's assertion and drew on the fact that he thought linear relationships must increase by a constant additive value—what he called a "consistent pattern."

Josh offered his reconstructive recap of the "two schools of thought" in the conversation, making this instance quite novel[15] for this classroom. Laurie argued that the relationship was linear and that the line will not be "perfect" because the data was based on the experimental data that they collected. PJ, in contrast, thought the relationship was not linear because the last point did not follow the linear pattern. He drew on his knowledge of what "linear" has meant: the pattern must increase additively by the same amount every time. Laurie seemed to be drawing on her knowledge of a "graph model": the additive pattern is going to be close to linear because experimental errors may occur. Josh stepped in and took Laurie's and PJ's assertions and created a contrast between the two ideas, juxtaposing their arguments and positioning the students in opposition.

This participant framework, then, "contains two hypotheses about an event and entails two originators of these hypotheses" (O'Connor & Michaels, 1996, p. 78). In setting up this discord, Josh has accomplished many things. He inducted Laurie and PJ:

> [...] into public versions of key intellectual roles [...] theorizer, predictor, hypothesizer [...which must be] stated in terms of other participants in the ongoing activity and in terms of the actual propositional content under discussion. Here the teacher has created a dramatic landscape for this event, featuring two protagonists. This provides an opportunity for other children (perhaps silently, perhaps vocally) to place themselves in relation to the [...] hypotheses, through [Laurie and PJ] as proxies (p. 78).

This occasion was taken vocally by Abram, who articulated how he defined the situation: that is, he seemed to be placing himself in relation to Laurie's hypothesis in that it can be "off a little bit" and still be linear.

Josh took this opportunity to assess what students were thinking by asking them, "How many think it's linear? ...not? ...not sure?" and students responded by a show of hands. This form, "How many think...?" served the other function I referred to at the beginning of this section. Josh used this question-form as a way of checking what a larger group of students thought. In this case, there were "two schools of thought" and Josh wanted to know where the rest of the class stood in relation to them. In this way, he informally assessed those students who were not vocally involved in the discussion.

The discussion in this example then took an interesting turn that will be discussed more in the following section related to issues of authority. Josh told students what he thought about the disagreement—he agreed with Laurie. Because Josh is in position of authority, it appeared that he has now established the "correct" answer by telling students that he thought it was linear and why he agreed with Laurie. He recognized that PJ made a valid point (the last point is off a little bit), but told the class that he thought the pattern was pretty linear. The conflict was resolved and Josh moved on by turning student's attention to the textbook so that they could talk about how to "show a linear pattern."

Summary

In this section, I have shown how Josh used many different forms of questions for many different functions. I have also shown how, in some cases, he used Reformulators to help students answer the question as he expected. Although many of his questioning sequences began with a wh- or how question as reformulators do, only some of these were reformulated after students were unsuccessful in producing the reasons for which Josh was searching. Rather than controlling the common knowledge through nonverbals, Josh "cued elicitation" by continually narrowing down the range of answers, and in some cases, even directing student attention to what the textbook stated.

Through his questions, Josh established part of the common knowledge of the classroom. These were especially apparent in the propositions that preceded a tag and those that preceded his "why" questions, which both seemed to function as joint knowledge markers because they were repeated discourse formats. The tags that Josh often followed his propositions with seem to be his way of being polite to students, especially when students were not answering questions in the manner that he hoped they would. He also seemed to use tags to soften additional information he gave when answers were not as precise as he seemed to want them to be.

When Josh followed a proposition with "why," he sometimes had a particular reason in mind. When students did not offer the reason that he had in mind, he often used a reformulator to direct student thinking. In this way, some of the questions that appeared open at first became more closed in nature.

Some of Josh's questions appeared to function as a way for Josh to let students know the kinds of sub-questions they should be asking themselves as he asked them a series of questions that were subcomponents of a larger task. When doing so, he may have been hoping that students would eventually internalize these questions and answer them independently. If Josh continued the school year by slowly withdrawing his vocalization of these questions and students began to ask themselves these questions, Josh may have been helping students work in their zone of proximal development. Subsequent observation and interviews with students have shown that students do eventually think about these smaller components that make up linear relationships, even though Josh did not really relinquish his role in asking those particular questions.

Some of the questions Josh asked implied that there were certain parts of the common knowledge that were fully established and were now decisions to be made rather than mathematical activity in which to be engaged: that is, these ideas were presupposed and implicit. The most prevalent appeared to be whether a relationship was linear or not. Related to this, students were expected to be able to determine whether a line's slope was positive or negative just by looking at it. The third thing that appeared to be a decision at this point (and not mathematical activity) was whether the rise or run had been found;

that is, when the process for finding the rise or run was being engaged in, students should have been able to name that process.

Through some of his forms of questioning, Josh allowed students to share their ideas, but as I will detail in the following section, he became the judge of whether the ideas were agreeable or reasonable in terms of the mathematics. Students shared their ideas, but the invitation to discuss these ideas was limited by the fact that Josh typically ended the conversation by telling students who he agreed with or by telling them which answers were reasonable.

One of Josh's primary functions as a teacher in this classroom was that he was the "asker of questions." In taking on this role, it became the student's job to respond to these questions. As I mentioned in Chapter 4, this was also one of the most apparent aspects of the textbook. Josh and the textbook seemed to share this common interrogative voice.

Goody (1978) speculates that questions from the teacher to the student are likely to be interpreted as 'examination' questions, holding the student responsible for knowing the right answer, seeking to control him by putting him publicly on trial. She indicates that control messages tend to be attributed to the questions of a superior, even when it was completely unintentional. Even though I have not addressed student's perceptions of Josh's questions here, what Goody speculates could have interesting implications for classroom dynamics. In addition, the very fact that Josh uses politeness strategies in the forms of his questions highlights authority issues in his classroom and the way that he mitigates his strong authority. In the next section, I will show how Josh makes his authority even more explicit through the use of particular markers such as "agree," "reasonable" and statements that follow "but."

Authority in Teacher Talk

> Teachers and students have grossly unequal power in the classroom. The teacher is the representative of adult authority, and backed up, at least in theory, by the power of force as well as by the traditions of schools. That difference in power extends to control of the dialogue itself, both its form and its content, that is, both the activity structure and the thematics. The teacher has the power to decide what will and will not be talked about, who has the right to speak at any given time, and what is the "correct" way to behave and to talk about the subject (Lemke, 1990, pp. 44-45).

Much of the literature related to issues of authority in education make a similar distinction between being an authority because of one's knowledge and being an authority because of one's position. This distinction has taken on many different names, for example, "rational authority" and "traditional authority" (Peters, 1966) or "sapiential (or "knowledge") authority" and "structural (or "positional") authority" (Skemp, 1979, following Paterson, 1966). Basically, "a teacher is *an* authority *in* authority" (Russell, 1983, p. 30, italics in original).

Skemp argues that these two types of authority are largely incompatible. He believes the authority by position to be like that of a "schooler": that is, the authority is the result of the role one plays and is imposed. In this position, the teacher commands, students obey; instructions are perceived as orders. In direct contrast, authority by knowledge is more like a "mentor." The authority is vested in the person by virtue of his own knowledge; his instruction is sought and is perceived as advice. This incompatibility, Skemp claims, can cause role confusion and conflict.

More recently, Forman, McCormick, and Donato (1998) have applied ideas from Wertsch (following Bahktin) to discuss authority patterns in reform-oriented classrooms. They find evidence that the "voice of decontextualized rationality" was privileged in one teacher's classroom, despite the teacher wanting to teach to the NCTM *Standards*. That is, the student-talk that was valued included mainly abstract talk that could be understood apart from the particular context in which it is taking place. The authors argue that the teacher asserts her authority through use of overlapping speech, vocal stress, repetition and expansion. The teacher was shown to overlap student explanations only when the student was not using the procedure that the teacher had recently taught. She also privileged the explanation of one student who used more "timeless, spaceless, impersonal, and algebraic" (p. 332) talk, even though two other students gave mathematically equivalent methods for solving the problem.

An Authority In Authority

In Josh's classroom, the patterns of authority in his talk were more apparent than the teacher discussed in Forman et al.'s article: that is, there were at least two words that were only "teacher" words: agree and reasonable. I will show how these words played a role in locating Josh as part of the locus of authority. Another word that functioned as a way for Josh to assert his authority was the word "but." Often, Josh would use the word "but" to announce that he was either about to make a mathematical point or that he was going to contradict something proposed by a student. In addition, Josh asserted the text as part of the locus of authority in often referring to and explicitly drawing from it as a regular part of the lesson.

"I agree...." "That sounds reasonable to me...": The teacher has the right to judge the reasonableness of a solution. In the previous section, I had mentioned that I would return to the issues of authority that appeared at the end of Example J-16. In that example, there was a disagreement between two students and Josh used revoicing to set up each hypothesis as an opposing viewpoint. In doing so, he was doing some very complex things in terms of the participation structures in the classroom. However, I then pointed out that the conversation took an interesting turn:

Example J-16 (Revisited)

J: Well, I tell you what, **I kinda agree with** what Laurie just said: that this is experimental data and in experimental data you're not always gonna get this perfect pattern. If I looked at this graph, I **would say** there looks to be a linear relationship. It looks to be pretty close to being in a line. I **agree**, PJ, that last point kinda screws things up a little bit because it's away from the rest of the data a little bit. But, **I think** that there is a linear pattern here. So, how are we going to show the linear pattern? Let's to ahead and take a look and read problem 1.2 where it says, "drawing graph models." Cory, would you read that please on p. 7.

This concluding statement to the ongoing discussion was fairly typical in Josh's classroom. He often asserted his viewpoint in terms of what he agreed/disagreed with when opposing viewpoints were given by students (either with respect to each other's views as above, or with respect to something Josh has said). However, Josh hedged (e.g. "kinda," "I think") and used modal verbs (e.g. "would") to soften his authoritative comments. This may indicate that Josh's authority was so strong that he tacitly thought it required words that softened his assertion. In fact, Josh revisited his viewpoint on this same issue a few minutes later. After a student went to the board and drew in the graph model that he thought best fit the data, Josh repeated that he "would agree" and affirmed that there was a linear pattern to the data. In asserting his authority, Josh again hedged and used modal verbs to soften his proposition.

Example J-17

J: **I would**. If I saw that data plotted from an experiment, **I would say** there's certainly a linear pattern to it. Now look where he drew in his line of best fit. Now, first of all, is that a line?

Ms: Yeah, it's supposed to be.

J: It's supposed to be a line? Um, of course if you draw your line thick enough, I guess it's gonna hit most of the dots, but, um, if I looked at this, **I would agree**. **I think** this one point, sometimes when you gather data something goes wrong and sometimes there's like one piece of data that's like away from the rest of the data. and that happens. But, if you take out that point, there's very much a linear pattern. Again, Laurie mentioned about experimental data, it's not going to be this perfect line, but does that line seem to fit the data? Are all the points fairly close to that line?

Ss: Yeah

J: I **would say** that's a pretty good line of best fit. How about in b? Would you go up please, Abram, and draw in what you think is a graph model for b.

In this example, a student was called to the board to draw in a graph model for the data the class had gathered during the bridge problem. As the student was drawing, Josh pointed out (from looking at the table of data) that he would say there was "certainly" a linear pattern to it. In doing so, he asserted a truth—there *is certainly* a linear pattern. This was further indicated in his use of present tense "is"—this is a fact. When drawing the line, the student wanted it to go through the one point that appeared to be off a little and drew in an unusually thick black line (thus, J's reactions of: It's supposed to be a line?). Josh again stated that he agreed that the pattern was linear. He continued by telling them why he would categorize this model as a linear one—sometimes there are errors in the data collection and one piece of data can be away from the others. He reminded the class about what Laurie had said and then differentiated "graph model" linear from "perfectly" linear. The hedges in the next two questions ("seem to fit" and "fairly close") seem to be part of differentiating "graph model" linear from perfectly linear (as was the case in Karla's classroom). Josh was pointing out that the line was not going to fit the points exactly and that they should just try to get as near to them as possible when they drew in the graph model. After students agreed with Josh, he repeated his affirmation; he believed the line to be a "pretty good" line of best fit, again implying his right to judge the correctness of an answer.

In the next example, instead of ending a dispute between two students, Josh asserted his position related to an answer with which he disagreed. Abram had drawn what he thought to be a "curve of best fit" for a group of points. However, Josh wanted him to think about the fact that this curve needed to be able to continue in a pattern that would be useful for prediction purposes. Josh asked Abram about how the curve should continue and how he knew that it would keep continuing in the same manner.

Example J-18

J: Where do you—it does what? It levels off like here?
Abram: Yeah, it- just like that
J: Then it goes back up?
Abram: Yep
J: I don't know. **I'm not so sure if I can say that with a lot of certainty**, Abram. See what I'm getting at?
Abram: I can
J: **It's gotta be** a curve or a line or a curve that I can extend that's gonna help me predict the pattern. First of all, I don't even know if I **agree** there's gonna be a curve there.

Here Josh began with the curve of best fit that Abram had drawn and asked Abram how the curve would continue, if it were drawn in further. Abram had drawn in what looked like a series of rolling hills upward and downward (sinusoidal in nature) and suggested that Josh predicted the continuing curve as he had imagined ("just like that"). Josh maintained that this continuation is not something that they could say with much certainty. Josh's use of "I" (rather than the royal plural "we") further asserted his position as being separate from Abram's, although he sufficiently hedged the disagreement so as not to insult Abram. The underlying message to Abram is, "This curve is not correct." Josh referred back to the definition they had read in the book when he relayed what a graph model was, using this as further evidence to convince Abram that the model he drew would not allow prediction. He then asserted that he did not even know if the data generated a curve model.

Another example of Josh asserting his authority when he disagreed with an answer a student gave included the following:

Example J-19

J: Oh, okay. Going, um, I don't know. Jennifer, I certainly think that, that if there is a pattern I think this curve models it better than the line does. I mean there's a couple of points that aren't even close to being on your model. Um, but the rest of them seem to fit pretty well. How many didn't draw anything on [letter] i- said there was no pattern?

[About 6-7 hands go up]

Alan: I got kinda a pattern. I didn't really know what to do because like, I was thinking about that but then there's that one point right, like on the lower part of the graph

J: Right here?

Alan: It just didn't make any sense to me

J: Yeah? Yeah, sometimes you can, sometimes you can almost force a graph model on a set of data. And I don't think that's what we want to do. I think you wanna, if there's an obvious pattern, you can draw in a graph model. But, **I agree** Jennifer, if there is a pattern, that's gonna be probably about as close as you're gonna get. Any questions on that? Okay, let's keep going then. And, that's all there is in problem 1.2, by the way. So, let's go ahead and take a look at problem 1.3

The graph that was being discussed consisted of two groups of somewhat vertically clustered points about a centimeter apart. Josh began the discussion of this by stating: "I don't know. Maybe I don't have much foresight here. I don't-I gotta really push myself to see any pattern in that data at all." Jennifer volunteered and told him that she had an idea. She went to the overhead and drew a parabola (opening downward) that went through many of the data points (in my observation notes I had written that I thought she had actually come up with a pretty nice fit of the data). Josh's response was one that hinted that Jennifer had a good idea, but he used hypothetical wording "if" to indicate that there really was not a pattern. When Josh continued by saying, "I mean there's a couple of points that aren't even close to being on your model," he seemed to further assert his own view, especially given the conversation earlier that took place between PJ and Laurie (see Example J-16). The conversation in Example J-16 ended with Josh stating that he agreed that the graph they discussed should have a linear model, even though one of the points did not fall on the line that they drew. He ended the discussion about this graph by telling Jennifer that he agreed, *if* there was a pattern (but there was not), that was probably (but not really) going to be as close as you were going to get. Thus, the underlying message is clear that Jennifer has made a nice attempt, but there is no graph model that can be drawn in for this data.

The other form that Josh used to interject his authority was typically heard at the end of a class discussion. "Reasonable" functioned in such a way that it allowed Josh to judge student contributions. In the following example, Josh asked students to generate equation models for the graph models they had drawn. After they had, he called on individual students and recorded their equations on the board. Students generated slopes that ranged from 4 to 6. He then turned to the graph model that he himself had drawn and determined the slope of the line on his graph.

Example J-20

J: Of course, it all depends where you draw your line. On mine, and I'm not saying my line of best fit is the perfect match, but on mine it looks like 10 is about, what, 59? And 59 divided by 10 is 5.9 slope? Anybody else? I **would say** this [equation with slope of 5.8] is a **reasonable** line of best fit. I'd say this [equation with slope of 6] is a **reasonable** line of best fit. [circles those on the board] Um, **I'd say** this [equation with slope of 4] is **not**. [crosses out that equation] Robert, where did you get y = 4x?

Robert: I went by 2's, um, on y [Robert shakes his head no]

J: You went by 2 on the y? Is that gonna make any difference though on the slope of your line?

Fs: No

J: [You're] still gonna need a rise divided by the run to find the slope. **I don't think this is reasonable, I think this one's gonna be a little bit too steep. I think this one's gonna be a little too steep**, this one's gonna be **a little iffy**. How many got a slope somewhere around 5.7, 5.8, 5.9, 6, 6.1, depending on where you drew your line of best fit?

After the equations were put on the board, Josh calculated the slope for his own graph model. Using his equation as an exemplar to test the other equations, he then told students which equations were reasonable and which were not. He qualified this statement with, "I'm not saying my line of best fit is a perfect match," making the judgement less bald. Josh then called on Robert to tell him how he came up with the equation of y = 4x, which Josh had deemed unreasonable. Robert stated that he used a different scale on the y-axis than Josh had—a scale of two instead of one. Josh asked if that would make any difference and a female student said it would not. Josh continued by telling Robert that he was still going to need rise divided by run to find the slope (suggesting, although not explicitly, that the ratio would still come out the same, no matter which rise and run you chose). Josh ended this part of the conversation by again pointing out which equations he thought were reasonable and why the others might not be (e.g. "this one's gonna be a little bit too steep"). In fact, he changed the focus of the discussion when he asked, "How many got a slope somewhere around…." implying that he was now assessing what people had done, which he, as the teacher, had the right to do.

"… But, …": As *an* authority of mathematical knowledge, one of my roles is to clarify the content

. Josh often used the word "but" together with "I think/would/am." In doing so, he set up a roadblock during the discussion to indicate that he had something he needed to clarify. When this occurred, Josh either contradicted a student idea or made a mathematical point.

The following examples show how Josh used "but" to cut off a mathematically incorrect idea that was posed by a student. By asserting himself in this manner, he seemed to be drawing on his authority of the knowledge of mathematics. He sometimes used this word to suggest that he was going to clarify himself, as in Example J-21 below.

Example J-21

J: How many have a graph model that looks something like what Christy just did? [a few students raise hands] Does anyone have one that's different?

Ss: Umm [inaudible]

J: Does anybody have one that's gonna be a little bit steeper? Why?

Laurie: I used a scale of two [referring to the fact that her line is steeper than Christy's, although it only *appears* to be because she used a different scale]

J: Oh, used a scale of two, oh, that would certainly make a difference as far as how steep your line is, yeah. **But**, what **I'm saying** is, even given this, these scales, does anybody have one that's a little bit steep that maybe is a little closer to this point?

Ss: Yeah

J: I'm not sure.

Ms: Oh [inaudible]

J: I'm not sure. Okay, Robert's is a little bit steeper so that this point is just slightly below the line and this point here is a little bit closer to the line? Remember what a graph model is—it's a line or a curve that shows the trend in the data, it fits the data. So that all the points are pretty close. Um, I don't know. Why do we do this? What's the purpose of a graph model? Abram, what's the purpose?

In this example, Josh had asked students if anyone had drawn in a line of best fit that was steeper than the one that was on the board so that it went closer to the last point (the point referred to in Example J-16). One student said that she had and showed Josh her drawing. Instead of using the scale that was on the board (which went up by ones on the x-axis), this student had used a scale of two's on the x-axis. By doing so, the graph looked steeper because the change of scale allowed the x-values to appear to increase more rapidly. However, the graph was really the same graph as the one that was given on the board: that is, the graph was not any closer to the outlying point. Josh possibly realized that the student did not understand this and told her that this did make the graph appear steeper. However, that was not what Josh had intended by his question—he had wanted a graph model that went "a little bit closer" to the last point using "these scales" (i.e. the ones that he had on the overhead).

Before moving on to another topic, Josh pointed out that Roger did have a graph model that was closer to that last point. He then reminded students of the definition of a graph model and pointed out that the line of best fit was close to the set of points with which they had been working. In doing so, he may have been indicated that the line that was closer to the last point was really a better line of best fit because it was closer to the last point. However. this was not made explicit. Josh then indicated that he was ready to move on to the reason for doing graph models by quietly saying. "I don't know…"

Josh also used the word "but" to indicate that he was making a contrastive point to what a student had asserted. In Example J-22. Abram had drawn in a graph model that he thought best fit the data. However. Josh knew that it would be difficult to use this graph model to make predictions. an important aspect in mathematical modeling. So. rather than state why Abram's graph was not the curve of best fit. Josh drew a graph that students were familiar with (an increasing exponential) and showed that he could extend it to predict future values. He then drew another curve that went up. then down and then back up again at a faster rate and asked Abram if that one could be extended.

Example J-22

J: Maybe a little like that [extends the increasing exponential he had drawn]?
Ss: Yeah.
J: **But.** I could extend it. [draws a curve that goes up. then down and then up at a steeper rate than the first part of the curve] Could I extend that?
Ss: No.

By using a curve with which Abram was already familiar and showing him that the graph could be extended and then providing a contrasting curve that could not be extended, Josh created a contrast to allow Abram to observe an important aspect of mathematical modeling: the curve of best fit had to be drawn so that it could be extended to allow predictions. If a curve of this type cannot be drawn, then a graph model cannot be drawn to fit the data. Rather than stating this property explicitly, Josh illustrated it, cueing the response for which he was searching. Also, the use of "but" separated an important contrasting proposition, "I could extend it."

The other function "but" seemed to have was to mark the fact that Josh was about to make a mathematical point related to the class discussion. Again, Josh seemed to be asserting his point of view as an authority on the subject of mathematics.

Example J-23

J: [...] Take a look at what they did. Now, their data was a little bit scattered, a little more scattered than ours was. **But**, they still were able to draw a line that seemed to fit the data pretty well. No point is really far away from the line, there's a couple of points below, you know, one or two slightly above. That is sometimes called a line of best fit. We're gonna use that term an awful lot. Cory read on.

In this example, Josh highlighted the idea that the points on the graph (in the book) appear scattered, but that a line of best fit could be drawn in to fit the data. His mathematical point appeared to be that even though the data does not look "perfectly" linear, a line of "best" fit can be drawn that "fits the data pretty well." Josh may have thought this an important point to make given the earlier discussion in which PJ thought the data they had graphed was not linear because one point appeared to be away from the others (see Example J-16). To PJ, this scattering may have implied that the data was not linear because some of the points were off.

Josh may have seen that a related idea might be problematic to students: once I extend the data, I can read predictions from the line of best fit. However, Josh seemed to want to make the point that this would give a prediction, but not indicate what would actually happen if they tested the data.

Example J-24

> J: Okay. Now, how did you guys- how many just extended their line? [about 6 hands go up] If you extend your graph model line or your line of best fit, that will help you make a prediction. Now, if I go ahead and test that data, are we gonna get exactly 51? Probably not, **but** it's a prediction. Okay, what I'd like to do now is this [...]

Again, Josh pointed out that the lines of best fit that they drew allowed them to make predictions, but the predictions did not represent what might actually happen. More than likely they won't get "exactly" 51.

Another way Josh used "but" was to suggest a more precise way of using the textbook. In this example, Josh asked the class how the students from Maryland had generated the slope for the equation model they wrote (which was given in the book). One student offered that he thought they had used "rise divided by run." Josh agreed with this, but wanted the class to pay particular attention to the actual points the Maryland student had used.

Example J-25

> J: Yeah, rise divided by run, **but** what points did they choose? Did they choose two of the data points that were actually in the, in the Maryland class data?

By drawing students attention to the actual points the students in the text had used, Josh directed students to attend to something that he believed to be important and he indicated that this statement was important by using the word "but." Josh may have tacitly shown that it was important to pay attention to the details in the textbook because they can help us think about what we are doing—in this case, finding the equation model for a graph model.

The word "but" often appeared when Josh was trying to get students to attend to details that he may have thought they were missing. In the following example, students had practiced drawing what they thought was the "line of best fit" for each set of data points they were given on a worksheet. It appeared that Josh wanted to make this definition more specific to help students be more precise in their drawing when he said:

Example J-26

J: Now, I'm not so sure if that's gonna be my line of best fit. It's a line, **but** does it best fit the data?
Ms: No
J: Cuz if I draw that line ahead, I'm just gonna have this point on it, it's gonna have this point on it, **but** all these points are gonna be quite a bit below it. Laurie, what'd you do?

Josh first used "but" to indicate that there was a particular piece of information that needed to be attended to: a line of best fit was not just *any* line, but a line that "best" fits the data. In his next turn, Josh used "but" to contrast something that a student had drawn in for his line of best fit. He pointed out that Alan drew a line, but that many of the points in the data were "quite a bit below it," suggesting that the line Alan had drawn was not the line of "best" fit. He called on Laurie to see if her line fit the data more closely.

The word "but" appeared to be similar to Karla's "so" in that both were repeated discourse formats that were marked with special enunciation. Often when Josh used the word "but," it was followed by "I." By choosing a contrastive word rather than a connective one, Josh may be setting up an environment where students view him as being an "external authority" to the community of the classroom. He was contrasting his own view with the ideas that students shared.

"Read on page...," "Look in your book...": The textbook is part of the mathematical authority and should be drawn from in many ways. Josh referred to the textbook in at least three different ways that helped define the role of the text in class. In the first, he would explicitly call on a student to read directly from the textbook or he himself would read something from the textbook—sometimes this defined the activity in the classroom. Secondly, Josh would refer back to the text when students had difficulty answering a question—allowing students to see that the text could be used as a resource when students were uncertain or unable to answer a question. Finally, he would sometimes refer students to the text to define what the homework was and what he expected students to do—asking them to pay particular attention to the directions that were given.

Example J-27

> J: Pretty linear? Let's find out. Uh, what I'd like you guys to do is this: **If you look in your books on page 7. [Ss open books] Problem 1.2 is called drawing graph models**. And what it has you look at, is it has you look at a classroom set of data from Maryland. And, they plotted the data and they tried to decide if it's linear or not. Now, we're gonna look at that in a couple of minutes, but I'd rather look at our data. So, what I'm gonna do is I'm gonna give you a sheet of graph paper and I'd like you and your seat partner to talk this over. And I'd like you to graph the classroom average.

In this example, Josh used the text to draw student's attention to what they were going to do next. It signaled that they were moving from talking about the data they had collected to actually looking at how to draw graph models. Not only did Josh indicate what page students should have their textbook open to, but he also interpreted the problem for students. By playing the role of interpreter, Josh may have positioned himself as an authoritative source that has the right to interpret what the text says for his students. This may be another way that Josh controlled the communication system in the classroom. In Example J-7, I had argued that Josh directed PJ to "say that again," acting as a mediator through which all ideas must pass. He may have also thought this was his role in terms of helping students make sense of the text. Instead of sometimes reading from the textbook, Josh interpreted it, as he is in Example J-27. Also, he deviated from the text and in his role as teacher, he has the right to redefine the task in terms of the data they collected first (even though the textbook may not indicated this).

In the next example, Josh called on students to read aloud from the textbook.

Example J-28 (see also Example J-23)

J: [...] So, how are we going to show the linear pattern? **Let's go ahead and take a look and read problem 1.2 where it says, "drawing graph models." Cory, would you read that please on p. 7**.

Cory: Yeah. A class in Maryland [...][reads from textbook]

J: Okay, first of all **look at their class average and then look at our class average. How do they compare?**

Cory: Theirs is like heavier or something

J: Okay. Looks like on every single layer, there's noticeably more pennies than ours.

Ms: [inaudible]

J: Maybe. Maybe it was thicker

Ms: Maybe their [inaudible] was thicker [inaudible]

J: I don't know, maybe they used construction paper or something. I don't know.

Ms: Looks like [inaudible]

J: Let's go ahead, read on

Cory: The class then made a graph of the data [...] [reads from textbook]

J: Okay, now, **let's look at that line again—[reading from the textbook] This line is a good MODEL for the relationship because for the thicknesses the class tested, the points on the line are close to the points from the experiment [re-reads part of what Cory just read]. Take a look at what they did.** Now, their data was a little bit scattered, a little more scattered than ours was. But, they still were able to draw a line that seemed to fit the data pretty well. No point is really far away from the line, there's a couple of points below, you know, one or two slightly above. **That is sometimes called [reads from textbook] a line of best fit.** We're gonna use that term an awful lot. **Cory read on.**

Cory: The line that the Maryland class drew was a graph model [...] [reads from textbook]

J: **Okay, I don't have a vocabulary chart yet** [typically has these hanging on the wall for each unit], I forgot to put it up, I'll get it up later. But, there's a **good definition for a graph model**, it's one of your **vocabulary words**. It's a straight line or curve that shows a trend in a set of data. It models the data, it shows the path. Now, what I'd like you guys to do, is I'd like you, there's a ruler on each table. I'd like you, if you think this is linear, if you don't think it's linear, you could draw something else, but if you think this is a linear relationship, I'd like you to draw a line that you think fits that data. It's gonna be a graph model that is a line that fits the data. Draw where you think it should go.

In this rather extended example, Josh called on Cory to read the page he had interpreted earlier. Josh asked students to compare their class average with the class average in Maryland. By doing so, he may have indicated that the numbers in the textbook were something by which they can compare their own work—it may be a standard to judge whether the answers they came up with were reasonable or not, especially in Josh's absence. After students shared why they thought the data was different, Josh asked Cory to read on. Josh pointed out the important idea students should take away from the textbook by re-reading a portion of the text that Cory just completed: "this line is a good MODEL for the relationship because for the thicknesses the class tested, the points on the line are close to the points from the experiment." After Josh talked about how this idea related to the Maryland student's data, he told students that this is sometimes called "a line of best fit." He then explicitly established this as how students should talk about this idea—it was something that "We're gonna use…an awful lot." At this point, students should know that they will be hearing this word and will be expected to use it, also. Josh then called on Cory to read more of the text so that Josh could introduce the next activity. Josh pointed out that they have a new vocabulary word now (that would usually be hanging on the classroom wall before the unit began) and gave a "good definition" as it was stated in the textbook. The next activity involved students in applying this "good definition" to the class average that they had computed.

In Example J-29, Josh again acted as interpreter for the textbook and told students that he wanted to make sure "we" understood what the class in Maryland had done. He was pointing out that students should notice the line of best fit (which he then renames as a graph model) and, more specifically, they should notice where the line was drawn in—that it matched the data "pretty well."

Example J-29

J: Would you **read where it says** finding an equation model. Would you **read that please**. Jennifer would **you read please**?

Jennifer: In your early work [inaudible] y = mx + b…[reads from textbook]

J: Now let's **make sure we understand what they, what this class in Maryland just did**. They drew in a line of best fit, they drew in a graph model. **Look where they drew their line**. Does that seem to match data pretty well?

As the discussion ensued, Josh turned to the text to discuss the purpose of the models that they were making.

Example J-30 (see also Example J-21)

J: I'm not sure. Okay, Robert's a little bit steeper so that this point is just slightly below the line and this point here is a little be closer to the line? **Remember what a graph model is—it's a line or a curve that shows the trend in the data, it fits the data**. So that all the points are pretty close. Um, I don't know. Why do we do this? What's the purpose of a graph model? Abram, what's the purpose?

[Abram's hand goes up right away]

Abram: To show the linear relationship

J: Yeah. I could maybe see that it's linear just from looking at the table or if just looking at the way the plots are pointed. Why did I draw the line in? Just to show the pattern? Christy?

Christy: To get a better look at what the data is trying to tell you

J: Well, maybe that's part of it. **Look back at your definition for a graph model. Look at your definition of a graph model. What does it say? Read that last paragraph to yourself on page 7.** Lance, what's the purpose here? Why do we even bother doing this?

Twice Josh asked about the purpose of graph models. Each time, he defined the question more specifically (see discussion pertaining to Example J-9). When students did not respond the way that he had hoped, he explicitly told students to look back at the definition of a graph model. When there was not an immediate response, he told them the exact page and paragraph to look on to answer the question in the "correct" manner. In doing so, he may have tacitly let students know that they should refer to the text when they were uncertain of something. In this case, the textbook established why they were engaging in the activity at all!

On a more simplistic level, Josh referred to the text explicitly when the upcoming activity was being determined. He pointed out that the directions were important and then referred back to a similar activity in which they had engaged earlier in the unit.

Example J-31

J: If you go over the Macallister bridge, it has those center spans. We would all agree that this bridge right here is not going to be as strong as this bridge right here. Let's test to see what kind of relationship there is. **If you look at the directions, it says we're**, you're gonna need eight four inch wide strips of paper with lengths 4 inches, 5 inches, 6, 7, 8, 9, 10, and 11 inches, two books of the same thickness, a small paper cup and about fifty pennies. Now this sounds very familiar to the problem we did about a week and a half ago, right? This time every bridge you make is only going to be one layer thick, one sheet of paper. What we're going to vary are the lengths. One thing you're going to change on this right now. Instead of bridge thickness, you're going to have bridge length. You're still going to have breaking weight. **The book says** to start with a bridge four inches long, we're going to start with bridge that is 5 inches long because when you do the one inch overlaps, you only have two inches in the middle and I'm not positive your cups would even fit on that. So, start with bridge five inches and then go six, seven, eight, where do we stop?

Ms: 11

J: 11 inches? Okay you're going to need to do some measuring, and a couple things to watch: make sure the only thing that differs from one test to the next is the length of the bridge, make sure you place your cups in the middle, make sure you place the pennies in, make sure you have one inch overlap on your books. Most people who screwed up last time, that's what you screwed up. Alright, any questions?

Cory: Yeah, um, what's the width of that [bridge]?

J: The width is just like last time, **make sure you read your directions** the width is four inches and you're going to fold it up one inch on each side so the bridge itself is only two inches wide. Questions? Okay, I'll let you guys get into your groups. I'll lay out materials [...]

In Josh's first turn in this example, it is interesting to note the pronouns Josh used. He separated the activity they had already engaged in by saying "we would all agree" (pointing to a joint understanding that they had already come to agree upon) from the activity that students would soon engage in as "you're gonna…." In addition, this was separate from the "book" because its role was giving direction to the activity. However, he again asserted his authority in deviating from the task as it was defined in the textbook—"the book says…," but "we're going to…." Josh seemed to be checking to see if students were listening when he asked where they were to stop. A male student implied that he was listening by answering this teacher-question. When a student then asked a question that Josh had already answered, Josh referred to the textbook again—"make sure you read the directions."

Summary

In this section, I have shown how Josh established himself as part of the mathematical authority through the use of two words that are "teacher-words": "agree" and "reasonable." In using these two words, he established what was correct and voiced his opinion regarding student ideas. He further established himself as a separate "knower" from his students by using the form "but" which is contrastive, rather than choosing a connective form when he was making important mathematical points.

Furthermore, Josh located the textbook as part of the authority from which students could draw when they needed. This was done by his explicit use of the textbook, which defined the tasks in the class, offered common ways of talking (to which Josh drew explicit attention) and defining mathematical terms, and directed students in what they should do outside of class. As Love and Pimm (1996) point out, there is an inherent authority to the text that cannot be ignored, "though [the teacher's] response to it may range from taking it for granted to seeing their role as challenging and criticising it" (p.

380). It appeared that Josh had taken his role to be a guide for students in how to interpret and use the textbook as part of the authority on what and how to do mathematics.

Sources of Knowing

Closely related to issues of authority in Josh's classroom was recognition of "knowing." As I maintained earlier, it appeared in Josh's use of "but" in which he may have indicated that he was a separate "knower" from the classroom. There were at least three particular sources of knowing that were implicit in Josh's talk: "model" students, the teacher and the textbook.

Knowing is Characteristic of "Model" Students, the Teacher and the Textbook

"...if I wanted to know...": Model Students Want to Know. In Josh's classroom, the word "know" appeared to be a teacher-word: that is, Josh was the only person to use the word "know" with words other than "I don't [know]." The only exception I have been able to locate was included in Example J-39 when Alan explained something and told Josh, "you know, the 33..." as he pointed to the last point that was graphed on the board. Often when Josh used the word "know" it was in conjunction with "if I wanted to [know]." This phrase seemed to act as Josh speaking as a "model" student who would do certain things.

In the following example, a student (Abram) drew in what he believed to be a curve of best fit for a set of data points. In drawing in this curve, he drew something that Josh believed would not allow him to make predictions. During the discussion, Josh offered some evidence to try to get students to see that they were not thinking about the idea correctly. While doing so, he used the words "wanted to know" many times.

Example J-38

J: You know, when you're talking about a curve, I've-we're talking- many of you have seen graphs that have curves that have something like this [draws an increasing exponential-like curve].

Ss: Yeah/Yes [overlapping, many students respond]

J: Or a curve that might look something like this [draws a decreasing exponential-like curve]. Okay, those are, those I could extend, right? **If** I extended my graph, **would you know** where to—here, let me give you an example. **If** I drew that much of a curve and **I wanted to know**, what the—if you're talking about breaking weight or something, **suppose I wanted to know** what the data was for this number right here [chooses a value that is beyond the curve that has been drawn] on my x-axis, whatever it was. Could you extend this graph [the one that Abram had drawn] to help you predict what the y- would be for this x-value? **If I wanted to know** what the y- was for this x-value?

Abram: Yeah, you could.

J: How?

Ms: I don't know.

Abram: You just go right up again.

Ms: You just keep going like that.

J: How do **I know** I go up? How do **I know** I don't keep going like this?

In this example, Josh appeared to be revealing what he knew about the mathematics through the voice of a model student. Instead of telling students that Abram's graph could not be extended in a way that "I know" will keep going, Josh masked his comments through saying, "If I wanted to know." Josh's "I" may be referring to anyone who "wanted" to know. However, the "I" using this pronoun (i.e. Josh) was modeling the type of wondering he would like students to be doing: that is, he was wondering aloud possibly in an attempt for students to see the kind of knowing they should "want" to be doing. Josh may have hoped that students would take on "wanting to know" and revealed how he was thinking about the situation. The mathematical message underlying his wondering was that a graph model must be drawn not only to "best fit" the data, but it must also allow the graph to be extended so that students can predict future values. If a set of data points did not allow both of these things to happen, then a graph model cannot be drawn.

Another way that Josh used "want to know" was in defining the components of a task. In the following example, students generated an equation model for their individual graph models and Josh asked them to talk about how they got the equation at which they arrived. He reminded them that they needed to know the slope of a line to write the equation, which is the first thing he advocated finding in writing the equation model.

Example J-39

J: And **we want to know** the slope of that line. What did you do? Alan, what did you do?

Alan: Uh, I took 33 as the rise and divided it by five.

J: Now where did you get 33 from?

Alan: From like our, um, the, bri- the paper, **you know**, the 33. It was the highest [point].

J: Right here?

Alan: Yeah.

In a sense, Josh was again modeling for students the type of thing they should "want to know" about finding an equation model: that is, they needed to know the slope. Alan responded by telling his what he used as the rise and the run (although did not name the run as such). He used 33 as the rise and five as the run. Josh probed further by asking Alan where he got 33. In the graph model, the last point on the table was (5, 33). Implicit in Alan's explanation was that he used the origin (0,0) as his second point to determine the rise and the run (i.e. rise/run = (change in y-values)/(change in x-values) = $(y_2 - y_1)/(x_2 - x_1)$ = (33 – 0)/(5 – 0) = 33/5). However, Alan did not explain that so Josh pushed his explanation to discuss where his values came from. When Alan replied, he may have thought that the value he had chosen was obvious because he said "you know, the 33." Josh clarified this further by pointing to the value on the board so students could see where Alan had gotten the 33 he had used.

The other two functions the word "know" played for Josh appear in Example J-40. In the first (referred to in other examples), Josh would assess how many students were thinking/doing some given thing. The second use appeared in four of Josh's turns during the two class periods. During this use, Josh said "should know" during a monologue. This typically took place when Josh was telling students what he expected them to know at that point in the school year: that is, during times when Josh was being explicit about parts of the common knowledge of the classroom.

Example J-40 (see also Example J-10)

> J: Right there, isn't it? [writes equation y = 1x + -3].So, y = 1x or just x plus a negative three or minus three is gonna be your equation. That's something you all should be able to do by now. In fact, it shouldn't even be that hard. **How many know** that if a slope of one, if the scale is a standard scale going up by ones on the x and y, **how many know** that a slope of one is a 45 degree angle going up? **Shouldn't you know** that a slope of one; over one up one, over two, up two; over three up three; is a 45 degree angle going up? That's something **you guys should know** by now. How about graph B? They gave you two points on the line in graph b. Jaylen, here, give me two points on the line for graph B.

This example took place as Josh was going through the Check-up with students. By this point in the school year, Josh was defining what students "*should* be able to do by now." They *should* be able to find the equation of a line. He changed direction a bit when he asked his students, "how many know that ...a slope of one...[is a line that goes up at a] 45 degree angle?" He was no longer asking about finding the equation of a line: he was now asking about a specific line and how that line appeared on the graph. He reminded students about what they "should" know. In choosing the modal verb "*should*," the underlying message may be "*should*, but *don't* [know]." When Josh read this bit of transcript (and two others like it) during our interviews, he told me, "This is me being pissed off." He indicated that he was upset because he did not think students were where they should be at in terms of what they knew about the mathematics they had covered.

"...what you know...": Knowing sometimes resides in the textbook. The other place "know" appeared was in Josh's utterances when he referred to information that was given to students in the textbook.

Example J-41

J: [...] Okay, let's take a look. So, what negative two means is if x increases by one, y drops by two. So, if **you know** x is 3, what's y?
Ss: [inaudible]
J: Three. Three. Right, x increases by 1, y decreases by 2. If you want to do it in the table, you should do it in a table. **I know** that at 2,5 slope is –2, so three would be three, four would be what?

Example J-42

J: Thirteen, isn't it? Or you could do it like this. For those of you who understood problem 1.5- that standard form of linear equation [writes y = mx + b on the board]. **You know** that point is on the line [circles 4, 5 in the table he just worked through]. Tell me what **you know**. Give me a y-, an m, or an x-value. What do **you know**? Tell me something. Jason Normington, tell me something. What are you given? What do **you know** in problem six? Do **you know** the slope?
[...] [conversation continued about what is given]
J: So I put in what **I know**. So, let's see, what's negative 2 times four?
Ss: Negative eight

In both of these examples, Josh used either "you know" or "I know." However, these statements both seem to indicate the same group of people—"I" seemed to be used as if it were "you." However, in using "I" Josh may be giving the statement of knowledge additional authority. In all of these examples, the place the information that was "known" came from was the textbook. In each of the examples, Josh seemed to be pointing students to what they "know" about the situation as defined by the textbook and then used what they knew to find something else.

In Example J-41, the x-value was known, so the y-value could be determined because the slope and a point on the line were given in the textbook. In Example J-42, Josh indicated that students again know a point on the line, he then requested that the students (first generally and then more particularly calls on Jason) to name the objects that they knew. In doing so, Josh may be trying to make a connection between the information that was given to the standard form of a linear equation. There was evidence that this connection was being drawn in the language that Josh used. After he wrote y = mx + b on the board, he then asked what was given in terms of these values "give me a y, an m, or an x-value," each of which holds a "slot" in this general equation. Josh used a Type 1 reformulator to make the question more precise and portrayed them as related to the terms in the equation. He called on Jason to tell him what he was "given" and then equated this with what "you know." When Jason does not respond right away, Josh used what appeared to be a Type 2 reformulator to indicate the incorrect choice. However, this reformulator was more closed than it appeared because Josh had previously narrowed down the range of responses to include three possible values: an x-, an m, or a y-value. After the values were recognized, Josh indicated what students were to do with them: "put in what I know," that is, substitute the values where they belong in the equation.

Summary

Knowing was attributed to Josh and to the textbook. These 'voices' of knowing had common ways of addressing the students through asking students multiple questions. "Knowing" was something that a model student "wants" to do. Josh was associated as someone who "knows" as he was the only person to use the word "know." The textbook appeared as a source of "knowing" by virtue of the fact that when students were asked, "What do you know?" the answer was to be found in what they were given by the textbook. What was in the textbook was absolute knowledge.

From Forms to Norms

Throughout the chapter, I have described the forms and functions of various discourse patterns in Josh's classroom, asserting that certain norms exist in the classroom as a result of the pervasive nature of these patterns. I now briefly address the norms in terms of the roles, rights, responsibilities and expectations the teacher, students and the content play in the classroom.

The broader forms that Josh has implemented included: asking questions, expressions that establish his authority and attributing thinking to sharing ideas and knowing to model students, himself, and the textbook. Through these Josh has established his role as the "asker of questions"; students are expected to answer them and give reasons for their answers. Josh has established his role as being part of the locus of authority, as a source of "knowing"; students are expected to generate ideas. Josh has also indicated that the textbook is an important source of knowing; students are expected to use and refer to it in many ways.

In the more specific forms that Josh used, many of the norms in his classroom seem to be directly related to the mathematical content:

- Pay attention to and agree with the common knowledge;
- There are sub-questions that allow you to focus in on the content;
- Strongly established common knowledge is a decision to be made; and
- Plurality is important in determining what is mathematically reasonable and what is not (limited invitation to diversity); and
- Know the reasons for the common knowledge.

The first thing Josh mentioned as important in influencing what he did in his classroom was his own content knowledge. He also said that the norms that he tried to establish in his classroom were embedded in the mathematical goals of the lesson. This seemed to also be quite apparent in the norms of his talk. The first three norms in this group have to do with what students should know; they are strongly associated with defining and drawing attention to the common knowledge.

One of the most important expectations Josh set related to the content was that students should be able to generate multiple solution methods.

> I'd always try to at least give the opportunity for kids to kind of talk about other ways that they might have solved the same problems. I think that's extremely important [...] I'm not really sure if there was ever a time when I explained that to them, that I want you guys to do this. I think it was more of an understood sort of thing and I tried to develop that by just pushing them a little bit, especially in the beginning to come up with different ideas. [...] I think kids realized that *how* they did things was every bit as important as the end product and the whys were very, very important. [...] They had to understand that there was a purpose to all this and there were some real key issues and key concepts that we were going to deal with. I tried to give them a picture, a bigger picture of where we were going (Interview J-1, 11/23/99).

In the above quotation from Josh's first interview, he also said that knowing the reasons for your ideas were "very, very important." This was something that he often insisted students do. He not only pushed students to justify their answers, but it appeared that students also felt that this was something they needed to be cued to do. Often, they would give an answer but rarely did they give reasons without Josh asking them to do so.

One norm was associated with Josh's expectations for students and what he believed their responsibility to be:

- Model students want to know.

Josh also seemed to allude to the fact that if students were "model" they should "want to know." He often phrased his questions in such a manner as to let students know that this is something they should do—"want" to know.

The last set of norms that appeared in the classroom discourse were directly related to Josh's and the textbook's roles in the classroom:

- As *an* authority of mathematical knowledge, one of the teacher's roles is to clarify the content;
- The textbook is part of the mathematical authority and should be drawn from in many ways;
- "Knowing" sometimes resides in the textbook; and
- The teacher has the right to judge the reasonableness of a solution.

The issues associated with the locus of authority were not directly addressed in the interviews with Josh. However, he did talk about his role was in the classroom:

> You could use that term 'facilitator,' I guess. But at least on paper I believe my role as, but again there were times when I found myself crossing the line and again being the presenter. "Did you guys think about doing it this way? This way? This way?"-as opposed to letting the kids generate their own ideas (Interview J-1, 11/23/99).

While Josh recognized that he pushed students thinking, he also recognized that he did not always let students generate their own ideas. He said that his years of teaching had offered many alternative strategies and that he often offered these when he struggled to get students to talk. Getting students to talk was more of a problem in this class than he remembered in other classes, so he felt that he ended up contributing like he would have liked the students to do:

> I found myself becoming a member of the class and me saying, "Geez, how about, I mean, look at it this way," or "look at it this way" (Interview J-1, 11/23/99).

In fact, the only statement Josh made about his own authority was more related to how it may have affected students:

> I really like to think that I try to make kids feel this successful and if the students gave an incorrect answer I'm really-I really don't think, at least I hope I didn't add to their negativism about mathematics. I hope I didn't embarrass them and I hope that in my classroom that getting an answer that was maybe incorrect, was not all that bad. That many times that the answer was incorrect that maybe if I looked hard enough and asked follow up questions that I could get a handle on some ideas that were actually right on target (Interview J-2, 12/02/99).

The issues of authority were fairly explicit in Josh's classroom talk. He had students contribute ideas and justify them through eliciting ideas. Once the ideas were on the table, however, he used his authority of the subject matter to determine which answers were reasonable. He also sometimes ended segments of topically related sequences by telling students with what ideas he agreed and disagreed.

Josh also explicitly drew from the textbook and offered it as a source of knowing in the classroom. The text was used to determine classroom activities, introduce and define mathematical language and as a reference source in which answers to Josh's questions were sometimes found. In addition, the information that was given in the textbook was associated with what students might "know."

Josh's initial reaction to the vignette I offered in the first teacher interview (see Appendix H) showed many of these same facets to which I have referred above:

> [...] it's extremely powerful being able to find the y-intercept in so many different ways and realizing the relationship between all the representations, between graph, table, and equation, um, that's one of the big ideas in the entire unit is being able to tie all that together [....] I would maybe take a minute to pull everyone together and say, you know, is that right? Is that how you did it? No, but is it still right?" (Interview J-1, 11/23/99).

In this reaction, Josh first focused on the mathematics in which the students were engaged. He said that he would bring everyone together and focus on whether the answers were "right." This response is a nice instance of two of Josh's central foci: the mathematical content and the determining of correct answers.

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[1] For a more extended look at a transcript from Josh's classroom talk, see Appendix O.

[2] For a timeline of these two days and the location of each example within those class periods, see Appendix K.

[3] This problem is included in Appendix L.

[4] Cazden (1986) considered the difficulties that researchers have discussed associated with classifying questions based on their presumed cognitive level. However, since I am not focusing on the cognitive difficulty of the questions, I will not explicate some of these here.

[5] The distinction being made here is that this type of question could be perceived as being factual because it elicits previously learned knowledge. However, it is open because a wide range of answers may be acceptable. A question of this nature presents an opportunity for students to describe observed phenomena for which they have not yet learned a name.

[6] Vacc (1993) explicates these further into each of the subsections. She defines closed reasoning-recalled sequences as those that "require students to develop one acceptable, logically organized response based on previously acquired knowledge" (p. 90); whereas non-recalled are not based on previously learned knowledge.

[7] This is a "form of language which tends to be associated with the expression of shared attitudes and opinions rather than of individual differences" (Britton, pp. 108-109).

[8] In this particular example, this is actually a metarequest.

[9] This is something that I have noticed in both classrooms—students seem to become comfortable with "a" pattern meaning that the function is linear. However, the pattern has to be of a particular type to be linear—it has to be a constant additive pattern in the table. That is, as the x values in the table increase by one, the y values increase by the same amount additively. It is possible that students take all patterns to be linear at this point because they have not discussed many contrasts and will soon get to non-linear in the next Investigation.

[10] The other two are social distance and rank of impositions. However, it appears that power is playing a larger role here, so that is my focal point.

[11] I am labeling this as "vague" because even as an observer in the classroom, I am unsure what she meant by the answer that she gave.

[12] In this lesson, students are being introduced to linear graph models. Some of the main mathematical ideas that students focused on in the lesson include drawing graph models (or lines of best fit). This is not to say that issues associated with scale are not important, they just weren't central to the topic of discussion at hand.

[13] Some examples come from the same segment of transcript or are overlapping. Where this is the case, I will indicate the other example in this manner.

[14] The values students got in the table were:

Bridge thickness:	1	2	3	4	5
Breaking weight:	5	11	17	23	33

[15] This claim is made based on the fact that there are no other instances of this type of juxtaposing of student ideas occurring in Josh's classroom in the data that I have analyzed.

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A TALE OI

HOW DISCOURSE STRUCTURES NORMS:
A TALE OF TWO MIDDLE SCHOOL MATHEMATICS CLASSROOMS

VOLUME II

By

Beth A. Herbel-Eisenmann

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

2000

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CHAPTER 8: CROSS-CASE STUDY

In the past two chapters, I described and interpreted the three most pervasive discourse patterns in Karla's and Josh's classroom talk. In this chapter, I look across the two case studies with the intent of pointing out similarities and differences. In Chapter 5, I showed that on the surface Josh and Karla are amazingly similar in terms of their background, experience, education and professional development activity and in terms of their students' understanding. They have even described the impact of their involvement with CMP as having similar implications for their teaching. However, a closer look at the discourse patterns in their classrooms reveals two different environments in which students are learning. Although I recognize that classroom formats change throughout any given lesson and that sometimes the activity that is being engaged in determines the format (Cazden, 1988), I will argue that in Karla's classroom students seem to be taking part in more discussions. In Josh's classroom, the students often participate in a recitation. The implications this might have for how students are engaging will be explored from a theoretical perspective at the end of this chapter.

Comparing Norms

I begin by explicitly comparing the norms in the two classrooms (see Table 8). The most noticeable aspect of this table is where the majority of the norms fall. Both Josh and Karla seem to carry in their discourse many sociomathematical norms. Some of these norms are very similar. For example, both teachers value multiple solution methods. In addition, they both address the fact that there are larger ways of addressing multiple problems. Josh used sub-questions to get students to focus on the components of their thinking; F

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thinking; Karla used Approximators to indicate class membership, possibly hoping students would see that some of the ways they addressed each mathematical relationship was going to be similar. In this way, it seemed that Josh focused his questions on past mathematical processes; Karla appeared to be working across time, referring to past, present, and future class memberships they have learned or may be working with soon.

Table 8
Josh's and Karla's Norms

Focus of Norm[1]	**Josh's Norms**	**Karla's Norms**
The Mathematical Content	Pay attention to and agree with the common knowledge; There are sub-questions that allow you to focus in on the content; Strongly established common knowledge is a decision to be made; Plurality is important in determining what is mathematically reasonable and what is not (limited invitation to diversity); and Know the reasons for the common knowledge.	There are connections in both the content (especially in terms of the multiple representations that are discussed) and the contexts that must be recognized; Plurality should be endorsed and acclaimed; Everyone should hypothesize the possibilities with respect to the mathematics; Mathematics is in the mind of the beholder; and There are relationships beyond what has already been discussed and explored, but the ideas for thinking about them are similar (e.g. you still need to think about the same things when you're trying to decide what is happening).

Students and Class	Model students want to know	The big ideas are stated and people must take responsibility for their own learning; Contributions will be valued and credited; Uncertainty is something that should be acknowledged; Mistakes are okay to make (or I'll protect you); and This classroom is a community with a common history.
The Teacher/Teaching	As *an* authority of mathematical knowledge, one of the teacher's roles is to clarify the content; The textbook is part of the mathematical authority and should be drawn from in many ways; "Knowing" sometimes resides in the textbook; and The teacher has the right to judge the reasonableness of a solution.	The teacher has the right to summarize and contribute as she sees fit.

The emphases that appear in the remaining sociomathematical norms indicate a few differences between Josh and Karla. For instance, Josh focused on the how's and why's of the content; Karla focused more on hypothesizing the possibilities of the mathematical c

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mathematical content. Josh used question forms to identify and draw attention to the common knowledge of the classroom. Karla developed the context and continuity of the common knowledge through telling stories about classroom events. Karla also drew student attention to the images and thinking processes in which they were engaged.

The differences that appear in the social norms are quite interesting. Most of Josh's social norms seem to be associated with the textbook and his own role in the classroom. Karla's attention seemed to be more directed at students and what she expected of them. This is consonant not only with the teacher interviews (e.g. Josh's concern with control; Karla's concern with listening to students) but also appeared to be evident in a closer look at the discourse forms in the classroom. For instance, the ways in which the two teachers viewed and drew on sources of knowledge in the classroom differed. In addition, the ways in which they defined their own roles in the classroom were different. These aspects (as well as others) are addressed further in the following sections.

Fine-Grained Discourse Analysis

Turn Length and Frequency

Proceeding to a micro-level of analysis, there is little difference in the length and frequency of turns each took in the two focus class periods. For a representation of these lengths, see Appendix P. Across the two class periods, Josh took 289 turns and Karla took 269. Josh's longest turn was 375 words; Karla's longest turn was 243 words. The largest difference in frequency (i.e. 12) exists in the 31-34 word range, followed by the 1-10 word range (i.e. 11 turn difference). In each of those categories, Josh took more turns. In the three categories in which Karla took more turns (i.e. 21-20 words, 66-100, 151-375 words), the d

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words), the differences were six, nine, and two respectively. Overall, there is very little difference[2] between the length and frequency of Josh's vs. Karla's turns.

When comparing the lengths and frequencies of student turns (see Appendix Q) in the two classrooms there appeared to be a larger difference between the number of short and long turns[3]. The longest turn one of Josh's students took consisted of 34 words. In Karla's classroom, there were two student turns that consisted of 50 words. This did not include turns in which students were explicitly asked to read from the textbook[4]. Josh's students had almost twice as many 1-word turns as Karla's students. In fact, Josh's students outnumber Karla's in one, two, three, four, and five word counts (where the total difference came out to be 213-151 = 62 turns). When focusing on the longer turns, Karla had 35 student turns that were 15+ words long compared with Josh's 14. In the middle range (6 to 14 words), Karla's student had more turns in every category except at 9 word-turns. Again, Karla's students took more turns in this range (87 compared with 50 in Josh's class). Overall, the actual number of turns students took in each class was not very different (278 in J's classroom vs. 273 in K's). Josh's students took more short turns and Karla's students took more long turns[5].

Not surprisingly, however, their classrooms were different places to be as I felt quickly in my work in them. In the next section, I address these differences. In doing so, I offer what I see in the previous two chapters as "meta-data," beginning at a larger structural level and then proceeding to focus on particular forms each teacher used. I will then turn to exploring what difference these differences might mean for students.

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Larger Structure of Discourse Format

One difference between Karla's and Josh's classroom talk occurred on a larger level. Most of the talk in Josh's classroom was of an I-R-E format with little deviation (for a detailed example see Example J-7, pp. 172-173). Throughout Josh's case, I pointed out many instances of the I-R-E sequence. When I realized how pervasive it was in Josh's talk, I used Mehan's categorization of Topically Related Sets (see Cazden, 1988, pp. 32-35) and place Josh's transcripts into this structure. I was quite surprised at how easily the classroom talk fit. To see a portion of this categorization from the October 6th transcript, see Appendix M.

This three-part sequence is the most prevalent style of classroom discourse at all grade levels. Cazden (1988) claims that this pattern is most obvious in a teacher-led lesson or recitation, "in which the teacher controls both the development of a topic (and what counts as relevant to it) and who gets a turn to talk" (p. 30). This idea of control of topic and turn appeared in my interviews with Josh. Josh said that he even though he wanted students to offer "multiple or varying paths," that he:

> still had the idea of where I wanted the kids to go [...] there was kind of an endpoint. And I think some CMP teachers are a little more student directed as to the direction they want to go and that's wonderful. I just don't know that I could go that route and I have the end product in mind (Interview J-1, 11/23/99).

Josh recognized that other teachers let students direct the conversation; however, he felt that he still needed to maintain control of the classroom talk. In fact, when I asked him what he thought was important to establish early in the school year, he said,

> I think I kinda wanted to get a general control of the classroom and understand that it's your classroom 49% of the time and mine 51%. I think that's important the first couple of weeks. And then, gradually, bringing out the joint ownership of the classroom (Interview J-1, 11/23/99).

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One way that he might have attempted to do this was to use (although probably not consciously) an I-R-E structure. Josh stated that he believed that this was his "teaching style" and he felt that he "had to have control over the classroom."

This was not to say that Karla did not have an I-R-E format in some segments of her classes. In fact, it appeared typically during times when she was going through homework or after she returned something she had graded. However, Karla often used another structure that Josh rarely used[6]-- she also used a lot of narrative storytelling. In some cases this appeared in the form of reconstructed dialogue (e.g. see Example K-7, p. 98). In other cases, Karla told stories about classroom happenings without using dialogue (e.g. see Example K-2, p. 89).

This type of narrative structure is more indicative of the type of talk that is valued outside of school (Edwards and Mercer, 1987). In fact, Cazden contends that there is a hidden curriculum in some classrooms which "denigrate such narratives and press for the substitution of other forms of explanation and justification" (p. 185). This did not seem to be the case for Karla as she often replayed stories and conversations in her classroom.

Common Functions Using Different Surface Forms

Although many of Josh and Karla's surface forms were different from one another[7] in the two cases, some of these forms fulfilled similar functions. For example, both Josh and Karla valued plurality. However, Josh used an interrogative form: "What do you think?" or "Does anyone have something else?" whereas Karla used narrative metacommenting, pointing out what "some people" did and what "we found." In addition, the way in which they responded to the multiple ideas asserted and established different pattern

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different patterns of authority. Where Josh often concluded the offering of strategies by directly telling students what he "agreed" with or what he considered "reasonable," Karla told students what she "would" do and shielded students from incorrect responses by using words like "people," "probably" and "seemed." In this section, I discuss how these different surface forms (while serving similar functions) may have helped to contribute to

Table 9
Common Functions using Different Surface Forms

Common Discourse Function	Karla's Forms	Josh's Forms
Plurality and Teacher Authority	• "Some people…other people"; "We found"; • "I would"; "Probably"; "Seemed"	• "Does anyone have something else?; • "I agree…"; "Reasonable"
Source of Valued Knowledge	• "They"; • "If you go too fast, I get lost"; "Help us out here"; • "Swoopy"	• "Read on page…"; "Look back in the book…"; "What do you know?" • Multiple Questions: Open → Closed questions • More official mathematical language
Drawing attention to the common knowledge	• "So"; • "Yesterday…"; "Remember…," "We found"; • "Can"	• "But"; • Tag questions; Proposition + "Why?"; • "Should"

different contexts in which students were learning. The different surface forms I discuss are summarized in Table 9.

Plurality and Teacher Authority

Karla and Josh both valued when students offered multiple solution methods. Both encouraged students to share their solutions. Josh elicited these by asking questions, e.g. "Does anyone have something else?" The form he chose (in keeping with much of his teaching style) was interrogative. Karla sometimes asked questions, but she more often used metacommenting to point out the different ways students were thinking, talking and solving problems. Her use of forms like "some people…other people," offered what various groups were doing. Some of the instances of this form may have been used to "shield" students either because they were incorrect or because she knew that they were not comfortable sharing their ideas.

Karla also often announced past conclusions by telling students what "we found." This form may have shown a joint construction of ideas and findings or it could have been a way for Karla to assert the common knowledge, using her role as authority in the classroom to tell students what they "know." In contrast, Josh's "What do you think?" seemed to be more about making private ideas public. Another form that Karla employed (that students also used) was to hypothesize possibilities by offering what "could" be.

In her interview, Karla said that she used this type of metacommenting to:

> […] bring out what a certain group of kids might have said […] sometimes I think I do it because I know the kids aren't going to say it [….] I try to paraphrase what this group did, what that group did, to try to help another group. Or, sometimes I use it just as a way to get things started, instead of "The right answer is: put these two things together and draw a line," you know, it's "This group did this type of thing" […] a lot of times that I do it [paraphrasing of talk or process], I think I've found a group that's really done a lot of different things with it [a problem], and so I want to make sure everybody sees that, but, I try not to make it, "I think you should do it this way" and "I think that you should do that" and "Now this is the right way." I just present a situation that occurred—this is what they said, "What do you think?" Or, "Do you agree?" (Interview K-3, 12/17/99).

In this quotation, Karla talked about a myriad of things. She shared student solutions and ideas because she knew students in the class were not comfortable doing this. She also did it in the manner that she did to avoid telling students the right answer: that is, she wanted to encourage students to make decisions and judgements about what other students in the class were doing. In a sense, she may have been trying to downplay her role as mathematical authority in the classroom, drawing students into doing this for themselves. In fact, in at least three of Karla's four interviews, she talked about this:

> I couldn't just answer all their questions. I still do, but I try not to say, "You're right, you're wrong—the answer's 36, now let's go onto the next one." It's, "Why do you think so?," "What do other people think?." I try not to be the overall judge all the time. I try to create an atmosphere where they'll do that. I don't always, but that's my goal [....] If I'm the all-knowing and I give all the answers, they don't feel like they have to do anything (Interview K-4, 1/07/00).

> I'm always asking, "What do other people think about that?" without myself saying, "I don't agree with this because." I don't want them to wait for me to say, "Yes, it's right" or that there is *the* correct answer [...] I have an expectation of, "Okay, three's the answer, why is it the answer? Where did you get three? Can you explain to somebody else how it's three?" (Interview K-1, 11/23/99).

In Josh's case, he stated many times that student's ideas were important, but that he did not want the conversation to stray too much. It was more imperative that he stay focused on the mathematical ideas he deemed significant.

> There are many times where we'll be going off on a tangent, but I kind of maybe grab them by the hand and kinda pull them back on track and I'm not saying that's completely right or wrong, that's more of a teaching style (Interview J-1, 11/23/99).

Typically, it was after the sharing of ideas that a deviation between Karla's and Josh's approaches to authority began. In Karla's classroom, once students offered ideas she sometimes told students what she "probably would" do (e.g. Example K-28). Other

times, she called on students to interpret what other students were saying, bringing students in touch with each others thinking, as in the following example:

Example K-18 (continued—see pp. 107-111)

K: Xavier, **help us out here.**
  Xavier: You do the same thing, except take 1.5 and times it by five and then divide that by [inaudible]
K: Why are you doing that?
  Xavier: Cuz it's the same thing. **What she did was** she found what 1.5 was or 5, wait now. [inaudible] Then, she divided, then she took away [inaudible]
K: Oh, **that's what she was actually doing**? [...]

Josh's authority was more explicit than this—he typically ended presentations of student ideas by telling them what he "agreed" with (e.g. see Example J-16, p. 191) or what he thought was "reasonable."

Example J-20 (see pp. 202-203)

J: [...] I would say this [equation with slope of 5.8] is a **reasonable** line of best fit. I'd say this [equation with slope of 6] is a **reasonable** line of best fit. [circles those on the board] Um, I'd say this [equation with slope of 4] is not. [crosses out that equation] Robert, where did you get y = 4x?
  Robert: I went by 2's, um, on y [Robert shakes his head no]
J: You went by 2 on the y? Is that gonna make any difference though on the slope of your line?
  Fs: No
J: [You're] still gonna need a rise divided by the run to find the slope. I don't think this is **reasonable**, I think this one's gonna be a little bit too steep. I think this one's gonna be a little too steep, this one's gonna be a little iffy. How many got a slope somewhere around 5.7, 5.8, 5.9, 6, 6.1, depending on where you drew your line of best fit?

Karla's use of " probably would" is suggestive, given her position in the classroom. However, it does not indicate the same type of certainty and force as Josh's statements. "I agree with what __ said..." or (simply an assertion that) "this is reasonable."

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In addition, there was not the same 'shielding' in Josh's classroom for incorrect answers that there was in Karla's classroom. Notice in Example J-20 above, Josh explicitly asked Robert where he got his slope from and then told him that his answer was not reasonable. This was in contrast to Karla's use of "people," which did not indicate specifically who was incorrect when someone (or a group) was wrong—"shielding" students from blame. Also, she hedged her talk about incorrect and missed answers:

Example K-33

> K: Some people just **seemed** to skip it [...] If you missed it, half a point on four, that's **probably** where it was, **probably** on the y-intercept. Because I could see that many people were **trying to** find their y-intercept they just used their slope backwards.

In her interviews, Karla had said that when she taught elementary school she like "to coddle students" and that she "[...] still do[es] that" (Interview K-2, 12/02/99). One way that she may have "coddled" students was to shield them, protecting them from the embarrassment of being wrong or sharing their ideas. This was a fact to which she referred multiple times in interviews—the fact that eighth grade was hard for students because they were so aware of their peers and what their peers thought about them.

In summary, even though both Karla and Josh valued and sought multiple solutions, the forms they used to elicit them were different. Even more variation existed in the way that these offerings were concluded—Josh often ended with giving his opinion about what was mathematically valid or reasonable; Karla sometimes did this, but often had students discuss and interpret one another's contributions. Also, instead of telling students what was reasonable, she stated what she "would" do, offering processes with less force. Thus, in Josh's classroom his role was positioned more as part of the locus of authority. In Karla's classroom, her role and authority did not seem quite as apparent.

Sources of Valued Knowledge

In addition to the teacher's knowledge and authority, there were other sources of knowledge from which can be drawn in the classroom. Two other possible sources included the textbook and the students. In Josh's and Karla's classrooms, the manner in which these sources were used and referred to varied quite a bit.

The Textbook. As I showed in Josh's case, he often drew explicitly from the textbook to: 1.) define the activity of the classroom (e.g. see Example J- 28, pp. 210-211: "read on page…"), 2.) offer the terminology that would be used (also see Example J-28, pp. 210-211: "this is sometimes called a line of best fit"), and 3.) define terms and what they meant to the answers students generated (e.g. see Example J-30, pp. 212-213: "What does it say?"). In addition, Josh attributed "knowing" to the textbook through his use of "What do you know?": that is, when students were given something in the textbook, Josh would refer to these as things students "knew." In Josh's classroom, the textbook was viewed as an authority and as a source of knowing.

After completing my textbook analysis, I see that Josh and the textbook have many similarities in their style of address. In fact, the textbook used prolonged sequences of questions and imperatives and so did Josh. This similarity may have attributed to the way that Josh used the textbook: that is, because the textbook sounded a lot like Josh, he may have felt comfortable having students read from it. Stylistically, Josh and the textbook have a common "voice," possibly contributing to the manner in which Josh used the textbook.

Josh had said (see pp. 71-72, Chapter 5) that the "teacher-talk" in the MGMP books was the most influential aspect in his changes as a teacher because:

> it would literally give you questions, sample questions to ask. And, they would list possible student responses of which there were many and it would ask-if the student was looking at it from this point, maybe ask this question (Interview J-1, 11/23/99).

So, there is the possibility that the earlier versions of the text may have influenced Josh to the point that he had begun to sound like the text because he believed that the types of questions they had suggested helped in "developing a true understanding" and that one way to do this was to ask "the right kinds of questions." Either way, the authoritative discourse appeared quite uniform: that is, both Josh and the textbook were portrayed as part of the locus of authority and they had common ways of talking.

In contrast, Karla rarely had students refer to the textbook in these ways. One possible reason for this was that the authoritative voice of the textbook was not like the voice with which Karla was more comfortable: that is, Karla seemed more content using stories and metacommenting. In most of these examples Karla more often interpreted what the text said and orally talked through the activities without referring students to the written text in the book (except when going through or assigning homework). This was not to say that Karla did not refer to the textbook. In fact, the manner in which she often referred to the text was quite unusual. Rather than refer to the textbook as "the book," Karla often used the pronoun "they" to refer to the book. For example:

Example K-10 (see p. 101)

> K: [Problem] 2A. I recommended using your graphing calculator. I could tell some of you did and some of you did not. **They** purposely set the equations up that way [...]

By referring to the authors of the textbook in this manner, Karla positioned herself as separate from the external mathematical authority. Even though she may have separated herself from the locus of authority, she sometimes seemed to be playing the role of interpreter who had access to their intentions. She called on a non-specific "they," rather than using the metonomy "the book gave you.." or the agentless, passive "the slope was given...," portraying the authority as a faceless entity that existed outside of the classroom.

When I asked Karla about her use of "they," she had not realized that she used this word to refer to the textbook. She assumed she had probably said something like "the book..." and thought she said "they" because she knew the authors. When I asked her about her relationship with "they," she said:

> I think in using CMP I'm always thinking about, nothing is arbitrary, that the authors had a reason for everything that's in there and so when I'm looking at something I'm thinking about that [...] I know they had a reason for it and I've got to figure out what that was [...] because they'll come back and ask me questions, too. So, it holds me accountable. I feel like they do hold me accountable and CMP is such a powerful curriculum that I'm afraid of not doing what the authors intended [....] I have so much respect for them and know what went into creating this [....] I know that it was well thought out, much more than I've thought about it, even began to think about it (Interview K-4, 1/07/00).

In a sense, it appeared that Karla viewed "they" as being an authority to which she must render—"I feel that much respect for them that I worry about, am I really trying to hold true to what their intent was?" (Interview K-4, 1/07/00). In fact, it almost seemed as if she used "they" as an Attribution Shield: that is, she attributed the "knowing" to the authors--a nameless group of people that she calls "they." By attributing this knowing to the authors, Karla seemed to be letting students know that there was a much bigger authority to w

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authority to which they all must answer. In doing so, she may have down-played her role as authority in the classroom.

<u>The Students</u>. In addition to the differences in the use of the textbook, there were also differences in the ways the teachers drew from another source of knowledge in the classroom—the students. There were at least two ways in which Josh and Karla differed in the manner in which they drew from students. The first way included how they engaged students with their own and each other's ideas. In the second, they each attended to student's ways of talking about mathematics in different ways. When engaging students, Josh asked multiple questions and many that appeared "open" were reformulated and became more "closed" possibly because he had a particular answer in mind. In contrast, Karla drew students into articulating their thinking by saying that she was "lost." In addition, she asked students to sometimes interpret each other's thinking when students were offering non-conventional ways of thinking about or solving mathematics problems.

In every one of Josh's interviews, he stated that he was not as student-directed as other teachers. He valued what students had to say, but also made the point that there were certain ideas that he needed to get to and that these were the priority in the classroom.

> I think going into a problem I kinda knew the direction we wanted to go and I'd let the kids throw some other ideas out there, but I was still pretty, pretty zeroed in. I think my tunnel vision was maybe more in place [....] I know where I want to go and I know where I think the kids need to be and sometimes I don't have the faintest idea how we're going to get there. Sometimes I feel like I really have to drag them along and sometimes I don't, we don't. And, it kinda depends on what you're doing as to how you get there, if that makes any sense [....] You just kinda have to see what happens and know what you want to get across to kids and know what you think is important (Interview J-3, 12/17/99).

Again, even though Josh valued student-contributions, he still felt he needed to keep a tighter leash on the direction of the conversation. In fact, he said that he had seen some intense mathematical conversations that were very student-centered, but that he just could not do that—it just was not a comfortable thing for him to do. He needed more control than that. One method of control that he incorporated that I have already mentioned was to use an I-R-E sequence. Another way that he controlled the classroom direction was through his questioning. As I showed in Chapter 7 (Josh's Case Study), Josh used many different forms of questions in his teaching[8]. However, I also showed that although many of his questions appeared "open," subsequent discussion proved that he had a predetermined answer in mind. Often he reformulated his questions until students came up with the answer for which he was searching. This may have made many of his questions appear test-like in nature because there was only one acceptable answer.

The use of questions in Karla's talk was really pretty slim, except when she returned something that she had graded and was going through it. In that case, her classroom format was fairly I-R-E in nature. Karla more often talked than inquired. And, when she did ask a question she hardly ever reformulated it (approximately once in each of the two focus lessons she did this). Also, in the two focus lessons she only once strung a series of questions together (which Josh often did). This string of questions occurred when students began to get off task from the conversation and they seemed to function as a way to draw student's attention to what she wanted them to be thinking about.

Example K-41 (not included in the case study)

K: [...] we've gotta decide which kind of average makes the most sense, a mean, median or a mode. Alex?

Alex: The mean would make the most sense.

K: Why would the mean make the most sense?

Alex: It's the one in the middle

K: Aren't they all in the middle, I get confused.

Alex: Naw, because the mean is like the top of something and something and you know, whatever. And I think you should do it that way because I said so and I'm a lot smarter [students laugh].

K: Back up to sixth grade

Kevin: Why do they call it such stupid names? Why don't they call it the middle?

K: That's not very helpful. [students talking] My question is. if we are going to find an average, we've got to take the average that makes the most sense. What we want to look at is what is the trend of the data. Is it a set of linear data? Is it not a set of linear data? Does it have some kind of pattern? So. we're trying to figure out as we shorten the bridge, what happens to the breaking point? So if we want to decide if that-what we're looking for. would a mean average make the most sense? Would a mode average make the most sense? Or a median?

Karla began by telling her class that they needed to decide what type of average they needed to find to take care of the differences that had occurred in the bridge data they had just collected. Alex offered the correct answer. but could not tell Karla why he suggested the mean average. In fact, he gave a rather ridiculous reason—"because I said so and I'm a lot smarter." When Karla tried to redirect the conversation by referring back to something they had learned in sixth grade. Ken chimed in with asking why "they" called it such stupid names. Karla pointed out that this was not very helpful to the discussion they were trying to have and then proceeded to refocus the conversation. In doing so. she posed a series of questions students should be considering. She did not seem to want students to answer these. she seemed to want them to know about what they should be thinking.

Many of Karla's other questions were revoicings of student contributions (about 20% of her questions) in which she would restate part of what the student said and ending

with rising intonation. It was as if she wanted to confirm her interpretation of what the student said correctly or wanted to indicate that they needed to explain what they said.

Karla mentioned this was one way she approached listening to students. She said that she began thinking about whether she was listening carefully to students during her involvement in piloting CMP. In fact, Marcie[9] had intensely pushed her thinking about this:

> I think Marcie pushing at me at going for where the kids are, know what you want in terms of a math goal, but use where they're at and then move them to your goals. So, she used to say that all the time, "Yes, you have a goal in mind, but you've got to figure out where they are." And, if they're using a word like "swooping" or if Jonah says this slightly different, try to take their words and push it farther. So that kind of started with probably Marcie more and wanting to have that ideal classroom that the *Standards* have set up, to me that was part of it [...] in peer coaching that's what I always look for: am I really listening to what they're saying or am I saying, "this is--, now do it" or am I listening to they're saying "it's swooping up" and move toward the word [...] (Interview K-4, 1/07/00).

Listening was something Karla said she was still improving. When we talked about her "ideal" classroom, she mentioned she had been peer coaching with a seventh grade teacher, Tammy, to work on listening to students because she felt Tammy did such a great job of it. In fact, Karla stated that Tammy's class was her "ideal" class because:

> She really listens to what kids are saying and because she knows the big idea of what we're supposed to be doing that day, she listens to them and knows what the next question is going to be, where it's supposed to be, what [...] she's really good at is hearing what they're saying [...] and she's really good at taking what they're saying and pushing it to where it's supposed to be (Interview K-2, 12/17/99).

Although Karla felt that she did not listen as well as she thought she should, it did appear that she often tried to make sense of processes students offered. As I showed in the extended Example K-18 (pp. 107-111), Karla sometimes stated that she was confused to get students to articulate themselves more clearly. And, when she still did not understand what students were saying, she also called on other students to explicate, putting them in the position of interpreting each other's thoughts. In Josh's classroom, he more often interpreted what students were saying and monitored the ideas that were being presented to keep the lesson focused on the big mathematical ideas.

The second way Josh and Karla differed in their use of student contributions was associated with the ways they talked about the mathematical content. Through my observations and analysis, I have attended to the multiple ways that the teachers and students talk about important mathematical ideas. In doing so, I have developed categories for the language these classrooms have used pertaining to linear and non-linear relationships. These categories include official mathematical language (OML), bridging language (BL), and contextual language (CL)[10]. I will briefly define these categories and illustrate them by offering examples of the language that has appeared in the classroom. I will then offer some differences I see in the ways that this range of language is used in each of the two classrooms.

Halliday (1975) defines a register as "a set of meanings that is appropriate to a particular function of language, together with the words and structures which express these meanings" (p. 195). He goes on to define the *mathematical* register, then, as being made up of the "mathematical use of natural language and meanings a language must express if it is being used for mathematical purposes" (p. 196). In short, these are the ways of talking and functioning mathematically that would be expected and appreciated by the mathematical community. Pimm (1987) extends this definition to include metaphorical meanings that people construct to make sense of mathematical ideas. For example, he refers to a common metaphor employed in many algebra classes: "a function is a machine." OML consists of language included in the mathematical register. This type of language is universal—it would be recognized by anyone in the mathematical community. Line, slope, y-intercept, rise and run would be examples of this type of language.

Two types of language make up BL: classroom generated language (CGL) and transitional mathematical language (TML). The first, CGL, consists of language that is student- or teacher- generated; it is more idiosyncratic in nature than OML. It still pertains to the mathematical object, but is part of the language of this classroom community. For example, Karla's class called decreasing exponentials "swoopy down curves." TML would be language that describes a location or process that is associated with the OML. TML include certain set phrases which are repeated often in these classrooms, e.g. "the number times x" (slope as its location in the equation), "what it goes up by" (slope as change in the y values in a linear table), or "the constant rate" (slope referred to in both the graph and the table as what makes the relationship linear).

CL is the language that students use that is dependent on specific contexts or situations the students have become familiar with through using this specific curriculum. The curriculum is problem centered and students work with re-occurring contexts. These include planning bike trips, being a part of fund- raisers, saving money, etc. Examples of the language students have used that would fall into this category would include rates such as "cost per person," "dollars per month," "meters per second" (all related to slope) or "the up-front money," "the down payment," and "the head start" (all related to y-intercept).

Both teachers draw on a range of language to talk about mathematical ideas and sometimes do this in a string. For example:

Example K-18 (see pp. 107-111)

K: The piece that we're trying to figure out is this **b**. And you said this **b** was your **up-front money** or your **up-front stuff.** or your **starting point** or **y-intercept**. Is this [the given point] my y-intercept? [...]

Example J-43 (not in the case study)

J: Sure it is. The **coefficient** is negative four// the **slope** / and the **y-intercept** again is zero/ [...] Some kids said/ like Jacob said there is no standard form. there wasn't a **coefficient**, there wasn't **a number multiplying by x**/ something of that nature//

In each of these examples, Josh and Karla were using multiple phrases to refer to the slope (coefficient. a number multiplying by x, slope) and to refer to the y-intercept (the up-front money, the up-front stuff. the b. the starting point. the y-intercept). However. a closer look at the type of language they use indicates a difference. In Josh's references to slope. he used two terms that were official mathematical language (OML): that is. "slope" and "coefficient" would be considered part of the mathematical register and would be recognized by the larger mathematical community. In contrast. Karla referred to the y-intercept by drawing on the story problem contexts students had used (i.e. "up-front money" or "up-front stuff"), relating the concept to more concrete situations with which students were familiar. In addition. she used TML (i.e. "starting point." "the b" (location in the equation)) and then one term that would be considered OML: "y-intercept."

There were multiple times in Chapter 6 that I referred also to Karla's use of unconventional mathematical terms; that is when Karla used CGL. For example

Example K-27 (see p. 128)

K: It does do a **swoopy**. Should I connect the dots? [...]

Karla indicated that she was not sure if she introduced "swoopy" or if students had, so during the 1999-2000 school year she made sure that she did not say it and students still suggested it. She realized that it appeared every year when they began to study exponential curves: she told me that typically students described exponential curves as looking like a "Nike swoosh." This phrase, in turn, became "swoosh" and then "swooshy" and then "swoopy" (Interview K-3, 12/17/99). When I asked Karla about this atypical language for mathematical ideas she said:

> I think they [the students] need common words, a common language and then build towards that [algebraic formalism]. Like slantiness[11], they don't use that anymore. Once in a while it'll still come up. [...] I think they needed it at first so they had something to talk about what was going on. To give them slope first and then what was going on I think would squash anything that they could do with it. [...] I hope it gives them a common language to start looking at things differently, especially like swoopies. Because then they can build from swoopies to know that this graph is swooping, it's non-linear, it's not a. I don't know what they were calling them last year, hills and u's[12] for quadratics and things. To first just recognize that there are different patterns and how to describe them using their language and then look at how algebraically we can describe the same thing. I think they need those building blocks in there (Interview K-3, 12/17/99).

This appropriation of student language for mathematical terms was quite common in Karla's classroom. In Josh's classroom, the focus was more on using official mathematical language. In fact, I have no instances of the type of classroom generated language in Josh's classroom that I have in Karla's classroom.

While Josh and Karla both drew on sources of knowledge in the classroom (i.e. the textbook and the students), they each did so differently. Josh explicitly drew on the textbook to define activity in the classroom, introduce and define mathematical language, and as a place to find answers. He also located the text as a place of "knowing." One of the reasons the way he interacted with the text made sense was that they have a common style of addressing students—using multiple questions and imperatives.

In contrast, Karla rarely had students read directly from the textbook (except when giving or going through an assignment). More often Karla used stories to interpret the activity in which the class engaged. Also, she referred to the textbook in an unusual way—as "they." I have argued that this word may have functioned as an Attribution Shield: that is, her reverence for the authors as more knowledgeable others about teaching and mathematics allowed her to attribute "knowing" to the authors of the textbook. They authors were an unnamed, knowing entity in her classroom.

Josh repeatedly stated in his interviews that, while it was important to have students share ideas, he needed to maintain control of the direction of the talk in the classroom. He allowed students to offer answers to his multiple questions, but in seeking a particular answer he reformulated many of his questions until they were closed in nature. By doing so, he may have indicated to students that his questions were really "test" questions rather than genuine inquiry into their thinking.

Karla's ways of drawing on students was quite different from Josh's. In her interviews, she said she tried to listen to students, take what they knew and push their thinking further. She wanted to give up some of the control and have students judge the solutions that were being discussed. This was what she valued in her "ideal" classroom. When students offered novel mathematical processes, she pushed them to articulate their thinking by saying that she was lost. She also engaged other students in interpreting student contributions, putting them in touch with each other's thoughts.

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Another way that Karla showed students that she valued their ideas was by appropriating the language they used for mathematical content. "Swoopy," "slanty," "hills" and "U's" were the classes' common ways of talking about exponential functions, slope, and quadratic functions. In Josh's classroom, these forms of talking did not exist: that is, official mathematical language and transitional mathematical language dominated the mathematical talk. These ways of talking are valued by the mathematical community, pointing outward to external sources as being valued.

Drawing Attention to Common Knowledge

The last set of surface forms that had a shared function in the two classrooms were the ones that drew attention to the common knowledge of the classroom. In Josh's case study, I delineated at least four forms that served this purpose: "But"; Tag questions; proposition + "Why?"; and "Should." In Karla's classroom, there were at least five such forms: "So"; "Yesterday…"; "Remember…"; "We found…"; and "Can."

Beginning with comparing "but" and "so," each was used as a marker to indicate an important mathematical point that was being made by the teacher. In the case of "but," the word was inherently contrastive. It set aside what had been said and contrasted it with the phrase that followed. For example, if I say, "I love you, but…." whether I love you or not was conditional; it was based on the condition that followed the word "but." When Josh said, "It's a line, but does it best fit the data?" in Example J-26 (p. 208), he contradicted the answer the student gave. Because his use of "but" was contrastive, it may have set Josh's viewpoint apart from the student's. In doing so, it may have further separated Josh

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"So" was inherently connective. When Karla used "so," she let students know that she, too, was going to make an important mathematical point. Her important mathematical points came after a series of contributions from students and the teacher and were more summative in nature. They did not contrast; they condensed the classroom talk, formulating what students should take away from the conversation. Karla said that she did this because she did not want students to get lost or to feel like they did not see the bigger picture. In a sense, she was drawing them into the common knowledge of the classroom, making it more concise and succinct than the entire classroom conversation that had occurred.

Both Josh and Karla also stressed the common knowledge through other forms. In doing so, Josh used tag forms and propositions followed by "why?." The tag form was not as assertive as a statement, but was more assertive than asking an outright yes-no question. Tags favor positive responses, making it easier for students to confirm what the person was saying and requiring explicit contradiction of what was being stated. When Josh would state a proposition and then ask, "why," he was letting students know that justification was important in mathematics.

The forms that Karla used to direct student attention to the common knowledge of the classroom were more narrative in nature. She told stories about "yesterday" or asked students to "remember" or reminded them what "we found." The common knowledge of the classroom were stories to be told about things in which the classroom community had engaged. In using phrases like "we found," students also had to tacitly agree with the findings Karla s

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findings Karla stated. However, the word "we" pointed to joint construction, offering the classroom community credit for the ideas summarized. Thus, Karla may have formulated the common knowledge as something she and the class did *together*. This is further substantiated in her use of question frames "that invite children to introduce their own reasons and experiences[13]" and "her terminating statement[s] that [...] make it clear that it is not only a decision they have arrived at together but that it is now to apply to all of them, including herself" (Mishler, 1972, p. 292)[14].

During interviews, I showed each teacher excerpts of Karla's use of "can" and Josh's use of "should." Both reacted in similar ways to the transcripts—Karla said she sounded "bitchy" and Josh said he was "pissed off." Their choice of modal verbs actually breached politeness. When I asked them what they were hoping to do with these statements, they both stated that they needed to let students know what they were responsible for knowing at that point in time. So, these two forms functioned in similar ways: that is, they indicated what would be considered "presupposed knowledge" from that point in time on. On closer inspection of the forms, however, there are interesting differences again. Karla's "can" indicated an ability to do something—she was "going to assume that you can." Implicit in Josh's "should know" was "but you do not." In other words, using "should" offered an underlying message that actually indicated that an expectation was not being reached. It offered a more negative connotation of student relationship to the common knowledge in the classroom.

Both Josh and Karla highlighted the common knowledge of the classroom. However, the surface forms they chose were different. Josh used words that were contrastive ("but") and that required affirmation (tag forms) and reasoning. Karla used words that were connective ("so") and that indicated that the common knowledge was a jointly constructed story to be told. When students were struggling with the common knowledge, both Josh and Karla used modals that breached politeness. Karla used "can" which indicated an ability to do something; Josh used "should" which inherently pointed out what students were *not* doing. Through these forms, Karla may have valued student ways of talking (i.e. narrative), pointed to the fact that the ideas in the classroom were jointly constructed (related to forming community) and let students know what she thought they were *able* to do. In contrast, Josh may have aligned himself with the external authority by positioning his ideas as contrastive to student's, constructed the common knowledge as something which should be agreed with, and let students know what they were *not* doing.

Discussion

Although the functions of the words/statements I discussed were the same, the surface forms indicated various interpretations that could be made. In fact, I believe that many of these surface forms contributed to the difference I felt when I sat in Josh's and Karla's classroom. I think these variations set up distinct environments in which students learned. The role each teacher played, the way each interacted and drew from students and the textbook and the manner in which they highlighted common knowledge was different.

Karla seemed to take on the role of protector and guide. She protected students through shielding their incorrect answers and sharing their ideas if they were too insecure to do so. She used student contributions to guide the talk in the classroom, signaling students to slow down and articulate themselves and other's ideas. She aligned herself with the students in the classroom through using narrative storytelling, appropriating student ways of talking about the mathematical content, offering jointly constructed knowledge as ideas that were formulated together, making connective, summative statements. The textbook authors were located as an external authority to which they all (including Karla herself) must be accountable.

Josh did many of these same things, but his forms may have contributed to a different environment. The interpretation of Josh's forms may have led to different positioning and authority patterns. In Josh's classroom, he and the textbook were viewed as a part of the external authority that made decisions about what was correct and what was mathematically reasonable. Using multiple questions that began open and were reformulated until they were closed may have indicated that the students were being tested on their mathematical knowledge. The focus on using more mathematically appropriate terms also aligned Josh more with the mathematical authority, rather than positioning him as part of the classroom community. The common knowledge in Josh's classroom was something to be justified and with which students were required to agree.

What Difference Might These Differences Make?

In this section, I explore what difference these differences might make in terms of how students are engaged with mathematical ideas. I do this in two parts. In this first section, I draw from the literature to discuss some of the differences that might exist in the two classrooms. However, the data that I have collected does not address many of these differences so I do not make claims about what *is* but about what *might be*. In the second section I offer: (1) compare/contrast of the two extended transcripts included in Appendix N and O; and (2) results from my preliminary analysis of student interviews.

Karla's dominant role seemed to be more like "Replying"; Josh's role may have been more like "Assessing" (Barnes, 1976, p. 112). When Replying, the teacher takes the students where they were, even though s/he may wish to broaden and shape the understanding; whereas when a teacher Assesses, s/he holds the student up to external standards. Barnes contends that replying sets up a more collaborative environment and gives the learner confidence in actively interpreting subject matter.

This distinction of Reply and Assess, Barnes recognizes, seem to be identical to Simon and Boyer's (1967 and 1970) understand vs. judge. These dimensions are defined by the following (Barnes, 1976, p. 112):

Figure 3

Understanding vs. Judging

UNDERSTANDING ←-------VERSUS	**--------→ JUDGING**
-accepts idea	-positive evaluation ('good')
-clarifies understanding	-negative evaluation ('wrong')
-reflects or paraphrases ideas	-counter proposals, suggest
-expands on someone's ideas	-implies judgements (should, should never, you always, everybody ought)

According to Barnes, these roles are essential parts of teaching because one values the student's ideas and his attempt to make sense of the world; the other puts his ideas up against the larger community's standards. However, he also recognizes that if a teacher stresses assessment over replying, the students may focus on "externally acceptable performances, rather than [...] trying to relate new knowledge to old" (p. 111).

Barnes connects these teaching roles to the roles that students play in communication. Teachers who assess/judge more often have students who "present"; teachers who reply/understand tend to find students "sharing." The difference is in the predominant form of communication students may employ. When students "present," they use "final draft" talk. Final draft is the style students use when they are showing the teacher that they have the "right" answer—it is "a formal completed presentation for a teacher's approval" (p. 108). In contrast, students who "share" use "exploratory talk" which is more improvised because students are rearranging thoughts. Barnes contends that exploratory talk was important in student problem solving because "tentativeness may for many children be a necessary condition for achieving hypothesis forming and testing" (p. 108).

The Assessment role of teaching is aligned with the domination of recitation in the classroom. In recitation:

> the teacher follows a pre-scripted checklist of questions, information and concepts, sticks closely to a preplanned list of test questions rather than responding to the substance of what students say [...] students typically give short, frequently tentative answers [...] topics shifts can be abrupt as the teacher moves down a checklist of important points, as it were, making sure students remember them... discourse is frequently choppy and lacks coherence (Nystrand, 1995, pp. 15-16).

In contrast, discussion is defined by its tightly interwoven comments and responses. Teachers make space for student ideas and the discourse is more balanced, although the teacher's voice is still crucial. Dialogic instruction involves fewer teacher questions and more conversational turns as teachers and students contribute ideas, make points, and ask questions. It is "less prescriptive since the actual conduct, direction, and scope of the discussion depend on what students as well as teachers contribute and especially on their interaction" (p. 17). Knowledge tends to unfold as the discussion proceeds; both personal and school knowledge are recognized and drawn upon. Student ownership is important.

More specific to mathematics education, Pirie and Schwarzenberger (1988) outline what they believe to be the components of a *mathematical* discussion. They define this as:

> *It is purposeful talk*
> i.e., there are well-defined goals even if not every participant is aware of them. These goals may have been set by the group or by the teacher but they are, implicitly or explicitly, accepted by the group as a whole.
>
> *on a mathematical subject*
> i.e., either the goals themselves, or a subsidiary goal which emerges during the course of the talking, are expressed in terms of mathematical content or process.
>
> *in which there are genuine pupil contributions*
> i.e., input from at least some of the pupils which assists the talk or thinking to move forwards. We are attempting here to distinguish between the introduction of new elements to the discussion and mere passive response, such as factual answers, to teachers' questions.
>
> *and interaction*
> i.e., indications that the movement within the talk has been picked up by other participants. This may be evidenced by changes of attitude within the group, by linguistic clues of mental acknowledgement, or by physical reactions which show that critical listening has taken place, but not by mere instrumental reaction to being told what to do by the teacher or by another pupil (p. 461, italics in original).

The authors use this definition to locate mathematical discussions and to make inferences about the mathematical understanding that the students are displaying in the talk. Later in this chapter, I look across the two extended transcripts that were provided in Appendix N and O, using the ideas of discussion and mathematical discussion to differentiate the talk that occurred in each classroom.

While neither Josh nor Karla fit "neatly" into one of these two categories (i.e. recitation vs. conversation), Karla tended to have more of the "discussion" attributes in her classroom and Josh tended to follow more of the "recitation" format. For example, Karla made space for student comments, but the discourse was not more balanced. Karla talked markedly more than students did in her classroom. There were fewer teacher questions in Karla's classroom than in Josh's, but the "big ideas" in Karla's classroom were used to guide the discussion. In Josh's classroom, the I-R-E format was pervasive but the actual flow of the lesson was not completely defined. Josh acknowledged that he did not always know how he was going to get where he was going, but he did know where he wanted to end up. As an observer, even though Josh proceeded through a "checklist" of topics, the discourse did not feel choppy and incoherent—it was all tied to the big idea that was the focus of the lesson.

When the organization of instruction is examined, we can find pedagogical "contracts" which define roles in the classroom. These instructional arrangements, Gutierrez (1992) shows, determine the "discourse patterns, rules of participation, and the nature of classroom interaction" (p. 15). In recitation, the I-R-E exchange so common in classrooms has several key provisions: the source of knowledge is the textbook and the teacher—it is given; the teacher monitors answers, initiates all topics, determines what is worth knowing; knowledge is fixed, objective, autonomous; the epistemic role of students is limited to remembering what others have said, not figuring things out or generating any knew knowledge. The discourse "contract" of discussion involves: on-the-spot grappling with ideas and problems which were not determined by the teacher; knowledge may come from students (and from their experiences) and is generated, and co-constructed among participants in the conversation; students figure things out; the teacher's role is to facilitate, guide, explore, anticipate, and analyze student responses.

How do these two things affect what students can learn? Nystrand claims that in recitation students are "merely doing school" (p. 17), going through the motions of trite and procedural interactions. This seems to be similar to Edwards and Mercer's "ritual understanding": students learn the rituals of what they are supposed to do and say rather than taking away the reasons for what they are doing.

Related to this point about how each portrays knowledge differently, Barnes recognizes that exploratory talk and final draft talk are:

> essentially a distinction between different ways in which speech can function in rehearsing knowledge—in exploratory talk, the learner himself takes responsibility for the adequacy of his thinking; final draft looks toward external criteria and distant, unknown audiences (p. 113).

He, too, contends that what is learned is different in each mode, making the difference between rote learning and understanding (i.e. ritual and principled understanding).

I believe one of the most interesting aspects associated with Karla using more discussions is the fact that, according to Edwards and Mercer's 'scale' of control, Karla actually asserted more control over the common knowledge than Josh. In fact, their categorization would interpret Josh's I-R-E structure as less controlling than many of Karla's discourse tools (see Table 10 below). So, even though students are discussing mathematical ideas, it appears that Karla actually maintained tight control over the common knowledge in the classroom. Discussion and control appear to be not mutually exclusive—they can, indeed, coexist.

Table 10
Karla's Control of the Common Knowledge

Karla's Discourse "Tools"	E & M's Method of Control of the Common Knowledge
Metacommenting (of which revoicing is a subset); Narrative retelling of stories	Paraphrastic interpretations of student-contributions and reconstructive recaps
Pronouns—"we"	Joint knowledge marker
Classroom-generated language—"swoopy"	Formulaic phrases
"So" to indicate important mathematical point	Significant marker with special enunciation
Telling students what they "can" and "will" do	Presupposed knowledge
Reference to re-occurring problem contexts	Repeated discourse formats

The use of this type of discussion in Karla's classroom was not unproblematic. In fact, there were many times when Karla had to remind students to be on task. She struggled not to let some students dominate the conversation. While many students were actively engaged in the conversation, there were always a few who were off task. This was apparent when students asked inappropriate questions (e.g. Kevin once asked, "Do you want to hear what I dreamed about last night?" and "Why don't we just talk in here like we did last year [during homeroom]?") or engaged in off-task behavior (e.g. making faces at other students across the room, drawing pictures unrelated to mathematics).

Another crucial aspect in the teacher's ability to lead discussions in the classroom is that of adequate and appropriate subject matter knowledge. Both Josh and Karla talked about how important their subject matter knowledge was in their teaching. In fact, Karla told me that she thought Josh's better subject matter knowledge allowed him to:

> [...] see a lot more, I think. He knows more math than I do and if a kid said something, sometimes I wouldn't even get what the kid was saying and he would. He would understand it better or he would see where it could lead and so he could do that so much better than I could. I've learned a lot more math from him because I'd go to him with, "So and so said this and that was so stupid" and then he'd say, "No, this is what they're saying. This is where it's going." And so I think he's so much better at knowing those kinds of things—where everything goes, how everything connects and I learned tons from him that way (Interview K-4, 1/07/00).

While all of the authors I have cited recognize the differences in the teacher's role, the students role, the basis of knowledge, etc., they also all point out that most classrooms operate somewhere between these two paradigms. Also, as Cazden contends, one lesson can move from recitation to conversation within a matter of moments and sometimes the activity that is being engaged in determines the format in which the lesson exists. For example, practicing times tables would require a different format than exploring the concept of multiplication in terms of repeated sets, area, etc. These two case studies offer illustrations of some of the shades of gray that exist between discussion and recitation.

Empirical Findings

I now draw from both the observations and the student-interviews to suggest similarities and differences related to students in the classrooms. I will use the extended classroom excerpts from Karla's and Josh's classrooms (given in Appendix N and O) to show the type of mathematical engagement that appeared in each classroom. By drawing on the student interviews, I will share some of the findings I have from my preliminary analysis of student mathematical understandings and ways of talking about mathematical concepts.

Classroom Observations

In this section, I look across Appendix N and O, which I included to allow the reader a more extensive view of the talk in the two classrooms. In this section, I draw on those analyses to offer some of the differences I see in the two classrooms in terms of the levels of "discussion" and "control."

Returning to Edwards and Mercer's idea of "common knowledge," there is evidence that Karla was using some aspects of control in this exploration of subject matter. In some cases, she ignored student contributions (e.g. Charity's "zero, zero and the negative zero") to continue with her summary and in other cases revoiced student contributions to make them more precise mathematically (e.g. Mark's contribution became "opposite of that coefficient of x"). She often used "we" to put forth joint understandings. However, what she controlled less was student options to speak and offer ideas. In fact, she made these ideas something to be explored and encouraged students to do so. She often ignored language that was not precise. This may have allowed students to offer their ideas as they were occurring: that is, students were allowed to engage in exploratory talk, rather than presenting their ideas in final draft form for Karla's approval. Also, as with much of her teaching, many of the ideas were attributed to the student from whom they came.

In Josh's classroom many of the same control mechanisms existed. He, too, revoiced student contributions to make them more mathematically precise. This was especially true related to the language that students used to express their ideas. There was also a large amount of controlling who got to speak and about what. This was most apparent in the I-R-E sequencing of the talk. In addition, Josh introduced every topically related sequence. Often he did this by calling on students to read about what they were "supposed" to do. Josh also sometimes used the pronoun "we," especially when he referred back to previous "common knowledge." For example, he said, "In the *Moving Straight Ahead* unit we said a y-intercept was where x was zero, right?" The "we" pointed to joint understandings. The option to disagree with these was further discouraged by Josh's use of a tag-form, which required affirmation from students.

From the more general perspective of "discussion" and "recitation," it appeared that students were engaging in discussion with their peers in Karla's classroom. Karla has taken on the role of summarizing and recording student ideas. She encouraged the class to check their answers on the graphing calculator, using the table and the graph for verification. The activity and discourse boundaries in this excerpt are significantly relaxed as students talk both to Karla and to each other. Student responses also build on previous student responses. Again, Karla still does not keep her utterances and intervention to a minimum, but she does not explicitly assert which ideas are correct.

The topics that were discussed in this excerpt were often introduced by students. Each of the cases were offered first by students and then taken up by Karla to be discussed. However, I must also point out that they did not always take place as quickly as some students would have liked. Mark advanced rapidly and continued to share cases that were ahead of the class discussion. At least two of Mark's contributions were not recognized by Karla while she worked with the class to explore and evaluate earlier ideas.

Students, in this discussion, were finding many cases of quadratics and exploring them. They were hypothesizing the solutions and using two representations on their graphing calculators to validate their choices. They were working together to articulate what they were thinking. They were sometimes questioning each other, offering other ways to think about the mathematical ideas, and helping each other understand what they were saying. In line with Karla's role in "replying" to students, students seem to be "sharing" rather than "presenting," especially before Karla said she needed to organize the ideas.

In contrast, Josh's role seemed to be "assessing/judging." This role became apparent in Josh's consistent evaluations of student responses, his counter proposals and suggestions and his use of "should." In response, students "presented" final draft talk. Josh's role was to offer topics, evaluate responses, ask questions, make connections (e.g. in his statements about what they learned in this and other units), etc. Students seemed to understand that the textbook was a place that also defined the activity in the classroom and that this was a place in which to find answers. Student's role seemed to be one of answering questions and offering confirmation. They also described how to do things and why. However, these were not offered until Josh requested them. In that sense, students seemed to have learned the "appropriate" way to respond. This also was true when Josh used tag-forms; students offered affirmation whenever Josh posed a tag-question, showing that they knew that this was the appropriate thing to say.

The discourse boundaries were not as relaxed as Karla's because Josh functioned within the I-R-E sequence most of the time. Josh seemed to control the topic and who got to speak. At least three times he called out the names of students who had their hands raised. At the end of each of these turns, Josh was the person who selected who spoke, rather than students self-selecting as they sometimes did in Karla's classroom. Rather than drawing on students as a source of knowledge, Josh and the textbook were defined in this manner. In fact, later in the conversation Chuck offered a generalization:

Example J-45 (Continued—beyond Appendix O)

J: Okay, Chuck, what do you think?
Chuck: So the one [inaudible] you're subtracting, it'll be a positive here, and if you're adding, it's gonna be negative?
J: You can formulate your own rules if you want to, yeah. I mean, that's one way of thinking about it. If this, if this x, if it's 4 + x, what would it have to be, though?
Chuck: It's gonna be—It'd be negative four
J: Negative four because four plus negative four is zero. Whatever x value sets it equal to zero. [...]

In this example, Chuck noticed that to find the x's, often you had to take the opposite of the operation given (e.g. in x + 3 you are adding, so it would result in –3). Josh responded by telling Chuck he could "formulate your own rules if you want to." However, unlike Karla's exploration of the rules Mark offered, Josh seemed to try to provide an example where Chuck's method did not work. The example that he gave, however, did work and affirmed Chuck's rule. I think Josh meant to say 4 –x as he tried to get Chuck to focus on "whatever x-value sets it equal to zero" rather than emphasizing the rule Chuck had offered where one might only attend to the operation that was given in the problem.

More particularly, in Karla's transcript, all the components for a "mathematical discussion" outlined by Pirie and Schwarzenberger (1988) exist. In the example, the talk is centered on a mathematical subject: solving quadratic functions by using the expanded form of the equation. While the idea of solving quadratics was introduced by the teacher, the process of using the expanded form to find the solution was introduced by one student, Mark. In this way, it appeared that the content was defined by Karla, while the process was defined by Mark. Many students proceeded by offering and then exploring and discussing multiple cases that were taken up by other students as well as the teacher. In doing so, there were "genuine pupil contributions" and "interaction." In fact, a few times "cross discussion" (Lemke, 1990) took place: that is, students actually talked to one another instead of all of the classroom talk being addressed at the teacher.

In Josh's classroom, it appeared that many of the components for a mathematical discussion also exist. The talk in his classroom is also focused on solving quadratic functions. However, the textbook seemed to define what they were "supposed" to do. Josh and the textbook seemed to define both the content and the processes that would be used. This was different from Karla's students defining the processes to be discussed.

When looking at Josh's transcript in terms of "genuine pupil contributions" and "interaction," there appeared to be further distinction between what happened in Josh's classroom as compared to Karla's. While students were contributing and answering Josh's questions, they sometimes seemed to be responding to rhetorical questions or tag-questions. In both instances, it seemed that a mere "okay" or "yep" was required. Other questions required students to focus on the "hows" and "whys" of the content. These are ones about which I am unsure. I would not consider these "mere passive response," yet they did not necessarily allow for the "introduction of new elements to the discussion." The answers to this type of questioning did help the "talk or thinking to move forwards," but they seem different from the contributions in Karla's classroom where students were offering many different cases of quadratics. One difference may be that Josh's student-talk was focused more often on operational statements; Karla's student-talk moved back and forth between operational and reflective. In Josh's classroom, I only have evidence of him making reflective statements; students merely confirmed the statements that he made.

The movement of the talk in Josh's classroom was picked up by other students, so there were "interactions." However, many of these interactions were not instances of "critical listening" taking place; instead, they were cued elicitations (i.e. the answers were cued because the form of the question only encouraged a positive response). In addition, there were no instances of cross discussion, as there were in Karla's classroom. Student responses were more often channeled through Josh. At one point, Josh may have wanted Mindy to respond to Chuck, but it was unclear to me whether Mindy understood that and was trying to explicate Chuck's idea or if she was introducing her own thinking about finding the solutions to the quadratic equation.

Recitation is more like what Lotman (1988) calls "univocal" in which the focus is on the "accurate transmission of information." In contrast, discussion treats utterances as "thinking devices." In schools, this means that teachers would use student statements for engaging and guiding the class discussion, rather than as a means for transmitting knowledge. What counts "is how teachers organize instruction" (p. 9).

According to Nystrand, the epistemology that accompanies these different formats of classroom interaction are that of "dialogism" and that of "objectivism." In dialogism "knowledge emerges from the interaction of voices" (p. 19) and the source of valued knowledge includes students' interpretation and personal experience. In this case, the focus is on the "transformation of understandings." In "objectivism" knowledge is "given." The sources of knowledge in the classroom are the textbook and the teacher. The focus is on "transmission" of knowledge from those who "know."

Student Interviews

During the course of my data collection, I spent time with students in small groups, interviewing them about the mathematics that they were learning. After a preliminary analysis of student interviews[15], it appeared that on the surface their mathematical understandings were quite similar. Generally, there appeared to be a few pairs that did very well, a large number of students who were still struggling with a few aspects of linearity (e.g. finding the slope given the graph of a line), and a few pairs who struggled with many aspects of linearity. All students proved to be quite proficient in their use and interpretation of the tabular representation. Most students also did well when asked to use and interpret graphical representations. The area that proved difficult for some students was the use of equations, especially the symbolic manipulation of equations and expressions.

Another interesting feature of the interviews was that students used very similar language to talk about the slope and y-intercept in linear relationships. Only one or two pairs from each classroom actually used the word "slope" without explicitly being asked to do so. They more often referred to the slope as "what it [the y-values in the table] goes up by." However, all students used the word "y-intercept" without being probed for it. These similarities were somewhat surprising to me, although I have observed this same phenomena in past work with Josh's and Karla's students. Given the fact that repetition is so important to people acquiring ways of talking (Tannen, 1989), I would have expected students in Josh's classroom to all use the word "slope." However, they did not and about the same number of Karla's students did not use the word slope. At a later time I hope to analyze these interviews to make more detailed and specific claims about student understandings and articulations across the two classrooms.

Conclusions

It appeared that one difference between the two teachers may be described in terms of "style." Josh had an interrogative style; Karla a discursive one. Josh considered himself to be more equal to the external authority and so positioned himself with it. In contrast, Karla was more deferential to it and located herself as closer to her students. This was particularly apparent in her forms of talking; she often appropriated student ways of talking and chose narrative as one of her predominant modes of expression. The way that Josh and Karla taught reflected these differences—it was part and parcel of the way they presented themselves to the classes they taught. Both teachers used control but the elements their control emphasized was different—Josh's emphasized control of turn and topic; Karla's emphasized control of the common knowledge.

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[1] The first row I labeled "sociomathematical norms" because they pertain to particular ways each classroom works with the mathematical content, the last two rows are considered "social norms."
[2] I am not using the word "difference" to indicate statistical significance.
[3] When counting turns there were instances where there were so many overlapping voices that making a word-count was impossible. In Karla's classroom, this discounted 40 turns; in Josh's classroom, 15 turns fell into this category.
[4] In Karla's classroom this consisted of one student turn consisting of 28 words; in Josh's classroom, it consisted of five student turns that ranged from 30 to 60 words each.
[5] "Long" relatively speaking. These turn lengths are still fairly short considering the length of the teacher's turns.
[6] The only instance of this that I've been able to find in my classroom recordings were when Josh asked students if they had ever driven over a bridge that was well known in the area and recounted a story about that bridge.
[7] This does not suggest that there are not also common forms with different functions. For example, both Josh and Karla use "__ said." In interviews, Karla immediately said she did this to give students ownership

for their ideas and thinking. Josh first said that he often did this to keep students on task and later included that he sometimes did this because students appreciated being credited for their ideas.

[8] In the two days worth of transcript, Josh asked approximately 394 questions. This included forms that did not end with rising intonation, but did require a response from students, e.g. ones where he restated part of an answer and ended with a drawn out word, indicating that students should finish what he was saying. In contrast, across the two class periods Karla asked about 100 questions.

[9] Marcie was a colleague (mentioned in Chapter 5) with whom Karla had collaborated while piloting the CMP curriculum. She was also involved in directing much of the professional development activity in which Karla was involved.

[10] For a more detailed description and more examples of these categories, see Appendix B.

[11] This word was commonly used to refer to slope because the slope was the "slantiness" of the line when you graphed it. It refers to the angle at which the line lies on the graph.

[12] "Hills" referred to quadratics with a negative term in front of the x^2 because the parabola would open downward, looking like a hill. "U's" referred to quadratics with a positive term in front of the squared term because the parabola would open upward, looking like a U. Even though students began by saying this was what the graph "looked like," they eventually dropped the look like and just referred to the graph as U's and hills.

[13] Even though Karla's use of questions was much less that Josh's, when she used them almost half (43%) were of the type that either pushed students to clarify (as in the revoicing examples I mentioned earlier) or else allowed students to offer their own perspective (e.g. What do you think…? How could you have…?).

[14] In this chapter, Mishler is looking closely at talk in first grade classrooms and showing how different cognitive strategies as well as different values and norms are carried in the language used by teachers (and the interactions between teachers and students). In the section I've quoted, he is claiming that one particular first grade teacher "shared responsibility for developing and confirming norms for behavior." However, I believe that this type of sharing of responsibility can also take place in developing and confirming what has been established as common knowledge in the classroom. When Karla used the word "we," she never differentiated her own point of view from the "we." Instead, the "we" seemed to be used as Mishler is describing it—as something in which they all took part. In addition, her use of "they" seems to further position herself as part of the classroom community in which joint activity takes place because she defers herself to this faceless external authority, making herself seem to be more part of the classroom community rather than part of the external authority who assesses the activity in which they took part.

[15] This preliminary analysis consisted of transcribing most of the student interviews from the Fall and beginning to look at the content in which students did well and where they struggled. I also attended to and categorized the language that they used for slope and y-intercept and began to write comparisons of the student understandings to fulfill class requirements in the some of the coursework that I completed in my program. It is not a comprehensive analysis, but I intend to pursue that as part of my research agenda.

CHAPTER 9: CONCLUSION/SUMMARY

> Mankind likes to think in terms of extreme opposites. It is given to formulating its beliefs in terms of *Either-Ors*, between which it recognizes no intermediate possibilities. When forced to recognize that the extremes cannot be acted upon, it is still inclined to hold that they are all right in theory but that when it comes to practical matters circumstances compel us to compromise. Educational philosophy is no exception. [...] At present, the opposition, so far as practical affairs of school are concerned, tends to take the form of contrast between traditional and progressive education.
>
> -John Dewey, 1938 from *Experience and Education*

The distinction between traditional and progressive (or, what I have been calling "reform-oriented") has been a focus for education since long before Dewey. This dissertation has truly been one focused on the "compromises" to which Dewey referred. We create categories in which to put things, painting the world as if it were black and white. The classrooms in which I have based my study both hold credentials as "reform-oriented"-the teachers in these classrooms not only agree with the reform proposed by the NCTM *Standards*, but also piloted and used a curriculum that was developed in the spirit of this reform—the Connected Mathematics Project (CMP). However, closer inspection of the discourse patterns in these classrooms showed that they were different from one another. In addition, further shades of gray appeared when looking at these classrooms through discussion vs. recitation. Neither classroom fit neatly into one of these two categories (as Cazden and others have recognized). However, the two classrooms offered instances of some shades of these dichotomous categories.

Since the release of the NCTM *Standards*, this debate over reform-oriented vs. traditional pedagogy has been much more volatile, especially in California. These "Math Wars" (as dubbed by articles in the *Notices of the American Mathematical Society*, see Jackson, 1997a, 1997b) have once again brought this dichotomy to the forefront of the discussion, ignoring the reality of classrooms and the possibility that there are shades of gray within each of these two paradigms. In calling a classroom "reform-oriented," we almost expect them all to look the same. On closer inspection, however, differences will and do appear within each of these paradigms.

I have offered case studies of difference in the context of similarity. I have not argued that one is *better* than the other; I have made the point that they are *different*. In doing so, I argue against the monolithic reform-oriented teacher/classroom that is so often portrayed in the media and in the literature. There is no uniform entity called a "reform-oriented" teacher/classroom. I re-introduce the notion of "reform-oriented," taking into account the subtleties of actual classroom practice.

In this concluding chapter, I address four topics related to this study. I revisit my original research questions and then discuss which of these I actually addressed in this dissertation. I then consider this work's implications, limitations and outline future research questions that this study has raised.

Research Questions

I begin this section by offering the *original* questions I had intended to answer in doing this dissertation. I then recap on the events that took place and how those shaped the questions that I *actually* answered in this dissertation.

My original intent was to answer the following questions:

A. What are the social and sociomathematical norms of the two classrooms?

- How are they explicitly (and implicitly) established and maintained?
- How are they embedded and carried in the classroom discourse?
- What understandings about mathematical content do they seem to encourage?

- What kinds of dispositions about learning and doing mathematics do they seem to encourage, as laid out in the NCTM *Standards*?

B. What are the things that influence the way the teachers think about and the way they work to establish and maintain their norms and sociomathematical norms in their classrooms?

C. What are the similarities and differences across the two classrooms (specifics will fall out of the above analyses)?

The questions with which I began this study sought to make sense of at least three aspects of the classroom: social norms, sociomathematical norms, and individual student understandings. The norms focused on student and teacher rights, responsibilities, expectations and roles. Social norms are not subject specific; they could exist in any classroom. Sociomathematical norms are ones that pertain specifically to the mathematical content: for example, how students are expected to justify their answers or form a mathematical argument are related to how these things are done in the larger mathematical community.

During the course of this study I encountered two critical decision points. In the first, I began to see that my focus both on student understandings and on the norms in the classroom could conclude as two separate dissertations. At that point, I had to decide which was actually most intriguing to me and chose to focus on the norms that were embedded and carried in the classroom discourse. While I illustrated and interpreted student contributions in terms of the mathematical content of which they were speaking, student understandings were no longer a primary focal point of this dissertation.

Originally, I had also planned to spend fall semester, 1999, in both teacher's classrooms to collect data on which to base my dissertation. In that data, the focus would have been on the students and their perceptions of the discourse patterns. However, during spring semester, 1999, Josh informed me that he had taken an administrative position that would begin the next school year. Due to this change, my data collection and focus had to change abruptly. I still sought to understand the discourse patterns and the norms in the classrooms but changed my focus to the teacher's perceptions of these rather than those filtered through student beliefs and understandings. I also continued to pursue similarities and differences in the two classrooms.

The questions that I have actually addressed in this dissertation include:

A*. What are the most prominent discourse patterns in each classroom?

- What are some of the social and sociomathematical norms embedded in and carried by them?
- What are some of the things that influence the way the teachers think about and the way they work to establish and maintain their social norms and sociomathematical norms in their classrooms?

B*. What are some of the similarities and differences across the two classrooms?

In undertaking this study, I have identified and analyzed some of the differences between these two teachers and their teaching as it appeared in the predominant discourse patterns in the classroom. Certain norms were apparent when discourse patterns were carefully considered. I focused on describing and illustrating the discourse patterns and the norms that were embedded and carried in them.

In Chapter 5, I described many aspects of the teacher's beliefs about subject matter, teaching and learning. While the CMP textbooks themselves influenced both teachers, Josh seemed to be impacted more by these than Karla. In fact, he described himself as "curriculum driven" in one of the interviews and said that the "teacher-talk" in the earlier versions of CMP had most influenced his teaching. The teacher-talk consisted of a series of questions the teacher might ask to encourage mathematical understanding. Josh said that using this type of questioning caused a huge change in his teaching and all of the professional development that followed merely helped him to "refine" his techniques. He believed that, by the time he finished his teaching career, his classroom was as close to his "ideal" classroom as he may have achieved.

The factor that played (and continues to play) a large role in Karla's shift in thinking about her teaching (and student-learning) happened through her peer interactions with other teachers. Karla collaborated with two other teachers in school, taking turns doing peer observations. When observing, Karla's colleagues focused on her teacher-talk and how she listened to students. They posed questions (e.g. What do your students understand?) that pushed Karla's thinking about her teaching and about what her students were learning. Karla was still working on listening to her students and continued to think about her role in the classroom in terms of how she could get students to talk more and not consider her to be the "all-knowing."

In Chapter 6, I described the three most prominent discourse patterns in Karla's classroom: phrases that created context and continuity for the common knowledge, metacommenting, and expressions of uncertainty. Through these repeated patterns, Karla tacitly established many social and sociomathematical norms (for the entire list, see pp.

84-85 or 143-145). Karla seemed to define her role as someone who told stories, summarized and contributed as she felt necessary. She encouraged students to share multiple solution methods, contribute whenever possible, acknowledge uncertainty, hypothesize possibilities and be responsible for their own learning. The classroom was seen as a community with a common history and the content was represented in a connected way (both in terms of the representations and the contexts in the textbooks).

Josh's most prominent discourse patterns were discussed in Chapter 7, including: questioning, expressions establishing authority, and talk about knowing. Through these repeated patterns, Josh also established certain social and sociomathematical norms in his classroom (for the entire list, see p. 152 or 221-223). Josh seemed to define his own role in the classroom as someone who "knew" and helped to determine correct and reasonable answers to the problems. Some of his forms drew attention to the common knowledge of the classroom (e.g. tags, sub-questions). Students were expected to know the reasons for the answers they gave and to offer multiple solution methods. If they were "model" students, then they should "want to know." The role of the textbook in the classroom was something to be referred to as a source of knowledge.

In Chapter 8, I began by comparing the focus of the norms in the classroom. While most of the norms were sociomathematical in both classrooms, the differences seemed to appear mainly in the social norms. Karla's attention seemed to focus on the students, whereas Josh was more concerned with his role in the classroom. I continued to highlight these differences (and others) by turning my focus to different surface forms that served similar purposes in the two classrooms. Differences in patterns of authority, how the teachers drew on sources of knowledge and directed attention to the common knowledge of the classroom were discussed. In Josh's classroom, he and the textbook were viewed as part of the locus of authority because they were drawn on as sources of knowledge. The common knowledge was highlighted through his use of: tag forms, "but," propositions followed by "why?" and "should." These various forms seemed to align Josh more closely with the locus of authority and separated him from the community of the classroom. In contrast, Karla seemed to align herself more with the classroom community through her use of: narrative story telling, "we" as jointly constructed knowledge, shielding of students through use of hedging and modals, and appropriation of student language (i.e. CGL). In addition, she may have downplayed her role as authority by rendering to the textbook authority by using "they" as an Attribution Shield and by offering what "I would" do (which was not as forceful as Josh's explicit contributions of what was "reasonable"). She drew on student knowledge and allowed students to control and offer the topic which then became the focal point of the discussion. Her ways of pointing to the common knowledge were more connective (i.e. "so") and were often in the form of a story. When she discussed presupposed knowledge, she used "can" a modal verb that indicated students had an ability to do mathematics.

The discourse analyses have helped me to identify prominent discourse patterns in the two classrooms in which different social and sociomathematical norms were encouraged. They have shown some of the ways that two "reform-oriented" classrooms may be different. As Josh said (see p. 231), some of these differences may be attributed to teacher "style." He felt more comfortable being in control of the classroom. Karla, in contrast, did not want to be the "all-knowing" and this surfaced in her discourse forms. Even though these two teachers have had many identical experiences in terms of professional development, education, certification and teaching assignment and very similar student understandings, there were things from which they had drawn that may have attributed to the different environments in the two classrooms. Josh drew mainly from the teacher-talk and this may have affected his teaching in ways different from the peer coaching in which Karla participated.

Implications

This work has value for both practical and theoretical reasons. At a practical level, this dissertation can inform professional development and teacher education programs, curriculum developers, and parents. At a theoretical level, this work might contribute an additional dimension to the knowledge base of teaching (Shulman, 1987) and offer insights into the way that shared intent (e.g. both teachers are dedicated to the new curriculum and what it stands for) does not necessarily translate into shared practice.

Practical Implications

Teacher educators and professional development leaders can use this information to increase teacher awareness about the array of language that could be used in the service of teaching mathematics, stressing not just multiple solution methods, but also multiple ways of talking about mathematical concepts (e.g. classroom generated language, transitional mathematical language). More importantly, this analysis could also be used to increase teacher understanding of the impact of tacit language patterns on the classroom in helping/hindering certain positioning.

In the context of a mathematics classroom, language is not something to which teachers typically attend. When I was a beginning junior high mathematics teacher, I would have never considered how my language might be promoting or undermining goals that I wanted to accomplish in my classroom. In the teacher interviews, when I offered particular patterns I had noticed in each classroom, the response was one of surprise. Both Josh and Karla, who were experienced teachers, did not seem to notice they were repeating these patterns of talk and found it difficult to articulate what they thought the talk patterns were accomplishing in the classroom. I think they would be somewhat surprised at how these patterns may have influenced the classroom environment, particularly around issues of control and authority.

I believe bringing a discourse perspective to the mathematics classroom can help teachers think about many things. For example, if we refer back to the NCTM *Professional Teaching Standards* (1991) document, it states:

> The discourse *embeds* fundamental values about knowledge and authority. Its nature is reflected in what makes an answer right and what counts as legitimate mathematical activity, argument and thinking. Teachers, through the ways in which they orchestrate discourse, convey messages about whose knowledge and ways of thinking are valued, who is considered able to contribute and who has status in the group (p. 20, italics added).

"Embeds"—what does that mean? What does it look like?

The discourse analysis that I have offered made evident how issues of positioning with respect to authority and epistemology might be "embedded" in teacher talk. It can help teachers see more clearly how using particular language forms in specific ways may be undermining or promoting the kind of discourse they may want to establish in their classroom. This type of analysis gives us a lens to think about *how* these things are inherent in the discourse.

Currently, I am constructing a course to engage experienced teachers[1] in thinking about some of these issues. To do so, I will draw from some of the readings I have done for this dissertation that focus on students (e.g. Tim Rowland (1999)) and that focus on teachers (e.g. the literature on questions). I plan to have the teachers read the articles and use them as a lens to view, discuss and analyze classroom data and/or interview situations from my database. My intent is to have them first think about these issues as an outsider, focusing on how they relate to someone else's classroom.

As they become attuned to language patterns and some of the issues I have dealt with in my dissertation, I will encourage them to use these to engage in action research in their own classrooms. They will be asked to choose an aspect in their classroom (e.g. student language or their own) and first study that aspect as it exists, collecting audio and/or video of their own teaching (or interviewing students, depending on the focus of their inquiry). After they study it as it exists, I will encourage them to compare the norms they think the existing patterns are fostering to those that they would like to occur in their classroom. This comparison will allow them to think about what their language patterns might be doing and will offer evidence of how their language patterns are playing out in the classroom. They will refine their talk patterns to align them with the goals they hope to promote, studying the classroom environment throughout these changes.

For publishers and curriculum developers, attention to specific language patterns (both pertaining to the content and to teacher positioning) could make the curriculum more effective and can help teachers understand that multiple ways of talking may allow students greater access to mathematical ideas and concepts. This study could also help parents become better "educated" about the variation in teaching in general, drawing the focus away from dichotomizing reform and traditional ideas about teaching and learning.

Theoretical Implications

From a theoretical perspective, this study can contribute to identifying aspects of the knowledge base of teaching and may offer insight into why teachers who went through similar educational and professional development experiences can come to develop different practices. This latter contribution is not apparent from a quick glimpse into classrooms and points to the importance of looking at teaching from a detailed perspective as it appeared that these two classrooms could have been very much alike.

The knowledge base outlined by Shulman (1987) began to specify what teachers need to know to do their job effectively. However, he also acknowledged that "much, if not most, of the proposed knowledge base remains to be discovered, invented, and refined" (p. 12). The work that I have offered here adds to the knowledge base in that knowledge of tacit language patterns used in the school teaching of mathematics does not exist in the categories he describes. For teachers to do their job effectively, they could also be drawing from sociolinguistics to think about their choice of words and how that may affect what is happening in their classrooms. Having knowledge of this literature could empower teachers at yet another level, helping them utilize their words and make choices about what they say. It could help them become aware of the implicit norms in their classroom and allow them to align their words with the goals they have set.

Shulman argued that there were "few descriptions or analyses of teachers that give careful attention [...] to the management of *ideas* within classroom discourse [...and are] needed if our portrayals of good practice are to serve as sufficient guides of better education" (p. 1). This dissertation offers a look at the ways that ideas were dealt with in the two classrooms and how those were different. These differences were embedded in and carried by the classroom discourse. This "pedagogical discourse knowledge," which scarcely overlaps pedagogical content knowledge apart from awareness of the mathematics register, is another base from which I believe teachers need to draw to do their job effectively, allowing them to align their practice more closely with their educational goals.

Limitations of This Study Suggestions for Future Research

As with any qualitative research, the findings in this dissertation are not generalizable to any other population. The teachers were chosen as ones that would be considered "model" for this curriculum by the authors (as evidenced by their strong involvement in the development of other teachers using the curriculum). While the case studies have offered insight into the plausible relationship between language patterns and the environment a teacher may establish, it is not causal in nature. There are other aspects about the classroom that are confounding variables and make it impossible for me to draw causal relationships. However, there does appear to be strong evidence that the forms the teachers used did create implicit structures that may have allowed for different patterns of authority and positioning. Furthermore, some of the talk in Karla's classroom often seemed to engage students in discussions rather than recitations.

As I have talked to many people about my work, they have almost always asked, "Did it make any difference to what the students learned?" The work that I have done thus far cannot answer this question. Connecting particular teaching strategies to facets of student understanding is difficult in general. However, I do intend to analyze and draw conclusions about student understanding in my future research. Whether I can ever claim that there are direct connections between these two things or not remains to be seen.

My sense of the two classrooms, informed by a surface analysis of the student interviews I carried out, was that students walked away with amazingly similar understandings of linear and non-linear functions. However, as I began to illustrate in the extended examples in Appendix N and O, there was a different feel to the type of interaction in which Karla's students were engaged. The discussions in which students took part seemed to have a quality that I am unsure how to quantify. This was one of the reasons I found these two classrooms so interesting and used them as sites for my research. I have not begun to explore the student interview data at the depth that it deserves to be investigated. While I have generally looked at how students talk about the content and what they seem to excel at (and not), I have not looked at whether the student's dispositions or discourse patterns were different in the interview setting. This is one route that I intend to take with the interviews: looking at linguistic patterns to see if there are congruencies in the teacher's forms with the student's forms. There is evidence to suggest that, at least in Karla's classroom, students do carry the classroom-generated language (CGL) that has been introduced and appropriated in the classroom to the interview setting. Are there other forms that students also carry out of the classroom? If so, what are they? What do those forms seem to say about the mathematical understandings and dispositions the students have developed?

Another area I have begun to consider is the impact of textbook materials on teacher discourse patterns. Josh said that the "teacher talk" had a strong influence on the changes he made in his teaching. I also found that Josh and the CMP text chose similar ways of addressing the students in the classroom. I am curious to know how textbooks directly impact the discourse forms teachers use. This dissertation indicates that, for Josh at least, it could have a very strong impact. The question also remains regarding whether the textbook influenced the way Josh addressed students or if Josh had already established the ways that he interacted with students and drew from the textbook in the manner that he did because the text 'sounded' like him. The links between features of the textbook and the way teachers draw from it (especially at a linguistic level) could offer interesting insights into the impact textbooks could have on teacher-talk.

A third area that needs further study is that of intention in teacher talk. When a teacher uses a particular form for a particular function, is that how it is being interpreted by the student? What does the literature say? How aware are teachers of the (actual and potential) impact of their forms on establishing environments? I addressed this minimally when I asked Josh and Karla to talk about some of the patterns that emerged from my early analysis. However, it was not addressed to the point that it could have been. If my analysis had been further along prior to the teacher interviews, I could have offered a lot more of the teacher's interpretation of the patterns. This issue also raises interesting implications for professional development of practicing teachers and is one that I intend to pursue.

Finally, I am again continually aware of the fact that student perceptions have once again been ignored. As Erickson and Schultz (1992) argued almost a decade ago, student's experience is virtually ignored in the literature. I had intended to offer a small contribution to this sparse body of literature. Due to circumstances, I have had to put that on my list of "things to do." This is one area that I find completely intriguing. How do students make sense of the discourse patterns in the classroom? Are they aware of, tacitly influenced by, and do they attend to the subtleties in the talk? Do the patterns seem to matter to the students? How are student interpretations similar and different from the ways the teachers interpret them? From the way the literature interprets them?

Ainley (1988) has already shown that there may be a mismatch between how teachers and students interpret questions. Is this true of other language forms?

Another issue related to students that I would like to investigate is how they interpret language forms in the textbook. The objective textual analysis that I offered in Chapter 4 showed the unit *Thinking with Mathematical Models* as having a fairly authoritative voice. It also constructed the student as a "scribbler" (Rotman, 1988) and the authors as a faceless presence. One may think that these might impact what the students do in the classroom. However, my classroom observations indicated that students were engaged and active in the classroom—they investigated and performed the experiments and seemed to enjoy doing them. They did not seem to be affected by the relationships and voice that the analysis uncovered. The analysis that I have presented here did not address this, but it would be interesting to pursue: addressing these issues from the student's perspective would offer a subjective analysis of the materials. Do students notice what the textual analysis showed? How do they interpret the language forms in the textbook? Does it seem to affect the environment and the ways students engage in the classroom?

It appears that I have raised more questions than I have answered. However, I believe that this is one of the main purposes of any inquiry. In raising the questions that I have, I am also defining the path of my future research agenda.

[1] I would like to recognize that I have chosen to focus on experienced teachers because I believe preservice and beginning teachers have many other concerns with which to attend. The type of introspective analysis I am advocating here would be better suited to teachers who are established and developmentally ready to take a detailed look at their practice.

APPENDIX A

CHECK-UP FROM *THINKING WITH MATHEMATICAL MODELS*

CHECK-UP

1. a. Circle the table or tables that show a linear relationship.

x	y
1	4
2	2
3	1.33
4	1

x	y
1	4
2	2
3	0
4	-2

x	y
1	2
2	4
3	8
4	16

b. Explain how you can recognize a linear relationship from a table.

2. a. Circle the equation or equations that show a linear relationship.

$y = 1/4\,x$ $\qquad y = 4/x$ $\qquad y = 4/x + 3$ $\qquad y = -4x$

b. Explain how you can recognize a linear relationship from an equation.

3. a. Circle the name of the graph or graphs that show a linear relationship, and write their equations.

b. Explain how you can recognize a linear relationship from a graph.

4. Find an equation of the line that passes through points A and B.

5. Find an equation of the line that passes through the point (0,5) and has slope 4.

6. Find an equation of the line that passes through the point (4,5) and has slope −2.

7. To plant potatoes, a farmer cuts each potato into about 4 pieces, making sure each piece has an "eye." The eye contains buds that will become the new plants. Each new plant will produce 5 potatoes. Thus, a single potato will yield 20 potatoes.

a. Make a table and a graph model that show how the number of potatoes grown depends on the number of potatoes cut and planted.

b. Write an equation that describes this situation.

APPENDIX B

ANALYTIC MEMO FOR TYPES OF MATHEMATICAL LANGUAGE

Analytic Memo 1
October, 1998

Working definitions of mathematical language (ML), bridging language (BL), and contextual language (CL). Right now, my thinking is "definition by example"...

I have a couple phrases/words that I am not sure where to put (i.e. coefficient, stair step) but have temporarily place in either ML or BL. I will break up the two BLs into classroom generated language (CGL) and transitional mathematical language (TML)

Mathematical Language	Transitional Mathematical Language	Classroom Generated Language	Contextual Language
• Slope • Y-intercept • Rise • Run • Linear • Parabola	• Difference of the x's/ difference of the y's • Starting point • Goes up/down by • Rate • Coefficient • Steepness • Increases/Decreases by • Where it crosses the y • Constant • Plus something • Where x is zero • What you add • Where it hits the y-axis • Number with the x	• Swooped down a little • Slantiness • Straight across • Little crisscross thing* • Stupid stores* • McDonald arches**	• Up-front money • Down payment • Dollars per month or week • Meters per second • Head start • Stair steps

*These examples aren't referring to slope and y-intercept, but are examples of this type of language arising in K's classroom.

**This example is the only type of CGL I could find in J's classroom. It was introduced by J to relate a picture of a parabola to something Ss were familiar with. It took place during Models, not during MSA.

It was not surprising to me that all but the McDonald arches came from K's classroom in the CGL column. The TML comes from both classrooms, but was more frequent in K's classroom, too. J's classroom worked more frequently in the ML domain.

I am not sure if all of the ones I have included in the TML should be included. In a sense, some of them feel more like describing where the slope/y-intercept are found in various representations (e.g. "plus something" is where you find the y-intercept in an equation). However, do Ss seem to use this language just when describing where to find these numerical values or do they use these terms metonymically (sp?) for these concepts? In other words, do they use it as a reference point or as the "what you add" as actually meaning "y-intercept" when they refer to it?

Now that both classes have started working with Models, I am not sure if I should classify something like "line of best fit" and "graph model". Even though they do not have to do with slope and y-intercept, I feel like they should be included. At this point, I am not really sure I know why—it is just a feeling I have. Both of these phrases were introduced by the text and J has used them frequently in the couple of lessons I have observed from this unit. I have not classified them, but have highlighted them to keep track of what happens with them...

As for the beginning questions, here are my thoughts so far:

Who generates the bridging language?

It seems, at this point, that it might be coming from the teachers, the texts, and the students. In the case of TML, it is more frequently introduced by the teachers and the texts. In some cases, it is a result of reading something in the text and it was picked up on and used by the teacher. Most often, through questioning, the teachers are getting students to adopt this language (especially in the examples that seem more like descriptions of where to find things in different representations). The CGL seems to be generated by Ss, although this has not occurred during any of the observations. I am conjecturing on this because of how I have seen K give credit to students for their language use or introduction of processes. It may be introduced by her at some points, too, but I will have to watch for these...

How does the teacher/students make this part of the classroom's way of talking about the mathematical concepts?

Not only does K use the language, but she also credits whoever created it when she uses it (especially during the earlier stages of introduction).. Need to look at this more..

What order does the language appear in when a combination of languages exist? How does this differ (if it does) between the two teachers?

I need to look into this more.. I did point out a couple of differences I noticed earlier which address early patterns in the language, but without exact counts. I have not looked at all for instances when combinations of language exist. I need to do that...

APPENDIX C

STUDENT INTERVIEW TASK 1

Task One

A new student has recently joined your class. Right now, you are talking a lot about non-linear relationships, but this student has never seen linear ones. She feels really lost and confused. Ms. Delmont (Mr. Muchinson) has asked you to work with this student to fill her in on what you know about linear relationships. What would you tell her about linear relationships?

APPENDIX D

STUDENT INTERVIEW TASK 2

Task Two

The Michigan Department of Natural Resources has been collecting data on three different species of animals. They found that these species show different patterns of population growth. Their growth patterns are given below. P represents the number of animals of each species after x years.

Species 1	**Species 2**	**Species 3**
$P_1 = 10{,}000 + 5x$	$P_2 = 10(2^x)$	$P_3 = 700 + 10x^2$

1. Describe the pattern of growth of each species. Explain how these patterns differ from each other.

2. Pick any two species. Is it possible that after some number of years the populations of these two species would be equal in number? If so, when? If not, why not? Explain how you would go about answering this question.

APPENDIX E

12 TASKS USED IN CARD SORT AND OTHER INTERVIEWS

Tasks

NAEP Table Problem

Observe the pattern in the following table:

A	B
2	5
4	9
6	13
8	17
14	?

A. If the pattern in the table were continued, what number would appear in the box at the bottom
of column B next to 14?

B. Describe the pattern in words.

C. Write an equation that represents the pattern. Explain what the variables and numbers in the equation mean.

Tim's Problems

1. If m represents a positive number, which of these is equivalent to $m + m + m + m$?

A. $m + 4$ B. $4m$

C. m^4 D. $4(m + 1)$

Answer: ____________________

2. Which one of the following is FALSE when a, b, and c are different real numbers?

A. $(a + b) + c = a + (b + c)$ B. $ab = ba$ C. $a + b = b + a$

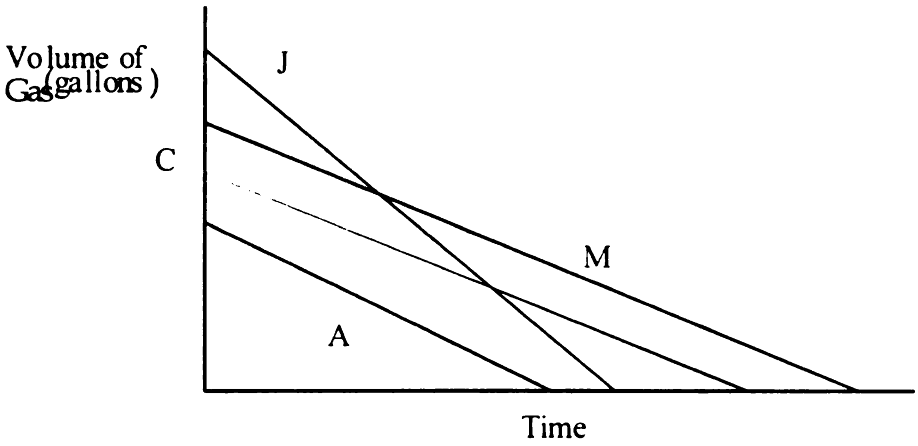
D. $(ab)c = a(bc)$ E. $a - b = b - a$

3. Find x if $10x - 15 = 5x + 10$.

4. If $x = 2$, what is the value of $\frac{7x + 4}{5x - 4}$?

Four Cars Problem

Jerry, Alicia, Calvin, and Marisa wanted to test their cars' gas mileage. After filling their cars with gas, they drove them on a test track at 65 miles per hour until they each ran out of gas. The graphs given below show how the amount of gas in their cars changed over time.

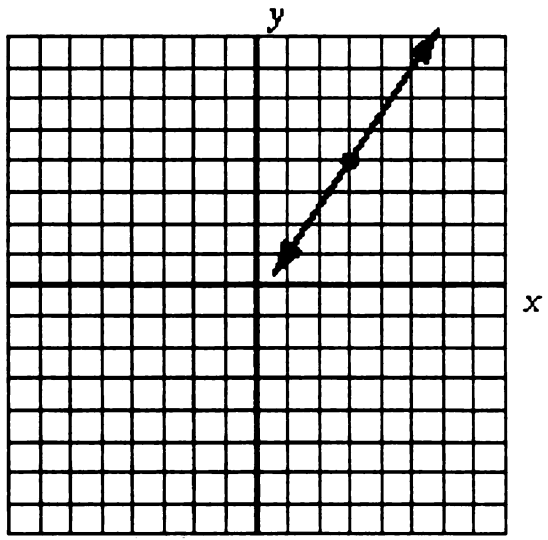


Answer each question below and explain how you used the graphs to decide your answer.

A. Whose car had the largest gas tank?
B. Whose car ran out of gas first?
C. Whose car went the farthest?
D. Whose car gets the worst gas mileage?

Slope & Point Problem

Part of a linear graph is shown below. It has a slope of 1.5 and goes through the point (3,4).

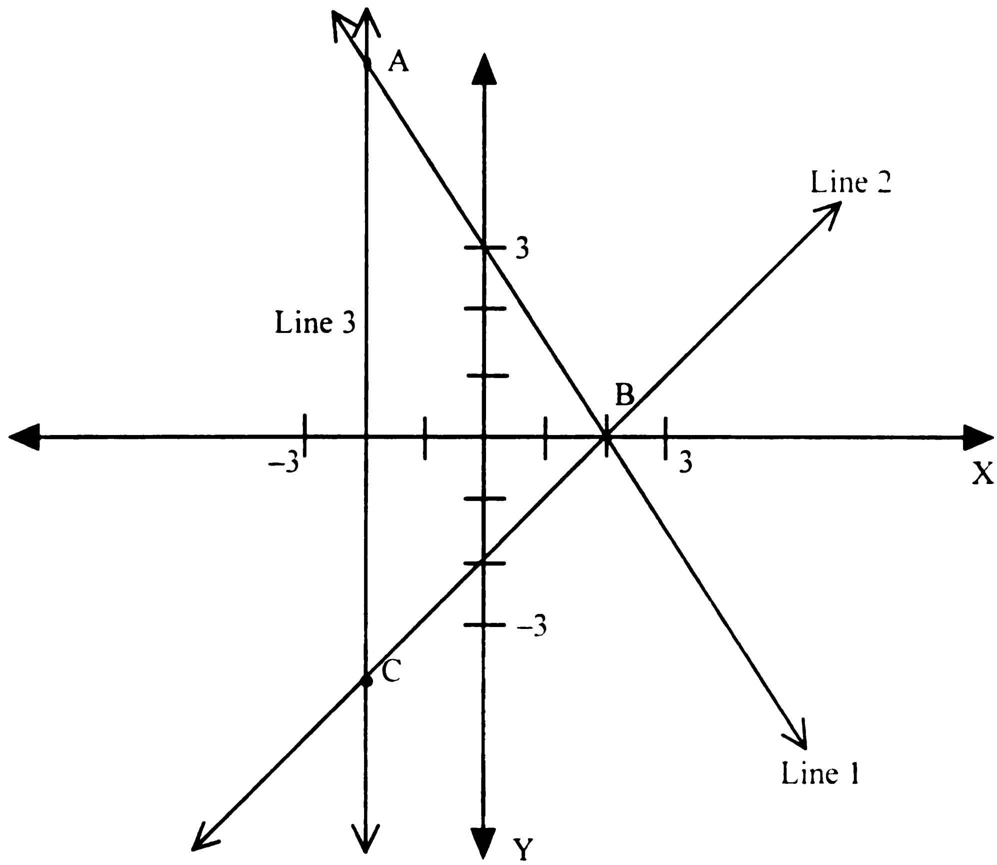


A. Find the coordinates of three other points on the line whose x and y-coordinates are both positive. How do you know that these points are on the line?
B. Find the coordinates of the y-intercept. Explain your methods.
C. Find the y-coordinate of the point whose x coordinate is –4. Explain your methods.
D. How would changing the slope of the line to 2 affect the y-intercept? Explain your reasoning.

E. How would the equation for Calvin's graph compare to the equation for Marisa's graph?

Intersecting Lines Problem

Three lines, Line 1, Line 2, and Line 3, are shown below on a two dimensional coordinate graph.



A. Write an equation for each line.
Line 1:
Line 2:
Line 3:
B. Explain how you figured out the equation for Line 1.
C. Find the perimeter of triangle ABC. Explain your reasoning.

Reception Costs Problem

The high school senior class wants to have a reception for parents and friends after graduation. They held fund-raisers during the year from 12th grade students to raise money for this event. One local restaurant told the students that they charge $150 for the room and $10 per person for food and drinks.
A. Write an equation for the cost of the event in terms of the number of people attending. Explain how your equation matches the information the students received from the restaurant.
B. If the class has $1620 to spend, how many people could come to the reception? Describe at least <u>two</u> methods you could use to answer this question.

C. A second restaurant told the students they charge groups according to the following equation:
$C = 25N$

C = cost and N = number of people attending.

Explain what this equation means in this context.
D. Which restaurant should the class choose if they want to keep their costs as low as possible? Explain your reasoning.
E. Is it possible that a third restaurant could charge the students according to the following equation, $C = -150 + 15N$? Explain why or why not.

Contractor Problem

Thomas is a flooring contractor. He sets floor tile in people's homes. As a contractor, he has to submit a "bid" for each new job. Each time he bids on a job, he measures the area of the floor that he will tile and then figures out how much material he will need. Here are the prices for the materials that he uses regularly.
Sub-Flooring: \$1.12 per square foot
(Sub-flooring is the wood that he lays down under the tile.)
Tile: \$4.29 per square foot
Adhesive: \$21.59 for one bucket (enough for a typical home)
Grout: \$34.95 for one bag (enough for a typical home)
Labor: \$125 plus \$0.68 per square foot
A. Thomas wants a formula to help him compute the total cost of his materials and labor for a typical home. Write a formula for Thomas' total cost. Explain what the numbers and symbols in your formula mean.
B. Write a formula that simplifies Thomas' cost calculations. Explain how your formula simplifies his calculations.
C. Suppose Thomas wants to make a 15% profit on each job. (In other words, he wants to charge each customer an additional 15% of the total costs to make a profit.) Write a formula that Thomas could use to compute how much he charges his customers, including his profit.

Phone Plans Problem

Your phone company offers three different monthly billing options for local phone service.

Option I: \$10.00 for up to 30 calls, plus \$0.20 for each additional call.
Option II: \$30.00 for an unlimited number of calls.
Option III: \$18.00 for up to 60 calls, plus \$0.05 for each additional call.

A. If Kelly makes about 23 local calls each month, what would be the best option for him? Explain your thinking.
B. If Tamika makes about 100 local calls each month, which would be the best option for her? Explain your thinking.
C. Suppose you want to help other people decide which is the best option for them. Make a table, a graph, <u>or</u> write a set of the equations for Options I – III. Explain how you would use your table, graph, or equations to select the best option for different people.

Pumping Water Problem

Suppose you turn a pump on and let it run to empty the water out of a pool. The amount of water in the pool (W, measured in gallons) at any time (T, measured in hours) is given by the following equation:

$$W = -350(T - 4).$$

Answer each question below and show how you used the equation to do so.

A. How many gallons of water are being pumped out each hour?
B. How much water was in the pool when the pumping started?
C. How long will it take for the pump to empty the pool completely?
D. Write an equation that is equivalent to $W = -350(T - 4)$. What does this second equation tell you about the situation?
E. Describe what the graph of the relationship between W and T looks like.

Soda Machine Problem

Lincoln High School has a pop machine near their athletic fields. It automatically counts how many cans of pop people buy each half-hour of the day. One Saturday in May, Lincoln has a Field Day where students, parents, and teachers participate in sports activities. The table below shows how much people used the machine in the morning (8 am to 11:30 am) on Field Day.

Time	8 to 8:30 am	8:30 to 9 am	9 to 9:30 am	9:30 to 10am	10am to 10:30	10:30 to 11 am	11 am to 11:30	11:30 to 12 noon	12 noon to 12:30	12:30 to 1:00 pm
Cans of Pop	4	9	13	18	24	28	34			

A. Graph the data carefully on the attached grid paper. Allow enough room to graph data from 11:30 to 1:00 pm.
B. Fit a graph or equation to the data. Explain how the graph or equation fits the data. In what way(s) does the model <u>not</u> fit the data?
C. Use your model to predict how many cans would be bought between 11:30 and 12 noon.
D. If the data turned out to be 44 cans for 11:30 to 12 noon, 58 cans for 12 noon to 12:30 pm, and 76 cans for 12:30 to 1:00 pm, would you change your model? Explain why or why not. If you think change is necessary, explain how you would change your model.

Gasoline Problem

The tables below show the amount of gasoline used by (demand) and the amount of gasoline available (supply) to drivers of cars and trucks the U.S. for the years 1993-1997.

Demand for gasoline

Year	Gas (billions of gallons)
1993	50
1994	60
1995	72
1996	86.4
1997	103.7

Supply of gasoline available

Year	Gas (billions of gallons)
1993	200
1994	230
1995	260
1996	290
1997	320

A. Describe the pattern of change for demand.
B. Describe the pattern of change for supply.
C. In what year will the demand be greater than supply? Explain your reasoning.

Populations Problem

The Michigan Department of Natural Resources has been collecting data on three different species of animals. They found that these species show different patterns of population growth. Their growth patterns are given below. P represents the number of animals of each species after x years.

Species 1	Species 2	Species 3
$P_1 = 10{,}000 + 5x$	$P_2 = 10(2^x)$	$P_3 = 700 + 10x^2$

A. Describe the pattern of growth of each species. Explain how these patterns differ from each other.
B. Pick any two species. Is it possible that after some number of years the populations of these two species would be equal in number? Explain how you would go about answering that question.

APPENDIX F

SEMI-STRUCTURED INTERVIEW FOR CARD SORT

Semi-structured Interview Questions
Sorting Task

(similar questions were asked of all groups; some deviation depending on the interview; some additional questions may have been asked of some groups)

What are the categories that you decided on?

How did you choose them?

Did you consider any other categories?

Could there have been any other possibilities?

What were/are some of the other possibilities?

Explain your categories. (define them)

How did you decide which problems to put where? Why did you put the problems you did in each category?

Did your categories change when the second group of problems were introduced?
- -Why/why not?
- -If they did, how?

If someone came in and saw your poster with your categories on it and they had never taken algebra before, what do you think your categories would tell them about what algebra is?

Do you think your categories would be helpful to someone who had never taken algebra before? Why/why not?

What do you think the categories would help them understand about algebra?

APPENDIX G

TEACHER INTERVIEWS 1-4

Teacher Interviews #1-4

Teacher Interview—Josh
Interview #1

1. How has your teaching developed or changed over the last ten years? What has played a role in this development or change?

2. You've taught many sections of students. Tell me about your first few weeks of school...

a. What mathematical ideas did you think were important for students to understand? How did you try to get students to understand the mathematics?

- What difficulties did you run into?
- What did you feel went smoothly?
- Were any of these (difficulties/not) unexpected?

b. What expectations and goals do you set for your students? How do you establish these classroom expectations and goals with your students?

- What difficulties did you run into?
- What did you feel went smoothly?
- Were any of these (difficulties/not) unexpected?

c. What expectations and goals do you see CMP setting? Are these ever helpful or in conflict with what you want to accomplish in your classroom?

3. Thinking about the two years of students I have worked with in your class... How is this year's class different from/similar to last year's? Why do you think those differences/similarities exist?

4. If you were working with a new teacher and he/she understood the mathematics and knew CMP, what issues would you think were important for her to know about teaching middle school mathematics?

- As a teacher, what role would you want to see her play in the classroom?
- What role do you want her to help students play?
- What do you want her to see her responsibilities to be with respect to her students? To the mathematics? To the curriculum?
- What kinds of expectations would you want her to set with her students?

4. I know that you also used to teach science classes. Could you tell me how your science classes were similar/different from your math classes? Did you try to make these classes different? Why/why not?

Teacher Interviews--Karla
Interview #1
(approximately 50 minutes?)

1. How has your teaching developed or changed over the last ten years? What has played a role in this development or change?

2. Tell me about these first few weeks of school...

a. What mathematical ideas did you think were important for students to understand? How did you try to get students to understand the mathematics?

- What difficulties did you run into?
- What did you feel went smoothly?
- Were any of these (difficulties/not) unexpected?

b. What expectations and goals do you set for your students? How do you establish these classroom expectations and goals with your students?

- What difficulties did you run into?
- What did you feel went smoothly?
- Were any of these (difficulties/not) unexpected?

c. What expectations and goals do you see CMP setting? Are these ever helpful or in conflict with what you want to accomplish in your classroom?

3. How is this year's class different from/similar to last year's? Why do you think those differences/similarities exist?

4. If you were working with a new teacher and he/she understood the mathematics and knew CMP, what issues would you think were important for her to know about teaching middle school mathematics?

- As a teacher, what role would you want to see her play in the classroom?
- What role do you want her to help students play?
- What do you want her to see her responsibilities to be with respect to her students? To the mathematics? To the curriculum?
- What kinds of expectations would you want her to set with her students?

5. I know that you also used to teach science classes. Could you tell me how your science classes were similar/different from your math classes? Did you try to make these classes different? Why/why not?

Interview #2—Follow up to #1

Most of the questions I would like to talk to you about today are follow ups from the first interview.

1. In talking with you the first time, you talked about "big ideas" quite frequently throughout the interview. I'm wondering if you could tell me:
-What the "big ideas" of algebra are that you are trying to get your Ss to understand?
-What role these big ideas play in terms of you thinking about your classroom environment?
-How do you decide if Ss understand these "big ideas"?

2. What would YOUR ideal mathematics classroom look like in terms of the classroom environment you want to establish? What would THE ideal mathematics classroom look like? What are the differences/similarities between these? Why?

3. We've been talking a lot about the classroom environment in terms of your and your student's roles, responsibilities, rights, expectations, etc. As we did so in the last interview, you stated that you don't really make these explicit to your students. Let's say that you decide that you want to do this and that you will be hanging a classroom poster on your wall that delineates what you want your students ROLES, RIGHTS, RESPONSIBILITIES to be.

-What would you list under each of these headings?
-If your students were asked to do this independent of your list, how do you think your students would be similar? Different?

4. Now that we've delineated some of these expectations, do you think that these are appropriate for other subject matter classrooms or do you think some/all of them are particular to mathematics classrooms?
-Ex. Students in your classes work with "big ideas" and offer "multiple solutions". Do you think these are specific to mathematics or would they be appropriate to other subject matter classes?

Josh's interview #3

1. What are some of the things that influence the way that you have come to teach?
--has mentioned: CMP; Ss (listening to their ideas; also, where they will be going next year); own understanding of mathematics

2. Are there particular things that influenced this this past year (working with that particular class or things that happened that influenced it (e.g. high school))?

3. Some of the things that I've noticed that you do include:

--crediting students
--Explicitness about where Ss are supposed to be
--a or b
--rise/run/slope/y-intercept/equation scaffold

-Have you noticed that you do this?
-If you were naming this strategy, what would you call it? (e. g. engaging students, getting students to listen)
-What is it you are hoping this strategy will do in your classroom?

4. How do these strategies tie back with the things that you said have influenced the way that you teach?

Karla's Interview 3

1. What are some of the things that influence the way you have come to teach? --has mentioned: colleagues (Y, MB, TK); NCTM Standards; CMP; Ss (developmental level and listening to their ideas; also, where they will be going next year); own understanding of mathematics

2. Are there particular things that influenced this this past year (working with that particular class or things that happened that influenced it (e.g. high school))?

3. Some of the things that I've noticed that you do include:

--recreating stories
--crediting students
--CGL
--Explicitness about where Ss are supposed to be
--using "they" (Who's 'they'? Who do you think Ss perceive 'they' to be? Why use 'they'? How does 'they' relate to you? Do you use 'they' to refer in this manner when teach science? Why/why not?)

-Have you noticed that you do this?
-If you were naming this strategy, what would you call it? (e. g. engaging students, getting students to listen)
-What is it you are hoping this strategy will do in your classroom?

4. How do these strategies tie back with the things that you said have influenced the way that you teach?

Josh's Interview #4

PART ONE—follow up questions from last interview

1. When thinking about your own preparation for teaching and then working throughout the launch, explore, summarize of the lesson, what does your knowledge of the subject matter help you do as you teach? How is a strong mathematical background helpful to you as a teacher?

2. How did the MGMP's "teacher talk" influence your teaching? Examples?

3. In a couple places in our last interview, you mentioned slight changes and refinements in your teaching which came after the initial big step of being involved in MGMP and CMP. Could you tell me more about these slight changes and refinements? What do you think these slight changes and refinements have been? Have they been more a matter of changes in how you think about the subject matter? How students interact? How you interact with students? Your expectations of your students? EXAMPLES?

J's responses from last interview:

I think, you know the MGMP is so closely aligned with the CMP, like I said that was the real big step and you know if I could look back, 10 years when I first started teaching the MGMP and then look at when I was teaching with CMP last year, **would there have been some changes? Yeah. I mean probably not incredibly drastic changes**, but I think the way I, what I expected of the kids in group situations maybe changed a little bit. (**EXAMPLES**?) And, how I interacted with the students in the group situations probably changed a little bit (**EXAMPLES**?) .

> I think it was, I said it was a gradual process, but I guess that now that I think back, it really wasn't as gradual—there was that real big bump when I went from AW to, and that was more content driven, I understand that, but it grew with MGMP. It was such a, such a leap um, that was the big step. And **I think from there, it's been more a matter of refining**.

4. You mentioned the Math project involvement in our last interview. Could you tell me a little more about this?

--What kinds of activities were you involved in? Examples? You mentioned "going over units". What did that entail?

--Did MB influence your teaching at all? How?

--What did you think of the discussions that were held?

--Did these experiences influence your teaching at all? If so, how? Examples? If not, why not?

PART TWO

Watch video of using balances during Say It with Symbols. Ask Josh to talk about:

--the mathematical ideas/procedures being taught

--the teaching strategies that he is using

--the goals that he has for his students—what he wants them to learn about the mathematics and about working in the classroom environment he has established (ASK MORE GENERALLY ABOUT HIS EXPECTATIONS OF THESE BY THIS POINT OF THE SCHOOL YEAR)

ENDING QUESTION: (if I have time)

I'm assuming that throughout the piloting and working with the two new teachers that you have hired in the last two years that you've also worked with and observed Karla teach. Could you describe how you see her classroom as being similar and/or different from yours?

Karla's Interview #4

PART ONE:

Finish last interview!

PART TWO—ask about these as follow up to the first part of the last interview:

1. When you focused on the word "discourse" in summer workshops:

--What does the word mean to you?

--What about discourse did your focus on in the workshops?

--How did studying this affect your teaching? Ex?

2. Question I asked:

So, what kinds of things do you think those kinds of questions do for kids?

Your response:

Puts more responsibility on them. It makes more of a conversation instead of I'm the one talking, write it all down, once in a while, say something. It **builds an actual true conversation between participants.**

Why do you think this is important? Do you think it affects your classroom environment? Why/why not?

3. In our last interview, you talked a little about the textbooks. You said:

We started with statistics with Use Numbers and then M's unit for fractions and fraction sense and since fraction sense was really the, the curriculum was slightly different, but the WAY that you were going to use that curriculum was going to be completely different, we had to add peer coaching. Because, M and Y knew that without it you could still go back with that and teach like you did if you had AW.

We couldn't find anything we put our hands on, especially if you weren't a math person. To ask them to think differently about it, it wasn't going to happen with AW.

In each of these, it sounds like you're describing a connection between how someone teaches and the textbook that they use. How do you think the textbook influences both WHAT and HOW you teach? Do you think it's similar/different for someone who has a strong math background vs. someone who doesn't? How?

PART THREE:

Watch video of using balances during Say It with Symbols. Ask Karla to talk about:

--the mathematical ideas/procedures being taught

--the teaching strategies that she is using

--the goals that she has for her students—what she wants them to learn about the mathematics and about working in the classroom environment she has established (ASK MORE GENERALLY ABOUT HER EXPECTATIONS OF THESE BY THIS POINT OF THE SCHOOL YEAR)

ENDING QUESTION: (if I have time)

I'm assuming that throughout the piloting and working with the two new teachers that you have hired in the last two years that you've also worked with and observed Josh teach. Could you describe how you see his classroom as being similar and/or different from yours?

APPENDIX H

STUDENT VIGNETTE FOR TEACHER INTERVIEWS

Vignette for Interview #1
Payson Middle School
11/16/99

Vignette #1

After completing Investigation 3.4 in Moving Straight Ahead (on standard form of a line), you have assigned some ACE problems for students to work on in small groups. As you are moving around the room, you see a group of three students who are trying to find the y-intercept of a given equation. Each student was suggesting that they use a different representation and they were disagreeing about which one would work to find the y-intercept. Student #1 was using the graph. He quickly located where the line intersected the y-axis and pointed to the graph, telling the other two students that they needed to find where the line started. Student #2 pointed to the equation and said that he used the number that there was no x attached to. In pointing this out, he mentioned that the first equation that they had done didn't have a y-intercept because you weren't adding anything to the x. Student #2 listened to what Student #1 was saying and agreed with him, so he ended up using the graph and answer that Student #1 had given him. There was no discussion between these two students, except an exchange of answers. In the midst of this, Student #3 mentioned that they could use the table and when the other two students told him he couldn't, he appeared to disengage from the conversation. Even though Student #3 wasn't talking to the other two, he continued to work on his graphing calculator. He entered the equation and then began to scroll on the table (away from where $x = 0$). Student #2 told him that he was going the wrong way because he needed to know where x was zero. Student #3 said "Oh, that's easy" and wrote the number down on his paper.

APPENDIX I

KARLA'S TIMELINES

Karla's Timeline
October 15, 1998

Main Idea of the Lesson: Data collection

Organization Time	Goes through ACE problem Number 1 as a class.	Investigation 2, p. 26.	Class discussion about the data on the board.	Small groups work on assigned Homework.
	Karla hands back the Check up. She talks about students grades (Example K-29) and then they go through the answers to the problems (Example K-21, K-22, K-32). Students are called on to give answers and describe methods of solution (Example K-9, K-11, K-23, K-24, K-34, K-39, K-40). Karla tells them some of the common mistakes and reminds them of events that happened during the quiz (Example K-7, K-10, K-33). She concludes by telling them what she expects in terms of the content from this point on (Example K-30)	Karla asks students to open their books to page 26. She describes the experiment that they will be doing in terms of the last bridge problem they did. A short discussion about what they think will happen when they change the length of the bridge (Example K-14). Karla talks about the accuracy of data collection in experiments in terms of what happened during the last bridge experiment. Students collect data in small groups and record their findings in a large table Karla has drawn on the white board.	Karla begins the discussion by pointing out how their accuracy improved in their data collection. She tells them where there were still problems (e.g. when people were considering the bridge "broken") (Example K-8, K-15). They decide whether they should take the mean, mode or median to help account for errors (Example K-26).	Karla asks them to find the mean average of the data on the board (Example K-31, K-35). Additionally, she assigns letters C-E on page 27 for

Karla's Timeline
October 20, 1998[1]
Main Idea of the Lesson: Modeling nonlinear functions

Organization Time	Grades ACE Problem 1 As a class.	Water Problem, p. 40.	Class discussion about the data on the board.	Small groups work on assigned Homework.
	Problem 1 is about Betty's Bakery. Karla asks them what each plan would do for raising cookie prices (ExampleK-25). This leads to a discussion about whether going up by 5% is the same as going up by 5 cents (Example K-1, K-3, K-4, K-5). They talk about whether each plan is linear or curvy and which plan they would choose. They discuss whether predications are accurate or not. Students grade one another's papers and hand them back.	Karla tells them they are on p. 40 She reminds them of some of the modeling activities they had done (Example K-16) and students tell her some they remember. Karla demonstrates a water experiment in which she pretends to pour water from one beaker to the next (Example K-17)—half of each amount goes into the next beaker (but she doesn't label how much water is in the beginning). She then tells students the first beaker has 32 ounces and then asks them how much water would be in each beaker as she pours the water from one to the next.	Karla records the number of ounces from each pour on the board. Students discuss whether they think the data is linear or not (Example K-12, K-37). Karla has students talk her through graphing the data. They discuss what to use for the scales on the x- and y- axis. After they graph the points, Karla asks them if they should connect the points or not. This leads to a discussion about continuous and discrete data (Example K-2, K-27, K-36). The discussion ends after they talk about whether the graph will cross the x-axis or not (Example K-13).	Karla assigns ACE problems 4-7 and describes what they will be doing in the problems (Example K-6, K-28, K-38). Students work in small groups on these problems until the end of the hour.

[1] In these timelines, two examples are not included. These are Example K-18 and K-20. They were from October 13, 1998 and were chosen because they were excellent examples of the functions I was describing.

APPENDIX J

ACE PROBLEMS 1 AND 3: BETTY'S BAKERY

1. Betty's Bakery sells giant cookies for $1.00 each. This price is no longer high enough to create a profit, so Betty decides to raise the price. She doesn't want to shock her customers by raising the price too suddenly or too dramatically. She considers these three plans:

- Plan 1: Raise the price by $0.05 each week until the price reaches $1.80.
- Plan 2: Raise the price by 5% each week until the price reaches $1.80.
- Plan 3: Raise the price by the same amount each week for 8 weeks, so that in the eighth week the price reaches $1.80.

    a. Make a table for each plan. How many weeks will it take the price to reach $1.80 under each plan?
    b. On the same set of axes, graph the data for each plan. Compare the shapes of the graphs and what they mean in terms of the changing cookie price paid by customers.
    c. Are any of the graphs that you drew linear? Explain.
    d. Which plan do you think Betty should implement? Give reasons for your choice (p. 41).

3. Since Betty raised her prices, cookie sales have fallen. Betty calls in a business consultant to help. The consultant suggests that Betty conduct a customer survey. Betty's customers are asked which of several amounts they would be willing to pay for a cookie. Here are the results:

Price	$ 1.75	$ 1.50	$ 1.25	$ 1.00
Customers willing to pay this price	100	117	140	175

a. Make a graph of these data, and draw a straight line or a curve that models the trend.
b. Use your graph model to predict the number of customers who would be willing to pay $1.35 and the number who would be willing to pay $2.00.
c. Do you think predictions based on your graph model are accurate? Explain.
d. The shape of this graph resembles the shape of another graph you have drawn. Look back at your work in this unit. Which situation has a graph similar to this one (p. 43)?

APPENDIX K

JOSH'S TIMELINES

Josh's Timeline
October 6, 1998

Main Idea of the Lesson: Drawing Graph Models and Writing Equation Models

Organization Time

Students put up data for bridge Problem and discuss it.

Students verbally give Josh their data and he records it on the board. Josh asks about the consistency across groups in collecting data (Example J-7). Josh then has students figure the averages and inquires about why they took the average of the data(Example J-7(con't)).

Class Introduction to graph models

Students open books to p. 7, Problem 1.2: Drawing Graph Models. Josh instructs them to graph the class averages. He then leads a discussion about the graph and the scale used to graph the data (Example J-2, J-8). Josh asks if students think the graph is linear or not and they discuss this (Example J-11). Josh calls on Corey to read p. 7 from the textbook (Example J-23, J-27, J-28). He emphasizes "graph model" by rereading part of the text. Students draw in graph models on their own paper and then Josh calls Christy up to the board to share her model so that they can discuss it (Example J-12, J-16, J-29). Student use this graph to predict a value for a bridge that is a thickness of nine (Example J-9).

Handout 1.2- Practice drawing graph models and Class discussion

Students work with seat partner to draw graph models in for nine sets of data (Example J-24). Josh then calls up individual students to put his/her graph model on the overhead projector (Example J-17). While going through the graph models, Josh revisits the definition of graph model, emphasizing that students must be able to extend and predict from the model (Example J-18, J-19, J-22).

Read and discuss Problem 1.3-find equation models and discuss as class

Josh calls on Jennifer to read Problem 1.3: Finding an Equation Model (Example J-28). He then asks how the students in the text generated their equation (Example J-25). Students use graphing calculator to investigate the given equation ($y = 8.7x$). Josh asks them to generate an equation for their own graph model and records these equations on the board. Josh leads a discussion about the equations that students generated (Example J-21, J-26). Josh figures out his equation from the model that he has drawn for the class. He then suggests that they look at the table for their equation to see how the numbers compare with their actual data. They decide if the data fit is reasonable or not (Example J-5, J-20).

Josh's Timeline

October 13, 1998

Main Idea of the Lesson: Introduction to Nonlinear Modeling

Organization Time (hands Back Check-up)

Goes through Check-up

Introduces Check-up by discussing what students "should" know by now. Some highlights of discussion about Check-up: doubling is a pattern (Example J-4), but not a linear one; how to tell if something is linear or not (from the table and graph)(Example J-13); writing the equation for a line (from two points and then finding rise, run, slope and y-intercept)(Example J-3, J-6, J-10, J-14, J-15).

Read Investigation #2 and discuss experiment

Corey reads Investigation 2 on page 26 entitled "Nonlinear Models". Discussion about which is stronger (and why), longer or shorter bridges. Josh describes the bridge activity and relates it to the one they did in Investigation 1 (Example J-30).

Students collect data in small groups

Students work in small groups to collect their data. They then record their data on the board in a large table.

Work on Problem 2.1

With 15 minutes left in class, Josh instructs students to complete problem 2.1 a, b, and c. He then tells them to get an average for the data that the class has collected.

APPENDIX L

PROBLEM 1.2 FROM *THINKING WITH MATHEMATICAL MODELS*

PROBLEM 1.2

Drawing Graph Models

A class in Maryland did the bridge-thickness experiment. They combined the results from all the groups and found the average breaking weight for each bridge. They organized their data in a table.

Thickness (layers)	1	2	3	4	5
Breaking weight (pennies)	10	14	23	37	42

The class then made a graph of the data. They thought the pattern looked somewhat linear, so they drew a line to show this trend. This line is a good *model* for the relationship because, for the thicknesses tested, the points on the line are close to points from the experiment.

[graph of the data is given]

The line that the Maryland class drew is a graph model for their data. A **graph model** is a straight line or a curve that shows a trend in a set of data. Once you fit a graph model to a set of data, you can use it to make predictions about values in between and beyond the values in your data.

Problem 1.2

A. Draw a straight line that seems to fit the pattern in the (thickness, breaking weight) data you graphed in Problem 1.1.

B. Based on your graph model, what breaking weights would you predict for bridges 6 layers thick and 7 layers thick?

C. Suppose you could use half-layers of paper to build the bridges. What breaking weights would you predict for bridges 2.5 layers thick and 3.5 layers thick?

APPENDIX M

JOSH'S TRANSCRIPT FROM OCTOBER 6TH

October 6, 1998

Josh's Classroom Transcript

Initiation	Response	Feedback/Evaluation
(begins with Josh taking a long turn in which he talks about the quiz that he just handed back) **TRS: Getting Data Back on the Board** Let's go ahead and take a minute (3 second pause) and get this data back up on the board. You were supposed to copy this down and then average it. U::h, let's get these groups up here real quick/ Somebody give me their group's data// Kara, what'd you have?	Kara: 4, 10	
I can't hear / 4, 10?	Kara: 16, 20, 36	
Another group/ Andy?	Andy: 4, 11, 21, 24, 32	36? Okay
Roger?	Roger: 6, 11, 14, 27, 34	Okay
Another group. I think there was—what was there, six groups?/ Jessica?	Jessica: 6, 11, 23, 27, 30	27, 34/ Okay, keep going//
Kyle, what'd you have for your group?	Kyle: 3, 7, 9, 16, 30	'Kay?/
	Amanda: 6, 14, 17, 25,	
Anybody I'm missing? Amanda, your group? The last one was? I couldn't hear you.	Amanda: 35	35?
Andy: Mr./ Manzini. It's 32, not 30		
For which one?	Ss overlapping: For 37	

37, 32?	Ms: Yeah	Thanks. Okay, so we had 6 different group's data here and um,
TRS: Why the data is inconsistent What- you were supposed to average them/ Now, before you average them/ take a look. How consistent/ were we/ the results?/ What would you say?/ Just by looking at it, was there any consistency in your data? Ms: (inaudible)		
I mean, as far as from one group to the next, do yo::u// notice anything?	Kristen: It pretty much stays the same.	Pretty much stay the same?/ Let's see (following the data on the board with his hand) 4 six's/ a little low, three 6's/ 10 and 11's primarily/ although, there's a 7 and a 14?/ This (at the 3rd bridge, one group had 9 which was much lower than the rest) one i::s/
This one kinda stands out, doesn't it?	Ss: Yeah	
Kinda away from the rest of the data?	Ms: that's (inaudible)	
Um/ these are pretty consistent, again this group/ for some reason had// some pretty lo::w numbers until they got to the::/ breaking part of [inaudible—Ss coughing] these I think were actually/ were very consistent/ um/ I wouldn't expect it to be all exactly the same// Why		

not?/ Why did we get all, why didn't all the groups get the same data for each/ and every layer? Andy?	Andy: 'Cuz, like, maybe, like, they didn't have an exact / inch or something.	Yeah, there's okay (overlap)
	Andy: They might have (inaudible)	That might have affected/ now I tried to make sure you guys were only having a one inch overlap/ but// as I was walking around some of you maybe didn't control that very well/
Anything else that might affect the data?	AJ: People may have used the same paper over and over again and the paper got weaker	You don't think so?/ Did you hear what he said?// Say that again, AJ
	AJ: That group probably ...(inaudible)	
Why? / What makes you think that?	AJ: Well, maybe they got the paper...	Okay. So, once it snapped/ the paper was weakened/ so it doesn't hold as many pennies when you use it?
Andy: It looks like that group dropped it from higher or something, put like more pressure on it		//I don't know (quietly)/ I don't know (louder)/

TRS: Looking at the averages Well, let me ask this question before we look at the averages/ Um/ or, (faster) let's go ahead and look at the averages first. What'd you guys get? What was the average breaking weight fo::r a bridge with thickness of o::ne layer? / Codi, what'd you find?	Codi: I got 4.8	Four point/ eight?
And we want to round to the nearest penny?	Andy: Yeah	So, approximately five?/
How about fo::r// two layers thick/ Andy, what'd you find?	Andy: I got 8	Rounded/ 8?
(faster) If anybody disagrees say something 'cuz I didn't do this/ How about for three? What'd you get?	Lindsay: I got 10.6 Ms: That's what I got	
You got what?	Lindsay: 10.6	10.6?/ Eight doesn't look reasonable, does it? // Because there's a seven and everything else is quite a bit higher (3 second pause)
Kara, what'd you get?	Kara: 10.6	10.6?/
Anybody agree?//	Ss: I agree	10.6, which is gonna round to::/ 11?/
How about fo::r three layers? (3 second pause) What'd you find for three layers? (faster) Guys, if you didn't do this like you were supposed to last night, do it now.// Three layers, what'd you find? Jordan, what's the		

average?	Jordan: Uh, I didn't get it	Didn't get it? Why not?/ No calculator at home?
Eric?	Eric: 33.1	For three layers? Doesn't look right//
	Eric: Oh, I/ was at the next one/ um/ 16.5 Fs: 16	16.5? (3 second pause)
Does it look// so, it's gonna round to 17?	Ss: Yeah, yes	If you got a 16.5?
Agree?	Ss: Yes.	
How about/ uh/ four layers?/ Codi, you did this.	Codi: Uh, 23	23? /
	Fs: Yeah	
I hear a yeah. How abou::t for five layers?	Carrie: 33	33? Carrie, that's what you got for the average? You rounded to the nearest penny?/ Okay.

APPENDIX N

EXTENDED TRANSCRIPT FROM KARLA'S CLASSROOM

The excerpt I have chosen to illustrate students' mathematical engagement in Karla's classroom occurred in March of the school while students were working in *Say It with Symbols*. I chose this instance because it is fairly representative of the type of discussions that took place whenever the class was "exploring[1]" something. In this unit, students have covered: 1.) order of operations, 2.) equivalent expressions (using an area model, which introduces the terms "factored form" and "expanded form"), 3.) algebraic properties (i.e. distributive, commutative), and 4.) writing quadratic expressions. During the section in which this observation took place, students had just finished solving linear equations and were beginning to solve quadratic equations. While the class actually spent over two class periods on this topic, I have chosen one "brief[2]" segment of transcript to illustrate the discussion that took place.

Prior to the transcript below, Karla had students using the tables and graphs on the graphing calculator to find where y = 0 (or the x-intercepts or solutions) for quadratic equations. They went through a couple of these as a class. Students shared how they found the solutions using the graph and/or table, depending on which representation they found most helpful. During this discussion, Mark told Karla that he could find the x-intercepts using just the expanded form of the equation.

Example K-42: Exploring Solutions to Quadratics Using the Expanded Form of an Equation[3]

K: [...] Mark has now thrown something out on table- thinking about, is there a way to do without the table and graph since the point of this unit, Max, is symbols. Mark, your idea was?

> Mark: That the 5x is, you, if you make that just the opposite, then that could be one of your y –intercepts. If it was –5x then that would be five.

K: So, you're wondering if maybe there's some way of looking from the equation, you could have gotten this [the solution]. [writing on the board] Dale?

Dale: In the $x2^4$, since there's like no number [meaning the coefficient of the term was one], you can kinda assume it's zero, zero.

K: [writing on the board] Since there's no number, you're assuming then that [one of the x-intercepts] might be zero?

Dale: Yeah [inaudible] equation x2 that's [inaudible]

K: Can you say that again?

Dale: When it's like y = x2, just that equation, that's the only spot where it went [inaudible]

[From fieldnotes: Dale is adding the fact that there needs to be another x-intercept (in addition to the one Mark just gave). He knows this from his experience of working with the graph of y = x^2].

Mark: It would have to be [overlapping-inaudible]

K: Oh, yeah. We graphed this one a lot, just the basic old parabola [i.e. y = x^2]. When it was just that x-squared, it did cross at zero, zero. That's the one I made you graph by hand before we got the graphing calculators out again. Hhmm [pause] Let's try some more and see if Dale and Mark are on to something here.

Without Karla eliciting the suggestion, Mark offered that he had a way to find the solutions without using the graphing calculator. Mark grounded his idea in the equation y = x^2 + 5x and stated that the opposite of the 5x gave a solution. In doing so, Mark stated that one of the "y-intercepts" would be the opposite of "5x." Technically, he should have used the term "x-intercepts" and the solution should have been the opposite of 5, not 5x. However, since students were in initial stages of trying to make sense of solving quadratics, Karla may not have thought it was an appropriate time to be concerned with the language because she was trying to understand Mark's ideas. Karla recast what Mark said, making the statement more general—what he was actually doing was trying to figure out a way to get the answer from the equation. Dale offered another, related, idea. He remembered manipulating the equation y = x^2 and that there was also another solution. That solution was (0,0). After Karla asked him to "say that again," Dale reminded the class that when the equation was y = x^2, the only x-intercept was (0,0), but now because there was something being added on (i.e. + bx, where b = Z+), they needed to consider that there might be a second x-intercept of (0,0). Karla reformulated what Dale said and then ended with a drawn out "Hhmm… Let's try some more and see if Dale and Mark are on to something here." Essentially, consonant with her use of "could," she seemed to be inviting students to explore possibilities. Charity became excited and exclaimed:

(Continued)—Exploring Quadratics of the Form x^2 + bx

Charity: They are! Ms. Delmont, they are.

K: What? Why are they on to something?

Charity: Cuz I just tried the same thing but with 3x and it's the same thing

K: So, let's go to our y = and let's try Charity's [x^2 + 3x] … [brief discussion about this being a parabola which indicated that the function was a quadratic] …With that in mind, Mark and Dale, tell me where you think it's gonna cross before I look at it.

Mark: -4x and zero, zero. [Mark continued by saying that he had gone on to another and then answered the original question]… 0,0 and 0, -3

Ariel: Ms. Delmont [inaudible] zero, zero [inaudible] squared

K: That's what Dale's suggesting. We've got just a plain old x-squared. Dale's suggesting it's gonna cross at zero, zero. And Mark's saying it's gonna do the opposite of this one [points to the coefficient "b" in the equation], so we're guessing, zero, zero and negative three, zero? And, Mark, you're also trying this one?

[…]

[Mark told her he had already done that one and moved on to x^2 – bx-- Karla stayed with the x^2 + bx form and had students check their guesses on the graph and table of their graphing calculators. Students confirmed that their guesses were correct.]

In this segment of discussion, Charity told Karla that the two boys were, indeed, on to something because she tried another equation of that form and their method worked for that one, too. Karla told the class that they were going to start with Charity's example and asked Mark and Dale to tell her where it was going to cross before they actually put the equation into the graphing calculator. Mark appeared to give the wrong answer, but then clarified that he had already moved on to another problem of that same form. He told the class that he did get -3 and (0,0) for Charity's equation and when he did x^2 + 4x that it worked there, too. Mark then told her he had moved on to another form, x^2 – bx.

Karla appeared to ignore Mark's last comment as she went back to the first form (i.e. x^2 + bx). She asked students to use either their graph or table to check their answer. Students confirmed that the answer was correct. At that point, Charity indicated that, like Mark, she had begun to explore the next form of quadratics. This became the next topic of discussion:

(Continued)—Exploring Quadratics of the Form x^2 – bx.

Mark: If you're adding a negative x-
Charity: If adding negative x, x will be positive
Mark: Ms. Delmont-
[other students talking, too-inaudible]
K: Tell me about negatives, Charity?
Charity: If you add-
[K interrupted and asked Kevin to be on task]
Charity: If it's a negative x, it's gonna be a positive.
K: Give me an example so I know what you're-
Charity: Um, x2 + -3x.
K: And, where is this one gonna cross?
Charity: Um, when y is at zero, it would be at 3, cross at positive 3
K: At positive three?
Charity: Yes.
K: What about this x-squared idea that Dale gave us? Is it still at zero, zero?
Charity/Mark: Yeah/Mmhmm.
K: Let's see if it does. Charity's probably right. I just inserted a negative and left everything else the same [in her equation on the overhead graphing calculator]. It's still a parabola?
Fs: Yeah
Max: You can't see until you graph it.
K: Charity thinks it's gonna cross at zero, zero and positive three.
Barbie: It does!
K: Looks like it. I'm going to check it on my table just to be sure.
[Students talking]
K: Hhhmm, it does.

Charity began this segment of discussion by telling Karla that "if it's a negative x, it's gonna be positive." Karla asked her to ground her claim in an example, so that she would have a better idea what Charity meant. Charity offered an example and then gave one of the solutions, positive three. Karla returned to Dale's idea and asked if zero, zero was a solution, too. Both Charity and Mark offered an affirmative answer. Karla used the overhead graphing calculator and checked the solution on both the graph and the table. Students agreed that their solutions were correct. Mark then moved the conversation to another case:

(Continued)—Exploring Quadratics of the Form –x^2 + bx

> Mark: If you have like a negative x-squared plus just a positive 4x, it's gonna be a positive 4 and zero?
> Max: Then why is it a zero with the x squared?
> Mark: Plus 5x, it'll still be a positive 5x

K: It just doesn't do the opposite?
> Mark: Mm-mm.

K: Let's look at that one
> Max: Could you just write down what she just said?
> Charity: It's going to be an upside down, like thingymajigger
> Ms: Negative [inaudible]

K: What do you mean, an upside down thingamajigger?
> Charity: Upside down [motions with finger]
> Barbie: Parabola?
> Charity: Yeah.
> [others talking, inaudible]
> Mark: A hill thing.

K: How do you know it's gonna be a hill thing?
> Barbie: Because-
> Mark: [inaudible]
> Charity: [laughs]

K: Yep, like this hill thing?
> Fs: Yep

K: They did something different, though.
> Fs: It crosses at zero, zero and-
> Ms: [inaudible]
> Charity: Zero.
> Kevin: The table, look at the table, it's way different.

K: Tell me about the table
> Kevin: It's like way different
> Fs: Yep
> Darla: So if it's a negative x-squared, then [inaudible]

By this point, Karla has had students make guesses about solutions from various forms of quadratic equations and then check them on their graphing calculator. She repeated this pattern with the next quadratic case that Mark offered: that is, the case –x^2 + bx. Mark offered this by asking Karla if the solution was going to be positive this time, instead of the opposite like they had found in the other cases. Karla did not offer an explanation. Instead, she proposed a question that made Mark's claim more general: that is, instead of thinking about the solutions only in terms of whether they were positive or negative, she pushed them to think about the solution as no longer being the "opposite" of the coefficient. She told students to "look at that one." Charity noticed right away that the graph was different—it was now an "upside down thingymajigger." Karla asked her to clarify what she meant. Charity used her fingers to motion the shape and another female student offered the word "parabola." Charity confirmed that that was what she had meant. Mark re-named the shape as "a hill thing" and Karla used this phrase in her next question. This word appeared in past class observations to indicate one type of quadratics (i.e. those with negative x-squared terms). This may have been the reason Karla appropriated this instead of "parabola," or she may not have heard the female student offer "parabola." Karla then turned the conversation to organizing all the cases that they had presented so that they could discuss them:

(Continued)—Organizing and Exploring the Cases

K: I think I gotta figure all of this out. Let me write down what you guys found. [overhead screen is put up so she can write on the board] It's kinda all over the place, let's try to put it all together, what we've got. Let's see, how about we look at the ones that had positive x-squared plus a positive x and find out what we've got

Charity: Zero, zero and the negative

K: An x-squared and 3x, x-squared and 4x, x-squared and 5x [writing on board]. I think those are the ones that we looked it, isn't it?

Ms: Yeah

K: And when those were equal to zero [writing-sets all three equations equal to zero], the x-intercepts, we always had a zero, zero [writing]. Why did we think we were always going to have a zero, zero?

Dale: Because of the x-squared.

K: Because of the x-squared? [writing] And we already knew the x-squared was gonna cross right there [writes intercept]? And then Mark, you started us out thinking about this one [the examples where the equation was of the form x^2 + bx]. How did you get those negative 3's, negative 4's and negative 5's?

Mark: Um, if it says negative, if it's x-squared plus negative?

[Mark was working on his graphing calculator and appeared not to be paying attention to what Karla was doing. Karla briefly let Mark know which case it was that they were discussing at that moment.]...

K: What did you say would happen?

Mark: The, one of the y-intercepts, or the x-intercepts, would be at the opposite of, like if it was a positive, like x, positive number and x, it'll be at negative that number x.

K: [writing on board] Opposite of that coefficient of x?

Mark: Yeah

[...] [one student defined "coefficient" for another]

K: Then Charity started looking at this

Fs: [inaudible]

K: Another group we might say, was x squared plus a negative coefficient in front of x? And what did we find out there for the x intercepts? [writing on the board] Am I going too fast?

Ss: No

K: I'm just- you guys came up with some interesting things that kept happening, but I had it all over the board and I needed to pull it all together somehow to see what we found. Charity started looking at what if we didn't add a positive x number for x squared, what if we had a negative x number? And, Charity, what did you find was happening to the x intercepts that way?

Charity: Um, well, it's gonna be zero, zero and then, um, it's gonna be, positive, like, it's not gonna be negative numbers

K: So kinda like Mark's thing over here, too?

Charity: Yeah

K: It's gonna be the opposite of that? This again was because of the x-squared. [writes (0,0) on the board] And then we found this. [writing on board] But then the weird thing that happened was when we threw a negative in front of the x-squared.

Max: It turned it over

K: Yeah. [student whistles] We only looked at one of those. [writes –x^2 + 5x = 0]. Max said it flipped over when the negative x-squared was there, the parabola became a hill.

Fs: Oh.

K: So, the ideas that Charity and Mark were bringing up with this opposite coefficient stuff fell apart right here. It wasn't the opposite of this coefficient, but the same.

Charity: Well, so that means the 5x would have to change, too because normally that, normally, in your intercepts it would be negative five, zero

K: That's [inaudible], you're right. Maybe it was just that one, what if it was like –x-squared and 6x?

Fs: Try it and see.

K: Try it and see?

[...]

Karla may have felt there were too many strategies being offered and decided to try to organize all of the suggestions. In doing so, she had students go through the type of problem they had discussed, the examples they had used, and the solutions they had found. Each time, Karla offered her own "paraphrastic interpretation" of what students offered, controlling the common knowledge of the classroom. This continuing discussion lasted for over two entire class periods. During that span of time, Karla (and some students) offered other forms of quadratics that included ax^2 + bx and ax^2 + bx + c (where a, b, and c were all nonzero integers). In following the textbook's lead and keeping in line with what students would need to know in algebra the following year, Karla tried to return to finding the solutions from the factored form. However, students continued to explore the solutions through using the expanded form of the equations.

What mathematical understandings do students seem to display? Using the parameters Pirie and Schwarzenberger have outlined, I will attend to: the focus of the discussion, the kind of language used, and the type of statements being made. The students have explored and discussed quadratic functions in a previous unit and again earlier in this unit. They have some understanding of these functions and this gives them something to talk about. The task that Karla has set before them is to think about particular points on the graph—not the entire graph. The points they are to focus on are when the x-values are zero which are the solutions or x-intercepts of the function. Students use some information that they already knew (e.g. that the equation y = x^2 has one x-intercept of (0,0)) and use their knowledge of technology (e.g. how to find points on a table or graph) to make inferences about the solutions from the expanded form of the equation.

The level of language varies quite a bit. Students seem to use the entire range of mathematical language that I introduced earlier in this chapter. Some of the OML students use included x-intercepts, negatives, x-squared, equation and parabola. In addition, Karla revoiced many of the student contributions and used terms like coefficient, parabola, and quadratic. She did not correct students who chose to use CGL. Some examples of this included the phrases "x2" and "hill-thing." When a student used a word that did not have as common a meaning as these two phrases (i.e. upside down thingymajigger), Karla pushed her to say what she meant. In response, Charity showed her using her fingers by tracing a parabola that opened upward in the air. Barbie then helped Charity name what she was talking about in OML: a parabola. The language early in the discussion is very much exploratory talk. Students are trying to figure out what is going on in the mathematics. When Karla turned to "figure all this out," student talk became more final draft. For example, Dale no longer used x2; he now used x-squared. In addition, Mark again made the mistake and said "y-intercept" for x-intercept. However, at this point he corrected himself and made his statement more mathematically correct. Karla again helped students make their contributions more mathematically correct and general when she revoiced Mark's statement, "if it was a positive, like x, positive number and x, it'll be at negative that number" to "opposite of that coefficient of x?" Karla also pointed out the discrepant event that took place when the squared value was negative: that is, the "opposite coefficient stuff fell apart right here." As the discussion ensued, students moved in and out of all levels of language. In some cases, some students lacked appropriate language. When they did, Karla sometimes revoiced making the contributions more appropriate and mathematically correct. In other instances, students offered the words or definitions. In other cases, students used ordinary language and mathematical language.

The type of statements being made were never "incoherent" (Pirie and Schwarzenberger, 1988, p. 467): that is, they did not appear to not make sense to other participants in the conversation. Some statements were "operational" or were about what to do or how to do it. Examples of these would include segments of transcript that were grounded in particular equations, rather than generalizing a rule. For instance, when Charity suggested Dale and Mark were on to something because she tried their method with another equation. This may have indicated that Charity thought she could generalize from just two equations. Also, Karla asked Charity to ground her idea "if it's a negative x, it's gonna be positive" in an example. This would be more operational. Other statements were "reflective" in that they offered explanations or attempted to move beyond the immediate task. Dan's suggestion that they needed to consider the solution (0,0) when the coefficient was one would be an example of this. He connected what they were doing to previous knowledge and suggested why that solution made sense. Also, students eventually[5] abstracted "rules" that they could follow for various forms of the equation. For example, when they discussed equations of the form y = ax^2 + bx, Mark said that they could take the opposite of the number with the x and divide it by the number in front of the x^2. Karla helped to connect this to the general form of the equation and showed them that they could write this as –b/a. I am unsure if they ever got to a point in their discussion, however, where they contemplated why this method worked. They did eventually connect these methods with factoring the expanded form to find the solutions.

Karla and I talked at length after each of the three class periods. Initially, she was quite surprised by the student suggestions. She had pursued student ideas and then tried to offer other cases if students had not considered them. She told me that she was not always sure what to do with the student contributions and had tried returning multiple times to the factored form of the equation because she knew that she was already more than two days behind Josh and did not want to fall further behind. In fact, according to the Teacher's Guide, this lesson should have taken only one day. Pacing was always an issue for Karla when she allowed students to explore their ideas.

Another issue Karla often had to confront during these discussions was that of her own mathematical content knowledge. During the class period that followed this Example K-42, I had written in my field notes: "I'm not exactly sure why Mark's method is working right now and I'm not having to orchestrate this discussion like Karla is!" The idea to which I was referring was Mark's suggestion that, given a quadratic of the form ax^2 + bx, all I had to do was "take the opposite of the x's number and divide it by the number in front of the x-squared." I returned to my office later that day and explored this by playing with the quadratic formula. I realized that Mark's strategy worked because when $c = 0$, the quadratic formula can be reduced to $-b/a$. However, students also went on to consider cases where c was not equal to 0. In that case, Karla told me she realized during the discussion that students were merely shifting the parabolas because adding/subtracting numbers caused the graph to shift. However, I spent more time looking through some algebra texts and realized that the shifts only took place when the equation was of a particular form: (ax + b)^2 + k. So, there was still a range of cases that had not been explored!

APPENDIX O

EXTENDED TRANSCRIPT FROM JOSH'S CLASSROOM

The segment of transcript that I chose from Josh's classroom is an instance of Josh teaching the same lesson as the one that was the focus in Karla's appendix. Again, this example is fairly representative of the way students were engaged with the mathematical content during the "explore" segment of the lesson. The mathematical goal of this lesson, like Karla's, was to solve quadratic equations. The transcript began at the beginning of the class period and students in this class have covered the same material as what I described in Karla's appendix.

Example J-44: Solving Quadratic Equations

J: [...beginning of tape...Josh is reviewing what they have been doing in this unit up to this point.] Those were all linear, equations, solving linear equations; taking two linear equations, setting them equal to each other and finding the value for x in which they're the same, right? We're doing this all symbolically. You've always had graphing calculators you can fall back on to help you make sense of things. Um, but there's a lot more than just linear. There's also [inaudible] quadratics. If you look at Problem 4.4, let's take a look at it. Would you read, please, where it says, "Solving Quadratic Equations"? It's on page 57. Would you read that please, Sandy would you read that please?

Sandy: [reads aloud]

J: Okay let's stop there for a second. (I-R-E) How do you know, first of all how do you know it's a quadratic equation[6]?

Ss: X-squared [multiple responses of this]

J: An x-squared. If I graphed it, I'm probably gonna get a parabola here. It's gonna go up and come down or come down and go back up. In the *Moving Straight Ahead* unit, we said a y-intercept was where x was zero, right[7]?

Ss: Mm-hmm

J: Where are the x intercepts for this? So, in other words if I set this equal to zero, if y [inaudible] over on this side because it's a y, it doesn't make any difference [referring to writing the equation either as y = the equation or the equation = y—pointing out that the order one writes it in doesn't affect the equation]. What are your x- intercepts? There might be one, there might be two or there might be zero. Use your graphing calculator. (I-R-E) Where are the x-intercepts? In other words, where is the parabola going to cross x-axis?

Alicia: It's in the book.

J: Pardon?

Robert: 0 and –5.

J: You looked that far ahead already [to Alicia]? Robert?

Robert: 0 and –5.

J: (I-R-E) How did you know that Robert?

Robert: Cuz I looked at zero and they're both zero and at –5, it's zero.

J: Okay, so, how many entered the equation into their graphing calculators and tabled it and looked to see what x was when y was zero? Now, if you didn't do that, obviously it's right in your book right there. (I-R-E) If you look at the table in your book, where are the two x-intercepts again?

Ss: Zero and negative 5

[They go through what the minimum, maximum and line of symmetry are using the graphing calculator and exploring the graph and table—Josh tells them that they know much more than they think. He also has them read that the x-intercept is the same thing as finding the "root" or the solution to the quadratic equations. Josh reminds them that the focus of this book is to do things with symbols and changes the topic to this subject at this point. Approximately 5-10 minutes of talk are omitted to get to the point where they are discussing solving quadratics.]

The class period began with Josh telling students that they have already solved linear functions symbolically. He reminded them that there were other functions that they had done that were non-linear that they were going to solve symbolically. To do this, he asked Sandy to read from the textbook. Sandy was asked to stop to make sure students remembered what determined a quadratic equation: that is, the equation had to have an x-squared term. Josh referred back to remind students what a y-intercept was, connecting this to the term x-intercept. Alicia seemed to understand that the textbook was a place to find answers as she told Josh that the answer to his question was in the book. Robert gave the answer for which Josh was searching. However, Josh first responded to Alicia, asking her if she had looked that far ahead and then called on Robert to respond.

Robert gave the same points he had mentioned earlier and Josh asked him to explicate how he knew that was the answer. Robert used the graphing calculator and described that these numbers were where the y-values were zero. Josh offered a positive evaluation to Robert (i.e. "okay") and asked how many others used graphing calculators to find the answer on the table. He referred back to Alicia's earlier comment and told students that they also could have seen it "right in your book right there." He then drew from the textbook and directed student attention to the table that was given there. Students replied by giving the same answer Robert had given[8]. Josh proceeded by having students talk about what then knew about parabolas, directing the focus to what the parabola looked like by inquiring about the minimum, maximum and line of symmetry. After they had gone through these, Josh asked Sandy to continue reading the definition of a "root." He moved the talk to focus on what he had defined the classroom activity to be earlier: solving quadratics symbolically.

(Continued): Solving Quadratics Symbolically—Writing Expanded Form in Factored Form

J: Okay, symbolic. Let's take a look at Problem 4.4. If you look on page 58, we're gonna look at another quadratic. In that, you look at--x-squared + 3x is an expanded form. Would you write that in factored form. In fact everyone should be able to do by now.

[Quiet overlapping talk while students worked on this]

J: If I was gonna draw a rectangle and the area of x-squared plus 3x, how could you do that [referring back to the area model they used to factor quadratics in the *Frogs* unit]?

[Ss talking quietly]

J: This is in expanded form? I'd like to have an equivalent expression in factored form. So, what could you factor out of both terms? (I-R-E) Who has [inaudible]. Jennifer, what did you do?

Jennifer: I, I, um, I know that x times x is x squared.

J: Okay.

Jennifer: And 3 times x is 3x, so put x + 3 in parenthesis.

J: Okay, how many have x times the quantity x + 3? [writes on the board] So, this is in factored form? (I-R-E) Could I draw an x by x+3 rectangle?

Ss: Yeah.

Josh began this segment by first redirecting the conversation to include what the focus was supposed to be (i.e. using symbols) and then he again referred to the textbook, indicating that the textbook was defining their classroom activity. He asked students to look at page 58 and the next quadratic they were going to solve. He reminded them this equation was in "expanded form" and requested that they write it in "factored form," telling students that this was something they "should be able to do by now" because it had been covered in the *Frogs, Fleas and Painted Cubes* unit. To aid their memory, Josh reformulated his question and referred back to the rectangle models that they had done when they learned how to factor quadratics in *Frogs*. He further reformulated his question by asking students what they could factor out of both terms. He called on Jennifer to describe what she had done. While Jennifer was answering the question, Josh offered a positive evaluation of her response to the first part of her solution and also to the second part of the solution. He drew the rectangle and again reminded the class that the dimensions of the rectangle gave them the factored form of the equation. The excerpt continued with Josh referring to the book again to define what "else we are supposed to do":

(Continued): Solving Quadratic Equations—Finding the Solutions

J: [Draws it on the board] Something like that? So I've got an x-squared piece and the 3x piece in expanded form? Okay? That's no big deal. What else are we supposed to do? It says, [reading from book] "find all possible solutions to the equation x^2 + 3x = 0." (I-R-E) Translate that for us, what are they asking for?

Sandy: The solutions.

J: The roots or the x-intercepts. In other words, I'm setting y equal to zero. What values of x, value or values of x give you a y-value of zero? Who can answer this for me? I don't care how [meaning they could use the graphing calculator or not], I'd just like an answer. What are the x-intercepts, what are the roots? Are there any? There might not be. You might, sometimes you get an equation where the parabola looks something like this and it never does cross the x-axis. PJ has an answer; Abram has an answer; Chuck has an answer. [naming all the people who have their hands raised] [long pause] How are you finding, Matt you know how to find the x-intercepts?

Matt: Umm.

J: Where are they? Alicia, Jennifer, Robert have an answer. (I-R-E) Carl, Jaylen, how? Jaylen, what are the x-intercepts?

Jaylen: Zero and negative three.

J: (I-R-E) How'd you get that, Jaylen?

Jaylen: I put it in the graphing calculator and found the zeros.

J: Okay, make sure you all can do that anyhow. (I-R-E) What equation did you enter into your graphing calculator?

Jaylen: I put x(x+3).

J: Which one?

Jaylen: [Inaudible—Josh pulled down the overhead screen]

J: So, you did the one that was in factored form?

Jaylen: Yeah.

J: Doesn't make any difference. Let's clear this out. And, if you don't mind, I'll do the first one, so x-squared plus 3x? Oops, try again.

Ms: You pushed the [inaudible].

J: Yeah, I know. X-squared plus three x. (I-R-E) Okay, so, you went to the table?

Jaylen: Yeah.

J: Okay, oh, geez. I used this one. (I-R-E) So, I'm looking for, what are you looking for, Jaylen?

Jaylen: The x-intercept is zero.

J: Okay. (I-R-E) Agree?

Ss: Yep.

J: The x-intercept is what x is when y is zero? So there's one, but—

Ms: Negative [inaudible]

J: There's another one, too, right?

Chuck: Yep

J: So, how many have the x-intercepts as being at –3 and 0? Okay, easy enough? [puts the overhead screen back up] What else were we supposed to do? We're gonna kinda walk ourselves through this problem. (I-R-E) What else were we supposed to do?

Jonah: It says find the y-intercept [inaudible].

J: Okay, looking at question b and question c, I'm a little confused.

Fs: [inaudible]

J: Aren't they the same thing? They're asking for all the possible solutions to that equation, um, and then they're also saying they're asking for the x-intercepts, isn't that the same thing?

Ss: Yeah.

J: Um, question d, though, says "which form makes it easier to find the roots"? Or the x-intercepts. What do you think? Is there one that makes it easier?

Abram: I can see it, well, in this problem--

J: (I-R-E) Does make any difference if you use your graphing calculator?

Fs: No.

Josh used the book text again to define what they were supposed to do: find all possible solutions to the equation. He called on students to "translate" what "they" were asking. By stating it in this manner, he first drew student attention to the fact that they should be able to take what the textbook said and translate it into their own language.

This may have given the impression that students need to be able to take textbook language and interpret it into their own. He possibly further indicated these different ways of talking by referring to the textbook as "they." Sandy, one of the more successful students in class, offered that they were looking for the solutions to the problem. Josh revoiced her response by stating the other two ways of referring to the solution: the roots or the x-intercepts. He then outlined the process one would go through to do this—"setting y equal to zero, what values of x, value or values of x give you a y-value of zero?"

Even though Josh had stated earlier that they were going to use symbols now, he told students that he did not care how they answered his question. In other words, they could use a graphing calculator if they had wanted. He may have thought they were not ready to move to symbols at that point and wanted more students to be successful doing what he asked. Matt may not have appeared to be attending to the question as Josh called on him and he responded with only "Umm." Josh continued by pointing out who *did* have an answer, calling out the names of students who had his/her hand raised. He chose Jaylen to give the answer and Josh requested an explanation, which Jaylen also supplied. Jaylen had used the graphing calculator. Josh evaluated Jaylen's answer by responding "okay," but may have indicated that he would have liked them to use symbols when he said "make sure you all can do that anyhow." This seemed to send the message that this was the "least" of what they would be doing. Josh had Jaylen detail the process he went through on the graphing calculator and reminded students that on the graphing calculator, it did not matter whether they used the expanded or factored form. Josh went through the process on the graphing calculator and then did a quick check of how many people came up with that answer. He again referred to the textbook using "they" and asked students what they were "supposed" to do. When he read question d., he asked students "which form makes it easier to find the roots?"

(Continued): Solving Quadratic Functions—Without Using the Graphing Calculator

J: It doesn't make any difference, does it? [inaudible] into your y equals, table it and look to see what x is when y is zero. But how about if you don't have a graphing calculator to use?

PJ: I'm dead.

J: We touched on this a little bit in the *Frogs* unit. See how good your memory is. Do you remember something?

Fs: [inaudible]

J: I know it was a long time ago, over a month. Alicia remembers something; Sandy remembers something; Chuck remembers something. Chuck, what do you remember?

Chuck: Uh, what you need to get to get zero at y, the three is zero and the x is zero

J: Hold on a second. (I-R-E) First of all, I know Jaylen said the x intercepts are- what were they –3 and 0? [writing on the board]

Chuck: Yeah.

J: Or negative three, zero and zero, zero as far as points are concerned. (I-R-E) Chuck, what did you, say what you said again, what'd you say?

Chuck: Uh, what you need to get to zero [inaudible] you got the three and the x

J: I-I think I understand what you're saying, but I'm not sure anybody else does. What you need to get to zero, okay. Do you understand what he's saying? (I-R-E) Jonah, do you understand what he's saying, "what you need to get to zero"?

Jonah: Not really.

J: Okay, I didn't think so. I don't think most people do. Chuck. Can you clear it up, what do you mean, "what you need to get to zero"?

Chuck: The number you need to get to zero. Like 3 times what to get to zero. Three times zero is zero and negative three, zero is zero

J: What are you looking at, the expanded form or the factored form?

Ms: I have no idea

Chuck: I looked at the expanded form

[Other students talking]

J: For both of them [to Chuck]? Okay. (I-R-E) Mindy, what do you think?

Mindy: [inaudible] think about what number you take times to get to be zero [inaudible] whatever the three times what equals zero.

J: [inaudible] for expanded form, I didn't hear that?

Mindy: Like the number three times whatever [inaudible].

J: 'Kay. But, what if didn't know what the x-intercepts were. How could you solve one of these questions for zero? Now, symbolically you could solve- because of the linear problem, we could solve it, right? We could solve it symbolically?

Ms: Yeah

J: How could you do that here? (I-R-E) You do an x-squared times zero?

Sandy: In factored form, um, in the parenthesis, x times zero, or x + 0 is zero and then x + -3 equals zero. So you need to use the negative or positive of that number.

J: It's a multiplication problem, isn't it? You're multiplying x times the quantity x plus three? Isn't any number times zero, zero?

Ss: [overlapping] Yeah

J: Just a little tiny bit about this. If you can get either of these terms equal to zero, since anything times zero is zero, (I-R-E) how can you get that term equal to zero, PJ, what would you put in for x to get to equal zero?

PJ: -3

J: Because -3 + 3 is zero and then negative three times zero is zero. (I-R-E) How can I get that term equal to zero?

Ss: Zero

J: Zero, (I-R-E) what's zero plus three?

Ss: Three

J: But what's zero times three?

Ss: Zero

J: Do you remember that?

Ss: Yeah

J: So, if I get either of these two terms, these are actually linear terms, equal to zero, then y will equal zero? And once you have your x-intercepts, you can do an awful lot. You can find a minimum, a maximum value, the line of symmetry is halfway between. Let's try it. Take a look at problem, first of all, let's finish this one. What I'd like you to do it this, I'd like you to, um, how should I ask this? Well, I'll let you use graphing calculators this time. I'd like you to sketch what the graph of that looks like. Where are the x-intercepts.

Fs: For what?

J: And what is the minimum or maximum value for this quadratic equation. Let's do that in your journal, please. Would you sketch a graph, give the x-intercepts and then give or show either the minimum or the maximum value. Does it go up to a point? Does it come down to a point before it starts going back up.

[Ss work quietly on this—minimal talking while they work]

Josh referred back for the *Frogs* unit again in asking students what they remembered about it. In doing so, he offered a very open question: "Do you remember something?" Three students raised their hands and Josh recognized them all and then called on Chuck to tell him what he remembered. Chuck may have offered a somewhat cryptic response as Josh asked him to "hold on a second." He then asked Jaylen what his x-intercepts were and wrote them on the board. Chuck agreed that these were the x-intercepts. Josh reformulated them into ordered pairs, making the answer more mathematically correct. He then let the class know that he thought he understood what Chuck was doing, but that he did not think other students knew what he meant. He asked the rest of the class if they understood what he was saying and three times restated what Chuck said, "what you need to get zero." Jonah told Josh that he did not really know what Chuck was saying and Josh asked Chuck to "clear it up." Chuck had a difficult time stating what it was he was doing. Josh tried to help him make his response more clear by asking him if he was looking at the expanded or factored form. One male student said he had "no idea," indicating that he was lost. Chuck said he was looking at the expanded form, which may have surprised Josh because he responded with the question, "for both of them?"

Instead of asking Chuck to articulate his process further, Josh asked Mindy what she thought. Mindy must have referred to using the answers that were already given (much of her turn was inaudible because she spoke very quietly) because Josh said, "but what if I didn't know what the x-intercepts were?"

Josh channeled the discussion to focus on the symbolic method and referred back to the fact that they already knew how to solve linear problems. He may have been hoping that the process for solving linear had built a foundation for students to see how they could solve quadratics. Sandy offered a way to solve the problem by looking at the factored form. However, her description was not clear to me as an observer. It may not have been clear to Josh, either, as he did not incorporate any of her suggestion into his next turn. He used a tag-question to suggest that the problem was a multiplication problem and to remind students that any number times zero is zero. He then scaffolded the process by going into a "tiny bit about this." He asked PJ what he could put in to make (x+3) equal to zero. PJ gave the correct response and Josh revoiced the response to show the process involved: that is, when you substituted –3 in for x, the result came out to be zero and –3 times zero gave them zero. Josh continued the process by asking students how to get the other term to be zero. Students talk him through this one as a class and when Josh asks, "Do you remember that?," students respond in the affirmative.

Josh seemed to be summarizing the conversation in his next term. He reviewed the fact that if either of the two terms equal zero, the y would equal zero. He also connected the parts of the factored form of the equation to the fact that they were really just dealing with linear terms. In other words, I could think about solving a quadratic as breaking the quadratic down into two linear equations, each of which must equal zero. By making this statement, he may have been hoping that students would connect their symbolic method for linear functions to the quadratic functions they were now tackling. He also told them what they could find once they have their x-intercepts: the minimum or maximum and the line of symmetry. Then, to give students further practice with doing this, Josh asked them to finish the problem by locating these on the problem they were currently doing.

What mathematical understandings do students seem to display? Again, I attend to: the focus of the discussion, the kind of language used, and the type of statements being made. The focus of the discussion is made when Josh called on Sandy to read from the textbook. After Sandy read, Josh inquired about things they had already learned about quadratics in previous units (i.e. *Moving Straight Ahead* and *Frogs, Fleas, and Painted Cubes*). By doing so, Josh defined the parameters of the classroom talk, using the textbook to determine what would be talked about and when. The talk moved from remembering what they had learned about quadratics in other units to factoring quadratics using an area model to solving quadratics to being focused on using symbolic manipulation to solve the problems. Each of these topics was defined and introduced by Josh. In some cases, he also defined the process for doing what he requested. For example, when he asked students to factor, he reformulated the question to ask them what the rectangle's dimensions would be. Some of the processes were defined by students. For instance, Josh asked students to find the x-intercepts or roots and told them, "I don't care how," insinuating that they could solve it symbolically, use the graphing calculator, etc. Jaylen used the graphing calculator and described what he did to solve the problem.

The range of language used in this transcript is fairly consistent. Josh and the students typically used official mathematical language (OML). In fact, Josh may have set this stage by immediately referring to the textbook and by modeling this type of language right away by asking, "How do you know it's a quadratic?" Some examples of this type of language included: quadratic, linear, parabola, y-intercept, x-intercept, minimum, maximum, line of symmetry, root, solution, expanded form, factored form. In fact, Josh further supported this type of talk in his revoicing of student contributions. For instance, when Jennifer said, "And 3 times x is 3x, so put x+3 in parenthesis," Josh revoiced her answer as, "…x times the quantity x+3," making her contribution more mathematically precise. Another example of this would include when one student gave the x-intercepts as –3 and 0. Josh first restated what he said and then revoiced himself, saying, "Or negative three, zero and zero, zero as far as points are concerned." There were also many times when Josh used OML and then continued by describing the process or definition that went with the term. For example, after he asked students to "translate the book," Sandy offered that they were trying to find the "solutions." Josh revoiced this as "the roots or x-intercepts" followed by what that meant: "In other words, I'm setting y equal to zero."

In attending to the type of statements being made, there is one instance of an "incoherent" statement. When Chuck tried to describe how he had come up with the solution he said, "Uh, what you need to get zero at y, the three is zero and the x is zero." However, the statement appeared to only be incoherent to other participants, but not to Josh. In fact, Josh pointed this fact out when he said, "I-I think I understand what you are saying, but I'm not sure anybody else does." He confirmed his suspicions by calling on Jonah and asked Chuck to "clear it up." Chuck tried to do this once and then Josh tried to help him make his contribution more clear by asking a directive question (e.g. What are you looking at, the expanded or the factored form?"). Josh may have attempted to have another student help with the explication of this method when he brought Mindy into the conversation by asking her what she thought. It was not clear to me whether Mindy was trying to help Josh understand Chuck's contribution or if she were offering her own method for solving the equation. However, it was clear that both Mindy and Chuck were attempting to use the expanded form to describe their solution. Josh pointed out that they needed to think about how to solve when they did not know what the x-intercepts were and proceeded by asking students to use what they knew about solving linear functions symbolically. He talked them through the process, filtering out the previous solutions methods that had been offered by students. He directed their attention to the factored form and using that to solve the quadratic equations. In fact, at one point, Josh read a question from the textbook that asked, "Which form makes it easier to find the roots?" Although the question is never answered explicitly, it became fairly clear that the factored form was the approved method.

Many of the student statements were "operational" in that they focused on how to do something. The segment of transcript that followed the above example would be an instance of this. Sandy's response to Josh, in fact, was about how to solve and only focused on the factored form as Josh had indicated it should. In many cases, the process of how to solve the problem was the focus of the conversation. There were also some "reflective" statements made by Josh. At least three times, Josh tried to get students to use what they had learned in previous units, attempting to make the common knowledge more connected for students. He referred back to: how they knew the equation was quadratic, how they had solved linear equations symbolically, how they could use the x-intercepts to find the minimum or maximum and the line of symmetry. Some of this information was related to what students had learned in previous units; other information was covered more recently in the current unit. In addition, explanations were given whenever Josh requested them.

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[1] "Explore" is part of the explicit pedagogical model that CMP encourages teachers to use: launch, explore, then summarize. The type of classroom talk that is in this example was something that I observed often when Karla was in the "explore" portion of the lesson.
[2] I use the word "brief" because the transcripts for this one observation was over 20 pages long. The actual discussion about this topic took place over two and a half class periods and it was difficult to choose a portion to be representative of the rich discussion that took place.
[3] This example is continuous except where I've noted otherwise. I have sectioned it off into topically related sequences for interpretation purposes.
[4] In this classroom, students say x-two and x-squared to both mean x to the second power. I have differentiated this for the reader by using x2 and x-squared. This would be another example of CGL that is

used in Karla's classroom. Notice this is not "corrected" by Karla, even though Dale is one of the highest achieving students in the class.

[5] Due to the length of this extended discussion, I was unable to continue to include these generalizations.

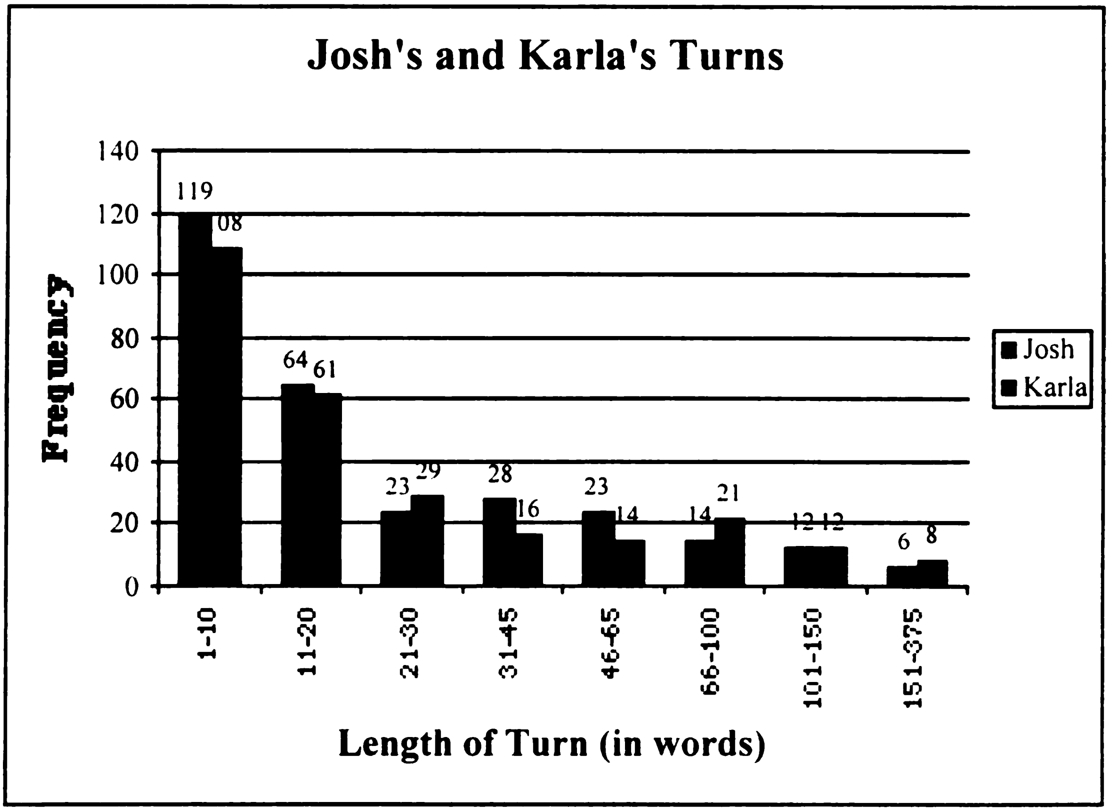
[6] I encourage the reader to again note the pervasive I-R-E pattern in this transcript, which begins at this point. Josh asked a question, students answered it and Josh affirmed the answer by repeating it. To show this, I have included (I-R-E) at the beginning of each of the exchanges that fit this pattern.

[7] This exchange could be considered I-R-E in nature. However, since the tag-form really encouraged merely an affirmation from students, I will not label these as such.

[8] This was the last thing I included in that segment of classroom talk so I could skip to the segment which focused on the mathematical point of the lesson (i.e. solving quadratics symbolically).

APPENDIX P

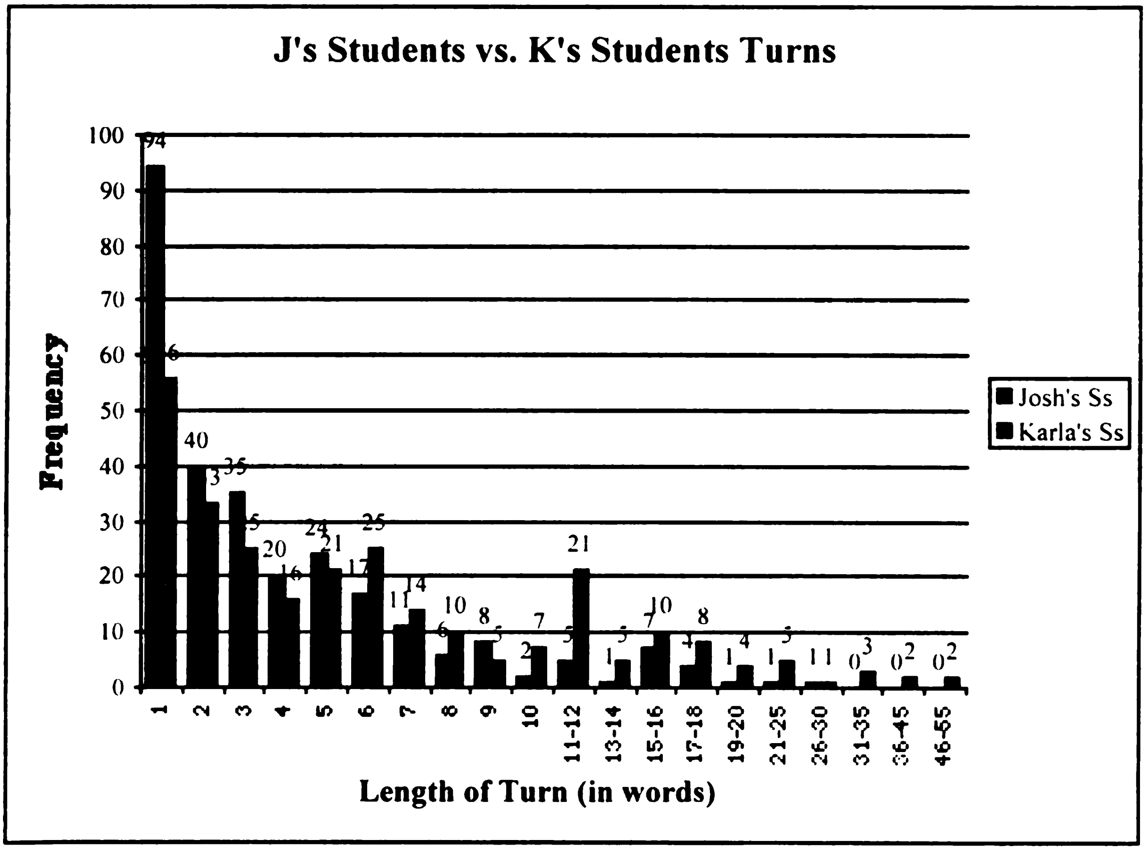
TEACHER TURN COMPARISONS



Josh's and Karla's Turns
140
120
100
80
60
40
20
0
Frequency
119
08
64
61
23
29
28
16
23
14
14
21
12 12
6
8
1-10
11-20
21-30
31-45
46-65
66-100
101-150
151-375
Length of Turn (in words)
Josh
Karla

APPENDIX Q

STUDENT TURN COMPARISON



J's Students vs. K's Students Turns
Frequency
100
90
80
70
60
50
40
30
20
10
0
Josh's Ss
Karla's Ss
1
2
3
4
5
6
7
8
9
10
11-12
13-14
15-16
17-18
19-20
21-25
26-30
31-35
36-45
46-55
Length of Turn (in words)

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