KNOT CONCORDANCES IN 3-MANIFOLDS

By

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ABSTRACT

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We deal with some questions regarding concordance of knots in arbitrary closed 3-manifolds. We first prove that, any non-trivial element in the fundamental group of a closed, oriented 3-manifold gives rise to infinitely many distinct smooth almost-concordance classes in the free homotopy class of the unknot. In particular, we consider these distinct smooth almost-concordance classes on the boundary of a Mazur manifold and we show none of these distinct classes bounds a PL-disk in the Mazur manifold. On the other hand, all the representatives we construct are topologically slice. We also prove that all knots in the free homotopy class of $S^1 \times pt$ in $S^1 \times S^2$ are smoothly concordant.

To the Gökova Geometry Topology Conferences.

ACKNOWLEDGEMENTS

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CHAPTER 1

INTRODUCTION

1.1 Overview

This thesis is focused on understanding different notions of knot concordance (topological concordance, PL-concordance, and smooth concordance) in general 3-manifolds, and classifying knots in terms of concordance.

Knot concordance and the resulting knot concordance group were first studied by Fox and Milnor [FM66] in the context of classical knot theory; that is, the theory of embeddings of a circle S^1 into S^3. Although the definition of concordance extends naturally to arbitrary 3-manifolds, several important differences arise when working in this broader setting, and until recently little systematic work had been conducted in this vein. Motivation for the development of concordance theory in a broader context is well motivated by a swath of beautiful applications of the classical case to the difficult subject of 4-manifolds. For instance, many constructions of exotic copies of $\mathbb{R}^4$ and the recent construction of Akbulut-Ruberman type of absolutely exotic compact 4-manifolds [AR16] rely on concordance in a fundamental way.

Concordance is an equivalence relation on knots in a 3-manifold. Specializing to the particular case of knots in S^3 leads to interesting further structure: concordance classes of knots in S^3 forms an abelian group under the operation induced by connected sum. This latter structure does not extend to concordance classes in a general 3-manifold; indeed, the connected sum operation changes the ambient manifold. There is, however, an action of the concordance group of knots in S^3 on the set of concordances classes of knots in an arbitrary 3-manifold given by connected sum. This action was studied by Celoria [Cel18], where he introduced the terminology *almost-concordance* to denote the quotient of the set of concordance classes of knots in Y by this action. Shortly after the notion of almost-concordance had been introduced, it was noticed that Almost-concordance and PL-concordance are equivalent definitions. In this thesis, we investigate natural classification problems for knots in 3-manifolds up to concordance and almost-concordance. We attack these problems with a two-pronged approach (1) by constructing concordances in 4-manifolds and (2) by finding obstruction to smooth or PL-surfaces in 4-manifolds using topological techniques.

1.2 Results and methods

Zeeman conjectured that there exists a contractible 4-manifold W^4 and a loop $\alpha \in \partial W$ such that α doesn't bound an embedded PL-disc in W [Zee63]. Akbulut verified the conjecture in [Akb91] using gauge theory. As a consequence of his theorem, we have a 3-manifold which is ∂W and a loop in ∂W which can't be almost-concordant to the unknot. Inspired by this work, we are able construct an infinite family of knots as in Figure 1.1, in the free homotopy class of the unknot with the same property. Furthermore we show that all these knots α_n bound a locally flat topologically embedded disk in the contractible ball W^4 by combining Freedman's result with a calculation of Alexander polynomials of knots.

Theorem 1.2.1 ([Yil18])**.** There is a contractable 4-manifold W^4 and family of knots $\{\alpha_n | n \in \mathbb{Z}^+\}$ in the boundary manifold ∂W, such that, α_n doesn't bound a PL-disk, but bounds a locally flat topological disk in W^4.

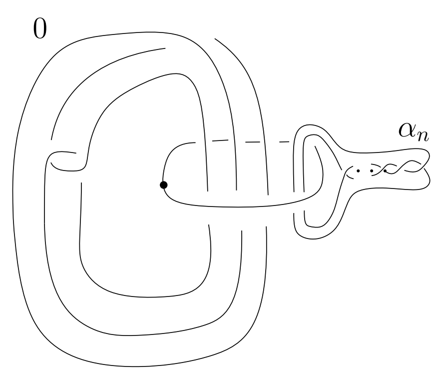

Figure 1.1 W^4

Next we are interested to distinguish these knots in terms of almost concordances. A particular concordance invariant introduced by Schneiderman [Sch03], denoted μ, stands out as a candidate tool for this question; indeed, μ applies to knots in arbitrary closed 3-manifolds and vanishes on knots in S^3. This invariant arises from a nice application of Wall's intersection number [WR99], lying in a quotient of the group ring on the fundamental group of a 4-manifold. The invariant is a perfect candidate to investigate almost-concordance and indeed we prove that it is an invariant of almost-concordance. This allows us to prove

Theorem 1.2.2 ([Yil18])**.** Given a closed 3-manifold Y, any non-trivial element $g \in \pi_1(Y)$ can be used to construct infinitely many distinct almost-concordance classes in the free homotopy class of the unknot. If $h \notin \{g, g^{-1}\}$, then the almost-concordance classes constructed using g and h are disjoint.

Using Theorem 1.2.2 we can distinguish all α_n's in Figure 1.1 in terms of almost-concordance. A similar result to Theorem 1.2.2 was obtained for lens spaces $L(p, 1)$ by Celoria [Cel18] using shifted τ-invariant coming from Heegaard Floer theory. His paper was the motivation for the work in [Yil18].

It is worth remarking that this theorem does not say anything about whether a knot in the boundary 3-manifold bounds a PL-disk in the 4-manifold or not. The theorem shows that the trivial homotopy class is represented by non-trivial almost-concordance classes in *any* closed 3-manifold (except for the 3-sphere). This raises a natural question. Can we find a 3-manifold with a free homotopy class that is represented by a unique knot, up to almost-concordance? The following theorem provides a positive answer.

Theorem 1.2.3 ([Yil18])**.** All knots in the free homotopy class of $S^1 \times \{pt\}$ in $S^1 \times S^2$ are smoothly concordant, i.e., $|C_x(S^1 \times S^2)| = 1$ where x represents $S^1 \times \{pt\}$ in $S^1 \times S^2$.

We give a constructive proof to the Theorem 1.2.3. We introduce a genus zero cobordism move and we use handlebody techniques for the construction.

Theorem 1.2.2 and Theorem 1.2.3 are also obtained in [FNOP19] in the topological category.

CHAPTER 2

PRELIMINARIES

2.1 Definitions, conventions, and notations

We consider manifolds that are smooth and oriented. Let Y be a closed, connected, oriented 3-manifold. A *knot* k in Y is an isotopy class of a smooth embedding $S^1 \hookrightarrow Y$. Two knots k_1 and k_2 are said to be *concordant* if there is a smooth proper embedding of an annulus $F : S^1 \times [0,1] \hookrightarrow Y \times [0,1]$, such that its boundary is $\partial F(S^1 \times [0,1]) = (-k_1) \times \{0\} \cup k_2 \times \{1\}$ where $(-k_1)$ is the same knot k_1 with the reversed orientation. If we allow F to have only finitely many singular points, all of which are cones over knots, then k_1 and k_2 are called *PL-concordant*. We call these knots *singularly concordant* if we allow F to be an immersion instead of an embedding. Two knots are singularly concordant if and only if they are freely homotopic. One can see this fact by using the Immersion Theorems and general position arguments (these can be found in [Hir12]) on the trace of homotopy. Concordance is an equivalence relation "$\sim$" on the set of oriented knots in Y. The set of equivalence classes is denoted by;

$$C(Y) = \{\text{Oriented knots in Y}\} / \sim .$$

Concordant knots k_1 and k_2 are freely homotopic, hence they are homologous. In [Cel18] Daniele Celoria defines the concept of almost-concordance of knots. Two knots k_1 and k_2 in Y are said to be *almost-concordant* if there are $k_1^{'}, \ k_2^{'} \subset S^3$ such that $k_1 \# k_1^{'} \sim k_2 \# k_2^{'}$, and this is expressed by $k_1 \dot{\sim} k_2$. Like concordance, almost-concordance is an equivalence relation, and it implies free homotopy of knots.

We denote almost-concordance classes by $\widetilde{C}(Y)$. More generally;

$$C_\gamma(Y) := \mathcal{K}_\gamma(Y)/\sim, \quad \widetilde{C}_\gamma(Y) := \mathcal{K}_\gamma(Y)/\dot{\sim},$$

where $\mathcal{K}_\gamma(Y)$ is the set of knots in free homotopy class γ in Y.

A knot $k \in Y^3 = \partial W^4$ is called *topologically slice* if it bounds a locally flat topologically embedded disk D in the 4-manifold W^4. It is called *smoothly slice* if D is smoothly embedded, and *PL slice* if D is a smooth embedding away from finitely many cone singularities.

2.2 Wall's Self Intersection Number, and a Concordance Invariant

There are many approaches to the knot concordance problem; here we focus on a classical technique. This technique is based on Wall's intersection number [WR99]. The application of this idea to knot concordance was studied in [Sch03] by Schneiderman.

Let k be a null-homotopic knot in Y; consider a singular concordance of k to the unknot u. After capping the unknot with a disk, we get a proper immersion of a disk $D \looparrowright Y \times I$ with $k = \partial D$. Let p be a transverse self-intersection of the immersion D; then any small neighbourhood of p looks like two surfaces intersecting at p. These surfaces are called *sheets*. The self-intersection number of k, defined as Wall's self intersection number of D, takes its value in the group ring $\mathbb{Z}[\pi_1 Y]$. To define this self-intersection number we first fix a path from the basepoint y_0 of $Y \times I$ to a basepoint of the immersed disk D, called a *whisker* of D. Now $g_p \in \pi_1(Y, y_0)$ is defined in the following way: it is a loop starting from y_0 going to the basepoint of D using the whisker, then to the self-intersection point p of D, then changing the sheet at the intersection point, going back to the basepoint of D, and finally to y_0 using the whisker. Then

$$\mu(k) := \mu(D) = \sum_p sign(p) \cdot g_p \in \mathbb{Z}[\pi_1 Y].$$

Since D is simply connected, the loop g_p does not depend on the path we choose while travelling on D as long as it stays away from self-intersection points. The value of $sign(p)$ is $+1$ if the orientation of $Y \times I$ at p matches with the orientation induced from sheets of D at p, and it is -1 otherwise. After fixing the whisker there is still an indeterminacy coming from the choice of the first sheet. Altering this choice changes the loop from g_p to g_p^{-1}. Also, self-intersection points coming from cusp homotopies give elements which are trivial in $\pi_1(Y)$. Since we are interested in a homotopy invariant, we also quotient out these elements, arriving at the abelian group

$$\widetilde{\Lambda} := \frac{\mathbb{Z}[\pi_1 Y]}{\{g - g^{-1} \mid g \in \pi_1(Y)\} \oplus \mathbb{Z}[1]}.$$

Here $\mathbb{Z}[1]$ is the abelian subgroup generated by the trivial element of $\pi_1(Y)$. Homotopy invariance in the above discussion follows from the following two Propositions.

Proposition 2.2.1 (Chapter 1.6 of [FQ14]). A homotopy between immersions of a surface in a 4−manifold is homotopic to a composition of homotopies, each of which is a regular homotopy or a cusp homotopy in some ball, or the inverse of a cusp homotopy.

Proposition 2.2.2 (Chapter 1.7 of [FQ14]). Intersection numbers and reduced self intersection numbers in $\widetilde{\Lambda}$ are invariant under homotopy rel boundary. The $\mathbb{Z}[1]$ component of the self intersection number is invariant under regular homotopy, and conversely two immersions of a sphere or disk which are homotopic rel boundary, and have the same framed boundary, are regularly homotopic rel boundary if and only if the $\mathbb{Z}[1]$ components of the self intersection numbers are equal.

Now we state and prove Schneiderman's knot concordance invariant.

Theorem 2.2.3 ([Sch03]). The map

$$\begin{array}{rccc} \mu: & C_1(Y) & \to & \widetilde{\Lambda} \\ & k & \mapsto & \mu(k) \end{array}$$

is well defined and onto.

Proof. We recall the proof from [Sch03].

Well Defined Let D and $D^{'}$ be singular null-concordances of a knot k, taking a singular sphere $S = D \underset{k}{\cup} D^{'} \subset Y \times I$ gives $S \in \pi_2(Y \times I) \cong \pi_2(Y)$. By [Hat00] Proposition 3.12, there exists a disjoint collection of embedded 2-spheres generating $\pi_2(Y)$ as a $\pi_1(Y)-$module. Tubing these generators together in $Y \times I$ we get an embedded sphere in $Y \times I$. This implies

$$\mu(S) = 0 = \mu(D) - \mu(D^{'}),$$

therefore $\mu(k)$ doesn't depend on D.

Concordance Invariance If $k_1, k_2 \in C_1(Y)$ and $k_1 \sim k_2$ then $\mu(k_2) = \mu(C \cup D) = \mu(D) = \mu(k_1)$ where C is a concordance from k_1 to k_2, and D is the singular concordance of k_1.

Surjectivity To construct $\pm g \in \mathbb{Z}[\pi_1(Y)]$ start with an unknot u and push an arc from u around a loop representing $g \in \pi_1(Y)$ and create a $\pm$ clasp as in Figure 2.1. Iterating this process one can get any desired element in $\mathbb{Z}[\pi_1(Y)]$ via connected summing of such knots. □

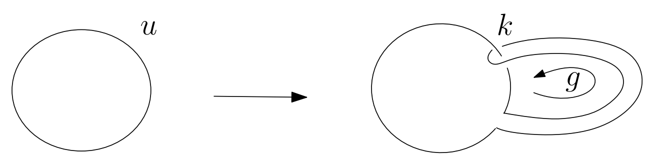

Figure 2.1 $\mu(k) = g$

CHAPTER 3

PROOF OF THEOREM 1.2.1

3.1 PL-Slice

The notion of almost-concordance is same as the PL-concordance in $Y \times I$. Indeed, if k_1 and k_2 are PL-concordant then we may assume, without loss of generality that the concordance has only one singular point which locally looks like a cone over a knot k. It is easy to see $k_1\#(-k)$ is smoothly concordant to k_2 by removing a ball around the cone point and connecting two boundary components by removing a neighbourhood of an arc lying on the concordance connecting k_1 to k. On the other hand if we have an almost concordance between k_1 and k_2 i.e., $k_1\#k_1'$ is concordant to $k_2\#k_2'$, then push the local knots inside the 4-manifold and take the cone over the knots in some local ball to get a PL-concordance. This tells us the family of knots we construct in Example 1 in particular in Figure 4.2 can not bound a PL-disk in the collar of the manifold. It can still, however, bound in a 4-manifold which Y bounds.

Next we see that none of these family members $\{\alpha_n\}$ in Figure 3.1 bounds a PL-disk in the Mazur manifold W^4.

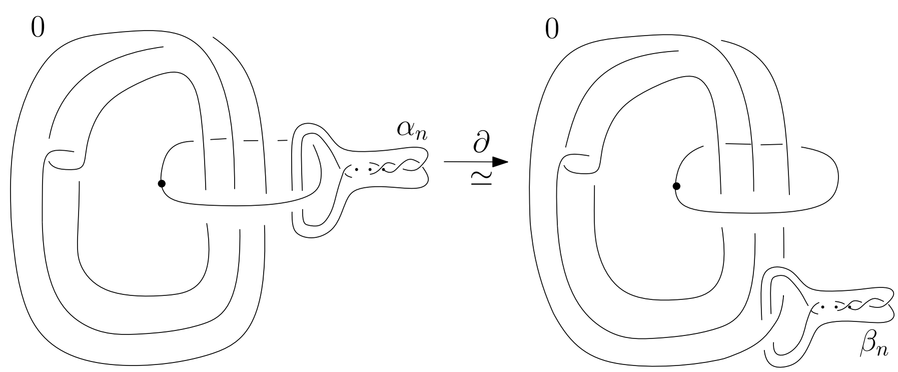

Figure 3.1 Boundary diffeomorphism

Here we use Akbulut's techniques [Akb16]. Observe that W^4 is a Stein domain by [Eli90]. Consider the boundary diffeomorphism which takes α_n to β_n as in Figure 3.1, using $0 \leftrightarrow \bullet$ exchange and symmetry of the link surgery diagram of Mazur manifold. The knot β_n is smoothly slice. To see that α_n is not slice we use the adjunction inequality as in [AM97]. Let $F \subset W^4$ be a properly embedded oriented surface in a Stein domain, such that $k = \partial F \subset \partial W^4$ is a Legendrian knot with respect to the induced contact structure.

Let f denote the framing of k induced from the trivialization of the normal bundle of F; then

$$-\chi(F) \geq (tb(k) - f) + |rot(k)|.$$

Recall that the rotation number $rot(k)$, and the Thurston-Bennequin number $tb(k)$ are given by the formulae

$$rot(k) = \frac{1}{2}(\text{number of “downward” cusps} - \text{number of “upward” cusps}),$$

$$tb(k) = bb(k) - c(k),$$

where $bb(k)$ is the blackboard framing (or writhe) of the front projection of k, and $c(k)$ is the number of right cusps.

Assume the curve α_n is slice, so $\chi(F) = 1$. By Figure 3.2, $tb(\alpha_n) = 2n - (2n - 1) = 1$, and $rot(\alpha_n) = 0$. The framing is $f = 0$. We have a contradiction: $-1 \geq 1$, hence α_n is not slice.

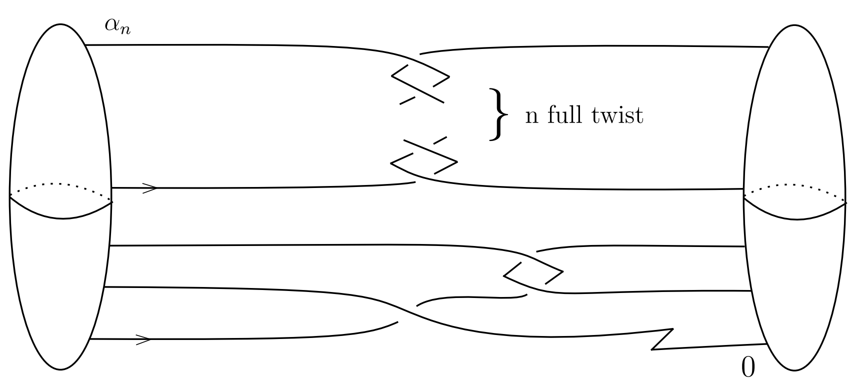

Figure 3.2 Stein handlebody of W^4

The same argument as in [[Akb91], Theorem 1] shows α_n does not bound a PL-disk in W^4.

3.2 Topologically Slice

Here we show that the family of knots that we constructed in the previous example are all topologically slice and therefore they are all distinct elements in the almost-concordance class of topologically slice knots on the boundary of the Mazur manifold.

A knot k in a homology sphere Y has well-defined Alexander polynomial $\Delta_k(t) \in \mathbb{Z}[t^{\pm}]$. Let F be a Seifert surface of k in Y and X be the knot complement. Then

$$\Delta_k(t) := \det(tS - S^T),$$

where S is an associated Seifert matrix of the bilinear form η

$$\begin{aligned} \eta : \quad H_1(F;\mathbb{Z}) \times H_1(F;\mathbb{Z}) &\rightarrow \quad \mathbb{Z}, \\ \eta(\alpha, \beta) &= \quad lk(\alpha^+, \beta). \end{aligned}$$

We adopt the convention that $\alpha^+ \in H_1(X - F)$ is the image of $\alpha \in H_1(F)$ via pushing α in the positive normal direction of F. As is seen in Figure 3.3, the Seifert surface F of k_n links the $0-$framed knot. One of its generators x links that knot. In this case $lk(x^+, x)$ is not a direct calculation, since we have to find a Seifert surface F_x (or F_{x^+}) of x (or x^+) to calculate $lk(x^+, x)$. On the other hand, using the lemma below we can calculate the Seifert matrix easily.

Lemma 3.2.1 (Lemma 7.13 of [Sav12])**.** Let $k \cup l$ be a boundary link (i.e., knots k and l bound disjoint Seifert surfaces) in a homology sphere Y, and Y' is a ± 1 surgery of Y along k. Then $\Delta_{l \subset Y}(t) = \Delta_{l' \subset Y'}(t)$, where $l' \subset Y'$ is the image of $l \subset Y$ under the surgery.

Since α and k_n have disjoint Seifert surfaces, see —Figure 3.3, left —we perform -1 surgery on α, and after some isotopy of k_n we get the right diagram. Therefore for the Seifert matrix $S = \begin{pmatrix} 0 & 1 \\ 0 & n \end{pmatrix}$ we have the corresponding Alexander polynomial

$$\Delta_{k_n \subset Y}(t) = \det\left(tS - S^T\right) = t \doteq 1.$$

Thanks to Freedman and Quinn's [FQ14] Theorem 11.7B these knots are all topologically slice.

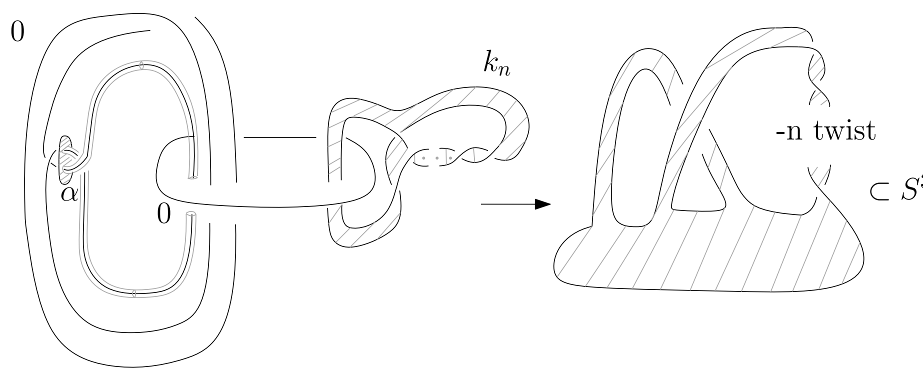

Figure 3.3 Alexander polynomial in homology sphere

CHAPTER 4

PROOF OF THEOREM 1.2.2

Lemma 4.0.1. For any knots $k \in \mathcal{K}_1(Y)$, $k' \subset S^3$ we have

$$\mu(k\#k') = \mu(k).$$

This implies that $\mu : \widetilde{C}_1(Y) \to \widetilde{\Lambda}$ is well defined and onto.

Proof. We will construct a singular disk which will give us the desired result. By definition, k bounds a proper immersion of a disk $D \subset Y \times I$, and similarly k' bounds $D' \subset S^3 \times I$. Any band sum $D \underset{b}{\#} D'$ where the interior of b is away from k and k' gives a proper immersion of a disk in $Y \times I$ bounded by $k\#k'$. Take the base point and the whisker of D as a base point and a whisker for $D \underset{b}{\#} D'$ so

$$\mu(D \underset{b}{\#} D') = \mu(D) + \beta\mu(D')\beta^{-1},$$

where $\beta \in \pi_1(Y)$ is determined by the band b and the whisker. On the other hand $\pi_1(S^3) = 1$ and D' lies entirely in $S^3 \times I$ therefore $\beta\mu(D')\beta^{-1} = 0 \in \widetilde{\Lambda}$ hence

$$\mu(D \underset{b}{\#} D') = \mu(D), \text{ and } \mu(k \underset{b}{\#} k') = \mu(k). \quad \square$$

This implies that Schneiderman's concordance invariant μ is also an almost-concordance invariant on freely null-homotopic knots.

Proof of Theorem 1.2.2. By Theorem 2.2.3 and Lemma 4.0.1, $\mu : \widetilde{C}_1(Y) \to \widetilde{\Lambda}$ is well defined, onto, and is an almost-concordance invariant on null-homotopic knots. For every non-trivial element $g \in \pi_1(Y)$ the target space $\widetilde{\Lambda}$ contains a subgroup isomorphic to $\mathbb{Z}$ generated by g. $\square$

Example 1. Let W^4 be a Mazur manifold as in Figure 4.1. There are various ways to see that the boundary is not the 3-sphere. Its fundamental group is known to be non-trivial [Lau79].

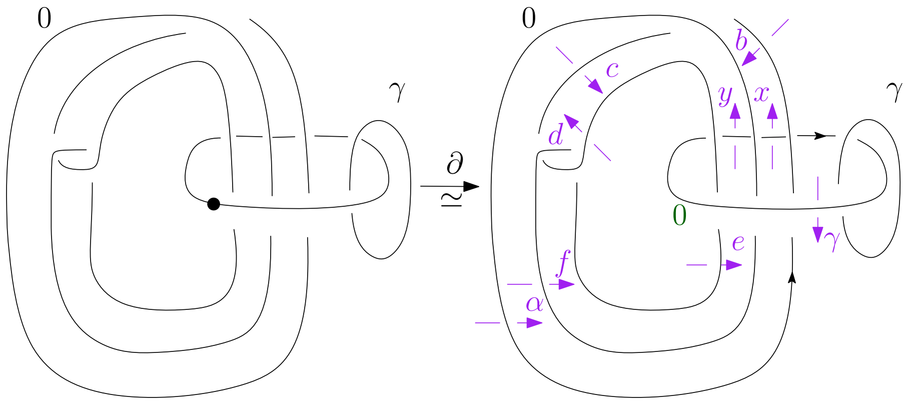

Figure 4.1 A homology sphere, Wirtinger presentation

The Wirtinger presentation gives 9 generators and 11 relations. Notice that last two relations come from the 0-surgeries.

$$\pi_1(Y) = \{\ \gamma, x, y, \alpha, b, c, d, e, f \mid \gamma\alpha\gamma^{-1}b^{-1}(1),\ \gamma f\gamma^{-1}\alpha^{-1}(2),\ \gamma e^{-1}\gamma^{-1}d(3),$$
$$dyd^{-1}\gamma^{-1}(4),\ \alpha y\alpha^{-1}x^{-1}(5),\ bxb^{-1}\gamma^{-1}(6),$$
$$b\alpha c^{-1}\alpha^{-1}(7),\ fdf^{-1}c^{-1}(8),\ dfd^{-1}e^{-1}(9),$$
$$d\alpha^{-1}b^{-1}(10),\ \gamma^{-1}\alpha f\gamma d\gamma^{-1}\alpha^{-3}(11)\ \}.$$

Using the relations 1, 2, 3, 4, 5, 6, 7, and 10 we can easily eliminate b, c, d, e, f, and x as

(1) $b = \gamma\alpha\gamma^{-1}$, (2) $f = \gamma^{-1}\alpha\gamma$, (10) $d = \gamma\alpha\gamma^{-1}\alpha$, (3) $e = \alpha\gamma^{-1}\alpha\gamma$, (4) $y = \alpha^{-1}\gamma\alpha^{-1}\gamma\alpha\gamma^{-1}\alpha$, (5) $x = \gamma\alpha^{-1}\gamma\alpha\gamma^{-1}$, (6) $x = \gamma\alpha^{-1}\gamma\alpha\gamma^{-1}$, (7) $c = \alpha^{-1}\gamma\alpha\gamma^{-1}\alpha$.

(8) and (9) give the same relation which is $\gamma^2\alpha\gamma^{-1}\alpha\gamma^{-1}\alpha^{-1}\gamma\alpha^{-1}\gamma\alpha^{-1}\gamma^{-1}\alpha\gamma^{-1}\alpha = 1$, and (11) is $\gamma^{-1}\alpha\gamma^{-1}\alpha\gamma^3\alpha\gamma^{-1}\alpha\gamma^{-1}\alpha^{-3} = 1$.

We describe the fundamental group:

$$\pi_1(Y) = \{\ \gamma, \alpha \mid \gamma^2\alpha\gamma^{-1}\alpha\gamma^{-1}\alpha^{-1}\gamma\alpha^{-1}\gamma\alpha^{-1}\gamma^{-1}\alpha\gamma^{-1}\alpha = 1,$$
$$\gamma^{-1}\alpha\gamma^{-1}\alpha\gamma^3\alpha\gamma^{-1}\alpha\gamma^{-1}\alpha^{-3} = 1\ \}.$$

Setting $\gamma = 1$ in this presentation would make this group trivial, hence γ is a nontrivial element of $\pi_1(Y)$. To construct an example corresponding to Theorem 1.2.2, take an unknot and push an arc along a nontrivial loop γ we get left diagram of Figure 4.2. Obviously $\mu(k_1) = \gamma^{\pm} \in \widetilde{\Lambda}$ is nontrivial. Hence it is not almost-concordant to the unknot. On the other hand by iterating this process (i.e. increasing the number of twists) we can construct infinitely many null-homotopic knots k_n with distinct μ invariant in the homology sphere, see the right diagram of Figure 4.2.

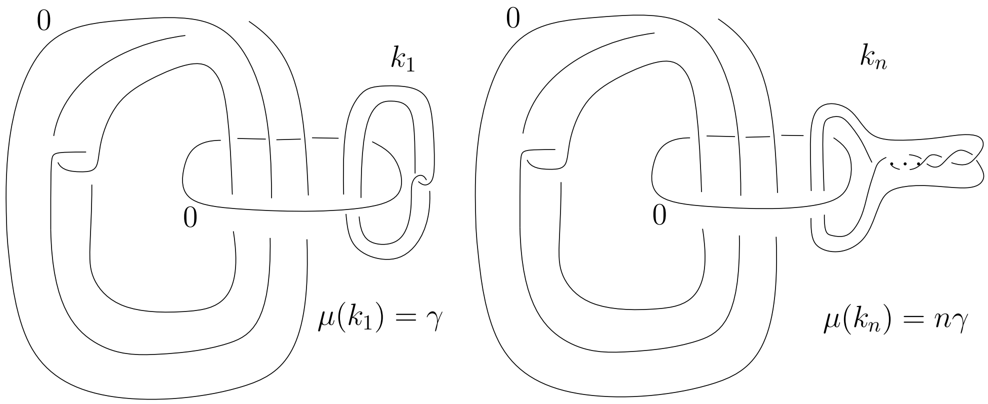

Figure 4.2 Distinct almost-concordant families

CHAPTER 5

PROOF OF THEOREM 1.2.3

In this chapter, we prove Theorem 1.2.3, which we restate here:

Theorem 5.0.1 ([Yil18]). All knots in the free homotopy class of $S^1 \times \{pt\}$ in $S^1 \times S^2$ are smoothly concordant, i.e., $|C_x(S^1 \times S^2)| = 1$ where x represents $S^1 \times \{pt\}$ in $S^1 \times S^2$.

Proof. First we introduce a (genus zero) cobordism move to a knot k, which starts with k, and ends with a two-component link, consisting of the knot obtained from k by changing one of its crossings union a small linking circle, as shown in Figure 5.1.

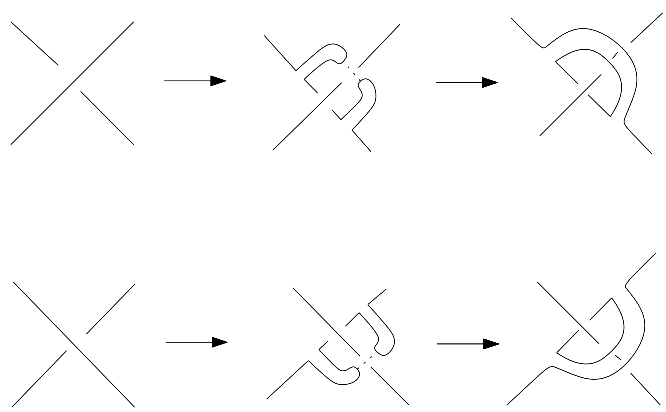

Figure 5.1 Crossing change

Let k be a knot freely homotopic to $k' = S^1 \times pt$ in $S^1 \times S^2$, one can go from k to k' by finitely many crossing changes and isotopies. Change all the necessary crossings of k by the cobordism described above. Notice that for every crossing change, we get a small linking circle to the resulting knot. See Figure 5.2 as an example. It is obvious from Figure 5.3 that all those small circles which link k' bound disks in $S^1 \times S^2$ disjoint from k'. We accomplish this by sliding over the 0−framed circle. By capping with disks these unknots we get a concordance from k to k' in $S^1 \times S^2$. □

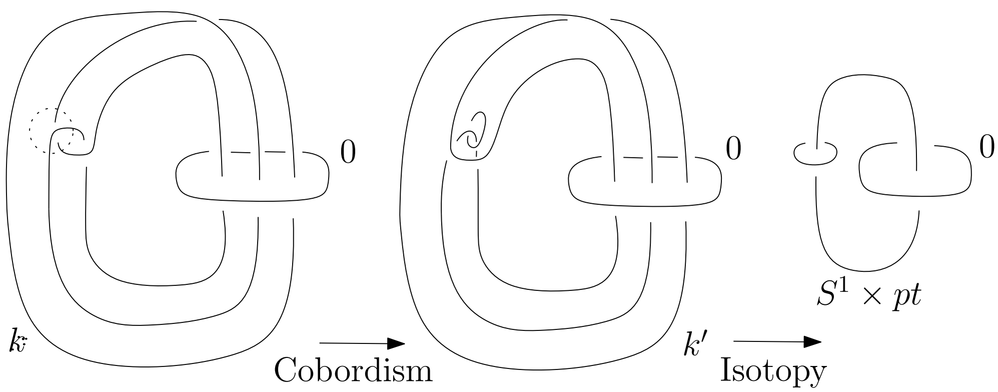

Figure 5.2 An example of crossing change

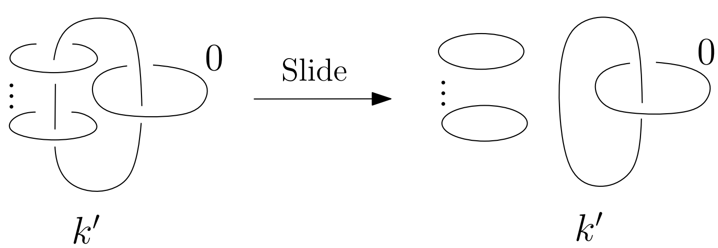

Figure 5.3 Sliding and capping with disks

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