This is to certify that the
dissertation entitled

WHAT IS MATHEMATICS?: STABILITY AND CHANGE IN PROSPECTIVE TEACHERS' CONCEPTIONS OF AND ATTITUDES TOWARD MATHEMATICS AND TEACHING MATHEMATICS

presented by

ELAINE ALLEN TUFT

has been accepted towards fulfillment
of the requirements for the

Ph.D. degree in Curriculum, Teaching, & Educational Policy

Helen J. Featherstone
Major Professor's Signature

Nov. 7, 2005

Date

WHAT IS MATHEMATICS?: STABILITY AND CHANGE IN PROSPECTIVE TEACHERS' CONCEPTIONS OF AND ATTITUDES TOWARD MATHEMATICS AND TEACHING MATHEMATICS

VOLUME 1

By

Elaine Allen Tuft

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

2005

ABSTRACT

WHAT IS MATHEMATICS? STABILITY AND CHANGE IN PROSPECTIVE TEACHERS' CONCEPTIONS OF AND ATTITUDES TOWARD MATHEMATICS AND TEACHING MATHEMATICS

By

Elaine Allen Tuft

In this study, I examined the conceptions of and attitudes toward mathematics and teaching mathematics of prospective elementary school teachers and whether they changed during the time they were in a course designed to help prepare them to teach mathematics to elementary-age students. There were 34 students in the class who served as subjects for the study. Four of those students also served as focus students, and I examined them as case studies. I used four main sources for data. The first was a mathematics inventory with Likert-type items and open-ended questions that the students took on the first day of class and again on the last day of class. The second source of data were math concept maps the students did at the first of the semester and the end of the semester. The third source were various journal entries the students wrote during the course including a "What Is Mathematics?" journal entry written at the beginning and end of the semester and a "Philosophy of Teaching Mathematics" entry written at the end of the semester. The final data source consisted of two interviews each of the four focus students. A colleague administered the first interview to the focus students during the course, and I administered the other interview to the focus students a semester after the class ended. I also developed a framework delineating different aspects of the students' conceptions of mathematics to use as a template for organizing the analysis.

The most important finding that became apparent from an analysis of the data is that during the course the students shifted their position in relation to

mathematics from that of an experienced student to that of a prospective teacher. It appeared that that shift in positioning served as a catalyst for change in the students' attitudes toward and some of their conceptions of mathematics and teaching mathematics. Instead of basing their conceptions of and attitudes toward mathematics and teaching mathematics on their own experiences with mathematics, they were able to think about how they wanted to teach it. Having a new relation to mathematics (as prospective teachers) the students were able to imagine it being taught in a way that was perhaps different and much more positive than they had experienced as students, and their attitudes toward mathematics and teaching mathematics became much more positive.

I found significant and noteworthy changes in the students' conceptions of the processes of mathematics and the usefulness of mathematics as well as in their conceptions of learning and teaching mathematics. I also conjecture that dissatisfaction plays a role in change. One of the major implications of this study for mathematics teacher educators is that by being aware of the role that the shift in positioning in the relation to mathematics plays in facilitating change in conceptions and attitudes of prospective teachers, mathematics teacher educators can provide experiences for their students in which they can experience thinking of themselves as teachers of mathematics. Also, knowing which areas are more likely to change, mathematics teacher educators can emphasize those areas.

The main contributions of this study to research in mathematics education were a study that showed there could be change, a new framework for conceptualizing mathematics, ideas for using concept maps in research, and understanding the relevance that the shift of positioning the students experience in relation to mathematics has to changes in conceptions and attitudes.

This work is dedicated to Greg, Hannah, Caleb, and Rachel.

ACKNOWLEDGEMENTS

If I would have known how difficult it would be to complete this dissertation, I may never have started it. It's a good thing I didn't know.

First and foremost, I must thank my husband, Greg Tuft, for his support. I would never have completed this dissertation without it. He supported me, encouraged me, and believed in me through all the years. He did all that he could so that I could fulfill this goal, including moving back to Michigan from Utah for a semester so that I could complete some critical work. He finally gave up several months of working so that he could care for our children full time while I worked on my dissertation full time. That was the only way I was ever able to finish it with three young children. I'm also thankful for my three children—Hannah, Caleb, and Rachel—who shared their mother with the computer and learned how to say words like "dissertation" and "campus" at very young ages. Realizing the influence my example could potentially have on my children provided great motivation for me to complete this work in spite of many obstacles that made it very difficult.

Helen Featherstone was a wonderful dissertation director, and many thanks go to her for her tireless efforts in my behalf. I could not imagine someone who could have been more competent, supportive, and kind in this role. Her willingness to persist with me through all the years that I was working on my dissertation off and on was much of the reason I was able to finish it. She always gave me excellent feedback and advice, and I learned a great deal from her. She is an outstanding teacher, and I feel blessed to have had the opportunity of working with her. My committee members—Jack Smith, Ralph Putnam, and Cheryl Rosaen—were also wonderful. I greatly appreciate their willingness to meet with me on different occasions, endure with me, teach me, and encourage me in such a great endeavor. I learned an immense amount from each of them, and they are each the kind of individuals I want to emulate as teachers, scholars, and people.

Thanks, also, to my parents-in-law, Don and Nadene Tuft, for their support and encouragement. They made many sacrifices of their own to help ensure that I would be able to finish this dissertation. Several times, they took my children for several hours or even a few days so that I would have time to work without interruption. They also made sure we never went hungry during that time.

I also appreciate the encouragement and support of my parents, Nard and Beth Allen, as well as all my brothers and sisters and their families and Greg's brothers and sisters and their families. They let me know they thought this was a worthy pursuit, and they believed I would be able to accomplish it. They did so many little things to help ensure my success. I especially appreciate the support of my brother and sister-in-law, Mike and Kaylene Allen, for allowing me to use their home as an office in which to work on my dissertation. It was just the workable arrangement I needed to have the time and isolation necessary to focus on this work.

Finally, I am grateful for the opportunities I have been given in my life to study, learn, and pursue this degree. I'm grateful for the teachings and examples of many who have instilled in me the desire to learn—especially my late mother, Melva Nielsen Allen. Her influence will forever be felt in my life.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1
INTRODUCTION/CONCEPTUALIZATION

Introduction

In 1967, in his essay, *I, Thou, and It*, David Hawkins wrote that when talking about teaching, people often make analogies and focus on the relationship between the teacher and the student, but he argued that there was a relationship between the teacher and the student and also a third thing, namely the subject, that was needed to complete the triangle. Hawkins wrote, "Adults and children, like adults with each other, can associate well only in worthy interests and pursuits, only through a community of subject-matter and engagement which extends beyond the circle of their intimacy." The relationship among these three entities can be visualized as depicted in Figure 1. The relationship between the teacher and the subject would include the teacher's knowledge of the subject as well as his or her conceptions of the subject and attitude toward it. Likewise, the line of the triangle from the student to the subject, would include the student's knowledge of, conceptions of, and attitude toward the subject.

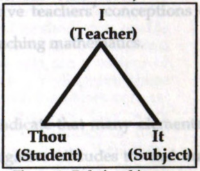

Figure 1: Relationship among the Three Entities

Focus of the Study

This study is an examination of one aspect of one piece of the relationship referred to by Hawkins, namely the relationship between the teacher and the subject (in this case mathematics) as highlighted in Figure 2. This study will look at the conceptions of and attitudes toward mathematics of a group of prospective elementary school teachers at the beginning of, throughout, at the end of, and soon after a course that is designed to help students prepare to teach mathematics. Specifically, I am interested in looking at these students' conceptions of what mathematics is (in the discipline of mathematics, in school, and outside of school) and at their attitudes toward mathematics (e.g., how they feel about themselves as mathematicians and learners of mathematics and how they think about the value of mathematics). I am also interested in looking at these prospective teachers' conceptions of teaching mathematics and their attitudes toward teaching mathematics.

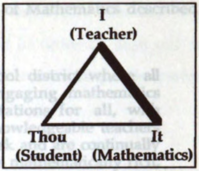

Figure 2: Relationship which is the Focus of this Study

Problem Statement

Research on teachers' attitudes and beliefs indicate that many elementary school teachers (and prospective teachers) have negative attitudes toward math and conceptions of what math is that are different from the view of math espoused by The National Council of Teachers of Mathematics and many other mathematics reformers (Becker, 1986; Bulmahn & Young, 1982; Dutton, 1954; Dutton, 1962; Fennema & Sherman, 1976; Kane, 1968; Kelly & Tomhave, 1985; National Research Council, 1989; NCTM, 1989; NCTM, 1991; NCTM, 2000; Pejouhy, 1990; Reys, 1968; Smith, 1964). These reformers and NCTM have advocated a particular vision of teaching mathematics that is based on the view that mathematics is more than just a collection of concepts and skills to be mastered; that it includes "methods of investigating and reasoning, means of communication, and notions of context;" and that being mathematically literate has a whole different meaning in this information age than it did in the industrial age (NCTM, 1989). On page 3 of its *Principles and Standards for School Mathematics*

(NCTM, 2000), The National Council of Teachers of Mathematics described a vision for mathematics education:

> "Imagine a classroom, a school, or a school district where all students have access to high-quality, engaging mathematics instruction. There are ambitious expectations for all, with accommodation for those who need it. Knowledgeable teachers have adequate resources to support their work and are continually growing as professionals. The curriculum is mathematically rich, offering students opportunities to learn important mathematical concepts and procedures with understanding. Technology is an essential component of the environment. Students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways until they find methods that enable them to make progress. Teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures. Students are flexible and resourceful problem solvers. Alone or in groups and with access to technology, they work productively and reflectively, with the skilled guidance of their teachers. Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it."

There is some evidence which I will review later in this chapter that suggests that teachers with ideas that are considered more traditional—or more absolutist—about what math is won't teach math in the ways that these reformers and the NCTM urge (Thompson, 1984; Thompson, 1992). Therefore, if we want teachers to teach math in the way the NCTM and other reformers urge, we may need to change some attitudes and conceptions of prospective elementary school teachers who will teach mathematics. In order to think about changing the beliefs of prospective teachers, we need to know more about what they are and something about their "natural history" as they learn about math teaching. In other words, if changing conceptions is important, it would be good to see what the process of change looks like. It would also be useful to learn about which conceptions and attitudes are more likely to change under particular circumstances and which stay the same. In what follows I will look in more detail at some of those issues and at the research cited in order to flesh out the argument summarized here. But first I will lay out the research questions which follow from the argument.

Research Questions

The overarching question that this study is designed to investigate is what conceptions of and attitudes toward mathematics prospective elementary teachers bring to a course designed to help prepare them to teach mathematics and how or whether these conceptions change/and or develop across the duration of the course.

Specifically, this study is designed to address the following questions:

1. What are the conceptions of mathematics of these prospective elementary teachers at the beginning of the course? What are their conceptions of mathematics at the end of the course? What is the difference between these conceptions at the beginning and end of the study? What might this change (or lack of change) look like in an individual student?
2. What are the conceptions of learning and teaching mathematics of these prospective elementary teachers at the beginning of the course? What are their conceptions of learning and teaching mathematics at the end of the course? What is the difference between these conceptions at the beginning and end of the study? What might this change (or lack of change) look like in an individual student?
3. What are the attitudes toward mathematics of these prospective elementary teachers at the beginning of the course? What are their attitudes toward mathematics at the end of the course? What is the difference between these

attitudes at the beginning and end of the study? What might this change (or lack of change) look like in an individual student?

4. What are the attitudes toward teaching mathematics of these prospective elementary teachers at the beginning of the course? What are their attitudes toward teaching mathematics at the beginning of the course? What is the difference between these attitudes at the beginning and end of the study? What might this change (or lack of change) look like in an individual student?

One way to visualize the overarching questions of this study is with the following table (See Table 1):

Table 1: Research Questions for this Study

	Math	Teaching Math
What are the Students' Conceptions of	at the beginning of the Semester?	at the beginning of the Semester?
	at the end of the Study?	at the end of the Study?
What are the Students' Attitudes toward	at the beginning of the Semester?	at the beginning of the Semester?
	at the end of the Study?	at the end of the Study?

Rationale for this Study

Central to the relationship between the teacher and mathematics are the teachers' conceptions of mathematics—what it is, what it means to do math, where and when it occurs, what counts as mathematical knowledge—and their attitudes toward mathematics—their confidence in their own abilities as well as their feelings of the worth of mathematics. Many educators and researchers believe that these ideas about mathematics influence the way prospective elementary school teachers think and feel about teaching math (Dossey, 1992; Ernest, 1991), arguing, for example, that prospective teachers who think of mathematics as a static body of knowledge consisting of isolated facts might be inclined to think it should be taught as arithmetic computation that is learned through repeated practice with exercises that are found in a textbook (Jesunathadas, 1990; Wilcox, Schram, Lappan, & Lanier, 1991). With this view, applications and connections with other subjects, mathematical concepts, or the world outside the classroom may not be made. On the other hand, a prospective elementary school teacher with a broader view of mathematics might have a different view of how mathematics should be taught. This teacher might find ways to help students see math as more than the exercises in their textbook and might provide experiences that would help them make other connections (Allen, 1991). Also, the experience of some mathematics educators suggests that even when teachers are given a curriculum to use that is designed to help them teach conceptually—or go to workshops to help train them to teach that way—they can still use those activities and materials to teach in a way that treats math as a set of rules even though their teaching may look, at first glance, more conceptual (Lappan, Personal Communication; Bouck, Personal Communication). According to these math educators, it was necessary for the teachers' conceptions of mathematics to change in order for them to be able to use the materials as they were intended.

Since there is much theoretical and empirical evidence, which will be reviewed later, to suggest that teachers' conceptions of mathematics affect the way they teach it, an examination of the conceptions of and attitudes toward mathematics of prospective elementary school teachers who will be teaching mathematics is a valuable undertaking for anyone interested in mathematics education in the elementary school, especially those who are involved in the preparation of these prospective teachers. Understanding prospective elementary teachers' conceptions of and attitudes toward mathematics is a crucial step in carrying out the reforms that have been advocated in recent years (NCTM, 2000; NCTM, 1995; NCTM, 1991; NCTM, 1989) because we cannot challenge these conceptions and attitudes without first knowing what they are.

Research literature is replete with findings that suggest most prospective elementary teachers in teacher education programs do not have positive attitudes toward mathematics and are not confident in their abilities related to mathematics (Ball, 1988; Becker, 1986; Dutton, 1962; Kane, 1968; Kelly and Tomhave, 1985; Larson, 1983; Reys and Delon, 1968; Smith, 1964). Many of these attitudes are related to the experiences prospective teachers had themselves as students of mathematics, and this view affects the way they ultimately teach math. Those who were successful in mathematics may think the patterns of teaching they saw are appropriate for teaching, and even those who struggled in math may still assume that what they experienced is how math should be taught (Ball, 1989; Jesunathadas, 1990).

If we want the reforms that are currently being advocated to be enacted, we have a long way to go with all who are involved with mathematics education, and particularly with prospective elementary teachers. One way of looking at where we want the prospective teachers to go is NCTM's *Curriculum and Evaluation Standards for School Mathematics* whose publication in 1989 had a profound impact on mathematics educators both in the U. S. and internationally (although the NCTM has published three more important and influential Standards volumes since 1989, these goals continue to shape the vision of many teachers and reformers). This volume identified five goals for students: (1) learning to value mathematics; (2) becoming confident in one's own ability; (3) becoming a mathematical problem solver; (4) learning to communicate mathematically; and (5) learning to reason mathematically. It seems reasonable to also expect these students' teachers to have these same characteristics, but there is evidence that they often do not—especially teachers of elementary students—and maybe part of their teacher preparation program should help them attain them. The last three goals identified are more connected to the subject matter knowledge of mathematics, and research indicates that most prospective elementary teachers do not know what they need to know about mathematics to be able to teach it as has been envisioned by the NCTM and others (Floden, McDiarmid, & Wiemers, 1990). The first two goals (learning to value mathematics and becoming confident in one's own ability) are directly related to this study.

In addition, Ernest (1991) and Dossey (1992) make a convincing theoretical argument that a philosophy of mathematics affects the way mathematics is taught. Most prospective elementary teachers have what Ernest describes as an absolutist view of mathematics—the view that mathematical knowledge consists of certain and unchallengeable truths—and that is inconsistent with the vision of teaching mathematics that is currently being recommended and commended as will be elaborated further in subsequent sections of this chapter.

Of course, it's difficult to have any effect on changing or challenging these prospective elementary teachers' ideas about mathematics without first understanding what they are. A major purpose of this study is to learn more about these conceptions and attitudes. Another purpose of this study is to learn about how or whether these conceptions and attitudes change and/or develop during the time these prospective teachers are taking a course that is designed to help prepare them to teach mathematics to elementary school children. The purpose of this study is not to disentangle cause and effect in relation to such a course and the questions that are addressed by this study. Rather, it is to look at the development of these conceptions and attitudes at different points across a semester in which these prospective teachers are talking, reading, writing, and learning about various perspectives on mathematics and mathematics teaching and learning—many of which may be different from perspectives they are familiar with. Teacher preparation is an opportune time to examine the development of these beliefs and attitudes since it is when these students are apt (forced, even) to think about mathematics and their relationship to it.

Background

In the past four decades, many studies have been conducted to learn about the attitudes toward mathematics of prospective elementary teachers. These works have laid a foundation for a part of my research. Additionally, in the past two decades a number of studies have been based on the assumption that teachers' beliefs (which are in essence what I am calling "conceptions") about the subject they are teaching influence their practice. This seems to be an easy claim for people to make since many would consider that notion to be "common sense." However, common sense is usually not enough basis for assumptions in educational research, and many have tried, through theory, philosophy, and research, to give this assumption more foundation. This assumption has been the basis for research examining teachers' beliefs and looking at change in these beliefs. These theoretical, philosophical, and empirical works all lay the groundwork for my research. The next few sections will present an overview of this work. I will begin by looking at the philosophical and theoretical foundations of this line of research and then review the related empirical research. I will follow each section with thoughts about what we have learned from this research and what questions it raises.

Philosophical and Theoretical Foundations of This Research

This line of research has at its foundation many philosophical and theoretical considerations. In this section, some of these considerations will be discussed including the distinction between beliefs and knowledge, various philosophies of mathematics, and various perspectives on teaching mathematics as well as some of the theoretical foundations of this research and my thoughts about these theoretical works.

The Question of Beliefs vs. Knowledge in Mathematics. A question related to this type of study that has been avoided by some but attacked by others is the question of distinguishing beliefs and knowledge. Some researchers have assumed that readers know what beliefs are; others have recognized that the distinctions between beliefs and knowledge are often fuzzy and that teachers frequently treat their beliefs as knowledge. While some have dismissed the distinction as being arguable for educational research or futile because of the broadness of their characterizations, others felt that some distinction is needed to provide common ground (Thompson, 1992). Some of the distinctions that have been offered have proven to be useful in grounding research on beliefs, and therefore, a discussion of some of the distinctions seems valuable here.

In everyday use *knowledge* is usually used when referring to familiarity or understanding of something generally thought of as "real" truth, whereas *belief* usually refers to a conviction of some idea that is not necessarily thought of as factually-based truth. One of the things that makes this distinction complicated is that at different times, something that is considered to be knowledge can be considered belief, and ideas that are considered to be beliefs can, at another time, be considered knowledge. An example of that is in Pre-Columbian days when the world was considered to be flat. In those days, that idea was considered knowledge by those who learned it, but now we regard that idea as having been a belief. There are other things that were once considered to be beliefs of people that are now considered to be knowledge. Another complicating aspect of this distinction is that even at the same time ideas can be considered to be belief or knowledge based on the person's perception. This is especially true of religious tenets which some consider to be knowledge, but others consider to be beliefs.

There are some distinctions between beliefs and knowledge that are quite generally accepted among those who distinguish them for research purposes. One of these is that beliefs can be held with varying degrees of conviction. Another is that belief carries with it some connotation of disputability, and the believer realizes that others may think differently. Most philosophers agree that truth or certainty is associated with knowledge and that disputability is associated with beliefs. Another distinction that is generally agreed upon is the idea that there are procedures for evaluating and validating knowledge, and that it must meet certain criteria involving canons of evidence. On the other hand, beliefs can be justified for reasons that do not meet these criteria (Thompson, 1992). Nespor (1987) explained belief systems this way,

> Belief systems often include affective feelings and evaluations, vivid memories of personal experiences, and assumptions about the existence of entities and alternative worlds, all of which are simply not open to outside evaluation or critical examination in the same sense that the components of knowledge systems are (p. 321).

Nespor (1987) attempted to provide a model of belief *systems* that was grounded in theories of cognitive psychology theories and that could serve as a framework for comparative and systematic investigations. She identified four features that could serve to distinguish beliefs from knowledge. These are *existential presumption*, (the idea that belief systems contain propositions or assumptions about the existence or nonexistence of entities such as God, assassination conspiracies, and even beliefs about students that become labels for entities such as ability, maturity, and laziness); *alternativity* (the representation of alternative worlds or alternative reality); *affective and evaluative loading* (feelings, moods, and subjective evaluations based on personal preferences); and *episodic structure* (stored material derived from personal experience or from cultural or institutional sources of knowledge transmission). She also identified two other features—*non-consensuality* and *unboundedness*—which she saw as being useful for characterizing the way beliefs are organized as systems. Non-consensuality refers to the principle of disputability associated with belief systems. Unboundedness refers to the characteristic that belief systems can be described as "loosely-bounded systems with highly variable and uncertain linkages to events, situations, and knowledge systems."

Philosophies of Mathematics. Many have tried to identify distinct philosophies of mathematics, and their categories have provided researchers a template for grouping teachers by their beliefs about mathematics. These philosophies can be thought of as conceptions of the nature of mathematics which include conscious or subconscious beliefs, concepts, meanings, rules, mental images, and preferences concerning the discipline of mathematics (Thompson, 1992). These elements may not be developed in such a way that they could be articulated by a given person, but they influence this person's philosophy nevertheless. The philosophies that have appeared to have the most impact on this line of research will be reviewed here.

Blaire (1981) identified four movements for mathematics: *logicism* in which mathematics is reduced to logic, *formalism* in which mathematics is treated as the manipulation of concrete objects by finite methods, *intuitionism* in which mathematical activity is explained as the mental construction of systems, and *hypothetical* which is a concentration on the hypothesizing feature of mathematics.

In an article published in the same journal as the article referred to in the preceding paragraph, Lerman (1983) criticized Blaire's ideas. He argued that there are only two distinct movements in philosophy of mathematics and two teaching perspectives that result from these. The first movement is what he calls the *Euclidean programme,* described as an attempt to base mathematics on firm foundations, which produces the *knowledge-centred* teaching perspective. The other movement is the *quasi-empirical programme* described as the recognition that mathematics progresses heuristically which leads to a *problem-solving* teaching perspective. He claims that Blaire's fourth category, *hypothetical*, is similar to the approach to mathematical knowledge described by Imre Lakatos in his book *Proofs and Refutations*. Lerman argues that this is not a fourth category of a philosophy of what mathematics is, but a way of thinking about and learning about mathematics.

Ernest categorizes philosophies of mathematics into two main areas. One area is the *absolutist* view that mathematical truth is absolutely certain and that mathematics is a realm of certain, unquestionable, and objective knowledge—perhaps the only one. The other area is the *fallibist* view that mathematical truth is improvable and can never be regarded as being above revision and correction. Ernest also wrote of other philosophies of mathematics including *progressive absolutism, Platonism, conventionalism, empiricism, quasi-empiricism,* and *social constructivism.*

Ernest further expanded on the two areas of the absolutist and fallibist views. He sees the absolutist view as including *logicism, formalism,* and *constructivism* strands. Logicism is the school of thought that regards pure mathematics as a part of logic. Formalism is the view that mathematics is a meaningless formal game played with marks on paper, following rules. The constructivism strand in the absolutist view is the idea that mathematical knowledge needs to be reconstructed in order to protect it from contradiction and loss of meaning.

Ernest proposes social constructivism as a new philosophy that he thinks describes mathematics more accurately than the other views. In formulating this philosophy, he draws on other philosophies, particularly conventionalism and quasi-empiricism. In this philosophy, mathematics is viewed as a social construction. It accepts the idea from conventionalism that human language as well as rules and agreement play an important role in establishing and justifying the truths of mathematics. It takes the fallibist epistemology of quasi-empiricism with the ideas that mathematics knowledge and concepts develop and change and that it does this through conjectures and refutations utilizing a logic of mathematical discovery. He notes three grounds he uses for describing mathematical knowledge as a social construction and for adopting that name:

1. The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction.
2. Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge.
3. Objectivity itself will be understood to be social.

Perspectives on Mathematics Teaching. Most of those who have provided categories of philosophy of mathematics have also offered corresponding categories of teaching perspectives.

Blaire (1981) suggests four movements of mathematics teaching: (1) *the teaching of mathematics as an art-form* (valuing mathematics in "all its beauty" and creating proofs to demonstrate that beauty); (2) *the teaching of mathematics as a game* (focusing on using games such as *Battleship* to teach coordinates and teaching that "each part of mathematics has rules for everything that one is allowed to do, and procedures, by which one can recognize that one has completed, not the 'whole game', but one part of it"); (3) *the teaching of mathematics as a member of the natural sciences* (using the experimental method by testing and modifying hypotheses), and (4) *the teaching of mathematics as technologically-oriented* (using diagrammatic solutions such as flow charts to solve problems, focusing on the application of the tools of one branch of mathematics to another, and emphasizing applicability of solutions). He also writes that perhaps there is a fifth perspective—mathematics as a language—but he writes that "[m]athematics can only be identified with a restricted use of the word 'language', for no set of symbols held under a tight syntactic structure can have the 'rich' semantic form one expects of ordinary language." Blaire prefers the technological-orientation since he claims that it meets the desires of both the pure mathematician and the industrialist.

Ernest (1991) identified five distinct ideologies of school mathematical knowledge. These are *industrial trainer, technological pragmatist, old humanist, progressive mathematics educator*, and *public educator*. These ideologies will be briefly described in the following paragraphs.

The *industrial trainer* ideology embodies the view that mathematics is a clear body of knowledge and techniques, that it is made up of facts and skills, and that it must be kept free from the taint of cross-curricular links and social values. In instruction, this translates into the transmission of knowledge as a stream of facts to be learned and consists of rote learning, memorization, practice of skills, and hard applications in school "work" of the subject.

The *technological pragmatist* ideology views mathematics as consisting of two parts: pure mathematical skills (in that mathematics is an unquestioned body of useful knowledge), procedures, facts, and knowledge as tools to be mastered and applications and uses of mathematics (with increasing complexity) as the vital part that justifies and motivates the study. Intrinsic values, creativity, and pattern are insignificant in this ideology because they are concerned with "the pursuit of science and craft, design and technology." In teaching mathematics, this implies skill instruction and teaching the art of applying math. The emphasis is on applied problem solving, and motivation is through work relevance.

The *old humanist* ideology embodies the view of mathematics as a pure, hierarchically structured, self-subsistent body of objective knowledge. The higher up, the more pure, rigorous, and abstract the mathematics, the more 'real' it is. The teacher in an old humanist perspective is a lecturer and explainer who uses various approaches, demonstrations, and activities to facilitate learning and understanding. The relationship between the teacher and student is one in which the master (teacher) transmits knowledge to the pupil.

The *progressive mathematics educator* sees mathematics as the vehicle for developing the whole child. Mathematics is considered a language and something that is creative with a human side. The processes of mathematical problem solving and investigating such as generalizing, conjecturing, abstracting, symbolizing, structuring, and justifying are prominent in this view as is the cross-curricular link. The teacher with this ideology provides encouragement, facilitation, and arrangement of structured environments and situations for explorations. The teacher is a facilitator of learning who non-intrusively guides her students.

The *public educator* reflects the nature of mathematics as a social construction. The teacher with this ideology uses genuine discussion, cooperative group work, autonomous projects, a learner-questioning educational environment, and socially relevant materials to teach that "all knowledge is culture-bound, value-laden, interconnected and based on human activity and [i]nquiry" and that the "genesis and the justification of knowledge are understood to be social, being located in human agreement." Ernest favors the public educator theory of teaching mathematics since it is consistent with his view that mathematics is socially constructed.

The idea of mathematics as something that is socially constructed and continually in the making is the view that is most currently espoused by prominent mathematics educators. This is evident in the National Council of Teachers of Mathematics' *Principles and Standards for Standards for School*

Mathematics (2000) which advocates a conception of teaching in which "doing" is more important than "knowing that" and in which students engage in purposeful activities that grow out of problem situations which require reasoning and creative thinking, gathering and applying information, inventing, discovering, communicating ideas, and testing those ideas through critical reflection and argumentation (NCTM, 2000; Thompson, 1992).

Theoretical Foundations of This Claim. Much of the evidence that educators and researchers use to support the claim that teachers' beliefs about the subject matter they are teaching influence the way they teach it has been theoretical in nature. Much of this has been written in sort of common-sensical ways such as when Blaire (1981) wrote,

> While no claim is to be made that there is a necessary connection between a particular philosophy of mathematics and a particular teaching perspective, it may be left to the individual to feel, that the acceptance of a particular philosophy of mathematics, may more readily lead to one teaching perspective rather than to another. (p. 148)

Blaire (1981), however, when writing of the connection between philosophies of mathematics and their connections to perspectives in mathematics teaching, also gave this caveat:

> It is clear that such tight logical connections cannot be drawn, when one discusses the complexities of successful teaching. The teacher may choose rightly to do what will motivate his class, rather than what will connect immediately to his long-term aim, of showing that mathematics rests on a formalist foundation.

Lerman (1983) had a view somewhat different from Blaire's. He argued that the relationship between a teacher's philosophy of mathematics and the teacher's teaching perspective is much stronger than Blaire suggests. He suggested that "one's perspective of mathematics teaching is a logical consequence of one's epistemological commitment in relation to mathematical knowledge, and not merely one of expediency in response to societal pressures, or of pedagogical convenience." Lerman wrote of the "serious situation" in mathematics teaching which was the result of most students not enjoying the subject or seeing its value and being unable to apply the mathematical knowledge they possess. Lerman argued that because of this "serious situation" that "the issues raised in considering the influence of the philosophy of mathematics on teaching style are of major significance in any attempt to alter this situation."

Nespor (1987) argued that teachers' beliefs play a major role for teachers in defining teaching tasks and organizing the knowledge and information considered to be relevant to those tasks. She argued that this is so because the contexts and environments in which teachers work, as well as many of the problems they encounter, are ill-defined and deeply entangled, creating a situation in which beliefs are particularly suited for making sense of such contexts.

Ernest (as reported by Thompson, 1992) identified three elements that influence the practice of mathematics teaching:

1. The teacher's mental contents or schemas, particularly the system of beliefs concerning mathematics and its teaching and learning;
2. The social context of the teaching situation, particularly the constraints and opportunities it provides; and,
3. The teacher's level of thought processes and reflection.

It is interesting to note the importance of the teachers' level of reflection on their mathematics teaching. This is something that has also been argued by others including Dewey (1933) and Schon (1987).

In a later work, Ernest (1991) again indicated his belief that teachers' performance as teachers depends fundamentally on their beliefs about mathematics in many ways. In fact, at the beginning of his book, he made the argument, "Much is made of the absolutist-fallibist distinction because. . . the choice of which of these two philosophical perspectives is adopted is perhaps the most important epistemological factor underlying the teaching of mathematics." One of the ways he pointed out how teachers' philosophies of mathematics affect their teaching was by describing how when teachers are given recommendations of curricular reforms, they assimilate those concepts into their own perspectives and understand them accordingly. He also acknowledges that there are several things that influence this relationship such as the opportunities and constraints afforded by the social context of teaching.

Thoughts about Theoretical Works. The theoretical works I read can be helpful in doing research looking at the relationship between teachers' beliefs about mathematics and their instruction, but not always easily. In my study, I am not looking at teaching practices, but these theoretical works assist me in thinking about the importance of the students' philosophies—whether they are likely to influence their practice or why they might influence (or fail to influence) practice. Instead of telling me how to do my research, these works support the value of it. One of the things that helps me feel confident about considering them is that although the theorists give different labels to their categories, there is much agreement about the characteristics of the different categories. They do not all have the same number of categories, but often, categories of one can be seen as subsets of categories of another. This seems to provide some validity for their categorizations.

Some of the categories seem like they could be used quite easily as a basis or template for doing empirical research, but some of the categories and definitions they write about seem slippery. For example, Ernest (1991) writes about two categories and then later writes of several categories. It's not clear how he categorizes different philosophies of mathematics. To use his categories one would have to presume what was intended by him. While many of the categories offered by the theorists seem like they could be used for empirical research, I am not aware if many of them have been used as a foundation.

Another thing that raises suspicion about some of the theories is that the writers claim research foundations, but they do not cite them. It would be more useful to know what evidence supports these categories.

Review of Empirical Research Related to My Research Questions·

Much research has been conducted in the past four decades that is related to my research questions. In the following sections, I will review much relevant research. I will begin by reviewing the research that has been done related to prospective teachers' attitudes toward mathematics and teaching mathematics. I will then review the research that has focused on teachers'/prospective teachers' beliefs about mathematics and teaching mathematics. The next section will be a review of research that has looked at both beliefs and attitudes of prospective teachers in relation to mathematics. Finally, I will summarize the main findings of these studies, and I will then highlight some of my observations about the strengths and weakness of the works I read.

A Historical Review of Research Relating to Attitudes. There was little research done on either beliefs or attitudes during the 1930s, 40s, and 50s. In the 1960s, however, several research projects investigated attitudes of elementary teachers toward math (Thompson, 1992). Dutton (1954) appears to have sparked some of this renewed interest when in 1954, he published an article entitled "Measuring Attitudes Toward Arithmetic" in which he described a procedure he used to determine attitudes toward arithmetic of prospective teachers. Through quantitative procedures he developed an attitude scale with scale values for each statement that was used for this purpose. This instrument was also used to determine when preservice elementary education majors reported that these attitudes toward arithmetic were developed. The article described the development of the scale as well as findings obtained using this scale. Dutton argued that attitudes toward arithmetic could be measured objectively and that significant data could be obtained that would be helpful in educating prospective elementary school teachers. He found statistically significant, unfavorable attitudes in the areas of not feeling secure in the subject, being afraid of word problems, and fear of the subject in general among his sample of 127 prospective elementary school teachers. He also argued that the chief factors that caused this dislike were lack of understanding, teachers who used inadequate methods or punished students when they didn't get the correct answer, difficulty in working arithmetic, and insecurity.

In the 1960s, four articles appeared in the National Council of Teachers of Mathematics' journal, *Arithmetic Teacher* (NCTM, 1988) that were related to prospective elementary teachers' attitudes toward mathematics. The first of these was another article written by Dutton in which he described a study he conducted to find new ways to apply the attitude scale he had previously made and to search for changes in attitudes of prospective elementary school teachers toward arithmetic from when he had previously used it in 1954. He concluded from this study that the attitudes of students toward arithmetic in 1962 were almost identical with attitudes held by students in his 1954 study and that attitudes toward arithmetic, once developed, are tenaciously held by prospective elementary school teachers. He also concluded that there had been no major improvement in the instructional programs of public and private schools that were directed at developing positive attitudes in students toward arithmetic since the prospective elementary teachers he studied reflected attitudes that he claimed are developed in a "traditionally-oriented arithmetic program."

Dutton's scale was the main instrument used for measuring prospective elementary teachers' attitudes toward arithmetic during the 1960s. Smith (1964) used Dutton's scale to compare the attitudes of prospective teachers to when Dutton first used the instrument—similar to what Dutton had done two years earlier. His findings were in agreement with the findings of Dutton. He concluded that "too many prospective teachers have negative attitudes toward a subject they will be required to teach" and that "[t]eacher-education programs should strive to change these attitudes." Then, in 1966, Reys and Delon again used this scale in a study that looked at prospective elementary teachers attitudes toward mathematics before and after completion of a course in the mathematics preparatory program for elementary education majors. From their findings, they concluded that the mathematics preparatory courses produced some improvement in the students' attitude toward arithmetic. The observed changes in attitude were slight, but the authors argued that it was not surprising that the changes were small after only a few months of instruction when the attitudes were "conceived at least five years prior to entering college and perhaps even cultivated through the years." However, these findings did indicate that change was possible, and the authors suggested that "high quality instruction in a continuous mathematics program for a longer period of time" would produce them.

Another article published in *Arithmetic Teacher* during the 1960s (Kane, 1968) reported on a survey not "obviously designed to sample attitudes toward mathematics" but, instead administered in a neutral setting by a neutral person who asked questions about English, science, and social studies as well as math. Unlike earlier attitude researchers, he found that the attitudes of the prospective teachers he studied were consistently higher toward mathematics and English than social studies and science. He also concluded that prospective teachers tended to have relatively favorable attitudes toward teaching mathematics in the elementary school. However, he also concluded that prospective teachers who have unfavorable attitudes toward mathematics tend to prefer teaching in grades K-3, while those who had the most favorable attitudes tended to prefer assignments in grades 4-6.

There appears to have been little done by attitude researchers in the 1970s, but by the 1980s, as "arithmetic" had given way to "mathematics," those concerned with attitudes toward mathematics began looking at "mathematics anxiety," and more studies appeared. One of them was Bulmahn and Young's (1982) study of the attitudes toward mathematics of prospective elementary school teachers. This study was justified on the premise that a person's environment has some effect on his or her mathematical ability and interest and that elementary school teachers are a significant part of any individual's early mathematical environment. Therefore, it was important to understand elementary school teachers' attitudes toward mathematics in case they needed to be changed. The study was also conducted based on the observation of the authors from over fourteen years of college teaching experience that suggested that the motivations and interests of elementary education majors was different from the motivation and interests of other majors. Their study consisted of two parts. One was a questionnaire asking questions about attitudes toward mathematics, and the other was a collection of essays written by over 200 elementary education students in their first course of a three-course mathematics sequence. They reported their findings rather broadly and generally, but they claimed to have found that many of these students considered mathematics to be their worst subject and that many of them considered their career options to have been limited by their mathematical abilities. Related to this finding was the finding that many beginning elementary education students feel that elementary teachers do not really need to be very good at mathematics beyond the basic computations.

Becker (1985) argued that Bulmahn and Young, with their broad generalities and little reported data, had made the problem of anxiety seem more severe than she judged it to be from her experience teaching mathematics to students majoring in elementary education. She administered a revised version of seven of the Fennema-Sherman Mathematics Attitude Scales (Fennema & Sherman, 1976) to eighty-one elementary education majors enrolled in a required mathematics course and to seventy-one students enrolled in a general astronomy course. In all areas of the scale she used, the education students' scores were similar to those of the astronomy students with the exception of the anxiety section where the education students' average was statistically significantly lower (i.e., more anxious). The astronomy students showed similar trends in the anxiety area, but their feelings were not as strong. Still, she concluded from her results that the sample of prospective elementary teachers she studied could not be classified as having an "alarming degree" of mathematics anxiety. She felt that the education students did not have negative attitudes toward mathematics and that their attitudes were comparable to those of a more general sample of students. She admitted that they were not "very positive," but she felt they were not as negative as had been implied by Bulmahn and Young. Still, she agreed that better prepared mathematics teachers would provide better instruction in mathematics, but she warned against blaming all the problems in the learning of mathematics by children on their teachers' anxieties about the subject.

Kelly and Tomhave (1986) studied the math avoidance of students who were afflicted with math anxiety, choosing a sample of students they believed would be populated by math avoiders: freshmen with no college preparatory mathematics courses, seniors who had had no college mathematics courses, freshmen who were enrolled in college algebra, students enrolled in a workshop for the math anxious, and 43 elementary education majors (only six of which had gone beyond college algebra in their mathematics preparation). To assess their mathematics anxiety, they administered the Mathematics Anxiety Rating Scale (MARS) which is a 98-item, self-rating scale that had been developed in 1972 by Richardson and Suinn and included normative data. They found that the elementary education majors scored higher (indicating higher math anxiety) than all other groups except those in the math anxious workshop. They also found that the female education majors were particularly math anxious compared to the other groups. They suggested that this high anxiety was caused by societal expectations more than ability, but they hypothesized that the high number of women elementary school teachers may be perpetuating math anxiety with young girls in their own classrooms. They recommended that immediate help be given the prospective elementary school teachers to help them understand and overcome their math anxiety.

From the latter 1990s up until this time, there has been some renewed interest in studying preservice teachers' attitudes toward mathematics —especially by doctoral students in research they have done for and reported in their dissertations. Some of those studies which are most relevant to my study will be reviewed in the following paragraphs. For example, in 2002, Patton also examined the mathematics anxiety and preparedness of preservice teachers at the University of Arizona, collecting data from 87 preservice teachers who were enrolled in a mathematics teaching methods class. She analyzed her data using statistical tests that assessed the relationship between mathematical anxiety and the factors of content knowledge, motivation, and perceptions of confidence and competence in conjunction with the *Arizona Mathematics Standards*. She found that students with "higher mathematical backgrounds" reported significantly lower levels of mathematics anxiety. The areas from the *Arizona Mathematics Standards* that were the sources of the most anxiety for these preservice teachers were functions in algebra, geometry, and measurement and discrete mathematics. The preservice teachers' perceptions of competence and confidence in teaching the *Arizona Mathematics Standards* were shown to be a significant predictor of their level of mathematics anxiety.

Three recent studies suggest that a college-level math/math methods course can influence students attitudes toward mathematics. Gregg (1998) conducted a study in which she looked at the attitudes toward mathematics and knowledge of mathematical concepts of preservice elementary, early childhood, and special education teachers *and* if these attitudes improved as the result of taking a specific mathematics course. She administered an "attitude instrument" to students who were enrolled in required mathematics courses for elementary education at the beginning and end of the semester in the fall of 1997 to determine their "self-efficacy in mathematics, their feelings about problem solving, and how they value mathematics." She also interviewed 15 students to determine their attitudes toward mathematics and when these attitudes were developed. She concluded that "students tend to improve their attitudes toward mathematics when taught by instructors who model appropriate strategies;" in other words, instruction influenced attitudes.

In a related study, Smith (1999) looked at preservice elementary teachers' attitudes toward mathematics and the teaching of mathematics in a "constructivist" classroom. This was a study of 28 students who were enrolled in a section of a geometry content course that was required for preservice teachers—and was taught using "constructivist" methods. Their attitudes were determined through the use of surveys, interviews, and questionnaires. Smith found that the attitudes of those preservice teachers were more positive after participation in a "constructivist" classroom, and she concluded that the results of the study "indicate that participation in a constructivist classroom does positively affect preservice teachers' attitudes toward mathematics teaching and learning."

Another study that looked at the effects of "constructivist instructional methods on preservice teachers' attitudes toward teaching and learning mathematics and science" was conducted by Gibson and Van Strat (2001). This study was a three-year study designed to track the changes in preservice teachers' attitudes toward teaching and learning mathematics and science while they were enrolled in an Urban Preservice Degree Articulation in Teacher Education (UPDATE) Project. This project was designed to provide a pathway for urban para-educators of color to become certified teachers. During the first year of the project, the preservice teachers were exposed to mathematics content using constructivist instructional approaches which they identified as "collaborative group work, problem solving, the use of manipulatives, and calculators" in three courses. The next year the preservice teachers took a biology course where the content was delivered through traditional methods of lectures and note taking. The following year, the preservice teachers took a redesigned introductory physical science course that was taught using constructivist instructional methods. The preservice teachers enrolled in Introduction to Physical Science completed three questionnaires that were each administered twice to the students—once during the beginning of the course and once at the end of the course. They also used journal entries, exit surveys, and interviews as data sources. The researchers found that the constructivist instructional methods that were used in science and mathematics courses had a positive influence on the preservice teachers' attitudes toward mathematics and science as well as their understanding of mathematical and scientific concepts while the more traditional methods resulted in a negative influence on the teachers' interest in teaching science.

Similarly, Couch-Kuchey (2002) reported on a study in which she examined the "effect of a constructivist mathematics methods course on the level of mathematics anxiety in preservice early childhood teachers." She argued that since teacher candidates often score above average on the Mathematics Anxiety Rating Scale (MARS), a mathematics teaching methods course "should be designed to build confidence, help alleviate mathematics anxiety, and promote effective teaching and learning" since such a course is "typically the last opportunity to influence preservice elementary teachers' attitudes towards mathematics." With that rationale, she designed this study to determine the effects of a "constructivist" mathematics methods course on 41 early childhood preservice teachers' level of mathematics anxiety. The participants completed the Mathematics Anxiety Rating Scale (MARS) at the beginning of the semester, after the course component, and at the completion of their fieldwork. Through results of a paired samples test, she found that there were significant differences in the paired samples from the pre-test to the post-test which indicated that the mathematics methods course was effective in reducing the level of anxiety among preservice elementary teachers. She also found through follow-up email questionnaires that "exposure to current, effective teaching strategies, materials, and manipulatives, and the applying of these components in lessons taught in the fieldwork" reduced the level of anxiety in preservice early childhood teachers.

Adkins and Robinson (2002) also conducted a study in which they examined the effects of mathematics methods courses on preservice teachers' attitudes toward mathematics and mathematics teaching. They surveyed preservice teachers at the beginning and end of four hours of coursework related to mathematics methods to determine their attitudes about mathematics and teaching mathematics. They were asked to rate their attitudes on a scale from 1 to 10 and to describe how their attitudes toward mathematics were formed. They followed up this initial research by assessing strategies that the preservice teachers used as they taught mathematics during their student teaching and their first year of teaching. They found that these teachers used strategies related to constructivist teaching. They also found that "students do change attitudes in the short term; however, long-term changes still need to be examined."

In another related study, Wilkinson (2001) looked at the attitudes of prospective elementary teachers enrolled in an elementary mathematics content course, but she also looked at the *foundations* of these attitudes toward mathematics learning and teaching. To learn about these foundations, Wilkinson permitted the students to share their "life-world perceptions as they learned mathematics," which were "perceptions that helped to form their attitudes about learning and teaching mathematics to children." She argued that while there was evidence that attitudinal changes can occur while preservice teachers are enrolled in mathematics methods courses, studies have shown that these changes may not be significant or lasting. The purpose of her study was to help students engage in a reflection process that asked the students to examine the foundations of their then-currently-held attitudes in hopes that this experience would facilitate lasting positive attitudinal changes. She selected nine students to be formally interviewed, and she used the transcripts of those interviews to write "life-world stories."

Allen (2001) also looked at factors that contributed to the math anxiety and avoidance behaviors of female elementary school preservice teachers. Allen examined 43 mathematics autobiographies and identified mathematical experiences in ten separate categories. He then used three of these categories—students' experiences with math teachers, math content, and math pedagogical practices—to develop experiences scales for those constructs. He also developed a fourth scale to measure experiences with math avoidance behaviors. In addition, he measured these female elementary school preservice teachers' math anxiety using the Revised Math Anxiety Scale. He analyzed his data with statistical tests. He found a strong correlation between math anxiety and the participants' mathematical content experiences. He also found moderate correlations for math anxiety and participants' experiences with math teachers. He identified the participants' experience with math content as a significant predictor of both math anxiety and math avoidance behaviors.

A Historical Overview of Research Relating to Beliefs. The 1970s brought the arrival of cognitive psychology which provided a place for the study of beliefs, and the work done by the attitude researchers seems to have laid some of the foundation for this line of study since much of that research directly or indirectly addressed teachers' beliefs and conceptions. In addition to a renewed interest in attitudes toward mathematics, the 1980s brought a renewed interest in beliefs and belief systems among scholars in education, but also among scholars in disciplines ranging from psychology to anthropology. This interest inspired the beginning of many studies that have focused on teachers' beliefs about mathematics and mathematics teaching and learning (Thompson, 1992). These researchers have worked from the premise that "to understand teaching from teachers' perspectives we have to understand the beliefs with which they define their work" (Nespor, 1987, p. 323). Most of the research on teachers' beliefs and conceptions of the past decade and a half has employed qualitative methods of analysis, and that has distinguished it from research done on teachers' attitudes in earlier decades which relied almost solely on data obtained through the use of a scale such as Dutton's scale or the MARS. The techniques they have used include Likert-scale questionnaires, interviews, classroom observations, stimulated recall interviews, linguistic analysis of teacher talk, paragraph completion tests, responses to simulation materials such as vignettes describing hypothetical students and classroom situations, and concept generation and mapping exercises (Thompson, 1992).

Some studies have been conducted to study the relationship between teachers' beliefs about the nature of mathematics and their teaching practice. Most of these have been conducted by examining the congruence between teachers' professed beliefs and their observed practice. The results from this research have indicated varying degrees of consistency between teachers' professed beliefs and their instructional practices. Other studies have been conducted to learn if teachers' beliefs about mathematics and/or teaching mathematics can or do change. These studies will be reviewed in the following paragraphs.

Thompson (1984) conducted case studies of three junior high school mathematics teachers to investigate the conceptions of mathematics and mathematics teaching that they held. The investigation sought to answer two questions: whether the teachers' professed beliefs, views, and preferences about mathematics and mathematics teaching were reflected in their instructional practices and whether the teachers' behavior was influenced by their conceptions. Each teacher was observed teaching mathematics daily over a four-week period. The first two weeks of each period the teachers were just observed, and the last two weeks daily interviews were conducted following the observed lesson. Each teacher also responded in writing to six tasks that were administered at different times throughout the case study. The purpose of five of the tasks was to learn about the teachers' views about various aspects of mathematics teaching. As described by Thompson, this information dealt with the teachers' views about: "the relative importance of various goals of mathematics instruction; the relative emphasis that should be given to several instructional objectives; the relative importance of several pedagogical practices; the more common reasons for students not making satisfactory progress in mathematics; and, the more valuable types of information in judging their own teaching effectiveness." The sixth task came from an instrument developed by Confrey (1978) that was used for assessing students' conceptions of mathematics. That task consisted of six bipolar dimensions that one might use to describe mathematics. This was used to obtain a description of the teachers' view of mathematics in terms of general characteristic qualities.

Differences among the teachers in the specific beliefs, views, and preferences that they held regarding mathematics and its teaching were evidenced through Thompson's case studies. There also seemed to be some relationship between their beliefs and their practices, but it was often subtle and not necessarily direct or simple. As she wrote, "Although the complexity of the relationship between conceptions and practice defies the simplicity of cause and effect, much of the contrast in the teachers' instructional emphases may be explained by differences in their prevailing views of mathematics." Two of the teachers she studied conceived of mathematics as a rather static body of knowledge, and both of these teachers presented the content as a finished product when teaching. However, one of them used a more "conceptual approach" that presented mathematics as "a set of integrated and interrelated topics," while the other one used a more "computational approach" that presented mathematics as a "collection of more or less arbitrary rules and procedures for finding answers to specific questions." The third teacher held a more dynamic view of mathematics and believed that the best way for the students to learn it was to be engaged in its creative generative processes.

Reflection seemed to play an important role in the formation of the beliefs of the teachers Thompson studied. The teacher who used a computational approach to teaching admitted her lack of reflectiveness upon her actions and their effects. The teacher who taught using a conceptual approach reflected mainly on the ease or difficulty of following her lesson plan or of getting her students to give the correct answers. In contrast, the teacher with the dynamic view of mathematics tended to reflect on her actions and their effects on her students. Through reflection she gained insights into possible causes of her students' difficulties and misconceptions, and this helped her become more aware of the subtleties inherent in mathematics.

Thompson also concluded that other conceptions that teachers possess about teaching that are general and not specific to the teaching of mathematics play a significant role in affecting their instructional decisions and behavior and for some teachers may take precedence over other conceptions specific to the teaching of mathematics. These could be conceptions about their students and the social and emotional make-up of their class or about how children learn.

In 1997, the *Journal for Research in Mathematics Education* published an article by Raymond in which she reported another study that looked at the relationship between a beginning elementary school teacher's mathematics beliefs and her teaching practice. Raymond gathered data for over 10 months through audio-taped interviews, observations, document analysis, and a beliefs survey. She analyzed the data by categorizing and comparing the beliefs and practices of this beginning elementary school teacher. She found that this teacher's beliefs and her practice were not wholly consistent. Instead, her practice was more closely related to beliefs about mathematics content than to her beliefs about how mathematics should be taught. Her beliefs about mathematics content had mainly influenced by her own experience as a student, and her beliefs about mathematics pedagogy were primarily influenced by her own teaching practice. She found that her teacher preparation program had little influence on her beliefs or her practice.

In a more recent study, Kyriakides, Philippou, and Charalambous (2002) from Greece also examined "teachers' philosophical beliefs about mathematics, the factors influencing the development of these beliefs, and their relation to teachers' beliefs and practices about teaching and learning mathematics." They analyzed data from 229 questionnaires and five interviews to create a "five-factor model, representing combination of the three dimensions of a model proposed by Ernest (1991)." They identified four groups of teachers and found a relative consistency between the teachers' philosophical beliefs and their beliefs regarding teaching and learning mathematics. However, they found inconsistencies between the teachers' philosophical beliefs and their teaching practices. They concluded that these inconsistencies could be partially attributed to factors that influenced the development of the teachers' philosophical beliefs. They recommended development of teachers' training programs that regarded the affective domain.

Some studies have not focused on the relationship between beliefs and practice, but have been conducted to learn about prospective elementary teachers' beliefs—and sometimes whether they are influenced by particular courses or other means of intervention. For example, Grant, Hiebert, and Weame (1994) reported on a related study that examined teachers' beliefs and their responses to reform-minded instruction in elementary mathematics. In this study, these researchers investigated the effect on teachers of observing or implementing an alternative form of instruction in an elementary mathematics class. They were interested in the relationships between the teachers' beliefs about mathematics and teaching mathematics and their "recognition and internalization of the principles of the alternative instruction." Some experienced teachers were hired to implement the alternative instruction, and the other teachers allowed the instruction to take place in their classroom while they observed.

The researchers interviewed the teachers to find out their views of what mathematics is of most value, how mathematics should be taught, their impressions of the project, and their reflections on how the project did or did not influence them. When the researchers analyzed the interviews, several patterns emerged. One was the differences between the teachers in their analysis of the main features of the instruction and the merits of those features. Another was a difference in the teachers' beliefs about mathematics and how to teach it. The third pattern they saw was the relationship between where the teachers stood in their analysis of the instruction and their beliefs about what is important in mathematics and how those things should be taught—in other words, the relationship between the previous two patterns. They found that this relationship was quite straightforward. The more the teacher's orientation resembled what they called the "skills/teacher responsibility" which is an orientation where teachers believe that what is to be learned and how it is to be learned is their responsibility and is usually skills and algorithms, the more they were focused on one particular aspect of the experimental instruction—and often on features the researchers believed were tangential. In contrast, the teachers whose orientations resembled what they called "process/student responsibility" which is an orientation where teachers believe they are responsible for creating an environment in which students have much of the responsibility for their own success, recognized what the researchers considered crucial aspects of the instruction, and they also understood the goals of the instruction. They concluded that the two most important influences on how teachers respond to alternative (more constructive) forms of instruction were the teachers' beliefs about mathematics and their beliefs about teaching and learning mathematics.

In a related study, Cooney, Shealy, and Arvold (1998) looked at the development of prospective secondary teachers' beliefs related to mathematics. In this study, the beliefs of four preservice secondary mathematics teachers were looked at as the students progressed through a four-quarter sequence in mathematics education that included student teaching. They found that "the ways in which the teachers structured their beliefs helped account for the fact that some beliefs were permeable whereas others were not." They also posited a scheme for conceptualizing the professional development of preservice teachers--naive idealist, isolationist, and connectionist.

In a similar study that focused on prospective elementary teachers rather than prospective secondary teachers, Vacc and Bright (1999) looked at preservice elementary school teachers' changing beliefs and instructional uses of children's mathematical thinking. In this study, 34 participants were introduced to Cognitive Guided Instruction (CGI) as part of a mathematics methods course. These participants were administered a belief-scale instrument, and their scores indicated that across the two-year sequence of professional course work and student teaching in their undergraduate program, there were significant changes in their beliefs and perceptions about mathematics instruction. However, their use of knowledge of children's mathematical thinking in their planning and teaching was limited. They concluded that while preservice teachers may acknowledge the tenets of CGI, they may be unable to use them in their teaching.

Campbell (1998) studied the changes in student teachers' thinking and beliefs about mathematics at a different time—during their student teaching experience. She measured these changes through the use of pre-teaching and post-teaching surveys, student teacher observations, and interviews. She considered four domains—personal attitudes and behaviors; perceptions about the nature or structure of mathematics; purposes for teaching mathematics; and the teacher's role in teaching and learning mathematics. She found no statistically significant differences in the mean scores in relation to total score, personal attitudes and behaviors, perceptions about the nature or structure of mathematics, and purpose for teaching mathematics. However, she did find a statistically significant difference in the area of the role of the teacher in teaching and learning mathematics. In this case, the changed moved away from viewing the role of the teacher as a facilitator to viewing the role of the teacher as an expert who shows or models.

Review of Research Relating to Both Beliefs and Attitudes. Recently, some research has been conducted that has looked at *both* the beliefs of and attitudes toward mathematics and teaching mathematics of teachers. One of these was a study done by Hare (1999) that looked at what urban early childhood teachers think about mathematics and how they teach it. In this study, 397 urban early childhood teachers from two school districts were administered a survey that examined their attitudes toward mathematics and mathematics teaching, their views of mathematics, views of teaching mathematics, and views of children learning mathematics. The surveys were analyzed with statistical tests. She found that teachers with 30 or more years of teaching experience had more positive attitudes toward mathematics than teachers with 1-3 years of experience. She also found that African American teachers had more positive attitudes toward mathematics and mathematics teaching than other ethnic groups and teachers who had a minor or major in mathematics had more positive attitudes than those who did not. She concluded that if instruction is to be transformed, reformers need to understand teachers' beliefs about mathematics, and that, as research had shown, since beliefs seldom change without significant intervention, "school districts must be informed about the changes necessary for the reform of mathematics teaching and identify and implement through staff developments and other measures what they perceive mathematics to be and how it should be taught."

In another study that looked at both beliefs and attitudes, Miller (1999) examined the mathematics knowledge of preservice elementary teachers within the context of a course that had been designed to be consistent with mathematics reform. She looked at questions related to,

> "(a) what preservice elementary teachers need to know to teach early childhood mathematics in a way that is consistent with the NCTM *Curriculum and Evaluation Standards* and the NCTM *Professional Standards for Teaching Mathematics*, (b) how preservice elementary teachers' mathematics knowledge can be categorized into Shulman's (1987) three types of knowledge), (c) what beliefs and attitudes about mathematics preservice elementary teachers bring with them to a mathematics course, and (d) how effective is a course that focuses on Shulman's types of knowledge and attitudes and beliefs in changing knowledge, attitudes, and beliefs."

(Shulman's three types of mathematics knowledge are subject matter, pedagogical content, and curricular.) She studied 28 students who were taking a first-semester "Mathematics for Elementary Teachers" content course. As data sources, she used documents from the course such as "initial activities and problem sets," journals, portfolios, task-based assignments, examinations, and course evaluations. She also interviewed six students five times to obtain collaborating data. Related to beliefs and attitudes, she found that most students identified school experiences and teachers as the primary source of their beliefs about mathematics; most students initially felt that mathematics was the study of numbers and that elementary teachers needed to know the "basics;" and a majority of the preservice teachers suffered from mathematics anxiety. She found that the course was effective in "changing some students' inhibiting beliefs and attitudes regarding mathematics and in reducing mathematics anxiety."

Krows (1999) also looked at the influence a teacher preparation program could have on "preservice teachers' belief systems and attitudes toward mathematics." She designed the study to answer three questions: "What are the attitudes and personal, pedagogical, and epistemological beliefs about mathematics held by preservice elementary and early childhood teachers before, during, and after the completion of teacher preparation coursework; what are the relationships among these beliefs and attitudes; and do these beliefs and attitudes differ as a function of the prospective teachers' educational experience." She studied a sample of 226 undergraduate and graduate students enrolled in the elementary education program. She surveyed 15 sections of six different classes over a three-semester period. She organized the participants into five groups based on their teacher education experience. The participants completed an 83-item Likert scale questionnaire in either a freshman-level mathematics class, an educational psychology class, or a mathematics methods class. She used statistical tests to examine the five groups. She found that regardless of the semester, the instructor, or the course, students who had participated in the elementary teacher education program held significantly different beliefs than those who had not. These preservice teachers had "significantly higher self-efficacy with respect to their ability to effectively teach mathematics." They also reflected a "strong adoption of the National Council of Teachers of Mathematics *Standards'* (1989, 1991) recommended constructivist orientation toward teaching mathematics," and they showed a "sophisticated epistemological view on the nature of mathematics as a dynamic, ever changing, problem driven branch of science."

Wilkins (2002) conducted a study in which he looked at the influence of teachers' content knowledge *and attitudes* on instructional beliefs and practices. He examined 407 in-service elementary teachers' level of mathematical content knowledge, attitude toward mathematics, beliefs about effective instruction, and use of reform-oriented instruction. He then modeled the relationship among those variables. He found that upper-elementary teachers (grades 3-5) had greater content knowledge and more positive attitudes toward mathematics than primary teachers (K-2). He found no difference in the teachers' beliefs about effective instruction, but he found that primary teachers used reform-oriented instruction more frequently than did upper-elementary teachers. He also found that teachers with a higher content knowledge were "less likely to believe in the effectiveness of reform-oriented instruction and less likely to use such instruction in their classrooms" and that "teachers with more positive attitudes towards mathematics were more likely to belief in the effectiveness of reform-oriented instruction and use it in their classroom."

Thoughts about These Empirical Studies. For the most part, the research that has been conducted in this area seems to have been reasonable. A variety of approaches such as questionnaires, observations, interviews, essays, and writing of short statements have been used that all provide valuable information. However, the research that has been done in this area has also been *limited* by the methods that have been used to obtain data. Some of them have relied only on one source of information to understand the teachers' beliefs of their practice. One source can give an incomplete picture, especially when that source is the teachers' professed views. A combination of sources would provide a more complete and accurate picture.

The empirical studies of the influence of mathematics teachers' beliefs on their practice that I looked at found a fairly consistent expected relationship, for the most part (Kyriakides et al, 2002; McGalliard, 1983; Raymond, 1997; and Thompson, 1984). Kyriakides and others (2002) and Raymond (1997) found what they characterized as variability. In her study of one beginning teacher, Raymond (1997) found much inconsistency between that teacher's beliefs and her practice. While Kyriakides and others found some relationship between beliefs and practice, they also found some inconsistencies that they thought could be partially attributed to factors that influenced the development of the teachers' philosophical beliefs. On the other hand, Thompson and McGalliard found high consistency. In fact, these researchers found more consistency in this relationship than researchers who looked at the relationship between beliefs about teaching (rather than about the subject they are teaching) and instructional practice. However, these researchers were careful to point out that the relationship was complex and could not be solely explained by a cause and effect relationship of beliefs on instructional practice. Another thing these particular researchers were careful about was using more than one source to measure or categorize beliefs. Although, again, this involved the paradox previously addressed, and these researchers did not mention how they dealt with that.

Research appears to support—though not unambiguously—the common-sense belief that teachers' views of mathematics influence their teaching practice. However, I also have to acknowledge that they aren't the only things that influence it and that the relationship is very complex. It seems clear that there is not a linear causality in this relationship and that they are more dialectically related than related by cause and effect. Another thing that complicates this relationship is that it is difficult to determine *how* and *how much* teachers' conceptions of mathematics influence their practice. Still, there is much contrast in teachers' instructional emphases when they are teaching mathematics, and it seems that much of that contrast can be explained by or attributed to differences in their views of mathematics.

Some of the research related to teachers' beliefs about mathematics looked at teachers' views of "reform-minded" or "constructivist" instruction (Grant, Hiebert, & Weame, 1994). Others looked at whether these beliefs change—especially during, and perhaps as a result of, a "math methods" course (Campbell, 1998; Cooney et al, 1998; Kyriakides et al, 2002; Vacc and Bright, 1999). Usually these researchers found a change in the teachers'/preservice teachers' beliefs, but often they found that this change was not visible in their teaching (Vacc & Bright, 1999). Campbell (1998) found an unwanted, but maybe not-so-surprising change in the student teachers' beliefs about the role of the teacher. In this case, the changed moved away from viewing the role of the teacher as a facilitator to viewing the role of the teacher as an expert who shows or models. The constraints associated with student teaching such as working with a specific cooperating teacher who will evaluate the student teacher and the student teachers' own inexperience teaching may have contributed to that change.

These researchers used a variety of methods to obtain their data such as having teachers observe alternative instruction, interviews, surveys, and observations.

The research that has been done related to teachers'—especially preservice teachers'—attitudes toward mathematics has also shown slight variability in results. However, virtually all of that research has shown that many preservice elementary teachers have negative attitudes toward mathematics (Allen, 2001; Becker, 1985; Bulmahn and Young, 1982; Dutton, 1954; Dutton, 1962; Kelly and Tomhave, 1986; Reys and Delon, 1966; Smith, 1964).

The only study of those just cited that looked at change in preservice teachers' attitudes toward math was the study done by Reys and Delon (1966). However, several other researchers have studied whether these attitudes can or do change. Usually they have tried to learn if there can be change in those attitudes during the time they are taking a "math methods" course. All of that research (of which I am aware) has shown at least a temporary improvement in the preservice teachers' attitudes (Adkins and Robinson, 2002; Couch-Kuchey, 2002; Gibson and Van Strat, 2001; Gregg, 1998; Hendrix, 2003; Miller, 1999; Reys and Delon, 1966; Smith, 1999; Wilkinson, 2001). Interestingly, a couple of researchers also found that teachers who have more favorable attitudes toward mathematics tend to prefer to teach the upper grades (4-6), while those with less favorable attitudes most often prefer to teach grades K-3 (Kane, 1968; Wilkins, 2002). In two of the studies, researchers tried to learn about factors that contributed to preservice teachers' negative attitudes (Allen, 2001; Patton, 2002).

Most of the research studies done related to teachers' attitudes toward math and/or teaching math used two or more sources for their data (Adkins and Robinson, 2002; Allen, 2001; Gibson and Van Strat, 2001; Gregg, 1998; Miller, 1999; Smith, 1999; Wilkins, 2002). However, some of them only used a survey or questionnaire that was administered two or more times and analyzed using statistical methods of quantitative research (Couch-Kuchey, 2002; Hare, 1999; Krows, 1999; Patton, 2002). Those who were able to show the same conclusions through the use of multiple data sources seem the most persuasive.

What these studies do not tell us is whether these changes are long lasting. They also don't tell us much about where the change is, what the change looks like, and why there was change. These are ideas that can be explored further in future research.

A few studies have looked at both attitudes toward and beliefs about mathematics of teachers or preservice teachers. Hare (1999) found that the more experience teachers had teaching mathematics, the more likely they were to have a positive attitude toward it. Miller (1999) found changes in both attitudes and beliefs in preservice teachers when they experienced a math methods course based on Shulman's framework. Krows (1999) also found changes in both attitudes and beliefs in preservice teachers after they had participated in the elementary preparation program—especially in comparison with students who had not. Wilkins (2002), found that upper elementary school teachers had greater content knowledge and more positive attitudes toward mathematics than primary grade teachers, but he also found that primary teachers used reform-oriented instruction more frequently than did upper elementary teachers. He also found that teachers with more positive attitudes towards mathematics were more likely to believe in the effectiveness of reform-oriented instruction and use it in their classroom. These research studies were mainly done with questionnaires and quantitative statistical analyses.

Although these researchers were careful about many things in conducting their research and coming up with their conclusions, there are other important questions that they did not address. One thing they did not address or account for is the idea that belief systems are not static. Of course, that is not easily done. Other important questions overlooked by these researchers is the extent to which teachers' and students' beliefs interact during instruction and the extent to which other influences such as school requirements, inexperience or insecurity, and lack of support have on instruction. Another possibility is that the way teachers know how to teach influences what they think math is. These are things that researchers should at least make an attempt to address and account for.

These are things that researchers who look at this relationship in the future should consider.

Some General Thoughts about This Literature

The research that has been done regarding the relationship of teachers' beliefs about mathematics and their teaching practice both helps us understand this relationship and raises questions about it. In the following sections I will note some of the reasons this is a complicated and a difficult line of research.

Difficulty of Defining and Categorizing Beliefs. The concept of beliefs has not been dealt with in any substantial way in the research. Researchers either give cursory and vague definitions of beliefs (e.g., as a synonym for thought), or they just seem to assume that the readers know what beliefs are. Most often they do not try to define beliefs, but they note characteristics of them, perhaps alluding to a vague definition (Nespor, 1987). This is not surprising, however; beliefs are very difficult to define and to categorize. This is also the case with the idea of belief systems (a metaphor for how beliefs are organized that is sometimes talked about in these studies rather than simply "beliefs"). There are many characteristics of beliefs and belief systems that cause this difficulty.

One of the most prominent characteristics that causes this difficulty is that beliefs and belief systems are dynamic. They are ever changing. They are not static entities to be uncovered. It's difficult to define something that is always changing.

Another characteristic associated with the difficulty of defining beliefs is that they are not held in total independence of all other beliefs. Particular beliefs that are held by the believer are influenced by other beliefs and sometimes modified to fit in with other beliefs also held by the believer. This characteristic supports the usefulness of the idea of thinking of them as "belief systems."

One final example of a characteristic of beliefs that makes them difficult to define and categorize is that beliefs are held with varying degrees of conviction which influence what the beliefs actually are and the extent of their effect on behavior. This makes looking at the relationship between beliefs and practice complicated since it's hard to define varying degrees of conviction and each degree's expected influence on behavior.

Difficulty of Measuring Beliefs. Another thing that makes it difficult to study the effects of teachers' beliefs about mathematics on their practice is that beliefs are difficult to measure. The researcher can't just use teachers' professed views since teachers may profess to believe what they think the researcher expects or wants them to believe. The teachers may feel that the researcher will think negatively toward them if they admit to believing something that is not popular or in line with the perceived agenda of the researcher. People may not even be sure what they believe. Their opinions may be conflicting enough that they do not feel like they can make a stand. It's also sometimes the case that beliefs are somewhat hidden—at least partially—even to the possessor because they may have never thought about what they believe about a particular topic except perhaps superficially or subconsciously. (This idea causes me to wonder if getting teachers or preservice teachers to think about, figure out, and confront what they believe is an important step in changing [or supporting] their beliefs?)

In addition to teachers' professed beliefs, researchers must look at other things such as the social context, the instructional setting, practices characteristic of the teachers, and the relationship between the teachers' professed views and their actual practice. Even in 1965, Scheffler noted that

> "with independent knowledge of the social context, we may judge belief as revealed in word and deed. Where these latter two diverge, we may need to decide whether to postulate weakness of will, or irrationality, or deviant purpose, or ignorance, or bizarre belief, or insincerity, and the choice may often be difficult. . . It will,

> in any case, never be reasonable to take belief simply as a matter of verbal response: belief is rather a "theoretical" state characterizing, in subtle ways, the orientation of the person in the world." (p. 89-90)

But this leads us to a perplexing paradox: it seems contradictory that in studies looking at the influence of teachers' beliefs on their practice, their practice is used as a measure of their belief. It's almost a circular or maybe tautologous logic. It seems like researchers already have to believe that what they are looking for is true if they consider teachers' practice a valid measure of their beliefs. Still, it's certain that more than a verbal response must be looked at to ascertain beliefs, and it's reasonable to believe that actions are at least often an indication of belief. If people modify new beliefs to fit into the beliefs they already hold, however, the beliefs may not be enacted in a way that would theoretically be expected. Actions, therefore, can be a statement of beliefs, but not fully; actions plus commentary by the person acting give a better, fuller picture of true beliefs. In a research situation, it would be just as weak to base a characterization of a person's beliefs only on his or her actions as it would be to base it only on his or her report.

Another thing that makes observation of practice a complicated measure of beliefs is that people see the same thing in different ways. The teacher might have a very different view of what happened than the observer did—even if the teacher sees a video-taped review of it. These are puzzling contradictions that researchers who look at this question have to deal with.

Relation of This Research to My Study

In this section, I will explain how I believe this research relates to and supports my dissertation questions. I will then discuss what I see as the relationship between this research and my study.

How Does This Research Support My Dissertation Question? In this study, I am looking at the question of how and whether the conceptions preservice teachers have about mathematics change and develop during a class designed to help prepare them to teach mathematics. Clearly this question only matters if teachers' conceptions of mathematics are important influences on how they will teach it.

There are certain ways to teach mathematics that research has indicated and the NCTM *Principles and Standards* (2000), among others, have suggested are more effective to help students gain a conceptual understanding of mathematics as opposed to a simple knowledge or rote memorization understanding. These methods are directly connected to certain philosophies of mathematics, and both the theoretical and the empirical work I have reviewed have lent support to the idea that teachers with particular philosophies of mathematics tend to teach in particular ways.

In all the research I am familiar with, there was no case in which someone who taught math in a constructivist way (i.e., as something that was created and discovered to make sense of patterns and relations in the world) didn't see math in a multiplistic/constructivistic/relativistic way. However, there was one case (Thompson, 1984) in which a teacher who thought of mathematics as a rather static body of knowledge taught with a "conceptual approach" that presented mathematics as "a set of integrated and interrelated topics." Seeing mathematics as a set of integrated and interrelated topics is certainly a part of teaching mathematics conceptually, but that can be done without teaching it in a constructivistic way. In contrast, the teacher in Thompson's study who held a more dynamic view of mathematics thought the students needed to be engaged in its creative generative processes to learn mathematics. On the other hand, *all* those who thought of math as arithmetic computation and memorizing algorithms (absolutistic or dualistic), taught it that way. There were also some who thought of mathematics in a multiplistic way who taught it in a way that would be expected of teachers with an absolutist view. The important point here is that if teachers have an absolutist view of mathematics, they do not teach it in a multiplistic way. So, I would argue that if we want teachers to teach mathematics in a conceptual way, it is necessary, but not sufficient, for teachers to think of math in a conceptual way. My contention is not that they have to learn to look at it that way from a "methods" course, but that something has to help them see math that way. However, if that is true, I think it justifies looking at prospective teachers' conceptions of mathematics in a "methods" course and striving to help them broaden their view—which gives me reason to care about my question.

Relation of this Study to Other Research. This study builds on much earlier research in teacher education. Some of the connections are worth noting. For decades much educational research has focused on prospective elementary teachers' attitudes about mathematics, especially whether they liked it or not or felt confident in their own abilities (Allen, 2001; Becker, 1985; Bulmahn and Young, 1982; Dutton, 1954; Dutton, 1962; Kelly and Tomhave, 1986; Reys and Delon, 1966; Smith, 1964). This research serves as a foundation for my study. However, this body of research did not look at the prospective teachers' attitudes toward the prospect of teaching mathematics, nor did it look at change in attitudes, as mine does. One thing this study may provide insights on is if there is a relation between students' attitudes related to mathematics and their attitudes about the idea of teaching it.

Some researchers did look at change in attitudes toward mathematics of preservice (or in-service) teachers (Adkins and Robinson, 2002; Couch-Kuchey, 2002; Gibson and Van Strat, 2001; Gregg, 1998; Hendrix, 2003; Miller, 1999; Reys and Delon, 1966; Smith, 1999; Wilkinson, 2001). My study will add to this body of research, and it will also add the dimension of looking at changes (or lack of changes) in conceptions (or beliefs) while looking at attitudes.

This study also builds on research that has been done related to beliefs about mathematics. The research that has been done to see if beliefs influence practice will provide a foundation for my research (Kesler, 1985; Kyriakides et al, 2002; McGalliard, 1983; Raymond, 1997; and Thompson, 1984). Others have researched whether these beliefs change (Campbell, 1998; Cooney et al, 1998; Hendrix, 2003; Kyriakides et al, 2002; Vacc and Bright, 1999), and their studies have similarities to my study, but without attitudes being looked at. My study will also add to this body of research.

Some research has been conducted recently that is similar to my study by novice researchers, such as myself, who looked at changes in both attitudes and beliefs—mostly for their dissertation studies (Hare, 1999; Krows, 1999; Miller, 1999; Wilkins, 2002). This research is similar to my study. However, they did not all study preservice teachers, and the combination of data sources that I am using is different from what was used in each of these studies. Again, my study will add to this body of research.

Other research (See for example, Ball, 1988) looked at the beliefs and attitudes of prospective teachers but not their development across a portion of the students' teacher preparation experience. This evolution is something that will be added with this study.

Another aspect that this study adds to the research related to prospective teachers' conceptions of and attitudes toward mathematics and teaching mathematics is the inclusion of case studies. Doing case studies is an addition to the research because it permits me to examine the ecology of what such a case looks like. Whereas Gregg (1998), Smith (1999), and others link the instructional approaches of the college teachers—or teacher educators—to changes in the prospective teachers' conceptions and attitudes, these cases allow me to look closely at both students who do change and students who do not change in the context of a class in which others are changing.

Preview of the Upcoming Chapters

In the next chapter, I will outline the methodology that I used in conducting my study. I will describe the overall design, the subjects and situation, the data sources, and how the data was collected, synthesized, and analyzed.

In Chapters 3, 4, and 5, I will look at the general trends of the group in relation to the changes and stability of the students' conceptions of and attitudes toward mathematics and teaching mathematics as many researchers have done before. These chapters provide a big picture of what ideas and attitudes might remain stable and what might change for a group of students in a methods class.

In Chapter 3, I examine the conceptions of these prospective elementary teachers in relation to the nature of mathematics and the usefulness of mathematics. Other research has looked at beliefs about mathematics of prospective teachers, but they have done it mainly by looking at the prospective teachers' views of the general characteristics of mathematics. I will look more closely at their conceptions related to content, processes, representations, and characteristics of mathematics. (These categories will be described in Chapter 2.) I know of no other research that has studied the prospective teachers' conceptions of the usefulness of mathematics.

Chapter 4 is an examination of these prospective elementary teachers' conceptions of learning and teaching mathematics. I analyze what I learn about how these prospective teachers think math is learned and how they think it should be taught. This is another aspect of prospective teachers' beliefs about mathematics that has not often been examined in previous studies.

In Chapter 5, I scrutinize the attitudes of these prospective elementary teachers toward mathematics and teaching mathematics. It is here that we see the most significant changes in the prospective teachers ideas over the course of the semester; the chapter analyzes which of these attitudes change and considers what the data indicates about the mechanism of change.

In Chapters 6 and 7, I examine the stability and change of the four focus students in relation to their conceptions of and attitudes toward mathematics and the teaching of mathematics. These cases permit me to look much harder at two students who didn't change much and two students who changed quite a bit. These are helpful in relation to understanding what this change or lack of change might look like—and how it looks to the students themselves.

Chapter 6 examines the cases of two students who entered the course with very negative attitudes toward mathematics and teaching mathematics. These students did not change their attitudes, and this lack of change is something that is not examined in the studies of large groups of prospective teachers who generally do show change.

In Chapter 7, I present the cases of two other students who also entered the course with very negative attitudes toward mathematics and teaching mathematics. However, their attitudes do change. This close-up look at these two students helps us understand what this change might look like and what factors may have influenced that change.

The final chapter—Chapter 8—is a summary of the major findings, implications, and contributions of this study.

CHAPTER 2
METHODOLOGY

Overall Design

The overall design of this study employed mainly qualitative methods of research, but also some quantitative methods. Prospective elementary school teachers who were senior students in their teacher training and were enrolled in a class designed to help prepare them to teach mathematics were the sources of information collected by the various data instruments and collection methods. They were administered an inventory at the beginning and end of their course; they made math concept maps at the beginning and end of their course; and they also wrote various journal entries that were used. These students didn't do any assignments they would not have done if this class were not used for this study, but their writings, concept maps, and inventories were used for research purposes. A few students served as focus students who were interviewed in addition to the writings and surveys done by the entire class. Most of the data were analyzed qualitatively with quantitative descriptors when appropriate, and the main part of the inventory was analyzed quantitatively.

Subjects and Situation

The students I studied in connection with this research were the 34 preservice elementary school teachers whom I taught in section 17 of TE 401, "Teaching Subject Matter to Diverse Learners," during Fall Semester 1995 at Michigan State University. Students in the two other sections of TE 401 for Team Two were also used as comparison groups for the inventory that was administered at the beginning and end of the semester.

Focus Students. Early in the course, I chose six students to serve as focus students. These students were interviewed on two separate occasions, and I used their interviews as well as their individual collections of all the data sources to do individual case studies of them. I selected these six students to be focus students and to be interviewed so that I could find out more in depth than I could from the other class assignments what they thought about math and teaching math and so I could have some individual, specific examples of how these conceptions and attitudes may or may not change.

There was not a lot of diversity in outward measures in my class such as gender, race, and age. Thirty-one of the 34 students in the class were female; one student had Asian ancestry, and the others were Caucasian; and all but two or three students were about 22 years old. However, I chose focus students to represent the diversity there was in the class in experience and background. In this particular class, there were only three students I would categorize as having a positive attitude toward math at the beginning of the semester, and the other 31 students had attitudes that were somewhat negative to very negative toward mathematics. I chose students who, based on their writings and the comments they made in class, were not afraid to express their opinion and did not appear to feel any obligation to say what they thought I might like to hear. One of the focus students was male, and the other five were female. The male had had positive experiences with math and had a positive attitude toward it. The five females had had negative experiences with math and had negative attitudes toward it. One of them was a 35-year-old woman who had returned to college to finish her degree, and the other five were 22 years old. I also had a large number of students who were majoring in child development—preparing to teach preschool—as well as elementary education in my class, and two of the focus students had this major.

In the end, in order to keep this dissertation to a manageable length, I chose to use four of these students in constructing case studies for this dissertation. Having begun analysis of all six cases, I was able to choose four who represented the full diversity of the group.

Teacher/Researcher Issue. Since this was a study of my own students, and I was both the teacher and the researcher, special considerations were made. In this section, I will try to outline why I chose to be both the teacher and researcher and what I did to alleviate some of the challenges associated with this type of research.

A major reason I felt justified in using my own class is that I did nothing in conjunction with this study that I would not have done as a part of my normal teaching practice with this class. For example, as a part of my normal teaching practice as a teacher educator, I have the students write in their own course journals about their prior experiences with mathematics and teaching mathematics and their expectations for the course. I also make audio records of each of the class sessions and of my interactions with students so that I have them available to review in planning and teaching the courses. These kinds of records play an important role in the personal professional development of teacher educators like me. I also make copies of student work in order to track the development of my students over the duration of the course. This is founded on the belief that the *evolution* of students is crucial in understanding their learning across the semester, and in adapting approaches and materials to meet students' needs. Another part of my normal teaching practice in this course was to have the students respond to an inventory the first day and the last day of the course. This helped me learn more about their experiences with mathematics as well as their ideas about mathematics and teaching mathematics. It also helped me learn if their ideas changed during the course. I also kept a teaching journal of my own practice as a teacher—in part, as a way of keeping track of student learning but also as a part of my own professional development.

These records—of the class sessions, of student work, and of the instructor's thinking—are things that I would have collected and used in my teaching of the course irrespective of any research agendas. Students were informed of these practices and if a student was uncomfortable with this style of teaching which involves this careful tracking of the course and of student evolution, I was willing to assist the student in finding an alternative section of the course to enroll in where the instructors do less of this kind of work. None of the students in the course opted to change sections.

These records were gathered for the purposes of *teaching* the course. At the end of the course, the students were asked if they were willing to allow me to use the records I gathered for research and educational purposes AFTER the course was over and grades were determined. Whether they granted or denied permission was kept confidential. The students were also informed that granting or denying permission would not influence their grades, field placements, or other decisions about their development and learning in their teacher education program. This was done in order to minimize feelings of coercion the students might have experienced about participating in a research study in which their instructor was involved, and to maximize the students' awareness of what they were granting me permission to study. The students were asked to sign consent forms if they were willing to allow me to use their work in my research after the semester was completed. All of the students in the class signed the consent forms and were willing to allow me to use their work for my research. (See Appendices A, B, and C.)

Another potential difficulty with being both the researcher and teacher of these students who serve as subjects of the study is that my own students might have reason to tell me what they think I want to hear, hoping to influence my opinion of them and/or their grades in a positive way. Some might argue that I might be able to learn more about students' conceptions and attitudes if I were to randomly select students from the teacher preparation program and administer my instruments to them. That would perhaps lessen the potential problem of the students saying or writing what they thought I wanted to hear. However, subjects of a study almost always form some opinion about what they believe is the agenda of the researcher, and there is always the possibility that they will say what they think the researcher or interviewer wants them to say—or something that they think contradicts that view—whether it is their opinion or not. Additionally, it cannot be assumed that just because a student/subject says what I would like to hear that it is *not* what the student thinks. Using my own students, I have the advantage of hearing them contribute to class discussions, seeing them work on math problems, seeing and hearing them talk and work in small groups, and reading all of their assignments. Being with these students in this type of classroom situation over the course of 15 weeks, I know them, and I have a bigger picture of them—and a bigger frame of reference in which to situate their comments and notice any hard-to-explain discrepancies—than I would if my only interactions with them were administering instruments and collecting data. I am also able to more fully see their development or lack of change in their conceptions of and attitudes toward math when I spend that much time with them, and I think that provides me with a much greater opportunity to learn more about my research questions listed in Chapter 1.

Data Collection and Analysis

I used three main sources of data and data collection to provide me with information that helped me answer my research questions. To look at the class as a whole, I used the students' own journal writings related to their reflections of their experience with mathematics and their ideas about mathematics and teaching mathematics, a mathematics inventory that I gave to the students at the beginning of the semester and the end of the semester, and the students' individual concept maps of math that they made at both the beginning and end of the semester.

To get more in-depth information about the focus students, another graduate student associated with another section of the same course (Sarah Theuele Lubienski) interviewed them during the semester, and I interviewed them a semester after the course was completed. I also used the collection of the individual journal entries and concept maps as well as their responses to the beginning and final Mathematics Inventory for each of the focus students to help me learn more about them, specifically. Another source I used to learn about both the group and the individuals was my own observation of their comments and participation in activities that were a part of TE 401. In the following paragraphs, more information about the four main methods I used to research my questions is provided.

Students' Journal Entries. During the semester, the students were given different topics to reflect and write about as part of their journal entries. The first day of class, the students were given the assignment to reflect on their own mathematical experience and to write about what stood out to them. They could write about one experience, their mathematical experience chronologically, or their mathematical experience as a whole. Their reflections were written as journal entries and were given to me the following class period. The second class period, which was also during the first week, the students were asked to write an entry about what they think mathematics is. They wrote this journal entry in class. At the end of the semester, the students were asked to write new (separate) entries about what they thought math is and another about how they thought it should be taught. These were the main journal entries I looked at, but other entries provided insight into my questions—especially in relation to the focus students—and were therefore used.

The analysis of these journal entries was qualitative. I analyzed them by reading them, looking for patterns and themes (and discrepancies), and then synthesizing and summarizing them by theme. To do this, I wrote each theme as a heading and under it typed each of the quotes from the students' journal entries that were related. I also analyzed these entries quantitatively by looking at the number of students who fell into the categories that were identified. Since I had looked at and analyzed these sorts of journal entries as part of a pilot study for this study, there were some themes I expected I might find such as influence of particular teachers, influence of another person, life experiences outside of school, classroom experiences, peer influences, and being good at reading vs. being good at math. I found all of these themes, but some of them were less prominent than they had been in my pilot study. I also found other themes that I summarized.

Mathematics Inventory. On the first day of class, I gave the students a Mathematics Inventory that focused on how the students think and feel about doing math themselves and about teaching math. A copy of this Mathematics Inventory is included as Appendix D. I had developed and piloted most of the items in the inventory in an earlier study (Tuft, 1994); a few of the items were adapted from a card sort task used by Ball (1988b). There were two parts to the Mathematics Inventory. The first part consisted of statements with a corresponding Likert-type scale ranging from one to five—1 corresponding to "not at all like me" and 5 corresponding to "very much like me." The students were instructed to circle the number that best describes how well the statement described them or something they might say. An example of one of these items is shown in Figure 3.

1. I enjoy doing mathematical tasks.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

Figure 3: Sample of Likert-type Item from the Mathematics Inventory

The second part of the Mathematics Inventory consisted of open-ended questions about the role or influence they perceive the teacher—specifically the teacher's attitude toward mathematics—playing in the mathematical success of their students and how they felt them about the prospect of teaching mathematics to elementary students. An example of one of these questions can be seen in Figure 4.

17. What concerns you the most about teaching mathematics to elementary students?

Figure 4: Sample of Open-ended Question from the Mathematics Inventory

The students completed this Mathematics Inventory on the first day of class and gave it to me when it was complete. They also completed it on the final day of class. I received a Mathematics Inventory from each of the 34 students in the class on the first day of class, but on the final day, due to absences caused by illnesses and other reasons, I only received 29. In all the analyses I did of the inventory I only used the 29 matched sets I had.

This Mathematics Inventory was also administered to two other sections of TE 401 at the beginning and end of the semester by their professors. The information obtained from these inventories was used to provide points of comparison for my own class (i.e., to see how typical my class was compared to others in relation to these items). The sample size was much smaller for each of the comparison classes than it was for mine (N=11 and N=18), so it was difficult to make a definitive comparison, but the range of responses of the students in my class compared seemed to not be atypical of the range of responses of the students in the other classes. A detailed comparison of the mean responses for each of the Likert-scale items for all three classes can be seen in Appendix E.

The Likert-scale portion of the inventory was analyzed quantitatively by using a matched pairs *t*-test. A t-tests for independent means was used to test the following null hypothesis:

H_0: $\mu_b = \mu_e$ *when* μ_b = *mean for the survey administered at the beginning of the semester, and* μ_e = *mean for the survey administered at the end of the semester.*

The open-ended questions were analyzed both qualitatively (summarizing responses by themes and patterns) and quantitatively (highlighting the number of responses that fell in each category). I looked at both types of questions together to help create a general picture of the class as well as specific pictures of the focus students.

Concept Maps. The second day of class (which was during the first week), I assigned the students to each draw a concept map with "math" being the central concept. I suggested that they first brainstorm and list everything they could think about associated with math and then look for patterns and ways they might want to categorize their list. The students were also asked to draw math concept maps of math at the end of the semester.

Since using concept maps as a source of information for this type of research is not typical, there was not a standard, conventional method of analyzing them. I read through all the research and literature I could find where people were using concept maps so I could see how they were analyzed. However, I could not find any examples of concept maps being used the way I was using them. The only examples I could find of analyses or evaluations were when concept maps were used as an assessment of learning some particular concept, and they were scored by the items they included and their placement. It didn't make sense to me to devise a system whereby I would give the concept maps a score. I was interested in learning about and describing the information contained in the concept maps, but I didn't want to make any kind of judgment that placed a value (e.g., good-bad) about what the students included.

Since there appeared to be no conventional method for analyzing concept maps, I tried to look at them in a couple of different ways to get a picture of what they told about the students' conceptions of and attitudes toward mathematics as a group and as individuals. The decisions about how I would analyze the concept maps emerged from my experience reviewing the use of concept maps and from conversations with members of my dissertation committee. One of the methods I used to analyze and synthesize the maps was to make a composite list of all the words the students used in their maps with their number of occurrences—separately for the beginning and the end of the semester. I then grouped these words in categories. I also made a separate list with all of the students' first-level connectors to the concept of "math." The other method I used to analyze and synthesize the concept maps was to create a descriptive matrix where I could record the different features of each concept map individually. To create this matrix, I identified different categories and subcategories of items that were included in the maps and made columns where I could mark whether each map had them or not. For some of the items, I just put an "X" if it was present. For others, I put an "L" for "low" if there were one or two related items, an "M" for "medium" if there were three to four related items, and an "H" for "high" if there were five or more related items. I also included a section where I coded the map construction. In this section, I looked at the number of levels of the map, the prominent level, if there were any sub-connections, and the number of first-level connectors. I numbered the students' concept maps 1-34 in alphabetical order, and coded the items for each map. I did this for each student's beginning and final math concept map and put them together on the matrix with the beginning one on top and the final one underneath. This way I could easily look at one student and compare the features of his or her beginning and final maps. I also made separate totals at the bottom of each column for the beginning group and the final group to help me see what was included by the group as a whole. A reduced sample of a portion of this matrix is shown in Figure 5.

	Map Construction				Everyday Life			School Subject								Parts					
Map Number	# of Levels	Predominant Level	Sub-connections	# 1st Level Nodes	Self	Others/Indirect	Careers	Courses	Basic	Advanced	Basic Operations	Other Content	Content	Processes	Characteristics	Numbers/Types	Steps	Symbols/Formulas	Theorems, etc.	Concepts	Functions
1	2.5	2	L	5	M						X			L	L						X
1	5	2	H	4	X						X		L	H		X				X	
2	2	2	0	4	M	M		X		X						X		X			
2					L	L		X	X	X	X	X									
3	2.5	2	L	3		L	M	X	X		X							X	X	X	
3	3	2	L	6	M	M	H				X		M	H	L	T		X			
4	1.25	1	0	15	N	L								L		X	X	X			
4	1.25	1	0	17	L	L		X		X	X		M	H		X					

Figure 5: Concept Map Analysis Matrix Sample

Both of the syntheses I created by analyzing the concept maps using these methods proved to be very useful.

Interviews. As noted previously, the focus students were each interviewed twice to provide more information about their conceptions and attitudes related to mathematics. These interviews were used to learn about the students' ideas relating to my dissertation questions and to track the evolution of their ideas across the semester. The students were NOT interviewed by me (the instructor) during the course but were interviewed by another graduate student associated with another section of the same course (Lubienski). These interviews were tape recorded so that I could carefully analyze them after the semester was over. The students were informed that granting or denying permission for an interview would not influence their grades, field placements, or other decisions about their development and learning in their teacher education program. The students were also reminded that they would have the opportunity, after the course was over, to grant or deny permission for these interviews to be used for research purposes. None of the focus students denied permission for using their interviews for research purposes. These interviews were NOT seen, heard, or used by me during the semester.

During the first interview, the focus students were shown a blank copy of the mathematics inventory they completed the first day of class and asked individually what they would answer for each question. The interviewer had a copy of each of the focus student's beginning Mathematics Inventory and asked them if they remembered what they answered that first day. If they didn't remember, she showed them. Then she asked them what they thought that question meant and why or why not their answer might have changed. The interviewer also asked them some general questions about their mathematics background. The interview guide for the first interview is included as Appendix F.

After the semester was over, these same focus students were invited to participate in follow-up interviews which I conducted. These were administered at the end of Spring Semester 1996 which was one semester after they had completed TE 401. During this interview they were asked specific questions

Table 2: Data Collection Summary

Data	When	Who	Purpose
Student Informational Consent Form	First day of class	All participating 401 students	To assure that students understand that I will be documenting the course and that after the course is over, I will be requesting their permission to use their materials for my study
Student Consent Form	End of Fall 1995 Semester	All participating 401 students	To get permission to use data obtained from students for study
"Math" Concept Maps (Beginning and Final)	First day of class/end of semester	All participating 401 students	To understand better what the students think is part of mathematics
Mathematics Inventory (Beginning and Final)	First day of class/end of semester	All participating 401 students	To learn about the attitudes and conceptions the students bring with them and have when they finish the class
Inventory Consent Form	First day of class	401 students from sections 15 and 16 Fall 1995	To obtain permission to use the inventories for the study
Mathematics Inventory (Beginning and Final)	First day of class/end of semester	401 students from sections 15 and 16 Fall 1995	To provide points of comparison
Reflections of Mathematics Experiences	First day of semester	All participating 401 students	To learn about the experiences, attitudes, and beliefs the 401 students bring to class
"What Is Mathematics?" Journal Entry (Beginning and Final)	Beginning and end of semester	All participating 401 students	To learn more about what the students think math is
"Philosophy of Teaching Mathematics" Journal Entry	End of semester	All participating 401 students	To learn about the students' conceptions of teaching mathematics that they have and are developing
Audio Tapes	Each class session	All participating 401 students	To have a record of what happened in the 401 class in case it is needed for reference
Interviews	Twice each focus student: once during semester, once at end of spring semester	4 focus 401 students	To find out more in depth about the conceptions and attitudes of a few 401 students

related to their conceptions of and attitudes toward mathematics and teaching mathematics. The guide for this interview is included as Appendix G.

These interviews were transcribed and analyzed qualitatively. Their responses were summarized in the relevant sections as part of cases of individual students.

A summary of the data collection used for this study is shown in Table 2.

Development of Conceptual Framework

When I designed this study, I expected to be able to determine a category that would most describe the students in relation to their conceptions of mathematics or as mathematics teachers at both the beginning and end of the semester. I anticipated using either Ernest's (1991) categories of *industrial trainer, technological pragmatist, old humanist, progressive mathematics educator,* and *public educator* or the descriptors proposed by Cooney and others (1998) of *naive idealist, isolationist,* and *connectionist.* However, as I began examining the data for the students as a group and for the focus students in particular, I decided that those labels would not really describe the many facets of their conceptions and attitudes I was seeing. I decided to frame my results in a way that would distinguish the different categories and conceptions I was seeing and would allow me to illustrate the variations that can be present in a single group or person with respect to their conceptions of and attitudes toward mathematics. So, to analyze the data, I developed a framework that emerged from the data as I was summarizing and examining it. This provided me with specific areas in which I could look for change or lack of change from the beginning of the semester to the end of the study. This framework with its categories and associated questions can be seen in Figure 6. I will describe each of the categories in the following paragraphs.

Framework for Analyzing the Conceptions of and Attitudes toward Mathematics of Prospective Elementary School Teachers

I. Nature of Mathematics

1. Content
What is the content of mathematics?

2. Processes
What processes are used in doing mathematics?

3. Representations
How is the content of mathematics represented?

4. Characteristics
What are the characteristics of mathematics?

II. Usefulness of Mathematics

5. General Usefulness of Mathematics
How is mathematics useful in the world?

6. Personal Utility
How is mathematics useful to the student personally?

7. Indirect Personal Utility
How does math that is done by others affect the student?

8. Utility for Others
How is mathematics useful to others?

III. Learning and Teaching Mathematics

9. Learning Math
How is math learned?

10. Teaching Math
How should math be taught?

IV. Attitudes

11. Attitudes toward Math
What is their attitude toward mathematics?

12. Attitudes toward Teaching Math
What is their attitude toward teaching mathematics?

Figure 6: Framework for Analyzing the Conceptions of and Attitudes toward Mathematics of Prospective Elementary School Teachers

The first three general categories in the framework are related to the students' conceptions of mathematics. The fourth category is devoted to the students' attitudes toward math and teaching math. The first category is *The Nature of Mathematics*. This is the category that addresses the conceptions of what mathematics is, what it's about, how it is done and represented, and what it is like. The first section in this category is *Content*. This is the section where the students' conceptions of the content of mathematics—or what mathematics is about—are analyzed. The second section in this category is *Processes*. This section refers to the students' ideas about the processes that are used in doing math. This would include thinking processes such as reasoning and problem solving, but it would also include processes such as talking or writing about math, using manipulatives, drawing a picture, making a chart or graph, or making connections with math to other subjects, things outside of school, or other content areas of math.[1]

The third section in the *Nature of Mathematics* category is *Representations*. This refers to the students' conceptions of how the content and sometimes the processes of mathematics are represented. It would include things like numerals, symbols, formulas, equations, theorems, proofs, drawings, graphs, charts, and even manipulatives. The final section of this category is *Characteristics*. This refers to the students' conceptions of what math is like or how they would characterize it. For example, if the students had the conception that only a few people could learn math, that math has only right and wrong answers, or that

[1] (I consider things like adding, subtracting, using logarithms, etc. content—even though the students are doing something, as they are with all math content. When I talk about processes, I'm referring to the *general* thinking processes, activities, etc. that can be used with all content. [Of course, some content has its own processes that are not used in learning every concept in math. For instance, it's not useful or relevant to make a chart or graph when doing some math, but it's very useful in other areas of the math content.])

math is a static (or dynamic) body of knowledge, these ideas would be included in the characteristics section.

The second major category in the framework is *Usefulness of Mathematics*. This area is where the students' conceptions of how mathematics is useful are categorized. There are four areas related to the usefulness of mathematics. The first section is *General Usefulness of Mathematics*. This refers to the students' general ideas of mathematics being useful without specific examples of how it is useful. The second section is *Personal Utility*. This refers to the students' conceptions of how they use math personally such as in shopping, balancing their checkbook, or cooking. The third section in the *Usefulness of Mathematics* category, *Indirect Personal Utility*, would include a recognition, for example, that math was used to create products such as computers, appliances, roads, etc. that the students use but for which they didn't do the math. The final section in this category is *Utility for Others*. This refers to the students' awareness that other people—often in their jobs—use math in a way that has no effect on the students' lives.

The third major category in the framework is *Learning and Teaching Mathematics*. This is the area where the students' ideas of how mathematics is learned and how mathematics should be taught are analyzed.

As noted, the fourth major category of the framework is related to attitudes. It includes a section for analyzing the students' attitudes toward mathematics and one for analyzing their attitudes toward teaching mathematics.

This framework has allowed me to code virtually every statement in the students' writings and every item in their concept maps. In the remaining chapters of this dissertation, I analyze and report the results of my study using this framework as an organizational template. In chapters 3-5, I look at the group as a whole; in chapters 6 and 7 I look at the four focus students.

CHAPTER 3

CONCEPTIONS OF MATHEMATICS

In this chapter, I will take a careful look at the students' conceptions of mathematics as they relate to the nature of mathematics and the usefulness of mathematics. For each related area, I will examine what the data tells us about their conceptions at both the beginning of the semester and the end of the semester, and I will then summarize the changes. This follows the overall questions of the study which were essentially "What were the students' conceptions at the beginning of the semester?"; "What were the students' conceptions at the end of the semester?"; and "How were they different?" Many have theorized that teachers' conceptions of mathematics influence the way they teach it (Blair, 1981; Dossey, 1992; Ernest, 1991; Lerman, 1983; and Nespor, 1987), and empirical studies have shown that, for most teachers, it is at least one factor that influences their teaching (Kesler, 1985; Kyriakides et al, 2002; McGalliard, 1983; and Thompson, 1984). There have been others who have looked at whether there is change in prospective teachers' beliefs related to mathematics after taking a math methods course (Campbell, 1998; Cooney et al, 1998; Kyriakides et al, 2002; and Vacc and Bright, 1999). In this chapter, I examine whether there is change for these prospective teachers and what that change looks like.

Nature of Mathematics

This section, focusing on the nature of mathematics, includes an examination of the students' ideas about the content of mathematics, the processes that are used in doing mathematics, the representations of the content and processes of mathematics, and the characteristics of mathematics.

Content

One aspect of these prospective elementary school teachers' conceptions of the nature of mathematics is what they consider to be the content of mathematics. They were not specifically asked their conceptions of the content of mathematics in any of the data sources, but the data sources provide a great deal of information about their ideas. The "What Is Mathematics?" journal entries and the mathematics concept maps were the main sources of this data.

Students' Conceptions of Content at the Beginning of the Semester. In the beginning "What Is Mathematics?" journal entries, twenty-seven of the 35 students (77%) identified number as a key part of what math is. In fact, four students wrote that "mathematics is the study of numbers." Twelve others expressed nearly identical views.

The manipulation of numbers was also a central part of what the content of mathematics is for some of the students. In fact, one student wrote, "I thought that it was just the manipulation of numbers." A similar response of another students was, "Math is the manipulating of numbers to find a solution." Other ways the manipulation of numbers as a central part of math was described by the students include the student who wrote, "Manipulating numbers and providing convincing rationales is mathematics;" the student who wrote, "I guess that math would be using numbers that can be manipulated in various ways to come up with answers;" and the student who wrote, "Mathematics is the learning of ways to manipulate numbers to solve problems which arise in our daily lives."

Some students wrote about the way numbers are used to come up with answers for mathematics. One example of that is the student who wrote, "When I think of math, I think of numbers. Math to me is working with numbers in various ways to arrive at a specific right answer." Another student wrote, "Mathematics is using numbers to come up with solutions to problems." Some other ways the students wrote about mathematics using numbers as a principal part of their description include the student who wrote, "Mathematics to me is usually anything involving numbers, but not strictly numbers. It can, and many times is numbers and words," and the student who wrote, "Mathematics is dealing with numbers and concepts."

The second most frequent area of the content of mathematics the students wrote about in the beginning "What Is Mathematics?" journal entries were the basic operations of mathematics (addition, subtraction, multiplication, and division). Nine students included something about these operations. A couple of examples of what the students wrote related to this are the student who wrote, "Basic math involves adding, subtracting, multiplying, and dividing numbers;" and the student who wrote, "It includes addition, subtraction, multiplication, and division. These are the basic operations that develop into problems." The other seven students wrote very similar statements.

Nine students wrote about other content of mathematics in their beginning "What Is Mathematics?" journal entries. One student wrote about different content by including some of the school mathematics subject areas: "I realized, from taking Math 202 that there's algebra, arithmetic, geometry (Euclidean, Non-Euclidean). Mathematics isn't just one single subject area." Others wrote about mathematics content that is usually not taught as a separate subject or course in math, but is part of different subject areas. One student wrote, "Other aspects of mathematics include measurement, creating some graphs to present/represent information, statistics, and telling time." Another wrote, "However, mathematics is more than just numbers. It includes shapes, angles, plains [sic], and degrees." Another student wrote, "For me, math involves numbers of course, but also shapes, direction, and spatial relation." One more example of this other content is the student who wrote, "Math is measurement, angles, probabilities, patterns, planes, calculations, fractions, money, chemistry. . . . Is there anything math doesn't touch?" (the student's ellipses).

A couple of students wrote about how the content of mathematics is related to other fields. One wrote, "I relate math to science—the two subjects go hand in hand." The other wrote, "As well, these things lay the foundation for understanding in chemistry, physics, and engineering. Without a good working knowledge of math, we wouldn't be able to accomplish much in these other related fields."

The concept maps also provided a great deal of information about the students' ideas of the content of mathematics. In fact, the students were more likely to list areas of content on their concept maps than they were to write about them in their "What Is Mathematics?" journal entries. The basic operations were the main content area that was included in them. In the beginning concept maps, 16 of the 34 students (not quite half) noted the basic operations. Seventeen students included "number" as an item in the beginning concept maps with three students including specific types of numbers (i.e., rational, irrational, real, imaginary, etc.).

Fifteen students listed different mathematics courses that are taken as a school subject as part of the content of mathematics in their beginning concept maps. Of those 15 who included names of courses, 11 included examples of basic math (up through algebra), and 13 included examples of more advanced courses (those taken after algebra). Algebra was the course most commonly included in the beginning set of concept maps with 10 occurrences, followed by geometry with six.

In the beginning concept maps, only five students of the 34 listed content other than the names of courses. This may be because the most significant connection the students had to mathematics was as a student, and it would be expected that they would think of the names of courses.

Students' Conceptions of Content at the End of the Semester. In the final "What Is Mathematics?" journal entries, students wrote about similar areas of the content of mathematics as part of the their conceptions of mathematics as they did in the beginning "What Is Mathematics?" journal entries, but the specific content of mathematics seemed to be less prominent in the journal entries as a group than they were in the beginning.

Number was again the area of the content of mathematics that was most frequently noted. However, it was noted much less frequently in the final "What Is Mathematics?" journal entries than in the beginning "What Is Mathematics?" journal entries. Fifteen students wrote about number in the final journal entries as opposed to 27 who included it in their beginning journal entries. In the final journal entries, only one student wrote that "math is the study of numbers." Three students wrote example of what students wrote about how numbers are used in mathematics are the following: "We use numbers as a standard way to talk about and explain math;" "Math involves not only working with numbers, but analyzing them also;" and "It is centered around numbers and numeration from which concepts and relationships are established." Another student also described how numbers are used this way:

> "I believe that mathematics consists of a great number of different ideas and concepts. First, and most obvious, math consists of the use of numbers. These numbers can be used in many ways. They can be used to do operations such as addition, but they can also be used to solve problems."

Another thing that seemed to change in the final "What Is Mathematics?" journal entries besides the less prominent noting of number was *how* students wrote about number. Fourteen of the 15 students who wrote about numbers wrote that math was not just numbers or was more than numbers. They all filled up the rest of their journal entry describing what they thought it was instead. One student wrote, "About 14 weeks ago, I believed that math was numbers, but now I know it is so much more . . . I now also see the way math helps to develop critical thinking skills." Another wrote, "My first response to this question involved the mention of the quantitative use of numbers. Now I see math in its broader sense." A similar example is the student who wrote, "I can now look at math as different concepts and ideas, not just 'some' numbers being put together for some reason that was not told to me." Another example of this sentiment was the student who wrote, "To sum it all up, math is more than working with numbers. It involves a deeper understanding and application of math related concepts." One final example of a student who expressed that sentiment is the student who wrote the following:

> "Math to me is not just numbers, symbols, confusion, frustration, working so hard to reach one correct answer, working step after step only to make a mistake and wonder, 'Where did I go wrong?' No, math now means something much different to me."

Seven students wrote about specific areas of the content of mathematics. Five of them included the basic operations as part of a greater list of content. These two examples are typical:

> Math is addition, subtraction, multiplication, division, functions, patterns, fractions, place values, measurement, probability, and statistics. . . They allow us to deal with shapes, distance, time, weight, money, calendars, maps, area, volume, music, and amount."

> "Mathematics is encompassing in that it's a subject that develops understanding in logical reasoning, statistics and probability, measurement, geometry, addition, subtraction, multiplication, division, and patterns and functions."

An example of a student listing different content related to mathematics without noting the basic operations is the student who wrote, "When I think of math, I think of measurement, balancing, weighing, algebra, geometry, physics, chemistry, probability, statistics, and just simply numerals. I think of calculators and rulers, scales, measuring tools, weight, height, and telling time."

There was also some change in the content of mathematics that was included in the final concept maps. While in the beginning concept maps, not quite half of the students (16 of 34) included the basic operations, in the final concept maps, 24 of the 34 students (almost 76%) included them. (This was *much* greater than the number of students who included basic operations in their final "What Is Mathematics?" journal entries—24 opposed to five.) The format of the concept maps seemed to lend itself to the inclusion of things that could be included in lists, and in many areas, the students seemed more likely to include those types of things as items in their concept maps than they were to write about them in sentences in their "What Is Mathematics?" journal entries. Number was included by 17 students in the final concept maps which was the same number of students who included it in their beginning concept maps. Seven students included specific types of numbers such as rational, irrational, integers, real, imaginary, and the like in their final maps as opposed to three students in their beginning maps.

The number of students who listed different mathematics courses that are taken as a school subject as part of the content of mathematics was the same with the final concept maps as it was with the beginning maps. In each set, 15 students included courses, although only about half of those students included courses in both their beginning and final concept maps while the other half only included them in either their beginning or final concept maps. In the final concept maps, 11 included examples of basic math, and all 15 included examples of advanced courses. This was very similar to the beginning concept maps where 11 included examples of basic math, and 13 included examples of more advanced courses. In the final set, geometry was the most common subject noted with 10 occurrences, followed by algebra with eight, and calculus with seven. (Algebra was the course most commonly included in the beginning set of concept maps with 10 occurrences, followed by geometry with six.) These seemed to be the names of the courses that the students were most familiar with and could most easily remember.

The biggest change in the concept maps was in the listing of other content areas of mathematics: whereas in the beginning set, only five students listed content other than names of courses or the basic operations, in the final set, 10 students listed other content. More students included something about content in their final concept maps than they did in their beginning maps. In the final maps, seven students had a low number of items relating to content, but five had a medium number (3-4), and one student had a high number (5+). This is in contrast to the beginning maps where seven students had a low number of items related to content (1-2), and none had a medium or high number. This perhaps suggests an expansion of their views of what content is part of mathematics.

Summary of Changes of Students' Conceptions of Content. There seemed to be some change in the way many of the students thought about content at the end of the semester as opposed to the beginning of the semester. This change was evidenced sometimes in their writing, but more often in their concept maps. One example of how content was included in the concept maps and changed somewhat from the beginning of the semester to the end of the semester can be found in the concept maps of student 18. In the beginning concept map, she listed different school courses of mathematics and areas related to mathematics (See Figure 7). In her final concept map, she had a major section on content in which she listed names of courses as well as specific content (See Figure 8.)

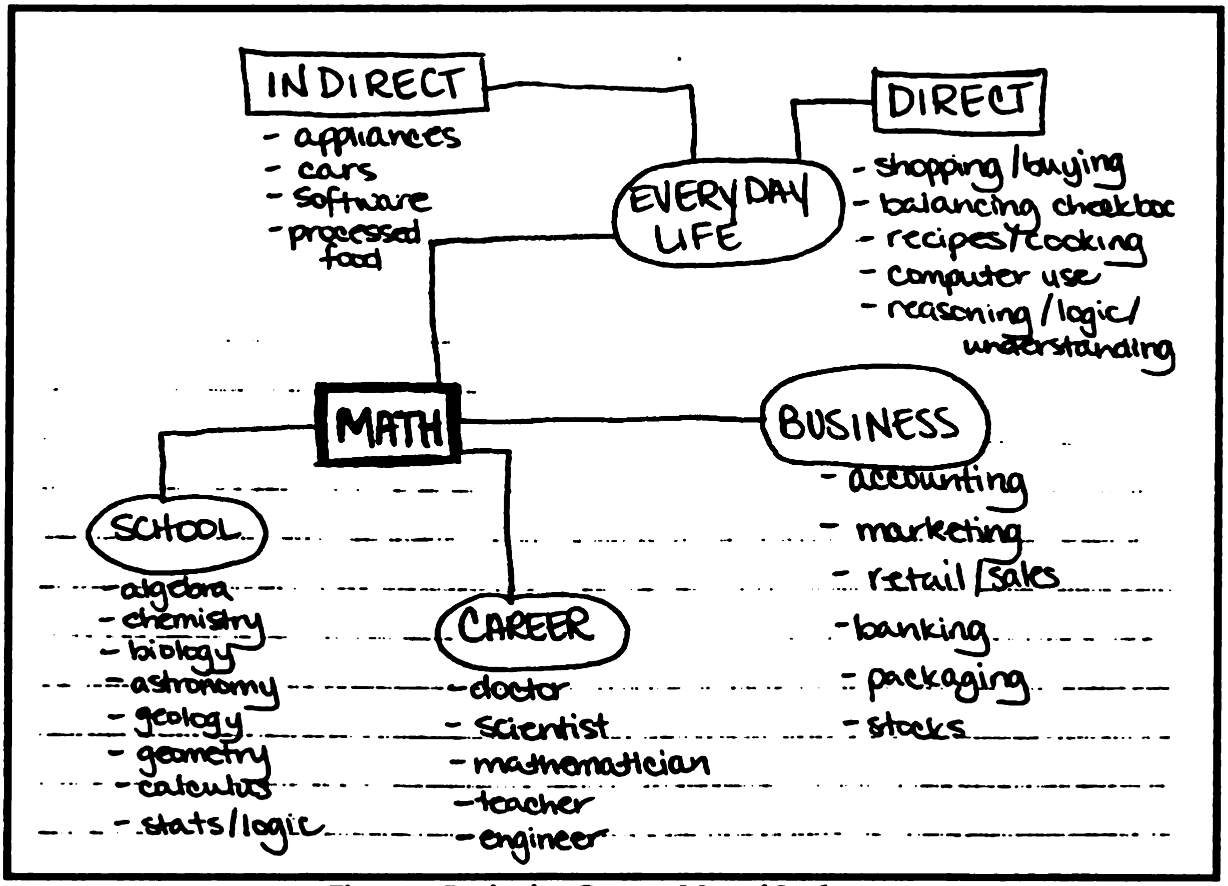

Figure 7: Beginning Concept Map of Student 18

As a group, the students' conceptions of the content of mathematics seemed to change somewhat over the course of the semester. Overall, there seemed to be an expansion of what the students considered to be included in the content of mathematics, and there seemed to be much less emphasis on only number as the content of mathematics. There was also more noting of the basic operations at the end in these data sources, but there was a much greater tendency to include them as part of a list of other mathematics content rather than separately as the sole or principal content of mathematics.

Processes

The processes that are used in thinking about and doing mathematics were another part of the students' conceptions of mathematics. These processes would include thinking processes such as reasoning and problem solving, but

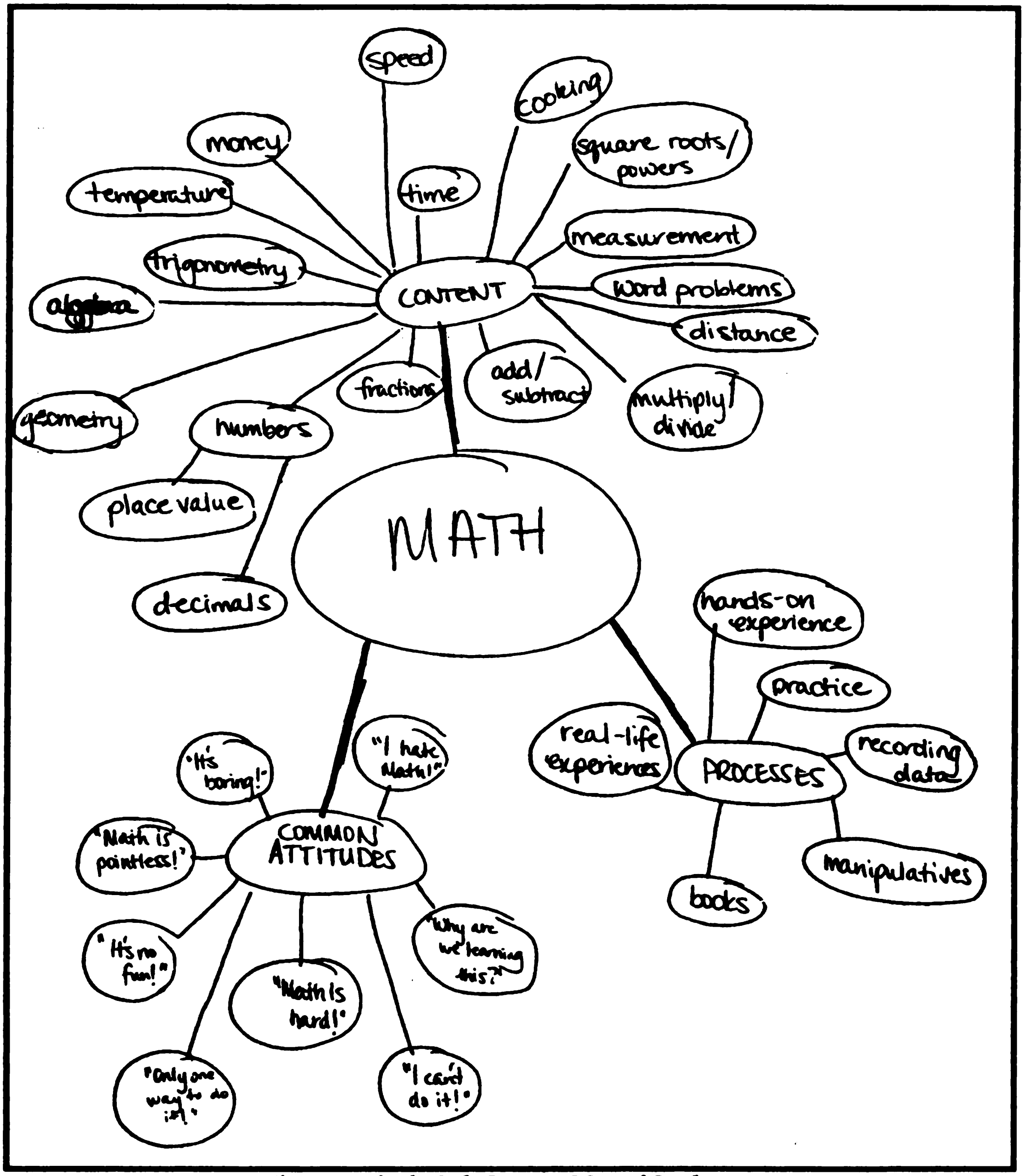

Figure 8: Final Math Concept Map of Student 18

they could also include things like writing, discussing, drawing a picture or table, and making connections within mathematics and with mathematics to other things. Again, the students were not specifically asked about their conceptions of the processes of mathematics in any of the data sources, but their writings in their journal entries and the items they included in their math concept maps provide much insight into their ideas.

Students' Conceptions of Processes at the Beginning of the Semester. At the beginning of the semester, the processes of mathematics were a part of quite a few of the students' conceptions of the nature of mathematics. In the beginning "What Is Mathematics?" journal entries, 14 students (40%) included something specifically about the thinking processes used in doing mathematics. In the beginning concept maps, mathematical processes were also quite prevalent. Thirteen students included a low number of items (1-2) related to processes, and six students included a medium number of items (3-4). This made a total of 19 out of the 34 concept maps (55.9%) that included something about processes. Six students included the actual word "process" in their concept maps.

Seven of the 14 students who wrote about processes in their "What Is Mathematics?" journal entries wrote about reasoning and logic. These were the processes the students wrote about most often. They were also the most frequently included processes in the beginning concept maps. Fifteen out of the nineteen students who included anything about processes in their concept maps used the word "reason" or "reasoning," and there were six occurrences of "logic." An example of what a student wrote about reasoning and logic in her beginning "What Is Mathematics?" journal entry was the student who, "With math comes logical and critical thinking. Knowing how to put it all together and also being able to take it all apart." Another student explained,

> "I believe that mathematics is the study of logic. It takes developed thinking skills in order to reason through many math problems. You need to know how to go step-by-step, logically, in order to obtain a correct answer... Finally, I believe that mathematics is the study of, and a major developer of, the entire thinking process."

Another student who also thought that reasoning and logic are a part of mathematics wrote, "I think that mathematics is a subject in which people learn reasoning and logic, and it just happens to use numbers to get the point across. Math can be seen as a process in which people solve problems using numbers and formulas."

One student explained how she thought the thinking processes of mathematics differed from those of other subjects. She wrote,

"I think that mathematics is simply another tool used to aid people with their thought processes. For example, the skills of analyzing and reasoning are not natural for most people. Mathematics can be used (Although often times it is not) to develop these important skills. While some may argue that there is only one correct answer for mathematics problems, I believe that how a person gets the correct answer is what math is all about. Mathematics is an opportunity to explore different venues that ultimately lead to a right answer.' Manipulating numbers and providing convincing rationales is mathematics. Mathematics offers a new way of thinking that is very different from history (interpretation), English (no correct answer), and science (experimentation)."

A few students wrote about problem solving or about thinking processes in general. When writing about that idea, one student even wrote about the idea that different processes can be used to get to an answer. She wrote,

"Mathematics is more than just operations. It involves problem-solving skills. It requires one to look at various numbers or information and find a solution. I believe mathematics is more than just memorization or computation of problems but is a skill that requires using a thought process. While it often seems that there is only one right answer, there are often different ways to get an answer. . . . It involves critical thinking."

Another student also emphasized that even being able to use memorized facts and formulas requires thinking. She wrote,

"I also think that mathematics is abstract thinking. Sure there are facts and formulas that need to be memorized, but you also have to be able to think about the problem in order to figure out which fact or formula you need to use."

A couple of students wrote about how story problems are used to help students use these thinking processes. One of these students wrote, "It is these story problems that help you to think in more complex ways. Math opens new doors to new ideas." The other wrote, "Story problems are a form of mathematics that allows students to explain what and why things occur quantitatively."

There were a few other words the students included in their beginning concept maps that were related to processes. For example, eight students included the word "understanding." Other examples of words the students used related to the idea of processes (with their number of occurrences) included "thinking" (4), "problem solving" (3), "trial and error" (3), "categorizing" (2), "sorting" (2), and "analytical thinking" (2). Of course, these processes are very connected to reasoning and logic.

Students' Conceptions of Processes at the End of the Semester. At the end of the semester, the attention the students gave to the processes of mathematics was much more prevalent and prominent than at the beginning of the semester. For example, 16 of the 30 students for whom I have final "What Is Mathematics?" journal entries wrote about math as a process or the processes used in mathematics. This was 53% of the final journal entries as opposed to 40% at the beginning of the semester. In fact, four students wrote that math is a process. One wrote simply, "Math is a process which involves numerous operations."

At the end of the semester there was a change in the amount of attention given to processes and an increase in describing different kinds of processes that are part of doing mathematics. As in the beginning "What Is Mathematics?" journal entries, the processes of reasoning and logic were written about—this time by six students as opposed to seven at the beginning of the semester—but they were not as dominant in the final journal entries as other processes of mathematics were included more frequently. An example of a student who emphasized the processes of reasoning and logic is the student who wrote,

> "However, I have also come to realize that mathematics is also related to logic. To be successful in math, one must understand the underlying logic surrounding specific problems. One must be capable of reasoning through a problem rather than relying on designated rules. To be able to reason mathematically is to be able to build on new mathematical ideas."

One student wrote about "logical knowledge" and problem solving, but she also added some other processes in her journal entry. She wrote,

> "In creating a definition of mathematics, I would conclude that it is a subject area that requires logical knowledge to solve problems. Problem solving makes up a big part of mathematics. . . . Comparing, measuring, classifying, and memorizing are just a few broad skills required in learning mathematics. . . Mathematical knowledge is a form of thinking that many times we don't believe ourselves to be engaged in."

Another student who also wrote about some different processes in her final journal entry was the student who wrote,

> "Math is sorting and classifying, forming graphs, and understanding various functions of numbers. . . . Math is making connections between concepts and numbers related to mathematics. Math is reasoning with numbers and math concepts."

Another excerpt from the final "What Is Mathematics?" journal entries that provides an example of a student who wrote about reasoning but also other processes was the following:

> "Math is: different ideas, different ways to attack a problem, accepting others' explanations, journals, justifying and explaining or reasoning out your thoughts, the process, not always the product. . . ."

In the final concept maps, the processes of mathematics were also more prominent than they were in the beginning maps. Four students included a low number of items related to processes, seven students included a medium number of items, and 15 included a high number (five or more) related to processes. This made a total of 26 out of the 34 students (76.5%) that included something about processes in their final concept maps. (This was in contrast to 55.9% in the beginning concept maps.) The increase in students who included a high number of items related to processes in the final maps versus the beginning maps was especially significant—zero in the beginning and 15 in the final maps. This was a huge jump. It seems to show that the students were thinking about processes—and multiple kinds of processes—much more at the end of the semester.

Eleven of the maps included the word "process" (or "processes"). "Problem solving" was the process included most often in the final concept maps. Twenty-four of the 34 maps included "problem solving" (70.6%). "Reasoning" was noted in 14 of the maps, and "logic" was included in 11. Some of the other processes that were included most frequently (with their number of occurrences in parentheses) were "comparing" (7), "thinking" (7), "trial and error" (5), "communicating" (4), "applying" (4), "making connections" (3), "understanding" (2), "experimentation" (2), "contrasting" (2), "analyzing" (2), "critical thinking" (2), and "thinking abstractly" (2). There were 37 other processes noted once.

Of the 15 students who included a high number of items relating to processes in their final concept maps, 13 of them either included no items or a low number of items relating to processes in their beginning concept maps. Three of these pairs of maps that show this contrast are shown below and on the following pages (See Figures 9 - 14):

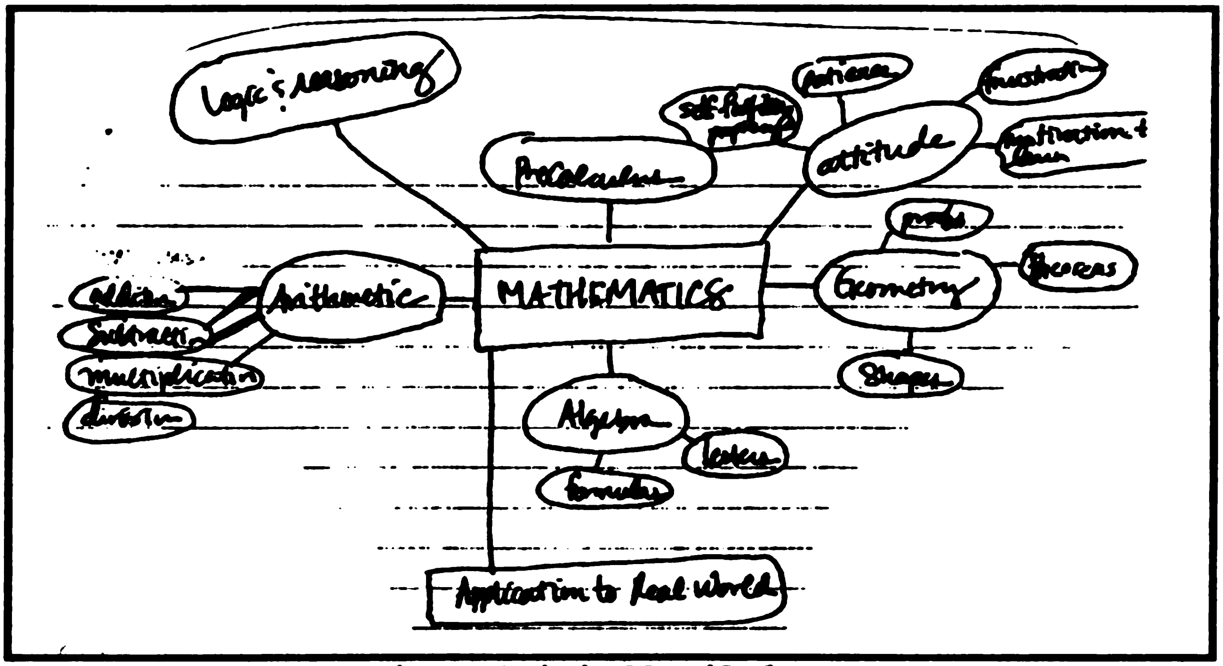

Figure 9: Beginning Map of Student 15

Student 15 had "logic and reasoning" in her beginning concept map (See Figure 9), but they were the only processes. In her final concept map, she had a whole section of processes that was much more extensive (See Figure 10). It's

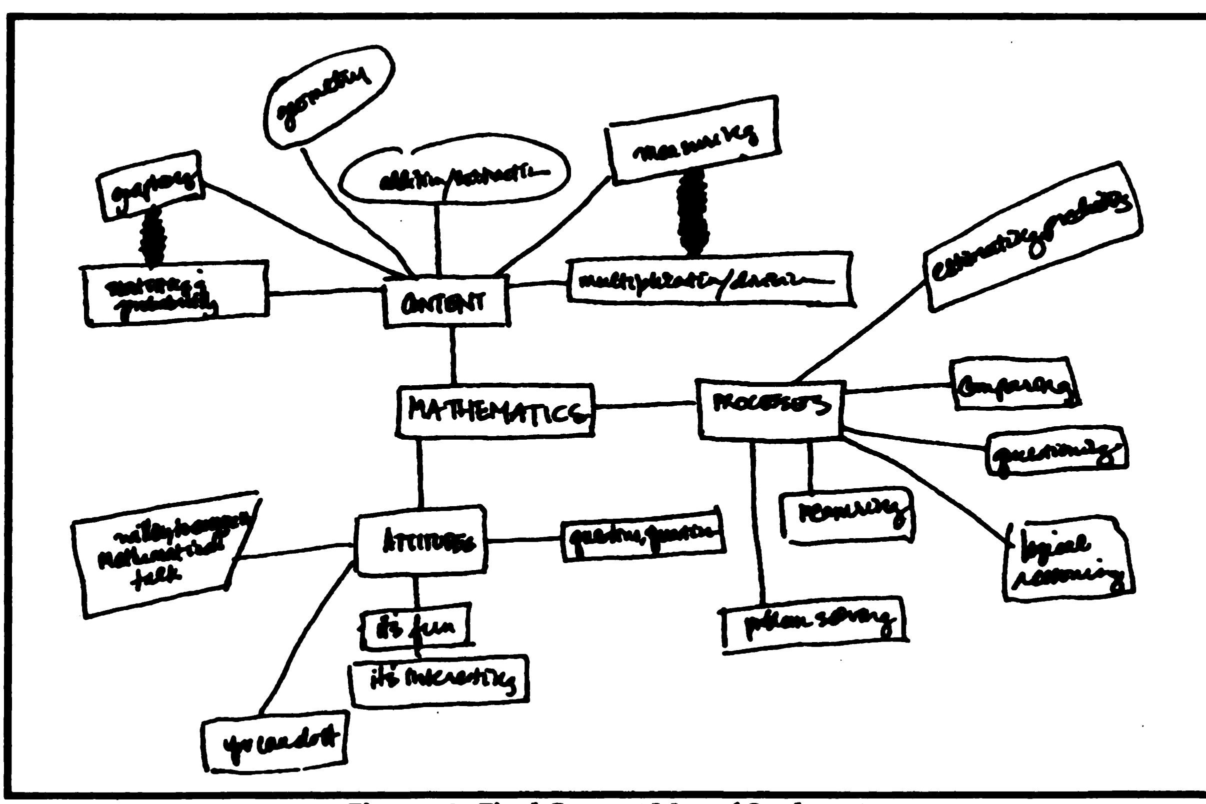

Figure 10: Final Concept Map of Student 15

Interesting to note that on both maps, the processes are off by themselves—separated from content and the other sections—which may be an indication that she recognizes that even though they are related, processes are not the same thing as content.

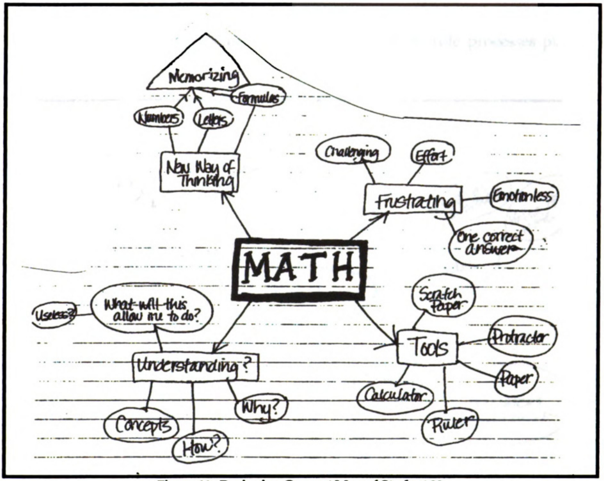

Figure 11: Beginning Concept Map of Student 29

The beginning concept map of student 29 (See Figure 11) is an example of a map that has no clear processes. The items "understanding?", "how?", and "why?" are possibly referring to processes, but it is not clear from her map. She may also be using them to refer to the usefulness of math. The item "new way of thinking" might be referring to processes, but it also might be a way to describe math as being different from other subjects. In her final concept map (See Figure 12), five of her six first-level connectors to the concept mathematics are processes or related to processes. The other connector, attitudes, even has "math as a process" in the next level. It's interesting to note that four of her director connectors are the four processes strands in the 1989 NCTM *Standards*—problem solving, reasoning, communication, and connections. It's also interesting that another of her direct connectors was "A way of developing thought processes." She seemed to really be focused on role processes play in mathematics.

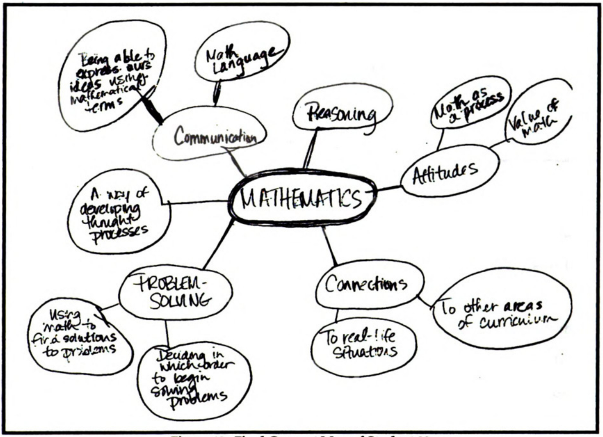

Figure 12: Final Concept Map of Student 29

Student 1 is another person who included very little about processes in her beginning concept map (See Figure 13 on page 89), but her final concept map was almost entirely about processes. Her four direct connectors to math were all processes—communicating, reasoning, problem solving, and making connections (See Figure 14 on page 90). Again, it is interesting to note that these

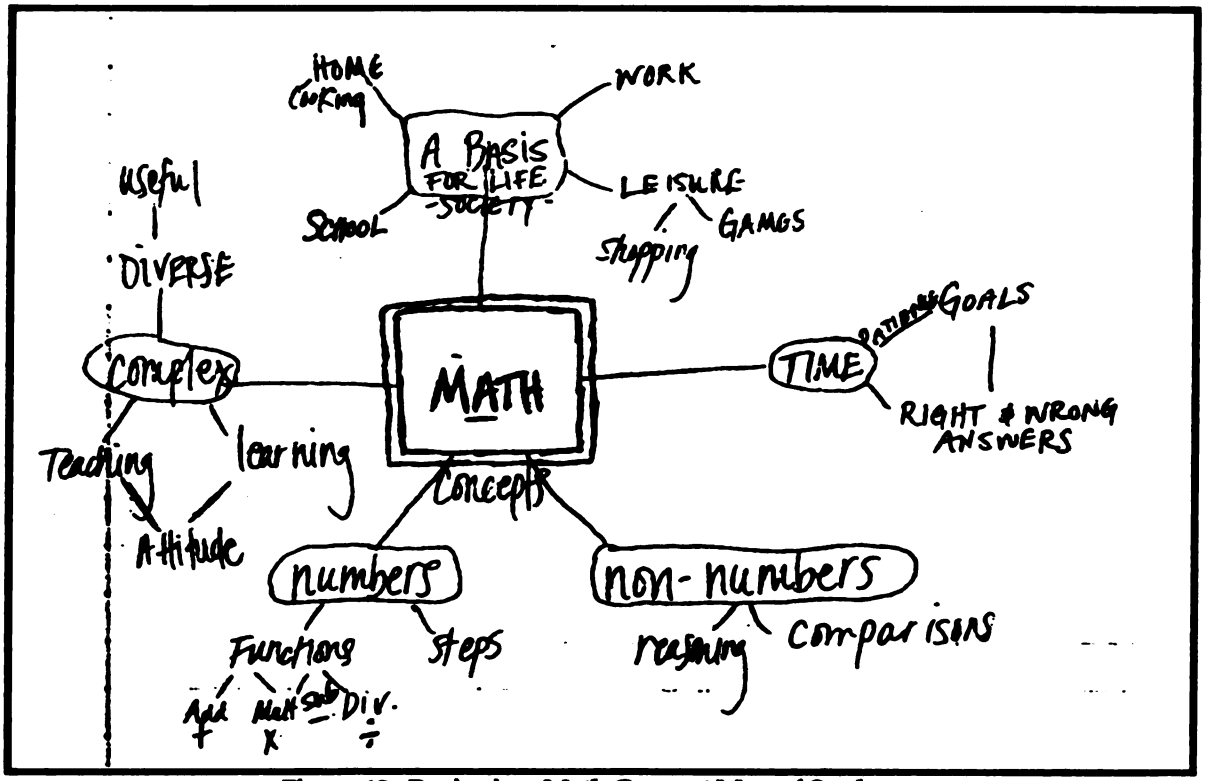

Figure 13: Beginning Math Concept Map of Student 1

are the four process strands identified by the 1989 NCTM *Standards*. (In the summary portion of this section, I will explain the role the 1989 NCTM *Standards* played in our class and why they may have had an influence on how these two students constructed their maps.)

Indeed, at the end of the semester, the idea of processes and their importance seemed to be a much more solidified part of the students' conceptions of mathematics than they were at the beginning of the semester. One last place this was seen was in the students' responses to item number 23 from the Mathematics Inventory, "Students' reasoning is more important than if they are able to get the answer that matches the answer key." There was a high level of agreement with that at the beginning of the semester with a class average of 4.138 (the second highest average of all items), but at the end of the semester, the average agreement was even slightly higher—4.379. This was not statistically significant (p=0.28), but perhaps indicates a trend of growing

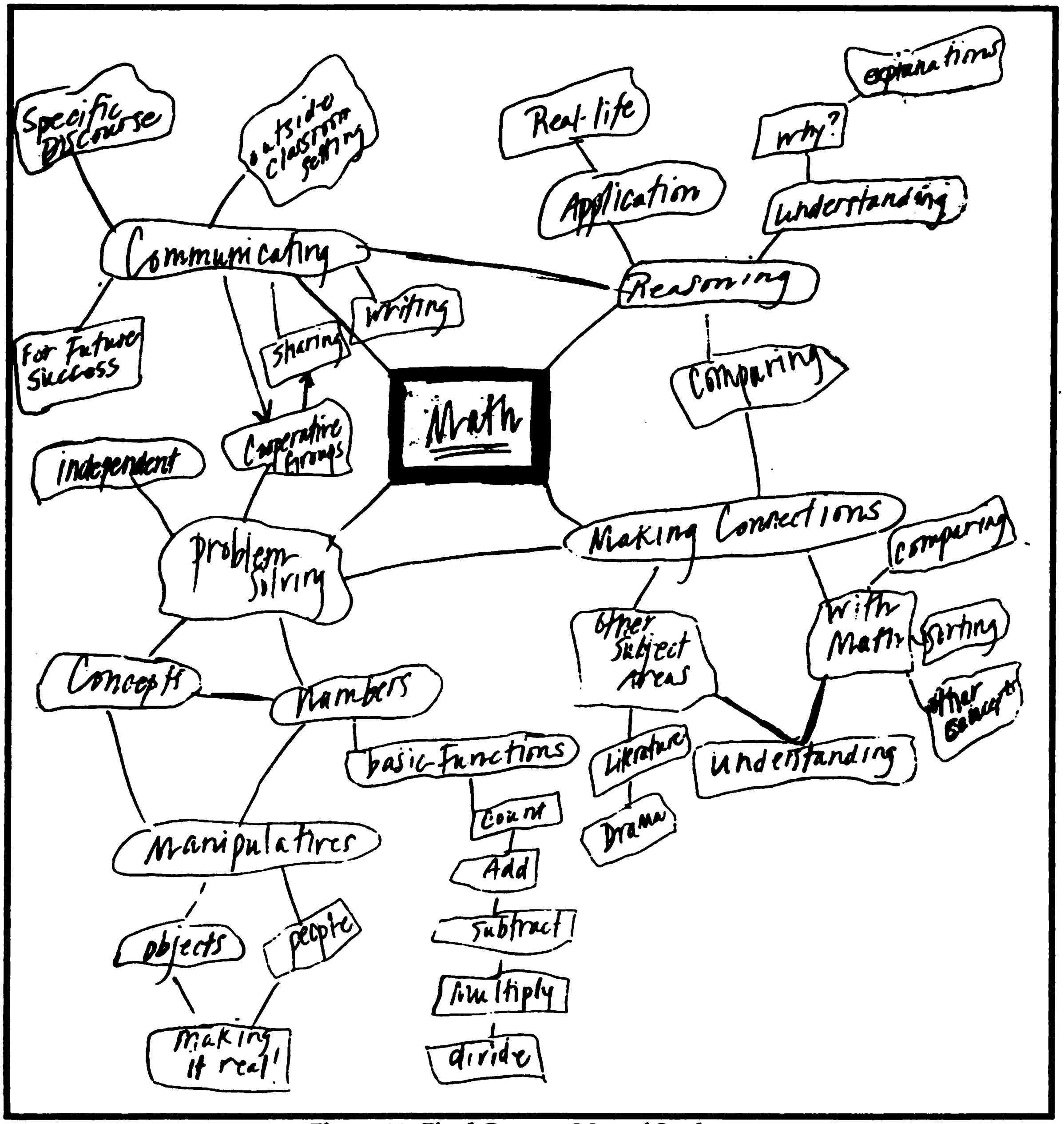

Figure 14: Final Concept Map of Student 1

agreement with that idea even though there was not much room for increase. One student exemplified an agreement with that idea when she wrote in her final "What Is Mathematics?" journal entry, "While math has often seemed to be just about getting answers, the trend seems to be switching to focusing on understanding and the processes and thought involved."

Summary of Changes in Students' Conceptions of Processes of Mathematics. The processes that are used in doing and thinking about mathematics received significantly more attention by the students as a group at the end of the semester than they did at the beginning. There was evidence that there was an increase in the prominence of processes in the students' conceptions of mathematics as well as in the awareness of different kinds of processes that are related to mathematics. This was shown most markedly in the contrast between the beginning and final concept maps, but it was also very evident in the difference between the beginning and final "What Is Mathematics?" journal entries (53% of the students wrote about processes in their final journal entries as opposed to 40% in their beginning journal entries).

Probably part of the reason there seemed to be more attention on processes at the end of the semester than at the beginning was because of the emphasis they received in class. We spent much time becoming familiar with the 1989 NCTM Standards, and in doing so emphasized that the first four were considered process standards—problem solving, communication, reasoning, and connections—while the others were considered content standards. (Again, it is worth noting that both students 29 and 1 included those four processes as major subcategories in their final concept maps [See Figure 12 on page 88 and Figure 14 on page 91]). Many of the assignments the students were given required that they look at the processes of mathematics. One assignment was a group presentation where the group talked about a specific area of content, and one of the things they had to do in the presentation was give examples of how the four process standards might be used in learning about and doing that particular content. The students were also assigned to plan a mathematics unit, and as part of that assignment, they had to think about the content they wanted the students to learn, the processes they wanted their students to use, and the attitudes related to that content that they wanted them to develop. They were assigned to make a concept map with those three areas that served as an overview of the unit. For the assessment they planned for their unit they were also expected to assess in those three areas—content, processes, and attitude. A few of the students even organized their final concept maps with content, attitude, and processes as the direct connectors to the concept of math. The final concept map of student 15 (highlighted in the previous section) was one example of this (See Figure 10 on page 86).

It seems that the idea of mathematical processes, as such, was not something the students had been explicitly taught about much in their experiences with mathematics as students. They didn't seem to be familiar with the idea that there were many processes that were used in doing mathematics—across content areas. Many students seemed to be aware of the processes of reasoning and logic being associated with math since those are probably the two that they were most likely to have heard about. Some students also associated the process of problem solving with math at the beginning of the semester. However, there were many processes used in mathematics that it seemed most students had never thought about—even though they may have used them. It seems that this was an area that was ripe for expansion in the students' minds because it was not an area they had heard much about or spent a lot of time thinking about.

Representations

The way the content of mathematics is represented was part of the students' conception of mathematics. This includes mainly the way content is represented—often on paper—by symbols, graphs, words, and the like. The students were not asked to describe how math is represented, but a couple of the data sources provide a little information about their conceptions.

Students' Conceptions of the Representations of Mathematics at the Beginning of the Semester. There was a little information about the students' ideas of how mathematics is represented in the "What Is Mathematics?" journal entries, but not a lot. There was more information in the concept maps. In the beginning "What Is Mathematics?" journal entries, three students wrote about math as equations or dealing with equations. Three other students wrote about math as being made up of formulas or working with formulas such as the person who wrote, "Mathematics is made up of formulas and data that are important in helping to figure out solutions."

In the beginning math concept maps, the students included several items that were related to representations of mathematics. The items that were listed most frequently were the following: "numbers" with 19 occurrences, "steps" with 10 occurrences, "formulas" with 8 occurrences, "equations," "theorems," and "letters" with six occurrences each, "functions" and "theories" with five occurrences each, and "proofs" with four occurrences. ("Number" can be considered content when talking about the idea of quantity, but it can also be considered a representation of the idea of quantity when, as is often the case, people use the word "number" when meaning "numeral"—the symbolic representation of a number.) The other words that were included in the beginning concept maps that were related to representations of mathematics with three or fewer occurrences were "symbols," "rules," "answers," "variables," "constants," "words," "equal," "laws," "terms," "algorithms," "non-numbers," "values," "number sets," "take away," and "in all." Overall, 22 of the beginning concept maps (65%) had at least one thing that could be considered as being related to representations. Thirteen of those 22 maps had three items related to representations, and two maps had four items. The students who gave the most attention to representations in their beginning concept maps were students 28 and 33 (See Figure 15 and Figure 16 on page 95).

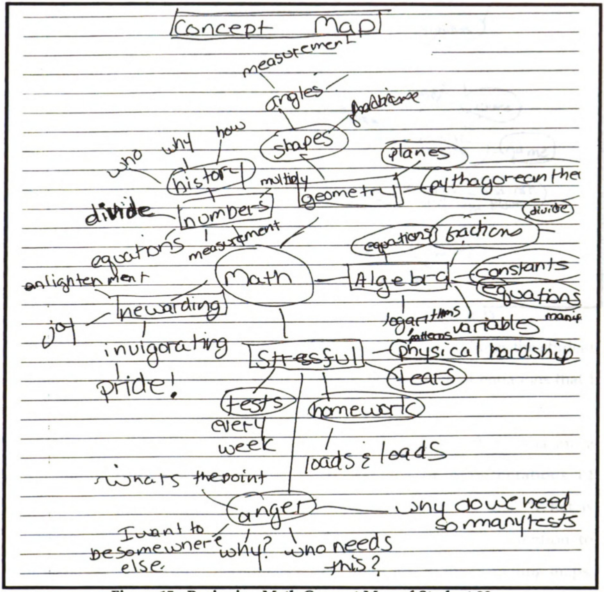

Figure 15: Beginning Math Concept Map of Student 28

Students' Conceptions of the Representations of Mathematics at the End of the Semester. Again, as with the beginning "What Is Mathematics?" journal entries, there was not a lot written about representations in the final "What Is Mathematics?" journal entries. Only two of the 30 journal entries included any mention of any representations, and they both wrote something to the effect that math was not just that. One student wrote, "I have learned that math is

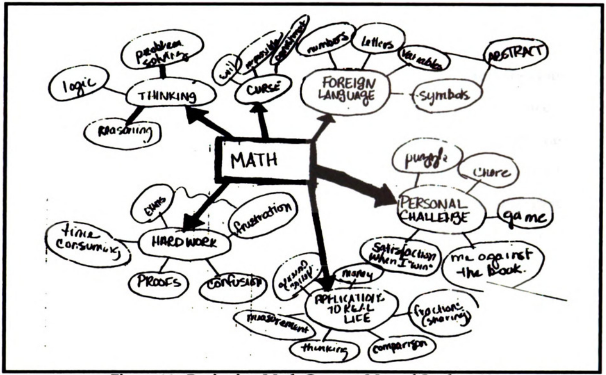

Figure 16: Beginning Math Concept Map of Student 33

much more than numbers and equations," followed by a list of things the student thought it is. The other student wrote, "Math is not a bunch of equations that I solve just because I can."

As was the case with the beginning concept maps, there was more information related to the students' conceptions of representations of mathematics in the final concept maps than there was in the final "What Is Mathematics?" journal entries. Overall there was a little less attention to representations in the final concept maps than there was in the beginning maps, but not a lot. Eleven students included the word "numbers," but seven additional students included different types of numbers (e.g., "whole numbers," "real," "non-real," "negative," "positive," "even," "odd," "rational," "irrational," "imaginary"). The most frequently included type of number was "whole numbers" with nine occurrences followed by "real" and "irrational" with four occurrences each. The other items that were related to representations included in the final concept maps were "formulas" with six occurrences, "word/story problems" and "equations" with four occurrences, "functions" with three occurrences, "steps," "algorithms," "variables," "solutions," "theorems," "symbols," and "operations" with two occurrences each, and the following with one occurrence each "problems," "x," "y," "unknowns," "rules," "theories," and "procedures." As was the case with the beginning concept maps, 22 students had at least one item related to representations on their map. Interestingly, only 13 students included something about representations in both their beginning and final concept maps. The other nine students who included something each time were a different set of students. In the final concept maps, there were six students who included three or more items related to representations as opposed to 15 students in the beginning maps.

Changes in Students' Conceptions of the Representations of Mathematics. There was not a lot of difference in the types of representations the students noted at the beginning of the semester as opposed to the end of the semester. For some reason, there was more attention to the different types of number in the final concept maps than there was in the beginning concept maps. One thing that seemed different in the "What Is Mathematics?" journal entries is that while in both the beginning and final sets there wasn't a lot written about representations, in the final set, the two students who wrote anything related to representations both wrote something to the effect that math was more than that. Again, the students were not specifically asked to list or write about the ways mathematics is represented, so the information that we do have about their conceptions of representations is only a glimpse of the group as a whole.

Characteristics

The data from this study provides some information about the students' conceptions of the characteristics of mathematics. There were three items on the

Mathematics Inventory that were related to students' conceptions in this area. The other data sources also provided some information. However, because of the open-ended nature of the "What Is Mathematics?" journal entries and the concept maps, and the fact that the students were not specifically asked about how they would characterize mathematics, there is not a great deal of information from these sources. It would not be unexpected for students to think of characteristics of mathematics when asked the question, "What is mathematics?", but it would have been more likely that they would answer with characteristics if they were asked to describe mathematics or asked the question, "What is mathematics like?"

Students' Conceptions of the Characteristics of Mathematics at the Beginning of the Semester. There were three items on the Mathematics Inventory that were related to the characteristics of mathematics. They were items 4, "I think of mathematics as a set of isolated facts that need to be memorized;" 11, "Mathematics is something that not everyone is able to understand;" and 17, "Mathematics is something at which everyone can be successful." When the students took this inventory on the first day of class, as a group, they didn't show a strong commitment to agreeing or disagreeing with any of these statements. The means for each of these statements were all close to 3 "somewhat like me." They were 2.48, 2.86, and 3.207, respectively. They were all leaning slightly in the direction that would be more consistent with a constructivist view, but they were all very close to the noncommittal 3. (In addition to describing a characteristic of math [i.e., it's something all can or cannot learn], items 11 and 17 might also be seen as telling us about how the students see the relationship between people and math.)

The concept maps also contained some information about what the students considered to be characteristics of mathematics. The most common characteristic included in the beginning concept maps was that math has "one right answer." Five students included that in their beginning concept maps. Other words or phrases included in the beginning concept maps related to this idea were that there were "concrete answer" and "right and wrong ways." Other characteristics in the beginning concepts (with one occurrence each) were "left-brained," "foreign language," "not similar to English," "tangible," and "abstract." In the beginning concept maps, there were 11 students who included at least one thing that could be considered a characteristic of mathematics.

We can also learn a little about the students' conceptions of the characteristics of mathematics from their "What Is Mathematics?" journal entries. In the beginning "What Is Mathematics?" journal entries, the students wrote about three main characteristics. One was the idea that mathematics is like a language as in the case of the student who wrote, "For some people, math is a language" or the student who wrote, "In my opinion, mathematics is a foreign language." Five students wrote of mathematics as being a language.

The second characteristic of mathematics written about by four students in the beginning "What Is Mathematics?" journal entries was the idea that mathematics is something that builds on previous knowledge. One student wrote, "Math is often about skills necessary to master concepts. These skills are usually built upon each other so that if one is not mastered, then it will be very difficult to understand the next." Another student wrote about this idea this way, "Through math, we can build on one problem and keep on building to discover new ideas, new solutions, and new formulas." One student wrote about the need to understand "basic math" in order to understand "upper-level math" with this sentence: "Understanding basic math concepts is the framework for understanding and succeeding in upper-level math courses." The only other characteristic one other student wrote about was that "[t]here is no leeway for mistakes in math." It is interesting to note that all of the students who wrote any characteristics of mathematics had a negative attitude toward mathematics (based on their own declaration and other measures). Still, we can't assume that they considered the characteristics they wrote about as something negative. They may have seen them as a neutral distinction.

The third characteristic of mathematics the students wrote about was that it is complex—or that it can be both simple and complex. For example, one student wrote, "Math can be very simple or very complex depending on the level you're working with or the reason that you are using it." Another student wrote, "Math can be basic, and it can also be incredibly complex depending on the problem."

Students' Conceptions of the Characteristics of Mathematics at the End of the Semester. For the items in the Mathematics Inventory related to characteristics, there were slight changes in the direction that is more consistent with a constructivist view of mathematics. None of these changes were quite statistically significant and may have been just a result of chance, but the apparent trend is still encouraging. (See Table 3.)

Table 3: Items Relating to *Characteristics of Mathematics* from the Mathematics Inventory

Item	Beginning Mean	Final Mean	p value
4. I think of mathematics as a set of isolated facts that need to be memorized.	2.48	2.138	0.18
11. Mathematics is something that not everyone is able to understand.	2.86	2.34	0.11
17. Mathematics is something at which everyone can be successful.	3.207	3.48	0.31

There seemed to be less of an emphasis on characteristics by the group in the final concept maps than there was in the beginning maps. Whereas five students included the idea that math has "one right answer" in their beginning concept maps, only one student included it in her final map. Two students wrote "right and wrong answers" on both the beginning and final maps. The other characteristics the students included in the final concept maps were "limited," "left-brained thinking," "simple to complex," "steps build upon each other" (on two maps), "interdependent," "exact science," and "just because." These had one occurrence each, except as noted. In the beginning concept maps, there were 11 students (32.4%) who included at least one thing that could be considered a characteristic of mathematics, but in the final concept maps, only three students (8.8%) included any characteristics. In both the beginning and final concept maps, all the characteristics the students included would probably be considered negative or neutral. It's hard to know for certain, though. For example, some might see math as having one right answer as something positive while others see it as something negative. However, based on the other information I have about the students who included those characteristics, I think they thought of them as at least mostly negative.

In the final "What Is Mathematics?" journal entries, eight students wrote something that could be considered related to the characteristics of mathematics. These students wrote about four different characteristics. Two students wrote about the idea of math being something that builds on itself. (This is something that three students wrote about in the beginning "What Is Mathematics?" journal entries.) These were the same two students who included that idea in their concept maps. One of them wrote, "Math skills and concepts are often interdependent and build upon one another. It is a giant puzzle that people must work to learn how to fit together." The other one wrote, "Math is something that builds on itself. In order to understand one concept you need to understand the concept before. Therefore, math is interrelated and connected throughout."

Three people wrote about the idea of math having a right answer. Interestingly, in these final journal entries, each of them also wrote about the idea that the correct answer can be found through different methods. One student wrote, "Math is memorizing a bunch of facts and doing several methods to get at the correct answer." Another student wrote, "Mathematics always leads to a certain answer—a concrete answer unlike many other subjects. One thing that I find fascinating about math is that many times you can achieve the correct answer through different ways." Felicia, one of the focus students, also expressed a similar idea. She was the only one who included that idea in both her math concept map and her journal entry.

Two students wrote about the accessibility of learning math. One wrote, "Though math is challenging, it is a subject that is accessible to anyone who is taught constructively." The other related quote was from a student who wrote what she believed was a conception of a characteristic held by some, but it isn't clear whether it is one of her own conceptions:

> "It is often thought of as extremely difficult or something that only certain people can do. Some even think that it is an inherited trait—that if their mom or dad were bad at math, then that is their excuse for not being highly successful with math."

In this case, the writer is avoiding taking responsibility for this idea, but she isn't disavowing it. It may be something she experienced and still believes or no longer believes. Or, it may be something she has observed or heard from others, and whether she agrees or not, she recognizes that there are those who have this conception.

Changes in Students' Conceptions of the Characteristics of Mathematics. In summary, it's hard to infer a lot about how the students' conceptions of the characteristics may or may not have changed over the course of the semester due to the small amount of data. However, it appeared in the final concept maps and "What Is Mathematics?" journal entries that there was less of a focus on what some seemed to consider as negative characteristics of math than there was at the beginning of the semester. This may have been a result of the students beginning to doubt their negative stereotypes of mathematics but having nothing to put in their place, or it may have simply been a result of them shifting their attention to other aspects of mathematics. It also seemed that some modified their belief that mathematics has only one right answer to include the idea that the right answer can be found in multiple ways. There was also some evidence from the Mathematics Inventory items 11 and 17 (see Table XX on page XX) and from the final "What Is Mathematics?" journal entries that there was an increase in the belief that mathematics is something that can be learned by everyone.

Usefulness of Mathematics

A significant part of the students' conceptions of mathematics was their ideas about whether and how mathematics is useful in this world. In the following sections, I will examine the conceptions of the students related to this area.

Four categories emerged from my analysis of the data: *general usefulness of mathematics, personal utility, indirect personal utility,* and *utility for others*. The category *general usefulness of mathematics* is for comments the students make about the usefulness of mathematics without giving specific examples of *how* it is useful. For example, if a student wrote simply, "Mathematics is useful;" that conception would be included in the *general usefulness of mathematics* category. *Personal utility* refers to the way that these students recognize that they use math themselves such as in balancing their checkbook, cooking, or figuring a discount when shopping. *Indirect personal utility* refers to the way these students recognize that math affects their lives, but for which they didn't do the math. For example, many students recognized that math was used in such things as the construction of buildings, elevators, and traffic lights, and they acknowledged that they use those things. But, they didn't do the math associated with their construction; so, the usefulness of math was indirect for them. *Utility for others* refers to the idea that they recognize math as something that is used in the lives of others such as by engineers, doctors, architects, carpenters, and scientists, but again it's not math that they do themselves. More examples and elaboration of these areas follow.

Students' Conceptions of the General Usefulness of Mathematics at the Beginning of the Semester. At the beginning of the semester, the students, as a group, held a fairly strong conception that mathematics was something that is generally useful in the world. This can be seen by their responses to the three items in the Mathematics Inventory that were related to the usefulness of mathematics in a general way. The first was item 3, "I think mathematics is an important subject that is useful in life." The mean response to that item on the first day of class was 3.966, indicating fairly strong agreement with the statement. The second item from the Mathematics Inventory related to the usefulness of mathematics was item 5, "I think mathematics is something very much related to this world." The mean response for that item on the first day of class was 3.931 which also indicated fairly strong agreement and was very similar to the mean for item 3. The final item from the Mathematics Inventory that was related to the usefulness of mathematics was item 13, "The only people who really need to understand math are those who choose a career which requires math skills." This item is less obviously related to the general usefulness of mathematics, but I included it in this section because it is related to the idea that mathematics is useful for people generally—not just for those who use it in their jobs. The mean response for that item on the first day of class was 1.517, indicating quite strong disagreement with the statement.

This general conception that mathematics is useful was also evident in other sources of data. For example, in the beginning group of "What Is Mathematics?" journal entries, 21 of the 35 students (60%) who wrote the entries wrote about the usefulness of mathematics. Most of them wrote specific examples of how math was used, but a few of their ideas were just about the general usefulness of math. For example, one student wrote, "Math is the basis for almost everything. Since the very earliest forms of writing, science, music, and even before that we see signs of man using math. Man and math are inextricably linked." Another wrote simply, "I recognize that math is everywhere."

In the beginning math concept maps, 26 of the 34 students (76%) of the students included something about the usefulness of mathematics. As was the case with the beginning "What Is Mathematics?" journal entries, most of what they included was related to how they use it personally, but some of it was related to the general usefulness of mathematics. Twenty-one of those 26 students had an item that was either "everyday life" or "real world" or "applications to real world." Two students had the item "not just school," and one student wrote "applications." Most of the students gave specific examples connected to those items, but five students just had those headers ("everyday life" or "real world applications") with no examples.

Students' Conceptions of the General Usefulness of Mathematics at the End of the Semester. At the end of the semester, the students still held the conception that mathematics was something that is generally useful in the world, but there was evidence to suggest that the degree to which they thought math was useful increased. This can be seen most clearly in Table 4 which shows the questions from the Mathematics Inventory that are related to usefulness of mathematics. While both item number 3, "I think mathematics is an important subject that is useful in life" and item number 5, "I think mathematics is something very much related to this world" had quite high beginning means, the final means were both statistically significantly higher. The final mean for item 3 was 4.379, and the p value was 0.035. The final mean for item 5 was 4.414, and the p value was 0.0079.

Table 4: Items Relating to *Usefulness of Mathematics* from the Mathematics Inventory

Item	Beginning Mean	Final Mean	p value
3. I think mathematics is an important subject that is useful in life.	3.966	4.379	0.035*
5. I think mathematics is something very much related to this world.	3.931	4.414	0.0079*
13. The only people who really need to understand math are those who choose a career which requires math skills.	1.517	1.483	0.84

*statistically significant

The data from the "What Is Mathematics?" journal entries also supports the idea that there was an increase in the degree to which the students thought of mathematics as generally useful. There was more attention to the usefulness of mathematics in the final "What Is Mathematics?" journal entries than there was in the beginning set. Twenty-eight of 30 students (93%) included something about math being a part of everyday life and/or math being everywhere. In fact, this was the idea written about most often in these journal entries. A few students claimed a change in their perception of mathematics being "everywhere" such as the student who wrote, "Math is a big picture of something that is everywhere; it is no longer a subject for me." Another student wrote,

> "For me, the meaning of math has changed drastically over the past several months. I used to see math as some boring thing that I always struggled to get through during my school years. I saw it as just addition, subtraction, multiplication, and division. I never realized how much mathematics was tied into my everyday life."

One more example of this declared change in perception is the student who wrote the following:

> "I used to think that math was something that we did for an hour a day in school that could be forgotten about as soon as the bell rang. Now, my views have changed. Instead of being something that can be easily forgotten, math is something that is involved in every aspect of my life."

Some students wrote generally of the usefulness of mathematics and its connection to the world. For example, one student wrote,

> "Math is a phenomenon of nature, which occurs everywhere and includes everything. . . .These classifications of math cover every aspect of life. They allow us to deal with shapes, distance, time, weight, money, calendars, maps, area, volume, music, and amount. They are the basis of computers and science. Humans are even mathematical—not only in the sense that we are capable of understanding mathematical concepts, but in the sense that our body functions are mathematical. For example, we have a heart rate and blood pressure, and we move because of forces (which are mathematical) which push against us. Furthermore, we each have a social security number, so you could even say that our identities are mathematical. So we can conclude that math is constantly surrounding us in every aspect of our life and without it nothing and nobody would exist!"

There was less inclusion of the general usefulness of mathematics at the end of the semester than there was at the beginning in the math concept maps, but the conception was still evident. For some individuals there was some change in the type and an increase in the amount of items relating to this area in their final math concept maps. For others, it seemed, usefulness of mathematics became less prominent as their concept maps were expanded to include other aspects of mathematics. This may be partially why the general usefulness of mathematics became less visible on the concept maps even when the Mathematics Inventory shows that they were more convinced of the general usefulness of mathematics at the end of the semester. Another reason the general usefulness of mathematics became less visible on the concept maps is because at the end of the semester, most of the items the students included related to usefulness of mathematics were specific examples rather than general references. Nine students included "everyday life" or "real world" in their final concept maps. One student each included the items "used in <u>everything</u>," "found everywhere," "application to our lives," and "infinite uses" in their final concept maps. A couple of examples of beginning and final concept maps which illustrate these types of changes related to both general and specific usefulness of mathematics follow:

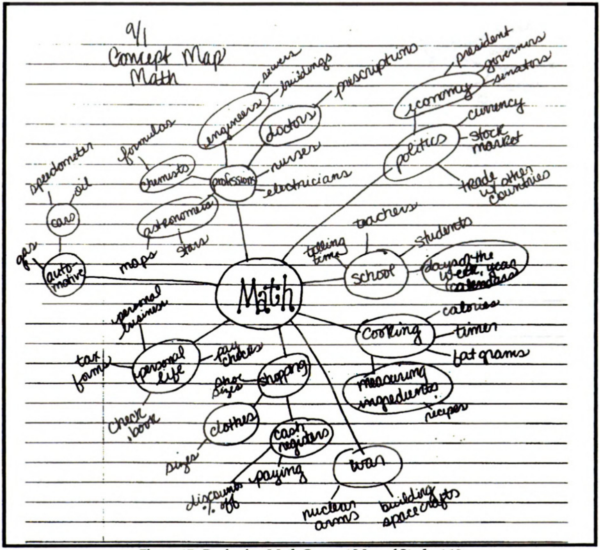

Figure 17: Beginning Math Concept Map of Student 10

One example of usefulness of mathematics being included in both the beginning and final maps, but less dominantly in the final map was the concept maps of student 10. Her beginning concept map was almost entirely about the usefulness of mathematics (See Figure 17). Her final concept map also contained a great deal about the usefulness of mathematics, but it had been expanded to include other aspects of mathematics (See Figure 18).

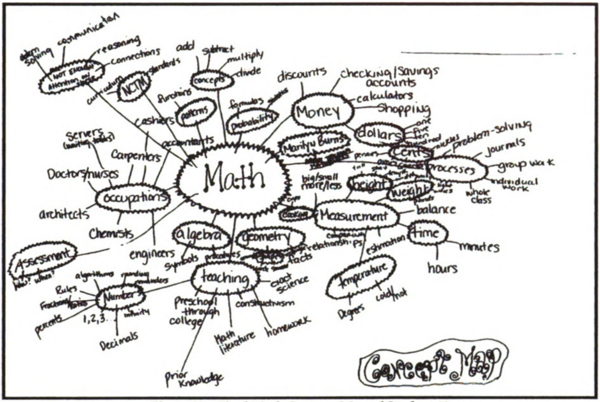

Figure Xx: Final Math Concept Map of Student 10

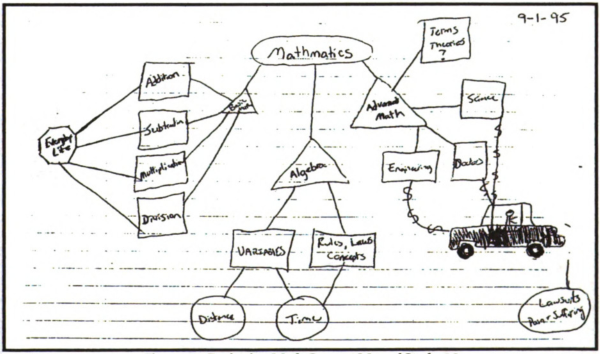

Figure 19: Beginning Math Concept Map of Student 3

Student number 3 is an example of a student who included a little of the usefulness of mathematics in his beginning concept map by noting two careers, an area where math are used under his advanced math sub-topic, and connecting "everyday life" to the four basic operations (See Figure 19 on the previous page). As can be seen in Figure 20, his final concept map was much more complex, and one of the areas that was elaborated was the usefulness of mathematics area.

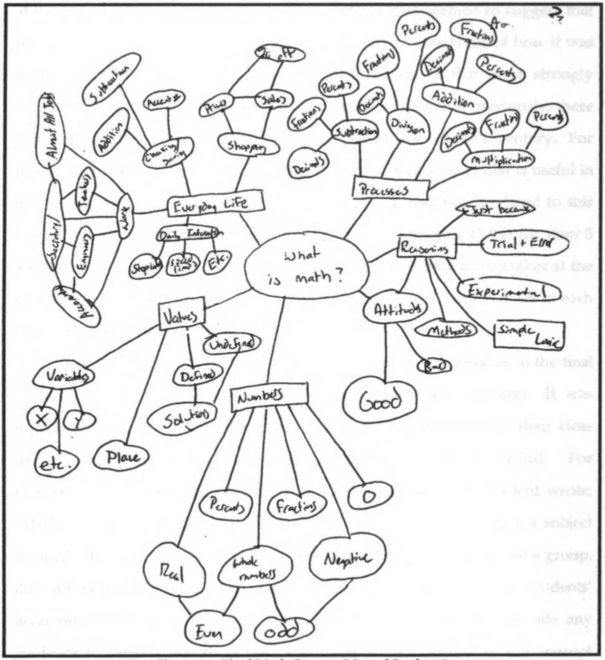

Figure 20: Final Math Concept Map of Student 3

Overall, at the end of the semester, the students seemed to consider mathematics as something that is generally useful in the world, and they seemed to believe it more strongly than they did at the beginning of the semester.

Summary of Changes in Students' Conceptions of the General Usefulness of Mathematics. Although the students seemed to believe that mathematics was useful in this world both at the beginning of the semester and at the end of the semester, there was some evidence that seemed to suggest that the degree to which they thought it was useful or their awareness of how it was useful increased during the semester. The data source that most strongly supports this assertion is the Mathematics Inventory. As noted previously, there were three items related to the usefulness of mathematics in the inventory. For two of them, item 3, "I think mathematics is an important subject that is useful in life," and item 5, "I think mathematics is something very much related to this world," the mean responses were quite high at the beginning (3.966 for item 3 and 3.931 for item 5), but the means were statistically significantly higher at the end of the semester (4.379 for item 3 and 4.414 for item 5) with $p < .05$ in each case (See Table 4 on page 104).

The increase in the attention to the usefulness of mathematics in the final "What Is Mathematics?" journal entries also supports this assertion. It was especially clear in the journal entries where the students declared that their ideas had changed regarding the usefulness of mathematics in the world. For example, in her final "What Is Mathematics?" journal entry, one student wrote, "Math is a big picture of something that is everywhere; it is no longer a subject for me." The change in the beginning and final math concept maps, as a group, did not noticeably show that there had been an increase in the students' awareness of the usefulness of mathematics, but it also did not provide any evidence to suggest that there *wasn't* an increase. The change in the concept maps for some individuals, however, did support the idea that they had a greater awareness of the usefulness of mathematics.

Students' Conceptions of the Personal Utility of Mathematics at the Beginning of the Semester. Not surprisingly, the area of usefulness of mathematics that the students referred to most often was the personal utility of mathematics—the contexts outside of school in which they actually needed to do math themselves. This is where the usefulness of mathematics is most evident to them in their lives. In the beginning "What Is Mathematics?" journal entries, several students (21 out of 35) included examples of how mathematics is useful to them personally. An example of a typical statement is, "We use math whenever we are dealing with money, and to me this is the most practical application." Using math to use money was the most common way students wrote about how math was useful to them personally. Another student wrote, "I think that it is very important to the tasks we complete in everyday life. For example, whenever you count money to pay for something or figure out how much time it will take you to do something." Of course, there were other personal uses of math the students wrote about as well. One example is the student who wrote, "It is the real-life experience of knowing how to halve the ingredients of a recipe or measure how much carpet is needed to cover the floor of a room. Math is one subject we use everyday." Another student wrote,

> "Through the years, we use math everyday of our lives. At the grocery store figuring out how much two pounds of grapes will cost at 49¢ per pound. Calculating 20% off your favorite sweater in the mall. Deciding how much flour you need to put in the cookie dough if you want to triple the recipe. Mathematics is a profound concept that is relevant to almost every aspect of our surroundings."

The recognition that math is useful to the students personally was also evident in the math concept maps. Twenty-six out of the 34 students included something in their beginning concept map that was a way they used math themselves. More than half of the students included three or more examples of ways they used math in their day-to-day lives. There was a variety of examples the students included as items of how they use math personally, but it's interesting how many of them relate to money (especially as it relates to shopping) or driving. Cooking was also an often-noted personal use of mathematics. Following is a list of the items the students included with their number of occurrences in descending order of frequency:

shopping (10)
cooking (9)
money (8)
checkbook and automotive/driving (7)
time and comparison (6)
gas/gas left and speedometer (4)
recipes, figuring out discount, and paying (3)
cars, distance, mileage, managing time, paychecks, tax, prices, sharing with friends, and games (2)
speed, how long a trip will take, mph, timer, fat grams, scheduling, paying bills, tax forms, gratuity, personal business, loans, selling, cash registers, shoe sizes, sizes, clothes, smallest to largest, leisure, race, and gpa (1)

The beginning math concept map of student 10 is a good example of someone who included many items related to the personal utility of mathematics (See Figure 17 on page 107). Most of the bottom half of her concept map is related to personal utility of mathematics. Another example with is the beginning concept map of student 14 (See Figure 21 on page 112). It's interesting that she connected the mathematical processes of "problem solving" and "reasoning" to situations where she uses math in her own life.

Students' Conceptions of the Personal Utility of Mathematics at the End of the Semester. At the end of the semester, the students' ideas, as a group, about how math is useful for them personally were quite similar to what they were at the beginning of the semester. There were slightly fewer students who included something related to the personal utility of mathematics in their final

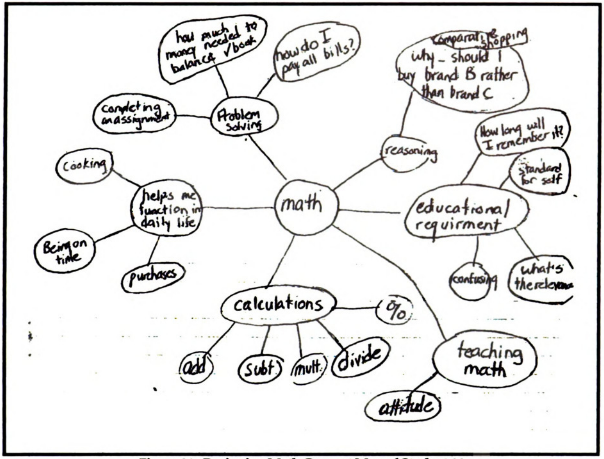

Figure 21: Beginning Math Concept Map of Student 14

concept maps than there had been in the beginning maps—23 in the final maps (67.6%) and 26 in the beginning maps (76.5%). There were a few more specific examples in the final concept maps, but the difference as a group was minimal. This was still, by far, the area that was included most often by the students in their concept maps. In contrast, there were *more* students who wrote about this idea in their final "What Is Mathematics?" journal entries than in the beginning set. Twenty-eight out of the 30 students for whom I have final journal entries included something about this idea in their final "What Is Mathematics?" journal entries whereas 21 out of 35 included something in this area in the beginning. Again, in both data sources, the personal uses related to money, driving, and cooking were the most frequently included. Here are two examples of what students wrote about the personal utility of mathematics in the final "What Is Mathematics?" journal entries:

> "I use mathematics everyday, whether it is basic arithmetic so I don't spend too much money at the grocery store, or algebra so I can figure out what the lowest grade is that I can get on an exam and still get an A."

> "Math is something that is used in daily life. I think people often don't realize how <u>much</u> we use math. This is probably because it isn't commonly taught that math is useful. Math is used in driving, cooking, telling time, shopping, banking, and many other things."

In the final concept maps, the students, again, included a variety of personal uses of mathematics. The uses related to money and cooking were very common as they were in beginning set of concept maps, but there seemed to be less of an emphasis on the uses related to driving and more of an emphasis on the uses related to time. The related words that the students most commonly included with their frequency were "checkbook" (9); "time/telling time" (9); "money" (8); "shopping" (8); "cooking" (7); and "work/job" (4). There were seven items included twice each and 46 other examples included by one student each. The final concept map of student 10 is an example of someone who included a high number of items related to the personal utility of mathematics (See Figure 18 on page 108). Much of the right side of her map is related to personal utility. She also included a high number of items in this area in her beginning concept map.

The 23 students who included something about the personal utility of mathematics in their final concept maps weren't a complete subset of the 26 students who included items in their beginning concept maps. A few students included items related to personal utility in either their beginning or final concept map. For example, there were two students who included a high number of items in this area in their beginning concept maps who didn't include anything in their final concept maps. There were also two students who didn't include anything in this area in their beginning concept maps but who included a high number of items in their final concept maps. There were only four students out of the 34 who never included anything related to personal utility in their concept maps. For most of the students, the difference in the inclusion of items related to personal utility between the beginning concept maps and the final concept maps seemed to reflect a change in the focus of their attention related to other aspects of mathematics rather than a change in their ideas about the personal utility of mathematics. In general, the final concept maps were much more detailed, complex, and elaborate than the beginning concept maps, and many students seemed to be thinking of math in a much broader sense than how they used it personally. For many of the students, while there was less focus on how they used math, there was more focus on aspects they would be thinking about if they were thinking of teaching math. I think if the students had been asked to include ways they use math on their concept maps that they would have all been able to include some examples of how they use it, and those examples wouldn't be that much different from examples they could have provided at the beginning of the class. However, with the open-ended prompt of making a concept map with "math" as the concept, they included what they were thinking about most at the time. An example of a student who included nothing related to personal utility of mathematics (or any other kind of usefulness of mathematics) in her beginning concept map but included a section in her final map is student 24 (See Figure 22). Her items were typical since she included the most common personal uses noted by the class as a whole of money, cooking, and time.

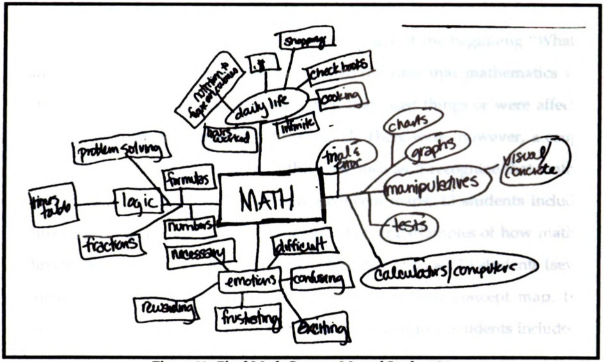

Figure 22: Final Math Concept Map of Student 24

Summary of Changes in Students' Conceptions of the Personal Utility of Mathematics. It was not surprising that there was not a lot of change in the students' conceptions of the personal utility of mathematics during the semester. Their notions of where they use math personally were obviously already a solid part of their conceptions of mathematics. There may have been a little increase in their awareness of how they use math in their daily lives as a group, but it was probably minimal. Almost all of the students included something about the personal utility of mathematics in their "What Is Mathematics?" journal entries—especially in the ending set. Most, but not quite as high as a percentage as with the journal entries, included items in their concept maps related to this area. However, my feeling is that if they had been asked specifically to provide examples of how they use math personally in their concept maps, probably all of them could have included examples similar to those that were noted.

Students' Conceptions of the Indirect Personal Utility of Mathematics at the Beginning of the Semester. There was some attention to the indirect personal utility of mathematics at the beginning of the semester, but only in the concept maps. There was no specific reference in any of the beginning "What Is Mathematics?" journal entries that referred to the idea that mathematics was useful to the students in an indirect way—that they used things or were affected by things for which mathematics had been used. There were, however, a couple of sentences that may have alluded to that idea such as "I recognize that math is everywhere." However, in the beginning concept maps, 13 students included indirect uses of mathematics, and they included several examples of how math is indirectly personally useful in their items. Over half of those 13 students (seven students) included a low number of items (1-2) in their concept map; two students included a medium number of items (3-4); and four students included a high number of items (five or more). An example of a concept map in which the student included a specific section for the indirect usefulness of math (as well as a section for the direct usefulness) was student 18 (See Figure 23).

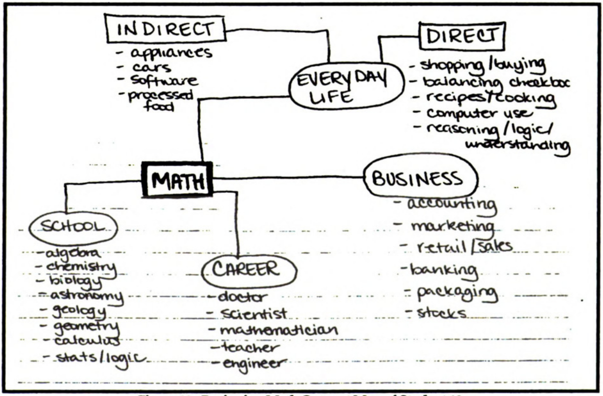

Figure 23: Beginning Math Concept Map of Student 18

Four students included the item "indirect" or "things unnoticed." Two students included "products we use." One student each included most of the examples, but three students included "traffic lights," and two students each included "economy," "stock market," "machines," and "appliances." There were 30 other examples listed once each, including "currency," "bridges," "painting the road," "parking spaces," "computer use," "software," "buildings," "TV," "VCR," "medicine," "prescriptions," "air travel," and "maps." These were all things that the students might use and for which mathematics was involved in their creation or use, but for which the students didn't do the math. At least some of the students seemed to have an awareness that they were affected indirectly by the way math was used for certain things in the world. It should be noted that on the first day of class, before the students did their beginning math concept maps, we had a discussion in class about some of the ways math is used that affect us indirectly like with traffic lights—not because that was part of my lesson plan, but because a student brought it up, and others joined in the discussion. So, that may have prompted some students to include some items related to this area in their math concept map that they may not have otherwise.

Students' Conceptions of the Indirect Personal Utility of Mathematics at the End of the Semester. At the end of the semester, the journal entries revealed a stronger conception of the students that mathematics was indirectly useful to them personally than there was at the beginning of the semester. This wasn't so evident, however, in their concept maps since the beginning and final sets were very similar in relation to this area. Twelve students included something related to indirect personal utility in their final concept maps while 13 did in their beginning concept maps. Ten of these students included related items on both their beginning and final maps. In the final set, six students had a low number of items (1-2), four had a medium number of items (3-4), and two had a high number of items (5 or more) related to the indirect personal utility of mathematics. This was similar to the beginning set in that most of those who included something in this area included a low number of items. The items they included were quite similar to what they were at the beginning of the semester, but there was some difference. For example, in the final math concept maps, four students each included "science" and "temperature," and three students included "computers," while only one student included each of those items in her beginning map. There was also some reference to music and art at the end of the semester that there wasn't at the beginning. Two students included "music" while one student each included "art," "mosaics," "symmetry," and "tessellations." These areas are related to the utility of mathematics because people use mathematics while they are composing music and often when they are creating art such as when they are making tessellations or mosaics or using symmetry. (The inclusion of these items may also suggest the possibility that some students were developing conceptions about math and aesthetics. Perhaps they were beginning to see mathematics as something that was more than useful—as something that was a source of beauty and aesthetic pleasure.) Similar to the first set, two students each included "traffic lights" and "speed limits." Again, there was a big variety in the other 22 items that were included in this area by one student each. Some examples of these are "populations," "building," "technology," "medicine," "government," "taxes," "international relations," "packaging," "pop cans and bottles," and "every time a product is sold."

The place where the change was most evident was in the final "What Is Mathematics?" journal entries. Whereas no student included any reference to the indirect personal utility of mathematics in his or her beginning "What Is Mathematics?" journal entry, 13 students included references to the way math was useful to them in an indirect way in their final "What Is Mathematics?" journal entries. The students included several specific examples of how math affects them indirectly. For example, one student wrote, "It is used daily in any given person's life whether the person is actually doing mathematical operations or using the products of other people's math operations." Another student wrote, "No matter where you go you will always see math in the buildings, cars, and so on. Mathematics is used to created just about every product of the world and is also used in its packaging." Another example of writing about this topic is the student who wrote,

> "I find it to be incorporated into just about everything. For example, traffic lights, ovens, recipes, calendars, time, day, etc. This list is never ending. Mathematics is a fact of life!!! . . . Everywhere you turn and look you could almost be sure is a result of some mathematical process."

Another example of a student writing about this realization is the student who wrote,

> "Math is present in our everyday lives in these basic ways, but it is also present in many complex and abstract ways. For example, this computer that I am using required mathematics to be created along with the house that I am sitting in."

In class, we talked about mathematics' connection to art (as in tessellations and design), music, and literature. We also talked some about math history and number theory, including Fibonacci and the Fibonacci numbers and how often these numbers appear in nature as they do in the spirals of pine cones. These connections of mathematics were examples that students often wrote about or talked about as new ideas to them such as the student who wrote, "Mathematics seems to somehow relate to everything. When I entered the class I knew math was related in manner of time and measurement, but the idea of art through tessellation and patterns was a new connection." Another student wrote much more extensively about the indirect personal utility of mathematics, including a reference to the Fibonacci numbers. He wrote,

> "Math encompasses life as we know it. Almost anything mass produced or created can be defined mathematically, or was created mathematically. . . Of course all of our lives we have been surrounded by math through the human world of engineering, as it is now called, and unknowingly through nature, e.g., the Fibonacci numbers in flowers or space examples. Although we never really pay tribute or homage to the concepts or people who have researched them, it is only fair to recognize that math is in, or very related to, everything—it is infinite in its uses, discussions, explanations, etc.—helping humans to understand and describe their lives. Whether material goods or driving your computer/human built automobile, math surrounds us as human beings, although we rarely acknowledge that fact."

Overall, the students as a group seemed to be much more aware of how math affected their lives indirectly at the end of the semester. There seemed to be a greater sense that "math surrounds us as human beings" as was expressed by the student in the just-noted passage and by many of the students.

Summary of Changes of Students' Conceptions of the Indirect Personal Utility of Mathematics. This is the area of usefulness of mathematics where there was probably the biggest change in the students' conceptions from the beginning of the semester to the end of the semester, and it's likely that the increase in their awareness of the indirect personal utility of mathematics was the biggest influence in the statistically significant increases in the items from the Mathematics Inventory that were related to the general usefulness of mathematics. These were items 3, "I think mathematics is an important subject that is useful in life;" and 5, "I think mathematics is something very much related to this world" (See Table 4 on page 105). At the beginning of the semester, some students seemed to realize that mathematics affected their lives even when they didn't do the math that they were benefiting from (perhaps especially because of the conversation we had on the first day), but at the end of the semester, it seemed that several more students recognized this and that as a group, the students had a greater awareness of how math affected them indirectly. This was most obvious in the "What Is Mathematics?" journal entries. In the beginning "What Is Mathematics?" journal entries, the students wrote quite a bit about how they use math and some about how math is used by people in certain jobs, but they didn't write *anything* specifically about how math affects them indirectly. In contrast, in the end-of-the-semester "What Is Mathematics?" journal entries, 13 students wrote specifically about the indirect personal utility of mathematics, and six specifically wrote that their ideas of mathematics had changed in that they now realize it is something that is everywhere and in everything.

Also, as speculated in the previous section, some of the students may have started seeing math as something that was more than useful—as something that was aesthetically pleasing—by the end of the semester. The data was insufficient to determine for sure whether that was the case, but the possibility is still worth noting.

Students' Conceptions of the Utility for Others of Mathematics at the Beginning of the Semester. The indirect personal utility of mathematics and the utility for others of mathematics are interconnected since an awareness of how math being done by others affects one indirectly implies an awareness that math *is done* by others. In these two areas of the usefulness of mathematics outlined by the framework, there is a focus on different subtleties of the same idea. While the indirect personal utility of mathematics is perhaps more focused on the products or services produced by other people doing math, the focus of the utility for others of mathematics is more the specific activities, jobs, or careers that the people who are doing the math are engaged in. Sometimes the math they do would affect them indirectly, and sometimes it would not. For example, the math that someone did across the country to build a deck for his home would be utility for others of mathematics for the students in this class, but there would be no indirect personal utility for them. In contrast, if the students talked about a civil engineer who used math in designing the road that leads to the building where we met as a class, their references to the engineer would be considered utility for others of mathematics, and their references to the road would be considered indirect personal utility using this framework.

Most students did not note the way math was used by others in activities, jobs, or careers in their beginning "What Is Mathematics?" journal entries or in their beginning math concept maps. In the beginning "What Is Mathematics journal entries, only five students wrote examples of math being used by other people, and in the beginning math concept maps, seven students included examples in this area. However, it can probably not be inferred that the other students didn't realize that other people use math. At the beginning of the semester, most of what the students included in their "What Is Mathematics?" journal entries and their math concept maps was related to the students' own relationships or experiences with math. They didn't seem to be as focused on math's relation to others and to the world in general. There were, of course, some exceptions. Most likely, if the students had been asked specifically how other people use math, they could have at least listed some examples of careers that require math.

An example of what one student who did write something about this area of mathematics' usefulness wrote in her beginning "What Is Mathematics?" journal entry related to the utility for others of mathematics was, "I guess math is also important in engineering, medicine, and cooking, but I would not know this first hand." Another student wrote, "Math is necessary in everyday life: at work, on computers, in grocery stores, in clothing stores, for doctors and nurses and accountants, etc. Every job in the world will use mathematics in some form."

Seven of the students included careers that require the use of math in their beginning concept maps. Three of the beginning concept maps that have been referred to earlier include examples of careers. These are the beginning concept maps of student 3 (See Figure 19 on page 108), student 10 (See Figure 17 on page 107), and student 18 (See Figure 23 on page 117). It's interesting that student 18 included a section for "careers" and a separate section devoted just to "business." Both of these are related to utility for others of mathematics. Another example of a beginning concept map that is entirely devoted to the usefulness of mathematics and includes some examples of careers is the beginning concept map of student 17 (See Figure 24).

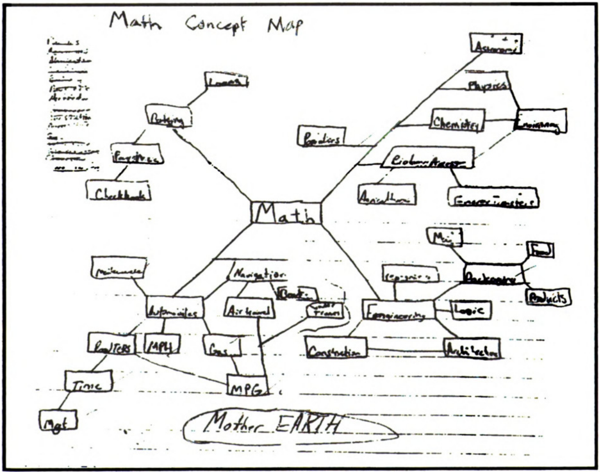

Figure 24: Beginning Math Concept Map of Student 17

The most common career requiring mathematics that the students included in their beginning math concept maps was "engineering" with six occurrences. Only one student who included careers *did not* include "engineering." The career included the next most frequently was "doctors" with three occurrences. Two students included "banking" and "packaging," and one student each included "nurses," "chemists," "electricians," "astronomers," "accounting," "marketing," "scientist," "mathematician," "teacher," and "proprietors."

Students' Conceptions of the Utility for Others of Mathematics at the End of the Semester. At the end of the semester, it seemed that the students' conceptions of the utility for others of mathematics was quite similar to what it was at the beginning of the class. It was something that was noted by a few students in the open-ended data sources of the "What Is Mathematics?" journal entry and the math concept map, but not by most of the students. In the final concept maps, eight students included examples of careers that use math (as opposed to seven in the beginning concept maps). Again, there was not a lot of change in this area. Three students wrote "occupations" or "careers," and one student wrote "almost all jobs." Again, the most frequently noted career was "engineers" with four occurrences. "Accounting," "doctors" and "chemist" each had two occurrences, and the following careers were noted by one student each: "teachers," "secretary," "nurses," "architects," "interior design," "chefs," "servers (waiting tables)," "carpenter," "cashier," and "astrology." The final math concept maps of student 10 (See Figure 18 on page 108) and student 3 (See Figure 20 on page 109) were examples of students who included some items in this area. Student 17 also included some examples of math's utility for others in his final math concept map (See Figure 25). While still prominent, the usefulness of mathematics was less dominant in his final math concept map than it was in his beginning math concept map since he also included a large section on school mathematics that focused more on the content of mathematics. His comment, "Math is in just about everything! If not, it is related!" written to the side of his final math concept map also indicates that he has a strong conception that mathematics is useful in the world.

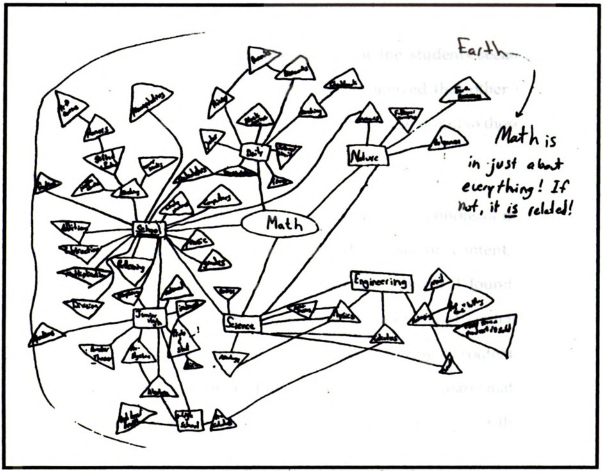

Figure 25: Final Math Concept Map of Student 17

Although many students wrote about how math that was done by others affects us indirectly in their final "What Is Mathematics?" journal entries, only four students referred specifically to the professions that use math. As was the case with the math concept maps, engineering was the career most frequently noted in the final "What Is Mathematics?" journal entries.

Summary of Changes of Students' Conceptions of the Utility for Others of Mathematics. The data related to this area of usefulness of mathematics was insufficient to determine for certain whether there was any change during the semester. From the information the data did provide, it appears there was little if any change in the students' conceptions of the utility for others of mathematics. Of course, as was noted previously, the increased recognition of the indirect personal utility of mathematics that the students seemed to gain during the semester implies that they also recognized that other people are doing the mathematics, and mathematics would thus be useful to those people.

Chapter Summary

In this chapter, I first examined the students' conceptions of mathematics related to the nature of mathematics in the areas of content, processes, representations, and characteristics. In relation to content, I found that there seemed to be some change in the students' conceptions of content over the semester—or at least their attention to it. When referring to content at the end of the semester, the students included a wider range of mathematics content than they did at the beginning of the semester, and there was not the emphasis on "number" as being the sole or principal content of mathematics. Other things they included as content at the beginning of the semester such as the basic operations or names of courses were still part of their conceptions at the end of the semester, but they were part of much larger "lists" of mathematics content.

There was a big change in the students' conceptions of and attention to processes at the end of the semester compared to the beginning. Mostly this was an increase in their focus on processes, in their recognition of many different processes that are used in mathematics, and their conclusions that the processes used in doing math are often more important than the product (or answer). This seemed to be an area where many students had a lot of room for expansion of their ideas because they had never had processes other than reasoning, logic, and problem solving pointed out to them.

There was not a lot of attention to representations in the open-ended data sources in either the beginning or final sets. The little attention there was did not show much change in the students' conceptions of the representations of mathematics. One small difference that was evident at times is that when students wrote about some of the representations at the end of the semester, they usually also that math was more than that (i.e., more than formulas, equations, theorems, symbols, etc.).

There was also not a lot of attention to the characteristics of mathematics—at either the beginning of the semester of the end of it. However, at the end of the semester, there seemed to be less of a focus on what some seemed to consider as negative characteristics of math than there was at the beginning of the semester. It also seemed that some students modified their belief that there is one right answer in mathematics to include the idea that there may be more than one way to get that answer.

I then examined the students' conceptions of mathematics related to the usefulness of mathematics in the areas of general usefulness of mathematics, personal utility of mathematics, indirect personal utility of mathematics, and utility for others of mathematics. In the area of general usefulness of mathematics, there was evidence that suggested that the students believed that mathematics was generally useful more strongly at the end of the semester than they did at the beginning of the semester. Part of this evidence was the statistically significant increase in the mean responses to two of the items on the Mathematics Inventory—item 3, "I think mathematics is an important subject that is useful in life," and item 5, "I think mathematics is something very much related to this world." There was also more attention to the usefulness of mathematics in the final "What Is Mathematics?" journal entries than in the beginning set, and some students even wrote that their ideas of the usefulness of mathematics had increased.

There did not seem to be a lot of change in their conceptions of the personal utility of mathematics—how they use math themselves—during the semester since they seemed to have pretty solid ideas of how they used it when the class began. The most common personal uses the students referred to were related to money (such as with shopping and checkbooks), cooking, and driving. There also didn't appear to be a lot of change in the students' conception of the utility for others of mathematics. There was some attention to that idea by some students at both the beginning and end of the semester, but there wasn't a lot of difference.

The area where there seemed to be the biggest change in the students' conceptions related to the usefulness of mathematics was in the area of indirect personal utility of mathematics, and I hypothesize that that the increase in the awareness of mathematics' indirect personal utility was the biggest influence in the statistically significant increase in the previously-noted items from the Mathematics Inventory. At the end of the semester, it seemed that several more students recognized that math that was done by others affected them than realized that at the beginning of the semester. This was especially evident in the

"What Is Mathematics?" journal entries. At the beginning of the semester, no one wrote anything about how math affects them indirectly, but at the end of the semester, 13 students wrote specifically about the indirect personal utility of mathematics. I think the reason that there seemed to be such a big change in this area of the usefulness of mathematics is simply that it was something the students had not thought much about before the class, and most of the students were able to recognize that they were indirectly affected by math that was done by others. In contrast, I think that there wasn't as much change in the students' conceptions of the personal utility of mathematics and the utility for others of mathematics because those were both ideas that the students were familiar with and had thought about before.

CHAPTER 4

CONCEPTIONS OF LEARNING MATHEMATICS AND TEACHING MATHEMATICS

In this chapter, I examine the students' conceptions of learning mathematics and teaching mathematics. I look at their conceptions at the beginning of the semester and the end of the semester, and I then summarize the changes or lack of change. The conceptions of learning mathematics and teaching mathematics were understandably very intertwined. Many statements the students made could be considered to be referring to both learning and teaching at the same time. Of course, implicit in one's ideas of how mathematics should be taught are in most cases that person's conceptions of how it is best learned. In the following analyses, I will make some attempt to separate the students' main conceptions of learning math and teaching math while still recognizing their interconnection.

In general, as the semester began, the students' conceptions of learning and teaching mathematics were similar to those other researchers have found in related studies (Cooney et al, 1998; Hare, 1999; Raymond, 1997; Vacc and Bright, 1999; and Wilkinson, 2001). This view usually included the ideas that math was learned through practice and memorization and by building on previous knowledge. Their view of teaching math was one in which the teacher modeled a few examples of how to do a problem and then assigned the students several exercises from the textbook to practice doing that type of problem. Most of these conceptions were based on their own experiences as students of math.

Learning Mathematics

Learning mathematics is something that all students had ideas about since it is something that they had all experienced as students. At the beginning of the semester, most of their conceptions seemed to be based on their own experiences as learners. For most of the students, by the end of the semester, it appeared these ideas had become integrated with ideas about learning mathematics that were presented to them in class or were part of their practicum experience.

Students' Conceptions of Learning Mathematics at the Beginning of the Semester. Conceptions of learning mathematics were not a huge focus of the students' ideas related to mathematics at the beginning of the study in the open-ended data sources, but there was some attention to it. Fewer than half of the students included anything related to learning math in their "What Is Mathematics?" journal entry or their math concept map. In the beginning "What Is Mathematics?" journal entry, 14 of the 34 students (41%) included ideas related to their conceptions of learning and/or teaching mathematics, and in the beginning math concept maps, 13 students (38%) included something related to learning math. One student had a specific section for learning in her beginning math concept map.

Nine of the 13 students who included something in this area in their beginning math concept maps included "memorization" or "memorized" as something related to how math is learned, and only four of those nine included anything related to learning besides "memorization." Another item included in the concept maps related to learning math was "games" with six occurrences, and an example of a game, "Around the World," was listed twice. "[D]ice game" was listed by one student. The inclusion of games could have been related to the idea of learning by memorization since many games are used to aid or motivate memorization. Other items that were related to memorization that were included by one student each in the beginning concept maps were "drills," "rote learning," and "mechanical." Six students wrote something about memorization in their beginning "What Is Mathematics?" journal entries. One student wrote, "When most people think of math, I think they think of the drills and memorization that went along with it when our parents were in school." Another wrote, "In my elementary school experience, math was a textbook which had problems that were solved with a definite set of memorized facts." Still, another wrote, "Mathematics is sitting down and memorizing information that you just do not need." This inclusion of the idea of memorization in their concept maps and "What Is Mathematics?" journal entries was probably related to their own experiences learning math at least as much as their conceptions of how math is learned or best learned—as was the case for the just-noted student whose sentence began, "In my elementary school experience, . . ."

There was also an item in the Mathematics Inventory related to memorization, and the students' response to it indicated that they didn't consider memorization as the way math is best learned. This was item 22, "The best way to learn math is to memorize the rules." The mean response for that item at the beginning of the semester was 2.069, indicating a fairly strong disagreement with that statement. One student also wrote a statement consistent with this idea in her beginning "What Is Mathematics?" journal entry. She wrote, "I believe mathematics is more than just memorization or computation of problems but is a skill that requires using a thought process."

Seven students wrote something about the importance of *understanding* in learning mathematics in the beginning "What Is Mathematics?" journal entries such as the student who wrote, "It's important for students to understand mathematics and grasp its meaning." Two of those seven students also wrote specifically about the idea that learning needed to go beyond mere memorization. One student wrote, "Mathematics is a science in itself. There are rules and guidelines to follow and memorize. Once this is done however, one has to learn how and/or when to use them. This seems to be the trickier part. Many people know the rules but may not know how or when to use them. This is the important aspect of mathematics."

Another student wrote about this idea this way,

"It's important for students to have a set of steps from which to work. But, they should understand what exactly is happening when you 'carry the one' and why you're doing it. Working with a set of memorized facts makes for a very static environment, which can't adapt to any new twist in the problems. Understanding, however, will allow students to push themselves and explore new ideas and concepts."

Also, as was noted in the previous chapter in the section on processes, eight students included the word "understanding" in their beginning math concept maps. The processes that are used in doing mathematics are often the same processes that are used in learning mathematics. There were a few other words the students used in their concept maps that were related to this idea. They were (with their number of occurrences) "thinking" (4), "problem solving" (3), "trial and error" (3), "categorizing" (2), "sorting" (2), and "analytical thinking" (2).

In the beginning "What Is Mathematics?" journal entries, two students also wrote about the idea that mathematics was something that everyone could do. One student wrote, "It is a very systematic way of thinking, and everyone can accomplish and master this type of thought if they are trained in a way that's useful to them." Another wrote, "With the proper instruction and the willfulness to learn, anyone can succeed in this subject area." While two students apparently thought that all students could learn math at this point in the study, few of their classmates agreed. This idea was related to two of the items from the Mathematics Inventory that were discussed in the characteristics section of this analysis (See page 97). These were items 11, "Mathematics is something that not everyone is able to understand," and 17, "Mathematics is something at which everyone can be successful." As noted in that section, the class did not show a strong commitment to agreeing or disagreeing with either of these statements at the beginning of the semester. The mean for item 11 was 2.86, and the mean for item 17 was 3.207.

Another conception of learning that three students referred to in their beginning "What Is Mathematics?" journal entries was the idea that math needed to be learned early in life. One student wrote, "Mathematics is learning the basic concepts that allow us to function in life, and it needs to be taught to children early in life." The other student wrote, "Mathematics builds its foundation during the early elementary school years." The other student's conception was that math may be learned more easily when a child is young because it is similar to learning a foreign language. She wrote, "Actually, being that young [in elementary school], math may have simply come like a foreign language does. Children can often pick up on languages other than their native language with much more ease than older children or adults."

There were two other items on the Mathematics Inventory that were related to learning mathematics. One of these was item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key." The mean response for that on the first day of class was 4.138 which indicated fairly strong agreement. Several students wrote about the thinking processes used in mathematics in their beginning "What Is Mathematics?" journal entries, but only one wrote specifically that the thinking processes were more important than the correct answer. She wrote, "While some may argue that there is only one correct answer for mathematics problems, I believe that how a person gets the correct answer is what math is all about."

The final item that was related to learning in the Mathematics Inventory was item 24, "In learning math, it is important to master topics and skills at one level before going on." The mean response for that was 4.172, again indicating fairly strong agreement. Four students wrote about this idea in their beginning "What Is Mathematics?" journal entries. One student wrote, "It is about a series of steps. Making sure you understand one concept before you move on to the next. It is the idea of 'building blocks.'" Another wrote, "Understanding basic math concepts is the framework for understanding and succeeding in upper-level math courses." One other student expressed the idea this way, "Math is often about skills necessary to master concepts. These skills are usually built upon each other so that if one is not mastered, then it will be very difficult to understand the next."

The other items that were included in the beginning concept maps that were related to learning only had one occurrence each. They were "motivation and learning," "manipulatives," "bars, cubes, blocks," "mechanical," "each child learns differently," "knowing why something works," "patterns," "group learning," "open listening," "books," "ditto sheets," "workbooks," "organization," "step by step," "answers in back of book," "how to get answers," "puzzle," and "getting down addition and subtraction." Some of those items were related to tools that are used in learning mathematics; a few of them were methods used in learning mathematics; and some were related to their own experiences in learning mathematics.

Students' Conceptions of Learning Mathematics at the End of the Semester. There was a lot more attention to learning mathematics at the end of the semester than there was at the beginning which is not surprising since the focus of the course was teaching mathematics. This was evidenced most noticeably in the final math concept maps and the final "What Is Mathematics?"

journal entries. In the final math concept maps, 19 out of the 34 students (56%) included something related to learning and/or teaching math as opposed to 13 students on the first day of class. The change was even greater in the final "What Is Mathematics?" journal entries. All but seven students wrote something related to this area (23 out of 30 or 77%) in contrast to 13 out of 34 (38%) at the beginning of the semester.

There was also a change in the focus of this attention. For example, there was much less attention to memorization, and the attention was different. In the beginning "What Is Mathematics?" journal entries, six students wrote something about memorization—usually as a memory of how they learned math in school. In the final "What Is Mathematics?" journal entries, only three students wrote anything about memorization, and two of them wrote that math is more than memorization. One wrote, "We should stop emphasizing the importance of memorizing math operations. This simply contradicts our purpose." The other wrote, "Too many students consider math the memorization of numbers, and in many ways it is. But, it is also much more." The other quote from the journal entries about memorization was from a student who copied exactly her beginning "What Is Mathematics?" journal entry and added a few new sentences. Her quote which was noted in the previous section was, "Mathematics is sitting down and memorizing information that you really do not need."

The change in the math concept maps was similar. While nine students included "memorized" or "memorization" in their beginning concept maps, only four students did in their final concept maps. Three students included "memorization" in both their beginning and final concept maps. One student included the item "games" in her concept map. There was also a change in the students' response to the item related to memorization in the Mathematics

Inventory. For item 22, "The best way to learn math is to memorize the rules," the final mean was 1.931 indicating even less agreement with that statement as a group than there was at the beginning of the semester when the mean was 2.069. The change was not statistically significant, but the direction of change was encouraging (See Table 5).

Table 5: Items Relating to *Learning Mathematics* from the Mathematics Inventory

Item	Beginning Mean	Ending Mean	p value
22. The best way to learn math is to memorize the rules.	2.069	1.931	0.55
23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.	4.138	4.379	0.28
24. In learning math, it is important to master topics and skills at one level before going on.	4.172	3.655	0.035*

* statistically significant

The importance of reasoning and the process over the product were ideas that were more prevalent at the end of the semester than at the beginning. Related to this, and often noted in the same thought, was the importance of understanding which received a little more attention at the beginning of the semester than did reasoning and the process over the product. One place we see a little change in this conception is in the means for item 23 from the Mathematics Inventory, "Students' reasoning is more important than if they are able to get the answer that matches the answer key." The final mean response was 4.379 which indicated quite strong agreement and was up from the beginning mean of 4.138 (See Table 5). Again, the change in the means for this item was not statistically significant, but the direction of change was promising. The final "What Is Mathematics?" journal entries are where we see the most attention to these ideas. An example of this attention was the student who explained,

> "To be successful in math, one must understand the underlying logic surrounding specific problems. One must be capable of reasoning through a problem rather than relying on designated rules. To be able to reason mathematically is to be able to build on new mathematical ideas."

One student focused on the idea that most mathematics requires thinking—more than a couple of seconds worth. She wrote,

> "Another thing that math is, is thinking. Students must realize that for most problems they cannot just sit down and work them out in two seconds. Most math problems should challenge the students' thinking. . . . students need to understand that most of the problems in their lives cannot be solved quickly, but that they require thought; math is not any different."

Another student explained that math required thought processes like conjecturing and exploring: "Math goes beyond the pencil and paper rote, drill style of teaching. Students need to conjecture, explore, and revise their thinking in math." Another:

> "Math is different ideas, different ways to attack a problem, accepting others explanations, journals, justify and explaining or reasoning out your thoughts, the process, not always the product, listening and then eliminating, a group activity, being open to non-traditional forms."

Another student who wrote about reasoning as well as the importance of the process over the product was the student whose final "What Is Mathematics?" journal entry included this thought, "Mathematics requires reasoning and problem solving. It makes people think about how and why things occur. There is a focus on the process and not just the right answer. I feel this is a very important aspect of mathematics." One student also noted the importance of the process over the product when she wrote, "While math has often been seen to be just about getting answers, the trend seems to be switching to focusing on understanding and the processes and thought involved."

With regard to the idea that understanding is an important part of learning mathematics, some students focused on the importance of understanding why mathematics operations work. A few students noted this at the beginning of the semester as well. In her final "What Is Mathematics?" journal entry, one student wrote, "To succeed in mathematics, one needs to practice the contributing exercises. More prevalent than this, students must understand the why and how of operations." Another student wrote,

> "Previously, I associated mathematics with computation of numbers and step-by-step processes. I have learned, however, that math can be better understood and better appreciated when concepts are explained and clarified."

The importance of processes such as reasoning was also much more prevalent in the final math concept maps. Again, as was noted in the processes section of the previous chapter, eleven of the maps included the word "process" (or "processes"). "Problem solving" was the process included most often in the final concept maps. Twenty-four of the 34 maps included problem solving (70.6%). "Reasoning" was noted in 14 of the maps, and "logic" was included in 11. Some of the other processes that were included most frequently (with their number of occurrences in parentheses) were "comparing" (7), "thinking" (7), "trial and error" (5), "communicating" (4), "applying" (4), "making connections" (3), "understanding" (2), "experimentation" (2), "contrasting" (2), "analyzing" (2), "critical thinking" (2), and "thinking abstractly" (2). There were 37 other processes noted once. Of this list, only "thinking" with four occurrences and "trial and error" with three occurrences were included in any of the students' beginning math concept maps.

A part of the students' conception of learning mathematics at the end of the semester that was not common at the beginning of the semester was the role of manipulatives in learning mathematics. At the beginning of the semester, one student included "manipulatives" and "bars, cubes, blocks" as items in her math concept map. No one wrote about manipulatives in their beginning "What Is

Mathematics?" journal entry. In the final math concept maps, 11 students included "manipulatives," and one included "Use manipulatives to aid in visualization." One student each included the items "dice," "counters," and "using numbers and objects" which were related to the use of manipulatives. A couple of students wrote about manipulatives in their final "What Is Mathematics?" journal entries. One wrote, "Math is manipulating numbers and objects. Actually handling material objects helps one to understand the math concept being learned. Manipulatives help make learning math seem more real and applicable." Another wrote, "Math in the schools is slowly getting more involved other than just textbook problems. More manipulatives are being used in the classroom which gives the learners who are not textbook learners a new perspective."

There was also some attention to the role of discourse in learning mathematics at the end of the semester while there was none at the beginning. In her final "What Is Mathematics?" journal entry, one student wrote, "Math involves a particular discourse . . . knowledge of a mathematical discourse is essential to understanding and communicating in math-related situations and a classroom environment" (the student's ellipses). Another wrote simply, "Math is also writing and speaking." One student explained her views of the importance of discourse in learning math this way, "I also have become a big proponent of having students explain their thinking. This expands their understanding as well as that of those listening." Another thought a student had about the need for talking about math concepts was, "One needs to be able to explain themselves as well as teach others their own ideas." About writing, one student wrote, "Finally, I believe that writing should be an integrated part of mathematics learning." Another student wrote, "I never saw the value of having students write in math class until we discussed it in class."

The influence that attitudes have on learning math was another idea that was noted by some students at the end of the semester. The most common idea about that influence can be represented by the student who wrote, "I think that all of the negative attention ('Oh, it's soo hard!') that math receives makes kids believe that it is too difficult for them and that they can't succeed at it."

The change in the mean response for item 24, "In learning math, it is important to master topics and skills at one level before going on," was statistically significant. The mean on the last day of class was 3.655 down from 4.172 on the first day. The *p* value for that change was 0.035 which, as was noted, is statistically significant (See Table 5 on page 138). A statistically significant change for this item—especially when the change was in the direction it was—is perhaps a little surprising. However, this is an item for which an argument can be made for both agreeing and disagreeing with it, and it seems that by the end of the semester, the students weren't really sure what they thought about it since the mean is closer to the middle of the Likert scale. Of course, most people realize that much of mathematics builds on itself, and for example, a person cannot manipulate calculus problems without first having some understanding of algebra. However, one reason why there may have been less agreement with that statement at the end of the semester than at the beginning is because in class we talked about how sometimes when students don't really "get" a concept such as addition or subtraction when it is taught alone, they will pick it up later when they are learning about other things like measurement or probability and statistics that include that concept. Another thing that may have influenced the change in the students' agreement with that statement is an article that we read and discussed (Flanders, 1987) that talked about how much time is spent in each grade repeating content that was covered in previous grades and suggested that eliminating some of that repetition could provide more time to increase both the breadth and the depth of the content covered in a math class. Much of the writing about math reform emphasizes that good problems have many points of access (NCTM, 1991), so that students do not necessarily have to have mastered math content A in order to attack problem B. However, this is very different from what students have experienced in school where all the problems on page 116 require the use of the algorithm that was explained on page 114. So, the change is not surprising.

Summary of Changes in Students' Conceptions of Learning Mathematics. One of the big changes in the students' conceptions of learning mathematics from the beginning of the semester to the end of the semester was the amount of attention those ideas received—especially in the open-ended data sources. There also appeared to be a change in the *way* students thought about learning and teaching. Most of the students' conceptions of learning mathematics at the beginning of the semester were related to how they had experienced learning mathematics themselves. By the end of the semester, the ideas they included about learning were more how they would think about learning math if they were teaching it. This showed a shift in the way the students positioned themselves in relation to mathematics. For the group as a whole, there was a shift in positioning from thinking of math as they experienced learning it and as it related to themselves as learners and users of math to thinking of math as a teacher would—a shift from experienced student to prospective teacher. This would be a hoped-for, and certainly not a surprising, effect of the course. When the students first entered the course, the only experiences most of them had had with math were their experiences learning it in school and their experiences using it in their daily lives. For most, this was the only association they had with math and therefore their only frame of reference from which to form conceptions of and attitudes toward math. Throughout the class, they were introduced to new ideas related to mathematics and teaching mathematics as preparation to help them become teachers of mathematics. These ideas offered them a vision of math and how it could be taught that was often different from their own experiences, and provided them a new frame of reference from which to form conceptions of and attitudes toward math. They spent fifteen weeks talking about, reading about, completing assignments related to, and thinking about teaching mathematics. It is not unexpected that at the end of the semester they would be thinking about math more as someone who was going to teach it than as someone who had only experienced it as a student and user. In the following paragraphs, I will highlight some of the areas that I think exemplify this shift in positioning.

One example that was illustrative of this shift in positioning was the inclusion of the role of manipulatives and the role of discourse in their ideas about how math is learned at the end of the semester while there was essentially no attention to these ideas at the beginning of the semester. These were ideas related to learning that perhaps a teacher would want to utilize while teaching math, but they are things that the students probably didn't experience much while they themselves learned math.

Another example of this shift in positioning was the increased attention to the idea that the process was more important than the product. Many students seemed to feel that only the right answer was valued in their own experiences as students, which was not a positive experience for most of them, and they seemed to want to emphasize the process more for their students when they taught. There was an item in the Mathematics Inventory about that idea—item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key." The class agreed with that statement quite strongly as the class began (4.138 mean), and they agreed with it even more strongly at the end of the semester (4.379 mean). Their comments in their final "What Is Mathematics?" journal entries showed an even greater increase in their attention to this idea. The students also paid more attention to specific processes that are used in learning such as reasoning, conjecturing, exploring, justifying, and revising. Related to this was the importance of understanding mathematics, and while there was some attention to this at the beginning of the semester, there was more at the end of the semester.

There was also more attention to the role that teachers' and students' attitudes play in learning. The prospective teachers seemed to be more aware that students' desire and confidence to learn mathematics is affected by their teacher's attitude toward math as well as their own attitude toward it.

The decreased emphasis on memorization as a way to learn mathematics could also be considered an example of a shift in the students' positioning related to mathematics. While the attention memorization received at the beginning of the semester seemed to be the students remembering how they learned mathematics, the focus related to memorization at the end of the semester was more that learning math went beyond memorization. However, at both the beginning of the semester and the end of the semester, most students didn't agree with the idea that memorization was the best way to learn mathematics as was illustrated by item 22 from the Mathematics Inventory, "The best way to learn math is to memorize the rules." As a group, the students disagreed with that statement quite strongly at the beginning of the semester and even more strongly at the end of the semester with mean responses of 2.069 and 1.931, respectively.

An interesting change in the students' conceptions of learning mathematics was the statistically significant change in the mean responses to item 24 of the Mathematics Inventory, "In learning math, it is important to master topics and skills at one level before going on." At the beginning of the semester, the students agreed with that statement quite strongly, and their mean response was 4.172. At the end of the semester, the students agreed with that statement less strongly, and their mean response was 3.655. As was noted previously, a possible reason for this change was because of ideas discussed in class that showed that there were arguments that could be made for agreeing or disagreeing with that statement.

For the group, there seemed to be a greater expansion of the students' conceptions of factors that are part of learning from the beginning of the semester to the end of the semester. There also seemed to be a shift in the way the students, as a group, positioned themselves in relation to mathematics. At the beginning of the semester, most of the students seemed to relate to mathematics in a way that was based on their personal experiences with math—most often as learners. By the end of the semester, most students seemed to be thinking about mathematics as if they were going to teach it.

Teaching Mathematics

As with the students' conceptions of learning mathematics, at the beginning of the semester, there was some, but not a great deal of information about the students' conceptions of teaching mathematics from the open-ended data sources (the "What Is Mathematics?" journal entry and the math concept map). However, there were a couple of open-ended questions in the Mathematics Inventory that were related to their conceptions of teaching mathematics that added some more information. Also, at the end of the semester, the students wrote a "Philosophy of Teaching Mathematics" journal entry that provided much information about their conceptions of teaching mathematics at that point. The conceptions the students included in the "Philosophy of Teaching Mathematics" journal entries related to *teaching* mathematics implied some of their conceptions of *learning* mathematics, but because they were explicitly asked to write about their conceptions of teaching mathematics, these ideas are only included in this section. Another reason I chose not to include the information that I could infer from the "Teaching Mathematics" journal entries in the learning mathematics section is because I was also interested in seeing how much attention the students gave these ideas at that point when they were not specifically asked to address them.

As was the case with their conceptions of learning, the students' conceptions of teaching at the beginning of the semester seemed very influenced by their own experiences as students of mathematics. By the end of the semester, they seemed to have incorporated other ideas that were influenced by their class and practicum experiences.

Students' Conceptions of Teaching Mathematics at the Beginning of the Semester. At the beginning of the semester, the students included some, but not a significant amount about their conceptions of teaching mathematics in the open-ended data sources of the math concept maps and the "What Is Mathematics?" journal entries. There was, however, more in the journal entries than the concept maps. Fourteen students included something related to teaching in their beginning "What Is Mathematics?" journal entry, and only three students included anything about teaching mathematics in their beginning math concept maps. In her math concept map, one student included "workbooks," "rote learning," and "group learning" which were probably ways she remembered it being taught. Another student included "needs to be taught well" and "needs to be displayed in positive way to learners" in his beginning math concept map. The student whose concept map included the most about teaching (and learning) is shown in Figure 26. She had a distinct section for "Instructing" with the items "make it known that it is o.k. to make mistakes," "show how you can learn from mistakes," "must show confidence," "must emphasize understanding vs. memorizing," and "each child learns in different ways." Some of these ideas were also part of the conceptions of other students at the beginning of the semester as was evidenced in the "What Is Mathematics?" journal entries.

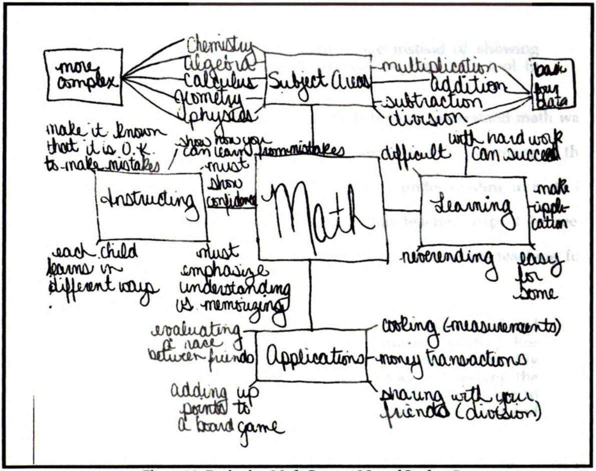

Figure 26: Beginning Math Concept Map of Student 5

The most common conception related to teaching mathematics that the students wrote about in their beginning "What Is Mathematics?" journal entries was the idea that math should be taught so that students can see how it's related to the world around us. Six students wrote about that idea. One student wrote, "It is important for students to grasp mathematics and understand its meaning and connections to the environment." Another wrote, "I think that it is important to teach math using realistic situations so that children can realize how relevant it is to their lives." Another student expressed her conception this way,

> "I think if basic math (adding, subtracting, etc.) is taught in ways that children can see how it is used in life, it won't seem foreign or as challenging. When you can see when you will be using or needing certain things then the process of learning it is valued."

Another student conveyed a similar idea when she wrote,

> "Many start to teach math for math's sake instead of showing students that mathematical knowledge can be used as a tool to solve real problems and help out in real life."

The idea that teachers should teach so that students understand math was another conception that was expressed by some students at the beginning of the semester. One student wrote, "Working towards an understanding in math is something students strive for and something that teachers expect to see." Another student elaborated more on the importance of teachers teaching for understanding. She wrote,

> "Teachers often fail to instruct students how to understand and analyze problems in a way to make them 'student friendly.' For example, an instructor who simply reads the problem and briskly writes the corresponding operations down does not convey the importance of interpreting what is actually being asked. Due to this common occurrence many children are left frustrated and clueless to what procedures had actually happened."

The only other comment that a student made in her beginning "What Is Mathematics?" journal entry related to conceptions of teaching mathematics is a student who wrote that teachers and memorization and drill can cause students to have a poor attitude. She wrote,

> "I think of the drills and memorization that went along with it when our parents were in school. That developed a poor attitude for those people. Now, these people are teaching people my age these same values which is why people who had older teachers when they were little don't like it now."

There were two items in the Likert-scale portion of the Mathematics Inventory that were related to conceptions of teaching mathematics. One of them was item 10, "Teaching mathematics should not be too difficult since a textbook already has the lessons prepared." On the first day of class the mean response was 2.034 indicating fairly strong disagreement with it. Most of the students probably realized—at least at some level—that teaching mathematics involved more than opening a textbook and reading through a prescribed lesson. The other was item 18, "Being good at mathematics is not required to be a good teacher of math." On the first day of class, the mean response for that item was 2.86 which was close to the middle, but leaning slightly in the direction of disagreement.

As was noted previously, the Mathematics Inventory included two open-ended questions that were related to the students' conceptions of teaching mathematics. The first of these questions asked, "What kind of influence do you think the teacher plays in the success of students in mathematics?" The most common responses from the students had to do with the idea that the teachers' attitudes toward mathematics affected the students. The idea written about most frequently was that teacher excitement and/or enthusiasm carries over to students. Eleven students wrote that. Ten different students wrote that teachers influence how students feel toward math. Related to that was that teacher confidence carries over to students which was written by four students, and three students wrote that the teachers' attitudes affect the students' attitudes which affects their performance.

A second open-ended question asked specifically about the role teachers' attitudes toward mathematics play in the their students' learning of mathematics: "Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not? How about a teacher's attitude about teaching it?

Explain." Thirty-three of the thirty-four students who answered that question wrote, "Yes." The one student who wrote "no" wrote that she doesn't like math, but she recognizes its importance and that students need to learn it. To explain why they thought teachers' attitudes affect their students' learning of mathematics, sixteen students wrote that the teacher's attitude rubs off on students. Nine students wrote that teachers' attitudes affect the way they teach it. Five students wrote that if teachers don't like math, they won't spend a lot of time or energy on it. Four students wrote that if a teacher has a bad attitude or is apprehensive, it will affect the students' attitude and learning. It seemed clear that almost all of the students had the conception that teachers' attitudes toward mathematics affected their teaching of mathematics—most often by influencing their students' attitudes toward math. This is something that they may not have thought about on their own, but because it was a question they were specifically asked in the open-ended questions of the Mathematics Inventory, they formed an opinion about it.

Students' Conceptions of Teaching Mathematics at the End of the Semester. Not surprisingly, the open-ended data sources provided much more information about the students' conceptions of teaching mathematics at the end of the semester than they did at the beginning. There was much more of a focus on conceptions of teaching mathematics by the students in their "What Is Mathematics?" journal entries as well as their math concept maps. Twenty-three out of the 30 students for whom I have final entries (76.7%) wrote something about teaching and/or learning in their final "What Is Mathematics?" journal entry compared to 14 out of 35 (40%) at the beginning, and 19 out of 34 students (55.9%) included something related to teaching and/or learning in their final math concept map compared to three (8.8%) at the beginning of the semester. Seven of those students included a high number of items (five or more) in this area while only one did in the beginning concept maps. (The one who included a high number of items—Student 5 whose beginning map is shown in Figure 26 on page 148—also had a high number in her final map.) The final "What Is Mathematic?" journal entries and math concept maps weren't the only sources of information regarding the students' conceptions of teaching mathematics at the end of the semester. Again, there were two Likert scale items, and there were two open-ended questions in the Mathematics Inventory that were related to conceptions of teaching mathematics which provided information. Also, as noted previously, the students each wrote a "Philosophy of Teaching Mathematics" journal entry at the end of the semester which, as would be expected, provided an enormous amount of information about their conceptions of teaching mathematics at that time. This became the main source of information for understanding their conceptions in this area. Of course, if people were asked the question, "What is mathematics?" we wouldn't necessarily expect them to respond with how they think math should be taught, but if they were asked, "What is your philosophy of teaching mathematics?" we would expect them to talk about how they think math should be taught.

There was not only an increase in the students' attention to teaching mathematics in the open-ended data sources of the final "What Is Mathematics?" journal entries and the final math concept maps, but, as was the case with their conceptions of learning mathematics, there also seemed to be a shift in positioning for these students from thinking about mathematics as it related to their own experiences to thinking about teaching mathematics as if they were the teacher or going to be the teacher. This shift was illustrated very clearly by one student's explanation of how she felt her conceptions of math had changed. In her final "What Is Mathematics?" journal entry, she wrote, "I am no longer looking at math as a subject. I look at it more in terms of how to teach it. . . I feel that I have shifted my belief in math from a personal to a professional level."

This shift in attention and positioning can be seen in several students by contrasting their beginning math concept maps with their final concept maps. At the end of the semester, these students seemed to be seeing math much more as a teacher would than a student would—and than they did at the beginning of the semester. I will illustrate this shift by comparing the beginning and final concept maps of four students.

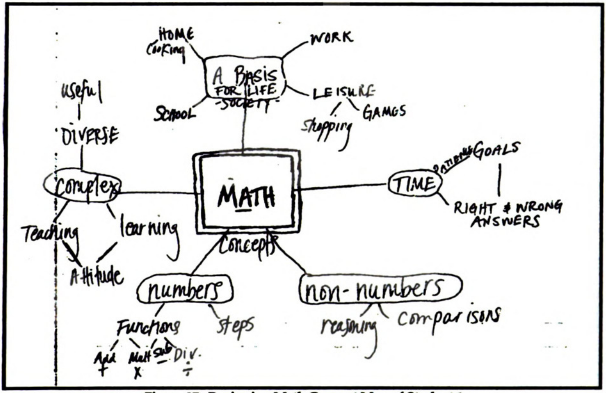

Figure 27: Beginning Math Concept Map of Student 1

Student 1 included "teaching" and "learning" both connected to "complex" and "attitude" in her beginning math concept map, but they were the only references to teaching and learning mathematics (See Figure 27). In her final math concept map, it seemed that virtually everything she included was related to teaching and learning mathematics and in a way that it seemed that she was thinking about teaching it herself (See Figure 28). Her four first-level connectors were the four process standards from the 1989 NCTM *Standards*, and it was clear from her journal entries that this student considered them to be very important when teaching mathematics. For example, in her "Philosophy of Teaching Mathematics" journal entry she wrote, "Students should learn to reason and make connections within math and apply it to real-life situations."

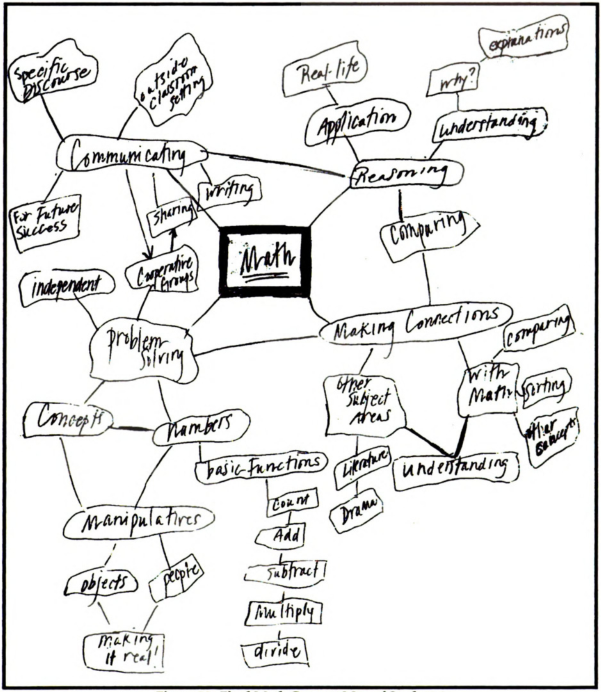

Figure 28: Final Math Concept Map of Student 1

All the items student 7 included in her beginning math concept map seemed very related to how she had used or experienced math herself (See Figure 29). It appears that she included memories of different types of subjects and problems she had experienced as a student. She even connected "tutor" to "calculus" which is again, most likely an experience she had while taking calculus. She even included a name of a teacher—Mr. Davey—again, most likely an actual teacher she had as a student. She also included a section on tests, and some of that was probably referring to her own experience, but at least one item, "interview for answer (video from TE 301)," was more likely a new way she had learned about assessing rather than a way she had been assessed herself. She also included some examples of how she could use math in the "real world," and she included some "attitudes" she had probably experienced in relation to mathematics.

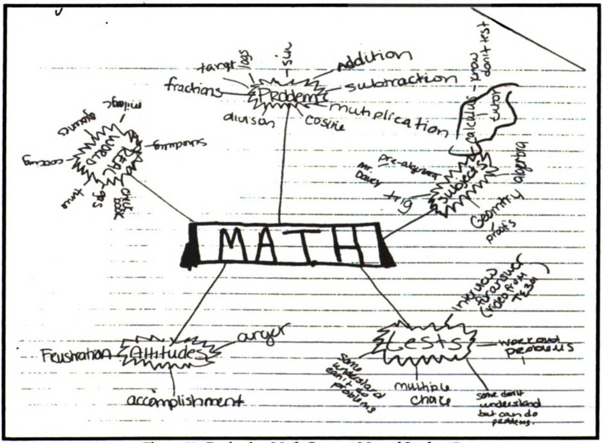

Figure 29: Beginning Math Concept Map of Student 7

Her final math concept map seems very focused on aspects of math that a teacher would be thinking of (See Figure 30). She included a category of "teaching" and also "philosophy" which appears to be some of her philosophy of teaching mathematics. She also included categories for "attitude," "processes," "assessment," and "real world." While she included "real world" in both her beginning and final math concept maps, there was a difference. In her beginning map, she connected ways she used math to "real world"—"scheduling," "check book," "gas," "time," "cooking," "groceries," and "mileage." In her final map, she connected "importance" and "relation" to "real world." This seemed to be a reference to the idea that when she taught math she wanted to teach students the importance of math in the "real world" and the relation of math to the "real world." The items that are connected to these categories seem closely related to conceptions of teaching mathematics.

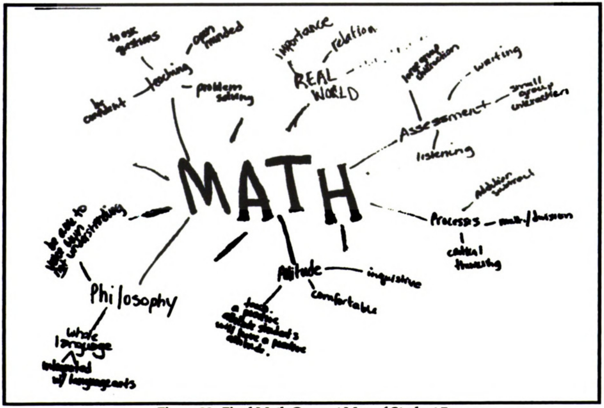

Figure 30: Final Math Concept Map of Student 7

The beginning math concept map of student 10 is entirely related to the usefulness of mathematics (See Figure 31).

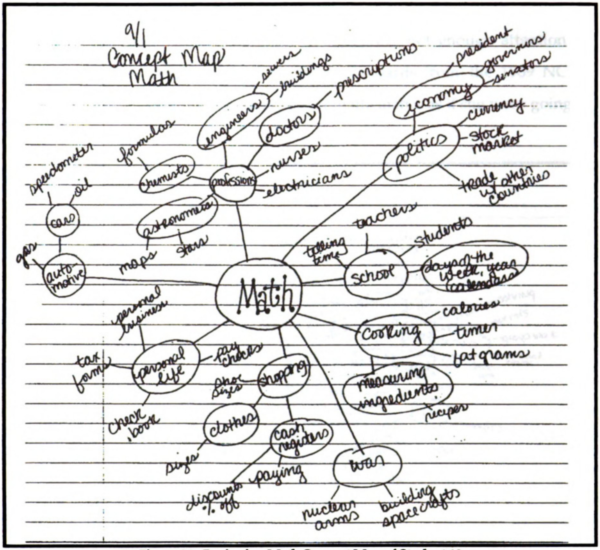

Figure 31: Beginning Math Concept Map of Student 10

The final math concept map of student 10 still includes a great deal related to the usefulness of mathematics, but it also included a significant number of items related to teaching mathematics and the way a teacher would look at mathematics (See Figure 32). For example, she included a category for "teaching." She also included a category of "assessment." She also included "Marilyn Burns" and "About Teaching Mathematics" which were the author and text that we used in the class. She also included "NCTM" with "curriculum" and "standards" connected to it. On the right side of her map she has "processes" with "problem solving," "journals," "group work," "individual work," and "whole class" connected to it. These are all related to teaching mathematics. In the upper left hand corner, she has a category of "not enough attention on these" and then connects the four process standards from the 1989 NCTM *Standards* to it. Clearly she is thinking about mathematics as if she were going to be teaching it.

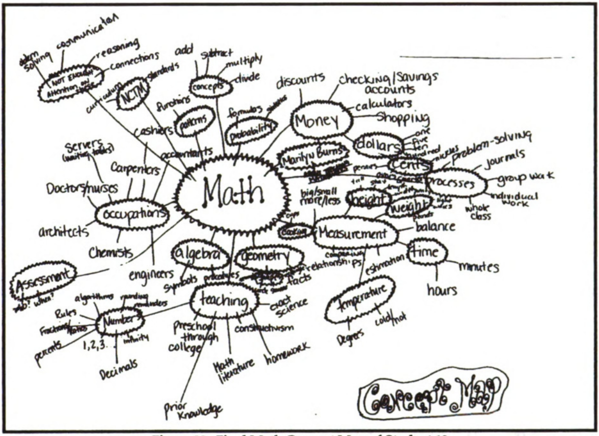

Figure 32: Final Math Concept Map of Student 10

In his beginning math concept map, student 21 included a little related to learning and teaching mathematics in the bottom right hand corner where he had a category of "important to learn" with "needs to be taught well" and "needs to be displayed in positive way to learners" connected to it. However, most of his map is related to his own personal relationship with math (See Figure 33).

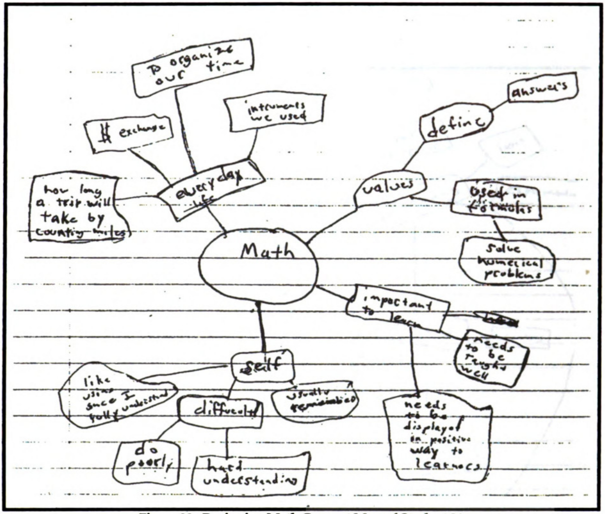

Figure 33: Beginning Math Concept Map of Student 21

The final math concept map of student 21 seems very related to his beginning map—as if he may have had it in front of him while he created his final map—but there were some differences that lead me to believe he had positioned himself more as a teacher than a learner of math by the time he did his final map (See Figure 34). New to this map were a variety of processes and content that are part of teaching school mathematics and a category of "effective learning needs" with "positive to learners," "creativity," "must see purpose of material being learned," "effective methods of teaching," and "continuous assessment of teaching processes." He also connected his "effective learning needs" category to another category of "reasoning." He definitely seemed to be focusing on conceptions related to teaching mathematics.

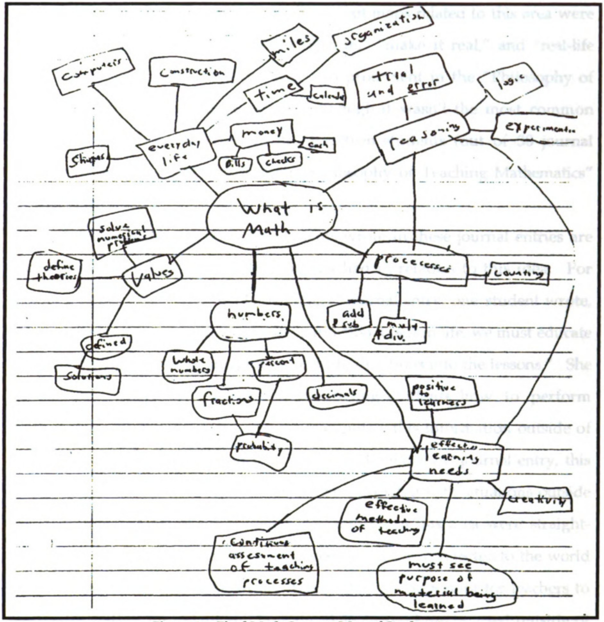

Figure 34: Final Math Concept Map of Student 21

Again, as was the case at the beginning of the semester, the most common conception related to teaching mathematics in the "What Is Mathematics?" journal entries and the math concept maps was the idea that math should be taught so that the students see its connection to their lives—inside of school and outside of school. Six students wrote about that in their final "What Is Mathematics?" journal entries, and seven students included items related to that conception in their final math concept maps. The types of items the students included in their math concept maps related to this area were "real world importance," "real world relation," "make it real," and "real-life experiences/situations." This idea was also prominent in the "Philosophy of Teaching Mathematics" journal entries, although it wasn't the most common conception the students wrote about. Fourteen students (out of 30 journal entries) wrote about that idea in their "Philosophy of Teaching Mathematics" journal entries.

A few examples of what the students wrote in these journal entries are illustrative of the conceptions the students had in relation to this idea. For instance, in her final "What Is Mathematics?" journal entry, one student wrote, "Due to the importance of math playing a functional part in life, we must educate students on this subject by integrating real-life situations into the lessons." She also wrote, "There is no point in instructing students how to perform mathematical operations if they cannot manipulate this information outside of the classroom." In her "Philosophy of Teaching Mathematics" journal entry, this same student wrote, "Math lessons should be associated with situations outside the classroom." Most of the students' declarations in this area were straight-forward statements that the math done in class should be connected to the world outside of school. One student wrote, "I think that it is important for teachers to teach why math is important and how the skills learned can be used outside of math time and in the everyday world." Another student wrote, "I think that the teacher's role in mathematics is to provide them with experiences that will help them gain math skills and experiences that relate to the world they know." One last example from the 14 who expressed this idea was the student who wrote, "Finally, I believe that the activities that are completed in class should be relevant to everyday life. Time should not be wasted on irrelevant activities that have no application to everyday life."

In addition to writing that math should be connected to the world outside of school, seven students wrote about the idea of connecting math to other subjects when teaching it in their "Philosophy of Teaching Mathematics" journal entries. One wrote simply, "It should be incorporated into all aspects of the curriculum." Often the students wrote about it being connected to language arts with writing and literature. One example of this is the student who wrote,

> "I am also in favor of connecting math to other subject areas. This helps students realize that math is everywhere! For example, incorporating math in language arts instruction with literature is something important. Using drama to teach math is another good way for students to 'see' math in action!"

Some students wrote about why they thought it was important to connect math to other subject—usually to help the students see where math is and to increase their interest in it. One student wrote, "By applying literature to math, students are able to see that math is full of ideas and concepts, not just memorization. I feel that students appreciate math when they can relate it to other subject areas." An example of another student who felt that connecting math to other subjects was a good way to show where math is used, and who even included connecting it to art, is the student who wrote, "Using journals or writing a story using math skills are ways to combine math and language arts. Also, art is often very mathematical, and I think that that should be showed to students (again relating mathematics to life)."

The students didn't write explicitly about this idea in their final "What Is Mathematics?" journal entries, but a few students did refer to that idea in their final math concept maps. For example, in her final math concept map, Student 1, whose map can be seen in Figure 28 on page 154, included "other subject areas" under "making connections" and "literature" and "drama" under "other subject areas." Student 7, whose final math concept map is shown in Figure 30 on page 156, also included "integrated with language arts" connected to "whole language."

The area that the students wrote about most often in their "Philosophy of Teaching Mathematics" journal entries was the idea that discourse was an important part of teaching mathematics. Twenty-Seven out of the 30 students for whom I have these journal entries (90%) wrote something about this idea. Some of them wrote at length about discourse. For my purposes, discourse includes both speaking and writing about mathematics. A couple of students also wrote quite extensively about that idea in their final "What Is Mathematics?" journal entries. For example, one student wrote, "I also have become a big proponent of having students explain their thinking. This expands their understanding as well as that of those listening." She also wrote, "I never saw the value of having students write in math class until we discussed it in class." Examples of what the other student wrote are, "One needs to be able to explain themselves as well as teach others their own ideas;" and "I believe that writing should be an integrated part of mathematics learning." Four students included "communicating" in their final math concept maps, and one student each wrote "specific discourse" and "writing."

As noted, the idea of using discourse to teach mathematics received significant attention in the "Philosophy of Teaching Mathematics" journal entries. A few of the student's quotes are exemplary of the many other quotes they wrote related to discourse. Some of the students simply wrote that they wanted discourse to be a part of their classroom such as the student who wrote, "I will have the children keep journals where they will explain their reasons for doing problems. I will have small and large group discussions;" or, the student who wrote,

> "One other thing that should be important in the mathematics is communication. Too often, students simply work on problems out of the book and turn them in. I think it is important that students have the chance to talk and write about the things they do in math class."

Most students, however, explained a little more about why they thought discourse was important. Usually they wrote something about the idea that it would increase the students' understanding and/or the teacher's insight into the students' understanding. One student wrote, "By working in groups and discussing the problems, students also become teachers trying to explain their reasoning to other students." Following are four more extensive quotes from the students' "Philosophy of Teaching Mathematics" journal entries:

> "I believe that a mathematics classroom should be a place where students can freely express their thoughts and ideas to their peers and the teacher. This expression should take place in both large group/whole class discussion and small group discussions. In addition, students should be encouraged to keep a journal expressing their feelings about math as well as any questions that they might have about something that they have come across during their exploration of the topic. The teacher should read these journal entries regularly and make comments or perhaps present a question or two that will cause the student to think deeper about a problem. Furthermore, the discussions which go on during class should be student led, with the teacher acting as a guide."

> "Students should gain and understand a mathematical discourse to communicate effectively with peers and others in a math community. Understanding and applying math discourse will help students understand and work with math concepts. Journal writing is a good way to put discourse into action, for students to be aware of what they know."

> "There are also a few aspects I feel are important to include in mathematics instruction. I believe communication is important both in writing and in discourse. The students should be able to effectively communicate what they are thinking about. This also gives an opportunity for assessment through what they write and say."

> "I believe very strongly in having students explain their reasoning with a correct or incorrect answer. Either way gives a great deal of

> insight into what students are understanding. By having students with wrong answers talk out their thinking they may correct themselves which will be more beneficial than me telling the right answer."

These quotes are all illustrative of the conception almost all the students seemed to have at the end of the semester that discourse—both writing and speaking about mathematics—is an important part of teaching mathematics.

The role of manipulatives was also a prominent part of the students' conceptions related to teaching mathematics at the end of the semester. As noted in the learning section, eleven students also included "manipulatives" in their final math concept map. In addition, two students wrote about manipulatives in their final "What is Mathematics?" journal entry. One wrote, "Math is manipulating numbers and objects. Actually handling material objects helps one to understand the math concept being learned. Manipulatives help make learning math seem more real and applicable." The other wrote, "Math in the schools is slowly getting more involved other than just textbook problems. More manipulatives are being used in the classroom which gives the learners who are not textbook learners a new perspective." The quotes from these two students and the huge increase in the inclusion of "manipulatives" in the final concept maps gives us evidence that the students were thinking about the role of manipulatives in teaching mathematics more at the end of the semester than they were at the beginning of the semester. However, the "Philosophy of Mathematics" journal entries are where the students really emphasize manipulatives. Twenty-six out of the 30 students (86.7%) wrote about manipulatives in those journal entries. Usually the students wrote that using manipulatives can be beneficial to the students' learning because of the hands-on experience or the concrete example they can provide. One example of this is the student who wrote, "Also, I would like to use many different resources. For example, I believe that manipulatives are very important for mathematical learning. They provide a hands-on opportunity as well as a concrete example." Another example of a student who stressed that benefit was the student who wrote, "There should be materials for the children to use as they work through problems. I feel manipulatives are an important aspect of the math curriculum. They give the children an opportunity to learn with hands-on experience." Another student wrote more extensively about the advantages of being able to handle objects when learning math. She wrote,

> "The use of manipulatives is one way to enhance children's problem solving and see math as something 'real.' Using manipulatives in a math environment also helps the students to make connections and reason with math. They develop understanding as they are able to visually handle objects and relate their work to math concepts and functions."

Some of the students also wrote that using manipulatives was a good idea because the hands-on approach they facilitated offered an alternative way to learn a math concept, and the students wanted to teach for diverse learning styles. One student wrote, "I want to use many manipulatives incorporated into the class. By having the many different approaches, it will give the students who may not grasp one way another chance by seeing it from a different point of view." Another example of a student who expressed this reason for using manipulatives was the student who wrote,

> "For mathematics, as well as for all other subjects, I want to be able to teach for many diverse learning styles. I think that this is best done by having manipulatives available to the students. I feel that hands-on experience is the most important aspect for learners. Hands-on experience makes math come to life, rather than just being an equation."

Another conception some of the students had about the benefits of using manipulatives is the idea that it can make math more fun and interesting for the students. For example, one student wrote, "In my math environment/

classroom, I hope to have many manipulatives for my students to work with. I think manipulatives help make math more fun and exciting to learn."

An additional role of manipulatives in teaching mathematics that some students wrote about is that having manipulatives in a math classroom provided the students with opportunities to explore and work together. An example of a student who wrote about this idea was the student who wrote,

> "Having counting bears, blocks, scales, interlocking cubes, Cuisenaire rods, etc. are things I will put in the room. This is part of my role because I will be there to explore with the children, also, and use the materials for lesson plans. The children's roles are to use the materials, together with friends and on their own without anyone else."

Some students realized that manipulatives could be helpful, but it depended on how they were used. Their conceptions included the recognition that while manipulatives could be useful, they lose their effectiveness if they are not used appropriately. For example, one student wrote,

> "I believe math classrooms should be very hands on and use manipulatives so children can construct their knowledge. It helps make abstract ideas more concrete. While manipulatives can be very effective, it's important not to rely on them alone and to evaluate their effectiveness. Some may only work to confuse others."

One of the focus students, Carolyn, also cautioned about the importance of using manipulatives appropriately if they are to be effective. She wrote,

> "Manipulatives are a good tool to use because some students may think about mathematical ideas more easily when they can physically see, touch, and work with the items. It is important to remember that not all students may benefit from this. When using manipulatives the teacher must make sure that the use of them makes sense in the activity and promotes the concept you are trying to teach."

One final example of a student who wrote that manipulatives should be used in teaching mathematics, but included a caveat was the student who wrote,

> "Activities used for teaching should revolve primarily around the use of manipulatives and concrete ways to represent mathematical topics. However, teachers must not assume that manipulatives will do the necessary teaching for them. Relying too heavily on manipulatives can be just as destructive as not using any manipulatives at all."

Overall, at the end of the semester, the students seemed to hold the conception that manipulatives should play an important role in teaching mathematics because of the concrete experience they provide, the opportunities they can offer for exploration, and the interest they can generate. However, several of the students also recognized that simply using manipulatives would not guarantee learning, especially if they were not used appropriately—in a way that doesn't match or make sense with the concept.[2]

The students also wrote about using tools other than manipulatives in teaching mathematics. Usually they wrote about using calculators or computers. Some of the students included tools used in teaching mathematics other than manipulatives in their final math concept maps as well. Eleven students included "calculators" and four students include "computers." Two students included "pen/pencil," and one student included "paper." The most common reason the students wrote about for using calculators and computers is that they were tools the students should be able to use outside the classroom. For example, one student wrote, "They are no longer forbidden from using calculators, since it is a tool they will likely use as adults." Another student wrote, "During this experience, students should be encouraged to use mental calculations, paper and

[2] We talked a lot about not just using manipulatives because they were fun and might increase interest, but making sure they were appropriate to the concept and had a direct correlation to symbolic representations. We also talked about the importance of talking a lot while they were used so the students could better learn/understand the concept that was being presented to them. We read Deborah Ball's "Magical Hopes: Manipulatives and the Reform of Math Education" which emphasized the idea that while manipulatives can be useful, they are not magical, and just because manipulatives are being used, it doesn't necessarily mean the students are learning.

pencil exercises, calculators, and computer methods. I want to provide practice for my students with all of the tools available outside the classroom."

A couple of the students expressed caution about using calculators. One of these was a focus student, Bridget, and her ideas will be examined later. The other student wrote,

> "In the recent age of technology, it is very easy to use a calculator or a computer to get an answer to a problem. If this occurs, the student may not have a clue how the answer was achieved and the only thing they truly learned was how to scam a teacher with a bogus answer. It is very important to see reasoning of how an answer is achieved so that the teacher can help the student if it is needed."

The next most common thing the students wrote about in their "Philosophy of Teaching Mathematics" journal entries was the importance of emphasizing processes when teaching mathematics. Twenty-one students wrote about that idea. As was noted in the learning section earlier in this chapter and processes section in the previous chapter, it was also something that received a great deal of attention in the final "What Is Mathematics?" journal entries and the final math concept maps. Sixteen students wrote something about processes in their final "What Is Mathematics?" journal entries, and 26 students included something related to processes in their final math concept map. Fifteen of them included a high number of items (five or more) related to that area.

In the "Philosophy of Teaching Mathematics" journal entries, the students wrote about the kinds of processes the students should be using/learning and why processes were so important in teaching mathematics. Many of the students wrote about the idea of processes being more important than the product. One student wrote, "Emphasis is on process rather than product. We want to see and understand students' thought processes rather than narrowing their perspectives by demanding one right answer." Another student wrote, "Along with this approach, an emphasis should be placed on the process used, and not the final product. When always focusing on the answer, the children learn how to give the teacher what they want, not the material." One student emphasized how emphasizing the process helps the teacher learn about the students' understanding. She wrote, "I feel the process of learning should be strongly emphasized rather than the product. It is the thought process of working on a problem, not the answer that reveals understanding." A few more examples of what students wrote relating to this idea follow:

> "Another important thing I would stress is the idea of process vs. product. I believe that the process is more important. A student needs to know how a problem works and not just how to quickly achieve a final product. I would like my students to be able to explain why something works."

> "I feel students should also use trial and error. This will help students understand how certain processes are useful or not useful. The learning emphasis would be more towards the process. If I teach students to emphasize the product, I feel they will see math as a right or wrong one-dimensional process. I want students to learn how to use processes to evaluate problems."

> "I want to emphasize more on process for which they learn the material as opposed to focusing more on the product. I think it is important to value the processes because children, in the end, will be more apt to take risks in mathematics, and it will motivate them to continue learning math with enthusiasm and interest."

The students also wrote about some specific types of processes they wanted to be part of their teaching of mathematics. For instance, one student wrote, "In all, I feel that math is just like the sciences, one needs to conjecture, explore, revise thinking, and conclude." Another student wrote, "Students should be encouraged to use any approach to get an answer and then to explain his/her line of thinking." She also wrote, "All lines of deep thinking should be valued regardless of his/her answer." Another student wrote about how she wanted different processes to be a part of her teaching this way:

> "Students' roles will be that of thinker, questioner, challenger, but they must also be able to support, explain, reason, or justify their concepts. It is important for both the teacher and the student to realize that support for a concept or a belief can come in many forms. Two such examples are through verbal explanation or use of an example to support or back up. An example could be in the form of numbers, word problems, or work with manipulatives, etc."

Another thing the students emphasized related to processes is that students shouldn't be expected to think about or work on a problem in a specific way—that they should be able to use their own thinking processes to search for a solution. For example, one student wrote, "Math is a process that each student goes through differently. It involves reasoning and justifying." Another student wrote more about the teacher's role related to this idea. She wrote, "I also think it is important to accept all ideas and to encourage children to take risks and just try the problems. Emphasize that there are various ways to do one problem." Related to this, in her final "What Is Mathematics?" journal entry, one student wrote, "There are many different ways to do problems. Math is a lot more open ended than I at first believed because of the rigid way that I was taught." A final example of a student who wrote about this idea in the "Philosophy of Teaching Mathematics" journal entries was the student who wrote,

> "I think that math teachers should shift their focus from teaching only basic skills and focusing on algorithms to teach a little more thinking, problem solving, and reasoning. This is not to say that basic skills should not be taught, because I think that they should. However, I don't think that students should be punished for finding answers in original ways."

In summary, with respect to the role that processes play in teaching mathematics, the students' conceptions included the ideas that there were various processes the students should learn such as reasoning and problem solving, that the process was more important than the product, and that students should be able to use their own processes to work on a problem. "There are many different ways to do problems. Math is a lot more open ended than I at first believed because of the rigid way that I was taught."

Related to the idea that it is important to emphasize the processes of mathematics when teaching mathematics is the idea that mathematics should be taught as more than drill, practice, and memorization. Ten students wrote about that in their "Philosophy of Teaching Mathematics" journal entries, and a few students wrote about that idea in their final "What Is Mathematics?" journal entries. As was noted in the learning section, one student wrote, "We should stop emphasizing the importance of memorizing math operations. This simply contradicts our purpose." The students didn't include a lot specifically about this idea in their concept maps, but there were a couple of items that were possibly related to this idea such as the item "not book or workbook" that one student included.

In their "Philosophy of Teaching Mathematics" journal entries, the students expressed this idea in more detail. One student wrote, "Mathematics should mean more than having students sit at their seats for 40 minutes to work on paper and pencil exercises." Another student wrote simply, "Math should not simply be dittos and homework." One student even implied that just using bookwork indicates some laziness on the part of the teacher. She wrote, "The different activities should be fun as well as educational. Book work all the time tells the students you don't really care and are too lazy to make up a real lesson plan."

A few of students wrote that thinking of teaching math as more than drill and practice was a new idea for them. One student wrote, "My philosophy about teaching math has broadened! I understand that math is a lot deeper than memorizing and computing basic math functions." Another student wrote, "My role and fear of teaching mathematics has changed and is still developing through this class. I no longer see myself assigning homework from the textbook, explaining examples, and calling it a day." Another student declared a change in her ideas related to this area and also wrote about the idea that she thought so many children feared math because of being taught through drill and practice. She wrote,

> "When I entered, I felt that the old system of explaining the concept, assigning problems, and correcting the work was the best way for students to understand and learn concepts. I have realized that this is the reason so many children fear the subject."

Another aspect of teaching mathematics that 16 students wrote about in their "Philosophy of Teaching Mathematics" journal entries was the idea that students should have the opportunity to work on mathematics in small groups as well as individually and as a whole class. This provides the students the opportunity to discuss the mathematics they were working on, teach other students, and learn from other students. Some students also included this idea in the final math concept maps. Two students included "cooperative groups." Student 7 whose final concept map is shown on page 154 included "large group interaction" and "small group interaction" on her map. Student 10 whose final concept map can be seen on page 158 included "group work," "individual work," and "whole class." Another student included "students help other students." A couple of students wrote about the importance of group work in their final "What Is Mathematics?" journal entries. One of these students wrote,

> "I also believe that math is collaboration. Students need to learn to be able to work together. They can learn the skills for group work by working in groups. They can learn to value other students' input, and they can learn to accept others' ways that may be different from their own. Learning to cooperate with other students is a lesson that students need to learn, and when group work is combined with math, it helps students to understand their task better. Sometimes other students can explain things to their peers in a way that the teacher can not, and this greatly improves students' understanding."

As noted, 16 of the students wrote something about this idea in their "Philosophy of Teaching Mathematics" journal entries. Usually they wrote about the same kind of benefits that the just-quoted student wrote in her final "What Is Mathematics?" journal entry such as learning the skill of collaboration, learning to value others' input, and being able to teach and/or learn from other students. For example, one student wrote, "I also want my students to work a lot in small groups so that they can have an opportunity to express, validate, and question their ideas." Another student wrote,

> "I want my students to work with math on an individual basis and in small groups. In small groups, students can learn from each other. Small groups also facilitate learning various problems-solving skills. Students can begin to understand other ways to solve a problem and methods of thinking."

Another student explained similar ideas more elaborately. She wrote,

> "I also believe that students should work both individually and in groups. Allowing children to work in groups gives them a chance to learn from one another. Students have a lot to teach one another and working in groups may assist them in learning a new way to do a problem. A lot of times students may be able to explain a concept better than the teacher, in a way that they can understand. Working in groups allows students to learn from another and also when students teach a concept to another student, they are also learning and reinforcing what they know."

Another student wrote about similar ideas and also the benefit of small groups being a less threatening environment. She wrote,

> "I also see value in having the students work in groups. This allows them to share ideas, question each other, and possibly teach others. It allows them to raise questions and gives everyone an opportunity to participate in a less threatening environment."

A few students wrote about having the students work in small groups in addition to working and discussing in whole or large groups. One example of this is the student who wrote, "In my classroom, the role of the student will be to take an active part in discussions, individual projects, group projects, whole class projects, and small group projects." Another student wrote,

> "I believe that a mathematics classroom should be a place where students can freely express their thoughts and ideas to their peers and the teacher. This expression should take place in both large group/whole class discussion and small group discussions."

One student wrote about the idea that since math is a social subject, it needs to be learned in a social setting. She wrote, "The activities should be organized around groups. Because of the scientific nature and math's abundance in the world, it is a social subject, it cannot be taught to students isolated from others for complete understanding."

In summary, many of the students' conceptions of teaching mathematics included the idea that students should have the opportunity to work in small groups as well as in large groups or individually. The benefits of this that they most often wrote about were that the students would have an opportunity to talk about the math, learn from and/or teach each other, and learn to work collaboratively.

With all of these conceptions of the students about what should be happening in a mathematics classroom, it's natural for them to think about the role the teacher would play. It wasn't something they wrote about explicitly in the final "What Is Mathematics?" journal entries, and the teacher's role was mostly implicit in all the items related to teaching math that the students included in their final math concept maps, but many of the students (17) did write explicitly about the role of the teacher in teaching mathematics in their "Philosophy of Teaching Mathematics" journal entries. Usually they described the teacher's role as that of a facilitator or a guide. For example, one student wrote, "My role as an educator in the classroom is to facilitate learning through providing materials and letting them explore them on their own." Another example of a student expressing an idea similar to this—and to that of many other students—was the student who wrote,

> "The role of the teacher is to guide the students through discoveries, give and provide the material needed for the students to resolve on their own the solution. This lets the students feel the sense of accomplishment with being the mathematician.

An example of how one student wrote about how this might look was the student who wrote,

> "The teacher would play two different roles throughout the teaching of a math lesson. At the beginning she would introduce the subject to the students and start them thinking about it. Then she would step back and allow the students to take an active role in their learning.

One student explained more fully how she envisioned the role of the teacher. She wrote,

> "Math has evolved significantly since my younger days as a student. The role of the teacher, for example, has shifted from that of dictator to that of a guide. Teachers guide students' thinking and reasoning process and encourage them to develop their own conclusions and solutions. Teachers, in an ideal classroom, are no longer giving the students stringent rules in which to use in order to solve a problem. Rather, the teachers stimulate students' prior knowledge and help them to formulate their own methods of

> solving certain problems. Teachers are now role models and think out loud with students in order to provide students with a multi-perspective on specific areas of math."

Another student also wrote about the teachers as a model of thinking about and doing math as well as allowing the students to do the same. She wrote,

> "The teacher will model for students how to try at math, but when performing mathematical tasks, the teacher will be the facilitator. He/she will ask a question, or something of that sort, to the students for them to solve, reason about, challenge, re-think, etc."

A few students wrote about how they felt like their conception of their role as a teacher had changed. For example, one student wrote,

> "My philosophy of teaching mathematics has changed a great deal. I now see the teacher's role as less integral in terms of what they say to the students, yet more important in their devising of activities. Students should be able to come to an understanding and reach a conclusion, propelling the lesson... Activities will not always be to explore an already-known concept, but may be to discover a concept on their own."

Item 18 from the Mathematics Inventory also had some relation to the students' conceptions of the role of the teacher. There was a statistically significant change in the mean response for this item which was, "Being good at mathematics is not required to be a good teacher of math." However, the direction of change was a little surprising. At the beginning of the semester the mean response was 2.86, and at the end of the semester there was more *agreement* with that statement as indicated by the mean response of 3.621. The p value for the change was 0.0065. This illustrates a paradoxical, but not surprising effect of our teaching as mathematics teacher educators. On the one hand, reformers believe that in order to teach elementary math well, teachers must know math far better than most now do—that this is important for them to see connections, to know which of the tangents that arise out of a math discussion should be followed up, and for many other pedagogical reasons (Ball, 1988b; Ball, 1989; Floden and others, NCTM, 1991; Schram and others, 1988). On the other hand, the kind of teaching we introduce them to, where the teacher is doing less presenting and explaining of the math and more facilitating and questioning, makes these prospective teachers feel their lack of mathematical knowledge will be less obvious and less of an impediment to the students' learning—because they won't be presenting the ideas, the *students* will. Sometimes, we, as teacher educators, do not communicate to them well enough that the way you select and design problems, respond to children's ideas, and decide to pursue some ideas and not others has to do with their subject matter knowledge. Is this perhaps because we do not want to communicate this? Are we enjoying their new-found enthusiasm for math teaching and the new image they have of themselves of math teachers? Also, as teacher educators, we want the students to believe that the fact that they didn't enjoy math as a student or feel successful at it does not mean that they cannot become a good teacher of mathematics. We want to instill some hope in them.

In summary, it seemed that for many of the students the role they had envisioned themselves taking as the teacher of mathematics had changed during the semester. Instead of seeing themselves as dispensers of knowledge, they saw themselves as facilitators of learning, and for some of them, it seemed they felt that kind of role would not highlight their lack of knowledge of mathematics.

Related to the role of the teacher and incorporated in the students' conceptions that have previously been discussed in this section are their conceptions of the role of the students in the mathematics classroom. In some way, all of the students wrote about or implied the role of the students, but in their "Philosophy of Teaching Mathematics" journal entry, 12 students wrote specifically about their ideas about the role the student should play. Usually they wrote something about the students being active participants in the learning process. For example, one student wrote, "I want students to be investigators in my classroom during math lessons." Another student wrote, "The students need to know that you as a teacher expect them to listen, ask questions, and learn math." Another student emphasized that the students need to take some responsibility for their learning. She wrote, "I want my students to take responsibility for their own learning in meaningful and appropriate ways. I feel it is important for students to take ownership of their own ideas about mathematics and indulge in student-invented projects." Another student wrote about the students' role in discussions: "Furthermore, the discussions which go on during class should be student led, with the teacher acting as a guide." One final example that is typical of what others wrote about the students' role in a mathematics classroom is the student who wrote,

> "In my classroom, the role of the student will be to take an active part in discussions, individual projects, group projects, whole class projects, and small group projects. It will not be a passive role. They will need to rationalize their answers etc. These concepts will be able to be used in other subject areas once the format is set, the concepts being teacher=guide, student=active investigator."

In sum, at the end of the semester, the prospective teachers widely held the conception that the students needed to take an active and responsible role in their learning of mathematics—in essence that they needed to act as if they were mathematicians.

Four of the students wrote some about the mathematical content they thought should be part of their mathematics teaching. One of them wrote about the importance of having "rich problems to solve." Two of the students listed different areas of content that should be taught. One of them emphasized the importance of the basic operations because they are the "foundation for higher-level mathematics." The other student focused more on some of the content in early elementary school. She wrote,

> "There are so many important things to learn, such as measurement. Children love to focus on themselves which makes learning about height and weight fun. Also, the concept of telling time is important. Also, adding and subtracting is something very important because it relates to everything, just about. Small children, such as kindergartners, love to learn about numbers and numerals and also geometry. Shapes are important and exciting for children when they learn the various kinds. Pictures of shapes and signs will be strewn around my classroom."

The other student wrote about the importance of adding some cultural diversity to the mathematics content. She wrote,

> "I believe also that teachers need to bring diversity into math. This can be done with the activities that they do. For instance, students could learn about Roman numerals. They can also learn numbers in different languages. There are many ways in which various cultural backgrounds can be incorporated into the math curriculum, and I believe that it should."

Two students also wrote about making sure their curriculum was developmentally appropriate for the students. For example, one student wrote, "I want to provide my class with a developmentally appropriate math curriculum that will challenge them and excite them early on so that, hopefully, math is not a dreaded subject. That is a goal I have decided on."

Additionally, in their "Philosophy of Teaching Mathematics" journal entries, two students wrote about different resources they would use to teach math. One student wrote,

> "I would try to use a wide variety of sources. I feel it is important to see and hear different viewpoints even if you do not agree with

> them. My sources would be different teachers in my school, materials from conferences, and books from authors such as Burns."

The other student focused mainly on using materials from the class as resources. She wrote,

> "I will use the books from our class as references for teaching mathematics. The Burns book will be a good one to use for activity ideas, the MCTM vignette book will be a good one for assessment ideas, and for ideas about how teachers view mathematics."

Eleven of the students—more than half of whom were child development majors—wrote about the physical classroom in their "Philosophy of Teaching Mathematics" journal entries. Many of them wrote about the classroom being a place that promotes learning. For example, one student wrote,

> "First, I think the classroom environment plays a strong role in the education of young children. It needs to be inviting to the children and encourage exploration and growth. There should be materials for the children to use as they work through problems."

Another student wrote some about the arrangement of the desks and the types of materials that should be in the classroom to create this environment:

> "The math classroom should be a friendly place. Student desks should be placed with partners or in groups to promote cooperative learning. Mathematics-related games and activities should be available for the students to use during their free time. It is important that students be comfortable in their math class, and that they are not intimidated by it."

Most of the 11 wrote about bulletin boards and other items that would be displayed in the classroom such as the student who wrote, "Their work should be displayed for others, not only to increase pride, but also to stimulate new ideas in others." Three other examples of students stressing this idea in their journal entries follow:

> "Bulletin boards in the class would revolve around themes of mathematics. The history of mathematicians is often overlooked

and bulletin boards provide a medium for the transfer of knowledge on these subjects. Another bulletin board might illustrate current news or discoveries that would have failed to exist without the aid of mathematics."

"My class will be full of numbers that the children can see and discuss. Basic 1, 2, 3, . . . etc. will be throughout the room and units that I hope to carry out with correspond with the décor of the room."

"I would like my classroom to have a math center in which there are manipulatives present. Perhaps once a week, I will propose a problem that is a little harder than what we're studying as a class. The children will have an opportunity to answer the question using the materials provided. I also want the room to be rich with math posters and poems about math, pictures of children involved with math learning, photos of famous mathematicians, etc."

From the just-discussed conceptions of the students related to teaching mathematics, we can see that many of them were thinking about some of the more logistical, but also philosophical, aspects of teaching math like the content they should teach, the resources they could use, and the physical mathematics classroom.

Another conception related to teaching mathematics that many of the students seemed to have at the end of the semester is the idea that teachers need to help students have good attitudes toward math so that they are able to learn it. Seventeen students wrote something related to this idea in their "Philosophy of Teaching Mathematics" journal entries. A couple of students wrote about that idea in their final "What Is Mathematics?" journal entries. One example is the student who wrote,

"I think that math is a useful tool and that math skills are important for children to adopt, but I think that we need to stop scaring children about math or we will raise yet another math-hating generation of kids."

Also, 17 students made references to attitudes in their final math concept maps. Some of these references were related to the students' own attitudes toward mathematics, but some of them were references to attitudes that they wanted their students to have. For example, in the bottom left corner, one of the focus students, Bridget, included "attitudes" in her final math concept map connected to "math" with two categories branching out from "attitudes"—"that we want" and "that we don't want" (See Figure 35). Connected to "that we want" she included "understanding," "answer questions," and "apply math to real life." She included "frustration," "confusion," "anger," and "I quit!" to "that we don't want." The students' references to attitude in their final math concept maps will be examined more fully in the next chapter which focuses on attitudes.

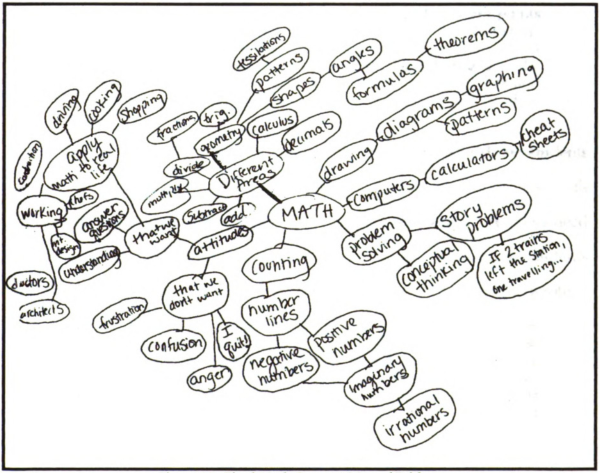

Figure 35: Final Math Concept Map of Bridget

The 17 students who wrote about students' attitudes in their "Philosophy of Teaching Mathematics" journal entries wrote mainly about the idea that students need to have positive attitudes toward math if they are going to succeed at learning it. For example, one student wrote, "Teachers also need to keep in mind the role that attitudes can play on students' success. They need to foster positive feelings and help develop confidence." Another wrote, "Math needs to be fun and exciting so that students are not afraid, they have confidence, and everyone feels they can do math." One final example that was similar to the ideas expressed by others was the student who wrote,

> "In order for a teacher or a student to be successful at mathematics, the key factor is the attitude or outlook. Students, as well as teachers, must look at mathematics with a positive attitude. Not like it's a chore, a boring task, or busy work in order to put points in the grade book. No, math must be thought of as interesting. Although one might not know a lot about math, their goal is to at least try it. But also accompanying the idea of just trying it, is the idea that one can't be afraid of it. The attitude of the teacher and the student will affect the outcome of the task and the ideas formed."

Some of the students also wrote about the importance of helping students have positive attitudes toward math because they often come to class with negative attitudes. An example of this is the student who wrote, "Teachers need to first of all, persuade students to see math positively. It is likely many students will view math in a negative way." Another example is the student who wrote,

> "I think that it is important for math teachers to create an environment in which students believe that they can succeed. Too often, students hear from their parents, friends, and even teachers that math is hard, and they believe that they can't succeed. Teachers need to work hard to make students feel good about their abilities, because the environment outside of school is hard on, and sometimes even hostile toward, mathematics."

Related to the idea that it's important for students to have positive attitudes towards math is the idea that it's important for teachers of mathematics to have positive attitudes toward mathematics. This is an idea that also received a significant amount of attention at the end of the semester. Three students wrote specifically about that in their final "What Is Mathematics?" journal entries. For example, one student wrote, "I think it is very important to be excited when teaching math and keep the students interested. If the children know that you care how they do and want them to succeed, they will try harder." There weren't any items in the final math concept maps that were clearly referring to teachers' attitudes, but this idea was very prominent in the students' "Philosophy of Teaching Mathematics" journal entries. Fifteen students (50%) wrote something about that. Some of them wrote about the importance of teachers having positive attitudes, and some of them wrote about their own attitudes being more positive. (The students' own attitudes toward teaching mathematics will be examined in the next chapter.) One student wrote simply, "I believe that is important from the beginning for not only the students to have a positive attitude toward math but also the teacher." Another student wrote, "My job as a teacher is to be excited about math and become interested in the topic I am teaching." A final example of a student expressing this conception—and that is illustrative of others' ideas—is the student who wrote,

> "I believe math is a useful and necessary part of life that needs to be taught in school. Math can be an intimidating or rewarding experience depending on how it is taught. I believe the teachers, as much as the students, influence one's attitudes."

The open-ended portion of the Mathematics Inventory also provides us some information about the students ideas related to the influence of the teachers' attitudes toward mathematics when teaching mathematics. At the beginning of the semester, the students were asked if they thought that teachers' attitudes toward mathematics influenced their students or the way they taught mathematics. Essentially all of the students thought that it did. At the end of the semester, it seemed to be an even more strongly held conception.

One of the open-ended questions from the Mathematics Inventory that was related to the students' conceptions of teaching mathematics was, "What kind of influence do you think the teacher plays in the success of students in mathematics?" At the end of the semester, the students' responses to that question mainly focused on the influence teachers' attitudes have on their students' attitudes toward mathematics. Twenty students out of the twenty-nine I received an inventory from wrote that the teacher determines or affects the students' attitudes. Five students specifically wrote that teachers' attitudes determines the students' success and/or confidence in math. Three students wrote that the teacher is a role model. A few other examples of what one student each wrote in response to this question were that teachers need to make math interesting so that students will want to learn, that teachers determine whether students think of math as a task or a fun experience, and that teachers can make math a chore.

When the students were asked the other open-ended question from the Mathematics Inventory that was related to their conceptions of teaching mathematics—specifically related to the influence of the teacher's attitude—their response was similar to what it was at the beginning of the semester. This question was, "Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not? How about a teacher's attitude about teaching it? Explain." Again, all but one student (28 out of the 29) wrote, "Yes" in response to the first question. The student who wrote "No" also wrote, "If you don't like it, you can pretend to. Just because I personally don't like math

I'll still stress it in my classroom." The most common explanation the students wrote for why they thought a teacher's attitude toward mathematics affected how he or she taught it was that the students model their teacher's attitude. Thirteen students wrote that. Nine students wrote that students can tell when their teacher does or doesn't like math. Six students wrote that a teacher who hates math or has a poor attitude toward it won't teach it well. Seven students wrote that if teachers are confident or have a good attitude toward math, they will be more effective teaching it or they will emphasize it more. On the other hand, three students wrote that if teachers are not excited about math, they won't put forth a good effort to teach it. Some other related ideas that one student each wrote in response to that question are, "The teacher's attitude will determine how far she will let the class's thinking expand to new ideas;" "If a teacher doesn't want to teach it, a student won't want to learn it;" and "If a teacher feels everyone can succeed, she will demonstrate that."

In summary, the students in this group seemed to almost unanimously hold the conception that teachers' attitudes toward mathematics influenced the way they taught math and also their students' attitudes toward mathematics. They all thought it was important for teachers to have positive attitudes toward mathematics if they were to teach it effectively.

A new idea related to conceptions of teaching mathematics that the students wrote about at the end of the semester that was not part of their writings or math concept maps at the beginning of the semester was the role of assessment. This wasn't so evident in the final "What Is Mathematics?" journal entries, but it was quite common in the final math concept maps and very prominent in their "Philosophy of Teaching Mathematics" journal entries. In the final math concept maps, four students included "assessment," one student included "authentic assessment," and one student included "assessment—not tests." One example of a concept map where assessment was prominent with its own section was the final math concept map of student 7 which can be seen on page 156 in Figure 30. Connected to "assessment," she included "large group interaction," "writing," "small group interaction," and "listening." The final math concept map of student 27 is another example of a map where assessment had its own category (See Figure 36). She included "writing," "written problems," "explaining," "group projects," "verbal answers," and "presentation" connected to "assessment." Seven students made specific references to assessment in their final math concept maps.

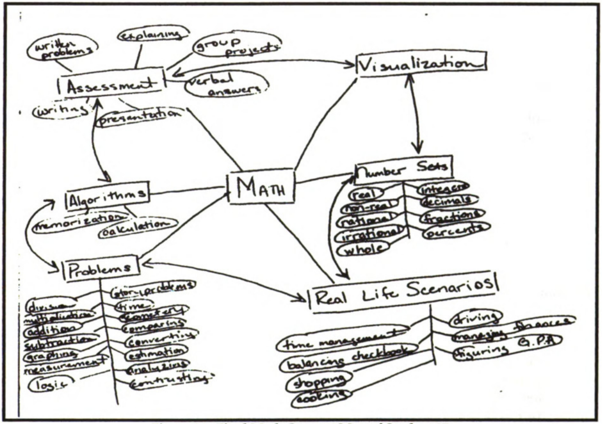

Figure 36: Final Math Concept Map of Student 27

Assessment was very prominent in the "Philosophy of Teaching Mathematics" journal entries with 18 students (60%) writing about that idea. Usually the students wrote about using several methods to assess, but they also wrote about the idea of making their assessment authentic or about the uses of assessment. One student expressed ideas about how she will assess the students' learning of math this way, "Watching them, observing, I am able to assess what they're learning. Listening tells me when I hear math used in the class." Other students listed the variety of ways they would assess in the mathematics classroom. For example, one student wrote,

> "To assess math, I will use many different methods. I will interact with my students, asking questions, discussing, and observing their work to make sure they understand the processes, not just getting the right answers. I will also assess my students' written work from journal entries, group work, etc."

Another student wrote about the different forms of assessment she would use this way:

> "I will assess the children's knowledge in a variety of ways. Each person learns differently, thinks differently, and comes to final answers in a different manner. As a result, it is important to assess their understanding in many different ways. I will have the children keep journals where they will explain their reasons for doing problems. I will have small and large group discussions. I will also work with children on an individual basis. I intend on watching children as they interact with each other and observe what they are doing. I will try my hardest to avoid basal textbook assignments and tests. I feel these do not reveal the children's understanding of the mathematical concepts. It is my goal to assess the true understanding of each unique individual."

A final example of students writing about the variety of ways they plan on assessing as teachers is the student who wrote,

> "I see great value in interviews that require students to explain their understanding. This can also be accomplished by keeping math journals. I also believe portfolios are a useful assessment tool that can be beneficial for both the teacher and student. It points out strengths and weaknesses along with growth in ideas and skills."

Other students wrote similar kinds of "lists." Some students focused on using listening to the students as a means of assessment such as the student who wrote, "The teacher needs to bring out the students ideas and questions. The teacher also needs to listen to the students. This listening can be used as another form of assessment." Another student wrote,

> "I also want my students to work a lot in small groups so that they can have an opportunity to express, validate, and question their ideas. I think this would be a good opportunity for the teacher to also assess the students' participation."

Another example of this is the student who wrote,

> "I feel the process of learning should be strongly emphasized rather than the product. It is the thought process of working on a problem, not the answer that reveals understanding. Therefore, I will pay close attention to the children's reasoning and comments when they are working on problems. Otherwise, I may never know what the children know and how they are thinking."

Some students emphasized the role that writing would play in their assessment. For example, one student wrote, "Within this, I feel that writing should be used as a way for the students to express their feelings towards math, and it can also be used as a way for teachers to evaluate their students." Another student wrote,

> "Although I have yet to experience the idea of students keeping math journals, it seems to me that writing about any experience is beneficial when attempting to understand concepts. Also, writing about mathematical understanding provides a useful means by which the teacher could discover any misconceptions that the students have."

In addition to using assessment to learn about the students, some students wrote about using assessment to assist the teacher in planning. One example of this is the student who wrote, "Assessment is important but can be difficult to do. It also requires analyzing results and using these to change or add concepts to your lessons. Another example is the student who wrote,

> "When teaching mathematics, a teacher must keep his/her goals for students as the focus when planning to teach any mathematical topic. I believe that the goals should center on the students' developing understanding of the material. It is important for

teachers to help students understand, and teachers also need to recognize when students are and are not comprehending mathematics."

At the end of the semester, many of the students were thinking about assessment in the mathematics classroom. They wrote about using a variety of methods such as listening to the students while they were discussing math and looking at their writing about math to assess whether the students understood the concepts. They also wrote about using assessment to inform or guide instruction. The fact that so many of the students were thinking of assessment also supports the idea that there had been a shift in positioning for the students in their relation to mathematics during the semester. How mathematical learning should be assessed is something that a teacher is concerned about. Someone who is just thinking of math in relation to how they use it would not be concerned with assessment. Someone who is thinking of math in relation to their own experiences with it might think of assessment in terms of how they were assessed as students such as with tests, but this is different from the way a teacher thinks about assessment—and from the way these students were writing about it at the end of the semester.

It seemed from all of the things the students wrote about teaching mathematics at the end of the semester, they were recognizing that there is a lot involved in teaching mathematics. This assumption was also supported by the students' response to a Likert-scale item from the Mathematics Inventory that was related to conceptions of teaching mathematics. This was item 10, "Teaching mathematics should not be too difficult since a textbook already has the lessons prepared." On the last day of class the mean response to that item was 1.828 indicating even less agreement with that statement than at the beginning of the semester when the mean was 2.034. The p value (0.35) wasn't statistically significant, but the direction of change was encouraging. Related to this, in her final "What Is Mathematics?" journal entry, one student wrote, "Remember this when teaching that there are many approaches to this thing called math, so don't just use textbooks."

At the end of the semester, the students were also asked in an open-ended question in the Mathematics Inventory if their ideas about mathematics or teaching mathematics had changed at all during the time they were in the course. The question was, "Have your ideas about mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how? Were there specific events or activities that helped your ideas change?" Twenty-nine students turned in their final Mathematics Inventory, and all 29 of those students wrote "yes" that their ideas had changed. One student wrote that her ideas had changed a great deal, and one wrote that they had drastically changed.

There was a big variety in the students' explanations for their "yes" responses. Most of what the students wrote about related to how they thought their ideas about teaching mathematics had changed were related to methods of teaching. Four students specifically wrote that they had gotten a lot of new ideas or learned alternative methods and approaches for teaching mathematics, and two others wrote that they were more open to new ideas. The most common responses were that they had been introduced to the idea of writing in math and/or would now put more emphasis on writing and that they feel that discussions are important. Five students each wrote about those ideas. Four students also wrote that they think that using manipulatives is important. Three students wrote that they believed it was important to incorporate literature in teaching math.

Only one student each wrote the other responses to this question. Some of them were related to the most common responses, but not quite the same idea. Some examples of the other ways the students wrote that their ideas about teaching mathematics had changed were being more focused on how students learn, learning how to incorporate language arts, gaining ideas about assessment, realizing the importance of students' attitudes, realizing the importance of everyone needing to understand math, and having more ideas about how to apply math to everyday situations.

Many of the students didn't respond to the part of the question about whether there were specific events or activities that influenced that change, but the most common response for those who did was the "in-class activities." Five students wrote that. Four students wrote the "readings," and four also wrote the "in-class discussions." Three students wrote "everything." None of the students wrote anything about their ideas of *mathematics* changing; they all focused on how their ideas of *teaching mathematics* had changed.

Summary of Changes in Students Conceptions of Teaching Mathematics. The students' conceptions of teaching mathematics at the end of the semester as included in their "Philosophy of Teaching Mathematics" journal entries are interesting and quite gratifying to me as their teacher, but because they weren't asked to write a "Philosophy of Teaching Mathematics" journal entry at the beginning of the semester, it's hard to make definitive conclusions about how their conceptions of teaching mathematics changed. However, there is other evidence that supports the conclusion that there was change, and it does seem that their attention related to mathematics was much more focused on ideas related to teaching mathematics at the end of the semester than it was at the beginning of the semester. Also, the fact that 29 out of 29 students wrote that their conceptions of teaching mathematics had changed when they were asked if they had in an open-ended question of the final Mathematics Inventory, supports this assumption.

From the beginning of the semester to the end of the semester, there was a definite shift in positioning for the students as a group in their relation to mathematics. The shift in positioning from thinking of math as it related to their own experiences to viewing it more as someone who is going to teach it was even more marked in relation to the students' conceptions of teaching mathematics than it was in relation to the students' conceptions of learning mathematics. At the beginning of the semester, when the students included anything about teaching mathematics in their math concept maps or "What Is Mathematics?" journal entries, most of them included something about teaching mathematics as it related to their own experience as a student. Of course, in the Mathematics Inventory, the students were specifically asked about some of their conceptions of teaching mathematics, and so in those cases, they wrote about their ideas related to teaching math that weren't necessarily referring to their own experiences as a learner of math.

The focus of the math concept maps for many of the students showed a shift in positioning when their final concept maps were compared with their beginning concept maps. It seemed evident from what the students included in their math concept maps at the beginning of the semester that most of the students were thinking about mathematics in relation to their own experiences as a learner and user of it. In contrast, at the end of the semester, many of these same students seemed to be thinking about mathematics as a teacher would, and the items they included in their final math concept maps showed that shift.

Another example of change that illustrated the shift in positioning of the students was the new emphasis on assessment at the end of the semester. This was an example of something a teacher would be concerned with in relation to mathematics that someone who had only experienced learning it or using it would not.

The biggest difference between the students' conceptions of teaching mathematics at the beginning of the semester compared to the end of the semester was the expansion of their ideas of how to teach mathematics. For example, at both the beginning of the semester and at the end of the semester, many students held the conception that mathematics should be taught in a way that it was understood. However, there were more students at the end of the semester who seemed to believe that. Another difference is that at the beginning of the semester when students wrote about math being taught so that it is understood, they didn't really offer any examples of *how* that might be done other than the teacher explaining it so the students could understand. In contrast, at the end of the semester, when the students wrote about teaching math so that it could be understood, they provided many examples of how this might be accomplished such as using discourse, manipulatives, small groups, and the teacher acting as a facilitator for learning rather than a dispenser of knowledge.

At the end of the semester, almost all the students seemed to believe that communicating about mathematical ideas is an important part of teaching. This seems to have been a new idea for them. No students wrote about that idea in any data source at the beginning of the semester. Also, when I first introduced the idea in class, many students made comments indicating it was a new idea and not something they had experienced. One student even commented that when she was in school, talking with each other during math was called "cheating." Other students agreed with that observation. The way that some students wrote about it at the end of the semester also indicate that it was a new idea such as the student who wrote, "I never saw the value of having students write in math class until we discussed it in class."

The students also focused more on the role that students' and teachers' attitudes play in teaching mathematics at the end of the semester. Many of them seemed to believe more strongly that attitudes play a big role in effective teaching of mathematics.

It seems clear from what the students wrote in their "Philosophy of Teaching Mathematics" journal entries that they had many new ideas about how to teach mathematics at the end of the semester. It would not be unreasonable to expect that having new ideas would help the prospective teachers feel more optimist about all children being able to learn and understand math. We also might expect them to feel more positive about the idea of teaching mathematics, which, as will be seen in the next chapter, seems to have happened.

In summary, at the end of the semester there was a lot more attention to teaching mathematics by the students, more focus on specific methods that might be used in teaching mathematics, and a shift in positioning to thinking of math as a teacher would.

Chapter Summary

In this chapter, I summarized the analyses of the students' conceptions of learning mathematics and teaching mathematics both at the beginning of the semester and at the end of the semester. At the end of the semester, there seemed to be an increase in the students' attention to ideas about learning and teaching mathematics compared to the beginning of the semester. There also seemed to be a shift in how many of the students positioned themselves related to mathematics. At the beginning of the semester, most of the students seemed to think about math as it related to themselves as learners and users of math, and at the end of the semester, most of the students seemed to be thinking about mathematics as if they were going to be teaching it. This shift in positioning seemed to be happening for nearly all of the students—and for all of them to some extent. It seems that there are many possible implications for this change in positioning. It is likely that this change would be one of the major influences on the students in changing both their conceptions of and attitudes toward both mathematics and teaching mathematics. It also seems like this shift made nearly all of the students feel more empowered in their relation to math and the prospect of teaching math.

The students' views of learning math and teaching math were very interrelated, and their consistency when expressing views about one in relation to the other supported the assumption of the validity and genuineness of their expressions.

CHAPTER 5

ATTITUDES TOWARD MATHEMATICS AND TEACHING MATHEMATICS

In this chapter, I examine the students' attitudes toward mathematics and teaching mathematics. I will first look at their attitudes toward mathematics at the beginning of the semester. I then examine the students' attitudes toward math at the end of the semester and analyze differences between attitudes at the beginning and end of the semester. I will then look at the students' attitudes toward teaching math in the same manner—at the beginning of the semester and at the end of the semester, followed by an analysis of the differences.

Across the last four decades, research has consistently shown than most prospective elementary teachers have negative attitudes toward math (Allen, 2001; Becker, 1985; Bulmahn and Young, 1982; Dutton, 1954; Dutton, 1962; Kelly and Tomhave, 1986; Reys and Delon, 1966; and Smith, 1964). More recently, researchers have tried to find out whether teacher preparation programs can improve these attitudes (Adkins and Robinson, 2002; Couch-Kuchey, 2002; Gibson and Van Strat, 2001; Gregg, 1998; Miller, 1999; Reys and Delon, 1966; Smith, 1999; and Wilkinson, 2001). In this analysis, I also look—through a different combination of data sources—at whether these attitudes improve, and I try to also examine how they change and what might influence that change.

Attitudes toward Mathematics

Students' Attitudes toward Mathematics at the Beginning of the Semester. Four data sources provided information about the students' attitudes toward mathematics at the beginning of the semester: the "Mathematics Reflections" journal entries (which is described in the next paragraph), the "What

Is Mathematics?" journal entries, the math concept maps, and the Mathematics Inventory.

At the beginning of the semester, most of the students in this class had a negative attitude toward mathematics. One place that this could be seen was in the "Mathematics Reflections" journal entries. On the first day of class, I assigned the students a do-at-home journal entry for which they were to write a reflection of their mathematics experiences. They were asked to bring this reflection to the next class period which was the next day. These reflections provided much information about the students' attitudes toward mathematics when the semester began—as well as some insight into what had influenced those attitudes (which will be reviewed later). Of the 33 "Mathematics Reflections" journal entries received, 14 were completely negative toward math, 16 included both sections that were positive and sections that were negative depending on the time in their life, and three were wholly positive toward math. Sometimes the students wrote about positive experiences they had had with math, but ten students wrote general comments related to their attitudes toward math, and all of them were negative. Examples of what four students wrote are, "Math has always been my least favorite subject ever since I was little;" "For some reason I had it in my head that math was this horrible, hard subject;" "In middle school math was easy for me, yet I still didn't like it. I was always afraid that at any moment, I would get totally confused;" and "My expectations for grades in math are so much different than for the rest of my classes." Others wrote similar things.

In the beginning "What Is Mathematics?" journal entries, 19 students also wrote about their attitudes toward math. Ten students wrote only about negative attitudes. Three students included references to both positive and negative attitudes. In each of these cases, the positive comment was that math was important or necessary. Six students wrote only something indicating a positive attitude. Most of what they wrote related to attitudes toward math was negative. For example, one student wrote, "Math to me is a pain. . . . just a hard subject." Another student wrote,

> "To me, the word mathematics is a bad one. It brings back many memories through the years both good and bad, but I seem to remember the bad ones better. Its meaning that is in my dictionary in my head is work. I always have to spend double the time on math as any other subject. It's kind of like a job you hate."

Four students described math as something that was unnecessary or not needed (especially when referring to higher math). For example, one student wrote,

> "I see no point in adding in all kinds of variables and formulas. That is complication, not math. . . . Sure, it might be 'fun' for some people to whip out some formula from their head to figure out the final cost of something, but it just doesn't seem necessary."

Another student expressed her feelings related to this attitude this way:

> "Mathematics is sitting down and memorizing information that you just do not need. Who in the world is going to stop you on the street and ask you a math problem. I don't think so. To me, it is something that builders and carpenters need to know. Of course you need to know the metric system and how to measure, but ordinary people just need to know the basics to balance their checkbooks. Everyone tells me that you need to know how to count. I can understand that, but who needs calculus unless you are going to teach it to a bunch of students. Like I said, know the basics and you will get by fine."

Five students described math as complex such as the student who wrote, "Mathematics is surrounded, made up of, many complex concepts." Thinking of something as complex doesn't necessarily imply a negative attitude, but in the case of the five students who wrote that it was complex, it seemed like they considered that something negative about math.

The majority of the beginning concept maps also reflected negative attitudes toward mathematics. At the beginning of the semester, 23 students included items that were related to their own attitudes toward math, and 21 of those 23 included negative items. Thirteen of those students also included items that were positive, but 11 of those students included only one or two positive items while one included three or four, and one included more than four positive items. There were two students who included a low number of items related to positive attitudes in their beginning math concept maps that did not include any items related to negative attitudes. Of the 21 students who included items related to negative attitudes in their concept maps, seven of them included a low number (1-2), nine of them included a medium number (3-4), and five included a high number. An example of a student who included a high number of items related to negative attitudes in her beginning concept map was student 33 (See Figure 37). At least half of her direct connectors to "math" head categories that are related to negative attitudes: "curse," "personal challenge," and "hard work." The only item she included that seemed positive was "satisfaction when I 'win'" that was connected to "personal challenge."

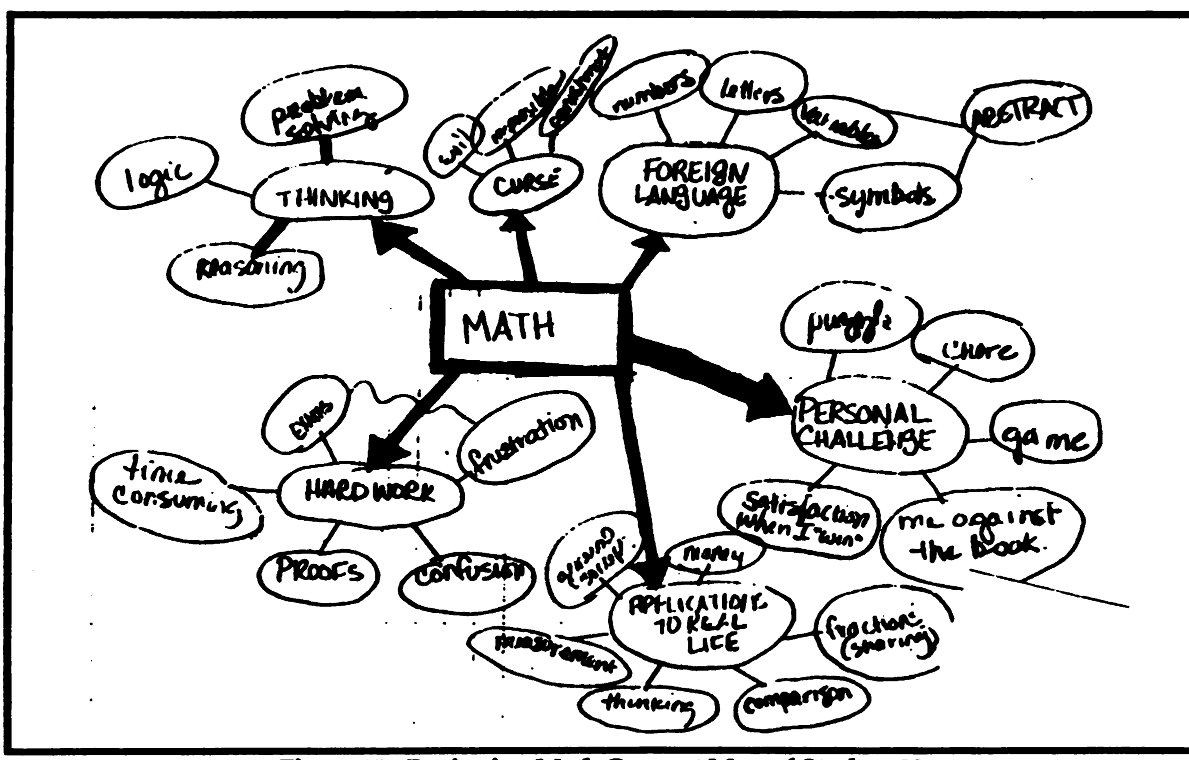

Figure 37: Beginning Math Concept Map of Student 33

An example of a student who included a large number of items related to negative attitudes, but also five items related to positive attitudes in her beginning concept map was student 28 (See Figure 38). The bottom section of her concept map is all related to negative attitudes, but there is that section around the eight o'clock position where she includes some positive attitudes: "rewarding," "enlightenment," "joy," "invigorating," and "pride!" There must have been at least a few moments in her life where doing math was a positive experience for her.

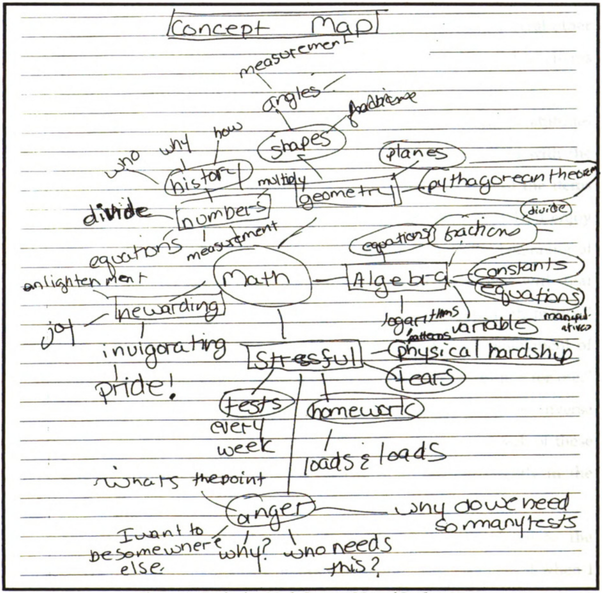

Figure 38: Beginning Math Concept Map of Student 28

The items that were included by the group in the beginning math concept maps that are related to negative attitudes show a wide range in the way they expressed these attitudes. Thirteen students wrote "frustration" or "frustrating;" nine students included "patience;" six students included "work" or "hard work;" five students included "difficult;" four students each included "confusion," "why do I need this?," and "time consuming." There were other examples that were included less often: three students each included "complex" and "personal challenge," and two students each included "need to know why," "hard to understand," "don't do well," "useless," and "anger." There were several other items that were included by one student each. Some examples of those items were "stressful," "curse," "struggle," and even "evil."

Six items in the Mathematics Inventory were related to students' attitudes toward math, and the students' responses to them were consistent with the majority of them having negative attitudes toward math. Three of the items were related to enjoyment of mathematics. One of these was item 1, "I enjoy doing mathematical tasks." The mean response for this item at the beginning of the semester was 2.55 which indicated fairly strong disagreement with that statement by the class as a group. Item 2, "I have always enjoyed mathematics," had an even lower mean response: 2.03. The other item related to enjoyment of mathematics was item 7, "Doing mathematics is usually very frustrating for me." Their mean response to that item was 3.24 which was consistent—in an inverse way—with their responses to items 1 and 2. The mean response to each of these items was close to the middle "3," but they were all leaning slightly in the negative direction.

One item from the Mathematics Inventory that was related to the students' confidence in doing mathematics was item 6, "I feel successful when I am engaged in mathematical tasks." The mean response for that item at the beginning of the semester was 2.62 which indicated more disagreement with that statement than agreement as a group, but it was pretty close to the middle of the scale.

Another item related to the students' confidence in their understanding of mathematics was item 15, "I understand elementary level mathematical concepts quite well myself." At the beginning of the semester, the mean response to that item was 3.517, which was an average response that leaned slightly to the positive side. Apparently, even though many students in the class had had negative experiences with math, many of them felt like they at least understood elementary level math. Item 25, "I can handle basic math, but I wouldn't do well at advanced mathematics," is also related to this idea. Their mean response to that was 3.17—again close to the middle. This is an item, that in hind sight, I wish had written as two separate items. It's hard to know if the students are agreeing with the entire statement or with just part of it. For example, they might agree that they can handle basic math but wouldn't do well at advanced mathematics. But, if the students think they can handle basic math and also advanced mathematics, it's hard to know what they would put. One part of the statement is true for them, and the other is not. It's also hard to know how students who didn't think they could do well at either basic math or advanced math would respond. Some of the students might not have been sure how to respond, and that might be why the mean is so close to the noncommittal "3."

Fortunately, there were a few expressions of positive attitudes toward mathematics in these beginning data sources, but they were definitely attitudes of the minority of the students. As noted, 15 students included items in their beginning math concept maps that were related to positive attitudes. However, 13 of them included a low number of items, and only two included a medium number of items. Both of the students who included a medium number of items in their beginning concept maps that were positive attitudes also included a much larger number of items that were indicative of negative attitudes. Only two students included any items that were positive attitudes without also including negative attitudes, and both of those students included a low number of items related to positive attitudes. All of the 13 students who included both positive attitudes and negative attitudes on their concept maps included more items that were related to negative attitudes. These items were included by two people each: "should be fun," "accomplishment," "with hard work can succeed," "relief," "rewarding," and "succeed." There were no positive attitudes that were included by more than two people.

In the "Mathematics Reflections" journal entries, there were some positive comments related to specific experiences with mathematics, but there weren't any general comments that reflected a positive attitude toward mathematics. However, in the beginning "What Is Mathematics?" journal entries, some students did express positive attitudes. As noted, six included only positive attitudes, and three students included reference to both positive and negative attitudes. Five students wrote that math can be fun, rewarding, and/or exciting—especially when you are able to get the right answer. One student wrote, "Mathematics can be fun like when you're learning something new or playing a game." Another wrote, "Math can be fun and exciting. When people are able to comprehend certain mathematical tasks, they can feel rewarded." A similar thought was expressed by another student: "It can be exciting, too, if you're engaged in the problem in question. It is also rewarding when you finally get the right answer." A final example of this type of positive experience and attitude is the student who wrote, "There is no better feeling than coming up with the correct answer to a difficult math problem."

Five of those who included positive attitudes toward math in their "What Is Mathematics?" journal entry wrote that math was important, essential, and/or necessary. Typical of the statements in that category, one student wrote, "Math is an important part of everyone's life." Another student wrote,

> "Math is very important, and there will never be any sort of substitute that could replace it. . . .Mathematics will always be one of the most important learning areas of all grade levels and in all sorts of life."

Three students wrote general comments about the importance of mathematics. For example, one student wrote, "To me, if one truly was afraid of everything related to math, they would be afraid to wake up in the morning. Math is such an integral part of life that everybody should be somewhat comfortable with it."

In summary, while there were a few students who had positive attitudes toward mathematics at the beginning of the semester, and some who had both positive and negative attitudes toward mathematics, the majority of the students in the class had negative or mostly negative attitudes toward math.

Students' Attitudes toward Mathematics at the End of the Semester. At the end of the semester, it seemed that the students, as a group, had much more positive attitudes toward mathematics than they did at the beginning of the semester. In their final "What Is Mathematics?" journal entries, 12 students wrote something that indicated a positive attitude toward math. Four other students wrote something indicating a positive attitude, but also included something indicating a negative attitude. Only one student included a reference to a negative attitude toward math without also including something that indicated a positive attitude. All four of the students who wrote references to both negative and positive attitudes wrote about the same positive aspect—that math was useful and necessary. For example, one of them wrote, "Math is a necessary part of life and cannot be separated from it. It has the potential to be fun and enjoyable if it is applied to daily experiences." Other students also had the attitude that mathematics was useful and wrote similar statements.

Some students felt like math could be fun and rewarding. One student wrote, "Math can be fun and exciting. When people are able to comprehend certain mathematical tasks, they can feel rewarded; they can now conquer many important tasks in life." Another student wrote, "[Math is] fun, exciting, something everyone can do, part of every aspect of one's life, what makes up the world. . ."

Some of the students wrote that their attitudes toward mathematics had changed and that they now felt more positively toward it. For example, one student wrote, "Now, I feel very confident when it comes to math. I realize that math is not there to trick me or to simply keep me busy." Another student wrote, "I can honestly say my attitude towards math has changed for the better."

> "I now look at mathematics in a different light. I never thought I would say this, but I actually think I am beginning to enjoy mathematics. It is very interesting to me. I find it to be incorporated in just about everything."

One student even wrote a haiku to describe the transformation in her attitude:

"What Is Math?: A Haiku
Confusion. Mayhem.
My brain is going crazy.
Oh! I get it! Cool!"

One student wrote that people's ideas of math change. She wrote, "Math is different for everyone but is constantly changing; the change that comes can be positive if math is looked at in new and innovative ways!! Such as TE classes."

As noted, there were still five students who included negative attitudes in their final "What Is Mathematics?" journal entries. All of them wrote about math in terms of their experience with it as a student. One student wrote, "To me, math is the source of great frustration and little confidence. I have a difficult time understanding the puzzles as I have not obtained all of the pieces that I should have as a young student." Another student wrote,

> "To me math is a more negative memory than positive. . .Those times table charts that had all the bright colored stars but never by my name has scarred me for life. . . . Math, as I'm sure you can tell, is a very stressful topic for me to remember and look back on."

One student wrote a statement about mathematics in his final "What Is Mathematics?" journal entry that didn't tell what his attitude was, but took a neutral position. He wrote, "Math is seen in many different ways. It can be seen as useful, boring, or interesting. No person sees math the same way as other people."

The students' responses to the items from the Mathematics Inventory that were related to attitude toward mathematics also showed some improvement in their attitude (See Table 6). While none of the changes in the mean responses were statistically significant, the trend of the change was in the more positive direction.

Table 6: Items Relating to Attitudes toward *Mathematics* from the Mathematics Inventory

Item	Beginning Mean	Final Mean	p value
1. I enjoy doing mathematical tasks.	2.55	3.07	0.069
2. I have always enjoyed mathematics.	2.03	2.41	0.21
6. I feel successful when I am engaged in mathematical tasks.	2.62	3.034	0.13
7. Doing mathematics is usually very frustrating for me.	3.24	2.86	0.25
15. I understand elementary level mathematical concepts quite well myself.	3.517	3.586	0.78
25. I can handle basic math, but I wouldn't do well at advanced mathematics.	3.17	3.31	0.67

As can be seen in the table, the biggest changes were in items 1, "I enjoy doing mathematical tasks," and 6, "I feel successful when I am engaged in mathematical tasks." Apparently, at least some students were feeling like doing math was more enjoyable than they thought it was when the class began.

There was also an open-ended question in the final Mathematics Inventory related to the students attitudes toward math: "Have your attitudes toward mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how? Were there specific events or activities that helped your attitudes change?" Twenty-seven out of 29 students wrote, "yes" to the first part of that question. Three students answered "not much"—that they started positive and still feel positive or that they had a good attitude all along. Eleven students wrote that their attitudes had improved, that they felt more positive, and/or that they were more optimistic. Eleven students also wrote that they felt more confident in their math abilities. Three students wrote that they realize its importance more, and two students wrote that they realize more that math is everywhere. One student each wrote that she now enjoys it, that she feels like math can be fun and interesting, and that she is more comfortable with math. No students wrote that their attitude toward math had changed in a negative way. In response to the question about what had influenced the change, three students wrote, "the readings" and/or "Marilyn Burns book." (The Marilyn Burns book, *About Teaching Mathematics* [Burns, 1992], was a textbook we used in the class for some readings and as a resource for the students when they did group presentations and wrote unit plans.) Three students wrote, "the in-class activities," and three wrote "everything." One student also wrote field experience, and another wrote, "activities observed in field experience." Of course, there is the possibility that since the inventories were not anonymous because the students wrote their student numbers on them, the students may have been trying to be polite and tell me what they thought I would want to hear. (In reality, the only inventories

I connected to the name of the student were those of the focus students. The others were anonymous to me.)

There were some interesting changes in the math concept maps related to attitude from the beginning of the semester to the end of the semester. One change in the concept maps was that there was a difference for some in the *way* they included those attitudes. Instead of including them in a way that looked like they were referring to their own attitudes, five of the students who included

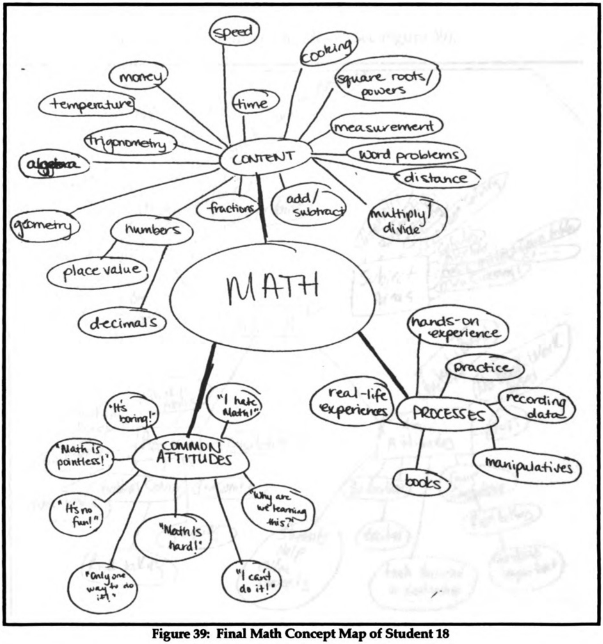

Figure 39: Final Math Concept Map of Student 18

attitudes in their final concept maps put them in categories that made them seem that they were not necessarily their own attitudes. In contrast, in the beginning concept maps, ALL of the inclusions of attitudes seemed to be references to the students' own attitudes. (I am assuming that the attitudes the students included in their beginning math concept maps were references to their own attitudes because they weren't written with headers such as "some people's attitudes" and they were consistent with the attitudes they expressed in their "Reflections of Mathematics Experiences" journal entries.) For example, student 18 had a section with the heading "common attitudes" (See Figure 39).

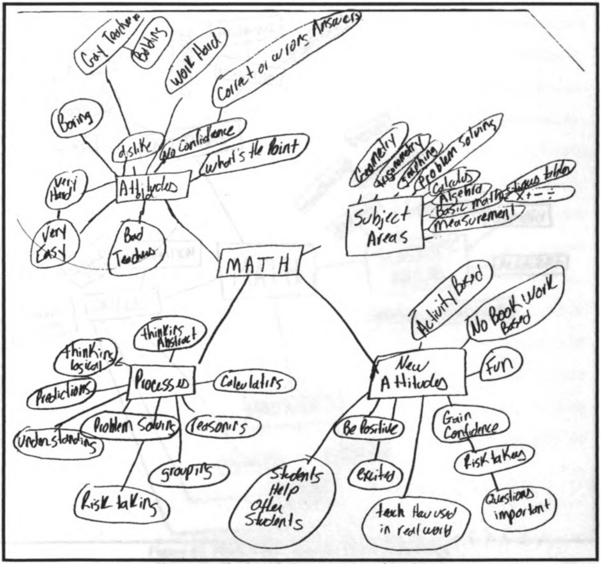

Figure 40: Final Math Concept Map of Student 9

Student 9 had one category for "new attitudes" which apparently were her current attitudes, and another category for "old attitudes" where she included the attitudes she used to have (See Figure 40). (On her first map, the only items she included related to attitude were negative.)

Branching out from her category of "attitudes," student 33 had two subcategories of "positive" and "negative" (See Figure 41) which could have been a reference to her own attitudes, but they may also have been her perception of two types of attitudes she sees.

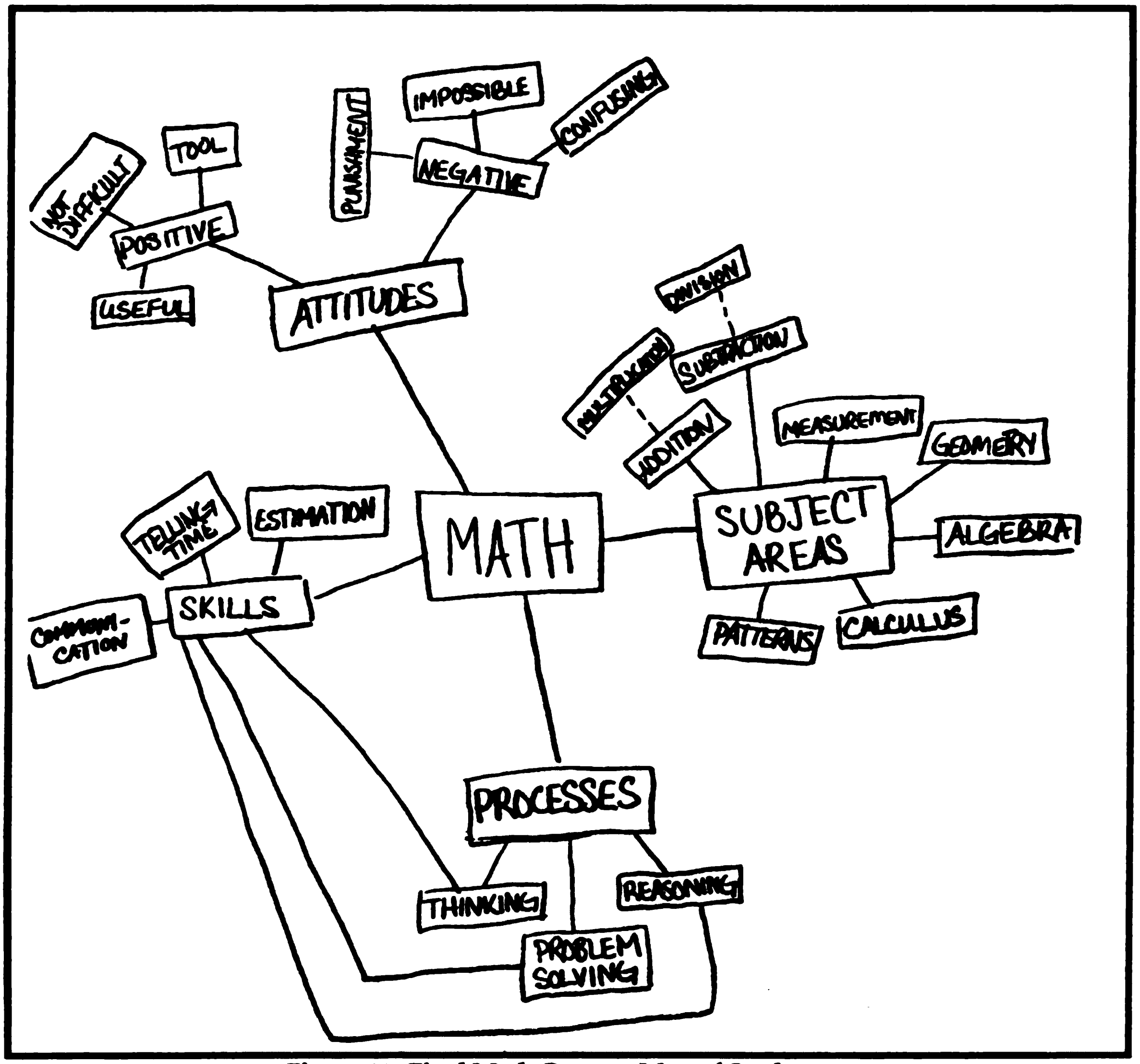

Figure 41: Final Math Concept Map of Student 33

The final concept map of Bridget, a focus student, which was shown in the last chapter in Figure 35 on page 183 also shows this change since she had categories for attitudes "that we want" and "that we don't want." So, even though these students included some negative attitudes and some positive attitudes in their final concept map, it seems that they are not necessarily their own—except in the case of student 9 where the attitudes included under "new attitudes" were probably her own. Instead, it seemed like they were again positioning themselves as teachers—or prospective teachers—and thinking about the attitudes they wanted to their students to have and not have.

Another change was that students focused less on attitudes—especially negative attitudes—in the final concept maps. In the beginning group, 21 students included negative attitudes, and 15 students included positive attitudes. In contrast, in the final concept maps, 10 students included negative attitudes, and 13 students included positive attitudes. The difference in those numbers is magnified when we separate the maps by those who seemed to be referring to their own attitudes and those who referred to different categories of attitudes. In the final math concept maps, of the 10 students who included negative attitudes, five of them included attitudes that seemed to be their own. Only one of those students included a high number of items (five or more) related to negative attitudes, and that was Felicia, a focus student whose development I will examine closely in the next chapter. Of the 13 students who included positive attitudes in their final concept maps, 10 included attitudes that seemed to be their own, and three of those 10 included a high number of items related to positive attitudes.

Again, of the negative attitudes that seemed to be the students' own, the most common one the students included in their final concept maps was "frustration" with five occurrences. (This was also the most common item in this category in the beginning concept maps, but there were 13 occurrences in that set.) Three students included "confusion," and two students each included "emotions," "patience," and "difficult." One student each included the other related items. They were "requires time, practice, and patience," "headaches," "why do I need to know it?," "insecurity," "struggle," "different brain waves," "bad," "bad teachers," "very hard," and "boring." Only one student each included the positive attitudes that seemed to be their own. Examples of these were "good," "good feeling when finish," "it's fun," "it's interesting," "you can do it," "not difficult," "appreciation," "intrigued," "enrichment," "achievement," "confidence," "excitement," "rewarding," and "comfortable." On the whole, the final concept maps seemed much more positive than the beginning concept maps when considering references to attitude toward math.

Summary of Changes in Students' Attitudes toward Mathematics. Overall, there seemed to be a more positive attitude toward mathematics for the group at the end of the semester than at the beginning. This was evident in the "What Is Mathematics?" journal entries, the Mathematics Inventory, and the math concept maps. In the final "What Is Mathematics?" journal entries, 16 students wrote something indicating a positive attitude, and five wrote something indicating a negative attitude. Four of the five who included references to negative attitudes also included references to positive attitudes. In contrast, in the beginning "What Is Mathematics?" journal entries, nine students wrote something indicating a positive attitude, and 13 wrote something indicating a negative attitude. (Three of those students were the same people.) Also, in the final Mathematics Inventory, 27 students wrote that their attitudes toward mathematics had changed during the time they had been in the course. Eleven students wrote that they felt more positive toward math, and 11 students also wrote that they felt more confident in their math abilities.

In the final math concept maps students paid less attention to attitudes, but put more emphasis on positive attitudes and less emphasis on negative attitudes. There was also a difference in the way that some students referred to attitudes when they did include them. Instead of listing them in a way that seemed to indicate they were their own, they made categories of attitudes such as "that we want" and "that we don't want." The mean responses for the items related to attitude toward mathematics in the Mathematics Inventory, at the end of the semester, also indicated a trend in the more positive direction, even though the change was not statistically significant for any of them.

It seemed that one of the reasons the students' attitudes were more positive is that they had had an opportunity to experience math in a way that was different from how they had experienced it as students and that they were more engaged and felt more successful with these new experiences. For example, in her final "What Is Mathematics?" journal entry, one student wrote, "I now look at mathematics in a different light. I never thought I would say this, but I actually think I am beginning to enjoy mathematics. It is very interesting to me." Another student wrote, "Now, I feel very confident when it comes to math. I realize that math is not there to trick me or to simply keep me busy."

Another apparent reason the students' attitudes were more positive is that now they were not thinking of their past experiences in math as much (positioning themselves as students), and they were thinking about math from the perspective of a prospective teacher. They seemed to be thinking more about how they wanted to teach it and how they wanted their students to experience it than they were thinking about their attitudes—mainly their beginning attitudes—toward it. One reason I conjecture this is that the attention of the students to their own attitudes greatly diminished when comparing the beginning math concept maps to the final ones. On the other hand, their attention to teaching mathematics greatly increased. Additionally, when I examined the students' conceptions of teaching mathematics, they seemed to have many more ideas of how to do this (teach so their students have a better experience than they did) at the end of the semester than they did at the beginning.

Attitudes toward Teaching Mathematics

Students' Attitudes toward Teaching Mathematics at the Beginning of the Semester. The Mathematics Inventory provided most of the information about the students' attitudes toward teaching mathematics at the beginning of the semester. Interestingly, even though some students included some of their ideas about teaching mathematics in their beginning "What Is Mathematics?" journal entries, no one wrote anything about his or her *attitudes* toward teaching mathematics in that journal entry. Likewise, the students included no items clearly related to attitudes toward teaching mathematics in their beginning math concept maps.

There were several items in the Mathematics Inventory related to attitude toward teaching mathematics. Some of them were related to enthusiasm or desire to teach. One example of an item in this category was item 8, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher." At the beginning of the semester, the mean response for that item was 2.93 which was close to the middle but leaning in the "not-at-all-like-me" direction. Another similar item was item 9, "I feel enthusiastic about mathematics and about teaching it." The mean response for that item was similar to that of item 8, but even a little lower; it was 2.724. Item 19, "I want to teach mathematics so that I can help students get excited about it," was also related to desire to teach. The mean response for this item was 3.310 which was again close to the middle, but this time leaning in the "very-much-like-me" direction.

There were also some items related to the students' confidence in their ability to teach mathematics. One of these was item 14, "I feel confident in my ability to explain mathematical concepts to children." The mean response for this item was a little below the mid-point of the scale—2.76. Another related item was item 16, "I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students." The mean response for this item, which was 2.14, indicated fairly strong disagreement with the statement.

Related to their confidence in their ability to teach mathematics was their confidence in their understanding of mathematics, and there were a few items related to this idea in the Mathematics Inventory. One of these was item 12, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school." The mean response for this item, which was 1.97, indicated fairly strong disagreement with that statement by the class. An item that was indirectly related to the students' confidence in their abilities to do math was item 20, "As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer." The mean response for that item was 2.66—a little below the mid-point of the scale.

Item 21, "I expect to regularly teach mathematics as an elementary teacher," was essentially neutral in relation to attitudes toward teaching math, but there were some students who maybe weren't planning on teaching elementary school or were hoping they could get in a team teaching situation where they wouldn't have to teach math. This item helped me know how relevant the other questions related to teaching math were to them. The mean response on the first day of class was a little lower than I would have expected.

It was 3.79. There were probably some students who were going through the teacher preparation program and teacher certification process so that they had the option of teaching elementary school, but they weren't completely sure that's what they were going to do.

Table 7: Students' Responses to Question 26 from the Beginning Mathematics Inventory

Question: "What do you consider to be your greatest strengths in relation to being a mathematics teacher?"

Category of Response	Response	Frequency
Influence of Difficulties the Students' Experienced as Students of Math	because they had difficulties as a student, they will be able to identify with and/or have compassion for students having challenges	11
	patient with students because they understand that children don't learn math the same all the time	3
	because of difficulties they experienced, they would understand how important support and encouragement of the teacher is	2
	because of difficulties she experienced, she thinks anyone can learn with patience, time, and support	1
	will have empathy for the different kinds of ways children understand math	1
	patience	3
	can understand both how frustrating and rewarding math can be	1
Methods of Teaching Mathematics	know different ways or methods to teach a concept	6
	can explain math well and/or in a way that children can understand	4
	can tie math into the children's immediate environment/real life	4
	will emphasize understanding vs. memorizing	3
	will use creative, hands-on lessons	2
	understand that there are ways to teach math other than with a textbook	2
Positive Attitude	their own enthusiasm	3
	have a good attitude	2
Influence of Own Success/ Knowledge of Mathematics	can relate to students who understand it well because she did	1
	has experienced success in math	1
	has a strong background in math	1
	enjoys solving problems and "the light bulb coming on"	1
	had experience tutoring others in math	1

Table 7 shows the students' responses by category to an open-ended question related to attitude toward teaching mathematics: "What do you consider to be your greatest strengths in relation to being a mathematics teacher?" As can be seen, the most common responses of the students related to the difficulties they experienced themselves as students. For example, 11 students responded that because they had difficulties as a student, they will be able to identify with and/or have compassion for students who are having challenges. Even though the students listed this as a strength, it is again evidence that the students were not that confident in their mathematical abilities. The next most common category had to do with the students' perceived abilities to teach mathematics such as the six students who wrote that they know different ways or methods to teach a concept. Five students thought that their own positive attitude would be a strength in teaching mathematics, and four felt that their own success in math would be a strength. Overall, the students showed some confidence about the idea of teaching mathematics, but again, the question asked them to write, in essence, what they felt most confident about.

Another open-ended question the students were asked in the Mathematics Inventory was, "What concerns you the most about teaching mathematics to elementary students?" The students' responses can be seen in Table 8. Again, the most common response alluded to their perceptions of their inadequacies in their mathematical knowledge. Eleven students were worried about explaining it in a way that could be understood by different learners. Some were concerned about the students—such as keeping them interested or making sure they understood. Some were concerned about how to teach math and, for example, find ways to teach it creatively. Two students were also concerned that their own negative attitudes would influence their students. These responses indicate that the prospective teachers did have concerns about the prospect of teaching mathematics, and as noted, many of these concerns were related to their own lack of mathematical knowledge. However, it should again be emphasized that the students were asked to write about their concerns for this question.

Table 8: Students' Responses to Question 27 from the Beginning Mathematics Inventory

Question: "What concerns you the most about teaching mathematics to elementary students?"		
Category of Response	**Response**	**Frequency**
Teaching/Explaining Mathematics Content	explaining it in a way that can be understood by different learners	11
	afraid a child would ask a question they couldn't answer	3
	math skills were not as strong as they should be	3
	not confusing the students or giving them incorrect information	2
	explaining something that she didn't understand herself	1
	a critical thinker and afraid he wouldn't be able to simplify things enough for the students	1
Responses of Students	making sure the students were interested and/or having fun	3
	making sure the students understand math	3
	afraid the students would quickly become frustrated and quit trying	2
	worried about the confidence students have towards math	2
	being a good enough support for those who need help	1
	challenging the quick learners	1
	helping students build confidence	1
	discouraging students about math as a whole	1
	frustrated students	1
	helping students who have a hard time understanding and memorizing math facts	1
How to Teach Math	finding ways to teach math creatively	3
	not knowing where to start or how to teach since students will be at different levels	2
	providing a variety of learning for different types of learners	1
	being able to relate math to real life	1
	won't be able to teach how everything is related to everything else	1
	limitations of workbooks and rote learning	1
	the lack of resources	1
	being prepared	1
Own Attitudes	afraid her attitude would be reflected in her students' work	1
	being able to be confident	1

The final open-ended question from the Mathematics Inventory that was related to attitude toward teaching mathematics was, "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." The students responses can be seen in Table 9. Twelve students thought their lack of expertise would be a strength, and again, the most common explanation for that position was that they would be able to relate with students who had difficulties. Ten students felt their expertise would be a strength, most commonly explaining that the more they understood math, the better they would be able to teach it and/or answer the students questions. Three students thought their lack of expertise would be a liability, and two thought their expertise would be a liability. Even though many of these students had not experienced much success in math and had negative attitudes toward math, most of them (22) still thought that either lack of expertise or their expertise would be a strength in teaching. This shows some confidence in their ability to teach mathematics.

Table 9: Students' Responses to Question 30 from the Beginning Mathematics Inventory

Question: "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain."

Category of Response	Response	Frequency
Lack of Expertise a Strength (12)	they would be able to identify with students who experienced difficulties	4
	would help them be more careful in making sure they understood what they would be teaching	2
	want to make math more exciting for their students than it had been for them	2
	could learn from other students	1
	would help them look for materials to help teach it	1
	would motivate them to get over their fears	1
	will be more open to students' ideas and thought processes	1
Expertise a Strength (10)	the more they understood math, the better they would be able to teach it and/or answer students questions	6
	would be easier to teach having positive feelings about math	1
	could be more creative	1
	have a desire to be a good teacher of math	1
	have a willingness to learn, try, grow, and make mistakes	1
	will present it in a way that all students will be interested	1
	are confident that they can help someone learn	1
Lack of Expertise a Liability (3)	has a lot to learn	1
	her students will also not like math unless she learns to enjoy it	1
	lack of love will make it more difficult, but that she hopes she finds ways to teach that she can be excited about	1
Expertise a Liability (2)	may assume others know more than they do	1
	might go too in-depth and confuse the students	1
	understands elementary math, but she wonders if she will be able to teach it to students	1
Lack of Expertise Either a Liability or a Strength (2)	need to look at it as a learning experience	2
	only a strength if it helps you be more determined to help the students learn	1

From the students' responses to the Likert-style items and the open-ended questions from the Mathematics Inventory, it is apparent that at the beginning of the semester, many students felt a great deal of apprehension about teaching mathematics, and many were not excited about the prospect. They were concerned about their own knowledge and abilities in math. They were also concerned about being able to explain the math to the students so that they could understand it, and they were concerned about just knowing how to teach it. Another concern they had was that their attitudes would affect their students' attitudes and learning. Of course, when they were asked what they considered to be their strengths in relation to teaching math, they listed what they felt most confident about, but the most common response for that also intimated their awareness of their inadequacies in their math abilities: they would be able to related to students who were having challenges.

Students' Attitudes toward Teaching Mathematics at the End of the Semester. Again, at the end of the semester, the main source of information related to the students' attitudes toward teaching mathematics came from the Mathematics Inventory. However, it wasn't the only source of information. While none of the students included anything related to their attitudes toward teaching mathematics in their beginning "What Is Mathematics" journal entries, a few did write about this in their final "What Is Mathematics?" journal entries. Again, as was the case at the beginning of the semester, the students did not include anything in their final math concept maps that was explicitly referring to their attitudes toward teaching math. They included much about their conceptions of teaching mathematics, and maybe some of their attitudes could be inferred from those inclusions, but there was nothing that was clearly related to their attitudes toward teaching mathematics. Maybe attitudes toward teaching mathematics did not lend themselves well to inclusion in a concept map, or maybe it just wasn't something the students were likely to focus on when they were thinking of the concept "math" as opposed to "teaching math."

Five students wrote something in their final "What Is Mathematics?" journal entries that indicated a positive attitude related to the idea of teaching mathematics. One of those five also indicated negative attitudes related to the idea of teaching mathematics. That student was Bridget, a focus student, and her attitudes will be examined in the next chapter. She was the only student who wrote anything indicating a negative attitude toward teaching mathematics. Of the students who wrote something positive, one student wrote, "I feel confident that I can teach math on an elementary level with the necessary skills to be successful." Another wrote, "Throughout this course, I have learned many new methods of teaching mathematics, ways to keep students interested and wanting to know more about the topic."

In the "Philosophy of Teaching Mathematics" journal entries, many students wrote about their attitude toward teaching mathematics. All of them indicated positive attitudes. They also all indicated that they were more positive than they had been. One student wrote, "This class has introduced me to many new aspects and perspectives of teaching math, enhancing my confidence and attitude toward math." Another wrote, "My role and fear of teaching mathematics has changed and is still developing through this class." A couple of them wrote about how they could be a good math teacher even though they aren't good at math themselves. One wrote, "The teacher must be a positive, professional role model for the student. I may not be good at math, but I'm going to try, and I won't be afraid. I will work hard." Another wrote,

> "I have not always enjoyed math in my life, but I do enjoy teaching it. I have watched the class I taught light up when learning about how tall they are or that they know five minutes have passed. That is exciting for me."

Again, the data from the Mathematics Inventory—both the students' responses to some Likert-style items and their responses to some open-ended questions—provides the most information about the students' attitudes toward teaching mathematics at the end of the semester.

Table 10: Items Relating to Attitudes toward *Teaching Mathematics* from the Mathematics Inventory

Item	Beginning Mean	Ending Mean	p value
8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.	2.93	3.38	0.12
9. I feel enthusiastic about mathematics and about teaching it.	2.724	3.31	0.033*
12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.	1.97	1.69	0.32
14. I feel confident in my ability to explain mathematical concepts to children.	2.76	3.276	0.048*
16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.	2.14	2.34	0.47
19. I want to teach mathematics so that I can help students get excited about it.	3.310	4.03	0.013*
20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.	2.66	2.45	.047*
21. I expect to regularly teach mathematics as an elementary teacher.	3.79	4.17	0.20

* statistically significant

The difference in the mean responses for four of the items from the Mathematics Inventory that were related to attitudes toward teaching mathematics was statistically significant from the beginning of the semester to the end of the semester (See Table 10). Three of these items with statistically significant differences were directly related to enthusiasm, confidence, and desire to teach—item 9, item 14, and item 19—and the changes indicated a more positive attitude toward teaching mathematics.

The other item from the Mathematics Inventory that showed a statistically significant change related to attitudes toward teaching mathematics was item 20, "As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer." Students agreed with that statement less at the end of the semester. The statistically significant change in that direction is probably again related to the students' new view of teaching math where they see themselves as a facilitator more than a dispenser of knowledge. They feel that if a student asks them something to which they didn't know the answer, they could say something like, "What do you think?" or "Who has any ideas about that?" They also feel that part of their role is to model mathematical learning and thinking, and if a student presents them with something new to learn, they have an opportunity to model it. This was also evidenced in the statistically significant change of another Likert-style item, "Being good at mathematics is not required to be a good teacher of math," which was discussed in Chapter 4. The students agreed with that statement more at the end of the semester. Comments that the focus students made in their interviews also support this conjecture and will be analyzed in the next two chapters.

The changes in the mean responses for the other Likert-style items from the Mathematics Inventory related to attitudes toward teaching mathematics were also mostly in the more positive direction, but were not statistically significant (See Table 10 on page 224). The direction of the change for item 12, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school," may have been an indication that the students were recognizing that there was a lot more to elementary school mathematics than the basic operations.

The students' responses to the open-ended questions in the Mathematics Inventory related to attitude toward teaching mathematics also reflect the more positive attitude many of the students seemed to have at the end of the semester about the idea of teaching mathematics. The first open-ended question was, "What do you consider to be your greatest strengths in relation to being a mathematics teacher?" The students' responses to that question can be seen in

Table 11. The most common response on the first day of class was also the most common on the last day. Seven people—down from 11 on the first day of class—wrote that since they had had difficulties as a student, they would be able to relate to their students. At the beginning of the semester, there were also 11 other responses related to that idea. In contrast, only the seven afore-noted students focused on their own difficult experiences. At the end of the semester, 13 students indicated that what they considered to be their strength was a positive attitude toward mathematics and/or teaching. Only five students wrote

Table 11: Students' Responses to Question 26 from the Final Mathematics Inventory

Question: "What do you consider to be your greatest strengths in relation to being a mathematics teacher?"

Category of Response	Response	Frequency
Influence of Own Difficulties in Math	since they had had difficulties as a student, they would be able to relate to their own students' difficulty and/or feelings of frustration	7
Positive Attitude	good attitude toward math	6
	love of math	4
	want their students to love math and be excited	2
	loves to teach	1
Teaching Ideas	have a lot of good ideas	2
	have resources/know where to find ideas	2
	would be facilitating students to explain their thinking	2
	can communicate in a way children can understand	2
	want to provide a rich learning environment including hands-on experiences and manipulatives	1
	learning to create fun activities	1
	can teach concepts in more than one way	1
	have the ability to probe children's thinking	1
	able to ask questions	1
	care more about how and why students got their answers than what the answer is	1
	can see connections between math and other subjects	1
	can relate math to the world outside of school	1
	think math is very related to daily life	1
	her philosophy of teaching math	1
	"I now see math in an entirely new light. I no longer see it as frustration and confusion, as sit down and do paper after paper, but as a justification and reasoning of ideas."	1
Interaction with Students	willing to spend extra time with students to make sure they understand	3
	patience	3
Knowledge of Math	understanding of elementary math topics	2
	strong math background and good grasp of the material	2
	willing to work extra hard to understand any areas of math	1

about a positive attitude for this question in the beginning Mathematics Inventory. This difference showed an increase in the students who felt positively about the idea of teaching mathematics. There were also many students who wrote about ways they would teach.

Table 12: Students' Responses to Question 27 from the Final Mathematics Inventory

Question: "What concerns you the most about teaching mathematics to elementary students?"

Category of Response	Response	Frequency
Teaching/Explaining Mathematics Content	won't be able to clearly explain a concept	4
	being clear and helping students understand	2
	not being able to explain the reasoning behind the steps of algorithms	2
	running out of ways to explain something	1
	teaching in the best way for students to learn while not knowing everything about math	1
Students	teaching diverse learners so that all could understand	3
	having students at different levels in the same class	2
	moving on to the next level when all the students are not ready	1
	having students who had been told their whole lives that math is difficult	1
	wouldn't be able to get the students excited enough	1
Lack of Mathematical Knowledge	not understanding the concepts and/or background of the concepts well enough to teach them	2
	not being able to answer a students' question	2
	losing the students' respect if she not able to explain a concept	1
	afraid of confusing the students	1
	feel incompetent	1
How to Teach Math	readjusting the texts to fit the *Standards*	1
	won't be able to cover everything, because want to go in depth	1
	not enough time	1
	thinking about the things students want to learn and how to help them achieve it	1
	active learning	1
	not knowing where to start	1
	need more experience	1
	need more lesson plans	1

In Table 12, the students' responses to another open-ended question can be seen. This question was, "What concerns you the most about teaching mathematics to elementary students?" Again, the students were concerned about not being able to explain everything, and some were concerned about their lack of mathematical knowledge. This was also the most common response on the first day of class, but this time four students wrote that they were concerned about that, and 11 wrote that the first day. Some students were also concerned about teaching diverse learners, and some were concerned about other concerns related to teaching math such as not having enough time, not being able to cover everything, and not knowing where to start. In contrast to the beginning of the semester, none wrote concerns about the effects of their own negative attitude toward math. Again, the students were asked to write what they were concerned about, and overall, the students' responses seemed to be things that even experienced teachers might have concerns about, such as adjusting texts to the *Standards*, accommodating diverse learners, and not being able to cover everything since they want to go in depth.

The other open-ended question from the Mathematics Inventory that students also answered on the first day of class was, "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." The students' responses to this question can be seen in Table 13. This time, eight students—as opposed to 12 on the first day—responded that they thought their lack of expertise would be a strength. Eleven students wrote that their expertise would be a strength—similar to the 10 who wrote that the first day. Four students—in contrast to 3 on the first day—wrote that their lack of expertise would be a liability. Five students wrote "both," and one student wrote neither.

Overall, the students seemed to feel some confidence in their ability to teach math since 19 of them considered either their expertise or lack of expertise to be a strength when they taught elementary mathematics. This was also the case at the beginning of the semester when 22 students considered either their expertise or lack of expertise to be a strength. This time, however, there were more who considered their expertise a strength and fewer who considered their lack of expertise a strength. If they felt like they were strong in mathematics knowledge, they thought that would help them in their teaching, and if they felt they were weak in mathematics knowledge, they felt like that would help them by motivating them to learn more, learning with their students, or relating with their students.

Table 13: Students' Responses to Question 30 from the Final Mathematics Inventory

Question: "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain."

Category of Response	Response	Frequency
Expertise a Strength (11)	understand elementary math	4
	have a good background in math	2
	can explain it well	2
	love math	2
	can help students understand	1
	can facilitate connections more easily	1
	will be able to teach it confidently	1
	will be able to successfully create and follow through unit plans	1
	have experience analyzing students' thoughts and reasoning	1
	extra knowledge is always helpful	1
Lack of Expertise a Strength (8)	would put more time in planning	3
	can explore with the students	2
	learning from own mistakes	1
	realizes it and will therefore work with it	1
	will be able to relate well with all students	1
Lack of Expertise a Liability (3)	will motivate them to learn more about math	2
	can do math, but worried about teaching it	1
	not planning on teaching it	1
Lack of Expertise Both a Liability and a Strength (5)	understand some concepts but not others	2
	not strong in math, but wants to learn and be a good teacher	1
	will be able to teach what she learns	1
	will understand what it's like to be behind and not understand	1
	understand what it's like to have boring teachers who don't care	1
Neither (1)	"My expertise will increase as I focus on certain subjects. I will have to re-teach myself and talk with others about it."	1

The last open-ended question from the final Mathematics Inventory that was related to attitude toward *teaching mathematics* was one that was referred to in the attitudes toward *mathematics* section: "Have your attitudes toward mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how? Were there specific events or activities that helped your attitudes to change?" As noted, 27 out of 29 students responded "yes" to the first part of the question. One student wrote, "no," and her explanation indicated that she was referring to the attitudes toward *teaching* mathematics portion of the question. She wrote, "I still don't feel completely successful at communicating ideas." Eleven students wrote that they felt more confident in their teaching abilities. Two wrote that they were more confident because they now had better ideas of how to approach teaching math. Two students also wrote that they now appreciate how hard it is to teach. One student wrote that she felt like she could have a positive influence on students, and another wrote that she looked forward to giving students a better education than she received. Only two students, both of them focus students—Bridget and Felicia—wrote that their attitude changed in a way that made them not want to teach. However, as a group, the students' responses to this question indicated that they had a more positive attitude about the idea of teaching at the end of the semester.

Summary of Changes in Students' Attitudes toward Teaching Mathematics. The students' attitudes toward teaching mathematics was the area where there appeared to be the biggest change in the students conceptions of or attitudes toward mathematics in the study—especially as measured by the Mathematics Inventory. The changes in the mean responses to several items related to attitudes toward teaching mathematics from the beginning of the semester to the end of the semester were statistically significant in the more positive direction. Also, in the open-ended questions related to attitude toward teaching mathematics, the students seemed to feel more positively about the idea of teaching mathematics (e.g., 27 of 29 students said their attitudes toward teaching mathematics had changed, and all indicated that change in a more positive way). Of course, they still had concerns about teaching math, but they were specifically asked to write about them, and they seemed mostly like concerns that are always a part of teaching math.

In the final "What Is Mathematics?" journal entries and in the "Philosophy of Teaching Mathematics" journal entries, several students wrote statements that indicated their attitudes toward teaching mathematics were more positive. Some examples of these statements were, "I feel confident that I can teach math on an elementary level with the necessary skills to be successful;" "This class has introduced me to many new aspects and perspectives of teaching math, enhancing my confidence and attitude toward math;" and "My role and fear of teaching mathematics has changed and is still developing through this class."

Again, it seems very likely that much of the reason the students seemed to feel more positively about teaching mathematics by the end of the semester is that they had more ideas of how it could be done. This seemed to be all part of their shift in positioning from thinking of math as an experienced student to thinking of it as a prospective teacher. Instead of feeling apprehensive about the idea of teaching mathematics because they could only imagine it being taught in the way they learned it, they were more confident because they had a new perspective of how math could be taught and were thinking about math as if they were going to be teaching it. This new vision also seemed to cause them to feel less concerned about their lack of mathematical knowledge since they imagined the students doing more of the explaining.

Chapter Summary

In this chapter, I examined the students' attitudes toward mathematics and teaching mathematics at the beginning of the semester and at the end of the semester. I then summarized the differences. I found that most, but not all, of the students in this class had had negative experiences with school math. At the end of the semester, I found a general improvement in the students' attitudes toward mathematics. This was evidenced in their journal entries, their math concept maps, and in their responses to the items from the Mathematics Inventory that were related to attitude toward mathematics.

I conjectured that part of the reason the students' attitudes toward math seemed to be generally more positive was that during the class, they had experienced math in a way that was new to them and that they didn't associate with the way they had learned it in school. I found evidence to support this idea in the changes in the students writings in the "What Is Mathematics?" journal entries, the math concept maps, and their responses to several items in the Mathematics Inventory. I also speculated that there had been a shift in the way the students positioned themselves in relation to mathematics, and that this shift allowed them to think of math more positively.

At the beginning of the semester, I found that many students also had negative attitudes toward *teaching* mathematics. Many of the students did not feel excited about the prospect of teaching math, and many of them did not feel confident in their abilities to do so. I found significant improvement in the students' attitudes toward teaching mathematics at the end of the semester. My analysis of the students' writing in their journal entries and in their responses to items from the Mathematics Inventory suggested that they had experienced math in a more positive way and had more ideas of how it could be taught, and this seemed to help them feel more confident and excited when thinking of the prospect of teaching math. There were also some students who became excited to teach math because they wanted to help their students have a better experience than they did. Again, I conjectured that part of what had influenced this improvement in their attitudes is that they were repositioning themselves in relation to math—they were no longer thinking of math and math teaching as they had experienced it in school; instead, they were now able to imagine how they wanted it to be in their classrooms when they were teachers.

WHAT IS MATHEMATICS?: STABILITY AND CHANGE IN PROSPECTIVE TEACHERS' CONCEPTIONS OF AND ATTITUDES TOWARD MATHEMATICS AND TEACHING MATHEMATICS

VOLUME 2

By

Elaine Allen Tuft

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Teacher Education

2005

CHAPTER 6

THE CASES OF TWO PROSPECTIVE ELEMENTARY SCHOOL TEACHERS: BRIDGET AND FELICIA

In this chapter, I continue the analysis of this study by taking a closer look at two of the focus students and their conceptions of and attitudes toward mathematics and mathematics teaching. Both students began the course with extremely negative attitudes toward mathematics and the idea of teaching mathematics. I chose to focus on them because they seemed to have exceptionally negative attitudes, and they were very "vocal" in both their comments in class and their class writings about those attitudes. I was interested in looking at what appeared to be some hard cases. They differed in the amount of success they had experienced in doing mathematics, but they both disliked it. They also each had their own interesting conceptions of mathematics. At the end of the semester, there seemed to be slight cracks for these students in different aspects of their conceptions of mathematics, but they both still did not like mathematics and did not want to teach it.

Bridget: A Discovery Leading to a Success of Sorts

Bridget was a 21-year-old senior. She had grown up in a suburb of Detroit and graduated from high school there before beginning her studies at Michigan State University. She was quite tall with blue eyes and blond, short hair that emphasized the multiple earrings she wore in each ear. Bridget was an English major, and when she began the course, she was mainly interested in teaching English, but she was also interested in learning about teaching other areas.

Bridget had experienced quite a bit of success in mathematics during her schooling, but it wasn't a subject she enjoyed. She described her background of mathematics in high school this way:

> "Freshman year I had a conceptual Geometry class. It was like an honors Geometry class. Sophomore year I had Honors Algebra 2. And, Junior year I was supposed to take Honors Trig and Precalculus, but it was too hard so I just took the regular trig/precalculus. So I was in a class with kids that were a year older than me. So, my senior year I had to choose between taking AP Calculus or not taking a math class, or they offered this new class which was called conceptual math which is what I ended up taking. It was actually really neat, because it was full of a bunch of kids that had all dropped out of the honors math course but wanted to take another math class. So, it was statistics, probability, and discrete math.

The conceptual math class she took her senior year was probably the most positive experience she had with math. She wrote more about that course in her "Reflections of Mathematics Experiences" journal entry. In fact, it was the main thing she wrote about:

> "When I was in 12th grade I decided to take some of the stress of my course load and I enrolled in a course called "Advanced Math Topics" instead of the next course on my track, AP Calculus. I arrived my first day and found myself in a room with about 15 other students, all of whom had previously been students in my honors math classes. The teacher began class by saying that she was aware we had all dropped off the honors track, probably because we all wanted to free up time for other things during our last year. Nonetheless, she told us, she still wanted us to learn the material she was going to be teaching, and had devised a plan wherein we could learn and not spend hours at night doing homework. The first half of class would be for lecture, which proceeded at a fairly rapid pace. The second half of class we were given an assignment which was to be completed and handed in by the end of the day. This turned out to be an excellent experience! I've never learned so much in a math class--I always paid attention in class and any questions I had on my homework could be answered immediately. Along with her work policy, my teacher brought in several aides to help us understand the more difficult material. We rolled dice doing probability, counted out candy in statistics, and constructed Legos during Discrete Math. As well, she

brought in other tools for learning that weren't as 'entertaining' as those above."

Bridget did relatively well in her math classes, except for Geometry and trigonometry. During her mid-semester interview, she described how she did in her high school classes this way:

> "I didn't do very well in Geometry. Algebra was easy. Trig was the same as Geometry. But, the conceptual math the statistics and probability was really easy for me, and the algebra was really easy. But, I don't think I did very well in Geometry. I always had problems with that. It was just a lot of memorization I think."

At Michigan State University, she only took the math classes that were required for the teacher education program, and she did well in them. She described her college experience with math classes this way:

> "Well, sophomore year I took like Math 103, and it was really easy because it was everything I had taken in high school, but that was all I needed. So, that's what I took. Then I took 201 and 202 for TE, and those were the exact same thing that I took in my statistics class. Again, so they were all really easy for me. But the math program here--I don't think I learned it as well here as I did in high school. If I hadn't known it already, I probably would have been in trouble."

Bridget's Conceptions of Mathematics

The data collected from Bridget throughout the study provide some insights into Bridget's conceptions of mathematics. They will be discussed in the subsequent sections following the format of the framework designed to analyze this study.

Bridget's beginning and ending concept maps are shown here (See Figures 42 and 43). They will be referred to in various sections of the analysis.

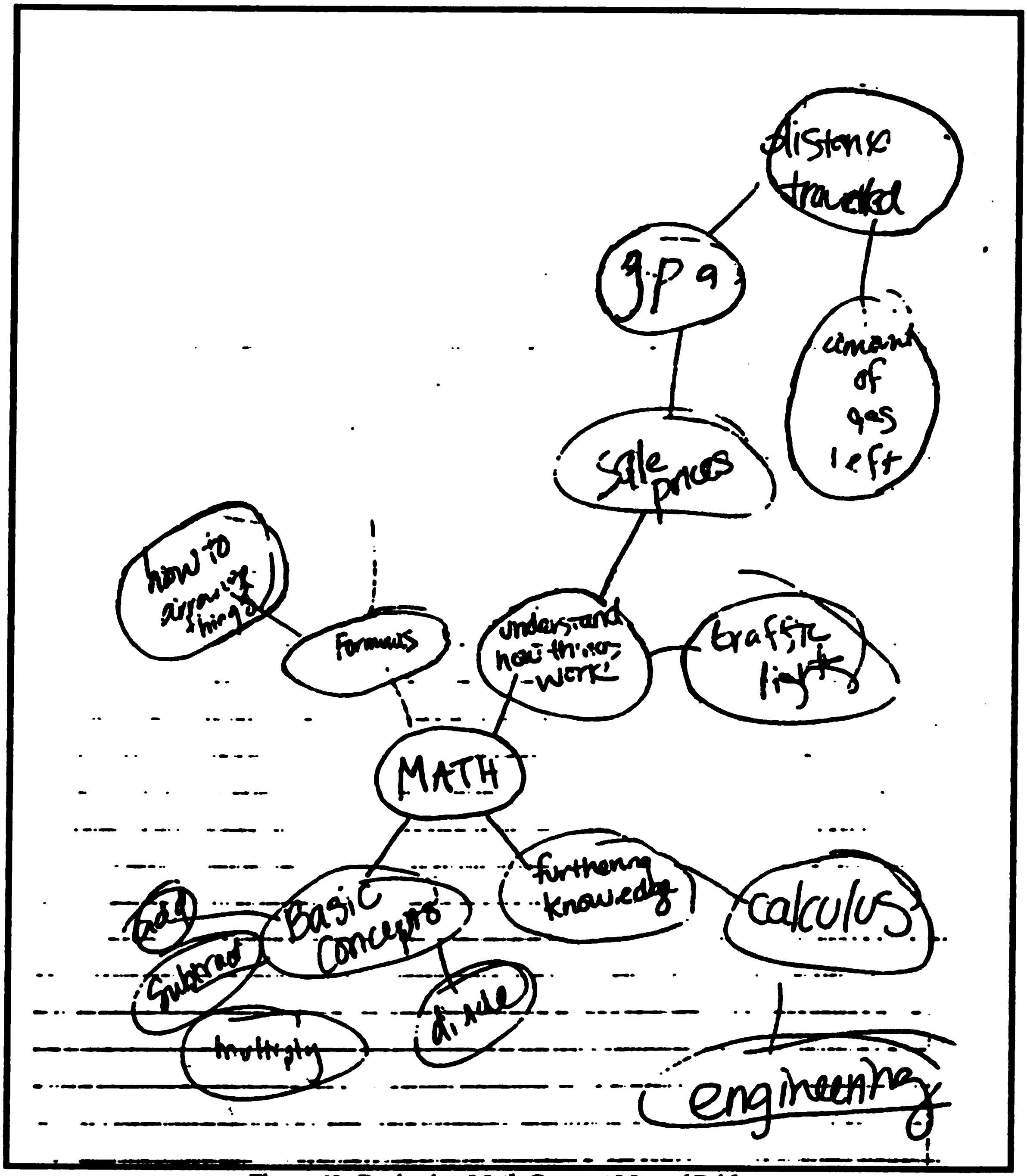

Figure 42: Beginning Math Concept Map of Bridget

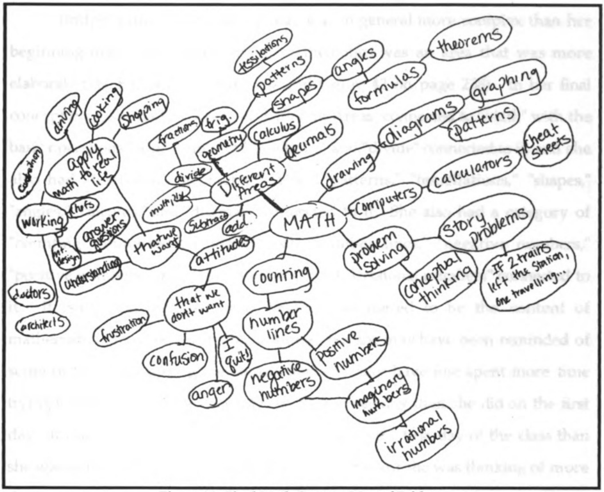

Figure 43: Final Math Concept Map of Bridget

Nature of Mathematics

Content. At the beginning of the semester, Bridget wrote about a few areas of the content of mathematics in her beginning "What Is Mathematics?" journal entry. She wrote that calculus, probability and statistics, and discrete math were part of math and that the basics were addition, subtraction, multiplication, and division. In her beginning math concept map, she also included the "basic concepts"--"add," "subtract," "multiply," and "divide." She also had "calculus" connected to a category she labeled "furthering knowledge" (See Figure 41).

Bridget's final math concept map was in general more complex than her beginning map, and the content of mathematics was an area that was more elaborate (See Figure 41 on page 237 and Figure 42 on page 238). In her final concept map, she had a concept of "Different Areas" connected to "Math" with the basic operations "add," "subtract," "multiply," and "divide" connected to it, and she also had "fractions," "geometry," "trig," "patterns," "tessellations," "shapes," "angles," "calculus," and "decimals" connected to it. She also had a category of "counting" connected to "math" with "number lines," "negative numbers," "positive numbers," imaginary numbers," and "irrational numbers" connected to it. It isn't clear whether what Bridget considered to be the content of mathematics increased during the semester, but she may have been reminded of some of the areas during the course, and she may have just spent more time trying to make a complete concept map on the final day than she did on the first day. It didn't seem that she was less engaged on the first day of the class than she was at the end of it; it just seemed that at that point she was thinking of more things related to the concept of "math" on the last day. After spending a semester talking about mathematics, it was probably easier to come up with a list of related concepts. In her final "What Is Mathematics?" journal entry, she didn't write anything about the content of mathematics.

Processes. Bridget didn't include much about her conceptions of the processes used in doing mathematics in her data sources. There was nothing about processes in her beginning or final "What Is Mathematics?" journal entries or in her beginning math concept map. However, she did include some things related to mathematical processes in her final math concept map. She included a first-level category of "problem solving" with "conceptual thinking" and "story problems" connected to it. She also included "drawing," "diagrams," and "graphing" which could be considered processes that are used in doing mathematics. Of course, we again can't assume that her ideas of mathematical processes increased during the semester, but it does appear that there was an increase of attention to them at the end of the semester.

Representations. The only things related to Bridget's conceptions of how mathematics is represented in all of the data sources were the subconcepts of "formulas" and "how to arrange things" written on her beginning concept map (See Figure 42 on page 237).

Characteristics. Bridget didn't include anything about her conceptions of the characteristics of mathematics in her beginning and final "What Is Mathematics?" journal entries or in her beginning and final math concept maps. However, we do learn some about her ideas of the characteristics of mathematics from three items on the Mathematics Inventory (See Table 14).

Table 14: Items Relating to *Characteristics of Mathematics* from the Mathematics Inventory with Bridget's Responses

Item	Beginning	Middle	Ending
4. I think of mathematics as a set of isolated facts that need to be memorized.	3	1	2
11. Mathematics is something that not everyone is able to understand.	4	4	3
17. Mathematics is something at which everyone can be successful.	2	3	2

For item four, "I think of mathematics as a set of isolated facts that need to be memorized," Bridget put a 3 on the first day of class. During the mid-semester interview she gave it a 1. The interviewer asked her if she thought she had changed at all in thinking about that. She said, "Ya, I guess so." The interviewer then asked her if she had any ideas why, and she responded,

> "Like I said before, I just I don't think I gave much thought to math before. It was just something that I did, and now I think I'm spending a lot more time thinking about how I feel about it and how I need to feel about it to be able to teach it to other kids. So, I think that that's maybe part of the reason why. I don't want them to think that it's facts that they have to learn."

At the end of the semester, she gave it a 2--right in between her beginning and mid-semester responses.

Bridget agreed quite strongly with item 11, "Mathematics is something that not everyone is able to understand." On the first day of class and during her mid-semester interview she put a 4 for that item. She explained what she was thinking during her mid-semester interview:

> "Well, you know, there's always one or two people who just don't get math no matter. . . . I have a cousin that since she's been in seventh grade, and since I've been in high school, I've been tutoring her in math, and like every week I would go to her house, and I would try to help her. When she was in high school and I was in college, it was like every couple of weeks that I saw her. I would help her and try to help her do algebra and geometry, and she just can't get it. It doesn't matter what I do, she just doesn't get it. And, uh, I don't think that--because other people when I explain things to them they understand it--and I don't think that it was that she wasn't understanding *me*, because she wasn't understanding her teachers either. She just doesn't get math. So I think that there are definitely some people that just don't get it, but then there's a lot of people that don't think that they get it but really could if they would just pay more attention."

The last day of class, Bridget gave that item a 3; so, she may not have agreed with that idea quite as strongly by the time the semester ended. But, item 17 was similar to that item in an inverse way, and her final response to it was more consistent with her first two responses to item 11. Item 17 states, "Mathematics is something at which everyone can be successful." She gave it a 2 on the first day of class, indicating that it was something she disagreed with--not completely, but quite strongly. During the mid-semester interview, she gave it a 3. Maybe she had been thinking about it more and wasn't sure what she thought. She explained why she gave it a 3 this way:

> "Well, again, like I said, I think that you have a 100 people. 50 of them can do math and you don't have to worry about them, and 30 of them need a lot of help doing math, and 15 of them think they can't do math, but really they can. They just need extra help, and maybe five of them just can't do it. So, I mean there's always

going to be people who can do it, but there's more people who think that they can't who just need more help to do it."

This seemed like quite a thoughtful answer even though it isn't *exactly* what we would like her to think. On the final day of class, she again gave it a 2. Even though items 11 and 17 are related, Bridget could have easily and reasonably thought of them a little differently. We wouldn't necessarily expect her answers to be consistent. Depending on what someone thinks of as "success," understanding mathematical concepts and being successful at math do not necessarily have to mean the same thing.

Usefulness of Mathematics

Throughout the study, Bridget saw mathematics as something that is useful. However, there seems to have been at least a slight increase during the semester in her views of where math is used and how it is related to the world.

General Usefulness of Mathematics. There were three items on the Mathematics Inventory that were related to the general usefulness of mathematics (See Table 15). Bridget's responses to these items at various times throughout the semester give us some insight into her ideas. Bridget agreed fairly strongly with item 3, "I think mathematics is an important subject that is useful in life," each time she took the inventory. On the first and last days of class, she gave it 4. During her mid-semester interview, she gave it a 5. During that interview, she said that she had been thinking about it more "because of what we've been studying in the TE class." She explained, "We've been doing a lot of talking about applications of math." When asked what kinds of applications they had been talking about, she said,

"Stuff like using math in recipes, and using math to figure out how far you're going, how many miles to the gallon you're getting in the car. Things I never really thought about before. I was always doing the math, but I didn't know. I was just thinking more about the fact that I'm using those things."

Table 15: Items Relating to *Usefulness of Mathematics* from the Mathematics Inventory with Bridget's Responses

Item	Beginning	Middle	Ending
3. I think mathematics is an important subject that is useful in life.	4	5	4
5. I think mathematics is something very much related to this world.	3	4	4
13. The only people who really need to understand math are those who choose a career which requires math skills.	1	1	2

For item 5, "I think mathematics is something very much related to this world," Bridget's response increased from a 3 the first day to a 4 during the mid-semester interview and on the final day of class. She explained that the reason she thought her response had changed a little bit was not because she didn't know those things before, but just because she had been thinking about it more. She said,

> "I've just been thinking more about it. So, I think probably I just. . . . It's not that I never knew these things before--like I never knew that math was what you used to figure out how much you took off your sweater if it was on sale or whatever. But I just never thought about it like explaining it to someone else probably."

Bridget disagreed with item 13, "The only people who really need to understand math are those who choose a career which requires math skills." On the first day of class and during the mid-semester interview, she gave it a 1, and on the last day of class, she gave it a 2. Regarding this item, she said, "Well, everybody needs to know how to do math, cause we all have to drive, we all have to buy stuff, etc."

Personal Utility. Bridget didn't write about how she used math personally in either of her "What Is Mathematics?" journal entries, but she did include some items related to personal utility on both of her math concept maps. On her beginning concept map, connected to a topic "understand how things work," she included "sale prices," "gpa," "distance traveled," and "amount of gas left" (See Figure 42 on page 237). On her final concept map, she included similar items, but with slight differences: "driving," "cooking," and "shopping" (See Figure 43 on page 238). These were connected to a category labeled "apply math to real life."

In her final interview, a semester after the course, Bridget talked a little more about how she and others could use math personally. She was asked if math existed differently or at all outside of a school classroom, and she said,

> "Well, ya, because like I was saying, I mean math is something that's like . . . . It's definitely separate from the school classroom, because you can do math outside of school, and you don't even realize that you're doing like math work. And, most people associate work with school, and you know, if you have to like figure percentages at your job, you know like if your job is you know working at The Limited or something, and you have to take 20% off all the shirts or something, then you would think that you're doing math like you would in school. But, it's like other stuff besides that like less obvious things like trying to figure out, you know, I don't know, how much money to put in the parking meter or something like that. But it's like math work that you don't even realize you're doing it."

Even though the data that was available related to Bridget's ideas of math's usefulness for her personally was quite limited, it appears that she recognized its usefulness both at the beginning of the semester and at the end of the study.

Indirect Personal Utility. Bridget included one item on her beginning math concept map that was related to indirect personal utility. That was "traffic lights." That was probably because on the first day of class someone said something about math even being used to build and operate traffic lights. She didn't include anything on her final concept map related to indirect personal utility. Bridget also didn't write anything about math's indirect utility for her in her beginning "What Is Mathematics?" journal entry, but in her final one, she seemed to be thinking of it more. She wrote, "I used to think that math was something that we did for an hour a day in school that could be forgotten about as soon as the bell rang. Now, my views have changed. Instead of being something that can be easily forgotten, math is something that is involved in every aspect of my life."

She seemed to still be thinking of this a semester after the course when she had her final interview. An example that illustrates a little of how Bridget understood the relation of mathematics to many things was a comment she made during her final interview when she was asked what it means to do math:

"Well, you're always doing math. . . .'Math is everywhere.' But really it is if you think about it like . . . . like I was sitting here and I was looking at these tables, and there's four tables put together, and they make a big rectangle. But, then like, I was thinking about how this rectangle of this table that we're sitting at is exactly proportionate to like this big rectangle that all four of them make up, and well I wasn't really thinking about doing math, but I guess that's doing math. And then, like, whatever, so you can't really tell if someone is doing math I think, because. . . unless they're writing it down. But, a lot of times you're doing math and not writing it down."

Also in her final interview, Bridget was asked if she considered mathematics as something valuable, and she responded, "Yes, it's valuable. It's valuable everywhere. Yes. I mean, generally speaking, math is something that's incorporated into like all the different parts of your life. So, it's pretty much valuable for everybody."

Utility for Others. Bridget also recognized that math was used by others, especially in specific professions. In her beginning "What Is Mathematics?" journal entry, she wrote, "[T]hese things lay the foundation for understanding in chemistry, physics, and engineering. Without a good working knowledge of math, we wouldn't be able to accomplish much in these other related fields." She also included "engineering" on her beginning concept maps connected to "calculus" (See Figure 42 on page 237). In her final concept map, she included even more professions that use math--"doctors," architects," "interior design," "chefs," and "construction" (See Figure 42 on page 238). These were connected to "working" which was connected to "apply math to real life." These were the only places she specifically noted how mathematics is used by others in a way that she might not use it herself.

Learning and Teaching Mathematics

Bridget's ideas about learning and teaching mathematics remained quite constant throughout the semester with slight changes. Some of what we learn about Bridget's ideas related to learning and teaching mathematics comes from her responses to some items from the Mathematics Inventory which can be seen in Table 16. These responses will be discussed in the following sections.

Table 16: Items Relating to *Learning and Teaching Mathematics* from the Mathematics Inventory with Bridget's Responses

Item	Beginning	Middle	Ending
10. Teaching mathematics should not be too difficult since a textbook already has the lessons prepared.	4	2	2
22. The best way to learn math is to memorize the rules.	2	2	1
23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.	4	3	3
24. In learning math, it is important to master topics and skills at one level before going on.	5	5	4

Learning Mathematics. Bridget had some definite ideas about how math is learned. She didn't think the best way to learn it was to memorize the rules, as can be seen with her responses to item 22, "The best way to learn math is to memorize the rules." She explained what she was thinking about that during the mid-semester interview:

> "The reason that I didn't put 1 is that I think there is some memorization involved in math. It's not the most important or best way to learn math, but you can't do calculus or you can't geometry without learning the theorems, and you can't do trig without learning your properties or whatever. All that stuff. They're like in little boxes in my geometry book. Anyway, so I think there are things that you do have to memorize, but it's not the best way to learn math. It's just how you do math."

For item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key," Bridget put a 4 the first day of class, indicating fairly strong agreement with the statement, but she put a 3--right in the middle--during her mid-semester interview and on the last day of class. During the mid-semester interview, she said she thought she had probably put a 3 the first day also, and when she was told she put a 4, she was asked if she had been thinking about that more, and she responded,

> "Um, ya. Um, I think it's important that kids know the process that they're going through to get the answer, but uh, and if they get that right and they get the answer wrong, it's definitely better than if they have the whole thing wrong, but I believe more and more that it's still really important to get the answer, because it's nice how it all sounds on paper, but in the real world, if you get the wrong answer, you get it wrong. I mean reasoning is important, but it definitely has an end you need to find."

Bridget strongly agreed with item 24, "In learning math, it is important to master topics and skills at one level before going on." She gave it a 5 on the first day of class. She also gave it a 5 during her mid-semester interview and explained why she thought that:

> "Well, because you can't do algebra if you don't know how to multiply, and you can't do trig if you don't know how to do algebra, and you can't do calculus if you don't know how to do trig, and you can't do probability if you can't add, and like, you know, if you go from here to there without knowing here, then you're lost when you get there."

This was consistent with what she wrote in her beginning "What Is Mathematics?" journal entry:

> "Understanding basic math concepts is the framework for understanding and succeeding in upper-level math courses. When we reach the upper levels of math, we begin working, instead of with 'meaningless' numbers, with concepts that will actually relate to our lives."

It's rather odd that she considered the concepts from the "upper-level math courses" as those that actually relate to our lives since the examples she gave of how she personally used mathematics were things like shopping which use low-level math. However, she further elaborated on her ideas related to math being a "stepping stone" in that same journal entry:

> "The key then, to success, is to be confident in the basic skills in math. Understanding addition, subtraction, multiplication, and division, things that seem like second nature to us, is essential to a child. Besides preparing them for later successes, strong math skills can lead to an improvement in self-esteem, as much of a child's "work" in a school is directly related to their achievements, particularly in math. So, math is not only a stepping stone to other areas, it is a destination for students. Working towards an understanding in math is something students strive for and something that teachers expect to see."

When Bridget responded to item 24 at the end of the semester, she gave it a 4. She apparently still strongly agreed with it, but her response left a little room for some other possibility.

In her final interview, Bridget was asked what she thought was the best way to learn math, and she said,

> "I guess, to start off with like when you're little you just have to do it over and over and over again because you've got to be exposed to all the ways of doing things and all the different like reasons for doing things. When I say like do it over and over, I don't mean like do a worksheet with a 100 multiplication problems on it, I mean like consistently working on different math things like story problems and that kind of stuff. Like my elementary school, they don't do math every day. I think that they just need to like do it all the time, and they need to recognize that are constantly doing it for other things, too, like dividing up groups of their friends to play baseball or whatever. Like, I think if they started to realize then that would be the way to start off to learn math. But, then I think that as you get older and can start to conceptually think, I think that learning math is like something you can only do if you're like. . . I don't know, like I have another example. I remember my Algebra II teacher was trying to teach us the concept of exponential expansion, and like we would just sit there in class and look at him like, "Huh?" No one understood what he was doing. And, then he did something, and I don't remember exactly what he did, but he was standing, and he was like standing across the room from something else, and he was saying like okay now, he said this however far away, and then he went over and he's like, "Now I'm this far away." And, then he's like now I'm half of this and this far

> away again. He was like, "Now I'm half of this. . ." and he's like measuring it out with a piece of tape so we could actually visually see him like exponentially getting farther away from this fixed object. And, like that's the kind of thing like when you try to learn harder math things, you have to be able to like visualize them. I think that's the problem I had with geometry. I can't like visualize those things, but I think that in order to learn things, you're going to have to see how they actually work. It makes sense, cause just paper doesn't make any sense.

From this response, we learn that a semester after the course was finished, Bridget's conceptions of how math is learned included the ideas that students needed to spend a lot of time doing math to learn it, that being able to visualize the concept—perhaps even with a physical representation—helps with learning, and that "in order to learn things, you're going to have to see how they actually work."

Teaching Mathematics. Bridget's ideas about teaching mathematics changed somewhat during the course of the semester. One area where Bridget's ideas about teaching mathematics seemed to change during the semester was the idea related to item 10 from the Mathematics Inventory, "Teaching mathematics should not be too difficult since a textbook already has the lessons prepared." On the first day of class she put a 4, but the next two times she took the inventory, she gave it a 2. During the mid-semester interview, the interviewer asked her if she had any ideas of "what happened there." She replied,

> "Again, this class has shown that you can't just open up a book and give a worksheet to someone and say, 'okay do this.' It's more like you have to understand it well enough to teach someone else what it is that they're doing before they can actually do it. So, it's not just handing out the ditto and opening the textbook."

The interviewer then asked her if there was anything in particular in the class that had made her think that way. Her response was,

> "Ya, everything about the class is. We're really spending a lot of time talking about how we can't just hand out a ditto and expect

> that they're going to get the concept—that we've got to actually explain and go through the whole process. I think that's something we've talked about—about how it's so important to do that."

Bridget recognized that a teacher's attitude can influence the success of the students in mathematics at the beginning of the semester and at the end of the semester. One of the open-ended questions the students were asked in the Mathematics Inventory was, "What kind of influence do you think the teacher plays in the success of students in mathematics?" On the first day of class, Bridget wrote, "The teacher (if she has a positive attitude) can give the class a good attitude by making sure that every student understands and feels confident with the information being taught." On the last day of class, she wrote something a little different, but not contradictory. She wrote, "If a teacher likes math, her students will like it more than if the teacher does not like math."

However, it seems like this is one place where Bridget's ideas about teaching mathematics changed at least somewhat during the semester. By the end of the semester, she seemed to have a greater recognition of the effect teachers' attitudes toward mathematics can have on their teaching it and their students learning it. Bridget's responses to another open-ended question from the Mathematics Inventory show how her conceptions may have changed to some extent. This question was, "Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not?" On the first day of class, Bridget wrote,

> "No--I don't like math, but I recognize its importance. Therefore, I will still teach it knowing that my students need to learn. A teacher's attitude about teaching math is important because that's her job--she must have a good attitude about it."

In response to this question on the last day of class, Bridget wrote, "Yes—if the teacher doesn't like math, the kids will know. They are very perceptive when it comes to things like that."

Bridget's "Philosophy of Teaching Mathematics" journal entry written at the end of the semester provides insight into her conceptions of teaching mathematics at that point:

> "I think the most important thing about teaching math at the elementary level is enthusiasm. In order for students to become interested in mathematics, I think that the teacher must also be excited, otherwise, students will pick up on her lack of enthusiasm and won't be interested themselves. Because I feel that it is extremely important for the teacher of math to be interested in math, I'm not quite sure that I am suited for the job. I like math only inasmuch as it can get me an answer, but I often find myself frustrated and bored with the subject.
>
> "I'm sure that a good teacher would do things like bring in concepts from real life to help to explain math. As well, she would allow students to explore different areas and methods for solving problems, so that they feel that they are making their own positive contributions to their learning. Students need to learn that they can help themselves to learn math. Every area of math has at least two angles from which it can be regarded, and the students need to learn to find these methods. The teacher should be there to explain and to guide explorations. If the teacher is actively involved in helping students to understand the concepts, then she must understand that she has to do more than just teach what is in the book. Materials from nature, business, and entertainment should all be used to help with comprehension. These things are useful not only to help students understand, they will help to increase enthusiasm.
>
> "There are many other things that are important for students' learning. They should be given the opportunity to reflect on their learning and to comment on the teaching. They should be actively involved in learning, using manipulatives and creating their own. Their work should be displayed for others, not only to increase pride, but also to stimulate new ideas in others. Wrong answers should be corrected through questioning, not lecturing. Students should be in an environment where they can work individually, in pairs, or in small groups.
>
> "I don't necessarily agree with calculators in the classroom, as they symbolize abandoning the basic concepts of learning. Just as I think students need to learn spelling, I think they need to master their multiplication tables. However, I do not necessarily think that these activities need to be done for grades. Rather, I think that these skills should be practiced so that their applications can be assessed. It is necessary to evaluate students on their ability to work through problems, not to penalize them for making a "stupid mistake."
>
> "Teaching math isn't just about working with numbers and formulas. It's about concepts and problem solving, about

> understanding and reasoning. I think that math is a truly valuable subject to teach and to learn, its rewards are innumerable. However, I don't feel confident in my ability or my interest in teaching mathematics.

It's interesting that at that point Bridget seemed to agree with many of the conceptions of how mathematics should be taught that were common in the class as a whole at the end of the semester. For example, she felt like it was important for the teacher to serve as a guide or facilitator in helping the students learn mathematics through questioning them and by having them explain their reasoning. She also believed that math should be connected to the world and that it should be taught using discussions and manipulatives and other materials—more than the textbook. Also, like others in the class, she believed that there was value in having the students memorize things like the multiplication table. Her sentence, "I don't necessarily agree with calculators in the classroom, as they symbolize abandoning the basic concepts of learning," gives some indirect evidence about what learning math means to her. She seems to think that the students need to learn to do all the operations that can be done with a calculator and that if they have calculators providing them the answers, they might not learn how to do them. She also felt that "math is a truly valuable subject to teach and learn." However, overriding all of her views about teaching mathematics was the idea that a teacher needed to be enthusiastic to teach math effectively, and that, as will be explored more fully in the following sections, made her believe she shouldn't teach math.

Attitudes toward Mathematics and Teaching Mathematics

Attitudes toward Mathematics

At the beginning of the semester, Bridget's attitude toward mathematics was quite negative even though she felt fairly confident doing it. She recognized its usefulness and importance, but she just didn't enjoy doing it. Her attitudes toward math did not appear to change during the semester. If anything, her attitude that she didn't enjoy math seemed to be solidified and strengthened. This was especially evidenced in her journal writing and interviews.

As noted, Bridget felt fairly confident in her ability to do mathematics. One of the places where a degree of Bridget's confidence related to mathematics could be seen was in a journal entry written three weeks into the semester. The students were asked to write a journal entry in which they looked at the five goals that the National Council of Teachers of Mathematics had identified for students in its 1989 *Curriculum and Evaluation Standards for School Mathematics*. These goals were (1) learning to value mathematics; (2) becoming confident in one's own ability; (3) becoming a mathematical problem solver; (4) learning to communicate mathematically; and (5) learning to reason mathematically. They were asked to think and write about whether they possessed those characteristics personally and if they thought it was important for teachers to possess them in order to help their students attain them. Following is Bridget's journal entry in response to that prompt:

> "In order to value mathematics, it is important that positive attitudes about math are developed at an early age. Instilling children with the belief that successes in later life are dependent on their ability to do math and do it well now, will increase their values as far as math goes. My math teachers, from as long ago as I can remember, have stressed the importance of math. Showing me not only how to do math, but how I will be able to apply it to my life. Directly related to valuing math, is the idea of being confident in math ability. I think that because the importance of math was always greatly stressed, I knew I had to do well. I

> usually have to put extra work in to be successful in math, so my confidence is a result of my determination. Confidence doesn't always come from an "A", although that helps, it comes from knowing how to approach and solve problems. Being a mathematical problem solver is not something that necessarily can be taught. The methods and means of problem solving can be explained to students, but that doesn't necessarily mean they will become problem solvers! I feel good about my problem solving, mostly, because it was an integral part of our learning since early elementary. Mathematical problem solving was combined with Future (social) Problem Solving in order to round out our understanding. Being able to communicate mathematically is something that I do not feel I have mastered. I can do math and understand math, but I'm not sure that I can talk about math effectively. I think that in order to teach math, it is essential that I be able to communicate well in that area. Therefore, it is a skill that I need to improve upon. However, since I am going to be teaching middle school to begin with, I will have a chance to improve my math communication skills before I am placed in that situation. In order to reason mathematically, one needs to be an efficient problem solver. I'm confident that my reasoning abilities are sufficient, although they could stand improvement. In order to teach or learn math, reasoning is key. It helps comprehension, explanation, and confidence in mathematics. I feel that I possess a lot of these skills, but there are areas where I still need a lot of work—the never-ending struggle to succeed at math."

In this journal entry, Bridget described herself as confident in her ability to do math. She thought this was a result of the value of math being stressed in her school experience and the fact that she put in extra work to do it since she realized it was important. She also described herself as confident in her ability to problem solve and attributed that to problem solving being an integral part of her learning since elementary school. She described her reasoning abilities as sufficient but with room for improvement. She also felt that teachers needed to be able to communicate mathematically to teach well, and she felt she needed to improve in that area. However, since she was not planning on teaching elementary school—at least at the beginning of her career—she seemed to think she had more time to acquire that skill. It seemed as though Bridget was trying to think hard about the questions since she was thinking about what experiences in her schooling had contributed to her attitudes and abilities. It also seemed that she was trying to be honest in her assessment of herself.

Another data source that gives insight into Bridget's attitudes toward math is the Mathematics Inventory. It contained several items related to attitudes toward mathematics, and Bridget's responses to these items at different times during the semester provide information about her attitudes toward mathematics and how or whether they were changing (See Table 17).

Table 17: Items Relating to Attitudes toward *Mathematics* from the Mathematics Inventory with Bridget's Responses

Item	Beginning	Middle	Ending
1. I enjoy doing mathematical tasks.	2	3	3
2. I have always enjoyed mathematics.	2	2	3
6. I feel successful when I am engaged in mathematical tasks.	4	4	4
7. Doing mathematics is usually very frustrating for me.	2	2	2
15. I understand elementary level mathematical concepts quite well myself.	4	5	4
25. I can handle basic math, but I wouldn't do well at advanced mathematics.	2	2	4

Bridget's responses to a few of the items from the Mathematics Inventory also support the conclusion that her negative attitude toward mathematics was not because she lacked confidence in her abilities to do mathematics, especially basic math. For example, for item six, "I feel successful when I am engaged in mathematical tasks," Bridget put a 4 each time she responded to the inventory. During her mid-semester interview, Bridget explained that being successful to her meant "that I can get the right answer." For item seven, "Doing mathematics is usually very frustrating for me," which is essentially an opposite statement of item six, Bridget put a 2 each time she responded to the inventory. That would be consistent with her response of 4 on item six. She qualified what she meant responding with a 2 to item seven during her mid-semester interview:

> "I can do math, but it takes so much time and so much effort. Too much memorization. Are we talking more a level like trig or calculus or like story problems and addition and subtraction,

multiplication. Because I think my answers would be a lot different. Because if it says 'doing mathematics is very frustrating for me' as far as multiplying, dividing, adding, and subtracting, I would say, 'No way.' because I can do any of those sorts of things."

The interviewer then asked her what would be frustrating, and she said, "Calculus and anything that has to do with calculus. Trigonometry." She was then asked why they were frustrating, and she explained,

> "I don't know. I guess I just have it in my mind that they're too complicated for me to learn. And, I'm sure probably if I actually sat down with them. . . . Like we spent like ten weeks in high school doing calculus in my senior year, and I couldn't get it. I just couldn't get it, and I don't know if I just had a mindset that said I wasn't going to be able to do it or what, but in any case I don't like it any more.

Bridget's responses to item 15, "I understand elementary level mathematical concepts quite well myself"--4, then 5, then 4--are consistent with her attitude that she can do basic math, but not necessarily higher math. For item 25, "I can handle basic math, but I wouldn't do well at advanced mathematics," Bridget put a 2 on the first day of class and during her mid-semester interview, but she put a 4 on the last day of class. Maybe she was agreeing with the part, "I wouldn't do well at advanced mathematics." During the mid-semester interview, the interviewer asked her what she would say is advanced math, and she said, "Calculus. Anything beyond calculus." The interviewer asked her why she saw that as the starting point, and Bridget said,

> "Because that was like the math I couldn't learn. That was where I got lost. I did fine all the way until I got up to calculus, and then I got to calculus, and I was lost. And, I don't really know what goes beyond calculus, but I'm sure there's other stuff."

Interestingly, the interviewer then noted to Bridget that "people seem to be defining where advanced math starts as where they're having trouble" and that other people had said that algebra was the beginning of advanced math.

Bridget didn't include anything related to attitudes in her beginning math concept map, but in her final concept map, she included an entire category of "attitudes" connected to the main concept of "math" (See Figure 43 on page 238). She subcategorized "attitudes" into "that we want" and "that we don't want." Connected to "that we don't want," she has "frustration," "confusion," "anger," and "I quit!". Connected to "that we want," she has "understanding," "answer questions," and "apply math to real life." She branched "apply math to real life:" further, and those connections are noted in the usefulness of mathematics section. She doesn't indicate specifically in the concept map which attitudes are hers, and it's possible that she has experienced each of those attitudes. Instead, this appears to be an example of a place where she has positioned herself as a teacher since the listing of attitudes "that we want" and "that we don't want" seems to be all about what she would want as an educator from her students. But of course, she has indicated her attitudes toward math in several other data sources.

A statement Bridget wrote in her "Philosophy of Teaching Mathematics" journal entry on the last day of class typified her attitude toward math throughout the study, but especially at the end: "I like math only inasmuch as it can get me an answer, but I often find myself frustrated and bored with the subject."

Attitudes toward Teaching Mathematics

Bridget wasn't particularly excited about the prospect of teaching mathematics to elementary school children at the beginning of the semester, and she was even less excited about it at the end of the semester. This was illustrated with the information obtained from several data sources. The main reason for this--as has been alluded to in previous sections—is because Bridget became convinced that she didn't enjoy mathematics enough or have enough enthusiasm for it to be a good teacher of it. She really bought into the idea that teachers' attitudes make a big difference in how well they can teach math, and she felt like her attitude was not positive enough.

One of the places this can be seen is in her responses to the items related to attitudes toward teaching mathematics from the Mathematics Inventory (See Table 18).

Table 18: Items Relating to Attitudes toward *Teaching Mathematics* from the Mathematics Inventory with Bridget's Responses

Item	Beginning	Middle	Ending
8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.	3	1	1
9. I feel enthusiastic about mathematics and about teaching it.	3	1	1
12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.	5	4	4
14. I feel confident in my ability to explain mathematical concepts to children.	4	2	3
16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.	1	5	4
18. Being good at mathematics is not required to be a good teacher of math.	2	2	3
19. I want to teach mathematics so that I can help students get excited about it.	2	1	3
20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.	2	2	2
21. I expect to regularly teach mathematics as an elementary teacher.	2	1	1

For item 8, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher," Bridget put a 3 the first day, but she put a 1--not at all like me--the second and third times she took the inventory. During the mid-semester interview, when she told the interviewer she was giving it a 1, she thought she thought she probably gave it a 4 or a 3 on the first day. The interviewer told her she put 3, and asked her what happened. She replied,

> "Um, I think that math—you have to like math to teach it, and like Elaine is really excited about math, and so it's really easy for her to teach us, and I'm sure in a classroom it would be easy for her to teach it. But, I don't like it at all, and I'm realizing as I go through this class that I have no interest in it really. It would be difficult for me to get other people excited about it. It just doesn't excite me. Same with science. I just—it's like *(apparently some facial expression)*. And, I've actually gone from thinking I wanted to teach elementary school to thinking I want to teach in a middle school just so I don't have to teach math and science."

The interviewer then asked her what subjects get her excited, and she responded, "English and social studies." The interviewer then asked her what was different about them. She explained,

> "They're more conceptual. You get to come up with your own opinions about them. Like when you write a paper for English, people don't tell you you're wrong. They might tell you that you put a comma in the wrong place or you needed to cut a sentence in half. But no one ever says that you went about it the wrong way or your idea is wrong. And, I like that there's a lot of freedom, and the same with history, I guess. Everyone's free to come up with their own opinions."

Bridget's analysis of how math differs from literature and social studies is really fascinating. We often hear people say that a reason they hate math is because it has one right answer, but it's uncommon for people to say so explicitly that what they like about literature and social studies is that their answer—no matter how illogical—is <u>never</u> flat out wrong.

Bridget's responses to item 9, "I feel enthusiastic about mathematics and about teaching it," were the same as they were for item 8. She said she felt like they were "pretty much the same question." Item 16, "I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students," is related in an inverse way to items 8 and 9. Her responses to that item changed dramatically from the first day of class to the mid-semester interview. She put a 1—"not at all like me"—on the first day of class, and a 5—"very much like me"—during the mid-semester interview. On the last day of class, she put a 4—still close to "very much like me." During the mid-semester interview, she explained why her opinion had changed so dramatically:

> "Again, I've been. . . My eyes have been opened. I don't think that I have the desire to teach math in an elementary school and maybe the ability. I don't know. I don't work well with numbers really so I don't . . . I would feel guilty going into a classroom and saying, 'Okay, I'm going to teach you math' when I hate this. I mean I don't really hate it that much believe it or not."

Saying her "eyes have been opened" seems to be really strong language that indicates that from Bridget's perspective there had been a change from the beginning of the semester in her desire or perceived qualifications to teach elementary school. She really seemed to believe that since she didn't like math, she shouldn't be teaching it.

Item 19, "I want to teach mathematics so that I can help students get excited about it," was related to items 8, 9, and 16, and she gave it a 2, then a 1, and then a 3. In the mid-semester interview, she said, "I mean if I was going to teach math, I think that I should get them excited about it." But, again, by that time she was already quite certain she wouldn't be teaching math. When she put a 3 on the final day of class, she may have been responding to it as if she were going to teach math.

Bridget's responses to item 21, "I expect to regularly teach mathematics as an elementary teacher,"—a 2, then a 1, and then another 1—show that she probably suspected she didn't want to or wasn't going to teach math even when the course began, but during the course, and by the end of the course, she had firmly decided she wasn't going to teach mathematics. At the beginning of the semester, it seemed her plan was to teach English in a middle school, but she said she was interested in learning about teaching other subjects and that is why she had enrolled in the elementary education program. She may have wanted to avoid a major in English or she may have simply wanted to find out if she would like teaching elementary school and give herself more options for employment after college.

Another illustration of Bridget's decision to not teach mathematics because her lack of enthusiasm for it is contained in her final "What Is Mathematics?" journal entry which was written the final day of class:

> "Throughout the course of this class, I have discovered that I do not have a lot of enthusiasm for math. I used to think that math was fun because I was always fairly successful at it. Now, I don't think that I really have enough enthusiasm or interest in it to be an effective teacher. I really think that in order for a teacher to be a good math instructor, she has to be very excited and confident in her mathematics abilities. If she is not, how can she expect her students to be? I feel that I will be doing my students an injustice by teaching something that I do not have a lot of interest in.
>
> "When I began this class, I was interested mostly in teaching English, but was also interested in learning about teaching other areas. Now however, I think that I really just want to teach English in the middle school. It is not because I am afraid of math that I have made this decision, but I do not feel that I am enthusiastic or capable enough to get a classroom full of intimidated students interested in mathematics."

During her final interview a semester after the course, Bridget still had the same attitude toward teaching mathematics. One of the questions was, "How do you feel about the idea of teaching mathematics in an elementary school?" Bridget responded,

> "I don't know how I feel about it. I don't really like the idea too much, and I think I said this maybe in a different interview, but I think that if you're going to be a good math teacher or a good teacher of anything, you have to be like truly interested and excited about whatever it is you're teaching. And, uh, I mean the best teachers are always the ones. . . . Like I had this government teacher that just LOVED government. And, so, like his excitement transferred to us, and we couldn't help but like the class. And, I don't think that I'm very excited about math. It's like not really something that I enjoy doing. I enjoy being able to prove to people that I know how to do it, but I don't really . . . . . Like if given the opportunity to do math or read a book, I wouldn't choose to do math. So, I don't think that I would be a good math teacher. I mean I think that I would be a good math teacher, but I don't think that it would be fair of me to be a math teacher, because it's not something that I can just like really get people excited over, and if it was like, 'Well, we can do math or we can you

> know, go have recess.' I would probably let everyone go have recess. Just cause I don't like it."

The reason Bridget didn't want to *teach* mathematics was not as much because she lacked confidence in her abilities to do and understand math as it was her lack of enthusiasm for it as was the case in her attitude toward mathematics itself. This is illustrated in her responses it item 12 of the Mathematics Inventory, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school." On the first day of class she put a 5, then she put a 4 the next two times she took the inventory (See Table 18). During the mid-semester interview she was asked if she thought she was changing at all on that, and she explained.

> "Possibly a little bit just because I'm a little worried about the whole idea of teaching math. I mean I've looked through the textbooks in my elementary school now, and I understand all the math that's in it, but there's always a chance. Well, there's probably a probability that I would have learned something in a different way than what I should have in what's accepted now to teach it in the way it's concepted now. I mean I can still do my fractions and stuff."

A few of the open-ended questions from the Mathematics Inventory also give us some insight into Bridget's attitude toward teaching mathematics--especially as it relates to her confidence. One of the questions was, "What do you consider to be your greatest strengths in relation to being a mathematics teacher?" On the first day of class, Bridget wrote,

> "I understand that learning and being successful in mathematics requires a lot of time and energy (for most people). Therefore, I wouldn't 'write off' a student who was struggling. I know what it feels like and can empathize with them."

On the last day of class, in response to that question, Bridget wrote, "My understanding of elementary math topics."

Another question was "What concerns you the most about teaching mathematics to elementary students?" On the first day of class, Bridget wrote, "That I will not be able to get the message across. I worry that students will not understand what I'm teaching, and I will have difficulty finding an alternate method of explanation." On the last day of class, she wrote, "My lack of enthusiasm." By the end of the semester, Bridget's lack of enthusiasm seemed to overshadow all other concerns she had related to the idea of teaching mathematics.

One of the open-ended questions on the Mathematics Inventory was "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." On the first day of class, Bridget wrote, "My knowledge will be a strength. I have gotten good grades all the way through calculus and statistics, so I'm confident that I have a good working knowledge that I can use to help my students." She didn't respond to that question on the last day of class. I can't know for sure why she skipped that question or if there's any significance to her not answering that question, but my guess is that she just overlooked it. When we put this omission alongside the fact that she no longer answered item 12—"I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school"—with a 5, we might hypothesize that she was no longer as confident that she understood al the math in the elementary curriculum.

By the end of the semester, Bridget seemed to realize there was more to being a good math teacher than having mathematical knowledge, and it was an area where she didn't feel as confident. In the Mathematics Inventory the students were given on the last day of class, there was a question asking them if their ideas about mathematics or teaching mathematics had changed at all during the time they had been in the course. Bridget wrote, "Yes—I don't want to teach math anymore. The more I thought about the role that attitude plays in being an effective teacher, the more I realized it's not for me."

Bridget still had this attitude toward teaching mathematics a semester after the course when she was interviewed for a final time. As noted previously, during that interview when asked how she felt about teaching mathematics, she said that a good math teacher needed to be excited about it, and she wasn't. She was also asked if her feelings about teaching mathematics had ever changed, and she responded,

> "I don't know. I think that I've always felt that. I think more after this year, after our 401 class, I started to realize that I didn't have enough excitement for math, and that me being involved in teaching math would not be a fair, like I wouldn't be giving my students a good opportunity just because, I don't know. Some people get so like keyed up over it, and I just don't. . . I would be concerned that I wasn't providing my students with an appropriate role model for learning math because I don't really care so much. So, I would probably feel like some of that was going to rub off on them.

By the time Bridget was interviewed a final time, she didn't even seem that committed to the idea of teaching English in a middle school. After the final interview, Bridget told me that she wasn't planning on teaching at all— anything —in the foreseeable future. She said that she was planning on moving to London and working for a magazine.

Major Themes in the Case of Bridget

The major themes in the study of the case of Bridget seemed to be a shift in her positioning related to math with an unusual result, her surprising dislike for math, and her realization or acceptance of the idea that in order to be a good teacher of mathematics, one must have a good attitude toward mathematics. This seemed to be a discovery for Bridget that led her to the conclusion that she should not teach mathematics.

A Shift in Positioning with a Different Result. It seems that much of what was going on with Bridget was what I observed with many of the students, and that was a shift in her positioning from thinking as an experienced student to thinking as a prospective teacher. This was evident in her final concept map where she included categories for attitudes "that we want" and "that we don't want." It was also evident in her "Philosophy of Teaching Mathematics" journal entry where she included many new ideas about teaching mathematics. For most of the students in the class, the shift from thinking about math as an experienced student to thinking of it in relation to being a prospective teacher seemed to help them feel more positive about math and teaching math, as well as help them see math as Krows (1999) described it as a "dynamic, ever changing, problem driven branch of science." However, for Bridget, it seems as though as she got closer and closer to the moment of teaching, she got clearer and clearer that she did not want to do this.

A Surprising Dislike of Math. In many ways, it seems surprising that Bridget would dislike math. She had experienced quite a bit of success learning it and using it; she loved her conceptual math class; and she was even able to help her cousin with math. Also, the fact that she spontaneously notes in her final interview that the table and the four tables are proportional suggests that she has a mind that is actually quite well suited to doing and enjoying math—to a kind of intellectual doodling with math. She used math as a recreation! And yet, she continues to hate the subject. This seems a bit odd, but for some reason, she seems to have *chosen* to not like math.

A Discovery Leading to a Success of Sorts. By the end of the course, Bridget strongly believed that for teachers to teach math well, they need to have an enthusiasm for the subject. This conviction was magnified greatly during the course of the study. In fact, shortly into the study, and especially by the end of the study, Bridget decided that even though she felt fairly confident in her ability to do math (especially at the elementary level), she felt like her dislike of it and lack of enthusiasm for it disqualified her from being a good teacher of it. This conclusion was evidenced in her journal entries, her responses to items on the Mathematics Inventory, and her interviews as noted throughout the preceding analysis.

It's hard to know for sure all the factors that contributed to Bridget's ideas about the need for enthusiasm. I would guess that my influence was at least part of it. I believe that teachers who have a good attitude toward math and are enthusiastic about it are better able to teach it, and it is an idea I brought up in class a few times and had them think about. I also forced them to think about that idea with some of the open-ended questions in the Mathematics Inventory. Other students seemed to also believe this, and it was something that many talked about in our class discussions. Many of the students were becoming visibly more enthusiastic about teaching mathematics as the course progressed, and perhaps Bridget felt isolated in her lack of enthusiasm.

It's also hard to know if Bridget's lack of enthusiasm for mathematics is the only reason she decided not to teach math. It may be a polite way to bow out. She may have realized that teaching is not what she wants to do, and this gives her a reason—that isn't her fault—for disqualifying herself from teaching. The fact that she was majoring in English and was planning on teaching middle school but "wanted to learn about teaching other subjects" when the class began, and then the fact that at the end of the study she had decided she didn't even want to teach English make me wonder if there were more reasons than her dislike of math for deciding not to teach.

Regardless of what reasons Bridget had for not wanting to teach or what caused her to believe that she needed to be enthusiastic about math in order to teach it well, if we take her declaration that she isn't going to teach because a teacher has a moral duty to be enthusiastic about what she teaches, her decision that she shouldn't teach mathematics given her attitude toward it can be considered a success of sorts.

Felicia: Slight Cracks, but a Strong Commitment to Already-Entrenched Negative Conceptions and Attitudes

Felicia was a 22-year-old second-year senior at Michigan State University who was majoring in "child development and teaching." She was a very attractive and petite young woman with long brown hair who had grown up in a small town in Michigan. The only classes she needed in order to graduate were TE 401 and TE 402 which were courses teacher education majors took in their senior year before they began their internship and focused on teaching math, language arts, science, and social studies to diverse learners. She felt like if her advisor had "organized her better" she would have graduated the spring before, and she wasn't very happy to still be taking classes.

Felicia had not had positive experiences with mathematics in her life, and she had developed a strong dislike for it. She described her experiences with mathematics during the mid-semester interview this way:

> "I don't remember ever liking it, but I don't remember having such a hard time with it until I got to high school. I mean, I always did well in elementary school with math. It wasn't fun. I didn't enjoy it, but I did well. I did okay. And, then junior high I actually did really, really well. In 7th grade was the only time I got straight A's because I got an A in my math class. And, then algebra started out really hard for me, and I remember failing my first test and going home in tears, and my dad who was the only person in the world who could explain math to me the way I understand it, started helping me understand it. So, everyday I'd sit in class, knowing that I had no clue what was going on, but that when I got home he'd explain it, and I would come back and do fine. So, I pulled a B out of that, and that actually felt fun to me when I figured out what x equaled. I felt really happy. I mean I wouldn't have chosen to do it. I'm not a motivated enough person to be like that, but it was, I

guess as far as math goes, a fairly positive experience. Then geometry just killed me--the logic, just everything about it. I just didn't understand what the purpose of it was. I just didn't understand anything about it. It was the hardest thing I've ever done."

The interviewer then asked her if she actually failed that class, and her response provides even more insight into the source of her extreme dislike for mathematics:

"No, I think I got a C. I mean, with my dad's help I always did okay. It was just a big struggle. I always spent hours at home working on it and struggling and struggling and you know, taking breaks to stomp upstairs and cry a while and then come back downstairs. It hurts my brain to do math--physically."

This account was very similar to what she wrote in her "Reflections on Mathematics Experiences" journal entry which was assigned the first day of class.

Felicia's experiences with mathematics once she got to college were also mostly negative and did nothing to improve her opinion of math. She described them this way in her "Reflections of Mathematics Experiences" journal entry:

"Upon entering MSU, I scored so poorly in my math placement tests, that I had to take a one or no credit (I don't remember) basic arithmetic class. This was somewhat humiliating, since I don't have a problem with basic math. I 4.0ed that class with ease. Then, before I could take 201 and 202, I had to take 103, college algebra. That was another struggle. This time my fiancé tutored me and I ended up with the same tears and headaches I'd had all through high school. I worked harder at that class than I'd ever worked in my life. I deserved a 4.0 for effort, and a 3.0 for the amount of problems I got right. However, I came out with a 2.5. Then came 201 and 202, which were so weird and un-mathlike, that I can't say whether I liked them or not. They didn't feel like math. They didn't make me cry, I didn't need a tutor, and I didn't get a stomach ache before every class. I didn't do exceedingly well in the first one. I think I got a 3.5 in that one, but in 202, I had a 4.0 all the way through. I knew what I was doing, and then wham, the final was nothing at all like the other tests, and I failed it completely. That gave me a 3.5, which I would have been thrilled with, but I knew I deserved a 4.0."

In her mid-semester interview, Felicia further explained her struggle with the college algebra class. She explained that the basic arithmetic class she had taken and "4.0ed" was not intended to prepare her for college algebra, but she was originally an English major, and her advisors didn't expect her to take any math beyond that first course. When she changed her major, she had to take college algebra before she took 201 and 202. Felicia wasn't happy to be taking this course on teaching mathematics because she thought she had already finished and survived all the math she would have to take. She told the mid-semester interviewer, "I guess I really had the attitude, which is bad, that I was completely done with math forever."

Felicia's Conceptions of Mathematics

The data collected from Felicia throughout the study provide much information about Felicia's conceptions of mathematics. These will be examined in the following sections as outlined by the framework designed to analyze this study.

Felicia's beginning and final math concept maps are shown here (See Figure 44 on page 270 and 45 on page 271). They will be referred to in various sections of the analysis.

Nature of Mathematics

Content. The specific content of mathematics was not a dominant focus of Felicia's "What Is Mathematics?" journal entries and her concept maps--both at the beginning of the semester and at the end. The other data sources provided no information about her conceptions of the content of mathematics. From the glimpses that are provided, it appears that there was no change in this area from the beginning of the semester until the end of the study. In her beginning "What Is Mathematics?" journal entry, she wrote, "I guess math is the study of numbers." She also wrote, "Counting is math, but so is calculus." She then

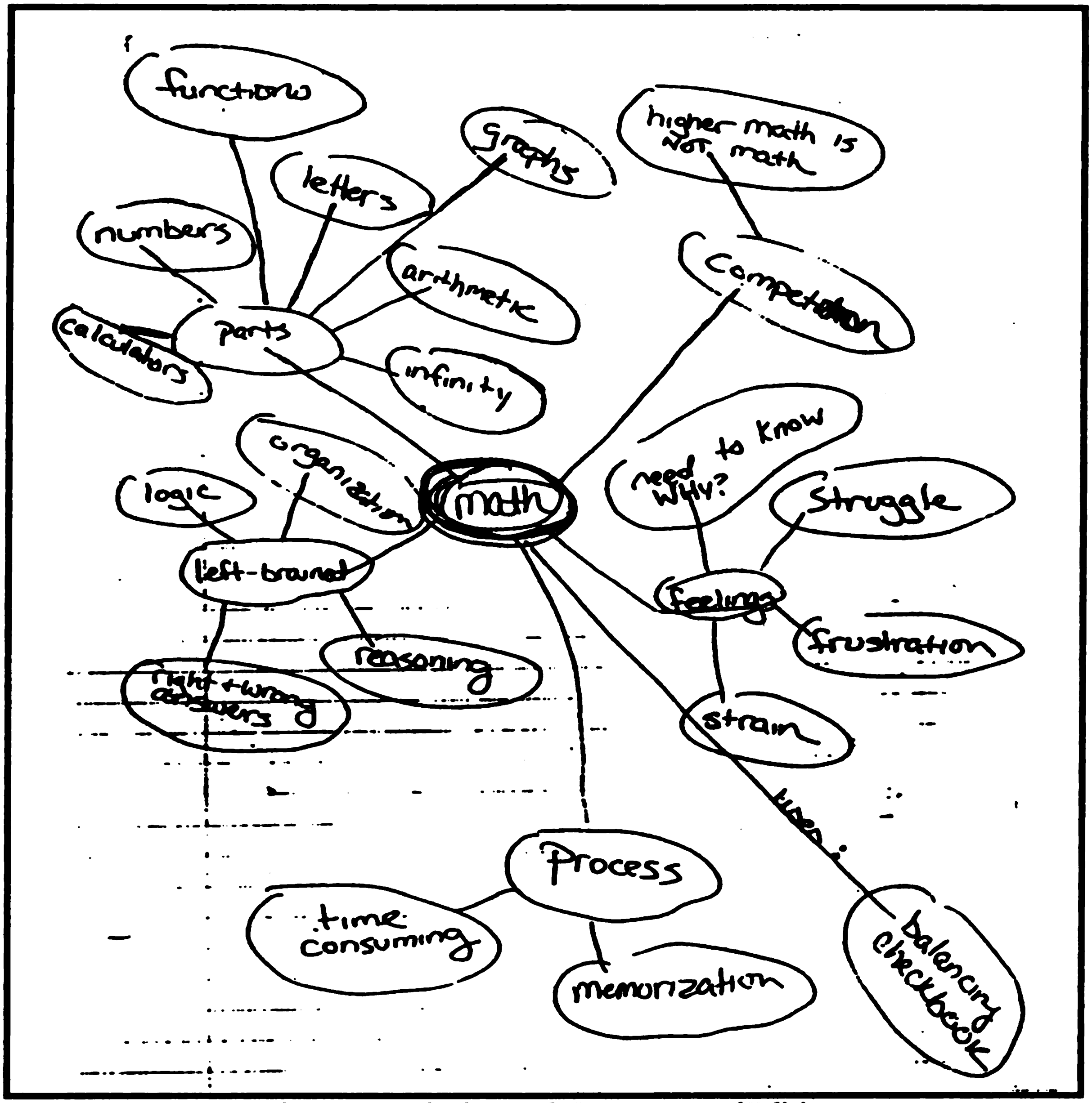

Figure 44: Beginning Math Concept Map of Felicia

added, "They seem to have no relation." In her final "What Is Mathematics?" journal entry, she wrote two sentences related to content, and one of them was essentially the same thing she wrote on the first day: "Math is the study of numbers;" and "Math is numbers and straight lines and logic." (This may be a connection to her idea of geometry which she didn't like since she mentioned logic when talking about it as noted earlier.) The only words she included in her beginning concept map related to content were "arithmetic," "infinity," and

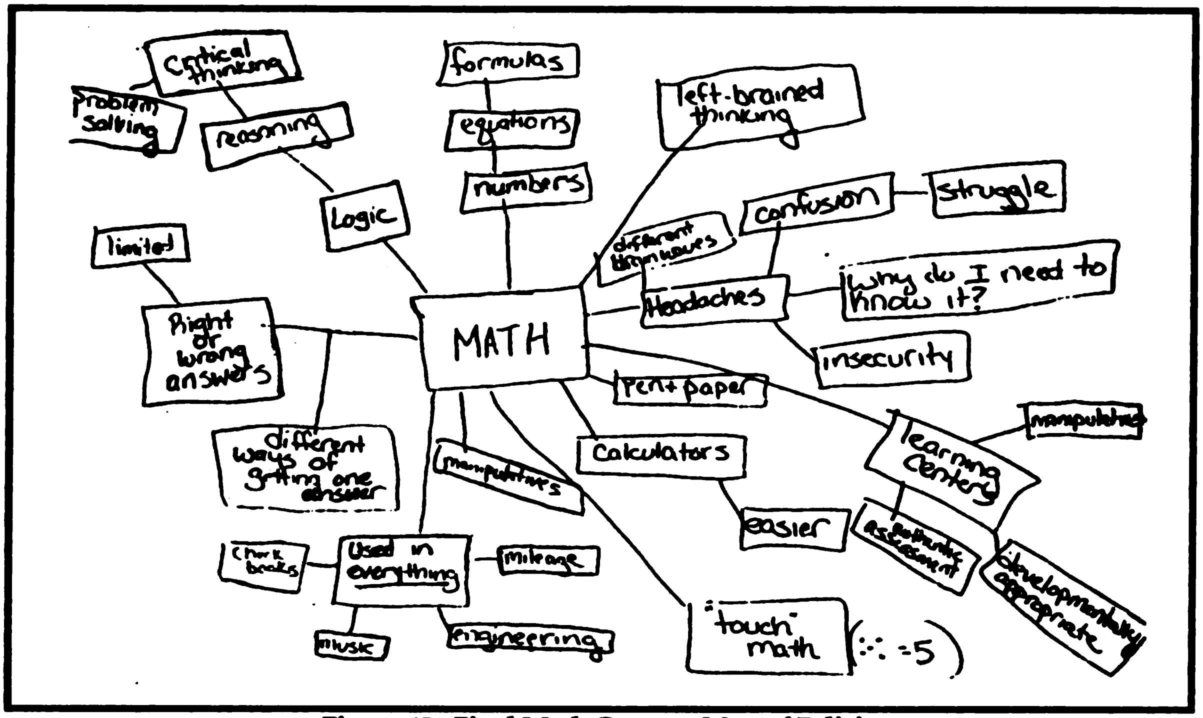

Figure 45: Final Math Concept Map of Felicia

"numbers." (It's interesting that she would include something as abstract as infinity.) In her final concept map there was no content included.

Processes. The data sources provide a little more information about what Felicia considered some of the thinking processes of mathematics, but again not a great deal. There may have been some expansion of what she considered as mathematical processes during the semester, but there isn't enough information to be certain. In her beginning concept map, she had the word "process" as well as "memorization," "reasoning," "logic," and "organization." "Memorization" was the only word linked to "process." The processes she included in her final concept map were similar, but she didn't include "organization" and "memorization," and she added "problem solving" and "critical thinking." Of course, there's no way to know if those were new thoughts or if she just hadn't included them in the first one. In her beginning "What Is Mathematics?" journal entry, relating to processes, Felicia wrote, "Math asks to prove things, to work problems backwards, to use formulas, to solve complicated equations." She also wrote, "Math is straight-forward logical thinking;" and, "Math is a way of solving problems with numbers." In her final "What Is Mathematics?" journal entry, Felicia wrote, "Math is a process that has to be thought about, manipulated, and explained."

Representations. These data sources also did not provide much information about how Felicia saw the content of mathematics as being represented. They also don't provide enough information to infer any change or lack of change. The only glimpses we get of her conceptions of the representations of mathematics are from her concept maps. In the beginning concept map, she included "numbers," "functions," "letter," and "graphs." In her final concept map, she included "numbers," "equations," and "formulas."

Characteristics. Felicia had some very strong ideas about what she considered to be the characteristics of math. In fact, she expressed these conceptions in her journal entries and concept maps more than anyone else in the class did. These ideas did not seem to change *much* during the semester, but it appears in some aspects, there may have been some slight cracks in her initial conceptions.

In her beginning "What Is Mathematics?" journal entry, Felicia provides some glimpses into what she considers some of the characteristics of mathematics. She wrote, "Math is straight-forward logical thinking that produces right or wrong answers. There is no in between. . . . There is only one answer—right or wrong." She also wrote, "Math is organized and definite and strict." Her beginning concept map also included some words that could be considered characteristics and that were consistent with what she had written in her beginning "What Is Mathematics?" journal entry: "left-brained," "organization," "right and wrong answers," and "time consuming" (See Figure 44 on page 270).

In her final "What Is Mathematics?" journal entry, she characterized math similarly to the way she did at the beginning of the semester, but there were slight differences that suggested that maybe she had been at least entertaining the idea that math could not be simply characterized in the way she had long thought. She wrote, "Math is a logical and straightforward subject," which is similar to what she wrote on the first day, but then she added, "yet it can be explored in ways that may not be logical or straightforward." Her idea that math only had right and wrong answers had also been expanded a little bit. She wrote,

> "Math is finding answers. Math only has right or wrong answers when it comes right down to it. There may be many ways to get those answers, but in the end, they must be the same. This is one strike against math, as far as I'm concerned. One can't be very individualistic in a math class when it comes to getting the right answer."

Her final concept map was also consistent in displaying this slight crack. She again included "right or wrong answers," but this time she included a connected subtopic of "different ways of getting one answer." She also connected the word "limited" to "right or wrong answers" which seems consistent with the attitude she expressed in the just-noted quote from her final "What Is Mathematics?" journal entry.

Another item she included on her final concept map that was on her beginning map was "left-brained thinking." It's interesting that Felicia wrote "left-brained" on both of her concept maps as a characteristic of mathematics because in her mathematics reflection she wrote, "I just know that my mind doesn't think of math the way most people explain it. I am left handed, and therefore right-brained." Thinking of mathematics as "left-brained" probably gave her some explanation or excuse and perhaps comfort for why she felt like she couldn't understand it.

Table 19: Items Relating to *Characteristics of Mathematics* from the Mathematics Inventory with Felicia's Responses

Item	Beginning	Middle	Ending
4. I think of mathematics as a set of isolated facts that need to be memorized.	3	3	3
11. Mathematics is something that not everyone is able to understand.	3	5	3
17. Mathematics is something at which everyone can be successful.	2	4	2

There were three items relating to characteristics of mathematics on the Mathematics Inventory (See Table 19). Regarding these items, there appears to have been some fluctuating of her ideas during the semester, but at the end of the semester she held pretty firmly onto her initial ideas—or did she? Even though the responses are the same, there may have been some cracks in some of her conceptions of the characteristics of mathematics.

For item number 4, "I think of mathematics as a set of isolated facts that need to be memorized," Felicia put a 3 each time she responded to the inventory. However, from Felicia's perspective, her idea of that had changed somewhat when she responded to that item during the mid-semester interview--even though her response didn't. When asked that item, she responded, "Um that's changed a little bit, too, probably. I'd say 3." When asked what she thought she put before she said that she didn't remember. When told she had also put a 3 then, she responded,

> "I guess—I don't know what you're looking for—but, I realize that people do think of it that way, and I can totally see how they would think of that way—as isolated facts—because that's what we've always done is memorize these isolated facts, and there really hasn't been a connection of them for me, but I realize that's not what's supposed to be, and I know there is a connection. So, I guess I'm seeing two pictures. I'm seeing my picture, and I know that it's wrong, and then I see the picture of what it should be, and I know that it's not isolated facts."

This is a fascinating quote for the fact that Felicia seems to be telling us what goes on her mind as she contemplates the difference between the way she has always thought of math and the way that we were talking about it in class. The fact that she says, "I know [my picture of math as a set of unconnected facts] is wrong certainly suggests a change in her conceptions, and yet she still talks about it as "my picture." She seems to be making a decision about whether she is going to keep on thinking the way that she always has or if she's going to add this new idea to what she also believes. This is an example that shows the role that choice plays in changing one's conceptions. (Part of my goal as a teacher of this course is to at least challenge some of the assumptions the students have about mathematics when they enter the class, and it seems this is an example of where this is happening.)

For item 11, "Mathematics is something that not everyone is able to understand," Felicia responded 3 at the beginning of the semester, 5 during the mid-semester interview, and 3 again on the last day of class. When she said a 5 during the mid-semester interview, she thought that was what she had probably put on the first day of class. When told she put a 3, she explained what she thought she might have been thinking:

> "I guess I was probably trying to be more positive. I mean, I would hope that everyone would be able to understand it, and maybe if you started in the beginning when they're young they can, but I just feel like my brain isn't designed that way. Like I wrote in a journal entry, I don't actually remember, like a lot of students do, a specific bad experience or a bad teacher who shot me down or said, 'Only do it this way.' I've had pretty decent math teachers in general, but I just feel like some people might not get it."

Although her final response to that item was a 3 which was her beginning response, there may have been some change. In contrast to the 5 she responded to it during the semester, there may have been partial, but not complete, resignation to the idea that *maybe* everyone can understand it if they are taught in the right way, or maybe she was just trying to be more positive.

Item 17, "Mathematics is something at which everyone can be successful," is not the same, but it is related in an inverse way to item 11. To that item she responded a 2 (close to not at all like me) the first day of class and then a 4 (close to very much like me) during the mid-semester interview. In that interview, she expressed some ideas similar to those she expressed when talking about item 11:

> "I guess, maybe I was thinking. . . . I don't know what I was thinking. I'm thinking right now that everybody can be successful at it probably if it's started in a certain way early--if it's really taught well when they're young. But, maybe not. I mean, I have no proof of that. Because to me, I feel like my brain just wouldn't take it no matter what, but I have no way of knowing that without going back and starting over when I was a little kid or something. "

At the end of the semester, she put a 2 again, indicating less agreement with that statement. Perhaps thinking of herself as an existence proof made her revert back to her initial conceptions. She didn't put a 1, though, which seems to leave some possibility in her mind that it might be true.

Usefulness of Mathematics

General Usefulness of Mathematics. Even though Felicia did not like math, she always held the conception that it was something that was useful in life. She recognized that it was used for many things in the world, and that was fine with her as long as she didn't have to do the math.

Table 20: Items Relating to *Usefulness of Mathematics* from the Mathematics Inventory with Felicia's Responses

Item	Beginning	Middle	Ending
3. I think mathematics is an important subject that is useful in life.	3	5	3
5. I think mathematics is something very much related to this world.	4	5	4
13. The only people who really need to understand math are those who choose a career which requires math skills.	2	2	3

It appears from Felicia's responses to the items from the Mathematics Inventory that are related to the usefulness of mathematics that there may have been a little fluctuating in her ideas during the semester, but by the end of the semester, her responses were very similar to what they were at the beginning (See Table 20). Her comments during her mid-semester interview provide us with a little more insight into what she was thinking. For item 3, "I think mathematics is an important subject that is useful in life," Felicia thought she might have been changing her opinion a little. When first asked that item, she said, "I think very much—a 5. For *me* it's not terribly important, but I think in life in general it is. I mean, obviously everything is done with math. I realize that. So, yeah." She was then asked if she thought that what's she put the first day, she responded, "Probably not. I was having a bad attitude day." She was then told she put a 3, and she said, "Yeah, well, we've talked a lot more about it in general—you know the big picture of math. So, I think that's changed." The interviewer then probed, "Like were there some things you didn't realize were part of math that now you think are?', and Felicia replied,

> "I think I've just been thinking about it more. Like I haven't been giving math that much thought in a long time. So now that we've been thinking about it more, I guess my feelings and my opinions have really changed a little bit."

Item number 5, "I think mathematics is something very much related to this world," is similar to item 3, and Felicia's thoughts about it in the mid-semester interview were also similar. She also gave it a 5 and when asked if she thought she had changed in thinking about it she said,

> "Yeah. Just the same as in the other stuff, you know, just that we've been talking about it more. I'm thinking in terms of kids being taught math, I guess, and that it should be related to the world. And, it *is*. I mean everything's done with math. A lot of things are."

She was then asked if there had been any specific things that she had done in class that had changed it. She replied,

> "Um, we just talk a lot about it a lot, you know, I mean, she had a film on how music is done with math and everything. And, I guess I've always known that, but I hadn't really thought about it, and I

> think if I had started when I was young and someone told me that and got me interested, math might have been fun. And, as it is, my opinion about everything being done with math, fine, I'm not against that, it's not like I want to go out and change it, but somebody else can do it. I don't want to do it."

When responding to item 13, "The only people who really need to understand math are those who choose a career which requires math skills," she didn't change her response from the beginning of the semester, but she explained,

> "I think that a lot of people have to know a lot of math in their careers, and in mine, if I just teach preschool, I don't have to know that much, but I have to know some. I have to have a basic understanding of it."

Personal Utility. The main way that Felicia saw math as being personally useful to her was in balancing her checkbook. She included that on her beginning and final concept maps, and she talked about it in her interviews: In her final interview, when asked if she considered mathematics as something valuable, she said, "[I]t's valuable for me when I balance my checkbook which is something I do very successfully." In her final "What Is Mathematics?" journal entry, she acknowledged that she used math every day, but she didn't provide specific examples of how she used it.

Indirect Personal Utility. One place where perhaps there was some change in Felicia's conceptions of math was in how math affected her indirectly and in how much it is used by others. In her beginning "What Is Mathematics?" journal entry she wrote, "Though 'school' math baffles me, I recognize that math is everywhere." In her final "What Is Mathematics?" journal entry she wrote more specifically about how math affects her indirectly. She wrote, "[M]ath is used to make and create the things I use."

Her two concept maps show some difference in her attention to how math is useful. In her beginning concept map, the only use she included was "balancing checkbook" with "uses" written on the line connecting it to "math" (See Figure 44 on page 270). In her final concept map, she had a category of "used in <u>everything</u>" that had "checkbooks," "mileage," "music," and "engineering" connected to it (See Figure 45 on page 271).

Another piece of evidence that shows there may have been a little expansion of her view of the usefulness of mathematics was a comment she made in her final interview. When asked if her ideas about math had changed at all during her life, she said, "I guess last semester thinking about applying—all the places that math is—changed [my view]."

Utility for Others. Felicia's view of mathematics' utility for others was similar to and connected with her view of its indirect utility for her. At the beginning of the semester she recognized that it was used by some in their professions. For example, in her beginning "What Is Mathematics?" journal entry she wrote, "Math is critical to many professions." As noted in the previous section, in her beginning concept map, she only included one use for mathematics which was a way she personally used it--"checkbooks." And, then in her final concept map, she included "music," "engineering," and "mileage"—ways it is also used by others (See Figure 44 on page 270 and Figure 45 on page 271).

In her final "What Is Mathematics?" journal entry Felicia expanded a little from her beginning journal entry by providing a few more examples of how math is used in a way that was very similar to her final concept map. She wrote, "Math is involved in many everyday processes. Math is used for music, for art, for engineering, and for everyday things such as figuring gas mileage." She also wrote, "Math of one level or another is used by nearly everybody, though not everyone realizes this."

One final example that shows how she continued to think of math's usefulness a semester after the course was in a comment she made during her final interview. When Felicia was asked if math existed differently or at all outside of a classroom. She said,

> "Oh, definitely. Um, like we talked about in class, like music involves math which I had never really thought about, but everything that's made, physically made, involves math somehow, with measurements and stuff like that. When they're figuring out how many of something there are. I think it's everywhere. I don't like it, but it's everywhere."

Learning and Teaching Mathematics

The data from this study provide some information about Felicia's conceptions of how mathematics is learned and how it should be taught. In the Mathematics Inventory there were four items related to conceptions of learning and teaching mathematics. These items along with Felicia's responses to them at various times during the semester are shown in Table 21. They will be referred to in the following sections.

Table 21: Items Relating to Conceptions of *Learning and Teaching Mathematics* from the Mathematics Inventory with Felicia's Responses

Item	Beginning	Middle	Ending
10. Teaching mathematics should not be too difficult since a textbook already has the lessons prepared.	2	1	2
22. The best way to learn math is to memorize the rules.	3	3	3
23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.	5	5	4
24. In learning math, it is important to master topics and skills at one level before going on.	5	5	3

Learning Mathematics. It appears from the information we had about Felicia's ideas of how mathematics is learned that these ideas didn't change much during the study. She didn't include anything about how math is learned on either her beginning or final math concept map or in final "What Is Mathematics?" journal entries. She did write one sentence related to learning (and teaching) math in her beginning "What Is Mathematics?" journal entry that gives us some information about her ideas: "Maybe it isn't taught this way, but I think math should be related to the world around us, not just memorizing multiplication tables." This was something she continued to believe at the end of the study. In her final interview she was asked what was the best way to learn mathematics, and she said, "Application, I guess, to real-life problems. That's the best I can think of."

A few items from the Mathematics Inventory give us a little more information about her ideas related to how mathematics is learned. For item 22, "The best way to learn math is to memorize the rules" Felicia put a 3--right in the middle--each time she responded to the item. In her mid-semester interview, she explained her thinking about that item:

> "I'm thinking more about it. I'm thinking about as far as memorizing say multiplication tables, because I was just talking about this the other day with someone that you need to understand the concepts of them, but then when you get up in higher numbers, you have to memorize how to do it, because you can't go drawing a bunch of you know 35 times 35 pictures of things. So, I think that it's a combination of both. I think that memorizing for me was a good tool, because I'm very good at memorizing, but I definitely needed to understand it, too. So, that's why I'm in the middle on that."

For item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key," Felicia put a 5 on the first day of class, and she also put a 5 during her mid-semester interview. She said it was "very much like me." When she was asked why she thought that, she said,

> "I think that reasoning and thinking about what makes sense is much more important because that's what my dad gave me when he was teaching me is "Why, why, why." And, if I couldn't explain to him why, then he started over which is part of my frustration. And, it just was more important that I understood that, because if I didn't understand that, I wasn't getting the right answer."

For some reason, Felicia's response dropped to a 4 when she took the inventory on the last day of class.

Item 24, "In learning math, it is important to master topics and skills at one level before going on," was a "weird" statement for Felicia. She wasn't quite sure what to think about it. When asked what she thought about it during the mid-semester interview, she said, "That's kind of weird question, but I think it's true—very much like me." She was then asked if she thought she put a 5 before, and she said, "Probably." The interviewer then asked her why she said it's a weird question and how she interpreted it. She said,

> "I don't know. I feel like the way it's phrased for some reason I just get this impression that it's like a trick question or something, and I know that they weren't supposed to be and that these weren't right or wrong answers, but it just gives me the feeling that there's something else, and I don't know about it. That maybe it isn't important, that maybe you can skip around, but to me it seems that you need to go from level to level."

The interviewer then said, "Why does it seem like there's some hidden thing? Because it's so obviously true?" Felicia responded, "Maybe because there's new ways of doing math that we're talking about, and maybe that's something that I just haven't heard about and that I should hear about." The last day of the semester, Felicia put a 3. She still may have not been sure what she thought about that idea, although this statement does make her sound more open to new ideas than she has sounded at some other points in the study. She sounds like she is at least considering the possibility that the "new methods" have some validity.

Felicia also believed that the teacher had a big influence on the students' learning of mathematics. She believed this at the beginning of the study and at the end of it. This was evidenced in part by a couple of the open-ended questions of the Mathematics Inventory. For the question, "What kind of influence do you think the teacher plays in the success of students in mathematics?" on the first day of class, Felicia wrote, "I think the teacher is an important influence. The teacher may determine by his or her teaching methods whether a child will like or hate math for the rest of their life." On the last day of class in response to this question, she wrote, "The teacher is a huge influence. They need to show confidence and value math."

The other open-ended question was one that asked the students if they thought a teacher's attitude toward mathematics or teaching mathematics affected how he or she taught it. On the first day of class, Felicia wrote, "It seems that it must affect it. I just couldn't be passionate about the importance of it like I could about language related lessons." On the last day of class, she wrote, "Yes. If you hate math and it frightens you, how can you teach it?"

Teaching Mathematics. There is more information about Felicia's ideas of how mathematics should be taught at the end of the semester than at the beginning; so, it is difficult to know how much these ideas changed. Probably the main difference was just the expansion of different ideas of how to teach it. Felicia was a child development major, and ideas that are emphasized in the discipline of child development were very much a part of her ideas about teaching. Examples of these are using learning centers, using manipulatives, and making sure that what is taught is developmentally appropriate. (I also earned a degree in early childhood education along with my elementary education degree, and the things she writes about are very much a part of what I learned about in my early childhood education and child development courses.) One example where this showed up was in her final concept map. She didn't include anything about teaching mathematics in her beginning concept map, but in her final map, she had a category of "learning centers" connected to "Math" with subtopics of "manipulatives," "authentic assessment," and "developmentally appropriate." In class we talked about manipulatives and authentic assessment, and little about concepts being developmentally appropriate, but we didn't talk at all about learning centers. So, it seems very likely that her views were influenced not only by this class, but also by her education in child development.

Felicia's responses to Item 10 from the Mathematics Inventory give some information about her ideas related to another aspect of teaching mathematics (See Table 21 on page 280). It read, "Teaching mathematics should not be too difficult since a textbook already as the lessons prepared." On the first day of class, she gave it a 1. During the mid-semester interview, she said,

> "Um, I disagree with that, because the textbooks don't always. . . I mean they give you a lesson, but there's a lot of kids who aren't going to understand it the way it's taught in that book. I was one of those kids. You sit back, and the majority says, 'Oh, sure we get it.' And, there's those few kids who are lost in the crowd going, 'I have no idea what you're talking about, but okay.' Because kids are afraid to say that they don't know when everyone else knows."

This statement by Felicia seems to indicate that at this point she has still very much positioned herself as a student in relation to math. The rest of the exchange between Felicia and the interviewer when talking about that item helps us understand her thinking. It follows:

> **Interviewer:** "So, would you say you're a one or a two on that?"
> **Felicia:** "I would say between a one or a two, but I think it makes it easier. I would definitely feel better about having something prepared, but I don't think it... I think it will be still difficult to teach. So, I guess 1-1 1/2, somewhere in there."
> **Interviewer:** "Okay, the first day, you said 2 on that. Do you think something's changed your mind at all?"
> **Felicia:** "I've thought it about it more. I guess I'd be a one now."
> **Interviewer:** "What's made you more . . . .?"
> **Felicia:** "Just talking more—just thinking more about it. I mean these are things that I probably had in my head already. You know, thinking about the kids who aren't going to get it that way, but I wouldn't have thought about it until we did this class."

On the last day of class, the inventory included a question asking the students if their ideas about mathematics or teaching mathematics had changed at all during the time they had been in the course. In response to this question, Felicia wrote, "Yes. We talked incessantly about how math should be taught."

Perhaps the best indication of Felicia's conceptions at the end of the semester of how math should be taught is found in her written philosophy of teaching mathematics. Of course, there is always the possibility that some of what she wrote was what she thought I, as her teacher, wanted to read. This philosophy in its entirety follows:

> "I believe that a classroom should be a positive place for learning math. Teachers should model an enthusiasm towards math and its processes so that students value math.
>
> "Since I intend to teach math in preschool, I will focus my philosophy in that direction.
>
> "I believe it is important for students to learn the meaning of numbers, how to use them, and real-life applications of what they learn.
>
> "The role of the teacher is to guide the students' learning by questioning everything so the students have to back up everything they say. True understanding is shown when a student can explain their work to another person. The teacher should be supportive and give positive feedback when needed.
>
> "The students' role should be to work in groups and individually to learn from each other and try things out on their own. They should feel comfortable questioning the teacher and trying various methods without fear of penalty.
>
> "Types of activities that should take place will vary according to topic and grade level. I would want my class to work in small groups, talk in one large group, explore in and outside of class, and use manipulatives extensively. However, I think there is something to be said for memorization of certain facts and tables too.
>
> "I believe learning should be emphasized through processes. The teacher and student should care about <u>how</u> a product is achieved.
>
> "The classroom should have plenty of math tools and manipulatives available to students. There should be learning centers with varying levels of difficulty to meet developmentally appropriate methods and standards.
>
> "If I had to teach an older classroom, I would definitely have the students keep math journals. There would be a lot of discussion where students are encouraged to support each other and assist each other so everyone can feel confident.
>
> "Assessment would be recorded on anecdotal cards. I would observe and question each student over time and gather samples of their work as they develop. I will assess my students using authentic assessment of what they have achieved over time in relation to themselves, not their classmates.

> "I will teach according to the developmentally appropriate principles I believe in. I will treat each child as an individual not a cohort group. I will respect my students and take into account the child development knowledge that I have."

In her final interview, a semester after the course, Felicia was asked how she thought math was most effectively taught, and she said,

> I guess it's effectively taught when. . . . It's hard to say, because I've had a couple good math teachers, but I'm trying to think of what they've done that made it effective. I guess when it's taught--when one concept is taught several different ways, because everyone has different understandings--different ways of learning. Um, if it's taught with pictures if possible, physical hands-on things that you can touch, that makes it a lot easier. Some math. I mean, some math you just can't. I don't think formulas really lend themselves to manipulatives. If they do I don't know about it. So, I guess that's probably the most helpful thing, and I guess that's what I would do if I were to teach math."

It's interesting that even after completing TE 401 Felicia thought first about her own experience as a student when asked about effective teaching math rather than the course she had just taken. Unlike the majority of the students in the class, she was thinking about math relative to her own experiences with it and had not shifted her position in relation to math to the kind of perspective someone who was going to teach math would have.

Felicia's Attitudes toward Mathematics & Teaching Mathematics

Attitudes toward Mathematics

Felicia did not have a positive attitude toward mathematics at the beginning of the semester, and this attitude did not appear to change during the semester. This was evidenced in every data source of the study. A negative attitude was her main connection with mathematics. In her "Reflections of Mathematics Experiences" journal entry which was assigned the first day of class, she explained much of the source and reason for this negative attitude. She began this reflection by writing, "In class, I stated that I had a bad attitude about math, but I didn't know where it came from. As other people talked about their experiences, I began moving back in time in my own mind, thinking about the hell I went through in the name of mathematics. I believe I was average in math during elementary school, and I even got an A in a seventh grade math class. I don't remember liking it even when I did well."

Felicia's father played a big role in helping her survive her school mathematics courses. He helped her experience glimpses of understanding, but still she didn't like it. She wrote about the role he played in her reflection:

> In high school, I hit algebra. Not the remedial algebra and not the advanced algebra, just the intermediate class. I received an E on my first test and went home in tears. I was a good student and E's were NOT acceptable. After that my dad took over my tutoring, and my algebra grades got steadily better. I found that even though it was a struggle, when I came out of the equation with the answer to what "X" equaled, I felt so good and so successful. I wanted to try another one. Yet if I had been allowed to quit at any time, I would have done so. Algebra was still difficult, and my dad and I spent literally hours going over it until I understood it. It took that long because I'd get so frustrated that I'd cry and leave the room with a headache. He was the only person who could explain math in a way I even remotely understood. He made sure I really understood the concepts behind the math.

Felicia's negative attitudes toward math were very evident in both her beginning and final math concept maps (See Figure 44 on page 270 and Figure 45 on page 271). In her beginning concept map, she had a topic of "feelings" connected to the concept "Math," and she had "struggle," "frustration," "strain," and "need to know why?" connected to "feelings." She also had "competition" connected to "Math" and "higher math is not math" connected to "competition." In her final concept map, she had a topic of "headaches" connected to "Math" and "different brain waves," "confusion," "struggle," "insecurity," and "why do I need to know it?" connected to "headaches."

In her beginning "What Is Mathematics?" journal entry, Felicia didn't write specifically about having a negative attitude toward math, but it seemed implied in some of her sentences:

"Though 'school' math baffles me, I recognize that math is everywhere."
"There is no appreciation on a test of the creative way a person tried to solve a problem."
"How did anyone come up with it?"
"Math is organized and definite and strict. Numbers are infinite, and so seems to be the universe. I can't comprehend either type of infinite without dizziness."
"Math is a way of solving problems with numbers. I find words easier to put together."

In her final "What Is Mathematics?" journal entry, Felicia also wrote several sentences that intimate her attitude toward math:

"Math of one level or another is used by nearly everybody, though not everyone realizes this. I know that I use math every day, and that math is used to make and create the things I use. This doesn't change my view of it, however. Those people can use their higher math. I'm happy with my "lower" math and my calculator."
"Math is finding answers. Math only has right or wrong answers when it comes right down to it. There may be many ways to get those answers, but in the end, they must be the same. This is one strike against math, as far as I'm concerned. One can't be very individualistic in a math class when it comes to getting the right answer."
"Math is numbers and straight lines and logic. It is headaches and time consuming effort. Math is useful and good and we can't live without it, but I, myself, will never love it."

Even when Felicia acknowledged some value or usefulness of mathematics, she seemed determined to not let those merits affect her attitude toward it. The just-noted statements and my observations of Felicia in class when we were engaged in mathematical tasks lead me to believe that there was an effort on Felicia's part to resist the temptation to enjoy math. Sometimes when we were doing a math task, Felicia would seem like she wasn't going to allow herself to like it even if it seemed fun and others were enjoying it. She wouldn't put much effort into trying it and would have a bored, disinterested, or even fearful expression. She seemed to have the attitude that "this is math, and it can't be fun." I also think that, as she admitted in her mid-semester interview, she was not happy about the idea of being in a "math" class when she thought she was finished with them. (She said, "But every time I start up again, like every time she's brought a problem into class, and we've had to do it, besides my bad attitude that says, 'Hey, I've finished my math credits.' I try it, and I just feel that shift take place in my brain, and it's just not comfortable.") Other times—usually if we were doing a problem geared to the lower grades—she seemed like she was engaged and was enjoying the task, but that still didn't improve her attitude toward it.

Felicia also did not have a lot of confidence in her ability to do and understand math. One place where this can be seen is in a journal entry she wrote three weeks into the semester in which the students were asked to look at the five goals that the National Council of Teachers of Mathematics had identified for students in its 1989 *Curriculum and Evaluation Standards for School Mathematics*. These goals were (1) learning to value mathematics; (2) becoming confident in one's own ability; (3) becoming a mathematical problem solver; (4) learning to communicate mathematically; and (5) learning to reason mathematically. They were asked to think and write about whether they possessed those characteristics personally and if they thought it was important for teachers to possess them in order to help their students attain them. This particular journal entry of Felicia's provides a lot of information about her attitude and confidence regarding math:

> "In regards to the first goal, valuing math, I don't think I have this characteristic, and I think I need to in order to help students value math. I guess I value math for other people, but not myself.
>
> "I definitely am not confident in my own abilities in math, and I need to be, for myself. However, I feel capable of helping students feel confident themselves. I think this because I recognize that math is useful and it is fun for some people. Therefore I would try to foster confidence in all of my students so they'd have an equal, fair chance at it.
>
> "I'm not sure if I'm a math problem solver. I think I may have a start on that. I do believe I'd need to be one in order to teach it.

"I don't think I communicate well mathematically. It seems that I would need to be able to do so to expect my students to communicate mathematically. The same goes for reasoning mathematically. I have a lot to learn before I can teach."

Another place where the lack of change in Felicia's attitude can be particularly seen is in her responses throughout the semester to the items related to attitude toward mathematics on the Mathematics Inventory (See Table 22). The only one there was any change on was item 15, "I understand elementary level mathematical concepts quite well myself." It appears she at least felt more confident in that area at the end of the semester. Some of her comments when responding to these items during the mid-semester interview provide additional insight into her attitudes and will be noted in the following paragraphs.

Table 22: Items Relating to Attitudes toward *Mathematics* from the Mathematics Inventory with Felicia's Responses

Item	Beginning	Middle	Ending
1. I enjoy doing mathematical tasks.	1	1	1
2. I have always enjoyed mathematics.	1	1	1
6. I feel successful when I am engaged in mathematical tasks.	1	1	1
7. Doing mathematics is usually very frustrating for me.	5	5	5
15. I understand elementary level mathematical concepts quite well myself.	2	3	4
25. I can handle basic math, but I wouldn't do well at advanced mathematics.	5	5	5

After responding a 1 on item number one, "I enjoy doing mathematical tasks," during the mid-semester interview, the interviewer asked Felicia if the reason there was no change is that there had been nothing that was really relevant to that or if she had had even more bad experiences. She responded,

"No, I haven't had any more bad experiences. I'm just like. . I guess I really had the attitude, which is bad, that I was completely done with math forever, and um . . . I don't remember what the questions were. If there are questions on there regarding teaching math, then that's probably changed. But, as far as my personal abilities and feelings about math, they probably haven't changed that much."

To explain why she didn't feel successful when engaged in mathematical tasks when asked item number six, Felicia responded,

> "Whenever I've had to do anything mathematical, I get a headache, and I usually don't know how to begin, and my answer is usually wrong, and it just doesn't interest me unless you count balancing your checkbook in which case I can do that quite well, and I feel very successful when I'm done with it, but that's just adding and subtracting."

For item number seven, "Doing mathematics is usually very frustrating for me," Felicia again talked about how doing math gave her a headache. In fact, this was something that Felicia talked about or wrote about several times during the study.

When Felicia was interviewed a semester after the course, she had still held on to her negative attitudes toward mathematics. She said that it was "valuable everywhere to everyone at some time or another." But, she also said that she didn't feel confident at all doing math "unless it's really simple." It was interesting that when she was asked if there was anything she wanted to add at the end of the interview, she indicated that she thought she was one of the stronger math haters and that she wouldn't want students to feel the way she did about math:

> "I wouldn't want students to feel like I do about math. I think I'm one of the stronger of the math haters, but I know, I mean, some people, they say, 'Oh, I'm not very good at math.' But, then they go into math class with me, and they do okay, and they understand it better, more quickly. I feel like everybody gets math more quickly than I do. I feel like nobody took the time to make sure I really understood it except for my dad. And, without him, I would have failed math altogether through my whole life."

Attitudes toward Teaching Mathematics. Felicia also did not have a positive attitude about the idea of teaching mathematics. These attitudes also did not seem to change during the semester. If anything, it seemed that her attitude about *not* wanting to teach mathematics was strengthened during the semester.

This lack of change can be seen in Felicia's responses to the items relating to attitudes toward teaching mathematics from the Mathematics Inventory as are shown in Table 23. Her responses are indicative of her lack of desire or excitement for teaching mathematics as well as her lack of confidence in her ability to teach mathematics to elementary school children.

Table 23: Items Relating to Attitudes toward *Teaching Mathematics* from the Mathematics Inventory with Felicia's Responses

Item	Beginning	Middle	Ending
8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.	1	1	1
9. I feel enthusiastic about mathematics and about teaching it.	1	1	1
12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.	1	1	1
14. I feel confident in my ability to explain mathematical concepts to children.	1	1	1
16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.	5	not answered	5
18. Being good at mathematics is not required to be a good teacher of math.	2	1	1
19. I want to teach mathematics so that I can help students get excited about it.	2	4	1
20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.	3	1	3
21. I expect to regularly teach mathematics as an elementary teacher.	4	4	2

Felicia definitely did not have much desire to teach mathematics. She was not excited about that thought. In fact, by the end of the semester she had decided that she was only going to teach preschool, partly so she wouldn't have to teach elementary-level mathematics. Actually, that was a decision she made early in the semester and perhaps already suspected when she entered the class. It wasn't just the math, though, that made her decide that she just wanted to teach preschool. In the mid-semester interview, when she was asked item 21, "I expect to regularly teach mathematics as an elementary teacher," she said that if she were going to be an elementary school teacher, she would expect to teach math, but at that point she wasn't expecting to teach elementary school. She was asked if math was the reason she didn't want to teach elementary school, and she said, "Really math has nothing to do with whether I'm going to teach elementary school. It's just the TE stuff in general." When probed further about the math teaching, she responded,

> "It's not a factor. I mean, if I decided. . . I would only choose to teach like kindergarten or first grade, and I think that I can handle that, and I think I would learn a lot more about teaching math if I knew I were going to teach it for sure. I mean, I'm trying, you know, really hard in this class to get math. I'm frustrated that I'm not explaining it well to kids. So, I want to do that better. I just didn't want to teach any higher."

She was again asked if it was because of math that she didn't want to teach higher grades, and she said, "Um, no, just in general. I'm not comfortable with older kids. I just don't like them."

Felicia's mid-semester response to item 19, "I want to teach mathematics so that I can help students get excited about it," might be interpreted that at that point she was more excited about teaching, but her explanation of why she responded 4 doesn't really indicate any change. She explained that she *would* like to be able to do that, if she could, but she didn't think she could. At the end of the semester, it appears from her response that she was focusing on the fact that she just didn't want to teach mathematics.

Felicia's responses to other items on the Mathematics Inventory related to attitude toward teaching mathematics consistently showed her lack of desire to teach mathematics. For example, for item 8, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher," Felicia put a one each time she responded to it. In her mid-semester interview, when she was asked that item, she said, "Not at all like me, and I probably actually won't teach mathematics as a teacher, because I will probably teach preschool since that's what I enjoy most." She was then asked if she thought there could be something that would help her change her attitude. Her response was,

> "I don't know... I guess someone would have to do something that I can't even imagine to make it more relevant to me. Um, make it matter that I should do math, because at this point I'm so happy with my life not doing a lot of math, and I would never in any way give this view to a child that I was teaching. I mean, I would be so careful to keep that from them, and I've done some math with the kids where I'm at the field placement now, and I have never. . . I mean, I go out of my way to make sure that I think it's fun and interesting and that I don't grumble along with them. I just think it matters that you start when you're young. I just can't image switching to loving math at any time in my life."

Felicia also didn't have a lot of confidence in her ability to teach math, and much of that she attributed to her dislike of math and her feeling that she was not successful in math. One of the places she expressed this lack of confidence was again in her mid-semester interview when she was talking about item 8, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher." She was asked when she thought about that question what made her not excited, and she said,

> "Because I don't feel like I have a deep enough understanding of it, and I don't think it makes me feel good enough to want to teach it to other kids. Like right now I'm supposed to teach a math lesson to either a group, an individual, or the whole class, and I'm choosing an individual simply because I feel like that third grade class actually in a sense is better at the math that they're doing right now than I am, because they've been doing it, you know. They're in school. They've been working up to it, and they have to do it, and it doesn't occur to them that 'I hate it. I'm not going to do it.' And, I don't feel comfortable enough teaching them, because I feel like they would challenge me with something—with a lot of things that I don't know how to do—that I don't remember or that they have a different way of doing it."

Felicia's responses throughout the semester to items 12 and 14, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school," and "I feel confident in my ability to explain mathematical concepts to children," respectively, also illustrate her lack of confidence to teach mathematics (See Table 23 on page 292).

Some of Felicia's responses to the open-ended questions from the Mathematics Inventory also give evidence that she did not feel much confidence in her ability to teach math. For example, the students were asked what concerned them the most about teaching mathematics to elementary students. On the first day of class, Felicia wrote, "TE classes have provided no methods. I wouldn't know how to teach math at all." On the last day of class, her response was different, but still indicates a lack of confidence. She wrote, "I feel incompetent in math!"

There was one thing Felicia had to offer as teacher of mathematics that she felt like was a strength and that was that she would be able to relate to students who were like her in relation to math. She felt this way at the beginning of the semester and at the end of it. One of the open-ended questions in the inventory asked the students what they considered to be their greatest strength in relation to being a mathematics teacher. On the first day of class for that question, Felicia wrote, "I would have empathy for the different kinds of ways children understand math." On the last day of class she wrote, "I can identify with students who take longer to understand and who understand differently."

Another piece of evidence supporting this was the open-ended question she was asked in the Mathematics Inventory, "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." On the first day of class, Felicia wrote, "A liability and a strength. I have a lot to learn, but I will understand kids who may feel as I do." On the last day of class, she wrote, "A liability. Therefore, I'm teaching preschool, which I can handle." This seems to be some progress—the kind that I wrote about as a success of sorts with Bridget.

Another place her desire to not teach mathematics was emphasized was in another question asked only on the last day of class in the open-ended section of the Mathematics Inventory. The students were asked if their attitudes toward mathematics or teaching mathematics had changed at all during the time they were in the course. Felicia wrote, "Yes. Since attempting to teach math, I have become even more certain that I shouldn't."

A semester after the course had ended, when Felicia was interviewed for a final time, she still had the same attitude toward teaching mathematics. When asked how she felt about the idea of teaching mathematics in an elementary school, she replied, "I don't plan on doing it at all, and I definitely wouldn't want to." When asked if her feelings about teaching math had ever changed, she said, "No. I mean, I feel comfortable teaching preschool kind of math, you know, just one-ton-one correspondence stuff, but that's as far as it goes."

During the final interview, Felicia was also asked about what concerns she would have about teaching mathematics if she were to teach it. She replied, "Um, my own knowledge of it. I would feel like I was teaching straight from the book, because I don't feel like I have a deep understanding, a good solid foundation of mathematical understanding."

Major Themes in the Case of Felicia

Three major themes stood out in the analysis of Felicia's conceptions of mathematics and attitudes toward mathematics during this study. One was her strong belief that her inability to like and do well in mathematics was physiological. Another theme was the slight cracks in her conceptions of the characteristics of math and the usefulness of math. Probably the strongest, overarching theme was her firm hold on her already-entrenched negative attitudes toward mathematics. These themes will be elaborated and summarized in the following sections.

A Strong Belief that Her Inability to Like and Do Well in Math was Physiological. Felicia had a strong belief that her inability to like and do well in math was physiological. She expressed that idea over and over in several data sources throughout the study. She never changed that belief at all. She often spoke and wrote about how doing math gave her headaches and how she felt like her brain wasn't made to think that way. She described math as "left-brained" and herself as "right-brained." She also talked and wrote about the physical pain she would feel when she was trying to do mathematical tasks—those beyond things that used basic arithmetic like balancing her checkbook. One example of this was her response to item number seven in the Mathematics Inventory, "Doing mathematics is usually very frustrating for me." During the mid-semester interview, she explained,

> "I said that it gives me a headache, but I feel like my brain shifts into a different gear when I try to do something in math when I haven't for a while. Like the beginning of all my math classes in college, I felt like I'd be focusing on a problem, and I physically felt something change like in my brain, and it hurt to do it, and when it was over, like if it was a test, I just felt physically exhausted like I had just run a marathon or something. It just really took a lot out of me. By the end of the class it wasn't that hard. My brain got used to thinking in those ways and thinking about math. But every time I start up again, like every time she's brought a problem into class, and we've had to do it, besides my bad attitude that says, "Hey, I've finished my math credits." I try it, and I just feel that shift take place in my brain, and it's just not comfortable."

This was typical of how she described her ability to do math throughout the study, regardless of the type of question or data source that was used.

Slight Cracks in Her Conceptions of the Characteristics of Math and the Usefulness of Math. There seemed to be some slight cracks in Felicia's conceptions of the characteristics of math and the usefulness of math by the end of the semester. For example, at the beginning of the semester, she described math as having "right and wrong answers." At the end of the semester, she still described math as having "right and wrong answers," but in both her final concept map and final "What Is Mathematics?" journal entry, she added that there was more than one way to get the right answer. She also seemed more open to the possibility that everyone could understand and/or be successful at mathematics *if* they were taught in the right way than she had been at the beginning of the semester. Other evidence that supports this assertion is that she said that she thought her conceptions were changing in how she saw math as useful in the world (but not for her personally) and the idea that she talked about on page 38 where she stated that she recognized that there were two pictures of math and she knew that hers was wrong.

A Firm Hold on Already-Entrenched Negative Attitudes. Felicia's main connections to mathematics were her negative attitudes toward it, and she firmly held onto these attitudes throughout the study. Felicia seemed very resolved not to give up her negative attitudes toward math—in spite of any merits or usefulness she acknowledged math had. She had had many negative experiences with math that had been the source of pain and frustration for her throughout her life, and she wasn't about to forget or change the attitudes those experiences had inspired. In her "Reflection of Mathematics Experiences" journal entry, she wrote,

> "Math has always been a struggle for me, whether I've done well or not. I don't like math. I know it's necessary, and I balance my checkbook quite well. But I don't want math again. My mind is set against it."

Felicia conceded some values and merits of math, but they didn't improve her attitude toward it. For example, even though she recognized that math is useful, it didn't affect her attitude toward it. Virtually every time she said or wrote anything about math's usefulness in the world, she always added something about still not liking it herself. For example, in her final "What Is Mathematics?" journal entry she wrote:

> "Math of one level or another is used by nearly everybody, though not everyone realizes this. I know that I use math every day and that math is used to make and create the things I use. This doesn't change my view of it, however. Those people can use their higher math. I'm happy with my 'lower' math and my calculator."

Also, her final two sentences of that journal entry were, "It is headaches and time consuming effort. Math is useful and good, and we can't live without it, but I, myself, will never love it."

Even a semester after the class, in her final interview, she said, "I think it's everywhere. I don't like it, but it's everywhere." It seems that Felicia is very committed to hold onto her negative attitudes toward mathematics eternally.

The notion of positioning may be helpful in understanding Felicia's lack of change. Felicia seemed to shift her position in relation to conceptions of teaching mathematics—maybe for the purposes of getting a good grade in the course—but she never seems to shift her position in relation to her attitudes toward math and teaching math. By the end of the semester, her attitudes still seemed to be wholly influenced by her experiences as a student. It seems as though Felicia's growing reluctance to see herself as a teacher of math might have made it difficult for her to see math differently—and especially to have a more positive attitude toward it. In other words, instead of seeing her decision not to teach as coming from her growing awareness of the potential ill effects of her attitude and lack of mathematical knowledge, it is also possible that her reluctance to see herself as a teacher of math made it hard for her to see math differently. Even as the semester began, she seemed pretty certain that she really wanted to teach preschool—not elementary school—and that determination may have hindered her imagining (or positioning) herself as an elementary teacher who would teach math. This example may support the idea that one of the factors that helps students think of math and teaching math differently—and have more positive attitudes toward math and teaching math—is that they are repositioning themselves in relation to math—as prospective teachers. Felicia didn't position herself as a future elementary school teacher, and her attitudes toward mathematics did not improve.

Chapter Summary

In this chapter I presented the cases of two students who began the semester with negative attitudes toward mathematics and teaching mathematics, and their attitudes did not improve during the semester. Felicia's negative attitude had been influenced by her negative experiences in learning mathematics. Bridget, on the other hand, was unusual in the fact that she had experienced some success in learning math and felt quite confident with it, but she still had negative attitudes toward it.

By the end of the semester, both of these students had decided—or confirmed their decision—that since they did not like math, they probably should not teach elementary school mathematics. I think both of them suspected they didn't want to teach elementary school when the class began, and not just because of mathematics. At the end of the study, Felicia was planning to teach preschool, and she felt she could handle the math in it. Her decision that she did not want to teach elementary school seemed to be in part because she didn't like math and had no substantial knowledge of the subject matter, but it also seems, as she admitted in her mid-semester interview, that part of the reason she chose not to teach elementary school is that she liked little children more than the older ones. At the end of the semester, Bridget was planning to teach middle school English and by the end of the following semester, she had decided she did not want to teach at all. If we accept her explanation, it certainly seems that the main reason for that was her lack of enthusiasm for math, but there may have been other reasons she decided she didn't want to teach, and her dislike of math gave her a reason to disqualify herself. Regardless, given their dislike of math, the fact that they decided they shouldn't teach it can be considered to a success to some degree. Their negative attitude toward mathematics and/or their lack of mathematical knowledge could have ill effects on their students' learning of mathematics.

The change—or lack of change—in these two students was not typical in the class. Of the 32 other students in my class, I could find only 2-3 others that I would classify as having attitudes that seemed as negative—or almost as negative—at the end of the semester as they did at the beginning. However, as previously noted, part of the reason I chose these two as focus students is because their attitudes seemed particularly negative, and I was interested in what might happen with and what I might learn from "hard" cases.

One thing I did learn from these "hard" cases is that it is not necessarily easy to change negative attitudes that have developed during 22 years of life—especially in a one-semester course. Another thing I learned is that dissatisfaction and sometimes choice play a role in change. These students both seemed content—even happy—with their attitudes toward math. Felicia often said or wrote things like, "I am happy with my lower math." Bridget expressed the idea that she saw herself as an English/social studies person, and it seemed that liking math didn't fit in with that image of herself. Their attitudes seemed to be part of their identity. They seemed to make the choice that they weren't going to change their attitudes. They did not seem to feel that they wanted or needed to change. For both of these students, having negative attitudes toward math seemed to serve at least one purpose for them—it gave them a reason for not wanting to teach elementary school, and if their attitudes were no longer negative, they would need to find another explanation.

It seems that for change to occur, there needs to be some level of dissatisfaction, and Bridget and Felicia did not seem dissatisfied with their feelings about math. On the other hand, most of the 401 students were unhappy with their attitudes toward math because they were committed to teaching elementary school and knew that teaching math would be difficult and painful unless they changed their attitudes. Therefore, they had reasons to change, but Bridget and Felicia did not. This notion also seems to connect to the idea that a shift in positioning—from experienced student to prospective teacher—is part of what influences the 401 students' changes in both conceptions of and attitudes toward mathematics and teaching mathematics.

CHAPTER 7

THE CASES OF TWO OTHER PROSPECTIVE ELEMENTARY SCHOOL TEACHERS: CAROLYN AND ELIZABETH

In this chapter, I analyze the cases of two other prospective elementary school teachers—Carolyn and Elizabeth. Both of these students, like Bridget and Felicia, began the semester with negative attitudes toward math and teaching math. However, unlike Bridget and Felicia, Carolyn and Elizabeth end the semester and the study with much more positive attitudes. There also seems to be a clear shift in positioning for both Carolyn and Elizabeth in their relation to mathematics—a shift from viewing it as a student would to viewing it as a teacher would. For Carolyn, this shift seems to gradually occur over the course of the semester; for Elizabeth, the shift happens early on and seems to influence all her conceptions of and attitudes toward mathematics.

Carolyn: An Increase in Confidence and Enthusiasm for Mathematics and for Teaching Mathematics

Carolyn was a 35-year-old divorced mother of two upper-elementary-age children. She had shoulder-length, dishwater-blond, permed hair that she usually wore pulled back at the top into a barrette with her bangs curled under. She wore wire-rimmed glasses, was shorter than average, and as is not unusual for a woman her age who had had two children, was probably carrying a few more pounds than she had in her younger years. She returned to college after several years away from school to earn her degree so that she could teach. Her major was elementary education with minors in English and history.

Carolyn's experiences with mathematics in her life had not been very positive, and she therefore did not have a very positive attitude toward mathematics. She thought her negative feelings toward math started probably around third grade when it came time to memorize the multiplication tables. She wrote, "I simply couldn't do it. This was a terrible experience for me. I felt really stupid and embarrassed." Another related negative experience from elementary school that stands out in her mind, she described this way:

> "I remember a few years later when my teacher sent me to the lower grade to help younger students learn their times tables. It must have been an attempt at peer teaching, trying to help me become more confident in my own skills. However, some kids realized I didn't know my times tables very well, and I got teased about it. My sister was in this class, and to this day, she remembers too."

Her training in mathematics was also quite limited. In high school she had one course in algebra and one in geometry. She claimed to have never been good at math and to have been an average student in those classes. At Michigan State University, she took Math 1825, which is intermediate algebra, and College Algebra. Since it had been several years since she had had a math class, she reported finding the first course especially difficult. About those classes she said,

> "Wo! Uh, since it had been such a long time since I had had any math, and Math 1825 was supposed to be like a review for I'm assuming like high school kids who just needed a little bit more before they went on, but to me it was like almost all new stuff, and it went really fast as a review. I was just like blown away, and I barely passed the class. And, then when I went on to 103, of course, it was a lot of the same stuff we did in 1825, and I did a lot better, because then it was a little bit of review for me. They were hard. I mean, it's tough stuff."

She didn't enjoy those classes because, as she said, "I didn't see any point in them, why I was taking it, why I was learning it, everything you use it for."

Carolyn had not taken the Math 201 and 202 courses which were required for elementary education majors before she took TE 401.

Carolyn's Conceptions of Mathematics

The data collected from Carolyn throughout the semester and in her final interview a semester after the course ended provide some glimpses into the conceptions that she had about mathematics and how and whether these conceptions changed at all during that period.

Carolyn's beginning and final math concept maps are shown here (See Figures 46 and 47). They will be referred to in various sections of the analysis.

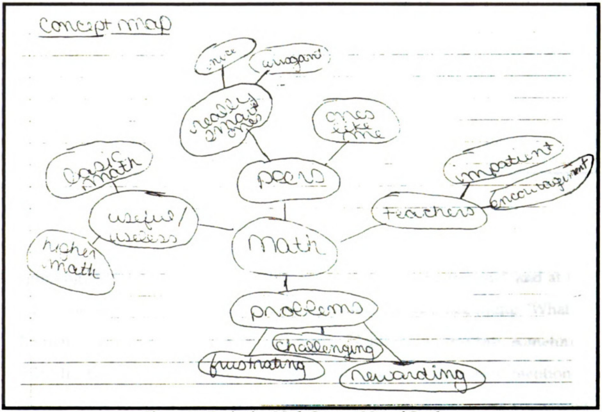

Figure 46: Beginning Math Concept Map of Carolyn

Nature of Mathematics

The data sources of this study provided information about Carolyn's conceptions of the nature of mathematics throughout the time of the study. Specific areas of the nature of mathematics will be discussed in the following sections.

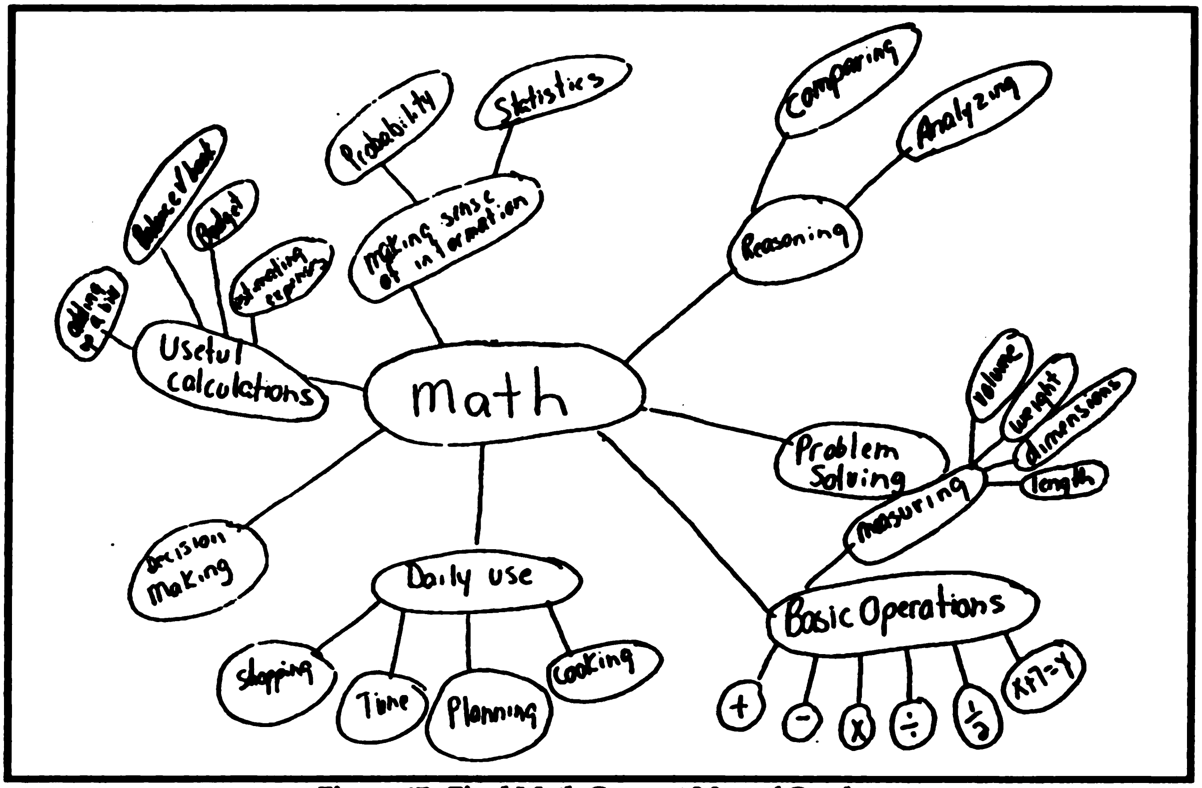

Figure 47: Final Math Concept Map of Carolyn

Content. The data sources for Carolyn didn't provide too much information about what she considered the content of mathematics. At the beginning of the semester, her attention to content was very minimal, and at the end of the semester it was only slightly greater. In her beginning "What Is Mathematics?" journal entry, she did write, "Adding, subtracting, sometimes multiplication are the concepts used most often." She also mentioned algebra—only to write that to her it was "not math" since it was not something that she used personally. (She was the only student who seemed to have this "definition" of math.) Her beginning concept map included no areas of content (See Figure 46 on page 305).

Although there was a little more about Carolyn's conceptions of the content of mathematics in the data she provided at the end of the semester, there still was not a lot of information. She didn't include any specific areas of content in her final "What Is Mathematics?" journal entry. Likewise, when she was asked to define or describe mathematics in her final interview, a semester after the course ended, she talked about it as being something that was useful in life for balancing her checkbook and things like that. However, in her final concept map, Carolyn included some specific areas of content: the basic operations, measuring (with subcategories of volume, weight, dimensions, and length), probability, and statistics (See Figure 47 on page 306).

Processes. Carolyn seemed to be thinking more about the processes of mathematics at the end of the semester than she was at the beginning. While she included nothing about processes in her beginning concept map, in her final concept map, she included the processes of problem solving, decision making, reasoning, comparing, analyzing, and making sense of information (See Figure 46 on page 305 and Figure 47 on page 306). This seemed to be an important change. Likewise, although there is nothing about processes in her beginning "What Is Mathematics?" journal entry, her entire final "What Is Mathematics?" journal entry seems to focus on processes. That complete journal entry follows:

> "Math is a logical way to think about life. It is reasoning, thinking, comparing, and decision making. Math is problem solving, a tool to use in daily life. That really says it all. Math is reasoning about information and making sense of it. It is comparing things and making good decisions because of it. Math helps us solve problems we encounter in life. It is definitely a tool we use to help us reason and think and solve problems. Math is more than a series of computations. It is a way to organize information and make decisions that will benefit us."

There was evidence, however, from the beginning of the semester that Carolyn valued the processes of mathematics more than their outcome. For instance, she wrote in her beginning "Reflection of Mathematics Experiences" journal entry,

> "My daughter has this memorization difficulty also. I figure as long as she can figure out 9 x 8 somehow, then she's okay. Sure she may not be very speedy, but at least she understands how to get the answer."

One more piece of evidence supporting the idea that Carolyn valued the processes of mathematics even from the beginning is an item from her Mathematics Inventory. For item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key," Carolyn put a five, indicating it was very much like something she believed, each time she responded to the inventory—on the first day of class, during her mid-semester interview, and on the last day of class. On the first day of class, this was one of only two items that she marked five on; so, it seems to be an idea that she especially agreed with and something that was quite important to how she thought about math.

There was also some evidence that Carolyn felt some confidence in her ability to use mathematical processes. Three weeks into the semester, the students were asked to write a journal entry in which they looked at the five goals that the National Council of Teachers of Mathematics had identified for students in its 1989 *Curriculum and Evaluation Standards for School Mathematics* and asked to think and write about whether they possessed those characteristics personally and if they thought it was important for teachers to possess them in order to help their students attain them.

> "I feel I can problem solve and reason through a situation mathematically. As a matter of fact, in my algebra class, we were given a story problem about time and distance. We were supposed to form an algebraic equation to solve this. Not me. I reasoned it out, the long way, but I understood it that way."

Representations. The only information related to Carolyn's conceptions of how mathematics is represented was in her final concept map where under her "Basic Operations" heading, she wrote the symbols for the basic operations, the fraction 1/2, and the equation x+7=y.

Characteristics. Carolyn didn't offer a lot of information about her conceptions relating to the characteristics of mathematics in the open-ended data sources, but her responses to the items related to characteristics of mathematics in the Mathematics Inventory provided some interesting information about her conceptions and how she felt they were changing. For each of the three items in this category, Carolyn's beliefs were modified as the semester progressed (See Table 24).

Table 24: Items Relating to *Characteristics of Mathematics* from the Mathematics Inventory with Carolyn's Responses

Item	Beginning	Middle	End
4. I think of mathematics as a set of isolated facts that need to be memorized.	4	2	1
11. Mathematics is something that not everyone is able to understand.	3	2	1
17. Mathematics is something at which everyone can be successful.	3	4	5

For item number 4, "I think of mathematics as a set of isolated facts that need to be memorized," her first response was a 4 indicating that it was close to very much like something she believed. When asked about this item in the mid-semester interview, she said, "I used to really think that. That's changing. So, I guess it's coming down to like 2." When asked what was happening here, she replied,

> "Um, just learning that you don't always have to rely on facts that you have to memorize. I mean even like I was telling you in algebra you have to memorize formulas and you have to memorize this. And, so it was even true in algebra for me, but I guess what I'm learning is that it doesn't have to be that way. It can be different. You can rely on other things rather than memorization."

Her response at the end of the semester to item number 4 was a 1 indicating that it was not something she believed at all.

When responding to item number 11, "Mathematics is something that not everyone is able to understand," Carolyn disagreed with it more each time she responded to it. In her mid-semester interview, she talked about this item this way, "I used to believe that very strongly. I probably put a 5, but now I'm thinking that I really don't believe that. So, I'm probably going to put a 2."

When she was told she put a 3 the first time, she said, "I thought I was way up there." And, when asked what was changing for her, she said,

"Well, because I think that everybody probably can understand it. Maybe a different way, and maybe they need to have some different experiences and be taught a little bit different than their neighbor does. So, if they get the right thing for that child, they can understand."

She disagreed with that statement even more strongly when she responded to the inventory at the end of the semester.

For item number 17, "Mathematics is something at which everyone can be successful," which is related in an inverse way to item number 11, Carolyn's response increased each time she responded to the inventory. In her mid-semester interview, she explained what she was thinking about regarding this question this way:

"Because successful is reaching a level of understanding. Success isn't always getting a right answer. Maybe that's why that's changed a little bit, because I took successful more as always getting the right answer—knowing the right answer. Now, I look at it more as understanding—understanding how to get there, even if you don't get to the right spot."

When asked what was making the difference in the definition for her, she replied,

"I think we talk a lot about process--the thinking about math--versus the right answers all the time. And, I think I related before, my daughter not knowing all the multiplication facts, but she can figure it. So, I see more value in being able to figure something out and at least knowing how you're supposed to figure something out and at least knowing how you're supposed to figure it out than just being able to give the right answer."

Usefulness of Mathematics

General Usefulness of Mathematics. As alluded to earlier, the usefulness of mathematics was something that was an important part of Carolyn's conception of mathematics. In her beginning "What Is Mathematics?" journal entry, she wrote,

> "Math is knowledge that people use everyday of their life. . . These everyday uses of math are what I consider to be basic math. . . In my opinion, mathematics is a useful tool which helps us to accomplish a specific task."

In fact, at the beginning of the semester, she only considered what was personally useful to her as "math." Among the subjects of this study, this was an idea that was unique to her. She wrote,

> "My opinion changes when we talk about things such as algebra. To me this is not math. It is not useful to me in my everyday life or even useful once a week or once a month. . . Higher math is simply not relevant to me in the real world."

In Carolyn's final interview a semester after the course, she had held on to this belief, but had modified it a little to make room for "two different kinds of math." When asked how she would define or describe math, she responded,

> "I still think of two different kinds of math, you know. I think of the kind of math that you use all the time--to balance your checkbook, to figure out how much money you have when you go to the store--you know, math like that as a useful part of everyday life. And, then there's this math that I don't know what it's for or what I'm supposed to use it for. You know, the algebra. What am I supposed to do with that besides forget it? You know? And, so, I kind of have two different ways to look at math."

When responding to the items in the Mathematics Inventory relating to usefulness of mathematics, Carolyn consistently indicated a strong belief that mathematics was useful in life each time she responded (See Table 25).

For item five, "I think mathematics is something very much related to this world," Carolyn responded with four throughout the semester, indicating strong agreement with the statement. For item three, "I think mathematics is an

Table 25: Items Relating to *Usefulness of Mathematics* from the Mathematics Inventory with Carolyn's Responses

Item	Beginning	Middle	End
3. I think mathematics is an important subject that is useful in life.	4	4	5
5. I think mathematics is something very much related to this world.	4	4	4
13. The only people who really need to understand math are those who choose a career which requires math skills.	2	2	1

important subject that is useful in life," Carolyn responded four on the first day of class and in her mid-semester interview, indicating it was close to something very much like her, and on the last day of class she moved up to five. She also seemed to have a strong belief that mathematics was something that is useful for everyone to understand—not just those whose career required it. This can be seen in her responses to item 13, "The only people who really need to understand math are those who choose a career which requires math skills." The first two times she responded to that item, she responded with a two, and on the last day of class, she responded with a one—not at all like her. Carolyn's final responses to items three and 13 suggest that her already-strong beliefs were strengthened during the course of the semester.

Personal Utility. As noted previously, personal utility of mathematics was something very much a part of Carolyn's conceptions of mathematics. This was the area of usefulness of mathematics that dominated her ideas of how math is useful in this world. An example of how mathematics was personally useful to her was in her beginning "What Is Mathematics?" journal entry where she wrote,

> "We use it to help us know how many minutes before we have to leave for school. We need to know if we have enough money for lunch and parking. We need to know if we give the clerk a $5, was it enough to cover the cost and how much change do we get back."

Another example was in her final interview, a semester after the course ended, when she was asked what it means to do math, and she responded, "Doing math is thinking about things like, 'I've got $20 in my purse, is that going to be enough to get the stuff I need at the store?' You know, that's math. And, budgeting your finances and things like that."

There didn't seem to be a lot of change in Carolyn's conception of the personal utility of mathematics during the course of this study. She felt like at least the basic operations of mathematics were something that were very useful to her personally at the beginning of the semester, and she still had that view in her final interview. It was always a dominant part of her conception of mathematics. As she wrote in her beginning "What Is Mathematics?" journal entry, "My opinion of what mathematics is: It is something that helps us to conduct transactions or simple calculations in our everyday life." Again, it should be noted, however, that in relation to the personal utility of mathematics, Carolyn saw mathematics as something beginning and ending with basic arithmetic.

Indirect Personal Utility and Utility for Others. In all of her writings and interviews, Carolyn never specifically addressed how mathematics was used in the world in a way that indirectly affected her. She also did not talk or write much about how math was useful for others. The only thing related to that idea and that showed some awareness that other people were using mathematics that she herself wasn't, was in her final interview when she was asked if what it means to do math depends on who's doing it or where it's done (like a mathematician vs. a student vs. a homemaker or in a mathematics department of a university vs. in a school classroom vs. outside of school). She responded,

"Well, it's all math. It's just different kinds of math, different uses for math. You know, where the engineer is whatever using math all the time in equations, that's just his kind of math. I guess an everyday person uses the common sense kind of math that we use everyday. And, like I said, I'm sure there are professions out there where they use higher math, and you really have to think about all those, you know, equations and stuff like that, but I think it's all math--doing math, but it's just a different type."

This statement seems to contradict what she says earlier about her defining math as only the math she uses. Obviously, she realized that there is a "higher math" that other people use, but apparently, for her own purposes, she only considered the math that she used as "real math"—perhaps it was a definition of math in her world.

Learning and Teaching Mathematics

This study provided a lot of insight into Carolyn's conceptions of how mathematics is learned and how it should be taught. Specific examples will be discussed in the following two sections.

Table 26: Items Relating to Conceptions about *Learning and Teaching Mathematics* from the Mathematics Inventory with Carolyn's Responses

Item	Beginning	Middle	End
10. Teaching mathematics should not be too difficult since a textbook already has the lessons prepared.	3	2	1
18. Being good at mathematics is not required to be a good teacher of math.	3	4	4
22. The best way to learn math is to memorize the rules.	1	2	2
23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.	5	5	5
24. In learning math, it is important to master topics and skills at one level before going on.	4	2	2

Learning Mathematics. The Mathematics Inventory provided information about some of Carolyn's conceptions of how mathematics is learned and how those conceptions may have been altered somewhat during the semester (See Table 26). For item 22, "The best way to learn math is to memorize the rules," Carolyn answered one at the beginning of the semester, but had moved to two the next two times she responded to that item. When asked how she would respond to that item during the mid-semester interview, she said,

"Oh no. I don't think that's at all like me. But, I'm not going to put one. I'm going to put two. Because I mean, I think you need to

> know some rules, but that's not always the best. I'm still going to stick with two, because I think at times it's best to just know some rules depending on what you're learning or need to learn."

When she was reminded that she put one the first time she did the inventory and was asked if she had changed, she responded, "When I said that, I probably went 'not at all,' because I thought that rules weren't important to help. But, now I see that sometimes you do need to have some rules to rely on." This change may have been influenced by discussions we had in class in which we talked about where standard algorithms came from, why they were the standard algorithm (because they were the most generalizable and efficient), why it is useful to know them, and why at some point--after students have gained an understanding of why they work--it is useful and desirable for the students to learn them.

Another item relating to learning mathematics was item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key." Carolyn strongly agreed with that idea throughout the semester.

For item 24, "In learning math it is important to master topics and skills at one level before going on," her response changed from a four at the beginning of the semester to a two at the middle and end of the semester. When asked why her ideas about that were changing during the mid-semester interview, she explained,

> "Well, I remember this one article we just read about not always having to start out the year with review and spending the first two months with things that they've done before. You can go on to something. Like you can jump ahead. You can jump to geometry, and you can teach them something about that, but you can still pull in addition and subtraction. That might not be a good example, but you can pull in other things that they might not be completely set and mastered, but they're going to be able to practice it some more through another topic, too."

The main way that Carolyn seemed to believe that math was learned was when it was applied to outside-of-school situations. As noted in the Usefulness of Mathematics section, that was when Carolyn considered it math. Also, when she was asked in her final interview what the best way to learn math was, she responded,

> "Um, using it. At first you have to, of course . . . . I'm a very, I guess I'd say I'm a real hands-on person, and I have to see things and see how they work and feel how they work. So, I think that's how I learn things and how I would learn math. And, other people probably wouldn't be that way. They might just learn math by memorizing things. That never works for me."

Of course, Carolyn's ideas of how mathematics is learned were directly connected to how she thought mathematics should be taught—some explicitly and some implicitly. These conceptions will be discussed in the following section.

Teaching Mathematics. Carolyn had some definite ideas about how mathematics should be taught. Some of them she had at the beginning of the semester, and some of them seemed to develop during the semester. However, her ideas were definitely not all influenced by the course. Carolyn had the interesting and unusual-for-this-group perspective of seeing how her own children were learning and being taught mathematics in elementary school while she was taking this course. One example of how this perspective influenced her ideas was her responses to item 10 on the Mathematics Inventory (See Table 26). For item 10, "Teaching mathematics should not be too difficult since a textbook already has the lessons prepared," Carolyn agreed with it less each time she responded to the inventory. At the beginning of the semester, she put a three; in her mid-semester interview, she responded with a two; and on the last day of class, she put a one. In her mid-semester interview, when she was asked about that item, she said,

> "Now? I'm going to say that that's really not like me. Maybe not a one. Maybe a two. Cause I might still want to rely on that

textbook somewhere, but I want to get away from that textbook. And, so I'm not going to want to use it. I want to look at some of these other things that we're learning as more important than that textbook."

When she was asked why she was changing on that, she said, "Well, there's things in class that we are doing it, but then my personal experiences with my kids, the things that are happening with them, too." When asked for a specific example, she said,

> "Well, like right now, my daughter is in a math group. They divide them up in groups, and they may or may not stay with their same teachers, and then they go to one of the other fifth grade teachers. So, she goes to one of the other fifth grade teachers, and they're in groups by ability. Her group, they have three math books. I don't know why they have three math books, and they use them--two of them--I think like every other day, and the third one every once in a while. All out of books. And, I just don't think that she's really getting it from that. I think she needs more than that. Like a couple years ago she went to a tutor—a special math tutor—and she did things like we do in class. She played with blocks, and she played with dice and different things like that. And, I think that's better than having to do just computations in the book. So, that's part of what's changing me, too."

Another conception that Carolyn had about teaching mathematics was illustrated by her responses to item 18, "Being good at mathematics is not required to be a good teacher of math." She put a three the first day of class, and then put a four during the mid-semester interview and on the last day of class. During the mid-semester interview, she said,

> "Okay, so I would probably agree with that, like a four. But, I mean you have to still understand math—have some basic—to teach elementary math. You don't need to have as much as you would to teach middle school and high school math, but you still need to know math, and you need to know how to think about math, and how to work through mathematical ideas and problems. And, you know, find answers."

When asked if she thought her idea of needing to be good at math to be a good math teacher was changing at all, she said,

> "Probably. Because I probably used to think that I had to know everything. You know, I had to know everything and all the answers, and if I didn't, I was in trouble. But, now I don't really think I have to know everything."

In the open-ended section of the Mathematics Inventory, there were a couple of questions related to conceptions of teaching and learning mathematics, especially the role that the teacher's attitude played in whether students were successful in math. At both the beginning of the semester and at the end of the semester, Carolyn felt strongly that the teacher's attitude had an effect on the students' learning. One of the questions relating to this was, "What kind of influence do you think the teacher plays in the success of students in mathematics? On the first day of class, in response to that question, Carolyn wrote, "Teacher attitude is very important. If the teacher can make math fun and useful for students, then they will put more effort into it and get more out of it." On the last day of class, Carolyn's response to this question was, "They have all the influence in the world. If the teacher is not excited about math, how can the students be?"

The other question related to the influence of the teacher's attitude was, "Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not? How about a teacher's attitude about teaching it? Explain." On the first day of class, Carolyn's response to this question was,

> "Yes. I'm kind of scared of math, since my childhood experience with math was difficult. I hated not being able to memorize the times tables and felt dumb and embarrassed. I want my students to feel good. I want math to be fun for them, not torture."

On the last day of class, for this question, Carolyn wrote, "Yes. If the teacher hates math, they could not teach it well. They would not be excited about it and won't put forth a real good effort."

Probably the thing that provides the best picture of Carolyn's views at the end of the semester of how mathematics should be taught was a journal entry that was assigned the last day of class. For this journal entry, the students were asked to describe their current philosophy of teaching mathematics. Carolyn's entry, which follows, provides much information about how she thought math should be taught:

> "I feel that a math classroom should be a happening place. Children should be working, talking, constructing, building, sharing, and thinking about a variety of ideas. Children should be sharing their ideas, thinking out loud, discussing ideas, and explaining their thinking. It should not be children sitting quietly in rows, completing problems #1-15 on page 76.
>
> "Rote memorization and page after page of calculations is not the way to create students who are excited about math and eager to explore new ideas. Activities that require them to think, reason, and make sense of math are important. Opportunities to do activities like we did in class or like I planned in my math unit are good. They give children the chance to think and problem solve.
>
> "Students can draw about math, write about it, talk about it, and these things will help students think about mathematical concepts. It helps them to remember things and have an organized way to think about information and ideas. Children remember things better when they have a variety of ways to think about it vs. simple rote memorization.
>
> "I have looked at a lot of books on math lately, and most of them seem to be simple worksheet kinds of books. I want to avoid these and find books that have activities more like the Marilyn Burns'. I like open-ended lessons where there is not necessarily one correct answer. This allows students to reason, to explain how they thought about a problem and what processes they went through in thinking about the problem. In other words, process is more important to me than product. A correct answer does not mean a student understands what they are doing and why. By the same token, a student could have an incorrect answer and still understand what they're doing and why.
>
> "Manipulatives are a good tool to use because some students may think about mathematical ideas more easily when they can physically see, touch, and work with the items. It is important to remember that not all students may benefit from this. When using manipulatives the teacher must make sure that the use of them makes sense in the activity and promotes the concept you are trying to teach.
>
> "As I stated in my math unit, I feel that my assessment techniques are weak and lacking something. Other than observing students, listening to them talk about mathematical thinking and looking at written work and pictures too. I could not think of any

> other ways to assess their math learning. I still feel there is more to use and want to consciously look for these ideas."

Carolyn still held on to some conceptions similar to those expressed in her philosophy of teaching mathematics on the last day of class a semester later. In her final interview, Carolyn was asked what her ideas were about how she thought math was most effectively taught and how she wanted to teach it. She responded,

> "Well, I kind of talked about that a little bit, too, how there's different ways to teach it. You know, it doesn't always have to be memorize this. Do this worksheet of 50 problems, and tomorrow we'll do another worksheet of 50 problems, you know. You can do some stuff like, you can work on some memorization stuff, but I like the idea of having the kids work in groups and solve problems and think about things. Like I remember some of those worksheets you brought in, some of the AIMS ones and stuff where the kids would work together as a group and think about things and work through a problem to help them understand. And, I think some of that kind of thing is good, and there's room for hands-on things and manipulatives, and there's times that they're going to want to work with that stuff. So, I think I want to give kids a lot of different ways to look at it, because I know from my own personal experience, I learn different than other people do, and one person might not be able to memorize, but they can understand how you get to that, and they can always figure it out if they have a manipulative or something to work with. Just presenting it in different ways. And, I think you'll reach more people that way, too."

Carolyn's Attitudes toward Mathematics & Teaching Mathematics

Attitudes toward Mathematics

As noted in the introduction of Carolyn, she began the semester with quite negative attitudes toward math. These seemed to be more a reflection of her confidence doing math—especially math other than basic math—than her feelings toward math as a subject and its usefulness in life. By the end of the semester, it seemed that her attitudes toward mathematics were much more positive.

Her beginning concept map is a good example of an expression of her attitudes at the beginning of the semester (See Figure 46 on page 305). Her entire map is focused on her experience with and attitude toward mathematics. One attitude it expresses is whether math is useful or useless. It appears from her configuration that she believed basic math was useful and higher math was useless. Another attitude on her map is her feeling toward problems. She described them as frustrating, challenging, and rewarding. This was not completely negative since apparently she found some problems rewarding. The other two sections of her concept map seem to be related to her experiences with math as a school subject which affected her attitude. She had one section for "teachers" which she described as either "impatient" or "encouraging." The other section was for "peers" which she subcategorized into "really smart ones" or "ones like me." Under "really smart ones," she broke them down further into "nice" and "arrogant." It is interesting to note that none of her final concept map is about her attitudes toward mathematics (See Figure 47 on page 306).

In the Mathematics Inventory, there were several items relating to attitudes toward mathematics. Carolyn's responses to these items at different times during the semester also provide information about her attitudes toward mathematics (See Table 27 on page 322) and how they were changing.

For item one, "I enjoy doing mathematical tasks," Carolyn put a three on the first day of class and at her mid-semester interview. To explain that answer, she said, "I don't think I'm ever going to really, really love doing math, but it'll be okay." However, she did raise her response a little on the last day of class to a four.

Item number two, "I have always enjoyed mathematics," is one that we wouldn't expect to change too much since it includes the past which cannot be changed, but Carolyn changed her response slightly throughout the semester.

Table 27: Items Relating to Attitudes toward *Mathematics* from the Mathematics Inventory with Carolyn's Responses

Item	Beginning	Middle	End
1. I enjoy doing mathematical tasks.	3	3	4
2. I have always enjoyed mathematics.	2	1	2
6. I feel successful when I am engaged in mathematical tasks.	3	3	3
7. Doing mathematics is usually very frustrating for me.	4	2	3
15. I understand elementary level mathematical concepts quite well myself.	3	4	4
25. I can handle basic math, but I wouldn't do well at advanced mathematics.	5	3	2

She first put a two, then a one, and finally a two again. However, during the mid-semester interview in which she responded one, she was asked if she had enjoyed math any more since she started the semester. To that question, she responded,

> "Um, yeah, yeah, because some of the little things she's done with us in class have been fun. It hasn't seemed like, 'Oh gosh, this is math. We're doing this really tough math thing.' It was just an activity, and that was more fun and something I could relate to. So definitely."

For item six, "I feel successful when I am engaged in mathematical tasks," Carolyn answered right in the middle each time she responded to the inventory. This seems consistent with other times when she talked about or wrote about feeling confident when doing basic math, but not higher math.

Carolyn's responses to item seven, "Doing mathematics is usually very frustrating for me," varied each time she responded to the inventory. On the first day of class, she put a four which was between "somewhat like me" and "very much like me." In the mid-semester interview, she put a two which was between "not at all like me" and "somewhat like me." On the last day of class she settled right in the middle with a three--"somewhat like me." When she was doing the mid-semester interview and asked that question, she said,

> "Now I would put probably a two because at least the math I'm doing now doesn't frustrate me." When asked what she thought she put before, she said, "When I did that I was probably thinking

about algebra or higher than that, and I was very frustrated with that. This isn't frustrating. This is more useful."

When asked if it was just that it's useful or if there were something else, Carolyn said,

> "No, it's just that it's useful. I have this thing about it has to be useful. If it's not useful, then I don't see a value in [it]. And, I really get frustrated if I don't know what I'm doing it for or what I'm going to use it for."

For item number 15, "I understand elementary level mathematical concepts quite well myself," Carolyn went from a three on the first day of class to a four during her mid-semester interview and on the last day of class. During her mid-semester interview when told that she had moved from a three to a four, she said, "I think I'm understanding better." The interviewer then asked her if she thought she had "seriously changed a little bit on that." She replied,

> "I don't know. I'm not sure if it is that I understand the concepts better than I did then. I'm just understanding that it's . . . well, it's different when you understand that there's a lot of different ways to approach them, and you don't have to get stuck in one--like the way I was taught. And, that might change that a little bit why I think I understand that."

She further explained,

> "Because there's different ways to look at. If you don't understand it one way, you can look at it through a different way. That's like something we did today about adding different ways instead of adding like the standard [algorithm--provided by interviewer]. Right. You can like add. Of course I can't remember an example. Like you can add the 400 and 200 together and get 600 at the hundreds, and then you can add the tens, and the ones, and you come out with the same answer. So, there's different ways, and that's a concept, and I understand it a little bit better by looking at it through other ways."

She was then asked what she considered to be elementary level mathematical concepts, and she said, "Of course, addition, subtraction, multiplication, division, um, some geometry, some statistics (you know, working with data and things like that), fractions."

Another place there was some change in Carolyn was with item 25, "I can handle basic math, but I wouldn't do well at advanced mathematics." On the first day of class, she put a five--that it was something very much like her. During her mid-semester interview she said that she was going to stick to the middle of the road and put a three. She thought that's probably what she had put before and was surprised that she had put a five. When told that she had put a five, she said,

> "Really? I really thought I'd do bad, huh? I don't know why that would change. I guess because when we first took this, I had just had the algebra class in the summer before we took this. So, I was probably still suffering from that bad taste in my mouth. I thought, 'Ooh, higher math. I can't do it.' But I look at it differently since then, I guess. Like I told you, I did really bad in the introductory review, but then I did a lot better in the second one. And, I didn't do great at it, but I did a lot better, and I guess that I just think that maybe if it's taught the right way, and it's not necessarily me, that I might be able to do advanced mathematics.

Carolyn told the interviewer that she considered algebra the break between basic math and advanced math. On the last day of class, she put a two, suggesting that she felt even less strongly that she wouldn't be able to learn advanced mathematics.

Attitudes toward Teaching Mathematics

The Mathematics Inventory and the final interview were the main sources of information about Carolyn's attitudes toward teaching mathematics. For each of the items relating to attitudes toward teaching mathematics from the Mathematics Inventory, Carolyn's responses changed somewhat from her responses on the first day of class (See Table 28). Overall she seemed to become more enthusiastic about the prospect of teaching mathematics as the semester progressed.

Table 28: Items Relating to *Attitudes toward Teaching Mathematics* from the Mathematics Inventory with Carolyn's Responses

Item	Beginning	Middle	End
8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.	3	4	4
9. I feel enthusiastic about mathematics and about teaching it.	3	4	4
12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.	3	2	1
14. I feel confident in my ability to explain mathematical concepts to children.	3	3	4
16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.	4	3	2
19. I want to teach mathematics so that I can help students get excited about it.	3	5	5
20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.	4	1	1
21. I expect to regularly teach mathematics as an elementary teacher.	4	5	5

One place this increase in enthusiasm was evident was in her mid-semester interview. Carolyn said that she was becoming more excited about teaching mathematics because she was "[l]earning different ways to teach it and learning more support for teaching it--resources and things you can do." This increase in excitement and enthusiasm was reflected in her answers to item numbers 8 and 9, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher" and "I feel enthusiastic about mathematics and about teaching it." For both of those items, she increased her response from a three the first time she took the inventory to a four the other two times.

Item number 12, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school," further supports the idea that there was in increase in Carolyn's confidence to teach math during the semester. Carolyn answered that it was less like her each time she responded to that item. To explain her change in response, she said,

> "I think I will encounter stuff that I don't understand, and that's probably going to be okay. I guess I'm feeling more comfortable that I don't have to understand it all. I don't have to be this authority and have every answer. That I can be confused, too. Every once in a while. As long as I learn something from it."

For item number 14, "I feel confident in my ability to explain mathematical concepts to children," Carolyn put a three on the first day of class and during the mid-semester interview, but she moved it up to a four on the last day of class. When asked that item in the mid-semester interview, Carolyn said,

> "After being in class today [her practicum class], I don't know. Today I had to try to explain for our group, the algorithm for multiplication, and it was two digits times two digits, and as we were writing this down on our overhead, we all got confused. You know? And I realized that sometimes it's really confusing to try to explain things. You can understand it in your head, but it's different to explain it verbally. So, I think that's going to be kind of difficult. So, I'm going to put me right in the middle at three."

The interviewer then asked her if she thought that the more she was thinking about that, the more unsure she was. Her response to that probe was,

> "No, I'm thinking that it's something that you need to work on. And, like tomorrow the two of us actually teach our math lesson in the class, and so that's going to be a real indication of how well we can explain things. I think that's something that just takes a lot of time and practice. You have to work at it, and if you're confused, and you're confused with the explanation, you have to look at it again, and you have to change it. But, I just think it's something that's going to take work."

Carolyn's responses to item 20, "As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer," suggested a pretty dramatic change from the first day of class and is consistent with and related to her thinking about item 14. On the first day, she put a four indicating that it was close to something very much like her. During the mid-semester interview and on the last day of class she put a one--not at all like her. During her mid-semester interview she said that she thought she had probably put something higher than that the first day, but guessed that it would have been a two or three. When she was told she had put a four, she said, "Wow! I would have really been embarrassed, huh?" She was then told that she had a five and crossed it out and then put a four. Her response to that was,

> "Yeah. You see I'm not going to be embarrassed now, because now it's going to be a way to show that that's what you do with math. You're confused, or you might be confused, and you might not understand, but that's how you're going to learn. And, it's okay to not know--even as a teacher."

Item number 16 was another one where Carolyn felt like she was changing. Each time she responded to this item, "I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students" she moved a step closer to "not at all like me." In her mid-semester interview, she explained why she thought her response to that item was changing:

> "I think coming from my really [sic] dislike and my bad feelings about math in the past, that that's one of the harder things to overcome, and I'm slowly overcoming it and getting more comfortable with it. It's just, it's going to take a while. I think math will probably be still—for a while at least—the hardest subject I teach. Because reading, you know, I always did good in reading and those kinds of things. So, reading, writing, social studies, and all that kind of stuff is going to be easier, I guess. And, math is still something that I'll struggle with a little bit more. But, I'm making progress."

Perhaps the most compelling data that supports the assumption that she became more excited about teaching mathematics during the semester was her responses to item number 19. In her mid-semester interview, regarding item number 19, "I want to teach mathematics so that I can help students get excited about it" Carolyn thought (correctly) that she had put a 3 the first time, but she said she wanted to go up to a 5 ("very much like me") for it. That attitude remained the same in the final inventory. She explained her changing attitude this way:

> "Yeah! Because, I'm seeing more how much fun you can have with math, and I was never excited about math, and now I think that I really want my students to like math. I really want them to enjoy

math and see it the way I did. Because the things that we're doing and learning about. You know, the little lessons and things that are helping me to realize that it really can be that way. And, I wasn't exposed to stuff like that when I was learning math. So, that's changing my opinion."

The open-ended questions of the Mathematics Inventory also provided some information about Carolyn's attitudes toward teaching mathematics and how they may have changed during the semester. For the question, "What do you consider to be your greatest strengths in relation to being a mathematics teacher?" on the first day of class, Carolyn wrote, "I feel math is important, and I can show students how it would be useful and relevant in their life." For the same question on the final day of class, Carolyn wrote, "My philosophy about teaching math. What I feel is important for children to learn. After this class I have learned a lot of good ideas and how to find good ideas."

For the question, "What concerns you the most about teaching mathematics to elementary students?" on the first day of class, Carolyn wrote, "Children who are like I was. They have a hard time understanding math concepts or memorizing math facts. How can I help them over these hurdles?" On the final day of class, her response to this question was, "Being clear. Knowing if students really understand or grasp the concepts."

For the two questions noted in the previous two paragraphs, it's hard to know if her responses represent any changes. At the end of the semester, she may very well have still believed what she wrote the first day since they were not ideas that are mutually exclusive to her final responses. Other ideas related to those questions may have just come to the forefront by that time.

Another of the open-ended questions from the Mathematics Inventory relating to attitude toward teaching mathematics was the question, "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." On the first day of class, Carolyn's response was, "I think my lack of a love for math will make it more difficult for me to teach math. I hope to find new creative ways to teach math that I am excited about, therefore my students will be also." On the final day of class, she wrote, "I think math will always be a subject I need to be conscious about my attitude. I will probably put more time into planning math than in other subjects, just to make sure I'm on track." Carolyn didn't show much change in her response to this question. Both at the beginning and the end of the semester she realized that her "lack of love for math" would make it more challenging to teach math than if she had a great deal of love for it, but she also seemed hopeful that it was something she could overcome and compensate for.

In the final Mathematics Inventory, the students were also asked, "Have your ideas about mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how?" For this question, Carolyn wrote, "Yes. I feel more confident in my abilities and don't feel like I'm hanging on a limb by myself." They were asked the same question with the word "attitudes" substituting for the word "ideas." For that question, Carolyn wrote, "same" with an arrow pointing to her response for the previous question.

In Carolyn's final interview which was a semester after the course ended, she was asked how she felt about the idea of teaching mathematics in an elementary school. She responded,

> "It still is a little scary, you know, because I'm not a great math person. Never have been. So, it's still kind of scary. And, I think a lot of people have this idea that teaching elementary math is going to be really easy. You know, you know that stuff, and it's going to be easy. But, from teaching our science lesson this last semester, I found that it's harder to teach things you know, because *you* know them. You understand them, and the way you might try to explain it to someone else, to a younger child, is too complicated, too hard, or you don't get through to them."

She was then asked if her feelings about teaching math had ever changed, and she said,

> "I'm not as scared as I used to be because, I mean, I've learned some different ways to teach math. And, just knowing that there's things out there to help you. There's resources or stuff you can look at. There's books you can read. You know, there's a lot of things that can help you teach math and that it's learning for me, too. You know, I'm not going to do it perfect the first time, but as long as I am aware of what I'm doing and realize when I need to do something different or might want to make a change here or there that I can grow with it. So, it doesn't seem as scary anymore, because I know I'm not perfect, and I'm learning, too. So, I feel more confident that way."

Major Themes in the Case of Carolyn

Three major themes emerge from a study of the case of Carolyn and her conceptions of and attitudes toward mathematics. One theme is the importance of the personal usefulness of mathematics for Carolyn. Another is the change in positioning of Carolyn. The third major theme is Carolyn's increase in her confidence and desire for teaching mathematics. These themes will be elaborated in the following summaries.

Importance of the Personal Usefulness of Mathematics. A major theme in the case of Carolyn was the importance of the usefulness of mathematics for her personally. Her ideas and attitudes related to this area did not seem to change much during the semester. They were very much a part of her at the beginning of the semester and when the study concluded. The theme of the personal usefulness of mathematics was a significant part of her ideas of the nature of math, her ideas about how math should be taught, and her attitude toward math and teaching math.

A Change in Positioning. One of the biggest changes evident in Carolyn during this semester is a change in her positioning from viewing mathematics as a student would to viewing it from the perspective of a teacher. This was particularly evident in her beginning and final math concept maps (see Figure 46 on page 305 and Figure 47 on page 306). Her beginning concept map was almost entirely focused on her experience with and relation to math as a student. It included sections for her views of her teachers and peers as well as her view of math problems and which math was useful and which was useless. Her final concept map portrays math more generally and does not focus as much on her personal relationship to math. It includes things that are much more like what would be expected from someone who was thinking about math generally or as someone who was going to teach math would look at it.

It also seems that the shift in her beginning and final "What Is Mathematics?" journal entries is evidence of a shift in positioning from a student mode to a teacher mode. Her beginning "What Is Mathematics?" journal entry was almost entirely about math as it was useful to her personally. In her final "What Is Mathematics?" journal entry, she again looks at math more generally in the world and not just its relation to her. She also shifts to a big emphasis on the processes used in mathematics which is something that teachers are more likely to be focused on.

More Confidence and Desire for Teaching Mathematics. At the end of the semester, Carolyn seemed to have much more desire to teach mathematics as well as more confidence in her abilities to teach math. There was much evidence at the beginning of the semester that Carolyn felt some trepidation about the thought of teaching mathematics, especially since learning mathematics had not been a very positive experience for her, overall. However, all the data sources—the Mathematics Inventory, the journal entries, the interviews, and to some degree the concept maps—seemed to suggest more confidence in the thought of teaching math and more desire to teach math. She claimed several times in her writings and interviews that she was gaining more confidence to teach mathematics than she had when she entered the class. She also expressed a strong desire to teach mathematics so that she could help students get excited about it and not have an experience like she did as a student. There seemed to be a change in Carolyn's idea of what she considered success—moving from getting the right answer to understanding, as was evidenced in the following quote from her mid-semester interview:

> "Because successful is reaching a level of understanding. Success isn't always getting a right answer. Maybe that's why that's changed a little bit, because I took successful more as always getting the right answer—knowing the right answer. Now, I look at it more as understanding—understanding how to get there, even if you don't get to the right spot."

It may be that this change in her conception of success was the engine that drove other changes. It may have helped her feel more successful in her mathematical thinking. It may have also helped her imagine a classroom where the process was more important than the final answer, and that was something she believed from the beginning of the semester, as was noted in the "processes" section of her analysis. She probably felt more enthusiastic about teaching in that type of a classroom than a classroom that was like the image she had from her own experiences as a student.

Elizabeth: Early Positioning as a Teacher

Elizabeth was a 22-year-old student with long, curly brunette hair. She was average height with a slender build. Elizabeth was majoring in child development and teaching. She had completed her internship in child development at the laboratory preschool the previous year, and she just had to take TE 401 and 402 to finish the teacher education part of her major. She described herself as having "been around a while."

Elizabeth felt fairly successful as a student in mathematics in elementary school. She felt especially successful when doing math that dealt with money—except in taking a percentage off a price. In her mid-semester interview, she described her experiences with math in elementary school this way:

> "Through elementary school, I had like the general eds., the basics, you know, starting off with addition and subtraction, and fractions, multiplication, and division. I remember math kind of in a hierarchy. Every year you learned kind of a new concept, and you review and go back a little bit and then build on that, and I remember being very successful in anything that dealt with money. They seemed to really make that worthy of our type of studying, but also I think we had more reason to enjoy it, you know, it was something that we could experience in our lives, that we had experience with money. And, the one thing I felt lacking was the area of, still now, percentages, and I go into a store, and I say, "Oh, 25% off." And, I can't quite even think what that means, so I always go from a base of ten, and that helps, and I think that maybe in elementary school if they would have helped me a little more, then I would have learned from that."

In junior high and high school, her success with math continued until she got to geometry. Her description of her experience with mathematics in junior high and high school follows:

> "In junior high, I was placed in one of the more advanced math classes. You start off with like algebra, and the advanced algebra, and then I went into geometry. But, in geometry, I started to get stuck. I started to have problems, so I went into the regular level. Then, in high school I was in geometry and algebra II, and I never got into calculus. I just figured I'm done here. I didn't have to take any more unless I wanted to, and I wasn't doing great in chemistry at the time, so I decided I didn't want to put any more on my plate than I've got. So, I basically went into general level. I was considered advanced when I was younger, but it got a little bit rough for me because I saw some of the teachers who wanted to really push the students who were excited and into it in the Math Olympiad and Science Olympiad, and they were really successful in it, and I felt that if I was the one who was struggling, and I would come in and ask for extra help, they had their attention sometimes more to the kids that really do well. So, that was disappointing, but I learned a lot in my gen. ed. classes, too. Except for one year we had a teacher that was gone the whole year, so we didn't really learn anything with all the substitutes. So, it was basically until algebra II, through geometry and into advanced algebra, and that really was pre-calc.

When asked how she did in her math classes, Elizabeth said,

"I was usually really successful. Mostly A's, some B's, and then when I was having that struggle with geometry, I was getting a C, and that's why I changed classes, because that wasn't acceptable to me. I said, 'No, I need to be in the B range.' In high school, that was always very important to me, and in college, I've had a little bit different agenda of things, but that was, you know, not successful for me. So, I wanted to make sure I was in the B range.

She was also asked what she thought made geometry such a bad experience for her, and she said,

"I think it was like postulates and theorems, and one problem for me would take up an entire page, and an entire night's worth of homework was just overwhelming. I was used to just cranking them out, you know, 24 problems a night you could go home and do your math, and my dad would help me out a lot. But, to think of applying all these theorems and postulates to these different figures, to one problem, to me just felt like too much. I mean, I understood a lot of it, and once I got done with theorems, I could explain them without having to use specific words. . . It seemed black and white to me. It was frustrating. But, once I really got the concepts down, then I really felt successful, and in the beginning, I was like, 'Gosh, this is overwhelming. I've never had math this way.' I've always thought it was plug into a formula, and this isn't really a formula to me."

In college, Elizabeth experienced more difficulties with math. She ended up only taking Math 201 and 202, but she had to take Math 202 twice to pass it. She also didn't feel like she was really learning math in her college classes. Following is her description of her experiences with math in college:

"I was told as a freshman I wouldn't have to take any math. But, then that changed, and there was Math 201, 202. First it was just 201, then it was 201a and 202, and I had another predicament when I was going into 201, they looked at my college entrance exams and said, 'Oh, you're only one point away of passing out of Math 116, entrance level math. So, I guess you'll have to go take that class, too.' So, instead of taking none, I've had to take three. So, I went and talked to the dean, he said, 'Oh, we'll override you, and you'll just have to take 201 and 202.' So, I took them, and I wasn't very successful here at all. I was in the C range. I passed out of 201 eventually. 202 I took in a semester I had some other hard classes, and I didn't receive a passing grade; so, I came back in the summer, and I took 202 over again. And, I was in a study group, and we all felt that we were struggling. They said, 'Oh, you've taken this before.' And, I said, 'Yes, it's one of the classes I've had to repeat.' And they were like, 'Oh, my gosh.' So, we helped each other. We all struggled through it and got 2.0s, and I was kind of glad to get that one over with. But, I was kind of disappointed the way I had been learning about this new way of teaching students and how it's not implemented at all in the way I've learned--at all. So, I was always looking at an example and trying to, um, solve all my problems compared to the example that was given, and not really learning the concepts and why it works. It was just like, 'Okay, I remember seeing that. I'll plug this in.' You know, what letters will fit into that. So, it wasn't really learning. I think it was more memorization.

As this study began, Elizabeth felt fairly confident with her ability to understand most basic math, but she didn't enjoy it, and she was particularly frustrated with her then-recent experience with mathematics while in college. In her "Reflections of Mathematics Experiences" journal entry which was assigned on the first day of class, Elizabeth wrote,

"I have always understood the basics of math. I've just never enjoyed it. I still have a struggle with percentages and taxes, things that I think should be taught more thoroughly in school in the elementary stages. The use of money was well taught I thought, but percentages always seemed like a tougher concept to me.
"Right now I am extremely frustrated with math, especially at this university and its requirements for elementary education majors: learning by formula and example and simply plugging in numbers is <u>not</u> learning. It seemed like that was all I could do in the time given to 'learn' a new unit and show my skills for the test and pass it accurately enough to meet the requirements of the College of Ed for graduation."

Elizabeth's Conceptions of Mathematics

The data collected from Elizabeth throughout the study provide some insights into her conceptions of mathematics. They will be discussed in the subsequent sections following the format of the framework designed to analyze this study. Elizabeth was very loquacious in her interviews, and although she seemed to wander a bit at times, she provided much information about what she was thinking.

Elizabeth's beginning and final math concept maps are shown here (See Figures 48 and 49). They will be referred to in various sections of the analysis.

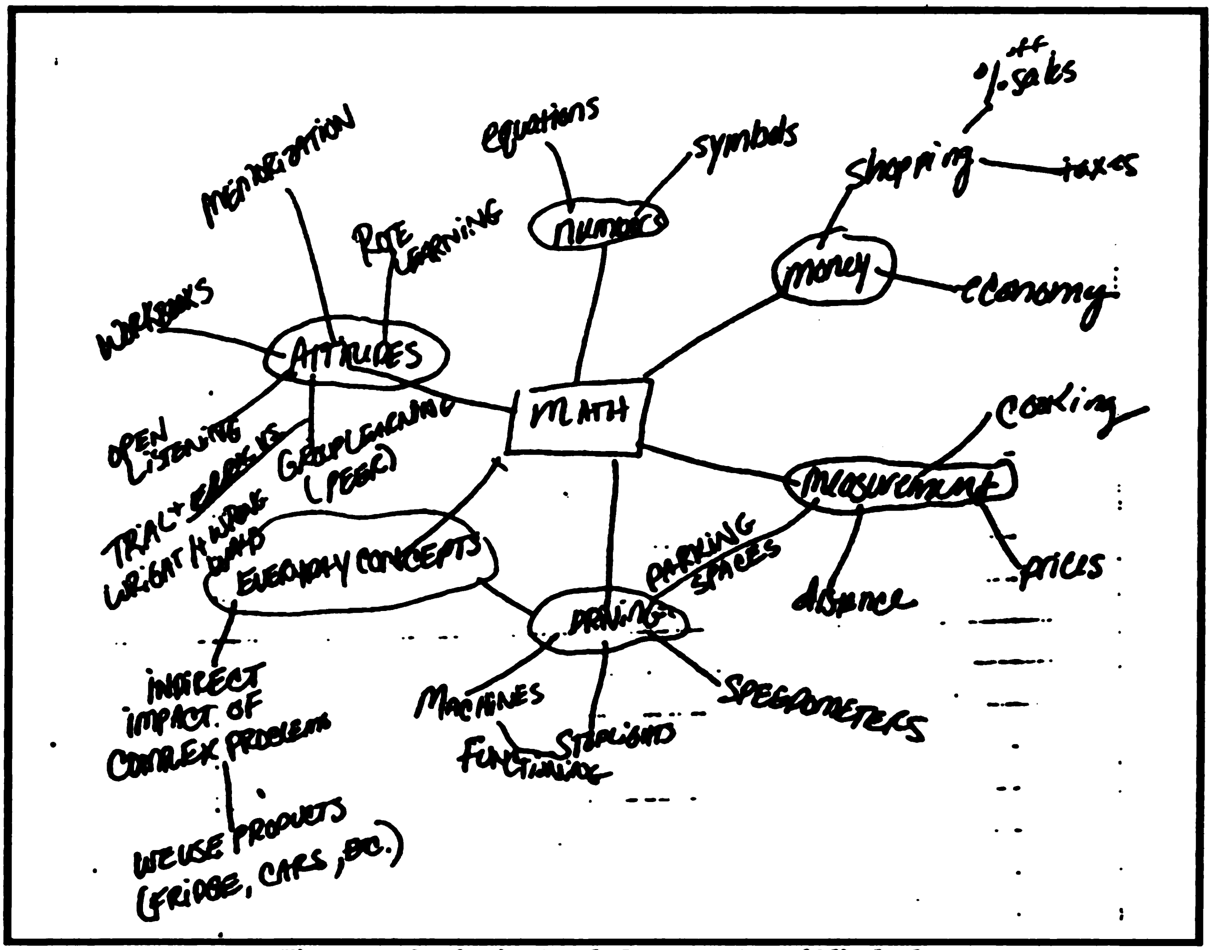

Figure 48: Beginning Math Concept Map of Elizabeth

Nature of Mathematics

In this section, I examine Elizabeth's conceptions of mathematics related to the nature of mathematics—its content, processes, representations, and characteristics—throughout the semester.

Content. In Elizabeth's beginning "What Is Mathematics?" journal entry, she included a sentence that gives some information about her ideas of the content of mathematics: "Strange words like ratio and quotient were used and it had many different faucets [sic], like multiplication, division, addition, subtraction, the times tables, algebra, geometry (with introduction of shapes and

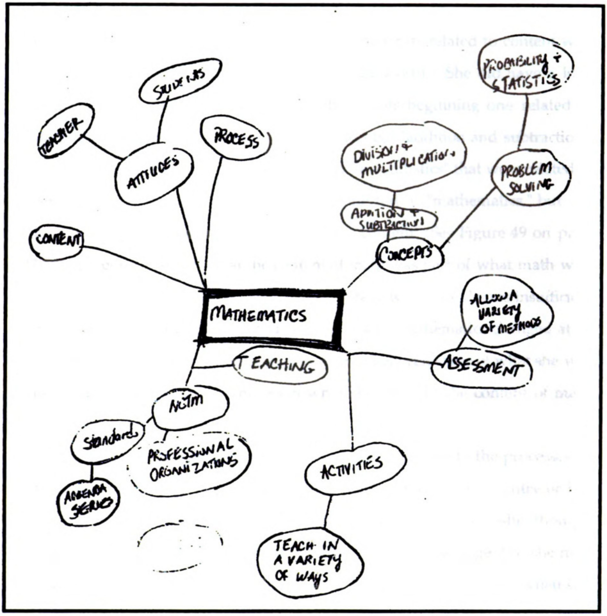

Figure 49: Final Math Concept Map of Elizabeth

theorems), then the really hard stuff like calculus came along." In her beginning concept map, she included many uses of mathematics which are related to content such as "measurement," but there was no listing of specific content of mathematics (See Figure 48 on page 336).

At the end of the semester, Elizabeth's focus of what math was changed somewhat in both her "What Is Mathematics?" journal entry and her final concept map compared to what they were at the beginning of the semester, but content was still at least a small part of that focus. In her final "What Is Mathematics?" journal entry, the only sentence she wrote related to content was, "Numbers are meaningful and have a place in the world." She did have a little more in her final concept map than she did in her beginning one related to content, though. She included the items "concepts," "addition and subtraction," "division and multiplication," and "probability and statistics" that were related to content. She also had the word "content" connected to "mathematics," but she didn't include any items that she connected to "content" (See Figure 49 on page 337). Apparently she knew that the content of math was part of what math was, but she didn't elaborate much on what that content was. The data is insufficient to determine if Elizabeth's ideas about the content of mathematics changed at all during the semester. The differences were more likely reflective of what she was thinking about at the time rather than what she thought the content of math was.

Processes. Elizabeth didn't include anything related to the processes of mathematics in either her beginning "What Is Mathematics?" journal entry or her beginning concept map. However, in her response to why she thought geometry had been such a hard subject for her, as noted on page 334, she may have given a glimpse of what she considered *a* process of mathematics when she said, "I've always thought it was plug into a formula, and this isn't really a formula to me." Elizabeth seemed to have been thinking a little more of the processes of mathematics at the end of the semester. In her final "What Is Mathematics?" journal entry, related to processes she wrote, "Problem-solving techniques should go beyond story problems and filter into everyday life, giving students the necessary skills to tackle everyday situations with math." In her final concept map, she had "process" connected to "mathematics," but again as with her item of "content," she didn't have anything else connected to "process." She did also have "problem solving" connected between "concepts" and "probability and statistics" (See Figure 49 on page 337).

Representations. Elizabeth included some of her ideas about the way math is represented in her beginning "What Is Mathematics?" journal entry. She wrote, "I've always thought math was equations and problems to be solved. The 'language' of math was written in symbols of >, <, or =; +; -; x; +." In her beginning concept map, she also included "numbers" with "equations" and "symbols" connected to it (See Figure 48 on page 336). Also, when she was talking about her mathematics background in her mid-semester interview, she talked about "postulates and theorems" in the quote on page 334 that was referred to in the previous section. This was probably the most attention anyone in the class gave to representations of math at the beginning of the semester. At the end of the semester, there was less emphasis on the representations of math in Elizabeth's conceptions of mathematics. She didn't include anything related to representations in either her final "What Is Mathematics?" or her final math concept map.

Characteristics. The only place we really get any information about Elizabeth's conceptions of the characteristics of mathematics is from the items related to characteristics of mathematics in the Mathematics Inventory (See Table 29). The only other thing in all of the other data sources that is somewhat related to characteristics of mathematics is an item she included on her beginning concept map. She wrote "trial and error vs. right and wrong ways" connected to "group learning" which was connected to "attitudes" (See Figure 48 on page 336). Apparently she realized that some considered math as having "right and wrong ways," but it is not clear in her concept map if that is *her* opinion of it.

One of the items related to the characteristics of mathematics in the Mathematics Inventory was item 4, "I think of mathematics as a set of isolated

Table 29: Items Relating to Conceptions of *Characteristics of Mathematics* from the Mathematics Inventory with Elizabeth's Responses

Item	Beginning	Middle	End
4. I think of mathematics as a set of isolated facts that need to be memorized.	3	2	2
11. Mathematics is something that not everyone is able to understand.	4	5	5
17. Mathematics is something at which everyone can be successful.	2	3	4

facts that need to be memorized." Elizabeth didn't agree with that characterization much, especially as the semester went on. She put a 3 for it on the first day and a 2 the next two times she took the inventory. When she gave it a 2 during her mid-semester interview, she thought she had put a 3 or 4 on the first day of class because she said, "I was more into that before." She explained what she was thinking about that statement this way,

> "I know that I wasn't learning things if I just memorized it. I was being successful on tests if I just memorized the facts and the formulas and the postulates, and I didn't really think I was applying them on the tests. I was just applying memorized facts. I knew which variables went in which spots and those kinds of things. So, it seemed like that was more memorized to me, and even in chemistry, all the different symbols that stood for the different gases and things like that. I had to memorize all that, but they had a place where they matter, and I think it's kind of important to do some memorization like your times tables and things like that to save time, but as long as you know the concept and what's behind it, where it can be useful, and not just this table that's in your head. So what if 4X4=16. What does that really mean. If you can apply it and draw it and show examples for it, it can save you a lot of time."

When Elizabeth began the course, she felt quite strongly that mathematics is something that not everyone is able to understand which was the statement for item 11 in the Mathematics Inventory. She put a 4 for that item on the first day of class. During the mid-semester interview and on the final day of class, she responded to that item with a 5, indicating that she agreed with it even more strongly. Apparently she came to that conclusion because of talking with and listening to her classmates. She found out others had struggled with it, too, and that was comforting to her. She explained to the mid-semester interviewer what she was thinking about that:

> "I think because the more I talk to other people and the more I've been in different discussions when you have time to actually get opinions from other people in class and not just sit there and hear what the professor says, hear what the teacher says, and then leave and go home and do your reading and try to apply what you're thinking to it. I think it's important to hear other people who will be your colleagues some day and your peers right now how they feel about it, what their backgrounds are, and to know that they're going to be contributing to other children's learning, too. And, if other people had struggles, it makes me feel like, okay, then what can we do about it? We have a common ground. You know, what's a way that you like to teach it, and what's a way that I am. And, if was in the school system, maybe I'd want to collaborate with that teacher and we could teach together because we've both had problems teaching, and I just feel good knowing that other people had problems, too, and I don't think of that in a negative way, but I think, 'Gosh, I'm not the only one out there.'"

The interviewer then probed further by asking her if she thought there was something about some people that they can't understand math. Elizabeth replied,

> "Yeah, I think there is a different like right-brain, left-brain kind of thing. The kind of person you are. If you've always been more into verbal expression and an extrovert, and really outgoing, I think those people to me have always been more language arts people, and then there's other people that they want to know the way things work. They're interested in the mechanics of things, and maybe more introverted, because they'd be excited just to sit home and work with a computer all afternoon. For me, I'm like, 'Oh, I've got to write that paper, and I have all these ideas in my head and I want to tell somebody about, but I've got to go work with this machine for a while to get it all looking perfect.' And, then I am excited with the end result, because it comes out looking much better than just the scrambled thoughts in my head. But, I think that some people will always be like, 'Well, I don't understand why that's important for me to know, because that's not the way I think.'"

Elizabeth's increasing belief that some children just can't learn math is interesting in the context of her other changes. It is especially interesting that this belief seems to be part of feeling better about the relation between herself and math.

It's also interesting to note that she frames this "not being able to understand" in positive terms that portray the people who can't learn math as more attractive, cooler folks (extroverts, really outgoing, etc.).

As a group, the class agreed with that statement ("Mathematics is something that not everyone is able to understand.") less at the end of the semester than they did at the beginning. (The mean was 2.86 at the beginning and 2.34 at the end of the semester.) But, unlike the majority of the students, at the end of the semester, Elizabeth thought even more strongly that not everyone can understand math. This conception appears to have been influenced by talking with her peers as well as considering herself as an example. It also seems to have been influenced by some other conceptions of characteristics of math that she revealed when talking about this item—that it's "left-brained" and that it's something that an introvert would more likely be good at. Evidently, Elizabeth believed that people think differently and that the way some people think doesn't allow for them to understand math. Part of Elizabeth's conceptions in this area were influenced by experiences she had in the course such as talking with her peers, but some of her ideas have no clear relation to any course experience. For example, the ideas she talked about related "left-brain/right-brain" and introverts vs. extroverts were not things we talked about in class. They may have been ideas she was introduced to in another course during the time she was in my class, but more likely they were part of her outlook before the course began even though she did not talk about them earlier.

Elizabeth's responses to item 17, "Mathematics is something at which everyone can be successful," seem inconsistent with her responses to item 11, especially as they change throughout the semester. Of course, it's very possible for someone to think of "understanding math" and "being successful at math" as different things. On the first day of class, she put a 2 for item 17, indicating fairly strong disagreement with the statement. During the mid-semester interview, she put a 3 which would indicate more ambivalence or uncertainty about what she thought about that. On the final day of class, she put a 4 for that item, indicating fairly strong agreement. She explained how she was thinking about it during her mid-semester interview:

> I think that's changing for me more, because if you give everybody a valid—like if you validate their reasons, and you show them that what they say is important, and a way that they can relate it to their life, that they can be successful with it, that it's going to make them feel more confident about it, and that they can be successful at it, and they're not always going to get a paper back that has a frowny face instead of a smiley face, you know. They knew some of the concepts, and it wasn't just that they got the right answer, but they could explain it. If they could stand up in the front of the class and explain it to someone else, that to me would mean that they're successful, not just whether I said they were right or wrong.

Somehow, even though Elizabeth's explanation of success in mathematics seems to indicate that it would include a level of understanding since she talked about the students being able to explain it, she was able to believe both these seemingly dichotomous conceptions of math or learning math—that not everyone can understand math and that everyone can be successful at math. This contradiction may be an important clue in what changes were happening for Elizabeth. She seems to be grappling with two seemingly contradictory views and trying to find a way to believe both. She may be accepting the idea that all can be successful at mathematics—and probably the part of her that is thinking of herself as a teacher wants to believe that—but at the same time, she did not want to give up the belief—that seemed based on her own experience—that not everyone can understand math.

Usefulness of Mathematics

In this section, I examine Elizabeth's conceptions of mathematics throughout the semester related to the usefulness of mathematics in the areas of general usefulness, personal utility, indirect personal utility, and utility for others.

General Usefulness of Mathematics. The usefulness of mathematics was a big part of Elizabeth's conceptions of mathematics—especially at the beginning of the semester, but also at the end of the study. Most of Elizabeth's beginning math concept map was related to the usefulness of math (See Figure 48 on page 336), but there wasn't anything related to the usefulness of math in her final concept map (See Figure 49 on page 337). Her beginning "What Is Mathematics?" journal entry also included some of her ideas of the usefulness of math, but as with her concept maps, her final "What Is Mathematics?" journal entry contained less.

Even though there seemed to be less attention on the usefulness of mathematics in the data sources referred to in the previous paragraph at the end of the semester compared to the beginning of the semester, there was some evidence that Elizabeth's ideas of how math is used in the world expanded somewhat during the study. For example, one of the questions the focus students were asked in the final interview was, "Have your ideas about what it means to know and do math changed at all during your life? If so, can you identify what has caused or affected that change?" Part of Elizabeth's response to that question indicates that she had a greater recognition of the utility of mathematics and where mathematics is at the end of the semester than she did at the beginning. She said,

> "Um, yeah. I think I have changed my mind about math. I mean I realized before that if you are balancing your checkbook, you're using math, and all those kinds of things, that it's everywhere all around you, but I wasn't able to recognize it as much before I kind of studied it in my TE class last fall.

Another place there can be seen slight bits of change in Elizabeth's conceptions related to the general usefulness of mathematics was in her responses to the relevant items from the Mathematics Inventory (See Table 30).

Table 30: Items Relating to Conceptions of the *Usefulness of Mathematics* from the Mathematics Inventory with Elizabeth's Responses

Item	Beginning	Middle	End
3. I think mathematics is an important subject that is useful in life.	3	3	4
5. I think mathematics is something very much related to this world.	4	3	4
13. The only people who really need to understand math are those who choose a career which requires math skills.	2	2	2

For item 3, "I think mathematics is an important subject that is useful in life," Elizabeth responded with a 3 the first two times she took the inventory which suggested some uncertainty in her agreement, but she put a 4 on the last day of class, indicating fairly strong agreement with the statement. During the mid-semester interview, she explained to the interviewer what she was thinking about related to that item:

> "Um, I'd say it's probably a 3, because I know math is useful in life and everything that you do everyday math is involved. That's something that I've had a kind of different look at from the class that you know, you stop at a light, and you're in your car. That light is working mechanically, and somebody has had to--you know an engineer or some type of a mechanic--to put that together, and that's using math. The car runs on math. I mean, putting gas in it, you have to know the price of gas. All the different gears and gadgets in your car someone worked out how they electronically fit together and work together. It's all mathematical. So, I know they're very important and useful, and it's a whole network of people who work together. I don't think it's like one person who's a master at math."

The interviewer then asked her what she thought she had put before, and she said, "Probably a 3." She was then asked if she thought anything about that had changed, and even though she responded with the same number on the scale as she did the first day of class, her response indicates that there had been at least *some* change:

"Um, I've seen more ways where math is useful in life. Before, I think there's nothing important about geometry. I'm never going sit home and think about shapes and things like that. But, now, when I think about chemistry and some of the things that I've learned and how that's involved with math and the way your body runs and, you know, temperature in the classroom and things like that. Math is everywhere, and it is really useful. And, sometimes if it's boring because you're reading it in a book, and you can't apply it to your life, doesn't mean that it's not used in your life. I think there's a parallel between the story problem in the book and how dull and boring that is to well, [how] you'd use this in life sometime if there was like a problem like, you know, "The Hedlunds were building a house. . . ." Well, why would I build a house as a third grader? If you have an aquarium in your classroom and say you have three fish in there. You want to buy six more fish, and these fish need this amount of water. How much bigger of an aquarium do you need to get if you're going to get more fish. That would mean something to them. They'd be like, 'More fish! More fish! We need a bigger aquarium!' And, they'd want to measure it, and things like that."

Most people who thought that "[m]ath is everywhere, and it is really useful" would probably answer that item with something higher than a 3, but for some reason Elizabeth did not. However, she did increase her response to that item to a 4 on the last day of class. It's interesting that Elizabeth thought of the usefulness of mathematics as it related to how she thought math should be taught. This connection will be highlighted more in the conceptions of teaching mathematics section of the analysis.

When Elizabeth was asked item 5, "I think mathematics is something very much related to this world," during the mid-semester interview, she gave it a 3 (See Table 30 on page 345). This item is related to item 3 discussed previously, "I think mathematics is an important subject that is useful in life," and she gave it the same response. Her response to item 5 was lower during the mid-semester interview than it was on the first day (a 4 vs. a 3), but Elizabeth's explanation during the interview indicated that there had been some change--and not in that direction. When read the item, she said, "I think I would probably say a 3 now, and I probably put a 2 before." When told that she put a 4, she was very surprised, but she said, "Oh, huh? Well, I still think I'd put 3 now. I realize it's related to this world, but not necessarily my world and what I think of." (This explanation may also clarify why Elizabeth gave only a 3 for item 3 as well.) When asked what she meant by her response, Elizabeth elaborated,

> "When I think of the rest of my life, I'm going to be teaching children. I'll be teaching them the math that's valuable to them at their age, and I won't be going on to physics or astronomy or calculus or anything like that with them. I'll give them a good basis what they might want to do with their life, but when I think about, the things I do, everything's kind of set out there for me. I don't really have to do math to try and run my car. I mean I have to add up the gas to put in my car, but I don't have to do those things. I know that someone else has already done them, and they're out there. So, they are related to this world, but, um, maybe not in the things that I do."

Again, Elizabeth was thinking about the usefulness of mathematics in relation to how she thought it should be taught. At the end of the semester, as with item 3, her response increased to a 4, suggesting more agreement with that statement.

The other item from the Mathematics Inventory that was related to the usefulness of mathematics was item 13, "The only people who really need to understand math are those who choose a career which requires math skills." Elizabeth put a 2 for it each time she took the inventory, indicating fairly strong disagreement with the statement.

Elizabeth's conceptions of the usefulness of mathematics as they relate to personal utility, indirect personal utility, and utility for others will be briefly addressed in the following sections.

Personal Utility. Elizabeth recognized the personal utility of mathematics for her throughout the study. In her beginning "What Is Mathematics?" journal entry she wrote, "Math is present in everyday life: e.g., shopping, % of sales and sales taxes, prices of products too. In time, numbers are used and are very important. Math is used in driving somewhat, with speed limits and gas prices." She also included a great deal related to personal utility of math on her beginning concept map. She included "money" with "economy" and "shopping" connected to it and "% off sales" and "taxes" connected to "shopping" (See Figure 48 on page 336). She also had a category of "measurement" connected to "math" with "cooking," "prices," "distance," and "parking spaces" connected to it. The item "driving" was connected to "math" and also "everyday concepts" and "measurement." She had "parking spaces" written on the line connecting "driving" and "measurement." "Speedometers" was another item she had connected to "driving" that was related to personal utility.

At the end of the semester, she didn't include anything related to the personal utility of math in her "What Is Mathematics?" journal entry or her concept map, but that was more likely due to a shift in her attention of what mathematics is more than because she thought differently about how mathematics is useful to her. A few of her comments in her final interview indicate that she still understood the personal utility of mathematics. For example, she said,

> "Doing math can mean a whole different number of things. It can either be like balancing your checkbook. If you see someone in the grocery line balancing their checkbook, they're doing math. If you're driving along, and you look at the speed limit, and you look at your speedometer, and you need to adjust to go only 45, you're going to do math if you let off the gas."

Or, when she was asked about if and how math existed outside of a school classroom, she said, "Like, dinner's at 5:00, and it's 4:30. How much time do I have between now and then?" Another example she gave during her final interview was,

> "You may not even be thinking that you're doing math if you're at home and you're doing laundry, and it says a full cup of laundry detergent. Well, it's all measured out there for you. You don't

> even have to think about it, but if you just pour it half way, then you're using math then, too."

Indirect Personal Utility. In her beginning "What Is Mathematics?" journal entry, Elizabeth included some ideas of how math affects her indirectly: "Math is everywhere and it is a means to regulate commerce, economy, population growth, measure things like the weather, and used in everyday life, probably everyday." She also included some ideas/items related to indirect personal utility on her beginning math concept map. Connected to "driving" she had "machines" and "stoplights" with "functioning" connected to both of them. She even had a category of "everyday concepts" with "indirect impact of complex problems" and "we use products (fridge cars, etc.)" branching off of it (See Figure 48 on page 336). So, it seems clear that she realized that math was used in a way that affected her indirectly. There weren't any specific examples of math's indirect personal utility in her final data sources, but in her final interview, she provided evidence that she still realized the indirect personal utility of math. For example, as noted earlier, she said,

> "I think I have changed my mind about math. I mean I realized before that if you are balancing your checkbook, you're using math, and all those kinds of things, that it's everywhere all around you, but I wasn't able to recognize it as much before I kind of studied it in my TE class last fall."

And, she also said in her final interview, "I think it's important for people to know that almost all of the material things that we have now are based on some sort of mathematical and scientific concept." Also, in her final "What Is Mathematics?" journal entry, the only thing Elizabeth wrote related to the usefulness of mathematics was, "Math is important and is used in everyday life to run the world." It seems to be a contradiction for Elizabeth to say that she wasn't able to recognize math being everywhere as much before she studied it in our class when she included some examples of the indirect personal utility of mathematics in her beginning math concept map and none in her final concept map. However, I think that the class did broaden how and where she saw math as being useful; my guess is that most of the change happened on the first day of class before she made her first math concept map. As was noted in the general section on indirect personal utility of mathematics in Chapter 3, on the first day of class, a student brought up the idea that math is all around us and in things we don't always think of. I think she really listened to that conversation. The reason I think that is because all of the items that she included in her beginning math concept map related to indirect personal utility were examples that we talked about in that discussion. Of course, there is the possibility that there was not as much change here as Elizabeth thought when she was asked about it a semester after the course ended.

Utility for Others. Elizabeth also recognized that math was used by others in ways that didn't affect her. She didn't refer much to mathematics being used by others in specific occupations, but she did make a couple of comments that implied her recognition of that use. For example, as noted earlier, when talking about her response to the Mathematics Inventory item 5, "I think mathematics is something very much related to this world," Elizabeth said,

> "When I think of the rest of my life, I'm going to be teaching children. I'll be teaching them the math that's valuable to them at their age, and I won't be going on to physics or astronomy or calculus or anything like that with them."

Physics and astronomy are especially directly related to specific careers. It can also probably be implied from the previously-noted sentence from her final "What Is Mathematics?" journal entry, "Math is important and is used in everyday life to run the world," that Elizabeth realized that math was something that was used by other people.

Learning and Teaching Mathematics

There seemed to be some expansion of Elizabeth's ideas about learning and teaching mathematics throughout the semester--especially her conceptions of teaching mathematics. Some of what we learn about her ideas related to learning and teaching mathematics comes from her responses to the relevant items from the Mathematics Inventory which can be seen in Table 31. These responses will be discussed in the following sections.

Table 31: Items Relating to Conceptions of *Learning and Teaching Mathematics* from the Mathematics Inventory with Elizabeth's Responses

Item	Beginning	Middle	End
10. Teaching mathematics should not be too difficult since a textbook already has the lessons prepared.	2	3	4
22. The best way to learn math is to memorize the rules.	3	not available	2
23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.	5	not available	4
24. In learning math, it is important to master topics and skills at one level before going on.	3	not available	3

Learning Mathematics

Elizabeth included some items in her beginning concept map that provide some insight into how she thought mathematics was learned at the beginning of the semester. She included "group learning (peer)," "trial and error vs. right and wrong ways," "open listening," "workbooks," "memorization," and "rote learning" (See Figure 48 on page 336). In the concept map, Elizabeth connected all of these to "attitudes," but to me they seem more related to learning than attitudes. Data from her interviews and "Reflections on Mathematics Experiences" journal entry suggest that she connected at least "workbooks," "memorization," and "rote learning" to "attitudes" because that was how she learned it, and since Elizabeth didn't like those methods, they contributed to her poor attitude toward math. For example, when she was talking about her own learning and her background in math in her final interview, she said,

> "Well, growing up I kind of was wary about math and kind of thought of it as being real[ly] just memorization and rote writing and book learning and just kind of take-the-test-and-move-on, and I didn't feel like I was building that much on all the skills that I gained each year."

She didn't include anything specifically related to learning mathematics in her final math concept map (See Figure 49 on page 337). She also didn't include anything about how mathematics is learned in her beginning "What Is Mathematics?" journal entry, but in her final "What Is Mathematics?" journal entry, she wrote quite a bit about teaching math, and part of that section was related to her conceptions of learning math:

> "I feel it is important to teach math in a manner that gives students choices: they have the opportunity to learn the same concept in many different ways correlating to the fact that students all learn in very different ways, at very different levels and speeds."

There was also some other evidence during the semester that Elizabeth thought that math was something that was learned in many different ways. During her mid-semester interview, when she answered item 7, "Doing mathematics is usually very frustrating for me," she talked about that idea. She said,

> "Um, I think it has always been very frustrating to me because it's seemed black and white and just right or wrong answer, and now I think I'm seeing more that there are different answers, there are different ways to get a problem. And, I think everything in the world is a shade of gray, and that there are exceptions to every rule, and there's different ways of going about things, and everyone can kind of be along the same flow of an answer and like getting the right idea, but nobody learns the same way, and everybody learns at their own pace and different levels. So, I think to expect everyone to—it's kind of a high expectation to expect everyone to think of one problem in the same way and get the same answer all the time, and that was frustrating. Now, I think it's starting to change a little bit in the way I see that people are looking at math differently, and hopefully learning to teach it that way. I'm hoping I'll erase some of the stigmas I've had and not teach it that way."

There were three items in the Mathematics Inventory that were related to the students' conceptions of how mathematics is learned. Due to an apparent tape recorder malfunction, we don't have Elizabeth's mid-semester responses to those items, but we do know what she put on the first day of the semester and on the last day. For two of those items, her thoughts seemed to change somewhat from the beginning of the semester to the end of the semester, but for the other one, they appeared to remain the same. For item 22, "The best way to learn math is to memorize the rules," Elizabeth responded right in the middle with a 3, which indicates neither agreeing or disagreeing, on the first day, but on the last day of the semester, she put a 2 which indicated that she disagreed with that statement at least somewhat. For item 23, "Students' reasoning is more important than if they are able to get the answer that matches the answer key," Elizabeth put a 5 on the first day, indicating very strong agreement with the statement, but on the last day of class she put a 4, indicating fairly strong agreement, but not quite as strong as she put the first day. We can only speculate as to why there was the slight change. It may have represented some change in her ideas, or she may have thought she felt the same about that idea. This was another item where Elizabeth's response changed in a direction that was different from the class as a whole. The beginning average response for the class on item 23 was 4.138, and the final average response was 4.379.

The last item related to conceptions of learning mathematics on the inventory was item 24, "In learning math, it is important to master topics and skills at one level before going on." Elizabeth put a 3 for that item on the first day and also on the last day of class. There may have been no change in her ideas. By putting a 3, Elizabeth was probably indicating some uncertainty about how she felt about that idea or perhaps some reluctance to commit to either agreeing or disagreeing with that statement.

During the final interview, a semester after the class, Elizabeth was asked what she thought was the best way to learn math or how math was learned, and she said,

> "If math isn't thought of as something scary, and you just get a ditto in the beginning, and it isn't all just a teacher standing there lecturing to you and writing on the board. But, I think it can be more exciting if you get to do activities that depend on math—if you see in the beginning what happens written down, and then you get to apply that and do some hands-on things. So, you're doing fractions. You've got something like a piece of pie. You get to cut it up into the different pieces. That will help you not just do the problem written on the board, but seeing it played out. Best way to learn math. I think to know what it looks like, a problem written down and really get to know numbers well, but then to be able to apply them and use your knowledge and show a finished result and say well, 'I can understand that this is a whole with no pieces missing because it makes a complete circle.' But, if there's a piece missing, that means there's 3/4 there and those kinds of things."

Prominent in Elizabeth's ideas of how mathematics is learned at the end of the study were the ideas that math is best learned when it can be applied to some experience and when it can be related to something concrete.

Teaching Mathematics

At the beginning of the semester, Elizabeth had some ideas about how math should be taught. She didn't seem to replace any of those ideas with other ideas during the semester, but there seemed to be some expansion of her ideas and some strengthening of her already-held conceptions.

The only item from the Mathematics Inventory specifically related to conceptions of teaching mathematics was item 10, "Teaching mathematics should not be too difficult since a textbook already has the lessons prepared." The changes in Elizabeth's responses to that item are surprising. The first day of class, she put a 2; during the mid-semester interview, she gave it a 3; and on the final day of class, she put a 4 (See Table 31 on page 351). It appeared that she agreed more strongly with that statement each time she took the inventory. This was again a change in a different direction from the class as a whole. The average for the class on the first day of class for that item was 2.034—almost exactly Elizabeth's response. But, the average for the class for that item on the last day of class was 1.828—quite different from her response of 4. During the mid-semester interview, after Elizabeth responded with a 3 for that item, she was asked what she thought she put the first day, and she thought she had put a 3 then as well. When told she put a 2 the first day, she explained what she was thinking about that statement:

> "A 2. I was thinking maybe a 2, but possibly a 3, because I'm excited that they have materials for you there, but I want to do research in that school before I even get into that school to see what type of math they have. If it's that old rote-learning dittoes, and you know teach to the MEAP test. The MEAP's getting reformed now, too, so that's good, but if it's just rote learning and times tables and just work books and math books everyday, I don't want to use that. That's when I will feel nervous that I'm on my own out there. I've got to do all these new things, and everyone's going to think I'm crazy like, 'Where's the A+ and the B+? I can't understand the way you evaluate. Your assessment here, you're saying well, he's saying he's excited about math, and that's really good. Well, who cares?' I'm like, 'That's very important at that stage.' That would be very important to me. I don't think it would be too difficult if there's something already there and I can maybe adapt it to my way of teaching, but if it's something that I consider to be valuable like from the skills I've learned in this class that will fit into that philosophy, then I'll be really excited."

From her explanation of her thoughts related to that item, it seems clear that Elizabeth doesn't plan on just mindlessly using the lessons that are prepared in a textbook to teach math, but that if she feels the lessons fit in with her philosophy of teaching math, she will use them, and that *will* make it easier to teach math. She also gives us a window on what is behind her apparently paradoxical response: she feels excited because she now knows that there are materials out there that can be used in ways that are different from the way workbooks of her school years were used. It's also interesting that Elizabeth talked some about her ideas related to assessment in that response—that she thinks it's important to assess attitude.

One part of Elizabeth's conceptions of teaching mathematics is that she believed that teachers' attitudes toward math and teaching math had an influence on how they taught it. This was part of her conceptions of teaching mathematics at both the beginning and end of the semester. In her "Reflections on Mathematics Experiences" journal entry written at the beginning of the semester, she wrote,

> "I also think that attitudes about math are important and can be passed down by a teacher. If the teacher has a negative or bad attitude, then that permeates to the students, so I know I need to work on mine."

Also, in the Mathematics Inventory, the students were asked an open-ended question, "What kind of influence do you think the teacher plays in the success of students in mathematics?" On the first day of class, Elizabeth wrote, "A high attitude influence, rewards verbally and in classroom for success." Her response is murky, but it seems clear that she is saying that a teacher's attitude has a major (or "high") influence on students. This is something that she seems to say over and over throughout the semester. On the last day of class, Elizabeth responded to this same question, "High--attitudes permeate." Another question they were asked was, "Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not? How about a teacher's attitude about teaching it? Explain." On the first day of class, Elizabeth wrote, "Yes—It permeates the teaching method, body movement, verbally. Yes—Not lethargic, lots of differences in ways of teaching and manipulatives." On the last day of class, she wrote, "Yes—It can hinder it and cause negative, dull vibes, or inspire and enlighten."

One place there seemed to be some change in Elizabeth's conceptions of how mathematics should be taught is in the expansion of ideas of how to do it. In the final Mathematics Inventory, the students were also asked if their ideas about mathematics or teaching mathematics had changed at all during the time they had been in the course. Elizabeth's response to that question was, "Yes. I am learning new ways of implementing math." Also, at the end of the semester, the students were asked to write their "Philosophy of Teaching Mathematics" for a journal entry, and Elizabeth's writing provides more insight into the expansion of her ideas of how math should be taught. Following is what she wrote for that assignment:

> "From my studies in Teacher Education and child development at MSU, I have developed a philosophy for teaching mathematics:
> "First, I will start off with a safe classroom for all students, adaptive to individual students' needs. Included will be developmentally appropriate, aesthetically pleasing surroundings such as a job chart (recycling center, water plants, feed pets, etc.), large group area, and math center. Included in the math center will be manipulatives for fine and gross motor development (puzzles and geoboards), counters, [C]uisenaire rods, a 0--99 chart, multiplication table (depending on the age group), and displayed work by individuals or class presentations. I plan to teach mathematics with a positive attitude, striving to make it 'fun' by using creative ideas to develop activities (such as popping popcorn--volume and size) that connect real-world situations. I will nurture children's natural curiosity and instill self-confidence. I will recognize the importance of affective and aesthetic development in children when planning lessons and setting up the classroom.
> "Drawing from readings taken from Marilyn Burns, the NCTM *Standards*, Pamela Scrantlon, and the addenda series from the NCTM, I have researched basic goals outlined in a math unit: <u>Attitude</u> goals of developing positive student attitudes, interviewing and expressing positive teaching attitudes. <u>Content</u> goals of the NCTM *Standards* such as challenging students with rich problems to solve and develop meaning for mathematical operations. <u>Process</u> goals of relating language and symbols to problems situations and informal language. Math will be taught using an active approach--hands-on methods and children's active learning through writing about math, speaking about, reading about math, and using math symbolically, which also connects math to language arts.

"Math should be thought of as fun in my classroom and a connection to real-world experiences!"

Although we talked somewhat in class about learning and teaching being developmentally appropriate, and we talked extensively about the role of manipulatives in teaching mathematics, making learning active, and connecting math to the world outside of school, I think much of her first descriptive paragraph was influenced by her training in child development. (Again, I am basing this assumption on my own experiences as an early childhood education major. The things she writes about—developmentally appropriate, aesthetically pleasing surroundings; a safe classroom for all students, adaptive to individual students' needs; a job chart; large-group area; math centers; and the like—were they types of things that were emphasized in my training. Also, I'm assuming this because she introduced her philosophy by writing, "From my studies in Teacher Education and child development at MSU, I have developed a philosophy for teaching mathematics:.") From the preceding journal entry, some of the main points that seem important in Elizabeth's conception of how mathematics should be taught include connecting mathematics to the world outside of school, using concrete materials, using active learning, and using discourse (discussing, reading, and writing). She also believes that the areas of content, processes, and attitudes should be addressed when teaching mathematics. (I don't know who the Pamela Scrantlon that she refers to is, but my guess is that it's a combination of Pamela Schram and Danise Cantlon since we read pieces by both of them in the class.)

Another place where Elizabeth echoed these ideas of how mathematics should be taught was in her final "What Is Mathematics?" journal entry. She wrote,

"- Teachers who have had negative experiences with math can still find positive ways to teach it.

> "- Math should be regarded with respect and taught conceptually. . . "I feel it is important to teach math in a manner that gives students choices: they have the opportunity to learn the same concept in many different ways correlating to the fact that students all learn in very different ways, at very different levels and speeds. Problem-solving techniques should go beyond story problems and filter into everyday life, giving students the necessary skills to tackle everyday situations with math."

In this passage, she emphasized the importance of teachers teaching math in a positive way even if they did not experience it that way. She also believes math should be taught conceptually and in a variety of ways so that different learners will have more opportunity to learn it. She also wrote about students learning problem-solving skills that they can use in "everyday situations with math."

In her final interview, a semester after the course, Elizabeth was asked how she thought math was most effectively taught, and teaching math with concrete things and connecting it to life outside of school were still both important parts of her conception of how math should be taught. In response to that question, she said,

> "I think it's most effectively taught with hands-on and real-world learning where I want to help the students see that it's not just numbers on the board, but you can apply that. If we're talking about measuring and measuring cups, then we'll actually make something in class, like bake pretzels or something. Then they can do science together with math and see the finished result—not just read about it, and it doesn't really mean anything to them. It has to kind of personalize it and help them see that it is a part of their lives, and it's something that they can do everyday."

One last place where we can get a glimpse of the expansion of Elizabeth's ideas related to teaching mathematics or the increase of her attention to teaching mathematics is in her concept maps. The only items related to teaching mathematics in Elizabeth's beginning concept map were those noted in the learning section that were mostly different ways that she had probably experienced learning mathematics as a student (i.e., "rote learning," "memorization," "workbooks," "trial and error," and "right and wrong ways," etc.). In contrast, teaching mathematics seemed to be the overriding theme of her final concept map. Related to teaching mathematics, she included "NCTM," "Standards," "Addenda Series," "professional organizations," "teaching," "activities," "teach in a variety of ways," "assessment," "allow a variety of methods," "concepts," "content," "attitudes," and "process" (See Figure 48 on page 336 and Figure 49 on page 337).

Elizabeth seemed to acquire some definite ideas about learning and teaching mathematics during the semester. She believed that math should be taught using concrete examples, discussions, connections to life outside of school, appropriate ready-made materials, and in a variety of ways so that different learners would have an opportunity to learn it. She also was thinking about assessment and that it should be done in a variety of ways as she wrote on her final math concept map (See Figure 49 on page 337). She also believed that teachers' attitudes have a big influence on students' learning, but she believed that teachers' attitudes could change and that "[t]eachers who have had negative experiences with math can still find positive ways to teach it." These conceptions seemed to have been influenced by a combination of her experiences in the class and her practicum, her training in child development, and her own experiences.

Elizabeth also seemed to be really thinking of mathematics as someone who was going to be teaching it would. This showed some shift in positioning in her relation to mathematics, and as previously suggested, this shift seemed to occur very early on in the semester since in many of her responses in the mid-semester interview, she seems to be talking about mathematics from the perspective of a teacher. The contrast between her beginning and final math concept maps also shows this shift in positioning. While her beginning concept map was mostly related to the usefulness of mathematics and her own experiences with math, the overriding theme and influence of her final map seemed to be teaching mathematics (See Figure 48 on page 336 and Figure 49 on page 337). The fact that she included "NCTM," "professional organizations," "Standards," and "Addenda Series" on her final map gives a strong indication that she was viewing math from the perspective of a professional teacher.

Attitudes toward Mathematics and Teaching Mathematics

Attitudes toward Mathematics

Elizabeth's attitude toward math was quite negative at the beginning of the semester. She valued math in realizing that it was useful in the world, but she didn't enjoy it much, and she didn't have a lot of confidence in her ability to do math. This negative attitude was fueled by her frustration with learning mathematics in school--especially once she got to geometry. As noted earlier, it also appears that her dislike for math had been greatly influenced by her then-recent experiences with math in college.

Another place that Elizabeth's lack of confidence in her mathematical abilities can be seen quite early in the study is in her response to the journal entry that was assigned a couple of weeks into the semester asking them to evaluate themselves in relation to the five goals that NCTM (1989) identified for students. Following is Elizabeth's complete journal entry in response to that prompt:

> "1) I feel I value math, to a point. I respect that I use it daily and it is involved in the intricate and modern machines and computers we use. But, I also feel that complicated math is inaccessible to me. I can see that it is a series of steps that a 'team' might use to work together and create a finished product.
> "Regarding my confidence in my ability in math, I know I am lacking. I often will work longer to get the right answer and am so frustrated if my final answer is not correct and my hard work to get there is not validated.
> "Being a problem solver is not a strong point for me either. I have often taken math home for studying long hours, and my attention span for this subject is quite short. I feel that if math is just a set of

rules then I should be able to apply them and get the answer QUICKLY--I never do!
"Communicating math and with math I feel is vital to modern understanding. I feel math's highly involved with the computer system I use and am learning more about every day. I follow a set of rules and read symbols and signs to move along the program, and I feel knowing this 'lingo' is vital today.
"Reasoning mathematically--I know I reason well and can break down a situation into a series of steps to obtain the goal or answer. I feel this is also related to science and the other areas of study in school--also getting yourself organized!
"2) I feel it is vital for me to have these goals and characteristics to be able to teach them to students. I feel all of my learning will bridge over to my students and my attitudes and beliefs will also come across. That is why I know it is important for me to be positive and try even harder in areas where I experienced frustration or trouble.
"I will get these goals by focusing on all these new concepts in TE 401, learning (from my classroom observation), and mostly getting to know my students and their struggles. I need to work harder on my weak areas so my students won't feel weak in those areas, and I need to remain open minded."

From this journal entry, it is evident that Elizabeth thought it was important to value math and to have confidence in your ability to do it. She also felt it was important to be able to problem solve, reason, and communicate mathematically. However, she felt lacking in each of these areas. However, she also conveyed a desire and willingness to do what she needed to in order to improve in these areas—as well as a hope that her experiences in the class and her practicum might help her do that.

By the end of the semester, Elizabeth seemed to enjoy math more and have more confidence in her ability to do it. This change was evidenced in all of the data.

One place this change can be seen is in her responses to the items on the Mathematics Inventory that are related to attitudes toward mathematics (See Table 32). She agreed with that statement, "I enjoy doing mathematical tasks," a little more each time she took the inventory. She explained how she interpreted

Table 32: Items Relating to Attitudes toward *Mathematics* from the Mathematics Inventory With Elizabeth's Responses

Item	Beginning	Middle	End
1. I enjoy doing mathematical tasks.	2	3	4
2. I have always enjoyed mathematics.	2	2	2
6. I feel successful when I am engaged in mathematical tasks.	1	2	3
7. Doing mathematics is usually very frustrating for me.	5	4	3
15. I understand elementary level mathematical concepts quite well myself.	3	3	4
25. I can handle basic math, but I wouldn't do well at advanced mathematics.	4	not available	3

what that item meant and how she would respond to it during her mid-semester interview:

> "It would mean that if I were to say now take out your math books or if we were going to do something mathematical, I would say I would enjoy it, and I would be excited about it, that I might be successful at it, and I would kind of scramble to get everything else out of the way and really concentrate on math. And, I don't think it's very much like me. I think I'm probably between a 2 and 3 range, because I have felt somewhat of a struggle in the past about math. It's been like a belly ache type of thing, and I know if I put my mind to it and do a lot of work with it, it doesn't come easy to me, but I can usually get around some of the obstacles that I've had in the past, and I can be successful, but not always right on task. I would have some of the right steps and not always the right answer. Or, I would get the right answer, and my way may not be acceptable, and they'll say, 'Oh, you need to do it this way.' I'm open to the new idea that there are many ways to do a problem, and they should all be validated, you know, not just the right answer and certain steps.

She was then reminded that she put a 2 the first day and asked if anything in the course had made her change. She responded,

> "I think I'm leaning more towards a 3, because, um, they're stressing so much more—in the course I've learned—that there's many acceptable answers, and you need to validate children's opinions and thoughts and ask them many questions about all the different ways they think they can get to this answer, and if an answer comes up on the board to ask their students their opinions like, 'Do you think that's right? Why do you think that's right?' Which uses much more emotion or feeling about it rather [than] just right/wrong. If the teacher just says 'Right;' or 'Wrong;' I think that kind of puts them in a dictator role. You need to have children more responsible for their actions and feeling, um,

> successful. 'Well, I'm not sure that's right.' And you say, 'Well, why don't you think so? Why doesn't it feel right to you. Maybe someone else has a comment.' And kind of have it as a group discussion rather than answer, '1-24, a, b, c, a, b, c,' and those are the answers. And, that's what math always was to me. It's kind of like an open and shut book all the time, and I'd always look at the answers in the back of the book, and I'd get it right, but I wasn't learning anything.

Part of the reason Elizabeth seems to be "enjoying doing mathematical tasks" more at this point is because she was having an opportunity to experience it in a way that was different from how she had experienced it growing up, which was a method where she "wasn't learning anything." She now could imagine a mathematics class where the teacher did not play a "dictator" role by giving the answers or saying, "Right;" or "Wrong." She seemed to like the idea of having the students explain their ideas and the teacher being more interested in the students' thinking than in if they can get the right answer. She seems to believe that this would get the students more invested in the problem and have "more emotion and feeling about it." It's interesting that even though Elizabeth was asked about her attitude toward math, she connected it to some ideas of teaching mathematics. This seems to show that she had positioned herself to look at math from the perspective of a teacher by that time.

Item 2, "I have always enjoyed mathematics," doesn't show any change in Elizabeth becoming more positive toward mathematics since she answered a 2 each time she took the inventory, but since the item was about history, we wouldn't expect the response to change—Elizabeth's consistent responses to item 2 can increase our confidence in other items on the inventory. She answers each of the other items in Table 32 in a way that indicates an improvement in her attitude toward herself as a doer of math, but we can see she isn't just circling positive answers—she is clearly thinking about each one separately.

Her responses to Item 6, "I feel successful when I am engaged in mathematical tasks," does show the same pattern of change as did item 1 (See Table 32 on page 363). She first responded with a 1, then a 2, and then a 3, indicating more agreement with that statement as the semester went on. Of course, she doesn't end up agreeing very strongly that she feels successful when she is engaged in mathematical tasks, but the trend is encouraging. She explained why she thought her response had changed during the mid-semester interview:

> "I think I'm feeling a little bit better, because I think if I was in school now as a younger student, some of my ideas might be validated more. If we talked about math more and not just sit and write out numbers, because I think it is kind of like another language--just symbols and signs--and if you don't know a language, you're not going to be successful unless you talk about it. In another language where you're like, 'I don't understand that;' or 'Can you show me a different way;' or 'I learn this way. This is the way I thought of it;' and if that's validated, then I feel like I would be more successful. And, I felt before that I wasn't successful, because I didn't get an A+ or 100% at the top of the paper. And, even the way that our primary teacher asks us to correct papers is more positive now. Instead of just putting x, x on the two that are wrong, we put stars on all the ones that are right, and it makes them feel more successful. I know I would if I was a student."

This is an interesting quote. Elizabeth's language seems to suggest that she was not *just* thinking of herself as a teacher, but that she was actually re-imagining herself as an elementary math student as she saw and learned about new ways of teaching. It seems that in her imagination she was re-experiencing being a young student in a math class and was then thinking, "Oh, I really could have been successful if they had taught me differently."

Item 7 from the Mathematics Inventory, "Doing mathematics is usually very frustrating for me," is related to items 1 and 6 in an inverse way, and just as her agreement with items 1 and 6 increased by one each time she took the inventory, her response decreased by one for item 7 each time she took the inventory, moving from 5—strong agreement—to 4 to 3. During the mid-semester interview, she explained why she was seeing doing math as less frustrating than she had when the course began:

> "Um, I think it has always been very frustrating to me because it's seemed black and white and just right or wrong answer, and now I think I'm seeing more that there are different answers-- there are different ways to get a problem. And, I think everything in the world is a shade of gray, and that there are exceptions to every rule, and there's different ways of going about things, and everyone can kind of be along the same flow of an answer and like getting the right idea, but nobody learns the same way, and everybody learns at their own pace and different levels. So, I think to expect everyone to--it's kind of a high expectation to expect everyone to think of one problem in the same way and get the same answer all the time, and that was frustrating. Now, I think it's starting to change a little bit in the way I see that people are looking at math differently, and hopefully learning to teach it that way. I'm hoping I'll erase some of the stigmas I've had and not teach it that way."

It seems from Elizabeth's responses to items 1, 6, and 7 that much of the reason she was feeling more positive toward math as the semester progressed is that she was seeing math and learning and doing math in a way that was less rigid than her original conception of what math was. The idea that a solution to a mathematics problem could be found in many valid ways seemed to appeal to her. Again, even though these items were focusing on her own experience with math, Elizabeth connected them to her ideas about *teaching* math.

Items 15, "I understand elementary level mathematical concepts quite well myself," and 25, "I can handle basic math, but I wouldn't do well at advanced math" both relate to the students' confidence in understanding and doing mathematics. For item 15, Elizabeth put a 3 the first two times she took the inventory and a 4 on the final day (See Table 32 on page 363). This increase by the final day would indicate that she felt more confident in her understanding of elementary level mathematical concepts. For item 25, "I can handle basic math, but I wouldn't do well at advanced mathematics," Elizabeth put a 4 on the first day and a 3 on the last day. Perhaps she felt somewhat more confident in her ability to understand advanced mathematics by the end of the semester.

Some of Elizabeth's final "What Is Mathematics?" journal entry is also illustrative of her more positive attitude toward math at the end of the semester. She wrote,

> "After studying mathematics in this course TE 401, I have come to many conclusions regarding math:
> "- Math is important and is used in everyday life to run the world.
> "- Math can be thought of as fun and even turned into a game.
> "- Teachers who have had negative experiences with math can still find positive ways to teach it.
> "- Numbers are meaningful and have a place in the world.
> "- Math should be regarded with respect and taught conceptually.
> "Many of these attitudes about math have changed for myself. I now see it in a brighter light."

Another place in the data that supports the idea that Elizabeth's attitude toward mathematics became more positive during the semester is in the final Mathematics Inventory given on the last day of class. The students were asked an open-ended question, "Have your attitudes toward mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how?" Elizabeth's response to this question was, "YES--They have improved. I have researched a new way of thinking about math and now enjoy it! (Then she drew a smiley face.)

A semester after the class, Elizabeth still seemed to have a more positive attitude toward mathematics than she did at the beginning of the study. For example, when she was asked, "Do you consider mathematics as something valuable? Where is it valuable? For whom is it valuable? Is it valuable for you personally?" she answered,

> "I do consider math valuable. It's valuable for everyone, and I think it's important for people to know that almost all of the material things that we have now are based on some sort of mathematical and scientific concept. And, I think it's valuable myself in that it's valuable to teach that math is important and not

just something that we have to learn and just a subject in school just like social studies and science, but that they can all work together."

She was also asked how confident she felt doing math, and she felt like she was more confident than she had been. She responded,

> "Um, my confidence level has risen somewhat. I'm not as nervous now if I don't always get the right answer, because I will always ask questions, and people around me can help me, and I'll have more resources than I think I had before. I was sort of nervous before about if students would ask me a question that I didn't know the answer to. Now, I can say, 'That's an excellent question. Let's write in our book of questions.' Or, 'Let's write that on the board.' I've done a k-w-l chart--what we already know, what we want to learn, and at the end of unit, what we have learned. So, we can put that in the middle of the questions. What do we want to know? 'We have this question, let's research this further. Let's investigate it.' And, sometimes I can even leave that study up to the students if I can't find the answer, they can help me find it, too.

Even though Elizabeth realized at the end of the study that she had limitations in her understanding of mathematics, she seemed to think of those limitations in a way that didn't cause her to have a negative attitude toward math or her ability to do it—or maybe to teach it. Once more, this is a case where Elizabeth was asked about her own attitudes toward math and responded by connecting it to her ideas of teaching mathematics.

Attitudes toward Teaching Mathematics

As with her attitude toward math, at the beginning of the semester, Elizabeth wasn't too excited about the idea of *teaching* mathematics to elementary school students, but again, as was the case with her attitudes toward math, by the end of the study, her attitudes toward teaching math also seemed to become more positive during the semester.

One place her trepidation about the idea of teaching mathematics was evident was in her "Reflections of Mathematics" journal entry which was assigned on the first day of class. In it she wrote, "If I had learned differently, I could be much more open to teaching it differently—but that would also mean years of retraining myself to think differently about math—and that is too frustrating to think of!"

Table 33: Items Relating to *Attitudes toward Teaching Mathematics* from the Mathematics Inventory with Elizabeth's Responses

Item	Beginning	Middle	End
8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.	3	3	4
9. I feel enthusiastic about mathematics and about teaching it.	2	3	4
12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.	2	3	2
14. I feel confident in my ability to explain mathematical concepts to children.	2	3	4
16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.	2	2	2
18. Being good at mathematics is not required to be a good teacher of math.	3	3	5
19. I want to teach mathematics so that I can help students get excited about it.	2	4	4
20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.	2	2	3
21. I expect to regularly teach mathematics as an elementary teacher.	3	3	4

Elizabeth's beginning responses to some of the items relating to attitudes toward teaching mathematics from the Mathematics Inventory also provide evidence of this apprehension (See Table 33). However, her responses to these items during the middle and at the end of the study indicate an overall, more positive attitude toward teaching mathematics by the end of the semester. For example, for item 8, "I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher," Elizabeth put a 3 the first day and during her mid-semester interview, and she put a 4 on the final day of class. When she said 3 for that item during the mid-semester interview, she thought she had put a 2 for it on the first day of class. She thought she was becoming more excited to teach mathematics at that point, and it appears that she was even more excited by the end of the semester since she put a 4 that day. During the mid-semester interview, she explained how she thought she was changing in relation to that statement:

> "I thought maybe I was hesitant about it. I mean 'somewhat' is kind of hesitant, but I thought I might have more negative thinking. How am I going to spend all the time to relearn this new way to teach it to students in a positive way because I have such a negative attitude? But, I am excited about some of my attitude changes myself and so many different ways that they're coming up with that we can show them, and I think students, like if I'm with an older group, and they already have some preconceived ideas about math, and I wanted to start from scratch. But, I think starting from scratch, too, can give them some really positive ways to start thinking about math and getting those ideas that I had.

Item 9, "I feel enthusiastic about mathematics and about teaching it," was similar to item 8, and Elizabeth's responses were also similar. She put a 2, then a 3, then a 4, respectively, the three times she took the inventory. She agreed with that statement a little more each time she took the inventory. She explained to the mid-semester interviewer how and whether she thought item 9 was different from item 8 this way and why she thought she was feeling more enthusiastic at that point than she did the first day of class:

> "It seems almost exactly the same, but enthusiastic and excited. . . You're more enthusiastic if you know you have a lot of really good ideas to share, that you've got a lot of materials, and you're really prepared. Excited would mean like, 'Oh, I can't wait 'til I have the opportunity when I get a chance to study this more and get prepared.' Enthusiastic I think would be, 'I'm ready.' You know, 'I've got these ideas.' Just from some of the different handouts and things that I've seen, I was like getting nervous at first reading about this new way of teaching math. I was like, that's just fine, but they've got that teacher's manual there that says, 'Do, it this way--the old way.' How am I going to come up with this whole new way without some sort of standard, like do a new lesson plan every night my first few years teaching and be exhausted trying to do it, but there are a lot of handbooks now and ways to teach children this kind of math, and I'm really interested in kind of like the revised way of these kind of problems that will use hands-on materials and prepare me and make me feel like I've got a back up if I haven't got something prepared for every single day. There are other ideas I can look at and say, 'Oh, well this is a developmentally appropriate way to implement this, and I'm going to use this in my

classroom, because I'm excited about this.' Not just I've got to create all these things off the top of my head."

The way that Elizabeth is distinguishing between being enthusiastic and excited about teaching math shows us that she is thinking carefully about the language in each item—more carefully that we might suppose given some of her phrasing.

Item 19, "I want to teach mathematics so that I can help students get excited about it," is also related to enthusiasm for teaching, and Elizabeth's responses to that item jumped from a 2 on the first day to a 4 during the mid-semester interview and on the final day of class (See Table 33 on page 369). This change in attitude seemed to have been influenced by her experiences in the class as well as her work in her field placement. She explained why she thought she had changed some to the mid-semester interviewer this way:

> "Um, maybe before I was thinking that I wouldn't be able to get students excited about it. I'm thinking more now that I've seen things in class, that I don't want to say that they're using it as a bribe or anything, but if they're using things that involve candy, and they're separating and sorting, and the end result, they get to actually eat it, then they're like, 'Oh, wow, I can't wait to do this one.' Or, if they get to use their bodies, like the younger students, and they get to rearrange themselves in the class or like classify themselves according to what they're wearing, they feel like, 'I've got a red shirt. Just because I've got a red shirt on, I'm important.' And that's important, too, not because the teacher picked me to go up first and write my name on the board, I think that can get them excited about doing math. Everyone gets to participate, and it's not a biased thing. You know, because they raised their hand, I'm not going to pick on them, it's something that I want to include all the students where they all get to use something that is familiar to them and important to them, and we can all participate in it. So, I'm excited about doing those kinds of things where like in school sometimes like where I am right now she'll say, 'Okay, it's time for reading workshop.' And, I'll hear these moans and groans, and I'm like, 'I don't understand why you guys are doing that.' I don't want to hear that, and it's going to happen. It's going to happen all the time, but hopefully there will be days when they're like, 'Oh, yeah, remember what we were doing yesterday. Let's go get our project. I can't wait to work on it again.' Or like they're ready to skip recess to keep on some type of math concept we're doing, we're working together."

Elizabeth's confidence in her ability to teach mathematics to elementary school students also seemed to increase a great deal during the semester. One place this can be seen is in a portion of her final "What Is Mathematics?" journal entry where she wrote,

> "Many of these attitudes about math have changed for myself. I now see it in a brighter light. I have come to realize the importance of instruction, and will strive to work harder to achieve a level of success due to my poor grasp on the subject. I feel confident that I can teach math on an elementary level with the necessary skills to be successful."

This increase in confidence is also clearly shown in her responses to item 14, "I feel confident in my ability to explain mathematical concepts to children." On the first day of class, she put a 2, during the mid-semester interview, she put a 3, and on the final day of class, she put a 4. She agreed with the statement a little more each time she took the inventory.

> I think I'm feeling more confident about explaining things, because I've had some experience this year in the classroom, and to not just say this is the concept and now do the problem, I like to open it up to the students, and if I've gotten to know them, I'm going to know what kinds of questions to expect. I'm starting to see some of that now. I know who's going to ask some of the questions, who will raise their hands first, I'll know the kind of question that is going to come from that type of student. And, I might know which students I can probe more from, and if I don't know the answer, sometimes this student in the back of the room is going to know the answer. So, that excites me, and I want to keep pushing that child, but I also see another child that is struggling with that, and I'm thinking, 'I'm wondering, you know, what you think of that.' And, I think it would be really exciting to use the students—more to be a facilitator for their learning rather than just explaining everything. But, if they have honest questions, I'm feeling more confident in being able to explain the reasons why, and they help me out, too. We kind of help each other out.

In this explanation, Elizabeth tells us specifically what is making her feel more confident about teaching math and that seems to be how she imagines her class working. She imagines herself as a facilitator of the students' learning rather than someone who just explains concepts, and she sees an active role for the students with them bringing up ideas and explaining their thinking—thus helping themselves and each other learn the math. This image of her classroom she had at that point was probably quite different from the image she had before she began the course, which was most likely the image of how she had been taught math, and it is one where she apparently sees her weaknesses in mathematics not being as evident.

Another related item was number 16, "I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students," and Elizabeth's comments about it also indicate that she was becoming more positive about the idea of teaching mathematics. Elizabeth put a 2 each time she took the inventory, but during the mid-semester interview, she told the interviewer that she thought she put a 3 on the first day. The interviewer asked her if she thought she was changing a little bit, and she said,

> "Yeah. I think I was feeling more comfortable, because I know how important it is in that I'm learning new concepts and ways that we can be positive and not make someone feel like they hate math and they're not going to want to use it in their lives, because it is very important, and I'm realizing some of the ways that you use it everyday. And, you really need it, and you need to be confident in those skills. Otherwise, you know, math's going to be with you all the way through college, and every time you have a test, you know, you'll dread going that day, and those kinds of things."

One item where the change in Elizabeth's responses seems a little surprising—at least at first glance—is item 18, "Being good at mathematics is not required to be a good teacher of math." On the first day of class and during the mid-semester interview, she responded a 3 to that item, but on the last day of class she put a 5, indicating strong agreement with the statement. During the mid-semester interview, she was asked what she was thinking about that item, and her answer provides some insight into why she was thinking that it wasn't that important to be good at mathematics to be a good teacher of math:

"Um, well maybe it's just a level I've chosen—like if I'm going to be working preschool and K-8. I would get a little more nervous a little bit more in 7th and 8th grade. I was successful in math at those levels, but anything beyond that, I feel you really have to have a good concrete understanding of what the geometry is, or what the algebra, or what pre-calc is, and I think you have to have a love for that, too, to be really interested in it to want to show people that they can be successful in that and what that means. But, I think, if I'm not that good at those kinds of concepts, I feel successful at some of the more basics, and if I screw up, I'm not going to feel totally intimidated because I know some of the students I can rely on in my class that are going to have the right answer. And, if I put a story problem up on the board, I might not have the answer, but I know somebody else is going to. And, I might have the teacher's manual, but I'd like to hear the way that they got to that answer. So, I don't think that being a little shaky on it is going to hold me back in some of those ways because I can learn from other students, and I think on a basic level I am successful, and I know that if I instill those same beliefs in other children, they can build on them to be more successful in the future, and hopefully, I can go back and learn more about the things I was weak in before to be successful, too, because I would hope that even if I was successful in the basics, if it was at a way that I learned those that made me unsuccessful later, you know was there something else I should have been doing to help be more successful in an 11th grade math course, or was it a teacher that I had a problem with their way of-- you know their concepts or the book we were using. I'd like to look at that more."

It seems that she felt she didn't need to be good at all math—just the math she was going to teach—and that if there were something she did get confused on that as a class they would be able to figure it out. This also seems to connect to her new view of the mathematics classroom and her role in it—that has been discussed previously—where she seems to feel her lack of knowledge will not be as big of an obstacle since the students would take a more active role and more responsibility in their learning.

Elizabeth's thinking about Item 12 from the Mathematics Inventory, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school," and item 20, "As a teacher, I would feel embarrassed if a student asked a question to which

I did not know the answer," was related to her thinking about the previously-addressed item 18, "Being good at mathematics is not required to be a good teacher of math." For item 12, "I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school," Elizabeth put a 2 the first day, a 3 during the mid-semester interview, and a 2 the final day of class (See Table 33 on page 369). There was a little fluctuation in her response, but it was a statement she didn't agree with much ever. She explained how she was thinking about it during the mid-semester interview this way:

> "Um, I'm feeling more, that's kind of like, if someone had an attitude, 'Oh, I'm going to have no problem with it, because obviously I passed.' And, I'd say every time when you get a new problem that you're going to teach to your children, you need to be sure that you're secure with it, and that you understand all facets of it, because if you're just going to explain them the problem and have them write their answer, that's one thing. But, if you want to open it up to the class and say, 'I'm wondering who can tell me what they think about this;' they're going to raise all sorts of questions that you may not have all the answers for, and you need to be prepared to say, 'Well, I'm not sure. Let's try and find out.' Where if you've done some research and you kind of know about that problem yourself, you're going to have some of those answers for them. So, I don't think that I'll know everything, and no one's going to know everything about everything, and kids will come up with some crazy answers, too, and some crazy questions. But, I feel that it may be a little more like me, because I've looked more in depth at some of the materials that I would be teaching. Before it was kind of like thinking, 'Um, I don't know what they mean by that. What will I be teaching in the third grade?' Now, I've seen it. I've been in a third grade class, and said, 'Oh, I know that. I remember that stuff. I remember this. I can do this.' But, I want to have the background and make sure that I'm going to be successful and not giving them misinformation."

For item 20, "As a teacher, I would feel embarrassed if a student asked a question to which I did not know the answer," Elizabeth put a 2 the first day of class, a 2 again during the mid-semester interview, and a 3 the final day. We don't know why she changed her response a little at the end of the semester, but it appears that she was not too worried about not knowing an answer to a question a student might ask. She explained her thinking on that during the mid-semester interview in a way that was consistent with her responses to items 18 and 12:

> "Yeah, I think that's pretty much the same where I'm not going to know all the answers. I know that going in now. I'm going to want to be very prepared, and I'm going to learn from the students in my class, but I want them to know that as a teacher, I'm not somebody who knows everything. I'm hopefully there to help us both learn together some of these things and to show them things that I think are important to learn and how to be more of a facilitator, but in the end, if I don't know, doesn't mean that there isn't an answer and that we can't find the answer together. I might be at first a little bit like, 'Hmmm.' You know, that's why I'm not quite a number 1, but I don't think I would have too much of a problem with it."

Elizabeth seemed to realize that a good understanding of mathematics is important for teaching mathematics, but she also seemed to think that not knowing everything about math didn't disqualify her from teaching it to elementary students. She also seemed to think that through teaching and talking with her students she could gain a better understanding of math. The conception of Elizabeth's that she can still be a good teacher of mathematics even if she doesn't have a complete knowledge of it also seems related to a sentence she wrote in her final "What Is Mathematics?" journal entry: "Teachers who have had negative experiences with math can still find positive ways to teach it." Again, feeling that her lack of mathematical knowledge would not be as big of an obstacle as she once feared—in the type of classroom she now imagined—seems to be much of the reason Elizabeth seemed to gain more confidence about the idea of teaching math as the semester progressed.

This position of Elizabeth's is interesting. It illustrates the previously-noted effect of our teaching as mathematics teacher educators where the prospective teachers feel that in the "guide on the side" instead of the "sage on the stage" role they will play that they don't need to have an extensive knowledge of mathematics. (This was discussed in Chapter 4 on page 177.)

Another data source that provides some information regarding Elizabeth's attitude toward teaching mathematics and how it may have changed are a few of the open-ended questions that were asked in the Mathematics Inventory. For example, the students were asked what they considered to be their greatest strengths in relation to being a mathematics teacher. On the first day of class, Elizabeth wrote,

> "-open minded,
> -try to portray a positive attitude,
> -patience--give it time,
> -go beyond workbooks and rote learning,
> -don't see math as black and white (right vs. wrong),
> -use of manipulatives and life learning experiences to put math to use (e.g., set up a store in the class and use math to 'buy items' and having duplicates)."

When Elizabeth responded to this question on the last day of class, her focus was more on the effects her attitude toward math would have on her teaching, and she wrote, "I am learning to have a positive attitude about it and create 'fun' activities."

In the Mathematics Inventory, she was also asked what concerns her the most about teaching mathematics to elementary students. On the first day of class, she wrote, "Frustrated students, workbooks, rote learning, time restrictions." On the last day of class, she wrote, "Re-adjusting texts to fit standards and active learning." Again, this response shows evidence of her new—to her more positive—vision of teaching mathematics where learning is more active and students aren't assigned multiple exercises from a textbook.

Another open-ended question in the Mathematics Inventory was, "Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain." Her answers were quite similar both times she responded to that question. On the first day of class, she wrote, "A strength—because I will try harder to make it a positive experience for the students." On the last day of class she wrote, "A strength—I am learning from my mistakes."

Elizabeth still seemed to feel more confident about teaching math a semester after the course than she did at the beginning of the study. In her final interview, Elizabeth was asked how she felt about the idea of teaching mathematics to elementary students, and she replied,

> "I'm still a little bit apprehensive, because I would like to see what sort of curriculum and materials the school is expecting of me and how much room I have to interpret with that and how much time I have to work with each concept so that I feel that student is confident with that concept and we can move on and build it on that. I think I've gained a whole bunch of skills on how to teach different concepts and really show hands on, but I'm a little bit nervous about maybe what teacher I'm working with in the beginning or what school I'm placed in, or in the future, what kind of job I might get where I may not have as much freedom to use the new ideas I have. They may just want to see, you know the parents may just want to see all 'A's' and all tests all the time and all of their written work, and I think it's more important or sometimes as important to have different kind of learning in the classroom where the whole class is involved, and they're building their community skills and dividing the class up into colors and groups and different things like that, too."

She was then asked if her feelings about teaching math had ever changed, and she replied, "Um, yeah. Since the beginning of this year, I was more apprehensive, and now I'm a little bit more confident. I mean I've gained all those skills. So, I'm looking to kind of dig in there." Again, although she had some apprehensions about teaching math which were typical and realistic concerns of most new teachers, Elizabeth still had more confidence about teaching mathematics at the end of the study than she did at the beginning.

Major Themes in the Case of Elizabeth

A major theme in the study of the case of Elizabeth seemed to be the change in her attitude toward mathematics and teaching mathematics. In both of those areas, her attitude changed from being quite negative to being quite positive. Another striking theme in this case is Elizabeth's positioning of thinking of herself as a teacher of mathematics and the way that she would connect almost everything she wrote about or talked about in the data sources to its implications for or connection to teaching math. These themes will be elaborated and summarized in the following sections.

Early Positioning as a Teacher. It seemed that there was a shift in positioning for Elizabeth in thinking of mathematics as it related to herself as a learner to thinking of it as it related to teaching mathematics and herself as a teacher of mathematics. This shift seemed to come more quickly for Elizabeth than it did for the majority of the students. While most of her beginning journal entries and her beginning concept map was related to her relation to mathematics as a learner, by the time she was responding to the questions in the mid-semester interview, the idea of teaching mathematics seemed foremost in Elizabeth's thoughts. Nearly every time she was asked anything about mathematics, Elizabeth included in her response how it connected to and what it implied for teaching mathematics.

This focus on teaching mathematics was evident in her response when she was talking about many of the Mathematics Inventory items during her mid-semester interview. For example, item 1, "I enjoy doing mathematical tasks," and item 6, "I feel successful when I am engaged in mathematical tasks" are all items that were included in the inventory to find out about the student's attitude toward mathematics, but as noted in the section on Elizabeth's attitude toward mathematics, when she was explaining her thinking for each of these, she talked extensively about teaching mathematics. Also, when she talked about the items related to the usefulness of mathematics, she connected them to her ideas of teaching mathematics. When she explained her thinking about item 1, "I enjoy doing mathematical tasks," and why she felt like she was enjoying it more than she had, she talked about the way she had experienced learning math in contrast to the way she was seeing it taught in the class and how she could imagine it being taught, and she felt that the new methods were much more enjoyable for her and would be for other students. When she responded to item 6, "I feel successful when I am engaged in mathematical tasks," she also talked about methods of teaching and how knowing about new methods of teaching helped her feel more successful. She said,

> "I think I'm feeling a little bit better, because I think if I was in school now as a younger student, some of my ideas might be validated more. If we talked about math more and not just sit and write out numbers, because I think it is kind of like another language—just symbols and signs—and if you don't know a language, you're not going to be successful unless you talk about it. In another language where you're like, 'I don't understand that;' or 'Can you show me a different way;' or 'I learn this way. This is the way I thought of it;' and if that's validated, then I feel like I would be more successful."

Elizabeth's final concept map showed a big emphasis on thinking about math as a teacher would. While the only items related to teaching mathematics in Elizabeth's beginning concept map were mostly different ways that she had probably experienced learning mathematics as a student (i.e., "rote learning," "memorization," "workbooks," "trial and error," and "right and wrong ways"), her final concept map seemed to be all more centered on teaching. She included such things as "professional organizations," "teaching," "activities," "teach in a variety of ways," "assessment," "allow a variety of methods," "concepts," "content," "attitudes," and "process" (See Figure 48 on page 336 and Figure 49 on page 337). There was evidence that Elizabeth was thinking of mathematics as a teacher of mathematics would at least by the time of the mid-semester interviews, and it seems that that positioning solidified during the semester.

More Positive Attitude toward Mathematics and Teaching Mathematics. An important story in Elizabeth's case is that her attitude toward mathematics and teaching mathematics became much more positive as the semester went on. This change was evidenced in many of her responses to the items in the Mathematics Inventory related to attitudes toward mathematics and teaching mathematics at different times during the semester. She also made several comments about her attitude improving in her journal entries and in her interviews. For example, in her final "What Is Mathematics?" journal entry, she wrote, "Many of these attitudes about math have changed for myself. I now see it in a brighter light." She also wrote, "I feel confident that I can teach math on an elementary level with the necessary skills to be successful." Also, when asked in her final Mathematics Inventory if her attitudes toward mathematics or teaching mathematics had changed at all during the time she was in the course, she wrote, "YES -- they have improved. I have researched a new way of thinking about math and now <u>enjoy</u> it!" During the final interview, a semester after the course, she said again that she felt her confidence in doing math and in the idea of teaching math had risen during the time she was in the course. It seemed that the main reason her attitude toward mathematics and teaching mathematics had improved was that she now had a new vision of a mathematics classroom—and her role as a teacher in it—and she felt that her lack of knowledge and positive experiences with math would not matter as much as she once feared.

Chapter Summary

In this chapter, I presented the cases of two other students who began the semester with negatives attitudes toward mathematics and teaching mathematics. These two also seemed like examples of "hard" cases that I was interested in looking at. Unlike the students presented as cases in the previous chapter, Carolyn and Elizabeth both had much more positive attitudes toward math and teaching math by the end of the semester. They both felt more enthusiastic about the idea of teaching math and hopeful that they could provide their students a better experience as students of math than they had experienced. It also seemed that there was a shift in positioning—for both Carolyn and Elizabeth—in their relation to math from that of a student to that of a teacher. For Elizabeth, the positioning as a teacher seemed to come early on, but for both of them, that position seemed to be quite solid by the end of the semester.

Another interesting observation about both Carolyn and Elizabeth is that it seemed that they had thought more about the readings and class activities than had Bridget or Felicia, which may have been an indication that they were more engaged in the class. For example, Carolyn talked specifically about some of the class activities such as the AIMS activities, and she referred to an article we read in class influencing her ideas about the order that math content needed to be taught. Elizabeth also refers to the reading by "Pamela Scrantlon." On the other hand, neither Bridget nor Felicia ever referred to a specific class reading.

Carolyn was very interested in math's connection and usefulness to the world and felt that connection should be the principal focus when teaching mathematics. There also seemed to be a change in what she saw as success—going understanding rather than getting the right answer. This change in her conception of success may have been the engine that drove other changes such as her more positive attitude toward mathematics and also toward teaching mathematics.

For Elizabeth, there seemed to be some unexpected changes in her conceptions of mathematics and teaching mathematics such as her growing view that she could teach math well without knowing it well, but when her explanations were examined, they didn't seem as surprising as they first appeared. Rather, they seemed examples of often-occurring, and sometimes paradoxical trends in teacher education.

The role of dissatisfaction in change also seemed to play a part in the changing conceptions and attitudes of Carolyn and Elizabeth—as it did with Bridget and Felicia. However, in contrast to Bridget and Felicia, Carolyn and Elizabeth seemed to be open to change and not as committed to their negative attitudes toward math. They may have even wanted to—or hoped to—change their attitude so that they would be more excited about teaching math. They did not need to hold onto their negative attitudes to preserve their identity or to provide them with an excuse for not wanting to teach elementary school.

CHAPTER 8
CONCLUSIONS, IMPLICATIONS, AND CONTRIBUTIONS

In this concluding chapter, I review some of the major findings from this study and propose some implications these findings might suggest. I also outline some contributions this study makes to the field of educational research.

Major Findings

There were several important findings that emerged from an analysis of the data collected for this study. I will review these major findings in the following sections. One of the major findings that I will review is the students' shift in positioning in relation to mathematics. The second and third findings I will review are the students' increased awareness of the usefulness of mathematics and their increased attention on processes used in doing mathematics such as thinking, writing, and representing. I will then review the change in the students' conceptions of learning and teaching mathematics. The next two major findings I will review are a more positive attitude toward mathematics and a more positive attitude toward teaching mathematics. Finally, I will review the role of dissatisfaction in change.

A Shift in Positioning. One of the major findings of this study was the way these prospective teachers shifted their position in relation to mathematics from the beginning of the semester until the end of the semester. Van Langenhove and Harr (1999) define positioning as "the discursive construction of personal stories that make a person's actions intelligible and relatively determinate as social acts and within which the members of the conversation have specific locations" (p. 16). The notion of positioning is related to the idea of personal identity and one's point of view in the world. It is also associated with the concept of a role, but it is more dynamic. Van Langenhove and Harre (1999) further describe the concept of positioning by writing, "[P]ositions can and do change. Fluid positionings, not fixed roles, are used by people to cope with the situation they usually find themselves in" (p. 17). This construct of positioning seems to be relevant to the change in the students' point of view in relation to mathematics that I observed in this study. As the study progressed, these students seemed to shift from looking at mathematics as experienced students to looking at it as prospective teachers. The shift in positioning that was evident in these students is not surprising. It's a result mathematics educators would hope for and even expect after the students took a course about teaching mathematics. However, it is something that is not typically noted—especially as a contributor to changes in the students' conceptions of and attitudes toward math and teaching math. It was interesting to be able to document this change.

This shift in positioning in relation to mathematics was especially apparent in the focus of the students' attention related to mathematics in the open-ended data sources of the "What Is Mathematics?" journal entries and the math concept maps. At the beginning of the semester, much of what students wrote was related to how they had experienced math themselves as learners and/or their relationship with math. At the end of the semester, their journal entries and concept maps showed much more attention to conceptions of learning and teaching mathematics than they did at the beginning of the study, and there was very little attention to their own experiences as students of mathematics. The students seemed to be thinking about math as if they were going to be teaching it. While in the beginning "What Is Mathematics?" journal entries, 14 out of 34 (41%) included something related to learning and/or teaching—almost exclusively related to how they learned it or how it was taught to them—in the final "What Is Mathematics?" journal entries, all but seven students included something about teaching mathematics (23 out of 30 or 77%)—at this point, all examples of how they thought mathematics should be taught. The attention to teaching was not only greater at the end of the semester, it was different. There was also one student who even specifically wrote that she was looking at mathematics differently:

> "I am no longer looking at math as a subject. I look at it more in terms of how to teach it. . . I feel that I have shifted my belief in math from a personal to a professional level."

Recognizing this shift in positioning proved to be very important because it seemed to be related to some of the other changes in the students' conceptions of and attitudes toward mathematics and teaching mathematics. The students made a change in their positioning in relation to mathematics and also in their attitudes toward and some conceptions of mathematics and teaching mathematics. One conjecture that seems to be especially supported by the case studies (see Chapters 6 and 7) is that for many students, shifting their position in relation to mathematics was the catalyst for changing their attitudes and conceptions related to mathematics. Having a new way to relate to and think about mathematics allowed these students to think of it more positively. Instead of basing their attitudes toward mathematics on only their personal experiences with mathematics—most often as a student—the students could imagine a different (and to many of them, better) way it could be taught and could think of it more positively. Positioning themselves as prospective teachers of mathematics also allowed the students to alter their conceptions of mathematics. For example, where they may have thought of mathematics as isolated bits of information to be memorized as it seemed to them when they learned it, after shifting their position to thinking of mathematics as a prospective teacher would, they could think of it as something that was connected to the world outside of school, to other subjects, and to other areas of mathematics (see Chapter 4).

Change in Conceptions of Learning and Teaching Mathematics. Related to the finding that the students shifted their position in relation to mathematics during this study was the finding that from the beginning of the semester until the end of the semester, there was a noticeable change in the students' conceptions of learning and teaching mathematics. Not surprisingly, most of the change was in the form of more ideas about how to teach mathematics. It was also manifest in the attention ideas related to teaching and learning mathematics received in the final open-ended data sources of the "What Is Mathematics?" journal entries and the math concept maps as was reviewed in the previous section.

One example of a change in the students' conceptions of learning mathematics is that at the beginning of the semester, many students wrote about learning mathematics by memorizing or included "memorizing" in their math concept maps. At the end of the semester, this idea was much less dominant, and when the students referred to memorizing, they were much more likely to write something like, "Math is more than memorizing." At the end of the semester, the students wrote about many ideas related to teaching and learning mathematics that were either not addressed or received minimal attention at the beginning of the semester. Examples of these ideas include the teacher as facilitator, the role of discourse (talking and writing), the role of manipulatives, the role of technology, connecting mathematics to other subjects and to life outside of school, and making sure the students understand the concepts associated with the operations and algorithms.

Increased Awareness of the Usefulness of Mathematics in the World. The analysis provided considerable evidence that there was an increase in the students' awareness of the usefulness of mathematics in the world at the end of the semester compared to the beginning of the semester. When the study began, most students identified ways in which they used math for such things as shopping, cooking, and driving. They also noted that other people used math—especially in their careers. However, many of the students did not include ideas about the way in which mathematics was used that affected them indirectly (what I termed "indirect personal utility") such as in the production of goods they use, and this was an area where there was a big increase.

One place that the increase in the students' awareness of the usefulness of mathematics could be seen was in the "What Is Mathematics?" journal entries. At the end of the semester, several students wrote comments indicating that they had a broader understanding of the usefulness of mathematics than they did when the course began. For example, one student wrote, "Math is a big picture of something that is everywhere; it is no longer a subject for me." Another student wrote,

> For me, the meaning of math has changed drastically over the past several months. I used to see math as some boring thing that I always struggled to get through during my school years. I saw it as just addition, subtraction, multiplication, and division. I never realized how much mathematics was tied into my everyday life.

One final example of what a student wrote at the end of the semester follows:

> I find [math] to be incorporated into just about everything. For example, traffic lights, ovens, recipes, calendars, time, day, etc. This list is never ending. Mathematics is a fact of life!!! . . . . Everywhere you turn and look you could almost be sure is a result of some mathematical process.

There was a statistically significant increase in the means of two items from the Mathematics Inventory that referred to the usefulness of mathematics generally. These were "I think mathematics is an important subject that is useful in life;" and "I think mathematics is something very much related to this world." It seems likely that this increase was largely due to the increase in the students' perception of the indirect personal utility of mathematics, i.e., the awareness that math was used in the production, packaging, distribution, etc. of things that they use.

Increased Attention to Processes. One of the areas that seemed to be ripe for change in the students' conceptions was the area of processes. The processes of mathematics, as such, seemed to be something that the students were often not familiar with—at least being thought of as processes. These processes include all types of activities and thought processes that are involved in doing mathematics such as problem solving, reasoning, logic, talking, writing, drawing pictures, making a graph or table, and connecting mathematical ideas to each other, other subjects, and the world outside of school. Being able to separate processes that are used in doing and learning mathematics from the content of mathematics itself is crucial for teachers to be able to think about mathematics themselves, plan lessons, and assess their students' understanding. When teachers understand that there are several processes that are used in learning all the content of mathematics, they can make learning goals and lesson plans that purposefully include opportunities to use specific, relevant processes. Likewise, when teachers assess their students' understanding of mathematics, they can assess the processes the students use and not just the content, thereby obtaining a more holistic view of the students' mathematical understanding.

At the end of the semester, the students seemed to be thinking much more about processes than they were at the beginning of the semester as evidenced in their "What Is Mathematics?" journal entries and their math concept maps. At the beginning of the semester, 40% of the students included some mention of at least one process in their "What Is Mathematics?' journal entry. In their final "What Is Mathematics?" journal entries, 53% of the students included something related to processes. There was not only a change in the number of students giving attention to processes at the end of the semester, but there was also a large increase in describing different kinds of processes that are part of doing mathematics. The processes of reasoning and logic were written about in both the beginning and final "What Is Mathematics?" journal entries, but they were not as dominant in the final journal entries because other processes of mathematics were included more frequently. At the end of the semester, the students were especially inclined to write about—or include in their math concept maps—the four process standards from the NCTM's 1989 *Curriculum and Evaluation Standards for School Mathematics* which are problem solving, communication, reasoning, and connections.

The change was even greater in the math concept maps. At the beginning of the semester, 56% included at least one process in the math concept map; none of these students included more than four processes. At the end of the semester, 77% of the students included something related to processes, and 15 of those students (58%) mentioned five or more processes. The increase in students who included a high number of items related to processes in the final maps versus the beginning maps was especially significant—zero in the beginning and 15 in the final maps. This was an enormous increase in attention to processes. It seems to show that the students were thinking about processes—and multiple kinds of processes—much more at the end of the semester.

A More Positive Attitude toward Mathematics. Overall, the students seemed to have a more positive attitude toward mathematics at the end of the semester than they did at the beginning of the semester. This was especially apparent in the students' declarations in their final "What Is Mathematics?" journal entries and in the focus of their final math concept maps. For example, in her final "What Is Mathematics?" journal entry, one student wrote, "I now look at mathematics in a different light. I never thought I would say this, but I actually think I am beginning to enjoy mathematics. It is very interesting to me."

Another student wrote, "Now I feel very confident when it comes to math. I realize that math is not there to trick me or to simply keep me busy," while a third declared, "I can honestly say my attitude towards math has changed for the better."

There was also an open-ended question in the final Mathematics Inventory related to attitudes toward mathematics, and the students' responses seemed to indicate that, as a group, they felt more positively toward math at the end of the semester than they did at the beginning. This question was, "Have your attitudes toward mathematics or teaching mathematics changed at all during the time you've been in this course? If so, how? If not, how?" Twenty-seven out of 29 students responded "yes" to that question. Three students responded, "not much" and explained that they started positive and still felt positive. (One student wrote both "yes" and "not much.") Eleven students wrote that their attitudes had improved, that they felt more positive, and/or that they were more optimistic. Eleven students also wrote that they felt more confident in their math abilities. Again, I conjecture that much of the reason the students were able to feel more positively toward mathematics at the end of the semester is that they had shifted their position in relation to mathematics from an experienced student to a prospective teacher. They had new ideas about how mathematics could be taught and how a mathematics classroom might look, and they didn't have only their own experiences as a student of mathematics on which to base their attitudes.

A More Positive Attitude toward Teaching Mathematics. As a group, the students also seemed much more enthusiastic about the idea of teaching mathematics at the end of the semester than they did as the study began. Additionally, they also voiced more confidence in their ability to teach mathematics. It seems that the main contributors to their more positive attitude were more ideas about how to teach mathematics—and in many cases a new vision of how it could be taught—and the students' shift in positioning in their relation to mathematics that allowed them to visualize teaching mathematics in a way that was perhaps different from what they had experienced as students but that they could imagine as a more positive experience for their own future students.

This more positive attitude was visible in all the data sources. (For example, See Table 10 on page 224 for significant changes on the mathematics inventory.)

The students also seemed to be less fearful of not understanding everything or making a mistake when teaching mathematics. Another statistically significant change in the students' mean responses to an item from the Mathematics Inventory was for the item, "As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer." The change was in the direction that indicated the students would feel less embarrassed in that situation. This was a somewhat paradoxical, but not surprising, result of the class. We want the students to realize that teaching in this new way where the teacher acts as a facilitator of learning requires a great deal of mathematical knowledge—to select problems, know which comments to pursue, what questions to ask, and for a variety of other pedagogical reasons—but the students feel that this kind of teaching will make their lack of mathematical knowledge less obvious. (For a more complete discussion on this paradox, see pages 177-178.)

The Role of Dissatisfaction in Change. This analysis also suggests that dissatisfaction plays a role in change. In general, for change to occur in people, there needs to be some level of dissatisfaction in their current state of thinking or emotion. If people are satisfied with how they feel, what they think, the results of their actions, etc., they are not likely to change. In contrast, if they are not satisfied with any of those things, they are much more likely to be willing to change their ideas, etc. and try something new. This assertion fits in with what Fullan (1993) describes as the lessons related to the complexity of the change process.

This role of dissatisfaction seemed very evident in many of the students, and it was especially marked in the four focus students. Carolyn and Elizabeth both wanted to teach elementary school and knew that they would need to be able to teach mathematics to do so. They also seemed to believe that it would be difficult for them to teach it effectively with the negative attitudes they held toward mathematics. Perhaps for this reason, they took considerable interest in learning to look at mathematics and teaching mathematics in ways that seemed more positive to them. They wanted to have better skills and attitudes (See Chapter 6). On the other hand, both Bridget and Felicia seemed content with their negative attitudes toward mathematics. They spoke as if their attitudes toward and conceptions of mathematics were part of their identities. They were happy not liking math. In fact, when Felicia was asked if she thought there could be something that would help her change her attitude toward math, her response was,

> "I don't know. I guess someone would have to do something that I can't even imagine to make it more relevant to me. Um, make it matter that I should do math, because at this point I'm so happy with my life not doing a lot of math."

By the end of the semester, both Bridget and Felicia had decided that they weren't going to teach elementary school. Therefore, they had no need or reason to change their attitudes toward mathematics while Carolyn and Elizabeth did have motivation to change their attitudes toward mathematics since they *were* planning on teaching elementary school.

Implications

This research provides some implications for mathematics teacher education that will be reviewed in this section. First, it indicates that students' shift in positioning in relation to mathematics can serve as a vehicle for changing their conceptions of and attitudes toward mathematics. Second, it provides insight into areas where change in prospective elementary teachers' conceptions and attitudes related to mathematics is more likely to occur. Third, it suggests that dissatisfaction may play a role in change.

Understanding the Role of the Shift in Positioning. Understanding what influences or contributes to changes in prospective teachers' attitudes and conceptions related to mathematics and teaching mathematics can help elementary mathematics teacher educators to provide experiences in their methods courses that will increase the likelihood that the students' attitudes and conceptions will change. For this example, if students' are able to change (improve) their attitudes toward mathematics when they think of teaching mathematics rather than focusing on their own experiences with mathematics, it would be important for teacher educators to provide opportunities through their course activities for the students to think of themselves as teachers and experience actually planning and teaching mathematics. It's hard to know for sure how this understanding should or might influence the planning and design of such a course since there are most likely subtle and obvious implications. However, some examples of activities to include in this type of course that may contribute to this result could be making lesson and unit plans, experiencing learning mathematics in a conceptual way, teaching mini-lessons to each other, and actually teaching elementary students in practicums. Another useful activity could be to have the students watch teachers who are effective at teaching mathematics—through visits to actual classrooms or through the use of videotapes. It may also be useful for students to talk with other class members about the process they have gone through in thinking differently about mathematics. This may help them realize that others have experienced changes in their attitudes and conceptions and that it is possible to think differently about mathematics. Recognizing that their conceptions and attitudes have change may improve their confidence in their efficacy to teach mathematics

Insight into Areas Where Change Is More Likely. This study provides some insight into where we are most likely to see changes in the prospective elementary school teachers' conceptions of and attitudes toward mathematics. If teacher educators are aware of these areas, they can put more emphasis on them by providing the students with information and activities that help them learn more about those areas. One of the areas where the teacher educators might expect change is in understanding the processes of mathematics and the role they play in doing and learning mathematics and should therefore play in the teacher's planning and assessment. Another area where change seems more likely is in understanding the usefulness of mathematics in the world, especially in the area of indirect personal utility. It particularly seems for these two areas—process and usefulness of mathematics—that the reasons these are areas where the students are more likely to change their conceptions is that they are areas where the students were less likely to have had much experience or information given to them. These were new ideas to them. Because they were able to add them to their conceptions of mathematics without challenging already-held ideas, change was easier and more likely.

These changes may have been related to specific activities I used in my teaching, and knowing that these are areas where change might occur may have implications in how others teach similar classes. The discussions we had as a class related to the students' conceptions of mathematics after they made their math concept maps and wrote their "What Is Mathematics?" journal entries, may have been helpful in facilitating the students' realization of the ways math is useful and related to their own lives. In addition, having the students learn about the process standards identified by NCTM (1989) and thinking about them for their Content Area Presentations and in planning their unit plans, seemed to help them realize the importance of processes. In planning courses intended to help students to think about the teaching of mathematics, math educators may want to consider including activities that help the students think about the role mathematics plays in their lives. They may also want to include activities that require the students to recognize the difference between content and processes and help them become familiar with NCTM's process standards (1989 and 2000).

Another area where change might be likely is in the students' attitudes toward mathematics and teaching mathematics. Again, this also seems to be very connected to the students' shift in positioning. When the students can experience mathematics in a different way and imagine it being taught in a way different from what they experienced in school (especially when their own experiences with it were negative), they are more likely to have a more positive attitude toward it and toward the idea of teaching it.

The Role of Dissatisfaction in Change. Finally, one implication of this study for teacher educators is that it is important to realize that dissatisfaction may play a role in the change of the students' conceptions of and attitudes toward mathematics and teaching mathematics. Teacher educators may have students who—for whatever reason—are not dissatisfied with their attitudes and conceptions, and no matter what the teacher educator does, those students may not change. Of course, it's also possible that students who do not feel any dissatisfaction regarding their conceptions and attitudes related to mathematics when they enter the class will develop that dissatisfaction that seems requisite for change and subsequently change, but it is not guaranteed. For those students who have negative attitudes and conceptions and feel no necessity to change, perhaps the greatest service the teacher educators can do is to help those students come to the conclusion that maybe they shouldn't teach mathematics as was the case with Bridget and Felicia.

Contributions of This Study

A Study that Shows There Can Be Change. Like a large number of other studies (Cooney et al, 1998; Krows, 1999; Miller, 1999; and Vacc & Bright, 1999), this study shows that prospective teachers' conceptions of and attitudes toward mathematics and teaching mathematics can change in the context of a mathematics methods course. It also identifies aspects of these conceptions and attitudes that are particularly likely to change. This study also documents the shift in positioning of these students in their relation to mathematics—from experienced student to prospective teacher—and provides evidence that that shift influences changes in the students' conceptions of and attitudes related to mathematics.

A New Framework for Looking at Conceptions of Mathematics. Through an examination of the data that was collected for this study, I was able to construct a framework that proved useful in framing the analysis of the data in relation to the students' conceptions of and attitudes toward mathematics and teaching mathematics. This framework provided a category to code all of the data gathered from the students. It could also be used as a template—or a starting point—in designing another study of prospective teachers' (or perhaps experienced teachers') conceptions of and attitudes toward mathematics and teaching mathematics.

Ideas for Using Concept Maps as a Data Source. Although many have used concept maps as an organizational tool to teach about concepts or as an assessment tool to see what students understand about a particular concept, using concept maps as a source of information for learning about prospective elementary teachers' conceptions of and attitudes toward mathematics seems to be an original idea—at least as a data source for a research study. This study shows the types of information that can be gained by using concept maps as a data source as well as ideas about how to analyze and synthesize groups of maps and evaluate individual maps more holistically.

Understanding the Relevance of the Shift in Positioning. One of the most important contributions of this study is the insight it provides regarding the relevance that the shift in positioning has to changes in the prospective elementary teachers' attitudes toward and conceptions of mathematics and teaching mathematics.

Summary

In summary, through this study, we were able to gain more understanding of the stability and change in prospective elementary teachers' conceptions of and attitudes toward mathematics and teaching mathematics. One of the most important findings of the study is that these students (prospective elementary teachers) shift their position in relation to mathematics between the beginning of the course and the end of the semester from that of an experienced student to that of a prospective teacher. This was important to understand, because it helps us understand why some of the change in the students' conceptions and attitudes occurs. We also learned that there are areas where change is more likely such as the students' conceptions of processes and usefulness of mathematics. The findings in this study also suggest that dissatisfaction may play a role in change.

One of the implications of this study is that knowing about the role the shift in positioning plays, mathematics teacher educators should provide experiences for their students where they can experience teaching mathematics or imagine teaching mathematics and all that it entails. Another implication is that knowing which areas are more likely to change such as the students' conceptions of processes and usefulness of mathematics, teacher educators should provide information, activities, and experience that will help their students increase their understanding in these areas. Finally, an implication for mathematics teacher educators would be to realize that dissatisfaction plays a role in change, and they may have some students who will not change regardless of what they do.

APPENDICES

APPENDIX A

Documentation of TE 401, section 17, Fall 1995

An important part of my work as a teacher involves examining our class discussions closely as well as looking carefully at what you write to me and to yourselves. Having records of our discussions and copies of your work is invaluable to me as a teacher. These help me to shape the instruction of the class to be responsive to you and also help me to continue to develop my own practice as a teacher. For myself, I write journal entries about each of our class sessions that give me a record of my own thoughts and reflections. I sometimes share what happens in our classes and talk with colleagues about teaching. These kinds of activities are an important part of learning in and from my own teaching.

I do these things because I find that they help me to teach well and to continue to develop my teaching. Although I have been teaching for several years now, there is always more to learn and new things to strive for. These activities that I describe above are also an important facet of being in a professional community and supporting the learning and development of other teachers. They are the same kinds of things that you will be encouraged to do as an intern next year -- filming your teaching, audio taping your lessons, collecting students' written work, writing journal entries about your class sessions, discussing your work with colleagues -- and, hopefully, you will have opportunities, resources, and support to do these kinds of things as a practicing teacher. In many ways, this is an opportunity for you to get a sense of how one teacher whom you get to watch closely (me) draws on these kinds of activities and resources to inform and develop her teaching practice.

The materials I make and collect around this course will only be used for the teaching of this course and will not be used for research or presentations during the semester. After the semester is over, I will use the records for research purposes only with your permission. At that time, you will be given the opportunity to grant or deny permission to use your work for this research.

....

I understand that the records that are being made of my TE 401 class--the audio tapes and written documents -- are part of the teaching practice of my instructor and cannot be used as data for research or as a part of a formal presentation without my consent. These documents will only be used to support the instruction of this course and the professional development of my instructor during the course. I understand that other teacher educators may be viewing these documents for these purposes during the duration of this course. I also understand that after the semester is over, I will be given the opportunity to grant or deny the use of these records for research purposes.

____

Signature Date

APPENDIX B

Informed Consent for Study of TE 401

This study will be used by me, Elaine Tuft, to fulfill the dissertation requirement for my Ph.D. in mathematics education through the Teacher Education Department at Michigan State University. The purpose of this study is to gain a better understanding of the beliefs about and attitudes toward mathematics that are held by prospective elementary and how these beliefs evolve across the course of TE 401. Because research has indicated that teachers' conceptions about mathematics influence they way they teach it, and since other research indicates that certain conceptions about mathematics and teaching mathematics are foundational to effective teaching of mathematics, I hope to learn more, through this research, about these conceptions and how they evolve during the time prospective students are enrolled in a course designed to help prepare them to teach mathematics. Because of this, I would like to analyze data collected in your section of TE 401. The things I would like to use to conduct this research are:

Audiotapes of classes: I will study the kinds of activities and interactions that occurred during the class that provide information related to my research questions. I might at times also allow other researchers, teacher educators, and teachers to listen to portions of the audiotapes in order to facilitate discussion of teaching and learning in teacher education settings. Having these kinds of concrete examples is an important part of creating courses for the education and professional development of teacher educators. I will not identify you by name to anyone who listens to these tapes; however, you could be identifiable to someone who knows you.

Copies of your writing and work for the course: Another important source of information about students beliefs and attitudes associated with mathematics is to look at students' writing and other work and instructors' comments. I would use students' work to study students' beliefs about and attitudes toward mathematics and the evolution of these conceptions. This would indude your journal entries and the inventory completed at the beginning and end of the semester as well as other assignments that were part of the course. These analyses are for research purposes only. Everything will be labeled with pseudonyms so that you are not identifiable, and the analyses I will conduct will not be used in any connection with grades, placements, or other decisions about your development and learning in your teacher education program.

Audiotapes of meetings with individual students during the semester: Informal interviews with individual students were taped at various points throughout the semester. Like the copies of student work, I would examine these interviews to better understand students' beliefs and attitudes related to mathematics and teaching mathematics. These analyses are for research purposes only. Everything will be labeled with pseudonyms so that you are not identifiable, and the analyses I will conduct will not be used in any connection with grades, placements, or other decisions about your development and learning in your teacher education program.

Audio tapes from your interview will be used for educational and research purposes if you give your permission.

If you desire further information about this study, you may call Elaine Tuft at 351-4690. Results of this research will be shared with you on request.

Three separate permissions are sought below: one, to use information collected about TE 401 from class sessions; two, to use copies of your class work from TE 401; and three, to use interviews in which you may have participated during fall semester.

Permission to participate and permission to use <u>TE 401 class data</u> for research and educational purposes:

I have read the above statement and agree to participate in this research by giving my permission for Elaine Tuft to analyze information collected during my TE 401 class for use in her doctoral dissertation: audiotapes of class sessions. I understand that my real name will never be used in any written reports of the research, and my responses will be treated with confidentiality.

Signature: ______________________________ Date: __________

In addition:

[ ] I give my permission for audio tape to be used for the educational purposes described. I do understand that although my name will not be provided to others who would listen to excerpts from the tapes, I might be identifiable to people who recognize me.

[ ] I do NOT give my permission for videotape and audio tape in which I can be identified to be used for the educational purposes described.

I understand that I am free to withdraw this consent at any time.

Signature: ______________________________ Date: __________

Permission to participate and permission to use <u>TE 401 class work</u> for research and educational purposes:

I have read the above statement and agree to participate in this research by giving my permission for Elaine Tuft to analyze photocopies of my class work from TE 401 for use in her doctoral dissertation. This will include my journal entries, the inventory completed at the beginning and end of the semester, and all other assignments. I understand that my real name will never be used in any written reports of the research, and my responses will be treated with confidentiality .

I understand that I am free to withdraw this consent at any time.

Signature: ______________________________ Date: __________

Permission to participate and permission to use <u>interviews</u> about TE 401 for research and educational purposes:

I have read the above statement and agree to participate in this research by giving my permission for Elaine Tuft to use tape-recorded interviews in which I participated during the fall semester for use in her doctoral dissertation. I understand that my real name will never be used in any written reports of the research, and my responses will be treated with confidentiality.

I understand that I am free to withdraw this consent at any time.

Signature: ______________________________ Date: __________

In addition,

[ ] I give my permission for audio tape in which I can be identified to be used for the educational and research purposes described.

[ ] I do NOT give my permission for audio tape in which I can be identified to be used for the education and research purposes described.

I understand that I am free to withdraw this consent at any time.

Signature: ______________________________ Date: __________

APPENDIX C

Mathematics Inventory Consent Form

To fulfill the dissertation requirement for my Ph.D. in mathematics education through the Teacher Education Department at Michigan State University, I, Elaine Tuft, am conducting a study with the purpose of gaining a better understanding of the beliefs about and attitudes toward mathematics that are held by prospective elementary and how these beliefs evolve across the course of TE 401. Because research has indicated that teachers' conceptions about mathematics influence they way they teach it, and since other research indicates that certain conceptions about mathematics and teaching mathematics are foundational to effective teaching of mathematics, I hope to learn more, through this research, about these conceptions and how they evolve during the time prospective students are enrolled in a course designed to help prepare them to teach mathematics.

Because of this, I would like to have you complete the attached mathematics inventory which will be used in the analysis of my research questions. You will also be invited to complete this same inventory at the end of the semester. At that time you will again be given the opportunity to complete it, thus giving your consent for it to be used, or decline completing it. These analyses will be for research purposes only. Everything will be labeled with pseudonyms so that you are not identifiable, and the analyses I will conduct will not be used in any connection with grades, placements, or other decisions about your development and learning in your teacher education program. During the semester, your instructor will not see these inventories and will not know whether you completed it or declined to. After the semester is over and grades have been given, your instructor will be allowed to see the results of this inventory summarized as a class aggregate if she desires.

If you desire further information about this study, you may call Elaine Tuft at 351-4690. Results of this research will be shared with you on request.

....

Permission to participate and permission to use <u>TE 401 class work</u> for research and educational purposes:

I have read the above statement and agree to participate in this research by giving my permission for Elaine Tuft to analyze responses to this mathematics inventory for use in her doctoral dissertation. I understand that by completing the inventory and returning it to her, I have given permission for her to use the information in her analyses. I understand that my real name will never be used in any written reports of the research, and my responses will be treated with confidentiality. I also understand that my instructor will not see my responses to this inventory until after the semester is over and grades have been given, and that then she may see (if she desires) results of this inventory summarized as a class aggregate.

Please check one of the following options::

[ ] I will complete this inventory and thus give my permission for it to be used in research purposes described above.

[ ] I choose NOT to complete this inventory.

I understand that I am free to withdraw this consent at any time.

Signature: ____ Date: ______________

APPENDIX D

Student #____________________

TE 401 Mathematics Inventory

On each scale, circle the number that corresponds to how much the following statements are like you or are things you would be inclined to say.

1. I enjoy doing mathematical tasks.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

2. I have always enjoyed mathematics.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

3. I think mathematics is an important subject that is useful in life.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

4. I think of mathematics as a set of isolated facts that need to be memorized.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

5. I think mathematics is something very much related to this world.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

6. I feel successful when I am engaged in mathematical tasks.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

7. Doing mathematics is usually very frustrating for me.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

8. I am excited that I will most likely have the opportunity to teach mathematics as an elementary teacher.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

9. I feel enthusiastic about mathematics and about teaching it.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

10. Teaching mathematics should not be too difficult since a textbook already has the lessons prepared.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

11. Mathematics is something that not everyone is able to understand.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

12. I don't expect to encounter any mathematics when I teach elementary school that I won't understand since I successfully completed elementary school.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

13. The only people who really need to understand math are those who choose a career which requires math skills.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

14. I feel confident in my ability to explain mathematical concepts to children.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

15. I understand elementary level mathematical concepts quite well myself.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

16. I would feel more comfortable if I knew that I would not have to teach mathematics to elementary students.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

17. Mathematics is something at which everyone can be successful.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

18. Being good at mathematics is not required to be a good teacher of math.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

19. I want to teach mathematics so that I can help students get excited about it.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

20. As a teacher, I would feel embarrassed if a student asked a question to which I didn't know the answer.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

21. I expect to regularly teach mathematics as an elementary teacher.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

22. The best way to learn math is to memorize the rules.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

23. Students' reasoning is more important than if they are able to get the answer that matches the answer key.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

24. In learning math, it is important to master topics and skills at one level before going on.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

25. I can handle basic math, but I wouldn't do well at advanced mathematics.

1	2	3	4	5
Not at all like me		Somewhat like me		Very much like me

Answer the following questions as completely as possible in the space provided.

26. What do you consider to be your greatest strengths in relation to being a mathematics teacher?

27. What concerns you the most about teaching mathematics to elementary students?

28. What kind of influence do you think the teacher plays in the success of students in mathematics?

29. Do you think a teacher's attitude toward mathematics affects how he or she teaches it? Why or why not? How about a teacher's attitude about teaching it? Explain.

30. Do you think your mathematical expertise (or lack of expertise) will be a liability or a strength when you teach mathematics in an elementary school? Explain.

APPENDIX E

Interview #1 Guide

Name________________________ Date_________________________

1. **Tell me a little about yourself.**
   a. **Age**
   b. **Where from**
   c. **Major/year**

2. **How much mathematics and which classes did you have in high school?**

   a. **How would you describe how you did in these classes? (feel successful?, grades?, like the class?, etc.)**

3. **How much mathematics and which classes did you have in college? Were they all at MSU?**

   a. **How would you describe how you did in these classes? (feel successful?, grades?, like the class?, etc.)**

4. **For the remainder of this interview, I am going to go over the mathematics inventory with you that you filled out the first day of class.** (Show it.) **Does this look familiar?**

   For each of the Likert-scale items ask:
   a. **What does this statement mean to you?**

   b. **Which number you would circle today and why?**

   (Circle the number they would circle today or have them circle it.)
   c. **Do you remember what you answered the first day of class?**

   (Show them their answer to that item on the Mathematics Inventory.)
   d. **If your answer has changed, why do you think it has changed? OR If your answer is the same, why do you think it hasn't changed?**

APPENDIX F

Elaine A. Tuft / Dissertation Data Collection

Interview 2 Guide

Name________________________ Date_________________________

Background in Mathematics

1. **Tell me a little about your math history?**

Conceptions of Mathematics

2. **How would you define or describe math?**

3. **What does it mean to know math?**

4. **What does it mean to do math? How can you tell when someone is doing math?**

5. **What is the best way to learn mathematics?**

6. **Does math exist differently or at all outside of a school classroom? If so, how?**

7. **Does what it means to do math depend on who's doing it or where it's done (i.e., a mathematician vs. a student vs. a homemaker or in a mathematics department of a university vs. in a school classroom vs. outside of school)?**

8. **Have your ideas about what it means to know and do math changed at all during your life? If so, can you identify what has caused or affected that change?**

Attitudes toward Mathematics

9. Do you consider mathematics as something valuable? Where is it valuable? For whom is it valuable? Is it valuable for you personally?

10. How confident do you feel "doing" math?

11. When have you felt the most confident doing mathematics? Can you think of what helped you feel that way?

Attitudes toward Teaching Mathematics

12. How do you feel about the idea of teaching mathematics in an elementary school (confident, apprehensive, excited, etc.)? Have your feelings about teaching mathematics ever changed?

13. What concerns do you have about teaching mathematics?

14. What are your ideas about how you think math is most effectively taught and how you want to teach it?

Other

15. Is there anything else related to any of the questions I have asked that you think might be important for me to know but I neglected to ask?

REFERENCES

REFERENCES

Adkins, G., & Robinson, S. (2002, November). *The effects of mathematics methods courses on preservice teachers' attitudes toward mathematics and mathematics teaching*. Paper presented at the Annual Meeting of the Mid-South Educational Research Association, Chattanooga, TN.

Allen, D. S. (2001). *Mathematics experience: Contributing factors to the math anxiety and avoidance behaviors of female elementary school pre-service teachers.* Unpublished doctoral dissertation, Texas Tech University, Lubbock.

Allen, E. (1991). *The effects of a video-based inservice workshop for middle school mathematics teachers*. Unpublished master's thesis, Utah State University, Logan.

Arp, K. C. (1999). *The relationship of preservice preparation to teachers' attitudes towards mathematics and teaching middle school mathematics.* Unpublished doctoral dissertation, Texas A&M University, College Station.

Ball, D. L. (1989). Breaking with experience in learning to teach mathematics: The role of a preservice methods course. *For the Learning of Mathematics,* 10 (2), 10-16.

Ball, D. L. (1988a). Unlearning to teach mathematics. *For the Learning of Mathematics, 8*(1), 40-48.

Ball, D. L. (1988b). *Knowledge and reasoning in mathematical pedagogy: Examining what prospective teachers bring to teacher education*. Unpublished doctoral dissertation, Michigan State University, East Lansing.

Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching* (Vol. 2), (pp. 1-48). Greenwich, CT: JAI Press.

Becker, J. R. (1986). Mathematics attitudes of elementary education majors. *Arithmetic Teacher, 33* (5), 50-51.

Blaire, E. (1981). Philosophies of mathematics and perspectives of mathematics teaching. *International Journal of Mathematics Education in Science and Technology, 12,* 147-153.

Bulmahn, B. J., & Young, D. M. (1982). On the transmission of mathematics anxiety. *Arithmetic Teacher, 30* (3), 55-56.

Campbell, B. J. (1998). *Changes in student teachers' thinking and beliefs about mathematics during the student teaching experience.* Unpublished doctoral dissertation, University of Maryland, College Park.

Confrey, J. (1978). *Conception of mathematics survey*. Unpublished manuscript, Mount Holy Oak College, South Hadley, MA.

Cooney, T. J., Shealy, B. E., & Arvold, B. (1998). Conceptualizing belief structures of preservice secondary mathematics teachers. *Journal for Research in Mathematics Education, 29*(3), 306-333.

Courch-Kuchey, D. L. (2002). *The effect of a constructivist mathematics methods course on the level of mathematics anxiety in preservice early childhood teachers.* Unpublished doctoral dissertation, University of Cincinnati.

Dossey, J. A. (1992). The nature of mathematics: Its role and its influences. In *Handbook of Research on Mathematics Teaching and Learning,* D. A. Grouws (Ed.). New York: Macmillan.

Dutton, W. H. (1954). Measuring attitudes toward arithmetic. *Elementary School Journal, 55,* 24-31.

Dutton, W. H. (1962). Attitude change of prospective elementary school teachers toward arithmetic. *Arithmetic Teacher, 9,* 418-424.

Ernest, P. (1991). *The philosophy of mathematics education*. Bristol, PA: The Falmer Press.

Fennema, E., & Sherman, J. (1976). Fennema-Sherman Mathematics Attitude Scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Psychological Documents*. Ms. No. 1225.

Flanders, J. (1987). How much of the content in mathematics textbooks is new? *Arithmetic Teacher, 35*(1), 18-23.

Floden, R. E., McDiarmid, G. W., & Wiemers, N. (1990). *Learning about mathematics in elementary methods courses* (Research Report 90-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Fullan, M., (1993). *Change forces: Probing the depths of educational reform.* Bristol, PA: The Falmer Press.

Gibson, H. L., & Van Strat, G. A. (2001). *A longitudinal study of the impact of constructivist instructional methods on preservice teachers' attitudes toward teaching and learning mathematics and science.* Paper presented at the Annual Meeting of the National Association of Research in Science Teaching, St. Louis.

Grant, T. J., Hiebert, J., and Weame, D. (1994, April). *Teachers' beliefs and their responses to reform-minded instruction in elementary mathematics.* Paper presented at the annual meeting of the American Educational Research Association, New Orleans.

Gregg, P. A. (1998). *Attitudes toward mathematics and knowledge of mathematical concepts of preservice elementary, early childhood, and special education teachers.* Unpublished doctoral dissertation, Clemson University, Clemson, SC.

Hare, A. Y. V. M. (1999). *Revealing what urban early childhood teachers think about mathematics and how they teach it: Implications for practice.* Unpublished doctoral dissertation, University of North Texas, Denton.

Hawkins, D. (1967). I, thou, and it. In *The informed vision: Essays on learning and human nature.* New York: Agathon.

Jesunathadas, J. (1990). *Mathematics teachers' instructional activities as a function of academic preparation.* Unpublished doctoral dissertation, Utah State University, Logan.

Kane, R. B. (1968). Attitudes of prospective elementary school teachers toward mathematics and three other subject areas. *Arithmetic Teacher, 15,* 169-175.

Kelly, W. P., & Tomhave, W. K. (1985). A study of math anxiety/math avoidance in preservice elementary teachers. *Arithmetic Teacher, 32* (5), 51-53.

Krows, A. J. (1999). *Preservice teachers' belief systems and attitudes toward mathematics in the context of a progressive elementary teacher preparation program.* Unpublished doctoral dissertation, The University of Oklahoma, Norman.

Kyriakides, L., Philippou, G., & Charalambous, C. (2002, July). Towards understanding teachers' philosophical beliefs about mathematics. In *Proceedings of the Annual Meeting of the International Group for the Psychology of Mathematics Education,* Norwich, England.

Larson, C. N. (1983). Techniques for developing positive attitudes in preservice teachers. *Arithmetic Teacher, 31,* 8-9.

Lerman, S. (1983). Problem-solving or knowledge-centered: The influence of philosophy on mathematics teaching. *International Journal of Mathematics Education in Science and Technology, 14*(1), 59-66.

McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In Douglas A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning.* New York: Macmillan Publishing Company.

Miller, J. L. (1999). *The mathematics preparation of elementary teachers.* Unpublished doctoral dissertation, University of Kentucky, Lexington.

National Council of Teachers of Mathematics. (1988). *Preparing elementary school mathematics teachers: Readings from the Arithmetic Teacher.* Reston, VA: Author.

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (1995). *Assessment standards for school mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

National Research Council. (1989). *Everybody counts--A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

Nespor, J. (1987). The role of beliefs in the practice of teaching. *Journal of Curriculum Studies, 19*, 317-328.

Patton, L. S. (2002). *A study of mathematics anxiety of the pre-service elementary teacher at the University of Arizona*. Unpublished doctoral dissertation, The University of Arizona, Tucson.

Pejouhy, N. H. (1990). Teaching math for the 21st century. *Phi Delta Kappan, 72* (1), 76-78.

Pittman, D. T. (2002). *A study examining the role of teacher beliefs and how these beliefs affect the teaching of mathematics*. Unpublished doctoral dissertation, George Mason University, Manassas, Virginia.

Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28* (5), 550-576.

Reys, R. E., & Delon, F. G. (1968). Attitudes of prospective elementary school teachers towards arithmetic, *Arithmetic Teacher, 15*, 363-366.

Schon, D. (1987). *Educating the reflective practitioner*. San Francisco: Jossey-Bass.

Schram, P., Wilcox, S., Lanier, P. & Lappan, G. (1988). *Changing mathematical conceptions of preservice teachers: A content and pedagogical intervention* (Research Report 88-4). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Smith, D. M. (1999). *Preservice elementary teachers' attitudes toward mathematics and the teaching of mathematics in a constructivist classroom.* Unpublished doctoral dissertation, Oklahoma State University, Stillwater.

Smith, F. (1964). Prospective teachers attitudes toward arithmetic. *Arithmetic Teacher, 11*, 474-477.

Thompson, A. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In Douglas A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning*. New York: Macmillan Publishing Company.

Thompson, A. (1984). The relationship of teachers' conceptions of mathematics teaching to instructional practice. *Educational Studies in Mathematics, 15*, 105-127.

Tuft, E. A. (1994). *A look at the mathematical beliefs of some prospective elementary school teachers*. Unpublished Manuscript. Michigan State University, East Lansing.

Vacc, N. N., & Bright, G. W. (1999). Elementary preservice teachers' changing beliefs and instructional use of children's mathematical thinking. *Journal for Research in Mathematics Education, 30*(1), 89-110.

van Langenhove, L. & Harre, R. (1999). Introducing positioning theory. In Rom Harre & Luk van Langenhove (Eds.), *Positioning theory*. Malden, MA: Blackwell Publishers Inc.

Waller, W. (1967). What teaching does to teachers. In *The sociology of teaching*. New York: John Wiley and Sons.

Wilcox, S. K., Schram, P., Lappan, G., and Lanier, P. (1991). *The role of a learning community in changing preservice teachers: Knowledge and beliefs about mathematics education* (Research Report 91-1). East Lansing: Michigan State University, National Center for Research on Teacher Learning.

Wilkins, J. L. M. (2002, October). The impact of teachers' content knowledge and attitudes on instructional beliefs and practices. In *Proceedings of the Annual Meeting [of the] North American Chapter of the International Group for the Psychology of Mathematics Education*. Athens, Georgia.

Wilkinson, M. E. (2001). *Foundations of attitudes toward mathematics learning and teaching held by preprofessional elementary school teachers*. Unpublished doctoral dissertation, University of Central Florida, Orlando.