

PLACEMENT OF PHASOR MEASUREMENT UNITS: OPTIMIZATION
APPROACHES AND APPLICATIONS

By

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ABSTRACT

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Phasor measurement units (PMUs) provide measurements with high precision at a high resolution (up to 50 samples per second). These measurements are synchronized and time-stamped using the Global Positioning System. Despite these advantages, the industry has been slow in adopting PMU technology due to the high cost of installing PMUs. Therefore, PMU locations must be judiciously selected through optimal placement of PMUs (OPP), which enables the minimization of installation cost.

This dissertation examines the OPP problem from several perspectives. First, the OPP problem definition is re-examined since most OPP literature associates the PMU installation cost with the PMU unit. Most techniques in the literature have proposed to minimize the number of PMUs while considering the complete observability of the system. However, PMUs require sufficient infrastructure to be in place before they can perform most of their intended functions. Therefore, the OPP problem should be reformulated to include the supporting infrastructure of PMUs. The proposed OPP formulation is implemented by using a bi-level framework. This framework can accommodate different varieties of OPP, such as single-stage, multistage, and application-based approaches. Moreover, the proposed framework achieves the optimal solution with the maximum observability in the case of multiple optima.

Second, this dissertation examines application-based OPP approaches, where specific technical benefits are prioritized over the cost of the OPP. Three applications are proposed. The first application is a fault-tolerance based OPP approach, where the network fault-

tolerance is enhanced by deploying PMUs to the vulnerable elements in the network. In the second application, a voltage stability criterion is developed and proposed, where the critical buses are identified and prioritized for PMU allocation. The third application addresses false data injection attacks (FDIAs), where the system topology is used by the adversary to bypass the bad data detection of state estimators. The proposed OPP approach enhances bad data detection against FDIAs by utilizing the PMUs as authenticators for each other.

To my parents, my brothers and my sisters.

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Chapter 1

Introduction

The first phasor measurement unit (PMU) prototype was developed by Virginia Tech in the early 1980s, and the first commercial PMU was manufactured in 1991 by Macrodyne and Virginia Tech. [1, 2]. However, at the time PMUs were perceived to be expensive solutions. Remote terminal units (RTUs) were used to monitor the power grid and provide sufficient situational awareness. RTUs were used to get the conventional measurements. These measurements include power injections and power flows, where the RTUs are sampled every few seconds for state estimation. Due to the type of measurements of RTUs, non-linear and iterative estimators were needed. Although the conventional measurements were not time-stamped and sampled at a relatively slow rate (once every few seconds), they were sufficient for system operations, since the system is assumed to be in quasi-steady-state.

Nevertheless, as the electrical industry is heading towards the smart grid technology, the quasi-steady-state assumption has become less valid. The penetration of renewable energy resources is increasing, and new smart controls are being introduced. Due to these reasons, better measurements are needed, thereby requiring more precise and faster measurement (units) [1]. The development of the time-synchronized PMUs has played a crucial role in addressing these challenges.

PMUs measure the phasor components of the voltage and the current at high rates (up to 50 Hz). These measurements are synchronized and time-stamped using the Global Positioning System (GPS) [3]. Currently, PMUs can measure the operating frequency and

frequency deviation along with the voltage and current phasors.

The advent of PMUs has enabled the development of many applications. For instance, [4–6] have used synchronized measurements to enhance wide area protection and control. Other applications have included estimating the dynamic states (speed and angle) of machines, small signal stability, and frequency stability [7–10]. Most applications of PMUs have depended on getting an accurate awareness of the system states. Therefore, state estimation has become the primary application of PMUs. State estimation usually refers to estimating the voltage magnitude and phase angle for every bus in the system. Moreover, when PMUs provide complete observability, state estimation becomes a linear process [11, 12]. Despite these advantages, the industry has been slow in adopting PMU technology due to the high cost of installing PMUs. Therefore, PMU locations must be judiciously selected through optimal placement of PMUs (OPP) which enables the minimization of installation cost.

1.1 Motivation and Challenges

Although PMUs offer huge advantages over RTUs, the industry has been slow in adopting this technology, due to the high cost of PMUs. This slow adoption, and the high cost associated PMU technology has driven the optimal PMU placement (OPP) problem. In OPP, the PMUs are judiciously selected such that maximum observability is achieved with the minimum cost.

The OPP literature can be classified into two categories: observability-based OPP and application-based OPP. The observability-based OPP aims to achieve complete observability for the power grid while minimizing the total cost of installing PMUs.

On the other hand, application-based OPP focuses on other benefits besides the system

observability, such as transient stability, state estimation, and bad data detection [13–15]. Most of the application-based OPP approaches use the traditional definition of the OPP problem, where the objective is to minimize the number of PMUs.

There have been several observability-based OPP approaches in the literature where the cost of installing PMUs is associated with the PMU itself. In these approaches, the number of PMUs is minimized while considering the observability of the grid [16–19]. Most OPP literature associates the PMU installation cost with the cost of the PMU unit. As a result, most of these techniques have been proposed to minimize the number of PMUs while considering the complete observability of the system [16–23]. However, the PMU unit cost only 5% of the installation of the PMU and 95% of the budget is spent on upgrading the supporting infrastructure for the PMU [24]. Therefore, the OPP problem should be reformulated to include the installation cost of PMUs.

1.2 Contributions

The contributions of the work presented here can be summarized as follows:

- It develops a comprehensive cost model for the OPP problem, including the cost of the PMUs as well as the infrastructure upgrade costs.
- It presents a flexible, multistage deployment plan, implemented over a period of time depending on the budget of the utility company.
- It affords the ability to prioritize PMU placement based on specific criteria such as bus criticality, thereby enabling application-based deployment.
- It improves the fault-tolerance of the system by enhancing the observability of critical

components, which are determined using vulnerability analysis. This analysis is based on the reliability indices of the composite system.

- It integrates the vulnerability analysis into the observability function. This integration enables cost-efficient OPP solutions while considering vulnerable buses. This integration of critical buses can be expanded to include multiple criteria for application-based OPP approaches.
- It uses a cost model for the installation of PMUs which is derived from the industry [24, 25]. Unlike most application-based OPP approaches which try to minimize the number of PMUs as in [13–15, 26], this work uses the installation cost to achieve a more realistic strategy.
- It can be used as a multi-stage process under a constrained budget, or as a single-stage process while minimizing the total cost.
- It can achieve the optimal solution with the maximum observability in the case of multiple optima due to the nature of the bi-level formulation.
- It enhances the security of the grid against false data injection attacks (FDIAs) through an OPP approach.

1.3 Organization of the Thesis

This thesis is organized as follows:

Chapter 2 focuses on observability-based OPP with a realistic scenario where PMUs are deployed over several budget periods. In this chapter, a comprehensive PMU installation cost

is used, where both the communication and the substation infrastructures are considered. This chapter focuses on the observability and the financial aspects of the OPP problem.

Chapter 3 introduces an application-based OPP to enhance the fault-tolerance of the network. In this application-based, the value of high assets is increased by deploying these assets on vulnerable elements of the network.

Chapter 4 introduces an application-based OPP approach, where voltage stability criterion is developed for prioritizing critical buses. This application-based approach assumes a multistage scenario, where PMUs are deployed over several budget periods.

Chapter 5 examines the security of the grid against false data injection attacks (FDIA). This chapter also introduces a secure OPP approach to enhance the resilience against such attacks. The proposed OPP formulation does not make any assumptions about the security of PMUs and enhances the bad data detection algorithms.

Chapter 6 provides concluding remarks and avenues for future research.

Chapter 2

Multistage Optimal PMU Placement Considering Substation Infrastructure

2.1 Introduction

In this chapter, a multistage PMU placement strategy is proposed which considers two factors: substation cost (including PMU cost) and communication infrastructures. The prioritization of critical buses can be integrated with the proposed approach. This approach can be used in an incremental way where PMUs are installed in multiple stages under a constrained cost. Unlike most multistage approaches, this approach does not consider a predetermined number of PMUs at each stage. Instead, the multistage installation maximizes the network observability and prioritizes critical buses while remaining within a predetermined budget for each stage.

In OPP, buses having one or more of the following criteria: high voltage buses, high impact on transient stability, or sensitive loads, are considered critical buses [13,14]. These buses are sometimes given higher priority to enhance system awareness from a stability perspective. Other researchers have proposed prioritizing buses based on different criteria such as reliability and state estimation [27–29]. Bus prioritization can be integrated with

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the proposed approach while considering both observability and the actual cost of the PMU installation. The major contributions of this work may be summarized as follows.

- It develops a comprehensive cost model for the OPP problem, including the cost of the PMUs as well as the infrastructure upgrade costs.
- It presents a flexible, multistage deployment plan, implemented over a period of time depending on the budget of the utility company.
- It affords the ability to prioritize PMU placement based on specific criteria such as bus criticality, thereby enabling application-based deployment.

2.2 PMU Installation Cost

This section discusses the cost of PMU installation. It presents the cost model for upgrading a substation and the cost of the communication infrastructure for PMU installation.

2.2.1 Substation Infrastructure

Most of the cost is associated with the installation process of the PMU and not the PMUs themselves. In fact, the PMUs cost about 5% of the total installation cost [24]. Most of the cost is spent on upgrading the substation and communication infrastructures. A report recently published by the U.S. Department of Energy (DOE) showed that the PMU installation cost ranges from \$40,000 to \$180,000 per PMU [24].

The cost varies depending on the infrastructure support for the PMUs. Typically, PMUs need sufficient communication infrastructure to send the measurement data to the PDC. The substation infrastructure also needs to be sufficient to utilize the functionalities of PMUs.

Formulating the installation cost for the PMUs is a complicated process. Although PMU installation requires the same infrastructure upgrades, such as communication, cyber-security, and other equipment upgrades, the approach to installing PMUs can differ depending on the utility and existing infrastructure support for the PMUs. For instance, a utility can install new stand-alone PMUs, or upgrade existing digital relays to enable PMU functionality [24]. Moreover, installing PMUs also depends on the availability of CTs and PTs [30]. Once the infrastructure of the substation is in place to support the PMUs, the installation cost can go down to 35% of the initial cost [24].

The proposed cost model of PMU installation in (2.1) considers the difference between prepared buses, where minimal upgrades are needed, and unprepared buses, by introducing g_i index. The index takes the value of 4.5 for unprepared buses and 1 for prepared buses. The model also includes the cost of adding additional measurement channels by including the cost of PTs and CTs.

$$\text{Cost} = \sum_{i=1}^N g_i (aP_i + b_i P_i) + K(P) \quad (2.1)$$

where

N number of buses in the system;

g_i prepared bus index;

a cost of installing PMU and basic upgrades at the substation;

b_i cost for installing additional PT or CT at substation i ;

$K(P)$ cost function for the communication infrastructure for

PMUs;

$$P = [P_1, P_2, \dots, P_i, \dots, P_N]^T;$$

where P_i takes the value of zero or one and indicates if a PMU is to be installed at substation i .

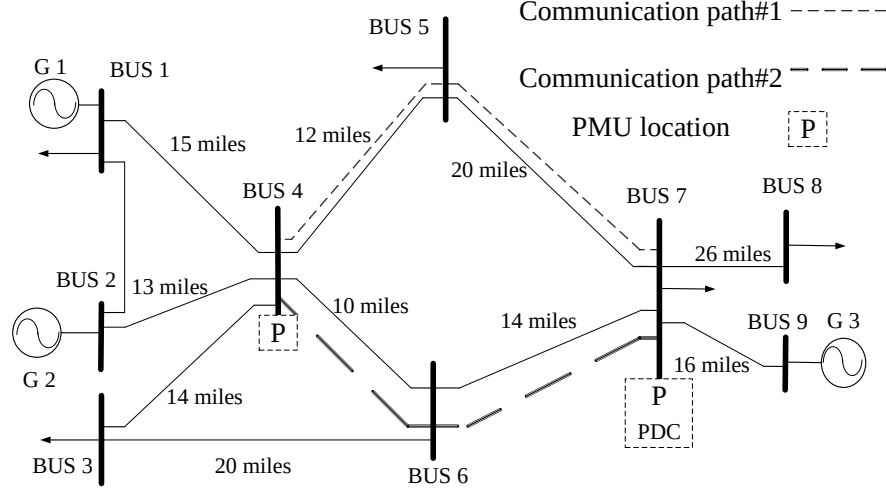


Figure 2.1: Sample 9-bus system with possible communication paths

2.2.2 Communication Infrastructure

The measurement data obtained by the PMUs are sent to the PDC, where the PDC sorts the data and processes it for other applications. Mohammadi *et al.* [31] have proposed to reduce the distance between the PMUs and the PDC to lower the total cost. In [31], it is also have proposed to place the PDC on a non-PMU bus to minimize the total communication distance. The work in [31], however, has not considered the cost of upgrading the substation for PMU installation. In this chapter, the PDC is assumed to be installed at one of the substations where a PMU is to be placed. Then, the path connecting all PMUs is minimized to lower the communication infrastructure cost (2.2).

There can be several communication paths connecting all PMUs at different substations. Consider the 9-bus system in Fig. 2.1; the PMUs are placed at buses 4 and 7 to make the system observable. However, there are two communication paths to connect both PMUs with

the PDC located at bus 7. As seen in Fig. 2.1, the first communication path is about 32 miles, and the second one is about 24 miles. Therefore, in order to minimize the communication infrastructure cost, the path connecting all PMUs with PDC needs to be minimized. The communication infrastructure is assumed to be passive optical network (PON) with optical ground wire (OPGW). The cost model in (2.2) is derived from [32, 33].

$$\min K(P) = \sum_{j=1}^n \text{len}_{i,j} \cdot \text{cc}_{i,j} + N_e \cdot P_j + N_b \cdot P_j + \text{PDC} \quad (2.2)$$

where n is the number of PMUs and $\text{len}_{i,j}$ is the length of the transmission line between buses i and j . The communication cost cc is either \$2,414 or \$0 per mile [34]. N_e represents the passive cost of the communication infrastructure such as the housing chassis, optical switch, wave filters. N_b , on the other hand, represents the cost per additional channel. N_e and N_b are assumed to be \$5,530 and \$125 respectively [32]. PDC is assumed to have a total cost of \$7,500 [35].

2.3 Proposed Approach

This section presents the approach for the multistage OPP. As discussed in the previous sections, the PMU installation cost plays a critical role in determining the installation process. In the multistage approach, maximizing the benefits of PMU installation takes higher priority over the cost function. This approach assumes that the utility sets a budget for the installation of PMUs and the first objective is to maximize the observability and priority buses while minimizing the installation cost and not exceeding the predetermined budget for the current stage.

2.3.1 Problem Statement

As mentioned earlier, the optimal placement for PMUs highly depends on the installation cost and available budget. In the proposed multistage approach, the observability is maximized, subject to the observability constraint described below, and the cost function is minimized. The observability in (2.3) needs to have enough redundancies for the desired observability conditions. For instance, under normal operating conditions, the observability constraint in (2.4) must be satisfied.

$$\mathcal{O} = H \times P \quad (2.3)$$

$$\mathcal{O} \geq I' \quad (2.4)$$

where P is a vector of length equal to the number of buses N , as described in section 2.2.1; I' is a vector of length N with all its elements equal to 1; and H is an $N \times N$ connectivity matrix. The entries for P and H are defined in (2.5) and (2.6) respectively.

$$P_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{if no PMU is at bus } i \end{cases} \quad (2.5)$$

$$h_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases} \quad (2.6)$$

In the proposed approach, the observability function in (2.7) is treated as a higher level objective function, and is subjected to minimizing the cost function. The cost function in (2.9) is treated as the lower level objective function, subjected to the higher level objective

(observability function). This setup allows maximizing the observability while minimizing the cost without violating the budget constraint, thereby reaching the optimal solution for the given budget. It should be noted that during the multistage process complete observability in (2.4) cannot be achieved; therefore the observability constraint is changed to the multistage condition in (2.8).

$$\max \sum_{i=1}^N \mathcal{O}'_i \quad (2.7)$$

subject to

$$\mathcal{O} \leq \bar{\mathcal{O}} \quad \text{“multistage observability condition”} \quad (2.8)$$

$$\min \text{Cost} = \sum_{i=1}^N g_i(a_i P_i + b_i P_i) + K(P) \quad (2.9)$$

subject to

$$C \leq C_{budget}$$

2.3.1.1 Priority Buses

For application-based OPP schemes, some buses are prioritized for PMU installation regardless of their contribution to the overall observability. These schemes range from stability criteria to reliability, and many others [13–15, 27]. In the proposed OPP scheme, priority buses can be chosen using any criterion.

The priority buses for the network are embedded in the observability function as bias using a priority vector R (2.10). The R vector has the length of the number of buses N . If all buses are treated equally then all elements of R are set to zero. The higher priority buses are determined based on the utility criteria, then arranged in descending order in a vector

L . Then the priority bias is assigned using the following algorithm.

Procedure 1 Priority Vector R

Initialize priority buses (L), $R = \text{zeros}_{1 \times N}$

N = number of buses

M = maximum number of branches in (H)

kk = length of L

for $j = 1 : kk$

$i = l(j)$

if $i \neq 0$ then

$r_i = M(kk - j + 1)$

else

$r_i = 0$

endif

endfor

Maximizing the modified observability vector $\mathcal{O}'$ gives bias to the higher priority buses. However, this vector cannot be used to test the observability of the network since the R vector skews the observability. As a result, the skewed observability in (2.7) is used for optimizing the OPP, and the original observability in (2.3) is used as a constraint for observability testing.

$$h'_{ij} = \begin{cases} 1 + r_i, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases} \quad (2.10)$$

where

$$\mathcal{O}' = H' \times P;$$

r_i bus i priority index;

N number of buses.

The cost of the overall PMU installation can be reduced by considering the effect of zero-injection buses (ZIB). Considering the effect of ZIBs improves the overall observability of the system, thereby reducing the number of PMUs needed to achieve the observability constraint. The effect of ZIB can be summarized into two points. If all buses connected to a ZIB are observable, the ZIB is considered observable by applying KCL. Also, an unobservable bus, when connected to an observable ZIB, is considered observable only if all of the other buses connected to the ZIB are observable.

2.3.2 Algorithm

The optimal placement problem in subsection 2.3.1 is a discontinuous bi-level problem. It also involves optimizing the communication infrastructure cost $K(P)$ within the installation cost in (2.9). A multisource Dijkstra algorithm is used to obtain the shortest path connecting all PMUs and PDC. Since the proposed model involves optimizing three objective functions (2.2), (2.7) and (2.9), evolutionary algorithms are the appropriate tools for solving such a problem. The proposed algorithm uses an opposition-based elitist binary genetic algorithm (O-BEBGA) to solve the bi-level OPP in subsection 2.3.1. The opposition element is added to the algorithm to enhance the overall performance since opposition-based methods have proven their superiority in terms of convergence speed and results [36–38].

2.3.2.1 Multisource Dijkstra

The design of the communication infrastructure involves finding the most cost effective path (2.2) between the PMUs and PDC. There are several algorithms that can be used to find this path such as Floyd-Warshall, Bellman-Ford, Johnson and Dijkstra algorithms. The Dijkstra algorithm is among the most efficient algorithms for single source undirected weighted graphs [39]. However, the communication network design problem is not a single source/destination problem; rather, it is a multisource single destination problem, or a single source/destination with a must pass nodes. The traditional Dijkstra algorithm can still be used to solve this problem. This entails using the Dijkstra algorithm n times to establish one communication line between two source nodes out of n source nodes. The multisource Dijkstra algorithm, on the other hand, can be used to pair up source nodes in one run.

The multisource Dijkstra is used to find the shortest paths P_{x1} connecting every source node s_i to the nearest source node s_j , where $(s_i, s_j \in S)$. This step generates a set $G = \{\phi_1, \phi_2, \dots, \phi_{n_1}\}$ with n_1 subsets, where $n_1 = \text{floor}(N_s/2)$ and N_s is the number of source nodes. Each subset ϕ has at least two connected source nodes. The next step is to connect the subsets in G to each other. First, the weights for the paths in P_{x1} are set to zero. The multisource Dijkstra is then used to obtain new paths P_{x2} that connect the subsets in G . It should be noted that the new paths P_{x2} , may have redundant routes, however these redundant routes have zero weight. The process of finding new paths and updating their weights is repeated until all the subsets in G are connected with one path. This path is the union of all paths P_{xi} obtained from the multisource Dijkstra algorithm.

The sample graph in Figure 2.2 demonstrates the implementation of multisource Dijkstra algorithm for the communication network design. The source nodes in S are a, d, f and j .

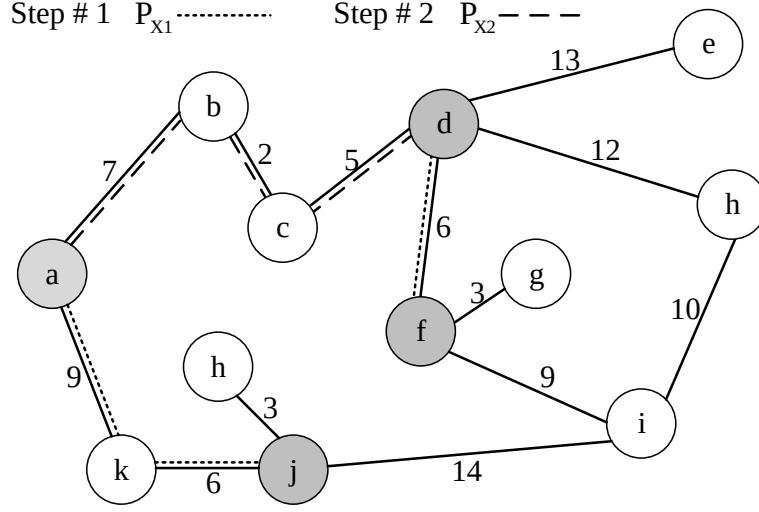


Figure 2.2: Sample graph to demonstrate the implementation of multisource Dijkstra algorithm.

The first loop of the multisource Dijkstra generates the G set with subsets ϕ_1 and ϕ_2 . The ϕ_1 subset contains the source nodes a and j ; the ϕ_2 subset contains source nodes d and f . The P_{x1} paths for the subsets in G are $\{a-k-j, d-f\}$. The next step is to connect the subsets ϕ_1 and ϕ_2 , which produce the path $P_{x2}=\{a-b-c-d\}$. The union of the paths P_{x1} and P_{x2} produces the shortest path connecting all the source nodes in S .

2.3.2.2 O-BEBGA

As discussed previously, multistage approaches can lead to higher cost for the overall installation of PMUs, since the solutions for each stage are often sub-optimal for the complete observability. This is because maximizing the observability at each stage increases the installation cost [40]. To overcome this issue, the O-BEBGA solves the OPP for the desired observability condition first. Then, the optimal solution for the complete observability (X_s) is used as optimal solution for the multistage installation. To enhance the performance further, the search space multistage installation is reduced to include only optimal location of

Candidates					Cost	$\mathcal{O}'$	Rank
Bus 1	Bus 2	Bus 3	Bus 4	...			
1	0	1	1	...	62000	18.3	1
1	0	1	0	...	54000	14.6	2
1	1	0	0	...	55000	14.6	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
0	0	0	1	...	3000	2.9	n-2
0	0	1	1	...	2400	2.2	n-1
0	1	0	1	...	1400	1.4	n

Feasible

Not Feasible

Figure 2.3: Sorting function.

PMUs in X_s .

The proposed algorithm solves the optimal placement problem in a parallel manner by initializing random candidates where each candidate x_i has the length of the number of buses N , thereby evaluating the candidate buses simultaneously instead of using systematic increments. By maximizing the observability function (higher objective), while minimizing the cost of PMU installation, the predetermined budget C_{budget}^k for each stage (k) is optimally utilized.

The higher and lower objectives share the same decision variables, meaning there are no decision variables exclusive to one objective or the other. The proposed approach exploits this advantage to evaluate both objectives simultaneously without using different search spaces for each objective. The proposed approach uses a sorting function to handle the simultaneous evaluation in the same search space. This function sorts all candidates according to their feasibility and fitness of the higher and the lower objectives, as seen in Fig. 2.3. As a result, the algorithm is guided towards the optimal solution where the cost is minimized and the observability is maximized.

The overall flowchart for the proposed algorithm is shown in Fig. 2.4. The crossover and mutation probabilities are $P_c = 0.7$ and $P_m = 0.3$ respectively. The O_{rn} donates the

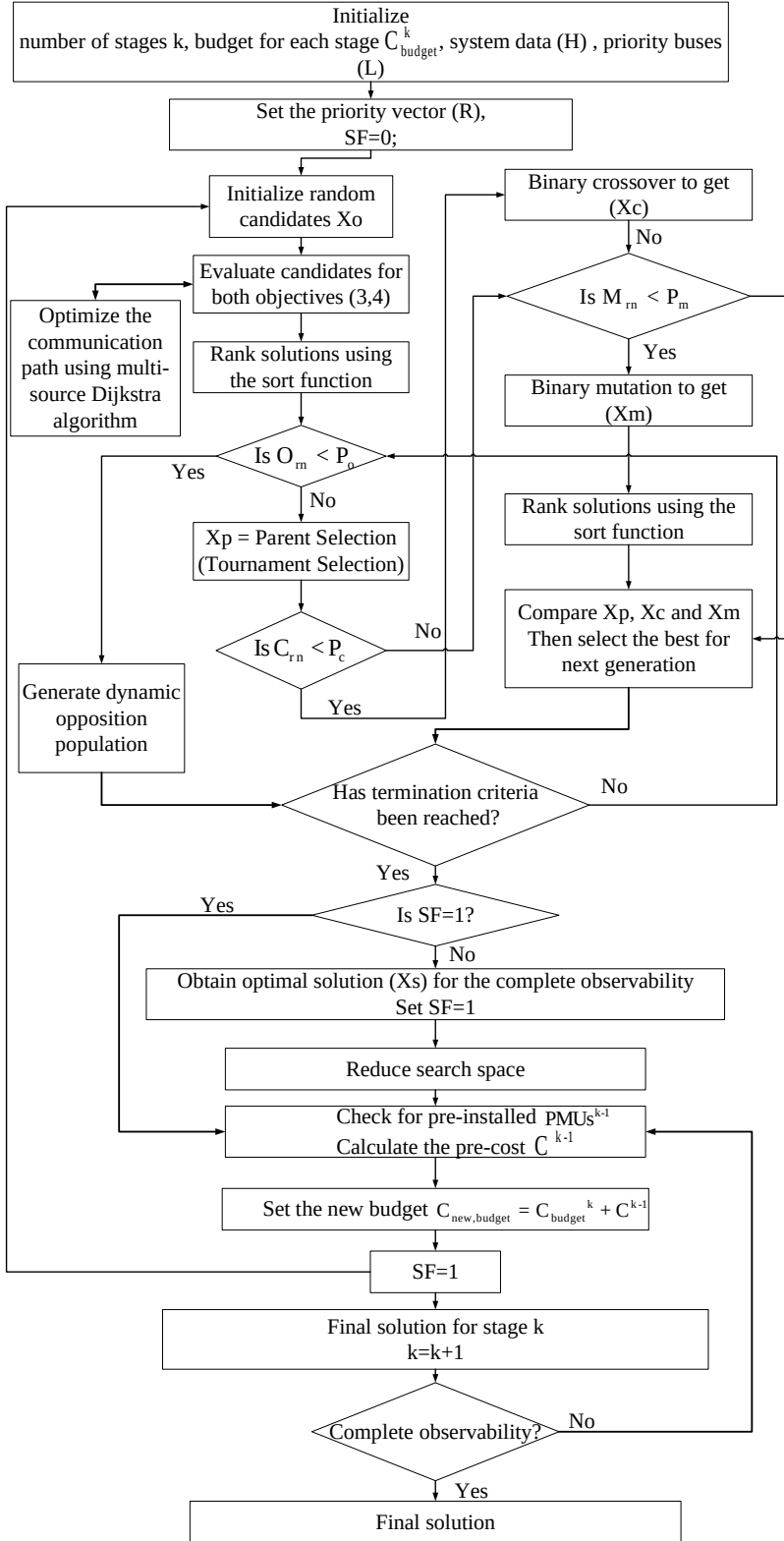


Figure 2.4: Flow chart of the proposed O-BEBGA algorithm.

opposition random variable; the crossover and mutation random variables are denoted by C_{rn} and M_{rn} respectively. Double point crossover is used to generate the offspring population X_c , and single point mutation is used to generate the mutated population X_m . The algorithm uses dynamic opposition with probability of $P_o = 0.4$.

There are many variations of opposition techniques in the literature. The two most common are the global opposition and the dynamic opposition. In the proposed algorithm, a modified dynamic opposition is used to generate the opposition population when applicable. Instead of generating the total opposite of the chosen individual X_i , only a third of the variables in X_i are selected for the opposition process (2.11).

$$X_i^{\text{opp}} = X^{\min} + X^{\max} - X_i \quad (2.11)$$

The algorithm is terminated if the conditions in (2.12) are met. The terms α and β are constants; where $\gamma(1)$ indicates how much of the current population is feasible and $\gamma(2)$ indicates if the population is converging to an optimum. The variance of the cost ($Var[C]$) is used to determine $\gamma(2)$, where k is the index for the current population.

$$\gamma = \begin{cases} \gamma(1) = \left| \overline{\mathcal{O}} - \max(\mathcal{O}) \right| \leq \alpha \\ \gamma(2) = \left| Var[C^{k-1}] - Var[C^k] \right| \end{cases} \quad (2.12)$$

2.3.3 Multistage Installation

In realistic scenarios, PMUs are installed in a multistage manner. The majority of existing multistage methods assume the minimum number of PMUs per stage. This assumption is unrealistic since PMU installation is restricted by the financial burden, substation infrastruc-

ture, and technical benefits at each stage. The proposed approach uses a financial capital C_{budget} , as the limit for each stage instead of using the number of PMUs as the limit.

At each stage k , the previously installed PMUs (PMU^{k-1}) are initiated, and the pre-installation cost C^{k-1} is calculated. The budget for each stage C_{budget}^k is set. Then the budget C_{budget}^k is modified to include the pre-installed PMUs cost C^{k-1} . The pre-installed PMU locations are maintained during the initialization of the random population X_o and during the mutation step. Since the pre-installed PMU locations are maintained for the initial and parent populations, the crossover population X_c maintains the pre-installed PMU locations by default.

$$C_{new,budget}^k = C_{budget}^k + C^{k-1} \quad (2.13)$$

2.3.3.1 Complete observability

Initially, the multistage approach cannot achieve complete observability; however as stages are added, or more money is added to the budget, the complete observability constraint in (2.14) is satisfied.

$$H \times P = \mathcal{O} \geq \dot{\mathbf{I}} \quad (2.14)$$

2.3.3.2 Observability under single line outage

It is required to have at least two measurement redundancies for every bus in the network to achieve observability under a single line outage. Therefore, the observability constraint is changed to (2.15).

$$H \times P = \mathcal{O}_s \geq \ddot{\mathbf{I}} \quad (2.15)$$

$$\mathcal{O}_{s,i} = \begin{cases} \sum(a_{ij} \times p_i) \\ 2, & \text{if a PMU is installed at bus } i \end{cases} \quad (2.16)$$

where $\mathcal{O}_s$ is the observability vector for all buses in the system and $\tilde{\mathbf{I}}$ is a vector of length equal to the number of buses, with all entries equal to 2.

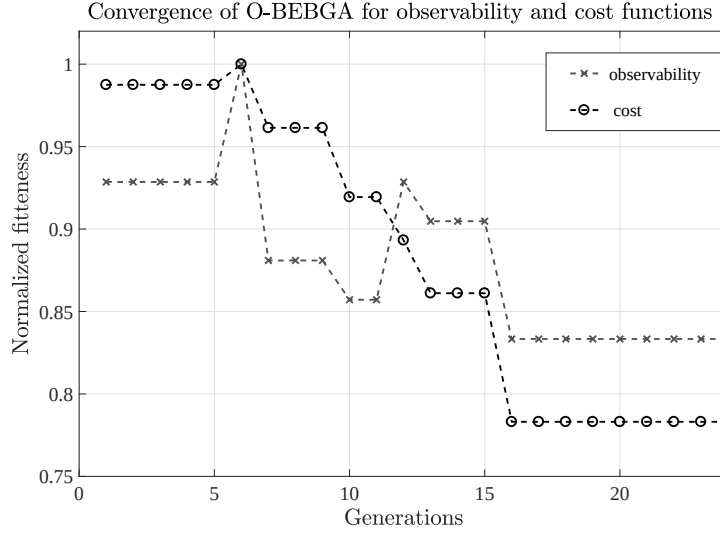


Figure 2.5: Convergence of O-BEBGA for the normalized observability and cost functions.

2.3.3.3 Single PMU outage

In a single PMU outage, every bus needs to have two independent measurements, either by two different PMUs or if ZIB effect is considered through KCL and a PMU. Therefore, the observability constraint in (2.8) is changed to the following:

$$H \times P = \mathcal{O}_p \geq \tilde{\mathbf{I}} \quad (2.17)$$

$$\mathcal{O}_{p,i} = \sum(a_{ij} \times p_i) \quad (2.18)$$

2.4 Simulation and Results

In this section, the proposed approach is tested on the IEEE reliability test system (RTS), IEEE 14-bus, 30-bus and 118-bus test systems. In subsection 2.4.1 all buses are treated equally and no prioritization is given to any bus. The higher priority buses and other observability conditions are tested in subsection 2.4.2. The buses are divided into two categories: prepared buses and unprepared buses. The prepared buses are assumed to have sufficient infrastructure, require basic security, network upgrades, and cost 75% less than the unprepared buses [24].

The base cost per PMU is assumed to be \$40,000, and the cost per additional PT or CT is assumed to be \$2,380 [41]. The cost of PDC is assumed to be \$7,500. The length of the transmission lines are obtained from [42]. The communication links are assumed to be running along the transmission lines where the cost of the communication links is assumed to be \$2,414 per mile [34] or \$0 if the communication link already exists.

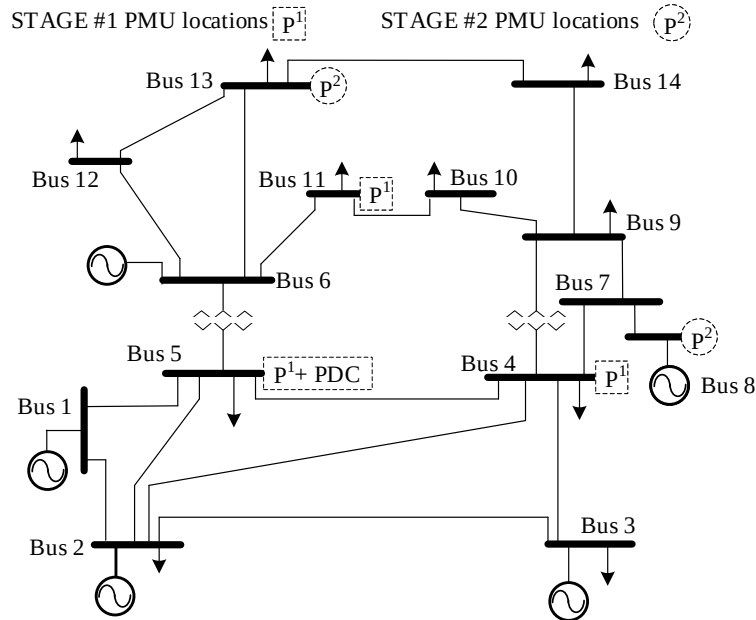


Figure 2.6: OPP solution for the IEEE 14-bus; not considering effect of ZIB.

Table 2.1: Multistage approach, not considering effect of ZIB

System	PMU locations	Stage budget C_{budget}^k	Cost	Remaining	Unprepared buses
First Stage ($k = 1$); Effect of ZIB Is Not Considered					
IEEE 14-bus	4, 5, 11	\$300,000	\$252,189	\$47,811	7, 9
IEEE RTS	15, 16, 21	\$350,000	\$213,522	\$136,478	10, 11, 17, 24
IEEE 30-bus	15, 16, 26	\$450,000	\$407,745	\$42,255	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 15, 19, 21, 30, 34, 45, 49, 66, 68, 77, 84, 89, 92, 105	\$4,000,000	\$3,910,268	\$89,732	—
Second Stage ($k = 2$); Effect of ZIB Is Not Considered					
IEEE 14-bus	4, 5, 8, 11, 13 ^a	\$200,000	\$121,656	\$78,344	7, 9
IEEE RTS	5, 6, 8, 15, 16, 21	\$350,000	\$286,227	\$63,773	10, 11, 17, 24
IEEE 30-bus	6, 7, 11, 15, 16, 22, 26	\$350,000	\$235,776	\$114,224	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 9, 15, 19, 21, 27, 30, 34, 40, 45, 49, 52, 56, 59, 66, 68, 71, 77, 80, 84, 89, 92, 105, 110	\$2,000,000	\$1,987,498	\$41,932	—
Third Stage ($k = 3$); Effect of ZIB Is Not Considered					
IEEE RTS	5, 6, 8, 9, 15, 16, 21, 23 ^a	\$350,000	\$266,245	\$83,755	10, 11, 17, 24
IEEE 30-bus	3, 6, 7, 11, 13, 15, 16, 20, 22, 26, 29 ^a	\$350,000	\$317,741	\$132,259	9, 12, 25, 27, 28
IEEE 118-bus	2, 5, 9, 12, 15, 19, 21, 27, 30, 31, 32, 34, 36, 40, 45, 49, 52, 56, 59, 63, 66, 68, 70, 71, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110, 118 ^a	\$2,000,000	\$1,022,043	\$977,957	—

^aComplete observability is achieved.

The proposed algorithm is used with a population size of $3 \times N$, where N is the number of buses. The performance of the O-BEBGA is shown in Fig. 2.5. Although the algorithm maximizes the observability, the minimization of the PMU installation cost drives the observability to a cost effective solution. It should be noted that maximizing observability often increases the cost. However, there exist cases where the same or better observability can be achieved at a better cost, as is the case for generations 5 and 12 in Fig. 2.5.

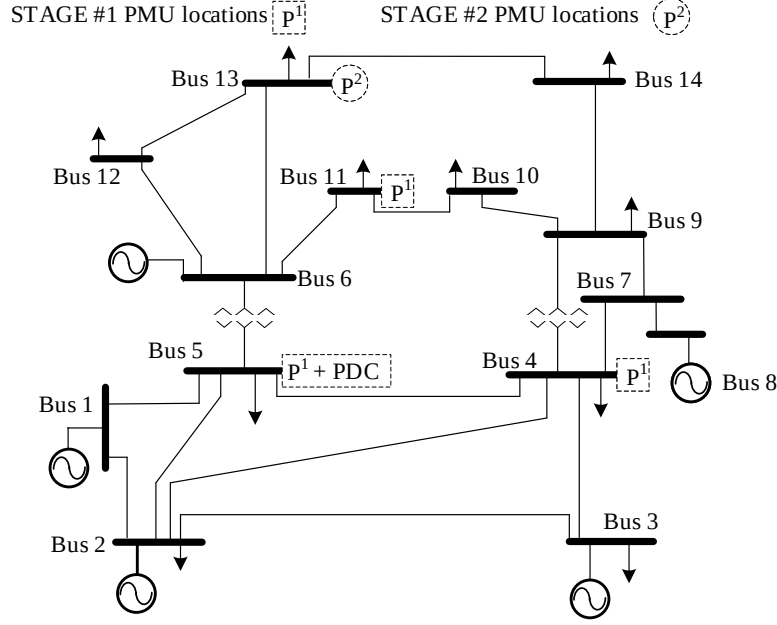


Figure 2.7: OPP solution for the IEEE 14-bus; considering effect of ZIB.

Table 2.2: Multistage approach, considering effect of ZIB

System	PMU locations	Stage budget C_{budget}^k	Cost	Remaining	Unprepared buses
First Stage ($k = 1$); Effect of ZIB Is Considered					
IEEE 14-bus	4, 5, 11	\$300,000	\$252,189	\$47,811	7, 9
IEEE RTS	5, 20	\$400,000	\$352,366	\$47,634	10, 11, 17, 24
IEEE 30-bus	3, 4, 10, 15, 20	\$500,000	\$426,236	\$73,764	9, 12, 25, 27, 28
IEEE 118-bus	45, 49, 53, 72, 80, 84, 86, 94	\$2,500,000	\$2,357,860	\$142,140	—
Second Stage ($k = 2$); Effect of ZIB Is Considered					
IEEE 14-bus	4, 5, 11, 13 ^a	\$150,000	\$73,448	\$76,552	7, 9
IEEE RTS	2, 5, 14, 20, 21	\$350,000	\$222,514	\$127,486	10, 11, 17, 24
IEEE 30-bus	3, 4, 10, 13, 15, 20, 29	\$350,000	\$238,861	\$111,139	9, 12, 25, 27, 28
IEEE 118-bus	2, 8, 12, 19, 21, 27, 34, 37, 45, 49, 53, 56, 68, 72, 75, 77, 80, 84, 86, 92, 94	\$2,000,000	\$1,924,513	\$75,487	—
Third Stage ($k = 3$); Effect of ZIB Is Considered					
IEEE RTS	2, 5, 8, 14, 20, 21 ^a	\$350,000	\$142,932	\$207,068	10, 11, 17, 24
IEEE 30-bus	3, 4, 7, 10, 13, 15, 16, 20, 29 ^a	\$350,000	\$230,720	\$119,280	9, 12, 25, 27, 28
IEEE 118-bus	2, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37, 40, 45, 49, 53, 56, 62, 68, 72, 75, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110 ^a	\$2,000,000	\$1,924,513	\$75,487	—

^aComplete observability is achieved.

Table 2.3: Cost Comparison Of Proposed Approach With Other Approaches

System	Ref. [30] ^a	Ref. [16]	Ref. [43]	Ref. [44]	Ref. [31]	Ref. [19] ^b	Proposed
Under Normal Operating Conditions; Effect of ZIB Is Not Considered							
IEEE 14-bus	2, 8, 10, 13 \$554,650	2, 6, 7, 9 \$870,400	2, 6, 7, 9 \$870,400	—	—	2, 6, 8, 9 \$679,320	4, 5, 8, 11, 13 \$373,845
IEEE RTS	3, 4, 7, 10, 13, 16, 20, 21 \$1,342,300	2, 3, 8, 10, 16, 21, 23 \$1,222,100	2, 3, 8, 10, 16, 21, 23 \$1,222,100	—	—	—	5, 6, 8, 9, 15, 16, 21, 23 \$765,994
IEEE 30-bus	3, 6, 7, 11, 13, 15, 17 20, 21, 24, 26, 30 \$1,236,400	1, 2, 6, 9, 10, 12, 15, 19, 25, 27 \$1,825,100	2, 3, 6, 9, 10, 12 15, 19, 25, 27 \$1,802,100	—	—	—	3, 6, 7, 11, 13, 15, 16, 20, 22, 26,29 \$961,262
IEEE 118-bus	—	—	1, 5, 9, 12, 13, 17, 21, 23, 26, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 71, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$8,224,600	—	—	2, 5, 11, 12, 15, 17, 21, 24, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 63, 68, 73, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$7,988,600	2, 5, 9, 12, 15, 19, 21, 27, 30, 31, 32, 34, 36, 40, 45, 49, 52, 56, 59, 63, 66, 68, 70, 71, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110, 118 \$6,890,379
Under Normal Operating Conditions; Effect of ZIB Is Considered							
IEEE 14-bus	2, 8, 10, 13 \$554,650	2, 6, 9 \$631,110	2, 6, 9 \$631,110	2, 6, 9 \$631,110	—	—	4, 5, 11 \$325,637
IEEE RTS	2, 8, 10, 15, 22, 23 \$1,076,000	2, 8, 10, 15, 20, 21 \$1,032,300	1, 2, 8, 16, 21, 23 \$941,410	—	—	—	2, 5, 8, 14, 20, 21 \$895,817
IEEE 30-bus	3, 7, 8, 10, 13, 15, 17, 19, 29 \$976,260	1, 2, 10, 12, 15, 19, 27 \$1,239,600	2, 3, 10, 12, 18, 24, 30 \$1,159,400	2, 4, 10, 12, 15, 18, 27 \$1,176,600	3, 7, 10, 12, 15, 20, 27 \$1,099,700	—	3, 4, 7, 10, 13, 15 16, 20, 29 \$717,812
IEEE 118-bus	—	—	1, 6, 8, 12, 15, 17, 21, 25, 29, 34, 40, 45, 49, 53, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 101, 105, 110, 114 \$7,840,900	2, 8, 11, 12, 15, 19, 21, 27, 31, 32, 34, 40, 45, 49, 52, 56, 62, 65, 72, 77, 80, 85, 86, 90, 94, 101, 105, 110 \$7,523,100	2, 8, 11, 12, 17, 21, 24, 27, 29, 43, 47, 49, 52, 56, 62, 71, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110, 114 \$8,173,500	—	2, 8, 11, 12, 19, 21, 27, 31, 32, 34, 37,40, 45, 49, 53, 56, 62, 68, 72, 75, 77, 80, 84, 86, 89, 92, 94, 100, 105, 110 \$6,063,617

^aHas two optimal solutions, one for minimum number of PMUs and the other for a cost model.

^bHas multiple optimal solutions, only the solution with the minimum cost is presented.

2.4.1 No Priority Buses

The model in section 2.2 is used and no priority is given to any bus. The multistage OPP is performed as a two-stage process for the IEEE 14-bus and a three-stage process for the IEEE RTS, the IEEE 30-bus and the IEEE 118-bus. Each stage is treated independently budget-wise, meaning the remainder of the budget from each stage is not added to the next stage budget. Complete observability is achieved for all systems within three stages. It should be noted that the number of stages in which complete observability is achieved depends on the budget specified by the utility.

The OPP is performed in two different cases. In the first case, ZIBs are treated as normal buses. The result of the multistage OPP is shown in Table 2.1. The OPP solution for the IEEE 14-bus is shown in Fig. 2.6 and Fig. 2.7. The ZIB effect is considered in the second case as shown in Table 2.2. A brute force approach was used to test the optimality of the results for the IEEE 14-bus test system for normal conditions and single line outage. The results of the brute force showed that the proposed approach found the global optima for the 14-bus IEEE test system.

The proposed approach is compared with some of the recent approaches in OPP literature, as seen in Table 2.3. These approaches include classical and evolutionary methods, mainly particle swarm optimization (PSO), Cellular Learning Automata (CLA) and binary imperialistic competition algorithm (BICA). The results show that the proposed approach achieves better overall cost for the OPP. This enhancement is partially due to the comprehensive installation cost model of the proposed approach, which considers the substation and communication infrastructure. Most of the other methods have not considered such a comprehensive model.

2.4.2 Using Priority Buses

The multistage PMU installation is performed on the IEEE 14-bus test system with predetermined priority buses. The higher priority buses for the IEEE 14-bus are chosen to be the high voltage buses, $L = [1, 2]$. The installation is performed as a four-stage process.

The first stage has a budget limit of \$400,000, and the remaining stages have budget limits of \$500,000 each. The first and second stages are used to achieve complete observability under normal conditions (2.14). The third stage is used to achieve observability for single line outage contingencies (2.16). The single PMU outage in (2.18) is chosen as the desired observability for the final stage. The results in Table 3.8 show the optimal PMU installation at each stage. The results show a comparison between treating all buses equally. The results show that prioritizing buses can drive up the PMU installation cost as seen in Table 3.8.

Table 2.4: Multistage PMU Installation for the IEEE 14-bus (Using Priority Buses), not considering Effect of ZIB

	All buses are treated equally $L = [\phi]$	Priority buses are used $L = [1, 2]$	Unprepared buses
Stage One $\mathcal{O} \leq \hat{\mathbf{I}}$	4, 6, 8 \$219,799	1, 2 \$185,777	7, 9
Stage Two $\mathcal{O} \geq \hat{\mathbf{I}}$	4, 5, 6, 8, 11, 13, 14 \$300,966	1, 2, 8, 10, 13 \$491,476	7, 9
Stage Three $\mathcal{O}_s \geq \hat{\mathbf{I}}$	2, 4, 5, 6, 8, 10, 11, 13, 14 \$133,860	1, 2, 4, 6, 8, 10, 11, 13, 14 \$255,764	7, 9
Stage Four $\mathcal{O}_p \geq \hat{\mathbf{I}}$	2, 4, 5, 6, 8, 10, 11, 13, 14 ^a \$0 ^b	1, 2, 4, 6, 8, 10, 11, 13, 14 ^a \$0 ^b	7, 9

^aNo additional installation of PMUs.

^bObservability already achieved at the previous stage.

Chapter 3

A Fault-Tolerance Based Approach to Optimal PMU Placement

3.1 Introduction

The concepts of reliability and fault-tolerance are closely related, yet distinct. While reliability is the ability of a system to perform its required function within a specific period, fault-tolerance is defined as the ability of a system to perform despite a failure (fault) event [45, 46]. Nevertheless, both concepts share some common metrics such as failure rate, repair rate, and availability. The proposed strategy of deploying PMUs in the proximity of higher probability contingencies increases the likelihood of more effective remedial actions, both preventive and corrective, thereby enhancing fault-tolerance. The work presented in this chapter is motivated by the idea that the value of a high cost asset such as PMU can be increased by deploying it so as to improve network fault-tolerance.

A new criterion for PMU placement is proposed that incorporates fault-tolerance into the OPP framework while minimizing the installation cost of PMUs. Reliability indices along with conditional probabilities are used to determine the critical components and assess the network vulnerability. The results from the vulnerability analysis are integrated into the

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OPP problem. Therefore, critical buses are given higher priority while considering both the network connections and the installation cost of PMUs. This enhances the situational awareness of network vulnerabilities and enables better operational control during faults.

The main contributions of the proposed approach are:

- It improves the fault-tolerance of the system by enhancing the observability of critical components, which are determined using vulnerability analysis. This analysis is based on the reliability indices of the composite system.
- It integrates the vulnerability analysis into the observability function. This integration enables for cost-efficient OPP solutions while considering vulnerable buses. This integration of critical buses can be expanded to include multiple criteria for application-based OPP approaches.
- It uses a cost model for the installation of PMUs which is derived from the industry [24, 25]. Unlike most application-based OPP approaches which try to minimize the number of PMUs as in [13–15, 26], this work uses the installation cost to achieve a more realistic strategy.
- It can be used as a multi-stage process under a constrained budget, or as a single-stage process while minimizing the total cost.
- It can achieve the optimal solution with the maximum observability in the case of multiple optima due to the nature of the bi-level formulation.

This chapter is organized as follows. Section 3.2 discusses the fault-tolerance and the vulnerability assessment. In Section 3.3, the cost of installing PMUs is presented. The

proposed approach is presented in Section 3.4. Sections 3.5 and 3.6 present the case studies and the conclusion respectively.

3.2 Fault-Tolerance: Vulnerability Analysis

The vulnerability analysis applied here has two primary steps. The first step is to identify the critical components of the system. The second step is to rank and validate monitoring these components. There are several approaches to assess the criticality of the system components. In [47, 48] Markov cut-set method along with DC-optimal power flow (DC-OPF) is used to assess the reliability of composite systems and to determine the critical components of the system. In [49], the critical components of the system are identified based on the impact of component outages on the interruption cost under predetermined loading conditions and forced outages.

The network vulnerability is determined by using AC-optimal power flow (AC-OPF) and determining the reliability indices of the conditional probability for the composite system S , given that component c_k has failed (3.2). The system failure is considered to be a load curtailment event, which can be considered as a *conditional* loss of load probability (LOLP), conditioned by a contingency, as shown in (3.2).

This approach assesses the direct impact of each component on the system. Each component c_k in the system has a failure rate λ_k and a repair rate μ_k . The probability of failure Q_k is shown in (3.1). To properly determine the criticality of a component c_k on the system, the failure rate of that component is inflated so that the probability of failure is almost one $Q_k \approx 1$. Then the reliability indices of the system are obtained by using AC-OPF along with Monte Carlo simulations. The results from this analysis represent the criticality of

component c_k to the system.

$$Q_k = \frac{\lambda_k}{\lambda_k + \mu_k} \quad (3.1)$$

$$\Gamma_k = P_{k,f}(S_f|c_{k,f}) \quad (3.2)$$

where

Γ_k criticality of component c_k ;

$P_{k,f}$ the probability of a system failure event given that

component c_k has failed;

S_f the event of the composite system failure;

$c_{k,f}$ the component c_k failure event.

However, since the purpose of this analysis is to prioritize the location of the PMUs for installation, ranking components by their criticality is not enough. The failure rate of the individual components needs to be considered. For instance, consider the 3-bus system in Fig. 3.1 which has one generator, two transmission lines and two loads. It can be observed that the generator and transmission line L_1 are the critical components for this system. However, if these components are highly reliable and transmission line L_2 is unreliable, monitoring the transmission line T_2 takes priority over transmission line T_1 . Therefore, it is not enough to consider the criticality of a component, but the reliability of each component needs to be considered too. The vulnerability indices in (3.3) consider both the criticality of each component and its failure rate. The pseudo algorithm for the vulnerability analysis is shown in Procedure 2.

$$V_k = \mathcal{N}(\Gamma_k, Q_k) \quad (3.3)$$

where

V_k the vulnerability index for component k ;

Q_k the failure probability of component k ;

$\mathcal{N}$ the cumulative distribution function (CDF)

for the normal distribution.

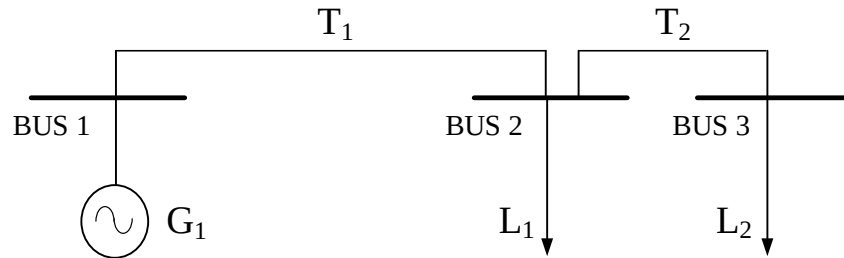


Figure 3.1: 3-bus sample system

Procedure 2 Network Vulnerability

Initialization load system and reliability data

N_c : the number of components

for $k = 1 : N_c$

Inflate the probability of failure Q_k for component c_k

Perform Monte Carlo simulation

Obtain Γ_k

Calculate $V_k = \mathcal{N}(\Gamma_k, Q_k)$, and reset Q_k

endfor

The results from this analysis are used to give the vulnerable buses higher priority by forming a bias matrix containing the vulnerability indices. The elements of the bias matrix are determined as in (3.4). When applying the vulnerability analysis to the sample system in Fig.1, the components of the system will get a vulnerability index V_k . These indices are

used to build the bias matrix, a 3×3 matrix in this case, where the elements of the bias matrix are determined using (3.4). These elements are incorporated into the observability in section 3.4.2.

$$v_{i,j} = V_k, \text{ if component } c_k \text{ is connecting bus } i \text{ to bus } j$$

$$v_{i,i} = \sum_{j=1}^{j=N} 1 + v_{i,j} - \sum_{j=1}^{j=N} H_{i,j} \quad (3.4)$$

where N is the number of buses and H is the $N \times N$ connectivity matrix defined in (3.5).

$$H_{ij} = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{else} \end{cases} \quad (3.5)$$

3.3 PMU Installation Cost

Most of the cost is associated with the installation process and the substation infrastructure rather than the PMUs themselves. A report published by the U.S. Department of Energy (DOE) has showed that the PMU installation cost ranges from \$40,000 to \$180,000 per PMU [24]. The cost varies depending on the infrastructure support for the PMUs. Therefore, it stands to reason that installation cost should be considered instead of minimizing the number of PMUs.

Although PMU installation requires the same infrastructure upgrades, such as communication, cyber-security and other equipment upgrades, the approach to install them can differ depending on the utility and the existing infrastructure support for the PMUs. For instance, a utility can install new stand alone PMUs, or upgrade existing digital relays to enable PMU

functionality [24]. Moreover, installing PMUs also depends on the availability of CTs and PTs [30]. Once the substation's infrastructure is in place to support PMUs, the installation cost can go down to 35% of the initial cost [24]. This chapter uses a model that includes the basic cost of each PMU unit in addition to the infrastructure cost, such as communication, security, PT and CT which may differ from one substation to another. The installation cost for PMUs is modeled as follows:

$$C = \sum_{i=1}^N (a_i + b_i + k(i))P_i + \text{PDC} \quad (3.6)$$

where

a_i unit cost for PMU at substation i

b_i cost for installing additional channels at substation i

$k(i)$ cost function for the communication network infrastructure

at substation i ;

PDC cost of phase data concentrator.

The P_i terms form a vector $P = [P_1, P_2, \dots, P_i, \dots, P_N]^T$, where P_i assumes the value of zero or one indicating if a PMU is to be installed at substation i . The cost coefficient for the communication network $k(i)$ at substation i , can take values between zero—if the communication infrastructure is sufficient for the installation—or \$8,000 per branch if the communication infrastructure are to be updated, as described in (3.7).

$$k(i) = \sum_{j=1}^n \text{len}_{i,j} \text{cc}_{i,j} \quad (3.7)$$

where n is the number of buses, and $\text{len}_{i,j}$ is the transmission line between buses i and j .

The communication cost cc is either \$8,000 or \$0.

The PMU installation cost model in (3.6) considers additional PMU channels and their associated costs, by considering the additional equipment needed for these channels such as PTs and CTs. It is assumed that the PMU will utilize all available channels. However, the cost of these channels is considered by the variable b_i , which differs from one substation to another depending on the number of channels and the type of available equipment. For instance, instead of using PTs for voltage measurements, capacitively coupled voltage transformers can be used. This can be accommodated by changing the value of the veritable b_i to correspond to the specific equipment available at substation i .

3.4 Proposed Approach

This section presents the approach for the optimal placement of PMUs. As discussed in the previous section the cost plays a critical role in determining the optimal installation process. In the proposed approach, the cost is given a higher priority over the network and vulnerability. The approach assumes that the utility sets a budget for the installation of PMUs and the higher objective is to minimize the cost and to make sure the optimal placement does not exceed the budget. In the lower objective, the network observability is maximized while given a bias for the vulnerable buses or branches. This bias is used to prioritize the vulnerable buses for the OPP.

3.4.1 Problem Statement

As discussed earlier, the OPP highly depends on the installation cost and the available budget. Therefore, the cost function in (3.8) is treated as higher level objective function. The observability function in (3.9) is treated as a lower level objective, subjected to the higher level objective (cost function). This bi-level setup allows maximizing the observability while

minimizing the cost without violating the budget constraint, thereby reaching the optimal solution for the given budget. Moreover, this bi-level formulation enables for the optimal solution with the maximum observability to be selected in the case of multiple optimal solutions. The vulnerability of the network is incorporated into the observability function as bias for vulnerable buses or branches as in (3.12). These vulnerability indices are obtained using Procedure 2 and (3.4).

$$\begin{aligned} \min C &= \sum_{i=1}^N (a_i + b_i + k(i))P_i + \text{PDC} \\ \text{subject to} \end{aligned} \tag{3.8}$$

$$C \leq C_{\text{budget}}$$

$$\max \sum_{i=1}^N \mathcal{O}'_i \tag{3.9}$$

where

$$\mathcal{O}' = H' \times B \tag{3.10}$$

$$B_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{else} \end{cases} \tag{3.11}$$

$$H'_{ij} = \begin{cases} 1 + v_{i,i}, & \text{if a PMU is installed at bus } i \\ 1 + v_{i,j}, & \text{if there is a branch connecting} \\ & \text{bus } i \text{ to bus } j \\ 0, & \text{else} \end{cases} \tag{3.12}$$

$v_{i,j}$ the vulnerability element (i, j) in the bias matrix of (3.4);

$v_{i,i}$ the vulnerability element (i, i) in the bias matrix of (3.4);

N number of buses.

Maximizing the modified observability vector $\mathcal{O}'$ gives priority to the vulnerable elements in the network. However, this vector cannot be used to test the observability of the network since the observability is skewed by the vulnerability indices. Therefore, the constraints in (3.13) and (3.14) are added to the second objective function in (3.9). The constraint in (3.13) is used to give high priority to vulnerable buses and the constraint in (3.14) is used to check for the desired observability.

$$\frac{\mathcal{O}'}{\mathcal{O}} \geq \gamma \quad (3.13)$$

$$\mathcal{O} \geq \mathbf{1} \text{ “complete observability constraint”} \quad (3.14)$$

where

$$\mathcal{O} = H \times B$$

γ priority index;

$\mathbf{1}$ a vector of length N , with all elements equal to 1;

$\mathbf{2}$ a vector of length N , with all elements equal to 2.

Zero Injection buses (ZIB) are buses with no load or generation units, and can be taken advantage of to reduce the number of PMUs required for achieving observability. Since the sum of all currents flowing to the ZIB is zero, Kirchhoff’s current law can be used to determine the current flow through the adjacent branches. The ZIB effect can be summarized as follows: A ZIB is considered observable if all adjacent buses of the ZIB are observable. An unobservable bus that is adjacent to a ZIB is considered observable if the ZIB and all of

the other buses adjacent to the ZIB are observable.

3.4.2 Algorithm

The optimal placement problem in subsection 3.4.1 is a discontinuous bi-level problem. Therefore, evolutionary algorithms are the appropriate tools for solving such a problem. Several evolutionary algorithms can be used for solving the OPP problem; the proposed algorithm uses opposition-based elitist genetic algorithm (O-BEGA) to solve the bi-level OPP in subsection 3.4.1. The opposition element is added to the algorithm to enhance the overall performance since opposition-based methods have proven their superiority in terms of convergence-speed and results [36, 37]. The overall flowchart for proposed algorithm is shown in Fig. 3.2.

The proposed algorithm solves the OPP problem in a parallel manner by initializing random candidates where each candidate X has the length of the number of buses N . However, this approach necessitates using of higher objective (cost objective); otherwise, the solution would converge to installing the PMUs on all buses. The complexity rises from minimizing the cost (higher objective) while maximizing the observability function.

The higher and the lower objectives share the same decision variables, meaning there are no decision variables exclusive to one objective or the other. The proposed approach exploits this advantage to evaluate both objectives simultaneously without using different search spaces for each objective.

The proposed approach uses a sorting function to handle the simultaneous evaluation in the same search space. The sorting function sorts all candidates according to the feasibility, and fitness of both the higher and lower objectives, as shown in Fig. 3.3. As a result, the algorithm is guided towards the optimal solution where the cost is minimized and the

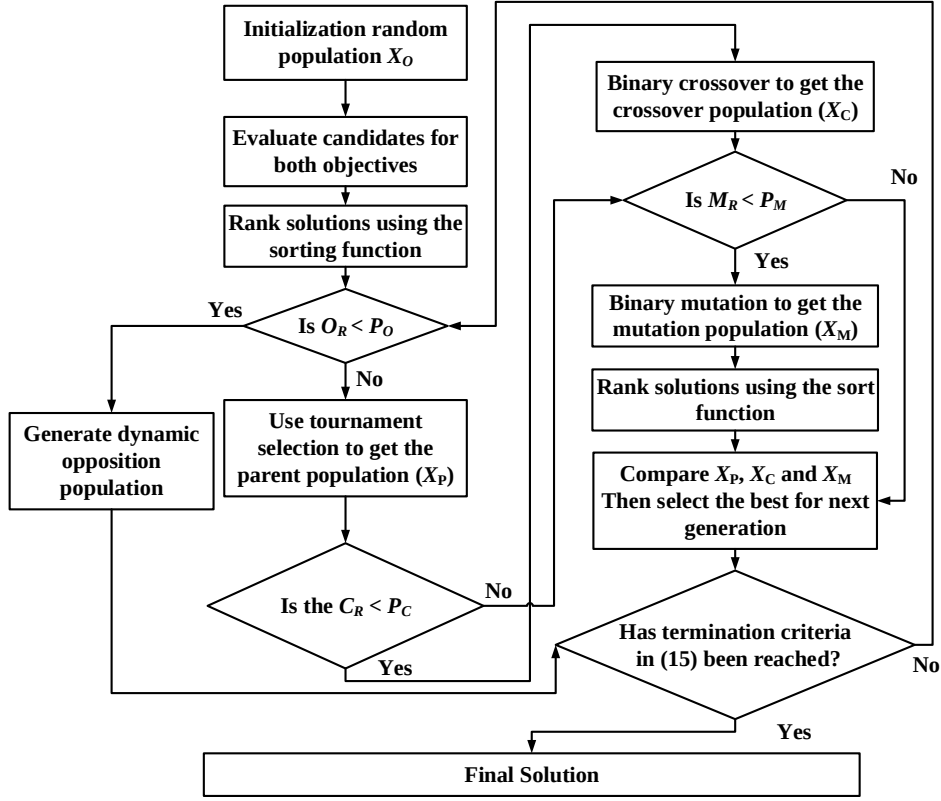


Figure 3.2: Flowchart of the proposed algorithm

observability is maximized.

The algorithm starts by randomly generating an initial population X_O , and then evaluating and ranking this population. The dynamic opposition is used when the random variable O_R is lower than opposition probability ($P_O = 0.4$); where the dynamic opposition is applied on a third of the randomly selected variables. The parent population (X_P) is selected using tournament selection, where two random candidates are matched against each other, and the winner is selected as a parent candidate for generating the crossover population (X_C). The probability of generating the crossover population P_C is 0.8, and the X_C is generated using binary double point crossover. The mutation population X_M is generated with a probability P_M of 0.1 using one-bit binary mutation, where one variable is randomly chosen and changed. The crossover and mutation steps are performed after comparing the

probabilities of P_C and P_M against the random variables C_R and M_R . After all the populations (X_p, X_M, X_c) have been generated, they are evaluated and ranked using the sorting function, then the termination criteria in (3.15) are checked. If the termination criteria have not been reached, the best among the current population is sent for the next iteration. The algorithm is summarized in Fig. 3.2

The algorithm is terminated if the conditions in (3.15) are met. In $\Upsilon(1)$, the mean of the population observability is measured against the member with the maximum observability in the current population. As the algorithm converges, the left-hand side of $\Upsilon(1)$ will converge to zero. Since the member with the maximum observability is more likely to be feasible due to the nature of the OPP problem, measuring the population mean observability against the maximum observability serves as an effective measure of feasibility.

In $\Upsilon(2)$, the mean of the installation cost is measured against the member with the maximum installation cost in the current population. Notice that the member with the maximum installation cost is more likely to represent a feasible yet sub-optimal solution. Nevertheless, as the population converges, this member will start drifting towards the mean of the population—since all the population members drift toward the optimal solution upon convergence. Therefore, the left-hand side of $\Upsilon(2)$ tends to zero as the algorithm progresses.

The terms α and β are constants. These constants are inversely proportional to the size of the system. Therefore, the α and β values that work for a big system will work for a smaller system, but the converse may not hold. The α and β are also proportional to the size of the population; the smaller the size of the population the smaller these constants

must be.

$$\Upsilon = \begin{cases} \Upsilon(1) = |\overline{\mathcal{O}'} - \max(\mathcal{O}')| \leq \alpha \\ \Upsilon(2) = |\overline{C} - \max(C)| \leq \beta \end{cases} \quad (3.15)$$

Variables					Cost	$\mathcal{O}'$	Rank
Bus 1	Bus 2	Bus 3	...	Bus N			
0	1	1	...	1	62000	17.4	1
0	1	0	...	0	54000	14.4	2
0	1	0	...	1	55000	13.2	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	0	0	...	0	3000	3.9	N-2
0	1	0	...	0	2400	2.6	N-1
0	0	0	...	1	1400	1.4	N

→ Feasible

→ Not feasible

Figure 3.3: Sorting function

3.4.2.1 Absence of feasible solutions

Normally the OPP problem has multiple feasible solutions. Therefore, once the algorithm is terminated the top-ranked members will have the same objective function values. More likely these top-ranked members will duplicate each other unless multiple optima exist with same the objective function values. Therefore, the scenario where the member ranked number 1 is infeasible should not occur.

A scenario where the top-ranked members are infeasible can happen if there are no feasible solutions for the problem. The nonexistence of feasible solutions is attributed to the constraints of the problem. For the OPP problem, there are two types of constraints observability constraints and cost constraint. The observability constraints are well established, and we know there are feasible solutions for these constraints. Therefore, infeasibility can only happen due to the cost constraint defined in (3.8). In the following, we consider the

possibility of infeasibility due to the cost constraint.

In the following example, we refer to the number of PMUs as the cost, and the objective is to minimize this cost. We use the number of PMUs for simplicity as this holds for the installation cost.

Suppose we solve for the IEEE 14-bus complete observability under normal conditions, and we chose to minimize the number of PMUs. And suppose that cost constraint in (3.8) is chosen to be $C \leq 3$. We know for a fact that no such solution exists since the optimal solution has a total number of PMUs equal to 4. In such a scenario, there are no feasible solutions, and therefore the upper part (feasible solutions) of the Sorting function in Fig. 3.3. would not exist. As this happens, the infeasible solutions can be divided into two groups. Group A in which the solutions are infeasible due to observability constraints. Group B in which the solutions are infeasible due to cost constraints. These two groups should be handled differently by the sorting function as follows:

- All members of group B must have a higher ranking than any member of group A.
- The members of group B are ranked according to the least cost constraint violation.
- The members of group A are ranked in descending order according to the observability.

These rules apply only to the infeasible solutions! By the using rules above, the solution ranked number 1 will be infeasible but satisfies the observability constraint, and this solution will have 4 PMUs (the optimal solution for the IEEE 14-bus). Notice that this solution still violates the cost constraint! Moreover, the algorithm will converge, and termination criteria in (3.15) will still work.

3.4.3 Single-Stage Installation

In a single-stage installation process, the OPP is solved for achieving the desired observability within one budget period. In other words, it is assumed that there are enough resources to deploy the PMUs at the required locations in a relatively short period. It should be noted that most of the OPP approaches are single-stage processes. The desired observability such as normal conditions, single line outage or a single PMU outage.

In any of the desired observability conditions the priority index γ of (3.13) is used. This index gamma assumes values in the interval $[1, 2]$. When γ is set to 1, all buses are treated equally from the vulnerability perspective. When γ is greater than 1, higher priority is given to the vulnerable buses. For the complete observability under normal conditions, the placement of PMUs must satisfy the observability constraint in (3.14).

As for observability under a single line outage, it is necessary to have at least two measurement redundancies for every bus in the network. Therefore, the observability constraint in (3.14) is changed to (3.16).

$$H \times B = \mathcal{O}^s \geq \mathbf{2} \quad (3.16)$$

where $\mathcal{O}^s$ is the observability vector for all buses in the system under single line outage contingency, as shown in (3.17).

$$\mathcal{O}_i^s = \begin{cases} 2, & \text{if a PMU is installed at bus } i \\ \sum(a_{ij} \times b_i), & \text{otherwise} \end{cases} \quad (3.17)$$

In a single PMU outage, every bus needs to have at least two independent measurements. Either by two different PMUs, or if ZIB effect is considered, through KCL and a PMU.

Therefore, the observability constraint in (3.14) is changed to (3.18).

$$H \times B = \mathcal{O}^p \geq \mathbf{2} \quad (3.18)$$

where $\mathcal{O}^p$ is the observability vector for all buses in the system under single PMU outage contingency, as shown in (3.19).

$$\mathcal{O}_i^p = \sum (a_{ij} \times b_i) \quad (3.19)$$

3.4.4 Multi-Stage Installation

In multi-stage installation OPP approaches, a more realistic scenario is assumed, where existing resources may not be enough to achieve the desired observability in one budget period. Therefore, the desired observability is achieved over several stages or years due to limited resources or budget.

The majority of existing multi-stage methods assume the minimum number of PMUs per stage. This assumption is unrealistic since PMU installation is restricted by the financial burden, infrastructure of the substations, and technical benefits at each stage.

The proposed approach uses a financial capital C_{budget} , as the limit for each stage instead of using the number of PMUs as the limit. The buses are prioritized using the vulnerability indices as in (3.12). The same bi-level model in section 3.4.1 is used, except the modified observability function $\mathcal{O}'$ is treated as the higher objective, and the cost objective is treated as the lower objective. In the final stage, the objectives are switched back to the original formulation with the observability as the lower objective and the installation cost as the

higher one. The observability constraint in (3.14) is changed to the following.

$$\mathcal{O} \leq \mathbf{1}$$

At each stage k , the pre-installed PMUs (PMU^{k-1}) are initiated, and the pre-installation cost C^{k-1} is calculated using (3.6). The budget for each stage C_{budget}^k is set. Then, the budget C_{budget}^k is modified to include the pre-installed PMUs cost C^{k-1} . The pre-installed PMU locations are maintained for the initial random population X_O and for the mutation population X_M . Since the pre-installed PMU locations are maintained for the initial and parent populations, the crossover population X_C maintains the pre-installed PMU locations by default. The multi-stage approach is summarized in Procedure 3.

$$C_{new,budget}^k = C_{budget}^k + C^{k-1} \quad (3.20)$$

Procedure 3 Multi-Stage Installation

Initialization number of stages (n), C_{budget} for each stage

for $k = 1$: number of stages

 set PMU^{K-1}, C_{budget}^k

 calculate $C^{k-1}, C_{new\ budget}^k$ using (3.20)

if $k =$ number of stages

 Switch the higher and lower objectives

endif

 Optimize the PMUs installation using the O-BEGA

 in section 2.3.2

$k = k + 1$

endfor

3.5 Simulation and Results

This section illustrates the application of the proposed OPP approach. In the first part of this section, the OPP approach is presented and compared with other approaches in the literature. Then, the OPP solutions for the cost criteria are presented. The second part of this section presents the vulnerability analysis and the OPP solution when incorporating this analysis. The algorithm is implemented in MATLAB, and the computation times for all three cases are under 2 seconds on a single processor.

Table 3.1: Comparison of The Proposed Solution With Other Solutions: Effect of ZIB is Not Considered

System	Ref. [43]	Ref. [16]	Ref. [30] ^b	Proposed
Under Normal Operating Conditions; Effect of ZIB Is Not Considered				
IEEE 14-bus	4 PMUs $\sum \mathcal{O} = 19$	4 PMUs $\sum \mathcal{O} = 16$	4 PMUs $\sum \mathcal{O} = 19$	4 PMUs $\sum \mathcal{O} = 19$
IEEE 30-bus	10 PMUs $\sum \mathcal{O} = 50$	10 PMUs $\sum \mathcal{O} = 50$	10 PMUs $\sum \mathcal{O} = 47$	10 PMUs $\sum \mathcal{O} = 52$
IEEE RTS	7 PMUs $\sum \mathcal{O} = 34$	7 PMUs $\sum \mathcal{O} = 34$	7 PMUs $\sum \mathcal{O} = 29$	7 PMUs $\sum \mathcal{O} = 34$
Single Line Outage; Effect of ZIB Is Not Considered				
IEEE 14-bus	—	7 PMUs ^a $\sum \mathcal{O} = 23^a$	7 PMUs $\sum \mathcal{O} = 25$	7 PMUs $\sum \mathcal{O} = 25$
IEEE 30-bus	—	15 PMUs ^a $\sum \mathcal{O} = 62^a$	16 PMUs $\sum \mathcal{O} = 59, 54$	16 PMUs, 15 PMUs ^a $\sum \mathcal{O} = 60^a, 62$
IEEE RTS	—	11 PMUs ^a $\sum \mathcal{O} = 45^a$	12 PMUs $\sum \mathcal{O} = 45, 48$	12 PMUs, 11 PMUs ^a $\sum \mathcal{O} = 57^a, 49$
Single PMU Outage; Effect of ZIB Is Not Considered				
IEEE 14-bus	—	—	9 PMUs $\sum \mathcal{O} = 36, 37$	9 PMUs $\sum \mathcal{O} = 39$
IEEE 30-bus	—	—	21 PMUs $\sum \mathcal{O} = 80, 81$	21 PMUs $\sum \mathcal{O} = 85$
IEEE RTS	—	—	14 PMUs $\sum \mathcal{O} = 65$	14 PMUs $\sum \mathcal{O} = 65$

^a Ignores radial buses in contingency cases.

^b Has multiple optimal solutions, maximum observability is shown.

3.5.1 Considering Installation Cost

In this section, the analysis is done without including the vulnerability analysis. The model in section 2.2 is used, and no bias is given to any bus. The buses are divided into two categories: prepared buses and unprepared buses. The prepared buses are assumed to have sufficient infrastructure and require basic security and network upgrades, which are covered under the basic PMU installation. The unprepared buses, on the other hand, require more upgrades such as additional PTs and CTs for the added measurements. Whereas the prepared buses are assumed to cost 35% of the initial cost. The base cost per PMU is assumed to be \$40,000, and the cost per additional PT or CT is assumed to be \$4,000. New communication lines and software upgrades are assumed to cost \$8,000, and PDC are assumed to cost \$8,000 [25].

Three test systems are used for the analysis: the IEEE 14-bus, IEEE 30-bus and IEEE

Table 3.2: Comparison of The Proposed Solution With Other Solutions: Effect of ZIB is Considered

System	Ref. [43]	Ref. [20]	Ref. [16]	Ref. [50]	Ref. [30] ^b	Proposed
Under Normal Operating Conditions; Effect of ZIB Is Considered						
IEEE 14-bus	3 PMUs $\sum \mathcal{O} = 16$	3 PMUs $\sum \mathcal{O} = 16$	3 PMUs $\sum \mathcal{O} = 16$	3 PMUs $\sum \mathcal{O} = 16$	3 PMUs $\sum \mathcal{O} = 16$	3 PMUs $\sum \mathcal{O} = 16$
IEEE 30-bus	7 PMUs $\sum \mathcal{O} = 47$	7 PMUs $\sum \mathcal{O} = 44, 47$	7 PMUs $\sum \mathcal{O} = 53$	7 PMUs $\sum \mathcal{O} = 55$	7 PMUs $\sum \mathcal{O} = 55, 56$	7 PMUs $\sum \mathcal{O} = 56$
IEEE RTS	6 PMUs $\sum \mathcal{O} = 38$	—	6 PMUs $\sum \mathcal{O} = 34$	—	6 PMUs $\sum \mathcal{O} = 33, 36$	6 PMUs $\sum \mathcal{O} = 40$
Single Line Outage; Effect of ZIB Is Considered						
IEEE 14-bus	—	7 PMUs $\sum \mathcal{O} = 27$	7 PMUs ^a $\sum \mathcal{O} = 35^a$	—	7 PMUs $\sum \mathcal{O} = 25, 25$	7 PMUs $\sum \mathcal{O} = 27$
IEEE 30-bus	—	13 PMUs $\sum \mathcal{O} = 57$	10 PMUs ^a $\sum \mathcal{O} = 55^a$	13 PMUs $\sum \mathcal{O} = 59$	14 PMUs $\sum \mathcal{O} = 58, 61$	13 PMUs, 10 PMUs ^a $\sum \mathcal{O} = 66$
IEEE RTS	—	—	9 PMUs ^a $\sum \mathcal{O} = 52^a$	—	10 PMUs $\sum \mathcal{O} = 50, 52$	8 PMUs $\sum \mathcal{O} = 46$
Single PMU Outage; Effect of ZIB Is Considered						
IEEE 14-bus	7 PMUs $\sum \mathcal{O} = 33$	7 PMUs $\sum \mathcal{O} = 35$	—	7 PMUs $\sum \mathcal{O} = 35$	7 PMUs $\sum \mathcal{O} = 33, 35$	7 PMUs $\sum \mathcal{O} = 35$
IEEE 30-bus	15 PMUs $\sum \mathcal{O} = 61$	15 PMUs $\sum \mathcal{O} = 63$	—	14 PMUs $\sum \mathcal{O} = 61$	15 PMUs $\sum \mathcal{O} = 65, 69$	14 PMUs $\sum \mathcal{O} = 72$
IEEE RTS	13 PMUs $\sum \mathcal{O} = 62$	—	—	—	12 PMUs $\sum \mathcal{O} = 59$	11 PMUs $\sum \mathcal{O} = 61$

^a Ignores radial buses in contingency cases.

^b Has multiple optimal solutions, maximum observability is shown.

RTS systems. First, the bi-level optimization approach is tested using minimum number of PMUs as the cost function. The results are compared with other approaches as seen in Table 3.1 and Table 3.2. The proposed algorithm always achieve better or comparable performance with the existing OPP schemes.

There can be several optimal solutions that satisfy the observability conditions while requiring same number of PMUs. However, these optimal solutions may have different observability levels (redundancies) $\sum \mathcal{O}$. The higher the number of redundancies, the better the solution is, as long as the cost of the solution is the same. For instance, the IEEE 14-bus single PMU outage in Table I shows two optimal solutions. Both solutions require the same number of PMUs (7 PMUs); however the proposed approach achieves a better observability ($\sum \mathcal{O} = 39$) with PMU locations $\{2, 4, 5, 6, 7, 8, 9, 10, 13\}$. The optimal solution of [30] achieves an observability of ($\sum \mathcal{O} = 37$) with PMU locations $\{1, 2, 4, 6, 7, 8, 9, 10, 13\}$.

Table 3.3: Cost From the Proposed Approach: Effect of ZIB is Not Considered

	Minimum number of PMUs		Proposed		Unprepared buses
	PMU locations	Cost From PMU installation	PMU locations	Cost of PMU installation	
Under Normal Operating Conditions; Effect of ZIB Is Not Considered ($\mathcal{O} \geq 1$)					
IEEE 14-bus	2, 6, 7, 9	\$224,800	2, 8, 10, 13	\$81,200	7, 9
IEEE RTS	2, 3, 8, 10, 16, 21, 23	\$239,600	2, 3, 5, 7, 14, 16, 21, 23	\$168,000	10, 11, 17, 24
IEEE 30-bus	2, 4, 6, 9, 10, 12, 15, 19, 25, 27	\$510,600	3, 5, 8, 11, 13, 15, 17, 19, 22, 26, 30	\$215,600	9, 12, 25, 27, 28
Single Line Outage; Effect of ZIB Is Not Considered ($\mathcal{O}^s \geq 2$)					
IEEE 14-bus	1, 3, 6, 8, 9, 10, 13	\$216,400	1, 3, 6, 7, 10, 12, 14	\$184,800	7, 9
IEEE RTS	1, 2, 3, 4, 7, 10, 11, 12 15, 16, 20, 21, 22	\$437,800	3, 4, 5, 6, 7, 9, 13, 14, 15, 18, 20, 22	\$247,800	10, 11, 17, 24
IEEE 30-bus	2, 3, 7, 8, 10, 11, 12, 13, 15 16, 19, 22, 24, 26, 27, 29	\$488,200	1, 3, 5, 6, 8, 10, 11, 13, 14, 15, 16, 19, 21, 24, 26, 29, 30	\$345,800	9, 12, 25, 27, 28
Single PMU Outage; Effect of ZIB Is Not Considered ($\mathcal{O}^p \geq 2$)					
IEEE 14-bus	2, 4, 5, 6, 7, 8, 9, 11, 13	\$329,800	1, 2, 3, 7, 8, 10, 11, 12, 13, 14	\$243,600	7, 9
IEEE RTS	1, 2, 3, 7, 8, 9, 10, 11, 15, 16, 17, 20, 21, 23	\$527,400	1, 2, 3, 4, 5, 6, 7, 8, 13, 14, 15, 16, 19, 21, 22, 23	\$333,200	10, 11, 17, 24
IEEE 30-bus	2, 3, 4, 6, 7, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24, 25, 26, 27, 28, 29	\$789,200	1, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 15, 16, 18, 20, 22, 24, 25, 26, 29, 30	\$640,000	9, 12, 25, 27, 28

Table 3.4: Cost From the Proposed Approach: Effect of ZIB is Considered

	Minimum number of PMUs		Proposed		Unprepared buses
	PMU locations	Cost of PMU installation	PMU locations	Cost of PMU installation	
Under Normal Operating Conditions; Effect of ZIB Is Considered ($\mathcal{O} \geq 1$)					
IEEE 14-bus	2, 6, 9	\$140,800	2, 8, 10, 13	\$81,200	7, 9
IEEE RTS	2, 8, 10, 16, 21, 23	\$234,000	2, 5, 7, 16, 22, 23	\$123,200	10, 11, 17, 24
IEEE 30-bus	2, 4, 10, 12, 19, 24, 27	\$314,600	3, 5, 10, 13, 15, 17, 19, 29	\$163,800	9, 12, 25, 27, 28
Single Line Outage; Effect of ZIB Is Considered ($\mathcal{O}^s \geq 2$)					
IEEE 14-bus	1, 3, 6, 8, 9, 11, 13	\$216,400	1, 3, 6, 8, 10, 12, 14	\$138,600	7, 9
IEEE RTS	1, 2, 7, 9, 10, 16, 20, 21	\$260,600	2, 5, 6, 7, 9, 16, 20, 21	\$170,800	10, 11, 17, 24
IEEE 30-bus	1, 2, 4, 7, 10, 11, 12, 13, 15, 16, 19, 23, 26, 30	\$330,600	1, 3, 5, 10, 11, 13, 14, 16, 18, 19, 23, 26, 30	\$256,200	9, 12, 25, 27, 28
Single PMU Outage; Effect of ZIB Is Considered ($\mathcal{O}^p \geq 2$)					
IEEE 14-bus	2, 4, 5, 6, 9, 11, 13	\$227,600	1, 2, 3, 8, 10, 11, 12, 13, 14	\$179,200	7, 9
IEEE RTS	1, 2, 3, 7, 8, 9, 10, 18, 19, 20, 21	\$322,200	1, 2, 4, 5, 6, 7, 8, 13, 15, 16, 19, 21	\$250,600	10, 11, 17, 24
IEEE 30-bus	1, 2, 4, 6, 7, 10, 12, 13 15, 16, 18, 19, 24, 27	\$460,200	1, 3, 5, 7, 10, 11, 12, 13, 15, 16, 18, 19, 23, 26, 29	\$386,600	9, 12, 25, 27, 28

The proposed approach always achieve higher redundancy level because of the bi-level formulation, where the observability is treated as a lower objective and is being maximized, while the being subjected to the higher objective (minimize the cost). Other OPP approaches treat the observability as a constraint and try to minimize the cost (number of PMUs) as follows:

$$\begin{aligned}
\min C &= \text{number of PMUs} \\
s.t. \quad \mathcal{O} &\geq 1
\end{aligned} \tag{3.21}$$

Using the formulation in (3.21) should lead to an optimal solution. However, in the presence of multiple optimal solutions, this formulation may not lead to the solution with

the higher number of redundancies as is evident from the results in Table 3.1 and Table 3.2.

After verifying the performance of the bi-level optimization, the cost model in section (2.2) is used for the OPP. The PMU placement is performed for two different cases. In the first case, the PMUs are placed under normal conditions, under a single line outage and single PMU outage. Also, ZIBs are treated as normal buses. The result of the placement is shown in Table 3.3. The ZIB effect is considered in the second case under normal operation, single line outage and single PMU outage as shown in Table 3.4.

The results show that the using minimum number of PMUs does not necessitate a lower installation cost, especially when each substation has its own cost factors. For instance, consider the IEEE RTS solution for normal conditions in Table III. When the minimum number of PMUs is used as the objective, the installation cost is about \$239,600 with a total of a 7 installed PMUs. On the other hand, when the installation cost is used as the objective more PMUs are installed (8 PMUs) at a lower cost (\$81,200). The same phenomenon can be observed for the IEEE 14-bus case for normal conditions in Table IV.

The impact of different approaches (minimum number of PMUs approach and the proposed approach) on the level of redundancy of the solution can be observed from the cost. Consider the IEEE 14-bus under normal conditions in Table III; both solutions have the same number of PMUs (4 PMUs). However, the cost obtained by the proposed solution of $\{2, 8, 10, 13\}$ with an observability of $\sum \mathcal{O} = 14$ is much lesser compared to the other solution of $\{2, 6, 7, 9\}$ which uses more channels and has a higher observability ($\sum \mathcal{O} = 19$).

3.5.2 Vulnerability Analysis

In this section, the vulnerability indices obtained from the vulnerability analysis are used to give bias to the vulnerable buses. Monte Carlo simulation along with AC-OPF is used

to obtain the conditional reliability for the IEEE RTS system [51]. Since the system failure is considered to be a load curtailment event, the loss of load probability (LOLP) is used as the indicator of the criticality of components. The results in Table 3.5 show the *conditional* LOLP of the transmission lines. The results also show the vulnerability indices of these components.

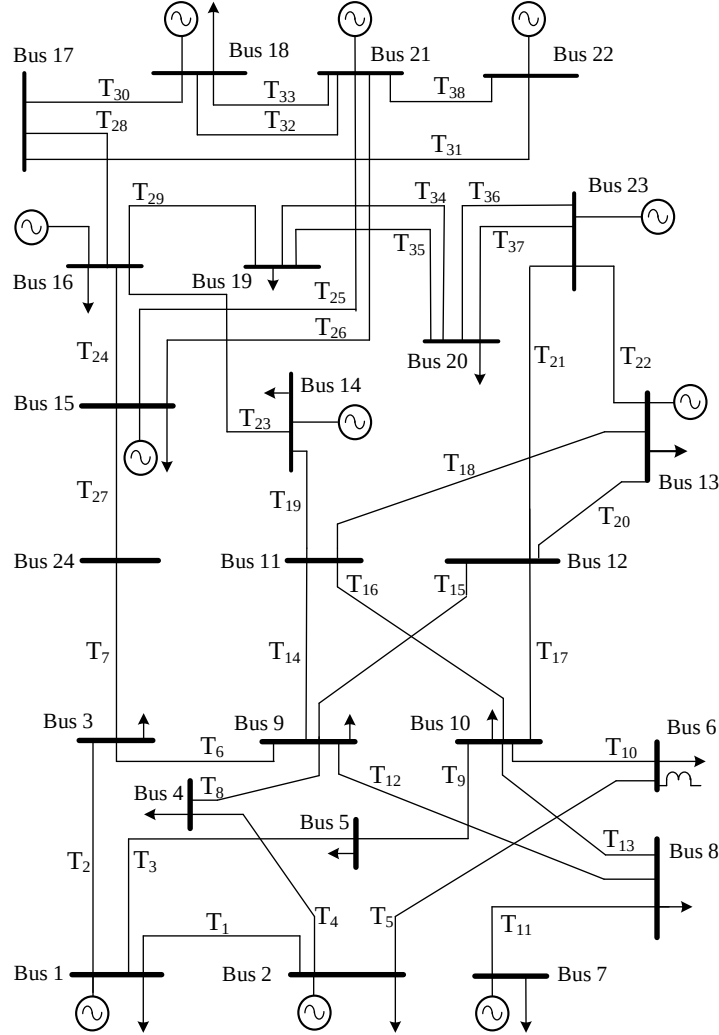


Figure 3.4: IEEE RTS system

The results from the conditional probability show that transmission lines T_{10} and T_{11} are highly critical to the system operation. Transmission line number T_{10} connects buses $\{6\}$ and $\{10\}$, and transmission line number T_{11} connects buses $\{7\}$ and $\{8\}$ as seen in

Fig. 3.4. To account for the reliability of the transmission lines: the probability of failure is multiplied by the conditional reliability indices. Then the CDF of the normal distribution is used to obtain the network vulnerability indices (V.I.) as in (3.3). The results are shown in Table 3.5.

Table 3.5: Conditional Reliability Indices –Vulnerability Indices (V.I.)

Transmission Line Number	<i>Conditional</i> LOLP, Γ_k	V.I.	Transmission Line Number	<i>Conditional</i> LOLP, Γ_k	V.I.
1	0.1034	0.40822	20	0.1105	0.42403
2	0.1061	0.40822	21	0.1141	0.45381
3	0.1072	0.4006	22	0.1040	0.43729
4	0.1058	0.41097	23	0.1125	0.42119
5	0.1074	0.42925	24	0.1067	0.40651
6	0.1069	0.40984	25	0.1096	0.42547
7	0.1187	0.65915	26	0.1086	0.42466
8	0.1046	0.40461	27	0.1156	0.43029
9	0.1088	0.40344	28	0.1093	0.41242
10	1.0000	0.99987	29	0.1068	0.40864
11	1.0000	0.82297	30	0.1050	0.40338
12	0.1082	0.42223	31	0.1134	0.45758
13	0.1105	0.42403	32	0.1047	0.40929
14	0.1053	0.62489	33	0.1038	0.40868
15	0.1075	0.63061	34	0.1048	0.41548
16	0.1074	0.63035	35	0.1057	0.41614
17	0.1126	0.64372	36	0.1111	0.41149
18	0.1057	0.42027	37	0.1052	0.40759
19	0.1054	0.41798	38	0.1065	0.43133

The vulnerability indices show that transmission lines T_{10} and T_{11} are still highly critical to the system operation. Therefore, the buses connected to these transmission lines are critical. The vulnerability indices are integrated into the observability function as described in subsection 3.4.1. By integrating the vulnerability indices into the observability, the OPP is solved to generate a more resilient solution while minimizing the total installation cost.

3.5.2.1 Single-Stage Installation

For the single-stage installation, the PMU placement is considered under all observability conditions. The results for the PMU placement are shown in Table 3.6 and Table 3.7. The results show that higher priority buses are monitored directly by the PMUs. However, the OPP cost is higher than normal PMU installation. This is due to the constraint on the priority index γ , which forces the OPP solution to a more secure and resilient solution, where the fault-tolerance of the system is enhanced. For instance, for normal conditions the critical components are directly monitored when the priority index is used by placing PMUs on buses $\{6, 9\}$. A better option, from fault-tolerance perspective, is to use buses $\{8, 10\}$, however this option has a much higher cost.

Table 3.6: Cost Comparison of the OPP; Effect of ZIB is Not Considered

	$\gamma = 1$	$\gamma = 1.5$	Unprepared buses
Normal Operating Conditions; Effect of ZIB Is Not Considered			
IEEE RTS	2, 3, 5, 7, 14, 16, 21, 23 \$168,000	3, 5, 6, 7, 9, 16, 21, 23 \$170,800	10, 11, 17, 24
Single Line Outage; Effect of ZIB Is Not Considered			
IEEE RTS	3, 4, 5, 6, 7, 9, 13, 14, 15, 18, 20, 22 \$247,800	2, 3, 5, 7, 9, 10, 13, 14, 15, 18, 20, 22 \$337,600	10, 11, 17, 24
Single PMU Outage; Effect of ZIB Is Not Considered			
IEEE RTS	1, 2, 3, 5, 6, 7, 9, 13, 14, 15, 16, 18, 19, 21, 22, 23 \$329,000	1, 2, 3, 5, 6, 7, 8, 9, 12, 14, 15, 16, 19, 21, 22, 23 \$338,800	10, 11, 17, 24

Table 3.7: Cost Comparison The OPP; Effect of ZIB is Considered

	$\gamma = 1$	$\gamma = 1.5$	Unprepared buses
Normal Operating Conditions; Effect of ZIB Is Not Considered			
IEEE RTS	1, 2, 8, 14, 20, 21 \$123,200	5, 6, 7, 9, 20, 21, 22, 23 \$162,400	10, 11, 17, 24
Single Line Outage; Effect of ZIB Is Considered			
IEEE RTS	2, 5, 6, 7, 9, 16, 20, 21 \$170,800	3, 4, 5, 6, 7, 9 16, 20, 21 \$190,400	11, 12, 17, 24
Single PMU Outage; Effect of ZIB Is Considered			
IEEE RTS	1, 2, 5, 6, 7, 9, 15, 16, 19, 21, 23 \$228,200	1, 2, 5, 6, 7, 8, 9, 14, 16, 20, 21, 22, 23, 24 \$339,400	11, 12, 17, 24

3.5.2.2 Multi-Stage Installation

The multi-stage PMU installation is performed as a three-stage process; each stage has its own budget. All the stages have a budget limit of \$70,000, where each stage is treated independently budget-wise. The results in Table 3.8 show the optimal PMU installation at each stage. The third stage presents the complete observability solution for both cases. The results show a comparison between treating all buses equally and using vulnerability indices. The results show that prioritizing vulnerable buses, produce a better observability and a more resilient OPP solution.

Table 3.8: Multi-Stage OPP; Effect of ZIB is Not Considered

	All buses are treated equally	Vulnerability indices are used	Unprepared buses
Stage one $C_{budget}^1 = \$7e4$	16, 21, 23 $\mathcal{O} = 13, \mathcal{O}' = 24.35$ \$64,400	9, 21, 23 $\mathcal{O} = 13, \mathcal{O}' = 27.39$ \$65,800	10, 11, 17, 24
Stage two $C_{budget}^2 = \$7e4$	1, 3, 8, 16, 21, 23 $\mathcal{O} = 25, \mathcal{O}' = 41.29$ \$63,000	2, 3, 8, 9, 21, 23 $\mathcal{O} = 19, \mathcal{O}' = 44.33$ \$63,000	10, 11, 17, 24
Stage Three $C_{budget}^3 = \$7e4$	1, 2, 3, 8, 14, 16, 21, 23 $\mathcal{O} = 32, \mathcal{O}' = 50.39$ \$40,600	2, 3, 5, 8, 9, 16, 21, 23 $\mathcal{O} = 34, \mathcal{O}' = 55.97$ \$42,000	10, 11, 17, 24

3.6 Conclusion

In this chapter, a new approach for application-based OPP is presented. Traditionally, for application-based OPP approaches, the objective is to minimize the number of PMUs. This chapter, however, considers the installation cost of PMUs, the observability, and the vulnerability of the system. The fault-tolerance aspect is enhanced through the prioritization of the critical buses for the installation of PMUs, where the criticality of the buses is quantified and integrated into the observability function using vulnerability analysis.

The proposed approach can be used as a single-stage or a multi-stage installation. The bi-level formulation enables the optimal solution with the maximum observability to be selected in the case of multiple optima. Moreover, the integration of the vulnerability indices into the observability function allows for a solution that can attain a trade-off between the technical benefits and the cost-effectiveness of the OPP. This process is controlled by a priority index, which can be determined by the utility at the planning stage. This incorporation of priority indices into the observability function can be used for other application-based OPP approaches or for multiple criteria application-based OPP approaches.

Chapter 4

Multi-stage Optimal PMU Placement to Benefit System Voltage Stability

4.1 Introduction

The OPP literature can be classified into two categories: observability-based OPP and application-based OPP. The observability-based OPP aims to achieve complete observability for the power grid while minimizing the total cost of installing PMUs. The application-based OPP, on the other hand, seeks to achieve other technical benefits out of the PMUs such as bad data detection, while minimizing the cost of PMU installation.

Transient stability has been considered for PMU deployment by [13], where generators are ranked by determining the individual machine trajectory and the proximity of this trajectory to the transient energy function stability margin. In this chapter, a multi-stage OPP approach is proposed, where voltage stability is prioritized. The proposed criterion can be used in addition to other criteria of PMU deployment. Voltage stability analysis is necessary to mitigate voltage collapse, which occurs when the voltage of a bus falls below acceptable margins, thus causing power system instability. This may happen due to several reasons including disturbance, sudden increase of load, or the inability of the system to supply sufficient

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reactive power to the load. This instability often lies with critical buses and transmission lines with heavy loads. Critical buses are identified using the proposed voltage stability criterion, and a cost model is used for each stage. There are several voltage stability indices that can be used to create the voltage stability criterion such as fast voltage stability index, line stability index and line stability factor [52–54]. The line stability factor is chosen for its superior performance compared to the other indices in dynamic operation [55]. The proposed voltage stability creation can be integrated with other OPP criteria for application-based OPP.

The rest of the chapter is organized as follows: section 4.2, describes the voltage stability criterion, section 4.3, describes the modeling approach for the OPP. The analysis for the case studies and conclusion are presented in sections 4.4 and 4.5, respectively.

4.2 Voltage Stability Criterion

In this section, the voltage stability criterion is developed using the line stability factor (LQP) in [54]. As mentioned earlier, the line stability factor is chosen for its accuracy in predicting the voltage collapse compared to other indices [52]. The LQP is derived for the real and reactive power of the system. Consider the sample system in Fig. 4.1, the apparent power at bus j is a function of the real and reactive power ($S = P + jQ$). The apparent power can also be expressed as a function of the current and voltage, $S_j = I_j^* V_j$. The current I can be expressed as follows:

$$I = \frac{V_i - V_j}{Z} = \frac{V_i - V_j}{R + jX} \quad (4.1)$$

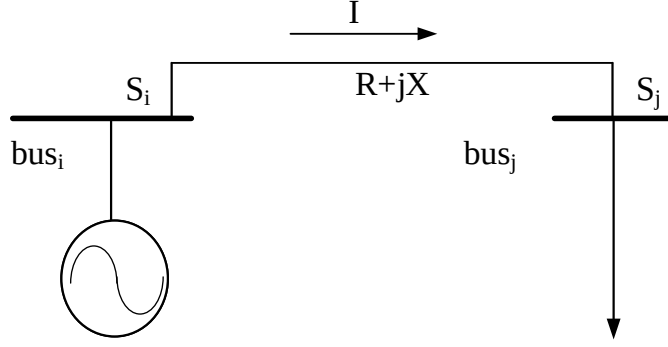


Figure 4.1: Sample system

By decomposing the current in (4.1) into its real and imaginary parts as

$$I = \frac{V_i \cos \delta_i - V_j \cos \delta_j + j(V_i \sin \delta_i - V_j \sin \delta_j)}{R + jX}, \quad (4.2)$$

the real power P_j and reactive power Q_j can be expressed as follows:

$$P_j = \left[(V_i \cos \delta_i - V_j \cos \delta_j) \frac{R}{R^2 + X^2} - (V_i \sin \delta_i - V_j \sin \delta_j) \frac{X}{R^2 + X^2} \right] V_j \quad (4.3)$$

$$Q_j = \left[(V_i \cos \delta_i - V_j \cos \delta_j) \frac{X}{R^2 + X^2} + (V_i \sin \delta_i - V_j \sin \delta_j) \frac{R}{R^2 + X^2} \right] V_j. \quad (4.4)$$

By making $\delta = \delta_i - \delta_j$, and assuming the network is predominantly inductive ($R \ll X$),

the P and Q are expressed as follows:

$$P_j = \left[\frac{V_i \sin \delta}{X} \right] V_j \quad (4.5)$$

$$Q_j = \left[(-V_i \cos \delta + V_j) \frac{1}{X} \right] V_j. \quad (4.6)$$

Now by using the trigonometric property $\sin^2\delta + \cos^2\delta = 1$ the relationships in (4.5) and (4.6) can be expressed as

$$\left[\frac{XP_j}{V_iV_j}\right]^2 + \left[\frac{XQ_j - V_j^2}{V_iV_j}\right]^2 = 1. \quad (4.7)$$

$$V_j^4 - (2XQ_j - V_i^2)V_j^2 + (X^2Q_j^2 + P_j^2X^2) = 0 \quad (4.8)$$

Therefore, V^2 is expressed as the quadratic relationship in (4.8). By assuming V^2 has a real solution, the discriminant $((2XQ_j - V_i^2) - 4(X^2Q_j^2 + P_i^2X^2))$ needs to be greater or equal to zero. As a result, $4\left(\frac{X}{V_i^2}\right)\left(Q_j + \frac{P_i^2X}{V_i^2}\right) \leq 1$, which is the LQP in (4.9).

$$LQP_{ij} = 4\left(\frac{X}{V_i^2}\right)\left(Q_j + \frac{P_i^2X}{V_i^2}\right) \quad (4.9)$$

where i and j are the subscripts for the buses. The higher the LQP, the closer the line is to voltage collapse since the discriminant gets closer to being imaginary. Since these LQP line indices, indicates the criticality of the lines in the system, these indices can be utilized to rank critical buses for the OPP. By summing up adjacent LQP indices of each bus, a critical bus index (Cr) is expressed as follows:

$$Cr_i = \frac{1}{\sum_{j=1}^{j=N} A_{i,j}} \sum_{j=1}^N LQP_{ij} \quad (4.10)$$

where N is the number of uses in the system, Cr_i represent bus i critical index. A is an $N \times N$ connectivity matrix of the system in (4.11). These indices in (4.10) are used to determine the critical buses for the OPP. These critical buses are incorporated into the observability

function, where these buses are given higher priority over other buses in the system.

$$A_{ij} = \begin{cases} 1, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases} \quad (4.11)$$

4.3 Modeling Approach

This section describes the observability criterion, incorporation of critical buses, and the overall model. The optimization approach is also presented.

4.3.1 Observability

The observability of the system can be measured using the connectivity matrix A and the vector of PMU locations as in (4.12). The system is considered observable under normal conditions if the constraint in (4.13) is met, which ensures that all the buses of the system are measured by the PMU, either directly or indirectly.

$$\mathcal{O} = H \times P \quad (4.12)$$

$$\mathcal{O} \geq \mathbf{I} \quad (4.13)$$

where P is a vector of length equal to the number of buses (N), $\mathbf{I}$ is a vector of ones of length N , and A is the $N \times N$ connectivity matrix defined in (4.11). The entries for the PMU locations vector P is defined in (4.14).

$$P_i = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 0, & \text{if no PMU is at bus } i \end{cases} \quad (4.14)$$

In the multi-stage approach, the complete observability constraint cannot be met at the initial stages. Therefore, this constraint is changed to the multi-stage condition in (4.15)

$$\mathcal{O} \leq \mathbf{I} \quad (4.15)$$

The PMUs can measure the buses of the network indirectly by considering the zero-injection buses (ZIB) effect. When the ZIB effect is considered, the overall cost for observability is reduced. This ZIB effect takes advantage of the PMU current measurements, to apply KCL thereby reducing the number of PMUs required for complete observability. This effect is summarized as follows:

- If all buses adjacent to a ZIB are observable, the ZIB is considered observable.
- If all buses adjacent to a ZIB are observable—except for one—and the ZIB is observable; then the unobservable bus is also considered observable.

4.3.2 Critical Buses

The voltage stability criterion in section 4.2 is used to rank the buses and determine the critical buses of the system. In the proposed approach, the critical buses criterion is embedded into the observability function (4.12), by integrating the priority vector $R = [Cr_1 Cr_2 \dots Cr_N]^T$. The critical buses criterion is embedded into the observability to prioritize the critical buses while enhancing the observability of the system. The higher priority

buses are determined based on the voltage stability criterion, then arranged in descending order in a vector L . Then the priority bias is assigned using the following algorithm.

Procedure 4 Priority Bus Vector L

Initialize priority buses (L)

N = number of buses

M = maximum number of branches in (A)

kk = length of L

$R = [Cr_1 \dots Cr_N]^T$

Sort R in descending vector

Normalize and filter R

for $j = 1 : kk$

$i = R(j)$

if $i \neq 0$ then

$L_i = M * (kk - j + 1)$

else

$L_i = 0$

endif

endfor

The priority vector L is incorporated into the connectivity matrix A , to set a bias for the critical buses, as in (4.16). However, by skewing the connectivity matrix A' , the observability tests in (4.13) and (4.15) are also skewed. Therefore, the skewed observability $\mathcal{O}'$ is used of the optimization of PMU locations, and the original observability $\mathcal{O}$ is used to check the

observability condition.

$$A'_{ij} = \begin{cases} 1 + L_i, & \text{if } i = j \\ 1, & \text{if there is a branch connecting bus } i \text{ and bus } j \\ 0, & \text{otherwise} \end{cases} \quad (4.16)$$

where

$$\mathcal{O}' = A' \times P$$

$$L_i = \text{bus } (i) \text{ priority index}$$

$$N = \text{number of buses}$$

4.3.3 Problem Statement

The proposed multi-stage OPP uses a PMU installation cost model to optimize the PMU locations. The cost model is based on the report published by the U.S. Department of Energy [24], which indicates that a PMU cost about \$40,000 on average. The cost model also considers the number of channels used by the PMUs, by considering the cost of additional measurements, \$4,000 per channel (4.19).

In the multi-stage OPP, the observability is maximized at each stage. However, maximizing the observability is limited by the budget of each stage. Therefore, the problem is set up as a bi-level optimization problem as follows:

$$\max \quad \mathcal{O}' = \sum_{i=1}^n \mathcal{O}'_i \quad (4.17)$$

Subject to

$$\mathcal{O} \leq \mathbf{I} \quad \text{“multi-stage observability condition”} \quad (4.18)$$

$$\min P_{Cost} = \sum_{i=1}^N (a \times P_i + b_i \times P_i) \quad (4.19)$$

Subject to

$$C \leq C_{budget}$$

where

P_i = the PMU at bus i , either 0 and 1

a = the cost of PMU installation

b_i = the cost of additional channels at bus i

4.3.4 Optimization

Several evolutionary optimization approaches can be applied to the bi-level optimization problem in 4.3.3. The opposition-based elitist binary genetic algorithm (O-BEBGA) is chosen for its performance since it uses opposition elements to enhance the convergence speed and the solution quality [36–38, 56].

Traditionally, bi-level problem objectives are solved separately, and the variables are exchanged between the objective functions to sort out the bi-level objectives. In this problem, however, both objective functions share the same search space; therefore, both functions can be evaluated at the same time, and the need for variable exchange is eliminated. This simultaneous evaluation can be done by using a sorting function which sorts the variables in terms of feasibility, higher objective maximization, and lower objective minimization. The

Variables					P_{Cost}	$\hat{O}$	Rank
Bus 1	Bus 2	Bus 3	...	Bus N			
0	1	1	...	1	62000	17.4	1
0	1	0	...	0	54000	14.4	2
0	1	0	...	1	55000	13.2	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
1	0	0	...	0	3000	3.9	N-2
0	1	0	...	0	2400	2.6	N-1
0	0	0	...	1	1400	1.4	N

→ Feasible
 Not feasible

Figure 4.2: Sorting function

sorting function is shown in Fig. 4.2.

The overall algorithm for the proposed approach is shown in Fig. 7. The algorithm solves for the complete observability, then uses the budget of each stage to determine the optimal solution for the stage, where the observability is maximized taking advantage of the available budget for the current stage. The algorithm assumes that each k stage is independent; therefore at each stage, the budget C_{budget}^k is utilized to the maximum as long as the observability objective justifies it. The SF in the flowchart is used to determine the current mode of the algorithm. When SF is set to zero, the algorithm solves for complete observability; and when SF is set to one, the algorithm solve for the optimal solution within the current stage k . The crossover probability (P_c) is set to 0.7 and the mutation probability (P_m) is set to 0.1. The O_{rn} , C_{rn} , and M_{rn} are random variables.

4.4 Case Studies

This section illustrates the application of the proposed OPP on the IEEE 14-bus and IEEE 30-bus test systems. First, the voltage creation results are presented; then the critical buses are incorporated into the multi-stage OPP framework.

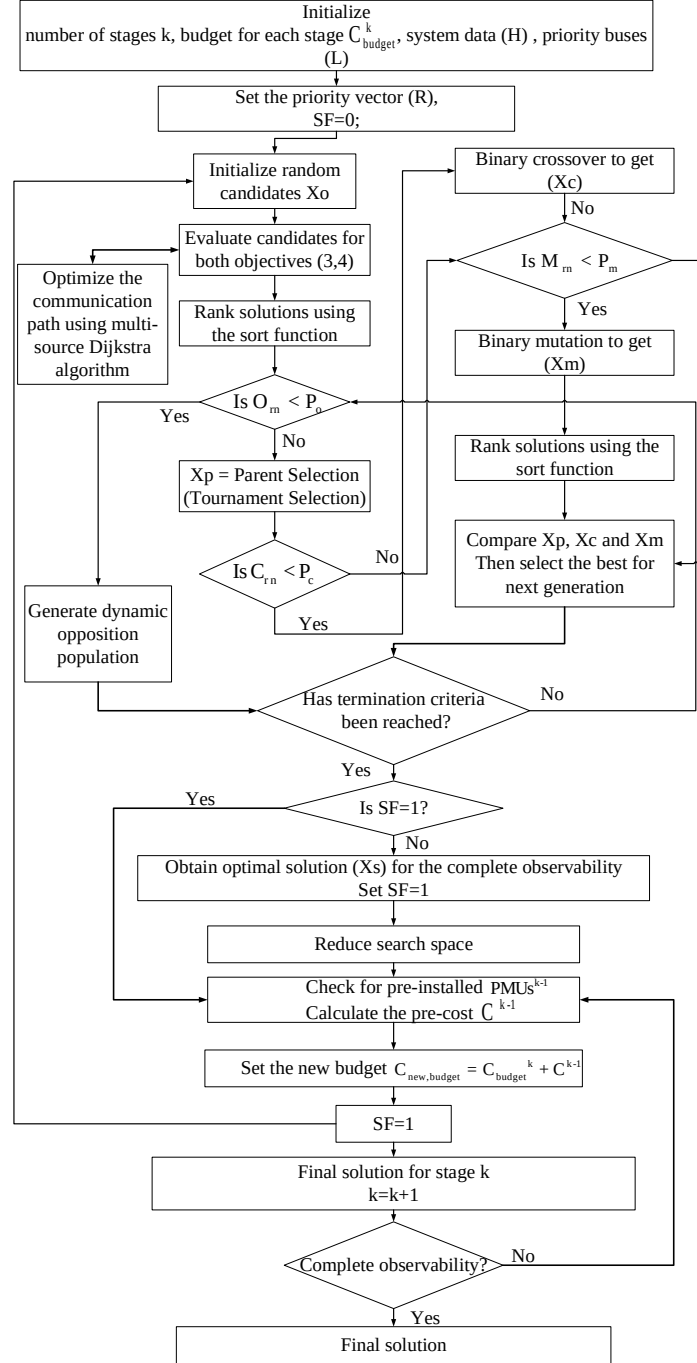


Figure 4.3: Flow chart of the proposed algorithm

4.4.1 Voltage Stability Analysis

The voltage stability criterion in section 4.2 is used to determine the critical buses of the system. First, the LQP is evaluated for the system, then the results are used to determine the critical bus index vector in the network as in (4.10). The peak loads for the systems are increased uniformly, until the system is on the verge of voltage collapse. By stressing out the system, the vulnerable branches can be identified using the LQP indices.

The results of the LQP for the IEEE 14-bus are shown in Table 4.1. The vulnerable branches for the IEEE 14-bus are the branches connecting buses 1-5 and 2-3, as shown in Table 4.1. The critical bus indices Cr for the IEEE 14-bus are shown in

Table 4.1: LQP Results for The IEEE 14-bus system

Line		LQP	Line		LQP
From Bus	To Bus		From Bus	To Bus	
1	2	0.649446	6	11	0.14243
1	5	0.978306	6	12	0.072266
2	3	0.884747	6	13	0.113537
2	4	0.26333	7	8	0.357901
2	5	0.16056	7	9	0.128808
4	3	0.440793	9	10	0.003097
5	4	0.102306	9	14	0.030256
4	7	0.240604	11	10	0.112265
4	9	0.225796	12	13	0.025232
5	6	0.723623	13	14	0.155987

Table II. It can be seen that buses 1 and 3 are buses with high criticality from a voltage stability perspective. The same analysis is applied for the IEEE 30-bus. The results for the LQP and the critical bus indices are shown in Tables 4.6 and 4.4 respectively. The results show that buses 1 and 2 are the critical buses for this system.

Table 4.2: The Critical Bus Indices for The IEEE 14-bus system

Bus	Cr_i	Bus	Cr_i
1	0.542584	8	0.17895
2	0.391616	9	0.077591
3	0.441847	10	0.038454
4	0.212138	11	0.084899
5	0.392959	12	0.032499
6	0.210371	13	0.073689
7	0.181828	14	0.062081

Table 4.3: LQP Results for The IEEE 30-bus system

Line		LQP	Line		LQP
From Bus	To Bus		From bus	To bus	
1	2	0.500684	15	18	0.040394
1	3	0.488809	18	19	0.012413
2	4	0.136807	20	19	0.014889
3	4	0.084614	10	20	0.061605
2	5	0.747487	10	17	0.024886
2	6	0.259297	10	21	0.066151
4	6	0.139397	10	22	0.06171
7	5	0.203464	22	21	0.002366
6	7	0.078182	15	23	0.065136
6	8	0.151653	22	24	0.056142
6	9	0.097725	23	24	0.049225
6	10	0.225193	25	24	0.015551
11	9	0.258252	25	26	0.086484
9	10	0.133004	27	25	0.039594
4	12	0.273633	28	27	0.306153
13	12	0.215200	27	29	0.075241
12	14	0.057810	27	30	0.112366
12	15	0.086120	29	30	0.03269
12	16	0.069457	28	8	0.130139
14	15	0.015862	6	28	0.003284
16	17	0.038807			

4.4.2 Multi-stage OPP

In the multi-stage OPP, the PMUs are installed in the system over a span of number of years. Each stage has its own independent budget, and the OPP utilizes the current budget

Table 4.4: The Critical Bus Indices for The IEEE 30-bus system

Bus	Cr_i	Bus	Cr_i	Bus	Cr_i
1	0.329831	11	0.129126	21	0.022839
2	0.328855	12	0.117036	22	0.030055
3	0.191141	13	0.1076	23	0.03812
4	0.12689	14	0.024557	24	0.03023
5	0.316984	15	0.041503	25	0.035407
6	0.119341	16	0.036088	26	0.043242
7	0.093882	17	0.021231	27	0.106671
8	0.093931	18	0.017602	28	0.109894
9	0.122245	19	0.009101	29	0.035977
10	0.081793	20	0.025498	30	0.048352
11	0.129126	26	0.043242	30	0.048352

within the framework presented in subsection 4.3.3. The critical indices obtained from the voltage stability analysis are used to prioritize the PMUs installation.

The OPP solution for the IEEE 14-bus is shown in Table 4.5. The complete observability is achieved in three stages. The budget for each stage is set to \$100,000; each stage has its own independent budget. The critical buses for the IEEE 14-bus are presented in the priority vector L , and the PMUs are installed on these buses first, as long as its feasible.

The OPP solution for IEEE 30-bus is shown in Table 4.6. The budget for each stage is set to \$200,000 and the critical buses from the voltage stability analysis are $L = [1, 2]$. The PMUs are installed on the higher priority buses first, since these buses provide the maximum skewed observability $\mathcal{O}'$.

Table 4.5: Multi-Stage PMU Installation for the IEEE 14-Bus System

	ZIB effect is NOT considered $L = [1, 3]$	ZIB effect is considered $L = [1, 3]$
Stage One P_{Cost}^1	1, 3 \$96,000	1, 3 \$96,000
Stage Two P_{Cost}^2	1, 3, 8, 10 \$92,000	1, 3, 9 \$56,000
Stage Three P_{Cost}^3	1, 3, 8, 10, 13 \$52,000 'Complete Observability'	1, 3, 6, 9 \$56,000 'Complete Observability'

Table 4.6: Multi-Stage PMU Installation for the IEEE 30-Bus System

	ZIB effect is NOT considered $L = [1, 2]$	ZIB effect is considered $L = [1, 2]$
Stage One P_{Cost}^1	1, 2, 10 \$168,000	1, 2 \$104,000
Stage Two P_{Cost}^2	1, 2, 6, 10, 11, 12 \$172,000	1, 2, 10, 24 \$116,000
Stage Three P_{Cost}^3	1, 2, 6, 10, 11, 12, 18, 23, 26, 29 \$188,000 'Complete Observability'	1, 2, 10, 12, 19, 24, 30 \$156,000 'Complete Observability'

4.5 Conclusion

This chapter presented a multi-stage OPP approach, where the voltage stability criterion is used to determine the critical buses in the system. The proposed voltage stability criterion is based on the line stability index. By using this criterion, the vulnerable branches and critical buses can be identified. These buses should be given higher measurement redundancy, since they represent the vulnerable parts of the system; thereby ensuring voltage collapse and system contingency are identified promptly to start remedial actions. The proposed creation

can be integrated with other criteria for application-based OPP.

Chapter 5

PMU Placement Against False Data Injection Attacks

5.1 Introduction

The false data injection attack (FDIA) is carefully designed to bypass the bad data detection (BDD) test, which is residually based, of the state estimators [57–60]. The BDD is used initially to detect anomalies in the measured data due to noise or malfunctioning devices. The most popular defense against FDIAs is to secure a minimum subset of network measurements to render FDIAs infeasible [58, 61, 62]. Switching the network topology as a new defense mechanism has been proposed by [63]. The authors of [64] proposed two detection algorithms for FDIAs, one based on the distribution of the measurements and the other is based on the probability over time. The authors of [65] used online data from the PMUs and load forecast to detect FDIAs but still assumed the PMU data to be uncompromisable. The references mentioned above used DC estimators and remote terminal units (RTU) measurements to evaluate their models and detection approaches.

The BDD of AC-estimators, on the other hand, is harder to bypass without detection [58]. The authors of [66] investigated the FDIAs on AC-estimators in which the authors concluded that the adversaries need to collect online data in addition to topology knowledge for a successful attack. In [67], the authors used graph theory to design an attack algorithm and

determine vulnerable measurements. These proposed approaches considered RTUs as the measurement devices for their models. Zhao *et al.* [68] used PMUs as a secure platform to develop a detection method for FDIA on nonlinear estimators.

The vulnerability of PMU to FDIAs has been introduced in [69, 70]. The ability to mask line outages via FDIA was investigated in [69]. The ability to spoof the global positioning system (GPS) signal of the PMUs and the consequential impact has been examined in [70]. In [71], a protection scheme against FDIAs was proposed, in which a small number of PMUs are deployed as a secure platform to render FDIAs infeasible, under a DC-estimator paradigm.

Most studies have used DC-estimator, which is a simplified linear estimator [57, 58, 60–64]. In the studies which consider nonlinear estimators, the PMU measurements and their effect were not considered. The studies that consider FDIAs under PMU paradigm did not consider the state estimation part of the process; they either assumed a DC attack scenario or proposed deploying PMUs as secure measurement units to guard against FDIAs [68–71].

As mentioned earlier, most FDIA literature considers PMUs to be a secure and uncompromisable platform. In literature that considers computerizing PMUs, no defense mechanism was presented. This chapter describes a formulation for the optimal placement of PMUs (OPP) to guard against FDIAs. This formulation, assumes that PMUs can be compromised, and the deployed PMUs serves as monitors and authenticators for each other.

5.2 Attack Model

This section briefly reviews the attack model for different state estimators and describes the attack model for PMU based state estimation.

5.2.1 RTU-based attack models

The FDIA rely on manipulating the measurements by an attack vector (a), where the attack vector is constructed by taking advantage of the system topology. Most of the FDIA literature use a DC-estimator model, however, the same strategy can be used for AC-estimators. Consider the following model

$$z = hx + v; \quad (5.1)$$

where z and v are vectors representing the measurement and noise respectively with a size of m by one. In DC-estimators, z consists of the real power flows and injections. In AC-estimators, the z consists of both real and reactive power measurements. The x vector consists of the bus angles for DC-estimators, and bus voltages and angles for AC-estimators. h is a constant Jacobian matrix with a size of m by n ; where m is the number of measurements and n is the number of states.

The attack vector a is constructed to avoid detection by either the largest normalized residual (LNR) or the chi-square test as follows:

$$z_{comp.} = z_{tru} + a; \quad (5.2)$$

$$\|z - hx\| \leq \tau_1; \quad (5.3)$$

$$J(\hat{x}) \leq \tau_2; \quad (5.4)$$

where τ_1 is a tolerance constant and τ_2 is the confidence level of the chi-square test. As seen in (5.5), by using such vector the regular BDD test can no longer detect the FDIA [61–63].

However, the adversaries need to know the topology of the grid, to use this vector.

$$\begin{aligned}\|z_{comp.} - hx_{comp.}\| &= \|z_{tru} + a - h(x + c)\| \\ &= \|z_{tru} + ch - hx - hc\| = \|z_{tru} - hx\| \leq \tau_1\end{aligned}\tag{5.5}$$

5.2.2 PMU-based attack model

The AC- and DC-estimators discussed in 5.2.1 rely on RTU measurements; therefore, the adversaries need to compromise several RTUs to launch a successful attack. However, the adversaries need only to compromise one PMU for a successful FDIA.

Unlike the RTU, a single PMU can have as many channels as the number of adjacent buses. As a result, the PMU can measure the current flow to all adjacent buses in phasor form. This feature makes state estimation a straightforward process, but creates a vulnerability where compromising a single unit is enough for a successful attack.

To launch a successful attack, the measurements vector in (5.2), which consists of the voltage and current flows in rectangular form, can be manipulated using the same strategy for DC-estimators, and the attack vector should use the grid topology to mask the data. Moreover, the adversaries do not need to know the complete topology of the grid (h); they need to know the local topology, which can be estimated by monitoring the PMU measurement data. Consider h_s to be a subset of h , ($h_s \in h$), corresponding to the local topology. The attack vector is constructed as follows:

$$\begin{aligned}z_{comp.} &= z_{tru} + a; \\ \text{where} \\ a &= c \times [0 \dots h_{s,1} \ h_{s,2} \dots h_{s,i} \ 0 \dots 0]^T; \\ z_{comp.} &= [z_{tru_1} \ z_{tru_2} \dots z_{comp_1} \ z_{comp_2} \dots z_{comp_i} \ z_{tru_{i+1}} \dots]^T.\end{aligned}\tag{5.6}$$

By using this attack vector the BDD tests in (5.5) and (5.4) can be bypassed without detection.

5.3 Optimal PMU placement against FDIAs

Most approaches propose securing a subset of the measurements to defend against FDIA [58]. Others deploy PMUs as the secure measurement subset, within a DC-estimator or a nonlinear estimator framework [68, 71, 72]. However, this chapter the proposes an OPP formulation to guard against FDIAs, while assuming PMUs to be compromisable. In this scenario, the adversaries can manipulate all channel of the compromised PMU. Thereby, manipulate several states instead of just one. Moreover, these states will not only pass the BDD test but behave as confirming errors, which makes harder to detect even when the adversaries do not have accurate knowledge of the local topology h_s .

Before discussing the proposed OPP approach, the complete observability for the system using PMUs is presented. The constraint in (5.7) must be met for achieving full observability, where $\mathbf{1}$ is a vector of length N with all its elements equal to 1, and N is the number of buses.

$$\mathcal{O} = A P \geq \mathbf{1} \quad (5.7)$$

where

$$P_i = \begin{cases} 1, & \text{if a PMU is installed at bus } (i) \\ 0, & \text{otherwise} \end{cases} \quad (5.8)$$

$$A_{ij} = \begin{cases} 1, & \text{if a PMU is installed at bus } i \\ 1, & \text{if there is a branch connecting} \\ & \text{bus } i \text{ to bus } j \\ 0, & \text{otherwise} \end{cases} \quad (5.9)$$

The proposed OPP formulation guards against a compromised PMU, by utilizing the other deployed PMUs. There needs to be enough independent redundancies, which would prevent the FDIA from bypassing the BDD tests in (5.4) and (5.5). However, these redundancies need to be kept at minimum to reduce the overall installation cost.

Suppose we have the deployed PMUs in a set $d = \{P_1, P_2, \dots, P_n\}$, where the attacked PMU is a member of this set d . The other deployed PMUs of this set serve as secure measurement units that can guard against the compromised measurements of the attacked PMU if and only if (5.10) holds [61].

$$\text{Rank}(H_p) = \text{Rank}(H_{p,2N-2}) + 2 \quad x_k \in X; \quad (5.10)$$

where H_p is the transition matrix with m rows corresponding to the p set of PMU measurements and $2N$ columns; and N is the number of the states in the system. The matrix $H_{p,2N-2}$ is the same as the H_p matrix except for the number of columns, where the columns corresponding to state x_k , ($x_k \in X$), are removed. It should be noted that the estimated states in X consist of the bus voltage magnitudes and angles in rectangular form. Therefore, each bus has two columns in the H_p matrix corresponding to the real imaginary parts of the bus voltage.

In this framework, the rest of the deployed PMUs (not-attacked) serve as members of

the p set (protected measurements), and the attacked PMU can be any member of the d set. The deployed PMUs in the d set are chosen in such a way that any member of the set can be removed—due to an attack—and the rest of the members will serve as the p set such that (5.10) holds for all buses that have PMUs.

To facilitate this scheme, the OPP in (5.11) is proposed. By using this criterion, the BDD of the system will be able to detect any FDIA on any single PMU. Since each PMU offers n_p independent measurements and the corresponding H_p will have a rank equals to n_p . The criterion in (5.11) guarantee that there will be at least one independent measurement supporting or contradicting any all-channel attack on a PMU.

$$\mathcal{O} \geq \check{\mathbf{l}} \quad (5.11)$$

where

$$\check{\mathbf{l}}_i = \begin{cases} 2, & \text{if a PMU is installed at bus } i \\ 1, & \text{otherwise} \end{cases} \quad (5.12)$$

5.4 Simulation and Results

In this section, the proposed approach is tested on the IEEE 14-bus and IEEE 30-bus test systems. To test the validity of the proposed approach, the FDIA is used on all the PMUs in the system, one at a time. The FDIA is used as an all-channel attack, where all channels of the attacked PMU are manipulated as in (5.6). The system states X are estimated using wighted least square (WLS) method. The significance level for the chi-square test is set to 0.05.

In this scenario, the adversaries take control of a single PMU and can manipulate all

the channels of the attacked PMU. Two schemes are used for deploying PMUs, the normal observability in (5.7), and the proposed scheme in (5.11). The FDIA is tested on all the deployed PMUs, one at a time.

Table 5.1 shows that deploying PMUs under normal conditions makes the PMUs at buses $\{2\}$ and $\{6\}$ vulnerable to FDIAs; therefore, the FDIAs can bypass the BDD test. However, for the same scheme PMUs at buses $\{7\}$ and $\{9\}$ are not vulnerable to FDIA, since the observability for these buses satisfies the criterion in (5.11).

As for the proposed approach, the FDIAs do not succeed on any of the deployed PMUs, since these attacks attempt to change at least one of the system states which are protected according to (5.10), (5.11). For the IEEE 30-bus, Table 5.2 shows that PMUs at buses $\{11\}$, $\{12\}$ and $\{19\}$ are vulnerable to FDIAs when deploying PMUs for normal conditions observability. The proposed approach, on the other hand, protects against FDIAs of a single PMU attack. Moreover, the new OPP scheme does not necessitate increasing the number of PMUs to guard against FDIAs as is the case with the IEEE 30-bus test system in Table 5.2.

Table 5.1: FDIAs on the IEEE 14-bus Under Different OPP Schemes

PMU Locations	Attacked PMU	$J(\hat{X})$	$\mathcal{X}(95\%)$	BDD Test
2, 6, 7, 9	2	003.15	003.94	Passed
	6	003.11	003.94	Passed
	7	169.91	003.94	Failed
	9	168.76	003.94	Failed
4, 5, 6, 7, 9 ^a	4	1270.3	12.338	Failed
	5	1246.2	12.338	Failed
	6	64.022	12.338	Failed
	7	240.501	12.338	Failed
	9	191.039	12.338	Failed

^aProposed OPP approach.

Table 5.2: FDIAs on the IEEE 30-bus Under Different OPP Schemes

PMU Locations	Attacked PMU	$J(\hat{X})$	$\mathcal{X}(95\%)$	BDD Test	PMU Locations	Attacked PMU	$J(\hat{X})$	$\mathcal{X}(95\%)$	BDD Test
1, 2, 6, 10, 11, 12, 19, 24, 25, 27	1	1,200.1	21.664	Failed	2, 4, 6, 9, 10, 12, 15, 18, 25, 27 ^a	2	261.592	29.788	Failed
	2	1,375.9	21.664	Failed		4	2713.91	29.788	Failed
	6	226.955	21.664	Failed		6	2833.82	29.788	Failed
	10	050.060	21.664	Failed		9	563.695	29.788	Failed
	11	010.229	21.664	Passed		10	446.976	29.788	Failed
	12	011.800	21.664	Passed		12	308.635	29.788	Failed
	19	010.010	21.664	Passed		15	311.102	29.788	Failed
	24	043.514	21.664	Failed		18	87.9443	29.788	Failed
	25	117.734	21.664	Failed		25	093.003	29.788	Failed
	27	097.362	21.664	Failed		27	098.649	29.788	Failed

^aProposed OPP approach.

As shown in Table 5.1 and Table 5.2, the proposed OPP approach guards against FDIA

for a single PMU all channels attack, since this attack is guaranteed to violate the conditions in (5.10) for at least one of the protected states. However, the proposed OPP approach might be susceptible to a single channel attack, where the adversaries take full control of a PMU but manipulate only a single channel. In this single channel attack, some of the adjacent buses of the attacked PMU might be susceptible to this attack. Yet, this single channel attack can be guarded against if the criterion in (5.11) is changed to $\mathcal{O} \geq \check{2}$, where $\check{2}$ is a vector of length N with all its elements equal to 2.

5.5 Conclusion

An attack on a PMU can have a high impact, as the adversary can manipulate the states of all connected buses. This chapter investigates the FDIAs on PMUs from an OPP perspective. The proposed OPP approach utilizes the PMUs as authenticators for each other. This approach does not require any modification to existing BDD tests or state estimation methods and can be used with new BDD algorithms.

Chapter 6

Conclusions and Future Work

This chapter provides a general conclusion about the methods that have been used, the general outcomes of this work and suggestions. Possible future developments that can be built on or added to the presented work are also provided.

6.1 Conclusions

This dissertation has presented a comprehensive approach to the OPP problem. In chapter 2, a realistic approach for the observability-based OPP has been presented. A comprehensive installation cost of PMUs has been used, where the substation and communication infrastructures are considered. Chapter 2 has also presented a multistage approach, where PMUs are deployed over several budget periods to achieve complete observability by the final period. The presented multistage approach achieves the optimal PMU allocation for the whole process, instead of targeting the optima for the current budget period. The results in this chapter show that OPP problems should be solved to minimize the installation cost, and not to minimize the number of PMUs. Solving for the minimum number of PMUs does not reflect the actual cost since it does not consider the infrastructure of the substations nor does it consider the additional cost of current channels. In cases where multistage OPP approach is chosen due to budget limitations, decision-makers should not target maximum observability for the current budget but aim for observability solution that is part of the targeted

observability at the final stage. The expectation to the previous statement is when decision makers are pursuing application-based OPP and not observability-based OPP approach.

In chapter 3 a fault-tolerance based OPP approach is developed in which the vulnerability of the network is assessed for PMU deployment. This strategy of deploying PMUs in the proximity of higher probability contingencies increases the likelihood of more effective remedial actions, both preventive and corrective. This framework achieves the optima with the maximum observability in the case of multiple optima. The results show that embedding the vulnerability into the connectivity matrix enable for cost-effective results. This approach of quantifying the targeted application and integrating this quantified application into the observability can be extended to handle multiple applications simultaneously in a cost-effective manner.

The rest of the dissertation focuses on the application-based OPP approaches while considering the installation cost of PMUs. Chapter 4 uses the multistage approach to solve the OPP problem while enhancing the voltage stability for the power grid. A criterion for voltage stability has been developed, where critical buses are identified and prioritized for PMU deployment. The results form the voltage stability criterion identified critical buses to be the PV buses (buses with generators). Installing PMUs at these buses can also be used to estimate and monitor the dynamics of the generators mainly the torque, rotor angle, and speed.

Chapter 5 addresses the emerging cyber-security risks of FDIAs from the perspective of PMU deployment. The proposed approach does not make assumptions about the security of the grid and utilizes the PMUs as authenticators for each other. This approach does not require any modification to existing BDD tests or state estimation methods and can be used with new BDD algorithms.

6.2 Future Work

The framework shows the potential for handling multiple criteria for application-based PMUs. As the PMU deployment requires substantial investment by the utilities, more benefits should be expected from the PMUs. Currently, most application-based approaches pursue one technical benefit besides the observability such as voltage stability. There is a need to investigate multi-application-based approaches where several applications of PMUs can be pursued cost-effectively.

One of the advantages of PMUs is the high resolution of measurement data. While this high rate of data sampling enables for better situational awareness and development of advanced controls, it creates a problem of handling large data efficiently without consuming vast resources. This big data issue emphasizes the need for faster computational tools to process this data efficiently.

Data analysis is another promising field for power systems using PMU data. The large volume of data being provided by PMUs can present new insights and opportunities for power grid operations. Data analysis can be used to forecast the lifetime of the grid components and their current conditions. New insights into the grid contingencies and the dynamics of the generators can also be used to enhance existing controls.

The cyber-security aspect of the grid is a significant concern. The FDIAs which has been presented in this work reflects one of these concerns. There is a need to develop new schemes to defend against such attacks. Moreover, there are other cyber-security attacks such as man-in-the-middle and denial of service. As the current power grid incorporates more and more smart grid technologies, these concerns will become increasingly critical and must be addressed.

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