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# A THEORY OF CONSUMER SURPLUS AND ECONOMIC RENT AND AN APPLICATION FOR MEASURING THE BENEFITS FROM THE MECHANICAL CUCUMBER HARVESTER

By

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A DISSERTATION

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ABSTRACT

A THEORY OF CONSUMER SURPLUS AND ECONOMIC RENT
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BENEFITS FROM THE MECHANICAL CUCUMBER
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The problems of measuring and interpreting consumer surplus, economic rent and economic surplus are explored. These concepts are then applied to measure the benefits associated with the adoption of the mechanical cucumber harvester. The problems associated with consumer surplus include estimating the Laspeyres, Paąsche, "utility", compensating and equivalent variation measures relative to various types of utility functions including homothetic and vertically parallel. The problem of economic rent includes measuring rent indirectly in the goods market relative to an industry supply curve which equals average cost excluding rent. The problems of economic surplus include welfare versus compensation interpretations and general equilibrium effects.

The theories of consumer surplus and economic rent are explored mathmatically, graphically and verbally. The Hausman and Willig techniques are compared and contrasted empirically relative to linear and exponential functional forms of compensated demand. The theoretical relationship between consumer surplus and economic rent is described. Economic rent in the resource and goods markets is compared for polar industry types. An array of techniques for presenting the results of benefit-cost analysis is presented. The general equilibrium impacts of a new technology are discussed.

Stephen Charles Cooke

The present value of consumer surplus associated with the mechanical cucumber harvester is between $35,321,000 and $35,355,000 relative to the Laspeyres and Paasche measures respectively. The present value of economic rent equals between $200,000 and $35,884,000 for a pivotal and covergent supply shift respectively. The internal rate of return is estimated to be between 55 and 75 percent. The benefit-cost rate of return ranges between -134 to -47 and 88 to 176 percent with and without displaced labor compensation respectively.

The Hausman and Willig techniques for estimating consumer surplus are best suited for "closed" and "open" form demand functions respectively. Only homothetic and vertically parallel utility functions result in a welfare rather than a compensation interpretation of consumer surplus. The kinked supply curve is consistent with the need to determine an industry supply curve excluding rent. Economic surplus can be reinterpreted as resources saved as well as a change in real income. Path independence assures an optimal restructuring of productive capacity with a minimum of transactions costs.

Dedicated to my mother

Laurette Quesnel Cooke

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# CHAPTER I

# INTRODUCTION, OVERVIEW AND BACKGROUND

## 1-1 Introduction

The changes in real income in an economy are measured in terms of consumer surplus, economic rent and economic surplus. This makes these measurements and their interpretation important. These measures imply an evaluation of non-marginal change in the structure and organization of the economy. One particular type of non-marginal change in the economy is the effect of the introduction of new technology in the production process of a commodity or group of commodities. In general, a new technology causes a shift in the production function and the aggregate supply function of a commodity. The shift in aggregate supply lowers the price of the commodity which increases the real income of consumers of the commodity and some resource owners. The shift in the supply function also implies a release of resources required in the initial production of the commodity. Resources are also saved in the production of less of the now relatively more expensive alternative commodities. These "saved resources" are equal in value to the change in real income to consumers and resource owners. In fact, these two measures are not only equal, but also mutually dependent upon each other in terms of increased consumption and production using the same resources, which is the essence of increased productivity in an economy.

The measures of consumer surplus and economic rent are examined in some detail in this study. There are serious theoretical questions associated with these indices of economic surplus. The use of consumer surplus and economic rent as measures of the change in real income has been the subject of considerable debate by many economists for a very long time. This study is in large part an attempt to understand the source of this controversy and to determine what difference it makes in estimating and interpreting changes in real income and resources saved particularly as they relate to measuring the impact of technological change. This evaluation of consumer surplus and economic rent is needed in light of recent articles which are claiming substantial breakthroughs in their measure if not their interpretation. Several titles are suggestive of the strength of the claims being made:

Hausman, J.A. "Exact Consumer Surplus and Dead Weight Loss." American Economic Review 71(4):662-676, September 1981.

Willig, Robert D. "Consumer's Surplus Without Apology." American Economic Review 66(4):589-597, September 1976.

It is important that applied economists keep abreast of these developments. The theoretical claims need to be examined carefully in an effort to determine whether too much or too little is being claimed. Beyond the theoretical claims of one author or another, however, it is important to carefully delineate the empirical considerations which are necessarily involved. Finally the matter of interpreting the results requires relating the empirical to the theoretical. Throughout this process there are a number of important assumptions which can distort as well as simplify. The process of tracking these interrelationships is tedious and involved but an important

exercise nonetheless. This study then is an attempt to systematically consider the theoretical, empirical and intuitive aspects of the problems associated with measuring change in real income particularly, but not exclusively, as it relates to the introduction of a new technology. The examination of the concepts of consumer surplus and economic rent concludes with an extended application of these concepts to the measurement of the economic impact of the introduction and adoption of the mechanical cucumber harvester in U.S. agriculture.

## 1-2 Consumer Surplus

The intention of the chapter on consumer surplus is to provide a coherent summary of the theoretical basis of the five measures of consumer surplus and to include, in some detail, the more recent developments in the theory. The problems of interpreting the measures of consumer surplus are also included particularly as they relate to the issues of separability, path independence, homothetic and vertically parallel utility functions. These concepts are central to whether consumer surplus (and economic surplus) is interpreted as a welfare measure or as a compensation measure. We return to these issues in the concluding chapter also.

Consumer surplus, as it is used in this study, is the genus, within which the Laspeyres, compensating variation, "utility," equivalent variation and Paasche measures are the major species. These five measures of consumer surplus are derived graphically and mathematically. First the assumptions associated with a simple two-good economy are described. Then the direct utility and expenditure minimization functions are presented. Then the compensated demand

function which is the first derivative of the expenditure function is presented. The expenditure and compensated demand functions are fundamental for understanding all of the measures of consumer surplus.

The Patinkin "all or none" demand function is described. The "all or none" demand function is very helpful in combination with the compensated demand function for understanding the relationship between price and income for a given level of utility. Using the insights based on the compensated demand function and the "all or none" demand function it is possible to describe first the compensating variation and the Laspeyres measures of consumer surplus. This is followed immediately with a description of the equivalent variation and the Paasche measure of consumer surplus. Finally the "utility" measure of consumer surplus associated with the ordinary demand function is presented.

The complications associated with using consumer surplus to measure the change in real income for a change in prices are presented. Problems in interpretating the "utility" measure of consumer surplus are described including a discussion of what Marshall meant when he first defined the notion of consumer surplus. Finally the developments by Hausman and Willig using the indirect utility function and Roy's identity are presented. These developments have contributed to solving some of the empirical problems of accurately measuring if not those of interpreting compensating and equivalent variation.

## 1-3 Economic Rent

The intention of the chapter on economic rent is to provide an overview and synthesis of several strands of rent theory. These

strands include the symmetry between economic rent and consumer surplus, the indirect measure of rent in the goods market relative to the Mishan criteria, and accurately measuring rent in the goods market relative to the Marshall-Viner, Lindner-Jarrett-Rose-Wise-Fell and Just-Hueth-Schmitz techniques. Economic rent can be defined and interpreted in the same way as consumer surplus. The symmetry between these two concepts is developed. Given this symmetric relationship between consumer surplus and economic rent, relatively less of this chapter is devoted to examining the relationship between price and welfare effects for a given level of utility and the associated compensated resource supply function. These relationships are presented in an abbreviated form. Rather, the main focus of attention in this chapter is an examination of the relationship between the indirect measure of economic rent in the goods market and the direct measure of economic rent in the resource market. The analysis of accurately measuring economic rent in the goods market relates to the idea that the industry supply curve must equal both average cost including rent and marginal cost excluding rent. We will refer to this as the "Mishan criteria" since he makes the case for this criteria in "What is Producer's Surplus" (1968, pp. 1274-1282).

As might be expected, it is very difficult to develop an industry supply function which meets the Mishan criteria. In an effort to understand the need for doing so, long-run supply curves for four types of industries are developed. The relationship between the type of resources required in production and their effect on the industry supply curve is examined. The four industries include firms which require the following types of resources in the production process:

(a) N abundant resources, (b) one absolutely scarce resource and N-1 abundant resources, (c) one super-abundant resource and N-1 abundant resources and finally (d) one relatively scarce resource and N-1 abundant resources. These industries represent the polar cases for examining economic rent in the factor market relative to the goods market. We are particularly interested in example (d) since this is the most common industry type and the one whose ordinary long-run supply function also violates the Mishan criteria. A futher variation on (d) would be an industry of firms requiring (e) a relatively scarce durable resource and N-1 abundant resources in the production process. This case provides yet a closer approximation to reality and the associated difficulty of measuring economic rent.

The discussion of these polar industry types is followed by an analysis of various means of estimating economic rent consistent with the Mishan criteria. The literature on the theory of accurately measuring economic rent has three more or less distinct schools of thought. These are the Marshall-Viner, the Linder-Jarrett-Rose-Wise-Fell and the Just-Hueth-Schmitz notions for measuring economic rent. These ideas seem to be consistent with and converge toward each other. More importantly they also are or can easily be modified to be consistent with the Mishan criteria. These approaches to measuring economic rent in the goods market are different in the way the intercept term is determined. Marshall-Viner suggest using cost of production data in the form of a particular expense curve to estimate rent including the intercept term. Linder-Jarrett-Rose-Wise-Fell suggest making an array of assumptions regarding the intercept term consistent with pivotal, parallel and convergent supply shifts. Finally

Just-Hueth-Schmitz suggest using econometric means to estimate the intercept term. Just-Hueth-Schmitz make an important distinction between ordinary and equilibrium supply functions. The equilibrium supply function is implicitly or explicitly common to all three approaches to measuring economic rent.

The chapter on economic rent concludes with an analysis of the impact of the introduction of new technology as a relatively scarce durable resource which causes a shift in the industry supply function. This is the analysis relevant to the introduction of the mechanical cucumber harvester. The associated measures of consumer surplus, economic rent and economic surplus are described.

Methodologically, this chapter is an all-encompassing examination of economic rent ranging from the substitution and welfare effects associated with vertically parallel and homothetic utility functions on the one hand to measuring shifts in the industry's supply function associated with a new technology on the other. It is so wide-ranging because the rent literature itself, a literature which seems to be in some disarray, is wide-ranging. Therefore it was deemed necessary to attempt a synthesis of ideas in this area. The section on the relationship between economic rent relative to consumer surplus is needed to preserve continuity and facilitate interpretation. The section on the polar types of industries is necessary as a means of exploring and coming to grips with both the Mishan criteria and the relationship between rent measured (directly) in the factor market and (indirectly) in the goods market. This section uses a simplifying device of assuming a shift in aggregate demand. This is done even though the

primary interest in this study is on the effects of a shift in aggregate supply. Hopefully the reader will not be confused when we then return to examining various alternative measures of consumer surplus, economic rent and economic surplus as proposed by Marshall et al.; Lindner et al. and Just et al. in terms of a supply shift.

## 1-4 Empirical Results

In the empirical chapter, the impact of the introduction and adoption of the mechanical cucumber harvester is estimated in terms of consumer surplus and economic rent under an array of assumptions and techniques. The empirical results include estimates of the five measures of consumer surplus using both linear and exponential demand functions. The linear and exponential functional forms are used to compare and contrast the Willig and Hausman techniques for estimating compensating and equivalent variation. Economic rent is also presented in terms of the Lindner-Jarrett-Rose-Wise-Fell theory by assuming various types of supply shifts including pivotal, parallel and convergent. A linear equilibrium supply function for cucumber pickles is used since it both meets the linearity assumption of the cross-multiplication technique and appears to be a reasonably good approximation of the supply function. Then the cost of the research and development for the mechanical harvester is presented. The economic surplus is estimated and presented in terms of a benefit-cost ratio, a benefit-cost rate of return and an internal rate of return. These rates are then compared with rates of return estimated for hybrid corn, poultry and U.S. agricultural research in general. Finally, the value of resources saved is estimated, particularly labor

resources. Then assumptions are made regarding the level of labor re-employment. Rates of return are then calculated which include these assumptions.

In order to estimate consumer surplus, economic rent and economic surplus, it is necessary to estimate a number of key values. These values include initial and subsequent equilibrium price and quantity, the own-price, cross-price and income elasticities of demand and own-price elasticity of supply. A measure of the supply shift which is attributable to the introduction of the mechanical harvester is also estimated. Once these values are determined, then they can be applied to the appropriate specific form equations to calculate the benefits and costs of research on the mechanical cucumber harvester.

The empirical chapter has essentially three purposes. First, we will illustrate the Willig and Hausman's contribution to consumer surplus theory as well as the differences and similarities between their claims and actual estimates. Second, we will estimate economic rent which meets the Mishan criteria in terms of the Lindner-Jarrett-Rose-Wise-Fell approach. Third, we will estimate economic surplus and the benefit-cost indices needed to compare the returns to research on the mechanical cucumber harvester with other agricultural research and development projects.

## 1-5 Historical Background

The mechanical cucumber harvester was selected as the subject for this thesis for a variety of reasons. The research on the cucumber harvester took place at Michigan State University in the Department of Agricultural Engineering. The people associated with

the development of the mechanical cucumber harvester are, for the most part, still on faculty. Michigan State University is still very active in research relating to the production and processing of cucumbers for pickles. The data on the research and development of the harvester were readily available and the research had a known audience interested in the results.

Furthermore, the mechanical cucumber harvester is a "labor-displacing" technology. Labor-displacing technology research has come under increasing criticism. A recent ruling by a California court in a suit brought by seasonal farm workers against the University of California required the University to conduct an economic impact analysis before undertaking labor-displacing technology research in the future. Therefore research on the methodology of estimating the economic impact of labor-displacing technology is timely.

Finally, the mechanical cucumber harvester was introduced in 1965, the same year as the mechanical tomato harvester. Schmitz and Seckler published an article on the economic impact of the tomato harvester entitled "Mechanized Agriculture and Social Welfare: The Case of the Tomato Harvester" (1970 pp. 569-577). A comparison of the economic impact of the two mechanical harvesters should be of some interest, and is included in this study.

Before turning to the theory and the data for estimating the benefits from technology, let us begin by describing the historical context of one technology. The story of the mechanical cucumber harvester represents a microcosm of the technological transformation since World War II of American agriculture in general and the fruit

and vegetable sector in particular. The setting for this story is Michigan. Michigan grows more pickling cucumbers than any other state in the Union, producing 126,000 tons of cucumbers for processing in 1978 or about 25 percent of the nation's cucumber production. The research on the mechanical cucumber harvester was done at Michigan State University (MSU) in the Department of Agricultural Engineering from 1956 to 1964. The research was directed by Professors Bill Stout of Agricultural Engineering and Stan Ries of Horticulture. Though the research itself was done by a few specific individuals, the story of the research, development and commercial adoption of the mechanical cucumber harvester is really about the coordinated efforts of many people from many different groups. In fact, the case of the mechanical cucumber harvester involves representatives from all the important agricultural groups: migrant workers, small farmers, processors, small machinery manufacturers, large agribusiness firms, large machinery manufacturers, large interest-group organizations; large research organizations including land grant institutions such as Michigan State University and finally the executive, legislative and judicial branches of the state and federal governments.

The purpose of this section is to provide the general chronological sequence of events in the mechanical cucumber harvester story. In the process of presenting this chronology, we will touch upon such issues as induced innovation; championing innovation; establishing research goals; private and public sector interaction; the processes of organizing, financing, and managing research; investment decision making and the productive and distributive impacts relative to the

mechanical cucumber harvester. In addition to the facts and figures of this case, this section is intended to include something of the sense of excitement and bold adventure as well as the pain and suffering of the unfolding events. In many ways this case touches on aspects of Americans and their institutions both at their best and worst, with all their strengths and limitations. Perhaps this is because Americans are most in tune with themselves when they are advancing on a new frontier. Finally, this chronology can be viewed as part of the MSU Agricultural Experiment Station is institutional memory through which it can know itself better.

Cucumbers were introduced in Michigan as a cash crop in the late 19th century for reasons which are now vague. Some say that after the pine trees were cut, some of the wealthy families of the lumber industry turned to pickle processing as a new growth industry. The climate, soils and proximity to markets in Chicago and Detroit were no doubt important factors too. The western border of Michigan along Lake Michigan has up to a 180-day growing season with 32 to 36 inches of annual rainfall. These may explain generally why Michigan ranked fifth in the nation in harvested acres of fresh vegetables and eight in processed vegetables in 1977. By far the largest number of harvested acres devoted to a single vegetable crop were the 26,000 acres of cucumbers, of which 24,000 acres were harvested for the processing market. In 1978, Michigan ranked first in the production of cucumbers for the processed market and ninth in the production for the fresh cucumber market. Over time, these two markets have become increasingly distinct with increased specialization regarding seed varieties,

cultural practices, harvesting and marketing. This study focuses on the processed market for pickling cucumbers exclusively.

Another important factor in the growth of the cucumber pickle industry was the ready supply of migrant labor available during harvesting. The sugar beet industry in Michigan began in the 1890s around Saginaw. A large number of migrant laborers who came to Michigan to work in the sugar beet fields were available to work in other commodities as well. Between the sugar beet thinning in May and its harvesting in mid-September, these migrant laborers were often employed harvesting other crops such as asparagus (harvested from May until mid-June), strawberries (harvested in late June), cherries (harvested in July), snap beans and cabbage (harvested in July and August), and cucumbers (harvested from August until mid-September).

From 1890 to 1920, migrant labor was supplied by the European immigrants coming to this country in large numbers. When legal restrictions on immigration in 1919 and 1921 reduced the flow of this source of labor, the producers turned to Mexican-American migrants from Texas. These Mexican-Americans provided migrant labor in Michigan from 1920 to the present. During World War II, the Bracero program was enacted. This program allowed Mexican nationals to enter the U.S. for the purpose of guaranteeing adequate wartime manpower in agriculture. Braceros were the best of Mexico's semi-skilled labor force--only healthy young men were employed. Wage rates for these laborers paid by growers were set by mutual agreement between the U.S. and Mexican governments. As part of the agreement, the laborers were to return to Mexico at the end of each growing season. A second phase of the Bracero program began in 1950. After World War II, the U.S.

government agreed to continue the program partly in response to the demand by growers for a secure and highly productive labor force and partly to give the various labor-intensive agricultural subsectors time to make alternative harvest arrangements (i.e., to mechanize). The Bracero program ended December 31, 1964.

The growth and development of the pickling cucumber subsector in Michigan was the result of a conjunction of more or less random forces: good markets; good growing conditions; and a large labor supply which was continuously available. The leaders of the pickling cucumber subsector were faced with the first major challenge to their growth and security with the mechanization of sugar beet production in the 1940s. The sugarbeet technology was developed primarily at the University of California at Davis. Cucumber producers and processors knew that once the war was over the Bracero program was scheduled to end. Domestic migrant laborers, it was feared, would no longer make the long trek from Texas to Michigan in sufficient numbers to assure a stable and adequate supply of labor to harvest cucumbers once sugar beets were mechanized. Or perhaps migrant laborers would only come for the few best-paying or least onerous crops. The pickling cucumber crop was neither and without migrant labor to harvest pickling cucumbers in August, the processed cucumber subsector was doomed.

With this scenario looming as a real possibility on the horizon, leaders within the industry began to explore the possibility of harvesting pickling cucumbers mechanically. As early as 1945, H.J. Heinz Company was experimenting with techniques to harvest cucumbers mechanically. In 1948, the National Pickle Packers Association (NPPA) (the

name was changed to Pickle Packers International in 1963), the pickle processor association, sponsored a graduate student named A.A. Banadyga at North Carolina State University, to do a complete search of the literature on all aspects of pickle production. This search revealed that no mechanical harvester research had taken place at any university including land grant institutions. NPPA then searched for all private entrepreneural research efforts to mechanize cucumber harvesting. There were several.

In the early 1950s, Ike Pierson was growing about 200 acres of pickling cucumbers near Ravenna, Michigan. This was a very large acreage for pickling cucumbers prior to the development of the mechanical harvester. Mr. Pierson had a large family and he was concerned for their safety given that there were necessarily a large number of migrant laborers on his farm for a substantial period of time each summer. He approached Lloyd Gilbert, who had a local reputation as an inventive genius, about developing a mechanical harvester. Gilbert did just that and his idea was so successful that it was patented and licensed to Chisholm-Ryder Manufacturing Company of Niagara Falls, New York. In all, about 25 to 30 of these machines were sold before the line was discounted in the mid-1960s. These early machines were all designed to conform to the prevailing cultural practices of the day. They were multi-pick machines which would harvest individual cucumber fruit as they reached the correct size without damaging the vine--at least this was the goal.

Other multi-pick innovators include Dolphus Grew of Saginaw, Michigan and Stewart Craig from Manawa, Wisconsin. Like Gilbert, each of these inventors tried to build a machine to emulate human picking

abilities. Such a machine would pick the crown fruit closest to the roots and all other fruits of the size most valuable for pickling, especially the small cucumbers, without injuring the vine. E.G. Humphries at North Carolina State University is still experimenting with multi-pick cucumber harvesters.

The leadership of NPPA, the processor association, and H. J. Heinz Company apparently believed that private entrepreneurial researchers would not be able to develop a commercially acceptable machine. A meeting of the research and development committee of NPPA decided to request help from agricultural engineers at a land grant university. Several universities were considered. Finally, it was decided to recommend to the NPPA executive committee that MSU's Agricultural Engineering Department be approached about doing research on a mechanical cucumber harvester which would be partially financed by NPPA. This decision was no doubt strongly influenced by the long list of cucumber research done at MSU since 1923. (To date MSU has about 70 M.S. and Ph.D. theses to its credit relating to some aspect of the cucumber pickle production process.) In the spring of 1956, a meeting with researchers in the Department of Agricultural Engineering was set and an agreement was reached to begin work on the mechanical cucumber harvester. The objectives of project 576 were three:

(1) to investigate the basic requirements of picking cucumbers mechanically,

(2) to evaluate and develop cucumber harvesters furnished by the industry, and

(3) to invent, design and develop a satisfactory cucumber harvester.

Project 576 was under the initial direction of B. A. Stout, an agricultural engineering graduate student; Professor W. F. Buchele of the Agricultural Engineering Department, power and machinery section; Professor I. J. Pflug of the Agricultural Engineering Department, food processing section; Professor S. K. Ries of the Horticulture Department, vegetable cultural practices section. This project, as it turned out, went through three distinct phases. The first phase from 1957 to 1959 was devoted to evaluating the multi-pick cucumber harvester as exemplified by the machines of Craig, Gilbert and Grew and to synthesize the best features of each into a best possible multi-pick machine. Stout and Ries concluded by 1960 that there was no future in the multi-pick idea. More specifically, they concluded that any machine that did not damage the vine would not pick the cucumbers.

The second phase of the research was more difficult. Stout and Ries wanted to try to develop a once-over destructive harvest concept. NPPA balked. This was a radical approach to harvesting cucumbers. Hand laborers could pick cucumbers from the same plant twice a week for six weeks or a total of twelve mini-harvests per season. Once-over destructive harvesting would, by contrast, yield only a single harvest per season. On the face of it this approach would only be $1/12^{th}$ as productive as the standard production practices.

The key to the new approach was to increase yield both through better seed varieties and more intensive plant populations and other improved cultural practices. This part of the plan was developed by Professors Ries and Peterson in the Horticulture Department. In 1960, Professor Peterson, a plant geneticist, had developed a new pickling cucumber hybrid called Spartan Dawn. Peterson's work led to increased

disease resistance in cucumber varieties as well as traits for the simultaneous maturation of fruit.

Finally, the NPPA accepted the feasibility of the new idea and provided further financial support. By this time Scott-Viner Company of Columbus, Ohio, also became interested in the research on the cucumber harvester going on at Michigan State University. Scott-Viner (merging with FMC in June 1962) was the farm machinery manufacturer that had supported MSU research on sugar beets earlier. Scott-Viner was also interested in producing a cabbage harvester also being developed at MSU by Stout and others. NPPA and Scott-Viner entered into a cooperative research agreement with MSU. The essence of the cooperative research agreement was that researchers at MSU would do the research and MSU and the researchers would share the rights to any resulting patents. The machinery manufacturer (i.e. Scott-Viner) would be granted an exclusive license to any patents in return for royalty payments to the patent holders. NPPA and Scott-Viner made a financial commitment totaling $16,000 extended over three years, 1961-1963, half for harvester and half for horticultural research.

On the 15th of September 1961, an agricultural engineering graduate student named Max De Long wrote in his journal,

> Roller test ... 2-inch diameter rubber washing machine rollers together, 0 clearance, only bottom roll driven. First plant took oversize, snapped off and broke one small no. 1 ....

This is the germ of the idea that became the mechanical cucumber harvester. This page in De Long's journal was signed and dated by inventors Max De Long, B. A. Stout, and S. K. Ries and notarized by

Virginia M. Yang on October 10, 1961. This signing and notarizing process is a procedure recognized by the U.S. Patent Office as a way to establish the date of discovery in order to protect the idea from "later discoveries." The inventors then had a year from the notarized date, or from October 10, 1961 to October 10, 1962 in which to apply for a patent.

The roller test ushered in the third phase of project 576. The idea had to be developed and tested before Scott-Viner would be interested in licensing the patent. This involved refinement of the concept and the building of a prototype for field testing. De Long tested the concept during the summer of 1962 and his M.S. thesis was based on his experience. In 1962, FMC purchased the Scott-Viner Company. FMC (these letters do not stand for anything, FMC is the company name) made it clear that they wanted to manufacture the mechanical cucumber harvester but they did not want two or three years of development time to come out of their five years of manufacturing protection. So, in May 1963, Don Pettengill was hired by the MSU Agricultural Engineering Department as a research assistant to develop the prototype harvester for field tests.

In June 1963, Public Law 78 (the Bracero program) was rejected by the U.S. Congress on a vote of 174 to 158. PL 78 was later extended for one year to cover the 1964 growing season. The race was on. During the spring and summer of 1963, Stout and Pettengill developed a pull-type mechanical cucumber harvester prototype. During field tests that summer the machine showed promise but the vine pick-up mechanism still posed a problem that was not resolved. Stout decided that the university had developed the harvester enough. Regarding the pick-up

mechanism he told FMC that it was a problem "to be solved by the industry and has been solved many times before."

In August 1963, the terms of the exclusive license between FMC and MSU were agreed upon. The terms were

(1) FMC would hold an exclusive license on the MSU cucumber harvester patent no. 3,347,031.

(2) FMC would have the right to sub-license the patent.

(3) The license agreement was for the life of the patent.

(4) Royalty payments to the MSU would be 3% of net retail sales on the first 100 machines.

(5) FMC was responsible for constructing an experimental model in 1964.

(6) FMC was responsible for improving the experimental model for five years.

(7) FMC was responsible for offering a mechanical harvester for sale within 12 months.

(8) FMC must pay a $200 minimum royalty per year after the first machine was sold.

(9) FMC must test or sell the mechanical cucumber harvester during each of the first five years or the contract is void.

(10) MSU had a right to examine FMC account books.

(11) FMC had exclusive rights including the right to sue patent infringers. MSU would "remit use of its name in all such suits" at FMC's expense and MSU would receive 10% above litigation expenses and would cooperate in such suits.

(12) MSU reserved the right to terminate the contract for non-payment of royalties or bankruptcy of the company.

(13) FMC could terminate the contract at any time.

In October 1963, the prototype built at MSU was shipped to FMC's regional plant in Hooperston, Illinois. Two machines were built for testing in 1964 in cooperation with H. J. Heinz Company in Holland, Michigan. Also in October 1963, Professor Stout left for FAO in Rome, Italy, to work for a year.

About this time, Vlasic Company purchased both Sheridan Pickle Company and Keat City Pickle Company to create a new corporate entity called Midwest Pickle Company. Dean Milk Company purchased Green Bay Foods (formerly Heifety Pickle Company) and Borden's purchased Aunt Jane's. In other developments, A&P had gained sufficient retail strength to require large pickle processors to meet its central purchasing specifications. Also, the fast food industry was developing rapidly and with it a new demand for bulk pickle "chips" or cross-sectional slices to complement hamburgers.

When the U.S. Congress extended the Bracero program for one more year to cover the 1964 growing season, the cucumber pickle subsector breathed a collective sigh of relief. Migrant labor was assured. FMC had another year to develop and test the new once-over cucumber harvester. In 1964, FMC developed two different mechanical cucumber harvester test models. Both machines were tested in Holland, Michigan, on cucumber acreage provided by the H. J. Heinz Company. Both machines were self-propelled and both showed promise. It was decided to manufacture five machines for commercial sale in 1965.

1965 was a critical year for the Michigan pickle subsector. The mechanical harvester was not yet commercially available. The supply function for migrant labor was assumed to be inelastic. The Bracero

program had ended New Year's Eve 1964. In addition, a long list of legislation had been passed to improve the life of migrants by the U.S. Congress and the Michigan legislature as part of and complementary to the Great Society programs initiated by President Lyndon Johnson. This package of legislation was the secondary stimulus or the stimulus for adoption as opposed to the earlier stimulus for research. At the federal level, the Farm Labor Contractor Registration Act was passed in 1964 and went into effect in 1965. This act was designed to eliminate the exploitation of laborers by crew leaders. The Migrant Health Act was passed in 1962. The Housing Act and the Equal Opportunity Act, each with provisions to aid migrant labor, were passed in 1964. The Fair Labor Standards Act was amended in 1966 to extend minimum wage coverage to farm laborers. At the state level, the Michigan legislature passed PA 154 in 1964, guaranteeing a minimum wage for farm laborers. Michigan's workmen's compensation was amended in 1965 extending benefits to agricultural workers. The Michigan legislature also passed that year the Migrant Housing Act, PA 289. Farm laborers were being granted a bundle of rights that had been extended to industrial laborers in the 1930s.

The 1965 cucumber pickle crop was harvested. Domestic migrant laborers, augmented by teenagers and housewives, took up the slack left by the Braceros. Workers' wages in competing crops such as tomatoes were bid up. In addition, about 5% of the cucumber crop was harvested mechanically. The number of acres planted in Michigan dropped from around 25,000 acres in 1964 to about 18,000 acres in 1965 or about a 28% decrease. Yields remained about five tons per acre while price per ton increased from about $67 in 1964 to $93 in 1965 or about

a 39% increase. This implies a unitary own-price arc elasticity of demand for pickling cucumbers. By 1967, the nominal price of cucumber pickles had increased to about $111 per ton in Michigan.

Behind the scenes another small drama was taking place. In 1963, Leon Dykehouse, owner of Dykehouse Pickle Company, Inc. in Vassar, Michigan, asked Bernie Wilde of Wilde Bros. Manufacturing Company in Bailey, Michigan, to build a pull-type mechanical cucumber harvester based on an idea Dykehouse had. This harvester was built, tested and failed. However, Wilde became interested in building a mechanical cucumber harvester, so he kept experimenting on his own. Wilde saw the FMC prototypes in Holland, Michigan that were being tested in 1964. Building on the once-over concept and associated roller mechanism, Wilde worked on perfecting the vine pick-up mechanism. Wilde developed a tractor-mounted unit with an Innis pick-up that carried the cucumber fruit and vines to the roller mechanism. Field tests of this Wilde machine in North Carolina in the spring of 1965 also failed. The remaining problem was that the cucumber vines got caught in the pick-up mechanism. Wilde solved this problem in the fall of 1965 by attaching a bean knife to the front of the harvester to cut the roots of the cucumber plant, thus freeing the vines to go up the pick-up belts easily. This was the final innovative breakthrough needed to harvest cucumbers successfully.

The rebuilt Wilde harvester was field tested in 1966. H. J. Heinz Company agreed to purchase the mechanically harvested pickling cucumbers from the field tests. Furthermore, H. J. Heinz Company agreed to purchase future perfected Wilde cucumber harvesters. In October 1966, H. J. Heinz and Green Bay Foods purchased two Wilde

machines each. That same year FMC sold 31 self-propelled harvesters and about 11% of the Michigan cucumber pickle crop was harvested mechanically. In September 1966, FMC notified MSU of the possibility of a lawsuit against two companies that might be infringing on MSU patents and FMC licenses. One was the Hume tomato harvester. The other was the Wilde cucumber harvester. Not until August 1969, however, was a patent infringement suit (MSU and FMC versus Wilde Manufacturing Co., civil action no. 6138) filed regarding patent no. 3,347,031 in U.S. District Court, Western District of Michigan, Southern Division, in Grand Rapids, Michigan. In August 1970, the case was settled by consent. Wilde agreed to pay $41,064 in five yearly installments in back royalties to FMC and to pay FMC 8% royalty on future machines built under the license after January 18, 1971.

In 1967, FMC sold no new machines, repossessed 6 of the 31 machines sold in 1966 and resold two of these as used machines. That same year, 55 Wilde machines were sold: 20 to the H. J. Heinz Company, 10 to Green Bay Food Company and 25 to others. Twenty percent of the cucumber pickle crop in Michigan was mechanically harvested, primarily by Wilde cucumber harvesters.

By 1968 the harvester had replaced the loss in production that was left by the departed Braceros. The next phase of adoption was the substitution of the mechanical cucumber harvester for the domestic migrant laborers. By 1970 over 80 percent of the Michigan cucumber pickle crop was mechanically harvested. The mechanical harvesters had been substituted for the labor of about 20,000 migrant workers in Michigan relative to the 1964 level.

The transition in cucumber production from hand harvesting to machine harvesting was not easy for the growers either. Cucumbers are very sensitive to changes in cultural practices. To offset the reduction in output through reduced pickings from 12 to 1, the cucumbers had to be managed more carefully. Many farmers accustomed to the multi-pick practices would not or could not change their management practices and thereby lost their contracts with the large pickle processors to raise cucumbers. Other farmers tried to "cheat" by hand harvesting once or twice and then using the mechanical harvester. This adoption strategy was also unsuccessful because cultural practices for hand harvesting were incompatible with those for mechanical harvesting. Then there were the farmers who had never grown pickling cucumbers before. The new farmers in general were the best adopters of the new technology because "they did what they were told." They had nothing to unlearn about cucumber production. The scale of cucumber production changed substantially with the introduction of the mechanical harvester. Prior to 1964 the average enterprise size was about 15 acres, while subsequently the "full utilization" enterprise size was about 150 to 200 acres. It is estimated that in Michigan about 2,000 small enterprise cucumber producers did not continue to produce cucumbers after the introduction of the mechanical harvester.

Today Wilde is the largest manufacturer of mechanical cucumber harvesters in the world. Only Cuke, Inc. still competes with Wilde. Cuke, Inc. is owned by Ken Purvis in Mendota, Illinois. FMC sold its last harvester in 1973, having sold about 50 self-propelled harvesters over an eight-year span. FMC discontinued its contract with MSU in December 1974 after paying about $20,000 to MSU in royalties.

The story of the mechanical cucumber harvester is not over. The current mechanical cucumber harvester is an imperfect substitute for hand harvesting in that the mechanical harvester is unable to distinguish small gherkin size cucumbers from the vine as the plant passes between the rollers. As a result there is specialization within the cucumber pickle subsector between the production of larger cucumbers using the mechanical harvester and the production of smaller cucumbers using hand harvesting. As a result, Ohio has increased its production of small cucumber pickles relative to Michigan because pickle production more fully utilizes the migrant labor force already in Ohio to harvest tomatoes. History is repeating itself to some extent.

Another result of the inability to harvest small pickles mechanically has been to simulate research at MSU to develop a "small pickle technology." Professor Larry Baker, who replaced Peterson at MSU as the cucumber geneticist in the Horticulture Department, was the project leader of this phase of research. Baker, building on Peterson's work, has developed an all-female, multi-node cucumber variety called Spartan Magic that can be induced to germinate asexually with the application of the chemical chlorflurenol. Baker left MSU in 1980 to work with Asgrow Seed Company (a subsidiary of Upjohn) in Kalamazoo, Michigan. In the Department of Agricultural Engineering, Professor Richard Lederbuhr is working on a "thresher" type mechanical cucumber harvester to harvest little cucumbers.

In a related development, processors noticed an increase in "bloaters" associated with mechanical harvesting and bulk handling

techniques. Bloating is the result of carbon dioxide build-up inside cucumbers during brining, causing damage to the internal structure of the cucumber and reducing its value. B;paters are useful only as relish. Increased handling weakened the cucumbers' internal structure and made the cucumber susceptible to bloating. Professor Ralph Costilow in the Microbiology Department at MSU developed a process to decrease the level of carbon dioxide in the brining tanks. This process called, sidearm purging was patented in 1974 and has saved processors from the loss in value of deteriorated cucumber stock. Costilow's contribution is just the latest in a long series of research projects at MSU on the microbiology of cucumbers. In fact, Costilow's major professor was Fred Fabian. Professor Fabian was a world-renowned expert in the microbiology of cucumber fermentation. Fabian began his work in cucumber fermentation as soon as he arrived at MSU in 1932 and continued until his retirement in the 1950s, publishing dozens of articles on the subject.

This brief case history of the research and development process gives some understanding of the complexity and significance of the mechanization of cucumber pickle harvesting. In summary, research on the once-over mechancial harvester was done at MSU from 1957 to 1963. The result was a patent on a mechanism to pinch the pickling cucumbers from the vine as it passed through a large "washing machine wringer" type device. FMC manufacturing company produced a prototype machine in 1964 and began commercial production in 1965. The self-propelled machines soon were eclipsed by a tractor-mounted type developed by Wilde Co. The machine was widely adopted in Michigan and to a lesser extent in surrounding North-Central states and in the

and in the Southern region. By 1978 about 30% of the harvested pickling cucumber acres were machine harvested. The direct stimulus for the development of the mechanical cucumber harvester was the imminent termination of the Bracero program. The indirect stimulus was the cycle of labor uncertainty and mechanization beginning with sugar beet production. Private entrepreneurs proved unable to develop a successful harvester nor is it likely would they have if given more time. The development and successful adoption of the mechanical cucumber harvester required a multi-disciplinary approach which included agricultural engineering, plant breeding, cultural practices and agricultural economics. Michigan State University was ideally suited to provide the necessary scientific manpower to successfully develop the components of the system referred to in this study as the mechanical cucumber harvester technology. The stimulus for research on the mechanical harvester came from outside the university, primarily from the National Pickle Packers Association and H.J. Heinz Company. A large farm machinery manufacturer is not well suited to provide low sales volume machines in a minor crop such as pickling cucumbers. Small farm machinery manufacturers seem to be best suited to produce and service such low sales volume machines.

The cucumber harvester is used only in those states where yields are sufficiently high and the supply of labor is uncertain. For these reasons, adoption of the harvester has been limited primarily to the North-Central Region of the U.S. Ohio has sufficiently high yields but, until recently at least, the labor supply available for the tomato crop has been used to hand-harvest cucumber pickles as well.

The future of hand-harvested cucumber pickles in Ohio, however, remains uncertain. Recent efforts to unionize seasonal workers will only result in the mechanization of the tomato harvest and as a result the cucumber pickle crop. This dynamic of labor uncertainty in a high value crop like tomatoes is simply the latest round of the labor uncertainty-mechanization cycle which indirectly led to the development of the mechanical cucumber harvester initially.

This historical background is intended to put a human face on the facts and figures of who wins and who loses with the introduction of a new technology. In the theory which follows consumers are reduced to "utility functions," cucumber farmers to "average and marginal cost functions," migrant workers to "resources saved" and the mechanical harvester to a "decrease in price resulting from a shift in supply." With this in mind, we turn next to a theory of consumer surplus.

# CHAPTER II

# A THEORY OF CONSUMER'S SURPLUS

## 2-1 Introduction

It is common sense to suppose that if the price of a good decreases then an individual's real income has increased accordingly. The same income will buy a little more of everything. Conversely it takes less income to keep one at the same level of satisfaction. Patinkin summarizes this common sense approach when he says:

> A given individual with a given "money" income achieves different levels of welfare under different trade situations. . . . (Patinkin, 1963, p. 90)

Both of these statements define the essence of consumer's surplus. Consumer's surplus has three essential ingredients: a level of utility, a change in price and a corresponding change in real income. These three components of consumer's surplus are referred to repeatedly in this chapter in relation to the theory of consumer's surplus. Consumer's surplus has at least five different measures, each with more or less exactness. But all the theory and measures of consumer's surplus relate back directly to the components of a level of utility, a change in price and a change in income.

The theory of consumer's surplus relates specifically to measuring the change in income associated with a change in the price of a good or service with respect to a given level of utility. The

five measures of this change in income are the Laspeyres, compensating variation, "utility," equivalent variation and Paasche measures.

In this chapter, the theoretical basis for each of these measures is presented. These measures of consumer's surplus are presented three ways: verbally, graphically and mathematically. The presentation of the general theory of consumer's surplus will be presented in the following order. First the assumptions associated with a simple two good economy are described. Then the direct utility and expenditure minimization functions are presented. Then the compensated demand function is developed. The compensated demand functions is the first derivative of the expenditure function. The expenditure and compensated demand functions are fundamental for understanding all of the measures of consumer's surplus.

Next the Patinkin "all or none" demand function is described. The "all or none" demand function is very helpful in combination with the compensated demand function for understanding the relationship between price and income for a given level of utility.

Using the insights based on the compensated demand function and the "all or none" demand function it is possible to describe first the compensating variation and the Laspeyres measures of consumer's surplus. This is followed immediately with a description of the equivalent variation and the Paasch measures of consumer's surplus.

Not until four of the five consumer's surplus measure are introduced and described is the "utility" measure and the associated ordinary demand function presented.

The complications associated with using the ordinary demand function to measure the change in income for a change in prices are

presented. Also, the problems of interpretation of the "utility" measure of consumer's surplus are described as well with a brief detour to show what Marshall meant when he first defined the notion of consumer's surplus.

Finally the latest and most recent developments by Hausman and Willig using the indirect utility function and Roy's identity which help solve some of the empirical problems associated with accurately measuring and interpreting compensating and equivalent variation are presented.

Much has been written about the theory of consumer's surplus. It is beyond the scope of this chapter to be comprehensive regarding all the aspects of theoretical economics which impinge on the theory of consumer's surplus. Rather it is my intention here to provide a coherent summary of the theoretical basis of the five measures of consumer's surplus. I have also included in some detail the most recent development in the theory as well as how the new approach relates both graphically and mathematically to the earliest notions of consumer's surplus.

## 2-2 The Direct Utility Function, the Expenditure Function and the Compensated Demand Function

Let us assume that a competitive economy contains only two goods--good Y and good Q. Further assume that good Y is the numeraire good in the economy. Therefore the price of good Y is always equal to one. The price of good Q is measured in terms of Y as is income. Good Y may be thought of as either a composite of N - 1 goods in the economy or as leisure. Let us also assume that a consumer in this economy has an initial endowment of good Y only.

The consumer is faced with the decision whether to trade Y for Q and if so how much. The consumer is assumed to make her/his decision as if on the basis of a continuous, downwardly concave, twice differentiable and ordinal direct utility function (Malinvaud, 1972, pp. 24-26). At each point on the consumer's direct utility function s/he is indifferent with respect to consuming the various combinations of Y and Q. The direct utility is a function of goods Y and Q, i.e.

(1) $U_o = f(Y,Q)$.

The initial endowment or income of the consumer is equal to income $Y_o$. Income $Y_o$ is equal to the sum of product of good Y (multiplied by its price = 1) and good Q multiplied by its price P.

(2) $Y_o = Y + PQ$

The decision to trade Y for Q is made on the basis of the results of the expenditure function. The expenditure function is the relationship of the price of Q and the given attainable level of utility. The decision rule of this consumer is to minimize expenditure of income Y in attaining a given level of utility U.

(3) $e(P,U)$ = minimize Y subject to $U(Y,Q) \geq U_i$

(Diamond and McFadden, 1974, p. 3)

The expenditure function is the least amount of income needed to attain a given level of utility for a given set of prices. By substituting equations (1) and (2) into (3) and setting up a Lagrangian function for a minimum then

(4) $e(p_t U_o) = Y + P_t Q + \lambda(U_o - f(Y,Q))$ $\quad t = 0$ to $n$

Equation (4) is the expenditure function to be minimized in order to find the quantities of Y and Q which maintains the consumer at utility level $U_o$ for a given price $P_t$. The solution to this function is then:

"If Y = f(X) is the quantity to be maximized or minimized, find those values of X for which $\frac{dy}{dx} = f'(X) = 0$ (Thomas, 1960, p. 130). Since equation (4) is the function of three variables (Q, Y and ) then the partial derivatives are set equal to zero such that

(5) $$\frac{\partial e(p_t U_o)}{\partial Y} = 1 - \lambda \frac{\partial f(Y,Q)}{\partial Y} = 0$$

(6) $$\frac{\partial e(P_t U_o)}{\partial Q} = P_t - \lambda \frac{\partial f(Y,Q)}{\partial Q} = 0$$

and

(7) $$\frac{\partial e(p_t U_o)}{\partial \lambda} = U_o - f(Y,Q) = 0.$$

Dividing equation (6) by (5) such that

(8) $$\frac{\partial f(Y,Q)/\partial Q}{\partial f(Y,Q)/\partial Y} = P_t.$$

Equation (8) is a solution for the consumer as to whether and how much Y to trade for Q. The solution states that the consumer should trade Y for Q until the ratio of the marginal utility of Q to Y equals the price of Q (Henderson and Quandt, 1971, p. 15). The tangent of the utility function to the expenditure function with the slope equal to $P_t$ is the only point that meets this condition. The assumptions regarding the nature of the utility function assure the second order conditions are positive (Malinvaud, 1972, p. 37). This is a sufficient condition to assure a minimum expenditure is reached.

Refer now to Figure 1. If P equals $P_o$ then $Y_o$ of Y and zero of Q are consumed. If the price of Q falls from $P_o$ to $P_2$ however then $Y_2'$ of Y and $Q_2$ of Q are consumed. It is important to note that with the decrease in the price of Q from $P_o$ to $P_2$ that the minimum expenditure necessary to maintain the level of satisfaction $U_o$ falls from $Y_o$ to $Y_2$.

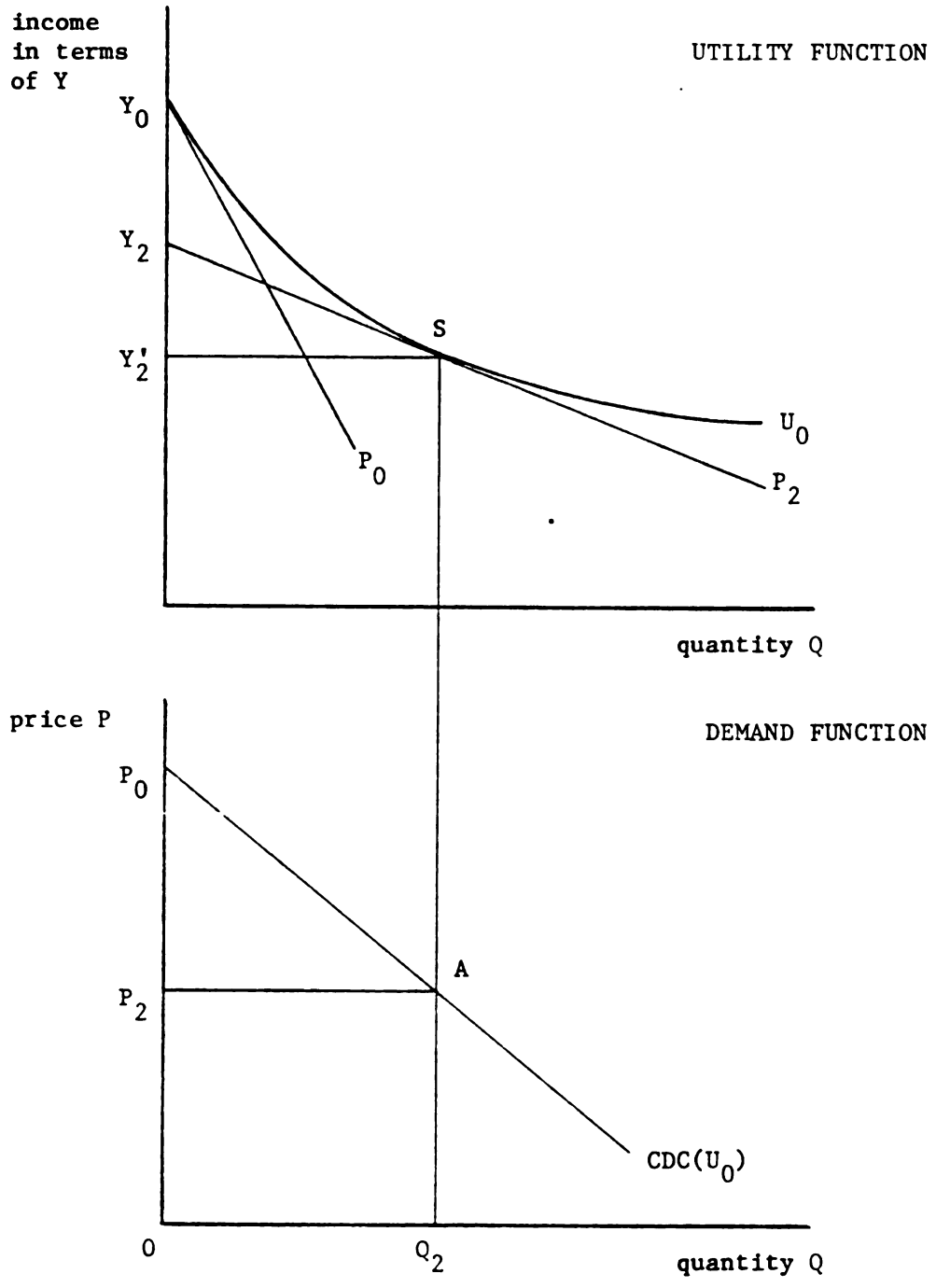


Figure 1. Compensated Demand Curve

The difference between $Y_o$ and $Y_2$ is a measure of increase in income resulting from the decrease in price for a given level of utility. This measure of the change in income can be measured directly in utility or demand space. In utility space the measure of the change in income is to subtract the subsequent income $Y_2$ from the initial income $Y_o$. In demand space the measure is the intergrad of the first derivative of the expenditure function with respect to Q over the range of prices from $P_o$ to $P_2$.

The first derivative of the expenditure function with respect to Q is called the compensated demand function. Equation (6) is the first derivative with respect to Q. Solving equation (6) for $\partial Q$ we see that Q is a function of $P_t$, and $U_o$. The compensated demand function is the inverse of the first derivative of the expenditure minimization function. The compensated demand function for Q with respect to utility level $U_o$ is

(9) $Q = \Psi(P,U_o)$

The compensated demand function is the relationship between own price, and the given level of utility. The compensated demand curve (CDC) is the schedule of price-quantity relationships which maintains a consumer at a given utility level. The integral of the compensated demand function from price $P_o$ to $P_2$ equals area $P_oP_2A$ in Figure 1. This area is equal to difference between expenditure functions $Y_o$ and $Y_2$ in relation to utility level $U_o$.

(10) $Y_o - Y_2 = P_oP_2A$

Intuitive the reason why the area under the compensated demand function is equal to the change in the expenditure function is as follows:

The definition of a derivative is "instantaneous rate of change in one variable per unit change in the other" (Thomas, 1960, p. 37). Integration on the other hand is "to indicate the whole of; to give the sum or total of" (Thomas, 1960, p. 37). It should be apparent then that the sum of the rate of change in one variable to another is equal to the total change in one variable to another. In this case the integral of the compensated demand function is equal to the sum of the total change in income to the total change in price. This is exactly what consumer's surplus attempts to measure.

## 2-3 The "All or None" Demand Function

In the last section the concepts of utility function, expenditure function and compensated demand function were introduced and described. In addition a measure of consumer's surplus was illustrated and described as measurable either in utility or demand space. In this section the idea of an "all or nothing" demand function will be used to reinterpret the meaning of consumer's surplus when and if it is measured.

Unlike the compensated demand function, the "all or none" demand function is not based on the assumption that price is determined as a result of competitive trade. In fact the "all or none" demand function is based on assumption that the good Q is provided by a monopolist. The monopolist sets the price of Q so that for any given quantity of Q the consumer is indifferent between buying each unit of Q at some average price $P_t$ and buying no units of Q at all. Patinkin has shown that the "all or none" price for a given level of utility is a vector from the intersection of the utility function and Y-axis to the

utility function (Patinkin, 1963, p. 86). Figure 2 shows that at price $P_o$ the expenditure for Q is equal to zero and total income equals $Y_o$.

(1) $Y_o = Y_o + P_oQ$ where $Q = 0$

If the consumer wants $Q_2$ of Q then s/he must pay $P_1$ for each unit. $P_1$ is an average price per unit set by the monopolist in relation to $Q_2$ units of commodity Q. Total income required to purchase commodity bundle ($Y_2'$ and $Q_2$) remains equal to $Y_o$.

(2) $Y_o = Y_2' + P_1Q_2$ Solving equation (2) for $Y_2'$.

(3) $Y_2' = Y_o - P_1Q_2$

$P_1Q_2$ is the maximum amount of income which can be extracted from a consumer who purchases $Q_2$ of Q. We know this is true because at $P_1$ the consumer is indifferent between consumption bundle ($Y_2'$ and $Q_2$) and bundle ($Y_o$ and $Q = 0$). If the price of Q is raised one penny more, then the consumer will choose only bundle ($Y_o$ and $Q = 0$). So if $Q_2$ is purchased in a monopoly market then $P_1$ is highest per unit price the consumer will pay.

$P_1Q_2$ is equal to area $P_1OQ_2W$. Therefore equation (3) can be rewritten as:

(4) $Y_2' = Y_o$ - area $P_1OQ_2W$.

Let us compare the monopoly case with the competitive case. Refer to Figure 1. In the competitive case we see that if price equals $P_2$ then $Y_2'$ of Y and $Q_2$ of Q are purchased. The total income required to purchase $Y_2'$ and $Q_2$ is $Y_2$.

(5) $Y_2 = Y_2' + P_2Q_2$. Solving for $Y_2'$

(6) $Y_2' = Y_2 - P_2Q_2$.

$P_2Q_2$ is the maximum amount of income that can be extracted from a consumer who purchases $Q_2$ of Q in a competitive market. We know this

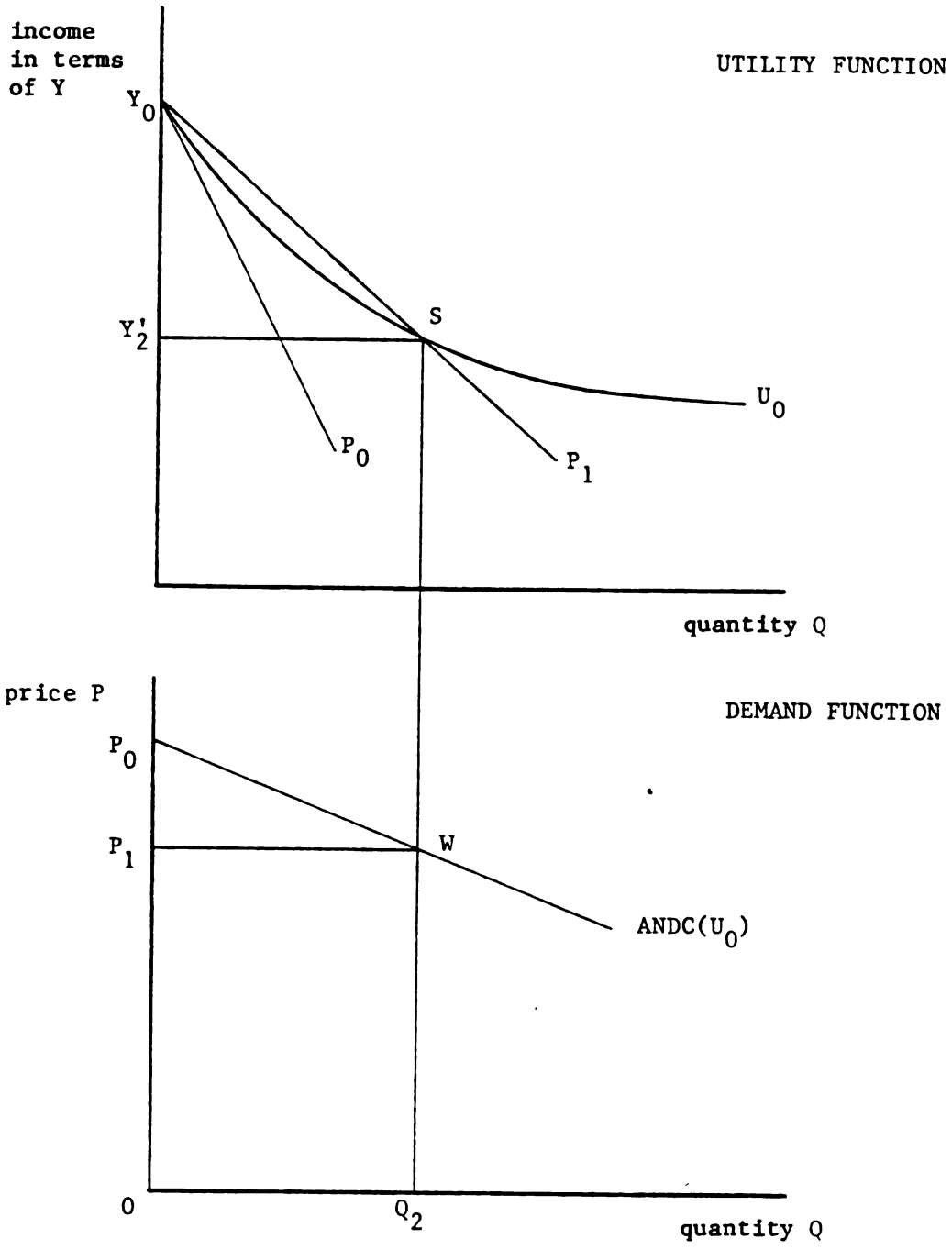


Figure 2. "All or None" Demand Curve

is true because if the price of Q increases the consumer will simply substitute the relatively less expensive Y for the more expensive Q and still maintain a position on utility function $U_o$. So, if $Q_2$ is purchased in a competitive market then $P_2$ is the highest price per unit the consumer will pay. $P_2Q_2$ is equal to area $P_2OQ_2A$. Therefore equation (6) can be rewritten as:

(7) $Y_2' = Y_2 - \text{area } P_2OQ_2A.$

Equations (4) and (7) are both equal to $Y_2'$. Therefore it is possible to compare the maximum amount a consumer would pay for $Q_2$ given competitive and monopoly prices and the same utility function $U_2$. Figures 1 and 2 are redrawn on the same graph in Figure 3 for ease of comparison.

Substituting equation (4) into (7)

(8) $Y_o - \text{area } P_1OQ_2W = Y_2 - \text{area } P_2OQ_2A$ Rearranging terms

(9) $Y_o - Y_2 = \text{area } P_2OQ_2A.$ We know that

(10) $\text{Area } P_1OQ_2W = \text{area } P_1P_2AW + \text{area } P_2OQ_2A$

Substituting equation (10) into (9) and simplifying

(11) $Y_o - Y_2 = \text{area } P_1P_2AW$

$Y_o - Y_2$ is the utility space measure of compensating variation. Area $P_1P_2AW$ is the maximum additional income that can be extracted from a consumer who purchases $Q_2$ under conditions of monopoly trade above that which can be extracted under conditions of competitive trade. In section 2-2 we proved that $Y_o - Y_2$ is equal to area $P_oP_2A$. Therefore equation (11) can be rewritten as

(12) $\text{area } P_oP_2A = \text{area } P_1P_2AW.$

Patinkin is the first to point out that these two measures are the same (Patinkin, 1963, pp. 87-88).

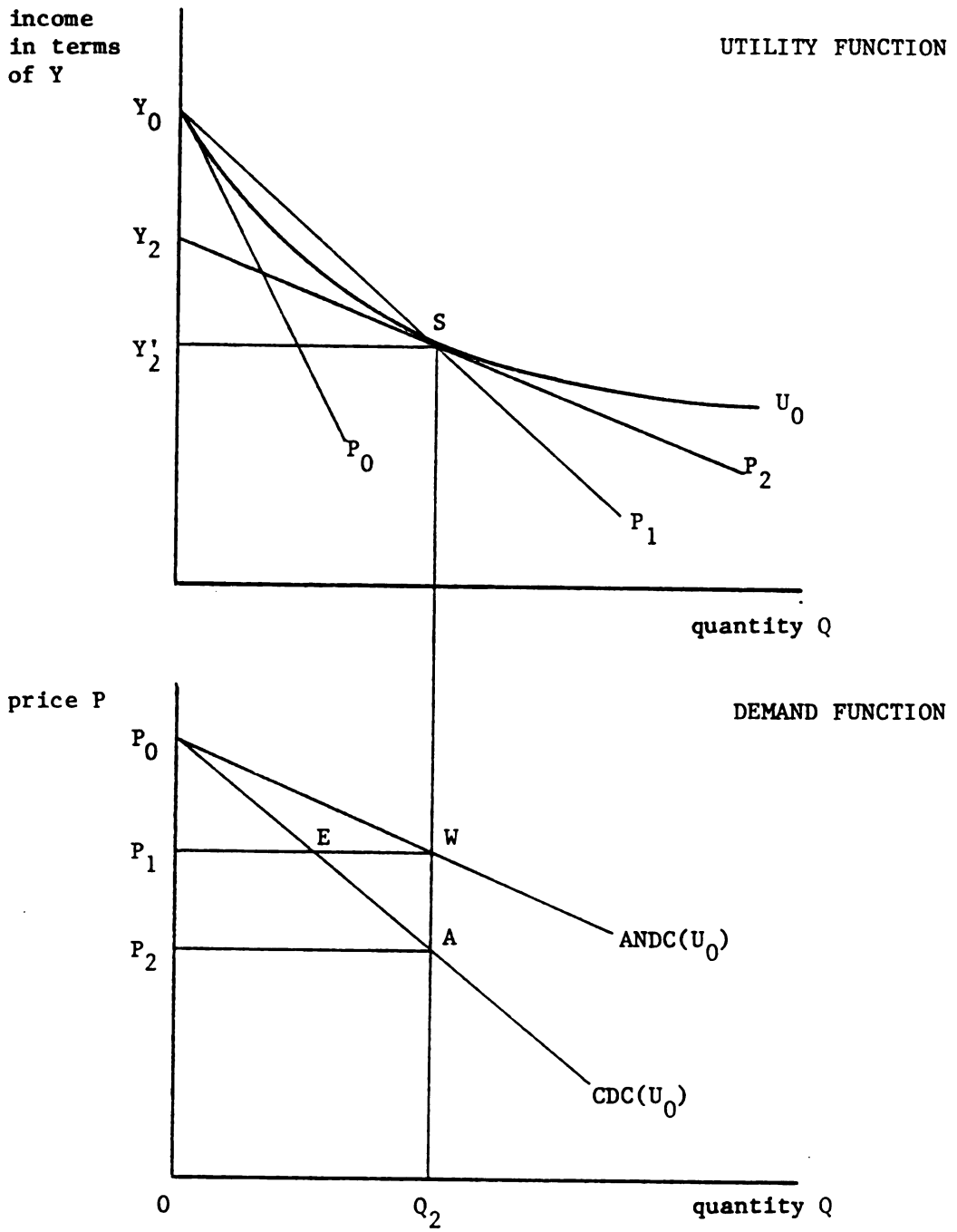


Figure 3. Compensated and "All or None" Demand Curves

Compensating variation is defined as the change in real income associated with a change in price relation to an initial utility level. Compensating variation can be interpreted as the additional income which can be extracted from a consumer who purchases a given quantity of a good under conditions of monopoly trade above that which can be extracted under conditions of competitive trade and still remain at the same level of utility. Typically compensating variation is interpreted as a welfare measure of the change in utility. The importance of these alternative interpretations of compensating variation will be discussed in section 2-10.

It is interesting to note that this additional income can be extracted either as an exogenous reduction in total income or as an additional per unit price which can be charged by a monopolist. The monopolist has two pricing alternatives. The monopolist can charge either an average "all or none" price for a given quantity of Q or charge a perfectly discriminating price for each unit of Q. Both of these alternatives extract the same additional income above that under the conditions of competitive trade (Patinkin, 1963, p. 87). A final interpretation of area $P_1P_2AW$ is its relation to taxation in a competitive trade economy. If an ad valorem tax of $P_1 - P_2$ is added to the price of $P_2$ for each unit of Q to $Q_2$ then the tax receipts would equal $P_1P_2AW$ or $P_oP_2A$. This is a possible policy alternative to publicly finance research and development which may result in lowering the price of yet other goods in the economy. This argument can also be made with respect to the monopolist who can afford to invest in research only if allowed to price discriminate.

## 2-4 Compensating Variation and Laspeyres Measure of Consumer's Surplus

The previous sections have described, illustrated and interpreted the general concept of consumer's surplus. This section will describe and explain two specific measures of consumer's surplus. These are compensating variation and the Laspeyres measures. In this section there is a modification of the assumption regarding initial endowment. Instead of assuming that the initial endowment is only of good Y, let us assume that initial endowment includes both Y and Q.

Refer to Figure 4. At $P_o$ the consumer purchases $Q_o$ of Q and has a total income equal to $Y_o$. When the price of Q decreases from $P_o$ to $P_2$ then the consumer purchases $Q_2$ of Q and requires an income of $Y_2$ to remain at the same level of utility. $Y_o$ is the initial income and $Y_2$ is the subsequent income necessary to remain at the same level of utility when the price of Q decreases from $P_o$ to $P_2$. The difference between the initial and subsequent income in relation to an initial utility level is called compensating variation (CV).

(1) Compensating variation (CV) $\equiv Y_o - Y_2$.

The direct measure of compensating variation is the difference between the initial and subsequent income in relation to the initial utility level. The measure of compensating variation in demand space is the integral of the appropriate compensated demand function $CDC(U_o)$ from $P_o$ to $P_2$. The measure of compensating variation in utility space is

(2) $CV\ (P_o, P_2, U_o) = Y_o - Y_2$.

The indirect measure of compensating variation is

(3) $CV(P_o, P_2, U_o) = LM$.

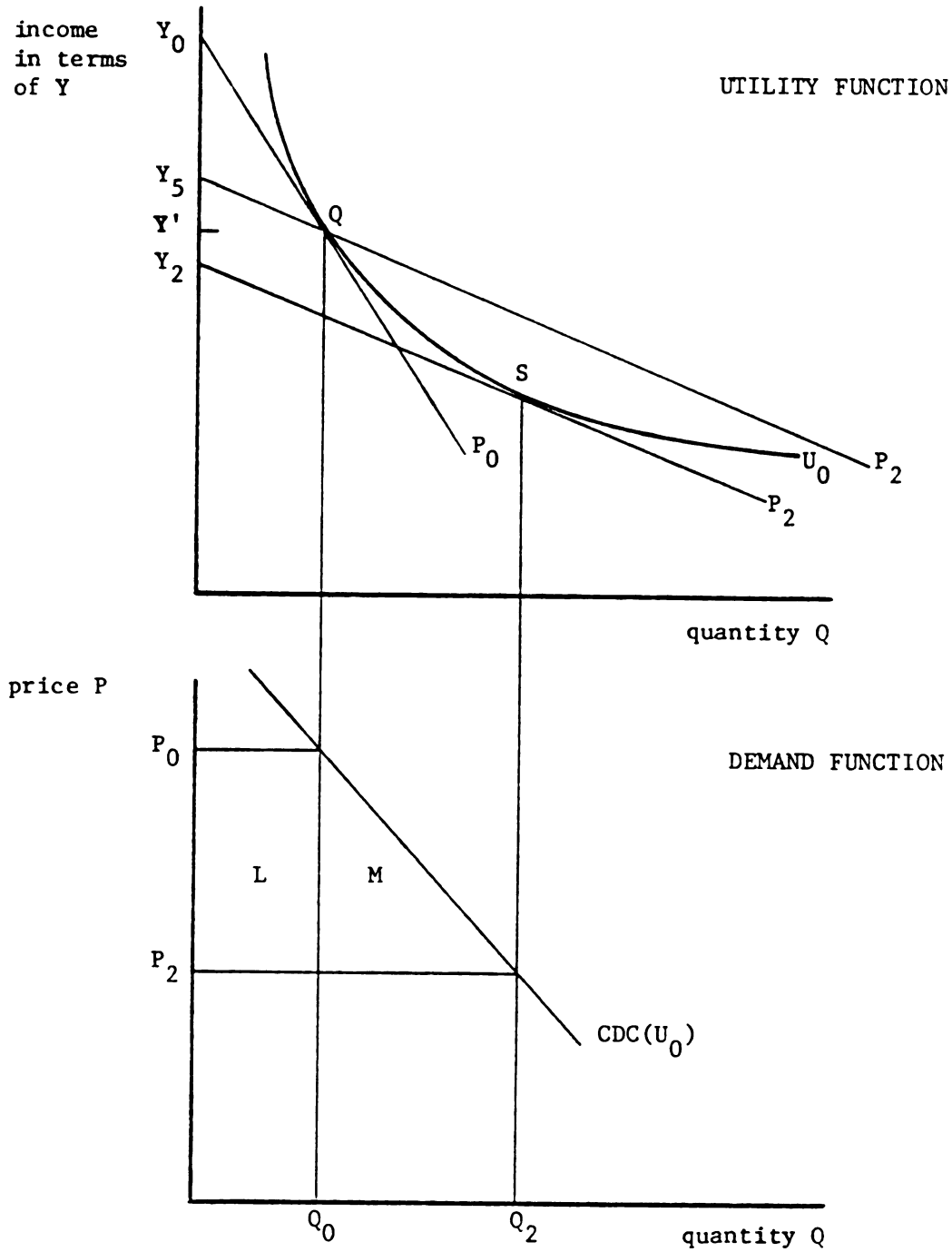


Figure 4. Laspeyres and Compensating Variation Measures of Consumer's Surplus

The problem with equation (2) is that $Y_2$ is not directly observable. The problem with equation (3) is that the compensated demand function $CDC(U_o)$ is not directly observable either. What is observable is $Y_o$, $P_o$, $P_2$ and $Q_o$. It is possible to approximate compensating variation using the observable data.

Let us assume that instead of purchasing $Q_2$ when the price of Q is $P_2$ that the consumer continued to purchase $Q_o$. This is not an expenditure minimizing decision on the part of the consumer but it is a possible decision none the less. The consumer's expenditure function is equal to $Y_5$ in this case.

(4) $Y_5 = Y' + P_2Q_o$

Since $Y_5$ is not the result of expenditure minimization when P equals $P_2$, then we know that

(5) $Y_5 > Y_2$

Therefore substituting (5) into (4)

(6) $Y_2 > Y' + P_2Q_o$

The initial expenditure function when P equals $P_o$ is

(7) $Y_o = Y' + P_oQ_o$

Dividing equation (6) by (7)

(8) $$\frac{Y_2}{Y_o} > \frac{Y' + P_2Q_o}{Y' + P_oQ_o}$$

The right side of equation (8) is the Laspeyres index. The Laspeyres index is defined as the ratio of subsequent to initial price multiplied by initial quantity (Tomek and Robinson, 1972, p. 204). Equation (8) shows that the Laspeyres index is greater than the inverse of the compensated variation index.

Equation (8) can be transformed into a simple Laspeyres measure of consumer's surplus while maintaining its relation to compensating variation.

Multiply and subtract $Y_o$ from both sides of equation (8).

(9) $Y_2 - Y_o < Q_o(P_2 - P_o)$.

Multiply both sides of (9) by minus one (-1)

(10) $Y_o - Y_2 > Q_o(P_o - P_2)$.

Substituting (2) into (10)

(11) $CV(P_o,P_1,U_o) > Q_o(P_o - P_2)$.

Compensating variation is greater than the Laspeyres measure of consumer's surplus.

(12) Laspeyres = $Q_o(P_o - P_2)$.

In Figure 4 we see that

(13) $Q_o(P_o - P_2)$ = area L.

By subtracting equation (4) from (7) we see that

(14) $Q_o(P_o - P_2) = Y_o - Y_5$.

Substituting (13) into (14)

(15) $Y_o - Y_5$ = L = Laspeyres measure.

Substituting (2) into (3)

(16) $Y_o - Y_2$ = LM = compensating variation.

It is apparent that the Laspeyres measure is a minimum measure of compensating variation. It is not hard to appreciate the idea that a consumer would purchase something other than the original bundle of goods (Y and Q) when the price of Q decreases. In fact the consumer would exchange the relatively more expensive Y for the relatively

less expensive Q when the price of Q falls to $P_2$. In Figure 4, area M is a measure of the change in income resulting from substituting Q for Y. The Laspeyres measure is a measure of the income reduction sufficient to still allow the purchase of the initial bundle of goods at the subsequent price of Q. The Laspeyres measure has the disadvantage of underestimating compensating variation on the one hand. On the other hand the Laspeyres measure is measurable since $Q_o$, $P_o$ and $P_2$ are all observable.

## 2-5 Equivalent Variation and Paasche Measures of Consumer's Surplus

The previous section described the compensating variation and Laspeyres measure of consumer's surplus as well as their relationship to each other for a price decrease. This section will describe and explain two additional and very similar measures of consumer's surplus. These are equivalent variation and Paasche measures of consumer's surplus. Again let us assume that initial endowments include both Y and Q.

Refer to Figure 5. The critical difference between the compensating variation-Laspeyres and the equivalent variation-Paasche measures is that the former are measured with respect to an initial utility level $U_o$ while the latter are measured with respect to a subsequent utility level $U_1$. Given subsequent utility level $U_1$, at $P_o$ the consumer purchases $Q_1$ of Q and requires a total income of $Y_1$. When the price of Q decreases from $P_o$ to $P_2$ then the consumer purchases $Q_4$ of Q and has a total income of $Y_o$. $Y_o$ is the subsequent income and $Y_1$ is the initial income required to attain the subsequent utility $U_1$ at the initial price $P_o$. The difference between the initial and

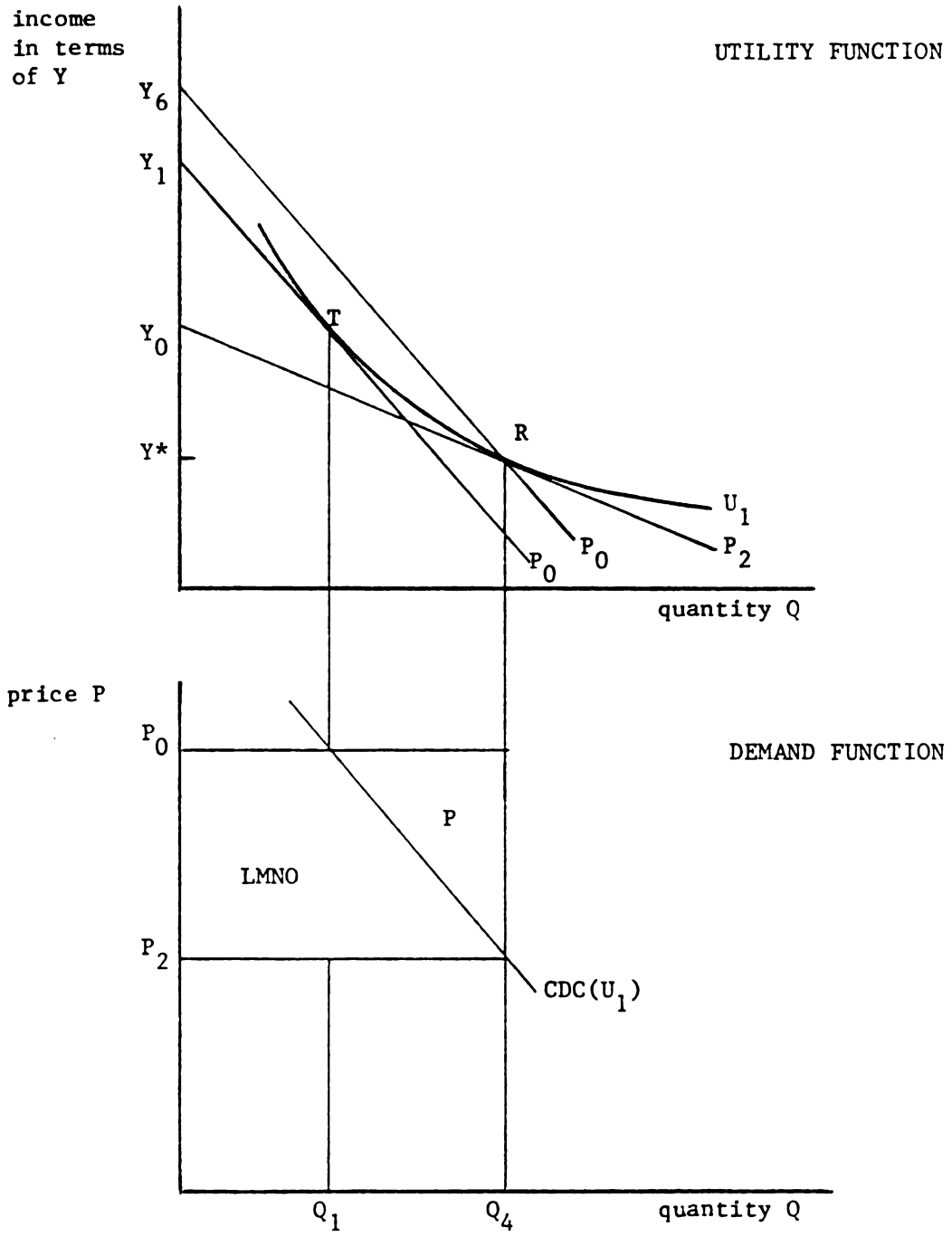


Figure 5. Paasche and Equivalent Variation Measures of Consumer's Surplus

subsequent income in relation to the subsequent utility level is called equivalent variation (EV).

(1) equivalent variation (EV) $\equiv Y_1 - Y_o$.

The measure of equivalent variation in utility space is the difference between the initial and subsequent income in relation to the subsequent utility level. The measure of equivalent variation in demand space is the integral of the appropriate compensated demand function $CDC(U_1)$ from $P_o$ to $P_2$. The utility space measure of equivalent variation is

(2) $EV(P_o,P_2,U_1) = Y_1 - Y_o$.

The demand space measure of equivalent variation is

(3) $EV(P_2,P_2,U_1) = LMNO$.

The problem with equation (2) is that $Y_1$ is not directly observable. The problem with equation (3) is that the compensated demand function $CDC(U_1)$ is not directly observable either. The only variables that are directly observable are $Y_o,P_o,P_2$, and $Q_4$. It is possible to approximate equivalent variation using the observable data.

Let us assume that instead of purchasing $Q_1$ where the price of Q is $P_o$ that the consumer purchases $Q_4$ This is not an expenditure minimizing decision on the part of the consumer but it is a possible decision (assuming the consumer can somehow increase his or her initial endowment). The consumer's expenditure function in this instance is equal to $Y_6$, such that

(4) $Y_6 = Y^* + P_oQ_4$.

Since $Y_6$ is not the result of expenditure minimization when P equals $P_o$, then we know that

(5) $Y_6 > Y_1$.

Therefore substituting (4) into (5)

(6) $Y_1 < Y^* + P_oQ_4$.

The initial expenditure function when P equals $P_2$ is

(7) $Y_o = Y^* + P_2Q_4$.

Dividing (7) by (6)

(8) $$\frac{Y_o}{Y_1} > \frac{Y^* + P_2Q_4}{Y^* + P_oQ_4}.$$

The right side of equation (8) is the Paasche index. The Paasche index is defined as the ratio of subsequent to initial price multiplied by subsequent quantity (Tomek and Robinson, 1972, p. 204). Equation (8) shows that the Paasche index is less than the inverse of the equivalent variation index.

Equation (8) can be transformed into a simple Paasche measure of consumer's surplus while maintaining its relation to equivalent variation. Dividing and subtracting $Y_o$ from both sides and cross multiplying the results are

(9) $Y_1 - Y_o < Q_4(P_o - P_2)$.

Substituting (2) into (9)

(10) $EV(P_oP_2U_1) < Q_4(P_o - P_2)$.

Equivalent variation is less than the Paasche measure of consumer's surplus.

(11) Paasche = $Q_4(P_o - P_2)$.

In Figure 5 we see that

(12) $Q_4(P_o - P_2)$ = LMNOP.

By subtracting (7) from (4) then

(13) $Y_6 - Y_o = Q_4(P_o - P_2)$.

Substituting (12) into (13)

(14) $Y_6 - Y_o$ = LMNOP = Paasche measure.

Substituting (2) into (3)

(15) $Y_1 - Y_o$ = LMNO = equivalent variation.

It is apparent that the Paasche measure is a maximum measure of equivalent variation. It is not hard to appreciate the idea that a consumer would purchase something other than the subsequent bundle of goods (Y and Q) with the price of Q increases. In fact the consumer would exchange the relatively more expensive Q for the relatively less expensive Y when the price of Q increases from $P_2$ to $P_o$. In Figure 5 area LMNO is a measure of the increase in income which is required to remain on the same utility level when the price of Q increases from $P_2$ to $P_o$ and Y is substituted for Q. Area LMNOP is the same measure as LMNO except Y is not substituted for Q. The Paasche measure overestimates equivalent variation on the one hand. On the other hand the Paasche measure is measurable, since $P_o$, $P_2$ and $Q_4$ are observable.

## 2-6 Utility Maximization Function, the Ordinary Demand Function and the "Utility" Measure of Consumer's Surplus

The previous sections have discussed measures of consumer's surplus purely in terms of the compensated demand function. The presentation of the "all or none" demand function was developed not as a measure but to provide additional theoretical depth to the interpretation of the area under the compensated demand function. But as has been stated twice so far in this chapter, the compensated demand

function is not directly observable. The compensated demand function is the derivative of the expenditure function which is minimized subject to a utility constraint. The "dual" approach to the decision of whether to substitute Y for Q and if so how much is called utility maximization. The utility maximization approach reverses what is maximized with that which is constrained. For expenditure minimization

(1) $e(P,U)$ = Minimize Y

subject to $U(Y,Q) \geq U_i$.

The utility maximization is

(2) $U(P,K)$ = maximize U where $U = f(Y,Q)$

subject to $Y \leq Y_o$ and $Y_o = Y + P_tQ$

(Diamond and McFadden, 1974, p. 3).

The utility maximization function is the relationship between the highest level of utility attainable for a given nominal income and a given set of prices. By rewriting equation (2) as a Lagrangian function for a maximum then

(3) $U(P,Y) = f(Y,Q) + \lambda(Y_o - Y - P_tQ)$.

Equation (3) is the utility maximization function used to determine the highest utility level, the attainment of which does not exceed total nominal income $Y_o$ given prices $(1,P_t)$ for (Y and Q) respectively.

To find the maximum utility, the partial derivatives of equation (3) with respect to variables Y, Q and are set equal to zero (Thomas, 1960, p. 130) such that

(4) $\frac{\partial U(P,Y)}{\partial Y} = \frac{\partial f(Y,Q)}{\partial Y} - \lambda = 0$

(5) $\frac{\partial U(P,Y)}{\partial Q} = \frac{\partial f(Y,Q)}{\partial Q} - \lambda P_t = 0$

and

$$(6)\quad \frac{\partial U(P,Y)}{\partial \lambda} = Y_o - Y - P_t Q = 0.$$

Dividing equation (5) by (4) results in

$$(7)\quad \frac{\partial f(Y,Q)/\partial Q}{\partial f(Y,Q)/\partial Y} = P_t.$$

Equation (7) is a solution for the consumer as to whether and how much Y to trade for Q or vice versa. The solution states that the consumer should trade Y for Q until the ratio of the marginal utility of Q to Y equals the price of Q (Henderson and Quandt, 1971, p. 15). The tangent between the budget constraint with slope to $P_t$ and the utility function is the only point which meets this condition. This is the necessary condition to assure maximum level of utility is reached. Interestingly, this is the same first order decision rule as reached above for expenditure minimization.

The second order conditions are sufficient to assure a maximum utility level. "If $d^2y/dx^2$ is negative when $dy/dx = 0$, y is a maximum" (Thomas, 1960, p. 130). We are interested in determining the sufficient conditions which assure utility maximization for a change in the consumption in Q. This is possible if equations (4), (5) and (6) are totally differentiated and solved simultaneously for dQ.

$$(7)\quad \begin{aligned} f_{11}dy + f_{12}dQ - d\lambda &= 0 \\ f_{21}dy + f_{22}dQ - P_t d\lambda &= \lambda dP_t \\ -dY - P_t dQ &= dY_o + QdP. \end{aligned}$$

Equation (7) is the matrix of second derivatives equations with respect to variables Y, Q and $\lambda$. The determinant of equation (7) is

$$(8)\quad D = \begin{vmatrix} f_{11} & f_{12} & -1 \\ f_{21} & f_{22} & -P \\ -1 & -P & 0 \end{vmatrix}.$$

It is possible to use the determinant of a system of equations to solve the system for one or more of the unknown values. In the case of utility maximization the unknown values are dY, dQ and dλ (Henderson and Quandt, 1971, p. 31). Solving the system of equation (7) using its determinant (8) for dQ gives

$$(9)\quad dQ = \frac{\lambda D_{22} dP + D_{32}(-dY_o + QdP)}{D} < 0.$$

Equation (9) is the second order condition for utility maximization given a change in Q. Since we are maximizing utility subject to a budget constant then $Y_o$ or nominal income remains constant. So let $dY_o = 0$ and divide both sides by dP, then

$$(10)\quad \frac{\partial Q}{\partial P} = \frac{\lambda D_{22}}{D} + Q\frac{D_{32}}{D} < 0 \text{ for a maximum.}$$

Equation (10) is the second order condition for utility maximization given a change in Q resulting from a change in P while holding income constant. Henderson and Quandt show that

$$(11)\quad \frac{\lambda D_{22}}{D} = \frac{\partial Q}{\partial P}\Bigg|_{\text{utility constant}}$$

and

$$(12)\quad \frac{D_{32}}{D} = -\frac{\partial Q}{\partial Y}\Bigg|_{\text{prices constant}}$$

(1971, p. 32).

Substituting (11) and (12) into (10)

$$(13)\quad \frac{\partial Q}{\partial P} = \frac{\partial Q}{\partial P}\Bigg|_{\text{utility constant}} - Q\frac{\partial Q}{\partial Y}\Bigg|_{\text{prices constant}} < 0.$$

Equation (13) is called the Slutsky equation. The expression on the left $\partial Q/\partial P$ is the slope of the ordinary demand function with respect to Q. The ordinary demand function is the first derivative of the utility maximization function. Equation (13) shows that the slope of an ordinary demand function is the composite of slope of the compensated demand function ($\partial Q/\partial P$ given constant utility) and an income effect ($Q(\partial Q/\partial Y_o)$ given constant prices). It is this latter influence on the slope of the ordinary demand function that makes the integral of the ordinary demand function a less than ideal measure of the change in income resulting from a change in price.

It is possible to determine whether an ordinary demand function is the result of utility maximization decision making on the part of a consumer by using equation (13). Equation (13) can be rewritten so that the measure of the second order conditions are observable.

$$(13)\quad \frac{\partial Q}{\partial P}\Bigg|_{\text{utility constant}} = \frac{\partial Q}{\partial P} + Q\,\frac{\partial Q}{\partial Y}\Bigg|_{\text{price constant}} < 0$$

The expression $\partial Q/\partial P$ is the slope of the ordinary demand function and observable. Q is the quantity consumed and observable. $\partial Q/\partial Y$ given constant prices is the measure of the income effect of the ordinary demand and it too is observable. If the sum of these measures is negative then the second order sufficient condition for utility maximization are met.

Let us examine a graphical presentation of utility maximization. Refer to Figure 6. If P equals $P_o$ then $Q_o$ of Q is consumed and income is $Y_o$. When P falls to $P_2$ then $Q_4$ of Q is consumed and nominal income remains fixed at $Y_o$. Now instead of minimizing expenditure, expenditure remains constant and the level of utility increases from $U_o$ to $U_1$.

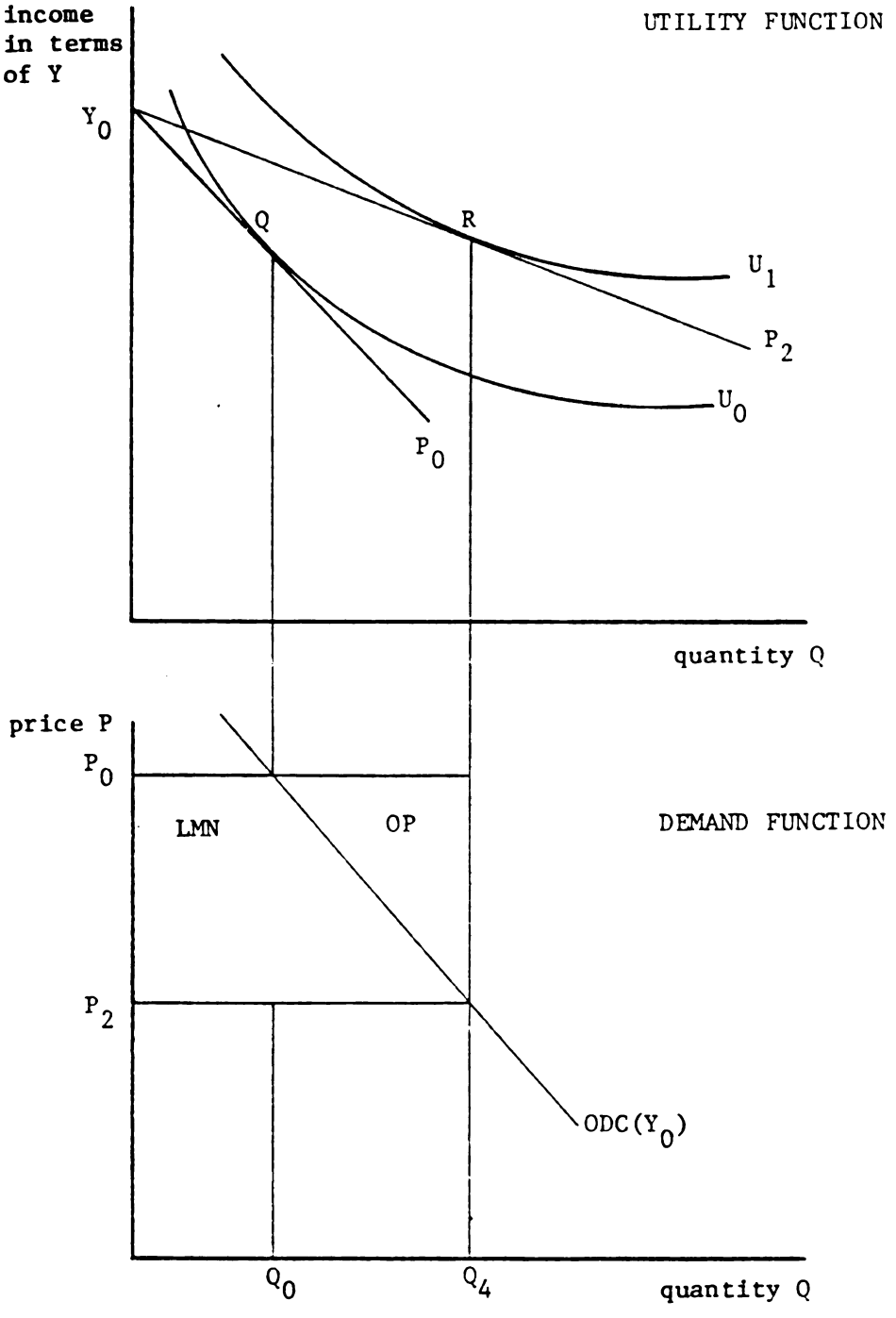


Figure 6. "Utility" Measure of Consumer's Surplus

Recall that the theory of consumer's surplus is defined as measuring the change in nominal income associated with a change in the price of a good or service with respect to a given level of utility. We see in the case of utility maximization that the level of utility is not fixed at all. The problem is compounded by the fact that nominal income is fixed while real income or utility is changing. Both of these results are the opposite of the case of expenditure minimization. The reason why the situation is particularly frustrating is due to the fact that the ordinary demand function is observable. Therefore it is important to examine the relation between the ordinary demand function and the compensated demand function.

The ordinary demand function is the relationship of a good to own price and given initial endowment of nominal income when Q is normalized in terms of Y.

(14) $Q = \emptyset\ (P_t, Y_o)$.

(Henderson and Quandt, 1971, p. 26). The ordinary demand curve (ODC) is the schedule of price-quantity relationships which maintains a consumer at the highest level of utility for a given level of nominal income. In Figure 7, we see the relationships that exist between the decision rules of expenditure minimization and utility maximization. When the price of Q decreases from $P_o$ to $P_2$ then the expenditure minimizing consumer purchases $Q_2$ of Q. The utility mazimizing consumer faced with a change in price of Q from $P_o$ to $P_2$ consumes $Q_4$ of Q. The utility maximizing consumer's actions can be broken down into two distinct steps. When the price of Q decreases from $P_o$ to $P_2$ the utility maximizing consumer might first act like an expenditure minimizing consumer by exchanging the relatively more expensive Y for

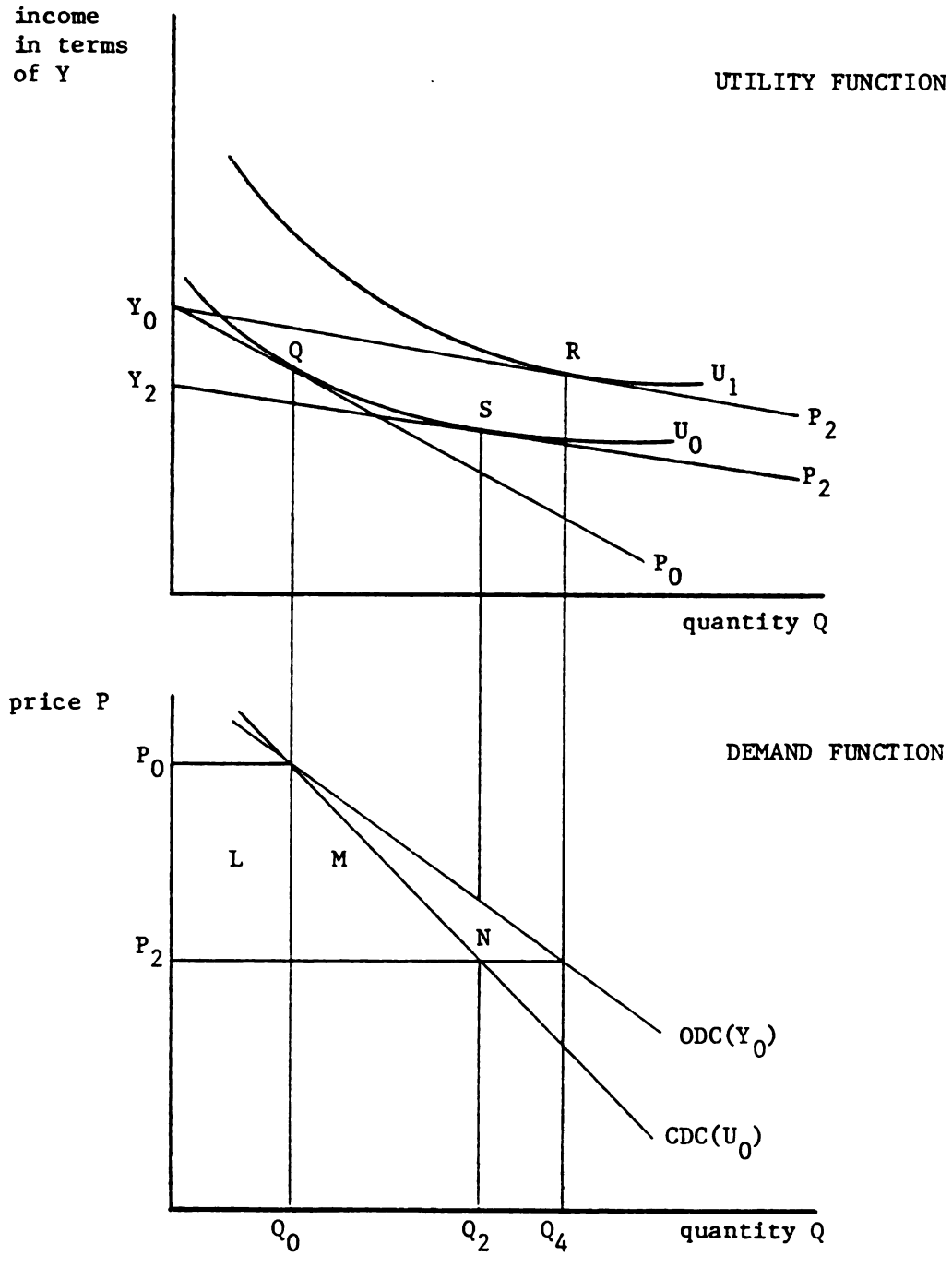


Figure 7. Ordinary and Compensated Demand Curves and Compensating Variation

relatively less expensive Q. The utility maximizing consumer thus moves from "Q" to S. This is called the substitution effect with respect to a constant level of utility. This is also a component of the Slutsky equation discussed above ($\partial Q/\partial P \mid$ constant utility). The utility maximizing consumer realizes a change in income equal to $Y_o - Y_2$ as does the expenditure minimizing consumer.

The second step taken by the utility maximizing consumer is unlike anything the expenditure minimizing consumer would do. The utility maximizing consumer uses a portion of the change in income ($Y_o - Y_2$) resulting from a decrease in the price of Q from $P_o$ to $P_2$ to purchase additional units of Q at price $P_2$. The second step moves the utility maximizing consumer from S to R. This is called the income effect with respect to constant prices (in this case $U_1$ and $P_2$). This is the other component of the Slutsky equation ($\partial Q/\partial Y \mid$ constant prices).

As shown in Figure 7, the change in income with the decrease in price of Q from $P_o$ to $P_2$ with respect to the initial level of utility level $U_o$ is equal to $Y_o - Y_2$. We have seen that

(15) $\quad Y_o - Y_2 = LM.$

The integral of the ordinary demand function from $P_o$ to $P_2$ (called $A_{P_2}^{P_o}$) is

(16) $\quad A_{P_2}^{P_o} = \int_{P_2}^{P_o} Qdp = LMN.$

Equation (16) is defined here as the "utility" measure of consumer's surplus. The reason for this name "utility" is discussed below.

(17) $\quad A_{P_2}^{P_o} = LMN =$ "utility" measure.

The "utility" measure overestimates $Y_o - Y_2$ which is the measure of compensating variation for a price decrease. The reason the "utility" measure overstates compensating variation is because there is a double counting of the change in income resulting from a change in price. The double counting associated with the "utility" measure vis-a-vis compensating variation is equal to the income effect portion of the Slutsky equation, i.e. $(Q(\partial Q/\partial Y) \mid$ constant prices). The income effect is equal to the increased consumption of Q from $Q_2$ to $Q_4$ purchased out of the change in real income when the price of Q decreased from $P_o$ to $P_2$. The difference between the utility measure (16) and compensated variation (15) equals the income effect.

(18) income effect = N

We have seen that compensating variation is not directly measurable in utility space. For a price decrease, we have established the compensated variation is a value bounded by the Laspeyres and "utility" measures. For a price decrease, compensating variation is greater than the Laspeyres measure and less than the "utility" measure. If the income effect is small then this range is also small, but still no exact measure of compensating variation is possible using observable data using techniques we have discussed so far.

Let us turn our attention to the relationship between the "utility" measure and equivalent variation. Refer to Figure 8. Assume that the consumer has initial nominal income of $Y_o$ and consumes $Q_o$ of Q at price $P_o$ at utility level $U_o$. $U_o$ is designated as initial utility level. When the price of Q decreases from $P_o$ to $P_2$ then the utility maximizing consumer purchases $Q_4$ of Q and achieves a subsequent utility level $U_1$ with now subsequent nominal income $Y_o$. The expenditure

minimizing consumer constrained to maintain subsequent utility level $U_1$ regardless of the price of Q would have purchased $Q_1$ of Q at the initial price $P_o$ requiring total income $Y_1$. The change in income resulting from a price change from $P_o$ to $P_2$ in relation to the subsequent utility level is

(19) $Y_1 - Y_o = LMNO.$

The area LMNO is equal to the substitution effect of the Slutsky equation. However, at the initial price $P_o$ the utility maximizing consumer, constrained by nominal income $Y_o$, could only purchase $Q_o$ of Q. The expenditure minimizing consumer is not constrained by nominal income but rather by real income in relation to a utility level. Therefore at price $P_o$ the utility constrained consumer purchases $Q_1$ of Q. The integral of the ordinary demand function from $P_o$ to $P_2$ is the "utility" measure.

(20) $A_{P_2}^{P_o} = \int_{P_2}^{P_o} Qdp = LMN.$

The integral of the compensated demand function in relation to subsequent utility $U_1$ equals equivalent variation.

(21) $Y_1 - Y_o = LMNO.$

The utility measure underestimates $Y_1 - Y_o$ which is equivalent variation for a price decrease. The reason the "utility" measure understates equivalent variation is because the income constraint associated with fixed nominal income $Y_o$ precludes the sufficient purchase of Q when its price is $P_o$ in order to achieve utility level $U_1$. This negative income effect is shown as the movement from "Q" to T in Figure 8. This undercounting of the change in real income resulting from a change in price with respect to subsequent utility level $U_1$

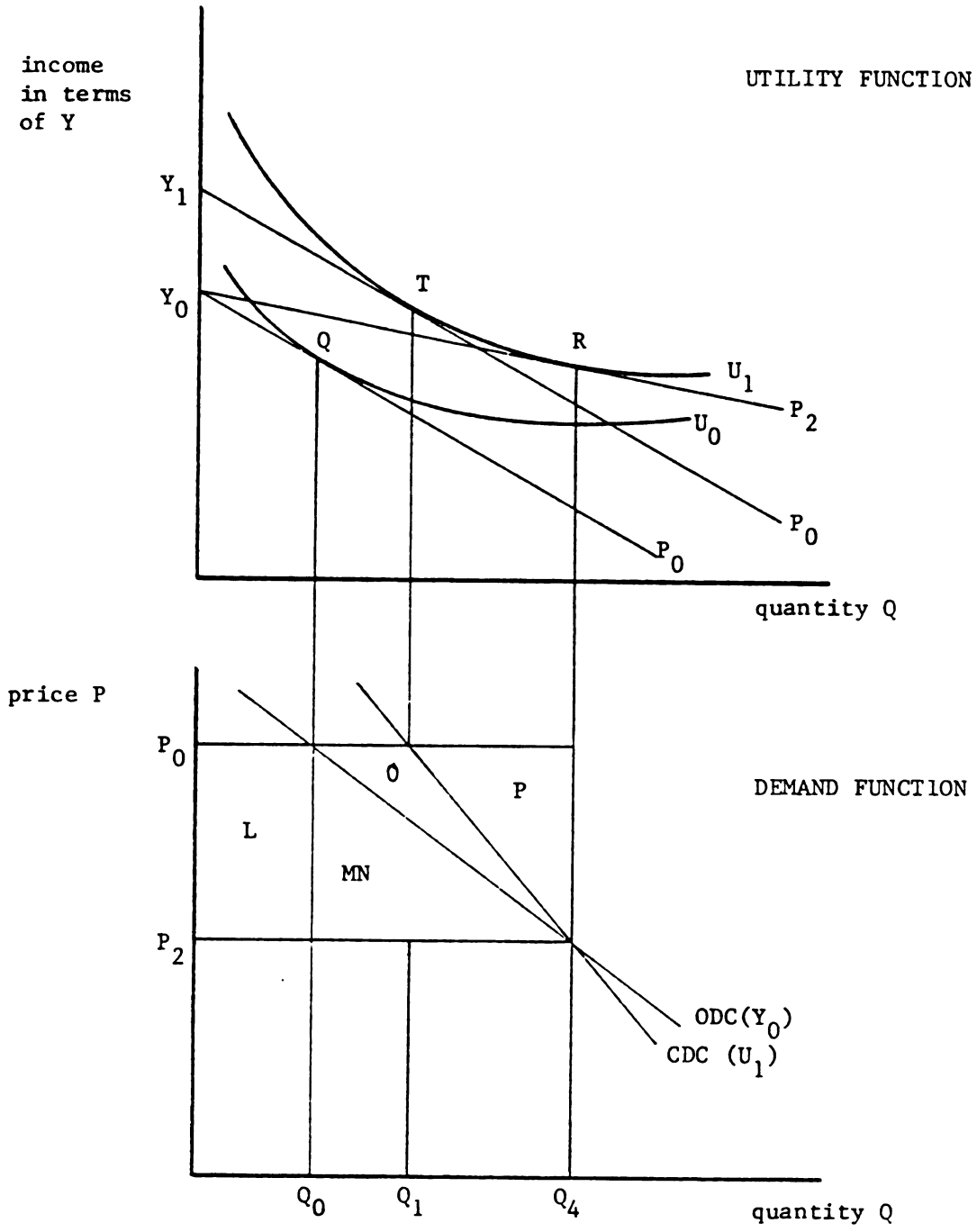


Figure 8. Ordinary and Compensated Demand Curves and Equivalent Variation

is a measure of the income effect. The difference between the utility measure (20) and equivalent variation (21) equals the income effect.

(22) income effect = area 0.

The equivalent variation like compensating variation is not directly measurable in utility space. For a price decrease, equivalent variation is a value bounded by the "utility" and Paasche measures. For a price decrease equivalent variation is greater than the "utility" measure and less than the Paasche measure. If the income effect is small then this range is also small, but still no exact measure of equivalent variation is possible using observable data using techniques discussed so far.

There is an important relationship between compensating and equivalent variation. Compensating variation is a measure of the substitution effect prior to the income effect. Equivalent variation is a measure of the substitution effect after the income effect has taken place. This difference in assumption regarding the income effect is suggestive of the "path dependency of consumer surplus" (Just, Hueth and Schmitz, 1982, p. 73; and Patinkin, 1963, p. 93).

By way of summary, the following relationships are established between the Laspeyres, compensating variation, "utility," equivalent variation and Paasche measure of consumer's surplus for a price decrease. For a price decrease

(22) Laspeyres < compensating variation < "utility" and

(23) "utility" < equivalent variation < Paasche.

These relationships are summarized graphically in Figure 9.

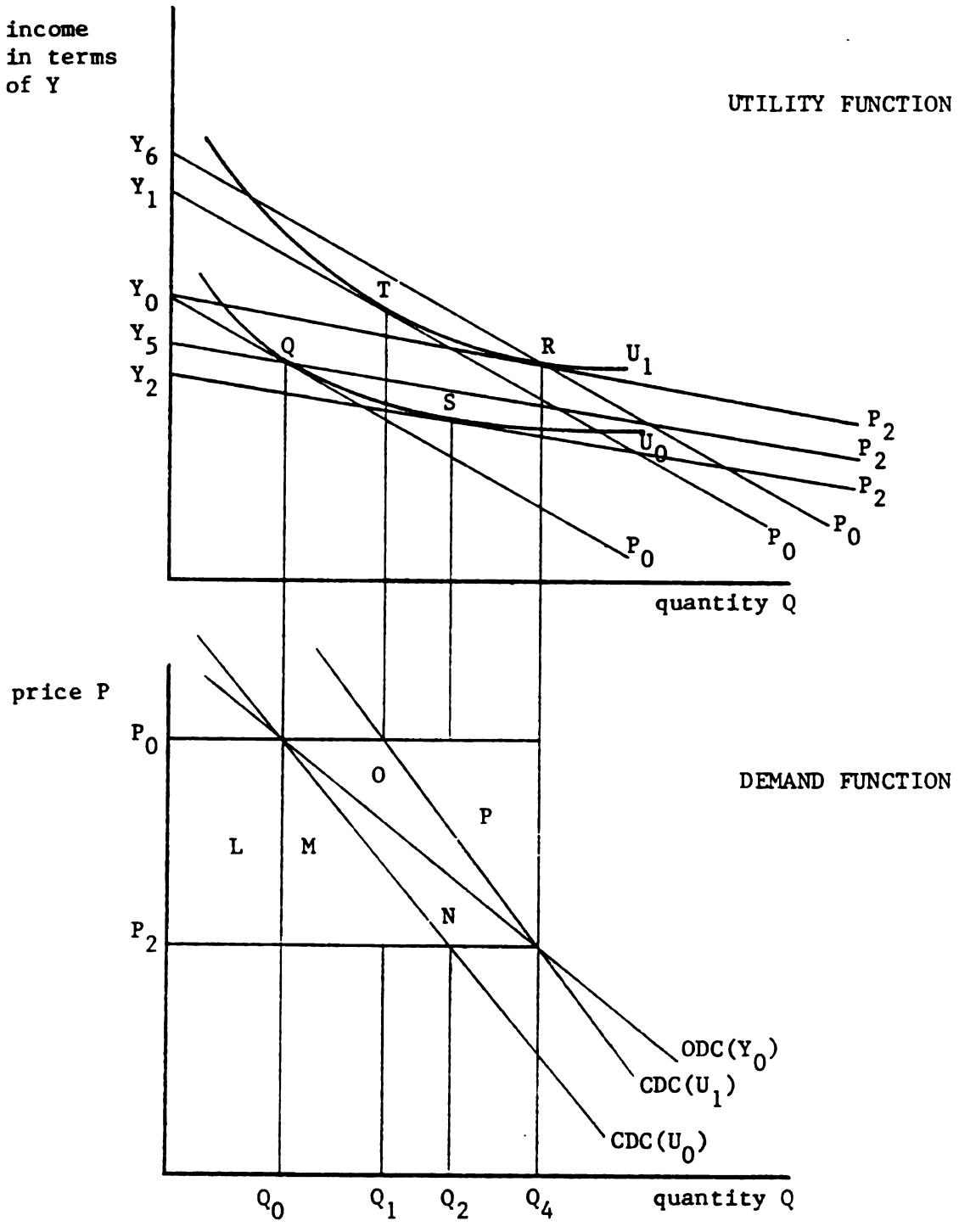


Figure 9. The Five Measures of Consumer's Surplus

## 2-7 Marshall's Notion of Consumer's Surplus

The previous sections have described the five measures of consumer's surplus. These measures are Laspeyres, compensating variation, "utility," equivalent variation and Paasche measures. The relationships between and among these measures of consumer's surplus was explained. The problem of measurability was also presented in relation to compensated and equivalent variation. Before going on to examine Willig and Hausman's recent suggestion for solving the measurability problems, I would like to take a brief detour to examine what Marshall meant when he first defined the notion of consumer's supply.

Refer to Figure 10. Assume that a utility maximizing individual has an initial endowment of $Y_o$ of good Y only. When the price of Q decreases from $P_o$ to $P_2$ the consumer purchases $Q_4$ of Q and achieves subsequent utility level $U_1$. There are two ways of measuring the change in income associated with the consumer's purchase of $Q_4$ at price $P_2$. The usual Walrasian measure is to assume price is given as $P_2$ and determine the quantity that would be purchased if the consumer were constrained to remain at the initial utility level $U_o$ in $Q_2$. The change in nominal income in this instance would be $Y_o - Y_2$ and equal to area $P_oP_2A$.

(1) $Y_o - Y_2 = P_oP_2A$ = Walrasian measure of compensating variation.

The other way of measuring the change in income associated with "consumer's purchase of $Q_4$" is called the Marshallian measure. The Marshallian measure assumes that quantity $Q_4$ is given and price is determined as the lowest price required to purchase $Q_4$ with respect

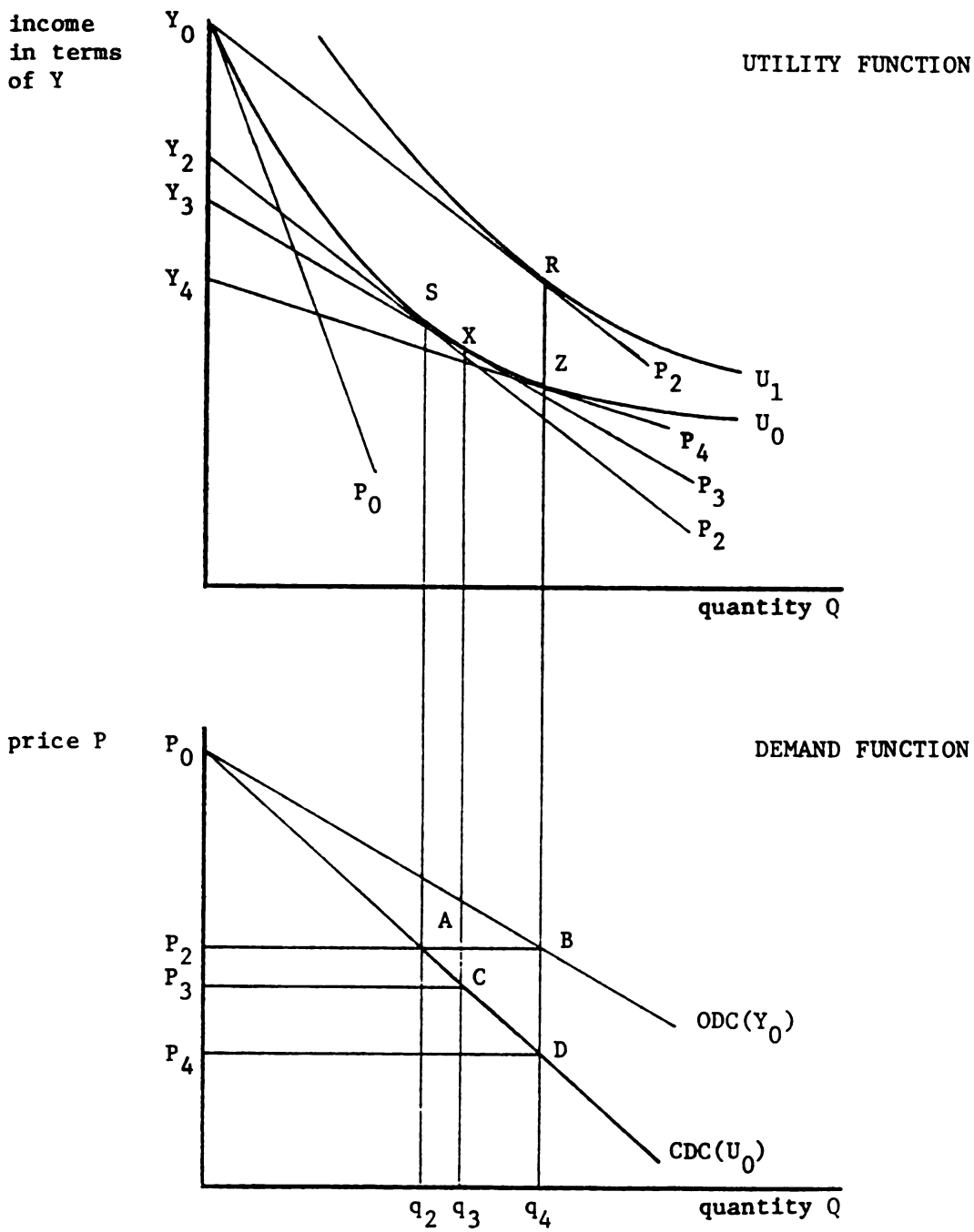


Figure 10. Marshall's Definition of Consumer's Surplus: The General Case

to initial utility level $U_o$, i.e., $P_4$. The change in income in this instance would be $Y_o - Y_4$ and equal to area $P_oP_4D$.

(2) $Y_o - Y_4 = P_oP_4D$ = Marshallian measure of compensating variation

Marshall's own notion of consumer's surplus seems to be "the excess of price [i.e., total expenditure] which he would be willing to pay rather than go without the thing, over that which he actually does pay" (Marshall, 1920, p. 124; and Patinkin, 1963, p. 92). We have already determined a measure of "the excess of price which he would be willing to pay rather than go without the thing." This amount is equation (2), the Marshallian measure of compensating variation. The measure of ". . . that which he already does pay" is more difficult to present since it involves two different utility levels and therefore lacks a common denominator with the Marshallian measure of compensating variation.

In order to establish a common denominator let us define the area associated with the income effect $P_oAB$ as being equal to area $P_2P_3CA$. Define

(3) $P_oAB = P_2P_3CA$.

The area $P_2P_3CA$ determines a quantity $Q_3$. $Q_3$ represents a point on the initial utility function $U_o$. The tangent to the initial utility function at $U_o(Q_3)$ equals an expenditure function $Y_3$ with slope equal to $P_3$. The change in income $Y_o - Y_3$ represents a measure of "that which he already pays" with $U_o$ as a common denominator.

(4) $Y_o - Y_3 = P_oP_3C$.

The Marshallian measure of consumer's surplus is equal to the difference between equations (2) and (4).

(5) $Y_o - Y_4 - (Y_o - Y_3) = P_oP_4D - P_oP_3C$.

Simplifying

(6) $Y_3 - Y_4 = P_3P_4DC$.

Equation (6) is an interpretation of the Marshallian notion of consumer's surplus (Patinkin, 1963, p. 94). It would only be by chance that the Walrasian and Marshallian measures of consumer's surplus would ever be equal.

(7) $Y_o - Y_2 \neq Y_3 - Y_4$.

The only instance when the Walrasian and Marshallian measures of compensating variation would be equal is when the income elasticity of Q is equal to zero (Samuelson, 1942, p. 85). If Figure 10 is redrawn so that the income elasticity of Q is zero then Figure 11 collapses into a special case in which the Walrasian and the Marshallian measures of compensating variation are equal.

(8) $Y_o - Y_2 = Y_o - Y_4$

Samuelson shows that the Marshallian assumption regarding constant utility of money is tantamount to assuming that the income elasticity of Q is equal to zero (Samuelson, 1942, pp. 84-86).

Finally in Figure 11 the change in income $Y_o - Y_2$ is equal to the distance of the vertical shift in the utility function. It is because of this coincidence in the special case that the integral of the ordinary demand function is called the "utility" measure of consumer's surplus. There are two reasons for putting quotation marks about the word utility when used this way. First the "utility" measure is in fact a measure of the vertical shift in the utility function only when the income effect is zero. This is true at most extremely rarely. The second reason is that we have assumed only ordinal utility functions

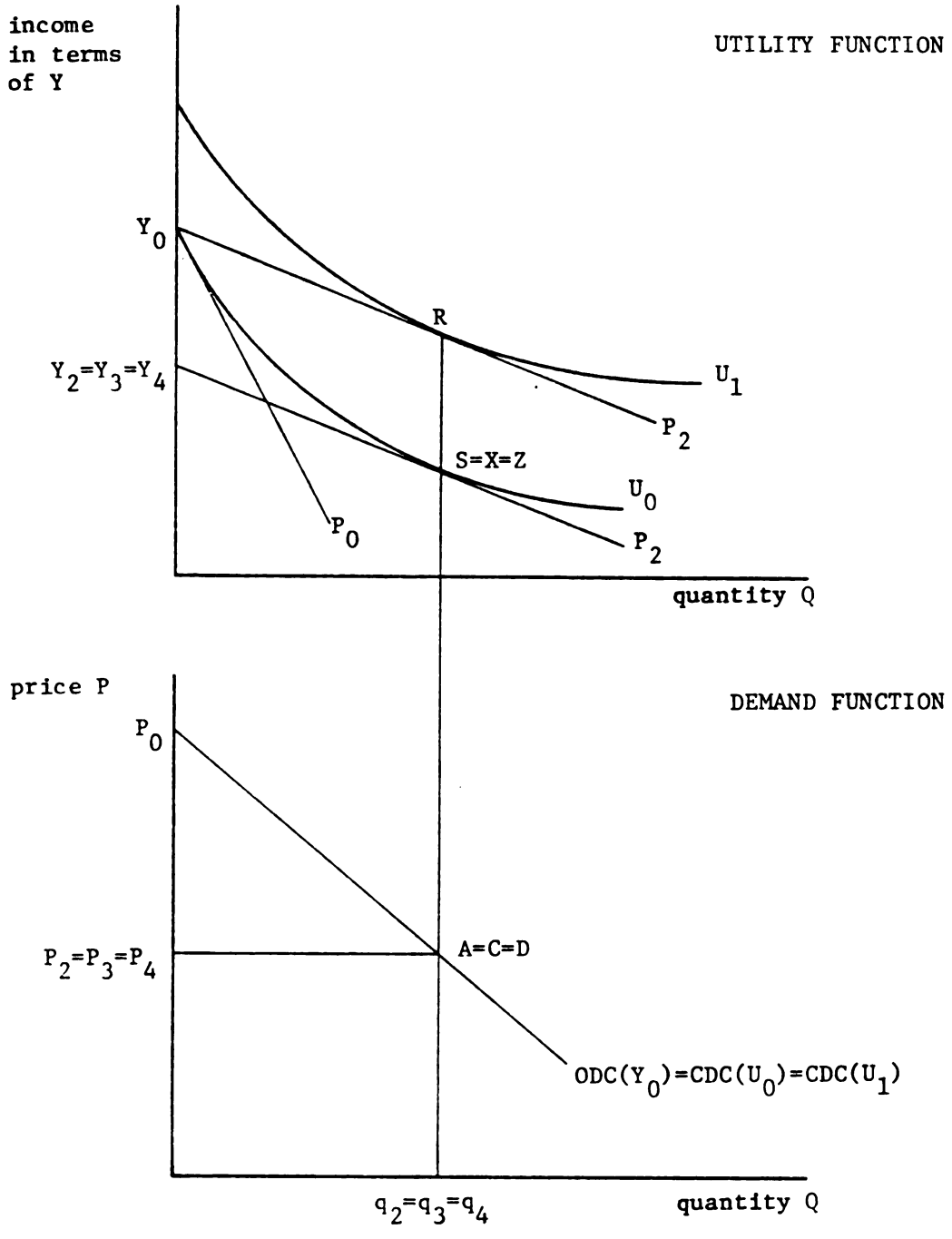


Figure 11. Marshall's Definition of Consumer's Surplus: A Special Case

in this analysis so far. The vertical distance between two ordinal utility functions lacks quantitative significance as a welfare measure (Samuelson, 1947, p. 198).

## 2-8 The Indirect Utility Function and Roy's Identity

In the previous sections the theory and directly observable measures of consumer's surplus were presented. The Laspeyres, "utility" and Paasche measures are based on directly observable data. These measures are useful in themselves in providing a range for the more empirically precise measures--compensating and equivalent variation. The problem still remains as to how estimate compensating and equivalent variation based on observable data. This question has been addressed and seemingly solves by two economists, Robert Willig and Jerry Hausman, using the concepts of the indirect utility function and Roy's identify.

In this section the indirect utility function will be described along with Roy's identify. These concepts are combined with the expenditure, utility, compensated demand and ordinary demand functions. The sum total of the interrelations between the various functions results in a way to measure compensating and equivalent variation based on the available observable data. The Hausman-Willig measure is a measure of compensating and equivalent variation in indirect utility space. The change in nominal income resulting from a change in prices with respect to a given utility is measured as the difference between the relevant expenditure functions. These measures of the change in nominal income are called compensating variation and equivalent variation and will be addressed in turn.

2-8-1 Compensating Variation

Compensating variation is defined as the difference between initial and subsequent nominal income with respect to an initial utility level for a given change in price. Refer to Figure 12. For a price decrease from $P_o$ to $P_2$, compensating variation is equal to the difference between initial nominal income $Y_o$ and subsequent nominal income $Y_2$ given initial utility level $U_o$.

(1) $CV\ (P_o,P_1,U_o) = Y_o - Y_2$.

Initial and subsequent nominal income are determined in relation to their respective expenditure minimization functions given utility level $U_o$. The initial real income $Y_o$ is equal to minimum expenditure when the price of Q is $P_o$ and initial utility level is $U_o$.

(2) $Y_o = e(P_o,U_o)$.

The subsequent nominal income $Y_2$ is equal to the minimum expenditure when the price of Q is $P_2$ and initial utility level is $U_o$.

(3) $Y_2 = e(P_2,U_o)$.

The expression for compensating variation for a price decrease from $P_o$ to $P_2$ given utility level $U_o$ can now be stated in terms of minimum expenditure functions by substituting (2) and (3) into (1)

(4) $CV(P_o,P_2,U_o) = e(P_o,U_o) - e(P_2,U_o)$.

Equation (4) is a direct measure of compensating variation using the expenditure functions for the appropriate values of real income. The value for the minimum expenditure function $e(P_o,U_o)$ is known apriori as equal to the observable value of initial nominal income $Y_o$.

(5) $CV(P_o,P_2,U_o) = Y_o - e(P_2,U_o)$.

The value of the subsequent expenditure function $e(P_2,U_o)$ is not known because there is no direct measure of the utility function $U_o$. An

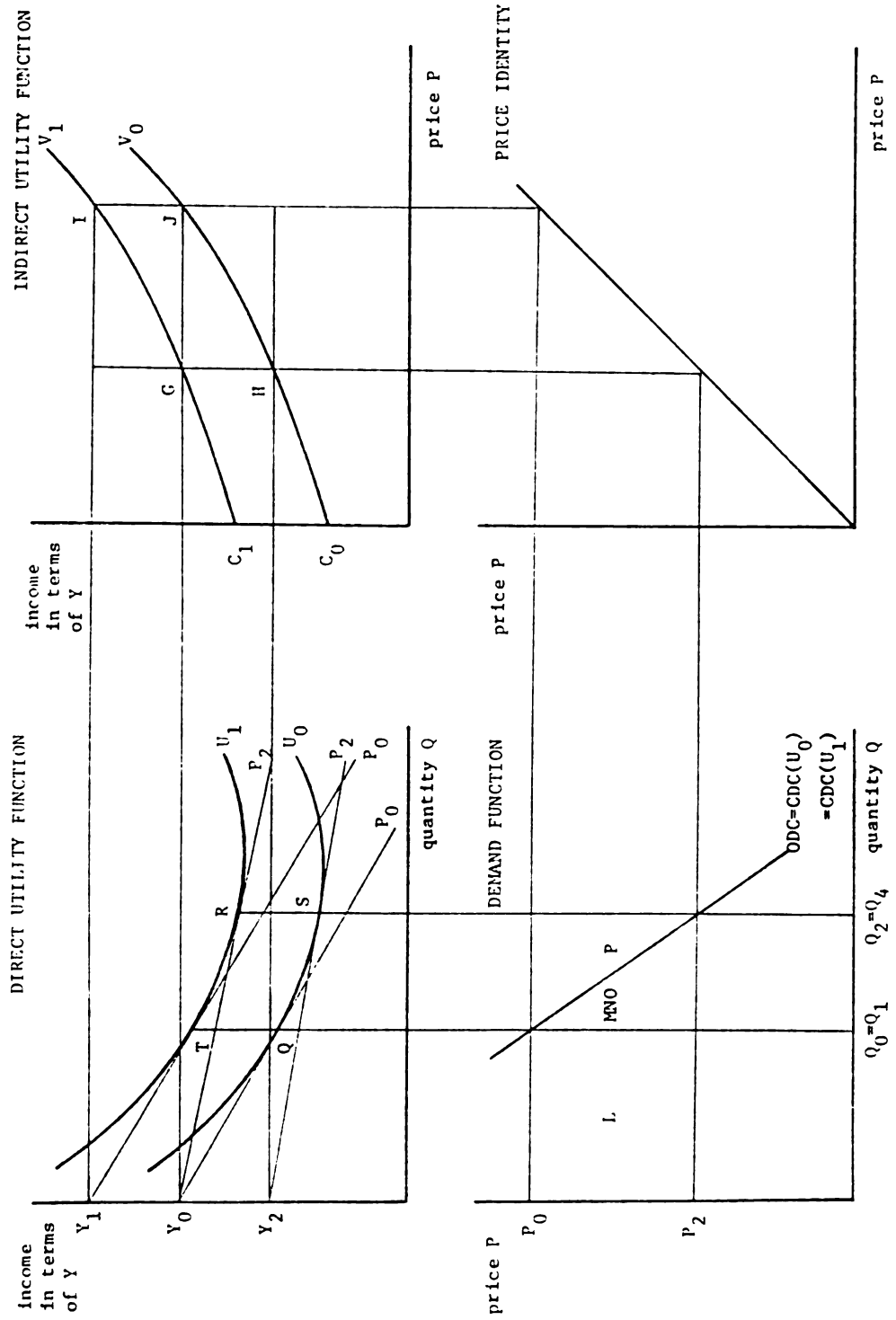


Figure 12. Vertically Parallel Direct and Indirect Utility Functions

indirect method of determining the value of the initial utility function $U_o$ does exist. It is possible to establish a cardinal measure of utility $U_o$ consistent with observable data by means of the indirect utility function and Roy's identity.

For any given utility function $\mu$ there exists a corresponding indirect utility function V where

(6) $U_o = V_o(P,Y)$

and

(7) $U_1 = V_1(P,Y)$

(Roy, 1947, p. 211).

The indirect utility function is defined as the relationship of nominal income to price necessary to maintain a given utility level (Just, Hueth and Schmitz, 1982, p. 95). The indirect utility function is the inverse of the expenditure minimization function for a given real income, price and utility level (Hausman, 1981, p. 665). For example the expenditure minimization function given price $P_o$ and initial utility level $U_o$ equals the inverse of the indirect utility function $V_o$ given price $P_o$ and initial real income $Y_o$.

(8) $e(P_o,U_o) = V_o(P_o,Y_o)^{-1}$

Similarly, the expenditure minimization function given price $P_2$ and initial utility level $U_o$ equals the inverse of indirect utility level $V_o$ for price $P_2$ and subsequent real income $Y_2$.

(9) $e(P_2,U_o) = V_o(P_2,Y_2)^{-1}$

Compensating variation can then be expressed in terms of the indirect utility function by substituting (9) into (5).

(10) $CV(P_o,P_2,U_o) = Y_o - V_o(P_2,Y_2)^{-1}$

Graphically compensating variation is equal to the vertical distance between J and H in Figure 12. J corresponds to the point $V_o(P_o,Y_o)$ on the indirect utility function $V_o$. H corresponds to point $V_o(P_2,Y_2)$ also on the indirect utility function $V_o$. The vertical distance JH is equal to GH which is equal to compensating variation for a price decrease from $P_o$ to $P_2$.

It is possible to determine the utility level $U_o$ and the subsequent nominal income $Y_2$ once the functional form of the indirect utility function $V_o$ is known. The point J on the indirect utility function $V_o$ corresponds to point Q on the direct utility function $U_o$. The relationships between J and Q is called Roy's identity and can be used to determine both the indirect and direct utility functions $V_o$ and $U_o$.

Roy's identity also relates the derivative of the indirect utility function at point J to the compensated and ordinary demand function at point $(P_o,Q_o)$ in demand space.

$$(11) \quad \frac{V_o(P_o,Y_o) \ / P}{V_o(P_o,Y_o) \ / Y} = Q_o$$

(Roy, 1947, p. 214; and Hausman, 1981, p. 667).

Roy's identify is the first derivative of the indirect utility function and is equal to the compensated demand function at point $(P_o,Q_o)$ given initial utility level $U_o$. Roy's identify is expressed in equation (11) in terms of $P_o$, $Y_o$ and $Q_o$ all of which are observable. Coincidentally at $Q_o$ the compensated demand function given utility level $U_o$ is equal to the ordinary demand function given nominal income $Y_o$.

$$(12) \quad Q_o = \psi(P_o,U_o) = \emptyset(P_o,Y_o).$$

This is particularly fortunate since the ordinary demand function $\phi(P,Y_o)$ is also observable. Substituting (12) into (11)

$$(13) \quad -\frac{\partial V_o(P_o,Y_o)/\partial P}{\partial V_o(P_o,Y_o)/\partial Y} = \phi(P_o,Y_o).$$

Equation (13) establishes the relationship between the indirect utility function and the ordinary demand function. This is the relationship which is needed to determine a cardinal measure of utility $U_o$ based on observable data. Setting equation (13) equal to zero assures expenditure minimization.

$$(14) \quad -\frac{\partial V_o(P_o,Y_o) / \partial P}{\partial V_o(P_o,Y_o) / \partial Y} - \phi(P_o,Y_o) = 0.$$

Equation (14) is a partial differential equation. "A differential equation is an equation that involves one or more derivatives or differentials" (Thomas, 1961, p. 875). By integrating this differential equation it is possible to find the function whose derivative is given (Thomas, 1961, p. 155). The integral of equation (14) is equal to the indirect utility function $V_o$. Simplifying and integrating (14).

$$(15) \quad V_o(P_o,Y_o) = \int(-\partial Y_o/\partial P_o - \phi(P_o,Y_o))/\partial P_o.$$

Hausman shows that (15) simplifies further if Q is normalized in terms of Y (1981, p. 668; also Hanemann, 1982, p. 2).

$$(16) \quad V_o(P_o,Y_o) = \int(dY_o/dP_o - \phi(P_o,Y_o))dP_o$$

Equation (16) implies that there are no cross price effects between Q and other goods in the economy associated with a homothetic utility function. The assumption that the ordinary demand is a function of own price and income only implies a separability assumption (Hausman, 1981, p. 667; and Burtless and Hausman, 1978, p. 1111).

Even assuming separability, the integration of a differential equation to find its closed form ranges in difficulty from easy to impossible. However it has been shown that for the more common functional forms used in econometrics to fit an ordinary demand function to consumption data, a closed form solution of the differential equation does exist (Hanemann, 1982, p. 9). Even when a closed form solution does not exist another method for approximating compensates and equivalent variation based on the indirect utility function approach does exist (Willig, 1976, p. 593; and Hanemann, 1982, p. 9). Once equation (16) is integrated there is an arbitrary constant of integration C in the equation. "The integral of the differential of a function U is U plus an arbitrary constant C" (Thomas, 1961, p. 159). The constant of integration is in fact the "Y intercept" of the indirect utility function when price is zero. The convention is to solve the indirect utility function for $C_o$ in terms of $P_o$ and $Y_o$. This solution is then set equal to the indirect utility function $V_o$ expressed in terms of $P_o$ and $Y_o$.

(17) $C_o = V_o(P_o,Y_o)$

The solution of the indirect utility function for $C_o$ in terms of $P_o$ and $Y_o$ transforms the ordinal indirect utility function into a cardinal indirect utility function. By substituting (17) into (6) results in a cardinal utility function for $U_o$.

(18) $V_o(P_o,Y_o) = C_o = U_o$

Equation (18) is a measurable function of the cardinal utility level $U_o$ in terms of the indirect utility function $V_o$, price $P_o$ and initial real income $Y_o$. Since $V_o(P_2,Y_2)$ is also a point on the indirect utility function $V_o$ then it too must equal $C_o$ and $U_o$.

(19) $V_o(P_2,Y_2) - C_o = U_o$

Solving equation (19) for $Y_2$ gives an expression for subsequent real income in terms of the indirect utility function $V_o$, expressed in terms of price $P_2$ and initial utility level $U_o$.

(20) $Y_2 = V_o(P_2,U_o)^{-1}$

Since (19) is an expression for the initial utility, it can be substituted in to (20) to give an expression for subsequent nominal income $Y_2$.

(21) $Y_2 = V_o(P_2,V_o(P_o,Y_o))^{-1}$

Equation (21) is a measurable expression of subsequent real income expressed in terms of $P_o$, $P_2$ and initial real income $Y_o$ all as a function of the indirect utility $V_o$. Substituting (21) into (1) gives a measurable expression for compensating variation for a price decrease from $P_o$ to $P_2$.

(22) $CV(P_o,P_2,U_o) = Y_o - V_o(P_2,V_o(P_o,Y_o))^{-1}$

Equation (22) is a general expression for Hausman's "exact consumer's surplus" measure of compensating variation. This "exact measure" of compensating variation is a direct measure of the change in income resulting from a decrease in the price of Q from $P_o$ to $P_2$ in relation to utility level $U_o$.

2-8-2 Equivalent Variation

The theory of equivalent variation is very similar to the theory described above for compensating variation. The crucial difference between these two measures of the change in real income is that for equivalent variation income $Y_o$ is assumed to be the subsequent nominal income while for compensating variation income $Y_o$ is assumed to be the

initial nominal income in relation to a minimum expenditure function. Consequently equivalent variation is defined as the difference between initial and subsequent nominal income in relation to the subsequent utility level given a change in price. Referring to Figure 12, for a price decrease from $P_o$ to $P_2$ equivalent variation is the difference between initial nominal income $Y_1$ and subsequent nominal income $Y_o$ in relation to level of utility $U_1$.

(23) $EV(P_o,P_2,U_1) = Y_1 - Y_o$.

Initial and subsequent nominal income are determined in relation to their respective expenditure minimization functions given utility level $U_1$. The initial nominal income $Y_1$ is equal to minimum expenditure when the price of Q is $P_o$ and the utility level is $U_1$.

(24) $Y_1 = e(P_o,U_1)$.

Subsequent nominal income $Y_o$ is equal to the minimum expenditure when the price of Q is $P_2$ and the utility level is $U_1$.

(25) $Y_o = e(P_2,U_1)$.

The expression for equivalent variation for a price decrease from $P_o$ to $P_2$ given utility level $U_1$ can now be stated in terms of minimum expenditure functions by substituting (24) and (25) into (23).

(26) $EV(P_o,P_2,U_1) = e(P_o,U_1) - e(P_2,U_1)$.

Equation (26) is a direct measure of equivalent variation using expenditure functions for the appropriate values of nominal income. The value for the subsequent nominal income $Y_o$ is known apriori and is equal to the expenditure function $e(P_2,U_1)$.

(27) $EV(P_o,P_2,U_1) = e(P_o,U_1) - Y_o$.

The value of the initial expenditure function $e(P_o,U_1)$ is not known because there is no direct measure of the utility function $U_1$.

An indirect method of determining the value of the utility function $U_1$ does exist. It is possible to establish a cardinal measure of utility function $U_1$ consistent with observable data by means of the indirect utility function and Roy's identity.

For any given utility function U there exists a corresponding indirect utility function V as a function of price and nominal income such that

(28) $U_1 = V_1(P,Y)$

(Roy, 1947, p. 211).

The indirect utility function is defined as the relationship between nominal income and price in maintaining a given utility level (Just, Hueth, and Schmitz, 1982, p. 95). The indirect utility function is the inverse of the expenditure minimization function for a given nominal income, price and utility level (Hausman, 1981, p. 665). The indirect utility function can be thought of as a real income function. The expenditure minimization function given price $P_o$ and utility level $U_1$ is equal to the inverse of the indirect utility function $V_1$ given price $P_o$ and initial real income $Y_1$.

(29) $e(P_o,U_1) = Y_1 = V_1(P_o,Y_1)^{-1}$.

The minimum expenditure function given price $P_2$ and utility level $U_1$ equals the inverse of the indirect utility level $V_1$ given price $P_2$ and subsequent nominal income $Y_o$.

(30) $e(P_2,U_1) = Y_o = V_1(P_2,Y_o)^{-1}$.

Equivalent variation can be expressed in terms of the indirect utility function by substituting (29) into (27).

(31) $EV(P_o,P_2,U_1) = V_1(P_o,Y_1)^{-1} - Y_o$.

Graphically equivalent variation is equal to the vertical distance between G and I in Figure 12. I corresponds to a point on the indirect utility function $V_1(P_o,Y_1)$. G corresponds to point $V_1(P_2,Y_o)$. The vertical distances between GI is equal to $^{JI}$ which is equal to equivalent variation for a price decrease from $P_o$ to $P_2$.

It is possible to determine the utility level $U_1$ and initial nominal income $Y_1$ once the functional form of the indirect utility function $V_1$ is known. The point G on the indirect utility function $V_1$ corresponds to point R on the direct utility function $U_1$. The relationship between G and R is called Roy's identity and can be used to determine both the indirect and direct utility functions $V_1$ and $U_1$.

Roy's identity relates the derivative of the indirect utility function at point G to the compensated and ordinary demand function at point $(P_2,Q_4)$ in the demand space. Roy's identity is presented in some detail later in this chapter.

$$(32) \quad -\frac{\partial V_1(P_2,Y_o)/\partial P_2}{\partial V_1(P_2,Y_o)/\partial Y_o} = Q_4$$

(Roy, 1947, p. 214; and Hausman, 1981, p. 667).

Roy's identity as the first derivative of the indirect utility function (also the first derivative of the inverse of the minimum expenditure function) is equal to the compensated demand function for utility level $U_1$. Roy's identity expressed in equation (32) is in terms of $P_2,Y_o$ and $Q_4$ all of which are observable. Coincidentally at $Q_4$ the compensated demand function for utility level $\mu_1$ is equal to the ordinary demand function for nominal income $Y_o$.

$$(33) \quad Q_4 = \psi(P_2,U_1) = \emptyset(P_2,Y_o).$$

This is particularly fortunate since the ordinary demand function $\phi(P_2,Y_o)$ is observable. Substituting (33) into (32)

$$(34) \quad -\frac{\partial V_1(P_2,Y_o)/\partial P_2}{\partial V_1(P_2,Y_o)/\partial Y_o} = \phi(P_2,Y_o).$$

Equation (34) establishes the relationship between the indirect utility function and the ordinary demand function. This is the relationship which is needed to determine a cardinal measure of utility $U_1$ based on observable data. Setting equation (34) equal to zero assures expenditure minimization.

$$(35) \quad -\frac{\partial V_1(P_2,Y_o)/\partial P_2}{\partial V_1(P_2,Y_o)/\partial Y_o} - \phi(P_2,Y_o) = 0.$$

Equation (35) is a partial differential equation. By integrating this differential equation it is possible to find the function whose derivative is given (Thomas, 1961, p. 155). The integral of equation (35) is equal to the indirect utility function $V_1$. Integrating and simplifying (35), then

$$(36) \quad V_1(P_2,Y_o) = \int(-\partial Y_o/P_2 - \phi(P_2,Y_o))dp.$$

Hausman shows that (36) simplifies further if Q is normalized in terms of Y (Hausman, 1981, p. 668; and Hanemann, 1982, p. 2).

$$(37) \quad V_1(P_2,Y_o) = \int(dY_o/dP_2 - \phi(P_2,Y_o))dp.$$

Equation (37) implies that there are no cross price effects between Q and other goods in the economy. This assumption reduces the ordinary demand function to the relationship between own price and income only associated with a homothetic utility function. This utility function implies a separability assumption (Hausman, 1981, p. 667; and Burtless and Hausmann, 1978, p. 1111).

Even assuming separability, the integration of a differential equation to find its closed form ranges in difficulty from easy to impossible. However it has been shown that for the more common functional forms used in econometrics to fit an ordinary demand function to consumption data, a closed form solution of the differential equation does exist (Hanemann, 1982, p. 9). However, even when a closed form solution does not exist another method for approximating equivalent variation based on Willig's interpretation of the indirect utility approach does exist (Willig, 1976, p. 593; and Hanemann, 1982, p. 9). One equation (37) is integrated, there is an arbitrary constant of integration C in the equation. "The integral of the differential of a function U of U plus an arbitrary constant C" (Thomas, 1961, p. 159). The constant of integration is in fact the "Y-intercept" of the indirect utility function $V_1$ when price is zero. The convention is to solve the indirect utility function for $C_1$ in terms of $P_2$ and $Y_o$. This solution is then set equal to the indirect utility function expressed in terms of price $P_2$ and subsequent income $Y_o$.

(38) $C_1 = V_1(P_2,Y_o)$

The solution of the indirect utility function $V_1$ equal to $C_1$ in terms of $P_2$ and $Y_o$ transforms the ordinal in direct utility function into a cardinal in direct utility function. A cardinal utility function is determined by substituting (38) into (28)

(39) $C_1 = V_1(P_2,Y_o) = U_1$

Equation (39) is a measurable cardinal utility function $U_1$ in terms of indirect utility function $V_1$, price $P_2$ and subsequent real income $Y_o$. Since $V_1(P_o,Y_1)$ is also a point on the indirect utility function $V_1$, it is therefore equal to $C_1$ and $U_1$.

(40) $V_1(P_o,Y_1) = U_1$

By solving equation (40) for $Y_1$ results in an expression for initial real income $Y_1$ in terms of the indirect utility function $V_1$ and $P_o$ and utility function $U_1$.

(41) $Y_1 = V_1(P_o,U_1)^{-1}$.

Since (39) is an expression for the utility function $U_1$, it can be substituted into (41) to give an expression for initial real income $Y_1$.

(42) $Y_1 = V_1(P_o,V_1(P_2,Y_o))^{-1}$.

Equation (42) is a measurable expression of initial nominal income $Y_1$ in terms of prices $P_o$, $P_2$ and subsequent nominal income $Y_o$, all as a function of the indirect utility $V_1$. A measurable expression for equivalent variation for a price decrease from $P_o$ to $P_2$ is now possible by substituting (42) into (24).

(43) $EV(P_o,P_2,U_1) = V_1(P_o,V_1(P_2,Y_o))^{-1} - Y_o$.

Equation (42) is a general expression for Hausman's "Exact Consumer's Surplus" measure of equivalent variation. This "exact measure" of equivalent variation is a direct measure of the change in income resulting from a decrease in the price of Q from $P_o$ to $P_2$ in relation to utility level $U_1$.

2-8-3 Roy's Identity

We have seen in the above discussion regarding the Hausman-Willig measure of compensating and equivalent variation that Roy's identity plays a crucial role. For this reason it might be beneficial to examine Roy's derivation of this identity in some depth to get at least an intuitive feel for the reasoning entailed in this expression.

Roy began by defining the indirect utility function and showing its relation to the direct utility function.

(44) $u = f(Y,Q) = V(P,Y_o)$ (Roy, 1947, p. 211).

Equation (44) implies that for every utility function U, expressed in terms of good Y and good Q, there exists a corresponding indirect utility function V, expressed in terms of price and nominal income. Roy defined the indirect utility function as "none other than the general form equation of the tangents to the indifference surface [of the direct utility function]" (Roy, 1947, p. 212, author's translation).

It is this relationship of minimum expenditure tangents to the direct utility function and the corresponding indirect utility function which allows for the development of Roy's identity. For example for a given minimum expenditure

(45) $Y' + P_oQ_o - Y_o = 0$

The point of tangency to the utility function $U_o$ is known to be equal to coefficients of the budget constraint.

(46) $(Y',Q_o,-1)$

(Roy, 1947, pp. 213-214).

The corresponding point to the minimum expenditure function (45) on the indirect utility function is

(47) $V_o(P_o,Y_o)$.

The point of tangency of the minimum expenditure function to utility function $U_o$ is equal to first derivative of indirect utility at point $V_o(P_o,Y_o)$ (Roy, 1947, p. 214). Setting the partial derivatives of the indirect utility function $V_o(P_o,Y_o)$ with respect to price $P_o$ and initial real income $Y_o$ equal to zero assure that these derivatives correspond to the minimum expenditure function solution in equation (45).

(48) $\partial V_o(P_o,Y_o)/\partial P_o = V_o(P_o,Y_o)/\partial Y_o = 0.$

Equations (46) and (48) represent the same point on different functions. Therefore the coefficients of the budget constraint of the minimum expenditure function should be proportional to the first derivatives of the corresponding indirect utility function (Roy, 1947, p. 214).

(49) $$\frac{\partial V_o(P_o,Y_o)/\partial P_o}{Q_o} = \frac{\partial V_o(P_o,Y_o)/\partial Y_o}{-1}$$

By rearranging terms equation (49) can be presented in the standard form known as Roy's identity such that,

(50) $$\frac{\partial V_o(P_o,Y_o)/\partial P_o}{\partial V_o(P_o,Y_o)/\partial Y_o} = Q_o$$

(Roy, 1947, p. 214).

## 2-9 Separability and Consumer Surplus

The previous section on the indirect utility function and Roy's identity illustrated the need to assume separability in order to obtain a cardinal welfare measure of utility associated with a price change. Therefore, it is important to explore the circumstances under which the separability assumption is met.

Separability can be viewed at least at one level as the mathematics of line integration between two functions. Essentially, the value of the integral between two functions depends on the path of integration between these two functions. The reason this is true is because the two functions are interrelated or functions of each other. The only instance when the value of the integral between two functions is independent of the path of integration is when the functions are exact differentials of each other. Thomas defines these conditions as follows:

> ". . . under certain conditions the line integral between two points $A_1$ and $A_2$ is independent of the path C joining them. That is, in circumstances soon to be described, the integral . . . has the same value for any two paths $C_1$ and $C_2$ joining $A_1$ and $A_2$. That happens when there is a function f(x, y, z) . . . such that df = Mdx + Ndy + Pdz
>
> then
>
> $$\int_C Mdx + Ndy + Pdz = \int_{A_1}^{A_2} df$$
>
> $$= f(x,y,z)\Big]_{(a_1b_1c_1)}^{(a_2b_2c_2)}$$
>
> $$= f(a_2,b_2,c_2) - f(a_1,b_1,c_1)$$
>
> is independent of the curve C joining the points $A_1$ $(a_1, b_1, c_1)$ and $A_2$ $(a_2, b_2, c_2)$. In this case we say that the expression
>
> $$M(x, y, z)\, dX + N(x, y, z)\, dY + P(x, y, z)\, dz$$
>
> is an exact differential. (1960, p. 716).

Thomas concludes with the following theorem.

> Theorem. Let M(x,y,z), N(x,y,z) and P(x,y,z) be continuous, together with their first order derivates, for all real values of x, y and z. Then a necessary and sufficient condition for the expression Mdx + Ndy + Pdz to be an exact differential is for the following equations all to be satisfied:
> $$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}, \frac{\partial M}{\partial z} = \frac{\partial P}{\partial x}, \frac{\partial N}{\partial z} = \frac{\partial P}{\partial y}$$
> (1960, p. 716)

Therefore the cross partial derivatives of the function must be equal.

The two possible economic interpretations of path independence are when the cross-price effects between goods in a consumer's consumption bundle are equal either to each other or to zero. Silberberg describes the economic interpretation of path independence as follows:

> . . . if say, $P_i$ changes, the demand curve for other commodities begins to shift at the rate $(\partial x_j/\partial P_i)$, $i \neq j$. If, however, some other price $P_j$ changes, the demand for commodity i shifts at a rate $(\partial x_i/\partial P_j)$. Since these rates are not in general equal, the way in which $P_i$ and $P_j$ are changed -- for example, first $P_i$, then $P_j$ or vice versa -- will affect the areas under the demand curves $\int P_i dx_i$ ... As it turns out, the condition $\partial x_i/\partial P_j = \partial x_j/\partial P_i$, i, j = 1, ..., N is precisely the condition for path independence. (1972, p. 944).

An example of path independence, when the cross-price effects are zero, is provided by Frisch in reference to commodities for which ". . . the marginal utility of using (or supplying) more electricity . . . can be safely regarded as independent of the quantity of Swiss cheese consumed" (Firsch, 1959, p. 178).

There are only two utility functions which assure separability or path independence. These two types of utility functions are vertically parallel and homothetic. A vertically parallel utility function

is one in which ". . . preferences are parallel with respect to commodity 1 (numeric good) on the relevant domain of the commodity space" (Chipman and Moore, 1980, p. 937). The property of such a function is that of ". . . yielding demand functions with zero income elasticities for all commodities other than commodity 1 (the numeric commodity) (Chipman and Moore, 1980, p. 938). A vertically parallel utility function is also one which results in equal cross-price elasticities the sum of which equal the own-price elasticity. See Figure 12. A vertically parallel function is path independent in the sense that compensating and equivalent variation are equal (GH = IJ) relative to a particular numeraire good # (Y) only. A homothetic utility function is one in which consumers partition or portion income to cover areas of expense regardless of relative price changes between categories. Additional income is distributed proportionately (Chipman and Moore, 1980, p. 937). Homothetic utility function has the property of yielding a demand function which has unitary own-price and income elasticities and zero cross-price elasticities (Chipman and Moore, 1980, p. 938; Samuelson, 1965, pp. 788-789). See Figure 13. A homothetic function is path independent in the sense that compensating and equivalent variation (GH and IJ respectively) are constant regardless of the good (Y or Q) selected as the numeraire.

The properties of homothetic and vertically parallel functions are so stringent that they are nearly impossible to meet empirically. Samuelson describes these two types of utility functions as ". . . interesting from a theoretical viewpoint but trivial for empirical application." (1965, p. 788). Therefore the utility functions

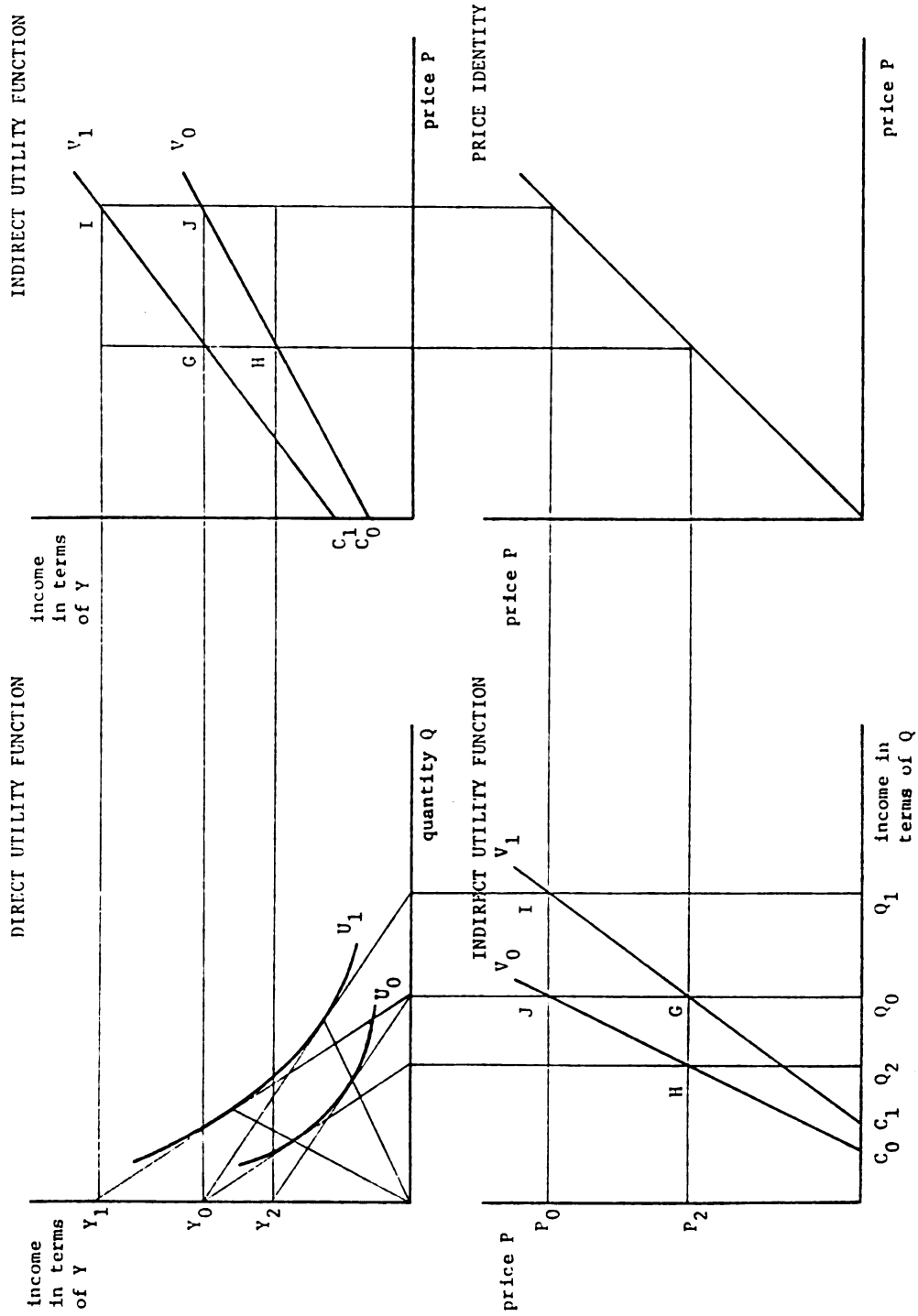


Figure 13. Homothetic Direct and Indirect Utility Functions

necessary to assure a unique welfare measure of economic rent measured indirectly with respect to the ordinary resource supply curve are very likely not to exist.

The implicit assumptions regarding the consumer's behavior are quite unrealistic in that all or nearly all additional increase in real income is spent on the commodity whose price decreased. We might well end our search for a measure of consumer surplus which is also a cardinal measure of welfare. A purist would end the search for any kind of measure of consumer surplus at this point. The pragmatist, however, would adjust the goal to that of finding a measure of consumer surplus which is a cardinal measure of compensation and continue the search. This is what we shall do. In the next section we explore the use of compensating variations as a compensation measure of consumer surplus.

## 2-10 Compensating Variation As A Cardinal Compensation Measure of Consumer Surplus

In the previous section we explored the circumstances under which the separability or path independence assumption is met. Path independence is met only for homothetic and vertically parallel utility functions. These types of utility functions are seldom found empirically since the former implies zero cross-price elasticities while the latter implies zero income elasticity. Therefore, it is important to understand the consequences of violating the separability assumption and its affect on the definition and interpretation of consumer surplus. In particular, we are now seeking a more modest goal of a cardinal compensation measure of consumer surplus.

In this case, we make no assumptions regarding the type of utility function other than the standard assumptions regarding continuity, convexity and differentiability. Let us begin by examining the explicit assumptions associated with use of compensating and equivalent variation as measures of consumer surplus. First, let us determine what these measures are not. We begin with the claim regarding the use of these measures made by Willig in his article "Consumer's Surplus Without Apology" (1976, pp. 589-597). Willig states:

> The purpose of this paper is to settle the controversy surrounding consumer's surplus and by so doing, to validate its use as a tool of welfare economics. I will show that observed consumer's surplus can be rigorously utilized to estimate the unobservable compensating and equivalent variations -- the correct theoretical measures of the welfare impact of changes in prices and income on an individual. (emphasis added) (1976, p. 589).

Willig then goes on to describe a technique for estimating compensating and equivalent variation. The implicit assumption is that the extent to which these measures are correctly estimated is the extent to which "the correct theoretical measures of welfare impact" are measured. Correct cardinal measures of welfare are implied. This assumption is inconsistent with the properties of separability and path independence. That is, a unique measure between two functions exists only when these functions are exact differentials of each other. This implies path independence which implies separability. These properties are only found in association with vertically parallel and homothetic utility functions. Willig is not considering the case of either the vertically parallel or homothetic utility functions. We know this is true because in the case of vertically parallel utility functions (a) compensation and equivalent variation

are equal, and (b) the indirect utility function approach is unnecessary since consumer surplus (or economic rent) can be measured accurately and simply as the integral of the ordinary demand (or supply) function over the relevant price range. Willig also cites Hicks: "In order that the Marshallian measure of consumer's surplus should be a good measure, one thing alone is needful - that the income effect should be small" (1976, p. 589, fn. 1). The homothetic utility function has an "income effect" by definition. Willig is clearly suggesting a wider application beyond these special case utility functions.

In the absence of either the vertically parallel or homothetic utility functions, an infinite number of "measures of welfare impact" can be described between two functions depending on the price adjustment path. Silberberg states:

> Clearly, an infinite number of consumer's surpluses can be generated, most of them uninteresting. Relative and absolute prices can be changed so as to hold $X_1$, fixed until U' is reached and then changed relatively so as to move the consumer along U' to X' . . . these paths generate values . . . which correspond to (but are not exactly equal to) Marshall's surpluses in which quantities are held constant. There is no way in general to relate these surpluses to other values . . . (1972, p. 945).

Perhaps the most insightful comment regarding the implicit assumption made by Willig comes from Chipman and Moore. In reference to the special case in which utility functions are vertically parallel such that the "Marshallian" and "Hickian" measures of consumer surplus are equal, they state:

> This is the case of Hicks zero "income effect" or equivalently, Mishan's zero "welfare effect." In Hicks words: "In order that the Marshall measure of consumer's surplus should be a good

> measure, one thing alone is needful -- that the income effect should be small." As we have seen, the same is true of the Hick measure. If the income effect is "small" so that preferences are approximately parallel, then of course a problem of approximation arises; but it is a problem not of the Marshall measure to the Hicks measure, but of both to a correct measure" (emphasis added). (Chipman and Moore, 1980, p. 38, fn. 17).

There is a "theoretically correct" cardinal welfare measure of consumer surplus and economic rent if and only if the utility functions are vertically parallel or homothetic. An accurate empirical estimate of compensating or equivalent variation associated with a utility function that is neither vertically parallel nor homothetic is not a "theoretically correct" cardinal measure of a welfare change associated with a change in the price of a good in the case of consumer surplus or a resource in the case of economic rent. There seems to be substantial confusion on this point. Willig implies that he has developed a "theoretically correct" cardinal welfare measure of consumer surplus when, in fact, he has developed an "empirically correct" cardinal compensation measure of consumer surplus. See Figure 14. This general case which relaxes the restrictions on the type of utility function results in "path dependent" measures of the change in real income. Unlike the vertically parallel utility case, there are an infinite number of measures of the vertical distance between $U_0$ and $U_1$ or $V_0$ and $V_1$ relative to a particular numeraire good. Unlike the homothetic utility case, the measures of compensating and equivalent variation are not constant regardless of the good selected as the numeraire.

Any claim under these circumstances as to which measure is "theoretically correct" is necessarily "ambiguous and normative"

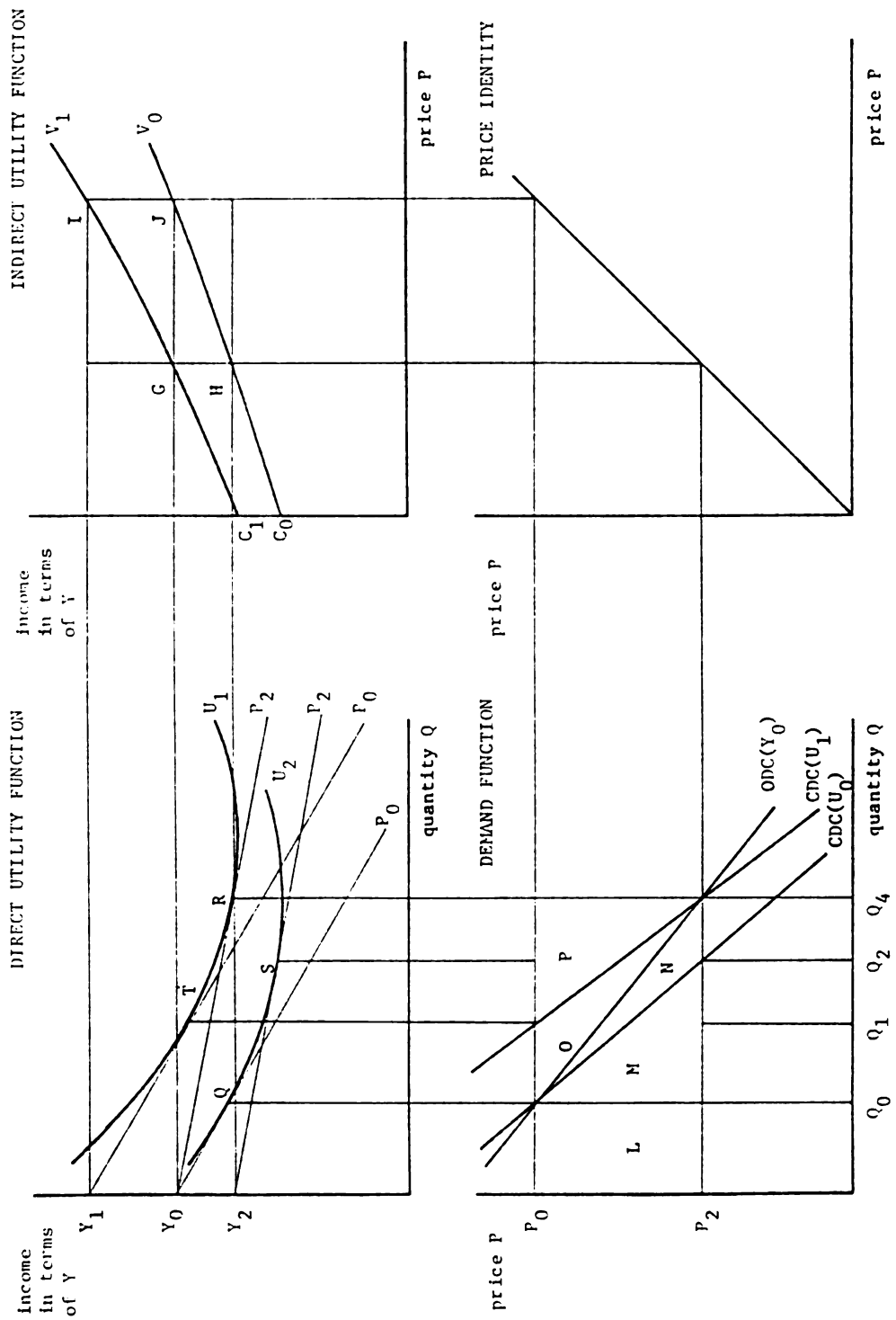


Figure 14. Generalized Direct and Indirect Utility Functions

(Silberberg, 1972, p. 942). Silberberg uses the following example to summarize the economic interpretation associated with the many measures of consumer surplus and economic rent when resource owners behave as if their utility functions are neither homothetic nor vertically parallel.

> Suppose a dam is constructed which lowers the marginal cost of electricity, local recreation and irrigation. In evaluating what benefit consumers would place on this project, some assumption must be made about the price adjustment path. Curiously paradoxical is the fact that a consumer will impute a different shadow price to the project if first the price of electricity is lowered, then that of irrigation and then of recreation, than if prices were lowered in some other sequence; yet the nagging truth remains that nothing can be done to correct such an inconsistency. It is therefore time, at least, for economists to abandon the term "welfare loss" [and welfare gain].. . . If the term were merely ambiguous, the analysis could be narrowed and at some cost the term could be meaningfully applied. However, the phrase simply has no meaning at all. (1972, p. 942).

Silberberg's comment is ironic in the sense that he is reiterating the comment Samuelson made in the 1940s (1942, pp. 75-91). Both Silberberg and Samuelson focus on the constancy of the marginal utility of money as a common denominator for a cardinal welfare measure. Samuelson also included the Chipman and Moore argument regarding the special types of utility functions--homothetic and vertically parallel--required to determine a unique welfare measure for consumer surplus and economic rent. Samuelson also recognized the relationship between a unique welfare measure and the mathematics of line integration (1947, p. 202). The major difference between Samuelson's earlier statements regarding a unique welfare measure and the later statements by Silberberg, Chipman and Moore is in the emphasis placed on the path independence properties of the line integral.

Samuelson underplayed the importance of the latter in the extreme. "In general, line integrals will replace simple integrals with the path of integration of the former a matter of no consequence" (Samuelson, 1947, p. 202). Silberberg, Chipman and Moore regard the path of integration of the line integral as all important. "This path-dependency is the raison d'etre of the many consumer surpluses" (Silberberg, 1972, p. 944). And "we shall say that a continuously differentiable (N + 1) vector-valued function v(p,m) furnishes an acceptable integral measure of welfare change if and only if . . . the line integral is path-independent . . ." (Chipman and Moore, 1980, p. 934). Thus the special cases when consumer surplus is a unique cardinal measure of welfare are assocated with very restrictive assumptions regarding consumer behavior and utility functions. When these restrictive assumptions are relaxed, then a unique cardinal measure of welfare does not exist. So the question remains 'is there anything an economist can say regarding the welfare effects of a price change in terms of economic rent or consumer surplus?'

A more modest goal is within our reach. Compensating variation has an interpretation that does not involve any claims of being "the correct theoretical measure of welfare impact." Compensating variation is defined as the change in income resulting from a change in price in relation to the initial level of utility $U_0$. See Figure 15. It is subject to the same interpretation as the "all or none" and compensated demand curve relative to an initial utility function. The compensating variation measure of consumer surplus can be understood as the additional income a consumer would receive under conditions of

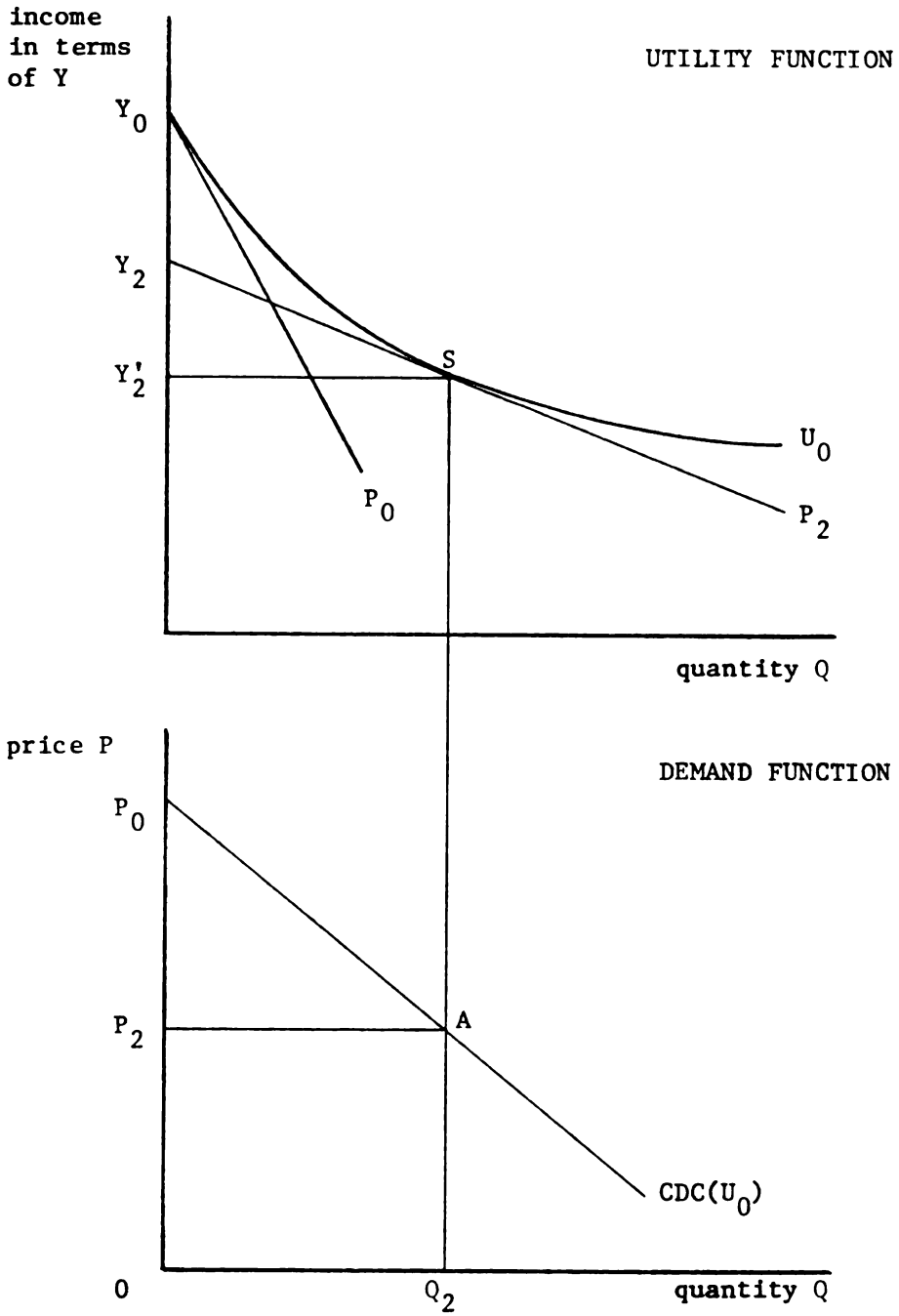


Figure 15. Compensated Demand Function

competitive trade above that under conditions of monopoly trade. See Figure 16. This is a compensation rather than welfare interpretation. Compensating variation is simply one of an infinite set of possible candidates for measuring consumer surplus. It has several characteristics besides the above interpretation which makes it a desirable choice as a standard unit of measure in "welfare" economics. Compensating variation cannot be construed as a cardinal welfare measure since no shift in utility is implied in the analysis. Compensating variation has the common denominator of the initial utility function. Compensating variation is well known, understood and widely discussed in the welfare literature. And, as with any other measure, as the income or welfare effect decreases, compensating variation is equal to a cardinal measure of welfare in those rare instances when one may exist. Finally, and again as with any other consistently selected measure, compensating variation can be used to measure compensation and thereby determine whether consumers and resource owners are better off with a particular price change for a particular good and/or resource than without it. Silberberg states:

> The following alternative . . . is within the realm of positive economics: statistically estimated ordinary demand curves, [and ordinary resource supply curves] . . . can be used to obtain approximations to the compensating variations in income. When a public-policy decision implies that some will be net losers, the compensating variations give well defined measures of how much money income would have to be transferred to those individuals to restore them to [an initial] just-as-preferred position. Similarly, estimates can be derived as to the amounts which could be extracted from the net gainers and still leave them at [an initial] just as-preferred position. Data of this sort can be provided, however, this is all that the economist, qua economist can provide. (1972, p. 951).

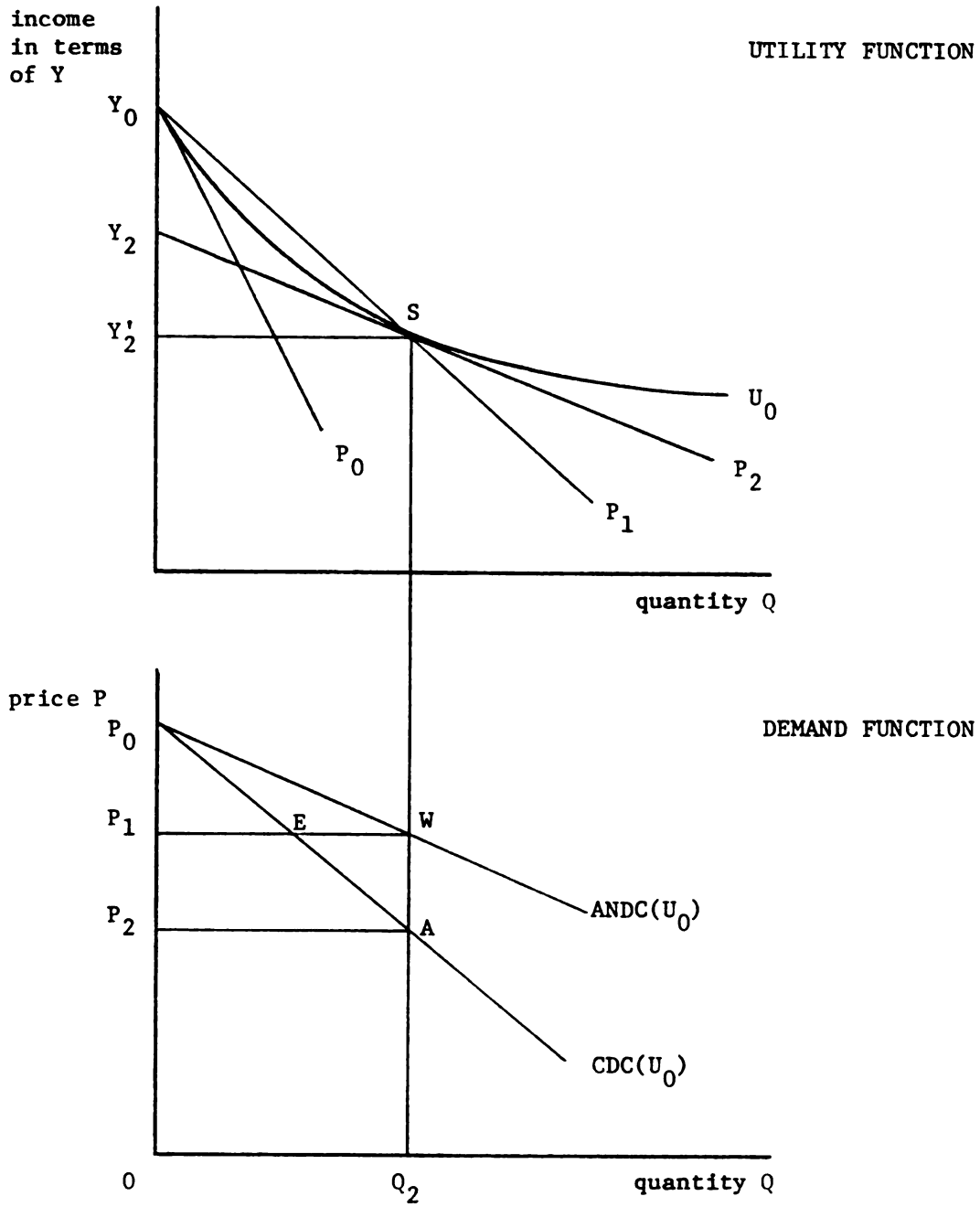


Figure 16. Compensated and "All or None" Demand Curves

The absence of a cardinal measure of welfare precludes an economist as an economist from making what Silberberg regards as political value judgments based on a change in "net social welfare" (1972, p. 951). For better or worse, a money measure of individual or group benefits does not, as a general case, directly correspond to a utility based social welfare measure. We are left with a cardinal measure of compensation rather than a cardinal measure of welfare. A cardinal measure of compensation determines the amount of money income consumers and resource owners are able (if not willing) to give up subsequently and still be as well off as initially. Meanwhile, the search goes on for a more general rationale for using the special case utility functions in order to derive the more powerful welfare measure (Houthakker, 1960 pp. 244-257; Samuelson, 1965, pp. 781-801; Gorman, 1961, pp. 53-56; Chipman and Moore, 1980, pp. 1-46).

# CHAPTER III

# A THEORY OF ECONOMIC RENT

## 3-1 Introduction

The similarities between economic rent and consumer surplus are very close. In fact, Mishan concludes that

> ... consumer's surplus and economic rent are both measures of the change in the individual's welfare when the set of prices facing him are changed... Any distinction between them is one of convenience only: consumer's surpluses have reference to demand prices, economic rent to supply prices (Mishan, 1959, p. 394).

Consequently, it can be shown that economic rent can also be defined as compensating variation. Compensating variation is the change in income associated with a change in resource price relative to an initial level of utility. The utility function can also be used to define a take it or leave it supply function. The difference between compensated and "take it or leave it" supply functions provides an interpretation of economic rent. Economic rent can be interpreted as the additional income a resource owner would receive for a given quantity of a resource under conditions of competitive trade above that which would be received under conditions of monopsony trade. Thus the utility function is important in both defining and interpreting economic rent.

Consumer surplus can be measured directly in the goods market given a compensated demand function or indirectly in the resource market. Conversely economic rent can be measured directly in the

resource market given a compensated supply function or indirectly in the goods market. The indirect measure of economic rent is "... derived with reference to the production function and seemingly without reference to people's preference function" (Mishan, 1968, p. 1275). The definition, interpretation and direct measure of economic rent is associated with the utility function while the indirect measure is associated with the production function and the industry supply function.

The causes of economic rent are related to the availability of the resource required in a production process. Resources can be categorized by the extent of availability such as:

(1) abundant
(2) absolutely scarce
(3) super-abundant
(4) relatively scarce

The lack of availability (or inelastic supply) of a resource results in a situation which allows for a claim on economic surplus by the resource owners. "The payment in excess of that necessary to maintain a resource in its current occupation" is the result. This payment is called economic rent (Mishan, 1959, p. 386). The problems of availability of a resource are compounded when the resource is a "lumpy" durable. Durable resources are often have an acquisition price which is greater than salvage value adjusted for depreciation and maintenance. These characteristics of a resource affect the elasticity of the input supply function faced by firms within a particular industry. The elasticity of resource supply affects the industry's output supply elasticity as well. This is the linkage between the cause of economic

rent and its direct measure in the resource market and indirect measure in the goods market.

The objectives of this chapter are the following. First, in sections 3.2 through 3.6, the symmetry between consumer surplus and economic rent is developed. This is done as a means to define, interpret and provide a direct measure of economic rent. This section builds upon the work of Mishan (1959) and Patinkin (1963). Second, in sections 3.7 through 3.12, the relationship between resource availability, long run industry supply curves and the indirect measure of economic rent is presented. The direct and indirect measures of economic rent are analyzed in terms of the "Mishan Criteria." This section builds upon the work by Mishan (1968) and Viner (1931). Finally in sections 3.13 through 3.14, we will review the recent developments in the literature regarding measuring economic rent and net benefits given a shift in the supply curve associated with a new technology. The ordinary and equilibrium supply curves as well as the particular expense curve are examined as alternative techniques for accurately estimating the slope and intercept of an industry's marginal cost curve exclusive of rent. This final section builds on the works by Marshall, Viner, Lindner, Jarrett, Rose, Wise, Fell, Just, Hueth, and Schmitz.

## 3-2 The Direct Utility Function, the Expenditure Function and the Compensated Supply Curve

Let us assume that a competitive economy contains only two goods, Y and R. Assume that Y is the numeraire good in the economy. Therefore the price of Y is always equal to one. The price of R and

income are measured in terms of Y. In fact, Y may be thought of as N-1 goods in the economy. Let us also assume that the consumer in this economy has an initial resource endowment of R only.

The owner of resource R faces the decision whether to trade R for Y and if so how much. Let us assume this decision is made as if on the basis of a continuous, downwardly concave, twice differentiable and ordinal direct utility function (Malinvaud, 1972, pp. 24-26). At each point on the direct utility function, the resource owner is indifferent with respect to consuming various combination of Y and R. Therefore the direct utility is a function of goods Y and R.

(1) $U_o = f(Y,R)$.

The initial resource endowment is equal to $Y_o$. $Y_o$ is the initial income of the resource owner which is equal to resource R forgone multiplied by its price P.

(2) $Y_o = -PR$.

Equation (2) is significantly different from the usual assumption regarding the expenditure function. Instead of assuming an exogenously determined nominal income budget constraint, here we are assuming an endogenously determined budget constraint.

> ...we require our individual in possession of given resources, or assets, to maximize such a function subject to $\sum P_r X_r = 0$. At least one of the X's is negative in order to indicate a quantity supplied per period by the individual of a good or service and of course, at least one of the X's is positive to indicate a quantity demanded per period of a good or service. The suggested constraint expresses nothing more than the proposition that, in all circumstances, the current earnings are equal to the current value of his expenditures. (Mishan, 1959, pp. 386-387).

Mishan's definition of the expenditure function is equally valid for the decision to either utility maximize or expenditure minimize.

Mishan's definition of the expenditure function also implies that each consumer is also a resource owner of at least one input required in the economy. The distinction between consumer and resource owner in this context is one of convenience only without a real difference. In this case, R is defined as "...one of the X's [which] is negative in order to indicate a quantity supplied per period by an individual of a good or service...." Y, on the other hand, is defined in this case as "...one of the X's [which] is positive to indicate a quantity demanded per period of a good or service...."

The decision to trade R for Y is made on the basis of the expenditure function. The expenditure function is the relationship between the price of R, income measured as Y and a given attainable level of utility. Let us assume that the decision rule of the resource owner is to minimize the expenditure of income measured as Y in attaining a given level of utility μ (Diamond and McFadden, 1974, p. 3).

(3) $e(P,U) =$ minimize Y subject to $f(Y,R) \geq U_1$.

The expenditure function is the least amount of income needed to attain a given level of utility for a given set of prices. The Lagrangian function for ependiture minimization is formed by substituting equations (1) and (2) into (3).

(4) $e(P_t,U_o) = Y + P_tR + \lambda[U_o - f(Y,R)]$

$t =$ time periods 1 to N.

Equation (4) is the expenditure function to minimize in order to determine the combination of Y and R which maintains the consumer at utility $U_o$ for a given price P at time t. The solution to this function is: If $Y = f(x)$ is the quantity to be maximized or minimized,

find the values of x for which $\partial Y/dx = f'(x) = 0$ (Thomas, 1960, p. 130). Since equation (4) is the function of three variables Y, R and $\lambda$, then the partial derivatives are set equal to zero.

$$(5) \quad \frac{\partial e(Y,R)}{\partial Y} = 1 - \frac{\lambda \partial f(Y,R)}{\partial Y} = 0,$$

$$(6) \quad \frac{\partial e(Y,R)}{\partial R} = P_t - \frac{\lambda \partial f(Y,R)}{\partial R} = 0,$$

$$(7) \quad \frac{\partial e(Y,R)}{\partial \lambda} = U_o - f(Y,R) = 0.$$

Then dividing (6) by (5) such that

$$(8) \quad \frac{\partial f(Y,R)/\partial R}{\partial f(Y,R)/\partial Y} = P_t.$$

Equation (8) is the solution for the resource owner as whether and how much R to trade for Y.

Equation (8) states that the resource owner should trade R for Y until the marginal utility of R to Y equals the price of R (Henderson and Quandt, 1971, p. 15). The tangent of the utility function to the expenditure function with the slope equal to $P_t$ is the only point that meets this condition. The assumptions regarding the nature of the utility function assures the second order conditions are positive (Malinvaud, 1972, p. 37). This is a sufficient condition to assure a minimum expenditure is reached.

This result can be shown graphically. See Figure 17. If the price of resource R equals $P_o$ then the resource owners income $Y_o$ is made up entirely of resource R. This happens only when the resource price is zero. At price $P_o$ (=0) the resource owner will not and cannot trade any R for Y. However even at price $P_o$ (=0), the resource

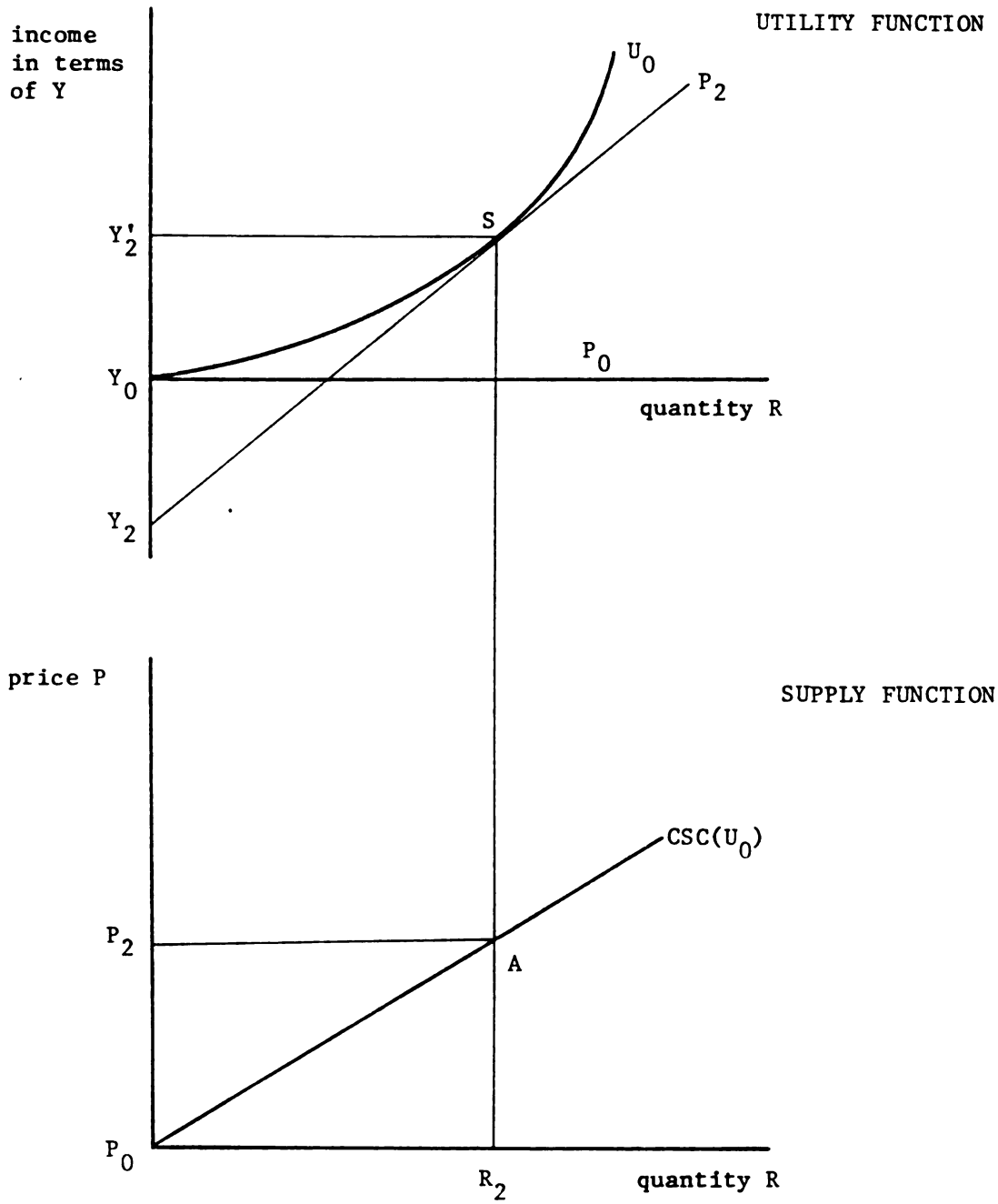


Figure 17. Compensated Supply Curve

owner is expenditure minimizing subject to a utility constraint since Po is tangent to $U_o$ at point $Y_o$ (=0).

If the price of resource R increases from $P_0$ to $P_2$, then $Y_2$ of good Y is purchased and $R_2$ of resource R is supplied by the resource owner. Notice that with the increase in the resource price from $P_0$ to $P_2$, the resource owner's minimum expenditure to maintain utility $U_0$ decreases from $Y_0$ to $Y_2$. The difference between $Y_0$ and $Y_2$ is a measure of the increase in income resulting from an increase in the price of R in relation to a given level of utility $U_0$. This measure of the change in income can be measured directly or indirectly relative to a given utility function. (The indirect measure associated with the industry output supply function is discussed later.) The direct measure of the change in income to the resource owner relative to a given utility function is to subtract the subsequent income $Y_2$ from the initial income $Y_0$. The direct measure of the change in income to the resource owner relative to a given utility function is the integral of the first derivative of the expenditure function (i.e. the compensated supply function) with respect to R over the range of prices from $P_0$ to $P_2$.

The first derivative of the expenditure function with respect to R is called the compensated supply function. Solving equation (6) for $\partial R$, we see that R is a function of $P_t$ and $U_o$. The compensated supply function is defined in this case as the inverse of the first derivative of the expenditure minimization function subject to a utility constraint with respect to R. The compensated supply function for resource R is equal to (9) $R = \psi(P_t, U_o)$.

The compensated resource supply function is the relationship between own price and a given level of utility. This relationship precludes a welfare effect in which the resource owner prefers to consume the resource itself as the resource price increases and less is needed to trade for other goods. The compensated supply function (CSC) is the schedule of price-quantity relationship of foregone resources endowment which maintains a resource owner at a given utility level. The intergral of the compensated supply function from $P_0$ to $P_2$ equals area $P_0P_2A$. This area is equal to the difference between expenditure functions $Y_0$ and $Y_2$ in relation to utility level $U_o$. Thus

(10) $Y_0 - Y_2 = P_0P_2A.$

Intuitively, the reason why the area above the compensated resource supply function is equal to the expenditure function is as follows. The definition of a derivative is the "instantaneous rate of change in one variable per unit change in the other" (Thomas, 1960, p. 37). Intergration on the other hand is "to indicate the whole of; to give the sum total of" (Thomas, 1960, p. 155). Thus the sum of the rate of change in one variable to another is equal to the total change in one variable to another. In this case, the integral of the compensated supply function is equal to the sum of the total change in income to the total change in price. This is exactly what economic rent attempts to measure. Thus economic rent is defined as the change in the resource owner's income associated with a change in price of the initial resource endowment for a given level of utility.

## 3-3 The "Take It or Leave It" Supply Function

In the previous section the concepts of the utility function, expenditure function and compensated supply function were introduced and described. Also the direct measures relative to a given utility function in utility and demand space were described and illustrated. In this section, the concept of a "take it or leave it" supply function is introduced and used to interpret the meaning of economic rent.

Unlike the compensated supply function, the "take it or leave it" supply function does not assume that resource price is determined as a result of competitive trade. In fact, the "take it or leave it" supply function is based on the assumption that resource R is sold to a monopsonist. The monopsonist pays an average price for a given quantity of the resource such that the resource owner is indifferent between selling each unit at this average price and selling no units at all. Patinkin has shown that the monopsonists' minimum price for a given quantity of a resource is an average price per unit equal to a price vector from the zero quantity to a point associated with the desired quantity on a given resource owners' utility function (1963, p. 86). Even though Patinkin's analysis deals with consumer surplus, his interpretation of the "all or none" demand curve associated with a monopolist is directly analogous to the "take it or leave it" supply curve associated with a monopsonist presented here in relation to economic rent.

Let us derive the take it or leave it resource supply curve. See Figure 18. At resource price $P_0$ (=0), the expenditure for good Y

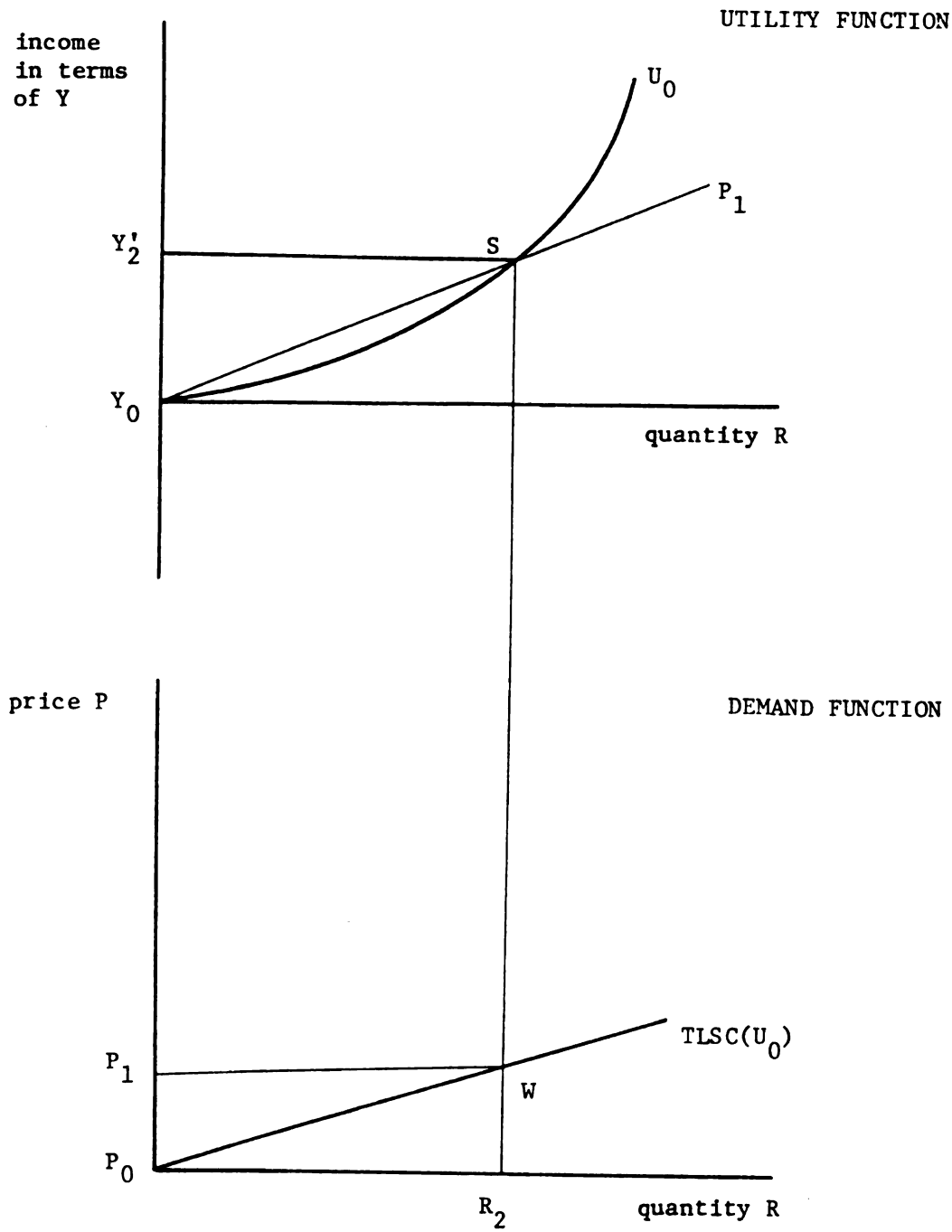


Figure 18. "Take It or Leave It" Supply Curve

equals zero and total income equal $Y_0$ (=0).

(1) $Y_0 = P_0R_0 = 0$

Equation (1) implies that the resource owner is unwilling and unable to sell any $R_0$ at price $P_0$. The resource owner must "consume" his entire initial resource endowment in order to remain at utility level $U_0$. For the resource owner to forgo $R_2$ units of the resource, the monopsonist must pay at least price $P_1$ per unit. If the resource price increases from $P_0$ to $P_1$, then the resource owner is indifferent between consuming $(Y_2', -R_2)$ bundle of goods and "consuming" the entire initial resource endowment$(0, R_o)$. In either case, the resource owner's income remains at $Y_0$. If the resource owner trades $R_2$ for $Y_2'$, then the resource owner's total income $Y_0$ can be stated in terms of $P_1$, $R_2$ and $Y_2'$.

(2) $Y_0 = -P_1R_2 + Y_2'$

$P_1R_2$ is the minimum payment the resource owner will accept to forego $R_2$ units of the initial resource endowment under conditions of monopsony trade. This is true because $P_1$ is the minimum resource price at which the resource owner is indifferent between consumption bundles $(Y_2', -R_2)$ and $(0, R_o)$. If the monopsonist lowers the per unit price for $R_2$ by even one penny, then the resource owner will prefer not trading any resource R at all. Therefore, if $R_2$ units of the resource are to be traded by the resource owner to the monopsonist, then price $P_1$ is the lowest per unit price the resource owner will "take" rather than "leave" the market altogether. A similar minimum per unit price

can be determined for each quantity of R in relation to utility function $U_0$ until the entire "take it or leave it" supply curve (TLSC) is mapped out. Area $P_1P_0R_2W$ is the direct measure in demand space of the minimum payment the resource owner will accept to forego $R_2$ units of initial resource endowment under conditions of monopsony trade.

Let us compare the monopsony with the competitive trade case. See Figure 19. Assuming condition of competitive trade and expenditure minimization subject to a utility constraint, if the resource price increases from $P_0$ to $P_2$, then $R_2$ is traded for $Y_2'$ and the minimum income required to remain on utility function $U_0$ decreases from $Y_0$ to $Y_2$. The resource owner's minimum income $Y_2$ can be stated in terms of $P_2$, $R_2$ and $Y_2'$.

(3) $Y_2 = -P_2R_2 + Y_2'$

$P_2R_2$ is the minimum payment the resource owner will accept to forego $R_2$ units of the initial resource endowment under conditions of competitive trade. This is true because $P_1$ is the price at which the ratio of the marginal utility of $(R_0 - R_2)$ and the marginal utility of $Y_2'$ are equal.

We can compare the effect of monopsony and competitive pricing on a resource owner's decision to trade some of the initial resource endowment relative to an assumed initial utility function $U_0$. Subtracting (3) from (2).

(4) $Y_0 - Y_2 = P_2R_2 - P_1R_2$

From Figure 19 we see that

(5) $P_2R_2$ = area $P_2P_0R_2A$

and

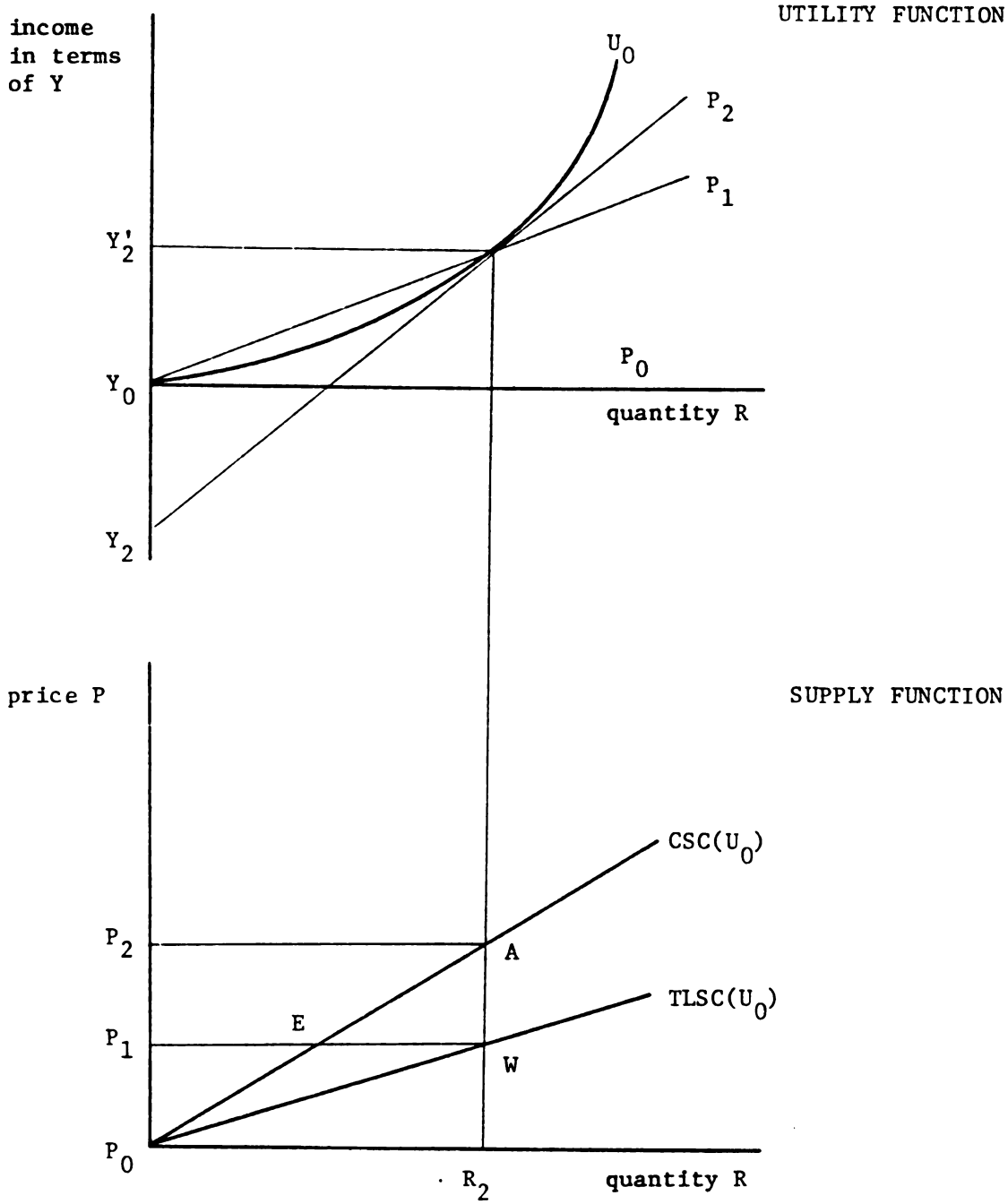


Figure 19. Compensated and "Take It or Leave It" Supply Curves

(6) $P_1R_2$ = area $P_1P_0R_2W$

Substituting (5) and (6) into (4) we see

(7) $Y_0 - Y_2$ = area $P_2P_0R_2A$ - area $P_1P_0R_2W$

Simplifying (7) we see

(8) $Y_0 - Y_2$ = area $P_2P_1WA$

Area $P_2P_1WA$ is the additional income the resource owner would receive for $R_2$ units of resource R under conditions of competitive trade above the minimum payment the resource owner will accept to forego $R_2$ under conditions of monopsony trade.

In the previous section we established the fact that $Y_0 - Y_2$ is the direct measure of economic rent associated with utility function $U_0$. Further we established that

(9) $Y_0 - Y_2$ = area $P_2P_0A$

Thus we can say that

(10) $Y_0 - Y_2$ = area $P_2P_0A$ = area $P_2P_1WA$

Therefore economic rent can be interpreted as area $P_2P_1WA$ or the additional income a resource owner would earn for a given amount of a resource under conditions of competitive trade above that which the resource owner would accept for these units of resource under conditions of monopsony trade. This difference in income can be extracted from the resource owner in one of two ways. This income can be extracted by the monopsonist by paying a perfectly discriminating price for each unit of the resource. This is comparable to extracting economic rent equal to area $P_2P_0A$. Alternatively the monopsonist can extract the same total rent from the resource owner(s) by paying the

minimum average price per units necessary to elicit the needed quantity of the resource. This is comparable to extracting economic rent equal to area $P_2P_1WA$.

Economic rent can also be extracted from resource owners by the state by means of taxation. If an ad volorem tax of $P_2 - P_1$ is added to the price of $P_1$ for each unit of $R_2$, then the total tax receipts paid by resource owners would equal area $P_2P_1WA$ (= area $P_2P_0A$). This is a policy alternative to publicly finance research and development which may be used to lower the price of yet other goods in the economy. Similarly, this argument can be made with respect to monopsonists who can afford to invest in research and development only if allowed to price discriminate.

It remains to be established under what conditions does equation (1) equal compensated variation. We turn to this question in the next section.

## 3-4 Utility Maximization Function and the Ordinary Supply Function

The previous sections have discussed measures of economic rent purely in terms of a given utility function and associated compensated supply function. The presentation of the "take it or leave it" supply function was developed not as a measure of economic rent but rather to provide additional theoretical insight into the interpretation of the area above the compensated supply function. Unfortunately, the compensated supply function may not be directly observable. It is the derivative of the expenditure function which is minimized subject to a utility constraint. A much more likely functional form is one which maximizes utility subject to an income constraint. Thus the utility

income constraint. Thus the utility maximization assumption simply reverses that which is maximized with that which is constrained. The functional form for expenditure minimization is

(1) $e(P,U) = \text{minimize } Y$

$$\text{subject to } U(Y,R) \geq U;$$

The functional form for utility maximization is

(2) $e(P,U)$ maximize $\mu\ (P,R)$ (where $U(P,R) = f(Y,R)$)

$$\text{subject to } Y \leq Y_0 \text{ (where } Y + P_t R = 0)$$

(Diamond and McFadden, 1974, p. 3).

Utility maximization is the relationship between the highest level of utility attainable for a given resource price and the initial resource endowment. Equation (2) can be rewritten as a Lagrangian function.

(3) $U(P,R) = f(Y,R) + \lambda(Y + P_t R)$

Equation (3) is the utility maximization function used to determine the highest utility level U, the attainment of which is not to exceed the resource owners total income ($= - P_t R$). Total income Y is determined by the resource price $P_t$ and initial resource endowment R.

To answer the questions of whether and how much R to trade for Y in order to maximize utility, the partial derivatives of equation (3) with respect to variables (Y, R and $\lambda$) are set equal to zero (Thomas, 1960, p. 130).

(4) $$\frac{\partial U(P,R)}{\partial Y} = \frac{\partial f(Y,R)}{\partial Y} + \lambda = 0$$

(5) $$\frac{\partial U(P,R)}{\partial R} = \frac{\partial f(Y,R)}{\partial R} + \lambda P_t = 0$$

(6) $$\frac{\partial U(P,R)}{\partial \lambda} = Y + P_t R = 0$$

Dividing (5) by (4)

$$(7)\quad \frac{\partial f(Y,R)/\partial R}{\partial f(Y,R)/\partial Y} = P_t$$

Equation (7) is the answer to the questions whether to and how much R to trade for Y in order to maximize utility. In fact it is the same answer as derived from the expenditure minimization functional form. That is the resource owner should trade R for Y until the ratio of the marginal utility for resource R to the marginal utility for good Y equals the price of R (Henderson and Quandt, 1971, p. 15). The tangent between the budget constraint with slope equal to $P_t$ and the utility function is the only point which meets this condition. This is a necessary condition to assure a maximum level of utility. This is the first order condition for a maximum.

The second order conditions are sufficient to assure a maximum. We are interested in determining the sufficient conditions which assure utility maximization given a change in the price of resource R,"If $d^2Y/dx^2$ is negative when $dy/dx = 0$, Y is a maximum" (Thomas, 1960, p. 130). Therefore we must total differentiate equations (4), (5), and (6) and then solve them simultaneously for R.

$$(8)\quad \begin{aligned} f_{11}\,dy + f_{12}\,dR + d\lambda &= 0 \\ f_{21}\,dy + f_{22}\,dR + P\,d\lambda &= \lambda\,dP \\ dy + P\,dR &= R\,dP \end{aligned}$$

Equation (8) is the matrix of second derivative with respect to variables Y, R, and $\lambda$. The determinate of the matrix of equation (8) is

$$(9)\quad D = \begin{vmatrix} f_{11} & f_{12} & 1 \\ f_{21} & f_{22} & P \\ 1 & P & 0 \end{vmatrix}$$

Using determinate (9) of the system of equation (8), it is possible to solve the system for one or more of the unknown values. In this case of utility maximization, the unknown values are dy, dR and $d\lambda$ (Henderson and Quandt, 1971, p. 31). Solving for dR gives the sufficient conditions for utility maximization relative to changes associated with resource R.

$$(10)\quad dR = \frac{\lambda D_{22}\, dP + RD_{32}\, dP}{D} < 0$$

Equation (10) is the second order condition for utility maximization given a change in R.

Dividing both sides by $\partial P$ holding the price of y constant, then

$$(11)\quad \frac{\partial R}{\partial P} = \frac{\lambda D_{22}}{D} + \frac{RD_{32}}{D} < 0 \text{ for a maximum}$$

Equation (11) is the second order condition for utility maximization given a change in R for a change in P. Henderson and Quandt show that

$$(12)\quad \frac{\lambda D_{22}}{D} = \frac{\partial R}{\partial P} \text{ given constant utility}$$

and

$$(13)\quad \frac{D_{32}}{D} = -\frac{\partial R}{\partial Y} \text{ given constant price of Y}$$

(1971, p. 32).

Substituting (12) and (13) into (11)

$$(14)\quad \frac{dR}{dP} = \frac{\partial R}{\partial P}\Bigg|_{\substack{\text{constant}\\ \text{utility}}} - R\,\frac{\partial R}{\partial Y}\Bigg|_{\substack{\text{constant}\\ \text{prices}}} < 0$$

Equation (14) is called the Slutsky equation (Burtless and Hansman, 1978, p. 1112). The expression $\partial R/\partial P$ on the left is the slope of the

ordinary supply function with respect to resources R. The ordinary supply function is the first derivative of the utility maximation function. Equation (14) shows that the slope of the ordinary supply function is the composite of the slope of the compensated supply function ($\partial R/\partial P$ given constant utility) and a "welfare effect" ($R\partial R/\partial Y$ given constant price of Y). It is this latter influence on the slope of the ordinary supply function that makes the intergral of the ordinary supply function a less than an ideal measure of the change in income associated with a change in resource price.

It is possible to determine whether an ordinary supply function is the result of utility maximizing decision making on the part of the resource owners (or as if they did) by using equation (14). It can be rewritten so that the measure of the second order conditions are observable.

$$(15)\quad \left.\frac{\partial R}{\partial P}\right|_{\substack{\text{constant}\\ \text{utility}}} = \frac{\partial R}{\partial P} + \left.\frac{R\partial R}{\partial Y}\right|_{\substack{\text{constant}\\ \text{prices}}} < 0$$

The expression $\partial R/\partial P$ (on the right) is the slope of the ordinary resource supply curve and is observable. R is the quantity of the resource foregone and is observable. $\partial R/\partial Y$ given constant prices is the welfare effect associated with the ordinary supply curve and it too is observable (Burtless and Hausman, 1978, p. 1123). The sign of the welfare effect may be positive, negative or zero (Mishan, 1959, p. 387). If the sum of these measures is negative the second order conditions for utility maximization are met.

A graphical presentation of utility maximization is present in Figure 20. Initially, the resource price equals $P_1$ and $R_1$ is traded. Initial income is equal to $Y_1$. When the price of the resource increases from $P_1$ to $P_2$, then $R_2$ is traded and nominal income remains at $Y_1$. Now instead of minimizing expenditures, nominal expenditures remains constant and the level of utility increases from $U_0$ to $U_1$. The theory of economic rent is defined as the measure of the change in real income associated with a change in the price for resource with respect to a given level of utility. We see in this case of utility maximization that the level of utility is not fixed at all. In this case only nominal income $Y_1$ is fixed.

The ordinary supply function associated with utility maximization is the relationship between a resource and its price and the level of initial endowment of R normalized in terms of Y (Burtless and Hausman, 1978, p. 1108).

(16) $R = \theta(P_t, Y)$

The ordinary supply curve (OSC) is the schedule of price-quantity relationships which maintain a resource owner at the highest level of utility for a given initial resource endowment. The reason why this situation is particularly frustrating is because the ordinary supply function $S_1$ associated with utility maximization is observable. Therefore it is important to examine the relationship between the ordinary supply function and the compensated supply function.

Now let us examine the relationship that exists between the decision rules of expenditure minimization and utility maximization. In

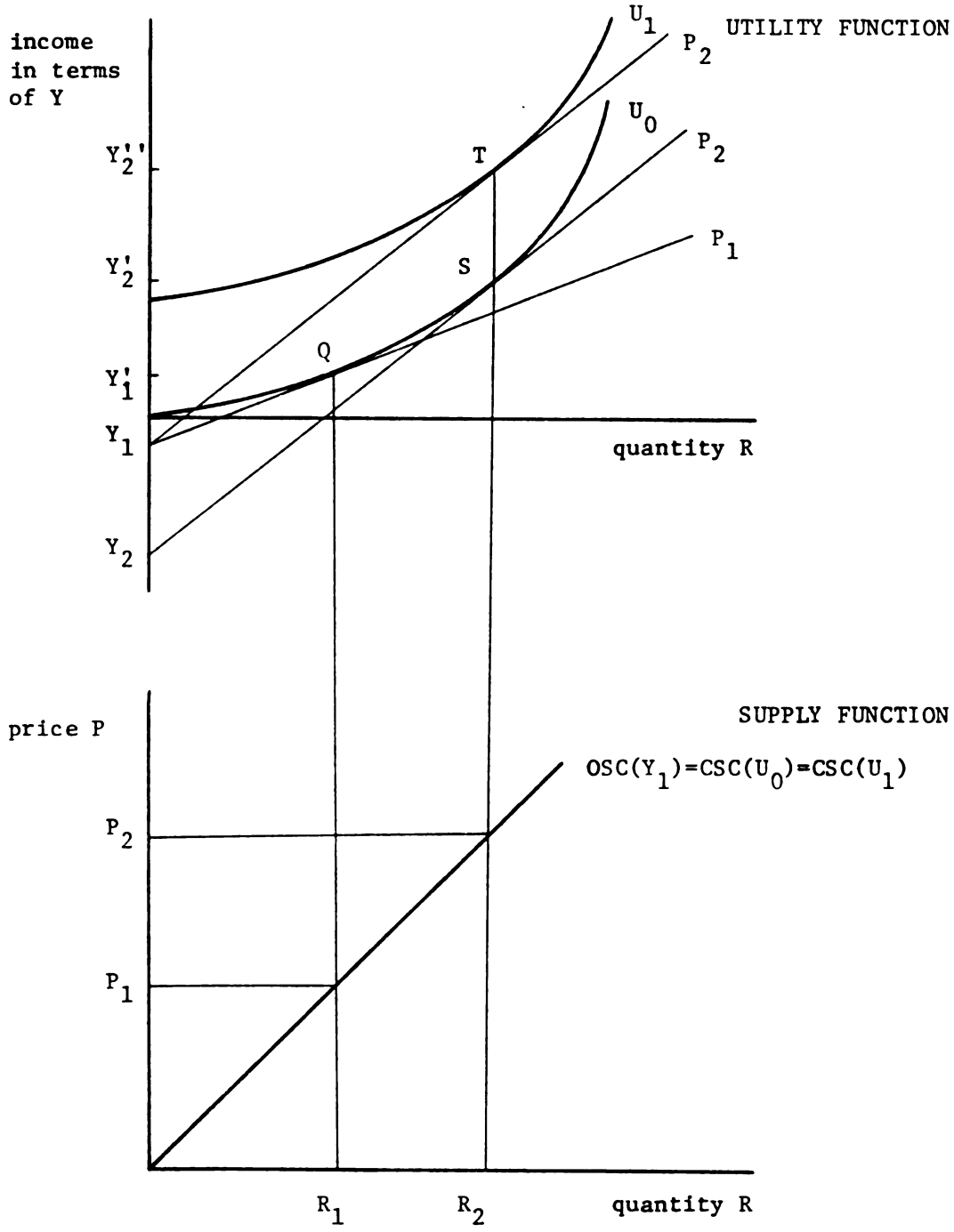


Figure 20. A Vertically Parallel Utility Function and the Compensated Supply Curve

particular, we are interested in the effect these decision rules have on their respective supply curves. See Figure 20. When the price of resource R increases from $P_1$ to $P_2$, then the expenditure minimizing resource owner trades $R_2$ units of R for $Y_2'$ units of Y. Nominal income for the expenditure minimizing resource owner can be decreased from $Y_1$ to $Y_2$ without being made worse off.

When the price of resource R increases from $P_1$ to $P_2$, the utility maximizing resource owner trades $R_2$ units of R for $Y_2$ units of Y. Nominal income for the utility maximizing resource owner remains the same at $Y_1$. However, real income for this individual has increased because initial nominal income $Y_1$ was associated with the low resource price $P_1$ and subsequently the same nominal income $Y_1$ is associated with the higher resource price $P_2$. When the price of resource R increases from $P_1$ to $P_2$, the utility maximizing resource owner might first act like an expenditure minimizing resource owner by trading more of the relatively more expensive R for the relatively less expensive Y. Thus the utility maximizing resource owner moves from point Q to point S on initial utility function $U_0$. This is called the substitution effect with respect a constant level of utility. This is also a component of the Slutski equation defined as $\partial R/\partial P$ given constant utility. The utility maximizing resource owner realizes a change in real income equal to $Y_1 - Y_2$ as does the expenditure minimizing resource owner.

The second step taken by the utility maximizing resource owner is unlike anything an expenditure minimizing resource owner would do.

The utility maximizing individual uses the change in real income ($Y_1 - Y_2$) resulting from the resource price increase ($P_1$ to $P_2$) to purchase additional units ($Y_2'' - Y_2'$) of good Y. The second step moves the utility maximzing resource owner from point S to point T and from $U_1$ to $U_2$. The vertical distance between $Y_2''$ and $Y_2'$ is equal to the vertical distance between $Y_1$ and $Y_2$. The horizontal distance between points S and T measured in terms of resource R is in fact a measure of the welfare effect component of the Slutski equation defined as $\partial R/\partial Y$ given constant prices in this case $P_2$. For the vertically parallel utility function, the horizontal distance and the welfare effect is zero.

The welfare effect can be positive, negative or zero. As long as the utility maximizing resource owner only purchases additional units of Y with the increase in real income then the quantity of resource R traded is uneffected. In this case, the ordinary supply curve $S_1$ is equal to the compensated supply curve. This is the situation illustrated in Figure 20. If the utility maximizing resource owner preferred to withhold resource R as real income increases (in effect consuming R in kind) then the horizontal distance measured as T-S would be negative. This would have the effect of making the ordinary supply curve more inelastic relative to the compensated supply curve. Economic rent measured with respect to the ordinary supply curve would over estimate the change in income associated with a change in resource price relative to a given and initial utility function.

On the other hand, if the utility maximizing resource owner preferred to trade yet more of resource R as real income increase (in

effect resource R is an inferior good from the point of view of the resource owner or subject to external economies), then the horizontal distance measured as T-S would be positive. This would have the effect of making the ordinary supply curve more elaslic relative to the compensated supply curve. Economic rent measured with respect to the ordinary supply curve would underestimate the change in income associated with a change in resource price relative to a given initial utility function.

In this section we established the relation between the observable ordinary supply curve and the theoretically necessary compensated supply curve. If the welfare effect is zero then the ordinary supply curve and the compensated supply curve are the same and represent a theoretically accurate (or separable) and observable means of measuring economic rent. The next question is 'what types of utility functions are associated with a zero welfare effect?' We address this question regarding the types of utility functions in the next section.

## 3-5 Separability and Economic Rent

In the previous section we analyzed the relationship between the expenditure minimization function and the utility maximization function particularly as these functional forms effect the resource supply function relative to measuring economic rent. We found that if the welfare effect is zero, then the ordinary supply curve equals the compensated supply curve. The latter is required to accurately determine the compensating variation measure of economic rent in the resource market. We turn now to examine the types of assumed utility functions which are associated with a zero welfare effect. Further, we are

interested in the consumption behavior implied by utility functions

The two possible economic interpretations of path independence are when the cross price effects between goods in a resource owner's consumption bundle are equal to each other or zero.

There are only two utility functions which assure separability or path independence. These two types of utility functions are vertically parallel and homothetic. A vertically parallel utility function is one in which ". . . preferences are parallel with respect to commodity 1 (numeric good) on the relevant domain of the commodity space" (Chipman and Moore, 1980, p. 937). The properties of such a function is that of ". . . yielding demand functions with zero income elasticities for all commodities other than commodity 1 (the numeraic commodity (Chipman and Moore, 1980, p. 938). A vertically parallel utility function is also one which results in equal cross-price elasticities the sum of which equal the own-price elasticity. A homothetic utility function is one in which resource owners partition or portion income to cover areas of expense regardless of relative price changes between categories. Additional income is distributed proportionately (Chipman and Moore, 1980, p. 937). Presumably, the resource owner also determines the quantity of resources traded (which determines the absolute level of income) independently of the partitioning of the income. Homothetic utility function has the property of yielding a demand function which has unitary own-price and income elasticities and zero cross-price elasticities (Chipman and Moore, 1980, p. 938; Samuelson, 1965, pp. 788-789).

The properties of homothetic and vertically parallel functions are so stringent that they are nearly impossible to meet empirically. Samuelson describes these two types of utility functions as ". . . interesting from a theoretical viewpoint but trivial for empirical application." (1965, p. 788). Therefore the utility functions necessary to assure a unique welfare measure of economic rent measured indirectly with respect to the ordinary resource supply curve is very likely not to exist.

We have found that two utility functions exist theoretically which, if they represent the actual utility function of a utility maximizing resource owner results in ordinary resource supply curve with a zero welfare effect. Unfortunately the implicit assumptions regarding the resource owners behavior are quite unrealistic in that all or nearly all additional increase in real income is spent on the commodity whose price decreased. In the next section we explore the use of compensating variations as a compensation measure of economic rent.

## 3-6 Compensating Variation as a Cardinal Compensation Measure of Economic Rent

In the previous section we found that vertically parallel and homothetic utility function assure that an ordinary resource supply function equals a compensated supply function. Given these utility functions the integral of the resource supply function over the relevant price range provides a cardinal welfare measure of economic rent. The implied behaviors associated with these utility functions are quite unrealistic as a general rule. Therefore, we are now seeking a more modest goal of a cardinal compensation measure of economic rent.

In this case, we make no assumptions regarding the type of utility function other than the standard assumptions regarding continuity, convexity and differentiality. Let us begin by examining the explicit assumptions associated with use of compensating and equivalent variation as measure of economic rent. The assumption of a welfare measure implies a utility function with the properties of separability and path independence. That is, a unique measure between two functions exists only when these functions are exact differentials of each other. This implies path independence which implies separability. These properties are only found in association with vertically parallel and homothetic utility functions.

In the absence of either the vertically parallel or homothetic utility function an infinite number of "measures of welfare impact" can be described between two functions depending on the price adjustment path. Silberberg states:

> Clearly, an infinite number of consumer's surpluses can be generated, most of them uninteresting. Relative and absolute prices can be changed so as to hold $X_1$, fixed until U' is reached and then changed relatively so as to move the consumer along U' to X' . . . these paths generate values . . . which correspond to (but are not exactly equal to) Marshall's surpluses in which quantities are held constant. There is no way in general to relate these surpluses to other values . . . (1972, p. 945).

An accurate empirical estimate of compensating or equivalent variation associated with a utility function that is neither vertically parallel nor homothetic is not a "theoretically correct" cardinal measure of a welfare change associated with a change in the price of a resource in the case of economic rent. There seems to be substantial confusion on this point. This can be shown with two sets of illustrations. Each set includes a pair of direct and indirect

utility function and associated ordinary supply curve. These illustration differ only with respect to the type of utility function. Figure 21 includes a vertically parallel pair of utility functions while Figure 22 relaxes this assumption.

A simple comparison of Figures 21 and 22 is useful to illustrate the relationship between a "theoretically correct" cardinal welfare measure and an "empirically correct" cardinal compensation measure of economic rent. Figure 21 illustrates the relationship between the direct and indirect utility functions as well as compensated and ordinary resource supply function given a vertically parallel utility function. Compensating variation (CV) is defined as the difference between initial and subsequent nominal income with respect to an initial utility level for a given change in price. Assuming a resource price increase, CV can be interpreted as ". . . the amount of compensation. . . received, that will leave the consumer in his initial welfare position following the change in price if he is free to buy any quantity. . ." (Currie et al., 1971, p. 746).

(1) $CV = Y_0 - Y_2$.

Equivalent variation (EV) is defined as the difference between initial and subsequent nominal income in relation to the subsequent utility level given a change in price. Assuming a resource price increase, EV can be interpreted as ". . . the amount of compensation paid. . . that will leave the consumer in his subsequent welfare position in the absence of the price change if he is free to buy any quantity. . ." (Currie et.al., 1971, p. 746).

(2) $EV = Y_3 - Y_0$.

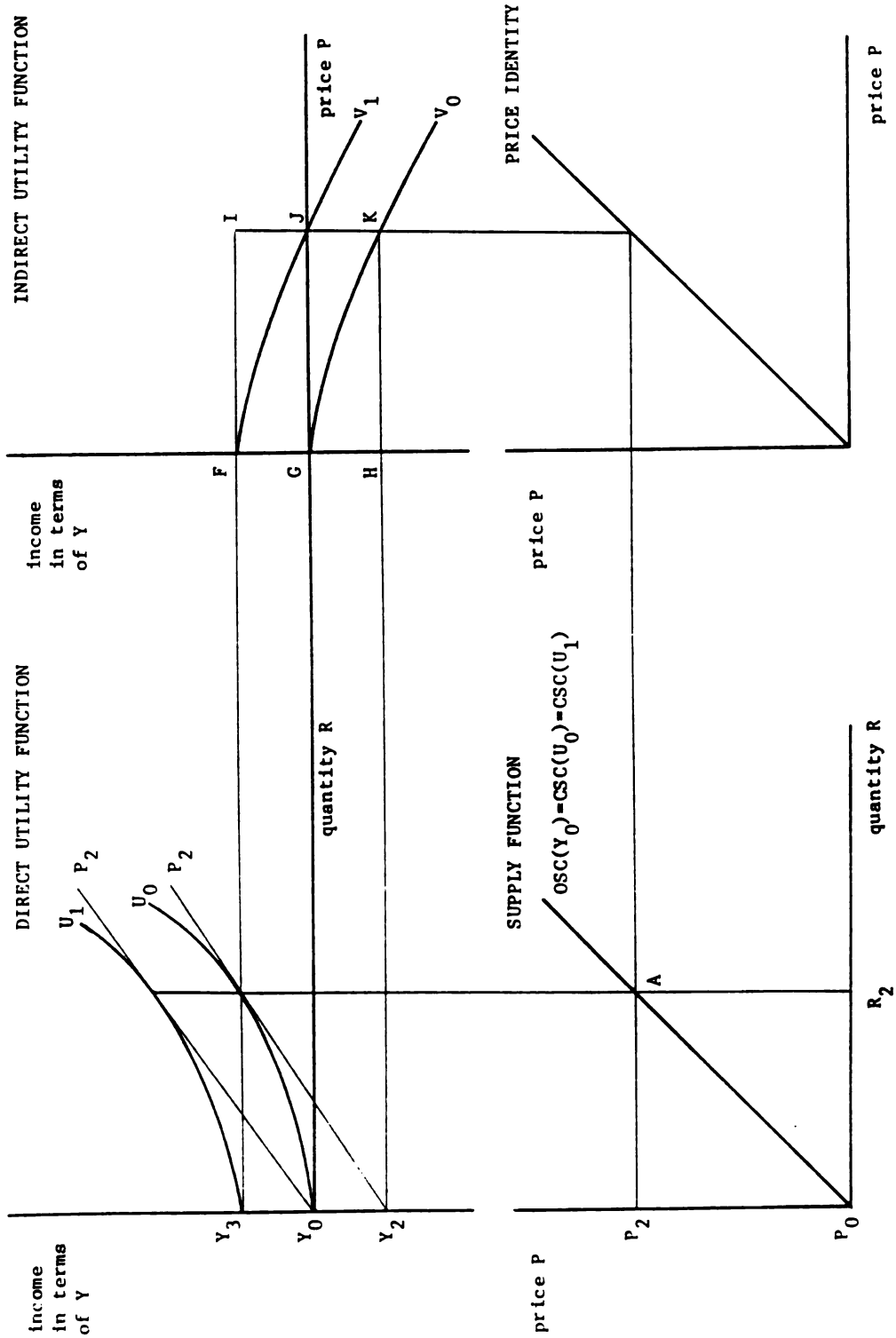


Figure 21. Vertically Parallel Direct and Indirect Utility Functions and Compensated Supply Function

The measure of economic rent in relation to the indirect utility functions ($V_0$ and $V_1$) can be defined such that

(3) $CV = Y_0 - Y_2 = J - K$

and

(4) $EV = Y_3 - Y_0 = F - G$,

It is also true that these measures of economic rent determined in relation to the compensated supply curve such that

(5) $CV = Y_0 - Y_2 = J - K =$ area $P_2P_0A$

and

(6) $EV = Y_3 - Y_0 = F - G =$ area $P_2P_0A$

therefore

(7) $CV = EV$.

Equations (3) through (7) illustrate the point that regardless of the path taken to measure economic rent associated with a shift in individual welfare from $U_0$ to $U_1$ or $V_0$ to $V_1$, the result is always the same. The measure of rent in this case is path independent. This is not a surprising result in light of our discussion on separability and path independence. The assumption of a vertically parallel utility function makes this result inevitable. This is the one of two special cases in which a unique cardinal welfare and compensation measures of a change in utility associated with a change in price can are equal.

There is a special relationship which exists between the direct and indirect utility functions. Samuelson has determined that ". . . when either [the indirect or direct utility functions] is homothetic so is its dual. 'Direct homogeneity'implies 'indirect homogeneity' and vice versa. If [the direct utility function] is homogenious of

degree M, [the indirect utility function] is homogeneous of degree = M" (1965, p. 785). A similar relationship holds for a vertically parallel direct utility function and its dual, the indirect utility function.

By way of comparison, Figure 22 illustrates the relationship between the direct and indirect utility functions when the assumption of separable preferences is relaxed. We shall assume that the utility function is neither vertically parallel or homethetic, such that resource R is withheld (consumed in kind) as income increases. The definitions of compensating variation and equivalent variation remain unchanged though in this case they are no longer equal. Their interpretations differ.

(8) $CV = Y_0 - Y_2$

and

(9) $EV = Y_3 - Y_0$.

The compensation measures of economic rent in relation to the indirect utility function can be defined such that

(10) $CV = Y_0 - Y_2 = J - K$

and

(11) $EV = Y_3 - Y_0 = F - G$.

It is also true that these measures of rent determine in relation to the compensated resource supply curves such that

(12) $CV = Y_0 - Y_2 = J - K$ = area LMNOP where $N = N_1 + N_2$

and

(13) $EV = Y_3 - Y_0 = F - G$ = area LM.

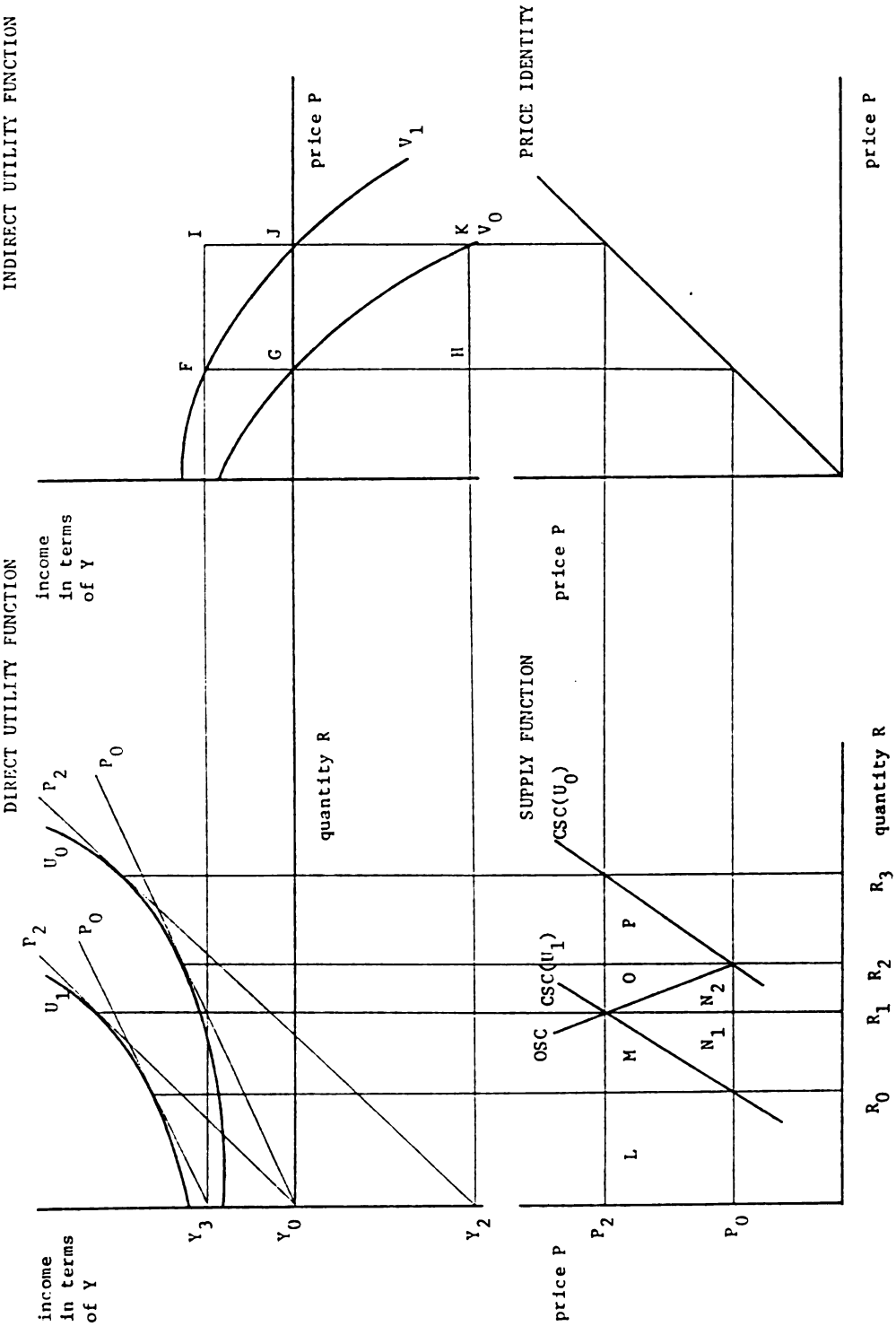


**Figure 22. Generalized Direct and Indirect Utility Functions and Compensated Supply Function**

Therefore

(14) CV > EV

The measure of economic rent associated with the Laspeyres and Paashe indices can be derived as well. The Laspeyres index (LI) is defined as the ratio of subsequent to initial price multiplied by initial quantity (Tomek and Robinson, 1972, p. 204).

(15) LI = L M $N_1$ $N_2$ 0

The Paashe index (PI) is defined as the ratio of subsequent to initial price multiplied by subsequent quantity (Tomek and Robinson, 1972, p. 204).

(16) PI = L M $N_1$

Therefore

(17) LI > PI

and

(18) CV > LI > PI > EV

As it turns out in the general case which relaxes restrictions on the type of utility function an infinite number of measures can be determined which estimates vertical distance between $U_0$ and $U_1$ or $V_0$ and $V_1$. Any claim under these circumstances as to which measures is "theoretically correct" is necessarily "ambiguous and normative" (Silberberg, 1972, p. 942).

Thus the special case when economic rent is a unique cardinal measure of welfare is assocated with very restrictive assumptions regarding resource owner behavior and utility function. When these restrictive assumptions are relaxed then a unique cardinal measure of welfare does not exist. So the question remains 'is there anything an

economist can say regarding the welfare effects of a price change in terms of economic rent or consumer surplus?'

A more modest goal is within our reach. Compensating variation has an interpretation that does not involve any claims of being "the correct theoretical measure of welfare impact." Compensating variation as it is defined is subject to the same interpretation of economic rent developed in relation to the "take it or leave" it and compensated supply curve relative to a given utility function. The compensating variation measure of economic rent can be understood as the additional income a resource owner would earn for a given amount of a resource under conditions of competitive trade above that which the resource owner would accept for these units of the resource under conditions of monopsony trade. This is a compensation rather than welfare interpretation. Compensating variation is simply one of an infinite set of possible candidates for measuring economic rent. It has several characteristics besides the above interpretation which makes it a desirable choice as a standard unit of measure in welfare economics. Compensating variation cannot be construed as a cardinal welfare measure since no shift in utility is implied in the analysis. Compensating variation has the common denominator of the initial utility function. Compensating variation is well known, understood and widely discussed in the welfare literature. And, as with any other measure as the income or welfare effect decreases, compensating variation is equal to a cardinal measure of welfare in those rare instances when one may exist. Finally, and again as with any other consistently selected measure, compensating variation can be used to

measure compensation and thereby determine whether consumers and resource owners are better off with a particular price change for a particular good and/or resource than without it.

The absence of cardinal measure of welfare precludes an economist from making what Silberberg regards as essentially political value judgments as to whether a policy should be implemented because group A receives net welfare benefits of X and group B loses net benefits of Y where X > Y such that net social benefits are positive (1972, p. 951). For better or worse a money measure of individual or group benefits does not, as a general case, directly correspond to a utility based social welfare measure. We are left with a cardinal measure of compensation rather than a cardinal measure of welfare. A cardinal measure of compensation determines the amount of money income consumers and resource owners are able (if not willing) to give up subsequently and still be as well off as initially.

## 3-7 Economic Rent and the Goods Market

As we make the transition from discussing economic rent in terms of utility function to discussing economic rent in terms of industry's marginal and average cost functions, it is helpful to analyze the consistency between the two approaches. Let us begin with a discussion of the definition of economic rent. Economic rent defined in terms of utility is simply compensating variation. Compensating variation is defined as "... the amount of compensation...received, that will leave the consumer in his initial welfare position following the change in price if he is free to buy any quantity..." (Currie et. al., 1971, p. 746). Compensating variation can also be defined as the

additional income a resource owner would earn for a given amount of a resource under conditions of competitive trade above that which a resource owner would accept for the resources under conditions of monopsony trade. Economic rent measured in terms of production is defined as "a payment in excess of that necessary to maintain a resource in its current occupation" (Mishan, 1959, p. 386). These two definitions are inconsistent if the production definition, implies an ordinary supply curve including welfare effects. Alternatively the production definition may imply a compensated supply curve. As such it then implies that the quantity of resource supplied is held constant. This in turn implies that then the Laspeyres rather than compensating variation measure of economic rent (Mishan, 1959, p. 992).

An alternative definition of economic rent measured in terms of production is "the difference between the current earnings of a resource and its transfer earnings" (Mishan, 1959, p. 386). This definition is also inconsistent with the compensating variation definitions. This production definition implies an upward sloping ordinary resource supply function including welfare effect (Mishan, 1959, p. 393). However, the ordinary supply function may have a negative slope over a portion of relevant prices if the negative welfare effect is greater than the substitution effect as is often true in the case of labor resources. This situation is aggrevated if labor suppliers actually prefer a lower paying job which they enjoy over a higher paying one which is stressful or monotonous such that the "substitution effect" appears to be negative as well (Mishan, 1959, p. 393).

For the purposes of this analysis, we shall use the definition of economic rent as compensating variation by assuming that the utility functions are vertically parallel such that all measures of economic rent are equal and that the measure of compensation equals the measure of welfare. This is done for ease of measuring economic rent indirectly in the goods market by relating the elasticity of resource supply to the industry's long run marginal cost curve excluding rent. Here is a note of caution regarding the simplifying assumption vertically parallel utility functions which is so often implied in applied work especially in relation to measuring consumer surplus. Mishan states:

> ...the two precise measures CV and EV...were initially suggested by [Hicks] indifference curve analysis. Important as these contributions were in clarifying our ideas on this tangled subject, it can be held that the tracing of these precise relationships assumes a far greater importance on the neglected supply side. For it is just these that we cannot reasonably suppose that a change in price has negligible effects on the welfare of the individual inasmuch as the supply of any one of his productive services enters significantly into his budget. To the extent it does so, the area above an individuals supply curve, especially in the case of only one productive service, is a much less reliable index of the surplus welfare than the area under his demand curve for any one good (1959, p. 39 ).

Even with this important simplifying assumption there are still substantial difficulties estimating economic remt indirectly in the goods market as the area below output price and above the industry's long run marginal cost curve excluding rent. We need to determine the circumstances which result in a change in resource price. The change in resource price is the cause of the change in economic rent. Thus we are interested in the types and characteristics of resources required in the production process of a set of firms which comprise an

industry and the effect these resources have on the long-run supply elasticity and economic rent of the industry. In this regard we are particularly interested in the possibility of measuring economic rent indirectly in the goods market as the area below output price and above the industry's long run marginal cost curve exclusive of rent. For example, if the marginal value product of a resource necessary in the production of a good increases because of a new technology, the resource price will increase only if the quantity of available resource is scarce relative to the increase in the industry's derived demand (Robinson, 1933, p. 110). Thus, from the point of view of an industry, a resource is scarce if the portion of aggregated resource supply needed by the industry is available in less than perfectly elastic supply.

Implicitly we can see that the causes of changes in economic rent are associated with the categories of availability of a resource required in the production process. As we shall see resources can be categorized by the extent of availability: (1) absolutely scarce, (2) relatively scarce, (3) abundant and (4) super-abundant. The degree of availability of a resource results in a situation which allows the resource owner to claim some of the economic surplus generated in the production process. The result is a "payment in excess of that necessary to maintain a resource in its current occupation" or rather payment in excess of its opportunity cost. This payment typically is measured as the area below output price and above an industry's long run marginal cost curve exclusive of rent. The problems of measuring economic rent in the goods market are compounded when the scarce

resource is also a durable. Durables are often subject to the situation in which acquition price is greater that salvage value adjust for depreciation and capital improvements. Thus the characteristics of a resource affect the elasticity of the compensated supply function faced by firms within a particular industry. The elasticity of supply affects the industry's long run marginal cost function. This is the linkage between the cause of economic rent (or a change in economic rent) as measured directly in the resource market or indirectly in the output market. In sections 3-8 through 3-12 we will examine the relationship between resource availability, the long run marginal cost curve excluding rent, for an industry and the indirect measure of rent is in the goods market. We will examine the Mishan criteria for measuring economic rent in the goods market relative to each of the resource types such that the industry supply curve must equal both the marginal cost excluding rent and the average cost including rent. The next five sections explore this relationship as well as the direct and indirect measures of economic rent in the resource and goods markets respectively for a given shift in aggregate demand. The analysis builds on the work of Viner (1931) and Mishan (1968).

## 3-8 Abundant Resources and a Constant Cost Industry

In this section, we will describe a constant cost industry whose firms require only abundant resources as inputs in production. First, we will specify the circumstances surrounding this industry. Then we will analyze the relationship between resource availability and the industry's long run marginal cost curves excluding rent. Finally, we ej€€ ®"v"!)j&" vg" mxx†/jmv"® ®j!"/v m&® j&®j!"/v )"mxh!"x †d "/†&†)j/

rent, consumer surplus and economic surplus for a given shift in demand.

In this constant cost industry, we assume atomistic competition in the commodity and resources markets. We assume that each firm in the commodity market is of equal size and efficiency which can be represented by a single "representative firm." All resources required in the production process are assumed to be available in abundance at a fixed price, independent of the quantity demanded by this particular industry. These abundant resources are assumed to include both variables and durables. The acquisition and salvage price of these abundant resources are assumed equal. Further it is assumed that internal size economies exist and have been fully exploited by each firm in the industry. It is therefore assumed that output expands at the extensive margin with the entry of additional firms of equal size and efficiency. We assume that there are neither external economies or diseconomies of large scale production of either a pecuniary or technological nature associated with this industry. It is assumed that there are no barriers to entry or exit in this industry. The number of firms in the industry are determined by the demand for the output. Finally, it is assumed that there is an exogenous shift in demand for this industry's output (Viner, 1931, pp. 62-64).

The process of adjustment to a shift in demand by the representative firm and the industry is illustrated in Figure 23. Initially, the representative firm's marginal cost $MC_0$ and average cost $AC_0$ are equal to equilibrium price $P_0$ at output $q_0$. Initially, the

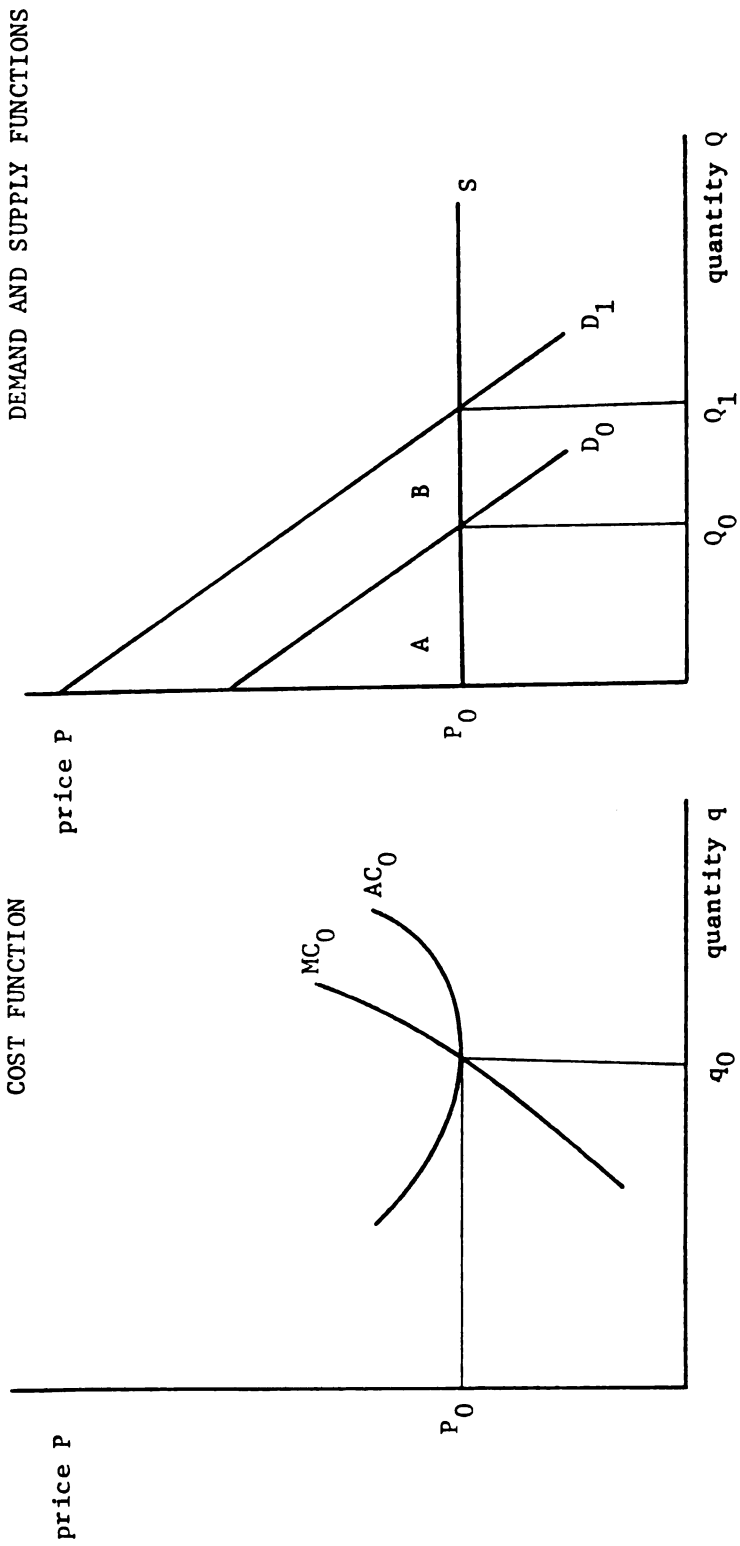


Figure 23. A Constant Cost Industry

industry is in long-run equilibrium at price $P_0$ and output $Q_0$ associated with demand $D_0$ and aggregate production of N number of firms in the industry.

With the subsequent increase in demand, M-N firm, attracted by short-run profits, enter the industry with size economies and efficiency equal to those of previously producing firms. Production increases from initial output $Q_0$ to the subsequent output $Q_1$ associated with M firms. Since the industry is not subject to external economies or diseconomies the subsequent average and marginal cost curves of the representative firm remain at $AC_0$ and $MC_0$ respectively. Since the price of all the resources remain constant as output expands, producers do not attempt to change the least cost combination of resource (Ferguson, 1975, pp. 90-92).

Let us examine the impact the shift in demand has had on the industry's long-run supply curve. Since all resources required in production are available in perfectly elastic supply, the slope of the industry supply curve is horizontal with infinite elasticity. There is no change in the relative intensity in the use of abundant resources, therefore there is no change in the slope or elasticity of the industry supply curve. The industry's supply curve in this case can be described either as the industry's average or marginal cost curve (with or without rent) (Robinson, 1933, p. 140). The industry's supply curve is also equal to the sum of the individual firms average and marginal cost. Total rent can be measured as the area above the industry's marginal cost curve excluding rent and below the equilibrium output price. Initially economic rent ($ER_0$) equals

(1) $ER_0 = 0$.

Subsequently economic rent ($ER_1$) equals

(2) $ER_1 = 0$.

The net change in economic rent equals

(3) $ER_1 - ER_0 = 0$.

We can see that there is no change in economic rent in the industry in which all resources are available in abundant supply.

Consumer surplus can be measured in the commodity market also. Initially consumer surplus ($CS_0$) equals

(4) $CS_0$ = area A.

Subsequently consumer surplus ($CS_1$) equals

(5) $CS_1$ = area A + B.

The net change in consumer surplus equals

(6) $CS_1 - CS_0 = A + B - A = B$.

The net change in economic surplus (ES) equals

(7) $ES_1 - ES_0 = ER_1 - ER_0 + CS_1 - CS_0 = B$.

The net change in economic surplus in an industry which requires only abundant resources in production equals the sum of two effects: The net change in economic rent (0) and the net change in consumer surplus (area B). Consumers are the sole beneficiaries in this case.

The process of adjustment to the shift in output demanded can be illustrated in the resource market for one of the required abundant resources. See Figure 24. Initially, the representative firm's marginal value product $MVP_0$ is equal to equilibrium resource price $P_0$ and quantity $q_0$. The derived demand curve $D_0$ is equal to the sum of the $MVP_0$ curves for N number of firms.

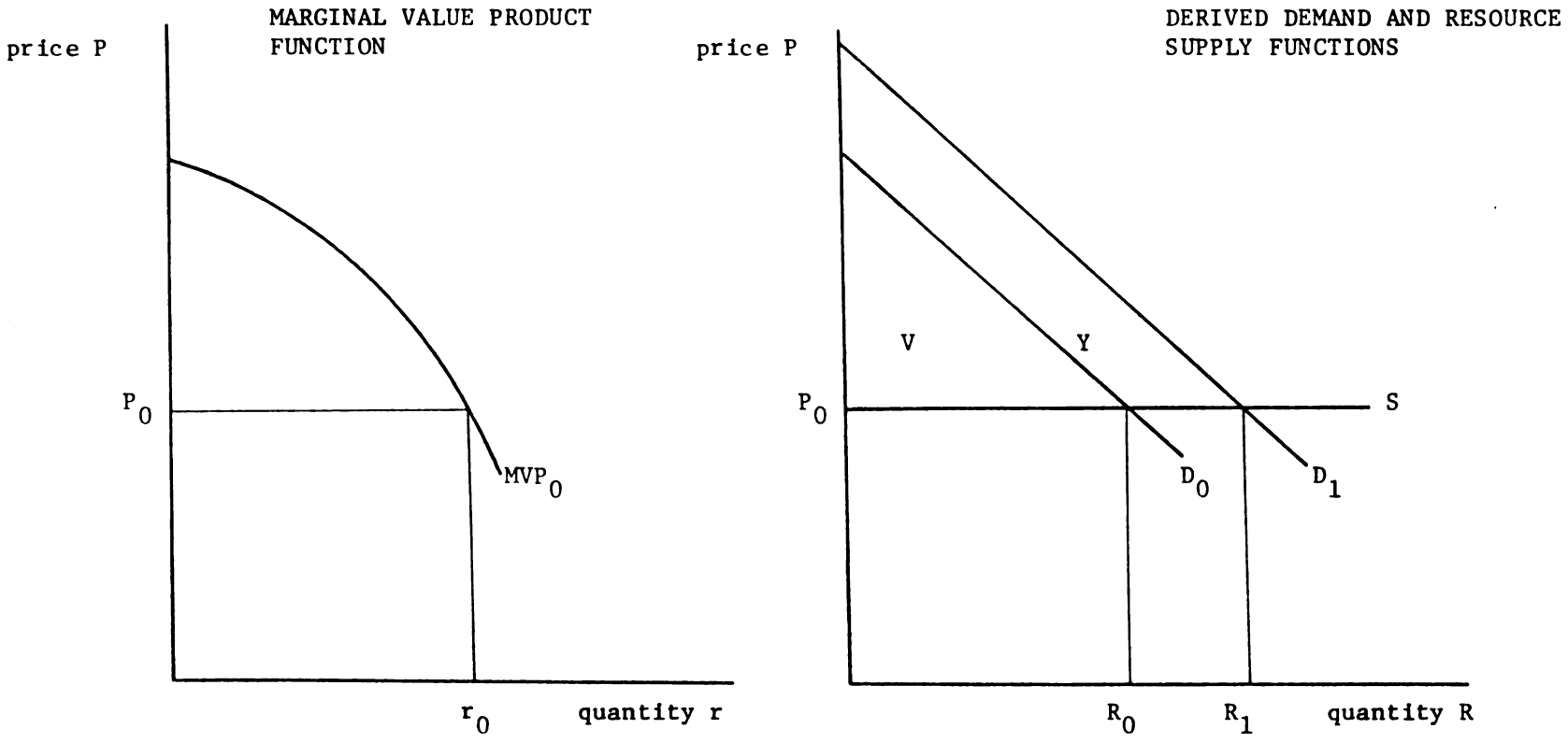


Figure 24. "Rent" on an Abundant Resource

With the subsequent increase in demand for output, M-N additional firms are attracted to the industry by short-run profits. These new firms enter the industry with the same production costs and size economies as previously producing firms. Subsequently, the industry's derived demand $D_1$ for the abundant resource is equal to $Q_1$ at equilibrium price $P_0$. The derived demand $D_1$ is associated with the sum of the $MVP_0$ curves for M number of firms in the industry. Because the derived demand for this resource by this industry is small relative to total supply of this resource to the economy, the resource appears to be abundant. Thus resource owners are not in a position to capture any economic surplus as output expands. The resource supply curve has a zero slope and infinite elasticity from the point of view of this industry.

The net change in economic surplus associated with a change in output can be measured in the resource markets of the inputs required in the production process. In the resource market for a particular abundant resource, we can illustrate the areas associated with economic rent, consumer surplus and economic surplus. Initially economic rent equals

(8) $ER_0 = 0.$

Subsequently economic rent equals

(9) $ER_1 = 0.$

The net change in economic rent measured directly in the resource market relative to this industry equals

(10) $ER_1 - ER_0 = 0.$

Initially consumer surplus equals

(11) $CS_0$ = area V.

Subsequently consumer surplus equals

(12) $CS_1$ = are V + Y.

The net change in consumer surplus measured indirectly in the resource market relative to this industry equals

(13) $CS_1 - CS_0 - V + Y - V = Y$.

The net change in economic surplus measured in the resource market relative to this industry equals

(14) $ES_1 - ES_0 = ER_1 - ER_0 + CS_1 - CS_0 = Y$.

Let us compare the net change in economic rent, consumer surplus and economic surplus as measured in the goods market as equations (3), (6) and (7) with these same changes measured in a particular abundant resource market as equations (10), (13) and (14). Based on the works of Just and Hueth (1979, pp. 948-951) we determine that the aggregate change in economic rent measured indirectly in the commodity market is equal to the sum of the change in economic rent measured directly in the resource market relative to this industry for each of the resources required in the production of a particular output.

(15) $ER_1 - ER_0 = 0$

There is no difference in economic rent whether measured in the commodity market or in the resource markets.

The aggregate change in consumer surplus measured directly in the commodity market is equal to the change in consumer surplus measured indirectly in the resources markets of each factor needed in the production of a particular commodity.

$$(16)\quad CS_1 - CS_0 = B = \sum_{i=1}^{k} (Y_i)$$

where i equals 1 to k resource markets for inputs required in the production process.

Finally, the aggregate change in economic surplus measured in the commodity market is equal to the sum of the change in economic surplus measured in the resource market for each factor needed for the production of that particular commodity.

$$(17)\quad ES_1 - ES_0 = ER_1 - ER_0 + CS_1 - CS_0 = B = \sum_{i=1}^{k} (Y_i)$$

where i equals the 1 to k resource markets for inputs required in the production process.

The entry of new firms into this industry is associated with profits. The role of profits in the adjustment process is illustrated in Figure 25. Initially, our representative firm has a total cost function ($TC_0$), marginal cost function ($MC_0$) and average cost function ($AC_0$) such that output $q_0$ is produced at equilibrium price $P_0$. After the exogenous shift in demand for output Q and before new firms enter the industry, the short-run industry output remains at $Q_0$ and output price increases to a price $P_2$. Previously producing firms earn a profit of at least B-A per unit. These previously producing firms will expand output until their marginal cost equals output price. At output price $P_2$, the representative firm produces $q_0'$ even though it means increasing production in the face of internal diseconomies of size.

On the point of producing at something other than the minimum cost point Robinson remarks:

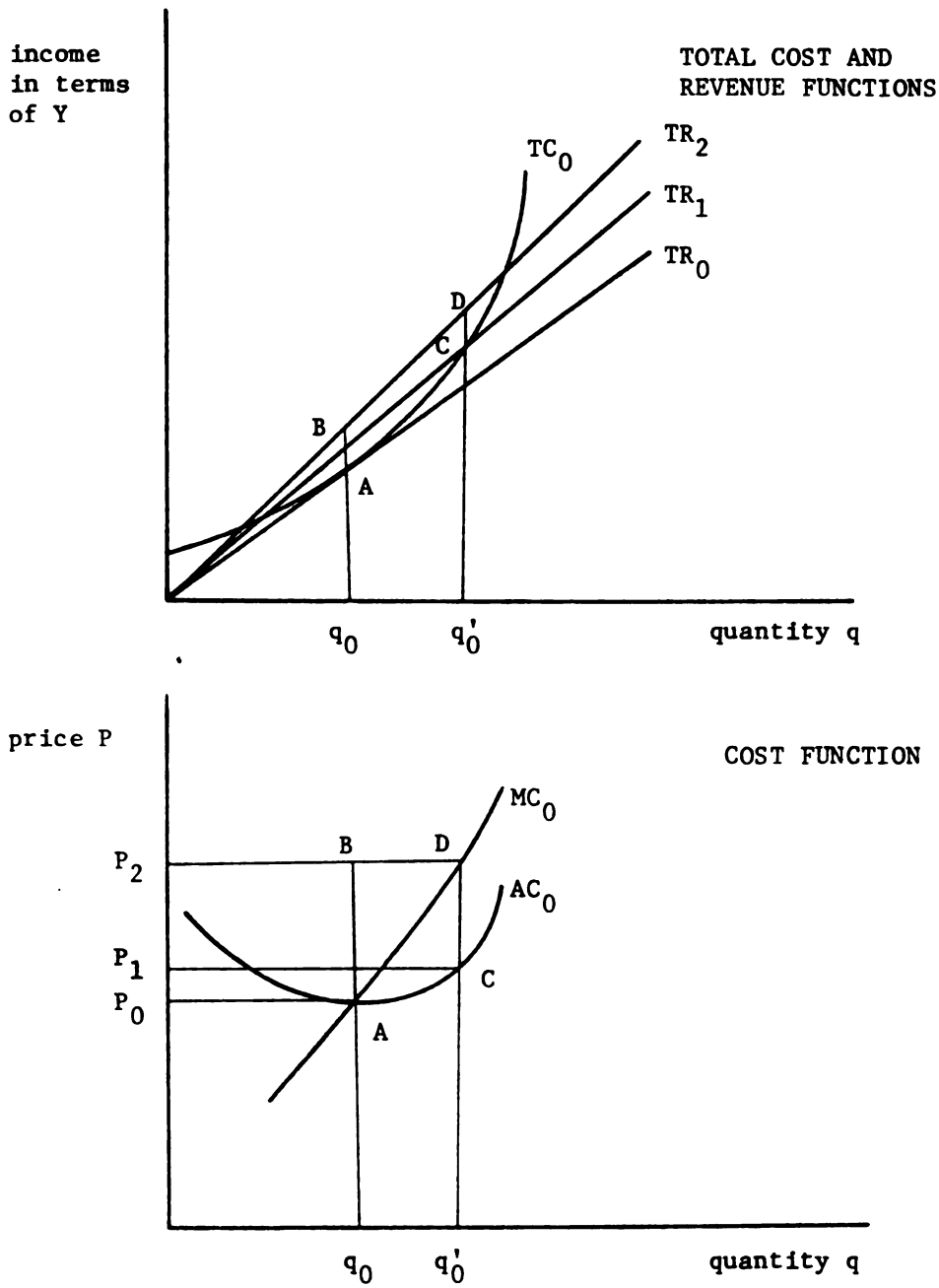


Figure 25. Profits and Industry Adjustment

> It is sometimes supposed that the optimum size of the firm is that which is most profitable to the entrepreneur, so that the entrepreneur has a motive for wishing his firm to be of optimum size. But this view is mistaken. It is no disadvantage to the entrepreneur to produce more than the optimum output. Indeed it is when profits are abnormally high (because new firms are failing to enter the industry to a sufficient extent to keep profits at a normal level) that the firms are of more than optimal size. (1933, p. 97).

As previously producing firms individually increase output from $q_0$ to $q_0'$ the output price $P_2$ becomes the equilibrium output for the industry before new firms enter. Profits per unit become equal to D-C As new firms enter they too produce $q_0'$. Now output further increases and output price decreases. As output prices decrease toward $P_0$ each firm moves back toward optimum firm size until only "normal profits" remain. Subsequent long run eqiulibrium price $P_0$ and output $Q_1$ are established for the industry with M firms producing $q_0$.

In this section, we have shown that in an industry in which only abundant resources are required in this production process, the industry's long-run supply curve is horizontal and equal to the industry's average and marginal cost curve (including and excluding rent). All economic surplus associated with the expanded production goes to consumers. The net change in rent is this case is zero. Consumer surplus can be measured accurately in either the commodity market or in the resource markets. . The role of short-run profits in attracting new firms to the industry is also illustrated.

## 3-9 An Absolutely Scarce Resource and an Increasing Cost Industry

In this section we will describe an industry whose firm requires one absolutely scarce resource as well as abundant resources as inputs in the production process. Once we have specified the circumstances

surrounding this industry, we will analyze the relationship between resource availability and the industry's long run marginal cost curve excluding rent as they relate to measuring economic rent, consumer surplus and economic surpluses for a given change in demand.

In this industry, we assume there exists some representative firm such that the efficiency of this firm is equal to any other firm in the industry. The production function of the representative firm is assumed to be such that it requires one absolutely scarce resource which is fully exploited by the industry. All other resources required in production are assumed to be in abundant supply available at a fixed price independent of the quantity demanded by this particular industry. These abundant inputs are assumed to include both variables and durables. The acquisition and salvage price of these abundant inputs are assumed equal. Further, we assume that there exists no external economies associated with large scale industry nor external diseconomies associated with relatively scarce resources. We assume that internal size economies exist and have been fully exploited. Output is assumed to expand at the intensive margin only and the number of firms in the industry is assumed to be constant. Finally, we assume an exogenous increase in demand for the industry' output (Viner, 1931, pp. 58-62).

The process of adjustment to a shift in demand by the representative firm and by the industry is illustrated in Figure 26. Initially, the representative firm's marginal cost ($MC_o$) and average cost ($AC_o$) are equal to equilibrium price $P_o$ and output $q_o$. Initially, the industry is in long run equilibrium at price $P_o$ and output $Q_o$

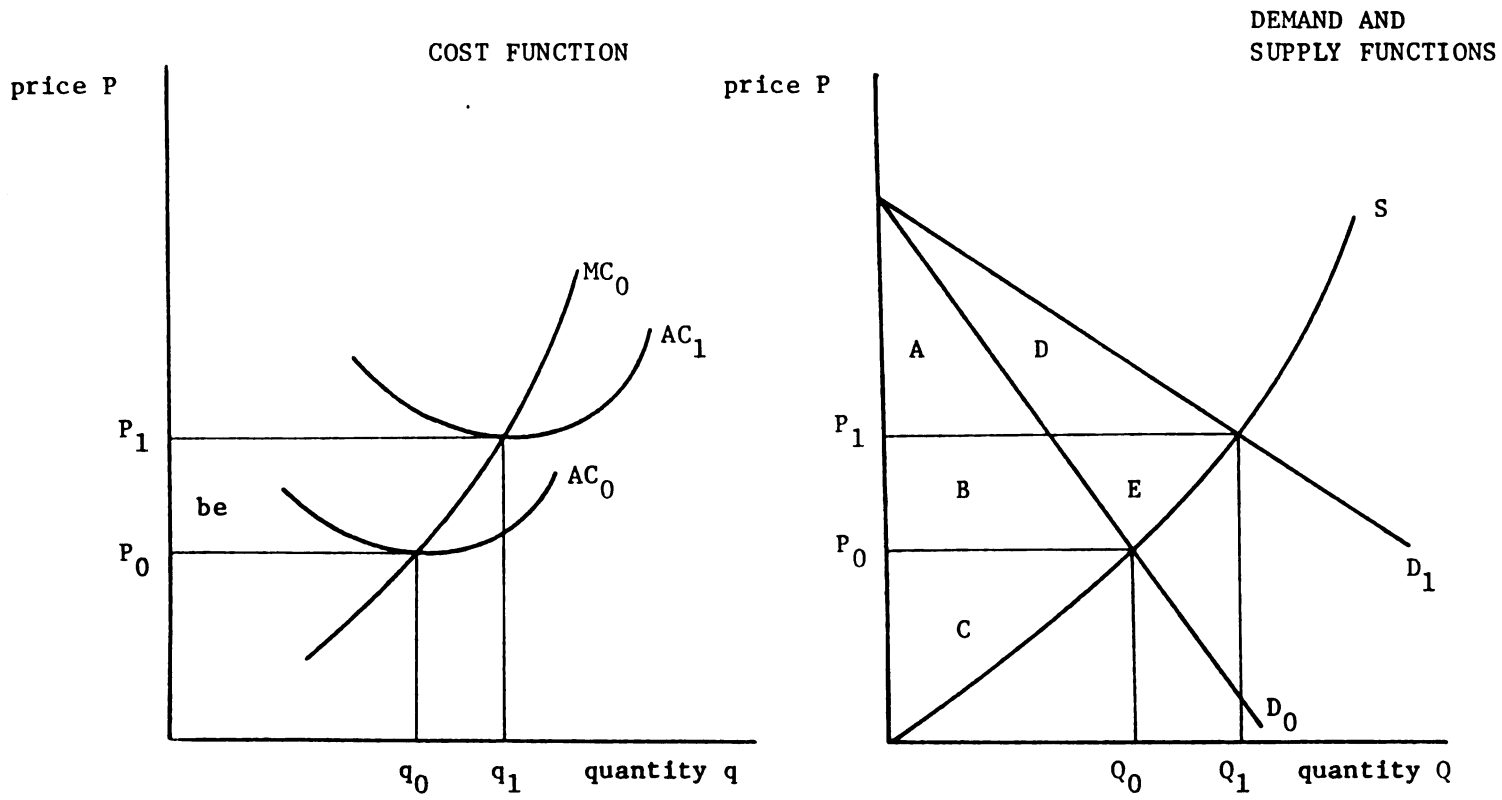


Figure 26. "Ricardian" Increasing Cost Industry

associated with the aggregate production of N number of firms in the industry.

With the subsequent increase in demand from $D_o$ to $D_1$, each previously producing firm expands output by using the absolutely scarce resource more intensively. Additional units of the abundant resources are combined with the absolutely scarce resource until each firm's output increases from $q_o$ to $q_1$. This expansion in output takes place along each firm's marginal cost curve $MC_o$ because the price of these inputs remain unchanged. Output increases until $MC_o$ equals the new equilibrium price. However, as the absolutely scarce resource is used more intensively, its price increases causing the average total cost to increase from $AC_o$ to $AC_1$.

The absolute scarce resource is often illustrated in terms of land used in agricultural production. On the point regarding a change in average total cost without affecting marginal cost Viner states:

> Given an absolutely limited amount of one of the factors, no change in the prices of the other factors, and a rise in the long-run demand for and in the long-run price of its product, and the long-run price of this absolutely scarce resource must rise. Let us suppose that the fixed factor is land. Its price or rent will rise until there ceases to be any excess of marginal over average cost . . . the increase in land-rent, however, will have no effect on marginal costs, and therefore on the long-run MC curve. . . (Viner, 1931, p. 60).

The presence of an absolutely scarce resource required in production results in an upward sloping supply function for the industry. The industry supply function in this case has an important set of characteristics. The industry long-run supply function is equal to the sum of the firm's average costs including rent. Also, the industry's long-run supply function is equal to the sum of the firm's

marginal costs excluding rent. (Viner, 1931, p. 61). This set of characteristics are critically important when attempting to measure economic rent indirectly in the goods market. In references to this case Mishan states:

> . . . if the factor-owner is concerned only with pecuniary advantage, this measure will be identical with the area above the supply curve reckoned as a sum of money. All of the Marshallian quasi-rent is to be imputed to the owners of specific capital equipment (at least in riskless situations). Ricardian rent is obviously to be imputed entirely to owners of nonmarginal land (1968, p. 1275).

The quasi-rent referred to by Mishan is the return to producers using temporarily undervalued capital assets in production. Quasi-rent in this case is equal to area be measured in relation to a representative firm's ex ante and ex post average cost functions. The sum of the quasi-rent across N firms in the industry is equal to area BE. Area BE is the sum of the quasi-rent which has been capitalized into the price of the absolutely scarce resource. This indirect measure of economic rent measured in the good market is determined as the integral of the industry marginal cost function excluding rent over the change in output price.

$$(1) \quad ER = \int_{P_o}^{P_1} S dP$$

where S is the industry's marginal cost excluding rent

Finally, we can measure the change in income to consumers and owners of the absolutely scarce resource by measuring the relevant acras in the goods market.

Initially, consumer surplus (CS) equals:

$$(2) \quad CS_o = A + B.$$

Initially, economic rent (ER) equals:

(3) $ER_o = C.$

Subsequently, consumer surplus equals:

(4) $CS_1 = A + D.$

Subsequently, economic rent equals

(5) $ER_1 = B + E + C.$

The net change in consumer surplus equals

(6) $CS_1 - CS_o = A + D - (A + B) = D - B.$

The net change in economic rent equals

(7) $ER_1 - ER_o = B + E + C - C = B + E.$

The net change in economic surplus (ES) where ES = CS+ER)

(8) $ES_1 - ES_o =$

$CS_1 - CS_o + ER_1 - ER_o = D - B + B + E = D + E.$

The net change in economic surplus for an industry which requires an absolutely scarce resource in production is the summation of two effects: the net change in consumer surplus (area D) and the net change in economic rent (area E). Area B represents a transfer of income from consumers to resource owners.

The process of adjustment to the shift in output demand can be illustrated in the resource market as well. See Figure 27. Initially, the representative firm's marginal value product ($MVP_o$) is equal to the equilibrium input price $P_o$ and resource quantity $r_o$. Initially, the industry's derived demand $D_o$ for the absolutely scarce resource, determines equilibrium price $P_o$ at quantity $R_o$. The derived demand curve $D_o$ is equal to the sum of the $MVP_o$ curves for N number of firms in the industry.

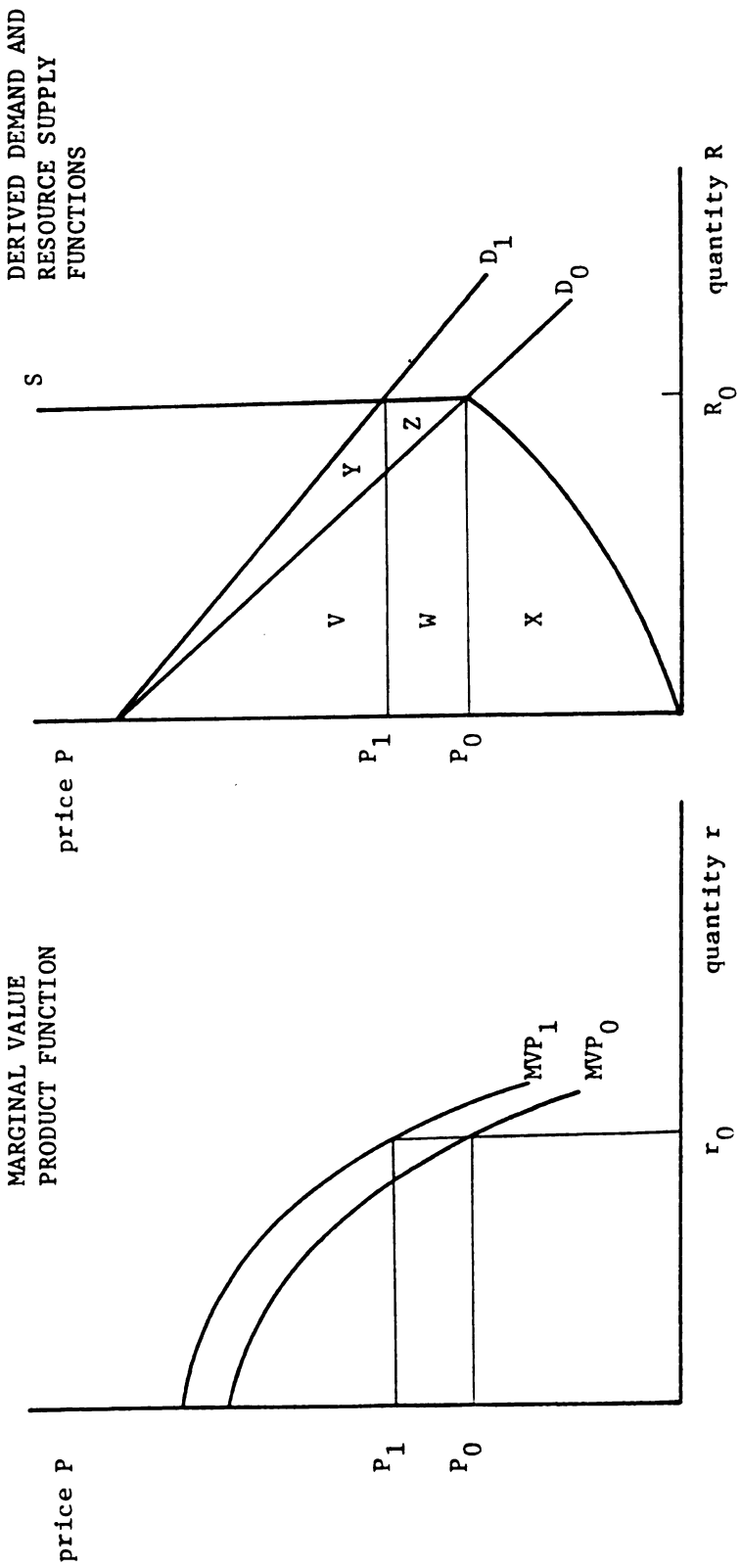


Figure 27. Ricardian Rent on an Absolutely Scarce Resource

With the subsequent increase in the demand for output, the marginal value product function of the firm increases from $MVP_o$ to $MVP_1$ as output price increases. The subsequent $MVP_1$ is equal to subsequent resource price $P_1$, and the absolutely scarce resource quantity $r_o$ is used more intensively. Subsequently, the industry's derived demand $D_1$ for the absolutely scarce resource determined the subsequent equilibrium resource price $P_1$ at quantity $R_o$. The derived demand curve $D_1$, is equal to the sum of the $MVP_1$ curves for N number of firms in the industry.

The change in economic surplus can be measured in the resource markets of the factors used in the production process. In the resource market for the absolutely scarce resource, we can illustrate the areas associated with changes in consumer surplus and economic rent.

Initial consumer surplus equals

(9) $CS_o = V + W.$

Initial economic rent equals

(10) $ER_o = X.$

Subsequent consumer surplus equals

(11) $CS_1 = V + Y.$

Subsequent economic rent equals

(12) $ER_1 = W + X + Z.$

The net cnange in consume surplus equals

(13) $CS_1 - CS_o = V + Y - (Y + W) = Y - W.$

The net change in economic rent equals

(14) $ER_1 - ER_o = W + X + Z - X = W + Z.$

The net change in economic surplus $(ES_1 - ES_o)$

(15) $ES_1 - ES_o = CS_1 - CS_o + ER_1 - ER_o = Y - W + W + Z = Y + Z.$

Let us compare the net change in consumer surplus, economic rent and economic surplus as measured in the goods market as equations (6), (7) and (8) with these same changes measured in the absolutely scarce resource market as equations (13, (14) and (15). Just and Hueth have shown that

> . . . the overall impact of introducing a distortion in some intermediate market is reflected by the sum of areas behind the general equilibrium supply and demand functions in that market (1979, p. 948).

and

> . . . the failure of perfect elasticity assumptions in a vertical market structure has no serious consequences so long as one is interested in aggregate welfare rather than the welfare of a particular set of producers or consumers (1979, p. 951).

Therefore the aggregate change in consumer surplus measured directly in the goods market is equal to the sum of the change in consumer surplus measured indirectly in the resource market of each factor needed in the production of that particular output.

(16) $CS_1 - CS_o = D - B = \sum_{i=1}^{k} (Y_i - W_i)$

where k equals the resources markets for the inputs required in production.

Similarly, the aggregate change in economic rent measure indirectly in the goods market is equal to the sum of the change in economic rent measured directly in the resource markets for each factor needed in production of that particular output.

$$(17)\quad ER_1 - ER_o = B + E = \sum_{i=1}^{k} (W_i + Z_i)$$

where k equals the resource markets for inputs required in production.

In this example, only the absolutely scarce resource has a non-zero rent or change in rent. Therefore,

$$(18)\quad ER_1 - ER_o = B + E = W + Z$$

Finally, the aggregate change in economic surplus measured in the goods market is equal to the sum of the change in economic surplus measured in the resource market for each factor needed in the production of that particular output.

$$(19)\quad ES_1 - ES_o = D + E = \sum_{i=1}^{k} (Y_i + Z_i)$$

where k equals the resource markets of the inputs required in production.

In this section, we have shown that the industry's long-run supply curve is equal to the industry's average cost including rent and its marginal cost excluding rent. We are able then to measure economic rent in the aggregate, indirectly and accurately in the goods market. This situation is circumscribed by a large set of assumptions. In the subsequent sections we will be searching for a more general case solution to the need to measure economic rent indirectly in the aggregate and accurately in the goods market. Identifying an industry's marginal and average cost functions excluding rent is the critical concern in this endeavor.

## 3-10 A Super-Abundant Resource and a Decreasing Cost Industry

In this section, we will describe an industry whose firms require one super-abundant resource as well as other abundant resources as inputs in production. First we will specify the circumstances surrounding this industry. Then we will analyze the relationship between resource availability and the industry's long run marginal cost curve excluding rent. Finally we will determine the associated direct and indirect measures of consumer surplus, economic rent and economic surplus in both the goods and resources markets for a given change in demand.

In this industry we assume atomistic competition in the goods industry and a representative firm such that the efficiency and size of this firm is equal to any other in the industry. The production function of this firm is assumed to require the use of one super-abundant resource and other abundant resources. The super-abundant resource is associated with external economies of large scale production. These external economies may be either pecuniary or technological in nature. A pecuniary external economy is a cost saving in required resources per unit of output resulting from a decrease in resource price associated with internal economies in the resource market brought about by the expansion of output by the industry as a whole. Technological external economies are a reduction in the quantity of required resources per unit of output resulting from an increase in resource quality associated with improved market performance made possible by expansion of output by the industry as a whole. All other resources in the production process are assumed to be

available in abundance at a fixed price independent of the quantity demanded by this particular industry. These abundant resources are assumed to include both variables and durables. The acquisition and salvage price of these abundant inputs are assumed equal. Further it is assumed that internal size economies exist and have been fully exploited by each firm in the industry. It is assumed therefore that output expands at the extensive margin with the entry of additional firms of equal size and efficiency. It is assumed that there are no barriers to entry or exit for firms in this industry. The number of firms are variable and determined in relation to demand for the output. Finally, it is assumed that there is an exogenous shift in demand for this industry's output. (Viner, 1931, pp. 69-71).

The process of adjustment to a shift in demand by the representative firm and by the industry is illustrated in Figure 28. Initially, the representative firm's marginal cost ($MC_0$) and average cost ($AC_0$) are equal to equilibrium price $P_0$ at output $q_0$. Initially, the industry is in long run equilibrium at price $P_0$ and output $Q_0$ associated with the aggregate production of N number of firms in the industry.

With the subsequent increase in demand, M-N additional firms, attracted by short run profits, enter the industry with production costs and size economies equal to those of the previously producing firms. Production increases from initial output $Q_0$ to subsequent output $Q_1$ associated with M firms. Since this industry is subject to external economies as output increases, the resource price of the super-abundant input decreases resulting in a decrease in both the

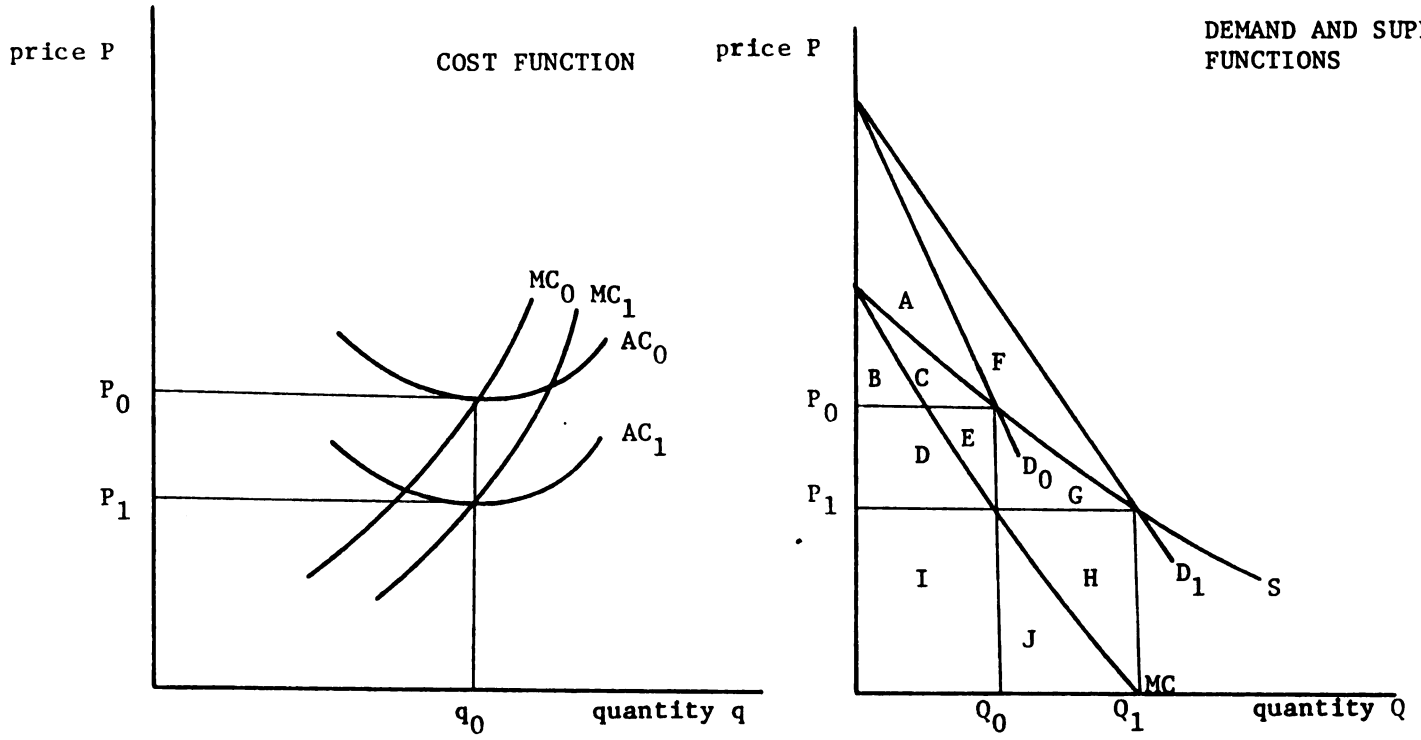


Figure 28. External Economies of Large Scale Production

average cost and marginal cost of the representative firm from $AC_0$ to $AC_1$ and $MC_0$ to $MC_1$. As production expands and the price of the super-abundant resource decreases while other resource prices remain unchanged, producers then begin to substitute the less expensive super-abundant resource for the relatively more expensive abundant resources in proprotions determined by the elasticity of substitution. The elasticity of substitution is defined as the "the proprotionate change in the ratio of the amounts of the factors employed divided by the proportionate change in the ratio of their prices to which it is due ... determined by the technical conditions of production" (Robinson, 1933, p. 256). The size of each firm remains unchanged however. "Irrespective of the change in factor proportions as we move along the long-period supply curve of the industry, the size of each firm in terms of output (as determined by the minimum average inclusive cost) is given by technology." (Mishan, 1968, p. 1276).

Let us examine the impact the decrease in price of the super-abundant resource has on the industry's long-run supply curve. Since average costs and marginal costs decrease as output increases, the industry's supply curve has a negative slope. Further this supply curve can be described as:

> The long-run average cost for each individual producer and therefore for the industry as a whole ... The long-run marginal cost curve for the industry as a whole ... would fall below the [S] curve. (Viner, 1931, p. 71).

Thus the industry's long-run supply curve is the industry's average cost curve including rent. It is also equal to the marginal cost of each firm though it is not the marginal cost curve for the industry as noted by Viner above. Therefore the industry's long-run supply curve

in the case of a super-abundant resource is not equal to the industry's marginal cost curve excluding rent. Robinson points out that when production does not include scarce factors but does include "economies of large scale industry" i.e. a super-abundant resource, then the industry's marginal cost curves excluding and including rent coincide and are both marginal to the industry's average cost curve (1933, pp. 137-138).

If we know the industry's average cost curve, then we can determine the industry's marginal cost curve because the average and marginal cost function have a unique relationship to each other. For example in Figure 28, we know that area D E I determine in relation to the average cost curve is equal to area B D I (Robinson, 1933, p. 29). Therefore we know that area E equals area B. Area E is that area above the industry's marginal cost curve excluding rent and below the initial equilibrium output price associated with initial output $Q_0$. Therefore

(1) $ER_0$ = area E = area B

Subsequent economic rent similarly defined and determined in relation to subsequent equilibrium price $P_1$ and quality $Q_1$ such that

(2) $ER_1$ = area H = area B + D

The net change in economic rent equals

(3) $ER_1 - ER_0 = B + D - B = D$

The integral of the industry's average cost function over the change in price $P_0$ to $P_1$ overestimates the change in economic rent such that

(4) $\int_{P_1}^{P_0} (S)dp$ = area D + E + G > area D

Area D is equal to the additional economic rent claimed by resource owner resulting from providing many more units of the resource for the small decrease in price. Resource owner of a super-abundant resource receive a rent associated with their ability to price discriminate over the elastic portion of their resource supply function. These resource owners can be viewed as a monopolist in the provision of the super-abundant resource the provision of which has net internal economies of large scale production.

Consumer surplus can be measured directly in the goods market also. Initial consumer surplus equals

(5) $CS_0$ = area A + B + C

Subsequent consumer surplus equals

(6) $CS_1$ = A + B + C + D + E + F + G

The net change in consumer surplus equals

(7) $CS_1 - CS_0$ = A + B + C + D + E + F + G - (A + B + C) = D + E + F + G

The net change in economic surplus equals

(8) $ES_1 - ES_0 = CS_1 - CS_0 + ER_0 - ER_0$ = D + E + F + G + D = D + F + G + H

We have explained the 'last' area D above. Area F is associated with the shift in demand from $D_0$ to $D_1$. Area D + E + G remains to be explained.

We know that

(9) Area D + E + G = area G + H

Robinson interpretes area G + H as

> "The divergence between [MC] and [S which] measures the difference between the cost of producing N units when N is being produced and the cost of producing N units when (N+1) is being produced. That is to say, it is the change in average cost, introduced by a unit increase in output, multiplied by the former

> output. This difference may be described as the <u>induced economies</u> due to the unit increase in output (1933, p. 138).

Thus the change in economic surplus associated with an industry which requires a super-abundant resource in production is the summation of three effects. The first is the net change in consumer surplus associated with the shift in demand (area F). Then there is the change in consumer surplus associated with 'induced economies' (area D + E + G). Finally there is the net change in economic rent (area D). Typically the net change in economic surplus is simply the sum of the net change in consumer surplus and economic rent (areas F + D). However the decreasing price of the super-abundant resource results in 'induced economies' which contributes to additional consumer surplus (area D E G).

The process of adjustment to the shift in output demand can be illustrated in the resource market as well. See Figure 29. Initially, the representative firm's marginal value product ($MVP_0$) is equal to the equilibrium input price $P_0$ and resource quantity $r_0$. Initially, the industry's derived demand for the super-abundant resource is $D_0$ with equilibrium price $P_0$ and resource quantity $R_0$. The derived demand curve $D_0$ is equal to the sum of the $MPV_0$ curves for N number of firms.

With the subsequent increase in demand for output, the producers use additional units of the super-abundant resource as its price decreases. The marginal value product function of the firm decreass from $MVP_0$ to $MVP_1$ as output price decreases. The subsequent $MVP_1$ is equal to subsequent resource price $P_1$, and the resource quantity $r_1$ is used. However, the lower price of the super-abundant resource does

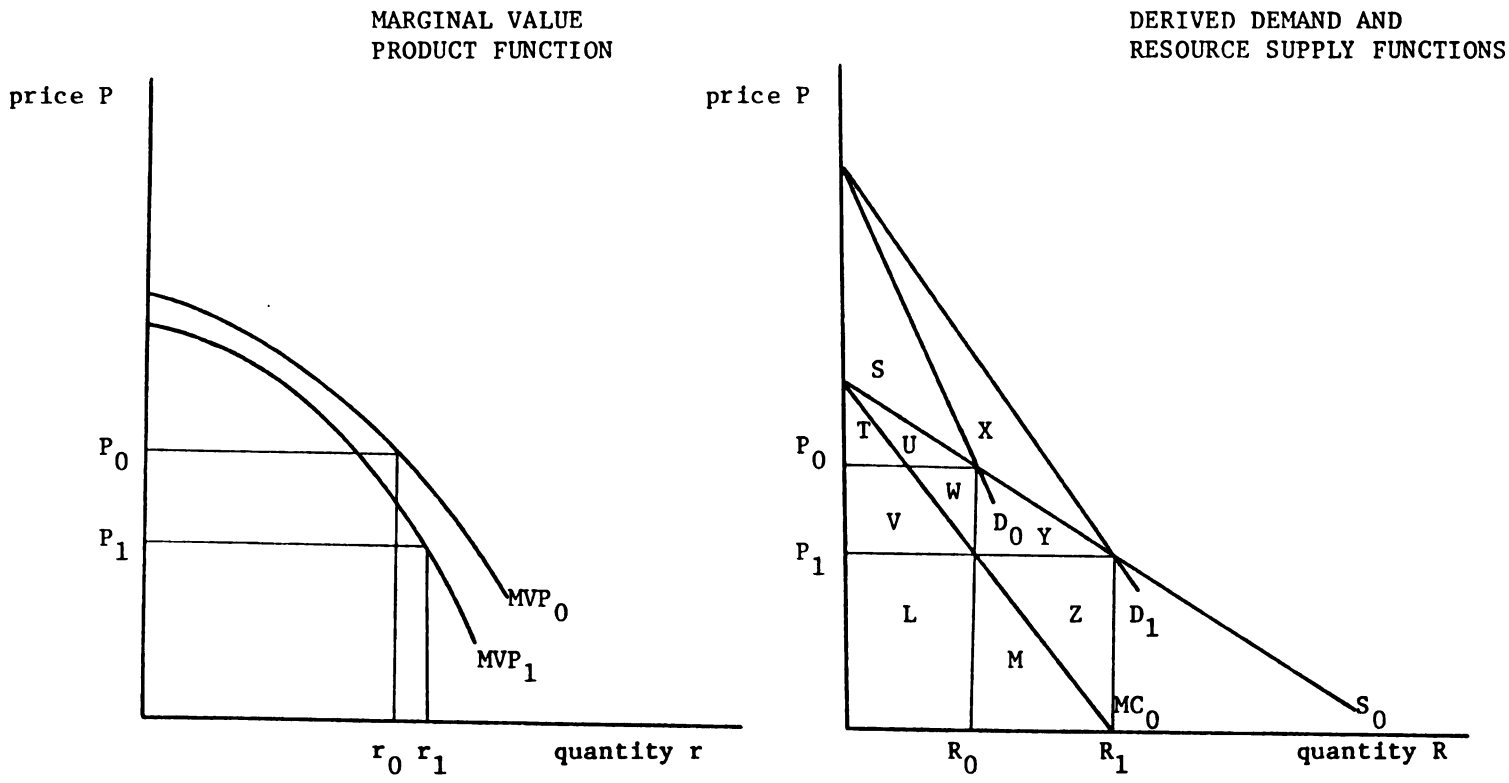


Figure 29. Rent on a Super-Abundant Resource

not cause production to expand in the previously producing firms in the long run. Size economies remain constant in the face of a change in relative input prices. M-N new firms are attached to the industry by the high short-run profits. These firms enter the industry equal in efficiency and size with the previously producing firms. Subsequently, the industry's derived demand $D_1$ for the super-abundant resource is equal to the sum of the $MVP_1$ curves for M firms in the industry. The subsequent equilibrium price for the resource is $P_1$ for output $R_1$. The resource market is assumed to be the source of external economies to the goods industry. These external economies to the good market are internal to the resource market. It is conceivable that the resource market associated with the super-abundant resource is simply a monopoly supplying the resource at an average "all or none" price with net internal economies of large scale production. These firms are sometimes called "natural" monopolies. The monopolist's long-run 'supply function' is equal to the firms average cost. The monopolists marginal cost function is uniquely related to its average cost function and his below the average cost function.

The net change in economic surplus can be measured in the resource markets of the factors in the production process. In the resource market for the super-abundant resource, we can illustrate the areas associated with consumer surplus, economic rent and economic surplus. Initial consumer surplus equals

(10) $CS_0$ = area S + T + U.

Subsequent consumer surplus equals

(11) $CS_1$ = area S + T + U + V + W + X + Y.

The net change in consumer surplus measured indirectly in the resource market equals

(12) $CS_1 - CS_0 = S + T + U + V + W + X + Y - (S + T + U)$

$= V + W + X + Y.$

Initial economic rent equals

(13) $ER_0 = W = T.$

Subsequent economic rent equals

(14) $ER_1 = Z = T + V.$

The net change in economic rent measured directly in the resource market equals

(15) $ER_1 - ER_0 = T + V - T = V.$

The net change in economic surplus equals

(16) $ES_1 - ES_0 = CS_1 - CS_0 + ER_1 - ER_0 = V + W + X + Y + V = V + X + Y + Z.$

Let us compare the net change in consumer surplus, economic rent and economic surplus as measured in the goods market as equations (3), (7) and (8) with these same changes measured in the super-abundant resource market as equations (12), (15) and (16). Based on the work of Just and Hueth (1979, pp. 948-951) we derive the aggregate change in consumer surplus measured indirectly in the resource market for each factor needed in the production of that particular output.

(17) $CS_1 - CS_0 = D + E + F + G = \sum_{i=1}^{k} (V_i + W_i + X_i + Y_i)$

where i equals the 1 through k resource markets for inputs required in the production process.

Similarly, the aggregate change in economic rent measured indirectly in the goods market is equal to the sum of the changes in economic

rent measured directly in the resource markets for each factor needed in production of that particular output.

$$(18)\quad ER_1 - ER_0 = D = \sum_{i=1}^{k} V_i$$

where i equals the 1 through k resource markets for inputs required in the production process.

In this case, only the super-abundant resource has a non-zero rent or change in rent. Therefore,

$$(19)\quad ER_1 - ER_0 = D = V$$

Finally, the aggregate change in economic surplus measured in the goods market is equal to the sum of the change in economic surplus measured in the resource market for each factor needed for the production of that particular output.

$$(20)\quad ES_1 - ES_0 = D + F + G + H = \sum_{i=1}^{k} V + X + Y + Z$$

where i equals the 1 through k resource markets for inputs required in the production process.

In this section, we have shown that in an industry in which a super-abundant resource is required in production, the industry long-run supply curve equals the industry average cost including rent but not the industry marginal cost excluding rent. In this case, it is possible to derive the industry's marginal cost curve excluding rent. It is then possible to measure economic rent, (as well as consumer surplus and economic surplus) in the aggregate, indirectly and accurately in the goods market. We also illustrated the relationship between these measures in the goods market and in the resource market.

## 3-11 A Relatively Scarce Resource and An Increasing Cost Industry

In this section, we will describe an industry whose firms require one relatively scarce resource as well as other abundant resources as inputs in production. First we will specify the circumstances surrounding this industry. Then we will analyze the relationship between resource availability and the industry's long run marginal cost curve excluding rent. Finally we will determine the associated direct and indirect measures of consumer surplus, economic rent and economic surplus in both the goods and resources markets for a given change in demand.

In this industry, we assume atomistic competition and a representative firm in the goods industry. We assume that the size and efficiency of each firm in this industry is equal to that of the representative firm. The production function of the representative firm is assumed to require the use of one relatively scarce resource and other abundant resources. The relatively scarce resource is associated with 'external diseconomies of large production'. These external diseconomies may be either pecuniary or technological in nature. Pecuniary external diseconomies are the additional cost of required resources per unit of output resulting from an increase in resource price associated with the expansion of output for the industry as a whole. An example of a pecuniary external diseconomy is a price increase for the purchase of large quantities of inputs in order to entice comparable resources from profitable employment elsewhere as output expands. Technological external diseconomies are the additional resources required per unit of output resulting from a

deterioration in resource quality brought about by expansion of output by the industry as a whole. An example of a technological external diseconomy is a need to attract increasingly less well adapted resources as output expands. (Robinson, 1931, p. 113). All other resources required in the production process are assumed to be available in abundance at a fixed price independent of the quantity demanded by this particular industry. These abundant resources are assumed to include both variables and durables. The acquisition and salvage price of the abundant inputs are assumed equal. Further it is assumed that internal size economies exist and have been fully exploited by each firm in the industry. It is therefore assumed that output expands at the extensive margin with the entry of additional firms of equal size and efficiency. It is assumed that there are no barriers to entry or exit for firms in this industry. The number of firms is variable and determined in relation to demand for the output. Finally, it is assumed that there is an exogenous shift in demand for this industry's output (Viner, 1931, 72-74).

The process of adjustment to a shift in demand by the representative firm and by the industry is illustrated in Figure 30. Initially, the representative firm's marginal cost ($MC_0$) and average cost ($AC_0$) are equal to equilibrium price $P_0$ at output $q_0$. Initially, the industry is in long run equilibrium at price $P_0$ and output $Q_0$ associated with demand $D_0$ and aggregate production of N number of firms in the industry.

With the subsequent increase in demand, M-N firms, attracted by short run profits, enter the industry with production costs and size

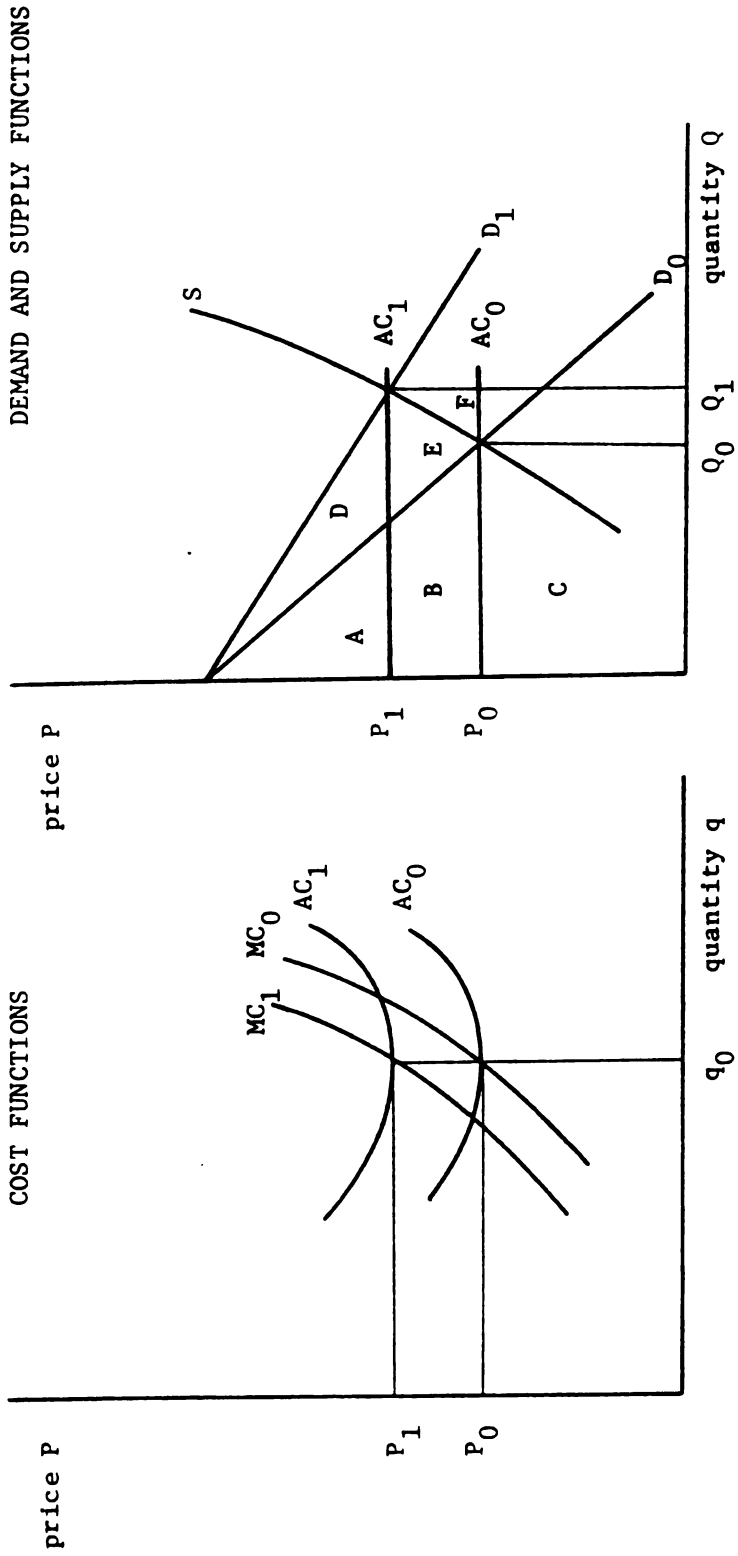


Figure 30. External Diseconomies of Large Scale Production

economies equal to those of the previously producing firms. Production increases form initial output $Q_0$ to subsequent output $Q_1$ associated with M firms. Since this industry is subject to external diseconomies as output increases, the resource price of the relatively scarce resource increases resulting in an increase in both the average cost and marginal cost of the representative firm from $AC_0$ and $MC_0$ to $AC_1$ and $MC_1$. As production expands, the price of the relatively scarce resource increases while other resource prices remain unchanged. Producers then begin to substitute the less expensive abundant resources for the more expensive relatively scarce resource in proportions determined by the elasticity of substitution. (Ferguson, 1975, p. 90-92). The size of each firm remains unchanged however.

> Irrespective of the change in factor proportions as we move along the long-period supply curve of the industry, the size of each firm in terms of output (as determined by the minimum average inclusive cost) is given by technology. (Mishan, 1968, p. 1276).

Let us examine the impact the increase in price of the relatively scarce resource has on the industry's long-run supply curve. Since average costs and marginal costs increase as output increases, the industry's supply curve has a positive slope reflecting increasing intensity in the use of the relatively scarce factor. Further this supply curve can be described as:

> The trend of the individual average (and also marginal) long-run costs and therefore also of the industry long-run average cost as the industry as a whole varies its output. This is also the long-run supply curve for the industry as a whole. The long-run marginal cost curve [including rent] for the industry as a whole ...would rise above the [S] curve. (Viner, 1931, pp. 73-74).

Thus the industry's long-run ordinary supply curve S is the industry's average cost including rent. It is also equal to the individual firms average and marginal cost including rent. But it is not equal to the marginal cost curve for the industry including rent as noted by Viner above. Nor is it equal to the marginal cost curve for the industry excluding rent. Mishan states:

> the industry supply curve is, admittedly, an average cost curve including rents...but it is not also a marginal curve excluding rent... (1968, p. 1277).

Relative to $AC_1$, the representative firm's initial average total cost curve $AC_0$ is equal to the firm's average cost excluding rent. Similarly since $AC_0$ equals $P_0$, the $P_0$ equals the industry's average cost curve excluding rent (and also the industry's marginal cost curve excluding rent). Total rent can be measured as average rent per unit of output ($AC_1 - AC_0$) multiplied by the units of output. (Robinson, 1933, p. 136, fn. 2). Similarly total rent can be measured as the area above the industry's marginal cost curve excluding rent and below the equilibrium output price. By either measure the change in economic rent is the same.

Initially economic rent equals (for reasons stated below)

(1) $ER_0 = 0$.

Subsequent economic rent equals

(2) $ER_1$ = area B + E + F.

The net change economic rent equals

(3) $ER_1 - ER_0 = B + E + F$.

The industry's ordinary supply function underestimates the change in rent because the supply function is equal to the integral of

individual firm's marginal cost functions from $P_0$ to $P_1$ which include the change in rent in the new resource price.

The integral of the industry's average cost function over the change in price $P_0$ to $P_1$ underestimates the change in economic rent such that

(4) $\int_{P_0}^{P_1} (S)dp$ = area B + E < area B + E + F

Area B E F is equal to the additional payment received by non-marginal units of the relatively scarce resource as the resource price increases in order to secure the services of the marginl units of the resource as output expands. Resource owners of the non-marginal units of the relatively scarce resource receive a rent by virtue of being necessary in production and available in less than perfectly elastic supply.

Area C does not enter into the initial estimate of economic rent because we assumed or inferred that $AC_0$ relative to $AC_1$ equals average cost excluding rent. Area C is an underestimate of subsequent economic rent already capitalized in the resource price of the relatively scarce resource in $AC_0$ relative to a prior and yet lower and unspecified average total cost function for some lower and unspecified industry equilibrium output. Stated another way, "... the area above the rising industry supply curve carries no economic significance" (Mishan, 1968, p. 1277).

Consumer surplus can be measured quite simply in the commodity market. Initially consumer surplus equals

(5) $CS_0$ = area A + B.

Subsequent consumer surplus equals

(6) $CS_1$ = area A + D.

The net change in consumer surplus equals

(7) $CS_1 - CS_0 = A + D - A - B = D - B$.

The net change in economic surplus equals

(8) $ES_1 - ES_0 = CS_1 - CS_0 + ER_1 - ER_0 = D - B + B + E + F = D + E + F$.

The net change in economic surplus for an industry which requires a relatively scarce resource in production is the summation of two effects: the net change in both consumer surplus (area D) and economic rent (area E + F). Area B represents a transfer of income from consumers to resource owners.

The process of adjustment to the shift in output demand can be illustrated in the resource market as well. See Figure 31. Initially, the representative firm's marginal value product ($MVP_0$) is equal to equilibrium resource price $P_0$ and resource quantity $r_0$. Initially, the industry derived demand for the relatively scarce resource is $D_0$ with equilibrium price $P_0$ and resource quantity $R_0$. The derived demand curve $D_0$ is equal to the sum of the $MVP_0$ curves for N number of firms.

With the subsequent increase in demand for output, the producers use additional units of the relatively scarce resource as its marginal value in production increases. The marginal value product function of the firm increases from $MVP_0$ to $MVP_1$ as output price increases. The subsequent $MVP_1$ is equal to subsequent resource price $P_1$ and resource quantity $q_1$ is used. However, the higher price of the relatively

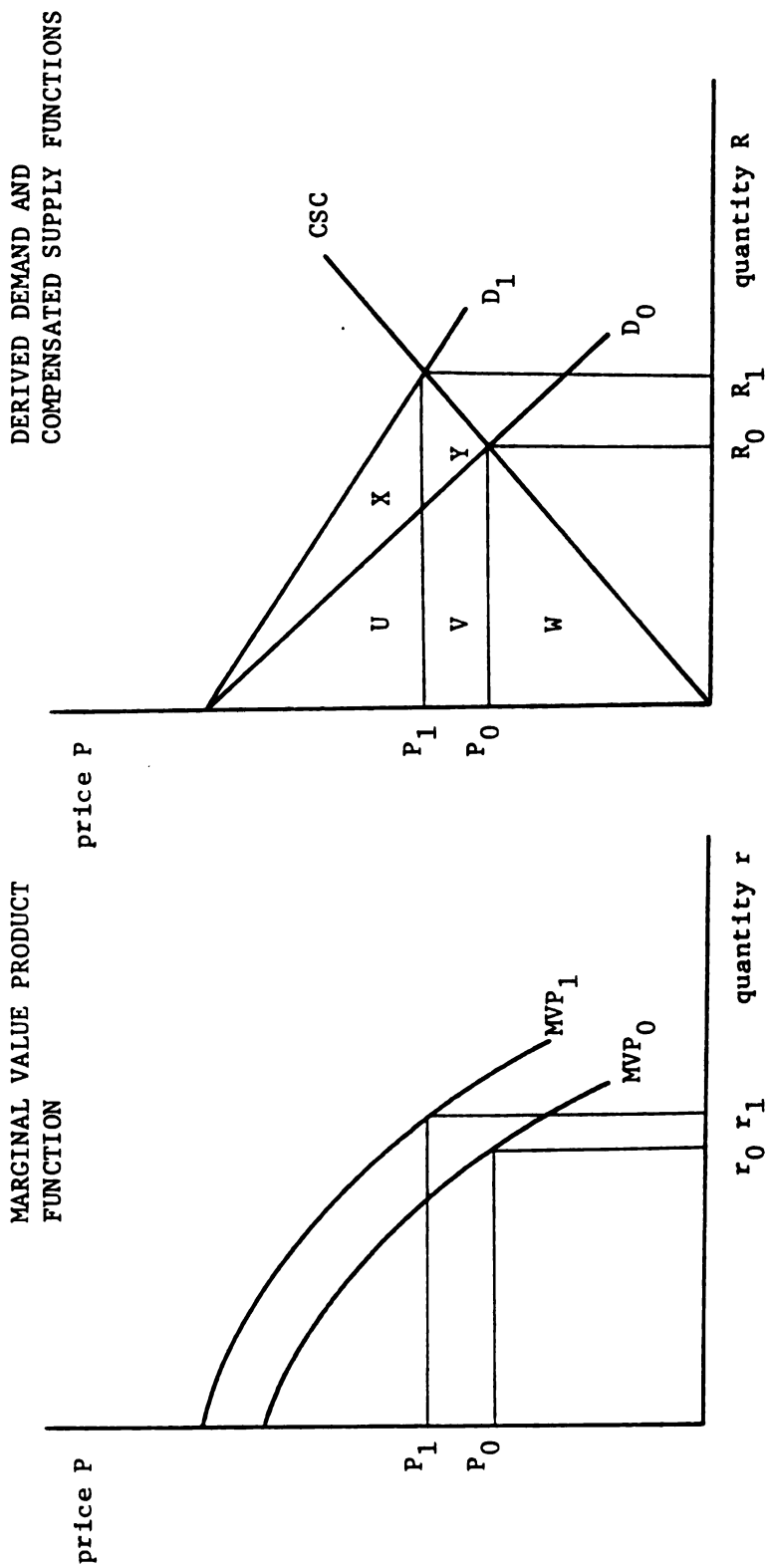


Figure 31. Rent on a Relatively Scarce Resource

scarce resource does not cause production to contract in representative firm in the long run. Size economies remain constant in the face of changing relative resource prices. M-N new firms are attracted to the industry by the high short-run profits. These firms enter the industry equal in efficiency and size with previously producing firms. Subsequently, the industry's derived demand $D_1$ for the relatively scarce resource is equal to the sum of the $MVP_1$ curves for N number of firms. The subsequent equilibrium price for the resource is $P_1$ for output $R_1$. The resource market is assumed to provide the relatively scarce resource primary to producers of the commodity in the industry under consideration. Thus the industry as a whole faces the supply function of the relatively scarce resource and not just a fixed resource price. However, individual producers act as if the industry did face a single price for the relatively scarce resource over the relevant range of quantities. As a whole these producers must pay a higher per unit price on the non-marginal resource units in order to secure the use of the marginal units as output expands. Non-marginal units receive a rent regardless of whether the units are homogeneous (pecuniary diseconomies) or heterogeneous (technological diseconomies). (Robinson, 1933, p. 113).

The net change in economic surplus can be measured in the resource markets of the factors required in the production process. In the resource market for the relatively scarce resource, we can illustrate the areas associated with economic rent, consumer surplus and economic surplus. Initially economic rent equals

(9) $ER_0$ = area W.

Subsequently economic rent equals

(10) $ER_1$ = area V + Y + W.

The net change in economic rent measured directly in the resource market equals

(11) $ER_1 - ER_0 = V + Y + W - W = V + Y$.

Initially consumer surplus equals

(12) $CS_0$ = area U + V.

Subsequently consumer surplus equals

(13) $CS_1$ = area U + X.

The net change in consumer surplus measured indirectly in the resource market equals

(14) $CS_1 - CS_0 = U + X - U - V = X - V$.

The net change in economic surplus equals

(15) $ES_1 - ES_0 = CS_1 - CS_0 + ER_1 - ER_0 = X - V + V + Y = X + Y$.

Let us compare the net change in economic rent, consumers surplus and economic surplus as measured in the goods market as equations (3), (7) and (8) with these same changes measured in the relative scarce resource market as equations (11), (14) and (15). Based on the work of Just and Hueth (1979, pp. 948-951) we determine that the aggregate change in economic rent measured directly in the resource market for each factor required in the production of a particular output.

$$(16)\quad ER_1 - ER_0 = B + E + F = \sum_{i=1}^{k} (V_i + Y_i)$$

where i equals the 1 through k resource markets for inputs required in the production process.

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In this case only the relatively scarce resource has a non-zero rent or change in rent. Therefore

(17) $ER_1 - ER_0 = B + E + F = V + Y.$

Similarly, the aggregate change in consumer surplus measured directly in the goods market is equal to the change in consumer surplus measured indirectly in the resource market for each factor needed in the production of that particular commodity.

(18) $CS_1 - CS_0 = D - B = \sum_{i=1}^{k} (X_i - V_i)$

where i equals the 1 through k resource market for inputs required in the production process.

Finally, the aggregate change in economic surplus measured in the commodity market is equal to the sum of the change in economic surplus measured in the resource market for each factor needed for the production of that particular commodity.

(19) $ES_1 - ES_0 = D + E + F = \sum_{i=1}^{k} (X_i + Y_i) = \sum_{i=1}^{k} (X_i) + Y$

where i equals the 1 through k resource market for inputs required in the production process.

In this section, we have shown that in an industry in which a relatively scarce resource is required in production, the industry's long-run supply curve equals the industry's average cost including rent but not the industry's marginal cost excluding rent. In this case, it is possible to derive the industry's average cost excluding rent. We found that using the industry supply curve, results in an underestimation of economic rent. However with the industry's average

cost excluding rent it is possible to measure economic rent (as well as consumer surplus and economic surplus) in the aggregate, indirectly and accurately in the commodity market. We also illustrated the relationship between these measures in the commodity market and the resource market.

## 3-12 A Relatively Scarce Durable Resource and An Increasing Cost Industry

In this section we will describe an industry whose firms require one relatively scarce durable resource as well as other abundant resources as inputs in production. First, we will specify the circumstances surrounding this industry. Then we will analyze the relationship between resource availability and the industry's long run marginal cost curves excluding rent. Finally, we will determine the associated direct and indirect measures of economic rent, consumer surplus and economic surplus in both the goods and resource markets for a given change in demand.

In this industry, we assume atomistic competition in the commodity and resource markets. We assume that firms in the commodity market are of equal size and efficiency which can be respresented by a single "representative firm." The production function of the representative firm is assumed to require the use of one relatively scarce durable resource and other abundant resources. The scarce durable resource is associated with external diseconomies of large production. These external diseconomies may be either pecuniary or technological in nature. Further it is assumed that the acquisition price of this

durable is greater than its salvage value adjusted for depreciation and maintenance. All other resources required in the production process are assumed to be available in abundance at a fixed price independent of the quantity demanded by this particular industry. These abundant resources are assumed to include both variables and durables. The acquisition and salvage prices of the abundant resources are assumed equal. Further it is assumed that internal size economies exist and have been fully exploited by each firm in the industry. It is therefore assumed that output expands at the extensive margin with the entry of additional firms of equal size though not necessarily equal efficiency. It is assumed that there are no barriers to entry for firms in this industry. There is however a barrier to exit for firms in this industry associated with the salvage value of relatively scarce durables. The number of firms entering the industry is determined in relation to the demand for the output. Finally, it is assumed that there is an exogenous shift in demand for this industry's output (Viner, 1931, pp. 72-74; Johnson and Quance, 1972, pp. 22-40).

The process of adjustment to a shift in demand by the representative firm and the industry is illustrated in Figure 32. Initially, the representative firm's marginal cost ($MC_o$) and average total cost ($AC_o$) are equal to equilibrium price $P_o$ at output $q_o$. Initially, the industry is in longrun equilibrium at price $P_o$ and output $Q_o$ associated with demand $D_o$ and aggregate production of N number of firms in the industry.

With the subsequent increase in demand, M-N firms, attracted by short run profits, enter the industry with size economies equal to

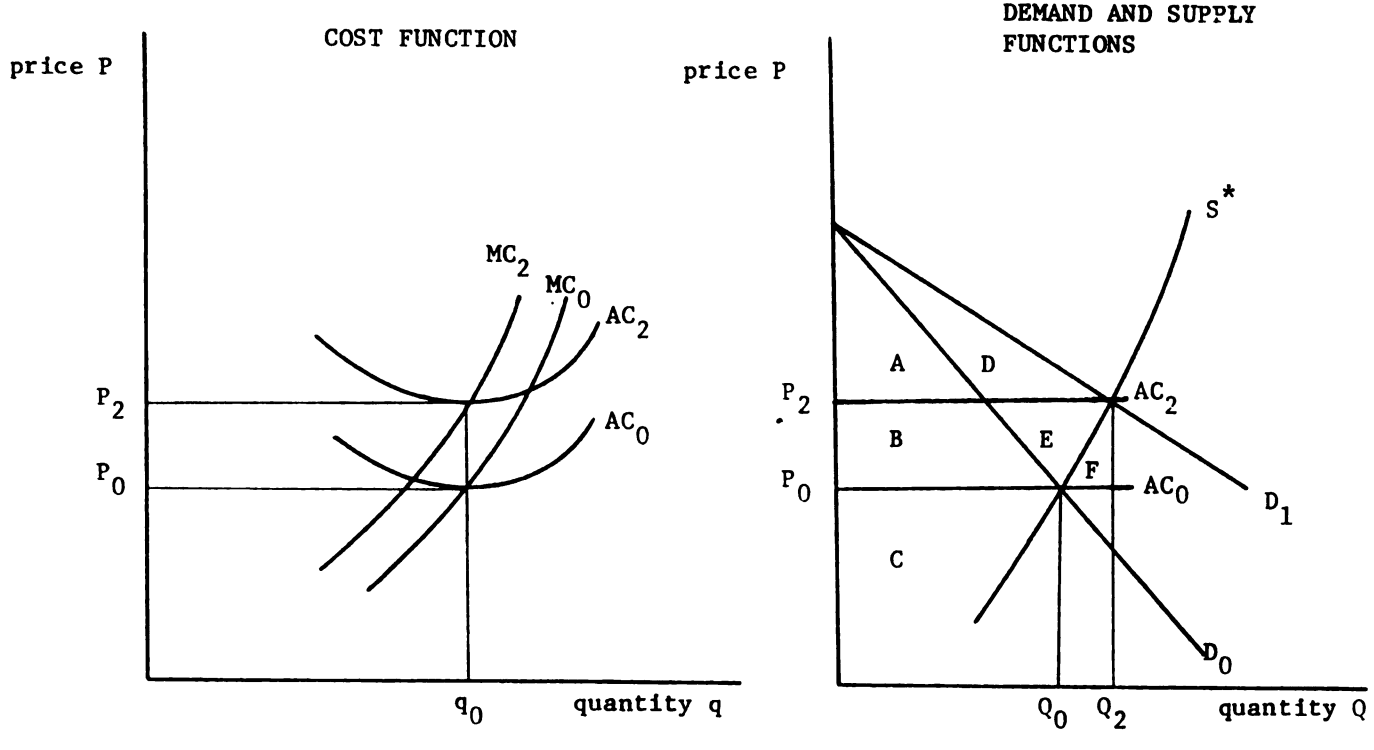



Figure 32. External Diseconomies of Large Scale Production

those of previously producing firms. Production increases from initial output $Q_o$ to subsequent output $Q_2$ associated with M firms. Since this industry is subject to external diseconomies as output increases, the resource price of the relatively scarce durable resource increases resulting in an increase in both average cost and marginal cost of the representative firm from $AC_o$ to $AC_2$. As production expands, the price of the relatively scarce durable resource increases while other resource prices remain unchanged. Producers begin to substitute the less expensive abundant resources for the more expensive relatively scarce durable resource in proportions determined by the elasticity of substitution (Ferguson, 1975, p. 90-92). The size of the firm remains unchanged (Mishan, 1968, p. 276).

Let us examine the impact the increase in price of the relatively scarce durable resource has on the industry's long-run ordinary supply curve. Since average cost and marginal costs increase as output increases, the industry supply curve has a positive slope reflecting increasing intensity in the use of the relatively scarce resource. The supply curve $S^*$can be described as the industry long run average cost curve including rent but not the marginal cost excluding rent. It is equal to the individual firms' average and marginal costs including rent. (Viner, 1931, pp. 72-74; Mishan, 1968, p. 1277). Relative to $AC_2$, the representative firm's initial average cost $AC_o$ is equal to the firm's average cost excluding rent. Similarly since $AC_o$ equals $P_o$, then $P_o$ equals the industry's average cost curve excluding rent as well as the industry's marginal cost curve excluding

rent. Total rent can be measured as average rent per unit of output ($AC_2 - AC_0$) multiplied by the units of output. Average rent per unit is the difference between average cost including rent ($AC_2$) minus average cost excluding rent ($AC_0$) determined for a given level of output (Robinson, 1933, p.136, fn. 2). Similarly total rent can be measured as the area above the industry's marginal cost curve excluding rent and below the equilibrium out put price. By either measure the change in economic rent is the same.

Initially economic rent equals

(1) $ER_o = 0.$

Subsequently economic rent equals

(2) $ER_1 = \text{area } B + E + F.$

The net change in economic rent equals

(3) $ER_1 - ER_o = B + E + F.$

The industry's ordinary supply function underestimates the change in rent because the supply function is equal to the sum of individual firms' marginal cost functions which include the change in rent associated with the higher resource price. The integral of the industry's average cost function over the change in price $P_o$ to $P_2$ underestimates the change in economic rent such that

(4) $\int_{P_o}^{P_2} (S)dp = \text{area } B + E < \text{area } B + E + F$

Area B E F is equal to the additional payment received by the non-marginal units of the relatively scarce durable resource as the resource price increases in order to secure the services of the marginal units as output expands. Resource owners of the non-marginal units of

the relatively scarce durable resource receive a rent by virtue of being required in production and available in less than perfectly elastic supply.

Consumer surplus can be measured in the commodity markets also. Initially consumer surplus equals

(5) $CS_o$ = area A + B.

Subsequently consumer surplus equals

(6) $CS_1$ = area A + D.

The net change in consumer surplus equals

(7) $CS_1 - S_o = A + D - A - B = D - B.$

The net change in economic surplus equals

(8) $ES_1 - ES_o = ER_1 - ER_o + CS_1 - CS_o = B + E + F + D - B = D + E + F.$

The net change in economic surplus for an industry which requires a relatively scarce durable resource in production is the summation of two effects: the net change in both consumer surplus (area D) and economic rent (area E F). Area B represents a transfer of income from consumers to resource owners. As we shall see in the subsequent discussions the transfer of income from consumers to resource owners as well as the economic rent to resource owners is less than would be the case if the relatively scarce durable were in fact a relatively scarce variable.

The process of adjustment to the shift in output demand can be illustrated in the resource market as well. See Figure 33. Initially, the representative firm's marginal value product ($MVP_o$) is equal to equilibrium resource acquisition price $P_{oA}$ and resource quantity $r_o$. Resource salvage price ($P_{oS}$) is some uniform percentage, less

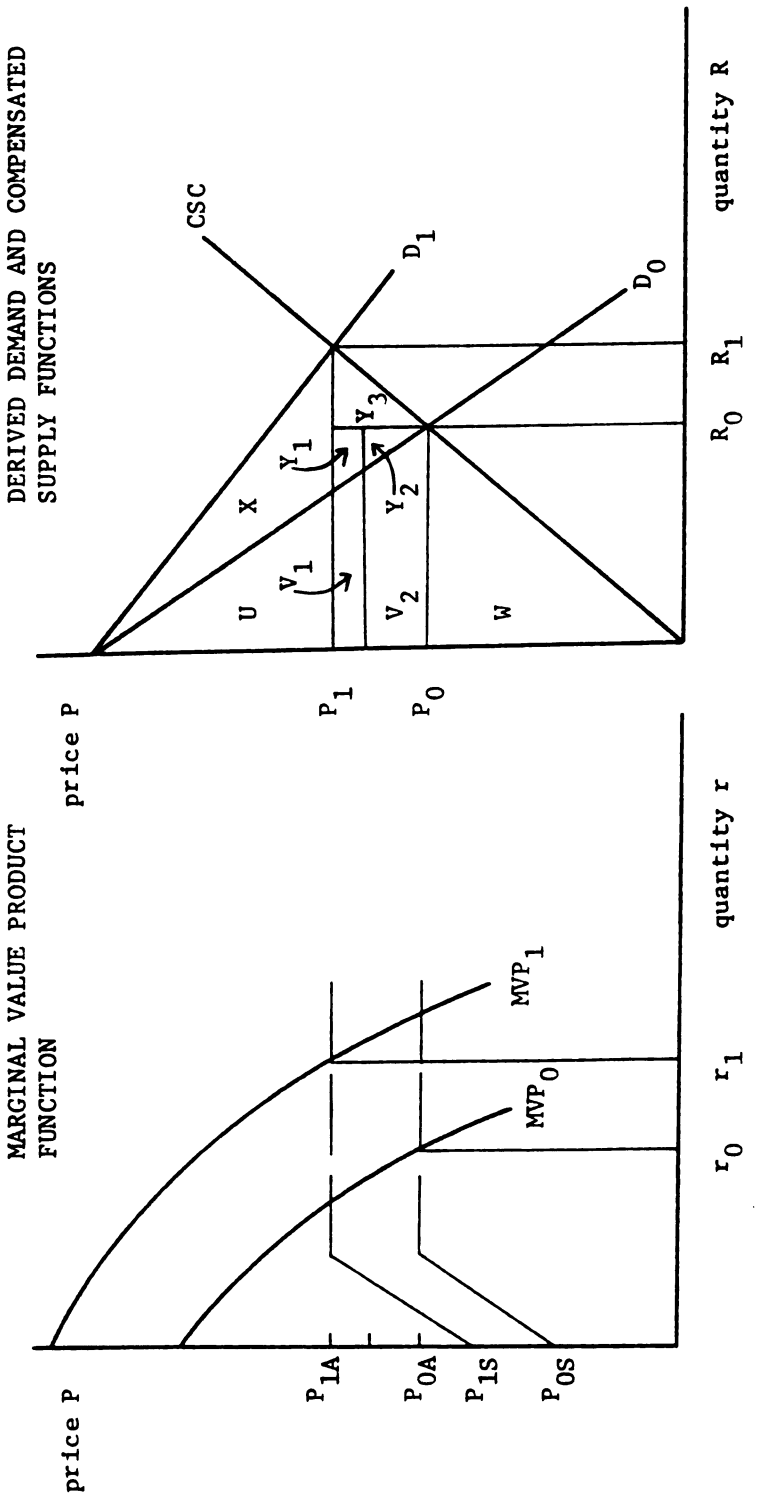



Figure 33. Quasi-Rent on a Relatively Scarce Durable Resource

than acquisition price. Initially, the industry's derived demand for the relatively scarce resource is $D_o$ with equilibrium acquisition price $P_o$ and resource quantity $R_o$. The derived demand curve $D_o$ is equal to the sum of the $MVP_o$ curves for N firms.

With the subsequent increase in demand for output, producers use additional units of the relatively scarce durable resource as its marginal value product increases. The marginal value product function of the firm increases from $MVP_o$ to $MVP_1$ as output increases. The subsequent marginal value product ($MVP_1$) is equal to the subsequent resource acquisition price $P_{1A}$ and resource quantity $r_1$ is used. However for the previously producing firms the subsequent marginal value product of the durable is greater than initial acquisition price on the initial units of durable r. Previous producers are in a position to receive a quasi-rent on the initial units of durable r. New entrants into the industry do not receive quasi-rent since they purchase the optimal units of the relatively scarce durable resource associated with the higher resource prices. Therefore the production costs of the entering firms are greater than the previously producing firms.

In this situation, previous producers may exercise the option of increasing the rate of extraction of services from a durable resource as the relatively scarce durable's resource price increases. By increasing the rate of extraction of services, however, producers implicitly decrease the useful life of the durable. Producers might also increase the rate of repair and maintenance services relative to

the rate of extracting services which would further increase the useful life of the durable. (Baquet, 1978, pp. 111-122). By increasing the useful life of the durable, the durable is replaced less frequently. The less frequently a durable is replaced the fewer opportunities there are for resource suppliers to realize the increased capital gains associated with each newly replaced relatively scarce durable resource. On the other hand, the newly producing firms have an "optimal" stock of a more expensive resource from which they are extracting service at a rate, the implicit price of which, equals the marginal value product. This implicit price is equal to acquisition price ($P_{1A}$) and above salvage value ($P_{1s}$).

In Figure 33, it is assumed that the representative firm owns two units of the relatively scarce durable resource. When the output price increases the aggregate demand for the scarce resource increases as M-N new firms are attracted to the industry by short-run profits. Subsequently, the industry's derived demand for the relatively scarce resource is equal to the sum of the $MVP_1$ curve for M number of firms. The subsequent equilibrium acquisition price for the resource is $P_{1A}$ for output $R_1$. Subsequent equilibrium salvage price ($P_{1S}$) for the resource is a uniform percentage less than acquisition price (adjusted for depreciation and maintenance) across all units of the relatively scarce durable resource. At the higher acquisition price, firms entering the industry purchase the number of units of the durable such that the optimal flow of services from the durable equals acquisition price ($P_{1A}$). Thus the newer entrants have an optimal stock of a more expensive resource. Therefore the new entrants have a higher

cost of production than the previously producing firms. Therefore, firms are of equal size but not of equal efficiency.

The resource market is assumed to provide the relatively scarce durable resource primarily to producers of the commodity in the industry under consideration. Thus the industry as a whole faces the supply function of the relatively scarce durable resource and not just a fixed resource price. However, individual producers act as if the industry did face a single price for the relatively scarce durable resource over the relevant range of quantities. As a whole, the new entrant producers are assumed to pay a higher per unit price on the infra-marginal resource units in order to secure the use of the marginal units as output expands. Infra-marginal units (owned by the supplier or previous producers) are assumed to receive a rent regardless of whether the units are homogeneous (associated with pecuniary diseconomies) or hetergeneous (associated with technological diseconomies). The process of paying resource owners the capitalized value of the relatively scarce durable by replacing durables at the end of an expected useful life is precluded by the previously producing firms. These firms have a stock of the durable, the initial acquisition price ($P_{oA}$) is less than its marginal value product. As long as the value of the flow of services from the durable is greater than its initial acquisition price then the previous producers may increase the rate of extraction of services until its implicit price equals the marginal value product in production. This results in shortening the life of the durable, which may be offset by an increase in maintenance and repairs. If the life of the durable is extended,

this deprives resource owners or suppliers of the associated capitalized value. The effect is to redistribute the net change in economic rent by redistributing the transfer of income from consumers to resource owners by an amount approximately equal to $V_1$ plus $V_2$. $V_1$ plus $V_2$ is quasi-rent captured by previous producers.

The net change in economic surplus can be measured in the resource markets of the factors required in the production process. In the resource market for the relatively scarce durable resource, we can illustrate the areas associated with economic rent, consumer surplus and economic surplus.

Initially economic rent equals:

(9) $ER_o$ = area W.

Subsequently economic rent equals (including quasi-rent)

(10) $ER_1$ = area $V_1 + Y_1 + V_2 + Y_2 + Y_3 + W$.

The net change in economic rent measured directly in the resource market equals

(11) $ER_1 - ER_o = V_1 + Y_1 + V_2 + Y_2 + Y_3 + W - W = V_1 + Y_1 + V_2 + Y_2 + Y_3$

Initially consumer surplus equals

(12) $CS_o$ = area $U + V_1 + V_2$.

Subsequently consumer surplus equals

(13) $CS_1$ = area $U + X$.

The net change in consumer surplus measured indirectly in the resource market equals

(14) $CS_1 - CS_o = U + X - (U + V_1 + V_2) = X - V_1 - V_2$.

The net change in economic surplus equals

$$(15)\quad ES_1 - ES_o = ER_1 - ER_o + CS_1 - CS_o = V_1 + Y_1 + V_2 + Y_2 + Y_3 + X - V_1 - V_2 = Y_1 + Y_2 + Y_3 + X.$$

Let us compare the net change in economic rent, consumer surplus and economic surplus as measured in the goods market as equations (3), (7), and (8) with these same changes measured in the relatively scarce durable resource market as equations (11), (14) and (15). Based on the work of Just and Heuth (1979, pp. 948-951) we determine that the aggregate change in economic rent measured indirectly in the goods market is equal to the sum of the change in economic rent measured directly in the resource market for each resource required in the production of a particular output.

$$(16)\quad ER_1 - ER_o = B + E + F = \sum_{i=1}^{k} (V_{1i} + Y_{1i} + V_{2i} + Y_{2i} + Y_{3i})$$

where i equals the 1 through k resource market for inputs required in the production process.

In this case only the relatively scarce durable resource has a non-zero change in rent. Therefore

$$(17)\quad ER_1 - ER_o = B + E + F = V_1 + Y_1 + V_2 + Y_2 + Y_3.$$

Similarly, the aggregate change in consumer surplus measured directly in the goods market is equal to the change in consumer surplus measured indirectly in the resource market for each factor needed in the production of the particular commodity.

$$(18)\quad CS_1 - CS_o = D - B = \sum_{i=1}^{k} (X_i - V_{1i} + V_{2i}) = \sum_{i=1}^{k} (X_i) + V_1 - V_2$$

where i equals the 1 through k resource markets for inputs required in the production process.

Finally, the aggregate change in economic surplus measured in the commodity market is equal to the sum of the change in economic surplus measured in the resource market for each factor needed for the production of that particular commodity.

$$(19)\quad ES_1 - ES_o = D + E + F = \sum_{i=1}^{k} (Y_{1i} + Y_{2i} + Y_{3i} + X_i)$$

$$= \sum_{i=1}^{k} (X_i) + Y_1 + Y_2 + Y_3$$

where i equals the 1 through k resource markets for inputs required in the production process.

In this section, we have shown that in an industry in which a relatively scarce durable resource is required in production, the industry's long-run supply curve equals the industry's average cost including rent, but not the industry's marginal cost excluding rent. In this case, it is possible to determine the industry's average cost excluding rent as well as the industry's marginal cost excluding rent. We found that using the industry supply curve results in an underestimation of economic rent. However with the industry's average cost curve excluding rent it is possible to determine economic rent as well as consumer surplus and economic surplus in the aggregate and accurately in the commodity market. We also illustrated the relationship between these measures in the commodity market and the resource market. In particular we determine that for a durable previously producing firms may adjust to the increase in resource price by increasing

the rate of extraction of service. This decreases the life of the durable which may be offset by an increase in maintenance, which reduces the turnover of capital thereby depriving resource owners of capitalized value. This situation redistributes the transfer of income from resource suppliers to resource owners. Previous producers are able to capture "long-run" quasi-rents.

## 3-13 Economic Surplus and New Technology

In sections 3-7 to 3-12, we have described, analyzed and illustrated the relationships between types of resources necessary in the production of a commodity and the change in economic rent, consumer surplus and economic surplus. We have discussed the condition under which these changes can be measured accurately in either the commodity or resource market. The analysis so far has assumed a shift in commodity demand. However, we are particularly interested in measuring the change in economic surplus associated with a shift in supply. This shift in supply is assumed to be the result of an introduction of a new technology in the firms' production process. The measure of change for a shift in supply is subject to theoretical and empirical difficulties. We have discussed in general terms the theoretical difficulties of accurately measuring economic surplus in the commodity market. We found that we needed to be able to identify the industry's marginal cost curve excluding rent or the uniquely related average cost curve excluding rent. To this point, we have not discussed the empirical difficulties related to determining either an industry's marginal or average cost curve excluding rent.

In this section, we will attempt to do several things. First, we will describe and illustrate the change in economic surplus, economic rent and consumer surplus for a given shift in supply. Then we will analyze the change in economic rent in light of our previous insights into the theoretical aspects of accurately measuring economic rent in the commodity market. Then we discuss how economic surplus can be measured empirically so that it is consistent with our theoretical understanding. To this end, a "kinked" supply function is presented as a means of measuring economic surplus which is empirically consistent with our theoretical understanding. The "kinked" supply curve is supplemented with the particular expense curve as a means of providing additional information regarding the Y-intercept term.

We begin by analyzing the change in economic surplus, economic rent and consumer surplus for a shift in supply resulting from the introduction of a new technology. We assume that the industry is the same as the one described in section 3-11. The industry in that section was one in which firms required a relatively scarce resource as well as abundant resources. We further assume a shift in supply when a new technology is introduced and adopted. This new technology is assumed to reduce the proportion of an abundant resource needed to produce a given level of output. As a result, the marginal productivity of the remaining required resources to produce a given quantity of a commodity increases. The average and marginal cost of a firm decreases and the industry average cost curve including rent shifts to the right or decreases. See Figure 34. Initially economic rent ($ER_o$)

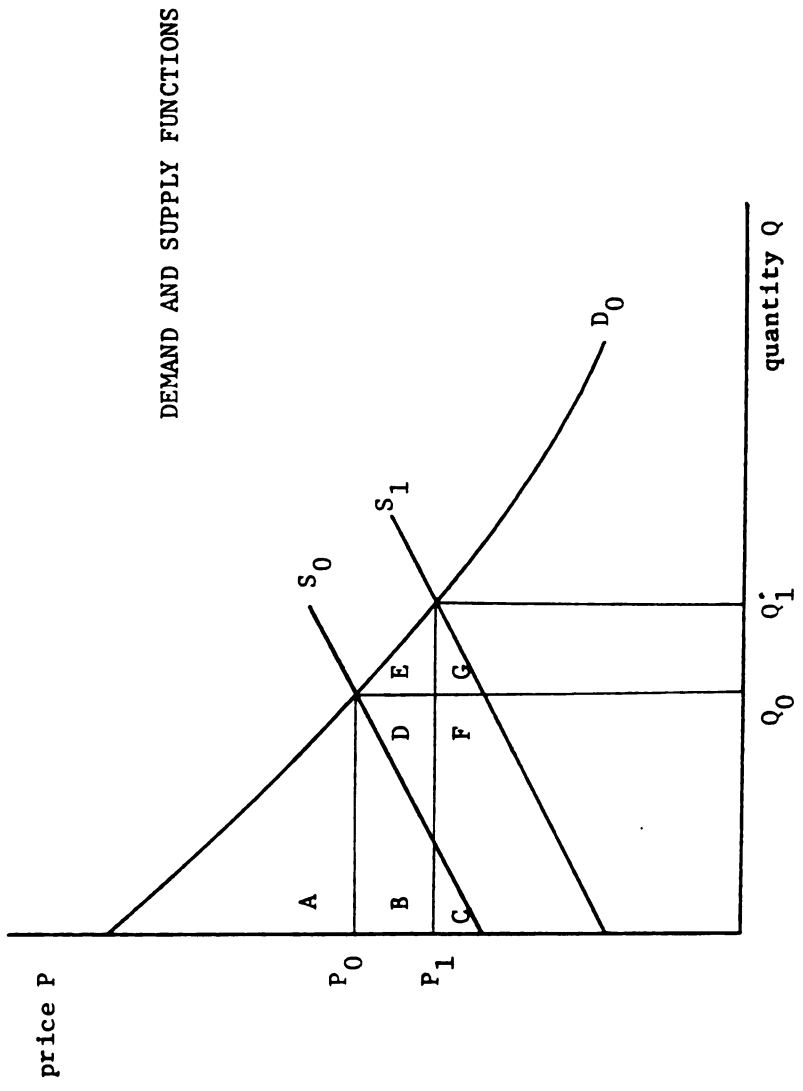



Figure 34. Economic Surplus and a Shift in Supply

approximately equals

(1) $ER_o \approx$ area B + C

Subsequently economic rent ($ER_1$) approximately equals

(2) $ER_1 \approx$ area C + F + G

able to capture "long-run" quasi-rents. In the goods market this

The net change in economic rent approximately equals

(3) $ER_1 - ER_o \approx C + F + G - B - C = F + G - B$

Equations (1), (2) and (3) are only approximations of economic rent because they are measured in relation to the industry ordinary supply curve which we have determined under estimates, the change in economic rent.

Initially consumer surplus ($CS_o$) equals

(4) $CS_o =$ area A

Subsequently consumer surplus ($CS_1$) equals

(5) $CS_1 =$ area A + B + D + E

The net change in consumer surplus equals

(6) $CS_1 - CS_o = A + B + D + E - A = B + D + E$

The net change in economic surplus approximately equals

(7) $ES_1 - ES_o = ER_1 - ER_o + CS_1 - CS_o$

$\approx F + G - B + B + D + E = F + G + D + E$

The net change in economic surplus is equal to the net change in economic rent approximated by (area F + G) and consumer surplus (area D + E). Area B is a transfer of income from resource owners to consumers.

Let us determine the change in economic surplus such that it can be accurately measured in the commodity market. See Figure 35. The

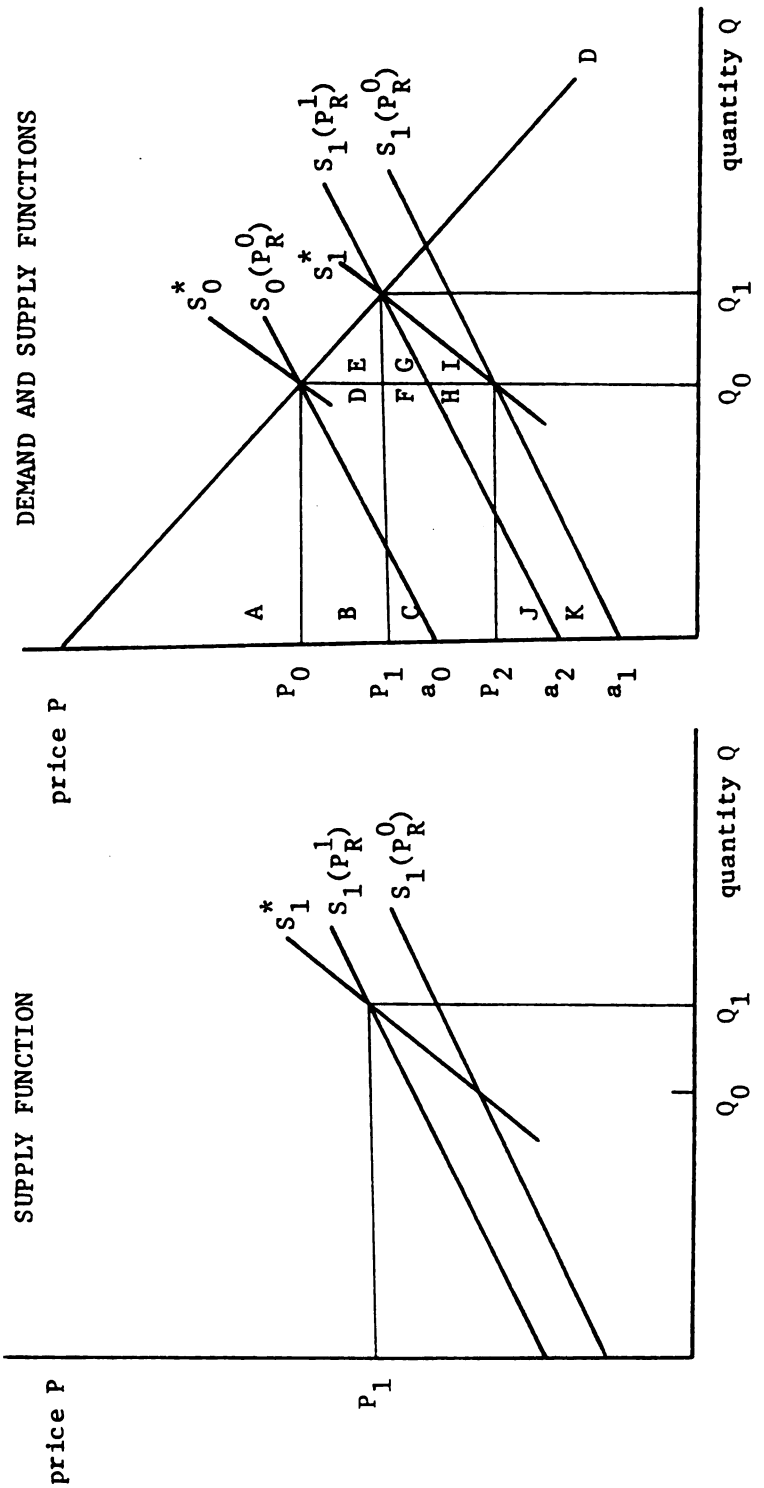



Figure 35. Ordinary and Equilibrium Supply Curves

subsequent ordinary supply curve for the industry equals $S_1(P_R^1)$. This is the industry's average cost curve including rent. The relatively scarce resource price $P_R$ has increased from $P_R^o$ to $P_R^1$ as output expands from $Q_o$ to $Q_1$. The commodity price has decreased from $P_o$ to $P_1$. The higher resource price is included in the industry's subsequent average cost curve $S_1(P_R^1)$. If the relatively scarce resource price had remained at $P_R^o$ as output expanded, then the industry's average cost curve excluding rent would have been equal to $S_1(P_R^o)$ (Just et al., 1982, pp. 178-179). $S_1^*$ is the equilibrium price and quantity combination associated with each ordinary supply curve. $S_1^*$ is therefore identified as the equilibrium supply curve.

> . . . $S_N^*$ may be interpreted as the competitive supply curve for the goods industry, which takes account of equilibrium adjustments in the output market . . . If input price is intimately related to output prices. . . then such a supply curve may be readily discernible; after all, only the locus of points along $S_N^*$ would be observed in response to any series of output price changes . . . For this reason, a supply curve, such as $S_N^*$, may be called an equilibrium adjustment of input use and input price as output price changes. The important property of such a curve is that the change in producer surplus defined with respect to an equilibrium supply curve measures the net change in quasi-rent for all affected producing industries. . . (Just et al., 1982, p. 181).

We can measure the net change in economic surplus as the average surplus per unit of output between the average cost curves $S_o(P_R^o)$ and $S_1(P_R^i)$ where $P_R^o \leq P_r^i \leq P_R^1$. The net change in economic surplus can be viewed as the sum of the net change in (a) the economic surplus to produce the original output $Q_o$ when $Q_1$ is produced and (b) the economic surplus to produce the expanded output $Q_1 - Q_o$ when $Q_1$ is produced. The measure of economic surplus (a) equals the average economic

surplus per unit $P_o - P_2$ multiplied by $Q_o$ units of output. Price $P_2$ is equal to the average cost of producing the initial output $Q_o$ excluding rent with the new technology. Therefore

$$(8)\quad ES_1 - ES_o \Big|_{Q_o} = Q_o(P_o - P_2) = \text{area } B + C + D + F + H$$

The measure of economic surplus (b) equals the average economic surplus per unit of $1/2\ (P_1 - P_2)$ and $1/2\ (P_o - P_1)$ multiplied by $(Q_1 - Q_o)$ units of output. It is assumed that D and $S^*$ are linear. Therefore

$$(9)\quad ES_1 - ES_o \Big|_{Q_1 - Q_o} = 1/2(P_1 - P_2)(Q_1 - Q_o) + 1/2(P_o - P_1)(Q_1 - Q_o)$$
$$= 1/2\ (P_o - P_2)(Q_1 - Q_o)$$
$$= \text{area } E + G + I$$

The net change in economic rent associated with the equilibrium supply shift from $S_o^*$ to $S_1^*$ given the introduction and adoption of a new technology equals the sum of equations (8) and (9) such that

$$(10)\quad ES_1 - ES_o = Q_o(P_o - P_2) + 1/2(Q_1 - Q_o)(P_o - P_2)$$
$$= P_o - P_2(Q_o + 1/2(Q_1 - Q_o))$$
$$= \text{area } B + C + D + E + F + G + H + I.$$

If we assume a parallel shift in the equilibrium and ordinary supply functions, then area B + C equals area J + K. Therefore equation (10) can be rewritten as

$$(11a)\quad ES_1 - ES_o = B + C + D + E + F + G + H + I$$

$$(11b)\quad ES_1 - ES_o = \qquad D + E + F + G + H + I + J + K.$$

Equations (11a) and (11b) equal the average and marginal cost measures of economic surplus respectively.

Furthermore we know that initially consumer surplus equals

(12) $CS_o$ = area A

Subsequently consumer surplus equals

(13) $CS_1$ = Area A + B + D + E

The net change in consumer surplus equals

(14) $CS_1 - CS_o = A + B + D + E - A = B + D + E$.

We can determine the net change in economic rent by subtracting equation (14) from equation (11). Therefore the net change in economic rent equals

(15a) $ER_1 - ER_o = B + C + D + E + F + G + H + I - (B + D + E)$

(15b) $= D + E + F + G + H + I + J + K - (B + D + E)$

(15c) $= C + F + G + H + I$

(15d) $= F + G + H + I + J + K - B$.

Equations (15C) and (15d) equal the average and marginal cost measures of economic rent respectively. From equation (15d) we can infer that initially economic rent equals

(16) $ER_o$ = area B + C

and that subsequently economic rent equals

(17) $ER_1$ = area C + F + G + H + I + J + K.

Equation (16) is an accurate measure of initial economic rent in that $S_o(P_R^o)$, the initial ordinary supply function, is taken as the industry's marginal cost function excluding rent. This is true in the sense that area B + C is taken as rent associated with initial output $Q_o$ before the supply shift which must be subtracted in order to

measure only the change in economic rent. Equation (17) is an accurate measure of subsequent economic rent in that the kinked line segment subtended by $S_1(P_R^o)$ and $S_1^*$ represents the industry's marginal cost excluding rent.

Therefore equation (16) can be rewritten as

$$(18)\quad ER_o = \int_{\alpha_o}^{P_o} \left(S_o(P_R^o)\right)dp = \text{area B + C}$$

and equation (17) can be rewritten as

$$(19)\quad ER_1 = \int_{\alpha_1}^{P_2} \left(S_1(P_R^o)\right)dp + \int_{P_2}^{P_1} \left(S_1^*\right)dp = \text{area C + F + G + H + I + J + K}$$

We have shown that for the purposes of measuring initial economic rent, the initial average cost curve including rent $S_o(P_R^o)$ equals the industry's marginal cost curve excluding rent. This is the same result we determined in the absolutely fixed resource-increasing cost industry case. For the purposes of measuring subsequent economic rent, the subsequent average cost curve excluding rent $S_1(P_R^o)$ and the subsequent equilibrium supply curve $S_1^*$ together define the industry's marginal cost curve excluding rent.

Let us turn our attention now to empirically estimating the measures of economic surplus which we have determined to be theoretically accurate. Typically, ordinary supply functions such as $S_o$ and $S_1$ are estimated using standard econometric techniques such as ordinary least squares or two stage least squares. An "ordinary" supply function is one in which the quantity of output of a commodity is a function of its own price and required resource prices (Tomek and Robinson, 1972, p. 66).

(20) $S_i : Q_t = a + b_1 P_t - b_2 P_{Rt} + e_t$

where $Q_t$ = quantity produced in time t

$P_t$ = commodity price in time t

$P_{Rt}$ = resource price in time t

$e_t$ = random error term in time t

a, $b_1$, and $b_2$ are the parameter of the equation

The inclusion of the resource price $P_R$ as an independent variable is a means of explicitly holding the price of the resource constant in an effort to determine the relationship between quantity and price of the output. For example, we are interested in determining the ordinary supply function $S_1$ assocaited with the new technology and resource price $P_R^o$. Unfortunately, the commodity price P and the resource price $P_R$ are not independent of each other. We know that in the case of a relatively scarce resource as output expands the resource price increases, which causes the commodity price to increase. Therefore P and $P_R$ are directly related to each other and therefore subject to multicollinearity.

Multicollinearity is a result of the violation of the assumption in multiple regression that "none of the independent variables are linear combinations of the others" (Kelejian and Oats, 1974, p. 122). Multicollinearity has the effect of making it very difficult to get reliable estimates of the ($b_1$) coefficient (slope of the ordinary supply curve) and ($b_2$) coefficient (the effect of changes in resource price on output). Reliable estimates of the ($b_1$) coefficient rather than accurate predictions is our primary concern for the supply model.

The estimation of the intercept term (a) also represents a problem. It is particularly difficult to accurately measure the intercept term using econometric means. There are typically very few, if any, data points near the Y axis on which to base an estimate of the intercept term. Consequently ". . . there is an increasing loss in precision in estimating a surplus change as the price limits diverge from the mean of the observed data" (Just et al., 1982, p. 169). On this same point, Lindner and Jarrett state:

> In general, we do not believe that econometrically estimated supply curves provide reliable estiamtes of the intercept term . . . because the available observations on P and Q used to calculate supply parameters are typically far removed from the point where the supply curve intercepts the vertical axis. Actual estimates of supply curves frequently involve or imply, negative intercept terms . . . it is clearly illogical as it imples that producers are prepared to supply positive quantities at zero price in the long run. (1978, p. 51).

Let us attempt to address these problems of estimating the parameters ($b_1$), and (a) in turn. A solution exists for accurately measuring ($b_1$), the slope of the supply curve, when perfect multicollinearity exists. The solution is simply to drop one of the two collinear independent variables and let the remaining independent variable estimte the combined effect (Kelejian and Oats, 1974, p. 127). Since we are interested in determining the elasticity of supply involving commodity price and quantity, then the remaining independent variable is the relativley scarce resource price. When we drop the relatively scarce resource price from the model, then the estimate of the ($b_1$) coefficient equals the slope not of the ordinary supply curve but rather of the equilibrium supply curve (Just and Hueth, 1979, p. 951-952). As long as we assume a parallel shift in the

equilibrium supply function, then the estimate of (b) is the same for $S_o^*$ and $S_1^*$. To assume a parallel shift in the equilibrium supply function does not presume a parallel shift in the ordinary supply function. In particular the parallel shift in the equilibrium supply function determines the vertical shift in supply measured at initial output $Q_o$. A shift in the ordinary supply function determines the vertical shift in supply measured at the Y intercept as (a).

From our discussion and analysis in preceding sections, we have shown that the slope of a commodity's ordinary supply function relates to the elasticity of substituting an abundant resource for a non-abundant resource. Further we determined that the elasticity of substitution is a function of the production technology. Therefore, a parallel shift implies that the new technology reduces the required units of an abundant resource without affecting the elasticity of substitution. However, it is also possible that a new technology reduces the required units of a non-abundant resource by changing the elasticity of substitution. There are four general types of ordinary supply shifts that are associated with the introduction of a new technology. These four types include parallel, proportional, pivotal and convergent. See Figure 36. Each of these types of shifts have a substantially different Y intercept term.

Thus a parallel shift implies no change in the elasticity of substituting abundant resources for the relatively scarce resource. A proportional shift implies an increase in the elasticity of substituting abundant resources for the relatively scarce resource. A pivotal supply shift implies an even greater increase in the elasticity of

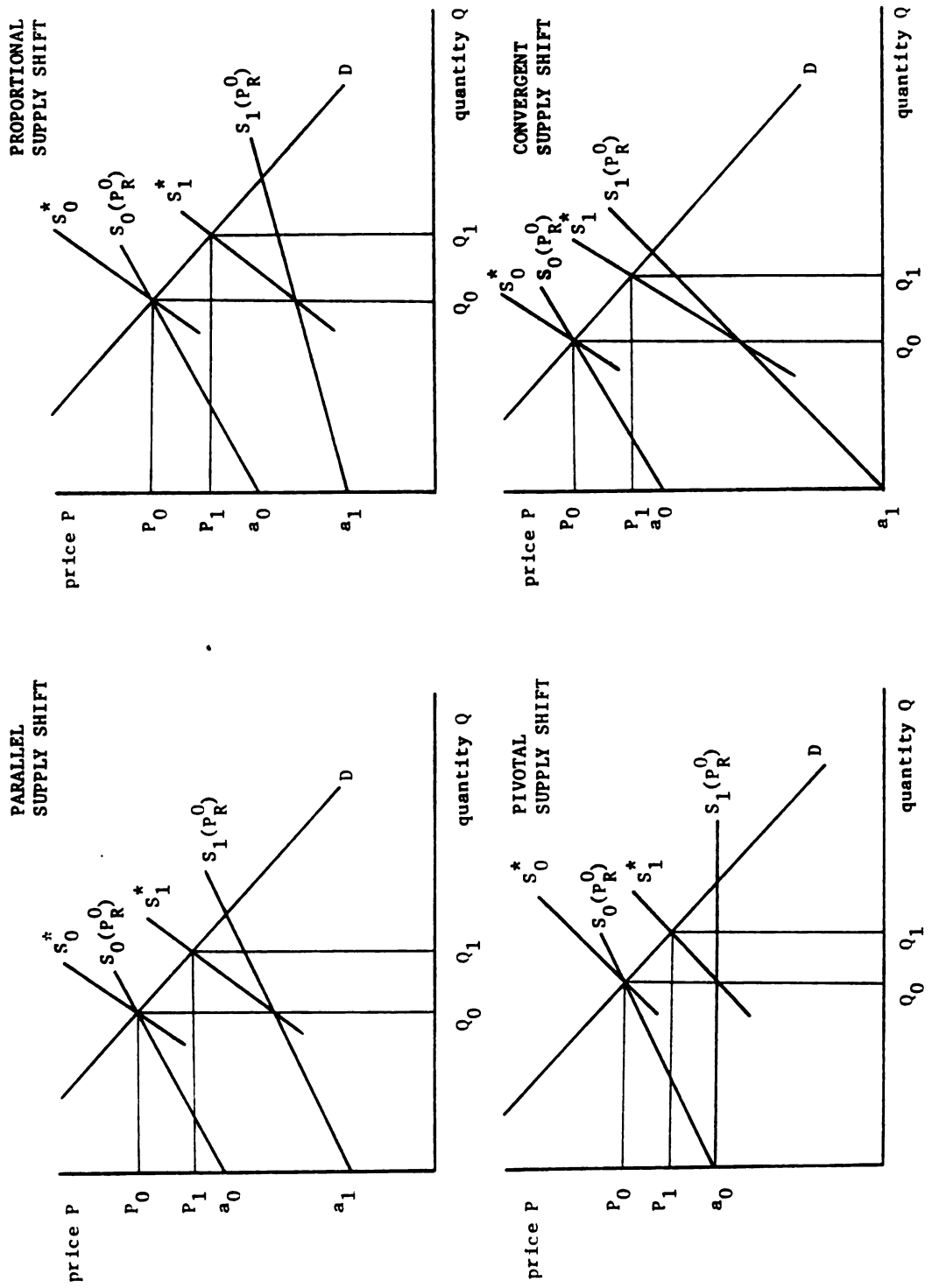



Figure 36. Types of Shifts in the Ordinary Supply Function

substituting the abundant resources for the relatively scarce resource. A pivotal shift also implies that the new technology has effectivley transformed the relatively scarce resource into an abundant resource or at least much less scarce resource from the point of view of the industry. A convergent shift implies a decrease in the elasticity of substituting abundant resources for the relatively scarce resource. From the point of view of the industry, the new technology has transformed the relatively scarce resource into an absolutely scarce resource or at least a much more relatively scarce resource. The type of supply shift is not only important in measuring net economic surplus but also important in measuring the incidence of benefits between consumers and resource owner as distinct groups (Duncan and Tisdell, 1971, pp. 124-129; Ayer and Schuh, 1972, pp. 564-566; Lindner and Jarrett, 1978, pp. 55-57). Rose has shown that the assumption regarding the type of shift has resulted in substantial overestimates of economic surplus (1980, p. 836). We have shown that the use of the average cost curve including rent to measure economic surplus underestimates actual economic surplus in equation (7). Rose's result implies that the assumption regarding the supply shift 'swamps' the underestimation from using the incorrect supply function for a net effect of overestimating economic surplus. Both sources of error are to be avoided.

In a series of articles by Lindner and Jarrett; Rose; Wise and Fell, a technique has evolved for estimating net economic surplus which implicitly incorporates additional assumptions regarding the shift in the intercept term into the supply model (1978, pp. 48-55;

1980, pp. 834-836; 1980, pp. 838-840). This work in turn builds upon a model developed by Pinstrup-Andersen, de Londono and Hoover (1976, pp. 132-34). The Lindner et al. model is designed as "a general formula for measuring the area of [economic surplus] which is independent of the nature of the supply shift, provided linear demand and supply can reasonably be assumed" (Lindner and Jarrett, 1978, p. 48).

We have determined in equation (11) that economic surplus associated with a parallel shift in the equilibrium and ordinary supply functions can be determined two ways - in relation to average or marginal cost functions. See Figure 37. Therefore, in this case the net change in economic surplus equals

(21) $ES_1 - ES_o = A + B + D = B + C + D.$

If the turning points of these areas can be determined, then it is possible to determine the areas using a technique referred to as "the rule of cross multiplication". (Lindner and Jarrett, 1980, p. 50).

Let us measure the contiguous areas of B C and D.

The coordinates of the turning points of area B C are

(22) $(0, A_o)$

(23) $(0, A_1)$

(24) $(Q_o, P_1(1-k)$

(25) $(Q_o, P_o)$ (Rose, 1980, p. 834).

The coordinates of the turning points of area D are

(26) $(Q_o, P_o)$

(27) $(Q_o, P_1(1-k))$

(28) $(Q_1, P_1)$.

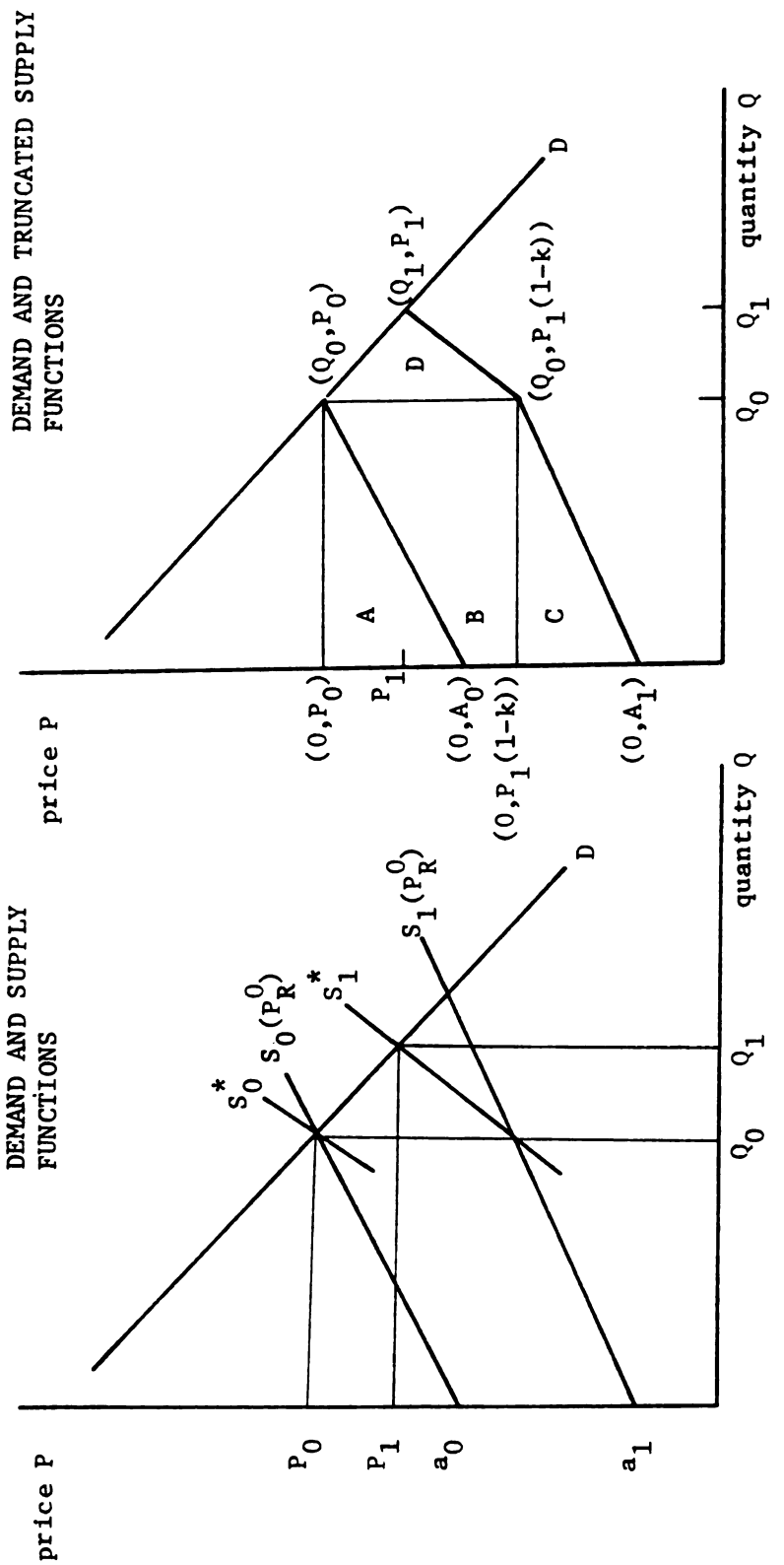



Figure 37. Economic Surplus and the Kinked Supply Curve

The empirical estimates of these coordinates determined ex post (i.e. after the supply shift has occurred) equals

(29) $P_1$ given

(30) $Q_1$ given

(31) $P_o = P_1 / [1 + (kE/(E + N))]$ (Pinstrup-Andersen et al., 1976, p. 134)

(32) $Q_o = Q_1 / [1 + (kEN/(E + N))]$ (Pinstrup-Andersen, et al. 1976, p. 134)

(33) $A_o$ by assumption

(34) $A_1$ by assumption

(35) $k = (A_o - A_1)/P_1$ : defined as a cost decrease excluding rent as a proportion of subsequent price, the difference in production cost $(A_o - A_1)$ measured empircally (Rose, 1980, p. 834).

Initial price and quantity $(P_o, Q_o)$ are defined as the equilibrium combination which would exist in the absence of the technology and subsequent supply shift other things remaining equal. The Pinstrup-Andersen et al. technique for estimating initial price and quantity depends on accurately estimating demand and supply elasticities N and E. The demand elasticity N is the absolute value of compensated demand function, associated with the subsequent equilibrium price and quantity $(P_1, Q_1)$. The supply elasticity E is the value of the subsequent equilibrium supply $S_1^*$ associated with the subsequent equilibrium price and quantity $(P_1, Q_1)$. It is assumed that the shift in equilibrium supply curves from $S_o^*$ to $S_1^*$ is parallel regardless of the associated shift in the ordinary supply function. This is a

necessary assumption in order to accurately estimate $P_o$ and $Q_o$ using equations (31) and (32).

Once the coordinates of the turning points are determined, it is possible to determine the equations for areas B C and D.

$$(36)\ \text{area B + C} = 1/2\ [0P_1(1 - k) + Q_oP_o + Q_oA_o + 0A_1 - 0A_o - 0P_o - Q_oP_1(1 - k) - Q_oA_1]$$
$$= 1/2\ Q_o[P_o - P_1 + kP_1 + A_o - A_1]$$

$$(37)\ \text{area D} = 1/2\ [Q_oP_1 + Q_1P_o + Q_oP_1(1 - k) - Q_oP_o - Q_oP_1 - Q_1P_1(1 - k)]$$
$$= 1/2[P_o(Q_1 - Q_o) + P_1(1 - k)(Q_o - Q_1)]$$

Adding equations (36) and (37) provides the general equation for net economic surplus.

$$(38)\ ES_1 - ES_o = \text{area B + C + D}$$
$$= 1/2\ Q_o[P_o - P_1 + kP_1 + A_o - A_1] + 1/2\ [P_o(Q_1 - Q_o) + P_1(1 - k)(Q_o - Q_1)]$$

We can see the intercept terms $A_o$ and $A_1$ appear explicitly in the estimation of area B C only. $A_o$ and $A_1$ are implicit in k as $(A_o - A_1)$. However, the difference in $A_o - A_1$ determined at the initial output $Q_o$ is different from determining $A_o$ and $A_1$ individually at the Y-intercept. Thus it is possible to estimate a cost decrease excluding rent $(A_o - A_1)$ without knowing the absolute values of $A_o$ and $A_1$. However, it is still possible to make some simplifying assumptions regarding the type of shift in the ordinary supply shift and make estimates of economic surplus accordingly.

Lindner and Jarrett made the following assumptions regarding the intercepts terms $A_o$ and $A_1$ in determining net economic surplus ex post.

| Type of shift | Intercept assumption | Intercept shift | $Q_o$ shift |
|---|---|---|---|
| (39) parallel: | $A_o = A_1 + kP_1$ ; | $k = \frac{A_o - A_1}{P_1}$ ; | $k = \frac{A_o - A_1}{P_1}$ |
| (40) proportional: | $A_o = A_1(1 + k)$ ; | $k = \frac{A_o - A_1}{A_1}$ ; | $k = \frac{A_o - A_1}{P_1}$ |
| (41) pivotal: | $A_o = A_1$ ; | $k = 0$ ; | $k = \frac{A_o - A_1}{P_1}$ |
| (42) convergent: | $A_o = A_1 + 2kP_1$ ; | $k = \frac{A_o - A_1}{2P_1}$ ; | $k = \frac{A_o - A_1}{P_1}$ |

(1978, p. 52).

By substituting the intercept assumptions of equations (39), (40), (41) and (42) in the general equation for economic surplus (38) we can develop the particular equations for economic surplus associated with a parallel, proportional, pivotal and convergent shifts in the ordinary supply functions respectively. Area D is not affected by the shift in the Y- intercept. Therefore the equations for net economic surplus for the various types of shifts are as follows:

| Type of Shift | Economic Surplus |
|---|---|
| (43) Parallel: | $ES_1 - ES_o = \text{area D} + 1/2\ Q_o[P_o - P_1 + 2kP_1]$ |
| (44) Proportional: | $ES_1 - ES_o = \text{area D} + 1/2\ Q_o[P_o - P_1 + kP_1 + kA_1]$ |
| (45) Pivotal: | $ES_1 - ES_o = \text{area D} + 1/2Q_o[P_o - P_1 + kP_1]$ |
| (46) Convergent: | $ES_1 - ES_o = \text{area D} + 1/2Q_o[P_o - P_1 + 3kP_1]$. |

These equations provide the correct estimates of economic surplus for an assumed shift in the ordinary supply function, in an industry with

firms requiring a relatively scarce resource. It is possible to use equations (43) through (46) to provide a range of possible estimates of economic surplus for a given assumption regarding the type of supply shift. It is ironic that Rose was the first to accurately specify these "kinked" supply equations to measure economic surplus yet he believed them to be theoretically invalid (Rose, 1980, p. 836). This study shows that these "kinked" supply estimates of economic surplus are theoretically valid in the case of an industry with firms which require a relatively scarce resource.

It is possible to use the Rose equations of economic surplus along with estimates of consumer surplus to determine economic rent to resource owners. The limitation of this approach is that the intercept terms themselves are assumed rather than actually estimated. The problem of estimating the initial and subsequent Y-intercept terms associated with a shift in the ordinary supply function remains.

An alternative to estimating the intercept term econometrically is needed. Let us begin by re-examining the assumptions associated with the industry's average cost curve including rent. In the cases analyzing the relationship between resource availability and the industry's average cost curve we consistently assumed "the size and efficiency of each firm in an industry is equal to that of a representative firm." Therefore, to know the average cost of the representative firm is to know the average cost of the industry. To know more than the representative firm's average cost is redundant given this assumption. The representative firm by definition cannot be construed

in any way as a marginal enterprise. The representative firm assumption implies, and it was explicitly stated as an assumption, that "internal economies exist and have been fully exploited by each firm in the industry" This in turn implies, and it was again explicitly assumed, that "output expands at the extensive margin with the entry of firms of equal size and efficiency." Finally, this implies and it was assumed, that there are "no barriers to entry or exit for firms in this industry."

This set of four interrelated representative firm assumptions was necessary to clearly specify the theoretically accurate measures of economic surplus, economic rent and consumer surplus in both the commodity and resource markets. However, these assumptions are not particularly realistic. Furthermore, these assumptions are not necessary to accurately measure the industry's marginal cost curve. When we relax this set of representative firm assumptions, the nature of the industry average cost curve excluding rent changes. An average firm in such an industry can be defined as a representative firm. However, an average firm does not equal the industry's average cost function but rather an average of the average cost function. Similarly, to know more than the average firm's average cost function is not redundant, it is useful. Additional information on the average costs of other firms provides useful information on marginal and non-marginal firms within the industry. A comparison of non-marginal, average and marginal firms by size provides information regarding size economies for instance. A comparison of the weighted average of these groups of firms by geographical regions provides information regarding

comparative advantage. As more and more information is known it becomes possible to array all the firms in an industry in increasing order of efficiency measured in terms of cost of production. The result would be a "particular expense" curve (Viner, 1932, pp. 74-78).

Viner defines the particular expense curve as "a curve representing the array of actual average costs of the different producers in the industry when the total output of an industry was a given amount, these individual costs being arranged in increasing order of size from left to right . . ." (1931, p. 76). The particular expense curve is also known as a 'bulk-line cost curve', 'accountants' cost curve' and 'statistical cost curve'. (Viner, 1931, p. 76). The particular expense curve is a short-run average cost curve. It is an average cost curve by definition. It is short-run because it measures

> transitory fluctuations in the efficiency of particular entrepreneurs or of particular units of factors, and it would neither be practicable nor sensible to recapitalize every unit of invested resources with every fluctuation units rate of yield. Even in the short run . . . there may be substantial variations as between the average costs, and therefore as between the net rates of return on original investments, of different producers (Viner, 1931, pp. 74-75).

This is not difficult to believe particularly in light of the differences which exist in initial resource endowments between producing areas as well as differences in the extent of fully exploiting size economies within a producing area. There is yet another very desirable characteristic of a particular expense curve. Individual firms in a competitive industry ignore their contribution to increasing or reducing costs to all other firms (Mishan, 1968, p. 1269, fn. 1). Therefore there is also a particular expense curve equal to actual short run average cost excluding rent ex post.

Marshall must have been aware of these characteristics of the particular expense curve because he explicitly describes the process of measuring economic rent in terms of "actual expenses of production" (1927, p. 810). Marshall defines the measure of producer's surplus as the area above "SS'" and below equilibrium price. Furthermore Marshall defines "SS'" as

> not a true supply curve adopted to the conditions of the world in which we live; but it has properties, which are erroneously attributed to such a curve. We will collect the particular expenses curve . . . For convenience the owners of differential advantages may be arranged in descending order from left to right; and thus SS' becomes a curve sloping upwards to the right (1927, p. 811, fn. 2).

Mishan is also aware of the use of the particular expense curve as a theoretically and empirically correct means of measuring economic rent.

> With commendable analytic finesse Marshall separates the rising supply curve resulting from differential advantages as between firms, "the particular expenses curve", from the possible external economies which may predominate and act, therefore, to lower the unit cost as the total output for the industry increased. . . This procedure is correct inasmuch as the area measures what the total number of firms take to be their rents or producers' surplus. (1968, p. 1269, fn. 1).

At another point in "What is Producer's Surplus" Mishan uses the particular expense curve to interpret Hick's use of the industry's ordinary supply curve excluding rent

> Hicks calls his SS' curve . . . a marginal curve for the industry. Unless he is thinking of the Ricardian land case he must be following Marshall's assumptions: Constant prices for all variable factors with the upward-sloping supply curve the result simply of ranking firms in order of efficiency. This construction is less general than the upward-sloping long period supply curve of the industry which results when firms of equal efficiency respond to an expanding market demand (1968, p. 1276, fn. 11).

Therefore, if we define the industry's average cost curve excluding rent as equal to the particular expense curve then the

problem of empirically estimating the Y-intercept becomes one of determining the average cost of production excluding rent of the most non-marginal producers, individually or as a group, within the industry. Similarly the type of supply shift associated with the introduction of a new technology becomes one of determining the change in cost of production excluding rent for the most non-marginal producers.

Let us examine what is involved in "estimating" a particular expense curve and how it would be used to measure economic surplus. See Figure 38. The estimation procedure surrounding the particular expenses curve is very data intensive. This procedure requires detailed cost of production on each firm or a significantly large sample of firms in the industry to determine relative efficiency. Furthermore, cost of production data at two points in time are required to establish a base and alternate periods against which changes in productivity and associated rent can be measured. Once this information is obtained and arrayed, it will be found that the cost of production for the marginal firms is greater than equilibrium price. The cost of production of the most marginal firm does not lie on the industry's equilibrium supply curve but on a high cost curve which is above and parallel to the equilibrium supply curve. The high cost curve is the locus of points associated with the average cost of production of the most marginal firm at each level of total output for the industry (Viner, 1931, p. 77). Lines $a_1c_1$ and $a_2c_2$ represent particular expense curves associated with output levels $Q_o$ and $Q_1$ respectively. $HC_1^*$ is the high cost curve associated with the industry's equilibrium supply curve $S_1^*$.

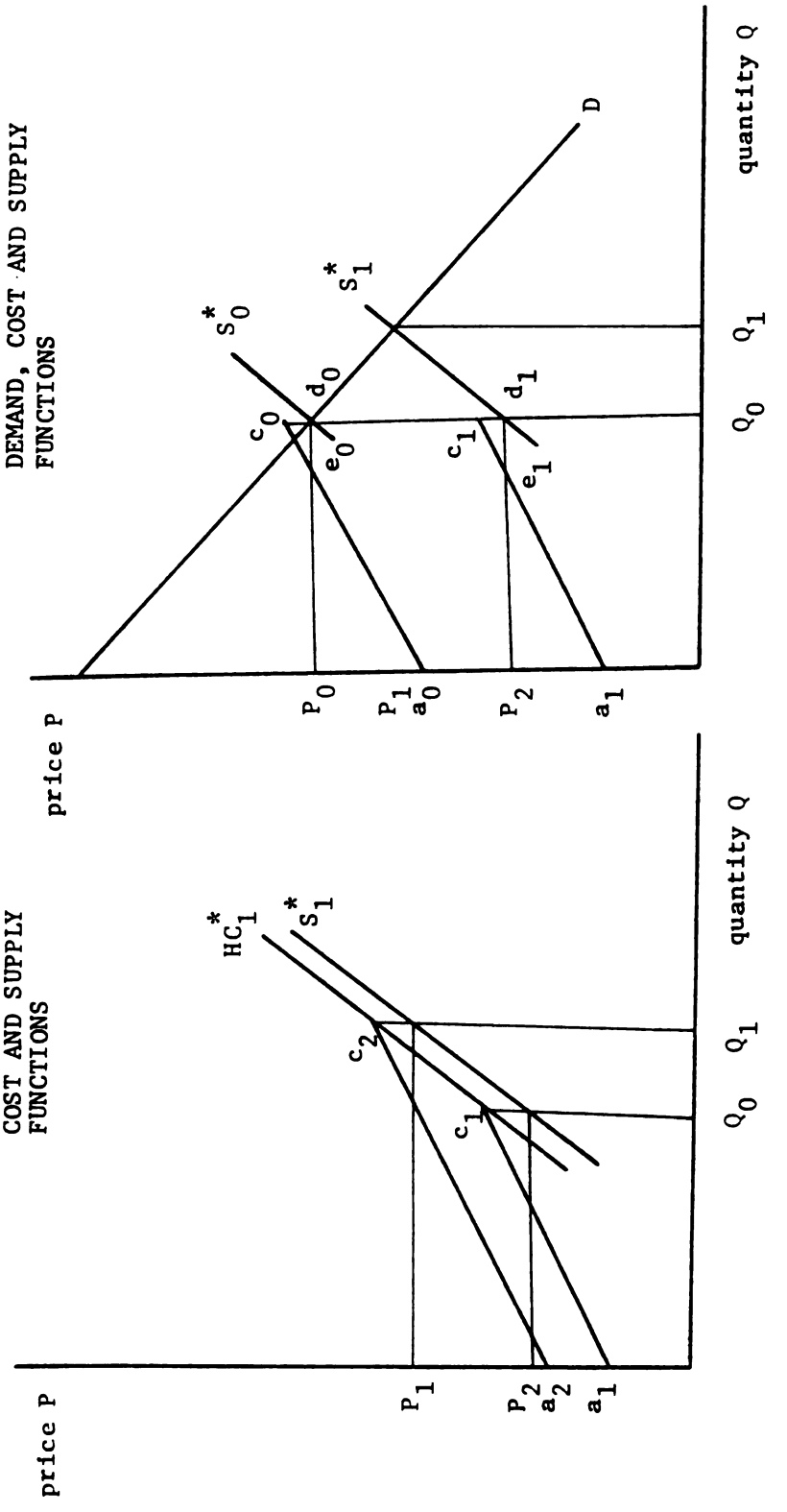



Figure 38. Economic Surplus and the Particular Expense Curve

The relationship between the particular expense curve and the industry's equilibrium supply curve provides a useful insight into the nature of relationship between individual firms and the industry as a whole. To this point Viner states:

> It is possible . . . to devise a theory of even long-run static equilibrium which still leaves room for an excess HC over the SS curves, and therefore for bankruptcy as a phenomenon consistent with long-run equilibrium. For such a theory, long-run equilibrium would apply only to the industry as a whole, and would be a sort of statistical equilibrium between rate of output and rate of consumption. None of the individual producers under this theory need be in long-run equilibrium at any time. At any moment, some producers would be enjoying exceptional profits, and others incurring heavy losses. The particular expenses curve could remain positive in its inclination and fixed in its locus, but there would be necessarily a constant process of shifting of their position on that curve or the part of the individual producers, and an equality in rate of withdrawal of producers from the industry through bankruptcy or otherwise, on the one hand, and of entrance of new producers into the industry, on the other hand. A theory of this sort would leave room for pure profits in a static state. (1931, p. 78).

The particular expense curve is therefore important not only to determine the intercept terms $a_o$ and $a_1$ but also to determine the additional transfer of economic rent from resources owners to consumers by way of the most marginal firms. This transfer of income is equal initially to area $c_o\ d_o\ e_o$ and subsequently by to area $c_1\ d_1\ e_1$. The difference between these two areas would be added to the measure of the net change in consumer surplus and subtracted from the measure of the net change in economic rent.

There do not appear to be any studies of economic rent or economic surplus which have used the particular expense curve. There are several studies however, which estimate particular expense curves for various industry for other purposes. For example, Taussig reports on his use of the particular expense curve as a tool to set prices to

assure sufficient supply of goods needed by the U.S. government during World War I. (1919, pp. 205-241). Also Secrist uses the particular expenses curve of the retail clothing industry in the U.S. to explore the efficiency of a firm over the life cycle of the proprietor (Secrist, 1923; 1924; 1933). More recently the U.S. Congress has published particular expense curves prepared by the U.S.D.A. for the major field crops in the U.S. These curves were presented as part of an analysis of the relationship between support prices and production costs in agriculture (1976).

In this section, we have done several things. First, we have defined economic surplus, economic rent and consumer surplus associated with a shift in supply given the introduction of a new technology. We have determined how these measures can be accurately measured in the commodity market. To this end the equilibrium supply function and the kinked supply function are presented as theoretically valid means of estimating economic surplus. The Lindner et al. model is used to estimate the kinked supply function associated with an array of assumptions regarding the Y-intercept. Finally, the particular expenses curve is presented as a means of estimating the ordinary supply curve in general and the Y-intercept in particular.

## 3-14 The Introduction of a New Technology as a Relatively Scarce Durable Resource

In this section, we will describe an industry whose firms initially require a relatively scarce resource $R_1$ (migrant labor) as well as other abundant resources as inputs in production. Subsequently this industry adopts a new technology (a mechanical harvester) in the

form of a relatively scarce durable resource $R_2$ which is an imperfect substitute for the relatively scarce resource $R_1$. We will specify the circumstances surrounding this industry prior to the introduction of the new technology. Then we will analyze the relationship between the new technology and the industry's subsequent long-run marginal cost curve excluding rent. Finally we will determine the associated direct and indirect measures of economic rent, consumer surplus and economic surplus in both the goods and resources markets for a given shift in supply associated with the introduction of a new technology.

In this industry, we assume atomistic competition in the commodity and resources markets. We assume that firms in the commodity market are of equal size and efficiency and can be represented by a single firm or "representative firm." The production function of the representative firm is assumed to require the use of one relatively scarce resource ($R_1$). The use of resource $R_1$ is associated with external diseconomies of large production. These external diseconomies may be either pecuniary or technological in nature. All other resources required in the production process are assumed to be available in abundance at a fixed price independent of the quantity demanded by this particular industry. These abundant resources are assumed to include both variables and durables. The acquisition and salvage prices of all of the resources are assumed equal. Further it is assumed that internal size economies exist and have been fully exploited by each firm in the industry. It is assumed that there are no barriers to entry or exit for firms in this industry. The number of firms entering the industry is determined in relation to the demand for the output.

Subsequently it is assumed that a new technology is introduced and adopted by firms in the industry. It is assumed that that the new technology is in the form of a relatively scarce durable resource. It is assumed that the relatively scarce durable resource is an imperfect substitute for the initial relatively scarce resource. We also assume that the new technology increases the internal size economies and these new internal size economies are fully exploited by all the firms in the industry. The number of firms remaining in the industry is determined in relation to the demand for the output. Finally, it is assumed that the initial relatively scarce resource supply is decreasing. It is assumed that the acquisition price of durable $R_2$ is greater than its salvage value adjusted for depreciation and maintenance.

The process of adjustment by the firms and the industry to the introduction of a new technology is illustrated in Figure 39. Initially, the representative firm's marginal cost $MC_0$ and average cost $AC_0$ are equal to equilibrium price $P_0$ and output $q_0$. Initially, the industry is in long-run equilibrium at price $P_0$ and output $Q_0$ associated with demand D and equilibrium supply $S_0^*$. Output $Q_0$ is the aggregate production of N firms in the industry.

With the subsequent introduction and adoption of the new technology, the representative firm's marginal and average total cost shift to $MC_1$ and $AC_1$ respectively. With the new cost structure, equilibrium commodity price decreases to $P_1$ while the quantity produced expands to $q_1$ for each firm. The industry is restructured such that N-M firms exit from the industry as the remaining M firms expand

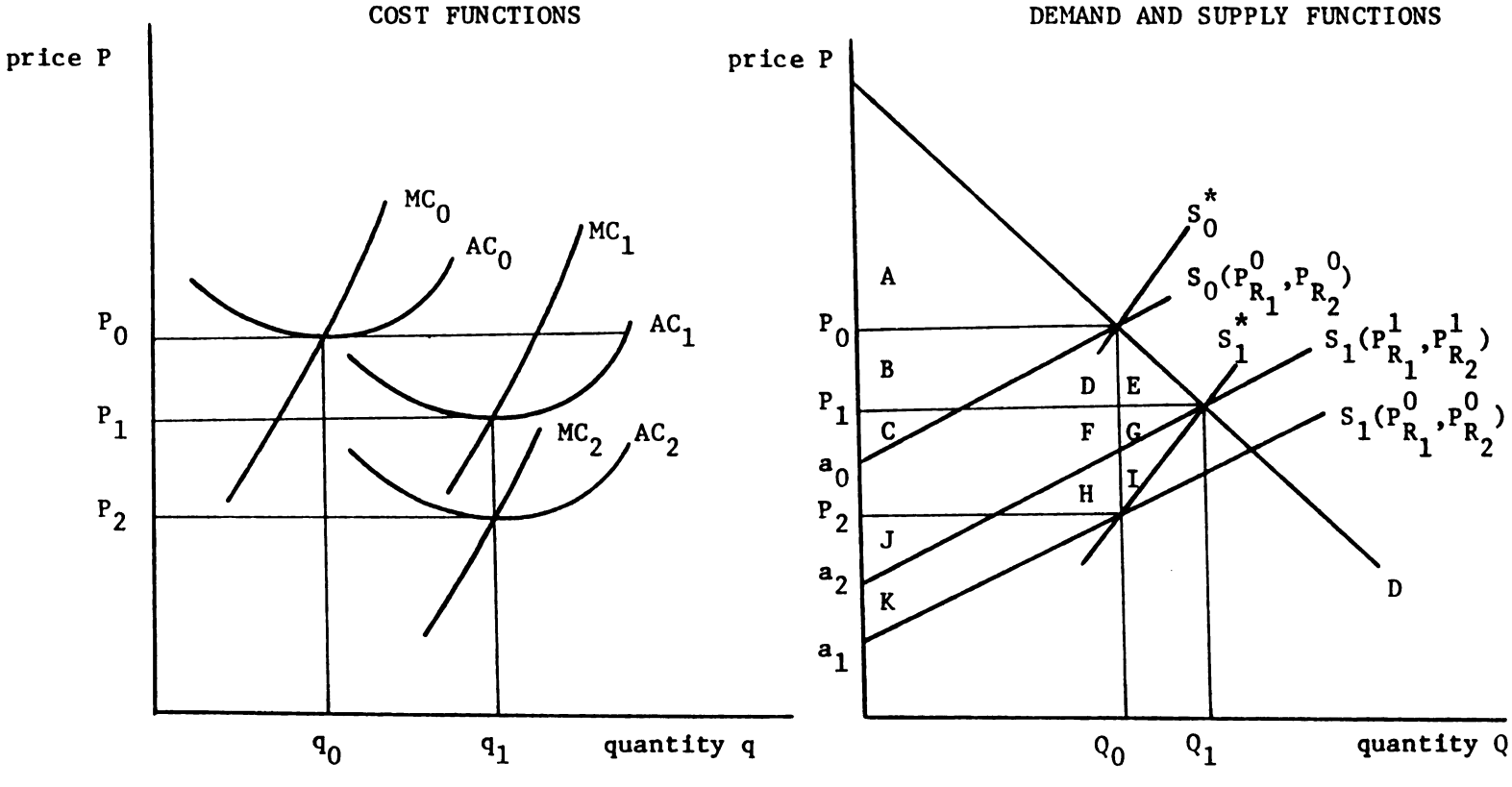



Figure 39. External Diseconomies of Large Scale Production Associated with the Introduction of a New Technology

individual production to fully exploit size economies associated with the new technology. The new technology is used initially because of the short-run profits associated with early adoption. The remaining firms adopt the technology to remain competitive with the early adopters. As a result, production for the industry expands to $Q_1$. Output $Q_1$ is the aggregate production of M firms in the industry.

The marginal and average cost curves $MC_1$ and $AC_1$ include the subsequent price of resource $R_1$ and the established price of durable $R_2$. The established price of durable $R_2$ is also its subsequent price in that the initial price of durable $R_2$ equals zero. Initial quantity of durable $R_2$ equals zero as well since initially $R_2$ was not an available factor in production. Durable $R_2$ only becomes available as a factor of production with its subsequent introduction and adoption as a new cost-saving technology. Therefore, with the introduction of durable $R_2$, previously producing firms substituted the relatively less expensive durable $R_2$ for the relatively more expensive resource $R_1$ in proportions determined by the elasticity of substitution (Ferguson, 1975, pp. 90-92). These firms avoid the problem of overinvesting in durable $R_2$ to the extent possible by having rational expectatons for the price associated with commodity output $Q_1$ and purchasing the optimal units $R_2$.

Let us examine the impact of the introduction and adoption of the new technology on the industry's long-run supply curve. Initially, the sum of N firm's marginal and average costs equal ordinary supply curve $S_0$. Initial ordinary supply curve is associated with inital resource $R_1$ and $R_2$ prices of $P_{R_1}^0$ and $P_{R_2}^0$. Subsequently, the sum of the M firms' marginal and average costs equal ordinary supply curve

$S_1$. Subsequent ordinary supply curve is associated with subsequent resource $R_1$ and $R_2$ price of $P_{R_1}1$ and $P_{R_2}1$. $P_{R_2}$ is greater than zero. Ordinary supply curves $S_0(P_{R_1}0, P_{R_2}0)$ and $S_1(P_{R_1}1, P_{R_2}1)$ intersect the equilibrium supply curves $S_0^*$ and $S_1^*$ at $(P_0, Q_0)$ and $(P_1, Q_1)$ respectively. There exist another subsequent ordinary supply curve $S_1$ associated with initial resource prices $P_{R_1}0$ and $P_{R_2}0$. This subsequent ordinary supply curve $S_1(P_{R_1}0, P_{R_2}0)$ is equal to the sum of the marginal and average cost $MC_2$ and $AC_2$ of M firms producing the initial output $Q_0$ with the new technology. The marginal and average cost curves $MC_2$ and $AC_2$ include the inital price $P_{R_1}0$ of scarce resource $R_1$ and the initial price $P_{R_2}0$ of scarce durable $R_2$. The kinked supply curve subtended by $S_1(P_{R_1}0, P_{R_2}0)$ and $S_1^*$ equals the industry's subsequent marginal cost excluding rent. Similarly $S_0(P_{R_1}0, P_{R_2}0)$ and $S_0^*$ equals the industry's initial marginal cost excluding rent.

Initially economic rent equals

(1) $ER_0$ = area B + C.

Subsequently economic rent equals

(2) $ER_1$ = area C + F + G + H + I + J + K.

The net change in economic rent equals

(3) $ER_1 - ER_0 = C + F + G + H + I + J + K - B - C$

$= F + G + H + I + J + K - B.$

Area F + G + H + I + J + K - B is equal to the net change in payment received by the non-marginal units of the relatively scarce resource $R_1$ and durable $R_2$ as the respective resource prices change in order to secure the services of the marginal units as output expands. The resource owners of the non-marginal units of the relatively scarce

resource $R_1$ and durable $R_2$ loss and receive rent by virtue of being required in production and available in less than perfectly elastic supply.

Consumer surplus can be measured in the commodity markets also. Initially consumer surplus equals

(4) $CS_0$ = area A.

Subsequently consumer surplus equals

(5) $CS_1$ = area A + B + D + E.

The net change in consumer surplus equals

(6) $CS_1 - CS_0 = A + B + D + E - A = B + D + E.$

The net change in economic surplus equals

(7) $ES_1 - ES_0 = ER_1 - ER_0 + CS_1 - CS_0$

$= F + G + H + I + J + K - B + B + D + E$

$= D + E + F + G + H + I + J + K.$

The net change in economic surplus for an industry associated with the introduction of a new technology and measured in the commodity market is equal to the summation of two effects: the net change in economic rent (area F G H I J K) and the net change in consumer surplus (area D E). Area B represents a transfer of income from resource owners to consumers.

The process of adjustment to the shift in output supply can be illustrated in the resource markets as well. See Figure 40. Initially, the representative firm's marginal value product for the relatively scarce resource $r_1$ equals $MVP_0$. At resource price $P_0$, then each firm use quantity $r_{10}$ of resource $r_1$. Initially, the industry's derived demand for the relatively scarce resource $R_1$ is $D_0$ with

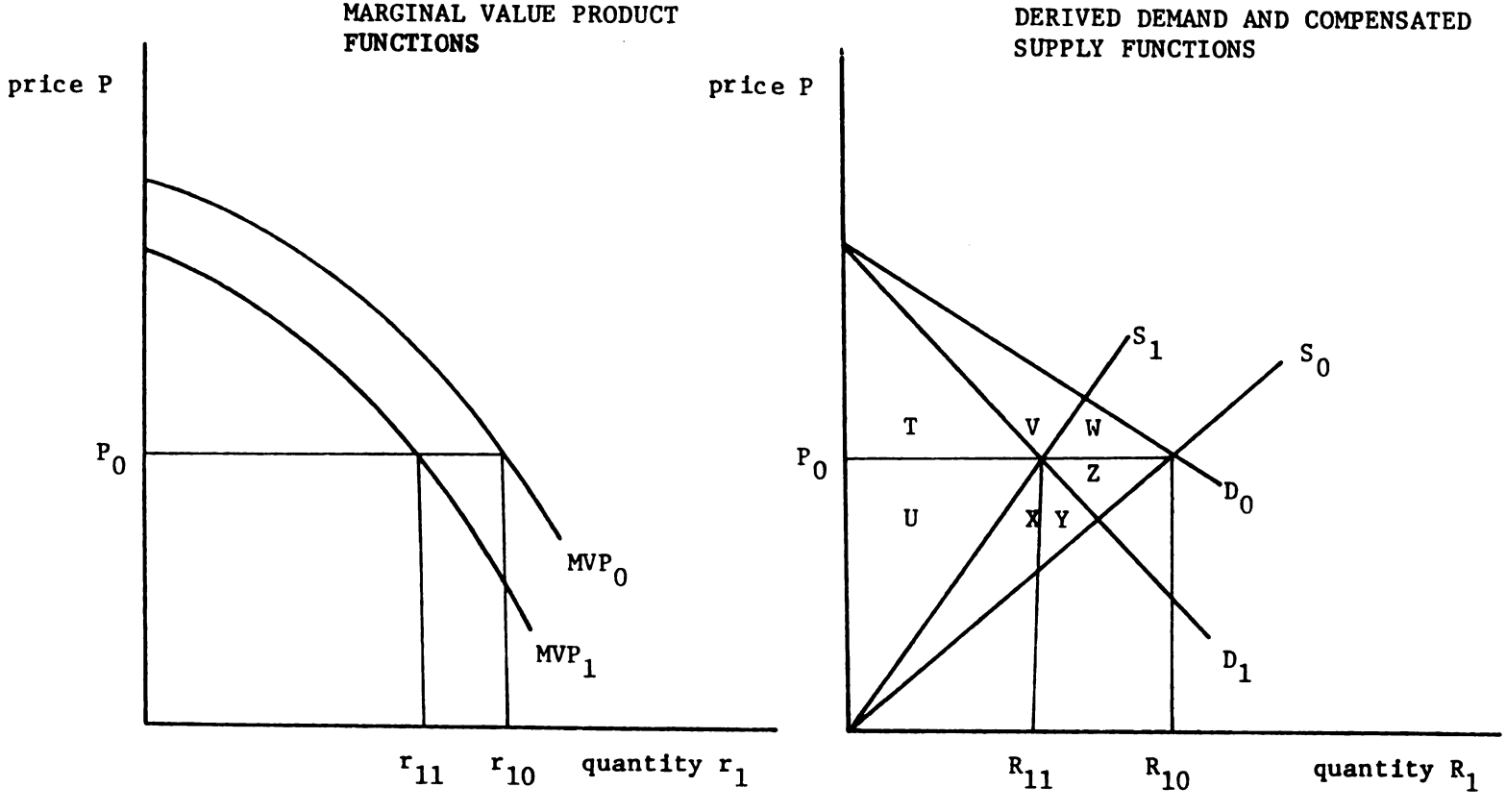



Figure 40. A Relatively Scarce Resource Displaced by a New Technology

equilibrium resource price $P_0$ and quantity $R_{10}$. The initial derived demand $D_1$ for resource $R_1$ is equal to the sum of the $MVP_0$ curves for N firms.

As commodity output expands to $Q_1$, the marginal value product for $r_1$ decreases to $MVP_1$ as producers substitute the relatively less expensive resource $r_2$ associated with the new technology for the relatively more expensive $r_1$. The sum of the marginal value product curves $MVP_1$ across M firms equals the subsequent derived demand $D_1$ for resource $R_1$ with subsequent equilibrium price $P_o$ and quantity $R_{11}$. The resource price remains unchanged because it is assumed that there has been a simultaneous and offsetting decrease in resource $R_1$ supply from $S_o$ to $S_1$ as derived demand for resource $R_1$ decreases from $D_o$ to $D_1$.

The process of adjustment to the shift in output supply can be illustrated in the resource market associated with the new technology. See Figure 41. With the introduction of the new technology as a relatively scarce durable $R_2$, producers substitute units of the relatively less expensive scarce resource $R_2$ for units of the relatively more expensive scarce resource $R_1$. As commodity output expands to $Q_1$, the marginal value product for $R_2$ equals $MVP_1$ with equilibrium acquisition price established at $P_A^1$ and each firm uses $r_{21}$ units of durable $r_2$. The scarce durable's salvage price $P_S^1$ is a uniform percentage less than acquisition price adjusted for depreciation and maintenance. The sum of the marginal value produce curves $MVP_1$ across M firms establishes the derived demand $D_1$ for the scarce durable $R_2$ with equilibrium price $P_A^1$ and quantity $R_{21}$.

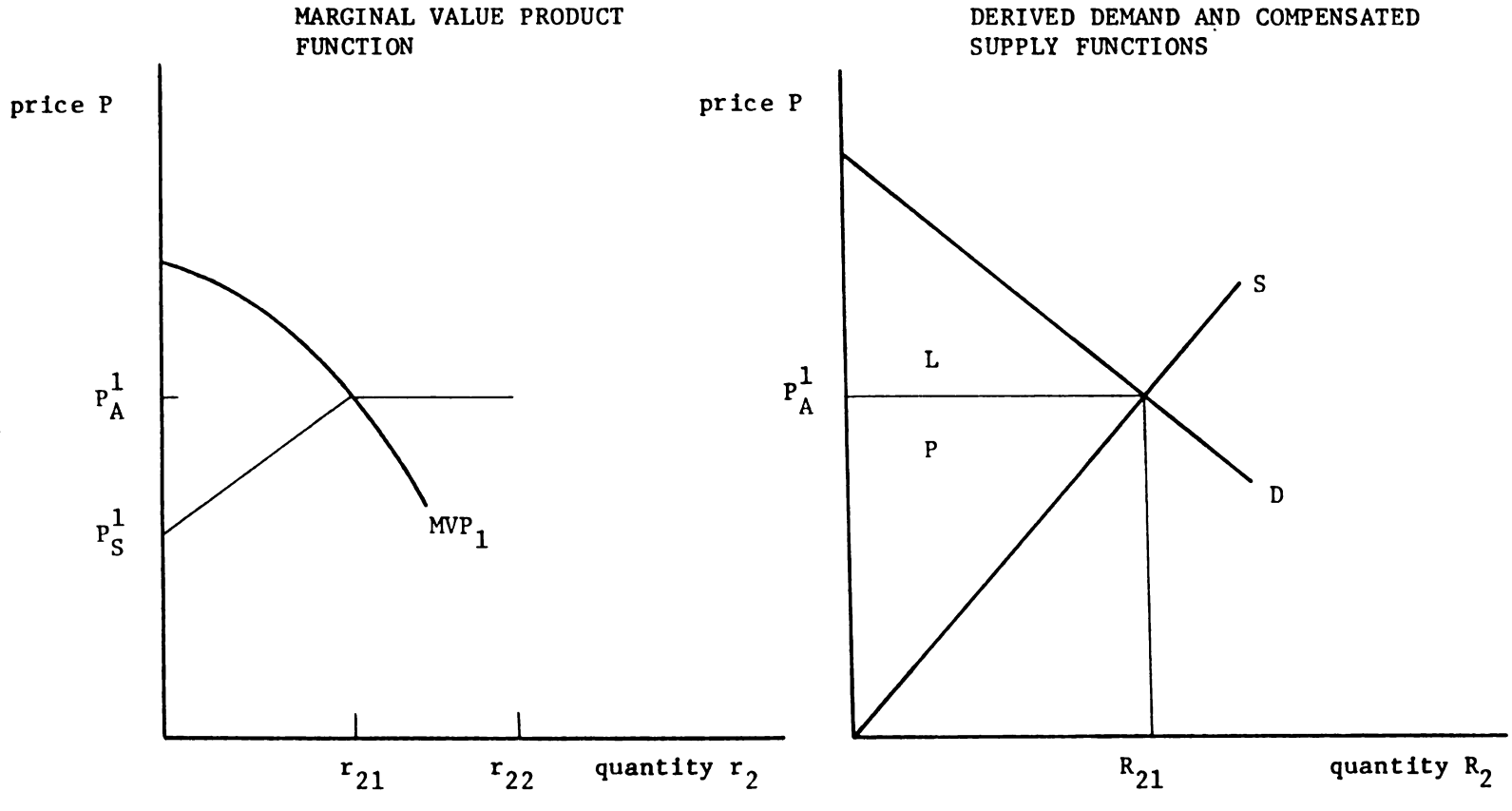



Figure 41. A New Technology Associated with a Relatively Scarce Durable Resource

The net change in economic surplus can be measured in the resource market of the factors required in the production process. In the resource market for the relatively scarce resource $R_1$, we can illustrate the areas associated with economic rent, consumer surplus and economic surplus. Initially economic rent for resource $R_1$ equals

(8) $ER_0 \Big| R_1 = \text{area } U + X + Y + Z$

Subsequently economic rent for resource $R_1$ equals

(9) $ER_1 \Big| R_1 = \text{area } U$

The net change in economic rent for relatively scarce resource $R_1$ equals

(10) $ER_1 - ER_0 \Big| R_1 = U - U - X - Y - Z = -(X + Y + Z).$

Initially consumer surplus equals

(11) $CS_o \Big| R_1 = \text{area } T + V + W.$

Subsequently consumer surplus equals

(12) $CS_1 \Big| R_1 = \text{area } T.$

The net change in consumer surplus for relatively scarce resource $R_1$ equals

(13) $CS_1 - CS_o \Big| R_1 = T - T - V - W = -(V + W).$

The net change in economic surplus for relatively scarce resource $R_1$ equals

(14) $ES_1 - ES_o \Big| R_1 = ER_1 - ER_o + CS_1 - CS_o \Big| R_1 = -(X + Y + Z) - (V + W)$
$= -(V + W + X + Y + Z).$

The net change in economic surplus can also be measured in the resource market for the relatively scarce durable $R_2$. Initially economic rent associated with zero units of durable $R_2$ equals

(15) $ER_o \Big|_{R_2} = 0.$

Subsequently economic rent associated with $Q_1$ units of durable $R_2$ equals

(16) $ER_1 \Big|_{R_2} = \text{area P}.$

The net change in economic rent for relatively scarce durable $R_2$ equals

(17) $ER_1 - ER_o \Big|_{R_2} = \text{area P}.$

Initially consumer surplus associated with zero units of durable $R_2$ equals

(18) $CS_0 \Big|_{R_2} = 0.$

Subsequently consumer surplus associated with $Q_1$ units of durable $R_2$ equals

(19) $CS_1 \Big|_{R_2} = \text{area L}.$

The net change in consumer surplus for relatively scarce durable $R_2$ equals

(20) $CS_1 - CS_o \Big|_{R_2} = \text{area L}.$

The net change in economic surplus for relatively scarce durable $R_2$ equals

(21) $ES_1 - ES_o \Big|_{R_2} = ER_1 - ER_o + CS_1 - CS_o \Big|_{R_2} = L + P.$

Let us compare the net change in economic rent, consumer surplus and economic surplus as measured in the commodity market as equations

(3) (6) and (7). With these same changes measured in the resource markets of relatively scarce resource $R_1$ and relatively scarce durable $R_2$ as equations (10, (13) (14) and (17), (20), (21) respectively. Based on the work of Just and Hueth (1979, pp. 948-951) we determine that the aggregate change in economic rent measured indirectly in the goods market is equal to the sum of the change in economic rent measured directly in the resource market for each resource required in the production of a particular output.

$$(22)\quad ER_1 - ER_o = F + G + H + I + J + L - B = \sum_{i=1}^{k} (ER_1 - ER_o)$$

where i equals the 1 through k resource markets for inputs required in the production process.

In this case only the relatively scarce resources $R_1$ and $R_2$ have a non-zero change in rent. Therefore

$$(23)\quad ER_1 - ER_o = F + G + H + I + J + K - B = -(X + Y + Z) + P.$$

Equation (23) states that the total change in economic rent measured in the commodity market to produce the expanded output $Q_1$ equals the decrease in rent $-(X + Y + Z)$ associated with resource $R_1$ and the increase in rent (P) associated with resource $R_2$.

Similarly, the aggregate change in consumer surplus measured directly in the commodity market and indirectly in the resource markets for each factor needed in the production process.

Therefore

$$(24)\quad CS_1 - CS_o = B + D + E = \sum_{i=1}^{k} (CS_1 - CS_o)$$

where i equals the 1 through k resource markets for inputs required in the production process.

Therefore

$$(25)\quad CS_1 - CS_o = B + D + E = \sum_{i=1}^{j} (CS_1 - CS_o) - L - V - W$$

where the 1 through j resource markets do not include those for $R_1$ and $R_2$.

Equation (25) states that the total change in consumer surplus measured in the commodity market associated with expanded output $Q_1$ equals the increase in consumer surplus associated with all the other abundant resources $\sum_{i=1}^{j} (CS_1 - CS_o)$ as well as the relatively scarce durable $R_2$ (L) and the decrease in consumer surplus associated with resource $R_1$ (-V-W).

Finally, the aggregate change in economic surplus measured in the commodity market equals the sum of the change in economic surplus measured in the resource market for each factor needed for the production process.

$$(26)\quad ES_1 - ES_o = D + E + F + G + H + I + J + K$$
$$= \sum_{i=1}^{k} (ER_1 - ER_o) + \sum_{i=1}^{k} (CS_1 - CS_o)$$

where i equals the 1 through k resource markets for inputs required in the production process.

Therefore

$$(27)\quad ES_1 - ES_o = D + E + F + G + H + I + J + K$$
$$= P - (X + Y + Z) + \sum_{i=1}^{j} (CS_1 - CS_o) + L - (V + W)$$
$$= L + P - (V + W + X + Y + Z) + \sum_{i=1}^{j} (CS_1 - CS_o).$$

Equation (27) states that the net change in economic surplus measured in the resource market associated with expanded output $Q_1$ equals the

sum of the net change in economic rent $(P - (X + Y + Z))$ and the sum of the net change in consumer surplus $(\sum_{i=1}^{j} (CS_1 - CS_o) + L - (V + W))$ measured in the resource market. Further, this equation states that the net change in economic surplus measured in the commodity market is equal to the net change in economic surplus measured in the resource markets.

In this section, we have shown that in an industry in which a new technology is introduced and adopted, the industry's marginal cost curve excluding rent is subtended by the ordinary supply curve associated with initial resource prices and the equilibrium supply curve. Further, we have determined the measures of economic rent, consumer surplus and economic surplus in the aggregate and accurately in the commodity market. We also determine the measures of economic rent, consumer surplus and economic surplus accurately in the resource markets. We also illustrated the relationship between those measures in the commodity market and resource markets. In particular we showed that the net change in economic rent equals the total economic rent of durable $R_2$ associated with the new technology less the change in economic rent of resource $R_1$. The net change in consumer surplus equals the net change in consumer surplus associated with the abundant resources plus the total consumer surplus associated with durable $R_2$ less the change in consumer surplus associated with resource $R_1$. Finally, we showed that the sum of the net change in economic rent and consumer surplus measured in the resource markets equals economic surplus measured in the commodity market.

# CHAPTER IV

# EMPIRICAL RESULTS

## 4-1 Introduction

The previous chapters dealt with the history of the mechanical cucumber harvester as well as the theory of consumer's surplus and economic rent needed to analyze the impact of the harvester. This chapter is a presentation of the empirical results relating to the estimation of consumer's and producer's surplus as well as the associated benefit-cost ratio and internal and benefit-cost rate of return to the research investment. In the context of estimating the "impact", it is necessary to present the specific form equations which are based on the general form equation presented in the preceding chapters. This is particularly true in the case of consumer's surplus.

Consumer's surplus results are presented in some depth in order to compare the deduced relationship in the theory chapter with actual empirical results. This then is not so much a proof of the theory as an illustration of the implicit or explicit assumption regarding consumer's surplus. The means used to illustrate the similarity (and dissimilarity) between Willig and Hausman is to estimate the demand for cucumber pickles assuming separability and use both the linear and exponential functional forms.

Economic rent is also presented as an illustration of the Rose; Wise and Fell theory by assuming various types of supply shifts

including pivotal, parallel and convergent. A linear supply function for cucumber pickles is used since it both meets the linearity assumption of the cross multiplication technique and appears to be a reasonably good approximation of the actual supply function.

Both consumer's surplus and economic rent are estimated in the retail goods market only. The final goods market appears to satisfy the conditions for estimating the returns to a vertical subsector in one industry as established by Just and Hueth. In addition, the geometric configuration of consumer's surplus and economic rent in the retail market are consistent with the measures developed in the previous chapters. Sadly, this limits the possibility of disaggregating the returns beyond the simple dichotomy of final consumers and resource owners. Further work needs to be done before complete disaggregation is possible.

In order to estimate consumer's surplus and economic rent as well as the net annual return, it is necessary first to establish a number of key values. These values include initial and subsequent equilibrium price and quantity, the price elasticity of demand and supply as well as the other coefficients of demand and supply equations. Another important value is the measure of supply shift which is traceable to the introduction of the mechanical harvester. Once these values are established, then they can be applied to the appropriate specific form equations to determine the benefits and costs of research on the mechanical cucumber harvester.

The first several sections of this chapter present the estimates for the demand and supply curve and the supply shift. These sections are followed by a second group which presents the estimates for

consumer's surplus and economic rent and the overall benefit-cost ratio and internal and benefit-cost rate of return. This chapter concludes with a section on the resources saved by the mechanical cucumber harvester including seasonal labor and small farm resources.

## 4-2 The Demand for Cucumber Pickles

In this section, the retail demand for cucumber pickles is estimated using linear and exponential functional forms. As indicated in the introduction, the purpose for using two types of functions to estimate the demand for cucumber pickles is to illustrate the effect the type of demand function has on estimating consumer's surplus. The functional form of the demand equation has an effect on the results obtained by the various measures of consumer's surplus.

In addition to estimating the demand, these functions are tested to determine whether the results are consistent with the concepts of utility maximization. The demand equations are also analyzed in terms of meeting the separability assumption. If there are no significant cross-price elasticities, then the demand for cucumber pickles is assumed to be "separable." Finally the values for own-price and income elasticity are estimated for each of the functional forms of the demand equations.

The demand for cucumber pickles can be expressed so that the quantity consumed is a linear function of price and income. The general form of such a linear demand equation is:

(1) $Q_t = B_0 + \alpha P_t + \delta Y_t + \mu_t$

where

$Q_t$ is the quantity consumed in time t.

$P_t$ is the price per unit in time t.

$Y_t$ is the disposable income in time t.

$\mu_t$ is the random disturbance in time t.

$B_o$, $\alpha$, $\delta$ are the estimated coefficients of the associated regression equation.

The estimated linear demand for cucumber pickles is:

(2) $DCPC_t = \beta_o + \beta_1 DV5558 + \beta_2 DV5963 + \alpha RPCPD_t + \delta DICD_t$

where

t = 1955 to 1978 and

DCPC is domestic civilian consumption of cucumber pickles in pounds per capita, retail weight.

DV5558 is a dummy variable 1 if 1955 to 1958 and 0 otherwise. (To correct for inconsistency in the price series, see (a) below.)

DV5963 is a dummy variable 1 if 1959 to 1963 and 0 otherwise. (To correct for inconsistency in the price series, see (b) below.)

RPCPD is the retail price of cucumber pickles in dollars per pound deflated by the consumer price index (1967 = 100).

This price series is a composite of the prices for:

(a) 7.5 oz. jar of sweet pickles, 1955-1958.

(b) 15 oz. har of sliced pickles, 1959-1963 and

(c) 12 oz. jar of sweet relish, 1964-1978

converted to dollars per pound.

DICD is the disposable income per capita in the U.S. deflated by the CPI (1967 = 100)

The absence of the price of complements or substitutes is discussed later in this section. The retail price of cucumber pickles sold through institutions is assumed to be included in the retail price series defined above. The estimates for the regression coefficients and the significance tests for equation (2) are as follows:

Demand For Cucumber Pickles (Linear)

$$(3)\quad DCPC_t = \underset{(2.04909)}{4.32143} - \underset{(-3.97462)}{2.3163\,DV5558} - \underset{(-2.92487)}{1.81798DV5963} - \underset{(-1.89662)}{4.46284\,RPCPD_t} + \underset{(8.34939)}{.00248926\,DICD_t}$$

where

adj $R^2$ = .962372 N = 24/DF = 19 F = 148.064

SE = .304028 SD = 1.56733 DW = 1.4829

Equation (3) is the linear demand function for cucumber pickles in the U.S. based on time series data from 1955 to 1978. The numbers in the parentheses under the coefficients are the "t" value. "t" values greater than 2 or less than -2 are considered significant. The t-value for the price coefficient is low but it has the correct sign. The adj $R^2$, the "F" statistic and the Durbin-Watson statistic all seem to indicate that equation (3) is at least a good estimate of the retail demand for cucumber pickles.

Before presenting the results of the exponential demand functions, the linear function will be tested in relation to the second order condition of the statistic equation to assure that the results are consistent with utility maximization and cost minimization. Recalling the results from the chapter on consumer's surplus, it is possible to determine whether an ordinary demand function is the result of utility maximization decision making on the part of consumers by the following:

$$(4)\quad \left.\frac{\partial Q}{\partial P}\right|_{\text{utility constant}} = \frac{\partial Q}{\partial P} + Q\left.\frac{\partial Q}{\partial Y}\right|_{\text{price constant}} < 0$$

where the expression $\partial Q/\partial P$ is the slope of the ordinary demand function. Q is the quantity consumed. $\partial Q/\partial Y$ given constant prices

is the measure of the slope of the income elasticity function. Therefore,

(5) $\left.\frac{\partial Q}{\partial P}\right|_{\text{utility constant}} = -4.46284 + (7.62143 * .00248926) < 0.$

$= -4.44387 < 0$

Since the sum of these measures is negative then the second order sufficient condition for utility maximization is met.

The demand for cucumber pickles can also be expressed so that price and income are an exponential function of the quantity consumed. The general form of an exponential demand equation is:

(6) $Q_t = e^{\beta o} P_t^{\alpha} Y_t^{\delta} e^{\mu t}$

where $Q_t$, $P_t$, $\mu_t$, $\beta$, $\alpha$ and $\delta$ are the same as for the linear equation. e is equal to the natural base number 2.718. The estimated exponential demand function for cucumber pickles is

(7) $\log DCPC_t = \beta o + \beta_1 DV5558 + \beta_2 DV5963 + \alpha \log RPCPD_t + \delta \log DICD_t$

where the definitions for t, DCPC, DV5558, DV5963, RPCPD, DICD are the same as for the linear demand function defined above. The interpretation for $\alpha$ and $\delta$ are significantly different than their counterparts in the linear demand function. The partial derivative of equation (7) with respect to price is equal to

(8) $\frac{\partial \log Q_t}{\partial \log P_t} = \alpha$

where $Q_t$ is equal to $DCPC_t$

$P_t$ is equal to $RPCPD_t$.

Equation (8) can be rewritten in terms of the chain rule so that

(9) $\alpha = \frac{\partial \log Q_t}{\partial Q_t} * \frac{\partial Q_t}{\partial P_t} * \frac{\partial P_t}{\partial \log P_t}$ (Chiang, 1974, p. 320)

where $\frac{\partial \log Q_t}{\partial Q_t} = \frac{1}{Q_t}$ (Chiang, 1974, p. 306)

and $\frac{\partial P_t}{\partial \log P_2} = (\frac{1}{P_t})^{-1} = P_t$ (Chiang, 1974, p. 306).

Therefore equation (9) can be rewritten so that

(10) $\alpha = \frac{\partial Q_t}{\partial P_t} \frac{P_t}{Q_t}$.

From equation (10), the coefficient of the price variable in an exponential demand function is equal to the price elasticity. Similarly it can be shown that the coefficient of the income variable ($\delta$) in an exponential demand function is equal to the income elasticity. This is in contrast to the definition of $\alpha$ and $\delta$ in the linear demand equation which are equal to $\partial Q_t/\partial P_t$ and $\partial Q_t/\partial Y_t$ respectively. Therefore

| Exponential Demand Function | Linear Demand Function |
|---|---|
| $\alpha = \frac{\partial Q_t}{\partial P_t} \frac{P_t}{Q_t}$ | $\alpha = \frac{\partial Q_t}{\partial P_t}$ |
| $\delta = \frac{\partial Q_t}{\partial Y_t} \frac{Y_t}{Q_t}$ | $\delta = \frac{\partial Q_t}{\partial Y_t}$. |

The estimates for the regression coefficients and the significance tests for equation (7) are as follows:

Demand for Cucumber Pickles (Exponential)

(11) $\log DCPC_t = \underset{(-5.6257)}{-6.58808} - \underset{(-4.37501)}{.478717\ DV5585} - \underset{(-3.15201)}{.372476\ DV5963}$

$- \underset{(-2.31892)}{.820024\ \log RPCPD_t} + \underset{(7.34322)}{1.07013\ \log DICD_t}$.

where

$adjR^2$ = .961367 N = 24/DF = 19 F = 144.087

SE = .0518963 SD = .264033 DW = 1.4634

Equation (11) is the exponential demand function for cucumber pickles in the U.S. based on time series data from 1955 to 1978. As before the numbers in parentheses below the regression coefficients are the "t" value. The t-test for all the coefficients are greater than 2 or less than -2 including the coefficient for the price variable. The $adjR^2$, the "F" statistic and the Durbin-Watson statistic indicate that the exponential functional form is also a good fit for cucumber pickle consumption data.

The price of substitutes and complements are not included in the estimation of the exponential demand function. The decision to exclude these valuables is presented at the end of this section.

The exponential demand function is also tested to determine whether it is the result of utility maximization decision making on the part of consumers. Again the general equation to test the second order condition is:

(12) $$\frac{\partial Q}{\partial P}\Big|_{\substack{\text{Constant}\\ \text{Utility}}} = \frac{\partial Q}{\partial P} + Q_t \frac{\partial Q}{\partial Y}\Big|_{\substack{\text{Constant}\\ \text{Prices}}} < 0$$

where $$\frac{\partial Q}{\partial P} = \alpha * \frac{Q_t}{P_t}, \text{ since } \alpha = \frac{\partial Q}{\partial P} * \frac{P_t}{Q_t}$$

and $$\frac{\partial Q}{\partial Y} = \delta * \frac{Q_t}{Y_t}, \text{ since } \delta = \frac{\partial Q}{\partial Y} * \frac{Y_t}{Q_t}.$$

Therefore equation (12) can be rewritten as

(13) $$\frac{\partial Q}{\partial P}\Big|_{\substack{\text{Constant}\\ \text{Utility}}} = \frac{\alpha Q_t}{P_t} + \frac{\delta Q_t^2}{Y_t} < 0$$

where $\overline{Q_t} = 7.63709$ $\alpha = -.820024$

$\overline{P_t} = .949625$ $\delta = 1.07013$

$\overline{Y_t} = 3028.27.$

Therefore,

$$(14)\quad \frac{\partial Q}{\partial P}\Big|_{\text{Utility}}^{\text{Constant}} = -.820024 * \frac{7.63709}{.949625} + 1.07013 * \frac{(7.63709)^2}{3028.27} < 0$$

$$= -6.5948 + .02061 < 0.$$

Since the sum of these measures is negative then the second order sufficient conditions for utility maximization is met. Both the linear and exponential demand for cucumber pickles equations satisfy the second order condition which assures results consistent with an assumed quasi-concave utility function. Consumers behave as if they chose to maximize utility subject to a budget constraint. It is critically important that the second order conditions be met for the entire Willig and Hausman analysis of consumer's surplus uses the idea of a quasi-concave utility function throughout their analysis. "If these conditions are violated over some or all of the price-income space in question, the exact or approximate welfare measures that one obtains are meaningless" (Hausman, 1981, p. 10).

The claim that Hausman makes is that a general equilibrium result is possible using his technique if the demand for the good in question has no significant cross price elasticities. This is called the separability assumption. The idea of separability is one in which "consumers follow a budget allocation process in such a way that, in the first stage, the total expenditure is divided into different subgroups and then, at the second stage, the amount allotted to each subgroup is allotted to individual commodities belonging to that

subgroup" (George and King, 1971, p. 24). In an extreme case a consumer may allocate a portion of the total expenditures to the purchase of commodity regardless of price, or as if the marginal rate of substitution between this and other commodities is zero. Since cucumber pickles require only about .2% of an average consumer's disposable income then consumers may purchase cucumber pickles irrespective of the price of complements or substitutes. Cucumber pickles may in fact meet the separability criteria necessary to assure a general equilibrium rather than a partial equilibrium result in relation to a change in the market equilibrium price and quantity. The empirical results below support this conclusion.

Cucumber pickles are a condiment often used to season hamburgers or as a garnish accompanying hamburgers. Therefore the consumption of cucumber pickles may well be inversely related to the retail price of hamburger. Similarly the consumption of cucumber pickles may be directly related to the retail price of processed tomatoes since catsup is also a condiment used to season hamburgers. The results of the demand for cucumber pickles including the price of hamburger and processed tomatoes is as follows:

Demand for Cucumber Pickles Including Complements and Substitutes (Linear)

$$(15)\quad DCPC_t = \underset{(2.0697)}{5.66105} - \underset{(-3.61429)}{2.87858}\ DV5558 - \underset{(-2.95992)}{2.2943}\ DV5963$$

$$\underset{(-2.12261)}{-6.30028}\ RPCPD_t - \underset{(7.14345)}{.00239625}\ DICD_t$$

$$\underset{(-.816482)}{-.0105926}\ RPHBD_t + \underset{(.887296)}{.670081}\ RPTMD_t$$

where t = 1955 - 1978

$adjR^2$ = .96078 N = 24 DF = 17 F = 94.4058

SE = .311192 SD = 1.56733 DW = 1.6057

and

RPHBD is the retail price of hamburger in cents per pound in the U.S. deflated by the consumer price index (1967 = 100)

RPTMD is the retail price of a 303 can of processed tomatoes in cents in the U.S. deflated by the consumer price index (1967 = 100)

The definition of the other variables are the same as above.

Equation (15) is the linear estimate of the demand for cucumber pickles in the U.S. from 1955 to 1978. This equation includes the retail price of hamburger and processed tomatoes as measures of a complement and substitute respectively. The sign of the estimated coefficients is less than 2 and greater than -2. This latter result suggests that the quantity of cucumber pickles consumed per capita is independent of the price of hamburger and the price of processed tomatoes. This does not prove that cucumber pickles are separable since there are other complements and substitutes for cucumber pickles. (The retail price of frankfurters was also used and not found to be significant.) However this result would appear to shift the burden of proof. It therefore can be said that cucumber pickles as a commodity has not been shown so far to be inconsistent with the assumption of separability. This result implies that to the extent that the separability assumption is true is the extent to which the measures of consumer's surplus and economic rent are a general rather than a partial equilibrium result.

## 4-3 The Supply of Cucumber Pickles

In the previous section the equations for the retail demand for cucumber pickles was presented. Both a linear and exponential function form were fitted to the consumption of cucumber pickle data. Each of these equations were tested using the Slutsky equation to check for consistency with the assumptions of utility maximization and both equations are consistent. An interpretation of the estimated coefficients for price and income for both equations was discussed. And finally the concept of separability was examined as it applies to possible cross-price elasticities of complements to and/or substitutes for cucumber pickles.

In this section the equations for the supply of cucumber pickles are developed and estimated. In particular the "equilibrium" supply curve as defined by Just and Hueth is presented. The equilibrium supply curve is defined as the supply response curve which includes "adjustments in input use and input price as output changes" (Just and Hueth, 1979, p. 952). The equilibrium supply curve is estimated for the farm level and the retail equilibrium supply elasticity is derived from the farm supply coefficient. In particular the price elasticity of supply is derived. The intercept term on the other hand is assumed to be positive (i.e., output is produced only at greater than zero prices, and each unit of output incurs a cost of productions). This section also presents a discussion of the "irreversible" supply. Irreversibility relates to Johnson's definition of the "long run" in which durable inputs are purchased or salvaged in relation to the change in output price (1958, pp. 74-93).

The equilibrium supply of a commodity is determined by the general form equation

(1) $Y_{jt} = b_o^* + b_1^* P_{jt}$

where $Y_{jt}$ are the units of output in market j at time t, $P_{jt}$ is the price of the output in market j at time t and $b_o^*$ and $b_{it}$ are the estimated coefficients of the intercept and slope respectively of the equilibrium supply curve.

Since pickling cucumbers are contracted in the spring before planting, output is assumed to be a function of price rather than the reverse. Also the functional form used to fit the farm supply data is linear since the cross-multiplication technique used to estimate net annual returns assume linearity between points.

The estimated linear supply of pickling cucumbers is

(2) $DCQCPC_t = B_1 ADOPT_t + B_2 FPCPD_t + B_3 (ADOPT_t * FPCPD_t) + \mu_t$

where t = 1965 to 1980

DCQCPC is the production of pickling cucumbers in pounds per capita, retail weight.

ADOPT is the percentage of pickling cucumber acres machine harvested.

FPCPD is the weighted average farm price for pickling cucumbers in dollars per pound, retail weight, deflated by the consumer price index (1967 = 100).

ADOPT * FPCPD is a slope shifter as technology is introduced.

The absence of input prices in the supply function is significant in that the coefficients of the equilibrium supply function are supposed to reflect the response of supply function as input price change as output changes. If input prices were included in the supply

function then the implicit assumption is that input prices are fixed regardless of the level of output. In the equilibrium supply model all input prices are allowed to vary with the level of output (Just and Hueth, 1979, p. 952). The estimates for the regression coefficients and the significance tests for equation (2) are as follows:

Supply of Pickling Cucumbers (Linear)

$$(3)\quad DCQCPC_t = \underset{(4.6)}{25.18\ ADOPT_t} + \underset{(18.16)}{113.7\ FPCPD_t} \underset{(-2.6)}{-304.5\ ADOPT_t * FPCPD_t}$$

where $ADJR^2$ = (.597) N = 16/DF = 13 F = (7.424)

SE = .43 SD = .65 DW = 2.4.

Equation (3) is the linear supply function of pickling cucumbers in the U.S. based on time series data from 1965 to 1980. The numbers in parentheses under the regression coefficients are the "t" values. The "t" values are either greater than 2 or less than -2 and are considered significant. The Durbin-Watson statistic indicates that there is no significant auto correlation. Auto correlation is defined as systematic error in the disturbance term $\mu_t$. "If no auto regression is indicated, we can retain the least squares estimates without fearing a loss of efficiency and a bias in the estimated standard error" (Kmenta, 1971, p. 296).

The F-statistic and the adjusted $R^2$ are in parentheses because they were estimated from a supply model which included the intercept term $B_o$. The intercept term proved not to be significantly different from zero and was dropped from the model. However, the statistical program used to estimate the coefficients will not calculate an

adjusted $R^2$ or F-statistic unless an intercept term is included in the model, hence the parentheses. The F-statistic is significant. The low adjusted $R^2$ implies that price and percentage of acres machine harvested explain only about 60% of the variability in the production of pickling cucumbers. This is not a surprising result since other independent variables such as weather, other technologies, policy changes might also explain some or not of the remaining 40% of the variability in output. The price of corn and soybeans were also included and found not to be significant in determining the production of pickling cucumbers.

Since it is the price elasticity of supply that is important in the subsequent analysis, it is only necessary that the estimate of the price coefficient is unbiased and efficient. The D-W statistic indicates that this is in fact the case. A simultaneous equation model is not used because it could introduce error into the estimate of the basically recursive system of price determination for cucumber pickles.

Once the supply curve is estimated it is possible to use the results to determine both the farm and retail supply elasticity. The price elasticity of supply for a linear supply curve is the slope multiplied by the ratio of price to output. The slope of the supply function is the partial derivative of the output with respect to price. Therefore the partial derivative of equation (3) with respect to price gives the following result:

$$(4) \quad \frac{\partial Q_t}{\partial P_t} = 113.7 - 304.5\ \text{ADOPT}_t$$

Equation (4) gives the slope of the linear supply curve for pickling cucumbers. The slope of the supply curve is the sum of the "normal"

slope and the change in the slope resulting from the adoption of the mechanical harvester. It is difficult to separate the effect the mechanical harvester had on the slope of the supply of pickling cucumbers from that of other external effects on the industry. The "Adopt" variable may be a poxy for the entire array of new technologies and other changes which were affecting the production of pickling cucumbers from 1965 to 1980. The "Adopt" independent variable reinforces the belief that the mechanical harvester changes the slope and intercept of the supply curve. There are however too many alternative explanations for why the supply curve was shifting coincidentally with the introduction of the mechanical harvester to rely on the "Adopt" variable as a measure of the shift in supply attributable to the mechanical harvester alone. For example from 1965 to 1980 several new seed varieties were introduced and the research on cultural practices was continually extended to producers during this time. In the next section, the means of measuring the shift in supply attributable solely to the mechanical harvester is discussed.

Equation (4) can be used to determine the price elasticity of supply by simply multiplying by the ratio of farm price to production. When this is done, the average farm supply elasticity is equal to 1.22695. This is not a surprising result since the intercept term $B_o$ was not significant. Therefore the supply function is initially a ray through the origin which by definition has an elasticity of 1 (Tomak and Robinson, 1972, p. 63). This result is also consistent with Cochrane's estimates of short run elasticity of supply for "certain individual vegetable crops (i.e., onions, tomatoes, cabbage)" as between "1.0 to 2.0" (Cochrane, 1955, p. 1164). Before estimating the

retail supply elasticity for cucumber pickles it is first necessary to consider the farm supply curve in light of the hypothesized irreversible supply function criteria.

Johnson has hypothesized that the supply response curve is consistent with his acquisition and salvage price definition of the long run (Johnson, 1962, pp. 131-143). Since durable goods are purchased or salvaged in relation to the output price, then output will itself reflect the purchasing or salvaging or durables as kinks in the supply response function. The supply function then is not a single continuous function over the range of prices but is rather a composite of supply curves with discrete turning points.

Johnson's hypothesized supply response curve is useful in short run price analysis as a means of explaining and estimating "turning points" in the data. Johnson's hypothesized supply response curve is also consistent with the Lindner, Jarrett and Rose notion of a "kinked" supply curve to estimate returns resulting from the introduction of a new technology. The "kink" in the Lindner, Jarrett and Rose supply function as well as Marshall's particular expense curve is a result of the industry's adjustment to the change in the relationship between input to output prices. Cochrane makes a similar distinction between the short run supply and supply response relationship in which the latter is irreversible (Cochrane, 1955, pp. 1162-1163).

There are some peculiarities with the pickling cucumber market which make testing the irreversible supply curve hypothesis difficult empirically. Pickling cucumbers are primarily a contracted crop and as such are less prone to wide fluctuation in price variability. Some range of price variability is required in order to induce producers to

purchase or salvage durable outputs. Another related fact associated with the pickling cucumber industry is that the trend in prices has been stable or slightly increasing in the time period studies from 1955 to 1978. The combination of stable prices on average and small variability makes the irreversible supply curve hypothesis difficult to test. Preliminary attempts at doing so were inconclusive for the reasons stated above.

The retail supply elasticity can be derived from the farm supply function by using the concept of marketing margins (Tomek and Robinson, 1972, p. 112). A marketing margin is defined as the "difference between the price paid by consumers and that obtained by producers" (Tomek and Robinson, 1972, p. 110). Therefore,

(5) $M = P_R - P_F$

where M is the marketing margin

$P_R$ is the retail price

$P_F$ is the farm price.

"The exact relationship between elasticities depends on how the primary and derived [supply] curves are related. Since the two curves are separated by a schedule of marketing margins, the problem reduces to one of how the marketing margin behaves" (Tomek and Robinson, 1972, p. 45). There are two relationships possible relating the farm price and the marketing margin. The margin may be a fixed dollar amount or it may be a fixed percentage of the retail price (Tomek and Robinson, 1972, pp. 45-46). Therefore the marketing margin can be expressed as:

(6) $M = C + aP_r$

where

C is the "dollar margin"

a is the "percentage margin".

If "a" is equal to zero then the margin is a constant dollar amount regardless of the amount marketed. In this case the two supply curves are parallel. The slope of parallel supply curves are equal and the elasticity of the two curves differ only by the price level. Therefore:

$$(7) \quad E_R = E_F * \frac{P_R}{P_F},$$

If "C" is equal to zero then the marketing margin is a constant percentage of the sales price. This implies that the elasticity of retail and farm supply are the same since the elasticity at the farm level is maintained at the retail level by the constant percentage change in the retail price associated with the constant percentage marketing margin. Therefore:

$$(8) \quad E_R = E_F$$

If neither "a" nor "C" are zero then the derived retail supply elasticity is given by the following:

$$(9) \quad E_R = E_F \left[1 + \frac{c}{(1-a)P_F}\right].$$

(Tomek and Robinson, 1972, pp. 46, 112). Equation (9) is consistent with both equations (7) and (8). If "a" is zero then (9) reduces to (7). If "c" is zero then (9) reduces to (8).

In order to determine which is the three equations for determining the derived retail supply elasticity of cucumber pickles it is first necessary to estimate the coefficients of equation (6). The results are as follows:

Retail Margin for Cucumber Pickles

(10) $MRGN_t = -.0814601 + .0155168\ DV5558$

$(-2.19339) \qquad (1.44403)$

$+ .021235\ DV5963 + 1.02536\ RPCPD_t$

$(1.83019) \qquad (26.25)$

$adjR^2 = .998429 \qquad N = 23/DF = 19 \qquad F = 4662$

$SE = .00510348 \quad SD = .128773 \qquad DW = 1.135$

An interpretation of the coefficients of "c" and "a" is in order.

(11) $c = -.0814601$

$a = 1.02536$

"c" is the constant dollar amount to convert a pound of pickling cucumbers into a pound of cucumber pickles regardless of the quantity of pickles supplied. A priori this amount should be positive or equal to zero. Similarly "a" is the fixed percentage of the retail price required to convert a pound of pickling cucumbers into a pound of cucumber pickles regardless of the quantity of pickles supplied. A priori this percent should be positive and between 0 and 1. Both the coefficients for "c" and "a" violate the a priori assumptions regarding the range of their value. These coefficients imply that the margin for cucumber pickles is 102.536 percent of the retail price minus 8.14601 cents.

Although this is an unanticipated result, it is consistent with the facts. The average deflated retail price for cucumber pickles from 1956 to 1978 is .856547 cents per pound. The average deflated farm price for pickling cucumbers is .0533024 cents per pound, retail weight. The average marketing margin over this time period is .8032

cents per pound. Using equation (10) to estimate the average marketing margin results in

(12) $\overline{MRGN} = -.814601 + (1.02536 * .856542)$

$= .7968 \doteq .8032$

where t = 1956 to 1978

MRGN is equal to the difference between the deflated retail price of cucumber pickles and the deflated farm price of pickling cucumbers per pound retail weight in the U.S.

DV5558 is a dummy variable if 1956 to 1958 and 0 otherwise

DV5963 is a dummy variable if 1959 to 1963 and 0 otherwise

RPCPD is the retail price of cucumber pickles in dollars per pound deflated by the consumer price index (1967 = 100)

Equation (10) is the estimated marketing margin between cucumber pickles and pickling cucumbers based on time series data from 1955 to 1978. The numbers in parentheses under the coefficients are the "t" values. "t" values greater than 2 or less than -2 are considered significant. The dummy variable though not significant are useful in correcting for the different price series used in compiling the retail price of cucumber pickles data series. These coefficients drop out for the time period 1965 to 1978, over which the retail supply elasticity will be determined. The adj$R^2$, and Durbin-Watson statistics, indicate that equation (10) is a good estimate of the marketing margin for cucumber pickles. The "t" statistic for "c" and "a" is considered significant. The actual and estimated average marketing margin are equal to 80 cents per pound rounded to the nearest cent. Therefore the marketing margin for cucumber pickles appears to be

strongly related to a fixed percentage of the retail price adjusted downward by a constant dollar amount per pound.

Using equations (7), (8), and (9), the array of retail supply elasticities are listed below for 1965-1978.

Retail Supply Elasticity for Cucumber Pickles

(13)

| | (a) $E_R = E_F$ | (b) $E_R = E_F\left[\frac{P_R}{P_F}\right]$ | (c) $E_R = E_F\left[\frac{c}{1 + (1-a)P_F}\right]$ |
|---|---|---|---|
| 1965 | 1.076 | 12.966 | 60.000 |
| 1966 | 1.193 | 9.242 | 63.322 |
| 1967 | 1.212 | 8.328 | 59.737 |
| 1968 | 1.275 | 8.070 | 64.760 |
| 1969 | 1.337 | 7.776 | 72.033 |
| 1970 | 1.164 | 7.020 | 64.550 |
| 1971 | 1.168 | 7.225 | 68.159 |
| 1972 | 1.197 | 6.556 | 71.461 |
| 1973 | 1.203 | 5.333 | 72.263 |
| 1974 | 1.493 | 4.602 | 75.662 |
| 1975 | 1.241 | 4.444 | 69.563 |
| 1976 | 1.286 | 3.114 | 77.920 |
| 1977 | 1.237 | 2.966 | 79.715 |
| 1978 | 1.088 | 2.854 | 73.106. |

## 4-4 Measuring the Vertical Shift in the Supply

In the previous section the farm level supply of pickling cucumbers was estimated. From this estimate the retail elasticity of supply of cucumber pickles was derived using the information regarding the

marketing margin. In this section the vertical shift in the supply of cucumber pickles is estimated. The vertical shift in the supply curve is determined using a cost of production study which compares the cost of producing pickling cucumbers using hand and machine harvesting techniques. This cost of production study is used to estimate the cost savings per pound of pickling cucumber produced using the machine harvesting. This cost saving is then multiplied by the percentage of the pickling cucumber crop harvested mechanically each year from 1965 to 1978. The product of the cost saving per unit and the percentage of units machine harvested provides an estimate of the vertical shift in the supply curve. This section then is a presentation of the data provided by the cost of production study along with data on the percentage of the pickling cucumber crop machine harvested.

Several studies have compared the cost of machine versus hand harvesting pickling cucumbers. These studies include:

1. "Economics of pickling cucumber production in Western Michigan," Stephen Harsh and Homer Patterson. Agricultural Economic Report No. 125, May 1969, Department of Agricultural Economics, Michigan State University, East Lansing, Michigan.

2. "Ohio crop enterprise budgets 1976: Horticulture crops," MM-363; Area and State Extension Farm Management Faculty. Department of Agricultural Economics and Rural Sociology, Ohio State University, Columbus, Ohio.

3. "Cost and returns of harvesting cucumbers with the once-over mechanical harvester," H. L. Liner, North Carolina Cooperative Extension Service Circular No. 560, April 1973, North Carolina Cooperative Extension Service, Raleigh, North Carolina.

The purpose of these studies is to help producers or potential producers of pickling cucumbers decide on the least cost means of producing and harvesting their product. For the purposes of this

study the "Ohio crop enterprises budgets 1976 . . ." is used to determine the average cost saving by a typical firm of machine harvesting their pickling cucumber crop.

Ohio is a particularly good state in which to compare the effect of the mechanical harvesting on the cost of producing pickling cucumbers. Refer to Figure 42. As was mentioned in Chapter I on the historical context of the invention of the pickle machine, the increasing uncertainty of seasonal labor in Michigan induced the search for the mechanical harvester. As Michigan declined in share of total U.S. production of pickling cucumbers, Ohio increased its share. In 1960 Michigan produced about 30% of the pickling cucumbers while Ohio produced about 2.5% of the U.S. total. Twenty years later Michigan produced only about 17.5% while Ohio increased its share from 2.5 to about 12%. Ohio can be seen as picking up almost the entire difference in Michigan declining share of pickling cucumber production during the time when the mechanical harvester was introduced in 1965.

Clearly Ohio farmers would be interested in a study comparing the cost advantage of machine versus hand harvesting during this period of rapid expansion pickling cucumber production. Pickling cucumbers and processing tomatoes are often grown in the same area as a means of extending employment opportunities for seasonal labor. Therefore it is not surprising that included in the "Ohio crop enterprise budgets 1976. . ." is a section comparing the costs of machine and hand harvesting processing tomatoes as well.

The "Ohio crop enterprise budget 1976 . . ." includes two sections on pickling cucumbers, one for "hand harvest" and the other entitled "machine harvest." In the hand harvest section production costs are

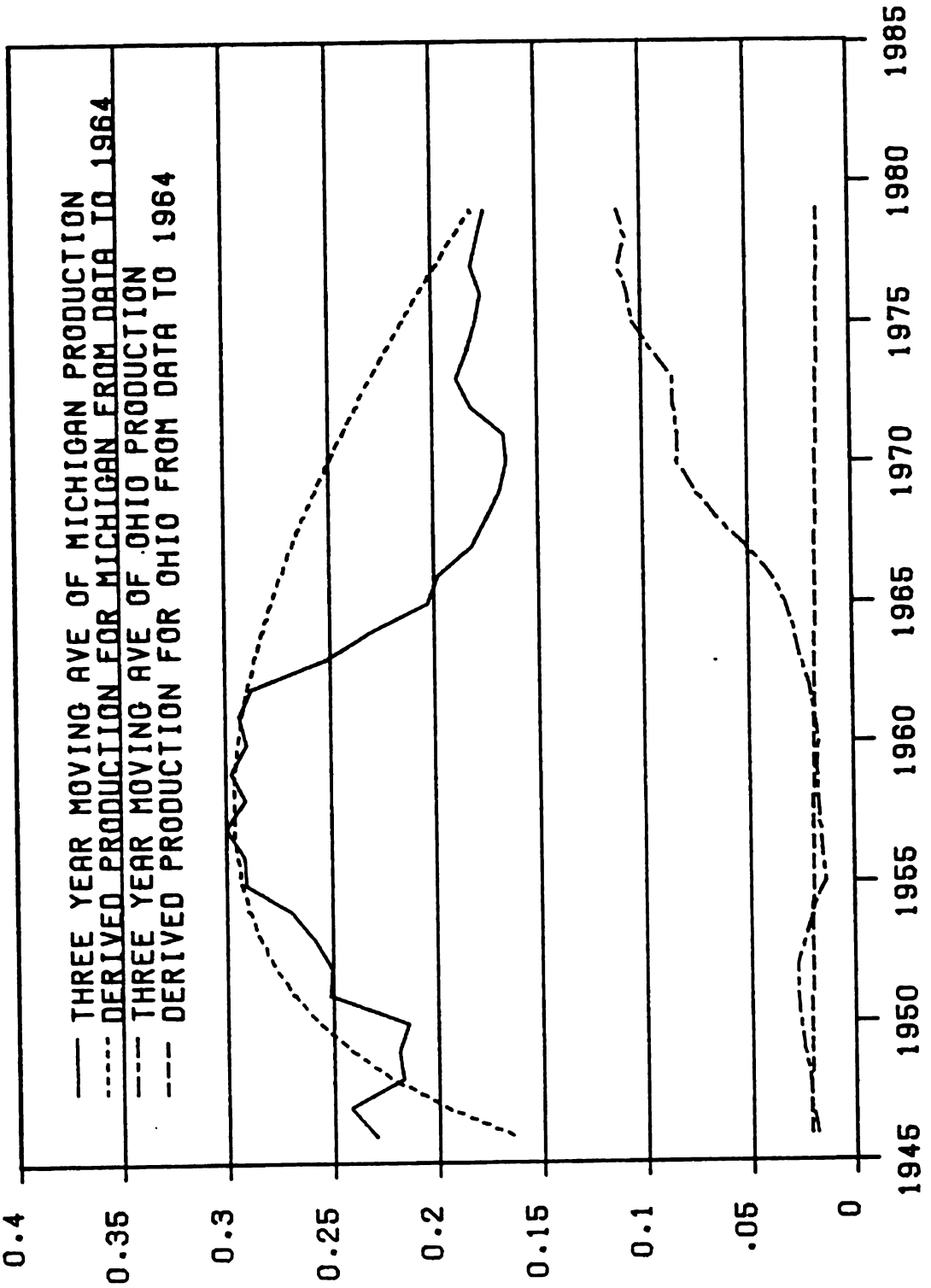



Figure 42. Michigan's and Ohio's Shares of U.S. Pickling Cucumber Production

estimated on a per acre basis for the three yield levels of 7, 10, and 13 tons per acre. Similarly in the machine harvest section production costs are estimated on a per acre basis for three yield levels of 2, 4.5 and 7 tons per acre. The difference in yield levels analyzed for hand and machine harvested pickling cucumbers reflects the substantially lower yields of "once-over destructive harvest" by the machine compared with the higher yield obtained by multiple hand picking of cucumbers.

The "Ohio crop enterprise budget 1976. . ." also includes an average value of a hundredweight of pickling cucumbers. Pickling cucumbers are priced in inverse relation to their diameter. Therefore three grade sizes are used in the Ohio study. These are grade sizes no. 1, 2, and 3. Table 1 lists the grades, the measure of grade size in inches, the price per hundredweight for each grade size and the assumed percentage of each grade size harvested by the machine and by hand as well as the implied "blend" price for a hundredweight of pickling cucumbers harvested by machine and by hand.

Table 1

Grade Size, Price per Hundredweight and Percent of Grade Size of Machine and Hand Harvested Pickling Cucumbers

| | | | % of grade harvested by | |
|---|---|---|---|---|
| grade | measure | price/cwt | machine | hand |
| 1 | up to 1-1/16" | $13.94 | 5% | 20% |
| 2 | 1-1/16" to 1-1/2" | $ 6.86 | 35% | 40% |
| 3 | over 1-1/2" | $ 3.86 | 60% | 40% |
| | "blend" price | | $5.41/cwt | $7.08/cwt |

The table above shows that not only does the machine harvest fewer tons per acre but that each ton is worth less as well. These are major disincentives for using a machine to harvest pickling cucumbers. The cost savings must be sufficient to offset these disincentives. In Tables 2 and 3 listed below is a summary of the yield per acre, price and cost of production for the various levels of yield associated with machine and hand harvesting pickling cucumbers.

Table 2

Yield per Acre, Price and Cost per Hundredweight of Machine Harvested Pickling Cucumbers

| yield | 2 tons/acre | 4.5 tons/acre | 7 tons/acre |
|---|---|---|---|
| price | $5.41/cwt | $5.41/cwt | $5.41/cwt |
| cost | $9.75/cwt | $4.77/cwt | $3.35/cwt |
| Returns to Management & Risk | $-4.34/cwt | $ .64/cwt | $2.06/cwt |

Table 3

Yield per Acre, Price and Cost per Hundredweight of Hand Harvested Pickling Cucumbers

| yield | 7 tons/acre | 10 tons/acre | 13 tons/acre |
|---|---|---|---|
| price | $7.08/cwt | $7.08/cwt | $7.08/cwt |
| cost | $8.04/cwt | $7.08/cwt | $6.55/cwt |
| Returns to Management & Risk | $-.96/cwt | $ 0/cwt | $ .53/cwt |

The machine harvest cost figures are based on harvesting an average of 175 acres of pickling cucumbers per year. The hand harvested cost figures on the other hand are based on harvesting an average of 15 acres of pickling cucumbers per year. The average of

175 acres for machine harvesting is not inconsistent with the 1974 Census of Agriculture in which for example Muskegon County reported 10 farm harvesting 1,452 acres of "cucumbers and pickles." Unfortunately the Department of Commerce groups harvested acres of pickling cucumbers with "slicer" cucumbers grown for the fresh market. "Slicers" are hand picked and grown on relatively small acreage per farm. Therefore the Census of Agriculture data on the structure of pickling cucumber farms is biased downward.

Next it is necessary to choose the level of yield which might be typical of machine and hand harvesting. Michigan's pickling cucumbers are about 90% machine harvested since 1972. Therefore Michigan pickling cucumber yields from 1972 to 1980 would provide a guide as to the expected yield per acre for machine harvested pickling cucumbers. Michigan's pickling cucumber yield from 1972 to 1980 is 4.49 tons/acre. Therefore the 4.5 ton/acre yield level is the one which is used to determine the typical cost of production for machine harvested pickling cucumbers.

The average level of yield for hand harvested cucumber pickles varies significantly by regions of the county. Figure 43 shows the difference in yield by region of the country. The "southern" region includes the states of Delaware, Maryland, North Carolina, Texas and Virginia. The "Great Lakes" region includes Indiana, Michigan, Ohio and Wisconsin. The "western" region includes California, Colorado and Washington. It is important to note that the yield in the southern region is about 3-4 tons per acre in spite of the fact that pickling cucumbers are still largely hand harvested. Southern yields are not sufficient to warrant the consideration of machine harvesting.

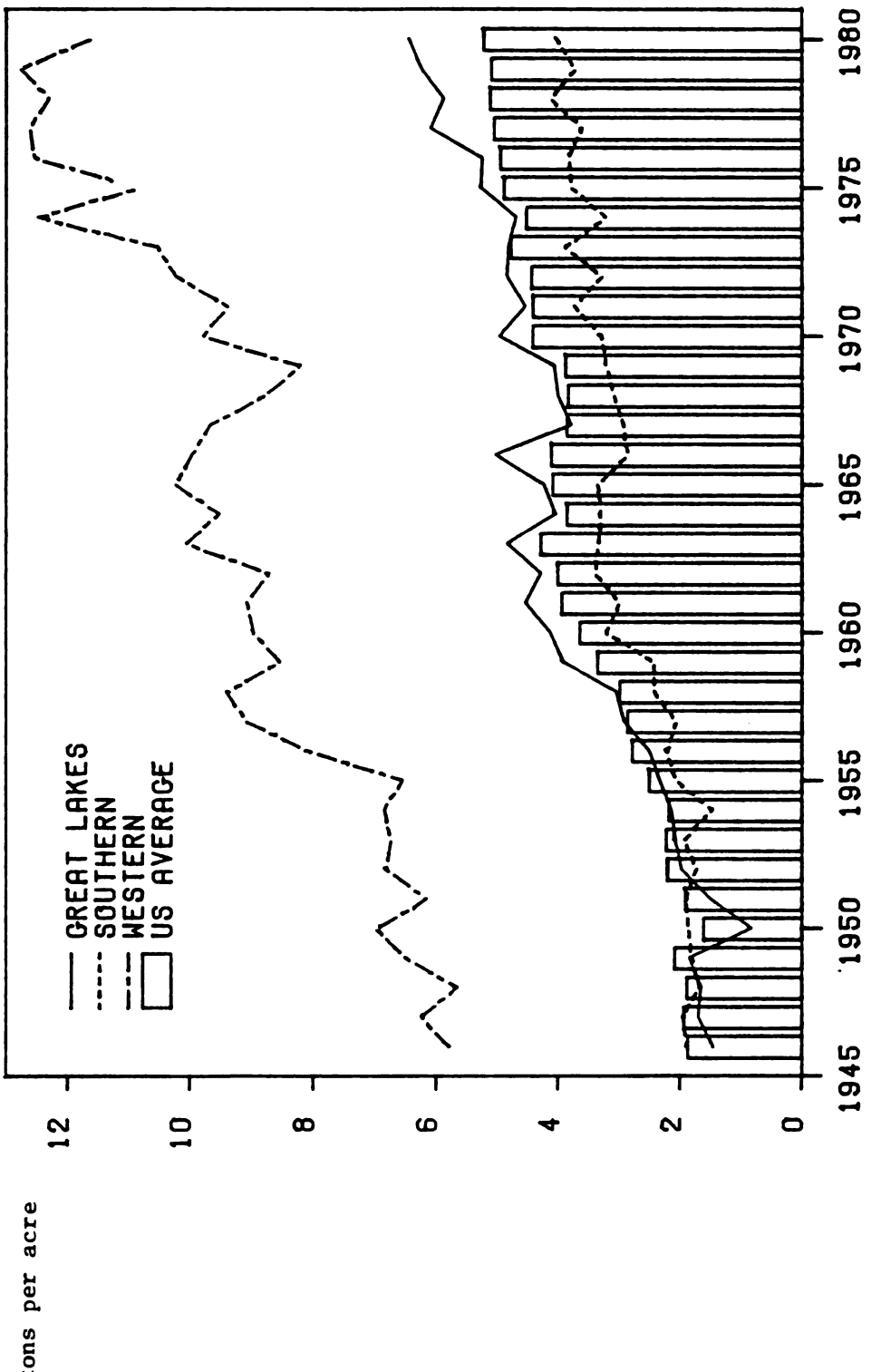



Figure 43. Yield of Pickling Cucumbers by Region

Figure 44 shows the relative importance of the "regions" to the total U.S. production. The "Great Lakes" have declined relative to the "southern" region, but again the large discrepancy in yields and the associated cost effectiveness of the machine harvester in the "Great Lakes" along with the desire on the part of processors to have a "smooth" supply of raw product from early spring to late fall has curtailed an exodus of pickling cucumber production from the "Great Lakes" to the "southern" region.

The western states have an unusually high level of yield due to double cropping. However the western states have not produced a significant percentage of total U.S. production. The bulk and spoilage associated with pickling cucumbers does not allow for economical transcontinental shipment of the low value raw product. Therefore the western states produce pickling cucumber primarily eventual consumption on the west coast.

This then leaves the Great Lakes region as the area from which to select the appropriate level of yield of hand harvested pickling cucumbers. In this case Ohio provides the best example of level of yield for hand harvested pickling cucumbers. Ohio, as was mentioned above, increased production of pickling cucumbers in inverse relation to Michigan decline in production. Seasonal labor was already in Ohio to harvest processing tomatoes. Therefore the increase in pickling cucumber production complemented the existing processing tomato industry by providing a more secure labor force for the producers and a long picking season in one location for the laborers. Ohio's pickling cucumbers from 1972 to 1980 were primarily hand harvested. The average yield in Ohio from 1972 to 1980 was 9.86 tons/acre or more than twice the average

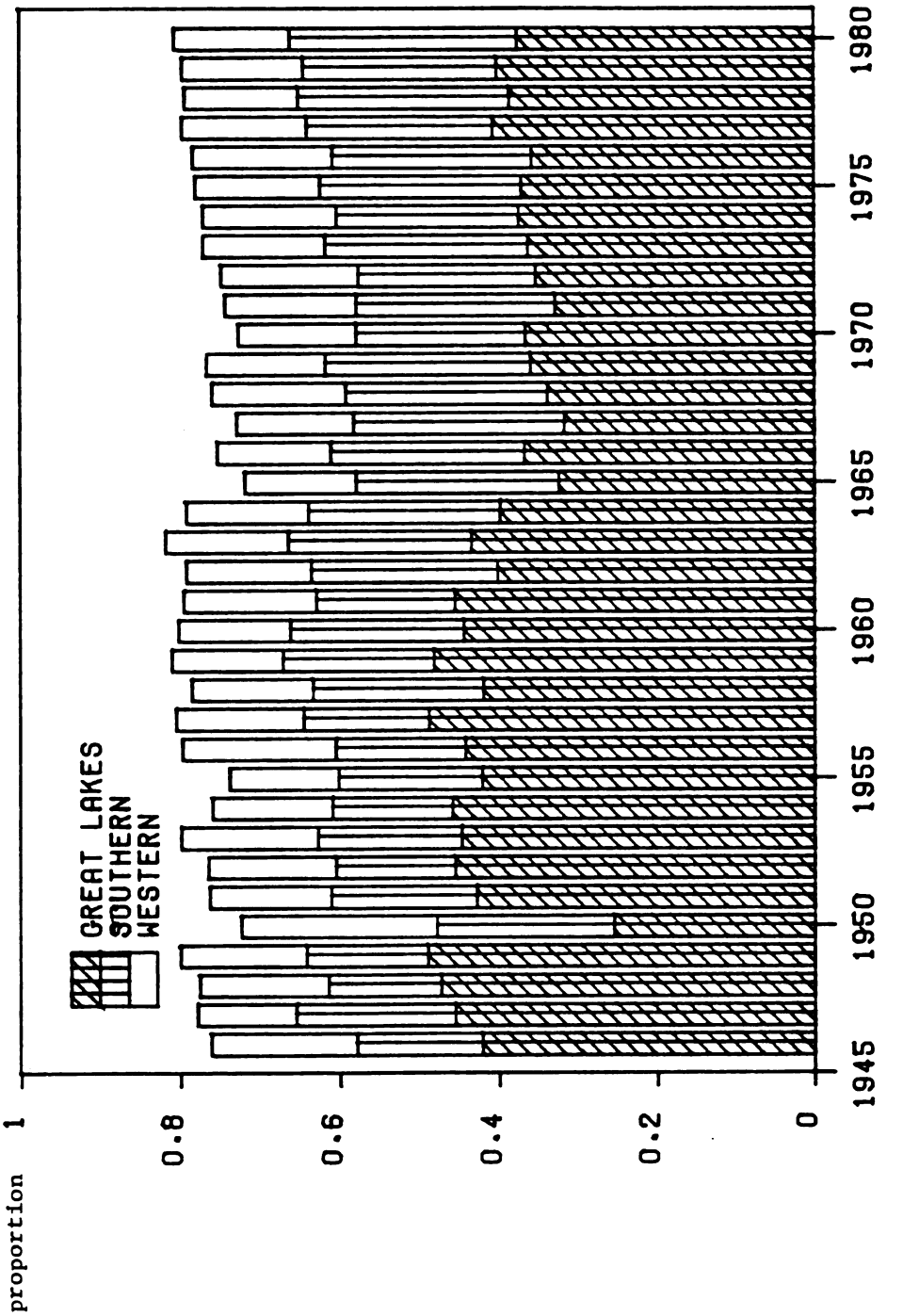



Figure 44. Proportion of Pickling Cucumbers Produced by Region

yield in Michigan during the same period. Therefore the 10 ton/acre yield level is the one which is used to determine the typical cost of production for hand harvested pickling cucumbers.

Therefore the cost saving between machine and hand harvested pickling cucumbers is the difference between the cost of machine harvested pickling cucumbers on 175 acres with an average yield of 4.5 tons/acre, receiving an average price of $5.41/cwt (1975 dollars), and the cost of hand harvesting pickling cucumbers on 15 acres with an average yield of 10 tons/acre, receiving an average price of $7.08/cwt (1975 dollars). This difference is equal to $.64/cwt farm weight (1975 dollars) in favor of machine harvesting. This figure then needs to be converted to dollars per pound, retail weight (1967 dollars).

Table 4

Cost Saving of Machine Harvesting
Pickling Cucumbers

| | |
|---|---|
| (a) | $.64/cwt farm weight, 1975 dollars |
| (b) | $.0064/lb. farm weight, 1975 dollars |
| (c) | $.0093776/lb. retail weight, 1975 dollars |
| (d) | $.0055/lb. retail weight, 1967 dollars |

The cost saving of the machine harvester would result in a retail saving of about .5 cents per pound retail weight in 1967 dollars for that percentage of the pickling cucumber crop machine harvested. Therefore it is necessary to determine the rate of adoption of the mechanical cucumber harvester in order to determine the annual effect the mechanical harvester had on decreasing the retail price of cucumber pickles. The rate of adoption of the mechanical cucumber harvester in the U.S. is reported in "Vegetable Situation," TVS-215, February 1980,

Economics, Statistics and Cooperatives Service, U.S. Department of Agriculture, Washington, D.C. The ESCS reports that in 1968 15% of the pickling cucumber harvested acres in the U.S. were machine harvested. Then, in 1975 and 1978, 30% of the pickling cucumber harvested acres were machine harvested (ESCS, 1980, p. 32). Before 1965 there were no successful mechanical cucumber harvesters commercially available. Therefore, using the above data it is possible to interpolate the percentage of pickling cucumber acres machine harvested for the intervening years. See Table 5.

## 4-5 Ex Ante Equilibrium Price and Quantity

In the previous section the cost savings from machine harvesting pickling cucumbers was presented along with the rate of adoption of the mechanical harvester. This information was used to determine a measure of the vertical shift in the retail supply of cucumber pickles resulting from the introduction of the mechanical cucumber harvester technology. In this section the estimate of the supply shift is used along with the price elasticity of supply and demand to determine the ex ante equilibrium price and quantity. The ex ante market equilibrium in this case is the price and quantity combinations which would have existed from 1965 to 1978 in the absence of the mechanical harvester, other things being equal.

In order to determine the ex ante equilibrium price and quantity it is necessary to transform two equations developed by Pinstrup-Andersen, Ruiz de Londono and Hoover as reported by Rose (Rose, 1980, p. 834). These are:

(1) $P_1 = P_0 [1 - (ke)/)e + n)]$

Table 5

Percentage of Units Machine Harvested, Cost Savings per Unit and Percentage of Cost Savings to Retail Price of Machine Harvested Pickling Cucumbers from 1964 to 1978

| year | (1) acres machine harvested (percent) | (2) acres machine harvested (1000) | (3) tons machine harvested (1000) | (4) tons machine harvested (percent) | (5) potential cost saving $/lb. retail (1967 = 100) | (6) actual cost saving $/lb. retail (1967 = 100) | (7) % of cost saving of retail price (percent) |
|---|---|---|---|---|---|---|---|
| 1964 | 00 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1965 | 03 | 3.282 | 13.769 | .033 | .0055 | .00018 | .02 |
| 1966 | 13 | 16.848 | 75.816 | .14 | .0055 | .00078 | .08 |
| 1967 | 15 | 23.205 | 104.423 | .18 | .0055 | .00096 | .10 |
| 1968 | 15 | 21.720 | 97.740 | .18 | .0055 | .00097 | .10 |
| 1969 | 17 | 22.049 | 99.221 | .20 | .0055 | .00110 | .12 |
| 1970 | 17 | 22.712 | 102.204 | .17 | .0055 | .00095 | .11 |
| 1971 | 20 | 25.520 | 114.840 | .20 | .0055 | .00112 | .12 |
| 1972 | 23 | 29.624 | 133.308 | .23 | .0055 | .00128 | .14 |
| 1973 | 25 | 31.475 | 141.638 | .24 | .0055 | .00130 | .14 |
| 1974 | 27 | 35.640 | 160.380 | .27 | .0055 | .00148 | .16 |
| 1975 | 30 | 41.520 | 186.840 | .27 | .0055 | .00152 | .15 |
| 1976 | 30 | 38.520 | 173.340 | .27 | .0055 | .00150 | .15 |
| 1977 | 30 | 37.140 | 167.130 | .27 | .0055 | .00147 | .15 |
| 1978 | 30 | 40.320 | 181.440 | .26 | .0055 | .00146 | .15 |

NOTES: The seven colums above are defined as follows:

(1) the percentage of pickling cucumber acres machine harvested

(2) the acres of pickling cucumbers machine harvested (in 1,000's of acres)

(3) the tons of pickling cucumbers machine harvested (in 1,000's of tons)

Table 5--continued

NOTES (continued):

(4) the percentage of pickling cucumbers machine harvested

(5) the potential cost saving from harvesting pickling cucumbers mechanically in dollars/lb. retail weight (1967 = 100)

(6) the actual cost saving from harvesting pickling cucumbers mechanically in dollars/lb. retail weight (1967 = 100)

(7) the percentage increase in the retail price of cucumber pickles in the absence of the cost savings resulting from the mechanical cucumber harvester.

Column (7) is the information needed to determine the ex ante equilibrium price and quantity. Column (7) represents the vertical shift in the retail supply curve resulting from the introduction of the mechanical cucumber harvester.

and

(2) $Q_1 = Q_0 [1 + (ken)/(e + n)]$.

Equations (1) and (2) are the equations for estimating ex post market equilibrium,

where

$P_1$ is the ex post equilibrium price

$Q_1$ is the ex post equilibrium quantity

$P_0$ is the ex ante equilibrium price

$Q_0$ is the ex ante equilibrium quantity

k is the cost decrease as a proportion of $P_0$

e is the supply elasticity

n is the absolute value of the demand elasticity.

Equations (1) and (2) are rewritten below for estimating ex ante market equilibrium such that:

(3) $P_0 = P_1/[1 - (ke)/(e + n)]$

and

(4) $Q_0 = Q_1/[1 + (ken)/(e + n)]$

where

k is the cost decrease as a proportion of $P_1$

and all other variables are the same as defined above.

In order to estimate the values for $P_0$ and $Q_0$, the values for $P_1$, $Q_1$, k, e, and n must first be determined. $P_1$ is the ex post equilibrium price and is set equal to retail price of cucumber pickles (1967 = 100) from 1965 to 1978. $Q_1$ is the ex post equilibrium quantity. The value of $Q_1$ is set equal to the estimated value of $\hat{Q}_1$ as determined by the exponential demand equation so that:

(5) $\hat{Q}_{1t} = e^{-6.58808} \quad RPCPD_t^{-.820024} \quad \overline{DICD}_t^{1.07013}$

where t = 1965 to 1978

$\hat{Q}_1$ is the estimated ex post equilibrium quantity in pounds per capita of cucumber pickles

e is the natural log base 2.71828. . .

RPCPD is the retail price of cucumber pickles (1967 = 100)

$\overline{DICD}$ is the average disposible income per capita (1967 = 100) from 1965 to 1978 and equals $3028.27.

k is the cost increase in the retail price of cucumber pickles as a proportion of the ex post retail price defined in the previous section as the percentage increase in the retail price of cucumber pickles in the absence of cost savings resulting from the mechanical cucumber harvester. e, in equations (3) and (4) is the retail supply elasticity of cucumber pickles defined in an earlier section by the equation:

(6) $E_R = E_F \quad 1 + \frac{c}{(1-a)\ P_F}$

where $E_F$ is given by the equation:

(7) $E_F = (113.7 - 304.5\ ADOPT_t) \quad \frac{FPCPD_t}{\overline{DCQCPC}_t}$

and "c" and "a" are the coefficients of the marketing margin equation:

(8) $MRGN_t = -.0814601 + 1.02536\ RPCPD_t$

where

ADOPT is the percentage of pickling cucumber acres machine harvested

$E_R$ is the retail supply elasticity of cucumber pickles

$E_F$ is the farm supply elasticity of pickling cucumbers

FPCPD is the weighted average farm price for pickling cucumbers in dollars per pound, retail weight deflated by the consumer price index (1967 = 100)

DCQCPC is the production of pickling cucumbers in pounds per capita, retail weight

n is the absolute value of the retail own-price elasticity of demand and is equal to .820024 given the exponential functional form of the demand equation estimated for the years 1955 to 1978.

The values for $P_1$, $Q_1$, k, e, n, $P_0$ and $Q_0$ for the years 1965 to 1978 are listed in Table 6 below.

Table 6

Ex Ante Equilibrium Price and Quantity of Cucumber Pickles Given Ex Post Exponential Demand and Linear Supply Functions

| year | $P_1$ | $Q_1$ | k | e | n | $P_0$ | $Q_0$ |
|---|---|---|---|---|---|---|---|
| 1965 | .947763 | 7.64312 | .02 | 60.0 | .82 | .947943 | 7.64193 |
| 1966 | .956688 | 7.62383 | .08 | 63.3 | .82 | .951460 | 7.61876 |
| 1967 | .941129 | 7.68728 | .10 | 59.7 | .82 | .942081 | 7.68091 |
| 1968 | .925020 | 7.79685 | .10 | 64.8 | .82 | .925982 | 7.79024 |
| 1969 | .906331 | 7.92848 | .12 | 72.0 | .82 | .907405 | 7.92079 |
| 1970 | .902127 | 7.95876 | .11 | 64.6 | .82 | .903071 | 7.05195 |
| 1971 | .916509 | 7.85620 | .12 | 68.2 | .82 | .917620 | 7.84841 |
| 1972 | .948519 | 7.63812 | .14 | 71.5 | .82 | .949791 | 7.62975 |
| 1973 | .939930 | 7.69531 | .14 | 72.3 | .82 | .941219 | 7.68668 |
| 1974 | .941341 | 7.68582 | .16 | 75.7 | .82 | .942811 | 7.67604 |
| 1975 | 1.017730 | 7.20951 | .15 | 69.6 | .82 | 1.101929 | 7.20077 |
| 1976 | 1.010580 | 7.25132 | .15 | 77.9 | .82 | 1.101207 | 7.24257 |
| 1977 | .983794 | 7.41281 | .15 | 79.7 | .82 | .985254 | 7.40381 |
| 1978 | .963284 | 7.54199 | .15 | 73.1 | .82 | .964726 | 7.53275 |

Table 6 above is a list of the ex post equilibrium price and quantity ($P_1$, $Q_1$) and the ex ante equilibrium price and quantity ($P_0$, $Q_0$) of cucumber pickles. Also listed is the percent of the vertical decline in retail supply of cucumber pickles (k) and the price elasticity of supply and demand (e, n) respectively.

It is also possible to estimate the values of the ex ante equilibrium price and quantity using the linear demand function and its

associated coefficients. By estimating two sets of values for the ex ante equilibrium price and quantity it will be possible to determine the effect functional form of the demand curve has on the estimate of net annual returns and the implied distribution of benefits between consumer's and owner's of factors of production. The linear demand function estimates are presented in Table 7.

Table 7

Ex Ante Equilibrium Price and Quantity of Cucumber Pickles Given Ex Post Linear Demand and Supply Functions

| year | $P_1$ | $Q_1$ | k | e | n | $P_0$ | $Q_0$ |
|---|---|---|---|---|---|---|---|
| 1965 | .947763 | 7.62987 | .02 | 60.0 | .554 | .947944 | 7.62906 |
| 1966 | .950688 | 7.61681 | .08 | 63.3 | .557 | .951463 | 7.61336 |
| 1967 | .941129 | 7.65947 | .10 | 59.7 | .548 | .942085 | 7.65521 |
| 1968 | .925024 | 7.73135 | .10 | 64.8 | .534 | .925986 | 7.72706 |
| 1969 | .906331 | 7.81477 | .12 | 72.0 | .518 | .907282 | 7.81053 |
| 1970 | .902127 | 7.83353 | .11 | 64.6 | .514 | .903075 | 7.82931 |
| 1971 | .916509 | 7.76935 | .12 | 68.2 | .526 | .917457 | 7.76513 |
| 1972 | .948519 | 7.62649 | .14 | 71.5 | .555 | .949628 | 7.62155 |
| 1973 | .939930 | 7.66482 | .14 | 72.3 | .547 | .941120 | 7.65953 |
| 1974 | .941346 | 7.65850 | .16 | 75.7 | .549 | .942706 | 7.65245 |
| 1975 | 1.017730 | 7.31762 | .15 | 69.6 | .621 | 1.101909 | 7.31156 |
| 1976 | 1.010580 | 7.34954 | .15 | 77.0 | .614 | 1.101207 | 7.34289 |
| 1977 | .983794 | 7.46907 | .15 | 79.7 | .588 | .985259 | 7.46255 |
| 1978 | .963284 | 7.56060 | .25 | 73.1 | .569 | .964731 | 7.55416 |

Table 7 presents the values of the ex ante and ex post equilibrium price and quantity associated with the linear functional form of the demand curve. In the next sections these data associated with both the linear and exponential demand curve are used to determine consumer's surplus, producer's surplus and net annual return. Subsequent sections present the present value of these measures of benefits as well as the costs and the associated benefit-cost and internal rates of return.

## 4-6 Estimating Consumer's Surplus

The information in the preceding sections develops the data required to estimate the benefits resulting from the introduction of the mechanical cucumber harvester. The theory of consumer's surplus as a means of estimating these benefits is developed in a preceding chapter. This section applies the data to the equations developed above to estimate consumer's surplus for the mechanical cucumber harvester technology consistent with the ideas presented in the chapter on the theory of consumer's surplus. This empirical application is presented in relation to the equation developed for both the linear and exponential demand curves. See Figure 45. Within each type of demand curve the Willig and Hausman approaches for estimating consumer's surplus are presented. These two approaches are presented in the context of the earlier measures of consumer's surplus, i.e. the Laspeyres, "utility" and Paasche measures. The first part of this section presents the estimates of consumer's surplus for the exponential functional form of the demand equation. The second part of this section presents the estimates of consumer's surplus for the linear functional form of the demand equation.

### 4-6-1 Exponential Demand

Recall from the chapter on consumer's surplus that a series of inequalities (and equalities) between the various measures of consumer's surplus were presented. These inequalities are of both a general form, i.e. for any functional form of the estimated demand curve, as well as those specific to the exponential and linear demand functional forms. The general relationships between the Laspeyres, compensating

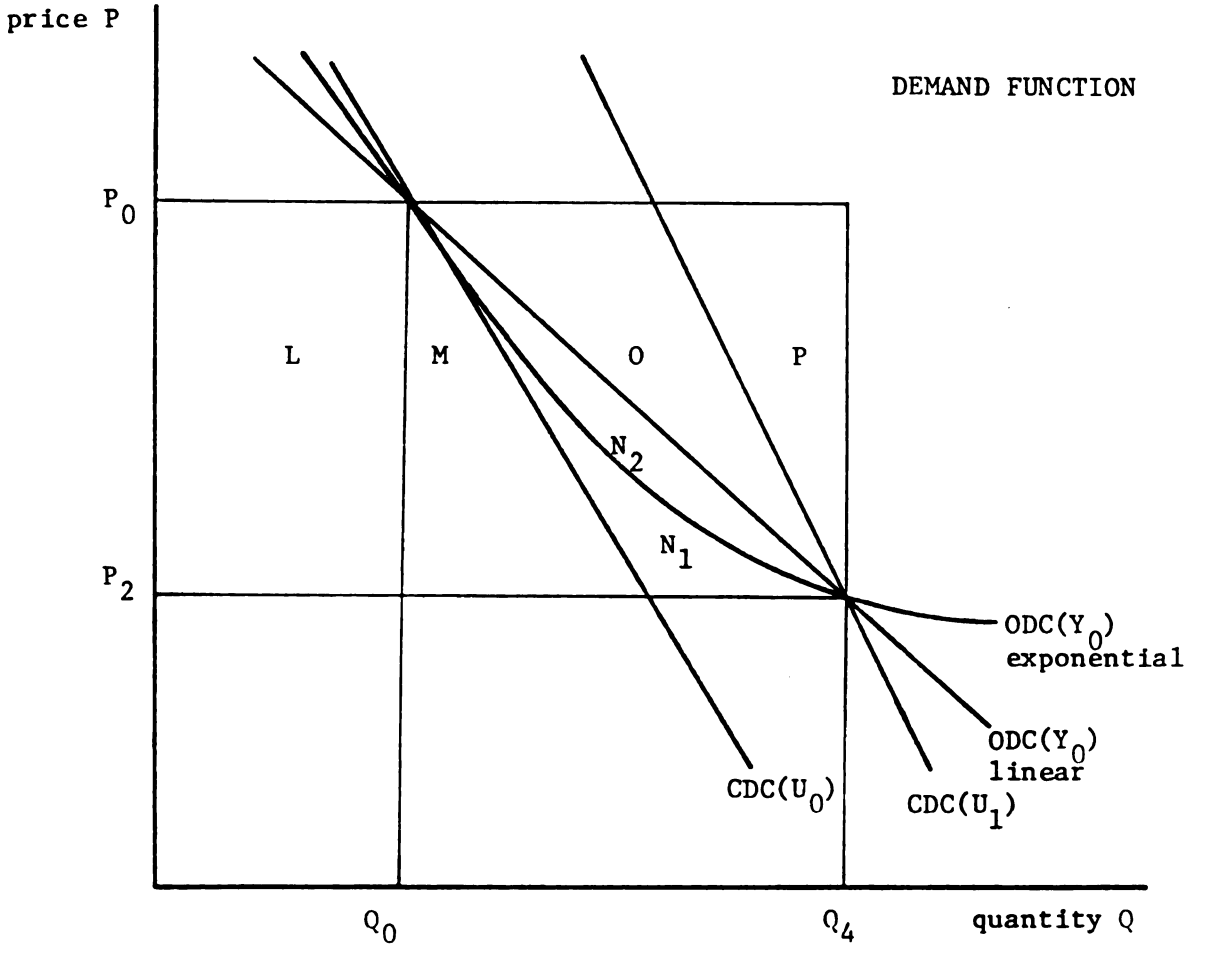



Figure 45. "Utility" Measure of Consumer's Surplus Using Linear and Exponential Ordinary Demand Functions

variation, "utility," equivalent variation and Paasche measures for a price decrease are as follows:

(1) Laspeyres < compensating variation < "utility"

and

(2) "utility" < equivalent variation < Paasche

The specific relationships between the Willig and Hausman's measure of compensating and equivalent variation given an exponential functional form of the demand curve are as follows:

(3) Willig's compensating variation = Hausman's compensating variation,

(4) Willig's equivalent variation = Hausman's equivalent variation

and

(5) "utility" < "utility" approximation.

The equations to estimate these measures of consumer's surplus given an exponential functional form of the demand curve are as follows:

(6) Laspeyres = $Q_0 (P_0 - P_1)$ = area L

(7) Paasche = $Q_0 (P_0 - P_1)$ = area $LMN_1N_2OP$

(8) "utility" approximation = $1/2 (Q_0 + Q_1)(P_0 - P_1)$ = area $NMN_1N_2$

(9) "utility" = $A_{P_1}^{P_0} = \frac{P_0Q_0 - P_1Q_1}{\alpha + 1}$ = area $LMN_1$

(10) Willig's compensating variation = $A_{P_1}^{P_0} \left| 1 - \delta A_{P_1}^{P_0} / 2Y_0 \right|$ = area LM

(11) Hausman's compensating variation

$$= Y_0 - \left| Y_0^{1-\delta} + Y^{-\delta} (1-\delta) A_{P_1}^{P_0} \right|^{\frac{1}{1-\delta}} = \text{area LM}$$

(12) Willig's equivalent variation = $A_{P_1}^{P_0} \left| 1 + \delta A_{P_1}^{P_0} / 2Y_0 \right|$ = area $LMN_1N_2O$

and

(13) Hausman's equivalent variation

$$= Y_0^{1-\delta} - Y^{-\delta}\,(1-\delta)\,A_{P_1}^{P_0}\;\frac{1}{1-\delta} - Y_0 = \text{area } LMN_1N_2O$$

where

$\alpha$ = -.820024, the price elasticity of demand for cucumber pickles given an exponential functional form

$\delta$ = 1.07013, the income elasticity for cucumber pickles given an exponential functional form

Y = 3028.27, the average disposable income in the U.S. from 1965 to 1978 (1967 = 100)

$P_0$, $P_1$, $Q_0$, $Q_1$, are the ex ante and ex post equilibrium price and quantity as described in the preceding section. And $Q_1$, $P_0$ and $Q_0$ are consistent with the parameter of the exponential demand function. The results of equations (6), (7), (8), (11) and (13) for the years 1965 to 1978 are shown in Table 8.

Table 8

Five Measures of Consumer's Surplus Given an Exponential Demand for Cucumber Pickles

| | Laspeyres (dollars) | Compensating Variation (dollars) | "Utility" Approximation (dollars) | Equivalent Variation (dollars) | Paasche (dollars) |
|---|---|---|---|---|---|
| 1965 | .137371E-02 | .137382E-02 | .137382E-02 | .137382E-02 | .137393E-02 |
| 1966 | .587857E-02 | .588052E-02 | .588053E-02 | .588053E-02 | .588248E-02 |
| 1967 | .731314E-02 | .731616E-02 | .731617E-02 | .731618E-02 | .731920E-02 |
| 1968 | .746380E-02 | .746695E-02 | .746696E-02 | .746697E-02 | .747013E-02 |
| 1969 | .850845E-02 | .851267E-02 | .851269E-02 | .851270E-02 | .851682E-02 |
| 1970 | .750426E-02 | .750747E-02 | .750748E-02 | .750749E-02 | .751069E-02 |
| 1971 | .871474E-02 | .871905E-02 | .871906E-02 | .871908E-02 | .872339E-02 |
| 1972 | .970176E-02 | .970707E-02 | .970709E-02 | .970710E-02 | .971241E-02 |
| 1973 | .990388E-02 | .990942E-02 | .990944E-02 | .990946E-02 | .991500E-02 |
| 1974 | .112412E-01 | .112484E-01 | .112484E-01 | .112484E-01 | .112556E-01 |
| 1975 | .108685E-01 | .108751E-01 | .108751E-01 | .108751E-01 | .108817E-01 |
| 1976 | .107968E-01 | .108033E-01 | .108033E-01 | .108034E-01 | .108099E-01 |
| 1977 | .108141E-01 | .108207E-01 | .108207E-01 | .108207E-01 | .108273E-01 |
| 1978 | .108612E-01 | .108679E-01 | .108679E-01 | .108679E-01 | .108745E-01 |

These results are consistent with the general form relationships presented in equations (1) and (2). These results are the various measures of the annual per capita increase in real income in dollars resulting from the introduction of the mechanical cucumber harvester. The benefits to the average person in the U.S. is about one cent per year. It is interesting to note that the Hausman and Willig measures of compensating and equivalent variations are consistent with results described by equations (3) and (4) as shown in Table 9 below.

Table 9

A Comparison of Hausman's and Willig's Measures of Consumer's Surplus Associated with the "Utility" Approximation Measure Given an Exponential Demand for Cucumber Pickles

| | Hausman's Compensating Variation (dollars) | Willig's Compensating Variation (dollars) | Hausman's Equivalent Variation (dollars) | Willig's Equivalent Variation (dollars) |
|---|---|---|---|---|
| 1965 | .137382E-02 | .137382E-02 | .137382E-02 | .137382E-02 |
| 1966 | .588052E-02 | .588052E-02 | .588053E-02 | .588053E-02 |
| 1967 | .731616E-02 | .731616E-02 | .731618E-02 | .731618E-02 |
| 1968 | .746695E-02 | .746695E-02 | .746697E-02 | .746697E-02 |
| 1969 | .851267E-02 | .851268E-02 | .851270E-02 | .851270E-02 |
| 1970 | .750747E-02 | .750747E-02 | .750749E-02 | .750749E-02 |
| 1971 | .871905E-02 | .871905E-02 | .871908E-02 | .871908E-02 |
| 1972 | .970707E-02 | .970707E-02 | .970710E-02 | .970710E-02 |
| 1973 | .990942E-02 | .990942E-02 | .990946E-02 | .990946E-02 |
| 1974 | .112484E-01 | .112484E-01 | .112484E-01 | .112484E-01 |
| 1975 | .108751E-01 | .108751E-01 | .108751E-01 | .108751E-01 |
| 1976 | .108033E-01 | .108033E-01 | .108034E-01 | .108034E-01 |
| 1977 | .108207E-01 | .108207E-01 | .108207E-01 | .108207E-01 |
| 1978 | .108679E-01 | .108679E-01 | .108679E-01 | .108679E-01 |

Only one year is not the same, 1969 for compensating variation. This discrepancy may be due to rounding error or to the probabilistic nature of the Willig measure of compensating and equivalent variation.

One anomaly does exist, however. The "utility" measure of consumer's surplus given an exponential demand function for cucumber pickles lies outside the range of possible measures of consumer's surplus as delimited by the Laspeyres and Paasche minimum and maximum measures respectively. Table 10 shows the relationship between the "utility" measure and the "utility" approximation measure for cucumber pickles given an exponential demand function.

Table 10

A Comparison of the "Utility" and "Utility" Approximation Measures of Consumer's Surplus Given an Exponential Demand for Cucumber Pickles

| | Laspeyres (dollars) | "Utility" (dollars) | "Utility" Approximation (dollars) | Paasche (dollars) |
|---|---|---|---|---|
| 1965 | .137371E-02 | .137490E-02 | .137382E-02 | .137393E-02 |
| 1966 | .587857E-02 | .590029E-02 | .588053E-02 | .588248E-02 |
| 1967 | .731314E-02 | .734681E-02 | .731617E-02 | .731920E-02 |
| 1968 | .746380E-02 | .749898E-02 | .746696E-02 | .747013E-02 |
| 1969 | .850856E-02 | .855445E-02 | .851269E-02 | .851682E-02 |
| 1970 | .750426E-02 | .753999E-02 | .750748E-02 | .751069E-02 |
| 1971 | .871474E-02 | .876279E-02 | .871906E-02 | .872339E-02 |
| 1972 | .970176E-02 | .976094E-02 | .970709E-02 | .971241E-02 |
| 1973 | .990388E-02 | .996565E-02 | .990944E-02 | .991500E-02 |
| 1974 | .112412E-01 | .113208E-01 | .112484E-01 | .112556E-01 |
| 1975 | .108685E-01 | .109418E-01 | .108751E-01 | .108817E-01 |
| 1976 | .107968E-01 | .108693E-01 | .108033E-01 | .108099E-01 |
| 1977 | .108141E-01 | .108872E-01 | .108207E-01 | .108273E-01 |
| 1978 | .108612E-01 | .109352E-01 | .108679E-01 | .108745E-01 |

In contradiction to the prescribed relationship, the "utility" measure is consistently greater than the "utility" approximation measure. In fact the "utility" measure is consistently greater than the maximum Paasche measure of consumer's surplus. For this reason the "utility" approximation measure is assumed to be correct since it is by definition

within the bounds prescribed by the Laspeyres and Paasche measures. The "utility" measure differs from the "utility" approximation measure by only a few 10,000's of a mil in absolute terms or about a difference of .5 percent. The cause of this discrepancy is unknown. However this difference does not change the relationship between the Hausman and Willig measures though there is a slight difference in the absolute amount of the return. Table 11 shows the results of the Hausman and Willig measures of consumer's surplus using the "utility" measure in their calculation.

Table 11

A Comparison of Hausman's and Willig's Measures of Consumer's Surplus Associated with the "Utility" Measure Given an Exponential Demand for Cucumber Pickles

| | Hausman's Compensating Variation (dollars) | Willig's Compensating Variation (dollars) | Hausman's Equivalent Variation (dollars) | Willig's Equivalent Variation (dollars) |
|---|---|---|---|---|
| 1965 | .137382E-02 | .137382E-02 | .137382E-02 | .137382E-02 |
| 1966 | .588052E-02 | .588052E-02 | .588053E-02 | .588053E-02 |
| 1967 | .731616E-02 | .731616E-02 | .731618E-02 | .731618E-02 |
| 1968 | .746695E-02 | .746695E-02 | .746697E-02 | .746697E-02 |
| 1969 | .851267E-02 | .851268E-02 | .851270E-02 | .851270E-02 |
| 1970 | .750747E-02 | .750747E-02 | .750749E-02 | .750749E-02 |
| 1971 | .871905E-02 | .871905E-02 | .871908E-02 | .871908E-02 |
| 1972 | .970707E-02 | .970707E-02 | .970710E-02 | .970710E-02 |
| 1973 | .990942E-02 | .990942E-02 | .990946E-02 | .990946E-02 |
| 1974 | .112484E-01 | .112484E-01 | .112484E-01 | .112484E-01 |
| 1975 | .108751E-01 | .108751E-01 | .108751E-01 | .108751E-01 |
| 1976 | .108033E-01 | .108033E-01 | .108034E-01 | .108034E-01 |
| 1977 | .108207E-01 | .108207E-01 | .108207E-01 | .108207E-01 |
| 1978 | .108679E-01 | .108679E-01 | .108679E-01 | .108679E-01 |

4-6-2 Linear Demand

In this part the measures of consumer's surplus are re-estimated using the equations associated with a linear functional form of the

demand curve. As before the general and specific relationship are presented using the parameter for the linear demand for cucumber pickles. The general relationship is the same as that presented in equations (1) and (2) for a price decrease, i.e.

(1) Laspeyres < compensating variation < "utility"

and

(2) "utility" < equivalent variation < Paasche.

The specific relationship between Willig and Hausman measures of compensating and equivalent variation are as follows:

(14) Willig's compensating variation
(using the minimum income elasticity)

> Hausman's compensating variation

> Willig's compensating variation
(using the maximum income elasticity)

(15) Willig's equivalent variation
(using the minimum income elasticity)

< Hausman's equivalent variation

< Willig's equivalent variation
(using the maximum income elasticity)

and

(16) "utility" = "utility" approximation.

The equations to estimate consumer's surplus for a price decrease given a linear functional form of the demand curve are as follows:

(17) Laspeyres $= Q_0 (P_0 - P_1)$

(18) Paasche $= Q_1 (P_0 - P_1)$

(19) "utility" approximation $= 1/2 (Q_0 + Q_1)(P_0 - P_1)$

(20) "utility" $= A_{P_1}^{P_0} = P_0 (\beta + \alpha P_0 / 2 + \delta Y) - P_1 (\beta + \alpha P_1 / 2 + \delta Y)$

(21) Willig's compensating variation $= A_{P_1}^{P_0} \left[ 1 - \delta A_{P_1}^{P_0} / 2Y \right]$

(22) Hausman's compensating variation =

$$Y - \left[ e^{\delta(P_1 - P_0)} \left[ Y + \frac{1}{\delta} (\beta + \alpha P_0 + \alpha / \delta) \right] - \frac{1}{\delta} (\beta + \alpha P_1 + \alpha/\delta) \right]$$

(23) Willig's equivalent variation = $A_{P_1}^{P_0} \left[ 1 + \delta A_{P_1}^{P_0} / 2Y \right]$

and

(24) Hausman's equivalent variation:

$$\left[ e^{\delta(P_0 - P_1)} \left[ Y + \frac{1}{\delta} (\beta + \alpha P_1 + \alpha / \delta) \right] - \frac{1}{\delta} (\beta + \alpha P_0 + \alpha/\delta) \right] - Y.$$

Where

$\alpha$ = -4.46284, the coefficient of price in the linear estimate of demand

$\delta$ = .00248926, the coefficient of income in the linear estimate of demand

$\beta$ = 4.32143 - 2.31634 DV5558 - 1.82798 CV5963, the constant term in the linear estimate of demand

Y = 3028.27, the average disposable income in the U.S. from 1965 to 1978 (1967 = 100)

$P_0$, $P_1$, $Q_0$, $Q_1$, are the ex ante and ex post equilibrium price and quantity and $Q_1$, $P_0$, $Q_0$, are normalized consistent with the parameters of the linear demand function as presented in the previous section.

The results of equations (17), (18), (20), (22), and (24) are given in Table 12 for the years 1965 to 1978. The results are consistent with the general form relationships presented in equations (1) and (2). These results are the various measures of the annual per capita increase in real income in dollars attributable to the introduction of the mechanical cucumber harvester. Again the benefits are about one cent per person per year for every person in the U.S. from 1966 to 1978. It is interesting to note that the income effect of the one cent increase

in income is about .0000001 cents or about a ten thousandth of a mil per person per year. The income effect is the difference between the compensating variations and the "utility" measures of consumer's surplus.

Table 12

Five Measures of Consumer's Surplus Given a Linear Demand for Cucumber Pickles

| | Laspeyres (dollars) | Hausman's Compensating Variation (dollars) | "Utility" (dollars) | Hausman's Equivalent Variation (dollars) | Paasche (dollars) |
|---|---|---|---|---|---|
| 1965 | .137742E-02 | .137748E-02 | .137749E-02 | .137750E-02 | .137756E-02 |
| 1966 | .589861E-02 | .589993E-02 | .589994E-02 | .589995E-02 | .590128E-02 |
| 1967 | .732155E-02 | .732357E-02 | .732358E-02 | .732360E-02 | .732562E-02 |
| 1968 | .743573E-02 | .743777E-02 | .743779E-02 | .743780E-02 | .743985E-02 |
| 1969 | .842325E-02 | .842582E-02 | .842584E-02 | .842585E-02 | .742843E-02 |
| 1970 | .742332E-02 | .742530E-02 | .742532E-02 | .742533E-02 | .742733E-02 |
| 1971 | .865641E-02 | .865915E-02 | .865917E-02 | .865919E-02 | .866194E-02 |
| 1972 | .972433E-02 | .972793E-02 | .9727956-02 | .972797E-02 | .973159E-02 |
| 1973 | .990413E-02 | .990782E-02 | .990785E-02 | .990787E-02 | .991158E-02 |
| 1974 | .112447E-01 | .112495E-01 | .112495E-01 | .112495E-01 | .112543E-01 |
| 1975 | .110647E-01 | .110698E-01 | .110698E-01 | .110698E-01 | .110748E-01 |
| 1976 | .109752E-01 | .109801E-01 | .109801E-01 | .109802E-01 | .109851E-01 |
| 1977 | .109315E-01 | .109362E-01 | .109363E-01 | .109363E-01 | .109410E-01 |
| 1978 | .109293E-01 | .109339E-01 | .109340E-01 | .109340E-01 | .109386E-01 |

Willig states that his measure of consumer surplus could be used to bound the "true" measure of consumer's surplus (Willig, 1976, p. 590). Willig's criteria for bounding the "true" or Hausman's "exact" measure is as follows:

(25) if $(E_{Y(min)} * A_{P_1}^{P_0}) / 2Y < .05$

(26) and if $(E_{Y(max)} * A_{P_1}^{P_0}) / 2Y < .05$

(27) and if $A_{P_1}^{P_0} / Y < .90$ (Willig, 1976, p. 589).

Then the Willig measures for compensating and equivalent variation as specified in equations (14), (15), (21), and (23) should bound the Hausman's respective equation as specified by equations (22) and (24). The Willig criteria is tested for the case of cucumber pickles. The results are presented in Table 13.

Table 13

Results of Testing Willig's Criteria for Bounds of the "Exact Measures" of Consumer's Surplus

| | Equation (26) (percent) | Equation (25) (percent) | Equation (27) (percent) |
|---|---|---|---|
| 1965 | .256300E-06 | .192219E-06 | .454877E-06 |
| 1966 | .109776E-05 | .823296E-06 | .194829E-05 |
| 1967 | .136265E-05 | .102196E-05 | .241841E-05 |
| 1968 | .138390E-05 | .103789E-05 | .245612E-05 |
| 1969 | .156774E-05 | .117577E-05 | .278239E-05 |
| 1970 | .138158E-05 | .103615E-05 | .245200E-05 |
| 1971 | .161115E-05 | .120833E-05 | .285944E-05 |
| 1972 | .181002E-05 | .135747E-05 | .321238E-05 |
| 1973 | .184349E-05 | .138257E-05 | .327179E-05 |
| 1974 | .209312E-05 | .156979E-05 | .371483E-05 |
| 1975 | .205968E-05 | .154471E-05 | .365549E-05 |
| 1976 | .204300E-05 | .153220E-05 | .362588E-05 |
| 1977 | .203484E-05 | .152608E-05 | .361139E-05 |
| 1978 | .203441E-05 | .152576E-05 | .361063E-05 |

The results in Table 13 show that the Willig criteria are met beyond question. Table 14 gives the results of Willig's measures for compensating variation bounding the Hausman "exact" measure.

The results in Table 14 show that in the case of cucumber pickles Willig's measure of compensating variation over-estimates the Hausman measure of compensating variation in seven of the fourteen years estimated. The Willig measure bounds the Hausman measure of compensating variation in six of the last seven years. This suggests the

Table 14

Willig's Measures of Consumer's Surplus as Bounding the "Exact Measure" of Compensating Variation

| | Willig's Compensating Variation e-minimum (dollars) | Hausman's Compensating Variation e-actual (dollars) | Willig's Compensating Variation e-maximum (dollars) |
|---|---|---|---|
| 1965 | .137749E-02 | .137748E-02 | .137749E-02 |
| 1966 | .589994E-02 | .589993E-02 | .589994E-02 |
| 1967 | .732358E-02 | .732357E-02 | .732357E-02 |
| 1968 | .743778E-02 | .743777E-02 | .743778E-02 |
| 1969 | .842583E-02 | .842582E-02 | .842582E-02 |
| 1970 | .742531E-02 | .742530E-02 | .742531E-02 |
| 1971 | .865916E-02 | .865915E-02 | .865916E-02 |
| 1972 | .972794E-02 | .972793E-02 | .972793E-02 |
| 1973 | .990784E-02 | .990782E-02 | .990783E-02 |
| 1974 | .112495E-01 | .112495E-01 | .112495E-01 |
| 1975 | .110698E-01 | .110698E-01 | .110698E-01 |
| 1976 | .109801E-01 | .109801E-01 | .109801E-01 |
| 1977 | .109362E-01 | .109362E-01 | .109362E-01 |
| 1978 | .109339E-01 | .109339E-01 | .109339E-01 |

possibility of rounding error which decreases in severity as the number of decimal places decrease. All errors occurred at the eighth place past the decimal while no error occurred at seven places past the decimal. Obviously the effect of the over-estimation is very small in both absolute and relative terms.

The results of Willig's measure for equivalent variation bounding the Hausman measure are presented in Table 15.

The results in Table 15 show that in the case of cucumber pickles Willig's measure of equivalent variation under-estimates Hausman's measure in two of the fourteen years. Conversely the Willig measures bound the Hausman measure in twelve of the fourteen years. The reason and effect of the under-estimation of equivalent variation is similar to those for over-estimation of compensating variation.

Table 15

Willig's Measures of Consumer's Surplus as Bounding the "Exact Measure" of Equivalent Variation

| | Willig's Equivalent Variation e-minimum (dollars) | Hausman's Equivalent Variation e-actual (dollars) | Willig's Equivalent Variation e-maximum (dollars) |
|---|---|---|---|
| 1965 | .137749E-02 | .137750E-02 | .137749E-02 |
| 1966 | .589995E-02 | .589995E-02 | .589995E-02 |
| 1967 | .732359E-02 | .732360E-02 | .732359E-02 |
| 1968 | .743780E-02 | .743780E-02 | .743780E-02 |
| 1969 | .842585E-02 | .842585E-02 | .842585E-02 |
| 1970 | .742533E-02 | .742533E-02 | .742533E-02 |
| 1971 | .865918E-02 | .865919E-02 | .865919E-02 |
| 1972 | .972797E-02 | .972797E-02 | .972797E-02 |
| 1973 | .990786E-02 | .990787E-02 | .990787E-02 |
| 1974 | .112495E-01 | .112495E-01 | .112495E-01 |
| 1975 | .110698E-01 | .110698E-01 | .110698E-01 |
| 1976 | .109801E-01 | .109802E-01 | .109802E-01 |
| 1977 | .109363E-01 | .109363E-01 | .109363E-01 |
| 1978 | .109340E-01 | .109340E-01 | .109340E-01 |

The comparison between the "utility" and the "utility" approximation measure given a linear demand for cucumber pickles is presented in Table 16.

The results in Table 16 show that the "utility" and the "utility" approximation measures of consumer's surplus are different in five of the fourteen years. Each of the five years differ by .000,001 mils or 1 millionth of a mil. The "utility" approximation measure over-estimates the "utility" for these five years yet each is less than the Hausman equivalent variation measure of consumer's surplus.

In summary, this section has presented the empirical results of estimating consumer's surplus using the equations associated with both the exponential and linear demand functions for cucumber pickles. For

Table 16

A Comparison of the "Utility" and "Utility" Approximation Measures of Consumer's Surplus Given a Linear Demand for Cucumber Pickles

| | "Utility" (dollars) | "Utility" Approximation (dollars) |
|---|---|---|
| 1965 | .137749E-02 | .137749E-02 |
| 1966 | .589994E-02 | .589994E-02 |
| 1967 | .732358E-02 | .732359E-02 |
| 1968 | .743779E-02 | .743779E-02 |
| 1969 | .842584E-02 | .842584E-02 |
| 1970 | .742532E-02 | .742532E-02 |
| 1971 | .865917E-02 | .865918E-02 |
| 1972 | .972795E-02 | .972796E-02 |
| 1973 | .990785E-02 | .990786E-02 |
| 1974 | .112495E-01 | .112495E-01 |
| 1975 | .110698E-01 | .110698E-01 |
| 1976 | .109801E-01 | .109801E-01 |
| 1977 | .109363E-01 | .109363E-01 |
| 1978 | .109340E-01 | .109340E-01 |

both functional forms, the general relation between the Laspeyres, compensating variation, "utility"/"utility" approximation, equivalent variation and Paasche measures were consistent with the theory for these relationships given a price decrease. Tables 17 and 18 are a comparison of the results of the five measures of the consumer's surplus for each of the functional forms.

There does not appear to be a clear pattern regarding the measure of consumer's surplus in relation to the exponential and linear demand functions. For some years the values associated with the exponential demand functions are greater than those associated with the linear demand function and vice versa.

Table 17

Consumer's Surplus Given an Exponential Demand Function

| | Laspeyres (dollars) | Hausman's Compensating Variation (dollars) | "Utility" Approximation (dollars) | Hausman's Equivalent Variation (dollars) | Paasche (dollars) |
|---|---|---|---|---|---|
| 1965 | .137371E-02 | .137382E-02 | .137382E-02 | .137382E-02 | .137393E-02 |
| 1966 | .587857E-02 | .588052E-02 | .588053E-02 | .588053E-02 | .588248E-02 |
| 1967 | .731314E-02 | .731616E-02 | .731617E-02 | .731618E-02 | .731920E-02 |
| 1968 | .746380E-02 | .746695E-02 | .746696E-02 | .746697E-02 | .747013E-02 |
| 1969 | .850856E-02 | .851267E-02 | .851269E-02 | .851270E-02 | .851682E-02 |
| 1970 | .750426E-02 | .750747E-02 | .750748E-02 | .750749E-02 | .751069E-02 |
| 1971 | .871474E-02 | .871905E-02 | .871906E-02 | .871908E-02 | .872339E-02 |
| 1972 | .970176E-02 | .970707E-02 | .970709E-02 | .970710E-02 | .971241E-02 |
| 1973 | .990388E-02 | .990942E-02 | .990944E-02 | .990946E-02 | .991500E-02 |
| 1974 | .112412E-01 | .112484E-01 | .112484E-01 | .112484E-01 | .112556E-01 |
| 1975 | .108685E-01 | .108751E-01 | .108751E-01 | .108751E-01 | .108817E-01 |
| 1976 | .107968E-01 | .108033E-01 | .108033E-01 | .108034E-01 | .108099E-01 |
| 1977 | .108141E-01 | .108207E-01 | .108207E-01 | .108207E-01 | .108273E-01 |
| 1978 | .108612E-01 | .108679E-01 | .108679E-01 | .108679E-01 | .108745E-01 |

Table 18

Consumer's Surplus Given a Linear Demand Function

| | Laspeyres (dollars) | Hausman's Compensating Variation (dollars) | "Utility" Approximation (dollars) | Hausman's Equivalent Variation (dollars) | Paasche (dollars) |
|---|---|---|---|---|---|
| 1965 | .137742E-02 | .137748E-02 | .137749E-02 | .137705E-02 | .137756E-02 |
| 1966 | .589861E-02 | .589993E-02 | .589994E-02 | .589995E-02 | .590128E-02 |
| 1967 | .732155E-02 | .732357E-02 | .732358E-02 | .732360E-02 | .732562E-02 |
| 1968 | .743573E-02 | .743777E-02 | .743779E-02 | .743780E-02 | .743985E-02 |
| 1969 | .842325E-02 | .842582E-02 | .842584E-02 | .842585E-02 | .842843E-02 |
| 1970 | .742332E-02 | .742530E-02 | .742532E-02 | .742533E-02 | .742733E-02 |
| 1971 | .865641E-02 | .865915E-02 | .865917E-02 | .865919E-02 | .866194E-02 |
| 1972 | .972433E-02 | .972793E-02 | .972795E-02 | .972797E-02 | .973159E-02 |
| 1973 | .990413E-02 | .990782E-02 | .990785E-02 | .990787E-01 | .991158E-02 |
| 1974 | .112447E-01 | .112495E-01 | .112495E-01 | .112495E-01 | .112543E-01 |
| 1975 | .110647E-01 | .110698E-01 | .110698E-01 | .110698E-01 | .110749E-01 |
| 1976 | .109752E-01 | .109801E-01 | .109801E-01 | .109802E-01 | .109851E-01 |
| 1977 | .109315E-01 | .109362E-01 | .109363E-01 | .109363E-01 | .109410E-01 |
| 1978 | .109293E-01 | .109339E-01 | .109340E-01 | .109340E-01 | .109386E-01 |

The specific relationships between the Willig and Hausman measures of compensating and equivalent variation for an exponential demand function were consistent with the predicted. For the linear function the Willig measures over-estimated the Hausman measure of compensating variation seven out of fourteen years by .000,000,1 cents in each case. Conversely the Willig measures under-estimated the Hausman measure of equivalent variation in two out of fourteen years again by .000,000,1 cents in each case. The possibility of rounding error cannot be ruled out.

The relationship between the "utility" and "utility" approximation measures for the linear demand function is off by .000,000,1 cents in five out of the fourteen years. This too may simply be the result of rounding error. However the discrepancy between the "utility" and "utility" approximation measures for the exponential demand function is a more serious matter. The "utility" measure is greater than the upper limit allowed by the Paasche measure. The reason for this inconsistency is unknown. The Willig and Hausman measures for the exponential demand function remained consistent with each other regardless of the utility measure used to estimate them. By default the "utility" approximation is the preferred measure in the case given an exponential functional form of the demand curve.

## 4-7 Estimating Net Annual Returns and Economic Rent

In the previous section the annual benefits to the retail consumer of cucumber pickles was estimated using consumer's surplus. Consumer's surplus was estimated using five different measures in relation to each of two functional forms of the demand for cucumber pickles--exponential

and linear. In this section the sum of the rent to owners of inputs available in less than perfectly elastic supply at each industry level in the cucumber pickle vertical sub-sector is measured. This measure of rent is estimated indirectly by first determining economic surplus as the net annual return (NAR) and subtracting from it a measure of consumer's surplus. Net annual returns are measured using three different assumptions regarding the "y-intercept" of the ex post supply function. These assumptions are that the supply shift is pivotal, parallel of convergent.

The equations for estimating net annual returns ex post is the sum of two areas x and y as discussed in section 3-13 such that:

(1) $NAR = x + y$

where

(2) $x = 1/2\ [P_0(Q_1 - Q) + (1 - k)\ P_1(Q_0 - Q_1)]$

and

(3) $y$ (pivotal shift) $= 1/2\ Q_0\ [P_0 - P_1 + k\ P_1]$

(4) $y$ (parallel shift) $= 1/2\ Q_0\ [P_0 - P_1 + 2\ k\ P_1]$

(5) $y$ (convergent shift) $= 1/2\ Q_0\ [P_0 - P_1 + 3\ k\ P_1]$

where

k is the percentage increase in the retail price of cucumber pickles in the absence of the cost savings resulting from the mechanical harvester.

$P_0$, $P_1$, $Q_0$, $Q_1$ are the ex ante and ex post equilibrium price and quantity and $Q_1$, $P_0$, $Q_0$ are normalized to be consistent with the parameters of the exponential and linear demand function.

As with consumer's surplus, the results for economic rent are presented in relation to the values of $P_0$, $Q_0$, and $Q_1$ normalized to be consistent

with the linear and exponential demand equation parameters. One objective is to see the impact these two functional forms have on the results. The estimates of equations (1) through (5) are presented below first assuming an exponential demand and linear supply and then assuming a linear demand and supply.

4-7-1 Exponential Demand and Linear Supply

The results of equations (2), (3), (4) and (5) assuming an exponential retail demand function and a derived linear retail supply function for cucumber pickles given the introduction of the mechanical cucumber pickle harvester are presented in Table 19.

Table 19

Values of x and y Associated with Net Annual Returns Given Exponential Demand and Linear Supply Functions for Cucumber Pickles

| | x (dollars) | y Pivotal Shift (dollars) | y Parallel Shift (dollars) | y Convergent Shift (dollars) |
|---|---|---|---|---|
| 1965 | .215055E-06 | .138297E-02 | .207908E-02 | .277519E-02 |
| 1966 | .393297E-05 | .591422E-02 | .889916E-02 | .118641E-01 |
| 1967 | .609934E-05 | .735958E-02 | .110626E-01 | .147656E-01 |
| 1968 | .636953E-05 | .750714E-02 | .112824E-01 | .150576E-01 |
| 1969 | .830013E-05 | .855012E-02 | .128460E-01 | .171418E-01 |
| 1970 | .646798E-05 | .754796E-02 | .113438E-01 | .151396E-01 |
| 1971 | .869151E-05 | .875909E-02 | .131608E-01 | .175625E-01 |
| 1972 | .107018E-04 | .974809E-02 | .146453E-01 | .195425E-01 |
| 1973 | .111709E-04 | .995142E-02 | .149509E-01 | .199504E-01 |
| 1974 | .143823E-04 | .112914E-01 | .169623E-01 | .226331E-01 |
| 1975 | .132626E-04 | .109216E-01 | .164089E-01 | .218962E-01 |
| 1976 | .131001E-04 | .108456E-01 | .162928E-01 | .217400E-01 |
| 1977 | .132042E-04 | .108617E-01 | .163162E-01 | .217708E-01 |
| 1978 | .133761E-04 | .109139E-01 | .163973E-01 | .218806E-01 |

It is interesting to note the relative insignificance of x in the estimate of net annual return. The assumptions regarding the type

of shift have a relative importance of 1 : 1.5 : 2 on the estimate of net presented in Table 20.

Table 20

Net Annual Returns Given Pivotal, Parallel and Convergent Supply Shifts and Exponential Demand and Linear Supply Functions for Cucumber Pickles

| | Net Annual Returns (pivotal shift) (dollars) | Net Annual Returns (parallel shift) (dollars) | Net Annual Returns (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | .138318E-02 | .207929E-02 | .277541E-02 |
| 1966 | .591815E-02 | .889309E-02 | .118680E-01 |
| 1967 | .736568E-02 | .110687E-01 | .147717E-01 |
| 1968 | .751351E-02 | .112888E-01 | .150640E-01 |
| 1969 | .855842E-02 | .128543E-01 | .171501E-01 |
| 1970 | .755443E-02 | .113503E-01 | .151461E-01 |
| 1971 | .876778E-02 | .131695E-01 | .175712E-01 |
| 1972 | .975879E-02 | .146560E-01 | .195532E-01 |
| 1973 | .996259E-02 | .149621E-01 | .199615E-01 |
| 1974 | .113058E-01 | .169766E-01 | .226475E-01 |
| 1975 | .109348E-01 | .164222E-01 | .219095E-01 |
| 1976 | .108587E-01 | .163059E-01 | .217531E-01 |
| 1977 | .108749E-01 | .163295E-01 | .217840E-01 |
| 1978 | .109273E-01 | .164106E-01 | .218940E-01 |

It is interesting to note that the above results show that the annual per capita returns to consumers and producers attributable to the mechanical cucumber harvester is approximately 1 cent, 1.5 cents and 2 cents assuming a pivotal, parallel or convergent supply shift respectively.

Economic rent can be derived from the results in Table 20 by subtracting a measure of consumer's surplus from the measures of net annual returns. It is possible to subtract each of the five measures of consumer's surplus from the three measures of net annual return

and derive a 3 x 5 matrix of the fifteen measures of economic rent each representing various assumptions regarding the consumer behavior and the type of supply shift. For simplicity however only the compensating variation measure of consumer's surplus is used to illustrate the range of results for economic rent. Economic rent (ER) is equal to the net annual return less consumer's surplus (CS).

(6) ER = NAR - CS

For a pivotal shift in the supply function the estimates of economic rent are presented in Table 21.

Table 21

Net Annual Returns, Compensating Variation and Economic Rent Given Exponential Demand and Linear Supply Functions and a Pivotal Shift in the Supply of Cucumber Pickles

| | Net Annual Returns (dollars) | Compensating Variation (dollars) | Economic Rent (dollars) |
|---|---|---|---|
| 1965 | .138318E-02 | .137382E-02 | .936365E-05 |
| 1966 | .591815E-02 | .588052E-02 | .376336E-04 |
| 1967 | .736568E-02 | .731616E-02 | .495261E-04 |
| 1968 | .751351E-02 | .746695E-02 | .465579E-04 |
| 1969 | .855842E-02 | .851267E-02 | .457461E-04 |
| 1970 | .755443E-02 | .750747E-02 | .469577E-04 |
| 1971 | .876778E-02 | .871905E-02 | .487292E-04 |
| 1972 | .975879E-02 | .970707E-02 | .517243E-04 |
| 1973 | .996259E-02 | .990942E-02 | .531663E-04 |
| 1974 | .113058E-01 | .112484E-01 | .574455E-04 |
| 1975 | .109348E-01 | .108751E-01 | .597522E-04 |
| 1976 | .108587E-01 | .108033E-01 | .553721E-04 |
| 1977 | .108749E-01 | .108207E-01 | .541750E-04 |
| 1978 | .109273E-01 | .108679E-01 | .594480E-04 |

The total rent to factor owners given a pivotal supply shift is equal to approximately .05 mils multiplied by the population of the U.S. Rent is therefore a small part of net annual return if the

supply shift is pivotal, i.e. .05 mils out of approximately 10 mils net annual return per capita or 1/2 of 1 percent of the total return.

For a parallel shift in the supply function the results for producer's surplus are presented in Table 22.

Table 22

Net Annual Returns, Compensating Variation and Economic Rent Given Exponential Demand and Linear Supply Functions and a Parallel Shift in the Supply of Cucumber Pickles

| | Net Annual Returns (dollars) | Compensating Variation (dollars) | Economic Rent (dollars) |
|---|---|---|---|
| 1965 | .207929E-02 | .137382E-02 | .705475E-03 |
| 1966 | .889309E-02 | .588052E-02 | .301257E-02 |
| 1967 | .110687E-01 | .731616E-02 | .375254E-02 |
| 1968 | .112888E-01 | .746695E-02 | .382180E-02 |
| 1969 | .128543E-01 | .851267E-02 | .434159E-02 |
| 1970 | .113503E-01 | .750747E-02 | .384278E-02 |
| 1971 | .131695E-01 | .871905E-02 | .445045E-02 |
| 1972 | .146560E-01 | .970707E-02 | .494894E-02 |
| 1973 | .149621E-01 | .990942E-02 | .505264E-02 |
| 1974 | .169766E-01 | .112484E-01 | .572827E-02 |
| 1975 | .164222E-01 | .108751E-01 | .554707E-02 |
| 1976 | .163059E-01 | .108033E-01 | .550256E-02 |
| 1977 | .163295E-01 | .108207E-01 | .550877E-02 |
| 1978 | .164106E-01 | .108679E-01 | .554277E-02 |

In this instance the total rent to factor owners is equal to approximately 5 mils or .5 cents multiplied by the population of the U.S. Economic rent is 5 mils out of a net annual return per capita of 15 mils or about 33-1/3% of the total return. This is a substantial increase in the contribution of rent to net annual return compared to the pivotal shift case.

For the convergent shift in the supply function the results for producer's surplus are presented in Table 23.

Table 23

Net Annual Returns, Compensating Variation and Economic Rent Given Exponential Demand and Linear Supply Functions and a Convergent Shift in the Supply of Cucumber Pickles

| | Net Annual Returns (dollars) | Compensating Variation (dollars) | Economic Rent (dollars) |
|---|---|---|---|
| 1965 | .277541E-02 | .137382E-02 | .140159E-02 |
| 1966 | .118680E-01 | .588052E-02 | .598750E-02 |
| 1967 | .147717E-01 | .731616E-02 | .745556E-02 |
| 1968 | .150640E-01 | .746695E-02 | .759704E-02 |
| 1969 | .171501E-01 | .851267E-02 | .863743E-02 |
| 1970 | .151461E-01 | .750747E-02 | .763861E-02 |
| 1971 | .175712E-01 | .871905E-02 | .885216E-02 |
| 1972 | .195532E-01 | .970707E-02 | .984615E-02 |
| 1973 | .199615E-01 | .990942E-02 | .100521E-01 |
| 1974 | .226475E-01 | .112484E-01 | .113991E-01 |
| 1975 | .219095E-01 | .108751E-01 | .110344E-01 |
| 1976 | .217531E-01 | .108033E-01 | .109497E-01 |
| 1977 | .217840E-01 | .108207E-01 | .109634E-01 |
| 1978 | .218940E-01 | .108679E-01 | .110261E-01 |

The total rent to factor owners in this instance is equal to approximately 10 mils or 1 cent multiplied by the population of the U.S. Economic rent is about 10 mils out of a net annual return per capita of about 20 mils or about 50% of the total return.

The rents associated with pivotal, parallel and convergent supply shifts are summarized in Table 24.

The approximate per capita economic rent for the pivotal, parallel and convergent supply shift are related as follows:

| | Pivotal ($R_1$) | | Parallel ($R_2$) | | Convergent ($R_3$) |
|---|---|---|---|---|---|
| (7) | .05 mils | < | 5 mils | < | 10 mils |

It is interesting to note that the ratio of $R_2 : R_3$ is 1 : 2, the ratio of $R_1 : R_2$ is 1 : 100, and $R_1 : R_3$ is 1 : 200. This seems to imply

Table 24

Economic Rent Given an Exponential Demand and a Linear Supply Function and Pivotal, Parallel and Convergent Shifts in the Supply of Cucumber Pickles

| | Economic Rent (pivotal shift) (dollars) | Economic Rent (parallel shift) (dollars) | Economic Rent (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | .936365E-05 | .705475E-03 | .140159E-02 |
| 1966 | .376336E-04 | .301257E-02 | .598750E-02 |
| 1967 | .495261E-04 | .375254E-02 | .745556E-02 |
| 1968 | .465579E-04 | .382180E-02 | .759704E-02 |
| 1969 | .457461E-04 | .434159E-02 | .863743E-02 |
| 1970 | .469577E-04 | .384278E-02 | .763861E-02 |
| 1971 | .487292E-04 | .445045E-02 | .885216E-02 |
| 1972 | .517243E-04 | .494894E-02 | .984615E-02 |
| 1973 | .531663E-04 | .505264E-02 | .100521E-01 |
| 1974 | .574455E-04 | .572827E-02 | .113991E-01 |
| 1975 | .597522E-04 | .554707E-02 | .110344E-01 |
| 1976 | .553721E-04 | .550256E-02 | .109497E-01 |
| 1977 | .541750E-04 | .550877E-02 | .109634E-01 |
| 1978 | .594480E-04 | .554277E-02 | .110261E-01 |

that relative danger of type 1 and type 2 error is greatest in relation to the assumption regarding pivotal vs. parallel shift.

4-7-2 Linear Demand and Linear Supply

The results of equations (2), (3), (4) and (5), assuming a linear retail demand function and a derived linear retail supply function for cucumber pickles given the introduction of the mechanical cucumber pickle harvester, are presented in Table 25. Again it is interesting to note the relative insignificance of x in the estimate of net annual return. The assumptions regarding the type of shift have a relative importance of 1 : 1.5 : 2 on the estimate of net annual return.

Table 25

Values of x and y Associated with Net Annual Returns Given Linear Demand and Supply Functions for Cucumber Pickles

| | x (dollars) | y (pivotal shift) (dollars) | y (parallel shift) (dollars) | y (convergent (shift) (dollars) |
|---|---|---|---|---|
| 1965 | .146094E-06 | .138365E-02 | .207859E-02 | .277352E-02 |
| 1966 | .268618E-05 | .592213E-02 | .889495E-02 | .118678E-01 |
| 1967 | .409248E-05 | .735140E-02 | .110420E-01 | .147327E-01 |
| 1968 | .414093E-05 | .746249E-02 | .112071E-01 | .149517E-01 |
| 1969 | .519673E-05 | .844736E-02 | .126831E-01 | .169188E-01 |
| 1970 | .401945E-05 | .744895E-02 | .111862E-01 | .149235E-01 |
| 1971 | .555379E-05 | .868279E-02 | .130374E-01 | .173920E-01 |
| 1972 | .727325E-05 | .975364E-02 | .146451E-01 | .195366E-01 |
| 1973 | .746894E-05 | .993358E-02 | .149151E-01 | .198966E-01 |
| 1974 | .964045E-05 | .112754E-01 | .169284E-01 | .225815E-01 |
| 1975 | .102336E-04 | .111036E-01 | .166748E-01 | .222681E-01 |
| 1976 | .997814E-05 | .110102E-01 | .165328E-01 | .220555E-01 |
| 1977 | .958174E-05 | .109636E-01 | .164615E-01 | .219593E-01 |
| 1978 | .934893E-05 | .109636E-01 | .164625E-01 | .219614E-01 |

The net annual return is presented in Table 26.

Economic rent can be derived from the measures of net annual return by subtracting a measure of consumer's surplus. It is possible to derive yet another 3 x 5 matrix of measure of producer's surplus by subtracting each of the five measures of consumer's surplus from each of the three measures of economic rent. Again for simplicity only the compensating variation measure of consumer's surplus is used to derive economic rent. The estimation of equation (6) given the three types of supply shifts is foregone and the rents associated with pivotal, parallel and convergent supply shifts are summarized in Table 27.

Table 26

Net Annual Returns Given Pivotal, Parallel and Convergent Supply Shifts and Linear Demand and Supply Functions for Cucumber Pickles

| | Net Annual Returns (pivotal shift) (dollars) | Net Annual Returns (parallel shift) (dollars) | Net Annual Returns (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | .138379E-02 | .207873E-02 | .277367E-02 |
| 1966 | .592481E-02 | .889764E-02 | .118705E-01 |
| 1967 | .735549E-02 | .110461E-01 | .147367E-01 |
| 1968 | .746663E-02 | .112113E-01 | .149559E-01 |
| 1969 | .845256E-02 | .126883E-01 | .169240E-01 |
| 1970 | .745296E-02 | .111902E-01 | .149275E-01 |
| 1971 | .868835E-02 | .130429E-01 | .173975E-01 |
| 1972 | .976091E-02 | .146524E-01 | .195439E-01 |
| 1973 | .994105E-02 | .149226E-01 | .199041E-01 |
| 1974 | .112850E-01 | .169381E-01 | .225911E-01 |
| 1975 | .111138E-01 | .166851E-01 | .222563E-01 |
| 1976 | .110202E-01 | .165428E-01 | .220655E-01 |
| 1977 | .109732E-01 | .164711E-01 | .219689E-01 |
| 1978 | .109729E-01 | .164718E-01 | .219707E-01 |

The approximate per capita economic rent given linear demand and supply functions for pivotal, parallel and convergent supply shifts are as follows:

| | Pivotal ($R_1$) | | Parallel ($R_2$) | | Convergent ($R_3$) |
|---|---|---|---|---|---|
| (8) | .04 mils | < | 5 mils | < | 10 mils |

The relative relationship between the measures of economic rent is nearly identical to that listed above involving an exponential demand function. The absolute amounts of economic rent in this case are lower than the previous one.

The next section presents the present value measures of consumer's surplus and economic rent as well as net annual returns as a means to facilitate comparison between the various measures and as part of the

Table 27

Economic Rent Given Pivotal, Parallel and Convergent Shifts in the Supply of Cucumber Pickles and Linear Demand and Supply Functions

| | Economic Rent (pivotal shift) (dollars) | Economic Rent (parallel shift) (dollars) | Economic Rent (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | .631077E-05 | .701250E-03 | .139619E-02 |
| 1966 | .248837E-04 | .299771E-02 | .597053E-02 |
| 1967 | .319271E-04 | .372255E-02 | .741318E-02 |
| 1968 | .288583E-04 | .377348E-02 | .751810E-02 |
| 1969 | .267423E-04 | .426248E-02 | .849822E-02 |
| 1970 | .276597E-04 | .376494E-02 | .750223E-02 |
| 1971 | .291953E-04 | .438378E-02 | .873837E-02 |
| 1972 | .329852E-04 | .492446E-02 | .981593E-02 |
| 1973 | .332266E-04 | .501474E-02 | .999625E-02 |
| 1974 | .355490E-04 | .568858E-02 | .113416E-01 |
| 1975 | .440521E-04 | .561529E-02 | .111865E-01 |
| 1976 | .400906E-04 | .556272E-02 | .110854E-01 |
| 1977 | .369627E-04 | .553483E-02 | .110327E-01 |
| 1978 | .389719E-04 | .553787E-02 | .110368E-01 |

process of estimating the benefit-cost ratio. Ideally it is the objective to compare variously defined measures of benefits to determine the effect on the absolute dollar amount of the mechanical cucumber harvester.

## 4-8 Comparing the Present Value of Benefits

In the previous section the estimates of net annual returns and economic rent associated with the mechanical cucumber harvester are presented. These measures of benefits along with consumer's surplus are based on several sets of assumptions regarding the functional form of the demand curve, the type of supply shift and separability of consumer goods. In order to compare the effect of the assumptions on

the measure of benefits it is necessary to establish a common denominator for these benefits across time.

In this section the estimates of consumer's surplus, economic rent and net annual returns are aggregated and corrected for the change in value across time. Once this is done it is possible to compare and contrast the measures of benefits in relation to the associated assumptions of functional form, type of supply shift and degree of separability. The benefits are aggregated by simply multiplying the annual per capita measure by the total population of the U.S. for that year. This changes the unit of analysis from per capita to total benefits per year.

To compare a flow of money across time it is necessary to determine the time value of the flow in relation to an arbitrary base period. Money has a time value equal to its opportunity cost. The opportunity cost of money is generally considered to be at least equal to the principle plus or minus a savings deposit interest rate on the principle per unit of time.

The base period is almost always selected as close to the present as possible. The idea being that the present facilitates evaluation of benefits and costs since it is most familiar. The present is also the time when decisions are made. The base period or "present" for this study is determined to be the year 1978. 1978 is chosen as the base year because this is the most recent year for which there are data on the retail price of cucumber pickles. Therefore the "present" value of benefits and cost in this study refers to 1978.

As stated above the opportunity cost of money is considered to be the principle plus or minus a savings deposit interest on the principle per unit of time. An ex post study with the base period being the most recent observation implies that the interest on the principle is added or compounded annually. Conversely an ex ante study with the base period at the present implies that the interest on the principle is subtracted or discounted annually in order to determine the present value of the money. Therefore since this is an ex post study of the impact of the cucumber harvester the present value of the benefits and costs are determined by compounding or adding the interest on the principle to the principle through time.

The time value of money assumes a minimum opportunity cost at which the money could be invested. The interest rate of 6 percent is used in this study as the minimum opportunity cost of capital. This is approximately the real rate of interest one would expect to receive from a bank savings deposit. This is also the interest rate used in a similar study of the tomato harvester (Schmitz and Seckler, 1970, p. 572). As a matter of consistency with that study the same interest rate is used in this study as well.

Two equations are used in this and later sections to estimate the present value of the stock and flow of benefits or costs. The present value of the stock of benefits or costs is equal to the sum of the present value of the benefits or costs. The present value of the benefits or costs are equal to the estimate of the benefit or cost multiplied by the expression $(1 + r)^N$ for an ex post study. The expression $(1 + r)^N$ is the means of compounding past benefits or cost forward by correcting for the time value of money. In this expression

r is the opportunity cost of capital (6 percent), and N is the number of years prior to the base period 1978 (such that 1978 = 0 . . . 1965 = 13, etc.). Therefore

(1) Present value of the stock of benefits or costs =

$$\sum_{i=0}^{N} (\text{benefit or cost})(1 + r)^{N}$$

The present value of the flow of benefits only is somewhat more complicated. In essence this is the process of transforming a flow to a stock and then back to a flow again. The present value of the benefits are determined as before by multiplying the estimates of benefits by the expression $(1 + r)^{N}$ for an ex post study. However instead of adding the present value of the benefits for all the years including the base year, the present value of all the benefits for all the years except the base year are added. This gives an expression of the stock of benefits from which a flow of benefit might be expected in the base year. The flow of benefits is determined to be this stock multiplied by the interest rate. This later estimate is added to the estimated benefits in the base year to provide a measure of the present value of the flow of benefits. Therefore

(2) Present value of the flow of benefits =

$$(\text{Benefits in base year}) + \left(\sum_{i=1}^{N} (\text{benefits})(1 + r)^{N}\right)(r)$$

The present value of the flow of cost can also be calculated with equation (2) but it is not used in the subsequent benefit-cost calculations presented in the next section so it is not presented here.

For examples of the estimate of the present value of the flow of benefits see the works of Griliches and Schmitz and Seckler (Griliches, 1958, p. 378; and Schmitz and Seckler, 1970, p. 572).

The economic interpretation of these two measures of the present value of benefits and costs is most easily understood in a benefit-cost context. The ratio of the present value of the stock of benefits to the stock of cost is equal to the benefit-cost ratio. The benefit-cost ratio is the number of time greater the actual return on the investment is compared to the opportunity cost of capital in the base year.

The ratio of the present value of the flow of benefits to the stock of costs is equal to the benefit-cost rate of return. The benefit-cost rate of return is equal to the estimated return on the investment from the base year in perpetuity given that the return on the investment up to the base year is only the minimum opportunity cost of capital. We will return to these definitions of benefit-cost ratios and rates of return along with a definition of the internal rate of return in the next section.

In the rest of this section the present value of the benefits associated with the cucumber harvester are presented. The present value of the stock and flow of total benefits given an exponential demand and linear supply and pivotal, parallel and convergent shifts in supply are determined. Then the present value of the stock of economic rent and consumer's surplus given an exponential demand and linear supply and pivotal parallel and convergent shifts in supply are determined. The present value estimates for the exponential demand case are presented in their entirety and for the linear demand case in summary form.

4-8-1 Exponential Demand and Linear Supply

The present value of the net annual returns or total benefits associated with the cucumber harvester given an exponential retail demand function, a derived linear retail supply function and pivotal, parallel and convergent shifts in supply are presented in Table 28.

Table 28

The Present Value of Net Annual Returns Given Pivotal, Parallel and Convergent Shifts in the Supply of Cucumber Pickles and Exponential Demand and Linear Supply Functions

| | Total Benefits (pivotal shift) (dollars) | Total Benefits (parallel shift) (dollars) | Total Benefits (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | 573,230 | 861,718 | 1,150,210 |
| 1966 | 2,341,210 | 3,518,090 | 4,694,960 |
| 1967 | 2,778,280 | 4,175,020 | 5,571,770 |
| 1968 | 2,700,530 | 4,057,440 | 5,414,350 |
| 1969 | 2,930,890 | 4,402,040 | 5,873,180 |
| 1970 | 2,467,120 | 3,706,760 | 4,946,400 |
| 1971 | 2,730,300 | 4,101,010 | 5,471,710 |
| 1972 | 2,890,430 | 4,340,910 | 5,791,400 |
| 1973 | 2,805,090 | 4,212,760 | 5,620,430 |
| 1974 | 3,024,520 | 4,541,570 | 6,058,620 |
| 1975 | 2,781,830 | 4,177,810 | 5,572,790 |
| 1976 | 2,625,620 | 3,942,740 | 5,259,860 |
| 1977 | 2,500,280 | 3,754,370 | 5,008,460 |
| 1978 | 2,389,800 | 3,589,010 | 4,788,210 |

The present value of the stock of total benefits associated with the estimates of the present value of benefits in Table 28 are $35,539,150; $53,381,220 and $71,223,410 given a pivotal, parallel and convergent supply shift respectively. The present value of the flow of total benefits associated with the estimates of the present value of benefits in Table 28 are $4,378,761; $6,576,543 and $8,774,322 given a pivotal, parallel and convergent supply shift respectively.

The present value of the economic rent associated with the cucumber harvester given an exponential retail demand function, a derived linear retail supply function and pivotal, parallel and convergent supply shifts are presented in Table 29.

Table 29

The Present Value of Economic Rent Given Pivotal, Parallel and Convergent Shifts in the Supply of Cucumber Pickles and Exponential Demand and Linear Supply Functions

| | Economic Rent (pivotal shift) (dollars) | Economic Rent (parallel shift) (dollars) | Economic Rent (convergent shift) (dollars) |
|---|---|---|---|
| 1965 | 3,880.56 | 292,369.00 | 580,857.00 |
| 1966 | 14,887.80 | 1,191,770.00 | 2,368,640.00 |
| 1967 | 18,680.90 | 1,415,430.00 | 2,812,180.00 |
| 1968 | 16,734.00 | 1,373,640.00 | 2,730,550.00 |
| 1969 | 15,666.10 | 1,486,810.00 | 2,957,950.00 |
| 1970 | 15,335.40 | 1,254,970.00 | 2,494,610.00 |
| 1971 | 15,174.40 | 1,385,880.00 | 2,756,580.00 |
| 1972 | 15,320.10 | 1,465,810.00 | 2,916,300.00 |
| 1973 | 14,969.70 | 1,422,640.00 | 2,830,300.00 |
| 1974 | 15,367.80 | 1,532,420.00 | 3,049,470.00 |
| 1975 | 15,201.00 | 1,411,180.00 | 2,807,160.00 |
| 1976 | 13,388.90 | 1,330,510.00 | 2,647,630.00 |
| 1977 | 12,455.60 | 1,266,540.00 | 2,520,630.00 |
| 1978 | 13,001.30 | 1,212,200.00 | 2,411,410.00 |

The present value of the stock of economic rents associated with the estimates of the present value of economic rent in Table 29 are $200,063, $18,042,130 and $35,884,330 given a pivotal, parallel and convergent supply shift respectively.

It is interesting to note that the assumption of a pivotal shift results in a measure of rent of only $200,000 or about .6 of one percent of the $35,539,150 estimated total benefits. For a parallel shift economic rent is about 34 percent of the $53,381,200 estimated

total benefits. For a convergent shift economic rent is about 50 percent of the $71,223,410 estimated total benefits. Conversely consumer's surplus measured as compensating variation must represent 99.4 percent, 66 percent and 50 percent of total benefits assuming a pivotal, parallel and convergent shift in the supply function respectively.

The present value of the consumer's surplus associated with the cucumber harvester given an exponential retail demand function is presented in Table 30. Note that the type of supply shift does not affect the measure of consumer's surplus.

Table 30

The Present Value of the Five Measures of Consumer's Surplus Given an Exponential Demand Function for Cucumber Pickles

| | Laspeyres (dollars) | Compensating Variation (dollars) | "Utility" (dollars) | Equivalent Variation (dollars) | Paasche (dollars) |
|---|---|---|---|---|---|
| 1965 | 569,305 | 569,349 | 569,349 | 569,349 | 569,393 |
| 1966 | 2,325,550 | 2,326,320 | 2,326,320 | 2,326,330 | 2,327,100 |
| 1967 | 2,758,460 | 2,759,600 | 2,759,600 | 2,750,600 | 2,760,740 |
| 1968 | 2,682,660 | 2,683,800 | 2,683,800 | 2,683,800 | 2,684,940 |
| 1969 | 2,913,820 | 2,915,230 | 2,915,230 | 2,915,240 | 2,916,650 |
| 1970 | 2,450,740 | 2,451,790 | 2,451,790 | 2,451,790 | 2,452,840 |
| 1971 | 2,713,790 | 2,715,130 | 2,715,130 | 2,715,140 | 2,716,480 |
| 1972 | 2,873,530 | 2,875,110 | 2,875,110 | 2,875,120 | 2,876,690 |
| 1973 | 2,788,560 | 2,790,120 | 2,790,130 | 2,790,130 | 2,791,690 |
| 1974 | 3,007,240 | 3,009,150 | 3,009,160 | 3,009,170 | 3,011,070 |
| 1975 | 2,764,960 | 2,766,630 | 2,766,640 | 2,766,640 | 2,768,320 |
| 1976 | 2,610,660 | 2,612,230 | 2,612,240 | 2,612,240 | 2,613,810 |
| 1977 | 2,486,320 | 2,487,830 | 2,487,830 | 2,487,840 | 2,489,340 |
| 1978 | 2,375,350 | 2,376,800 | 2,376,810 | 2,376,810 | 2,378,260 |

The present value of the stock of consumer's surplus associated with the estimates of the present value of consumer's surplus in Table 30 are as follows:

| | |
|---|---|
| Laspeyres: | $35,320,990 |
| Compensating Variation: | $35,339,080 |
| "Utility": | $35,339,090 |
| Equivalent Variation: | $35,339,220 |
| Paasche: | $35,355,170 |

It is interesting to note that the variance in the measures of consumer's surplus are less than that for economic rent.

A summary of the present value of the stock and flow of total benefits as well as the present value of the stock economic rent and consumer's surplus given exponential demand, linear supply and pivotal, parallel and convergent supply shifts are presented in Table 31.

Table 31

The Benefits Associated with the Mechanical Cucumber Harvester in 1978 Given Exponential Demand and Linear Supply Functions and Pivotal, Parallel and Convergent Shifts in Supply

| 1978 Stock of Total Benefits | | |
|---|---|---|
| (Pivotal Shift) | (Parallel Shift) | (Convergent Shift) |
| $35,539,150 | $53,381,220 | $71,223,410 |
| **1978 Flow of Total Benefits** | | |
| (Pivotal Shift) | (Parallel Shift) | (Convergent Shift) |
| $4,378,761 | $6,576,543 | $8,774,322 |
| **1978 Stock of Economic Rent** | | |
| (Pivotal Shift) | (Parallel Shift) | (Convergent Shift) |
| $200,063 | $18,042,130 | $35,884,330 |

| 1978 Stock of Consumer's Surplus | | | | |
|---|---|---|---|---|
| (Laspeyres) | (Compensating Variation) | (Utility) | (Equivalent Variation) | (Paasche) |
| $35,320,990 | $35,339,080 | $35,339,090 | $35,339,220 | $35,355,170 |

Before comparing and contrasting the measures of benefits a summary of the benefits given a linear demand function is presented in the following section.

4-8-2 Linear Demand and Linear Supply

The process of estimating the present value of the flow of total benefits as well as the present value of the stock total benefits, economic rent and consumer's surplus given a linear demand function is the same as the one described above and illustrated given an exponential demand function. A summary of the benefits with the mechanical cucumber harvester in 1978 given linear demand and supply functions and pivotal, parallel and convergent shifts in supply is present in Table 32.

Table 32

The Benefits Associated with the Mechanical Cucumber Harvester in 1978 Given Linear Demand and Supply Functions and Pivotal, Parallel and Convergent Shifts in Supply

| | | | | |
|---|---|---|---|---|
| 1978 Stock of Total Benefits | | | | |
| (Pivotal Shift) | | (Parallel Shift) | | (Convergent Shift) |
| $35,533,370 | | $53,345,540 | | $71,157,710 |
| 1978 Flow of Total Benefits | | | | |
| (Pivotal Shift) | | (Parallel Shift) | | (Convergent Shift) |
| $4,387,787 | | $6,576,972 | | $8,786,149 |
| 1978 Stock of Economic Rent | | | | |
| (Pivotal Shift) | | (Parallel Shift) | | (Convergent Shift) |
| $128,714.70 | | $17,940,880 | | $35,753,020 |
| 1978 Stock of Consumer's Surplus | | | | |
| (Laspeyres) | (Compensating Variation) | (Utility) | (Equivalent Variation) | (Paasche) |
| $35,392,260 | $35,404,640 | $35,404,750 | $35,404,830 | $35,417,290 |

A comparison of the measures of benefits given exponential and linear demand functions and pivotal, parallel and convergent supply shifts is presented next.

### 4-8-3 Comparing the Measures of Benefits

The importance of estimating the benefits using both an exponential and linear function form of the demand curve is to determine the effect of each on the results. We are now in a position to compare the effect of functional form on the estimate of benefits. Recall that both equations satisfy the Slutsky equation in which consumers act as if they utility maximize. The results from Tables 31 and 32 are presented together in Table 33. The last column in Table 33 gives the difference between the estimate of benefits determined by subtracting the "linear estimates" from the "exponential" ones.

Table 33 shows that stock of total benefits associated with an exponential demand function are greater than those associated with a linear demand function by a difference of $5,780 to $65,700 or .02 to .09 of one percent of the stock of total benefits.

Interestingly the flow of total benefits associated with the exponential demand function are less than those associated with a linear demand function by a difference of $429 to $11,827 or .07 to .13 of one percent of the flow of total benefits. This reversal in the sign of the stock and flow estimates results from the fact that in 1978 the yearly benefit associated with the linear function more than outpaced the interest on the 1965 to 1977 stock associated with the exponential function. On average however the flow of benefits associated with the exponential demand is greater than that associated with the linear demand.

Table 33

A Comparison of Estimated Benefits Associated with the Mechanical Cucumber Harvester Given Exponential and Linear Demand Functions

| Type of Supply Shift | Exponential Demand Linear Supply | Linear Demand Linear Supply | Difference |
|---|---|---|---|
| 1978 Stock of Total Benefits | | | |
| Pivotal | $35,539,150 | $35,533,370 | $ 5,780 |
| Parallel | 53,381,220 | 53,345,540 | 35,680 |
| Convergent | 71,223,410 | 71,157,710 | 65,700 |
| 1978 Flow of Total Benefits | | | |
| Pivotal | $ 4,378,761 | $ 4,387,787 | $ -9,026 |
| Parallel | 6,576,543 | 6,576,972 | -429 |
| Convergent | 8,774,322 | 8,786,149 | -11,827 |
| 1978 Stock of Economic Rent | | | |
| Pivotal | $ 200,063 | $ 128,715 | $ 71,348 |
| Parallel | 18,042,130 | 17,940,880 | 101,250 |
| Convergent | 35,884,330 | 35,753,020 | 131,310 |
| 1978 Stock of Consumer's Surplus | | | |
| Laspeyres | $35,320,990 | $35,392,260 | $-71,270 |
| Compensating Variation | 35,339,080 | 35,404,640 | -65,560 |
| "Utility" Equivalent Variation | 35,339,090 | 35,404,750 | -65,660 |
| Paasche | 35,339,220 | 35,404,830 | -65,610 |
| | 35,355,170 | 35,417,290 | -62,120 |

Perhaps the most interesting effect of the choice of functional form of the demand curve is the implied effect upon the distribution of benefits between consumers and owners of scarce resources. An exponential demand function in this case attributes more of the total benefits to resource owners and less to consumers than does a linear demand function. Conversely a linear demand function in this case attributes more of the total benefits to consumers and less to resource owners.

The range of difference in total benefits between the exponential and linear demand function is only between $5,780 and $65,700 in favor of the exponential function, yet the implicit distributive difference associated with the exponential demand function is between $136,908 and $196,870 saving in benefits in favor of resource owners. This is a difference of between .39 and .27 percent. Therefore the redistributive effects of the choice of the functional form of the demand curve are 3 to 20 times greater than the effect on the total stock of benefits.

Therefore in this case the choice of functional form of the demand curve does make a difference both on the total and relative distribution of benefits. The effect of choice of functional form of the demand equation is greater on the distribution than on the total though the former amounts to less than .5 of one percent in distribution with respect to the total benefits.

Besides the effect of function form there are also the effects of assumptions regarding the type of supply shift and separability which can also be analyzed. A comparison of the differences between the measures of total benefits of the mechanical cucumber harvester associated with assumptions on the type of supply shift and separability is presented in Table 34.

The most obvious difference in Table 35 is the nearly $36,000,000 saving in the total stock of benefits between the assumptions of either a pivotal or a convergent supply shift. All of the change in benefits of the supply shift impacts the measures of economic rent. The type of supply shift has no effect on the measure of consumer's surplus. Therefore the assumption regarding the supply shift has a very pronounced impact on the total stock of benefits in general and

Table 34

A Comparison of the Differences Between the Measures of Total Benefits of the Mechanical Cucumber Harvester Given Pivotal, Parallel and Convergent Supply Shifts

| | Exponential Demand Linear Supply Difference | Linear Demand Linear Supply Difference |
|---|---|---|
| 1978 Stock of Total Benefits Difference in Supply Shift | | |
| 1. pivotal vs. parallel | 17,842,070 | 17,812,170 |
| 2. parallel vs. convergent | 17,842,190 | 17,812,170 |
| 3. pivotal vs. convergent | 35,684,260 | 35,624,340 |
| 1978 Flow of Total Benefits Difference in Supply Shift | | |
| 1. pivotal vs. parallel | 2,197,782 | 2,189,185 |
| 2. parallel vs. convergent | 2,197,779 | 2,209,177 |
| 3. pivotal vs. convergent | 4,395,561 | 4,398,362 |
| 1978 Stock of Economic Rent Difference in Supply Shift | | |
| 1. pivotal vs. parallel | 17,842,070 | 17,812,160 |
| 2. parallel vs. convergent | 17,842,200 | 17,812,140 |
| 3. pivotal vs. convergent | 35,684,270 | 35,624,310 |
| 1978 Stock of Consumer's Surplus Separability | | |
| 1. Laspeyres vs. compensating variation | 18,090 | 12,380 |
| 2. Paasche vs. equivalent variation | 15,950 | 12,460 |
| 3. "Utility" vs. compensating variation | 10 | 110 |
| 4. "Utility" vs. equivalent variation | 130 | 80 |
| 5. Laspeyres v. Paasche | 34,180 | 25,030 |

is all important to the stock of economic rent in particular. A pivotal shift is nearly equal to assuming that there is no change in rent to resource owners associated with a new technology. A convergent shift on the other hand is nearly equal to assuming that the benefits

of a new technology are nearly twice that of a parallel shift and all the additional benefits go to scarce resource owners. This holds for the flow of total benefits as well. Therefore the assumption regarding the type of shift of the supply function associated with the introduction of a new technology is critically important both in relation to total benefits and to economic rent.

The separability assumption states that decisions regarding the purchase of a commodity or group of commodities are made independently of the change in relative price or other goods. Separability implies a budgeting process in which income is allocated to purchase goods irrespective of relative prices. Cucumber pickles is a good candidate to meet this criteria because it is a very small percentage of an individual's total consumption bundle in a year. Statistically the price of hamburger and processed tomatoes did not appear to influence the quantity of pickles purchased in a year.

Despite these conditions, cucumber pickles were in fact substituted for some other more expensive item for a total effect in 1978 in the U.S. of between $12,380 and $18,090. The decision to purchase cucumber pickles does not appear to be made independently of relative price changes. However the substitution effect for cucumber pickles seems to be very small and may not be observed by estimating an ordinary demand function for cucumber pickles. The substitution effect is estimated in this case as the difference between either the Laspeyres and compensating variation or the Paasche and equivalent variation measures of consumer's surplus.

It is interesting to note in passing that the income effect is very small indeed. The income effect ranges between $10 and $130 in

1978 for the U.S. This is measured as the difference either of the "utility" and compensating variation or the "utility" and equivalent variation measures of consumer's surplus.

These results seem to indicate a number of considerations when using consumer's surplus and economic rent to estimate total benefits. The case of cucumber pickles seems to meet as nearly as any commodity ever would the separability assumption required to estimate the general equilibrium effects of the introduction of a new technology. Yet even cucumber pickles are not purchased irrespective of relative prices. Therefore it is doubtful if the separability assumption is ever met but rather only approximated. The income effect in this case is negligible. This however may not often be true, a fact which will compound the separability assumption problem. The extent to which the separability assumption is violated is the extent to which a general equilibrium result is in fact only a partial equilibrium result.

Of critical importance to either a general or partial equilibrium result is the assumption regarding the type of supply shift. A means of determining the type of supply shift associated with a new technology deserves additional attention in the literature building on the work of Duncan and Tisdell (Economic Record, 1971, pp. 124-129).

## 4-9 Research and Development Costs and Impact Measures

In the previous section the present value of the various categories of benefits were estimated. These estimates of benefits were compared and contrasted based on the assumptions regarding the type of supply shift, the income and substitution effects and the form of the demand equation. The net return or the range of total benefits

attributable to the mechanical cucumber harvester is established. The next objective is to determine general measures of impact of the cucumber harvester using benefit-cost ratios as well as benefit-cost and internal rates of return. To achieve this objective however the present value of the cost of research, development and extension education associated with the mechanical cucumber harvester must first be established. Fortunately this is a simpler process than estimating the benefits.

In this section the cost of research, development and extension are presented along with their associated present value of the stock of total cost in 1978. These cost estimates are then used in relation to the previously derived estimates of the stock and flow of total benefits in 1978 to determine the three measures of impact.

The sequence of events associated with the research and development of the mechanical cucumber harvester are presented in an earlier chapter. In this section the costs or contribution of the various groups and farm machinery manufacturers are present. The nominal costs are listed in Table 35.

The information on the farm machinery manufacturers comes from phone conversations with Ken Pervis of Cuke Inc., Bernie Wilde of Wilde Manufacturers, Jane Odle of FMC, and Charles Burton of Chisholm-Ryder, Inc. The funds listed for Michigan State University are from the personal files of Dr. Bill Stout, the co-inventor of the mechanical cucumber harvester. The funds listed for the USDA are reported in CRIS (current research information system) of the USDA listed under the categories of sub-commodity 1220 (melons and cucurbits) activity 5200 (mechanication, improvement of physical efficiency).

Table 35

Nominal Research and Development Costs of the Mechanical Cucumber Harvester by Source of Funding

| | Chisholm Ryder | MSU | FMC | Wilde | Cuke | USDA | Total |
|---|---|---|---|---|---|---|---|
| 1958 | 35,000 | 12,000 | | | | | 47,000 |
| 1959 | 5,000 | 6,000 | | | | | 11,000 |
| 1960 | 30,000 | 1,200 | | | | | 31,200 |
| 1961 | | 2,000 | | | | | 2,000 |
| 1962 | | 7,000 | | | | | 7,000 |
| 1963 | | 7,000 | | | | | 7,000 |
| 1964 | | | 200,000 | | | | 200,000 |
| 1965 | | | 200,000 | | | | 200,000 |
| 1966 | | | 100,000 | | | | 100,000 |
| 1967 | | | 50,000 | 240,000 | | | 290,000 |
| 1968 | | | 50,000 | 25,000 | 200,000 | | 270,000 |
| 1969 | | | 50,000 | 25,000 | 20,000 | 3,733 | 98,733 |
| 1970 | | | 50,000 | 25,000 | 20,000 | 88,093 | 183,093 |
| 1971 | | | 50,000 | 25,000 | 20,000 | 160,452 | 255,452 |
| 1972 | | | 50,000 | 25,000 | 20,000 | 193,477 | 288,477 |
| 1973 | | | 50,000 | 25,000 | 20,000 | 216,163 | 311,163 |
| 1974 | | | | 25,000 | 20,000 | 276,001 | 321,001 |
| 1975 | | | | 25,000 | 20,000 | 297,279 | 342,279 |
| 1976 | | | | 25,000 | 20,000 | 328,426 | 373,426 |
| 1977 | | | | 25,000 | 20,000 | 327,706 | 372,706 |
| 1978 | | | | | | 233,703 | 233,703 |

Some funds invested in the development of a mechanical harvester by private individuals such as Dolphus Grew, Ike Pierson, Lloyd Gilbert and Stewart Craig and by Fox, Inc. are not included. This information was not available and presumed relatively small and only indirectly related to the funding of the machine concept which was ultimately

adopted. However in a present value framework early funds are weighted much more heavily than more recent funds. The exclusion of these earlier investments will bias the research returns upwards.

On the other hand the substantial USDA funds since 1969 is primarily funded for the second generation of mechanical cucumber harvesters and other "improvements of physical efficiency." An example of the former is the research begun in 1977 by L. M. Pike at Texas A & M University. The title of Pike's research is "Development of Cucumber Cultivars for Mechanical Harvesting." The objectives of the research are to:

> . . . develop high quality, high yielding, disease resistant pickling cucumbers which can be mechanically harvested. These cultivars will include all pistillate flowering types of which some will be parthenocarpic. (CRIS, 1982, ID NO. 224220-1/RMS)

Parthenocarpic cucumbers are ones induced through a chemical response other than sexual generation. This line of research is called the "little pickle" technology which is not yet available for commercial application.

Lower yields have precluded the use of the mechanical harvester in the South to a large extent; therefore research in this area in South Carolina is necessarily in anticipation of possible returns to research in the future. Including these data will tend to bias the returns to the mechanical harvester downward. It is difficult to know with any degree of certainty whether these two sources of bias are offsetting. Implicitly the following results assume that they are offsetting.

In order to compare the cost of research associated with the mechanical harvester through time it is necessary to establish a common

denominator with respect to the general price level and a point in time. The results are presented in Table 36.

Table 36

Nominal, Real and Present Value of Real Research and Development Costs on the Mechanical Cucumber Harvester

| | Nominal R&D Cost (dollars) | Real R&D Cost (dollars) | Present Value of Real R&D Cost (dollars) |
|---|---|---|---|
| 1958 | 47,000.00 | 54,272.50 | 174,059.00 |
| 1959 | 11,000.00 | 12,600.20 | 38,123.20 |
| 1960 | 31,000.00 | 34,949.30 | 99,757.10 |
| 1961 | 2,000.00 | 2,232.14 | 6,010.65 |
| 1962 | 7,000.00 | 7,726.27 | 19,627.40 |
| 1963 | 7,000.00 | 7,633.59 | 18,294.30 |
| 1964 | 200,000.00 | 215,285.00 | 486,739.00 |
| 1965 | 200,000.00 | 211,640.00 | 451,413.00 |
| 1966 | 100,000.00 | 102,881.00 | 207,016.00 |
| 1967 | 290,000.00 | 290,000.00 | 550,507.00 |
| 1968 | 270,000.00 | 259,117.00 | 464,039.00 |
| 1969 | 98,733.00 | 89,920.80 | 151,919.00 |
| 1970 | 183,093.00 | 157,432.00 | 250,922.00 |
| 1971 | 255,452.00 | 210,595.00 | 316,651.00 |
| 1972 | 288,477.00 | 230,229.00 | 326,584.00 |
| 1973 | 311,163.00 | 233,781.00 | 312,852.00 |
| 1974 | 321,001.00 | 217,333.00 | 274,378.00 |
| 1975 | 342,279.00 | 212,332.00 | 252,891.00 |
| 1976 | 373,426.00 | 219,018.00 | 246,089.00 |
| 1977 | 372,706.00 | 205,348.00 | 217,669.00 |
| 1978 | 233,703.00 | 119,602.00 | 119,602.00 |

The annual cost of research and development on the mechanical cucumber harvester, compounded forward at a 6 percent rate of interest from 1958 to 1977 inclusive is equal to the present value of the stock of total cost of the research in 1978 of $4,985,153. It is possible to use this estimate of the present value of the stock of total cost in conjunction with the estimate of the stock and flow of total benefits

to determine a benefit-cost ratio and rate of return. These measures can be determined in relation to both the exponential and linear demand curves and for the associated pivotal, parallel and convergent supply shift.

4-9-1 Benefit-Cost Ratio

The general equation for a benefit-cost ratio is the ratio of the present value of the stock of benefits to the present value of the stock of costs.

$$(1)\quad \text{Benefit-Cost Ratio (BCR)} = \frac{\text{Present Value of the Stock of Benefits}}{\text{Present Value of the Stock of Costs}}$$

The meaning of this ratio is the number of times greater the actual return on the investment is compared to the opportunity cost of capital in the base year.

Therefore applying equation (1) to the measures of the present value of the stock of total benefits estimated in the previous section along with the present value of the stock of total costs estimated above gives the results presented in Table 37.

Table 37

The Benefit-Cost Ratios Associated with the Mechanical Cucumber Harvester Given an Exponential and Linear Demand Function and Pivotal, Parallel and Convergent Supply Shifts

| Type of Supply Shift | Exponential Demand Linear Supply | Linear Demand Linear Supply |
|---|---|---|
| Pivotal | 7.128999 | 7.12784 |
| Parallel | 10.70804 | 10.70088 |
| Convergent | 14.28711 | 14.27393 |

Table 37 shows that in 1978 the investment in the mechanical cucumber harvester resulted in a return of 7 to 14 times greater than a savings deposit alternative investment of 6 percent real interest. The range in the cost-benefit ratio reflects the assumptions associated with the type of shift in the supply function and the functional form of the demand equation.

4-9-2 Benefit-Cost Rate of Return

The general equation for the benefit-cost rate of return is the ratio of the present value of the flow of benefits to the present value of the stock of costs.

(2) Benefit-Cost Rate of Return (BCRR) =

$$\frac{\text{Present Value of the Flow of Benefits}}{\text{Present Value of the Stock of Costs}}$$

The meaning of this expression is the rate of return on the investment from the base year in perpetuity given that the return on the investment up to the base year is equal to a minimum opportunity cost of capital. Therefore applying equation (2) to the measure of the present value of flow of total benefits estimated in the previous section along with the present value of the stock of total costs estimated above gives the results presented in Table 38.

Table 38 shows that the average return on the money invested in the mechanical cucumber harvester from 1978 in perpetuity will be between 87.83604 and 176.2463 per cent. The average rate of return on money invested in the mechanical cucumber harvester from 1965 to 1977 inclusive is assumed to be equal to 6 percent. The range in the benefit-cost rate of return reflects the assumptions associated with

the type of shift in the supply function and the functional form of the demand equation.

Table 38

The Benefit-Cost Rates of Return Associated with the Mechanical Cucumber Harvester Given an Exponential and Linear Demand Function and Pivotal, Parallel and Convergent Supply Shifts

| Type of Supply Shift | Exponential Demand Linear Supply (percent) | Linear Demand Linear Supply (percent) |
|---|---|---|
| Pivotal | 87.83604 | 88.0171 |
| Parallel | 131.9226 | 131.9312 |
| Convergent | 176.0091 | 176.2463 |

4-9-3 Internal Rate of Return

Another measure of the average rate of returns to the mechanical cucumber harvester is an internal rate of return. An internal return is the implicit interest rate at which the present value of the stock of benefits is equal to the present value of the stock of costs. This interest rate is determined by a process of trial and error in which the range of possible interest rates is narrowed through successive iterations. A tolerance of 5 percentage points was assumed to be sufficient to provide a sense of the actual internal rate of return. This tolerance is sufficiently wide as to include the actual rates of return for either a linear or exponential demand function. The internal rates of return are presented in Table 39.

Table 39 shows that for the years 1958 to 1978 the average rate of return on the money invested in the mechanical cucumber harvester is equal to between 55 and 75 percent. The range in the internal rate

of return reflects the assumptions associated with the type of shift in the supply function only.

Table 39

The Internal Rate of Return Associated with the Mechanical Cucumber Harvester Given Either an Exponential or Linear Demand Function and Pivotal, Parallel and Convergent Supply Shifts

| Type of Supply Shift | Exponential or Linear Demand Linear Supply (percent) |
|---|---|
| Pivotal | 55-60 |
| Parallel | 65-70 |
| Convergent | 70-75 |

4-9-4 Comparing the Rates of Return

So far we have determined three expressions for the returns to the mechanical cucumber harvester. These expressions are the benefit-cost ratio, the benefit-cost rate of return and the internal rate of return. It is interesting to compare these measures of return and their interpretations. The benefit-cost ratio can be expressed as a rate of return since the ratio is an expression of the number of times greater the actual return to the investment is above its assumed opportunity cost of 6 percent. A summary of the three expressions of returns is presented in Table 40.

Table 40 shows the three sets of rates of return for the mechanical cucumber harvester. Each set of expressions for the rate of returns has a different interpretation. The benefit-cost ratio is defined above as an expression of the number of times greater the actual return to investment is above its assumed opportunity cost in

Table 40

The Benefit-Cost Ratio "Rate of Return", The Benefit-Cost Rate of Return and the Internal Rate of Return Associated with the Mechanical Cucumber Harvester Given a Pivotal, Parallel and Convergent Supply Shift

| Type of Shift | Benefit-Cost Ratio "Rate of Return" (percent) | Benefit-Cost Rate of Return (percent) | Internal Rate of Return (percent) |
|---|---|---|---|
| Pivotal | 43 | 88 | 55-60 |
| Parallel | 64 | 132 | 65-70 |
| Convergent | 86 | 176 | 70-75 |

the base year. Therefore in 1978 the average rate of return on the money invested in the mechanical cucumber harvester is between 43 and 86 percent.

The benefit-cost rate of return has a different meaning. This expression gives the average rate of return which can be expected in 1978 and every year thereafter in perpetuity and is equal to between 88 and 176 percent. Of course this expression assumes that the rate of return from 1965 to 1978 is only 6 percent.

The internal rate of return has yet a different meaning. The internal rate is the implicit interest rate over a prescribed time period which makes the stock of benefits equal to the stock of costs. Therefore the implicit interest rate from 1958 to 1978 is equal to between 55 and 75 percent. An internal rate of return can be calculated only for a time period with fixed end points. Conversely an internal rate of return does not exist in perpetuity.

Another interesting comparison can be made between the internal rate of return for the cucumber harvester and the internal rate of

return found in the literature for other agricultural research and development projects. The internal rates of return for the mechanical cucumber harvester compares favorably with those estimated by Griliches, Peterson and Evenson for hybrid corn, poultry and total agricultural research in the U.S. respectively (Fishel, ed., 1971, p. 116). A summary of their results is presented in Table 41.

Table 41

Internal Rates of Return Reported for Agricultural Research and Development Projects

| Research and Development Project | Internal Rate of Return (percent) |
|---|---|
| Griliches; hybrid corn, 1910-1955 | 35-40 |
| Peterson; poultry, 1915-1960 | 21-25 |
| Evenson; U.S. Agricultural Research, 1938-1963 | 46-48 |
| Cooke; cucumber harvester, 1958-1978 | 55-75 |

The relatively high internal rate of return associated with the mechanical cucumber harvester is the result of two factors. First only seven years elapsed (1958-1964) before the investment in research and development on the cucumber harvester began to realize a return. Also the stock of total cost for the first six of those seven years was a modest $356,000. Therefore the short lead time and the low initial investment results in a relatively high internal rate of return. The relatively wide range in the internal rate of return for the cucumber harvester reflects the assumptions associated with the type of shift in the supply function.

4-10 Labor Compensation

In the previous section the estimates of the benefit-cost ratio and the benefit-cost and internal rates of return were estimated. Implicit in the estimation of these rates of return is the assumption that the resources saved can produce output elsewhere in the economy and are valued at their factor price. Since the mechanical cucumber harvester is a labor saving technology then it is important to consider whether the labor was in fact employed elsewhere in the economy. To the extent that the labor resources are not employed elsewhere then these resources are wasted and must be considered as a cost to the process of introducing the mechanical harvester.

In this section the amount of labor in terms of man-hours saved by the mechanical harvester is estimated. The value of the man-hours saved is also estimated. Then the rates of return are estimated in relation to the various possible levels of alternative employment for the labor resources saved. Finally the opportunity for alternative employment of the man-hours saved is determined in relation to the growth which took place in cucumber production of the introduction of the mechanical harvester.

In order to determine the man-hours of labor saved it is necessary to establish a base line of man-hours required to harvest pickling cucumbers prior to the introduction of the mechanical harvester. This is done by determining the average bushels of pickling cucumbers harvested. This number is then divided by the average number of bushels picked per hour per worker in the years just prior to the introduction of the mechanical harvester. The average bushels of pickling cucumbers hand harvested between 1962 and 1965 inclusive is 17,437,680 bushels.

The average number of bushels picked per hour per worker during this time is estimated to be 1.28 bushels per hour ("Labor Used to Produce Vegetables, Estimates by States, 1959," Economic Research Service, USDA, Statistical Bulletin No. 341; "An Evaluation of the Worker Yield Return Formula for the Michigan Pickle Industry with Recommendations for the Industry," Lloyd Gallardo, Dale Hathaway, Glen Johnson and Noel Stuckman, Department of Agricultural Economics, Michigan State University, E. Lansing, MI, A.E. No. 745, January 1959). Therefore the base line number of man-hours of labor used to harvest cucumber pickles for the years 1962 to 1965 inclusive was estimated to be 13,623,200 man-hours per year. The man-hours saved can then be estimated in relation to this estimate of total man-hours required to harvest pickling cucumbers prior to the introduction of the mechanical cucumber harvester.

To estimate the man-hours saved, the total man-hours required is multiplied by the percentage of the pickling cucumbers hand-harvested each year after the introduction of the mechanical cucumber harvester. The difference between this latter estimate of man-hours required to pick cucumbers after the introduction of the mechanical harvester and the estimate of man-hours required to pick cucumbers before the introduction of the mechanical harvester provides a measure of the man-hours saved. This measure of man-hours saved can then be multiplied by the average hourly wage rate for hired farm workers to provide an estimate of the total value of the man-hours of labor saved by the introduction of the mechanical cucumber harvester. A wage rate of $1.65/hr. is used for consistency with the Schmitz and Seckler analysis. The results of the estimates are presented in Table 42.

Table 42

Man-hours and Labor Cost Saving of the Mechanical Cucumber Harvester Assuming Cucumber Production Remained at the 1962-1965 Average

| | Average Number of Man-hours Required from 1962 to 1965 (1) | Man-hours Required with Harvester Without Growth (2) | Man-hours Saved with Harvester Without Growth (3) | Value of Man-hours of Labor Saved by Mechanical Harvester (dollars) (4) |
|---|---|---|---|---|
| 1965 | 13,623,200 | 13,171,900 | 451,255 | 744,570 |
| 1966 | 13,623,200 | 11,688,800 | 1,934,370 | 3,191,710 |
| 1967 | 13,623,200 | 11,234,900 | 2,388,300 | 3,940,700 |
| 1968 | 13,623,200 | 11,222,500 | 2,400,710 | 3,961,170 |
| 1969 | 13,623,200 | 10,936,400 | 2,686,740 | 4,433,120 |
| 1970 | 13,623,200 | 11,258,500 | 2,364,720 | 3,901,780 |
| 1971 | 13,623,200 | 10,844,800 | 2,778,350 | 4,584,270 |
| 1972 | 13,623,200 | 10,443,500 | 3,179,690 | 5,246,490 |
| 1973 | 13,623,200 | 10,401,100 | 3,222,050 | 5,316,380 |
| 1974 | 13,623,200 | 9,963,410 | 3,659,780 | 6,038,630 |
| 1975 | 13,623,200 | 9,848,090 | 3,775,090 | 6,228,900 |
| 1976 | 13,623,200 | 9,897,340 | 3,725,850 | 6,147,650 |
| 1977 | 13,623,200 | 9,973,520 | 3,649,660 | 6,032,950 |
| 1978 | 13,623,200 | 10,017,100 | 3,606,090 | 5,950,040 |

Columns (1) through (4) are further defined below:

Column (1) is the average number of man-hours required to harvest pickling cucumbers from 1962 to 1965 inclusive.

Column (2) is the number of man-hours required to harvest pickling cucumbers annually from 1965 to 1978 with the introduction of the mechanical harvester if the production of cucumbers remained at the 1962-65 average.

Column (3) is the number of man-hours of labor saved by the introduction of the mechanical cucumber harvester annually if the production of cucumbers remained at the 1962-65 average.

Column (4) is the dollar value of the man-hours of labor saved by the introduction of the mechanical cucumber harvester annually is the production of cucumbers remained at the 1962-65 average.

Table 42 shows the annual cost savings in wages associated with the mechanical cucumber harvester. Again it is necessary to correct these annual estimates of cost saving for the time value of money. The present value of the labor cost saving is presented in Table 43.

Table 43

The Present Value of the Labor Cost Savings Associated with the Mechanical Cucumber Harvester Assuming Cucumber Production Remained at the 1962-1965 Average

| | Labor Cost Savings (dollars) |
|---|---|
| 1965 | 1,588,110 |
| 1966 | 6,422,340 |
| 1967 | 7,480,620 |
| 1968 | 7,093,860 |
| 1969 | 7,489,670 |
| 1970 | 6,218,840 |
| 1971 | 6,893,050 |
| 1972 | 7,442,240 |
| 1973 | 7,114,510 |
| 1974 | 7,623,630 |
| 1975 | 7,418,720 |
| 1976 | 6,907,500 |
| 1977 | 6,383,260 |
| 1978 | 5,950,040 |

The sum of the present value of the labor cost savings is the present value of the stock of the labor cost savings and is equal to $92,026,420 in 1978. The present value of the flow of labor cost savings is determined the same way as for the present value of the flow of total benefits described in section 4-8 and is equal to $11,114,621 in 1978.

The present value of the stock and flow of labor cost savings can be subtracted from the present value of the stock and flow of total benefits respectively. The result is the estimate of the stock

and flow of total benefits if 100 percent labor compensation is assumed. The benefit-cost ratio and benefit-cost rate of return can then be reestimated under this and various other assumptions regarding the level of labor compensation. The benefit-cost rates of return associated with various levels of compensation paid to displaced labor is presented in Table 44.

Table 44

The Benefit-Cost Rates of Return Associated with Various Levels of Compensation Paid to Displaced Labor

| Percentage of Displaced Wage Bill Paid in Compensation (percent) | Flow of Compensation Required in 1978 (dollars) | Benefit-Cost Rates of Return | | |
|---|---|---|---|---|
| | | Type of Supply Shift | | |
| | | pivotal (percent) | parallel (percent) | convergent (percent) |
| 0 | 0 | 88 | 132 | 176 |
| 25 | 2,778,655 | 32 | 76 | 121 |
| 50 | 5,557,311 | -23 | 21 | 65 |
| 75 | 8,335,966 | -79 | -35 | 9 |
| 100 | 11,114,621 | -134 | -91 | -47 |

Table 44 shows that the range of benefit-cost rates of return for the introduction of the mechanical cucumber harvester is from -134 to 176 percent. The range of return reflects the assumptions associated with the type of supply shift and the percent of labor compensation paid. Schmitz and Seckler also show that the returns to the tomato harvester could be negative also if 100 percent compensation were paid. Their estimate of the range of benefit-cost rates of return are between -8 to 1,288 percent (Schmitz and Seckler, 1970, p. 574). The above results demonstrate the desirability of determining the opportunity

for alternative employment of the labor. However, if labor can produce output elsewhere in the economy at the wage rate of 1.65 or more then the laborers are no worse off than before the introduction of the harvester minus the cost of the transition to the place of alternative employment.

An obvious place for displaced labor to get employment is in the pickling cucumber industry itself. If the growth in production is greater than the pace of machine harvesting then the residual must be hand-harvested. The total man-hours required to hand-harvest cucumbers can be estimated by multiplying the bushels of pickling cucumbers produced by the percentage of production hand-harvested and divided by the average bushels harvested per hour per worker. This is a measure of the actual man-hours required to harvest pickling cucumbers from 1965 to 1978. The growth in man-hours required from 1965 to 1978 can be determined by subtracting the number of man-hours required for hand-harvesting from 1965 to 1978 if the production of cucumbers remained at the 1962-1965 average from the actual man-hours required to harvest pickling cucumbers from 1965 to 1978. If the results are greater than the number of man-hours saved by the introduction of the mechanical harvester if the production of cucumbers remained at the 1962-1965 average then all disemployed workers can be re-absorbed picking cucumbers with little cost in transition. Table 45 presents the comparison between additional man-hours required and the man-hours saved.

As it happens the man-hours from the growth in the level of production offsets the man-hours of labor saved in each year except 1965 and 1969. 1965 may not be accurate since this year was included

Table 45

A Comparison of Additional Man-hours of Labor Required and the Man-hours of Labor Saved Associated with the Mechanical Cucumber Harvester

| | Additional Man-hours Required with Harvester with Growth | Man-hours Saved with Harvester without Growth |
|---|---|---|
| 1965 | 299,973 | 451,255 |
| 1966 | 2,627,870 | 1,934,370 |
| 1967 | 4,115,660 | 2,388,300 |
| 1968 | 3,055,650 | 2,400,710 |
| 1969 | 1,684,790 | 2,686,740 |
| 1970 | 3,947,650 | 2,364,720 |
| 1971 | 3,163,280 | 2,778,350 |
| 1972 | 3,239,060 | 3,179,690 |
| 1973 | 3,887,060 | 3,222,050 |
| 1974 | 3,680,960 | 3,659,780 |
| 1975 | 5,383,470 | 3,774,090 |
| 1976 | 4,492,040 | 3,725,850 |
| 1977 | 4,298,980 | 3,649,660 |
| 1978 | 5,733,210 | 3,606,090 |

as part of the pre-mechanical harvester average man-hours required since about 97 percent of the crop was hand-harvested in 1965. In 1969 about 1,000,000 man-hours less were employed than the 1962-65 average. This would reduce benefits by only about $100,000 in 1978 and would have little effect on the benefit-cost rates of return.

Since labor can be absorbed in the economy at its value as a factor of production then it need not be included in the calculations of the returns to the mechanical harvest ex post. Ex ante the range of benefit-cost rates of return should include the possibility of 100 percent compensation given that alternative employment opportunities are unknown. Therefore, ex ante the range of benefit-cost

returns are -134 to 176 percent, and ex post the range of returns are 88 to 176 percent.

A similar argument can be made for the approximately 2,000 small pickling cucumber producers who were displaced by the mechanical harvester. The growth in the production of cucumber pickles could have absorbed the cucumber producers displaced by the mechanical harvester. The transition cost would probably prohibit this however. Since the production of cucumber is perhaps only one of several enterprises for a farmer producing 15 acres of cucumbers, and since entire regions of the country such as Michigan were almost entirely mechanized, then it is unlikely a Michigan pickling cucumber farmer displaced by the mechanical harvester would move to Ohio just to continue growing cucumber pickles. Therefore in aggregate there was sufficient growth to absorb displaced small producers but in fact little of this took place. It can be assumed either that these farmers employed their resources elsewhere at their opportunity cost equal to that for pickling cucumber production or that the benefits of the mechanical harvesters are overstated by the amount of the wasted resources of the approximately 2,000 displaced small cucumber producers. It is unlikely that even 100 percent compensation to these small producers would affect the ex post benefit-cost rate of return substantially.

# CHAPTER V

## CONCLUSIONS REGARDING THE GENERAL EQUILIBRIUM IMPACTS

### 5-1 Consumer Surplus and Economic Rent Theory

We introduced this research with the following statement. The changes in real income in an economy are measured in terms of consumer surplus, economic rent and economic surplus. This makes these measurements and their interpretation important. These measures imply an evaluation of non-marginal change in the structure and organization of the economy. One particular type of non-marginal change in the economy is the effect of the introduction of new technology in the production process of a commodity or group of commodities. In general, new technology causes a shift in the production function and the aggregate supply function of a commodity. The shift in aggregate supply lowers the price of the commodity which increases the real income of consumers of the commodity and some resource owners. The shift in the supply function also implies a release of resources required in the initial production of the commodity. Resources are also saved in the production of less of the now relatively more expensive alternative commodities. These "saved resources" are equal in value to the change in real income to consumers and resource owners. In fact, these two measures are not only equal, but also mutually

dependent upon each other in terms of increased consumption and production using the same resources, which is the essence of increased productivity in an economy.

We have not yet discussed the concept of resources saved. In fact there are several aspects of the theories of consumer surplus and economic rent which should be highlighted or explored further. We will begin by reinterpreting consumer surplus and economic rent in terms of resources saved particularly as it relates to the substitution effect, (associated with a price decrease due to technological change). The resources saved concept is a matter of redefining the change in income to a dollar value of the units of resource inputs released for additional productive use within the economy. After we have defined economic surplus as resources saved, then we will relate the micro and macro measures of resources saved, called economic surplus and net national income respectively. The discussion of national income accounts leads (as always) to the issue of separability and path independence.

We shall see that without path independence, the net national income measure of resources saved (or increased productive capacity) lacks a common denominator needed for comparisons of changes in real income across time. At the macro level of analysis, the absence of path independence seems to imply an array of alternative types of restructuring of the productive capacity of an economy when relative prices of commodities change. The extent of the restructuring is directly related to the importance these commodities relative to the associated total budget expenditure. The type of restructuring, however, depends on the extent of the separability of one commodity to

alternative commodities as saved resources are redirected toward increased productive capacity in the economy. We will then examine the resource savings associated with the mechanical cucumber harvester measured in terms of consumer surplus, economic rent and economic surplus. Finally we will discuss whether the benefits from the mechanical cucumber harvester are those of welfare or compensation.

5-1-1 The Substitution Effect and Resources Saved

The substitution effect, relative to the purchase of final goods which embody consumer surplus, is defined as the increase in income associated with the substitution of a less expensive good(s) for a more expensive good(s) while remaining on the same utility function. See Figure 46. Assume an economy has two goods available, Y and Q. Assume that good Y is the numeral good (or leisure) with a price equal to one. Further assume that a consumer's initial consumption bundle includes $Y_o$ only. The consumer's utility function is assumed to be quasi-concave (Y and Q are substitutes at each level of utility) monotonic (insatiable wants) and twice differentiable (expenditure minimized at each level of utility). As the price of Q decreases from $P_o$ to $P_2$, the consumer is able to substitute units of the more expensive Y for the less expensive Q along utility function $U_o$ such that the subsequent consumption bundle is $(Y_2', Q_2)$. $Y_oY_2$ is the measure of consumer surplus associated with the price decrease from $P_o$ to $P_2$. $Y_oY_2$ is also equal to the value of the resources saved in the production of $Y_o - Y_2$ fewer units of good Y which are not in the consumer's subsequent consumption bundle while remaining on utility function $U_o$. $Y_2Y_2'$ are the units of Y necessary to trade for $Q_2$ relative to utility

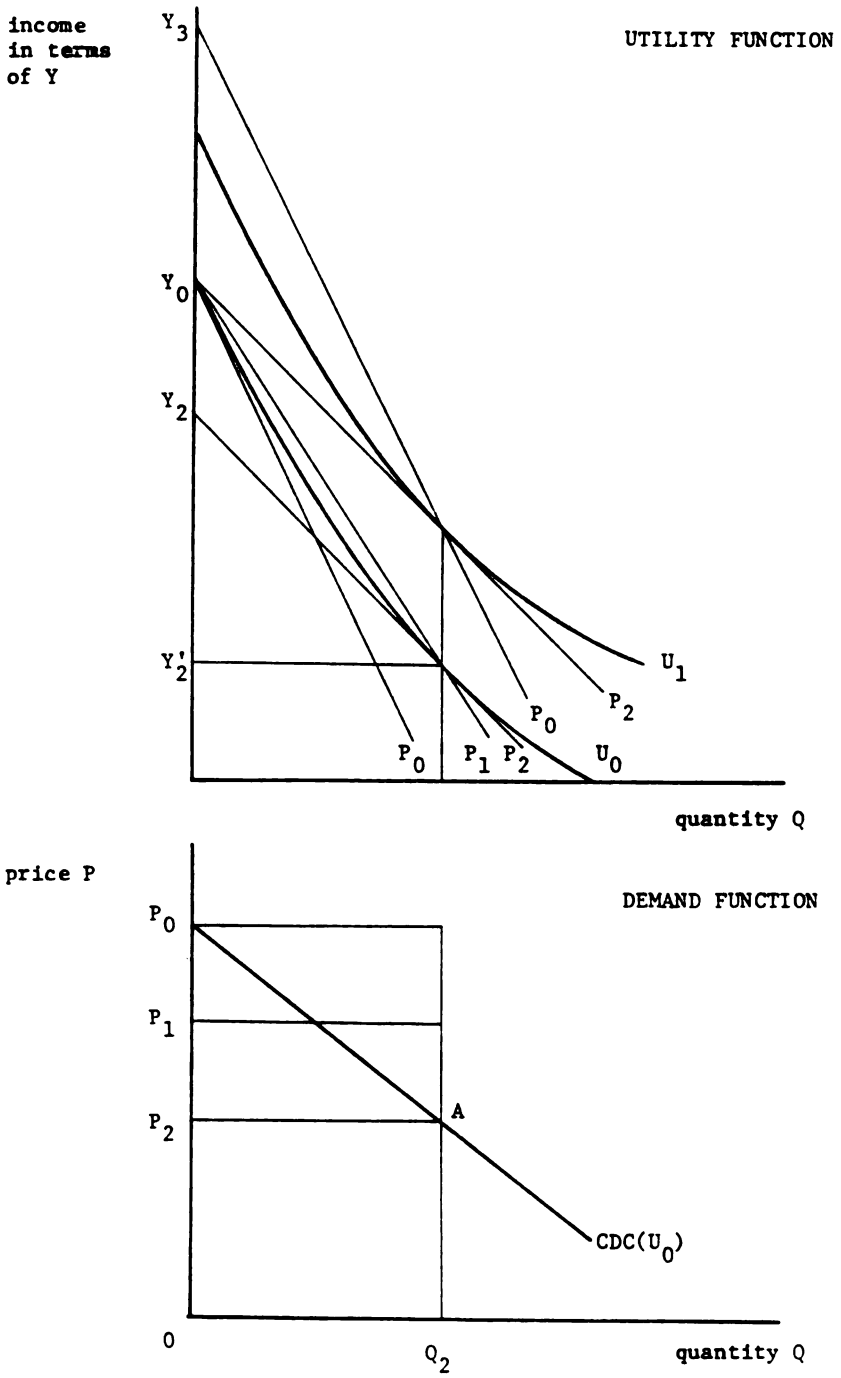



Figure 46. Compensated Demand and the Substitution Effect

function $U_o$. Therefore $Y_oY_2$ are the resources saved and $Y_2Y_2'$ is the opportunity cost of $Q_2$ associated with the price decrease from $P_o$ to $P_2$. Measured in goods space consumer surplus $Y_oY_2$ equals area $P_oP_2A$ or $(P_1-P_2)Q_2$. $P_1$ is defined as the "all-or-none" price for $Q_2$ units of Q. Opportunity cost $Y_2Y_2'$ equals $P_2Q_2$. $Y_2Y_2'$ and $P_2Q_2$ can also be defined as market value. Total ability to pay relative to utility function $U_0$ equals $Y_oY_2'$ or area $P_oAQ_20$ or $P_1Q_2$.

The measure $Y_oY_2$ seems to imply that there is no increase in income since less of Y is consumed in order to increase the consumption of Q. This appears to be a zero-sum gain. This is not the case however. There is an additional "gift from technology" equal to $Y_0Y_2$ which exists subsequent to the price decrease only. Even if the consumer was required to pay $Y_oY_2'$ for $Q_2$ then someone else (a monopolist, perhaps) would be better off subsequently by an amount equal to $Y_oY_2$ while the consumer remains on the initial utility function $U_o$ and is no worse off. There will be a zero-sum gain to the economy if and only if none of the saved resources can be used elsewhere in the economy. There is generally a positive-sum gain overall but the distribution question remains unresolved. Who has the right to the surplus is fundamentally a political question. A measure of the surplus subsequently available is fundamentally an economic one.

The substitution effect, relative to the purchase of resources which embody economic rent, can also be interpreted in terms of resources saved. The substitution effect in this case can be defined as the ability to substitute an initial endowment of resources for commodities relative to a given level of utility. See Figure 47.

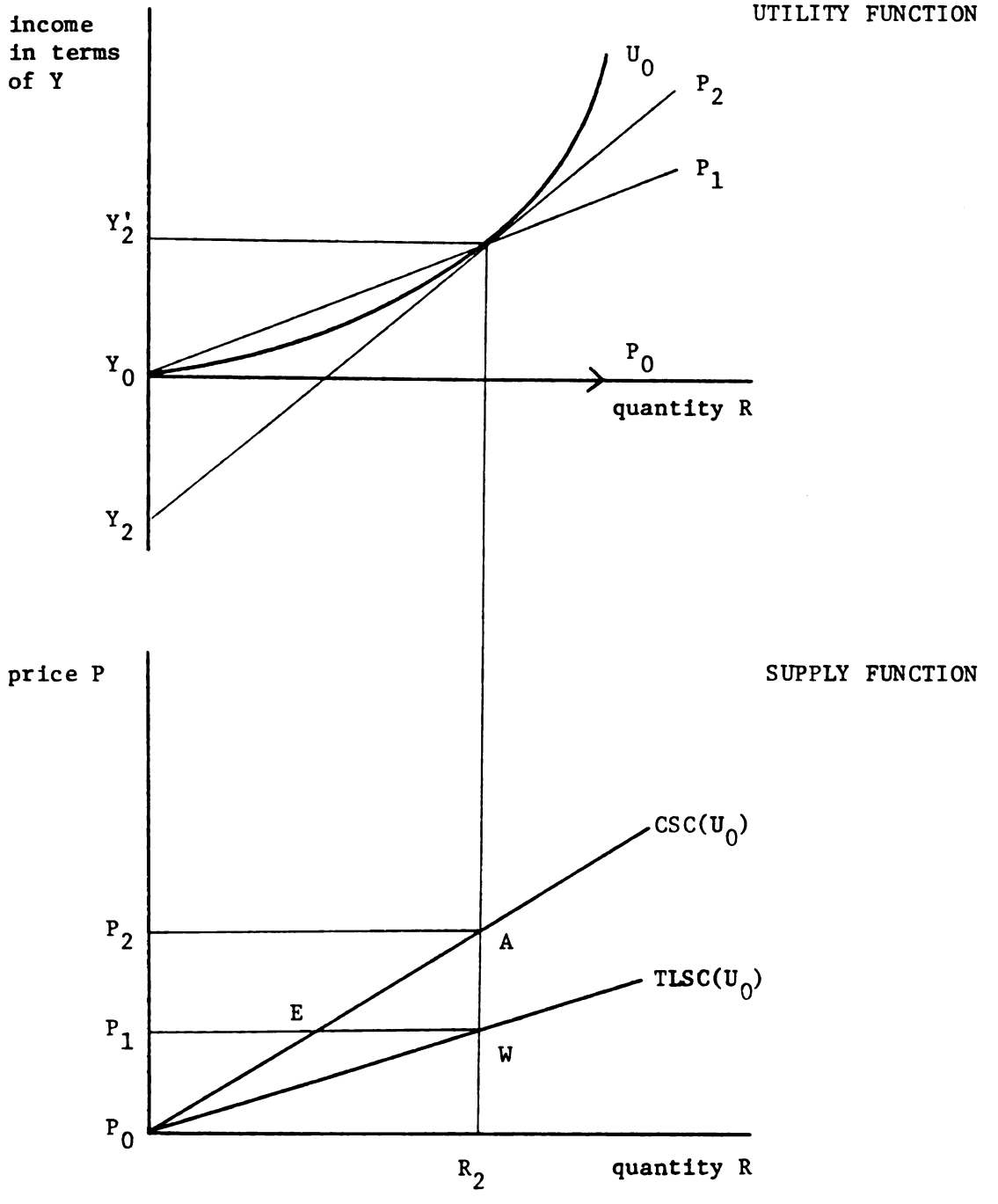



Figure 47. Compensated and "Take It or Leave It" Supply Curves

Assume an economy has one good (Y) available. Assume a consumer has an initial resource endowment of labor (R) in form of foregone leisure. Assume the price of labor is zero, so that initially no labor is provided. The consumer's initial consumption bundle is (0,0) with respect to the quantity of Y consumed and the quantity of labor provided.

Subsequently the price of labor increases from $P_0$ to $P_2$ as a result of the shift in the marginal value product of labor with the introduction of a new technology used to produce Y. The consumer is now willing to supply $R_2$ units of labor in return for $Y_oY_2'$ units of Y. $Y_0Y_2^1$ is the opportunity "cost" of $R_2$. (It is perhaps more realistic to suggest that $R_2$ units of labor is the opportunity cost for $Y_oY_2'$ units of good Y.) The market value of $R_2$ units of labor is not $Y_oY_2'$ however, but rather the substantial larger amount $Y_2Y_2'$. Economic rent is equal to market value minus opportunity cost. Therefore economic rent on $R_2$ units of labor equals $Y_oY_2$. The consumer is therefore able to forgo the consumption of $Y_oY_2$ units of Y subsequently without being worse off than initially. The resources saved when the consumption of $Y_oY_2$ units of Y is forgone is equal in value to $Y_oY_2$. Therefore the interpretation of economic rent is resources saved in foregone subsequent consumption by the resource owner relative to a given level of utility.

Measured in resource space, the market value (and ability to pay) for $R_2$ units of labor $Y_2Y_2'$ is equal to $P_2R_2$. The opportunity cost of $Y_oY_2'$ is equal to area $P_0AR_2$ or $P_1R_2$. ($P_1$ is the "take it or leave it"

price for $R_2$ units of labor.) Economic rent $Y_oY_2$ is equal to area $P_oAP_2$ or $(P_2-P_1)R_2$.

The measure $Y_oY_2$ of economic rent associated with an increase in the price of resource R seems to imply that there is no increase in income since more Y is consumed as the supply of R is increased. This appears to be a zero-sum gain. This is not the case. There is an additional "gift from technology" equal to $Y_0Y_2$ which exists subsequent to the resource price increase only. Thus the resource owner is better off by amount $Y_oY_2$ subsequently which is not the case initially. Even if the supplier of resource $R_2$ were not paid $Y_oY_2$, then someone else (a monoponist perhaps) would be better off subsequently by an amount equal to $Y_oY_2$ while the supplier would be no worse off. As before there will be a zero-sum gain if and only if none of the saved resources can be used in the economy. There is a positive-sum gain over-all though the question regarding distribution remains problematic.

Both consumer surplus and economic rent can be defined in terms of resources saved in the production of one or more commodities, the reduced consumption of which, after a relative price change, leaves the consumer or resource owner as well off as initially. There are a set of relationships which exist between the consumer surplus and economic rent interpretation of resources saved. See Figure 48. If we consider a simplified Robinson Crusoe world we can see these relationships. AH is a consumer's utility function for goods Y and Q. CF is the production possibilities curve in the production of goods Y and Q. The consumer has an initial resource endowment which can be used

UTILITY AND PRODUCT - PRODUCT
TRANSFORMATION FUNCTIONS

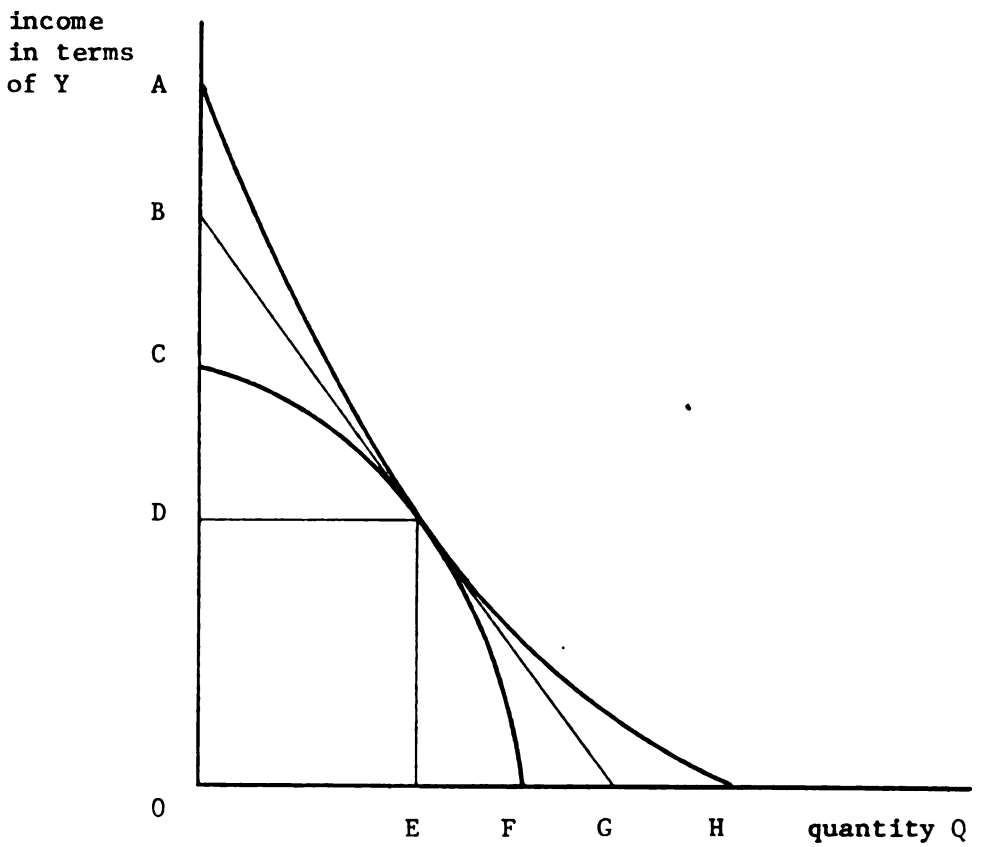



Figure 48. Marginal Rates of Substitution
and Transformation

to produce Y and Q in least cost combinations along the CF curve. The optimal allocation of the initial resource endowment is one which results in the output of D units of Y and E units of Q. The implicit price ratio between Y and Q is equal to BG. In this economy, we can assume that Y is the numeraire good with a price equal to one.

The various measures of the "price" of good Q can be measured in terms of good Y. AD equals the consumer's total ability to pay for E units of Q. BD equals the market price for E units of good Q. BD also equals the opportunity cost for E units of good Q from the point of view of the individual as consumer whose initial consumption bundle includes A units of Y and zero units of Q. CD equals the opportunity cost for E units of good Q from the point of view of the individual as resource owner (Hirschleifer, 1976, pp. 192-193). BC equals the economic rent captured by the individual as a resource owner for sufficient units of the relatively scarce initial endowment required to produce E units of good Q. Therefore we can see that the opportunity cost to the consumer includes rent to the resource owner. The implication is that if the resource owner were not able to capture BC units of rent that the consumer would be better off by an amount equal to BC at a minimum. Consumer surplus and economic rent represents a positive-sum gain for the economy as a whole but the distribution of these benefits to society is at best a zero-sum gain between consumers and resources owners. As long as consumers and resource owners are the same people this does not pose a problem. However the reality in a modern complex economy is that resource owners are typically a very small subset of all consumers.

The nature of the distributional question can be seen in Figure 49. Consider the case in which initially the supply functnon $S_o^*$ is such that no units of Q are purchased at $P_o$. Subsequently the supply function shifts from $S_o^*$ to $S_1^*$ as a result of the introducton of a new technology. Subsequently, $Q_2$ units of Q are purchased at price $P_2$. The total change in resources saved or economic surplus is equal to $P_oBC$. $P_oP_2C$ is the measure of consumer surplus. $P_2BC$ is the measure of economic rent. There are several important features of this example. First, there are no resources saved associated with new technology in the production of $Q_2$ units of Q since none were produced initially, therefore no resources were required. Second all resources saved in this case are derived from the goods which appear in the consumer's initial consumption bundle but do not appear in the consumer's subsequent consumption bundle while remaining on the same utility function. Third, the owners of relatively scarce resources in the production of $Q_2$ units are in a position to capture benefits which might otherwise go to consumers of good Q except for the fact that the resource they own is necessary in the production of good Q and availabile in less than perfectly elastic supply. Fourth, we can see that the resource which is relatively scarce has an initial price of $P_{Ro}$ which is less than its subsequent price $P_{R1}$. A supply function which holds the resource price constant at $P_{Ro}$ is the initial ordinary supply function $S_1(P_{Ro})$. Similarly, the supply function which holds the resource price constant at $P_{R1}$ is the subsequent ordinary supply function $S_1(P_{R1})$. The initial ordinary supply function overestimates economic rent by amount BCD. The subsequent ordinary supply function

DEMAND AND SUPPLY
FUNCTIONS

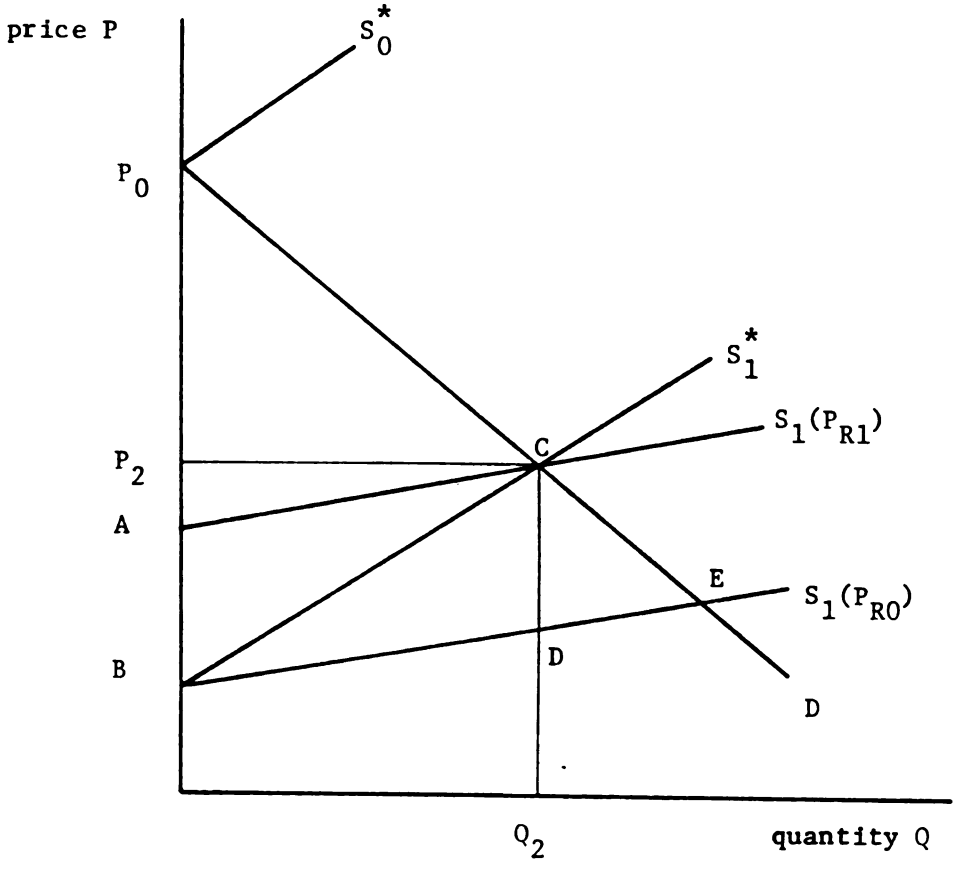



Figure 49. Equilibrium and Ordinary Supply Functions

underestimates economic rent by amount ABC. The correct measure of economic rent equals $P_2BC$ relative to the subsequent equilibrium supply curve $S_1^*$. Finally, area BCE is the measure of the additional resources which could have been saved with the introduction of the new technology if the production of good Q did not require a scarce resource. The competition for benefits between consumers and resource owners is in fact a negative-sum gain, in that as more benefits are captured by the resource owners, the lower the total of additional saved resources available to the economy as suggested by area BCE. Conversely as fewer benefits are captured by resource owners, the greater the total additional saved resources available to the economy. This negative-sum situation with respect to distribution decreases in severity as the supply function becomes more elastic such that the ordinary and equilibrium supply functions merge.

### 5-1-2 Net National Income and Economic Surplus

The concepts of consumer surplus and economic rent have applications at both the micro and macroeconomic levels. At the microeconomic level, we have determined that the sum of consumer surplus and economic rent equals economic surplus and can be interpreted as the change in real income or resources saved (Schmid, 1984, pp. 219-225). At the macroeconomic level, consumer surplus alone is used as an approximation of economic surplus. Economic surplus at the national level is called net national income which is equal to gross national income less depreciation. In the U.S. the change in net national income is approximated by using a Paasche quantity measure of the change in real income. The relationship between the micro and

macroeconomic measures of economic surplus will be clearer by analyzing the various indicies and measures of change in real income. See Table 46 and Figure 50.

In the microeconomic context, the Laspeyres and Paasche prices measures of the change in real income are used to bound the other measures of consumer surplus but not economic surplus which includes economic rent. In the macroeconomic context, the Laspeyres and Paasche price indices are used as a measure of the change in the absolute price level i.e. inflation. Thus the consumer price index, probably the most widely recognized measure of inflation, is in fact

Table 46. The Laspeyres and Paasche Price and Quantity Indices Measured in Utility and Demand Space.

| | Utility Space | | | |
|---|---|---|---|---|
| | Price | | Quantity | |
| | Laspeyres | Paasche | Laspeyres | Paasche |
| Base Period | $Y_o(U_o)$ | $Y_6(U_1)$ | $Y_5(U_o)$ | $Y_o(U_o)$ |
| Alternate Period | $Y_5(U_o)$ | $Y_o(U_1)$ | $Y_0(U_1)$ | $Y_6(U_1)$ |
| Index | $Y_5/Y_o$ | $Y_o/Y_6$ | $Y_o/Y_5$ | $Y_6/Y_o$ |
| Consumer Surplus | $Y_o - Y_5$ | $Y_6 - Y_o$ | $Y_5 - Y_o$ | $Y_o - Y_6$ |
| | Demand Space | | | |
| Base Period | $P_oQ_o(U_o)$ | $P_oQ_4(U_1)$ | $P_1Q_o(U_o)$ | $P_oQ_o(U_o)$ |
| Alternate Period | $P_1Q_o(U_o)$ | $P_1Q_4(U_1)$ | $P_1Q_4(U_1)$ | $P_oQ_4(U_1)$ |
| Index | $P_1Q_o/P_oQ_o$ | $P_1Q_4/P_oQ_4$ | $P_1Q_4/P_1Q_o$ | $P_oQ_4/P_oQ_o$ |
| Consumer Surplus | $(P_o-P_1)Q_o$ | $(P_o-P_1)Q_4$ | $P_1(Q_o-Q_4)$ | $P_o(Q_o-Q_4)$ |

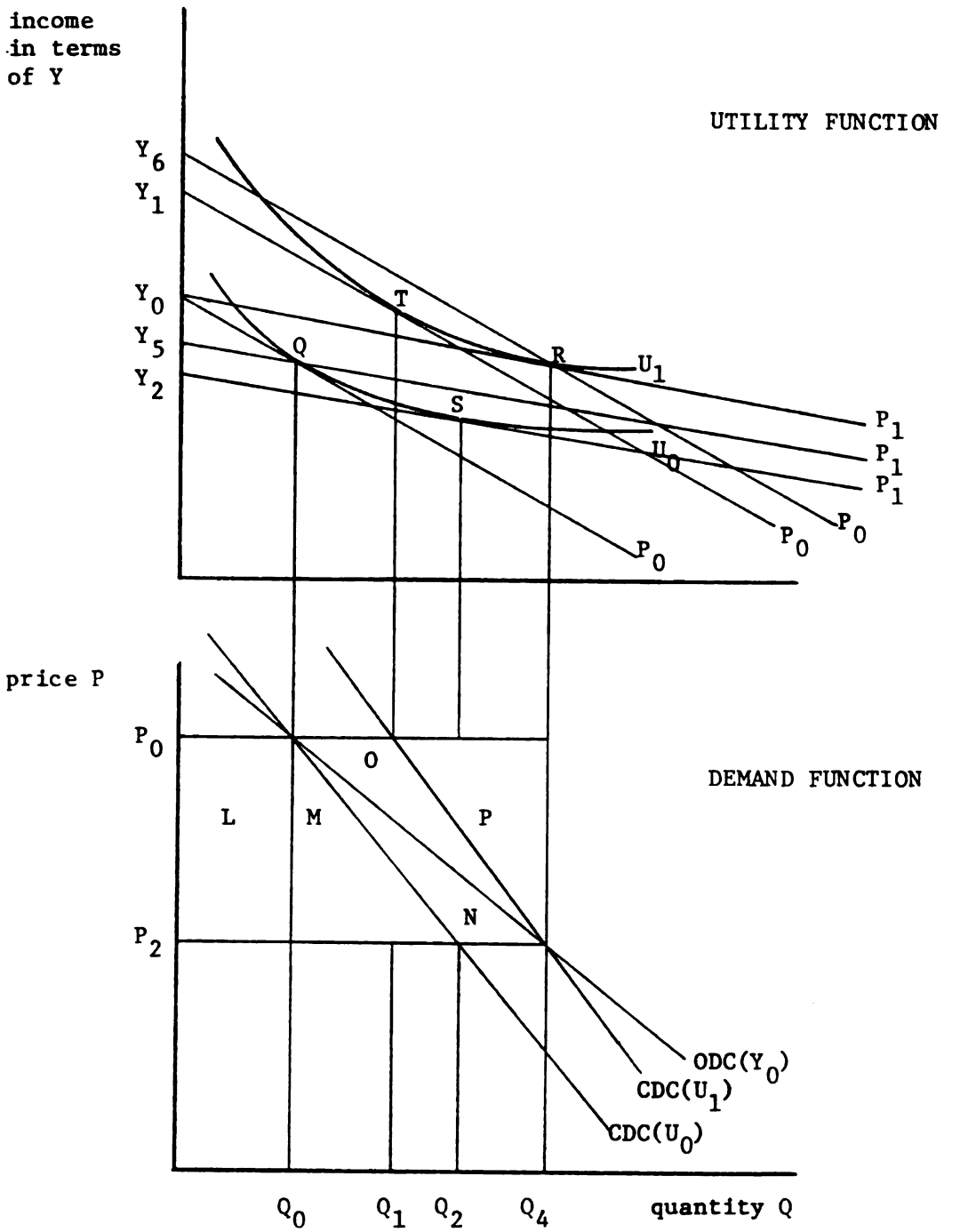



Figure 50. The Five Measures of Consumer's Surplus

a Laspeyres price index. Similarly, the GNP deflator is a Paasche price index. The micro and macro Laspeyres and Paasche measures appear to be worlds apart yet a clear relationship exists.

There are two types of Laspeyres and Paasche measures. Those which measure changes in prices and those which measure changes in quantity. All indices or measure of the change in real income have elements in common. Each has a designated base period. The base period is used to establish the "real" price or base quantity against which "nominal" prices or alternative quantities are compared. All price measures hold quantity constant and equal to the base period. Similarly all quantity measures hold prices constant and equal to the base period. The base period quantity always appears in the denominator of a price measure. Similarly the base period price always appears in the denominator of a quantity measure. The measure of the change in real income is equal to the denominator minus the numerator for any index.

The definition of the Laspeyres and Paasche measures are the same at both the micro and macroeconomic level. The units of analysis are different however. At the macroeconomic level, these measures refer to an aggregate of all final goods, or the value added to all goods (and services) within an economy for which a market exists per capita. The macro measures also refer to the change in aggregate wages, salaries, proprietor's income, rental income and corporate profits and net interest divided by the population of the nation. The micro measures on the other hand, typically refers to the change in income of a single representative consumer relative to the change in price of a single commodity or group of commodities.

The uses of the Laspeyres and Paasche price and quantity measures is different at micro and macro level. At the macroeconomic level, the Laspeyres and Paasche price indices are used to measure inflation as indicated above. The Laspeyres and Paasche quantity measures on the other hand are used to measure the change in real income as the change in physical output of the economy (Dornbusch and Fischer, 1981, pp. 39-44). At the microeconomic level, the Laspeyres and Paasche price measures are used to bound the change in consumer surplus. The Laspeyres and Paasche quantity measures are also used to measure the change in consumer surplus in microeconomic studies. These quantity indices are used less often in microeconomic analysis since it implies unitary elasticity of demand and a homothetic utility function. (Peterson, 1966, pp. 666-669; Griliches, 1958, pp. 419-431). All prices used in microeconomic studies for estimating Paasche and Laspeyres price or quantity measures of real income change are divided by a macro price index as a means of adjusting for differences in the absolute price level (i.e. inflation) at different points in time.

The question then is - when are the micro price and quantity measures of the change in real income equal to each other and when are they equal the macro quantity measures of the change in real income. The relationship between the micro and macro measures of changes in real income is based on the assumption that the change in real income in the aggregate equals the sum of the change in consumer surplus associated with each commodity. We will analyze this set of relationships in the next section regarding separability.

There is an important point of divergence between the micro and macro measures of real income change we have discussed so far. The micro measure of economic surplus equals consumer surplus plus economic rent. The macro measure of economic surplus equals consumer surplus only. Therefore the macroeconomic measure of economic surplus assumes that the national aggregate supply function is perfectly elastic. The Paasche measure of economic surplus is a bound for consumer surplus but it does not have any relationship to economic surplus as a whole except in the case in which economic rent is zero (i.e. a perfectly elastic supply function).

### 5-1-3 Path Independence, Resources Saved and National Income Accounting

Silberberg's concluding remarks on separability provide a point of departure for a further examination of the issue of separability and path independence. Silberberg states

> Curiously paradorical is the fact that a given consumer will impute a different shadow price to the [dam] project if first the price of electricity is lowered, then that of irrigation, and then of recreation, than if prices were lowered in some other sequence; yet the nagging truth remains that nothing can be done to correct such an inconsistency. [1972, p. 950]

What is the cause of this inconsistency and what difference does it make? Silberberg relates this paradox to the lack of path independence associated with the implicit social utility function. Path independence is a mathematical term for an economic reality. In order to get a better intuitive understanding for path independence, we will reinterpret path independence in terms of resources saved. There are three tasks. The first is to relate resources saved to vertically parallel and homothetic utility functions which meet the separability

and path independece criterion. Next, we must relate resources saved to a utility function which does not meet the separability criterion. Finally, we will discuss the relationship between national income accounting and resources saved.

In general, we know that the quantity demanded for any kind of good or service ($q_1$) is a function of own price ($p_1$), the price of substitute and complementary goods and services ($p_2$) and income (4).

(1) D: $q_1 = f(p_1, p_2, y)$

This relationship holds for all goods in a society's consumption bundle. The result is a matrix of quantities, prices and income which can be expressed as own-price, cross-price and income elasticities.

(2)

| | $P_1$ | $P_2$ | Y |
|---|---|---|---|
| $q_1$ | $e_{11}$ | $e_{12}$ | $e_{1y}$ |
| $q_2$ | $e_{21}$ | $e_{22}$ | $e_{2y}$ |

This matrix of elasticities for a simple two good economy of $q_1$ and $q_2$ is helpful in discussing the effect the type of utility function has on interpreting changes in relative prices in terms path independence and resources saved. A vertically parallel utility function for good $q_1$ implies that "the price of a numeraire ($q_2$) is held constant...in which case it is necessary and sufficient for the income elasticity of demand to be zero for all non-numeraire commodities ($q_1$)..." (Chipman and Moore, 1980, p. 1) From the homogeneity condition for a system of demand equations we know that the sum of the own-price, cross-price and income elasticities must add to zero (George and King, 1971, p. 8). Therefore, in the case of a vertically parallel utility function,

the income elasticity equals zero (by definition) and the sum of the own price and cross-price elasticities must add to zero (from the homogeneity condition).

We need to examine the effect of a decrease in the price of $q_1$ (resulting from the introduction of a new technology) has on $q_1$ and $q_2$ assuming a vertically parallel utility function. This is then the general equilibrium effect of a price change in this two good economy. The partial equilibrium effect on $q_1$ is the result of the substitution and income effects on $q_1$ when $p_1$ changes. The general equilibrium effect includes this plus the effect on $q_2$ when $p_1$ changes. In the context of the system of demand equations, the general equilibrium effects are transmitted across commodities in terms of cross-prices elasticities. As it happens, cross-price elasticities are also a function of the substitution and income effects. (Hicks, 1939, p. 51). (Cross-price effects can be viewed as the income and substitution effects of $p_2$ on $q_2$, if instead of assuming the price of $p_1$ decreases with $p_2$ constant, we assume that the price of $p_2$ has increased with $p_1$ constant.) We are assuming that $q_1$ and $q_2$ are substitutes over a certain range of relative prices.

Let us describe the substitution and income effect on $q_1$ and $q_2$ associated with a decrease in $p_1$ and a vertically parallel utility function. Relative to $q_1$, the substitution effect is positive as the relatively less expensive $q_1$ is substituted for the relatively more expensive $q_2$. The income effect on $q_1$ is zero by definition. Relative to $q_2$, the substitution effect is negative. This implies that

the income effect on $q_2$ is also negative such that the overall cross-price elasticity is positive and equal to the absolute value of own-price elasticity.

(3) $| e_{11} | = e_{12}$

and

$$e_{11} + e_{12} = 0$$

Further, we know that a positive cross-price elasticity implies a own-price elasticity absolute value greater than one (Hicks, 1939, p. 48). Therefore as $p_1$ decreases, the total demand for $q_2$ decreases, given an elasticity of demand for $q_1$ greater than one in absolute terms associated with a vertically parallel utility function.

Now we can analyze the resources saved, given a new technology in the production of $q_1$ which lowers $p_1$ relative to a vertically parallel utility function. We will assume that some quantity of $q_1$ is being produced before the price decrease and that the production of $q_1$ only requires abundant resources both before and after the adoption of the technology. Consequently the supply function is horizontal and no economic rent is captured. The value of resources saved are derived from two sources. First, resources are saved in the production of the initial quantity of $q_1$ using the new technology. The amount of resources saved is equal to the Laspeyres measure. Second, resources are saved in the production of fewer units of the now relatively more expensive $q_2$. This is the same result described in the section on the substitution effect. However, the significance of the vertically parallel utility function lies in the fact that there is a once and

for all restructuring of the saved resources in one particular direction only. In particular, even though the income effect for $q_1$ is zero, the substitution effect is such that, the subsequent consumption bundle contain fewer units of $q_2$ than initially. Therefore all of the resources saved in the production of the initial quantity of $q_1$, as well as some of the resources saved in the production of the subsequent quantity of $q_2$, are available to produce the subsequent quantity of $q_1$. The question is whether some other better use for the saved resources could have been made. This question is relevant to the issue of path independence. Path independence implies that these saved resources can be used to produce $q_1$ and the economy can and will be restructed to take optimal advantage of these resources on a once and for all basis. The movement from the initial to subsequent consumption bundles for a society from one vertically parallel function to a higher one represent an optimal cardinal shift in welfare as measured by net national income regardless of whether it is measured in a two good or N-good economy.

Next we need to examine the effect of a decrease in the price of $q_1$ (resulting from the introduction of a new technology) has on $q_1$ and $q_2$ assuming a homothetic utility function. Again we are interested in analyzing the general equilibrium effects manifested through own-price, cross-price and income elasticities. A homothetic utility function for goods $q_1$ and $q_2$ implies that the absolute value of own-price elasticity equals one, cross-price elasticity equals zero and income elasticity equals one (Samuelson, 1965, pp. 788-789). The homogeneity condition holds in that the sum of own-price, cross-price and income elasticities equals zero.

Let us describe the substitution and income effect on $q_1$ and $q_2$ associated with a decrease in $p_1$ and a homothetic utility function. Relative to $q_1$, the substitution effect is positive as the relative less expensive $q_1$ is substituted for the relatively more expensive $q_2$. The income effect on $q_1$ is positive by definition. Relative to $q_2$, the substitution effect is negative. This implies that the income effect on $q_2$ is positive and equal to the substitution effect such that the overall cross-price effect is zero. Finally, we know as $p_1$ decreases, the total demand for $q_2$ is unchanged, given unitary, own-price and income elasticity of demand for $q_1$ associated with a homothetic utility function.

We can now analyze the resources saved, given a new technology in the production of $q_1$ which lowers $p_1$ relative to $p_2$ and in conjunction with a homothetic utility function. As before, we assume that some quantity of $q_1$ is being produced before the price decrease and that the production of $q_1$ requires abundant resources both before and after the adoption of the technology. Consequently the supply function is horizontal and no economic rent is captured. The value of resources saved are derived from two sources. First, resources are saved in the production of the initial quantity of $q_1$ using the new technology. The amount of resources saved is equal to the Laspeyres measure. Second, resources are saved in the production of fewer units of the now relatively more expensive $q_2$. The significance of the homothetic utility function is that there is a once and for all restructuring of the saved resources in a particular direction. Even though the cross-price effect is zero, the substitution and income effects of $q_2$ are

such that the subsequent consumption of $q_2$ equals the initial consumption. Therefore all resources saved relative to $q_2$ are used subsequently to produce $q_2$ in the same proportion. Similarly all resources saved in the production of $q_1$ with the new technology are available to produce additional subsequent units of $q_1$ in direct proportion. Path independence associated with a homothetic utility function implies that these resources can be used to produce $q_1$ and $q_2$ with minimal restructuring of the economy and still achieve the optimal advantaged from the use of these resources. The movement from the initial to the subsequent consumption bundles for a society from one homothetic utility function to a higher one also represents an optimal cardinal shift in welfare as measured by national income regardless of the number of goods in the economy.

Let us summarize the implications of vertically parallel and homothetic utility functions on resources saved. Both types of utility functions imply additional production of $q_1$. The vertically parallel function results in the reduced production of $q_2$ while the homothetic functions implies no change in the production of $q_2$. The vertically parallel function implies a once and for all restructuring of the productive capacity of the economy away from $q_2$ in favor of $q_1$. The homothetic function implies a minimum of restructuring of productive capacity since production expands for $q_1$ and $q_2$ proportionately to resources saved for each of these commodities. In analyzing the path dependent utility function, it seems clear that the key elements associated with path dependences include the extent of restructuring productive capacity relative to resources saved and the once and for all optimal nature of this restructuring.

We will now examine the implication of a utility function which does not meet the separability criterion. In particular, we will examine the effect of a decrease in the price of $q_1$ (resulting from the introduction of a new technology) has on $q_1$ and $q_2$ assuming a more general utility function. We are interested in analyzing the general equilibrium effects manifested through own-price, cross-price and income elasticities. In this case we will assume that this utility function implies an absolute value of own-price elasticity of less than one, a positive income elasticity and a negative cross-price elasticity. The sum of these elasticities equals zero.

Let us describe the substitution and income effect on $q_1$ and $q_2$ associated with a decrease in $p_1$ and a general utility function. Relative to $q_1$, the substitution effect is positive as the relatively less expensive $q_1$ is substituted for the relatively more expensive $q_2$. The income effect on $q_1$ is positive in that $q_1$ is a normal good and more is consumed as real income increases. Relative to $q_2$, the positive income effect is greater than the negative substitution effect such that the overall cross-price elasticity is negative. We know that a negative cross-price elasticity implies a own-price elasticity of less than one (in absolute terms) (Hicks, 1939, p. 48). Finally, as $p_1$ decreases, the total demand for $q_2$ increases for this general utility function.

We can now analyze the resources saved, given a new technology in the production of $q_1$ which lowers $p_1$ relative to $p_2$ and in conjuction with a general utility function. We assume that some quantity of $q_1$ is being produced before the price decrease and that the production

of $q_1$ requires abundant resources both before and after the adoption of the technology. Consequently the supply function is horizontal and no economic rent is captured. The value of resources saved are derived from two sources. First, resources are saved in the production of the initial quantity of $q_1$ using the new technology. The amount of resources saved is equal to the Laspeyres measure. Second resources are saved in the production of fewer units of the now relatively more expensive $q_2$. There is a restructuring of these saved resources away from the production of $q_1$ and toward the production of $q_2$. The inelastic own-price elasticity implies that a portion of the resources saved to produce $q_1$ are released to produce $q_2$. All of the saved resources relative to $q_2$ are reused to produce $q_2$ subsequently. In this two good case, path dependence does not differ significantly from the vertically parallel case, at least as far as resources saved are concerned. Both functions result in a restructuring of the productive capacity. In the vertically parallel and homothetic cases all restructuring takes place in favor of the good associated with the new technology ($q_1$) and away from the alternative good ($q_2$). In the general case, the restructuring is taking place in favor of the alternative commodity. This is suggestive of the important difference between the general and the special utility functions.

For the special case functions, the addition of other commodities into the analysis would not change either the direction of the restructuring of productive capacity or the value of the cardinal measure of the change in welfare. Thus the special case functions imply an optimal restructuring of the economy. For the general case

function, however, the addition of other commodities into the analysis could change both the direction of restructuring of productive capacity and the value of the "cardinal measure of the change in welfare." In fact, the value of the measure of "welfare" is dependent upon the path or choice for restructuring the productive capacity of the society in the general function case. Since the welfare measure is path dependent it looses all meaning as a welfare measure per se and as an optimal restructuring. In effect the yard stick changes length depending upon what is being measured.

Let us return to Silberberg's "curious paradox" of the dam. The dam is, in effect, the new technology in the production of three goods - electricity, recreation and irrigation. The relative prices of these goods will change with the completion of the dam.. The reduction in the price of these commodities results in the saved resources associated with an initial production of these goods themselves plus the alternative goods which are substitutes for these goods. These saved resources are then available to be used as inputs into a subsequently restructured economy. The priority of the restructured economy is determined by the order in which the reduction in the price of electricity, recreation and irrigation takes place. The restructuring is once and for all in that asset fixity often precludes multiple restructuring of the saved resources. There is no cardinal measure of a societal optimal restructuring in the general case for lack of a common denominator across various levels of utility. Consequently the national income accounting of the dam can be interpreted as the compensation that winners could pay without being worse off

subsequently than initially (rather than welfare) with the introduction of the dam given a particular ordering in the reduction of relative prices of electricity, recreation and irrigation and the associated restructuring of productive capacity which reflects that ordering.

The question remains as to whether the national income account measure of the change in real income is equal to the sum of the change in consumer surplus across all commodities during the same period. Both the national income accounts and measures of consumer surplus imply a particular "ordering in the change in relative prices and associated restructuring of productive capacity." Both measures are path dependent. These measures will equal each other as long as the ordering in the change is relative prices for the individual consumer surplus measure is the same as the ordering of the change in relative prices implicit in the national income accounts. This is the same as saying the values will be equal if the measures are the integral of the same path between the initial and subsequent social utility functions. This criterion is not quite as unattainable as it might at first seem. Work in the area of consumption economics has resulted in various determination as to the extent of cross-price effects between commodities over an entire matrix of goods and services. (Frisch, 1959; Barten, 1964, 1967; Goldman and Uzawa, 1964) In particular this work has revealed that commodities can be designated by groups. The cross-price elasticities within and among these groups can be estimated. If no cross-price elasticities can be determined between commodities or groups of commodities they are assumed to be separable.

There are categories of separability including, "weak" and "strong." Boutwell and Simmons describe the distinction between strong and weak separability with the following example:

> Suppose we have a universe of commodities which can be divided into three groups: food I, clothes J, and entertainment K. Weak separability implies that the marginal rate of substitution of two goods in I, say corn and beans, is unaffected by a change in the quantity of a good J, say neckties. Strong separability implies that the marginal rate of substitution between corn and neckties is unaffected by an increase in the quantity of movies. (1968, p. 377).

This work on the separability between and among commodities and commodity groups provides information regarding the path of general equilibrium effects for a given change in relative prices. In welfare economics, this information is useful in predicting for a given project the magnitude of the compensation which could be paid for a given adjustment path. These separability estimates would also provide some insight into the probability that resources saved could be redirected toward increased production in a restructured economy. Finally separability information would be useful in comparing the magnitude of compensation and the extent of restructuring for two different public investment projects. Next we will summarize the results of the introduction of the mechanical cucumber harvester in light of this general equilibrium understanding and interpretation of economic surplus.

## 5-2 Empirical Results

The theories of consumer surplus and economic rent have been applied in this study to measure the impact of the introduction and adoption of the mechanical cucumber harvester technology. The case of the mechanical cucumber harvester provides an extended example of the

empirical consideration required to actually estimate benefits under various assumptions. The consumer surplus estimates involved estimating the retail demand for cucumber pickles using both a linear and exponential functional form for the demand equation. Then for each of these functional forms the Willig and Hausman techniques for estimating compensating and equivalent variations were applied, and compared with each other as well as with the Laspeyres, Paasche and "utility" measures of consumer surplus.

The economic rent estimates of benefits associated with the introduction of the harvesting technology was also determined. The assumptions regarding the nature of the supply shift as pivotal, parallel or convergent were used. The type of supply shift had a substantial impact on the estimate of benefits attributable to the introduction of the harvester technology in absolute terms. The variance in the supply shift assumption on economic rent proved to be far more important than the variance associated with any of the assumption or combination thereof on consumer surplus.

The final task remains that of interpreting these measures of economic surplus in terms of welfare, conpensation and general equilibrium.

### 5-2-1 Consumer Surplus Results

The Hausman and Willig measures of compensating and equivalent variation measures of consumer surplus associated with an exponential demand for cucumbers are identical as predicted by Hanemann (1981). The Hausman and Willig measures of compensating and equivalent variation associated with the linear retail demand for cucumber

pickles are not equal nor does Willig's equation for an estimate bound the Hausman estimate as predicted by Willig (1976). This is in spite of the fact that cucumber pickles meet the Willig criteria requiring the value of the good be a small percentage of an individual's disposable income (with a vengence). Willig's equation for estimating compensating variation overstates the Hausman measure in seven of the fourteen years estimated. However the Willig equation for approximating compensating variation does bound the Hausman measure of compensating variation in six of the last seven years estimated. The differences in question are very small but the discrepancy between Willig's claims and the actual empirical estimates remain.

The present value of consumer surplus associated with the mechanical cucumber harvest is equal to between $35,320,990 and 35,355,170 based on the Laspeyres and Paasche measures respectively.

5-2-2 Economic Rent Results

The contribution of this study on economic rent are both theoretical and empirical. Theorically, the three notions for measuring economic rent by Lindner-Jarrett-Rose-Wise-Fell, Just-Hueth-Schmitz and Marshall-Viner are shown to converge and to be consistant with the Mishan criteria. These techniques for estimating rent are different in the way by which the intercept term is determined (by assumption, econometically and cost of production respectively). Each requires, implicitly or explicitly, the use of the equilibrium supply curve however.

The variance in estimated rents associated with the pivotal, parallel and convergent supply shifts are as dramatically large as the

variance in the estimates of consumer surplus are dramatically small. Curiously, the importance attributed to economic rent and consumer surplus in the literature seems to be inversely related to the magnitude of these various measures on the net result. The empirical problem associated with measuring economic rent relates to estimates of the "Y-intercept" term. The best technique for estimating the Y-intercept for the purposes of estimating economic rent is the one suggested by Marshall and Viner using estimates of cost of production to determine the particular expense curve. The data requirement however makes this technique very costly. There is the added benefit of being able to attribute the ability of capturing rent to various types of firms and resource owners associated with these firms. In this study the intercept term was assumed to be either pivotal, parallel or convergent and the effect of the assumptions were compared.

The present value of economic rent associated with the mechanical cucumber harvester is equal to between $200,063 and 35,884,330 based on pivotal and convergent supply shift respectively.

5-2-3 Economic Surplus Results

The present value of the annual flow of benefits from the introduction of the mechanical cucumber harvester technology in 1978 is between $4.4 and $8.8 million depending on the assumption made regarding the nature of the supply shift. The present value of the total cost of research and development on the mechanical cucumber harvester is about $5 million dollars in 1978. This translates into an internal rate of return between the years 1958 to 1978 of 55 to 75%.

This internal rate of return compares favorably with estimates of the returns to research on hybrid corn, poultry and aggregate U.S. agricultural research. This internal rate implies that the resources saved (primarily seasonal labor) was re-employed elsewhere in the economy at a comparable wage rate.

The labor saved through the introduction of the mechanical harvester is about 3.6 million man-hours per year. The annual value of the labor saved in 1978 is about $11 million. If none of the labor resource saved is re-employed elsewhere in the economy, then the benevit cost rate of return is -135 to -47% compared to 88 to 176% if all the labor is re-employed. Sufficient growth in the pickling cucumber industry along was sufficient to re-employ this amount of saved labor resource. Approximately 2,000 small pickling cucumber producers were released from production with the introduction of the mechanical harvester as the optimal enterprise size increased from about 15-20 acres to about 150-175 acres or about a ten fold increase.

### 5-2-4 Welfare or Compensation?

The interpretation of benefits as we have discovered repeatedly depends on separability and path-independence. These in turn are determined in relation to the own-price, cross-prices and income elasticity coefficients. The nearly unitary value of the income elasticity suggests that consumer's relate to cucumber pickles as if they have a homothetic utility function. However, the homothetic utility function also implies unitary own-price elasticity and zero cross-price elasticity. Neither the linear nor exponential functional form of the demand for cucumber pickles suggest unitary own-price

elasticity. The cross-price elasticity is derived based on the homogeneity condition. No statistically significant cross-price elasticities were derived using the price of hamburger, hot dogs and tomatoes as possible complements with or substitutes for cucumber pickles.

Table 47. Estimates of Own-Price, Cross-Price and Income Elasticity of Demand for Cucumber Pickles

| functional form | elasticity | | |
|---|---|---|---|
| | own-price | cross-price | income |
| linear | -.52 | (-.46) | .98 |
| exponential | -.82 | (-.25) | 1.07 |

The demand elasticities of cucumber pickles are consistent with the more general utility function case described above. Cucumber pickles are associated with $q_1$ and all the substitutes for cucumber pickles are associated with $q_2$. We found in the general case that the income effect of $q_1$ implies it is a normal good in that more is consumed as real income increases. The negative cross-price elasticity implies an own-price elasticity of less than one in absolute terms. Finally as $P_1$ (the price of cucumber pickles) decreases, the total demand for $q_2$ (all the substitutes for cucumber pickles) increases.

The value of resources saved are derived from three sources. First, resources are saved in the production of the initial quantity of $q_1$ using the new technology. This amount of resources saved is equal to the Laspeyres measure of consumer surplus. The Laspeyres measure in this case has a present value in 1978 of about $35.3 million. The second source of saved resources is in the production of

fewer units of the now relatively more expensive substitutes. This amount of resources saved is equal to the compensating variation minus the Laspeyres measure. This difference has a present value in 1978 of about $18,090. The third source of resources saved associated with economic rent and is a combination of resources saved in the production of the initial quantity of $q_1$ using the new technology plus resources saved in forgone subsequent consumption of the now more expensive substitutes by the resource owners. Economic rent in this case has a present value in 1978 of between $200,000 and $35.8 million.

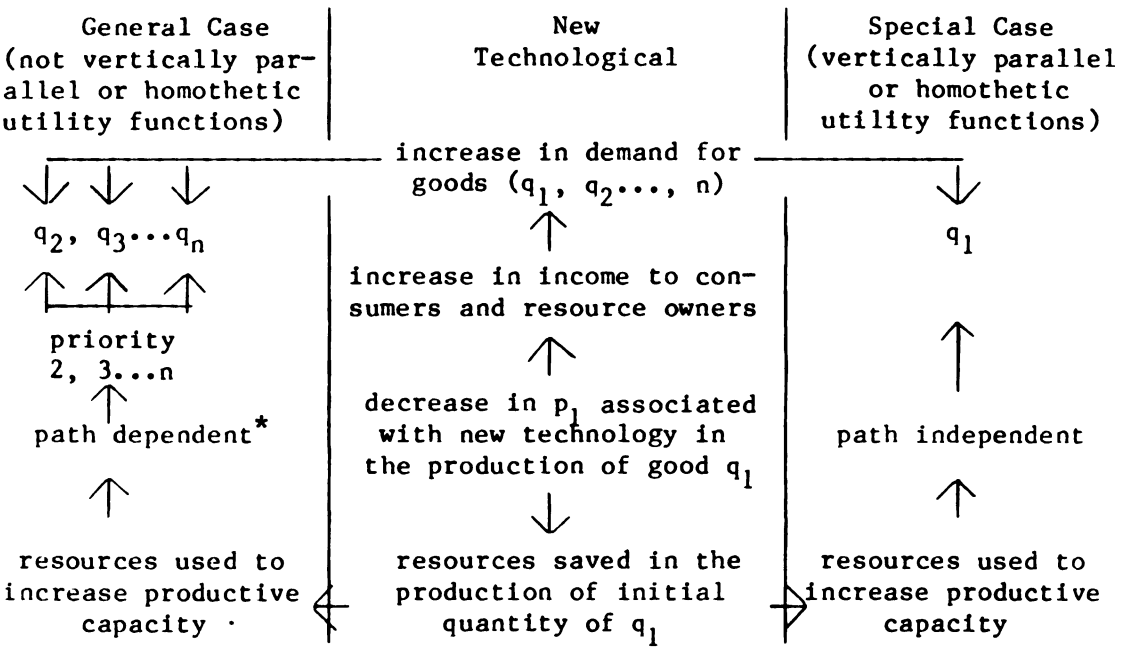
The total present value of resources saved in 1978 equals between $35.5 and $71.2 million. There is a restructuring of these saved resources away from the production of cucumber pickles (assuming no change in demand) and toward the production of substitutes for cucumber pickles. The inelastic own-price elasticity implies that a portion of the saved resources are released for use elsewhere in the economy. All of the saved resources relative to alternative substitutes are reused to produce these commodities subsequently. The positive-sum gains from the technology therefore depend primarily on the ability of the released resources to increase productive capacity. If this does not happen then the change in "real" income to consumers by the price decrease in cucumber pickles will be just offset by a price increase as the demand for substitute commodities increases without an accompanying increase in productive capacity.

Is the $35.5 to 71.2 million present value of total resources save a measure of welfare or compensation? This measure of total

benefits is not a measure of welfare because the demand for cucumber pickles does not imply that consumer act as if they have a vertically parallel or homothetic utility function in relation to cucumber pickles. The absence of either of these utility functions implies path dependence in restructuring the productive capacity of the economy. Path dependence relates to the priority to which the restructuring the productive capacity for alternative commodities with saved resources takes place. It is assumed (perhaps heroicly) that these resources could be successfully transferred across commodities.

The total present value of resources saved is the maximum measure of compensation consumers and resource owners could pay after the introduction of the new technology and associated relative price changes and still be as well off subsequently they were initially. This is a maximum in the sense that it assumes that all resources saved are re-employed to increase the productive capacity of substitutes for cucumber pickles. If the demand for cucumber pickles had implied either a vertically parallel or homothetic utility function then the resources saved in the production of the initial quantity would have all been used to increase the production of cucumber pickle only. See Table 48. The general case utility function is more realistic in that resources are freed to increase the production of alternative commodities. The general case also points up the potential difficulty of transferring productive assets across commodities. The special case utility functions are less realistic in that all freed resources from $q_1$ are used to produce $q_1$. However, the special case functions also solves the resource transfer problem to a large

Table 48. Path Dependence and Economic Restructuring Associated with Technological Change




*The restructuring process proceeds in a "clockwise" direction if the elasticity of demand for $q_1$ is less than one (shown here) and "counter-clockwise" if the elasticity of demand for $q_1$ is greater than one. The direction in the unitary elastic case depends on income and cross-price elasticity.

extent since no saved resource (or very few) need cross over to increase the production of other commodities. Nor is there any question as to the subsequent restructuring of the productive capacity in the special case since only one option is possible. Further this one option has relatively low transactions costs and therefore a high probability of taking place. The general case functions have many restructuring options and associated high transformation costs and lower probability of taking place.

On this point of the difference between welfare and compensation interpretation of consumer surplus and economic rent there is substantial confusion. In particular, Willig's claim regarding compensating and equivalent variation as "the correct theoretical measures of the welfare impacts of changes in prices and incomes on an individual" (1976, p. 589) is quite misleading. It is true only in the case of vertically parallel or homothetic utility function. In all other instances Willig's technique provides somewhat more accurate empirical estimates of compensation. The returns to mechanical cucumber harvester should be thought of as a measure of compensation and compensation only. If this is true for cucumber pickles then it is very likely to be true for every other commodity in the economy.

Consumer surplus can be defined as compensation and interpreted as the addition income a consumer, receives under conditions of competitive trade relative to monopoly trade. Similarly economic rent is the addition income a resource owner receives under conditions of competitive trade relative to monopsony trade. Economic surplus is equal to both the change in real income and the resources saved. The extent to which the maximum change in real income is realized is determined by the extent to which saved resources are successfully re-employed in expanding productive capacity in the economy. Separability and Path independence imply zero or very low transformation costs such that the maximum change in real income is likely to be realized.

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