CONSTRAINTS ON LORENTZ-INVARIANCE VIOLATION WITH THE HAWC OBSERVATORY

By

Samuel Stephens Marinelli

A DISSERTATION

Submitted to Michigan State University in partial fulfillment of the requirements for the degree of

Physics — Doctor of Philosophy

2019

ABSTRACT

CONSTRAINTS ON LORENTZ-INVARIANCE VIOLATION WITH THE HAWC OBSERVATORY

By

Samuel Stephens Marinelli

Lorentz invariance is one of the fundamental symmetries assumed by the Standard Model (SM) of particle physics. However, many extensions to the SM allow or predict Lorentzinvariance violation (LIV), in which this symmetry is either spontaneously broken or is only approximately valid at low energies. Under models in which LIV occurs in the photon sector, photon decay to electron/positron pair becomes allowed at sufficiently high energies. Astrophysical γ (gamma) rays propagate over sufficient distances before reaching Earth that very nearly all of them should decay above the energy at which the decay becomes allowed. The detection of high-energy photons can therefore be used to constrain such models.

The High-Altitude Water-Cherenkov (HAWC) observatory is a γ -ray telescope located on the Sierra Negra volcano in the state of Puebla, Mexico. The detector consists of 300 waterfilled tanks instrumented with four photomultiplier tubes each, which detect the Cherenkov light produced by charged particles in extensive air showers initiated by TeV cosmic γ rays. This thesis presents an analysis of high-energy emission from several HAWC γ -ray sources and sets new constraints on LIV-induced photon decay. This is done using a new algorithm for estimating the energies of γ rays detected by HAWC, utilizing a neural network (NN) trained on Monte Carlo (MC) simulations of the detector. The NN uses a multilayer perceptron architecture and maps several geometric features of the air shower to an estimate of the primary photon's energy. HAWC's high-energy sensitivity combined with its improved energy resolution using the NN allows an LIV-induced cutoff in γ -ray spectra below 209 TeV to be ruled out with 95% confidence. This corresponds to limits on the quantum-gravity energy scale of 8.74×10^{30} or 4.27×10^{22} eV for LIV perturbing the photon dispersion relation at first or second order respectively. These limits constitute a new best constraint on photon-sector LIV.

Copyright by SAMUEL STEPHENS MARINELLI 2019

ACKNOWLEDGMENTS

Thanks to my thesis advisor and mentor, Jim Linnemann. You taught me about all of the technical facets of the work: astrophysics, particle physics, HAWC, statistics, machine learning—the list goes on. But you also taught me how to conduct myself as a scientist and how to communicate my ideas and results. Thanks for all your patience and guidance.

Thanks to my thesis committee, Kirsten Tollefson, Gus Sinnis, Ed Brown, and Scott Pratt. You helped me plan out a thesis about SNRs and didn't bat an eye when I came back with a thesis about LIV.

Thanks to Tilan Ukwatta and Tolga Yapıcı, who were postdocs in our group while I was here. You set an example for me as a scientist that I still strive to emulate to this day.

Thanks to my predecessor, Udara Abeysekara. You were my first guide to HAWC, and you left some seriously big shoes to fill. And thanks to my successors, Joe Lundeen, Alison Peisker, and Dan Salazar-Gallegos. You tolerated all my jargon and acronyms. Now go fill my shoes (size $8\frac{1}{2}$).

You're welcome to all the young people in the collaboration who got to hang out with me at collaboration meetings and read my musings on Slack. I don't know what you'll do without me.

Thanks to everyone in the HAWC collaboration. You were the giants whose shoulders I got to stand on. Thanks to John Pretz and Andy Smith for all the wisdom and advice on data, algorithms, and everything else. Thanks to Humberto Martínez-Huerta and Pat Harding for letting your LIV theory knowledge diffuse into my head (albeit slowly).

Thanks to my parents, George and Kathy Marinelli. You encouraged me literally since birth, and it's no exaggeration to say none of this work would have been done without you. And thanks to Xueying Huyan, who put up with the travel, the nights and weekends spent working late, and the endless rants about software. You were a constant source of comfort and support while I was writing this document. Now go write your own!

TABLE OF CONTENTS

LIST (DF TABLES
LIST (DF FIGURES
Chapte	er 1 Lorentz-invariance violation
1.1	The Lorentz transformation
1.2	Violation phenomonology
1.3	Photon decay
Chapte	er 2 γ -ray production at PWNe and SNRs $\ldots \ldots \ldots \ldots $ 9
2.1	Pulsar properties
2.2	Pulsar evolution
2.3	Mechanisms of hadronic acceleration
2.4	Models of leptonic acceleration
Chapte	er 3 The HAWC observatory 22
31	Atmospheric air showers 22
3.2	Water-Cherenkov detectors
3.3	Data-acquisition system
3.4	Science goals
Chapte	$er 4 \text{Air-shower reconstruction} \dots \dots$
4.1	Calibration
4.2	Edge finding
4.3	Core reconstruction
4.4	Angle reconstruction
4.5	Background rejection
4.6	MC simulations of air showers
Chapte	er 5 Energy estimation
5.1	$Method \qquad \dots \qquad 48$
0.1	5.1.1 Input Variables
	5111 Multiplicity 49
	5112 Containment 50
	5113 Atmospheric Attenuation 50
	$5.1.1.5$ Atmospheric Attenuation $\dots \dots \dots$
	5.1.2 Iranning
к o	Derformance
0.2	5.2.1 Other Energy Variables
	5.2.1 Other Energy variables
	5.2.1.1 Fraction Hit/Fraction-Hit NN
	5.2.1.2 Likelihood Energy Estimator

		5.2.1.3	Ground Para	meter Energ	gy Es	tima	tor										57
		5.2.1.4	Ground Ener	gy Estimate	or												57
	5.2.2	Linearity															58
	5.2.3	Bin Over	lap														59
	5.2.4	Bias				• •											63
	5.2.5	Resolutio	m														69
	5.2.6	RMS Err	or														69
	5.2.7	Bin Cont	amination .														71
	5.2.8	Agreeme	nt of Estimate	ors										•			71
Chapte	er 6	Crab Ne	bula spectra	l reconstru	ictio	n .								•			75
6.1	Skyma											•	•	•		•	75
6.2	Binnec	l-likelihoo	d formalism			• •				•		•	•	•		•	77
6.3	Flux-p	oints calc	ulation											•			85
6.4	Fit res	ult				• •		• •		•		•	•	•		•	86
	-																0.0
Chapte	r''	LIV limi	ts			• •	• •	• •	• •	·	• •	·	·	•	•••	·	92
7.1	Analys	sis techniq	ue			• •	• •	• •	• •	·	• •	·	·	•	••	•	92
7.2	Result	s and inte	rpretation .			• •	• •	• •		·	•••	•	·	•	••	•	99
	0	a 1 ·	1.6.4														105
Chapte	er 8	Conclusi	ons and futu	ire prospe	cts .	• •	• •	• •	• •	•	• •	·	•	•	•••	·	105
	NDIV																100
APPE	NDIA					• •	• •	• •	• •	·	• •	·	•	•	•••	·	100
BIBLI	OGRA	PHY															125

LIST OF TABLES

Table 5.1:	NN input variables that characterize energy. The annuli are about the shower axis. Annuli $0 - 8$ are each 10 m thick. "Annulus" 9 extends from 90 m to infinity.	49
Table 6.1:	Best-fit values of log-parabola spectral parameters with statistical and sys- tematic uncertainties. Systematic uncertainties are the sum in quadrature of the deviations from the nominal result produced by performing the anal- ysis using the systematics MCs as described in Section 6.3.	87
Table 6.2:	Energy (in TeV) and energy squared times flux (in TeV/(cm ² · s)) for each point in Figure 6.7. Statistical errors are propagated from the uncertainties on the flux normalization as refit in each energy bin as described in Section 6.3. Systematic errors are sums in quadrature of deviations of the flux points computed using systematics MCs from the nominal flux points	88
Table 7.1:	Top-hat radii for the seven sources	94
Table 7.2:	Best-fit values of the spectral parameters in the LIV model. The σ parameter is fixed for each source.	102
Table 7.3:	p-values and 95%-confidence limits on $E_{\rm LIV}$. Limits are expressed in TeV. The limit using the nominal MC is provided along with a systematic error obtained by adding in quadrature the deviations of the limits calculated using the systematics MCs. The systematics MCs are those having a lower than nominal charge smearing, higher charge smearing, lower QE, higher QE, lower PMT threshold, and no broad-pulse effect. The systematics-degraded limit is then computed as the nominal limit minus its lower error. See Section 4.6 for more details on the systematics MCs.	103
Table 7.4:	95%-confidence lower limits on the QG energy $E_{\rm QG}$ for <i>n</i> values of 1 and 2. All limits are expressed in eV. These limits can be computed from the ones given in the nominal degraded column of Table 7.3 using Equation (1.18).	103
Table 7.5:	Systematics-degraded limits on E_{LIV} at various confidences. Limits are expressed in TeV. A $Z\sigma$ limit is defined to have confidence $F(Z)$ where F is the standard-normal cumulative distribution. As a result, the 3σ , 4σ , and 5σ limits correspond to confidences of 99.87%, 99.9968%, and 99.999971% respectively.	104

Table 7.6:	Observed and expected bin contents from the 2HWC J1825–134 analysis,	
	as plotted in Figure 7.4. Columns are central reconstructed bin energy, ob-	
	served count, background expectation, signal-plus-background expectation	
	with E_{LIV} set to its 95% lower limit of 259 TeV, and signal-plus-background	
	expectation with E_{LIV} set to infinity.	104

LIST OF FIGURES

Figure 3.1:	An atmospheric air shower. Dots indicate shower particles. Image taken from [1] with permission.	25
Figure 3.2:	Left: a HAWC tank with a charged particle passing through (the thick line coming from above) and the resulting Cherenkov cone (the thinner lines). Right: the layout of HAWC's 300 tanks (large circles) and their PMTs (smaller filled circles). The gap in the center is the location of the counting house, which holds the electronics. Image taken from [2]	27
Figure 3.3:	The output voltage from the PMT (top) and the resulting FEB output. From this output the time over threshold (ToT) for each threshold can be extracted and used to estimate the number of PEs produced in the PMT. Image taken with permission from [1].	29
Figure 3.4:	Diagram of the HAWC data-acquisition system [3]	32
Figure 4.1:	Slewing times shown as functions of low and high ToTs. The curves show the fit function from Equation (4.1).	37
Figure 4.2:	The effective charge in each PMT in a γ -ray event. Image taken from [2].	41
Figure 4.3:	The arrival time of each hit used in reconstructing the direction from which the example event in Figure 4.2 originated. Image taken from [2].	43
Figure 4.4:	The lateral distribution of charge within a background event and a signal event. Image taken from [2].	45
Figure 5.1:	Dependence of NN energy estimate on each input variable, with the others frozen at their median values.	54
Figure 5.2:	Correlation between energy variables and MC-truth energy. \ldots . \ldots	60
Figure 5.3:	Gaussians representing distributions of MC-truth energies in reconstructed- energy or energy-proxy bins. The resulting bin purity (fraction of events not migrating into adjacent bins) is shown in Figure 5.8.	64
Figure 5.4:	Distribution of MC-truth energies in energy bins	66
Figure 5.5:	Bias of energy variables.	68
Figure 5.6:	Resolution of energy variables	70

Figure 5.7:	RMS error of energy variables	72
Figure 5.8:	Fraction of MC events reconstructed in each NN bin whose MC-truth energies actually lie in that bin.	73
Figure 5.9:	Correlations of three energy variables	74
Figure 6.1:	The skymap for fraction-hit bin 3 with NN energies between 3.16 and 5.62 TeV.	78
Figure 6.2:	The background skymap for fraction-hit bin 3 with NN energies between 3.16 and 5.62 TeV. This bin uses the DI background calculation.	79
Figure 6.3:	The skymap for fraction-hit bin 9 with NN energies between 56.2 and 100.0 TeV.	80
Figure 6.4:	The background skymap for fraction-hit bin 9 with NN energies between 56.2 and 100.0 TeV. This bin uses background randomization.	81
Figure 6.5:	The 68% quantile of the distribution of the angular distance between the true and reconstructed directions of events in data and MC	84
Figure 6.6:	The Crab log-parabola spectrum fit using HAWC data binned in NN energy, along with similar measurements made by the H.E.S.S. [4] and HEGRA [5] experiments. Errors on the HAWC spectrum are statistical only. See Figure 6.7 for systematic errors.	88
Figure 6.7:	The Crab energy spectrum produced by the flux-points analysis described in Section 6.3, along with measurements by H.E.S.S. [4] and HEGRA [5]. Errors are statistical and systematic added in quadrature.	89
Figure 6.8:	Pulls (residuals divided by their expected fluctuations) for each of the 2D bins included in the Crab analysis. The x axis indicates NN bins; the color scale indicates fraction-hit bins.	90
Figure 7.1:	A log-parabola energy spectrum subject to a hard cutoff. The blue curve shows the true-energy spectrum; the orange curve shows the reconstructed-energy distribution.	93
Figure 7.2:	95%-confidence lower limit on E_{LIV} using 2HWC J1825-134 data as a function of top-hat radius.	95
Figure 7.3:	Two times the natural logarithm of the profile likelihood defined in Equation (7.4), plotted as a function of E_{LIV} for the Crab Nebula.	98

Figure 7.4:	Expected and observed contents of the reconstructed-energy bins from the 2HWC J1825–134 limit-setting procedure. The orange points show the expected distribution under the signal-plus-background model in which the LIV cutoff energy is set to infinity and the other parameters are optimized. The green points show the expected distribution under the signal-plus-background model with the hard cutoff set to its 95%-confidence lower limit and the parameters optimized. The red points indicate the expectations from background only.	100
Figure 8.1:	Mean free path of photons subject to the CMB [6]. \ldots \ldots \ldots	107
Figure .1:	Data/MC comparison of fraction hit. $p = 0.0997$	110
Figure .2:	Data/MC comparison of fraction of tanks hit. $p = 1.10 \times 10^{-5}$	111
Figure .3:	Data/MC comparison of log of core-fit amplitude. $p = 0.0538.$	112
Figure .4:	Data/MC comparison of core distance from array center. $p=0.0538.\ $	113
Figure .5:	Data/MC comparison of cosine of zenith angle. $p = 0.715.$	114
Figure .6:	Data/MC comparison of fraction of charge contained in annulus 0 . $p = 0.0762$	115
Figure .7:	Data/MC comparison of fraction of charge contained in annulus 1 . $p = 0.116$	116
Figure .8:	Data/MC comparison of fraction of charge contained in annulus 2 . $p = 0.550$	117
Figure .9:	Data/MC comparison of fraction of charge contained in annulus 3 . $p = 0.929$	118
Figure .10:	Data/MC comparison of fraction of charge contained in annulus 4 . $p = 0.165.$	119
Figure .11:	Data/MC comparison of fraction of charge contained in annulus 5 . $p = 0.0118$	120
Figure .12:	Data/MC comparison of fraction of charge contained in annulus 6 . $p=0.11$.121
Figure .13:	Data/MC comparison of fraction of charge contained in annulus 7 . $p = 0.0196$	122

Figure .14:	Data/MC comparison of fraction of charge contained in annulus 8 . $p = 1.34 \times 10^{-7}$	123
Figure .15:	Data/MC comparison of fraction of charge contained in annulus 9. $p = 4.54 \times 10^{-15}$.	124

Chapter 1

Lorentz-invariance violation

Symmetry principles are fundamental to the modern understanding of the laws of physics. Many diverse properties of the Universe can be seen as arising out of the symmetries that characterize it. Momentum conservation is guaranteed by space-translation invariance; energy conservation is guaranteed by time-translation invariance; the gauge bosons' existence is necessitated by their corresponding gauge symmetries. And one such fundamental symmetry assumption is that the Universe obeys Lorentz symmetry, a symmetry group corresponding to invariance of the laws of physics under rotations and changes of reference frame. Lorentz invariance is an assumption of the Standard Model (SM) of particle physics, but there are reasons to believe it may be violated, as described in Section 1.2. The subject of this thesis is a search for evidence of the violation of the Lorentz symmetry group, carried out using the High-Altitude Water-Cherenkov (HAWC) observatory, described in Chapter 3.

1.1 The Lorentz transformation

The Lorentz group is the group of linear transformations that can be used to transform quantities in one inertial reference frame into the values they assume in another. Let the spacetime interval between two events, or locations in spacetime, be defined as

$$s \equiv \sqrt{\left(c\Delta t\right)^2 - \left|\Delta \boldsymbol{r}\right|^2} \tag{1.1}$$

where c is the speed of light, t is time, \mathbf{r} is the position 3-vector, and Δ represents a difference taken between the two events. It follows from the axioms of special relativity that the interval between two events is invariant under a change of reference frame [7]. The transformations of the Lorentz group must therefore have the property that they leave the interval between any two events unchanged. The only such transformations are translations and rotations (where rotations involving the time direction must be hyperbolic as described below) [7]. Since translations are affine transformations rather than linear transformations, they do not fall into the Lorentz group, and so the Lorentz group consists of rotations in 4-dimensional spacetime¹.

The following derivation of the functional form of the Lorentz transformation follows [7]. An arbitrary rotation in 4 dimensions can be expressed as a composition of $\binom{4}{2} = 6$ rotations such that each rotation occurs in a plane containing a different pair of the axes of the coordinate system (i.e. the tx-, ty-, tz-, xy-, xz-, and yz-planes). The xy-, xz-, and yz-rotations are ordinary circular rotations and can be interpreted as reorientations of the spatial coordinates of the reference frame. For example, a rotation by an angle θ in the xy-plane can be represented as

$$r' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} r$$
(1.2)

where r is the 4-position $\begin{bmatrix} ct & x & y & z \end{bmatrix}^{T}$ and the prime indicates a transformed quantity. However, the minus sign in Equation (1.1) implies that, in order to leave the interval unchanged, rotations involving the time direction must be hyperbolic rather than circular. For

¹The full group of transformations which leave the interval invariant and therefore represent changes of inertial reference frame is called the Poincaré group or the inhomogeneous Lorentz group and consists of translations and rotations (and compositions thereof). These transformations are all affine.

example, the transformation in the tx-plane is

$$r' = \begin{bmatrix} \cosh \psi & -\sinh \psi & 0 & 0\\ -\sinh \psi & \cosh \psi & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} r$$
(1.3)

for some real parameter ψ , called the rapidity of the transformation. Such a transformation is known as a Lorentz boost and corresponds to a change from one inertial frame to another which has some (constant) velocity with respect to the first. Let the velocity in question be V, directed in the x-direction. We can attempt to find the relation between ψ and V by considering the movement of the spatial origin of the boosted frame in both coordinate systems. Substituting $r = \begin{bmatrix} ct & Vt & 0 & 0 \end{bmatrix}^T$ and $r' = \begin{bmatrix} ct' & 0 & 0 & 0 \end{bmatrix}^T$ into Equation (1.3) and considering the x-component of the resulting vector equation produces

$$0 = -ct \sinh \psi + Vt \cosh \psi, \tag{1.4}$$

which implies that the boost speed and rapidity are related by

$$\tanh \psi = \frac{V}{c} \equiv \beta, \tag{1.5}$$

and so the boost transformation, in terms of the scaled boost velocity β , is

$$r' = \begin{bmatrix} \Gamma & -\beta\Gamma & 0 & 0\\ -\beta\Gamma & \Gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1 \end{bmatrix} r$$
(1.6)

where Γ is the Lorentz factor, defined by

$$\Gamma \equiv \frac{1}{\sqrt{1-\beta^2}}.\tag{1.7}$$

1.2 Violation phenomonology

In general, Lorentz-invariance violation (LIV) can be introduced by the addition of a not Lorentz-invariant term into the free-particle Lagrangian of a theory [8]. This results in a modification to the theory's free-particle dispersion relation of the following form:

$$E^2 - p^2 c^2 + f(q) q^2 = m^2 c^4$$
(1.8)

where q is either pc or E and f is an arbitrary dimensionless function. In the ultrarelativistic limit, where $p \gg mc$, the choice of either pc or E for q is equivalent. f should be small, at least at low energies, so that the low-energy physics remains unperturbed, reproducing the Lorentz-invariant Universe we observe. In order to consider the behavior in the energy regime in which LIV is just becoming perceptible, it is desirable to simplify Equation (1.8) by Taylor-expanding f about q = 0 and keeping only the lowest-order nonzero term in q, resulting in

$$E^2 - p^2 c^2 \pm \alpha_n q^{n+2} = m^2 c^4 \tag{1.9}$$

where n is the order of the lowest-order nonzero term in the Taylor expansion, henceforth called the order of the violation, and α_n is a positive real number called the LIV coefficient of order n. Models of quantum gravity (QG) that induce LIV are expected to produce a coefficient

$$\alpha_n \sim E_{\rm QG}^{-n},\tag{1.10}$$

where $E_{\rm QG}$ is the energy at which quantum-gravitational effects become nonnegligible. $E_{\rm QG}$ is in turn expected to be of order 10^{28} eV, corresponding to the Planck energy scale [9].

For photons, m = 0, and the sign of the LIV term in Equation (1.9) therefore determines

whether the particle's energy or momentum is greater. This has a qualitative effect on the allowed physics; in particular, choosing the negative sign means that the photon's energy exceeds its momentum, and when they differ sufficiently, the extra energy can be used to create a electron/positron pair as described in Section 1.3.

Thus far we have characterized LIV generically. However, a theoretical framework called the minimal Standard Model Extension (mSME) has been developed that specifically examines the functional form of the Lagrangian density associated with such Lorentz-symmetrybreaking phenomena [10]. The mSME Lagrangian adds to the SM Lagrangian the minimal set of non-Lorentz-invariant terms expressible as products of SM and gravitational fields that maintain gauge invariance and renormalizability. Some of these break CPT symmetry. Since we will be concerned with LIV in the photon sector, we will assume only the photon-sector contributions to the Lagrangian, which, in addition to the SM terms, are the following:

$$\mathcal{L}_{\text{LIV}} = -\frac{1}{4} (k_F)_{\rho\lambda\mu\nu} F^{\rho\lambda} F^{\mu\nu} + \frac{1}{2} (k_{AF})^{\rho} \epsilon_{\rho\lambda\mu\nu} A^{\lambda} F^{\mu\nu}$$
(1.11)

where k_F and k_{AF} are constant coefficients which are free parameters of the model, A is the photon field, $F^{\mu\nu} \equiv \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$, and ϵ is the Levi-Civita symbol. The left term of Equation (1.11) is CPT-even, and the right is CPT-odd [9].

1.3 Photon decay

Under LIV models in which Lorentz-invariance-violating term is preceded by a minus sign in Equation (1.9), photon decay to an electron-positron pair becomes possible. This is only allowed at very high energies, and so this statement is compatible with the fact that photon decay is apparently forbidden at all energies thus far observed. Here we will derive the energy at which such a decay becomes allowed. The derivation follows [9].

The dispersion relation for a photon with energy ω and wavenumber k under the generic LIV framework described in Section 1.2 is

$$\omega^2 = k^2 \left(1 + \alpha_n k^n \right). \tag{1.12}$$

Let p be the momentum of the positron produced in the decay, and let θ be half the angle between the emitted positron and electron. Then energy conservation implies

$$k\sqrt{1+\alpha_n k^n} = \sqrt{m_e^2 + p^2} + \sqrt{m_e^2 + |\mathbf{k} - \mathbf{p}|^2}$$
(1.13)

$$=\sqrt{m_e^2 + p^2} + \sqrt{m_e^2 + p^2 + k^2 - 2pk\cos\theta}.$$
 (1.14)

Squaring this equation, rearranging, and squaring again gives the condition

$$\left(\alpha_{n}k^{n} + 2\sin^{2}\theta\right)p^{2} - 2\alpha_{n}k^{n+1}p\cos\theta + 2m_{e}^{2} + \alpha_{n}k^{n}m_{e}^{2} = 0, \qquad (1.15)$$

which admits solutions

$$p_{\pm} = \frac{\alpha_n k^{n+1} \cos\theta \pm \sqrt{\alpha_n^2 k^{2n+2} \cos^2\theta - 4\left(\sin^2\theta + \alpha_n k^n\right)\left(1 + \alpha_n k^n\right)m_e^2}}{2\alpha_n k^n + 2\sin^2\theta}.$$
 (1.16)

In order for the decay to be allowed, the positron momentum must be real, and so the discriminant must be nonnegative. For a given value of the LIV coefficient α_n and a given photon momentum k, this may be possible at all θ in $[0, \pi]$, at some values of θ , or at no values. In the last case, the decay is forbidden. The critical relationship between the LIV

coefficient and photon momentum in order for the decay to be allowed is

$$\alpha_n \ge \frac{4m_e^2}{k^n \left(k^2 - 4m_e^2\right)}.$$
(1.17)

When the left- and right-hand sides of Equation (1.17) are exactly equal, the discriminant of Equation (1.16) assumes a value of $-4m_e^2k^4\sin^2\theta/(k^2-4m_e^2)^2$, which is negative for all opening angles except 0 and π , where it is 0, and so the decay is allowed only for particles emitted in exactly the same direction or in opposite directions. When the greater-than relationship is satisfied, more angles are allowed around 0 and π .

This behavior, in which photon decay is prohibited below a particular α_n -dependent momentum, allows the value of α_n to be constrained by observations of high-energy photons because the decay probability of photons from relevant astrophysical sources is precisely zero below the critical momentum and, as will be shown in Chapter 7, very nearly one above it. Such a limit in turn implies a constraint on the QG energy scale via Equation (1.10):

$$E_{\rm QG} \ge k \sqrt[n]{\frac{k^2}{4m_e^2} - 1}.$$
 (1.18)

The following chapter, Chapter 2, describes pulsar wind nebulae (PWNe) and supernova remnants (SNRs), two related classes of astrophysical sources capable of producing γ rays of sufficient energies to set tight constraints on α_n . Chapter 3 discusses the HAWC observatory, a detector with unprecedented sensitivity to γ rays of such energies, and Chapters 4 and 5 detail the algorithms by which γ -ray event data are analyzed and properties of the particles are reconstructed. Chapter 6 describes a spectral analysis of the Crab Nebula, the brightest source of TeV γ rays, and Chapter 7 shows that the highest-energy measurements of photons derived from the Crab imply a tighter constraint on photon-sector LIV than has been set by any past experiment. Chapter 8 contains some closing remarks about the results and possible future extensions of the analysis.

Chapter 2

γ -ray production at PWNe and SNRs

Pulsars are a class of fast rotating neutron stars. Some pulsars accelerate electrons and positrons to form PWNe, nebulae that surround them and can in turn radiate TeV γ rays. PWNe are some of the most common sources of TeV γ rays, and one PWN, the Crab Nebula, is the brightest such source and will be the subject of the analysis carried out in Chapter 6 and one of the subjects of the analysis discussed in Chapter 7. Pulsars are formed during supernova explosions, and these explosions often produce SNRs, which consist of supernova ejecta and interstellar matter accumulated by these ejecta as they propagate.

2.1 Pulsar properties

The neutron was discovered by James Chadwick in 1932 [11]. Lev Landau predicted based on the existence of the neutron that there would exist very dense stars composed primarily of neutrons. Walter Baade and Fritz Zwicky published a similar prediction in 1934 [12]. Pulsars were later discovered in 1967 via their radio emission by Antony Hewish and Jocelyn Bell [13].

Pulsars are products of supernova explosions. They are minute in size compared to their progenitors. The radius of a neutron-star progenitor is greater than 10^6 km whereas that of a pulsar is approximately 10 km. This shrinking results in a pulsar with a core density in excess of 2×10^{14} g/cm³, making pulsars approximately as dense as atomic nuclei. The high

density allows for production of neutrons and neutrinos via the process

$$p + e^- \to n + \nu_e. \tag{2.1}$$

The resulting neutrons are stable in the pulsar. If they β -decayed, they would produce electrons with energy no greater than 0.78 MeV, but the Fermi energy of the surrounding electron gas in the pulsar is hundreds of MeV, and so the β -decay process is disallowed by the Pauli exclusion principle because there are no electron states unoccupied with a suitable energy [14].

The high rotational frequencies of pulsars can be understood in terms of angular-momentum conservation. Prior to the supernova, the star's radius is around six orders of magnitude larger than it is after collapsing into a pulsar, and so, assuming that the star possesses some nonzero angular momentum in its initial state, the frequency of rotation will be magnified by orders of magnitude by the collapse to compensate for the shrinking radius. Since the angular momentum is $L = I\omega \propto mR^2\omega$ (where I is the star's moment of inertia, ω is its rotational frequency, m is its mass, and R is its radius), the pulsar frequency is approximately

$$\omega_{\text{pulsar}} = \frac{R_{\text{star}}^2}{R_{\text{pulsar}}^2} \omega_{\text{star}}, \qquad (2.2)$$

corresponding to a period of

$$T_{\rm pulsar} = \frac{R_{\rm pulsar}^2}{R_{\rm star}^2} T_{\rm star}, \qquad (2.3)$$

assuming that a negligible fraction of the stellar mass and angular momentum are lost during collapse. A representative pulsar period of order 1 ms is then obtained by assuming a stellar radius of 10^6 km, a pulsar radius of 20 km, and a stellar period of 1 month [14].

An order-of-magnitude estimate of the magnitude of the pulsar's magnetic fields can be obtained in a similar fashion. It is reasonable to assume that the magnetic flux through the surface of the northern or southern hemisphere of the star is approximately conserved as the star collapses into a pulsar:

$$\int_{\text{pulsar}} \boldsymbol{B}_{\text{pulsar}} \cdot d\boldsymbol{A} = \int_{\text{star}} \boldsymbol{B}_{\text{star}} \cdot d\boldsymbol{A}.$$
 (2.4)

Then the magnetic field scales like

$$B_{\text{pulsar}} = \frac{R_{\text{star}}^2}{R_{\text{pulsar}}^2} B_{\text{star}}.$$
(2.5)

For a representative stellar magnetic field of 1000 G, a pulsar magnetic field of 2×10^{12} G is computed.

If the axis of symmetry of the pulsar's magnetic field differs from its axis of rotation, the magnetic field will be time-dependent as a result of the rotation. This time-dependent magnetic field then begets an electric field. The speed with which the magnetic field rotates at the pulsar surface is

$$v = \frac{2\pi R}{T},\tag{2.6}$$

which gives a value of 4×10^6 m/s for a pulsar with radius 20 km and period 30 ms. The magnetic field at the pulsar surface should be approximately perpendicular to the velocity of rotation, so the electric-field magnitude is simply

$$E = vB, (2.7)$$

which produces a representative value of 10^{15} V/m. An electron or positron subject to such a field would gain energy at a rate of 1 PeV/m [14].

A pulsar's rotational kinetic energy is

$$E = \frac{1}{2}I\omega^2, \qquad (2.8)$$

which gives a value of 4×10^{61} eV for a pulsar with period 30 ms, mass 2×10^{30} kg, and radius 20 km. A pulsar that converts its kinetic energy into the energy of accelerated particles with an efficiency of 1% over a lifetime of 10^{10} years must convert that energy at a time-averaged rate of 10^{42} eV/s [14].

2.2 Pulsar evolution

Since the pulsar's rotational kinetic energy is

$$E = \frac{1}{2}I\omega^2 = \frac{2\pi^2 I}{T^2},$$
(2.9)

its spin-down luminosity, defined as $\dot{E} \equiv -dE/dt$, is related to its period and the time derivative of its period by

$$\dot{E} = \frac{4\pi^2 I \dot{T}}{T^3}.$$
(2.10)

This quantity represents the rate at which the pulsar's rotational energy is transformed into other forms of energy. Measured spin-down luminosities vary by several orders of magnitude, from 3×10^{28} erg/s for PSR J2144-3933 to around 5×10^{38} erg/s in the case of the Crab pulsar and PSR J0537-6910 [15]. Pulsars with spin-down luminosities exceeding about 4×10^{36} erg/s

normally produce perceptible PWNe [16, 17].

If the assumption is made that the time derivative of the pulsar's frequency of rotation has a power-law relationship with the frequency itself, i.e.

$$\dot{\omega} = -k\omega^n,\tag{2.11}$$

where n is called the braking index, then solving this differential equation for $\omega(\tau)$ (where τ is the pulsar age) and inverting the relationship to find the age as a function of the period gives

$$\tau = \frac{T}{(n-1)\dot{T}} \left[1 - \left(\frac{T_0}{T}\right)^{n-1} \right]$$
(2.12)

for $n \neq 1$ [18]. Observations are consistent with the statement that the braking index assumes a value between 2 and 3 [19]. A value of 3 would be expected assuming the rotational energy is converted entirely into magnetic-dipole radiation. For the case where n = 3 and the pulsar has spun down enough that $T \gg T_0$, Equation (2.12) simplifies to

$$\tau_{\rm c} = \frac{T}{2\dot{T}} \tag{2.13}$$

where $\tau_{\rm c}$ is called the pulsar's characteristic age. Measurements of T, \dot{T} , and τ have shown that the true age often exceeds the characteristic age, indicating that the assumption $T \gg T_0$ is violated for those pulsars [20].

For a pulsar producing a purely dipole magnetic field (i.e. neglecting the contribution from accelerated particles), the constant of proportionality k in Equation (2.11) is related to the dipole moment by

$$k = \frac{2M_{\perp}^2}{3Ic^3} \tag{2.14}$$

where M_{\perp} is the component of the magnetic dipole moment perpendicular to the pulsar's axis of rotation. This implies that the magnitude of the magnetic field at the pulsar's equator is

$$B = \left(3.2 \times 10^{19} \,\mathrm{G/s^{1/2}}\right) \sqrt{T\dot{T}}.$$
(2.15)

Millisecond pulsars have magnetic fields of order 10^8 G while the fields of slower rotating pulsars called magnetars can exceed 10^{15} G. For pulsars with perceptible PWNe, the magnetic field estimated via Equation (2.15) is usually in the range 1×10^{12} G $- 5 \times 10^{13}$ G [17].

If n is time-independent, the pulsar's spin-down luminosity evolves in time according to

$$\dot{E} = \dot{E}_0 \left(1 + \frac{t}{\tau_0} \right)^{-(n+1)/(n-1)}$$
(2.16)

where \dot{E}_0 and \dot{T}_0 are the spin-down luminosity and period time derivative at the pulsar's birth and

$$\tau_0 \equiv \frac{T_0}{(n-1)\,\dot{T}_0}.\tag{2.17}$$

The spin-down luminosity is approximately constant for $t \ll \tau_0$; for $t \gg \tau_0$, it decays according to a power law with index (n+1)/(n-1).

2.3 Mechanisms of hadronic acceleration

A charged particle moving in the nonuniform magnetic field of the pulsar experiences a time-dependent magnetic field. The particle's motion in the direction perpendicular to the field is approximately circular. According to Faraday's equation, the time dependence of the magnetic field induces an electric field perceived by the particle:

$$\boldsymbol{\nabla} \times \boldsymbol{E} = -\frac{1}{c} \frac{\partial \boldsymbol{B}}{\partial t}.$$
(2.18)

This electric field varies the perpendicular component v_{\perp} of the particle's velocity. Specifically $E_{\perp} \equiv m v_{\perp}^2/2$ changes by

$$\Delta E_{\perp} = \int q \boldsymbol{E} \cdot d\boldsymbol{l} = q \int (\boldsymbol{\nabla} \times \boldsymbol{E}) \cdot d\boldsymbol{S} = -\frac{q}{c} \int \frac{\partial \boldsymbol{B}}{\partial t} \cdot d\boldsymbol{S}.$$
 (2.19)

The time derivative of the magnetic field can be approximated in terms of the Larmor period $T_{\rm L}$:

$$\Delta E_{\perp} = \frac{q}{c} \frac{\Delta B}{T_{\rm L}} \pi R_{\rm L}^2. \tag{2.20}$$

Since $T_{\rm L} = 2\pi/\omega_{\rm L}$, $R_{\rm L} = v_{\perp}/\omega_{\rm L}$, and $\omega_{\rm L} = qB/(mc)$,

$$\Delta E_{\perp} = \frac{1}{2} m v_{\perp}^2 \frac{\Delta B}{B} = E_{\perp} \frac{\Delta B}{B}.$$
(2.21)

The fractional change in E_{\perp} is equal to the fractional change in B; in other words, $E_{\perp} \propto B$. In the observer reference frame, no electric field is present, only a magnetic field, which does no work on the particle. Since the particle's total energy is conserved, when it moves into a region of greater magnetic field and E_{\perp} increases, E_{\parallel} decreases so that $E_{\perp} + E_{\parallel}$ is constant, and vice versa. As a result, a particle encountering an increasing magnetic field will slow down and eventually reverse direction. Such a region of heightened magnetic field is thus known as a magnetic mirror [6].

A particle reflected by a magnetic mirror that is approaching the particle will be reflected

with a speed greater than its initial speed (because the initial and final speeds are equal in the rest frame of the mirror). In the following derivation, the particle's energy is E, the component of its momentum in the direction towards (or away from) the magnetic mirror is p_{\parallel} , the magnetic mirror approaches with speed U, unprimed and primed quantities refer to the observer and mirror rest frames respectively, and pre- and post-reflection quantities are indicated with i and f respectively. The transformed initial energy and momentum are given by

$$E'_{i} = \Gamma\left(E_{i} + Up_{\parallel i}\right); \tag{2.22}$$

$$p'_{\parallel \mathbf{i}} = \Gamma\left(p_{\parallel \mathbf{i}} + \frac{U}{c^2}E_{\mathbf{i}}\right). \tag{2.23}$$

In the mirror rest frame, the reflection simply flips the sign of the momentum:

$$E'_{\rm f} = E'_{\rm i};$$
 (2.24)

$$p'_{\parallel f} = -p'_{\parallel i}, \tag{2.25}$$

and so the final energy and momentum in that frame are given by

$$E'_{\rm f} = \Gamma \left(E_{\rm i} + U p_{\parallel \rm i} \right); \tag{2.26}$$

$$p'_{\parallel f} = -\Gamma \left(p_{\parallel i} + \frac{U}{c^2} E_i \right).$$
 (2.27)

Performing the inverse Lorentz transformation back into the observer frame gives the final

particle energy:

$$E_{\rm f} = \Gamma^2 \left[\left(1 + \frac{U^2}{c^2} \right) E_{\rm i} + 2Up_{\parallel \rm i} \right] = \Gamma^2 E_{\rm i} \left(1 + \frac{2Uv\cos\theta}{c^2} + \frac{U^2}{c^2} \right)$$
(2.28)

where v is the particle's starting speed and θ is the angle between its initial velocity and the direction toward the mirror [6].

Taylor-expanding to first order in U/c, which will be small in astrophysical environments, produces

$$E_{\rm f} \approx E_{\rm i} \left(1 + \frac{2Uv\cos\theta}{c^2} \right).$$
 (2.29)

When Equation (2.29) is averaged with respect to θ from 0 to $\pi/2$, the result is

$$E_{\rm f} = E_{\rm i} \left(1 + \frac{4U}{3c} \right), \tag{2.30}$$

which gives the factor by which the particle's energy increases when it interacts with the mirror. If the particle traverses a distance d between two approaching mirrors and is repeatedly reflected between them, its energy grows in time according to the equation

$$\frac{dE}{dt} = \frac{\Delta E}{\Delta t} = \frac{\frac{4UE}{3c}}{\frac{d}{c}} = \frac{4UE}{3d}$$
(2.31)

assuming $v \approx c$. So the energy increases exponentially with time with a time constant of

$$\tau_{\rm F} \equiv \frac{3d}{4U}.\tag{2.32}$$

This acceleration process is known as first-order Fermi acceleration, in reference to the fact that $\tau_{\rm F}$ is proportional to the first power of 1/U. There is also an analogous process in which

particles scatter off of a single moving region of increased magnetic field (presumably a gas cloud); this is called second-order Fermi acceleration due to the proportionality of its time scale to $1/U^2$ [6].

The CR diffusion equation, in the absence of sources, assuming a steady-state solution, and with an energy-gain rate of $E/\tau_{\rm F}$, reads

$$\frac{d}{dE} \left[\frac{E}{\tau_{\rm F}} n(E) \right] + \frac{n(E)}{\tau_{\rm esc}} = 0$$
(2.33)

where n is the number density of particles per unit energy and τ_{esc} is the timescale on which particles escape from the system in which they are being accelerated. Performing the energy derivative and rearranging gives

$$\frac{dn(E)}{dE} = -\left(1 + \frac{\tau_{\rm F}}{\tau_{\rm esc}}\right)\frac{n(E)}{E},\tag{2.34}$$

which admits solutions of the form

$$n(E) \propto E^{-(1+\tau_{\rm F}/\tau_{\rm esc})}.$$
 (2.35)

Fermi acceleration produces a power-law energy spectrum [6].

Because $\tau_{\rm F} \propto 1/d$, first-order Fermi acceleration is efficient in regions in which magneticfield inhomogeneities occur on short distance scales. This condition is satisfied in astrophysical shock waves, and first-order Fermi acceleration occurring at such sites is known as diffusive shock acceleration (DSA) [6]. The DSA hypothesis is relevant to SNRs because a PWN's wind expands supersonically and drives a shock wave into the surrounding SNR [17].

As the SNR initially expands after the supernova, it accumulates mass from the inter-

stellar and circumstellar media and eventually enters the Sedov-Taylor phase, in which the energy of the SNR is approximately conserved. The outer edge of the SNR is a shock, called the forward shock, that compresses and heats these media; there is also a reverse shock at the inner boundary of the SNR, which initially moves outward but eventually begins moving back in towards the pulsar and PWN. At a time several thousand years after the supernova, still prior to the beginning of the Sedov phase, the SNR reverse shock and the PWN forward shock meet. Until this happens, the two shocks constitute a pair of converging magnetic mirrors capable of accelerating the hadronic CRs [17].

In the presence of molecular clouds, the accelerated CRs can interact with hadronic matter to produce very-high-energy γ rays that are detectable by instruments such as HAWC.

2.4 Models of leptonic acceleration

In the synchrotron self-Compton (SSC) model of γ -ray emission at sites of leptonic acceleration such as PWNe, synchrotron radiation produced by accelerating electrons interacts with the same electron population again via inverse Compton (IC) scattering to produce γ rays. The synchrotron photons have energies in the X-ray band. The parent electrons lose energy to synchrotron radiation at a rate of

$$-\frac{dE_e}{dt} = \frac{\sigma_{\rm T} E_e^2 B^2}{6\pi m_e^2 c^3}$$
(2.36)

where $\sigma_{\rm T}$ is the Thompson cross section. Assuming a power-law electron energy spectrum with index α_e , the spectral energy distribution (SED) of synchrotron photons is then

$$E_{\gamma}^{2} \frac{dN_{\gamma}}{dE_{\gamma}} = -\frac{dE_{e}}{dt} \frac{dN_{e}}{dE_{e}} \frac{dE_{e}}{d\nu_{\gamma}}$$
(2.37)

where N represents a particle flux (particles per unit area per unit time) and ν_{γ} is the linear frequency of a photon. Using Equation (2.36), $dN_e/dE_e \propto E_e^{-\alpha_e}$, and the relation

$$E_e = m_e c^2 \sqrt{\frac{2\pi m_e c \nu_\gamma}{eB}} \tag{2.38}$$

between an electron's energy and the energy of a resultant synchrotron photon, the SED is found to be

$$E_{\gamma}^2 \frac{dN_{\gamma}}{dE_{\gamma}} \propto E_{\gamma}^{-(\alpha_e - 1)/2} \tag{2.39}$$

[6].

These synchrotron photons are then subject to IC scattering with the same PWN electrons. The energy density of the synchrotron photons in the rest frame of an electron with Lorentz factor Γ is

$$u' = n' E'_{\gamma} = \Gamma n \Gamma E_{\gamma} = \Gamma^2 u \tag{2.40}$$

where u is the photon energy density, n is the photon number density, and primes indicate the electron rest frame. One factor of Γ comes from the effect of length contraction on the number density; the other comes from the Lorentz transformation of the photon's 4momentum. The rate at which an electron loses energy to IC scattering is then

$$-\frac{dE_e}{dt} = \frac{4}{3}\sigma_{\rm T}cu' = \frac{4}{3}\sigma_{\rm T}c\Gamma^2 u.$$
(2.41)

The factor of 4/3 comes from averaging over the angle of collision of the particles. In the ultrarelativistic Klein-Nishina regime, where $\Gamma E_{\gamma} \gg m_e c^2$, the average energy of the produced IC photon is simply determined by the electron energy according to

$$\overline{E_{\gamma}^{\rm IC}} = \frac{1}{2} E_e. \tag{2.42}$$

Combining this result with the per electron energy-loss rate (Equation (2.41)) via a derivation similar to the one leading to Equation (2.39) yields

$$E_{\gamma}^2 \frac{dN_{\gamma}}{dE_{\gamma}} \propto E_{\gamma}^{-\alpha_e} \ln E_{\gamma}.$$
(2.43)

Notably, in the Klein-Nishina regime the photon spectral index approaches the electron index [6].

IC scattering can also occur via other varieties of target photons. These include the photons that comprise the cosmic microwave and infrared backgrounds.

The mechanisms of particle acceleration and γ -ray production described in this chapter can yield the very-high-energy photons needed to test for the signature of Lorentz-invariance violation. The following three chapters discuss the HAWC observatory and the algorithms by means of which it reconstructs the properties of these photons.

Chapter 3

The HAWC observatory

The HAWC observatory is located at 4100 m above sea level on the side of the Sierra Negra volcano in the state of Puebla, Mexico. The observatory detects cosmic γ rays, with the goal of investigating their sources and the mechanisms by which they acquire such high energies.

3.1 Atmospheric air showers

When γ rays and charged CRs strike the Earth's atmosphere, they produce showers of lower-energy particles known as extensive air showers. The HAWC observatory detects some of the shower particles that reach the ground and uses information about their locations, energies, and arrival times to reconstruct information about the initial cosmic particle, which is referred to as the primary particle. This reconstruction process is described in detail in Chapters 4 and 5.

The air showers produced by γ -ray, electron, or positron primaries consist almost exclusively of photons, electrons, and positrons and are called electromagnetic showers. The five varieties of particle interaction most relevant to the propagation of these showers are pair production, brehmsstrahlung, ionization, Cherenkov radiation, and trident (three-lepton) production. Pair production occurs when a photon strikes the nucleus of an atom of an atmospheric molecule such as nitrogen and is converted to an electron/positron pair. Brehmsstrahlung occurs when an electron or positron is slowed down by the electromagnetic
field of the nucleus of an atmospheric molecule and emits a photon due to this acceleration. Ionization occurs when an electron or positron strikes a molecule and knocks off an electron. Cherenkov radiation occurs when an electron or positron moving faster than the speed of light in the medium in which the shower is propagating radiates photons by analogy with a sonic boom. Finally, trident production occurs when an electromagnetic interaction between a shower electron or positron and the nucleus of an atmospheric molecule causes the production of an electron/positron pair. Above 80 MeV (an energy known as the critical energy), brehmsstrahlung is the dominant interaction of shower electrons and positrons with the atmosphere, such that the number of particles participating in the shower. Below the critical energy, the electrons' and positrons' energy is quickly dissipated into the atmosphere via ionization [21].

An approximate form for the longitudinal evolution of the total number of particles in the shower is given by Greisen [22]:

$$N(t) \approx \frac{0.31}{\sqrt{\ln(E_0/E_c)}} \exp\left\{ t \left[1 - \frac{3}{2} \ln \frac{3t}{t + 2\ln(E_0/E_c)} \right] \right\}.$$
 (3.1)

Here t is the shower depth (defined as the distance the shower has propagated in units of 37.1 g/cm^2 , the radiation length of electrons and positrons in air [21]), E_0 is the primary energy, and E_c is the critical energy. According to this formula, the depth at which the number of particles is maximized, referred to as shower maximum, is

$$t = \ln(E_0/E_c)$$
. (3.2)

In order to determine the lateral width of the shower, it is necessary to consider also the Coulomb scattering of electrons and positrons off atmospheric molecules. This process dominates the transverse spreading out of the shower particles as the shower propagates. The characteristic length scale for this Coulomb-induced spreading is the Molière radius, defined as

$$r_1 \equiv \frac{2m_e c^2 X_0}{E_c} \sqrt{\frac{\pi}{\alpha}} = 9.3 \,\mathrm{g/cm^2},$$
 (3.3)

where m_e is the electron mass, X_0 is the radiation length, and α is the fine-structure constant. At HAWC's altitude this corresponds to 124 m. Greisen [22] and Kamata and Nishimura [23] provide the following expression, called the NKG function, for the lateral shower distribution:

$$\rho(r) \propto \left(\frac{r}{r_1}\right)^{s-2} \left(1 + \frac{r}{r_1}\right)^{s-4.5} \tag{3.4}$$

where s is the shower age, a parameter appearing in the Mellin-transform solution to the differential equations governing shower evolution. The age has a value of 0 at the start of the shower and increases as it propagates.

The NKG function is sharply peaked at r = 0. This peak is called the shower core. The core propagates along a line called the shower axis. The region of space containing the shower particles is approximately an elliptic paraboloid but is often referred to as the shower plane. The shower core, axis, and plane are depicted in Figure 3.1.

3.2 Water-Cherenkov detectors

The HAWC observatory is made up of 300 water-filled tanks that utilize the water-Cherenkov technique to detect the charged particles produced in air showers. These tanks occupy 57% of



Figure 3.1: An atmospheric air shower. Dots indicate shower particles. Image taken from [1] with permission.

an area of 22000 m² of a flat region constructed on the side of the Sierra Negra volcano. Each steel tank is cylindrical in shape with a diameter of 7.3 m and a height of 5 m and contains a light-tight plastic bladder filled to a depth of 4.5 m with water. The water is purified to the point that its attenuation length is around 10 m, such that Cherenkov photons can propagate unattenuated for distances of order the tank size [24]. The bottom of the tank is instrumented with three Hamamatsu R5912 photomultiplier tubes (PMTs), referred to as the A, B, and D PMTs, and one high-quantum-efficiency (high-QE) Hamamatsu R7081 PMT, referred to as the C PMT. The R5912 PMTs are 8 in in diameter, and the R7081 is 10 in [2]. The former were repurposed from the Milagro experiment to reduce the cost of constructing HAWC. The bladders contained by the tanks are black to minimize reflection of Cherenkov photons so that the PMTs only detect photons emitted directly by the shower particles [25].

When a cosmic γ ray interacts with the atmosphere above the detector, it induces an air shower. Once the shower propagates down to the altitude of the detector, shower electrons and positrons continue to shower in HAWC's tanks and also produce Cherenkov light if they are moving faster than the speed of light in water ($2.26 \times 10^8 \text{ m/s}$). Shower photons also enter the tank and continue to shower into electrons and positrons in the water, which can produce Cherenkov light as well. The Cherenkov photons are then detected by the PMTs. This process and the layout of the tanks and PMTS are shown in Figure 3.2. Since the Cherenkov threshold for positrons and electrons in water is 1 MeV and the ionization energy loss of positrons and electrons in water is 2 MeV/cm [26], positrons and electrons entering the water with energies below 900 MeV will drop below their Cherenkov threshold before they exit the tanks if they pass through the full 4.5 m of water. However, only a small fraction of this energy loss is via Cherenkov radiation; most is via ionization (as with a



Figure 3.2: Left: a HAWC tank with a charged particle passing through (the thick line coming from above) and the resulting Cherenkov cone (the thinner lines). Right: the layout of HAWC's 300 tanks (large circles) and their PMTs (smaller filled circles). The gap in the center is the location of the counting house, which holds the electronics. Image taken from [2].

shower propagating in air). Conversely, the median energy of ground-level muons, which are relatively rare in γ -induced showers but common in hadron-induced showers, is such that they will not go below their Cherenkov threshold of 213 MeV before exiting the tanks. Therefore such a muon will usually pass closer to one PMT than to the rest, producing an excess signal in one PMT, which can be used to discriminate hadronic showers from photonic ones [24]. Such algorithms are described in more detail in Section 4.5.

The 10-in PMTs located at the centers of HAWC's tanks have a quantum efficiency (QE, the probability of producing a photoelectron (PE) that reaches the first dynode of the PMT) about twice that of the 8-in PMTs. This fact is taken into account by the various

reconstruction algorithms described in Chapter 4. The PMTs are provided with a voltage of approximately 1700 V, but the exact values are chosen per PMT in order to match their gains, which are approximately 1.6×10^7 [24]. The 8-in and 10-in PMTs both contain 10 dynodes each.

If a photon shower with primary energy 1 TeV originates from directly above HAWC, on average 7% of that primary energy will reach the ground in the form of the energy of the shower particles. For 100-TeV photons, this number is 28%. Overhead γ showers with primary energy above 1 TeV are detected with nearly 100% efficiency, where a shower is considered to trigger the detector if it produces hits in at least 6.7% of the active PMTs in the event. Those of lower energies can also be detected if their first interactions occur at lower than average altitudes [2].

3.3 Data-acquisition system

Each PMT is connected by an RG-59 cable to one of the front-end boards (FEBs), which prepare the signals to be fed into time-to-digital converts (TDCs). The cables are all of the same length, 610 ft, so that signals from all PMTs require the same time to propagate to the counting house, the building located in the center of the HAWC array containing the data-acquisition system (DAQ). Each FEB is actually two boards connected to a Versa Module Europa (VME) crate (Wiener 6023x610) by which they can interact. A FEB has 16 input channels. It amplifies and integrates each input signal. The signal is then sent to two comparators with two different voltage thresholds. The comparators' outputs are multiplexed by the digital half of the FEB, producing an ECL signal with two or four transitions depending on whether the voltage ever reached the higher of the two thresholds.



Figure 3.3: The output voltage from the PMT (top) and the resulting FEB output. From this output the time over threshold (ToT) for each threshold can be extracted and used to estimate the number of PEs produced in the PMT. Image taken with permission from [1].

See Figure 3.3. The low threshold corresponds to a charge of about 1/4 PE deposited in the PMT; the high threshold corresponds to about 4 PEs. The edge times contain information sufficient to reconstruct the time and deposited charge of the original PMT pulse. The process by which this information is extracted is described in Section 4.1. The analog portion of the FEB also distributes the high voltage (HV) required by the PMTs; each FEB provides HV to the 16 PMTs from which it receives data [3].

The times are measured by a CAEN V1190A-2eSST TDC, which can handle up to 128 PMTs' worth of FEB signals. The complete detector uses 10 TDC units. The TDCs receive an external 40-MHz clock signal, with 8 output bits used to represent 256 subdivisions of the 25-ns clock period, for a timing resolution of 98 ps. TDC output is triggered every 25 μ s; at each trigger, each TDC writes out 26 μ s of edge-time data (the overlap avoids dropping

edges on the time-window boundary) along with other peripheral data such as time stamps and error information. The resulting data rate for each TDC is 45 MB/s, for a per PMT data rate of 350 kB/s [3].

Each TDC module is controlled by a General Electric XVB602-13240010 single-board computer (SBC). The SBC runs a custom program called the Readout Process on the CentOS operating system that receives the TDC output and sends it via Gigabit Ethernet using ZeroMQ [27] to the online-reconstruction farm. The readout rate of TDC data is about 60 MB/s, limited by the clock speed of the VME chip on the TDC. When a TDC has taken 25 TDC events' worth of data, corresponding to about $625 \,\mu$ s, it sets a data-ready bit, which is read by the SBC, initiating the data transfer to the online-reconstruction clients. The SBC recombines 42 such blocks of data, the last two of which must be redundant with the following set of 42 due to occasional TDC firmware errors, and pushes them to the Data Queue, which feeds data to the Reconstruction Clients [3].

The Reconstruction Client is a program that runs on several dedicated computers, collectively known as the online-reconstruction farm, and reconstructs various parameters characterizing the air showers detected by HAWC. See Chapter 4 for the details of the reconstruction process and its outputs. A Reconstruction Client polls the Data Queue for a block of 25 ms of data and runs the reconstruction algorithms on them [3].

The various processes running on the DAQ and online-reconstruction farm are coordinated by a system called Experiment Control. Experiment Control monitors and adjusts several settings of the DAQ and starts and stops data-taking runs. It also reports and logs significant errors in the operation of the detector and will attempt to restart the experiment when a failure causes it to crash. In the modern configuration of the experiment, these attempts are almost always successful although crashes and subsequent restarts often occur frequently during lightning storms. The restart process takes several minutes. Experiment Control communicates with the processes that it oversees by serializing data to the JSON format via the cJSON library [28]. Experiment Control is also responsible for sending alerts to external systems which provide alerts regarding possible astrophysical transients such as γ -ray bursts (GRBs), among them the GRB Coordinates Network (GCN) [3].

Monitoring of the experiment is performed by a dedicated Python application. This package obtains data directly from some monitored components and indirectly from others via Experiment Control. These data are stored via MySQL and copied to servers at the University of Maryland (UMd) and the National Autonomous University of Mexico (UNAM). A web page provides plots generated via the Google Charts API to shifters, the collaboration members assigned to monitor and control the operations of the experiment [3].

Figure 3.4 shows the flow of data among all components of the DAQ and online-reconstruction system.

3.4 Science goals

The HAWC observatory was constructed with the goals of studying TeV γ -ray emission from Galactic and extra-Galactic astrophysical sources as well as various possible new particlephysics processes occurring in the Universe. The detector is also sensitive to charged CRs, including their energy spectrum and anisotropy.

PWNe, described in Chapter 2, are the most common TeV sources in the Galaxy [17]. These include the Crab Nebula, the brightest source of TeV γ rays. The AGILE and Fermi-LAT experiments have observed flares from the Crab between 100 MeV and 100 GeV [29, 30]. The ARGO-YBJ experiment has found evidence of possible TeV time variability [31], but



Figure 3.4: Diagram of the HAWC data-acquisition system [3].

no imaging air-Cherenkov telescopes (IACTS) have seen indication of Crab flare events in the TeV. According to the Fermi-LAT analysis, the observed Crab flare represented, at its peak, a 50-fold increase in the synchrotron emission from the Crab. The suggestion has been made, in order to explain the relatively short time scale on which such large flares can occur, that the underlying particle-acceleration mechanism is magnetic reconnection and not shock acceleration [32]. Among the motivations of HAWC's construction was its sensitivity to a possible TeV inverse-Compton component to these flares: HAWC would be able to detect a 10-fold Crab flare in 10 min. The experiment also monitors other PWNe for flares [33]. Crab time dependence on the time scale of its rotational period has also been observed [34], but HAWC is unlikely to be sensitive to this.

SNRs, also described in Chapter 2, are another class of sources of great interest in the study of TeV γ rays. They are a leading candidate for the source of Galactic CRs. The

observation of TeV γ rays from SNRs could offer evidence of hadronic acceleration at such sources. This is because an SNR accelerating hadrons near a molecular cloud would cause the production of γ rays via a reaction such as

$$2p \to 2p + \pi^0 \to 2p + 2\gamma \tag{3.5}$$

The spectral shape of such γ -emission would be distinct from that of inverse-Compton emission. IC443 is an interesting SNR from which TeV emission has been observed by the VERITAS [35] and Fermi telescopes [36]. Specifically Fermi claims to have detected the distinct shape of π^0 -decay in the IC443 energy spectrum. The possibility of observing the source with HAWC at higher energies was a particularly strong motivation for HAWC's construction because of Milagro's 3σ detection of IC443 at 35 TeV [37], suggesting the spectrum was harder than measured by VERITAS [33].

Another motivation for the construction of HAWC was the prospect of observing active galactic nuclei (AGNs). AGNs are prone to large flares, sometimes up to a factor of 10 in flux. Because AGNs are extra-Galactic sources, γ rays detected from them must propagate over large distances, and so they can also be utilized for the study of axions [38]. HAWC's status as an all-sky TeV detector is of particular significance because it implies sensitivity to "orphan" flares, inverse-Compton AGN flares which occur in the absence of contemporaneous synchrotron flares [39]. These suggest hadronic acceleration [40, 41]. As the only current all-sky TeV γ -ray observatory, HAWC has the unique ability to detect these flares despite the lack of lower-energy counterparts on which to trigger [33].

HAWC is also sensitive to dark-matter-induced TeV γ -ray emission. Objects with high mass-to-light ratios are the primary targets for such an analysis since relatively little γ - emission induced by CR acceleration is expected from these sources [42, 43, 44]. The peak γ production rate from dark-matter decay or annihilation typically occurs at an energy roughly an order of magnitude below the dark-matter mass, so HAWC's dark-matter sensitivity is competitive for a mass above 10 TeV [33].

Lastly, HAWC was constructed with the intent of searching for GRBs in the TeV. The Fermi Large-Area Telescope (LAT) observed a GRB emitting a photon with energy 95 GeV [45]. Milagro's predecessor Milagrito found 3σ evidence for TeV γ rays from GRB 970417A [46]. Milagro continued to search for TeV emission from GRBs but did not find any [47]. HAWC was designed to have sufficient TeV GRB sensitivity to determine whether such sources radiate in the TeV. The experiment's all-sky nature implies that triggered searches can be performed retroactively simply by looking back at existing HAWC data from the time and location of a known burst seen at lower energies. Determining whether the GRB spectrum cuts off at TeV energies has implications regarding the propagation of γ rays through the interstellar medium. A cutoff is expected based on models of the interaction of γ rays with the Extragalactic background light (EBL) [48], and so a measurement of a higher than predicted cutoff (or none) would suggest physics beyond the SM, such as axions [49].

Chapter 4

Air-shower reconstruction

HAWC uses dedicated software to perform event reconstruction, the process by which raw data are analyzed and quantities such as shower-core location, shower direction, primary-particle species, and primary energy are extracted. These shower features can then be used to do high-level data analysis of astrophysical sources. The event-reconstruction software is part of HAWC's Analysis and Event-Reconstruction Integrated Environment (AERIE) C++ framework.

4.1 Calibration

Two varieties of calibration must be performed on HAWC data in order for it to be usable for event reconstruction. These are the timing calibration, which corrects for PMT-to-PMT timing variations and for the slewing time needed for a PMT's voltage to reach the low threshold, and the charge calibration, which produces an estimate of the number of PEs produced in a PMT from the time its output voltage spends over each of the two thresholds.

The timing calibration begins with the taking of laser calibration data. A laser is fired into each HAWC tank through each of 68 neutral density filters with transmittances ranging from 22% to 87%. The resulting data are stored in a ROOT [50] file. A function of the form

$$S(T) = \exp \frac{-T - p_0}{p_1} - \exp \frac{T - p_2}{p_3} + p_4 - p_5 T$$
(4.1)

is fit to the data, where T is the time over threshold (ToT), S is the average slewing time, and the quantities p_i are real numbers that float in the fit. Figure 4.1 shows the slewing time as a function of ToT in the laser calibration data. The relationship is approximately linear. The upturn at small ToT, modeled by the decaying exponential in Equation (4.1), occurs for pulses that barely exceed the threshold because of the curvature near the top of these pulses. The growing exponential accomodates the very small downward curvature at large ToT.

Several PMT-specific timing corrections are applied. Since the diffuser that emits laser light into the tank is located at the center of the tank's roof over the C PMT, the laser light reaches the C PMT earlier than the A, B, and D PMTs by on average 2.2 ns, so the slewing times for C PMTs are corrected by this amount. A per tank correction must also be made for the lengths of the optical cables that lead from the laser in the counting house to the tanks. These corrections can be as large as 10 ns. Finally a per PMT time-residual correction is applied. This is determined by iteratively reconstructing batches of 2×10^5 events and computing a correction for each PMT equal to its time residual averaged over the sample of events. Three iterations of this procedure are performed. The result is an additive correction to p_4 for each PMT.

Charge calibration is also carried out using the laser data. The laser is fired several times on each filter-wheel setting, and the fraction of laser pulses resulting in at least one hit in the PMT is computed. A PMT is considered to have triggered on a laser pulse if it registers a hit within 2μ s of the pulse. The fraction of pulses causing a PMT to trigger is an estimate



Figure 4.1: Slewing times shown as functions of low and high ToTs. The curves show the fit function from Equation (4.1).

of the probability that laser light and/or a CR causes a hit in that PMT during the 2- μ s time window. This can then be related to the probability of a laser trigger:

$$P_{\rm trig} = P_{\rm laser} + P_{\rm CR} - P_{\rm laser} P_{\rm CR} \tag{4.2}$$

where P_{laser} is the occupancy (i.e. the probability of triggering on the laser by production of a PE passing the low threshold), P_{CR} is the probability of triggering on a CR, and P_{trig} is the probability of triggering on either. P_{CR} can be measured from data taken while the laser is not firing. Assuming that the number of PEs produced in the PMT is a Poisson variate, the Poisson mean for a given laser intensity can be computed using

$$P_{\text{laser}} = P(n > 0) = 1 - P(n = 0) = 1 - \frac{e^{-\mu}\mu^0}{0!} = 1 - e^{-\mu}$$
(4.3)

where *n* is the number of PEs produced and μ is the mean. So, combining Equations (4.2) and (4.3), the measured mean number of PEs from a laser pulse of a given intensity is computed as

$$\mu = \ln \frac{1 - P_{\text{CR}}}{1 - P_{\text{trig}}} \tag{4.4}$$

[51].

This technique is only useful when the occupancy P_{laser} is signifantly less than 1; for an arbitrarily high laser intensity, the occupancy saturates at 1, meaning that the occupancy no longer contains useful information about the laser intensity. However, since μ should be directly proportional to the laser intensity, this relationship is fit for occupancies between 0.05 and 0.90 and then extrapolated [51].

For each filter setting applied to the laser, a simple Monte Carlo (MC) simulation is

performed in which a Poisson distribution with the appropriate mean μ for that laser configuration is sampled and Gaussian noise is added. This noise represents the PMTs' charge resolution. The resulting PE-count distribution is compared with the ToT distribution from the laser data, and nine ordered pairs of PE count and ToT are generated by matching the tenth through nintieth (in increments of 10%) percentiles of the two distributions. These pairs are aggregated across different laser intensities to produce a curve relating ToT with PE count [51].

4.2 Edge finding

As discussed in Section 3.3, larger ("four-edge") hits will produce four TDC edges as the voltage exceeds first the low threshold and then the high threshold and then falls below the high threshold and then the low threshold. Smaller ("two-edge") hits never exceed the high threshold. There is the potential for ambiguity between a single four-edge hit and a pair of two-edge hits because in each case four edges are present. These cases can be distinguished using the time between the first two edges. The FEB forces the low ToT for any hit to be at least 53 ns by delaying the trailing low-threshold edge if necessary. But the time between the rising low- and high-threshold edges is normally much smaller than this. So four-edge hits can be distinguished from two-edge hits based on whether the time between the first two edges is greater than 53 ns [51].

The edge finder also identifies and flags hits that need to be removed prior to reconstruction. Two very small pulses can create a spurious two-edge hit, which the edge finder marks with a "bad" flag. PMT afterpulsing can also produce unusable data. An afterpulse that occurs during a four-edge hit and is strong enough to cause the voltage to rise back above the high threshold after having already fallen below it results in a "six-edge" hit, which is marked using an "ambiguous" flag. Bad and ambiguous hits are not used by the reconstruction algorithms described below [51].

4.3 Core reconstruction

The location of the shower core is used by several other reconstruction algorithms, including the angle fitter, background-rejection variables, and energy estimators. In order to fit the location of the shower core, a model of the lateral distribution of particles within the shower must be assumed. In principle the correct model is the NKG function (Equation (3.4)). However, properly normalizing the NKG distribution requires computing Γ functions, which are relatively expensive to evaluate and also require numerical differentiation [2].

Instead a more practical functional form has been developed:

$$\rho(r) \propto \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) + \frac{N}{(0.5 + r/r_1)^3}$$
(4.5)

where $\sigma \equiv 10 \text{ m}$ and $N \equiv 5 \times 10^{-5}$. This function is a linear combination of a twodimensional Gaussian and a power-law-like function meant to approximate the behavior of the NKG function. However, there is no free shower-age parameter; only the normalization and the x- and y-coordinates of the core float in the fit. Equation (4.5) is also analytically differentiable and does not diverge at the core like the NKG function [2].

This function is fit to the charge observed in each PMT in order to estimate the location of the shower core. Figure 4.2 shows the effective charge in each PMT for an example event. The core fitting algorithm is known as the Super Fast Core Fit (SFCF). The median error of



Figure 4.2: The effective charge in each PMT in a γ -ray event. Image taken from [2].

the SFCF core estimate is approximately 4 m for smaller air showers and 2 m for larger ones, among showers whose cores land on the array. Core reconstruction is less precise for off-array showers; a shower with a core 50 m from the edge of the detector has a core resolution of approximately 35 m [2].

Prior to the execution of the SFCF, a simple estimate of the core location is performed. This estimate is the average of the PMT locations weighted with their estimated charges (the "center of charge") and is used as the initial guess for the core location in the SFCF minimization [2].

4.4 Angle reconstruction

As shown in Figure 3.1, shower particles in a shower originating not directly from zenith arrive at the detector at different times because some must propagate a greater distance than others. The arrival times of the hits can therefore be used to estimate the direction from which the primary particle came. These times are shown for an example event in Figure 4.3. The simplest model of the propagating shower consists of a plane of particles. Two considerations complicate this model: the differences in distances travelled by particles at different distances from the shower axis and the fact that each PMT measures the time of the first Cherenkov photon to strike it [2].

Because the shower begins with the interaction of the primary γ ray at one point in the atmosphere, it propagates as a cone, with particles far from the shower axis therefore falling behind those closer to it. This curvature effect results in the nonplanar surface shown in Figure 3.1 [2].

Furthermore, the stochastic nature of the particle interactions between shower particles and the atmosphere means that even particles at the same distance from the axis travel different distances since they scatter multiple times and take nonlinear paths to the ground. The resulting time delay between the earliest and latest particles in the shower is of order several ns near the axis and greater far from the core. When several shower particles reach the water of a tank at different times, the resulting Cherenkov light can produce overlapping hits in the PMTs. The recorded time is that of the first light. The difference in shower thickness between particles near the axis and those far from it necessitates a position-dependent sampling correction since the problem is more severe far from the axis [2].

These two effects are corrected for together. MC simulations of showers and their in-



Figure 4.3: The arrival time of each hit used in reconstructing the direction from which the example event in Figure 4.2 originated. Image taken from [2].

teractions with HAWC were used to deduce a suitable functional form for the correction. This function was then fit to a sample of actual HAWC data events and verified on another disjoint population. Fitting the function to data events is necessary because the MC simulation is not sophisticated enough to adequately capture the subtleties of the effects discussed above. A χ^2 fit is then used to fit a plane to the corrected times of the hits [2].

4.5 Background rejection

The primary background to the γ rays detected by HAWC consists of various hadronic CR species, mostly protons. In order to distinguish between CR events and γ events, two variables have been developed that exploit the differences in the signatures of these classes of events in HAWC data. As discussed in Section 3.1, γ -induced air showers consist mostly of electrons, positrons, and photons. Muons occur uncommonly in these showers. By contrast, hadronic showers contain muons and mesons such as pions, and the subsequent electromagnetic sub-showers produced by hadronic secondaries appear as inhomogeneities in the shower's lateral distribution [2].

The background-rejection variable known as compactness is defined by

$$\mathcal{C} \equiv \frac{N_{\text{hit}}}{\max_{i \in F} q_i} \tag{4.6}$$

where N_{hit} is the number of hits within 20 ns of the reconstructed shower plane, q_i is the effective charge in the *i*th PMT, and F is the set of indices of PMTs greater than 40 m from the reconstructed core of the shower. Because hadronic showers contain muons and subshowers, they are likely to have large hits beyond the 40-m radius. This is not the case for



Figure 4.4: The lateral distribution of charge within a background event and a signal event. Image taken from [2].

 γ events, wherein the lateral distribution is fairly smooth. Compactness is therefore larger on average for γ events than for hadronic ones [2].

The second variable used to discriminate between hadronic and photonic showers, known as the Parameter for Identifying Nuclear Cosmic rays (PINCness), is defined as

$$\mathcal{P} \equiv \frac{1}{N_{\text{hit}}} \sum_{i=1}^{N_{\text{hit}}} \left(\frac{\zeta_i - \langle \zeta_i \rangle}{\sigma_{\zeta_i}} \right)^2 \tag{4.7}$$

where $\zeta_i \equiv \log_{10} q_i$, $\langle \zeta_i \rangle$ is the average log charge among PMTs with radial distance within 5 m of the *i*th PMT's radial distance, and σ_{ζ_i} is the expected fluctuation of the log charge in the *i*th PMT as measured on a sample of events near the Crab Nebula. Figure 4.4 shows the log charges and the local average $\langle \zeta_i \rangle$ used in the PINCness computation. In hadronic showers, such as Figure 4.4a, the charges tend to differ more from their local averages than in γ showers, such as Figure 4.4b. This is again due to a combination of muons and secondary sub-showers and results in a higher PINCness value for hadrons than for photons.

4.6 MC simulations of air showers

A detailed MC simulation of the HAWC experiment is used to optimize and evaluate the reconstruction algorithms described above and in Chapter 5. This same simulation is also used in spectral likelihood analyses such as the ones described in Chapters 6 and 7. The simulation of a single event consists of propagating the air shower through the atmosphere, simulating the interactions of the shower particles with the detector on the ground, and running the reconstruction algorithms on the resulting pseudodata.

The shower propagation is performed using version 7.4000 of the CORSIKA package [52]. GEANT version 4.10.00 [53] is used to simulate the propagation of the shower particles and Cherenkov photons through the water in the tanks. The simulation of the PMTs is performed using custom software. This software models each PMT as having an efficiency chosen to match vertical-muon rates in data. The PMT signal is then fluctuated according to a chargesmearing parameter, and single-PE noise is added. The width of the charge smearing and the distribution of single-PE noise are chosen such that distributions of reconstructed quantities in the simulation match the distributions of the same quantities in data. The simulated PMT signals are then reconstructed using the same reconstruction software used with data [2].

Any inaccuracies in the assumptions made in modeling the detector in these simulations will produce systematic errors in analysis results that depend on them. It is therefore necessary to estimate uncertainties on several quantities in the simulation, perform analyses using several variants of the simulation, and cite the spread in the estimates of the relevant physics parameters as a systematic error. The quantities in the simulation for which this procedure is performed are the charge-smearing parameter described above, the QE of the PMTs, the PE threshold in the PMT, and the presence or absence of an effect called "broad pulse" in the simulation, described below.

The width of the charge smearing is expressed as a fraction of the recorded charge. The nominal value of this quantity in HAWC's simulations is 0.1100, with 0.0935 - 0.1265 being the range used for evaluating systematic errors.

A QE of 55% has been chosen as the best value for the A, B, and D PMTs; 80% is used as the nominal QE for the high-QE C PMTs. The A/B/D QE is varied between 44% and 66% when studying systematics, and the C QE is varied between 64% and 96%.

The PMT threshold is assigned a nominal value of 0.55 PE. A lower PMT threshold of 0.30 PE has been shown to match data event rates better than the 0.55 PE value but shows poorer agreement with respect to the distributions of hadron-rejection variables, and so the threshold is varied to 0.30 PE to study systematics.

Because calibration data are produced using laser pulses, HAWC's charge calibration assumes photons arrive at PMTs promptly, with arrival times differing by less than 10 ns. However, in actual air showers photons do not arrive simultaneously at the PMTs and as a result produce larger than expected ToTs and reconstructed charges. To compensate for this, a broad-pulse effect is introduced into the simulation, in which PE estimates in PMTs are increased in order to resemble the values observed in data. This effect is part of the nominal simulation; to study systematics, it is removed.

The uncertainty ranges for these MC parameters are felt to be somewhat conservative. Studies of the HAWC detector and simulation are ongoing and are expected to result in more precise measurements of these quantities in the near future.

Chapter 5

Energy estimation

The Neural-Net (NN) Energy Estimator is a γ -ray energy-estimation algorithm developed for HAWC with the dual goals of calculating an accurate and precise energy estimate while still being efficient enough to be run on all HAWC data as part of event reconstruction. It uses the Toolkit for Multivariate Analysis (TMVA) [54] NN implementation.

The MC-based plots presented throughout this thesis (with the exception of the Appendix) were made using the HAWC MC simulation internally referred to as DAQSim 4. HAWC collaborators can find more details about this simulation in HAWC internal memo 2397.

5.1 Method

An NN is a function mapping several quantities associated with an event (input variables) to some regression target or output variable, in this case $\log_{10} E$ where E is the primary energy of the shower. A detailed description of the functional form of the NN can be found in [54]. This function is characterized by many (479 in this implementation) free parameters called weights. The optimal values of the weights are determined in a process called training.

Name	Shower Characteristic Described
Fraction of channels hit	
Fraction of tanks hit	Multiplicity
Log core amplitude	
Core distance	Fraction of shower contained in the detector
Cosine zenith angle	
Fraction of charge in annulus 0	
Fraction of charge in annulus 1	
Fraction of charge in annulus 2	
Fraction of charge in annulus 3	
Fraction of charge in annulus 4	Atmospheric attenuation of shower
Fraction of charge in annulus 5	
Fraction of charge in annulus 6	
Fraction of charge in annulus 7	
Fraction of charge in annulus 8	
Fraction of charge in annulus 9	

Table 5.1: NN input variables that characterize energy. The annuli are about the shower axis. Annuli 0 - 8 are each 10 m thick. "Annulus" 9 extends from 90 m to infinity.

5.1.1 Input Variables

Fifteen input variables are used to characterize the shower energy. These variables have been chosen in order to capture three qualities of the shower: the multiplicity in the detector, which is a crude measure of how much energy reaches the ground, the containment of the shower within the detector, which indicates by how much the multiplicity in the detector needs to be scaled up, and the atmospheric attenuation of the shower, which indicates how much energy was lost on the way to the ground. The input variables are summarized in Table 5.1.

5.1.1.1 Multiplicity

Three variables are used to quantify the overall multiplicity of the shower. The first, the fraction of channels hit, is defined here as the number of channels hit in the event divided

by the total number of functioning channels in the detector. Channels with bad hits are conventionally excluded from the denominator in this quantity in other contexts in HAWC analysis. However, in practice both choices have been found to yield comparable estimator performances, and the calculation is faster when bad channels are not excluded.

The second variable relating to the shower multiplicity is the fraction of tanks hit. A tank is considered hit if at least one of its PMTs receives a hit.

The third multiplicity variable is the logarithm of the overall normalization parameter from the SFCF core fit. This quantity should in principle be more robust against offarray showers than the fraction of channels or tanks hit since, with an ideal fitter, the fit normalization would be independent of the containment of the shower. In practice, this parameter is one of the least important NN inputs, perhaps indicating that the fit normalization parameter is a poor estimate of the integral of the shower's lateral distribution.

5.1.1.2 Containment

One input variable is used to infer the fraction of the shower contained within the array: the distance of the reconstructed core from the array center. This is somewhat crude because the detector is not rotationally symmetric. The "core fiducial scale" C, defined as 100 times the factor by which the detector area would have to be scaled in order to barely contain the reconstructed core, has been evaluated as an alternative measure of the containment of the shower but does not yield a better energy estimate.

5.1.1.3 Atmospheric Attenuation

Eleven variables are used to account for the attenuation of the shower by the atmosphere on its way to the ground: the zenith angle (in cosine) and ten annular variables representing the lateral distribution of charge in the shower front. The zenith angle θ is included because showers at higher zenith must pass through more atmosphere. Training was attempted using θ , $\cos \theta$, or $\sec \theta$ as an input variable; $\cos \theta$ proved to yield the best performance.

The annular variables are defined as follows. For i between 0 and 8 inclusively, the ith annular variable is the summed charge of all hits, excluding those in bad PMTs, landing a distance between (10 m)i and (10 m)(i+1) from the shower axis, divided by the total charge in the event. The distance is measured in the reconstructed shower plane, not the plane of the detector. The ninth is defined as the fraction of the charge landing more than 90 m from the shower axis. These ten variables capture the lateral energy distribution within the shower.

The purpose of the annular variables is to estimate the first-interaction height of the shower, which is also relevant in determining how much atmosphere the shower has passed through. One way of estimating the first-interaction height is via the shower-age parameter of the NKG function, which is higher for "older" showers. However, introducing the age parameter from an NKG fit as an NN input does not improve the energy reconstruction. It may be a poor estimate of the age. Since higher-age showers are wider, there should be some information about the first-interaction height in the lateral distribution of the shower, and this is why the annular variables are included: as a more empirical way of mapping the lateral distribution to the first-interaction height.

Using the fraction of charge deposited in an annulus instead of the total charge desensitizes the NN algorithm to systematic errors involving the charge scale as modeled by the MC. Three of the known systematic uncertainties affecting HAWC's MC are uncertainties in the PMT threshold, charge fluctuations, and QE, all of which impact estimates of PE counts in the MC. The NN is made more robust against these sources of uncertainty by eliminating the dependency on the total PE count in an event in favor of utilizing only the shape of the lateral distribution.

5.1.2 Training

The goal of the training process is to choose the vector of NN weights \mathbf{w} that minimizes the error function, defined as

$$D(\mathbf{w}) \equiv \frac{1}{2} \sum_{i=1}^{N_{\text{events}}} u_i \left[\log_{10} \hat{E}(\mathbf{x}_i; \mathbf{w}) - \log_{10} E_i \right]^2$$
(5.1)

where u_i is the weight of the *i*th event (the event's relative importance in the training, unrelated to \mathbf{w} , which are also called weights), E_i is the MC-truth energy of the *i*th event, \mathbf{x}_i is the *i*th event's vector of input variables, and \hat{E} is the energy estimate for the event. This minimization is performed by repeatedly iterating over the sample of training events, with a small update to \mathbf{w} performed at each event. The details of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) algorithm for updating the weights can be found in [54]. The MC events are weighted (the u_i are set) to resemble an $E^{-2.63}$ (Crab Nebula-like) power-law spectrum during the training. This weighting has been observed to produce a relatively flat RMS error between 1 and 100 TeV (see Section 5.2.6).

The event weighting manipulates the thrown E^{-2} spectrum into the desired $E^{-2.63}$ spectrum but also corrects the distributions of two other quantities that are thrown with unphysical distributions in the MC: the zenith angle and distance from array center. The zenith distribution, like the energy distribution, will be different for different varieties of sources (e.g., an isotropic angular distribution for cosmic rays or a delta function for a point-like γ source), and so its effective distribution in modeling a specific source must be determined using weights. The core distance from the array center is importance-sampled, with events near the array center being oversampled because they more often pass eventselection cuts. The weighting removes this bias in the distribution.

5.1.3 Resulting Functional Form

The function mapping inputs to outputs that is produced by the training process is difficult to visualize because its domain is 15-dimensional. One way of doing this is to plot the output as a function of each individual input while freezing the others at their respective median values. This is done in Figure 5.1. The medians are chosen using MC, and the input variables are plotted over the ranges they span in this sample.

The dependence of the output on fraction of channels hit is surprising: the slope is mostly negative. This is because the other inputs, including the fraction of tanks hit, have been frozen: among events with a given fraction of tanks hit, those with a higher number of channels hit have a higher density (number of hits per unit area) in the detector. These events probably have their cores on the array, and such events have a lower energy on average.

With other inputs fixed, the energy estimate is an increasing function of core distance. This is because an off-array event must have a greater primary energy in order to produce the same number of hits in the detector.

The inner annular variables produce low energy estimates when their values are very high. This occurs because only a small, low-energy event can fit almost entirely within the central region of the detector.



Figure 5.1: Dependence of NN energy estimate on each input variable, with the others frozen at their median values.

Figure 5.1 (cont'd)



5.2 Performance

In this section the performance of several energy estimators is evaluated using photon MC. Events are weighted to resemble an $E^{-2.63}$ spectrum as in training, and the cuts applied are the same as those used in training as well. If the NN becomes "overtrained" on the training sample, it will perform anomalously well on that sample, better than on independent MC samples or on data, and so the evaluation MC sample is chosen to be a disjoint set of events from the training sample in order to avoid overestimating the performance of the estimator.

This analysis is performed only using events that pass background-rejection cuts. Although the MC sample consists only of γ -ray events, standard hadron-rejection cuts are still imposed to ensure that only events that are liable to be used in an actual analysis are considered. These cuts (on compactness and PINCness) were chosen to optimize the expected statistical significance of the Crab Nebula using γ MC and data background events. Each energy estimator is evaluated without and with a fiducial cut designed to select events well contained within the HAWC array:

$$C < 100.$$
 (5.2)

5.2.1 Other Energy Variables

Several energy or energy-proxy variables are evaluated in this section. In addition to the NN energy, the following will be considered.

5.2.1.1 Fraction Hit/Fraction-Hit NN

Here fraction hit is defined as the fraction of PMTs that are hit within an event, excluding bad hits. This variable represents the shower multiplicity and is therefore positively correlated with the event's primary energy. In order to determine what an explicit energy estimate based on fraction hit would look like, an NN was trained using only fraction hit as an input variable. This is the fraction-hit NN.

5.2.1.2 Likelihood Energy Estimator

The Likelihood (LH) Energy Estimator uses the maximum-likelihood (ML) formalism to estimate the shower energy. The energy estimate is chosen to maximize the likelihood function associated with the observed event, which is the product over all hits of single-hit likelihood functions. More detail about this reconstruction algorithm can be found at [1].

5.2.1.3 Ground Parameter Energy Estimator

The Ground Parameter (GP) Energy Estimator uses the value of the shower energy density estimated at some fraction-hit-dependent distance from the shower core to derive an energy estimate. This is corrected for zenith as well.

5.2.1.4 Ground Energy Estimator

It is worth considering whether any energy-estimation algorithm could perform better than a mapping from energy deposited on the ground to primary energy, corrected for zenith angle. To address this question, a neural net has been trained using two input variables: MC-truth ground energy and cosine of reconstructed zenith angle. This will be referred to as the Ground Energy Estimator. This estimator cannot actually be implemented since it uses MC-truth information. It simply represents the best achievable performance by an estimator seeking to use only the zenith angle and some measure of the energy deposited on the ground by a shower.

5.2.2 Linearity

Figure 5.2 contains the joint distributions of the energy variables with MC-truth energy. The identity line is drawn in black for variables with a dimension of energy. The NN Estimator (Figure 5.2a) displays good linearity above 1 TeV with a bias at the lowest energies due to the threshold set by the fraction-hit cut applied in the MC sample. Applying the fiducial cut (Equation (5.2)) removes some poorly reconstructed events at high energies, but otherwise the NN performs quite well even on off-array events. This is because the NN input variables already model the containment of the shower within the array (see Table 5.1), allowing the NN to handle off-array events reasonably well. The most noticeable change is an improvement in resolution at high energies.

Fraction hit is much less linear than the NN for the following reason. All low-energy events are inherently low-multiplicity events and therefore have low fraction hit. However, the converse is not true: a high-energy event, although its shower contains a large number of particles, may have an arbitrarily low multiplicity in the detector depending on where it lands. As a result, Figures 5.2c and 5.2e show that at all values of fraction hit, high-energy events are present. This effect is only somewhat lessened when the fiducial cut is applied (Figures 5.2d and 5.2f). Because the fraction of PMTs hit cannot exceed one, this variable saturates at an energy around 30 TeV and cannot distinguish among higher energies.

The LH Estimator (Figure 5.2g) is generally linear above 10 TeV but with a greater thickness (resolution) to the distribution than the NN displays. This improves somewhat
with the addition of the fiducial cut (Figure 5.2h).

Figure 5.2i shows that without the fiducial cut, the GP Estimator suffers from highenergy contamination at low values of the energy variable due to events reconstructed off the array. Adding the fiducial cut removes most of these events (5.2j).

The Ground Estimator displays a linearity comparable to that of the NN (Figure 5.2k). Its performance is not changed much by imposing the fiducial cut (Figure 5.2l) because the MC-truth ground energy is not sensitive to the fraction of the event contained within the array. The only change is due to the better zenith-angle estimate for well contained events.

5.2.3 Bin Overlap

Figure 5.3 shows the extent of the overlap in the MC-truth energy content of bins in the various energy variables. Each bin is represented by a Gaussian with a mean and standard deviation equal to the mean and standard deviation respectively (in log space) of the MC-truth energies of the events reconstructed in that bin.

The NN energy bins (Figure 5.3a) show some overlap. The extent of the overlap is relatively unchanged by the fiducial cut (Figure 5.3b) for the reason described in Section 5.2.2, namely that the NN explicitly handles off array events using the core distance from the array center. The fraction-hit bins (Figure 5.3c) show a much greater overlap especially at low energies due to the high-energy contamination of all bins. This also decreases the dynamic range of the variable. The LH variable (Figure 5.3g) possesses a greater dynamic range than fraction hit, but there is still significant bin overlap at low energies. Its dynamic range is also worsened slightly by the fiducial cut (Figure 5.3h). There is substantial overlap in the GP bin contents without the fiducial cut due to the presence of high-energy events whose energies are substantially underestimated at high energy (Figure 5.3i). This is improved significantly



(a) NN Estimator correlation without fiducial cut. Correlation coefficient = 83.6%.



(b) NN Estimator correlation with fiducial cut. Correlation coefficient = 79.9%.



(c) Fraction hit correlation without fiducial cut. Correlation coefficient = 59.3%.



(d) Fraction hit correlation with fiducial cut. Correlation coefficient = 66.6%.

Figure 5.2: Correlation between energy variables and MC-truth energy.

Figure 5.2 (cont'd)



(e) Fraction-hit NN correlation without fiducial cut. Correlation coefficient = 58.9%.



(g) LH Estimator correlation without fiducial cut. Correlation coefficient = 69.1%.



(f) Fraction-hit NN correlation with fiducial cut. Correlation coefficient = 66.7%.



(h) LH Estimator correlation with fiducial cut. Correlation coefficient = 64.7%.

Figure 5.2 (cont'd)



(i) GP Estimator correlation without fiducial cut. Correlation coefficient = 54.7%.



(k) Ground Estimator correlation without fiducial cut. Correlation coefficient = 76.0%.



(j) GP Estimator correlation with fiducial cut. Correlation coefficient = 63.0%.



(l) Ground Estimator correlation with fiducial cut. Correlation coefficient = 70.2%.

by the fiducial cut although the problem persists (Figure 5.3j). Even the Ground Estimator (Figures 5.3k and 5.3l) has a smaller dynamic range than the NN, especially at low energies, because it tends to overestimate the energies of low-energy events more than the NN does. The raw histograms used to create these distributions can be seen in Figure 5.4.

5.2.4 Bias

Figure 5.5 shows the log-energy bias of the energy variables (except for fraction hit, which does not have a dimension of energy). Here bias is defined as the mean deviation of the estimated log energy from its MC-truth value:

$$b \equiv \left\langle \log_{10} \hat{E} - \log_{10} E \right\rangle$$

= $\left\langle \log_{10} \frac{\hat{E}}{E} \right\rangle$ (5.3)

where E is the MC-truth energy and \hat{E} is the estimate thereof. The estimator bias is a way of quantifying the average offset of the estimator as a function of MC-truth energy. The ideal bias of 0 is shown as a black line on the plot.

All of the estimators examined here suffer from a positive bias at low energies and a negative bias at high energies (Figure 5.5a). The former is due to the threshold cut used in selecting the training and evaluation MC samples. The latter is caused by saturation of the detector at high energies. The LH estimator displays the least bias of all estimators above 1.5 TeV.

Imposing the fiducial cut has a small effect on the NN performance, worsening the bias between 1.5 and 10 TeV and improving it elsewhere (Figure 5.5b).



(a) NN bin overlap without fiducial cut.



(c) Fraction-hit bin overlap without fiducial cut.



(e) Fraction-hit-NN bin overlap without fiducial cut.



(b) NN bin overlap with fiducial cut.



(d) Fraction-hit bin overlap with fiducial cut.



(f) Fraction-hit-NN bin overlap with fiducial cut.

Figure 5.3: Gaussians representing distributions of MC-truth energies in reconstructedenergy or energy-proxy bins. The resulting bin purity (fraction of events not migrating into adjacent bins) is shown in Figure 5.8.

Figure 5.3 (cont'd)



(k) Ground-energy bin overlap without fiducial cut.



(h) LH bin overlap with fiducial cut.



 (\mathbf{j}) GP bin overlap with fiducial cut.



(l) Ground-energy bin overlap with fiducial cut.



(a) NN bin content without fiducial cut.



(c) Fraction-hit bin content without fiducial cut.



(e) Fraction-hit-NN bin content without fiducial cut.



(b) NN bin content with fiducial cut.



(d) Fraction-hit bin content with fiducial cut.



(f) Fraction-hit-NN bin content with fiducial cut.

Figure 5.4: Distribution of MC-truth energies in energy bins.

Figure 5.4 (cont'd)





(k) Ground-energy bin content without fiducial cut.



(h) LH bin content with fiducial cut.



(j) GP bin content with fiducial cut.



(l) Ground-energy bin content with fiducial cut.



(b) NN estimator with and without fiducial cut.

Figure 5.5: Bias of energy variables.

5.2.5 Resolution

Figure 5.6 shows the energy variables' log-energy resolution, defined as the standard deviation of the log energy estimate:

$$r \equiv \sqrt{\left\langle \left(\log_{10} \hat{E} - \left\langle \log_{10} \hat{E} \right\rangle \right)^2 \right\rangle}.$$
(5.4)

This quantity is a measure of the width of the estimator's distribution. Figure 5.6b shows that the NN energy resolution is improved by the addition of the fiducial cut, particularly at high energies. With the cut, the NN displays a significantly smaller resolution than the LH Estimator, GP Estimator, or Ground Estimator between 100 GeV and 68 TeV (Figure 5.6a). All the estimators' resolutions become small at low energies. This should not be understood to mean that the energy reconstruction is best at low energies, for all estimators are also characterized by substantial biases in this regime (see Figure 5.5).

5.2.6 RMS Error

Figure 5.7 shows the log-energy RMS error of the different energy variables. This is defined as the RMS deviation of the estimated log energy from the MC-truth log energy:

$$\rho \equiv \sqrt{\left\langle \left(\log_{10} \hat{E} - \log_{10} E \right)^2 \right\rangle} \\
= \sqrt{\left\langle \log_{10}^2 \frac{\hat{E}}{E} \right\rangle}.$$
(5.5)





(b) NN estimator with and without fiducial cut. $\Sigma_{i} = \sum_{i=1}^{n} C_{i} \sum_{j=1}^{n} D_{i} = 1$

Figure 5.6: Resolution of energy variables.

This quantity is also equal to the sum in quadrature of the bias and resolution:

$$\rho = \sqrt{b^2 + r^2}.\tag{5.6}$$

Without the fiducial cut, the NN displays an RMS error that is roughly constant, with a value around 0.17, in the interval from 1 to 32 TeV (Figure 5.7b). The RMS error is smaller for the NN than for all other estimators at all energies.

5.2.7 Bin Contamination

In a spectral analysis, there is the concern that the highest-energy bins may have a low purity (i.e. they may contain mostly misreconstructed lower-energy events). This can happen if the spectrum is sharply decreasing, meaning that there are far more lower-energy events, which may fluctuate upwards, than there are higher-energy events. Figure 5.8 shows, for the NN, LH, and GP Estimators, the fraction of events reconstructed in each half-decade energy bin whose MC-truth energies are also within the bin, assuming an $E^{-2.63}$ spectrum. With the fiducial cut (Figure 5.8b), the NN bin purity is highest at all energies and increases with energy. The purity is 86% in the highest-energy NN bin with the fiducial cut.

5.2.8 Agreement of Estimators

As a check on the consistency of the various energy variables, it is interesting to plot them against each other. This is done in Figure 5.9. A fiducial cut is applied. The estimators are all fairly correlated with each other. There are classes of events that lie away from the identity line, and these can be understood in terms of the similar classes of events in Figures 5.2h and 5.2j.



(b) NN estimator with and without fiducial cut.

Figure 5.7: RMS error of energy variables.



Figure 5.8: Fraction of MC events reconstructed in each NN bin whose MC-truth energies actually lie in that bin.



(c) Correlation coefficient = 93.3%.

Figure 5.9: Correlations of three energy variables.

Chapter 6

Crab Nebula spectral reconstruction

The Crab Nebula is a PWN located at 5 hours, 34 minutes, and 32 seconds right ascension (R.A.) and 22 degrees, 0 arcminutes, and 52 arcseconds declination. It is the brightest γ -ray source seen by HAWC, and a spectral analysis of this source using the NN energy variable is therefore an excellent proof of principle for the NN technique. Specifically the application of the NN energy-estimation method to the Crab allows the Crab's energy spectrum to be resolved at unprecedentedly high energies.

6.1 Skymaps

The first step in HAWC's high-level data analysis is the production of skymaps. A skymap is a kind of two-dimensional histogram representing the distribution of the reconstructed directions of events detected by HAWC. The bins in this histogram are referred to as pixels, and the pixelization of the sky is determined using the HEALPix [55] software. Each pixel subtends a solid angle of 0.00328 square degree. Skymap files are stored using the FITS [56] data format.

The skymaps are produced using 836 sidereal days of HAWC data. These data are subjected to cuts on PINCness and compactness that are chosen to optimize the expected statistical significance of the Crab Nebula, where this expectation is computed using γ MC and background data. Events are binned using a two-dimensional binning scheme with rectangular bins in fraction hit and NN energy, and one skymap is produced for each analysis bin. The fraction-hit bin edges are 0.067, 0.105, 0.162, 0.247, 0.356, 0.485, 0.618, 0.740, 0.840, and 1.000. The NN-energy bins are each one quarter decade wide and span the range from $10^{-1/2}$ TeV to $10^{5/2}$ TeV (316 GeV to 316 TeV). These are sometimes referred to by letter names ranging from *a* to *l*. Since the analysis technique described below relies on being able to characterize the distributions of photon and hadron events using MC and data respectively in each bin, it is necessary to exclude from the analysis bins that are very poorly populated, which are generally bins with high fraction hit and low NN energy or vice versa. Fortunately these low-statistics bins contribute only negligibly to the statistical power of the analysis. Only 37 of the 108 two-dimensional bins are expected to contribute nonnegligibly to the Crab's statistical significance. The other bins are not included.

Some hadron events survive the compactness and PINCness cuts, and so it is necessary to estimate the amount of background in the maps in order to correctly model the expected event count in each pixel during the spectral analysis. For bins with reasonably good statistics, this is done using a technique called direct integration (DI) [57, 58]. The background rate is modeled as a function of hour angle (H.A.) and declination, estimated for each twohour interval by integrating the data over that period while excluding known bright γ -ray sources such as the Crab, Markarians 421 and 501, and the Galactic plane.

In bins with lower statistics, DI becomes imprecise. For bins with total counts below 5×10^5 , an approach called background randomization is used instead. For each bin, a two-dimensional histogram of H.A. and declination is populated using the entire data set. This distribution is then sampled 10000 times, and each resulting (H.A., declination) pair is combined with the time from one randomly chosen event from the data set, and the pixel

in which this pseudoevent would have fallen is incremented. The background map is then scaled to have the appropriate normalization.

Following DI or background randomization, an additional smoothing is performed on the background maps using a flat, circular kernel with radius 0.5°. This further damps out local fluctuations in the background maps.

Figures 6.1 and 6.2 show the near-Crab regions of the count and background maps respectively for the 2D analysis bin consisting of events with reconstructed energies between 3.16 and 5.62 TeV from fraction-hit bin 3. This bin's background is estimated using DI.

Figures 6.3 and 6.4 show the count and background maps for events with energies reconstructed between 56.2 and 100 TeV coming from fraction-hit bin 9. This bin's background is computed using background randomization. The background map shows that the per pixel background expectation is a fraction of an event in all pixels in this lower-statistics bin.

6.2 Binned-likelihood formalism

The Crab energy spectrum is modeled as a log parabola:

$$\frac{dN}{dE} = \Phi_0 \left(E/E_0 \right)^{-\alpha - \beta \ln(E/E_0)}$$
(6.1)

where Φ_0 , α , and β are free parameters. E_0 , the pivot energy, is fixed. Its value is chosen to decorrelate Φ_0 and α in the spectral fit. 7 TeV has been empirically determined to be a suitable value for E_0 for the Crab Nebula.

The log parabola's name refers to the fact that the logarithm of the flux depends quadrat-



Figure 6.1: The skymap for fraction-hit bin 3 with NN energies between 3.16 and 5.62 TeV.



Figure 6.2: The background skymap for fraction-hit bin 3 with NN energies between 3.16 and 5.62 TeV. This bin uses the DI background calculation.



Figure 6.3: The skymap for fraction-hit bin 9 with NN energies between 56.2 and 100.0 TeV.



Figure 6.4: The background skymap for fraction-hit bin 9 with NN energies between 56.2 and 100.0 TeV. This bin uses background randomization.

ically on the logarithm of the energy:

$$\ln\left(\frac{dN/dE}{\Phi_0}\right) = -\alpha \ln \frac{E}{E_0} - \beta \left(\ln \frac{E}{E_0}\right)^2 \tag{6.2}$$

The three free parameters in Equation (6.1) are determined using a maximum-likelihood analysis of the skymaps. The number of events falling into a given pixel is modeled as a Poisson variate with a mean equal to the background expectation in that pixel plus the signal expectation under the log-parabola model of the Crab spectrum:

$$L(\Phi_0, \alpha, \beta) = \prod_i \frac{e^{-(b_i + s_i)} (b_i + s_i)^{c_i}}{c_i!}$$
(6.3)

where L is the likelihood and b_i , s_i , and c_i are the background mean, signal mean, and observed count in the *i*th combination of skymap and pixel.

Calculating the expected signal s_i in a given pixel requires knowledge of the detector response. This consists of the distribution of reconstructed energies produced by a given true energy and the distribution of reconstructed shower directions produced by a given true direction. The latter is known as the point-spread function (PSF). These distributions are built using MC and are stored in a ROOT file called a detector-response file. For each analysis bin, the file contains the distribution of reconstructed energies produced by a nominal trueenergy distribution of $E^{-2.63}$, which can then be reweighted for an arbitrary spectrum in order to evaluate its likelihood. The file also contains for each bin a histogram of the angular distances between MC events' true and reconstructed directions and a double-Gaussian fit to this distribution:

$$\rho(r) = \frac{A}{2\pi} \left\{ \frac{f}{\sigma_1^2} \exp\left[-\frac{1}{2} \left(\frac{r}{\sigma_1} \right)^2 \right] + \frac{1-f}{\sigma_2^2} \exp\left[-\frac{1}{2} \left(\frac{r}{\sigma_2} \right)^2 \right] \right\}$$
(6.4)

where ρ is the number of events per unit reconstructed solid angle at a distance r from the true direction and the normalization A, the radii σ_1 and σ_2 , and the coefficient f float in the fit. The double-Gaussian function is chosen to accomodate two classes of events: well reconstructed and poorly reconstructed. Figure 6.5 shows the 68% quantiles of the double-Gaussian PSFs fit to data and MC. In most bins, the MC underestimates the PSF width. This is treated as a systematic error, described in Section 6.3.

It is useful to define a log-likelihood-ratio (LLR) test statistic (TS) comparing the signalplus-background hypothesis described above to a background-only hypothesis with no signal ($\Phi_0 = 0$). The likelihood of the latter model is

$$L_{\rm bkg} = \prod_{i} \frac{e^{-b_{i}} b_{i}^{c_{i}}}{c_{i}!}.$$
(6.5)

The TS is then defined as

$$D \equiv 2 \ln \frac{\max_{\Phi_0,\alpha,\beta} L(\Phi_0,\alpha,\beta)}{L_{\text{bkg}}}$$
(6.6)

$$= 2\sum_{i} \left[c_i \ln\left(1 + \frac{s_i}{b_i}\right) - s_i \right].$$
(6.7)

This TS indicates the extent to which the signal-plus-background hypothesis better models the data than the background-only hypothesis. According to Wilks' theorem [59], the TS is distributed under the background-only hypothesis according to a χ^2 distribution with



Figure 6.5: The 68% quantile of the distribution of the angular distance between the true and reconstructed directions of events in data and MC.

three degrees of freedom in the high-statistics limit. The TS can therefore be employed to compute the statistical significance with which HAWC detects an excess of events from the source above background.

Statistical uncertainties of the spectral parameters can also be computed by means of Wilks' theorem. Considering for instance Φ_0 , the LLR

$$2\ln \frac{\max_{\Phi'_0,\alpha',\beta'} L(\Phi'_0,\alpha',\beta')}{\max_{\alpha',\beta'} L(\Phi_0,\alpha',\beta')}$$
(6.8)

should be asymptotically distributed according to a χ^2 distribution with one degree of freedom if Φ_0 assumes its true value. This LLR can therefore be evaluated for a given value of Φ_0 to test whether that value is compatible with the observed data, and the set of values of Φ_0 at which the *p*-value derived from this χ^2 is not less than 0.32 represents the 68%containment (1 σ) error band for Φ_0 . The same procedure can be carried out for α and β as well.

6.3 Flux-points calculation

It is also desirable to be able to compute a flux estimate in a way that is less reliant on the assumption that the true Crab emission spectrum is exactly a log parabola. A second style of analysis is carried out in addition to the one described in Section 6.2. In this analysis, an additional fit is done for each NN-energy bin using only the two-dimensional bins falling into that energy bin and allowing only Φ_0 to float in the fit. The result is an estimate of the flux in each energy bin that is obtained assuming that the shape of the spectrum in that bin is still approximately log-parabolic but allowing the event rate in that bin to deviate

from that predicted by the log-parabola fit from Section 6.2. The computed flux for each reconstructed-energy bin is plotted at the mean true log energy contributing to that bin assuming the best-fit log-parabola shape.

This choice places the point at an energy that is representative of the true energy at which the flux is being measured only if the functional form of the source's true energy spectrum is approximately log-parabolic in shape. It is not sufficient for detecting substantial, qualitative deviations from the log-parabola functional form, such as an LIV-induced hard cutoff. Chapter 7 will discuss a technique for doing this.

The calculation of expected counts in skymap pixels is reliant upon the MC simulation of HAWC described in Section 4.6. The computed spectrum is therefore subject to systematic uncertainties deriving from the uncertainties in the MC simulation described in Section 4.6. To compute these uncertainties, the spectral analysis is performed using dedicated systematics MC simulations in which the charge uncertainty, quantum efficiency, PMT threshold, and broad-pulse effects have been assigned values at the extremes of the ranges described in Section 4.6. In order to account for the data/MC discrepancy in the angular resolution, an additional iteration of the spectral analysis is performed in which the PSF is modeled using the double-Gaussian parameters measured using data. Each of these effects is varied one at a time, and the resulting uncertainties on the flux points are added in quadrature with each other and with the statistical errors derived from the likelihood fitting procedure.

6.4 Fit result

Figures 6.6 and 6.7 show the results of the log-parabola and flux-points analyses of HAWC Crab data respectively. Table 6.1 contains the best-fit values of the log-parabola spectral

Parameter	Value	Stat. error	Syst. error
$\Phi_0[10^{-13}/(\mathrm{cm}^2\cdot\mathrm{s}\cdot\mathrm{TeV})]$	3.27	0.05	+1.10/-0.76
α	2.623	0.014	+0.081/-0.109
β	0.08	0.01	+0.06/-0.05

Table 6.1: Best-fit values of log-parabola spectral parameters with statistical and systematic uncertainties. Systematic uncertainties are the sum in quadrature of the deviations from the nominal result produced by performing the analysis using the systematics MCs as described in Section 6.3.

parameters along with their uncertainties, and Table 6.2 contains the values of the quantity energy squared times flux at each energy plotted in Figure 6.7 along with uncertainties. The error band in Figure 6.6 shows the statistical uncertainty on the flux propagated from the uncertainties on the fit parameters. The error bars in Figure 6.7 are statistical errors added in quadrature with the systematic errors described in Section 6.3. Figure 6.7 shows that the HAWC measurement of the Crab flux at energies between 10 and 50 TeV exceeds that measured by H.E.S.S. and HEGRA. The result also constitutes the highest-energy observation of the Crab γ -ray spectrum by any experiment. The production of γ rays by the Crab at these energies has been predicted by multiple theoretical models [60, 61]; however, this measurement is the first empirical measurement.

Figure 6.8 shows the "pulls" for the log-parabola fit to the Crab. The pull for a bin is defined as the residual divided by the expected fluctuation of that residual under the best-fit model:

$$t \equiv \frac{c-s-b}{\sqrt{s+b}} \tag{6.9}$$

where c, s, and b are the observed count, expected signal, and expected background in the bin added over pixels that lie within a certain angular distance of the Crab. The distance is chosen for each bin to be the radius that would optimize the sensitivity of a hypothetical aperture-based analysis (i.e. it maximizes the quantity s/\sqrt{b}). The resulting quantity t



Figure 6.6: The Crab log-parabola spectrum fit using HAWC data binned in NN energy, along with similar measurements made by the H.E.S.S. [4] and HEGRA [5] experiments. Errors on the HAWC spectrum are statistical only. See Figure 6.7 for systematic errors.

Energy	Energy squared times flux	Stat. error	Syst. error
1.2	37.8	0.8	+2.7/-1.4
2.2	29.6	0.6	+5.1/-3.9
3.8	22.8	0.5	+7.1/-4.6
6.9	15.8	0.4	+6.5/-4.1
12.6	11.0	0.4	+5.5/-2.9
22.7	7.4	0.4	+2.9/-2.0
40.4	4.7	0.4	+2.3/-1.3
72.8	1.8	+0.4/-0.3	+1.9/-0.5
126.0	1.1	0.4	+0.6/-0.3

Table 6.2: Energy (in TeV) and energy squared times flux (in TeV/(cm² \cdot s)) for each point in Figure 6.7. Statistical errors are propagated from the uncertainties on the flux normalization as refit in each energy bin as described in Section 6.3. Systematic errors are sums in quadrature of deviations of the flux points computed using systematics MCs from the nominal flux points.



Figure 6.7: The Crab energy spectrum produced by the flux-points analysis described in Section 6.3, along with measurements by H.E.S.S. [4] and HEGRA [5]. Errors are statistical and systematic added in quadrature.



Figure 6.8: Pulls (residuals divided by their expected fluctuations) for each of the 2D bins included in the Crab analysis. The x axis indicates NN bins; the color scale indicates fraction-hit bins.

should have a mean of zero and a standard deviation of one if the data are compatible with the model within statistical errors. However, a substantial negative bias can be seen in the pulls. This results from the systematic underestimation of HAWC's angular resolution by the MC (Figure 6.5). Because a fixed radius is predicted by the MC to contain a larger fraction of events than it actually does in data, the MC-based prediction exceeds the observed count. This source of systematic error is accounted for via the angular-resolution systematic described in Section 6.3.

The results presented in this chapter demonstrate that the HAWC experiment can make

unprecedentedly high-energy measurements of γ -ray flux using the NN energy variable. However, measurements of the kind shown in Figure 6.7 are not in and of themselves sufficient to constrain LIV. The flux-points technique relies on the assumption that true emission spectrum of the source is at least approximately log-parabolic in shape. In particular, the kind of hard cutoff that would be expected under LIV would qualitatively violate this assumption. A bin containing an excess of events reconstructed at high energies would still be assigned a positive flux and a high energy even if all of these events were lower-energy events whose energies were overestimated by the NN. Chapter 7 will present a technique by which HAWC data reconstructed using the NN algorithm can be used to set constraining limits on photon-sector Lorentz-invariance violation and demonstrate the presence of high-energy flux.

Chapter 7

LIV limits

HAWC's observations of unprecedentedly high-energy γ rays provide a vantage point from which to search for and constrain the sort of hard spectral cutoff that might occur in the presence of photon-sector LIV as described in Section 1.3.

7.1 Analysis technique

According to Section 1.3, a photon dispersion relation like Equation (1.12) (in which the LIV perturbation contributes with the same sign as the k^2 term) allows photon decay to electron/positron pair for a photon momentum k satisfying Equation (1.17). For photons propagating on astrophysical distance scales, the decay probability above the energy cutoff is very nearly one [9]. As a result, if such a decay is allowed, the observed energy spectrum from an astrophysical source of very-high-energy γ rays should have the distinct feature of a hard cutoff, i.e. an energy above which there is absolutely no emission. The blue curve in Figure 7.1 shows the effect of such a cutoff on a log-parabola spectrum.

However, because HAWC's energy resolution is imperfect, an event with true energy below the cutoff energy still has a nonzero probability of being assigned a reconstructed energy above the cutoff. This is shown in the orange curve in Figure 7.1. As a result, it cannot simply be concluded that there is no LIV-induced cutoff below the highest energy assigned to any photon in HAWC data. Instead a profile-likelihood technique is used to



Figure 7.1: A log-parabola energy spectrum subject to a hard cutoff. The blue curve shows the true-energy spectrum; the orange curve shows the reconstructed-energy distribution.

determine a 95%-confidence lower limit on any LIV cutoff that may be present.

In this analysis the data are binned in a manner similar to that described in Section 6.1 with two modifications. First, each of the top two reconstructed-energy bins, the one spanning 100 TeV to 178 TeV and the one spanning 178 TeV to 316 TeV, is subdivided into three bins of equal size in log space. This finer binning is done to increase the precision with which the high-energy spectral shape can be resolved. Second, the pixel-by-pixel analysis is replaced with an aperture- or "top-hat"-based analysis: instead of treating each pixel as a separate bin in the likelihood, we add the contents of all pixels within a certain radius of the center of the source, and the likelihood is constructed using only these summed counts.

Source name	Top-hat radius ($^{\circ}$)
Crab	0.6
2HWC J1825-134	0.4
J1839 - 057	3.2
2HWC J1844-032	1.1
2HWC J1908+063	0.9
2HWC J2019+367	0.8
2HWC J2031+415	0.4

Table 7.1: Top-hat radii for the seven sources.

This reduces the systematic dependency of the result on the modeling of the PSF, which is known to be poor in the MC (as shown in Figure 6.5).

The top-hat radius is chosen independently for each source and used in all bins in the analysis of that source. The limit-setting procedure is carried out using successively larger values of the top-hat radius until the value levels off; the limit improves as the radius is increased until most or all of the high-energy signal photons are within the top hat. The chosen top-hat radii are shown in Table 7.1. Figure 7.2 shows the top-hat optimization procedure for 2HWC J1825-134.

Observations of seven sources are used. These have been chosen for the presence of high-energy emission in HAWC data. The sources are the Crab Nebula, 2HWC J1825–134, J1839–057, 2HWC J1844–032, 2HWC J1908+063, 2HWC J2019+367, and 2HWC J2031+415. The 2HWC sources are all in the vicinities of known PWNs or SNRs and are described in the 2HWC catalog [62]. J1839-057 is an unidentified source that is near the SNRs Kes 73 and G26.6-0.1 [62]. The Crab Nebula is modeled as a point source with a log-parabola spectrum as in Chapter 6. The remaining sources are treated as extended; the morphology is modeled as a two-dimensional Gaussian distribution centered at the location of the source (known from other experiments and shown in Table 7.3). The width σ of this Gaussian is fixed to a value determined by a prior HAWC analysis of these sources. This analysis used a pixel-


Figure 7.2: 95%-confidence lower limit on $E_{\rm LIV}$ using 2HWC J1825-134 data as a function of top-hat radius.

by-pixel likelihood technique and was therefore able to estimate the sources' morphologies. These sources' energy spectra are modeled as exponentially cut off power laws:

$$\frac{dN}{dE} = \Phi_0 \left(\frac{E}{E_0}\right)^{-\alpha} \exp\left(-\frac{E}{E_{\exp}}\right)$$
(7.1)

where E_{exp} is the energy at which the spectrum cuts off.

Since in this analysis we are concerned only with the LIV cutoff energy, the parameters of the sources' emission spectra will be treated as nuisance parameters. These are Φ_0 , α , and in the case of the Crab β or in the case of the other sources E_{exp} . The nuisance parameters are dealt with using the profile-likelihood formalism [63]. Let $\boldsymbol{\theta}$ be the vector of nuisance parameters ($[\Phi_0 \ \alpha \ \beta]^{\text{T}}$ or $[\Phi_0 \ \alpha \ E_{\text{exp}}]^{\text{T}}$). Again using Wilks' theorem, we can consider an LLR of the form

$$D(E_{\rm LIV}) \equiv 2 \ln \frac{\max_{\boldsymbol{\theta}', E_{\rm LIV}'} L(\boldsymbol{\theta}', E_{\rm LIV})}{\max_{\boldsymbol{\theta}'} L(\boldsymbol{\theta}', E_{\rm LIV})}$$
(7.2)

where E_{LIV} is the LIV-induced hard-cutoff energy. This will be asymptotically distributed as a χ^2 with one degree of freedom. This LLR can be rewritten as

$$D(E_{\rm LIV}) = 2\ln \frac{\max_{E_{\rm LIV}'} L_{\rm p}(E_{\rm LIV})}{L_{\rm p}(E_{\rm LIV})}$$
(7.3)

where $L_{\rm p}$ is the so called profile likelihood, defined as

$$L_{\rm p}(E_{\rm LIV}) \equiv \max_{\boldsymbol{\theta}'} L(\boldsymbol{\theta}', E_{\rm LIV}).$$
(7.4)

The equivalency of Equations (7.2) and (7.3) implies that we can treat the profile likelihood as if it were the ordinary likelihood for a one-parameter model.

Under the hypothesis that there is no LIV cutoff $(E_{\text{LIV}} = \infty)$, the TS $D(\infty)$ can be used to perform a likelihood-ratio test to determine the significance with which the data deviate from their expectations under the null (no-cutoff) hypothesis. Under the null hypothesis, Wilks' theorem states that $D(\infty)$ should be distributed as a χ^2 with one degree of freedom. A *p*-value quantifying the statistical significance of the deviation from the null hypothesis is computed by integrating the χ^2 distribution with one degree of freedom from the value of $D(\infty)$ to ∞ . These values are reported in Table 7.3.

Since a 95%-confidence lower limit is simply the low edge of a 90%-confidence uncertainty interval, such a lower limit on E_{LIV} can then be computed by solving

$$D(E_{\rm LIV}) = F^{-1}(0.9) = 2.71 \tag{7.5}$$

where F is the cumulative distribution for a χ^2 variable with one degree of freedom. Such limits are computed for the nominal and systematics MCs for each source and listed in Table 7.3. A systematic uncertainty for each nominal limit is calculated by adding in quadrature the deviations of the systematics limits from the nominal limits. The final result, a degraded limit, is then computed as the nominal limit minus its lower systematic error.

The 95%-confidence limit is computed in two parts, shown in Figure 7.3. First the likelihood is maximized over all parameters ($E_{\rm LIV}$ and the nuisance parameters). The profile likelihood is then computed at successive lower values of $E_{\rm LIV}$ until the profile likelihood is less than the maximum likelihood minus 2.71. The value of $E_{\rm LIV}$ at which the profile likelihood equals the maximum likelihood minus 2.71 is then found by repeatedly bisecting the resulting interval.

Because the hard cutoff parameter has a negligible effect on the low-energy bins in the



Figure 7.3: Two times the natural logarithm of the profile likelihood defined in Equation (7.4), plotted as a function of $E_{\rm LIV}$ for the Crab Nebula.

analysis, their expectations are the effectively the same under the null and alternative hypotheses. As a result, they nearly cancel in the likelihood ratio. The limit is therefore driven by the content of the high-energy bins, whose expected contents can change considerably between hypotheses.

7.2 Results and interpretation

Figure 7.4 shows the contents of the reconstructed-energy bins using 2HWC J1825-134 data. It also shows the expected background distribution as well as the expected counts under the signal-plus-background model in which the hard cutoff is set to its 95%-confidence lower limit. The limit derived from these data, prior to degrading for systematics, is 259 TeV. This value lies in the second to last bin in the data. These bin contents and expectations are also shown in Table 7.6.

Table 7.3 contains the *p*-values and 95%-confidence lower limits on E_{LIV} for each of the seven sources. p = 1 when the likelihood is maximized at $E_{\text{LIV}} = \infty$. The lowest *p*-value of 0.082 occurs for 2HWC J1844-032; after correcting for seven independent statistical tests, this corresponds to a *p*-value of 0.450. We therefore conclude that we have not observed statistically significant evidence of an LIV-induced cutoff.

The strongest systematics-degraded limit of 209 TeV is given by 2HWC J1825-134. By far the dominant source of systematic uncertainty is the QE, which allows the limit to vary between 209 and 327 TeV. The QE systematic dominates for the other sources as well. Increasing the PMT QE in the MC increases the number of PMT hits expected per event, thereby increasing the expected event rate at high estimated energy and making the fixed data resemble a deficit compared to the null-hypothesis model. Conversely, a lower QE cor-



Figure 7.4: Expected and observed contents of the reconstructed-energy bins from the 2HWC J1825-134 limit-setting procedure. The orange points show the expected distribution under the signal-plus-background model in which the LIV cutoff energy is set to infinity and the other parameters are optimized. The green points show the expected distribution under the signal-plus-background model with the hard cutoff set to its 95%-confidence lower limit and the parameters optimized. The red points indicate the expectations from background only.

responds to a lower expected high-reconstructed-energy event rate, making the data counts high compared to their model expectations and reducing the possibility of a cutoff. This is exacerbated by the large QE uncertainties: as described in Section 4.6, the A, B, and D PMTs' QE is allowed to vary between 44 and 66%, and the C PMTs' QE is varied between 64 and 96%.

As an additional systematic check, the Crab limit was computed modeling the Crab spectrum instead as an exponentially cut off power law. The result was 137 TeV, somewhat lower than the log-parabola-derived limit. Incorporating this value in the systematics for the Crab limit inflates the overall systematic by 4 TeV.

Equation (1.18) can be used to compute 95%-confidence lower limits on $E_{\rm QG}$ from the $E_{\rm LIV}$ limits given in Table 7.3. Table 7.4 shows the resulting limits. These substantially exceed the previous best limits on $E_{\rm QG}$. An analysis [9] of HEGRA data set $E_{\rm QG}$ lower limits of 1.5×10^{29} eV for n = 1 and 2.8×10^{21} eV for n = 2. The limits from all seven HAWC sources are more constraining than the HEGRA limits. The strongest limits are set by 2HWC J1825–134: 8.74×10^{30} eV for n = 1 and 4.27×10^{22} eV for n = 2.

These limits exceed the previous best photon-decay-based LIV limits by more than an order of magnitude. Limits set by searching for vacuum birefringence (such as [64]) exceed these limits by several orders of magnitude; however, they are not applicable to all possible photon-sector LIV scenarios [9]. With respect to models in which birefringence is disallowed, the limits presented here are the most constraining.

Table 7.5 shows higher-confidence limits on the hard-cutoff energy. These have been degraded to reflect systematic uncertainties as well. 2HWC J1825–134 provides 3σ evidence for γ emission continuing up to 141 TeV. A cutoff below 55 TeV can be ruled out at the 5σ level. The highest existing 5σ measurement of γ -ray flux was performed by HEGRA at an

Source	$\Phi_0[(\mathrm{cm}^2\cdot\mathrm{s}\cdot\mathrm{TeV})^{-1}]$	α	β	$E_{\rm exp}({\rm TeV})$	$E_{\rm LIV}({\rm TeV})$	$\sigma(^{\circ})$
Crab	3.6×10^{-13}	2.63	0.10		5160	
2HWC J1825-134	3.6×10^{-13}	1.95		134	792	0.55
J1839-057	4.0×10^{-13}	2.29		131	104	1.01
2HWC J1844-032	2.6×10^{-13}	2.34		9904	105	1.29
2HWC J1908+063	1.2×10^{-13}	2.22		185	1166	0.64
2HWC J2019+367	0.7×10^{-13}	1.65		48	214	0.29
2HWC J2031+415	0.7×10^{-13}	1.89		86	251	0.42

Table 7.2: Best-fit values of the spectral parameters in the LIV model. The σ parameter is fixed for each source.

energy of 42 TeV [5]. The HAWC measurement of 2HWC J1825–134 therefore constitutes the highest-energy 5σ measurement of cosmic γ rays and a discovery of γ -ray flux above 55 TeV.

Source	p	Nom.	Syst. err.	Low CS	High CS	Low QE	High QE	Low	No BP	Nom.
								thresh.		degraded
Crab	0.412	151	+44/-24	154	145	194	131	139	162	127
2HWC J1825-134	0.998	259	+68/-50	263	259	327	211	245	266	209
J1839 - 057	0.997	79	+23/-12	82	83	101	70	71	82	67
2HWC J1844-032	0.082	81	+23/-20	81	88	103	63	75	75	61
2HWC J1908+063	1.000	214	+60/-46	217	212	273	168	208	222	168
2HWC J2019+367	0.791	125	+30/-17	128	123	155	108	124	129	108
2HWC J2031+415	0.709	150	+36/-24	153	152	186	126	148	153	126

Table 7.3: *p*-values and 95%-confidence limits on E_{LIV} . Limits are expressed in TeV. The limit using the nominal MC is provided along with a systematic error obtained by adding in quadrature the deviations of the limits calculated using the systematics MCs. The systematics MCs are those having a lower than nominal charge smearing, higher charge smearing, lower QE, higher QE, lower PMT threshold, and no broad-pulse effect. The systematics-degraded limit is then computed as the nominal limit minus its lower error. See Section 4.6 for more details on the systematics MCs.

Source	n = 1	n=2
Crab	1.96×10^{30}	1.58×10^{22}
2HWC J1825-	$-134 \mid 8.74 \times 10^{30}$	4.27×10^{22}
J1839-057	2.88×10^{29}	4.39×10^{21}
2HWC J1844-	-032 2.17 × 10 ²⁹	3.64×10^{21}
2HWC J1908+	-063 4.54 × 10 ³⁰	2.76×10^{22}
2HWC J2019+	$-367 \mid 1.21 \times 10^{30}$	1.14×10^{22}
2HWC J2031+	-415 $ 1.92 \times 10^{30} $	1.55×10^{22}

Table 7.4: 95%-confidence lower limits on the QG energy E_{QG} for *n* values of 1 and 2. All limits are expressed in eV. These limits can be computed from the ones given in the nominal degraded column of Table 7.3 using Equation (1.18).

Source	95%	3σ	4σ	5σ
Crab	127	64	54	48
2HWC J1825-134	209	141	75	55
J1839 - 057	67	58	51	45
2HWC J1844-032	62	54	48	41
2HWC J1908+063	168	118	78	53
2HWC J2019+367	108	71	55	46
2HWC J2031+415	126	42	32	26

Table 7.5: Systematics-degraded limits on E_{LIV} at various confidences. Limits are expressed in TeV. A $Z\sigma$ limit is defined to have confidence F(Z) where F is the standard-normal cumulative distribution. As a result, the 3σ , 4σ , and 5σ limits correspond to confidences of 99.87%, 99.9968%, and 99.999971% respectively.

E (TeV)	Obs.	В	S + B (limit)	S + B (no LIV)
1.33	225124	209268.32	225468.61	225211.97
2.37	76029	66813.93	76423.32	76502.31
4.22	19175	14233.04	19082.14	19154.73
7.50	6271	3780.80	6298.96	6306.56
13.34	1733	759.90	1761.70	1734.13
23.71	633	196.44	623.18	591.83
42.17	212	57.06	204.23	199.08
74.99	43	13.31	25.53	54.02
110.07	6	2.49	2.53	8.10
133.35	5	1.92	1.92	5.68
161.56	5	1.28	1.28	3.48
195.73	2	0.64	0.64	1.72
237.14	0	0.36	0.36	0.86
287.30	1	0.20	0.20	0.37

Table 7.6: Observed and expected bin contents from the 2HWC J1825-134 analysis, as plotted in Figure 7.4. Columns are central reconstructed bin energy, observed count, background expectation, signal-plus-background expectation with $E_{\rm LIV}$ set to its 95% lower limit of 259 TeV, and signal-plus-background expectation with $E_{\rm LIV}$ set to infinity.

Chapter 8

Conclusions and future prospects

In addition to constraining models of LIV, the results presented in this thesis constitute the first measurement of γ -ray flux above 100 TeV. This measurement suggests the existence of a wealth of new science that can be carried out in this new energy window. It also demonstrates the currently unparalleled ability of the HAWC experiment to carry out this science. As the only γ -ray observatory able to resolve these energies, HAWC will be uniquely poised to carry out measurements of γ -ray sources in this energy band. Several such analyses are already underway and will be published within a year of this writing. Furthermore, the lower limit on E_{LIV} of 209 TeV derived from 2HWC J1825-134 implies that with 95% confidence, there exist photons of this energy in the Universe and they can propagate on Galactic distance scales.

The HAWC detector is currently undergoing a major enhancement. Smaller water tanks called outriggers, each housing a single PMT, have been deployed and are currently taking data. Once the development of software for calibration, reconstruction, and analysis of outrigger data is complete, the background-rejection and reconstruction capabilities of the experiment will improve further. A future iteration of the analysis presented here might gain substantial sensitivity from the incorporation of outrigger data.

Another avenue for improvement is the reduction of systematic errors. The systematic uncertainties on the MC described in Section 4.6 are deliberately conservative and reflect the relatively wide ranges considered plausible for the parameters in question at the time when those ranges were determined. A new iteration of HAWC's MC and accompanying systematics simulations are currently underway, and ongoing studies of these parameters generally tend to suggest that some of their uncertainty ranges can be significantly reduced although this work is not yet complete.

If a future version of this analysis is able to rule out an LIV-induced cutoff at even higher energies, the attenuating effect of the cosmic microwave background (CMB) may need to be taken into account. Figure 8.1 shows the mean free path of photons subject to CMB interactions as a function of energy. At 259 TeV, the highest limit set in this thesis, the mean free path is approximately 60 kpc. The distance to 2HWC J1825–134, the source from which the most constraining limit is derived, is 4 kpc, and so only 6% of photons ought to be lost to CMB interactions. However, Figure 8.1 shows that the mean free path drops steeply at these energies. This suggests that a future analysis might need to take into account the CMB interaction in modeling the sources' energy spectra.

While the work presented in this thesis on the topic of LIV has ruled out certain physics models, the work on energy reconstruction will also provide a new lens through which HAWC can explore the high-energy universe. For this reason and others, the near future will be a time of great scientific productivity for the HAWC experiment.



Figure 8.1: Mean free path of photons subject to the CMB [6].

APPENDIX

Appendix

Data/MC agreement of neural-net input variables

In Section 5.2, the NN Energy Estimator was shown to outperform the other available HAWC γ -ray energy-reconstruction algorithms in MC. In order for the same performance to be achieved in data, the 16-dimensional joint distribution of the 15 inputs and the output must be correctly modeled in the MC. This is difficult to verify because of the curse of dimensionality. A first-order check and necessary condition is to see that the one-dimensional distributions of input variables are consistent between data and MC.

Of course the data also contain an admixture of cut-passing hadrons. Instead of modeling these with MC, their distributions can be built from data and subtracted off. To this end, two different locations in the sky must be considered: the location of the Crab Nebula (the γ -source used for this comparison) and a location 4° away from the Crab at the same declination. The cuts used for this analysis are the hadron-rejection cuts described in Chapter 5, the fiducial cut (Equation (5.2)), and a zenith cut at 40°. The MC is weighted to resemble a Crab-like spectrum with spectral index 2.63 and normalized to the data. The latter is done to deconvolve the question of the data/MC agreement of the overall event rate, which is not a function of the energy-estimation algorithm, from the question of the probability distributions obeyed by the NN inputs. This analysis was performed using the HAWC-250



Figure .1: Data/MC comparison of fraction hit. p = 0.0997.

MC simulation that was current as of March, 2016, and used in the first publication of HAWC's measurement of the Crab Nebula energy spectrum, published in 2017.

The resulting distributions are shown in the figures in this appendix. To quantify any disagreement in the distributions, a Kolmogorov-Smirnov (KS) test is performed for each variable. The *p*-values from these tests are displayed in the captions of the respective figures.

Three input variables show a discrepancy at the 3σ level: the fraction of tanks hit and the outermost two annuli. The uncertainty in the modeling of these variables and the resulting uncertainty in the the mixing matrix of the NN energy variable are accounted for by performing spectral analyses using a variety of different simulations as described in Section 4.6.



Figure .2: Data/MC comparison of fraction of tanks hit. $p = 1.10 \times 10^{-5}$.



Figure .3: Data/MC comparison of log of core-fit amplitude. p=0.0538.



Figure .4: Data/MC comparison of core distance from array center. p = 0.0538.



Figure .5: Data/MC comparison of cosine of zenith angle. p=0.715.



Figure .6: Data/MC comparison of % p=0.0762.



Figure .7: Data/MC comparison of fraction of charge contained in annulus 1 . p=0.116.



Figure .8: Data/MC comparison of fraction of charge contained in annulus 2 . p=0.550.



Figure .9: Data/MC comparison of fraction of charge contained in annulus 3 . p=0.929.



Figure .10: Data/MC comparison of fraction of charge contained in annulus 4 . p=0.165.



Figure .11: Data/MC comparison of $\mbox{ fraction of charge contained in annulus 5}$. p=0.0118.



Figure .12: Data/MC comparison of fraction of charge contained in annulus 6 . p=0.11.



Figure .13: Data/MC comparison of $\mbox{ fraction of charge contained in annulus 7 . } p = 0.0196.$



Figure .14: Data/MC comparison of fraction of charge contained in annulus 8 . $p=1.34\times 10^{-7}.$



Figure .15: Data/MC comparison of fraction of charge contained in annulus 9. $p=4.54\times 10^{-15}.$

BIBLIOGRAPHY

BIBLIOGRAPHY

- [1] Z. Hampel-Arias. Cosmic Ray Observations at the TeV Scale with the HAWC Observatory. PhD thesis, The University of Wisconsin - Madison, 2017.
- [2] A. U. Abeysekara et al. Observation of the Crab Nebula with the HAWC Gamma-Ray Observatory. *The Astrophysical Journal*, 843(1):39, 2017.
- [3] A. U. Abeysekara et al. Data Acquisition Architecture and Online Processing System for the HAWC gamma-ray observatory. *Nucl. Instrum. Meth.*, A888:138–146, 2018.
- [4] M. Holler, D. Berge, C. van Eldik, J.-P. Lenain, V. Marandon, T. Murach, M. de Naurois, R. D. Parsons, H. Prokoph, D. Zaborov, and for the H. E. S. S. collaboration. Observations of the Crab Nebula with H.E.S.S. Phase II. arXiv e-prints, September 2015.
- [5] F. Aharonian et al. The Crab nebula and pulsar between 500-GeV and 80-TeV. Observations with the HEGRA stereoscopic air Cerenkov telescopes. Astrophys. J., 614:897–913, 2004.
- [6] M. Spurio. *Particles and Astrophysics: A Multi-Messenger Approach*. Astronomy and Astrophysics Library. Springer International Publishing, 2014.
- [7] L.D. Landau and E.F. Lifshitz. *The Classical Theory of Fields*. Course of theoretical physics. Butterworth-Heinemann, 1975.
- [8] Sidney Coleman and Sheldon L. Glashow. High-energy tests of Lorentz invariance. *Phys. Rev. D*, 59:116008, Apr 1999.
- [9] H. Martínez-Huerta and A. Pérez-Lorenzana. Restrictions from Lorentz invariance violation on cosmic ray propagation. *Phys. Rev. D*, 95:063001, Mar 2017.
- [10] D. Colladay and V. Alan Kostelecký. Lorentz-violating extension of the standard model. *Phys. Rev. D*, 58:116002, Oct 1998.
- [11] James Chadwick. The existence of a neutron. Proceedings of the Royal Society of London Series A, 136, Jun 1932.
- [12] W. Baade and F. Zwicky. Cosmic Rays from Super-Novae. Proceedings of the National Academy of Sciences, 20(5):259–263, 1934.
- [13] A. HEWISH, S. J. BELL, J. D. H. PILKINGTON, P. F. SCOTT, and R. A. COLLINS. Observation of a Rapidly Pulsating Radio Source. *Nature*, 217(5130):709–713, 1968.

- [14] C. Grupen. Astroparticle Physics. SpringerLink: Springer e-Books. Springer, 2005.
- [15] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs. The Australia Telescope National Facility Pulsar Catalogue. *The Astronomical Journal*, 129:1993–2006, April 2005.
- [16] International Astronomical Union. Symposium, F. Camilo, and B.M. Gaensler. Young Neutron Stars and Their Environments: Proceedings of the 218th Symposium of the International Astronomical Union Held During the Iau General Assembly XXV, Sydney, Australia, 14-17 July 2003. IAU Symposium Series. Astronmical Society of the Pacific, 2004.
- [17] Bryan M. Gaensler and Patrick O. Slane. The evolution and structure of pulsar wind nebulae. Ann. Rev. Astron. Astrophys., 44:17–47, 2006.
- [18] R.N. Manchester and J.H. Taylor. Pulsars. A Series of books in astronomy and astrophysics. W. H. Freeman, 1977.
- [19] Margaret A. Livingstone, Victoria M. Kaspi, and Fotis P. Gavriil. Long-Term Phasecoherent X-Ray Timing of PSR B0540–69. The Astrophysical Journal, 633(2):1095, 2005.
- [20] J. M. Migliazzo, B. M. Gaensler, D. C. Backer, B. W. Stappers, E. van der Swaluw, and R. G. Strom. Proper-Motion Measurements of Pulsar B1951+32 in the Supernova Remnant CTB 80. *The Astrophysical Journal Letters*, 567(2):L141, 2002.
- [21] T.K. Gaisser. *Cosmic Rays and Particle Physics*. Cambridge University Press, 1990.
- [22] Progress in Elementary Particle and Cosmic Ray Physics. Number v. 3 in Series in physics. North Holland Publishing Company, 1956.
- [23] Koichi Kamata and Jun Nishimura. The Lateral and the Angular Structure Functions of Electron Showers. *Progress of Theoretical Physics Supplement*, 6:93–155, 1958.
- [24] Joshua Wood. An All-Sky Search for Bursts of Very High Energy Gamma Rays with HAWC. PhD thesis, Maryland U., 2016.
- [25] I. G. Wisher. Real-time Transient Monitoring With the HAWC Detector: Design and Performance. PhD thesis, The University of Wisconsin - Madison, 2016.
- [26] M.J. Berger, J.S. Coursey, M.A. Zucker, and J. Chang. Stopping-Power & Range Tables for Electrons, Protons, and Helium Ions. https://www.nist.gov/pml/ stopping-power-range-tables-electrons-protons-and-helium-ions.
- [27] ZeroMQ home page. http://zeromq.org/.

- [28] cJSON GitHub repository. https://github.com/DaveGamble/cJSON.
- [29] M. Tavani et al. Discovery of Powerful Gamma-Ray Flares from the Crab Nebula. Science, 331:736, February 2011.
- [30] A. A. Abdo et al. Gamma-Ray Flares from the Crab Nebula. Science, 331(6018):739– 742, 2011.
- [31] G. Aielli et al. Enhanced TeV gamma ray flux from the Crab Nebula observed, 09 2010.
- [32] Benoit Cerutti et al. Extreme Particle Acceleration in Magnetic Reconnection Layers: Application to the Gamma-Ray Flares in the Crab Nebula. *The Astrophysical Journal*, 746(2):148, 2012.
- [33] A. U. Abeysekara et al. Sensitivity of the high altitude water Cherenkov detector to sources of multi-TeV gamma rays. Astroparticle Physics, 50:26–32, December 2013.
- [34] S. Ansoldi et al. Teraelectronvolt pulsed emission from the Crab pulsar detected by MAGIC. Astron. Astrophys., 585:A133, 2016.
- [35] Thomas Brian Humensky. VERITAS Studies of the Supernova Remnants Cas A and IC 443. AIP Conf. Proc., 1085:357–360, 2009.
- [36] M. Ackermann et al. Detection of the Characteristic Pion-Decay Signature in Supernova Remnants. Science, 339(6121):807–811, 2013.
- [37] A. A. Abdo et al. Milagro Observations of Multi-TeV Emission from Galactic Sources in the Fermi Bright Source List. *The Astrophysical Journal Letters*, 700(2):L127, 2009.
- [38] Dieter Horns et al. Hardening of TeV gamma spectrum of AGNs in galaxy clusters by conversions of photons into axion-like particles. *Phys. Rev.*, D86:075024, 2012.
- [39] H. Krawczynski et al. Multiwavelength Observations of Strong Flares from the TeV Blazar 1ES 1959+650. The Astrophysical Journal, 601(1):151, 2004.
- [40] Francis Halzen and Dan Hooper. High energy neutrinos from the TeV Blazar 1ES 1959+650. Astroparticle Physics, 23(6):537 542, 2005.
- [41] A. Reimer et al. Neutrino Emission in the Hadronic Synchrotron Mirror Model: The "Orphan" TeV Flare from 1ES 1959+650. *The Astrophysical Journal*, 630(1):186, 2005.
- [42] Pat Scott and et al. Direct Constraints on Minimal Supersymmetry from Fermi-LAT Observations of the Dwarf Galaxy Segue 1. *JCAP*, 1001:031, 2010.
- [43] E. Aliu et al. VERITAS deep observations of the dwarf spheroidal galaxy Segue 1. Phys. Rev. D, 85:062001, Mar 2012.

- [44] J. Aleksić et al. Segue 1: the best dark matter candidate dwarf galaxy surveyed by MAGIC. In Proceedings, 32nd International Cosmic Ray Conference (ICRC 2011): Beijing, China, August 11-18, 2011, volume 5, pages 149–152, 2011.
- [45] M. et al. Ackermann. Fermi-LAT Observations of the Gamma-Ray Burst GRB 130427A. Science, 343(6166):42–47, 2014.
- [46] R. Atkins et al. The High-Energy Gamma-Ray Fluence and Energy Spectrum of GRB 970417a from Observations with Milagrito. *The Astrophysical Journal*, 583(2):824, 2003.
- [47] A. A. Abdo et al. Milagro Constraints on Very High Energy Emission from Short-Duration Gamma-Ray Bursts. The Astrophysical Journal, 666(1):361, 2007.
- [48] R. C. Gilmore, P. Madau, J. R. Primack, R. S. Somerville, and F. Haardt. GeV gammaray attenuation and the high-redshift UV background. *Monthly Notices of the Royal Astronomical Society*, 399:1694–1708, November 2009.
- [49] M. A. Sánchez-Conde, D. Paneque, E. Bloom, F. Prada, and A. Domínguez. Hints of the existence of axionlike particles from the gamma-ray spectra of cosmological sources. *Physical Review D*, 79(12):123511, June 2009.
- [50] R. Brun and F. Rademakers. ROOT: An object oriented data analysis framework. Nucl. Instrum. Meth., A389:81–86, 1997.
- [51] Hao Zhou. Search for TeV Gamma-Ray Sources in the Galactic Plane with the HAWC Observatory. PhD thesis, Michigan Technological University, 2015.
- [52] D. Heck, J. Knapp, J. N. Capdevielle, G. Schatz, and T. Thouw. CORSIKA: a Monte Carlo code to simulate extensive air showers. February 1998.
- [53] S. et al. Agostinelli. GEANT4—a simulation toolkit. Nuclear Instruments and Methods in Physics Research A, 506:250–303, July 2003.
- [54] Andreas Hoecker, Peter Speckmayer, Joerg Stelzer, Jan Therhaag, Eckhard von Toerne, and Helge Voss. TMVA: Toolkit for Multivariate Data Analysis. *PoS*, ACAT:040, 2007.
- [55] K. M. Górski, E. Hivon, A. J. Banday, B. D. Wandelt, F. K. Hansen, M. Reinecke, and M. Bartelmann. HEALPix: A Framework for High-Resolution Discretization and Fast Analysis of Data Distributed on the Sphere. *The Astrophysical Journal*, 622:759–771, April 2005.
- [56] D. C. Wells, E. W. Greisen, and R. H. Harten. FITS a Flexible Image Transport System. Astronomy and Astrophysics, 44:363, June 1981.

- [57] R. Atkins, W. Benbow, D. Berley, E. Blaufuss, J. Bussons, D.G. Coyne, R.S. Delay, T. DeYoung, B.L. Dingus, D.E. Dorfan, R.W. Ellsworth, and A. Falcone. Observation of TeV Gamma Rays from the Crab Nebula with Milagro Using a New Background Rejection Technique. *The Astrophysical Journal*, 595(2):803–811, 2003.
- [58] D. E. Alexandreas, D. Berley, S. Biller, G. M. Dion, J. A. Goodman, T. J. Haines, C. M. Hoffman, E. Horch, X.-Q. Lu, C. Sinnis, G. B. Yodh, and W. Zhang. Point source search techniques in ultra high energy gamma ray astronomy. *Nuclear Instruments and Methods in Physics Research A*, 328:570–577, May 1993.
- [59] S. S. Wilks. The Large-Sample Distribution of the Likelihood Ratio for Testing Composite Hypotheses. Ann. Math. Statist., 9(1):60–62, 03 1938.
- [60] O. C. de Jager, A. K. Harding, P. F. Michelson, H. I. Nel, P. L. Nolan, P. Sreekumar, and D. J. Thompson. Gamma-Ray Observations of the Crab Nebula: A Study of the Synchro-Compton Spectrum. *The Astrophysical Journal*, 457:253, January 1996.
- [61] D. Horns and F. A. Aharonian. The Crab Nebula: Linking MeV Synchrotron and 50 TeV Inverse Compton Photons. In V. Schoenfelder, G. Lichti, and C. Winkler, editors, 5th INTEGRAL Workshop on the INTEGRAL Universe, volume 552 of ESA Special Publication, page 439, October 2004.
- [62] A. U. et al. Abeysekara. The 2HWC HAWC Observatory Gamma-Ray Catalog. The Astrophysical Journal, 843:40, July 2017.
- [63] G. Cowan. Statistical Data Analysis. Oxford science publications. Clarendon Press, 1998.
- [64] D. Gotz, S. Covino, A. Fernandez-Soto, P. Laurent, and Z. Bosnjak. The polarized Gamma-Ray Burst GRB 061122. Mon. Not. Roy. Astron. Soc., 431:3550, 2013.