SUPPORTING MAINTENANCE IN MATHEMATICS USING THE VIRTUAL-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL SEQUENCE INTERVENTION PACKAGE

By

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A DISSERTATION

Submitted to
Michigan State University
in partial fulfilment of the requirements
for the degree of

Special Education–Doctoral of Philosophy

2019

ABSTRACT

SUPPORTING MAINTENANCE IN MATHEMATICS USING THE VIRTUAL-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL SEQUENCE INTERVENTION PACKAGE

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Despite the growing attention being paid to teaching mathematics for students with disabilities, the existing research tends to focus on mathematical skill acquisition, but not on skill maintenance. Maintenance of mathematical skills is especially important as mathematics has applications in daily life and is directly related to important life skills such as purchasing, calculating tips, and budgeting. In addition, maintenance of basic mathematical skills is necessary for advancing to higher grade-level contents because mathematical contents build upon previous contents.

This dissertation is comprised of three stand-alone but inter-related studies that explored the maintenance of mathematical skills among students with disabilities. The researcher conducted a systematic review of the literature regarding attention to maintenance when exploring mathematical interventions for students with intellectual disability and autism. The researcher then conducted two original studies, each which examined the effectiveness of an intervention package—the virtual-representational-abstract (VRA) instructional sequence with fading support and the VRA with overlearning—to promote maintenance of basic operations skills among students with disabilities.

In the first study, to determine the extent to which researchers focused on skill maintenance in teaching mathematics, the researcher reviewed all studies from 1975 to 2018 that involved teaching mathematics to individuals with developmental disabilities. A total of 128

studies met inclusion criteria but only 46 studies involved a maintenance phase (35.9%). Of the studies that included a maintenance phase, there was no consensus among researchers on the standards for conducting a maintenance phase. The most widely taught mathematical content was numbers and operations. All studies employed intervention packages which included more than one instructional method and/or materials and the most widely used instructional method was prompting while the most widely used instructional materials were visual supports.

The second study was an experimental research study which used a multiple probe across participants design to examine the effectiveness of the VRA instructional sequence with fading support in teaching subtraction with regrouping to four students with disabilities, including intellectual disability and/or autism. A functional relation was found between the VRA instructional sequence with fading support and students' accuracy in solving the problems. Students also maintained the skill up to six weeks after the intervention.

The third study also utilized a multiple probe across participants design to evaluate the effectiveness of the VRA instructional sequence with overlearning in teaching multiplication to three students with disabilities. A functional relation existed between the VRA instructional sequence with overlearning and accuracy of solving multiplication problems. Students also maintained the skill up to eight weeks after the intervention.

Overall, in this dissertation, intervention packages including more than one instructional, or a combination of instructional methods and materials were not only effective for skill acquisition, but also for skill maintenance. The review of literature identified explicit instruction with visual supports, manipulatives, or task analysis as potentially beneficial packages for promoting maintenance while the experimental studies demonstrated the effectiveness of the VRA instructional sequence with fading support or overlearning.

ACKNOWLEDGEMENTS

First, I would like to thank my dissertation chair and academic advisor, Dr. Emily Bouck, for her continued support, critical feedback, and guidance over the past four years. Thank you for your guidance, as it has truly contributed to my development as an educator and a researcher. It would be a lie if I said there were no difficult moments, but every time you helped me overcome adversity, I found that I matured both personally and professionally. I think it was possible because of your support and faith in me. I would also like to convey my appreciation to the members of my dissertation committee, Dr. Marisa Fisher, Dr. Jack Smith, and Dr. Rand Spiro. Your guidance and every piece of advice you have given me have truly made this dissertation even stronger.

I would also like to thank my family. Thank you for your encouragement, endless support and prayer while I completed this journey. Mom, thank you for guiding me and always being a model of a warm, thoughtful, and wise educator. You are forever an inspiration to me, and you will always be my role model. Dad, thank you for always giving me courage, for trusting me, and guiding me to become who I am today. I appreciate that you’ve taught me to put 100% effort into everything I do. To my sister, thank you for always being a wonderful little sister, staying by my side, and cheering me on. To my best friend, Dr. Keunchul Lee, thank you for your warm heart, support, kindness, and for understanding me.

Glory be to God.

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CHAPTER 1

INTRODUCTION

Mathematics learning is critical as it is directly related to functional living as well as later academic outcomes (Cihak & Grim, 2008; Edwards, Rule, & Boody, 2017; Watt, Ducan, Sieglar, & Davis-Kean, 2014). However, according to the National Assessment of Educational Progress (NAEP, 2017), 84% of fourth-grade students with disabilities and 91% of eighth-grade students with disabilities performed below proficiency level in mathematics. As students age, mathematics becomes progressively challenging and students can struggle to a greater extent (NAEP, 2017). Researchers found students who have difficulties in mathematics at an earlier grade keep struggling with mathematics through upper grade levels (Nelson & Powell, 2017). Nelson and Powell (2017) conducted a systematic review of the literature which included longitudinal research studies on progression of mathematics learning. The authors found that even though students with disabilities made improvements over time, they could not perform at grade level in mathematics. Such data highlight the need for research on effective instructional methods and materials for teaching mathematics to students with disabilities.

Instructional Methods and Materials for Students with Disabilities in Mathematics

To combat the challenges of students with disabilities, including learning disabilities and developmental disabilities, who need significant instruction in learning mathematics, researchers identified evidence-based practices (EBPs) in teaching mathematics. EBPs for students with disabilities include technology-assisted instruction, explicit instruction, and manipulatives (Bouck & Park, 2018; Kiru, Doabler, Sorrells, & Cooc, 2018; Miller & Hudson 2007; Spooner, Root, Saunders, & Browder, 2018).

Technology-assisted instruction

Technology-assisted instruction is defined as the use of electronic devices, applications, or virtual networks to support students' learning (Odom et al., 2015). For example, iPads are used to display videos in which a model performs a task that the student can imitate; this method is known as video modeling, which is an EBP for students with disabilities (Park, Bouck, & Duenas, 2019; Wong et al., 2015). More recently, researchers are also using virtual manipulatives on iPads to teach mathematics to students with disabilities (e.g., Bouck, Bassette, et al., 2017). Spooner et al. (2018) found 9 out of 36 studies used technology to teach mathematics (i.e., numbers, operations, geometry, algebra) for students with developmental disabilities. Hudson, Rivera, and Grady (2018) also found technology and multimedia components (e.g., computer-based, video-based instruction) are effective methods to teach mathematics to students with developmental disabilities (e.g., Ayres, Langone, Boon, & Norman, 2006; Hudson, Zambone, & Brickhouse, 2016, Spriggs, Knight, & Sherrow, 2015). In addition, Kiru et al. (2018) as well as Ok, Bryant, and Bryant (2019) found technology-based instruction including desktop, laptop, and handheld devices (e.g., iPads) has positive effects in teaching mathematics for students with learning disabilities.

Explicit instruction

Explicit instruction is a sequence of instructional supports, which includes clear demonstration of skills and feedback until students master the skill (Archer & Hughes, 2011). Explicit instruction in mathematics typically includes three parts—modeling, guided instruction, and independent practice (Doabler & Fien, 2013; Knight, Smith, Spooner, & Browder, 2012). During modeling, a teacher uses think-aloud, which is verbally stating the procedure of solving problems while doing so. While the teacher models how to solve the problem using think-aloud, s/he should use appropriate and consistent vocabulary (Archer & Hughes, 2011; Doabler & Fien, 2013). During the guided practice, the teacher provides cues or prompts to the student as needed. The teacher allows the student to take initiative to solve problems. If not, the teacher provides verbal prompts such as “what do you need to do first to solve the problem?” Also, if the student makes mistakes, the teacher provides immediate feedback. During independent practice, the student is asked to solve the problems independently without any assistance from the teacher. Independent practice is used to assess whether the student is able to solve the problems without prompting or guiding. If the student continues to struggle with the concept, the teacher can go back and re-teach as necessary (Agrawal & Morin, 2016). Spooner et al. (2018) analyzed 36 articles published between 2008 and 2016 and found explicit instruction to be an EBP and the most widely used instructional method for students with developmental disabilities. The National Center on Intensive Instruction also suggests that explicit instruction is an effective method for teaching students with disabilities.

Manipulatives

Manipulatives, which are generally concrete objects (e.g., base-10 blocks, pattern blocks, fraction tiles), are commonly used in teaching mathematics (Carbonneau, Marley, & Selig, 2013). Manipulatives provide opportunities for students to physically manipulate the objects in learning mathematical concepts such as basic operations, algebra, and geometry (Cabonneau et al., 2013). For example, students with disabilities can use base-10 blocks while solving addition or subtraction with regrouping, algebra tiles while learning the concept of algebra, or Cuisenaire rods while solving word problems (Bouck, Satsangi, Doughty, & Courtney, 2014; Browder, Jimenez, & Trela, 2012; Marsh & Cooke, 1996;). The use of manipulatives facilitates conceptual understanding of mathematical components (Bouck & Park, 2018). The National Council of Teachers of Mathematics (NCTM, 2000) encourages the use of manipulatives.

Although concrete manipulatives are one of the most used instructional practices and an effective tool for teaching mathematical concepts to students with disabilities (Bouck & Park, 2018; Spooner et al., 2018), concrete manipulatives can be stigmatizing if they are used to teach middle or high school students with disabilities (Satsangi & Bouck, 2015). Secondary students with disabilities may feel embarrassed when they use concrete manipulatives like base-10 blocks in front of their same grade peers without disabilities. This is because concrete manipulatives sometimes involve materials that are not considered age-appropriate or differ from what their peers without disabilities use in the classroom (Bouck et al., 2012).

As an alternative, researchers explored virtual manipulatives, which are online or app-based manipulatives, in place of concrete manipulatives (Bouck et al., 2014; Bouck, Chamberlain, & Park, 2017; Root et al., 2017). Satsangi and Bouck (2015) suggested virtual manipulatives are more socially acceptable, more age-appropriate, and less stigmatizing compared to concrete manipulatives for secondary students. Satsangi & Miller (2017) emphasized virtual manipulatives can be readily available through computers or portable electronic devices for teachers and students to choose from. This also provides more autonomy and self-determination for students and allows teachers to accommodate their students' needs.

Comparative studies of concrete and virtual manipulatives found both manipulatives to be effective (Bouck et al., 2014; Bouck, Chamberlain, et al., 2017; Root et al., 2017; Satsangi, Bouck, Taber-Doughty, Bofferding, & Roberts, 2016). Bouck, Chamberlain, et al. (2017) evaluated the effectiveness of concrete and virtual manipulatives in teaching basic operation (i.e., subtraction) using base-10 blocks. The researchers found both manipulatives were effective but two of the three participants (i.e., middle school students with mild intellectual disability or learning disabilities) preferred virtual manipulatives. Also, Bouck et al. (2014) found both types of manipulatives are effective in teaching subtraction with regrouping for three students with autism spectrum disorder (ASD). With the virtual manipulatives, students with ASD achieved independence sooner than with the concrete manipulatives and achieved slightly higher accuracy on the problems. Root et al. (2017) also found both types of manipulatives are effective with schema-based instruction in teaching problem-solving to three students with developmental disabilities. The authors found the students with developmental disabilities solved the problems more independently with virtual manipulatives than with concrete manipulatives and they preferred to use virtual manipulatives. Finally, Satsangi et al. (2016) found both virtual and concrete manipulatives were effective in teaching algebra to three secondary students with learning disabilities. They discussed that using virtual manipulatives provided more autonomy when solving problems as the students needed less prompting while using virtual manipulatives as opposed to using concrete manipulatives.

Manipulative-Based Instructional Sequences for Mathematics

When manipulatives are used to teach mathematics to students with disabilities who need significant instructions, they are generally used in a sequential learning process (Bouck & Park, 2018). Bouck and Park (2018) reviewed studies involving manipulatives and identified that 29 out of 36 studies employed concrete manipulatives in the concrete-representational-abstract (CRA) instructional sequence.

Concrete-representational-abstract

Concrete manipulatives for students with disabilities are commonly used as a part of the concrete-representational-abstract (CRA) instructional sequence (Bouck & Park, 2018). The

CRA is an instructional sequence that gradually moves from the use of concrete manipulatives that represent abstract concepts (i.e., numbers) toward the use of numerical strategies (Agrawal & Morin, 2016). The CRA instructional sequence consists of a total of three phases. In the concrete phase, students use concrete manipulatives to solve mathematical problems (e.g., basic operations, algebra, fractions). In the representational phase, students use pictorial representations or drawings to solve problems, for example, drawing a line for ten and a dot for one (e.g., 23 equals to two lines and three dots). Finally, in the abstract phase, students use numerical strategies to solve the problems (Agrawal & Morin, 2016). During each phase, explicit instruction is provided (i.e., modeling, guided instruction, and independent practice; Doabler & Fien, 2013).

The basic idea of the CRA instructional sequence is fading concreteness, which was originally proposed by Bruner (1966). He suggested a three-part progressive sequence for acquiring abstract mathematical concepts—the use of physical or concrete objects, the use of pictorial or graphic representations (i.e., iconic form), and an abstract model of the concept (i.e., a symbolic form); all three are crucial in learning a new concept. For example, to teach students the concept of "three," it could be represented first by any three physical objects (e.g., beans, beads, marbles, blocks), then a drawing of three dots, finally the Arabic numeral 3 (Fyfe, McNeil, Son, & Goldstone, 2014). This progression allows students to move from concreteness to abstractness for conceptual understanding (Fyfe et al., 2014).

The CRA instructional sequence is an EBP for students with learning disabilities (Bouck, Satsangi, & Park, 2018). Bouck et al. (2018) conducted a systematic review of literature assessing the use of the CRA instructional sequence in teaching mathematics for students with learning disabilities. The review included 20 studies that used the CRA instructional sequence between 1975 and 2015. They found the CRA instructional sequence is an EBP for teaching mathematics, especially basic operations with regrouping.

The CRA instructional sequence has also been found to be effective for students with developmental disabilities. However, while extensive research exists on teaching mathematics using the CRA instructional sequence to students with learning disabilities, fewer studies have explored its use among students with developmental disabilities. Since 1998, five studies have used the CRA instructional sequence to teach mathematical concepts to students with developmental disabilities. Morin and Miller (1998) used the CRA instructional sequence to teach multiplication and problem-solving for secondary school students with mild or moderate intellectual disability. In the single case study, the authors found a functional relation between the CRA instructional sequence and the accuracy of solving mathematical problems. Flores, Storizer, Hinton, and Terry (2014b) utilized the CRA instructional sequence to explore its effectiveness in teaching addition and subtraction to students with developmental disabilities using a group design. The group who received the CRA instructional sequence showed greater improvement than the control group. Stroizer, Hinton, Flores, and Terry (2015) also used the CRA instructional sequence to teach basic operations including addition, subtraction with regrouping, and multiplication via a single-case study. The authors found the CRA instructional sequence was effective in teaching the targeted skills by observing the improvement between baseline and intervention. Bouck, Park, and Nickell (2017) explored the use of CRA instructional sequence in teaching functional mathematics (i.e., making change money with coins). They found the CRA instructional sequence was effective in teaching money-related mathematics for secondary school students with intellectual disability. Finally, Yakubova, Hughes, and Shinaberry (2016) used the CRA instructional sequence in conjunction with video modeling to teach addition, subtraction, and number comparisons to four students with ASD. The authors found all participants showed immediate improvement in acquisition of the skills as well as they maintained after three weeks of the intervention using a single case design.

Although the use of CRA instructional sequence is effective for students with disabilities (Bouck, Park, et al., 2017; Flores et al., 2014a; Flores et al., 2014b; Maccini & Hughes, 2000; Strickland & Maccini, 2012; Stroizer, Hinton, Flores, & Terry, 2015; Yakubova, et al., 2016), as noted earlier, concrete manipulatives could be stigmatizing for older students with disabilities (Bouck et al., 2012; Satsangi & Bouck, 2015). While emerging, researchers have attempted to replace concrete manipulatives with virtual manipulatives within the CRA instructional sequence (e.g., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, et al., 2018).

Virtual-representational-abstract

In the virtual-representational-abstract (VRA) instructional sequence, concrete manipulatives are replaced by virtual ones during the first phase of the instructional sequence (Bouck, Bassette, et al., 2017). Although the VRA is an emerging instructional sequence with limited research, two published studies have examined the effectiveness of the VRA instructional sequence (i.e., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, Bassette, & Whorley, 2018). Bouck et al. (2018) explored the effectiveness of the VRA instructional sequence in teaching basic operations (i.e., place value, addition, subtraction, and multiplication) for two secondary students with intellectual disability. Bouck et al. (2018) used app-based manipulatives such as base 10 blocks for place value, addition, and subtraction, and color tiles for multiplication. They found a functional relation between the use of the VRA instructional sequence and the accuracy of solving the targeted mathematical problems. Bouck, Bassette, et al. (2017) also explored the effectiveness of the VRA instructional sequence in teaching equivalent fractions for secondary students with disabilities. In Bouck, Bassette, et al. (2017), the participants used fraction tiles in the virtual phase, drew fraction tiles in the representational phase, and used only numbers in the abstract phase. Overall, the participants achieved higher accuracy during intervention compared to their baseline and the authors concluded the VRA instructional sequence is effective in teaching fractions. Both studies using the VRA instructional sequence included a maintenance phase consisting of two sessions, but most of the students struggled to maintain the skills two weeks after the intervention ended. These results suggest that interventions should be designed to target learning beyond acquisition of skills.

Learning Stages for Students with Disabilities

For students with disabilities, learning is generally seen as involving four stages—acquisition (i.e., acquiring skills), fluency (i.e., using precisely the skills learned in a short time), maintenance (i.e., ability to continue using the learned skills without learning again), and generalization (i.e., being able to apply learned skills with other forms of stimuli; [Alberto & Troutman, 2009; Shurr, Jimenez, & Bouck, 2019]). However, the existing research on teaching mathematics to students with disabilities tends to focus on acquisition of mathematical skills (Lafay, Osana, & Valat, 2019; Spooner et al., 2018). Historically, educators and researchers have focused less on maintenance of acquired skills, as fluency and maintenance will not be feasible without skill acquisition (Dekeyser, 2009; Wachsmuth, 1983). Due to the emphasis placed on skill acquisition, researchers have identified effective interventions to support acquisition of mathematical skills and knowledge for students with disabilities (e.g., explicit instruction, CRA, VRA; [Browder et al., 2008; Bouck, Bassette, et al., 2017; Doabler & Fien, 2013; Spooner et al., 2018; Satsangi, Hammer, & Hogan, 2018]), yet limited attention has been paid to instructional methods and materials that support maintenance (Lafay et al., 2019).

Maintenance

Maintaining mathematical concepts is crucial for learning further mathematics as well as independent living, since each domain of mathematics does not stand alone (Powell, Fuchs, & Fuchs, 2013). Learning in-depth mathematics occurs by connecting concepts across mathematical domains (NCTM, 2018). For example, in elementary school level, students learn basic operations, and in middle school level, students are expected to use basic operations while solving linear algebra. Also, maintaining mathematical skills allows students to apply acquired skills later in everyday life (Spooner, Saunders, Root, & Brosh, 2017; Szekely, 2014). Although existing literature tends to focus on skill acquisition rather than maintenance, researchers have recommended strategies—distributed practice, fading support, and overlearning—for improving maintenance in general (Alberto & Troutman, 2009; Kim, Ritter, & Koubek, 2013; Shurr et al, 2019).

Distributed practice. In distributed practice, trials are spread across lessons and across days (Kim et al., 2013; Shurr et al., 2019). The optimal spacing of lessons is individualized for learners (Kim et al., 2013). Hence, instruction is planned to provide a quick reminder of concept knowledge efficiently to move into skill maintenance (Shurr et al., 2019). Specifically, for mathematics, Rohrer and Taylor (2006) conducted a comparison study on whether distributed practice is more effective for skill maintenance than massed practice in teaching college mathematics. The authors found distributed practice was more effective for students to maintain mathematics knowledge when comparing retention knowledge after 4 weeks.

Fading support. Fading support typically involves gradual fading of instruction provided by the teacher, which is considered appropriate for supporting maintenance of skills (Shurr et al., 2019). Although research on fading support in mathematics instruction is limited, many instructional methods and EBPs are used in conjunction with fading support for students with disabilities (Banda, Dogoe, & Matuszny, 2011; Burton, Anderson, Prater, & Dycher, 2013). One of the EBPs that fading support is combined with is video-based instruction, which is an EBP for both students with ASD and intellectual disability (Park, Bouck, & Duenas, 2019; Wong et al., 2015). Burton et al. (2013) used video self-modeling to teach money-related mathematical skill (i.e., purchasing) to students with intellectual disability and/or autism using single case design. The authors used fading support to promote students' skill maintenance and found that it is effective in maintaining the acquired skill. Wu, Cannella-Malone, Wheaton, and Tullis (2016) also used video prompting with fading support to teach daily living skills for individuals with developmental disabilities. The authors explored two fading supports, which are fading within intervention and fading after acquisition. The participants maintained the skills at a higher accuracy after receiving both fading supports.

The idea of fading emerged because other instructional methods which include teacher's modeling and guiding, such as explicit instruction and step-by-step procedures, may prohibit skill maintenance (Sigafoos et al., 2007). In such methods, every session of an intervention involves some kind of instruction or prompting, and students are not provided the opportunity to solve problems without supports. Hence, they may not maintain the skill when no instruction or prompting is provided. For example, explicit instruction within the CRA and the VRA instructional sequences is essential to gradually move towards thinking about concepts abstractly (Agrawal & Morin, 2016; Fyfe et al., 2014). However, if explicit instruction is provided every time, it is difficult to expect the student to maintain the skill without explicit instruction. Hence, gradual fading support along with explicit instruction is necessary for students to maintain acquired skills.

Overlearning. In overlearning, learned skills are practiced repeatedly for an extended period of time until students have fluency (Alberto & Troutman, 2009). Overlearning promotes long-term learning in mathematics and maintenance of knowledge (Kim et al., 2013; Wang & Beck, 2012). Although a potential drawback of overlearning is that it needs additional time (Rohrer, 2009), it could be effective for students with disabilities who require a lot of time and effort to transfer their knowledge from working memory into long term memory (Steele, Minshew, Luna, & Sweeney, 2007).

In instructional sequences such as the CRA and the VRA, researchers recommend repeating the same lesson when students have failed to meet the mastery criterion (e.g., Bouck et al., 2018). Therefore, it could be said that overlearning has already been incorporated within instructional sequences to achieve the mastery criterion, which is usually 80% or higher in three sessions. However, this mastery criterion is limited to the acquisition of skills. Conducting additional sessions even after the students achieve the mastery criterion for acquisition could be one way of maintaining the skills (Baldwin & Ford, 1998). Increasing the number of sessions can be interpreted as providing more opportunities to become fluent, which is important for maintenance in terms of automaticity and mastery (Hagman & Rose, 1983; Wang & Beck, 2012). In other words, to achieve long-term maintenance, increasing the mastery criterion allows additional sessions of learning even after the acquisition of skills (i.e., in three consecutive sessions; [Wang & Beck, 2012]).

Purpose of the Study

For students with disabilities who need significant instruction in learning mathematics, acquiring mathematical skills and maintaining those skills is important for further mathematics education and independent daily living (Spooner et al., 2017; Szekely, 2014). As shown in previous literature, the CRA instructional sequence is effective in teaching mathematics to students with disabilities (Flores et al., 2014a, 2014b; Stroizer et al., 2015; Satsangi et al., 2016; Yakubova et al., 2016). However, the use of concrete manipulatives in the CRA instructional sequence may be stigmatizing to secondary students with disabilities (Bouck et al., 2012; Satsangi & Bouck, 2015). Research involving the VRA instructional sequence, which uses virtual manipulatives instead of concrete manipulatives, is emerging. In addition to the use of the VRA instructional sequence, fading support and overlearning can be used to improve maintenance of mathematical skills among students with disabilities (Shurr et al., 2019).

Study 1 – Systematic review of the literature

For study 1, a systematic review of the literature was conducted on research which involved teaching mathematics to students with developmental disabilities and target maintenance. As previously noted, maintenance in mathematics is important for students with developmental disabilities (Alberto & Troutman, 2009; Shurr et al., 2019). The purpose of the systematic review of literature was to explore the extent to which research studies from 1975 to 2018 included a maintenance phase or targeted maintenance. The researcher coded studies based on whether maintenance occurred, number of maintenance sessions or tests, latency between end of intervention and maintenance, mathematical contents taught, and specific instructional methods and materials used to maintain the acquired mathematical skills. Results of these previous studies were analyzed and discussed to inform the current state of the literature on maintenance in mathematics for students with developmental disabilities.

Study 2 – VRA and fading support intervention package

Although emerging, previous researchers explored the effectiveness of the VRA instructional sequence in teaching mathematics for secondary students with disabilities (Bouck, Bassette, 2017; Bouck, Park, 2018). Yet, all participants in these previous studies failed to maintain the acquired skills. The purpose of this study was to examine the effect of the VRA instructional sequence with fading support on acquisition and maintenance of mathematical skill (i.e., subtraction with regrouping) among students with developmental disabilities. The study involved a multiple probe across participants single case design to examine if a functional relation exists between the intervention package (i.e., the VRA instructional sequence with fading support) and students acquisition as well as maintenance of the basic operations skill (i.e., subtraction with regrouping)

Study 3 – VRA and overlearning intervention package

Although previous researchers explored the effectiveness of overlearning in mathematics, no researchers examined overlearning in mathematics for students with disabilities. The purpose of study 3 was to explore the effectiveness of an intervention package, which included the VRA instructional sequence and overlearning, in teaching mathematical skills to students with disabilities. The researcher used a single case multiple probe across participants design to examine the effectiveness of the intervention package. This study provided information on whether the VRA instructional sequence with overlearning is effective in acquiring mathematical skills and/or in maintaining them.

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CHAPTER 2

A SYSTEMATIC REVIEW OF LITERATURE

Mathematics is a core subject that affects life for all students after graduation, including aspects of employment, financial skills, purchasing, and cooking (Edwards, Rule, & Boody, 2017; Jordan et al., 2013). Despite the importance of mathematics, a significant number of students find mathematics difficult and, as the grade level increases, more students struggle (Shanley, 2016; The Nation's Report Card, 2016). Although this is true for all students, students with developmental disabilities tend to experience significantly greater challenges with mathematics (Kearns, Towles-Reeves, Kleinert, Kleinert, & Thomas, 2011). Kearns et al. (2011) found less than 50% of students with developmental disabilities were able to solve problems that required basic computational skills with or without a calculator. In addition, only 4% to 8% of the 12,649 students across seven states in their study could use computational procedures for everyday life skills (e.g., purchasing, cooking).

Students with Developmental Disabilities in Mathematics

Mathematics instruction for students with developmental disabilities (i.e., intellectual disability and/or autism) tends to focus on teaching numbers and operations (i.e., addition, subtraction, multiplication, division), geometry, and algebra (Hudson, Rivera, & Grady, 2018). Different researchers conducted systematic reviews of the literature to identify instructional method and materials for teaching mathematics to students with developmental disabilities (Browder, Spooner, Ahlgrim-Delzelll, Harris, & Wakeman, 2008; Butler, Miller, Lee, & Pierce, 2011; Hart Barnett & Cleary, 2015; Hudson et al., 2018; King, Lemons, & Davidson, 2016; Spooner, Root, Saunders, & Browder, 2018). These methods include: systematic instruction (e.g., time delay, system of least prompts, most-to-least prompt, prompting/fading), direct instruction (i.e., teacher oral presentation in a group format), explicit instruction (i.e., sequential supports divided into steps; modeling, guiding, testing), graphic organizers (i.e., diagrams), and visual representations (e.g., manipulatives, pictures, number lines).

The existing reviews of the literature for mathematics education and students with developmental disabilities tend to highlight students' ability to acquire mathematical skills (Browder et al., 2008; Butler et al., 2011; Hart Barnett & Cleary, 2015; Hudson et al., 2018; King et al., 2016; Spooner et al., 2018), but learning is not limited to acquisition (Shurr, Jimenez, & Bouck, 2019). Learning for students with disabilities, including developmental disabilities, occurs in four stages: acquisition (i.e., acquiring targeted skills), fluency (i.e., performing skills with speed and accuracy), maintenance (i.e., performing skills without re-teaching), and generalization (i.e., applying skills to other forms of stimuli; Alberto & Troutman, 2009; Burns, Codding, Boice, & Lukito, 2010; Shurr et al., 2019). Although each of these stages are important in learning a skill, few researchers tend to focus on skill maintenance for students with developmental disabilities in general (e.g., Spooner, Knight, Browder, & Smith, 2012). In a review of EBPs for teaching core content (i.e., literacy, mathematics, and science) to students with developmental disabilities, Spooner et al. (2012) found only eight of 18 studies included a maintenance phase. Despite receiving less attention, maintenance is crucial in using acquired skills independently for further education and applying skills to daily living (i.e., generalization), which is the ultimate goal for students with developmental disabilities (Kellems, Rickard, Okray, Sauer-Sagiv, & Washburn, 2017).

Maintenance in Mathematics

Maintenance is especially important in mathematics because it has a hierarchical and sequential structure in which content builds on previous content (Geary, Nicholas, Li, & Sun, 2017; Powell, Fuchs, & Fuchs, 2013). For example, mathematical content taught in higher grades (e.g., algebra) requires mathematical content taught in earlier grades (e.g., addition, subtraction, multiplication, division; [NCTM, 2018]). In other words, advanced mathematics often deepens the mathematics that students have learned in previous schooling (Powell et al., 2013). The nature of mathematics makes student maintenance of earlier mathematical knowledge and skills critical (Clements, Fuson, & Sarama, 2017; Geary et al., 2017).

For students with developmental disabilities, maintenance in mathematics is also important for their independent living, which includes skills such as purchasing, cooking, budgeting, and time management (Burton, Anderson, Prater, & Dyches, 2013; Patton, Cronin, Bassett, & Koppel, 1997; Spooner, Saunders, Root, & Brosh, 2017). These skills require basic, and sometimes advanced, mathematical abilities, and students need to perform them without reteaching (Spooner et al., 2017). It is important students maintain the mathematical skills that they acquired so they can generalize the skills and apply them in their daily lives for an extended amount of time even after instruction has ended. If students with developmental disabilities have acquired mathematical skills but do not maintain them, they will not be independent in performing everyday life skills which may require reteaching (Kellems, Cacciatore, & Osborne, 2019; Kellems et al., 2016; Xin, Grasso, Dipipi-Hoy, & Jitendra, 2005).

Despite the importance of maintenance in mathematics (Burton et al., 2013; Xin et al., 2005), there has been little attention paid to maintenance for students with developmental disabilities (Spooner et al., 2012). Therefore, the purpose of this review is to identify research studies that examine maintenance of mathematical skills for students with developmental disabilities. This review will identify intervention methods and materials that promote maintenance for mathematics. The following research questions are used to guide this review: (a)

How frequently do studies involving mathematics interventions for students with developmental disabilities include a maintenance phase?; (b) Of the studies involving maintenance, how was the maintenance phase implemented?; (c) Of the studies involving maintenance, what were the participant characteristics and the study designs?; (d) Of the studies involving maintenance, what mathematical content was taught to students?; and (e) What instructional methods and materials were used for maintenance of mathematics for students with developmental disabilities?

Method

Inclusion criteria

To be included, studies had to meet pre-determined criteria. First, the studies evaluated the effectiveness of an intervention for teaching mathematics. Second, the studies used either a single-case experimental design or a group design. Third, participants were diagnosed as having developmental disabilities, including autism or intellectual disability. Last, all studies were published in English in a peer-reviewed journal between the years of 1975 and 2018.

Literature search procedures and screening

The researcher employed a comprehensive approach to identify all the studies that met the inclusion criteria. First, electronic databases (i.e., ProQuest, ERIC, and PsycINFO) were used to search for the literature. For the initial search, two categorized terms were used: mathematics and a term that reflects disabilities. Within the disability category, specific terms were used including intellectual disab*, mental impairment, mental retardation, cognitive impairment, autis*, developmental disabilit*. During each search, the term from the first category (i.e. mathematics) was combined with the terms from the other category (i.e., disabilities) until all permutations were exhausted. All permutations included math* AND intellectual disa* OR mental impairment OR mental retardation OR cognitive impairment OR autis* OR developmental disabilit*. A total of 3,277 studies were located via the keyword search. A screening was conducted by reading the title, abstract, and, if needed, the method section to eliminate articles that did not include any of the following categories: mathematics, disability categories (i.e., intellectual disability, autism, developmental disabilities), and the effectiveness of intervention. If an article was not an experimental design study (i.e., single-case experimental design, group design), the article was excluded (e.g., a review of literature). Also, studies that did not include at least one student with developmental disabilities were eliminated. Finally, studies focused on other content areas (e.g., literacy, science) in addition to mathematics were excluded.

A manual search of relevant articles was conducted in select journals published between 1975 and 2018: *Journal of Autism and Developmental Disorders, Focus on Autism and Other Developmental Disabilities, Research in Developmental Disabilities, Educational and Training in Autism and Developmental Disabilities,* and *Research and Practice for Persons with Severe Disabilities*. During the manual search, the researcher searched for the same terms as the keyword search and determined whether each article met the inclusion criteria. The researcher found a total of 108 studies met the inclusion criteria from both the database and manual searches. Once inclusion criteria were applied to the articles retrieved from the electronic database search and manual search, an ancestral search was used on the 108 studies to locate studies cited in each of the articles that met the inclusion criteria of this review. During the ancestral search, 20 articles met the inclusion criteria, bringing the total to 128 articles.

An independent observer, who was a doctoral student in special education, served as a second reviewer for 23% of the 3,277 studies collected after the electronic database search to provide reliability to the screening process before determining inclusion. For the articles collected through the keyword search, the second reviewer reviewed 763 articles, and interrater reliability (IRR) was 99.7%. For the articles collected through the manual search, the second reviewer reviewed 20% of the relevant journals in the field, and IRR was 100%. For the ancestral search, the second reviewer reviewed the references list of 25% of the 108 studies which met the inclusion criteria from both the electronic and manual searches, and IRR was 100%. The first author and the second rater discussed discrepancy and used consensus data for the final analysis.

Coding of studies

Each individual study was coded relative to the research questions based on the following categories: participant characteristics, intervention characteristics, methodological design characteristics, mathematical domains, and maintenance characteristics.

Participant characteristics. Participant characteristics involved disability categories (i.e., autism, intellectual disability, autism with intellectual disability, developmental disabilities) and age (e.g., elementary school, middle school, high school, post-secondary school). Also, IQ score was coded. When information was not mentioned in the article, it was coded as unknown.

Methodological design. Methodological design characteristics were coded for whether each study used single-case experimental design or group design (i.e., quasi-experimental design, pre- and post- design) to examine the effectiveness of the intervention.

Mathematical contents. Academic mathematical content was coded based on the National Council of Teachers of Mathematics (NCTM, 2018): numbers and operation (i.e., place value, addition, subtraction, multiplication, division), measurement (i.e., time, money), data analysis and probability (i.e., collect, organize, display relevant data), geometry (i.e., two- or three-dimensional shape, length, width, area, volume), and algebra (e.g., solve for x; $2x + 1 = 5$).

Intervention. Based on a recent review of mathematical literature for students with developmental disabilities (i.e., Spooner et al., 2018), two intervention categories were included in this review: instructional method used and instructional materials used. Some examples of instructional methods were prompting, fading, instructional sequence, time-delay, explicit instruction, and video-based instruction. Additionally, instructional materials were iPad, graphic organizer, task analysis, flash cards, number lines, touch points, calculator and manipulatives. As the studies used a variety of interventions, the methods and materials were not coded into binary variables. Rather, all the intervention methods and materials used have been listed in a table.

Maintenance. Maintenance was defined as phases or sessions where researchers examined participants' acquisition of mathematical skills after all the instruction or intervention was over (Alberto & Troutman, 2009). Maintenance was coded for inclusion of a maintenance phase or post-intervention (i.e., yes or no), latency between last instruction or end of intervention and beginning of maintenance (i.e., number of weeks or days), length of maintenance, number of maintenance sessions or post-intervention tests, and accuracy level of maintenance.

IRR for coding. The second reviewer randomly selected 25% of 128 articles to code for IRR. To calculate the percentage of agreement, following formula was used: $\frac{agreements}{agreements+disagreements} \times 100$ for the coding of each categories for each individual study. The IRR of the coding procedures was 97%. In case of disagreement, the first and second reviewers discussed and used consensus for coding.

Results

Maintenance

A total of 128 studies met the inclusion criteria after the keyword, manual, and ancestral searches. Of those studies, only 46 studies included a maintenance phase (35.9%). Studies that did not include a maintenance phase were eliminated from further analysis. A total of 312 students participated across the 46 studies. Ages varied from 47 months to 19 years old and IQ from 36 to 107 (see Table 1). Of these 46 studies, 39 used a single-case experimental design and 7 used a pre- and post-test group design.

Among the studies that used a single-case experimental design, researchers conducted a range of one to 10 maintenance sessions (μ = 3.1 sessions). In these studies, the time period between the last intervention session and the first maintenance session ranged from 1 day to 22 weeks. Among the studies that used a pre- and post-test design (n=8), maintenance was evaluated by conducting one or more follow up tests after the post-test (see Table 1). These maintenance phases were conducted between two weeks and five months after instruction. Of the 46 studies, eight did not specify the duration of time between the end of the last intervention and the beginning of maintenance (i.e., Bouck, Satsangi, Taber-Doughty, & Courtney, 2014; Everhart, Alber-Morgan, & Park, 2011; Jimenez, Courtade, & Browder, 2008; Smeets, & Oliva, 1987; Rao & Kane, 2009; Root, Browder, & Saunders, 2017; Sheriff & Boon, 2014; Yakubova & Bouck, 2014). Finally, 28 out of the 46 studies indicated the average maintenance level was 80% or higher, which is the criterion for maintenance mastery (see Shurr, Jimenez & Bouck, 2019). Students in fourteen studies received less than 80% in the maintenance phase and an additional four studies did not indicate accuracy of maintenance.

Mathematical content

Of the 46 studies that included a maintenance phase, 44 studies contained one academic mathematical content domain (e.g., algebra or geometry), whereas two studies focused on more than one mathematics domain (i.e., Browder, Jimenez, Trela, 2012; Jimenez & Staples, 2015). The most-taught content was numbers and operation (71.4%), followed by measurement (18.4%), algebra (4.1%), geometry (4.1%), and data analysis/probability (2%; see Table 2).

Among the 28 studies that met the mastery criterion for maintenance (i.e., 80% or higher average maintenance level), 20 studies involved teaching numbers and operations (i.e., four studies involved teaching numerals, 10 studies involved teaching addition and subtraction, three studies involved teaching multiplication and division, two studies involved teaching operations, and one study involved teaching fractions). In addition, five studies involved teaching measurement, one study involved teaching geometry, and two studies involved teaching algebra.

Interventions

All included studies involved intervention packages consisting of multiple instructional methods and/or instructional materials (see Table 2). The most frequently used instructional method was prompting (i.e., simultaneous, system of least prompting; n=20), followed by explicit instruction (n=10), feedback (n=10), time delay (n=6), fading (n=6), and instructional sequence (i.e., concrete-representational-abstract, virtual-representational-abstract; n=6), technology-based instruction (n=6), video-based instruction (n=4), modeling (n=5), touch-math (n=3), cognitive strategy instruction (n=3), self-instruction (n=2), mnemonic strategy (n=2), next dollar strategy (n=2), fluency instruction (n=1), and error correction (n=1). With regard to the instructional materials, flash cards (n=12), manipulatives (n=10), task analysis (n=10), graphic organizers (n=8), calculator (n=6), mnemonics charts (n=1), number lines (n=1), and touch point (n=1) were used. In addition, researchers in three studies (i.e., Jimenez & Staples, 2015; Tzanakaki et al., 2014; Whitby, 2012) used curriculum (i.e., scripted lessons) to teach numeracy skills and operations. In addition to reviewing the interventions used to teach mathematics contents in general, it is important to understand which mathematical contents are supported by which instructional methods and materials. Hence, the instructional methods and materials used to teach different mathematical contents are discussed below:

Number identification. Researchers in five studies taught number identification to students with developmental disabilities (Akmanoglu & Batu, 2014; Everhart et al., 2011; Jowett, Moore, & Anderson, 2012; Skibo, Mims, & Spooner, 2011; Tzanakaki et al., 2014). Researchers in the five studies all used prompting as the intervention method and flash cards as the intervention material. Students in three of the five studies in number identification maintained the skills they learned.

Addition and subtraction. Thirteen studies taught addition and/or subtraction to students with developmental disabilities (Bouck et al., 2014; Braten & Throndsen, 1998; Calik & Kargin, 2010; Chung & Tam, 2005; Flores, Hinton, Strozier, & Terry, 2014; Hasselbring, 1988; Horton, Lovitt, & White, 1992; Poncy, Skinner, & Jaspers, 2007; Rao & Kane, 2009; Rockwell, Griffin, & Jones, 2011; Smeets, Lancioni, & Striefel, 1987; Whalen, Schuster, Hemmeter, 1996; Yikmis, 2016). The instructional methods used to teach addition and subtraction were explicit instruction, simultaneous prompting, modeling, immediate or delayed feedback on incorrect responses, schema-based instruction, mnemonic strategy, and touch math. With regard to the instructional materials, manipulatives (i.e., concrete and virtual), task analysis, graphic organizers, flash cards, and touch points were used. Despite the varied instructional methods and materials used in these studies, all the students in the studies maintained the skills they learned. Of the 13 studies, four (i.e., Bouck et al., 2014; Horton et al., 1992; Poncy et al., 2007; Yikmis, 2016) allowed students to use intervention materials during the maintenance phase.

Multiplication and division. Researchers in six studies taught multiplication and division to students with developmental disabilities (Bouck, Bassette, Taber-Doughty, Flanagan, & Szwed, 2009; Irish, 2002; McCallum & Schmitt, 2011; Rao & Mallow, 2009; Singer-Dudek & Greer, 2005; Zisimopoulos, 2010). Of these, only one study mentioned the complexity of the multiplication and division problems that they were teaching (i.e., Singer-Dudek & Greer, 2005; double-digit multiplication). Different instructional methods (i.e., technology-based instruction, modeling, prompting, fading, time delay, mnemonic strategy, fluency instruction, and corrective feedback) were used to teach multiplication and division. Only two studies (i.e., Rao & Mallow, 2009; Zisimopoulos, 2010) used an instructional material, both were flash cards (see Table 2). Students in three of the six studies maintained the skills.

Multiple operations. Researchers in seven studies taught problems that required a combination of at least three operations (e.g., addition, subtraction, multiplication) to students with developmental disabilities (Bouck, Park, Shurr, Bassette, & Whorley, 2018; Lancioni, Smeets, & Oliva, 1987; Root et al., 2017; Sheriff & Boon, 2014; Whitby, 2012; Yakubova & Bouck, 2014; Yakubova, Hughes, & Shinaberry, 2016). These studies involved explicit instruction, prompting, error correction, instructional sequence, technology-based instruction, schema-based instruction, and video modeling as instructional methods, and manipulatives (i.e., virtual and concrete), graphic organizers, calculators, task analysis, and flash cards as instructional materials. Of these seven studies, three studies demonstrated that students' maintenance level was above 80%.

Fractions. Researchers in three studies taught fractions to students with developmental disabilities (Bouck, Bassette, et al., 2017; Bouck, Park, Sprick, et al., 2017; Yakubova, Hughes, & Hornberger, 2015). Researchers used sequential learning with virtual manipulatives, explicit instruction, and video-based instruction. With regard to the instructional materials, two of the three studies used virtual manipulatives. Of these studies, only one study demonstrated student maintenance levels above 80% (Yakubova et al., 2015).

Measurement. Researchers in eight of the studies taught measurement, and all were related to money (Bouck, Park, & Nickell, 2017; Burton et al., 2013; Cihak & Grim, 2008; Colyer & Collins, 1996; Denny & Test, 1995; Frederick-Dugan, Varn, & Test, 1991; Trace, Cuvo, & Criswell, 1977; Waters & Boon, 2011). Researchers in these eight studies used instructional methods such as concrete-representational-abstract (CRA) instructional sequence with explicit instruction, video self-modeling, next dollar strategy, and touch math. With regard to the instructional materials, manipulatives, task analysis, calculators, and visual supports were used. Participants in six of these studies maintained skills.

Geometry. One study focused on geometry (i.e., solving the Pythagorean theorem in 32 steps) to students with developmental disabilities (Creech-Galloway, Collins, Knight, & Bausch, 2013). The intervention package involved simultaneous prompting (i.e., verbal, modeling, physical prompts), calculators, videos, and task analysis. Three out of four students in this study maintained the skill.

Algebra. Researchers in one study taught algebra to students with developmental disabilities (Jimenez et al., 2008). The researchers used systematic prompting with fading, concrete representations, and task analysis. Students' performance was measured based on nine steps of task analysis, and two out of three students maintained the skill.

Multiple contents. Researchers in two studies taught multiple mathematics content (Browder et al., 2012; Jimenez & Staples, 2015). Both studies used math stories (i.e., word problems) and graphic organizers. Browder et al. (2012) taught geometry, measurement, algebra, and data analysis. The researchers used the least intrusive prompting necessary (i.e., indirect verbal prompting, direct verbal prompting, modeling, physical guidance). Students in this study only had an average maintenance level above 80% for algebra. Jimenez and Staples (2015) taught early numeracy skills in algebra and geometry. They used least-to-most prompting, constant time delay, and task analysis. Students in this study maintained their numeracy skills at least two weeks after the intervention ended.

Discussion

This systematic review of literature examined the characteristics of experimental research that included a maintenance phase when teaching mathematics to individuals with developmental disabilities. Of the studies that met the inclusion criteria, only 35.9% included a maintenance phase. Among these studies, the number of maintenance sessions as well as the duration between the last intervention session and beginning of the maintenance phase varied. Of the studies that included a maintenance phase, addition and subtraction were the most frequently taught mathematical content. All the studies with a maintenance phase used intervention packages that included more than one instructional method. The most widely used instructional method was prompting followed by explicit instruction, feedback, time delay, and instructional sequence, and the most widely used instructional material was flash cards followed by manipulatives, graphic organizers, and task analysis.

Inclusion of maintenance

In this review, only 46 out of 128 studies included a maintenance phase, suggesting researchers tend to focus more on acquisition of mathematical skills rather than maintenance. This is consistent with the findings from a previous review of literature, which suggested researchers did not pay much attention to the maintenance phase in teaching core content to students with developmental disabilities (Spooner et al., 2012). A hypothesis for the lack of inclusion of a maintenance phase in many studies may be the lack of quality indicators that focus on maintenance for both single-case and group design (see Cook et al., 2014; Kratochwill et al., 2013). Researchers use quality indicators to determine in designing a study to ensure it is methodologically sound (Cook et al., 2014). If a maintenance phase is not necessary for a study to be determined methodologically sound, researchers may focus less on it.

Across the studies, a consistent definition of maintenance was missing. For example, the Bouck, Chamberlain, and Park (2017) study was not included in the current review as they used the term ‘generalization’ instead of the term ‘maintenance’ while measuring the extent to which students maintained the skills after the intervention ended. The authors used this term because the numbers in the problems were different from those in the intervention. However, other studies that used different numbers during and after the intervention referred to the post-intervention phases as the maintenance phase. Future researchers may first need to consider what the application of maintenance in mathematics is and under what conditions it should be determined.

There are also no consistent criteria regarding the number of sessions to be included in the maintenance phase as well as the duration between the intervention and maintenance phases. According to Alberto and Troutman (2009), the definition of maintenance is “the ability to perform a response over time without reteaching” (p. 43). Studies in this review involved between one and 10 maintenance sessions. Studies also varied in length of the maintenance phase and many of them measured maintenance at only one point in time after the intervention, which does not capture the maintenance of skills over time. Defining the adequate number of sessions and the length of maintenance phase needed to accurately measure maintenance is important. Some studies in this literature review also do not mention the latency between intervention and maintenance phases at all and two studies conducted the maintenance phase less than a week after the intervention ended. Future researchers may need to consider defining the adequate latency between phases for accurately measuring maintenance rather than simple memory retrieval.

Mathematical content

According to findings from the current review, students maintained lower grade-level mathematical skills (e.g., addition without regrouping) with greater ease than more advanced skills such as multiplication and division. This finding is consistent with previous studies that suggested students find it easier to acquire lower grade-level skills such as number identification than more complex skills such as algebra (Shanley, 2016). Mathematical performance is associated with working memory, which is the mental space where one holds information while executing cognitive tasks such as solving mathematical problems (Ashcraft & Krause, 2007; Raghubar, Barnes, & Hecht, 2010). Working memory plays an important role in solving mathematical problems whenever the process involves more than simple retrieval (Ashcraft & Krause, 2007; Passolunghi & Costa, 2016). For example, working memory is more crucial when regrouping is necessary (e.g., 34 + 18) rather than without regrouping (e.g., 2 + 3). Hence, reliance on working memory increases when solving multi-step mathematical problems (Ashcraft & Krause, 2007; Ayres, 2001). As working memory mediates the relation between IQ and mathematical performance (Passolunghi, Mammarella, & Altoe, 2008), students with developmental disabilities tend to struggle with complex mathematical problems (e.g., English, Barnes, Taylor, & Landry, 2009).

A limited number of studies (26%) examined the effectiveness of interventions in teaching mathematics outside of numbers and operations. As higher grade-level mathematical content builds upon basic contents such as numbers and basic operations (Geary et al., 2017; Powell et al., 2013), researchers may have focused on the acquisition of basic mathematical skills for students with developmental disabilities. As maintenance of basic mathematical skills (e.g., addition, subtraction) impacts students' performance with higher grade-level mathematical contents (e.g., algebra; Powell et al., 2013), researchers need to focus more on maintenance while teaching basic mathematical skills and focus on teaching higher grade-level contents after the basic skills are maintained.

Instructional methods and materials

All of the studies involved an intervention package, which included more than one instructional method or combined instructional methods with instructional materials. The most widely used instructional method was prompting, which includes simultaneous prompting, system of least prompts, verbal prompts and visual prompts. However, the exact effect of prompting is unknown because many of these studies used prompting in conjunction with other methods and/or materials. When the same prompting instruction was used with different instructional materials and/or to teach different math contents, maintenance effects varied.

While the exact intervention effects of each instructional method cannot be determined, the current review suggests some intervention packages such as prompting with fading and explicit instruction with visual supports or manipulatives may be effective for skill maintenance. Prompting along with fading was used in six studies (i.e., Jimenez et al., 2008; Jowett et al., 2012; Smeets et al., 1987; Trace et al., 1977; Tzanakaki et al., 2014; Zisimopoulos, 2010). Of these six studies, students in five studies maintained the skills, even when teaching more complex mathematic concepts such as algebra (i.e., Jimenez et al., 2008). In addition, four studies involved explicit instruction as an instructional method and visual support and/or manipulatives as an instructional material(s) in teaching addition or subtraction (Braten &

Throndsen, 1998; Calik & Kargin, 2010; Flores et al., 2014; Rockwell, et al., 2011). All students in these studies not only acquired but also maintained the skills.

Implications for practice

From this systematic review, practitioners can glean implications regarding effective mathematics instructional methods and materials for students with developmental disabilities. Practitioners should consider using intervention packages, pulling from multiple methods and/or materials to support acquisition and maintenance (Spooner et al., 2018). From previous research, prompting with visual supports was effective in supporting maintenance of number identification while addition and subtraction skills were maintained when explicit instruction, prompting, visual supports and/or manipulatives were used. The current review suggests prompting and explicit instruction as well as visual supports and manipulatives could enhance content mastery and skill maintenance.

Another implication is that students with developmental disabilities can acquire not only numbers and operations skills, but also higher grade-level mathematical contents such as algebra. While there is limited research on strategies that support acquisition and maintenance of advanced mathematical skills for students with developmental disabilities, practitioners could employ existing methods and materials such as prompting with fading, visual supports and manipulatives to support acquisition and maintenance of higher grade-level skills (e.g., algebra; Brower et al., 2012; Jimenez et al., 2008)

Limitations & future directions

There are some limitations in interpreting the results of this review. First, as with other reviews, it is possible that some articles were missed during the search. During the search procedure to determine which articles met the inclusion criteria, the author did not consider peer-reviewed journal articles written in a language other than English, chapters from books, or dissertations. In addition, only five journals were selected for the hand search, so studies from other journals that would have met the inclusion criteria may have been missed. Results from these excluded research studies have not contributed to the findings of this review. Second, the author excluded studies in which targeted dependent variables included functional content within academic content instructions in mathematics along with other subjects, such as science or literacy (e.g., Collins, Hager, & Galloway, 2011; Karl, Collins, Hager, & Ault, 2013). The current review concentrated on studies that solely involved teaching academic mathematics. Third, the average maintenance levels calculated in this review may not be accurate since not all studies provided exact maintenance scores.

In terms of future research, as few studies evaluate interventions for teaching mathematical contents other than numbers and operations, information on effective instructional methods and materials for teaching higher grade-level mathematical contents is limited. This review suggests additional research on interventions for teaching more advanced mathematical contents, such as fractions and algebra, is needed for students with developmental disabilities. Findings from the current review showed that all studies used intervention packages. Given the effectiveness of intervention packages in promoting maintenance, future research should pay more attention to intervention packages and explore questions such as which instructional methods and/or materials should be in packages, which packages are effective for different mathematical skills, and which packages work across disability categories. In addition, as the use of intervention packages increases the difficulty of identifying individual effects of each instructional factor, future researchers need to seek ways to delineate the effects of various instructional factors in intervention packages. Also, more experimental studies should explore specific instructional methods and materials that support maintenance of skills (e.g., overlearning, fading support). Finally, the current review focused on skill maintenance in mathematics for students with developmental disabilities. However, another important stage of learning is generalization (Shurr et al., 2019). Hence, another area of future research is to identify what instructional methods support students' generalization (e.g., applying acquired skills in a community setting, authentic activity in a school setting, solving word problems).

Table 1. Characteristics of Studies

Studies	Maintenance			Design		Math Content	Disabilities	Age/Grade	IQ
	Inclusion	Duration	# sessions/tests	SCE	Group				
Akmanoglu & Batu (2004)	Y	1, 2, 4 weeks	3 sessions	X		numerals	3, autism	6, 12, 17 years	N/A
Bouck, Bassette, et al. (2017)	Y	2 weeks	2 sessions	X		fraction equivalence	3, LD, OHI, ID	13, 14, 13 years	80, NA, 70
Bouck, Park, Nickell (2017)	Y	2 weeks	2 sessions	X		money related	4, ID, LD, ID, LD	12, 13, 12, 12 years	68, 92, 56, 74
Bouck et al. (2009)	Y	min. 1 weeks	2 sessions	X		multiplication	3, ID (mild)	12 years	63, 62, 62
Bouck et al. (2014)	Y	N/A	3 sessions	X		subtraction (single/double)	3, autism	6-10 years	N/A
Bouck, Park, Sprick, et al. (2017)	Y	1- 2 weeks	2 sessions	X		fractions (addition)	4, OHI, ID, ID, ID	14, 13, 13, 13 years	N/A, 68, 53, 70
Bouck, Park, et al. (2018)	Y	2 weeks	2 sessions	X		basic operation	2, ID	12, 13 years	NA
Braten & Throndsen (1998)	Y	2, 4 weeks	3 post-tests		X	addition	1, developmental delay	8 years	N/A
Browder et al. (2012)	Y	2-3 weeks	vary across participants	X		geometry, measurement, algebra, data analysis	4, ID	11-13 years	30-41
Burton et al. (2013)	Y	1, 1, and 1 week	3 sessions	X		money related	4, autism (3) & id (1)	13, 14, 15, 13 years	85, 76, 61, 66
Calik & Kargin (2010)	Y	10-20days	2 sessions	X		basic addition skills	3, ID	8 years	73, 68, 75
Cihak & Grim (2008)	Y	6 weeks	1 session	X		money related purchasing	4, autism & id	15, 16, 17 years	50, 45, 47, 35 (moderate/severe)
Chung & Tam (2005)	Y	2 weeks	2 post-tests		X	word problems	10, ID	10 years	55-70
Colyer & Collins (1996)	Y	1 week	0, 1, 5, 6 sessions	X		money related	4, ID	12, 15, 14, 14	60, 46, <40, <40
Creech-Galloway et al (2013)	Y	1 week	0-5 sessions	X		Pythagorean theorem	4, ID	15-17 years	43, 48, 57, 41

Table 1 (cont'd)

Denny & Test (1995)	Y	1, 4, 10 weeks	2, 3, 2 sessions	X		money related (counting)	3, DD	17 years	43, 72, 39
Everhart et al. (2011)	Y	N/A	4 sessions	X		numbers	2, TBI & Down syndrome	9 & 6 years	N/A
Flores et al. (2014)	Y	after 2 weeks of instruction & after instruction ended	2 CBM		X	basic addition and subtraction	11, ASD, DD, ID	5-12 years	below 55 -100
Frederick-Dugan et al. (1991)	Y	4 weeks	1 session	X		money related (purchasing)	2, ID	20, 18 years	36, 40
Hasselbring (1988)	Y	4 months	pre-post-and follow up		X	basic addition	160, ID	7-14 years	N/A
Horton et el. (1992)	Y	7 days, 26 days	2 sessions	X		subtraction	7, ID	13-15	44-70
Irish (2002)	Y	2 weeks	3-6 sessions	X		basic multiplication	3 LD, 3 ID	9-11 years	83, 62, 70, 80, 74, 105
Jimenez & Staples (2015)	Y	at least 2 weeks	1 session	X		early numeracy skills (within algebra & geometry)	3, ID	10, 11, 11 years	40, 45, 40
Jimenez et al. (2008)	Y	N/A	1 session for 2 students and none for 1	X		algebra	3, DD	15-17 years	45
Jowett, Moore, & Anderson (2012)	Y	2, 3, 4, 5, 6 weeks	5 sessions	X		numeracy skills	1, ASD	5 years	72
Lancioni et al. (1987)	Y	N/A	vary across participants	X		operations	4, ID	9, 7, 7, 8 years	58, 66, 66, 62
McCallum & Schmitt (2011)	Y	1, 2 weeks	2, 3, 4 sessions	X		division fact fluency	1, ID	13 years	59
Poncy et al. (2007)	Y	2 weeks	1 session	X		basic addition (without regrouping)	1, ID	10 years	44

Table 1 (cont'd)

Rao & Kane (2009)	Y	N/A	10 sessions	X		decimal subtraction	2, ID	7^{th} or 8^{th} grade	50, 47
Rao & Mallow (2009)	Y	1 day, 1-3 weeks	13, 4 sessions	X		multiplication facts	2, ID	7^{th}, 8^{th} grade	62, 49
Rockwell et al. (2011)	Y	6 weeks	3 sessions	X		word problems (one step addition & subtraction)	1, ASD	10 years	79
Root et al. (2017)	Y	N/A	3 sessions	X		problem solving	3, ASD	11, 9, 7 years	58, 46, 55
Sheriff & Boon (2014)	Y	N/A	6, 6, 4 sessions	X		one-step word problem	3, ID	14, 14, 13 years	65, 65, 59
Singer-Dudek & Greer (2005)	Y	1 month, 2 months	2 post-tests		X	multiplication (2 digit by 2 digit)	4, Developmental delays	adolescents	N/A
Skibo et a. (2011)	Y	2weeks	2, 2, & 1 sessions	X		number identification	3, ID	10, 7, 8 years	less than 20, 44, less than 20
Smeets et al. (1987)	Y	44 - 48 days	6-9 sessions	X		operations (addition, subtraction)	4, NA	9-13 years	73, 76, 66, 86
Trace et al. (1977)	Y	1 week, 1 month	2 post-tests		X	money related (counting)	14, ID	14-18 years	46-70
Tzanakaki et al. (2014)	Y	5 months	follow up test		X	numeracy skills	6, ASD	47-81 months	83, 61, 77, 89, 51, 93
Waters & Boon (2011)	Y	5days	6, 5, 1 sessions	X		money related (computation)	3, ID & ASD	15, 14, 16 years	64, 61, 64
Whalen et al. (1996)	Y	22 weeks	1 session	X		basic addition	2, ID	6 & 9 years	50, 69
Whitby (2012)	Y	4.5 weeks	3 sessions	X		word problem (operation)	3, autism	14, 13, 13 years	90, 94, 107
Yakubova & Bouck (2014)	Y	NA	3 sessions	X		operation	5, ID	11 years	57-68
Yakubova et al. (2015)	Y	1 week	3 sessions	X		fractions	3, ASD	17, 19, 18 years	72, 81, 70
Yakubova, Hughes, Shinaberry (2016)	Y	3 weeks	3 sessions	X		operation	4, ASD	5, 6, 6, 6 years	N/A
Yikmis (2016)	Y	7, 14, 21 days	3 sessions	X		basic addition	3, ASD	8, 9, 10 years	N/A
Zisimopoulos (2010)	Y	1, 3, 10 weeks	3 sessions	X		multiplication	2, ID	11, 12 years	N/A

Table 2. Instructional Methods and Materials Categorized by NCTM Mathematical Contents

Studies	Maintenance (Average percent)	Mathematical skills	Instructional method	Instructional material
		Numerals (5)		
Akmanoglu & Batu (2004)	95.60%	numerals	simultaneous prompting (modeling & verbal prompts)	number cards
Everhart et el. (2011)	93.75%	numbers	computer-based instruction (digital verbal prompt), feedback	flashcards
Jowett, Moore, & Anderson (2012)	100%	numeracy skills	video modeling, prompting, gradual fading prompts	number cards
Skibo et a. (2011)	77.32%	number identification	least to most prompting	response card
Tzanakaki et al. (2014)	75.17%	numerals	math recovery numeracy curriculum, prompting, fading	number lines, number cards
		Addition/ Subtraction (13)		
Bouck et al. (2014)	95.5%	subtraction (single/double)	system of least prompts	concrete/virtual manipulatives
Braten & Throndsen (1998)	95.80%	addition without regrouping	explicit instruction & self-instruction	unifix cubes
Calik & Kargin (2010)	100%	basic addition skills	touch math, explicit instruction, immediate feedback	Touch points
Chung & Tam (2005)		word problems (addition, subtraction)	cognitive strategy instruction	graphic organizer
Flores et al. (2014)	4.45 (CBM Mean)	basic addition and subtraction	explicit instruction, CRA+SIM (strategic instruction model: Mnemonic DRAW), feedback (10 sessions more for the abstract for the fluency)	graphic organizer, post organizer
Hasselbring (1988)	80%	basic addition	recall training (drill)	fast facts
Horton et el. (1992)		subtraction	modeling, corrective feedback	calculator, cue cards, graphic organizer
Poncy et al. (2007)	100%	addition without regrouping	cover copy compare, taped problem, feedback	
Rao & Kane (2009)	100%	decimal subtraction	simultaneous prompting/ 0s time delay (modeling)	task analysis
Rockwell et al. (2011)	100%	word problem (addition & subtraction)	schema-based instruction, direct instruction, mnemonic strategy, explicit instruction	schematic diagrams
Smeets et al. (1987)	94.12%	operations (addition, subtraction)	stimulus manipulation (visual prompting/fading) & delayed feedback only	graphic organizer

Table 2 (cont'd)

Whalen et al. (1996)	85.00%	basic addition	CTD (0 s & 3 s), unrelated instructive feedback (sight words)	flash card
Yikmis (2016)	100%	basic addition	Touch math, simultaneous prompting	task analysis
		Multiplication / Division (6)		
Bouck et al. (2009)	63.30%	multiplication	pentop computers, computer hint	
Irish (2002)	38%	basic multiplication	pegword mnemonics & CAI	
McCallum & Schmitt (2011)	52.20%	division fact fluency	taped problems intervention (& time delay), self-monitoring, audio-recording - she was told to cross out the incorrect answer and write the correct answer provided by the CD	
Rao & Mallow (2009)	100%	multiplication facts	simultaneous prompting	flash cards
Singer-Dudek & Greer (2005)	fluency 100% (1month) & 90% (1 month); mastery 100% (1 month) & 35% (2 month)	multiplication (2 digit by 2 digit)	fluency instruction vs. mastery instruction, faster rate of correct responding was reinforced (after receiving instruction on skills)	
Zisimopoulos (2010)	81.20%	multiplication	picture fading, immediate modeling, corrective feedback	flash card pegword mnemonic form
		Operations (7)		
Bouck, Park, et al. (2018)	56.6%	four operations (place value, add, sub, multiplication)	VRA instructional sequence, explicit instruction	virtual manipulatives
Lancioni et al. (1987)	99.70%	four operations (picture/word problem)	modeling, prompting, error correction	calculator
Root et al. (2017)	96.6%	operation	schema-based instruction, least to most prompting (indirect, direct verbal, modeling)	concrete & virtual manipulatives, graphic organizers
Sheriff & Boon (2014)	72.83%	one-step word problem (add, sub, multi)	computer-based instruction	graphic organizer, calculator
Whitby (2012)	58.30%	word problem (operation)	Solve it! Curriculum (scripted lessons)	cue cards, strategy posters
Yakubova & Bouck (2014)	90%	operation		calculator
Yakubova, Hughes, & Shinaberry (2016)	74.3%	operation (sub, add, comparison)	video modeling, CRA instructional sequence, explicit instruction	concrete manipulatives, task analysis

Table 2 (cont'd)

		Fractions (3)		
Bouck, Bassette, et al. (2017)	76.6%	fractions (equivalence)	VRA instructional sequence, explicit instruction	virtual manipulatives
Bouck, Park, Sprick, et al. (2017)	77.50%	fractions (adding)	VA, explicit instruction	virtual manipulatives
Yakubova et al. (2015)	86.60%	fractions	video-based instruction	task analysis
		Measurement (8)		
Bouck, Park, Nickell (2017)	75%	money related	CRA, explicit instruction	concrete manipulatives
Burton et al. (2013)	93.65%	money related	video-self modeling	task analysis
Cihak & Grim (2008)	100%	money related purchasing	counting on, next dollar strategy, 0s second, 3s second, least-to-most prompt	task analysis
Colyer & Collins (1996)	100%	money related	least to most prompting, time delay, instructive feedback, next dollar strategy	visual flash card
Denny & Test (1995)	97.6%	money related	modeling, one more than technique	
Frederick-Dugan et al. (1991)	100%	money related	progressive time delay, prompting	calculator, picture prompt, money card
Trace et al. (1977)	$p<0.01$; $p<0.025$	money related (counting)	modeling, verbal prompts, fading	visual presentation of target values
Waters & Boon (2011)	76%	money computation (3digit)	touch math, explicit instruction, corrective feedback	
		Geometry (1)		
Creech-Galloway et al (2013)	96%	Pythagorean theorem	simultaneous prompting (modeling+verbal & verbal+physical prompt)	calculator, video, task analysis
		Algebra (1)		
Jimenez et al. (2008)	100%	algebra	systematic prompting with fading (0s time delay; 4s time delay)	concrete representation, task analysis
		Multiple (2)		
Browder et al. (2012)	geo: 62.5%; algebra: 80%; data: 54.2%; measurement: N/A	geometry, measurement, algebra, data analysis	math stories (word problem), least intrusive prompting	graphic organizer, task analysis,
Jimenez & Staples (2015)	90.46%	early numeracy skills in algebra & geometry	Early numeracy curriculum (systematic instruction, a least of most prompting, constant time delay)	graphic organizers, manipulatives, task analysis

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CHAPTER 3

USING THE VIRTUAL-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL SEQUENCE WITH FADING SUPPORT TO TEACH MATHEMATICS TO STUDENTS WITH DEVELOPMENTAL DISABILITIES

Students with developmental disabilities learn in four stages: acquisition, fluency, maintenance, and generalization (Shurr, Jimenez, & Bouck, 2019). Each stage is important, and, while linear, the stages build upon each other (Alberto & Troutman, 2009; Shurr et al., 2019). Within the realm of academic learning for students with developmental disabilities, the greatest attention is typically given to the stage of acquisition, which involves the initial acquisition of skill (Dekeyser, 2007). Much less attention is paid to how students maintain the skills and concepts they acquire (Spooner, Knight, Browder, & Smith, 2012). Maintenance, the ability to continue using what students learned without learning it again, occurs after students develop both accuracy and fluency (i.e., can perform the skills accurately within a reasonable amount of time; Alberto & Troutman, 2009; Shurr et al., 2019). Maintenance is especially important in mathematics because mathematics builds on previous concepts as it advances (Powell, Fuchs, & Fuchs, 2013).

Maintenance in mathematics is important for students with disabilities as mathematics supports students' preparation for independent living (Saunders, Spooner, & Davis, 2018). Basic mathematical skills, such as computational skills (e.g., addition, subtraction, multiplication, and division), are used in daily life (Saunders et al., 2018). If individuals with developmental disabilities do not maintain what they acquire, it will be challenging for them to be independent, especially in activities such as purchasing and budgeting that involve computation (Cihak & Grim, 2008). Although research on interventions for students' maintenance in mathematics is limited, many researchers have studied mathematics instruction to support individuals with developmental disabilities (e.g., Spooner, Saunders, Root, & Brosh, 2017).

Instructional Methods and Materials in Mathematics

Spooner, Root, Saunders, and Browder (2018) conducted a systematic review and evidence-based synthesis of the literature examining mathematical interventions for students with developmental disabilities. They found systematic instruction (e.g., system of least prompt, time delay, error correction), graphic organizers, manipulatives, technology assisted instruction, and explicit instruction were evidence-based practices (EBPs) in teaching mathematics to students with developmental disabilities. These EBPs include both instructional methods, such as explicit instruction, as well as materials (e.g., manipulatives; Spooner et al., 2018). The authors also found most researchers generally used both instructional methods and materials as intervention packages (Spooner et al., 2018). One of the most frequently used combinations to teach mathematics to students with developmental disabilities was explicit instruction and manipulatives (Spooner et al., 2018).

A common approach to using manipulatives in mathematical instruction for students with disabilities in general is the concrete-representational-abstract (CRA) instructional sequence (Bouck, Satsangi, & Park, 2018). The CRA is a sequence in which concreteness is gradually faded toward abstract thinking, an effective method in mathematics education (Agrawal & Morin, 2016; Fyfe, McNeil, Son, & Goldstone, 2014). When using the CRA instructional sequence, the targeted mathematics area or skill is first taught using concrete manipulatives, then representation (e.g., drawings, pictures), and finally numerical strategies without any manipulatives or drawings (Agrawal & Morin, 2016). The CRA instructional sequence uses explicit instruction (Agrawal & Morin, 2016; Bouck, Satsangi, et al., 2018).

While the CRA is an EBP for students with learning disabilities, researchers have increasingly applied the CRA to students with developmental disabilities, including students with intellectual disability and/or autism (Bouck, Park, & Nickell, 2017; Flores, Hinton, Strozier, & Terry, 2014; Stroizer, Hinton, Flores, & Terry, 2015; Yakubova, Hughes, & Shinaberry, 2016). Bouck, Park, et al. (2017) used the CRA instructional sequence to teach making change with coins in a single case study. The authors found a functional relation between the CRA instructional sequence and the accuracy of the skill learned. The four students with intellectual disability all acquired the skills within 9 to 11 sessions with an average of 15 minutes per session. Flores et al. (2014) also used the CRA instructional sequence to teach single-digit addition and subtraction (i.e., zero to nine) for 11 students with developmental disabilities via a group design study. They found significant increases in accuracy of solving problems across baseline, intervention, and post-intervention. Stroizer et al. (2015) found the CRA instructional sequence was effective in teaching addition and subtraction with regrouping as well as multiplication to three students with autism. In their single case study, the researchers found a greater increase in accuracy when the CRA instructional sequence was introduced compared to students' baseline. Finally, Yakubova et al. (2016) found a functional relation between the CRA instructional sequence and the accuracy of four students with autism in solving addition, subtraction, and number comparison problems.

Virtual Manipulatives in Mathematics

Although the CRA instructional sequence is effective, researchers were concerned that use of concrete manipulatives stigmatizes middle and high school students (Bouck et al., 2012; Satsangi & Bouck, 2015). Researchers explored the use of virtual manipulatives (i.e., online or app-based manipulatives)—in place of concrete manipulatives—to support students with disabilities in mathematics (Bouck, Chamberlain, & Park, 2017; Bouck Satsangi, Doughty, & Courtney, 2014; Root, Browder, Saunders, & Lo, 2017). Existing studies comparing virtual and concrete manipulatives found virtual manipulatives as effective as concrete manipulatives, and students preferred to use virtual manipulatives (e.g., Bouck et al., 2014; Bouck, Chamberlain, et al., 2017; Root et al., 2017; Bouck, Shurr, Bassette, Park, & Whorley, 2018). In addition, Bouck, Chamberlain, et al. (2017) found students were more independent with virtual manipulatives.

One means of using virtual manipulatives in place of concrete manipulatives is by adapting the CRA instructional sequence to the virtual-representational-abstract (VRA) instructional sequence (Bouck & Sprick, 2019; Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, Bassette, & Whorley, 2018). In the VRA instructional sequence, virtual manipulatives are used instead of concrete manipulatives, but the representational and abstract phases remain the same as in the CRA instructional sequence (Bouck, Bassette, et al., 2017). Bouck, Park, et al. (2018) explored the effectiveness of the VRA instructional sequence in teaching basic operations including place value, addition, subtraction, and multiplication to two secondary students with intellectual disability. They found immediate effects when the VRA instructional sequence was introduced and the students achieved higher accuracy during intervention than baseline. Bouck, Bassette, et al. (2017) used the VRA instructional sequence in a single case study to teach equivalent fractions to three secondary students with disabilities, including intellectual disability, who were successful in acquiring the skill.

Supporting Maintenance in Mathematics

The majority of the studies using the CRA and VRA instructional sequences support the efficacy of the learning sequences to support mathematical skill acquisition (Bouck, Bassette, et al., 2017; Bouck, Park, et al., 2017; Flores et al., 2014; Stroizer et al., 2015). However, not all of the students who participated in the studies maintained the skills even though the students showed higher scores than baseline (e.g., Bouck, Park, et al., 2018; Bouck, Bassette, et al., 2017). To date, few studies have examined strategies to target maintenance in mathematics for individuals with developmental disabilities. However, some researchers suggested fading support to be an effective method for promoting students' maintenance (Alberto & Troutman, 2009; Collins, 2012; Shurr et al., 2019; Snell & Brown, 2011).

Fading support is a method in which instructions or prompts provided by the teacher are gradually faded to allow students to initiate solving problems by themselves (e.g., Burton, Anderson, Prater, & Dyches, 2013; McNeill, Lizotte, Krajcik, & Marx, 2006; Morton & Flynt, 1997; Paine, Carnine, White, & Walters,1982). Paine et al. (1982) conducted two single case studies to examine the effects of fading teacher instruction to support for problem-solving skills, including multiplication, with four elementary school students without disabilities. In the first study, the authors found traditional instruction—high structure chalkboard demonstration and high structure worksheet—without fading support was insufficient to improve students' performance level; however, when fading support was introduced, students showed improvement and finally maintained the skill. In the second study, the authors only used high structure chalkboard demonstration and students still acquired and maintained the skill only after fading support was provided. McNeill et al. (2006) also explored the effectiveness of fading using written instructional support to teach writing scientific explanations to 331 seventh-grade students without disabilities via group design study. They found significant improvement for all students with or without receiving fading support. However, students in a treatment group who received fading support had significantly higher scores compared to those in the control group. McNeill et al. (2006) concluded students who receive fading support are more likely to maintain the knowledge they acquired without any instructional support. Burton et al. (2013) used video-self modeling with fading support in teaching purchasing skills to individuals with developmental disabilities. They found video-self modeling was effective for mathematical skill acquisition as well as that the systematic fading of the video model across five maintenance sessions was effective for students to maintain the purchasing skills.

Although previous researchers examined the effects of fading support, research on the effects of fading support in teaching mathematics for students with developmental disabilities is limited. This study sought to investigate the effectiveness of VRA instructional sequence with fading support in teaching mathematics, particularly in terms of maintaining the mathematical skills after acquisition. As previous literature suggested fading support is an effective method for acquiring skills (e.g., McNeill et al., 2006; Paine et al., 1982), this study examined whether the intervention package of the VRA instructional sequence and fading support enhances student acquisition and maintenance of basic mathematical skill (i.e., subtraction with regrouping). The specific research questions involve: a) To what extent do students with developmental disabilities acquire mathematical skills using the VRA instructional sequence with fading support?; b) To what extent do students with developmental disabilities maintain the skills they learned using the VRA instructional sequence with fading support? and c) What is the perception of students with developmental disabilities and their teacher regarding the VRA instructional sequence with fading support?

Method

Participants

Four middle school students participated in the study. Each student received special education services in pull-out settings from their school district. The students were taught by special education teachers. Participants were chosen according to the following criteria: (a) teacher suggestion of the student struggling with subtraction with regrouping; (b) confirmation of students' struggles with basic operations (i.e., below grade level) when the subtests of the KeyMath-3 assessment were administered by researchers (i.e., numeration, mental computation, addition and subtraction, and multiplication and division); and (c) successful demonstration of the ability to perform addition or subtraction without regrouping on the KeyMath-3 assessment. Any students who did not meet the aforementioned criteria did not participate in the study.

Harry. Harry was a 14-year-old, seventh-grade, Caucasian student. He was identified as a student with intellectual disability based on his Wechsler Preschool and Primary Scale of Intelligence II (WPPISI-II). Harry's achievement scores on math, reading, and writing were 68, 66, and 63, respectively. On the KeyMath-3 assessment administered by the researcher, Harry's numeration score was 17 (3.1 grade equivalency), addition and subtraction score was 13 (2.6 grade equivalency), and his total operations score was 28 (2.8 grade equivalency).

Emma. Emma was a 14-year-old, seventh-grade, Caucasian student. She was identified as a student with autism according to her Individualized Education Program (IEP). Due to a recent move, limited information was available for Emma; however, according to her IEP, she had difficulties with regrouping in mathematics. On the KeyMath-3 assessment, Emma's numeration score was 15 (2.5 grade equivalency), addition and subtraction was 13 (2.6 grade equivalency), and her total operations score was 25 (2.6 grade equivalency).

Brett. Brett was an 11-year-old, sixth-grade, Caucasian student. He was identified as having autism according to his IEP. According to Brett's standard scores on the Wechsler Individual Achievement Test IV (WIAT-IV), he scored in the low range for basic reading skills (79), math calculation (76), written expression (57), and written language (75). In addition, he scored in the very low range for reading comprehension (<40) and reading fluency (55). On the KeyMath-3 assessment (Connolly, 2007) administered by the researcher, his numeration was scored at 12 (1.8 grade equivalency), addition and subtraction at 13 (1.4 grade equivalency), and his total operations score was 34 (3.2 grade equivalency).

Mateo. Mateo was a 14-year-old, eighth-grade, African American student. He was identified as having autism according to his IEP. On the Woodcock Johnson III test of achievement, his grade equivalency was 1.1 in applied problems in math, 3.6 in math fluency, and 1.7 in calculation. On the TOWL-4, he scored at 3rd grade level on story composition and below 3rd grade level on contextual conventions. On the researcher-administered KeyMath-3, Mateo's numeration score was 7 (i.e., K.8 grade equivalency), addition and subtraction was 5 (1.2 grade equivalency), and his total operations score was 10 (1.4 grade equivalency).

Setting

The researcher conducted a study in two middle schools and one elementary school in the Midwest. Harry and Emma were in the same middle school, located 30-minutes away from the public research university. Brett was in an elementary school located 1-hour away from the same university, and Mateo was in a middle school located 15-minutes away from the same university. The study occurred in a school's empty classroom or hallway, which had one table and two chairs that enabled the researcher and a student to sit next to each other. Both Harry and Emma worked individually with the researcher at a table in the hallway just outside their special education classroom. The researcher worked with Brett individually in an empty room connected to his special education classroom. Finally, the researcher worked with Mateo in his self-contained classroom, where he typically received his mathematics instruction. All baseline, intervention, and maintenance sessions occurred in the same setting for each student.

Materials

The study included materials such as learning sheets, pencils, and an iPad with a virtual manipulative. Consistent with typical VRA instructional sequence (e.g., Bouck, Bassette, et al., 2017), explicit instruction was administered, which involved modeling, guided instruction, and independent practice. For each virtual, representational, and abstract phase, two problems for modeling and two problems for guided instruction were provided. For independent practice, the researcher provided five problems with targeted skills (i.e., subtraction with regrouping). Each sheet for independent practice involved unique sets of problems and none of the problems from modeling and guided practice were repeated in the independent portion of the same learning sheet. The researcher listed all possible problems and then randomly assigned them to learning sheets.

During the virtual phase, students were provided an iPad on which manipulative apps were available. The app used in this study was *Base 10 Blocks*, which involved two different colors of base 10 blocks to distinguish between the subtrahend and minuend, developed by Brainingcamp (2018). The app presented a place value chart and the pictorial representations are presented on top of the chart (i.e., virtual base 10 blocks). The place value chart had two sections for the subtrahend and minuend in the problem. Students set up the numbers represented in the problems by dragging and dropping the virtual base 10 blocks into their respective ones, tens, or hundreds place on the chart (see Figure 1).

Independent and dependent variables

The independent variable for the study was the use of VRA instructional sequence with fading support. Students used the Brainingcamp (2018) app during the virtual phase, drawing images (e.g., lines, dots) during representational phase, and numerical strategies during the abstract phase and the extended abstract with fading support phase. The dependent variable was the percentage of accurate answers among five problems during independent practice.

Experimental design

In order to gauge the effectiveness of VRA instructional sequence with fading support, a multiple probe across participants design was employed. A total of three phases (i.e., baseline, intervention, and maintenance) was included (Gast & Ledford, 2014). Consistent with the previous VRA study (i.e., Bouck, Bassette, et al., 2017; Bouck, Park, et al., 2018), which used a multiple probe across participants design, all participants started their baseline simultaneously. All participants had at least five baseline sessions. When the first student (i.e., Harry) met the baseline criteria—80% of the data falling within 25% of the median without an accelerating trend (Gast & Ledford, 2014)—he began intervention. When Harry completed the first intervention session, the researcher conducted a baseline session for the other students. When Harry achieved 80% or higher on three sessions of virtual phase, he moved to the representational phase. At that time, the researcher conducted an additional baseline session for other students, and then the second student (i.e., Emma) subsequently entered the first intervention session. When Emma entered the first intervention session, the third student (i.e., Brett) and the fourth student (i.e., Mateo) completed one additional baseline session. When Emma achieved mastery criterion for three sessions of the virtual phase, the researcher conducted another baseline session for the remaining students, and Brett entered the first intervention session. When Brett achieved mastery criterion for three sessions of the virtual phase, the researcher conducted a final baseline session for the last student (i.e., Mateo) before he entered the intervention.

Generally, each virtual, representational, and abstract phase involved at least three sessions (i.e., a total of at least 9 sessions). After each student's final abstract session, the faded support abstract phase was conducted for six sessions. Finally, the maintenance phase was conducted for four weeks, one session per week respectively, after the final fading support session. If a student scored less than 80% accuracy, s/he repeated that lesson with the same learning sheet at the next session (e.g., Mancl et al., 2012). If a student refused to solve problems for five consecutive sessions, the intervention was discontinued for that student.

Procedures

Baseline. Students were asked to solve subtraction with regrouping problems independently. Each session involved five problems. Prompts or cues were not provided to students. Baseline data were measured for a minimum of five sessions, or until data were stabilized (i.e., 80% of the data falling within 25% of the median; Gast & Ledford, 2014). In order to enter the intervention phase from the baseline, each student needed a zero-celeration or decelerating trend with stability (Gast & Ledford, 2014).

Intervention. In this study, VRA instructional sequence was used along with fading support to teach subtraction with regrouping. In each session, explicit instruction was provided (i.e., modeling, guided instruction, and independent practice, see Figure 2 for instructions used in each portion). During modeling, the researcher demonstrated how to solve two problems using a think-aloud, which was verbalizing the mathematical approach to solve the problems. During guided instruction, the researcher asked each student to solve two problems. If students made a mistake or did not initiate solving the problem, the researcher engaged and provided prompts or cues (e.g., "What's next?" "Did you set blocks correctly?"). During independent practice, each student solved a total of five problems on their own without any help or prompts. These independently solved problems were calculated for accuracy as the dependent variable.

In accordance with the typical VRA instructional sequence administration, each phase (i.e., V, R, and A) involved at least three sessions (i.e., a total of at least nine sessions in the VRA sequence; Bouck, Park, et al., 2018). Additionally, this study included a fading support abstract phase, which was provided after the final session of the abstract phase. In this phase, the researcher gradually faded the explicit instruction, from two modeling problems and two guided instruction problems to only one guided instruction problem over six sessions (i.e., two sessions for one modeling and two guided; two sessions for one modeling and one guided; and two sessions for no modeling and one guided).

Virtual. In this phase, students used the virtual app (e.g., *Base Ten Blocks*) by Brainingcamp (2018) on the iPad to solve subtraction with regrouping problems. In each virtual session, the researcher first modeled how to solve problems using the virtual manipulative. The modeling involved how to set up the numbers corresponding to the problem on the place value chart, how to ungroup one larger block into smaller value blocks (e.g., one ten block into ten one blocks), how to count the total number of blocks, and how to write the answer. After modeling two problems, each student solved two problems with the researcher's guidance (e.g., "Did you count the ones correctly?" "Do you set blocks correctly?" "what's next?"), when necessary. After two problems of guided instruction, each student solved five problems independently using the virtual manipulative.

Representational. In the representational phase, students solved problems using drawings. First, the researcher drew squares, lines and dots to represent hundreds, tens and ones place values respectively, and demonstrated how to solve two problems using think-aloud. Next, the researcher provided prompts or cues as needed when each student used drawings to solve two problems. Finally, each student solved five problems independently using drawings.

Abstract. In this phase, the researcher asked students to solve problems abstractly without using any virtual manipulative or drawings. During the modeling portion of the abstract phase, the researcher showed how to solve two problems with regrouping using decomposition and counting-on (with fingers) mathematical strategies (Siegler, 1987). Then, students solved two problems with prompting as needed. Finally, students completed five problems independently without using any virtual manipulatives or prompts.

Faded support abstract. After each student achieved 80% accuracy or higher in the abstract phase for three sessions, the researcher provided fading support. During the extended abstract phase, a total of six sessions was provided with fading support. For the first two sessions, the researcher provided one modeling instead of two modeling and two guided instructions. For the next two sessions, the researcher provided one modeling, and one guided instruction. For the last two sessions, the researcher only provided guided instruction for one problem. Students completed five problems independently using numerical strategies during all the faded support abstract sessions. If students failed to achieve 80% or higher accuracy, then the lesson was repeated in the next session.

Maintenance. A total of four maintenance sessions were conducted spread across four weeks after each student's last intervention session. Some sessions for some students were delayed because of weather and winter breaks for up to two weeks. During maintenance, no prompting, cueing, or using virtual manipulatives was allowed for the students. Each student was asked to solve five problems independently, the same as during baseline.

Treatment fidelity and inter-observer agreement

The researcher created a detailed checklist to evaluate treatment integrity. The checklist included whether the students received all the materials corresponding to each phase (i.e., virtual, representational, or abstract phase) and if the explicit instruction was appropriately implemented (i.e., modeling, guided instruction, independent practice). The researcher collected treatment fidelity data for a minimum of 30% of the intervention sessions across all phases for all students. The treatment fidelity was 100% for all students across all phases.

Interobserver agreement (IOA) was calculated based on two independent observers' scores of the data across all phases (i.e., baseline, virtual phase, representational phase, abstract phase, extended abstract phase, and maintenance). Agreements or disagreements on accuracy were calculated. The percentage of IOA was calculated as following: total agreements were divided by total agreements plus disagreements and multiplied by 100 to obtain a percentage (Gast & Ledford, 2014). IOA data was checked for a minimum of 30% across all phases for all students. IOA was 100% for all students.

Social validity

The researcher conducted brief interviews after intervention with the teachers and the students. In the interview, the researcher asked students about their perceptions about each phase (i.e., virtual, representational, and abstract phase) and which phase they preferred. Also, students were asked about fading support, whether reducing modeling and guided practice had helped them solve the problem independently during maintenance. Additionally, the teachers were interviewed about their mathematics instructional methods, their students' mathematics abilities, mathematic skill maintenance, and their opinion of using VRA instructional sequence with fading support to teach subtraction with regrouping compared to their traditional teaching method.

Data analysis

The researcher conducted visual analysis to analyze the data. The researcher calculated level by determining if 80% of the data points for each phase or condition fall within 25% of the median value (Gast & Ledford, 2014). To determine a trend, the split-middle method was used (White & Haring, 1980). The researcher divided data within each condition in half, found the intermediate point of the mid-rate and mid-date for each half, and then drew a line through the data which passed through the intersections to find whether the trend was zero-celerating, decelerating, or accelerating (Gast & Ledford, 2014). To determine the effectiveness of the intervention, the researcher used Tau-U, which combined phase AB (i.e., baseline and intervention) non-overlap with the trend from B, which was the intervention phase (Parker, Vannest, Davis, & Sauber, 2011). The researcher used an online-calculator to calculate the effect size (see http://www.singlecaseresearch.org/calculators/tau-u). If the Tau-U score was less than or equal to 65%, it indicated a small effect, 66-92% indicated a medium effect, and more than 92% suggested a large effect (Parker, Vannest, & Brown, 2009).

Results

Overall, the VRA instructional sequence with fading support resulted in increasing accuracy in solving subtraction with regrouping problems from baseline to intervention (see Figure 3). Results from the visual analysis demonstrated a functional relation between the VRA instructional sequence with fading support and accuracy in solving problems. Students also maintained their skill up to four weeks after instruction ended.

Harry

Harry answered zero problems correctly during the five baseline sessions. His baseline data were stable and zero-celerating. When Harry entered the intervention phase, he showed an immediate effect in his first intervention session using the virtual manipulative. During the virtual phase, he repeated only one session and met the mastery criterion (i.e., three sessions of 80% or higher). However, during the representational phase, although he received higher scores than baseline phase, he struggled with drawing. Harry refused to work during all sessions in the representational phase. The researcher encouraged him to solve problems with drawing for five sessions, but the student refuse to draw; the researcher discontinued with his intervention.

Emma

During baseline, Emma's accuracy in solving subtraction with regrouping was zero. The data were stable with a zero-celeration. Emma experienced an immediate effect from her baseline to her first intervention session (i.e., 0% to 100%). Emma only repeated two sessions, both during the abstract phase. The intervention data were stable with an accelerating trend. The average accuracy during intervention was 89.4% (i.e., virtual phase 100%, representational phase 80%, abstract phase 76%, and fading support phase 100%). Emma maintained the skill over the five weeks after the final intervention session. The Tau-U effect size was 100% for the intervention phase, and 100% for the maintenance phase, which suggested the VRA instructional sequence with fading support was a highly effective intervention package for Emma.

Brett

During the baseline, Brett's average accuracy in solving subtraction with regrouping was 2.5%. The data were stable with a zero-celeration. Brett demonstrated an immediate effect from his baseline to first intervention session (i.e., 0% to 100%). Brett only repeated one session during the representational phase. The intervention data were stable with an accelerating trend. The average accuracy during intervention was 76% (i.e., virtual phase 93.3%, representational phase 75%, abstract phase 93.3%, and fading support phase 93.3%). Brett maintained the skill over the six weeks after the final intervention session. The Tau-U effect size was 100% for the intervention phase, and 100% for the maintenance phase, which suggested the VRA instructional sequence with fading support was a highly effective intervention package for Brett.

Mateo

During baseline, Mateo scored 0% on all baseline sessions in solving subtraction with regrouping problems. The data were stable with a zero-celeration. Mateo experienced an immediate effect from his last baseline session to first intervention session (i.e., 0% to 100%). Mateo repeated one session during representational phase and one session during abstract phase. The intervention data were variable with an accelerating trend. The average accuracy during intervention was 90.5% (i.e., virtual phase 93.3%, representational phase 80%, abstract phase 90%, and fading support phase 96.6%). Mateo maintained the skill over the four weeks after the final intervention session. The Tau-U effect size was 100% for the intervention phase, and 100% for the maintenance phase, which suggested the VRA instructional sequence with fading support was effective for both skill acquisition and maintenance.

Social validity

All four students stated they enjoyed participating in the study and all expressed a preference for using the virtual manipulative to solve problems. Harry and Mateo said they preferred to use the app—expressing that they could solve problems faster when using the app. Mateo added that everything about the iPad was awesome. Emma mentioned that using the numerical strategy was quite difficult but once she knew how to do it, it was easier than using the virtual manipulative. In the future, she said she would like to use numerical strategy. She also wants to use numerical strategy to teach other students so they will be able to solve problems too.

When the teachers were interviewed, one of them discussed that she wishes this intervention was conducted with other students as well because they would have enjoyed it. She also said that her students really like using technology in the classroom. All three teachers said their students had benefitted from learning the mathematical skill using the VRA intervention package. When asked about their students' maintenance of skills, the teachers said they have concerns that students easily forgot what they have learned. So, usually they have to reteach again and again. However, as the results of the study show that the students maintained their skills after a month of using the VRA package, the teachers mentioned that they would like to incorporate this package in their teaching settings to support students' maintenance in the future.

Discussion

This study explored the efficacy of the VRA instructional sequence with fading support in teaching subtraction with regrouping to students with intellectual disability and/or autism. Based on the visual analysis, a functional relation was found between the VRA intervention package and the students' accuracy in solving problems. The intervention had a strong effect on students' skill acquisition as well as maintenance. Three of the students' accuracy level improved from baseline to intervention; Harry was discontinued from study participation. The three students also maintained the skill for four-to-six weeks after the intervention ended.

Findings from this study support and extend research on teaching mathematics using the VRA instructional sequence. Similar to the existing studies (e.g., Bouck, Bassette, et al., 2017; Bouck, Park, et al., 2018), students experienced immediate effects and acquired skills in a short period—a maximum of 11 sessions. Students in the current study expressed enjoyment and preference for using virtual manipulatives to solve problems, which aligns with findings from previous literature (e.g., Bouck, Park, et al., 2018; Root et al., 2017). They also mentioned learning mathematics was easier using virtual manipulatives. Given that the use of virtual manipulatives can be less stigmatizing and more socially appropriate for older students with disabilities (Bouck et al., 2014; Satsangi & Miller, 2017), the VRA instructional sequence can be an effective and appropriate method to teach mathematics to older students with disabilities.

The current study added a fading support phase to the typical VRA instructional sequence to support students' skill maintenance with gradual fading of explicit instruction. In previous studies that used the VRA instructional sequence, some participants failed to maintain the skills after the intervention ended, although the students improved their skills during intervention and met the mastery criterion of acquisition (e.g., Bouck, Bassette, et al., 2017; Bouck, Park, et al., 2018). Yakubova et al. (2015) suggested students need more scaffolded instruction and may benefit from extending the duration of the intervention phase, which would provide more opportunities to practice skills. In the current study, the typical duration of the intervention phase was extended by six sessions by adding the fading phase (i.e., total of 15 sessions). Fading is a common intervention used to improve students' independence (Edwards et al., 1995; Jimenez et al., 2008; Rock & Thead, 2007; Tzanakaki et al., 2014); although there is wide variation in how instruction is faded, fading instruction in general is known to be effective to promote errorless learning (Jimenez et al., 2008). Although it is unknown whether the students benefited from the longer duration of the intervention phase or from the fading support, the results of this study suggested that the combination of these methods promotes maintenance of skills.

While the VRA instructional sequence with fading support was generally effective, one of the students found the representational phase to be challenging. Harry performed well with virtual manipulatives but struggled in solving problems during the representational phase. He refused to draw to solve triple-digit subtraction with regrouping problems. One hypothesis was that the regrouping required him to draw too many representations; however, in conversations with the teacher, she said Harry also displayed refusal behaviors in the classroom during instruction. Previous researchers discussed the challenges of the representational phase (Bouck, Bassette, et al., 2017) and some studies successfully eliminated the phase when the targeted skills were difficult to draw (e.g., fractions, algebra, geometry; Bouck, Park, et al., 2019; Bouck, Park, Sprick, et al., 2017; Cass, Cates, Smith, Jackson, 2003). As Cass et al. (2003) found the representational phase was not a vital component in the CRA in terms of students acquiring and maintaining area and perimeter skills, future researchers could consider teaching mathematics without the representational phase in sequential learning using virtual manipulatives.

Implications for practice

The findings from this study possess implications for practice. First, educators could use the VRA instructional sequence with fading support to support students' acquisition as well as maintenance of skills. As mentioned earlier, all students in this study demonstrated higher accuracy in solving subtraction with regrouping problems and maintained the skill with more than 80% accuracy for a minimum of four weeks using the VRA instructional sequence with fading support. Often, students with disabilities who struggle with mathematics in earlier grades tend to continue struggling in later grades as well (Nelson & Powell, 2018). It could be hypothesized that lack of maintenance in earlier grades contributes to the difficulties in learning higher grade-level mathematical contents (Powell, Fuchs, & Fuchs, 2013). Maintaining conceptual understanding of basic mathematical skills is crucial as these skills need to be applied while learning more complex skills (Powell et al., 2013). If students with developmental disabilities do not maintain what they acquired in mathematics, teachers may need to reteach the concept over and over (Shurr et al., 2019). In such situations, educators could promote maintenance of basic operations skills using the VRA instructional sequence along with systematic fading.

Another implication is that educators could use virtual manipulatives in classrooms to motivate students in learning mathematics. Some students with and without disabilities have negative feelings toward mathematics and this can lead to math anxiety and affect mathematical performance (Foegen, 2008; Latterell, 2005; Park & McLeod, 2018). However, the social validity data from this study suggests that all students enjoyed using the virtual manipulative as a tool to learn the mathematical skill. In addition, students expressed they would like to use virtual manipulatives to learn mathematical concepts in the future. Given that the availability of technology has grown substantially in schools (U.S. Department of Education, 2016), educators could use virtual manipulatives, available through computers or tablets, to make mathematics more engaging to reluctant students with disabilities.

Limitations and future directions

There are some limitations to this study. First, one of the students—Harry—failed to move on to the abstract phase, as he refused to draw during the representational phase. As this study followed the VRA instructional sequence, he was discontinued from the study and the researcher could not explore whether he can solve problems using numerical strategies. Previous researchers suggested representing higher-level mathematical concepts through drawing may be challenging for students with disabilities (e.g., Bouck, Bassette, et al., 2017). This study indicates some students may also face difficulty in drawing problems involving basic operations. Future researchers should explore if a VA instructional sequence or other adaptation (e.g., either virtual phase or abstract phase) result in students' acquisition and maintenance of basic operations.

The researcher only used one type of app-based manipulative (i.e., Brainingcamp, 2018), which has been used in previous research studies (Bouck, Chamberlain, et al., 2017; Bouck, Park, et al., 2018). However, not all teachers have access to iPads and the app has a cost. Researchers may seek to explore other app options such as free ones, as well as the use of virtual manipulatives via other means of technology (e.g., National Library of Virtual Manipulatives). Also, since the researcher delivered intervention in a one-on-one setting, it is unknown whether app-based interventions are effective in small group or whole class instruction settings. Future research needs to examine the effectiveness of the VRA intervention package using a pre- and post-test design.

Given this study focused on skill acquisition and maintenance, researchers did not conduct a generalization phase. Generalization is another important learning stage out of four (i.e., acquisition, fluency, maintenance, and generalization; Shurr et al., 2019). As previous research in this field has mainly focused on skill acquisition (Spooner et al., 2012), for this study researchers sought to examine the effectiveness of interventions specifically targeting the maintenance of skills. Future researchers should conduct a generalization phase in addition to maintenance where students solve other types of problems (e.g., solve word problems, functional mathematics) or solve problems in a different environment (e.g., in a typical classroom).

Figure 1. Virtual Manipulatives for Subtraction

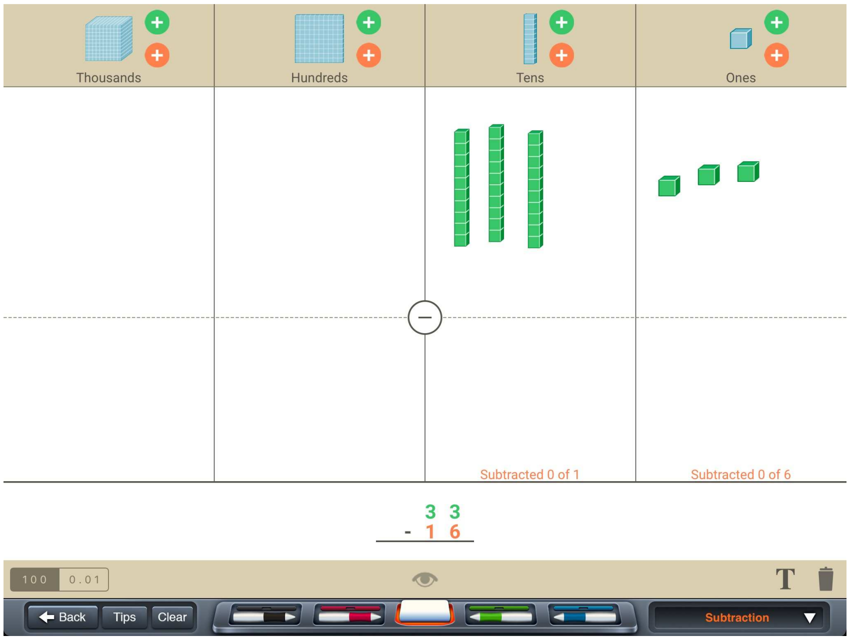


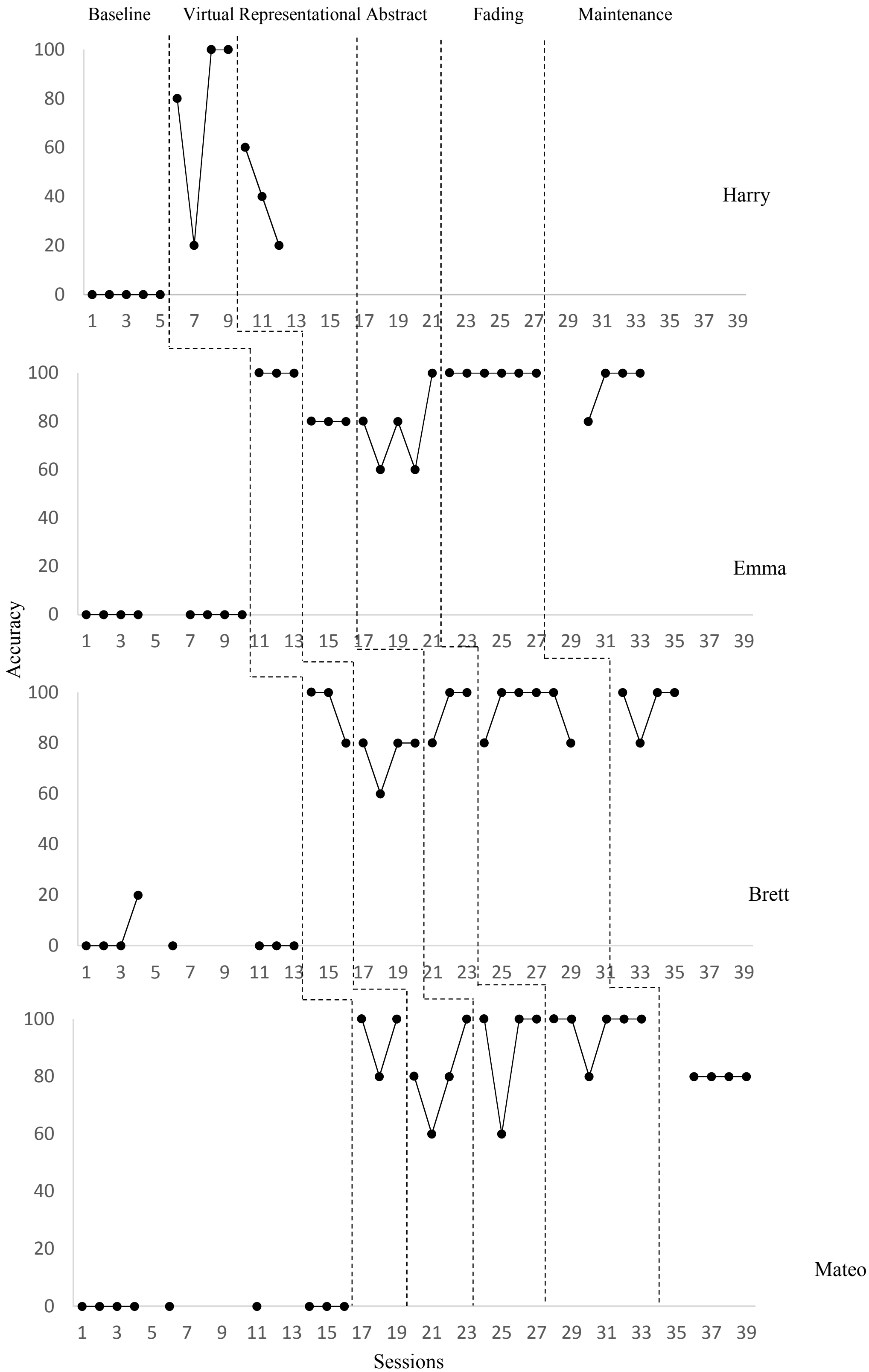
Figure 2. Explicit Instruction

Virtual Phase		
Modeling	Guided instruction	Independent practice
This problem is $34 - 17$. To solve this problem, we are going to use the *Base 10 Block* App. First, can I take away 7 from 4 and make a positive number? No, I need to ungroup (or regroup) my one ten into ten ones. Now I have 14 ones in my ones column and can take 7 away. Next, I can take one ten block from two ten blocks. So, how many ones are there in the ones column? I have 7 ones and one ten. So, my answer is 17.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need, but try to solve them on your own, alright? [Student begins. If, for example, a student does not regroup one ten block into ten one blocks, the instructor might say "what do you do next?"]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best! I am still here, but I will not provide any help.

Representational Phase		
Modeling	Guided instruction	Independent practice
Now, I am not going to use iPad, but use drawings. This problem is $25 - 9$. I am going to draw my place value chart. The lines represent tens and the circles represent ones. I am going to set up my problem correctly. Can I take 9 away from 5 to make a positive number? No, I can't. So, I need to ungroup my one ten into ten ones. Now, I can take 9 away from 15. How many ones are there in the ones column? I have 6 ones. How many tens are left? I have 1 ten. So, my answer is 16.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need but try to solve them on your own. [Student begins. If, for example, a student draws more circles than the ones in the problem, the instructor might point the number on the learning sheet.]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best. I am still here, but I will not provide any help. [If a student says, s/he is not confident, then provide one more problem in guided instruction]

Abstract Phase		
Modeling	Guided instruction	Independent practice
Now, I am not going to use an iPad or drawings. This problem is $23 - 17$. To solve this problem, I am going to use numerical strategy. First, can I take away 7 from 3 and make a positive number? No, I can't. So, I need to ungroup my one ten into ten ones. Now I can take 7 away from 13. I am going to use my fingers to count on—8, 9, 10, 11, 12, 13—so I have 6 ones and no tens left. So, the answer is 6.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need, but try to solve them on your own, alright? [Student begins. If, for example, a student ungroups a ten but does not write down the number of tens left in the tens place, the instructor might point to the tens place.]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best. I am still here, but I will not provide any help.

Figure 3. Accuracy Percentage of Solving Subtraction with Regrouping Problems



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CHAPTER 4

USING THE VIRTUAL-REPRESENTATIONAL-ABSTRACT INSTRUCTIONAL SEQUENCE WITH OVERLEARNING TO SUPPORT STUDENTS WITH DISABILITIES IN MATHEMATICS

Mathematics education is a key area for students with disabilities and is essential for success in their post-school outcomes (i.e., independent living, employment, post-secondary education; Browder et al., 2018; Collins, Hager, & Galloway, 2011; Szekely, 2014). Despite the importance of teaching mathematics to students with disabilities, the most studied academic content in teaching students with disabilities has historically been literacy (Szekely, 2014). Mathematics has received less attention in research and in practice compared to other aspects of educating this population (Allsopp & Haley, 2015; Geary & Hoard, 2002; Spooner, Knight, Browder, & Smith, 2012).

According to the National Assessment of Educational Progress (NAEP), 84% of fourth-grade students with disabilities and 91% of eighth-grade students with disabilities are below grade level in terms of mathematics proficiency (National Center for Education Statistics, 2017). The percentage of students with disabilities who performed below grade level in mathematics in both fourth-grade and eighth-grade did not change significantly since 2011. Such data on the poor performance of students with disabilities suggest educators need to use effective methods to support mathematics learning.

Concrete-Representational-Abstract Instructional Sequence

Previous researchers have identified concrete manipulatives and explicit instruction to be evidence-based practices (EBPs) for teaching mathematics (Carbonneau, Marley, & Selig, 2013; Doabler & Fien, 2013; Spooner et al., 2018). Concrete manipulatives are objects designed to help students learn mathematics (e.g., base 10 blocks, unifix cubes) and explicit instruction is a process in which the instructor first models solving a problem and then guides the student in doing the same (Bouck & Flanagan, 2010; Doabler & Fien, 2013). Concrete manipulatives are generally combined with explicit instruction through a framework called the concrete-representational-abstract (CRA) instructional sequence (Bouck & Park, 2018).

The CRA instructional sequence involves a gradual fading of concreteness (Agrawal & Morin, 2016). The CRA allows students to move from solving mathematical problems with concrete manipulatives (e.g., base ten blocks), to representations (i.e., drawings, pictures), and finally using numerical strategies (Agrawal & Morin, 2016). This instructional sequence has shown to be effective in teaching various mathematical skills (e.g., fractions, addition, subtraction) to students with various disabilities, including autism and learning disabilities (Bouck, Park, & Nickell, 2017; Flores, Hinton, & Schweck, 2014; Stroizer, Hinton, Flores, & Terry, 2015; Yakubova, Hughes, & Shinaberry, 2016). Bouck et al. (2017) explored the effectiveness of the CRA instructional sequence in teaching money-related problems (i.e., making changes with coins) to four students with intellectual disability or learning disabilities. They found a functional relation between the CRA instructional sequence and the accuracy of solving problems. Flores et al. (2014) used the CRA instructional sequence to explore its effectiveness in teaching multiplication to four students with learning disabilities using a single case design. The researchers found a functional relation between the CRA instructional sequence and the number of correct digits in the answer. Stroizer et al. (2015) examined the effects of the CRA instructional sequence in teaching basic operations (i.e., addition with regrouping, subtraction with regrouping, and multiplication facts) to three elementary school students with autism using a single case design. The participants achieved higher accuracy in solving problems in each skill when the CRA instructional sequence was introduced. Finally, Yakubova et al.

(2016) used the CRA instructional sequence along with video modeling to teach numbers and basic operations (i.e., one- and two-digit addition, subtraction, and number comparison) to four students with autism in a single case study. The authors concluded the CRA instructional sequence with video modeling is effective in teaching the mathematical skills.

Despite the effectiveness of the CRA instructional sequence in supporting mathematical concepts for students with disabilities, research has emerged on the use of virtual manipulatives in place of concrete manipulatives. Virtual manipulatives are digital versions of concrete manipulatives that can be used online or in an app on a tablet (Bouck, Working, & Bone, 2018). Researchers found virtual manipulatives (e.g., web-based, app-based) to be similarly effective to concrete manipulatives (Bouck, Chamberlain, & Park, 2017; Bouck, Satsangi, Doughty, & Courtney, 2014; Root, Browder, Saunders, & Lo, 2017). Further, virtual manipulatives are preferable to concrete manipulatives, especially among secondary students with disabilities, as virtual manipulatives are more socially desirable and less stigmatizing for older students (Bouck et al., 2012; Satsangi & Miller, 2017).

Virtual-Representational-Abstract Instructional Sequence

One way to use virtual manipulatives is through an instructional sequence similar to the CRA: the virtual-representational-abstract (VRA) instructional sequence (Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, Bassette, & Whorley, 2018). The VRA is an adaptation of the CRA, so the gradual movement from a manipulative toward numerical strategy remains the same (Bouck, Bassette, et al., 2017). The difference between the VRA and the CRA is the use of virtual manipulatives in the first phase instead of concrete manipulatives (Bouck, Bassette, et al., 2017). Although limited, research regarding the efficacy of the VRA for supporting mathematics instruction for students with disabilities is emerging. Bouck, Bassette, et al. (2017) conducted a single case study using the VRA instructional sequence to teach equivalent fractions to three middle school students with disabilities including learning disabilities. The authors found a functional relation between the VRA instructional sequence and the acquisition of finding equivalent fractions. Bouck, Park, Shurr, et al. (2018) also examined the VRA instructional sequence to teach place value, addition, subtraction, and/or multiplication to two secondary students with mild intellectual disability. Using a single case design, the authors found the use of VRA was effective in acquiring the mathematical skills as both participants acquired all three of their targeted mathematical skills.

Maintenance in Mathematic Skills

Although previous research on the CRA and the VRA instructional sequences often included a maintenance phase to assess if students maintained their skills or knowledge after a short period of time, the primary focus of these interventions was acquisition (e.g., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, et al., 2018; Flores et al., 2014). Yet, learning mathematics is more than acquisition; acquiring the skills does not guarantee continued use (Shurr, Jimenez, & Bouck, 2019). Learning for students with disabilities can be conceptualized in four phases: acquisition, fluency, maintenance, and generalization (Shurr et al., 2019). However, researchers generally pay greater attention to the acquisition phase rather than the maintenance phase while teaching mathematics (Lafay, Osana, & Valat, 2019; Spooner et al., 2012). Further, the studies which include a maintenance phase do not assess the effects of EBPs on maintenance of mathematical skills among students with disabilities (Lafay et al., 2019).

Maintenance (i.e., the ability to use acquired skills without re-learning; Alberto & Troutman, 2009) in mathematics is particularly crucial since domains of mathematics are interconnected across grade levels and mathematical skills need to be applied in everyday life

(Common Core States Standards Initiative [CCSS], 2019; Saunders, Spooner, & Davis, 2018). For example, elementary students learn mathematical concepts such as counting, operations, and fractions, whereas secondary students learn more advanced mathematics which build on these basic concepts (e.g., ratios and proportional relation, algebra, and modeling; CCSS, 2019; Powell, Fuchs, & Fuchs, 2013). In addition, basic operations such as addition, subtraction, multiplication, and division are applicable to daily life (Cihak & Grim, 2008; Szekely, 2014). Maintaining acquired basic computation skills is important for individuals with disabilities to lead a successful post-school life (Nelson & Powell, 2018).

Some researchers suggested overlearning is an effective method to support maintenance of skills (Dougherty & Johnston, 1996; Shurr et al., 2019; Singer-Dudek & Greer, 2005). Overlearning is defined as the “repeated practice over time past the point of initial independent performance” (Shurr et al., 2019, p. 42). With overlearning, students have an opportunity to practice the same skills consistently (Alberto & Troutman, 2009). Driskell, Willis, and Copper (1992) conducted a meta-analysis regarding the benefits of overlearning in long-term retention of information for students in general. They found 15 studies that reported the effectiveness of overlearning for maintenance and concluded overlearning had a moderate effect on skill maintenance (Driskell et al., 1992). To date, no research studies have explored the effectiveness of overlearning to support maintenance in mathematics for individuals with disabilities.

Given that maintenance of mathematical skills is important for students with disabilities, more research is needed on methods that promote maintenance. As a part of this effort, the purpose of the current study is to evaluate the effectiveness of the VRA instructional sequence with overlearning in acquiring and maintaining basic operation skills (i.e., multiplication). The following research questions will guide this study: (a) What are the effects of the VRA instructional sequence with overlearning on students' mathematical acquisition?; (b) What are the effects of the VRA instructional sequence with overlearning on students' skill maintenance one, two, three, and four weeks after the intervention ends?; and (c) What perceptions do students with disabilities and their teachers have of the VRA instructional sequence with overlearning?

Method

Participants

Three students with disabilities (i.e., one student with autism and two students with learning disabilities) participated in this study. At the time of data collection, three of the students were receiving their mathematics instruction in a special education classroom. All of them were taught by special education teachers in a Midwestern state. Participants were chosen based on the following inclusion criteria: (a) teacher recommendation for students who were struggling with mathematics (i.e., below grade level); (b) parent consent and student assent; (c) demonstration of struggles with basic operations (i.e., below grade level) as shown on the KeyMath3 assessment administered by the researcher; and (d) fine motor ability to use iPad app manipulatives when a student is asked to drag and drop objects on the tablet screen. The researcher excluded any student who did not meet the above inclusion criteria.

Jake. Jake was sixth-grade Caucasian student who was 11-years-old at the time of data collection. He was identified as a student with an autism. Jake's standard scores on the Woodcock-Johnson Tests of Achievement IV (WJ-IV; Schrank, Mather, & McGrew, 2014) were as follows: basic reading skills (79), reading comprehension (<40), reading fluency (55), math calculation (76), written expression (57), and written language (75). According to the KeyMath-3 assessment (Connolly, 2007) administered by the researcher, Jake's numeration score was 12 (i.e., 1.8 grade equivalency) and his total operations score was 34 (i.e., 3.2 grade equivalency).

Owen. Owen was a 13-year-old, seventh-grade Caucasian male. According to his IEP, he was eligible to receive special education services as he was identified as having learning disabilities, especially in reading, reading comprehension, and math calculation. Owen's IEP indicated he also had ADHD. On his most recent WISC-IV assessment (Wechsler, 2004), Owen's full-scale IQ was 71. According to the most recent mathematics achievement data available in Owen's folder, he was in the 5th percentile on the Test of Early Mathematics Ability (TEMA-3; Ginsburg & Baroody, 2003). On the KeyMath-3 assessment (Connolly, 2007) administered by the researcher, Owen's numeration score was 23 (i.e., 4.8 grade equivalency) and his total operations score was 39 (i.e., 3.6 grade equivalency).

Henry. Henry was sixth-grade Caucasian student who turned 12-years-old during the time of the study. He was identified as a student with learning disabilities in his IEP. According to the most recent assessment, WISC-V (Wechsler, 2014), his Full-Scale IQ was 75. Henry's standard scores were not available on the Woodcock Johnson III test of achievement (Woodcock, McGrew, & Mather, 2011). However, his grade equivalency in the sub-tests were as follows: early reading skills (1.2), word reading (below 1.0), reading comprehension (below 1.0), spelling (K.5), numerical operations (2.0), math problem solving (2.1), math fluency—addition (1.9), math fluency—subtraction (1.4). On the researcher-administered KeyMath-3 (Connolly, 2007), Henry's numeration score was 17 (i.e., grade equivalency of 3.1) and his total operations score was 24 (i.e., grade equivalency of 2.5).

Setting

The participants were enrolled in three different public elementary or middle schools which were located at a distance of 1-hour (i.e., Jake), 30-mintues (i.e., Owen), and 15-mintues (i.e., Henry) from a public research university in a midwestern state. All sessions with Jake occurred in an empty classroom connected to his special education classroom. Owen's sessions were conducted in a hallway, which was equipped with one big table and chairs, in front of his special education classroom. For Henry, the sessions occurred in the classroom where he typically received his mathematics instruction while his classmates received other lessons. The researcher worked one-on-one with each student throughout the study (i.e., baseline, intervention, and maintenance).

Materials

Researcher-developed learning sheets constituted one of the materials used for this study, along with an iPad with app manipulatives, and pencils. In each session, the researcher used a learning sheet containing a modeling portion (two problems), a guided instruction portion (two problems), and an independent practice portion (five problems) consistent with prior studies of the CRA or VRA instructional sequence (Bouck, Park, Shurr, et al., 2018; Mercer & Miller, 1992). Each of the portions were on different sheets of paper stapled together. To develop learning sheets, the researcher listed all possible single-digit by single-digit and/or double-digit by single-digit multiplication problems and randomly assigned the problems to learning sheets. The problem sets presented in the independent practice portion of each learning sheet were unique. The set of problems used during modeling and guided instruction was not used again during the independent practice on the same learning sheet.

For the virtual phase, the iPad app, *Color Tiles* by Brainingcamp (2018) was used. This app was a virtual version of color tiles or polyomino concrete manipulatives. The *Color Tiles* (Brainingcamp, 2018) app included four different colors of tiles that could be manipulated (see Figure 4). In addition, four different colored pens were on the bottom of the iPad screen. The tiles could be arranged and grouped in the white space on the screen and the colored pens were used to write the corresponding numbers beside the tiles.

Independent and dependent variables

The VRA instructional sequence with overlearning was used as the independent variable. In the virtual phase, students used a virtual manipulative to solve targeted mathematical problems. During the representational phase, students used drawings to represent numbers to solve problems. In the abstract phase, students solved problems using a numerical strategy. The percentage of accuracy on solving the five multiplication problems during the independent portion of each learning sheet was used for the dependent variable.

Experimental design

The researcher used a multiple probe across participants design to evaluate the effectiveness of VRA instructional sequence with overlearning for acquisition and maintenance (Gast & Ledford, 2014). Each participant started baseline simultaneously, and a minimum of five sessions for baseline was conducted. When the first student (i.e., Jake) had a zero-celeration or a decelerating trend with stability across a minimum of five sessions (Gast & Ledford, 2014), he started with the virtual phase of the VRA instructional sequence and other students completed another baseline session. When Jake met mastery criterion for acquisition using the virtual manipulative (i.e., three sessions of 80% or 100%), the second student (i.e., Owen) entered the intervention phase. Likewise, once Owen met the mastery criterion, the last student, Henry completed a final baseline session and entered the intervention phase.

When each participant achieved 80% or 100% on three sessions during each of the virtual, representational, and abstract phases, the student completed two more sessions as part of overlearning. In previous VRA studies, researchers conducted three sessions per phase (i.e., virtual, representational, and abstract phases) resulting in a total of at least nine sessions. This study added *two more sessions* per phase (i.e., a total of at least 5 sessions in each phase and at least 15 sessions overall). The mastery criterion for these additional sessions was 100%. Once each participant acquired 80% or higher on the first three sessions and 100% on the last two sessions (i.e., overlearning), s/he could enter the next phase. If a student did not achieve 80% accuracy for a lesson on the first three sessions, then the student repeated the lesson where s/he scored lower than 80% during the next session (e.g., Mancl, Miller, & Kennedy, 2012). Likewise, if a student did not achieve 100% during overlearning sessions, the student repeated the lesson during the next session. Finally, four maintenance sessions were to be conducted one, two, three and four weeks after the last intervention session in the abstract phase.

Procedures

Baseline. The baseline for each participant involved at least five sessions. During baseline, each student answered five multiplication problems. No manipulatives, prompts, or cues were provided to students. The criterion for moving from baseline to intervention was zero-celeration or a decelerating trend with stability during the baseline (i.e., 80% of data falling within 25% of the median; Gast & Ledford, 2014).

Intervention. During intervention, the researcher used explicit instruction along with the VRA instructional sequence (see Figure 5), consistent with previous VRA studies (e.g., Bouck, Bassette, et al., 2017). During the modeling portion, the researcher demonstrated how to solve two problems either using an iPad app manipulative, drawings, or numerical strategy. During modeling, the researcher used a think-aloud, which is verbally stating the process of solving the problem. Next, the student solved two problems and, if needed, the researcher provided prompts or cues. For example, if a student made a mistake in counting tiles, then the researcher provided cues such as "did you count your tiles correctly?" If a student did not try to solve the problems, then the researcher gave a gesture prompt such as pointing to tiles on the iPad or a verbal prompt such as "set up tiles on your iPad." The researcher also provided feedback. If the student solved the problem correctly, then the researcher made a positive statement for the student (e.g., you solved it correctly). If the student did not solve the problem correctly, then the researcher noted where the error occurred and showed him/her how to perform it correctly during guided instruction (Mercer & Miller, 1992). During independent practice, the student solved five problems independently, meaning the researcher did not provide any help, prompts, or cues.

Virtual. During the virtual phase, the researcher used a virtual manipulative—*Color Tiles* (Brainingcamp, 2018)—to model setting up the problem, counting the tiles, and writing the answer. If the problem was 4 x 2, the researcher drew four circles on the iPad screen and dragged two tiles into each circle. Then, the researcher counted all the tiles to get the answer. After modeling two problems, each student was asked to solve two problems and the researcher guided (e.g., did you count tiles correctly? did you set tiles correctly? what's next?) as necessary. After this, each student solved five problems independently using virtual manipulatives.

Representational. During the representational phase, the researcher modeled how to solve problems using drawings. For multiplication, lines, squares, or circles were used to represent numbers on the learning sheet. For example, if the problem was 4 x 8, then four circles were drawn to represent the groups and eight squares or lines were drawn to represent the items. The researcher started by modeling two problems using drawings and then she provided prompts or cues as needed while the student solved two problems. Last, each student solved five problems independently using drawings.

***Abstract*.** The next five sessions involved the student using a numerical strategy to solve problems without any manipulatives or drawings. During the modeling portion, the researcher used repeated addition (e.g., 2 x 3 = 2 + 2+ 2) as a numerical strategy. Then, for the guided instruction, each student was provided two problems to solve using the numerical strategy and the researcher provided either prompts or cues if needed with feedback. Finally, the student moved to independent practice and solved five problems.

Maintenance. During maintenance, students solved problems for a total of four sessions. Starting one week after the students' last intervention session, one session was conducted for maintenance once a week for four weeks. Due to winter breaks and inclement weather, some students faced multiple delays of up to two weeks in between maintenance sessions. For each maintenance session, students were asked to solve problems using the numerical strategy independently, which was consistent with the baseline phase.

Treatment fidelity and inter-observer agreement

The researcher used a checklist to assess treatment fidelity for the intervention. The checklist included whether a student received correct materials per phase (i.e., virtual, representational, and abstract phase), and whether the researcher provided explicit instruction appropriately (i.e., modeling, guided instruction, independent practice). The researcher found 100% treatment fidelity for all students across all phases.

Interobserver agreement (IOA) was collected for a minimum of 30% for baseline, intervention, and maintenance for each student by two independent observers. The scores were calculated based on the accuracy of solving the mathematical problems (i.e., multiplication). In order to obtain an IOA percentage, total agreements were divided by the sum of agreements and disagreements, then multiplied by 100 (Gast & Ledford, 2014). IOA was 100% for all students.

Social validity

Social validity was assessed by interviewing the students and their teacher(s). During the interview, the students were asked about their perceptions of the VRA instructional sequence with overlearning. They were also asked which phase they preferred. The teacher was also interviewed about her perception of the VRA instructional sequence with overlearning as compared to the teachers' typical instruction method for teaching basic mathematics. Also, the teacher was asked about students' learning and maintenance of the skills they learned.

Data analysis

Visual analysis was the primary method for the analysis of the data in this study. The researcher analyzed the data using three steps to identify functional relation between the independent variable (i.e., VRA instructional sequence with overlearning) and the dependent variable (i.e., accuracy in solving problems). First, to determine the stability, the researcher calculated if 80% of the data fell within 25% of the median of each phase (Gast & Ledford, 2014). Second, the researcher determined the trend (i.e., zero-celerating, decelerating, accelerating trend) by using the spilt-middle method (White & Haring, 1980). The researcher found the mid-rate, mid-date, and middle point of the mid-rate and mid-date for each phase (i.e., baseline, intervention, and maintenance), and drew a line between the middle point of the mid-rate and mid-date to determine the trend (Gast & Ledford, 2014). Third, the researcher used Tau-

U to determine the effect size (Parker, Vannest, Davis, & Sauber, 2011). The Tau-U combines nonoverlap between phases (i.e., baseline and intervention) with the trend in the intervention phase, which means that it could be able to control for an undesired positive baseline trend (Parker et al., 2011). An online calculator was used to calculate the effect size (see http://www.singlecaseresearch.org/calculators/tau-u). If the effect size was 93% or higher, it suggested there was a large effect, 66-92% suggested a medium effect, and less than 66% suggested a small effect (Parker, Vannest, & Brown, 2009).

Results

The VRA instructional sequence with overlearning increased the percentage of accuracy for all students in solving problems independently. For all students, a functional relation was found between the VRA instructional sequence with overlearning and solving multiplication problems. All of them achieved scores at 80% or higher during the entire intervention and maintenance phases except for Owen's third virtual session (i.e., 60%).

Jake

During baseline, Jake answered one problem correctly across all five sessions. His baseline data were stable with a decelerating trend (see Figure 6). He experienced an immediate effect from the last baseline session and the first intervention session (i.e., 0% to 100%). Jake's overall average accuracy during intervention was 92.5% (i.e., virtual phase 90%, representational phase 96%, and abstract phase 92%). Jake repeated only one session during the intervention, which was observed during overlearning for the virtual phase. The intervention data were stable with zero-celeration. Jake maintained the skill for eight weeks after the final intervention session. The Tau-U effect size was 100% for his intervention phase and maintenance phase.

Owen

During baseline, Owen's data were stable with a zero-celeration trend (see Figure 6). His highest accuracy was 20% during the baseline. He experienced an immediate effect from the last baseline to the first intervention (i.e., 0% to 80%). Owen's overall average accuracy during intervention was 90.9% (i.e., virtual phase 84.4%, representational phase 96.6%, and abstract phase 94.2%). Owen repeated a total of seven sessions across all phases. Of the seven, six sessions were repeated during overlearning sessions: three during virtual, one during representational, and two during abstract phase. His intervention data were stable with an accelerating trend. Owen maintained the skill four weeks after the final intervention session. The Tau-U effect size for his intervention phase was 100%, and 100% for the maintenance phase.

Henry

Henry's baseline data were variable with a zero-celeration trend (see Figure 6). He could not solve multiplication problems correctly with consistency during baseline. However, Henry experienced an immediate effect from the last baseline session to the first intervention session (i.e., 0% to 100%). Henry's overall average accuracy during intervention was 96% (i.e., virtual phase 96%, representational phase 96%, and abstract phase 96%). Henry did not repeat any session during the entire intervention phase. His intervention data were stable with zero-celeration. Henry maintained the skill over the five weeks after the last intervention session. The Tau-U effect size was 100% for both intervention phase and for the maintenance phase.

Social validity

All students preferred to use the virtual manipulative as they enjoyed using the iPad and discussed that the virtual manipulative was easier to use for solving problems. Jake repeatedly asked where the iPad was during the representational phase. Henry mentioned that solving problems was easier when he could just drag tiles on the iPad. In addition, he said that using just the numerical strategy in the abstract phase also helped him in solving problems. Owen mentioned that learning multiplication was fun and that he needs to use this in everyday life. He also provided some examples of everyday uses such as when purchasing something in the market. Both Henry and Owen expressed that they would like to use an iPad to learn other mathematical skills in the future.

When the teachers were interviewed, all teachers stated that the VRA instructional sequence with overlearning helped their students to acquire and maintain the mathematical skill. In particular, one teacher mentioned that overlearning is useful and effective to support maintenance, given she is also using this method in her classroom as part of *Connecting Math*. Another teacher said that she finds overlearning is effective and she would like to use the method to support maintenance of mathematical skills, given that her students need a lot of reteaching to remember what they learned. With regard to the virtual manipulatives, two of the three teachers have access to iPads in their classrooms, but they were not sure what apps are effective in teaching mathematics. The teachers said that they would like to use *Color Tiles* app (Brainingcamp, 2018) in conjunction with this VRA intervention package and they would recommend this method to other practitioners.

Discussion

This study explored the effectiveness of VRA instructional sequence with overlearning to teach multiplication to three students with disabilities (i.e., autism and learning disabilities) and to promote maintenance. Consistent with previous studies that examined the VRA instructional sequence (i.e., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr, et al., 2018), all students showed immediate effects in the first session of the virtual phase. Jake and Henry improved from

0% in the last baseline session to 100% in the first intervention session while Owen showed an improvement from 0% to 80%. Based on visual analysis, a functional relation was found between the VRA instructional sequence with overlearning and students' accuracy of solving multiplication problems. All three students solved the problems with higher accuracy during the intervention as compared to baseline. In addition, all students maintained the mathematical skill at high levels of accuracy (i.e., 80% or 100%) for at least four weeks after the intervention ended.

In previous studies that examined the VRA instructional sequence, students acquired mathematical skills but many of the students struggled with maintaining them (i.e., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr et al., 2018). In the Bouck, Park, Shurr et al. (2018) study, the researchers used the VRA instructional sequence to teach basic operations to two middle school students with intellectual disability. The two participants acquired a combination of three mathematical skills (i.e., place value, addition, subtraction, and/or multiplication). However, the students struggled to maintain most of these skills. One of the students received 0% on both maintenance sessions for multiplication, which was the same as his baseline. Bouck, Bassette et al. (2017) also used the VRA instructional sequence to teach equivalent fractions to two students with developmental disabilities and one with learning disabilities. They found two of the students maintained the skill. However, both studies which explored the VRA instructional sequence (i.e., Bouck, Bassette, et al., 2017; Bouck, Park, Shurr et al., 2018) included only two maintenance sessions conducted two weeks after the intervention ended. Hence, it is unknown whether the students in the previous studies would have maintained the skills for a longer time. In the current study, due to multiple delays, the maintenance phase extended between five to eight weeks for all of the students and students maintained the skill with 80% or 100% accuracy even at eight weeks.

In this study, the maintenance of multiplication skills for a longer period was supported by the overlearning method added to the typical VRA instructional sequence. Previous researchers discussed overlearning could be achieved by practicing the skill even after achieving the predetermined criterion by adding more intervention sessions (Baldwin & Ford, 1998; Binder, 1987). Hagman and Rose (1983) suggested students' maintenance may be supported by setting the criterion for the additional practice sessions higher than the acquisition criterion. However, despite discussions on the use of overlearning to support maintenance of skills, limited empirical research existed on the use of overlearning in teaching mathematics prior to the current study. In this study, at least two sessions were added to each phase of the VRA instructional sequence and the criterion for overlearning sessions was increased from 80% to 100%. As the results of this study suggest, including additional sessions with a higher mastery criterion can result in students maintaining the mathematical skill. While overlearning supported the maintenance of multiplication skill, one of the students also expressed frustration about repeating sessions. This suggests a possible trade-off between the effectiveness and social validity of overlearning.

Implications for practice

This study suggests the VRA instructional sequence is a viable option to support mathematical skill acquisition among secondary students with autism and learning disabilities. Previous research supported the effectiveness of the CRA instructional sequence for students with learning disabilities (Bouck, Satsangi, & Park, 2018). However, practitioners may be concerned that students who need instruction in basic operations even at the secondary level would find concrete manipulatives stigmatizing (Satsangi & Bouck, 2015). Educators could use the VRA instructional sequence to teach basic mathematical skills such as multiplication to secondary students with disabilities.

Another implication involves maintenance of skills using the VRA instructional sequence with overlearning. When students with disabilities face difficulties in maintaining skills, teachers need to reteach concepts over and over (Shurr et al., 2019). Further, maintenance of basic operation skills is important for the acquisition of higher grade-level mathematical skills (Powell et al., 2013). So, educators who are concerned about their students' maintenance of mathematical skills may consider using the overlearning to build foundational skills, in- and of-itself as well as to promote learning of grade-level contents, such as algebra and geometry, among secondary students with disabilities (Powell et al., 2013).

Limitations and future directions

Although this study found positive results, there are still limitations. First, given this study occurred in a one-on-one setting with a researcher, the effect of the intervention in other settings, such as small group instruction or whole group classroom setting, is unknown. To generalize the effectiveness of VRA intervention package, future researchers should replicate this study in other settings. Second, since this study focused on skill acquisition and maintenance, it is unknown if students can generalize the skills into actual daily living settings or word problems. Future researchers need to assess whether students can generalize the skills into other forms of problems. Third, in this study, the dependent variable was measured based on researcher-created work sheets. Future researchers need to assess student accuracy using standardized tests to establish the reliability of VRA intervention package. Fourth, the current study only used overlearning in conjunction with the VRA instructional sequence, so future research needs to explore what other methods can support students' acquisition as well as maintenance. In addition, this study explored only one numerical strategy (i.e., repeated addition). Future researchers could assess the effectiveness of other numerical strategies such as partial product multiplication to offer a variety of strategies to students. Finally, although one maintenance session was scheduled per week, due to out-of-control circumstances (e.g., snow days, absences), the duration between maintenance sessions was not the same for all students.

Figure 4. Virtual Manipulatives

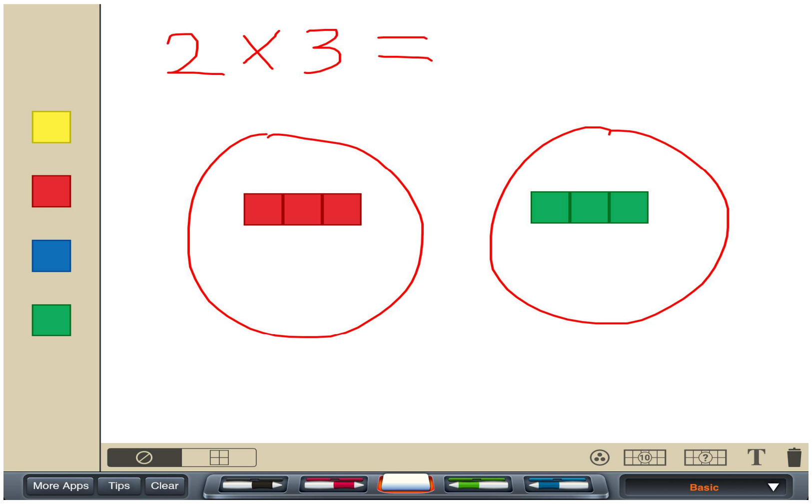
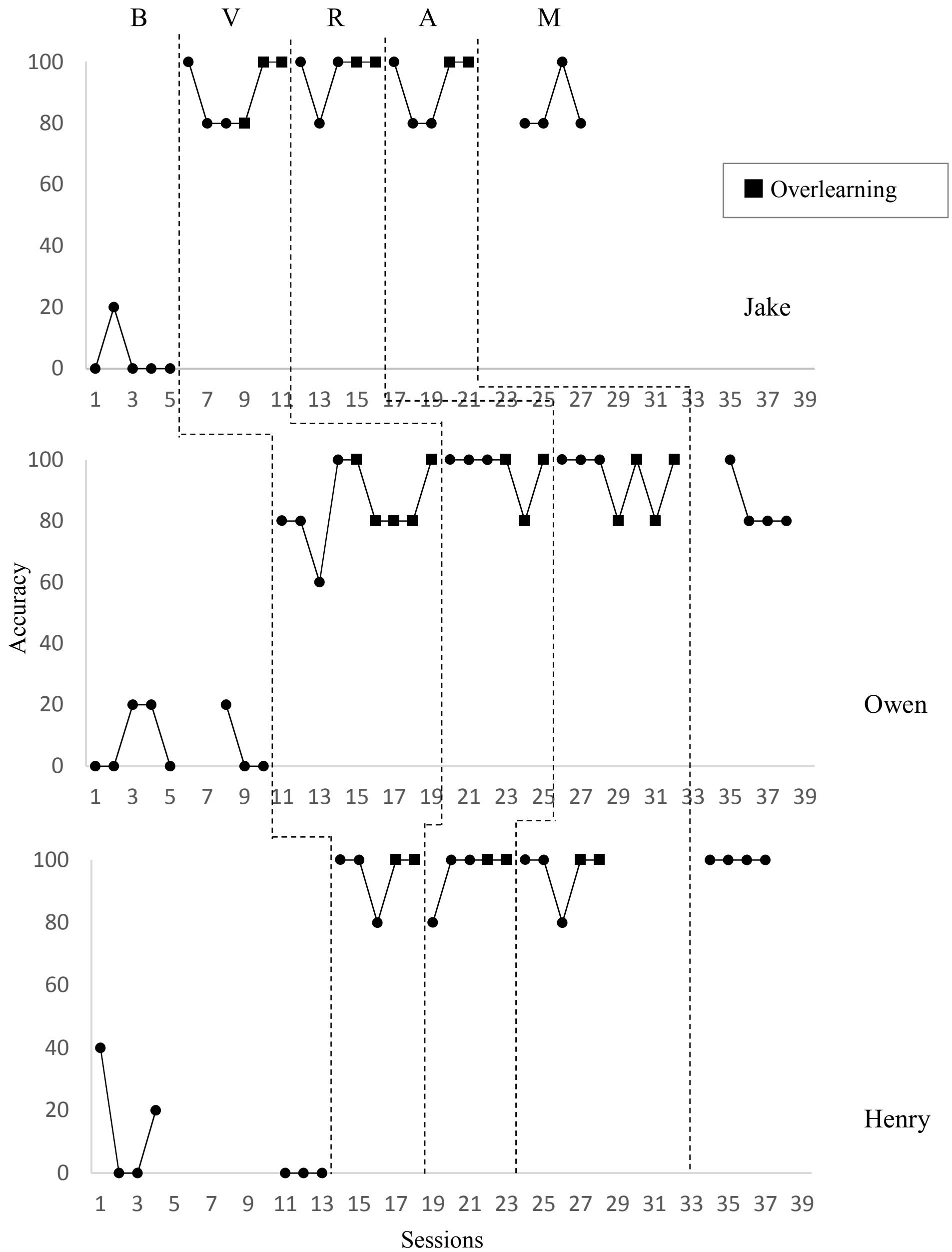


Figure 5. Explicit Instruction

Virtual Phase		
Modeling	Guided instruction	Independent practice
This problem is 3 × 8. To solve this problem, we are going to use the *Color Tiles* App. Multiplying three and eight means 3 groups of 8. First, I need three groups, so I am going to draw three circles to represent my three groups. Then, I am going to pull out 8 color tiles and put in each group. Now, do I have three groups of 8? Yes, I do. So, how many total color tiles have I used? (count them all) I have 24 color tiles. So, my answer is 24. Also, I can do repeated addition for multiplication. I know I have 3 groups of 8. So, I can do 8+8+8 to get the answer.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need, but try to solve them on your own, alright? [Student begins. If, for example, a student does not set up the color tiles accurately – such as having only 7 color tiles in a problem of 3 x 8 – the instructor might cue "Check your number of tiles. Do you have the correct number of color tiles?"]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best! I am still here, but I will not provide any help.
Representational Phase		
Modeling	Guided instruction	Independent practice
Now, I am not going to use iPad, but use drawings. This problem is 4 × 2. This means I have four groups of 2. I am going to draw two circles which represent number 2. I have four groups, so I am going to draw two circles three more times. Now I have one, two, three, four groups of two. How many circles do I have? One, two, three, four, five, six, seven, and eight. So, my answer is 8.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need but try to solve them on your own. [Student begins, If, for example, a student draw circles more than the problem, the instructor might point the number on the learning sheet]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best. I am still here, but I will not provide any help.
Abstract Phase		
Modeling	Guided instruction	Independent practice
Now I am not going to use an iPad. This problem is 7 × 6. To solve this problem, I am going to use numerical strategy. First, 7 x 6 means 7 groups of 6. Multiplication is repeated addition. So, I am going to add 6 seven times. (6+6+6+6+6+6+6). The answer is 42.	Now that I have showed you how to solve the two problems, it's your turn! I am going to have you try two problems yourself. I am here to provide any help that you may need, but try to solve them on your own, alright? [Student begins. If, for example, a student omits numbers – such as counting 6 only six times in a problem of 7 x 6 – the instructor might cue "Check your number. Did you add 6 seven times?"]	You have tried a couple of problems. Do you feel confident to solve these five problems on your own? Try your best. I am still here, but I will not provide any help.

Figure 6. Accuracy Percentage of Solving Multiplication Problems



Note. B refers to baseline; V refers to virtual; R refers to representational; A refers to abstract; M refers to maintenance

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CHAPTER 5

DISCUSSION

The three studies in this dissertation explored the maintenance of mathematical skills for students with disabilities. The dissertation includes a review of literature and two intervention studies using the VRA instructional sequence—one with fading support and one with overlearning. The systematic review of literature in Chapter 2 explored the extent to which researchers focused on skill maintenance in teaching mathematics for students with developmental disabilities, including intellectual disability and autism. A limited number of researchers included a maintenance phase after the mathematics intervention ended in published research. Chapter 3 presents an intervention using a single-case experimental design (i.e., multiple probe across participants), which examined the effectiveness of the VRA instructional sequence with fading support in teaching subtraction with regrouping to four secondary students with intellectual disability and autism. Students acquired the skills and maintained them for up to six weeks. Chapter 4 also reports on a single-case experimental design (i.e., multiple probe across participants) that explored the effectiveness of the VRA instructional sequence with overlearning in teaching multiplication to three middle school students with autism and learning disabilities. The VRA instructional sequence with overlearning was effective in teaching multiplication as all students acquired the skill and maintained it up to eight weeks after the intervention ended. Both intervention studies showed a functional relation between the intervention packages and students' acquisition and maintenance of the mathematical skills (i.e., subtraction with regrouping and multiplication).

The main result of this dissertation is that intervention packages, which include more than one instructional method or combine an instructional method with an instructional material, are effective in supporting maintenance for students who need significant instruction in mathematics. Across the dissertation, intervention packages were found to not only support student acquisition of mathematical concepts or skills, but more importantly, maintenance. When students needing significant support in mathematics received an intervention package targeting a mathematical skill or concept, the maintenance data were more positive. Given the importance of maintenance in teaching and learning of students, inclusive of students with disabilities, researchers and practitioners should focus on intervention packages in mathematics.

Current State of Research on Skill Maintenance in Mathematics

Despite increasing attention on teaching mathematics to students with disabilities (e.g., Hudson, Rivera, & Grady, 2018; Kiru, Doabler, Sorrells, & Cooc, 2018; Spooner, Root, Saunders, & Browder, 2018), the existing research for students with developmental disabilities tends to focus on mathematical skill acquisition rather than skill maintenance. Maintenance is especially important in teaching mathematics given that mathematical content builds upon previous content and mathematical skills are widely applied in everyday life (Geary, Nicholas, Li, & Sun, 2017; Powell, Fuchs, & Fuchs, 2013; Szekely, 2014). Researchers need to focus more on supporting maintenance of skills among students with disabilities.

In this dissertation, the researcher identified that intervention packages support students' maintenance of skills. Although individual contributions of each method or material in the package toward the enhancement of mathematical skill maintenance is unknown, the results of this dissertation suggest that intervention packages are promising for acquisition as well as maintenance for students who need significant instruction in learning mathematics. More experimental research needs to focus on explicitly examining methods and materials in intervention packages to support maintenance of skills among students with disabilities.

Effective Instructional Methods to Support Students' Maintenance

As noted, maintenance of mathematical concepts is important for students with disabilities; it is not sufficient to just acquire mathematical contents and skills, they also need to be able to apply those skills when instruction is not available (Geary et al., 2017; Powell et al., 2013; Shurr, Jimenez, & Bouck, 2019). As the systematic review suggests, certain intervention packages may lend themselves useful in supporting maintenance of mathematical skills among students with disabilities, such as prompting with fading support and explicit instruction with visual supports or manipulatives, although the effects of these interventions varied across mathematical contents.

Another effective method for supporting maintenance of mathematical skills for students with disabilities is the use of a graduated sequence of instruction in combination with overlearning or fading support, as suggested by the experimental studies in this dissertation. Specifically, both intervention packages (i.e., the VRA instructional sequence with overlearning and the VRA instructional sequence with fading support) resulted in student maintenance of basic operation skills up to six or eight weeks after intervention ended. Findings from the systematic review suggested the use of technology in teaching mathematics was not generally effective to support maintenance of skills. For example, previous researchers using the VRA instructional sequence found that students struggled to maintain basic operations and fractions skills (Bouck, Bassette, et al., 2017; Bouck, Park, et al., 2018). However, when fading support or overlearning were added to the VRA instructional sequence, all the students maintained the skills. This suggests fading support and overlearning in combination with the VRA instructional sequence could contribute to the maintenance of mathematical skills among students with disabilities.

Fading support

Fading support could contribute to maintenance by facilitating a gradual shift from teacher-initiated supports to student-initiated learning. The systematic review suggests one widely used instructional method in teaching mathematics was prompting, and when prompting was paired with fading, students were more likely to maintain with higher accuracy (Jimenez, Courtade, & Browder, 2008; Jowett, Moore, & Anderson, 2012; Smeets, Lancioni, & Striefel,1987; Trace, Cuvo, & Criswell, 1977; Zisimopoulos, 2010). Although prompting generally refers to methods such as simultaneous prompting and the system of least prompts, the modeling and guided instruction portions of explicit instruction could be regarded as prompting as they involve demonstrations, explanations, and guidance (Kiru et al., 2018). Hence, explicit instruction could also be faded like prompting in order to achieve a gradual shift from teacher-initiated supports to student-initiated learning (Shurr et al., 2019). To increase independency and accuracy in solving problems, gradually fading the explicit instruction is the key for students becoming more independent and maintaining the skills after the instruction is no longer available.

Another mechanism through which fading support contributes to maintenance of skills is by providing additional opportunities to practice the skills (Yakubova, Hughes, & Hornberger, 2015; Shurr et al., 2019). Given that maintenance involves students consistently performing what they have acquired over time without relearning (Shurr et al., 2019), previous researchers suggested students with disabilities may need longer durations of intervention (Yakubova et al., 2015). By adding a fading support phase in addition to the typical VRA instructional sequence, the total number of sessions was extended from nine to 15 sessions. While it is unknown whether the results were associated with the opportunity to practice the skill for a longer period (than typical) or the gradual fading of explicit instruction, adding fading support was an effective method for supporting students' maintenance of mathematical skills.

Overlearning

Overlearning is defined as the practice of a skill beyond successful performance (Baldwin & Ford, 1998). Bloom (1986) suggested overlearning could develop students' automaticity and result in improved maintenance. However, there is some contention on the definition of overlearning among researchers. Some researchers discussed that a higher number of practice sessions (not the level of performance) can be referred to as overlearning (e.g., Hagman & Rose, 1983), while others suggest overlearning sessions could have a higher mastery criterion (Baldwin & Ford, 1998). As noted above, Yakubova et al. (2015) suggested students with disabilities may require additional practice to maintain skills, which indicates maintenance could be supported by simply increasing the number of intervention sessions. On the other hand, students in this dissertation received additional practice opportunities while attempting to meet the higher mastery criterion. For example, one of the students in the VRA instructional sequence with overlearning study repeatedly solved the same learning sheet for three sessions (i.e., to achieve 100% accuracy) although the student consistently received 80% accuracy. So, it is challenging to discern whether increasing the number of sessions or the higher mastery criterion contributes to maintenance of skills.

Implications for Practice

This dissertation offers some overall implications for practice. Given the VRA intervention packages in this dissertation were effective for students' maintenance of skills up to eight weeks after the intervention ended, practitioners may want to consider incorporating the fading support or overlearning methods in their classrooms to support skill maintenance. In the social validity interviews from the experimental studies, teachers mentioned they felt frustrated that students easily forgot what they learned, especially when breaks occurred in the instruction (e.g., winter break, summer break). If students are struggling with maintaining basic operations skills, using these VRA intervention packages could be beneficial.

Another implication is that the VRA intervention packages used in this dissertation (i.e., the VRA instructional sequence with fading support or overlearning) are efficient methods. These intervention packages require a minimum of 15 sessions per skill, which can be implemented in about 3 weeks if practitioners teach the skill daily. As numbers and basic operations form a majority of annual math goals for students with disabilities (e.g., Kurth & Mastergeorge, 2010), practitioners could teach basic mathematical skills to students with disabilities in a relatively short amount of time using these VRA intervention packages.

A final implication is related to the use of technology in instruction to promote engagement in mathematics learning. All participants mentioned they preferred to use virtual manipulatives on an iPad. Given that students enjoy working with an iPad, practitioners may want to increase students' motivation and engagement in learning mathematics by using virtual manipulatives. If practitioners have access to virtual manipulatives, the VRA intervention packages used in this dissertation (i.e., VRA instructional sequence with fading support or overlearning) could be effective methods to teach mathematical skills. If teachers do not have access to iPads, they can use the National Library of Virtual Manipulatives website on computers. This website allows online access to virtual manipulatives for free (see nlvm.usu.edu).

Limitations and Future Directions

There are several limitations of this dissertation. The first limitation is that it is difficult to generalize the results of this dissertation to all students with disabilities. The systematic review of literature focused only on studies conducted with students with developmental disabilities and the experimental studies only explored the effectiveness of fading support and overlearning methods with a limited number of students with autism, learning disabilities, and intellectual disability. Future research needs to explore the inclusion and characteristics of maintenance for students with other disabilities and needs to assess the effectiveness of intervention packages targeting maintenance using group design. Second, this dissertation could not identify individual effects of instructional methods and materials on skill maintenance as researchers employed several instructional methods and/or materials as a part of intervention packages. The individual effects of fading support and overlearning methods on maintenance are also unknown as they were offered as a part of intervention packages. Future researchers should seek ways to identify individual effects of methods and materials used in packages. Third, this dissertation mainly focused on basic operations. Although basic operations are foundational mathematical skills that support the learning of more advanced mathematics, more experimental research is needed to examine effective strategies to support mathematical contents other than numbers and operations. Finally, this dissertation only assessed the effectiveness of two methods—fading support or overlearning in conjunction with the VRA instructional sequence—in supporting students' maintenance. Future research should identify other methods and materials which can support students' maintenance of mathematical skills.

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