TWO BEGINNING TEACHERS' INTERVENTION IN SMALL GROUPS IN FIGURED WORLDS OF MATHEMATICS CLASSROOMS

By

Byungeun Pak

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ABSTRACT

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The purpose of this dissertation study is to understand beginning teachers' intervention in small groups. In this qualitative study, drawing on the notion of figured worlds (Holland, Lachicotte, Skinner, & Cain, 1998), I examined how beginning elementary teachers intervene in small groups and for what purposes in mathematics classrooms through the lenses of teacher noticing, teacher identity, and perceptions of students in mathematics classrooms. Working with two beginning teachers, I collected data including survey responses, video-recordings, and interview transcripts.

Building on thematic analysis of these data, I present three findings in relation to in terms of teacher noticing, teacher identity, and perceptions of students. First, I offer a noticingmediated intervention framework that contributes to understanding how beginning teachers may make decisions related to intervention in small groups. This framework provides several ways to (re-)construct individual teachers' decision-making process related to intervention. Second, I demonstrate how current and designated teacher identities shape their intervention in small groups. I detail multifaceted aspects of current and designated teacher identity that each beginning teacher invoked to explain, make sense of, and reason about their intervention in small groups. Third, I illustrate how teachers' recognition of students in terms of categories shapes their intervention in small groups, sometimes through mediation by their recognition of power and authority dynamics at the micro-interaction level. Building on these findings, I present four points to discuss. First, this dissertation study can serve as a call for research on extending the understanding of novice teachers' intervention in diverse contexts. Second, this study suggests one possible example related to how to draw upon professional noticing to examine intervention in small groups in a detailed way. Third, this study details potential resources (e.g., knowledge, expectations, and experiences; teacher identity; and categories of students) beginning teachers may use when they make decisions to intervene in small groups. Fourth, this study supports the field to understand figured worlds of mathematics classrooms from beginning teachers' perspectives of intervention in small groups.

This dissertation study has implications for research as well as for teacher education. For research, this study contributes to understanding how teacher noticing can be used to explore teachers' intervention in small groups. It also holds promise for ways to understand mathematics classrooms as figured worlds. For teacher education, teacher educators can use this study to design instructional activities that help novice teachers explore their intervention in small groups in relation to the noticing-mediated intervention framework, the multiple aspects of their current/designated identities, and their recognition of students in terms of different categories in mathematics classrooms. This study broadens and deepens the understanding of how beginning teachers intervene in small groups and for what purpose by relating it to teacher noticing, to teacher identity, and to figured worlds of mathematics classrooms.

Copyright by BYUNGEUN PAK 2020 Once my younger brother told my mother that he wished the sun stayed up longer. My mother replied to him, "Do not wish the sun to set slowly. The sun must go down here to shine enough daylight on your brother and his family in the U. S." To my mother, father, and mother-in-law in South Korea.

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CHAPTER 1. INTRODUCTION

At the heart of this dissertation study is understanding how elementary beginning teachers intervene in small groups in mathematics classrooms. Intervention, which means in this dissertation study making comments or asking questions, in small groups is a teaching activity for teachers, including beginning teachers. This dissertation study was guided by my teaching experience as an elementary school teacher and motivated by questions emerging from my observations during field instruction of prospective teachers when they engaged in this teaching activity in mathematics classrooms. Among these questions are: How do prospective teachers intervene in small groups in mathematics classrooms and why? How do they learn what they do when they intervene in small groups? Delving more deeply into these questions in relation to beginning teachers invited me to think that this teaching activity could be challenging for beginning teachers because it is not always certain how their intervention will influence student learning. One of the reasons for this uncertainty would be, I assumed as a former teacher, that intervention is very often mediated by what teachers see or hear in small groups. This assumption led me to wonder about beginning teachers' intervention in terms of what they notice and how this noticing shapes intervention. I also came to realize that this intervention is highly unlikely to escape from the influence of how they as teachers themselves and how they perceive students. That is, teachers' intervention shapes and is shaped by teacher identities and teachers' perceptions about students. As such, I investigated beginning teachers' intervention through the lens of teacher noticing, teacher identity, and perceptions of students in mathematics classrooms.

Research has provided evidence that there may be benefits and challenges for students from participating in small group work. Possible benefits for students include academic achievement and developing social skill (Boaler, 2016; Cohen & Lotan, 2014; Davidson, 1990).

There may also be challenges for students, such as getting stuck on a problem or having some students dominating discussion or getting disrupted emotionally when making mistakes (Bishop, 2012; Esmonde & Lange-Osuna, 2013; Langer-Osuna, 2011; McCaslin, Vriesema, & Burggraf, 2016).

Given these potential benefits and challenges, intervening in small groups may not be an easy task for teachers, especially for beginning teachers. Since teachers are not always present with small groups, it may not always be clear for teachers to figure out what is going on in small groups. Furthermore, given the uncertainty, there are likely not single right or wrong ways to intervene. For example, when students in a small group are off-task, it may not be clear how teachers can help them get on-task. When students in a small group get stuck on a math task, teachers may not know how to help them without giving them answers or step by step solutions.

Despite the potential contribution of intervention, research on teachers' use of small groups has emphasized their planning for small group work more than their interventions in small groups (Webb, 2009; 2013). However, some researchers have begun to investigate intervention in small groups on the part of practicing teachers (e.g., Chiu, 2004; Cohen & Lotan, 2014; Dekker & Elshout-Mohr, 2004; or Gillies & Boyle, 2006) and prospective teachers (Pak, 2017). Considering the complexity of prospective teachers' intervention in small groups (Pak, 2017) in particular, I argue that novice teachers' intervention deserves more research attention. Building on these prior studies, I investigated in this dissertation study how beginning teachers intervene in small groups in mathematics classrooms in relation to the three constructs discussed above - teacher noticing, teacher identity, and teachers' perceptions of students.

First, I investigated how teachers notice before and during intervening in small groups. Prior research on teacher noticing has suggested the possible relationship of teacher noticing to

teaching actions (e.g., Amador, Males, Earnest, & Dietiker, 2017; Roth McDuffie et al., 2014; Wager, 2014; Wells, 2017), even though I have not seen any published research that applies the noticing construct to understand how teachers, particularly beginning teachers, notice before intervening. However, my own prior research (Pak, 2018) suggests that how teachers notice before intervening in small groups may be related to how teachers intervene in small groups. Building on Pak (2018), thus, in this dissertation study, I investigated how beginning teachers notice before and during intervening in small groups to understand how noticing is related to intervention.

Second, I explored how teacher identity is related to teachers' intervention in small groups. In this dissertation study, I define teacher identity as the interrelated collection of individuals' views of themselves as teachers, as well as their beliefs, knowledge, and/or dispositions (Drake, Spillane, & Hufferd-Ackles, 2001; Holland, Lachicotte, Skinner, & Cain, 1998; Spillane, 2000). I recognize that exploring teacher identity in a certain context (e.g., mathematics classrooms) in which teaching takes place may help researchers understand that particular teaching. Considering this well-known relation between teacher identity and teaching activities in a general sense, I argue that beginning teachers' identities as teachers may be related to ways of interventions in small groups in mathematics classrooms. As such, in this dissertation study I investigated how beginning teachers' identities are related to intervention in small groups.

Third, I examined how teachers' perceptions of students shape intervention in small groups in mathematics classrooms. Prior research suggests that teachers position students in terms of certain categories of students, which can shape their teaching practice accordingly (Horn, 2007). Given that teachers' recognition of students is related to teaching practice,

intervention as a teaching activity would not be an exception. Since intervention involves an intimate interaction between teachers and students, how teachers perceive some students (e.g., academically low students or disengaged students) are highly likely to shape the ways they intervene in small groups with those students. As such, in this dissertation study I investigated ways that categories of students recognized by beginning teachers are related to intervention in small groups.

As described above, teachers' intervention in small groups may be more than a simple verbal behavior. It deserves closer examination. The purpose of this dissertation study is to understand how beginning teachers intervene in small groups in particular relation to how they notice before and during intervening and to their teacher identities and perceptions of categories of students. In the reminder of this chapter, I pose four research questions in relation to intervention, professional noticing, teacher identity, and categories of students. In the next chapter, I offer a literature review to present my argument building on prior research in teacher education in general and in mathematics teacher education in particular with respect to intervention, teacher noticing, teacher identity, and categories of students. Next, I present a theoretical framework building on figured worlds (Holland et al., 1998), which can be helpful for understanding how teacher identity and teachers' perceptions of categories of students both shape and are shaped by intervention in small groups. Fourth, I describe how I collected and analyzed data, including on-line survey, observation, and interview data, from beginning teachers. Fifth, I present a framework that contributes to constructing an individual teacher's decision-making process related to intervening in small groups. Sixth, I illustrate how beginning teachers' teacher identities are linked to their intervention in small groups. Seventh, I demonstrate how teachers' recognition of students in terms of categories of students is related to

intervention in small groups. Last, I provide discussions, implications, limitations and conclusion of this dissertation study.

This dissertation study can contribute to understanding how novice teachers intervene in small groups. The understanding can lead to thinking of diverse ways to support novice teachers to be productive in intervening in small groups, which may contribute to raising benefits and reducing challenges for students in small groups.

Research Questions

This dissertation study highlights how beginning teachers intervene in small groups, how they notice before and during intervening in small groups, and how teacher identity and categories of students are related to intervention in small groups. To guide this dissertation study, I pose four research questions;

(1) How do beginning teachers intervene in small groups in mathematics classrooms and for what purposes?

(2) How is beginning teachers' noticing before and during intervening in small groups related to their intervention in small groups in mathematics classrooms?

(3) How are beginning teachers' teacher identities related to how they intervene in small groups in mathematics classrooms?

(4) How do beginning teachers' perceptions of students shape their intervention in small groups in mathematics classrooms?

CHAPTER 2. LITERATURE REVIEW

In relation to these four research questions, in this chapter, I describe the evidence research has provided around these questions and what the evidence may mean for this dissertation study. This chapter consists of five sections. Each section is related to a literature review on (1) benefits and challenges for students participating in small group work, (2) teachers' intervention in small groups, (3) teacher noticing in general and professional noticing in particular, (4) teacher identity, and (5) categories of students.

Benefits and Challenges for Students from Participating in Small Group Work

In this section, I describe potential benefits and challenges for students in small groups. The focus of this dissertation study is not on investigating these benefits and challenges, which is also an important research topic. I describe them because they can be reasons for and/or outcomes of teachers' intervention in small groups. Small groups in this dissertation study generally refer to any group work assigned by teachers in which a group of two to four students cooperate to solve mathematical tasks (Blunk, 1998; Chapin, O'Connor, & Anderson, 2009; Davidson, 1990).

Potential benefits for students

In small groups, students can "practice and refine their growing ability to communicate mathematical thought process and strategies" when small groups become a space "for asking questions, discussing ideas, making mistakes, learning to listen to others' ideas, offering constructive criticism, and summarizing discoveries in writing" (NCTM, 1989, p. 78). The quote above indicates that in small groups, while solving a given mathematical task, students may have

opportunities to explain, justify, or revise their mathematical ideas by communicating with other students, thereby both making sense of mathematical concepts and developing social skills.

Mathematics education research has acknowledged the intellectual and social benefits of participating in small groups for student learning (e.g., Davidson, 1990; Lindquist, 1989; Yackel, Cobb, & Wood, 1991). Lindquist (1989) suggests benefits for small groups in mathematics classrooms on the part of students such that small groups can "increase students' responsibility for their own learning," "encourage students to work together, a social skill that all persons need," and "increase the possibility of students solving certain problems or looking at problems in a variety of ways" (pp. 629-630). Similarly, outside of mathematics education, Cohen and Lotan (2014) note that in small groups, students can learn academically, including conceptual learning and creative problem solving, and that they can also learn to listen to or work with other students. What these researchers have in common is their claim that participating in small groups can be effective for intellectual and social learning benefits.

Potential challenges for students

Just because students are in small groups does not mean that students work well together (Boaler, 2016; Cohen & Lotan, 2104; Johnson & Johnson, 1990). Working together with other group members in small groups, students may experience challenges that prevent them from learning from each other. For example, Langer-Osuna (2011) described how as a cooperative project in a small group proceeded, a female group leader was positioned as being "bossy" by other group members and thus became less engaged in project-related talk. Bishop (2012) showed that in a partner talk, a particular student positioned herself as a "dumb" learner and another student as a "smart" learner. Esmonde and Langer-Osuna (2013) illustrated that students in small groups had difficulty in communicating with each other because they engaged in

different discourses (e.g., mathematical learning vs. friendship and romance). Ding, Li, Piccolo, and Kulm (2007) note that students in small groups may experience some challenges when no group member can answer the question, students exhibit problems communicating with each other, and/or some students dominate group work. Similarly, outside of mathematics education, Cohen and Lotan (2014) also warn that small groups may "develop hierarchies where some members are more active and influential than others" (p. 23). These challenges that students may experience in small groups may make them come to view themselves in a particular way (e.g., dumb, smart, or bossy), which affects their mathematics learning.

In this dissertation study, I propose that teachers' intervention in small groups is one way in which teachers try to deal with the benefits and challenges for students. Understanding these benefits and challenges can help me understand how and why teachers intervene in small groups.

Teachers' Intervention in Small Groups

In this section, I describe teachers' intervention in small groups and its relationships to student learning. More specifically, I show what is known from prior research about how teachers, both practicing and prospective, intervene in small groups. In particular relation to practicing teachers, I also address a potential relationship between teachers' intervention and benefits for students.

How practicing teachers intervene in small groups

Even though the role of the teacher in both setting up small group work and managing effective small group interactions has been regarded as "a rich area for further research" (Blunk, 1989, p. 210), researchers have tended to pay attention more to setting up small groups than to managing or intervening in small groups (Webb, 2009; 2013). When researchers have

specifically addressed intervention in small groups, they have recommended that teachers should intervene in small groups only when it is necessary (Cohen & Lotan, 2014; Ding, Piccolo, & Kulm, 2007; Johnson & Johnson, 2003).

Several researchers have attended to different intervention approaches in small groups: for example, evaluation (Chiu, 2004); mediating students' thinking (Gillies & Boyles, 2006); progress of group interaction process (Dekker & Elshout-Mohr, 2004); or treating status-issues (Cohen & Lotan, 2014; Featherstone et al., 2011). More specifically, in the evaluation approach, teachers make comments and questions to agree/disagree with students' ideas in small groups (Chiu, 2004). In the mediating students' thinking approach, teachers offer comments and questions to scaffold students' thinking (Gillies & Boyles, 2006). In the group interaction-related approach, teachers make comments to facilitate the communication process among students in mathematics classrooms (Dekker & Elshout-Mohr, 2004), in ways that "are not concerned with students' reasoning and products, but with their interaction" (p. 43). For the treating status-issues approach, teachers make comments publicly to assign competence to low-status students (Cohen & Lotan, 2014). Research on these approaches has broadened understanding of teachers' intervention in small groups. However, they focused solely on practicing teachers who had training, a team meeting, or professional development on intervention in small groups and each investigated a single intervention approach.

A potential relationship between intervention and benefits for students

The research on how practicing teachers intervene in small groups demonstrates how teachers' intervention can support student learning in small groups in many ways. For example, Chiu (2014) showed that teachers' evaluative comments or questions supported students in small groups to sustain or change their mathematical understanding embedded in their work. Gillies

and Boyle (2008) suggested that teachers' intervention by mediating students' thinking might "challenge students' understanding and encourag[ing] them to think more deeply and reflectively about the issues under discussion" (p. 1345). Dekker and Elshout-Mohr (2004) illustrated that intervention by the process-help comments supported students to participate in the discussion by motivating students to look critically at the other students' work. Cohen and Lotan (2014) showed that by treating status issues, teachers might be able to put a low-status student's contribution in public to raise the status of students.

These intervention approaches may be useful in this dissertation study for identifying and analyzing interventions in small groups. However, I suggest that more work is necessary to understand how novice teachers intervene in small groups in real teaching contexts without the benefits of professional development focused on a particular intervention approach. In prior work, I investigated prospective teachers' approaches to intervention in small groups.

How prospective teachers intervene in small groups

Building upon findings and limitations of these prior studies on practicing teachers' interventions, Pak (2017) investigated ways prospective teachers in elementary mathematics methods courses intended to intervene in small groups and for what purposes. Drawing upon an analytic framework built on the four intervention approaches described above, this study explored diverse ways prospective teachers intended to intervene in small groups based upon responses of the participants to four different scenarios (Appendix A). The study found that prospective teachers planned to intervene in diverse and complicated ways with a range of purposes. More specifically, prospective teachers intended to make comments or ask questions that would (1) extend students' mathematical thinking; (2) elicit students' mathematical thinking; (3) acknowledge students' potential contribution; (4) encourage students to work together; and

(5) provide content help. When making those interventions, prospective teachers had purposes including (1) evaluating students' understanding; (2) mediating students' mathematical thinking;(3) making group interaction go smoothly; and (4) assigning competence to low status students. In relation to relationships between intervention actions and purposes, Pak (2017) also found that a single intervention action was not always related to a single purpose, which meant that there were diverse combinations of intervention actions and purposes.

Pak's (2017) study emphasized in its findings the complexity of prospective teachers' intervention approaches. There is little empirical evidence, however, on interventions on the part of beginning teachers in mathematics classroom teaching contexts. Thus, I build on prior research with practicing teachers and my work with prospective teachers to try to understand beginning teachers' intervention.

Teacher Noticing and Intervention in Small Groups

In this section, I explore prior research related to a potential relationship between how teachers notice before intervening and how they intervene in small groups. More specifically, first I introduce general understandings of teacher noticing. Second, I address ways researchers have used professional noticing of children's mathematical thinking (Jacobs, Lamb, & Philipp, 2010) as a teacher noticing framework. Last, I address a potential relationship between professional noticing and teachers' intervention in small groups.

General views of teacher noticing

Like other professions that develop their own professional vision (Goodwin, 1994), teachers develop perceptual frameworks that allow them to view complex classroom events in particular ways (Jacobs et al., 2010). In classrooms, "multidimensionality", "simultaneity", and

"unpredictability" are salient for teachers (Doyle, 1977, p. 52). Teachers face a "blooming, buzzing confusion of sensory data" (Sherin & Star, 2011, p. 69). They often select some features that they think are crucial for students' learning while ignoring others. Research on how teachers notice in mathematics classrooms captures what teachers actively, intentionally select or ignore in complex classroom events (Jacobs et al., 2010; Sherin, 2017; van Es & Sherin, 2008). In particular, prospective and beginning teachers tend to notice different features of classroom events than experienced teachers do (Amador, Weston, Estapa, Kosko, & De Araujo, 2016; Star & Strickland, 2008).

Teacher noticing can be understood in many ways. Teacher noticing generally involves two processes: identifying particular events and making sense of those events (Sherin, Jacobs, & Philipp, 2011). Some researchers view making sense as interpreting and deciding how to respond (Jacobs et al., 2010). Building on Jacobs and colleagues (2010), I view teacher noticing as a set of processes- attention, interpretation, and decision about how to respond. I intentionally use *how teachers notice* in this dissertation study to emphasize the process of noticing, instead of the phrase of *what teachers notice*, the phrase commonly used by others (e.g., Sherin, 2017; Star & Strickland, 2008). *What teachers notice* seems to indicate that teacher noticing focuses solely on what they attend to, rather than the process. I also view the processes of attention, interpretation, and deciding how to respond as occurring simultaneously in practice, following Jacobs and colleagues' (2010) acknowledgement that they are complicatedly interrelated.

Professional noticing of children's mathematical thinking

Jacobs and colleagues (2010) conceptualize teachers' noticing as professional noticing of children's mathematical thinking. As described above, they view professional noticing as a set of

three processes: attending to children's strategies, interpreting children's understandings, and deciding how to respond on the basis of children's understanding.

Some researchers have used professional noticing of children's mathematical thinking in different contexts to investigate the development of teachers' noticing ability (e.g., Jacobs, Lamb, Philipp, & Schappelle, 2011; Fernández, Llinares, & Valls, 2012; Schack et al., 2013). Other researchers have extended professional noticing in different ways. For instance, professional noticing has been extended beyond children's mathematical thinking to investigate how teachers notice children's equitable participation (Wager, 2014), to study how teachers use curriculum materials (Amador et al., 2017), or to inform teachers of "indicators that a teacher should try to notice in a classroom setting which related to the understanding of a group of students" (Wells, 2017, p. 184). Others also extended it to investigate how prospective teachers notice children's competencies related to children's mathematical thinking and children's funds of knowledge, or children's multiple mathematical knowledge bases (Roth McDuffie et al., 2014). In common, these researchers drew upon the three processes to investigate teachers' noticing of classroom events beyond children's mathematical thinking.

Similar to these prior studies that extend professional noticing to investigate their own research purpose, I extend professional noticing to investigate how beginning teachers notice before and during intervention in small groups in this dissertation study. Among those studies, this dissertation study looks similar to Wells (2017) in that both are interested in teachers' intervention in small groups. Wells (2017), however, used professional noticing to inform teachers of when to intervene in small groups by noticing students' gestures and conversation. This dissertation study differs from Wells (2017) in that it is about *how* novice teachers intervene in small groups.

How teachers notice before and during intervention in small groups

The three processes of professional noticing (Jacobs et al., 2010) can be useful to delve into teachers' ways of attention and interpretation that shape decisions about how to intervene in small groups. However, previous literature on teachers' intervention in small groups (Chiu, 2004; Cohen & Lotan, 2014; Dekker & Elshout-Mohr, 2004; Gillies & Boyles, 2006) has typically not explored how teachers notice before intervening in small groups. They tend to concentrate instead on ways teachers respond to what students are doing in small groups.

Teachers' intervention literature, however, can still offer an insight into how exploring novice teachers' attention, interpretation, and decisions can broaden our understanding of teachers' interventions in small groups. For example, suppose that a beginning teacher responds to the low status of students who offer a potential idea for discussion (Cohen & Lotan, 2014; Featherstone et al., 2011). The teacher might make such a particular response because he or she can "catch a child with low status offering an idea that no one in her [the child's] group notices" (Featherstone et al., 2011, p.88). It can be inferred indirectly from the situation that the teacher noticed something. In more detail, the beginning teacher responds to the idea of the low-status child because the teacher attended to the status of the child in the small group.

Building upon this insight inferred from literature on practicing teachers' intervention, Pak (2018) analyzed how prospective teachers notice before planning to intervene in small groups, using a lens of the three processes of professional noticing, and how the three processes are interrelated to one another. This study found that in relation to attention, the prospective teachers attended to different features of small groups across scenarios, such as students' mathematical thinking, interaction between group members, and status of students. It also identified that for decisions about how to respond, their decisions were diverse across scenarios,

such as engaging students' mathematical thinking, encouraging students to work together, and acknowledging students' potential contribution.

Given Pak (2018), it seems that professional noticing may shape how teachers intervene in small groups. More specifically, decisions about how to respond, which can be shaped by teachers' interpretations, may affect how teachers intervene in what students are doing in small groups. Building on that, I investigated in this dissertation study how the three processes of professional noticing are interrelated and particularly how the interpretations the novice teachers made were related to attention and decisions about how to respond.

On the other hand, professional noticing happens at or is influenced at the group level as well as the individual. Louie (2018) extended teacher noticing to a teaching activity that was influenced by certain dominant discourses normalized in a school, which were "socially and culturally produced" (p. 41). This study reported that when a female teacher noticed difficulty her students had in understanding and solving mathematical problems, the teacher felt she held deficit mindsets towards those students. Tracing the sources of deficit mindsets, Louie (2018) found that professional noticing "belong not to individuals but to groups of people who interact with one another and their environments to code, highlight, produce and articulate material representations, and in other ways create and maintain systems for seeing" (p. 41). Based on this understanding of the relation of contexts to teacher noticing, Louie (2018) suggests that research on teacher noticing needs to take into consideration contexts in which professional noticing takes place.

Even though Louie's study is not about how teachers intervene in small groups, it is suggestive for this dissertation study. That is, what teachers notice could be shaped strongly by perspectives or ideas from outside of classrooms. Building on that, I suggest in this dissertation

study that investigating professional noticing could be related not only to what individual teachers notice but also to what shapes their attention or decisions through interpretation.

Teacher Identity in Relation to Teaching Practices

In this section, I discuss the relationships between teacher identity and teaching practices. First, I explain how teacher identity can be conceptualized for this dissertation study. Second, I address the relationships between teacher identity and teaching practices in general and intervention in small groups in particular, sometimes meditated by professional noticing.

Conceptualization of teacher identity.

In a literature review on teacher identity, Beauchamp and Thomas (2009) note that teacher identity has been used "as a frame or an analytic lens through which to examine aspects of teaching" (p. 176). Even though many researchers have tried to understand teacher identity, however, their definitions of teacher identity vary from one another (Beijaard, Meijer & Verloop, 2004; Olsen, 2011).

In general, teacher identity is defined as how teachers view themselves as teachers in certain contexts. Horn, Nolen, Ward, and Campbell (2008) have investigated teacher identity development building on anthropologists' view of identity, such as Holland and colleagues (1998). According to these anthropologists, identity is "the way a person understands and views himself, and is often viewed by others, at least in certain situations—a perception of self that can be fairly constantly achieved" (p. 68). By this definition, they mean that the person's view of him/herself would shape and be shaped by others in certain situations. In a study that explored how teacher identity of prospective teachers develops across contexts, such as coursework and field placements, Richmond, Juzwik, and Steele (2011) view teacher identity as how teachers

"tell about themselves" in addition to how they "are told by, to, and about" themselves (p. 1866), drawing on the constructs of actual and designated identity by Sfard and Prusak (2005). These studies highlight that understanding teacher identity is inseparable from contexts. This definition of teacher identity helps researchers "make sense of the relationships individuals develop with the contexts they encounter in teacher education" (Horn et al., 2008, p. 62).

In particular, researchers in mathematics teacher education have conceptualized teacher identity in different ways. Spillane (2000) views teacher identity as "an individual's way of understanding and being in the world of work" (p. 308). Teacher identity does not only include knowledge and beliefs, but also encompasses, "dispositions, interests, sense of efficacy, locus of control and orientations toward work and change" (p. 308). Beyond the notion of teachers' sense of selves as teachers, this conceptualization adds more complexity to teacher identity. Building on the conceptualization of teacher identity by Spillane (2000), Drake and colleagues (2001) suggest that "teachers construct storied identities" that "serve as the lens through which they understand themselves personally and professionally and through which they view the content and context of their work" (p. 2). Aguirre, Mayfield-Ingram, and Martin (2013) also define mathematical teacher identity as "an identity that consists of knowledge and lived experiences, interweaving to inform teaching views, dispositions, and practices to help children learn mathematics" (p. 27). This mathematical teacher identity is also understood as "the stories that people tell about themselves and what they view as important to them: their understanding of their place in the world and their core belief" (p. 27). In common, these researchers view teacher identity as more than self-understandings, including beliefs, knowledge, and/or dispositions, constructed in personal and professional lives.

Building on several views of teacher identity in this subsection, I define teacher identity as an interconnected collection of teachers' individual self-understandings as teachers, beliefs, knowledge, and/or dispositions. In the next section, I describe research on how teacher identity may be related to teaching practices in general and intervention in small groups in particular.

Relationships between teacher identity and teaching practices

Across research on teacher identity, how teacher identity can shape teaching practice has been investigated. In particular, in mathematics teacher education, researchers mentioned above (Aguirre et al., 2013; Drake et al., 2001; de Freitas, 2008; Spillane, 2000) have shown that teacher identity can shape and be shaped by teaching practices. Spillane (2000) investigated how a reform-oriented teacher's identity as a teacher and as a learner about teaching differed in two subject matter contexts, mathematics and literacy, and how these differences in her identity shaped her teaching practices. He found that after analyzing tasks and discourse patterns of her mathematics lessons, the teacher's teaching practices depended largely upon memorization and procedure, which contrasted with her literacy instruction with an emphasis on students' reasoning. On the basis of understanding that subject matter contexts influence teaching practices, Drake and colleagues (2001) also investigated how elementary teachers' identities played a role in their learning and teaching practices. They analyzed 10 elementary teachers' stories of learning experiences and of teaching practices in mathematics and literacy contexts. They compared the narrative differences in their identities in both subjects. They found that these teachers' stories reflected their identities as mathematics and literacy learners and teachers, which shaped their teaching practices. In the study, unlike literacy stories that were uniform among these 10 teachers, mathematics stories were "dominated by disappointing and discouraging experiences learning mathematics in school" (p. 10). Both studies suggest that

teachers may have differences in their identities depending upon subjects. Mathematics teacher identity is "one of many disciplinary identities" that teachers develop (Aguirre et al., 2013, p. 10). These studies also emphasize ways teacher identity in particular relation to the mathematics teaching context can inform and shape teaching practice.

There have been many studies on teacher identity to understand teaching practices in the field of mathematics teacher education. However, the field does not know enough about this particular relationship between teacher identity and beginning teachers' intervention. As such, I propose that investigating the relationship deserves more attention because of a possible contribution to understanding the nature of intervention in small groups.

Professional noticing mediating between teacher identity and intervention

The research reviewed above suggests there might be an interrelationship among noticing, identity, and teaching practice. Few studies, however, have investigated these relationships. Wager (2014) is suggestive for this dissertation study. Wager (2014) investigated the relationships between teacher positionality and teacher noticing. She found that teachers' positioning themselves as equitable mathematics educators shaped what they noticed about students' participation in elementary mathematics classrooms, which also guided their equityoriented teaching practices. Wager's study is on teacher noticing of students' participation in elementary mathematics classrooms, not on how teachers notice before intervening in small groups. However, Wager's finding still suggests that one form of the relation between how teachers view themselves and intervention may be the one mediated by professional noticing before intervening in small groups.

In this dissertation study, I do not focus on this relationship between teacher identity and professional noticing because the relationship is not the main focus of this study. I provided this

subsection to suggest, in relation to this dissertation study, that professional noticing may sometimes mediate the relationship between beginning teachers' identities and how they intervene in small groups.

Categories of Students in Small Groups in Mathematics Classrooms

In this section, I explain the potential relationships between categories of students and teachers' teaching practices, potentially intervention in small groups. First, I describe categories of students identified by researchers and how these categories are related to teachers' teaching practices. Second, I address how teachers' perceptions of students in terms of categories could shape teachers' teaching practices. I present three studies in mathematics education to suggest that categories of students may be related closely to teachers' intervention in small groups.

Multiple categories of students in schools

Many education researchers have identified different categories of students (e.g, Good & Power, 1976; Veldman & Worsham, 1983; Barett & Connot, 1986). For example, Good and Power (1976) developed a typology of students where five student types were identified (success students, social students, dependent students, alienated students, and phantom students). These researchers found that teachers responded to each of these types of students in different ways (e.g., for success students, the teacher was likely to ask challenging questions). Veldman and Worsham (1983) analyzed ratings made by classroom observers in relation to students' behaviors. These researchers identified four types of students in junior high schools: Good students, Outgoing students, Rebellious students, and Withdrawn students and suggested the relations of students in each category to their teachers in terms of their interactions (e.g., Rebellious students have poor relations with teachers). This body of literature relied on the

researchers to identify categories of students and suggest "their relation to the teacher and to classroom events" (Veldman & Worsham, 1983, p. 203).

To be clear, these studies did not examine the process that teachers used to construct categories of students or ways their recognition of students in terms of certain categories shaped their teaching practice, which is relevant to this dissertation study. However, these studies still could provide implications for this dissertation study. There are numerous categories of students that teachers may recognize in (mathematics) classrooms regardless of whether they are aware of it.

Categories of students in mathematics classrooms

Several studies in mathematics education have identified categories of students recognized by teachers as well as their peers. For example, Horn (2007) examined ways teachers in two different high schools understood the discrepancy between "students' perceived abilities and the intended school curriculum through conversational category systems" (p. 37). In this study, the teachers perceived some students in terms of certain categories of students (e.g., fast kids, slow kids, and lazy kids). The researcher showed that how teachers recognized certain students seemed to shape the ways the teachers implemented equity-geared reforms in terms of curriculum as well as teaching practice. In a meeting where teachers talked about creating courses aligned with a rigorous reform-based curriculum, a teacher explained her perception of power dynamics between "the kids who are fast learners" and "kids who are slow learners" (Horn, 2007, p. 50). The teacher expressed a plan to find group-worthy activities that would allow the slow students to participate positively to reduce the dynamics. This recognition of categories of students in her teaching shaped her view of her role as a teacher who should "vary the curricular activities as a way of addressing issues in his or her classroom" (Horn, 2007, p.

51). This shows that recognizing students as fast learners and slow learners affected her view of curriculum and potentially her teaching practice. Furthermore, this recognition of students in terms of certain categories of students (e.g., academically lower students or disruptive students) may not be credited solely to individual teachers. This recognition may be shaped by perspectives, views, and ideas coming from outside mathematics classrooms, similar to the findings of Louie (2018). This suggests for this dissertation study that teachers' recognition of students may shape teachers' intervention in small groups, sometimes mediated by perspectives, views, and ideas coming from outside mathematics classrooms.

Other research has focused on how categories of students are recognized by their peers in small groups. Some researchers examined how some students' positioning of their peers shaped their participation in small group work (Bishop, 2012; Langer-Osuna, 2011), as discussed in the previous section. Langer-Osuna (2011) illustrated how in a small group, three boys recognized a girl as being "a group leader" in the beginning of the group project. But later the male students stated, "You can't let her be the boss" (Langer-Osuna, 2011 p.212). They consistently positioned her as being "bossy" later then. This category given by her peers to her resulted in her exclusion from the group work. In a study to characterize meaningful discourse patterns in small-group interactions, Bishop (2012) investigated the ways that two female students jointly enacted their identities through their positioning of each other. The researcher suggested that in a partner talk, a particular student positioned herself as a "dumb" learner and another student as a "smart" learner. These categories (e.g., dumb and smart) reflected their sense of "superiority and inferiority," which showed how they perceived each other as mathematics learners. The female student recognized herself as "dumb" and took up another student's positioning of herself as "stupid." The focus of these two studies was not to identify categories of students but to use

these categories recognized by other members to examine how this positioning shapes students' participation in small groups and their learning. However, these two studies are suggestive for this dissertation study in that they highlight that students' participation and learning in small groups are shaped by how they are recognized by others, including by peers.

To be clear, these studies above are not related to how teachers' perceived categories of students shape their intervention in small groups. There may be an important suggestion I could make for this dissertation study. That is, if peers' recognitions matter for students' participation in small groups, teachers' recognitions also matter for students' participation in small groups. Building on that, in this dissertation study, I investigate categories of students beginning teachers may recognize in their math instruction and their potential relationships to teachers' intervention in small groups.

Summary

In this literature review chapter, first, I illustrated potential benefits and challenges for students from participating in small groups. Second, I detailed how teachers intervene in small groups as potential ways to promote intellectual and social benefits for students in small groups. I also described four intervention approaches. Third, I explained how teacher noticing can be related to teachers' intervention in small groups. I highlighted professional noticing of children's mathematical thinking (Jacobs et al., 2010) as a lens to investigate how teachers notice before and during intervening in small groups. I suggested that professional noticing and teachers' intervention are interrelated in that both inform each other. Fourth, I addressed how researchers in mathematics teacher education conceptualize teacher identity and how teacher identity is related to teaching practices in general to suggest the potential relationships between teacher

identity and teacher' intervention in small groups, sometimes through the mediation of professional noticing. Fifth, I presented several studies to show how researchers have identified different categories of students in classroom contexts and how these categories of students were related to learning and teaching.
CHAPTER 3. THEORETICAL FRAMEWORK

In the previous chapter, I noted the potential relationships between teachers' intervention in small groups and professional noticing, teacher identity and teachers' perceptions of categories of students. In this chapter, I introduce the notion of figured worlds (Holland et al., 1998) as a lens to investigate these relationships. First, I describe Holland and colleagues' (1998) conceptualization of figured worlds, particularly their depiction of the elements that make up figured worlds, and why mathematics classrooms can be considered as figured worlds. Second, I describe the relationship between practices and activities and figured worlds. Building on these relationships, I introduce my view of intervention in small groups as a day-to-day activity specific to figured worlds of mathematics classrooms and the potential relationships of this activity to teacher noticing with an emphasis on interpretation, teacher identity and to categories of students in mathematics classrooms. Last, I present diagrams that conceptualize these relationships.

Figured Worlds and Mathematics Classrooms

Figured worlds are "socially and culturally constructed realm[s] of interpretation in which particular characters and actors are recognized, significance is assigned to certain acts, and particular outcomes are valued over others" (Holland et al., 1998, p. 52). I describe the elements of figured worlds to explain why I view mathematics classrooms as figured worlds from the perspective of teachers.

What makes up figured worlds?

To show figured worlds in contexts, Holland and colleagues (1998) describe the example of girls who enter college where they encounter figured worlds of romance where romance, love,

or attractiveness matter the most. In figured worlds of romance, attractive women, boyfriends, or lovers are recognized as actors to these women. Certain acts like putting on makeup, flirting, and falling in love are meaningful to them. These women value receiving proper treatment from their boyfriends. The women spend much of their time and thoughts on attractiveness and romance.

This example related to figured worlds of romance suggests several elements that make up figured worlds- practices and activities, interpretation, artifacts, and power. First, figured worlds are "socially produced, culturally produced, culturally constructed activities" (Holland et al., 1998, pp. 40-41). Practices and activities are what people do on a regular basis in ways specific to figured worlds. In figured worlds of romance, the women need to decide if they are eager or indifferent to engaging in certain practices and activities, such as beautification or putting on makeup. Many women devote much time and efforts to participating in such practices and activities. Figured worlds of romance are realized through the women's participation in practices and activities specific to the worlds.

Second, figured worlds provide people in the world with contexts for interpretation. Interpretation is defined as a process to give meaning to actors, acts, and outcomes. In figured worlds of romance, some women take a great amount of time to give meaning to actors (e.g., boyfriends and lovers), acts (e.g., falling in love), and outcomes (e.g., developing intimate relationships).

Third, figured worlds are "evinced in practice through the artifacts employed by people in their performance" (Holland et al., 1998, p. 61). Artifacts refer to materials and concepts that help people enter the figured worlds. In figured worlds of romance, the sexy clothes and certain ways of wearing makeup could be examples of these artifacts. These artifacts have been developed as social and cultural means and these artifacts could become a pivot that actors use to

enter these figured worlds of romance where the female students value learning to use these artifacts to participate in the worlds.

Fourth, Holland and colleagues emphasize the notion of power in relation to how larger institutional, societal, and cultural ideas are embodied by people through continual participation in practices and activities. Power indicates the various degrees of access to the ideas that are perceived by people to be valuable in figured worlds. In figured worlds of romance, through participation in practices and activities, these women develop expertise on making them be more attractive. The women with more expertise may have more power in the figured worlds of romance, which means that they come to embody the broader societal and cultural idea underlying the attractiveness. This idea is aligned with male/female relations and matters in the figured worlds of romance. In these relations, attractiveness is an important capital to the women because it is more likely to help them build and sustain more powerful relationships with men.

Teachers' figured worlds of mathematics classrooms

Mathematics classrooms in the U.S. can be considered figured worlds, according to Boaler and Greeno (2000) and Esmonde and Langer-Osuna (2013). These researchers identified figured worlds of mathematics classrooms that were constructed by students. Figured worlds constructed by students would be a partial image of figured worlds of mathematics classrooms. These worlds are also constructed by teachers, which is my focus in this dissertation study. Furthermore, there have been few published studies on teachers' figured worlds of mathematics classrooms. Some researchers have taken a step to investigate figured worlds from teachers' perspectives (e.g., Horn et al., 2008; Ma & Singer-Gabella, 2011), but these studies were not related to figured worlds of mathematics classrooms and to novice teachers engaging in a day-today teaching activity.

For the purpose of informing what figured worlds of mathematics classrooms constructed by teachers may look like, I draw on Munter, Stein, and Smith's (2015) descriptions of what direct and dialogic teaching looks like within mathematics classrooms. I acknowledge that these descriptions did not draw on a figured worlds framework, but their descriptions are still useful for demonstrating the elements of figured worlds.

According to Munter and colleagues (2015), in mathematics classrooms in which teachers rely more on direct teaching, teachers view mathematics as having a right answer or a procedure to follow. On the other hand, in mathematics classrooms in which teachers rely more on dialogic teaching, teachers encourage students to interact with one another about multiple ways of problem solving and reasoning.

It seems that both teaching styles are related to the elements of figured worlds. First, teaching activities in each teaching style have been socially and culturally constructed. For example, dialogic ways to ask questions (e.g., explanation/justification) have been developed collectively by the mathematics education community.

Second, when teachers rely on either teaching style, their interpretation matters in relation to perceptions of students, meaningful acts (e.g., mathematical talk), and valuable outcomes (e.g., multiple ways of problem solving or a right answer). In particular, interpretation very often guides teachers to notice something important in relation to students, behaviors, and outcomes in mathematics classrooms.

Third, classroom norms as artifacts shape teachers' expectations and interpretations. Each teaching style may have different classroom norms. For example, teachers relying more on dialogic teaching may establish classrooms norms where students are expected to respect others' ideas.

Fourth, each of these teaching styles is aligned with different ideas or perspectives on teaching and learning. Depending on the teaching styles (e.g., dialogic teaching), the relevant idea (e.g., valuing students' sense-making process) may shape ways teachers interact with students and expect students to interact with peers (e.g., encouraging students to learn from each other). Whether direct or dialogic teaching, the relevant ideas have been constructed and supported by education communities. To support the idea, teachers employ their authority as resources to control over students in their classroom instruction. Teachers recognize how this authority plays out in the interactions with students (Herbel-Eisenmann & Wagner, 2014; Herbel-Eisenmann, Wagner, & Cortes, 2008). For example, in an empirical study on teachers as authority (Wagner & Herbel-Eisenmann, 2014), one teacher recognized himself as disciplinary authority while wanting students to develop a sense of authority within the discipline. In a new school, the teacher tried to renegotiate his authority in the interactions with students to support students develop the sense. Given these studies, it is possible that it is their recognition of themselves as authority that have teachers keep navigating the power and authority dynamics among teachers and students.

In a nutshell, these potential relationships between direct/dialogic teaching and the elements of figured worlds suggest ways to show that teachers construct figured worlds of mathematics classrooms.

Broadly speaking, there are close relationships between teaching activity and the elements of figured worlds. These relationships can be ways to investigate figured worlds of mathematics classrooms constructed by teachers. That is, it is worth exploring teaching activity in terms of the elements of figured worlds of mathematics classrooms. In this dissertation study,

therefore, I explore figured worlds of mathematics classrooms beginning teachers construct through their mathematics instruction in relation to elements of the figured worlds.

Practices, Activities, Identity, and Types of Actors in Mathematics Classrooms

In the previous section, I explained the elements of figured worlds and related them to figured worlds of mathematics classrooms. In this section, I describe in more detail my view of intervention in small groups as a teaching activity in the sense of Holland and colleagues (1998). I also describe the relationships of intervention in small groups to teacher noticing with emphasis on interpretation, to teacher identity or their perception of themselves as teachers, and to teachers' recognition of students in terms of categories of students in figured worlds of mathematics classrooms. Along with this description, I also draw on the elements of figured worlds to provide the rationale for examining the relationships of intervention to teacher noticing, to teacher identity, and to categories of students in figured worlds of mathematics classrooms.

Intervention as a day-to-day teaching activity

As mentioned earlier, one of the elements of figured worlds is socially and culturally constructed practices and activities. Holland and colleagues (1998) illustrate examples related to these practices and activities in particular figured worlds. In figured worlds of romance, many women did eagerly engage in practices, such as beautification or putting on makeup. In the figured world of Alcoholics Anonymous, participants participated in practices of telling stories about themselves before joining the organization. Also, Holland and colleagues (1998) drew on the ethnographic work of Favret-Saada (1980) in the figured world of witchcraft where people joined "the practice of witchcraft by the simple act of questioning" (p. 55). What Holland and

colleagues emphasized was that figured worlds, like the world of witchcraft, happen moment by moment. These activities, like the simple act of questioning, are important because people form their identities in the process of participating in practices and activities specific to figured worlds and also because particular types of actors are established through these practices and activities.

Given that teaching is a socially and culturally constructed activity (Leontiev, 1978), diverse teaching activities in mathematics classrooms could be considered to be socially and culturally constructed activities. In relation to this dissertation study, one such teaching activity is intervention in small groups in mathematics classrooms. This is because teachers may learn ways to intervene that have been produced collectively by education-related communities that consist of teachers and researchers. For example, in some studies, teachers participated in professional development regarding intervention in small groups and enacted in their classroom particular ways of intervening in small groups (Chiu, 2004; Cohen & Lotan, 2014; Dekker & Elshout-Mohr, 2004; Gillies & Boyle, 2006). In addition to professional development, teachers may have had chances to observe, experience, and learn ways of intervention in small groups in their K-12 schooling, their teacher preparation program, or their teaching context (e.g., Horn et al., 2008; Ma & Singer-Gabella, 2011). In short, teachers do not invent ways of intervention in small groups in a vacuum. Rather, ways of intervention are likely to be socially constructed and culturally developed over time and then individually learned and enacted on a regular basis by teachers in their classrooms.

Building on my view of practices and activities in figured worlds, I argue for exploring intervention in small groups as one of the teaching activities specific to figured worlds of mathematics classrooms. I shed light on intervention in small groups because it is day-to-day

activities like intervention in small groups that reveal the figured worlds of mathematics classrooms.

Teacher noticing mediated by interpretation of participation in activities

As mentioned earlier, figured worlds are contexts for people to engage in interpretation. The definition of interpretation mentioned earlier suggests that people's interpretation is tied to their participation in practices and activities because actors, acts, and outcomes would not be separable from their contexts. For example, in figured worlds of witchcraft, people should be able to understand the significance of certain acts in specific time and space. To live in the worlds as actors, they should be able to answer questions embedded in specific contexts, such as, "Was that person's intent gaze a sign of witchcraft directed at me, a mark of interest in the color of my dress, or simply an accidental glance, looking through me to the thought beyond?" (Holland et al., 1998, p. 56). Responding to questions like the one above needs people to develop interpretation in ways specific to figured worlds, which happen moment by moment.

This interpretation depends to some degree on how people develop "the ability to sense (see, hear, touch, taste, feel) the figured world" through the "continual participation" in practices and activities in figured worlds (Holland et al., 1998, p. 52). The ability to sense figured worlds supports people to interpret their participation in ways specific to figured worlds. In relation to figured worlds of witchcraft, people need to be able to sense recognizable actors, meaningful acts, and valuable outcomes. For example, to answer the questions like the one above, they should be able to sense a person's gaze before interpreting, or giving meaning to, the intent gaze act of others as actors in the figured worlds. The ability to sense the figured worlds allows people to interpret actors, acts, and outcomes in practices and activities specific to the figured worlds.

Broadly speaking, interpretation happens in any figured world in relation to actors, acts, and outcomes. This notion of interpretation is important because it suggests teachers engage in interpretation to recognize students, to assign significance to certain behaviors, and to value certain learning goals in figured worlds of mathematics classrooms. Given people's ability to sense figured worlds is related closely to interpretation, teachers' attention to students, behaviors, and outcomes would also be connected to interpretation in figured worlds of mathematics classrooms.

As mentioned in the literature review, this interpretation is at the center of mathematics teacher noticing. The leading researchers on teacher noticing view interpretation as essential parts of teachers' perceptual framework on which teachers depend as deciding what (not) to notice. Specifically, interpretation is one of the three processes of the professional noticing (attention, interpretation, and decisions of how to respond) (e.g., Jacobs et al., 2010; Jacobs, Lamb, Philipp, & Schappelle, 2011; Fernández, Llinares, & Valls, 2012; Roth McDuffie et al., 2014; Schack et al., 2013). In connection to the view of interpretation as one of the elements of figured worlds, I argue that it is the interpretation that connects figured worlds to teacher noticing. In other worlds, interpretation allows teachers to construct the worlds of mathematics classrooms through teacher noticing.

However, when it comes to intervention in small groups as a day-to-day teaching activity, studies have not provided how interpretation works in professional noticing in figured worlds of mathematics classrooms. Given the close relationships between intervention in small groups and figured worlds, it is necessary to explore how teachers engage in interpretation to figure the worlds of mathematics classrooms. As such, I investigate professional noticing in terms of how

interpretation shapes attention and decisions of how to respond as beginning teachers intervene in small groups in figured worlds of mathematics classrooms

Teacher identity and participation in practices and activities

In figured worlds of mathematics classrooms, I view identities as outcomes, resources, and lenses for actors participating in practices and activities. First, identities are formed through participating in practices and activities on a daily basis. For Holland and colleagues (1998), identities, which means how individuals view themselves, are "important outcomes of participating in activities" organized in figured worlds (Holland et al., 1998, p. 57). In figured worlds of romance, flirting with and falling in love with male students, and putting on makeup to be attractive, are among these activities. For these college women, "their mastery" of romantic practices and activities contributes to developing "a concept of themselves as actors in the world of romance" (Holland et al., 1998, p. 99). It is identities that "trace our [their] participation" in practices and activities in figured worlds of romance (Holland et al., 1998, p. 40).

Second, identities are sometimes resources that people draw on to shape their participation in practices and activities. Depending on the degree of their mastery, they perceive themselves as more or less romantic actors and draw on their identities to inform their participation in the figured worlds of romance. For example, when they perceived themselves as being less attractive, the college female students tried to be indifferent to beautification practices. A perception of themselves as romantic actors made by their parents or relatives in their childhood could also be used to shape ways for them to participate in practices and activities. In a nutshell, their identities may serve as resources that they may use to participate in figured worlds of romance now and in the future.

Third, identities are utilized by actors as lenses to explain, make sense of, or reason about their participation in practices and activities. I think that this concept of lenses differs from identities as resources in that it highlights ways people try to understand their participation in practices and activities. In figured worlds of romance, these college women invoked identities to make sense of their participation in practices and activities. For example, one female student who viewed themselves as being very attractive tried to reason about and made complaints about ill-treatment given by her boyfriend. This student used her identity as an attractive woman to make sense of her participation in romantic activities.

As explained above, in any figured world, participating in practices and activities, people form and develop their identities. Also, they use these identities as resources to shape and as lenses to make sense of their participation in practices and activities in figured worlds. This is an important point for this dissertation study. Given that people's identities are related closely to how they participate in practices and activities, it is possible to assume certain links between teacher identity and intervention in small groups as a day-to-day teaching activity in figured worlds of mathematics classrooms. As such, I argue for investigating the relationships between teacher identity and intervention. Building on that, in this dissertation study, I explore how beginning teachers use their identities as outcomes, resources, or lenses to engage in intervention in small groups as a teaching activity in figured worlds of mathematics classrooms.

Types of actors and participation in practice and activities

As mentioned above in the description of figured worlds, figured worlds serve as contexts for interpretation of recognizable actors, meaningful acts, and valuable outcomes. I emphasize actors in particular because figured worlds "rest upon people's [actors'] abilities to form and be formed in collectively realized 'as-if' realms" (Holland et al., 1998, p. 49).

These abilities include how people recognize others as certain types of actors in figured worlds. For example, the college female students in the figured world of romance know and talk about a number of words for types of men, such as "jerks," "jocks," "cowboys," "fratty baggers," "brains" (Holland et al., p. 102). These gender-marked types are cultural artifacts, which are socially produced, individually learned, and recognized by women as actors in the world of romance. Women position men in terms of certain types of men, such as jerks. Drawing on jerk types of men, they assume the taken-for-granted progress of male/female relations in interactions with other women. With respect to how such relationships should develop, they would avoid developing the relationship with "jerks" because these types of men are "figures who cause such relations to go awry" (Holland et al., 1998, p. 103). These types also reveal that they are related closely to who has a high/low power in the male/female relationships in the figured worlds of romance. How a woman positions a potential lover in terms of these gender-marked types means that the man has power, more or less, in the world of romance where the male/female relations matter. As such, these gender-marked types are hardly separable from power. Like in the figured world of romance, any figured world is likely to have a certain category system to position actors that people use to position each other on a daily basis. Also, power dynamics among actors, which result from differences in power, are likely to play out in the interactions and relationships among actors positioned into different categories.

As a whole, viewing mathematics classrooms in terms of figured worlds calls for understanding ways to understand people's recognition of others and its relationships to participation in practices and activities and to power and authority dynamics in the figured worlds. Several studies related to categories of students (e.g., Bishop, 2012; Horn, 2007; Langer-Osuna, 201) suggest that teachers may recognize students in terms of different categories. These

differences, such as being academically higher or lower in mathematical understanding, could be a basis of power and authority dynamics because of the differences in power. Studies on teachers as authority (Herbel-Eisenmann & Wagner, 2014; Herbel-Eisenmann, Wagner, & Cortes, 2008; Wagner & Herbel-Eisenmann, 2014) suggest that teachers recognize authority dynamics that play out in the interactions among teachers and students. In other words, teachers may recognize ways to use their power and authority to minimize the power and authority dynamics among students. Broadly speaking, teachers participate in teaching activities (e.g., intervention in small groups) and use their power and authority to deal with the dynamics among students in mathematics classrooms.

Given that, it is possible that categories of students that teachers might recognize allow teachers to see the power and authority dynamics playing out in the interactions among students and among teachers and students in small groups. As such, I investigate in this dissertation study categories of students recognized by beginning teachers and their relationships to intervention and to power and authority dynamics operating in figured worlds of mathematics classrooms.

Conceptualizing the Relationships

In this chapter, I described how I conceptualize figured worlds as a lens to explore the relationship of intervention in small groups to professional noticing, to teacher identity, and to categories of students. Building on that, I present below a representation of the theoretical framework (Figure 3.1). When beginning teachers intervene in small groups in mathematics classrooms, they notice something in relation to students, behaviors, and outcomes in small groups. They also bring into figured worlds of mathematics classrooms their sense of how they recognize themselves or their identities as teachers. They also use categories of students, which

means how they recognize students, to make sense of what is going on in small groups and to decide how to intervene in small groups and of understanding power and authority dynamics in small groups. Their recognition of themselves as teachers (teacher identity) and of students (in terms of categories of students) is more likely to be shaped by participation in teaching activities on a daily basis in figured worlds of mathematics classrooms.



Figure 3.1 Figured worlds as a lens for understanding the relationships

In Figure 3.1, arrows indicate what I focus on in this dissertation study. To understand how beginning teachers intervene in small groups and for what purposes, I intend to examine in the context of figured worlds of mathematics classrooms how professional noticing is related to intervention and how teacher identity is related to this teaching activity, sometimes through mediation by professional noticing. I also intend to investigate how categories of students recognized by teachers shape their intervention in small groups in connection with power and status that play out in the interactions among others.



Figure 3.2 Three relationships I analyzed in this dissertation study

Three relationships to investigate in this dissertation study

Figure 3.1 indicates that there are many relationships I planned to potentially investigate in this dissertation study. Among those relationships, I explored three sets of relationships. The first are the ones between professional noticing and intervention in small groups. The second are the ones between teacher identity and intervention. The third are the ones between categories of students and intervention, very often mediated by power and status in figured worlds of mathematics classrooms. The three arrows highlighted in yellow in Figure 3.2 show these three relationships. I investigated these relationships because the data collection and analysis, which is described in Chapter 4, allowed me to see them the most.

CHAPTER 4. METHODS

As I noted earlier, the purpose of this dissertation study is to investigate how beginning teachers intervene in small groups in relation to professional noticing, to teacher identity, and to categories of students recognized by teachers in figured worlds of mathematics classrooms. In this chapter, I describe how I investigated these relationships. First, I describe a larger research context. Second, I introduce two beginning teachers as participants. Third, I explain how I collected data using data collection tools drawing to some extent from prior studies discussed in the literature review. Fourth, I introduce how I analyzed data, again based in part on data analysis from those prior studies. Fifth, I explain how my own subjectivity as a researcher might have shaped this dissertation study.

Research Context

The participants in this dissertation study were selected from a larger group of participants in a longitudinal large-scale research project titled Development of Ambitious Instruction (DAI). The purpose of this DAI research was to explore the relationships between teacher preparation and ambitious instruction in elementary mathematics and English Language Arts (ELA). This DAI research recruited participants from five teacher preparation programs in three states. For the dissertation study, I only recruited participants teaching in one Midwestern state. The data collection tools for this DAI research included online surveys, observations, and interviews. For observation, this DAI research used the Mathematics Scan (M-Scan) classroom observation instrument for mathematics and the Protocol for Language Arts Teaching Observation (PLATO) for ELA and mathematics. For this dissertation study, I collected additional data including involving small groups, stimulated recall interview, and survey responses, as described in detail below.

Participants

Three beginning teachers participated in this dissertation study initially. They were beginning teachers in the sense that they were in their second or third year in teaching careers in 2018-2019. They reported that they implemented small groups on a regular basis as an instructional structure to teach mathematics, which would allow me to pursue my interest in this dissertation study. Since I was able to collect the whole set of data from only two teachers, whom I call Leslie and Marva hereafter, I introduce the background information related to their teaching grade, school, teaching subjects, and student population. All students' and teachers' names used in this dissertation study are pseudonyms.

Leslie

She was in the third year of her teaching career in 2018-2019. She taught at a private Catholic school for K-8 students, which was located in an urban city area. She taught 20 fourth grade students in her own classroom. Students in her classroom as well as in the school were predominantly White. Her teaching subjects included Social Studies, English Language Arts, and Science. Mathematics was not a subject area assigned for her to teach. However, she had integrated mathematical concepts into Social Studies lessons, which were labeled Social Students/Mathematics lessons in the larger DAI research project.

Marva

Marva was in the second year of her teaching career in 2018-2019. She taught at a public charter school for K-12 students, which was located in an urban city area. This area was highly diverse in race/ethnicity, culture, and language because of the increase in population of immigrants and refugees from different countries, which was reflected in the school as well. She taught 23 first grade students in her classroom. Seventy-three percent of the students were

English Language Learners. Their parents were from Bengal, Yemen, Bosnia, and Poland. Her teaching subjects included Mathematics, Social Studies, English Language Arts, and Science.

Data Collection

In this section, I describe the recruiting process, data collection process, and data collection tools. All processes below were begun after receiving approval from the Institutional Review Board (IRB) at the university.

Participant recruitment process

First, I identified potential participants among beginning teachers in the DAI project. I watched project video recordings of beginning teachers in the state who implemented small groups in mathematics instruction. More specifically, based on Pak (2017), I looked for beginning teachers who made comments or asked questions that agreed or disagreed with students' work, engaged in students' mathematical thinking, focused on interaction between group members, or acknowledged students' potential contribution to group discussion (e.g., status). As a result, I identified four beginning teachers as potential participants in this dissertation study.

Second, I asked these four beginning teachers about their willingness to participate in this additional study. I contacted them via an email that included: the introduction of myself as a researcher, the reason I was contacting them, the introduction of this study, the relationships between this study and the larger DAI study; the focus of this study (intervention in small groups in their mathematics teaching); the expectations of this study (one online open-ended survey, three video recordings, and four interviews); the benefits from participation (25 dollars per each participation), and the contribution of their participation in this study to teaching, learning, and

teacher education. As a result, I recruited three beginning teachers who were willing to participate in my dissertation study.

Third, I asked them to give me permission by a consent agreement form. This form included the purpose of the study, time required for participation, risks/benefits, confidentiality, and so on. The three teachers gave me the form signed.

Data collection process

I emailed them to ask them to take the open-ended online survey and to set up dates and times for observations and interviews. I then asked them to inform me of dates and times when they were going to implement small groups in mathematics instruction. I was able to complete setting up schedules for observation and interviews. Among the three teachers, one teacher did not continue to participate in my dissertation study after the first stimulated recall interview for a personal reason. As such, I was able to collect the whole set of data from the two teachers, Leslie and Marva.

Data collection tools

For data collection, I used an online survey, a follow-up interview protocol, an observation for video-recording, and a stimulated recall interview protocol.

Online survey.

I used the online survey used in Pak (2017). The survey consisted of four scenario items (Appendix A) and was delivered online. The purpose of the survey was to explore how teachers would intervene and for what intervention purposes.

Each item was linked to one of the four intervention approaches of teachers to small groups mentioned in the literature review: (1) evaluating students' work (Chiu, 2004); (2) mediating students' mathematical thinking (Gillies & Boyle, 2006); (3) progress of group

interaction process (Dekker & Elshout-Mohr, 2004); and (4) treating status-issue (Cohen & Lotan, 2014). Each item featured a hypothetical scenario describing interactions among students in a small group who potentially needed the teacher to intervene.

Follow-up interview protocol.

The follow-up interview protocol was semi-structured (Appendix B). The purpose of the interview was to understand some of the teachers' responses in a more detailed way since the responses of the participants to the online survey were limited because of the short length of responses to the survey. The interview protocol consisted of three portions. The first portion was to make more sense of their responses to the four scenarios. The second portion was to explore how participants thought about small groups in mathematics teaching (e.g., benefits, challenges, expectations, or reasons for using small groups) and their experiences in relation to small groups in their personal and professional lives. The third portion was to understand ways teachers intervened in small groups using seven questions with brief hypothetical situations (e.g., What would you do when no student in a small group can answer a question they need to solve? or What would you do when group members treat one another with authority and no true dialogues exist?). I expected this portion to inform me of certain elements of figured worlds, such as power, culture, or artifacts, in which teachers participated and teacher identity (Holland et al., 1998). I conducted this one-time interview in approximately 60 minutes per participant. The interview data were audio-recorded and fully transcribed for analysis.

Video-recordings of mathematics teaching.

I video-recorded the full length of three different mathematics lessons for each teacher. The lessons lasted about 51 minutes on average and ranged from 41 minutes to 60 minutes. The number of small groups was four groups of four to five students in Leslie's lessons and eight

groups of two or three students in Marva's lessons. I located the video-camera in a place where I could video-record the whole classroom at once. For video-recordings, I asked the teachers to wear a microphone to record clearly what teachers and students said as they interacted with one another. The video-camera followed the teachers zooming in to capture body movement or facial expressions of teachers and students in small groups when teachers' intervention took place. All students gave permission for this study.

Stimulated recall interview protocol.

For follow-up interviews in relation to their mathematics instruction, I conducted stimulated recall interviews. For stimulated recall interviews, I identified specific episodes with potentially productive intervention approaches in video-recordings of mathematics lessons. These approaches included asking students to explain their mathematical thinking, asking students to evaluate other students' work, mediating students' thinking, and encouraging students to work together. The stimulated recall interview protocol was semi-structured (Appendix C). It included in the moment questions for how beginning teachers intervened in small groups, and questions about how and what they noticed. I also asked the participants about possible learning sources for specific interventions. I conducted each interview in about 60 minutes in the afternoon on the same day I observed the lesson. The interview data were audio-recorded and fully transcribed for analysis.

In both the follow-up interview protocol and the stimulated recall interview protocol, there were some questions that prompted beginning teachers to tell stories in relation to intervention in small groups. As I noted above, teacher identity could hardly be separate from certain contexts in which it was and is formed. I asked beginning teachers to tell their stories, like Aguirre and colleagues (2013), Drake and colleagues (2001), and Spillane (2000), because

one way to understand teacher identity might be through stories within certain contexts. By stories, I did not mean structured style of personal life stories, but personal and professional stories in relation to beginning teachers' experiences in certain practices and activities, such as small group work. Table 4.1 shows an overview of data collection and its relation to the research questions.

Research questions	Data collection tools	
(1) How do beginning teachers intervene in small groups in mathematics classrooms and for what purposes?	 Online survey Follow-up interview protocol Video-recordings Stimulated recall interview protocols 	
(2) How is beginning teachers' noticing before intervening in small groups related to their intervention in small groups in mathematics classrooms?	 Online survey Follow-up interview protocol Video-recordings Stimulated recall interview protocols 	
(3) How are beginning teachers' teacher identities related to how they intervene in small groups in mathematics classrooms?	Follow-up interview protocolsStimulated recall interview protocols	
(4) How do beginning teachers' perceptions of students shape their intervention in small groups in mathematics classrooms?	 Follow-up interview protocol Video-recordings Stimulated recall interview protocols 	

|--|

Data Analysis

In this section, I describe how I analyzed the data sources in three different ways that led me to the three findings, which address the four research questions. Each analysis is described in a separate section below because each had a distinct analytic approach.

First, I address the first and second research questions, which focus on the relationships

between professional noticing and intervention in small groups. I present how I developed

codebooks and used them to identify certain relationships between professional noticing and intervention. Building on this analysis, I propose a noticing-mediated framework that contributes to understanding individual teachers' decision-making process.

Second, I address the third research question, which is related to the relationships between teachers' identities and their intervention in small groups. Through a closer examination of the interview transcripts, I reconstructed how both teachers invoked their identities when explaining their intervention in small groups. Particularly, I highlight similarities and differences in the ways the two teachers invoked aspects of both their current teacher identities and their designated teacher identities.

Third, I address the fourth research question, which focuses on identifying the relationships between teachers' perceptions of certain categories of students and power and authority dynamics in figured worlds of mathematics classrooms. I present some patterns I identified from teachers' explanations of their recognition of categories of students in combination with power and authority dynamics.

Analyzing relationships between professional noticing and intervention in small groups

I examined the relationships between teacher noticing and intervention using a thematic analysis (Glesne, 2011) in which I sorted and categorized the interview excerpts, identified certain themes and compared themes to find patterns. I explain the analytic process in four phases.

Identifying intervention episodes to analyze

In this first phase, I identified 46 intervention episodes as the unit of analysis. Each intervention episode referred to a specific portion of the interview related to the teachers' talk about a specific intervention in a small group. Particularly, in the stimulated recall transcripts,

the episode began with having the teachers watch their own intervention. It also included the teachers' explanation, elaboration on, and reasoning about what they noticed, what they thought went on in the small group, where they learned the intervention, and how they viewed themselves as teachers when intervening in the small group.

Development of codebooks

In this second phase, to develop the current codebooks in this dissertation study, I went through three steps- 1) identifying initial codes by reading half of the interview episodes, 2) conducting an interrater reliability process, and 3) applying refined codes to all intervention episodes. I began from codebooks I had developed working with prospective teachers in prior studies (Pak, 2017; 2018). But they had two differences. First, beginning teachers in this dissertation study had more authentic classroom experience than the prospective teachers in prior studies. Second, I collected additional data from observations and interviews besides data from the open-ended survey. The codebooks in prior studies were based solely on responses to the open-ended survey. For these reasons, I remained open to new codes emerging from analyses of these data sources.

Identifying initial codes by reading half of interview transcripts.

The previous codebooks included the four categories of attention, decisions, responses, and interpretation. The definitions of all categories remained the same in these current codebooks, except for interpretation, as discussed below. In both codebooks, attention meant events to which teachers attend, such as certain students' dominance of group work or off-task behaviors or bodily expression. Decision meant what their intervention aims for, such that the teacher tells students to work together for the purpose of helping the group work process go

smoothly. Responses meant types of verbal actions teachers make when intervening in a small group, such as asking students to explain their mathematical thinking.

To identify codes of attention, I began with the previous codes of attention, such as students' mathematical thinking, interaction between students, power dynamics, and status of students. I integrated two of the previous codes (students' mathematical thinking, interaction between students) into the current codes of attention (dominance of group work, group work progress, and approaches to mathematical tasks). I obtained new codes (e.g., students' voice level, facial expression, or body posture) that emerged from the initial readings of the intervention episodes. One reason for different codes between the current and previous codes of attention was related to the events to which teachers were asked to attend. In the prior studies, prospective teachers were asked to attend to short hypothetical scenarios, which might make them attend in general ways (e.g., interaction between students). In this dissertation study, Leslie and Marva watched their own teaching video footage of specific events in small groups in which they interacted with specific students they had known in their own classrooms. It seemed that this difference made a distinction between the current codes of attention and those of attention in the previous codebooks.

To identify codes of decisions, I began with the previous codes of decisions. These previous codes included mediating students' mathematical thinking, making sure students work together, and acknowledging students' potential contribution. In my current analysis, I identified two of these codes, including mediating students' mathematical thinking and making sure students work together. There were also new codes (e.g., providing language support and increasing on-task behaviors) that were very specific to classroom contexts on the part of both teachers and sometimes only Marva.

To identify codes of responses, I began with the previous codes of responses that included extending students' mathematical thinking; eliciting students' mathematical thinking; acknowledging students' potential contribution; encouraging students to work together; or providing content help. Each code in the previous codebooks had subcodes. In my current analysis, I identified some of the sub-codes of these codebooks. These codes included asking students to explain their thinking, asking students metacognitive questions, or asking students to comment on peers' strategies. I identified new codes such as explaining to English Language Learners how to use language correctly. These new codes were specific to both teachers' contexts and sometimes only to Marva.

To identify codes of interpretation, I had to create new codes that emerged from the intervention episodes. This was because these previous codes of interpretation did not tell much about teachers' decision-making process through what they notice, which is one of the interests in this dissertation study. In the previous codebooks, the category of interpretation was prospective teachers' reasonings about interventions they reported they would do. I coded these reasonings in terms of two codes: General and Specific. Instead of these codes, in this dissertation study, I defined teachers' interpretation as connections teachers might make between attention, decision, and responses, and different kinds of resources recognized by both teachers. In my current analysis, I identified new codes of interpretation. These current codes of interpretation were related to knowledge, group norms, teachers' roles, and learning experiences, which are detailed later.

Conducting interrater reliability test.

To measure the interrater reliability, which dealt with the reliability of my coding process using the current codebooks, I asked a graduate student with coding experience to double code

responses. I pulled 20 percent of excerpts from each category. I made sure these excerpts were included equally in the number of each category (Table 4.2). Before coding the data, I asked the coder to read the codebooks of categories and codes with me. We then coded independently and compared each other's coding. As shown in Table 2, the agreement rate in the step was 85 percent for attention, 83 percent for decisions, 86 percent for responses, and 84.4 percent for interpretation.

We disagreed on whether some excerpts were an example of specific codes. For example, in the category of attention, we disagreed on some excerpts regarding whether they were *(group work) approaches to mathematical tasks* or *(Learning materials) playing with learning materials*. After discussing our reasonings behind our coding decision, I changed the descriptions of the former code so that it did not include the excerpts related to learning materials.

In the category of decisions, we disagreed on some excerpts regarding whether they were *managing students' behaviors*. We resolved the disagreement by integrating the first code into *increasing on-task behaviors*.

In the category of responses, we disagreed on some excerpts regarding whether they were *encouraging students to work together* or *providing content help to students*. We resolved the disagreement by changing the descriptions of the second code so that it did not include the excerpts related to the first code.

In the category of interpretation, we disagreed on some excerpts regarding whether they were (general) knowledge of ways to have students work together or (teachers' roles) fostering students' capacity to work together with peers or (Learning experiences) facilitating small group work. After discussing our reasonings behind our coding decision, we resolved the disagreement

by changing the descriptions of codes so that each code shows its emphasis on knowledge or teachers' roles or learning experiences.

Section	Attention	Decisions of how to respond	Responses	Interpretation
The numbers of whole excerpts	47	81	68	125
The numbers of excerpts used for interrater reliability	10	16	14	25
Agreement rate	85%	83%	86%	84.4%

Table 4.2 Numbers of responses for the interrater reliability test

Applying the refined codes to the whole interview transcripts.

After refining the descriptions of codes, I applied the refined codes to all of the intervention episodes. I engaged in three rounds of reading of the intervention episodes to analyze them in three ways- 1) finding new codes, 2) condensing some codes into one code, 3) expanding initial codes to include additional instances. These ways of coding happened simultaneously in the three rounds of readings.

First, I looked for new emerging codes. But I did not find new codes from reading the intervention episodes. Second, I looked for codes that needed condensing into one code. As a result, I found several codes that needed to condense into one code. For example, the initial codes of decisions of how to respond had *increasing on-task behaviors* and *directing students to group work*. As I coded more interview transcripts, I found that both codes indicated teachers' aims at increasing students' on-task behaviors. So, I integrated the latter code into the former

code *increasing on-task behaviors*. For another example, the initial codes of interpretation had *(teacher roles) being culturally responsible for students' needs* and *(teacher roles) being flexible with students' needs*. I viewed the phenomenon in the first code as a specific example of the second code. I integrated the first code into the second code.

Third, I looked for codes that needed to expand their descriptions. For example, one of the initial codes of attention was "showing little group work." As I coded more interview transcripts, I found more excerpts that made me see the need to alter the code into "group work progress." The initial code of "showing little group work" could include only one aspect of group work progress small groups have made on their own. However, the code of "group work progress" could include all types of progress that small groups might make, including the phenomena related to little progress of group work.

Establishing the current codebooks

In this third phase, as a result of these three steps above, I developed the current codebooks (Appendix D). I acknowledge that these codebooks are subject to modification by inclusion of new codes or removal of some codes in the future, because these current codebooks are an outcome from the knowledge and beliefs I held around the time of analyzing the data. They are also limited by the data, which means that if I collected more data from other teachers in other contexts, other codes might emerge. Keeping this in mind, I introduce the current version of codebooks that I used to further the analysis.

Codes of attention.

The attention category consists of codes related to 1) group work, 2) bodily expression, 3) learning materials, and 4) language learners (Table 4.3). Each code has its own subcodes. I explain each code below in more detail.

Table 4.3 Codes of attention

Codes	Group work	Bodily expression	Learning materials	Language learners
Sub codes	 Dominance of group work Group work progress Approaches to mathematical tasks 	 Students' body posture Students' voice level Students' facial expression 	 Mathematically improper use of manipulative Playing with learning materials 	 English Language Learners' language use

(Group work) dominance of group work. This code was used to code excerpts in which teachers mentioned they saw some students take over group work. For example, "My one student just kind of took it and ran with it. Kind of took the whole group" (T1A2E2) and "Ruby was writing, doing all the work and trying to talk to them" (T2A3E2). I used this code to code excerpts like these ones when they clearly suggested that teachers attended to some students dominating the group work.

(Group work) group work progress. This code was used to code excerpts in which teachers talked about group work progress students had made collectively in the moment of intervening in the small group. For example, Leslie said, "They just weren't doing what they were supposed to do" (T1A1E4). Marva said, "They didn't have a whole lot written down so I wasn't really sure what was getting done" (T2A3E3). I used this code to code these excerpts because they showed clear instances of teachers' attention to the lack of progress the students made in the moment of intervention.

(Group work) approaches to mathematical tasks. This code was used to code excerpts in which teachers said that they paid attention to mathematical approaches students were using to solve the group task. This code was very often used to code approaches made by the whole group

members. For example, Leslie said, "I looked at their numbers and what they had. And they kind of had one written like off to the side" (T1A1E1). Marva said, "In the small group, the students are not understanding that both sides need to have the same value." (T2H1E1). These excerpts show that both teachers talked about their attention to certain mathematical ways the students had to solve the group work. This code was also used to code excerpts with individual students' approaches to the group work. For example, Leslie said, "I saw that one problem on his paper that said 8 + 3 = 7" (T2A1E2). In this excerpt, she talked about an individual student's approach to the problem, which was incorrect. I used this code to code excerpts like these ones when they clearly suggested that teachers attended to certain mathematical approaches to group tasks.

(Bodily expression) students' voice level. This code was used to code excerpts in which teachers talked about their attention to students' voice level (e.g., loud or quiet). For example, Leslie said, "I heard a lot of bickering and I kinda heard, 'you're not listening, that's not what I said, well that's wrong" (T1A1E2). Marva said, "They're both quiet. Well, Ben is a little bit louder. They're both very quiet still" (T2A1E4). These excerpts clearly show that both teachers attended to the degree of students' voices in terms of whether they were loud or quiet.

(Bodily expression) students' body posture. This code was used to code excerpts in which teachers talked about their attention to students' body posture (e.g., sit back or heads down). For example, Leslie said, "Malek was the one the kind of sitting back in his seat, un-engaged, looking at his pencil or looking at his hands or something" (T1A2E4). Marva said, "The one little boy that was sitting there, he had his head down" (T2A3E2). These excerpts show that both teachers saw some students sitting in a right body posture. Some students were sitting back. Some had their head down. I used this code to code excerpts like these ones that clearly showed their attention to students' body posture.

(Bodily expression) students' facial expression. This code was used to code excerpts in which teachers talked about their attention to students' facial expression. In my analysis, I used this code only one time. It was when Leslie said, "The look on Yasemin's face kind of said it all to me because I can tell when she's just - when she doesn't get it and she's upset and she's frustrated and so just being able to read her face I think kind of said the majority of it" (T1A2E1). Even though there was only one instance to use this code, this code remained in the current codebooks because students' facial expressions tended to be noticed easily by teachers (Wells, 2017).

(Learning materials) mathematically improper use of manipulative. This code was used to code excerpts in which teachers mentioned that they attended to *mathematically* inappropriate ways students used manipulatives. This code was specific to Marva because only Marva implemented the measurement-related lesson where students used manipulatives to measure several items. For example, Marva said, "they were using it but they were using it in almost a messy way" (T2A2E3). She also said, "They're not measuring fully because they're leaving too many gaps" (T2A2E4). These excerpts show that she saw some students improperly using nonstandard measuring tools. I used this code to code excerpts like these ones that clearly showed their attention to improper ways students used manipulative.

(Learning materials) playing with learning materials. This code was used to code excerpts in which teachers mentioned that they attended to inappropriate ways students used mathematical manipulatives. This code differs from the code *improper use of mathematical manipulative* in that it was used to code excerpts where students used materials in a playful manner. This code was specific to Marva because only Marva implemented the lesson where students used manipulatives or learning materials (e.g., blocks or counters). For example, Marva

said, "Not many of the groups had been working with the blocks the way they were. The other groups that were working with blocks, it was more so in a playful manner" (T2A2E2). She also said, "I had other kids that all of the sudden started playing" (T2A2E1). These excerpts show that she saw some students playing with learning materials. I used this code to code excerpts like these that clearly showed their attention to some students using learning materials as toys.

English Language Learners' language use. This code was used to code excerpts in which teachers mentioned that they attended to how English Language Learners (ELLs) used English in writing and in speech. This code was specific to Marva whose students were dominantly ELLs. For example, Marva said, "I looked at the paper and saw it. I saw that she had two one instead of one, two" (T2A1E3). She attended to the order of the numbers because it looked like 12 when it was supposed to be 21. In this excerpt, the girl was an English Language Learner from an Arabic country where she knew the numbers were "always flipped." I coded this excerpt with this code because she attended to how the girl wrote a two-digit number in a flipped way. I used this code to code excerpts like these that clearly showed their attention to ELLs' language use.

Codes of decisions of how to respond.

As a reminder, this code, decisions of how to respond, implies certain purposes/intentions that are related to certain ways for teachers to respond. I applied this category to the excerpts where teachers explicitly talked about their purposes/intentions. The decision category consists of codes related to 1) making sure students work together, 2) increasing on-task behaviors, 3) mediating students' mathematical thinking, 4) understanding group work progress, and 5) providing language support.

Making sure students work together. This code was used to code excerpts where teachers talked about their intentions to help students work together with peers in small groups. For

example, Leslie said, "So I wanted to make sure, as a group, that they were as focused as they could be and they were working on it" (T1A2E3). In another interview, she said, "I have to get her back on track because otherwise the whole group is just - I mean those two girls aren't gonna talk the boys would be the only ones talking" (T1A2E1). Marva said, "I had to find a way to get the group to recognize, hey, you're responsible for bringing him in, too" (T2A1E1). These excerpts were given this code because they showed clear intentions related to making sure students work together when they said, "I wanted to make sure, as a group, that they were as focused as they could be" or "I had to find a way to get the group to recognize, hey, you're responsible for bringing him in."

Increasing on-task behaviors. This code was used to code excerpts where teachers talked about their intentions to increase students' on-task behaviors. For example, Leslie said, "And so I walked over there and I said, 'Okay. Well, what are you guys doing?' I wanted to make sure that he was on task" (T1A1E4). Marva said, "I had to stop him from the tipping because it's safety thing and I had to get him to see that, 'Hey, this isn't a choice, you have to work."" (T2A1E1). These excerpts were given this code because they showed clear intentions to increase these students' on-task behaviors. In the excerpts above, Leslie expressed her intention to make sure the boy was on-task. Marva wanted to stop the boy from tipping his chair and she wanted him to pay attention to the group work.

Mediating students' mathematical thinking. This code was used to code excerpts where teachers talked about their intentions to promote the mathematical thinking process of individual students as well as small groups. For example, Leslie said, "This way the students could physically see the difference between the two equations" (T1H1E1). In another intervention, she said, "They [the students in a small group] would stop what they were doing and really think

about, 'Okay, why am I saying these numbers?' Not just, 'Okay, what numbers can I throw around in the air right now?'" (T1A2E3). Marva said, "I wanted him to also see his mistake too because I could already kind of tell based on his body language that he wasn't going to get that he had a mistake" (T2A1E2). These excerpts were given this code because they showed certain intentions related to promoting students' thinking process ("the students could physically see the difference" or "They would stop what they were doing and really think" or "I wanted him to also see his mistake too").

Understanding group work progress. This code was used to code excerpts where teachers talked about their intentions to figure out progress students in small groups made in relation to problem solving or communication processes. Excerpts that received this code included teachers' sense of uncertainty related to progress made by individual students or small groups. For example, Leslie said, "I had to figure out the reason why the group wasn't communicating effectively; was because she didn't feel like her voice was heard or she just didn't like the group or what was going on" (T1A1E2). Marva said, "I stopped to figure out what was going on because of that answer that was so blatantly wrong" (T2A1E1). I gave this code to these excerpts because they showed clear intentions to get a better sense of where students were at in either communication or problem solving. Leslie wanted to figure out a reason why the group did not communicate well. Marva looked for what was going on in the small group.

Providing language support. This code was used to code excerpts where teachers talked about their intentions to support students' language needs. This code was specific to Marva whose students were predominantly ELLs. There were a few intervention episodes where this code was used. For example, Marva said in an intervention episode (T2A3E4):
I was hoping that she would recognize a problem and in the future start or at least for today start going electric. So I figured she'll probably need reinforcement later on just because it's-- I mean, she's learning the language and that's part of the language's structure. So just at least start-- she'll be able to start recognizing that she's flipping it. She'll be able to monitor herself.

This excerpt suggests that Marva recognized the girl wrote the two-digit number flipped and she had intentions that are related 1) to helping the girl recognize her problem and 2) to be able to monitor herself in the future. I gave this code to the excerpt because it showed clear intentions related to language support.

Codes of responses.

The response category consists of nine codes. These codes are: 1) encouraging students to work together, 2) providing content help to students, 3) asking students to explain their thinking, 4) asking students to provide reasons, 5) evaluating other students' ideas, 6) asking students metacognitive questions, 7) asking students to comment on peers' strategies, 8) explaining to English Language Learners how to use language correctly, and 9) telling students not to play with manipulatives.

Encouraging students to work together. This code was used to code excerpts where teachers motivated students to work together with others in small groups. For example, Leslie said, "What are you guys-- are you guys working together?" (T1H1E7) or "You need to be working as a group. We need to be moving on from this" (T1A3E2). Marva said, "Oh he's helping you count them. Okay that's good. So you guys are working together to count them out" (T2A1E4). I gave this code to these excerpts because they clearly showed that both teachers asked students to work together.

Providing content help to students. This code was used to code excerpts where teachers helped students make sense of mathematical tasks the students worked on. In excerpts that received this code, teachers asked students questions that required yes/no or simple answers to draw attention to important ideas. Leslie asked, "So you're taking just the dollars and adding them up?" (T1A1E1). Marva also asked, "Is seven bigger or smaller than eight?" (T2A1E2). In excerpts that received this code, teachers also gave students essential information helpful to solve problems. For example, Leslie asked, "So you said eighteen plus thirty-six, right? You said nine plus nine is eighteen, and nine plus nine plus nine plus nine is thirty-six. Eighteen plus thirty-six, eight plus six is? There you go. So, make sure you double-check your work okay?" (T1A1E3). Marva also provided some information that might be helpful for students. "Eight is bigger. So how is your answer seven? I think you need to try and draw it out and see if you can figure it out." (T2A1E2). I gave this code to these excerpts because she helped students see important information to solve the problems correctly.

Asking students to explain their thinking. This code was used to code excerpts where teachers asked students to explain what they did and how they solved the group task. For example, Leslie said, "Mary, explain to me what the group's doing" (T1A1E2) or "Come over here and explain it to the group" (T1A3E2). Marva said, "Well, see I had another group measure this way and get seven. Please show me how you got seven? Show me" (T2A2E4). I gave this code to these excerpts because they asked the students to explain what they did ("explain it to the group" or "show me").

Asking students to provide reasons. This code was used to code excerpts where teachers posed questions that asked why they solved and thought in certain ways. They also asked the reasons typically after confirming what students said. For example, Leslie said, "Why did you

bring the five up there" (T1A1E3). In this excerpt, she asked the students to reason about their answer. Marva said, "So why do you go from twenty to-- from forty to fifty? So twenty--" (T2A2E2). In this excerpt, she asked the student to explain their reason behind their solution.

Evaluating other students' ideas. This code was used to code excerpts where teachers asked the students to evaluate their peers' ideas. Both teachers asked the students if they agreed with the ideas. For example, Leslie said, "So then write down your answer if your whole group agrees. Okay? What does your group say?" (T1A2E2). In this excerpt, she asked the students if the whole group agreed to the answer. Marva said, "I started asking about what they noticed and specifically Nadia, the Bengali girl in the group, was the one who told me the most but I clarified and asked if they all agreed" (T2A3E3). In this excerpt, she asked the students "if they all agreed."

Asking metacognitive questions. This code was used to code excerpts where teachers asked students questions that helped them think about what and how they were doing in relation to problem solving. For example, Leslie said, "Okay. Well, why are you thinking this? Who has an idea about where we go next? Do you guys think this is the right idea?" (T1H1E9). Marva said, "Do you know, or no? Do you know what you're supposed to be doing?" (T2A3E1). I gave this code to these excerpts because they showed both teachers asked the students metacognitive questions that help the students assess their work.

Asking students to comment on peers' strategies. This code was used to code excerpts where teachers asked the students to make comments on their peers' ideas. For example, Leslie said, "Finally ask both partners what they thought about Robin's [a student in the third hypothetical scenario]" (T1H1E3). In this excerpt, she asked the students to comment on Robin's ideas. Marva said, "How about-- can you come over and look at his measurement? Do you like

his measurements? Go-- you use it way too much, though. What do you li-- do you like his measurement?" (T2A2E3). In this excerpt, she asked the students to comment on a way a student measured.

Explaining to English Language Learners how to use language correctly. This code was used to code excerpts where Marva helped ELLs use written and spoken English correctly. A few excerpts received this code. For example, Marva said, "You keep flipping your numbers, Honey. I know in Arabic you go right to left. But remember, when we're writing in English, which way do we write from? We start on this side and go that way" (T2A1E3). This excerpt was given this code because it showed she tried to explain how to write the two-digit number in a U.S. context. When she said, "Do you know what accurate means? It means exact" (T2A2E2), I gave this code to this excerpt because it showed she explained the meaning of "accurate" to an ELL.

Telling students not to play with mathematical manipulatives. This code was used to code excerpts where Marva told the students not to use manipulatives in a playful manner. This code was specific to Marva because only Marva implemented the lesson where students used manipulative or learning materials (e.g., blocks or counters). There were a few excerpts that received this code. For example, Marva said, "I told him that we're not playing with them because they're tools" (T2A3E4). I gave this code to this excerpt because it showed she asked the boy to use counters as tools to learn mathematics.

Codes of interpretation.

As a reminder, interpretation in this dissertation study is defined as connections teachers might make between attention, decision, and responses, and different kinds of resources recognized by both teachers. This category of interpretation consists of codes related to 1)

knowledge, 2) group norms, 3) teacher roles, and 4) learning experiences. Codes related to this category sometimes involve in teachers' judgements and assumptions. Particularly, knowledge-related codes involve judgments and assumptions both teachers had towards students based on their observations. Table 4.4 shows each code and its subcodes. I explain each code below in more detail.

Codes		Subcodes
Knowledge	Specific knowledge	 Students' mathematical proficiency Students' engagement Students' at-home environment Students' language
	General knowledge	 Importance of creating supportive environment Students' anxiety of being wrong Ways to have students work together Students' different learning styles
Group norms		Working with peersUsing manipulative in a mathematically proper way
Teacher roles		 Fostering students' capacity to work with peers Motivating students to think for themselves Being flexible with students' needs
Learning experiences		 Understanding of students' learning Students' anxiety Facilitating small group work Redirecting students

Table 4.4 Codes of interpretation

(Knowledge) specific knowledge of students' mathematical proficiency. This code was used to code excerpts where teachers talked about what they know about specific students in terms of their mathematical proficiency. This mathematical proficiency indicated how high or low students' academic achievement was, what students could (not) do in mathematics or

whether students understand mathematical concepts well or not. For example, I used this code when Leslie said, "So it's normally the students that are a little lower" (T1A1E1) and "it seems like he especially had an understanding of how place value works." (T1HE2). These excerpts showed that Leslie drew on knowledge of those specific students in terms of their academic achievement (e.g., higher or lower) or the levels of their understanding of certain mathematical concepts (e.g., place value). I also used this code when Marva said, "the little boy that he's working with is academically lower" and "he's already beginning multiplying" (T2A1E4), which indicated that she had a specific knowledge of some students' mathematical proficiency in relation to their academic achievement (e.g., lower) or the levels of their understanding of certain mathematical mathematical concepts (e.g., already beginning multiplying).

(Knowledge) specific knowledge of students' engagement. This code was used to code excerpts where teachers talked about what they knew about specific students in terms of their engagement pattern. This engagement pattern indicated ways students behaved when they participated in small groups. For example, I used this code when Leslie said, "That particular student has a habit of kind of singling himself out" (T1A2E4) because this excerpt showed she used knowledge of how the specific student participated in small groups (e.g., a habit of kind of singling himself out). I also used this code when Marva said, "those two have a tendency just to do their own thing instead of working together" (T2A2E1). The reason for using this code was because this excerpt showed she had a specific knowledge of those two students" "tendency just to do their own things."

(Knowledge) specific knowledge of students' at-home environment. This code was used to code excerpts where teachers talked about what they knew about specific students in terms of their home environment, particularly parent support. This at-home environment indicated how

much support students received from their parents in relation to doing homework at home. This code was specific to Marva. For example, I used this code when Marva said, "A lot of times it's the Bengali parents that are coming to check in on the student's progress. They're the ones who you can see the parents are working with them at home" (T2A1E4). The reason for using this code was because this excerpt showed she had a specific knowledge of parental support the student received.

(Knowledge) specific knowledge of students' language. This code was used to code excerpts where teachers talked about what they knew about specific students in terms of their language. This code was specific to Marva whose students were predominantly ELLs. For example, I used this code when Marva said, "Yeah, her language skills are still developing." (T2A2E2) and "And the little boy, well, he's a native English speaker. He almost might as well be an ESL student" (T2A3E2). The reason for using this code was because this excerpt showed she had a specific knowledge of those students' language development levels.

(Knowledge) general knowledge of the importance of creating a supportive environment. This code was used to code excerpts where teachers knew that they should create a supportive environment in their own classrooms. This code was specific to Marva. For example, I used this code when Marva said, "Just the whole idea that you don't want this storm cloud over your classroom. You want to create this positive environment and just a positive, pleasant culture where you want to encourage each other [to learn from each other]." (T2A2E3). This excerpt shows that she emphasized how important she had to create a supportive environment by bringing in "positivity" and "pleasant culture."

(Knowledge) general knowledge of students' anxiety of being wrong. This code was used to code excerpts where teachers knew that some students were anxious about making mistakes or

being wrong. This code was specific to Leslie. It also appeared only in hypothetical intervention episodes. For example, I used this code when Leslie said, "From my personal experiences, kids are afraid to be wrong because they don't want to be made fun of or seen as less smart as others" (T1H1E3). This excerpt shows that she knew that some students were afraid to be wrong in working together with others.

(Knowledge) general knowledge of ways to have students work together. This code was used to code excerpts where teachers knew ways to support students to be able to work together. For example, I used this code when Leslie said, "So by asking her and then asking another student and then asking her to repeat it and asking different strategies I kinda -- by being there I kinda got them to open up and all communicate" (T1A1E2). I used this code for this excerpt because she has known of ways to facilitate interactions among students ("asking her and then asking another student and then asking her to repeat it"). I also used this code when Marva said, "A lot of teachers use the Ask Three Before Me. So it's like ask three other students before you come to me" (T2A3E1). Her way, "the Ask Three Before Me," could be her own way to have students work together before they asked her.

(Knowledge) general knowledge of students' different learning styles. This code was used to code excerpts where teachers knew that students had different learning styles. There were several learning styles recognized by both teachers. For example, I used this code when Leslie said, "I'm a very visual learner. I know some kids are auditory, some kids are visual, some kids are kinesthetic. So I know kids have to learn different ways" (T1H1E1). I also used this code when Marva said, "sometimes maybe orally the students might not understand what a student's saying, but when they put it down on paper and they can see the reasoning and the process

behind what another student did, then it makes more sense to them" (T2H1E2). These excerpts show that both teachers acknowledged different learning styles between students.

(Group norms) working with peers. This category of group norms involved expectations both teachers had about students in small groups. This specific code was used to code excerpts where teachers talked about their expectation or anticipation towards students, that is, expecting students to work with peers in small groups. For example, I used this code when Leslie said, "What I was really listening for was are they giving an explanation and does everyone agree?" (T1A2E2). I also used this code when Marva said, "If they're working in a group, they should be working together and not just--and helping each other, not just sitting there and doing it on their own" (T2A2E1). These excerpts show certain norms in which both teachers expected students to work with their peers ("are they giving an explanation" or "helping each other, not just sitting there").

(Group norms) using manipulative in a mathematically proper way. This code was used to code excerpts where teachers talked about their expectation or anticipation towards students, that is, expecting students to use mathematical manipulative in a mathematically proper way. This code was specific to Marva and used in a few intervention episodes related to lessons where mathematics tools were used. For example, I used this code when Marva said, "It's actually exciting that she's taking that [the idea of tools not toys] on and helping kind of put it into the classroom and making it more of a norm for the class. It helps having other students out there in the classroom who might still be using tools as toys" (T2A3E4). This excerpt shows certain norms in which she expected students to use manipulative as a mathematical tool.

(Teachers' roles) fostering students' capacity to work with peers. This category of teachers' roles involved expectations about themselves as teachers. This code was used to code

excerpts where teachers talked about their views of teachers' roles as facilitating students to develop their ability to work with others. For example, I used this code when Leslie said, "I didn't want to lead the kids right to the answer. I wanted them to kind of fool around with it and give them a chance to communicate and discuss their own ideas [with others in small groups]" (T1A1E4). I also used this code when Marva said, "I like having students work cooperatively and be able to learn from each other. And if they're not working together, they're not going to learn from each other" (T2A2E1). I gave this code to these excerpts because they showed their views of teachers' roles ("give them a chance to communicate and discuss their own ideas" and "having students work cooperatively and be able to learn from each other" (Signe them a chance to communicate and discuss their own ideas" and "having students work cooperatively and be able to learn from each other").

(Teachers' roles) motivating students to think for themselves. This code was used to code excerpts where teachers talked mostly about their views of teachers' roles as encouraging students to think for themselves without depending on teachers. This differs from the code *fostering students' capacity to work together with peers* in that it highlights students being mathematical thinkers. This code was specific to Marva. For example, I used this code when Marva said, "I'm really trying to get them to use different, just think about things and try to make sense of them on their own. Even if it might not be right but they're making sense of it. It makes sense to them" (T2A1E2). I gave this code to these excerpts because they showed Marva's view of teachers' roles in helping students to be independent thinkers ("try to make sense of them on their own").

(Teachers' roles) being flexible with students' needs. This code was used to code excerpts where teachers talked mostly about their views of teachers' roles as being as flexible as possible with students' needs. This code was specific to Marva. For example, I used this code when Marva said, "it's just me being flexible and so hopefully just trying to be flexible. It's just me

finding, and knowing the students, and what's happening and just trying and going with the flow and trying to figure out what's happening" (T2A2E1). I gave this code to this excerpt because it showed her view of teachers' roles as a teacher being flexible ("It's just me finding, and knowing the students, and what's happening and just trying and going with the flow").

(Learning experiences) Understanding of students' learning. This code was used to code excerpts where teachers talked about specific learning experiences about the importance of making sure that they understand what students are learning or understanding. This code was specific to Leslie. For example, I used this code to code an excerpt below that Leslie said about her learning from a mathematics methods course (T1A2E3).

I think it was senior year of college and there was a mathematics course we had to take and it was all about the number talk and making sure that you understand what the kids are saying and they're communicating effectively to you. So that whole class, I mean every single day, we had a new number talk where a new group would present and they would have misconceptions on purpose and they would have to clarify them. And so I think that class was a huge help in really trying to understand what the kids are actually saying, what are they actually understanding? where do they need to go next so that they fully understand the concept?

This excerpt showed that she learned about the importance of understanding students' learning in her teacher preparation program ("that class was a huge help in really trying to understand what the kids are actually saying, what are they actually understanding?"). Even though the context was implementing number talk with the whole class, she seemed to take this learning to the small

group context. I used this code to code excerpts like this one in which the teacher talked about learning experiences related to understanding students' learning in different contexts.

(Learning experiences) students' anxiety. This code was used to code several excerpts where teachers talked about specific learning experiences about what it looked like to work with students with anxiety. This code was applied more to Leslie than Marva. For example, I used this code to code an excerpt below that Leslie said about her learning from a professional development and her own experience (T1A2E1).

Last year actually, I chose to go to one that was all about the psychology of students when they learn. So they were breaking down this is what's happening in their brain. Their anxiety, their fears flaming up which actually can cause their brain to just kind of to a point shut down. And so I had a student two years ago that anytime I called on him, he would just get angry and he would just lash out and it didn't matter. I would prep him beforehand and say I'm going to call on you for number four, you have the right answer. I just need you to read it. And he just could not do it and I didn't know what was going on, I thought maybe he's just being defiant, maybe he just doesn't want to participate. I could not figure it out until they said that and then that just kind of clicked for me like oh, this is what's going on in his brain. And they kind of gave you tools to help him out which was nice, so just kind of learning from that experience.

This excerpt consisted of Leslie's recollection of her learning about anxious students. She talked about her learning in a professional development on the psychology of students in the mode of learning. She also detailed how she worked with an anxious student in her teaching. She

emphasized this learning experience as "tools" she could use to support a student to begin to overcome his anxiety.

(Learning experiences) facilitating small group work. This code was used to code multiple excerpts where teachers talked about their learning experiences about how to have students work with their peers in small groups. This code pays attention to how to facilitate small group work. This code was applied more to Leslie than Marva. For example, I used this code to code an excerpt below that Leslie said that she learned how to support students to work with others from her co-partner teacher in her school (T1A1E2).

I think the biggest one was my co-partner because she does small groups a lot and so I would tell her, "Oh this problem was so unsolvable and I didn't know what to do." And she'd say, "Oh, well, try to have them work with this, try to pull them aside, don't pull them out of the group, make sure that you're trying to keep them involved, engage, try to make a joke about it. Try to get them to say what they think. Try to get them to agree or disagree or explain why and get their thinking, their logic more vocal so that way you can kind of understand where they're coming from and what the root of the problem is so you can kind of get rid of it.

In this excerpt, Leslie told that the co-partner teacher emphasized how to support students in small groups to engage in the group work in many ways. What the teacher advised as shown in this excerpt was an important learning opportunity for Leslie ("the biggest one was my copartner").

(Learning experiences) redirecting students. This code was used to code excerpts where teachers talked about how to help students reengage in group work. I gave this code to excerpts

where teachers talked about learning experiences of redirecting students who were off-task or were struggling in solving the group work or in communicating effectively. This code differs from the code *facilitating small group work* in that it deals with issues that students in small groups might have in the moment of intervention. This code was applied to both teachers. For example, I used this code to code an excerpt below that Leslie said that she learned from her observation of a mentor teacher about how to get students back to their group work (T1A2E2).

Well I guess I saw that it works with the whole group when she kind of mentions the kid's name and whatnot, so I tried to use the same thing in a small group and it's easier to kind of insert a bunch of kids' names in a small group than it is to put everybody's name. At some point during the whole group discussion I had four students that were looking down playing with a pencil or something, and it's hard to say well, Alice, Jacob, Sally, and so then you're just saying their name. You're interrupting your teaching. Whereas in a small group, if I just had one student that was off-task I would say, "hey Spencer, by the way have you guys noticed this? Have you thought about that?" And you don't skip a beat, you just keep going. So it's easier to do in small groups and I think I saw her [Leslie's mentor teacher] use it and it works in whole groups so I kind of just applied it to my small groups as well.

In this excerpt, Leslie told a way her mentor teacher used to have students re-engage in their work ("she kind of mentions the kid's name"). Even though the teacher did so when working with the whole group, Leslie took it to intervening in small groups. When she saw an off-task student, she used this way of redirecting students. I used this code to code excerpts like this one

that both teachers talked about their learning experiences related to how to redirect students' group work.

Exploring the relationships among categories and codes

In the previous phase, I detailed the current codebooks. In this fourth phase, I examined categories and codes to understand how the categories and codes in the current codebooks could be related to one another. My purpose in doing this analysis was to find ways to understand teachers' decision-making process related to intervention in small groups. There were two ways to analyze the relationships- 1) putting categories and codes back into intervention episodes in a table, and 2) creating diagrams to visualize relationships among categories and codes.

Putting categories and codes into intervention episodes in a table.

Using a table, I integrated categories and codes into 38 individual intervention episodes. I used only 38 of 46 intervention episodes (24 actual intervention episodes and 14 hypothetical intervention episodes) in the table. I did not include eight hypothetical intervention episodes in my analysis. This was because they were missing codes in at least one of the four categories (attention, decisions, responses, and interpretation). For example, the fifth hypothetical intervention episode in the follow-up interview (T2H1E5) was not included because it did not have any code assigned in the category of both decisions and interpretation.

Following ways to help researchers make comparisons, suggested by Glesne (2011), I created a table where I put 38 intervention episodes and four categories (attention, decisions, responses, and interpretation). Creating the table helped me analyze the individual categories and codes in the codebooks to understand how these categories and codes were related to one another in the context of intervention in small groups in two ways.

Looking into the table invited me to begin to raise questions about how categories and codes were related to one another in relation to specific intervention episodes. The questions include, for example, how would Leslie's specific knowledge of students' engagement be related to her attention to group work progress? How would her attention to group work progress shape her response by providing content help to students in a small group? How would her view of teachers' roles (fostering students' capacity to work together with peers) be related to her decisions of how to respond (understanding problem solving progress)?

This first way segued me into the second way in which I tried to (re-)construct what both teachers were noticing and thinking in each intervention episode. This way of analyzing led me to make connections among categories and codes. For example, building on her actual responses in the table, I (re-)constructed how Marva intervened in a small group during an observed lesson. I found that she attended to a student dominating the group work over her peers. She approached the group and encouraged the students in the small group to work together. She decided to intervene in this way for the purpose of making all students work together. This decision-making was also based on her view of teachers' roles, her specific knowledge of students' language, and/or learning experiences related to how to redirect students.

Creating diagrams to visualize relationships.

In addition to the table, I also analyzed the relationships among categories and codes by creating comparison charts (Glesne, 2011). I created several different kinds of diagrams to visualize the relationships among categories and codes in a more holistic way. Figure 4.1 is the final version of the diagrams that I created to understand the relationships.



Figure 4.1 A diagram to explore the relationships among categories and codes

I created diagrams for each of the 38 intervention episodes. These diagrams allowed me to explore the relationships in a holistic way along with the table I created. By these diagrams, I also asked myself questions that I asked above working on the table. To answer these questions, I needed to go back and forth between the table and these diagrams and sometimes videorecordings, which helped me understand how these categories and codes were related to one another.

Among those questions, the main question for me to explore was, How would all these categories and codes be related to one another as components of individual teachers' decision-making process in relation to intervention in small groups? I pursued answering this specific question because the components (attention, decisions, responses, and interpretation) in intervention episodes allowed me to answer this question. As a result, I developed the noticing-mediated framework that allowed me to construct both teachers' decision-making processes as they intervened in small groups, which is described in the first finding chapter.

Analyzing aspects of teacher identities on the part of both teachers

I explored teacher identity on the part of Leslie and Marva using a thematic analysis (Glesne, 2011). I explain the analytic process in three phases. First, reading the whole interview

transcripts, I identified excerpts that provided evidence related to teachers' teaching identities. Second, I conducted a closer examination of the excerpts in relation to teacher identity to find similarities and differences among them in terms of teacher identity. Third, I investigated the relationships between teacher identity and intervention in small groups.

Identifying excerpts related to teacher identity

In this first phase, in search of excerpts that clearly showed instances of teacher identity, I read the whole interview transcripts to identify excerpts with instances of teacher identity. I looked for several cues that might be related to teacher identity. The cues included 1) I-statements in which teachers said who they were, 2) use of strong adjectives, adverbs, or verbs (e.g., very, highly, wonderful, or hate), 3) characters who had a strong impact on the beginning teacher's teaching, or 4) certain words, such as want, expect, believe, or (dis)like. These cues helped me find their identities as teachers.

With these cues in mind, I looked for excerpts where teachers explicitly invoked who they were in relation to intervening in small groups. As a result of the search, I obtained 33 excerpts with clear instances of teacher identity. One of the excerpts as an example is, "As a teacher I want to see [the students] learning from each other and not just, 'This is how you do it. Do it'" (T2A1E4). This quote included a clear instance ("as a teacher"), which suggested that she wanted to become a teacher who could help the students learn from each other. To make sure these 33 excerpts are clear instances of teacher identity, I asked a mathematics educator to examine whether she saw clear instances in the excerpts. She had an extensive qualitative research experience, including research on teacher identity. She confirmed that all 33 excerpts provided evidence of teachers' identities as teachers.

The data analysis led me to focus more on self-understandings, one of the interrelated components in the definition of teacher identity discussed in the literature review chapter. As a reminder, I defined teacher identity as an interrelated collection of teachers' self-understandings, beliefs, knowledge, and disposition. During this current analysis, I found that the excerpts in the whole interview transcripts did not offer clear examples of the initial definition of teacher identity. Instead, the excerpts showed self-understandings on the part of both teachers the most. So I used the excerpts related to teacher identity with specific focus on both teachers' self-understandings in further analysis.

Finding similarities and differences

In this second phase, using the 33 excerpts, I read back and forth between these excerpts to find similarities and differences within each teacher. The exploration guided me to look for teacher identity in terms of their talk about who they currently were and sometimes who they wanted to be in the future, which was also built on the idea of the actual and designated teacher identity (Sfard & Prusak, 2005). As such, first, I looked for teacher identity in terms of whether teacher identities were related to their current teacher identity or designated (future/aspirational) teacher identity. The current teacher identity meant their current views of themselves as a teacher ("as a person I've always been more comfortable in small groups" (T1A1E1)). The designated teacher I'd rather be just almost like eavesdropping and watching them interact and see them learn from each other" (T1A1E1)).

Second, using the 33 excerpts, I examined different aspects of current and designated teacher identity on the part of both teachers. Reading the excerpts within and across individual teachers allowed me to realize both teachers very often emphasized how their past experiences

had shaped their teacher identity. For example, "I guess part of my personal experience was me personally. I'm one of eight kids and I always, my older brothers and sisters were always very loud, very outspoken, very confident. And I was the more shy, quiet one" (T1H1). In the same interview, she talked about personal aspects that shaped her current teacher identity ("I internalized it [what might happen in a child's mind] into the classroom.") Like in this example, I investigated multiple aspects of current and designated teacher identity on both teachers' part.

Looking for the relationships between teacher identity and intervention

In this third phase, building on the analysis of different aspects of both teachers' current and designated teacher identities, I explored how their teacher identities were related to their intervention in small groups. I read again all excerpts and analyzed corresponding intervention in small groups in intervention episodes. For example, I read an excerpt where Leslie invoked her teacher identity (T1A3E4).

I was one of eight kids, ... I knew I could go to my one sister and she was really good at quizzing me, versus I know I could go to my other sister and she was really good with helping me with writing.

I identified this excerpt as an instance of Leslie's current teacher identity that was related to her experience as a child. By going back to its corresponding intervention, I looked into how she intervened in a small group. In her actual intervention, she made sure that the students learned from each other to solve the group task. She intervened in the small group this way because she wanted her students to see and use each other as a learning source, like she used her older siblings as her learning source. This was a connection I constructed between her intervention and the personal aspect of her current teacher identity. Like in this example, I looked back and forth

between the excerpts related to teacher identities and intervention in small groups to identify how both might be related.

Analyzing the relationships between categories of students and power and authority dynamics recognized by both teachers

Using a thematic analysis (Glesne, 2011), I examined how categories of students recognized by both teachers are related to power and authority dynamics. I explain the analytic process in three phases. First, reading the whole interview transcripts, I analyzed excerpts to identify instances of certain categories of students both teachers recognized. As a result, I obtained excerpts that included different categories of students. Second, I examined the excerpts in relation to categories of students to identify power and authority dynamics that both teachers recognized. Third, I investigated the relationships between categories of students and power and authority dynamics.

Identifying excerpts related to categories of students

In this first phase, building on ideas that the teachers recognized the diverse types of students in Horn (2007) and that these students were important actors who occupy figured worlds of mathematics classrooms (Holland et al., 1998), I identified excerpts with instances of certain categories of students. Reading the whole interview transcripts to construct categories of students, I looked for several cues that might indicate categories of students. These cues included, but were not limited to, certain verbs (e.g., outperform, take over, sit back, argue, or control), adjectives or adverbs (e.g., higher, lower, shy, or outspoken), or characters who were related to students (e.g., parents, teachers, siblings, or peers). These cues helped me find excerpts related to diverse categories of students Leslie and Marva recognized as they explained their intervention in small groups.

As a result, I identified 31 excerpts where categories of students were recognized by teachers, which I used in the further analysis. These categories were related to academic achievement, engagement, personality, language development, and at-home support. For example, when Leslie said, "two students that were pretty high. And the two students that were pretty high" (T1A1) and when Marva said, "He's one of the highest in the class" (T2A2E2), I considered this to be a cue related to categories of students related to academic achievement. The category-related cues allowed me to establish categories of students both teachers recognized. I illustrate these categories of students as one of the findings in Chapter 7. To make sure these 31 excerpts show clear instances of categories of students, I asked a mathematics education researcher to examine whether she saw clear instances in the excerpts. She confirmed that all excerpts showed explicitly both teachers' categorizations of students.

These categories are related closely to some subcodes in the category of interpretation, which I introduced earlier in this chapter. These subcodes are specific/general knowledge of students' at-home environment, language, engagement, mathematical proficiency, and anxiety of being wrong. These kinds of knowledge seemed to be used by teachers as resources to make connections to their ongoing noticing (attention, decisions, and responses). Since the knowledge was mostly related to specific students, the knowledge could be embedded into categories of students recognized by teachers. For example, Marva had a specific knowledge that Ben was very high at his mathematical proficiency, which was one of the subcodes in specific knowledge. The categories of students related to academic achievement were constructed from Marva's talk about students who were high or low in their mathematical understanding. As such, unsurprisingly, these categories of students were related to some codes in the interpretation category.

Examining the relationships between categories of students and intervention

In this second phase, using categories of students in the excerpts, I investigated how categories of students recognized by teachers were related to their intervention in small groups. To understand these relationships, I read the 31 excerpts and analyzed corresponding actual interventions where the teachers and students interacted with one another in small groups. For example, if I read an excerpt in which Leslie recognized some students in a small group as being "the lower ones" (T1A1E1), I went to the corresponding actual intervention where she intervened in a small group. I noticed that in actual intervention, she interrupted one student initiating his explanation to give the "lower" students a chance to learn and she called on some students who were "lower" ones to check out whether they understood. This way I tried to identify the explicit connections between categories of students and specific intervention in small groups.

Looking for the relationships between the dynamics and categories of students

In this third phase, I tried to find where and how power and authority dynamics played out. I did this analysis because excerpts related to categories of students sometimes embedded power and authority dynamics. My main purpose of doing this kind of analysis was to understand how these dynamics might play out in the interactions and relationships between students and between teachers and students. I looked through those excerpts in two ways.

First, I examined the individual excerpts to find out levels these dynamics played out. Since the data were related to intervention in small groups, these dynamics very often played out at the interaction level between students and between students and teachers ("I know that my one student [who were one of the higher ones] just kind of took it and ran with it. Kind of took the

whole group" (T1A2E2). I also anticipated additional levels, such as classroom level or macro level, in terms of how these dynamics might play out in those excerpts.

Second, I explored the individual excerpts to see how these dynamics shaped and were shaped by categories of students in different levels. Reading the 31 excerpts, I created a table to understand how these levels of power and authority dynamics are related to the five categories of students in those excerpts. Using the table, I paid attention to patterns in three ways. The first way was to figure out the frequency of categories of students in individual excerpts. I took a closer look at the categories of students in the table to understand patterns of which categories of students appeared more frequently. The second way was to examine relationships between frequent categories and power and authority dynamics. I examined the table to figure out patterns related to where these dynamics were related to the more frequent categories of students. The third way was to look for differences in these relationships between Leslie and Marva. I compared how Leslie was different from, or similar to, Marva.

Researcher Subjectivity

Through the entire research process, including the analytic process, I tried to keep in mind the idea of Peshkin (1988) that subjectivity is inevitable in doing research. Instead of pretending no subjectivity, thus, I have tried to keep an eye on subjectivity I might bring to the research and its influence on the current research. Peshkin's idea has encouraged me to be aware of benefits and drawbacks caused by subjectivity in doing research.

For the benefits, first, my teaching experience as an elementary school teacher in South Korea led me to the topic of the dissertation study. As a head teacher in an elementary school, for instance, I had opportunities to observe the teaching of beginning teachers in their early career and noticed how intervening in small groups did not always work well for them. It led me to

think that an important practice beginning teachers need to keep developing in their teaching could be how to intervene in small groups. Second, my observation experiences as a field instructor, as a teaching assistant in elementary mathematics methods courses, and as a M-scan rater of the DAI project also invited me to think of what research questions I had to ask to investigate ways to understand certain decision-making processes behind intervening in small groups.

On the other hand, subjectivity can be a detriment for the research (Peshkin, 1988). For drawbacks, I was inclined to judge the responses of the beginning teachers to my questions in the interviews in terms of my intention I had when I made the interview protocols. For instance, when I analyzed the interview excerpts where Marva provided some students with content-help, I initially judged her intervention in terms of whether it was right or wrong because the intervention was different from my expectation towards intervention. I came to view her intervention with a deficit view because I thought she intervened in that way because those students happened to be ELLs. This kind of judgment could never be helpful for the dissertation study. Tendency to judge their responses according to my expectations and beliefs could distort the results of the dissertation study. In other words, the findings cannot reveal a reality of decision-making processes related to how the beginning teachers intervened in small groups.

Being aware of both benefits and drawbacks, I tried to manage the detriment in the current research. First, I tried to keep aware of the drawback in action. To do this, I tried to be honest by explaining the judgment tendency coming from the subjectivity in meeting with my advisor so that I could get support from her. It helped me keep open to any possibilities of having the drawback when I analyzed the responses of the beginning teachers. Second, I tried to find out where the judgment tendency came from. I came to realize that the tendency was tied to

experiences as a former elementary school teacher for almost two decades in which I should decide one of many solutions in every classroom matter. So, when I read the responses of the beginning teachers in the dissertation study, it was natural for me to judge their responses, especially when I had certain expectations. To avoid the tendency, I tried to be aware of why I made decisions in the analysis.

CHAPTER 5. THE NOTICING-MEDIATED INTERVENTION FRAMEWORK

In this chapter, I introduce the noticing-mediated intervention framework as a lens that can provide a way to (re-)construct teachers' decision-making process in relation to intervening in small groups in elementary mathematics classrooms. First, I explain the framework by detailing its two components: 1) ongoing noticing and 2) interpretation of small group work in particular connection with teachers' resources. Second, I use one intervention episode as an example to demonstrate how this framework works. Third, I describe patterns common to both teachers in terms of the two components of the framework. Fourth, I provide the summary of this chapter.

What Is the Noticing-Mediated Framework?

Figure 5.1 represents the noticing-mediated intervention framework. This framework consists of two interrelated components: 1) ongoing noticing and 2) interpretation of small group work in particular connection with teachers' resources, shortly interpretation. These two components are distinct in terms of their role in a teacher's decision-making process. The first component has three subcomponents, 1) attending, 2) deciding how to respond, and 3) responding. Ongoing noticing describes what a teacher does before and at the moment of intervention in small groups. For example, a teacher observes a student dominating small group work (attending). The teacher approaches the group with the goal of making sure students work together (deciding how to respond). The teacher then asks students in the small group to explain their thinking and to evaluate other students' mathematical ideas (responding). Teachers' actions in each of these three subcomponents are chained together in the context of teachers' intervening

in small groups as the steps of this process occur nearly simultaneously in the moment of intervening.



Figure 5.1 The Noticing-mediated intervention framework.

The second component, interpretation, has four subcomponents, or resources: 1) knowledge, 2) group norms, 3) teachers' roles, and 4) learning experiences. These resources are related to what teachers draw on to interpret, make sense of, or make connections to the three subcomponents of the ongoing noticing. For example, in relation to the aforementioned case, the student's dominance of the small group work (*attention* in ongoing noticing) may remind the teacher of what the teacher knows about the dominant student's and other students' engagement patterns (knowledge). The teacher may decide to encourage students in the small group to work together (*decision of how to respond* in ongoing noticing) because the teacher thinks that she as a teacher should foster students to develop capacity to work together (teachers' roles). These connections do not necessarily imply that this teacher is aware of these moment-to-moment connections, but they are still important to understand because they help explain a decisionmaking process a teacher goes through while intervening in small groups.

Besides their distinct role, there is a key difference between these two components, ongoing noticing and interpretation. The first component highlights what a teacher may have in mind at the moment when the teacher intervenes in small groups. The second component concerns the historical sense of the intervention. It is historical in that an individual teacher has built, developed, and accumulated these resources, which are connected to the teachers' ongoing noticing, in different contexts, such as teacher preparation programs, mentor teachers' classrooms, and/or their own teaching. Both components are tied closely to each other in that as a teacher engages in ongoing noticing (attention, decision of how to respond, and responses), the teacher's experiences may become integrated into resources (knowledge, group norms, teachers' roles, and learning experiences) for future ongoing noticing.

This framework integrates the construct of professional noticing (Jacobs et al., 2010) and teachers' intervention in small groups. The construct of professional noticing, which consists of attention, interpretation, and decisions of how to respond, is drawn upon in this framework building on my previous work (Pak, 2018), which suggests that professional noticing can serve as a lens for exploring how teachers make decisions when intervening in small groups in mathematics classrooms. Using these three processes of professional noticing in the main parts

of the framework contributes to understanding a teacher's decision-making process in the context of intervening in small groups. I used my analysis of the interview data to build this framework to elaborate what may constitute the two beginning teachers' attention, decisions of how to respond, responses, and interpretation.

Constructing A Teacher's Decision-Making Process in Intervention

In this section, I use a beginning teacher's intervention in a small group as an example to illustrate the re-construction of the teacher's decision-making process using the framework. I chose this example because it includes the range of the common patterns identified in the data and described in the next section. I examine this example in terms of the two components of the framework: 1) ongoing noticing and 2) interpretation.

Background information of the intervention episode

The example I use in this section is the fourth intervention episode from a Social Studies/Mathematics lesson in Leslie's class. Prior to the lesson from which this episode came, students had learned economics terminology, such as scarcity and opportunity cost, in Social Studies. This particular lesson was focused on reviewing this terminology. The mathematical components the teacher integrated in this lesson and group activity were addition, money, and decimals in the context of teaching scarcity and opportunity cost.

In this lesson, the students engaged in a group activity based on a hypothetical scenario regarding students' background knowledge about Black Friday shopping. To borrow the teacher's own words, the scenario was that students had to "go through Black Friday fliers and hypothetically come up with what they would buy their family or their group members if they were to receive X amount of money and they had to then adjust once they've found out how

much they had in the family, they had to readjust how much that they would spend on each person and who would get what item, and kind of go from there" (T1A3).

In her teaching, Leslie used small groups as an instructional organization on a regular basis. In this lesson, she had students work together with peers in four groups of five, which was a typical group formation in her teaching. The group formation was based on mixed ability, with two or three students who Leslie considered to be higher in mathematical competency in each small group.

Constructing Leslie's ongoing noticing

I (re-)construct Leslie's ongoing noticing (see left side of Figure 5.2) drawing on examples from the fourth episode of the third stimulated recall interview (T1A3E4). According to the interview, Leslie paid attention to the progress that students in a small group were making on the group activity. Looking at the students' worksheet, she saw that the small group did not get their group work done (attending to group work progress). She decided to respond to the small group with the purpose of "seeking to understand" whether the students "were thinking of total as the whole sum altogether." By understanding, the teacher meant to figure out the degree of progress that students had made solving the group task in the small group (deciding to understand the group's problem-solving progress). The teacher approached the small group and told the students her expectation to find exact costs that they should obtain by adding three totals, pointing out what students missed in solving the group task (responding by providing content help to students).

This example shows how three subcomponents (attention, decision of how to respond, and response) could be linked to one another as Leslie intervened in a small group. Taking a

close examination of a teacher's ongoing noticing can help explain part of the teacher's decisionmaking process in the context of the teacher's intervention in small groups.

This example is simpler than other intervention episodes. It has a single event the teacher attended to, a decision of how to respond, and a response the teacher made. Some intervention episodes have more complicated links among these aspects. For example, a teacher may give her attention to several events, have two or more decisions, and make several responses to the event in a small group.



Figure 5.2 An example of constructing a teacher's decision-making process

Constructing Leslie's interpretation in connection with her resources

This intervention episode shows resources (see right side of Figure 5.2) that Leslie reported having in mind in relation to this particular intervention in a small group. I (re-)construct from this particular example ways she interpreted, or made connections, between resources (specific knowledge, group norms, teachers' roles, and learning experiences) and each of the three subcomponents of the ongoing noticing. I describe this construction in terms of the four subcomponents of interpretation in the framework.

Specific knowledge and ongoing noticing.

In this example, Leslie had developed specific knowledge about the students in her classroom. She knew how particular students engaged in small groups in her classroom. The teacher knew that there were some students who made their peers annoyed when they worked together in small groups. This specific knowledge was connected to the teacher's attention, her decision of how to respond, and her response. First, this specific knowledge of students' engagement shaped the teacher's attention to the group work progress as she monitored small groups in her teaching. The slow progress the small group had made with the task reminded the teacher of her knowledge of the students' prior engagement patterns. Second, this specific knowledge of students' engagement influenced the teacher's decision to understand the problemsolving progress of the small group as she began to intervene in the small group. The teacher did not know what went on in the small group before intervening but knew well of the students' previous engagement patterns. This knowledge influenced her decision to respond to the small group in such a way. Third, this knowledge informed her response, which was guiding the small group to see what they were missing in solving the group task. They had not figured out what they should have done to solve the task- they "need[ed] three totals." The teacher thought that it

would be helpful to provide students with a clear direction to solve the task given their engagement patterns.

Group norms and ongoing noticing.

Leslie expected her students to be responsible for working cooperatively with peers. This expectation she held towards students' cooperation in small groups had an influence on what she was looking for among many things, one of which is the degree of the progress the particular small group should make at a given time. In this intervention episode, this group norm had an influence on her attention to the slow progress of the small group on the group task.

Teachers' roles and ongoing noticing.

Only one of the teachers' roles appeared in this example. The role was fostering students' capacity to work together with peers. Leslie said that her students would live in a world where being able to work well in a group was a valuable competency. Her responsibility as a teacher, therefore, was to prepare her students for the future world. This role was connected to the teacher's attention and decisions of how to respond. First, this role shaped her attention more to the group work progress of the particular small group. Like in the group norm above, this role made her more sensitive to the group work progress in the small group. Second, this role encouraged her to decide to understand the small group's progress on the group task. To foster students' capacity to work with others, the teacher needed to make sense of what the students had done or understood in the small group.

Learning experiences and ongoing noticing.

A particular learning experience in relation to understanding students' learning appeared in this example. Leslie had learned about the importance of understanding students' learning in her teacher preparation program. As noted in Figure 5.2, Leslie said that one of the contexts in

which she came to learn the importance of understanding students' learning was her mathematics methods course where she learned to implement number talks. This learning experience was connected to Leslie's decisions of how to respond and her responses. First, this learning experience may have shaped her decision to understand the small group's problem-solving progress. Leslie learned that it was important to understand what students meant by what they explained and justified through doing number talks. The importance of understanding students' thinking might have influenced her decision to figure out what students were doing in relation to problem solving progress. Second, this learning experience seems to have influenced her response of providing content help to students. This learning experience related to "a clarification of what they did" might have shaped her response in which the teacher led the small group to figure out what they were missing in solving the group task.

Patterns of Ongoing Noticing and Interpretation

In this section, I detail common patterns that both beginning teachers, Leslie and Marva, demonstrated in relation to ongoing noticing and interpretation. First, I present common patterns that I found in relation to each of the three subcomponents of ongoing noticing: 1) attention, 2) decision of how to respond, and 3) responses. I also present patterns that were unique to each teacher in relation to ongoing noticing. Second, I illustrate common patterns with respect to interpretation resources, including knowledge, group norms, teachers' roles, and learning experiences. I also present patterns that were unique to each teacher in relation to interpretation.

I present these common and unique patterns because beginning teachers' decisionmaking processes may be constrained by the range of ongoing noticing in which they engage and the resources they draw on to make connections to their noticing. More generally, presenting

these patterns can also extend understanding of teachers' decision-making processes, particularly beginning teachers.

Patterns in relation to ongoing noticing

Beginning teachers' attention.

There were three common events that drew both beginning teachers' attention. These common events are 1) group work progress, 2) students' voice level, and 3) students' body posture. First, both teachers attended to how much small groups made progress on the small group task. For example, in the third intervention of the third stimulated recall interview (T2A3E3), Marva talked about what she attended to. She said, "I saw the paper and I was like, it doesn't look like they've got a whole lot done. I was like, are they doing any work?" Levels of progress received both teachers' attention in the context of intervening in small groups.

Second, both teachers gave attention to students' voice levels in small groups. They attended not only to loud voice levels, indicating students' arguments, but also quiet voice levels, indicating either a productive communication among students or off-task behaviors. For example, in the fourth intervention of the first stimulated recall interview (T2A1E4), Marva talked about what she attended to in a small group. She said, "I saw them both writing. And they're both quiet. Well, Ben is a little bit louder. They're both very quiet still." In similar ways, students' voice level received both teachers' attention.

Third, both teachers talked about their attention to students' posture. They attended to students lying on the table and/or with their heads down. For example, in the fourth intervention of the second stimulated recall interview (T1A2E4), Leslie said, "Malek was definitely just, was not interested, was more interested in just laying back." Students' body posture, like in the interview excerpt, received both teachers' attention.
Beginning teachers' decisions of how to respond.

There were three common decisions of how to respond that both beginning teachers made when intervening in small groups. First, both teachers decided to understand students' problemsolving progress. For example, in the third intervention of the second stimulated recall interview (T2A2E3), Marva indicated her decision to "see where they were."

Because I wanted to see where they were because I know the one little girl Rosa, she's very hit or miss with what she does and can do so I never really know what she can or can't do. Because sometimes it looks like she can do something, and other times I'm like What happened? You did this perfectly yesterday. So, I wanted to stay to make sure that they were getting it and understood it.

In this excerpt, Marva wanted to "make sure that they were getting it and understood it." This is a typical decision made by both teachers. They wanted to figure out what was going on with the group's problem-solving progress.

Second, both teachers decided to make sure students were working together with each other in small groups. For example, in the first intervention of the second stimulated recall interview (T1A2E1), Leslie talked about her decision to help the students "teach the other two."

I was trying to get her back into the group because in that group like I said I had two students that were shy and I had two students that were pretty high up in their understanding so I wanted them to kind of teach the other two. The teacher in this excerpt decided to encourage the two students who "were high up in their understanding," to teach the other two, who "were shy." In similar ways, both teachers made sure students worked together in small groups.

Third, both teachers decided to mediate students' mathematical thinking as they intervened in small groups. For example, in the second intervention of the second stimulated recall interview (T2A2E2), Marva talked about her decision that she had in mind, which was "giving you [students] information."

When I do it like that I'm not working as a facilitator, I'm more so-- now I'm actually-now I'm giving you information. Now how can you use it? So I'm still-- now, I still want them to use it. It's not just okay now you know how to do this. Just now you got it. It adds to what you've already been doing.

When deciding to provide information, the teacher intended students to use the information to solve the group task. Like this teacher, both teachers decided to give mathematical information for the purpose of mediating students' mathematical thinking.

Beginning teachers' responses.

There were three common responses that the two beginning teachers made as they intervened in small groups. First, both teachers provided content help to students in small groups. For example, in the second intervention of the third observation lesson (T1A3E2), Leslie made a response in relation to providing content help to a student in a small group. In this excerpt below, the teacher tried to lead students to compare their strategies.

Leslie: So, start from your right and then add to your left?

A student: Oh wait. You can group these and then these. Leave these two out so 18 times

2.

Another student: 18 times 2. I just sometimes -- I add all them like 9, 10, 11, 12, 13, 14, [counting to 54] No...

Leslie: Okay so my question is, we had three or four different ways of doing this. Are any of them wrong?

A student: No.

Leslie: No. Any of them are going to make the right answer, right? Okay.

In this excerpt, the teacher pointed out that despite different ways of solving the task, students could have the same answer. By offering the content-related help, the teacher tried to help the students think of strategies that they used with the group task. As in this excerpt above, both teachers made suggestions of what to do to solve the mathematical task at hand.

Second, both teachers asked students to explain their mathematical thinking. For example, in the fourth intervention of the second observation lesson (T2A2E4), Marva asked a student to show how the student got her answer. "Well, see I had another group measure this way and get seven. Please show me how you got seven? Show me." Just as the teacher asked students how the student got seven as an answer, both teachers asked students to explain their mathematical thinking.

Third, both teachers encouraged students to work together. For example, in the first intervention of the second observation lesson (T2A2E1), Marva said to students in a small group,

"I'm not going to tell you how to do it, you need to figure out how it works for you where you are working together as a group." In these excerpts, both teachers asked students in small groups to work together with their peers.

Patterns unique to each teacher in ongoing noticing.

There were also patterns unique to each teacher in relation to the three subcomponents of the ongoing noticing. First, for attention, besides the three common events mentioned above, some events, such as approaches to mathematical tasks and students' facial expression, received Leslie's attention, while some events, such as playing with learning materials and improper use of math manipulatives, and ELLs' language use, received Marva's attention. Second, for decisions of how to respond, apart from the three common decisions above, two decisions - increasing on-task behaviors and providing language support - received Marva's attention. Third, for responses, apart from the three common decisions above, one response related to evaluating other students' ideas appeared in Leslie's responses. Some other responses, such as asking students to provide reasons, asking students to metacognitive questions, explaining to ELLs how to use language correctly, and asking students to comment on peers' ideas, appeared in Marva's interventions.

Patterns in relation to interpretation

There were common patterns in relation to connections both teachers made between resources (knowledge, group norms, teachers' roles, and learning experiences) and the three subcomponents of ongoing noticing - attention, decisions of how to respond, and responses. In terms of knowledge, both teachers had specific knowledge of students' engagement and students' mathematical proficiency that could be connected to all subcomponents of ongoing noticing noticing. For group norms, both teachers had a particular group norm of working with peers.

This group norm was connected more to attention in the ongoing noticing. For teachers' roles, two types of teachers' roles, 1) fostering students' capacity to work together and 2) motivating students to think for themselves, were connected to all three subcomponents of ongoing noticing. Lastly, two types of learning experiences, experiences through which these teachers learned about 1) redirecting students and 2) facilitating small group work, were connected to decisions of how to respond and responses.

Specific knowledge.

Specific knowledge and attention.

Two particular types of specific knowledge, 1) students' engagement and 2) students' mathematical proficiency, were most often related to what teachers attended to.

First, specific knowledge of students' engagement appeared in relation to both teachers' attention to students' voice level, dominance of group work, and students' body posture. For example, in the second intervention of the first stimulated recall interview (T1A1E2), Leslie "heard a lot of bickering, "you're not listening, that's not what I said, well that's wrong", and "so just that one talking over the other, talking over the other." This excerpt shows that she attended to the students' voice level in a small group. In relation to these students' voice level, she "knew something was up because that one student in particular just kinda -- has a little bit of difficulty working in small groups." She used her specific knowledge of a particular student's engagement pattern to interpret the students' voice level to which she attended.

Second, specific knowledge of students' mathematical proficiency appeared in relation to teachers' attention to group work progress, approaches to mathematical tasks, and students' voice level. For example, in the second intervention of the first stimulated recall interview (T2A1E2), Marva attended to a group's approach to the group task. "I didn't hear him say

anything, it's just I saw that one problem on his paper that said 8 + 3 = 7." She saw a student having an incorrect answer on his paper. She related the answer 7 to what she knew of her students' tendency in relation to addition:

I wasn't noticing it as much earlier in the year but now that we've started-- since we started unit three I'm noticing a bunch of them that, for some reason, they just, all of a sudden, their answers are smaller than what they began.

Marva recollected a recent tendency that students' answers "were smaller than what they began" in addition. This excerpt shows how her specific knowledge of students' mathematical proficiency informed her attention to students' approach to the addition problem.

Specific knowledge and decisions of how to respond.

Two particular types of specific knowledge, 1) students' engagement and 2) students' mathematical proficiency, were related to both teachers' decisions of how to respond. First, specific knowledge of students' engagement appeared in relation to making sure students work together and understanding problem solving progress. For example, in the third intervention of the second stimulated recall interview (T2A2E3), Marva saw two students measure the same book simultaneously using their own paper clips. The teacher had asked students that only one student should have been doing the measuring instead of everyone doing it all at the same time. She decided to understand the students' problem-solving progress. She "wanted to stay to make sure that they [the students] were getting it and understood it." In the interview, she talked about her knowledge of the two students' engagement in the group.

[H]e's fairly new. So, he has his friends but-- well, he's not new, he was here last year but he's not good with new people. And she's a little controlling amongst the other Arabic speakers. So, it's a little funny. He's an Arabic speaker but they don't interact like she does with the other Arabic speakers

She knew that the female student of the two students tended to interact with other students in Arabic. She also knew that the two students did not seem to talk to each other. This specific knowledge informed her interest in wanting to know what was going on in the small group.

Second, specific knowledge of students' mathematical proficiency was related to understanding problem solving progress and mediating students' mathematical thinking. For example, in the second intervention of the second stimulated recall interview (T2A2E2), attending to the students' quiet voice, Marva "wanted to see and make sure there were no struggles going on too" because she knew "they were still close to the beginning of what was happening." She wanted to understand what was happening in the small group in relation to the group task. In relation to this decision to understand the group's problem-solving, the teacher used her specific knowledge of students' mathematical abilities. She "knew if Ben wasn't getting it, that was going to be a struggling area for the whole group, so I used it as a quick moment." Knowing Ben's mathematical proficiency was higher than others in her classroom made her curious about the slow progress the students had made and informed her decision to understand their understanding.

Specific knowledge and responses.

Two particular types of specific knowledge, 1) students' engagement and 2) students' mathematical proficiency, were related to both teachers' responses. First, specific knowledge of

students' engagement appeared in relation to responses, such as encouraging students to work together, providing content help to students, and asking students to explain their thinking. For example, in the fourth intervention of the first stimulated recall interview (T2A1E4), Marva asked the students in a small group if they were working together:

Marva: Are you guys doing it together? Because I hear you talking. Are you copying what he's doing or are you-- or you were actually talking with him? Ben: He's helping me count.

Marva: Oh, he's helping you count them. Okay that's good. So, you guys are working together to count them out.

In this excerpt, the teacher expressed her curiosity about their cooperation to solve the group task. In the same intervention-related interview, the teacher said what she knew about the students' engagement in the small groups. "Brice is eager to learn and I think that makes him and that other little boy a good partner" but "Ben-- so sometimes I fear that he's taking over." This specific knowledge of these students' engagement patterns influenced her response "to check in" with the students to know how they were doing at the moment.

Second, specific knowledge of students' mathematical proficiency was related to responses, such as providing content help to students, asking students to explain their thinking, and asking students to provide reasons. For example, in the third intervention of the second stimulated recall interview (T1A2E3), attending to the progress a small group had made, Leslie heard that the students "were just throwing numbers around." The teacher tried to guide the students to see the meaning regarding the numbers as she intervened in the small group.

Leslie: Well, if Maria wants to buy a glass of lemonade from me, right? She's going to

give me?

Students: 60 cents.

Leslie: She's going to pay 60 cents for a cup of lemonade? What did we say I'm selling them for?

Students: Oh, \$1.

Leslie: So, you're going to give me a dollar, okay? Dollar, please. [crosstalk]... Do I get to walk away with that dollar one happy, lucky girl?

Students: No.

Leslie: What do I have to do?

A student: You have her 60 cents back.

In this small group, she had a concern regarding the students' mathematical proficiency. "I was concerned. This was the group that I was originally most concerned with because it has a few students that are, not low, but just a little bit on the lower side." This specific knowledge of the students' "low" mathematical proficiency influenced her responses by leading the students to see change they needed to give back to Leslie.

Group norms.

Among two group norms, only one type, working with peers, was connected to both teachers. This particular group norm appeared in relation to teachers' attention to group work progress and students' body posture.

In relation to the connection between the group norm and group work progress, for example, in the fourth intervention of the first stimulated recall interview (T1A1E4), Leslie attended to group work progress in a small group. She said, "they [the students in the small group] had drawn a bunch of pictures on the paper" and "they were rushing, figuring out the answer and then not discussing it." In the same intervention-related interview, the teacher talked about an expectation of students working with peers in small groups. "I think it's really important because teamwork and working as a team and having a group partner and being able to do that effectively, is something that is going to be carried out through your whole life." She emphasized the importance of being able to work with others through students' lives in the future. This group norm had the teacher pay attention to the small group's progress on the group task because the progress shows whether students work actively with each other in small groups.

In relation to the connection between the group norm and students' body posture, for example, in the first intervention of the first stimulated recall interview (T2A1E1), Marva talked about her attention to a student's body posture.

I could see that Jordan was on his own-- I could see him rock. He was back in his chair, his paper was up on the desk so when he's leaning back, I could see that he couldn't see the paper, and he was too far away to be able to hear and listen to them."

She saw Jordan's body posture, which indicated his off-task. In the same intervention-related interview, the teacher talked about her expectation towards students' working with peers.

[It]'s usually a privilege to work in groups because they like working with each other and nobody likes sitting there working quietly. So for them, it's being able to be responsible with that ... So I see he wasn't really working, he was off task then the expectation is you're engaged in it. And it's not responsible to just be sitting there because now you're not learning. And they know that the reason they're in school is to learn and that's one of the things I always go back to is, "We're here to learn. You have to be trying, if you're not trying you're not learning.

This excerpt shows part of the expectation she has tried to help her students hold. She expected students to engage in what they should be working in small groups. The teacher's expectation seems to influence her attention to Jordan's off-task and her reasoning about his behavior.

Teachers' roles.

Among three teachers' roles, two types, 1) fostering students' capacity to work together and 2) motivating students to think for themselves, were connected to both teachers' attention. In relation to teachers' decisions of how to respond, one teachers' role, fostering students' capacity to work with peers, was linked to understanding problem solving progress and mediating students' mathematical thinking. In relation to teachers' response, another teachers' role, motivating students to think for themselves, was connected to providing content help to students and asking students to explain their thinking.

Teachers' roles and attention.

Two particular types of teachers' roles were connected to both teachers' attention to group work progress. For example, in the first intervention of the second stimulated recall interview (T2A2E1), looking at the students' measuring worksheet, Marva noticed that the

students were not doing much of the group work (attending to group work progress). Instead, they were working independently. "When I walked up it looked like they were each just doing it by themselves on their own instead of working as a group." In the same intervention-related interview, she talked about her role as a teacher in relation to implementing small groups. "[A]s a teacher, I like having students work cooperatively and be able to learn from each other. And if they're not working together, they're not going to learn from each other." This excerpt shows her view of teachers' role that teachers are responsible for fostering students to work with others and to learn from each other. This teachers' role had an influence on her attention to the small group's progress on the group task because the students did not work together with each other.

Teachers' roles and decisions of how to respond.

Among three types of teachers' roles, only one type, fostering students' capacity to work with peers, was connected to both teachers' decisions of how to respond. The decision to respond was increasing on-task behaviors. For example, in the fourth intervention of the first stimulated recall interview (T1A1E4), attending to students in a small group "rushing, figuring out the answer and then not discussing it," Leslie decided "to get them back on-task" by asking students to "work[ing] on more of the discussion about their answer" as she intervened in the small group. This decision of increasing on-task behaviors seems to be connected to Leslie's view of her role, which is fostering students to develop an ability to work with others.

I wanted the students to also be the teachers. That was one of the big beliefs that I always have. And so, keeping that in mind, I didn't want to lead the kids right to the answer. I wanted them to kind of fool around with it and give them a chance to communicate and discuss their own ideas.

An idea underlying this excerpt is that having more chances to "communicate and discuss their own ideas" with peers enables students to develop their capacity to learn from each other. This seems to influence the teacher's decision, a decision to increase students' on-task behaviors by helping them participate actively in discussion.

Teachers' roles and responses.

Among three types of teachers' roles, only one type, motivating students to think for themselves, was connected to both teachers' responses. This particular teachers' role appeared in relation to teachers' responses, such as providing content help to students and asking students to explain their thinking. For example, in the second intervention of the first stimulated recall interview (T2A1E2), seeing a student having an incorrect answer on his paper. Marva led the student to see a mistake the student had made.

Marva: Okay. You have to draw that out though. You can't just say, "Okay, I did fingers."I need to see it on paper. Because... Is seven bigger or smaller than eight?A student: Bigger.

Marva: Eight is bigger. So how is your answer seven? I think you need to try and draw it out and see if you can figure it out. Sabia's already started. I'm going to come back to you guys for this one.

In this excerpt, the teacher helped the student see that one of the two addends (8 and 3) could not be bigger than the sum (7). She did not provide the answer to the students. In the same

intervention-related interview, in relation to this response, she talked about her view of her role as a teacher who should motivate students to think for themselves.

I'm really trying to make thinkers. I'm trying to make them learn how to think for themselves and not rely on just telling me. Because I know I've got a couple who just... when they have a question they need the answer. If they have a problem they need an answer. They don't want to or can't sometimes solve it on their own. I have one little girl, you ask her a question and she shuts down. Like she just won't think for herself. She won't go through the process. Another little boy who ruins his day if he doesn't get an answer to a question. So, I'm really trying to get them to use different things, just think about things and try to make sense of them on their own. Even if it might not be right but they're making sense of it. It makes sense to them. And so with that, it's me not giving them the answers. It's making them think. It's hard sometimes.

This excerpt shows that she had faith in students' ability to make sense of the problem and the mistake. To do that, she would like to keep motivating them to "learn how to think for themselves." This motivation seems to have an influence on her responses by providing content help to the student without telling the answer or how to solve the problem.

Learning experiences.

Among four types of learning experiences, two particular types, experiences related to redirecting students and facilitating small group work, appeared in relation to both teachers' responses, such as providing content help to students. Both teachers talked about the two types of learning experiences in different contexts, such as teacher preparation programs and their field

placements. For example, in the third intervention of the third stimulated recall interview (T1A3E3), Leslie helped students in a small group figure out a mistake they had made.

Marva: Are you guys adding in that [inaudible] on the side or something so that you can make sure-

A student: We're adding 14 [inaudible].

Marva: Are you doing it in your head or are you doing it to the side? Because I already see a mistake.

A student: I know.

Marva: Andrew, did you see our mistake?

A student: It's, I do.

Marva: What's our mistake?

A student: [inaudible] there? No, we didn't add the [crosstalk]. We add--

Marva: You need to add the exact cost, okay?

She provided content help to the students by pointing directly to the mistake. In relation to this way of response, she talked about what she learned from her teaching in the past.

I teach the lesson to one fourth-grade class and then to another fourth-grade class, so sometimes I forget to say one thing to one class then sometimes I have to stop the other class and say, "Guys, I'm sorry I forgot to say this. Make sure that you're not rounding. Make sure you're using the actual numbers." So just reiterating the directions and making sure that you're very explicitly clear with them. Her point from this excerpt was that in her own teaching she has redirected students whenever she saw students doing something incorrect in solving a group task. It seems that frequent experiences of redirecting students have informed her of pointing out the students' mistake in her response.

Patterns unique to each teacher in interpretation

There were patterns unique to each of them in relation to the four subcomponents of interpretation. First, for knowledge, other aspects of specific knowledge, such as students' language and students' at-home environment, appeared in Marva' intervention-related interview. In relation to general knowledge which did not appear commonly in both teachers' interviews, general knowledge of the importance of creating a supportive environment appeared in Marva's interview, while other elements of general knowledge, such as ways to have students work together and students' anxiety of being wrong, appeared mostly in Leslie's interpretations. Second, in relation to group norms, another group norm, using manipulatives in a mathematically proper way, was related to only Marva's interview. Third, in relation to teachers' roles, another teachers' role, such as being flexible with students' needs, appeared only in Marva's intervention interview. Fourth, in relation to learning experiences, some learning experiences, such as experiences related to understanding of students' learning, facilitating small group work, and students' anxiety, were related only to Leslie's interview.

Summary

In this chapter, I illustrated the noticing-mediated intervention framework (Figure 3.1) by taking as an example Leslie's intervention episode. The framework consists of two components:

ongoing noticing (attention, decisions of how to respond, and responses) and interpretation resources (knowledge, group norms, teachers' roles, and learning experiences). I also detailed patterns Leslie and Marva had in relation to ongoing noticing and interpretation resources and how interpretation resources were related to ongoing noticing. Table 5.1 below summarizes the relationships between interpretation resources and ongoing noticing.

Interpretation resources	Ongoing noticing				
Specific knowledge of	Attention	Students' voice level			
students' engagement		Dominance of group work			
		Students' body posture			
	Decisions of how	Making sure students work together			
	to respond	Understanding problem solving progress			
	Responses	Encouraging students to work together,			
		Providing content help to students			
		Asking students to explain their thinking			
Specific knowledge of	Attention	Group work progress			
students' mathematical		Approaches to mathematical tasks			
proficiency		Students' voice level			
	Decisions of how	Understanding problem solving progress			
	to respond	Mediating students' mathematical thinking			
	Responses	Providing content help to students,			
		Asking students to explain their thinking			
		Asking students to provide reasons			
(Group norms) working	Attention	Group work progress			
with peers		Students' body posture			
(Teachers' roles) fostering	Attention	Group work progress			
students' capacity to work	Decisions of how	Increasing on-task behaviors			
together	to respond				
(Teachers' roles) motivating	Attention	Group work progress			
students to think for	Responses	Providing content help to students			
themselves		Asking students to explain their thinking			
(Learning experiences)	Responses	Providing content help to students			
redirecting students					
(Learning experiences)	Responses	Providing content help to students			
facilitating small group					
work					

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CHAPTER 6. RECONSTRUCTING WAYS TEACHERS INVOKE THEIR IDENTITY IN THE CONTEXT OF INTERVENING IN SMALL GROUPS

In this chapter, I (re-)construct aspects of teacher identity that the two beginning teachers, Leslie and Marva, invoked to explain their intervention in small groups. By aspects of teacher identity, I suggest that teacher identity is not a single construct but multifaceted, and each teacher used multiple facets, or aspects, as resources for their reasoning about their intervention in small groups. In this chapter, I shed light on those aspects of current and designated teacher identity that the teachers used to explain their intervention in small groups. To that end, first, I demonstrate aspects of current teacher identity used by both teachers to make sense of their intervention in small groups. I present my reconstruction of more frequent aspects of their current teacher identity followed by less frequent aspects of their current teacher identity. Second, I illustrate certain aspects of designated teacher identity. These aspects of their designated teachers in combination with their current teacher identity. These aspects of their designated teacher identity provide insight into who the two beginning teachers want or hope to be in the future. Third, I provide the summary of this chapter

The Two Beginning Teachers' Current Teacher Identity

In this section, I elaborate on certain aspects of current teacher identity invoked more or less frequently by both teachers as they explained their interventions in small groups. First, I begin with more frequent aspects of current teacher identity invoked by Leslie. Leslie most often drew on aspects of her current teacher identity shaped by her personal experiences. Second, I go on to introduce more frequent aspects of Marva's teacher identity. Marva often invoked aspects of her current teacher identity related to her view of teachers' role. These distinct aspects of both teachers were linked closely to their reasoning about their intervention in small groups. Following each teacher's frequent aspects of current teacher identity, I also introduce other aspects of teacher identity invoked less frequently by each teacher.

Leslie's frequent aspects of current teacher identity: Personal aspects

Leslie invoked aspects of her current teacher identity related to her personal experiences as a child and a learner. I label these aspects as the personal aspects of teacher identity. These aspects were tied closely to Leslie's intervention in small groups. These aspects of her current teacher identity were distributed across her reasoning about intervention episodes and were invoked more frequently than other aspects of her identity. However, I do not claim that each of these aspects appeared multiple times in all four interviews and in all intervention episodes. As shown below, some of these aspects appeared one time in a particular intervention episode. Even though I acknowledge these examples below include ones that appeared only one time, I present these examples as *frequent* aspects because they each relate to Leslie's personal experiences as a child and a learner, which she used to explain her interventions related to hypothetical and actual situations.

"I was the more shy, quiet one" (Survey follow-up interview).

In the follow-up interview to the open-ended online survey (T1H1E3), Leslie invoked a personal aspect of her current teacher identity when she talked about her plan to intervene in the small group in hypothetical Scenario 3 (see Appendix A.). The scenario depicted a situation where a small group of students had difficulty working together to identify line-symmetric figures and draw lines of symmetry. In her responses in the open-ended online survey, she noted she understood the situation that, "Robin [one of the students in the small group in Scenario 3] is

feeling self-conscious that her way was critiqued." She went on to note that she would intervene in the small group by helping them get back to working well with each other.

I would go to the group and ask them to review their discussion with me. I would specifically ask Robin what he/she thought about each of them and finally ask both partners what they thought about Robin's. I would praise the criticism and remind them that sometimes the best way that we learn is through making mistakes.

In explaining why she would intervene in the small group in this way, she wrote, "from my personal experiences, kids are afraid to be wrong because they don't want to be made fun of or seen as less smart than others. I would reiterate that being wrong isn't a bad thing." This quote suggests that there is a tight link between her personal experiences and her intervention in the small group.

When asked to talk more about her personal experiences in the follow-up interview, Leslie elaborated on her personal experience as a child in her family. She had seven siblings, some of whom were older than Leslie. She was shy and quiet while her siblings were always very outspoken. When she made mistakes, her older siblings teased her for her mistakes.

I'm one of eight kids and I always, my older brothers and sisters were always very loud, very outspoken, very confident. And I was the more shy, quiet one. And so, being one of eight kids, I mean, if you say something that's wrong someone is about to harp on you and tease you a little bit. Which was, I mean, that's just my family, that's just what we do. But I think it-- I internalized it into the classroom. So even though I wasn't really made fun of if I got a question wrong, I was hesitant to answer a question if I didn't think it was 100% right. Because I was a little fearful that someone was going to make fun of me in the classroom. And I see that with a lot of my kids.

What Leslie highlighted in this excerpt was that having experiences as an introverted child with extroverted siblings made her understand how a child may feel when the child makes mistakes and others critique the mistakes. These experiences also shaped her sense of empathy toward those kids who make mistakes. In a response to another hypothetical intervention episode (T1HE11), Leslie said, "having that experience I kind of relate to the kids that kind of go under the radar and so I want to pull them out what I wished someone would've pulled out of me a long time ago." She "internalized" this personal experience as a child into "the classroom." In her plan to intervene in the small group above, this internalization seemed to explain her plan to intervene in the small group by "ask[ing] Robin what he/she thought about each of them and finally ask[ing] both partners what they thought about Robin's." She invoked her teacher identity related to her experience as a child and furthermore used this personal aspect to reason about her intervention in this specific small group.

"As a person I've always been more comfortable in small groups" (First stimulated recall interview).

Leslie also invoked a personal aspect of her current teacher identity to explain the first intervention episode in the first stimulated recall interview (T1A1E1). Being asked to talk about her intervention, the teacher expressed her personal sense of comfort being in and working with small groups. She contrasted this comfort with small groups to the sense of intimidation she felt when teaching by lecture in front of the whole class.

I guess a large part of it is through the university and using small groups through the university and during my internship and whatnot. But part of it, I guess, it also comes down to... as a person I've always been more comfortable in small groups. I don't like getting in front of large classrooms and whatnot. In Jeonju when I had to teach in front of 32 kids it was just intimidating to me. I'd rather do small groups because I'd rather talk to individual students and get to know where they're at and try to figure out what step they're stuck on so I can help them more. Rather than just "Okay you're one in 32, I hope you understand it.

This excerpt shows that her sense of comfort with small groups seems to have been supported through the teacher preparation program at the university (e.g., university coursework and field experiences). As she worked with other preservice teachers in small groups at the university, she had developed the personal aspect of her teacher identity as a person who felt comfortable working in small groups.

In addition to explaining why she uses small groups in her teaching, the excerpt above suggests a general way that she intervenes in small groups. She prefers to interact with individual students in small groups, get a better sense of what the students do and do not understand, and support them to understand mathematical concepts.

In the lesson for which this aspect of her current teacher identity was invoked, the students had worked in small groups on figuring out the cost of dinner by adding the cost of ingredients that they would need to make the meal. While monitoring the classroom, the teacher looked at the numbers on the worksheet of the students in one small group. "I was wondering if they were estimating or I was wondering if they were just taking the number, the dollar values, or what they were going on." She mentioned that her main concern in the small group was

related to "the [dominant] students that [were] going to kind of take over and just run with things." She did not want the other two "shy" students to passively copy answers of the dominant students. She hoped that "those [two] students especially were understanding and that they knew what was going on and where those numbers were coming from and why we were adding the two together at the end." She intervened in the small group to clarify what they were doing. She called on the shy students to make sure they understood (e.g., "Alice, what's this? What are all the numbers over here?").

Though it seems general, this personal aspect invoked by the teacher in the excerpt above seems to support the understanding of her intervention in the specific small group. The teacher expressed being more comfortable in small groups. The teacher was comfortable at "talk[ing] to individual students and get[ting] to know where they're at and try[ing] to figure out what step they're stuck on." In her actual intervention, the teacher wanted to know more of what the two "shy" students understood by asking questions for clarification for what they were doing. Her attempt to "know more of what students are stuck on" drives the teacher's habit of practicing to understand the students' current understanding and struggles. The habit seems to influence her intervention in a specific way in which she asked those "shy" students clarifying questions because she felt they needed more help because they "seemed to be struggling."

"I was one of eight kids" (Third stimulated recall interview).

Leslie also invoked a personal aspect of her current teacher identity when explaining the fourth intervention episode in the third stimulated recall interview (T1A3E4). In this lesson, the students in small groups had worked on addition, money, and decimals in the context of teaching scarcity and opportunity cost. The students engaged in a group activity based on a hypothetical scenario regarding students' background knowledge about Black Friday shopping.

Looking at the students' worksheet, the teacher attended to slow progress they were making on the group work. "I saw they only had one answer written down." She knew that some "higher side" students in the small group tended to take over the small group work and "lower side" students joined passively. "Seeking to understand" whether the students "were thinking of total as the whole sum altogether," the teacher approached the small group and told the students her expectation to find exact costs that they should obtain by adding three totals, pointing out what students were missing in solving the group task.

Being asked to talk more about the intervention, the teacher invoked the personal aspect of her current teacher identity that has been shaped by experiences as a child in her family. As mentioned above, she had seven siblings, some of whom were older. In this excerpt, the teacher explained her older siblings as a source of learning for her within her family:

I think that every student has strengths and weaknesses. And I think they vary across the board. So, I have some students that are really good at drawing, and some students that are really good at reading comprehension. So, when we do a reading comprehension activity that's all about drawing, I try to pair those two students up together so they can kind of build off each other, and help each other's strengths and weaknesses and help them grow. And I think that-- I mean I was one of eight kids, so I-- my brothers and sisters were constantly-- I knew I could go to my one sister and she was really good at quizzing me, versus I know I could go to my other sister and she was really good with helping me with writing. So, I knew that I had all these resources that I could go to. So, I really want to build that community in my classroom too, where I'm not the only resource you have to go to. If I'm working with a student, there are 17 other kids here that you can

go to help you. Now, they're not supposed to give you the answer, but they're supposed to help you kind of understand it too.

She emphasized in this excerpt that everyone has "strengths" and "weaknesses." This acknowledgment was tied to her experience as "one of eight kids." Her older siblings were able to help her with things she was not good at or could not do alone, such as quizzing and writing. Her experience as "one of eight kids" shaped her reasoning about her intervention.

What the teacher did in her intervention was make sure that the students learned from each other to find the exact costs of the totals. She intervened in the small group this way because she wanted her students to see and use each other as a learning source. This idea of other students as a learning source "that you can go to help you" explains her intervention in the particular small group. In another intervention episode (T1A1E4), the teacher also emphasized this idea. She "wanted the students to also be the teachers" instead of leading the students "right to the answer." Her emphasis on students being teachers of each other was built on her personal experience as one of eight kids.

"I was always that student" (Second stimulated recall interview).

Leslie invoked another personal aspect of her current teacher identity when explaining the first intervention episode in the second stimulated recall interview (T1A2E1). However, in this example Leslie also invoked professional aspects of her current teacher identity along with the personal aspect.

In this lesson, the students in small groups had worked on solving a story problem in which students "were reviewing how to find the profit using the cost that it takes to make an object or good or service and the cost that it takes to sell the object or service." While monitoring the classroom, Leslie noticed the look on Yasemin's face in a small group. Yasemin's facial

expression informed Leslie that Yasemin might be upset and frustrated. The teacher also knew that Yasemin always "did not want to be wrong" and that when she made mistakes, she tended to have a "mental block." The student ended up being "afraid to try anything else."

In her response to this small group, the teacher was talking to get Yasemin back into the group. "I have to get her back on track because otherwise the whole group is just - I mean those two girls aren't gonna talk the boys would be the only ones talking." The teacher tried to help the students, particularly Yasemin, make more sense of what they could do with the story problem by providing more real-life examples (e.g., "I have to go to Walmart and buy that right [inaudible], Ok? One empty cup costs 40 cents. The two things I have to go to Walmart and buy are what?").

Being asked to talk more about her intervention, the teacher invoked the personal aspect of her identity as a learner. She used to be the student who "got nervous," and she used that experience to interpret the behavioral responses of students in her class:

I was always that student that got nervous, but I still answered. So, for me, it was always, "They're being defiant. They're not doing what I'm asking them to do. They're not participating in class." And I would drop their conduct grade because I didn't understand.

This excerpt suggests that building on her experience as a learner, the teacher seemed to expect her students to participate actively in small groups. With that expectation, she used to interpret students' non-participation behaviors as "being defiant" and as a result would have dropped Yasemin's conduct grade.

In addition to the personal aspect, however, in the same interview, Leslie called on a professional aspect of her current teacher identity building on learning experiences in a

professional development. She talked about a new professional orientation that changed her view of seemingly "defiant" students.

But for them [instructors in a professional development] to send someone to sit down and talk to me and say, "No, this [being defiant] isn't what's going on. This [being nervous and anxious] is what's going on in their brain." I mean, it gives you a whole new perspective and you treat that student a little bit differently than you would if you just thought, "Oh, they're being defiant."

This excerpt shows that learning from the professional development invited the teacher to think alternatively about what might be going on in students' minds. The teacher came to view the students as "being nervous" instead of as "being defiant." This alternative professional orientation shaped the teacher's intervention in a way to "treat that student a little bit differently." Through her actual intervention in the small group, the teacher tried to "get her [Yasemin] back on track" and help the student participate in the talk by providing real-life examples that might make sense to Yasemin.

These excerpts above show that Leslie's current teacher identity does not always stand alone with one aspect, the personal in this example, in her reasoning about intervention. Rather, Leslie sometimes invoked a mixture of different aspects of her identity. In these excerpts, Leslie invoked a mixture of aspects of her current teacher identity shaped not only by personal experiences but also by learning experiences as a professional. Leslie called on the personal aspect of her current teacher identity, which was mediated by the new orientation towards students' behaviors from professional experiences to shape her actual intervention.

Leslie's less frequent aspects of current teacher identity

Another aspect of Leslie's identity that she invoked was related to her disposition towards mathematics. This aspect of her current teacher identity appeared in one intervention episode.

"I love teaching math" (Third stimulated recall interview).

The aspect of Leslie's identity related to her disposition toward mathematics appeared as she explained the second intervention episode in the second stimulated recall interview (T1A2E2).

This example comes from the same lesson described above, beginning with the quote, "I was always that student." As a reminder, I briefly introduce the lesson again here. This lesson required the students in small groups to work on solving a story problem. The students "were reviewing how to find the profit using the cost that it takes to make an object or good or service and the cost that it takes to sell the object or service."

The teacher "was concerned about the two students" in a different small group from above. She had known they were diagnosed as "ADHD" and often "unfocused." She wanted to know their understanding because "they kind of were teetering between working well and not working well." In her intervention, she checked out students' understanding ("Does that make sense, Spencer?"). She also asked students in the small group to give an explanation and decide if they agreed with others' explanations ("make sure you and your group all agree on the explanation as well, okay?").

Being asked to talk more about the intervention, she invoked the aspect of her teacher identity related to her disposition toward mathematics. She viewed herself as someone who was comfortable with "teaching mathematics." To highlight her positive disposition towards

mathematics, she contrasted her disposition with that of a mentor teacher she had worked with in her field placement.

I think it was just... she [the mentor teacher] wasn't very comfortable teaching math so I think when you do one-on-one work, it's a lot more cognitively with mathematics because if a student doesn't understand it, it's not... Do I just understand what the process is? Or do I understand what they're saying and how do I get them to where they need to be? So, I'm not saying she wasn't good at it, I mean obviously she was an amazing teacher, but it was just something that she wasn't as comfortable with as a subject so she tried to stay away from as much one-on-one. Whenever I taught math she was always, "oh do small groups! You get this! If a kid needs help, go see Ms. Leslie." It was always just kind of deflected to me, which was fine. I love teaching math so I didn't mind but I think if a teacher's not as comfortable with a subject, he or she is less likely to do small, more intimate work with the students because it might not make sense to them and then they don't want to sound foolish as well.

In this excerpt, using the mentor teachers as a contrasting example, the teacher highlighted her sense of comfort with mathematics and a strong affiliation with teaching the subject ("I love teaching math"). She also expressed her confidence in working with a student who did not understand and needed the teacher's help related to the process of problem-solving. For Leslie, this kind of strong disposition is an essential foundation for a teacher to do "more intimate work with the student."

It seems that this "more intimate work" could help explain Leslie's intervention in which she asked the students to explain what they understood and to know if they (dis)agreed with

others' ideas. She emphasized that this kind of intimate work involved one-on-one work with students in small groups. This intimate work is not what some teachers like the mentor teacher might be strongly comfortable with. By making this contrast, the teacher highlighted the dispositional aspect of her teacher identity to explain her "intimate" kind of intervention in the specific small group.

Marva's frequent aspects of current teacher identity: Role-related aspects

Marva invoked aspects of her current teacher identity related to her view of teachers' roles as a teacher. I label this aspect as role-related aspects of teacher identity. These role-related aspects were linked frequently to Marva's intervention in small groups. These aspects were distributed across her reasoning about intervention over other aspects and invoked more frequently than other aspects. This is not to say that each of these aspects appeared multiple times in all four interviews and in all intervention episodes. However, I present these examples as *frequent* aspects because of their relation to Marva's role-related experiences as a teacher. She used these aspects to explain her intervention related to actual situations.

"For me it fits into me as a teacher" (First stimulated recall interview).

Marva invoked the role-related aspect of her current teacher identity for making sense of the first intervention episode in the first stimulated recall interview (T2A1E1). The goal of this lesson was "follow procedures of addition and to recognize that addition is two parts making a whole." In this lesson, the students in small groups had worked on simple addition problems with a missing part in the addends, such as 5 + [] = 9.

Marva recognized the "typical" body posture and action of Jordan, a male student in a small group. While other group members were working together in the small group, this student

was "back in his chair" so "he couldn't see the paper, and he was too far away to be able to hear and listen to them."

In Marva's intervention, "without putting it [the off-task behavior] on him," she encouraged the group members to work together ("are you guys working together?"). The teacher also wanted the group to include Jordan in the group work ("Who are you supposed to be working with right now? You three are supposed to be working together, not just [inaudible] and Rosa, you can't leave Jordan out.") By doing this, the teacher wanted Jordan to realize that he had to get back to work with his group members.

Being asked to talk more about her intervention, the teacher detailed a view of her role as a teacher. In connection to her view, she seemed to have a certain expectation towards students working together with each other.

I think for me it fits into me as a teacher because I try to give them that opportunity to work in groups and it's usually a privilege to work in groups because they like working with each other and nobody likes sitting there working quietly. So, for them, it's being able to be responsible with that because a lot of people say, "Oh, they're only in first grade." But I'm like, "Yeah, now's the time to teach them that responsibility and to listen and follow directions." Because if you wait it's going to be that much harder to get them to do it. If you set the expectation, they can do it. It's just setting that expectation and sticking to it. And so for me-- with that in mind because of how I am as a teacher because of that, So I see he wasn't really working, he was off task then the expectation is you're engaged in it. And it's not responsible to just be sitting there because now you're not learning. And they know that the reason they're in school is to learn and that's one of the

things I always go back to is, "We're here to learn. You have to be trying, if you're not trying you're not learning."

When Marva said, "it fits into me as a teacher," the "it" in this excerpt meant for the teacher to give the students a chance to work with others in small groups, which was her view of her role as a teacher. This excerpt shows her view in a strong connection to an expectation and belief that she held in relation to students' cooperation. This particular view of her role as a teacher seemed to shape the way she intervened in the small group. In her intervention, when she saw Jordan "off-task," she tried to help him and his group peers recognize that they had to work together with others "to learn" from each other. What Marva did in her intervention was to provide the students with an opportunity to work with others. By this intervention, she seemed to put her view of teachers' role in intervening in small groups into practice. Marva used this role-related aspect of her current teacher identity to explain the way she intervened in the small group, particularly in relation to Jordan's typical off-task behavior.

"It's just me being flexible" (Second stimulated recall interview).

Marva also invoked a role-related aspect of her current teacher identity to explain the fourth intervention episode in the second stimulated recall interview (T2A2E4). The goal of this lesson was "figure out how to use the measuring tools and the ideas behind accurately measuring." In this lesson, the students in small groups had to measure the length of their desk, a marker, their book, and then a fish figure that was in the book using connecting blocks, paper clips, rulers and tape measures.

While monitoring small groups, she heard a student saying "stop" to another student in a small group. She stepped in with an intent to "cut the attitude" and to understand what was going on in the small group.

In her intervention, she asked the students to explain how they measured the book with paper clips because the students did not have the same measurements ("Let's see. How did you measure it? Show me"). She found that a student said "stop" because the student "didn't like the way she [another student] was measuring with the book open." The student measured the book and got eight, while another student got seven because of leaving "too many gaps" between paper clips. The teacher tried to facilitate them to help them figure out what was wrong on their own ("What do you think that [leaving gaps] means for a measurement?").

Being asked to talk more about her intervention, her role as a teacher trying to "be flexible" was invoked several times in relation to this particular intervention episode. Marva invoked the role-related aspect of her current teacher identity depending on students' needs she perceived at the moment. The excerpts below show her view of teachers' role with respect to flexibility.

A lot of it is-- it's just me being flexible and so hopefully just trying to be flexible. It's just me finding, and knowing the students, and what's happening and just trying and going with the flow and trying to figure out what's happening.

Because if I was just, it's got to be this way, it would never get done correctly. Just because if I went with what was planned and stuck to it, either half the kids would never get it, or things would fall apart and kids would probably get hurt, because they'd start fighting or something out of nowhere. So just being able to adjust and know what the needs are at the moment and just finding ways to meet those needs.

According to these excerpts, in general, when Marva perceived a need to intervene in a small group, she first tried to find out what was going on in the small group. She also tried to "go with

the flow" in the group. This role related to flexibility emphasizes what the teacher should do. The teacher is responsible for "knowing what the needs are at the moment and just finding ways to meet those needs." This flexibility exists in "all the subjects, even just and now even in the classroom just being in a school."

There might be several events happening simultaneously in a small group that might need Marva's attention and intervention in a flexible way. "Being flexible" as one of the role-related aspects of her current teacher identity influenced ways for Marva to intervene in the small group. In her intervention in this interview, she needed to respond to the students' argument. She also had to help them revise their strategies to measure the book without telling them what was wrong. Even though she had an intention to "cut the attitude" in the beginning, her actual intervention was more about leading the students to thinking about their measuring strategies. This actual intervention related to learning mathematics seems to be an example of her "just being able to adjust and know what the needs are at the moment." She might feel a need to help the students learn mathematics at the moment by "going with the flow." This way Marva called on the aspect related to flexibility to reason about her intervention in the specific small group.

"I'm more of just a monitor." (Third stimulated recall interview).

Marva also invoked the role-related aspect of her teacher identity as being a monitor. This teachers' role as a monitor appeared in two intervention episodes. This example below comes from the second intervention episode in the third stimulated recall interview (T2A3E2).

The goal of this lesson was "using a double tens frame as a strategy to solve a story problem." In this lesson, the students in small groups needed 1) to find things that they had noticed about double tens frames and 2) to solve a story problem using double tens frames.

While monitoring small groups, the teacher saw Jordan, the male student who was introduced earlier, "laying on the table with his head down" and another female student, Annie, "sitting there playing with counters," and the other female student, Ruby, "doing all the work and trying to talk" to Jordan and Annie. The teacher felt that Ruby was "defeated" because the other two students "weren't listening to her."

In her short intervention, the teacher reminded Jordan and Annie that they had to help Ruby find what they noticed about double tens frames ("I hope you guys are working, helping her.") The teacher also told the two students to realize what they were doing ("I don't see you working and I see you playing.")

Being asked to talk more about her intervention in this interview, she invoked her role as a teacher trying to be "a monitor."

At that point, I'm more of just a monitor. I'm monitoring them and letting them know, "You're not doing what you should be doing. This is what you should be doing." I'm not really going to step in and babysit them. I'm just monitoring, say, and alerting them.

By being "a monitor," in general, Marva perceived her role as someone who lets students know what they are doing wrong and warns them to keep them from doing it. In relation to this specific small group, the teacher felt "no point in staying that long." The teacher did not need to "babysit" the students because she had "already told them that they needed to work." In her short intervention, instead of providing content help or mediating the students' mathematical thinking, the teacher tried quickly to help them see what they were doing and asked them to work together. The teacher seemed to draw on the role-related aspect of her current teacher identity as being a monitor to make sense of her short intervention in the small group. In another intervention episode, Marva invoked this role as a monitor to explain her intervention in another small group in a different way. In the fourth intervention episode in the first stimulated recall interview (T2A1E4), she talked about a "refreshing" moment when she watched and listened to some students in a small group work well together. This was refreshing because "that's one of the things I would really like to see. I don't get to see it as much as I want to." Helping them in the small group while saying little of what to do resonated with her role as a monitor. "As a teacher I want to see [the students] learning from each other." This way the teacher used this role-related aspect of her teacher identity to make sense of her intervention in small groups.

Marva's less frequent aspects of current teacher identity:

Marva invoked another less frequent aspect of her teacher identity. This aspect was related to her personal experiences related to working in groups. This personal aspect of her current teacher identity appeared two times in the same intervention episode as she reasoned about her intervention in small groups. I introduce this aspect drawing on the intervention episode where Marva invoked this aspect of her current teacher identity.

"He called me a control freak." (Second stimulated recall interview).

Marva invoked this personal aspect of her teacher identity when making sense of the first intervention episode in the second stimulated recall interview (T2A2E1).

As I mentioned earlier, in this lesson, the students had to "figure out how to use the measuring tools and the ideas behind accurately measuring." As a group activity, the students in small groups had to measure the length of several items around them.

Marva saw two students measuring using different tools. A male student was using the tape measure to measure the table. Another female student was measuring the same table with
the ruler. Although Marva had asked students to work together, she thought, "Well maybe they're using both the tools for some weird reason?"

In her intervention, the teacher told the students her expectation ("You guys are supposed to be working as a group.") She had them recognize the task instruction ("You do it together. Like one of you guys might measure while the other one writes. Or you guys can maybe one person hold something at that end, while the other person holds it at this end".) But the teacher did not tell them more explicitly how to work together ("I'm not going to tell you how to do it, you need to figure out how it works for you where you are working together as a group.")

Being asked to talk more about her intervention, the teacher recollected her personal experience. In her recollection, as a middle school student, she did "a lot of things" in the small group to get a group work done when another group member "slacked off." Her teacher called her "a control freak" because she "took over" the group work.

I guess growing--when I was going through school, when you were working in a group, it was expected you would work with your group. Because I guess looking back, I do specifically remember one teacher--I kind of took over and did a lot of things in a group because I didn't trust group members, because there happened to be one specific person in the group who slacked off and didn't like doing things, so I took over and just did their part. And he called me *a control freak*, and it's a group project and you need to work as a group. I guess it kind of does go back to at least middle school, the idea of working with the people in your group. Just because they're there doesn't mean they're actually working as a group.

In this excerpt, Marva highlighted the personal aspect of her teacher identity. In the same interview, the teacher said, "I hated the way he [the teacher] called me a control freak for wanting to do well and take over the other parts." She disliked the teacher's negative view of her as a learner. From this personal experience, Marva knew that just telling students to work together does not make them work together. She also seems to reason that with such an intention as hers, some students may do a lot of things to get group work done. The excerpt suggests that instead of making those students view themselves as a control freak, she wants to intervene in small groups in a way to help students to figure out how to work together with peers.

This personal experience seemed to influence Marva's intervention in the specific small group. Her intervention above was that she told the students her expectations and gave them a chance to think about how to work together with others on their own. In the same interview, she said, "That's why I wanted to give them the choice and just say, 'you need to figure it out, I'm not telling you how. Figure it out, but you're working--you need to work together." This quote shows that she wanted the students to figure out how to work together on their own. The personal aspect of her current teacher identity as a learner called "a control freak" seems to mediate her explanation about her intervention in the particular small group.

The Two Beginning Teachers' Designated Teacher Identity

In the previous sections, I illustrated how both teachers' current teacher identities were used by the teachers to explain their intervention in small groups. In this section, I describe how both teachers also invoked their designated teacher identity in explaining their intervention in small groups. In particular, I describe the role-related aspects both teachers drew on as a common aspect to explain their interventions in small groups. Both teachers expressed who they want to be as a teacher in relation to intervention in small groups in the future.

Leslie: "I don't want to be the teacher that's constantly harping on the kids." (Second stimulated recall interview)

This example appeared in the second intervention episode in the first stimulated recall interview (T1A2E2), which was described in the earlier section. I re-introduce briefly the information I described in an earlier section for the purpose of showing how Leslie called on the personal aspect to reason about her actual intervention.

This lesson required the students in small groups to work on solving a story problem. The students reviewed "how to find the profit using the cost that it takes to make an object or good or service and the cost that it takes to sell the object or service." Since the two students in the small group were "ADHD" and often "unfocused," the teacher wanted to figure out their understanding. In her intervention, she checked out the students' understanding. She also asked students in the small group to explain their ideas and to make sure they agree.

Being asked to talk about what this particular intervention meant for her as a teacher in this interview, Leslie drew on the role-aspect of her designated teacher identity.

I think it gives the kids a chance to kind of express their-- it challenges the older kids, or not the older kids, the higher kids to verbalize what they are saying. Because some kids were, "oh! I know this, I know this, I know this, but I just can't say it". My teachers always told me growing up if you can teach it, you know it. If you can't, then you're kind of teetering on that. I know it but I don't really know it, and so I guess giving the other kids-- I don't want to be the teacher that's constantly harping on the kids. You need to be refocused, you need to be refocused, you need to be refocused. And so giving the other kids a chance to kind of bring them into conversation and help them out. That way they don't feel like I'm constantly nagging them. It also gives them a chance to kind of kind of prove to me that, "Hey I can focus, I can get along with my friends. But I can also learn at the same time," so it's not just playtime, it's also work time.

In this excerpt, Leslie used her teachers as an example to suggest that students need to have a chance for them to verbalize what they know by interacting with others in small groups. To give the chance, Leslie did not want to "be the teacher that's constantly harping on the kids," which was the role-related aspect of Leslie's designated teacher identity. When she tells them what to do, working with others on their own is less likely to happen, which leads them to depend on the teacher. Instead she needed students to have a chance to "teach" each other what they were learning.

Leslie invoked this role-related aspect of her designated teacher identity to make sense of her actual intervention. In her intervention, she tried to give the students more chances to explain their strategies to each other. The teacher wanted her students to recognize that they could focus and learn from each other on their own, which could be benefitted by "nagging them" less. This shows that Leslie sometimes makes sense of her intervention by calling on her designated teacher identity.

Marva: "As a teacher I'd rather be just almost eavesdropping and watching them interact." (Second stimulated recall interview)

This episode was described in the earlier section related to an intervention episode (T2A2E4) where I demonstrated Marva invoked her current teacher identity. Additionally, Marva invoked designated teacher identity to explain her intervention. In this episode, Marva

invoked her current and designated teacher identity to make sense of her intervention in small groups.

I re-introduce briefly the information I described in an earlier section for the purpose of showing how Marva's designated teacher identity is related to her intervention. In this lesson, the students were expected to "figure out how to use the measuring tools and the ideas behind accurately measuring." In the lesson, the students in small groups were asked to measure the length of several items around them using non-standard measuring tools (e.g., paper clips).

The teacher noticed students arguing with each other about how to measure a book. The teacher tried to "cut the attitude" and to understand what was going on in the small group. The students argued because a student did not like the way another student was measuring with the book open. In her intervention, the teacher asked the students to explain how they measured the book with paper clips because the students did not have the same length of the book as measuring it. She found that a student measured the book and got eight with "too much space" between paper clips. The teacher tried to facilitate them to help them figure out what was wrong on their own.

Being asked to talk about what this particular intervention meant for her as a teacher in this same interview, Marva called on the role-related aspect of her designated teacher identity as a teacher. This aspect suggests that the teacher wanted to be the one who watches students learn from each other and step back.

[A]s a teacher I'd rather be just almost eavesdropping and watching them interact and see them learn from each other and almost step back. I'm just hovering, and just when I need to, I step in and I'll intervene. I don't like to just be all-knowing, because I'm not. And

they know that I don't know everything and I tell them that, "I don't know everything." In the beginning, they're shocked, but then they're like, "Wow, well, we all know something." And it's funny because it's their way too, because they kind of teach me, and it makes it fun for them because we're all learning from each other and they do like to try and teach me little things.

In this excerpt, intervening in small groups, Marva would rather be "almost eavesdropping and watching them interact and see them learn from each other and almost step back," which would be the role-related aspect of her designated teacher identity. This is because "as a teacher [she] want[s] to see [students] learning from each other," she said in the same interview (T2A1E4). Marva also acknowledged that there were times she had to "step in" and "intervene," instead of almost stepping back. With the particular small group, however, she decided to intervene because of the argument due to different ways of using non-standard measuring tools in the group. Her intervention seemed to reinforce her ideas of what she wants to be as a teacher intervening in small groups.

In the excerpt, the teacher devoted more talk to benefits ("we're all learning from each other and they do like to try and teach me little things.") These benefits come from students' cooperation in small groups when the teacher almost steps back. By telling more of what the teacher wanted to see in small groups, the teacher seemed to try to highlight her vision of who she wants to be in the future in the context of intervening in small groups. She would like to be the teacher who minimizes interventions in small groups unless necessary.

When Marva explained her intervention in this episode, she also used the aspect of her current teacher identity related to "being flexible." She viewed herself as a teacher who was flexible in a way to try and go "with the flow" and try "to figure out what's happening" in the

moment. This example suggests that making sense of her intervention in a particular small group sometimes involved her designated teacher identity as well as her current teacher identity.

Summary

In this chapter, I (re-)constructed multiple aspects of the current and designated teacher identity that Leslie and Marva invoked to explain their intervention in small groups. I illustrated aspects of current teacher identity used more or less frequently by both teachers to make sense of their intervention in small groups. Leslie more frequently drew on personal aspects of her current teacher identity as a learner and a child ("I was the more shy, quiet one" and "I was one of eight kids"). Leslie less frequently invoked her disposition-related aspects of her current teacher identity towards mathematics ("I love teaching math.") Marva very often invoked teachers' rolerelated aspects of her current teacher identity ("For me it fits into me as a teacher" and "It's just me being flexible.") Marva' less frequent aspect of her teacher identity was related to her personal experiences related to working in groups ("He called me a control freak.") On the whole, their reasoning about their intervention in small groups were tied closely to these distinct aspects of both teachers' current teacher identity. I also described how both teachers invoked their designated teacher identity in explaining their intervention in small groups. I described the role-related aspects both teachers drew on to explain their interventions in small groups ("I don't want to be the teacher that's constantly harping on the kids" and "as a teacher I'd rather be just almost eavesdropping and watching them interact."). This chapter suggested that making sense of intervention in a particular small group sometimes involved teachers' current and designated teacher identity.

CHAPTER 7. CATEGORIES OF STUDENTS AND THEIR ASSOCIATIONS WITH POWER IN FIGURED WORLDS OF MATHEMATICS CLASSROOMS

In this chapter, I construct relationships between categories of students and power and authority dynamics. These categories and dynamics were recognized by Leslie and Marva when they provided their elaboration, explanation, and reasoning about their intervention in small groups.

A figured world happens as a social process (Holland et al., 1998) in which power and authority dynamics play out in the interactions and relationships among everyday actors on a daily basis. Teachers and students are everyday actors who occupy figured worlds of mathematics classrooms. Students are actors who are expected to produce certain meaningful acts and valuable outcomes in relation to small groups. Negotiations with their peers happens as a natural process in working together. Some students have more authority in these negotiations that other students. To support these negotiations between students, teachers very often intervene in small groups. It is very often teachers that dominate these negotiations. Their intervention is aligned largely with certain expectations, values, and beliefs from both within and outside of the mathematical classrooms, including teachers' recognition of different categories of students as actors and of power and authority dynamics among actors deserves attention in figured worlds of mathematics classrooms.

To that end, first, I describe categories of students both teachers recognized as they explained their intervention in small groups. This description is my construction of more or less common categories recognized by both teachers. Second, I demonstrate the relationship between categories of students and intervention in small groups. I explain how these categories are related

to both teachers' intervention patterns through their talk and sometimes practice. Third, I elaborate on how both teachers recognized power and authority dynamics in connection with certain categories of students at three levels: micro-interaction level, classroom level and macro level. Particularly, I illustrate their recognition of these dynamics in relation to micro-interaction level (e.g., power dynamics between different categories of students and between teachers and students). Fourth, I provide the summary of this chapter

Categories of Students Recognized by Two Beginning Teachers

In this section, I detail certain categories of students Leslie and Marva recognized as they explained their intervention in small groups in figured worlds of mathematics classrooms. These categories include academic achievement, engagement, personality, language development, and at-home support. I explain common categories to both teachers. Leslie and Marva both recognized categories of students related to academic achievement, engagement, engagement, and personality. Sometimes I demonstrate differences between teachers within these common categories. I end this section with categories of students related to language development and at-home support, which were recognized only by Marva.

Academic achievement-related categories of students

Leslie and Marva represented students in two categories, academically higher and lower students, in most hypothetical and actual intervention interviews. In the survey follow-up interview, both teachers recognized students as 1) academically higher students or 2) academically lower students. For example, Leslie said about a student, "this kid's really good at math" (T1H1E7). She also said, "the kids don't do high. ... the kids that didn't do so well" (T1H1). Marva positioned students as having high or low mathematical understanding ("he's

very high" and "the ones who may not be where they should be at") (T2H1). Like in these examples, both teachers used these categories of higher and lower students to describe certain students in hypothetical intervention interviews.

In actual intervention interviews, both teachers also drew on these categories of students to explain certain students. Observing students working in a small group, Leslie recognized "students that seemed to be going through it no problem" (T1A1E1). In the same interview, she also recognized "the two students that I called on I knew were some of the lower ones." In these quotes, she recognized certain students as being higher and lower in mathematical understanding. Marva also recognized students in terms of high and low students. "If he's not getting it, I know the rest of them probably won't get it in this group." (T2A2E2). In this quote, Marva positioned one student as having a high mathematical competency and rest of the students as lower in their problem solving. Through their interviews, as a whole, both teachers repeatedly used these categories in their explanation of intervention and positioned certain students in small groups as academically higher and lower students.

Engagement-related categories of students

Leslie and Marva recognized categories of students related to engagement through their explanation of intervention in small groups. This engagement includes four categories- 1) good teammates, 2) taking-over students, 3) disengaged students, and 4) leaders. There was one noticeable difference between these two teachers. Leslie recognized students using all categories. Marva recognized all categories except the category of leaders. In common, however, both teachers represented students in terms of engagement in most hypothetical and actual intervention interviews.

Good teammates.

Both teachers invoked the "good teammates" category to explain students' participation in small groups. Leslie tended to identify students in terms of whether they were good teammates in small groups. She recognized students being "good teammates" or "good team workers" as an important ability that students must develop for the future workplace (T1H1). Being in the good teammates category means to be students who "work very well socially with other kids" (T1A3E2). To be good teammates, some students "really need to work on their social skills" (T1A3E2). Marva also viewed some students as "a good partner," who happened to be her "bestbehaved students in the class" (T2A1E4).

Taking-over students.

Both teachers used this category to describe some students who dominate group work. Leslie described some students as someone who "will just kind of take over" (T1H1) when explaining her role as a teacher in intervening in small groups. These students tended to dominate group work over other students. "The students that are going to kind of take over and just run with things" (T1A1E1). In many cases, she described students in small groups as someone who "just kind of took it and ran with it" (T1A2E2). Marva also recognized this category of students in small groups. Indicating some students who dominated group work, she described those students in terms that "they're Mr. Tough guy" who does "boss people around" (T2H1E8). Marva recognized this category in her actual observation of some students. "She's a little controlling amongst the other Arabic speakers" (T2A2E3). Marva acknowledged a tendency for the girl to take over group works in working with other students in small groups. As a reminder, she talked about a time when her teacher called her "a control freak" (T2A2E2)

because she took over a group task. This experience suggests that Marva recognized some students as taking-over students even though she did not use this term for her students.

Disengaged students.

Both teachers used this category of disengaged students to describe students. Leslie recognized some students as being disengaged. For example, these disengaged students are the students who "are not as focused as they should be" (T1A2E2). They are also disengaged because they are distracted. According to her, they are "the kids that want to goof off" (T1H1). Marva described some students as "the student who is usually the one who's off-task" and "He's very busy, always everywhere, everyone's business" (T2H1). Students within this category of disengaged students seem to be also "the ones who haven't responded to anything" that was required to do in small group work (T2A3E4). Like in these examples above, the category of disengaged students is recognized by both teachers in relation to intervening in small groups.

Leaders.

As mentioned earlier in this subsection, only Leslie used this leader-related category to explain some students' behaviors. She used this category to position a student as a leader. "He's very much the leader in the class" (T1A3E1). According to her, students as a leader are one natural pathway related to the development of students in fourth grade as a transition year (T1H1):

I think in fourth grade, especially, it's a transition year. And so it's kind of hard for some of the kids to come out of their shell. And so, if a kid is coming out of their shell that normally doesn't, and they're starting to be authoritative, you don't want to squash that

authority, but you want to make sure that it's put in the right place. So if a kid wants to be a leader, I want them to still have that chance.

This excerpt suggests that she recognized some students as "the kids that are natural leaders" (T1H1). For those students, she wanted to give students the chance for students to develop authority as a leader. When she said, "multiple students that want to be the leader" (T1H1E8), she acknowledged students in terms of this category.

Personality-related categories of students

Leslie and Marva recognized categories of students related to personality through their explanation of intervention in small groups. This personality includes three categories, 1) shy students, 2) outspoken students, and 3) anxious students. This does not necessarily mean that both teachers recognized and used the three categories. Leslie recognized students using all categories. Marva recognized shy students only. In common, however, both teachers represented students in terms of personality in most hypothetical and actual intervention interviews.

Shy students.

Both teachers recognized the category of shy students as they explained students' participation in small groups. Leslie put some students into this category. Some students are "a kid that's super quiet" and "the kid that's a little bit quieter." (T1H1). She knew that some students wouldn't speak up as much" because they "were a little bit shyer" (T1A2E1). This category was also used by Marva as she described some students. She described a student as a shy student. "He's very shy, because I know his mom says he doesn't even like talking, like asking his dad questions or having his dad ask him things because he's so shy and he kind of shuts down" (T2A1E1). This quote indicates that she recognized some students in terms of this

category. It also suggests that she has formed the category of shy students not only from her own experience with students but also from interactions with their parents.

Outspoken students.

Only Leslie used this category of outspoken students to explain ways students work with others in small groups. Students in this category are on the opposite side of shy students. "One student that I know is not very shy. So they're very forward and they'll be the speaker if they need to" (T1H1). Since they tend to be "really confident in what they're doing" in small groups (T1H1E7), they look very "impulsive" and "competitive" (T1A1).

I know there's a lot of students in my class that are very competitive. And so they always want to be right and they want to be the person that discovered it the first time. They want to be the person that can kind of brag about it.

In this quote, she elaborated on what it looks like for outspoken students to work with others in small groups. Outspoken students wanted to be doing group tasks quicker and better than their peers.

Anxious students.

Only Leslie used this category of anxious students. She invoked this category to describe some students as being anxious. She stated that some students "constantly come to school anxious or nervous that they're going to get called on" or "have testing anxiety" (T1A2E1). A quote (T1H1E3) related to her experience working with a student also suggests her recognition of students in relation to this category.

I see that with a lot of my kids. My first year I actually had a student that-- I didn't know exactly what was going on until I went to a conference about it. But he had the deer in headlights. He would just freeze whenever you asked him a question unless you gave him time to prepare it ahead of time. Because he had that anxiety that he was going to be wrong.

This quote suggests that she positioned one student as being anxious. The student had an anxiety that "he was going to be wrong" (T1H1E3). Their anxiety and their fears flare up and can cause "their brain just kind of to a point shut down" (T1A2E1). This quote above also suggests that she frequently recognized this category of students ("I see that [anxiety] with a lot of my kids") (T1H1E3). All examples above suggest that she recognized students in terms of this category.

Categories of students recognized only by Marva

Language development-related categories of students.

Only Marva recognized students in terms of 1) language learners or 2) fluent speakers. Even though these categories did not always refer to English Language Learners (ELLs), she used these categories to describe mostly ELLs because they happened to be about 75 percent of her student population in her classroom. She positioned some students as language learners because they "can't read the tasks because of language" (T2H1). Students as language learners were the ones who needed "the kid-friendly language" which made mathematical concepts "more accessible to them" (T2H1). A quote (T2A3E2) suggests that she positioned students into this category. I mean, fully consider all the students' needs - and usually I try to consider all the needs of the students, and in that group - I didn't account for, while Nellie does speak fluent English, her other language is Bengali. And the little boy, well, he's a native English speaker. He almost might as well be an ESL student because he's pretty much just learning how to talk, after that having a speech delay all the way up until the end of kindergarten. And the other girl, Nadia, is an ESL student but her first language is Arabic.

This quote shows Marva recognized some students as fluent speakers. A native speaker could be viewed as a language learner (ESL: English as Second Language) due to "a speech delay." All the quotes combined, she seemed to assume differences in the language development among students (some students who need "the kid-friendly language" and some students who "speak fluent English"). Like shown in these examples, this language development category is one category of students she kept in mind when trying to understand students' needs.

At-home support categories of students.

Only Marva recognized this category of students related to at-home support through her explanation of intervention in small groups. She recognized students in terms of the degree of parental support of students' learning at home. "We do have some parents who don't teach these things at home. So, on top of those skills, the strategies, different ways of thinking through it, the social skills too" (T2H1). In this quote, she recognized some students as the ones who do not receive enough parent support. A quote below (T2A2E2) shows how she recognized one student in relation to this category of students.

He's just going to keep falling further and further behind. We've seen it with his siblings too, because I had his older sister last year, who's now in fourth grade. He has an older brother who's in second grade but should be in third grade, and they're just in reading, math, everywhere, they're all falling behind because they're not getting support at home. And actually the sister, she used to really really care about school and she would try, but just wasn't getting support at home and then got to third grade and it was like a switch just flipped.

This quote suggests that she had seen the lack of appropriate parental support of the student. It seems that she views some students as students who are not "getting support at home," which made them "keep falling further and further behind."

Relating Categories of Students to Intervention in Small Groups

In this section, I demonstrate how categories of students recognized by Leslie and Marva were related to their intervention in small groups in figured worlds of mathematics classrooms. Both teachers used different categories of students, with varying degrees, in their explanation of intervention and their actual intervention. There were two patterns I identified in relation to the relationships between categories of students and intervention. First, among different categories of students, both teachers used academic achievement-related categories of students (academically high or low students) more frequently than other categories. Many intervention episodes had these academic achievement categories in common regardless of whether both teachers intervened to support a mathematical learning process (e.g., asking students to explain their mathematical thinking) or a social process (e.g., encouraging students to work together) in

small groups. Second, these academic achievement-related categories (e.g., academically low students) were used in combination with other categories of students (e.g., shy students or language learners). It was specific to Marva that academic achievement-related categories (e.g., academically high students) were also combined with language development (e.g., fluent speakers) and/or at-home support (e.g., students who get parental support).

Building on these patterns, in this section, I present two examples to highlight the relationships between categories of students and intervention in small groups. First, I demonstrate what the relationships looked like when both teachers intervened in a small group to support a mathematical learning process. I present one example from Leslie where she used categories of students related to academic achievement in combination with engagement and personality. Second, I illustrate what the relationships looked like when both teachers intervened in a small group to support a social process. I present one example from Marva where she used categories of students related to academic achievement in combination with engagement, and at-home support. I chose these examples because Leslie intervened in small groups to support a mathematical learning process more than Marva and vice versa.

Example 1: Leslie's use of categories of students and its relationship to intervention with focus on a mathematical learning process

This example comes from Leslie's first lesson. As a reminder, in this lesson, the students were asked to work together on choosing ingredients to make the meal from the list on the worksheet. They had to figure out the cost of dinner by adding the cost of ingredients that they chose to make the meal. This intervention happened when, as she was monitoring the classroom, Leslie looked at the numbers on the worksheet that a group of four students (Alice, Jacob, Fred and Bob) had worked on. The teacher began to initiate conversation with the small group by

asking questions (T1A1E1). I numbered each utterance in the excerpt to refer to in my explanation that comes after the excerpt. I also included descriptions of what Leslie and the students were doing in parenthesis to help readers visualize what was happening.

- Leslie: (Points to some numbers on the worksheet) So what's going on over here? What's this?
- 2. Fred: This is number of- (interrupted by Leslie).
- 3. Leslie: Alice, what's this? What are all the numbers over here?
- 4. Alice: This is, uh, addition problem ... for this.
- 5. Leslie: What's two plus six? What's that?
- 6. Alice: Eight (answers quietly.)
- 7. Leslie: But where did you get those numbers from? I don't see two and six.
- 8. Alice: (Points numbers the group chose on the worksheet) From the dollars...

[inaudible] ... two numbers.

- 9. Leslie: Okay. So you're taking just the dollars and adding them up?
- 10. Alice: Yeah.
- 11. Leslie: Okay. Jacob, do you think that's the best way of doing this?
- 12. Jacob: Yes (nods confidently).
- 13. Leslie: So what do you have to do after you add up all the dollars? (asks all students)
- 14. Fred: We have to add up all the dollars (voluntarily). [inaudible].
- 15. Leslie: And then what do you do after that?
- 16. All students: We add them all together (in unison).
- 17. Leslie: Okay. Can you try? (stands up to leave the small group.)

18. Students: Yes (Students nod slightly and Leslie leaves the small group.)

This excerpt shows two ways Leslie intervened in the small group for supporting a mathematical learning process-1) asking students to explain their mathematical thinking and 2) evaluating other students' ideas. In more detail, when she looked at the numbers on the group's worksheet, she was wondering "if they were estimating" or "if they were just taking the number, the dollar values, or what they were going on." In Line 2, Fred tried to answer Leslie's question from Line 1. It would have been possible for her to not interrupt Fred so that he could continue to provide his explanation. Instead, however, she interrupted his initiation to talk. She chose to call on Alice first to ask questions. The teacher was asking Alice to explain the group's strategy to solve an addition problem they created. From Line 3 to Line 10, she intervened in the small group by asking students to explain their mathematical thinking. After listening to Alice's explanation, she shifted her attention from Alice to Jacob to ask him to evaluate Alice's explanation (Line 11 and 12). This was where the second way of intervention (evaluating other students' ideas) occurred. She continued to ask four students to explain what to do based on what they understood from Alice's explanation. From Line13 to 17, she intervened in the small group by the first way of intervention in this small group.

Leslie recognized the students in this specific small group in terms of academic achievement, engagement, and personality. These categories of students recognized by Leslie were related to the two ways of intervention. The excerpt below suggests that her intervention was influenced by her recognition of Alice and Jacob in terms of academic achievement-related categories of students.

My biggest concern was making sure because I had two students in that group and the two students that I called on I knew were some of the lower ones. So that's-- normally when I call on a student, I try to choose a student that I want to make sure they understood.

In this excerpt, the "two students" indicate Alice and Jacob, who are the ones that she positioned as academically lower students. Fred and Bob were academically higher students given that they "seemed to be going through it no problem." Since she recognized Alice and Jacob as low students who seemed to be struggling, she decided to call on Alice (Line 3 to 10) and then Jacob (Line 10 to 11) to check on with his understanding. This is a reason why she did not choose to call on Fred and Bob and why she interrupted Fred's explanation in the beginning of the intervention. This way these academic achievement-related categories influenced her intervention in the group of four students.

Leslie also used categories of students related to engagement and personality to explain her intervention in the small group. "By now I know the students that are going to kind of take over and just run with things, and so I try to choose the other students that are kind of shy or a little less outspoken." This quote suggests that she has recognized the four students in the small group in terms of categories of students related to both engagement and personality. She has positioned Fred and Bob as taking-over students ("the students that are going to kind of take over.") On the other hand, she has positioned Alice and Jacob as shy students ("the other students that are kind of shy or a little less outspoken.") This way these engagement- and personality-related categories influenced her intervention in the group of four students.

This recognition of students in terms of categories of students seems to be shaped by her concern, which might be influenced by the taken-for-granted process of the relations between

students with different categories. She might have an idea of what happens when power dynamics play out between the high/low, taking-over/outspoken/shy students. Alice and Jacob "could have been sitting there the whole time" and Fred and Bob "could have just taken over and been writing the answers." This was her concern that made her intervene in the ways she did (Line 2, 3, and 10). It seemed that she had the normative idea of the relations between different categories of students and her intervention was shaped by this taken-for-granted process.

At the end of the intervention, she became certain that they understood what they were doing because "neither one of them stuttered or took a moment of silence." Her actual intervention from the beginning to the end was also influenced by her concern informed by her recognition of certain students in relation to engagement and personality.

As illustrated above, Leslie continued to recognize students in terms of categories of students and made strongly intentional talk moves while intervening in small groups with focus on a mathematical learning process. Even though categories of students were constructed from their explanation about their own intervention in small groups, this example suggests that their recognition of students in terms of categories of students was tied closely to their actual intervention in small groups.

Example 2: Marva's use of categories of students and its relationship to intervention with focus on a social process

This example comes from Marva's first lesson. As a reminder, the goal of this lesson was "follow procedures of addition and to recognize that addition is two parts making a whole." In this lesson, the students in small groups had worked on simple addition problems with a missing part in the addends, such as 4 + [] = 8. This intervention occurred when Marva monitored two students (Ben and Brice) in a small group and she heard a student (Ben) say to another student

(Brice), "Yeah, count with me, count with me" solving addition problems on the worksheet. The teacher began to initiate conversation with the small group by asking questions (T2A1E4). I numbered each utterance in the excerpt to refer to in my explanation that comes after the excerpt. I also included descriptions of what Marva and the students were doing in parenthesis to help readers visualize what was happening.

- Marva: (Sits down on a chair at the group.) Are you guys doing it together? Because I hear you talking. (Talks to Brice) Are you copying what he's doing or are you-- or you were actually talking with him?
- 2. Ben: He's helping me count.
- Marva: Oh he's helping you count them. Okay that's good. So you guys are working together to count them out.
- 4. Ben and Brice: Yeah (in unison).
- 5. Marva: All right, can I watch you guys do one?
- 6. Ben: Yeah. Five (writes numbers on his worksheet) [inaudible]
- 7. Marva: Okay what goes in there? (talks to Brice pointing the worksheet)
- 8. Ben: Five. (looks at Bill smiling.) Brice, count with me. [inaudible] Five, six (writes numbers on his worksheet) [inaudible]
- 9. Marva: (talks to Ben) He's writing it out on the paper. (talks to Brice) You got to say it out loud though so he knows (meaning that Ben knows Brice counts numbers with him).
- 10 Brice: (writes numbers on his worksheet) Seven, eight, nine (Ben writes the number repeating Brice says each number)

11. (Brice's pencil was broken and Marva leaves the group to get them pencils)

This excerpt shows one way Marva intervened in the small group for supporting a social processencouraging students to work together. In the beginning of her intervention, she wanted to make sure that Ben was not doing all the work and Brice was just copying what Ben was doing. In Line 1, she asked the two students (Brice and Ben) if they were working together to count the numbers. After listening to Brice having worked with Ben (Line 2), she became certain that the two students were actually counting numbers (Line 3). In Line 7 to 10, she watched them count numbers together to solve an addition problem on the worksheet. In Line 9, she helped the two students work together by asking Brice to say out loud numbers so that Ben knew Brice was working with him. Encouraging students to work together occurred throughout her intervention as shown in this excerpt.

Marva recognized Ben and Brice in this specific small group in terms of academic achievement, engagement, language development, and at-home support. These categories of students recognized by Marva were related to her intervention by encouraging students to work together. In relation to this intervention, the excerpt below suggests that her intervention was influenced by her positioning Ben and Brice into these categories of students. First, this excerpt below suggests that Marva recognized a boy (Ben) in terms of engagement-related categories of students.

All right it initially looked like Ben, the one little boy who kept saying "Yeah, count with me, count with me" it looked like he was doing all the work and I know sometimes he does, because the little boy that Ben was working with is academically lower but Brice,

the one little boy, he does want to learn. He gets upset when he doesn't know things but he is eager to learn and I think that makes him and that other little boy a good partner. So he-- so sometimes I fear that he's taking over so I want to check in and make sure that he wasn't just taking over that and when I stopped I realized he wasn't and just his voice -he's a little bit louder so it sounded like he was but he wasn't

She usually positioned Ben as a good teammate or partner ("that makes him [Ben] and that other little boy [Brice] a good partner.") In the same interview, she said that she had had "Ben help some of her students who behaviorally maybe aren't so nice," which also implies her positioning of Ben as a good teammate. In this excerpt above, however, in the beginning, she also positioned Ben as a taking-over student ("It looked like he was doing all the work and I know sometimes he does.") It seems that this recognition made her check out if they were working together (Line 1: "Are you guys doing it together?") because she had "fear" that he was taking over even if it turned out he was not actually taking over. These quotes above suggest that teachers may position individual students in terms of multiple, sometimes conflicting, categories of students, as in the case of Ben as a good teammate and a taking-over student. This also suggests that teachers intervene in different ways from moment to moment shaped by the movement of their positioning of individual students from one category (a good teammate) to another category (a taking-over student).

Second, she recognized the two students in terms of academic achievement-related categories of students. In the same interview, she positioned Ben as an academically high student ("Ben is so academically high especially in math"). She also positioned Ben as someone who "outperformed the students." In the excerpt above, she viewed Brice as an academically low

student ("the little boy that Ben was working with is academically lower.") This recognition related to academic achievement seems to make her check whether Ben as a higher student was taking over and Brice as a lower student was learning passively (Line 1 and 9) which would not help Brice learn how to do addition.

Third, she recognized the two students in terms of at-home support and language development. In the same interview, Ben was positioned as a student who got high parental support ("the ones who you can see the parents working with at home.") On the other hand, Brice was viewed as a student who did not get parental support at home ("He's got a lot of siblings so he doesn't get a lot of attention.") In relation to language development, she recognized Brice in terms of language learners ("he's in -- he also -- language wise we're trying to get him recommended for speech.") In the same interview, she also positioned Ben as a language learner who did "come in with less language skills." It is not clear in this excerpt how her recognition of Ben and Brice in terms of these categories influenced her intervention. However, Lines 5, 7, and 9 suggest that she decided to stay with them to watch them work together. She particularly tried to help Brice find where to write and count numbers out loud with Ben. I infer that her consideration of these two students in relation to language development and at-home support could have shaped her intervention that encouraged Ben and Brice to keep working together as shown in Line 8 and 10.

As shown in the excerpt and explained above, Marva kept recognizing students in terms of categories of students and made intentional talk moves while staying with small groups with focus on a social process. This particular example suggests that their recognition of students in terms of categories of students was also tied closely to their actual intervention in small groups.

Two Beginning Teachers' Recognition of Power and Authority Dynamics

In this section, I construct ways Leslie and Marva recognized power and authority dynamics in connection with certain categories of students in explaining their intervention in small groups. When they used categories of students to explain students' participation in small groups, these dynamics played out at the three levels of the interactions and relationships among actors in figured worlds of mathematics classrooms. These three levels are 1) micro-interaction level, 2) classroom level, and 3) macro level. First, at the micro-interaction level, both teachers talked about these dynamics between students (e.g., academically high students and low students) and between teachers and students. Second, at the classroom level, both teachers explained an imposition of their power and authority over small groups through the establishment of norms and expectations (e.g., making sure that everybody is participating in a respectful way in a small group work). Third, at the macro level, both teachers acknowledged an impact of certain broader ideas from outside of the classrooms on themselves as teachers (e.g., understanding some students as leaders in terms of a developmental perspective in which the fourth grade is a transition year and some fourth students are starting to be authoritative).

Further analysis suggests two patterns related to the relationships between categories of students and power and authority dynamics, which I focus on in this section. First, both teachers recognized these dynamics mostly at the micro-interaction level. Second, they recognized these dynamics across all categories of students recognized by both teachers. In other words, these dynamics played out in all categories regardless of whatever categories of students both teachers recognized in explaining students' participation in small groups.

In this section, I present three examples from Leslie and Marva to shed light on these patterns. First, I share an example from Leslie where she recognized power and authority

dynamics among students in a small group. In this example, she positioned the students in the small group in relation to categories of students in relation to academic achievement, engagement, and personality. Second, I present another example from Leslie in which she recognized power and authority dynamics between students and students and between herself as a teacher and the students in a small group. In this example, she positioned a girl in the small group in relation to categories of students in relation to academic achievement, engagement, and personality. Third, I take an example from Marva in which she recognized power and authority dynamics among students and between herself as a teacher and the students and between herself as a teacher and the students in a small group. In this example, she positioned a girl in the small group. In this example, she positioned a boy in the small group in relation to categories of students in a small group. In this example, she positioned a boy in the small group in relation to categories of students in a small group. In this example, she positioned a boy in the small group in relation to categories of students in elation to engagement, personality, and language development. I chose these particular examples because they show ways the power and authority dynamics played out across all categories of students recognized by these teachers and sometimes only by Marva.

Example 1: Leslie's recognition of power and authority dynamics between students and students

Leslie recognized power and authority dynamics that played out between students and students, at the micro-interaction level, with different categories of students. An excerpt in the follow-up interview (T1H1) suggests how these dynamics played out at this micro-interaction level. This excerpt was her response to an interview question related to the reason why she used a mixed ability group.

I guess the hardest part is trying to find the right mixture between the kids because sometimes the kids have social issues. You think, okay I could pair this kid that really gets it and this kid that doesn't get it so much, but then there are social conflicts that are kind of in the background. I guess behaviors would also kind of clash sometimes because I have a few kids, especially this year, that they are a little bit impulsive. So, they just kind of shout out the answer. And so that mixed with a student that doesn't really talk, that student just is going to take the conversation and run with it versus the student that doesn't really talk. They're just going to sit back and it might not be that they don't know it. It might just be that they don't get a chance to put their two cents in.

In this excerpt, she was talking about how she formed mixed ability groups. Her talk about "social issues" and "clash" suggests that she recognized the power and authority dynamics that "sometimes" played out "in the background" between students. She acknowledged that these "conflicts" happen between students even though she tried to form small groups in "the right mixture between students."

In this excerpt, she used different categories of students she recognized in relation to academic achievement, engagement, and personality. In a mixture of students in small groups, there were students who she positioned as academically high or low students in their understanding ("really gets it" and "doesn't get it so much"). Some students were viewed as taking-over students ("take the conversation and run with it") or disengaged students ("just going to sit back"). Some were considered as shy students ("doesn't really talk") and as outspoken students ("a few kids, especially this year, that they are a little bit impulsive. So, they just kind of shout out the answer.")

Mixing students in terms of different categories in small groups is always "the hardest part" of implementing small groups. This is because different categories of students "sometimes" cause these dynamics between students. "They're just going to sit back and it might not be that

they don't know it. It might just be that they don't get a chance to put their two cents in." This quote suggests that she recognized that shy students or academically low students may not have a chance to contribute to the small group and as a result become disengaged students ("sit back"). What she said happens in the background in small groups is that these dynamics happen between students, particularly in this excerpt, between shy or academically low students and outspoken or academically high students who have different degrees of power and authority.

Example 2: Leslie's recognition of power and authority dynamics between students and between teachers and students

Leslie recognized power and authority dynamics that played out at the micro-interaction level taking place between students and between teachers and students with different categories of students. An excerpt in the second intervention in the first stimulated recall interview (T1A1E2) suggests how these dynamics played out at the micro-interaction level.

But I had to figure out the reason why the group wasn't communicating effectively; was because she didn't feel like her voice was heard or she just didn't like the group or what was going on. So by asking her and then asking another student and then asking her to repeat it and asking different strategies I kinda -- by being there I kinda got them to open up and all communicate but as soon as I walked away I think she closed right back up and right back out. So she said that she didn't think her voice was being heard but I'm -- in that group especially I think she's one of the lower ones so I think she was just maybe a little conscious of that and she was a little nervous about putting herself out there. She was being out there and she was incorrect and they were trying to correct her and she just wasn't responding well to it.

This excerpt suggests that she used different categories of students she recognized to describe a girl in the small group. She recognized the girl in relation to academic achievement, engagement, and personality. In this small group, the girl was positioned as an academically lower student in her understanding ("she's one of the lower ones so I think she was just maybe a little conscious of that."). She was viewed as a disengaged student ("she just wasn't responding well to it.") She was also recognized as an anxious student ("she was a little nervous about putting herself out there.")

In this excerpt, it seems that Leslie recognized the girl in terms of certain categories of students to understand power and authority dynamics between the girl and her peers. Before her intervention in the small group, she "heard a lot of bickering, 'you're not listening, that's not what I said, well that's wrong," and "so just that one talking over the other, talking over the other." In this excerpt, she offered her guess related to why this verbal disagreement happened on the part of a girl in the small group. "She didn't feel like her voice was heard or she just didn't like the group." To reason why these dynamics happened between the girl and her peers in the small group, she used her positioning of the girl as academically lower, anxious, and disengaged.

This excerpt also suggests her recognition of imposing her authority as a teacher over the students in the particular small group. She intervened in the small group by "asking her and then asking another student and then asking her to repeat it and asking different strategies." She had a clear purpose of her intervention, that is, she wanted to get these students "to open up and all communicate." Since she knew once the girl was self-aware of her low academic ability and anxious about "putting herself out there" in the small group, she tried to make intentional talk moves to push the girl's voice to be heard by her peers. Recognizing the girl in terms of certain

categories of students shaped her recognition of how she used her authority to deal with power dynamics between the girl and her peers. This shows her recognition of power dynamics between herself as a teacher and her students.

Example 3: Marva's recognition of power and authority dynamics between students and between teachers and students

Marva recognized power and authority dynamics that played out at the micro-interaction level taking place between students and between teachers and students with different categories of students. An excerpt in the first intervention in the first stimulated recall interview (T2A1E1) suggests how these dynamics played out at the micro-interaction level.

Before, I saw he wasn't working and he was-- I saw typical, or our typical Jordan behavior, like, hey, he's not working, he doesn't want to be doing this. I have to step in and tell him he needs to work or else. And I'm really-- we have a field trip coming up in the next few weeks, and I've been kind of threatening that he won't go, because he really wants to go, and the time-- it'll work for a short period. But it's hit or miss if it works. So I was like, how do I do this without putting it on him? So I stepped in and I asked, "Why aren't you working?" And normally, when I ask, whenever he gets in trouble or gets called out for a behavior, he just kind of stares at me. He's very shy, he-- because I know his mom says he doesn't even like talking, like asking his dad questions or having his dad ask him things because he's so shy and he kind of shuts down. But-- and that's one of the things we're just-- he's still working on just talking because he's really-- he's almost, he's ESL, almost. Well, not ESL. He's ELL. Because not a second language. But he's learning English still, and just how to talk and communicate. So I was trying to figure out how-- I knew I had to step in because otherwise, he'd be on the floor soon. And I'm glad it didn't get to that point, where he was on the floor, but I knew I had to find a way to get the group to recognize, hey, you're responsible for bringing him in, too.

This excerpt suggests that she used categories of students related to engagement, personality, and language development to describe a boy in a small group. She recognized the student in relation to academic achievement, engagement, and personality. In this small group, the student was positioned as a disengaged student ("he's not working, he doesn't want to be doing this.") He was positioned as a shy student ("he's so shy and he kind of shuts down.") She also recognized him as a language learner ("he's learning English still, and just how to talk and communicate.") She recognized the student in terms of multiple categories.

In this excerpt, it seems that she used her recognition of the student in terms of certain categories of students to understand power and authority dynamics between the student and his peers in the small group. Before her intervention in the small group, she "saw he wasn't working and he was-- I saw typical, or our typical Jordan behavior" This excerpt suggests that Jordan was excluded by his peers because of his disengaged behavior. It seems that his disengagement was also explained by the teacher in relation to him as a language learner and shy student. Since he often shut down (shy student) and was still learning English (language learner), his peers had difficulty in communicating with him and understanding his thinking.

This excerpt suggests that recognizing Jordan in terms of several categories helped Marva find ways to use her power and authority as a teacher to deal with these dynamics between Jordan and his peers. She intervened in the small group in two ways. First, she stepped in and asked him, "Why aren't you working?" Intervening this way was shaped by her positioning of

Jordan as a disengaged student. "I knew I had to step in because otherwise, he'd be on the floor soon." She did try to make him feel less threatened by her questions. She felt a need for her to use her authority to bring him back to the group work. This shows her recognition of these power dynamics between herself as a teacher and the student. Second, she also told Jordan's peers to include him in the group work. In her actual intervention, she said, "You three are supposed to be working together" and "you can't leave Jaden out." She used her authority as a teacher to help his peers recognize that they were responsible for bringing Jordan back to the task. Intervening this way means her imposition of her authority over the individual students in small groups. It is not hard to tell how her intervention was related to which categories of students. However, it is clear that this particular intervention seems to be shaped by her recognition of Jordan as a mixture of language learners, shy students, and disengaged students.

Summary

In this chapter, I demonstrated ways teachers recognize students in terms of categories of students as actors and of power and authority dynamics among actors in figured worlds of mathematics classrooms. I described categories of students both teachers recognized as they explained their intervention in small groups. These categories include academic achievement, engagement, personality, language development, and at-home support. Leslie and Marva both recognized categories of students related to academic achievement, engagement, and personality. Only Marva recognized students in terms of categories of students related to language development and at-home support. I also showed how teachers' recognition of students in terms of categories shaped their intervention in small groups. For example, in Example 1, Leslie's recognition of Alice and Jacob as academically low students and shy students shaped her

interruption of Fred's initiation to explain and her asking Alice to explain her understanding. I also elaborated on how teachers recognized power and authority dynamics. In the elaboration, I showed that both teachers recognized these dynamics mostly at the micro-interaction level and that they recognized these dynamics across all categories of students recognized by both teachers.

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CHAPTER 8. CONCLUSION

In this chapter, I explain how this dissertation study contributes to the field. To explain the contributions, first, I raise four points to discuss. Second, I describe implications of this dissertation study in relation to research and teacher education. Third, I describe the limitations of this study. Fourth, I conclude and summarize this dissertation study.

Discussion

I discuss what this dissertation study may mean for the field in four points. The four points are 1) the complexity of intervention in small groups; 2) connecting teacher noticing to intervention in small groups; 3) resources teachers may use in their decision-making processes; and 4) mathematics classrooms as figured worlds revealed through day-to-day teaching activities.

The complexity of intervention in small groups

This dissertation study described how beginning teachers intervened in small groups in mathematics classrooms and why they intervened in specific ways. As detailed in Chapter 5 in relation to the noticing-mediated intervention in small groups, Leslie and Marva intervened in diverse ways (*hows*) with multiple purposes (*whys*). This finding related to *hows* and *whys* of intervention is similar to and different from my previous work on prospective teachers' intervention in small groups, which was built on earlier studies related to practicing teachers' intervention after professional development (e.g., Chiu, 2004; Dekker & Elshout-Mohr, 2004; Gillies & Boyles, 2006).

This dissertation study is similar to some extent to Pak (2017) in that both identify similar *hows* and *whys* of intervention. One example of the similarity is that novice teachers in this
dissertation study and Pak (2017) intervened by encouraging students to work together. Their purpose alike was to make sure students work together or sometimes mediate students' mathematical thinking. Pak (2017) suggested that when prospective teachers intervene in small groups, they might have several purposes in mind and choose one of them. This is consistent with the finding in this dissertation study. Building on this similarity, therefore, I argue that understanding this teaching activity requires researchers to view *hows* and *whys* of intervention as being multifaceted in their relationships, instead of seeing them as being straightforward.

On the other hand, this dissertation study is different from Pak (2017) in that it provides additional sets of *hows* and *whys* of intervention in small groups. This difference was expected because the findings in Pak (2017) were based on the response data from an open-ended online survey, which might cause *hows* and *whys* to be limited to students' mathematical thinking. In addition to the survey responses, on the other hand, this dissertation study collected data from actual classrooms (video-recordings and interview transcripts). As such, there were new *hows* and *whys* of interventions on the part of beginning teachers. For example, they had purposes like increasing students' on-task behaviors or providing language support. They also made comments/questions such as explaining to ELLs how to use language correctly or telling students not to play with materials. I do not claim that this dissertation study provides a comprehensive understanding of *hows* and *whys* of intervention. But I argue that this dissertation study can serve as a call for research on extending the understanding of novice teachers' *hows* and *whys* of intervention to include *hows* and *whys* of intervention in diverse contexts.

Connecting teacher noticing to intervention in small groups

In Chapter 2, the literature review, I discussed that researchers have extended the notion of professional noticing to investigate diverse phenomena (e.g., Amador et al., 2017; Roth

McDuffie et al., 2014; Wager, 2014) and argued that intervention in small groups could be one of the areas that could be better understood using a professional noticing framework. In Chapter 5 related to the noticing-mediated framework, I found ways the two beginning teachers' interventions in small groups were shaped by their decision-making processes through professional noticing. This finding supports the argument in terms that it shows one possible example related to how to use professional noticing to examine an individual teacher's decisionmaking process for them to take in intervention in small groups.

Furthermore, this dissertation study differs from a few studies that, to date, have used professional noticing in connection with intervention in small groups. Wells (2017) is a study that used professional noticing in the context of intervening in small groups. Wells (2017) conducted a detailed and rigorous analysis of students' group work in mathematics classrooms to examine when to intervene in small groups. Pak (2018), which was based on data from an open-ended online survey, found that professional noticing may contribute to understanding intervention in small groups on the part of novice teachers. Pak (2018) investigated how prospective teachers noticed before planning to intervene in small groups, using the three components of professional noticing- attention, interpretation, and decisions of how to respond. The important difference of this dissertation study from these two studies is that this dissertation study shows that intervention in small groups is shaped largely by interpretation in relation to their professional noticing.

I emphasize this point related to the difference because it supports my argument in Chapter 3 that interpretation may be the link to connect professional noticing to figured worlds. The finding related to the noticing-mediated intervention framework in Chapter 5 clearly suggests that interpretation was essential for the two beginning teachers in making decisions

related to intervention in small groups. Also, interpretation is at the center of the definition of figured worlds in relation to recognizable actors, significant acts, and valuable outcomes in figured worlds. The central position of interpretation in figured worlds guided me to shed more light on interpretation in an individual teacher's decision-making process related to intervention in small groups in mathematics classrooms.

Building on the findings in Chapter 5, Figure 8.1 shows how this decision-making process is part of professional noticing. Chapter 5 shows the close interactions and relationships among attention, interpretation, and decisions of how to respond, which is consistent with what research on professional noticing found in relation to the interrelationships among the three components. Particularly, as I defined in Chapter 2 in relation to interpretation as making connections, the findings in Chapter 5 suggest that interpretation allows teachers to make connections between several resources related to students, behaviors, and outcomes and attention and decisions of how to respond. The elaboration of how interpretation works in professional noticing in particular. I consider decisions of how to respond to be purposes teachers may have as they intervene in small groups. This dissertation study suggests how an individual teacher may process their decision-making in relation to intervening in small groups in mathematics classrooms.

This dissertation study shows one possible example related to how to use professional noticing to examine intervention in small groups in a detailed way. Even though I do not claim that this dissertation study shows all of the ways that professional noticing happens in the context of intervention in small groups, I argue that this study could be considered a starting point.



Figure 8.1 The relationships among resources and the three components of professional noticing **Resources used in teachers' decision-making process**

This dissertation study suggests potential resources beginning teachers might use when they make decisions to intervene in small groups. These resources include 1) knowledge, expectations (group norms and teachers' roles), and experiences; 2) teacher identity; and 3) categories of students. I describe in this section what this dissertation study may mean for what the field knows about resources teachers may use in their intervention in small groups. Along with the descriptions, I also present how these resources are related to the elements of figured worlds

Knowledge, expectations, and learning experiences.

This dissertation study found that Leslie's and Marva's ongoing noticing (attention, decisions, and responses) were based on resources both teachers used when they interpreted to make decisions. These resources included specific/general knowledge, group norms

(expectations towards students), teachers' roles (expectations towards themselves as teachers), and learning experiences in different contexts. These resources were not highlighted in the previous studies related to intervention in small groups (e.g., Chiu, 2004; Dekker & Elshout-Mohr, 2004; Gillies & Boyles, 2006). These studies suggested that when practicing teachers intervene in small groups, they have certain ways to intervene and certain purposes to achieve. These studies did not answer a question related to what teachers' responses were based on. This dissertation study provides an answer to the question by including deeper understandings related to decision-making processes related to intervention in small groups. These processes underlie interpretation that makes connections between resources and teachers' attention, and decisions of how to respond. In relation to interpretation, this idea of knowledge, expectations, and experiences as resources informs ways to understand how novice teachers intervene in small groups because these resources shaped to some degree novice teachers' decision making particularly through connecting to their attention and decisions of how to respond (purposes).

Again, as above, the framework of figured worlds draws attention to interpretation as something that teachers engage in in mathematics classrooms. As a reminder, interpretation is defined as a process for people to give meaning to actors, acts, and outcomes in figured worlds. Chapter 5 suggests that in mathematics classrooms, these resources seem to support the two beginning teachers to engage in the process of giving meaning to certain students, students' certain behaviors, or certain learning outcomes as they intervened in small groups. For example, in the intervention episode illustrated in Figure 5.2, Leslie's specific knowledge of a student's engagement patterns allowed her to decide to understand the progress of the small group the student was part of. This example shows that the resource (specific knowledge of the student's engagement) supported Leslie to give more meaning to certain learning outcomes (the progress

of students' mathematical understanding) in the group. Like in this example, the figured worlds framework pays attention to interpretation in ways to link the resources to actors, acts, and outcomes in mathematics classrooms.

Teacher identity.

This dissertation suggests in Chapter 6 that both teachers invoked their current and designated teacher identity to explain, make sense of, and reason about their intervention in small groups. In this dissertation study, teacher identity was used in two ways. First it was an analytic lens for me as a researcher to understand teachers' intervention in small groups. Second, teacher identity served as a resource for teachers to use to make intervention-related decisions.

In a broad sense, research on teacher identity has suggested the close relationships between teacher identity and teaching practice in many contexts (e.g., Drake et al., 2001; Spillane, 2000). In these studies, it seemed that teacher identity was used as an analytic tool for these researchers to examine the relationships. For example, Drake and colleagues (2001) analyzed 10 elementary teachers' stories of learning experiences and of teaching practices in mathematics and literacy contexts to understand the relationships between teacher identity and teaching practices. In this research, teacher identity was a crucial tool that Drake and colleagues used to investigate the relationships. I do not claim that teachers in these studies did not use their identity as a teacher to explain their teaching practices. Rather, these researchers shed light on teacher identity to examine teaching practices.

The idea of teacher identity as a resource for teachers is similar to the way mathematical teacher identity is discussed by Aguirre and colleagues (Aguirre et al., 2013). Even though these researchers did not label teacher identity as a resource, they illustrated multiple aspects of teacher identity that are built on teachers' experiences as learners or persons. These aspects of

teacher identity could be seen as resources that teachers took with them and used to make sense of their teaching practices. To be similar, beginning teachers in this dissertation study drew on multiple aspects of teachers' current and designated teacher identity to explain their intervention in small groups. For example, Marva called on the role-related aspect of her current teacher identity to explain her specific intervention in a small group. In this way, her teacher identity was a resource for her to make sense of her intervention in the small group. As such, I argue that these aspects of current and designated teacher identity could be resources for beginning teachers in intervening in small groups.

In Chapter 3, I noted that in any figured world, people use their identities as resources to make sense of their participation in practices and activities in figured worlds. This notion is important because figured worlds guided me to pursue the investigation of Leslie's and Marva's teacher identity in relation to how they drew on their identities as resources to make sense of their intervention in small groups. As a result, Chapter 6 suggests that teachers draw on their current and designated teacher identity as resources to make sense of their intervention in small groups as a day-to-day teaching activity in mathematics classrooms.

Categories of students.

In this dissertation study, I found that Leslie and Marva often recognized students in terms of certain categories related to academic achievement, engagement, or personality. Further, both teachers seemed to use these categories of students as resources to decide what to do in their intervention in small groups. This finding is particularly consistent with educational studies that suggested categories of students in mathematics classrooms, particularly Horn (2011). She showed how teachers' teaching practices could be shaped by how teachers perceived students. She constructed certain categories of students (e.g., fast kids, slow kids, and lazy kids)

recognized by the teachers, suggesting that this recognition shaped ways the teachers implemented certain curricular activities (e.g., group-worthy activities) in mathematics classrooms.

This dissertation study also suggests that both teachers recognized these categories of students as resources that shaped her specific way of using her power and authority to minimize power and authority dynamics among students by intervening in this small group. For example, as appeared in Chapter 7, Leslie looked at the numbers on the worksheet that a group of four students (Alice, Jacob, Fred and Bob) had worked on and asked questions to the small group (T1A1E1). Fred was the first student who responded to the question. She interrupted Fred's talk and called on Alice to explain her mathematical thinking. This intervention by interruption of Fred's talk and seeking Alice's explanation seemed to be shaped by her recognition of students in the small group. She perceived Fred as an academically higher student and taking-over students, and Alice as an academically lower student and shy students who are dominated by taking-over students. This example suggests that both teachers recognized their power and authority dynamics among themselves as teachers and students in small groups. This point is consistent with studies on teachers as authority (Herbel-Eisenmann & Wagner, 2014; Herbel-Eisenmann, Wagner, & Cortes, 2008; Wagner & Herbel-Eisenmann, 2014) in terms that teachers recognize their power and authority dynamics. This dissertation study still could contribute to educational studies on categories of students in mathematics classrooms in terms of ways that teachers use their power and authority dynamics over students through intervention in small groups.

As I described in Chapter 3, the theoretical framework, the notion of figured worlds emphasizes types of actors recognized by people and power (the impact of the larger ideas on the

actions on the part of people as actors). In Chapter 7, in figured worlds of mathematics classrooms from teachers' perspectives, teachers recognized students in terms of categories of students and power and authority dynamics among students and among teachers and students. The notion of power suggests that categories of students may be related to the ideas coming from institutional, societal, and cultural practices. For example, positioning Fred mentioned above as academically higher students may be shaped by the academic achievement related to their test scores administered by schools, districts, or states. As I emphasized in Chapter 3, these different categories of students are also one reason for teachers to use their power and authority to reduce the power dynamics among students. As such, mathematics classrooms as figured worlds contribute to drawing attention to recognition of power and authority dynamics that operate in the interactions among students and among teachers and students in small groups.

Mathematics classrooms as a figured world(s)

This dissertation study suggests an important point in which figured worlds of mathematics classrooms could be revealed, at least in part, through a day-to-day teaching activity, such as Leslie's and Marva's intervention in small groups. This point is consistent with Holland and colleagues' view of figured worlds as happening as a social process and in time. For example, in the figured world of witchcraft mentioned in Chapter 3, the simple questioning, which people did moment by moment, led people to the witchcraft-related figured world. In mathematics classrooms, figured worlds also happen moment by moment through teachers' intervention in small groups.

In more detail, this dissertation study could provide a detail of how figured worlds could happen on a day-to-day basis through teachers' intervention in small groups in mathematics classrooms. This teaching activity is related to the elements of figured worlds, such as

interpretation, power, or practices and activities. First, engagement of teachers in intervention in small groups shows how mathematics classrooms become a space subject to interpretation of actors, particularly by teachers in this dissertation study, as shown in categories of students recognized by teachers. Second, this engagement shows how power and authority dynamics play out in the interactions between actors. As mentioned earlier in Chapter 7, categories of students sometimes went hand in hand with these power and authority dynamics. Third, engagement in intervention in small groups means for teachers to participate in socially and culturally constructed practices and activities in that knowledge, expectations (group norms and teachers' roles), and learning experiences related to intervention in small groups show that intervention is not an individual teacher's creation. These points all together contribute to understanding what figured worlds of mathematics classrooms look like and how the figured world happens and when. As such, I argue that the elements of figured worlds come into play through teachers' intervention in small groups on a day-to-day basis.

In relation to educational research in the context of mathematics classrooms, this dissertation study supports Boaler and Greeno (2000) and Esmonde and Langer-Osuna (2013) who also viewed U.S. mathematics classrooms as figured worlds. These researchers investigated figured worlds of mathematics classrooms building on what students experienced in mathematics classrooms. This dissertation study differs from them in terms that it pays attention to figured worlds of mathematics classrooms from the teachers' perspectives. However, it still explains teacher-led figured worlds of mathematics classrooms, which was not the main focus of Esmonde and Langer-Osuna (2013). To be clear, this dissertation study did not intend to fill up what Esmonde and Langer-Osuna (2013) did not detail in their study. It did not intend to explore figured worlds of mathematics classrooms in a holistic way. However, this dissertation study

paid great attention to phenomena of how figured worlds of mathematics classrooms happened whenever teachers intervened in small groups. As such, I argue that this dissertation study could contribute to encouraging the field to deepen a sense of what teacher-led figured worlds of mathematics classrooms might look like in the context of small groups.

More importantly, this dissertation study contributes to a body of literature on figured worlds that researchers constructed from the teachers' perspectives (e.g., Horn et al., 2008; Ma & Singer-Gabella, 2011). This contribution is related to the possibility to draw on figured worlds as a conceptual and analytic lens to understand intervention in small groups as a day-to-day teaching activity (e.g., intervention in small groups) on the part of novice teachers. By drawing on the notion of figured worlds in this dissertation study, I was able to learn deeply about how the two beginning teachers' participation in intervention in small groups shaped and was shaped by professional noticing with a great focus on interpretation; teacher identity; and categories of students and power and authority dynamics that went along with the categories in mathematics classrooms. That is, interpretation (giving meaning to actors, acts, and outcomes) was related to professional noticing as one of the components in the noticing-mediated intervention framework, and to categories of students (e.g., academic achievement or engagement) recognized meaningfully by both teachers. It was also related to teacher identity in terms that both teachers invoked certain aspects of current and designated teacher identities (e.g., personal aspects of Leslie's current teacher identity), which seems to be meaningful for these teachers in the moment of intervening in small groups. Without the notion of figured worlds, I might not consider how professional noticing, teacher identity, and categories of students are related together to interpretation and to intervention in small groups in mathematics classrooms. Figured worlds helped me investigate professional noticing, teacher identity, and categories of students in

relation to intervention in small groups as activities specific to mathematics classrooms. This dissertation study could be a starting point to move forward in relation to research on intervention in small groups. As such, I argue for exploring intervention in terms of figured worlds of mathematics classrooms from teachers' perspectives because the exploration could contribute to understanding how beginning teachers may participate in a day-to-day teaching activity (e.g, intervention in small groups) from different perspectives, including, but not limited to, professional noticing, teacher identity, and categories of students.

This dissertation study also calls on different looks at the theoretical framework (Figure 3.1) in Chapter 3. Building on the framework, I chose the particular relationships of intervention to professional noticing, to teacher identity, and to figured worlds. I chose them in this dissertation study because these three relationships were conspicuous more than others. As a result, this framework could contribute to understanding the relationships. However, as I analyzed, many questions emerged. Particularly, I wondered how professional noticing would play out in the relationships of intervention to teacher identity and to figured worlds (teachers' recognition of categories of students). For example, I analyzed the relationships between categories of students and intervention along with power and authority dynamics. The question was how what the teachers noticed would mediate power and authority dynamics though categories of students. However, I also acknowledge that these categories of students are one of the elements of figured worlds. As such, it is possible to keep examining the relationships

Implications

This dissertation study has several implications. First, it has implications related to several areas of research around teachers' intervention in small groups. Second, it has

implications with respect to practice and ways to contribute to intervention in small groups on the part of beginning teachers.

Implications for research

There are contributions to three research areas, including teacher noticing, teacher identity, and figured worlds. First, this dissertation study opens up how teacher noticing can be used to explore teachers' intervention in small groups. In particular, as shown in Chapter 5, it seems that the notion of professional noticing could work well with intervention in small groups. This dissertation study helps the researchers with interest in intervention in small groups examine teachers' intervention in combination with their decision-making processes. The potential areas that deserve more attention include: 1) identifying a more comprehensive set of attention, decisions, responses, and interpretation resources, 2) examining how the noticing-mediated framework works in diverse contexts, and 3) constructing a full story of specific intervention in small groups with students and teachers who are racially, culturally, and linguistically diverse, building on the framework.

Second, this dissertation study offers ways teacher identity shapes teachers' ordinary teaching activities. This study found that there were multiple aspects of their teacher identity on which beginning teachers drew to explain their intervention in small groups. It turned out that intervention in small groups was shaped by their current teacher identity (who they were) and designated teacher identity (who they wanted to be as a teacher in the future). These points suggest at least two implications for research on teacher identity. The first implication is to examine diverse aspects of teachers' current/designated teacher identity that shape and are shaped by their intervention in small groups in mathematics classrooms. The second implication is to investigate how current and designated teacher identity are related to each other and how the

relationships shape their intervention in small groups. For example, in Chapter 6, Marva invoked her current teacher identity related to "being flexible" to explain her intervention in which she asked the students to explain how they measured the book with paper clips. It seemed that she was not satisfied with this intervention and this dissatisfaction reminded her of who she wanted to be "as a teacher." She would "rather be just almost eavesdropping and watching them interact," which was her designated teacher identity. As shown in this example, I suggest examining the relationships between current/designated teacher identity and further how they shaped her intervention.

Third, this dissertation study holds promise for ways to understand mathematics classrooms from the notion of figured worlds. As mentioned earlier, the studies on figured worlds of mathematics classrooms were based on students' experiences. This dissertation study suggests figured worlds of mathematics classrooms revealed through teachers' day-to-day teaching activity, which shows the elements of figured worlds (e.g., practices and activities, power and authority, or recognition of students). This dissertation study has two implications for research on figured worlds of mathematics classrooms. The first implication is to investigate how teachers' day-to-day teaching activities are related to the elements of figured worlds. For example, ways Leslie and Marva intervened in small groups (e.g., encouraging students to work together) could be considered artifacts that have been collectively constructed by teachers and researchers. These ways could be viewed as artifacts for the teachers to use for the purpose of achieving students' cooperative learning. These two teachers learned and used those ways of intervention as artifacts to work with certain categories of students (e.g., taking-over students or disruptive students). The second implication is to understand how power and authority dynamics

play out at classroom level and macro level (e.g., how teachers' intervention in small groups are shaped by broader ideas and beliefs coming from outside classrooms).

All points combined, another important implication is to explore the trajectories of how beginning teachers' intervention in small groups develops over time. This developmental exploration could allow the field's understanding to be much richer when their decision-making processes and teacher identity are considered along with the development of intervention in small groups. One more dimension to be considered would be new emerging figured worlds of mathematics classrooms. Given teachers take a journey from novices to experienced teachers, the development of an ordinary teaching activity may go hand in hand with figured worlds of mathematics classrooms. As such, this dissertation study has an implication for understanding teachers' intervention in small groups in combination with professional noticing, teacher identity, and the elements of figured worlds.

Implications for university courses and professional development

This dissertation study has potential implications in the context of mathematics methods courses and professional development.

Mathematics methods courses.

This dissertation study can be used to support teacher educators in mathematics education to help their students learn how to intervene in small groups. One way for teacher educators is to have prospective teachers approximate intervention in their learning activity. For example, they could design a classroom activity where prospective teachers engage in role-playing based on scenarios, like the ones in Appendix A, as a small case for prospective teachers to analyze, reflect, and discuss intervention in small groups on the part of their cooperating teachers as well as their own.

To support prospective teachers to learn diverse ways to intervene in small groups, teacher educators could also make different combinations between the pedagogies of practices and findings in this dissertation study (components of the noticing-mediated intervention framework, teacher identity, and categories of students along with power and authority dynamics). For example, teacher educators could have prospective teachers who are on their practicum record their own intervention in a small group. They could give a course assignment to analyze their own intervention in terms of their knowledge, expectations, and learning experiences; multiple aspects of their current/designated teacher identities they invoke; and the relationships between categories of students they recognized and their intervention in the small group. In particular, given both teachers' designated teacher identity was related to their roles as teachers, teacher educators may focus more on role-aspects of teachers' designated teacher identities in relation to intervention in small groups. On the whole, this dissertation study could support teacher educators to view intervention in small groups as an ordinary teaching activity that consists of multiple dimensions of teaching.

Professional development.

This dissertation study can also serve as part of professional development or a short workshop on teacher noticing and/or implementing small groups. First, teacher educators could design an activity that can be used in two or more sessions in professional development on teacher noticing in general and professional noticing in particular. For example, teacher educators begin with analysis of a video case where a teacher asks a student to evaluate other students in a small group from the three components of professional noticing (attentioninterpretation-decisions of how to respond). Having participants relate to their own intervention and communicate with other teachers, teacher educators guide them to discuss what the teacher's

intervention might be based on from several resources (knowledge, expectations, learning experiences). They could go further the discussion by extending to teacher identity and categories of students they recognize.

Second, drawing on this dissertation study, teacher educators could design an activity as part of professional development or a short workshop that focuses on how to implement small groups in mathematics classrooms. This activity could serve as a tool for participants to think deeply of intentions they had (e.g., improving communications skills and social skills) when they decide to implement small groups and how these intentions play out in the interactions with students. Teacher educators could start the activity by having participants watch a teacher's intervention in a small group(s). They could have the participants relate what they notice to their own teaching by making a comparison and contrast. They could also have the participants connect to their teacher identity and to categories of students they perceive in mathematics classrooms.

Limitation

Even though this dissertation study has several implications for research as well as teacher education and professional development, it still has limitations related to its research design. In this section, I describe four limitations- 1) the small number of participating teachers; 2) the short period of time; 3) stimulated recall interviews; and 4) students as co-constructors of figured worlds of mathematics classrooms.

The small number of beginning teachers

The small number of beginning teachers in this dissertation study might limit the understanding of intervention in small groups. I analyzed the data from two beginning teachers.

It is not clear whether this dissertation study would have similar findings if I analyzed the data from several beginning teachers with different grade levels. The findings might be different from the current findings. For example, participation of more beginning teachers in this study might allow me to obtain a more diverse set of attention, decisions, or responses and interpretation resources across different grades, which might affect the noticing-mediated intervention framework in Chapter 5. I could also see more various categories of students these teachers might recognize depending on their different grades. Even though this difference in findings might be the case, I argue that this dissertation study can still contribute to the field in terms of unearthing what might happen behind beginning teachers' intervention in small groups. Regardless of the small number of participating teachers, this dissertation study still allows the field to deepen its understanding of this ordinary teaching activity on the part of beginning teachers.

A short period of investigation

A single semester investigation of beginning teachers' intervention in small groups might limit the understanding of intervention in small groups. Even though I said that data collection occurred in a single semester, the actual period for me to collect from the beginning teachers was about two months apart from the first interview to the last interview. Furthermore, I did not collect data in the beginning and end of the year. Thus, it is not clear whether and how their intervention would be different from year to year. The findings might be different when I took a developmental perspective on intervention in small groups. I could understand how their identity as a teacher could develop along with change in ways to intervene in small groups coming from more chance to participate in professional development. I could also see how categories of students recognized by beginning teachers develop from year to year and how they could learn to

negotiate with students to reduce impacts of their power and authority on students in small groups. As such, taking a view of the learning trajectory to understand intervention in small groups would allow me to find much richer data sources. However, even though this dissertation study was conducted in about two months, I was able to provide a solid understanding of intervention in small groups in combination with several important constructs, such as professional noticing, teacher identity, and figured worlds of mathematics classrooms. As my future research, I would examine intervention in small groups in combination with those constructs from developmental perspectives.

Stimulated recall interviews

This dissertation study used a stimulated recall interview as a way to collect data related to their noticing, identity, and categories of students in figured worlds of mathematics classrooms. To conduct this stimulated recall interview, I identified four intervention episodes that could allow Leslie and Marva to pay attention to ways they engaged in intervention during the interviews. I intended to choose diverse ways of intervention in small groups (e.g., explain students' mathematical thinking, asking students to evaluate other students' work, mediating students' thinking, and encouraging students to work together). I also conducted each interview on the same day I observed the lesson because I thought teachers might not remember the details of what they did and why. For time constraint, I did not have Leslie and Marva identify and choose intervention episodes significant and meaningful for them. Their own choice of intervention episodes would motivate them to talk about what they notice, who they are, and their figured world of mathematics classrooms. Further, this alternative way might have allowed me to have the data that were more authentic to them. This is my future research. I would have

teachers in the future research spend some time identifying intervention episodes and reflecting their intervention from the lens of professional noticing, teacher identity, and figured worlds.

Students as co-constructors of figured worlds of mathematics classrooms

I paid great attention to examining figured worlds of mathematics classrooms through interactions during teachers' intervention in small groups. This dissertation study suggested what figured worlds of mathematics classrooms looked like in the two teachers' mathematics instruction, particularly, through examining their day-to-day teaching activity. I did not include students' experiences related to teachers' intervention in small groups in mathematics classrooms. This exclusion was because of my perceptions related to the lack of research focus on teachers' engagement in ordinary teaching activity. As I acknowledged earlier in Chapter 3, teachers and students live in figured worlds of mathematics classrooms they construct together. In the future, I would investigate figured worlds of mathematics classrooms co-constructed by both teachers and students through their engagement in an ordinary teaching and learning activity, particularly teachers' intervention and students' interactions with their peers and teachers in small groups.

Conclusion

This dissertation study was motivated by questions related to how novice teachers intervene in small groups and why, relationships between teacher identities and intervention, and relationships between categories of students and intervention in small groups. Working with two beginning teachers, I analyzed the survey, video, and interview data related to intervention from three lenses- professional noticing with focus on interpretation, multiple aspects of current and

designated teacher identity, and teachers' recognition of students in terms of categories of students.

As a result, I was able to answer the question related to how beginning teachers intervene in small groups and why. I suggested a noticing-mediated intervention framework that details the *hows* and *whys* of intervention and furthermore explains an individual teacher's decision-making process in the context of intervention in small groups in mathematics classrooms. As an answer to the research question related to the relationships between teacher identities and intervention, I suggested that teacher identities were linked closely to intervention in small groups. Given teachers invoked different aspects of their current and designated teacher identity to explain their intervention in small groups, multiple aspects of teachers' current teacher identity and their designated teacher identity shaped specific ways of intervention in small groups. As an answer to the research question related to the relationships between categories of students and intervention in small groups, I found that this day-to-day teaching activity revealed figured worlds of mathematics classrooms. Their intervention was shaped by their recognition of students in terms of categories of students as actors in the figured worlds. These categories of students were never separable from power and authority in the figured worlds of mathematics classrooms.

Despite several limitations related to research design, this dissertation study is significant for research as well as teacher education. It opens up multiple research areas that deserve the researchers' attention in relation to professional noticing, teacher identity, and figured worlds. It also provides the field of mathematics teacher education with ways to support beginning teachers to learn intervention in small groups in elementary mathematics methods courses as well as in professional development.

On the whole, in this dissertation study, I looked deeply into intervention in small groups, thinking of intervention as a day-to-day teaching activity. This dissertation study broadens and deepens the understanding of how beginning teachers intervene in small groups and for what purpose by relating teachers' interventions to teacher noticing, to teacher identity, and to figured worlds of mathematics classrooms.

APPENDICES

APPENDIX A: An Open-ended Online Survey

(1) The first scenario: Please read through the following scenario, modified from Chiu (2004, pp. 379-380), and make responses to the prompt below

You are teaching second grade students about the meaning of the equal sign. You asked students in small groups to work on a math problem: 8 + 5 = 1 + 7. When you are circulating around the classroom, you hear what students in a small group are saying.

Dan: What are we supposed to do?

Ada: [Shrugs her shoulders] add 8 to 5 [counting on with her fingers]. So we get 13. It goes into the square.

Dan: But how about 7?

Ada: I don't know...

Kay: I think we...need to add 7 to the square... [mumbles something quietly but unclear]

Dan: Teacher! I have a question! [You walk over.] I think I need to put 13 in the square [pointing to the square]

Please write down briefly what you think is going on in the small group.

<u>Please write down what comments you would make and/or questions you would ask. For each comment or question, write a brief statement of *why* you would intervene in that way. If you would not say anything to this group at this time, explain *why not*.</u>

[comments and/or questions]

(2) The second scenario: Please read through the following scenario, modified from Chapin, O'Connor, & Anderson (2009, p. 64), and make responses to the prompt below.

You are a fifth grade teacher. Your students are working together in small groups on decimal addition problems. One of the small groups is working on a decimal problem: "5 + 0.4". You are hearing their talk as follows as you monitor.

Anna: I don't think it matters which way you do it.

- Andy: Let's use one of the word problems to see if it makes a difference. [she reads] "Andy put 5 gallons of gasoline into the gas can. He kept filling and adding another fourtenths of a gallon. How much gas is now in the can?"
- Jesse: I think maybe Hank is right. I thought you just lined up the numbers but if you add five and point four like this [Kei writes the problem down], the answer is wrong; it's too small. Like five gallons of gas plus four-tenth more is more than five.

5 +0.4_____

Hank: I think it is because we have to add the same things- like we add hundreds and hundreds with big numbers so now we have to add tenths and tenths or ones and ones. Anna: But where are the tenth in five? [Raises her hand for your help and talks to you] We need your help.

Please write down briefly what you think is going on in the small group.

<u>Please write down what comments you would make and/or questions you would ask. For each comment or question, write a brief statement of *why* you would intervene in that way. If you would not say anything to this group at this time, explain *why not*.</u>

[comments and/or questions]

(3) The third scenario: Please read through the following scenario, modified from Dekker et al. (2004, p. 58), and make responses to the prompt below.

You are teaching fourth grade students about lines of symmetry for two-dimensional figures. You are asking them to work together on a task: Identify line-symmetric figures and draw lines of symmetry. On a one-page handout, there are six figures, four of which are line-symmetric figures. At the introduction of the lesson, you clarified to the whole class your role and expressed your expectations. You will not provide any help with the content. Your expectation is that students need to work collaboratively and discuss, showing each other their work, giving each other explanations, and critiquing each other. As you monitor, you listen to the interactions between students in a small group. You decide to keep listening to their talk.

Kathy: Do you think that your way works out? Robin: Yes... You did it first like this (pointing to a line of symmetry Kathy drew on the handout).

That's not the way I thought. Ebbie: Stop it now, man... You need to explain... Kathy: We have to talk a lot, so Robin: Okay, then I will listen... Ebbie: Okay, we now have to choose one of the figures and fold it in halves...

Some time later, Kathy and Ebbie discuss closely together and Robin doesn't seem to participate. You notice this and you decide to intervene in this small group.

Please write down briefly what you think is going on in the small group.

Please write down what comments you would make and/or questions you would ask. For each comment or question, write a brief statement of *why* you would intervene in that way. If you would not say anything to this group at this time, explain *why not*.

[comments and/or questions]

(4) The fourth scenario: Please read through the following scenario and make responses to the prompt below.

You are a third grade teacher. You asked students in small groups to solve an open-ended task: Jane had [8, 9, 15] candies. Her mother gave her [3, 6, 18] candies. How many candies does Jane have now? You assigned individual students in small groups to roles: recorder, facilitator, checker, and cleanup. Now you are monitoring small groups. You are listening to students talking in a small group.

Jimmy (facilitator): How should we start to solve it? RuAnn (checker): Pick two numbers and....I don't know... Minjun (cleanup): ... (shrugs his shoulder.) Jimmy (facilitator): What do you think (stares at group members.) Ashley (recorder): I don't know. Minjun (cleanup): ... RuAnn (checker): What do you think, Minjun? Minjun (cleanup): I think... we need to... first...make numbers easy to add...(shrugs his shoulder, looks away, and, as usual, mumbles something quietly. But nobody can hear.)

Please write down briefly what you think is going on in the small group.

Please write down what comments you would make and/or questions you would ask. For each comment or question, write a brief statement of *why* you would intervene in that way. If you would not say anything to this group at this time, explain *why not*.

[comments and/or questions]

APPENDIX B: A Follow-up Interview Protocol

This interview is part of a research study on beginning teachers. The purpose of the study is to examine how elementary teachers make comments and ask questions when they intervene in small groups in relation to teacher noticing and teacher identity.

I am going to show you the survey you took last time. For a moment, I would like you to take look at the survey responses you made. I will ask you to explain in more detail your responses.

- Talk me through your responses to the first scenario through the fourth scenario and add any additional information about your understanding of the scenario, how you would respond, and why.
- What do you think of the role of the teacher when students are working in small groups?
- What are the benefits and challenges of using small groups, particularly in math teaching ?
- Tell me about your expectations for students when they are working in small groups.
 - What do you think students learn from working together with others in small groups?
- When you use small groups in mathematics teaching, what are your reasons for using them?
 - In general, when you intervene (ask questions or make comments) in small groups while they are working, why do you do so?
- Can you describe your experiences working in small groups with peer students in your K-16 schooling?
 - How do those experiences shape your teaching now?
- Would you tell me in more detail what you learned in methods courses in terms of implementing small groups in general, working with small groups in particular?
- Can you describe ways you saw your mentor teacher facilitate and intervene in small groups in your internship?
- Do the teachers in your school use small groups in teaching math very often?
 - Why do you think that they use small groups? Why do they not use?
 - Have you talked with or seen other teachers in your schools implementing small groups?
 - Have you talked about what they do for small groups?

I am going to ask you to describe how you would intervene in small groups in a hypothetical situation from now on.

- What would you do when no student in a small group can answer a question they need to solve?
- What would you do when students have difficulty communicating with each other?
- What would you do when only one student of a group is working on the task?
- What would you do when group members treat one another with authority and no true dialogues exist?
- What would you do when a group has "gotten stuck" on a problem and doesn't seem to be getting anywhere?
- What would you do when you see that a group, after considerable trial and error, has solved a difficult problem?
- What would you do when one student approaches you with a question about procedure that is really answerable from the task instructions?

APPENDIX C: A Stimulated Recall Interview Protocol

This interview is part of a research study on beginning teachers. As part of this study, we are interviewing beginning teachers who are teaching mathematics in grades K-5 in 2018-19. The purpose of the study is to examine how elementary teachers make comments and ask questions when they intervene in small groups in relation to teacher noticing and teacher identity.

- Please describe the today's lesson. Please tell me mathematical aspects of the lesson.
- What is a learning goal for today's lesson in relation to mathematics?
- Please describe the task or activity students are working in small groups.
- Can you tell me why you use these kinds of activities when using small groups?
- How did you form small groups when you made small groups for this lesson? Mixed ability groups or ability groups? Why?

Prior to the interview, I identified portions of video recordings where you intervened in small groups, which I will show you for stimulated recall interview. In this interview, showing the portions, I will ask you questions regarding your intervention in small groups.

- Talk me through your intervention in that moment.
 - What did you think was going on in the small group you intervened in?
 - Can you possibly tell me what you said to yourself in that moment?
 - What did you notice, see or hear, in more detail, before intervening in the small group?
 - What were you hoping would happen in the small group after your intervention?
 - What do you think happened as a result of your intervention?
 - (for classroom management-related intervention) What did you attend to when you made comments for classroom management? Why?
 - Tell me why you stayed longer or shorter in this small group than the other groups.
- How did you learn or know this way of intervention?
 - Where and how did you learn to intervene in that way?
 - Did you get the idea for this particular intervention from someone who you observed or learned from? Who are they?
 - How does the observation or learning help you intervene in that way?
 - What does it mean to you as a teacher?
- When students are working in small groups, what are you paying attention to?
 - Is there something that you have in mind when you intervene in small groups?
 - Why is it meaningful for you?
 - Where does it come from?

APPENDIX D: Codebooks of Attention, Decisions, Responses, and Interpretation

Categories	Codes	Descriptions	Examples
Attention	(Group work) Dominance of group work	Teachers attend to particular students doing all the work in small group work and other students engaging passively in the group work.	(T2A3E2) And she was writing, doing all the work and trying to talk to them but she was they weren't listening to her. (T2A1E4) All right it initially looked like the one little boy who kept saying "Yeah, count with me, count with me" it looked like he was doing all the work
Attention	(Group work) Group work progress	Teachers attend to the degree of progress that small groups are expected to make at the moment teachers interact with them. In this code, teachers talk about students off-tasks or miscommunication.	(T1A3E1) on his paper, he'd already written down what they were supposed to do. (T2A3E3) They looked like they were thinking about it and going through it, but they didn't have a whole lot written down so I wasn't really sure what was getting done.
Attention	(Group work) Approaches to mathematical tasks	Teachers attend to how students solve the group task often struggling to identify proper solutions.	(T1A1E3) I did see a mathematical error to which I mean, we were dealing with a lot of nines and a lot of eights and whatnot so it was expected. (T2A1E2) I didn't hear him say anything, it's just I saw that one problem on his paper that said $8 + 3 = 7$
Attention	(Bodily expression) Students' voice level	Teachers attend to students' voice level. The voice level is sometimes too quiet or loud. The loud voice level may indicate arguments between students emerging from different opinions and behavioral issues	(T1A1E3) I kind of heard while I was working with the other group up front what they were saying and they were communicating well. (T1A1E2) I could hear even just in the background that something wasn't right. (T2A1E4) I saw them both writing. And they're both quiet. Well,he is a little bit louder. They're both very quiet still
Attention	(Bodily expression) Students' body posture	Teachers attend to ways students' bodies are posed. Some instances may be salient, such that some students may have their head down. Some may sit laying back or lie on the floor.	(T2A3E2) The one little boy that was sitting there, he had his head down. You can't even see him in the video because he's hiding behind everyone. Has his head down (T1A2E4) But he was definitely justwas not interested, was more interested in just laying back. Kind of staying back letting the group do the work and "I'm just going to kind of sit here and take up space".

Table 9.1 Codebooks of attention, decisions, responses, and interpretation

Table 9.1 (cont'd)

Attention	(Bodily expression) Students' facial expression	Teachers attend to students' facial expression, which reveals the students' affection (e.g., embarrassment or anxiety or anger).	(T1A2E1) The look on her face kind of said it all to me because I can tell when she's just - when she doesn't get it and she's upset and she's frustrated and so just being able to read her face I think kind of said the majority of it.
Attention	(Learning materials) Mathematically improper use of manipulative	Teachers attend to students using manipulatives in a mathematically improper way. This code appears particularly when the lesson is about measurement.	(T2A2E3) so I wasn't really sure if they were both working on the same on the same one or if they were working on different ones, but I could just see stuff was kind of like it was they were using it but they were using it in almost a messy way, so it wasn't yeah.
Attention	(Learning materials) Playing with learning materials	Teachers attend to students who are playing with learning materials, such as counters, blocks, or rulers, as toys.	(T2A3E4) before I had walked over there, I had seen the one little boy with the glasses on, yeah because there 're the boy who has eyes glasses on, he was playing with counters,
Attention	English Language Learners' language use	Teachers attend to potential issues that English Language Learners use English in solving problems. These issues include the meaning of certain words or different number notations because of cultural difference.	(T2A1E3) I asked her what that number was and she said it was 12. But it was really written 21. And so when I said that to her she had that oh moment and like yeah, I do everything else right but there are two digits. I could see on her face that she recognized that her two-digit numbers were always flipped.

Table 9.1 (cont'd)

Decisions of how to respond	Making sure students work together	Teachers decide to ensure that all students work together in small groups. Teachers may intend students to include off- task students in group work. It differs from the code below, "Increasing on-task behaviors," in that its focus is on on-task students as the ones who teachers expect to include off-task students.	(T1A2E1) So my thinking at that point was I have to get her back on track because otherwise the whole group is just - I mean those two girls aren't gonna talk the boys would be the only ones talking. (T2A1E1) I'm just trying to get, I'm just trying to make them realize that, hey, you're responsible for working with him, too. It's not just him that's at fault here. Because I did tell them, "It's not just you two, he's part of the group."
Decisions of how to respond	Providing language support	Teachers decide to scaffold language use on the part of students, particularly English Language Learners.	(T2A1E3) I was hoping that she would recognize a problem and in the future start or at least for today start going electric. So I figured she'll probably need reinforcement later on just because it's I mean, she's learning the language and that's part of the language's structure. So just at least start she'll be able to start recognizing that she's flipping it. She'll be able to monitor herself.
Decisions of how to respond	Mediating students' mathematical thinking	Teachers decide to help students pay attention to mathematics in the small group work.	(T2A2E2) When I do it like that I'm not working as a facilitator, I'm more so now I'm actually now I'm giving you information. Now how can you use it? So I'm still now, I still want them to use it. It's not just okay now you know how to do this. Just now you got it. It adds to what you've already been doing.
Decisions of how to respond	Understanding group work progress	Teachers decide to check in what and how students work in relation to problem solving progress or communication progress. They are curious about collective progress and a particular student's progress.	 (T2A1E2) So it was more so I stopped to figure out what was going on because of that answer that was so blatantly wrong. (T1A2E3) the student that I was talking to most of the time, she's actually a new student. So I'm still trying to get a feel for where she's at academically so I know how I can help catch her up to where everyone else is at. So that's why I chose her over the other students to talk.
Decisions of how to respond	Increasing on- task behaviors	Teachers decide to manage students' behavioral issues, including off-task behaviors, in small groups so that they engage in group work. In comparison to the code, "Making sure students to work together," this code focuses on off-task students.	(T1A1E4) I have to get them back on-task (T2A1E1) So I just knew I had to get him I had to stop him from tipping because it's a safety thing and I had to get him to see that, "Hey, this isn't a choice, you have to work."

Table 9.1 (cont'd)

Responses	Encouraging students to work together	Teachers tell students that they have to work as a group to solve the group task. Teachers often ask students to specify what to do to work together.	(T2A1E4) T: Are you guys doing it together? Because I hear you talking. Are you copying what he's doing or are you or you were actually talking with him?/ A student: He's helping me count./ T: Oh he's helping you count them. Okay that's good. So you guys are working together to count them out.
Responses	Providing content help to students	Teachers help students make sense of the mathematical task in small groups. In this code, teachers lead students to certain ways to solve the task especially when students are struggling with the task.	(T2A1E3) T: Is that working on the addition problems?/ A student: I was only trying to understand when it applies./ T: We need a single number in those math problems written down, okay? (T1H1E1) When they answered that 8+5=13 I would answer with that's right! I would then write down 8+5=13 and 8+5=13+7 I would ask if these two equations were the same.
Responses	Asking students to explain their thinking	Teachers ask students to describe how they got the answer. Teachers sometimes want students to explain their thinking to other students in small groups.	 (T1A2E3) T: So, explain it to me, please./ A student: So, 60I'm going to explain it. (T2A2E4) T: Well, see I had another group measure this way and get seven. Please show me how you got seven? Show me (T1H6E1) "Well, how did you get that? What do you mean by this one?"
Responses	Asking students to provide reasons	Teachers ask students to provide why they think what they did works.	(T1A1E3) T: Why did you bring the five up there, Madeline.(T2A2E2) T: Wait. Right. So why do you go from twenty to from forty to fifty? So twenty
Responses	Evaluating other students' ideas	Teachers ask students if they agree or disagree with other students' answers or strategies. The focus of this evaluating response is on pressing students to evaluate each other's ideas.	(T1A3E2) T: So do you guys agree on this number? Let's go back to it./ A student: five minus/ T: Everybody agrees on this number?/ A student: No. [Why do?]/ T: Why do you not agree?[you need a four?]/ T: Okay. We're not arguing about what people are saying. We're making sure that the math is correct, which it is. Keep going
Responses	Asking students metacognitive questions	Teachers ask students to think of what they do to solve the task, to help students be self-aware of task-related behaviors, or to monitor what they are learning.	(T2A1E1) I asked them "What are you doing? Or what strategy are you using?" (T1H5E1) Okay, they have to understand the question. "Okay, well what do you guys think is the first step? Well it could be any idea. What do you think we're supposed to do? What are they asking us to do here?"
Responses	Asking students to comment on peers' strategies	Teachers ask students to make comments on other students' solutions to the mathematical task.	(T2A2E3) how about can you come over and look at his measurement? Go you use it way too much, though. What do you li do you like his measurement? What do you like about it?

Table 9.1 (cont'd)

Responses	Explaining to English Language Learners how to use language correctly	Teachers scaffold English Language Learners to understand the correct use of language.	(T2A1E3) T: 12. Is that how you write a 12? No. You keep flipping your numbers, Honey. I know in Arabic you go left or right to left. But remember, when we're writing in English, which way do we write from? We start on this side and go that way. So it's one-two, not two-one, okay? Can you fix that?
Responses	Telling students not to play with manipulative	Teachers discourage students from using mathematical manipulatives in a playful manner.	(T2A3E4) T: Why are you playing with these tools, tools not toys?/ A student: He is playing, not me I told him, tools not toys. (T1A2E3) Leave it away. Put it away.
Interpretation	Specific knowledge of students' at- home environment	Teachers talk about specific students' learning environment at their home. This knowledge includes parental support for the students and their siblings in the at-home environment.	(T2A2E2) We've seen it with his siblings too, because I had his older sister last year, who's now in fourth grade. He has an older brother who's in second grade but should be in third grade, and they're just in reading, math, everywhere, they're all falling behind because they're not getting support at home. And actually the sister, she used to really really care about school and she would try, but just wasn't getting support at home and then got to third grade and it was like a switch just flipped
Interpretation	Specific knowledge of students' language	Teachers talk about the level of language proficiency individual English Language Learners are developing.	(T2A3E2) while she does speak fluent English, her other language is Bengali. And the little boy, well, he's a native English speaker. He almost might as well be an ESL student because he's pretty much just learning how to talk, after that having a speech delay all the way up until the end of kindergarten. And the other girl is an ESL student but her first language is Arabic.
Interpretation	Specific knowledge of students' engagement	Teachers talk about ways particular students participate in small group work, such that some students tend to work independently when they work together in small groups.	 (T1A2E4) That particular student has a habit of kind of singling himself out and being like, oh they won't listen. (T2A2E1) I looked at them and "They're working by themselves again. They're ignoring each other again."Whenever we work in groups, those two have a tendency just to do their own thing instead of working together.
Interpretation	Specific knowledge of students' mathematical proficiency	Teachers talk about students' different mathematical abilities. In this code Teachers tend to label students "academically lower" or "higher."	(T1A2E3) I was concerned. This was the group that I was originally most concerned with because it has a few students that are, not low, but just a little bit on the lower side (T2A1E4) I know sometimes he does, because the little boy that he's working with is academically lower but the one little boy, he does want to learn.

Table 9.1 (cont'd)

Interpretation	General knowledge of importance of creating supportive environment	Teachers talk about how important it is to create a classroom culture where students can receive differentiated support from teachers and schools.	(T2A2E3) I think it's just promoting positivity in the classroom. And just the whole idea that you don't want this storm cloud over your classroom. You want to create this positive environment and just the positive, pleasant culture where you want to encourage each other and not bring each other down.
Interpretation	General knowledge of students' anxiety of being wrong	Teachers talk about students being anxious about making mistakes or taking test or getting called to explain their thinking.	(T1H1E3) From my personal experiences, kids are afraid to be wrong because they don't want to be made fun of or seen as less smart as others
Interpretation	General knowledge of ways to have students work together	Teachers talk about their knowledge of different pedagogical ways they help students work together with peers in small groups.	 (T1A1E2) So by asking her and then asking another student and then asking her to repeat it and asking different strategies I kinda by being there I kinda got them to open up and all communicate (T2A3E1) It kind of goes a lot of teachers use the Ask Three Before Me. So it's like asking three other students before you come to me, because if the students can't help you, then I know it's my fault. I need to clarify something. And it was more of a modified version of that because it was a group, and obviously, all the other groups are all working and doing what they should be doing. So they probably know what they're doing.
Interpretation	General knowledge of students' different learning styles	Teachers talk about students' different ways of learning mathematical concepts.	(T1H1E1) So I know kids have to learn different ways. So, being a visual learner, I would as a kid, I would probably look at the equations and say, "Okay. They want $8 + 5 = 13$." And I wouldn't really look at the seven, it would just kind of be there. Versus if I see these two equations side by side, I'm like, "Well, wait. There is a difference between these two." So being able to physically see the difference and seeing, in fact. These two equations are not the same and kind of keeping the equal sign as the balance.
Interpretation	(Group norms) Working with peers	Teachers talk about their expectations of students learning from each other by helping each other.	(T2A2E1) So does thatI meanso that if they're working in a group, they should be working together and not justand helping each other, not just sitting there and doing it on their own, because they don't know, they might be doing it completely wrong.
Table 9.1 (cont'd)

Interpretation	(Group norms) Using manipulative in a mathematically proper way	Teachers talk about their expectations of students' use of manipulative in mathematically proper ways rather than playing with manipulative.	(T2A3E4) I think it's actually exciting that she's taking that on and helping kind of put it into the classroom and making it more of a norm for the class and so it's it kind of helps having other students out there in the classroom who might still be using tools as toys.
Interpretation	(Teachers' roles) Fostering students' motivation to work together with peers	Teachers talk about their view of teachers' roles as having students work together with peers in small groups. They emphasize what they as a teacher should do to support students to work effectively in groups.	(T1A1E4) I didn't want to lead the kids right to the answer. I wanted them to kind of fool around with it and give them a chance to communicate and discuss their own ideas. (T2A2E1) as a teacher, I like having students work cooperatively and be able to learn from each other. And if they're not working together, they're not going to learn from each other
Interpretation	(Teachers' roles) Motivating students to think for themselves	Teachers talk about their view of teachers' roles as encouraging students to become an independent thinker while working in small groups. In this code, teachers emphasize individual students' learning mathematics with understanding.	(T2A1E2) I'm really trying to get them to use different, just think about things and try to make sense of them on their own. Even if it might not be right but they're making sense of it. It makes sense to them. And so with that, it's me not giving them the answers. It's making them think. It's hard sometimes. (T2A3E4) And kind of back to the last intervention we had talked about, where it's important just making them think about the math and be independent thinkers, and me not telling them what to do with the problem solving,
Interpretation	(Teachers' roles) Being flexible with students' needs	Teachers talk about their view of teachers' roles as being flexible with meeting stude nts' diverse needs. Teachers try to meet students' cultural differences.	(T2A2E4) A lot of it is it's just me being flexible and so hopefully just trying to be flexible. It's just me finding, and knowing the students, and what's happening and just trying and going with the flow and trying to figure out what's happening.
Interpretation	(Learning experiences) Understanding of students' learning	Teachers talk about their learning experiences in different contexts (e.g., teacher preparation programs or professional development or classroom placement) in relation to the importance of making sure that they understand what students are learning.	(T1A2E3) I think it was senior year of college and there was a mathematics course we had to take and it was all about the number talk and making sure that you understand what the kids are saying and they're communicating effectively to you. So that whole class, I mean every single day, we had a new number talk where a new group would present and they would have misconceptions on purpose and they would have to clarify them. And so I think that class was a huge help in really trying to understand what the kids are actually saying, what are they actually understanding? where do they need to go next so that they fully understand the concept?

Table 9.1 (cont'd)

Interpretation	(Learning experiences) Students' anxiety	Teachers talk about their learning experiences in relation to an anxiety students may have when making mistakes. This code differs from Code "General knowledge of students' anxiety of being wrong" in that it focuses on what teachers learn from learning in different contexts (e.g., teacher preparation programs or professional development). Even though what teachers experience results in knowledge of students' anxiety, they talk more about their sense-making in the contexts.	(T1H3E1) Because he had that anxiety that he was going to be wrong. And so a lot of it had to do with building the environment, as well as knowing your kids. And knowing, okay, I have to ask him this question personally ahead of time. I have to have him write down his answer so he can just go and read it. And he knows he's going to be right, he's going to build up his confidence. And it got towards the end of the year he would take a risk once in a while. But knowing that kids do have that anxiety. It's not just that they're nervous that their friends are going to giggle or laugh at them. And most of the time you can play it off, "oh, that's a silly answer." Or, "oh, good try but let's try it again. What else do you think? Try this. What have you thought about this before?" But the times where the kid has a lot of anxiety you just have to really work with them about it
Interpretation	(Learning experiences) Facilitating small group work	Teachers talk about their learning experiences in different contexts (e.g., teacher preparation programs or professional development or classroom placement) in relation to ways to motivate students to engage in small group work.	(T1A1E2) I think the biggest one was my co- partner because she does small groups a lot and so I would tell her, "Oh this problem was so unsolvable and I didn't know what to do." And she'd say, "Oh, well, try to have them work with this, try to pull them aside, don't pull them out of the group, make sure that you're trying to keep them involved, engage, try to make a joke about it. Try to get them to say what they think. Try to get them to agree or disagree or explain why and get their thinking, their logic more vocal so that way you can kind of understand where they're coming from and what the root of the problem is so you can kind of get rid of it.
Interpretation	(Learning experiences) Redirecting students	Teachers talk about their learning experiences in different contexts (e.g., teacher preparation programs or professional development or classroom placement) in relation to having students re-focus on their group work.	(T2A3E2) I feel like one of my mentor teachers through college was very just she would give a quick snappy comment that was meant to redirect them. And a lot, I wouldn't say it was always, but I felt a lot of times it was effective just to give them that quickly. You're not doing this, but you need to be doing this. And norm certain students it worked for. I should put it that way. Certain students it worked well for, so.

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