This is to certify that the

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A NEW APPROACH TO SIMILARITY RELATIONS
IN SET THEORY ANALYSIS

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A NEW APPROACH TO SIMILARITY RELATIONS IN SET-THEORY ANALYSIS

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ABSTRACT

A NEW APPROACH TO SIMILARITY RELATIONS
IN SET-THEORY ANALYSIS

By

Alexis Turkalo

A Set-Theory approach to analysis attempts to identify, define and explain the pitch content of atonal music, which cannot be explained easily by "traditional" methods. The Structure of Atonal Music, by Allen Forte, is probably the best known text that deals with this theory.

The Problem

While some of the relationships as defined by Forte are fascinating as number-theory, they sometimes fail to be useful in musical analysis for the simple reason that they deal with elements that are often not present in a given score. Many relationships are left unexplored, ignored and, for the most part, considered to be of marginal importance.

The Procedure

The system for relating sets of like cardinality that is presented in this dissertation provides

information left unexplored by Forte. It is based on the premise that sets can and should be compared on the basis of which subsets they have in common. Therefore, a series of charts, bound in a separate volume (the Index), relating pairs of sets, from those of cardinality 4 to those of cardinality 8, was prepared. Whereas, formerly only a minority of pairs of sets could be compared by similarity relations, this new approach allows for all pairs of sets of equal cardinality to be compared in a meaningful way.

To illustrate the use of this method, the dissertation presents a sample analysis; that of the second movement of Three Pieces for String Quartet by Igor Stravinsky. This short movement was analyzed by Forte in The Structure of Atonal Music. He used the orchestral version of this movement.

Conclusions

When used in the analysis of music, the charts in the Index will direct the analyst's attention toward the investigation of certain specific subsets. These subjects will, in turn, be found in the music as important structural units, either as vertical ("chords" or "harmonies") or horizontal ("melodic") elements.

The analysis of the movement of the Three Pieces by this approach clearly points to the importance of several subsets which relate most of the sections of this composition to each other.

The charts also contain statistical information: they point out which sets have a greater or less frequency of occurrence as subsets.

Finally, the Index also provides composers with a convenient aid for the organization of sound structures in their own compositions. In this way, the Index is a useful tool for analysis as well as for composition.

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CHAPTER I

A BRIEF REVIEW OF THE THEORY OF SETS

Set Theory, when it is used for the analysis of musical pitch structures, contains only 12 elements, corresponding to the integers 0 through 11, with the integers assigned to the 12 notes of our equal-tempered scale, starting with c-natural which equals 0, c-sharp which equals 1, etc., to b-natural which equals 11. Any collection of notes, isolated and labeled for easy reference, is called a pitch-class set (abbreviated pcs). Although there may seem to be a limitless number of pc sets, in actuality there are 220. The set-theory approach does not recognize octave transpositions of pc sets. (C-natural, or for that matter, b-sharp or d-double-flat, is always assigned the integer 0, regardless of the octave in which it is located.)

Further, all pc sets are reduced to normal form, in which all of the pitches are arranged in ascending order (mod 12) in such a way that the entire structure is contained within the smallest possible interval. Under these conditions, there are only 6 distinct types of 2-pc sets; 12, 3-pc sets; 29, 4-pc sets; 38, 5-pc sets; 50, 6-pc sets; 38, 7-pc sets; 29, 8-pc sets; 12, 9-pc sets; and 6, 10-pc sets. (Since there is only one type each of 1-pc, 11-pc and 12-pc sets, they need no formal classification.)

Apart from the convenience of brevity in assigning integers to pitch classes, the main value of integer notation lies in the ease of finding transpositions and inversions of sets. For illustration, let us take a set from Webern's <u>Five Movements for String Quartet</u>, Op. 5, no. 5:

Example 1. Set A

3 9 8 7 2

normal form
A: [2,3,7,8,9]

Transpositions of pc sets are achieved through the addition of the same integer to each element of the set: e.g., to transpose the set A [2,3,7,8,9] up a minor 7th, one must add the integer 10 (the interval, in consecutive half-steps) to each element of the set. This addition is in modulo 12: for any value more than 11, it is necessary to subtract 12, or multiples of 12, in order to reduce the value to one between 0 and 11.

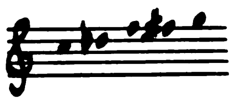
If one adds 10 to the elements of set A, the result is [12,13,17,18,19] which, by the subtraction of 12 from all the elements becomes [0,1,5,6,7], to be referred to as set B. The same transposition could have been accomplished by the subtraction of 2 from all elements of set A. However, transpositions are always noted as additive operations. For this reason, set B is a transposition of set A, with the value of transposition, or transposition operator, being 10. This can be expressed as B=T(A,10), which is read "set B is a transposition of set A at level 10."

Inversion of a pcs is accomplished by the subtraction of each element of a set from 12. Let us again use set A: [2,3,7,8,9], and subtract each element from 12. The result is set C: [10,9,5,4,3]. In order that we use consistent notation, these elements of set C are arranged in ascending order: [3,4,5,9,10]. The inversion of set A to its form as set C can be expressed as C=I(A,0), which is read "the set C is the inversion of set A transposed 0 times."

If a set is not a literal inversion (I=0) of another set, but is a transposed form of the inversion, one must go through two steps to find the transposition operator. The inversion is always done first, followed by the transposition. Let it be assumed that set D: [8,9,10,2,3] is an inverted and transposed form of set A: [2,3,7,8,9]. How can one find the transposition operator? First, he should determine the inversion, and then the transposition operator that will provide the given elements of set D. Set C, the inversion of set A, has already been found above. The integer 5, when added to the elements of set C, yields the desired result, or set D. Therefore, D=T(I(A),5), which is read "set D is the inversion of set A, transposed to level 5." The following example recapitulates the operations discussed above, with their graphic representations by notes on a staff:

Example 2.

set A: [2,3,7,8,9]  "original"

set B: [0,1,5,6,7]  "original" set transposed, t=10

set C: [3,4,5,9,10]  "original" set inverted, I=0

set D: [8,9,10,2,3]  "original" set inverted and transposed, I=5

None of these four sets, A through D, is in prime form. The prime form of a set is a specific configuration of the normal order, which is transformed in such a way that not only are all the elements of the set contained within the smallest possible interval, but the first element is always 0, with the other elements arranged with the smallest interval at the "lower" end (nearest to 0). In most cases, this would involve either a transposition, or an inversion and transposition of the normal order.

In the previous examples, the prime form of sets A through D is (0,1,2,6,7), and the label by which each of these sets is represented is 5-7, that is, the 7th set in the list of sets containing 5 elements, or having a cardinality of 5, in the set charts, to be introduced later. Set A is an inverted and transposed version of the prime form of 5-7: set A is actually 5-7 in the form I^9, that is inverted and transposed to level 9. (The notation I^9 automatically implies that a transposition has taken place: it is not necessary to notate IT^9, as the T would be redundant.)

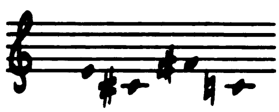
Throughout the course of this dissertation it will not be necessary, generally, to indicate the inversion or transposition of a prime form of a set: the label of the set, in this case, 5-7, will suffice. A set, as it appears in the music (in normal form) will be enclosed in brackets, as in set A: [2,3,7,8,9], while the prime form will be enclosed in parentheses, as in 5-7: (0,1,2,6,7).

Once the prime form of any pcs is found, the interval content is examined to determine the kind and number of interval classes (abbreviated ic) that are contained therein. Since the interval between any two pitches is found by taking the difference (mod 12) between them, only 6 ics are possible, from 0 to 6 (7=5, 8=4, 9=3, 10=2, 11=1, and 12=0). The interval vector is a simple representation of the interval content of a set, written as an ordered array; i.e., the first (left-most) number is always the number of occurrences of ic 1 (half-steps or major 7ths), the next number to the right is always the number of occurrences of ic 2 (whole-steps or minor 7ths), etc. If some interval class is not represented, the entry 0 is written in the appropriate position in the vector. Thus, each interval vector is written as a series of integers, six integers in all, each representing the number of each ic present in the set. Once again, let us use our example of set 5-7: (0,1,2,6,7). The interval vector for this set is written thus: [310132], meaning that in set 5-7, ic 1 has 3 entries, ic 2, 1 entry, etc. A complete chart of all the prime forms and interval vectors of the pc sets can be found in <u>The Structure of Atonal Music</u> by Allen Forte (hereafter to be abbreviated as Forte, <u>SAM</u>), Appendix 1, p. 179. A complete set of charts with this information, and more, is also included within the Index of this dissertation, which will be explained fully in the following chapters.

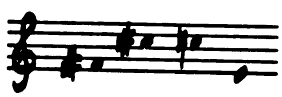
Two other aspects of interval content of sets are the basic interval succession (bis) and the basic interval pattern (bip). The basic interval succession is simply a numerical representation of intervals found in succession in an arrangement of pitch classes; e.g., the set 4-15, if arranged in a melody as the pitches [0,1,4,6] would have a bis of [3-5-6] in Example 3a, or [5-1-4] in Example 3b:

Example 3.

a



b



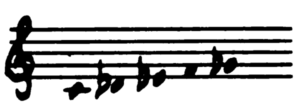
4-15:	[4 1 6 0]	[6 1 0 4]
bis:	[3-5-6]	[5-1-4]
bip:	356	145

The basic interval pattern (bip) is a further distillation of this concept. The bip is a "normalization of the interval succession, such that the numbers of the latter are grouped together in ascending numerical order without intervening spaces or hyphens" (Forte, SAM, p. 64). Taking the sets in Example 3 once again for illustration, we find that the bips for these sets are 356 and 145, respectively.

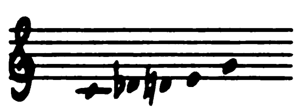
Some sets have a large number of bips possible, while others have relatively few. This ability is closely related to the appearance of the interval vector. Further, we shall see that this ability is also related to a set's frequency of occurrence as a common subset. This is illustrated in great detail in Chapter III.

There are pairs of sets, as illustrated below, that have identical interval vectors, yet different prime forms, which are given the name "Z-related pairs" (the letter Z has no special significance: it is simply an arbitrary convention). This coincidence of interval vectors will be discussed in greater detail during the following chapter. In the example below, the sets 5-12 and 5-36 in Forte, SAM are labeled 5-Z12 and 5-Z36. The interval vector for both sets is [222121].

Example 4.



5-12: (0,1,3,5,6)



5-36: (0,1,2,4,7)

For every set there is a complementary set, e.g., a set consisting of all the pitch classes not contained in the first. In a universe of 12 pitch-classes this means that for each of the 3-pc sets there is a corresponding 9-pc set, for each 4-pc set, a corresponding 8-pc set, etc. Each complementary pair of sets has the same ordinal number: 4-12 is the complement of 8-12, and vice versa; 5-7 is the complement of 7-7, and vice versa, etc. In the 6-pc sets, the complement is either an identical set (transposed, or inverted and transposed) or another set which is Z-related to the first.

Furthermore, there is a distinct correspondence between the interval vectors of a set and the interval vector of its complementary set, since the concept of the complement is a purely mathematical one, within a strictly limited universe of only 12 elements.

Compare, for example, the vectors of 5-13 and 7-13:

5-13 [221311]
7-13 [443532]

With the exception of the entry for ic 6, each entry in the vector of 7-13 is greater by exactly 2 than the corresponding entry in the vector of 5-13. This invariant correspondence is a general property of complementary pc sets (see Forte, SAM, p. 77, for a fuller discussion of these properties).

When one compares a set and its complement with another set and its complement, one speaks of complex (K,Kh) relations. The K- and Kh- relations are based on the condition of a set's being either a subset or superset of another set and/or its complement. As given in Forte, SAM, pp. 93-96, both relations have the same preliminary conditions:

1. $2<\#(S)<10$ & $2<\#(T)<10$

The cardinal numbers of both sets S and T are greater than 2 and less than 10,

and

2. $\#(S)\neq\#(T)$ & $\#(S)\neq\#(\overline{T})$

The cardinal number of set S does not equal the cardinal number of either set T or the complement of set T.

Given these preconditions, the definition of the relation K is:

$S/\overline{S}\ \varepsilon\ K(T,\overline{T})$ iff $S\supset\subset T \mid S\supset\subset\overline{T}$

Set S and its complement ($\overline{S}$) are members of the set-complex K about set T and its complement ($\overline{T}$) if and only if set S can contain or be contained in either set T or its complement ($\overline{T}$).

Under the same two preconditions, the definition of the relation Kh is:

$S/\overline{S} \in Kh(T,\overline{T})$ iff $S \supset\subset T$ & $S \supset\subset \overline{T}$

Set S and its complement ($\overline{S}$) are members of the set-complex Kh about set T and its complement ($\overline{T}$) if and only if set S can contain or can be contained in <u>both</u> set T <u>and</u> its complement ($\overline{T}$).

CHAPTER II

CERTAIN ANOMALIES IN THE FORTE CONCEPT OF SIMILARITY RELATIONS

Whereas the K and Kh relations compare sets of different cardinalities, Similarity Relations compare sets of like cardinality. As presented by Forte, Similarity Relations are based on whether or not the interval vectors of the sets involved are similar or dissimilar. Arbitrarily, similarity (or dissimilarity) has been determined by how many of the six interval vector entries are identical. According to this logic, the more entries that are identical, the more similar the sets are considered to be. If all six vector entries are alike (the Z-relation), the sets are considered the most similar; conversely, if none of the vector entries is alike, the sets are considered to be least similar. Forte (*SAM*, p. 49) gives four categories of similarity relations. By adding the Z-relation, and ranking the others in descending order of similarity, one can define similarity relations as follows:

Z = all six vector entries alike.
R_1 = four vector entries alike, with the remaining two entries being identical. only interchanged.
R_2 = four vector entries alike.
R_0 = no vector entries alike.
R_p = having maximum similarity with respect to pitch class. (This really means that the two sets of cardinality n have a common subset of cardinality n-1.)

Note: it is impossible for two sets of identical cardinality to have five vector entries in common, since in every case, the sum of all the vector entries must be constant for each cardinality.

As seen above, Forte ignores those pairs of sets that have one, two or three vector entries in common. These are simply dismissed as being undefined for analytical purposes.

The fifth similarity relation, R_p, is actually not dependent at all on the interval vector, but is based on common subsets, and can be considered either separately, or in conjunction with, the relations Z, R_1, R_2 and R_0.

Therefore, pairs of sets of like cardinality can have the following relationships (as defined by Forte) to each other, ranked from most similar to least similar (it is unclear where R_p belongs in the ranking):

1. ZR_p
2. R_1R_p
3. R_2R_p
4. R_0R_p
5. Z
6. R_1
7. R_2
8. R_p
9. R_0

Unfortunately for musical analysts, the nine types of relations listed above account for only a small portion of possible comparisons of pairs of sets, except for the R_p relations, which Forte thinks are too common to have value as distinct relations. He simply chooses to dismiss all the other possibilities as "undefinable" or "not significant." In addition, the relations based on only one factor (such as R_p) are given less weight than the relations containing two factors (such as R_1R_p), and this weight is then used as a basis for determining the relative importance of sets within the context of music. It will be seen that this approach is arbitrary, and can ignore important relationships between sets which fall into the rather large "undefined" category.

Even within the defined categories, important information can be overlooked under the above method.

Let us take an example from Forte (SAM, p. 55), which provides an illustration of the subset concept by comparing a pair of sets of like cardinality, 6-16 and 6-9 from Webern's Five Pieces for Orchestra, Op. 10, no. 4. Set 6-16 is the "melody" and 6-9 is the set constituting the first beat of the measure after the bar-line:

Example 5.

	normal form	prime form	interval vector
6-9:	[1,3,5,6,7,8]	(0,1,2,3,5,7)	[342231]
6-16:	[0,2,3,4,7,8]	(0,1,4,5,6,8)	[322431]

On a purely visual level, one sees that the two sets have the notes of the triplet figure (8,7,3) in common. This means that the pitch classes 3, 7 and 8 are included in the normal forms of both sets. In addition, from the chart in Forte (SAM, p. 177), we find that the sets 6-9 and 6-16 are in relation R_1, but not R_p. This means:

a. the interval vectors share 4 of the 6 entries.

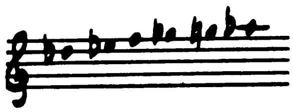
b. the sets do not have a 5-pc set in common, either in actual pitches, or some transposition or inversion.

Quoting Forte (SAM, p. 51), "Pc sets 6-16 and 6-9 are in R_1, but not in R_p, with the result that melody and 'accompaniment' are at once similar and dissimilar."

Perhaps the above example is a trivial case, but we shall use it to explore in greater detail what constitutes similarity. The two sets have no 5-pc set in common (they are not in R_p). If we look to the "next lower level," as it were, to investigate how many 4-pc sets they have in common, we find the following group of eight sets: 4-2, 4-4, 4-5, 4-14, 4-15, 4-16, 4-22 and 4-29: i.e., out of a possible 29 4-pc sets, fully 27.6% of them are common to both 6-16 and 6-9. This is illustrated graphically in Example 6.

Sets 6-9 and 6-16 are shown below in normal form with the pcs numbered; the eight 4-pc sets are shown with their prime forms in parentheses, as well as those corresponding pitches (beneath sets 6-9 and 6-16) which constitute the sub-sets within the two 6-pc sets. It is clear that the similarity (R_1) of the interval vectors of the pair of 6-pc sets is due, in great part, to the large group of sub-sets they have in common, which naturally contributes to the similar interval content shown in the vectors.

Example 6.



		6-9:	1	3	5	6	7	8	6-16:	0	2	3	4	7	8
4-2:	(0,1,2,4)			3	5	6	7			0	2	3	4		
4-4:	(0,1,2,5)			3		6	7	8			2	3	4	7	
4-5:	(0,1,2,6)		1		5	6	7				2	3	4		8
4-14:	(0,2,3,7)		1		5	6		8		0	2	3		7	
4-15:	(0,1,4,6)		1	3		6	7				2		4	7	8
4-16:	(0,1,5,7)		1	3			7	8		0	2			7	8
4-22:	(0,2,4,7)		1	3	5			8		0	2		4	7	
4-29:	(0,1,3,7)		1		5		7	8		0	2	3			8

Of the 4-pc subsets, 4-14 seems to be the most prominently positioned within the two 6-pc sets illustrated in Example 5: in set 6-16 (the melody), set 4-14 constitutes the four highest notes, shown isolated below (compare to Example 5):

Example 7.

 4-14: [0,2,3,7]

In set 6-9, 4-14 is the vertical structure beginning at the bar-line, shown below (again, see Example 5):

Example 8.

4-14: [1,5,6,8]

It will not be necessary to illustrate all of the other 4-pc subsets as musical entities, though there may be reason to do so in actual analysis if some of these other subsets are significant structures. However, it can be very helpful to have this information, rather than the relatively uninformative R_1, because it allows the analyst to discuss actual structural elements which appear in the music. These elements are, after all, what the composer works with, not interval vectors. (It will be explained in the next chapter why we will not deal specifically with subsets of cardinality n-3, n-4, etc., in our expanded subset concept.)

Given that a system of subsets can be substituted for the relations R_1, R_0, R_1R_p, etc., would this system be at least analogous, in theory, to these relations, i.e., would ZR_p (the highest ranked relation, with identical vectors plus at least one common subset of cardinality n-1) contain the highest number of subsets? Would the "lowly" R_0 (no identical vector entries, and no common n-1 subsets) contain the lowest number of subsets? To question further, should there actually be a ranking of similarity relations from "most similar" to "least similar," based on the interval vector? In the following examples, there are no real patterns in the number of subsets to warrant such a ranking. There is, however, a great deal of useful analytical information revealed, especially for pairs of sets that have "no defined relation." This information is contained in the charts in the Index of this dissertation.

Because this information is in chart form (to be read by cross-indexing one set with another, much like a multiplication table), the following examples will be presented with one set on the left, and the other set above. In the space where the two sets "cross," the sets of cardinality n-1 and n-2 which are held in common will be listed thus:

	6-9
6-16	
	4-2,4,5, 14,15, 16,22, 29

Between sets 6-9 and 6-16 (discussed in Examples 5 and 6), there are no 5-pc sets in common, but there are eight different 4-pc sets: 4-2, 4-4, 4-5, 4-14, 4-15, 4-16, 4-22 and 4-29.

In the Index, the cross-reference of the two 6-pc sets is found on page 6:8, i.e., the eighth page of the chart dealing with the 6-pc sets. The layout of the Index will be explained fully in the next chapter.

Let us go through the list of eight types of similarity relations for sets of cardinality n, ranked from "highest" to "lowest," giving examples of subsets of cardinality n-1 and n-2 held in common. There is no reason to assume that these rankings have a real meaning, because within each category of relation there is a wide range in the number of common subsets. The pairs of sets in the following examples are chosen at random, the only precondition being to show two pairs of the same cardinality, one yielding a relatively large number of subsets, the other yielding a minimal number of subsets. The range of difference in the number of subsets shows quite clearly that the categories of "defined" similarity relations listed on page 14 may be unimportant, at least so far as assigning weight to them for musical analysis if concerned.

First, consider two Z-related pairs:

1a.	6-19	1b.	6-23
6-44	5-21,22	6-45	
	4-7,8,17, 18,19, 20		4-12,13, 27

While Z-related pairs would seem to be all very similar, because their interval vectors are identical, pair a, 6-19/6-44, has two 5-pc subsets and six 4-pc subsets, while pair b, 6-23/6-45 has no 5-pc subsets and only three 4-pc subsets. Pair a is in R_p, while pair b is not.

Secondly, compare two pairs of sets in R_1R_2:

2a.	8	2b.	8-6
8-27	7-29	8-8	7-7
	6-11,12, 17,18, 24,25, 31,33, 46,47, 50		6-5,6,7, 18,38, 41,43

Pair a, 8-13/8-27, has one 7-pc subset and eleven 6-pc subsets, while pair b, 8-6/8-8, also has one 7-pc subset, but only seven 6-pc subsets.

Thirdly, consider two pairs of sets in R_2R_p:

3a.	7-11	3b.	7-18
7-27	6-14,24	7-32	6-31
	5-3,10, 11,17, 21,23, 26,27, 29,30, 37		5-18,21, 22,25, 26,27, 30,32

Pair a, 7-11/7-27, has two 6-pc and eleven 5-pc subsets, while pair b, 7-18/7-32, has one 6-pc and eight 5-pc subsets.

The fourth comparison is of two pairs of sets in R_0R_p:

4a.	5-30	4b.	5-1
5-35	4-22	5-16	4-3
	3-6,7,9, 11		3-2,3

Pair a, 5-30/5-35, has one 4-pc subset and four 3-pc subsets, while pair b, 5-1/5-16, has one 4-pc subset also, plus two 3-pc subsets.

Fifthly, we see two pairs of sets in R_1 relation:

5a.	5-13	5b.	5-1
5-14		5-35	
	3-1,3,4, 5,8		3-6

Pair a, 5-13/5-14, has five 3-pc subsets in common, while pair b, 5-1/5-35, has only one 3-pc set in common; of course, neither pair has any 4-pc sets in common, since they are not in R_p.

Next, compare two pairs of sets in R_2:

6a.	7-12	6b.	7-2
7-14		7-35	
	5-5,14, 29,36		5-23,24

Pair a, 7-12/7-14, has four 5-pc sets in common, while pair b, 7-2/7-35, has two 5-pc sets in common.

Seventhly, compare two pairs of sets in R_p:

7a.	4-4	7b.	4-13
4-27	3-7	4-28	3-10
	2-2,3,4, 5		2-3,6

Pair a, 4-4/4-27, has four 2-pc sets in common, while pair b, 4-13/4-28 has only two 2-pc sets in common. Both pairs have one 3-pc set in common (R_p).

In the final example of this group, two pairs of sets in R_0 are shown:

8a.	6-10	8b.	6-20
6-50		6-30	*
	4-17,18, 29		

Pair a, 6-10/6-50 has three 4-pc sets in common, while pair b, 6-20/6-30, has neither 5-pc nor 4-pc subsets in common. That is the significance of the asterisk.

If one compares the pairs of sets in 1b (page 21) and 8a (page 23), one will note that both involve sets of cardinality 6. In 1b, the sets are Z-related and thus share the same vector, yet contain exactly the same number of subsets as pair 8b, which have least similar vectors, designated by the relation R_0. This is an extreme case, showing that the vector-dependent relationship does not give a clear picture of the number of possible subsets held in common between pairs of sets of like cardinality. Yet, in analysis based on methods as presented in Forte, SAM, the relationship shown in 1b would be weighted as much more significant than the relationship shown in 8a, even though both examples yield the same result, namely the same number of subsets in common.

As mentioned earlier, the number of pairs of sets related to each other by the "defined" relations is a small percentage when compared to the total number of pairs of sets related to each other by "undefined" relations. The majority, in Forte's method, would be ignored.

Examples 9a-b and 10a-b illustrate just what type of information would be ignored. Examples 9a and 10a show "defined" relations, while 9b and 10b show "undefined" relations which reveal a greater number of common subsets than in the "defined" pairs:

"defined" (Z-related) pair		"undefined" pair	
9a.	5-18	9b.	5-31
5-38	4-18	5-38	4-18,27
	3-3,4,5, 8,10, 11		3-3,5,7, 8,10, 11

In the above examples, the "undefined" pair, 5-31/5-38, has two 4-pc subsets, while the "defined" pair, 5-18/5-38, has only one. Both pairs contain six 3-pc subsets.

"defined" (R_2R_p) pair		"undefined" pair	
10a.	6-33	10b.	6-41
6-47	5-35	6-47	5-14,29
	4-13,14, 16,22, 23,26, 27		4-4,6,13, 14,15, 16,23, 27

In the above examples, the "undefined" pair, 6-41/6-47, has more subsets, both in the 5-pc and 4-pc categories, than the "defined" pair. Similar pairs ("defined" vs. "undefined") could be cited in great numbers, since they are quite common. In analysis based on the interval vector method, the relations expressed in 9b and 10b would not be considered (they would not be listed in the charts).

The next chapter will introduce the charts found in the Index. It will describe the layout of these charts and give statistical information on the various subsets appearing on those charts. The charts show similarity relations; not those defined in the first chapter (based on interval vectors), but rather on the subsets held in common between pairs of sets. Examples 1a-b through 10a-b, discussed above, are, in essence, samples of the kind of information contained therein. None of the "defined" relations is labeled there.

CHAPTER III

A NEW WAY OF LOOKING AT SIMILARITY RELATIONS

An examination of similarities based on subsets reveals a wealth of information not obtainable by a simple comparison of interval vectors. On page 4:i begins a compilation of these similarity relations, starting with pairs of 4-pc sets, and ending with 8-pc sets. (Comparisons of sets of 1-pc, 2-pc, 3-pc, 9-pc, 10-pc, 11-pc and 12-pc are not included in the compilation, hereafter called the Index, for the small sets reveal too many supersets and the large sets show too many subsets to have any relevance.)

For each category of cardinality n, subsets of cardinality n-1 and n-2 are given. Because the information contained in the Index cannot be included on one page for any cardinality, it is organized in the following manner: each cardinality has a separate index (the 4-pc index is on page 4:i; the 5-pc index is on page 5:i, etc.). The first diagram of the index is a Pasqual triangle, with the numbers down the left-hand side and those listed diagonally from upper left to lower right being the ordinal numbers of the sets in question: e.g., on page 4:i, the Pasqual triangle for the chart of similarity relations among pairs of 4-pc sets contains the numbers 1 through 29 in the manner specified above, since there are 29 different 4-pc sets.

The Pasqual triangle is partitioned into sections, set apart by horizontal lines and vertical columns of asterisks. Within each section is an underlined number (such as 14), which is the page number on which that particular section will be found in the following pages. (For readers familiar with the standard manner of partitioning maps in an atlas, this is a similar technique. Imagine a map partitioned into sections, with each section appearing on the following pages "blown up" in larger scale. This is the way the Pasqual triangles are handled: each underlined page is a blow-up of that part of the chart.) If one were looking for the similarity relation between sets 4-12 and 4-24, one would look, therefore, on page 4:14, the page that contains the cross-references of sets 4-11 through 4-15 with sets 4-21 through 4-25. The similarity of set 4-12 to set 4-24 will be shown thus:

3-8

2-2,4,6

This entry represents the common subset 3-8 as well as subsets 2-2, 2-4 and 2-6. It is typical of the way the information of common subsets will be portrayed in the Index: for each pair of sets of cardinality n being compared, the subset(s) of cardinality n-1 will appear first, followed below by the subsets of cardinality n-2. If there are no subsets of cardinality n-1, the appropriate place in the chart will remain blank. In certain rare cases, there will be no subsets of either cardinality n-1 or n-2, in which case the entire entry will be marked by a single asterisk.

The reader may well wonder why these charts do not contain common subsets of cardinaltiy n-3 (or n-4, etc.). The reason for this is that most pairs of sets of cardinality n will contain most of the possible subsets of cardinality n-3 and lower, so that there would not be enough significant differentiation to be useful for analytical purposes.

Within the body of the charts, certain sets appear more often than others as subsets, certain sets appear rarely, and others appear within these extremes. Within the indexes described above and after the appearance of the Pasqual triangles, other information follows:

a. The complete listing of the sets of the cardinality in question, giving prime forms and interval vectors.

b. The complete listing of the subsets in question, giving prime forms, interval vectors and the number of occurrences of these subsets in the chart, followed by the percentage of occurrence of these subsets.

See page 4:ii. The list of 3-pc and 2-pc sets shows how many times that particular subset appears among pairs of 4-pc sets. In this case, it shows that set 3-1 appears 10 times, or 2.5% of the time, while set 3-2 appears 36 times, or 8.9% of the time, within the chart for 4-pc sets.

While the interval vector is not a completely valid criterion for establishing similarity relations, it does have a bearing on the percentage of occurrences of subsets within the charts of the Index.

The percentage of occurrence of subsets within the charts is closely related to the arrangement of entries within the interval vector. A set whose interval vector is of the type which has "equal or nearly equal distribution of entries" (Forte, SAM, p. 16) will have a high percentage of occurrence in the charts. Conversely, a set having "unique vector entries," or "maximum number of some ic" will have a low percentage of occurrence.

Furthermore, there is a general correspondence between subsets of high percentage of occurrence (as listed in the indexes) and the counts of the basic interval patterns (see Forte, SAM, p. 69). High percentage of occurrence corresponds quite closely to high bip counts, and low percentage of occurrence corresponds closely to low bip counts. This is illustrated in the examples below.

An analysis of this concept of "percentage of occurrence" follows. When subsets are of a cardinality of two less than their "parent group," they have the most varied values of percentages of occurrence. Consider the percentage of occurrence for 2-pc subsets of 4-pc sets, 3-pc subsets of 5-pc sets, etc.:

2-pc subsets of 4-pc sets	Percentage of occurrence among pairs of 4-pc sets
a. 2-4	56.9%
b. 2-1,2,3,5	51.7
c. 2-6	25.9

Set 2-4 occurs as a common subset of pairs of 4-pc sets more often (56.9%) than the other 2-pc sets. Set 2-6 occurs as a common subset only 25.9% of the time. The rest of the 2-pc sets appear 51.7% of the time.

3-pc subsets of 5-pc sets	Percentage of occurrence among pairs of 5-pc sets
a. 3-2,3,4,7,8 11	42.7%
b. 3-5	39.3
c. 3-6,10	19.3
d. 3-1,9	17.1
e. 3-12	4.0

The most common 3-pc subsets of pairs of 5-pc sets are the group of sets 3-2, 3-3, 3-4, 3-7, 3-8 and 3-11, occurring 42.7% of the time, while the lone subset of least occurrence (4.0%) is set 3-12, the augmented triad. The other 3-pc subsets occur with intermediate frequency, as shown.

4-pc subsets of 6-pc sets	Percentage of occurrence among pairs of 6-pc sets
a. 4-15,29	30.9%
b. 4-4,5,11,12, 13,14,16,18, 27	22.5
c. 4-2,22	18.9
d. 4-3,7,8,10, 17,19,20,26	
e. 4-6	8.6
f. 4-1,21,23,24	6.4
g. 4-9,25	3.7
h. 4-28	1.2

The example above gives information which can be compared directly with the chart listing bip counts of pc sets of cardinality 4 in Forte, SAM, p. 69; i.e., sets 4-15 and 4-29 occur with the highest frequency as common subsets of pairs of 6-pc sets. They also are the two sets which have the most numerous different basic interval patterns. At the other end of the scale, set 4-28, the diminished 7th chord, has the lowest percentage of occurrence, and it also has the fewest possible basic interval patterns of all the 4-pc sets.

5-pc subsets of 7-pc sets	Percentage of occurrence among pairs of 7-pc sets
a. 5-9,11,18,24,36,38	21.8%
b. 5-4,13,26,29,30	19.3
c. 5-3,6,10,16,20,21,25,27,32	17.1
d. 5-2,5,14,23,28	14.9
e. 5-19	11.1
f. 5-7,8,12,17,22,34,37	9.4
g. 5-15	7.8
h. 5-33	6.4
i. 5-1,35	5.1
j. 5-31	3.0

See Forte, SAM, p. 69 for comparison with bip counts of the 5-pc sets.

The next two charts are arranged in a similar way, and need no further comment:

6-pc subsets of 8-pc sets	Percentage of occurrence among pairs of 8-pc sets
a. 6-11,40	25.9%
b. 6-5,9,15,16,18,31	22.4
c. 6-3,10,12,17,24,25,39,41,43,46	19.2
d. 6-36,47	16.3
e. 6-2,14,19,21,22,27,33,34,44	13.5
f. 6-4,6 ,8,13,23,26,28,29,30,37,38,42,45,48,49,50	8.9
g. 6-1,32	5.2
h. 6-7	3.7
i. 6-20	1.5
j. 6-35	.7

7-pc subsets of 9-pc sets		Percentage of occurrence among pairs of 9-pc sets
a.	7-13,26,30,36	54.5%
b.	7-4,9,11,18, 24,29,38	42.4
c.	7-5,14,19,28, 31	31.8
d.	7-2,3,6,8,10, 12,16,17,20, 22,23,25,27, 32,34,37	22.7
e.	7-7,15	15.2
f.	7-1,21,35	9.1
g.	7-33	4.5

Note that the chart of 7-pc subsets of 9-pc sets is included here even though there is no formal segment of the Index dealing with 9-pc sets. The reason is simple: the "percentages of occurrence" are quite high. Fully 16 of the 38 possible 7-pc subsets (42%) occur in over 30% of all pairs of 9-pc sets.

CHAPTER IV

SOME THOUGHTS ON ANALYTICAL METHODS IN SET THEORY

Analysis which makes use of set-theory techniques can be deceptive. In the past, it had a tendency to provide a great deal of information of dubious value, since much of this information was derived from comparisons of elements (sets) whose only relationships to each other were generated by the system of analysis used (essentially, number theory) and not on structural, compositional relationships in the score. By the former methods, it has been the standard practice to isolate and label, first, every conceivable set which the analyst feels he should label. This "isolation" has been done for vertical as well as horizontal sets. Horizontal sets are further broken up by the process called imbrication, in which segments of melodic sets are labeled. Common sense must be used, of course, for one to determine what groupings of notes will be considered sets. In music outside of the "common practice" orbit, there is a substantial "margin of error," or difference of opinion, which can

govern how various analysts will isolate these sets. The number and the types of sets isolated and labeled can vary substantially from the work of one analyst to that of another. Because the essence of this type of analysis has been to compile charts of all the sets used in the composition (or movement, or section), then to find the K, Kh or K* relations among sets, and finally to count the K-relations in order to determine "nexus-sets," anyone who is skeptical of this type of work can ask, legitimately, "of what use is the composition itself, other than as a source of raw materials (set-labels) for the determination of certain mathematical results?" Truly, there has been much of this type of analysis recently in which a K-relation chart is the final result.

This is not to say that one must not identify the elements of music. Yet, just as one could not consider analysis to be the counting of triads, seventh-chords, etc., in common-practice music for the compilation of such information, one should not accept this type of compilation of information as analysis of non-tonal music.

For these reasons, the following analysis will not be based on compilations. It will not deal with K-relations as described above. The charts which have been prepared for the Index will be used as a guide to direct our attention towards pcs relationships existing within the music. "Weighted values" will not be assigned to similarity relations. It will be, rather, simply pointed out how the composer has used the elements in his composition.

The composition to be analyzed is the second movement of Igor Stravinsky's "Trois Pièces pour Quatuor à Cordes" (Three Pieces for String Quartet). This work has been chosen for two major reasons. The first is that Allen Forte, in his Structure of Atonal Music, has analyzed the orchestral version of this movement (Forte, SAM, pp. 130-139). The second reason is that the orchestral version has some essential differences from the original string quartet version which the reader may now compare.

This movement consists of short sections, or fragments, which are strung together in a way which lend themselves quite naturally to comparative analysis. In the text, "composite sets" will be mentioned. A composite set is the total pitch-content of each section, or, in a few instances, subsection.

Rather than a score completely covered with set-labels, a relatively "clean" score is provided, with the sections labeled (pp. 39-41). In this way, the reader will be able to follow the method used in the subsequent analysis: he will see how the information derived from the Index can lead to the discovery of information about the set-content of the composition. This will serve as a guide to the use of the Index. Even though the Index is a source of a great deal of potential information, many of the subsets listed therein will not necessarily be relevant to the composition, while others often will. This kind of situation was illustrated in Chapter II. If a subset found in the chart has limited or no structural significance to the sections being compared, it will be so stated. If, however, a subset listed in the Index does have structural significance, it will be illustrated (as on pp. 16-18).

It is hoped that the system of pc-set relationships will enlighten the reader's understanding of harmonic practice in music not based on the "common practice" period.

Figure 1.--Second movement of <u>Trois Pièces pour Quatuor à Cordes</u> by Igor Stravinsky.

Figure 1 (cont'd.).

Figure 1 (cont'd.).

CHAPTER V

IGOR STRAVINSKY: TROIS PIÈCES POUR QUATUOR À CORDES, SECOND MOVEMENT

This movement is made up of seven different sections. Some appear several times, while others appear only once. These sections have been labeled A through G (Figure 1, pp. 39-41). When the sections are given the same letter, it means that they are similar in content. A brief outline of the movement is given below:

A_1 Measures 1-12: Two sets alternating in such a way that the second of each pair is a kind of "resolution" of the first, followed by one melodic motive stated alone (mm. 4-5), and in conjunction with the alternating sets (mm. 9-10). The composite set is 6-6: [3,4,5,8,9,10].

B_1 Measures 13-14: A short melodic fragment (1st violin and cello). The composite set is 6-3: [10,11,0,1,3,4].

C_1 Measures 15-16: An abruptly accented motive, in low tessitura. The composite set is 4-6: [5,10,11,0].

B_2 Measures 17-19: An expanded version of B_1 (joined by the 2nd violin and viola) ending once again with the interval [1,10]. The composite set is 8-1: [9,10,11,0,1,2,3,4].

C_2 Measures 20-21: An abruptly accented motive as in C_1. Same composite set, 4-6, although the vertical structures within the composite set are changed slightly.

(Sections B_1+C_1 have a parallel construction to B_2+C_2).

D_1 Measure 22: Identical vertical sets in a high tessitura. The composite set is 6-17: [2,3,4,6,9,10]. This section contains the first appearance of pc [6].

C_3 Measure 23: Again, an abruptly accented motive, in low tessitura, this time piano. The same composite set as in C_1 and C_2, 4-6, but the vertical sets are not identical to those in the previous C sections.

D_2 Measure 24: The same composite set as in D_1, 6-17, eliding into the next section.

E This section is in three parts.

E_1 Measure 25: The composite set is 10-2.

E_2 Measures 26-28: The composite set is 9-1.

E_3 Measures 29-30: The composite set is 7-1.

The composite set of the combined E sections is 10-2, all pcs except 5 and 7. This section has greater pc density and greater rhythmic activity, as well as ostinati in E_2.

F_1 Measures 31-32: A slow, pianissimo melody over a pedal. The composite set is 8-12: [9,10, 0,1,2,3,4,6].

F_{1a} Measure 33: An arpeggio "cadence." The composite set is 8-23: [9,11,0,2,4,5,6,7]. The first appearance of pitch [7].

F_2 Measures 33-35: A kind of "Klangfarbenmelodie," serving as a final cadence for section F. The composite set is 10-2, though a different one from section E (i.e., all pcs but [10,0]).

G Measures 36-43: Two phrases of violin melody plus accompaniment. Again, great pc density.

G_1 The composite set is 11-1: all pcs but [10].

G_2 The composite set is 12-1: universal set.

F_{1a2} Measures 44-47: A link between the middle sections (E, F and G) and the rest of the movement, using a fragment of F_2, the arpeggio of F_{1a}, and the melodic motive from A_1. The composite set is 8-6: [4,5,6,7,9,10,11,0].

A_2 Measures 48-51: A return of the alternating sets in A_1. The composite set is, again, 6-6. If this section is combined with m. 46, in order to include the melodic motive first associated with section A (mm. 4-5), then the composite becomes 8-14: [3,4,5,7,8,9,10,0].

B_3 Measure 52: A slightly different version of this melodic motive. The composite set is 7-4: [9,10,11,0,1,3,4].

D_3 Measure 53: One abrupt vertical set. The composite set is identical to D_1 and D_2, 6-17.

B_4 Measures 54-55: Again, an unaccompanied melodic fragment; this time the composite set is 6-1: [10,11,0,1,2,3]. The four B sections have had different composite sets, but each statement ended with the unifying interval (10,1), set 2-3.

A_3 Measures 57-59: A final appearance of the alternating sets. The composite set is 6-6 again. The final set is left without its "resolution."

C_4 Measures 60-61: The final statement of the abruptly accented motive. The composite set is, again, 4-6.

Thus, this movement uses section A, B, C and D as the frame for the middle sections E, F and G. The middle section has the higher pc density. Also, it contains the sections which are not re-used, while sections A, B, C and D are used in various juxtapositions among themselves. The set-theory approach will first be used to explore juxtapositions of the outer sections.

A_1 and B_1

Sections A_1 and B_1 are composed of sets 6-6 and 6-3, respectively. Because these sets are of an identical cardinality, they lend themselves to investigation by Similarity Relation. From the Index, p. 6:2 (henceforth labeled I-6:2), the similarity relation of set 6-6 to 6-3 is found to be 5-6, 4-4, 4-5, 4-7, 4-8 and 4-15. This type of relation will be notated thus: SIM 6-3/6-6 = 5-6;4-4,5,7,8,15. This information leads us to investigate the 4-pc sets in sections A_1 and B_1. Section A_1 is composed of two 4-pc sets, which are both forms of the same set, 4-8:

Example 9.

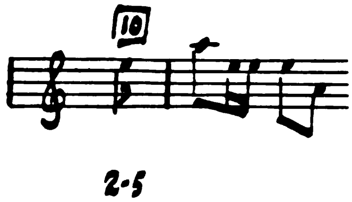
Set 4-8 is one of the sets found in the Index relating set 6-6 to 6-3. It appears prominently in set 6-6 (A_1). Upon examination of B_1, one does not find 4-8 as a prominent set. However, another set from the Index does appear in B_1. That set is 4-7:

Example 10.

In this way, two of the subsets from the relation of sections A_1 and B_1 appear prominently within the music. The other common subsets play no motivic role, and are simply hidden within the pitch-content of the two sections. They do contribute to the intervallic similarity of the two 6-pc sets.

Further investigation of the two 4-8 sets in A_1 reveals that they have the two pcs 4 and 9 in common. This intersection of the two sets is prominently illustrated by Stravinsky in that he uses this set, 2-5: [4,9] as the melodic motive in mm. 4-5 and 9-10:

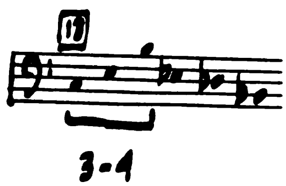
Example 11.



(The remaining set 4-23 acquires significance later.)

What is the relation of set 4-8 (A_1) to 4-7 (B_1)? From I-4:4, SIM 4-7/4-8 = 3-4; 2-1,4,5. Set 3-4 appears in both 4-8 and 4-7 (Example 12). Set 3-4, in section

Example 12.

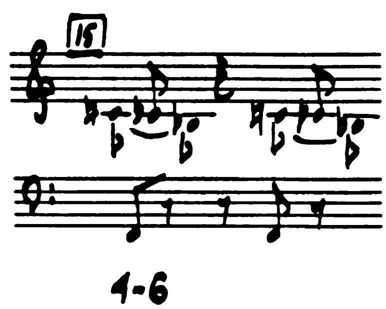


A_1, is the "resolution" of the top three voices over the pedal [9]. In section B_1, set 3-4 constitutes the first three notes of the melody. These three notes are further emphasized in the second appearance of section B (m. 18), in the viola part.

B_1 and C_1

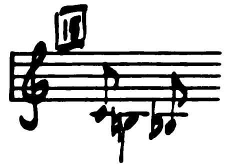
Sections B_1 (set 6-3) and C_1 (set 4-6) are not composed of sets of the same cardinality, so the similarity-relation method is not used directly here:

Example 13.



Instead, the two sections are seen to contain three pcs in common, 10, 11 and 0, which comprise the set 3-1. In section C_1, set 3-1 does occur once as a simultaneity at the end of m. 15:

Example 14.



3-1

However, otherwise there is no significant relation between the pitch-contents of B_1 and C_1. Set 4-6 (C_1) is not a subset of 6-3 (B_1) but it is a subset of 6-6 (A_1), though there is no appearance of 4-6 as a musical element in A_1.

Now 4-6 (C_1) is compared to 4-7 (B_1) for similarity relations:

Example 15.

4-7

B_1

4-6

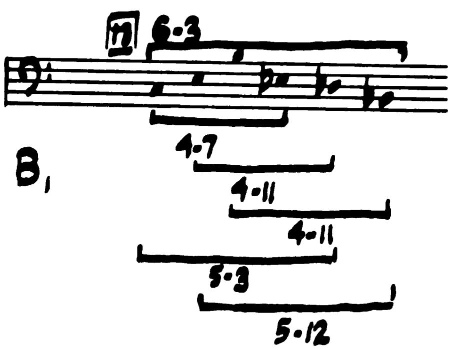
C_1

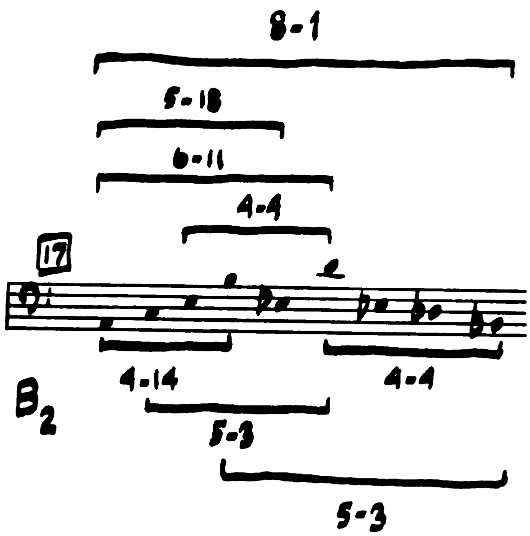
From I-4:4, SIM 4-6/4-7 = 2-1,5; i.e., the two sets have the interval of the minor 2nd and perfect 4th (or 5th) in common. There are no trichords in common, however, between 4-7 and 4-6. Thus, sections B_1 and C_1 are not closely related (with regard to intervallic content) which is why section C is used as an interruption between B_1 and B_2.

<u>C_1 and B_2</u>

Since section B_2 is an expansion of B_1, the relation of section C_1 to B_2 will be the same as its relation to B_1, as described in the paragraph above. Of greater interest, at this point, is the comparison of section B_2 to its cousin B_1:

Example 16.

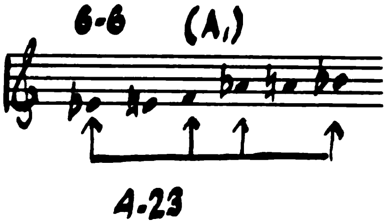


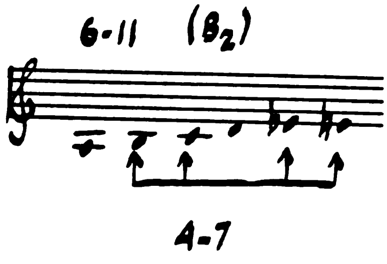


Shown above are some of the sets which the ear might grasp within the melodic fragments of sections B_1 and B_2. (Please note that all of the sets of section B_1 appear in B_2, and thus have not been labeled again.) In B_2, the addition of the opening (9) and the high-point (2) to the 4-7 of B_1 yields a new set 6-11, which can now be compared via similarity relation to both the composite set 6-6 of section A_1 and the composite set 6-3 of section B_1.

By comparing 6-6 to 6-11 (Example 17), one finds (from I-6:6): SIM 6-6/6-11 = 5-14; 4-4,6,7,15,16,23. Only one of these subsets is of structural significance in A_1, and that one might be difficult to find. That set is 4-23 (Example 17 and see Example 11), consisting of the pcs remaining from the composite set of A_1 after the removal of the pc set (4,9) which is used as a melodic motive, illustrated in Example 11.

Example 17.





Only one of the subsets mentioned (4-7; see again Example 17) occurs in section B_2 (and likewise in B_1). Thus the similarity of 6-6 or 6-11 is not strong in this specific case.

By comparing 6-3 (B_1) to 6-11 (B_2; see Example 16), one finds a stronger similarity than between 6-6 and 6-11 illustrated above. In I-6:3, SIM 6-3/6-11 = 5-3,10; 4-2,3,4,7,10,11,12,13,15. There are three forms of 5-3 within B_1 and B_2, as well as occurrences of 4-4, 4-7 and 4-11 (again, see Example 16), and none of these sets contain tritones (ic 6).

Within the composite set of B_2 (8-1) the sets 5-3 and 4-4 appear again. Thus, Stravinsky uses a segment of the "chromatic scale

Example 18.

in such a way as to emphasize the ics 3, 4, and 5. There are only two occurrences of ic 1, [2,3]:

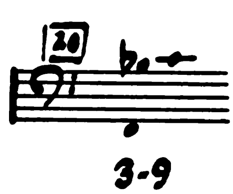
Example 19.

In this example, the bis is [4-5-4-6-3-3-5-4-1-1-2-3].

B_2 and C_2

Section C_2 is quite similar to C_1. An additional set (appearing in m. 20) formed from the same composite set 4-6 is set 3-9 (Example 20). This is in addition to the 3-1 seen already in Example 14. Comparisons of the composite set of C_2 (4-6) to the 4-pc sets in B_2 reveal that only sets 3-1 and 3-9 are common 3-pc subsets (Example 21).

Example 20.



Example 21. These comparisons can be found in I-4-2, I-4:4 and I-4:6.

	4-4	4-7	4-14
4-6	3-1		3-9
	2-1,2,5	2-1,5	2-1,2,5

The above example shows, in chart form, the common subsets between set 4-6 (C_2) on the left-hand side, and the 4-pc sets found in B_2 across the top. SIM 4-4/4-6 = 3-1; 2-1,2,5. SIM 4-6/4-7 = 2-1,5. SIM 4-6/4-14 = 3-9; 2-1,2,5. (References to the location of this information in the Index will be discontinued henceforth, as the reader is now expected to be familiar with the organization of the Index.) In addition, section C_2 (as well as C_1) shows itself to be related in a subtle way to the 4-pc sets within the melodic line of section B_2 (which are illustrated in Example 16). Set 3-1 relates 4-6 to 4-4, and set 3-9 relates 4-6 to 4-14, while all three comparisons have 2-1 and 2-5 in common, and two comparisons have 2-2 in common.

The study of similarity relations by the Forte method, illustrated on page 55, reveals that 4-6 is related to 4-4 and 4-14 by the Rp relation, while the relation of 4-6 to 4-7 is not mentioned. For smaller sets such as these, this lack of information is not critical, but in comparisons of larger sets, the paucity of information is often of crucial importance, and examples of this sort of lack will be illustrated later. Again in Example 22, 4-6 shows no pertinent relation to 4-7, while the subset method (Example 21) shows, at least, that the intervals 1 and 5 are common to both sets.

Example 22.

	4-4	4-7	4-14
4-6	R_p		R_p

C_2 and D_1 (and C_3 and D_2)

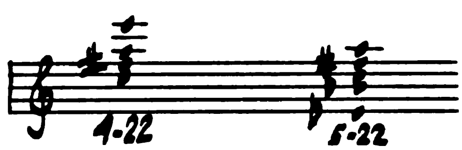
Sections C and D alternate. Since the pitch-content of these sections is identical, respectively, C_2C_3 may be compared to D_1D_2. The composite set of C_2C_3 is, once again, 4-6. The composite set of D_1D_2 is 6-17:

Example 23.

Set 4-6 is a subset of 6-17 but is not represented in any special way in the D sections. However, set 4-8, the important set from the A sections (see Example 9), is also found within set 6-17 (Example 23), as the outermost notes of the vertical structure. In this way, the chord of D sections contains strong ties to the A sections and a slightly weaker, but still important, structural relation to the C sections. There are, however, none of the 4-pc sets found in sections B_1 and B_2 (Example 16) contained in the pc set of D_1D_2. This is a further contrast between the melodic sets of the B sections and the vertical sets of the outer sections so far.

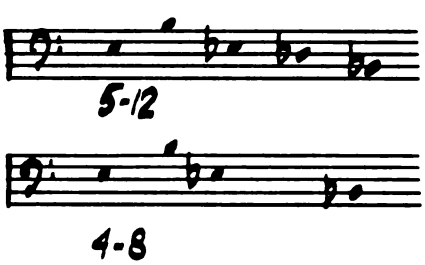
A further look at the chords of the D sections reveals that they are composed of an arco grouping (4-22) and a pizzicato grouping (5-22):

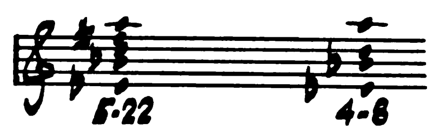
Example 24.



The set 4-22 appears later as a melodic fragment in section G_1. A similarity comparison between 4-22 and the other prominent 4-pc sets heretofore encountered reveals that it has the least similarity to 4-8 (A_1) and 4-7 (B_1), but is more closely related to 4-6 (because of the 3-9 in section C) and to 4-4, 4-11 and 4-14 (the B sections, Example 16). The set 5-22 (Example 25) compares most favorably to set 5-12 of section B. Both sets contain set 4-8, which is the most important of the 4-pc sets in this movement, and from which section A is constructed. In this case pcs 3 and 10 are common to both.

Example 25.



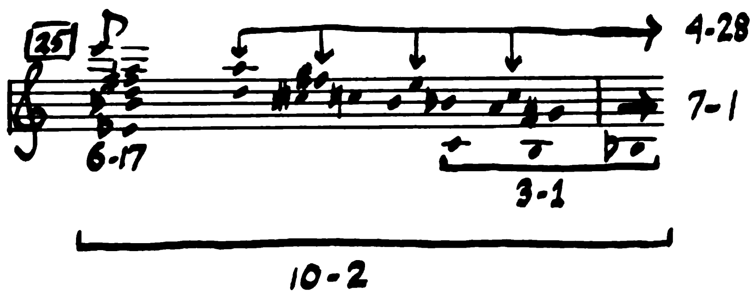


The Middle Sections

Since the middle sections of this movement (E, F and G) are composed of large composite sets (for the most part of more than 8 pcs), an analysis, from the

Index, based on common subsets will not be fruitful. There are too many common subsets. Therefore, sets will simply be labeled in the following examples, and it will be pointed out if they can be related to the sets of the outer sections.

Example 26.



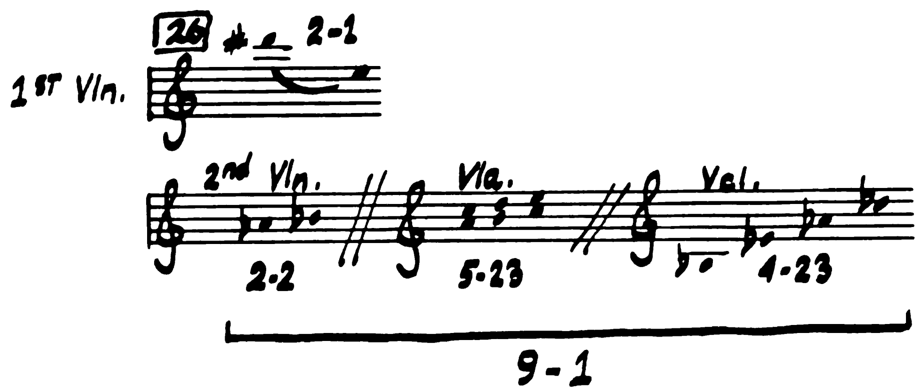
Section E_1 begins with set 6-17, which is in section D and has already been described. In addition, set 4-28 (the diminished 7th chord) makes its only appearance superimposed on set 7-1 (Example 26).

Section E_2 (Example 27) is a statement of three ostinati patterns in the lower three parts with the 1st violin presenting notes related in character to the 1st violin part of the D sections.

The set 4-23 in the cello part is the same as that found in section A_1 as the "remaining notes" when the melody was removed from the accompaniment (see

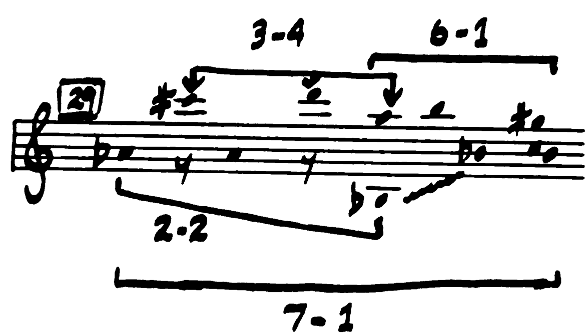
Example 11). Thus section E_2 is constructed of elements from the outer sections.

Example 27.



An abrupt change of character occurs in E_3.

Example 28.



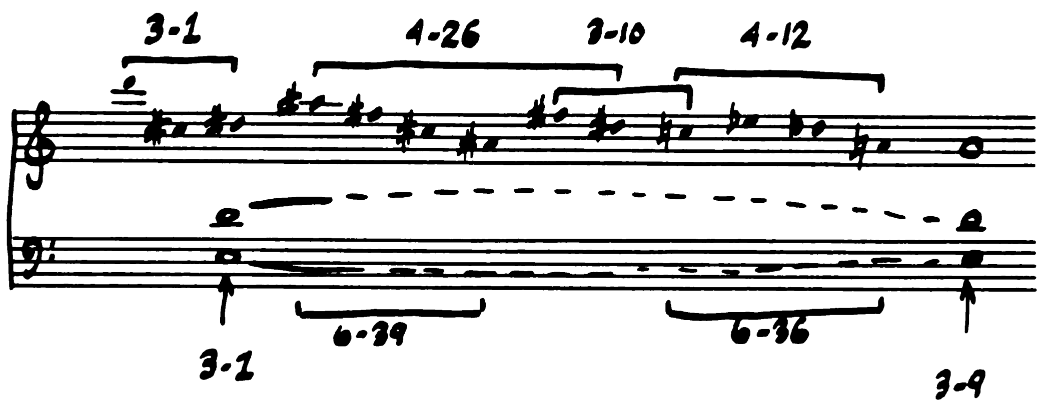
This is too short to be a section, and so has been labeled as a part of E. It emphasizes the "chromatic scale" as is clear from its composite set

7-1. However, the set 3-4 (see earlier in section B), appears here, though in a different form.

$\underline{F}_1$

Section F_1 is illustrated in the next example, showing its component sets. Set 3-1 was found in the C sections, as was set 3-9.

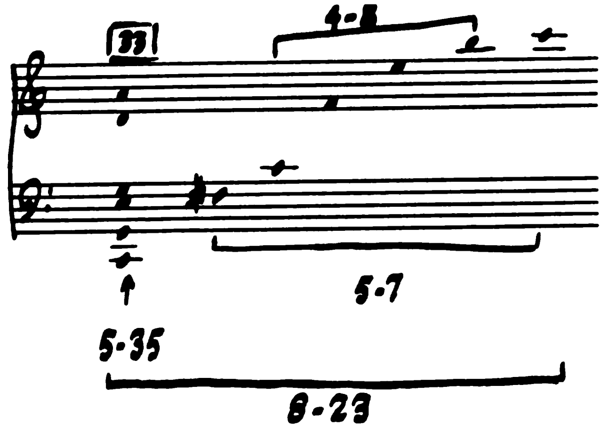
Example 29.



$\underline{F}_{1a}$

The downbeat and arpeggio has several prominent features. First is the set 5-35, a diatonic structure which is composed of the notes of the "pentatonic scale." This structure appears melodically in section G_2.

Example 30.

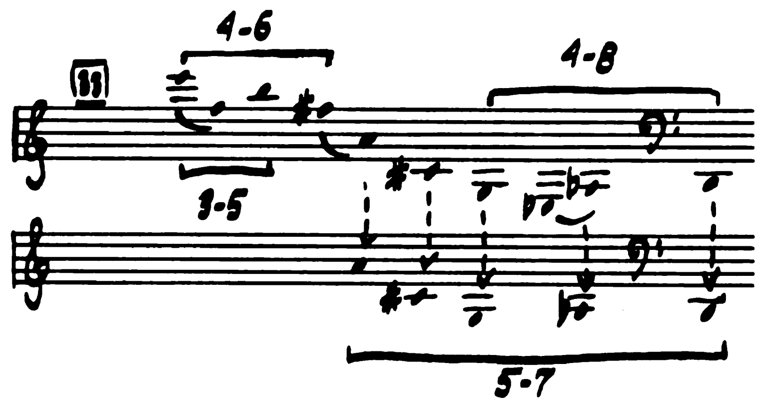


The arpeggio itself, including the downbeat, is set 8-23, the complement of the set 4-23 which was the cello ostinato in E_2. Also, set 4-8, already very prominent in A_1, is easily noticeable in the 1st violin part. With pc 6 added (in the viola part), the set 5-7 is formed. Set 5-7 had no significance up to this point, but appears here and in F_2. It is also a feature of the subset analysis at the end of the movement, though only as a set relating two other sections.

$\underline{F}_2$

Section F_2 contains two important sets from previous sections: 4-6 from C and 4-8 from A, as well as the set 5-7, mentioned above. The set 5-7 here is formed from the pitches which are <u>not</u> grace notes, and can be heard as an unornamented melody.

Example 31.



Both sets 4-6 and 4-8 appear as parts of the melodic line. The grace notes in this section are reminiscent of those in sections D and E.

Set 3-5, labeled above, appears twice in that form at the end of section G, and in another form as a prominent harmony in section G.

G_1, G_2 and F_{1a2}

Sections G_1, G_2 and the following section, F_{1a2}, are labeled in Figure 2, page 63. See especially the vertical sets shown under each beat of the G sections.

Sections G_1 and G_2 are two parallel phrases of a violin tune, accompanied by the other instruments in a chordal accompaniment. In Figure 2, the accompanimental figure, plus the notes of the melody, are labeled below the score, giving the type of set occurring within each beat. The types of harmonies which occur are mostly

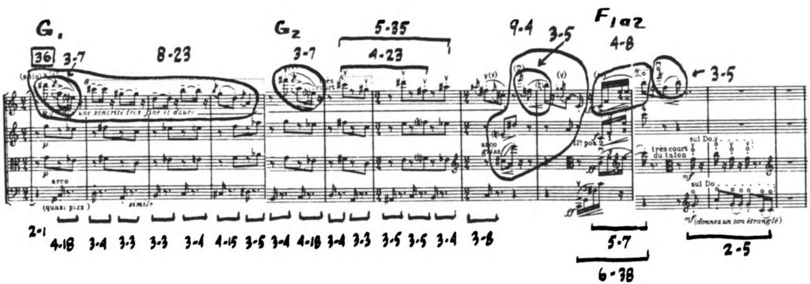


Figure 2.--The G sections and F_{1a2}.

3-pc sets: 3-3, 3-4 and 3-5 (each of which contains one ic 1). The two 4-pc sets, 4-15 and 4-18, are related in that both have the common subset 3-11.

A look at the sets of the melody in the G sections show a close relation to sets seen already in the movement. The set 8-23, starting in m. 36, and its complement 4-23, beginning in m. 41, have been seen already in Examples 29 and 26, respectively. Further, set 5-35 (the notes of the pentatonic scale) appeared as a vertical set in section F_{1a}. In the G section, set 5-35 appears in a melodic form (mm. 41-42).

Set 3-7, at the head of both phrases, has as yet not had a prominent role, but, as its use in the G sections provokes some curiosity as to other appearances, a further search reveals that this structure, with the prime form (0,2,5), does indeed appear in section B_1, B_2 and B_3 as the final three notes of each section (Example 32).

Example 32.

Section G_2 ends with set 9-4, the complement of the various forms of set 3-4 in the accompaniment to the G sections (again, see Figure 2). Set 3-5 (in m. 43) also appears in the accompaniment to the G sections as well as in section F_2 (Example 31 and m. 33 in the score).

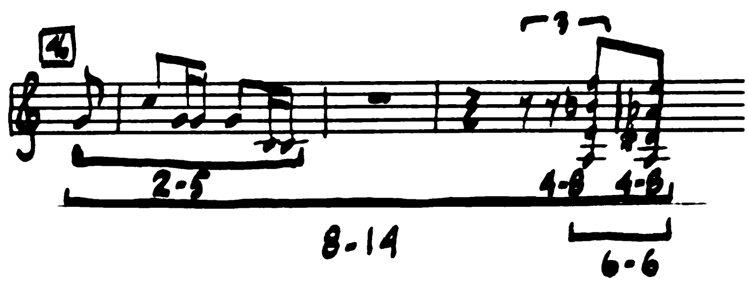
The importance of set 3-5, used as a melodic motive before and after section G proper (m. 33, and again in mm. 44 and 45) is one of a "punctuation point" just after the cadential arpeggio described in Example 30. In m. 33, the arpeggio was followed by set 3-5 as the first part of a short "Klangfarbenmelodie" which led to section G. In m. 45, set 3-5 is used once again as the beginning of another transitional passage.

Example 33.

This time, set 3-5 is followed immediately by set 2-5 which first appeared motivically in section A_1. Now, after a measure of silence, section A does indeed return.

The movement is brought to a conclusion with a return to the materials of the beginning, namely sections A, B, C and D, in slightly different juxtapositions. As stated above, the cadential arpeggio in m. 44 leads directly into the set 2-5, which had been an integral element of A1. In the return, this motive (m. 46) appears alone, but is followed two measures later by 6-6 of section A_2. Unlike its appearance in A_1, set 2-5 is not made up of pitches which are included in A_2 (Example 34) so the composite set of A_2 plus the "melody" is now 8-14: [3,4,5,7,8,9,10,0].

Example 34.

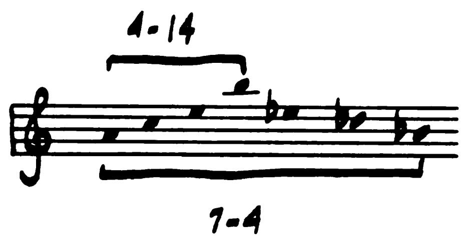


<u>A_2 and B_3</u>

The set 6-6 of A_2 is identical in pitch content to set 6-6 of A_1, and is also composed of the same two forms of set 4-8.

Section B_3 follows A_2 just as B_1 followed A_1. However, B_3 has one additional note at the beginning (as compared to B_1) making the composite set 7-4: [9,10,11,0,1,3,4].

Example 35.

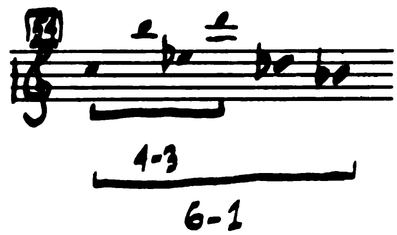


The relationship of A_2 to B_3 is the same as that of A_1 to B_1. As seen above, the addition of one note to B_3 forms a new 4-pc set, 4-14. Pitch class set 4-14 is the complement of pcs 8-14, which was illustrated in Example 34; this complementary relationship could possibly signify an additional relationship between A_2 and B_3. Further, a comparison of 4-14 (B_3) to 4-8 (A_2) reveals once again that set 3-4 is common to both (see Example 16 for a similar appearance of 4-14 in section B_2).

D_3 and B_4

Sections B_3 and B_4 are interrupted by the set 6-17; this interruption constitutes section D_3 (Example 36).

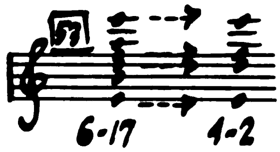
Example 36.

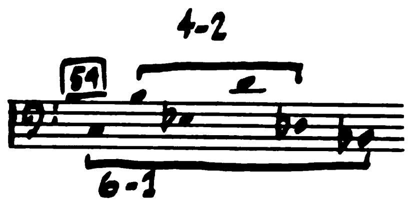


Section B_4 is of the same character and texture as the other B sections, but as a final statement of this idea it is slightly more intense. The composite set is 6-1. The "chromatic scale" nature of this set is less softened than the "chromatic scale" of B_2 (see again Examples 18 and 19) by the statement of two consecutive ics 1. Also, B_4 does not end with set 3-7 (Example 32) as did the other B sections.

A check for similarity relations between the B and D sections shows that set 6-17 of D_3 and set 6-1 of B_4 have only one relative subset between them listed in the Index, and that is 4-2, but this set has no structural significance.

Example 37.

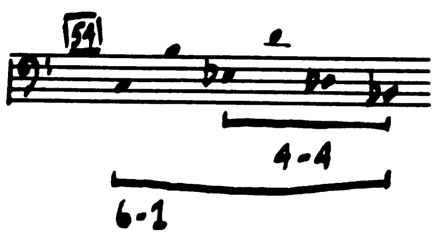


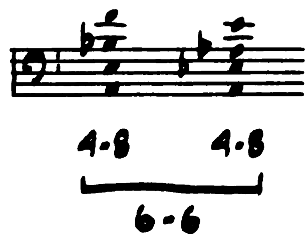


<u>B_4 and A_3</u>

Sets 6-1 (B_4) and 6-6 (A_3) have the subsets 4-4 and 4-7 in common.

Example 38.





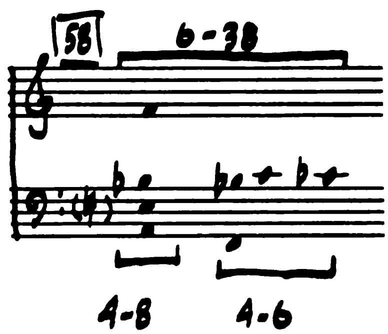
Whereas set 4-4 does appear in B_4 as part of the melody, neither 4-4 nor 4-7 appears as a distinctive, musically recognizable element in A_3. The A and B sections show no direct link.

$\underline{A_3 \text{ and } C_4}$

The relation of these two sections is structurally interesting in that this is the only time that an A section is juxtaposed with a C section. This happens at the end of the movement, when, for the first and only time, the alternating forms of set 4-8 stop in an unresolved way: the final appearance of 4-8 in A_3 does not have a "resolution" (a 4-8: [3,4,8,9] does not follow the last 4-8: [4,5,9,10]).

Further, the last appearance of 4-8 contains two of the pitches found in the following section, C_4 (Example 39).

Example 39.



By combining the last 4-8 of A_3 with C_4, one can form another composite set, 6-38 (see already in section F_{1a2}, Figure 2), which can be compared with 6-6 of A_3. SIM 6-6/6-38 = 5-7; 4-5,6,8,9,16. We know that 4-6 and 4-8 must be present, of course, since they are the sets that sections C_4 and A_3 are made of; the other 4-pc sets have had no significance. We have seen set 5-7 in sections F_{1a} and F_2 (Examples 30 and 31). Thus the "mysterious" appearance of set 5-7 in the middle section of the movement is evoked once more in the only juxtaposition of section A_3 and C_4, but it is not "explained."

Figure 3.--Second movement of Trois Pièces pour Quatuor à Cordes by Igor Stravinsky with all pertinent sets labeled.

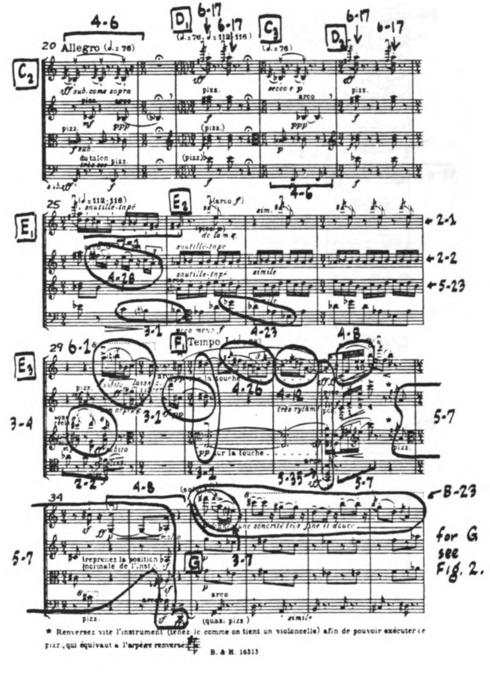


Figure 3 (cont'd.).

Figure 3 (cont'd.).

CHAPTER VI

CONCLUSIONS

With the preparation of charts from which the analyst will be able to determine easily which subsets are common to any two sets of like cardinality, the author feels that he has made a small, yet valuable, contribution. The work represented by the 169 pages in the charts of the Index is, however, only a start in this direction. These charts only make comparisons between sets of the same cardinality (for instance one 6-pc set compared to another 6-pc set).

Further work in this area could explore the subsets common to pc-sets of <u>unlike</u> cardinality (for example, what 4-, 5- or 6-pc sets do sets 8-14 and 7-8 have in common?).

Another purely technical aspect not answered in the Index is: "When two sets of like cardinality have a common subset, and both sets are in normal form, is (are) the common subset(s) also in normal form, or is (are) the subset(s) inverted in relation to the supersets? In addition, do the subsets appear in forms which are both inverted and not inverted?" Further, the charts only state that the subset exists at least once; they do not tell the reader whether the subset appears more than once within either of the supersets, or both.

The Index of this dissertation was prepared without the aid of computers. It was painstakingly put together using very helpful charts of subsets of prime forms which were published in 1974 in the Appendix of a dissertation entitled The Invariance Matrix by Bo Harry Alphonce.

The charts in the Index are by no means musical analysis as such: they are meant as an aid to analysis of pitch-content in musical compositions which use a system of "harmony" not easily dealt with by traditional means. By using these charts to help search for relationships among sets of a given composition, the analyst will be directed towards the investigation of certain specific subsets, eliminating most of the drudgery of the search. In addition, the complete list of subsets and their prime forms, interval vectors and "percentage of occurrence" in the charts provide a handy reference, giving further insight into the make-up of the sometimes very complex 20th century sonorities.

There is, of course, no guarantee that the insight thus gained will reveal the composer's thought processes completely, but it could reveal something of the composer's subconscious ability to produce unity and contrast in a musical composition, through pitch organization.

In conclusion, the charts in the Index, bound separately, may now be used by all who wish to aid their own analysis. The chapter on the second movement of the _Trois Pièces pour Quatuor à Cordes_ by Stravinsky was included as an example of how the information contained in the Index can be used in actual practice. It does by no means claim to answer all questions about the structure of the movement, but it does reveal the pervasive use of several key sets in establishing a distinctive sound to this movement. The relationships found within this movement represent but a very small part of the total body of information in the Index, which is indeed extensive, including virtually all combinations of pitches possible within our equal-tempered 12-tone scale.

Finally, the Index has a further use not mentioned hitherto, namely as an aid for composers. This is a convenient way for any composer of music to help organize for himself a sound universe in which he knows quite easily what types of sets have certain specific relationships to each other.

BIBLIOGRAPHY

BIBLIOGRAPHY

Alphonce, Bo Harry. "The Invariance Matrix" Ph.D. dissertation, Yale University, 1974. This work contains a computer print-out of all supersets and subsets for every pcs.

Forte, Allen. The Structure of Atonal Music. New Haven: Yale University Press, 1973.

Stravinsky, Igor. Trois Pièces pour Quatuor à Cordes. Paris: Edition Russe le Musique (Russischer Musikverlag) for all countries. Printed by arrangement by Boosey & Hawkes, Inc., New York.

A NEW APPROACH TO SIMILARITY RELATIONS IN SET-THEORY ANALYSIS

Volume II

By

Alexis Turkalo

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

DOCTOR OF PHILOSOPHY

Department of Music

1980

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Index for Charts of Similarity
Relations Among Pairs of 4-pc Sets

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12       *      * 12
13       *      *  13
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** Underlined numbers (e.g., 14) indicate page numbers whereon a particular quadrant of the chart will be found. In the case of 14, this shows that on (4:14) is located that part of the chart where sets 4-11 through 4-15 are cross-indexed with sets 4-21 through 4-25.

Pcs	Prime Form	Interval vector
4-1	0,1,2,3	321000
4-2	0,1,2,4	221100
4-3	0,1,3,4	212100
4-4	0,1,2,5	211110
4-5	0,1,2,6	210111
4-6	0,1,2,7	210021
4-7	0,1,4,5	201210
4-8	0,1,5,6	200121
4-9	0,1,6,7	200022
4-10	0,2,3,5	122010
4-11	0,1,3,5	121110
4-12	0,2,3,6	112101
4-13	0,1,3,6	112011
4-14	0,2,3,7	111120

(continued)

Pcs	Prime Form	Interval vector (continued)
4-15	0,1,4,6	111111
4-16	0,1,5,7	110121
4-17	0,3,4,7	102210
4-18	0,1,4,7	102111
4-19	0,1,4,8	101310
4-20	0,1,5,8	101220
4-21	0,2,4,6	030201
4-22	0,2,4,7	021120
4-23	0,2,5,7	021030
4-24	0,2,4,8	020301
4-25	0,2,6,8	020202
4-26	0,3,5,8	012120
4-27	0,2,5,8	012111
4-28	0,3,6,9	004002
4-29	0,1,3,7	111111

Subsets of 4-pc Sets, cardinality 3- and 2-

Pcs	Prime Form	Interval vector	Number of occurrences/ Percentage of occurrences among pairs of 4-pc Sets
3-1	0,1,2	210000	10 / 2.5%
3-2	0,1,3	111000	36 / 8.9
3-3	0,1,4	101100	36 / 8.9
3-4	0,1,5	100110	36 / 8.9
3-5	0,1,6	100011	36 / 8.9
3-6	0,2,4	020100	10 / 2.5
3-7	0,2,5	011010	36 / 8.9
3-8	0,2,6	010101	36 / 8.9
3-9	0,2,7	010020	10 / 2.5
3-10	0,3,6	002001	10 / 2.5
3-11	0,3,7	001110	36 / 8.9
3-12	0,4,8	000300	1 / .2
2-1	0,1	100000	210 / 51.7%
2-2	0,2	010000	210 / 51.7
2-3	0,3	001000	210 / 51.7
2-4	0,4	000100	231 / 56.9
2-5	0,5	000010	210 / 51.7
2-6	0,6	000001	105 / 25.9

(4:1)

	4-1	4-2	4-3	4-4	4-5
4-1					
4-2	3-1,2 2-1,2,3				
4-3	3-2 2-1,2,3	3-2,3 2-1,2,3,4			
4-4	3-1 2-1,2,3	3-1,3 2-1,2,3,4	3-3 2-1,2,3,4		
4-5	3-1 2-1,2	3-1 2-1,2,4	2-1,2,4	3-1,4 2-1,2,4,5	

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	4-1	4-2	4-3	4-4	4-5
4-6	3-1 2-1,2	3-1 2-1,2	2-1,2	3-1 2-1,2,5	3-1,5 2-1,2,5,6
4-7	2-1,3	3-3 2-1,3,4	3-3 2-1,2,4	3-3,4 2-1,2,4,5	3-4 2-1,4,5
4-8	2-1	2-1,4	2-1,4	3-4 2-1,4,5	3-4,5 2-1,4,5,6
4-9	2-1	2-1	2-1	2-1,5	3-5 2-1,5,6
4-10	3-2 2-1,2,3	3-2 2-1,2,3	3-2 2-1,2,3	3-7 2-1,2,3,5	2-1,2,5

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	4-1	4-2	4-3	4-4	4-5
4-11	3-2 2-1,2,3	3-2,6 2-1,2,3,4	3-2 2-1,2,3,4	3-4,7 2-1,2,3,4,5	3-4 2-1,2,4,5
4-12	3-2 2-1,2,3	3-2,3 2-1,2,3,4	3-2,3 2-1,2,3,4	3-3 2-1,2,3,4	3-8 2-1,2,4,6
4-13	3-2 2-1,2,3	3-2 2-1,2,3	3-2 2-1,2,3	3-7 2-1,2,3,5	3-5 2-1,2,5,6
4-14	3-2 2-1,2,3	3-2 2-1,2,3,4	3-2 2-1,2,3,4	3-4 2-1,2,3,4,5	3-4 2-1,2,4,5
4-15	2-1,2,3	3-3 2-1,2,3,4	3-3 2-1,2,3,4	3-3,7 2-1,2,3,4,5	3-5,8 2-1,2,4,5,6

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	4-6	4-7	4-8	4-9	4-10
4-6					
4-7	2-1,5				
4-8	3-5 2-1,5,6	3-4 2-1,4,5			
4-9	3-5 2-1,5,6	2-1,5	3-5 2-1,5,6		
4-10	2-1,2,5	2-1,3,5	2-1,5	2-1,5	

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	4-1	4-2	4-3	4-4	4-5
4-16				3-4	3-4,5,8
	2-1,2	2-1,2,4	2-1,2,4	2-1,2,4,5	2-1,2,4,5,6
4-17		3-3	3-3	3-3	
	2-1,3	2-1,3,4	2-1,3,4	2-1,3,4,5	2-1,4,5
4-18		3-3	3-3	3-3	3-5
	2-1,3	2-1,3,4	2-1,3,4	2-1,3,4,5	2-1,4,5,6
4-19		3-3	3-3	3-3,4	3-4
	2-1,3	2-1,3,4	2-1,3,4	2-1,3,4,5	2-1,4,5
4-20				3-4	3-4
	2-1,3	2-1,3,4	2-1,3,4	2-1,3,4,5	2-1,4,5

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	4-6	4-7	4-8	4-9	4-10
4-11	2-1,2,5	3-4 2-1,3,4,5	3-4 2-1,4,5	2-1,5	3-2,7 2-1,2,3,5
4-12	2-1,2,6	3-3 2-1,3,4	2-1,4,6	2-1,6	3-2 2-1,2,3
4-13	3-5 2-1,2,5,6	2-1,3,5	3-5 2-1,5,6	3-5 2-1,5,6	3-2,7 2-1,2,3,5
4-14	3-9 2-1,2,5	3-4 2-1,3,4,5	3-4 2-1,4,5	2-1,5	3-2 2-1,2,3,5
4-15	3-5 2-1,2,5,6	3-3 2-1,3,4,5	3-5 2-1,4,5,6	3-5 2-1,5,6	3-7 2-1,2,3,5

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	4-1	4-2	4-3	4-4	4-5
4-21		3-6			3-8
	2-2	2-2,4	2-2,4	2-2,4	2-2,4,6
4-22		3-6		3-7	
	2-2,3	2-2,3,4	2-2,3,4	2-2,3,4,5	2-2,4,5
4-23				3-7	
	2-2,3	2-2,3	2-2,3	2-2,3,5	2-2,5
4-24		3-6			3-8
	2-2	2-2,4	2-2,4	2-2,4	2-2,4,6
4-25					3-8
	2-2	2-2,4	2-2,4	2-2,4	2-2,4,6

(4,8)

	4-6	4-7	4-8	4-9	4-10
4-16	3-5,9 2-1,2,5,6	3-4 2-1,4,5	3-4,5 2-1,4,5,6	3-5 2-1,5,6	2-1,2,5
4-17	2-1,5	3-3 2-1,3,4,5	2-1,4,5	2-1,5	2-1,3,5
4-18	3-5 2-1,5,6	3-3 2-1,3,4,5	3-5 2-1,4,5,6	3-5 2-1,5,6	2-1,3,5
4-19	2-1,5	3-3,4 2-1,3,4,5	3-4 2-1,4,5	2-1,5	2-1,3,5
4-20	2-1,5	3-4 2-1,3,4,5	3-4 2-1,4,5	2-1,5	2-1,3,5

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	4-11	4-12	4-13	4-14	4-15
4-11					
4-12	3-2 2-1,2,3,4				
4-13	3-2,7 2-1,2,3,5	3-2,10 2-1,2,3,6			
4-14	3-2,4 2-1,2,3,4,5	3-2 2-1,2,3,4	3-2 2-1,2,3,5		
4-15	3-7 2-1,2,3,4,5	3-3,8 2-1,2,3,4,6	3-5,7 2-1,2,3,5,6	2-1,2,3,4,5	

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	4-1	4-2	4-3	4-4	4-5
4-26				3-7	
	2-2,3	2-2,3,4	2-2,3,4	2-2,3,4, 5	2-2,4,5
4-27				3-7	3-8
	2-2,3	2-2,3,4	2-2,3,4	2-2,3,4, 5	2-2,4,5, 6
4-28					
	2-3	2-3	2-3	2-3	2-6
4-29	3-2	3-2	3-2		3-5,8
	2-1,2,3	2-1,2,3, 4	2-1,2,3, 4	2-1,2,3, 4,5	2-1,2,4, 5,6

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	4-6	4-7	4-8	4-9	4-10
4-21	2-2,6	2-4	2-4,6	2-6	2-2
4-22	3-9 2-2,5	2-3,4,5	2-4,5	2-5	3-7 2-2,3,5
4-23	3-9 2-2,5	2-3,5	2-5	2-5	3-7 2-2,3,5
4-24	2-2,6	2-4	2-4,6	2-6	2-2
4-25	2-2,6	2-4	2-4,6	2-6	2-2

	4-11	4-12	4-13	4-14	4-15
4-16	3-4	3-8	3-5	3-4,9	3-5,8
	2-1,2,4, 5	2-1,2,4, 6	2-1,2,5, 6	2-1,2,4, 5	2-1,2,4, 5,6
4-17		3-3		3-11	3-3
	2-1,3,4, 5	2-1,3,4	2-1,3,5	2-1,3,4 5	2-1,3,4 5
4-18		3-3,10	3-5,10	3-11	3-3,5
	2-1,3,4, 5	2-1,3,4, 6	2-1,3,5, 6	2-1,3,4, 5	2-1,3,4, 5,6
4-19	3-4	3-3		3-4,11	3-3
	2-1,3,4, 5	2-1,3,4	2-1,3,5	2-1,3,4, 5	2-1,3,4, 5
4-20	3-4			3-4,11	
	2-1,3,4, 5	2-1,3,4	2-1,3,5	2-1,3,4, 5	2-1,3,4, 5

	4-6	4-7	4-8	4-9	4-10
4-26	2-2,5	2-3,4,5	2-4,5	2-5	3-7 2-2,3,5
4-27	2-2,5,6	2-3,4,5	2-4,5,6	2-5,6	3-7 2-2,3,5
4-28	2-6	2-3	2-6	2-6	2-3
4-29	3-5 2-1,2,5,6	2-1,3,4,5	3-5 2-1,4,5,6	3-5 2-1,5,6	3-2 2-1,2,3,5

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	4-11	4-12	4-13	4-14	4-15
4-21	3-6 2-2,4	3-8 2-2,4,6	2-2,6	2-2,4	3-8 2-2,4,6
4-22	3-6,7 2-2,3,4,5	2-2,3,4	3-7 2-2,3,5	3-9,11 2-2,3,4,5	3-7 2-2,3,4,5
4-23	3-7 2-2,3,5	2-2,3	3-7 2-2,3,5	3-9 2-2,3,5	3-7 2-2,3,5
4-24	3-6 2-2,4	3-8 2-2,4,6	2-2,6	2-2,4	3-8 2-2,4,6
4-25	2-2,4	3-8 2-2,4,6	2-2,6	2-2,4	3-8 2-2,4,6

(4:15)

	4-16	4-17	4-18	4-19	4-20
4-16					
4-17	2-1,4,5				
4-18	3-5 2-1,4,5 6	3-3,11 2-1,3,4, 5			
4-19	3-4 2-1,4,5	3-3,11 2-1,3,4 5	3-3,11 2-1,3,4, 5		
4-20	3-4 2-1,4,5	3-11 2-1,3,4, 5	3-11 2-1,3,4, 5	3-4,11 2-1,3,4, 5	

	4-11	4-12	4-13	4-14	4-15
4-26	3-7		3-7	3-11	3-7
	2-2,3,4,5	2-2,3,4	2-2,3,5	2-2,3,4,5	2-2,3,4,5
4-27	3-7	3-8,10	3-7,10	3-11	3-7,8
	2-2,3,4,5	2-2,3,4,6	2-2,3,5,6	2-2,3,4,5	2-2,3,4,5,6
4-28		3-10	3-10		
	2-3	2-3,6	2-3,6	2-3	2-3,6
4-29	3-2	3-2,8	3-2,5	3-2,11	3-5,8
	2-1,2,3,4,5	2-1,2,3,4,6	2-1,2,3,5,6	2-1,2,3,4,5	2-1,2,3,4,5,6

	4-16	4-17	4-18	4-19	4-20
4-21	3-8				
	2-2,4,6	2-4	2-4,6	2-4	2-4
4-22	3-9	3-11	3-11	3-11	3-11
	2-2,4,5	2-3,4,5	2-3,4,5	2-3,4,5	2-3,4,5
4-23	3-9				
	2-2,5	2-3,5	2-3,5	2-3,5	2-3,5
4-24	3-8			3-12	
	2-2,4,6	2-4	2-4,6	2-4	2-4
4-25	3-8				
	2-2,4,6	2-4	2-4,6	2-4	2-4

(4:18)

	4-16	4-17	4-18	4-19	4-20
4-26	 2-2,4,5	3-11 2-3,4,5	3-11 2-3,4,5	3-11 2-3,4,5	3-11 2-3,4,5
4-27	3-8 2-2,4,5,6	3-11 2-3,4,5	3-10.11 2-3,4,5,6	3-11 2-3,4,5	3-11 2-3,4,5
4-28	 2-6	 2-3	3-10 2-3,6	 2-3	 2-3
4-29	3-5,8 2-1,2,4,5,6	3-11 2-1,3,4,5	3-5,11 2-1,3,4,5,6	3-11 2-1,3,4,5	3-11 2-1,3,4,5

	4-21	4-22	4-23	4-24	4-25
4-21					
4-22	3-6 2-2,4				
4-23	 2-2	3-7,9 2-2,3,5			
4-24	3-6,8 2-2,4,6	3-6 2-2,4	 2-2		
4-25	3-8 2-2,4,6	 2-2,4	 2-2	3-8 2-2,4,6	

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	4-21	4-22	4-23	4-24	4-25
4-26	2-2,4	3-7,11 2-2,3,4,5	3-7 2-2,3,5	2-2,4	2-2,4
4-27	3-8 2-2,4,6	3-7,11 2-2,3,4,5	3-7 2-2,3,5	3-8 2-2,4,6	3-8 2-2,4,6
4-28	2-6	2-3	2-3	2-6	2-6
4-29	3-8 2-2,4,6	3-11 2-2,3,4,5	2-2,3,5	3-8 2-2,4,6	3-8 2-2,4,6

	4-26	4-27	4-28	4-29
4-26				
4-27	3-7,11 2-2,3,4,5			
4-28	2-3	3-10 2-3,6		
4-29	3-11 2-2,3,4,5	3-8,11 2-2,3,4,5,6	2-3,6	

**Index for Charts of Similarity
Relations Among Pairs of 5-pc Sets**

```
 1   1
 2    2
 3     3
 4   1  4
 5_______5
 6       *  6
 7       *   7
 8       *    8
 9   2   *  4  9
10_______*_____10
11       *       *11
12       *       * 12
13       *       *  13
14   3   *  6    * 9 14
15_______*_______*____15
16       *       *       *16
17       *       *       * 17
18       *       *       *  18
19   5   *  8    * 12    *16 19
20_______*_______*_______*____20
21       *       *       *       *21
22       *       *       *       * 22
23       *       *       *       *  23
24   7   *  11   * 15    * 20    *24 24
25_______*_______*_______*_______*____25
26       *       *       *       *       *26
27       *       *       *       *       * 27
28       *       *       *       *       *  28
29   10  *  14   * 19    * 23    * 27    *30 29
30_______*_______*_______*_______*_______*____30
31       *       *       *       *       *       *31
32       *       *       *       *       *       * 32
33       *       *       *       *       *       *  33
34   13  *  18   * 22    * 26    * 29    * 32    *34 34
35_______*_______*_______*_______*_______*_______*____35
36       *       *       *       *       *       *       *36
37   17  *  21   * 25    * 28    * 31    * 33    * 35    * 37
38       *       *       *       *       *       *       *  38
                                                          36
```

Pcs	Prime Form	Interval vector
5-1	0,1,2,3,4	432100
5-2	0,1,2,3,5	332110
5-3	0,1,2,4,5	322210
5-4	0,1,2,3,6	322111
5-5	0,1,2,3,7	321121
5-6	0,1,2,5,6	311221
5-7	0,1,2,6,7	310132
5-8	0,2,3,4,6	232201
5-9	0,1,2,4,6	231211
5-10	0,1,3,4,6	223111
5-11	0,2,3,4,7	222220
5-12	0,1,3,5,6	222121
5-13	0,1,2,4,8	221311
5-14	0,1,2,5,7	221131
5-15	0,1,2,6,8	220222
5-16	0,1,3,4,7	213211
5-17	0,1,3,4,8	212320
5-18	0,1,4,5,7	212221
5-19	0,1,3,6,7	212122
5-20	0,1,3,7,8	211231
5-21	0,1,4,5,8	202420
5-22	0,1,4,7,8	202321
5-23	0,2,3,5,7	132130
5-24	0,1,3,5,7	131221
5-25	0,2,3,5,8	123121
5-26	0,2,4,5,8	122311
5-27	0,1,3,5,8	122230
5-28	0,2,3,6,8	122212
5-29	0,1,3,6,8	122131
5-30	0,1,4,6,8	121321
5-31	0,1,3,6,9	114112
5-32	0,1,4,6,9	113221
5-33	0,2,4,6,8	040402
5-34	0,2,4,6,9	032221
5-35	0,2,4,7,9	032140
5-36	0,1,2,4,7	222121
5-37	0,3,4,5,8	212320
5-38	0,1,2,5,8	212221

Subsets of 5-pc sets, cardinality 4- and 3-

Pcs	Prime Form	Interval vector	Number of occurrences/ Percentage of occurrence among pairs of 5-pc sets
4-1	0,1,2,3	321000	6 / .9%
4-2	0,1,2,4	221100	26 / 4.0

(continued)

Pcs	Prime Form	Interval vector	Number and percentage of occurrences in pairs of 5-pc sets (cont'd)
4-3	0,1,3,4	212100	10 / 1.4%
4-4	0,1,2,5	211110	28 / 4.0
4-5	0,1,2,6	210111	28 / 4.0
4-6	0,1,2,7	210021	6 / .9
4-7	0,1,4,5	201210	6 / .9
4-8	0,1,5,6	200121	10 / 1.4
4-9	0,1,6,7	200022	1 / .1
4-10	0,2,3,5	122010	6 / .9
4-11	0,1,3,5	121110	28 / 4.0
4-12	0,2,3,6	112101	28 / 4.0
4-13	0,1,3,6	112011	28 / 4.0
4-14	0,2,3,7	111120	28 / 4.0
4-15	0,1,4,6	111111	28 / 4.0
4-16	0,1,5,7	110121	28 / 4.0
4-17	0,3,4,7	102210	6 / .9
4-18	0,1,4,7	102111	8 / 4.0
4-19	0,1,4,8	101310	21 / 3.0
4-20	0,1,5,8	101220	6 / .9
4-21	0,2,4,6	030201	10 / 1.4
4-22	0,2,4,7	021120	28 / 4.0
4-23	0,2,5,7	021030	6 / .9
4-24	0,2,4,8	020301	6 / .9
4-25	0,2,6,8	020202	3 / .4
4-26	0,3,5,8	012120	10 / 1.4
4-27	0,2,5,8	012111	28 / 4.0
4-28	0,3,6,9	004002	0 / 0.0
4-29	0,1,3,7	111111	28 / 4.0
3-1	0,1,2	210000	120 / 17.1%
3-2	0,1,3	111000	300 / 42.7
3-3	0,1,4	101100	300 / 42.7
3-4	0,1,5	100110	300 / 42.7
3-5	0,1,6	100011	276 / 39.3
3-6	0,2,4	020100	136 / 19.3
3-7	0,2,5	011010	300 / 42.7
3-8	0,2,6	010101	300 / 42.7
3-9	0,2,7	010020	120 / 17.1
3-10	0,3,6	002001	136 / 19.3
3-11	0,3,7	001110	300 / 42.7
3-12	0,4,8	000300	28 / 4.0

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	5-1	5-2	5-3	5-4	5-5
5-1					
5-2	4-1,2 3-1,2,3,6				
5-3	4-2,3 3-1,2,3,6	4-2,4,11 3-1,2,3,4,6,7			
5-4	4-1 3-1,2,3	4-1,4 3-1,2,3,4,7	4-4 3-1,2,3,4,7		
5-5	4-1 3-1,2	4-1 3-1,2,4	3-1,2,4	4-1 3-1,2,4,5,8	

	5-1	5-2	5-3	5-4	5-5
5-6		4-4	4-4,7	4-4,5	4-5
	3-1,3	3-1,3,4, 7	3-1,3,4, 7	3-1,3,4, 5,7,8	3-1,4,5, 8
5-7				4-5	4-5,6
	3-1	3-1,4	3-1,4	3-1,4,5, 8	3-1,4,5, 9
5-8	4-2		4-2	4-12	
	3-1,2,3, 6	3-1,2,3, 6	3-1,2,3, 6	3-1,2,3, 8,10	3-1,2,8
5-9	4-2	4-11	4-2,11	4-5	4-5
	3-1,2,3, 6	3-1,2,3 4,6,7	3-1,2,3, 4,6,7	3-1,2,3, 4,5,7, 8	3-1,2,4, 5,8
5-10	4-3		4-3	4-12,13	
	3-2,3	3-2,3,7	3-2,3,7	3-2,3,5, 7,8,10	3-2,5,8

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	5-1	5-2	5-3	5-4	5-5
5-11	4-2 3-1,2,3,6	4-2,4 3-1,2,3,4,6,7	4-2,4 3-1,2,3,4,6,7	4-4 3-1,2,3,4,7	4-14 3-1,2,4,9,11
5-12	3-2,6	4-11 3-2,4,6,7	4-11 3-2,4,6,7	4-13 3-2,4,5,7,10	3-2,4,5
5-13	4-2 3-1,2,3,6	4-2 3-1,2,3,4,6	4-2 3-1,2,3,4,6	4-5 3-1,2,3,4,5,8	4-5,29 3-1,2,4,5,8,11
5-14	3-1,3	4-4 3-1,3,4,7	4-4 3-1,3,4,7	4-4 3-1,3,4,5,7,8	4-6 3-1,4,5,8,9
5-15	3-1	3-1,4	3-1,4	4-5 3-1,4,5,8	4-5 3-1,4,5,8,9

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	5-6	5-7	5-8	5-9	5-10
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5-7	4-5,8 3-1,4,5,8				
5-8	3-1,3,8	3-1,8			
5-9	4-5,15 3-1,3,4,5,7,8	4-5 3-1,4,5,8	4-2,21 3-1,2,3,6,8		
5-10	4-15 3-3,5,8	3-5,8	4-12 3-2,3,8,10	4-15 3-2,3,5,7,8	

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5-17	4-3		4-3		4-14
	3-2,3	3-2,3	3-2,3	3-2,3,4	3-2,4,5, 11
5-18			4-7	4-12	4-14
	3-2,3	3-2,3,4	3-2,3,4	3-2,3,4, 5,8,10	3-2,4,5, 8,9,11
5-19				4-13	4-29
	3-2,3	3-2,3,7	3-2,3,7	3-2,3,5, 7,8,10	3-2,5,8, 11
5-20					4-14,29
	3-2	3-2,4	3-2,4	3-2,4,5, 8	3-2,4,5, 8,9,11

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5-12	3-4,5,7	4-8 3-4,5	3-2,6,10	4-11 3-2,4,5,6,7	4-13 3-2,5,7,10
5-13	4-5 3-1,3,4,5,8	4-5 3-1,4,5,8	4-2 3-1,2,3,6,8	4-2,5 3-1,2,3,4,5,6,8	3-2,3,5,8
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5-15	4-5 3-1,4,5,8	4-5,16 3-1,4,5,8,9	3-1,8	4-5 3-1,4,5,8	3-5,8

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5-22	3-3	3-3,4	3-3,4	3-3,4,5,10	3-4,5,11
5-23	3-2,6	4-10,11 3-2,4,6,7	4-11 3-2,4,6,7	3-2,4,7	4-14 3-2,4,9,11
5-24	3-2,6	4-11 3-2,4,6,7	4-11 3-2,4,6,7	3-2,4,5,7,8	4-29 3-2,4,5,8,9,11
5-25	3-2	4-10 3-2,7	3-2,7	4-13 3-2,5,7,8,10	4-29 3-2,5,8,11

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5-16	3-3,5,8	3-5,8	4-12 3-2,3,8,10	3-2,3,5,8	4-3,12 3-2,3,5,8,10
5-17	3-3,4	3-4,9	3-2,3	3-2,3,4	4-3 3-2,3
5-18	4-7 3-3,4,5,8	4-16 3-4,5,8,9	4-12 3-2,3,8,10	3-2,3,4,5,8	4-12 3-2,3,5,8,10
5-19	4-15 3-3,5,7,8	4-9 3-5,8	3-2,3,8,10	4-15 3-2,3,5,7,8	4-13,15 3-2,3,5,7,8,10
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5 -14	4-4 3-1,3,4,7,9	3-4,5,7	3-1,3,4,5,8		
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5-27		4-11	4-11		4-14
	3-2,6	3-2,4,6,7	3-2,4,6,7	3-2,4,7	3-2,4,9,11
5-28				4-12	4-29
	3-2,3	3-2,3,7	3-2,3,7	3-2,3,5,7,8,10	3-2,5,8,11
5-29				4-13	4-14
	3-2	3-2,4,7	3-2,4,7	3-2,4,5,7,8,10	3-2,4,5,8,9,11
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5-22	4-8 3-3,4,5	4-8 3-4,5	3-3,10	3-3,4,5	3-3,5,10
5-23	3-4,7	3-4,9	3-2,6	4-11 3-2,4,6,7	4-10 3-2,7
5-24	3-4,5,7,8	4-16 3-4,5,8,9	4-21 3-2,6,8	4-11,12 3-2,4,5,6,7,8	3-2,5,7,8
5-25	3-5,7,8	3-5,8	3-2,8,10	3-2,5,7,8	4-10,13 3-2,5,7,8,10

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5-17	4-14 3-2,3,4,9,11	3-2,4	3-2,3,4,11,12	3-3,4,9	3-4,9
5-18	4-14 3-2,3,4,9,11	3-2,4,5,10	3-2,3,4,5,8,9	4-16 3-3,4,5,8,9	4-16 3-4,5,8,9
5-19	3-2,3,11	4-13 3-2,5,7,10	4-29 3-2,3,5,8,11	4-15 3-3,5,7,8	3-5,8
5-20	4-14 3-2,4,9,11	4-8 3-2,4,5	4-29 3-2,4,5,8,11	4-16 3-4,5,8,9	4-16 3-4,5,8,9

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5-31				4-12,13	
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5-32					
	3-3	3-3,7	3-3,7	3-3,5,7,8,11	3-5,8,11
5-33					
	3-6	3-6	3-6	3-8	3-8
5-34					
	3-6	3-6,7	3-6,7	3-7,8,10	3-8,9,11
5-35					
	3-6	3-6,7	3-6,7	3-7	3-9,11

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	5-6	5-7	5-8	5-9	5-10
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5-27	3-4,7	3-4,9	3-2,6	4-11 3-2,4,6,7	3-2,7
5-28	4-15 3-3,5,7,8	3-5,8	4-12 3-2,3,8,10	4-15 3-2,3,5,7,8	4-12,15 3-2,3,5,7,8,10
5-29	3-4,5,7,8	3-4,5,8,9	3-2,8,10	3-2,4,5,7,8	4-13 3-2,5,7,8,10
5-30	4-15 3-3,4,5,7,8	4-16 3-4,5,8,9	3-3,6,8	4-15 3-3,4,5,6,7,8	4-15 3-3,5,7,8

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	5-11	5-12	5-13	5-14	5-15
5-21	4-17 3-3,4,11	3-4	4-19 3-3,4,11,12	3-3,4	3-4
5-22	3-3,4,11	4-8 3-4,5,10	4-19 3-3,4,5,11,12	3-3,4,5	3-4,5
5-23	4-14 3-2,4,6,7,9,11	4-11 3-2,4,6,7	3-2,4,6,11	4-23 3-4,7,9	3-4,9
5-24	4-22 3-2,4,6,7,9,11	4-11 3-2,4,5,6,7	4-29 3-2,4,5,6,8,11	4-16 3-4,5,7,8,9	4-16 3-4,5,8,9
5-25	3-2,7,11	4-13 3-2,5,7,10	4-29 3-2,5,8,11	3-5,7,8	3-5,8

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5-17	4-3			
	3-2,3,11			
5-18	4-12,18	4-14		
	3-2,3,5,8,10,11	3-2,3,4,9,11		
5-19	4-18,29		4-18	
	3-2,3,5,8,10,11	3-2,3,11	3-2,3,5,8,10,11	
5-20	4-29	4-14	4-14,16	4-29
	3-2,5,8,11	3-2,4,9,11	3-2,4,5,8,9,11	3-2,5,8,11

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5-36	4-2	4-2	4-2	4-13	4-6
	3-1,2,3,6	3-1,2,3,6,7	3-1,2,3,6,7	3-1,2,3,5,7,10	3-1,2,5,9,11
5-37		4-4	4-4	4-4	
	3-1,3	3-1,3,4,7	3-1,3,4,7	3-1,3,4,7	3-1,4,11
5-38		4-4	4-4	4-4,5	4-5
	3-1,3	3-1,3,4,7	3-1,3,4,7	3-1,3,4,5,7,8,10	3-1,4,5,8,10

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5-31		4-16	4-12		4-12,13
	3-3,5,7,8	3-5,8	3-2,3,8,10	3-2,3,5,7,8	3-2,3,5,7,8,10
5-32	4-15			4-15	4-15
	3-3,5,7,8	3-5,8	3-3,8,10	3-3,5,7,8	3-3,5,7,8,10
5-33			4-21	4-21	
	3-8	3-8	3-6,8	3-6,8	3-8
5-34			4-21	4-21	
	3-7,8	3-8,9	3-6,8,10	3-6,7,8	3-7,8,10
5-35	3-7	3-9	3-6	3-6,7	3-7

	5-11	5-12	5-13	5-14	5-15
5-26	3-2,3,4, 6,7,11	4-11 3-2,4,6, 7,10	4-19,24 3-2,3,4, 6,8,11 12	3-3,4,7, 8	3-4,8
5-27	4-14,22 3-2,4,6, 7,9,11	4-11 3-2,4,6, 7	3-2,4,6, 11	3-4,7,9	3-4,9
5-28	3-2,3,7, 11	3-2,5,7, 10	4-29 3-2,3,5, 8,11	4-15 3-3,5,7, 8	4-25 3-5,8
5-29	4-14 3-2,4,7, 9,11	4-13 3-2,4,5, 7,10	3-2,4,5, 8,11	4-16,23 3-4,5,7, 8,9	4-16 3-4,5,8, 9
5-30	4-22 3-3,4,6, 7,9,11	3-4,5,6, 7	4-19 3-3,4,5, 6,8,11, 12	4-15,16 3-3,4,5, 7,8,9	4-16 3-4,5,8, 9

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5-22	4-18 3-3,5,10, 11	4-19 3-3,4,11 12	4-18 3,3,4,5, 10,11	4-18 3-3,5,10, 11	4-8 3-4,5,11
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5-24	4-29 3-2,5,8, 11	3-2,4,9, 11	4-16 3-2,4,5, 8,9,11	4-29 3-2,5,7, 8,11	4-16,29 3-2,4,5, 8,9,11
5-25	4-29 3-2,5,8, 10,11	3-2,11	3-2,5,8, 10,11	4-13,29 3-2,5,7, 8,10, 11	4-29 3-2,5,8, 11

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5-37	4-4 3-1,3,4,7	3-1,4	3-1,3	3-1,3,4,7	3-3,5
5-38	4-4,5 3-1,3,4,5,7,8	4-5 3-1,4,5,8	3-1,3,8,10	4-5 3-1,3,4,5,7,8	3-3,5,7,8,10

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5-32	4-17			4-15	
	3-3,7,11	3-5,7,10	3-3,5,8,11	3-3,5,7,8	3-5,8
5-33			4-24		4-25
	3-6	3-6	3-6,8,12	3-8	3-8
5-34	4-22				
	3-6,7,9,11	3-6,7,10	3-6,8,11	3-7,8,9	3-8,9
5-35	4-22			4-23	
	3-6,7,9,11	3-6,7	3-6,11	3-7,9	3-9

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5-27	3-2,11	4-14 3-2,4,9, 11	4-14 3-2,4,9, 11	3-2,7,11	4-14,20 3-2,4,9, 11
5-28	4-12,29 3-2,3,5, 8,10, 11	3-2,3,11	4-12 3-2,3,5, 8,10, 11	4-15,29 3-2,3,5, 7,8,10, 11	4-29 3-2,5,8, 11
5-29	3-2,5,8, 10,11	4-14 3-2,4,9, 11	4-14,16 3-2,4,5, 8,9,10, 11	4-13 3-2,5,7, 8,10 11	4-14,16 3-2,4,5, 8,9,11
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5-24	3-4,11	3-4,5,11	4-11,22 3-2,4,6,7,9,11		
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5-36	4-2,22	4-13		4-6	
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5-37	4-4		4-19	4-4	
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5-38	4-4		4-5	4-4	4-5
	3-1,3,4, 7,11	3-4,5,7, 10	3-1,3,4, 5,8,11	3-1,3,4, 5,7,8	3-1,4,5, 8

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5-34					
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5-35	3-11	3-9,11	3-9,11	3-7,11	3-9,11

	5-21	5-22	5-23	5-24	5-25
5-26		4-19	4-11	4-11	4-27
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5-27	4-20		4-11,14, 22	4-11,22	4-26
	3-4,11	3-4,11	3-2,4,6, 7,9,11	3-2,4,6, 7,9,11	3-2,7,11
5-28				4-29	4-27,29
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5-29			4-14,23	4-16	4-13,27
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5-30	4-19	4-19	4-22	4-16,22	
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5-36	3-2,3,5,10,11	3-2,3,9,11	4-18 3-2,3,5,9,10,11	4-13,18 3-2,3,5,7,10,11	3-2,5,9,11
5-37	3-3,11	4-19 3-3,4,11,12	3-3,4,11	3-3,7,11	3-4,11
5-38	4-18 3-3,5,8,10,11	3-3,4,11	4-18 3-3,4,5,8,10,11	4-18 3-3,5,7,8,10,11	4-20 3-4,5,8,11

	5-21	5-22	5-23	5-24	5-25
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5-32	4-17	4-18			4-26,27
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5-33				4-21	
	3-12	3-12	3-6	3-6,8	3-8
5-34			4-22	4-21,22	4-27
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5-35			4-22,23	4-22	4-26
	3-11	3-11	3-6,7,9, 11	3-6,7,9, 11	3-7,11

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5-30	4-19,24 3-3,4,6,7,8,11,12	4-22 3-4,6,7,9,11	4-15 3-3,5,7,8,11	4-16 3-4,5,7,8,9,11	

	5-21	5-22	5-23	5-24	5-25
5-36		4-18	4-22	4-22	4-13
	3-3,11	3-3,5,10, 11	3-2,6,7, 9,11	3-2,5,6, 7,9,11	3-2,5,7, 10,11
5-37	4-19	4-19			4-26
	3-3,4,11, 12	3-3,4,11, 12	3-4,7,11	3-4,7,11	3-7,11
5-38	4-20	4-18			4-27
	3-3,4,11	3-3,4,5, 10,11	3-4,7,11	3-4,5,7, 8,11	3-5,7,8, 10,11

	5-26	5-27	5-28	5-29	5-30
5-31	4-12,27		4-12,27	4-13,27	
	3-2,3,7, 8,10, 11	3-2,7,11,	3-2,3,5, 7,8,10, 11	3-2,5,7, 8,10, 11	3-3,5,7, 8,11
5-32	4-27	4-26	4-15,27	4-27	4-15
	3-3,7,8, 10,11	3-7,11	3-3,5,7, 8,10, 11	3-5,7,8, 10,11	3-3,5,7, 8,11
5-33	4-24		4-25		4-24
	3-6,8,12	3-6	3-8	3-8	3-6,8,12
5-34	4-27	4-22	4-27	4-27	4-22
	3-6,7,8, 10,11	3-6,9,11	3-7,8,10 11	3-7,8,9, 10,11	3-6,7,8, 9,11
5-35		4-22,26		4-23	4-22
	3-6,7,11	3-6,7,9, 11	3-7,11	3-7,9,11	3-6,7,9, 11

	5-26	5-27	5-28	5-29	5-30
5-36		4-22		4-13	4-22
	3-2,3,6,7,10,11	3-2,6,7,9,11	3-2,3,5,7,10,11	3-2,5,7,9,10,11	3-3,5,6,7,9,11
5-37	4-19	4-26			4-19
	3-3,4,7,11,12	3-4,7,11	3-3,7,11	3-4,7,11	3-3,4,7,11,12
5-38	4-27	4-20	4-27	4-27	
	3-3,4,7,8,10,11	3-4,7,11	3-3,5,7,8,10,11	3-4,5,7,8,10,11	3-3,4,5,7,8,11

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	5-31	5-32	5-33	5-34	5-35
5-31					
5-32	4-12,27 3-3,5,7,8,10,11				
5-33	3-8	3-8			
5-34	4-27 3-7,8,10,11	4-27 3-7,8,10,11	4-21 3-6,8		
5-35	3-7,11	4-26 3-7,11	3-6	4-22 3-6,7,9,11	

	5-31	5-32	5-33	5-34	5-35
5-36	4-13,18 3-2,3,5, 7,10, 11	4-18 3-3,5,7, 10,11	3-6	4-22 3-6,7,9, 10,11	4-22 3-6,7,9, 11
5-37	3-3,7,11	4-26 3-3,7,11	3-12	3-7,11	4-26 3-7,11
5-38	4-18,27 3-3,5,7, 8,10, 11	4-18,27 3-3,5,7, 8,10, 11	3-8	4-27 3-7,8,10, 11	3-7,11

	5-36	5-37	5-38
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5-37	3-1,3,7, 11		
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Relations Among Pairs of 6-pc Sets

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6–10	2	4								
11–15	3	6	9							
16–20	5	8	12	16						
21–25	7	11	15	20	25					
26–30	10	14	19	24	30	35				
31–35	13	18	23	29	34	39	43			
36–40	17	22	28	33	38	42	46	49		
41–45	21	27	32	37	41	45	48	51	53	
46–50	26	31	36	40	44	47	50	52	54	55

(The fragment of the chart above is really the extension of the lower right-hand corner.)

Pcs	Prime Form	Interval vector
6-1	0,1,2,3,4,5	543210
6-2	0,1,2,3,4,6	443211
6-3	0,1,2,3,5,6	433221
6-4	0,1,2,4,5,6	432321
6-5	0,1,2,3,6,7	422232
6-6	0,1,2,5,6,7	421242
6-7	0,1,2,6,7,8	420243
6-8	0,2,3,4,5,7	343230
6-9	0,1,2,3,5,7	342231
6-10	0,1,3,4,5,7	333321
6-11	0,1,2,4,5,7	333231
6-12	0,1,2,4,6,7	332232
6-13	0,1,3,4,6,7	324222
6-14	0,1,3,4,5,8	323430
6-15	0,1,2,4,5,8	323421
6-16	0,1,4,5,6,8	322431
6-17	0,1,2,4,7,8	322332
6-18	0,1,2,5,7,8	322242
6-19	0,1,3,4,7,8	313431
6-20	0,1,4,5,8,9	303630
6-21	0,2,3,4,6,8	242412
6-22	0,1,2,4,6,8	241422
6-23	0,2,3,5,6,8	234222
6-24	0,1,3,4,6,8	233331
6-25	0,1,3,5,6,8	233241
6-26	0,1,3,5,7,8	232341
6-27	0,1,3,4,6,9	225222
6-28	0,1,3,5,6,9	224322
6-29	0,1,3,6,8,9	224232
6-30	0,1,3,6,7,9	224223
6-31	0,1,3,5,8,9	223431
6-32	0,2,4,5,7,9	143250
6-33	0,2,3,5,7,9	143241
6-34	0,1,3,5,7,9	142422
6-35	0,2,4,6,8,10	060603
6-36	0,1,2,3,4,7	433221
6-37	0,1,2,3,4,8	432321
6-38	0,1,2,3,7,8	421242
6-39	0,2,3,4,5,8	333321
6-40	0,1,2,3,5,8	333231
6-41	0,1,2,3,6,8	332232
6-42	0,1,2,3,6,9	324222
6-43	0,1,2,5,6,8	322332
6-44	0,1,2,5,6,9	313431
6-45	0,2,3,4,6,9	234222
6-46	0,1,2,4,6,9	233331
6-47	0,1,2,4,7,9	233241
6-48	0,1,2,5,7,9	232341
6-49	0,1,3,4,7,9	224322
6-50	0,1,4,6,7,9	224232

Subsets of 6-pc sets, cardinality 5- and 4-

Pcs	Prime Form	Interval vector	Number of occurrences/ Percentage of occurrence among pairs of 6-pc sets
5-1	0,1,2,3,4	432100	6 / .5%
5-2	0,1,2,3,5	332110	21 / 1.7
5-3	0,1,2,4,5	322210	21 / 1.7
5-4	0,1,2,3,6	322111	21 / 1.7
5-5	0,1,2,3,7	321121	21 / 1.7
5-6	0,1,2,5,6	311221	21 / 1.7
5-7	0,1,2,6,7	310132	21 / 1.7
5-8	0,2,3,4,6	232201	6 / .5
5-9	0,1,2,4,6	231211	21 / 1.7
5-10	0,1,3,4,6	223111	21 / 1.7
5-11	0,2,3,4,7	222220	21 / 1.7
5-12	0,1,3,5,6	222121	6 / .5
5-13	0,1,2,4,8	221311	21 / 1.7
5-14	0,1,2,5,7	221131	21 / 1.7
5-15	0,1,2,6,8	220222	6 / .5
5-16	0,1,3,4,7	213211	21 / 1.7
5-17	0,1,3,4,8	212320	6 / .5
5-18	0,1,4,5,7	212221	21 / 1.7
5-19	0,1,3,6,7	212122	21 / 1.7
5-20	0,1,3,7,8	211231	21 / 1.7
5-21	0,1,4,5,8	202420	21 / 1.7
5-22	0,1,4,7,8	202321	6 / .5
5-23	0,2,3,5,7	132130	21 / 1.7
5-24	0,1,3,5,7	131221	21 / 1.7
5-25	0,2,3,5,8	123121	21 / 1.7
5-26	0,2,4,5,8	122311	21 / 1.7
5-27	0,1,3,5,8	122230	21 / 1.7
5-28	0,2,3,6,8	122212	21 / 1.7
5-29	0,1,3,6,8	122131	21 / 1.7
5-30	0,1,4,6,8	121321	21 / 1.7
5-31	0,1,3,6,9	114112	15 / 1.2
5-32	0,1,4,6,9	113221	21 / 1.7
5-33	0,2,4,6,8	040402	6 / .5
5-34	0,2,4,6,9	032221	6 / .5
5-35	0,2,4,7,9	032140	6 / .5
5-36	0,1,2,4,7	222121	21 / 1.7
5-37	0,3,4,5,8	212320	6 / .5
5-38	0,1,2,5,8	212221	21 / 1.7

(continued)

Pcs	Prime Form	Interval vector	Number of occurrences/ Percentage of occurrence among pairs of 6-pc sets
4-1	0,1,2,3	321000	78 / 6.4%
4-2	0,1,2,4	221100	231 / 18.9
4-3	0,1,3,4	212100	120 / 9.8
4-4	0,1,2,5	211110	276 / 22.5
4-5	0,1,2,6	210111	276 / 22.5
4-6	0,1,2,7	210021	105 / 8.6
4-7	0,1,4,5	201210	120 / 9.8
4-8	0,1,5,6	200121	120 / 9.8
4-9	0,1,6,7	200022	45 / 3.7
4-10	0,2,3,5	122010	120 / 9.8
4-11	0,1,3,5	121110	276 / 22.5
4-12	0,2,3,6	112101	276 / 22.5
4-13	0,1,3,6	112011	276 / 22.5
4-14	0,2,3,7	111120	276 / 22.5
4-15	0,1,4,6	111111	378 / 30.9
4-16	0,1,5,7	110121	276 / 22.5
4-17	0,3,4,7	102210	120 / 9.8
4-18	0,1,4,7	102111	276 / 22.5
4-19	0,1,4,8	101310	120 / 9.8
4-20	0,1,5,8	101220	120 / 9.8
4-21	0,2,4,6	030201	78 / 6.4
4-22	0,2,4,7	021120	231 / 18.9
4-23	0,2,5,7	021030	78 / 6.4
4-24	0,2,4,8	020301	78 / 6.4
4-25	0,2,6,8	020202	45 / 3.7
4-26	0,3,5,8	012120	120 / 9.8
4-27	0,2,5,8	012111	276 / 22.5
4-28	0,3,6,9	004002	15 / 1.2
4-29	0,1,3,7	111111	378 / 30.9

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6-1				
6-2	5-1,2			
	4-1,2,3,4,10,11			
6-3	5-2,3	5-2,4,10		
	4-1,2,3,4,7,10,11	4-1,2,3,4,5,10,11,12,13,15		
6-4	5-3	5-9	5-3,6	
	4-2,3,4,7,11	4-2,3,4,5,11,15,21	4-2,3,4,5,7,8,11,15	
6-5		5-4	5-4,6	5-6
	4-1,4,7	4-1,4,5,12,13,15	4-1,4,5,7,8,12,13,15	4-4,5,7,8,15

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	6-1	6-2	6-3	6-4	6-5
6-6			5-6	5-6	5-6,7
	4-4,7	4-4,5,15	4-4,5,7, 8,15	4-4,5,7, 8,15	4-4,5,6, 7,8,9, 15
6-7	*				5-7
		4-5	4-5,8	4-5,8	4-5,6,8, 9,16
6-8	5-2	5-2	5-2		
	4-1,2,4, 10,11	4-1,2,4, 10,11	4-1,2,4, 10,11	4-2,4,11	4-1,4,14
6-9	5-2	5-2,9	5-2		5-5
	4-1,2,4, 10,11	4-1,2,4, 5,10, 11,15, 21	4-1,2,4, 5,10, 11,15	4-2,4,5, 11,15, 21	4-1,4,5, 6,14, 15,16, 29
6-10	5-3	5-8	5-3	5-3	5-18
	4-2,3,4, 7,11	4-2,3,4, 11,12, 21	4-2,3,4, 7,11, 12,15	4-2,3,4, 7,11, 21	4-4,7,12, 14,16, 18,29

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	6-1	6-2	6-3	6-4	6-5
6-11	5-3	5-10	5-3,10	5-3	5-18
	4-2,3,4,7,10,11	4-2,3,4,10,11,12,13,15	4-2,3,4,7,10,11,12,13,15	4-2,3,4,7,11,15	4-4,6,7,12,13,14,15,16,18
6-12		5-9	5-12	5-9	5-7,19
	4-2,11	4-2,5,11,13,15,21	4-2,5,8,11,13,15	4-2,5,8,11,15,21	4-5,8,9,13,15,16,18,29
6-13		5-10	5-10		5-19
	4-3,10	4-3,10,12,13,15	4-3,10,12,13,15	4-3,15	4-12,13,15,18,29
6-14	5-3		5-3	5-3	
	4-2,3,4,7,11	4-2,3,4,11	4-2,3,4,7,11	4-2,3,4,7,11	4-4,7,14
6-15	5-3		5-3	5-3	
	4-2,3,4,7,11	4-2,3,4,5,11,12	4-2,3,4,5,7,11,12	4-2,3,4,5,7,11	4-7,12,18,29

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	6-6	6-7	6-8	6-9	6-10
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6-7	5-7 4-5,6,8, 9,16				
6-8	4-4,23	*			
6-9	5-14 4-4,5,6, 15,23	4-5	5-2,23 4-1,2,4, 10,11, 22,23		
6-10	4-4,7	4-16	5-11 4-2,4,11, 14,17, 22	5-24 4-2,4,11, 14,16, 21,22, 29	

	6-1	6-2	6-3	6-4	6-5
6-16			5-6	5-6	
	4-2,4,7	4-2,4,5, 15	4-2,4,5, 7,8,15	4-2,4,5, 7,8,15	4-4,5,7, 8,14, 15,16, 29
6-17					5-7,19
	4-2	4-2,5,13, 15	4-2,5,8, 13,15	4-2,5,8, 15	4-5,6,8, 9,13, 15,16, 18,29
6-18					5-7,19
	4-4	4-4,5,13, 15	4-4,5,8, 13,15	4-4,5,8, 15	4-4,5,6, 8,9,13, 14,15, 16,18, 29
6-19					5-18
	4-3,7	4-3,12	4-3,7,8, 12	4-3,7,8	4-7,8,12, 14,16, 18,29
6-20		*			
	4-7		4-7	4-7	4-7

	6-6	6-7	6-8	6-9	6-10
6-11	5-14		5-23	5-14,23	5-3,18
	4-4,6,7, 15,16, 23	4-6,16	4-2,4,11, 14,22, 23	4-2,4,6, 10,11, 14,15, 16,22, 23	4-2,3,4, 7,11, 12,14, 16,18, 22
6-12	5-7	5-7		5-9,24	5-24
	4-5,6,8, 9,15, 16	4-5,6,8, 9,16	4-2,22	4-2,5,6, 11,15, 16,21, 22,29	4-2,11, 16,18, 21,22, 29
6-13					5-16
	4-9,15	4-9	4-10,17	4-10,15, 29	4-3,12, 17,18, 29
6-14		*	5-11		5-3,11
	4-4,7		4-2,4,11, 17,22	4-2,4,11, 14,22	4-2,3,4, 7,11, 14,17, 22
6-15					5-3,16
	4-4,5,7	4-5	4-2,4,11, 17	4-2,4,5, 11,29	4-2,3,4, 7,11, 12,17, 18,29

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	6-1	6-2	6-3	6-4	6-5
6-21		5-8		5-9	
	4-2,11	4-2,5,11, 12,15, 21	4-2,5,11, 12,15	4-2,5,11, 15,21	4-5,12, 15,29
6-22		5-9		5-9	
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	4-4,5,7, 8,15, 16	4-5,8,16	4-2,4,14, 17,22	4-2,4,5, 14,15, 16,22, 29	4-2,4,7, 14,16, 17,22, 29
6-17	5-7	5-7			
	4-5,6,8, 9,15, 16	4-5,6,8, 9,16	4-2,22	4-2,5,6, 15,16, 22,29	4-2,16, 18,22, 29
6-18	5-7,14	5-7		5-14	
	4-4,5,6, 8,9,15, 16,23	4-5,6,8, 9,16	4-4,14, 23	4-4,5,6, 14,15, 16,23, 29	4-4,14,16, 18,29
6-19					5-16,18
	4-7,8,16	4-8,16	4-14,17	4-14,16, 29	4-3,7,12, 14,16, 17,18, 29
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	4-3,10	4-3,10, 12,13, 15	4-3,10, 12,13, 15	4-3,15	4-12,13, 15,18, 29
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		4-12,13, 15	4-12,13, 15	4-15	4-9,12, 13,15, 18,29

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	4-5,15	4-5,25	4-2,11	4-2,5,11, 15,21, 29	4-2,11, 12,21, 29
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	4-5,15, 16	4-5,16, 25	4-2,11, 22	4-2,5,11, 15,16, 21,22, 29	4-2,11, 16,21, 22,29
6-23					
	4-15	4-25	4-10	4-10,15, 29	4-3,12, 19
6-24			5-23	5-23	
	4-15,16, 23	4-16	4-10,11, 14,22, 23	4-10,11, 14,15, 16,22, 23	4-3,11, 12,14, 16,22
6-25			5-23		
	4-8,16, 23	4-8,16	4-10,11, 14,22, 23	4-10,11, 14,16, 22,23, 29	4-11,14, 16,22, 29

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	6-11	6-12	6-13	6-14	6-15
6-16	4-2,4,7,14,15,16,22	4-2,5,8,15,16,22,29	4-15,17,29	5-11,21 4-2,4,7,14,17,19,20,22	5-13,21 4-2,4,5,7,17,19,20,24,29
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6-18	5-14 4-4,6,13,14,15,16,18,23	5-7,19 4-5,6,8,9,13,15,16,18,29	5-19 4-9,13,15,18,29	4-4,14 20	5-38 4-4,5,18,20,29
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6-40	5-36	5-36		5-27	5-38
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 7        *  7
 8        *   8
 9  2     * 4  9
10________*_____10
11        *        *11
12        *        * 12
13        *        *  13
14  3     * 6      *9  14
15________*________*_____15
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7-1	0,1,2,3,4,5,6	654321
7-2	0,1,2,3,4,5,7	554331
7-3	0,1,2,3,4,5,8	544431
7-4	0,1,2,3,4,6,7	544332
7-5	0,1,2,3,5,6,7	543342
7-6	0,1,2,3,4,7,8	533442
7-7	0,1,2,3,6,7,8	532353
7-8	0,2,3,4,5,6,8	454422
7-9	0,1,2,3,4,6,8	453432
7-10	0,1,2,3,4,6,9	445332
7-11	0,1,3,4,5,6,8	444441
7-12	0,1,2,3,4,7,9	444342
7-13	0,1,2,4,5,6,8	443532
7-14	0,1,2,3,5,7,8	443352
7-15	0,1,2,4,6,7,8	442443
7-16	0,1,2,3,5,6,9	435432
7-17	0,1,2,4,5,6,9	434541
7-18	0,1,2,3,5,8,9	434442
7-19	0,1,2,3,6,7,9	434343
7-20	0,1,2,4,7,8,9	433452
7-21	0,1,2,4,5,8,9	424641
7-22	0,1,2,5,6,8,9	424542
7-23	0,2,3,4,5,7,9	354351
7-24	0,1,2,3,5,7,9	353442
7-25	0,2,3,4,6,7,9	345342
7-26	0,1,3,4,5,7,9	344532
7-27	0,1,2,4,5,7,9	344451
7-28	0,1,3,5,6,7,9	344433
7-29	0,1,2,4,6,7,9	344352
7-30	0,1,2,4,6,8,9	343542
7-31	0,1,3,4,6,7,9	336333
7-32	0,1,3,4,6,8,9	335442
7-33	0,1,2,4,6,8,10	262623
7-34	0,1,3,4,6,8,10	254442
7-35	0,1,3,5,6,8,10	254361
7-36	0,1,2,3,5,6,8	444342
7-37	0,1,3,4,5,7,8	434541
7-38	0,1,2,4,5,7,8	434442

(subsets, next page)

Subsets of 7-pc sets, cardinality 6- and 5-

Pcs	Prime Form	Interval vector	Number of occurrences/ Percentage of occurrence among pairs of 7-pc sets
6-1	0,1,2,3,4,5	543210	3 / .4%
6-2	0,1,2,3,4,6	443211	15 / 2.1
6-3	0,1,2,3,5,6	433221	15 / 2.1
6-4	0,1,2,4,5,6	432321	6 / .9
6-5	0,1,2,3,6,7	422232	15 / 2.1
6-6	0,1,2,5,6,7	421242	3 / .4
6-7	0,1,2,6,7,8	420243	1 / .1
6-8	0,2,3,4,5,7	343230	3 / .4
6-9	0,1,2,3,5,7	342231	15 / 2.1
6-10	0,1,3,4,5,7	333321	15 / 2.1
6-11	0,1,2,4,5,7	333231	15 / 2.1
6-12	0,1,2,4,6,7	332232	15 / 2.1
6-13	0,1,3,4,6,7	324222	3 / .4
6-14	0,1,3,4,5,8	323430	15 / 2.1
6-15	0,1,2,4,5,8	323421	15 / 2.1
6-16	0,1,4,5,6,8	322431	15 / 2.1
6-17	0,1,2,4,7,8	322332	15 / 2.1
6-18	0,1,2,5,7,8	322242	15 / 2.1
6-19	0,1,3,4,7,8	313431	15 / 2.1
6-20	0,1,4,5,8,9	303630	0 / 0.0
6-21	0,2,3,4,6,8	242412	15 / 2.1
6-22	0,1,2,4,6,8	241422	15 / 2.1
6-23	0,2,3,5,6,8	234222	6 / .9
6-24	0,1,3,4,6,8	233331	15 / 2.1
6-25	0,1,3,5,6,8	233241	15 / 2.1
6-26	0,1,3,5,7,8	232341	6 / .9
6-27	0,1,3,4,6,9	225222	10 / 1.4
6-28	0,1,3,5,6,9	224322	3 / .4
6-29	0,1,3,6,8,9	224232	3 / .4
6-30	0,1,3,6,7,9	224223	3 / .4
6-31	0,1,3,5,8,9	223431	15 / 2.1
6-32	0,2,4,5,7,9	143250	3 / .4
6-33	0,2,3,5,7,9	143241	15 / 2.1
6-34	0,1,3,5,7,9	142422	15 / 2.1
6-35	0,2,4,6,8,10	060603	0 / 0.0
6-36	0,1,2,3,4,7	433221	15 / 2.1
6-37	0,1,2,3,4,8	432321	3 / .4
6-38	0,1,2,3,7,8	421242	3 / .4
6-39	0,2,3,4,5,8	333321	15 / 2.1
6-40	0,1,2,3,5,8	333231	15 / 2.1

(continued)

Pcs	Prime Form	Interval vector	Number and percentage of occurrence in pairs of 7-pc sets (cont'd)
6-41	0,1,2,3,6,8	332232	15 / 2.1%
6-42	0,1,2,3,6,9	324222	3 / .4
6-43	0,1,2,5,6,8	322332	15 / 2.1
6-44	0,1,2,5,6,9	313431	15 / 2.1
6-45	0,2,3,4,6,9	234222	3 / .4
6-46	0,1,2,4,6,9	233331	15 / 2.1
6-47	0,1,2,4,7,9	233241	15 / 2.1
6-48	0,1,2,5,7,9	232341	3 / .4
6-49	0,1,3,4,7,9	224322	6 / .9
6-50	0,1,4,6,7,9	224232	3 / .4
5-1	0,1,2,3,4	432100	36 / 3.0%
5-2	0,1,2,3,5	332110	105 / 14.9
5-3	0,1,2,4,5	322210	120 / 17.1
5-4	0,1,2,3,6	322111	136 / 19.3
5-5	0,1,2,3,7	321121	105 / 14.9
5-6	0,1,2,5,6	311221	120 / 17.1
5-7	0,1,2,6,7	310132	66 / 9.4
5-8	0,2,3,4,6	232201	66 / 9.4
5-9	0,1,2,4,6	231211	153 / 21.8
5-10	0,1,3,4,6	223111	120 / 17.1
5-11	0,2,3,4,7	222220	153 / 21.8
5-12	0,1,3,5,6	222121	66 / 9.4
5-13	0,1,2,4,8	221311	136 / 19.3
5-14	0,1,2,5,7	221131	105 / 14.9
5-15	0,1,2,6,8	220222	55 / 7.8
5-16	0,1,3,4,7	213211	120 / 17.1
5-17	0,1,3,4,8	212320	66 / 9.4
5-18	0,1,4,5,7	212221	153 / 21.8
5-19	0,1,3,6,7	212122	78 / 11.1
5-20	0,1,3,7,8	211231	120 / 17.1
5-21	0,1,4,5,8	202420	120 / 17.1
5-22	0,1,4,7,8	202321	66 / 9.4
5-23	0,2,3,5,7	132130	105 / 14.9
5-24	0,1,3,5,7	131221	153 / 21.8
5-25	0,2,3,5,8	123121	120 / 17.1
5-26	0,2,4,5,8	122311	136 / 19.3
5-27	0,1,3,5,8	122230	120 / 17.1
5-28	0,2,3,6,8	122212	105 / 14.9
5-29	0,1,3,6,8	122131	136 / 19.3
5-30	0,1,4,6,8	121321	136 / 19.3
5-31	0,1,3,6,9	114112	21 / 3.0
5-32	0,1,4,6,9	113221	120 / 17.1

(continued)

Pcs	Prime form	Interval vector	Number and percentage of occurrence in pairs of 7-pc sets (cont'd)
5-33	0,2,4,6,8	040402	45 / 6.4%
5-34	0,2,4,6,9	032221	66 / 9.4
5-35	0,2,4,7,9	032140	36 / 5.1
5-36	0,1,2,4,7	222121	153 / 21.8
5-37	0,3,4,5,8	212320	66 / 9.4
5-38	0,1,2,5,8	212221	153 / 21.8

(7:1)

	7-1	7-2	7-3	7-4	7-5
7-1					
7-2	6-1,2 5-1,2,3, 4,8,9, 10				
7-3	6-1 5-1,2,3, 4	6-1,36 5-1,2,3, 4,5,11, 16,36			
7-4	6-2,3 5-1,2,3, 4,6,8, 9,10	6-2,10, 36 5-1,2,3, 4,5,8, 9,10,11, 16,18, 24,36	6-36 5-1,2,3, 4,5,11, 16,36		
7-5	6-3,4 5-2,3,4, 6,9, 10,12	6-9,11 5-2,3,4, 5,9,10, 14,18, 23,24, 36	5-2,3,4, 5,36	6-3,5,12 5-2,3,4, 5,6,7, 9,10, 12,18, 19,24, 36	

	7-1	7-2	7-3	7-4	7-5
7-6		6-36	6-36,37	6-5,36	6-5
	5-1,4,6	5-1,4,5, 11,16, 18,36	5-1,4,5, 11,13, 16,17, 21,36	5-1,4,5, 6,7,11, 16,18, 19,36	5-4,5,6, 7,18, 19,36
7-7				6-5	6-5,6
	5-4,6	5-4,5,14, 18	5-4,5,38	5-4,5,6, 7,18, 19	5-4,5,6, 7,14, 18,19
7-8	6-2	6-2	6-39	6-2	
	5-1,2,4, 8,9,10	5-1,2,4, 8,9,10	5-1,2,4, 13,25, 26,37	5-1,2,4, 8,9,10	5-2,4,9, 10
7-9	6-2	6-2,9	6-37	6-2	6-9
	5-1,2,4, 8,9,10	5-1,2,4, 5,8,9, 10,14, 23,24	5-1,2,4, 5,13, 17,26	5-1,2,4, 5,8,9, 10	5-2,4,5, 9,10, 14,23, 24
7-10	6-2	6-2,36	6-36,40	6-2,36	
	5-1,2,4, 8,9,10	5-1,2,4, 5,8,9, 10,11, 16,36	5-1,2,4, 5,11, 16,25, 27,36 38	5-1,2,4, 5,8,9, 10,11, 16,36	5-2,4,5, 9,10 36

	7-1	7-2	7-3	7-4	7-5
7-11	6-3	6-8	6-14,39	6-3	6-3
	5-2,3,4,6,10,12	5-2,3,4,10,11,23	5-2,3,4,11,13,17,21,25,26,27,37	5-2,3,4,6,10,11,12	5-2,3,4,6,10,12,23
7-12		6-36	6-36	6-36	
	5-1,4	5-1,4,5,11,14,16,36	5-1,4,5,11,16,36	5-1,4,5,11,16,36	5-4,5,14,36
7-13	6-4	6-10	6-15	6-10	6-4
	5-3,6,8,9	5-3,8,9,11,16,18,24	5-3,11,13,16,21,26,38	5-3,6,8,9,11,16,18,24	5-3,6,9,18,24
7-14		6-9	6-40	6-12	6-9,12
	5-2,9,12	5-2,5,9,14,23,24,36	5-2,5,25,27,36,38	5-2,5,7,9,12,19,24,36	5-2,5,7,9,12,14,19,23,24,36
7-15				6-12	6-12
	5-9,12	5-9,24,36	5-13,36	5-7,9,12,19,24,36	5-7,9,12,19,24,36

(7,4)

	7-6	7-7	7-8	7-9	7-10
7-6					
7-7	6-5,38 5-4,5,6, 7,18, 19,20				
7-8	5-1,4,13	5-4,28			
7-9	6-37 5-1,4,5, 13,17, 30	6-41 5-4,5,14, 15,28, 29	6-2,21 5-1,2,4, 8,9,10, 13,26, 28,33		
7-10	6-36 5-1,4,5, 11,16, 36	5-4,5,38	6-2 5-1,2,4, 8,9, 10,25	6-2 5-1,2,4, 5,8,9, 10	

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	7-1	7-2	7-3	7-4	7-5
7-16	6-3		6-15,39	6-3	6-3
	5-2,3,4,6,10,12	5-2,3,4,10,16	5-2,3,4,13,16,21,25,26,37,38	5-2,3,4,6,10,12,16	5-2,3,4,6,10,12
7-17	6-4		6-14		6-4
	5-3,6,9	5-3,9,11	5-3,11,17,21,27,37,38	5-3,6,9,11	5-3,6,9
7-18			6-39,40	6-5	6-5
	5-2,4,6	5-2,4,5,18,36	5-2,4,5,13,21,25,26,27,36,37,38	5-2,4,5,6,7,18,19,36	5-2,4,5,6,7,18,19,36
7-19				6-5	6-5
	5-4,6	5-4,5,14,18	5-4,5,38	5-4,5,6,7,18,19	5-4,5,6,7,14,18,19
7-20					6-6
	5-6	5-11,14,36	5-11,13,21,36,37,38	5-6,7,11,19,36	5-6,7,14,19,36

	7-6	7-7	7-8	7-9	7-10
7-11	6-16 5-4,6,11,13,17,20,21,30	5-4,6,20,29	6-39 5-2,4,10,13,25,26,37	6-24 5-2,4,10,13,17,23,26,29,30	5-2,4,10,11,25,27
7-12	6-36 5-1,4,5,11,16,36	6-41 5-4,5,14,15,28,29	5-1,4,28	6-41 5-1,4,5,14,15,28,29	6-36 5-1,4,5,11,32,36
7-13	6-16 5-6,11,13,16,18,20,21,30	6-43 5-6,15,18,20,28,38	6-21 5-8,9,13,26,28,33	6-21,22 5-8,9,13,15,24,26,28,30	5-8,9,11,16,38
7-14	6-38 5-5,7,19,20,36	6-18,38 5-5,7,14,19,20,29,38	5-2,9,25	6-9 5-2,5,9,14,23,24,29	6-40 5-2,5,9,25,27,36,38
7-15	6-17 5-7,13,19,22,36	6-7 5-7,15,19	5-9,13,33	6-22 5-9,13,15,24,30,33	5-9,36

(7:7)

	7-1	7-2	7-3	7-4	7-5
7-21	5-3,6	5-3,11, 16,18	6-14,15 5-3,11, 13,16, 17,21, 26,27, 37,38	5-3,6,11, 16,18	5-3,6,18
7-22	5-6	5-16,18	5-16,17, 21,37, 38	5-6,16, 18	5-6,18
7-23	5-2,9	6-8,9 5-2,5,9, 11,14, 23,24, 36	6-40 5-2,5,11, 25,27, 36,38	5-2,5,9, 11,24, 36	6-9 5-2,5,9, 14,23, 24,36
7-24	5-2,4,9	6-9 5-2,4,5, 9,14, 23,24	6-39 5-2,4,5, 13,25, 26,37	5-2,4,5, 9,24	6-9 5-2,4,5, 9,14, 23,24
7-25	5-3,8,10	6-10,11 5-3,8,10, 11,14, 16,18, 23,24, 36	5-3,11, 16,25, 36	6-10 5-3,8,10, 11,16, 18,24, 36	6-11 5-3,10, 14,18, 23,24, 36

	7-6	7-7	7-8	7-9	7-10
7-16	5-4,6,13, 16,21, 22	5-4,6,38	6-39 5-2,4,10, 13,25, 26,37	5-2,4,10, 13,26	6-42 5-2,4,10, 16,25, 31,32, 38
7-17	5-6,11, 17,21, 22	5-6,38	5-9,37	5-9,17	6-46 5-9,11, 27,32 34,38
7-18	6-5,17 5-4,5,6, 7,13, 18,19, 21,22, 30,36	6-5 5-4,5,6, 7,18, 19,38	6-39 5-2,4,13, 25,26, 37	5-2,4,5, 13,26, 30	6-40 5-2,4,5, 25,27, 32,36, 38
7-19	6-5 5-4,5,6, 7,18, 19,20	6-5,18, 41,43 5-4,5,6, 7,14, 15,18, 19,20, 28,29, 38	5-4,28	6-41 5-4,5,14, 15,28, 29	6-42 5-4,5,31, 38
7-20	6-16,17 5-6,7,11, 13,20, 21,22, 30,36	6-6,18 5-6,7,14, 19,20, 29,38	5-13,37	5-13,14, 29,30	5-11,31, 36,38

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	7-11	7-12	7-13	7-14	7-15
7-11					
7-12	6-4,11, 29				
7-13	6-16 5-3,6,11, 13,20, 26,30	5-11,15, 16,28			
7-14	6-25 5-2,12, 20,23, 25,27, 29	5-5,14, 29,36	5-9,20, 24,38		
7-15	5-12,13	5-15,36	6-22 5-9,13, 15,24, 30,33	6-12 5-7,9,12, 19,24, 36	

	7-1	7-2	7-3	7-4	7-5
7-26		6-10	6-15	6-10	
	5-3,8,9	5-3,8,9, 11,16, 18,24	5-3,11, 13,16, 21,26, 27,38	5-3,8,9, 11,16, 18,24	5-3,9,18, 24
7-27		6-11	6-14		6-11
	5-3,10	5-3,10, 11,14, 18,23, 36	5-3,11, 17,21, 26,27, 36,37	5-3,10, 11,18, 36	5-3,10, 14,18, 23,36
7-28				6-12	6-12
	5-8,9,12	5-8,9,24, 36	5-13,26, 36	5-7,8,9, 12,24, 36	5-7,9,12, 19,24, 36
7-29				6-12	6-12
	5-9,12	5-9,11, 12,14, 23,24, 36	5-11,25, 27,36, 38	5-7,9,11, 12,19, 24,36	5-7,9,12, 14,19, 23,24, 36
7-30					
	5-6,9	5-9,11, 18,24	5-11,18, 21,26, 27,38	5-6,9,11, 18,24	5-6,9,18, 24

	7-6	7-7	7-8	7-9	7-10
7-21	6-16,19				
	5-6,11, 13,16, 17,18, 20,21, 22,30	5-6,18, 20,38	5-13,26, 37	5-13,17, 26,30	5-11,16, 27,32, 38
7-22	6-19	6-43			
	5-6,16, 17,18, 20,21, 22	5-6,15, 18,20, 28,38	5-28,37	5-15,17, 28	5-16,32, 38
7-23				6-9	6-40,46
	5-5,11, 36	5-5,14, 29,38	5-2,9,25	5-2,5,9, 14,23, 24,29	5-2,5,9, 11,25, 27,34, 36,38
7-24		6-41	6-39	6-9,22, 41	
	5-4,5,13, 30	5-4,5,14, 15,28, 29	5-2,4,9, 13,25, 26,28, 33,37	5-2,4,5, 9,13, 14,15, 23,24, 26,28, 29,30, 33	5-2,4,5, 9,25, 34
7-25					6-27,45
	5-11,16, 18,36	5-14,18, 29	5-8,10, 25	5-8,10, 14,23, 24,29	5-8,10, 11,16, 25,31, 32,34, 36

	7-11	7-12	7-13	7-14	7-15
7-16	6-3,39 5-2,3,4,6,10,12,13,21,25,26,37	5-4,16,32	6-15 5-3,6,13,16,21,26,38	5-2,12,25,38	5-12,13,22
7-17	6-14 5-3,6,11,17,21,27,37	5-11,32	6-4 5-3,6,9,11,21,38	5-9,27,38	5-9,22
7-18	6-39 5-2,4,6,13,21,25,26,27,30,37	5-4,5,32,36	5-6,13,18,21,26,30,38	6-40 5-2,5,7,19,25,27,36,38	6-17 5-7,13,19,22,30,36
7-19	5-4,6,20,29	6-41 5-4,5,14,15,28,29	6-43 5-6,15,18,20,28,38	6-18 5-5,7,14,19,20,29,38	5-7,15,19
7-20	6-16 5-6,11,13,20,21,29,30,37	6-47 5-11,14,29,32,35,36	6-16 5-6,11,13,20,21,30,38	6-18 5-7,14,19,20,29,36,38	6-17 5-7,13,19,22,30,36

	7-1	7-2	7-3	7-4	7-5
7-31				6-13	
	5-10	5-10,16	5-16,25	5-10,16, 19	5-10,19
7-32					
	5-10,12	5-10,16, 18,23	5-16,17, 21,25, 26,27	5-10,12, 16,18	5-10,12, 18,23
7-33					
	5-8,9	5-8,9,24	5-13,26	5-8,9,24	5-9,24
7-34					
	5-10	5-10,23, 24	5-17,25, 26	5-10,24	5-10,23, 24
7-35					
	5-12	5-23,24	5-25,27	5-12,24	5-12,23, 24

	7-6	7-7	7-8	7-9	7-10
7-26			6-21	6-21	6-46
	5-11,13, 16,18, 21	5-18,28, 38	5-8,9,13, 26,28, 33	5-8,9,13, 24,26, 28,30, 33	5-8,9,11, 16,27, 32,34, 38
7-27				6-24	
	5-11,17, 18,21, 30,36	5-14,18, 29	5-10,26, 37	5-10,14, 17,23, 26,29, 30	5-10,11, 27,32, 36
7-28	6-17		6-21	6-21	6-45
	5-7,13, 19,22, 30,36	5-7,19, 28	5-8,9,13, 26,28, 33	5-8,9,13, 24,26, 28,30, 33	5-8,9,31, 34,36
7-29		6-18			6-46
	5-7,11, 19,20, 36	5-7,14, 19,20, 29,38	5-9,25	5-9,14, 23,24, 29	5-9,11, 25,27, 34,36, 38
7-30	6-16	6-43		6-22	6-46
	5-6,11, 13,18, 20,21, 30	5-6,15, 18,20, 28,38	5-9,13, 26,28, 33	5-9,13, 15,24, 26,28, 30,33	5-9,11, 27,32, 34,38

	7-11	7-12	7-13	7-14	7-15
7-21	6-14,16		6-15,16		
	5-3,6,11, 13,17, 20,21, 26,27, 30,37	5-11,16, 32	5-3,6,11, 13,16, 18,20, 21,26, 30,38	5-20,27, 38	5-13,22, 30
7-22		6-49	6-43		
	5-6,17, 20,21, 37	5-15,16, 28,32	5-6,15, 16,18, 20,21, 28,38	5-20,38	5-15,22
7-23	6-8			6-9,40	
	5-2,11, 23,25, 27,29	5-5,11, 14,29, 32,35, 36	5-9,11, 24,38	5-2,5,9, 14,23, 24,25, 27,29, 36,38	5-9,24, 36
7-24	6-39	6-41	6-22	6-9	6-22
	5-2,4,13, 23,25, 26,30, 37	5-4,5,14, 15,28, 29,35	5-9,13, 15,24, 26,28, 30,33	5-2,5,9, 14,23, 24,25, 29	5-9,13, 15,24, 30,33
7-25		6-47	6-10		
	5-3,10, 11,23, 25,29	5-11,14, 16,29, 32,35, 36	5-3,8,11, 16,18, 24	5-14,23, 24,25, 29,36	5-24,36

	7-16	7-17	7-18	7-19	7-20
7-16					
7-17	6-44 5-3,6,21, 22,32, 37,38				
7-18	6-39,44 5-2,4,6, 13,21, 22,25, 26,32, 37,38	6-44 5-6,21, 22,27, 32,37, 38			
7-19	6-42 5-4,6,31, 38	5-6,38	6-5 5-4,5,6, 7,18, 19,38		
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 8       *   8
 9  2    * 4  9
10_______*_____10
11       *     *11
12       *     * 12
13       *     *  13
14  3    * 6   * 9 14
15_______*_____*____15
16       *     *     *16
17       *     *     * 17
18       *     *     *  18
19  5    * 8   * 12  *15 19
20_______*_____*_____*___20
21       *     *     *     *21
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26       *     *     *     *     *26
27       *     *     *     *     * 27
28  10   * 13  * 16  * 18  * 20  *  28
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Pcs	Prime Form	Interval Vector
8-1	0,1,2,3,4,5,6,7	765442
8-2	0,1,2,3,4,5,6,8	665542
8-3	0,1,2,3,4,5,6,9	656542
8-4	0,1,2,3,4,5,7,8	655552
8-5	0,1,2,3,4,6,7,8	654553
8-6	0,1,2,3,5,6,7,8	654463
8-7	0,1,2,3,4,5,8,9	645652
8-8	0,1,2,3,4,7,8,9	644563
8-9	0,1,2,3,6,7,8,9	644464
8-10	0,2,3,4,5,6,7,9	566452
8-11	0,1,2,3,4,5,7,9	565552
8-12	0,1,3,4,5,6,7,9	556543

(continued)

Pcs	Prime Form	Interval Vector (continued)
8-13	0,1,2,3,4,6,7,9	556453
8-14	0,1,2,4,5,6,7,9	555562
8-15	0,1,2,3,4,6,8,9	555553
8-16	0,1,2,3,5,7,8,9	554563
8-17	0,1,3,4,5,6,8,9	546652
8-18	0,1,2,3,5,6,8,9	546553
8-19	0,1,2,4,5,6,8,9	545752
8-20	0,1,2,4,5,7,8,9	545662
8-21	0,1,2,3,4,6,8,10	474643
8-22	0,1,2,3,5,6,8,10	465562
8-23	0,1,2,3,5,7,8,10	465472
8-24	0,1,2,4,5,6,8,10	464743
8-25	0,1,2,4,6,7,8,10	464644
8-26	0,1,2,4,5,7,9,10	456562
8-27	0,1,2,4,5,7,8,10	456553
8-28	0,1,3,4,6,7,9,10	448444
8-29	0,1,2,3,5,6,7,9	555553

Subsets of 8-pc sets, cardinality 7- and 6-

Pcs	Prime Form	Interval Vector	Number of occurrences/ Percentage of occurrence among pairs of 8-pc sets
7-1	0,1,2,3,4,5,6	654321	3 / .7%
7-2	0,1,2,3,4,5,7	554331	10 / 2.5
7-3	0,1,2,3,4,5,8	544431	10 / 2.5
7-4	0,1,2,3,4,6,7	544332	10 / 2.5
7-5	0,1,2,3,5,6,7	543342	10 / 2.5
7-6	0,1,2,3,4,7,8	533442	10 / 2.5
7-7	0,1,2,3,6,7,8	532353	10 / 2.5
7-8	0,2,3,4,5,6,8	454422	3 / .7
7-9	0,1,2,3,4,6,8	453432	10 / 2.5
7-10	0,1,2,3,4,6,9	445332	10 / 2.5
7-11	0,1,3,4,5,6,8	444441	10 / 2.5
7-12	0,1,2,3,4,7,9	444342	3 / .7
7-13	0,1,2,4,5,6,8	443532	10 / 2.5
7-14	0,1,2,3,5,7,8	443352	10 / 2.5
7-15	0,1,2,4,6,7,8	442443	3 / .7
7-16	0,1,2,3,5,6,9	435432	10 / 2.5
7-17	0,1,2,4,5,6,9	434541	3 / .7
7-18	0,1,2,3,5,8,9	434442	10 / 2.5
7-19	0,1,2,3,6,7,9	434343	10 / 2.5
7-20	0,1,2,4,7,8,9	433452	10 / 2.5

(continued)

Subsets of 8-pc sets, continued

Pcs	Prime Form	Interval Vector	Number and percentage of occurrences
7-21	0,1,2,4,5,8,9	424641	6 / 1.5%
7-22	0,1,2,5,6,8,9	424542	3 / .7
7-23	0,2,3,4,5,7,9	354351	10 / 2.5
7-24	0,1,2,3,5,7,9	353442	10 / 2.5
7-25	0,2,3,4,6,7,9	345342	10 / 2.5
7-26	0,1,3,4,5,7,9	344532	10 / 2.5
7-27	0,1,2,4,5,7,9	344451	10 / 2.5
7-28	0,1,3,5,6,7,9	344433	10 / 2.5
7-29	0,1,2,4,6,7,9	344352	10 / 2.5
7-30	0,1,2,4,6,8,9	343542	10 / 2.5
7-31	0,1,3,4,6,7,9	336333	10 / 2.5
7-32	0,1,3,4,6,8,9	335442	10 / 2.5
7-33	0,1,2,4,6,8,10	262623	3 / .7
7-34	0,1,3,4,6,8,10	254442	3 / .7
7-35	0,1,3,5,6,8,10	254361	3 / .7
7-36	0,1,2,3,5,6,8	444342	10 / 2.5
7-37	0,1,3,4,5,7,8	434541	3 / .7
7-38	0,1,2,4,5,7,8	434442	10 / 2.5

6-1	0,1,2,3,4,5	543210	21 / 5.2%
6-2	0,1,2,3,4,6	443211	55 / 13.5
6-3	0,1,2,3,5,6	433221	78 / 19.2
6-4	0,1,2,4,5,6	432321	36 / 8.9
6-5	0,1,2,3,6,7	422232	91 / 22.4
6-6	0,1,2,5,6,7	421242	36 / 8.9
6-7	0,1,2,6,7,8	420243	15 / 3.7
6-8	0,2,3,4,5,7	343230	36 / 8.9
6-9	0,1,2,3,5,7	342231	91 / 22.4
6-10	0,1,3,4,5,7	333321	78 / 19.2
6-11	0,1,2,4,5,7	333231	105 / 25.9
6-12	0,1,2,4,6,7	332232	78 / 19.2
6-13	0,1,3,4,6,7	324222	36 / 8.9
6-14	0,1,3,4,5,8	323430	55 / 13.5
6-15	0,1,2,4,5,8	323421	91 / 22.4
6-16	0,1,4,5,6,8	322431	91 / 22.4
6-17	0,1,2,4,7,8	322332	78 / 19.2
6-18	0,1,2,5,7,8	322242	91 / 22.4
6-19	0,1,3,4,7,8	313431	55 / 13.5
6-20	0,1,4,5,8,9	303630	6 / 1.5
6-21	0,2,3,4,6,8	242412	55 / 13.5
6-22	0,1,2,4,6,8	241422	55 / 13.5

(continued)

Subsets of 8-pc sets, continued

Pcs	Prime Form	Interval Vector	Number and percentage of occurrences
6-23	0,2,3,5,6,8	234222	36 / 8.9%
6-24	0,1,3,4,6,8	233331	78 / 19.2
6-25	0,1,3,5,6,8	233241	78 / 19.2
6-26	0,1,3,5,7,8	232341	36 / 8.9
6-27	0,1,3,4,6,9	225222	55 / 13.5
6-28	0,1,3,5,6,9	224322	36 / 8.9
6-29	0,1,3,6,8,9	224232	36 / 8.9
6-30	0,1,3,6,7,9	224223	36 / 8.9
6-31	0,1,3,5,8,9	223431	91 / 22.4
6-32	0,2,4,5,7,9	143250	21 / 5.2
6-33	0,2,3,5,7,9	143241	55 / 13.5
6-34	0,1,3,5,7,9	142422	55 / 13.5
6-35	0,2,4,6,8,10	060603	3 / .7
6-36	0,1,2,3,4,7	433221	66 / 16.3
6-37	0,1,2,3,4,8	432321	36 / 8.9
6-38	0,1,2,3,7,8	421242	36 / 8.9
6-39	0,2,3,4,5,8	333321	78 / 19.2
6-40	0,1,2,3,5,8	333231	105 / 25.9
6-41	0,1,2,3,6,8	332232	78 / 19.2
6-42	0,1,2,3,6,9	324222	36 / 8.9
6-43	0,1,2,5,6,8	322332	78 / 19.2
6-44	0,1,2,5,6,9	313431	55 / 13.5
6-45	0,2,3,4,6,9	234222	36 / 8.9
6-46	0,1,2,4,6,9	233331	78 / 19.2
6-47	0,1,2,4,7,9	233241	66 / 16.3
6-48	0,1,2,5,7,9	232341	36 / 8.9
6-49	0,1,3,4,7,9	224322	36 / 8.9
6-50	0,1,4,6,7,9	224232	36 / 8.9

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	8-1	8-2	8-3	8-4	8-5
8-1					
8-2	7-1,2 6-1,2,3, 4,8,9, 10,11, 36				
8-3	7-1 6-1,2,3, 4,36	7-1,3 6-1,2,3, 4,14, 15,36, 37,39, 40			
8-4	7-2,4 6-1,2,3, 5,8,9, 10,11, 12,13, 36	7-2,3,11 6-1,2,3, 8,9,10, 11,14, 15,16, 24,25, 36,37, 39,40	7-3 6-1,2,3, 14,15, 36,37, 39,40		
8-5	7-4,5 6-2,3,4, 5,6,9, 10,11, 12,13, 36	7-9,13 6-2,3,4, 9,10, 11,15, 16,21, 22,24, 36,37, 41,43	6-2,3,4, 15,36, 37	7-4,6 6-2,3,5, 9,10, 11,12, 13,15, 16,17, 18,19, 24,36, 37,38	

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	8-1	8-2	8-3	8-4	8-5
8-6	7-5 6-3,4,5, 6,9, 11,12	7-36 6-3,4,9, 11,23, 25,40, 41,43	6-3,4,40	7-14 6-3,5,9, 11,12, 18,25, 26,38, 40	7-5,7 6-3,4,5, 6,7,9, 11,12, 18,38, 41.43
8-7	6-1,5,36	7-3 6-1,14, 15,16, 36,37, 39,40	7-3 6-1,14, 15,36, 37,39, 40,44	7-3,6 6-1,5,14, 15,16, 17,19, 36,37, 38,39, 40	7-6 6-5,15, 16,17, 19,36, 37,38
8-8	6-5,6,36	6-16,36, 37,41	6-36,37, 44	7-6 6-5,16, 17,18, 19,36, 37,38	7-6,7 6-5,6,7, 16,17, 18,19, 36,37, 38,41, 43
8-9	6-5,6	6-41,43	6-42	6-5,18, 38	7-7 6-5,6,7, 18,38, 41,43
8-10	7-2 6-1,2,8, 9,10, 11,36	7-2 6-1,2,8, 9,10, 11,36, 40	7-10 6-1,2,27, 36,40, 42,45, 46	7-2 6-1,2,8, 9,10, 11,36, 40	6-2,9,10, 11,36

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	8-1	8-2	8-3	8-4	8-5
8-11	7-2	7-2,3,9	7-3	7-2,3	7-9
	6-1,2,8,9,10,11,36	6-1,2,8,9,10,11,14,15,21,22,24,36,37,39,40,41	6-1,2,14,15,36,37,39,40,46	6-1,2,8,9,10,11,14,15,24,36,37,39,40	6-2,9,10,11,15,21,22,24,36,37,41
8-12	7-4	7-8	7-10,16	7-4	7-4
	6-2,3,5,10,12,13,36	6-2,3,10,15,21,23,36,39,40	6-2,3,15,27,28,36,39,40,42,44,45,46	6-2,3,5,10,12,13,15,17,36,39,40	6-3,5,10,12,13,15,17,21,36
8-13	7-4	7-36	7-10	7-4	7-4
	6-2,3,5,10,11,12,13,36	6-2,3,10,11,23,25,36,40,41,43	6-2,3,27,36,40,42,45,46	6-2,3,5,10,11,12,13,18,25,36,40	6-2,3,5,10,11,12,13,18,36,41,43
8-14	7-5	7-11	7-17	7-11	7-5
	6-3,4,5,6,8,9,11,12	6-3,4,8,9,11,14,16,24,25,39,40	6-3,4,14,39,40,44,46	6-3,5,8,9,11,12,14,16,17,18,24,25,39,40	6-3,4,5,6,9,11,12,16,17,18,24
8-15		7-9	7-10	7-6	7-6,9
	6-2,5,9,12,36	6-2,9,16,21,22,24,25,36,37,40,41,43	6-2,27,28,36,37,40,42,45,46	6-2,5,9,12,16,17,18,19,24,25,26,36,37,38,40	6-2,5,9,12,16,17,18,19,21,22,24,36,37,38,41,43

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	8-6	8-7	8-8	8-9	8-10
8-6					
8-7	6-5,38, 40				
8-8	7-7 6-5,6,7, 18,38, 41,43	7-6 6-5,16, 17,19, 36,37, 38,44			
8-9	7-7 6-5,6,7, 18,38, 41,43	6-5,38	7-7 6-5,6,7, 18,38, 41,43		
8-10	6-9,11, 40	6-1,36 40	6-36	6-29,42	

	8-1	8-2	8-3	8-4	8-5
8-16	6-5,6,9,12	6-9,16,22,25,39,40,41,43	6-39,40,44,46	7-14 6-5,9,12,16,17,18,25,26,38,39,40	7-7,15 6-5,6,7,9,12,16,17,18,22,38,41,43
8-17	6-3,8	7-11 6-3,8,14,15,16,24,25,39	7-16 6-3,14,15,27,28,39,42,44	7-11 6-3,8,14,15,16,19,24,25,39	6-3,15,16,19,24
8-18	6-3,5,11,13	7-36 6-3,11,15,23,24,25,39,40,41,43	7-16 6-3,15,27,28,39,40,42,44	7-38 6-3,5,11,13,15,17,18,19,24,25,39,40	7-38 6-3,5,11,13,15,17,18,19,24,41,43
8-19	6-4,10	7-13 6-14,15,16,21,22,43	7-17 6-4,14,15,44,46	7-37 6-10,14,15,18,19,26	7-13 6-4,10,15,16,19,21,22,43
8-20	6-6,11,13	6-11,14,15,16,24	6-14,15,44	7-38 6-11,13,14,15,16,17,18,19,24	7-38 6-6,11,13,15,16,17,18,19,24

	8-6	8-7	8-8	8-9	8-10
8-11		7-3	7-12		7-2,23
	6-9,11, 40,41	6-1,14, 15,31, 36,37, 39,40	6-36,37, 41,47, 48,49	6-41	6-1,2,8, 9,10, 11,32, 33,36, 40,46, 47
8-12		7-18			7-10
	6-3,5,12, 23,40	6-5,15, 17,31, 36,39, 40,44	6-5,17, 36,44, 49	6-5,30, 42	6-2,10, 27,36, 40,42, 45,46
8-13	7-36		7-12	7-19	7-10,25
	6-3,5,11, 12,18, 23,25, 40,41, 43	6-5,36, 40	6-5,18, 36,41, 43,47, 49	6-5,18, 29,30, 41,42, 43	6-2,10, 11,27, 29,33, 36,40, 42,45, 46,47
8-14	7-5	7-18			7-23
	6-3,4,5, 6,9,11, 12,18, 25,40	6-5,14, 16,17, 31,39, 40,44	6-5,6,16, 17,18, 44,47, 48	6-5,6,18	6-8,9,11, 32,33, 40,46, 47
8-15	7-14	7-6	7-6	7-19	7-10
	6-5,9,12, 18,25, 26,38, 40,41, 43	6-5,16, 17,19, 31,36, 37,38, 40	6-5,16, 17,18, 19,36, 37,38, 41,43	6-5,18, 29,30, 38,41, 42,43	6-2,9,27, 29,36, 40,42, 45,46

	8-1	8-2	8-3	8-4	8-5
8-21		7-8,9			7-9
	6-2,9	6-2,9,21,22,23,24,37,39.41	6-2,37,39	6-2,9,24,37,39	6-2,9,21,22,24,37,41
8-22		7-11,36		7-11	
	6-3,8,9,11	6-3,8,9,11,14,16,22,23,24,25,40,41,43	6-3,14,39,40,46	6-3,8,9,11,14,16,24,25,26,39,40	6-3,9,11,16,22,24,41,43
8-23				7-14	
	6-8,9,12	6-8,9,25,40	6-40,46	6-8,9,12,18,25,26,38,40	6-9,12,18,38
8-24		7-13			7-13
	6-4,10	6-4,10,15,16,21,22,43	6-4,15,46	6-10,15,16,26	6-4,10,15,16,21,22,43
8-25					7-15
	6-12	6-21,22	6-28,45	6-12,17	6-7,13,21,22

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	8-6	8-7	8-8	8-9	8-10
8-16	7-7,14	7-18	7-7,20	7-7	
	6-5,6,7, 9,12, 18,25, 26,38, 40,41, 43	6-5,16, 17,31, 38,39, 40,44	6-5,6,7, 16,17, 18,38, 41,43, 44,47, 48	6-5,6,7, 18,38, 41,43	6-9,33, 40,46, 47
8-17		7-21			
	6-3,25	6-14,15, 16,19, 20,31, 39,44	6-16,19, 44	6-29,42	6-8,27, 29
8-18	7-36	7-18	7-22	7-19	
	6-3,5,11, 18,23, 25,40, 41,43	6-5,15, 17,19, 31,39, 40,44	6-5,17, 18,19, 41,43, 44	6-5,18, 29,30, 41,42, 43	6-11,27, 29,40, 42
8-19		7-21	7-22		
	6-4,26, 43	6-14,15, 16,19, 20,31, 44	6-16,19, 43,44, 49	6-43	6-10,46
8-20		7-21	7-20		
	6-6,11, 18	6-14,15, 16,17, 19,20, 31,44	6-6,16, 17,18, 19,44, 47,48	6-6,18	6-11,32, 47

	8-11	8-12	8-13	8-14	8-15
8-11					
8-12	7-26 6-2,10, 15,21, 31,34, 36,39, 40.46, 49				
8-13	7-12 6-2,10, 11,33, 36,40, 41,46, 47,49	7-4,10 6-2,3.5, 10,12, 13,23, 27,30, 36,40, 42,45, 46,49, 50			
8-14	7-23,27 6-8,9,11, 14,24, 31,32, 33,39, 40,46, 47,48	7-18 6-3,5,12, 17,31, 39,40, 44,46, 50	7-29 6-3,5,11, 12,18, 25,33, 40,46, 47,50		
8-15	7-9 6-2,9,21, 22,24, 31,34, 36,37, 40,41, 46	7-10,28 6-2,5,12, 17,21, 27,28, 30,31, 34,36, 40,42, 45,46	7-10,19 6-2,5,12, 25,27, 29,30, 36,40, 41,42, 43,45, 46	6-5,9,12, 16,17, 18,24, 25,31, 40,46	

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	8-1	8-2	8-3	8-4	8-5
8-26				7-37	
	6-10,11	6-10,11, 14,16, 21,22, 23,24, 25,36, 37	6-14,27, 28,45	6-11,14, 19,24, 25,26	6-10,11, 19,24
8-27					7-38
	6-10,11, 12,13	6-10,11, 15,21, 23,24, 25	6-15,27, 28,45, 46	6-10,11, 12,13, 15,17, 18,19, 24,25	6-10,11, 12,13, 15,17, 18,19, 21,24
8-28					
	6-13	6-23	6-27	6-13	6-13
8-29	7-5	7-13	7-16		7-5,13
	6-3,4,5, 6,9,10, 11,12	6-3,4,9, 10,11, 15,16, 21,22, 39,41, 43	6-3,4,15, 27,28, 39,42, 44,45	6-3,5,9, 10,11, 12,15, 16,17, 18,39	6-3,4,5, 6,9, 10,11, 12,15, 16,17, 18,21, 22,41, 43

	8-6	8-7	8-8	8-9	8-10
8-21	6-9,23,41	6-37,39	6-37,41,48	6-41	6-9,33
8-22	7-36 6-3,9,11,23,25,26,40,41,43	6-14,16,31,39,40	6-16,41,43,47,48	6-41,43	7-23 6-8,9,11,32,33,40,46,47
8-23	7-14 6-9,12,18,25,26,38,40	6-38,40	6-18,38,47	6-18,38	7-23 6-8,9,32,33,40,46,47
8-24	6-4,26,43	6-15,16,31	6-16,43,49	6-43	6-10,46
8-25	6-7	6-17	6-7,17	6-7,30	6-45

	8-11	8-12	8-13	8-14	8-15
8-16		7-18	7-29	7-18,20,29	7-14,30
	6-9,22,31,33,34,39,40,41,46,47,48	6-5,12,17,31,34,39,40,44,46,50	6-5,12,18,25,33,40,41,43,46,47,50	6-5,6,9,12,16,17,18,25,31,33,39,40,44,46,47,48,50	6-5,9,12,16,17,18,22,25,26,31,34,38,40,41,43,46
8-17		7-16		7-11	7-32
	6-8,14,15,24,31	6-3,15,27,28,31,39,42,44	6-25,27,29,42,	6-3,8,14,16,24,25,31,39,44	6-16,19,24,25,27,28,29,31,42
8-18		7-16,18,31	7-19,31,36	7-18	7-19,32
	6-11,15,24,31,39,40,41,49	6-3,5,13,15,17,23,27,28,30,31,39,40,42,44,49,50	6-3,5,11,13,18,23,25,27,29,30,40,41,42,43,49,50	6-3,5,11,17,18,24,25,31,39,40,44,50	6-5,17,18,19,24,25,27,28,29,30,31,40,41,42,43
8-19	7-26	7-26		7-17	7-30
	6-10,14,15,21,22,31,34,46,49	6-10,15,21,31,34,44,46,49	6-10,43,46,49	6-14,16,31,44,46	6-16,19,21,22,26,31,34,46
8-20	7-27			7-20,27	
	6-11,14,15,24,31,32,48	6-13,15,17,31,44	6-11,13,18,47	6-6,11,14,16,17,18,24,31,32,44,47,48	6-16,17,18,19,24,31

	8-6	8-7	8-8	8-9	8-10
8-26					7-25
	6-11,25, 26	6-14,19, 31	6-19,47, 48	6-29	6-10,11, 27,29, 32,33, 45,47
8-27					7-25
	6-11,12, 18,23, 25	6-15,17, 19,31	6-17,18, 19,47, 49	6-18,29, 30	6-10,11, 27,29, 33,45, 46,47
8-28		*			
	6-23		6-49	6-30	6-27
8-29	7-5		7-20	7-19	7-25
	6-5,25	6-5,15, 16,17, 39,44	6-5,6,16, 17,18, 41,43, 44,47, 48	6-5,6,18, 29,30, 41,42, 43	6-9,10, 11,27, 29,33, 42,45, 47

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8-21	7-9,24 6-2,9,21,22,24,33,34,37,39,41,48	7-8 6-2,21,23,34,39	6-2,23,33,41	6-9,24,33,39,48	7-9 6-2,9,21,22,24,34,37,41
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	6-5,6,9,12,16,17,18,22,33,34,39,41,43,44,47,48	6-3,15,16,27,28,29,39,42,44	6-3,5,11,15,17,18,27,28,29,30,39,41,42,43,44	6-4,10,15,16,21,22,34,43,44	6-6,11,15,16,17,18,44,47,48

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