TEACHERS' REPORTED DIGITAL TOOL USE WITHIN AN INQUIRY-BASED, PROBLEM-CENTERED MIDDLE SCHOOL MATHEMATICS CURRICULUM

By

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ABSTRACT

TEACHERS' REPORTED DIGITAL TOOL USE WITHIN AN INQUIRY-BASED, PROBLEM-CENTERED MIDDLE SCHOOL MATHEMATICS CURRICULUM

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Abstract: Mathematics teachers use various types of digital tools, including common mathematics-specific digital tools and general-purpose digital tools, in ways to support their instruction, student learning, and curricular goals. This study examined teachers' reported use of digital tools within units of an inquiry-based, problem centered curriculum to determine: (1) what digital tools teachers report using within problems in a unit and the manner of the use and (2) how those digital tools were used within problems and across problems in a unit. Inquiry-based, problem centered curricula provide opportunities to examine teachers' inclusion of digital tools within the mathematics classroom around a specific, common instructional method.

Eight middle school teachers were selected based on self-reported moderate to high level of digital tool use and using the desired curriculum series. Each teacher described their use of digital tools on researcher selected problems during a single interview lasting approximately 1.5 hours. Digital tools were primarily used to: (1) support understanding the problem context through videos, (2) carry out straight-forward mathematical tasks through calculators and shared Google applications, (3) facilitate whole-classroom interactions through document cameras, other digital display devices, and shared Google applications, and (4) support summative use for recall, feedback, and artifact creation through video creation, written feedback applications, and shared Google applications. The four uses were strongly rooted in teachers' reported goals for the Launch-Explore-Summarize-Reflect instructional model and desired classroom-interaction characteristics. No notable differences in the use of digital tools were observed across problems

at the investigation or unit level. The analysis suggests that teachers use digital tools to strengthen classroom-interaction of the instructional model. This work provides guidance for future consideration of the creation and inclusion of digital tools within inquiry-based, problem-centered mathematics curricula by outlining the types of digital tool uses and their purpose that are reported on through the various phases of individual problems.

I dedicate this to all my former students and teachers.

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CHAPTER 1: INTRODUCTION

Mathematics teachers use numerous types of tools to support their instruction, student learning, and curricular goals. Tools have provided ways for teachers and students to communicate mathematical ideas for years, from using an abacus for calculations, compass for constructing circles in a plane, and protractor for measuring angles. As time has moved forward and moved beyond these tools, many of these same actions can be done with more modern tools, such as digital calculators for calculations and the use of dynamic geometry software for creating circles and measuring angles. The type of tool to be used for a particular action may be predicated on where students are in their mathematics learning experience (Niess et al., 2009). Digital calculators may not make sense to use in the early grades when students are trying to build conceptual understanding of numeric operations, yet an abacus could help those students with creating mental models of arithmetic. On the other hand, using a digital calculator for computation at the high school level makes sense over the use of tools that support conceptual understanding of computations.

How teachers *think about and use* support materials that are either curricular-provided or from external sources for use in their teaching of the mathematics curriculum is of great interest. Different tools can be used for the same action but whether they are appropriate or strategic depends on the context of the classroom: the mathematics being taught, students' prior knowledge, the curricular materials used, and the instructional model used, etc. These are aspects that teachers take into account when deciding on which materials to use to support their instruction, student learning, and curricular goals. Digital tools are a type of support material.

Researchers have investigated how teachers implement curricular materials (Choppin, 2011a; Choppin, 2011b; McMaken & Porter, 2012), including how curricular resources are represented in the written curriculum (Freeman & Rudnick, 2012), how teachers view the written curriculum during planning (Behm & Lloyd, 2009), how the curriculum is implemented in the classroom (Heck, Chval, Weiss, & Ziebarth, 2012; Remillard & Heck, 2014), and associated student learning outcomes with the new curricula (Stein, Grover, & Henningsen, 1996; Wilson & Roseman, 2012). Overall, how curricular materials are used to teach K-12 mathematics has been an interest of recent research in mathematics education (Remillard, 2009; Lloyd, Remillard, & Herbel-Eisenmann, 2009).

As a subcategory of these support materials, digital technologies, or digital tools, that come with the curriculum may be tailored to tasks within the curriculum (Asay, 2016). Digital tools from sources external to the curriculum may be less likely to be tailored to the written tasks in the curriculum but may provide opportunities of use to attend to aspects of instruction, student learning, and curricular goals. Given the rapid growth in technology over the past few decades, a plethora of new tools are now omnipresent in today's mathematics classroom, including web-based tools, software, and new hardwares. Recent curriculum revisions and the creation of new mathematics curricula have provided updated digital tools for teacher and student consumption as well as exposure to digital tools not directly associated with the curricula. What digital tools teachers are using within curricula, how they utilize technology relative to the curricula, and why they are using technology with these curricula has had minimal recent exploration and needs to be examined.

The use of digital tools within a written curriculum may enhance the mathematical experience for students (Zbiek & Hollebrands, 2008). Digital tool presence and suggested use of digital tools within a set of curricular materials varies across curricula and schools (Rock & Brumbaugh, 2013). The tools themselves do not promote or guarantee an effective mathematical experience, but the ways in which tools are used with the content of the curriculum in the classroom by the teacher and with students can have a profound impact on the learner (Laborde, 2016). Teachers mediate experiences that students have with using digital tools in the classroom (Remillard, 2016). Teachers are finding implementation of digital tools with newer hardwares into the mathematics classroom to be much more challenging than expected (Johnson, 2016).

Technology as Described in Policy Documents

Over the past two decades, numerous policy documents have been developed which emphasize the use of digital tools. Four of most interest are the *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), the *Common Core State Standards for Mathematics* (CCSSI, 2010), and NCTM's position statements (2015). Given the spread of years of these policy documents, their stated uses of digital tools provides some insight as to what was deemed to be important or budding at that time.

For instance, in *Adding It Up* (2001), the use of calculators and computer are discussed in generalities within the five intertwined strands of proficiency in which technology is not one of the strands but also recognizes that more research needs to be done in the area on their effective use within the mathematics classroom to attend to the five strands of mathematical proficiency. *The Principles and Standards for School Mathematics* (2000) suggests that the use of calculators and computers will make for classrooms where student can "focus on decision making, reflection, reasoning, and problem solving" (p. 24). Both of these early documents take the stance that the use of calculators and computers can be beneficial in the ability to create and examine different representations as well as provide efficiency in the creation of these representations. Efficiency in representation construction would provide teachers and students the ability to interact with more examples or offload time to examine the representations.

The two most recent documents of the four provide instances of using more specific digital tools. For instance, the fifth Standard for Mathematical Practice (SMP5) from the Common Core State Standards for Mathematics (2010) suggests the use of “a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software”. The description within SMP5 goes beyond “calculators and computers” and focuses on the specific softwares that can be used on computers or other devices. SMP5 suggests that students should be able to decide among which digital tools to use and their affordances and limitations, meaning that they have multiple options to choose from. In the description of digital tools uses, actions associated with use include using estimation to detect possible errors, visualize, analyzing assumptions and their consequences, and compare to hypotheses. These are actions that go beyond those mentioned in the early policy document, suggesting a shift from representation building and efficiency to actions that provide deeper meaning of the mathematics to be learned.

The NCTM position statement (2015) on strategic use of technology suggests the use of “both content-specific and content-neutral technological tools”. Content-specific mathematics technologies include many of the same previously mentioned as well as the use of interactive applets. Content-neutral technologies provides a slight shift in the types of digital technologies mentioned in these documents, which are described as “communication and collaboration tools, adaptive technologies, and web-based digital media”. These tools are meant to “increase students’ access to information and ideas and enhance student-student and student-teacher interactions to support and enrich sense making”. In addition, strategic use within either of these two categorizations of digital tools has the possibility to develop students’ by-hand procedures, among other instructional and learning outcomes.

In short, these policy documents view strategic and appropriate technology use as being an important pillar to high-quality mathematics instruction and mathematics learning experiences. Digital tool use in the mathematics classroom is no longer optional but needs to be purposeful and needs to provide support for instruction, student learning, and curricular goals. However, these policy documents describe a shift in how digital tools are to be used in the mathematics classrooms and the types of actions associated with their use. Content-specific tools are to be more than just answer-seeking or representation-creation devices, but to be used to provide opportunities for deeper exploration. The use of newly developed content-neutral technologies within mathematics classrooms has potential impact on classroom interactions. Each of these categorizations of technologies and their uses in the classroom will be a focus of this dissertation.

Curriculum for the Teaching and Learning of Middle School Mathematics

The National Science Foundation released *Materials for Middle School Mathematics Instruction: Program Solicitation* (1989), which provided a framework for proposals for the funding of new, innovative middle school mathematics curriculum to overhaul the state of mathematics education in the United States. Two guidelines for prospective curriculum from the NSF call were to "explore and improve on teaching methods, possibly including new uses of technology and new applications of mathematics, appropriate for presenting the new material to middle school students" and to "develop strategies and materials for teachers to improve their understanding of mathematics and introduce them to more effective methods of instruction" (Senk & Thompson, 2003), among other objectives. One such NSF-funded curriculum that made its arrival in 1991 was Connected Mathematics 1 (Lappan, Fey, Fitzgerald, Friel, & Phillips, 1991). Designed as an inquiry-based, problem-centered curriculum, CMP1 has been updated twice over for the most up-to-date edition of CMP3 (Lappan, Phillips, Fey Friel, 2014).

Of the eight guiding principles which were used in the development of CMP curricular materials over the years, two of the principles relate to instruction and technology. One principle promotes inquiry-based instruction. The principle states that “Classroom instruction focuses on inquiry and investigation of mathematical ideas embedded in rich problem situations through rich classroom discourse and collaboration” (CMP, 2019). This type of instruction is different than direct instruction, which is a transmission instructional model where teachers model procedures and provide information to students and then the students mimic and focus on procedural skills.

A second principle focuses on effective use of technology within CMP. The principle states that “The curriculum reflects the information-processing and delivery capabilities of calculators and computers and the fundamental changes such tools are making in the way people learn mathematics and apply their knowledge of problem solving to new tasks” (CMP, 2019). This statement reflects the use of two types of technology to leverage the roles of learners in the process of learning mathematics.

Along with these two guiding principles, the curriculum suggests defined roles for both the teacher and students at various phases of navigating through problems within the curriculum. The three phases of navigating through a problem within CMP is called Launch-Explore-Summarize, or LES. The LES instructional model is said to support inquiry-based teaching and is drastically different than direct instruction. The two guiding principles and the instructional method suggest possible interactions among each other. How does teachers’ use of technology provide “rich classroom discourse and collaboration”, which are tenants of inquiry-based instruction? How does inquiry-based instruction support the use of digital tools? How does the instructional method impact the uses of digital tools within the curriculum?

Purpose of Dissertation

The purpose of this dissertation is to systemically investigate how teachers of an inquiry-based, problem-centered mathematics curriculum use digital tool for their teaching of middle school mathematics. This study will attempt to uncover what digital technologies teachers are using and how they are using them at various points of the written curriculum through interviews with teachers covering select problems within a single unit of mathematics.

The digital tools used by teachers and students at different phases of a lesson and across a series of lessons have potential impact on how students learn mathematics. Jackson, Garrison, Wilson, Gibbons, & Shahan (2013) suggest that within inquiry-based lessons that the quality of the summary that students are able to give at the end of a lesson or a unit is dependent on how well the lesson or unit was initialized. This suggests that the ways curricular materials are implemented, in this case digital technologies, within the storyline of a lesson or series of lessons may play an important role in the opportunities to provide quality learning experiences for students within a lesson and across a series of lessons.

Research Questions

This dissertation study will attempt to answer the following two research questions through self-reporting by teachers from problem-centered mathematics classrooms:

1. What digital tools do teachers report using and what are the purposes of their use?
2. How do teacher purposes for digital tool use unfold at the problem level and at the unit level?

The following chapters will provide a review of relevant literatures on digital tool use and aspects of problem-centered mathematics classrooms, description of the study setup and participant selection process, the analysis of data provided by the participants, interpretation of the data, and conclusions to the study.

CHAPTER 2: LITERATURE REVIEW

Much research has examined the usefulness of curricular resources as it pertains to the teaching of school mathematics. Recent focus on the Common Core State Standards for Mathematics (CCSSI, 2010), which has a vision of providing coherence across the grade levels and changes in teaching practices with different curricula (Schmidt & Houang, 2012; NCTM, 2014), provides an opportunity to examine digital tool use under updated content and practice standards. Teacher predominately rely on adopted mathematics textbook curricula as their focus for curricular resources (Grouws, Smith, & Sztajn, 2004). Middle school mathematics teachers rely on the use of district-adopted textbooks as part of their planning and instruction (Choppin, Davis, Drake, & Roth McDuffie, 2013; Davis, Choppin, Roth McDuffie, & Drake, 2013). However, textbook curricula vary in their intended roles for teachers and students, the context in which the mathematics is presented, and supporting curricular materials to fit those roles and contexts (Choppin, McDuffie, & Davis, 2015; Stein, Remillard, & Smith, 2007).

Mathematics curricula have been described as either being either a delivery mechanism or a thinking device (Roth McDuffie, Choppin, Drake, & Davis, 2018; Choppin, Roth McDuffie, Drake, & Davis, 2015). To understand these two categories better, they should be viewed as actions and not nouns so that a thinking device is not thought of as the same as a digital tool device. A delivery mechanism curriculum can be thought of as an “organization of delivery”, which consists of a set of classroom practices where information is being delivered by an expert (teacher) to novices (students). These classroom practices are often associated with that of a traditional classroom. A thinking device curriculum can be thought of as an “organization of thinking”, which consists of dialogue among students and teacher that elicit student thinking and classroom interactions among students to help generate student understanding. These classroom practices are often associated with that of a problem-centered classroom. This study will investigate the actions associated with the use and purpose of digital tools within a single "organization of thinking" curriculum with known digital tools associated with the curriculum.

To help inform the actions associated with digital tool use within the problem-centered curriculum, a literature review on the uses of digital tools and their purpose for use is needed. In addition, since the instructional interactions between teachers and students differ when "organization of delivery" and "organization of thinking" curricula are used, it is important to look at the literature base on instructional models used for problem-centered curricula.

This literature review will focus on the following areas:

- Reported uses of digital tools within the mathematics classroom and their purpose
- Problem-centered lessons within inquiry-based classrooms.

Digital Tool Use and Purpose

The description of digital tool use mentioned in the policy documents in Chapter 1 provide a picture for how research on digital tool use within the mathematics classroom has developed over the past 30 years. First, there seemed to be a lumping of all digital tools used in the mathematics classroom by two basic classes, referred to as "calculators and computers", that was focus on their use compared to their non-use. Later on, research on these two types of digital tool categories split and became more refined. Research on computers focused on specific softwares that were used on them that gave different outputs and different user interactions. Research on calculators moved towards the use of graphing calculators within middle school and high school mathematics classrooms with a focus on their particular functionalities (ex. representations, dynamic capabilities, CAS) and their impact on instruction and learning.

Calculators and Computers

Focus of early research on digital tool use looked broadly at the inclusion of calculators and computers in the mathematics classroom. Research performed in this area 20 to 30 years ago brought to light the impact in which calculators and computers may have on the mathematics classroom. Bright and Prokosch (1995) reported through a study within 13 middle school mathematics classrooms that the use of one of these two technologies held students' attention to being off task only 3% of the time. The use of computers in the classroom provided slightly more engagement (or less time being off task) than when calculators were used, showing that different technologies had different effects on student engagement with their mathematical tasks.

However, the presence of technology-infused classrooms, or classrooms which exhibited use of calculators and computers during instructional time, was not very common in the 1990s. An analysis of videos of enacted eighth-grade mathematics lessons in the years 1995 and 1999 provided data on digital tool use (Jacobs et al., 2006). For the videos from 1995, 4% of the lessons observed included the use of graphing calculators and 4% showed evidence of computer use. In the videos from 1999, 6% showed graphing calculator use and 1% showed computer use. Across the years, computational calculator use (use of calculators without access to graphing capabilities or the use of graphing capabilities on graphing calculators) increased from 32% in 1995 to 39% in 1999. Although the purpose of this study was to find out how these enacted lessons aligned to the various NCTM standards established in 2000, the low frequency of technology use in these videos was far from the goals established in the *Principles and Standards* (2000).

While no claims were made about the appropriateness of their use, it can be said that technology did not play a major role in enriching students' problem-solving opportunities prior to the establishment of the new standards. The low frequency of technology use suggested that teachers were still teaching with traditional methods, which dominated the previous non-technological decades. How these two types of digital tools are used more broadly within problem-centered curricula, which have become more prevalent in classroom use since 2000, are of interest in this study.

Calculators

Researchers have found multiple uses of graphing calculators to be impactful on teacher instruction and student learning. However, the use of graphing calculators in the mathematics classroom has been controversial among teachers (Milou, 1999). In a survey of 146 middle school and high school teachers from 51 school districts, teachers indicated that graphing calculator use within first-year algebra courses was much less than the reported use in a second-year algebra course. Teachers reported use of graphing calculators as a motivational tool for student learning and engagement and reported modifications to their algebra curriculum to incorporate the use of graphing calculators in algebra instruction. Teachers were unsure as to when graphing calculators were to be used, as many were unsure of its role in developing algebraic concepts and procedural skills.

While Milou (1999) reported on teachers' use of graphing calculators as being a motivational tool, Doerr and Zangor (2000) identified five modes of graphing calculator use within a pre-calculus mathematics classroom (with additional empirical studies supporting these uses): computational tool (Graham & Thomas, 2000; Berry & Graham, 2005), transformational tool (Lee, 2010; Ruthven, Deaney, Hennessy, 2009), data collection and analysis tool (Choi-koh, 2003), visualizing tool (Paschal, 1995; Martin, 2008), and checking tool (McCulloch, Keene, Kenney, 2012). From this list of categorizations of uses of graphing calculators, some uses exhibit basic use (computational, checking) and other uses bring an experience that may not be done easily without the use of the graphing calculator (transformational, data collection and analysis, visualizing). Categorizing graphing calculator uses within this pre-calculus classroom highlights their impact on instruction and student learning.

Aside from the five categorizations of graphing calculator use, who uses the graphing calculators has an impact on communication in the classroom. When graphing calculators were used as a personal device, communication within small group settings was inhibited. This would have implication on the guiding principle for the use of technology to promote “rich discourse and collaboration”, which are tenants of inquiry-based instruction. In this dissertation study, categories were created that capture how digital tools are used within the changing classroom interactions among students and teachers across the phases of individual lessons.

The influence of graphing calculator use within classroom instruction has implications on student learning experiences. McCulloch (2011) found that the use of graphing calculators helped six high school calculus students maintain productive pathways for problem solving. Forster and Taylor (2003) found that students in an all-girls year 12 calculus class where graphing calculators were routinely used developed communicative competence. In this classroom, the teacher asked students for three guesses along with explanations from the students. This allowed for students to articulate their ideas, develop their ideas with an expert to scaffold, and develop skills in public debate. It was important for the teacher to value all three of the guesses and explanations, as it was noted that the creation of communicative competence could be sabotaged by teachers who evaluated students’ guesses and explanations prior to the investigation with the graphing calculators. This dissertation study examined when these types of instances occur within the phases of individual lessons of a problem-centered curriculum.

Computers

Vahey, Takar, and Roschelle (2004) saw the potential of computers in the classroom having similar impact on instruction and learning to the use of graphing calculators that preceded it. They saw the potential in communication capabilities and representational infrastructure within computers that could support multiple effective learning strategies, from collaborative strategies to those that are practice focused. To show that this is the case, a group of eighth grade students with computers outperformed high school students on AP Calculus exam items, showing the power of the various strategies when using computers can have on learning complex mathematics.

More recent research (Spires et al., 2008) showed that students are aware of the link between use of technology and their own academic engagement. Interviews with nearly 4,000 North Carolina middle school students showed that many had used computers for the use of creation and sharing of information both in and outside of school. 78% of the students reported to have used PowerPoint, 83% used painting, drawing, and design programs, and 48% created webpages on computers. In addition, many of these students reported use of computer technology outside of school for different purposes than information creation and sharing, such as video games, music, email, and messaging. When it came to these students finding general information, 86% reported that they would favor the use of the internet over finding the information in a book. Given students' exposure to technology in the schools and their use of technology outside of school, students notice when classroom instruction lacks technological interactions and that has implications on their classroom engagement. This dissertation study examined how teachers and students used digital tools that are external to the curriculum, such as those listed above.

Yet, how computers are used within the mathematics classroom needs further research. Dunleavy and Heinecke (2007) studied the effects of 1:1 computer access for at-risk middle school students on mathematics and science standardized test scores. Results showed significant gains for science, with boys outpacing girls. Yet, there were no significant effects for mathematics. While the unit of analysis was pretest and posttest scores by students, the study did not focus on how the computers were used by the students nor have a focus on how they were used in instruction. This dissertation study examined teachers who reported using 1:1 computing and how they used these ubiquitous tools within the mathematics classroom.

Although students showed experience with using computers both in school and out of school (Spires et al., 2008), how they are implemented in the classroom for positive mathematical experiences matters. Dunleavy and Heinecke (2007) showed that just their use did not produce gains in standardized test scores. Yet Vahey, Takar, and Roschelle (2004) showed that the thoughtful implementation of computers played an important role in students' learning of advanced mathematics. What this dissertation study explored was how computers were used for the ability to connect students to grade-level mathematics through instructional practices by teachers and through collaborative practices by students.

Technology with Curriculum

Among the overlap of research areas in this dissertation is the mixture of technology use along with curriculum. Okita and Jamalian (2011) noted that developing curriculum and instruction are often done separately from the creation of technological environments. It is suggested that they should be done together to combine the strengths of technology to the types of mathematical performance and conceptual understanding desired by the curriculum.

For example, Roschelle et al. (2010) examined how the use of interactive representational technology, a paper curriculum, and teacher professional development to replacement units with mathematics that was advanced for middle school students impacted student learning. They found that their approach provided positive learning outcomes with the advanced mathematics, regardless of teacher and student characteristics. In other words, the thoughtful inclusion of technology and curriculum provided students opportunities to learn more advanced mathematics compared to the replaced unit, where teacher and student characteristics may play a role in student success with at-grade-level mathematics.

Beyond the mathematical gains that digital tools can afford, there are other outcomes that emerge when technology is successfully integrated with instruction for any mathematics curriculum. When used appropriately with instruction, digital tool use can develop positive student attitudes towards learning, build student confidence in mathematics, promote student engagement with mathematics, student achievement, and better access to conceptual understanding (Guerrero, Walker, & Dugdale, 2004). However, these outcomes are dependent on "teachers' skills in integrating technology into the mathematics curriculum according to sound pedagogical practices" (p. 5). This dissertation study examined how digital tools played a role in promoting student engagement and access to conceptual aspects of problem-based tasks.

Information and Communication Technology

Information and Communication Technology, or ICT, is the collection of all digital technologies that provide the ability to create and communicate information. Computers and graphing calculators can be considered part of this collection, along with types of hardwares (ex. digital projector, SmartBoard, document camera) and softwares (ex. Desmos, spreadsheets, dynamic software) that can be found on them (Leung, 2006). The presence and use of ICT in the mathematics classroom has influence on instruction, student opportunities to learn, and mathematical goals (Hegedus & Moreno-Armella, 2009). Use of these tools have been shown to help not only students' mathematical skills, but also their communication and literacy skills (Genlott & Grönlund, 2016).

Villarreal, Esteley, and Mina (2010) suggest that the use of ICT is an "empowering partner" in modeling mathematics and that the absence of ICT would have made work "more difficult, less rich, or impossible" (p. 417). Yet, teachers have reported that they are not suffering from a shortage of ICT (De Witte & Rogge, 2014). Thus, available ICT seems to play an important role in the production of student work and teacher instruction within problems or sets of problems that provide modeling opportunities and opportunities for students to communicate their work to their peers and teacher. This dissertation examined the use of ICT and how their use played out within lessons.

Summary of Digital Tool Use and Purpose

The previous section looked at the literature on two major digital tools that are part of the middle school mathematics instruction, computers and calculators. For calculators, the review of literature focused on the use of graphing calculators, as their functionalities provide opportunities past computation that non-graphing calculators are primarily used for. Empirical studies on graphing calculator use in the classroom showed multiple purposes for their use, including uses as a computational, transformational, data collection and analysis, visualizing, and checking tool. In addition, other studies showed the use of graphing calculators by students as a tool for communication and being motivated and productive in problem solving.

The use of computers and other ICT provide opportunities for students to work collaboratively within mathematics classrooms. Although some studies have shown mixed results on their impact on student achievement, there is a need for investigating how ICT are used within certain curricula type, with an emphasis on the actions associated with their use and the purpose of their use.

Doerr and Zangor (2000) stated various uses of graphing calculators, which have multiple actions associated with their use. For the same reason, the assortment of ICT used in the mathematics classroom can have variety of actions associated with their use.

Notable frameworks that help characterize types of digital tool use include SAMR (Puentedura, 2006) and RAT (Hughes, Thomas, & Scharber, 2006). Each are used to help describe how technology acts with respect to an original task. For example, from Table 2.1, digital tools can be evaluated on their impact on the dimensions of instruction, student learning, and curriculum goals. If a digital tool has impact on multiple themes across dimensions, it is said to have either amplified or transformed the tasks. Similarly, SAMR provides a framework that describes how much the use of a digital tool distances the task from the task when digital tools are not used. What neither of these two frameworks provide is a focus on the classroom actions associated with digital tools in the process of learning mathematics.

Instructional Methods	Student Learning Processes	Curriculum Goals
	… include …	
teacher's role	activity task	"knowledge" to be gained, learned, or applied
interactions with students	thinking process – mental process	"experience" to be gained, learned, or applied
assessment of students	task milieu (individual, small group, whole class, others)	
professional development	student attitude	
preparation		
administrative tasks		

Table 2.1: Dimensions (with themes) of the RAT Framework (Hughes, Thomas, & Scharber, 2006)

This dissertation study is interested in the state of digital tool use within problem-centered mathematical tasks, to understand their use and purpose through their actions in the classroom that are closely related to select themes of the RAT Framework than to the three dimensions.

Problem-Centered Mathematics

This study provides a snapshot into digital tool use within a problem-centered curriculum. Therefore, it is important to describe aspect of the curriculum and the literature around its use. Confrey and Maloney (2007) and Geiger, Faragher, and Goos (2010) suggest that tasks offering technology-enhancements are best used with an inquiry-based instructional approach. How an inquiry-based, problem-centered mathematics curriculum incorporates digital tool use, or its users incorporate digital tool use within the curriculum, is of interest in this study.

The middle grades offer opportunities for students and teachers to use similar tools that are appropriate at given stages (Hollenbeck & Fey, 2012), such as using a digital geoboard to explore the maximum area of a triangle within a rectangular grid during an initial exploration of the task and eventually moving to a digital app that shows the shearing effect on area. In addition, they suggest that different phases of a lesson allow for students and teacher to have different types of interactions, such as a teacher introducing a new concept at the beginning of a lesson or students communicating their understanding at the end of a lesson. These suggest different actions by different people in the classroom at various phases of a lesson. The setup of this dissertation study was intentional in determining who was using digital tools and when they were being used within lessons.

LES Instructional Model – Launch, Explore, Summarize

All lessons have some form of a beginning, middle, and an end. The same holds true for tasks within problem-centered, inquiry-based curricula. For tasks within inquiry-based curricula, teachers generally provides students a problem in which to explore. The teacher may provide some basic groundwork in terms of content and context. In the middle, students explore the problem and attempt to extract the mathematics from the context and make strides towards formalizing the mathematics of the task. This is typically done through some guiding questions included in the text or provided by the teacher. The end is when both the students and the teacher work together to solidify what was discovered during the task and connect it to the mathematical world. This study will be using an instructional model that uses specific terminology from an inquiry-based, problem centered curriculum to communicate the beginning, middle, and end of a lesson, or a Problem, within the curriculum. They will be referred to as the Launch, Explore, and Summarize, respectively, within any Problem from the third edition from the Connected Mathematics Project (Lappan, Phillips, Fey, & Friel, 2014), or CMP.

The phases of the instructional model are interconnected and important for maintaining the cognitive demand of problem-based instruction. In a study of 165 middle school teachers using the second edition of CMP (Jackson et al, 2013), it was found that when the Launch supported students ability to develop a common language to describe the context and mathematical features of the Problem and the cognitive demand of the Problem was maintained through the Launch, then the resulting Summarize were considered to be of high-quality during whole-classroom discussions. In cases where the Launch did not meet these conditions, then the resulting Summarize session was not considered to be of high quality. The importance of this study is to show how critical it is to have a good introduction to the Problem while not lowering the demand of the Problem during the Explore portion of the lesson.

Although the study by Jackson et al. (2013) does not focus on the use of digital tools, the choice of digital tool use by teachers to be used during any phase of the LES instructional model of a Problem has implications on overall student learning. Furthermore, the collective use of digital tools within a Problem or a sequence of Problems within a Unit has implications on student engagement and learning of mathematics and the contexts in which they are presented. Digital tool use within Problems will have connections to the themes from the RAT Framework. However, the explicit actions in which digital tools are used within are the focus of this dissertation study.

Summary

This literature review was intended to provide some background relative to the two research questions of this dissertation study. The first portion was to examine the use and purpose of digital tools in the mathematics classroom. Two major types of digital tools commonly used and reported on in the literature are computers and graphing calculators. The literature review provided an overview of empirical studies that highlighted the various uses of each of these digital tools as well as shed some light on their purpose associated with their use. In addition, the overarching category of ICT was introduced to discuss the wide varieties of hardwares and softwares that are present and evolving in today's mathematics classroom, often used in tandem with computers and calculators.

The review of literature on the second research question gave a brief introduction to the dynamics of an inquiry-based, problem-centered curriculum and the instructional model used. Although more on the curriculum type and instructional model will be discussed in the methods chapter, it was important to give a baseline for the work of Jackson et al. (2013) to link back to digital tool use and purpose mentioned in the first half of the literature review.

Chapter 3 will provide a description of the study setup that highlight aspects discussed in Chapter 2. An inquiry-based, problem-centered curriculum used in the study will be discussed and analyzed for its instructional characteristics and suggested digital tool use. Chapter 3 also provides how classroom actions associated with digital tool use will be examined through the interview of teachers.

CHAPTER 3: METHODS

The purpose of this chapter is to describe the setup of the study: the curriculum used, a unit analysis of the Units and Problems used within the study, the participant selection process, a description of the interview protocol, and an overview of analysis procedures. Interviews of teachers from a problem-centered mathematics curriculum report on their use of digital tools within select problems from a selected unit is the mechanism used for collecting data to answer the following research questions:

1. What digital tools do teachers report using and what are the purposes of their use?
2. How do teacher purposes for digital tool use unfold at the problem level and at the unit level?

Connected Mathematics Project

The Connected Mathematics Project (CMP), an NSF-funded middle school mathematics curriculum for both students and teachers, is a problem-centered curriculum that promotes an inquiry-based classroom environment. Each grade (6 through 8 & 8^{th}/Algebra 1) within CMP is composed of six to eight Units, each which has four to five Investigations. Each Investigation has four to five Problems. Each Problem within CMP is designed for the teacher and students to engage with the LES (Launch, Explore, Summarize) instructional method and generally take one to one and a half days to complete. A set of exercises (ACE) are located at the end of each Investigation and are aligned to specific Problems within the Investigation, with the alignment indicated in the teacher's guide (TG).

The names Problem, Investigation, and Unit and Launch, Explore, Summarize use capital letters to indicate a specific chunk of tasks within the CMP3 curriculum. It is a convention used to show a difference between a specific set of lessons within CMP3 covering, say, three-dimensional measurement in the Unit called Filling and Wrapping compared to what teachers may casually call a unit on three-dimensional measurement, which may vary across curricula.

Unit Analysis

The following is to provide a background on the resources available to teachers who use the CMP3 curriculum. This will help inform the types of digital tools that are discussed in the interview protocol and provide some context into the design of Units and the selected Problems that teachers are to discuss during the interview.

Curriculum-Provided Digital Tools

Each Unit of CMP3 provides opportunities for teachers and students to explore various types of digital tools explicitly identified or provided by the curriculum. In the printed and digital student edition (SE) of each Unit, there are icons within the margins of the text that indicate a type of curriculum-provided digital tool, such as a Launch Video, Digital Math Tools, or Student Activities, which are available online. Users have to go to DASH or Realize to determine the specific digital tools, as the icons in print only indicate that a curriculum-provided digital tool exists.

The printed and digital teacher's guide (TG) of each Unit indicate the availability of curriculum-provided digital tools within each Investigation and each Problem through the inclusion of a Unit Planning Chart and an Investigation Planning Chart, respectively. Within these planning charts, the existence of Launch Videos are identified, along with the existence of Digital Math Tools or Student Activities that may align to individual Problems. After the Investigation Planning Chart, the TG provides a description of each Problem, providing an overview of a Problem followed by instructions for the teacher as they progress through each phase of the LES. Here, a more complete explanation is given of how a suggested curriculum-provided digital tool may be used within the Problem, particularly with the use of Launch Videos as they are explicitly described within the Launch section of these Problem descriptions. The digital TG provides the same planning charts and descriptions as the printed version, with hyperlinks to each of the curriculum-provided digital tools described in the TG for quick access for the teacher. One small variation between the digital and printed planning charts is that the print versions do not include the existence of a Launch Video in the planning chart but do mentioned them in the written Problem description when they are available. In general, the Teacher's Guide provides explicit mentions of Launch Videos, Math Tools, and Student Activities that align to individual Problems.

Launch Videos are described within the TG to provide opportunities to familiarize students with the context of the Problem. However, there are different types of context in which Launch Videos provide familiarization relative to the Problem. Some videos provide context to the nouns within the Problem. For example, Filling and Wrapping Problem 1.1 provides a Launch Video on shipping containers, as the real-world context of shipping containers is used to introduce students to the idea of finding volume. Other Launch Videos, such as Problem 2.2 from It's In The System, provides a Launch Video that echoes the problem as written in the text, where two brothers purchase six tacos and two drinks for $9 and another set of siblings purchase four tacos and two drinks for $7. Here, the context of the Launch Video is the mathematical problem itself, compared to the previous example emphasizing familiarity with the nouns (ex. shipping containers) within the Problem. Distinguishing the two types of contexts within Launch Videos: real-world context (RWC) and mathematical context (MC). In addition to these two codes for Launch Videos, there also exists videos that help give students context to the activity they will be participating in, such as videos that portray students in the collection of data through jumping jacks in Problem 1.1 in Variables and Patterns. These can be described as videos that provide support to the activities students will be engaged in within the Problem, so they provide Problem context (PC).

The curriculum-provided Math Tools (MT) are tools that can be used at various points within the curriculum. They include digital tools such as algebra tiles, coordinate grapher, geoboard, integer chips, number lines, and pattern blocks, among others. None of the curriculum-provided Math Tools are specific to any Problem within the curriculum or provide any type of real-world familiarity.

The curriculum-provided Student Activities (SA) offer interactive experiences that help deepen students' understanding and provide opportunities for exploration. In some cases, there are activities that are aligned to specific Problems within the curriculum, such as the Locker Problem, Virtual Bridge Experiment, and Pouring and Filling. For these digital activities, they can be used to replace the activity as written in the text or in tandem to augment the experience. For example, the Virtual Bridge Experiment provides a digital environment in which the length and thickness of a bridge can be manipulated and chips can be placed in a red cup resting upon the top of the bridge until the bridge breaks. Compared to the activity as written in the text, which includes paper, cups, pennies, and books, the virtual environment provides opportunity to conduct the experiment without the set up and the mess associated with classroom lab experiences.

Some Student Activities give the opportunity to explore abstract concepts that cannot easily be done without the use of technology. Such examples are the Virtual Box, which provides the ability to make any box with integer lengths less than 10, display as a net, and place unit cubes inside of the net and reassemble. Another is Painted Cubes, which allows for a rectangular prism to be painted on each side be rotated around the screen and have the box expand into unit cubes that can be individually explored. In both examples, the ability to create and show what happens on the inside of a rectangular prism is easily accessible in the virtual environment and would be very tedious with a physical model.

Other Student Activities provide opportunities for students to use strategies in the form of games that can be played against the computer or another student, such as the Factor Game, Product Game, Fraction Game, among many others. These games promote student thinking skills through strategies that will benefit them in terms of some point scheme while at the same time limiting their opponent's ability to gain as many points. These games are not necessarily part of any particular Problem within the curriculum but provide additional opportunities for students to learn mathematics in a gaming setting.

Collectively, the Launch Videos, Math Tools, and Student Activities are digital tools that are provided by the curriculum. In the discussion of analysis of digital tool use by teachers, teachers' use of curriculum-provided digital tools and digital tools that come from outside the curriculum will be discussed.

It is worth mentioning a few things about the treatment of the digital tools within the TG. First, when a Launch Video is available to use, it is explicitly written about within the Launch description of the Problem. No other curriculum-provided digital tool receives this type of treatment within the TG. All other digital tools, such as the Math Tools or the Student Activities, are simply mentioned at the beginning of the Problem Overview prior to the LES descriptions, under a heading called "Using Technology". In this section, the use of the curriculum-provided digital tools are described in a way where the technology is an option. Phrases such as "If your students have a computer, ..." assumes that having this type of hardware is not a requirement for this curriculum type, but recognizes that some students will have access and the TG is making the teacher aware of this. Other words, such as “can” and “may” are often found in this description, which also enforces the optional nature of digital tools use as written in the TG.

Standards for Mathematical Practice

Outside of the explicit instances of Launch Videos within the Problem descriptions, the promotion of teacher use of curriculum-provided digital tools is not evident in the TG. Each Investigation within the curriculum explicitly states a subset of the eight Standards for Mathematical Practice (CCSSI, 2010), covering between two to four of the practice standards for any one Investigation. One particular practice standard, MP5: Use Appropriate Tools Strategically, is often dubbed as the technology practice standard. Although there is no guarantee that all eight Standards for Mathematical Practice are mentioned within a Unit (a collection of consecutive Investigations), the instances of when the TG explicitly addresses MP5 and the suggested use of digital tools within the Problem may provide extra emphasis on the use of digital tool use for that Problem.

Within the six selected Units for this study, seven instances of MP5 were mentioned with description as to the nature of how the Problem(s) address MP5. Although these Investigations may not be the first or final Investigations of the selected Units (which include the chosen Problems discussed during interviews), these are the instances where the curriculum writers decided it was important enough to write about and, conversely, not write about in other Investigations. The seven instances of explicit attention to MP5 are in Table 3.1.

A few characteristics from the seven descriptions of instances of MP5 within the six selected Units become apparent. First, a representation is considered a tool. The ability to use a graph, drawing, or a grid are explicit uses of tools in these descriptions, regardless of the source

Grade	Unit	INV #	Description
6	Covering & Surrounding	3	In Problem 3.4, students use many different tools to help them find the areas of the polygons. Some students may subdivide the polygons on the grid into rectangles and triangles. They may also use a coordinate grid and subtract the coordinates of the vertices to find the lengths of vertical and horizontal sides. Other students may use a grid to enclose each polygon in a rectangle.
6	Variables and Patterns	3	In Problems 3.2 and 3.3, students find the value of a variable given a value of the other variable. They may use a graph as their method. Students may choose to trace one of the values to find the other value.
7	Comparing and Scaling	2	As students work with concepts of proportionality, they use appropriate tools. Students use calculators in Investigation 2 so that they can focus on reasoning critically rather than computing fractions and decimals. In Problem 2.2, students begin to use grid paper to graph proportional relationships.
7	Comparing and Scaling	3	Students continue to use tools appropriately in this Investigation by graphing on grid paper and using calculators so that they can focus on proportional reasoning rather than strict computation.
7	Filling and Wrapping	2	In order to visualize the figures that result from slicing a prism or pyramid in Problem 2.3, students may use a variety of tools, such as a drawing of the figure, a three-dimensional object, and a computer animation of the three-dimensional figure.
8	Thinking with Mathematical Models	2	Students use multiple representations of algebraic ideas, including tables, graphs, and equations. Students may also use the Trace feature on a graphing calculator to check the reasonableness and accuracy of their graphs or solutions to equations, such as in Problem 2.3.
8	It's In The System	3	In this Investigation, students use graphs to visualize and find the solutions to the systems. In Problem 3.1, students use a graph to find the solution to a system of inequalities. They compare the cost from two security companies to determine which company is more cost-effective based on the number of days of service. Using graphs of the expressions to solve this Problem shows the graphical relationship of one expression being "less than" the other, or one line being below the other.

Table 3.1: Instances of MP5 explicitly covered for any Investigation within the six selected CMP3 Units

of the representation or how the representation is constructed.

Second, in the instance within Thinking with Mathematical Models, they make explicit mention of multiple representations of tables, graphs, and equations. They follow this up by stating the possible use of the trace feature on a graphing calculator for checking the accuracy of their representations, obviously not created on the graphing calculator. This gives the sense that the graphing calculator is not a primary tool for the construction of these representations.

Third, there are a few explicit instances of how students “use calculators”, where once it was mentioned how the use of calculators helped offload computational work so that students could focus on the context of the Problem. It is interesting to note that the use of calculators is never explicitly mentioned in either Unit or Investigation planning charts. However, there is a feeling that calculators are omnipresent in the curriculum.

Fourth, the use of the curriculum-provided digital tools is never mentioned in these seven cases, which were all in Investigations of Units that this dissertation is not covering. The implications this has to this research project include teachers having to be the decider on what digital tools to use, when to use them, and why they should be used without any prompting by a practice standard to tell them that they should use a digital tool. However, this also means that when digital tools are available, there were other mathematical practice standards that the authors felt more inclined to explicitly mention over those that had the opportunity to address MP5 with a curriculum-provided digital tool.

Overall, the explicit descriptions of the seven cases where MP5 is addressed within the six selected CMP3 Units suggest that the word ‘tools’ does not necessarily infer ‘digital tools’ even when digital tools are present, there are other tools or representations that are described prior to mentioning the digital tool(s). This has possible implications on teachers who follow the recommendations and practice standards aligned to Problems within the written text.

Selected Problems within Units

Each grade level has two predetermined Units from which teachers can choose to participate in prior to their interviews, with a goal of having two teachers participate in each of the six total Units, for a total of 12 participants. The purpose of selecting these six Units was to get a mix of the various mathematical strands that are present during the middle grades, so that digital tools that are specific to particular strands of mathematics, such as using graphing calculators when exploring algebra concepts or dynamic geometry environments when studying geometry concepts, do not dominate the study. The intent is to uncover teachers' use of digital tools within this type of mathematics curriculum. The six selected Units are summarized in Table 3.2, which provides the range of Problems within the Unit being covered and the suggested digital tools by the curriculum.

Table 3.3 provides an analysis of the curriculum-provided digital tools suggested by the TG within the first and last Investigations in each of the six Units selected in this study. Teachers will be asked about the Problems which come from the first and final Investigations of the Unit. Furthermore, each Investigation is partitioned into first and final Problem(s). Within each indication of a digital tool use within a Problem, there are codes for the various types of Launch Videos and indicators of the curriculum-provided Student Activities and Math Apps.

An important take away from Table 3.3 is the explicit mention of a digital tool for all but five of the Problems included across the six Units. Therefore, the availability of some type of curriculum-provided digital tool is available in most Problems used within this study. Table 3.3 does not indicate in which phase of the LES instructional model the digital tools should be used within a Problem, but the suggestions are within the TG for each of the Problems.

Grade	Unit	Initial Investigation	Final Investigation	Curriculum-Suggested Digital Tools
6	Covering and Surrounding: Two-Dimensional Measurement	INV 1 (1.1-1.3) Designing Bumper Car Rides	INV 4 (4.1 & 4.3) Measuring Surface Area and Volume	Launch video, Areas and Perimeters of Shapes and Images, Pattern Blocks, Data and Graphs
6	Variables and Patterns: Focus on Algebra	INV 1 (1.1-1.4) Variables, Tables, and Graphs	INV 4 (4.1 & 4.5) Expressions, Equations, and Inequalities	Coordinate Grapher, Data and Graphs, Number Line
7	Comparing and Scaling: Ratios, Rates, Percents, and Proportions	INV 1 (1.1-1.4) Ways of Comparing: Ratios and Proportions	INV 3 (3.1 & 3.3) Markups, Markdowns, and Measures: Using Ratios, Percents, and Proportions	Coordinate Grapher, Number Line, Expression Calculator, Data and Graphs, Mug Wumps
7	Filling and Wrapping: Three-Dimensional Measurement	INV 1 (1.1-1.4) Building Smart Boxes: Rectangular Prisms	INV 4 (4.1 & 4.5) Cylinders, Cones, and Spheres	Virtual Box, 3D Geometry Tool, Virtual Cylinder, Pouring and Filling
8	Thinking with Mathematical Models: Linear and Inverse Variation	INV 1 (1.1-1.3) Exploring Data Patterns	INV 5 (5.1 & 5.5) Variability and Associations in Categorical Data	Virtual Bridge Experiment, Data and Graphs, Coordinate Grapher (no INV 5 suggestions)
8	It's In the System	INV 1 (1.1-1.3) Linear Equations with Two Variables	INV 4 (4.1 & 4.4) Systems of Linear Inequalities	Launch Video, Coordinate Grapher, Graphing Calculator (optional)

Table 3.2: Selection of Units within CMP3 with Investigations and digital tools mentioned

Grade		6		7		8	
Unit		Covering and Surrounding	Variables and Patterns	Comparing and Scaling	Filling and Wrapping	Thinking with Mathematical Models	It's In The System
First Investigation	First Problems	1.1 Launch Video (RWC), Areas and Perimeters of Shapes and Images (SA), Pattern Blocks (MT) 1.2 Pattern Blocks (MT), Data and Graphs (MT)	1.1 Launch Video (PC) 1.2 Coordinate Grapher (MT), Data and Graphs (MT)	1.1 Launch Video (PC), Mug Wumps (SA), Data and Graphs (MT) 1.2 NONE	1.1 Launch Video (RWC), Virtual Box (SA) 1.2 Virtual Box (SA)	1.1 Launch Video (RWC), Virtual Bridge (SA), Data and Graphs (MT) 1.2 Virtual Bridge (SA), Data and Graphs (MT)	1.1 Launch Video (MC), Coordinate Grapher (MT) 1.2 NONE
	Final Problem(s)	1.3 Pattern Blocks (MT), Data and Graphs (MT)	1.3 Launch Video (RW), Coordinate Grapher (MT), Data and Graphs (MT) 1.4 NONE	1.3 Launch Video (MC), Number Line (MT) 1.4 Launch Video (MC), Coordinate Grapher (MT), Mug Wumps (SA), Expression Calculator (MT)	1.3 NONE 1.4 Launch Video (PC)	1.3 Launch Video, Data and Graphs (MT)	1.3 Coordinate Grapher (MT)
Final Investigation	First Problems	4.1 Launch Video (RWC), Virtual Box (SA), 3D Geometry (MT)	4.1 Launch Video (RWC)	3.1 Launch Video (MC)	4.1 3D Geometry (MT)	5.1 Launch Video (PC)	4.1 Launch Video (MC), Graphing Calculator (optional)
	Final Problem	4.3 Virtual Box (SA), 3D Geometry (MT)	4.5 Launch Video (MC), Number Line (MT)	3.3 Launch Video (MC), Coordinate Grapher (MT)	4.5 3D Geometry (MT), Filling and Pouring (SA)	5.3 NONE	4.4 Graphing Calculator (optional)

Table 3.3: Digital tools explicitly listed in planning charts of six selected CMP3 Units

Participant Call

The participants of the study were current teachers of the Connected Mathematics Project (CMP) mathematics curriculum who had used CMP3 (Lappan, Phillips, Fey, & Friel, 2014), the most recent published version, for at least one school year prior to their participation in the study. These teachers had access to Pearson's online Math Dashboard through either an older platform known as Dash or a refreshed platform called Realize. Both of these online digital components provide teachers access to electronic teacher editions, student electronic books, teacher e-book with note taking abilities, printable worksheets, general digital math tools, digital student activities specific to problems within the CMP3 curriculum, and videos aligned to particular problems. The student versions of both Dash and Realize provide a subset of these features, sans electronic teachers editions.

The selection of participants targeted members on Facebook's CMP Discussion Group for Teachers and Teacher Leaders private group page, which is administered and moderated by the staff at the Connected Mathematics Project at Michigan State University, and Twitter through the use of the Connected Mathematics Twitter page. The online community served as a professional learning community where teachers and teacher leaders support each other in various aspects of teaching with the CMP curriculum. At the time of the study, there were approximately 780 members who interact within the Facebook group and 1350 followers on Twitter. Some teachers within the group used earlier editions of CMP at the time of this study, so the need to ensure that only teachers who use CMP3 were screened for during the initial screening and the follow-up email to schedule interviews.

The general call through both social media sources consisted of a link to a Google Form, which asked potential participants for an email address to contact them, the number of years taught using CMP3, a Likert scale from 1 to 5 for a self-assessment of digital tool use within the CMP3 curriculum with 1 being little to no digital tools used and 5 being digital tools used frequently, and grade level(s) taught using CMP3. Those who responded to the Google Form who had at least one year of teaching with CMP3 and rated themselves a 3 or higher on the self-assessment on digital tool use were considered for interviews. Each interview participant was compensated a $50 Amazon gift card for their time in the study.

The targeted number of participants for this study was 12. This number of participants ensured that all three grade levels of CMP3 would be represented and two Units within each grade level would be covered. Since the focus of this study is on the curriculum type used by middle school teachers and not a particular grade level nor a particular strand of mathematics across the three grade levels, spreading the participants over each grade with various mathematical strands helped to focus on the inherent characteristics of the inquiry-based, problem-centered curriculum. Each of the three grade levels had two predetermined Units of interest, with the intention of having two participants for each of these Units. The redundancy of participants in these Units provided evidence of possible similarities and differences of their digital tool use.

The willing participants engaged in one-on-one interviews with the researcher, which were conducted through an online video conference using either Zoom, Skype, or Facetime. Participants were audio recorded for transcription purposes. No participants were video recorded. The interview provided insight into the reason(s) for their technology decisions that pertain to a single chosen Unit and certain Investigations within that Unit. Participants were initially chosen by their grade level taught and the need for participants of certain Units. Some participants taught across multiple grades using CMP3, which provided the researcher flexibility in choosing a grade level or Unit of need as interviews progressed. For the interviewees, they had the ability to choose between two Units within their grade level as long as Unit interviews had not been previously scheduled by two other participants. Participants who scheduled early interviews had the ability to choose Units that they felt most comfortable in discussing their digital tool use.

The willing participants engaged in one-on-one interviews with the researcher, which were conducted through an online video conference using either Zoom, Skype, or Facetime. Participants were audio recorded for transcription purposes. No participants were video recorded. The interview provided insight into the reason(s) for their technology decisions that pertain to a single chosen Unit and certain Investigations within that Unit. Participants were initially chosen by the grade level that they teach. Some participants taught across multiple grades using CMP3, which provided the researcher some flexibility in choosing a grade level or Unit of need as interviews progressed and for the interviewees a Unit to choose a grade level or Unit that they feel most comfortable with in discussing their technology decisions and uses. Descriptions of participants will be in Chapter 4.

Interview Protocol Description

The intent of the interview protocol in Appendix A for those participating in one-on-one interviews over one of the six predetermined CMP3 Units was to extract participants' use of digital tools and how their use impacted classroom dynamics at the Problem level and at the Unit level. Semi-structured interviews were performed over other means of data collection (e.g. survey) because of the needed flexibility to talk about the various digital tool types across Units and to attend their purposes of use.

Prior to prompts about specific Problems within their selected Unit, participants were asked how they generally see digital tool use through the phases of the LES instructional model, through consecutive Problems (an Investigation), the Mathematical Reflection at the end of Investigations, and across Investigations (i.e. a Unit). Teachers were asked about their use of the Teacher's Guide as a source of support for their technology use and for their mathematical and personal goals for each of the Investigations covered in their Unit.

Once the preliminary field of questions were completed, each participant was asked to describe their teaching progressions through Problem 1.1 of their respective Unit with an emphasis on describing when digital tools were used during the Problem relative to the LES instructional model. Follow-up questions were asked if their response was unclear as to 1) when digital tools were used within the LES instructional model, 2) how digital tools were being used, 3) the digital tools' purpose of use, and 4) if curriculum-provided digital tools that were mentioned in the planning chart of the Teacher's Guide of the Problem were not mentioned for their use in the Problem.

If participants did not mention digital tools during a phase of a Problem, participants were asked about their reasoning for not using a digital tool. The importance of asking about non-use was to 1) identify reasons for their non-use and 2) provide a way to ensure that digital tools were not used in case the participant forgot to mention digital tool use.

The same line of questioning was used for all of the Problems within Investigation 1 of their respective Units and the Mathematical Reflection at the end of Investigation 1 (which does not have a LES instructional model associated with it). Before continuing the same prompt for the first and final Problem of the final Investigation of their Unit, participants were asked about the mathematical and personal goals for the final Investigation. At the completion of the two Problems and the Mathematical Reflection for the final Investigation, participants were asked what a dream digital tool for their respective Unit and the curriculum would look like.

Participants were also allowed to make a final statement about digital tool use relative to their Unit or the curriculum.

Analysis

The analysis of interview data was shaped by two sources: 1) a two-participant pilot study and 2) the dimensions of the RAT Framework that focus on instructional methods, student learning processes, and curriculum goals (Hughes, Thomas, & Scharber, 2006). The initial portion of this analysis section will provide a description of the development and reasoning behind the categorization scheme used within the analysis that attend to the two research questions. Within each of the dimensions of the RAT Framework are themes that became of greater interest during the analysis from the pilot study, which were more focused on the actions within the dimensions (Table 2.1).

Although the RAT Framework had the potential to be used to categorize instances of digital tool use in this study, many of the themes within the dimensions were not purposeful for this study (ex. professional development, preparation). A new framework was created that captured how teachers' descriptions of digital tool use fit in with the attributes of an inquiry-based, problem-centered mathematics classroom. The four categories used for the analysis are: Understand/Learn, Doing, Classroom, and Recall. A brief description of each category will provide meaning to each and separation from the other categories.

The Understand/Learn category of actions pertains to actions that help students understand the mathematics within the Problem and allowing students to provide cognitive actions as related to the mathematics. For example, participants from the pilot study reported "access context" multiple times as an action. This action attempts to provide students contextual understanding in either the real-world or mathematical scope of the Problem. Other actions within the Understand/Learn category of actions refer to students' actions of working with the Problem, in which they may "struggle, hypothesize, predict, and verify".

The Doing category of actions refers to actions that describe students working with a Problem. Coded action words such as "calculate, build, count, sketch, create representations, create by hand, and draw" are all action words associated with a physical action.

The Classroom category of actions refers to those actions where it is evident that the participant is not talking about actions at the individual or small group level, but those that occur at the whole-class level by either students or the teacher. Actions such as "demonstrate, extend, move forward, engage, ask, and observe" are all in the context of interactions that occur within a larger grouping of students. Given the context of participants' responses, they may have discussed how one student demonstrated on a document camera to the rest of their peers or how the teacher used a document camera to demonstrate several cases in which a Problem could be solved. Either of these situations would be coded as demonstrate and would be appropriate to say that they were shared with the majority of the classroom.

The Recall category of actions refers to those actions where students or the teacher use an artifact to look back on or to create a new artifact that can be possibly used in future Problems. Actions such as "refer" indicate that something has been created in the past that either the student or teacher referred back to. Other action words such as "print" and "save" suggest an artifact being constructed for future use. Other actions such as "notes' can be ambiguous but either way serve as an artifact.

The framework for categorizing actions with digital tool use above, which will be called the UDCR from here on out, provides an important piece to answer the two research questions:

1. What digital tools do teachers report using and what are the purposes of their use?

   a. What digital tools do teachers report using?
   b. How do the actions associated with digital tool use relate to UDCR?
2. How do teacher purposes for digital tool use unfold at the problem level and at the unit level?
   a. How does the actions associated with UDCR play out within the LES of Problems?
   b. How does the actions associated with UDCR play out across multiple Problems?

Analysis for Research Question 1

The analysis for the first research question will consist of two parts. The first being a status report on the use of digital tools, namely answering the question *"What digital tools do teachers report using from a problem-centered mathematics classroom?"*. The second part looking at the continuation of the research question *"... and what are the purpose of their use?"*.

For the first part, participants were asked through the interview protocol about the digital tools that they used within the classroom during the selected Problems of their respective Unit. The overarching prompt "Tell me about how you progress through the LES of the Problem, and when you use digital tools in this Problem, why you choose to these digital tools, and how you and/or your students use them" provides the opportunity for teachers to discuss which digital tools they are using. Teachers reported on their use of both curriculum-provided and external digital tools. Given that the participants were asked the same set of questions over the set of Problems within their Unit, the types of digital tools used were often repeated across Problems. The focus on this portion of the analysis is on which digital tools teachers are using, not on the frequency of their use. Like the study by Jacobs et al. (2006), this portion of the analysis is simply trying to identify which digital tools are being used.

The procedure to obtain this data was through transcribing each of the teacher interviews. Instances of digital tool use for each teacher were identified and coded by phases within the LES of a Problem or the Mathematical Reflection. Those digital tools that are curriculum-provided were identified and coded as curriculum-provided through the Teacher's Guide and the online resources that are stated in the unit analysis discussed previously. Any other digital tool not on that list was identified as being an external digital tool. Aggregate data across teachers will be reported on which digital tools were used, the frequency in which they were used within phases of the LES instructional model for Problems and uses in the Mathematical Reflection, how many different teachers were using the tools (to contrast with the frequency numbers), and the percent of teachers from the study who report on using a digital tool.

For the second part, participants were asked through the interview protocol about why they choose to use digital tools within the classroom during the selected Problems of their respective Unit. The overarching prompt "Tell me about how you progress through the LES of the Problem, and when you use digital tools in this Problem, why you choose to these digital tools, and how you and/or your students use them" provides the opportunity for teachers to discuss why they choose to use digital tools within the Problem. If teachers did not explain why they selected digital tools they reported using, the follow-up question "Can you elaborate more on why you choose to use these technologies?" followed their initial response to the prompt.

The procedure for analysis is through summative examination of the coding of participant interviews. For each coding of digital tool use in the prior portion, each use of digital tools was placed in one of the four mutually-exclusive categories of purpose. After all use codes were placed in the UDCR categories, each category was examined for patterns of purpose. Findings will report on the different purposes aggregated across teachers and provide supporting examples from individual teacher interviews.

Analysis for Research Question 2

The analysis for the second research question comes from the structure of the selected Problems and the interview protocol. Within CMP3, the LES instructional model helps in identifying when digital tools are used within the phases of a Problem. The inclusion of Mathematical Reflections that look across sequences of Problems (an Investigation) provide purpose of digital tool use as a summary to a sequence of Problems. Relative to the interview protocol, teachers provided instances of digital tool use within Problems and the Mathematical Reflection of the first Investigation and cover the first and final Problems within the final Investigation of their respective Units along with the Mathematical Reflection within the final Investigation. This provides opportunity to see how purposes of use unfold at the Unit level. The use of digital tools reported by teachers within Problems and across Problems provides an opportunity to understand actions at the micro (Problem) and macro (Unit) level.

Teachers reported out on digital tool use by responding to the prompt "Tell me about how you progress through the LES of the Problem, and when you use digital tools in this Problem, why you choose to these digital tools, and how you and/or your students use them" for Problems and "Tell me about how you choose to use the Mathematical Reflection, and if you and/or your students use digital tools during this Mathematical Reflection, why you or your students choose these digital tools, and how you and/or your students use them" for Mathematical Reflections. There is a slight language difference between the two prompts because teachers were known to modify the Mathematical Reflection from the written version, either by shortening the amount of questions or modifying the questions themselves. Regardless, there was some sort of summary of Problems at the end of an Investigation that took on the nature of a Mathematical Reflection, which did not have an LES instructional model associated with it.

The procedures of coding to attend to the second research question is through transcribing each of the teacher interviews and identifying either digital tool use or phases of LES at the Problem level. If a digital tool was mentioned, then it had to be identified to a particular phase of the LES of a Problem. If a phase of LES was identified, then digital tool use was sought out. There were instances of a phase of LES being reported with no digital tool use. The reporting of data will be the creation of an aggregate teacher data with the categories of the UDCR framework as rows and the phases of the LES instructional model and Mathematical Reflection as columns.

At the Unit level, a follow-up analysis looked at the sequence of Problems or grouping of Problems within Investigations (first and final, similar to Table 3.3). Given that is an unequal amount of Problems in each of these subgroupings, a frequency of digital tool use within these subgroupings of Problems relative to the Unit will provide data for the issue of scaling. Patterns in the data will be explored. Data will be reported as a table of aggregate teacher data with UDCR as rows and the subgroupings of Problems relative to the Unit as columns.

CHAPTER 4: FINDINGS

The purpose of this chapter is to describe the major findings from the data analysis of the coded transcripts of interviewed participants in this study. This chapter will focus solely on instances when teachers describe digital tool use within their respective Units. This chapter will not capture teacher-reported instances when digital tools were not used and their associated specifics as to why the participants choose not to use certain digital tools at various points within their chosen Unit. These instances of digital tool non-use will be discussed at the end of Chapter 5, where observations from the data that go beyond the research questions will be discussed. Given that the research questions center around teachers' uses and purpose of digital tools, it is important to keep the focus of when digital tools were used here in Chapter 4. Topics covered in this findings chapter include participant descriptions and the major findings attending to the actions associated with the reported digital tool use and the interplay between digital tool use, UDCR, and LES.

Participant Descriptions

Multiple research participant calls through the Facebook group, the CMP Facebook page, and through various Twitter postings netted 40 willing participants. Of those participants, 11 rated themselves below a 3 (on a scale of 1 to 5) on their frequency of digital tool use and were not considered for moving forward in the interview process. Three of the 40 respondents reported teaching only one year of CMP3 and were also not considered. Of the remaining respondents, eight were either participants within a particular grade level or Unit of need for the researcher and were able to proceed with a one-on-one interview. Interviews with eight of candidates took place through various video conference platforms (ex. Skype, Facetime, Zoom) or by voice only (ex. phone, Zoom but with no video feed). All of the interviews were audio recorded, transcribed, and coded.

An initial goal of this research was to obtain interviews from two teachers for each of the two selected Units within each of the three grade levels for a total of 12 participants. The reasoning behind this was to have common Units described by more than one teacher and to have multiple Units at each grade level. The realization of obtaining only eight willing participants meant that this would not be the case for all the intended Units. However, the arrangement of participants within their selected Units provided double coverage of three of the Units, with each of the Units in different grade levels. All eight participants will be included in analysis of Chapter 4, but broader claims across grade levels cannot be made.

This interviewed participant pool provides two teachers covering each of the following Units: Variables and Patterns (Grade 6), Comparing and Scaling (Grade 7), and Thinking with Mathematical Models (Grade 8). In addition, two singleton participants for the Units of Covering and Surrounding (Grade 6) and Comparing and Scaling (Grade 7) will be included. An intended Grade 8 Unit for the study was Function Junction, which none of the 40 participants indicated that they teach or were willing to be interviewed about. One possible reason for no participants for this Unit is that it is part of an extended offering for Grade 8, where schools use it if they are trying to offer an algebra option to their Grade 8 students. Since not all Grade 8 teachers use the curriculum this way, finding teachers who use this Unit was difficult. Function Junction was replaced with It's In The System as a viable Grade 8 Unit. However, the timeliness of this decision relative to the holiday break and semester switch during December and January yielded zero participants for this Unit.

The remainder of the section will provide information on participants and their general claims about digital tool use within Problems, across Problems, and their stated mathematical and teacher goals for the first and final Investigations of their respective Units.

Participant Demographics

Table 4.1 provides a rundown of all eight of the interviewed teachers. All participants were given aliases for anonymity purposes. The table also provides the state in which they teach, years teaching CMP3, the grade levels they have taught using CMP3, their self-ranking of digital tool use, and their selected Unit for their interview.

Alias	State	Years Teaching CMP3	Grade Levels Taught	Self DT Ranking (1-5)	Selected Unit (Grade)
Ms. Lambeau	California	3	6	5	Variables and Patterns (6)
Ms. Lombardi	Michigan	4	6, 7	3	Variables and Patterns (6)
Mr. Nitschke	Oregon	4	6, 8	5	Covering and Surrounding (6)
Ms. Canadeo	Illinois	4	7	4	Comparing and Scaling (7)
Ms. Hutson	Oregon	5	6, 7, 8/ALG	4	Comparing and Scaling (7)
Ms. Starr	Texas	3	7, 8/ALG	4	Filling and Wrapping (7)
Ms. White	Delaware	3	8/ALG	3	Thinking with Mathematical Models (8)
Ms. Favre	New York	3	7, 8/ALG	4	Thinking with Mathematical Models (8)

Table 4.1: Participant demographics

Prior to the interview portion over each teachers' Unit of choice, teachers were asked some general questions about their use of the resources related to the CMP3 curriculum and a general digital tool use with the curriculum. The purpose of this is to get some background information about the participants' digital tool use and habits prior to entering the interview over their selected Unit.

Reported General Digital Tool Use by LESMR

Teachers were asked about the LES instructional model and which digital tools they generally use within each phase of the LES and the Mathematical Reflection. Table 4.2 provides a list of the digital tools teachers reported to use generally within a Problem. There are three major takeaways that can be seen in their responses. First, all of the participants reported some kind of video being used during the Launch of a Problem, either the curriculum-provided Launch Videos or from an outside source such as YouTube. Second, almost all of the teachers report some type of class-wide viewing digital tool, such as a document camera, SmartBoard, or AppleTV. Third, almost all of the teachers reported using some type of connected document system (ex. Google Docs, Padlet) or centralized learning management system (ex. Schoology, Google Classroom) past the Launch of a general Problem.

Although this list provides teachers' reported use of digital tools not specific to teachers' selected Unit, it provides a partial answer to the first research question. Teachers reported uses of digital tools at almost every phase of the LES and MR. What Table 4.2 does not provide is a connection to actions when using these tools. The intent of the prompt during the interview was to establish a baseline for what digital tools teachers may report on. However, teachers provided insight into what digital tools they generally use that did not necessarily play out in talked about the selected Problems within their Unit.

Alias	Launch	Explore	Summarize	Mathematical Reflections
Ms. Lambeau	Launch Videos	iPads (Educreations)	Google Forms	Google Classroom
Ms. Lombardi	Videos (Launch, YouTube)	Desmos, CMP tools	AppleTV, iPads	Google Docs
Mr. Nitschke	Launch Videos	CMP apps, Juno	Document Camera	SeeSaw
Ms. Canadeo	Launch Videos, Google Slides	No DT	Google Slides, Google Classroom	<Does not use MR>
Ms. Hutson	Launch Videos	SmartBoard, Calculators	Document Camera	Document Camera
Ms. Starr	PowerPoint, Launch Videos	PowerPoint, CMP Apps, Desmos Activities	Powerpoint, Google Forms, Google Classroom, Desmos Activities	Google Forms
Ms. White	Videos (Launch, YouTube)	Virtual Manipulatives, Calculators	SmartBoard, Schoology	Schoology, Google Docs
Ms. Favre	Videos (Launch, YouTube)	Desmos, Document Camera, Camera	Desmos, Document Camera, Camera, Kahoot	Padlet, Images, Videos, Pictures

Table 4.2: Teachers' general claims of digital tool use by LES instructional model and Mathematical Reflections

Reported General Digital Tool Use Within a Unit

After the teachers thought about the types of digital tools they used in general, they were asked about digital tool use for selected Problems across Investigations within a Unit. Table 4.3 provides a synopsis of teachers' responses, which ranged widely, with some various reasoning. Three participants were unsure of how their use of digital tool changed over time, while others were able to make claims about the shifts in the use of digital tools.

Three of the participants mentioned more digital tools towards the end of an Investigation or the end of the Unit because the students will have better conceptual understanding of the mathematics and may be looking towards more efficient means through digital tools. Others stated that the end Problems of an Investigation or the end Investigation of a Unit would have more digital tool use by default, because the teacher found the beginning Problems in an Investigation or the beginning Investigation with the Unit to be an opportunity to let students learn mathematics with hands-on opportunities.

One teacher, Ms. Hutson, stated more digital tools towards the beginning of Investigations and the beginning Investigations of a Unit. Her thinking was to use digital tools as a form of contextual engagement and to model what the teacher expects to see in students' written work.

The combination of Table 4.2 and Table 4.3 provides some interesting questions, most notably when these digital tools are used within the Unit. Table 4.2 provides general information stripped of when these digital tool uses occur within a Unit. Table 4.3 provides teachers' beliefs about when digital tool use may be more prevalent in certain parts of a Unit than in others. Table 4.3 does, however, provide some justification as to the shifts of focus that occur within the text of a problem-based mathematics curriculum.

Three teachers made connections as to what learning shifts occur within Units of CMP3 and how they would support using digital tools differently over time. Ms. White made the connection that digital tools used earlier on in the Unit are generally used by the teacher, whereas digital tools used later on in the Unit are more student-focused uses of digital tools. Ms. Favre made the connection of early digital tool use conforming to conceptual understanding where digital tool use in the latter portions of a Unit lead to more mathematical skill-based practice.

Alias	First Problems	End Problems	First Investigation	Final Investigation
Ms. Lambeau	Unsure	Unsure	Unsure	Unsure
Ms. Lombardi	Less Tech (beginning to learn concepts)	More Tech (have greater understanding than at beginning)	Less Tech (more hands on while they learn the concept)	More Tech (concepts are solidified, students are looking for efficiency)
Mr. Nitschke	Less tech (focus of concepts)	More Tech (focus on algorithms/numerical solutions)	Less Tech (more open-ended prompts, less	More Tech (using Juno for computerized assessments)
Ms. Canadeo	Unsure	Maybe more Tech (comparable to first Problems)	Unsure	Unsure (states still uses the same instructional technologies)
Ms. Hutson	Most Tech (contextual engagement; model what teacher wants to see in students' graphs)	Less Tech (teacher says students are doing more hands on)	Most at first because teacher is modeling on the SmartBoard. Uses more Launch Videos towards the beginning.	Less tech (students are working in their notebooks by hand)
Ms. Starr	Unsure (developing at beginning)	Unsure (solidifying at end)	Unsure (developing at beginning)	Unsure (solidifying at end)
Ms. White	Less Tech (by hand)	More Tech (as they learn what equations mean & what graphs should look like)	Tech Used by Teacher	Tech Used by Students
Ms. Favre	Tech related to conceptual understanding	Tech related to skill practice	Tech related to conceptual understanding	Tech related to skill practice

Table 4.3: Synopsis of teachers' general claims of digital tool use by location within Investigations and Unit

Table 4.3 is interesting because of the varied reasoning across all participants. For two teachers who mentioned their digital tool use was dictated by hands-on experiences for their students, Ms. Lombardi and Ms. Hutson, these opportunities occur on opposite ends of

Investigations: Ms. Lombardi stated the hands-on opportunities to occur early in an Investigation and Unit whereas Ms. Hutson explicitly states that hands-on opportunities were to occur at the end of Investigations and a Unit. This is just an example of the various lines of reasoning for digital tool use within a Unit.

Mathematical and Teacher Goals

Teachers reported on their mathematical and teacher goals of the first and final Investigations of their selected Unit. Table 4.4 provides a listing of the teachers' intended mathematical goals for the first and final Investigations of their selected Unit along with additional teacher goals for each. This provides evidence of sets of teacher beliefs on how digital tools use may play out within the teachers' selected Units. For the mathematical goals, teachers described the major storylines of the Investigations and used that as their mathematical goal for the Investigation. A few teachers looked at their own lesson plans or a version the Teacher's Guide to look up goals listed in these documents. The teacher goals made them think a bit, as they knew they had to address something other than the mathematical goals they just stated. Many of the teacher goals focused on student shortcomings that they have experienced in the past, either in previous Units or with previous cohorts of students. Such goals as "appropriate scaling and labels on graphs" and "paying attention to the units attached to a number" provide insight as to what their students need to work on that goes beyond the mathematical goals of their respective Units. Other teacher goals have a focus on classroom climate and culture, making connections to other areas of study, and becoming students who persevere when they encounter mathematics that they may initially struggle with.

Alias	Mathematical Goals	Teacher Goals
Ms. Lambeau	INV 1: Connecting real life to representations. INV 4: Translating among tables, graphs, and equations	INV 1: Appropriate scaling and labels on created graphs. INV 4: How pieces relate when building various representations.
Ms. Lombardi	INV 1: Sense of patterns, model and understand INV 4: Write an expression/equation	INV 1: Appropriate scaling and labels on created graphs. INV 4: Distributive Property
Ms. Canadeo	INV 1: Understanding of ratios related to Juice Problem INV 3: Connecting rates with proportions and different units of measurement.	INV 1: Emphasize the foundational skills within this Unit and how they will be important for future Units. INV 3: Paying attention to the units connected to the number; appreciate the metric system.
Ms. Hutson	INV 1: Write and compare ratios; apply to real-world scenarios INV 3: Be able to connect ratios to proportions; not be dependent on calculators.	INV 1: Connect the use of ratios to a cooking unit the teacher does independent of CMP3. INV 3: Connect the use of ratios to a finance unit the teacher does independent of CMP3.
Ms. White	INV 1: Collecting data in two variables and being able to make sense of it. Create representations INV 5: Defining categorical data and creation of two-way tables. How categorical data differs from numerical data.	INV 1: Establishing classroom culture at the beginning of the school year. INV 5: Having students provide detailed answers using words.
Ms. Favre	INV 1: Identify linear and nonlinear patterns. INV 5: Construct and analyze a two-way table of categorical data.	INV 1: Persevere (MP1)* in identifying patterns. INV 5: Engaging in productive struggle. Understanding that students are generally comparing two different relative frequencies.
Mr. Nitschke	INV 1: Relationship between perimeter and area. INV 4: Relationship between surface area and multiple areas. Relationship between volume and stacking layers.	INV 1: Use of algebraic expressions/formulas. INV 4: Perseverance (MP1)* in solving problems. Precision (MP6)*, attention to detail.
Ms. Starr	INV 1: Connections between Volume and Surface Area. INV 4: Volume and Surface Area of 3D shapes	INV 1: Drawing 3D shapes in perspective, Efficiency of boxes, perseverance (MP1)* in solving Problems. INV 4: Visualizing, Drawing 3D shapes, Relationship among volume formulas for various 3D shapes.

Table 4.4: Teachers' stated mathematical goals and teacher goals of their selected Unit

Absent from the either of the two types of goals are mentions of the use of digital tools. This is not a surprising finding, as Table 3.1 showed that of the Units selected, the curriculum did not explicitly state MP5 being covered in Investigation 1 or the final Investigation of these selected Units. All instances of explicit MP5 were in the middle Investigations of the Units. Digital tools could possibly be used to help attend to these teacher goals but are not the primary driver in reaching these goals. In having digital tools come up zero times for both the mathematical goals and teacher goals of all the teachers, the researcher feels confident that these teachers will report out what may be considered typical digital tool use in the classroom. These will possibly be teachers who will think thoughtfully about what digital tools they will use at various points of their Unit, how they will use them, who in the classroom will be using them, and if the juice is worth the squeeze of using a digital tool.

Using Digital Tools to Attend to Other Mathematical Practices

Three participants stated either mathematical or teacher goals that aligned to one of the Standards for Mathematical Practice (CCSSI, 2010). A way to uncover underlying reasons to why teachers choose to use digital tools can possibly be found in the Standards for Mathematical Practice from the Common Core State Standards. The mathematical practices provide potential purposes for digital tool use, as the use of digital tools that attend to practice standards may provide reasoning for their use. Of the eight practice standards, two of the standards are rooted in the curriculum and the scope of this study. MP4: Model with Mathematics is at the heart of the curriculum as a majority of the CMP3 Problems could have some aspect of modeling.

From reviewing the transcripts for possible explicit or implicit reasoning of digital tool use as related to the Standards for Mathematical Practice, three of the remaining six mathematical practices stood out. Table 4.5 provides a synopsis of these instances of digital tool use that attended to MP1: Making Sense of Problems and Persevere in Solving Them, MP3: Construct Viable Arguments and Critique the Reasoning of Others, and MP8: Look For and Express Regularity in Repeated Reasoning.

From Table 4.5, it can be seen that these teachers are using digital tools to perform actions that are related to some of the other Standards for Mathematical Practice. The use of video to connect student to the various contexts is common for six of the eight teachers. Seven teachers indicate various digital tools for the sharing out of student work. Five teachers discussed digital tool use at the end of Investigations to attend to providing generalities. Each teacher had at least one implied use of a digital tool that aligned to one either MP1, MP3, or MP8. More on the implied connection to the SMPs will be examined in Chapter 5.

Summary of Participants

The above provided some descriptions of the participants of this study. Through the initial interview questions, teachers discussed their general digital tool use within a general Problem (Table 4.2) and the variation of digital tool use within a Unit (Table 4.3). Teachers also described their intended goals for each of the two Investigations that were discussed during their Unit (Table 4.4), where three teachers had teacher goals that aligned to one of the eight Standards for Mathematical Practice from the Common Core. In an analysis of teacher interviews, some digital tool uses had implied connections four of the mathematical practice standards.

Alias	MP1 (Sense & Persevere)	MP2 (Reason)	MP3 (Argument & Critique)	MP8 (Repeated Reasoning)
Ms. Lambeau	various forms of context (video)		Share/present work to class (SmartBoard with Google Slides)	End of Unit video to discuss specific problems or generalities (FlipGrid)
Ms. Lombardi	various forms of context (video)		Share/present work to class (document camera; AppleTV with iPads)	screenshots of graphs in final MR w/ explanations/ generalities (Desmos with Google Docs)
Ms. Canadeo	various forms of context (video)		Share/present work to class (document camera)	
Ms. Hutson	various forms of context (video)		Share/present work to class (document camera; SmartBoard)	End of Investigation video stating generalities (SeeSaw w/Pisa)
Ms. White	various forms of context (video)	Written explanations of representations of data (Google Docs)	Share/present work to class (document camera)	
Ms. Favre	various forms of context (video)			Students answer MR questions, provide examples, provide generalizations (Padlet)
Mr. Nitschke			Share/present work to class (document camera)	End of Investigation video stating generalities (SeeSaw w/Pisa)
Ms. Starr			Make claims about relationships (Pouring and Filling App)	

Table 4.5: Instances of implied Mathematical Practices from digital tool use

Findings for Research Question 1

This analysis of data will be broken up into two parts, the first focused on what tools are used with the second part being on their purpose of use.

Wide Variety of Digital Tool Use

The following analysis will look at the first part of the first research question on the digital tools that teachers report using. The purpose is to give the status of digital tools that are reportedly being used by teachers in an inquiry-based, problem-centered mathematics classroom.

From Table 4.6 on digital tool use, it is easy to see that the teachers reported out on a wide variety of digital tools. What is noticeable is the limited amount of curriculum-provided digital tool use beyond Launch Videos. Videos from external sources were also used. The use of videos of some format were prevalent across all teacher in the study, as all but one teacher, Ms. Starr, reported on using some form of video in their instruction over at least one of the selected Problems in their respective Unit.

All teachers reported out that their classrooms were equipped with some form of 1:1 computing technology for their students and were utilized in varying degrees during instruction. Not surprisingly, 100% use of some form of computing device lead to an array of digital tool use in the classroom. We can see that these eight teachers are using digital tools and computer use is high. In addition, the growth of computers in the classroom can be seen from data from 20 to 25 years ago by Jacobs et al. (2006), whose work showed single-digit percentage computer use by teachers in reviewed classroom videos from 1995 and 1999. Many types of digital tools were reported due to the varieties of digital tools that are currently accessible on computing devices.

	Digital Tool	Total count of uses within phases of LES	Number of teachers reporting use of digital tool	% of teachers using digital tool
Curriculum Provided	Launch Videos	15	5	62.5%
	Data and Graphs	1	1	12.5%
	Virtual Bridge	1	1	12.5%
	Pouring and Filling App	1	1	12.5%
External	YouTube	5	3	37.5%
	Other Videos	2	2	25%
	Calculator	4	2	25%
	Desmos	1	1	12.5%
	Desmos Number Line	2	1	12.5%
	Document Camera	18	6	75%
	Digital Projector	1	1	12.5%
	SmartBoard	14	2	25%
	SmartNotebook	9	1	12.5%
	SmartBoard Number Line	2	1	12.5%
	iPad	3	1	12.5%
	Educreations	1	1	12.5%
	AppleTV	1	1	12.5%
	PowerPoint	16	2	25%
	Google	1	1	12.5%
	Google Docs	5	2	25%
	Google Slides	5	3	37.5%
	Google Forms	3	2	25%
	Google Classroom	2	1	12.5%
	Schoology	2	1	12.5%
	Padlet	3	1	12.5%
	SeeSaw	4	2	25%
	Pisa	2	1	12.5%
	Juno	1	1	12.5%
	Sketchpad.io	2	1	12.5%
	Flipgrid	1	1	12.5%

Table 4.6: Teacher-reported digital tool use

The use of calculators and other related calculator technologies reported by teachers was relatively low, with two teachers reporting using calculators a total of four instances within Problems. Other calculator-type technologies were also present, such as Desmos and SmartBoard Number Line, but not used by multiple teachers within the study. Reported calculator use by teachers in this study hover near the same rate of roughly 30% as Jacobs et al. (2006) reported in their study.

Although the focus of this portion of the analysis was on what digital tools teachers report using, numbers are provided to make general comparisons of the data to each other digital tool. However, not all numbers have the same meaning. For instance, PowerPoint was mentioned 16 times for its use within individual phases of the LES of Problems among two teachers. One teacher reported use of PowerPoint 14 times, often using it for the Launch, Explore, and Summarize of the same Problem. A similar story plays out for reported SmartBoard use. On the other hand, the use of document cameras had a high count and was mentioned by six of the eight teachers in the study. This is a digital tool that was widely reported by teachers and multiple times through their interviews.

In summary, teachers report on using a wide variety of digital tools. While some of the digital tools may be specific to the computer hardware available to the teachers and/or their students, teachers report uses of digital tools of the same type regardless of that specificity. A good example is the consistent use of video from various sources. Outside of Launch Videos, few curriculum-provided digital tools were reported on for their use. Teachers reported a wide variety of digital tools external to the curriculum, including tools for calculations (calculator, Desmos, etc.), collaboration (Google suite of tools, Padlet, SeeSaw, etc.), communication (Google suite of tools, document camera, digital projector, AppleTV), and artifact creation (Educreations, Sketchpad.io, Flipgrid, etc.). These types of digital tools are not available through curriculum-provided sources yet are frequently used by teachers in this study.

Providing Purpose for Digital Tool Use

As teachers reported out their digital tool use within their selected Units, it became apparent that none of these teachers are using digital tools for the sake of using digital tools.

They had very pointed reasons as to why they chose to use a certain technology at specific instances of time within a Problem and across Problems. Even with the variations of reasoning that occurred in Table 4.3 where the teachers generally described their digital tool uses across the Unit, their reasoning to use digital tools within CMP3 Units closely aligned with four categories of digital tool use. Teachers at the various phases of the LES of a Problem looked at the possible use of digital tools through the lens of:

- How does the use of a digital tool help my students understand the Problem?
- How does the use of a digital tool help my students carry out the mathematical task of the Problem?
- How does the use of a digital tool help with the way the teacher can interact with the students and the students interact with their peers?
- How does the use of a digital tool help with attending to the big picture of a Problem/Investigation/Unit?

The next four subsections will provide evidence from interview transcripts supporting these categories arising from the interviews and connect to the literature.

Teachers' Focus on Problem Familiarity

Evidence that teachers use digital tools to help students understand the Problem can be seen in the following quotes by Ms. Starr and Ms. White when they described how they negotiate the use of digital tool at the beginning of Problems:

> To set the stage and so they can see a picture of something, um, whether it's, I mean I'm thinking about like the Juice Box or they change from a cylindrical can into a rectangular prism. Like I show that video to show them this is the Problem and now you're going to solve it. (Ms. Starr, Grade 7 Teacher)

Ms. Starr suggests the use of video to set the stage of the Problem, which students are unfamiliar with the mathematical aspect of the Problem.

> There are some [Launch] videos that I use from CMP3 because they align with what the math is, but then I find that there's some launch videos that do not align with what the math question is asking so I don't use them. (Ms. White, Grade 8)

They each discussed aspects that teachers pay attention to at the beginning of lessons: alignment to context of the Problem. Ms. Starr discussed the use of a photo or video for students to gain familiarity with the Problem. Ms. White discussed the alignment of curriculum-provided videos to the Problem and made mention when she found instances when the curriculum-provided video does not align to the Problem, then Ms. White made the decision on not to use that particular digital tool because it does not help with student understanding of the Problem. As will be mentioned in Chapter 5, teachers focused on the Launch of a Problem in order for inquiry-based instruction to be successful at the end of a lesson. These teachers are supporting the claim by using digital tools (or not using them) to get the students familiarized with the Problem.

Consistently throughout the interviews with the teachers, teachers were often worried about their students having a good understanding of the context of the Problem prior to engaging with the Problem. One of the most frequently used digital tools used by teachers were videos. Often times, teachers chose to use curriculum-provided Launch Videos or they used videos from external sources, such as YouTube, to address the context of the Problem (Boster et al, 2007; Bransford et al., 1986). Teachers made explicit their various feelings about the curriculum-provided Launch Videos, as many felt that they do not adequately address the context of the

Problem, so they decide to still find a video that will better address the context when they feel addressing the context prior to engagement with the Problem is important. All teachers reported on the importance of problem familiarity, with a total of 17 explicit instances across all interviews.

Teachers' Focus on Carrying Out Tasks

Evidence that teachers use digital tools to help students carry out the mathematical task of the Problem can be seen in the following three quotes:

> I use the technology of graphing technologies and things like that to model graphing. (Ms. Hutson, Grade 7)

Ms. Hutson suggest that graphing utilities help modeling the act of graphing. Ms. Lombardi discussed the use of Desmos for its graphing features.

> "My students will use like Desmos graphing features a lot, especially in algebraic units." (Ms. Lombardi, Grade 6)

Ms. Favre highlighted the use of Desmos to attend to multiple representations, which also offloads student work.

> "I find myself using programs like Desmos because I can produce this seamlessly go back and forth between a table, a graph, and equation and show things kind of side by side." (Ms. Favre, Grade 8)

An interesting aspect of these comments by teachers is how they are using graphing utilities to make going between representations more efficient for their students. In similar comments on non-graphing use of calculators, teachers discussed computational efficiency so that students can attend to the mathematics of the tasks and not spend their time computing numbers. Whether it is using digital tools for quicker computations or quicker construction of multiple representations, these digital tools help students by getting them into the context of the Problem and not spending their time on monotonous calculations and construction of representations. These are similar findings to two of the five calculator uses stated by Doeer and Zangor (2000), where graphing calculators are used as a computational tool and a visualization tool. The degree in which they were helpful for real-world problem in context beyond offloading computation and representation creation, as stated by Bardini, Pierce, and Stacey (2004) was not found beyond these two uses of graphing calculators.

Teachers' Focus on Classroom Interactions

Evidence of the use of digital tools for interactions among teachers, students, and their peer can be seen throughout all interviews. A few statements by teachers supporting this claim are:

> They'll be in charge of presenting C4 and what they'll do is they'll go on their Chromebook through Google classroom, I'll post a shared set of Google slides where they can click on it on their Chromebook and every student has editing access to those slides and then they have to find the slide that they're in charge of presenting and they add their work at their reasoning and their answer and then they're the ones up at my board presenting that problem to the class. So they worked on it in their Chromebook, creating the slide and then they're presenting the slide through my projection to the rest of the class. That's often the summarize that I do. (Ms. Canadeo, Grade 7)

Here, Ms. Canadeo discussed how students are using Google Slides to leverage classroom discussions towards the end of Problems. Ms. Lombardi suggests similar actions with a similar projection tool.

> Our students do have one to one devices, we have AppleTV, so that promotes the use of that, like students sharing, sharing their screens at the end of class, like when we're summarizing, it brings class discussion in, so that's just like sharing out their work. (Ms. Lombardi, Grade 6, Variables and Patterns)

Mr. Nitschke describes a third digital hardware that provides opportunites for students to share out their work to the rest of the classroom.

> So now that you had, you had mentioned the document camera, that's a big part of our sharing out of student ideas. (Mr. Nitschke, Grade 6)

The quotes by Ms. Canadeo, Ms. Lombardi, and Mr. Nitschke provide support for sharing out student work and for facilitating classroom discussions through the use of digital tools. The culminating moment for all Problems is the Summarize portion of the LES. Teachers expressed their desire to make sure that all students are active participants at this stage and tried to find ways to leverage digital tools so that could happen. Teachers felt it was important that students were able to express their work with their peers as well as with the teacher. Therefore, the use of digital tools that allowed for students to display and share their work and thinking to others and allowed for feedback and refinement were frequently reported by teachers. The works of Vahey, Takar, and Roschelle (2004), Forster and Taylor (2003), and Genlott and Grönlund, (2016) point to the classroom communication that can take place when computers and computer accessories are available for use. All teachers described some form of projective technology for the sake of students sharing out their ideas to the entire classroom.

Teachers' Focus on Showcasing Understanding

The final portion of evidence is on how teachers use digital tools for students to see the big picture at multiple levels within a Unit. Ms. Lambeau discussed how she used FlipGrid to allow students to capture the big picture of a Unit.

> FlipGrid is, I don't know if you've ever heard of it, but it's where you, um, it's kind of like a message board that you create... Um, so that would be a good, good chance to kind of have them see the whole, the whole big picture. (Ms. Lambeau, Grade 6)

Ms. Lombardi discussed how the students in her class used Google Docs to share their ideas and provide feedback.

> We use Google Docs so they'll have like a share it with a partner to like peer edit or to have a partner read over it. Um, so like between one other person and then they'll share it to me to like review and add comments to it, like any questions that I might have and then offer feedback. (Ms. Lombardi, Grade 6)

Ms. Hutson described how students create videos to explain their mathematical thinking.

> So they're explaining the reasons through verbal, through SeeSaw, so it's like a video recording. Um, and then they are picking example that they have in their notebook they're proud of, um, and showcasing kind of like how to solve. For instance, this one is how to solve a proportion and to give an example. So then they were going back through their notebook from Unit 1 and finding a place where they solved a proportion and explaining how they solved it. Um, and then so it's all kind of through a video format, um, and then showing documentation of their learning. (Ms. Hutson, Grade 7)

These are examples of how teachers are had their students use digital tools to help document their learning. In addition to Ms. Hutson's comment, students also uploaded their SeeSaw videos to a portfolio system that they could access at any time. In particular, teachers discussed ways in which they used digital tools as a form of a student-created artifact. They saw the power in using the digital artifacts as a way for students to easily access their past thinking and as a way of revising their mathematical thinking over time. Many teachers found the creation of digital artifacts by students at the end of Investigations or Units as a way to submit to school-wide student portfolio systems as well. This points to classroom work in creating artifacts that Spires et al. (2008) alluded to when describing student experiences in using various computer softwares to showcase their mathematical understanding. All but one of the teachers expressed the desire to use digital tools to allow students to express their overall understanding of a particular strand of mathematics.

Connecting Guiding Questions to the Four Categories

Digital tool uses were sorted for common purposes of use through the four categories: Understand/Learn, Doing, Classroom, and Recall. Table 4.7 provides an exhaustive list of teacher-reported actions with digital tools associated with the four categories of actions. These categories of actions support the purpose of teachers' use of digital tools.

From Table 4.7, a few actions are evident that play out in each of the categories. For Understand/Learn, the action associated with digital tool use for the access to context was pervasive, with 28 instances across Problems of digital tool use for that purpose from six different sources. The majority of these uses to access context came in the form of video.

For the Doing category, the leading action is associated with understanding students' role in what they are doing in the Problem or Mathematical Reflection, with eight instances of digital tool use from four different sources. What is interesting about these instances is they are teacher-based uses of digital tools and not student use. In the cases when PowerPoint and Schoology were used, the teachers reported presenting information to the students on their roles within the

Understand/Learn	Doing	Classroom	Recall
Context (28) (Launch Video, Youtube, Document Camera, other video, PowerPoint, Digital Projector) Representation (3) (Document Camera, SmartBoard)	Understand Role (8) (PowerPoint, Launch Video, Schoology, iPad) Draw (5) (Educreations, Sketchpad.io, Google Slides, SB # Line) Create Representation (4) (SmartBoard, Desmos # Line, Google Classroom, SB w/SN) Calculate (4) (Calculators, Google) Reporting (3) (Google Docs, Google Slides, SB w/SN) Collect Data (2) (Data & Graphs, Virtual Bridge) Discover Relationship (1) (Pouring and Filling App) Create Video (1) (Flipgrid)	Sharing/Present (26) (Document Camera, SB w/SN, PowerPoint, SmartBoard, AppleTV, Pouring & Filling App) Collect Data (2) (SmartBoard) Write (2) (Google Docs, SB w/SN) Organizing (1) (Google Slides) Create Representation (1) (Desmos # Line) Comparing (1) (Desmos) Gallery Walk (1) (Sketchpad.io) Draw (1) (SB # Line)	Artifact (8) (SB w/SN, Google Docs, Google Forms, SeeSaw, Schoology w/ Google Slides & Google Docs, Padlet) Feedback (4) (Google Forms, Google Docs, Google Classroom) Create Video (2) (SeeSaw w/Pisa) Crowd Source (2) (Padlet) Camera (1) (iPad)

Table 4.7: Actions sorted by categories of actions with frequency and digital tools associated with action

tasks when working in small group settings. The iPad was used as a timer for student to know how much time they had left with the task. All of the other actions in this category are student-led actions when working with the Problem.

Sharing and presenting was the dominate action within the Classroom category, with 26 reported instances of digital tool use from six difference digital tools. Other actions were present within the Classroom category but none as frequent as those for sharing and presenting. The digital tools associated with sharing and presenting were projective tools for more than a single or small number of people to observe. The use of document cameras, SmartBoards, and AppleTV allow for other application to be projected to a larger number of people, such as the Pouring and Filling App and PowerPoint.

Actions coined as artifacts were the most frequent within the Recall category, with a total of eight instances with six different digital tool sources or combination of them. A close second place to artifact was the action of feedback, which took advantage of the Google suite of tools that allow for collaboration with others in the classroom. Among digital tools used for artifacts and feedback, collaborative digital tools dominate their use.

Summary of Findings for Research Question 1

Key findings related to the first research question are that teachers use a variety of digital tools within their classroom, where all teachers reported having classrooms with 1:1 computing. From this:

- The consumption of video for students to understand or learn about problem context was critical to all teachers in the study.
- The sharing of ideas and work was central for classroom interactions and teachers used projective digital tools to attend to this interaction.
- Teachers reported the use of digital tools for the creation of artifacts for students to show their summative mathematical understanding.

From the perspective of curriculum-provided digital tools, Launch Videos were frequently used. Of the few opportunities that other curriculum-provided digital tools were suggested to be used within the Problems to be explored in this student, they were used a total of three time.

Findings for Research Question 2

This analysis of data will focus on when the use of digital tools occurred within the phases of Problem (LES) and across Problems (Investigation, Unit). Categorization of actions are a focus from the data analysis from the first research question with specific digital tools playing a less significant role. Similar to the findings of the first research question, this will be presented in two parts. The first part highlighting the four categories of actions, which will be referred to as the UDCR framework, and the second part will look at grouping of consecutive Problems.

Findings at the Problem Level

The following analysis is a separate planned analysis of Table 4.7, where actions along with the digital tools that were associated with those action were placed into one of four categories of actions. Table 4.8 will concentrate on when these actions with digital tools occurred within the phases of all Problems (LES) or during the Mathematical Reflection. For simplicity, the digital tools will be left off the actions that are listed.

The major takeaway from this table is the concentration of actions within specific phases of the LES of Problems and the Mathematical Reflections. Although there do exist a few outliers, the majority of digital tool uses fall under a pattern that relates categories of actions (and their purpose of use) to the phases within Problems and the Mathematical Reflection.

	Launch	Explore	Summarize	Mathematical Reflection
Understand/ Learn	Context (28) Representation (3)			
Doing	Understand Role (2) Create Representation (1) Calculate (1)	Understand Role (6) Draw (5) Create Representation (3) Calculate (3) Reporting (3) Collect Data (2) Discover Relationship (1)		Create Video (1)
Classroom	Sharing/Present (2)		Sharing/Present (24) Collect Data (2) Write (2) Organizing (1) Create Representation (1) Comparing (1) Gallery Walk (1) Draw (1)	
Recall			Artifact (1)	Artifact (7) Feedback (4) Create Video (2) Crowd Source (2) Camera (1)

Table 4.8: Teachers' reported use by categories of actions across LES/MR

In general, the dominate use of digital tools that were reported fall under the diagonal of Table 4.8. Digital tools whose purpose was for understanding and learning were generally used during the Launch of Problems. Digital tools whose purpose was for the execution of mathematical tasks occurred during the Explore of Problems. Digital tools whose purpose was for whole-classroom interactions were used during the Summarize of Problems. And finally, digital tools whose purpose was for the ability to recall mathematical knowledge gained were used during the Mathematical Reflections at the end of Investigations. More on the implication of this table will be discussed in Chapter 5.

Findings at the Unit Level

The analysis of data to determine how digital tools were used at the Unit level, or when digital tools were used across Problems. Table 4.9 provides an aggregated account of actions of digital tool use along with where Problems were positioned within the two selected Investigations of the Unit that were explored. The counts represent the times within phases of LES and MR in which the actions were mentioned.

The aggregate data of teachers' use of digital tools across the Unit shows that actions associated with digital tool use are relatively uniform across the Unit. The need to use a digital tool to cover context of Problems decreases over the course of a Unit, as can be seen in the progression of the Understand/Learn row across the sequence of Problems across a Unit. Similarly, the use of digital tools for the purpose of sharing and presenting are concentrated at the Problems of the first Investigation.

Although the frequencies of these actions with digital tool use seems to be higher for those sets of Problems in the first Investigation, remember that the setup of the study included all of the Problems in Investigation 1. For the selected Units, the number of Problems in the first Investigation had either three or four Problems within Investigation 1. For the final Investigation, only the first and final Problems of that Investigation were used in the study, so those sequences of Problems within the Unit only consist of one Problem.

	First Problems, First INV	Final Problems, First INV	First MR	First Problem, Final INV	Final Problem, Final INV	Final MR
Understand/ Learn	Context (10) Representation (2)	Context (9)		Context (6) Representation (1)	Context (3)	
Doing	Understand Role (5) Reporting (1) Create Representation (1) Collect Data (1)	Understand Role (2) Calculate (1) Collect Data (1)		Draw (4) Reporting (2) Understand Role (1) Calculate (1)	Draw (1) Calculate (1) Discover Relationship (1)	Create Video (1)
Classroom	Share/Present (10) Collect Data (1) Write (1) Organizing (1)	Share/Present (11) Collect Data (1) Comparing (1)		Share/Present (2) Write (1) Gallery Walk (1)	Share/Present (3) Create Representation (1) Draw (1)	
Recall			Artifact (2) Feedback (2) Create Video (1)			Artifact (6) Feedback (2) Create Video (1)

Table 4.9: Actions reported by category and location within Unit

Both of the Mathematical Reflections had similar digital tool use for the creation of an artifact or for feedback purposes. The digital tools use was more prominent for the final Mathematical Reflection possibly due to the summative nature of the entire Unit, whereas the Mathematical Reflection of the first Investigation leads into the second Investigation of a Unit. Teachers reported in the interviews that they felt the final Investigation to be more important to attend to than the Mathematical Reflections at the end of preceding Investigations.

Summary of Findings for Research Question 2

Two key findings related to the second research question are:

- The purposes are tightly linked to the phases of the LES instructional model. Teachers reported uses of digital tools closely linked to understanding and learning context through the use of videos during the Launch, various digital tool uses for carrying out the mathematical tasks during the Explore, uses of digital tools for sharing and presenting during the Summarize, and digital tools that allow for the creation of artifacts and feedback during the Mathematical Reflections. Therefore, the uses of digital tools are purposeful towards specific phases of the LES instructional model and the Mathematical Reflection.
- The categories of actions are well disbursed across beginning and end Problems within Units. Greater frequencies of digital tool use are reported within the first few Problems of the first Investigation of a Unit. Digital tool use still persists though the end of a Unit and during the final Mathematical Reflection.

These findings suggest that teachers are using digital tools in very precise ways. Many of the teachers discussed using digital tools at particular points within Problems such that it allowed for their students to have insight into Problems, help them carry out the task, communicate with others, and document their overall learning. Table 4.8 provides evidence that teachers are using digital tools in ways that have particular actions during the phases of the LES of Problems and the Mathematical Reflection. Whereas Table 4.3 may have suggested that there may have been possible digital tool use that was either at the beginning of Units or at the end, Table 4.9 suggests that digital tool use was uniform throughout Units, suggesting that the actions associated with digital tool use are more dependent on the instructional method than the progression of learning of a particular mathematical strand.

CHAPTER 5: DISCUSSION OF FINDINGS

This discussion chapter will take a deeper look into the findings described in Chapter 4. The purpose is to add a layer of synthesis to this study as to what the data tells us about the use of digital tools within an inquiry-based, problem-centered mathematics curriculum. Later in the chapter, the discussion will focus on items not directly related to the two research questions, but that gives some insight for possible future research.

Overall, teachers reported a variety of digital tools within their inquiry-based, problem-centered mathematics classroom. Major findings include:

- The consumption of video for students to understand or learn about problem context was critical to all teachers in the study.
- The sharing of ideas and work was central for classroom interactions and teachers used projective digital tools to attend to this interaction.
- Teachers reported the use of digital tools for the creation of artifacts for students to show their summative mathematical understanding.
- The categories of actions (UDCR) are tightly linked to the phases of the LES instructional model.
- The categories of actions have uniform during the beginning and end Problems within Units.

These bullet points serve as a synopsis of findings from the data relative to the two research questions from this study. They will collectively be examined in the upcoming sections.

Variety of Digital Tools Used

Table 4.6 provided a listing of all digital tools that teachers reported for use within their selected Problems of their Unit. All teachers in the study reported some use of 1:1 computing technology that their students had access to (Chromebooks, laptops, MacBooks, iPads, BYOD). Within their specific domain of computing, they used tools to provide contextual understanding of Problems, allowed students to communicate and collaborate, and allowed them to construct artifacts. Teachers provided their students these opportunities regardless of the computing device needed. For example. Ms. Lombardi reported using iPads and allowing students to share their screens out through the use of AppleTV while other teachers discussed using Chromebooks and Google suite of tools to share out on SmartBoards or digital projectors. Either way, these digital tools allowed for student to communicate their findings. What should not be alarming is the length of the list of external digital tools (relative to the curriculum-provided digital tools), as many serve the same functional features but are specific to the computing technology available.

A more concise categorization of these digital tools may look like the following in Table 5.1:

Digital Tool Category	Instances of use within LESMR
Videos (YouTube, other)	7
Calculating (Calculator, Desmos)	7
Projective Hardware (SmartBoard, document Camera, AppleTV, etc.)	39
Projective Software (SmartNotebook, PowerPoint, etc.)	29
Communication and Collaboration Tools (Google suite of tools, Padlet, Schoology, etc.)	42

Table 5.1: Condensed categories of external digital tools reported by teachers

The digital tool categories with high counts of use, such as projective softwares and communication and collaboration tools, teachers have to turn to external tools because those types of digital tools are not offered by the curriculum. However, teachers choose to use these tools for purposes related to the LES instructional model. Although teachers found little use for curriculum-provided digital tools outside of the Launch Videos within the Problems of this study, they are using external digital tools to help them within the LES instructional model of Problem beyond the Launch.

Non-Math-Specific Softwares

Many of the external digital tools that are not specific to being mathematical tools reported on are Web 2.0 tools (Dohn, 2009) which allow for students to share their thinking in a digital format (Spires, Hervey, Morris, & Stelpflug, 2012) and give them a sense of ownership with their work (Blau & Caspi, 2009). Classroom pages such as Google Classroom (Iftakhar, 2016), Schoology (Biswas, 2018), and Pisa are forms of digital repositories that allow for teachers to distribute and/or collect digital works.

Teachers repeatedly discussed the use of various forms of communication tools, such as the use of Google Docs, Padlet, and other platforms. Providing students the opportunity to communicate their mathematical thinking to others and to receive feedback were main tenants of use during the Explore, Summarize, and Mathematical Reflection. So other classification of digital tools received the same treatment as communication tools, as they offered multiple actions which allowed for them to be used during multiple phases of the LES of a Problem.

Many of these Web 2.0 tools afford teachers the ability to provide feedback right within the tool, which allows for students to make appropriate revisions to their work. Mr. Nitschke stated that when using SeeSaw “They can give feedback to others and I can give feedback to them. So one thing that I been using with it is giving them feedback and saying, okay, you haven't explained this or there's not enough detail.” Mr. Nitschke is explaining how SeeSaw works as a mechanism of providing feedback to his students but also as a communication and collaborative tool, which provides students the opportunity to revise their thinking and their work through SeeSaw with input from their peers and teacher.

Math-Specific Softwares

Math-specific digital tools reported by teachers that are used on user hardwares include Desmos and curriculum-provided apps, such as the Virtual Bridge App, Data and Graphs, and Pouring and Filling. Teachers reported using Desmos for the creation of various mathematical representations (tables, graphs). Use of the three curriculum-provided apps teachers reported out on were used for the sake of data collection within the context of the Problems. For example, Ms. Favre reported using the Virtual Bridge app for students to gather data for Problem 1.2, but only after students collected data by hand for a similar context in Problem 1.1. In this case, the digital tool was used for the purpose of efficiently collecting data.

Of all the classes of digital tools examined in this study, the use of math-specific digital tools was possibly the most interesting, mainly for its non-use. At the same time, it is difficult to say they were not used, as they occupied the longest column in Table 4.7. What can be seen in this table is that there was not a dominate math-specific digital tool or action associated with a group of math-specific digital tools that were experienced in the other phases. It appears that math-specific digital tools could have a short shelf life within particular Units and are not useful in other Units. The tendency for teachers to use math-specific tools during the Explore runs into another issue that teachers often reported: the desire for hands-on opportunities for students. A major limitation of this study was not having teachers report out on all of the Investigations in a Unit, which the likelihood of math-specific digital tools being reported to being used may have increased for the middle Investigations of a Unit.

Calculator Use

Teachers throughout the study reported on the use of calculators throughout their respective Units, as a way of offloading work so students can focus on the context of the

Problem and not the construction of a representation. Ms. Starr stated that:

> In general, I allow students to use calculators quite a bit. Uh, just because by seventh grade, I kind of assume they should be able to do a lot of the calculations, you know, like if I asked them to do it without, they would be able to. And sometimes it can help us focus on the underlying math concepts.

Calculators were reported as being present in classrooms but were rarely the focus of digital tool use for Problems. Participants described their general use as being available to students who may struggle with the numbers and their calculations in the Problem (Bouck, 2008). Teachers indicated more frequent use of software-based calculators, such as Desmos, when it came to creating representations. Given that students had 1:1 computing devices with the capability to do calculation elsewhere, it is not surprising that teachers reported on actions of calculating with digital tools other than physical calculators. In addition, the ability to create multiple representations (Lesh, Post, & Behr, 1987) with one tool provides opportunities to connect to the majority of students in the classroom.

Digital Tools Used Summary

The amount and varieties of digital tools used in the classroom are increasing at a rapid rate. Hardware is becoming more ubiquitous, even in our pockets, and the softwares available are becoming easier to use (Hedberg, 2011). This all has implications on classroom practices. The data suggests that this is true in the mathematics classrooms. Teachers are using digital tools that support individual, small group, and whole classroom interactions. They are using compatible softwares on these devices that allow for students to be content creators, create representations efficiently, and participate in whole class discussions. Teachers are aware of digital tools external to the curriculum and think through various lenses in how they can be used. Curriculum-provided digital tools are almost always Problem-specific and are far less likely to be used than digital tools that have non-math specific qualities. Teachers are finding digital tools from any source that is the right tool for their instruction with problem-centered tasks. This has implications on how curriculum designers can plan for teachers use of digital tools and the digital tools in which curriculum can make available to teachers.

Purpose of Digital Tool Uses

Teachers constantly discussed using tools that allowed for their students to understand the Problem, carry out the task of the Problem, interact with their peers and the teacher about the Problem, and help express the big picture of Investigations and Units. The creation of the four categories of actions from these statements shaped this study in a way that the actions provided the purpose of their use. In addition, they had close alignment to the phases of the LES instructional method, which provided even better reasoning for their purpose of use.

Placing digital tool uses into these categories provided clarity as to why some tools were frequently reported on, such as the use of videos and projective devices. What was maybe more surprising is how much more frequently some digital tools were used over digital tools that ranked second in these categories. In general, these digital tools are non-math-specific digital tools, and as mentioned in the previous section, they have the ability to be used during numerous Problems within a Unit. Because of this flexibility of use of non-math-specific digital tools, they are able to be used in more than one of the categories representing different sets of actions, such as the use of the Google suite of tools mentioned for multiple actions within the Doing, Classroom, and Recall categories of actions.

It was important to focus on the actions, or purpose, of the digital tool uses rather than the digital tool itself. Focusing on the use(s) of the digital tool allowed for actions to be placed in a category mutually-exclusive to the other categories of actions. Digital tools that have multiple action types, such as the Google suite of tools, provide opportunities to see their use in multiple phases of the LES of Problems.

From the literature, researchers discussed the various ways in which graphing calculators were used within observed classrooms. Milou (1999) discussed the use of graphing calculators as a motivational tool, while Doerr and Zangor (2000) discussed its use in five modes: computational, transformational, data collection and analysis, visualizing, and checking. Beyond graphing calculators, reported uses of digital tools meet some of these modes of tool use. For example, the use of the Virtual Bridge app as a way for data collection and analysis. However, not all of these are met through the reported uses of digital tools. As for motivation, Spires et al. (2008) reported numbers on students' interactions with certain digital tools inside and outside of their school experiences, but not one teacher described their purpose of digital tool use as being motivational. This may be because of how omnipresent technology is in today's world and how that impacts students viewing its use as something special.

The digital tool used does not dictate where it may fit within a lesson, its purpose of use does. For the case of this study, teachers used numerous digital tools that were used in specific phases of the LES instructional model or the Mathematical Reflections (sans Launch Videos and YouTube Videos, which dominate the digital tools used in the Launch phase of LES) (Krauskopf, Zahn, Hesse, 2012). Many digital tools are finding their way into the Explore, the Summarize, and possibly the Mathematical Reflections at the end of Investigations. What varies here are how they are used and, subsequently, who are the ones using it. For example, the Google suite of tools were reported on by teachers for their use within the Explore, Summarize, and Mathematical Reflection. Although the same tools were used, their uses changed and who they were being used with depending on phase of the LES of the Problem.

Close Connection Between LES and UDCR

Table 4.8 provided the biggest take away from this study: the relationship between actions with digital tools and the LES instructional model. Although the table does show some outlying instances, the bulk of the digital tool action running down the main diagonal of the table shows how actions are related to the LES.

Jackson et al. (2013) established that the quality of the summary within an inquiry-based lesson depends on how well the problem was initiated. The results of this study suggest that teachers are finding ways to initiate Problems through the use of videos. Teachers choose to use videos and other digital tools to attend to contextual characteristics within problem-centered lessons at the beginning of the Problem. In order to have a quality summarize, they use digital tools frequently that allow for communication and collaboration among students and the teacher. The ends of the LES instructional model are well supported by purposeful digital tool use.

What about the Explore? The actions associated with the Explore are meant for students as they go through the enactment of the task. Although there are numerous digital tool actions reported in the Explore phase, a few things stick out: 1) there is no frequency leader like context for the Launch and sharing and presenting for the Summarize, and 2) the uses of digital tools for the most frequently reported action understanding role are digital tools that the teacher used, not the students. The uses of PowerPoint, Launch Video, Schoology, and iPad were used for classroom management purposes. Except for the action: understanding role, most actions were associated with digital tool use that occurred no more than five times across all eight teachers in the study. This makes it difficult to making any major claims about what digital tool uses occur during the Explore of a Problem. From the data, digital tool use within the Explore varies across seven different digital tool actions, where none of them are common among the majority of teachers in the study.

Uniform Use of Digital Tools Across Multiple Lessons

Table 4.9 suggested that there is some uniformity of digital tool uses across lessons within a Unit. For example, attending to context of Problems had multiple instances in all reported Problem sequences within Units. Sharing and presenting was constant for classroom interactions in regardless of where Problems were located within Units. As previously mentioned, there are an unequal amount of Problems within each of these sequences of Problems but scaling the frequencies of actions shows that actions with digital tools are increased a tad at the beginning of Units, but still maintain their uses throughout.

This indicates that no matter how deep into the Unit a teacher may go, no matter where students are at with the unpacking of the mathematics within the Unit, there is a need to hold the structure of the LES instructional model and the actions with digital tools that support each of its phases. Although Jackson et al. (2013) make their claim focused on individual tasks, there seems to also be an inherent claim that this is also true for the sequence of tasks within a Unit. This is clear the actions of digital tool use in the final Investigation, final Problem category of Table 4.9 and from the notable uptick in actions with digital tool use when comparing the final Mathematical Reflection to the Investigation 1 Mathematical Reflection.

Explore

Teachers' inclusion of curriculum-provided math apps was limited in this study, as only three different CMP-provided digital tools were mentioned once by all teachers for their use: Virtual Bridge App, Data and Graphs, and Pouring and Filling. Of all the math-specific digital tools mentioned in this study, these digital tools had a unique characteristic: they were mentioned in the planning chart in the Teacher's Guide as to suggestions on when to use them and also offered brief descriptions on how they may be used.

The setup of this study is somewhat problematic for making general claims about curriculum-provided math-specific apps (and even external math-specific apps) within the Explore. Although the use and purpose of certain digital tools seem apparent for the Launch, Summarize, and Mathematical Reflection regardless of Problem location relative to the Unit, the study does not capture the possible use of math-specific digital tools that could be used during the middle Investigations, once students have had those "hands on" opportunities that teachers found to be purposeful during their reporting of actions in Investigation 1. Overall, the use of digital tools during the Explore is less clear than those in the other mentioned phases and that could be because of the difference in tool type, as the digital tool types reported to be used during the Launch, Summarize, and Mathematical Reflection tend to be general digital tools which allows for flexibility of use across multiple mathematical strands.

The explicit mention of their existence and possible use given in the Teacher's Guide suggests that teachers should use them. The data suggest that it has less to do with the quality of these digital tools and more to do with where teachers felt they would fit within the LES instructional model. These CMP-provided digital math tools are suggested to be used within the Explore of a Problem and the teachers in this study agreed with that location of use when it was brought up that they were not using digital tools suggested by the Teacher's Guide. The main issue is that the teachers in this study reported that they were reluctant to use any type of technology, outside of a calculator, during the Explore. To them, the Explore was a moment for students to have "hands on" or "by hand" experiences with limited technology use during the early Problems of their respective Units.

Of the curriculum-provided digital tools that were explicitly mentioned in the Teacher's Guide and were not mentioned by the teachers for use in their reported Problem descriptions during the interview, the interviewer followed up with questions on the non-use of these digital tools. No participant said that they use them later in the Unit, but that does not mean that they do not use them as the focus of the follow-up question is why they do not use that digital tool for the particular Problem being reported on.

Teachers wanting their students to have "hands-on" experiences was a major theme in teachers responses to the non-use of curriculum-provided digital tools. Grade 8 teachers were pressed as to why they did not choose to use the Virtual Bridge App, Data and Graphs App, or the Coordinate Grapher when exploring in Problem 1.1 and 1.2. Their responses were centered around the ability of students to make sense of their collected data and for them to think about how they would have to construct various representations, such as tables and graphs, and to give proper labeling to their representations. Teachers were aware that some of these digital tools do not offer students those experiences, as they are already formatted to fit the expected data. Ms. White said that "it's cool that they could do the bridge thing online, but that is really taking away a lot of stuff from the kids being purposeful in it", meaning that the Virtual Bridge App takes away some of these implicit secondary opportunities for student learning. Teachers found this to be a meaningful skill for students to engage without the initial use of digital tools. Ms. Lambeau suggested a reason for not using the Number Line App in Grade 6 was because students needed to create the number line on an assessment, so the skill of making the number line from given data was as important as understanding the data itself.

Overall, it is difficult to make any other claims about the curriculum-provided math apps and teachers' willingness to use them in Problems. Other reasons that teachers chose not to use those mentioned in the Teacher's Guide were that they found them to be a contextual mismatch (ex. Mug Wumps App in Grade 7 Problem not about Mug Wumps) or that they would slow down the students working on the Explore. Teachers reasoning of students needing to have "hands on" experiences during the first Investigation makes many of the curriculum-provided digital tools that have an intended focus on student use in the Explore to be dead on arrival.

Reported Digital Tool Use and Policy

This portion of Chapter 5 is turning away from a being about findings from the data and will turn towards being a discussion of the data in realms that are not relevant to the two research questions of the study but are important to discuss, as they have implications on digital tool use in the classroom.

In Chapter 1of this dissertation, a few policy statements on technology were discussed, including *Principles and Standards for School Mathematics* (National Council of Teachers of Mathematics, 2000), *Adding It Up* (Kilpatrick, Swafford, & Findell, 2001), the *Common Core State Standards for Mathematics* (CCSSI, 2010), and *NCTM's position statements* (2015).

Specific attention was given to the CCSSM Standards for Mathematical Practices (SMPs), since the curriculum was revised with these standards in mind. Table 4.5 showed that attending to context attended to MP1: Sense and Persevere, sharing and presenting work attended to MP3: Critiquing the Reasoning of Others, and creating final projects at the end of a Unit attended to MP8: Repeated Reasoning. These instances of purpose of digital tool use aligned with the Launch, Summarize, and Mathematical Reflection, respectively. This provided evidence that the actions associated with digital tool use not only met the MP5: Use Appropriate Tools Strategically practice standard but allowed for access to other practice standards as well through common digital tool uses across the LES instructional model and the Mathematical Reflection.

In addition, CMP's guiding principle on technology was mentioned in the introduction of this dissertation. It is important to look at this because: 1) it's a guiding principle of the curriculum these teachers use, 2) these teachers all reported digital tool use, and 3) it would be helpful to know if the data suggests the type of digital tool use that the curriculum says should occur.

The guiding principle on technology states:

> "The curriculum reflects the information-processing and delivery capabilities of calculators and computers and the fundamental changes such tools are making in the way people learn mathematics and apply their knowledge of problem solving to new tasks" (CMP, 2019).

There is plenty of evidence in the data that teachers are using information-processing (context) and delivery capabilities (sharing/present) when using computers in the classroom. Although calculators may be a harder sell, the use of other tools for calculation purposes, such as those on other softwares on computers such as Desmos, show that those two categories are somewhat linked, much like the combined section in the literature review suggested. How those two digital tool categories are used to address the way "people learn mathematics" and "apply their knowledge of problem solving to new tasks" can be supported through the data. The use of video for students to learn about real-world contexts of Problems, the use of sharing and presenting tools to communicate and collaborate with their peers, and the use of tools for creation to build artifacts of learning and the ability to receive feedback and revise mathematical thinking are evidence that the data supports the latter half of that statement.

In total, it appears that the data suggest that teachers using this inquiry-based, problem-centered mathematics curriculum are adhering to the technology principle of the curriculum.

This dissertation helps address is to make when that occurs within and across Problems in Units more evident to teachers, especially teachers who are new to teaching with a problem-centered curriculum and/or have only experienced mathematics from some other avenue.

On the Non-Use of Digital Tools

The nature of this study was built around teachers use of digital tools. The expectation of this study was that teachers would be judicious in what digital tools they would select to use at various points of their Unit. Whether it be curriculum-provided digital tools, digital tools external to the curriculum, digital tools that were suggested to be used by the Teacher's Guide, or digital tools for other reasons, teachers make decisions on which digital tools to include in their classroom experience for student learning.

Inherently, teachers also make decisions to <u>not</u> use digital tools at various points of their Units. Whether these decisions are based on accessibility to the digital tool, its ease of use, or its connection to the Problem, teachers are making just as much of a decision to not use digital tools as they are when they choose to use digital tools. Both decisions are purposeful in the teacher's mind. There were multiple scenarios in which this played out in the interviews in which teachers reported on the non-use of digital tools. Teachers explicitly reported out non-use of digital tools when a digital tool was suggested in the Teacher's Guide, or teachers reported on non-use of digital tools for reasons related to beliefs in pedagogy. Regardless of which of these scenarios played out, the interviewer would ask follow-up questions to receive additional feedback from the interviewee as to why they made the decision to not use digital tools or use an alternative digital tool. The following subsections will look at a few cases in which multiple teachers reported on the non-use of digital tools.

Launch Videos

The way teachers discussed their views on curriculum-provided Launch Videos provided a puzzling piece for the researcher in the study since, on one hand, they were the most widely used type of digital tool reported in this study! Teachers shared their various issues with Launch Videos, as Launch Videos vary greatly in the context in which they seek to support and in the style of the videos themselves. The purpose of this section is to show some shortcomings that teachers reported in the use, or potential non-use, of Launch Videos.

As mentioned in the unit analysis section, there were three types of contexts that videos generally support: real world, Problem, and mathematical context. In general, Launch Videos would support exactly one of these context types. Interviews were coded for these context types for individual teachers (found in individual interview coding in Appendix B) but were not reported on in Chapter 4 for context type.

Many teachers stated that showing Launch Videos that occurred early on in their respective Units were essential for students to be able to make connections to the context of the initial Problems in Investigation 1, similar to findings of Jackson, Garrison, Wilson, Gibbons, & Shahan (2013). Grade 7 teacher Ms. Hutson reported that “at the beginning, I use a lot of Launch Videos because students need high interest levels and things like that.” Using videos to provide motivation and context that allow for the possibility of high quality summarize at the end of Problems.

However, teachers reported issues with certain Launch Videos during follow-up questions from the researcher when asked about their non-use of a Launch Video. For instance, Ms. White reported that “when I look at a Launch Video and sometimes even the question at the end of the Launch Video does not even align to what the math is or it is a completely separate question than what the math is in the book." For this reason, Ms. White only showed a subset of the Launch Video, the bit that applied to the Problem as written in the textbook.

Others who showed Launch Videos stated issues of student confusion which trickled past the Launch portion and into the Explore. Grade 8 teacher Ms. Favre discussed the Launch Video that described how triangles are used to make strong bridges yet noted that the first two Problems in Thinking with Mathematical Models deal with bridge length and bridge thickness using paper. Creating a rigid bridge with triangular constructions was never an option in the Unit.

Grade 6 teacher Ms. Lambeau stated that "the kids are usually confused afterwards and sometimes I feel like it kind of slows down when we could just kind of jump into the math work." She additionally stated that "they do not need a whole lot of background or anything like that as much unless it is like they just need to understand the context." This gives support to showing Launch Videos that address the real-world context of the Problem or support the Problem context.

Other reasons related to Problem context for skipping Launch Videos were when context has already been established. Grade 6 teacher Ms. Lambeau stated that her reason for skipping the Launch Video in Problem 1.3 was that the context had not changed from the previous Problem and therefore needed no context introduction. Ms. White made the same reasoning statement for skipping the Launch Video in Problem 1.3 of her Unit.

Some teachers substitute or supplement with YouTube videos or other videos during the Launch. In general, these videos help support the real-world context of the Problem. Teachers reported that external videos did not have a direct connection to the Problem context as written in the textbook. In cases of substitution, teachers reported not agreeing with the aim of the Launch Video compared to the aim of the Problem, but still understood that the context within the

Problem needed to be addressed so they sought alternative videos during the Launch to meet this need of connecting their students to the context of the Problem.

The location and context type of Launch Videos within the Unit played a role in whether they were used or not. Of the six teachers used in the analysis of this study, only the Grade 7 teachers using the Unit Comparing and Scaling showed Launch Videos that focused on the mathematical context of the Problem at the end of the Unit. The Grade 6 and Grade 8 teachers did not show their Launch Videos at the end of their Units, which focused on real world and Problem context, respectively. The context for both the Grade 6 and Grade 8 Launch Videos for the first Problem in the final Investigation centered around rollercoasters. The Grade 6 teachers reported that their students were already familiar with rollercoasters so no need to show them.

A final reason that teachers reported for not using Launch Videos was on their general appeal. Teachers reported on the various nature of videos, where some were made to inform about the real-world context and would be something that one would see on television. Other videos were reported to be "cartoony" or offer a "silly jingle" that teachers do not find to be appealing to show their students. Given the smorgasbord of the various types of curriculum-provided Launch Videos, an encounter with an early Launch Video that was considered unfavorable by a teacher had implications for the rest of the Launch Videos within that Unit.

Overall, this study found that these teachers used Launch Videos more than any other digital tool, curriculum-provided or external, as can be seen in Chapter 4. However, that does not mean that they were not critical of them and their use related to the Problem. Some teachers found Launch Videos to be appropriate but not worth the class time to show them due to the context already being established. Teachers also found some Launch Video to not be properly aligned to the questions that were asked in the textbook, especially those Launch Videos that ended with a question posed to the students. This misalignment made entering the Explore portion problematic for students, as they were not sure whether they should focus on the question posed at the end of the Launch Video or those in the Problem in the textbook. Also, Launch Videos were less likely to be used later in the Unit if they were not related to the mathematical context of the Problem. Launch Videos that focused on repeated or known real-world context were less likely to be used, which teachers used the time savings for student work during the Explore and classroom discussions. Finally, teachers found many of the Launch Videos to be subjectively poor, which made them not use them and possibly impacted their use of future Launch Video within that Unit.

Summary

The data analysis provided an opportunity to look deeper into what digital tools are being used and how they are used. The aggregate data on teachers' reported use of digital tools showed a variety of digital tools used. However, that was dictated by the various 1:1 computing scenarios across these teachers' classroom, as they used different but functionally similar Web 2.0 tools for the means of communication and collaboration. Identifying the purpose of these digital tool uses helped in managing the multiple tools reported that performed similar actions in the classroom.

Teachers tend to stick to digital tools that are relevant to the instructional phases of the LES instructional model. Teachers often reported using digital tools during the Launch of Problems, which is to help ensure a quality start to the Problem so that a quality summary can be had at the end of a task (Jackson et al., 2013). The use of digital tools in both the Launch and Summarize helped teachers manage the contextual and whole-class interactions aspects that comes with a problem-centered curriculum. What is still unclear from the study is how teachers generally manage the Explore with digital tool use. Future studies that focus on the enactment of entire Units and across multiple teachers within the same Unit would help in addressing how digital tools are utilized during the Explore.

From analysis of the transcripts, teachers also discussed their reasoning for non-use of certain digital tools. Digital tools of interest were the curriculum-provided digital tools, such as Launch Videos or any of the math-specific apps provided by the curriculum, that were mentioned in the Teacher's Guide for possible use during the Problems that teachers were interviewed on. Although Launch Videos were one of the more frequently used digital tools reported for use in this study, teachers also provided various reasons as to not use them, which include Problem alignment and context type.

Some recommended digital tools were reported to be passed on due to teachers wanting their students to have "hands on" experiences. Many of the recommended digital tools that were suggested to be used during the Explore were not used due to teachers wanting students to have "hands on" interactions with the mathematical tasks. From the setup of this study, it is unclear if those digital tools remain to be unused during the Unit, as the focus was on Problems in the first and final Investigations only. This observation is not an indictment on math-specific tools overall, but that they were generally not used during the first Investigation.

CHAPTER 6: CONCLUSIONS, LIMITATIONS, NEXT PLANS

Key findings from Chapter 4 and Chapter 5 related to the research questions are:

- The consumption of video for students to understand or learn about problem context was critical to all teachers in the study.
- The sharing of ideas and work was central for classroom interactions and teachers used projective digital tools to attend to this interaction.
- Teachers reported the use of digital tools for the creation of artifacts for students to show their summative mathematical understanding.
- The consumption of video for students to understand or learn about problem context was critical to all teachers in the study.
- The sharing of ideas and work was central for classroom interactions and teachers used projective digital tools to attend to this interaction.
- Teachers reported the use of digital tools for the creation of artifacts for students to show their summative mathematical understanding.
- The purposes are tightly linked to the phases of the LES instructional model. Teachers reported uses of digital tools closely linked to understanding and learning context through the use of videos during the Launch, various digital tool uses for carrying out the mathematical tasks during the Explore, uses of digital tools for sharing and presenting during the Summarize, and digital tools that allow for the creation of artifacts and feedback during the Mathematical Reflections. Therefore, the uses of digital tools are purposeful towards specific phases of the LES instructional model and the Mathematical Reflection.
- The categories of actions are well disbursed across beginning and end Problems within

Units. Greater frequencies of digital tool use are reported within the first few Problems of the first Investigation of a Unit. Digital tool use still persists though the end of a Unit and during the final Mathematical Reflection.

Faithfulness to the LES Instructional Model

Staying faithful to the LES instructional model meant that certain people in the classroom are going to have defined types of interactions at distinguishable points of the enacted Problem in the classroom. In the Launch, teachers choose ways in which they want to set the context of the Problem, with many teachers choosing Launch Videos to help students understand the context within the Problem better before moving towards the student-led Explore of the Problem (Jackson et al, 2013). The Launch of Problems were generally reported to be teacher led and hinted at some student participation.

During the Explore, teachers described how they allowed students to explore the Problem, either individually or in small group settings. All teachers described that the Explore should provide students with hands-on interactions with the Problem. Due to the setup of the study which only looked at the beginning Investigation and two Problems within the final Investigation, "hands on" during the initial Problems of a Unit often meant "hands off" digital tools. which led to an overall minimal use of digital tools during the Explore. The Explore, with or without digital tools, was described as being student-led and the teacher has a minimal role during this time, as teachers reported circulating the classroom. If teachers were to intervene with the use of digital tools during the Explore, it would take away from the focus of the students being the ones working on the mathematics. Interject the teacher and/or a digital tool at this phase of the LES, and the focus of the Problem is now gone. Given the teachers' minimized role during the Explore, so too does the role of digital tools within the Explore (sans calculators).

Teachers described the Summarize of Problems as being as space that brought the teacher and students together, as an opportunity for reporting out their findings during the Explore and to refine students' thinking and understanding of the Problem. Teacher presence in this phase of the LES provided a plethora of digital tools uses, from using document cameras to show students' static analog work to using Google Slides, Docs, and Forms for sharing students' thinking through written words or pictures. Teachers had the ability to use digital tools to facilitate types of whole classroom discussions that are part of LES instructional model, whether digital tools are used or not.

Of the teachers who had their students participate in the Mathematical Reflections (or in engaging in questions similar to them), the focus was on solidifying the mathematics learned over an Investigation or a series of Investigations. When digital tools were used, teachers generally used digital tools that had student create some type of content that allowed for students to articulate their mathematical understand through written or spoken words. These digital tools also offered opportunities for the teacher and/or students to offer feedback to the creators to offer opportunities of revision of both student thinking and of their created content.

Teachers have decisions to make every step of the way through a Problem. During teachers' descriptions of how they attended to the LES of selected Problems within their Unit, they described how a digital tool may fit within particular parts of the LES or how particular digital tools do not fit in with particular parts of the LES. The constant here is the LES, which the teachers of this study remained faithful to. Teachers implicitly used digital tools in ways that helped in attending to MP1, MP3, and MP8 during the classroom interactions at various stages of the LES instructional model (Table 4.5). This provides additional evidence that teachers' reported purposeful use of digital tools would also be considered using appropriate tools strategically (MP5) as it pertains to the instruction of tasks.

Recommendations for Users, Creators, and Researchers

This study has implications on two groups of people related to the curriculum: users (teachers) and creators (content creators and curriculum developers). This study should help teachers in determining which tools are purposeful to use within a problem-based curriculum. For creators, it should help identify spots within and across Problems in which digital tools can be used to help support teachers and, therefore, help support student learning.

Recommendations for Teachers

The findings from this study provide support for teachers who teach using an inquiry-based, problem-centered curricula who are looking to improve their instruction, improve their students' learning experiences, or are simply looking to incorporate digital tools into their instruction. Teachers reported on their digital tool use for this study in ways that allowed them to justify why they were using digital tools, which fell into one of the four categories of actions that eventually was associated with particular phases of the LES instructional model. While this study was not comparative to show outcomes when digital tools were used versus when not used, teachers discussed their purposes such that the use of digital tools helped either their instruction or student learning.

If a teacher new to using an inquiry-based, problem-center curricula were to ask about what digital tools to use, the data suggests videos to support problem context during the initial phase of problems, projective technologies (ex. SmartBoard, document camera) and communication tools (ex. Google suite of tools) for whole-class discussions at the end of problems, and digital tools that offer students the ability to create constructions of their thinking that have feedback capabilities at the end of sequences of problems.

The data does not suggest that this is the best way, it simply suggests that these are the most common ways in which teachers are incorporating digital tools within phase of problem-based tasks. However, the support teachers provided during the interviews as to why they are using the digital tools gives purposeful evidence in how these digital tool uses are supportive to the enactment of the tasks.

A difficult area to provide reasonable suggestions for digital tool use is when students were to engage with the mathematical tasks (in the case of this study, during the Explore). One reason this study did not come away with conclusive digital tool use for this particular phase is that the mathematics that students engage with varies by strand of mathematics. Given that this would be a phase in which math-specific digital tools would possibly be used (as supported by the few times they were used within this study), looking at Problem in which concepts were being introduced (first Investigation) and when mathematical concepts and skills are being solidified (final Investigation), the opportunities to witness that reported transition by teachers was lost in this study.

A study that could possibly help in identifying digital tools for the explore would include reporting out on all Problems within a Unit and for multiple teachers being interviewed over the same Unit. From this study, however, the array of Units captured the nature of digital tools during the instructional phases of Problems and did not provide details on individual strands of mathematics, which was by design. Therefore, there were scattered uses of digital tools reported by teachers within the Explore portion across Units. As mentioned in Chapter 5, the desire for hands-on opportunities for students during the early Problems of Units is prevalent, but it is not clear how this may change in the middle of Units where math-specific tools may be used.

Teachers reported use of few math-specific digital tools during their interviews. A major limitation of this study was on the selection of Problems within the Units that teachers discussed. Reporting on only Problems within the first and final Investigations of a Unit provided little clarity on what digital tools were used during the middle Investigations. Although the actions associated with digital tool use for the beginning and end phase of tasks appeared to be uniform regardless of where Problems were located in the Unit, how digital tools may be treated during the student exploration phase of a task may not be uniform throughout a Unit. The possibility of math-specific digital tools and those that are provided by the curriculum may have a greater chance of being used during the middle Investigations of a Unit.

Nonetheless, this study provides evidence of digital tools used during the Launch, Summarize, and Mathematical Reflections that have purposes tied to instruction. Where teachers new to a problem-centered curriculum may be wondering how they would use digital tools related to the mathematical strand, the teachers in this study showed that purposeful digital tool use was more related to the instructional model, which has more implications on classroom management than mathematical content.

Recommendations for Content Creators and Curriculum Developers

For content creators and curriculum developers of problem-based curricula, this study provides evidence of digital tools teachers are choosing to use and the purposes as to why they are using them. The need to combine these two groups together in this section is so they can work in tandem, as suggested by Okita and Jamalian (2011). The data suggests a few recommendations for future creation and functionality of digital tools.

Expanding Launch Videos by Context Type

Teachers overwhelmingly discussed the use of videos for connecting students to the context of Problems, with these tools being utilized during the Launch of Problems. At the same time, teachers were often critical of the curriculum-provided Launch Videos, as discussed in Chapter 5, that often lead to the non-use of those videos and sent teachers looking for videos to support their students' needs from external sources. The data analysis in Chapter 4 was shown in a way to make the data more concise. However, it lacks the specificity of coding that occurred when initially coding teacher interviews. The coding of context, often associated with the use of video, was coded for its relation to problem context, real world context, or mathematical context of the Problem. This can be seen in the coding for individual teachers in Appendix B.

Although these digital tool uses were coded for context, not all contexts are the same. This may be a reason why some teachers chose to not use some of the Launch Videos or could not find videos to address the context type they wanted to address for the Problem. Therefore, teachers found some Launch Videos to be "not helpful".

A suggestion for content creators and curriculum developers who wish to provide teachers with videos for the introduction of problem-centered tasks is to create three videos for each tasks that 1) connects to the real-world context of each task, 2) connects to the problem-context of the task, and 3) connects to the mathematical context of the task. This would allow teachers to find a context type match they desire to convey to their students prior to student engagement on the task. The goal would not be for teachers to show all three videos but would allow them to find a single video that would be of purposeful use for the teacher. The additional videos could serve as support for students who need help with the context in the other areas.

Math Tools Within Collaborative and Communication Tools

Teachers reported digital tools use during the Summarize and the Mathematical Reflection (a summary of consecutive lessons) that promoted the use of digital tools that offered collaborative and communication features. This allowed for the creation of digital artifacts with feedback mechanisms incorporated in. It may not be in the best interest of a content creators or curriculum developers to build their own platforms to make these types of actions, given the robust sets of ubiquitous tools already available, such as the Google suite of tools that teachers often reported on use during multiple phases of instruction. What could be more beneficial in promoting the use of math-specific digital tools, which were reported for use in only three instances as a collective group of tools, would be if they could be integrated into the collaborative and communication tools already used by teachers.

For example, if students were able to collect data from the Virtual Bridge Experiment and export their data to Google Sheets or Microsoft Excel for efficient creation of representations and regression equations during Thinking with Mathematical Models. This would allow for more opportunities for the Virtual Bridge Experiment digital tool to be use within the Unit, beyond the suggested use of Problems 1.1 and 1.2. Teachers may find that functionality useful in subsequent Problems in the Investigation or in later Investigations in the Unit.

For the Pouring and Filling app, if students were able to download GIFs of their pours and place them into a Google Doc, they could write about what they are seeing along with a non-static representation within their document that could be shared with their peers or their teacher for feedback.

Teachers did not mention the use of curriculum-provided digital tools that went outside the recommended phases of Problems as mentioned in the planning charts of the Teacher's Guide. If a goal of content creators and curriculum designers is to give teachers and students the ability to use digital tools in more places, finding ways of allowing teachers to choose to use them purposefully in more than a single task or two could be met if ways of integrating curriculum-provided digital tools within commonly used external digital tools meant for collaboration and communication can be found. It is understandable from a creator point-of-view that if a math tool only has a short window of being used, then it is unmotivating to create many of these types of digital tools. However, if the functionality of math tools increased such that they could be used over additional Problems and also in other phases of the LES instructional model, such as when Google suite of tools are often reported to being used during the Summarize, there may be better motivation in offering them to teachers and teachers choosing to use them.

Recommendations for Researchers

For researchers, this dissertation served as a stepping stone for making connections between the actions/purpose of digital tool use and their connection to instructional methods. The purpose of this dissertation was to examine the actions/purpose of digital tools within an inquiry-based, problem-centered mathematics curriculum. From the findings, it appears that the use of digital tools has more to do with the instructional method than the mathematical content. From this, we can learn about how actions/purpose of digital tool use play out in other curricula and determine if their uses adhere to different instructional methods.

This dissertation showed that there were dominate uses of digital tools during the Launch, Summarize, and Mathematical Reflection, yet there was an interesting hole in dominate use of digital tools during the Explore. There were multiple factors as to why this could be the case, including teachers wanting their students to explore by hand or that the math-specific digital tools were used during the middle Investigations of a Unit, which were not examined in this study. Regardless, the heavy use of non-math specific digital tools shows that there is a need to look at the actions associated with digital tool use, as math-specific digital tools do not provide the same types of opportunities for learning mathematics. Yet, there exists curricula that rely heavily on the use of math-specific tools. What actions do these curricula depend on to be successful? Relative to this dissertation study, do they hold the same categories of purposes of use?

Given the alignment of actions that the UDCR has to phases of the LES instructional model, it would suggest that there is a close correspondence between particular actions/purpose and phases of a lesson. However, there were opportunities, and some did show up in the data, where actions were out of phase with the LES. What does current research say about these other categories of digital tool use that are used in non-aligned phases of the LES? Do they make sense in other curricula with different instructional methods? How are math-specific digital tools used in other curricula?

The creation of categories of actions that were inspired from RAT (Hughes, Thomas, & Scharber, 2006) and SAMR (Puentedura, 2006) provide a finer look as to what makes the use of digital tools different. The point of this study was not to describe how transformative the use of digital tools would be, but to describe how they impact classroom dynamics. These two frameworks state that there is a change to various degrees when digital tools are used without focusing in on the dynamics of the classroom. This study focused on the classroom dynamics through the lens of purposeful digital tool use, but not to the degree of identifying how transformative the uses of digital tools are. For an inquiry-based, problem-centered mathematics curriculum, there is good reason for that: there are too many different digital tools used within the same Problem to identify which digital tools provide transformative opportunities for student learning. What is best is to determine how each individual digital tool impacts the task through classroom dynamics.

Limitations

This section will look at limitations in two different manners. The first will be limitations on the design of the study. The second will look at limitations on claims able to be made from the collected data and analysis.

Limitations of Study Design

The limitations within this study are numerous. From a study setup standpoint, the low number of participants was problematic for many claims that the researcher wished to have been able to make. The initial goal was to have 12 participants to cover six Units across the three grade levels, with each Unit having two participants and two Units for each grade level. Through the call for participants, the number of participants that met the minimum requirements and were willing to move forward with a one-on-one interview were lower than expected. In the end, eight participants were interviewed, with two participants for the Unit Variables and Patterns and one for Covering and Surrounding in Grade 6, two for Comparing and Scaling and one for Filling and Wrapping in Grade 7 and Thinking with Mathematical Models for Grade 8.

A reason for Grade 8 having participants for only one Unit was a misfire by the researcher. The intention was to have two participants for the Grade 8 Unit Function Junction. When Grade 8 teachers were willing to participate, the first two agreed to work with Thinking with Mathematical Models, so no more for that Unit were needed. Finding two teachers to discuss Function Junction was difficult, as it is an optional Unit for schools that are looking to fulfill what is considered the first-year algebra course during the course of Grade 8. It comes to be that very few schools attempt to fulfill this requirement while using CMP3 materials and therefore created difficulty in finding teachers willing to discuss that particular Unit. Halfway into the interview call, I changed the second intended Grade 8 Unit from Function Junction to

It's In the System. Finding willing Grade 8 participants for this Unit at that time were inhibited by semester breaks, holiday vacations, and a nationwide cold snap. The well of willing participants for interviews in the months of November and early December froze over by the end of December through January.

From this, a decision had to be made on how to handle this subset of interviews from the initial goal of 12 teachers and having only eight. An initial goals was to be able to compare teachers' digital tool uses across grade levels, given that having two Units covered by two teachers each from each grade level would possibly strip away the mathematical strands having a significant impact on findings, since each grade has a focus on more than two mathematical strands from the CCSSM. From this, it was decided that claims across grade levels could not be made given the 3-3-2 participant arrangement of teachers across the grades, but that looking at the study as eight teachers collectively could inform the Grade 6-8 experience as a whole. Other issues in trying to make claims across grade levels is not all the mathematical strands are represented in the selected six Units. For example, Units that covered statistics and probability were not included in this study yet are mathematical strands in CCSSM for each of the three grade levels.

Now from the lack of certain Units included in the study, there is another level of lacking within all of the selected Units. For all six of the selected Units, none of the middle Investigations were unpacked during the interviews. This was purposeful, as the intent was to hopefully show stark contrast between digital tool use in early Problems of a Unit, such as those in Investigation 1, compared to the Problems in the latter portion of the Unit, such as those in the final Investigation of the Unit. The goal of this study was never to identify possibly transition points in digital tool use within Units but that a transition did occur, and that digital tool use may look different comparatively at both ends of a Unit because of this transition. The theory was that digital tool uses for early Problems that are contextually based in a real-world scenario that focus on developing conceptual understanding may lead to different uses for Problems in the final Investigation, which typically have a focus on mathematical context that focus on utilizing procedural understanding. However, as it was pointed out by a few teachers in the study, the final Investigation sometimes acts as a new mini Unit and is not the Investigation that is the solidifying Investigation compared to the previous Units. Sometimes the final Unit is the closely related but not the same topics that go within the same mathematical strand. The ability to make claims about digital tool use across the beginning and end Investigations could no longer be made. However, the data suggests that teachers still use the same digital tools for similar purposes of instruction across these Problems.

Due to the middle Investigations are not unpacked, claims about what digital tools are used and how they are used as it relates to the entire Unit cannot be made. Early on in the interviews, prior to asking teachers about their digital tool use within the selected Problems within the first and final Investigations, the teachers were asked about what digital tools they generally use within the particular Unit, as mentioned in Table 4.3. All of the digital tools they mentioned that they generally use within that Unit were mentioned during their reporting of Problems in Investigation 1 and the final Investigation. What this does not conclude is that no additional digital tools were used within the Unit or that the digital tools they listed were used in the same or different way during the middle Investigations. The only claims that can be made is the types of digital tools used and their uses at the beginning and end of Investigations.

An initial intention was to compare teachers’ reported digital tool use to the Arc of Learning (Edson et al., 2019) for the shifts that occur in student engagement over the span of a

Unit. However, due to not having data on the middle Investigations and the issue of final Investigations not necessarily being connected directly to the previous Investigations, this seemed to be an unproductive endeavor. Changes in the study would have to be made, such as attending to each of the issues listed, which would make for much longer interviews if all Investigations of a Unit were reported on (which would also limit the number of willing participants for the study).

Of the Investigations reported on in this study, some things cannot be said about the transitions within them due to the design of the study. Although all of the Problems from Investigation 1 are reported on in every interview for each Unit, these Investigations vary. Of the Units covered in this study, Investigation 1 has three to four Problems. Some Investigation 1 in Units have up to five Problems (none of which were used in this study). What constitutes an Investigation, a sequence of Problems, is a moving target within the context each Unit. The transition from moving from the last Problem of Investigation 1 to the first Problem of Investigation 2 is manufactured by the curriculum writers. To say that an Investigation constitutes a unique subunit of the Unit is subjective, as typically the following Investigation closely relates to the previous Investigation. The sequence of Problems would likely remain unchanged, however.

Problems in early Investigations generally addressed conceptual understanding and Problems in the lattermost Investigation generally attended to the mathematical world. However, there were times where this is not always the case. For instance, in the Grade 7 Unit Comparing and Scaling, the last two Problems of Investigation 1, Problems 3 and 4, offer digital tools that attend to mathematical context of the Problems. Therefore, it is difficult to say that Investigation 1 addresses conceptual understanding opportunities outright given that the curriculum-provided digital tools, such as Launch Videos which teachers reported are one digital tool that are generally shown for any Unit, are focused on the mathematical context.

Similarly, there were no guarantee that the Problems in the final Investigation of a Unit address the context from the mathematical world. From the unit analysis in Chapter 3, Launch Videos are included in five of the six Units in the analysis and only two of those Launch Videos attend to the mathematical context of the Problem, meaning that they either attended to either the Problem context or real-world context. The teachers in this study provided good reason for why this is the case: the end Investigations seem to be whatever closely related standards are left to cover for a mathematical strand. For example, both Grade 8 teachers discussed how of the first five Investigations of Thinking with Mathematical Models, the first four Investigations were about modeling and comparing with numerical data. The fifth and final Investigation is about categorical data and the creation of two-way tables. Comparing Investigation 1 where students were collecting numerical data in the form of the two bridge experiments to the final Investigation where students were introduced to categorical data, it is not surprising to see a jump in the types of digital tool uses, as they both appear to be two concepts that are students' first exposure to them at that moment. Therefore, teachers treat the final Investigation like a new mini Unit, in which they plan for purposeful digital tool use much like they would in Investigation 1.

Additionally, this study looked at only the first and final Problem of the final Investigation, as opposed to all of the Problems in Investigation 1. Given the premise of the study would be to show different digital tool uses across and within a sequence of Problems, picking on the first and final Problems of the final Investigation seemed reasonable at first. The first Problems from Investigation 1 and the final Investigations could be compared, as well as the final Problems of the two Investigations. However, when the final Investigation is not the culmination of the previous Investigations but a collection of a closely related topic that are placed at the end of the Unit, it makes for inconclusive evidence of what happens with digital tool use across Investigations under the setup of this study. If the study were to be redone by looking at two Investigations only, it would look at the first and the second to last Investigation of these Units or an Investigation that completes a sequence relative to the context of the initial Investigation.

Outside of Unit considerations and Problems to probe, additional limitations are placed on the process of the data collection. This study collected data from what was reported by teachers when revisiting what they did in their classroom during their teaching of the selected Unit. The timing of the interviews did not necessarily align with when the Units were being taught in the classroom. Some teachers, such as the Grade 8 teachers, had already taught their selected Unit early in the school year. The Units for Grade 6 and Grade 7 were Units that were typically taught towards the middle of the school year and given that interviews were mostly done prior to mid-December, many of these teachers had to think back to the previous school year as to what they did. During the interviews, many teachers looked back at their Teacher's Guide, lesson plans they may have written, and other notes to remind them as to what they did during each phase of LES for each Problem.

Limitations on Reported Claims

The claims that can be made from the data have their own limitations. To start, the organization of the coding for teacher use of digital tools in the tables in Appendix B bring up a few issues. The second table for each participant that organizes their digital tool use compared to the four themes of digital tool use and LES does not capture individual Problems but the

Problems collectively reported on within the study. It is unclear when these uses of digital tools occur, even when cross referencing with the third table that looked at digital tool use of the four themes of digital tool use and the grouped Problems and Mathematics Reflections that were reported on in the Unit. For example, if a YouTube video was used in the Launch to attend to real world context and there is only one report of this in the grouped Problem table under "Final Problems, First Investigation", it is unknown from the tables if this occurred in Problem 1.3 or Problem 1.4. Creating a table for the LES of each Problem would address this yet would possibly take away from the table that shows digital tool use across all Problems reported on within a Unit, which shows the concentration of use down the main diagonal of the table.

The number of participants being less that the initial target was problematic for the ability to make claim through the data. Although scaling up can always be mentioned to describe what can be done differently in the future for any research, obtaining less participants than the intended amount led to multiple issues when it came to making claims across teachers, within and across grade levels, and the overall CMP3 curriculum. Therefore, only claims about the group of participants as a whole could be made.

Given the small number of teachers involved in the study, claims about how one rates themselves on the pre-interview Likert scale for their self-reported frequency of digital tool use and their actual digital tool use was not useful. The reason for this pre-interview item was to identify teachers who report at least some digital tool use while teaching. Using the scale for anything more than that for this study would be problematic and was possibly problematic in participants who were not included in the round of interviews.

For example, Ms. Lombardi rated herself a 3 for digital tool use. Looking at the table of codes for Ms. Lombardi, she was well over the average instances of digital tools used in the

Problems reported on in this study. On the other hand, Mr. Nitschke rated himself a 5, yet used minimal amounts of digital tools within the Problems of his selected Unit. It makes one wonder if participants who rated themselves as a 2 (not chosen to participate in interviews) were closer to a 3 or 4. It is unclear what metric teachers used to compare themselves with in this self-rating, possibly they use the most digital tools among their fellow math faculty at their school so they rate themselves high or they just do not feel tech savvy in general so they rate themselves lower, but it is problematic for the researcher in getting the number of participants needed to make stronger claims.

It turns out from the data of the eight participants that the use of digital tools probably had more relevance to the Unit than it did to the teachers' self-reported rankings. If the data tells us anything, it probably says more about the pre-interview Likert scale prompt, which could probably be changed to "Do you use digital tools? Check yes or no." That could have fielded more possible participants for the interview round to make stronger claims at the end of this study.

Next Plans

This section will discuss how to take this research further and how changes would need to be made in order to obtain deeper understanding than what digital tools are used and their purpose of use.

Observations vs. Interviews

Ideally, this research would have performed in a classroom for a selected Unit in lieu of teacher interviews. This would have provided opportunities for collecting different type of data as well, especially on the observed interactions among the teacher and students and within student groupings. Of the four themes of digital tool use coded in the interviews, those classified as Classroom seemed to be of the most interest because they were tools that were used by both the teacher and students during the Summarize portion of a Problem, whereas digital tool use during the Launch was teacher directed and easy for the teacher to report out on in an interview format.

Reporting on the minimal amount of digital tool use for the Explore was also problematics, given that is a student-led activity. The data suggested that teachers chose to use minimal digital tools during the Explore portion of a Problem and often citing "hands on" experiences when digital tools were available. Observing students working through a portion of a Problem where digital tools were not being used (or minimally used) that was bookended by two phases of the instructional model that typically always had some form of digital tool use would have been interesting to observe and would possibly have generated follow-up questions for the teachers based off of these observed student experiences. From the interviews, teachers were charged with explaining what goes on during each phase of the LES instructional model for a Problem, but reporting by the teacher seemed to lack what their students were experiencing during the Explore, as their descriptions felt second hand compared to the descriptions for the Launch and Summarize. As teachers reported student use of digital tools during the Explore, they generally did not elaborate on students' experiences, feelings, and struggles while working through the Explore portion. Teachers were able to talk a bit about student experiences during the Summarize portion, as teachers are an active participant within that phase of the Problem.

Classroom observations would also help in attending to the entire Unit, not only the first and final Investigations and only the first and final Problems in the final Investigation. One particular issue with the interview format and having teachers reporting out on their teaching is the amount of time it took to discuss the preliminary questions, the Problems within the first and final Investigations, and the closing questions of the interview protocol. A typical interview took 50 to 70 minutes depending on the number of Problems in the first Investigation of the respective Unit and the depth of the interviewees' responses. For instance, Grade 6 Variables and Patterns and Grade 7 Comparing and Scaling had four Problems in Investigation 1, whereas Grade 8 Thinking with Mathematical Models had only three Problems in Investigation 1, for a total between five and six Problems discussed during the interview. To discuss all Problems within a Unit would take roughly three to four hours and the ability to land participants for a multiple-hour interviews would be problematic in finding willing participants.

In addition, shortchanging the final Investigation was problematic. Variables and Patterns & Comparing and Scaling both had five Problems in the final Investigation, which means that the middle three Problems were not discussed in the interview protocol, which could have shown a shift in digital tool use compared to Investigation 1 or a shift within the Investigation itself. Having classroom observations for the entire Unit would address the hole of middle Investigations, which may address digital tool use transition points within the Unit, and the hole of middle Problems in the final Investigation.

Overall, if this study were to be extended, it would cover entire Units and include classroom observation of what takes place during the Problems. The reporting by teachers only produced data that was specific to the two research questions. It met the need of this exploratory study, but deeper understanding can be made by looking at entire Units being taught in the classroom and having an understanding on how digital tools are incorporated with the various mathematical strands. With this, there would be the possibility of extending this research to other research areas, such as comparing to the Arc of Learning (Edson et al., 2019) to observe how teachers' use of digital tools interplays with the evolution of student learning through the progression of a Unit.

Evolving Digital Tools Used in the Mathematics Classroom

From the reported data, one of the holes of digital tool uses seems to be the use of math-specific digital tools, compared to the frequent use of non-math digital tools. That seems so odd to me. There were times during the creation of tables with codes where I would look at a table and wondered if someone else looked at it, if they knew the context was a mathematics classroom. Could it be an English or science class? I could not tell a difference of where the math was within my table of codes in some cases.

How can math-specific tools become appealing to be used when the data suggests three things currently: 1) teachers do not use many math-specific digital tools and do not use them frequently, 2) teachers are frequently using non-math digital tools for collaboration and communication purposes within the classroom, and 3) teachers repeatedly stated that they wanted their students to have "hands on" experiences?

"Hands on" was a common theme by teachers on why not to use a digital tool. Even during the interviews, teachers would open curriculum-provided digital tools and get a bit excited about them. When Mr. Nitschke was playing around with the Virtual Box App and creating nets from the box on his Chromebook, he said:

> "So I would definitely use that with my students now that we have these, um, these computers. Yup. But I will not do it instead of the cutting out of the paper. So I would want them to do the paper and the full thing. They really enjoy doing that and kind of, you know, understanding the three dimensional, you're not going to get from, from this kind of a thing."

Even when excited about a digital tool use, it is still trumped by the “hands on” experience. He did not say he would not use it, just that it would possibly be a supplement to the “hands on” experience.

Since this excitement does not guarantee that the digital tool will be used, how can future math-specific tools be created so that they have a higher likelihood of being used in the mathematics classroom (not necessarily needing to be a replacement for “hands on” experiences)? I think the potential answer to this question lies in the digital revolution we are currently in. We are moving out of Web 2.0 which brought us the collaborative and communication tools much like Google Docs and Google Slides that many of the teachers reported using within Problems into Web 3.0, which is transforming user experiences on handheld devices (smartphones, tablets) to produce new types of user experiences. In particular, the addition of augmented reality (AR) and artificial intelligence (AI) into the mainstream.

I will focus on AR, as I think a lot about the possibilities that may produce. Consider Mr. Nitschke’s take on the use of Virtual Box app relative to the experience of doing it by hand. Consider taking a smartphone and scanning a 3D box with the camera. AR software understands that you are looking at a box. From this, there are actions that can be taken on the box, such as peeling back the sides to make a net. In addition to this, the ability of AR to recognize dimensions of the box and calculate attributes such as surface area and volume can be utilized. Would Mr. Nitschke consider this to be “hand on” enough for him to at least use it, if not replace the paper method he uses in his classroom?

As of right now, the iPhone has a measurement tool that is able to measure distances between two points in a room and areas of two-dimension objects, such as the surface area of a rectangular desk. And that is the technology that is sitting in people’s pockets currently. It will become more robust over time. Currently there are math-specific apps that allow for augmented reality experiences, such as GeoGebra AR. Here, the user is able to place a 3D geometric object in the room, scale it, move around and into it. It currently lacks the feature of recognizing 3D object. However, that type of technology already exists, as AR is able to scan entire rooms within buildings to construct 3D models where one can go in and out of the building and into room within it.

Could AR prove to be a "hands on" replacement and could AR be used in ways that attend to the categories of purposeful actions in a problem-centered mathematics classroom? I think AR is getting close to that. Users can record their experience so that they produce an artifact for future consideration. Much like the reason why teachers choose to use digital tools like Google Docs and Google Slides, the possibility of using AR for mathematical experiences in the measurement strands are ripe for future use.

In final, I would like to do this type of research study again, but years down the road where the possibility of seeing this type of Web 3.0 technology play out in the classroom to fill the void of math-specific digital tool uses that are currently reported in the data and to see if they replace or supplement what is considered today to be "hands on" experiences of student learning of mathematics. With the current transition of printed textbooks to digital textbooks (Rosenblatt, 2019), teachers are going to be interacting with computing technology in the problem-centered mathematics classroom more than ever. The digital tools that they will be able to choose from will be numerous.

Future Questions

From the analysis, findings, and conclusions of the study, more questions arise related to the topic of digital tool use within inquiry-based, problem-centered mathematics curricula that need to be considered through future research. They include:

- How are math-specific digital tools utilized within problem-centered tasks? (This study did not uncover much use and therefore needs a better understanding of their use)
- How do students interact with digital tools within inquiry-based, problem-centered mathematics curricula? (Attends to students interactions with digital tools that teachers choose for them to use, classroom observations warranted)
- What are indicators of transitions of digital tool use across sequences of problem-centered tasks? (Studies over entire units can address possible transitions in digital tool use as tasks addressing conceptual understanding shift to procedural skills with a unit.)
- How are Web 3.0 tools, along with Web 2.0 tools, used in the instruction of problem-centered mathematics curricula? (In time, more interactive digital tools, that may be math-specific, will be used. How do they interact with the collaborative and communication tools from the Web 2.0 era?)

APPENDICES

APPENDIX A: INTERVIEW PROTOCOL

Interview Protocol

>>> You are being asked to participate in a research study. The purpose of the study is *to examine middle school mathematics teachers' decisions on their choice and use of digital technologies within the classroom*. You will be asked to answer some questions. It should take about *60* minutes. Your participation is voluntary. If you do not want to answer any of the questions, please let me know. You can also ask me to stop at any time. You must be 18 or older to participate. Is it okay if I record our interview?

>>>If you have any questions after the interview, please contact *Brin Keller* at *brin@msu.edu*. You indicate that you voluntarily agree to participate in this research study by proceeding with the interview.

>>>Today, we are going to look at the Problems and Mathematical Reflection of Investigation #1 and two Problems in Investigation #<final Investigation number> and its Mathematical Reflection of <Unit name> in Grade <Grade number>. In particular, I am interested in the types of digital tools you use during these Investigations, why you choose these uses these technologies, and how you use them within and across Investigations. I'm going to ask you questions about technologies that you use in your teaching during this Unit, for each Problem and Mathematical Reflection, and through each phase of the Launch, Explore, and Summarize, or LES, of each Problem.

Define Digital Tools###

>>>>Before we get to the Problems within this particular Investigation, I'm going to ask you some general questions about your technology use within CMP3, in general.

Does the Launch of Problems in a Unit promote a particular type of technology use? How do you find yourself using digital tools within the Launch and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me more about how the use of technology seems similar across all Launches within this Investigation? Can you elaborate on the possible differences in the use of technology across Launches in this Investigation? Does the Launch promote certain types of technology use, in general? (RQ1, RQ2)

>>>>Does the Explore of Problems in a Unit promote a particular type of technology use? How do you find yourself using technology within the Explore and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me more about how the use of technology seems similar across all Explores within this Investigation? Can you elaborate on the possible differences in the use of technology across Explores in this Investigation? Does the Explore promote certain types of technology use, in general? (RQ1, RQ2)

>>>>Does the Summarize of Problems in a Unit promote a particular type of technology use? How do you find yourself using technology within the Summarize and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me more about how the use of technology seems similar across all Summarize within this Investigation? Can you elaborate on the possible differences in the use of digital tools across Summarizes in this Investigation? Does the Summarize promote certain types of digital tool use, in general? (RQ1, RQ2)

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(General Problem-Level questions)

>>>>Does the Problem location within an Investigation promote a particular type of

technology use? For example, do you select and use technologies differently in Problem 1 compared to Problem 4? How do you find yourself using technology related to the Problem location within the Investigation and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me about how the formalizing of the mathematical content as one gets deeper into an Investigation influences the types of technology choices and their uses? (RQ2, RQ3)

*****

(General Mathematical Reflection questions)

>>>>Does the Mathematical Reflection within an Investigation promote a particular type of technology use? For example, do you select and use technologies in the Investigation because of the Mathematical Reflection? How do you find yourself using technology related to the Mathematical Reflection within an Investigation and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me about how the formalizing of the mathematical content as one gets to the end of an Investigation influences the types of technology choices and their uses? (RQ2, RQ3)

*****

(General Investigation-Level questions)

>>>>Does the Investigation location within a Unit promote a particular type of technology use? For example, do you use technology differently in Investigation 1 compared to Investigation 4? How do you find yourself using technology related to the Investigation location within the Unit and why do you use them in this manner? (RQ1, RQ2, RQ3)

Can you tell me about how the formalizing of the mathematical content as one gets deeper into a Unit influences the types of technology choices and their uses? (RQ2, RQ3)

>>>>Before we begin to look at the Problems of Investigation #1 of <Unit name> in Grade <Grade number>, what do you believe are the intended mathematical goals of this particular Investigation?

>>> What other goals do you have as a teacher of this Investigation that may be different than the mathematical goals you just mentioned?

>>>In general, what technologies do you use during this Investigation to help you reach those goals? I am looking for a short list of technologies so that it can help guide this interview. It does not have to be complete list, as we will be diving in to each Problem within the Investigation and you may remember more about the technologies that you use when we get to these points. Also include any digital tools that you use from outside of the curriculum.

>>>>Do you find yourself generally following the technology recommendations within the planning chart in the teachers' edition or do you occasionally follow the planning chart's technology recommendations? (RQ1, RQ2, RQ3, IM, SLP, CG)

(The following will be used for each Problem within the first Investigation of the selected Unit and the first and last Problem of the selected terminal Investigation of the selected Unit)

>>>>We will look at Investigation #____, Problem #_____. Tell me about how you progress through the LES of the Problem, and when you use digital tools in this Problem, why you choose these digital tools, and how you and/or your students use them. (RQ1, RQ2, RQ3)

Primary Follow-Up Questions:

-Can you elaborate more on why you choose to use these technologies? (RQ3, possible IM, SLP, CG)

- Can you tell me about how the use of these technologies influence your instructional methods? (IM)

- Can you tell me about how the use of these technologies influence student learning processes? (SLP)

- Can you tell me about how the use of these technologies influence the curriculum goals? (CG)

Secondary Follow-Up Questions:

-Can you tell me more about how the use of these technologies influences the task(s) as written? How does the use of your chosen technologies modify the written task(s), if at all? (RQ2, SLP, CG)

-Have you taught this Problem without using these technologies? How is the experience different? How is the experience similar? (IM, SLP, CG)

-Can you tell me more about how the use of these technologies influence the intended mathematical content? Does it allow you and/or your students to focus on particular aspects of the mathematical content? Does it allow you to focus on aspects of mathematics that is not a focus within the written Problem? (CG)

-You mentioned that you use the same technologies across the LES. Can you highlight the differences and similarities among the types of uses and outcomes across the Launch, Explore, and Summarize when using these technologies? (RQ2)

If technology is not used during a Problem:

-What are the reasons that you choose to not use any technology for this problem?

-What would a digital tool have to offer for you to consider using a digital tool for this Problem?

*****

(The following will be used for each Mathematical Reflection at the end of the first Investigation of the selected Unit and at the end of the last Investigation of the selected Unit)

Let's now look at the Mathematical Reflection at the end of Investigation #____. Tell me about how you choose to use the Mathematical Reflection, and if you and/or your students use digital tools during this Mathematical Reflection, why you or your students choose these digital tools, and how you and/or your students use them. (RQ1, RQ2, RQ3)

Primary Follow-Up Questions:

-Can you elaborate more on why you choose to use these technologies? (RQ3, possible IM, SLP, CG)

- Can you tell me about how the use of these technologies influence your instructional methods? (IM)

- Can you tell me about how the use of these technologies influence student learning processes? (SLP)

- Can you tell me about how the use of these technologies influence the curriculum goals? (CG)

Secondary Follow-Up Questions:

-Can you tell me more about how the use of these technologies influences the task(s) as written? How does the use of your chosen technologies modify the written task(s), if at all? (RQ2, SLP, CG)

-Have you assigned this Mathematical Reflection without using these technologies? How is the experience different? How is the experience similar? (IM, SLP, CG)

-Can you tell me more about how the use of these technologies influence the intended mathematical content? Does it allow you and/or your students to focus on particular aspects of the mathematical content? Does it allow you to focus on aspects of mathematics that is not a focus within the written Mathematical Reflection? (CG)

-You mentioned that you use the same technologies as you mentioned for the prior Problems in the Investigation. Can you highlight the differences and similarities among the types of uses and outcomes across the previous Problems when using these technologies during the Mathematical Reflection? (RQ2)

If technology is not used during a Problem:

-What are the reasons that you choose to not use any technology for this Mathematical Reflection?

-What would a digital tool have to offer for you to consider using a digital tool for this Mathematical Reflection?

>>>>>What would be a dream digital tool to use during this particular Unit? What would its uses look like?

>>>>>What would be a dream digital tool for the entire CMP3 curriculum? What would its uses look like?

>>>>>Do you have anything more to say about digital tool use within this Unit or the

CMP3 curriculum?

>>>>>Thank you for participating in this interview today!

APPENDIX B: TABLES OF CODING FOR INDIVIDUAL PARTICIPANTS

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Problem, Context Math			
Calculator			Comparing	
Document Camera				
SmartBoard	Understand Graph (3)	Create Graph	Collect Data, Sharing	
iPad		Create Video	Collect Data	
Non-Math Specific Tools (GDocs, LMS)		Create Graph, Draw Scenario		Artifact (2) (Google Slides, Google Forms)
Curriculum-Specific Apps				

Table A2.1: Ms. Lambeau's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Problem (video) Understand Graph (SmartBoard) Context Math (Launch Video)	Create Graph (SmartBoard)		
Explore	Understand Graph (2 SmartBoard)	Create Graph (Google Classroom) Draw Scenario (2 Google Slides)		
Summarize			Collect Data (2 SmartBoard) Sharing (SmartBoard) Comparing (Desmos)	Artifact (Google Forms)
Mathematical Reflection		Create Video (FlipGrid)		Artifact (Google Forms) Feedback (Google Classroom)

Table A2.2: Ms. Lambeau's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Problem (Video) Understand Graph (SmartBoard)	Create Graph (SmartBoard)	Collect Data (SmartBoard)	Artifact (Google Slides)
Final Problems, First Investigation	Understand Graph (2 SmartBoard)	Create Graph (Google Classroom)	Sharing (2 SmartBoard) Comparing (Desmos)	
First MR				Artifact (Google Forms) Feedback (Google Classroom)
First Problem, Final Investigation		Draw Scenario (2 Google Slides)		
Final Problem, Final Investigation	Context Math (Launch Video)			
Final MR		Create Video (FlipGrid)		

Table A2.3: Ms. Lambeau's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Real World (2 Launch), Context Real World (YouTube), Context Math (Launch)	Understand Role		
Calculator		Graph Inequalities (Desmos Number Line)	Graph Inequalities (Desmos Number Line)	
Document Camera	Understand graph (2) Context Real World		Class Discussion, Sharing	
SmartBoard				
iPad		Timer, Draw (Educreations)		
Non-Math Specific Tools (GDocs, LMS)	Understand graph (2 PowerPoint)		Sharing (AppleTV)	Artifact (Google Docs) Feedback (Google Docs)
Curriculum-Specific Apps		Collect Data (Data & Graphs)		

Table A2.4: Ms. Lombardi's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Understand graph (2 Doc Cam) Context Real World (2 Launch Video) (YouTube) (Doc Cam) Context Math (Launch Video)	Understand Role (Launch Video) Timer (iPad)	Class Discussion (doc camera)	
Explore		Collect Data (Data & Graphs App) Draw (iPad-Educreations) Graph Inequalities (Desmos # Line)		
Summarize	Understand graph (2 PowerPoint)		Sharing (AppleTV) (Doc Cam) Graph Inequalities (Desmos # Line)	
Mathematical Reflection				Artifact (Google Docs) Feedback (Google Docs)

Table A2.5: Ms. Lombardi's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Understand graph (2 Doc Cam) (2 PowerPoint)	Understand Role (Launch Video) Timer (iPad)		
Final Problems, First Investigation	Context Real World (Launch Video) (YouTube)	Collect Data (Data & Graphs App)	Sharing (AppleTV) Class Discussion (Doc Cam)	
First MR				Artifact (Google Docs) Feedback (Google Docs)
First Problem, Final Investigation	Context Real World (Launch Video) (Doc Cam)	Draw (iPad- Educreations)		
Final Problem, Final Investigation	Context Math (Launch Video)	Graph Inequalities (Desmos # Line)	Graph Inequalities (Desmos # Line)	
Final MR				Artifact (Google Docs) Feedback (Google Docs)

Table A2.6: Ms. Lombardi's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Real World (2)			
Calculator				
Document Camera			Sharing (5)	
SmartBoard				
iPad				
Non-Math Specific Tools (GDocs, LMS)			Assessment	Artifact (2)
Curriculum-Specific Apps				

Table A2.7: Mr. Nitschke's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Real World (2 Launch Video)			
Explore				
Summarize			Sharing (5 Doc Cam) Assessment (Juno)	
Mathematical Reflection				Artifact (2 SeeSaw)

Table A2.8: Mr. Nitschke’s claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Real World (Launch Video)		Sharing (2 Doc Cam)	
Final Problems, First Investigation			Sharing (Doc Cam)	
First MR				Artifact (SeeSaw)
First Problem, Final Investigation	Context Real World (Launch Video)		Sharing (Doc Cam)	
Final Problem, Final Investigation			Sharing (Doc Cam) Assessment (Juno)	
Final MR				Artifact (SeeSaw)

Table A2.9: Mr. Nitschke's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Problem (Launch), Context Problem (YouTube), Context Math (3 Launch)			
Calculator		Decimal Approximations, Calculate		
Document Camera			Sharing	
SmartBoard				
iPad				
Non-Math Specific Tools (GDocs, LMS)	Organization (Google Slides)	Reporting Out (Google Slides), Converting (Google) Draw (Sketchpad.io)	Organizing Similar Methods (Google Slides), Gallery Walk (Sketchpad.io)	
Curriculum-Specific Apps				

Table A2.10: Ms. Canadeo's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Problem (Launch Video) (YouTube) Context Math (3 Launch Video)	Decimal Approximations (Calculators)		
Explore		Reporting Out (Google Slides) Converting (Google) Draw (Sketchpad.io) Calculate (Calculator)		
Summarize			Organizing (Google Slides) Sharing (Doc Cam) Gallery Walk (Sketchpad.io)	
Mathematical Reflection				

Table A2.11: Ms. Canadeo's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Problem (2 Launch Video)	Reporting Out (Google Slides)	Organizing (Google Slides)	
Final Problems, First Investigation	Context Math (2 Launch Video)	Converting (Google) Decimal Approximations (Calculators)	Sharing (Doc Cam)	
First MR				
First Problem, Final Investigation	Context Math (Launch Video)	Draw (Sketchpad.io)	Gallery Walk (Sketchpad.io)	
Final Problem, Final Investigation		Calculate (Calculators)		
Final MR				

Table A2.12: Ms. Canadeo's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Problem (Launch), Context Real World (Other), Context Math (3 Launch)			
Calculator				
Document Camera			Sharing (2)	
SmartBoard		Multiple Representations, Create Fraction Sticks, Present, Annotate	Present (3), Write, Conversation, Annotate	Export
iPad				Create Video (2)
Non-Math Specific Tools (GDocs, LMS)				
Curriculum-Specific Apps				

Table A2.13: Ms. Hutson's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Problem (Launch Video) Context Real World (Other Video) Context Math (3 Launch Video)		Class Discussion (SmartBoard w/ SmartNotebook)	
Explore		Multiple Representations (SB w/ SN) Create Fraction Sticks (SB w/ SN) Present (SB w/ SN) Annotate (SB # Line)		
Summarize			Present (3 SB w/ SN) Write (SB w/ SN) Sharing (2 Doc Cam) Annotate (SB # Line)	Export (SB w/ SN)
Mathematical Reflection				Create Video (2 SeeSaw w/ Pisa)

Table A2.14: Ms. Hutson's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Problem (Launch Video) Context Real World (Other Video)	Multiple Representations (SB w/ SN)	Present (SB w/ SN) Write (SB w/ SN) Conversation (SB w/ SN) Sharing (Doc Cam)	Export (SB w/ SN)
Final Problems, First Investigation	Context Math (Launch Video)	Create Fraction Sticks (SB w/ SN)	Present (SB # Line) Sharing (Doc Cam)	
First MR				Create Video (SeeSaw w/ Pisa)
First Problem, Final Investigation	Context Math (Launch Video)	Present (SB w/ SN)	Present (SB w/ SN)	
Final Problem, Final Investigation	Context Math (Launch Video)	Annotate (SB w/ SN)	Annotate (SB w/ SN)	
Final MR				Create Video (SeeSaw w/ Pisa)

Table A2.15: Ms. Hutson's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)				
Calculator				
Document Camera				
SmartBoard				
iPad				Camera
Non-Math Specific Tools (GDocs, LMS)	Slide Show (5 PowerPoint)	Slide Show (5 PowerPoint)	Slide Show (4 PowerPoint)	Feedback (2 Google Forms)
Curriculum-Specific Apps		Discovering Relationships (Pouring & Filling App)	Discussing (Pouring & Filling App)	

Table A2.16: Ms. Starr’s claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Slide Show (5 PowerPoint)			
Explore		Slide Show (5 PowerPoint) Discovering Relationships (Pouring & Filling App)		
Summarize			Slide Show (4 PowerPoint) Discussing (Pouring & Filling App)	
Mathematical Reflection				Camera (iPad) Feedback (2 Google Forms)

Table A2.17: Ms. Starr's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Slide Show (2 PowerPoint)	Slide Show (2 PowerPoint)	Slide Show (2 PowerPoint)	
Final Problems, First Investigation	Slide Show (2 PowerPoint)	Slide Show (2 PowerPoint)	Slide Show (2 PowerPoint)	
First MR				Feedback (Google Forms)
First Problem, Final Investigation	Slide Show (PowerPoint)	Slide Show (PowerPoint)		
Final Problem, Final Investigation	Slide Show (PowerPoint)	Discovering Relationships (Pouring & Filling App)	Discussing (Pouring & Filling App)	
Final MR				Feedback (Google Forms)

Table A2.18: Ms. Starr's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Real World (YouTube)			
Calculator		Compute (2)		
Document Camera			Sharing (2)	
SmartBoard				
iPad				
Non-Math Specific Tools (GDocs, LMS)		Writing (Google Docs)	Writing (Google Docs)	Artifact (Google Slides, Google Docs)
Curriculum-Specific Apps				

Table A2.19: Ms. White's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Real World (YouTube)			
Explore		Compute (2 Calculators) Writing (Google Docs)		
Summarize			Sharing (2 Doc Cam) Writing (Google Docs)	
Mathematical Reflection				Artifact (2 Schoology w/ Google Slides & Google Docs)

Table A2.20: Ms. White's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Real World (YouTube)			
Final Problems, First Investigation		Compute (Calculators)	Sharing (Doc Cam)	
First MR				Artifact (Schoology w/ Google Slides & Google Docs)
First Problem, Final Investigation		Compute (Calculators) Writing (Google Docs)	Writing (Google Docs)	
Final Problem, Final Investigation			Sharing (Doc Cam)	
Final MR				Artifact (Schoology w/ Google Slides & Google Docs)

Table A2.21: Ms. White's claims of actions when digital tools are used during various stages of their respective Unit

	Understand/Learn	Doing	Classroom	Recall
Videos (Launch, YouTube, other)	Context Real World (YouTube), Context Problem (YouTube)			
Calculator				
Document Camera			Sharing (2)	
SmartBoard				
iPad				
Non-Math Specific Tools (GDocs, LMS)	Project (Digital Projector)	Access (Schoology)		Crowd Source, Artifact (2 Padlet)
Curriculum-Specific Apps		Collect Data (Virtual Bridge App)		

Table A2.22: Ms. Favre's claims of actions when digital tools are used related to digital tool type

	Understand/Learn	Doing	Classroom	Recall
Launch	Context Real World (YouTube) Project (Digital Projector) Context Problem (YouTube)			
Explore		Collect Data (Virtual Bridge App) Access (Schoology)		
Summarize			Sharing (3 Doc Cam)	
Mathematical Reflection				Crowd Source (2 Padlet) Artifact (2 Padlet)

Table A2.23: Ms. Favre's claims of actions when digital tools are used during the LES instructional model and Mathematical Reflection during their respective Units

	Understand/Learn	Doing	Classroom	Recall
First Problems, First Investigation	Context Real World (YouTube) Project (Digital Projector)	Collect Data (Virtual Bridge) Access (Schoology)	Sharing (2 Doc Cam)	
Final Problems, First Investigation			Sharing (Doc Cam)	
First MR				Crowd Source (Padlet) Artifact (Padlet)
First Problem, Final Investigation	Context Problem (YouTube)			
Final Problem, Final Investigation				
Final MR				Crowd Source (Padlet) Artifact (Padlet)

Table A2.24: Ms. Favre's claims of actions when digital tools are used during various stages of their respective Unit

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