# CONNECTING REFLECTION AND PRACTICE: TRANSFORMING A MATHEMATICS CLASSROOM CULTURE OF PARTICIPATION

By

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## ABSTRACT

# CONNECTING REFLECTION AND PRACTICE: TRANSFORMING A MATHEMATICS CLASSROOM CULTURE OF PARTICIPATION

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Unlike previous research on participation in mathematics classrooms that focuses on the teacher's role eliciting student talk, the purpose of this dissertation is to study the interactive process through which a teacher connected her reflection and practice, fostering transformations in the culture of participation in a third-grade Spanish immersion mathematics classroom. I draw on hermeneutic listening (Davis, 1996) to understand the teacher's departure from listening for predetermined ways of participating. Methods from participatory design research with children (Bang & Vossoughi, 2016; Groundwater-Smith et al., 2014) and analytical tools from social semiotics (van Leeuwen, 2005) informed the collaborative design and analysis of interventions intended to promote a more inclusive culture of participation. Findings indicate that bringing the students' and the teachers' perspectives on participation into dialogue helped the teacher broaden her ideas on participation. A prolonged, iterative process was necessary to connect these reflections to a practice where the teacher learned to listen hermeneutically for participation. This process started with teacher-researcher co-designed interventions and continued as the teacher recognized participation in unexpected places and moments. The culture of participation evolved toward embracing multiple ways of participating in cooperative mathematical activity in which the students and the teacher addressed unanticipated ideas that emerged in interactions. I describe the semiotic innovations that characterized these transformations and the contextual influences that mediated semiotic innovations. I discuss implications for mathematics education research and professional development.

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#### **Chapter 1 Introduction**

The topic of my dissertation was motivated by a three-year collaboration with Valery Abad (all names are pseudonyms) in her third-grade mathematics class. Valery and I discussed and tried different approaches to respond to her interest in what she called "abrir la conversación" [*to open up discussions*]. The issue of not all students participating became a recurrent concern in our discussions. For example, after one lesson where the teacher and I thought most students had been noticeably enthusiastic, addressing each other during a wholeclass discussion, Valery mentioned that Arnie's participation had puzzled her because of how quiet he had been. As I was leaving the school that day, I saw Arnie outside waiting for his parents to pick him up. Without any prompting, Arnie told me how much he had enjoyed the class and that he thought one of his classmate's strategies during the number talk was very smart (field note, December 6, 2016).

When I shared Arnie's comment with Valery, our conversation shifted from whether Arnie participated to how he participated. Specifically, Valery mentioned he tended not to speak up in whole-class discussions but he seemed to be "un buen oyente" [*a good listener*]. Valery said that the idea of speaking up in class could be off-putting for Arnie (field note, December 8, 2016). Observations and discussions like these motivated me to question what it means to participate in a mathematics classroom. Specifically, I began to wonder how students' and teachers' ideas about participation may differ, and how teachers' ideas of what it means to participate may shape what counts as participation.

Research on participation in mathematics classrooms has primarily studied the role of the teacher in eliciting student talk. Few studies have focused on how students actually participate and what they have to say about their own participation (Dallimore, Hertenstein, & Platt, 2004;

Kane & Maw, 2005). The tendency to overlook students' perspectives in favor of teachers' and researchers' perspectives is particularly marked in studies involving elementary school children (Groundwater-Smith, Dockett, & Bottrell, 2014). My dissertation addresses this issue by drawing on students' perspectives on participation to support a teacher reflect on what counts as participation and adjust her teaching accordingly. Children's perspectives challenged adults' initially narrow conceptualizations of participation.

The purpose of this dissertation is to study the interactive process through which a teacher connected her reflection and practice, fostering transformations in the culture of participation in a third-grade Spanish immersion mathematics classroom. I focus on the following research question:

How did a teacher connect her reflection on what it means to participate in mathematical activity with her practice in order to transform the classroom culture of participation? I focus on this interactive connection between reflection and practice to make sense of how a classroom culture of participation evolved.

Sociocultural learning theory (Rogoff, Paradise, Mejía Arauz, Correa-Chávez, & Angelillo, 2003; Vygotsky, 1978; Wenger, 1998) informs my conceptualization of participation. I focus on the classroom culture of participation as a situated phenomenon where the cultural system that the classroom constitutes influences what is considered legitimate participation, including which ways of participating are overlooked or dismissed. Rather than promoting a culture of participation that imposes the teacher's perspectives or the students' perspectives on participation, I focus on transformations in the culture of participation grounded on mutually informed reinterpretations of these perspectives. These reinterpretations involve the teacher's commitment to reflecting on and questioning preconceived ideas about participation. To

understand these reinterpretations, I bring hermeneutic listening (Davis, 1996; 1997) to the study of participation. Hermeneutic listening attends to the transformation from rigid and preset mathematics teaching to embracing unanticipated and surprising ways of engaging with mathematical ideas (Davis, 1996). In addition to guiding my exploration of how the class reinterpreted what counts as participation, this framework also informed the interventions designed to foster a classroom culture of participation inclusive of diverse ways of participating. A Participatory Design Research approach (Bang & Vossoughi, 2016) and methods from research with children (Veale, 2015; Groundwater-Smith et al., 2014; Punch, 2002) informed the design and analysis of these interventions. Analytical tools from social semiotics (Halliday, 1978; O'Halloran, 2005; van Leeuwen, 2005) informed the analysis of the transformation of the culture of participation of this classroom.

This dissertation contributes new knowledge about mathematics research and practice. My study involves both the students' and the teacher's perspectives in the conceptualization of participation and in promoting and making sense of the transformation of the classroom culture of participation. In doing so, my study helps us reconsider what it means to participate and whose perspectives on participation should inform our research. This involves practice speaking to theory, as definitions of participation emerged from and considered participants' specific context, needs, and experiences. My study also expands previous research on teachers' roles on participation by informing how teachers can learn from students to transform the culture of participation. This involves connecting teacher reflection and practice, highlighting a teacher's commitment and support to question preconceived notions on participation, and creative teaching strategies to embrace and support complex and diverse practices that students bring to the

classroom. Ultimately, this dissertation contributes an approach to mathematics teaching and research that honors student diversity.

# **Overview of Chapters**

In the next chapter, I situate this study in the literature on student participation in mathematics classrooms, contrasting prevailing views on participation as student talk with the conceptualization of participation as a complex situated phenomenon that guides this dissertation. I discuss hermeneutic listening as a framework informing the teacher's reinterpretation of situated participation. In Chapter 3, I describe the research design, including how the participants and I prepared our research collaboration. Drawing from Participatory Design Research and principles from research with children, the teacher and I designed intervention cycles as part of a methodology that supported us to elicit students' perspectives on participation. I introduce social semiotics as an analytical framework consistent with both the theoretical framework and the purpose of the study. Closing this chapter is a description of the data generation and analysis. Chapters 4, 5 and 6 present findings at the level of reflection, practice, and reflection-practice connections, respectively. Chapter 4 presents findings about the teacher's initial reflections on participation, as she brought into dialogue the students' and her perspectives on what it means to participate. Chapter 5 describes findings regarding the initial process of preparing to recognize multiple ways of participating supported by the iterative codesign of flexible interventions. Chapter 6 focuses on findings about the ongoing connections between reflection and practice that helped the teacher engage in hermeneutic listening for participation to transform the culture of participation. Finally, in Chapter 7 I bring into my final discussion the three levels of findings with a focus on the connections between reflection and

practice that promoted the transformations of the culture of participation. I address implications for research and practice.

## **Chapter 2 Literature Review and Theoretical Framework**

This study is situated on the literature on participation in mathematics classrooms, with a focus on how participation has been conceptualized. I pay particular attention to conceptualizations of the role of the teacher and the classroom culture on participation. I then define participation from a sociocultural theory of learning and discuss how I draw on hermeneutic listening to explore the teacher's shift toward transforming her understanding of participation.

## **Literature Review**

Previous research on participation in mathematics classrooms has mainly equated participation with student talk. Accordingly, the role of the teacher has frequently been characterized in terms of attempting to distribute talk time uniformly among students. Here, I review these previous views on participation and then discuss the classroom culture of participation as a concept that focuses on social aspects of participation, including meanings associated with participation that are simultaneously shared and contested.

**Previous views on participation in mathematics education research.** Participation is nearly equated with student talk in mathematics education research. Many studies use the terms participation and student talk interchangeably (For a few examples see Brown, 2017; Franke et al., 2015; Ing et al., 2015; Jansen, 2006; Krummheuer, 2007; O'Connor, Michaels, Chapin, & Harbaugh, 2017; H. Smith & Higgins, 2006; Takeuchi, 2015; Webb et al., 2017; 2014). For example, when studying students' participation in mathematics classrooms Jansen (2006) focused on students "talking about mathematics" (p. 412). Similarly, when examining how social structures influence student participation in mathematics classrooms, Civil and Planas (2004) focused on how some student talk influences how classes unfold "while other voices are not

heard" (p. 8). Most studies that acknowledge diverse ways of participating in mathematics classrooms do so on a parenthetical note preceding the announcement of the studies' focus on student talk (Foote & Lambert, 2011; Reinholz & Shah, 2018; M. B. Wood & Hackett, 2018). The few studies that have explored participation beyond student talk consider listening (Lack, Swars, & Meyers, 2014; Moschkovich, 2018), reflecting (Rodd, 2003), gesturing (Williams-Pierce et al., 2017), and drawing (O'Halloran, 2015). Although student talk is an important dimension of participation in mathematics classrooms, the overemphasis on student talk ignores less vocal but equally relevant ways of participating. In this study, I consider these and other spontaneous ways of participating, acknowledging students' multiple ways of being part of mathematical activity.

Previous research that views participation as student talk frequently focuses on this type of participation occurring in whole class discussions (Baxter & Woodward, 2001; Esmonde & Langer-Osuna, 2013; Lo, Wheatley, & Smith, 1994; Lubienski, 2000). These studies highlight differences in the amount of student talk during discussions, exploring individual and classroom characteristics that prevent certain students from speaking up. In this dissertation, in addition to exploring multiple ways in which participation occurs, I also explore multiple types of activities where it takes places. Specifically, I explore participation in whole class discussions, as well as in individual and small group work.

Mathematics education research has also frequently treated participation as a quantifiable individual phenomenon. In some cases, this quantification is framed as absence or presence of participation. For example, when studying differences between male and female students' participation, Fassinger (1995) stated that "some students participate daily and others by the end of the semester have not uttered a word" (p. 82). Similarly, Foote and Lambert (2011)

problematized some students' "lack of participation" (p. 248). The quantification of participation is concerned with increasing the number of utterances per student (Foote & Lambert, 2011; H. Smith & Higgins, 2006) or "the amount of detail students provide" (Webb et al., 2017, p. 6).

One assumption of this view on participation that is rarely interrogated is that more student talk is better. The locus of participation is assumed to be in individual students. Students who talk more are active and gain opportunities to learn, and those who talk less are judged as passive and in need of more opportunities to speak up (Baxter & Woodward, 2001; Lack et al., 2014; Webb et al., 2014). Given the limited time available for students to talk, one student's participation means other students' non-participation (Hardy, 2007; Moschkovich, 2018; Skott, 2013; Wager, 2014; M. B. Wood & Hackett, 2018). In this dissertation, I contribute to alternative conceptualizations that, rather than determining who participates more and who participates less, consider different ways in which student participation may occur simultaneously as one student's participation interacts with others'. Although some students may benefit from expanding their ways of participating, there is the risk of forcing quieter students to come closer to what may be perceived as a norm for amount of talk based on their outspoken peers. Instead, I focus on understanding the complex and diverse ways in which participation unfolds during mathematical activity. This consideration challenges normative views of participation as a predetermined kind and quantity of student talk and opens up possibilities for the class to recognize ideas that may otherwise go unnoticed.

In addition to exploring who participates and how, previous research has also explored what students participate in. The focus has been on specific mathematical practices established by the Common Core Standards for Mathematical Practice (Common Core State Standards Initiative, 2010) and NCTM's mathematical processes (National Council of Teachers of

Mathematics NCTM, 2000). These practices include articulating problem-solving strategies, making and justifying claims, attending to precision, and attending to others' ideas (Brown, 2017; Esmonde & Langer-Osuna, 2013; Franke et al., 2015; Krummheuer, 2007; Moschkovich, 2018; Turner, Dominguez, Maldonado, & Empson, 2013). In addition to these mathematical practices, in this dissertation I consider practices that may emerge in the context of diverse ways of participating, such as posing problems (Dominguez, 2016), listening (Goldin, O'Neill, Naik, & Gomez Zaccarelli, 2019), and helping others (Jansen, 2006). I also consider the specific content of the mathematical tasks the class participates in. Some of this content emerges unexpectedly and may deviate from the predetermined curriculum.

The teacher's equalizing role. Previous research on mathematics education has characterized the teacher's role regarding participation mainly in terms of eliciting student talk. Specifically, teachers strive to uniformly distribute talk time among students, disrupting the tendency to attend to mathematical ideas uttered by a few dominant students. This role involves developing classroom norms conducive to equitable student talk (Franke et al., 2015; Lack et al., 2014; Wager, 2014; Webb et al., 2014). As Webb et al. (2014) put it, "Techers can set ground rules and guidelines for desired participation, promoting greater student talk" (p. 81). These norms influence student talk by signaling to students the desirable ways to add to discussions.

To foster these classroom norms, teachers intervene during discussions so that the class hears from students who usually do not speak up. Some associated teacher moves include inviting quiet students' contributions (O'Connor et al., 2017; Wager, 2014); asking probing questions (Franke et al., 2015; Ing et al., 2015); validating marginalized students' mathematical ideas (Turner et al., 2013; M. B. Wood et al., 2019); legitimizing the use of multiple languages (Takeuchi, 2015); using multiple interaction structures, such as whole-class discussions, small-

group work, and student-student conversations (Webb et al., 2017); and guiding students' reflection on their own participation (Lack et al., 2014).

An argument that these studies develop is that the teacher practices that elicit participation cannot be prescribed (Esmonde & Langer-Osuna, 2013; Franke et al., 2015; Skott, 2013). Rather than preplanning a sequence of specific moves, teachers elicit participation when they responsively draw on this repertoire of moves as they mediate classroom interactions. Teachers observe for patterns in students' amount of talk and whose utterances the class is considering. Teachers draw on these observations and their knowledge of their class to decide on moves that encourage specific students to voice their ideas. Over time, students pick up on those moves and become aware of who is expected to talk and when (Hardy, 2007). When teachers fail to develop these classroom norms, a cycle is perpetuated where a few outspoken students monopolize discussions while the ideas of quieter students remain unheard.

In addition to drawing on specific teacher moves, teachers foster participation-related classroom norms through the tasks that they implement. Previous research has argued that teachers can elicit talk by piquing student interest through mathematically challenging tasks (Emanuelsson & Sahlström, 2008; Jansen, 2006; Lo et al., 1994). When these tasks require interaction, they promote the need for different students to contribute to discussions (Boaler, 1999; Cohen, 1994; Emanuelsson & Sahlström, 2008; Featherstone et al., 2011). In this way, the teacher orchestrates students' opportunities to contribute to mathematical discussions. Even when these types of tasks are in place, however, the teacher still observes during small-group work, drawing on a repertoire of teaching moves when necessary to equalize the amount of student talk.

Instead of focusing on the role of the teacher as equalizing the amount of student talk, in this dissertation I focus on connections between reflection and practice that are foundational to fostering an inclusive culture of participation. Connecting reflection that destabilizes taken for granted perspectives on participation with a practice that honors student participation involves teachers assuming the role of learners of what it means to participate in a specific classroom. In a bidirectional, iterative process, practice informs further reflection to make sense of the evolving culture of participation. I explore a teacher's process of connecting reflection and practice that helped her see participation in spaces and moments that were hiding from her perspective.

The classroom culture of participation. Educational researchers have theorized culture as fluid, dynamic and pluralistic (Ladson-Billings, 2014; Paris, 2012; Paris & Alim, 2014). This theorization challenges the tendency to attribute predetermined static cultural traits to specific groups (for example, Latinos or women). Rather than assuming a monolithic and static culture associated with specific races, ethnicities or languages, this theorization acknowledges that students are members of several communities with distinct cultures and evolving practices. Students bring these multiple cultural ways of being and practices to the classroom. The role of the teacher involves understanding, honoring and exploring students' multiple cultural practices, instead of attempting to replace them with practices from the dominant culture of the school or the discipline (Paris, 2012). This theorization of culture fosters pluralism and orients teachers "toward the idea that learners can be sources and resources of knowledge and skills" (Ladson-Billings, 2014, p. 79). In mathematics education, Louie (2017) drew on this theorization to argue that teachers unintentionally exclude students who do not align with the dominant culture of mathematics education from learning opportunities. She called for more research on how teachers may disrupt this culture of exclusion. This theorization informs my conceptualization of

a mathematics classroom culture. The culture of a classroom, like culture at other social levels, is heterogeneous and participants value different practices and ways of making sense of mathematical ideas (Gresalfi, Martin, Hand, & Greeno, 2009; Moschkovich, 2018; T. Wood, Williams, & McNeal, 2006). Teachers and students continuously reconfigure these values during mathematical activity (Bishop, 1988; 1991; Cobb & Yackel, 1998; Nasir, Hand, & Taylor, 2008). This conceptualization brings attention to the interaction between the individual and the social: individuals influence the development of the class culture and the class culture influences how individuals engage in activity (Gresalfi et al., 2009; Hodge & Cobb, 2016; Lo et al., 1994; Nasir et al., 2008).

I refer to the culture of participation of a mathematics classroom as the relatively shared, yet continuously negotiated, sense of what counts as participation. Previous research suggests that teachers promote a classroom culture that elicits and supports a specific type of participation through explicit classroom norms and through favoring ideas that emerge in particular ways (Civil & Planas, 2004; Nickson, 1994; O'Connor et al., 2017). Over time, students learn to recognize the ways of participating that the teacher values (Hardy, 2007; Skott, 2013; Turner et al., 2013). While some students align their participation to match teacher's expectations, others do not (Hardy, 2007; Hodge & Cobb, 2016; Lubienski, 2000).

As discussed above, teachers frequently attend to the ideas of the most vocal students, missing out on others' ideas. In this process, a class culture of participation that overvalues student talk gets reinforced. In this dissertation, instead of attempting to change how individual students participate to match the teacher's expectations, I explore a way of broadening what counts as participation by focusing on understanding the multiple meanings that participation has for students and teachers. In turn, considering mathematical ideas that emerge through these

multiple ways of participating may contribute to challenging the overvaluing of student talk as the ultimate form of participation.

Researchers have called for the exploration of insiders' perspectives on participation to inform the development of an inclusive classroom culture (Brown, 2017; Hodge & Cobb, 2016; Pimm, 1994). Previous studies have focused primarily on teacher's insights (as argued by Esmonde & Langer-Osuna, 2013; Lack et al., 2014). Few studies have asked students about their own participation in mathematics classrooms (for exceptions, see Jansen, 2006; Lack et al., 2014; Turner et al., 2013). These studies suggest that students who see their participation as influencing the ideas the class considers see themselves as competent. Conversely, those whose ways of participating are not valued or not acknowledged see themselves as less competent (Lack et al., 2014). These studies also suggest that just as teachers influence the culture of participation, so do students, if we care to learn from what they have to say. For example, Jansen (2006) discusses the potential implications on teaching of considering students' willingness to participate in specific ways and the types of activity they prefer to participate in. My contribution to this line of research consists of gaining a deeper understanding of students' perspectives on participation and drawing on their perspectives to inform the transformation of the classroom culture of participation.

The study of the students' and the teacher's participation-related perspectives and practices implies a focus on the context-specific nature of participation. What counts as participation may differ across classrooms (Hodge & Cobb, 2016; Pimm, 1994). Moreover, the school and social context where the classroom is situated may influence the participation notions that are valued, as well as teachers' and students' perspectives on participation (Civil & Planas, 2004; Esmonde & Langer-Osuna, 2013; Lubienski, 2000; Matthews, 1984). This situated nature

of participation is consistent with the previous discussion on the impossibility of predetermining teacher moves to promote participation. Accordingly, in this dissertation I explore how the teacher's reconceptualization of participation continued to develop in dialogue with the evolving culture of participation.

## **Theoretical Framework**

In mathematics education research, a predominant conceptualization of participation highlights student talk. In this dissertation, I expand this conceptualization by considering time and space as two relevant dimensions of participation. This means locating participation within longer stretches of time, in more varied spaces, and across a myriad of artifacts so as not to miss what participation means for students. I begin by discussing the theoretical grounding of my conceptualization of participation and how I use this concept to understand the culture of participation in a mathematics classroom.

**Participation as a complex situated phenomenon.** I draw on sociocultural theory to define participation as an evolving social experience of engaging in meaningful shared activity, that combines doing, talking, thinking, feeling, and belonging (Lave & Wenger, 1991; Rogoff et al., 2003; Vygotsky, 1978; Wenger, 1998). Participation is a situated phenomenon in which the individual and the social interact as members of a bounded social community, such as a classroom, coordinate the pursuit of a shared enterprise. It is situated because the context influences both the activities that are considered meaningful and the ways of becoming part of an activity (R. Thomas, Whybrow, & Scharber, 2012b). The shared history and culture of the community also influence how different ways of participating are valued (Mafra Goulart & Roth, 2006; Rogoff et al., 2003; Wenger, 1998). Participation involves an interaction between the individual and the social because, rather than being a static set of practices, members of a

community permanently negotiate what it means to participate (Wenger, 1998). As the community develops over time, the activities in which members engage and how they participate in these activities evolve, too. Different members may introduce novel activities or ways of engaging in activity. When an individual's ways of participating are legitimized and influence how others participate, the member experiences a sense of belonging.

An implication of this conceptualization is that participation involves developing a sense of belonging. Engaging in shared activity constitutes participation when students feel they are influencing the activity (R. Thomas, Whybrow, & Scharber, 2012a; 2012c; Wenger, 1998). For example, although some teachers may dismiss silence as a form of participation, silent students may be joining in shared activity through intent observation and thought (Moschkovich, 2018; Rogoff et al., 2003; Schultz, 2010). When studying participation in terms of mutuality and influence, von Wright (2006) argued that students feel they are participating when they perceive others as taking their ideas seriously. Similarly, students may not feel they are participating even if they engage in talk, unless they feel others consider their contributions. In this conceptualization of participation, what matters is not so much a student's individual contribution at a specific point in time but the impact that such contribution may have over time and across the complex space constituted by the classroom . Students' recognition of their own sense of participation highlights my interest in eliciting students' perspectives on participation.

This conceptualization also has implications on the role of the teacher. What counts as participation varies from classroom to classroom (Anderson, 1998; Hammel et al., 2009; Hickey, 2003). Even within each classroom, teachers' and students' meanings of what counts as participation may differ (Mafra Goulart & Roth, 2006; Moguel, 2004; Rocca, 2010). Teachers may decide to focus on helping students participate a certain way. Although this approach has

the potential of helping students expand the ways in which they participate, it can also exclude children whose preferred ways of participating are overlooked. In contrast, when the classroom allows multiple ways of participating to influence activity, the community expands the ways in which students may make sense of ideas and more students experience a sense of belonging in mathematical activity (Dallimore et al., 2004; Davitti & Pasquandrea, 2017; Norton, 2001). In this dissertation, the teacher's role includes connecting this reconceptualization to her practice by fostering a transformation of the culture of participation grounded on students' actual ways of participating. This conceptualization also pays attention to ways of participating that are spatial an temporal in nature, including gestures, gaze, movement, writings, and drawings. I consider the role of the teacher as someone learning to see participation in multiple locations and across time.

Hermeneutic listening for student participation. As discussed before, rather than focusing on a teacher-imposed view of what participation is, my study focuses on a teacherresearcher collaboration to help the teacher reconceptualize what counts as participation. To guide the study of this reconceptualization, I draw on hermeneutic listening's attention to "imaginative participation in the formation and transformation of experience through an ongoing interrogation of the taken-for-granted" (Davis, 1996, p. 53). Hermeneutic listening refers to a concerted and participatory type of interaction through which teachers and learners influence the unfolding of classroom possibilities (Davis, 1996; 1997). Hermeneutic listening is a departure from listening for predetermined mathematical understandings expressed in preconceived ways. I bring this concept to understand a teacher's departure from listening for predetermined ways of participating, attending to the multiple situated ways in which students participate.

Adding to previous research on mathematics education that has drawn on hermeneutic listening (Arcavi & Isoda, 2007; Crespo, 2000; Doerr, 2006; Hintz & Tyson, 2015; Johnson &

Larsen, 2012), I focus on three concepts that are relevant to the exploration of participation as a situated phenomenon: listening to more than spoken language; the teacher as a full participant in the learning activity; and listening as involving both the individual and the social. First, hermeneutic listening transcends spoken language as it involves attending "not just to the voices and actions of students but also to the... cultural situation in which the teaching is located" (Davis, 1997, p. 374). I draw on hermeneutic listening to interpret students' embodied actions, silence and reflection as possible ways of participating in mathematical activity.

Second, hermeneutic listening positions the teacher as a full participant in the learning activity. Instead of conceptualizing the teacher as a transmitter of knowledge or a removed facilitator of student activity, both teacher and students question, interpret and transform the direction of a lesson (Davis, 1997). The teacher does not engage in surveillance of students to ensure they enact specific actions—such as formal talk—and not others (Davis, 1996). The teacher joins students in interpreting mathematical ideas and redirecting collective action accordingly. Situating the teacher as an active participant requires that the teacher no longer searches for predetermined ways of speaking about mathematical concepts, but instead interprets and responds to students' ideas developed and expressed in spontaneous and diverse ways.

Third, hermeneutic listening is concerned with the individual and the social. In line with my focus on the transformation of the culture of participation of the classroom, hermeneutic listening involves attending to how the social group influences what individuals do and vice versa (Davis, 1997). The interdependence of agents and setting, self and other, and individual and collective draws attention to how the collective moves across ideas and actions. That is, the focus is on the possibility of interpretations of classroom phenomena that facilitate the emergence of mathematical ideas. Additionally, hermeneutic listening involves a commitment on

the part of the researcher and the teacher to engaging with students in exploring mathematical ideas.

Hermeneutic listening applied to the study of participation draws attention not only to formal ways of expressing ideas but also to divergent and creative ways of engaging in mathematical activity. Transforming the classroom culture of participation toward attending to alternative ways of participating involves students, the teacher and the researcher coming together to listen for difference (Davis, 1997). Interactions in which participants listen for difference are grounded on a teacher's belief in students' ability to generate mathematical ideas, and a commitment to approaching students' ideas with authentic curiosity (Davis, 1997). Teaching strategies that operationalize hermeneutic listening include: asking genuine questions—questions for which the teacher does not know the answer—, directing students' listening—drawing attention to what others do and say—, and amplifying students' participation—building on what students do and say to inform how lessons unfold—(Hintz & Tyson, 2015). These strategies informed the collaborative design of interventions intended to support the transformation of the culture of participation.

The above theoretical commitments conceive of participation as a complex situated phenomenon that can be understood through hermeneutic listening. Aiding the process of recognizing participation more broadly are the ongoing collaboration to raise perspectives on participation to awareness, the evolving role of the teacher, the types of lessons and mathematical activities, and the flexible adjustment of tasks, lessons and units in response to student participation. Considering these contributing factors sheds light on how the teacher learned to recognized unexpected situated participation and how she fostered an inclusive culture of participation.

#### **Chapter 3 Context of the Study and Methodology**

In this chapter, I discuss the participatory-design research with children methodology and the social semiotics analytical framework that guided the study of the transformation of the culture of participation in this mathematics classroom. I begin describing the research context.

# **Site and Participants**

The school system. This K-12 school system consists of five campuses located in western Michigan. More than 2,200 students from various Christian denominations attend these religious schools, founded almost 100 years ago. A quarter of the student population is non-White. The vision and mission statements mention a commitment to expanding the diversity of the student body, with a focus on increasing affordability for students from diverse socioeconomic backgrounds. The schools' financial aid program grants partial scholarships to 50% of the student population. Additionally, through a variable tuition system, what each family pays depends on their unique financial situation. The school system attributes its high (94%) retention rate in part to these efforts toward affordability. More than 90% of high school graduates enter college, and many of them received a partial scholarship from the school system.

Teachers and staff periodically attend professional development sessions on how to serve students from diverse cultural, ethnic, denominational and socioeconomic backgrounds. Teachers also engage in professional development on the principles and practices of the Learning without Limits project developed by scholars in the Faculty of Education at the University of Cambridge (Hart, Dixon, Drummond, & McIntyre, 2004). This professional development focuses on shifting away from ideas of student ability as a fixed individual characteristic to the notion of transformability. Transformability refers to a conviction that teachers and students can develop transformational learning opportunities. To help students take up these opportunities, teachers are

expected to communicate high expectations for all students and to help students experience a sense of confidence when putting effort into classroom activities. These teaching practices include asking questions that foster student reasoning, and publicly acknowledging students' effort and perseverance, and rejecting ability-related labels and stratified grouping arrangements.

Common Core State Standards, the Michigan Curriculum Framework, and the school system's goals for graduates inform curriculum and assessment design. Goals for graduates are intended to permeate teaching at all levels, and they include developing creative problem solving, teamwork, and effective communication skills. In line with the schools' efforts toward diversity, goals for graduates include demonstrating a willingness to engage with a range of people and ideas in respectful ways and to become justice seekers actively involved in community issues. Teachers are encouraged to weave these goals in their lessons and to explicitly discuss these goals with students.

The language immersion program. Since 2010 and in response to the interest of several parents, the school system offers a Spanish language immersion program. The goals of the immersion program are to support students to simultaneously develop functional proficiency in Spanish, cultural competence, and mastery of grade level curricular standards. According to a survey the school administers to parents annually, the main motivations for enrollment in the immersion program are for students to eventually enjoy the professional opportunities of speaking a language other than English, and to encourage cross-cultural understanding. The majority of the children of Latino Spanish-speaking parents in the school system attend the immersion program. These children represent 20% of students in the immersion program and their parents choose the program mainly to support their children's bilingualism and biculturalism.

The program follows a language enrichment full immersion model (Brisk, 2011) where all instruction is in Spanish in the beginning grades and some classes are taught in English in upper elementary grades. From PK to first grade all instruction is in Spanish. Second to sixth grade include 60 minutes a day of instruction in English. Mathematics is taught in Spanish, one hour a day, five days a week. PK-5 students have a class that focuses on Spanish grammar and communicative skills, and on the traditions and history of Spanish speaking countries. After fifth grade, students have the option of joining an immersion cohort that offers Spanish language arts, social studies, and Bible studies in Spanish. For all other classes, students join the mainstream English-speaking classrooms.

Students in the immersion program frequently engage in activities with the local Spanishspeaking community. This includes field trips to local organizations, attending cultural events, interacting with guest speakers, engaging in community outreach, and engaging with events that involve the Spanish-speaking community. All teachers and teacher aides in the language immersion program are native or near-native speakers of both Spanish and English. Teachers are encouraged to use Spanish at all times and to use their discretion to identify moments when students' emotional or physical well-being may require the use of English.

The elementary school. The elementary school serves nearly 650 PK-4 students, with an average class size of 22 students. There are 91 faculty and staff members. The Spanish immersion program for the entire school system originated in this school. Table 3.1 shows the number of classrooms per grade, including the number of Spanish immersion classrooms, for the 2017-2018 academic year. In addition to homeroom teachers, there are teachers for the co-curricular classes (arts, music, physical education, and Spanish). The school principal encourages collaboration among teachers and among teachers, coaches, and subject specialists. These

collaborations include co-planning lessons, peer observation, and co-developing co-curricular activities and events. The school also encourages collaboration with the teacher preparation program at a local college. During the spring semester, some teachers serve as mentors to student teachers, including Spanish-English bilingual student teachers placed in the immersion classrooms.

The school provides pacing guides that include the sequence of topics for each teaching unit, the beginning and finishing dates for the unit, and standards and learning goals. Each teacher decides how to plan lessons, the activities to include and the amount of time devoted to different topics within the unit. The school selected the textbook that teachers are expected to use for mathematics lessons. The textbook includes a teacher's version with lesson plan ideas and materials, and there are English and Spanish editions, so both immersion and mainstream classrooms use the same textbook. Each teacher independently designs on-going assessments for the content area. Teachers from each grade level get together to design an end-of-unit test. Immersion teachers translate the test to Spanish. Each teacher decides when to administer the test within the dates established for the unit. The school also administers an end-of-year test for reading and math, designed by an external organization.

Table 3.1 Number of classrooms per grade

Number of Spanish	Number of mainstream	Total number
immersion classrooms	classrooms	of classrooms
1	5	6
1	3	4
2	3	5
3	3	6
	Number of Spanish immersion classrooms 1 1 2 3	Number of SpanishNumber of mainstreamimmersion classroomsclassrooms15132333

Table 3.1 (cont'd)

Third	2	2	4
Fourth	2	3	5
Total	11	19	30

The students. My study took place in one of the third-grade Spanish immersion classrooms, during the 2017-2018 academic year. Initially, there were 21 students: 15 female students (one Latina, one Asian American, two African American, 11 White) and six male students (two Latino, one African American, three White). One student moved to a mainstream classroom in the middle of the year. All students spoke English at home primarily, including the three Latino/a students whose use of Spanish at home was limited. Most students (18) attended the language immersion program since either PK or kindergarten, and the other three students started in first grade. All students seemed comfortable sustaining conversations, as well as following directions and discussions in Spanish. Three of the students were on Individualized Educational Plans (IEPs). All three of them were pulled-out of mathematics class three times a week to work with the immersion program support aide in Spanish.

The teacher. The teacher, Valery Abad, is a US-born Latina who considers both English and Spanish as her native languages. Growing up in Florida, she spoke Spanish at home exclusively with her Cuban parents and siblings. She holds a bachelor's degree in bilingual Spanish education and a master's degree in education, both from liberal arts colleges in the same city where the school is located. At the time of this study, she was in her fifth year teaching, and all her teaching experience had been in the third-grade Spanish-immersion classroom at this school. She was actively involved in school activities and frequently volunteered to support various programs and events and to serve as mentor teacher to student teachers.

**Researcher role and positionality.** Acknowledging that students can influence the culture of participation if teachers are willing to carefully consider their perspectives meant that rather than approaching this site as a knowledge-bearer, I joined the students and the teacher in ongoing dialogue about their participation. I assumed the role of a participant-observer (Glesne, 2011) working with the students and the teacher with a genuine interest in understanding their participation-related perspectives and practices. The synergy between Valery's desire to learn with her students and my commitment to "openness, curiosity, desire and willingness to interact in collaborative ways" (Glesne, 2011, pp. 157-158) invigorated our interest in this collaboration. Our common ground as Latina/o, bilingual educators facilitated rapport building, and our differences—Valery as a Cuban-America teacher and I as a Colombian international graduate student—enriched our collaboration.

I made sense of the culture of participation in this classroom, while being aware of my influence in this context, as I was moving along the insider-outsider researcher positionality continuum (Foote & Bartell, 2011; Glesne, 2011). Over time, I became familiar with classroom norms, participated in classroom routines and, in some ways, students interacted with me as an insider to this classroom. Together, the students, the teacher, and I gradually developed trust that facilitated the dialogues we had about participation and about the connections between reflection and practice. Working together with the students and the teacher in their mathematics lessons, I adopted practices related to the culture of participation of this classroom and I needed to reflect on my involvement in these practices to bring to my awareness my interactive role in the evolving culture of participation.

The types of relationships we developed in this classroom along with my dissertation's focus on joining students in reinterpreting what it means to participate required challenging hierarchies that favor researchers' or teachers' knowledge. Accordingly, the methodological decisions I made included using a participatory design research with children methodology to study a classroom issue that was meaningful to the teacher, the students, and the researcher. The issue of transforming the culture of participation emerged through several conversations and after two years preparing the collaboration, which I discuss in the following section.

#### **Preparing the Research Collaboration**

My initial research on this site focused on the interplay between mathematical activity and language use in classrooms where the language of instruction is different from students' first language. In the spring of 2016, I observed two mathematics units, and Valery and I co-analyzed teaching episodes. In the fall of 2016, I observed Valery's new third-grade math class, and she and I discussed possible issues for further research. Valery got interested in whole class discussions, noting that students tended to focus on giving correct answers. She envisioned students explaining their ideas and addressing each other. I shared in this interest and we started studying and incorporating number talks (Parrish, 2010) in her lessons. Initially, we decided together on a topic and I facilitated the number talks. We took this opportunity to explicitly discuss with children practices such as explaining their strategies, listening to each other's ideas, and justifying reasons to agree or disagree. After I conducted three number talks, Valery began to select and conduct number talks on her own.

Once students seemed more comfortable talking about their mathematical ideas, Valery and I focused on designing tasks and orchestrating discussions that would support the types of interactions Valery envisioned. We read and discussed articles from practitioner-oriented journals

(Breyfogle & Williams, 2008; M. S. Smith, Hughes, Engle, & Stein, 2009), and we studied groupworthy tasks (Featherstone et al., 2011). From November 2016 to June 2017, we designed and implemented tasks that had multiple possible solutions and multiple possible strategies, and that asked students to work collaboratively and explain their ideas. For each task, we anticipated a few strategies that were likely to emerge during small group work. We planned open-ended questions to probe and extend student thinking. Either one of us led the launching of each task and the whole class discussion that followed. We both supported student work in small groups.

During the fall of 2017, I observed math lessons in Valery's new third-grade group. I also occasionally observed recess, science, social studies, and the end of day reflection. For the mathematics class, the teacher and I continued using number talks and groupworthy tasks that I either observed or co-taught. Over time, Valery thought that although students seemed comfortable engaging in these tasks and discussing mathematical ideas, there were noticeable differences in participation. She was concerned with the pattern of a few students tending to dominate discussions and others tending to be quieter or limit their spoken contributions to agreeing or disagreeing with their peers. In my conversations with students, the notion of participation as involving more than talk emerged, which motivated my interest in bringing the students' and Valery's perspectives into dialogue.

## Methodology

In the process of developing a methodology that would cohere with the participatory design of my dissertation, I recognized the key role of the students, the teacher and the researcher coming together to reinterpret what it meant to participate in this classroom. At the same time, I was also aware that translating these emerging reinterpretations of participation into teaching practice would involve rethinking the role of the teacher and ongoing reflection, and

reconsidering the types of lessons and mathematical activities, as discussed in the theoretical framework. Accordingly, I wanted to develop synergies between two related methodological processes: the process of data generation and analysis and the collaborative process of fostering transformations in the culture of participation. Data generation and analysis needed to simultaneously provide evidence of the evolving culture of participation, orchestrate opportunities for the teacher to question preconceived ideas of what it means to participate, and support her in revising tasks, lessons and teaching strategies. For this purpose, I used two complementary methodologies: participatory design research (PDR) and research with children. I draw on PDR because of its simultaneous focus on structuring collaboration and learning from the collaboration. I draw on research with children because of its attention to children's perspectives that are foundational to the design of a collaborative research. The resulting methodology, participatory design research with children, is consistent with my conceptualization of participation as a phenomenon influenced by all involved and it supported the teacher's process of learning to listen hermeneutically for participation.

**Participatory design research.** Participatory design research is consistent with this study's interest in maintaining partnerships that facilitate collaborative design, implementation, and analysis of interventions that address local (i.e. classroom) issues (Bang & Vossoughi, 2016). Design based research—one of the frameworks that PDR integrates—defines interventions as a set of commitments and actions to address a classroom issue (Bell, 2004; Engeström, 2011). Design based research involves cycles of planning, implementing and reflecting interventions based both on theory and on teachers' perceptions of the issue of interest. In this dissertation, this methodology involved iterative cycles of designing, analyzing and refining interventions to transform the culture of participation. The overall purpose of these
cycles of intervention was to provide recurrent spaces for all participants—teacher, students, and researcher—to engage in hermeneutic listening. By coming back to these spaces, each time with broader, more complex ways of listening and seeing, the teacher and I expected to provoke a departure from listening for predetermined forms of participation toward acknowledging and fostering ways of participating previously ignored.

Participatory research—the second framework that PDR integrates—seeks to challenge power hierarchies, including the positioning of the researcher as an expert that comes to the site to execute a predetermined agenda (Cornwall & Jewkes, 1995; Fals-Borda, 1987). Participatory approaches do not regard the researcher as the producer of knowledge, and the researched as the object of analysis and the recipient of knowledge. Instead, participatory researchers move from working on or for participants to working with them (Berg, 2004; Bergold & Thomas, 2012; Hansen, Ramstead, Richer, Smith, & Stratton, 2001; Powell, Jeffries, & Selby, 1989). Participatory methodologies are consistent with this dissertation's theoretical framework that focuses on the teacher reinterpreting with students what it means to participate in this classroom.

In PDR, the process of partnering—establishing, maintaining and reflecting on the partnership—is foundational to the research (Bang & Vossoughi, 2016; Gutierrez, Engeström, & Sannino, 2016; Vakil & de Royston, 2016). This process contributes to making interventions likely to be legitimate and relevant to participants (Ehret & Hollett, 2016; Gutierrez et al., 2016). In this dissertation, the design, implementation, and analysis of the interventions did not depend solely on the researcher. Instead, Valery and I collaborated in these iterative processes, drawing on students' perspectives. This approach contributed to the robustness of interventions—by including the perspectives of those most directly affected—and the robustness of the analysis, as we co-developed different insights and interpretations. The PDR approach also provided

opportunities for the teacher to engage in hermeneutic listening for participation. Each intervention cycle involved reflecting on taken for granted ideas about participation and about the teacher's role on participation, as well as engaging in participatory interactions where the students influenced the unfolding of the lessons. Teaching strategies associated with hermeneutic listening, such as asking genuine questions, directing students' listening and amplifying students' participation, informed the design and implementation of interventions.

Research on mathematics education that uses PDR has focused primarily on issues involving adults (Steinberg & Kincheloe, 2010). Examples include parents, teachers and researchers collaborating around issues of reform-based mathematics teaching (Booker & Goldman, 2016) and the collaboration between an instructor and his adult students to transform an undergraduate class pedagogy (Powell et al., 1989). Given that this dissertation involves children, in the following section I synthesize concepts from research with children that inform my study.

**Research with children.** Research with children methodologies recognize that children are knowledgeable about their own experiences and that they can play a role in transforming their circumstances. Doing research for or on children frequently assumes that children are empty vessels or unfinished individuals in transit to adulthood (Clark & Moss, 2011; Cook-Sather, 2002; Groundwater-Smith et al., 2014; Punch, 2002). Instead, doing research with children capitalizes on the belief that children act according to and have insight into the cultures of their age groups (Kirk, 2007; Rudduck, 2007). In this dissertation, the design and analyses of interventions involved children's perspectives, as a methodological innovation for revising current views on the issue of mathematics class participation.

This dissertation embraces methodological commitments that are sensitive to children's expertise and needs, including being flexible (Clark & Moss, 2011; Jacquez, Vaughn, & Wagner, 2013), suspending control at different stages of the process (Boyden & Ennew, 1997; Jacquez et al., 2013; N. Thomas & O'Kane, 1998), and attending to participants' perspectives (Kirk, 2007; Punch, 2002). These commitments echo tenets of hermeneutic listening that focus on participatory interactions that foster mutual influence, as teachers learn with students to broaden the ways they see, hear, and feel participation. A way to share control and attend to children's perspectives is to flexibly draw on multiple methods that are appropriate for particular children (Hill, 1997; Kirk, 2007; Punch, 2002). Children's comfort participating in methods traditionally used with adults varies depending on the topic of discussion, their age, the researcher-interviewee relationship, self-confidence, and language skills (Punch, 2002). Accordingly, I drew on multiple data generation methods, as I describe next.

### **Data Sources**

There were two stages of data generation. The first stage focused on supporting the teacher to bring into dialogue the students' and her initial perspectives on what it means to participate. The second stage focused on connecting the teacher's reflection to her practice.

**Reflections on what it means to participate.** Bringing the students' and the teacher's perspectives on participation into dialogue to begin to reinterpret participation involved eliciting students' perspectives. I conducted multi-methods focus groups with students to discuss participation in general and in the mathematics class specifically (see Appendix A). The structure of focus groups resonated with the consideration of both the individual and the social influencing participation, as individual contributions frequently prompted conversations where the group co-

developed ideas. Additionally, working with small groups and using multiple methods helped me attend to all students, as opposed to the most outspoken only.

I conducted six focus groups with three or four students in each group, during the second week of April 2018. Consistent with tenets from research with children methodologies, the structure of these focus groups was flexible. I continuously reiterated that participation was voluntary and that students could choose whether to join a focus group or not. I informed students that I was going to share their ideas with the teacher. Students chose their own groups, an important consideration to help the children feel comfortable sharing their ideas. Students could also choose to leave the group at any moment and join another group later, or not join at all. Two students took up the option of leaving a group and came back a different day. Although I started the groups in Spanish, I asked students to express their ideas in either Spanish or English. In most groups we ended up using both languages at different times.

Students could also choose among multiple methods to share their ideas. I initially asked students to think about what it meant to participate to them and how they participated in the mathematics class. Each student individually decided whether to draw or write about these ideas or to think and then share during the conversation. Paper and markers were available and all but one student used these resources. After this individual part, each student could decide whether to share ideas with the rest of the group or to share privately with me after the focus group. All students shared their drawings and writings and explained what they represented. I asked follow up questions to clarify or to ask for specific examples. In all groups, students frequently addressed each other, adding to what others said or proposing alternative ideas. Each focus group lasted about 30 minutes. I audio recorded and transcribed each group and I collected students' work.

Acknowledging the teacher's role as a full participant in classroom activity and as an insider on the culture of the classroom, I also conducted an interview with Valery (see Appendix B). This 45-minute semi-structured interview took place after all the focus groups, and the teacher and I discussed her ideas about participation in her mathematics class. We also discussed context-specific situations that she thought influenced her perception about participation and how she addressed participation-related issues. I audio recorded and transcribed the interview. Recognizing my role as a participant-observer directly engaged in the classroom culture of participation, I wrote an analytical narrative reflecting on my own perspective on participation in general and in this mathematics classroom.

After I conducted an initial analysis of the focus groups and teacher interview data (as described in the Data Analysis section below) Valery and I co-analyzed some of the responses to bring into dialogue the students' and the teacher's perspectives on participation. This was a reflection opportunity where the teacher began to expand what she considered participation. I took field notes during this conversation. Valery and I also discussed how these emerging reflections of participation could inform the design and implementation of intervention cycles, which I discuss next.

**Connections between reflections on participation and practice.** Between April and June of 2018, the teacher and I engaged in the intervention cycles. Informed by the PDR methodology, the cycles included co-designing, implementing and reflecting on interventions. In the co-designing phase that was informed by the teacher's reflection, we engaged in an iterative process of planning and revising teaching plans. As the teacher implemented and reflected on these plans, we revised subsequent ones in response to the directions each unit took. I took field notes documenting the ideas from the first stage of data generation that guided each intervention,

the practitioner-oriented articles we read, and the ideas for lessons that we discussed. These designs were flexible and we made adjustments based on unanticipated ways of participating and mathematical ideas emerging in interaction with the students.

During the implementation phase, I video recorded 14 mathematics lessons (two lessons per week, for seven weeks), using two cameras. One was a handheld camera following the teacher and connected to the lapel microphone she wore. The other one was a wide-angle camera set on a tripod recording the entire classroom. I set an audio recorder at one of the students' tables to capture audio from different parts of the classroom. This set up helped see and hear both the teacher and the students. I used the Final Cut Pro software to create a unified video for each lesson, with a split screen showing the recording from both cameras. I transcribed all videos within 24-hours of the recording. Transcripts included spoken utterances in their original language, as well as descriptions of gaze, gestures, and movement related to the exploration of mathematical ideas. I also wrote analytic field notes (Glesne, 2011) to document tentative preliminary interpretations and questions about the transformation of the culture of participation.

The teacher and I reflected on the interventions and discussed adjustments to lessons, possible teaching strategies to try, and questions or insights about participation. I also had conversations with students whose participation continued to puzzle the teacher and me. In these conversations, I asked specific students about their participation, including what they were doing during a task, and how other students, the teacher and I helped them participate, or come up with or understand an idea. I had six of these conversations with different students, and I started each conversation letting the student know I had questions for my research, and that I would share notes from our conversation with the teacher. I took field notes about these conversations. In addition to these ongoing conversations both with the teacher and the students, during the

summer of 2018, the teacher and I discussed how the dynamic connection between reflection and practice facilitated a process of reinterpreting what it meant to participate. We conducted four 45-minute sessions co-analyzing data (as described below). I audio recorded and transcribed these sessions.

This methodological design of the PDR with children generated multiple sources of data that elicited the students', the teacher's and my participation-related perspectives. These sources of data are also consistent with the focus of hermeneutic listening on participatory interactions to transform classroom experience. Additionally, the co-analysis of interventions was part of the process of learning to listen hermeneutically for diverse student participation.

### Social Semiotics Analysis of Connections Between Reflection and Practice

I draw on social semiotics (Halliday, 1978; van Leeuwen, 2005) as an analytical framework to interpret the connections between the teacher's reflection and practice because understanding these connections is key to understanding the transformation of the culture of participation in the classroom. Acknowledging the influence of social contexts in meaning making, social semiotics focuses on the processes of mutual influence where culturally situated meanings emerge (Morgan, 2006; van Leeuwen, 2005). In this study, social semiotics guided the exploration of the mutual influence between the students' and the teacher's perspectives on participation that supported a transformation of what it means to participate in a mathematics classroom.

Instead of assuming that pre-established static meanings reside in specific semiotic resources (i.e. resources for meaning-making), social semiotics is concerned with the way meanings emerge in interaction and how social contexts influence these meanings (Halliday, 1978; van Leeuwen, 2005). Semiotic resources include language, images, symbols, gestures, and

artifacts that interact to support meaning-making (Vannini, 2007). I explore how the teacher learned to recognize and validate participation in students' use of multiple semiotic resources that transcended spoken language.

Social semiotics has been used in mathematics education research to make sense of multiple issues, including: how teachers and students continuously rearrange their relationships with each other, with mathematical ideas, and with multiple semiotic resources (Hoffmann, 2006; O'Halloran, 2005; Sáenz-Ludlow & Presmeg, 2006); how teachers' perceptions of students' socioeconomic backgrounds influenced classroom discourse (Atweh, Bleicher, Cooper, 1998, 1998); or how social factors influenced secondary students' construction of the nature of school mathematical activity (Morgan, 2006). Previous research has also explored how social semiotics can help teachers become aware of dominant discursive practices (de Freitas & Zolkower, 2009; 2011) or of the value that students see in using specific semiotic resources (Hoffman, 2006). In this dissertation, I draw on social semiotics to study the connections between reflection and practice, a process that began by bringing into dialogue the students' and the teacher's perspectives on participation, which I explain in the following section.

**Different meanings of participation.** In spite of the relative stability and cohesion that characterizes cultural groups, they are not monolithic and the meanings and values that different members attribute to specific practices differ. In social semiotics, connotations are variations within the culturally shared meanings, values, and practices of a social group. Frequently, children's connotations of specific aspects of a culture are ignored or suppressed (van Leeuwen, 2005, p. 22). Counter to this social practice, in this dissertation it was important to elicit what it meant to participate for both the students and the teacher. The first stage of data generation

focused on eliciting these meanings, to provide material for the teacher's reflections on participation.

Understanding transformations in the culture of participation. Over time, certain practices and meanings associated to cultural phenomena tend to go unnoticed and unquestioned. Semiotic innovation occurs when specific practices and resources take on new meanings in a social group (van Leeuwen, 2005). Members of the group may not put in words the transformations of the practices, but they recognize the novelty and respond to it in different ways, from adoption to resistance (van Leeuwen, 2005). I draw on semiotic innovation to explore the evolving transformations in the culture of participation that emerged not from the teacher's reflections or her practice but from the connections between reflection and practice.

Semiotic innovation occurs when different processes interrupt cultural traditions, static roles and overreliance on one semiotic resource that contribute to the stagnation of meanings associated to a cultural phenomenon (van Leeuwen, 2005). For example, teachers frequently assume that their role is to ensure the observance of norms that traditionally regulate classroom interactions, such as having students raise their hand to get a turn to talk in a class discussion. Semiotic innovation can occur when the teacher rearranges the way that students socialize ideas, thinking of alternatives to traditional discussions where teachers assume the role of moderators relying primarily on spoken language. In this dissertation, the second stage of data generation the design and implementation of cycles of interventions—sought to provoke the occurrence of semiotic innovation.

I focus on the interaction among challenging the authority of traditions, role shifting, and reassigning saliency—all of which contribute to semiotic innovation (van Leeuwen, 2005)—to explore transformations in the culture of participation that developed as the teacher connected

reflection and practice. Engaging in the hermeneutic listening process of interrogating taken for granted perspectives on participation involves challenging the authority of traditions that prevent teachers from thinking differently about participation, and assuming the role of a participant in mathematical activity as a listener that makes room for multiple ways of participating. Teaching strategies inspired by hermeneutic listening, including attending to more than spoken language and amplifying students' participation, helped the teacher reassign saliency to the use of multiple semiotic resources she was learning to see as legitimate participation.

#### **Collaborative Data Analysis**

Collaborative data analysis served to expand opportunities for the teacher to reflect on participation and to connect this reflection to practice. In the following section, I discuss the two stages of data analysis in which the teacher and I collaborated.

Analysis of perspectives on participation. This stage of data analysis focused on what it meant to participate for the students and for the teacher, as well as on bringing these different perspectives into dialogue. I did an initial transcript analysis of the student focus groups and the teacher interview. I annotated transcripts focusing on nuances in characterizations of what it means to participate. My experience as a participant observer in this classroom allowed me to make sense of students' responses and to follow up with them when necessary. Some students' statements were salient because they clearly represented my perception of the student's usual ways of participating, or because they were unexpected and surprising. For example, some outspoken students made comments problematizing whole class discussions. These comments were salient and intriguing for me because they revealed unknown participation related issues that even students who were apparently comfortable and skillful at navigating discussions experienced. I also annotated statements where students agreed, disagreed or built on others'

responses during a focus group, identifying convergence and divergence of perspectives on participation among students. I analyzed annotations across transcripts to identify recurrent ideas, which I interpreted as indicating relatively shared values about specific ways of participating. I contrasted recurrent ideas in the students' and the teacher's transcripts to identify divergence in the ways of participation that they valued.

Consistent with the collaborative nature of this dissertation, my initial transcript analysis was tentative and exploratory and the teacher and I engaged in co-analysis. This collaborative data analyses served as a reflection opportunity to carefully consider students' perspectives on participation. I selected vignettes of responses from the focus groups and from the teacher interview to co-analyze. Some vignettes illustrated the contrasting ideas that the students and the teacher had about participation. Other vigenttes included ideas that were unique and intriguing, having the potential of helping us make sense of participation in more inclusive ways. For example, while the idea of helping others as a legitimate way of participating was recurrent in the focus groups, only one student mentioned asking for help. Considering both helping and asking for help as complementary ways of participating eventually became the focus of one of the intervention cycles that I describe in Chapter 5.

In the co-analysis of these vignettes, I asked the teacher to relate the students' perspectives to her own perspectives on participation. I shared my initial annotations on the vignettes and frequently refined my tentative interpretations. During this co-analysis, the teacher and I began to discuss how to address ideas from this emerging reflection on participation in the intervention cycles. As these ideas evolved in the iterative process of designing and revising interventions, I analyzed the connections that the teacher drew among these initial ideas, her ongoing reflections on participation and her practice.

Analysis of connections between reflection and practice. The second stage of data analysis took place in the summer following the school year and focused on video data from the intervention cycles. Similar to the first stage, I conducted an initial analysis of the entire video data. I focused on interactions that illustrated evolving reinterpretations and legitimization of what it meant to participate in this classroom. I drew on my knowledge of the culture of the classroom to identify these interactions. For example, I noticed that when students would ask a question about a topic that was not included in the third-grade pacing guides, the teacher would explain that the topic was beyond their grade level. During the implementation phase of this study, however, the teacher began exploring some of these questions, as I describe in Chapter 5. I also drew on the students' and the teacher's comments regarding how they were feeling that something new or unusual was happening. That was the case when students asked to engage in one of the co-designed teaching strategies that Valery implemented, or when the teacher shared her excitement about students' positive response to a way of participation that was the focus of an intervention cycle and that the class had not considered before.

After identifying interactions that illustrated transformations of the cuture of participation, I preselected three videos (one from each intervention cycle) that I considered included pivotal moments in the transformations of the culture of participation as candidates to co-analyze with the teacher. Valery and I discussed these candidates and changed one of the videos for another one she was particularly interested in because of its focus on a teaching strategy that she was curious about.

From each of the videos, we watched excerpts that I had annotated. We focused on how the teacher recognized and drew attention to multiple ways of participating and how students responded and specific mathematical ideas emerged. We discussed our perceptions of

transformations in the ways of participating that influenced the unfolding of classroom interactions. Using these insights, I went back to the entire video data set to refine my interpretations. I attended to challenging the authority of tradition, role shifting and reassigning saliency to multisemiotic resources as contributing to semiotic innovations to promote the legitimization of multiple ways of participating. I also added notes and quotes from the coanalyses sessions related to school influences on classroom participation, and notes from conversations with children during the interventions. Considering multiple data sources that elicited different participants' perspectives strengthen the robustness of the interpretations.

Finally, the teacher and I co-analyzed the connections between her reflections on participation and her practice. We focused on her learning to listen differently, more hermeneutically, and how this different way of listening allowed her to see participation more broadly, including her role in amplifying multiple ways of participating. This collaborative data analysis facilitated the emergence of factors that influenced the connections the teacher established between reflection and practice, including raising perspectives on participation to awareness, taking the role of a learner of student participation, and flexibly revising tasks, lessons and units that could support multiple ways of participation that students valued. Collaborative data analysis is also consistent with the participatory research framing of validity in terms of the collaboration's usefulness for all participants (Bergold & Thomas, 2012) and its relevance to both research and practice (Creswell & Miller, 2000, p. 128).

#### Chapter 4 Broadening the Teacher's and Researcher's Perspectives on Participation

Beginning in this chapter and continuing in Chapters 5 and 6, I present findings that illustrate the interactive and evolving process of reflecting, preparing to, and engaging in transformative practice. In this chapter I describe how bringing the students' and the teachers' perspectives on participation into dialogue helped the teacher broaden her ideas on what it meant to participate. In Chapter 5 I describe how reflections informed the teacher-researcher co-design of interventions, preparing the teacher to promote diverse ways of participating and to address issues that stifled participation. In Chapter 6 I describe ongoing connections the teacher developed between reflection and practice, as she engaged in hermeneutic listening for participation that fostered transformations in the culture of participation.

I begin by discussing how opening up to alternative ways of participating involved letting go of deep-seated ideas regarding what counts as participation. Understanding nuances and diversity within the situated, context-specific culture of participation laid the ground for the teacher to begin engaging in hermeneutic listening for participation. In this reflection phase, Valery began to broaden her perspective on participation by learning to see beyond quantity and instead focusing on qualities of participation. She also began to expand her ideas by learning to locate participation in time and space, thus allowing herself to consider the lasting effects of participation and the students' multiple ways of enacting participation in the classroom. Unanticipated student responses piqued her curiosity, thus confronting her with tensions that motivated her to reimage her practice. All transcripts are presented in their original language, with English translation provided in italics.

## Learning to See Participation as Qualifiable

Valery began to question her perspective on participation as primarily quantifiable by considering her students' perceptions as something qualifiable. I illustrate this connection with an episode from a student focus group where a binary of either presence or absence of participation came up and was challenged by the group (Table 4.1).

	Student perspectives	Teacher perspectives
Paige:	If you won't care and try to get the answers right and	Valery: Los uso [los palitos] porque así [los estudiantes] saben
	read and if you're just being lazy and cheating,	que cualquiera puede participar, no sólo los que siempre
	that's not participating You need to read the	levantan la mano No siempre funciona porque pueden
	problems and try. Then you're participating.	pasar si no quieren participar, y entonces yo escojo otro
Willie:	Yo no estoy de acuerdo con este porque yo no puedo	nombre, pero me gustaría ver que todos participen
	estar aquí y no participar. Si yo estoy aquí, estoy	Porque a veces algunos no participan. Y hay varias
	participando.	formas de participar, no tiene que ser todos igual, pero
	I don't agree with that because I cannot be here and	que todos participen.
	not participate. If I'm here, I'm participating.	I use them [the sticks] because then they [the students]
Mackenzie: Mío es lo mismo. Yo no intentó escribir qué es		know anybody can participate, not only those who
	participar y yo vio el papel de Paige, pero yo estoy	always raise their hand. It doesn't always work because
	participando porque yo estoy aquí Yo estoy	they can pass if they don't want to participate, and then I
	escuchando y estoy viendo ahora.	draw another name, but I would like to see everybody
	Same here. I didn't try to write what it is to	participating Because sometimes some [students] don't

# Table 4.1 Example of participation as qualifiable and as quantifiable

# Table 4.1 (cont'd)

participate, and I did look at Paige's paper, but I'm	participate. And there are different ways of
participating because I'm here I'm listening and	participating, it doesn't have to be the same way for
I'm looking.	everyone, but [what's important is] that everyone
	participates.

In the student responses, Paige depicts a binary of participation and non-participation and she associates specific behaviors with either category in the binary. In contrast, for Willie and Mackenzie their presence in the classroom constitutes a valid form of participation. As described before, there were two stages in the focus groups: in the first stage, students chose whether to write, draw or think about participation, and in the second stage they shared their ideas. In the excerpt, Makenzie explained that in the first stage she did not write what it meant to participate and she looked at Paige's paper, which are behaviors that Paige associated with nonparticipation. Mackenzie, however, argues that she was participating. Her comments suggest that rather than identifying participation or non-participation in specific moments (i.e. in the first or in the second stage of the focus group), for her participated during the second stage but not in the first stage of the focus group, Mackenzie preferred to locate her participation in the whole focus group in ways that included the hermeneutic abilities of listening and observing.

Valery's characterization of participation was closer to the absence or presence binary that Paige mentioned. For Valery, the presence of participation—students sharing an idea or answering a question—implied a simultaneous absence of participation in that not every student could actually participate during whole class discussions. When she called on a student, the student chose between participation and non-participation by either accepting or declining Valery's invitation to talk. Since students' response to her invitation could vary every time, for Valery the status of participation or non-participation was not static but changeable over time. Yet, unlike Mackenzie who considered listening and observing as legitimate ways of participating, Valery's perspective was grounded primarily in student talk.

Considering these contrasting connotations of participation, Valery began to broaden her perspective on participation as she began to consider the situated ways in which participation occurred in her classroom. She experienced a tension between her curiosity about students' responses and apprehension about the implication of these responses on her teaching. Reinterpreting non-spoken participation as legitimate participation, Valery questioned her goal of trying to distribute talk time uniformly among students. This, however, conflicted with her commitment to preventing the monopolization of discussions by a few students. Navigating this tension involved reconsidering the relationship between participation and talk, which I illustrate in the contrasting student and teacher comments in Table 4.2.

<i>Table 4.2</i>	Examples	of prob	lematizing	participation	as talk
		- J P	a	p	

Student perspectives	Teacher perspectives
Calum:Tú puedes participar [cuando] muestras a todos qué hiciste	Valery: Casi siempre hay alguien que quiere participar,
y explicas tu estrategia. Pero si no escuchan o si no te	alguien que levanta la mano para explicar algo o
entienden o si Señora Abad es como 'oh sí. Siguiente,'	alguien que quiere mostrar qué hicieron. Es pocas
entonces es como, ¿para qué hablaste?	veces que pasa que nadie quiere [participar],
You can participate [when] you show everybody what you	entonces hay que darles más tiempo y tengo que
did and explain your strategy. But if they don't listen or if	pensar si están distraidos o a lo mejor están
they don't understand or if Mrs. Abad is like 'oh yes. Next, '	confundidos.
then it's like, why speaking out?	There's almost always someone who wants to
Lia: Puedes participar si dices la respuesta. Pero esto es posible	participate, someone raising their hand to explain
no deja que todos piensan porque ya saben la respuesta. O	something or someone who wants to show what they
es posible tú no sabe por qué es la respuesta. Entonces	did. There are few times when no one wants to
puedes participar pero es un poquito mal participación. Es	[participate], then I have to give them more time and
buen participación si tú explica la respuesta o si tú espera	I have to think whether they are distracted or maybe
que todos piensan.	they are confused.

### Table 4.2 (cont'd)

You can participate if you tell the answer. But maybe this doesn't let others think because they already know the answer. Or maybe you don't know why that is the answer. So, you can participate but it's a little bit bad participation. It's good participation if you explain the answer or if you wait for everybody to think. Students' perspectives on participation transcended talk and included the effect of such talk. Calum's comments problematized the tendency of equating talk with participation as, for him, talking did not necessarily mean desirable participation. Instead, he qualified participation in relation to the effects that his talk could have on others' thinking. Talk that did not influence others' thinking or the direction of a lesson was pointless participation for Calum. Lia's comments added complexity to the relationship between talk and participation. Rather than valuing talk in itself as a legitimate way of participating, how she qualified such participation depended on whether talk supported or hindered other people's participation. She problematized talk that denied others the opportunity to think or that did not illuminate how to develop a mathematical idea. Both Calum and Lia valued participation that positively influenced others.

In contrast with students' perspectives on participation as influencing others, Valery's perspective suggests a concern in the delay of talk rather than the effect of talk. While Lia related time to think to productive participation for all, Valery considered time to think as absence of participation associated with misbehavior or lack of understanding. This suggests that Valery perceived participation as immediate responses to her questions, regardless of if and how the class took up those responses.

As the teacher expanded her perspective to include qualifying participation, she was well on her way to rethinking what counted as participation. Initially, she showed some resistance to the idea of participation as being present without speaking out, focusing on what she perceived as a tension between acknowledging non-spoken participation and disrupting the monopolization of class discussions. Recognizing that listening and observing could be valid ways of participating and that uniformly distributed talk could be experienced as irrelevant or counterproductive, Valery was left with many questions that needed to be addressed. She

wondered about how she could bring all students' ideas into meaningful dialogue where they could influence each other's thinking in the limited time she had for each lesson. She also wondered about what else students saw as enhancing or stifling participation that she had not considered before.

I interpreted these wonderments as a productive destabilization of Valery's perspective on participation that could inform her hermeneutic listening for student participation. Rather than reducing participation to student talk, Valery began to question what, besides spoken language, she could interpret as participation, and how she could engage with students in a participatory interaction to influence the unfolding of lessons. These wonderments motivated Valery to consider how she could join the students in connecting ways of participating such as listening, observing and thinking with the idea of mutual influence.

### **Relocating Participation as a Dynamic Interaction Between the Individual and the Social**

While Valery's initial interpretation of participation foregrounded individual student characteristics and preferences, she began to consider the role that social interactions played in how students experienced participation. Table 4.3 shows perspectives on participation that relate the social and individual as equally important aspects.

Student perspectives	Teacher perspectives
Emma:si tú eres como 'yo, yo, yo' [levantando la mano],	Valery: Algunos aprenden mejor hablando sobre lo que están
entonces, Señora Abad va a call on you y puedes aprender	pensando, entonces a veces casi que es inmediato
más porque puedes explicar tus ideas.	que termino una explicación y van a empezar a
if you are like 'me, me, me'[raising her hand], then, Mrs.	participar. Pero hay otros que no aprenden así y tal
Abad will call on you and you can learn more because you	vez participan sólo cuando yo escojo quién responde
can explain your ideas.	una pregunta o resuelve un problema, y ni así Yo
Calum: Tienes que ser rápido. Levantas tu mano rápido para que	sé que a veces Daniel va a estar haciendo otra cosa y
Señora Abad te vea primero y entonces tú dices [tu idea]	entonces luego no va a poder decir nada cuando le
primero. Porque si no haces esto, entonces ella no te va a ver	pregunte algo. O Emi A ella no le gusta pasar a la
o ella va a elegir a otra persona. Esto es bueno porque tú	pizarra y contarnos sus estrategias.
puedes escuchar a un otra persona pero entonces tú no	Some learn better by talking about what they are
puedes decir tu idea. También a veces, si muchas personas	thinking, so sometimes it's almost immediately after I
dicen antes que tú, entonces cuando tú dices, ya no va a ser	finish an explanation and they'll start participating.
nuevo o como interesante.	But there are others who don't learn like that and

# Table 4.3 Example of over-competitiveness in whole class discussions

You have to be fast. Raise your hand fast, so that Mrs. Abad	maybe they participate only when I pick who gets to
sees you first and then you can tell [your idea] first. Because	answer a question or solve a problem, and not even
if you don't do that, then she won't see you and she will pick	then I know that sometimes Daniel will be doing
someone else. That's good because you can listen to	something else and then he won't be able to say
someone else but then you cannot tell your idea. Also	anything when I ask him a question. Or Emi She
sometimes, if many people tell [their idea] before you do,	doesn't like going to the board and telling us about
then when you tell yours, it won't be new or, like, interesting.	her strategies.

Emma and Calum's comments suggest that they experienced whole class discussions as over-competitive situations where students had to engage in a race to be the first one to capture the teacher's attention. Rather than personal preferences or individual characteristics determining participation, a social arrangement of competitive interactions stifled their participation. Students declared how they strategized interactions in whole class discussions in order to secure participation that was influential to others. Students perceived that their participation could be inconsequential to others depending on when they got to share their ideas. For example, although Calum valued listening that came from others, for him listening meant not sharing his ideas and not influencing others' thinking. During data analysis, Calum's comments on whole class discussions were salient because he was a talkative student who seemed comfortable frequently speaking up in discussions. Learning how he perceived whole class discussions was revealing and drew my attention to the role that different types of interactions played in how students experienced participation. Other students shared similar strategizing to optimize their participation in whole class discussions, including raising their hand quietly to increase the chance of being called on or waiting to raise their hand until they had something original to share. While some students tried sharing early rather than later so classmates would not get bored and stopped paying attention, others tried sharing when the teacher was asking complex questions that allowed for generative answers.

Unlike the students' perspectives, Valery's perspective on participation foregrounded individuality. Her comments suggest that rather than seeing interactions among students' participation, she attributed participation and non-participation to students' individual preferences. For her, these preferences were predictable, stable and unrelated to how others' participated. Valery interpreted absence of student talk in whole class discussions as specific

students' aversion to be on the spotlight, rather than related to an intimidating type of social interaction. She valued participation in the form of the quick responses from talkative students and she problematized what she perceived as the non-participation of quieter students.

Analysis of these contrasting perspectives on participation in whole class discussions helped Valery to relocate participation into the students' meaningful social interactions. Students' comments surprised her, as she was more concerned with the different amount and frequency of student talk during discussions. Rather than discarding her knowledge about specific students' preferences, she began to rethink participation as a dynamic interaction between the individual and the social, considering how the arrangement of social interactions influenced participation. She became increasingly interested in devising whole class discussions where students could experience a sense of mutual influence without feeling they had to compete for opportunities to participate in ways they liked.

Relating individual and social influences on participation helped Valery reflect on social arrangements that could support participation. Table 4.4 shows perspectives that illustrated this interplay.

Table 4.4 Example	'e of	partici	pation as	bel	longing	to a	group
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	Student perspectives		Teacher perspectives
Marta:	I like it when we're like [clutching hands]. So, we need to	Valery:	Algunos son más líderes y pueden organizer el
	help each other and then we're all participating together and		grupo y otros son más como de apoyar al grupo y
	doing stuff. Usually, that would help you participate because		no toman la iniciativa en las estaciones, a
	then you get lots of ideas or, like, cool stuff to do.		veces tomo eso en cuenta para asignar roles o
Coretta	: When someone does everything and says like 'OK, we have		partes de la actividad y que luego junten sus
	to finish this page and I'm going to do this,' and the rest of us		partes, pero que yo pueda ver lo que cada uno
	only copy, I feel a little discouraged or almost as if the group		sabe, lo que cada uno hizo.
	is scattered so, we're all participating but it feels like we're		Some are more leaders [than others] and they
	not in it together.		can organize the group and others are more like
Stacey:	I like helping others because I can help everyone feel part of		they support the group but they don't take
	the team.		initiative in the stations, sometimes I take that
Jimmy:	Sometimes you can participate too much because you're not		into account to assign roles or parts of the
	really helping but telling others 'this is the answer.'		activity and then they bring their parts together,
Rose:	Yeah. You have to think and you have to listen first.		but in a way that I can tell what each one knows,

# Table 4.4 (cont'd)

Jimmy: And you can say like 'I think we can try this way because I

what each one did.

draw it here and I think it works. What do you think?'

Students' perspectives suggest that they understood participation as membership to a social group. They valued participating in harmoniously coordinated activity characterized by collaboration, inclusion, and consideration of others' ideas, which is in stark contrast with the over-competitiveness of discussions. Valery's response suggests her concern with individuality, both in terms of which roles to assign to students depending on their individual characteristics and in terms of individual contributions to the task.

Of the multiple ways of participating that Valery was learning to see, the idea of helping resonated with her earlier interest in encouraging students to help each other. This convergence between her students' and her own perspectives served as an entry point for Valery to consider helping each other as a valid form of participation. She related students' interest in helping to her interest in promoting whole class discussions that could amplify rather than limit participation. Some of her previous efforts to encourage helping frequently involved asking students who finished a task first to help classmates who needed help. Reflecting on this idea, Valery commented that requesting students to help others finish a task seemed too prescriptive and limited compared to the spontaneous and creative ways of helping that students described.

### **Rethinking the Teacher's Role in Participation**

Reflecting on the students' and her perspectives on participation, Valery became aware of her perceptions on her role in student participation. While she initially saw her role as equitably eliciting student talk, she started to see her role as inviting students into meaningful mathematics tasks that acknowledged multiple ways of participating in cooperative interactions. The examples in Table 4.5 illustrate the contrast between students' perception of how the types of tasks influenced their participation and Valery's focus on eliciting student talk.

	Student perspectives	Teacher perspectives
Stacey: [Participate in] what? Like in charlas de números [number		Valery: Tengo que estar mirando. No sólo en las discusiones
	talks], or when Señora Abad is explaining something, or	sino también cuando están trabajando en sus grupos.
	when we work at our table?	Si veo que alguien no está participando, a veces me
Jimmy	: It's different but it's the same. It's the same because	siento con ellos y le pido a alguien del grupo que no
	sometimes what we're doing doesn't matter. Like, I love to	está participando que haga algo, como leer parte de
	ask questions all the time.	un problema que les puede servir para pensar en una
Juan:	Cuando no sabes la respuesta [a un problema] sí puedes	estrategia.
	participar. Cuando no sabes la respuesta tú debes participar.	I have to keep on watching. Not just during
	Tienes que hacer tu mejor. Tienes que preguntar y pedir	discussions but also when they are working in
	ayuda, y escuchar y pensar.	groups. If I see that someone is not participating,
	When you don't know the answer [to a problem] you still	sometimes I sit with them and I ask someone in the
	can participate. When you don't know the answer you	group who is not participating to do something, like
	should participate. You have to do your best. You have to	reading part of a problem that can help them think of
	ask questions and ask for help, and listen, and think.	a strategy.

Students' perspectives suggest that how they participated varied according to the specific types of activities. For Stacey, my initial question about how they participated in mathematics class was too general, as she saw specific ways of participating according to the activities. Although Jimmy recognized that some ways of participating transcended the type of activity, student responses in different focus groups indicate they tended to associate talk with participation in whole class discussions. In Jimmy's example, although asking questions may involve student talk, it is not a kind of talk where students explain or justify an idea but elicit others' ideas and it may express curiosity, confusion, or the need for help. Juan's response suggests that questions foster multiple ways of participating, including asking for help, thinking and listening. Although several students mentioned helping, Juan was the only student who mentioned asking for help. This idea was new to Valery and me and we eventually reconsidered the role of both helping and asking for help as a way of promoting collaboration among students, as I describe in Chapter 5.

When discussing activities other than whole class discussions, students mentioned drawing, observing, writing, reading, and working with specific materials (e.g. rulers and base ten blocks) as valid ways of participating. Students' comments did not seem to favor one way of participating over others or to subordinate one way to others. For example, instead of referring to listening or observing as simply facilitating talk, students described them as part of multiple, alternative and/or complementary ways of participating. During co-analysis, although Valery was intrigued by the idea of multiple ways of participating as alternative and complementary, she continued to see a tension between validating non-spoken participation and supporting students in talking about their ideas. She seemed to value non-spoken participation as long as it eventually translated into student talk, which initially obscured her ability to envision ways to recognize and

incorporate alternative ways of participating into lessons. This became a recurrent topic of discussion that we revisited and addressed in subsequent stages of the collaboration.

Valery's comments in Table 4.5 above suggest that she saw her role as interrupting what she saw as non-participation, rather than wondering how students participated in a specific activity and how to support or amplify their participation. Learning about students' perspectives on how the types of activities influenced how they participated helped Valery consider that she could influence participation through the activities she planned.

Valery continued to reconsider her role in supporting participation when she learned about the contrast between the students' and her perspectives on pacing participation. The examples in Table 4.6 illustrate these perspectives.

# Table 4.6 Example of pacing participation

	Student perspectives	Teacher perspectives
Harper:	Yo quiero hacer otra cosa cuando ya sé lo qué hacer	Valery: Les pido que me muestren con sus pulgares si
	si yo sé qué debo hacer en el libro o yo sé qué hacer	entendieron, o más o menos, o no mucho. Y ahí, si
	cuando tenemos como un problema, si Señora Abad	muchos están confundidos, explico de una manera
	explica más y más y más, yo ya quiero hacer la cosa.	diferente o a lo mejor me devuelvo un poquito a algo
	I want to do something else when I already know what I	que necesitamos repasar otra vez [A veces] se
	have to do. Like, if I know what I have to do in the	distraen o se cansan y preguntan '¿qué tengo que
	textbook or I know what to do when we have like a	hacer? ¿cómo se hacía? ¿cómo era?' Entonces si no
	problem, if Mrs. Abad keeps on and on and on	están prestando atención, yo les pregunto a ellos:
	explaining, I just want to go for it.	'¿cómo vamos a saber qué hacer si no tengo todos los
Emma:	Me gusta cuando hacemos lo difícil porque es más	ojos aquí?' y ahí ya puedo volver a explicar.
	divertido pero a veces Señora Abad dice: 'No pases	I ask them to show me with their thumb if they
	la página. No haces esa página hoy. Haces ésta.' Y a yo	understood well, or more or less, or not so much. And
	no me gusta porque sólo necesitas repetir y repetir y	then, if too many of them are confused, I explain in a
	repetir algo que ya sabes. No necesitas pensar.	different way or maybe I go back a little bit to

I like it when we do difficult things because it's more fun... but sometimes Mrs. Abad says 'Don't turn the page. Don't do that page today. Do this one.' And I don't like that because you just need to repeat and repeat and repeat something that you already know. You don't have to think. something we need to review again... [Sometimes] they get distracted or tired and ask 'what do I have to do? How do you do this? How was it?' So, if they are not paying attention, I ask them 'how will we know what to do if I don't have all eyes here?' and right then and there I can explain again. Students concurred that excessive teacher explanations held them back and prevented them from engaging in more meaningful participation. For Harper, her sense of participation was compromised when she was expected to listen to Valery when she was ready to engage with the mathematics problem on her own. Emma's comment suggests a desire to enliven participation by being allowed to move to more complex topics ahead in the book. For these students, participation promoted by students' sense of autonomy and curiosity was often disfavored by the culture of participation based on repetitive and unappealing activities.

While some students saw some repetitions as redundancies that undermined their participation, Valery saw those repetitions as a way to support participation. In contrast to students' appreciation of autonomous exploration of mathematical ideas at their own pace, Valery seemed to discourage what she perceived as discordant involvement in mathematical tasks. For Valery, her role involved ensuring a uniform pace that allowed her to verify whether students were keeping up with specific mathematical ideas before proceeding to others. This meant bringing all students' attention to the same explanations and activities to ensure everybody was ready to participate.

As Valery's awareness of these contrasting perspectives on pacing increased, she began to develop a more flexible consideration of the role she played in students' participation. In addition to taking into account the ways of participating that specific activities could enable, Valery began to think of the content of those activities. Rather than understanding her role as pacing participation around predetermined ideas, she started to wonder how she could join students in understanding and developing the multiple ideas that could emerge in interactions.

Valery's openness to learn from students helped her broaden her perspectives on participation, approaching it with a willingness to expand what she saw as participation and what
she considered influences on participation. Rather than having definitive visions of what participation looked like, Valery began to acknowledge that there were multiple interpretations of where participation was located, from participation identified in specific individual talk at specific moments, to participation through observation, thinking, gesturing and drawing that was part of social interactions at different paces during the span of a meaningful activity. The ways Valery began to talk about participation resonate with ideas from hermeneutic listening for participation as she highlighted how her students' perspectives were influencing her own perspectives. Curiosities, questions, tensions and ideas about this classroom's situated participation motivated and informed our collaboration to connect Valery's reflections on participation with how she prepared to recognize and validate multiple unanticipated ways of participating, as I describe in Chapter 5.

# **Reflections on My Evolving Perspectives on Participation**

Recognizing that my own connotations of participation could influence the culture of participation and how I made sense of the students' and the teacher's perspectives, I engaged in a reflection to bring to my awareness my own perspectives on participation.

Initially, I valued listening and observation, which might have made students' listening and observation in Valery's classroom salient to me. As a student, my preference was not to speak up in classes as much as possible but to add to discussions if and when I thought I had an idea or question worth sharing. Although I never felt that I was not participating or that my more talkative peers showed more or better participation, I was aware of the importance that peers and instructors tended to place on talk. As an instructor in mathematics teaching methods courses at the college level, I noticed differences in student talk and I explicitly brought this issue to my students. In different semesters, I administered a survey. Reflecting on the questions, I used

talking and participating interchangeably and I was looking at participation in terms of present or absent student talk. In Valery's classroom, I might have reinforced the existing culture of participation that favored student talk when I facilitated number talks while trying to manage talking time among students.

My researcher role influenced my perspectives on participation, as I had read about classroom participation and how others have studied this phenomenon. Like other researchers (see for example Civil & Planas, 2004; M. B. Wood & Hackett, 2018), however, a definition of participation as students talk failed to help me make sense of participation as it emerged in Valery's classroom. Additionally, I had researched the interplay between multiple semiotic resources and language in Valery's classroom in the first year of our collaboration. This may have contributed to the impact that students' comments on non-spoken participation had on me.

This brief but important self-reflection helped me acknowledge my own influence on the culture of participation in Valery's classroom. I also became aware that my perspectives on participation seemed closer to Valery's than to the students', which added to the synergies between our curiosity about and commitment to exploring students' perspectives.

# **Chapter 5 Preparing to Recognize Diverse Participation**

The process of learning to listen hermeneutically for situated participation began with Valery's reflection on her and the students' perspectives on participation described in the previous chapter. This process continued with the co-design of iterative cycles of interventions to connect Valery's reflection to her practice. This iterative co-design relates to factors that supported the teacher to learn to listen hermeneutically for participation, such as the role that different types of lessons and mathematical activities played in how students participated. It also fostered the teacher's flexible adjustment of tasks, lessons and units in response to our growing understanding of student participation.

Consistent with PDR principles, each intervention cycle included a set of commitments and actions intended to transform the culture of participation toward being more inclusive. We approached each intervention as an evolving opportunity to join students in reinterpreting participation. The design of the interventions was flexible and each cycle informed the following one by responding to ongoing reflections and discussions with students. Valery used the first lesson of each unit to elicit students' understandings of the topic and to observe initial ways of participating, which informed the development of the rest of the unit.

The focus of each cycle responded to Valery's interests and how she was making sense of the evolving culture of participation. For the first cycle, she prioritized addressing aspects that students thought stifled their participation. For the second cycle, Valery and I talked about how the culture of participation was still overemphasizing student talk, so the focus of this cycle was on non-spoken participation. For the third cycle, Valery was interested in addressing feelings that students associated with participation and she brought up the idea of the positive feelings that

students mentioned when they described helping others. Collaboration among students became the focus of the third cycle.

For each cycle, I discuss motivations and commitments, resources we drew on, the content of the unit, and focal teaching strategies intended to address different aspects of the culture of participation. Some of these strategies emerged in interaction, as Valery recognized and amplified participation in the classroom. We named those strategies after seeing them emerge across several interactions, and Valery continued to draw on them throughout the unit. Other strategies were designed and named as Valery and I discussed ways to address questions and tensions that emerged during her initial reflection. These designed strategies in each intervention cycle, we used them cumulatively. For example, the strategies from the first cycle were also part of the second and third cycles. I also discuss how the design of interventions became progressively more open-ended, as Valery's evolving conceptualization of participation and connections to practice became increasingly responsive to moment-to-moment interactions.

# First Intervention Cycle: Addressing Aspects that Stifled Participation

In this intervention cycle, the teacher wanted to address the two main issues that students reported as stifling participation: overdoing certain activities and topics, and the competitiveness dominating whole class discussions. The goals of the intervention were to provide opportunities for students to extend their mathematical thinking and to better support students to share ideas with the whole class.

Ideas from practitioner-oriented articles and knowledge from Valery's previous professional development informed the design. Regarding the goal of providing opportunities to extend mathematical thinking, we initially talked about planning extension activities (Gearhart &

Saxe, 2014). Valery also shared ideas from a professional development workshop about allowing time for students to make sense of problems on their own without the teacher explaining or directing. Regarding the goal of supporting students to share ideas with the class, when preparing the research collaboration, Valery and I had discussed ideas about orchestrating mathematical discussions (M. S. Smith, Hughes, Engle, & Stein, 2009). Some of these ideas, however, focused on student talk sequenced by the teacher, which conflicted with our focus on multiple ways of sharing ideas that may emerge in unexpected places and times. We discussed modifications and alternatives to whole class discussions, including using writing to engage students with others' ideas (Butman, 2014), and strategies to "narrow participation gaps" in whole class discussions (Hand, Kirtley, & Matassa, 2015).

To make sense of how ideas from these articles could help us address Valery's concern with the aspects that students reported as stifling participation, we analyzed the teaching unit as described in the textbook and the pacing guides. The pacing guides allotted two weeks for the ten lessons in this unit on multiplication. This was the second unit on multiplication in the academic year. Understanding the expected learning goals became important for the design of this and subsequent intervention cycles, as Valery began to think of including learning goals not included in the pacing guides as part of the extension activities. The first multiplication unit took place during the fall and it included three main objectives: (1) understanding meanings of multiplication, specifically repeated addition and counting objects in a number of groups of equal size; (2) developing proficiency multiplying two single-digit numbers and multiplying a singledigit number and a multiple of 10; and (3) understanding the commutative, associative and distributive properties of multiplication. This second multiplication unit included lessons that addressed these three goals and also lessons on one and two-step word problems.

For the first lesson of this unit, we planned on identifying ideas that could help us address students' comments about exploring what they called "interesting topics." The class revisited meanings of multiplication in a whole class discussion that the teacher initiated asking "¿Qué es multiplicación?" [*What is multiplication?*]. The class continued with a number talk intended to elicit multiplication strategies. We anticipated that students would use direct modeling and number facts to explain how they solved the problems. We considered prompting the use of derived facts to extend students' thinking. After this lesson, Valery and I decided to integrate the following two strategies, inspired in hermeneutic listening, in the rest of the unit.

**Focal strategy I:** Addressing wonderment questions. During the first lesson of the unit a student asked about multiplying two two-digit numbers which started an impromptu whole class discussion (presented in the next chapter). After the lesson, Valery and I discussed that asking questions was one salient way of participating that students brought up in the focus groups. I shared with Valery the idea of wonderment questions. Instead of asking factual information or confirmation on the correctness of an answer, wonderment questions indicate curiosity about an unexplored topic (Aguiar, Mortimer, & Scott, 2010; Chin & Brown, 2010). This includes concepts not yet understood, and 'what if' questions about changing the conditions or context of a concept.

We modified our initial idea of developing extension tasks and instead decided to address wonderment questions related to the topic of the unit as they emerged. Following the student's question about multiplying two two-digit numbers, addressing wonderment questions meant reframing the learning objectives of the unit to include multiplying two two-digit numbers. We modified the initial unit outline, starting with a lesson inspired in a number talk where students are presented a 8x25 grid (Parrish, 2010) and answered the question "¿Cómo podemos saber

cuántos cuadritos hay?" [*How can we know how many squares are there*?]. To lessen students' perception of over-competitiveness during whole class discussions, Valery and I designed the following teaching strategy.

**Focal strategy II: Gallery walks.** As an alternative to whole class discussions, Valery used gallery walks where she displayed student work around the classroom, usually in posters that grouped similar solutions or strategies. Students walked around the classroom, observing the posters. There were sticky notes and pencils by the posters so that students could write their reactions, including asking questions, expressing agreement or disagreement, noting connections between strategies in different posters, or explaining the work displayed. Finally, students went back to their own posters and read others' reactions. We tried different approaches to authorship, from asking all students to write their names on the papers to be displayed, to telling students they could decide whether or not to write their names, and whether or not to have their work displayed.

Gallery walks were intended to mitigate the over-competitiveness that students reported experiencing in whole class discussions. Instead of regulating the order of sharing and limiting the number of students who got to share their work, in gallery walks all students had opportunities to consider everybody else's mathematical ideas. Sometimes there was a brief whole class discussion following a gallery walk, where students shared what they noticed or what strategies they found interesting or surprising. Other times, a task followed where students were asked to try a strategy that they had seen in the gallery walk.

# Second Intervention Cycle: Recognizing Multiple Ways of Participating

This intervention cycle focused on students' non-spoken ways of participating, including gestures, drawings, writing, and movement. The goal of the intervention was to support the

teacher to learn to see students' non-spoken participation as valid and contributing to the ongoing mathematical activity in the classroom. At this point, Valery was committed to recognizing multiple ways of participating besides student talk. Ideas about gesturing and using specific materials emerged in conversations with two students during the first intervention cycle. During a whole class discussion in the multiplication unit, one student, Daniel, was folding a piece of paper under his desk. Valery noticed it and we discussed that Daniel could give us ideas for a multiplication task using paper folding. In my following visit to the classroom, Daniel was folding paper under his desk again. When I asked him if I could see what he was doing, he quickly responded "nada" [*nothing*], as he put the paper in his desk and opened his mathematics textbook. It took some talking to convince Daniel that I was not trying to catch him out of task, that I was authentically curious about what he was doing, and that the teacher and I thought paper folding could be part of a mathematics lesson he could help us launch. When I described this interaction to Valery, she became increasingly interested in validating the exploration of mathematical concepts associated with non-spoken actions.

Valery and a student, Coretta, had another conversation that further reinforced this interest in exploring non-spoken ways of participating. During a lesson, Coretta stretched her fingers and looked at them quietly before answering questions about word problems. When Valery approached her and asked if looking at her fingers helped her with the problems, Coretta said "Sí. Para ver la respuesta. A veces mis manos hacen cosas que yo no sé qué son" [*Yes. To see the answer. Sometimes my hands do things that I don't know what they are*]. This comment added to Valery's interest in exploring non-spoken ways of engaging with mathematical tasks.

We drew on practitioner-oriented articles on non-spoken mathematical communication (Hand et al., 2015; J. N. Thomas, 2018; Wilson, 2012). We discussed gesturing, drawing and

movement as ways to generate and animate mathematical ideas, instead of incidental to or a replacement of spoken communication. Additionally, we discussed examples of classroom interactions when Valery had explicitly referred to students' gestures and handling of objects. For example, I brought up a geometry lesson from the previous year, where Valery drew attention to a student's swiping movement with an arm to describe the tracing of an angle. Valery was particularly interested not only in her own recognition of students' non-spoken participation but also on students' recognizing each other's gestures and body movement as part of the exploration of mathematical ideas.

The pacing guides allotted two weeks for the ten lessons in this unit on fractions. There were four main objectives: (1) understanding a fraction as a part-whole relationship and as a number in the number line; (2) representing fractions using set models, area models and fraction bars; (3) comparing fractions, including comparing whole numbers to fractions and equivalent fractions; and (4) solving word problems that involved representing and comparing fractions. The unit focused on proper fractions and representing whole numbers as fractions. During the first lesson of the unit, Valery showed the fraction 1/4 written in a poster sheet and asked the class "¿Qué sabes de fracciones?" [*What do you know about fractions?*]. She asked students to write or draw anything they wanted to share about fractions on sticky notes and to place their notes on the poster. Following this activity, she read out loud some of the ideas on the sticky notes related to definitions of fractions. She then asked "¿Qué son las fracciones?" [*What are fractions?*] in a whole class discussion. The class revisited this poster throughout the unit, sometimes adding or revising the responses on the sticky notes.

We initially planned using materials such as paper and pencil, counters, geoboards, and unifix cubes to foster the use of gestures, drawings, and movement as ways of participating. In

the initial lessons of the unit, however, some students did not use paper to draw fractions, as Valery intended. Instead, they folded the paper to represent a fraction. In the second part of the unit, we designed lessons around a representation of Michigan demographics (see Appendix C). Both in the paper folding and in the demographics tasks, Valery drew on the following teaching strategy that related to hermeneutic listening's idea of amplifying participation.

**Focal strategy: Spotlighting gestures, drawings, and movement.** One way in which Valery spotlighted gestures, drawings and handling of objects involved showing these actions to the class to focus attention on specific concepts. For example, when students folded paper to represent and compare fractions, Valery drew attention to how a student was trying to be precise, making sure the paper was "doblado perfecto" [*folded perfectly*]. A follow up task emerged about the notion of the parts of a fraction being the same size. Another way Valery spotlighted nonspoken participation involved showing students' actions as part of a problem-solving strategy, which I describe in chapter 6.

#### **Third Intervention Cycle: Encouraging Collaboration Among Groups**

This intervention focused on helping as a legitimate way of participating. The purpose of the intervention was to validate and foster collaboration, which was informed by students' focus group responses, reflections on previous intervention cycles, and a conversation with a student. The idea of helping that emerged during focus groups became a recurrent topic of discussion during reflections. When Valery and I revisited ideas from our co-analysis of focus group responses, Valery mentioned that the first two interventions had addressed the issue of multiple ways of participating and relating the individual and the social. She wondered whether and how we could still address feelings associated with participation and the qualification of participation. The idea of supportive and productive help, endorsed by students, became relevant. In this

discussion, I brought up listening and observing as ways of participating that students had mentioned and that we had not elicited or recognized.

A conversation with one of the students during the second intervention resonated with these reflections. During the second intervention, Valery thought that Calum was not interested in one of the tasks because he left his group and stared out the window. Valery was particularly intrigued because this was unusual of Calum, and we decided that I would talk with him about his participation in that task. When I asked Calum about the moment when he left his table, he explained he was not bored. He thought the task was difficult, which he liked. He explained he was not just staring out the window: he was "más o menos como haciendo trampa" [*kind of cheating*]. His group was stuck and he thought he'd take a look at how other groups were approaching a problem. He thought that if he had looked at a group's work, that would have been considered cheating. He decided to stand close to a group to eavesdrop, looking out the window to avoid appearing as if he were cheating. When I shared what Calum said to me with Valery, we discussed ways to transform perceptions of what Calum did from cheating to collaborating.

We drew on practitioner-oriented articles about geometry teaching that resonated with ideas of multiple ways of participating (Mack, 2007; Tent, 2001). We also discussed ideas about collaboration in mathematics classrooms (Ghousseini, Lord, & Cardon, 2017) and we revisited the idea that one of the students, Juan, had mentioned during the focus groups about not only helping but also asking for help as an important way of participating. There were 12 lessons in this unit and the main objectives were: (1) categorizing shapes according to relevant attributes (e.g. number of sides) and characteristics (e.g. absence or presence of parallel sides); (2) solving problems about the area and perimeter of rectangles; and (3) classifying and comparing angles. For the first lesson of the unit we adapted Mack's (2007) task to elicit students' understanding of

two-dimensional shapes and foster collaboration among students. Each student received a card with a shape in it. Students walked around the classroom finding others whose shape belonged in the same category. A whole class discussion ensued, where Valery elicited students' reasons for their categorizing and naming of the shapes. The unit continued with lessons about the names and characteristics of different polygons (triangles, rectangles, pentagons, hexagon, heptagon, octagon). In the initial lessons, the teacher and I planned tasks where students would collaborate within small groups to make, name and describe shapes.

As the content of the unit deviated from the pacing guides and became progressively more unpredictable than in previous cycles, Valery and I relied on each other's expertise and ideas to refocus the lessons. Motivated by students' wonderment questions, in the second part of the unit the class explored properties of shapes. One lesson focused on the triangles of a polygon, with students working in groups exploring the relationship between the number of triangles that made up a polygon and its number of sides. The final part of the unit, that originally focused on defining and exploring strategies to calculate the area and the perimeter of rectangles, evolved to focus on exploring circles (as discussed in the next chapter). Throughout the unit, Valery continued to refine how she was integrating the idea of collaboration, observation and listening through the following focal strategy, related to hermeneutic listening's idea of engaging in participatory interactions to influence lessons.

**Focal strategy: Group ambassadors.** Valery proposed the idea of group ambassadors, as a teaching strategy to foster collaboration both within and among groups. Students started working in groups and once Valery considered groups had come up with strategies that they wanted to try, she asked each group to choose an ambassador. The ambassador went to another group and observed, while groups resumed their work. After an observation period, sometimes

Valery left it to each ambassador to decide if or when to intervene, ask questions or contribute ideas to the host group. Other times, she asked ambassadors to share how their original group was approaching the problem. Ambassadors then reported back to their original groups. Each group decided whether to adjust their approach to the problem taking into account the ideas the ambassador reported. With this strategy, Valery acknowledged observation and listening in as valid and legitimate ways of participating, while simultaneously fostering opportunities to help and ask for help without stifling participation.

# **Chapter 6 Connecting Reflection and Practice**

During the implementation and ongoing analysis of interventions, the connections between reflection and practice grew stronger and more evident, creating a synergy benefiting both processes. The intervention designs, described in the previous chapter, reflected Valery's vision for transforming the classroom culture of participation. Rather than understanding her role as following prescribed steps intended to facilitate participation, she responsively drew on her evolving repertoire of teaching strategies. Students responded by amplifying each other's ways of participating, joining Valery in legitimizing them. Creative ways of exploring unanticipated mathematical ideas emerged, intensifying Valery's openness when interpreting participation in times and spaces that she had not considered before. She continued to add complexity to her reconceptualization of participation, simultaneously recognizing the role of meaningful and flexible activity, multiple semiotic resources, and the influence of school demands.

In the following sections, I describe three transformations that the teacher, the students, and I experienced as we entered the implementation phase of the mathematics units: (1) expanding the scope of the mathematical ideas available for exploration, (2) using multiple semiotic resources to develop mathematical ideas, and (3) participating fluidly in mathematical activity. I draw on classroom examples from the video-recorded lessons that Valery and I co-analyzed, as well as examples from other lessons that helped understand each transformation. I discuss temporal and spatial dimensions of the kind of participation Valery was learning to see as well as factors influencing her hermeneutic listening for participation, including Valery's evolving role, the types of lessons and mathematical activities, and the flexible adjustment of tasks, lessons and units in response to student participation. After discussing interpretations of each example, I describe transformations from a semiotic innovation perspective. I describe

Valery's ongoing reflection on participation and her identification of tensions among her evolving reconceptualization of participation, and her evolving practice and demands from the school context. To do this, I draw on quotes from the co-analysis sessions.

# Expanding the Scope of Mathematical Ideas Available for Exploration

Over time, Valery joined students in exploring mathematical ideas they were interested in. An interaction from Fall 2016, where the class was developing different representations of 1/10, illustrates Valery's prior approach to exploring mathematical ideas:

	Utterances	Translation	Actions	
Billy	¿Puede ser al revés, como	Can it be the other way	With arms	
	no uno y diez pero diez y	around, like, not one and	stretched out	
	uno? Como, ¿una fracción	ten but ten and one? Like,	toward the board	
	puede algo así como tiene el	can a fraction like have a	where 1/10 was	
	numerador, cómo se dice	numerator, how do you say	written, moves	
	greater than, el	greater than, the	hands up and	
	denominador?	denominator?	down.	
Valery	Ah! Eso lo vas a ver no el	Oh! You'll study that not		
	próximo sino el siguiente	next year but the year after		
	año con Señora M.	with Mrs. M.		
Claire	Estamos en fracciones	We're studying unit	Looking at Billy.	
	unitarias, Billy.	fractions, Billy.		
Valery	Por ahora estamos viendo	Right now we're studying	Nodding.	
	fracciones unitarias.	unit fractions.		

*Table 6.1 Valery's previous approach to topics to explore* 

Valery's response to Billy and Claire's reminding him of the topic to be studied indicate that the class as a whole had established a border separating the mathematical ideas they could explore at a given time from the ones that were off-limits. During the intervention cycles, Valery blurred this border by expanding the scope of ideas the class considered as possible and relevant for their learning. Instead of pacing guides mandating mathematical content and Valery assuming the role of an enforcer of this content, alternative unexpected topics emerged in the class. The perspectives from the student focus groups regarding feeling ready and curious about topics beyond the scope of a lesson sensitized Valery to notice and explore unexpected ideas with her students. As Valery developed lessons from these ideas, students became increasingly explicit in their suggestion of ideas. Although Valery and I initially co-planned lessons that addressed topics predetermined in the pacing guides and the textbook, topics evolved in response to students' declared interests.

In the following sections, I describe how addressing students' curiosities and recognizing mathematical ideas in off-task comments motivated participation in mathematical activity that involved ideas meaningful to both students and Valery.

Addressing students' curiosities. In addition to using questions to elicit students' understanding, Valery engaged in interpreting the wonderings that students' questions communicated. In turn, students recognized that certain questions could influence the ideas the class explored. Considering students' wonderment questions became a legitimate way of participating that steered the direction of a unit beyond the scope of the content predetermined in the pacing guides. This process started in the first lesson of the first intervention, when the following interaction emerged during a whole class discussion on what it means to multiply:

	Utterances	Translation	Actions
Calum	Tengo pregunta. ¿Qué es cien	I have a question. What's one	Looking at Valery.
	por cien? ¿Es diez mil?	hundred times one hundred? Is	
		it ten thousand?	
Jimmy	Mil.	One thousand.	
Valery	¿Escuchas la pregunta de	Can you hear Calum's	Addressing the
	Calum?	question?	class.
Calum	¿Qué es cien más cien? Es	What's one hundred plus one	Speaking up,
	diez mil.	hundred? It's ten thousand.	addresses the class.
Jimmy	Mil.	One thousand.	
Lia	No, no es mil.	No, it's not one thousand.	
Calum	Yo pienso es diez mil.	I think it's ten thousand.	
Valery	¿Por qué?	Why?	
Calum	Porque no puede ser mil	Because it can't be one	
	porque mil es cien por diez.	thousand because one	
		thousand is one hundred times	
		ten.	
Halley	Tú sabes la respuesta.	You know the answer.	Points at Valery.
	Y tú. Dinos.	And you. Tell us.	Points at José.
Valery	Pero, ¿cómo sabes la	But how do you know the	
	respuesta? Vamos a tener que	answer? We'll have to do	
	hacer más de esto después.	more of this later.	

Table 6.2 Example of wonderment question

Unlike her earlier concern for isolating content by grade level, Valery engaged the class with Calum's wonderment question. As more students showed curiosity and shared ideas, Valery became increasingly interested in this topic. After the lesson, Valery and I discussed how Calum's question could inform future lessons. She decided to introduce multiplying a two-digit number by a single-digit number, which was beyond the scope of the pacing guides. The class engaged in a task figuring out the number of cells in a 25x8 grid. Using a gallery walk—one of the strategies we co-designed as an alternative to whole class discussions— students compared different strategies. They assessed the efficiency of counting by ones, repeated addition, and splitting the problem into smaller parts. Eventually, the class converged to the idea of using facts they knew to split the problem into smaller parts (e.g. 4 x 25 equals 100 and 2 x 100 equals 200).

Toward the end of the unit, Valery revisited Calum's question, asking the class to figure out ways to know what one hundred times one hundred is. During this whole class discussion, Calum was quieter than usual. As students where putting away materials immediately after the lesson, the following interaction took place:

Tahle	63	Calum	ı's	sense	of	nartici	natic	n
ruoic	0.5	Cuium	5	schse	<i>v</i> <sub>j</sub>	parici	punc	<i>''</i> 1

	Utterances	Translation
Halley	Esto era muy difícil. A mí me gusta	This was very hard. I like that a lot. That's
	mucho. Esto es por qué yo mostré tantas	why I showed so many strategies. What
	estrategias. ¿Qué de ti, Calum? Hoy no	about you, Calum? You didn't participate
	participaste.	today.
Calum	Todo que hicimos hoy fue para responder	Everything we did today was to answer
	mi pregunta 'qué es cien por cien' que yo	my question 'what is one hundred times
	quería saber.	one hundred' that I wanted to know.

Drawing on the idea from focus groups of considering qualities instead of quantity of participation, Valerie and I reflected on Calum's statement in terms of the relative importance of the number and length of his speaking turns vis-à-vis the influential reach of his question by reshaping the topic of the entire lesson. For him, influencing the topic the class was studying constituted a personally important way of participating. This way of participating highlights the temporal dimension of participation extending beyond the utterance or lesson level. Calum's question had enduring effects for the class and for him, as he experienced a sense of mutual influence even in this last lesson of the unit when he was quiet. This co-interpretation of this brief interaction encouraged Valery to continue exploring wonderment questions as a way of challenging her interpretations of participation on the basis of quantity.

Although I focused on Calum's example, there were other instances of wonderment questions that the class took up. In one lesson where the class was working on a task about recognizing fractions as numbers on the number line, Lia asked whether fractions could be added, since they were numbers. The topic of adding fractions became part of the unit, extending topics from the pacing guides that considered representing and comparing fractions only. In another lesson, Rose asked what were "esos números antes de cero" [*those numbers before zero*], referring to the negative numbers on a number line displayed on the wall above the board. Negative numbers became the exploration of a series of number talks.

Students picked up on Valery's interest in addressing their curiosities and Valery perceived an increase in the number of wonderment questions asked. Valery and I wondered whether this perceived increase in student questions was related to students noticing that Valery was now taking up and exploring their questions, or if the number of wonderment questions was stable but Valery recognized them more often now than before. My analysis of the entire video

data set suggests that students gradually became more insistent when asking these types of questions. Two examples include Juan asking if they could study the abacus and Coretta asking how it was possible for her older sister to add and subtract letters and numbers. Valery briefly entertained their questions, asking students what they knew about the abacus and asking their conjecture about what letters represented in mathematical operations. She announced she would take note of the questions and would try to address them when they had time. A few weeks later, Juan insisted on his question suggesting he could bring an abacus to school for an activity. Stacey and Marta brought up Coretta's question, asking if they could talk about her question "como hablamos sobre que Calum quería" [*the way we talked about what Calum wanted*]. In both cases, students drew attention to wonderment questions as possible sources of topics to explore. In Coretta's case, other students showed interest in her question and directly asked for the question to be given the same importance that Calum's question received. Calum's question and how Valery addressed the question continued to influence students' participation over time beyond that specific interaction.

Semiotic innovation that facilitated addressing students' curiosities involved the interaction among challenging the authority of tradition, shifting roles and reassigning saliency to questions. Sticking to the goals predetermined in the pacing guides, even if they did not reflect the students' interests, was a strong tradition that Valery used to observe. Valery dared to challenge this tradition by expanding the learning goals. This involved Valery shifting away from her role of safeguarding predetermined content and, instead, looking for connections between predetermined content and the ideas students were interested in exploring. Students' roles also shifted, as they started to become co-participants in determining the content of lessons.

Valery reassigned saliency to wonderment questions as a legitimate way of participating and students responded favorably by engaging in this way of participating.

When we discussed the process of addressing wonderment questions, Valery expressed a tension related to school expectations, specifically assessment. Valery could decide some of the assessment for each unit, but other parts were predetermined by the school. Although she was curious about and thought the class benefitted from addressing wonderment questions, she frequently felt the need to filter those questions in light of the school assessments. Valery felt that she should address only student ideas that would help them during the school assessments, even if unanticipated explorations were more sophisticated than the predetermined content. As she joked: "no va a importar que saben hacer un edificio entero, si la prueba sólo quiere ver que sepan abrir la puerta del edificio" [it won't matter if they know how to build an entire building if the test only cares about whether they can open the building's door]. Valery looked for ways to connect the explorations of students' wonderment questions to opportunities for them to better understand topics that would be assessed. In the multiplication unit, for example, she explicitly expressed that multiplying a two-digit number by a one-digit number could involve multiplying two one-digit numbers, which was the topic in the assessment. In other cases, however, she felt the class would need to use their time to address wonderment questions besides studying predetermined topics.

**Recognizing mathematical ideas in "off-task" behavior.** Throughout the interventions, Valery recognized mathematical ideas in seemingly off-task behavior. These ideas pushed the boundaries of the mathematical content that the class explored. Over time, students began to bring attention to mathematical ideas in jokes, side conversations and games. I illustrate this transformation with an example from the third intervention cycle. The planed content of the

geometry unit focused on defining and calculating the area and perimeter of rectangles. In this lesson, students had solved examples of the area and perimeter of rectangles on the board. As Valery turned to her desk to get materials to transition to a new task, Willie went to the board and grabbed a marker:

	Utterances	Translation	Actions	
Willie	Y el círculo es como un	And the circle is like a zero	Smiling as he draws a	
	cero porque no tiene área y	because it doesn't have	circle on the board and	
	perímetro.	area and perimeter.	writes a number zero in the	
			middle of the circle.	
Valery	Amigos, ¿vieron lo que	Friends, did you see what	Quickly turns around to	
	hizo Willie?	Willie just did?	see what Willie drew.	
			Facial expression of	
			surprise, as she addresses	
			the class.	
Class			Some students giggle,	
			others say no.	
Valery	Mira lo que hizo aquí.	Look what he did here.	Pointing at the circle.	
	No todos te oímos.	Not everybody heard you.	Looking at Willie.	
Willie	¿Esto?	This?	Stops smiling. Points at the	
			rectangle problem he	
			explained earlier.	
Valery	Aquí.	Here.	Pointing at the circle.	

Table 6.4 Willie's strategy for the area of circles

*Table 6.4 (cont'd)* 

	Lo que acabas de decir.	What you just said.	
Willie	¿El círculo es como un	The circle is like a zero	Looking at Valery.
	cero porque no tiene	because it doesn't have	
	perímetro y área?	perimeter and area?	
Class			A student gasps, others
			giggle, others exclaim
			'what?!'
Valery	¡Qué interesante!	How interesting!	Addressing the class.

Valery and I interpreted this episode in three parts: in the first part, Willie seemed to be joking, smiling as he said something he found funny. In the second part, when Valery draws attention to the circle and to Willie's comment, he seemed confused and tried to redirect the teacher's attention to one of the problems he had explained earlier. In the third part, some students started considering what Willie said and expressed surprise. What started as a joke during the few seconds when Valery turned to her desk, evolved into a mathematical idea that Valery spotlighted and that generated surprised and interest.

This interaction shifted the focus of the unit to the topic of circles. Willie's comment motivated a series of lessons that started with a heated debate about whether the circle was a shape. Some students argued the circle was not a shape because a shape was enclosed by sides and circles did not have sides. These students argued that sides had to be straight lines. A new debate arose, with other students saying sides could be curved, to accommodate the case of the circle. Eventually, these debates cycled back to redefining perimeter as "la distancia alrededor de una figura plana" [*the distance around a flat shape*]. Students thought this definition worked for both quadrilaterals and circles, as opposed to the textbook definition of perimeter as the sum of the lengths of a polygon's sides, that they thought did not work for circles. As these debates and redefinitions evolved, the class engaged in figuring out the area and the perimeter of circles using their own definitions.

When co-analyzing this episode and its influence on the direction the unit took, Valery reiterated her interpretation of specific interactions as sources of meaningful mathematical activity that enabled participation. She mentioned: "Ellos mismos dijeron que les gusta participar en cosas interesantes. Entonces hay que encontrar las cosas interesantes" [they themselves said they like to participate in interesting things. So we have to find the interesting things]. In the example, the ideas about circles, sides, area and perimeter that the class explored were topics that students considered interesting and that generated debates they did not want to stop. Referring to the initial outline of the unit, Valery mentioned: "Lo que seguía era más lecciones de área y de perímetro y ¿cómo iban a participar ahí? A lo mejor bien aburridos y sin creatividad porque no era nada nuevo" [What was coming up was more area and perimeter and, how were they going to participate in something like that? Maybe all bored and with no creativity because it wasn't anything new]. In this quote, she acknowledges the relationship between the content of activities and how students participated. In contrast with her initial perspective of participation, instead of framing participation in terms of the absence-presence binary, she qualified participation, predicting they would have participated in apathetic and uncreative ways. Flexibly adjusting tasks, lesson and units in response to student participation emerged as a characteristic of the evolving culture of participation that fostered the sense of mutual influence that students valued.

There were other instances of mathematical topics that emerged from what could initially be perceived as off-task behavior. That was the case when Valery noticed that Daniel was folding paper during a whole class discussion and Valery and I talked about approaching him to learn about what he was doing. The following time I visited the classroom, Daniel was folding paper again. When I asked him to share what he was doing, Daniel hid the paper quickly thinking I was trying to catch him off-task. Eventually he shared what he was doing and Valery became interested in using paper folding for a task on multiplication. Daniel helped introduce this task to the class, as he showed his paper folding and Valery asked students to tell the number of parts there would be after each folding and to explain their conjectures. Another example comes from the fractions unit when a lesson using coins as fractions of a dollar emerged as Gina overheard two other students playing as if they were buying items from each other. In their game, Ada was buying from Mackenzie and she needed one dollar in change. As they both looked at the play money, Ada asked Mackenzie to give her "two quarters and five dimes" in change. Gina brought this to Valery's attention, saying: "hay fracciones en monedas" [there are *fractions in coins*]. Valery eventually used play money in a lesson on fractions addition. Valery looked for and recognized participation in Daniel's paper folding and in Ada and Mackenzie's play, instead of isolating participation as an occurrence located in the space of a class discussion or small group work when students talk about mathematical ideas.

Similar to addressing students' curiosities, semiotic innovation also influenced the recognition of mathematical ideas in off-task behavior by reassigning saliency to students' ideas, promoting role shifting, and challenging the authority of tradition. The exploration of circles emerged as Valery assigned saliency to a comment that Willie intended as a joke and not as a mathematical idea worth exploring. Instead of adopting the classroom management role of

enforcing classroom norms and minimizing off-task comments, Valery's role shifted toward interpreting what Willie did as a way of participating that involved meaningful mathematical ideas. Willie's role shifted too, from a joker or a disruptive student to authoring a mathematical claim that the class debated in several lessons. Valery noted that students at this school began to explore the area and perimeter of circles in fifth grade. Instead of delaying this exploration, Valery challenged the authority of tradition established by the pacing guides by joining her students' interest in exploring circles. As she expanded the scope of the unit in this direction, she thought that exploring ideas about circles helped the class develop nuanced understandings of the concepts of sides, area and perimeter in polygons.

This transformation presented a tension for Valery. In several instances, such as Willie's example, Valery felt she put students on the spot to showcase their ideas. Valery wondered if this made some students uncomfortable, especially when students thought she was drawing attention to them because they were doing something unrelated to class. Valery related this concern to the social aspects associated to participation that students expressed during focus groups. She mentioned a tension between spotlighting mathematical ideas in off-task behavior and avoiding students' embarrassment.

#### **Using Multiple Semiotic Resources to Develop Mathematical Ideas**

In the previous examples, Valery drew on two forms of student talk, wonderment questions and "off-task" comments, to expand the mathematical content the class considered. In addition to student talk, she began to recognize and draw attention to gestures, drawings and movement that she considered as relevant to the development of ideas. Both Valery and the students started to value these semiotic resources in mathematical activity. Recognizing multiple semiotic resources meant relocating participation from seeing participation in students'

utterances to seeing participation in students' gestures, movements, gaze, drawings, and handling of objects.

One example of the recognition and uptake of multiple semiotic resources comes from the unit on fractions, in the second cycle of interventions. Valery handed out copies of graph bars representing different Michigan demographics (see Appendix C). Students worked in small groups, figuring out what fraction of the population corresponded to each category in the graphs (for example, which fraction of the population spoke a language other than English at home and which fraction spoke English). Because of the quality of the printing, the fill-in color of each bar appeared not as a solid color but as if it had been printed out in smaller sections, likely due to an ink cartridge that needed replacement. Valery and I did not notice this coloring issue but the students did, and many groups interpreted the broken coloring as showing each section partitioned into equal parts. To figure out the fractions, students counted the number of these parts in each bar. For the first graph, representing women and men, students counted more than 50 equal parts. Valery tried unsuccessfully to elicit alternative strategies. In the following example, Ada and Willie were leaning over observing as Jimmy counted out loud the parts in the women's bar and Marta counted the parts in the men's bar.

Table 6.5 Stu	dents' initia	l strategies to	<i>compare fractions</i>
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	Utterances	Translation	Actions
Ada	Señora Abad, ¿hay más	Mrs. Abad, are there more	Addressing Valery.
	mujeres o hombres?	women or men?	
Valery	Mmm.	Mmm.	
Willie	¿Quieres contarlos?	Do you want to count them?	
Valery	No,	No,	Shakes head, smiling.

	y por eso tengo una	and that's why I have a		
	pregunta, Marta y Jimmy.	question, Marta and Jimmy.	Marta and Jimmy stop	
	¿Cómo van a saber las	How will you know the	counting.	
	fracciones?	fractions?		
Ada	Contando los partes.	By counting the parts.		
Valery	¿Qué mas podemos hacer	What else can we do		
	además de contar?	besides counting?		
Jimmy	Yo no sé, porque está difícil	I don't know, because		
	contarlos.	counting them is difficult.		
Marta	Contar despacio y contar	Counting slowly and		
	dos veces.	counting twice.	Goes back to counting.	

In this example, Valery attended to what students were doing to make sense of how they were approaching the problem. She drew on spoken language to try to redirect the students' approach. Students referred back to what they saw as parts in the graphs and continued their approach. Other questions Valery tried with other groups included asking for a strategy that would be more efficient than counting and asking if they could compare the two bars without counting the parts. In both groups students insisted in counting parts.

Valery was having one of these conversations with a group when she saw Emi use her index fingers to compare the bars. Emi put the tip of one finger where the women's bar started and used her other finger to indicate where the bar ended (as represented in Figure 6.1). Keeping this gesture (left finger stretched and right finger indicating where the bar ended), she slid her hands to the bar representing men. She then took a pencil and wrote 1/2 above the women's bar and 1/2 above the men's bar. On a loud and excited voice, Valery called on the class and asked all students to congregate around Emi's table. Valery handed a new sheet with the demographics graphs to Emi and asked her to show her strategy to the class: "haz de nuevo lo que acabas de hacer" [*do again what you just did*]. Quietly, Emi repeated her hand gestures and movements and looked up to Valery, without writing down the fractions this time. Valery remarked: "¡Qué interesante!" [*How interesting!*]. Students responded with unintelligible chatting as they went back to their groups and adapted Emi's strategy. One group marked the length of the men's bar on a piece of paper and slid the piece of paper to compare it to the length of the women's bar. Another group used a ruler to measure each bar. Another group folded the graph in the middle where the women's and the men's bars met. They held the paper up, backlighting the graph to compare the two bars.

Figure 6.1 Emi's gestures on the graph



Emi contributed a generative strategy that influenced how others approached this problem. While Valery relied on spoken language in the interaction with Ada, Marta, Jimmy and Willie, Emi's strategy relied on the interaction between her gestures and the graph. Instead of asking Emi to put in words what she did and how it helped her figure out the fractions, Valery spotlighted Emi's gestures on the graph. Students uptake of Emi's strategy, adapting it in multiple ways, also involved interactions among students, the graph, and resources such as paper, rulers, and movement. Later in the lesson, students used similar strategies to compare bars in the other demographics graphs.

When Valery and I discussed the interactions around Emi's strategy, Valery established connections to her reconceptualization of participation. She recognized that Emi made sense of the problem through her quiet participation, which highlighted the idea of multiple ways of participating besides talk. As Valery put it, "no tuve necesidad de preguntarle [a Emi]" [*I didn't have to ask her*]. This was particularly meaningful to Valery, as Emi tended to be quiet in whole class discussions. Valery and I also discussed ideas from the focus groups about the interplay between the social and the individual. Valery recognized that she used Emi's gestures and writing to gain insights into Emi's sense-making, but also to help Emi influence how the rest of the class approached this problem. She stated that the strategy stopped being just Emi's and became part of the whole class: "ni tuve que preguntarles si lo que Emi hizo les iba a servir para la actividad. Ellos solitos se fueron a intentar" [*I didn't even have to ask them if what Emi did would help them to do the activity. They, pretty much on their own, went on to try*]. She added:

No era solamente como ver si Emi estaba participando o si estaba callada. Tenía que ver con que yo me diera cuenta cómo estaba participando, y que los niños se dieran cuenta y que hicieran algo con la idea de Emi.

It wasn't just like seeing whether Emi was participating or if she was quiet. It had to do with me noticing how she was participating, and with the children noticing as well, and that they did something with Emi's idea. In this statement, Valery connected ideas of multiple ways of participating, such as Emi's gestures and writing, with qualifying participation (i.e. "how she was participating"), and with the interplay between the individual and the social.

Beyond Emi's example, recognizing and taking up multiple semiotic resources became part of the culture of participation. For example, in a lesson when the class discussed if 5/4 was a possible fraction, Valery saw Emma's gestures. Emma traced in the air an arc that would correspond to a quarter of a circle, stopping momentarily before tracing the second arc in the air, then the third and the fourth. She paused a little bit longer this time and then traced a fifth arc in the air, as if retracing the first one she gestured. Valery interpreted this to mean Emma thought of four fourths making up a whole circle and then an additional fourth. Valery mimicked Emma's gestures to bring Emma's idea to the attention of the class. A student, Lia, connected these gestures to the idea of time: "cinco cuartos de hora es algo que existe. Es como setenta y cinco minutos. Entonces cinco cuartos es algo que existe." [*five quarters of an hour is something that exists. It's like seventy-five minutes. So, five quarters is something that exists*]. Emma's gestures helped Lia develop an idea about whether the fraction 5/4 was possible or not. Valery trace participation not only in Emma's gestures or Lia's talk but in the interaction among utterances, gestures and the numbers on the board.

Reassigning saliency to multiple semiotic resources contributed to semiotic innovation. In Emi's example, Valery assigned saliency to gestures and movement by quickly bringing the class together around Emi's strategy. Assigning saliency to gestures and movements helped the class recognize Emi's strategy as a valid way to approach the task. How Emi participated in this activity influenced how the group participated. This included students, as they took up and adapted Emi's strategy. It also included Valery's participation, as she did not mandate a specific

procedure to come up with an answer. She worked with students to understand the multiple ways and resources they used to make sense of the fractions.

Valery learned to see how students who tended to be quiet in class were indeed generating multiple strategies and contributing ideas by using multiple semiotic resources. She, however, expressed a tension, as she questioned whether she was allowing students to remain silent, denying them opportunities to learn to talk about their mathematical ideas. For example, in Emi's case, Valery wondered if she should have facilitated a whole class discussion to give students opportunities to articulate ideas about fractions related to Emi's strategy. Valery's awareness of the culture of the school influenced this concern. She thought that in other grades teachers may not recognize and validate non-spoken participation, misinterpreting silence as lack of interest or understanding. Simultaneously, Valery thought that forcing students who tended to be quiet to speak up could be intimidating and off-putting.

# Fluid Participation in Mathematical Activity

Valery came to value a flexible structure of mathematical activity where students developed diverse strategies, coordinated materials, and engaged in fluid ways of participating to explore different tasks. An example of this transformation comes from the third intervention cycle in the geometry unit. The class spent a few lessons exploring whether circles had perimeter and area, eventually agreeing that they do. In one of the lessons, two student strategies to figure out the area of a circle emerged and motivated Valery to try a flexible structure for a task related to those strategies. I describe the two student strategies first and then I discuss the flexible tasks that followed.

While Valery was launching a task to generate strategies to establish the perimeter of a circle, Juan grabbed a piece of graph paper and started to draw a circle that he then erased (figure

6.2a). He grabbed a compass and drew several circles on other sheets (figure 6.2b, 6.2c and 6.2d). He paused and observed the last two circles he drew (shown in figure 6.2d). Putting away the other sheets, Juan grabbed a new sheet, looked back and forth between the circles and the new sheet, and finally drew the circles in Figure 6.2e.





Noticing what Juan was doing, Valery approached him after launching the task:

Table 6.6 Juan's strategy for the area of a circle

	Utterances	Translation	Actions
Valery	Deja ver, Juan. ¿Qué	Let me see, Juan. What	Leaning over, moves paper
	hiciste?	did you do?	toward her and observes.
Juan	Hizo círculos.	I made circles.	Looking at the paper.
Valery	¿Querías hacer círculos?	You wanted to make	Looking at Juan.
		circles?	
Juan	Es más o menos una idea.	That's more or less the	
	Quiere ver la área de esto.	idea. I want to see the	Points at circles.
		area of this.	
Valery	Ajá. Queremos ver cómo	Uh huh. We want to see	Moving papers back
	saber el área y el	how to figure out the area	toward Juan. Stands back
	perímetro de un círculo.	and perimeter of a circle.	up.
	Muy bien.	Very well.	
Juan	Es como así.	It's like this.	Gestures as if inscribing
			one of the circles in a
			square his hands formed.
Valery	¡Ahh! ¡Qué interesante!	Oh! How interesting!	

Instead of asking Juan to put away the graph paper and pay attention to the launching of the task, Valery approached him willing to interpret what he was doing as valid participation in mathematical activity. She was tracing Juan's participation in what could have been misconstrued as off-task behavior. In the excerpt above, Valery initially interpreted Juan's spoken explanations and the drawings as indicating his thinking about the area and perimeter of circles, which Valery validated, stating that was what the class was working on. When Juan gestured inscribing the circle in a square, Valery understood he was trying to relate the area of the circle to the area of the square. Making sense of Juan's idea involved the interaction of multisemiotic resources, including the circles, the gestures, and the spoken language.

Later in the lesson, Valery assigned saliency to Juan's idea. Juan went back to his group to work on the task generating strategies to figure out the perimeter of a circle. This activity included the group ambassadors strategy, where a representative of each group visited and helped or got help from another group. When Juan's group was hosting an ambassador, Valery asked Juan to share his circles on the graph paper, writing "para el área" [*for the area*] as a title. Juan used a red marker to trace over the square that inscribed one of the circles. After the ambassadors reported back to their original groups, students briefly shared what they found interesting, as a whole class. Paige was one of the students who visited Juan's group. Stacey, who heard about Juan's idea through Paige, commented:

Me gustó como lo hizo el otro grupo que Paige estuvo porque el área del círculo es así casi como área de este cuadrado [levanta y señala el papel donde Paige explicó la estrategia de Juan]. Esto es un estimación. Es un buen estimación.

I liked how the other group that Paige was visiting did it because the area of the circle is almost like the area of this square [holds up and points to the paper where Paige explained Juan's strategy]. It's an estimation. It's a good estimation.

In this case, Valery used the ambassadors' strategy to assign saliency to Juan's work that coordinated the circle, the inscribing square, the smaller squares making up the inscribing square,

and the title "*for the area*." Through collaboration among groups, Paige and Stacey became fluid participants in this multisemiotic process, understanding Juan's strategy as a "*good estimation*" to calculate the area of a circle. Rather than focusing on participation occurring in present time during a discussion or small group work, Valery facilitated students' interaction with participation initiated in the past, thus making Juan's circles influence others' thinking.

Halley generated the second strategy that the class explored. Responding to Stacey's comment, Halley suggested an alternative. She went to the board and drew a square and a circle next to it:

	Utterances	Translation	Actions
Halley	Si tienes como algo, como	If you have like something, like	
	slime* aquí, adentro de este	slime here, inside this circle,	Points at the
	círculo,	you can put it inside this square,	circle.
	puedes ponerlo adentro de este		Points at the
	cuadrado,	and find the area here.	square.
	y encontrar el área aquí.		Points at the
		And it's still, it'll be the same,	circle.
	Y todavía es, va a ser el mismo,	how do you say amount?	
	¿cómo se dice amount?	Amount.	
Valery	Cantidad.	Amount of slime.	
Halley	Cantidad de slime.		

Table 6.	7	Hallev's	strategy	for	the	area	of a	circl	e
			~	, ~ .			-,		-

\*Halley refers to the elastic, gooey arts and crafts material. In this class, students made slime using glue, water, paint and glitter.
Halley added to Juan's idea, using spoken language, the drawings of a circle and a square, and the imagined slime she mentioned. Halley typically felt comfortable speaking in whole class discussions and articulating her ideas. In the past, her ideas frequently influenced the strategies that Valery asked the entire class to use. This time, however, the class showed interest in both Juan's and Halley's strategies.

The class discussed both Juan's and Halley's ideas and different students wanted to try their strategies. In the following lesson, Valery asked students to form groups according to the strategy they wanted to use. She provided graph paper with a circle on it. Additionally, the groups that wanted to try Halley's strategy received sugar to cover the circle. Figure 6.3a shows sample work from a group that tried Halley's strategy. These students used a ruler to make sure the sugar was uniformly distributed along the circle and to rearrange the sugar in the squares. They counted the number of squares that the sugar covered to determine the area of the circle. Figure 6.3b shows sample work from a group that tried Juan's strategy. This group counted the number of squares the circle covered, approximating the corners of the inscribing figure as half a square. They added fractions (half squares) and whole squares to figure out the area of the circle. *Figure 6.3 Figuring out the area of a circle* 



In the gallery walk that followed, students used sticky notes to comment on the two strategies. Not only did students comment on the strategies, but they also reacted to each other's comments. For example, reacting to the sugar's strategy, a student wrote "¡Oh, brillante! Cuando necesitas saber el área de círculo, sólo necesitas ir por azúcar. ¡Que una estrategia más eficiente! [*Oh, brilliant! When you need to find out the area of a circle, all you need to do is get some sugar. What an efficient strategy!*]. Another student drew an arrow from this note to another that read: "Pero esto es exacto, no estimación" [*But this is exact, not an estimation*]. Similarly, reacting to Juan's inscribing square strategy, a student commented: "Me gusta" [*I like it*] and put a sticker with a thumbs up. Another student placed a sticky note overlapping, commenting: "Esto toma para siempre sumando todas las fracciones" [*This takes forever, adding all the fractions*].

When co-analyzing this lesson, Valery focused on the flexible structure of the mathematical activity, where different students tried different materials and approaches to engage with the idea of area of a circle. Valery mentioned "para que puedan participar tiene que haber variedad" [*for them to be able to participate, there has to be variety*], emphasizing the word "variety" by opening up both her arms and hands, as if releasing something she was previously holding. Valery recognized that in this lesson there were different ways of participating that were beyond her control. She mentioned she could have organized this lesson as two tasks, one with all students trying Halley's strategy and one with all students trying Juan's. Thinking that this arrangement would replicate the kind of repetitive exploration that students criticized in the focus groups, Valery decided to integrate both strategies in one task, with students choosing which strategy to explore. Valery used a lesson that was driven by students' ideas and interests, instead of following predetermined topics and rigid lesson plans.

She flexibly adjusted the lesson to allow students to explore ideas they were interested in. This type of lesson and activities facilitated students' fluid participation in mathematical activity.

Rather than using a class discussion for students to explain their work and converge to an answer or an assessment of the strategies' efficiency, Valery used a gallery walk where opinions, arguments and counterarguments coexisted around students' displayed work. As Valery commented: "es como si las cosas tan variadas que hicieron de maneras tan diferentes estuvieran hablando por ellos. Mejor que eso. Más claro" [*It's as if the things so diverse that they did, in such different ways, were speaking in their behalf. Even better than that. More clear*]. Having come a long way from her initial perspectives on participation as individual talk, Valery now seemed attuned to fluid and generative participation, where students, materials, and ideas interacted to support meaning-making. Engaged in this hermeneutic listening for participation, the students and Valery in interaction seemed to have developed answers to her earlier questions about how to bring all students' ideas into a meaningful dialogue of mutual influence.

Students noticed the fluidity of participation that the class was moving toward and they explicitly suggested moments when they wanted groups to work in different ways. Earlier in the geometry unit, for example, students tried different strategies and materials to figure out the perimeter of circles. These strategies included using yarn to cover the circumference of a circle and then stretching and measuring the yarn, as well as arranging popsicle sticks on the circumference to figure out the sum of the lengths of all the sticks. One of the students, Harper, referred to a lesson in the fractions unit to suggest that different groups should explore these two strategies. When the class was discussing both the yarn and the stick strategies to decide which one to use, Harper mentioned they could try both, in a way similar to when they had used unifix cubes and paper and scissors to add fractions. This suggestion, along with the multiple evidence

presented in this study, indicates how students rightfully appropriated the transformed culture of participation. It was a rightful appropriation because they actively contributed to this transformation.

Valery expressed tensions between the flexible structure of activities and the school demands. As the end-of-year student assessment approached, the school administration wanted to increase the number of hours that students on Individualized Educational Plans (IEPs) spent outside the mainstream classroom working with the IEP teacher aide. Valery's students would join students from the other Spanish immersion third-grade classroom. The school administration wanted both teachers to synchronize the textbook pages they studied each day. This school expectation prompted Valery to completely devote some lessons to topics and problems from the textbook. This strict structure was in stark contrast with the flexibility that she and her students had built around their teaching and learning and that allowed them to engage in fluid ways of participating. Valery explicitly discussed with the students that some days they would exclusively work on the textbook, explaining that it was only in preparation for the end-of-year test. Valery was determined to protect the space for diverse participation that we—the students, Valery, and I—had forged together.

#### Chapter 7 Discussion: What It Took to Transform the Culture of Participation

When Valery and I began discussing participation in this mathematics classroom, we saw what seemed to us as a well-defined problem: How to level student talk. Listening carefully to students, however, we realized we needed to understand what participation meant and for whom. Over time, we went from asking why some students were not participating to asking how they were participating. This was a question that neither of us alone could answer and it required collaborating with students. In this chapter, I focus on two lessons that Valery and I learned through this collaboration that informed transformations in the culture of participation: (1) Reconsidering what counts as participation, and (2) approaching participation hermeneutically. I make connections to previous literature on participation in mathematics classrooms as a way to better understand our learning about participation. The chapter ends with a discussion regarding implications for research and practice.

#### **Reconsidering What Counts as Participation**

To think about participation in generative and inclusive ways, we need to first unlearn reductionist and exclusionary views on participation. Initially, the normalization of traditional views on participation hindered our ability to see alternatives, even though those alternatives were hidden in plain sight. As a cultural group, a classroom is a place for a teacher to raise to awareness taken for granted cultural meanings, values, and practices as a first step to question and potentially transform them (Louie, 2017; Paris, 2012). In this case, bringing the students' and the teacher's perspectives into dialogue helped the teacher reinterpret her own perspectives on participation. Informed by a participatory research methodology, the teacher and I assumed the role of learners, supporting each other in making sense of what participation meant in this classroom.

Learning to see beyond quantity to focus on qualities of participation enabled the teacher to recognize the important role that social interactions, mathematical tasks and teaching strategies play in how students experience participation. Confronting unintended yet problematic consequences that the culture of participation had on students' participation motivated the teacher to reimagine her practice in light of the situated meanings of participation in this classroom. These meanings of participation that emerged from the dialogue among different perspectives serve to add nuances to the views on participation prevalent in the mathematics education literature. Contrary to previous studies that have foregrounded the role that individual preferences or characteristics play in how students participate (see, for example, Lubienski, 2000), for these students the type of social interactions influenced their participation. Collaborative, fluid interactions where students experienced a sense of mutuality supported participation in this classroom.

Rather than taking on a single form, this fluid participation emerged as observation, listening, thinking, helping, drawing, movement and talk. Students infused mathematical activity with these ways of participating. This means that to learn to see participation differently, teachers and researchers need to listen and observe in moments and places that have not been associated with participation, including for example, private gestures, that is, gestures related to mathematical activity that students intended for themselves and not for an external audience. It also means attending not only to individual instances of participation but also to the effect that participation has on other students' thinking and on the unfolding of the lessons. Learning to see participation in this fluid way involved expanding where and when we look for participation, as it extends beyond single utterances throughout the span of meaningful mathematical activity.

Unlike many studies that see the teacher's role in participation in mathematics classrooms as equitably distributing student talk (Foote, 2011; Jansen, 2006), in this dissertation the teacher's role involved joining students as a participant with an intense curiosity to learn about context-specific ways of participating. Only from this role was the teacher able to challenge her unexamined involvement in favoring adult perspectives on participation over those of her students.

# **Listening Hermeneutically for Participation**

No longer satisfied with looking for predetermined ways of participating, Valery wondered what her teaching could look like if she recognized and validated multiple ways of participating. Our collaboration helped her connect this reflection with practice as we co-planned units. In this planning stage she reimagined and revised the types of activities and teaching strategies that could help her see, in situ, the kind of participation she was envisioning. Rather than a prescriptive list of steps for the teacher to follow, as she implemented lessons and began to recognize participation in practice, she flexibly and responsively drew on her evolving teaching repertoire.

Teaching strategies that previous research has associated with hermeneutic listening informed the strategies that Valery brought into her teaching practice, including asking genuine questions, directing students' listening and amplifying participation. Valery not only asked genuine questions but she also directed attention to students' wonderment questions, which became a way of participating that allowed for the sense of mutual influence that students valued. Valery also provided multiple opportunities for students to amplify each other's participation, including students observing and listening in during gallery walks and as group ambassadors. These strategies fostered the type of collaborative interaction that students

mentioned in focus groups, illustrating how teachers can foster social goals that students associate with participation (Jansen, 2006). To make sense of and amplify the ingenious and multiple ways in which students participated, Valery needed to be responsive to the multiple ideas that emerged. Previous research on mathematics class discussions has suggested predetermined strategies that "give teachers control over what is likely to happen in a discussion" (M. S. Smith, Hughes, Engle, & Stein, 2009, p. 550). In contrast, Valery shared control of what happened in discussions and elsewhere with students, responding to their interest in complex mathematical ideas and honoring their fluid participation.

Valery also learned to see both spoken and non-spoken participation as equally valid instead of dismissing them as off-task behaviors. Consistent with previous research that has reframed off-task behavior as possible participation that can open up resources for mathematical meaning-making (Langer-Osuna, 2018), Valery's learning to listen hermeneutically allowed her to interpret these seemingly off-task behaviors differently. These strategies facilitated participatory interactions that drew participants—students and the teacher—to learn to listen to one another hermeneutically.

Listening hermeneutically for participation positioned the teacher to recognize mathematical ideas in spaces and moments she had not considered before. She revisited and revised teaching plans to make room for the flexible exploration of these ideas that interested the students. In turn, the tasks she used facilitated the emergence of additional mathematical ideas beyond the narrow scope of the textbook topics in unexpected ways of participating that the teacher continued to reflect on. This synergistic connection between reflection and practice contributed to the development of a dynamic culture of participation influenced by both the students and the teacher.

Over time, the culture of participation evolved toward embracing multiple ways of participating in cooperative mathematical activity in which students and the teacher addressed unexpected ideas that emerged in interactions. Semiotic innovations characterized the transformation of the meanings and practices associated with participation, including challenging the tradition of studying mathematical ideas prescribed by pacing guides, reassigning saliency to non-spoken participation and wonderment questions, and shifting the role of the teacher from an equalizer of student talk to a participant joining students in their fluid and dynamic understanding of participation. Although this culture of participation conflicted with some school demands, the teacher's excitement about the ways of participating that she was learning to see and students' responsiveness to transformations in her teaching outweighed the influence that these restrictive school demands had on the culture of participation.

Valery fostered a dynamic and inclusive culture of participation. As she learned to see participation, students appropriated strategies for listening hermeneutically to what peers had to say in words or otherwise. This evolving culture of participation enhanced the opportunities that the students and the teacher had to develop the often neglected need for a sense of belonging and mutual influence that is at the core of human activity, including mathematical activity.

#### **Expanding Research on Participation**

The culture of exclusion in mathematics education has been described as the marginalization of students who do not align with the dominant culture of mathematics education (Louie, 2017). The tendency in the literature to conflate participation with student talk may unintentionally contribute to this culture of exclusion by focusing on talkative students. The conceptualization of participation as a complex social phenomenon that informed this dissertation allowed me to consider participation beyond student talk, including gestures,

observation, listening, and writing. Bringing this conceptualization into dialogue with hermeneutic listening informed the focus on learning how students participate in a specific culture of participation, as opposed to perpetuating marked and unmarked participation by looking at individual student characteristics.

The focus on the culture of participation highlighted the role that the classroom as a cultural group played in participation. Recognizing that what constitutes participation varies across and within classrooms brought up questions about what counts as participation and who participates in the making of that decision. Findings from this dissertation highlight the possibility of exploring through innovative research designs the situated meanings of participation in different contexts, using methodologies that elicit both teachers' and students' meanings of participation. In this dissertation, I used Participatory Design Research (PDR) with children as a methodology that guided this exploration. PDR's iterative process of designing, implementing and reflecting on co-designed interventions provided multiple opportunities for the teacher to connect her reflection on participation with her practice. PDR also provided multiple opportunities to co-create spaces for the teacher to engage in hermeneutic listening with her students in order to transform the culture of participation. Future research could draw on participatory and inclusive methodologies to foster collaborations among multiple participants who influence participation in mathematics classrooms, including, students, teachers, parents, and school administrators.

The social semiotics analytical framework that informed this dissertation drew attention to how social interactions and social contexts influence the culture of participation. By illustrating some context-specific tensions, this dissertation extends previous research that argues that tensions between the classroom and the school play a role in how students participate

(Brown, 2017). In this case, the teacher navigated assessment practices that demanded attention to narrow mathematical ideas that often conflicted with the more connected mathematical ideas that interested students. Assessment practices also reinforced the authority that predetermined pacing guides exerted on participation. These contextual influences that attempted to counteract semiotic innovation mediated the teacher's engagement with hermeneutic listening for participation. These findings suggest that future research on hermeneutic listening for participation could attend to and address contextual factors that mediate how teachers connect their emerging reinterpretation of participation with their practice.

#### **Reimagining the Teacher as a Learner of Student Participation**

Engaging in hermeneutic listening for participation involved a transformation in the teacher role from an observer monitoring and disrupting silence to a participant joining students to learn to reinterpret their participation. Willingness to elicit and carefully consider students' perspectives on their own participation could help teachers learn about their students' ways of participating and classroom dynamics that influence or stifle their participation. Instead of attempting to bring students to an idealized standard of student talk, a more productive and generative approach could be to promote a classroom culture of participation that honors students' multiple ways of participating.

Rather than prescriptive steps to follow, in this dissertation the connections between reflection and practice were dynamic. These connections were informed by our teacherresearcher collaboration where we planned tentative focal strategies intended to enable and amplify multiple and unanticipated ways of participating. By attending to non-spoken participation and bringing it to the attention of the class, the teacher validated these contributions as important for the learning of mathematical concepts by the whole group. Pedagogical

approaches to help teachers expand and flexibly draw on a teaching repertoire that responsively addresses multiple ways of participation could strengthen the connections between teachers' reflection on participation and their practice.

Tensions between students' perspectives on participation and Valery's beliefs regarding her role in supporting participation posed a challenge as she initially tried to connect reflection and practice. These tensions included how to spotlight ideas in off-task behavior without embarrassing students, and how to accentuate students' non-spoken participation while simultaneously providing opportunities to talk about their mathematical ideas. Teachers could engage in exploring similar tensions in order to move their practice to a space where they and their students can reinterpret what counts as participation.

Rather than having students participate in predetermined ways, in this dissertation the teacher and I worked together to move our understanding of participation closer to the multiple ways in which students experience and perceive participation. It took a collaborative effort to unlearn normative views on what it means to participate, to make sense of the richness and complexity of students' perspectives on participation. A broader, more inclusive perspective on participation, however, did not make its way into the teacher's practice smoothly. A prolonged, iterative process was necessary to connect the teacher's reflection to a practice where she learned to listen hermeneutically for participation. Many were the moments in which the teacher found herself surprised by (re)discovering participation through gestures, movements, observation and comments. Many were the spaces—physical, intellectual, and temporal—in which she found forms of participation previously unaccessible within the culture of participation in which her practice had developed. As we—students, teacher and researcher—came together as co-

participants in this study, we worked as a group on shaping the important decision of what counts as participation in a mathematics classroom.

**APPENDICES** 

# **APPENDIX A—Initial Focus Group Questionnaire**

We will be exploring your ideas about participation in math class. Each one of you can choose how to express your ideas. You can draw, you can talk or you can write about your ideas at any moment. [Paper and pencil provided.]

Let's talk about participation in math class. Take five minutes to draw, write or think about how you participate in math class.

[After five minutes.] Let's share your ideas. You can show us what you draw or wrote and you can talk about your ideas:

What does it mean for you to participate in class?

When do you like to participate in math class? When don't you like to participate?

Why do you participate in math class?

How do you decide when to participate in math class?

What does your teacher do that helps you participate in math class?

# **APPENDIX B—Initial Teacher Interview Questionnaire**

For you, what is student participation in math class?

How does participation matter in math class?

How do different students participate in math class?

What do you do to help your students participate in math class?

How do you respond to students' participation?

How does what you know about your students inform how you support participation in math class?

What school guidelines or expectations do you think influence how you address participation?

What examples about discussions with other teachers, with parents or supervisors that influenced how you view or respond to participation can you give?

# INFORMACIÓN SOBRE MICHIGAN

https://www.census.gov/quickfacts/MI

Género		
Mujeres	Hombres	
Idiomas que se hablan en el ho	ogar	
Idioma diferente a inglés	Inglés	
Edad de los niños menores de 18 años		
0 a 5 años	6 a 12 años 13 a 17 añ	ios
Familias		
Con un padre	Dos padres	

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