THREE ESSAYS IN INDUSTRIAL ORGANIZATION

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ABSTRACT

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Chapter 1: Optimal Use of Patents and Trade Secrets for Complex Innovations

In this paper, the optimal protection of “complex" innovations that involve multiple complementary components is considered. The innovator can protect each component through either patent or trade secret protection, so that the complete innovation may be protected through a mix of patents and trade secrets. The innovator faces potential entrants who might acquire the components either by costly imitation or through licensing. It is shown that: (i) secrecy is optimal when the patent length is relatively short; otherwise, a combined use of patents and trade secrets is optimal. (ii) The innovator is (weakly) over-rewarded compared to an inventor with a simple innovation (with identical profitability and imitation cost). (iii) A “Strict Utility Requirement" policy (that precludes the use of a patents-secrets mix) enhances allocative efficiency ex-post but stifles R&D incentives ex-ante.

Chapter 2: Welfare Effects of Certification under Latent Adverse Selection (with Anthony Creane and Thomas D. Jeitschko)

Asymmetric information is a classic example of market failure that undermines the efficiency associated with perfectly competitive market outcomes: the “lemons" market. Credible certification, that substantiates unobservable characteristics of products that consumers value, is often considered a potential solution to such market failure. This paper examines welfare effects of certification in markets in which there is asymmetric information, but without an adverse selection problem. We analyze the market equilibrium when the certification technology becomes available and contrast this with the equilibrium without certification. We find that despite an improvement in allocative efficiency, overall welfare may decrease due to the possibility of certification when such certifi-

cation is either costly or inaccurate. In fact, most of these results are not derived from the direct welfare cost of certification, but rather from certification's effect on the market(s).

Chapter 3: A Role of Non-practicing Entity (NPE) when Patent Ownership is Fragmented

This paper presents a mechanism by which a non-practicing entity (NPE) enhances returns to individual inventors. Assuming an environment in which: (i) a product, manufactured by a practicing entity (PE) must adopt all complementary innovations patented and owned by individual inventors; and (ii) a threat of litigation by an individual inventor is not credible so that patents are sold through a sequential auction, the following two games are studied in turn. First, I consider the game played by two PEs in which they compete to acquire the patents, and then negotiate licensing fees based on their patent portfolio in the shadow of litigation. Next, the same game played by a PE and a NPE. I highlight the two key findings: (i) in the former game, the two PEs buy all patents at 0 prices, and thus the aggregate payoff to the individual inventors is also 0 (for almost all parameter values); and (ii) in the latter game, on the contrary, if total number of patents is sufficiently large, the existence of the NPE induces the PE to buy some patents at a positive price and thereby increases the aggregate payoff to the individual inventors.

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CHAPTER 1

OPTIMAL USE OF PATENTS AND TRADE SECRETS FOR COMPLEX INNOVATIONS

1.1 Introduction

Firms protect their innovations not only by patents but also by secrecy.[1] These two protection modes help the firms appropriate returns from their innovations through different mechanisms. Patents grant patents holders a legal right to exclude others from using protected innovations for a fixed period of time but require disclosure of all information. In contrast, trade secrets do not have a fixed time period and do not require any disclosure of information but they are vulnerable to independent reverse engineering.[2]

In the intellectual property rights literature, initially, patents received far more attention than trade secrets. Subsequently, several authors have highlighted trade secrets as an important appropriability method (e.g., Levin et al., 1987; and Cohen et al., 2000), and analyzed the secrecy mechanism as a viable alternative to patents (e.g., Friedman et al., 1991). However, most of this work does not consider a potential for a combined use of patents and trade secrets, the focus of this paper.[3]

To fill this gap in the literature, this paper considers a situation in which an innovator has a "complex" innovation that involves multiple complementary components. The innovator can protect each component either by a patent or by secrecy, i.e., the innovation may be protected through a mixed use of patents and trade secrets. In such a setting we ask the following questions. Under which condition, does the innovator opt for the mix use? Would the innovator with the

[1] More generally, Hall et al. (2014) point out that there are two different types of intellectual property rights (IPRs): formal IPRs including patents, trademarks, designs and copyright, and informal IPRs including secrecy, confidentiality agreements and lead time advantage. In this paper, I focus only on patents and secrecy.

[2] Coca-Cola's secret formula, which has been locked away in a vault for more than 90 years, is a good example of well guarded trade secrets.

[3] Hall et al. (2014) offer a comprehensive survey on firms' choice between formal and informal IPRs including both theoretical and empirical work.

complex innovation earn more than an innovator with a simple innovation when the mix use of patents and trade secrets is feasible? Is restricting her choice set to the binary one, either patenting or relying on secrecy, socially desirable?

Substantial anecdotal evidence suggests that patents and trade secrets are combined in many settings. Premarin, which has been sold by Wyeth (merged with Pfizer in 2009), has been the most popular hormone-therapy drug for menopausal women since 1942. Wyeth filed two patents to protect the innovation in the early 1940s. However, the company effectively extended the duration for its exclusive position in the industry beyond the length of the patents. The reason for this success was that no rivals could uncover closely secured trade secrets, more specifically the so-called Brandon Process: a secret chemical process extracting conjugated estrogen from pregnant mare urine (Jorda, 2007; and Orly, 2013).

Jorda (2007) also attributes a success story of GE in the business of industrial (or synthetic) diamonds to the use of patents in conjunction with trade secrets. GE patented a part of its technology for making diamonds, and therefore the disclosed information was publicly available to its rival companies after the expiration of patents. However, as GE also closely kept the remaining part secret, it could maintain a dominant position in that market. Pan (2010) reports another example; an Illinois based energy company, Coskata, applied for patents to protect technology on a bio-reactor while keeping the identity of the micro-organism fed into the bio-reactor secret. In addition, there are many other examples showing a similar strategy pattern in the food industry (see the introduction in Belleflamme and Bloch, 2013) and in the organic chemical industry in the 19th century (Arora, 1997). In fact, the web pages of many patent attorneys and law firms advise their customers to use this type of strategy. [4]

This paper first examines the best protection mode for an innovator facing potential imitators after the discovery of a complex innovation. To this end, it is important to figure out the inventor's ability to appropriate returns from the innovation under each protection mode: the all-by-patent

[4] For example, visit http://www.peloquinlaw.com/combining_protection.html. (retrieved on 8/4/2016)

mode, the all-by-secrecy mode and the optimal patent-secret mix. Instead of imposing ad hoc assumptions in that regard, my framework extends Henry and Ponce (2011)'s work in the context of complex innovations. In this model, entry times of the imitators under each protection modes are endogenously determined in equilibrium. More specifically, the length of patents (T) and the rates of entry by the imitators (λ), which are driven by imitation costs (κ), are key variables governing the entry times of the imitators under patent and trade secret protection respectively.

The first key finding of the paper is that the all-by-secrecy is optimal when the patent length is relatively short, otherwise, the optimal combined use of patents and trade secrets is optimal. To understand this result, it should be first noted that the all-by-patent is dominated by any patent-secret mix because the inventor can effectively extend the duration of monopoly beyond T by mixing the two protection mechanisms; due to the perfect complementarity of the components, patented components make entry by the imitators infeasible during the patent period, and the remaining components kept by secrecy delay entry after T. Moreover, the optimal patent-secret mix requires patenting just the component with the minimal imitation cost because the prolonged monopoly duration is determined by the aggregate imitation costs of all components kept under secrecy. Therefore, the optimal protection mode has a binary feature: depending on T, choosing either the all-by-secrecy or the optimal patent-secret mix; the optimal patent-secret mix is more advantageous than the all-by-secrecy in the sense that the mix takes advantage of the patent system, but it is also less advantageous in the sense that it requires the inventor to disclose a part of the innovation, thereby increasing the rates of entry.

Next, this paper compares the optimal decision and the payoff to the innovator with the complex innovation (the "complex innovator" hereafter) with that to another inventor with a "simple" innovation comprised of a single component (the "simple innovator" hereafter). *Ceteris paribus* (i.e., with identical profitability and imitation costs), the complex innovator is more inclined to patent because she can take advantage of the patent system with only a partial disclosure of the whole innovation. Moreover, the complex innovator is (weakly) over-rewarded compared to the simple innovator. When T is relatively short, so that both inventors rely only on secrecy, they earn the same expected payoff regardless of whether the innovation can be fragmented or not, but if T falls into the region in which the complex innovator opts for the optimal patent-secret mix, the complex innovator earns strictly more than the simple innovator does.

In addition to the positive analysis of ex-post innovation problem, this paper further derives ex-ante and ex-post policy implications of imposing a "Strict Utility Requirement" (SUR). For ex-ante welfare analysis, we also take into account ex-ante investment incentives of the innovator in the R&D stage. Under the SUR, the patent office turns down all patent applications that discloses only a part of innovation in compliance, and thus the inventor cannot use a patent-secret mix. The complex inventor's payoff schedule converts into one that is exactly same as that of the simple inventor's. Whereas the SUR always enhances ex-post welfare, the ex-ante welfare implication is ambiguous. This is because the availability of the combined use of patents and trade secrets might boost the inventor's ex-ante investment incentive.

1.2 Related Literature

There is a large body of literature viewing patents and trade secrets as mutually exclusive alternatives in a variety of contexts. Anton and Yao (2004) study the innovator's choice between patenting and relaying on secrecy, focusing on signaling (or strategic disclosure) aspects. Kultti et al. (2007) and Kwon (2012) consider a similar problem in a patent race model in which two or more innovators compete to develop the same innovation simultaneously, and they make some normative arguments regarding socially desirable patent policies. Denicolo and Franzoni (2004) focus on whether granting a prior user right to the first inventor (i.e., the subsequent innovator is able to patent and exclude the first inventor when the first inventor resorted to trade secrets) increases social welfare or not. Erkal (2005) and Bhattacharya and Guriev (2006) consider a cumulative innovation framework. More specifically, Erkal (2005) considers two consecutive R&D races between two firms in which the winner of the first race has an option to protect it either by a patent or by secrecy, and compares several patent policy regimes. Bhattacharya and Guriev (2006) build a model in which the upstream inventor has two potential licensees in the downstream market. They show which of the two protection modes is more efficient from the inventor's and the social planner's point of view. All of these papers offer great insights in understanding distinctions between patents and trade secrets as substitutes for each other, but neglect the possibility of the combined use of those two mechanisms.

Although there is a nascent strand of literature exploring the patent-secrecy mix mechanisms, research papers on this topic are still sparse. Lee (2015) envisions secrecy as a mechanism for entry deterrence, but patents as a more nuanced instrument of transferring entrants' surplus to the inventor or deterring entry. He shows that the mix use of protection can arise in equilibrium when the two mechanisms perform different functions. Unlike his model, I show similar results that the two mechanisms could be complements with the opposite assumption that knowledge covered by trade secrets can be transferred or licensed to the potential imitators while patents only deter entry. The two papers most closely related to my own approach are Cugno and Ottoz (2006) and Belleflamme and Bloch (2013). Both models start with a situation in which the inventor has come up with a complex innovation, and show that the mix of patents and trade secrets may provide optimal protection. They, however, consider an innovation which can be fragmented continuously. Cugno and Ottoz (2006) consider a framework in which it is a prerequisite for an imitator's entry, given a patent-secrecy portfolio, to not only uncover a part protected by secrecy but also find a way to invent around the other part protected by patents. Belleflamme and Bloch (2013) study a broader set of competitive configurations (e.g., an imitator may be able to get some surplus even with access to only a small portion of the innovation) as well as protection modes including dynamic patenting. Albeit my paper considers complex innovations with finite number of perfectly complementary components, it is based on a micro-foundation which determines the strength of each protection mode endogenously, and thus provides a detailed mechanism by which the evolution of the imitation game and inventor's profit is fully described. Moreover, my framework also explicitly considers the ex-ante investment incentive of the inventor, which is absent in their works.

The remainder of the paper is organized as follows. In section 2, the base model and assumptions are introduced. Section 3 provides the main analysis including the calculation of the inventor's expected payoffs under each protection mode, to derive the inventor's optimal decision and to compare her payoff to the one of the simple innovator. In Section 4, policy implications regarding SUR are drawn. Section 5 contains two possible extensions and checks the robustness of the main results, and the final section presents the conclusion.

1.3 The Model

This section introduces the basic setup of the model, which builds upon the framework in Henry and Ponce (2011). Consider an infinite horizon model in which Δ represents the length of each time period. More specifically, we consider the continuous time framework which could be understood as the limit of a discrete time framework as Δ approaches to 0.

There are three players: an innovator, denoted by i, and two potential imitators, indexed by $b \in \{j,k\}$. All parties are assumed to be risk neutral agents maximizing their (expected) discounted payoffs, and have the same per-period discount factor, $\delta := e^{-r\Delta}$, where $r > 0$ represents a discount rate.

The innovator already has a "complex" innovation at time period $t = 0$ allowing her to introduce a new product to the market. Here, the complex innovation is comprised of two perfectly complementary components of knowledge: x and y, which differ by the extent of difficulties in reverse engineering.[5] In the beginning, the innovator can protect each component by a patent or by secrecy; there are four possible choices: protecting all components by a patent, all by trade secrets, x by secrecy and y by a patent, and y by secrecy and x by a patent. Note that the choice set of innovation protection incorporates some implicit assumptions about patent examination process. I assume that the novelty requirement is strict in the sense that the patent office treats an invention already on sale as a prior art and thus rejects a patent application for that.[6] Consequently, the

[5] We relax this assumption and consider N-component model in section 5.

[6] After the America Invents Act (AIA) was enacted in 2011, some have argued that invention claims on a "secret" sale is no longer considered as a prior art, based upon a new interpretation

inventor cannot rely on a dynamic protection method combining patents and trade secrets, protecting x by a patent and y by secrecy initially, for example, and then at some point later trying to get another patent for y. However, I assume the weak utility requirement, which allows the inventor to get a patent even when hiding part of the invention and disclosing only the other part.

An imitator can "enter" the market and start to earn profits in the market only with access to both knowledge components. There are two different ways for the imitators to "adopt" (or learn) the knowledge components; they can either buy a component from the innovator through a contract or imitate the component through a reverse engineering process. The imitation costs of x and y are given by κ_x and κ_y respectively. Without loss of generality, I assume that $\kappa_y \geq \kappa_x > 0$. Put differently, the component y is more difficult or costly to imitate.

Regardless of whether a firm is the innovator or an imitator, the per-period profit is the same for all selling in the market and is determined only by the number of firms in the market; π_n represents a single firm's per-period profit with n firms in the market. Then, the present value of stream of all profit flows per firm with n firms in the market is equal to $\Pi_n \equiv \pi_n/r$. All parameter values, including profits given the number of firms (π_n), imitation costs (κ_x, κ_y and κ) and the discount rate (r), are common knowledge. Throughout the paper, I maintain the following assumption stating that joint industry profits are dissipated by more entrants.

Assumption 1: $\Pi_1 > 2\Pi_2 > 3\Pi_3$.

Furthermore, to make the analysis non-trivial, I assume that the sum of imitation costs is not so high that both imitators have an incentive to enter the market.

Assumption 2: $\Pi_3 > \kappa := \kappa_x + \kappa_y$, *where* $\kappa_y \geq \kappa_x > 0$.

The game unfolds as follows under each protection mode chosen by the innovator. First, when the inventor protects both components by the patent, it confers a temporal monopoly to the inventor for a fixed term T in exchange for disclosing information about the innovation. As a result, the

of an amended clause about the prior art. There have been considerable number of debates supporting or opposing the view. Here, I accept the old meaning. For more details, see for example, http://patentlyo.com/patent/2012/10/did-the-aia-eliminate-secret-prior-art.html.

imitators become aware of how the innovation works without incurring any costs, but have to wait to enter the market until the patent expires. In summary, the imitators choose their entry times after T without incurring any costs in all-by-patent scenario.

In the second scenario of all-by-secrecy protection, as soon as an imitator uncovers how to "adopt" the two components through licensing or imitating, he is able to enter the market. Henry and Ponce (2011) suggest two possible types of contracts: transferable and nontransferable. Under a transferable contract, unlike a non transferable one, an imitator can resell the knowledge to the remaining imitator. Since Henry and Ponce (2011) show that offering transferable contracts is optimal and the only way for the inventor to earn a stream of profits similar to the one obtained through patent protection, I will focus only on transferable contracts.[7] Transferable contracts specify the price levels offered by the seller (the inventor) to the buyers (the imitators) before the first purchase takes place and by the sellers (the inventor and the first buyer) to the remaining buyer after that moment for each knowledge component.

A more detailed description of the game under all-by-secrecy mode is given below. In any time period t from the beginning until the first knowledge adoption occurs for one of the two components, the game unfolds as follows: (i) The innovator i announces prices for x and y, p_{ib}^{xt} and p_{ib}^{yt}, to buyer $b \in \{j,k\}$. (ii) Given the offered prices, the imitators simultaneously choose to learn each component by accepting the contract (c_{ib}^{t}), to learn each component by imitating (m_{b}^{t}) or to wait (w_{b}^{t}). Thus, the action set of an imitator b at time t is given by $A_{b}^{t} = \{c_{ib}^{t}, m_{b}^{t}, w_{b}^{t}\} \times \{c_{ib}^{t}, m_{b}^{t}, w_{b}^{t}\}$ in which the first and second elements represent his action for component x and y respectively. Suppose now that the first knowledge adoption arises by one of the two imitators. Without loss of generality, say imitator j learned the component x. In that case, there would then be two sellers, i and j, for component x while there is still only one seller, i, for component y.

[7] It is not difficult to find a technology transfer agreement in which a licensor grant a licensee a right to sublicense or resell the licensed technology (a transferable contract). Such technology transfer agreements can be found in the archive of the U.S. Securities and Exchange Commission webpage. (For example, see https://www.sec.gov/Archives/edgar/data/1223112/000119312510190208/dex1074.htm. Retrieved: Oct/12/2016.)

From this period on, until somebody acquires another component, the game proceeds as follows in each period: (i) Two sellers of x offer prices, p_{ik}^{xt} and p_{jk}^{xt}, to k, and the sole seller i for component y announces prices, p_{ij}^{yt} and p_{ik}^{yt}. (ii) Given the price offers, the imitator j chooses $a_j^t \in A_j^t = \{c_{ij}^t, m_j^t, w_j^t\}$ for the remaining component y, while, at the same time, the imitator k chooses $a_k^t \in A_k^t = \{c_{ik}^t, c_{jk}^t, m_k^t, w_k^t\} \times \{c_{ik}^t, m_k^t, w_k^t\}$. Note that now k has three different options for getting component x: by buying from i (c_{ik}^t), by buying from j (c_{jk}^t) and by imitating (m_k^t). The game goes on in a similar manner until everybody possesses both knowledge components and enters the market.

At time period t, every agent observes and recalls the history up to $t - \Delta$. The equilibrium concept, which will be used for the analysis, is symmetric Markov perfect equilibria (MPE) in which each agent's strategy is optimal given the state variable and other agents' strategy. The state variable is denoted by $\theta^t := (\theta^{xt}, \theta^{yt})$ at time t in which θ^{xt} (θ^{yt}) indicates which imitator has obtained the component x (y) in addition to the inventor, i.e., θ^{xt}, $\theta^{yt} \in \{\{i\}, \{i,j\}, \{i,k\}, \{i,j,k\}\}$.

The last case to be considered is that of the patent-secret mix. For expositional simplicity, assume that x is protected by a patent and y by secrecy. Then, the imitators can use x freely after T, while they still need to adopt y for entry. Then, the imitators will adopt y in a similar way as was described in the all-by-secrecy mode. But now the state variable θ^t should also specify whether the current time period t is before or after the patent expires. If $t \geq T$, the imitators start to sell as soon as they adopt y. If $t < T$, however, the remaining duration until patent expiration is different at each t, which results in different payoff structures given the same entry time after T. Thus, under this scenario I derive a symmetric MPE of the game in which the state variable is $\theta^t := (\tau, \theta^{yt})$. Here, the first element indicates whether it is before or after the expiration of the patent. If it is before T, then, furthermore, the first element specifies at which period the players are currently. (i.e., $\tau = t$ if $t < T$ and $\tau = T_+$ otherwise).

1.4 Optimal Use of Patents and Trade Secrets

This section explores how the innovator would optimally protect her complex innovation by using patents and trade secrets. To this end, I solve the innovator's profit maximizing problem depending on modes of protection: patenting both components, relying only on secrecy, and mixing patents and trade secrets. A comparison of profits is provided next, followed by an illustration of how the payoff of the complex innovator relates to that of the simple innovator.

1.4.1 Expected Payoffs in Each Protection Mode

1.4.1.1 Patenting Both Components

I study the case in which all components are protected by the patent first. Here, notice that we only consider ironclad patents which grant the inventor an exclusive right to use the innovation until T. In addition, the imitators do not need to pay to use the innovation (i.e., both components) once the patent expires. Consequently, at time T, it is optimal for the imitators to enter the market immediately and simultaneously. Hence, the discounted expected payoff of the inventor under protecting all by patent mode is given by

$$V_i^P = (1 - e^{-rT})\Pi_1 + e^{-rT}\Pi_3 - f$$

in which f represents a patent application fee.

1.4.1.2 Relying only on Secrecy

How much can the innovator appropriate from her innovation when she relies only on trade secrets? Working backward, I consider cases after the first adoption of a knowledge component takes place. Before doing that, for expositional simplicity, let us define a subgame, in which there is one seller for a component, as a monopoly game with respect to that component. Similarly, define a subgame, in which there are two sellers for a component, as a competitive game with respect to that component.[8] Thus, for example, if there are two sellers for one component and there is only the original innovator for the other component, then it is called a monopoly game with respect to the former and a competitive game with respect to the latter.

Lemma 1 *(i) In the unique MPE of a competitive game with respect to both components or with respect to only one last component, the component(s) with two sellers is (are) sold to the remaining buyer(s) at a zero price(s) immediately; and (ii) In an MPE of a game which is competitive with respect to one component and monopolistic with respect to the other, the component with two sellers is sold to the remaining buyer at a zero price, and the MPE is unique up to equilibrium payoff regardless of the timing of its sale.*

Whenever there is competition in selling a knowledge component, it is sold to the remaining buyer at 0 price, and it can be assumed that the timing of the sale, in general, is immediate. The intuition behind result (i) is straightforward. For expositional ease, suppose $\theta = (\{i,j\},\{i,j\})$. Since price offers are stationary in MPE and entry is profitable for k, entry by him is immediate if he buys from one of the sellers or imitates both components. Anticipating that, i and j compete in prices to sell the two components, and thus equilibrium prices are driven to 0. Now, take a step back and consider a subgame in result (ii). For expositional ease, say j has just adopted x, i.e., $\theta = (\{i,j\},\{i\})$. Note that since x and y are perfect complements, the stream of market profits for each firm depends entirely on the timing of the first adoption of y after which the entry by both imitators follows promptly. Then, it is never optimal for k to buy x at some positive prices or to imitate x now taking price offers and imitators' adoption decision for y as given. This is because k can wait instead until the first adoption of y arises and buy it at 0 price. Therefore, k buys x from one of the sellers at 0 price in equilibrium, which allows one to assume that a subgame with $\theta = (\{i,j\},\{i\})$ proceeds to a subgame with $\theta = (\{i,j,k\},\{i\})$ instantly. The remaining analysis for the subgame directly follows from the main results of Henry and Ponce (2011) in which they consider a single component innovation. Even if, for simplicity, the following lemma characterizes

[8] Recall that as only transferable contracts are considered, the first adopter becomes an additional seller for the corresponding component.

a unique MPE for only the case in which the first adoption has occurred for x, it is applicable in a similar way to the other case as well.[9]

Lemma 2 *(Henry and Ponce (2011)) In a unique symmetric MPE in a monopolistic game with respect to y only (i.e.,* $\theta = (\{i,j,k\},\{i\})$*), (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for the remaining component y by setting* $p^y_{ib} = \kappa_y$ *for* $b \in \{j,k\}$*; (ii) entry times of each imitator follow the exponential distribution with hazard rate,* $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$*; and (iii) the expected equilibrium payoffs of the inventor and of the imitators are* $V_i^{y|x} = \mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3+\kappa_y)$*, where* $\mu^{y|x} := r/(r+2\lambda^{y|x}) \in (0,1)$*, and* $V_b^{y|x} = \Pi_3 - \kappa_y$ *respectively.*

Given the occurrence of the first adoption of x, the discounted expected payoff of the inventor consists of three parts; she will earn monopoly profits for a random duration until the first adoption for y arises ($\mu^{y|x}\Pi_1$), a licensing revenue from selling y when the first adoption for y takes place ($(1-\mu^{y|x})\kappa_y$), and triopoly profits thereafter ($(1-\mu^{y|x})\Pi_3$) in which $y|x$ denotes a variable associated with a monopolistic game with respect to y given x was adopted by both imitators. A delay in the imitators' entry can be attributed to the equilibrium outcome of balancing the benefits and costs of immediate entry as next explained. From Lemma 1 and result (i) in Lemma 2, we know that the first and second adopter pay κ_y and 0 to learn component y respectively while they end up obtaining the same market profits because of almost simultaneous entry. Thus, on the one hand, an imitator has an incentive to wait to save the cost of knowledge acquisition. On the other hand, delaying entry entails costs as well: they need to sacrifice market profits which would have been earned by immediate entry. Hence, in symmetric equilibrium, the imitators mix waiting and buying y from the inventor in a way that such benefits and costs are offset exactly. A speed of entry is described by the hazard rate of entry, $\lambda^{y|x}$, and $\mu^{y|x}$ represents the natural duration of monopoly

[9]For $\theta = (\{i\},\{i,j\})$ or $(\{i\},\{i,k\})$, instead the inventor sets $p^x_{ib} = \kappa_x$ for $b \in \{j,k\}$; hazard rate is $\lambda^{x|y} = r(\Pi_3 - \kappa_x)/\kappa_x$; the expected payoffs of the inventor and the imitators are $V_i^{x|y} = \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3+\kappa_x)$, where $\mu^{y|x} := r/(r+2\lambda^{y|x})$ and $V_b^{x|y} = \Pi_3 - \kappa_x$.

under secrecy protection.[10] Simple comparative statics shows that the higher imitation costs of y (κ_y), the later the entry by the imitators.

Now consider the game starting at $t=0$, in which the inventor is the only seller of both knowledge components, x and y. Unlike the single innovation component case, the inventor can induce several different sequences of adoption by manipulating price offers, which will result in different speeds of entry and timings of knowledge sale.

Lemma 3 *In the monopoly game with respect to both components, the inventor never charges* $p_{ib}^{xt} > \kappa_x$ *and* $p_{ib}^{yt} > \kappa_y$ *for* $b \in \{j,k\}$.

Lemma 3 states that it is never optimal for the inventor to charge such high prices for both components, letting the imitators incur imitation costs rather than buying from her. Once one rules out the price structure in Lemma 3, there remain three candidates for the symmetric equilibrium price structures: (1) $p_{ib}^{xt} \leq \kappa_x$ and $p_{ib}^{yt} > \kappa_y$, (2) $p_{ib}^{xt} > \kappa_x$ and $p_{ib}^{yt} \leq \kappa_y$ and (3) $p_{ib}^{xt} \leq \kappa_x$ and $p_{ib}^{yt} \leq \kappa_y$ for $b \in \{j,k\}$. The price structure (1) induces a step-by-step adoption sequence (denoted by $x \to y$) in which the imitators adopt x first and then y next (i.e., mixing *cw* and *ww* in the current stage). Similarly, (2) induces the imitators to adopt y first and then x next (denoted by $y \to x$). Lastly, (3) induces an all-or-nothing adoption sequence (denoted by *xy*) in which the imitators adopt x and y together (i.e., mixing *cc* and *ww*). To see why, suppose we have the price structure (1). The imitators are aware that if a first adoption occurs for x, they promptly enter a subgame with $\theta = (\{i,j,k\},\{i\})$ in which $p_{ib}^{y} = \kappa_y$ and they end up getting $\Pi_3 - \kappa_y$. Therefore, the imitators are willing to adopt only x for now.[11]

It turns out that inducing the imitators to follow the sequence *xy* is optimal for the inventor.

[10] In some other papers, e.g., Denicolo and Franzoni (2004) and Belleflamme and Bloch (2013), $\mu^{y|x}/r$ is instead defined as the discounting adjusted duration of monopoly under secrecy protection.

[11] One might ask why they do not mix *cm* and *ww* instead. In fact, the imitators are indifferent to the choice between mixing *cm* and *ww* and mixing *cw* and *ww* because we are assuming the continuous time framework. I assume the tie-break rule here is to choose knowledge sale rather than imitation.

The proposition below summarizes the results.[12]

Proposition 1 *In a unique symmetric MPE when the inventor only relies on secrecy, (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for both components by setting* $p^x_{ib} = \kappa_x$ *and* $p^y_{ib} = \kappa_y$ *for* $b \in \{j,k\}$*; (ii) entry times of each imitator follow the exponential distribution with hazard rate,* $\lambda^{xy} = r(\Pi_3 - \kappa)/\kappa$*; and (iii) the expected equilibrium payoffs of the inventor and of the imitators are* $V_i = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3+\kappa)$*, where* $\mu^{xy} := r/(r+2\lambda^{xy}) \in (0,1)$*, and* $V_b = \Pi_3 - \kappa$ *respectively.*

In the appendix, I show that the optimal payoffs of the inventor under sequence xy and $y \to x$ are given by, respectively,

$$V_i^{xy} = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3+\kappa)$$

where μ^{xy} and λ^{xy} are as in result (ii) in Proposition 1, and

$$V_i^{y\to x} = \mu^{y}\Pi_1 + (1-\mu^{y})\left\{\kappa_y + \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3+\kappa_x)\right\}$$

where $\mu^y = r/(r+2\lambda^y)$, $\lambda^y = r(\Pi_3-\kappa)/\kappa_y$, $\mu^{x|y} = r/(r+2\lambda^{x|y})$ and $\lambda^{x|y} = r(\Pi_3-\kappa_x)/\kappa_x$. Here, y stands for variables associated with an adoption game for y given optimally chosen price structure (2), conditional on not having any adoption for both, and $x|y$ for variables associated with an adoption game for x, given that the adoption of y occurs. Subtracting $V_i^{y\to x}$ from V_i^{xy} shows the trade-offs between the two mechanisms.

$$\begin{aligned} V_i^{xy} - V_i^{y\to x} &= \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}(\Pi_1 - \Pi_3) \\ &\quad -(\mu^{xy}-\mu^y)\kappa_y - \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}\kappa_x \end{aligned}$$

One can readily see that the expected time until entry is longer under xy than under $y \to x$, i.e., $\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y} > 0$. This is because, roughly speaking, the imitators are more aggressive

[12] There exists a trivial asymmetric MPE as well. However, in the literature, papers touching on a variant of wars of attrition games (or timing games more generally) with the continuous time framework like mine mostly focus only on symmetric equilibrium. For example, see Choi (1998), Fudenberg and Tirole (1991) p.117~130, footnote 7 in Henry and Ponce (2011) p.966 and the cited papers therein.

in entering in an $y \rightarrow x$ sequence, in which they may have a chance to split the licensing fees, than in xy. Thus, the advantage of xy relative to $y \rightarrow x$ results from a longer duration of monopoly (the first term on RHS). However, there is a disadvantage associated with xy: the inventor collects revenues from knowledge sales later (the second and third terms on RHS). Given assumptions 1 and 2, though, the advantages outweigh the disadvantage, and hence the sequence xy is optimal.

Let V_i^S denote the maximized payoff of the inventor under protecting all-by-secrecy mode. Then,

$$V_i^S = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa).$$

It is worth noting that the profit flow of the innovator under secrecy-only protection resembles the one under patent-only protection in that both mechanisms yield a monopoly profit for some time and a triopoly profit after the simultaneous entry by the imitators.

1.4.1.3 Mixing Patents and Secrecy

The final category to analyze is the case where the innovator patents one component and keeps the other secret. The analysis is analogous in either of the two possible variations in this category, i.e., x patented and y kept secret, or y patented and x kept secret. For expositional simplicity, let us examine the former case of the two, in which case the imitators cannot use x until time T but can do so freely afterwards. Thus, due to the perfect complementarity of x and y, the imitators' entry during the patent term is not feasible. However, the timing of entry after T is closely related with the timing of adoption of component y.

Let us consider a subgame starting at T. There are two scenarios to investigate: (i) a competitive game with respect to y, (i.e., the first adoption of y has occurred before T), $\theta = (T_+, \{i, j\})$ or $(T_+, \{i,k\})$, and (ii) a monopoly game with respect to y, $\theta = (T_+, \{i\})$. Notice that these subgames have a stationary property as true of the games analyzed using the secrecy mechanism, and thus the analysis therein is directly applicable. From Lemma 1 and 2, we can infer that under scenario (i), at time T, the discounted expected payoffs of the inventor and the imitators are the same as discounted sum of triopoly profits (i.e., $V_i|_{\theta=(T_+,\{i,j\})} = V_b|_{\theta=(T_+,\{i,j\})} = \Pi_3$) since the entry by the second imitator is immediate. In contrast, under scenario (ii), the adopting game for y begins from this period on, and thus, $V_i|_{\theta=(T_+,\{i\})} = (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3 + \kappa_y)$ and $V_b|_{\theta=(T_+,\{i\})} = \Pi_3 - \kappa_y$ as in Lemma 2.

With the results above in mind, we will derive the maximum payoff of the inventor. Observe first that eliminating the chance of the first adoption of y during the patent term strictly increases the inventor's payoff as it extends the duration of monopoly even beyond T. Second, the inventor is able to make the imitators to preclude the imitators from adopting y before T by setting p_{ib}^{yt} relatively high for $b \in \{j,k\}$ and $t < T$. To understand this point, conditional on not having the first adoption, at time $t < T$ an imitator j would get $e^{-r(T-t)}(\Pi_3 - \kappa_y)$ in a worst-case scenario in which no adoption takes place until time T either by himself or by the rival imitator, k. Instead, if j decides to obtain y now, his payoff would be $e^{-r(T-t)}\Pi_3 - \min\{p_{ij}^{yt}, \kappa_y\}$ which is strictly less than his minimum possible payoff unless the inventor offers quite low price for y. Put differently, if $p_{ib}^{yt} > e^{-r(T-t)}\kappa_y$ for all $t < T$, adopting y earlier than T is strictly dominated by waiting for the imitators.

The discussion above is summarized in the following proposition.

Proposition 2 *Suppose the inventor protects x by patenting and y by relying on secrecy. In any symmetric MPE, (i) it is optimal for the inventor to induce the imitators not to adopt y until the patent expires ($t < T$) by setting high enough prices and to set $p_{ib}^{y} = \kappa_y$ for $b \in \{j,k\}$ thereafter ($t \geq T$); (ii) entry times of each imitator follow the exponential distribution with hazard rate, $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$ starting from T; and (iii) the expected equilibrium payoffs of the inventor and of the imitators are $V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\{(1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3 + \kappa_y)\} - f$, where $\mu^{y|x} := r/(r+2\lambda^{y|x}) \in (0,1)$, and $V_b = e^{-rT}(\Pi_3 - \kappa_y)$ respectively.*

Proposition 2 states that under the patent-secret mix the monopoly period is extended beyond the term of the patent in the expected sense, depending on the imitation cost of the component kept by secrecy. It is also certain that the component protected by secrecy is not adopted by the imitators until T for sure as the adopting game for the component begins after T.

It remains to examine which mixed use is more profitable: either patenting x and keeping y secret as in the previous discussion or patenting y and keeping x secret. As shown in Corollary 1, the inventor (weakly) prefers the former over the latter because the more costly the component kept secret, the longer duration of monopoly beyond T while the term of patent is invariant regardless of which component to patent.

Corollary 1 *When the inventor uses a patent-secret mix, (i) it is optimal for her to keep the component y, which is the more costly to imitate, secret if* $\kappa_y > \kappa_x$*; and (ii) she is indifferent between protecting x by a patent and y by secrecy and protecting y by a patent and x by secrecy if* $\kappa_y = \kappa_x$.

If we denote the maximum payoff of the inventor under the optimal mixed protection mode by V_i^M,

$$V_i^M = (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3+\kappa_y)\right\} - f.$$

This result is comparable to that found by Anton and Yao (2004). They show that an innovator may patent small innovations but keep secret big innovations with the view that patents and trade secrets are mutually exclusive substitutes. Though my model also suggests that the inventor may optimally protect the more important y (more costly to imitate) by secrecy and x with a patent, such behavior results from the combined use of those two protection mechanisms here. In addition, the reasons for doing that in the two models are different. Here, keeping y secret is to generate a longer duration of monopoly, whereas, in their framework, where the strategic disclosure works as a signaling devise, the inventor faced with limited legal protection is reluctant to disclose big innovations as they are valuable enough to make the imitator want to imitate.

1.4.2 Optimal Protection Decision and Complex vs. Simple Innovations

In this section, I derive one of the key findings of this study by comparing the inventor's payoffs. If the patent filing fee is too high, the inventor would always avoid patenting. In order to exclude those trivial cases, we impose the following assumption.

Assumption 3: $f < \Pi_1 - V_i^S$.

First, note that

$$V_i^M - V_i^P \quad = \quad e^{-rT}\left\{\mu^{y|x}(\Pi_1 - \Pi_3) + (1-\mu^{y|x})\kappa_y\right\} > 0$$

for all $T > 0$. There are two different elements which make using a patent and a trade secret together more appealing than using a patent alone. After the patent expires, not only can the inventor be a monopolist for some more random periods (the first term in the brace), but she also can collect revenues from the sale of y (the second term in the brace). Consequently, the innovator never protects both components by a patent. Then, by comparing V_i^S and V_i^M, we obtain the following proposition.

Proposition 3 *(i) There exists a unique patent length $\overline{T}^{Com}$ such that the inventor protects the complex innovation through the optimal patent-secrecy mix (as described in Corollary 1) if the patent length is relatively long ($T \geq \overline{T}^{Com}$) and otherwise only by secrecy ($T < \overline{T}^{Com}$); and (ii) holding all other variables constant, $\overline{T}^{Com}$ decreases as the composition of the imitation costs become more asymmetric, i.e., κ_y increases and κ_x decreases by the same amount.*

There are two facets of optimal protection worth noting. First, the inventor's optimal protection has a binary feature: either all-by-secrecy or patenting x and keeping y secret. Second, *ceteris paribus* the inventor becomes more aggressive in patenting (i.e., $\overline{T}^{Com}$ gets smaller) under a more asymmetric structure of the imitation costs. The intuition behind this result is as follows. V_i^S depends not on the composition of the imitation costs but on the aggregate imitation costs (see Proposition 1), and thus it is invariant to such cost changes. However, V_i^M increases as κ_y increases and κ_x decreases by the same amount because term T is given the same as before while the expected duration for monopoly beyond T will be lengthened. As a result, the threshold patent length at which V_i^S and V_i^M are equal gets shorter.

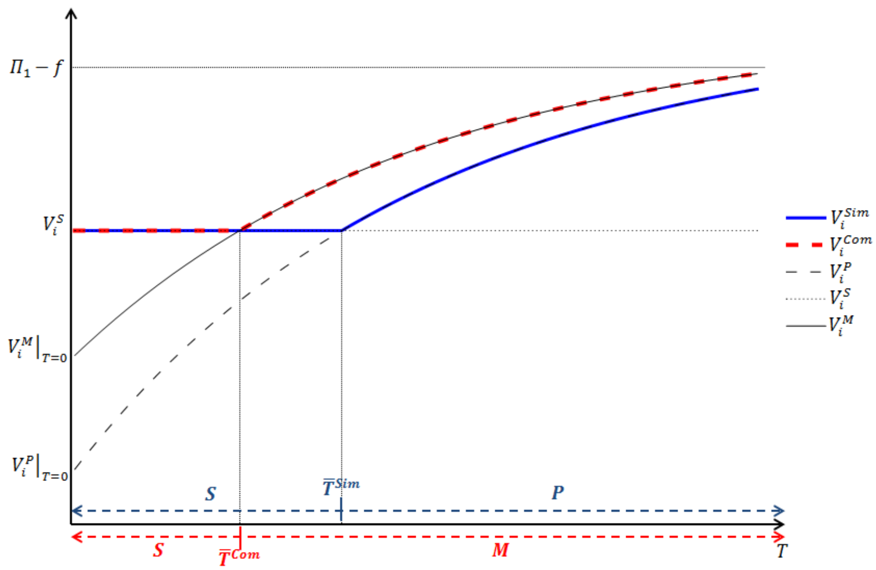
What if the discovery rate of secrecy (λ) is exogenously given regardless of what portion of an innovation is kept secret? When λ is exogenous and constant, as long as at least one component has been protected by secrecy, keeping either x or y secret generates the same λ. Likewise, patenting either x or y makes the inventor a monopolist until T. Given that x and y are perfect complements, to take advantage of both mechanisms, the inventor would always choose to combine both of them in use. In addition, determining which component to patent and which to keep secret is never a concern for the inventor because she is indifferent to the choice between the two possible mixing methods. Lastly, an imitator may uncover the inventor's trade secrets even during the patent term, which is in sharp contrast to the prediction of this paper that under a patent-secret mix entry never takes place until the patent expires.

Now, let us compare the payoffs of inventors having the complex innovation (the "complex innovator" hereafter) and a simple innovation (the "simple innovator" hereafter) respectively. The simple innovation consists of a single and not separable component of which imitation cost is κ. We also assume that the identical profit structure for the simple innovation. Note that, the available protection for the simple innovator is binary: either protecting all-by-patenting or all-by-secrecy. Then, it is easy to see that the simple innovator's equilibrium payoff is $V_i^{Sim}(T) = \max\{V_i^S, V_i^P(T)\}$ while the complex innovator's is $V_i^{Com}(T) = \max\{V_i^S, V_i^M(T)\}$ in which V_i^P, V_i^S and V_i^M are defined as in section 3.1.

Proposition 4 *There exists a unique patent length $\overline{T}^{Sim}$ such that the inventor protects the simple innovation only by a patent if the patent length is relatively long ($T \geq \overline{T}^{Sim}$) and only by secrecy otherwise ($T < \overline{T}^{Sim}$). Moreover, $\overline{T}^{Com} < \overline{T}^{Sim}$.*

As shown in figure 1, both the complex innovator (red dashed line) and the simple innovator (blue line) have the same payoff schedules until $\overline{T}^{Com}$. If T is intermediate i.e., $T \in [\overline{T}^{Com}, \overline{T}^{Sim})$, the payoff of the complex innovator starts to grow, while the simple innovator's is still fixed. When

Figure 1.1: Payoff Comparison between Complex Innovator and Simple Innovator



Note: Vertical and horizontal axes represent the innovator's payoffs and length of patent, respectively. The two dashed lines near the horizontal axis show the optimal protection modes for the simple innovator (upper blue) and for the complex innovator (lower red).

T exceeds $\overline{T}^{Sim}$, the payoff of the simple innovator also begins to increase but never catches up to the complex innovator's. From the optimal protection choice depicted in the bottom, we also find that the complex innovator is more inclined to apply for a patent.

One important observation is that without the patent system the inventor would get the same payoffs as the simple inventor would. To understand this, recall that the equilibrium payoff of the inventor under secrecy protection does not depend on the composition of imitation costs but only on the aggregate costs (Proposition 1). This implies that it is not *per se* the existence of a number of perfectly complementary components which may make the complex innovator more profitable. Coexistence of the patent system and the secrecy mechanism is crucial to have that result.

1.5 Strict Utility Requirement

Up to this point, only the positive analysis has been conducted. In particular, it was shown that the inventor with a complex innovation is (weakly) over-rewarded. However, if the patent examination office imposes a strict utility requirement (SUR), under which the innovator needs to disclose both components to get a patent, it can transform the inventor's payoff schedule into that of another investor having a simple innovation. This section addresses policy implications of SURs whether they are socially desirable or not.

There are numerous research studies investigating optimal patent policy regarding various policy variables (e.g., the patent scope or the term of patent). However, there is no clear consensus on this topic, and results heavily depend on underlying environments and assumptions.[13] For the sake of a thought experiment, we only consider the length of patent as the policy instrument of interest. We then consider a simple model in which the social planner decides whether to impose an SUR or not after the discovery (ex-post welfare implication) and before the discovery (ex-ante welfare implication) of the complex innovation in that order. For simplicity, throughout this section we assume the two components are symmetric in terms of imitation costs. (i.e., $\kappa_x = \kappa_y = \kappa/2$)

1.5.1 Ex-post Welfare Implication of SUR

In order to first calculate ex-post welfare after the discovery of the innovation with and without SUR, given the patent length, T, we denote the ex-post welfare functions under each protection mode, P, M and S, by $\widetilde{W}^P(T)$, $\widetilde{W}^M(T)$ and $\widetilde{W}^S$ as follows

$$\begin{aligned}
\widetilde{W}^P(T) &= (1-e^{-rT})W_1 + e^{-rT}W_3 \\
\widetilde{W}^M(T) &= (1-e^{-rT})W_1 + e^{-rT}\left\{\mu^1 W_1 + (1-\mu^1)W_3\right\} \\
\widetilde{W}^S &= \mu^2 W_1 + (1-\mu^2)W_3
\end{aligned}$$

[13] For example, Gilbert and Shapiro (1990) and Gallini (1992) show sharply contrasting results.

where μ^1 and μ^2 are the natural duration of monopoly generated by keeping 1 component and 2 components by secrecy, respectively.[14] As is usually assumed in the literature, W_1 and W_3 represent social welfare, defined as the sum of industry profits and consumer surplus under monopoly and triopoly market structure respectively, and social welfare is higher under a triopoly situation than under a monopoly one, i.e., $W_1 < W_3$.

As the inventor's optimal decision is opting for S if $T \leq \overline{T}^{Sim}$ and P otherwise under SUR, ex-post welfare with SUR is

$$\widetilde{W}^{SUR}(T) = \begin{cases} \widetilde{W}^S & \text{if } T \leq \overline{T}^{Sim} \\ \widetilde{W}^P(T) & \text{otherwise} \end{cases}.$$

Similarly, ex-post welfare without SUR is given by

$$\widetilde{W}^{NO}(T) = \begin{cases} \widetilde{W}^S & \text{if } T \leq \overline{T}^{Com} \\ \widetilde{W}^M(T) & \text{otherwise} \end{cases}.$$

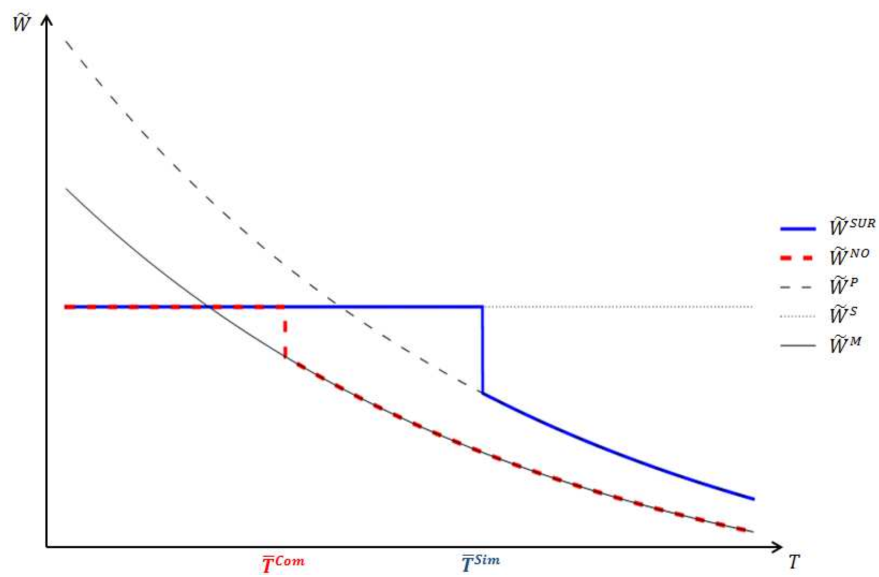
The schedules of the two welfare functions can be compared by first identifying their characteristics. Observe first that $\widetilde{W}^M$ is always less than $\widetilde{W}^P$ because the patent-secret mix always entails a longer duration for a monopoly. Next, while $\widetilde{W}^S$ does not vary by the patent length T, $\widetilde{W}^M$ and $\widetilde{W}^P$ decrease as T increases. Last, and most interestingly, there is a sudden drop in the two welfare functions. More specifically, $\widetilde{W}^{SUR}$ suddenly decreases at the threshold, $\overline{T}^{Sim}$. The reason for this discontinuity is that while the optimal payoff of the inventor remains the same in the two protection modes at $\overline{T}^{Sim}$, the expected monopoly duration is shorter under secrecy protection. To see this, it is useful to compare the natural duration of the secret, μ^2 and the natural patent length, $(1-e^{-r\overline{T}^{Sim}})$ where $\overline{T}^{Sim} = (1/r)\ln[(\Pi_1 - \Pi_3)/\{(1-\mu^2)(\Pi_1 - \Pi_3 - \kappa) - f\}]$ by the symmetric imitation cost structure and the definition of $\overline{T}^{Sim}$.[15] Then, it is easy to see that

$$(1-e^{-r\overline{T}^{Sim}}) - \mu^2 = \frac{(1-\mu^2)\kappa + f}{\Pi_1 - \Pi_3} > 0.$$

[14] $\mu^1 = r/(r+2\lambda^1)$, $\lambda^1 = r(\Pi_3 - \kappa/2)/(\kappa/2)$, $\mu^2 = r/(r+2\lambda^2)$, $\lambda^2 = r(\Pi_3 - \kappa)/\kappa$.

[15] See the proof of Proposition 4.

Figure 1.2: Ex-post Welfare with and without SUR



Note: Vertical and horizontal axes represent respectively social welfare and length of patent

The intuition is simple: under secrecy protection as the inventor has another channel of appropriating her innovation, i.e., selling the knowledge components, a shorter monopoly is required for her to achieve the expected payoff she would have earned under patent protection, with the threshold $\overline{T}^{Sim}$. This implies when the inventor is indifferent between the secrecy and patent protection modes, ex-post welfare is higher under secrecy protection, i.e., $\widetilde{W}^S > \widetilde{W}^P(\overline{T}^{Sim})$. A similar logic can be applied to show $\widetilde{W}^S > \widetilde{W}^M(\overline{T}^{Com})$.

From the discussion above, it is clear that $\widetilde{W}^{SUR}$ and $\widetilde{W}^{NO}$ must be drawn as depicted in figure 2. The figure shows that $\widetilde{W}^{NO}$ is equal or less than $\widetilde{W}^{SUR}$ for all T, implying that availability of a patent-secret mix decreases ex-post welfare, and thus, imposing SUR is advisable.

Proposition 5 *Imposing the strict utility requirement is socially desirable ex-post (strictly desirable for $T > \overline{T}^{Com}$).*

1.5.2 Ex-ante Welfare Implication of SUR

This section turns from the above examination of ex-post welfare implications of SUR to the ex-ante ones, taking into account the inventor's investment incentives. To this end, the first question is to determine which T will be chosen by the social planner facing an innovator when only the patent protection mode is available. Then, such T is a solution of the following problem:

$$\max_{T} W^{P}(T) = \hat{I}^{P} \cdot \widetilde{W}^{P} - D(\hat{I}^{P})$$

subject to

$$\hat{I}^{P} = \arg\max_{I} I \cdot V_{i}^{P} - D(I)$$

where I is the probability that the innovation process turns out to be successful; $D(I)$ is a cost function of such R&D process given I; $\hat{I}^{P}$ is the optimal investment decision of the inventor given T. Furthermore, let us assume that the objective function subject to the optimal investment decision is strictly quasi-concave in T, and so the solution exists uniquely. There is a trade-off of giving more incentive for the inventor to invest by increasing T, which increases the probability that the innovation project is successful but also decreases ex-post social welfare by extending the duration for which the inventor is a monopolist. Thus, if we denote the solution of the above problem by $T^{P*} \in (0,\infty)$, it is set by the social planner in such a way that those two marginal effects are balanced. We can consider the same problem with mixed protection. Let $T^{M*} \in (0,\infty)$ be the solution of such social planner's problem.

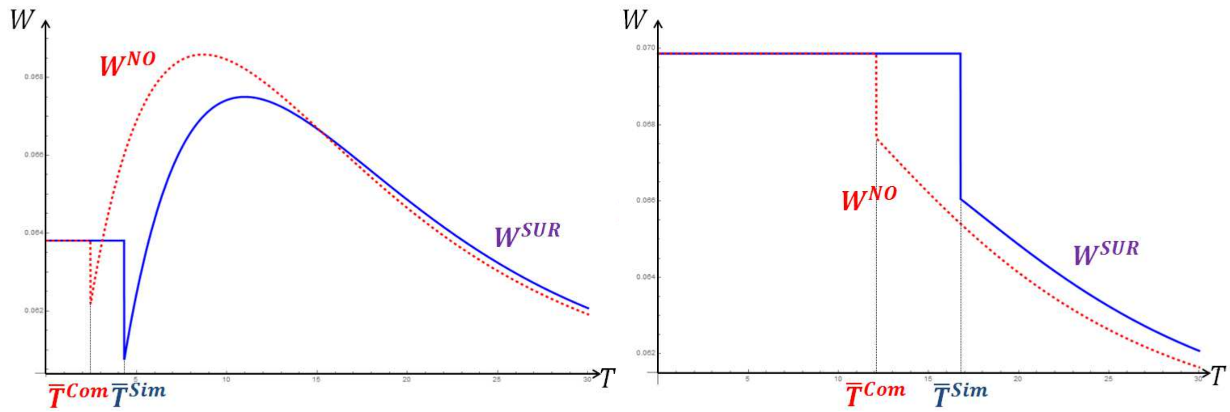
Then, the ex-ante social welfare functions with and without SUR are defined again by the optimal decision rules of the innovator:

$$W^{SUR}(T) = \begin{cases} W^{S} & \text{if } T \leq \overline{T}^{Sim} \\ W^{P}(T) & \text{otherwise} \end{cases}$$

and

$$W^{NO}(T) = \begin{cases} W^{S} & \text{if } T \leq \overline{T}^{Com} \\ W^{M}(T) & \text{otherwise} \end{cases}.$$

Figure 1.3: Example of Ex-ante Welfare with and without SUR



(a) Case (I): $\kappa = .04$

(b) Case (II): $\kappa = .085$

Note: Vertical and horizontal axes represent social welfare and length of patent, respectively. W^{SUR} and W^{NO} are the ex-ante welfare functions given T with and without SUR respectively.

Even if the socially optimal length of patent is well defined for P and M, a complete analysis for all underlying primitives is somewhat challenging. First, given T, optimally chosen investment levels under P, M and S are all different. Moreover, due to the discontinuity nature of the ex-post welfare functions introduced in the previous subsection, the ex-ante welfare functions are also not continuous and even, at times, not monotone. Therefore, in what follows, I will demonstrate, by offering an example, that policy implications are sensitive to parameter values of the model and ambiguous in general.

Example: Consider the following two cases which only differ by the imitation costs: for case (I), $\kappa = .04$ and for case (II), $\kappa = .085$. Except for that difference, the two cases share the same modeling assumptions: $D(I) = I^2$, $\Pi_1 = .4$, $\Pi_3 = .1$, $W_1 = .5$, $W_3 = .8$, $r = .1$ and $f = 0$. Figure 3 shows welfare functions for the two cases. Note that the policy implication to be drawn is that imposing SUR is socially desirable whenever $W^{SUR} > W^{NO}$. First, consider case (I) in which the strength of secrecy mechanism (relatively low κ), by which the lower bound of ex-ante investment incentives is determined, is relatively low. In this situation, $\overline{T}^{Com}$ and $\overline{T}^{Sim}$ are less than T^{M*} and

T^{P*}, respectively, and this implies the secrecy only protection is not giving enough investment incentives to the innovator. Hence, in this case, for some relatively intermediate T, allowing the inventor to choose M may increase the social welfare. However, the policy implication is not monotone in T. Roughly speaking, imposing SUR increases social welfare over either the lower or the upper tail range of T but decreases it over the intermediate range of T. In contrast, in case (II), the strength of secrecy is so high that even without the patent system, the inventor is willing to invest aggressively. Therefore, adding M option to the innovator's choice set always lowers social welfare. This example illustrates that the policy implication of SUR is, in general, ambiguous.

1.6 Extensions and Robustness

This section considers two extensions to check the robustness of the main findings. The complex innovation is comprised of $N(>2)$ components rather than 2 components is first discussed, followed by an exploration of the case in which patent rights are probabilistic.

1.6.1 *N* Component Innovation

We have assumed so far, for simplicity, that the complex innovation has just two complementary components. Here, suppose we have a complex innovation which is comprised of N different complementary components. The imitation costs of each component are given by $\kappa_1 \leq \ldots \leq \kappa_n \leq \ldots \leq \kappa_N$ for $n \in \{1,2,\ldots,N\}$, and let $\kappa \equiv \sum_{n=1}^{N} \kappa_n < \Pi_3$ denote the aggregate imitation cost. I show that the main results we had for the model with 2 components carry over to this situation.

When the inventor applies for a patent containing all N components, her payoff remains the same as before:

$$V_i^P = (1-e^{-rT})\Pi_1 + e^{-rT}\Pi_3 - f.$$

Regarding the secrecy only mode and the patent-secret mix, equilibria in each scenario also shows a pattern that is similar to the 2 component model. We formally characterize the equilibria in the next two lemmas.

Lemma 4 *In the N component model, when the inventor relies only on secrecy, the unique symmetric MPE is characterized as follows: (i) it is optimal for the inventor to induce the imitators to mix adopting and waiting for all N components by setting* $p_{ib}^n = \kappa_n$ *for all* $n \in \{1,2,\ldots,N\}$ *for* $b \in \{j,k\}$*; (ii) entry times of each imitator follow the exponential distribution with hazard rate,* $\lambda = r(\Pi_3 - \kappa)/\kappa$*; and (iii) the expected equilibrium payoff of the inventor is* $V_i^S = \mu\Pi_1 + (1-\mu)(\Pi_3 + \kappa)$ *where* $\mu = 1/(r+2\lambda)$.

Lemma 5 *In the N component model, (i) the optimal patent-secret mix is patenting only component 1, which is the least costly to imitate, and keeping all the other components by secrecy; (ii) the inventor induces the imitators not to adopt all components except component 1 until the patent expires (*$t < T$*) by setting high enough prices and sets* $p_{ib}^n = \kappa_n$ *for all* $n \in \{2,3,\ldots,N\}$ *for* $b \in \{j,k\}$ *thereafter (*$t \geq T$*); (ii) entry times of each imitator follow the exponential distribution with hazard rate,* $\lambda^{-1} = r(\Pi_3 - \kappa_{-1})/\kappa_{-1}$ *starting from T; and (iii) the expected equilibrium payoff of the inventor is* $V_i^M = (1-e^{-rT})\Pi_1 + e^{-rT}\{\mu^{-1}\Pi_1 + (1-\mu^{-1})(\Pi_3 + \kappa_{-1})\} - f$ *where* $\mu^{-1} = 1/(r+2\lambda^{-1})$ *and* $\kappa_{-1} = \sum_{n=2}^{N} \kappa_n$.

V_i^S is the same expression as in the 2 component model, and V_i^M is also similar to the counterpart of the 2 component model. Hence, we reach to the same conclusion that the optimal protection mode is either *S* or the optimal *M* because *P* is always dominated by the optimal *M*, and thus the complex innovator weakly earns more than the simple innovator.

The *N* component model also offers some additional insights through a simple comparative static analysis. Assuming symmetric imitation costs, as the innovation becomes more complex (larger *N*), both the expected duration of prolonged monopoly (higher λ^{-1}) and the inventor's expected payoff (higher V_i^M) increase in the optimal patent-secrecy mix, and, as a result, the threshold equating V_i^S and V_i^M ($\overline{T}^{Com}$) gets shorter.

Proposition 6 *In the N component model, (i) there exists a unique patent length* $\overline{T}^{Com}$ *such that the inventor protects the complex innovation through the optimal patent-secret mix (as described in Lemma 4) if the patent length is relatively long (*$T \geq \overline{T}^{Com}$*) and only by secrecy otherwise*

($T < \overline{T}^{Com}$); and (ii) when all components are symmetric in terms of imitation cost, i.e., $\kappa_1 = \ldots = \kappa_n = \ldots = \kappa_N$, $\overline{T}^{Com}$ shortens as the innovation becomes more complex (i.e., as N rises).

1.6.2 Probabilistic Patents

How much of the previous results would be changed when we assume that patents are probabilistic rather than ironclad? Put differently, what if filing for a patent no longer absolutely assures the inventor of T-period monopoly position, but only provides some probability? Here, assuming patents are probabilistic, we determine under which conditions the combined use of patents and secrecy turns out to be optimal, at least for some values of patent length, and thus the complex innovator is still (weakly) over-rewarded.

To formalize the idea, consider the following simplified version of a post-grant review in the U.S. patent system or an opposition proceeding in the European patent system.[16] Right after a patent is granted to the inventor, the imitators have a chance to challenge it at the patent office. For simplicity, assume that the challenge process takes place instantly without any delay at 0 cost, which implies it is optimal for the imitators to challenge every patent granted. Let us further assume that $f = 0$ and that the two components are symmetric in terms of imitation cost $(\kappa_x = \kappa_y = \kappa/2)$ and the probability that each component will be upheld as valid through the trial proceeding when the components are challenged. Denote by α the probability that at least one component survives the challenge and is upheld as valid when the inventor has chosen P. Similarly, define $\rho\alpha$ as the probability that a component is upheld as valid when the inventor has chosen M where ρ is a measure of the inverse degree of complementarity between the two components in obtaining a valid patent and takes a value in $[1/2, 1]$. [17]

[16] United States Patent and Trademark Office states "Post grant review is a trial proceeding conducted at the Board to review the patentability of one or more claims in a patent on any ground that could be raised under §282(b)(2) or (3). Post grant review process begins with a third party filing a petition on or prior to the date that is 9 months after the grant of the patent or issuance of a reissue patent." (For more details, visit https://www.uspto.gov/patents-application-process/appealing-patent-decisions/trials/post-grant-review. Last retrieved on Aug/19/2016.)

[17] To see this, note that the joint distribution of trial results for the two components is as follows

Then, the equilibrium payoff of the inventor under S remains as in the basic model, i.e., $V_i^S = \mu\Pi_1 + (1-\mu)(\Pi_3 + \kappa)$, whereas the equilibrium payoffs under the other two protection modes change. When the inventor opts for P, she earns monopoly profit until T and triopoly profit afterwards only with probability α, and with the complementary probability, $1-\alpha$, the imitators can enter the market from the beginning. Hence,

$$V_i^P = \alpha\left\{(1-e^{-rT})\Pi_1 + e^{-rT}\Pi_3\right\} + (1-\alpha)\Pi_3.$$

When the inventor chooses M, she becomes a monopolist until T and then the adopting game for the remaining component by the imitators follows right after T if the patent is upheld as valid (with probability $\rho\alpha$). Otherwise (with the complementary probability $1-\rho\alpha$), the same adopting game starts from the beginning. Thus,

$$V_i^M = \rho\alpha\left\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\right\} + (1-\rho\alpha)V^1,$$

where $V^1 = \{\mu^1\Pi_1 + (1-\mu^1)(\Pi_3 + \kappa/2)\}$, $\mu^1 = r/(r+2\lambda^1)$ and $\lambda^1 = r(\Pi_3 - \kappa/2)/(\kappa/2)$.[18]

since x and y are symmetric in terms of validity:

Component $x \setminus y$	Valid	Not valid
Valid	A	B
Not valid	B	C

Then, $\alpha = A + 2B$ and $\rho\alpha = A + B$, and thus, $\rho = (A+B)/(A+2B)$ which takes a value between $1/2$ and 1. For example, depending on whether the trial results of x and y are perfectly positively correlated ($B = 0$), independent ($B = \rho\alpha(1-\rho\alpha)$) or perfectively negatively correlated ($B = \alpha/2$), ρ equals to 1, $(1-\sqrt{1-\alpha})/\alpha$, or $1/2$ respectively.

[18] Alternatively, we can assume that patent rights are probabilistic in the sense that patent examiners at the patent office reject a claim (component), which is included in a patent application and does not meet one of the patentability requirements, with a certain probability. (In the U.S. patent law, the patentability requirements are patentable subject matter, novelty, non-obviousness and utility). If rejected claims are disclosed to the public, the equilibrium payoffs will be the same as those described in the post-grant review scenario. If rejected claims are not disclosed to the public, and thus if the imitators still needs to incur imitation costs or to buy from the inventor in order to adopt those components, the equilibrium payoffs with P and M are modified as follows respectively: $V_i^P = (2\rho-1)\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}\Pi_3\} + 2(1-\rho)\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\} + (1-\alpha)V_i^S$ and $V_i^M = \rho\alpha\{(1-e^{-rT})\Pi_1 + e^{-rT}V^1\} + (1-\rho\alpha)V_i^S$. The qualitative results do not, however, change under this alternative scenario.

In order to find the optimal protection mode depending on the patent length, first observe that if $T=0$, for all α and ρ,

$$V_i^S > V_i^M|_{T=0} > V_i^P|_{T=0}.$$

This tells us that when T is 0, it is better to protect as many components as possible by secrecy so that the inventor can have a longer random monopoly period regardless of α and ρ. This is somewhat obvious: recall that we had the same ranking between the payoffs when $T=0$ even in the basic model, and patenting, moreover, now became less attractive due to the probabilistic nature of patent rights. By the continuity of payoff functions, when patent duration is relatively short, S would be still optimal compared to M or P. However, as T increases, the ranking between the payoffs may be reversed since, for all α and ρ,

$$\frac{\partial V_i^P}{\partial T} = \alpha r e^{-rT}(\Pi_1 - \Pi_3) > \frac{\partial V_i^M}{\partial T} = \rho \alpha r e^{-rT}(\Pi_1 - V^1) > \frac{\partial V_i^S}{\partial T} = 0$$

where the first inequality follows from the fact that $\rho \leq 1$ and $V^1 > \Pi_3$. In fact, the full optimal schedule of protection mode over $T \in [0, \infty)$ crucially depends upon the ranking of V_i^P, V_i^M and V_i^S in the limit (i.e., $T \to \infty$). Thus, to fully characterize equilibrium constellation, let us define the following thresholds: α^*, $\hat{\rho}(\alpha)$ and $\tilde{\rho}(\alpha)$ such that respectively

$$\begin{aligned}
\lim_{T\to\infty} V_i^P(\alpha) \gtreqless V_i^S &\iff \alpha \gtreqless \alpha^*, \\
\lim_{T\to\infty} V_i^M(\alpha,\rho) \gtreqless V_i^S &\iff \rho \gtreqless \hat{\rho}(\alpha), \\
\lim_{T\to\infty} V_i^M(\alpha,\rho) \gtreqless \lim_{T\to\infty} V_i^P(\alpha) &\iff \rho \gtreqless \tilde{\rho}(\alpha).
\end{aligned}$$

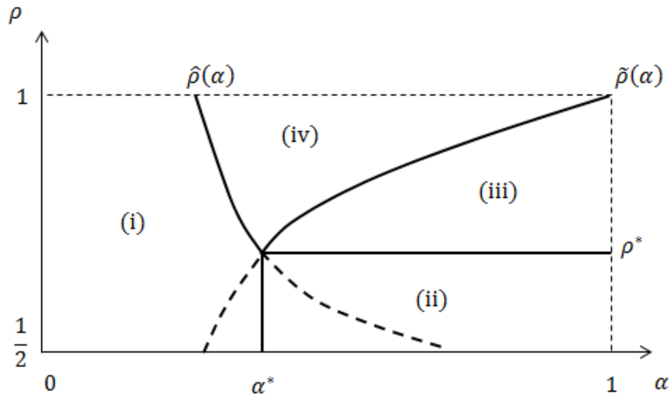
Then, the equilibrium protection constellations are as shown in Proposition 7 and illustrated in figure 4.

Proposition 7 *In the probabilistic patents model, given* $(\alpha,\rho) \in (0,1] \times [1/2,1]$*, the optimal protection modes chosen by the inventor are as follows:*

(i) if $\alpha \leq \alpha^*$ *and* $\rho \leq \hat{\rho}(\alpha)$*, S always;*

(ii) if $\alpha > \alpha^*$ *and* $\rho \leq \rho^*$*, S if T is small and P otherwise;*

Figure 1.4: Equilibrium Protection Constellations



(iii) if $\alpha > \alpha^*$ *and* $\rho^* < \rho < \tilde{\rho}(\alpha)$*, S if T is small, M if T is intermediate and P otherwise;*

(iv) if $\tilde{\rho}(\alpha) \leq \rho$ *and* $\hat{\rho}(\alpha) < \rho$*, S if T is small and M otherwise*

where $\alpha^* = \frac{V_i^S - \Pi_3}{\Pi_1 - \Pi_3} \in (0,1)$, $\hat{\rho}(\alpha) = \frac{V_i^S - V^1}{\alpha(\Pi_1 - V^1)}$, $\tilde{\rho}(\alpha) = \frac{\alpha(\Pi_1 - \Pi_3) - (V^1 - \Pi_3)}{\alpha(\Pi_1 - V^1)}$ *and* $\rho^* = \tilde{\rho}(\alpha^*) = \hat{\rho}(\alpha^*) \in (1/2, 1)$.

Proposition 7 confirms that in the probabilistic patents model the combined use of patents and trade secrets (for some T) arises in equilibrium as before insofar as the probability of getting a valid patent with the two components is relatively high (i.e., relatively high α), and the gap between such probabilities with the two components and with a component is not that high (i.e., relatively high ρ). That corresponds to the area (iii) and (iv) in figure 4, and in this region it is also robust that the complex innovator is weakly over-rewarded.

It is noteworthy that now there is a pair (α, ρ) under which P dominates M for all T (area (ii)). Recall that in the main analysis, either M or P ensured obtaining a patent, and, as a result, P was always inferior to M as it only made the inventor give up a chance to prolong the monopoly period beyond T by disclosing all information. That is no longer the case since P has an advantage over M in the sense that it has a higher probability of acquiring a patent, and the degree of that advantage depends on ρ. More specifically, for each $\alpha > \alpha^*$, the optimal pattern of the inventor's strategy changes, as ρ increases, from the one in (ii) of Proposition 7 to the one in (iii) and finally to the one in (iv). Another interesting point is that some dubious innovations, which would not have been patented at all if they were a simple innovation, might be patented if ρ is high (corresponding to the subarea of (iv) such that $\alpha < \alpha^*$).

1.7 Conclusion

This paper studies the optimal use of patents and trade secrets by an inventor who has a complex innovation. I introduce the mechanism of how the patent-secret mix can extend the duration of monopoly beyond the length of patent, and thus dominate the patent protection. For that reason, the inventor's optimal choice has a binary feature: either opting for the optimal patent-secret mix or secrecy only. The optimality of patent-secret mix is carried over to the case of probabilistic patent rights as well on a broad set of parameters.

As one may easily expect, with a richer set of protection modes, including the mixed one, the inventor who has the complex innovation is over-rewarded compared to another inventor who has a simple innovation. However, the policy implication of imposing SUR considering the ex-ante investment incentives is ambiguous and sensitive to underlying parameters. Therefore, unfortunately, this paper informs us that designing optimal intellectual property rights policies calls for a case-by-case scrutiny with the patent-secret mix as the inventor's protection strategy. In this regard, it would be also beneficial to conduct a similar analysis to illuminate how and when firms optimally use a patent-secret mix in different innovation frameworks, such as, a cumulative innovation setting, a patent portfolio setting and a complementary innovation setting associated with patent pooling.

APPENDIX

Appendix for Chapter 1

Proof of Lemma 1

Result (i): I explore all possible competitive games with respect to a knowledge component in turn. First, consider a competitive game with respect to both components. There are two possible scenarios: (1) an imitator is a seller for both components besides i, i.e., $\theta = (\{i,j\},\{i,j\})$ or $(\{i,k\},\{i,k\})$; (2) each imitator is a seller for different components in addition to i, i.e., $\theta \in (\{i,j\},\{i,k\})$ or $(\{i,k\},\{i,j\})$. Assume scenario (1) first and say j the seller without loss of generality, i.e., $\theta = (\{i,j\},\{i,j\})$. I will show that both of components are sold immediately at 0 prices in equilibrium. The proof for this claim consists of the following three steps, and these can be applied to the rest of three cases in a similar manner.

Step 1: Given price offers, k's best respond is

$$\begin{cases} c_{\underline{s}k}c_{\underline{s}k} & \text{if } p^x_{\underline{s}} \leq \kappa_x \text{ and } p^y_{\underline{s}} \leq \kappa_y \\ c_{\underline{s}k}m_k & \text{if } p^x_{\underline{s}} \leq \kappa_x \text{ and } p^y_{\underline{s}} > \kappa_y \\ m_k c_{\underline{s}k} & \text{if } p^x_{\underline{s}} > \kappa_x \text{ and } p^y_{\underline{s}} \leq \kappa_y \\ m_k m_k & \text{otherwise.} \end{cases}$$

where $\underline{s}$ denote the seller who offers the lower price. This is straightforward from the fact (1) that including w_k for any components is never a best response because the immediate entry, which is possible only when k adopts x and y, is profitable for k and (2) that it is optimal for k to pick the cheapest way of adopting a component.

Step 2: In any MPE, all components are sold to k through contracts offered by one of the two sellers. Suppose not, which implies either $p^x_{\underline{s}} > \kappa_x$ or $p^y_{\underline{s}} > \kappa_y$. For simplicity, say $p^x_{\underline{s}} > \kappa_x$. Then, it is profitable for either i or j to set $p^x = \kappa_x$ at which k buys x rather than imitating because it increases her or his payoff by κ_x without changing the timing of entry by k at all since it takes place anyway in that period.

Step 3: In a unique MPE, $p^x_{ik} = p^x_{jk} = 0$ and $p^y_{ik} = p^y_{jk} = 0$. Again, suppose it were not the case. If both sellers offer a pair of positive prices for a component, at least one of the two has an incentive to set a price just below the rival seller's and collect more revenue without affecting the present value of each firm's profit stream form the market. Similarly, if one of them offers a strictly positive price, but the other offers 0, then the latter has an incentive to increase her price. Thus, in any competitive game with respect to both components, there is a unique MPE where all remaining buyers obtains knowledge components still not in their possession at 0 price immediately.

Next, consider another class of competitive games in which there are two sellers for a component, and the other was obtained by all firms. For simplicity, assume that all firms learned x. Then, there are two possible cases to consider, $\theta = (\{i, j.k\}, \{i, j\})$ or $(\{i, j.k\}, \{i, k\})$. The analysis is analogous to the previous one but simpler than that because it is now sufficient to consider only one remaining component to be sold.

Result (ii): Consider a game that is competitive with respect to a component but monopolistic with respect to the other. I show that the remaining buyer of the former component purchases it at a 0 price in an MPE. For simplicity, say x was adopted by j at time t, i.e., $\theta = (\{i, j\}, \{i\})$. First, note that until the first adoption takes place for y as well, i is the monopolist in the market because of perfect complementarity of x and y. If k chooses to wait for x, he starts to earn triopoly profit once the first adoption for y takes place since he can get x for free almost immediately (from result (i)). Conversely, if k chooses either buy or imitate at time t, he enters a subgame of $\theta = (\{i, j, k\}, \{i\})$ right away in which he ends up earning the same profit stream with imitators' actions for y remaining the same as in the previous case. As so far as the minimum price of x offered to k is positive, imitating or buying x at time t is dominated by waiting, in any MPE of this subgame, x should be sold to k at 0 price and the equilibrium payoff is independent of the timing of sale of x. ■

Proof of Lemma 2

Together with Lemma 1 in this paper, see Lemma 2 and Proposition 2 in Henry and Ponce (2011) except calculating the buyers' payoffs in equilibrium. Note that

$$V_b^{y|x} \quad = \quad \frac{\lambda^{y|x}(\Pi_3 - \kappa_y) + \lambda^{y|x}\Pi_3}{r + 2\lambda^{y|x}}.$$

Once $\lambda^{y|x}$ in Lemma 2 is plugged into the above equation, it follows that $V_b^{y|x} = \Pi_3 - \kappa_y$. ■

Proof of Lemma 3

Suppose there is an MPE in which the innovator offers $p_{ib}^x > \kappa_x$ and $p_{ib}^y > \kappa_y$ for $b \in \{j,k\}$. Then, in any equilibrium it must be the case that the first adoption of a component by an imitator[19] is associated with imitation by himself rather than buying knowledge through a contract. Given that if the seller offers a price equal to its imitation cost for that component, the only thing changes would be the mode of adoption of it from imitating to buying with all other aspects of the equilibrium unchanged. Then, clearly by doing this the seller will be strictly better off because now she additionally obtains revenue from selling knowledge. This contradicts the supposition that $p_{ib}^x > \kappa_x$ and $p_{ib}^y > \kappa_y$ for $b \in \{j,k\}$ are MPE price offers. ■

Proof of Proposition 1

Step 1: First, let us calculate the expected payoffs to the innovators and imitators given different price combinations. If $p_{ib}^x \leq \kappa_x$ and $p_{ib}^y > \kappa_y$, m_b^x is never optimal because at least b can resort to c_{ib}^x to adopt x allowing b to save imitation cost, keeping all other aspects of the game unchanged. By the same token, c_{ib}^y is never optimal as b can instead choose m_b^y. Then, a candidate for an equilibrium is among $c_{ib}^x w_b^y$, $w_b^x w_b^y$, $c_{ib}^x m_b^y$ and $w_b^x m_b^y$. However, if the equilibrium should be associated either with $c_{ib}^x m_b^y$ or $w_b^x m_b^y$, i always has a profitable deviation of adjusting $p_{ib}^y = \kappa_y$, which

[19] Depending on an equilibrium strategy profile, a component and an imitator need to be modified as components and the imitators respectively.

increases i's revenue from selling y without changing other aspects of the game. Since that is a clear contradiction, it must be the case that b chooses either $c^x_{ib}w^y_b$ or $w^x_b w^y_b$ if this is an equilibrium. This implies that knowledge adoption takes place for x first and for y later in turn in equilibrium. $(x \to y)$

Let $F_b : [0.\infty) \times [0, \kappa_x] \to [0,1]$ refer to the distribution function of entry times of b. Taking F_{-b} as given and conditional on no adoption for x and y until time t, the expected benefit of waiting infinitesimal dt measure of time more (comparing to purchasing x now) is given by $f_{-b}p^x_{ib}dt/(1-F_{-b})$ since b can get x freely instead of paying p^x_{ib} if $-b$ enters during this time window. However, there are also costs associated with waiting. b needs to sacrifice some surplus which he would otherwise have been able to earn, $r(\Pi_3 - \kappa_y - p^x_{ib})dt$. [20] In equilibrium the above two terms should be equal, which yields

$$\frac{f_{-b}}{1-F_{-b}} = \frac{r(\Pi_3 - \kappa_y - p^x_{ib})}{p^x_{ib}}.$$

Note that $\lambda_{-b} := f_{-b}/(1-F_{-b}) = r(\Pi_3 - \kappa_y - p^x_{ib})/p^x_{ib}$ is stationary, and thus adoption strategies of the imitators follow an exponential distributions with the hazard rates.

The timing of the first adoption for x is a random variable which follows exponential distribution with the hazard rate of $\lambda_j + \lambda_k$ since it is the minimum of adoption times of the two imitators which are independently and exponentially distributed with hazard rates of λ_j and λ_k respectively given $p^x = (p^x_{ij}, p^x_{ik}) \in [0, \kappa_x]^2$. By applying the technique used by Henry and Ponce (2011) along with Lemma 2 [21], if we write down the expected payoff of the inventor,

$$V_i^{x\to y}(p^x) = \frac{r\Pi_1}{r+\lambda_j+\lambda_k} + \frac{(\lambda_j+\lambda_k)V_i^{y|x}}{r+\lambda_j+\lambda_k} + \frac{\lambda_j p^x_{ij} + \lambda_k p^x_{ik}}{r+\lambda_j+\lambda_k}.$$

One can easily check that $V_i^{x\to y}(p^x)$ is strictly increasing in both price levels. In order to see this, observe

$$\frac{\partial V_i^{x\to y}}{\partial p^x_{ij}} = \frac{\lambda_j}{\phi} - \frac{1}{\phi^2}\frac{\partial \lambda_j}{\partial p^x_{ij}}\left\{r\Pi_1 - rV_i^{y|x} - (r+\lambda_j)p^x_{ik} + \lambda_j p^x_{ij}\right\}$$

[20] Recall from Lemma 1 and 3 that once the first adoption takes place for x, they enter the subsequent subgame immediately in which imitators' expected payoffs in equilibrium is $\Pi_3 - \kappa_y$.

[21] See the proof of Proposition 2 in Henry and Ponce (2011) for more details

where $\phi := r + \lambda_j + \lambda_k$. Since $\partial\lambda_j/\partial p^x_{ij} < 0$, it suffices to check the term inside of the squared bracket is positive. Plugging in the expression for $V_i^{y|x}$ from Lemma 2 and $r(\Pi_3 - \kappa_y - p^x_{ik})$ for $\lambda_j p^x_{ik}$ and collecting terms,

$$\begin{aligned} & r\Pi_1 - rV_i^{y|x} - (r+\lambda_j)p^x_{ik} + \lambda_j p^x_{ij} \\ = \; & \frac{r(2\Pi_1 - 4\Pi_3 - \kappa_y)(\Pi_3 - \kappa_y)}{(2\Pi_3 - \kappa_y)} + \lambda_j p^x_{ij} \end{aligned},$$

and the above term is positive as $2\Pi_1 - 4\Pi_3 - \kappa_y > 0$ by assumption 1 and 2. Hence $V_i^{x\to y}$ is maximized when $p^x_{ib} = \kappa_x$ and $p^y_{ib} > \kappa_y$ for $b \in \{j,k\}$.

It is easy to apply the same logic for the rest of price structures to find out optimal pairs of prices. If the innovator wants a component to be adopted by the imitators in the current state, she charges a price equal to its imitation cost. Otherwise, a price for that component is higher than its imitation cost.

Step 2: Candidates for symmetric MPE and corresponding maximized expected payoffs of the innovator are as follows. If $p^x_{ib} = \kappa_x$ and $p^y_{ib} > \kappa_y$ in the beginning (inducing an adoption sequence of x first and y later, $x \to y$),

$$V_i^{x\to y} = \mu^x \Pi_1 + (1-\mu^x)\left\{\kappa_x + \mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3 + \kappa_y)\right\}$$

where $\mu^x = r/(r+2\lambda^x)$, $\lambda^x = r(\Pi_3 - \kappa)/\kappa_x$, $\mu^{y|x} = r/(r+2\lambda^{y|x})$ and $\lambda^{y|x} = r(\Pi_3 - \kappa_y)/\kappa_y$. If $p^x_{ib} > \kappa_x$ and $p^y_{ib} = \kappa_y$, (inducing an adoption sequence of y first and x later, $y \to x$),

$$V_i^{y\to x} = \mu^y \Pi_1 + (1-\mu^y)\left\{\kappa_y + \mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3 + \kappa_x)\right\}$$

where $\mu^y = r/(r+2\lambda^y)$, $\lambda^y = r(\Pi_3 - \kappa)/\kappa_y$, $\mu^{x|y} = r/(r+2\lambda^{x|y})$ and $\lambda^{x|y} = r(\Pi_3 - \kappa_x)/\kappa_x$. If $p^x_{ib} = \kappa_x$ and $p^y_{ib} = \kappa_y$, (inducing simultaneous adoption for both components, xy),

$$V_i^{xy} = \mu^{xy}\Pi_1 + (1-\mu^{xy})(\Pi_3 + \kappa)$$

where $\mu^{xy} = r/(r+2\lambda^{xy})$ and $\lambda^{xy} = r(\Pi_3 - \kappa)/\kappa$. One can verify that an imitator's payoff is the same in all cases as $V_b^{x\to y} = V_b^{y\to x} = V_b^{xy} = \Pi_3 - \kappa$.[22]

Step 3: Now we show that in the monopoly game with respect to both components, it is optimal for the inventor to induce a simultaneous adoption by setting $p_{ib}^{x*} = \kappa_x$ and $p_{ib}^{y*} = \kappa_y$ since $V_i^{xy} > V_i^{y\to x} \geq V_i^{x\to y}$.[23] (The last inequality is strict if and only if $\kappa_x < \kappa_y$.)

First, let us show $V_i^{xy} > V_i^{y\to x}$. If we take $V_i^{y\to x}$ from V_i^{xy}, we get

$$V_i^{xy} - V_i^{y\to x} = \left\{\mu^{xy} - \mu^y - (1-\mu^y)\mu^{x|y}\right\}(\Pi_1 - \Pi_3 - \kappa_x) - (\mu^{xy} - \mu^y)\kappa_y.$$

By replacing μ^{xy}, μ^y and $\mu^{x|y}$ with the expressions in step 2, we have

$$\begin{aligned}
V_i^{xy} - V_i^{y\to x} &= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)}(\Pi_1-\Pi_3-\kappa_x) \\
&\quad -\frac{(2\Pi_3-\kappa-\kappa_x)\kappa-(2\Pi_3-\kappa)\kappa_y}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa)}\kappa_y \\
&= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)(\Pi_1-\Pi_3-\kappa_x)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} - \frac{2(\Pi_3-\kappa)\kappa_x\kappa_y}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa)} \\
&= \frac{2\kappa_x\kappa_y\left[(\Pi_3-\kappa)(\Pi_1-\Pi_3-\kappa_x)-(\Pi_3-\kappa)(2\Pi_3-\kappa_x)\right]}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} \\
&= \frac{2\kappa_x\kappa_y(\Pi_3-\kappa)(\Pi_1-3\Pi_3)}{(2\Pi_3-\kappa-\kappa_x)(2\Pi_3-\kappa_x)(2\Pi_3-\kappa)} > 0
\end{aligned}$$

since all terms in the numerator and the denominator are positive by assumption 1 and 2.

---

[22] In order to derive this, plug in corresponding hazard rates into the following three: $V_b^{x\to y} = \lambda^x\{(\Pi_3-\kappa)+(\Pi_3-\kappa_y)\}/(r+2\lambda^x)$, $V_b^{y\to x} = \lambda^y\{(\Pi_3-\kappa)+(\Pi_3-\kappa_x)\}/(r+2\lambda^y)$ and $V_b^{xy} = \lambda^{xy}\{(\Pi_3-\kappa)+\Pi_3\}/(r+2\lambda^{xy})$.

[23] In fact, when $p_{ib}^x = \kappa_x$ and $p_{ib}^y = \kappa_y$, not only the sequence *xy* but any other sequences can take place because buyers are indifferent among the following actions: *cc*, *cw*, *wc* and *ww*. If *cw* or *wc* is included in an optimal behavioral strategy of *b* given $p_{ib}^x = \kappa_x$ and $p_{ib}^y = \kappa_y$ with strictly positive probability, *i* would lower prices for each component by ε in order for *b* to mix only adopting or waiting for both and thereby to increase her expected payoff. (p_{ib}^x and p_{ib}^y approach to κ_x and κ_y from below, but cannot hit that point.) With that said, if we do not specify optimal behavior of the imitators in this case, a price equilibrium may not exist. To guarantee the existence of equilibrium, we assume that the imitators behave in such a way that the inventor's expected payoff is maximized when they are indifferent among several actions. Another simple justification, which yields a similar consequence, would be that the inventor can sell knowledge components through a bundling.

Next, to show $V_i^{y\to x} \geq V_i^{x\to y}$, write the difference between the two as follows:

$$\begin{aligned} V_i^{y\to x} - V_i^{x\to y} &= \left\{(1-\mu^x)(1-\mu^{y|x}) - (1-\mu^y)(1-\mu^{x|y})\right\}(\Pi_1 - \Pi_3) \\ &\quad +(1-\mu^y)\left\{\kappa_y + (1-\mu^{x|y})\kappa_x\right\} - (1-\mu^x)\left\{\kappa_x + (1-\mu^{y|x})\kappa_y\right\}. \end{aligned}$$

Substitution of μ^x, $\mu^{y|x}$, μ^y and $\mu^{x|y}$ given in step 2 into the above equation gives, after a similar tedious algebra as shown above,

$$V_i^{y\to x} - V_i^{x\to y} = \frac{4\kappa_x\kappa_y(\Pi_1 - 3\Pi_3)(\Pi_3 - \kappa)(\kappa_y - \kappa_x)}{\{2(\Pi_3-\kappa)+\kappa_x\}\{2(\Pi_3-\kappa)+\kappa_y\}(2\Pi_3-\kappa_x)(2\Pi_3-\kappa_y)},$$

which is 0 if $\kappa_x = \kappa_y$ and strictly positive otherwise.

Step 1 through 3 completes the proof since we only consider symmetric MPE. However, one might wonder whether there exists an asymmetric sequence generating higher payoff than V_i^{xy}. Step 4 demonstrates that when the inventor induces asymmetric sequences, the inventor's payoff is a weighted average of some two of the three expected payoffs: V_i^{xy}, $V_i^{y\to x}$ and $V_i^{x\to y}$ implying it is never optimal for her to do so.

Step 4: Here, I only offer a proof for the case in which the inventor induces j to choose $x \to y$ ($p_{ij}^x = \kappa_x$, $p_{ij}^y > \kappa_y$) and k to choose $y \to x$ ($p_{ik}^x > \kappa_x$, $p_{ik}^y = \kappa_y$), which gives enough description on how to prove the remaining cases. Let $V_i^{x\to y/y\to x}$ denote the expected payoff of the investor gets from this pricing strategy. Then,

$$\begin{aligned} V_i^{x\to y/y\to x} &= \frac{r}{r+\lambda^x+\lambda^y}\Pi_1 + \frac{\lambda^x}{r+\lambda^x+\lambda^y}\left\{\kappa_x + (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3+\kappa_y)\right\} \\ &\quad + \frac{\lambda^y}{r+\lambda^x+\lambda^y}\left\{\kappa_y + (1-\mu^{x|y})\Pi_1 + \mu^{x|y}(\Pi_3+\kappa_x)\right\}. \end{aligned}$$

Rearranging terms and writing it again in terms of $V_i^{x\to y}$ and $V_i^{y\to x}$ completes the proof.

$$\begin{aligned} V_i^{x\to y/y\to x} &= \frac{1}{2(r+\lambda^x+\lambda^y)} \times \Big[r\Pi_1 + 2\lambda^x\left\{\kappa_x + (1-\mu^{y|x})\Pi_1 + \mu^{y|x}(\Pi_3+\kappa_y)\right\} \\ &\quad + r\Pi_1 + 2\lambda^y\left\{\kappa_y + (1-\mu^{x|y})\Pi_1 + \mu^{x|y}(\Pi_3+\kappa_x)\right\}\Big] \\ &= \frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)}V_i^{x\to y} + \frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)}V_i^{y\to x} \end{aligned}$$

where $\frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)}$ and $\frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)} \in (0,1)$, and $\frac{r+2\lambda^x}{2(r+\lambda^x+\lambda^y)} + \frac{r+2\lambda^y}{2(r+\lambda^x+\lambda^y)} = 1$. ■

Proof of Corollary 1

If the inventor patents x and keeps y secret, from Proposition 2,

$$V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{y|x}\Pi_1 + (1-\mu^{y|x})(\Pi_3+\kappa_y)\right\} - f.$$

On the contrary to this, if the inventor chooses the opposite mix use,

$$V_i = (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{x|y}\Pi_1 + (1-\mu^{x|y})(\Pi_3+\kappa_x)\right\} - f.$$

If we subtract the second expression from the first, we have

$$\begin{aligned} & e^{-rT}\left\{(\mu^{y|x}-\mu^{x|y})(\Pi_1-\Pi_3) + (1-\mu^{y|x})\kappa_y - (1-\mu^{x|y})\kappa_x\right\} \\ = \quad & \frac{2\left\{(\Pi_1+\Pi_3-2\kappa)\Pi_3+\kappa_x\kappa_y\right\}(\kappa_y-\kappa_x)}{e^{rT}(2\Pi_3-\kappa_x)(2\Pi_3-\kappa_y)} > 0 \end{aligned}$$

by assumption 1 and 2. ■

Proof of Proposition 3

Result (i): In the text, we have already shown that protecting all by patent is dominated by the optimal mix use of a patent and secrecy. Thus, we only need to show the existence of $\overline{T}^{Com}$. Note that $V_i^M|_{T=0} < V_i^S$, $\lim_{T\to\infty} V_i^M > V_i^S$ by assumption 3 and V_i^M is strictly increasing in T. Therefore, there exists a unique $\overline{T}^{Com} \in (0,\infty)$ which solves $V_i^M = V_i^S$. More specifically,

$$\overline{T}^{Com} = \frac{1}{r}\ln\frac{(1-\mu^{y|x})(\Pi_1-\Pi_3-\kappa_y)}{(1-\mu^{xy})(\Pi_1-\Pi_3-\kappa)-f}.$$

Result (ii): Given $\overline{T}^{Com}$, note that as κ_y increases and κ_x decreases by the same amount, $\mu^{y|x}$ increases. Thus, the numerator of the log on RHS decreases while all other things remain constant, which causes $\overline{T}^{Com}$ to decrease. ■

Proof of Proposition 4

For the simple innovation, the innovator's optimal payoff is $\max\{V_i^P, V_i^S\}$. Since $V_i^P|_{T=0} < V_i^S$, $\lim_{T\to\infty} V_i^P > V_i^S$ by assumption 3 and V_i^P is strictly increasing in T, there exists a unique $\overline{T}^{Sim} \in$

$(0,\infty)$ which solves $V_i^P = V_i^S$, and

$$\overline{T}^{Sim} = \frac{1}{r} \ln \frac{\Pi_1 - \Pi_3}{(1-\mu^{xy})(\Pi_1 - \Pi_3 - \kappa) - f}.$$

Hence, it follows that the optimal protection mode is patenting if $T < \overline{T}^{Sim}$ and relying on trade secrets otherwise. Taking away $\overline{T}^{Com}$, which was obtained in Proposition 3, from $\overline{T}^{Sim}$, we have

$$\overline{T}^{Sim} - \overline{T}^{Com} = \frac{1}{r} \ln \frac{\Pi_1 - \Pi_3}{(1-\mu^{y|x})(\Pi_1 - \Pi_3 - \kappa_y)}.$$

Then, since $\Pi_1 - \Pi_3 > (1-\mu^{y|x})(\Pi_1 - \Pi_3 - \kappa_y)$, it follows $\overline{T}^{Sim} > \overline{T}^{Com}$. ■

Proof of Lemma 4

Let us start from the case $N = 3$. Once we rule out the price structure in which the inventor charges higher than imitations costs for all components by the same logic in Lemma 3, equilibrium candidate price structures fall into one of the following three categories: trying to sell (1) one component (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} > \kappa_2$ and $p_{ib}^{3t} > \kappa_3$), (2) two components (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} \leq \kappa_2$ and $p_{ib}^{3t} > \kappa_3$) and (3) all three components (for example, $p_{ib}^{1t} \leq \kappa_1$, $p_{ib}^{2t} \leq \kappa_2$ and $p_{ib}^{3t} \leq \kappa_3$) for $b \in \{j,k\}$ given that no adoption has occurred. Note that (1) is analogous to the 2 component model because once an imitator adopts a component, they enter a subgame in which the remaining two components are sold in the all-or-nothing adoption fashion as in Proposition 2. Since we showed that in Proposition 2 that the step-by-step adoption sequence in the two component model is less profitable than the all-or-nothing adoption sequence, the inventor prefers price structure (3) to (1). Also, with a very similar reason, (3) will be chosen over (2).

Applying this logic recursively, it is straightforward to see that for all $N > 3$, the inventor will sell knowledge in all-or-nothing fashion by setting $p_{ib}^{nt} = \kappa_n$ for all $n \in \{1,\ldots,N\}$ and $b \in \{j,k\}$. Hence, slight modification of Proposition 2 allows us to have Lemma 4 as stated in the body. ■

Proof of Lemma 5

Define components kept by secrecy in a patent-secret mix as N_S. Then, N_S should be an element in $2^{\{1,\ldots,N\}}$, where $2^{\{1,\ldots,N\}}$ is a power set of $\{1,\ldots,N\}$, excluding two extreme protection modes: patenting all ($N_S = \emptyset$) and keeping all-by-secrecy ($N_S = \{1,\ldots,N\}$). We know that if all components in N_S are not adopted by the imitators until T, the inventor's discounted expected payoff starting from T is given by

$$\mu^{N_S}\Pi_1 + (1-\mu^{N_S})(\Pi_3 + \kappa_{N_S})$$

where $\kappa_{N_S} = \sum_{n\in N_S}\kappa_n$, $\mu^{N_S} = 1/(r+2\lambda^{N_S})$ and where and $\lambda^{N_S} = r(\Pi_3 - \kappa_{N_S})/\kappa_{N_S}$ from the Lemma 4. Also, note that it is optimal for the inventor to choose $p_{ib}^{nt} > e^{-r(T-t)}\kappa_n$ for all $n \in N_S$ by the same logic in Proposition 2. Then, the optimal patent-secret mix is a solution to the following problem:

$$\max_{N_S\in 2^{\{1,\ldots,N\}}\setminus\{\{1,\ldots,N\},\emptyset\}} (1-e^{-rT})\Pi_1 + e^{-rT}\left\{\mu^{N_S}\Pi_1 + (1-\mu^{N_S})(\Pi_3+\kappa_{N_S})\right\} - f.$$

It is easy to see that the solution N_S^* maximizes κ_{N_S}, and so $N_S^* = \{2,\ldots,N\}$, i.e., it is optimal patenting only component 1 least costly to imitate and keeping all the other components as trade secrets. ■

Proof of Proposition 6

As the proof for the point (i) is exactly the same as the one in Proposition 3, here only the proof for the point (ii) is provided. When $\kappa_1 = \kappa_2 = \ldots = \kappa_N = \kappa/N$,

$$\overline{T}^{Com} = \frac{1}{r}\ln\frac{(1-\mu^{-1})(\Pi_1 - \Pi_3 - (1-\frac{1}{N})\kappa)}{(1-\mu)(\Pi_1-\Pi_3-\kappa)-f}.$$

Only the numerator in the log is a function of N. Since μ^{-1} increases in N, $\overline{T}^{Com}$ gets shorter as N increases. ■

Proof of Proposition 7

We will first show (α^*, ρ^*) is in the interior of $(0,1]\times[0,1/2]$, which implies each area is not an empty set, and then investigate equilibrium protection mode in each area in turn.

Step 1: We show that (α^*, ρ^*) is in the interior of $[0,1]\times[1/2,1]$. Since $\alpha^* = (V_i^S - \Pi_3)/(\Pi_1 - \Pi_3) \in (0,1)$ is obvious, it is sufficient to show that $\rho^* = \tilde{\rho}(\alpha^*) = \hat{\rho}(\alpha^*) = \frac{(V_i^S - V^1)(\Pi_1 - \Pi_3)}{(V_i^S - \Pi_3)(\Pi_1 - V^1)} \in (1/2, 1)$. First, $\rho^* < 1$ since $(V_i^S - \Pi_3)(\Pi_1 - V^1) - (V_i^S - V^1)(\Pi_1 - \Pi_3) = (\Pi_1 - V_i^S)(V^1 - \Pi_3) > 0$. Next, observe $\rho^* > 1/2$ is equivalent to

$$2(V_i^S - V^1)(\Pi_1 - \Pi_3) - (V_i^S - \Pi_3)(\Pi_1 - V^1) \quad > \quad 0.$$

After some algebra, one can easily verify that the above inequality is equivalent to the following condition:

$$(\Pi_1 - \Pi_3)(V_i^S - V^1) - (\Pi_1 - V_i^S)(V^1 - \Pi_3) > 0.$$

Since $\Pi_1 - \Pi_3 > \Pi_1 - V_i^S$, it remains to show $V_i^S - V^1 > V^1 - \Pi_3$. Now to conclude, note that

$$\begin{aligned} V_i^S - V^1 - (V^1 - \Pi_3) &= \\ (\mu^2 - 2\mu^1)(\Pi_1 - \Pi_3) - (\mu^2 - \mu^1)\kappa &= \\ \frac{(\Pi_1 - 3\Pi_3)\kappa^2}{(4\Pi_3 - \kappa)(2\Pi_3 - \kappa)} &> 0. \end{aligned}$$

where the second equality follows from replacing μ^2 and μ^1 with the expressions given in the body and rearranging terms.

Step 2: Patterns of optimal protection in each area are as follows.

Area (i): For all T, $V_i^S > V_i^P$ and $V_i^S > V_i^M$ given $\alpha \le \alpha^*$ and $\rho < \hat{\rho}(\alpha)$. Hence, S is trivially optimal for the inventor for all T.

Area (iv): Given $\tilde{\rho}(\alpha) \le \rho \le 1$, $V_i^M > V_i^P$ for all T given α, which implies P is never optimal. Also, since we consider the area $\hat{\rho}(\alpha) < \rho \le 1$ where $V_i^S > V_i^M|_{T=0}$ but $V_i^S < \lim_{T\to\infty} V_i^M$ given α, there exists a threshold $\overline{T}^M$ such that $V_i^S = V_i^M|_{T=\overline{T}^M}$ by the Intermediate Value Theorem. Moreover, since V_i^M is monotonically increasing in T the threshold is unique. Thus, the optimal protection mode is S for $T \le \overline{T}^M$ and M for $T > \overline{T}^M$.

Area (ii) and (iii): First, consider the following subarea where $\rho < \hat{\rho}(\alpha)$ and $\alpha > \alpha^*$. Here, applying a similar logic used in a proof for area (iv), we can show that the optimal protection mode is S for $T \leq \overline{T}^P$ and M for $T > \overline{T}^P$ where $\overline{T}^P$ is such that $.V_i^S = V_i^P|_{T=\overline{T}^P}$. Next, consider the rest of the area. Here, for any (α, ρ) $\lim_{T\to\infty} V_i^P > \lim_{T\to\infty} V_i^M > V_i^S$, and recall that $V_i^S > V_i^M|_{T=0} > V_i^P|_{T=0}$. Therefore, by continuity of payoff functions we know that S is optimal when T is small and P is optimal when T is large. One remaining question is whether there are some intermediate T values under which M is optimal. The previous statement is true if and only if $\overline{T}^P > \overline{T}^M$ where $\overline{T}^P$ and $\overline{T}^M$ are defined as above. That is because in that case as T increases V_i^M surpasses V_i^S first while V_i^P is still less than V_i^S even if it is overtaken by V_i^P eventually. To see $\overline{T}^P > \overline{T}^M$ is equivalent to $\rho > \rho^*$, observe the following:

$$
\begin{aligned}
& \overline{T}^P > \overline{T}^M \\
\iff & \frac{1}{r}\ln\frac{\alpha(\Pi_1-\Pi_3)}{\alpha(\Pi_1-\Pi_3)-(V_i^S-\Pi_3)} > \frac{1}{r}\ln\frac{\rho\alpha(\Pi_1-V^1)}{\rho\alpha(\Pi_1-V^1)-(V_i^S-V^1)} \\
\iff & \frac{\alpha(\Pi_1-\Pi_3)-(V_i^S-\Pi_3)}{\alpha(\Pi_1-\Pi_3)} < \frac{\rho\alpha(\Pi_1-V^1)-(V_i^S-V^1)}{\rho\alpha(\Pi_1-V^1)} \\
\iff & \rho > \frac{(V_i^S-V^1)(\Pi_1-\Pi_3)}{(V_i^S-\Pi_3)(\Pi_1-V^1)} = \rho^*
\end{aligned}.
$$

In sum, in area (iii) the optimal protection mode is S if T is small, M if T is intermediate and P otherwise whereas S if T is small and P otherwise in area (ii). ■

CHAPTER 2

WELFARE EFFECTS OF CERTIFICATION UNDER LATENT ADVERSE SELECTION (WITH ANTHONY CREANE AND THOMAS D. JEITSCHKO)

2.1 Introduction

Asymmetric information is a classic example of market failure that undermines the efficiency associated with perfectly competitive market outcomes—the "lemons" market (Akerlof, 1970). This issue has recently regained more detailed scrutiny with the rise of "organic," "fair trade," "local," "sustainable" etc. products. In markets for such products, the concept of product quality in consumer preferences has extended to include the process of production and distribution. When such properties are claimed by firms, along with specific corporate philosophies or policies, it is very hard or impossible for consumers to ascertain the quality or the process of production of those products even after their consumption experience in many cases. Due to the aforementioned credence good nature,[1] quality assurance mechanisms, such as offering warranties (e.g., see Spence, 1977) and building up a reputation (e.g., see Klein and Leffler, 1981; and Shapiro, 1983), are not expected to function well to address the information problem in those markets.[2] Accordingly, credible (third-party) certification is often considered the only potential solution for overcoming informational asymmetries. In addition, while "quality" in the traditional, lemon's sense (vertical differentiation) may be hard to measure, consumers' valuation of the production process, as in the examples above, makes certification more easily quantifiable. Although the degree to which such claims are backed up by formal certification or regulated by law varies, various certification schemes are in use in many marketplaces.

In spite of the seeming attractiveness of credible certification, it is still important to substantiate whether introducing certification increases or decreases social welfare through a formal analysis,

[1] For more detailed definition and discussion of credence goods see, for example, Darby and Karni (1973) and Dulleck and Kerschbamer (2006).

[2] See Bonroy and Constantatos (2015) and Dranove and Jin (2010) for detailed discussion on this point.

especially considering its popularity. To this end, in this paper we consider markets in which goods have unobservable characteristics that consumers value. We analyze the market equilibrium when a technology is available that credibly verifies the relevant attributes of products and contrast this with the equilibrium without certification. We assume that the certification technology could be costly and imperfect. One effect the certification technology has is to potentially create two markets (market segmentation)—one with certified products and the other without—while without certification there is only one pooled market. As a result, one has to consider both the firm's decision as to whether to seek certification as well as the consumer's choice as to which market to patronize.

We find that certification may be welfare-reducing when the certification technology is costly or imperfect. This implies that even when the certification technology is perfectly accurate—and therefore is able to resolve informational asymmetries in the market place—overall welfare may decrease due to the possibility of certification when such certification is costly. Similarly, even when the certification technology is costless, welfare may decrease when such certification is quite imperfect.

The underlying reason for this is that while the certification process admits better information and therefore increases efficiency by reallocating high quality goods to consumers with relatively higher valuations, it also brings a negative impact on the average quality of non-certified goods. The latter effect can push some high quality producers out of the non-certified market or the entire group (a market collapse). In contrast, absent any certification, the increased presence of products with desirable characteristics provide a sufficiently strong positive externality to sustain an equilibrium that entails a larger number of high-quality products. When the certification technology is relatively costly or imperfect, only a small portion of high type producers are present in in the certified market in equilibrium, and this may still cause the non-certified market to collapse, which results in a quite low welfare level. Most of these results are not a result from the direct welfare cost of certification, but rather from certification's effect on the market(s).

2.2 Related Literature

This paper contributes to the literature on certification (or labeling), or more broadly, quality disclosure. There are two streams of the certification literature. The first strand focuses on the role of strategic certification intermediaries. Some studies aim to investigate how competition between certifiers affects market outcomes including optimal pricing schemes, the amount of information transmitted to consumers or quality provision (e.g., Albano and Lizzeri, 2001; Fischer and Lyon, 2014; Hvide, 2009; and Lizzeri, 1999). Some other papers (e.g., Benabou and Laroque, 1992) explore whether a reputation concern can mitigate the incentive problems of certification intermediaries. Some of these papers have something in common with ours in that they show a potential source of inefficiency associated with certification. However, while they emphasize the role of strategic certifiers as a source of inefficiency, we are more interested in sellers' incentives for getting certified and their incidence on markets taking certification environments as given.

The other strand of this literature focuses on a seller's incentive for quality disclosure. Since the "unraveling result"[3] was presented by Grossman (1981) and Milgrom (1981), many subsequent studies have explored the subject of when and why unraveling fails to hold (e.g., see De and Nabar, 1991; Guo and Zhao, 2009; Grubb, 2011; Hotz and Xiao, 2013; Shavell, 1994; and Viscusi, 1978). Closely related with such work, several authors also studied whether mandatory disclosure laws enhance or hinder efficiency when voluntary disclosure fails to occur (e.g., see Bar-Isaac et al., 2012; Board, 2009; Gavazza and Lizzeri, 2007; Harbaugh et al., 2011; Jovanovic, 1982; and Matthews and Postlewaite, 1985). Though these papers share several features with us, the most important difference in ours is that we compare the situations with and without certification, rather than comparing a situation in which sellers voluntarily choose to get certified with another situation in which sellers must get certified to sell.[4]

[3]The unraveling result refers to the situation in which each seller voluntarily discloses her quality to consumers for differentiating herself from other sellers with a lower quality if quality disclosure is credible and costless.

[4]For more examples, see cited papers in Dranove and Jin (2010).

There are only a relatively small number of studies that focus on welfare effects of certification. The authors of these studies point that welfare may decrease with the availability of certification mostly in the context of Eco-labeling, as we do in this paper. Among these studies, Baltzer (2012), Bonroy and Constantatos (2008) and Zago and Pick (2004) differ from ours in that the underlying mechanisms that derive welfare decreasing results in their work are other than exacerbated adverse selection problem by certification. The most closely related papers to this one are Mason and Sterbenz, 1994 and Mason, 2011. Taking into account sellers' incentive for opting for certification, the above two papers show that certification can aggravate adverse selection, and thus certification maybe welfare reducing. However, the main drivers of the exacerbated adverse selection problem in their approaches and our approach are different: unlike they are more focused on the mimicking incentives of low type sellers, we are more interested when certification causes the collapse of the non-certified market. More importantly, we assume a downward sloping demand curve in order to seriously consider allocative efficiency, associated with certification, which is an important factor when examining welfare effects of certification.

The remainder of this paper proceeds as follows. Section 2 describes the base model. To explain the value of information, in section 3, we compare two benchmarks: full information and no information case. We derive a certification equilibrium and conduct a welfare analysis in Section 4. Finally, Section 5 concludes. All omitted proofs are in the appendix.

2.3 The Base Model

We consider markets in which there is asymmetric information between consumers and firms about the product quality.

2.3.1 Consumers

In order to introduce certification (which implies a segmentation of the market and hence sorting of demand according to distinct quality/grades) later in the model, a richer modeling structure for demand is needed. Following Mussa and Rosen (1978), we adopt the vertical differentiation model of gradations g with a continuum of consumer types each of whom has a unit demand. Consumers are of total mass 1 and distributed on $[\underline{\theta}, \bar{\theta})$ according to the strictly increasing cdf, $F(\theta)$. Then, a type-θ consumer's net utility, when paying price p, is given by

$$U(g, p|\theta) = \theta g - p,$$

while a consumer gets 0 when buying nothing. All consumers are price takers.

2.3.2 Firms

On the supply side suppose there is a mass (measure) n of firms which are also all price takers. A firm produces (or serves) either high ($\bar{g}$) or low ($\underline{g}$) quality product (or service). Since we do not focus on a moral hazard problem but on an adverse selection problem, we assume these quality levels are exogenouly given. A proportion of high grade producers is indexed by $\gamma_0 \in (0,1)$, and $\gamma_0 n$ firms can produce a unit of high quality at cost of $\bar{c}$. The rest of producers, $(1-\gamma_0)n$, can produce a unit of low quality at cost of $\underline{c}$. Each seller knows the true quality of her own product, but consumers do not. In order to make the analysis of the lower grade market simpler when a segmentation of the market arises (under full information or with certification), let us normalize $\underline{g} = \underline{c} = 0$.

2.3.3 Two Benchmarks

In the rest of this section, we study the equilibrium configurations when there is no certification technology (No Information/No Certification) and when the quality of product is observable to consumers (Full Information) in turn as two benchmark cases before delving into equilibrium configurations with certification. By comparing welfare under the two benchmarks, we will explain why certification can potentially be valuable to the society.

2.3.3.1 No Information (No Certification) Equilibrium

Without any quality certification the market is subject to the law of one price, therefore only overall market demand for different compositions of quality on offer is required. Consider the supply side first. No high quality goods are offered if the price is below their production costs. More specifically,

$$q^S(p) = \begin{cases} \kappa(1-\gamma_0)n,\ \kappa \in [0,1] & \text{if } p = 0, \\ (1-\gamma_0)n & \text{if } p \in (0, \bar{c}), \\ (1-\gamma_0+\kappa\gamma_0)n,\ \kappa \in [0,1] & \text{if } p = \bar{c}, \\ n & \text{if } p > \bar{c}. \end{cases} \tag{2.1}$$

Since consumers rationally anticipate the supply schedules, the demand structure is given by

$$q^D(p) = \begin{cases} 1 - F\left(p/\gamma\bar{g}\right) & \text{if } p \geq \bar{c}, \\ 0 & \text{if } p < \bar{c}, \end{cases}$$

in which γ is beliefs of consumers, and so $\gamma\bar{g}$ represents ex-ante average quality in the market. To make the analysis non-trivial, we take the following assumption.

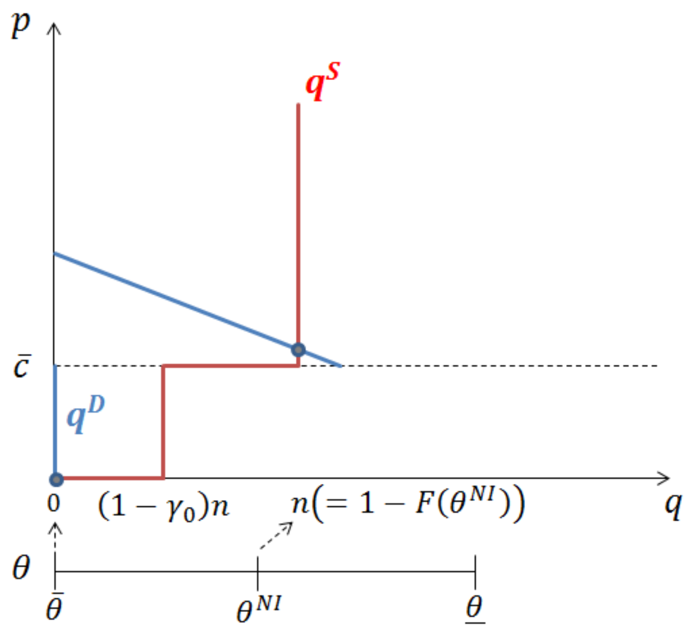
Assumption 1: $0 < \bar{c} < \gamma_0\bar{g}\bar{\theta}$

Assumption 1 specifies cost structure under which adverse selection need not necessarily happen. The last inequality implies that at least some trade of high quality given the prior belief is efficient. The potential for adverse selection comes from the first inequality, which implies that all high producers are driven out if consumers believe that there are no high type in the market. This is because in that case even for the highest type the consumer valuation would be lower than $\bar{c}$.

Since we are interested in inquiring about if certification can decrease social welfare, we further assume that the number of firms is relatively small so that there is asymmetric information in the market but not necessarily an adverse selection problem.

Assumption 2: $n \leq 1 - F\left(\bar{c}/\gamma_0\bar{g}\right)$

Figure 2.1: No Information Equilibrium



Note: The demand curves need not necessarily be linear.

Under Assumption 2, in any equilibrium in which a positive quantity is traded (at an equilibrium price greater than $\bar{c}$), rational expectation implies that consumer beliefs must be the same as prior beliefs, i.e., $\gamma = \gamma_0$, and all n firms serve consumers.[5] As seen in Figure 1, although there exists a trivial one in which no transaction occurs ($p^{NI} = 0$ and $q^{NI} = 0$) as well in general, we select the equilibrium with the higher quantity and the higher level of welfare.

Thus, in equilibrium

$$q^{NI} = n; \qquad p^{NI} = \gamma_0 \bar{g} H(1-n)$$
$$\pi_H^{NI} = p^{NI} - \bar{c}; \qquad \pi_L^{NI} = p^{NI}$$

where $H(\tau) \equiv F^{-1}(\tau)$ a quantile function defined on $[0,1)$.[6]

[5]In principle, γ could differ from γ_0 if n is sufficiently large because there may be some high quality producers staying out of the market in equilibrium when the market price is equal to $\bar{c}$. See Creane and Jeitschko (2016) for more on this point.

[6]$q^{NI} = n$, or $\theta^{NI} = H(1-n)$.

2.3.3.2 Full Information Equilibrium

Under full information assumption, firms will be sorted into two distinct groups. And thus, we have to examine the two separate markets. In the low grade market (market L), it is optimal for low grade producers to produce as long as the market price (p_L) is greater than their production cost $\underline{c} = 0$. All low quality firms are indifferent between producing or not when $p_L = 0$. Therefore, the supply correspondence in market L is given by,

$$q_L^S(p_L|n) = \begin{cases} \kappa(1-\gamma_0)n,\ \kappa \in [0,1] & \text{if } p_L = 0, \\ (1-\gamma_0)n & \text{if } p_L > 0. \end{cases}$$

However, it is optimal for any type of consumer, θ, not to purchase in this market if p_L is positive because $U\left(\underline{g}, p_L|\theta\right) = -p_L < 0$ for all consumer types. This implies the demand correspondence in market L is given by,

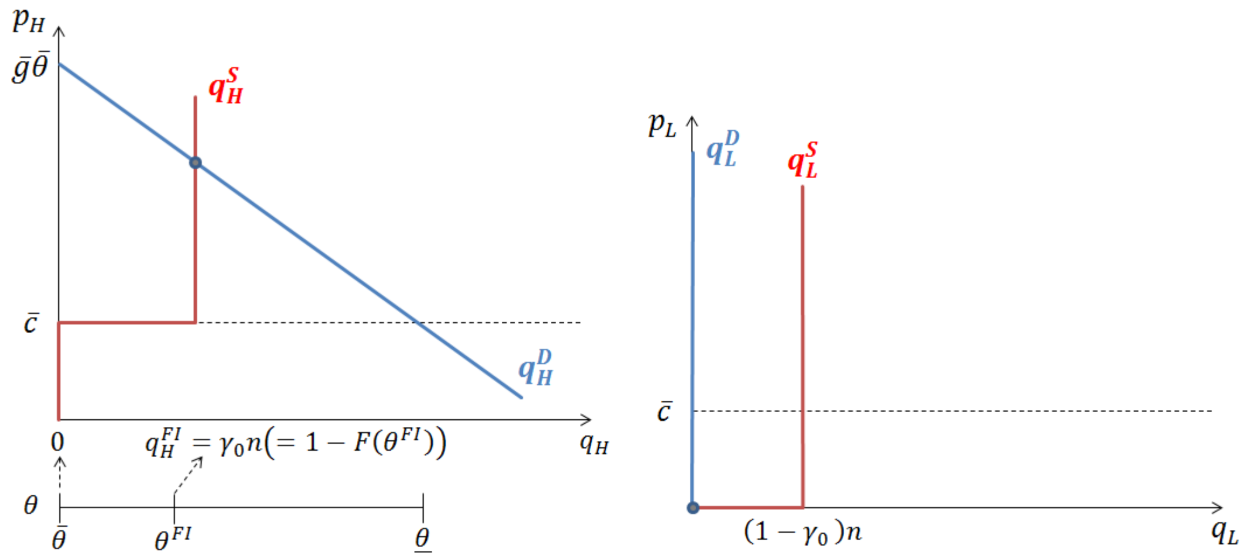
$$q_L^D(p_L|n) = \begin{cases} \in [0,1] & \text{if } p_L = 0, \\ 0 & \text{if } p_L > 0. \end{cases}$$

In principle, the equilibrium quantity can vary though the unique equilibrium price is 0 ($p_L = 0$). In all such situations, we assume there is no transaction by setting $q_L^{FI} = 0$ and $p_L^{FI} = 0$. Note that the normalization assumption in market L ($\underline{g} = \underline{c} = 0$) results in the normalization of surplus in that market as well. [7]

Now consider the high grade market (market H). The supply schedule can be derived in a

[7]This do not harm generality much for total welfare analysis. However, when it comes to consumer surplus, this simplifying assumption makes a difference. With the normalization assumption, the surplus of the marginal consumer in market H is 0. In contrast, if we assume $\underline{g}\bar{\theta} > \underline{c}$, so that if some lower quality products can be served in an equilibrium, the marginal consumer of the higher grade market should get some surplus since now she has an outside option of buying from the lower grade market. Therefore, consumer surplus from the high grade market may decrease with the normalization in market L.

Figure 2.2: Full Information Equilibrium



(a) High Grade Goods Market

(b) Low Grade Goods Market

Note: The demand functions need not necessarily be linear.

similar way as we did for market L:

$$q_H^S(p_H|n) = \begin{cases} 0 & \text{if } p_H < \bar{c}, \\ \kappa\gamma_0 n,\ \kappa \in [0,1] & \text{if } p_H = \bar{c}, \\ \gamma_0 n & \text{if } p_H > \bar{c}. \end{cases}$$

In order to derive demand, note that a type-θ consumer would purchase given p_H if and only if

$$U(\bar{g}, p_H|\theta) = \bar{g}\theta - p_H \geq 0.$$

If we define the marginal consumer who is indifferent between buying or not as $\theta_H^{FI} \equiv p_H/\bar{g}$, consumers with $\theta \geq \theta_H^{FI}$ buy high grade goods, while the remainder ($\theta < \theta_H^{FI}$) do not. The demand schedule is thus given by

$$q_H^D(p_H) = \begin{cases} 1 - F(p_H/\bar{g}) & \text{if } p_H \in [0, \bar{g}\bar{\theta}], \\ 0 & \text{if } p_H > \bar{g}\bar{\theta}. \end{cases}$$

Assumption 2 implies that under full information all high types serve in the market,[8] and thus the equilibrium outcome can be summarized as follows,

$$\begin{aligned} q_H^{FI} &= \gamma_0 n; & q_L^{FI} &= 0; \\ p_H^{FI} &= \bar{g}H(1-\gamma_0 n) > \bar{c}; & p_L^{FI} &= 0; \\ \pi_H^{FI} &= p_H^{FI} - \bar{c}; & \pi_L^{FI} &= 0, \end{aligned}$$

where π_H^{FI} and π_L^{FI} stand for equilibrium profits of high and low quality firms under full information respectively.[9]

2.3.3.3 Welfare Comparison

Having the equilibrium configurations in the two benchmarks, we show that information increases social welfare. We take social welfare as the sum of firm profits and consumer surplus. Since price paid by consumers is just transferred to firms, total welfare under no information and full information can be written as gross consumer benefits net of production costs:

$$\begin{aligned} W^{NI} &= \int_{\theta^{NI}}^{\bar{\theta}} \gamma_0 \bar{g} \theta dF(\theta) - \gamma_0 n \bar{c}, \\ W^{FI} &= \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g} \theta dF(\theta) - \gamma_0 n \bar{c}. \end{aligned}$$

Proposition 1 *(Information Increases Welfare) Welfare is strictly greater under full information than under no information regardless of a proportion of high grade producers, i.e.,*

$$W^{FI} > W^{NI}, \; \forall \gamma_0 \in (0,1).$$

[8]Mathematically, this is because Assumption 2 implies $n < 1 - F(\bar{c}/\gamma_0 \bar{g}) < (1/\gamma_0)[1 - F(\bar{c}/\bar{g})]$ and thus, $n \leq (1/\gamma_0)[1 - F(\bar{c}/\bar{g})] \iff p_H^{FI} = \bar{g}H(1-\gamma_0 n) \geq \bar{c}$.

[9]$q_H^{FI} = \gamma_0 n$, or $\theta^{FI} = H(1-\gamma_0 n)$.

Proof: Subtracting W^{NI} from W^{FI} and rewriting it shows a trade-off between the two situations as follows:

$$\begin{aligned} W^{FI} - W^{NI} &= \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g}\theta dF(\theta) - \int_{\theta^{NI}}^{\bar{\theta}} \gamma_0 \bar{g}\theta dF(\theta) \\ &= \underbrace{\left\{ \int_{\theta_H^{FI}}^{\bar{\theta}} \bar{g}\theta dF(\theta) - \int_{\theta_H^{FI}}^{\bar{\theta}} \gamma_0 \bar{g}\theta dF(\theta) \right\}}_{\substack{\text{Gains from reallocating} \\ \text{high quality to } \theta \geq \theta_H^{FI}}} - \underbrace{\int_{\theta^{NI}}^{\theta_H^{FI}} \gamma_0 \bar{g}\theta dF(\theta)}_{\substack{\text{Losses from not served} \\ \text{consumers } (\theta^{NI} < \theta < \theta_H^{FI})}} . \end{aligned}$$

Now we show that the above expression is always greater than 0. Note that $\int_{\theta_1}^{\theta_2} \theta dF(\theta) = \theta_2 F(\theta_2) - \theta_1 F(\theta_1) - \int_{\theta_1}^{\theta_2} F(\theta) d\theta = \int_{\tau_1}^{\tau_2} H(\tau) d\tau$ for any arbitrary $\theta_1 < \theta_2$ and corresponding $\tau_1 < \tau_2$. The first equality follows from integration by parts and the second from the definition of $H(\cdot)$. This implies,

$$W^{FI} - W^{NI} = \bar{g} \left[(1-\gamma_0) \int_{\tau_H^{FI}}^{1} H(\theta) d\theta - \gamma_0 \int_{\tau^{NI}}^{\tau_H^{FI}} H(\theta) d\theta \right],$$

where $\theta_H^{FI} = H(\tau_H^{FI})$ and $\theta^{NI} = H(\tau^{NI})$. Moreover, since $(\tau_2 - \tau_1)H(\tau_1) < \int_{\tau_1}^{\tau_2} H(\tau) d\tau < (\tau_2 - \tau_1)H(\tau_2)$ for any increasing $H(\cdot)$[10], we have

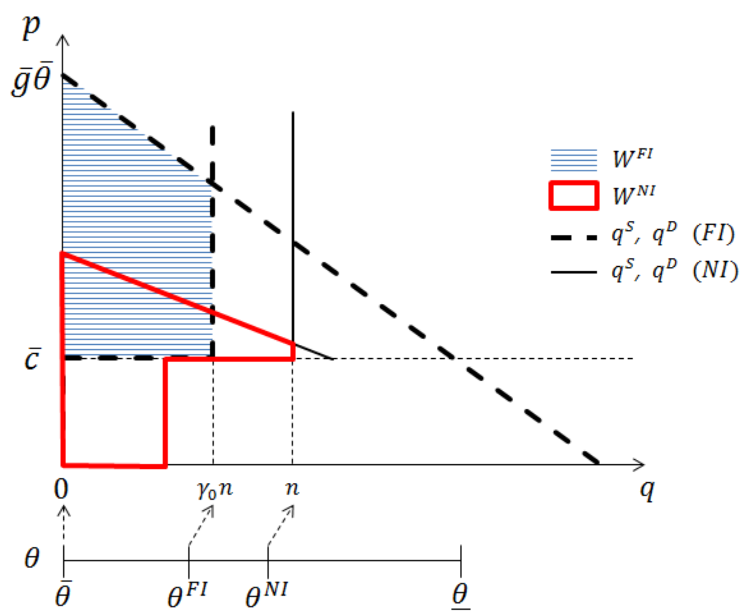
$$\begin{aligned} W^{FI} - W^{NI} &> \bar{g} \left[(1-\gamma_0)(1-\tau_H^{FI})\theta_H^{FI} - \gamma_0(\tau_H^{FI} - \tau^{NI})\theta_H^{FI} \right] \\ &= \bar{g} \left[(1-\gamma_0)\gamma_0 n \theta_H^{FI} - \gamma_0(1-\gamma_0) n \theta_H^{FI} \right] \\ &= 0. \end{aligned}$$

■

Proposition 1 states that restoring full information improves welfare. The intuition behind this result is straightforward. Even though the market is not subject to adverse selection under no information, the market still suffers from an allocation inefficiency associated with asymmetric information. This is because those with relatively higher valuations may obtain a low quality

[10]Since $F(\cdot)$ is strictly increasing, so is $H(\cdot)$.

Figure 2.3: Welfare Comparison between NI and FI Benchmarks



Note: The demand function needs not necessarily be linear.

product and vice versa. When we move from the no information case to the full information case, while the aggregate production costs remain the same, allocative efficiency is restored because now high quality goods are allocated only to the group of consumers with relatively higher valuations. To understand this point further, refer to Figure 3. The measure of $\gamma_0 n$ consumers above θ_H^{FI} gain at least $\bar{g}(1-\gamma_0)\theta_H^{FI}$ each from the reallocation of goods, while the measure of $(1-\gamma_0)n$ consumers between θ^{NI} and θ_H^{FI} lose at most $\gamma_0\bar{g}\theta_H^{FI}$. Hence, welfare under full information is always greater.

2.4 Certification Equilibrium

The previous section shows that information increases social welfare. One immediate implication of the result is that perfect and cost-less certification, which coincides with the full information benchmark, is welfare enhancing. However, what if the certification technology is costly or imperfect? Are there any cases in which welfare with certification is even lower than that under no information? In order to answer these questions, we derive a price equilibrium given certification decisions of firms and then characterize a certification equilibrium. The welfare analysis follows next and shows that certification may actually decrease social welfare.

For simplicity and tractability of the model, we employ further assumptions. Throughout this section, let us assume that consumers are uniformly distributed on the line segment $[0,1]$, i.e.,

$$F(\theta) = \theta$$

on $[0,1]$, and $H(\tau) = \tau$. Also, for simplicity we normalize $\bar{g} = 1$. To reduce the equilibrium constellations to consider, we also restrict our interests to the case in which Assumption 2 binds and denote such measure of producers by $\bar{n}$. With the uniform distribution assumption, note that $\bar{n} = 1 - \bar{c}/\gamma_0$.

2.4.1 Certification Environment

We consider a certification market in which all certifiers are homogeneous in their certification costs and accuracy of certification test. More specifically, each certifier needs to incur the same fixed cost of certification, $z \geq 0$. We further assume that the certification market is perfectly competitive so that the equilibrium price for certification is always equal to z.

Certification technology may be imperfect in the following sense. Let $\widetilde{g}$ denote the grade discovered by the certification test. Then,

$$Pr\{\widetilde{g} = \bar{g} | g = \bar{g}\} = x; \qquad Pr\left\{\widetilde{g} = \bar{g} | g = \underline{g}\right\} = 0;$$
$$Pr\left\{\widetilde{g} = \underline{g} | g = \bar{g}\right\} = 1 - x; \qquad Pr\left\{\widetilde{g} = \underline{g} | g = \underline{g}\right\} = 1,$$

where $0 < x \leq 1$. So, the test is informative in the sense that the test perfectly weeds low grades out and identifies a high type with some precision. Put differently, the certification test commits a type-I error with probability $1 - x$ but not a type-II error. However, x can be lower than $1/2$. Certification is imperfect unless $x = 1$.

If the test reveals that a seller is of high type ($\tilde{g} = \bar{g}$), she gets certified (Crt). Otherwise ($\tilde{g} = 0$), the seller can only sell in the pool of sellers who did not apply for certification and applied but failed to get certification (*NC*). Put differently, consumers can only condition on whether the good is certified or not certified. Finally, the sellers cannot shop around for certification, and so the first certification result sticks. This assumption precludes the possibility that the sellers use an imperfect and cost-less certification repeatedly in such a way that full information outcome may be replicated.

We consider the following two-period model when certification is an available option for sellers:

Period 1 (Certification Decision): In the first period each seller makes a decision whether to opt for certification test or not.

Period 2 (Walrasian Price Formation): In the second period, with there being asymmetric information partially resolved through the certification (given sellers' certification decisions and outcomes) process, trade takes place in which Walrasian mechanism forms prices in both markets: certified (p^{Crt}) and non-certified (p^{NC}) market. Depending on offered prices in each market, high and low grade producers decide whether to produce at production costs of $\bar{c}$ and 0 respectively (produce to demand) or to shut down, and consumers choose to buy in the *Crt* market or in the *NC* market or not to buy.

In fact, firms need to go through a certification process before producing in many cases. For example, to get the USDA Organic label, a farm must develop and implement an organic system plan first and then hire a USDA-accredited certifying agent to get inspected before producing. Note that in this case the certification cost, z, can include all expenses for adopting the organic system as well as an agency fee.[11]

2.4.2 Price Equilibrium

Given the assumptions on certification environment, there is no chance that the low type is identified as the high type by the test. Thus, without loss of generality, we can assume that the low

[11] See https://www.ams.usda.gov/services/organic-certification/becoming-certified (last retrieved on May 1, 2017).

type never seeks certification, and the price equilibrium constellations depend only on the high producers' certification decisions. Let α denote the proportion of high type sellers who choose to opt for certification. Then, there are three cases to consider: a pooling equilibrium ($\alpha = 0$), a semi-separating equilibrium ($\alpha \in (0,1)$) and a fully separating equilibrium ($\alpha = 1$). In a pooling equilibrium in which no high type seeks certification ($\alpha = 0$), the resulting outcomes coincide with the no information benchmarks, and there is nothing to be analyzed further. In a semi-separating equilibrium or a fully separating one, certification environment gives rise to the following two interesting features. First, third party certification results in market segmentation. Second, if a large portion of high grade firms congregate in one of the two markets, this may give rise to the collapse of the other market due to the intensified adverse selection even if there is no adverse selection problem in the beginning when all firms are pooled together in one market.

In order to derive a price equilibrium for each $\alpha \in (0,1]$, write down the supply schedules in each market which are similar to (2.1) with the modification of population of high and low grade producers. In the Crt market there are $\alpha x \gamma_0 \bar{n}$ measure of high producers deciding whether to produce or not. Similarly, in the NC market, the rest of producers, the pool of $(1-\alpha x)\gamma_0 \bar{n}$ measure of high types and $(1-\gamma_0)\bar{n}$ of low types, make the same decisions. And thus, supply schedules in the two markets are given by

$$q^{Crt/S}(p^{Crt}) = \begin{cases} 0 & \text{if } p^{Crt} \in [0,\bar{c}), \\ \rho \alpha x \gamma_0 \bar{n},\ \rho \in [0,1] & \text{if } p^{Crt} = \bar{c}, \\ \alpha x \gamma_0 \bar{n} & \text{if } p^{Crt} > \bar{c}; \end{cases} \tag{2.2}$$

$$q^{NC/S}(p^{NC}) = \begin{cases} \kappa(1-\gamma_0)\bar{n},\ \kappa \in [0,1] & \text{if } p^{NC} = 0, \\ (1-\gamma_0)\bar{n} & \text{if } p^{NC} \in (0,\bar{c}), \\ [(1-\gamma_0) + \kappa(1-\alpha x)\gamma_0]\bar{n},\ \kappa \in [0,1] & \text{if } p^{NC} = \bar{c}, \\ [(1-\gamma_0) + (1-\alpha x)\gamma_0]\bar{n} & \text{if } p^{NC} > \bar{c}. \end{cases} \tag{2.3}$$

Since consumers rationally expect the supply schedules, the posterior belief for the *Crt* market is given by

$$\gamma^{Crt} \equiv \gamma(\bar{g}|Crt) = \frac{\alpha x \rho \gamma_0}{\alpha x \rho \gamma_0} = 1$$

as long as $p^{Crt} \geq \bar{c}$ and $\gamma(1|Crt) = 0$ otherwise. Note that in the *NC* market low grade producers are willing to sell at all positive prices while high types would only do so when $p^{NC} \geq \bar{c}$, which implies in any equilibrium associated with some transaction and with shutting down by some firms, the exiting firms must be of high type. From this argument we know that,

$$\gamma^{NC} \equiv \gamma(\bar{g}|NC) = \frac{(1-\alpha x)\,\kappa\gamma_0}{(1-\alpha x)\,\kappa\gamma_0 + (1-\gamma_0)} \leq \frac{(1-\alpha x)\,\gamma_0}{(1-\alpha x)\,\gamma_0 + (1-\gamma_0)}$$

in which κ is 1 for $p^{NC} > \bar{c}$, any number in $[0,1]$ for $p^{NC} = \bar{c}$ and 0 for $p^{NC} < \bar{c}$. Thus, $\gamma^{Crt} = 1 > \gamma_0 \geq \gamma^{NC}$ for all $\alpha \in (0,1]$, which basically means that certification allows high quality producers to differentiate themselves from low quality producers with some probability while lowering the average quality in the *NC* market .

Given these beliefs and two price levels - p^{Crt} and p^{NC}, consumers are divided into three separate groups depending on the following decision rule: a type-θ consumer

$$\begin{cases} \text{buys in market } i & \text{if } \gamma^i\theta - p^i \geq \max\{\gamma^j\theta - p^j, 0\}, \\ \text{does not buy} & \text{if } 0 \geq \max\{\gamma^{Crt}\theta - p^{Crt}, \gamma^{NC}\theta - p^{NC}\}, \end{cases} \tag{2.4}$$

where $i, j \in \{Crt, NC\}$ and $i \neq j$.

After defining a price equilibrium formally, we will derive some preliminary results.

Definition 1 *A price equilibrium in the second period, given high types' certification decision (α) and certification precision (x), is a quadruple of $(p^{Crt}, \gamma^{Crt}, p^{NC}, \gamma^{NC})$ such that*

1. *Walrasian Market Clearing in Both Markets: given p^{Crt} and p^{NC}, the quantities supplied, determined by (2.2) and (2.3), equal to the quantities demanded, derived by (3.4) and beliefs: γ^{Crt} and γ^{NC}, in each market.*

2. *Consistent Beliefs: there exists* ρ *(and* κ*)* $\in [0,1]$ *describing high grade firms' production decisions in (2.2) (and (2.3)) which implies equilibrium quantity in the certified market (and non-certified market) and* γ^{Crt} *(and* γ^{NC}*).*

Note that there could be two potential types of price equilibria: with only one active market and the collapse of the other, and with two active markets. Let's denote the former one as "1-market" equilibrium and the latter as "2-market." To verify the equilibrium characteristics and its uniqueness, first we derive some conditions that equilibrium prices should meet.

Lemma 1 *In any price equilibrium in which some high quality goods are traded,*

1. *the equilibrium price in the Crt market is always greater than that in the NC market (i.e.,* $p^{Crt} > p^{NC}$*);*

2. *if some high quality producers shut down, the equilibrium price for that market must be equal to* $\bar{c}$*.*

There are several important implications of Lemma 1. First, since p^{Crt} is higher than p^{NC} in equilibrium (and $\gamma^{Crt} > \gamma^{NC}$), there is no 1-market equilibrium in which transaction occurs only in the *NC* market. This implies that 1-market and 2-market equilibria have to result in the price structures $p^{Crt} \geq \bar{c} > p^{NC} = 0$ and $p^{Crt} > p^{NC} \geq \bar{c}$ respectively.

Second, there exists a marginal consumer who is indifferent between buying in *Crt* market and *NC* market. The cutoff consumer's type θ^{Crt} is derived from the following condition:

$$\gamma^{Crt}\theta^{Crt} - p^{Crt} = \gamma^{NC}\theta^{Crt} - p^{NC}. \tag{2.5}$$

Without considering the possibility of not buying, all consumers with $\theta \geq \theta^{Crt}$ are affiliated with the *Crt* market while the remainder belongs to the *NC* market.

Third, all high types with a certification should serve ($\rho = 1$) in both 1-market and 2-market equilibria. To see this, recall that the equilibrium price under no information with $\bar{n}$ is equal to

$\bar{c}$ implying the margin is 0 for the high grade producers. Now, there are only $\alpha x \gamma_0 \bar{n}$ number of producers in the *Crt* market, and they serve consumers with relatively high valuations from the top (i.e., $\theta \in [\theta^{Crt}, 1]$). Also, note that each consumer's willingness to pay is also higher than that under no information because $\gamma^{Crt} > \gamma_0$. Therefore, it must be the case, $p^{Crt} > \bar{c}$ and $\rho = 1$.

Fourth, in any 2-market equilibria, $p^{NC} = \bar{c}$ and $\kappa < 1$. In no information benchmark, θ^{NI}-type consumer's willingness to pay, $\gamma_0 \theta^{NI}$, was equal to $\bar{c}$. Now since $\gamma^{NC} < \gamma_0$, his willingness to pay is lower than $\bar{c}$, and so some high types have to shut down. It is optimal that some high types choose to sell and the others do not only when those two actions yield the same payoffs of 0 to them, which implies $p^{NC} = \bar{c}$.

If both type of equilibria coexist, even though the previous claim implies that given α, x and $\bar{n}$, social welfare from the *Crt* market is invariant with the equilibrium quantity equal to $\alpha x \gamma_0 \bar{n}$, we may have multiple price equilibria due to the nature of the *NC* market. Hence, equilibrium selection is necessary to make a prediction about optimal certification decisions. To be consistent with the no information benchmark and as we want to show welfare under certification might be lower than under no information, we select the most favorable equilibrium in terms of welfare. Note that if there exist both 1-market and 2-market equilibrium, the welfare maximizing one is the 2-market equilibrium. Otherwise, the welfare maximizing equilibrium is the 1-market one. The lemma below characterizes such price equilibria.

Lemma 2 *The price equilibrium constellations are as follows.*

1. *If the production cost of high quality good is relatively high (i.e., $\bar{c} \in [\gamma_o^2, \gamma_0)$), the NC market always collapses ($\kappa = 0$), so a 1-market equilibrium is the unique welfare maximizing price equilibrium for all $\alpha \in (0,1]$ and $x \in (0,1]$; and*

2. *if the production cost of high quality good is relatively low (i.e., $\bar{c} \in (0, \gamma_o^2)$), there exists a threshold ξ such that*

   a) *for $\alpha x \in (\xi, 1]$, the NC market always collapses, so a 1-market equilibrium is the* *unique welfare maximizing price equilibrium; and*

b) *for* $\alpha x \in (0, \xi]$, *a 2-market equilibrium with* $\kappa(\alpha, x) \in (0, 1)$ *is the unique welfare maximizing price equilibrium where* $\kappa(\alpha, x)$ *is the larger root solving* $p^{NC} = \bar{c}$.

The lemma states conditions under which we have 1-market and 2-market equilibria. In order to grasp a deeper understanding, first we explain two different negative effects of high type's departure, from the pool of entire producers to the *Crt* market, on equilibrium price in the *NC* market. Not only it causes the average quality in the *NC* market (γ^{NC}) to fall (rotation in the *NC* market demand curve), but also it skims consumers with highest valuations from the pool (left shift of the *NC* market demand curve). However, that does not necessarily result in the collapse of the *NC* market if some portion of high type shut down. With a downward sloping demand curve, this exit by high grade producers has two countervailing effects on equilibrium price down in the *NC* market: p^{NC} might increase due to the decrease in quantity supplied (move along the *NC* market demand curve) but decrease due to the further drop in the average quality in the *NC* market (further rotation in the *NC* market demand curve). If the former effect dominates the latter, 2-market equilibria may emerge. Therefore, as stated in Lemma 2 price equilibrium configurations crucially depend on the magnitude of $\bar{c}$. 2-market equilibria do not emerge at all when $\bar{c}$ is relatively large (or $\bar{n}$ relatively small) because, roughly speaking, the smaller $\bar{n}$, the less potential for such positive (quantity) effects on p^{NC}.

Given any certification decisions (characterized by α), the corresponding welfare-maximizing price equilibrium is unique and well defined. Corollary 1 formally summarizes price equilibrium outcomes.

Corollary 1 *The welfare maximizing price equilibrium outcomes are as follows (given* α *and x).*

1. *When only 1-market equilibria exist,*

$$\begin{aligned}
\gamma^{Crt} &= 1; & \gamma^{NC} &= 0;\\
\theta^{Crt} &= 1-\alpha x\gamma_0\bar{n}; & \theta^{NC} &= \theta^{Crt};\\
p^{Crt} &= \theta^{Crt}; & p^{NC} &= 0;\\
\pi_H^{Crt} &= p^{Crt}-\bar{c}; & \pi_H^{NC} &= \pi_L^{NC} = 0.
\end{aligned}$$

2. *When 1-market and 2-market equilibria coexist,*

$$\begin{aligned}
\gamma^{Crt} &= 1; & \gamma^{NC} &= \gamma^{NC}\left(\kappa(\alpha,x)\right);\\
\theta^{Crt} &= 1-\alpha x\gamma_0\bar{n}; & \theta^{NC} &= 1-\left[1-(1-\alpha x)\left\{1-\kappa(\alpha,x)\right\}\right]\bar{n};\\
p^{Crt} &= (1-\gamma^{NC})\theta^{Crt}-p^{NC}; & p^{NC} &= \gamma^{NC}\theta^{NC} = \bar{c};\\
\pi_H^{Crt} &= p^{Crt}-\bar{c}; & \pi_H^{NC} &= 0;\\
& & \pi_L^{NC} &= p^{NC};
\end{aligned}$$

where $\kappa(\alpha,x)$ is defined as in Lemma 2, and θ^{Crt} and θ^{NC} represent the marginal consumer in the Crt and NC market respectively.

2.4.3 Certification Equilibrium and Welfare Analysis

Assuming all aspects and results of the model up to now are common knowledge to the agents, now we explore optimal decisions of high types on certification and conduct welfare analysis. As stated earlier, we are interested in checking whether availability of certification is socially beneficial or not. In order to highlight the welfare decreasing result without involving 2-market equilibria, first we consider the case 1 in Lemma 2 where the collapse of the *NC* market arises whenever some high types get certification. Next, we confirm that similar results also hold for the other cases in which 2-market equilibria may emerge.

2.4.3.1 Analysis when only 1-market Equilibria Exist

Throughout this subsection, assume that $\bar{c} \in [\gamma_0^2, \gamma_0)$ is relatively large so that even an arbitrarily small exit of high quality triggers the collapse of the *NC* market. In period 1, each high type will decide whether to get certified or not given a pair of a certification cost and a test precision, $(z,x) \in \Omega \equiv \mathbb{R}_+ \times (0,1]$. From the two benchmark cases and Corollary 1, given all other agents' strategies (given α), an individual high type's payoff of opting for certification is given by

$$E\Pi_H(\text{Cert}|\alpha) = x\{(1-\alpha x\gamma_0\bar{n}) - \bar{c}\} - z$$

because she gets π_H^{Crt} only with probability x and 0 with the complementary probability, but pays z regardless of the certification outcome. In contrast, the payoff of not getting certified is always given by $E\Pi_H(\text{No Cert}|\alpha) = 0$ regardless of α because in either case she ends up getting $\pi_H^{NC} = 0$.

The optimal decision rule for a high type is derived by comparing the payoffs with and without certification. However, since in our model each agent is atomic, technically it is hard for us to imagine how each individual's deviation affects the equilibrium prices. To this end, we define the certification equilibrium as follows which is particularly useful for the analysis associated with 2-market equilibria:

Definition 2 *Given (z,x), α^* characterizes a certification equilibrium if there is no profitable ε-deviation among the high grade producers. A profitable ε-deviation means that there exists an arbitrarily small positive number $\varepsilon \approx 0$ such that $E\Pi_H(Cert|\alpha^*) > E\Pi_H(No\ Cert|\alpha^* - \varepsilon)$ and $E\Pi_H(No\ Cert|\alpha^*) > E\Pi_H(Cert|\alpha^* + \varepsilon)$.*

Since here $E\Pi_H(\text{Cert}|\alpha)$ and $E\Pi_H(\text{No Cert}|\alpha)$ are monotone and constant with respect to α respectively, we have three different certification equilibrium configurations depending on the costs of certification fixing the certification precision. Given x, when the certification cost is relatively small, a fully separating equilibrium arises ($\alpha^* = 1$). When the certification cost is relatively high, a pooling equilibrium arises ($\alpha^* = 0$). When the certification cost is intermediate, a semi-separating equilibrium arises with $\alpha^* \in (0,1)$ in which all high quality producers are indifferent between getting certified or not, i.e., $E\Pi_H(\text{Cert}|\alpha^*) = E\Pi_H(\text{No Cert}|\alpha^*)$. The preceding arguments are summarized in the following proposition.

Proposition 2 *Suppose only 1-market equilibria exist (i.e., $\bar{c} \in [\gamma_0^2, \gamma_0)$). Then, given $x \in (0,1]$, in equilibrium all high types apply for certification ($\alpha^* = 1$) when z is relatively small, only some of them ($\alpha^* \in (0,1)$) when z is intermediate and no high types ($\alpha^* = 0$) when z is relatively large. Specifically,*

$$\alpha^* = \begin{cases} 1 & \text{if } z \in [0, \tilde{z}(x)], \\ \frac{1}{x\gamma_0\bar{n}}\left[1 - \left(\frac{z}{x} + \bar{c}\right)\right] & \text{if } z \in (\tilde{z}(x), \hat{z}(x)), \\ 0 & \text{if } z \in [\hat{z}(x), \infty), \end{cases}$$

where $\tilde{z}(x) \equiv x\{(1 - x\gamma_0\bar{n}) - \bar{c}\}$ and $\hat{z}(x) \equiv x(1 - \bar{c})$.

As each $(z,x) \in \Omega$ has a unique certification equilibrium, we can define three mutually exclusive subsets of Ω in which fully separating, semi-separating, and pooling equilibria arise respectively:

$$\Omega^{FS} \equiv \{(z,x) \in \Omega | z \leq \tilde{z}(x) \text{ and } x \in (0,1]\},$$
$$\Omega^{SS} \equiv \{(z,x) \in \Omega | \tilde{z}(x) < z < \hat{z}(x) \text{ and } x \in (0,1]\},$$
$$\Omega^{P} \equiv \{(z,x) \in \Omega | \hat{z}(x) \leq z \text{ and } x \in (0,1]\}.$$

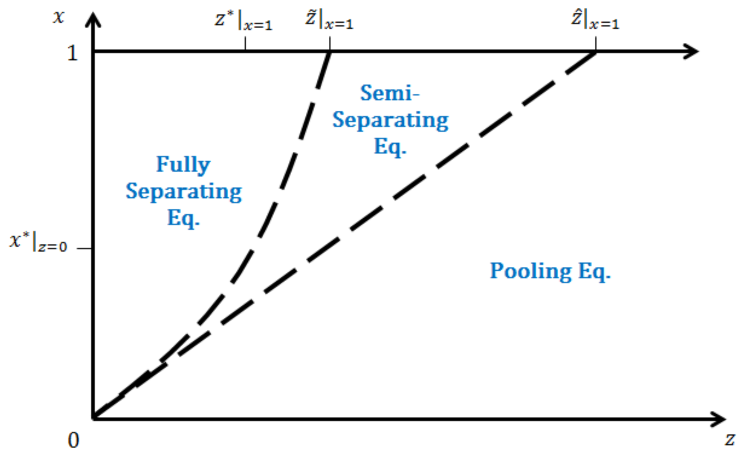
Figure 4 illustrates the three subsets of Ω on the $z - x$ plane. A simple comparative statics analysis suggests that more high quality producers seek certification, ceteris paribus, as z decreases on parameter spaces in which a semi-separating equilibrium emerges (i.e., $^\forall(z,x) \in \Omega^{SS}$).

For (z,x) in Ω^{FS} or Ω^{SS} , the welfare function defined as the sum of consumer surplus and firms' expected profits net of certification costs can be written as

$$W^{Crt}(z,x) = \int_{\theta^{Crt}}^{1} \theta d\theta - \alpha^* x\gamma_0\bar{n}\bar{c} - \alpha^*\gamma_0\bar{n}z$$

where $\theta^{Crt} = 1 - \alpha^* x\gamma_0\bar{n}$ and α^* as defined in Proposition 2. Otherwise, i.e., $(z,x) \in \Omega^P$, $W^{Crt}(z,x) = W^{NI}$. Depending on the parameter values, we have three qualitatively different re-

Figure 2.4: Certification Equilibrium Constellations

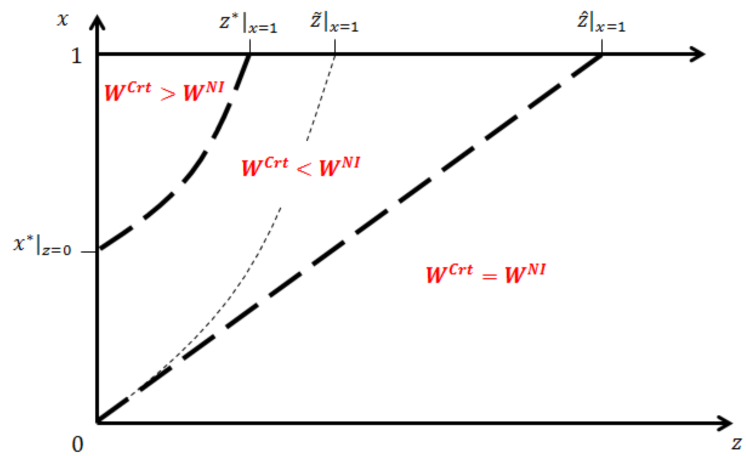


Note: Vertical and horizontal axes represent accuracy of the certification technology and the certification cost respectively.

sults. If certification environments are similar enough to the full information benchmark (put differently, if z is low, and x is high), then welfare with certification is greater than welfare under no information ($W^{Crt}(z,x) > W^{NI}$). In this situation, certification is profitable for all high type producers. Even though certification causes the NC market collapse and accrues certification costs, allocative efficiency enhanced by certification outweighs such loss. In the opposite extreme cases, if the cost of certification is too large or the certification technology is too inaccurate, nobody has an incentive to opt for certification. Thus, welfare with certification coincides with welfare under no information ($W^{Crt}(z,x) = W^{NI}$).

The most interesting case is the intermediate area in between the above two extremes in which welfare with certification is lower than welfare under no information ($W^{Crt}(z,x) < W^{NI}$). In order to formally state these results, let us define $\Omega^{NI} \equiv \left\{(z,x) \in \Omega^{FS} | W^{Crt}(z,x) = W^{NI}\right\}$ and upper contour set of welfare level W^{NI} as $\Omega^{NI+} \equiv \left\{(z,x) \in \Omega^{FS} | W^{Crt}(z,x) > W^{NI}\right\}$.

Figure 2.5: Welfare Comparisons



Note: Vertical and horizontal axes represent accuracy of the certification technology and the certification cost respectively. $z^*|_{x=1}$ represents a value such that $W^{Crt}(z^*,1) = W^{NI}$. In a similar fashion, $x^*|_{z=0}$ represent a value such that $W^{Crt}(0,x^*) = W^{NI}$.

Proposition 3 *Suppose only 1-market equilibria exist (i.e., $\bar{c} \in [\gamma_0^2, \gamma_0)$). Then,*

$$W^{Crt}(z,x) \begin{cases} > W^{NI} & \text{if } (z,x) \in \Omega^{NI+}, \\ = W^{NI} & \text{if } (z,x) \in \Omega^{NI} \cup \Omega^{P}, \\ < W^{NI} & \text{otherwise}, \end{cases}$$

where Ω^{NI+} is a convex proper subset of Ω^{FS} which is located around the point $(0,1)$ as shown in Figure 5.

Figure 5 shows a graphical illustration of Proposition 3. It is worth noting that it is not per se certification cost which makes W^{Crt} lower than W^{NI}. As we discussed in the full information benchmark, certification brings welfare gain from reallocating high quality to consumers whose value for the good is relatively high. However, if only a small measure of high grade producers are selected into the *Crt* market, such welfare gain with certification will be negligible. Moreover, given that we consider a parameter space on which the *NC* market always collapses with certification, welfare loss will be huge because most of high type producers now shut down. In other words, the main driving force for a welfare-decreasing result is adverse selection caused by a negative impact of certification on the *NC* market. To see this point clearly, consider the vertical axis in Figure 5. If the certification technology is relatively inaccurate ($x < x^*|_{z=0}$), we see $W^{Crt} < W^{NI}$ even though the certification cost is 0.

2.4.3.2 Analysis when 1-market and 2-market Equilibria Coexist

We now turn our attention to the other case $\bar{c} \in (0, \gamma_0^2)$ so that 2-market equilibria may exist. For simplicity, we investigate only the following two special cases in turn: "a cost-less ($z = 0$), but imperfect ($x \in (0,1]$) test" and "a costly ($z \in [0,\infty)$), but perfect ($x = 1$) one".

Cost-less ($z = 0$), but Imperfect ($x \in (0,1]$) Test

In this case, from Corollary 1 one can easily see that $\pi_H^{Crt} > 0$ and $\pi_H^{NC} = 0$ for all $\alpha > 0$ (i.e., regardless of whether there exists a 2-market equilibrium or not). Then, since certification is free here, expected payoffs of high types are always higher with certification, i.e., $E\Pi_H(\text{Cert}|\alpha) = x\pi_H^{Crt} > E\Pi_H(\text{No Cert}|\alpha) = 0$ for all $\alpha > 0$, which implies the unique certification equilibrium is a fully separating one ($\alpha^* = 1$).

What remains unknown for welfare calculation is to check when we would have a 2-market equilibria. From Lemma 2, 2-market equilibria are expected for only small x values. Let κ^- and κ^+ the smaller and larger solutions solving for

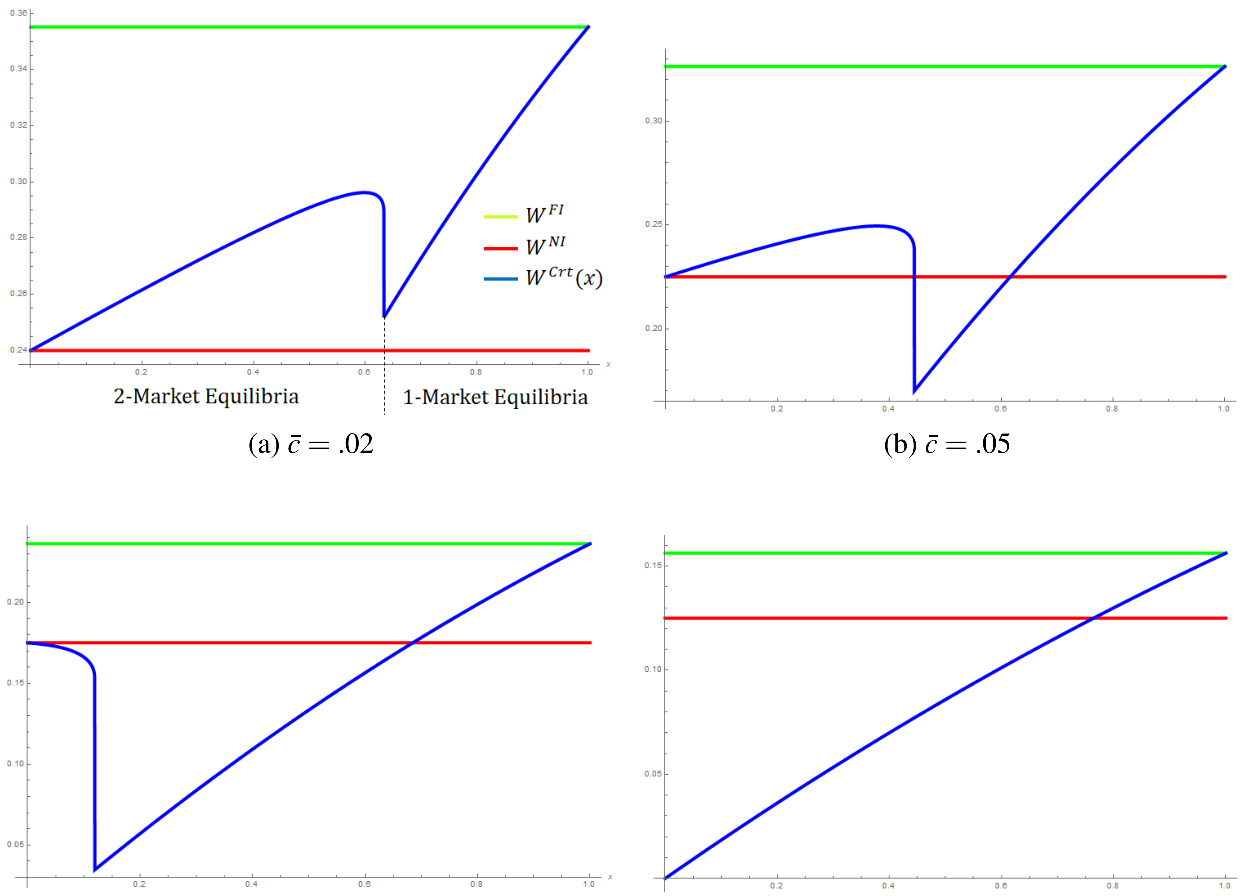
$$p^{NC}(\kappa|\alpha^* = 1, x\cdots) = \bar{c}$$

respectively. Then, if real roots of the above equation exist, then according to our equilibrium selection criteria, we choose $\kappa(\alpha^* = 1, x) = \kappa^+$. Hence, welfare in this case is given by

$$W^{Crt}(x) = \underbrace{\int_{\theta^{Crt}}^{1} \theta d\theta - x\gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } Crt \text{ market}} + \underbrace{\int_{\theta^{NC}}^{\theta^{Crt}} \gamma^{NC} \theta d\theta - (1-x)(1-\kappa^+)\gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } NC \text{ market}},$$

where all corresponding variables are defined as in Lemma 2-2 with $\kappa(\alpha^* = 1, x) = \kappa^+$. If we

Figure 2.6: Welfare Comparisons under Cost-less, but Imperfect Certification when $\gamma_0 = .5$



(a) $\bar{c} = .02$

(b) $\bar{c} = .05$

(c) $\bar{c} = .15$

(d) $\bar{c} = .25$

Note: Vertical and horizontal axes represent welfare levels and the certification accuracy respectively.

have just 1-market equilibria, welfare is given by

$$W^{Crt}(x) = \int_{\theta^{Crt}}^{1} \theta d\theta - x\gamma_0 \bar{n}\bar{c},$$

where all corresponding variables are defined as in Lemma 2-1.

We demonstrate welfare comparisons of a 2-market equilibrium through simulations when $\gamma_0 = .5$ for various values of production cost $\bar{c}$ in Figure 6. In all 3 panels except $\bar{c} = .25 (= \gamma_0^2)$, we have cutoffs of test precision determining before which 2-market equilibria emerge and otherwise 1-market equilibria. Put differently, if test is more precise than the cutoffs, certification results in the collapse of the *NC* market in equilibrium. The simulation shows that W^{Crt} could be lower than W^{NI} here too as long as production costs are not too small.

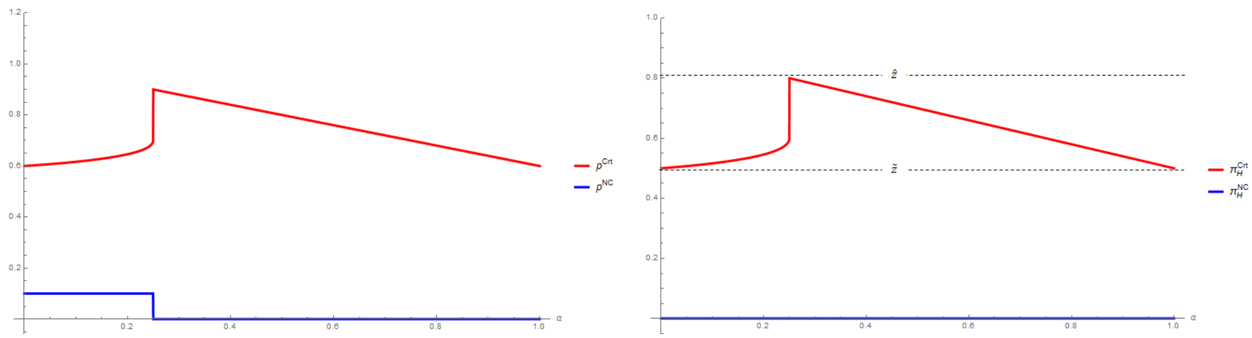
Costly ($z \in [0, \infty)$), but Perfect ($x = 1$) Test

Now we study the case of costly but perfect certification. More specifically, consider the case where $\gamma_0 = .5$ and $\bar{c} = .1$. Then, depending on certification decisions, whole trajectories of equilibrium prices and profit levels of high grade producers are as shown in Figure 7. As seen in the case of cost-less and imperfect certification, 2-market equilibria prevail if only small number of high types choose to get certified (relatively small α). This causes discontinuities in the price and profit functions.

Given the price equilibrium outcomes, optimal certification decisions depend on the sizes of $E\Pi_H(\text{Cert}|\alpha) = \pi_H^{Crt} - z$ and $E\Pi_H(\text{No Cert}|\alpha) = 0$. Note that if the certification cost is relatively small ($z \leq \tilde{z}$), a fully separating equilibrium is obtained (refer to the right panel of Figure 7). Similarly, if the certification cost is relatively large ($z \geq \hat{z}$), a pooling equilibrium results. Otherwise ($\tilde{z} < z < \hat{z}$), a certification equilibrium candidate must satisfy the following condition: $\pi_H^{Crt}(\alpha^*) = z$. Note that for values of z slightly above $\tilde{z}$, there might be two equilibrium candidates (α) making high types indifferent between opting for certification and not doing so. However, the certification equilibrium associated with the smaller candidate of the two is not ε-stable because high types currently not opting for certification expect the equilibrium price and their profits to increase if a small measure of them deviate. Therefore, the certification equilibrium is characterized by α^* such that $\pi_H^{Crt}(\alpha^*) = z$ and $\partial \pi_H^{Crt} / \partial \alpha |_{\alpha = \alpha^*} < 0$.

One thing to note is that here the highest certification cost supporting a semi-separating equilibrium, $\hat{z}$, is lower than the one we would have obtained if we did not consider 2-market equilibria (which would have been given as $\hat{z} = 1 - \bar{c} = .9$ as in Proposition 2). Whenever 2-market equilibrium exist, the price in *Crt* market should be set in such a way leaving the exact same amount of surplus to the marginal type consumer (θ^{Crt}) which he would get when purchasing in the *NC* market instead. For this reason, when 2-market equilibria exist, the upper bound supporting a

Figure 2.7: Price Equilibrium Outcomes under Costly, but Perfect Certification when $\gamma_0 = .5$ and $\bar{c} = .1$



(a) Equilibrium Prices in *Crt* and *NC* Markets (b) Equilibrium Profits of High Types

Note: Vertical and horizontal axes represent price and profit levels and measure of high grade producers opting for certification respectively.

semi-separating certification equilibrium gets smaller.

Welfare calculations again depend on the type of equilibrium. If we have 2-market equilibria,

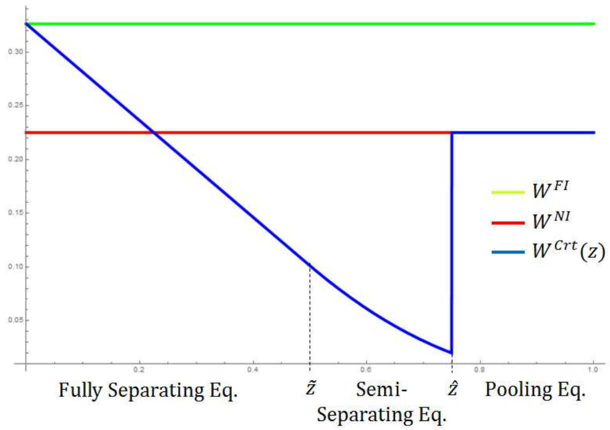
$$W^{Crt}(z) = \underbrace{\int_{\theta^{Crt}}^{1} \theta d\theta - \alpha^* \gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } Crt \text{ market}} + \underbrace{\int_{\theta^{NC}}^{\theta^{Crt}} \gamma^{NC} \theta d\theta - (1-\alpha^*)(1-\kappa^+)\gamma_0 \bar{n}\bar{c}}_{\text{Welfare from } NC \text{ market}} - \underbrace{\alpha^* \gamma_0 \bar{n} z}_{\text{Certification Costs}},$$

where all corresponding variables are defined as in Lemma 2-2 with $\kappa(\alpha^*, x = 1) = \kappa^+$ and Proposition 2. If we have just 1-market equilibria,

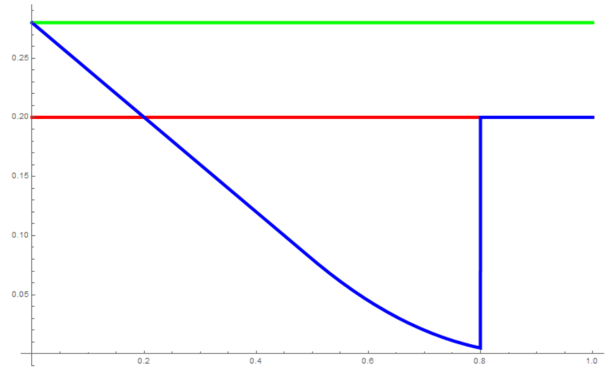
$$W^{Crt}(z) = \int_{\theta^{Crt}}^{1} \theta d\theta - \alpha^* \gamma_0 \bar{n}(\bar{c} + z),$$

where all corresponding variables are defined as in Lemma 2-1 and Proposition 2. Figure 8 summarizes simulation results regarding welfare levels for various values of production cost. It confirms once again that welfare could be lower with certification. In all cases, as the certification cost, z, increases from 0 to a certain threshold, welfare with certification monotonically decreases from the full information level to the one lower than the no information level. Beyond the threshold, it becomes the same as the no information level.

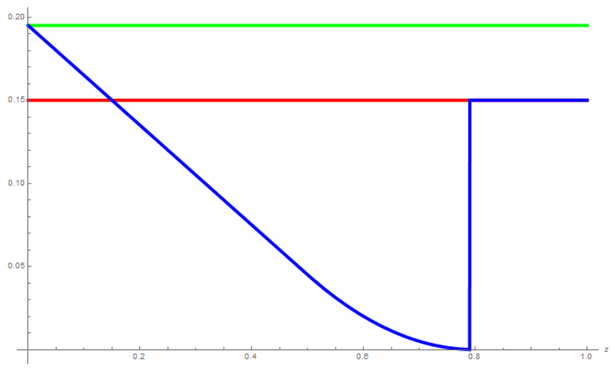
Figure 2.8: Welfare Comparisons under Costly, but Perfect Certification when $\gamma_0 = .5$



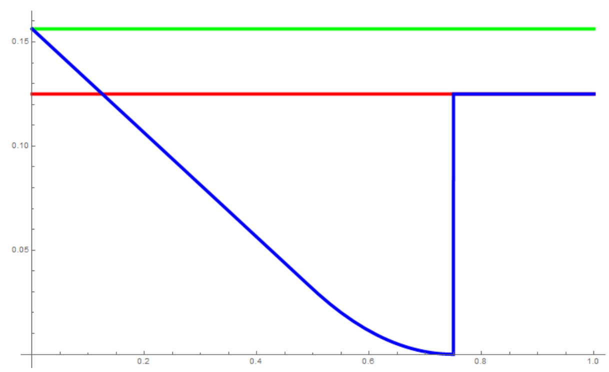
(a) $\bar{c} = .05$



(b) $\bar{c} = .10$



(c) $\bar{c} = .20$



(d) $\bar{c} = .25$

Note: Vertical and horizontal axes represent welfare levels and certification costs respectively.

2.5 Conclusion

It is well-known that markets with asymmetric information about quality or other vertically differentiating goods characteristics suffer from allocative inefficiencies. Overcoming these asymmetries can lead to welfare increases. However, this understanding overlooks the close relationships between markets. Allowing for certification may create the co-existence of both a certified and a non-certified market. Both consumers and firms subsequently self-select into one market, the other market, or opting out. Total welfare is a function of these self-selections, and thus welfare implications cannot be considered in isolation. If certification is imperfect or costly, then positive externalities of high quality in non-certified markets can be destroyed without a complete offsetting through added welfare in the certified markets. This implies subtle policy-implications when evaluating the introduction of government certification programs and regulations of industry self-governance with respect to certification programs.

We close this paper by discussing one feasible extension that may offer additional interesting insights. It would be interesting to add a first stage to the game in which firms make their entry decisions. This will allow us to see long-run consequences of certification depending on different certification environments, which have received little interest in most of existing studies. We leave this idea for future research to consider.

APPENDIX

Appendix for Chapter 2

Proof of Proposition 1

Proof of proposition 1 is in the text of the paper. ■

Proof of Lemma 1

Let us show $p^{Crt} > p^{NC}$ first. Suppose there exist a price equilibrium where at least some high quality goods are traded and $p^{Crt} \leq p^{NC}$. Then, since $\gamma^{Crt} \geq \gamma^{NC}$,

$$\gamma^{Crt}\theta - p^{Crt} > \gamma^{NC}\theta - p^{NC} \forall\theta \in [0,1].$$

Then, all consumers $\theta \geq P^{Crt}/\gamma^{Crt}$ wants to buy in Crt. Having some equilibrium output of high quality goods, it must be the case that the cutoff type should be in the interior of support of consumer types i.e., $P^{Crt}/\gamma^{Crt} \in (0, \bar{\theta})$. Otherwise, there is no demand for both markets, and such an equilibrium does not exist. Thus, it is sufficient to only consider the following range of $p^{Crt} \in (0, \gamma^{Crt})$: If $0 < p^{Crt} < \bar{c}$, there is no supply in Crt market even though there is demand, so such prices are not market clearing. If $\bar{c} \leq p^{Crt} < \gamma^{Crt}$, there are always some low quality producers willing to sell in NC since $p^{NC} \geq p^{Crt} \geq \bar{c} > 0$ while there is no demand at all for that market. Hence, these prices are not market clearing either, which implies $p^{Crt} > p^{NC}$ must be true.

The second argument is obvious from the binary feature of firm's optimal decision rule. ■

Proof of Lemma 2

Given $\bar{n}$ (or $\bar{c}$) and $\alpha \in (0,1)$, for a 2-market equilibrium to exist, the following conditions must hold:

1. Existence of marginal types (θ^{Crt}) between buying in Crt market and buying in NC market:

$$\gamma^{Crt}\theta^{Crt} - p^{Crt} = \gamma^{NC}\theta^{Crt} - p^{NC};$$

2. Existence of marginal types (θ^{NC}) between buying in *NC* market and not buying:

$$\gamma^{NC}\theta^{NC} - p^{NC} = 0;$$

3. Market clearing condition in *Crt* market

$$q^{Crt/D} \equiv 1 - \theta^{Crt} = \alpha\gamma_0\bar{n} \equiv q^{Crt/S};$$

4. Market clearing condition in *NC* market

$$q^{NC/D} \equiv \theta^{Crt} - \theta^{NC} = [(1-\alpha x)\,\kappa\gamma_0 + (1-\gamma_0)]\,\bar{n} \equiv q^{NC/S};$$

5. Consistent beliefs: $\gamma^{Crt} = 1$, $\gamma^{NC} = \frac{(1-\alpha x)\kappa\gamma_0}{(1-\alpha x)\kappa\gamma_0+(1-\gamma_0)} \leq \frac{(1-\alpha x)\gamma_0}{(1-\alpha x)\gamma_0+(1-\gamma_0)}$ where $\kappa \in [0,1]$.

6. From high grade firm's indifferent condition between selling in *NC* and being inactive and the definition of $\bar{n}$, for all $\kappa \in [0,1]$

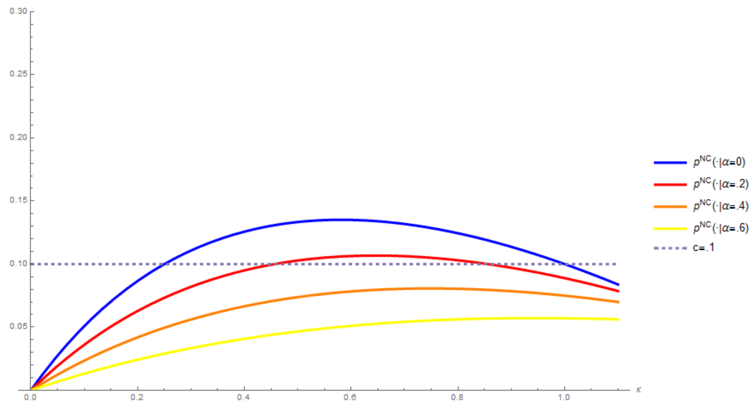
$$p^{NC} = \bar{c}.$$

From the condition 3, $\theta^{Crt} = 1 - \alpha\gamma_0\bar{n}$. Plugging this in the condition 4, we end up having $\theta^{NC} = \{1 - [1-(1-\alpha x)(1-\kappa)\gamma_0]\,\bar{n}\}$. Since p^{Crt} can be adjusted according to the condition 1 once p^{NC} is determined, the existence of a 2-market equilibrium pins down to find a proper κ satisfying $p^{NC} = \bar{c}$. More specifically, from the condition 2, 5 and 6, if there exists $\kappa \in [0,1]$ which solves the following equation,

$$p^{NC}(\kappa|\alpha, x\cdots) \equiv \frac{(1-\alpha x)\,\gamma_0\kappa}{(1-\alpha x)\,\gamma_0\kappa + (1-\gamma_0)}\{1 - [1-(1-\alpha x)(1-\kappa)\gamma_0]\,\bar{n}\} = \bar{c},$$

a 2-market equilibrium exists. Before solving the equation, let us show that welfare is maximized with the largest value $\kappa(\alpha, x\cdots)$ satisfying the above equation. First, note that welfare generated from the *Crt* market will be the same no matter what kind of equilibrium we have because q^{Crt} and γ^{Crt} are invariant. What only changes in the *Crt* market is p^{Crt} which is a net transfer from consumers to producers over different kinds of equilibria. Therefore, whenever a 2-market equilibrium exists, a 1-market equilibrium is not a welfare maximizing one. Then, it remains to pick up

Figure 2.9: Certification Equilibrium Constellations



Note: We assume that $\gamma_0 = .5$, $\bar{c} = .1$ and $x = 1$. Vertical and horizontal axes represent p^{NC} and κ respectively.

a 2-market equilibrium resulting in the highest welfare level, which is obviously the largest value $\kappa(\alpha, x\cdots)$ such that $p^{NC} = \bar{c}$.

The first and second derivative of $p^{NC}(\kappa|\alpha, x\cdots)$ with respect to κ are respectively,

$$\frac{\partial p^{NC}}{\partial \kappa} = \frac{\left\{ \begin{array}{c} \gamma_0(1-x\alpha) \times \Big[1-\gamma_0 - \Big\{\gamma_0^2(1-x\alpha)^2\kappa^2 \\ +2\gamma_0(1-\gamma_0)(1-x\alpha)\kappa + (1-\gamma_0)\left(1-\gamma_0(1-x\alpha)\right)\Big\}\bar{n}\Big] \end{array} \right\}}{\left[1-\gamma_0+\gamma_0(1-x\alpha)\kappa\right]^2}$$

$$\frac{\partial^2 p^{NC}}{\partial \kappa^2} = -\frac{2\gamma_0^2(1-\gamma_0)(1-\gamma_0 x\alpha\bar{n})(1-x\alpha)^2}{\left[1-\gamma_0+\gamma_0(1-x\alpha)\kappa\right]^3} < 0.$$

Note that $p^{NC}(\kappa|\alpha,\cdots)$ has a similar shape with a concave quadratic curve passing through the origin around which the slope of the curve is positive. Furthermore, as α increases from 0 to 1, this curve gets closer to the horizontal axis fixing κ and has a (weakly) larger argmax of the function (i.e., $\partial p^{NC}/\partial\alpha < 0$ given κ for all $\kappa \in [0,1]$ and $\partial(\arg\max_\kappa p^{NC})/\partial\alpha \geq 0$). Even though, Figure 9 is drawn for the case where $\gamma_0 = .5$, $\bar{c} = .1$ (so $\bar{n} = .8$) and $x = 1$, the shape of p^{NC} is similar for all other parameter values. Also, we will have the same properties when it comes to x rather than α because $p^{NC}(\kappa|\alpha, x\cdots)$ is symmetric with respect to α and x.

Note that $p^{NC}(\kappa = 1, \alpha = 0) = \bar{c}$ always by construction. So, if

$$\left.\frac{\partial p^{NC}}{\partial \kappa}\right|_{\alpha=0,\kappa=1} \geq 0,$$

2-market equilibria do not exist. Now observe that

$$\begin{aligned} sign\left(\left.\frac{\partial p^{NC}}{\partial \kappa}\right|_{\alpha=0,\kappa=1}\right) &= sign\left(\left[1-\gamma_0-\left\{\gamma_0^2+2\gamma_0(1-\gamma_0)+(1-\gamma_0)^2\right\}\bar{n}\right]\right) \\ &= sign\left(1-\gamma_0-\bar{n}\right). \end{aligned}$$

Therefore, $\left.\frac{\partial p^{NC}}{\partial \kappa}\right|_{\alpha=0,\kappa=1} \geq 0 \Longleftrightarrow 1-\gamma_0-\bar{n} \geq 0$ which is equivalent to $\bar{c} \geq \gamma_0^2$.

Even though we may have some 2-market equilibria if $\bar{c} < \gamma_0^2$, that is the case only for small α. Finally, whenever a 2-market equilibria exists, the welfare maximizing $\kappa(\alpha)$ is the larger root of the equation $p^{NC}(\kappa|\alpha,\cdots) = \bar{c}$. Now it completes the proof to define ξ as a positive number making $p^{NC}(\kappa|\alpha x = \xi,\cdots) = \bar{c}$ have only one solution. ■

Proof of Proposition 2

Obvious from the discussion in the text and thus omitted. ■

Proof of Proposition 3

We will walk through a sequence of steps leading to the desired results.

Step 1: $W^{Crt}(z,x)$ is strictly decreasing in z and strictly increasing in x on Ω^{FS} since $\partial W^{Crt}(z,x)/\partial z = -x\gamma_0\bar{n} < 0$ and

$$\begin{aligned} \partial W^{Crt}(z,x)/\partial x &= (1-x\gamma_0\bar{n})\gamma_0\bar{n} - \gamma_0\bar{n}\bar{c} \\ &= \{(1-x\gamma_0)-\bar{c}(1-x)\}\gamma_0\bar{n} \\ &> (1-\bar{c})(1-x)\gamma_0\bar{n} \\ &> 0 \end{aligned}$$

where the first inequality follows from $\gamma_0 < 1$, and the second inequality follows from Assumption 1.

Step 2: If x is too small, $W^{Crt}(z,x) < W^{NI}$ regardless of z on Ω^{FS}. To see this, fix $z = 0$. Since $W^{Crt}(0,1) = W^{FI} > \lim_{x\to 0} W^{Crt}(0,x) = 0$ and $\frac{\partial W^{Crt}(z,x)}{\partial x} > 0$, there exists a unique x^* such that $W^{Crt}(0,x) \lesseqqgtr W^{NI}$ if and only if $x \lesseqqgtr x^*$by the Intermediate Value Theorem. Then, for all $(z,x) \in \Omega^{FS}$ such that $x < x^*$, $W^{Crt}(z,x) < W^{NI}$ since $W^{Crt}(z,x)$ strictly decreases in z.

Step 3: For each $x > x^*$ on Ω^{FS}, there exists $z^*(x) < \tilde{z}(x)$ such that $W^{Crt}(z,x) \lesseqqgtr W^{NI}$ if and only if $z \gtreqqless z^*(x)$. First, let us show that $\lim_{z\to\tilde{z}(x)} W^{Crt}(z,x) < W^{NI}$.

$$
\begin{aligned}
\lim_{z\to\tilde{z}(x)} W^{Crt}(z,x) - W^{NI} &= (1/2)\left[1-(1-x\gamma_0\bar{n})^2\right] - x\gamma_0\bar{n}\bar{c} - \gamma_0\bar{n}\tilde{z}(x) \\
&\quad - \left\{(1/2)\gamma_0\left[1-(1-\bar{n})^2\right] - \gamma_0\bar{n}\bar{c}\right\} \\
&= \gamma_0\bar{n}\left\{(1/2)\left[x^2\gamma_0\bar{n} - (2-\bar{n})\right] + \bar{c}\right\} \\
&< \gamma_0\bar{n}\{(1/2)[\bar{n} - (2-\bar{n})] + \bar{c}\} \\
&= -\gamma_0\bar{n}(1/\gamma_0 - 1)\bar{c} \\
&< 0
\end{aligned}
$$

where the second equality follows from arranging terms; third equality follows from arranging terms after plugging in $1-(\bar{c}/\gamma_0)$ for $\bar{n}$; the first inequality follows from $x^2\gamma_0 < 1$. Along with this results and the fact that $W^{Crt}(0,x) > W^{NI}$ for all $x > x^*$, we proved the point again by the Intermediate Value Theorem.

Step 4: From Step 2 and 3, $\Omega^{NI+} = \left\{(z,x) \in \Omega^{FS} | z < z^*(x) \text{ and } x > x^*\right\}$ and $\Omega^{NI+} \subsetneq \Omega^{FS}$.

Step 5: For all $(z,x) \in \Omega^{SS}$, $W^{Crt}(z,x) < W^{NI}$. The welfare was defined as the sum of consumer surplus. Compared to the fully separating case, now less amount of consumers are served in equilibrium given x while consumer valuation for the good remains the same as before so that consumer surplus decreases. In addition to this, the aggregate expected profits are lower in semi-separating equilibrium because by definition producers earn 0 now while their profits are positive in fully separating equilibria. In sum, $W^{Crt}(z,x)$ is lower than $W^{Crt}(\tilde{z}(x),x)$, which implies

$W^{Crt}(z,x) < W^{NI}$ for all $z \in (\tilde{z}(x), \hat{z}(x))$ given x. ■

CHAPTER 3

A ROLE OF NPE WHEN PATENT OWNERSHIP IS FRAGMENTED

3.1 Introduction

A non-practicing entity (NPE)[1] is a company or an individual who amasses a few or large number of patents only so as to monetize its patents through a licensing negotiation or litigation, without exerting any efforts to commercialize, develop and sell a product adopting technologies covered by its patents. According to the RPX report, the proportion of NPE lawsuits had rapidly increased from 35 percent to 66 percent from 2010 to 2012 and remained between 66–70 percent since then until 2015.[2]

This remarkable rise of NPE lawsuits, seen in recent years, have aroused considerable controversy on roles of NPEs. Some scholars expressed their concern for negative impacts of a NPE on incentives of practicing entities (PEs) to develop a new product by underscoring opportunistic patent acquisition and assertion strategies employed by NPEs (e.g., see Chien, 2010; and Lemley and Melamed, 2013). On the contrary, a common defense of NPEs is that NPEs offer small individual inventors a better means of enforcing their patents and thereby increase the payoff to the individual innovators (e.g., see Hagiu and Yoffie, 2013; and Risch, 2012). Even though a number of studies started to analyze strategic behavior of NPEs and its incidence on innovation market outcomes, to the best of my knowledge, there is still no research on how NPEs can increase the payoff of the individual innovators and why that can be an important issue.

In order to address the above question, this paper considers an environment in which a product, manufactured by a PE must adopt all complementary innovations patented and owned by individual inventors; and a threat of litigation by an individual inventor is not credible so that patents are sold

[1] Sometimes, it is also called a "patent troll" or a patent assertion entity (PAE).

[2] Source: "2015 Report, NPE Litigation, Patent Marketplace, and NPE Cost" by RPX (last retrieved on May 1, 2017 from https://www.rpxcorp.com/wp-content/uploads/sites/2/2016/07/RPX-2015-Report-072616.FinalZ.pdf).

through a sequential auction. The following two games are studied in turn. First, I consider the game played by two PEs in which they compete to acquire the patents, and then negotiate licensing fees based on their patent portfolio in the shadow of litigation. Next, the same game played by a PE and a NPE is analyzed.

In the former game, it turns out equilibrium outcomes significantly depend on whether there exists a state of "patent acquisition stalemate (*PAS*)." A *PAS* state refers to an intermediate patent allocation in which none of the two PEs litigation threat becomes credible eventually regardless of the results of the remaining patent auctions from that point on. It is shown that whenever a *PAS* state exists, the two PEs buy all patents at 0 prices, and thus the aggregate payoff to the individual inventors is also 0 in equilibrium. This implies that in fact patent ownership fragmentation can lead to sharp decrease in the individual innovators' payoff.

In sharp contrast to the above results, in the latter game, if total number of patents is sufficiently large, the existence of the NPE induces the PE to buy some patents at a positive price and thereby increases the aggregate payoff to the individual inventors. Therefore, this implies that fragmented patent ownership can significantly lower the payoff to the individual innovators, and in this situation the NPE can play an important role of helping such small innovators get paid for their innovations.

3.2 Related Literature

This paper contributes to a burgeoning literature on roles of NPEs. As in this model, there are some papers that touch on strategic patent acquisitions by PEs and NPEs (Choi and Gerlach, 2017; Cosandier et al., 2014; and Scott Morton and Shapiro, 2014). In a different vein, there are some other papers that study how NPEs (or PEs) strategically assert their patents in order to maximize their licensing revenues from an established patent portfolio (Choi and Gerlach, 2015; Hovenkamp, 2013; Lemley and Melamed, 2013; and Scott Morton and Shapiro, 2014).

Though the previous studies offer keen insights regarding various effects of NPEs on innovation market outcomes, this paper is different from the previous studies in either of the following two aspects: First, this paper is the only one attempting to study the mechanism through which NPEs can increase the payoff of the small individual innovators. Another distinction of this paper is patent acquisitions with endogenous valuations. While a patent acquisition takes place at once, if there is any, in most of previous studies, patent acquisitions take place sequentially in this paper. As a result, a value of an ex-ante identical patent can vary depending on in which stage of the sequential auction, which is characterized by each party's intermediate patent portfolio, the patent is sold.

This paper proceeds in the following way. Section 2 presents the model. In Section 3 and 4, I analyze the game played by two PEs and that by a PE and NPE respectively. Section 5 concludes.

3.3 Model

While I consider the two games played between two PEs and between a PE and a NPE[3] in turn, for simplicity, here I delineate the model as for only the former game. Necessary model modifications are elaborated later (right before the latter game is to be analyzed.)

There are two firms (PEs), 1 and 2 , who produce a complex technology product adopting $N > 2$ complementary functionalities. The functionalities are complementary in the sense that the product is not marketable without incorporating those features. All N technologies, which enable respective functionalities, are protected by patents owned by N different individual inventors. Thus, both of PEs take a risk of infringing N patents by selling the product unless they get a right to use them through a license contract with patent owners or buy those patents. The duopoly profit is denoted by π.[4]

[3]Usually, a NPE refers to all entities that own patents but do not produce any goods using technologies covered by their patents. In this sense, individual inventors are also sometimes called as NPEs. However, I make a distinction between individual inventors and the NPE as a patent aggregator throughout the paper.

[4]The product market competition is abstracted by assuming that each PE earns π.

In the beginning, each individual innovator and each PE engage in licensing in the shadow of litigation. Let $\alpha(\in (0,1])$ denote the probability that a patent is upheld valid in court. This probability is assumed to be the same and independent across all N patents. Define $R(>0)$ as a per patent damage award for infringement and $L(>0)$ as a litigation cost. The damage award per a patent is also the same for all patents, and all agents incur the same litigation cost. If a litigation threat of an individual inventor is credible against a PE,[5] i.e., $\alpha R \geq L$, they negotiate on a licensing fee through a Nash bargaining with equal bargaining power. They are settled out of court in order to avoid litigation costs and agree on a licensing fee of αR.[6] In this situation, there is nothing to further analyze, and the aggregate payoff to individual innovators is $2N\alpha R$ and the payoff to each firm $\pi - N\alpha R$. By assuming that the product market is lucrative enough to cover licensing fees and the litigation cost i.e., $\pi - N\alpha R - L > 0$, I abstract from analyzing the firms' entry or exit decision.

If the threat of litigation by an individual innovator is not credible, i.e., $\alpha R < L$, each innovator is not able to collect licensing revenue. However, even in this scenario, if the bundle of all patents gave them a credible litigation threat (i.e., $N\alpha R \geq L$), the individual innovators would have formed a patent pool (or a licensing coalition) to profitably license their patents. If this is possible, there is no room for a third party NPE to play a role of increasing returns to the individual innovators. I rule out this scenario by making the following assumption to study the role of the NPE when cooperation is not an available option for the individual inventors.[7]

Assumption 1: ***(Information Structure)*** *(i) An individual innovator is aware of what kind of product is going to be released by PEs and how much licensing revenues (αR) will be determined in*

[5] In this paper, a credible litigation threat is always in regard to a single defendant because "Leahy-Smith America Invents Act," which was enacted in 2011, precludes a plaintiff to file a patent suit against multiple defendants,

[6] Assuming equal bargaining power, in a Nash bargaining between an individual inventor and a PE, the individual inventor's payoff is $\alpha R - L + (1/2)2L = \alpha R$ while the PE's is $-\alpha R - L + (1/2)2L = -\alpha R$.

[7] Note that the assumption that I need in this paper needs not be asymmetric information between the individual innovators and PEs. It is sufficient to assume that forming a licensing coalition is not feasible for some reason.

court in expected sense, but does not know which other $(N-1)$-patents are going to be potentially infringed by the product and who are the owners of those patents ex-ante. (ii) On the contrary, two PEs and a NPE know all that information.

In fact, this is not an unrealistic assumption because many NPEs are founded by former lawyers of a PE or patent attorneys.[8] Given Assumption 1, which implies that ex-ante coordination among individual inventors is impossible, the individual innovators put their patents up for a sale through an auction, and PEs engage in the patent acquisition game. Finally, given a patent allocation, a licensing fee between PEs are negotiated and transferred in the shadow of patent litigation.

Put up for sale, N patents are sold through a sequence of second-price auctions. Thus, the setup for the patent acquisition game introduced here is similar to the model used in Gale and Stegeman (2001). Since all patents are symmetric in terms of expected licensing fee ex-ante so that the order of sequence does not have any effect on equilibrium characteristics.[9] Any stage of the game is described by its current state of patent allocation, which is denoted by (n_1, n_2) where n_1 and n_2 are the number of patents possessed by firm 1 and 2 respectively. Let $\mathscr{F} = \{(n_1, n_2) \in \mathbb{I}_+^2 | n_1 + n_2 = N\}$ and $\mathscr{A} = \{(n_1, n_2) \in \mathbb{I}_+^2 | n_1 + n_2 < N\}$ denote a collection of all final and intermediate patent allocations respectively where $\mathbb{I}_+ = \{0, 1, 2, ...\}$. Firm i's bidding strategy is a function $b_i : \mathscr{A} \to \mathbb{R}_+$. Let $b_i(n_1, n_2)$ be the firm i's optimal bidding in $(n_1, n_2) \in \mathscr{A}$.

Once a final patent allocation is determined through the patent acquisition game, firms negotiate about licensing fees in the shadow of litigation. For an arbitrary final allocation (i.e., $(n_1, n_2) \in \mathscr{F}$), let $V_i(n_1, n_2)$ denote the net licensing revenue of firm i in the shadow of litigation. Then, the payoff

[8] For example, Intellectual Ventures, a well-known NPE, was founded by Nathan Myhrvold and Edward Jung, both of whom once held important positions at Microsoft, among many others (e.g., see Ewing and Feldman, 2012).

[9] This does not necessarily mean that equilibrium prices are all alike on the equilibrium path due to the endogenous valuation nature of this game. As well documented in auctions with endogenous valuations literature (e.g. Krishna 1993, 1999), patents, that are the same ex-ante, could have different values to different buyers depending on a patent allocation.

of firm i is

$$\Pi_i(n_1,n_2) = \pi + V_i(n_1,n_2) - (\text{sum of payments for obtaining } n_i \text{ patents}),$$

where $i = \{1,2\}$, and the aggregate payoff of individual innovators is the sum of auction revenues.

The equilibrium concept is Subgame Perfect Nash equilibrium (SPNE).

3.4 Equilibrium Analysis: PE vs. PE

In this section, I solve the game between two PEs by backward induction confining our attention to the case in which the threat of litigation by an individual innovator is not credible (i.e., $\alpha R < L$), but the sum of individual patent's licensing revenue is greater than the licensing cost (i.e., $N\alpha R \geq L$). The game between a PE and a NPE is analyzed in the next section.

3.4.1 Licensing in the Shadow of Litigation

In terms of calculating equilibrium licensing fees, the model shares many things in common with Lemus and Temnyalov (forthcoming). Since the validity of a patent (α) is determined independently across all N patents, the probability that firm i's product infringes on x number of patents in court out of firm j's patent portfolio of size n_j follows the following binomial distribution:

$$\begin{pmatrix} n_j \\ x \end{pmatrix} \alpha^x (1-\alpha)^{n_j - x}$$

and vise versa. Thus, the expected damage awards that firm i needs to pay to firm j is $n_j\alpha R$ once litigation takes place. However, because firm i can counter-litigate without incurring additional costs, she also collects $n_i\alpha R$ from firm j in expected sense. Taking this into consideration, one of the two will initiate litigation only when the net transfer is greater or equal to the litigation cost, i.e., $|n_i - n_j|\alpha R \geq L$. If this is the case, as similarly discussed in section 2, a symmetric Nash bargaining results in an out-of-court settlement in which the net payment of firm i is $(n_i - n_j)\alpha R$.

In all other cases in which no PE has any incentives to initiate litigation, there are no licensing payments.

The following lemma, which is basically a reminiscence of Proposition 1 in Lemus and Temnyalov (forthcoming), summarizes the above arguments and formally characterizes $V_i(n_1,n_2)$ in equilibrium for an arbitrary final allocation.

Lemma 1 *PEs always cross-license their patents through an out-of-court settlement (no litigation) whenever one of the two has a credible litigation threat, and the net licensing revenue of firm i is given by*

$$V_i(n_1,n_2) = \begin{cases} (n_i - n_j)\alpha R & \text{if } |n_1 - n_2|\alpha R \geq L, \\ 0 & \text{otherwise.} \end{cases} \tag{3.1}$$

For notational simplicity, define a minimum gab in numbers of patents owned by the two PEs, which makes one of the two PE's litigation threat credible, K as follows:

Definition 1: *K is a positive natural number such that $(K-1)\alpha R < L \leq K\alpha R$.*

Also, let us divide the set of final allocations ($\mathscr{F}$) into two mutually exclusive subsets: a set of allocations with a credible litigation threat,

$$\mathscr{L} \equiv \{(n_1,n_2) | (n_1,n_2) \in \mathscr{F}, |n_1 - n_2| \geq K\}$$

and without a credible litigation threat, $\mathscr{F} \backslash \mathscr{L}$.

3.4.2 Patent Acquisition

With the equilibrium characterization in the licensing stage in mind, now let us turn to the subgame perfect Nash equilibrium of the patent acquisition game. It should be noted that this game can be considered as auctions with endogenous valuations first studied by Krishna (1993, 1999). Put differently, even though all patents have identical expected damage awards ex-ante, when they are auctioned sequentially, a value of such an ex-ante identical patent can vary depending on a patent allocation in which it is sold.

As similarly defined for final allocations, for an arbitrary intermediate patent allocation (i.e., $(n_1,n_2) \in \mathscr{A}$), let $V_i(n_1,n_2)$ denote the net continuation valuation of firm i in the subgame starting from that patent allocation. Thus, on an equilibrium path, $V_i(n_1,n_2)$ represents the sum of a net licensing fee which will be determined by a final allocation and payments for obtaining additional patents in firm i's patent portfolio from that time on.

Before solving the game, equilibrium selection and its implications are briefly discussed.[10] Equilibrium selection is needed because the second-price auction under complete information has multiple equilibria including seemingly quite unreasonable ones. Following the approach taken in many previous studies in the literature, I select the equilibrium in which each bidder bids the marginal increase in valuation of winning the patent given a patent allocation, (n_1,n_2), i.e.,[11]

$$b_1(n_1,n_2) = V_1(n_1+1,n_2) - V_1(n_1,n_2+1), \tag{3.2}$$

$$b_2(n_1,n_2) = V_2(n_1,n_2+1) - V_2(n_1+1,n_2). \tag{3.3}$$

Given the above optimal bidding strategies, if $b_1(n_1,n_2) > b_2(n_1,n_2)$, firm 1 wins the patent at the payment of firm 2's bid, and the game proceeds to (n_1+1,n_2). Therefore,

$$V_1(n_1,n_2) = V_1(n_1+1,n_2) - b_2(n_1,n_2), \tag{3.4}$$

$$V_2(n_1,n_2) = V_2(n_1+1,n_2). \tag{3.5}$$

Similarly, if $b_2(n_1,n_2) > b_1(n_1,n_2)$, firm 2 wins the patent, and the game proceeds to (n_1,n_2+1), and

$$V_1(n_1,n_2) = V_1(n_1,n_2+1), \tag{3.6}$$

$$V_2(n_1,n_2) = V_2(n_1,n_2+1) - b_1(n_1,n_2). \tag{3.7}$$

[10] Most of these preliminary results follow directly from Gale and Stegeman (2001).

[11] For more detailed explanations, see, for example, Dudey (1992), Gale and Stegeman (2001) and Krishna (1993, 1999). The above equilibrium selection is usually justified by applying iterated elimination of weakly dominated strategies in the literature.

When bids are tied, i.e., if $b_1(n_1,n_2) = b_2(n_1,n_2)$, it is immediate to see that (3.4) and (3.6) coincide with (3.5) and (3.7) respectively by using (3.2)~(3.3). This implies that equilibrium payoffs are uniquely determined even when the game involves multiple equilibria due to some ties. An equilibrium price is given by $\min\{b_1(n_1,n_2), b_2(n_1,n_2)\}$.

One important remark is that on an equilibrium path, the game must proceed towards the state with the higher total valuation due to the optimal bidding rules ((3.2) and (3.3)). To see this, define the total valuation function, $V(n_1,n_2) := V_1(n_1,n_2) + V_2(n_1,n_2)$. Then, firm 1 wins the auction at (n_1,n_2) if and only if $V(n_1+1,n_2) \geq V(n_1,n_2+1)$ while firm 2 wins the auction if and only if $V(n_1+1,n_2) \leq V(n_1,n_2+1)$. Following Gale and Stegeman (2001), I call this property of the equilibrium path the "principle of maximizing local valuation (*MLV*)."

Given the above preliminary results, one can easily analyze the last stages in which the N-th auction takes place (i.e., at allocations $(n_1,n_2) \in \mathscr{A}$ such that $n_1+n_2 = N-1$). There are three cases to consider: (i) regardless of the result of this auction, the game moves on to a state with credible litigation threat; (ii) regardless of the result of this auction, the game moves on to a state without credible litigation threat; (iii) depending on the result of this auction, the game moves on to either a state with credible litigation threat or a state without it.

For expositional ease, assume that firm 1 has more patents in its patent portfolio ($n_1 \geq n_2$). In case (i), i.e, $(n_1+1,n_2), (n_1,n_2+1) \in \mathscr{L}$, by (3.1), (3.2) and (3.3), both firms bid $2\alpha R$ in equilibrium, i.e., $b_i(n_1,n_2) = 2\alpha R$ for $i = \{1,2\}$. Then, since $n_1+n_2 = N-1$, (3.4) - (3.7) imply the payoff to the individual innovator selling her patent increases by this much, i.e., $V_i(n_1,n_2) = (2n_i - N)\alpha R$ for $i = \{1,2\}$. In case (ii), $(n_1+1,n_2), (n_1,n_2+1) \in \mathscr{F}\backslash\mathscr{L}$. Hence, no firm has an incentive to obtain an additional patent at a positive price, which gives rise to $b_i(n_1,n_2) = 0$ and $V_i(n_1,n_2) = 0$. Lastly, in case (iii), the result of the last auction determines whether firm 1 ends up having a credible litigation threat and collecting a licensing fee from firm 2. Thus, firms bid most aggressively here compared to the previous two cases. To see this, note that $b_i(n_1,n_2) = [(n_1+1)-n_2]\,\alpha R$ because $(n_1+1,n_2) \in \mathscr{L}$ but $(n_1,n_2+1) \in \mathscr{F}\backslash\mathscr{L}$. Also, since only the cases in which $K \geq 2$ are considered, $b_i(n_1,n_2) \geq 2\alpha R$, which implies $V_1(n_1,n_2) = 0$ and $V_2(n_1,n_2) =$

$-[(n_1+1)-n_2]\,\alpha R \leq -2\alpha R$. I call this case a "critical patent allocation."

Definition 2: *Suppose $n_1 \geq n_2$ without loss of generality. Then, $(\hat{n}_1, \hat{n}_2)$ is a "critical patent allocation" if and only if $\hat{n}_1 + \hat{n}_2 = N-1$, and $(\hat{n}_1+1, \hat{n}_2) \in \mathscr{L}$ but $(\hat{n}_1, \hat{n}_2+1) \in \mathscr{F} \backslash \mathscr{L}$.*

By taking a similar approach recursively, I can characterize equilibria of subgames starting from patent allocations that are relatively close to a final allocation in the following lemma.

Lemma 2

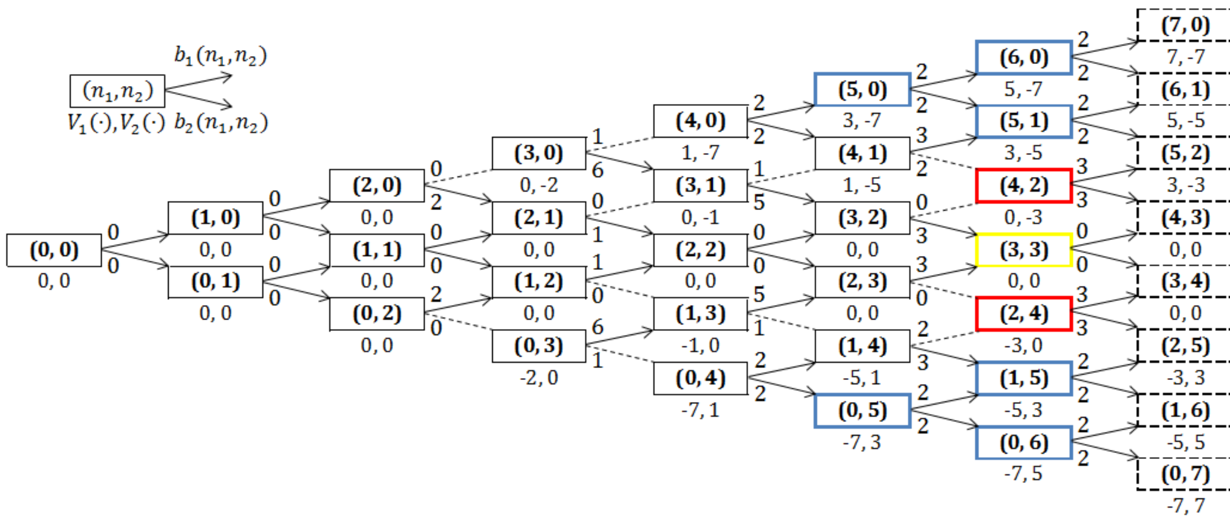
1. ***(Active Patent Acquisition)*** *In any subgames starting from $(n_1, n_2) \in \mathscr{A}$ such that $|n_i - n_j| - (N - n_i - n_j) \geq K$, $b_i(n_1, n_2) = 2\alpha R$ for all remaining patents, and thus $V_i(n_1, n_2) = (2n_i - N)\alpha R$ in equilibrium where $i = \{1, 2\}$.*

2. ***(Patent Acquisition Stalemate** (PAS))* *In any subgames starting from $(n_1, n_2) \in \mathscr{A}$ such that $(N - n_i - n_j) + |n_i - n_j| < K$, all remaining patents are sold at 0 prices in equilibrium, i.e., $b_i(n_1, n_2) = 0$, and thus $V_i(n_1, n_2) = 0$ where $i = \{1, 2\}$.*

3. *A critical patent allocation is never reached on the equilibrium path if a state of PAS exists.*

First, if a patent allocation is fairly asymmetric, firms bid $2\alpha R$ (result 1). In this case, the litigation threat will be eventually credible, and thus winning an additional patent has both offensive (by increasing n_i by 1) and defensive values (by not letting n_j increase by 1). In contrast, if a patent allocation is relatively symmetric, all remaining patents are auctioned off at 0 prices (result 2). Here firms have no incentive to acquire additional patents at positive prices because the patent acquisition game will end in a final state without a credible threat of litigation anyway. Lastly, result 3 explains the property of the critical patent allocations. If a *PAS* allocation exists, the last patent determines whether the leading firm will be eventually able to earn a licensing fee (greater than $2\alpha R$) or not, and thus both firms bid most aggressively at the critical patent allocations. The principle of *MLV* implies that anticipating this, firms would try to avoid reaching the critical patent allocations in the previous stage.

Figure 3.1: PE1 vs. PE2 Game for the Case of $N = 7$ and $K = 3$



Note: Numbers in dashed line rectangles indicate a final patent allocation $(n_1, n_2) \in \mathscr{F}$. Numbers in and below rectangles indicate a patent allocation $(n_1, n_2) \in \mathscr{A}$ and continuation payoffs of players, divided by αR, at that allocation respectively. Numbers above or below arrows or dashed lines indicate firm 1's bid and firm 2's, divided by αR, respectively. An arrow shows to which direction the game proceeds at a patent allocation in equilibrium. Finally, blue, red, and yellow rectangles represent a state of active patent acquisition, a critical patent allocation, and a state of *PAS* respectively.

Example 1 Consider the case of $N = 7$ and $K = 3$. As shown in Figure 1, a set of states of active patent acquisition is $\{(5,0),(5,1),(6,0),(0,5),(1,5),(0,6)\}$ at each of which $b_1(\cdot) = b_2(\cdot) = 2\alpha R$; a set of states of *PAS* $\{(3,3)\}$ at which $b_1(\cdot) = b_2(\cdot) = 0$. A set of critical patent allocations $\{(4,2),(2,4)\}$.

The results in Lemma 2 are not only useful for deriving the equilibrium of the entire game (as will be formulated in Proposition 1 shortly after) but also have an interesting implication in themselves. The aggregate payoff of individual innovators crucially depends on the initial patent allocation. To see this point, instead of assuming that all N patents are owned by the individual innovators, suppose that firm 1 and 2 have some patents in the beginning. Then, when the initial patent allocation is an active patent acquisition state, individual innovators sell their patents at the price of $2\alpha R$. On the other hand, when the initial allocation falls in the *PAS* region, individual innovators earn nothing.

Since Lemma 2 studies only for the subgames starting from patent allocations relatively close to the final allocations, a further analysis regarding all preceding subgames is necessary in order to find the equilibrium of the entire game. In the appendix, I show that only two qualitatively different type of equilibria can emerge depending on whether a state of *PAS* exists or not.

Proposition 1

1. *If a state of PAS does not exist ($K = 2$ and N is even), each patent is sold at $2\alpha R$ in any equilibrium, and thus the aggregate payoff to the individual inventors is the same as they would have earned through individual licensing contracts ($2N\alpha R$).*

2. *If a state of PAS exists ($K \geq 3$, or $K = 2$ and N is odd), the last auction (N-th patent) takes place at one of the PAS states, and each patent is sold at 0 in any equilibrium. Therefore, the aggregate payoff to the individual inventors is 0 .*

Proposition 1 tells us that when a *PAS* state exists (corresponding to most cases), surprisingly, PEs acquire all patents at 0 prices despite the competition between the two firms. Put differently, the existence of a *PAS* state enables PEs to pay nothing to use patented technologies and settle down a final allocation without a credible litigation threat. Loosely speaking, this is because whenever a *PAS* state exists, in an earlier stage of the game the losing firm is able to catch up in number of patents by bidding more aggressively in any equilibrium. To better understand this point, see Figure 1 which illustrates the analysis for the case of $N = 7$ and $K = 3$. For example, the firm 2, who is losing at $(2,0)$ or $(2,1)$, is able to win the patent by paying 0 and thereby avoid the game moving on closer to a state of active patent acquisition. Anticipating this, there is no incentive for any firm bids positive amount to be the leader in preceding stages.

Another implication of Proposition 1 is that the aggregate payoff of individual innovators decreases dramatically when patent ownership is fragmented whenever a *PAS* state exists. To see this, suppose all patents are owned by a single innovator instead. In sharp contrast to the above results, when the entire patents are sold all together at once, that is there is no chance that the firms can acquire a portion of patents, this all-or-nothing feature of the auction leads each firm to bid $2N\alpha R$. Thus, the innovator can earn $2N\alpha R$ either through an auction or a licensing negotiation.

3.5 Equilibrium Analysis: PE vs. NPE

In this section, I solve for the equilibrium of the game played between a PE and a NPE and contrast this with that of the game between the two PEs. As in the previous section, taking final patent allocations as given, I study the licensing problem first, and then the patent acquisition problem later.

3.5.1 Licensing in the Shadow of Litigation

A few modifications to the previous model are needed. Let (n,m) denote a patent allocation in which n and m represent the number of patents possessed by the PE and the NPE respectively. Since the NPE does not produce a product, the only source of profit for the NPE is licensing fees. Thus, NPE's payoff function is given by

$$\Pi_{npe} = V_{npe}(n,m) - (\text{sum of payments for obtaining } m \text{ patents}),$$

where $V_{npe}(n,m)$ regarding a final and an intermediate patent allocations is defined as in the previous model.

The main difference between the two games is that the NPE is not subject to counter-litigation because the NPE produces nothing and thus will not infringe any patents of PE. This implies the one-way nature of litigation between them: only the NPE can have a credible litigation threat when $m \geq K$. Thus, now the set of patent allocations with a credible litigation threat is $\mathscr{L} \equiv \{(n,m)|(n,m) \in \mathscr{F}, m \geq K\}$.

It is straightforward to show Lemma 3, the counterpart to Lemma 1 through an analogous analysis.

Lemma 3 *A PE and a NPE always settle out-of-court (no litigation) whenever a litigation threat of a NPE is credible, and the net licensing revenues of the PE to the NPE are given by*

$$V_1(n,m) = \begin{cases} -m\alpha R & \text{if } m \geq K, \\ 0 & \text{otherwise;} \end{cases} \tag{3.8}$$

$$V_{npe}(n,m) = \begin{cases} m\alpha R & \text{if } m \geq K, \\ 0 & \text{otherwise.} \end{cases} \tag{3.9}$$

3.5.2 Patent Acquisition

In order to solve the subgame perfect Nash equilibrium of the patent acquisition game, I take a similar strategy as that used in the game between the two PEs. A critical state is defined first. Lemma 4, the counterpart of Lemma 2, follows. Finally, based on Lemma 4, I derive Proposition 2, the counterpart of Proposition 1.

Definition 3: $(\hat{n},\hat{m})$ *is a critical state if* $\hat{n}+\hat{m}=N-1$, $(\hat{n},\hat{m}+1)\in\mathscr{L}$ and $(\hat{n}+1,\hat{m})\in\mathscr{F}\backslash\mathscr{L}$.[12]

Lemma 4

1. *(**Active Patent Acquisition**) In any subgames starting from* $(n,m)\in\mathscr{A}$ *such that* $m\geq K$, $b_1(n,m)=b_{npe}(n,m)=\alpha R$ *for all remaining patents, and thus* $V_1(n,m)=-(N-n)\alpha R$ *and* $V_{npe}(n,m)=m\alpha R$ *in equilibrium.*

2. *(**Patent Acquisition Stalemate**) In any subgames given* $(n,m)\in\mathscr{A}$ *such that* $(N-n)<K$, *all remaining patents are sold at 0 prices in equilibrium, i.e.,* $b_i(n,m)=0$, *and thus* $V_i(n,m)=0$ *where* $i=\{1,npe\}$.

[12]Note that in the previous game, assuming $\hat{n}_1\geq\hat{n}_2$, at the critical state, $\hat{n}_2$ is an integer such that $(N-K)/2-1<\hat{n}_2\leq(N-K)/2$. Here, $(\hat{n},\hat{m})=(N-K,K-1)$. Thus, $\hat{m}$ is constant when N changes while all other numbers vary.

3. *A critical patent allocation,* $(N-m,m-1)$*, is never reached on the equilibrium path.*

The results in Lemma 4 are similar to those in Lemma 2 in that in a state of active patent acquisition, both the PE and the NPE bid positive amount (result 1); in a state of *PAS*, they bid 0 (result 2); and in a critical patent allocation they bid most aggressively (result 3).

However, Lemma 4 differs significantly from Lemma 2 in two respects. First, as already pointed out, the NPE is the only one who can earn positive licensing revenue. More importantly, it is not how much a patent allocation is symmetric but how many patents the PE has that determines a *PAS* state. By the same token, it is not how much a patent allocation is asymmetric but how many patents the NPE has that determines that determines an active patent acquisition state.

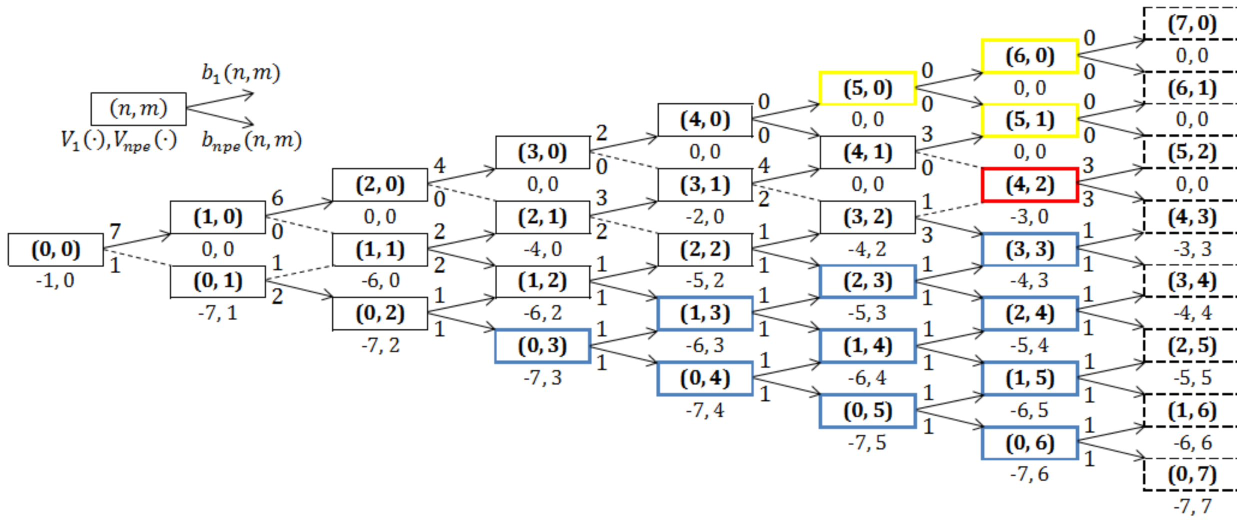
Example 2 Consider the case of $N=7$ and $K=3$. As shown in Figure 2, a set of states of active patent acquisition is $\{(0,3),(1,3),(0,4),(2,3),(1,4),(0,5),(3,3),(2,4),\ (1,5),(0,6)\}$ at each of which $b_1(\cdot)=b_{npe}(\cdot)=\alpha R$; a set of states of *PAS* $\{(5,0),(6,0),(5,1)\}$ at each of which $b_1(\cdot)=b_{npe}(\cdot)=0$. A set of critical patent allocations $\{(4,2)\}$.

In order to fully characterize the equilibrium of this game, it remains to analyze what happens at the patent allocations not studied in Lemma 4. To this end, given N and K, I only consider allocations $(n,m)\in\mathscr{A}$ such that $n\leq N-K$ and $m\leq K$. Assuming N large enough,[13] K different classes of allocations are studied in turn where class-i ($i\in\{1,\cdots,K\}$) allocations are defined as $(n,K-i)$ where $n\in\{N-K,\ldots,0\}$. That is, by backward induction I study first for class-1 allocations from $(N-K,K-1)$ to $(0,K-1)$, next class-2 allocations from $(N-K,K-2)$ to $(0,K-2)$, and so on all the way up to class-K allocations. Before reporting results for general cases, it is helpful to see the example below first.

Example 3 Consider the case of $N=7$ and $K=3$ in Figure 2 as an example again. Note that an equilibrium path must be a sequence of patent allocations from the initial state $(0,0)$ to a final

[13] See the proof for Proposition 2 for the reason why we can assume this without loss of generality.

Figure 3.2: PE vs. NPE Game for the Case of $N = 7$ and $K = 3$



Note: Numbers in dashed line rectangles indicate a final patent allocation $(n,m) \in \mathscr{F}$. Numbers in and below rectangles indicate a patent allocation $(n,m) \in \mathscr{A}$ and continuation payoffs, divided by αR, of players at that allocation respectively. Numbers above or below arrows or dashed lines indicate PE's bid and NPE's, divided by αR, respectively. An arrow shows to which direction the game proceeds at each patent allocation in equilibrium. Finally, blue, red, and yellow rectangles represent a state of active patent acquisition, a critical patent allocation, and a state of *PAS* respectively.

allocation linked by arrows. Consider class-1 allocations (i.e., $(4,2),(3,2),\ldots,(0,2)$) first. As seen in the figure, the NPE first strictly wins a patent at the allocation of $(3,2)$, which is lead to a final allocation with a credible litigation threat (i.e., $(4,3)$ or $(3,4) \in \mathscr{F}$) in equilibrium. Note that it takes two stages for the NPE to strictly win an auction for the first time from the back. A similar pattern is observed for class-2 allocations (i.e., $(4,1),(3,1),\ldots,(0,1)$) as well. While when n is relatively large ($(4,1)$, $(3,1)$, or $(2,1)$), the PE wins the patent, and the game leads to an allocation without a litigation threat, when n is relatively small, an auction is tied or won by the NPE. At $(0,1)$, NPE strictly wins the auction, and the game reaches a final allocation with a litigation threat. Since the second and the 6th patent are auctioned off at $(0,1)$ and $(3,2)$ respectively, it takes 4 additional stages for the NPE to strictly win an auction for the first time from the back. I denote such additional stages for class-i allocations as $\Delta(i)$. Also, it is shown that in any equilibrium, the PE only pays αR to acquire the first auction; the all the rest of patents are sold at 0; and the game reaches one of a final allocations without a litigation threat.

It turns out that $\Delta(i)$, defined as in the above example, is an important variable in characterizing the equilibrium of this game. By formally constructing a sequence $\{\Delta(i)\}_{i=1}^{K-1}$, I report the subgame perfect Nash equilibrium for general N and K in the following proposition.

Proposition 2 *The game ends in a final allocation without a credible litigation threat (i.e., the auction for N-th patent always takes place at one of the PAS states) in equilibrium. Moreover,*

1. *if the number of patents is sufficiently large ($N > \sum_{i=1}^{K-1}\Delta(i)$) given K, PE wins each patent a few auctions during earlier stages (up to the ($N - \sum_{i=1}^{K-1}\Delta(i)$)-th auction) at αR, and then the remaining patents are sold at 0. Thus, the aggregate payoff to the individual inventors is $\left[N - \sum_{i=1}^{K-1}\Delta(i)\right]\alpha R$;*

2. *Otherwise, all patent is sold at 0 in any equilibrium. Therefore, the aggregate payoff to the individual inventors is 0.*

Here, $\Delta(i)$ is a sequence constructed recursively as follows: for $i = 2$, $\Delta(1) = 2$, and for $i \geq 2$, $\Delta(i)$ is the smallest integer Δ satisfying the following inequality

$$\Delta > \frac{2(K-i)+1+\sum_{s=1}^{i-1}\Delta(s)+\left[V_{npe}\left(\tilde{n}(i-1),K-i\right)/\alpha R\right]}{K-i}; \tag{3.10}$$

where $V_{npe}\left(\tilde{n}(i-1),K-(i-1)\right)$ is a sequence constructed recursively as follows: for $i = 2$, $(K-1)\alpha R$, and for $i \geq 3$,

$$V_{npe}\left(\widetilde{n}(i-2),K-(i-2)\right)+\Delta(i-1)\left[K-(i-2)\right]-\sum_{s=1}^{i-1}\Delta(s)-2(K-i)-3. \tag{3.11}$$

Though the NPE never acquires any patent in equilibrium, the existence of the NPE induces the PE to purchase a few patents at αR in earlier stages if the total number of patents is relatively large. This is in sharp contrast to the result obtained in the game between PEs because the

Table 3.1: Minimum Threshold Number of Patents

K	2	3	4	5	6	7	8	9	10	$\cdots$
$\sum_{i=1}^{K-1} \Delta(i)$	2	6	12	20	30	42	56	72	90	$\cdots$

aggregate payoff to the individual innovators may be positive even with a *PAS* state. The main discrepancy between the results of the two games stems from the different litigation nature between the two games. In contrast to the previous game in which PEs amass patents for both offensive and defensive purposes, in this game, PE acquires patents for only defensive purposes while the NPE does so for only offensive purposes.

The amount paid to the individual innovators depends on the relative magnitudes of N and $\sum_{i=1}^{K-1} \Delta(i)$. The intuition behind this result is as follows. In each class-i allocations, as illustrated in Example 3, the PE tries to prevent the game moving towards a state of active patent acquisition by acquiring a patent. However, this defensive patent acquisition is costly for the PE as well. Thus, the PEs bid aggressive to win only when an allocation is relatively close to a *PAS* state. Loosely speaking, $\sum_{i=1}^{K-1} \Delta(i)$ captures the upper limit to which the PE can use such strategy when the game is analyzed up to class-$(K-1)$ allocations. On the other hand, when N increases, the NPE has more opportunities to acquire a patent for offensive purposes. This is because when N increases, the area of *PAS* remains the same as before while the area of active patent acquisition is extended. Therefore, only when N is greater than $\sum_{i=1}^{K-1} \Delta(i)$, the aggregate payoff to the individual innovators is given by the difference between the two terms. Interestingly, the minimum threshold number of patents $\sum_{i=1}^{K-1} \Delta(i)$, which guarantees a positive payoff to the individual innovators, is not proportional but increases more rapidly along with an increase in K. I report the values of $\sum_{i=1}^{K-1} \Delta(i)$ using (3.10) and (3.11) for a couple of different values of K in Table 1.

3.6 Conclusion

This paper considers a situation in which the threat of litigation by an individual innovator is not credible, and thus individual innovators' patents are sold through a sequential auction. The two games are studied in turn: the game played by the two PEs in which they compete to acquire the patents, and then negotiate licensing fees based on their patent portfolio in the shadow of litigation, and then, the same game played by PE and NPE.

It is shown that in the former game, all patents are sold at 0 prices whenever there exists an intermediate state of patent acquisition stalemate (*PAS*). This is surprising considering that if all patents were possessed by an innovator and auctioned together, they would have been sold at a price equal to the highest attainable licensing fees ($2N\alpha R$). In contrast, in the latter game, even with a *PAS* state, the existence of NPE induces PE to purchase some patents at positive prices as long as the total number of patents are sufficiently large. Therefore, in sum, this paper suggests that fragmented patent ownership can make it hard for the individual innovators to monetize their patents, and in this situation NPE can play an important role of increasing the aggregate payoff to the individual innovators without any opportunistic behaviors though such role of NPE is limited in the sense that the amount paid to the individual innovators is less than the highest attainable licensing fees ($N\alpha R$).

Since this analysis is based on many stylized assumptions, one can think of many natural extensions of the current framework. For example, one can check to what extent the key findings of this paper are robust when the model allows for injunction possibilities in addition to damage awards. Another important extension is to consider the game played by multiple PEs and a NPE. One technical difficulty associated with such extension is that continuation payoffs of each player in an intermediate patent allocation may not be uniquely defined because multiple equilibria with different winners may emerge in many cases. These remain interesting venues for future research.

APPENDIX

Appendix for Chapter 3

Proof of Lemma 1

The proof directly follows from Proposition 1 in Lemus and Temnyalov (forthcoming). ■

Proof of Lemma 2

Assume $n_1 \geq n_2$ without loss of generality.

Result 1: when the N-th auction takes place in this class of subgames, $V_1(n_1,n_2) = (2n_1 - N)\alpha R$ and $V_2(n_1,n_2) = (2n_2 - N)\alpha R$. Let us go one stage back, and consider only patent allocations being lead to one in $\mathscr{L}$. Then, in every auction for the $(N-1)$-th patent, $b_i(n_1,n_2) = 2\alpha R$. By doing this recursively back to the $(N-t)$-th patent auction, it is easy to see that for any $(n_1,n_2) \in \mathscr{A}$ such that $n_1 + n_2 = N - (t+1)$, $b_1(n_1,n_2) = b_2(n_1,n_2) = 2\alpha R$, $V_1(n_1,n_2) = (2n_1 - N)\alpha R$, $V_2(n_1,n_2) = (2n_2 - N)\alpha R$ and $V(n_1,n_2) = 2\left[(n_1+n_2) - N\right]\alpha R$.

Result 2: when the N-th auction takes place in this class of subgames, $V_1(n_1,n_2) = V_2(n_1,n_2) = 0$, which immediately implies $b_1(n_1,n_2) = b_2(n_1,n_2) = 0$. Then, backward induction implies result 2.

Result 3: note that $V_1(\hat{n}_1,\hat{n}_2) = 0$ and $V_2(\hat{n}_1,\hat{n}_2) = -\left[(\hat{n}_1+1) - \hat{n}_2\right]\alpha R$. Also, note that there are only two allocations which can be lead to $(\hat{n}_1,\hat{n}_2)$: $(\hat{n}_1,\hat{n}_2 - 1)$ and $(\hat{n}_1 - 1,\hat{n}_2)$. Thus, it is sufficient to show that (i) firm 1 always wins the auction at $(\hat{n}_1,\hat{n}_2 - 1)$, which is true if and only if $V(\hat{n}_1+1,\hat{n}_2-1) > V(\hat{n}_1,\hat{n}_2)$ by *MLV*, and (ii) firm 2 at $(\hat{n}_1 -, \hat{n}_2)$, which is true if and only if $V(\hat{n}_1 - 1,\hat{n}_2 + 1) > V(\hat{n}_1,\hat{n}_2)$. To prove claim (i), let us compare the magnitudes of $V(\hat{n}_1+1,\hat{n}_2 - 1)$ and $V(\hat{n}_1,\hat{n}_2)$. Since $(\hat{n}_1+1,\hat{n}_2-1)$ is an allocation of 'Active Patent Acquisition,'

$$V(\hat{n}_1+1,\hat{n}_2-1) = 2(\hat{n}_1+1+\hat{n}_2-1-N)\alpha R = -2\alpha R.$$

However, whenever a *PAS* state exists, i.e., either when $K \geq 3$ or when $K = 2$ and N is odd,

$$V(\hat{n}_1,\hat{n}_2) = -\left[(\hat{n}_1+1) - \hat{n}_2\right]\alpha R < -2\alpha R,$$

which implies $V(\hat{n}_1+1,\hat{n}_2-1) > V(\hat{n}_1,\hat{n}_2)$. The proof of claim (ii) is analogous. ■

Proof of Proposition 1

Result 1: When $K=2$ and N is even, at all final allocations but $(N/2,N/2)$, litigation threat is credible (i.e., $\mathscr{F}\setminus\{(N/2,N/2\} = \mathscr{L}$). This implies that $V_i(n_1,n_2) = (n_i-n_j)\alpha R$ for all $(n_1,n_2) \in \mathscr{F}$ and $b_i(n_1,n_2) = 2\alpha R$ for all $(n_1,n_2) \in \mathscr{A}$ such that $n_1+n_2 = N-1$ where $i=\{1,2\}$. By doing this recursively, it is easy to see that $b_i(n_1,n_2) = 2\alpha R$ for all $(n_1,n_2) \in \mathscr{A}$.

Result 2: Consider the case $K \geq 3$ and N is an odd number. The proof for the other cases ($K=2$ and N is odd, or $K \geq 3$ and N is even) are analogous. I walk through the following two steps.

Step 1: First, I prove that result 2 holds when $N \geq K = 3$ by induction. It is straightforward to check that result 2 holds by backward induction when $N=K=3$. Now it suffices to check that if result 2 holds for $N>3$, then so does result 2 for $N+2$. To prove this, define $N' = N+2$ and denote all corresponding allocations associated with N' by $(n_1',n_2') \in \mathscr{A}' \cup \mathscr{F}'$ where $\mathscr{A}' = \{(n_1',n_2')|n_1'+n_2' < N+2\}$ and $\mathscr{F}' = \{(n_1',n_2')|n_1'+n_2' = N+2\}$. Note that

$$\mathscr{F}' = \left\{(n_1',n_2')|(n_1',n_2') = (n_1+1,n_2+1), {}^{\forall}(n_1,n_2) \in \mathscr{F}\right\} \cup \{(N',0),(0,N')\}.$$

Then, the game associated with N' is equivalent to the one with N in terms of payoff structure for all final allocations except extreme allocations: $(N,0)$ and $(0,N)$, i.e.,

$$\begin{aligned} V_i'(n_1',n_2') &= \begin{cases} (n_i'-n_j')\alpha R & \text{if } |n_i'-n_j'|\alpha R \geq L \\ 0 & \text{otherwise} \end{cases} \\ &= V_i(n_1,n_2) \end{aligned}$$

for all $(n_1',n_2') \in \mathscr{F}'\setminus\{(N',0),(0,N')\}$ where $n_1' = n_1+1$ and $n_2' = n_2+1$ since $n_i'-n_j' = n_i-n_j$. This implies that the equilibrium consequences of the subgame including only allocations in which both firms have at least 1 patent ($(n_1',n_2') \in \mathscr{A}' \cup \mathscr{F}'$such that n_1', $n_2' > 0$) and the entire game with N are exactly the same.

Suppose that result 2 holds for the case of N but not for the case of N'. Then, since the game ends in *PAS* given N, it must be the case, by Lemma 2-3, that there exists $(n_1^{*\prime},1)$ on the equilibrium path of the subgame starting from $(1,1)$, which leads the firms to *PAS* in the end. Note that $(n_1^{*\prime},1)$ must be such that $n_1^{*\prime} \leq \hat{n}_1' - 1$, $b_1(n_1^{*\prime},1) > b_2(n_1^{*\prime},1)$ and $V_1(n_1^{*\prime},1) = V_2(n_1^{*\prime},0) = 0$ where $\hat{n}_1'$ is the firm 1's patent share in the critical allocation given N'. Then, since all $(n_1',1)$ such that $n_1' \leq n_1^{*\prime}$ are lead to *PAS*, at $(n_1^{*\prime},0)$ it must be the case $b_1(n_1^{*\prime},0) \geq b_2(n_1^{*\prime},0)$, or $V(n_1^{*\prime}+1,0) \geq V(n_1^{*\prime},1) = 0$. However, if the game could end in an allocation in $\mathscr{L}'$, some patents will be sold at a positive price in equilibrium of the subgame starting from $(n_1^{*\prime}+1,0)$. If this is the case, then it must be the case $V(n_1^{*\prime}+1,0) < 0$ because the price paid weakly decreases along any equilibrium path, by Theorem 1A in Gale and Stegeman (2001), which is a contradiction.

Step 2: It completes the proof to show that given an arbitrary odd number N, result 2 holds for all $K \in \{3,4,\cdots,N\}$. By step 1, given N, result 2 must hold for the case $K = 3$. Now it remains to check if result 2 holds for $K > 3$, then so does result 2 for $K+1$. This can be shown following an approach analogous to that used in step 1. ■

Proof of Lemma 3

The proof is analogous to that of Lemma 1 and omitted. ■

Proof of Lemma 4

The proof is analogous to that of Lemma 2 and omitted. ■

Proof of Proposition 2

Given K and N, I formalize the idea discussed in the text by considering and studying allocations $(n,m) \in \mathscr{A}$ such that $n \leq N-K$ and $m \leq K$. First, I claim that assuming any arbitrary N given K is innocuous. Similar to the PE vs. PE game, the game associated with $N' = N+1$ is equivalent to the one with N in terms of payoff structure for all final allocations except extreme allocations: $(0,N)$. Put differently, if $(n',m') \in \mathscr{A}' \cup \mathscr{F}'$ where $\mathscr{A}' = \{(n',m')|n'+m' < N+1\}$ and $\mathscr{F}' = \left\{(n',m')|(n',m') = (n+1,m)^{\forall}(n_1,n_2) \in \mathscr{F}\right\} \cup \{(0,N')\}$, the game associated with N' is equivalent to the one with N in terms of payoff structure for all final allocations except the extreme allocation: $(0,N')$, i.e.,

$$V_1'(n',m') = \left\{ \begin{array}{ll} -m'\alpha R & \text{if } m' \geq K \\ 0 & \text{otherwise} \end{array} \right\} = V_1(n,m)$$

$$V_{npe}'(n',m') = \left\{ \begin{array}{ll} m'\alpha R & \text{if } m' \geq K \\ 0 & \text{otherwise} \end{array} \right\} = V_{npe}(n,m)$$

for all $(n',m') \in \mathscr{F}' \backslash \{(0,N')\}$ where $n' = n+1$ since n' is independent of credibility of NPE's litigation threat. This implies that the equilibrium consequences of the subgame including only allocations in which PE has at least 1 patent ($(n',m') \in \mathscr{A}' \cup \mathscr{F}'$such that $n' > 0$) and the entire game with N are exactly the same. Furthermore, regarding newly added allocations such that $(0,m)$ and $m > K$, the analysis is straightforward by result 1 in Lemma 4.

Now consider the following three cases in turn: (i) class-1 allocations, (ii) from class-2 to class-$(K-1)$ allocations, and (iii) class-K allocations.[14]

Case (i): By Lemma 2, it is immediate to see that NPE wins the $(N-2)$-th auction since $b_1(n,K-1) = \alpha R < b_{npe}(n,K-1) = K\alpha R$, but all the other auctions are tied at $b_1(n,K-1) = b_{npe}(n,K-1)$. This implies that for all class-1 allocations, all patents are sold at αR, and thus, $V_1(n,K-1) = -(N-n)\alpha R$ and $V_{npe}(n,K-1) = (K-1)\alpha R$.

Case (ii): First, consider class-2 allocations: $(n,K-2)$ to $(n,K-2)$ where $n = \{N-K, N-K-1, \ldots 0\}$. When $n = N-K$, by Lemma 4 we know that PE wins the auction by paying 0, and the game proceeds to a *PAS* state. Hence, $V_1(N-K,K-2) = V_{npe}(N-K,K-2) = 0$. Now consider the case $n = N-K-1$. From the previous result and the result in case (i), NPE's bid is $(K-1)\alpha R$ while PE's bid is $(K+1)\alpha R$, and thus PE wins this auction. However, as n decreases, it becomes

[14] Note that when $K = 2$, only case (i) and (iii) are relevant.

too costly for PE to win the auction to avoid entering into a state of active patent acquisition. Given N is large enough, NPE wins the auction finally at some n and thereafter the winning bid becomes αR until we reach the case $n = 0$.

In a similar way, one can demonstrate that similar properties hold for all the other classes. From applying a similar approach shown above for the class-2 allocations and using (3.2) - (3.7), one can construct a sequence $\{\Delta(i)\}_{i=1}^{K-1}$, which characterizes an important equilibrium consequence for these class of allocations. $\Delta(i)$ denotes additional stages, needed for NPE to win the auction for the first time from the back when considering class-i allocations, on top of the total number of such stages up to class-$(i-1)$ allocations, i.e., $\sum_{s=1}^{i-1}\Delta(s)$. It turns out that how much PE pays to the individual innovators in equilibrium crucially depends on $\sum_{i=1}^{K-1}\Delta(i)$, we focus on how to derive this sequence.

For the case of $i = 1$, from the result in case (i), we know that $\Delta(1) = 2$. For all the other cases, $k \geq 2$, one can easily verify that $\Delta(i)$ is constructed recursively as follows:

(ii)-A. For a relatively large n such that $N-(K-i)-\sum_{s=1}^{i-1}\Delta(s) \leq n \leq N-K$, the winning bid is always 0, and thus, $V_1(n,K-i) = V_{npe}(n,K-i) = 0$;

(ii)-B. For a relatively small n such that $0 \leq n < N-(K-i)-\sum_{s=1}^{i}\Delta(s)$, the winning bid is always αR, and thus, $V_1(n,K-i) = -(N-n)\alpha R$ and $V_{npe}(n,K-i) = (K-i)\alpha R$;

(iii)-C. For an intermediate n such that $N-(K-i)-\sum_{s=1}^{i}\Delta(s) \leq n < N-(K-i)-\sum_{s=1}^{i-1}\Delta(s)$,

$$b_1(n,K-i)\begin{cases} < b_{npe}(n,K-i) & \text{if } n = N-(K-i)-\sum_{s=1}^{k}\Delta(s), \\ \geq b_{npe}(n,K-i) & \text{otherwise.} \end{cases}$$

For notational simplicity let $\widetilde{n}(i) \equiv N-(K-i)-\sum_{s=1}^{i}\Delta(s)$. With this definition, $N-(K-i)-\sum_{s=1}^{i-1}\Delta(s)$ can also be simply rewritten as $\widetilde{n}(i-1)+1$.

Now, to figure out $\Delta(i)$, let us calculate the value of $b_{npe}(n,K-i)$ first in this range of n. Recall that,

$$b_{npe}(n,K-i) = V_{npe}(n,K-(i-1)) - V_{npe}(n+1,K-i).$$

Note that in this range of n, $V_{npe}(n+1,K-i)=0$ by (ii)-A. Therefore, considering (ii)-B,

$$b_{npe}(n,K-i)=\begin{cases} V_{npe}(n,K-(i-1)) \leq (K-(i-1))\,\alpha R & \text{if } n=\widetilde{n}(i-1), \\ (K-(i-1))\,\alpha R & \text{otherwise.} \end{cases} \tag{3.12}$$

Then, this implies that for n ranging from $\widetilde{n}(i-1)-1$ to $\widetilde{n}(i)$,

$$V_1(n+1,K-i)=-\left[\sum_{s=n+1}^{\widetilde{n}(i-1)} b_{npe}(s,K-i)\right] \tag{3.13}$$

since by construction PE wins or ties as long as $n>\widetilde{n}(i)$, and by (ii)-A $V_1(n,K-i)=0$ for $n \geq \widetilde{n}(i-1)+1$. On the other hand, for n ranging from $\widetilde{n}(i-1)-1$ to $\widetilde{n}(i)$, by (ii)-B,

$$V_1(n,K-(i-1))=-(N-n)\alpha R.$$

Therefore, here

$$b_1(n,K-i)=(N-n)\alpha R-\left[\sum_{s=n+1}^{\widetilde{n}(i-1)} b_{npe}(s,K-i)\right]. \tag{3.14}$$

From the discussion so far, it must be the case that $\Delta(i) \equiv \widetilde{n}(i-1)-\widetilde{n}(i)+1$ is a smallest positive integer Δ such that

$$\begin{aligned} & b_{npe}(\widetilde{n}(i-1)-\Delta+1,K-i)-b_1(\widetilde{n}(i-1)-\Delta+1,K-i) \\ = & \left\{ \begin{array}{l} [K-(i-1)]\,\alpha R-[N-\widetilde{n}(i-1)+\Delta-1]\,\alpha R \\ +V_{npe}(\widetilde{n}(i-1),K-(i-1))+(\Delta-2)\,[K-(i-1)]\,\alpha R \end{array} \right\} > 0 \end{aligned}$$

where the equality follows from (3.12) and (3.14). This is equivalent to, by the definition of $\tilde{n}(i)$,

$$\Delta > \frac{2(K-i)+1+\sum_{s=1}^{i-1}\Delta(s)-V_{npe}(\widetilde{n}(i-1),K-(i-1))\,/\,\alpha R}{K-i}.$$

Now it is sufficient to figure out $V_{npe}(\widetilde{n}(i-1),K-(i-1))$. If $i=2$,

$$V_{npe}(\widetilde{n}(i-1),K-(i-1))=(K-1)\alpha R.$$

If $i \geq 2$, since NPE wins at $(\tilde{n}(i-1), K-(i-1))$,

$$
\begin{aligned}
& V_{npe}\left(\tilde{n}(i-1), K-(i-1)\right) \\
= & V\left(\widetilde{n}(i-1), K-(i-2)\right) - V_1\left(\widetilde{n}(i-1)+1, K-(i-1)\right) \\
= & \left[\widetilde{n}(i-1) - N + K - (i-2)\right] \alpha R \\
& + b_{npe}\left(\widetilde{n}(i-2), K-(i-1)\right) + \left(\Delta(i-1)-2\right)\left(K-(i-2)\right)\alpha R \\
= & V_{npe}\left(\widetilde{n}(i-2), K-(i-2)\right) \\
& - \sum_{s=1}^{i-1} \Delta(s) + \Delta(i-1)\left(K-(i-2)\right) - 2(K-i) - 3
\end{aligned}
$$

where the second equality follows from (ii)-A and (3.13), and the third equality follows from rearranging terms and (3.13).

Case (iii): Having all the results drawn in case (i) and (ii), one can investigate $(n,0)$ allocations in a similar way. It turns out that if $n > \widetilde{n}(K-1)$, an auction is tied, or PE wins the auction at 0 price, leading to a *PAS* state. Otherwise, i.e., $n \leq \widetilde{n}(K-1)$, PE wins the auction by paying αR. Hence, if $N > \sum_{i=1}^{K-1} \Delta(i)$, the aggregate payoff to the individual innovators is given by $\left[N - \sum_{i=1}^{K-1} \Delta(i)\right] \alpha R$. ■

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