READING TO LEARN MATHEMATICS: EXPLORING READING STRATEGY IMPLEMENTATIONS OF MIDDLE SCHOOL STUDENTS THROUGH THEIR READING OF MATHEMATICAL TEXTS

By

Gregory Benjamin Beaudine

A DISSERTATION

Submitted to
Michigan State University
in partial fulfillment of the requirements
for the degree of

Curriculum, Instruction, and Teacher Education – Doctor of Philosophy

2020

ABSTRACT

READING TO LEARN MATHEMATICS: EXPLORING READING STRATEGY IMPLEMENTATIONS OF MIDDLE SCHOOL STUDENTS THROUGH THEIR READING OF MATHEMATICAL TEXTS

By

Gregory Benjamin Beaudine

The project detailed in this document explores the reading strategies used by seventh grade students as they navigate a three-page passage from a grade-level appropriate, nationally distributed textbook. Broadly speaking, I explore what mathematical reading look like for middle school students reading a passage from a nationally published and adopted textbook? More specifically, I seek to identify the reading strategies used by students as they read the selected passage, the ways these strategies help students engage with the text, and the purpose served by the implementation of said reading strategies. In pursuit of these answers, I conducted 22 semi-structured, one-on-one interviews, utilizing a verbal reading protocol at two separate research sites. During these discussions, I asked a group of seventh grade students to read a three-page passage from a nationally distributed mathematics text book, stopping them periodically to ask "what are you thinking about right now?". Collectively, these 22 students used 23 of the 25 focus strategies at least one time, demonstrating that middle school student utilize reading strategies as they explore mathematical text. Finally, this group of 22 students each utilized a different collection of reading strategies, but they gravitated toward six: *Paraphrase text, Plan a solution or predict a result, Self-check, Question or critique the text, Pause to reflect,* and *Read Aloud*.

This dissertation is dedicated to both friends and family who made this document possible. This dedication applies, in particular, to DocAllie who's advice never went unheard or unappreciated (though I'm certain it often felt that way).

ACKNOWLEDGEMENTS

Before continuing to my description of this project, I must first acknowledge those persons and organizations that most aided in my effort. The first and fore most is my partner, Dr. Allison Levine, who modeled the path from practicum project to PhD dissertation. Without her willingness to engage in my inquiries, I could not have created and completed such a project.

Similarly, Dr. Niral Shah and Dr. Corey Drake were instrumental in guiding me from first year PhD student to PhD recipient. Each played the role of my academic advisor, where Dr. Shah saw my work through my practicum project, and Dr. Drake took the reins from that point through the dissertation revisions. They both provided support when I was distraught, encouragement as I pondered ideas, and challenged me to improve my writing, my research, and my approach to this type of work.

The assistance of Dr. Tanya Wright, Dr. Beth Herbel-Eisenmann, and Dr. Jack Smith, as members of my dissertation committee, cannot go without mention. They were all quite instrumental in guiding the development and summary of the project. Without their assistance, I would likely still be attempting to explain the link between middle school students' mathematical reading comprehension and their enacted mathematical knowledge.

Finally, I need to express my appreciation for the other students in my PhD Cohort. Collectively, their brilliance ranges from critical race and gender studies to literacy to science to social studies, and everything in between. We shared highs and lows; the loss of a classmate and friend, Maximillion Monroy-Miller; and regularly challenged each-others' ideas. I can honestly say I am a better and more well-rounded individual, and a more just teacher, because of my interaction with these individuals. Thank you to each of you.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

CHAPTER 1: Only the Beginning

The toy in 1992. Mattel, the company that brought the world Hot Wheels®, Fisher-Price®, and Mega-Bloks™ (Mattel, 2020), introduced one of the more notorious toys in human history. Teen Talk Barbie was introduced to the world in the summer of 1992. Each doll was programed with four randomly selected sayings from a pool of 270 phrases, among which was "Math class is tough" (Livingston, 2018). After months of objection from the American Association of University Women, Mattel agreed to remove the one single phrase from the bank of 270 sayings and offered to trade a newer version of the doll with anyone in possession of the offending doll (Associated Press, 1992).

The comedian in 2006. Then President George W. Bush gave a speech in New Mexico in which he said, "You know, a lot of people probably think math and science isn't meant for me – it kind of seems a little hard, algebra" (Bumiller, 2006). Days later, the comedian Stephen Colbert dedicated a two-and-a-half-minute segment of his show to the difficulties found in mathematics, specifically in algebra. It began with an exasperated effort to understand the letter in the algebraic equation $2x=4$, because "the x could be anything" (Colbert, 2006). In the same segment, he continued the bit by declaring "I've said it before, equations are the devil's sentences" and characterized the quadratic equation as "an infernal salad of numbers, letters, and symbols." He closes the segment by paraphrasing the President's point as "mathematics is impossible."

The daily cartoon in 2012. Syndicated cartoonist Dan Piraro created *Bizarro*, a daily comic that stands as a "unique concoction of surrealistic imagery, social commentary, and witty plays on words" (Piraro, 2020). In one 2012 cartoon (Figure 1-1), he depicts a male customer purchasing two books, for $16.99 each (Wayno & Piraro, 2012). The books are entitled

Figure 1-1

Bizarro: Math for Dummies

"Math for Dummies." The female sales clerk tells the man he owes $50, not the $33.98 (plus tax) the man should owe.

The animated movie in 2018. Set in the 1960s (Acuna, 2018), the Incredibles movies (Bird, 2004; 2018) depict a family of superheroes doing everything they can to remain unnoticed and fit into society, because the job of a superhero was made illegal over a decade earlier. In the second installment of the movies, during a 30 second segment, Mr. Incredible is seen helping his oldest son, Dash, with mathematics homework. As the homework session begins to go awry, Mr. Incredible complains "Why would they change math? Math is math."

A quarter of a century has passed between the introduction of Teen Talks Barbie and the release of Incredibles 2, and the trope is still the same – math is hard, math is confusing. Math class can be tough, but it is my assertion that one component of this confusion is based on the perceived inaccessibility of the discipline's text. Mr. Colbert highlighted the struggles found in Algebra from simply adding letters to an equation, the bookstore clerk exploited a perceived inability to double the price of a book, and both Dash and Mr. Incredible would likely have fewer issues with the homework if they could read the text. Improved reading skill may not alleviate all errors in mathematics, but could very well lead to an improved conceptual understanding, allowing math class to be far less “tough.”

Comedic Fodder

The previous examples all work on a comedic level because of a societal trope that “math is hard”, accentuated by the complexity of mathematical notation that utilizes words, symbols, and visual images to convey the presented information (Adams, 2003; Hillman, 2014). Basic literacy skills rarely provide the same fodder for comedians. As a society, we tend to view one’s lack of literacy as an embarrassing or shameful thing. We expect adults to effectively communicate in their jobs and elsewhere in their daily lives, which usually means they can read, write, and speak clearly. When we identify a student with low literacy skills, we make a concerted effort to remediate their learning. Students who struggle with their development of reading skills may “feel ostracized from academia, avoid situations where they may be discovered or find themselves unable to fully participate in society or government” (Gunn, 2018).

This inability or unwillingness to participate within society, due to lower literacy skills, has multigenerational implications that may affect both economic and health viability for individuals and their families (Gunn, 2018; Proliteracy.org, 2020). To rescue a student from a lifetime of struggles with literacy skills, states across the country have taken steps to make certain students read at a 3rd grade level before moving on to 4th grade (Florida Legislature, 2016; Michigan Education Agency, 2017; Texas Education Agency, 2016). Illiteracy is, largely, not a topic to be joked about.

Would the purchase of “Science for Dummies” be equally ripe for exploitation? Would the outcry be the same if Teen Talk Barbie said “History class is tough”? Animators often make very intentional decisions about their representations (Beaudine et al., 2017), and I believe the discipline of mathematics was chosen for this homework scene to utilize the “math is hard” trope. Would Mr. Incredible’s homework scene be as comical if the context were reading? Why does this trope exist? Why are we so willing to accept that mathematics is just hard? Why do we accept the division of populations into math people and non-math people? Why do we not hold the disciplines of mathematics and literacy, or mathematics and history, or mathematics and science, at the same level of reverence? Why are mathematics textbooks so difficult to read, for content and understanding?

While the questions outlined above drive this inquiry, their answers are not found in this text. I do, however, present an exploration into one potential way to shift this perspective of mathematics – through explicit focus on mathematical reading. Much of the mathematical text students encounter in their classes differs from other text in that it combines multiple representations, all of which work in concert to convey the messages intended by the author. The reading of mathematical texts presents a challenge for students (and their families) who have not been taught how to synthesize the meaning of each of these representations. It is my belief that we can avoid the nightly experience modeled by the Incredibles (Bird, 2018) through the explicit instruction of mathematical reading skills.

Life Imitates Art Imitating Life

Students in our public-school systems are regularly sent home with nightly mathematics homework. During these nightly homework sessions, the challenge for the students (and their families) is to remember what happened in class hours (years?) earlier, a challenge willingly accepted in the pursuit of mathematical and academic success. One need not look far down their Facebook wall to see a parent at the end of their proverbial rope, much like Mr. Incredible, attempting to explain the mathematics to their child while simultaneously trying to understand the solution sought by the present text.

In the 2018 film, Mr. Incredible's struggles with his son's homework stemmed from the shift to New Mathematics, in the 1960's. In New Mathematics, the curriculum set aside rote memorization in favor of a more conceptual understanding of mathematics (Knudson, 2015). This movie moment comes at an interesting time in mathematics education history, as it parallels the contemporary parents' panic caused "by their sudden inability to help their children with their [Common Core-aligned] homework" (Knudson, 2015). Mr. Incredible was right about one thing, math is indeed math. While we continue to learn more about teaching and learning and educational priorities and pedagogies change, the scene remains the same – parents and students working together to complete the nightly homework, at times unable to access the information on the textbook pages and exasperated conversation ensues.

We currently live in atypical times. Schools across the country closed their doors during the Spring 2020 semester (Nagel, 2020), and teachers were asked to shift their traditional face-to-face instruction to meet the virtual need. In the past, parents were asked to aid their students with their nightly homework, as modeled by Mr. Incredible. During the Coronavirus closures of 2020, students missed more class as instruction moved from the face-to-face format to an online setting (Goldstein et al., 2020), and the burden to facilitate learning has shifted from the teachers in the classroom to the home.

Districts tried to provide much needed support for families in the form of food (e.g., Axelrod & Moret, 2020; Peikes, 2020) and technology (e.g., Nguyen, 2020; Thomas, 2020) because, in many districts, instruction is expected to continue through the end of the 2019-20 academic year. During these struggles to provide food (e.g., Norwood, 2020) and technology (e.g., Rauf, 2020), districts are being praised. US Secretary of Education DeVos (2020) highlighted one district's "solution-oriented innovation," using school busses as mobile Wi-Fi-hubs. The busses' mobile nature and various technological "snags" (Sheppard, 2020) make these efforts less solution-oriented and more of a work-around to address the current need.

Through all the shifts and novel work-arounds brought about in 2020, we had a nation of students and parents at home, social-distancing, and waiting for news that our lives can return to "normal." In the meantime, these students tried to complete their academic work, and I imagine the scene so brilliantly designed and illustrated by Bird (2018) was playing out over and over in American homes. Mr. Incredible pondered why mathematics changed, a flawed inquiry because the math has never changed. A better inquiry, perhaps, is to explore how mathematics education could shift in such a way that students and parents had more complete access to the information presented in their text. This shift would allow parents and teachers to play the role of the helpful guide and not the master of all knowledge.

In an animated scene that lasts all of 25 seconds, Mr. Incredible can be seen working on Dash's homework, being told the math is to be completed differently, being presented with the book, and ranting about the changes in mathematics. What if Mr. Incredible's ability to decipher Dash's mathematical textbook was stronger and he had the skills necessary to read and understand the text Dash handed him? What if Dash had the skills to read, process, and understand the mathematical text for himself? What keeps students and parents, alike, from reading a mathematical passage and implementing the ideas discussed within? I will argue, in the text that follows, that improved mathematical literacy skills in general, and proficient mathematical reading specifically, would provide the access needed by both student and parents to make sense of mathematics texts.

A Personal Note

Even those of us labeled as “math people” have our struggles, many of which could have been alleviated through a focus on mathematical reading instruction. I grew up in a small, predominantly white city, and attended the city public schools for the entirety of my K-12 education. In second grade, the schools began to separate students into high and low mathematics classes. In this initial sorting of students, I was placed in the lower mathematics class and worked hard throughout the year to move into the upper mathematics, a position I would not relinquish until AP Calculus, my Senior year. I failed my first semester of AP Calculus, dropped the course, and returned to mathematics during my first year of college.

After high school, I attended two universities in pursuit of a teaching certification. The first was a university defined by its regional and commuter status, the second was a State flagship, 4-year institution, categorized as an R1 research institution. In the math classes that followed, passing grades were easy to secure if I went to class and did the homework. When I missed classes, as many college students do, I found it very difficult to successfully complete the homework, and had trouble connecting the content from the missed course with either the previous or future lessons.

I sought help for those missed classes, but when I did, I often found roadblocks. Course instructors had research and service obligations may have placed remediating a student who missed class at a lower priority. Other options were to seek help from the TAs, many of whom had never taught and held their studies and research at a higher priority than teaching. Finally, even when making it to class or TA sessions, the adjustment to unfamiliar accents led to misunderstandings.

I continue to be frustrated by my undergraduate work, looking back, because I had all the necessary instruction and solutions in the textbooks. During my K-12 studies, the book loaned to me was used to assign practice problems for nightly homework. The required texts at the collegiate level were used similarly, though I spent hundreds of dollars on those each semester. The explanations for all the concepts presented in class, the direction needed to complete the homework, all the information needed to prepare for each exam could have been found between the covers of my textbook. I should have been able to read the text to supplement or remediate my learning, except I could not. I did not know how. The text, written in the English, the only language I have ever known, was beyond my comprehension.

Years later, I was working in a Southern Association of Colleges and Schools accredited 2-year college as a tutor, tutoring lab supervisor, adjunct professor, and student advocate for the mathematics department. In this role, I realized that I was not the only one who struggled with reading mathematics. Over my two-year tenure as a student advocate, I fielded several student concerns related to the mathematics department and the struggles of these students.

Several of these concerns echoed my experience, outlined above. Students missed class. They struggled with new accents. They were frustrated by tutor explanations that aligned with the instructors' teachings. They found that instructors prioritized their own tenure pursuits, placing service and grant projects above teaching or tutoring. In each of these cases, the students were asked to purchase a textbook for the class, and were largely unable to use the book as an effective instructional tool.

I share my story, and the echoes of students 15 years later, to highlight an opportunity lost. We ask K-12 schools to spend thousands of dollars adopting and purchasing new mathematics curriculum and associated textbooks. Similarly, we ask college students to spend hundreds of dollars on textbooks each semester. In both settings, the textbook purchase is made to gain access to practice problems and little more.

In other subject areas, the texts are well read. I remember instruction, at an early age, on how to read a history or science book for content. I received reading assignments for language arts, history, and science classes. I found that a missed history class, for example, could be made-up, in part, by reading the chapters discussed in class. From this reading, the big ideas can still be learned, even if the instructor's nuanced view is absent. The same could be said for many language arts or science classes.

I believe a missed mathematics class should operate in a similar manner. My experience, though, found the textbook only useful for homework assignments. I was not mathematically literate. This all led to a line of research that explored mathematical literacy, and more specifically, the ways students read the mathematical text presented to them. Literacy, as considered for this project, "involves the integration of listening, speaking, reading, writing, and critical thinking and includes the cultural knowledge that enables a speaker, writer, or reader to recognize and use language appropriate to different social situations" (Gibbons, 2009, p. 7). This project aims to focus closely on the reading component of literacy, and in mathematics contexts. Ultimately, this project will identify reading strategies used by middle school students reading mathematical passages, explore the ways these reading strategies are implemented, and the purpose for their implementation. This project is guided by the following research questions:

What does mathematical reading look like for middle school students reading a passage from a nationally published and adopted textbook?

- What reading strategies do students use when engaging mathematical text?
- In what ways do students use these strategies as they engage with the text?

- What purpose do these strategies serve?

Chapter Previews

Chapter 2 introduces the RAND reading comprehension model, identifying reading comprehension as the interplay of the reader, the text, and the activity of reading. The chapter then seeks to identify reading strategies and their implementations through the perspectives of the research in Content Area Literacy, Disciplinary Literacy, and Mathematical Literacy, then explores definitions of literacy and mathematical literacy. I offer the definition of mathematical literacy that guides this study, highlight a need to explore mathematical reading, and identify the reading strategies with which this study began.

In Chapter 3, I outline the methodological influences for this project, the verbal reading protocol design used for each interview, and protocol to analyze the students' interview data. Through this chapter, I identify the 25 reading strategies explored in the study. The chapter closes with a description of how this study protocol was implemented.

Chapters 4, 5, and 6 are used to share details about the outcomes of the study, each answering the three sub-questions, to better understand what mathematical reading looks like for middle school students. In Chapter 4, I identify the reading strategies used by the 22 study participants and use the words of one student to help define reading strategies. Chapter 5 focuses on the three reading tasks, Task 1: Explain It!, Task 3: Try It! (the first), and Task 6: Try It! (the third), that asks students to find a solution, and the work completed by students as they sought a solution. The work of 11 students are shared in this fifth chapter to model the ways reading strategies are commonly implemented during their work. The focus of Chapter 6 is the work of a trio of students. Through their work, I highlight the ways the reading strategies allow students deeper understanding of the content.

Chapter 7 brings this work to a close. In doing so, I summarize the key points from each chapter, connect the findings to the larger body of research, offer a formal answer to the research questions posed above. I also offer implications and suggestions for teachers, teacher educators, and those in charge of designing, selecting, or implementing curriculum. Finally, I close the last chapter by identifying limitations from this study and avenues I see as ripe for future research.

CHAPTER 2: An Exploration of Reading in Mathematics

This study investigates the ways students engaged in mathematical reading through an observation of reading strategies employed by the study participants as they read from a mathematics textbook and solved the related problems. In this study, reading strategies are understood to be intentional efforts made by the reader to better understand the text, building on the work of Afflerbach, et al. (2008). To effectively explore how students read, I first had to understand how reading comprehension has been studied in prior research, the ways literacy practices vary across disciplines, and what literacy practices look like in mathematics. Once a clear picture of each was created, I looked closely at the reading strategies proposed by three different veins of literacy instruction used in mathematics classes – content-area literacy (CAL), disciplinary literacy (DL), and mathematical literacy (ML). The reading comprehension literature was critical in connecting those reading strategies exhibited by each student to the decisions that student made as they began their mathematics work. This effort allowed for an assessment of students' understanding of the mathematical text.

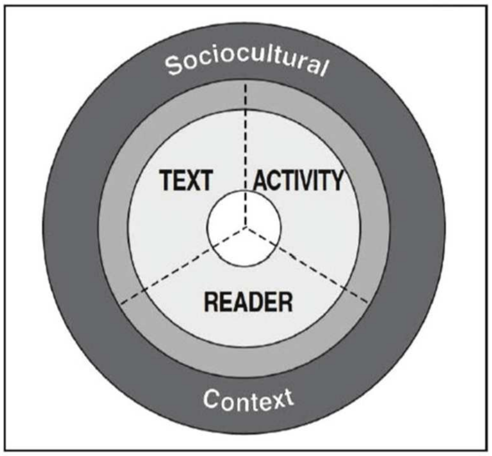
In this chapter, I discuss key literature and definitions related to reading comprehension, literacy practices across disciplines, reading strategies found in mathematics, and mathematical reading. I begin this chapter with the exploration of reading comprehension and text genres and provide an explanation of how I amended a years-old literature review relating to common literacy practices in mathematics spaces to ensure new material was not omitted from this work. I then outline literacy practices commonly found within CAL, DL, and ML instruction. The chapter continues with a definition of mathematical literacy that best bridges across each of the aforementioned literacy perspectives, a description of the role reading plays in mathematical literacy, and a list of reading strategies found throughout this review of literature.

Reading Comprehension

The RAND Reading Study Group (RRSG) identified "*reading comprehension* as the process of simultaneously extracting and constructing meaning through interaction and involvement with written language" (Kirby, 2003, p. 1, emphasis in original). In doing so, they identify the comprehension of reading as an interaction of three components – the reader, the text, and the activity – with the added consideration of the socio-cultural context surrounding each (Snow, 2002, Figure 2-1). Throughout the remainder of this section, I introduce the key components of the RAND model for reading comprehension, exploring how the roles of the readers, the text, and the process of reading affect the reading of mathematical text. In later chapters, I use these three components, and the socio-cultural setting for the interviews, to explore the ways 7th grade students read a passage from a nationally distributed mathematics textbook.

Figure 2-1

RAND Model for Reading Comprehension



The Reader

Researchers have long sought to find ways to address student individuality within mixed-level classrooms through differentiation (e.g., Dixon et al, 2014; Tomlinson & Jarvis, 2009), an effort allowing teachers to individualize the instruction for their students. Similarly, the RRSG's assessment of reading comprehension depended, in part, on the individual doing the reading (Snow, 2002). When considering the reader, the RRSG identified the individual's capabilities, motivation, knowledge, and prior experience as factors to consider when attempting to assess one's reading comprehension. Through this effort, they suggested that the wider ranging one's abilities, the better prepared they are to comprehend a textual passage (Kirby, 2003; Snow, 2002).

Snow (2002) explained that fluency, knowledge, and reading capability all change as one gains experience, where with more fluency and knowledge, one's ability to read is improved. In this sense, fluency is the translation of text with speed and accuracy (Fuchs et al, 2001) and knowledge is a student's understanding of the contents' "vocabulary and topic knowledge, linguistic and discourse knowledge, knowledge of comprehension strategies" (Kirby, 2003, p. 2). Fuchs et al (2001) explained that one's reading fluency develops throughout their elementary school experience, with a negative acceleration trend identified through middle and high school (i.e., secondary students are still developing their reading fluency, but at a slower rate than during their elementary education years). A student's ability to quickly and accurately decipher a text is ever increasing and dependent on exposure and experience.

A reader's knowledge of a subject, and motivation to read said content, also grows with exposure and experience. "As a reader begins to read and completes whatever activity is at hand, some of the knowledge and capabilities of the reader change" (Snow, 2002, p. 13). In this sense, one would expect a reader effectively comprehending the text to be more efficient, employ fewer comprehension or reading strategies, and gain greater understanding of their text as they work. The sum of these experiences may ultimately encourage or discourage student interest in the process of reading a discipline's text. As Kirby (2003) suggested, "these attributes vary considerably among readers and even within an individual reader as a function of the particular text being read and the reading activity" (p. 2).

Through this study, I explore the reading strategies employed by middle school students, as well as the related work completed for each reading passage, to identify how an individual might approach the text found within a grade-appropriate mathematics textbook. To do so, each individual interview, as explained in detail in Chapter 3, was structured similarly, but with enough free space that the interview could accommodate the needs of the individual participant or react appropriately to the introduction of "outside" knowledge brought forth by each student. These shifts will become more clear with the more in depth analysis found in Chapters 5 and 6.

The Text

As a second component of reading comprehension, the RRSG clearly stated the importance of assessing the textual features present in any reading passage (Snow, 2002). They identified three pieces – "the surface code (the exact wording of the text), the text base (idea units representing the meaning of the text), and the mental models (the way in which information is processed for meaning)" (Kirby, 2003, p. 2). Each of these three pieces must be considered, and change with the genre and purpose of the text.

Written genres, as defined by (Purcell-Gates, et al., 2007), "are differentiated and identifiable written text types" (p. 11). More broadly, genres can be summarized as "socially constructed language practices, reflecting community norms and expectations" (Purcell-Gates, et al., 2007, p. 11), practices that shift to match the current socio-cultural context. The socio-cultural context surrounding the reading varies by discipline texts and text structure. For example, the text of a mathematics book is quite different from that found in a novel or newspaper article. Gibbons (2009) outlines a dozen genres of texts students may encounter or create in a classroom, and places the readers into four roles: A code-breaker, a participant in the text, a user of the text, and a text analyst. The socio-cultural context of each discipline dictates the common genre for text, and the expected role of the reader.

Given that “the language used in mathematics textbooks is a distinctive form of ordinary English” (Watkins, 1979, p. 216), the genres found in mathematics textbooks are of the utmost importance for this study. Solomon and O’Neill (1998) discuss the genres of text traditionally found within mathematics, concluding that written text *about* mathematics often follows the characteristics of a narrative text – “written in a personal style and the past tense, in narrative fashion … and it describes the circumstances around the solution to the problem” (p. 213). They contrast narrative text about mathematics with those texts written *of* mathematics – constructed with both “words and pictures to tell the reader how to solve the problem; it is impersonal, it is written in report style and it uses simple present tense and short sentences” (Solomon & O’Neill, 1998, p. 213).

Mathematics textbooks are often written to fit this latter, more succinct form. While each of these texts possess the voice of their author (Herbel-Eisenmann & Wagner, 2007), the third person narration of the mathematics lacks the time referent found in traditional narrative writing (Solomon & O’Neill, 1998). Dietiker (2013) proposed a third perspective linking text of mathematics to narrative texts, a mathematical *fabula*, substituting the mathematical logic found within the text for the traditional narrative timeline. Ultimately, however, most mathematical textbooks are not written to be narrative texts, outright. While mathematics textbooks may adopt some narrative features, they tend to follow informational or procedural text structures and purposes, like those found in science classrooms (Purcell-Gates, et al., 2007).

The Activity

The activity of reading is where the reader and the text begin their interaction. During this process, the reader is expected to "expand and unstuff meaning" (Fuentes, 1998, p. 81) found within the condensed and largely symbolic text structure found in mathematics. Several factors surrounding this connection must be considered when trying to understand students' reading and comprehension processes. In outlining these factors, the RRSG suggested that "the reading activity includes one or more purposes or tasks, some operations to process the text, and the outcomes of performing the activity, all of which occur within some specific context" (Kirby, 2003, p. 2).

The purpose of any activity is either internally or externally generated. In the context of the classroom, and particularly a mathematics classroom, the request to read is generated by the instructor, not the student-reader. When students struggle with comprehending the teacher-assigned reading passage, "we see instructors in the process of constructing the understandings for their students and handing them over to them" (Fuentes, 1998, p.82). This effort by teachers might allow students some understanding of the topic at hand, but does little to improve students' reading in that it forgoes student reading all together in favor of explanation and student understanding. In this example, the internal desire to read or understand is lost and only an external, teacher generated desire to understand is left.

When reading, students should possess some skill or strategy to decipher and interpret the text. Fuentes (1998), for example, explained how the FLIP – Friendliness, Language, Interest, and Prior knowledge – approach to pre-reading was encouraged and utilized by the students in their classroom. In this effort, Fuentes asked their students to identify the friendliness of the text, assess the language used by the author, state their level of interest in the passage, and activate any related prior knowledge held by the students. Some of these pre-reading strategies are found in the list in Chapter 3, as some of the 7th graders in this study did attempt to employ some pre-reading strategies as they read.

The study described in this dissertation explicitly explores the reading strategies used by middle school students as they worked through a selected text book passage. To this end, I recognize a distinction between reading skills and reading strategy has been drawn, in which Afflerbach et al (2008) wrote:

> *Reading strategies* are deliberate, goal-directed attempts to control and modify the reader's efforts to decode text, understand words, and construct meanings of text. *Reading skills* are automatic actions that result in decoding and comprehension with speed, efficiency, and fluency and usually occur without awareness of the components or control involved (p. 368, emphasis added).

Additionally, I recognize the students in this study had varying strengths in reading, and each new task posed changed the difficulty for each student. In this sense, students may use each of the strategies outlined in Chapter 3 both skillfully and strategically. Through this document, I make no effort to differentiate between a reading skill or a reading strategy, as my primary intent is to identify which strategies are utilized, when they are used, and the purpose the strategy implementation may serve. In my effort to identify the reading strategies used by these students, I did not attend to the level of automaticity or the intention of students, evidence that is necessary to distinguish between a strategy and a skill. As such, each implementation was considered a strategic effort, as opposed to a skillful one.

The Context

As discussed above, reading comprehension hinges on the reader, the text, and the interaction between the two. Each of these components “occurs within a larger sociocultural context that shapes and is shaped by the reader and that interacts with each of the three elements”

(Snow, 2002, p. 11). Snow explained the need to consider the origins of the reader's motivation and instruction that may influence strategy selection and implementation, as well as both the long- and short-term implications for their effort, as each of these components can affect the approach taken by the student-reader.

For these interviews, I selected the text they would be reading and I brought that text to the students. Each student and I sat either in a conference room or empty classroom, and in each case both a video and audio recorder were present. This setting is unlike anything they might experience on a regular school day, and the topic in question, mathematical reading, was not something heavily attended to by either classroom teacher, though both teachers expressed an interest in the work. The interview held no academic implications for the volunteering participants. Based on the RAND model, each of the three components, the reader, the text, and the activity, are affected by the setting in which the interaction takes place. For this study, both I, as the interviewer, and the setting of the interview were unfamiliar, which may have led the participants to perform differently than they would within their class or during homework.

For this study, each of these three components identified by the RRSG – the reader, the text, and the activity – are explained in more detail throughout Chapter 3, detailing how this study fits within this framework. In the text that completes this chapter, I outline my search for, and exploration of, reading strategies and literacy instruction in mathematics spaces through Content-Area Literacy, Disciplinary Literacy, and Mathematical Literacy. I then explore ideas of mathematical reading, broadly speaking, and identify the initial set of reading strategies selected for this project.

Literacy Document Search

In preparation for this study, I reviewed 57 documents spanning literacy practices within content-area, disciplinary, and mathematical literacies. I added these 57 newly reviewed documents to those previously reviewed during the construction of a literature review for related projects. The search was conducted through the ProQuest-ERIC database, beginning with the keyword searches for “content area literacy”, “disciplinary literacy”, and “mathematical literacy”, respectively. Additional filters, described below, were applied and the full tally of results was 98 documents.

The search for “content area literacy” returned 1349 peer-reviewed results, and was narrowed by adding ‘mathematics’ to the search, returning 166 peer-reviewed journal articles. One final filter, published since 1 January 2015, reduced the number of articles to 61 peer-reviewed articles from a scholarly journal relating to content area literacy and mathematics. The same process was followed with disciplinary literacy – adding mathematics to the search and narrowing the dates to include only articles since 1 January 2015 – reducing the initial 629 results to 26 peer-reviewed articles from a scholarly journal.

For mathematical literacy, because it already included a reference to mathematics, the process was amended slightly. The initial search for mathematical literacy returned 619 peer reviewed documents. The additional search term was “middle school”, and like the other two searches, the date narrowed the search to articles published since 1 January 2015. This process narrowed the list to 28 peer-reviewed articles from a scholarly journal.

The three lists were compiled and duplicates were removed, leaving 98 articles. The review continued by assessing the abstracts for relevance to this project. Three guiding questions were used to narrow the span of documents further – (1) Did the document explicitly mention content area literacy, disciplinary literacy, or mathematical literacy? (2) Was the study conducted in a mathematics classroom? (3) Were middle school students or their teachers the participants in the study? Articles with three "no's" were dismissed, further narrowing the pool to the 57 peer-reviewed articles from a scholarly journal that formed the basis of this review.

Two final steps were taken in creating the basis for this review. First, a close review of each of the 57 articles was made, seeking definitions and clear explanations of literacy, content area literacy, disciplinary literacy, mathematical literacy, or implemented reading strategies. Finally, eight articles were repeatedly referenced by multiple authors, and were added to this collection of peer reviewed work.

Exploring Literacy Practices

Literacy practices largely fall into the RRSG's "process" category of reading comprehension. Each reader will possess a unique set of skills and understanding that aid their reading process, and the text may be structured to be interpreted through a particular lens. That said, most literacy practices would fall within the process of reading – the interplay between the reader and the text. For each of the three literacy frameworks, I will present literature that explores the values of the framework, the reading strategies used by teachers or observed by researchers, and close with an assessment of how the RAND model might contribute to understanding the current practices of CAL, DL, and ML.

Literacy, itself, is an "interplay of meaning-making systems (alphabetic, oral, visual, etc.) that teachers and students should strive to study and produce" (NCTE 2005). To this end, students spend much of early elementary schooling learning how to read. With age, the pressure to be "at level" for reading increases for these students. As mentioned in Chapter 1, many states (e.g., Florida, Michigan, Texas) have tied a student's promotion from one grade to the next with their performance on annual reading and/or mathematics standardized assessments, beginning in third grade (Florida Legislature, 2016; Michigan Education Agency, 2017; Texas Education Agency, 2016). These policies, when implemented, negatively affect specific population groups (e.g., English Language Learners, students with special needs, and ethnic minorities) more than other populations, and tend to negatively affect the educational trajectory of those students who are retained (National Education Agency, 2019). They also tend to focus solely on reading in an English-Language Arts (ELA) context, instead of a broader set of content-area literacy skills or discipline-specific literacy practices.

This recognition of the important role played by reading instruction and assessment, traditionally dictated by school administrators or school boards, can be found within national education expectations (e.g., ESSA, U. S. Department of Education, n.d.b; CCSS, 2020). According to the New London Group (1996), society's concept of literacy has historically been strictly tied to reading and writing of particular text – that is, text found in "page-bound, official, standard forms of the national language" (p. 61). It has been common for US public schools to pursue an education heavily focused on the three R's – reading, writing, and arithmetic. Two of these three (reading and writing) were central in the past to an individual's identification as "literate," as they were identified as "tools that equip people for intelligent participation in daily life" (Hildreth, 1947, p. 1). Our societal understanding of literacy has shifted greatly since Hildreth's writing. Gibbons (2009), for example, writes that "literacy involves the integration of listening, speaking, reading, writing, and critical thinking and includes the cultural knowledge that enables a speaker, writer, or reader to recognize and use language appropriate to different social situations" (p. 7). For students within the United States, this idea of literacy still tends to focus on the reading and writing of English text.

These same literacy practices, listening, speaking, reading, writing, and critical thinking, are also found within the study of mathematics. The Organisation for Economic Co-operation and Development (OECD; 2013) describes a need for mathematically literate students to formulate, employ, explain, describe, and interpret a variety of mathematical texts. These five criteria are very similar to those listed in Gibbons' (2009) description of literacy – listen and read (interpret), speak and write (explain, describe) and think critically (formulate, employ). Some standards, like the Common Core State Standards (CCSS, 2020), ask students to engage with a task, arrive at a solution, then communicate some justification for their solution. Mathematics education research, as I have previously observed (Beaudine 2018), tends to focus on the study of literacy in two ways – through students' mathematical writing and their mathematical discussion. Students' pursuit of mathematical literacy, and our instruction of such literacy, should not ignore the need to interpret written mathematical text, whether through reading formally written mathematics, or interpreting another's mathematical writing.

NCTE (2020) believes that the exploration of different modes of literacy - reading, writing, speaking, and listening - is critical for students as they become more familiar with how these literacies support one another. With this in mind, I chose to focus on three lines of literacy research, those surrounding Content-Area Literacy (CAL), Disciplinary Literacy (DL), and Mathematical Literacy (ML), and how students can leverage each to better facilitate the study of mathematics. In an effort to better define each type of literacy, a thorough exploration of content-area, disciplinary, and mathematical literacies follow below. While these three perspectives are regularly framed as instructional approaches to literacy, I used the research surrounding each to identify the reading strategies commonly associated with these instructional practices, and the implications of the students' implementation of said strategies.

Content-Area Literacy

Content-Area Literacy (CAL; also referred to as “subject area” or “subject-matter” literacy) begins with the premise that “every teacher is a teacher of reading” (Brozo et al., 2013) and promotes “the use of generic strategies in order to understand texts" (Armstrong et al, 2018). In this sense, instructors are asked to incorporate a generic set of literacy skills (Moje, 2008; Shanahan & Shanahan, 2012). The only thing that stands out as unique, from a CAL perspective, is the student themselves. From this perspective, the generic reading strategies taught through CAL should help students read all texts, including those found in mathematics spaces (Doerr & Temple, 2016). While the genre of text, the action of reading, and the socio-cultural context may shift between classes, or between passages within a single text, the CAL perspective introduces a series of reading strategies intended to be effective across each of these shifts.

This CAL approach to literacy is based on a belief that when common literacy and reading strategies are supported across each course, students have more and better opportunities to read, write, and talk about the discipline (Carter & Dean, 2006). Literary technique is the central focus of a CAL focused program (Moje, 2008; Shanahan & Shanahan, 2012). The studies highlighted below use CAL to explore student reading (Carter & Dean, 2006), student writing (Martin & Polly, 2016), and teacher-implemented instructional strategies (Armstrong et al., 2018).

In their study, Carter and Dean (2006) “examine whether mathematics teachers incorporated reading strategies for decoding, vocabulary, and comprehension into their lessons and to explain how these strategies are used to help students understand mathematical concepts” (p. 131). Their study focused on 14 students enrolled in a three-week individualized, summer institute for students going in 5th through 11th grade, taught by Mathematics Education focused graduate students pursuing a Curriculum and Instruction degree. The investigation into reading strategies was unknown by the study participants. Carter and Dean (2006) analyzed the instruction practices around decoding, vocabulary, and comprehension. They found that "[70] of the 101 instances of reading instruction found in this study were vocabulary strategies" (p. 143).

Carter and Dean (2006) suggest that because proven literacy strategies exist, "effective reading strategies [should be incorporated] into their discipline-specific instruction because content area readers must cope with increasingly denser reading material and unfamiliar, technical vocabulary." (p. 127). The advantages of such an approach are clear. Students would receive the same instruction regarding the implementation of CAL strategies used in their English course as they receive in mathematics, science, or social studies classes (Massey & Riley, 2013). Simply put, a CAL approach to literacy provides a set of strategies that are intended to be universally helpful as students engage with, and attempt to make sense of, any text they are asked to read. Carter and Dean (2006), ultimately, conclude that mathematics teachers must understand their role in the instruction of reading mathematics, proactively finding ways to include reading strategies into their lessons.

In their project, Martin and Polly (2016) stated that by using "content-area literacy strategies, students enhance their ability to internalize course content and develop conceptual understanding of a particular subject" (p. 60). Their specific approach to this investigation focused on the analysis of the teacher-prompted student writings produced over the course of one month, contributed by 51 fourth grade students in three classrooms. Specifically, Martin and Polly wanted to better understand how students began to utilize writing when solving problems, and follow the evolution of said writing as the students gained more experience in their mathematical writing.

Martin and Polly (2016) found that their study participants were relatively new to writing in mathematics spaces. Because of the students' collective lack of experience, the teacher modeling and scaffolding were heavily used by students in the early responses, but limited the students’ creativity and variety when responding to the prompt. Martin and Polly also stated that, as students gained experience in writing in mathematics, their responses began to vary more widely from the model presented by the teacher.

Armstrong et al (2018) define CAL as “the ability to use listening, speaking, reading, writing, and viewing to gain information within a specific discipline (p. 85). Through their work, Armstrong et al. share “six practical [instructional] strategies to help build students’ content skills in the mathematics classroom” (Armstrong et al., 2018, p. 85). The specific instructional strategies shared by Armstrong et al. aim to help mathematics students internalize vocabulary, generate meaningful questions while reading, utilize and create visual supports, think-aloud as they work, communicate through mathematical writing, and engage in reading their mathematical text. Each of these six goals will be revisited, as they lead directly to the developed list of reading strategies shared below.

While the studies outlined above found a great deal of value in CAL, others have suggested that teaching only these common literacy practices falls short of exploring all the literacy practices needed to be successful in a specific discipline. Shanahan et al. (2011), for example, remind readers that these strategies are not as universally applicable as each discipline becomes more nuanced. Additionally, this focus on generic strategies can, at times, receive resistance from instructors, as they are not teachers of reading, and their discipline is what they most value (Moje, 2008). Griffo et al. (2015) observed that literacy skills and strategies look different across the various disciplines. They suggest that “when we read and when we write, we read and write about something in particular and for a particular purpose" (Griffo et al., 2015, p. 47-8). Anderson et al (2018) recognize this as well, but suggest that quality CAL instruction would leave room for specific disciplines to customize the generic strategies.

Additionally, the CAL perspectives shared above all discussed the instruction or literacy practices, and did not address the action of reading. Carter and Dean (2006) explored the reading strategies taught in a summer institute, Martin and Polly (2016) addressed the writing process found in their study, and Armstrong et al (2018) were concerned with teaching their students specific strategies intended to improve their reading. An assessment of the process of reading is absent, and mathematics-based CAL studies in the future would benefit from a closer inspection of the students, the text, and just how the students are reading the text provided.

Some have suggested generic literacy practices may be insufficient for one to effectively explore the literacy practices within a specific field. In mathematics, for example, "reading requires a precision of meaning, and each word must be understood specifically in service to that particular [mathematical] meaning" (Shanahan & Shanahan, 2008, p. 49). Molina (2012) adds that "language struggles are embedded in mathematics" (p. 1), brought forth, in part, by the ways instructors present the symbolic and linguistic complexities of the discipline, and how these characteristics depart from a layman's idea of literacy. Given the perceived shortcomings of the CAL perspective, I found it was also necessary to explore discipline-specific instructional practices and identifying commonly taught reading strategies.

Disciplinary Literacy

Shanahan and Shanahan (2014) define disciplinary literacy as "the idea that we should teach the specialized ways of reading, understanding, and thinking used in each academic discipline, such as science, history, or literature" (p. 636). In other words, disciplinary literacy

(DL) makes “the assumption that each discipline has its own unique ways of reading, writing, and communicating, for meaning making, and that knowledge of the discourses of each discipline allows [students] full participation” (Dostal & Robinson, 2018, p. 2). Most commonly, the emphasis in a DL model is on the specialized knowledge and abilities used to “uncover these field-specific ways of approaching language tasks and to find methods to explicitly teach them to students” (Mongillo, 2017, p. 331), allowing students the opportunity to learn with the tools used by experts within the field (Shanahan & Shanahan, 2012). As Lent (2016) points out, disciplinary literacy is constructed through “anything that helps us make meaning, whether in visual, audio, or multimodal format” (p. 4).

It is important to remember, however, that the development of students’ DL practices should attend to the language patterns students may see in a later career and not tied to immediate goals such as a homework assignment or standardized exam (Croce & McCormick, 2020). Students should practice using literacy tools in meaningful ways within that discipline, similar to the ways in which a disciplinary expert would use the same literacy tools. Unlike CAL, which tends to utilize the same strategies across all courses, DL perspective is quite flexible, allowing discipline instructors the ability to shift their teaching as new literacy practices are accepted (Moje, 2008). The studies shared below explore DL from the perspective of teachers across content areas (Shanahan et al., 2011) and secondary mathematics teachers (Doerr & Temple, 2016; Temple & Doerr, 2018).

Shanahan, et al. (2011) recognized the existence of CAL practices, but sought to explore the literacy practices unique to different disciplines. The study participants were set into teams of six - two disciplinary experts, two teacher educators, and two high school teachers from each discipline. In conducting this study, Shanahan et al. employed both a think-aloud protocol, for the disciplinary experts, and focus group conversations within each of the three teams of six. They coded each interview and compared and contrasted the aspects of literacy that relate to each of these fields - mathematics, social studies, and science. Through this work, Shanahan et al. crafted a profile of readers within the disciplines of chemistry, history, and mathematics.

When investigating the reading processes of full professors of mathematics, specifically, Shanahan et al. (2011) looked for evidence related to reading strategies used while reading an unfamiliar text. They also investigated the strategies employed by expert readers in chemistry and history. The mathematicians in their study made a concerted effort to consider the writing without being influenced by the author’s positionality or the time in which the document was written. The mathematicians, in this case, sat in contrast to the historians, who felt both author and era were critical to the understanding of the text. Additionally, Shanahan et al. found that the mathematicians folded their new information in with the prior knowledge held, used the text structure to cue understanding and identify important passages, and attended to the accuracy of the document with little consideration to the credibility of the author. The mathematicians in this study were observed attending to both the visual images and the written text, making no distinction about one being more valuable than the other; read and reread the text, as needed, in search for a more complete understanding; and chose a text to read that was of interest to them and their work.

From a DL perspective, students reading mathematical text should strive to focus more on the logic of the content presented within the passage than the author or time-period within which the text was written. The research surrounding DL also suggests that readers of mathematics seek to update existing knowledge with the new information and use text structure to cue the identification of critical information. A proficient reader of mathematics, from a DL perspective, will be able to assess a document for accuracy, consider both the visual and written text present within the reading, read and reread as needed, and select texts that are of interest to their work, because these are the skills of a mathematician. These DL characteristics of a mathematical reader are included in the list of reading strategies explored in this project.

In a study of two mathematics teachers, Doerr and Temple (2016) chronicle the initial rejection of reading in a mathematics class and the teachers' shift toward embracing the importance of such a practice. The study took place over a four-year period, during which the researchers worked with two middle school mathematics teachers interested in the mathematical writing associated with the *Connected Mathematics Project* (CMP) curriculum. Throughout the first three years of the study, Doerr and Temple report the classroom teachers' frustration with the reading heavy curriculum. In year four, the research group shifted their focus from mathematical writing to mathematical reading, as it relates to CMP.

Through this work, Doerr and Temple (2016) found that all study participants, literacy experts included, began the investigation into mathematical texts through a CAL perspective. The teachers, early in the study, pre-taught concepts that were needed to complete the investigations, as opposed to allowing the students to discover said concepts through the investigations, as the text was designed. The teachers also questioned the wordiness of the CMP text due to the "the large amount of mathematically irrelevant information they contained" (Doerr & Temple, 2016, p. 19). By the end of the project, the two teachers shifted toward a DL lens, recognizing that reading in ELA and mathematics spaces was different, but many of the same strategies could be utilized. As such, a "3PAS" strategy was designed and implemented in both classrooms, asking students to establish a purpose for reading, picture the problem being described, pause to reflect and check the work being done, answer the question, and verify the solution makes sense (Doerr & Temple, 2016).

Even after national shifts to include either CAL or DL practices within classrooms, Griffo et al. (2015) suggest that literacy practice and achievement among US students should be concerning, as test performances vary widely between students of varying racial and socio-economic backgrounds. The narrative is similar in mathematics, where both Programme for International Student Assessment (PISA) and Trends in International Mathematics and Science Study (TIMMS) data suggests the US mathematics achievement trails international peers (Carr, 2016a, 2016b). Nationally, "members of historically disadvantaged race/ethnic and socioeconomic (SES) groups lag behind White or Asian, middle-SES, advantaged groups" (Cameron et al., 2015, p. 789).

One possible explanation for these continued struggles is DL's introduction and focus in secondary classrooms. The aforementioned international tests tend to focus on middle and high school students and our CCSS begins to define disciplinary specific reading goals in sixth grade (Shanahan & Shanahan, 2014), which gives our students a very short period of time to gain much needed experience. While many of their very own studies have taken place at the secondary level (Shanahan & Shanahan, 2008, 2015; Shanahan et al., 2011), Shanahan and Shanahan (2014) suggest young children can participate in disciplinary literacy practices such as exploring two different views on a subject or making inferences about an object or image, closing with "it is never too early" (p. 639) to explore disciplinary literacy practices.

Disciplinary literacy does not have the long history of research found in the CAL literature. Much progress, largely through the work of Shanahan and Shanahan, has been made in researching DL perspectives, particularly in the establishment of expert practices. It is not yet clear what components of DL students engage with, adopt, and discard within the study of mathematics, and the dearth of DL studies relating directly to mathematics might be explained by the increased focus on mathematical literacy practices. Much like the CAL research, DL research focuses much more on how these reading strategies are taught to students than how students are reading and understanding the text through the strategies they choose to use.

Mathematical Literacy

Much like CAL and DL, mathematical literacy (ML) has been defined by many. Research surrounding ML adopts a DL influenced perspective, where the goal is to identify the practices of experts and craft instruction to teach and utilize those practices. Chen and Chui (2015) combined the policy-driven perspectives of both the OECD and the U.S. Department of Education. In doing so, they explained that students exhibiting ML characteristics can create, utilize, and interpret a range of mathematical representations and can "analyze and communicate ideas as they pose and interpret solutions to mathematical problems" (p. 265). Firdaus et al. (2017) suggested "mathematical literacy is about usability or mathematical functions that have been learned by the students in the school to everyday life in order to compete in a globalized world" (p. 213).

In prior work, I suggested any effort to develop the ML skills of our students meant attending to three components – classroom discussion, written communication, and mathematical reading (Beaudine, 2018). Over time, my perspective of literacy in general, and ML specifically, has evolved and become more nuanced. I now believe an adapted version of Gibbon's (2009) definition of literacy encompasses each of the above perspectives. That is, ML pertains to one's ability to listen, speak, read, write and think critically about mathematics, allowing the literate individual to understand and use language appropriate for the mathematics setting. In this view, it is important to consider ML as a math-centric focus on both the general CAL skills and more discipline-specific DL skills used by mathematicians. Studies within the realm of ML have explored teachers developing materials (Herbel-Eisenmann et al., 2015), students' written and verbal discourse within the classroom (Sigley & Wilkinson, 2015; Moschkovich & Zahner, 2018), and students' views of reading in mathematics (Beaudine, 2019a).

Herbel-Eisenmann et al. (2015), through the development and implementation of professional development materials, focused on the classroom discourse inside secondary mathematics spaces. In doing so, they worked with classroom teachers to investigate student and teacher positioning and to increase understanding and mastery of the mathematics register for both teachers and students. The study included a discussion amongst teachers with a focus on the discursive practices found within their classrooms. The analysis completed for this project was accomplished through the study of both teacher field notes and classroom videos.

Through their study, Herbel-Eisenmann, et al. (2015) explored the attention paid toward the development of vocabulary in mathematics, language used in and out of the classrooms, the different usages of parts of speech across different disciplines, the ways reading a mathematics textbook relates to reading a novel, and the analysis of written mathematics explanations. They found that, over time, teachers shifted their educational goals away from exclusively teaching the mathematical vocabulary to a broader understanding and appreciation for how mathematical vocabulary relates to or mirrors the everyday vocabulary used by their students. They ultimately suggest "that engagement with readings and analyzing student work and textbooks have promise toward developing nuance in teachers' understandings and talk about the mathematics register" (Herbel-Eisenmann et al., 2015, p. 40).

Similarly, Sigley and Wilkinson (2015) and Moschkovich and Zahner (2018) explored student discourse in mathematics spaces. For Sigley and Wilkinson, through a case study of a bilingual middle school student, Ariel, they explored his interaction with both oral and written forms of mathematical communication. In doing so, they sought to identify signs of understanding during the problem-solving process, the ways this student interacted with the mathematical register while solving the problem, and the communication skills the student brought to the process.

After working with Ariel over a span of 18 months, Sigley and Wilkinson (2015) demonstrated that Ariel's solution process became more sophisticated as his understanding of mathematical language and mastery of the math register improved. They suggested that teachers create situations in their mathematics classrooms that encourage students to implement a wide variety of mathematical skills and knowledge. They suggest that doing so "students should be encouraged to communicate those understandings by employing all of their resources – linguistic, symbolic, and gestural" (Sigley & Wilkinson, 2015, p. 85). They close by suggesting any effort to water down the mathematical instruction or vocabulary does a disservice to the students in that room, leading more toward rote memorization than dynamic problem solving.

Moschkovich and Zahner's (2018) project also attended to the communicative efforts of bilingual students as they worked in groups to solve the given mathematics problems. Moschkovich and Zahner introduced and applied an academic literacy in mathematics (ALM) framework within their work with two separate groups of students – one group in eighth grade, the other in sixth. The ALM framework attended to three components – proficiency, practice, and discourse within mathematics. A student's reading of a mathematics textbook has holds in each of these three components, where the knowledge gained falls in proficiency, the act of reading in practice, and the communication process that is writing and reading mathematical text is found within the discourse component.

Their purpose for this study was to investigate the ways the two groups of bilingual students engaged in problem solving, talked through their disagreements, and the role that academic literacy in mathematics played throughout their research. Through their analysis, Moschkovich and Zahner (2018) reiterate that teacher-led discussion allowed the teacher to moderate and model interactions, but when left to a group of students, the mathematical support for claims made during problem solving were lacking. They also suggest that the ALM framework used in this project "highlights that academic literacy in mathematics is multidimensional and cannot be reduced to, for example, helping students acquire static meanings for words provided by the teacher or a textbook" (Moschkovich & Zahner, 2018, p. 1009).

In a mathematical reading-specific study, Beaudine (2019a) asked each of the ten participants how they felt about reading and about mathematics, and how they saw the two disciplines overlapping. The responses to these questions identified reading in mathematics as a component of solving word problems (9 students), understanding equations (2 students), and reading directions and definitions (1 student each). While two participants identified equations as a necessary component to mathematical reading, the students generally characterized reading in mathematics as attending to text written in word, sentence, and paragraph formats.

In each of the studies above, the ML research spends a great deal of time exploring the discussion within mathematics classrooms. Many of these ideas fit within the exploration of mathematical reading, as well. The process of reading, particularly as described by the RRSG (2002), is a complex system in which the student, the text being read, and the action of reading contribute to the reader's understanding of the text. Sigley and Wilkinson (2018) described a need to challenge students to apply a wide range of skills and mathematical knowledge in their work. I argue that work includes an embrace of the reading of mathematical texts, whenever possible.

As discussed above, prior research into ML has explored the classroom discourse in and the written work created by students in mathematics spaces (e.g., Herbel-Eisenmann et al., 2015; Moschkovich & Zahner, 2018; Sigley & Wilkinson, 2015). ML has a reading component as well, which generally gains less attention than either mathematical discourse practices, broadly speaking, and mathematical writing and discussion more specifically (Beaudine, 2018). Furthermore, the CCSS (2020) views technical reading as an important part of a student's education and ML, if nothing else, requires a technical approach to reading. For example, Shanahan and Shanahan (2008) report on a discussion with a mathematician in which the potentially critical difference in the function words "the" and "a" are discussed. As this text moves forward, however, the RAND model of reading comprehension, focusing on the reader, the text, and the action of reading will sit at the forefront of the analysis.

Mathematical Reading

Texts, in mathematics, can take a great many forms. Adams (2003) first described the process of mathematical reading as an effort to "acquire comprehension and mathematical understanding with fluency and proficiency through the reading of numerals and symbols [all non-numeric symbols], in addition to words" (p. 786). Hillman (2014) later added images such as diagrams, graphs, and tables to the list of texts that must be interpreted when reading in mathematics. Mathematical reading, then, is the interpretation of written and graphical text, in a way that allows the reader to expand upon the information presented (Adams, 2003; Adams et al., 2015; Hillman, 2014).

Adams et al. (2015) identified a paradox, of sorts, in the way mathematics is taught and what it could mean to read mathematical text. The practice of mathematics is often about simplifying and condensing information, whereas the process of reading tends to expand the text on the page into a representation packed with meaning (Adams et al., 2015). In a previous study (Beaudine, 2019b), I chose to bound the investigation to the written texts that consist of words, numbers, and symbols, much like Adams' (2003) initial exploration of mathematical reading. In doing so, study participants had an opportunity to read mathematical texts that were written in strictly sentence-paragraph form, written entirely in numeric-symbolic forms, or written using a combination of the two that resembles that of a mathematics textbook passage. For this current study, I have expanded my gaze to consider the reading of images, as well.

Tendencies of Expert Readers

Many of the previous studies researched mathematical reading through the eyes of researchers, graduate students, or classroom instructors – those considered to be the experts in a mathematics classroom (e.g., Doerr & Temple, 2016; Shanahan et al., 2011; Shepherd & van de Sande, 2014). Their goal in doing so was to identify reading practices utilized by expert readers of mathematics. The goal for Shanahan et al. (2011), as explored above, was to compare reading practices used by experts across a variety of fields.

Like Shanahan et al., Shepherd and van de Sande (2014) situated their mathematical reading investigation in a higher education space, and sought a comparison between readers. Instead of comparing readers across disciplines, however, Shepherd interviewed only individuals studying mathematics. They interviewed six participants, three faculty members and three graduate students in mathematics. Each individual participated in a 2-hour session consisting of both a read aloud and interview segment that explored how their participants read unfamiliar mathematics texts.

Shepherd and van de Sande (2014) found that each of the faculty members read their mathematics passages in very similar ways, and it was noticeably different from the reading processes used by both undergraduate (from an earlier study) and graduate students. The study found that the faculty members skimmed much more of the text, compared to the reading done by both undergraduate and graduate students, and the more proficient the reader, the more they could "read-the-meaning" (Shepherd & van de Sande, 2014). This is to say that the more advanced the reader, as observed by Shepherd and van de Sande, the more fluent the reader's ability to vocalize the meaning of the written symbols, as opposed to reading the literal symbols on the page. As an example, consider the expression $0.4 + 5$. The more proficient or literate the reader, the more likely they would "read" the expression as "adding four-tenths and five" as opposed to "zero point four, plus five" as may be observed in an elementary classroom.

As described above, Doerr and Temple (2016) aided two 6th grade teachers in a four-year design-based research project attending the implementation of the CMP curriculum. Throughout their study, Doerr and Temple documented a clear shift from frustration about unreasonably wordy text the students were asked to read to an embrace of mathematical reading when working with the CMP textbooks. While the study was conducted in a middle school setting, the participants at the center of the investigation were the mathematics teachers, the disciplinary experts in that setting.

In each of these studies, the student perspective is missing from the conversations. Knowledge of what experts do in any field is important, particularly when exploring teaching from a DL perspective, as it provides a target for less experienced individuals and can further inform the work of researchers, teachers, and curriculum developers, among others. We have little information, however, about how novice mathematicians, like the middle school students included in this study, read mathematical text, and what they identify as reading in mathematical spaces. Student voice, as explained by Zheng et al. (2014), is both important and rarely considered when contemplating changes to schooling. Experts regularly expound ways to improve the educational process in our schools, all the while “the perspective of students are recognized as valuable, but not often queried or considered” (Zheng et al., 2014, p. 279).

In a previous study (Beaudine, 2019a), I found similar results to those of Zheng et al. (2014), namely that students are not often asked about their opinions related to mathematical reading, and may not view the process in the way experts or policy makers might. Experts, as explored above, have stated mathematical reading shows up in many forms (e.g. Adams, 2003; Adams et al., 2015, Hillman, 2014), is done in specific ways (e.g., Shepherd & van de Sande, 2014), and is important in pursuit of mathematical understanding (e.g., Doerr & Temple, 2016). The study I conducted (Beaudine, 2019a) found that those students did generally believe reading is found in mathematics, but the ten students overwhelmingly suggested that reading, in mathematics, consists of word problems and nothing else.

The current study, described in detail below, extends this study of mathematical reading to different styles of mathematical text. The focus of the project is the actions, thinking, and opinions of middle school students. With this focus, I hope to establish how students use specific reading strategies in certain ways to accomplish particular goals.

Reading Strategies

From the work outlined above, it is evident that CAL, DL, and ML are highly active areas of research. It can also be stated that reading is important for disciplinary success and is conducted differently within each discipline. As such, explicit instruction related to the reading practices of disciplinary experts would benefit our students greatly. There is evidence that students in middle school are already attempting to utilize reading strategies as they engage with mathematical texts (Beaudine, 2019b). Because of this, mathematical reading, particularly the reading conducted by students, exists as an area ripe for exploration.

In an earlier study (Beaudine, 2019b) and like Bergeson and Rosheim (2018), I leaned on the expertise of Pressley and Afflerbach (1995) and Hilden and Pressley (2011). In their earlier work, Pressley and Afflerbach outlined a list of 40 characteristics exhibited by constructively responsive readers (CRR). The separated these 40 items into three components of reading - actions taken before, during, and after reading a passage of text. The effort to adopt this list, outright, as done in my earlier projects, is akin to applying a CAL perspective of literacy. Both the DL and ML perspectives suggest that a strictly CAL perspective is insufficient as disciplines become more complex and nuanced. Through this review of literature, a reflection on past projects, and building from the CRR characteristics first presented by Pressley and Afflerbach (1995), I identified 20 reading strategies that students would likely utilize as they engaged with the mathematical text presented to them (Table 2-1), as well as the option to not use any strategy.

These reading strategies comprised the starting point in my coding process. More information about each of the reading strategies outlined above can be found within the codebook (Appendix A). This initial list of codes, discussed further in Chapter 3, left open the possibility of adding strategies that were exhibited by students while they participated in the

Table 2-1

Initial Reading Strategies

Strategy	Source
Use no strategy	
Preview text	Doerr & Temple, 2016
Apply prior knowledge	Adams et al, 2015; Bergeson & Rosheim, 2018; Brozo & Crain, 2018
Read Aloud	Pressley & Afflerbach, 1995; Hilden & Pressley, 2011
Plan a solution or predict result	Bergeson & Rosheim, 2018; Brozo & Crain, 2018
Modify a plan or prediction	Bergeson & Rosheim, 2018; Doerr & Temple, 2016
Make notes while reading	Armstrong et al., 2018; Bergeson & Rosheim, 2018; Doerr & Temple, 2016
Paraphrase text	Bergeson & Rosheim, 2018; Brozo & Crain, 2018
Read text closely	Armstrong et al., 2018; Doerr & Temple, 2016
Read entire passage	Adams et al., 2015
Read symbols as words	Armstrong et al., 2018; Doerr & Temple, 2016
Decode the text	Armstrong et al., 2018
Attends to prose and equations equally	Shanahan et al., 2011
Seek clarification or external assistance	Brozo & Crain, 2018

Table 2-1 (cont'd)

Strategy	Source
Pause to reflect and self-check	Bergeson & Rosheim, 2018; Brozo & Crain, 2018; Doerr & Temple, 2016
Reread the text	Armstrong et al., 2018; Harkness & Brass, 2017; Shanahan et al., 2011
Seeks important information	Bergeson & Rosheim, 2018
Selective reading	Harkness & Brass, 2017
Skims the text	Harkness & Brass, 2017
Uses text clues	Armstrong et al., 2018; Doerr & Temple, 2016
Create analogy or metaphor	Pressley & Afflerbach, 1995

study. It must also be noticed that the reading literature differentiates clearly between a reading strategy and a reading skill. A reading strategy is a process one willfully and intentionally implements, whereas a reading skill is something done naturally and without intent (Afflerbach et al., 2008).

Closing Thoughts

From the studies presented above, several things are evident. First, some believe that CAL is critical for the success of students in mathematics spaces (Armstrong et al, 2015) because "by using content-area literacy strategies, students enhance their ability to internalize course content and develop conceptual understanding" (Martin & Polly, 2016, p. 60). Others direct attention to the resistance to CAL found in secondary classrooms, where instructors are specialists of the discipline they teach, because instructors "perceived reading strategy instruction as representing pedagogy outside the disciplines" (Doerr & Temple, 2016, p. 9). These same teachers gravitated more naturally to Disciplinary Literacy practices because they better appealed to "those who never felt qualified to teach 'reading'" (Lent, 2016, p. 2).

It is my contention that the answer to literacy in mathematics classrooms is not "either/or" but rather "both/and," where the "and" is Mathematical Literacy. When exploring mathematical literacy, studies tend to pull from both CAL and DL practices. The research studies, however, tend to focus on student discourse and mathematical writing and setting reading practices aside for the moment. This may be further evidence of mathematics instructors resisting reading practices, as suggested by Moje (2008), for several reasons ranging from a lack of confidence or expertise in teaching reading to the demands of the discipline and related testing leaving no space for expanded instruction.

No matter the reasoning, reading in mathematics cannot be overlooked. Students may view mathematical reading as pertaining only to the sections of the text that are written in sentences and paragraph form (e.g., directions, word problems), but experts suggest it is much more. Adams (2003) and Hillman (2014) clearly stated that mathematical reading consists of a need to decipher equations, symbols, and images in addition to the sentence and paragraph structure found in an expository text. To do so, expert readers do, and students should, rely on a wide range of reading strategies to understand the text they are attempting to read. An exploration into the ways middle school students utilize these reading strategies is the crux of this study, and will be reported on as this text moves forward.

It is my belief that students are using reading strategies learned in other academic spaces to decipher the mathematics text through which they are working. Following the model of many of the studies shared above, I have selected an unfamiliar reading passage, designed a verbal reading protocol, and constructed a discourse analysis coding scheme to identify the reading strategies used by students as they read through the presented textbook passage. More about the methodology of this study follows in Chapter 3.

CHAPTER 3: This Current Study

This chapter outlines the planned and the implemented methodology used to gather and analyze data about the ways students read selected mathematical passages. In doing so, I describe the methodological theory guiding the study, share the interview protocol, introduce the study participants, and outline the text used for the reading exercise. I also share information related to data analysis including the student-created artifacts and the coding structure used to analyze the interviews. This chapter closes with a description of how the planned methods unfolded over the course of 22 individual student interviews. Additionally, this chapter introduces and explores how the methods for this study were framed by the student, text, and sociocultural portions of the RAND reading comprehension model. The action of reading, and how students understand and apply that which is read, will be explored throughout the three chapters that follow.

The decision to interview students in middle school was made for three reasons. Several studies have explored the value of reading in mathematics, though they tend to explore the reading of expert mathematicians (Shepherd & van de Sande, 2014) or instructors (Doerr & Temple, 2016). Rarely have they considered the reading conducted by students. Having information about the way experts read mathematical text is helpful, as it provides a model of successful reading approaches toward which student reading strategies can be scaffolded. “The perspective of [elementary and secondary] students are recognized as valuable, but not often queried or considered” (Zheng et al., 2014), leaving researchers and administrators to unilaterally make decisions about instruction. While research suggests students as young as second grade can successfully report their thoughts while reading, few studies have explicitly sought the input of middle school students. As such, there is an opening to explore the reading processes of middle school students as it relates to mathematical text.

Second, I found, through previous studies, that middle school students have very clear thoughts related to mathematical reading that need to be more deeply explored than the structure of my previous studies allowed (Beaudine, 2018, 2019a). Finally, my teaching experience during my K-12 tenure was at the middle school level. Having taught both 7th and 8th grade mathematics, I have a great appreciation for the ability and brilliance of these students. With more, and more accurate, information about the ways middle school students currently read mathematics texts, and the profiles of expert readers that have already been established, we can begin to craft a path from novice to expert mathematical reader.

Method

This study was designed to answer questions related to the ways middle school students use reading strategies as they read mathematical texts. To do so, I conducted a qualitative study of middle school students reading mathematical passages from a nationally distributed textbook. The goals for this study included the identification of reading strategies used by the study participants, analysis of the students' discourse surrounding these solutions, and exploration of both how the strategies were used and the purpose of each strategy implementation. As such, I adopted a qualitative discourse analysis framework and utilized a semi-structured clinical interview protocol to collect the data through a series of one-on-one interviews.

Ginsburg (1997) suggested there were two routes to identifying what a child knows: an assigned exam or task (e.g., IQ test) or clinical interview designed to explore the responses of the participant. For this study, I chose the latter, a semi-structured clinical interview, because it is "deliberately nonstandardized" (Ginsburg, 1997, p. 3). Each interview followed the same broad interview protocol, for standardization purposes, but with space to follow the unique interpretations and thoughts of students who participated in the study as they read through the provided mathematics passage and solved the related questions. “People are actually quite good at reporting the contents of their working memory” (Hilden & Pressley, 2011, p. 427), and Pressley and Afflerbach (1995) suggested that a Verbal Reading Protocol (VRP) is an effective way to solicit information about a reader’s thinking. As such, a VRP was designed and implemented in such a way that the interview participants had sufficient leeway to explore any ideas that came throughout the interview – a semi-structured verbal reading protocol.

Wyatt et al. (1993) conducted such a study of the reading of behavioral and social science professors deemed experts in their field based on their advanced academic training. Each participant was asked to self-select three research articles they would be interested in reading through only the articles’ author(s) and title. Wyatt et al. (1993) then directed the participants to read one of the selected articles “as they normally would” (p. 53). Each interview was audio recorded and notes about reading strategies were taken as the researchers followed along with the reader. In this study, each interview was scored across four categories: never occurring, occurring once, occurring 2 to 4 times, or occurring 5 or more times. They made this decision to minimize debate over the exact number of times, suggesting “although the two raters might disagree whether 7 or 8 instances of a behavior occurred, this made no difference when the response classification was that the behavior occurred ‘5 or more times’” (Wyatt et al., 1993, p. 55).

In a more recent study, and one focused on the reading of mathematics, Shepherd and van de Sande (2014) asked their participants – three mathematics professors and three mathematics graduate students – to read and work aloud through their chosen textbook. The interviews were scheduled to take place over two hours’ time, in which the researchers would sit and observe the reader over the first half of the interview, and debrief with the reader during the second half.

Each participant was given scratch paper, in addition to the text, so they had the opportunity to take notes if they so desired. Through their work, Shepherd and van de Sande (2014) established that the more advanced the mathematician, the more robust and effective their approach to reading. That is, Shepherd and van de Sande (2014) found the more accomplished the mathematician, the more likely it was the individual would skim familiar text passages, read the meaning of the symbols presented as opposed to reading each individual, made a deliberate effort to gauge and remediate their own understanding as they read, and extended ideas found within the text to situations beyond what was presented.

The clinical VRP designed for this study was patterned after the work of Wyatt et al. (1993) and Shepherd and van de Sande (2014), with guidance from Hilden and Pressley (2011), and influenced by knowledge gained during a previous iteration (Beaudine, 2019a). That is, each student was offered a grade-appropriate reading passage from a nationally distributed text book, along with space to take notes, and asked to read the mathematics passage as they usually would. Hilden and Pressley suggested that readers as young as second grade could be successful in reading and reporting their thoughts through a VRP. Additionally, they suggested keeping the instructions simple, limiting direct influence on the participants' processing to every extent possible, and carefully considering whether or not to pause participants' reading to cue a reflection (Hilden & Pressley, 2011).

Finally, I asked students in an earlier study to read the text aloud (Beaudine, 2019a). For the purposes of the current study, I chose to remove the requirement that students read aloud as they worked to limit the performative nature of reading aloud, leaving that option for the participating students to decide. This decision stems from two considerations. First, reading aloud is an identified reading strategy of interest for this study and asking that one read aloud directly influences each participant's decision to utilize that strategy. This goes against the second of Hilden and Pressley's guidelines, listed above. Secondly, requiring participants to read aloud pulls an already synthetic situation farther from these students' normal act of reading. In some cases, a participant is so attentive to the process of reading aloud that they are unable to comprehend the message found within the words as they read.

Student Interviews

Protocol

Each interview was divided into nine tasks, seven of which were coded for use in this study - the pre-interview and post-interview tasks were neither coded nor included in this analysis because they were not directly related to the participants' reading of the selected textbook passage. The pre-interview task of each interview acted as a "training" passage, a space in which the participant and I practiced the read and report process. The students were explicitly asked to read the text, reflect on what was read, and imagine how a solution might be found. I used this training task to encourage students to be as descriptive as possible when describing their thinking. After this pre-interview task was complete, each student was given clear instructions to describe their own thinking as clearly as possible throughout the remainder of the interview. They were also informed, at this time, that the only prompts offered from me would be "What are you thinking about?", "Is there anything else you are thinking about?", or "Can you say more about that?" The VRP used in this project was chosen to allow each participant the space to describe what they were thinking in their own words and allowed me to explore the brilliance shared by these middle school students without projecting strategies that were not used upon them.

The next six interview tasks, Tasks 1 through 6, consisted of reading one three-page passage from a mathematics textbook (Appendix B), reflecting on what was read, and completing the associated exercise. The text used was marked with Post-it® flags to identify the predetermined stopping points, with each flag placed at naturally occurring pauses within the text (i.e., at printed borders within one reading passage or between two different tasks), dividing the three-page passage into six tasks. The purpose of each of these points was twofold: to prompt each student's reflection at the same point in each interview and to avoid overloading the participants' working memory. At each of the stop points, the participants were asked "What are you thinking about?" and "Is there anything else you are thinking about?" before moving forward with the interview. Once the participants' thoughts were exhausted in one task, they were asked to continue reading the next section of text.

After each of the six reading tasks were complete, the students and I participated in a debrief conversation, Task 7. The first half of the debrief asked participants to reflect upon the reading they just completed. Each debrief began by asking the students to "Reflect on the process of reading, broadly, how did it go?" Other questions asked the students to identify the hardest passage, the easiest passage, sections they know they read, sections they know they did not read (e.g., Could you point out a passage you know needs to be read? Can you identify a section you did not read?). The more detailed and focused questions varied from student to student, but generally explored ideas or questions they brought up during the six reading sections. The debrief was critical in offering each participant a chance to clarify actions that I may have misunderstood, or identify strategies used that were unclear in the moment. This seventh task was also coded and used as data for the following chapters.

The final, post-interview task discussed the participants' perspectives on reading, mathematics, reading in mathematics, and mathematics in reading. This portion of the student interviews was not included within the analysis presented below because it did not directly relate to the reading of the selected passage of text. It, instead, allowed students to explore their thoughts about reading, mathematics, and how the two disciplines may work together. This presents a line of study that is not addressed within this document, but will likely be explored in future iterations of this work.

The interviews were designed to last approximately 30 minutes. On average, the interviews lasted 37 minutes – 22 minutes reading and working through six text sections and 15 minutes reflecting on the process of reading the selected text. Twenty-one of the 22 students were able to complete all 9 interview segments. The other student did not complete Task 6, the final reading task, due to time constraints. Throughout the remainder of this document, there are seven tasks that will be regularly referenced. Tasks 1 through 6 represent each of the six reading passages, as well as their associated work and analysis. Task 7 the coded portion of the debrief conversation held between myself and each participant. Table 3-1 outlines what is covered in each of these seven tasks.

Table 3-1

Interview Task Descriptions

Task Number	Page	Task Description
Task 1	1	Explain It! - Basketball free-throw competition, invites students to critique the reasoning of characters in the problem
Task 2	2	Example 1 - Lifeguard hourly pay, Dan and Nathan, asks students to explore two models and find Dan and Nathan's hourly pay

Table 3-1 (cont'd)

Task Number	Page	Task Description
Task 3	2	Try It! (the first) - Lifeguard hourly pay, Jennifer, allows students to employ ideas found in Task 2
Task 4	3	Example 2 - Feeding Brian's dogs, demonstration of scaling ratios down to unit rate then up to common denominator
Task 5	3	Example 3 - Jumping rodents, comparing unlike ratios through scaling of given rates
Task 6	3	Try It! (the second) - Filling a 3-gallon bucket, allows student opportunity to utilize the ideas demonstrated in Task 4 and Task 5
Task 7	NA	Debrief conversation reflecting on the process of reading

Participants

The participants for this study came from two different schools in the upper Midwest. Two teachers, in two different schools, were recruited to nominate students to participate in this project. Each teacher nominated at least ten students. In alignment with the IRB approval, students could participate in the study only if both the parental permission form was returned and the student agreed to participate on the day of their interview. In all, 22 students participated. Each student was in seventh grade at the time of the study and all names are self-chosen pseudonyms. The teachers were asked to recruit as representative a sample of their school's demographics as possible, taking into consideration the participants' achievement, gender, and race. The students, themselves, were not asked to self-identify. Twelve students from Site 1, a predominantly white school sitting on the edge of suburban and rural space, agreed to participate. Ten students from Site 2, a suburban school with a much more racially diverse student body, participated in the interviews.

Interviews with both teachers, after the completion of the student interviews, provided some insight into each of the sites chosen for the study. The following two subsections outline the teachers' perceptions of the participating schools. This is the only time the instructors will be referenced so they will be simply referred to as Ms. H, from Site 1, and Ms. C, from Site 2. The following is provided as evidence of the difference as sociocultural context between the two schools, and will be recalled throughout the next three chapters, as needed, to support the RAND reading comprehension framework.

Site 1. Site 1 is a traditional middle school, traditional in that it housed sixth, sevenths, and eighth grade students. Ms. H identified their school as being "a suburban school, a little bit rural too." Ms. H went on to explain that Site 1 is a high performing school, scoring above other schools in the area on standardized assessments. The school is not ethnically diverse, but "we have a pretty big diversity in terms of socioeconomic status."

To select the students invited to participate in the study, Ms. H and her colleagues used scores from a school-implemented screener assessment to group students into one of five categories: low, low average, average, high average, and high. Ms. H, in her judgement, intentionally selected students in each category that she believed would be willing to participate in the study. "And then we looked at from that- … making sure we had- kind of um, male and female representation and then also asking for students that would be comfortable and willing to meet and to talk and people that would be, you know, able to explain or kind of have a conversation.". When asked if reading level was considered, Ms. H responded "since it was on math, we looked for the most part on math," but did consider reading "a little bit."

Site 2. When speaking to Ms. C about Site 2, she shared the school is a STEM-focused private school within the public-school system. Students in this school applied and were selected through a lottery system. No tuition is collected from these families, as the school operates within the local public school system. Each grade level has approximately 150 students, ranging from grades three through eight, and the students follow a looping schedule which allows them to be in classes with the same instructor for two consecutive years. Ms. C identifies the students as being “on the higher socioeconomic scale for sure,” and “very racially diverse.”

When selecting participants for this study, Ms. C explained they “offered [the interview] up and just said, ‘Hey, if you guys are interested, it'd be really helpful.’” They went on to say they offered volunteers a pizza party at the end of the study. From the initial group that returned a parental permission slip, “I narrowed it down to the 10 who I thought would be best to choose from.” When selecting the ten students, Ms. C stated “based off the conversations I hear of them with their classmates, whether they're shy, if I can look at their work and see whether they're elaborating or just doing bare minimum,” seeking to get a wide variety of student work and explanation.

The RAND Reading Study Group (RRSG) model, from Chapter 2, consists of three components – the reader, the text, and the act of reading – encapsulated by the sociocultural context within which the reading takes place. Per the RRSG, each reader brings their cognitive capabilities, motivation, knowledge, and experiences (Snow, 2002). In this project, as described above, the two teachers were asked to nominate a wide range of students to participate in the interviews. Based on Ms. H’s assessment, the Site 1 students were largely at grade level, as the school did not have an accelerated course of study. Most of the Site 2 students came from the accelerated courses, and many mentioned working on eighth and ninth grade mathematics in their seventh-grade year. School structure has a great deal to do with the prior knowledge held by the students and the difference is important to identify at this time, though will be difficult to distinguish in Chapters 4, 5, and 6.

In accordance with the RAND analysis of the reading text, I looked “at all the categories of texts and the dimensions on which they vary” (Snow, 2002, p. 24). This includes a description of the text that includes difficulty, the style of writing, the level of engagement intended by the text, and the different representations of text included in the passage. I then identified the top reading strategies implemented by the students for that task and summarized the students’ solutions to the task. Finally, I followed the problem descriptions and solutions with a closer look at the work of some of the students. At the end of these descriptions, I include a RAND “activity” component assessment, considering the purpose, the operation, and the consequences (Snow, 2002) of these participants’ reading effort.

Mathematics Text

A 3-page passage from Pearson Publishing’s *enVision Mathematics* (Appendix B) was selected based on the prominent role Pearson plays in education. While the *enVision Mathematics* series is widely adopted across the United States, it was not the primary text for either research site. An exploration of Pearson as a company, and the *enVision Mathematics* text as the specific example, is a necessary component of my analysis conducted through the RAND reading comprehension model.

As a company, Pearson has overseen the scoring of the SAT exam (Pearson, 2003); holds close ties to the Partnership for Assessment of Readiness for College and Careers (PARCC; Gewertz, 2017), an annual, Common Core-aligned exam given annually to American public school students; and markets itself as a company that produces “world-class tools, content, products, and services [that] are designed to help people adapt to our changing world, navigate its challenges and opportunities, and ultimately make progress in their lives" (Pearson, 2019). The Pearson Publishing empire was quite active within American K-12 education at the time of this text selection.

One of these "world-class tools" is the enVisionmath line of textbooks, a line that includes the elementary, middle (e.g., enVisionmath 2.0); and high school (e.g., enVision Integrated Mathematics) levels. Due to Pearson's prevalence within the US education structure, outlined above, and their efforts to publish a continuous mathematics series of texts that spans the entirety of the K-12 system, this project uses the enVisionmath 2.0, 7th grade text, as it best fits the level of the project's participants.

The passage selected was drawn from the enVisionmath2.0 text for 7th-grade (Forseman, 2017). In all, the participants read through three pages of this text which included an introductory "Explain It!" activity, three examples complete with a solution, and two "Try It!" activities where the students could attempt some of the ideas they were reading about. The questions I posed to each student during the readings were intended to prompt the recall of the students' working memory. The debrief questions changed from student to student, but were directly related to the participants' overall impression of the reading and observations I made while each student was reading. The content selected relates to rates of change, a key concept that is encountered repeatedly throughout the 6-12 mathematics curriculum, between its initial introduction as unit rates, rate of change and slope in middle school, through the uses of the first derivative in Calculus (with applications across STEM courses).

Lesson 2-1 is formatted similarly to the other lessons found within this textbook, and is written in a style and with vocabulary one might expect from a widely adopted seventh-grade mathematics textbook. Lesson 2-1: Connect Ratios, Rates, and Unit Rates, in the student version of the enVisionmath2.0 text (Forseman, 2017), states, “I can … use ratio concepts and reasoning to solve multi-step problems” (p. 85). Each student was asked to read the same three-page passage and report their thinking through a verbal reading protocol. The three-page passage read by each student shows a wide variety of visual cues for students to utilize as they work. In it, the text begins each task with a description, written primarily in English sentences. All but the final task includes some sort of visual beyond the written words, and each example has a string of mathematical statements written in symbols. In all, students reading the text have written words, tables, diagrams, graphic organizers, images, symbolic mathematical statements, academic goals and standard codes, and whitespace to read in a search for understanding and comprehension. Task 1, as one example, is written in a less technical manner than Task 4, but all six tasks work together to develop the understanding sought by any one reader.

In the context of the RRSG model, the features inherent in the text play a key role in the understanding of the reading passage. The RRSG identifies these textual representations in three categories, “the surface code (the exact wording of the text), the text base (idea units representing the meaning), and a representation of the mental models embedded in the text” (Snow, 2002, p. 14). The RRSG also suggests that a reader’s ability to decipher the text may shift as the reading progresses and as the text changes. Much of Chapter 6 is spent discussing the ways three students engage with the text, and how differences in representation, particularly between tasks 4, 5, and 6, affect the readers’ understanding of the text.

Data Sources

The data sources for this project consist of three components – an audio recording, a video recording, and student-created artifacts. Each of the three had a role to play in the design of the analysis of the participants’ reading.

Audio

For each interview, an audio recorder was started immediately after the training task. This recording served as the primary source for the interview data, transcripts, and analysis. At the completion of each interview, the audio file was uploaded and sent off for transcription. The audio files for each interview were processed through online transcription software (Temi.com). The transcripts were “cleaned” by listening to the audio file while following along with the raw transcription. The effort to clean the transcripts allowed for the identification of the two parties participating in the interview, as well as a chance to break the interview up into distinct utterances, and to clarify any wording that was misunderstood in the initial transcription process. These transcripts formed the foundation of the analysis described below.

Video

The video recordings were started as soon as the participant affirmed their consent to participate and were the last thing to be turned off once the interview was over and the participant was leaving. The camera was focused only on the participants’ workspaces, and was used to identify or clarify moments in the audio files’ transcript. The video recording was helpful in offering an illustration of how each participant approached the reading, tracking when students marked on the page, and identifying the “this” and “that” objects referred to by each participant.

Written Artifacts

Each participant was presented with a clean packet used for the project. Each stack of paper included the consent form, a practice task used in a previous study, the three pages they were expected to read, and three pages of practice problems that close the Lesson 2-1 section of the book. The students were encouraged to write on the packets, as needed, as any notes made would be helpful when the analysis began.

Researcher Notes

I took notes during the interview, immediately after each interview, and while each video was watched and re-watched. The hand-written notes were broken into the interview sections found above, transcribed into a digital file, and paired with the transcript. They were useful in highlighting moments in each interview where a participant did something that was not captured by either the audio or video file (e.g., a participant glanced at models in a previous section seeking help with the current task).

Analysis

The text that follows describes the analysis process. The coding schemes described below were informed by an earlier research project (Beaudine, 2019a), though several changes were made based on the results of that previous project, and a need to include a broader literacy perspective. Three major changes included longer reading passages; the inclusion of disciplinary literacy practices within the analysis; and the recruitment of a second individual to code the data to strengthen the reliability of the work. The analysis protocol includes two components: student artifacts produced during the interview and the coding structure designed to assess those artifacts.

Interview Coding

The first goal of this study was to identify the reading strategies used by middle school students as they read an unfamiliar mathematics passage, as explored in depth in Chapter 4. After the codes were identified, I then explored patterns of usage across all participants (Chapter 5) and the ways in which these strategies might be used by a student (Chapter 6).

Codebook

The final codebook (Appendix A) for this project grew from the initial 21 codes outlined in Chapter 2 to a final number of 25 codes. The first step in this process was identifying the reading strategies most likely to arise during these interviews. Mathematics read aloud, disciplinary literacy, and content-area literacy studies informed the first draft of the codebook. In all, 21 codes were initially selected – the 20 reading strategies outlined in Chapter 2 and the decision not to employ a strategy – as they were present in multiple studies, and more codes were to be added as needed. This collection of 21 codes was not intended to be an exhaustive list, but rather a best guess of the strategies that were most likely to be utilized by middle school mathematics students reading unfamiliar texts. Once the initial codes were selected, an iterative process of coding, reflecting, and amending was used to clarify the codebook.

A second coder, a colleague who specializes in literacy, was invited into the coding process after the codebook's third revision. The second coder was trained, and participated in two cycles of coding and reflection, adjusting the codebook as needed. We then coded one interview in its entirety, at the utterance level, switching with each change of voice between the text, the reader, and myself. In all, the interview had 1776 opportunities to assign a code at the utterance level, a code was assigned by either myself or my colleague, and all disagreements of the utterance level coding were mediated through consulting the codebook and discussion. At the end of this second step, 24 codes were in the codebook.

Through this effort to enhance the reliability of code applications, one of the original codes was separated into two, and two new codes were added to the list, stretching the original code structure from 21 to 24 codes. Through our conversations, we found that the original *Pause to reflect* and *Self-check* code did not always fit, as students often paused without clearly self-checking their work, and would self-check their work without pausing. As such, this one code became two different codes – *Pause to reflect* and *Self-check*.

The two new codes that were introduced at this time were *Question or critique the text* and *Making connections across the text*. During the reliability discussions, my colleague and I identified passages of the interview in which these students were openly asking questions about the text in search of clarification, or critiquing the relevance or presentation of the information provided. We also saw that students were making connections across different portions of the text, comparing and contrasting components of the six different tasks.

The final step for codebook development came from the coding of the 22 interviews. The codebook remained amendable because there was a strong likelihood that more codes would arise through the more than 13 hours of interviews with these students. One more code was added to the study through interview coding and student observation – *Create a mental image*. Students creating a mental image explicitly explained their effort to visualize or imagine the situation being described in the text. This third step led to the final list of 25 reading strategies included in this project (Table 3-2).

Table 3-2

Reading Strategies

	Strategy	Source
Strategy 01	Use no strategy	Original Design
Strategy 02	Preview text	Original Design
Strategy 03	Apply prior knowledge	Original Design
Strategy 04	Read aloud	Original Design
Strategy 05	Plan a solution or predict result	Original Design

Table 3-2 (cont'd)

	Strategy	Source
Strategy 06	Modify a plan or prediction	Original Design
Strategy 07	Make notes while reading	Original Design
Strategy 08	Paraphrase text	Original Design
Strategy 09	Read text closely	Original Design
Strategy 10	Read entire passage	Original Design
Strategy 11	Read symbols as words	Original Design
Strategy 12	Decode the text	Original Design
Strategy 13	Attends to prose and equations equally	Original Design
Strategy 14	Seek clarification or external assistance	Original Design
Strategy 15	Pause to reflect	Seeking Reliability
Strategy 16	Reread the text	Original Design
Strategy 17	Seeks important information	Original Design
Strategy 18	Selective reading	Original Design
Strategy 19	Skims the text	Original Design
Strategy 20	Uses text clues	Original Design
Strategy 21	Create analogy or metaphor	Original Design
Strategy 22	Question or critique the text	Seeking Reliability
Strategy 23	Make connections across the text	Seeking Reliability
Strategy 24	Self-check	Seeking Reliability
Strategy 25	Create a mental image	Student Interview

Coding Transcripts

Once the codebook was finalized and all the transcripts were complete, cleaned for accuracy, and formatted to include speakers and timestamps, the transcripts were placed in a spreadsheet. Each utterance, a talk-turn or direct reading of the text, was given its own line and every code had a column unto itself. An example of a coded spreadsheet can be found in Appendix C. A second coder, as mentioned above, was asked to code one interview to improve upon the reliability of the coding and analysis. Through the independent dual coding process of a single interview, my colleague and I found 92 discrepancies in 1776 possible coding decisions, and each disagreement was discussed in detail, and in most cases, resolved through agreement. There were only three instances, of 92 discrepancies, where we were unable to come to agreement.

The full coding was done with the codebook and related discussion between myself and the second coder in mind. The 22 interviews in this study were all coded, at the utterance level, one final time over a two-week timeframe. The utterance level coding was completed for each of the six reading tasks – Explain It!, Example 1 Try It! (the first), Example 2, Example 3, and Try It! (the second) – as well as the debrief conversation where each participant was offered a chance to explain their thoughts and work one final time. The final four questions in each interview were not coded because they did not directly relate to the process of reading the text included in this study.

While the basis for the coding was at the utterance level, the analyses presented in Chapters 4 and 5 were completed at the task level. This was done with the understanding that several utterances exist within a single task, and in many cases, the single reading strategy (e.g., reading out loud, critiquing the text, pausing to reflect) was utilized multiple times within a single task. The interest in this study was which strategies were used by students as they read the presented text. As such, the totaling the strategies used across the individual tasks seemed most reasonable.

Conclusion

The purpose of this study was to identify the reading strategies used by the 22 seventh-grade interview participants and how those strategies are implemented. Additionally, the study sought to explore the purpose served by these reading strategies through the work of individual students' interviews. To accomplish these three goals, a semi-structured clinical interview, utilizing a modified verbal reading protocol, was chosen. Such a structure allowed for the flexibility to explore avenues of discussion uniquely presented by an individual, but offered a standard line of questioning that presented opportunities to compare across interviews.

The analysis began with a codebook based heavily upon literacy, disciplinary literacy, and content-area literacy research, and evolved throughout the course of the project. Much of this evolution came with the introduction of a second coder, and the discussions held throughout the training and practice processes. Once the codebook was finalized, each interview was coded one final time at the utterance level, with the analysis that followed done at the task level.

In Chapter 4, I share strategy-level analysis, identify the strategies used by students and provide comparisons of the interviews across participants. In short, Chapter 4 presents the reading strategies used by these students. Chapter 5 brings about a description of how the reading strategies were used within each task and a more in-depth analysis of common combinations of strategies, answering the question of how these strategies were implemented by these 7th-grade students. Three students, Eggy, Morgan, and Henry, are the focus of Chapter 6. This focus on each student’s use of reading strategies to read, understand, and complete a single page of mathematical text allows for a much more fine-grained exploration of what purpose these reading strategies may serve.

CHAPTER 4: The Strategies Used While Reading

In Chapter 3, I outlined the process used to both collect the data, and analyze said data. In this fourth chapter, I share the overall, and very broad findings from the 13 hours and 40 minutes of interview data. The primary goal of this chapter is to address the first research question, identifying the reading strategies used by the participants in this study. The findings are presented first by reading strategies, tallying the total number of students who used each of the reading strategies. I then analyze data at the task level, identifying the number of participants who used specific reading strategies for each task.

In doing so, over the course of the next three chapters, I apply focus on the "activity" component of the RAND reading strategies model, tying the student process of reading to the work done to complete the task. In this chapter, I focus on identifying the reading strategies used by these students. In Chapters 5 and 6, more attention will be placed on the activity of reading, and the connection between what was read and the work that was done.

Beyond these lists of reading strategies collectively used by the students in this study, this chapter identifies trends in strategy usage by the tasks completed and study site. The chapter ends with a reflection on the difficulty of identifying certain reading strategies "in the moment," which highlights the importance of sitting with study participants and debriefing components of each interview.

Findings

During this study, 22 middle school students were asked to read through a section of a widely adopted textbook, one not used in their own classroom, and complete any related tasks on the three pages of selected text. Overall, students spent about 22 and a half minutes reading the tasks and working through the related activities. Students from Site 1 spent an average of 23 and a quarter minutes reading, whereas students from Site 2 worked a bit more swiftly, averaging 21 and a half minutes.

Interview Segments Completed and Coded

In all, 22 students participated in the study, and 21 of the 22 completed all interview tasks. The only exception was Figglestein, who did not finish Task 6, the second Try It! exercise, due to time constraints. In all, these students completed 131 tasks and 22 debriefing conversations for a total of 153 coded segments. In the subsections that follow, I identify the reading strategies that were implemented the most and least across the seven coded sections and then the strategies used at least once by the most and fewest students.

Reading Strategies Used by Students

When I centered attention on the strategies explored in this project, I found 23 of the 25 possible strategies were used by at least one of the 22 study participants at least once while reading (Table 4-1). On average, 12.6 students used each strategy. Two strategies, *Use no strategy* and *Create analogy or metaphor*, went entirely unused by all 22 participants. Additionally, the strategies *Read symbols as words* (15 students), *Use text clues* (16), *Read aloud* (18), *Seek important information* (19), *Modify a plan or prediction* (20), *Pause to reflect* (21), *Question or critique the text* (21), *Self-check* (21), *Plan a solution or predict result* (22), and *Paraphrase text* (22) were all utilized at least one time by at least two-thirds of the study participants.

Table 4-1

Number of Participants Implementing Each Strategy

	Strategy	Number of participants
Strategy 01	Use no strategy	00
Strategy 02	Preview text	12
Strategy 03	Apply prior knowledge	08
Strategy 04	Read Aloud	18
Strategy 05	Plan a solution or predict result	22
Strategy 06	Modify a plan or prediction	20
Strategy 07	Make notes while reading	06
Strategy 08	Paraphrase text	22
Strategy 09	Read text closely	09
Strategy 10	Read entire passage	07
Strategy 11	Read symbols as words	15
Strategy 12	Decode the text	02
Strategy 13	Attends to prose and equations equally	11
Strategy 14	Seek clarification or external assistance	14
Strategy 15	Pause to reflect	21
Strategy 16	Reread the text	08
Strategy 17	Seeks important information	19
Strategy 18	Selective reading	22
Strategy 19	Skims the text	07
Strategy 20	Uses text clues	16
Strategy 21	Create analogy or metaphor	00
Strategy 22	Question or critique the text	19

Table 4-1 (cont'd)

	Strategy	Number of participants
Strategy 23	Make connections across the text	14
Strategy 24	Self-check	21
Strategy 25	Create a mental image	02
	Average number of students per strategy	12.6

Reading Strategies Used in Tasks

When the data is viewed by task (Appendix D), a picture of the reading strategies most and least used by the study participants becomes more clear. The five reading strategies most used by the students in this study, across all tasks, were *Read aloud* (104 implementations), *Plan a solution or predict result* (97), *Pause to reflect* (89), *Paraphrase text* (86), and *Self-check* (77), each employed in over half of the completed interview tasks. Six strategies were used in fewer than 10 tasks across the 153 completed tasks: *Use no strategy* (0), *Create analogy or metaphor* (0), *Decode the text* (2), *Create a mental image* (4), *Skims the text* (7), and *Make notes while reading* (7).

Overall, the study participants used an average of 38.68 strategies across all seven tasks analyzed for this study, for an average of 5.53 strategies per task. The final row in Appendix D shows a decrease in reading strategies used by an average participant per task. In Task 1: Explain It!, readers in this study used just over seven strategies each. Between Task 1 and Task 5: Example 3, a decrease in the average number of strategies used per participant can be seen before a small increase between Task 5 and Task 6: Try It! (second).

A more thorough description of this note will follow below, but it is important to mention that the reading strategies during the first six tasks - Explain It!, Example 1, Try It! (first), Example 2, Example 3, and Try It! (second) - should be viewed differently than those in Task 7: Debrief. The analysis across the first six tasks was based on observation – that is, I recorded those strategies that I observed students attempting to use. I attributed strategy usage to what was seen and heard while the students read through the text. In Task 7, the strategies reported were openly discussed between the student and myself, and the coding was based on what each participant said they were doing during their reading.

Student Reading Strategy Selection

The unique strategies used by each student (Appendix E) and the total number of tasks in which a student used specific strategies (Appendix F) are also shared within this text. Fred, who used 19 different reading strategies at least once, used the largest number of unique strategies, while Jeff used just eight unique strategies as he completed the reading and interview. The only other participant to use fewer than ten unique strategies was Carolynn (9). Broadly speaking, the mean number of strategy implementations used by these students was 38.7 total strategies employed per student interview. They used an average of 14.4 unique strategies when working through the seven tasks.

Student Strategies per Task

A grand total of 851 reading strategies were implemented by the students as they worked through the 153 completed interview tasks. Of the 22 students, Fred used the most unique strategies (19) of any participant in the study, and he implemented a reading strategy 45 times while reading and completing the seven tasks. There were three students, however, who had a higher total number of strategy implementations while reading and working through the selected textbook passage – Dave (59 total strategies), Eggy (50), and Johnny (46). Hannah, in her work, employed the same number of reading strategies as Fred.

Jeff and Carolynn implemented the fewest number of unique strategies, eight and nine respectively. Jeff used far fewer strategies (17) in their reading pursuit than their next closest peer, Carolynn (26). Only one other student, Rachel (29), chose to use fewer than 30 strategies as they read through and discussed the selected reading.

Each of the students in this study utilized at least eight of the 25 reading strategies, and no student used more than 19 unique strategies as they read and worked through the text. Two strategies were used by all 22 students, and two went unused. Of the 23 reading strategies that were used by at least one student, ten were used by at least two-thirds of the study's participants and nine were used by fewer than ten students.

As the chapter progresses, I outline some trends relating to the selection of the reading strategies used in the reading and explore how the strategy usage was different across sites. This chapter closes with a more detailed discussion of why the reading strategies identified during the first six tasks must be framed differently than those found during the debrief. In chapter 5, I investigate the potential causes of the decreasing trend of average strategies used per task, as highlighted by Table 2 and explore the combinations of strategies that helped students work through Tasks 1, 3, and 6. Chapter 6 explores the reading strategies used by three students and their effort to understand the text. In both chapters, I make an explicit effort to connect the participants' reading to their work through the RAND model for reading comprehension.

Reading Strategies Trends

Through the above discussion of reading strategies, I highlighted big-picture trends from Tables 4-1 and 4-2. In the sections that follow, I explore possible reasons for the participants' decline in average strategies used per task. The remaining text focuses on the most and least used strategies. This is done in two sections and utilizes the words of Charlotte, a Site 1 student. The first section explores the strategies with the highest number of implementations and those used by the most students. The second section will investigate the strategies with the fewest number of implementations and by the fewest students.

Average Reading Strategies

As mentioned above, the table in Appendix D shows a clear decrease in the average number of student implemented reading strategies used to complete each task. Each student used an average of 7.2 reading strategies to complete their work for Task 1. A decrease is seen on the second page where the same students used an average of 6.4 and 5.4 reading strategies to complete Task 2 and Task 3, respectively. The rate of decrease slowed for Task 4 and Task 5, where students used an average of 4.6 and 4.2 reading strategies per task, respectively. The only uptick in reading strategies was between Task 5 and Task 6, where students used an average of 4.8 reading strategies to complete their work related to Task 6. The question, then, is why? What caused the decrease in the number of reading strategies used between Task 1 and Task 6? I propose three possible explanations for the shrinking number of reading strategies implemented – comfort, demand, and space. By this, I am suggesting that readers' strategic approach may be affected by their level of comfort with a text, the demands made by the text, and the space provided for them to work.

The participants in this study knew the reading came from a textbook, but were unfamiliar with Forseman's (2017) *enVision Mathematics* text. As stated in Chapter 3, some students asked what they were expected to read and I suggested they read the text as they normally would, however they so choose to read it. The first page brought with it a very open space, a single introductory problem, and some guiding questions. The second page held Task 2 and Task 3, the page had less whitespace and more structure than the first page. On the third page, the students found Task 3, Task 4, Task 6, and comparatively little whitespace. As each student became more familiar with the stylistic layout, with the reading protocol, and with the audio and video recordings, they seemed to become less reliant on strategies to solve the problems.

The demand of the tasks likely played a role, as well. Each student was asked to work through an exercise on each page. On the first page, many students were relying on knowledge gained from past years to address Janie's claim, because they had no related model on the same page. On the second page, many students worked through Task 2, finding the unit rates for both lifeguards on their own, before realizing the work was complete and modeled for them in the second column. When they were asked to find Jennifer's pay, they needed only to follow along with the model from Task 4. With more structure came less cognitive demand, and therefore fewer reading strategies employed. By the time they reached the middle of Task 4, all 22 students realized the work was done for them. With the work complete, the participants read the examples, but their deliberate effort to understand each component was slightly reduced. With this explanation, the uptick in strategy implementation between Task 5 and Task 6 problem might be explained by the difference in demand. Task 5 only needed to be read, Task 6 needed to be solved.

Finally, the difference in strategy usage may be based on the design of each page. The first and second pages have a lot of white space with ample room to work or take notes. The third page is much more crowded that either the first or second page, and in particular, Task 4: Example 2 has the least amount of whitespace. This crowding may deter students from making an effort to mark on the page, to take notes. The third page also comes with a cluster of equivalence statements (Task 4), as well as a diagram and a table (Task 5). These require a different style of reading, and the information may be more easily attained than a text that is predominantly written prose (Task 1). Text that is more easily deciphered would require fewer reading strategies.

As stated above, I believe the characteristics of a given text may affect the number of reading strategies an individual may use when reading for comprehension and understanding. In the case of this study, the readers tended to use fewer reading strategies as they progressed through the three-page passage. A reader becomes more comfortable with the style of writing as they progress through text, and in doing so, they may not need to utilize as many reading strategies to further their comprehension of the presented text. Similarly, the less a specific mathematical task asks a reader to do, the less effort they may place into understanding that section of text, since the work has already been completed. Finally, the amount of space provided may limit the type of engagement from some readers.

Students' Most and Least Used Reading Strategies

For this analysis, I look more closely at the most used and least used reading strategies. The first list consists of the five most used reading strategies when considering the number of students using an individual strategy and the number of tasks within which the strategy was used. The second list holds the five least used strategies across the same slices of data. The text below explores these lists in more detail.

Most Used Strategies

Consider the strategies found in Table 4-2. When looking at those strategies that were used by the most students and comparing that list to the strategies used in the most tasks, the lists are quite similar. Four of the five strategies on each list show up on the other. The strategy that appears the most across all the tasks, *Read Aloud*, was used by 18 of the 22 students. Similarly, *Questioning or Critiquing the Text*, was implemented in 53 tasks, the sixth most of the 24 strategies implemented. The other four strategies in both columns are identical, and will be discussed in more detail below

Table 4-2

Most Used Strategies

Strategy (students)	Strategy (separate implementations)
Paraphrase text (22)	Read aloud (104)
Plan a solution or predict result (22)	Plan a solution or predict result (97)
Self-Check (21)	Pause to reflect (89)
Question or critique the text (21)	Paraphrase text (86)
Pause to reflect (21)	Self-Check (77)

Paraphrase the Text. Pressley and Afflerbach (1995) identified this strategy as "Repeating/restating text just read to hold in working memory … [or] repeating/restating a thought that occurred during reading" (p. 35). All 22 students took time to paraphrase some component of their reading, and did so on a total of 86 tasks. All but three of these 86 tasks were observed during the students' reading and working segment. Twenty students paraphrased during task 1, 13 in Tasks 2, 3, and 5, and 12 paraphrased parts of Tasks 4 and 6.

This strategy was employed in two distinct ways - either to remind the reader what was said before taking the next step toward a solution or to simplify more complicated representations. Charlotte did both in her interview. While finalizing a solution to the Explain it! problem (Task 1), she looked back to the text to find out what the solution wanted, to which she announced "Out of the times-, like, okay. Who made more out of how many attempts they had?" In this moment, Charlotte seemed to be repeating the text to remind herself about the goal of the task at hand.

In Example 3 (Task 5), Charlotte paraphrased the table presented in the text. She reported, “Oh, okay. So, [pause] okay, so the ratios, [pause] okay. So the rabbit jumps three jumps in eight meters. The kangaroo is five jumps in 12 meters. So you keep, you keep like adding on onto the table until you find, um, [pause] the distance.” In this effort, Charlotte translates the information found in the table into meaning that helps her better understand the solution presented in Example 3.

Plan or Predict. “Generating an initial hypothesis about what the text is about, one that can be revised or refined” (Pressley and Afflerbach, 1995, p. 33). Much like the paraphrasing, all 22 students planned a solution or predicted the outcome before doing any work. The reading strategy of planning and predicting was utilized in 97 of the completed tasks. Nineteen students planned or predicted an outcome in Task 1, 20 in Task 2. In two tasks, Task 5 and the debrief, fewer than ten participants chose to plan a solution or predict an outcome.

In an ELA study, one might see a student make a guess about the presented reading or sharing some ideas about “what comes next” in the text. The study participants were rarely seeking to state a hypothesis at any point in the readings, and almost never guessed at the end result unless otherwise directed by the text. In this study of mathematical reading, however, students chose to talk through the steps they believe they needed to take to find a solution.

As an example, Charlotte could be observed planning a solution in Task 4. After reading the details of the example and being asked what she’s thinking about, she shares “So I'm mostly thinking that, um, [whispering] okay, we did this before [/whispering]. Um, I need to find like how many, like I need to divide this by nine to find how many per hour?” In the sentence following her whispering she is formulating a plan to find the pay rate for one of the two lifeguards.

Pause to Reflect and Self-Check. At the beginning of this project, these two strategies were one. Shanahan et al. (2011) stated, “When explaining how he thought about the ideas in a text, one mathematician said that he asks himself questions. ‘Did I see this fact before? Did I see a special instance of this fact before? Do I know if the statement is correct? Can I prove it?’” (p. 418), and based on this definition the code began as “pause and self-check.” Through the reliability coding effort, my colleague and I discovered that, while the two are closely related, they are not universally entwined. One may very well pause and not make any noticeable effort to correct or assess their own work. Similarly, one may realize a mistake as they are working, make the correction, and not pause while doing so.

The two were still very closely associated with one-another, as evidenced by the same 21 students using both strategies during their interviews. One student, Gracie, chose not to use either of the two strategies. The two strategies were clearly not always used in tandem, as the different counts demonstrate. In several cases, however, either the pause was followed by an assessment or “check” of the participant’s work or the pause occurred in the midst of a self-check.

Charlotte provided an example of each. In one moment, she stated, “All right, so zero. Okay. Three gallons. We have to label that. [pause] no. [pause] all right, now I got it,” where the pause and the self-check were closely related to each other, with the audible pause coming before the audible self-check. In other moments, Charlotte’s pause could be found after the self-check had begun. For example, “Um, okay. So seven days. Okay. So, and this one you need to do. Okay. So I think we needed to do 128 times seven. Wait. [pause] Okay. And then you need to find out how much each dog, like, eats per day.”

Least Used Strategies

When looking at those reading strategies used the least by the students in this study (Table 4-4), the two lists, the number of students using a particular reading strategy and the number of tasks in which the strategy was used, include the same five strategies. I will set *Decoding* and *Create a mental image* aside for the time being, as neither were easily observed. A section of this text, below, is dedicated to strategies that were difficult to identify through observation. The remaining three strategies were either seen in my previous work or commonly prescribed as a test-taking strategy.

Table 4-3

Least Used Strategies

Strategy (students)	Strategy (separate implementations)
Creates a Mental Image (2)	Decoding (2)
Decoding (2)	Creates a Mental Image (4)
Make Notes While Reading (6)	Make Notes While Reading (7)
Skim the Text (7)	Skim the Text (7)

Skim the Text and Making Notes While Reading. These two reading strategies may seem like an odd pairing, and none of the 22 participants used the two in tandem. That is, in no single utterance did a student both skim the text and seek to write notes. The two strategies do share one characteristic, however, as they are often presented as effective mathematics standardized test taking strategies (e.g., Penn State Learning, 2020; Todd, 2020). Just like a textbook, the text found within any standardized assessment must be read by the test taker. When one considers that both strategies are often found within the Content Area Literacy literature, encouraging students to skim, take notes, and define a purpose for their reading process

(Harkness & Brass, 2017), their application in test taking spaces is a logical extension of mathematical reading practices.

Shepherd and van de Sande (2014) found, through their work, that the more proficient a mathematician, the more likely it was the individual would skim a familiar passage of text. Penn State Learning (2020) suggests that skimming could be used in a different manner, encouraging test takers to “Skim test and do those questions you know immediately.” Todd (2020) phases it a bit differently, but in both cases the intent is to quickly prioritize the questions so a test taker can quickly complete the problems they know, to maximize the points earned throughout the assessment. When seeking evidence of skimming, I was looking for sections of text where the participants moved more quickly, possibly because they found passages containing information with which the student was already familiar (Shepherd & van de Sande, 2014).

It was difficult to identify, through observation, whether the text was being skimmed, or selectively read by the participants. As such, six of the seven times *Skim the text* was assigned can be found in the debriefs. Charlotte, for example, stated, “Well it, [pause] I just thought like you can make it a little like, it's confusing cause like, um, like I could figure it out if I just like look at it, but like, um, just by looking at it really quickly.” In this passage, she suggested that if she spent more time looking at Example 2, she might have been able to more clearly understand the information presented. Instead, she just skimmed Example 2.

There are clearly several reasons one might choose to skim a passage of text. Shepherd and van de Sande (2014) report that expert mathematicians may skim over passages of text they recognize and understand, in lieu of closely reading each passage. Standardized test taking strategies, some taught as early as elementary school, encourage students to skim the text to find familiar information. Charlotte demonstrated that students may use this strategy in a different manner, skimming past sections perceived as confusing or unnecessary.

Examples of note taking while reading might include “highlighting, underlining, circling, making notes, outlining, or somehow flagging important points in text, including important examples” (Pressley & Afflerbach, 1995, p. 44). Students, beginning in elementary school, are often encouraged to highlight important information within the test question through underlining or circling (Todd, 2020). Bergeson and Rosheim (2018) identify the note-taking function found on iPads as advantageous for students who can use their mathematical text electronically. They suggest this function would allow students to make notes or highlight a section of text so they could ask their classmates later. Additionally, at the middle school level, in an early conversation with Ms. H, she pointed to a mnemonic device written on her board that addressed how one should engage with a test question. One of the priorities was to take notes, to find and circle information critical to the problem.

Charlotte identified a need to take notes while reading on several occasions. On one such occasion, during the first task, she reported, “I'm gonna like, I feel like I should, like, record, like, I should, like, record it on paper- Like, write it down, like, the number of free throws.” The information she needed is represented on the page, and yet the act of taking notes was employed by Charlotte to better understand what was being asked of her or more clearly identify the numbers with which she needed to work.

Coding Difficulties

Earlier in this chapter, I suggested that the first six tasks and the debrief should be viewed somewhat differently. In the first six tasks, the students were asked to read, then prompted to share what they were thinking. The coding during this time was based on observation. In other words, I identified reading strategies I felt were clearly enacted, without consulting the individual doing the reading. Through this process, reading strategies that were explicitly enacted (e.g., *Read aloud, Paraphrase the text, Skim the text* etc.) were easy to identify. Table 4-2 shows a definite shift between codes commonly identified while the students were reading and those requiring student input to attribute.

This shift matches the task at hand, whether student responses were “in the moment work” or “after the fact reflections.” Students, during the read- and work-aloud, were asked to share their thoughts as they relate to the text read or work to be done. At this time, students generally reflected on the immediate need, because reading and solving the question was their priority. During Task 7, however, each student and I spent time focused on specific passages or responses provided. This “second look” provided space for students to follow lines of thought that they may have squashed, earlier, because their problem was not yet done.

There were moments where the reading strategies were a bit subtler, and fell between two possible codes. Consider the difficulty in distinguishing between a student skimming the text and selectively reading the text, for example. Pressley and Afflerbach (1995) identify selective reading as reading “only particular sections and which particular sections … to read particular sections before reading others … [or] quit the reading because the content in the reading is not relevant to the current reading goals” (p. 32-3).

The definition above, by Pressley and Afflerbach, sounds a great deal like Charlotte’s description of her effort to read Example 2. The difference in coding, however, is two-fold. During the interview, she attempted to read the entire problem, including the difficult text, so selective reading would not have been appropriate. At that time, though, Skim the Text was also not appropriate because she had not yet explained her intent as she read quickly. That explanation did not come until after the reading was complete, and at that time the Skim the Text code was applied.

Two other reading strategies were similarly difficult to assign while observing the students read. *Seeks important information* and *Uses text clues* were each discussed by over two-thirds of the participants during the debrief, making up nearly half of all the occurrences of the two codes - 15 of 34 and 15 of 31, respectively. In one moment, I witnessed one participant's eyes dart from the Try It! at the bottom of page two to Example 1 above to better understand the models presented in both. Outside of that one observation, the assignment of these two codes hinged on the participants' words as they explained their thinking or reflected on their work.

Reflection and Implications

Through the closer investigation of lists of reading strategies outlined above, I arrived at several realizations. First, I was reminded that there is a discernible difference between reading strategies and reading skills. Afflerbach et al. (2008) described reading strategies as an intentional effort taken to better understand the text being read, whereas a reading skill requires no additional effort as the skill has become second nature. While I am confident that each of the reading strategies discussed above were used strategically, I recognize there were other passages of the interviews where the same strategy was used skillfully.

Consider the *Read aloud* strategy that was seen in 104 of the 153 completed interview tasks. In most cases, the students proceeded to read the text out loud, recognizing that the interview was to be recorded. This reading presented by these students was largely smooth and confident, with only moments of hesitation when shifting from one style of text to another. There were other segments of the interview, however, where participants returned to the presented problem, re-read the text out loud, and then proceeded to complete the task. In these later cases, I argue they were using the read aloud strategically.

Beyond the strategy versus skill discussion, many of these strategies are implemented in a variety of ways, those explicitly taught were not widely used, and some strategies are difficult to identify through mere observation. Charlotte modeled many of the strategies explored above, and in doing so, used some strategies differently from one implementation to the next. For example, she paraphrased text to refocus her solution and to simplify an image. Charlotte also paused to reflect which prompted an assessment of her work in one instance. Later in the interview, she recognized the need to check her work, which prompted the pause.

Research has demonstrated that both *Skim the text* and *Make notes while reading* are used by expert mathematicians as they engage with unfamiliar texts, are encouraged as effective test taking strategies, and are explicitly taught by instructors like Site 1's Ms. H. In this study, however, Charlotte was one of only four students who tried to take notes and skimmed the text presented. I will note that these seventh-graders are not expert mathematicians, and the text chosen did not resemble a standardized test of any kind. That said, it is curious that more students did not utilize either of these strategies during their respective interviews.

One possible explanation for the low number of implementations recorded for *Skim the text* may be that participants made a concerted effort to *Read entire passage*, though the latter strategy was only observed in 11 completed interview tasks. Another possibility is that it is difficult to identify *Skim the text* through observation, alone. I, as the interviewer, was following along as the students read and took notes about moments that seemed important. I was not always able to follow their eye movements, and the camera was trained on their workspace, not their face. This combination made it difficult to decide whether the participant was skimming the text or selectively reading the passage, choosing to engage with some text, but not others. It was only during the debrief, for most instances, that it became clear that the student chose to skim through the text.

Conclusion

The goal of this chapter was to outline the reading strategies used by the seventh-grade students who participated in this study. The 22 students in this study implemented an average of 14 unique strategies as they worked through three pages from a nationally distributed textbook. As the participants became more familiar with the process, and the text began to use less space for each example, the number of reading strategies used per task declined. *Read aloud*, *Plan a solution or predict result*, *Pause to reflect*, *Paraphrase text*, *Self-check*, and *Question or critique the text* were the most commonly used strategies and *Decode the text*, *Create a mental image*, *Skim the text*, and *Make notes while reading* are found among the least commonly used strategies. Two strategies, *Use no strategy* and *Create an analogy or metaphor*, went unobserved and unmentioned.

In the next chapter, I explore in greater depth how these reading strategies were used differently by these students as they worked toward solutions in Task 1: Explain It!, Task 3: Try It! (the first), and Task 6: Try It! (the second). Through this process, I introduce more students' voices to the project, and explore how these efforts aided student comprehension of the text, leading them to the strategy chosen. A similar approach will be taken in Chapter 6, where I describe the ways three students work through a single page of the selected reading passage.

CHAPTER 5: Investigating How Reading Strategies Were Used

Chapters 3 and 4 outlined the intent and methods of this investigation into students' mathematical reading and identified the reading strategies used by the 22 study participants. In this fifth chapter, I present an in-depth analysis of students' strategies and solutions for three of the six tasks included within each interview. I also, through the application of the RAND reading comprehension model, explore trends about how the chosen reading strategies were utilized, seeking to better understand the ways students applied the understandings gained from the reading effort. The analysis below focuses primarily on those strategies most used for the three open-ended tasks students were asked to complete as part of their interview.

The analysis in this chapter falls in three sections, following the three tasks explored – Task 1: Explain It!, Task 3: Try It! (the first), and Task 6: Try It! (the second). Within each section, I begin with a thorough description of the task the students were asked to complete, attending to the RAND reading comprehension model's "text" component. As explained in Chapter 3, and in accordance with the RAND analysis of the reading text, I looked "at all the categories of texts and the dimensions on which they vary" (Snow, 2002, p. 24). This includes a description of the text that includes difficulty, the style of writing, the level of engagement intended by the text, and the different representations of text included in the passage. A more thorough investigation utilizing the RAND reading comprehension model will be included with the three students in Chapter 6. The chapter closes with a reflection on what these findings suggest about how reading strategies are, or could be, used to facilitate a student's mathematical reading. In all, representative examples of ten students' work are shared within this chapter.

One final note before I present the more detailed exploration of these findings. There are several strategies that may be considered both a reading strategy and a solution strategy, *Uses* *prior knowledge*, for example. I agree that a student who recognizes a basketball situation and immediately ties it to their game later in the day, as seen in Task 1, could use that information to both further their understanding of the text being read and to lead their solution down a logically sound path. Because the two are so closely related and difficult to parse, I make every attempt to be explicit about the components that are reading strategies, those that are solution strategies, and how the former informs the latter.

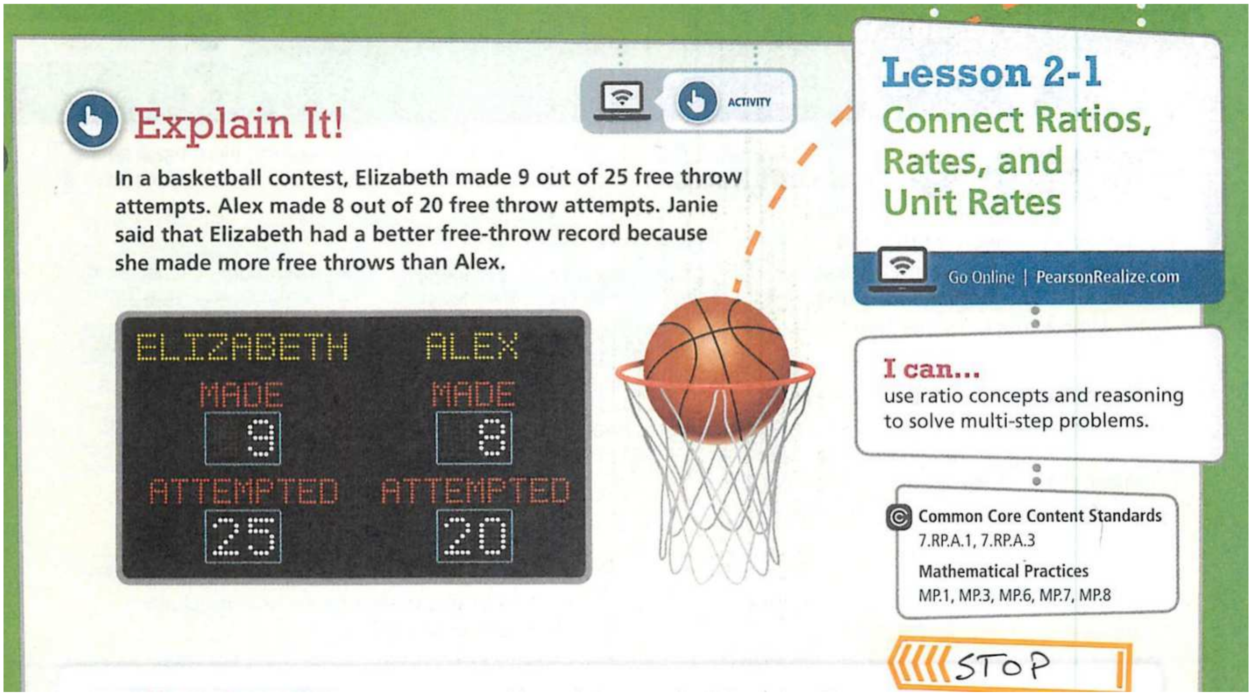
Task 1: Explain It!

Task 1 opened Lesson 2-1: Connect Ratios, Rates, and Unit Rates (Forseman, 2017, pp. 85-7) with a story about a free-throw contest between two people. This task was the students' first introduction to Forseman's EnVisionmath 2.0 textbook and their first opportunity to read without direction from me, as the researcher. The task was completed in two parts. Each student was asked to read until the tab labeled stop (Figure 5-1), at which point they were asked, "What are you thinking about?" Once they described their thoughts, they were asked to work through the two questions that followed (Figure 5-2).

There is a lot of information presented in Task 1 (Figure 5-1). On the right side of the page are the section title, the "I can" statement, and the codes for the Common Core State Standards and Mathematical Practices addressed in the section. The prose of the problem presents a "realistic" situation surrounding a free-throw competition between two students, Alex and Elizabeth. I characterized this task as "realistic" because it represented an event that could happen, but is not "real-life" in the sense that the contest is not actually happening. In this realistic event, a third student, Janie, claimed Elizabeth performed better than Alex. There are two images that relate to the story being told, a scoreboard with the number of made shots and total attempts and a basketball graphic. Students were asked to stop reading at the line dividing the problem and the related questions.

Figure 5-1

Task 1: Explain It! - Part 1



Explain It!

ACTIVITY

In a basketball contest, Elizabeth made 9 out of 25 free throw attempts. Alex made 8 out of 20 free throw attempts. Janie said that Elizabeth had a better free-throw record because she made more free throws than Alex.

Lesson 2-1

Connect Ratios, Rates, and Unit Rates

Go Online | PearsonRealize.com

I can...

use ratio concepts and reasoning to solve multi-step problems.

Common Core Content Standards
7.RP.A.1, 7.RP.A.3

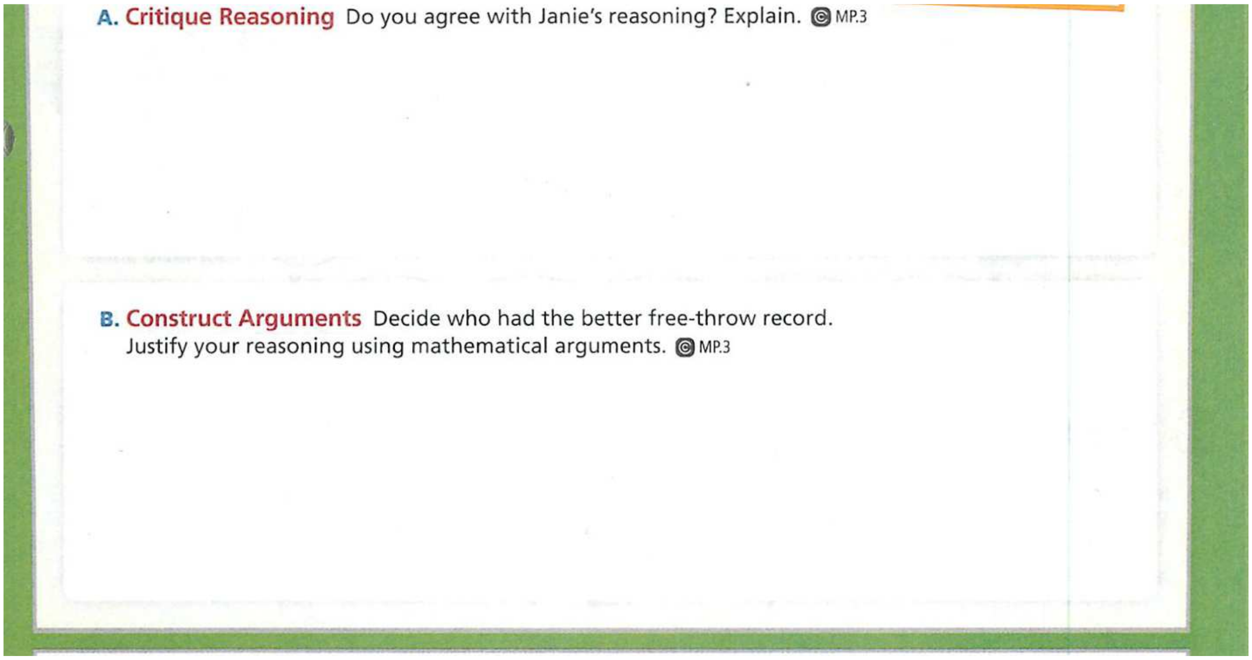
Mathematical Practices
MP.1, MP.3, MP.6, MP.7, MP.8

STOP

In the second part of the task (Figure 5-2), the students were presented with two questions related to Janie’s claim. The book provided more whitespace than text, leaving ample room for any work the students felt they needed to provide. The initial question asked students to reason through the realistic situation and decide whether they agree with Janie. To complete Part A, students were expected to decide if they agree with Janie or not, and then explain why. This question does not require any sort of calculations on the part of the student, at this time, though some chose to work the mathematics during Part A. The second question asked them to mathematically defend their answer from the first question.

Figure 5-2

Task 1: Explain It! - Part 2



A. **Critique Reasoning** Do you agree with Janie's reasoning? Explain. MP.3

B. **Construct Arguments** Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

Task 1 Text Analysis

The text used on this first page had quite a variety of engagement across all participants. Each read the narrative about the basketball context, some were quite diligent about reading the contents on the top right-hand side of the page, but the utility of that information was brought into question several times. Very few attempted to read the common core standard codes, and those who did questioned why they were there. As stated above, there were multiple representations of text, including written sentences, standard codes, and images.

Each student read the prompts to both Part A and Part B, when asked to continue working down the page. Many students, while working through the second half of the page, answered the question posed in Part B as they were trying to justify their response in Part A. Additionally, the text in this second section of Task 1 included no images or guide as to the path to a solution. This first page of text had a short narrative portion, but the purpose of the text was to help students remember concepts taught in lessons past. Like most texts written of mathematics, the purpose here was descriptive and expositional in nature (Solomon & O'Neill, 1998), but well within the participants' ability to read and comprehend.

Outcomes

Over 90% of the students, 20 out of the 22 participants, decided Janie was not correct in her assertion. In the first study site, 11 of the 12 students said Janie was incorrect and one never settled on an answer. At the second study site, nine of the ten participants disagreed with Janie and the tenth student agreed that Elizabeth had a better record than Alex.

Mathematical Solutions

There was a large number of solutions presented as the students worked toward a resolution. The three most common approaches sought to establish a unit rate for both Alex and Elizabeth; compare fractions made from the made shots and total shot; and compare the percentage of made shots for both shooters. Other solutions included sums and a comparison of made shots. In all, four students used unit rates, ten compared fractions, and four more students sought out each shooter's free-throw percentage.

Top Reading Strategies

Of the 26 reading strategies, four of them were observed in the work done by two-thirds of the study participants - *Paraphrase the text* (20 students), *Plan a solution or predict the result* (19), *Read aloud* (18), and *Pause to reflect* (17). Three other reading strategies were observed more in Task 1 than any other task - *Seek clarification* (11), *Preview the text* (10), and *Uses prior knowledge* (5). As the analysis is presented, below, the latter three strategies will be the central focus, as they proved to be somewhat unique to Task 1.

Solving Task 1

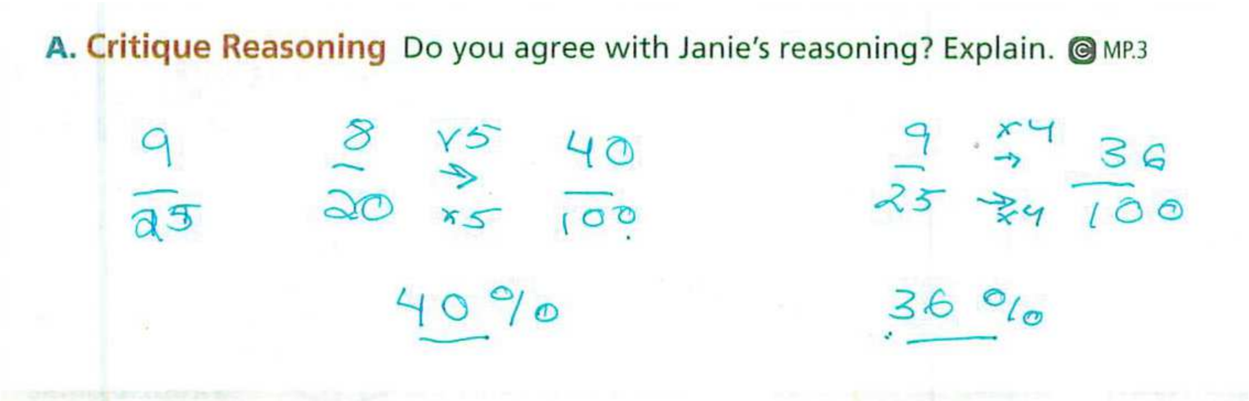
The text in this section explores the reading strategies used during this section of the interview through the students' reading and work toward a solution. In doing so, I have chosen to share student work that represents the most common solutions used to disagree with Janie's claim, and the solution of the participant who agreed with Janie. Through this exploration, I explicitly focused on the implementation of three reading strategies - Seek clarification or external assistance, Preview the text, and Use prior knowledge - since they were used more during Task 1 than during any other task.

Solution Approach 1 - Percentage Comparison

Five of the study participants sought out percentages to justify who won the free-throw contest described in Task 1. Four of the five students who chose to use a percentage comparison were from Site 2. Hannah, one of the students from Site 2, was "thinking of a basketball game that I have later on." The solution she shared (Figure 5-3) shows her work "calculating, um, converting the fractions so we can have Alex and Elizabeth shots like proportional [pause] with like percentages of how many they made right out of [100]."

Figure 5-3

Hannah's Task 1 Solution



A. Critique Reasoning Do you agree with Janie's reasoning? Explain. Ⓒ MP.3

$\frac{9}{25}$ $\quad \frac{8}{20} \xrightarrow[\times 5]{\times 5} \frac{40}{100}$ $\quad \frac{9}{25} \xrightarrow[\times 4]{\times 4} \frac{36}{100}$

40%

36%

On paper, Hannah has a solution that clearly shows that Alex's eight made shots out of 20 (40%) is a higher percentage than Elizabeth's nine made shots out of 25 (36%). When asked to answer part B, Hannah shared:

> Elizabeth had a better free- wait, no, Elizabeth. Yeah. Elizabeth had a better free throw record even though we had the math- Uh mm. No, I was thinking that since Ella Elizabeth tried more shots and she actually made more physically and it would be that, but Alex is a better free throw record because he, he took less shots and made less- He made more shots than that amount in less shots. Like he made more than Elizabeth made with his amount that he took because he took less shots and made less and um, made less of them. But that's not, it doesn't matter because Elizabeth took the more amounts of shots made around the same as him.

Hannah would go on to change directions two more times before deciding that "Janie is wrong because you can look and Alex made more of less than Elizabeth did."

In her interview, Hannah immediately mentioned that she had a basketball game later in the day, so she could relate to the setting. This statement serves as evidence that Hannah brought in and *Used prior knowledge* to better understand and solve the task at hand. Hannah also stated that, to establish a free throw percentage, she needed to find something out of 100, an example of Hannah's effort to bring previously learned mathematical concepts into the problem, as well. In this sense, the prior knowledge strategy is both a reading and a solution strategy, as stated above. For this project, however, I focus on the reading portion of their strategy implementations.

Within Hannah's solution, there is evidence that she paused to reflect on her thoughts, she *Planned a solution or predicted the result* as a route toward a solution (e.g., finding the free-throw percentage), and she spent a great deal of time *Self-checking* her answer, attempting to decide between Elizabeth's nine baskets or Alex's 40 percent. Each of these strategies allowed her to settle on Alex as the free-throw contest winner, meaning Janie was incorrect.

Analysis of Activity”. Hannah’s engagement began by reading the written words provided within the top section of page 1. While doing so, she alludes to a familiarity between the realistic problem and her real-life experiences, as she has a basketball game scheduled for later in the day. Through this effort to *Use prior knowledge*, Hannah demonstrated how prior knowledge may aid a student’s search for understanding as they read through a given problem. She recognized the setting, related it to her own lived experience, and could design a solution based on this understanding.

As mentioned earlier in this chapter, the strategy of *Using prior knowledge* also presents as a solution strategy. Her prior knowledge led directly to the solution strategy implemented – finding a free-throw shooting percentage. She knew that, in the game of basketball, points matter and Elizabeth had more points. She also seemed to recognize that a better record will have a better percentage of successfully made baskets. After weighing both prior knowledge-based ideas carefully, Hannah settles on the assessment that Janie was incorrect and that Alex actually holds the better free-throw record. To verify her understanding, Hannah spent time double checking her work to make certain the solution she found answered the question presented in the problem, another example of *self-check* being both a reading strategy and a problem-solving strategy used by students.

Solution Approach 2 - Fraction Comparison

In the example above, Hannah found fractions with equivalent denominators in her solution, but took the extra step to change the fractions to percentages before answering the questions posed in Task 1. Ten students chose to use fractions to make their determination about Janie’s claim. In doing so, they took two different routes – taking an educated guess or scaling the fractions to find a common denominator.

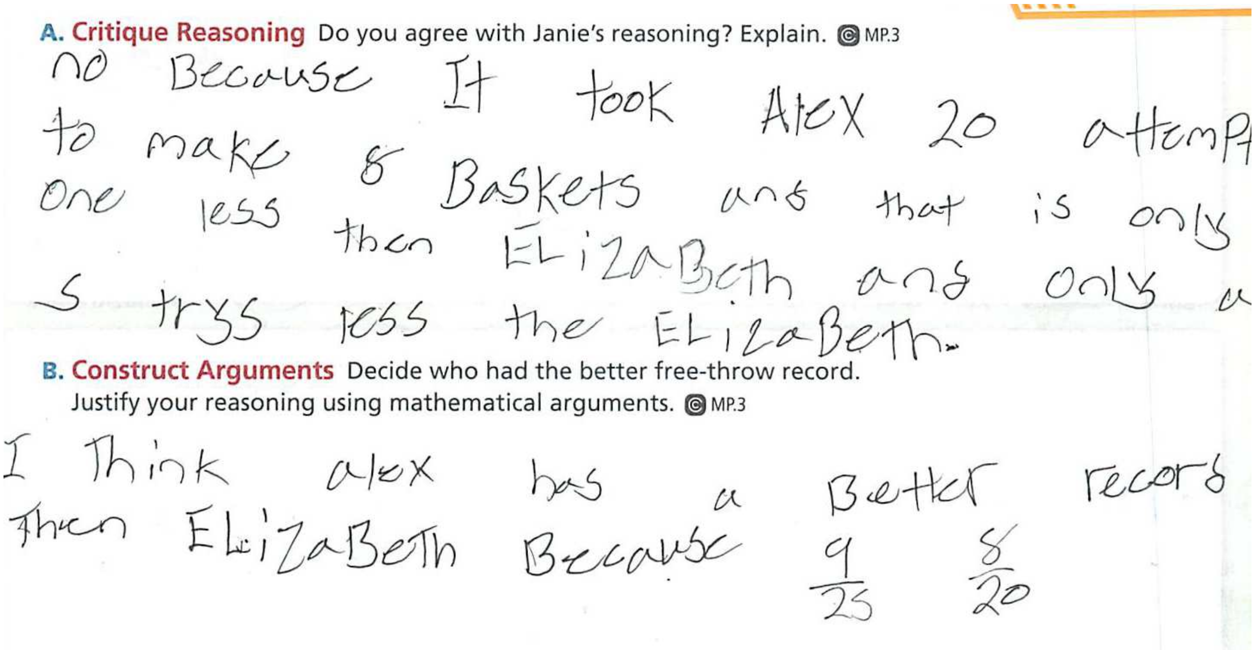
Educated Guess. Three of the ten students chose to decide based on an educated guess. They each had a sense that eight made shots out of 20 attempts was better than nine made baskets out of 25 attempts, but they could not find the words to describe their thinking. Take Alex, a Site 1 student, as an example. In his work (Figure 5-4), Alex concluded that Janie's reasoning is incorrect and that Elizabeth has a worse free-throw record. When asked about Janie's thinking, Alex responded:

> I'm thinking that, uh, Elizabeth, they're saying who, um, they say, uh, Elizabeth had a better record, better record than Alex because she made more. But I think Alex has a better record because he only attempted eight or he only attempted 20 and he made eight.

As he continued his work, Alex was asked to "Justify your reasoning using mathematical arguments" (Forseman, 2017, p. 85), to which he replied, "I don't know how to justify using mathematical arguments. What does it mean by mathematical arguments?" Ultimately, Alex went on to decide that Janie was wrong "because 9/25 8/20" (Figure 4).

Figure 5-4

Alex's Task 1 Solution



A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

no Because It took AleX 20 attemp
to make 8 Baskets and that is only
One less then ELiZaBeth and only a
5 tryss less the ELiZaBeth.

B. Construct Arguments Decide who had the better free-throw record.
Justify your reasoning using mathematical arguments. MP.3

I Think alex has a Better record
then ELiZaBeth Because $\frac{9}{25}$ $\frac{8}{20}$

Alex read aloud, *Paraphrased the text*, and *Paused to reflect* on his work on several occasions throughout this one task. At one point, he stated that he did not know how to proceed, an indication he was *Self-checking* his knowledge and chose to seek clarification from an outside source, namely me. Early in his solution for Task 1, it was clear that Alex understood that eight shots made from 20 attempted shots was better than the nine out of 25 made by Elizabeth, because Alex made his eight shots in "five less tries than Elizabeth." He did not find a way, however, to move beyond his initial educated guess.

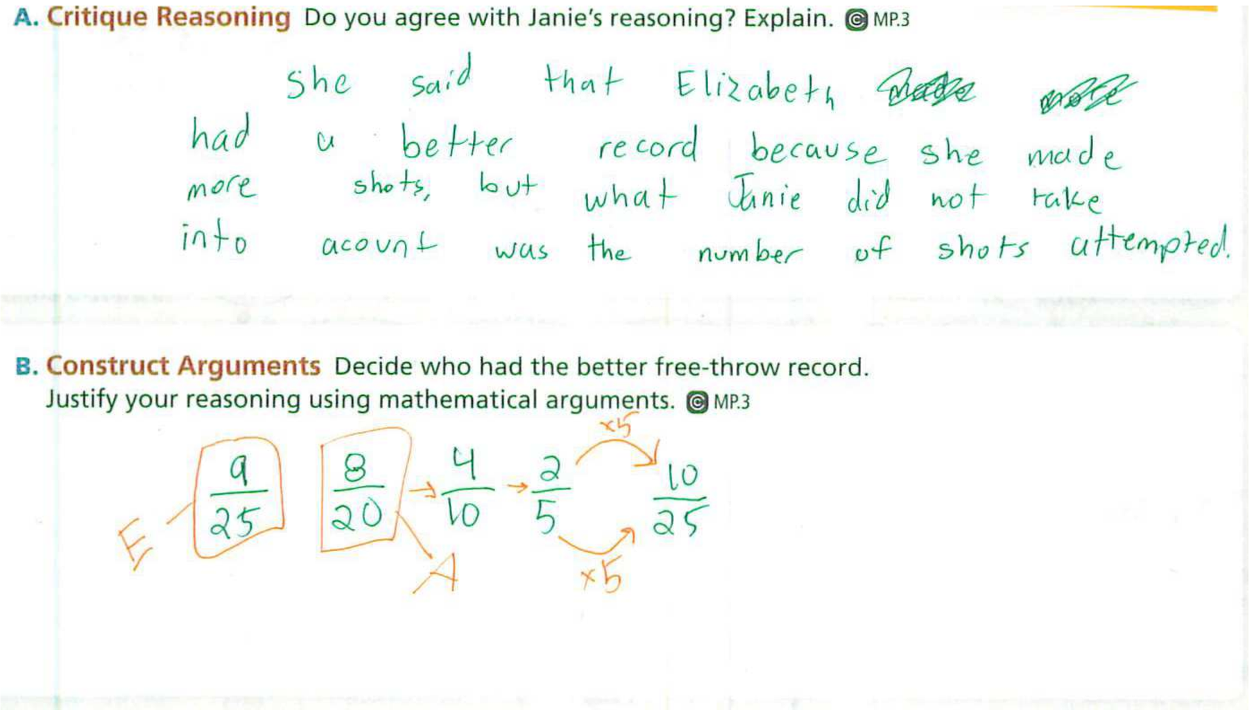
Analysis of Activity. Alex represents students who understood enough about what was read to make a logical guess at the answer, but did not have as thorough a solution as presented by Hannah. In Hannah's work as explained above, she applied her personal knowledge of the realistic situation described, she planned a solution and self-checked her understanding as she went along. Alex's search for understanding what was read began with an effort to *Paraphrase the text* of the problem, demonstrating he understood what was explained in the text. He then *Sought clarification or external assistance* when the text requested he use "mathematical arguments" to justify his reasoning. In both cases, these strategies were used by Alex to further his understanding of the text presented in the book.

When pursuing the solution to Task 1, Hannah connected her experience as a basketball player to connect the written text to a solution path. This effort left Hannah with the answer that Janie was incorrect, and the mathematical means to justify that reasoning (40% is better than 36%). Alex's effort, as described above, offered his assessment of the problem based on what he was thinking as he read, but he was ultimately unable to identify a clear solution path or justify his response to Part B. Ultimately, Alex had the reading strategies necessary to decipher the text presented in Task 1 and arrive at an answer. He did not, however, have the solution strategies needed to assess the validity of that answer.

Scaling Fractions. Seven other students looked to fractions to provide their justification, four from Site 1 and three from Site 2. Each of these seven students sought two fractions with common denominators, but three routes were taken. Four students – all three Site 2 students and one from Site 1 – used the denominator of 25, one chose 100ths, and the other two scaled the two fractions to 500ths. The examples below come from Henrick, a Site 2 student who chose the denominator of 25, and Charlotte, a Site 1 student who scaled the denominators to 500. In both cases, the students scaled two fractions, but in very different ways.

Figure 5-5

Henrick's Task 1 Solution



A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

She said that Elizabeth ~~made~~ ~~more~~ had a better record because she made more shots, but what Janie did not take into acount was the number of shots attempted.

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

E $\boxed{\frac{9}{25}}$

$\boxed{\frac{8}{20}} \rightarrow \frac{4}{10} \rightarrow \frac{2}{5} \xrightarrow[\times 5]{\times 5} \frac{10}{25}$ A

Henrick. The work shown by Henrick (Figure 5-5) was done in two different segments. The green print was completed during the initial reading, the orange markings came afterward, during the debrief conversation held between the two of us. For this analysis, both are useful, but the priority is the green text.

When reading Part A, Henrick was observed paraphrasing the text and seeking clarification. In contrast to Alex, however, Henrick's search for clarification hinged on process, not content. Through the passage below, Henrick both *Paraphrased the text* and *Sought clarification or external assistance* as he worked toward his solution to Task 1, Part A.

> Janie said that so that Elizabeth had a better record because, because than Alex because she made more free throws. That isn't necessarily correct because she's not taking into account the amount that was, that were attempted by both of them because Elizabeth made more. But she also took more shots. Alex made less but also took less shots. So yeah. So should I write that down?

After asking "so should I write that down?" Henrick recorded his answer to Part A and turned his attention to Part B.

In answering Part B, Henrick shared that he "can simplify this down to four over 10 should simplify that down to two over five." This statement demonstrated that he both *Planned a solution or predicted the result* when reducing Alex's fraction from 8/20 to 4/10, but also that he *Modified the plan* as he simplified the fraction further. Henrick then looked at Elizabeth's fraction of 9/25, reporting, "so Elizabeth- don't think I can simplify that down anymore cause 25 divided by, yeah, 25 divided by five is I think divided by one but yeah. So yeah. So she made nine 25th, he made two-fifths." In this step, Henrick offered a more formal phrasing of the fraction, where Henrick states "she made nine-25ths, he made two-fifths" as opposed to reading his work as nine over 25 and two divided by five. This is an example of a student reading symbols as text.

Additionally, Henrick's plan changed again, because 9/25 did not simplify down to fifths. Henrick's solution was to "multiply [the two and the five] by five cause that'll get [both fractions] to have a common denominator that would get that to 10 over 25," ultimately identifying that Alex would have had more made baskets than Elizabeth, had they shot the same number of free-throws.

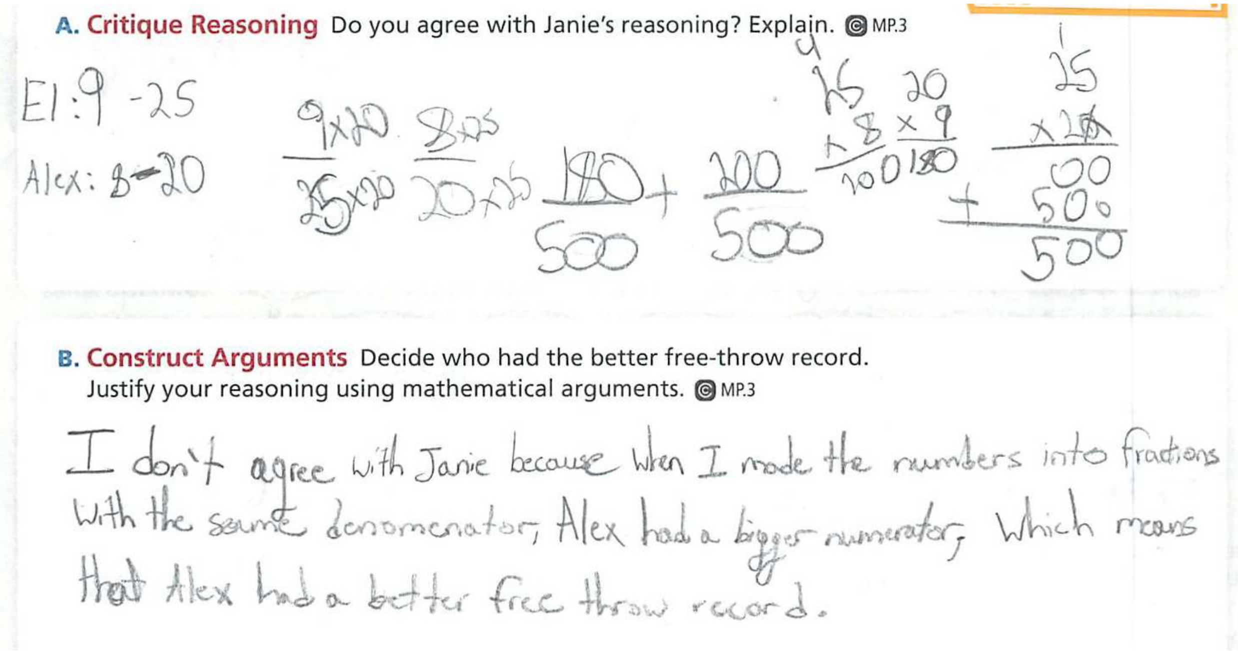
Analysis of Activity. Like most of the participants in this project, Henrick's reading process began with a *Read aloud*. After his initial read of the text, Henrick *Paraphrased the text* and *Sought clarification or external assistance*. Henrick then *Planned a solution or predicted the result* and twice *Modified the plan or prediction* based on newly realized information. Each of these actions were made with the intention of clarifying his own understanding of the text found in Task 1, and formulating a solution to address Janie's claim.

The latter two strategies, planning a solution and modifying that plan, also lead directly into the work done to find an answer, suggesting they are both solution strategies as well as strategies used to help understand the reading. In his solution, Henrick took the words describing the problem, translated them into a mathematical approach toward a solution, then modified the plan as he realized that the process was not going to flow as simply as he had initially planned. His search for a common denominator represented an understanding of the problem combined with prior knowledge of similar tasks.

Charlotte. Through his process, Henrick scaled two fractions. He began by scaling 8/20 down to 2/5, then scaled 2/5 up to 10/25 before identifying Alex as the winner. Charlotte also scaled two fractions in her work (Figure 5-6). As she began her work on Task 1, Charlotte *Read aloud* and concluded her reading of the free-throw contest by *Seeking clarification or external assistance*, asking "Okay. And then what else do I have to read?" There was other text she could have addressed, but chose to read only the text describing the problem.

Figure 5-6

Charlotte's Task 1 Solution



A. Critique Reasoning Do you agree with Janie's reasoning? Explain. © MP.3

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. © MP.3

I don't agree with Janie because when I made the numbers into fractions With the same denomenator, Alex had a bigger numerator, Which means that Alex had a better free throw record.

Like Henrick, Charlotte placed both made baskets to attempted baskets into fraction form, then sought a common denominator between them. Her first thought was to simplify both Alex and Elizabeth's fractions but stated "Oh look, you can't do that. Okay. So I'm going to do a different one … I'm going to say- it is a big number, but I think I should, like, multiply them." Through this thought process Charlotte was observed *Planning a solution or predicting the result*, in route to a solution. She then modified her prediction. Ultimately, Charlotte decided to scale both denominators to 500, the product of 20 and 25, as seen in her work in Task 1, Part A. In her work, as discussed in Chapter 4, Charlotte also *Paused to reflect* and *Made notes while reading*.

Analysis of Activity. Charlotte's approach was very similar to Henrick, with the big difference the direction of the scaling. Henrick scaled his fractions down, initially, then up to a common denominator. After reading the problem out loud, Charlotte *Sought clarification or* *external assistance*, *Paused to reflect* multiple times as she processed the text and *Planned a solution or predicted the result*. Through this effort, she created enough of an understanding of Task 1 that she was able to adjust her solution process, as needed, in route to a final assessment of Janie's claim.

Her initial solution was to simplify both fractions, similar to Henrick's' initial decision, but later scaled both fractions up to a common denominator. Her efforts to pause to reflect, as a solution strategy, allowed her to adjust and find an alternate solution that best fit the task described in the reading. She was also diligent about writing notes throughout her reading and her work so ideas did not slip away as she worked.

Solution Approach 3 - Unit Rates

Of the 22 students participating in this study, four chose to solve Task 1 by comparing unit rates. Three of the four students attended the Site 1 school. The process taken by these four students took one of two routes. They each saw a need to find the total number of shots taken for each successful free-throw.

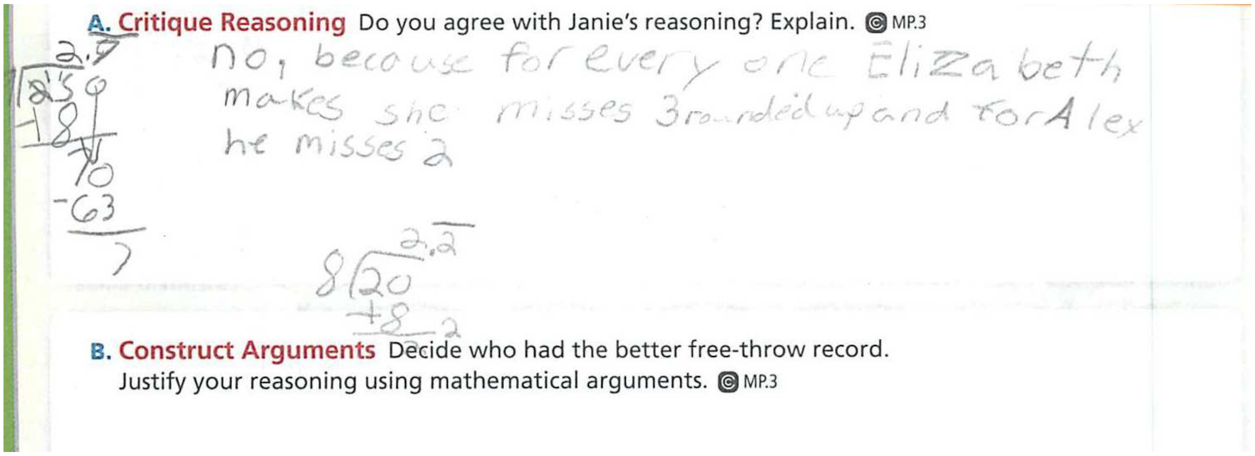
Tim opened the reading segment of his interview by asking, "Do I read these, too?" evidence that he *Previewed the text* and wanted to clarify what he was expected to read. He then proceeded to read aloud all the text included in the Explain It! section. He began with the description of the basketball contest, then read the lesson title, the "I can" statement, and attempted to read the Common Core State Standards.

Tim's initial thought was, "Well, we'll have to find how many, well- Elizabeth made nine out of 25 free throws. So, you'll have to find the unit rate of the nine. So how many misses did she have per one you made." When asked if he was thinking anything else, Tim said, "Oh, how Janie was said that that was kind of confusing cause it's a little bit obvious to me that Alex." In these two statements, Tim has shared both a *Plan for a solution* and *Predicted the result* related to Janie's claim.

Tim's stated goal was to compare the ratio of missed shots to made shots for both Alex and Elizabeth. His work (Figure 5-7), however, showed an effort to divide the total number of attempted shots by the number of made free-throws for both characters. Ultimately, Tim found that Janie's reasoning is incorrect "because for everyone, Elizabeth makes, she misses three baskets, rounded up. It was 2.7 infinitely. And for Alex, he misses two rounded down 2.2 infinitely. This is my work."

Figure 5-7

Tim's Task 1 Solution



A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

no, becouse for every one Elizabeth makes she misses 3 rounded up and for Alex he misses 2

2.7
8)25 0
-18
70
-63
7

$\overline{2.2}$
8)20
+8
2

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

Through most of this study, I looked at how these students were reading and interpreting the text provided by Forseman (2017). Tim provided a moment to consider how students may read their own work or that of a peer. In his work, Tim intended to divide the number of missed shots by the number of successful free-throws. His two division problems yielded two unit rates, "2.7 infinitely" and "2.2 infinitely." He then rounds 2.7 up to 3 missed shots for every shot Elizabeth made, and 2.2 down to 2 misses for every successful shot for Alex. To clarify this decision, Tim says, "In basketball you can't really have missing 2.7 and 2.2." He worked through the mathematics, arrived at answers that did not fit his understanding of the problem, and made what he deemed a logical decision to address his *Self-check* concern.

Analysis of Activity. Tim began by *previewing the text* presented to him. In doing so, he decided to *seek clarification or external assistance*, as demonstrated by his effort to identify the portions of text I felt he needed to be read. He asked to *identify the information important* for his work, and potentially gauging how important some portions of the text might be for the project as a whole. After reading the passage, he identified a route to a solution and offered a prediction about what the end result would be. Much like the four students above, Tim used each of these strategies to build a clear understanding of the text, itself, before heh ventured toward a solution.

Tim's solution was to identify the unit rates for both content participants. In doing so, he divided the number of total shots by the number of made free-throws, as described above. Once completed, he double checked his work to make certain the unit rates made sense for the problem described. His rounding effort did not change his ultimate answer, but the unit rates were also not signals that the solution found matched his understanding of the problem posed. His initial effort was to find the number of shots made per shot taken. When he took a moment to check the work he had completed, Tim clarified that, based on his understanding of basketball, you cannot miss 2.7 and 2.2 shots, respectively, so the rounding helped him make sense of the established unit rates.

Task 1 Summary

All 22 students worked through Task 1, 20 of them decided that Janie was incorrect, one was indecisive. One student recognized basketball as a game about total points, and concluded that since Elizabeth made more free-throws, scoring more points, she had the better record. Of the 20 students who disagreed with Janie, 16 students presented a solution like those shared above. In doing so, they utilized three reading strategies more in Task 1 than any of the remaining five tasks – *Seek Clarification*, *Preview the Text*, and *Use Prior Knowledge*.

The strategies used by the most students throughout Task 1 were Paraphrase (20 students), Plan or Predict (19), Read Aloud (18), and Pause to Reflect (17). Of these strategies, reading aloud, planning or predicting, paraphrasing, and pausing to reflect continued to be used by over half of the students in the study for each of the three tasks explored in this chapter. In addition to these four strategies, the Self-Check strategy was used by at least 12 students in Tasks 1, 3, and 6.

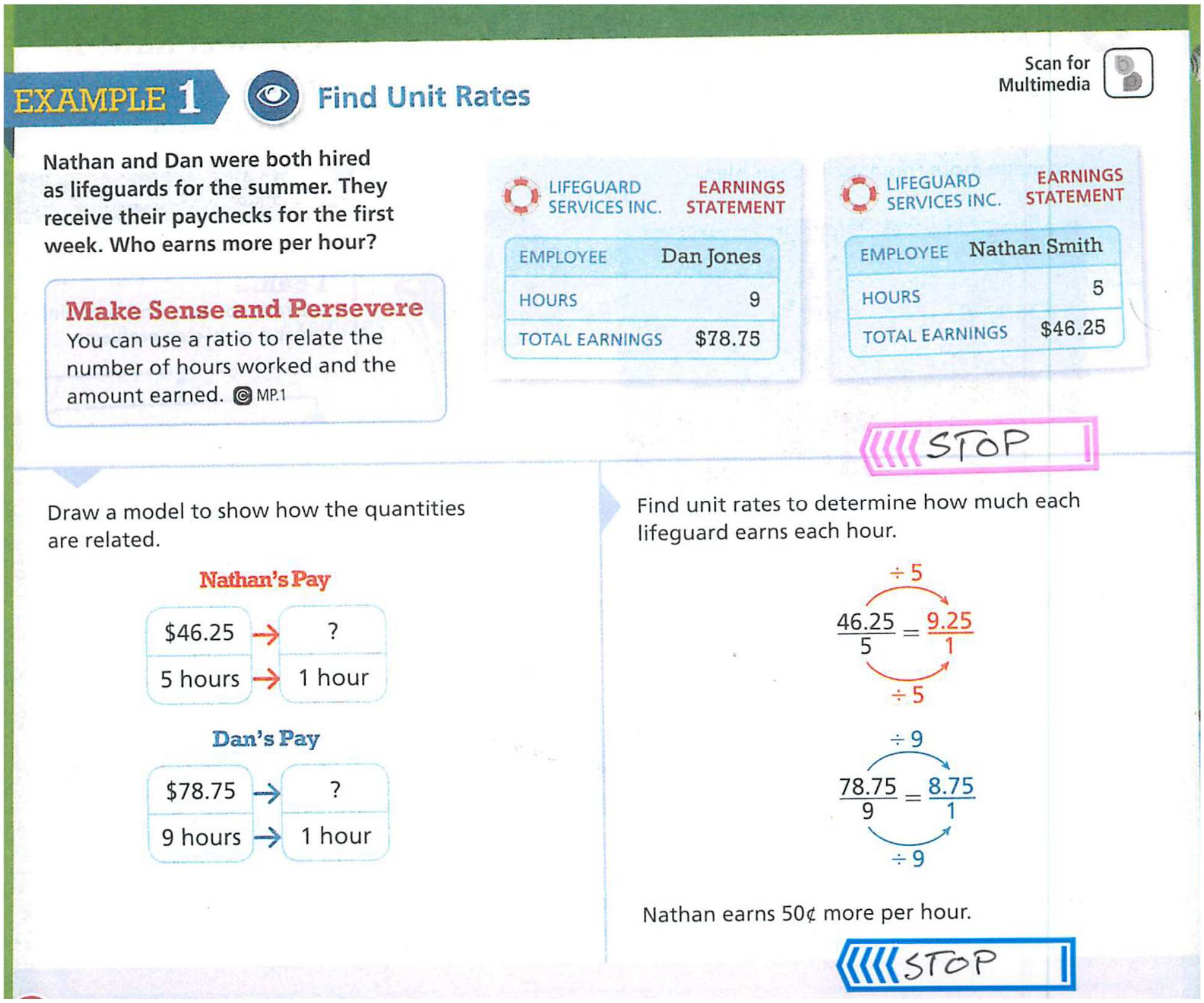
Chapter 4 hypothesized that one reason for the decrease in the number of reading strategies used in each task might be the increasing comfort felt by the participants as they settled into the interview and related reading. The three strategies that were largely unique to Task 1, Seek Clarification (11), Preview the Text (10), and Use Prior Knowledge (5), may be further evidence that, as students become more comfortable with their work, they begin to use fewer reading strategies, and employ them more skillfully.

Task 3: Try It! (the first)

Task 2: Example 1 (Figure 5-8) fell between Task 1 and Task 3, and provided each participant a chance to read a similar solution to the one they would ultimately pursue in Task 3. In Example 1, text demonstrates how to find the hourly rate for two lifeguards, then compare their rate of pay. Example 1 takes a sample pay-stub from two lifeguards, Dan and Nathan, and utilizes the two graphic organizers found in Task 3. The example then shows the subtraction between the two hourly rates for Dan and Nathan to identify which lifeguard gets paid more.

Figure 5-8

Task 2: Example 1



EXAMPLE 1 **Find Unit Rates**

Scan for Multimedia

Nathan and Dan were both hired as lifeguards for the summer. They receive their paychecks for the first week. Who earns more per hour?

Make Sense and Persevere
You can use a ratio to relate the number of hours worked and the amount earned. MP.1

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Dan Jones
HOURS	9
TOTAL EARNINGS	$78.75

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Nathan Smith
HOURS	5
TOTAL EARNINGS	$46.25

STOP

Draw a model to show how the quantities are related.

Nathan's Pay

$46.25	→	?
5 hours	→	1 hour

Dan's Pay

$78.75	→	?
9 hours	→	1 hour

Find unit rates to determine how much each lifeguard earns each hour.

$$\overset{\div 5}{\underset{\div 5}{\frac{46.25}{5} = \frac{9.25}{1}}}$$

$$\overset{\div 9}{\underset{\div 9}{\frac{78.75}{9} = \frac{8.75}{1}}}$$

Nathan earns 50¢ more per hour.

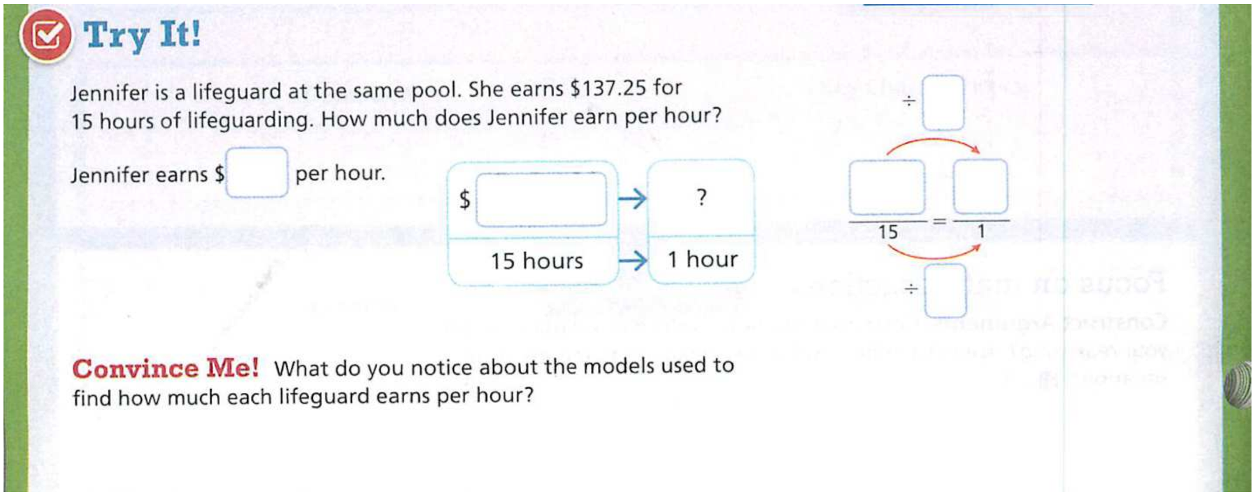
STOP

In Task 3 (Figure 5-9), the reader is asked to identify Jennifer's hourly pay using the same models presented further up the page. There is a small passage of prose, two pay reports presented in a table form, and a pair of models through which a participant may organize their work. There was much less white space in Task 2 than on the previous page, but the work was done for the reader.

In this example, the textbook demonstrated how to find the hourly pay for two lifeguards working at the same pool. On the left side of the solution, the text presented a model that sets up a proportional reasoning exercise. The right side shows the work done by the textbook's author to get the final answers of $9.25 and $8.75, respectively. Task 3 asks the reader to complete a similar process.

Figure 5-9

Task 3: Try It! (the first)



Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $ ☐ per hour.

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

Task 3 Text Analysis

The third task of these interviews (Figure 5-9) follows the mathematics and story from Task 2 directly. Task 3 introduces a third lifeguard, Jennifer, and offers both a total pay and total number of hours worked. Just like Task 2, the graphic organizers are in place for the students to utilize in their solution.

The representations on the page for Task 3 included written sentences and the graphic organizers. There were no images associated with this passage section of the reading. Many students demonstrated a close connection with what they read in Task 2, and their solutions for Task 3, a tendency that made sense given the presentation of the graphic organization in Task 3 that mirrors that of Task 2. Those students who did not utilize all the graphics, tended toward a long division problem of 137.25 by 15.

Outcomes

In Task 3, the students largely used long division to divide Jennifer's total pay by the number of hours worked. Seventeen of the 22 participants settled on an answer that was approximately \$9.15 per hour – 14 answered \$9.15, one answered \$9.16, and two others answered \$9.17. Each of these 17 students found their answer by dividing \$137.25 by 15 hours. Three other students worked through the same division process and arrived at answers that had three decimal places. One student settled on \$9.56 and another offered no answer.

Implemented Strategies

The most used strategies throughout Task 3 were *Reading aloud* (17 students), *Pause to reflect* (17), *Plan a solution or predict the result* (13), and *Paraphrase the text* (13). Additionally, two strategies were used more in Task 3 than in any other task – *Make connections across the text* (10) and *Reading symbols as words* (9). The analysis presented below explores these six reading strategies as utilized by three students. Two of the students arrived at \$9.15 for Jennifer's rate of pay, the other was derailed by an unexpected third decimal place.

Solving Task 3

As stated above, most students used long division to find Jennifer's hourly pay. The lone exception was a student who asked if they could use a calculator, though one other student did suggest they would usually use a calculator for a problem like Task 3. Setting the calculator discussion aside, the differences in the students' solutions fell along two lines. One difference stemmed from the final answers, outlined above. The other was related to the ways students utilized the graphic models provided by the textbook intended to help guide the solution process.

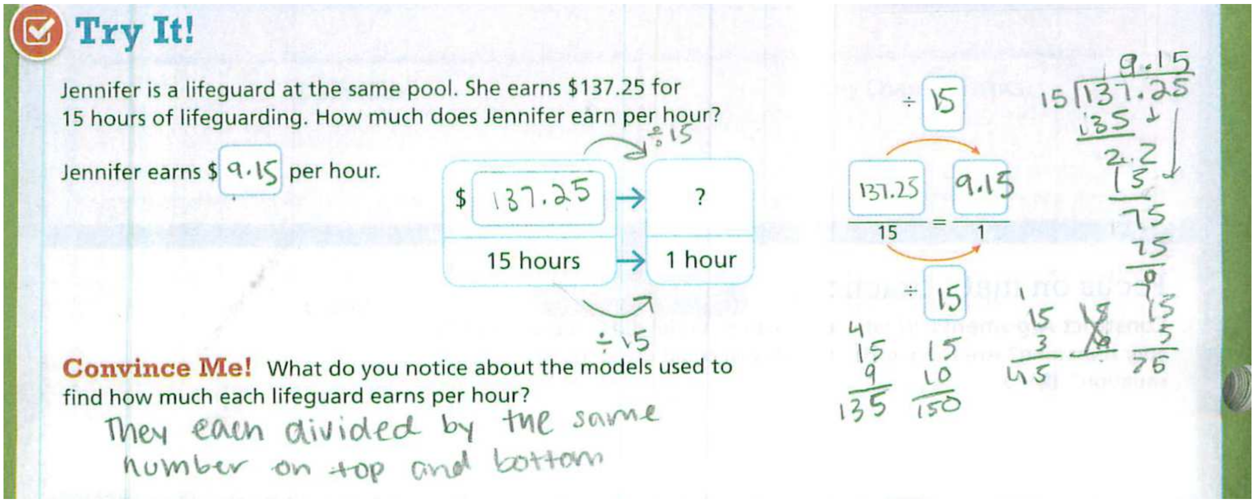
In the analysis that follows, I share the solutions of Maya and Dave, the latter of whom also participated in a conversation about the relevance of three decimal places and money problems.

Model Usage

Maya (Figure 5-10) and Dave utilized the first graphic model, but unlike Dave, Maya also used the second model. I observed Maya add two curved arrows and indicated a need to divide by fifteen between the 15 hours and the 1 hour. She then recorded the same information in model two and structured her solution as 137.25 divided by 15. Through the process of long division, Maya found that Jennifer earns $9.15 per hour while lifeguarding.

Figure 5-10

Maya's Task 3 Solution



Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $ 9.15 per hour.

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

They each divided by the same number on top and bottom

In her solution, Maya began by *Reading aloud*, then spent about three minutes working and speaking to herself. In the passage provided below Maya can be observed *Planning a solution or predicting a result*, *Pausing to reflect*, and *Self-checking* her work which led her to *Modify her prediction* until she settled on her answer.

> [Working aloud] 25 [pause] divide by fifteen, oh. divide by fifteen, fifteen. oh and then [pause] fifteen, fifteen times nine, 137. [pause] then nine, 135, no, it wouldn't be 150. [pause] ten, yeah, [pause] oh yeah, fifteen oh [pause] two, two, one fifteen seven five seventy-five zero five so she makes [inaudible] and 15 cents per hour.

She followed this passage by *Paraphrasing the text*, then reading through her solution.

In her explanation, she shifted between reading the symbols in the text as they are presented, and reading the meaning of the symbols. Paraphrasing both the problem and her work, Maya stated that Jennifer "earns 137 point two five for 15 hours of lifeguarding." She later explained her work results in "9 point one five, so nine dollars and 15 cents." In the first statement and the beginning of the second statement, Maya read the raw numbers character by character – 137.25 becomes "one hundred thirty-seven point two five" and 9.15 becomes "nine point one five – void of context. She closed her response, however, by reading the meaning of the number in context – 9.15 as "nine dollars and fifteen cents."

Maya's interview also demonstrated that students look across their reading to make connections across the text. In this case, Example 1 is the text that immediately precedes Task 3 and demonstrates how one might utilize the two models found in the Try It! problem. Through the 22 interviews, a few students could be seen glancing from their work in Task 3 up to the models in Example 1. Maya explained that because of her experience in math class, she's accustomed to "[looking] at previous, like, examples to see if there's, like, easier ways than what you're thinking. Or is there certain, like, ways that you can do the problem or solve a problem." This illustrates Maya's concerted effort to use the information found in the text to guide her solution process.

Analysis of Activity. Maya relied greatly on her prior experience in math class to structure her approach to Task 3. She read aloud and immediately began working through Task 3. She later explained that her experiences in math classes have prepared her to use as much known information as possible when engaging with a new problem, and in this case, that meant to refer to Task 2 as she worked. Her effort to read this problem consisted of reading aloud and applying relevant prior knowledge.

During her solution, Maya regularly paused to reflect on the work she completed and referred to Task 2 to get further clarification. This effort led her to approach the solution similarly to the textbook solution. Many students tried to fill in some of the provided boxes, but Maya was one of the few who used each box of the graphic organizer, and used the boxes to help justify her work.

She was also observed pausing, repeatedly. During these moments, she would shift her responses slightly, moving closer to a final answer with each shift. It was during this unprompted verbal work-aloud that Maya provided the best evidence of her work, and how it related to what was read. She said "divide by fifteen, oh. divide by fifteen" as she filled in the boxes above and below the right graphic organizer. Then while setting up the long division, Maya shared "fifteen. oh, and then [pause] fifteen," a moment where she double checked the text to make certain she had the correct numbers before proceeding.

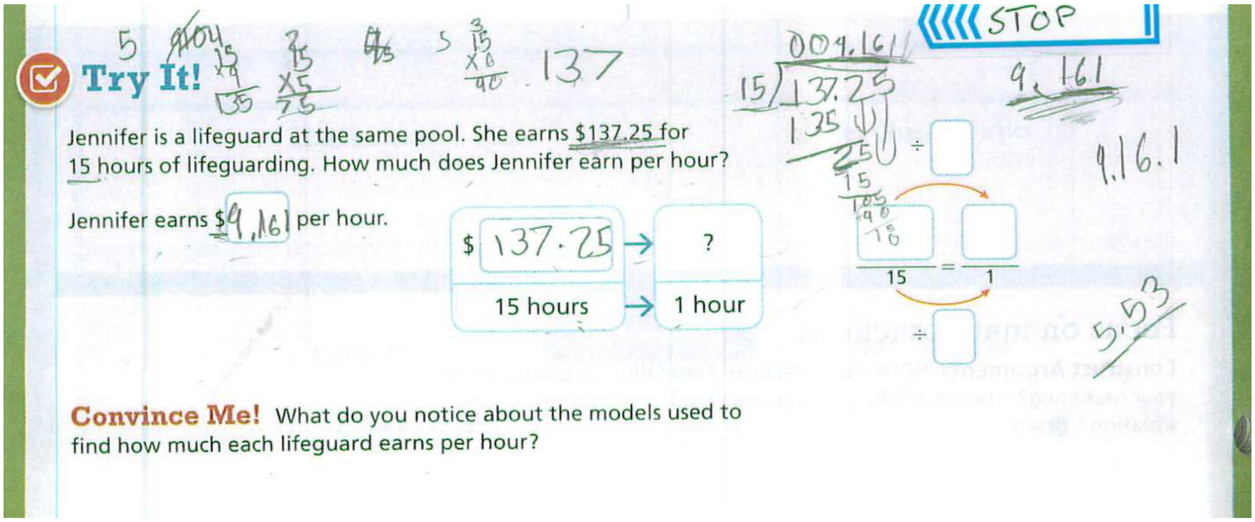
Money with Three Decimal Places

Dave began his work (Figure 5-11) with a *Read aloud*, then *Made connections across the text*, between Task 2 and Task 3, specifically. He pointed out "then there's the, and then there's that ratio again from like up [in Example 1] here for the fraction stuff." He also pointed out, in reference to the second model, "there's like a weird little thing over here, which I have no clue what that is." In doing so, he demonstrated the same effort to identify connections across the text that Maya showed, above.

While Dave did not use the second model or make the same extra lines Maya made, he did use the same long division process to find his solution. Ultimately, Dave found that Jennifer made $9.161 for each hour lifeguarding. The error for Dave was two-fold. After subtracting 135 from 137, he indicated he needed to bring down the 2, but wrote a 5, instead. In the next step, he showed that 15 goes into 105 a total of 6 times, though a seventh would have worked. He finds that additional 15, which leads to the third decimal place. With just the first error, Dave should have arrived at $9.17. The second error provided the opening for the discussion that follows.

Figure 5-11

Dave's Task 3 Solution



Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $9.161 per hour.

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

Dave identified "my answer for it was 9 doll- 9 dollars and one 61- one hundred 61 cents." This immediately prompted a Self-Check, because, "Which I don't- wait, how's that work?" He went on to ponder:

> If you have $9 in it, then it says one 61 cents- Is that like a, isn't that a dollar and 61? But this is like a decimal and it's not actual real wor- world stuff. [pause] Wait, so it's 9 dollars and then point 1 dollar. [pause] What? That's weird. [pause] I wish they taught, like, real world, so like you couldn't have the decimal. This is the 9 dollars and then 161 cents I guess.

Over the next five minutes, Dave and I discussed "real world" problems and the role a third decimal might play in a money problem. I asked him what he felt the 9.161 would be in real life money, and Dave thought it would be "it would be like $10 and 16 cents or 61," which was quickly amended to be “nine 61 and then it would be 0.1,” with the tenth of a cent causing him some unease. I observed Dave attempt to make a connection beyond the text that was presented, so I asked about the signs at a gas station and the nine-tenths added to each gallon of gas. Dave is a middle school student, but he does not drive, and he was not aware. He did ask, however, “How do you, how do you spend nine tenths of a cent?” A wonderful question to which I had no immediate answer.

Analysis of Activity. After reading both Task 2 and Task 3, Dave attempted to address Jennifer’s pay similarly to the method demonstrated in Task 2. Like Maya, Dave could be seen trying to connect the work done in Task 2 to the work he was doing in Task 3. He did not exhibit the same level of understanding, however, as Maya used each of the diagrams provided, and Dave only used the first graphic organizer.

Dave’s issue was not in understanding the mathematics, however. He made two simple mistakes, outlined above. Upon the completion of his long division process, Dave spent a great deal of time and effort to understand how a problem that deals with money, following a similar example that had two nice dollar and cent amounts, could possibly have three decimals. While he was not the only participant who came to an answer of three decimal places, he is the only one who actively worked to understand what 9.161 really meant in the context of the problem. This effort allowed us to discuss real-life financial situations where three decimals may be reasonable.

Task 3 Summary

Just like more than half the students participating in this study, Maya and Dave read the problem out loud, formulated a plan, paraphrased parts of the text, and paused to reflect on their work. Despite utilizing the models differently, they each used long division to solve the problem. Both students referred to Example 1, the task that preceded the search for Jennifer’s hourly pay.

Maya's interview also highlighted the difference between reading the numbers as they are written, and reading the meaning of the symbols on the page.

Two mistakes made by Dave opened a door to an entirely different conversation. In his search to better understand his answer, $9.161, he self-checked his work and questioned the validity of his answer. Dave and I took his work on Task 3 and attempted to move the information from a "realistic" to a "real-world" setting. My real-world connection to the third decimal place is found at the gas pump, but Dave is a middle school student who does not drive and has no need to purchase gasoline.

Task 6: Try It! (the second)

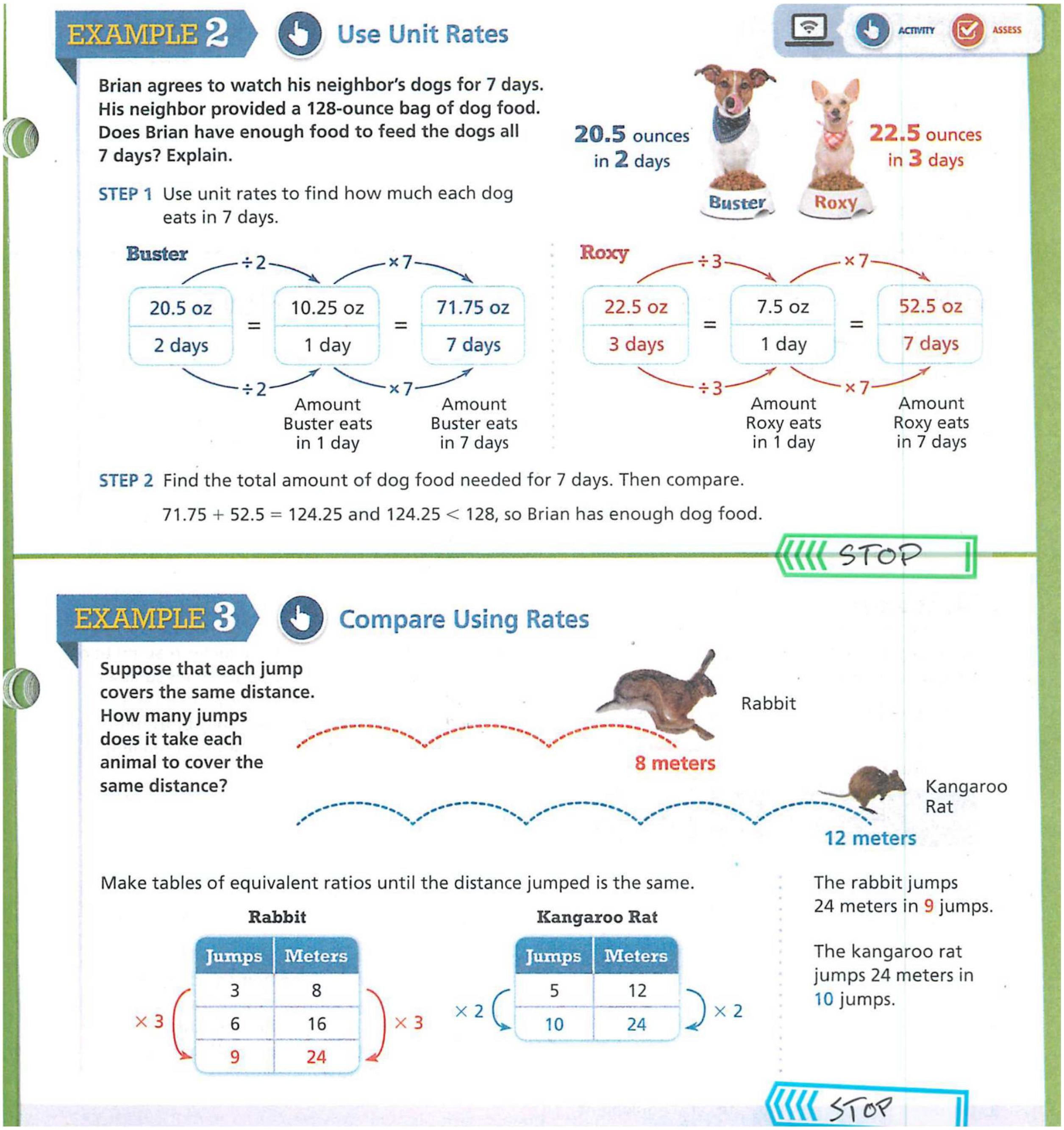
The final analysis section of this chapter came from the third page each student was asked to read. On this page, there were three passages, Task 4: Example 2, Task 5: Example 3, and Task 6: Try It! (the second). The second page of the reading discussed hourly wages for lifeguards at a single pool. This third page presented three different problems that are mathematically similar, but the story context of each of the problems is independent of the next.

Task 4: Example 2 and Task 5: Example 3

Like Example 1's placement between Task 1 and Task 3, Examples 2 and 3 (Figure 5-12) fall between Task 3 and Task 6. In Example 2, the book describes a realistic setting where the caregiver to two dogs has asked a neighbor to take care of two dogs for a week, providing a single bag of food. The question posed and the completed answer lead the reader through a process to determine whether the bag is large enough to feed the two dogs for the seven days. Example 3 explores the jumping power of two rodents. The example has a combination of written prose, diagrams of the two rodents jumping, and two tables to help demonstrate the answer. Each rodent jumped at a different rate, and the solution explains how many jumps it would take each of the two rodents to jump the same distance. In both Examples 2 and 3, the textbook scales up a unit rate to find the solution.

Figure 5-12

Task 4: Example 2 and Task 5: Example 3



EXAMPLE 2 Use Unit Rates

Brian agrees to watch his neighbor's dogs for 7 days. His neighbor provided a 128-ounce bag of dog food. Does Brian have enough food to feed the dogs all 7 days? Explain.

20.5 ounces in **2** days

22.5 ounces in **3** days

STEP 1 Use unit rates to find how much each dog eats in 7 days.

Buster

÷ 2, × 7

$$\frac{20.5\text{ oz}}{2\text{ days}} = \frac{10.25\text{ oz}}{1\text{ day}} = \frac{71.75\text{ oz}}{7\text{ days}}$$

÷ 2, × 7

Amount Buster eats in 1 day

Amount Buster eats in 7 days

Roxy

÷ 3, × 7

$$\frac{22.5\text{ oz}}{3\text{ days}} = \frac{7.5\text{ oz}}{1\text{ day}} = \frac{52.5\text{ oz}}{7\text{ days}}$$

÷ 3, × 7

Amount Roxy eats in 1 day

Amount Roxy eats in 7 days

STEP 2 Find the total amount of dog food needed for 7 days. Then compare.

71.75 + 52.5 = 124.25 and 124.25 < 128, so Brian has enough dog food.

STOP

EXAMPLE 3 Compare Using Rates

Suppose that each jump covers the same distance. How many jumps does it take each animal to cover the same distance?

8 meters

12 meters

Make tables of equivalent ratios until the distance jumped is the same.

Rabbit

Jumps	Meters
3	8
6	16
9	24

× 3 × 3

Kangaroo Rat

Jumps	Meters
5	12
10	24

× 2 × 2

The rabbit jumps 24 meters in 9 jumps.

The kangaroo rat jumps 24 meters in 10 jumps.

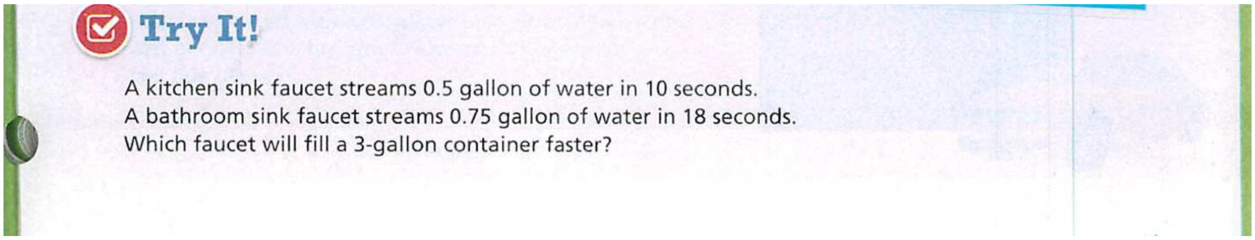
STOP

Task 6: Try It! (the second)

In Task 6 (Figure 5-13), the students were asked to read about two faucets, and determine which of the faucets would most quickly fill three-gallons. Of the six tasks each participant was asked to read, this is the only one presented without any sort of graphical representation. The task consists only of the written words and whitespace for students to work toward a solution.

Figure 5-13

Task 6: Try It! (the second)



Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

Task 6 Text Analysis

Throughout this three-page section of text, the activities presented for student participation have an increasingly narrow representation of text. Task 1 presented a collection of sentences describing the competition being held and the assessment of one character (Janie). The text also included a scoreboard with the contest results and a basketball graphic for those who may not know it was a basketball game. Task 3 also provided both a paragraph of text outlining the task and a graphic intended to aid the students. Each student also had a very clear guide for the Task 3 solution, because the example shown on the same page had an identical context and process. Task 6 had only a paragraph describing the problem to be solved.

Conceptually, the three problems are quite different. Task 1 speaks of students in a free-throw shooting context, a situation to which several students related to directly, and one that went unquestioned by all 22 participants, suggesting they understood what was being discussed. Task 3 discussed the pay of lifeguards, and each of the 22 participants seemed to have at least a workable understanding of money and the idea of being paid. Generally, the only critique was that the three lifeguards were paid differently.

The third page of the textbook gets farther from realistic middle school life with each task. Task 4 was about pet sitting, which is a job quite possibly offered to middle school students, but the requirements to feed the animals in ounces will not fit the experience of most. When writing directions for feeding my own dogs, we talk about the number of scoops to provide, as opposed to the ounces of food.

Task 5 centers on the length of the jumps made by two rodents, a rabbit and a kangaroo rat. It is reasonable to think that most middle school students are familiar with a rabbit, given all the cartoon representations found throughout one's childhood and their prevalence across most of North America. A desert dwelling kangaroo rat, however, is an animal unlikely to be observed by a Midwestern student. Beyond the knowledge of each animal's existence, one might question the relevance of the effort to establish the number of jumps needed for both animals to travel the same distance.

In task 6, the students were asked about filling a 3-gallon bucket with either the kitchen or bathroom faucet. No further context was provided, and no images exist. This presentation, both in context and visual presentation made the problem difficult for the participants to relate. Why would students need to fill a 3-gallon bucket?

Outcomes

Task 6 was the final reading passage students were asked to work through. Because it is preceded by five other tasks, the students were more focused on ratios, rates, and unit rates than they were in Task 1. This final task had far less structure than Task 3. The solutions shared by the students were less wide ranging than those from Task 1, but more varied than from Task 3.

Twenty-one of the students attempted the task, 18 found the kitchen sink would fill the bucket quicker, one said the bathroom faucet would be quicker, and two gave no definite answer. One student, Figglestein, did not have an opportunity to complete Task 6 because of time constraints. The three most common solutions used division to find unit rates, fractions seeking a common numerator or denominator, and skip counting the given ratios.

Top Strategies

While working through this final reading task, over half of the 21 students planned or predicted (16 students), paused to reflect (16), read aloud (15), self-checked (14), and modified their prediction (13). The student work shared in this section largely explores groupings of strategies, as the students were left with little structure other than the written words on the page. The self-check and pause strategies were closely associated, as were self-check and modifying predictions. Additionally, I share ideas related to the questioning and critiquing the text strategy used by three students in Task 6.

Solving Task 6

The goal for this section is to explore the combinations of strategies used in Task 6, because the 21 students tended to use fewer unique reading strategies when solving this problem. Specifically, I highlight the relationships between pause to reflect and self-check, as well as self-check and modify the plan or prediction. Additionally, I discuss ways in which students question or critique the text as they read and work through Task 6.

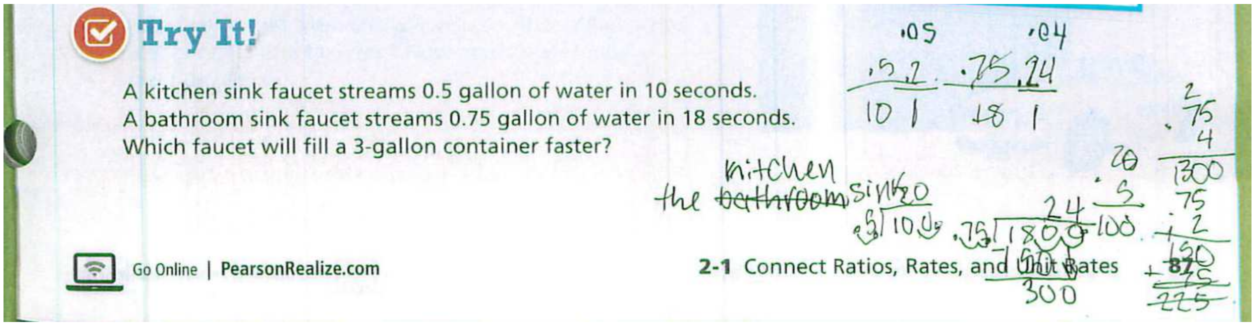
To accomplish these goals, I share the work of four students. Carolynn solved the problem with unit rates, like the solution some students used for Task 1. Joe and Hannah used fractions, Joe sought a common denominator and Hannah found a common numerator. Fred used a table to solve the problem.

Solution Approach 1 - Unit Rates

Carolynn, who did not seek a unit rate in Task 1, chose to take that route in this final task. After I directed her attention to this sixth and final reading passage, she spent over three minutes reading the problem to herself and constructing a solution. Her work (Figure 5-14) demonstrates an effort to divide the number of seconds by the number of gallons, establishing a unit rate of seconds per gallon. Carolynn found that the kitchen faucet takes 20 seconds for each gallon, and the bathroom faucet takes 24 seconds for a gallon. Her notation, however, caused some confusion.

Figure 5-14

Carolynn's Task 6 Solution



Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

kitchen
the ~~bathroom~~ sink

Go Online | PearsonRealize.com

2-1 Connect Ratios, Rates, and Unit Rates 87

When Carolynn reported her answer, she stated "I decided the bathroom sink. Um, because [pause] wait- actually I think it might be the kitchen sink." In this segment alone, we can see Carolynn *Pause to reflect* and *Self-check* her response, ultimately changing her final answer to the kitchen faucet. Her written work mirrors this change in direction as she crossed out the bathroom sink and wrote "kitchen" above it. When asked to explain her math, Carolynn reported:

> So, I did point five over 10 and then point seven five over 18 and figured out how much it like streams in one second. So, I got point two over one for um, for the kitchen sink and point seven five or point two four over one for the bathroom sink. Um, and then I decided

> and then I decided that the bathroom sink or the kitchen sink is, the um, going to fill it up the fastest.

The "point two over one" and "point two four over one" are not representative of the division problems she wrote in her solution, and she was observed trying to reason through what she did and what the results meant.

Carolynn closed her thoughts on Task 6 by stating, "I'm thinking about if I'm right or wrong because I don't really remember if point two four is bigger or smaller than point two," again self-checking her work. We agreed to continue with the interview and return to discuss this problem at the end of the interview. We discussed that, with the way she represented the ratios – the kitchen faucet as 0.2/1 and the bathroom faucet as 0.24/1 – it looked like the bathroom poured more gallons for each second. The division work done, however, shows the kitchen sink takes less time to fill a gallon of water, 20 seconds versus 24 seconds. In the end, she arrived at the correct answer because she took a moment to pause and check her own work.

Analysis of Activity. Carolynn's effort to read silently provided little information about her reading process. As such, the evidence about her reading, understanding, and how she connected the text to her work came from her responses to interview prompts. In these moments, Carolynn reviewed and explained her completed solution, Carolynn *Paused to reflect*, *Self-checked* her work, and *Modified her solution* all after she completed her work. This suggests that her search for understanding included the text she added to the page. This text, like that of the printed textbook, was read and contributed to her comprehension and understanding.

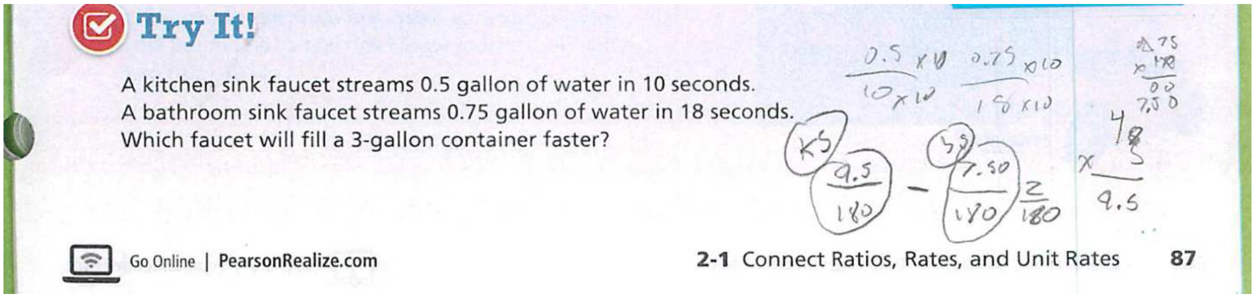
Carolynn's approach to her solution was to use unit rates. This solution combined with her Task 1 solutions, which did not include a unit rate, hints that she made *Connections across the text* as she read the three-page passage. She did not begin reading the three-page textbook passage with unit rates in mind (in Task 1). Task 3 encourages students to find a unit rate in order to identify the hourly wage of the lifeguard, Jennifer. Both Tasks 4 and 5 identify the unit rates used in their problems, so by the time Carolynn arrived at Task 6, it made sense she would lean that direction for her solution.

Solution Approach 2 - Fractions

Several students utilized fractions to solve the given problem. In each case, they set the ratio as gallons over seconds. Joe sought to identify a common denominator for his fractions. Hannah, on the other hand, looked for the three gallons poured by finding a common numerator for the two ratios.

Figure 5-15

Joe's Task 6 Solution



Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

Go Online | PearsonRealize.com

2-1 Connect Ratios, Rates, and Unit Rates 87

Common Denominator. In Joe's work (Figure 5-15) he began by placing the numbers given in the problem into fractions that represent gallons over seconds. He then decided to seek a common denominator for both fractions, scaling the denominators up to 180, multiplying the ten from the kitchen faucet and the 18 from the bathroom faucet. His notation seems to demonstrate that he multiplied both numerators and denominators by ten, but the result clearly demonstrates both values in the Kitchen ratio were multiplied by 18, and the bathroom values were multiplied by ten. He arrived at nine and five-tenths over 180 for the kitchen and seven and five tenths over 180 for the bathroom.

Early in his work, Joe expressed confusion about what to do with the problem. He explained that he was, “Kind of trying to, kind of wanting to like, set this up as a fraction, but I don't know how to, I don't think it's possible.” After a *Pause to reflect*, he offered the following:

> [pause] it just doesn't make sense because it doesn't have the tables that I could read, really. So I can, even though it has it like right in front of me. [pause] And then, I mean I guess you could put zero point five over 10 and zero point seven five over 18. But then you'd have to find the common denominator, which is 180- Wait, So you could make it a fraction. So if you could change that to- Wait, does 18 go to any other number that makes it even before 180?

In this passage, Joe expressed frustration and proceeded to *Question or critique the textbook* because this task “doesn’t have the tables that I could read.” Joe persevered and *Planned a solution*, setting up the two fractions and finding the common denominator of 180. He *Paused to reflect* on his reasoning twice to *Self-Check* his process, stating “wait, so you could make it a fraction” and “wait, does 18 go into any other number.” In this action, Joe is used his self-check to *Modify his plan* for a solution.

Analysis of Activity. One of the more notable observations made of Joe was his effort to Q*uestion or critique the text*. While reading the third page, each student was presented with sentences, diagrams, and tables associated with Tasks 4 and 5. In Task 6, as mentioned above, there were only written words from which the participants worked. Joe lamented the lack of a table of some sort to help with his analysis, and in doing so, he demonstrated the value of visual representations in the creation of mathematical meaning.

Joe worked through the problem without the table, of course, and arrived at the intended answer through is use of common denominators. This final answer hinged on Joe’s effort to *Create a plan or predict a result*, *Pause to reflect* upon the text provided, and *Modify his plan* to utilize the presented information. He realized a table is not present and that he had to take a different route toward the solution. Joe also shifted his thinking with regard to the denominator, searching for another number to go into 18. In the end, he simply multiplied ten and 18 to find his denominator, and established that the kitchen faucet would be quicker than the bathroom.

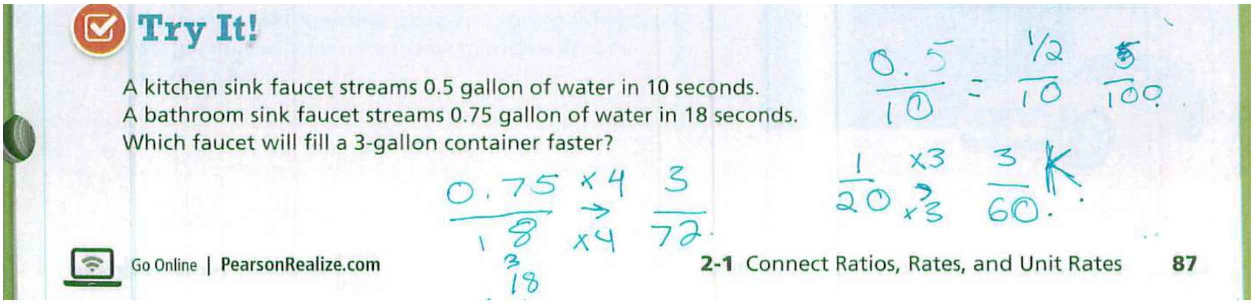
Like the students' work outlined above, Joe used the identified reading strategies to better understand the task at hand. As he progressed toward his solution, Joe was observed using similar strategies to solidify his solution. These efforts by Joe further demonstrate that reading strategies and solution strategies are closely tied to one-another.

Common Numerator. After *Reading aloud*, Hannah's work (Figure 5-16) demonstrated her path to identify which of the two faucets would fill three-gallons more quickly. Like Joe, Hannah's first order of business was to set up the fractional representation of the gallons to seconds ratio. She began with the Kitchen faucet, explaining:

> The first instinct I had was to put the zero point five over 10 to figure out the unit rate. How many water- how many per second I would do. So I would put to figure out the kitchen sink, we had zero point five gallons of water that would equal the same as one half over 10. If we multiply to each side by 10 maybe we get five that wouldn't help though. [pause] I'll try it anyways.

Figure 5-16

Hannah's Task 6 Solution



Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

$\frac{0.5}{10} = \frac{1/2}{10} \quad \frac{5}{100}$

$\frac{1}{20} \xrightarrow[\times 3]{\times 3} \frac{3}{60}$ K

$\frac{0.75}{18} \xrightarrow[\times 4]{\times 4} \frac{3}{72}$

$\overset{3}{18}$

Go Online | PearsonRealize.com

2-1 Connect Ratios, Rates, and Unit Rates 87

In the passage above, Hannah spent time outlining her process of reading and describing the information she found through that process. Through this effort, she provided clues about how she used reading strategies to comprehend the text. Hannah *Planned a solution or predicted* *the result*. She structured one of the ratios into a fraction, thought through what she could do with it, *Self-checked* her idea, *Paused to reflect* for a moment, then proceeded to try out her plan. As suspected, her plan did not give her the desired solution, but Hannah's work did highlight how these three strategies were used together in search of a solution.

After scaling the kitchen faucet ratio up to five and realizing the bathroom faucet ratio would not conveniently arrive at 5, Hannah *Modified her plan*. Having established that the kitchen sink fills five gallons in 100 seconds, Hannah worked backward to establish how many seconds it would take to fill one gallon. She shares that "if we had a hundred fits 20 in five times. fits 10 in 10 times. So, it would fit five and 20 times. So, it'd be one 20th [pause] and 10 so fill one gallon in 10 seconds, 20 seconds." Hannah then multiplied both the numerator and denominator by three to establish that the kitchen fills three-gallons in sixty seconds, three over 60.

The mathematics for the bathroom faucet was similar. Hannah began with the gallons over seconds fraction and sought a multiplier that would make the numerator, 0.75, a whole number, "So that would be, if we multiply it by two, we could get one point fit- five. If we multiply it by four, we would get three." Hannah then multiplied both the numerator and denominator by 4, arriving at 3 over 72 or three gallons in 72 seconds. "So, the kitchen faucet would be faster because we had 60 seconds for that, for it to fill up for, to stream three gallons compared to [72 seconds for] the bathroom sink."

Analysis of Activity. Hannah did not outwardly or explicitly display many of the reading strategies explored in this study. Her effort to explain explaining her process of however, did hint at the collective usage of several strategies, following her initial *Read aloud*. Through this explanation, there is evidence that Hannah also *Planned a solution or predicted a result*, *Self-checked* her ideas, *Paused to reflect* on the work done, and *Modified her plan* when needed.

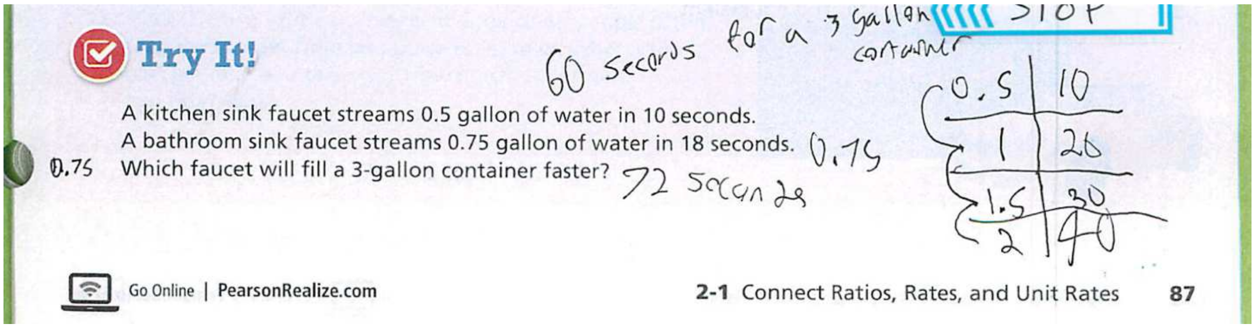
After her initial *Read aloud*, Hannah stated a desire to find a unit rate, dividing 0.5 by 10, indicating that she identified the information with which she chose to work, and formulated a plan to do so. With a little more thought, provided using the *Pause to reflect* strategy, she created a pair of fractions based on water volume to seconds. As her solution progressed, Hannah was observed combining the *Pause to reflect* reading strategy with the *Self-check* strategy, which lead her to *Modify her plan*. The combination of the three reading strategies helped Hannah move from a step she felt might not work to a final solution in line with most of her classmates.

Solution Approach 3 - Tables

Fred was the only student to utilize tables in his solution to Task 6 (Figure 5-17), an interesting fact because the author used tables in their solution to Example 3 (Forseman, 2017), the task that immediately preceded this Try It! problem. In The first table, Fred placed the given information for the kitchen faucet into the first row, then counted by a half and ten, respectively. After completing the fourth row, Fred recognized every two rows adds one full gallon and 20 seconds to the table.

Figure 5-17

Fred's Task 6 Solution - Part 1



Try It!

60 seconds for a 3 gallon container

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds. 0.75
0.75 Which faucet will fill a 3-gallon container faster? 72 seconds

0.5	10
1	20
1.5	30
2	40

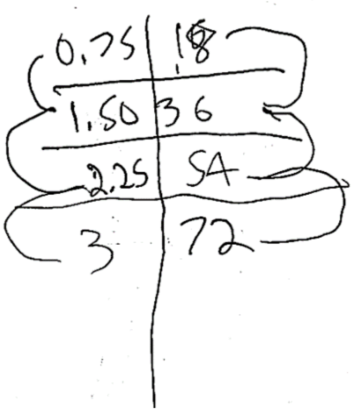
Go Online | PearsonRealize.com

2-1 Connect Ratios, Rates, and Unit Rates **87**

In his second table, Fred placed the bathroom faucet information into row one and again counted up four times. In this case, the fourth row is the three gallons Fred was seeking. He then writes out that one faucet takes 60 seconds for a 3-gallon container and the other takes 72 seconds.

Figure 5-18

Fred's Task 6 Solution - Part 2



0.75	18
1.50	36
2.25	54
3	72

In his solution, Fred was observed *Reading aloud*, *Paraphrasing the text*, and *Pausing to reflect* on the text and his work. In the passage that follows, Fred also demonstrates an effort to *Seek out important information*. Fred states:

> what I did is I started by using 0.05 and 10 so then I moved down and another one and I balanced added another 0.5 so that's one and 0.5 plus that one is 1.5 and then after that it's two and then it keeps going with that. Now on the other side, you start at 10 and then you add another ten is 20 another 10, 30 another 10, 40 and you'd keep going up. And then like it could become 60.

When asked about this solution, his reasoning for using the tables, Fred stated, "Um, I decided, I don't really know. I'd try early how to do the unit right. One as much as I feel like this one was kind of easier." I suggested above that Fred's use of tables was interesting because he was the only one and the preceding passage used tables. It seems that even Fred, who intentionally used tables, did not consciously make that connection with another portion of the text.

Analysis of Activity. Fred's efforts to understand the problem consisted of a *Read aloud*, an effort to *Paraphrase the text*, the *Identification of important information*, and a use of the *Pause to reflect* strategy. The initial three strategies were done before Fred began his solution. The latter strategy, the *Pause to reflect* strategy, was used both during Fred's initial reading and throughout his solution, to make certain he did not stray from his solution process.

Fred took a unique route to this solution and utilized a self-made table to help organize his work. This solution process was like that displayed in Task 5, as the book explored the jumping abilities of both a rabbit and a kangaroo rat. Fred was the only one of the 22 individuals who made tables while solving Task 6, and likely did so because that solution process was explained inches above this faucet problem. The solution process Fred showed demonstrates that he, like many of his classmates, tried to use information found elsewhere in the reading to complete the current task.

Task 6 Summary

This final task had the fewest reading strategies used when compared to the other two problems students were asked to both read and work. One explanation is the lack of graphical information presented in this sixth task. Another possibility is the students were all much more comfortable with the circumstances surrounding the interview and reading, allowing them to simplify their approach. The work of these four students is used to highlight strategies that were commonly combined by all the participants, and identify some strategies that could have been used, but were not.

Once setting a plan for the problem's solution, the students above worked through their strategy until they recognized a problem. At that point, they employed a combination of pausing to reflect, checking their own work, and reformulating a plan to solve the problem. In some cases, like Carolynn, that meant a concerted effort to reconsider what the numbers represent. In other cases, like Hannah, the student simply adjusted the problem's "starting point" and progressed from there.

Chapter Reflection

This chapter presented 11 student examples of solutions for three tasks. The variety of approaches on Task 1 and Task 6 were greater than the efforts made in Task 3. One might guess that the close relationship between Example 1 and Task 2, working with lifeguards at the same pool and exploring their hourly pay, would explain the more focused approach. I believe the structure provided in Task 3, however, could also be credited for crafting the 22 solutions presented.

The variety of solutions did not end with the overall strategies. Even when employing the same solution strategy (e.g., using fractions to compare two rates), Joe and Hannah took very different approaches to their solution. One sought a common denominator for the two ratios, identifying how much water was run through the faucet in a set amount of time, and the other choosing to find a common numerator to identify how long it took for each faucet to run three gallons.

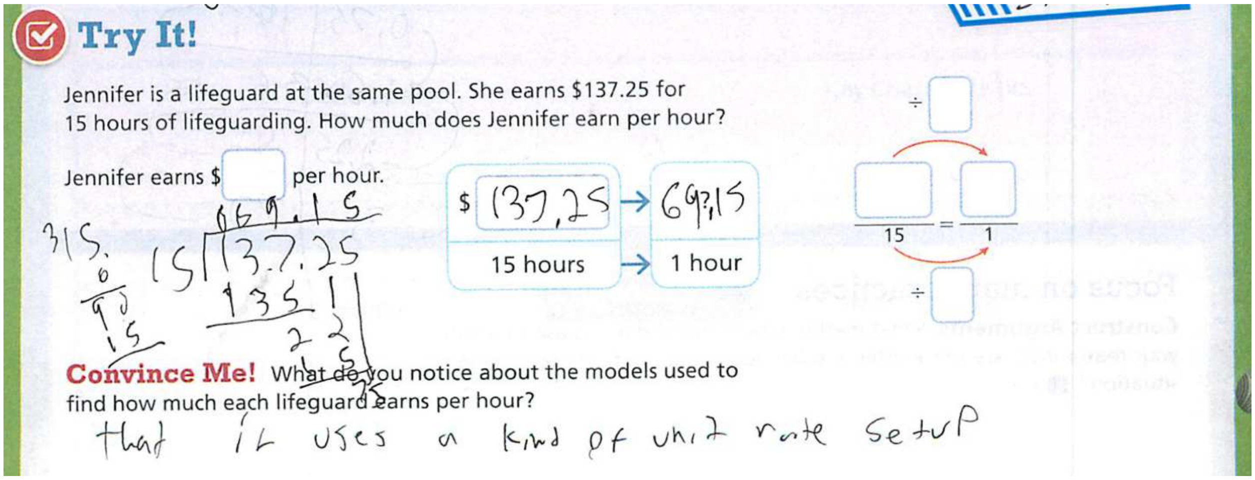
Through Task 1 and Task 3, I used student work and interview responses to demonstrate how these students used particular reading strategies. Across the entirety of interviews, the students read aloud, planned or predicted a solution, paraphrased portions of the text, paused to reflect on their reading, checked their own work, and modified their predictions as they identified issues. There were moments where they previewed what they were supposed to read, connected this work to prior knowledge, sought clarification either of the task or the text, and questioned or critiqued the text as they needed.

The one reading strategy I most expected and observed the least was the identification of connections across the text. In Task 3 it seemed obvious, as some students were seen looking up the page toward Example 2 as they worked through Task 3. In Task 6, where two examples were explained above, the students chose alternate solutions. Fred did pursue a solution like Example 3, but did not make that connection when asked about his decision to use tables in his solution to Task 6.

Several other ideas emerged as I reflected on the work done for this chapter. First, when reading and working through a textbook, particularly one designed to be consumable, students are reading both the author's writing and their own. In Task 3, Fred reported his Jennifer's pay as $69.15 per hour. When looking at his work more carefully (Figure 5-19), it is evident that a poorly formed '0' was the cause of his incorrect solution.

Figure 5-19

Fred's Task 3 Solution



Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $☐ per hour.

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

that it uses a kind of unit rate setup

Second, the content of the problem matters to the students. Several students reported an appreciation of the basketball problem. For example, Henry found it relatable because he identifies as a basketball player and Hannah mentioned a basketball game she had later in the day. Other problems were a source of frustration because they were not "real-world" problems. Dave, for example, took to critiquing the text, stating, "I wish they taught, like, real world, so like you couldn't have the decimal."

Finally, during the debrief, each student and I talked about what they know they read and what they know they did not read. In many of the interviews I would point to a section of text and ask if they read that portion. Each of the students reported reading the text structured in sentences and paragraphs. When asking about equations and images, the responses varied. Some students declared they did read the equations being discussed and others said they skimmed the work. With images, they either read the image or looked at it. Geoff, for example, read the text of Example 3 and "looked at the tables." I do not know where "just looked at" falls on a "I read" to "I skimmed" continuum, but it is a question I hope to explore in future projects.

Conclusion

Chapter 3 outlined the methodological influence and design of this study, including the verbal reading protocol. Chapter 4 identified the reading strategies used by the students during their interviews, and used student words to help define the more commonly used strategies. In this fifth chapter, the goal was to present examples of how students are using single reading strategies while working toward a solution. In Task 6, an effort was made to demonstrate how students may group reading strategies in pursuit of a solution. In Chapter 6, the focus is on three students, Eggy, Morgan, and Henry, with a closer look at how each implemented their reading strategies as they worked through a single page of the reading.

CHAPTER 6: The Work of Three Readers

Chapter 4 identified each of the reading strategies used throughout this project and highlighted those strategies used most and least by the 22 study participants. In Chapter 5, I used several students' work to exemplify how the more commonly used strategies were implemented by these seventh-grade students, and how the strategy choices changed when the task changed. In this sixth chapter, I share the work of only three students, one for each page of the textbook passage all students were asked to read. I also link my analysis of the three students' work to the RAND reading comprehension model, at the end of each students' work. I chose these students' work because of the decisions they made while working, their description or explanation of the decisions, and the advantages gained with each decision as it related to the task they worked through.

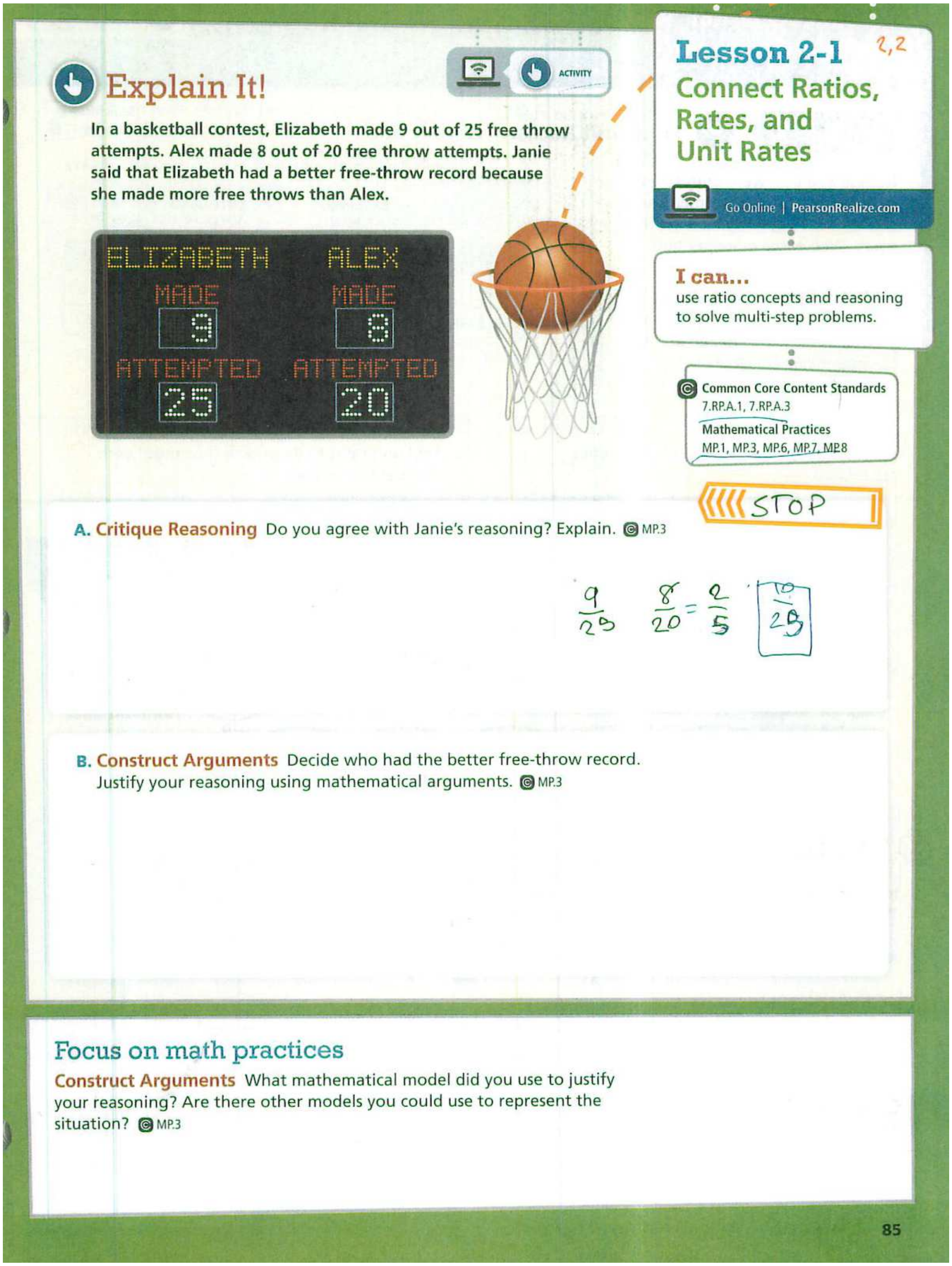
Eggy's work was chosen based on her effort to read everything, as well as the relative dearth of writing throughout Task 1. Morgan's work through Task 2 and Task 3 was chosen to highlight the connections she made between reading the example and the work shown during the exercise. Henry's work on the third page was chosen because Task 4 and Task 5 each present a solution that could be utilized when solving Task 6, at the bottom of the page. Henry, though, did not connect his solution to that of either Example 2 or 3. This chapter closes with the three students' debriefs, using their own reflections to accentuate and support some of the claims made earlier in the chapter. Remember, all names are pseudonyms.

Page 1: Eggy's Explain It! Work

Eggy, a student at Site 2, and I spent nearly five and a half minutes reading, working, and talking through the Explain It! problem. During this time, Eggy used ten unique reading strategies as she formulated and explained her solution. The first 90 seconds of Eggy's Task 1

Figure 6-1

Eggy's Page 1 Work



ACTIVITY

Explain It!

In a basketball contest, Elizabeth made 9 out of 25 free throw attempts. Alex made 8 out of 20 free throw attempts. Janie said that Elizabeth had a better free-throw record because she made more free throws than Alex.

Lesson 2-1

2,2

Connect Ratios, Rates, and Unit Rates

Go Online | PearsonRealize.com

I can…
use ratio concepts and reasoning to solve multi-step problems.

Common Core Content Standards
7.RP.A.1, 7.RP.A.3
Mathematical Practices
MP.1, MP.3, MP.6, MP.7, MP.8

STOP

A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

$\frac{9}{25}$ $\frac{8}{20}=\frac{2}{5}$ $\boxed{\frac{10}{25}}$

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

Focus on math practices

Construct Arguments What mathematical model did you use to justify your reasoning? Are there other models you could use to represent the situation? MP.3

85

efforts included eight of the ten reading strategies being implemented. I focus on those strategies that demonstrate Eggy's interaction directly with the text as she evaluated the components of text that were most useful for her purpose. In other words, I explored the strategies that most directly allowed Eggy to understand the problem posed by the textbook. Segments of her interview are shared in this section and the full transcript for Eggy's Explain it! work (Appendix G) is included at the end of this text.

Initial Read and Report

Eggy's initial effort to read the problem and respond to the "what are you thinking about?" prompt tallied 90 seconds. During the first 40 seconds, Eggy was observed trying to read all the text she could find above the orange "Stop" sign on her paper. During this Task 1 read, she stated the following:

> Explain it! In a basketball contest, Elizabeth made nine out of 25 free throw attempts. Alex made eight out of 20 free throw attempts. Janie said that Elizabeth had a better free throw record because she made more free throws than Alex. Lesson one: connect ratios, rates and unit rates. Go online, Pearsonrealize dot com. I can use ratios, ratio concepts and reasoning to solve multi step problems. [pause] Um, Common Core standards, blah, blah, blah. Umm–

In this passage alone, there are some interesting decisions made by Eggy. First, she chose to read all the words provided by the text, evidence that she tried to *Read the entire passage*. She read in a left to right, top to bottom order, as one would normally read text written in English. In a mathematics textbook setting, however, it might have been more useful to read the section title and the information below outlining the purpose of the lesson prior to reading the story about the basketball contest. Eggy began her reading with the problem description, then moved on to the lesson title, the "I can" statement, and finally the Common Core standards.

Second, Eggy stopped reading the codes for the Common Core standards and did not read the graphic of a scoreboard out loud. These actions exemplified her *Selective reading* of the text. This focus allowed Eggy to focus on that which she deemed most critical. The logical implication is that, for Eggy, the standards codes and the scoreboard illustration did not require the same level of attention paid to the written words.

Finally, Eggy clearly *Paused to reflect* on the text she was about to read out loud before proceeding to read the Common Core standards. She then "blah'd" out the specific standards and practices codes cited in the textbook. The underlining of the Common Core standards is evident on Eggy's paper (Figure 6-1), but it must be noted the underlining came well after this reading passage. During our debriefing conversation, Eggy underlined the Common Core codes to add emphasis to her statement that they were unnecessary and meant little to her, personally. Like Eggy, many students questioned the inclusion of the Common Core codes, suggesting they might be useful for teachers, but not necessarily in the student version of a text.

In just this first 40 seconds, Eggy *Read aloud*, *Read the entire passage*, and *Paused to reflect* before reading the Common Core standards and practices, and omitted the scoreboard and "blah'd" the associated standards from her read aloud, demonstrating she *Selectively read*. When asked to report her thinking, Eggy responded:

> Um, do I agree with Janie's reasoning? [pause] Um, Janie said that Elizabeth had a better free throw record because she made more free throws and Alex. It could be, [pause] it could be that Elizabeth was lucky too this time, you'd need more to make like an actual ratio. It depends really, because they were really close and it could be like– [pause] I mean Alex– For one, um, Elizabeth attempted 25 times, made nine of them, and then Alex attempted 20 times, made eight of them. So if we were putting, putting those in fractions, then nine over 25 and eight over 20, well not even, not good ratios to compare to. So- … You'd have to try- You'd have to, not transform, you'd have to convert those.

In Eggy's second talk turn, she *Paraphrased* the question being posed and the information found within the problem. Eggy also *Paused to reflect* three times to both reflect on what she wants to share and assess her understanding of the problem. She continued her reflection by *Identifying important information* found in the original problem and *Planning a solution or predicting the result*, then *Questioning or critiquing the text* as it related to her solution strategy, because the book provided numbers that were "not good ratios to compare."

Through the first 90 seconds of her five-minute encounter with the Explain It! problem, Eggy used eight of the ten reading strategies she used during this task overall, and tallied 12 total reading strategy implementations. As Eggy moved on to the questions posed by the textbook, she *Previewed the text* to better understand what was being presented. Throughout her work on Part A and Part B, she was observed *Self-checking* her work and understanding.

What Should Be Read?

Over the first 40 seconds, the initial reading, Eggy chose to read all the words and symbols creating the description for the Task 1: Explain It! section. As she read out loud, a notable change in tone was observed as Eggy shifted her attention from the description of the problem to the lesson title, I can statement, and Common Core Standards. Eggy read the description of the basketball contest in an expressive tone one might use while reading a story. She read the remaining text in a more monotone manner. She chose not to read the codes for the Common Core standards or the image of a scoreboard during this initial read.

I believe Eggy's efforts to *Read the entire passage*, demonstrated by the first 40 seconds of her work on Task 1, set the stage for her to summarize the problem and respond quickly. After she tried to read nearly all the text available, Eggy followed the reading by quickly summarizing the purpose Task 1 – "do I agree with Janie's reasoning?" She then identified the information critical to finding her solution – “Elizabeth attempted 25 times, made nine of them, and then Alex attempted 20 times, made eight of them.”

Through her initial reading effort, Eggy identified information that was not vital to the question posed in the story about the basketball contest. She then identified the relative importance of these passages. Eggy said that, “ideally looking back at it, ratios, rates and unit rates, I mean, I should have read that,” then shared an intentional effort to not read “like, just some tiny little, like, self-promo things.” Eggy, like others, suggested some components of the text were not necessary, particularly in a student version of the textbook. During her read aloud, she demonstrated the text she found important (e.g., her expressive read of the basketball contest description) and that which she did not (e.g., common core standards, blah, blah, blah).

Throughout her work on this task, she never revisited the lesson title, the “I can” statement, or the Common Core standards. Much later in the interview, she openly suggested some of that information was unnecessary and should not have been read. However, these distinctions could not be made without having tried to read all the provided text.

Pausing to Reflect

Eggy’s four pauses were followed by three different actions. The first pause came during her initial read of the text included at the top of the first page, the moment when Eggy reached the codes for the Common Core standards and mathematical practices. In this moment, Eggy may have been deciding whether to include the Common Core standards passage in her reading. After her pause, Eggy began to read those out loud but chose to “blah blah blah” over the number and letter codes that lead the reader to a specific standard or practice.

The three other pauses came when Eggy reflected on her initial reading. In the first of these three instances, she asked herself “do I agree with Janie's reasoning?” Eggy then paused for four seconds, and restated the important information from her previous reading, “Janie said that Elizabeth had a better free throw record because she made more free throws than Alex.” Her initial comments after the pause suggest Eggy used those four seconds to refresh her understanding of the problem, highlighting the information she would likely need to formulate a solution.

The final two pauses came while Eggy was formulating her response. As she continued her explanation, Eggy paused, mid-sentence. In the first instance, Eggy shared “It could be, [pause] it could be that Elizabeth was lucky too this time …” In the second instance, Eggy said “It depends really, because they were really close and it could be like– [pause] I mean Alex– For one, um, Elizabeth attempted 25 times, made nine of them, and then Alex attempted 20 times, made eight of them.” In both cases, Eggy seemed to pause her explanation to clarify her explanation. In the first example, Eggy began to talk, paused, then seemed to continue down the same line of reasoning. The second example found her pausing briefly, then following a different line of reasoning.

For Eggy, each of the pauses seemed to help her process the text that she read, but they were utilized in different ways. She paused during her initial read of the problem, seemingly deciding whether to read a passage. Three more pauses came as Eggy described her initial thoughts relating to the problem. In doing so, she seemed to use the pauses to locate relevant information and to adjust or affirm her verbal response, as needed.

Eggy’s Solution

Eggy spent the final 3-and-a-half minutes of her interview working toward a solution, presenting her answer, and justifying her response. She began her solution by paraphrasing the question being asked, then stating, “I disagree because one– you need more data to see if it's actually, like, if it's actually consistent and if it's there, consistency." As she worked through her solution, Eggy first identified that "it could be that Elizabeth was lucky too this time," then chose to work with Alex's free-throw shooting effort. She took Alex's ratio of shots made (8) to shots attempted (20) and simplified it to two-fifths. She then scaled the ratio to 25 attempted shots, establishing:

> Janie's statement is incorrect. Um, because if Elizabeth were to stay– if Alex were to stay consistent and you converted the– if Alex were to stay consistent and Alex took 25 attempted shots, Alex would make 10 sho- Um, 10, 10 free throws or 10 free throw attempts.

Throughout her work, Eggy used a total of ten unique reading strategies. Early in her work, Eggy took time to pause and think about what she was reading, what she was being asked to do, and what her response to the prompt may be. She paused 4 times in the first 90 seconds of her work, and did not pause again throughout the remainder of the page.

Beyond simply pausing to reflect on the reading passage, Eggy made decisions that seemed to relate to what she was going to read. In the first 40 second of her interview, she chose to read all the written words and ignore the diagrams. As she continued, she remained strategic about where her attention was placed, focusing on the information identified as important (e.g., Elizabeth and Alex's free-throw records) as opposed to information that seemed ancillary (e.g., the Common Core standards).

Page 1 Summary

Like each of the students in this study, Eggy was a seventh grader. She attended a suburban public charter school that housed students between third and eighth grade, and she was very open and willing to participate in the study. When asked to rate herself on a one to five scale, where five is the most proficient, Eggy said she was a five in both reading and mathematics.

The text, itself, came from a seventh-grade textbook, and as Eggy suggested, was not a problem for her to read. She reported that she found nothing particularly easy or difficult, stating, “I mean I've just sometimes just fumbled my words, but otherwise it's easy to read.” Additionally, she found that some of the smaller, self-promotion components of the book were unnecessary and not helpful for her purposes.

While reading, Eggy made use of several reading strategies. The most notable was her variation of the *Pause to reflect* strategy. She used this one strategy four times and it seemed to serve three different purposes. The pauses slowed her reading down, allowed her to re-assess any information that might be important, and to shift her reasoning, as needed.

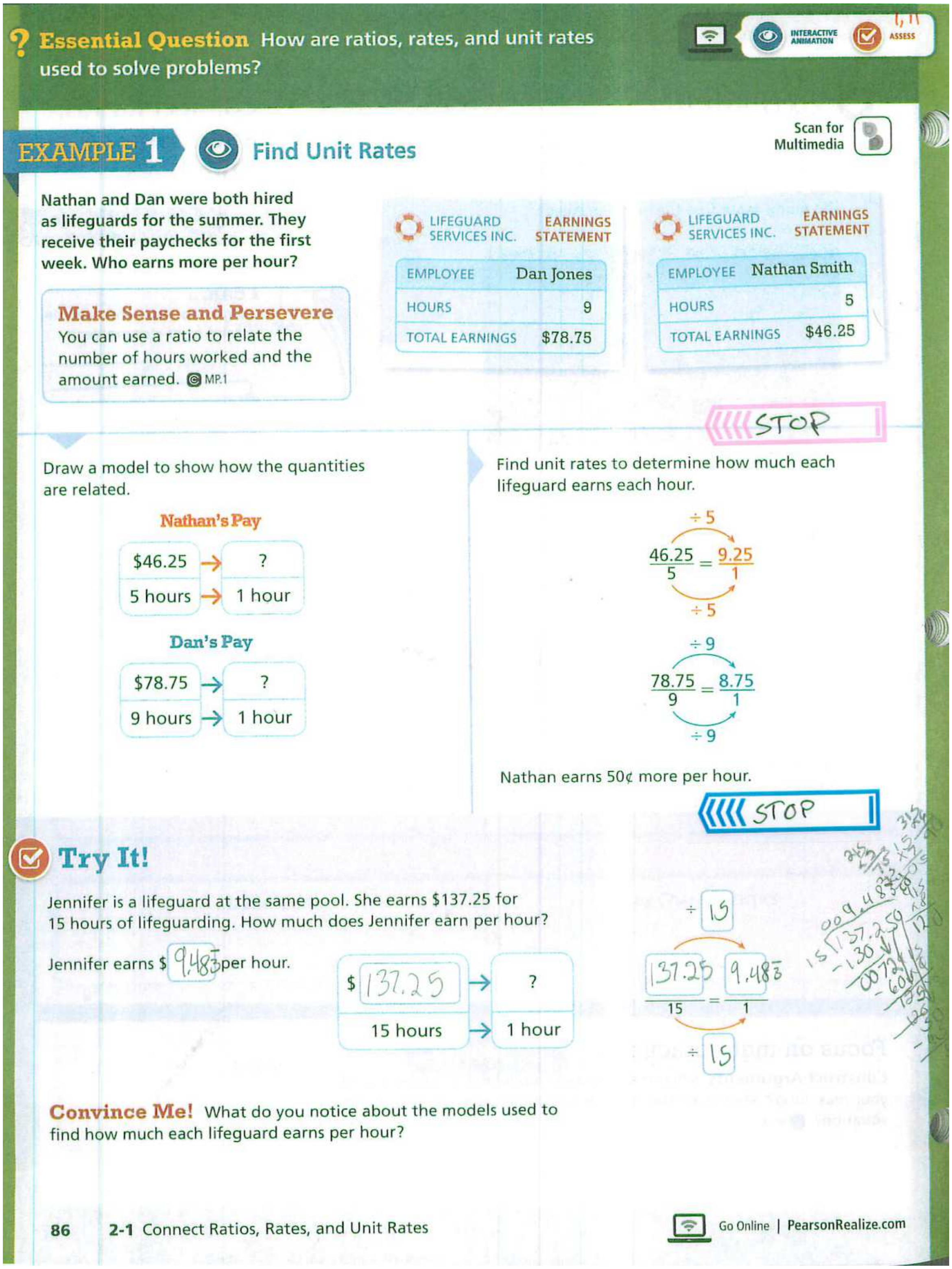
It is interesting to note Eggy’s regular mentions of consistency. She began by implying more data was necessary to make a firm statement about who had the better free-throw record, citing the possibility that Elizabeth may have just been lucky when the three characters reported their data. She went on to make sure that “if Alex were to stay consistent and Alex took 25 attempted shots” he would have made more shots than Elizabeth. In her initial read of the problem, Eggy identified that the two characters did not take the same number of attempts. She was also steadfast that an assumption of consistency was necessary to make a claim about which of the two students held the best free-throw record. This demonstrates that Eggy, while attending to the words on the page, is folding this information in with her prior knowledge of ratios.

Page 2: Morgan’s Reading of Example 1 and Try It! (the first) Work

Morgan, a Site 1 student, and I spent almost eight minutes reading Task 2, working through Task 3, and reflecting on both tasks. Task 2: Example 1 was divided into two sections (Figure 6-2), the first of which introduced the problem, while the second section modeled a solution to the task. Task 3: Try It! (the first) followed, and was closely related to Example 1,

Figure 6-2

Morgan's Page 2 Work



? Essential Question How are ratios, rates, and unit rates used to solve problems?

INTERACTIVE ANIMATION
ASSESS

EXAMPLE 1 Find Unit Rates

Scan for Multimedia

Nathan and Dan were both hired as lifeguards for the summer. They receive their paychecks for the first week. Who earns more per hour?

Make Sense and Persevere
You can use a ratio to relate the number of hours worked and the amount earned. MP.1

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Dan Jones
HOURS	9
TOTAL EARNINGS	$78.75

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Nathan Smith
HOURS	5
TOTAL EARNINGS	$46.25

STOP

Draw a model to show how the quantities are related.

Nathan's Pay

$46.25 →	?
5 hours →	1 hour

Dan's Pay

$78.75 →	?
9 hours →	1 hour

Find unit rates to determine how much each lifeguard earns each hour.

$$\frac{46.25}{5} \xrightarrow[\div 5]{\div 5} \frac{9.25}{1}$$

$$\frac{78.75}{9} \xrightarrow[\div 9]{\div 9} \frac{8.75}{1}$$

Nathan earns 50¢ more per hour.

STOP

Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $ $\boxed{9.48\overline{3}}$ per hour.

$ $\boxed{137.25}$ →	?
15 hours →	1 hour

$$\frac{\boxed{137.25}}{15} \xrightarrow[\div \boxed{15}]{\div \boxed{15}} \frac{\boxed{9.48\overline{3}}}{1}$$

$$15\overline{)137.250}$$

Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

86 2-1 Connect Ratios, Rates, and Unit Rates

Go Online | PearsonRealize.com

adding a third lifeguard character at the same pool and offering similar graphic organizers to help with the solution. Segments of her interview are shared in this section and the full transcript for Morgan's Page 2 work (Appendix H) is included at the end of this text.

Morgan used seven unique strategies when reading Task 2, and used six unique strategies while solving Task 3. Of those, she used *Read aloud*, *Modify her plan or prediction*, *Paraphrase the text*, and *Self-check* the most in her work. Through her reading of Task 2, Morgan chose not to make any notes on the page. She did show her work on Task 3, as is evident by Figure 6-2, above. In the text below, I explore Morgan's effort to predict or plan a solution, and the factors that led to a modification of that plan, itself.

Initial Plans of Action

In both Task 2 and Task 3, Morgan offered an initial route to a solution. In Task 2, as she continued to read the problem and the textbook's solution to the question, her ideas about the problem changed. With Task 3, she encountered some issues with her solution, but without more information from the text, she did not seek another solution.

Task 2: Example 1

In her initial read of Task 2, Morgan, like Eggy in Task 1, made an intentional effort to read all of the words constructed into sentence and paragraph form. In Eggy's work, there was no clear sign the images were read along with the text. Morgan leaves no such mystery.

> Okay. "Nathan and Dan were both hired as lifeguards for the summer. They received their paychecks for the first week. Who earns more per hour?" So, "make sense and persevere. You can use a ratio to relate the number of hours worked and the amount earned." So, Lifeguard Services, Inc. So, um, his total earnings and– okay. Yeah

In this initial read, Morgan read all sentences present, including the "Make Sense and Persevere" box ignored by most of the 22 participants. She also made a reference to the tables outlining the pay for both lifeguards, though she did not read them aloud. It should be noted that she did not read the Essential Question at the top of the page, and made no mention of the reference to the Common Core Mathematical Practice, MP.1 found in the make sense and persevere box.

When asked what she was thinking about, Morgan reported, “I'm thinking about, like, dividing and trying to, like, divide them even they again, like, but just as mainly when I'm thinking about.” When asked about the division, she said “Maybe three for Dan Jones. And I was thinking, cause it's kinda hard for that one, I was thinking, like, five maybe.” At this moment, she spoke about finding a unit rate through division, though she was not clear about which numbers were to be divided.

Morgan, based on her reading of the first part of Task 2, suggested that the route to a solution goes through division. When prompted to provide more detail, she stated that Dan Jones’ hours and pay should be divided by 3, presumably because both 9 and $78.75 are divisible by 3. She suggested the only thing that could work for Nathan’s pay would be five. Through these first 54 seconds, Morgan read the problem aloud, paraphrased a portion of it, and outlined what she thought would be a viable solution for both Dan and Nathan – division by 3 and 5, respectively.

As Morgan continued her reading, she found the example suggested she “draw a model to show how the quantities are related” (Forseman, 2017). She concluded that “they did per hour, so they divided it per hour. So, they just divided it per hour basically.” This new information led Morgan to abandon her initial plan of dividing Dan’s numbers by three, but allowed her to keep her plan of dividing by five for Nathan’s pay. Morgan allowed new information presented by the text, in this case a model of a solution, to change her initial ideas about the problem, leading her to modify the solution she initially outlined.

Task 3: Try It! (the first)

Following Morgan’s work from Task 2 to Task 3 allows for a comparison of her reading processes between the two related reading passages. In Task 2, Morgan was not asked to identify the two lifeguards’ hourly pay on her own, though she did demonstrate that she was formulating a plan to do so. Once the textbook demonstrated the path to a solution, she modified her own plan based on the information found further down the page.

When Morgan reached Task 3, she used many of the same reading strategies. She began with a *Read aloud*, “So, ‘Jennifer is a lifeguard at the same pool. She earns 137 and tw- dollars and 25 cents for 15 hours of lifeguarding. How much does Jennifer earn per hour?’” She continued her work by *Paraphrasing the text*, then *Planned a solution or predicted the result*, “You'd have to find that by doing 137 point 25 divide that by 15.” Throughout this process, she utilizes some strategies as both a reading strategy and a solution strategy. Morgan, as she sought to understand Task 3, read aloud, paraphrased what was read, and identified the solution path she selected. As she worked toward her solution, Morgan paraphrased her own planned process.

Unlike her experience with Task 2, where she initially stated a plan to find the answers, then later adjusted the plan when more information became available, Morgan never stops to revisit her plan to find Jennifer’s hourly pay. Morgan spends just over 2 minutes working through a long-division problem and expresses uncertainty about her answer. After being prompted to revisit the question and explain her answer, she shares:

> So what I did is, cause I know 15 divided by 15 is one so I'd have to divide one 37 point 25 by 15 and I got nine Oh nine I'm point 483 cents basically said it's 9 dollars eighty- and 483 cents I got if you did dollars. But– so, my answer, then, would be 900.483, okay. With the line above it.

Morgan then records her answer, on paper, as 9.483, where the 3 is repeated.

The remainder of Task 3 conversation centered on Morgan's uneasiness with the three decimal places and the repeating decimal. She recognized the question relates to “dollars and stuff,” but an hourly wage “feels like it doesn’t, like, repeat, like, over and over and over again.” This struggle is an indication that Morgan is attempting to settle her prior understanding of money with the representation found while establishing Jennifer’s pay. This self-check leads her to reconsider the validity of her work, but she never revisits her initial plan.

Connecting One Section to the Next

When reading Task 2, Morgan stated an initial route to a solution, then updated the plan as the book shared more information. In Task 3, Morgan stated an initial plan to find Jennifer’s hourly pay and received new information in the form of a number that did not fit her context. In the first instance, Morgan willingly embraced the new information and formed a new understanding. In the second, Morgan stayed true to her initial solution because of the work in Task 2.

Task 2 highlighted how one could find the hourly pay for a lifeguard. The text demonstrated it twice, with both Dan and Nathan’s pay, and used two graphic organizers in pursuit of the answer. Task 3 held the same context, a lifeguard at the same pool, and was presented with the same graphics that were used above. Because Morgan had read Task 2, she knew how to find Jennifer’s hourly pay. When asked about the relationship between Task 2 and the following Task 3, Morgan said:

> Um, [Task 2] was just, like, helpful because it just showed me the steps to do it. And like when I'm doing this, it's sometimes hard for me to, like, picture exactly like what I'm supposed to do the first time. But like, once my teachers, like, show me how, like, how it's done, um, then I understand it a lot better. Just, it just helps me I guess.

Morgan suggested that developing the knowledge of mathematics requires some speculative planning on her part, but also the willingness to accept the expertise of either her instructor or the text being used. Once that expertise was shared, and the plan was updated, there seemed to be no reason for Morgan to reconsider the strategy for finding Jennifer's pay. Her unease with more than two decimal places was logical, because that does not fit any real-life script for money known to these middle school students, but in Morgan's case, she deemed the error to be related to her long division, not her expressed solution path.

Page 2 Summary

Morgan is a seventh-grade student from the Site 1 school. She attended a public school structured in a traditional elementary-middle-high school system found at the edge of Midwestern suburban and rural space. When asked about how well she thinks she does in both math and reading, Morgan suggested that both were a 3.5 on the same five-point scale seen above with Eggy.

The text on this second page is more typical of traditional mathematics textbook writing, in that it presented a problem, a full solution was presented for the reader, and followed by a practice exercise that follows the same solution process. For Morgan, the text explained how to find the hourly pay of two lifeguards, and the exercise asked her to do the same for a third lifeguard. Task 3, the problem Morgan was asked to solve, is presented with the same graphical organization used by the textbook author during Task 2, above.

As outlined above, Morgan employed similar reading strategies on both Task 2 and Task 3. She *Read aloud*, *Planned her solution or predicted the result*, *Modified the plan* as needed, *Selectively read* the information she deemed important, *Paraphrased the text*, and *Self-checked* her understanding as she worked. The biggest shift between Task 2 and Task 3 was the modification of her planned solution. In Task 2, she initially suggested a need to divide one lifeguard's pay by three, because both the hours and pay were divisible by three. As she continued reading the solution to Task 2, she saw the textbook use nine, instead of 3, to find a unit rate. Morgan took this same idea into Task 3, dividing both Jennifer's pay and hours by 15, as opposed to three or five which are both common factors, establishing Jennifer's hourly pay.

Morgan arrived at an incorrect answer, but her effort to engage with the text shifted her approach between the two similar activities. During her initial read of Task 2, she believed dividing both pay and hours by a common factor might provide the answer she sought. After reading the textbook's solution to Task 2, and beginning the work for Task 3, Morgan sought the greatest common factor between 15 and 137.25. This shift suggests that she applied information found in one section of the text to another.

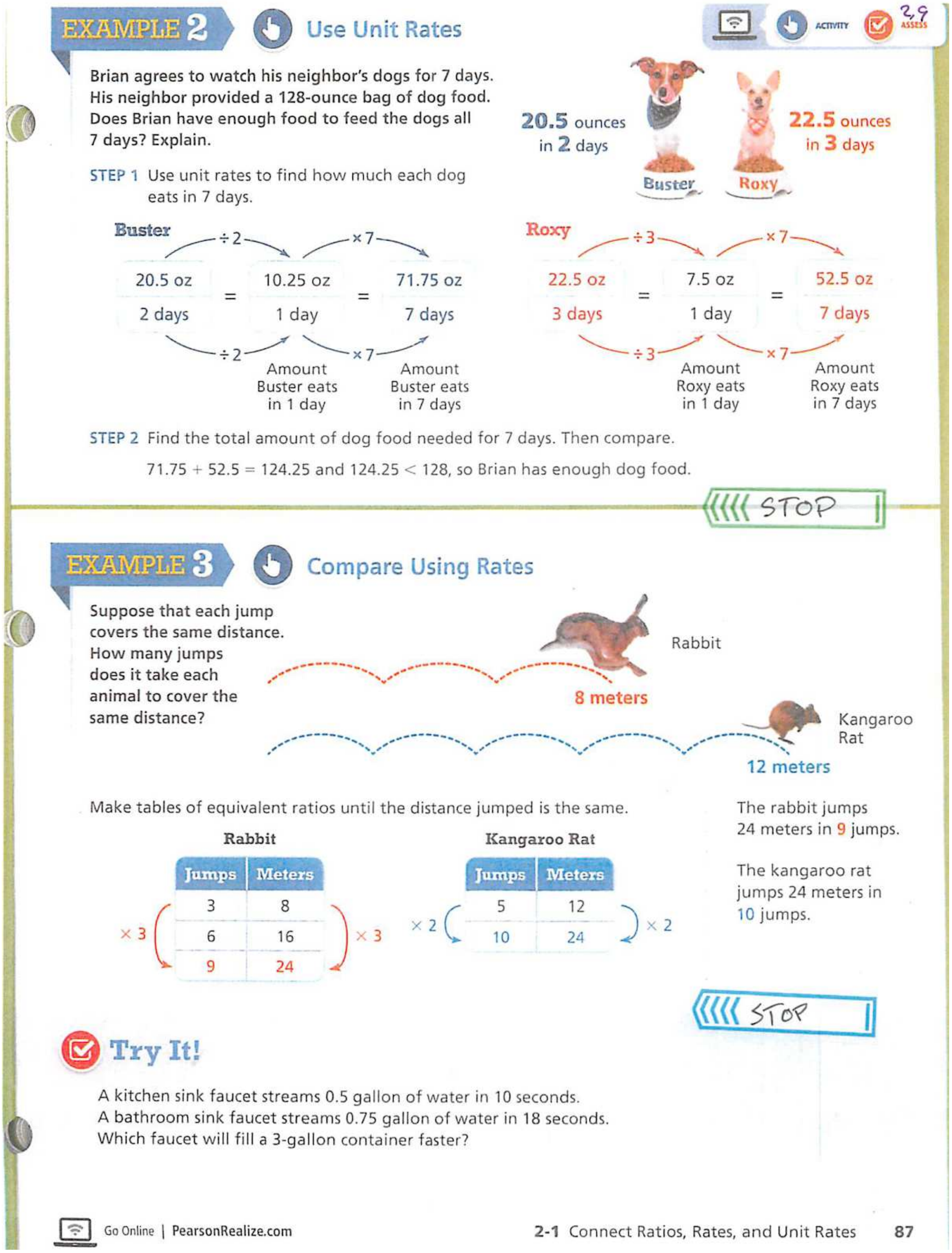
Page 2 of the selected reading passage held both an example and an exercise that were closely related to one another. The close ties between Task 2 and Task 3 allowed Morgan, and other participants, a direct model to follow as they worked to find Jennifer's hourly pay. On page 3 of the selected passage, the three tasks are related by mathematical content, but the context of each problem varies. The students in this study, largely, did not have the same experience when reading and working through page 3. That is, very few students used a solution presented by the textbook on page three to solve the final task at the bottom of that page.

Page 3: Henry's Reading of Example 2, Example 3, and Try It! (the second) Work

The analysis of the third page of the reading passage focuses on my conversation with Henry, a Site 2 student. Students used, on average, fewer than five unique reading strategies in each of the three tasks on page 3, an interesting finding considering the increase in technicality presented in Examples 2 and 3. Henry used seven, four, and five unique reading strategies, respectively, in his reading of the three tasks. Like Eggy and Morgan, Henry *Read aloud*,

Figure 6-3

Henry's Page 3 Work



ACTIVITY 2,9 ASSESS

EXAMPLE 2 Use Unit Rates

Brian agrees to watch his neighbor's dogs for 7 days. His neighbor provided a 128-ounce bag of dog food. Does Brian have enough food to feed the dogs all 7 days? Explain.

20.5 ounces in **2** days — Buster

22.5 ounces in **3** days — Roxy

STEP 1 Use unit rates to find how much each dog eats in 7 days.

Buster

$$\frac{20.5\text{ oz}}{2\text{ days}} \underset{\div 2}{\overset{\div 2}{=}} \frac{10.25\text{ oz}}{1\text{ day}} \underset{\times 7}{\overset{\times 7}{=}} \frac{71.75\text{ oz}}{7\text{ days}}$$

Amount Buster eats in 1 day

Amount Buster eats in 7 days

Roxy

$$\frac{22.5\text{ oz}}{3\text{ days}} \underset{\div 3}{\overset{\div 3}{=}} \frac{7.5\text{ oz}}{1\text{ day}} \underset{\times 7}{\overset{\times 7}{=}} \frac{52.5\text{ oz}}{7\text{ days}}$$

Amount Roxy eats in 1 day

Amount Roxy eats in 7 days

STEP 2 Find the total amount of dog food needed for 7 days. Then compare.

71.75 + 52.5 = 124.25 and 124.25 < 128, so Brian has enough dog food.

STOP

EXAMPLE 3 Compare Using Rates

Suppose that each jump covers the same distance. How many jumps does it take each animal to cover the same distance?

Rabbit — 8 meters

Kangaroo Rat — 12 meters

Make tables of equivalent ratios until the distance jumped is the same.

Rabbit (× 3)

Jumps	Meters
3	8
6	16
9	24

Kangaroo Rat (× 2)

Jumps	Meters
5	12
10	24

The rabbit jumps 24 meters in 9 jumps.

The kangaroo rat jumps 24 meters in 10 jumps.

STOP

Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds. A bathroom sink faucet streams 0.75 gallon of water in 18 seconds. Which faucet will fill a 3-gallon container faster?

Go Online | PearsonRealize.com 2-1 Connect Ratios, Rates, and Unit Rates 87

Planned a solution or predicted the result, *Modified his plans* as needed, and occasionally *Paused to reflect* on the task at hand. His work was unique, however, in that he provided running commentary throughout his effort to read, understand, and solve the tasks provided. He also leaned more on text clues within the text to craft his understanding of each passage. As seen in Figure 6-3, Henry wrote nothing down, but he explained his work verbally, as he went. I share segments of his work in the text that follows, and Henry's full Page 3 interview can be found in Appendix I.

Task 4: Example 2 and Task 5: Example 3

In Tasks 4 and 5, as the text became more technical and required more synthesis, Henry provided a running discussion about the components of the text he found helpful while making sense of each passage. This running commentary asked questions of, complemented, and critiqued the value of the text. Through this effort, Henry could identify textual clues that directed him to information vital to his understanding.

Task 4: Example 2

In Task 4, Henry took ten seconds between being asked to read the example and beginning his read aloud. This pause likely gave him time to sort through the information he felt needed to be read, and the order in which he was going to proceed. It also allowed Henry to identify clues within the text that helped his understanding after he was done reading. Consider the following excerpt:

> "Brian agrees to watch his neighbor's dog- dogs for seven days. His neighbors provided a 128-ounce bag of their dog food. Does Brian have enough food to feed all seven days?" And it's so it says there's two dogs. Well actually it does not say how many dogs there are … We can figure this out by right here though where it says the dogs and it has good information like when it says 20 point five ounces in two days, which can help us determine how much one dog we eat. Um, and then the other one is 22 point five ounces in three days …

In this passage, Henry read the problem out loud. Based on the images provided, he assessed that “dogs” referred to just two animals and identified the importance of knowing how much each dog eats. Henry was also observed thinking through his solution as he read the solution presented in the textbook. While he arrived at a slightly different answer than the text, 123.25 versus 124.25, he was quick to acknowledge that “I think that's a good, that's a good question. And it's pretty- it's pretty clear.”

Task 5: Example 3

His work through Task 5 was similar. Henry read the introductory sentences, “So suppose that each jump covers the same distance. How much- how many jumps does it take each animal to cover the same distance?” then set out to describe the tables associated with the problem. As he assessed the table, Henry explained:

> You can see how much, how many jumps it takes them to reach the same. Um, what do you say? Distance. And so for the kangaroo rat, it takes ten jumps to get to 24 meters and it takes nine jumps for the rabbit to get to 24, which is good. So shows, and also right here, this is very good too

In doing so, he acknowledged the information found in the table is needed for forming an answer to the question posed at the beginning of the example.

Henry circled back to the diagram of the rabbit and kangaroo rat after establishing the answer to the problem. In doing so, he presented an alternate route to a solution one could take, mirroring the solution presented in Task 4. He stated:

> [the diagram] shows the- it shows, like, the distance per hop, which is just- With this be divided by three. With this be divided by five, which is good. It's a good thing. And if you wanted to go more, go more, you could see how much they jump. um, how- the distance for every jump they take.

This description offered by Henry echoed the solution presented in Task 4, where the dog’s total food intake was divided by the number of days to find a unit rate. That unit rate was then scaled up to account for the seven days. Henry stated, however, “that's not what they're asking,” and the solution presented in Task 4 “has a lot of information that- … that shows that it's good.”

Through the two examples on Page 3, Henry was observed working through the problems in his head as he read the text. During that process, he identified important information presented in both written word and image forms and synthesized the information into an understanding of each explanation. In Task 5, Henry began to outline a solution like that presented in Task 4, though he conceded the solution in Task 5 better fit the question being asked. Throughout this pursuit, he continued to highlight things he liked about the text, and the clarity with which it was written or explained. His running commentary made the identification of some strategies (e.g., seeking important information and using text clues) much easier to observe.

Task 6: Try It! (the second)

As stated above, Henry chose not to write anything down on his paper as he worked. He did, however, present a thorough assessment of the faucet problem, arriving at the intended result, “the kitchen sink faucet will fill gallons container, like, fill the three-gallon container faster.” While working toward his solution, Henry was observed reading aloud, creating a plan and modifying his solution, like Morgan’s approach to her Try It! problem at the end of the second page. Much like he did in reading Tasks 4 and 5, Henry identified characteristics of the text that led him to find information critical to the solution of the problem.

In his first comment, below, Henry read the problem aloud and pointed out that graphical features from both Tasks 4 and 5 were absent because they were not necessary to answer the question. He reported:

> “A kitchen sink faucet streams, zero point th- five gallons of water in ten seconds. A bathroom sink faucet streams zero point seven five gallons in 18 seconds. Which faucet will feel a three-gallon container faster?” Alright, so this one you don't really need all the diagrams and stuff because it does have enough information in the problem.

In the passage above, Henry used the textual cues provided to identify the information vital to the solution. The lack of imagery told him that the information he needs is found within the text of the problem.

In his next series of statements, below, Henry again assessed the work of the textbook author, offered a possible plan, then immediately revised that plan:

> So that's very good. You can, it's, you can tell how much- you just have to, you know, multiply this by, so-and-so, like- So, to get this to three, you can divide it- Well, you can divide three gallons by zero point five, which'd you six. And then you multiply six by 10 which would get you 60.

Throughout his read of the text, Henry was very complimentary of the problems posed and explanations given. While working through the faucet problem, Henry mentioned some critique or compliment of the text on three different occasions, and used the practice as a demonstration that he understood what was being asked of him. At one point, he gently critiqued the lack of images associated with the Try It! Problem at the bottom of the third page. He shared that he prefers “diagrams and stuff … [and] maybe you can put like a picture of each faucet, like saying that it drips 18 per second.” He recognized the effort was unnecessary, “But me, I like seeing sti- things, like, visually.”

We spent a total of one minute and forty seconds solving and discussing this final task, demonstrating the ease with which Henry located the important information and worked toward a solution. He initially stated the solution could be found by multiplying, then quickly adjusted his plan to divide the number gallons sought, three, by the number of gallons that flow out of the kitchen cycle in one cycle of ten seconds, five-tenths. The result of the division was six cycles, so he multiplied the ten seconds per cycle by six to find the kitchen faucet fills a three-gallon container in 60 seconds. His solution for the bathroom faucet mirrored the solution above, establishing the bathroom faucet fills a three-gallon container in 72 seconds.

Page 3 Summary

Henry was a Site 2 student, attending the same school as Eggy. He, when asked, rated himself a four out of five in mathematics, and a 3.5 out of five with regard to reading. Throughout his interview, he was very talkative, and spent much of the time critiquing and commenting on the text chosen for this study. This running commentary provided a great deal of insight about his thought process as he worked through the third page of the selected reading passage.

The text presented three different problems, on this third page. The passages provided less whitespace than either of the first two pages, and the three stories were not related to one-another in any way other than the mathematical principles being applied. Task 4, at the top of the third page, presents the most complex string of equations, Task 5 has very graphic-heavy representations, and Task 6 is completely void of imagery. Additionally, as middle school students read down the page, each example gets further from their reality.

In each of the three sections read by Henry, he was observed critiquing or complimenting the text as he went and using text clues to identify important information. In Example 2, he identified the diagram of the two dogs as helpful for establishing that Brian had two dogs. He leaned on the table to help his understanding of Example 3, but used the diagram of the jumping rodents to offer an alternate solution path one could take. In the Try It!, the lack of images told Henry the information he needed to use was found within the paragraph itself, and no further information was needed, though he shared a preference for pictures. One question I wish I asked Henry is why, if he preferred a picture on the page, he did not draw a representation of either faucet.

Debrief Conversations

The information shared above chronicles the observations made of each of these three students while they read through the text. After the reading was complete, each of the students participated in a short debrief of the activity. During this time, they each shared information about their approach to reading and identified decisions they made while they were attempting to understand the presented text.

Henry continued to compliment the problems in the book, finding them relatable based on either first or second hand knowledge, *making connections* with the text. He shared “I think that these questions are really good. They make you, like- Cause these questions to relate to actual, like, people.” For him, the basketball question was most relatable, because he enjoyed keeping his own statistics, but he shared knowledge that “a lot of people have dogs. I don't personally, but a lot of people have dogs and they love their dogs.”

Henry also explained how he *selectively read* the text by identifying sections of the text he did not read, such as the “Make Sense and Persevere” box on the second page. For him, “this is kind of helping you, like, [if] I don't know what to do. I did know what to do, so I didn't, like, need it.” This sits in contrast with Morgan, who read that call-out box as she read through Example 1.

Morgan did not mention the call-out box in Example 1, but she did question the need to read the lesson title, *I can* statement, and Common Core standards. She reported, “I didn't, like, think these (gesturing to the lesson title, I can statement and Common Core standards), like, the- Like, the lessons were very important to read. I just felt like the pictures were important to read.”

Like Henry, *text clues* such as images were deemed helpful, and some of the other components found in the text were identified as irrelevant to the task at hand.

Eggy also questioned the inclusion of the Common Core standards, "blah-ing" her way through the standard and mathematical practice codes. When asked to highlight or point out areas she did not need to read, she immediately underlined the Common Core codes for standards and mathematical practices. She later identified a need to read the "ratios, rates and unit rates" but felt the self-promotion was an unimportant component of the chapter.
Beyond *selectively reading* the text, using *text clues* to support understanding, and *locating important information*, these three readers identified connections both across and outside the text. Both Eggy and Morgan suggested the text started off as a little confusing, but each component built on the previous one. Eggy stated the "I feel like they, they were like slowly transitioning" and Morgan found the examples helpful as she worked through the related Try It! problems. Henry found the problems related to real people, and therefore, "maybe I'm bored in math class and that and then we're talking about this question. I'm like, Oh, that might get me, like, into the class."

Each of these themes were difficult to identify through observation and self-report, alone. The debrief was a critical component of the design of this project, and the enactment of said discussion made it possible to highlight reading processes that may have otherwise been overlooked.

Closing Thoughts

In Chapter 4, I outlined the strategies used by the 22 students who participated in this study. Chapter 5 investigated how some of the more common strategies were used by the students. This chapter described the advantages gained by three readers - Eggy, Morgan, and

Henry - as they read the text presented to them. The three readers *Selectively read* the text, *Used text clues*, *Identified important information*, *Paused to reflect*, *Planned a solution or predicted a result*, then *modified the plan or solution*, *Questioned or critiqued the text*, and *Made connections across the text* and beyond the task. Each of these actions seemed to help these three students stay engaged, informed, and working toward the solutions they provided.

Eggy made an intentional decision about what should be read, and what should not - *Selective reading*. Her effort, when asked to "read until that stop point," demonstrates what she considered important when reading, and provides a bit of insight into what Eggy considered "reading" in a mathematics space. As she began her work, Eggy read all the text that included phrases, sentences, and paragraphs, an example of *Reading the entire passage*, but did not include the diagrams in this initial read. She also paused briefly to identify whether she needed to read through the Common Core standards found below the *I can* statement in the text, began reading them, then decided they were unimportant to her final goal so she "blah-ed" out the identifying standard or mathematical practice codes.

The Common Core standards were important enough that Eggy attempted to read them, then quickly decided they were not critical to her effort to solve the problem at hand. Similarly, when asked to read the passage, she made no effort to read the scoreboard graphic and made no mention of the basketball falling through the hoop. For Eggy, reading focused on passages deemed essential to her understanding of the context of the problem. To that end, neither the Common Core standards nor the included graphics added to Eggy's understanding of the problem.

Eggy also paused to reflect four times in the first 90 seconds of her solution, and not again until she reached Example 1. The four pauses served three different purposes. The first, just before reading the Common Core standards, was an effort to decide the importance of that passage. The second pause allowed Eggy an opportunity to revisit and paraphrase, refreshing her understanding of Janie's opinion. The third and fourth pauses served the same purpose, allowing Eggy a moment to consider and shift her response in the middle of a thought, as needed.

With Morgan's work, she utilized the *Planned a solution or predicted the result* strategy in conjunction with the *Modify the plan or prediction* strategy in Example 1, while she was attempting to understand the task at hand. When she arrived at the exercise at the bottom of the page, she constructed a plan, "You'd have to find that by doing 137 point 25 divide that by 15" and chose not to modify this route to her solution even when her work led to an uncomfortable result, 9.483. The reading strategy tandem in an earlier passage made the combination unnecessary during the Try It! problem, because Morgan already read, planned, and modified her plan to find an hourly wage for the lifeguards and knew what was expected of her when calculating Jennifer's hourly wage.

Henry read through two examples and worked one problem. The three tasks had a loose relationship, as they each related to ratios, rates, and unit rates, but the connection was not as close knit as those identified by Morgan. Henry commented readily about his likes and dislikes related to each task, *Questioning or critiquing the text* as he worked. He identified a preference or images over just the written sentences, but overall appreciated the relatable nature of each problem. He used the *text cues*, in the form of visual images, found within to help make sense of the problems. He also showed little trouble *Identifying the important information*, even when the images were absent.

Collectively, this trio seemed to *Made connections across the text* and beyond the text as they read. They also *Read selectively*, choosing not to spend much time or effort on components they either felt were unnecessary for all readers, or not needed for their own purposes. Finally, the structure of the text and inclusion or omission of images were identified as valuable. Both Morgan and Henry expressed appreciation for the visual text cues present within the images.

CHAPTER 7: A Reflection

As I look to close this project and consider the design of the next, I offer this final chapter as a summary of the preceding six. In doing so, I review the key components of the project, offer a detailed exploration of the findings and their place in the current body of literature, and provide implications for where and how this information can be best used. The chapter closes with potential limitations to this work's application and an outline of future research lines that should be followed.

Project Summary

This project is rooted in a series of observations surrounding the practice of reading mathematical texts. It began with my own education, in the mathematics courses that provided the greatest struggle. I felt the only way I could be successful in a mathematics course was to be present and attentive. Reading the textbook, after the fact or in lieu of class attendance, caused more frustration and confusion than assistance.

I noticed a similar frustration from both my middle school and junior college students. Neither group of students were asked to regularly read their textbook, and found it difficult to catch up when a class was missed. I find this particularly unfortunate for the college students who spend hundreds of dollars a year on mathematics texts, only to find out the books are used for the practice problems or online access, and rarely for the expertise found within the authors' words.

This study falls at an opportune time in education, as schools across the world close and students are being asked to complete their work virtually, and with less teacher guidance each week. It is not unreasonable to think homework scenes like that from the second Incredibles movie are playing out in homes everywhere, where students and parents are struggling to understand the assignment and the instructional text is difficult for both parties to understand.

Research Questions

A better understanding, and better instruction, of reading mathematics would be helpful for students and parents alike. This project seeks to better understand the ways students approach reading, which in turn may lead to more productive, explicit instruction related to reading mathematical texts. Specifically, this project explored the following questions:

> What does mathematical reading look like for middle school students reading a passage from a nationally published and adopted textbook?
>
> - What reading strategies do students use when engaging mathematical text?
> - In what ways do students use these strategies as they engage with the text?
> - What purpose do these strategies serve?

Methodology

To explore these questions, I pursued a qualitative study consisting of semi-structured clinical interviews of 22 seventh grade students in two different schools. The student participants were volunteers from two different research sites, and were interviewed over the course of a month in the Fall 2019 semester. Each interview followed the same verbal reading protocol, asking students to read through the same three-page passage from a seventh-grade mathematics textbook.

Each interview transcript was analyzed and coded for the 25 reading strategies identified in Chapter 3. This initial coding provided the answer to the first question, “What reading strategies do students use when engaging mathematical text?” The answer to the next two questions, “In what ways do students use these strategies as they engage with the text?” and

“What purpose do these strategies serve?” required more analysis, so the RAND reading comprehension model was utilized to provide answers. These analyses can be found throughout Chapters 4, 5, and 6.

Major Findings

The findings for this project come in three groups, and are discussed in more detail below. It can be said each of these students used reading strategies as they engaged with mathematical texts. The strategies chosen varied from student to student, and at times, between implementations from the same student within the same passage. Additionally, some strategies tended to be linked to one another, as students worked through their text.

Detailed Findings

Through this next section, I dive more deeply into the major findings presented above. In doing so, I review some of the literature surrounding content-area, disciplinary, and mathematical literacies and the RAND Reading Study Group (Kirby, 2003; Snow, 2002). In doing so, I highlight the list of strategies most used by these 22 students (Table 7-1) and outline the transition from strategy implementation to mathematical understanding through the RAND reading comprehension model. This section closes with some quick observations that were not sufficient to be “major findings” but intriguing enough to be considered in future and related work.

Reading Strategies

Between Adams (2003) and Hillman (2014) it was established that mathematics is, indeed, read. Others, like Gibbons (2009), have identified genres of writing found in academic spaces. While there are some that might identify components of mathematics as narrative, even absent a timeline (Dietiker, 2013), most researchers would classify mathematical writing as informational or expository texts. In any case, it is certain that mathematical text is read, and students must be prepared to engage with all presented representations.

To compile a list of reading strategies, I consulted a range of literacy perspectives. In doing so, I paid close attention to studies that utilized a similar verbal reading protocol (e.g., Pressley & Afflerbach, 1995; Shepherd & van de Sande, 2014), in which the participants read and reported their thoughts. I also looked for projects that included the researcher working with the classroom teachers as they explored their ideas of literacy, together (e.g., Doerr & Temple, 2016; Herbel-Eisenmann et al., 2015) and those that compared the reading perspectives of experts in their respective fields (e.g., Shanahan et al., 2011).

This exploration covered literature from traditional literacy research to those more focused on the nuances found within disciplinary spaces (e.g., content-area literacy, disciplinary literacy, and mathematical literacy). Through this review of literature, I sought explicit examples of reading strategies being implemented within their work. This process, in conjunction with previous projects (Beaudine 2018, 2019a), led to an initial list of 21 reading strategies. With the help of a colleague and the study participants, that list grew to the 25 reading strategies found in Chapter 3. This effort set the stage to identify the reading strategies used by these 22 participants, the first of the three research questions.

Most commonly used

Of the 25 strategies identified for this project, 23 of them were used by at least one student. This suggests that these 22 students were using reading strategies as they engaged with mathematical text. The questions for this project were not whether or not students were using reading strategies, however, but rather which strategies they used and how they used said strategies.

In all, the 22 participants completed a total of 153 interview segments in which they read, worked through the mathematics, and reflected on the process. Six strategies were either used by at least 21 of the 22 students, or were used in over half of the interview segments. These strategies are outlined in Table 7-1 and form the basis of the strategic approach taken by the students who volunteered in this study.

Table 7-1

List of Most Used Reading Strategies

Strategy (students)	Strategy (separate implementations)
Paraphrase text (22)	Read aloud (104)
Plan a solution or predict result (22)	Plan a solution or predict result (97)
Self-Check (21)	Pause to reflect (89)
Question or critique the text (21)	Paraphrase text (86)
Pause to reflect (21)	Self-Check (77)

Least used strategies

The strategies most used by these middle school students form a nice point from which to begin the discussion, as they were utilized by nearly all the students, and often. These six strategies, alone, may be insufficient, however, as text often varies and reading approaches may change with more familiarity and practice. Because of this, I suggest that the strategies found on the least used list (Table 7-2) be carefully weighed before they are set aside.

Instruction related to test taking and studying often includes suggesting students *Skim the text* to preview the passage and *Make notes as they read* through the work to identify important information (Penn State Learning, 2020; Todd, 2020). Additionally, there is evidence that efforts to slow down and *Decode the text* (Armstrong et al., 2018) or *Create a mental image* (added by students in this study) also aid mathematical readers in their pursuit of understanding.

Table 7-2

List of Least Used Reading Strategies

Strategy (students)	Strategy (separate implementations)
Creates a mental image (2)	Decode the text (2)
Decode the text (2)	Creates a mental image (4)
Make notes while reading (6)	Make notes while reading (7)
Skims the text (7)	Skims the text (7)

All of this suggests that students are reading their mathematical text, and in doing so, are employing reading strategies. This study demonstrates that the 22 students from these two Midwestern sites gravitated toward a collection of six reading strategies, with at least 21 students using the strategies or the strategy was implemented in at least half of the interview segments. There is also evidence, whether through the work of these students or previous literature, that the strategies that went largely unused by these students hold a great deal of potential value. The introduction of these lesser used strategies may complement the strategies naturally implemented by students as they read their mathematics.

RAND Work

To better understand the implementation of these reading strategies, the work of the RAND Reading Study Group (RRSG) was utilized. In their work, they discuss a framework for reading comprehension that consists of three individual components that work together within the socio-cultural context of the classroom – the reader, the text, and the action of reading (Kirby, 2003; Snow, 2002). It is through this lens that I sought to explore the second and third research questions – “In what ways do students use these strategies as they engage with the text?” and “What purpose do these strategies serve?”

Readers

When exploring “the reader,” Snow (2002) includes an individual's capabilities, motivation, knowledge, and prior experience as factors to consider when attempting to assess one’s reading comprehension. As explained throughout the earlier chapters of this text, the participants were all seventh-grade students, nominated by their instructor. In this selection, both instructors considered the students’ reading and mathematics abilities, and the level to which they felt their students would participate within the interview. That is to say, each of the students in the study were deemed capable by their classroom instructor, and nothing done in this project would suggest differently.

The motivation of these students is largely external. The project was my design based on my inquiry. I presented the documents to be used, I asked the questions of the students, and their participation was rewarded, in part, by a gift card and pizza party upon completion. In one instance, when asked if they had any unanswered questions at the end of the interview, a student did ask for more details about the project and why it was being done. All the students showed some form of engagement and worked to do well, but much of the motivation for reading this passage in this way was externally driven.

The knowledge and prior experiences are key to consider in any study. The RRSG identified both as being critical to one’s reading comprehension (Snow, 2002). This study interviewed seventh grade students reading seventh grade mathematics materials. The content covered in this passage, ratios and unit rates had not yet been taught in this academic year. Even so, some of the participants mentioned knowing how to approach Task 1, the exercise coming prior to any instructional text, because they saw the information the year before, in sixth grade.

Text

The RRSG identified “the surface code (the exact wording of the text), the text base (idea units representing the meaning of the text), and the mental models (the way in which information is processed for meaning)” (Kirby, 2003, p. 2) as important components to be considered when assessing the text. For the participants in this project, neither the surface code nor the text base seemed to be an issue. Eighteen of the 22 read the text out loud, with minimal hesitations, pauses, or other issues. The other four read to themselves, but seemed to have a similar understanding of the content presented.

In only one area was the exact wording of the text questioned. In Task 5, several students thought the written words “Suppose each jump covers the same distance. How many jumps does it take each animal to cover the same distance” (Forseman, 2017, p. 87). This introduction uses “the same distances” in successive sentences to mean two different things. In the first sentence, it refers to each individual animal’s jump being the same size. In the second sentence, the book is asking how many jumps must be taken for each animal, knowing they jump different lengths, to reach a common distance. The solution, for most students, was to use the diagram to the right of this text to clarify the meaning of the written words. Across the remainder of the text, the written words were clear, and the diagrams provided were identified by students as providing clarification (e.g., the picture of Brian’s two dogs) or guidance (e.g., the graphic organizers in Task 3).

One component of the text I had not considered prior to this project, but one that played an important role in the work of at least one student, is the participants' own writing. The textbook author provided space enough for students to work through the problems posed, or take notes as needed. In one instance during a Task 3 solution, Fred was working through a long division problem and misread his solution. In doing so, he reported his solution of 009.15 as

$69.15 per hour. The issue, in this case, was his printing of the second zero that looked like a 6 combined with not self-checking his solution. This mistake demonstrates there is more to reading a mathematics text than the printed text, itself.

Activity

This is the moment where the reader and the text begin to interact. The action of reading, as explained by the RRSG, "includes one or more purposes or tasks, some operations to process the text, and the outcomes of performing the activity, all of which occur within some specific context" (Kirby, 2003, p. 2). In this sense, the reader employs a series of skills and strategies to uncover the information presented within the printed text. The effort to do so begins with a purpose, whether generated by the student, the text, or an outside observer. In this case, the driving purpose stemmed from my inquiry and the request to read the text provided. As discussed above, a secondary purpose certainly could be their desire to complete the task to a satisfactory degree, though that also is dependent on me, as the researcher.

Each student worked through seven tasks – six from the text and one a conversation reflecting upon the work completed. In this, I observed and identified the actions taken by the students, and summarized those, above. The detailed exploration of this activity was presented in Chapters 5 and 6, where I spent time looking more closely at the decisions made by students as they worked through the text.

Doing so led to a pair of findings. First, the student, throughout their work, approached the problems from a wide variety of directions. Most notably Task 1 and Task 6 offered several different solution routes, as explored in Chapter 5. In doing so, these students utilized the reading strategies they deemed necessary in ways that helped them move from reading to understanding to solution. Eggy, for example, paused multiple times in the first 90 second of her Task 1 explanation. Chapter 6 explored how these four pauses seemed to operate in three different ways. She stopped once to decide whether the next section of text should be read; she paused to refresh her understanding of the problem; and she paused while offering her explanation, clarifying her answer as she went along.

Second, not only do students utilize similar strategies as they worked, as described above, but they regularly group some strategies together. In Chapter 5, I outlined the effort of four students as they combined three strategies, together, in search of a final solution. Each of the four *Paused to reflect*, *Self-checked* their understanding of the text, and *Modified a plan or prediction* based on the information gained during their self-assessment. In doing so, each moved closer to the intended solution and settled on the kitchen faucet as the faster way to fill the 3-gallon bucket.

One conversation that came up, regularly, involved the reading of text versus the skimming of text versus just looking at something. This was a component of the project that sufficed, unexpectedly, and one I was unprepared to deeply pursue – a task for another day. My current understanding, after speaking with these students, is “just looking at” is like “skimming,” and is not “reading.” I am excited to pursue this idea further in future work.

Socio-Cultural Setting

When considering the three components of reading outlined above, the RRSG was careful to remind educators that each component “occurs within a larger sociocultural context that shapes and is shaped by the reader and that interacts with each of the three elements” (Snow, 2002, p. 11). It is difficult to generalize the socio-cultural context surrounding the 22 interviews conducted for this study. The premise of each interview was the same, as was the text utilized and the questions posed as students were working. The two schools were quite different, as were the approaches to their reading, and the solution paths followed by each of the students.

The setting of both schools was outlined in Chapter 3. Generally speaking, Site 1 was a typical public middle school housing sixth, seventh, and eighth grade students. Site 2 was a public charter school serving students from third grade through eighth grade. In both sites, the seventh-grade students who participated in this study, and their classmates, sat with their mathematics teacher for a single period each day before moving on to other courses. Students in both schools were expected to take the State’s annual standardized assessment.

The socio-cultural setting for each of the interviews conducted for this study did not match that of these students’ school experience. The interviews were conducted one-on-one, in an empty classroom or conference room, and were conducted by an individual unfamiliar to each student. Through the interview, I attempted to build rapport with each participant, make the process as simple and pleasant as possible. Even with this effort, the students were recorded, asked to read and work through an unfamiliar textbook passage, and do so in a setting unlike their day-to-day academic experience.

When viewing the students and their work, however, having gone through a similar experience, there are a handful of things that stand out. First, middle school students are not afraid to share their opinions, or criticize the task at hand. I found the participants of this study to be quite open and willing to answer questions about their thought processes. Additionally, when they were confused or put off by something in the text, they willingly offered that critique.

Secondly, these middle school students were using reading strategies as they engaged with the text. As stated earlier, all 22 students used at least 8 reading strategies in their work, and 23 of the 25 reading strategies chosen for this study were used by at least one student, at least one time. While both instructors mentioned an interest in mathematical reading throughout our various conversations, neither regularly taught reading strategies to their students, but the strategies were evident within the collection of interviews. The students were learning the strategies elsewhere, and bringing them to mathematics.

Finally, as the students were using a variety of strategies in pursuit of understanding and successful solutions to the problems posed, they grouped several strategies together, regularly. there were pairs of strategies that the students utilized together. The most common pairings were the *pause to reflect* and *self-check*, and *planning a solution or making a prediction* and *modify the plan or prediction*. While both pairings were used by multiple students, it was most common for a participant to pause to check their work or locate an error then pause, reflecting on where their understanding went awry.

Implications and Limitations

Through this project, I interviewed 22 middle school students. A previous and related project adds ten more students to that count, for a total of 32 middle school students ranging from sixth-grade to eighth. From this work, I can state that students do read mathematical texts, and when they engage with said text, they employ a wide range of reading strategies. These strategies vary greatly from task to task, depending on the style of text presented and the connections made by the reader to other parts of the text.

Curriculum Development

From a curriculum development perspective, we must recognize that students do attempt to read the text provided, and they do so using reading strategies learned elsewhere. With this knowledge, mathematics curricula should seek out reading strategies that are consistently and successfully employed by students reading mathematical texts. Once identified, these practices can be presented to younger students as they begin to utilize their mathematics textbooks.

When considering the writing of the mathematical text, the instructional purpose must be considered. Morgan found the continuity between Task 2 and Task 3 helpful for her work, allowing the explanation of the first to guide her solution for the second. The structure provided by the author operated as a guide for Morgan. Conversely, there were wide ranges of solutions demonstrated on both Task 1 and Task 6, neither of which had such an explicitly designed solution path. The more structure provided in a problem, the narrower the solution set across a group of students.

Additionally, those creating the mathematical content used in classes should consider what they know about how students read mathematics, and bring those components into their curriculum. For example, Henry suggested that images were very helpful in building his mathematical understanding. He recognized that there was sufficient information in Task 6 to solve the problem, but questioned why the textbook did not at least include a picture of a faucet along with the printed text. When adopting textbooks, curriculum specialists should closely consider the presentation of content, not just the validity of the content.

For Teachers

No matter the quality of the text, or the variety of representations provided, mathematical reading will take practice and guidance. When considering the overlap of mathematics and reading, the ten students from my earlier study suggested that reading was found in mathematics. When I asked the students what reading looked like, those students provided examples like directions, definitions, and word problems (Beaudine, 2019a).

The students participating in this study viewed reading similarly. They primarily saw reading as written sentences and phrases. Some students did highlight their effort to read the diagrams and equations. When pressed to elaborate on that process they tended to back away from “reading” and move more toward “skimming” or “looking at it.”

Research suggests that each of these forms of text is read (Adams, 2003; Hillman, 2014), and we, as teachers, have a responsibility to communicate this to our students. Reading, particularly in mathematics, is more than just interpreting the written word. Reading involves the analysis and interpretation of all the printed text provided. Students should receive instruction that provides a model for what is and is not helpful within the text, and ways to interpret and synthesize all forms of textual representations.

With this, it would be helpful for teachers to know common reading strategies that effectively aid their students in their effort to understand their text. Through this process, teachers should adopt strategies they see within the classroom that are effective, introduce new strategies known to be effective for students, and model each strategy clearly and consistently. If we take this project as an example, a class may not utilize all 25 strategies presented, but a core set of strategies will surface, and improving students’ efforts to implement these core strategies could help as the mathematics and their texts get more complex.

For Teacher Educators

The responsibility to prepare the teacher of tomorrow is one of great importance. The challenge, particularly for secondary teacher education, is to work across disciplines to prepare teachers to utilize the skills and strategies taught in another's classroom within their own lessons. We see mathematical principles used regularly in science classrooms. Similarly, to do history well requires a great deal of reading and writing, content traditionally lumped in with English-language arts courses. The challenge for teacher educators is to remind preservice teachers that the learning of one discipline does not end at the door. The classrooms have walls around them, the disciplines should not.

For mathematics education, specifically, an effort to openly embrace the fact that mathematics is read would benefit both the preservice teachers, but their future students. A great deal of attention is placed on students' mathematical writing and providing space in class so students can have discussions about mathematics, but little effort, comparatively, is put toward the instruction of mathematical reading. Teacher preparation programs should explore this component of mathematical literacy, and prepare their preservice teachers to teach these skills to their students.

The good news is, students are already using reading skills learned in other classrooms to access their mathematical content. The findings of this study demonstrate that 23 of the 25 focal reading strategies were used by at least one student, and the 22 students gravitated toward a collection of strategies. This suggests that new strategies may not always need to be taught, but rather that teachers should be able to highlight and model these oft used strategies for their students. This ability should be cultivated throughout one's preservice teacher preparation.

Finally, it must be recognized that these preservice teachers will be making curricular decisions related to mathematics. They will choose or design units. They will highlight what is important for their students. They will model how one might approach the discipline. As teacher educators, we have a responsibility to open our thought process up to future teachers. We should model our approach to curriculum design and selection, weighing factors deemed important, embracing influence from other disciplines, and deliberately choosing what is and is not included in our lessons. Our decisions affect these preservice teachers as much as their decisions will affect their own students. We would be wise to prepare them to make those decisions, and our modeling, our encouragement of meta-conversations surrounding the action of teaching or reading mathematics, will help guide their future teaching and curricular efforts.

Limitations

There is not, to my knowledge, one set of reading strategies that all students studying mathematics can employ to be successful. This is because there is not a single type of student. Each student will find a different collection of strategies advantageous, and that could shift from problem to problem. I can say with certainty that this group of students gravitated toward a set of strategies (Table 7-1), and largely ignored others. This process might be one of elimination, as opposed to identification, as we could slowly pair down the list of reading strategies to those most often employed by students. A group of 22 participants, however, is insufficient to do so.

This project, also, cannot make claims related to any trends related to the type of school, or the demographics of the students. The project was not designed to do so. The goals of this project related to identification of reading strategies commonly used by middle school students. I sought out two different locations to collect as much data as I could. I did not seek to differentiate the two sites from one another beyond a trivial description of their setting. Likewise, I did not ask students to identify their own demographic information, therefore any assessment about race, gender, or socio-economic status is inappropriate. This study was not designed for that pursuit.

While this study provided some much-needed information, and has provided ideas for my own instruction, it serves as a starting point from which to build. The suggestions presented above seem logical, and will guide my instruction and curriculum analysis, moving forward, but there is a great deal of information still unknown relating to students' process for reading. I do believe that, as we learn more about how students read mathematics, some critical work into the differences in academic setting, gender, race, special education identification, and socio-economic status is needed. More research, along these lines, is needed.

Future Research

This line of research is relatively new, with much of the mathematical reading literature dating back at most 20 years. Previous studies have outlined the difficulty students may have reading mathematical texts, identified multiple representations used in mathematical writing, addressed both the verbal discussions and mathematical writing found within the classroom, and explored the way expert mathematicians approach their reading. This study sought to better understand the approach of middle school students while identifying the reading strategies used by these students.

As this body of research progresses, it would be beneficial to take a more critical look at the processes used by students as they read their mathematics. This would help identify possible differences in instruction across school settings. Through studies focused on differences in gender, race, socio-economic status, or special education identification, as they relate to both instruction and achievement, would allow the field to problematize the role of reading in the mathematics classroom and the instruction taking place. This effort would almost certainly advance the research surrounding mathematical reading.

The critical studies proposed above would almost certainly lead to an exploration of mathematics teachers' perspectives of reading within their classrooms. These beliefs would undoubtedly affect the content presented in the classroom. For example, a teacher who finds the textbook to be a useful tool for instruction might spend time encouraging students to engage with the text, directly. Alternatively, a teacher who is primarily focused on the amount of content they must work through in any given year may find little time to help their students develop their mathematical reading skills. Teacher perspectives about mathematical reading is an area ripe for exploration.

One final avenue for future research involves a study into the different representations found across mathematical reading. This study utilizes a single section from one textbook. There are certainly other textbooks that represent the same content in a different manner. Some textbook authors might choose a more narrative and wordy approach to the discipline. Others might opt for a more symbolic and succinct representation. Students will likely encounter both in their academic careers, and should be equally prepared. Additionally, should one style better cater to the needs of students, the textbook industry might opt to shift in that more student-friendly direction.

In Closing

This project has shifted my perspective of reading in mathematics. When teaching in middle school, I found myself to be amongst the teachers who felt content-area literacy practices, in mathematics spaces, were unnecessary, and took time away from the mathematics being discussed in class. I have slowly come to believe that reading practices would be useful for students engaging in mathematics, particularly if they were to miss a class and had only the textbook to "catch up." As this project wraps, I am convinced that mathematical reading is not simply useful, but a critical component to teaching mathematics, and the explicit instruction mathematical reading practices is as necessary as teaching students the roles of addition or multiplication. Students, with no regular mathematical reading instruction are successfully utilizing reading strategies found outside of the mathematics classroom. They are grouping the strategies together, and allowing this reading process to guide their mathematical understanding. The more we understand about these practices, the better we can prepare students for all components of mathematical literacy – mathematical discussion, mathematical writing, and mathematical reading.

APPENDICES

APPENDIX A

Codebook

“a *reading strategy* – a deliberate, conscious, metacognitive act” (Afflerbach et al., 2008, p. 368)

The passages read by the study participants consisted of six components – Explain it, Example 1, Try it! (the first), Example 2, Example 3, and Try it! (the second). The coding done should consider each of the segments, individually. In this light, a participant who displays zero reading strategies across all segments read would be credited with “Absence of Strategy” six times. Similarly, a participant who reads each segment out loud would be credited with “Reading Aloud” six separate times.

Unlisted strategies will be added, as needed, and defined at that time.

- Strategy 01 – Use no strategy
  - Looks like: Participant does not utilize a reading strategy of any kind

- Strategy 02 – Preview text
  - Noting text characteristics, important parts, gathering information about what it might be about, determining what to read, determining order, determining areas to be read in detail, determining what to ignore. “These activities are carried out in anticipation of a more careful reading of the text” (Pressley and Afflerbach, 1995, p. 33)
  - Looks like: Participant takes a moment to consider the text prior to the first read of each section
  - Examples
    - Flip through pages
    - Read bold or highlighted text
    - Look at images
    - Asks clarification questions about text or process before beginning

- Strategy 03 – Apply prior knowledge
  - “There are many recommendations in the literature … that readers should activate prior knowledge related to a new text before attempting to read it. Similarly, prior knowledge is generally seen as enhancing the interaction of reader and text” (Pressley & Afflerbach, 1995, p. 33)
  - Looks like: Participant links content read to something already known
    - May occur prior to reading
    - May occur during reading
  - Examples
    - Process knowledge: Participants may bring knowledge of Unit Rate from their 6th grade classroom

        - Situational knowledge: Participants may suggest they know of playing basketball, paychecks, feeding animals, and rodents may all be found useful for any individual participant

- Strategy 04 – Read aloud
    - "Another tactic for dealing with difficulty is to read aloud. Although the effects of this tactic are not clear, such reading does force more conscious attention and slower processing of text than typically occurs during silent reading" (Pressley & Afflerbach, 1995, p. 37)
    - Looks like: participant chooses to read passage out loud
    - Examples:
        - Participant reading to themselves, then begins to read aloud
        - Participant reads out loud without prompting

- Strategy 05 – Plan a solution or predict the result
    - "Generating an initial hypothesis about what the text is about, one that can be revised or refined" (Pressley and Afflerbach, 1995, p. 33)
    - Looks like: Participant offers a hypothesis as to what might be coming later in the text or how they might find a solution
    - Examples
        - Participant offers initial thoughts about what text might be about
        - Participants describe what they might have to do to find a solution
        - Participant offers idea of final result before that result has been established

- Strategy 06 – Modify a plan or prediction
    - "The earlier a reader is in a reading, the less definite the reader is about whether the current hypothesis is the meaning of the text … as the reading proceeds, there is a continuous evaluation of whether the provisional macrostructure currently in place is consistent with information being encountered in the text" (Pressley and Afflerbach, 1995, p. 39)
    - Looks like: Participant changes the way they view the text, recognizes new information changes understanding of what is being asked or approach to solution
    - Examples
        - Updates understanding or approach to the problem
        - Reader may initially believe they are comparing paychecks of the lifeguards. As they read, they see the quest is for pay rate, not total pay
        - Agree or disagree with Janie's assertion, only to change it later in the work

- Strategy 07 – Make notes while reading
    - "Highlighting, underlining, circling, making notes, outlining, or somehow flagging important points in text, including important examples (Pressley & Afflerbach, 1995, p. 44)
    - Looks like: Participant makes notes <u>as</u> they read
    - Examples
        - On the text being read

        - On a separate page or device
        - NOTE: These notes are separate from the work requested by the text book

- Strategy 08 – Paraphrase text
    - "Repeating/restating text just read to hold in working memory … repeating/restating a thought that occurred during reading" (Pressley and Afflerbach, 1995, p. 35)
    - Looks like: Participant revoices some component of the text, may relate effort to help with remembering
    - Examples
        - Reader may restate text verbatim or offer paraphrased version of text just read
        - Participant may recall a thought had during the reading process
        - Reader may offer reason for revoicing

- Strategy 09 – Read text closely
    - Closely attending to each word
        - "By close reading, the mathematicians meant a reading that thoughtfully weighed the implications of nearly every word" (Shanahan, Shanahan, & Misischia, 2011, p. 420).
        - "The" and "a" holding very different definitions for more experienced readers (Shanahan & Shanahan, 2008)
    - Spending time to understand calculations
        - Individuals with less experience spend more time attempting to understand the calculations being presented in the text (Shepherd & van de Sande, 2014).
    - Decipher an image, diagram, graph or table
    - Looks like: Participant slows pace, attends carefully to each image, word, symbol, or step in a passage
    - Examples
        - Participant states they are spending time understanding specific words, number sentences, symbols, or diagrams
        - Participant slows down reading cadence or re-reads a section more slowly the second time
        - Participant may take reading passage in "chunks"

- Strategy 10 - Read entire passage
    - "He points out, in addition, the importance of certain whole expressions (locutions as he calls them), ones whose meanings cannot necessarily be understood merely by knowing the meanings of the individual words; that is, the expressions function as semantic units on their own" (Pimm, 1987, p. 86)
        - Examples
            - From Mathematics
                - If and only if

        - Square root
    - An idiom found in common language
  - Looks like: Participant suggests that words mean something different as a whole than of considered each individually
  - Examples
    - "Brian agrees to watch his neighbor's dogs for 7 days" (enVisionMathematics 2.0, 2018, p. 87). Implies that Brian is tending to the creatures, not literally watching said dogs for 7 straight days

- Strategy 11 – Read symbols as words
  - "The first difference is that the more mathematically advanced readers are more likely to skim over portions of the material that they are familiar with and often read-the-meaning, instead of reading the symbols verbatim." (Shepherd & van de Sande, 2014, p. 85)
  - Looks like: Participant reads symbolic phrase as opposed to the individual symbols themselves
  - Examples
    - 3+5 as "the sum of three and five" as opposed to "three plus five"
    - $7.25 as "seven dollars and 25 cents" as opposed to "seven point two five dollars"

- Strategy 12 – Decode the text
  - "Encountering symbols in mathematical exposition can therefore present a stumbling block to readers who are not familiar with them or not experienced in rapidly decoding them or assigning them meaning" (Shepherd & van de Sande, 2014, p. 77).
    - Describe students in their work asking about how to pronounce particular words
    - Did not have any mathematical symbols that needed extra care in their study
  - Looks like: Participant uses decoding knowledge to decipher unknown text passages
  - Examples
    - Recognizing letter grouping to help decipher word
    - Allow context of word usage to help define word

- Strategy 13 – Attends to prose and equations equally
  - "The mathematicians did not make a distinction among the text features, in this case between mathematical equations and prose … Both prose and equations were referred to as sentences or concepts by the mathematicians" (Shanahan, et. al., 2011, p. 418)
  - Looks like: Participant makes no distinction between importance of prose passages and the symbolic ones
  - Examples

        - Participant spends time reading both prose and equations, as opposed to attending to primarily one or the other
        - Participant attends to images, diagrams, and tables, in addition to the equations and prose

- Strategy 14 – Seeks clarification or external assistance
    - "if the reader is unable to adequately comprehend some part of a passage alone or seeks additional information on the topic, there are resources that can be used to search outside the passage" (Shepherd & van de Sande, 2014, p. 83)
    - Looks like: Participant seeks or explores text outside of the passage being read in order to improve understanding
    - Example
        - View more examples
        - Consults outside sources (e.g., interviewer, calculator, text)

- Strategy 15 – Pause to reflect
    - "these mathematicians [were] much more engaged in identifying error and internal inconsistency" (Shanahan, Shanahan, & Misischia, 2011, p. 422)
    - Looks like: Participant pauses while reading the text
    - Examples
        - Reflection may be indicative of reader taking a moment to process or think about the text, reflection before proceeding
        - May be a participant strategizing how they would approach a problem

- Strategy 16 – Reread the text
    - "After going through the text once, there can either be mental review, for example, through recitation or self-testing, or physical review through rereading. Rereading can result in information not noted as important previously being attended to more carefully" (Pressley and Afflerbach, 1995 p. 59)
    - Looks like: Reader progresses in at least 3 ways
        - Linearly/non-selectively
        - To specific passage to find information
        - Skimming for keyword/concept
    - Examples
        - Reader returns to text and begins reading from the beginning
        - Participant returns to selective part of text to read, or bounces around text
        - Reader skims for keywords or concepts before settling into re-reading

- Strategy 17 – Seeks important information
    - "text implicit [questions], in which the information to respond to the questions is located in the text, but requires integration across sentences, paragraphs, or pages" (Raphael & McKinney, 1983, p. 68)
    - Looks like: Participant looks to identify specific points within the text to aid with understanding

  - Examples
    - Participant returns to a specific passage (or collection of passages) because they believe those to hold the important information
    - Participant discusses an effort to find specific information and discard items deemed unnecessary

- Strategy 18 – Selective reading
  - Readers decide "to read only particular sections and which particular sections … to read particular sections before reading others … [or] quit the reading because the content in the reading is not relevant to the current reading goals" (Pressley & Afflerbach, 1995, p. 32-3).
  - Looks like: Any effort to omit one section of text
  - Examples
    - Participant omits at least one portion of the passage being read

- Strategy 19 – Skims the text
  - "Skimming passages of familiar information were often skimmed and quickly summarized by the mathematicians. One mathematician skimmed an entire (textbook) page of the initial material about vector spaces and metric spaces, giving an outline summary as he went" (Shepherd & van de Sande, 2014, p. 78)
  - Looks like: Participant quickly progresses through portions of the passage, with limited engagement. May be sign of information already understood
  - Examples
    - Participant progresses through one section more swiftly than another
    - Reader may indicate information already known or understood

- Strategy 20 – Uses text clues
  - "The mathematicians, in contrast, were more focused on using text structure to help determine what the problems and solutions were and were less aimed at author awareness or rhetorical interpretation" (Shanahan et. al., 2011, p. 417)
  - Looks like: Participant makes note of page lay-out or structure of text during initial reading or solution
  - Examples
    - The selected text has call-out boxes, diagrams, tables. Reader may point at these as important or helpful to their understanding
    - Participant knew what do to next based on the page lay-out
    - In this case, participant may use layout of Try It (the first) because it matches Example 1

- Strategy 21 – Create analogy or metaphor
  - "At a deeper level, metaphor is involved in the way mathematicians discuss their objects of interest and discovery; for instance, by talking of expressions *vanishing*, functions *obeying* a rule or being *well-behaved*, the *inheritance* of mathematical properties or discovering mathematical *laws*" (Pimm, 1987, p. 95, emphasis not added)

  - Looks like: Participant uses an analogy or metaphor to aid in understanding
    - Extra-mathematic (e.g., graph is a picture, modular mathematics is clock mathematics)
      - "*extra-mathematical metaphors*. These [metaphors] attempt to explain or interpret mathematical ideas and processes in terms of real-world events, and such metaphors can involve everyday objects and processes. (Pimm, 1987, p. 95, emphasis not added)
    - Structural (embedded in notation, more in chapter 8)
      - "*structural metaphors* and address in the next section, involves a metaphoric extension of ideas from within mathematics itself. Such metaphors are frequently also embodied in the notations of written mathematics" (Pimm, 1987, p. 95, emphasis not added)
  - Examples
    - For Extra-mathematical metaphors - "a graph is a picture, modular arithmetic is clock arithmetic or a linear equation is a gear" (Pimm, 1987, p.95)
    - For structural metaphors - the very wide use of the 'x' sign or exponential notation." (Pimm, 1987, p. 95)

Added through coding

- Strategy 22 – Question or critique the text
  - During interviews, it has become evident that students are questioning the wording of the text or critiquing the layout or work of the text.
  - Examples
    - Participant asks question to text looking for answer "How many dogs does he have?"
    - Participant expresses insight on problem "15 hours a day is a lot" or "Dan's a lot harder worker"
    - Critiques writing of the text "that was vague, confusing, doesn't make sense" or "I don't think that really had to be there"

- Strategy 23 – Make connections across the text
  - During the interview, participant discusses connections between one passage and another
  - Examples
    - "They're all the same problems, just different stories."
    - "They're doing the same thing as …"

- Strategy 24 - Self-Check
  - "When explaining how he thought about the ideas in a text, one mathematician said that he asks himself questions. 'Did I see this fact before? Did I see a special instance of this fact before? Do I know if the statement is correct? Can I prove it?'" (Shanahan, Shanahan, & Misischia, 2011, p. 418)

        - Self-check is an explicit effort to gauge understanding, may be followed with an affirmation or critique of work done

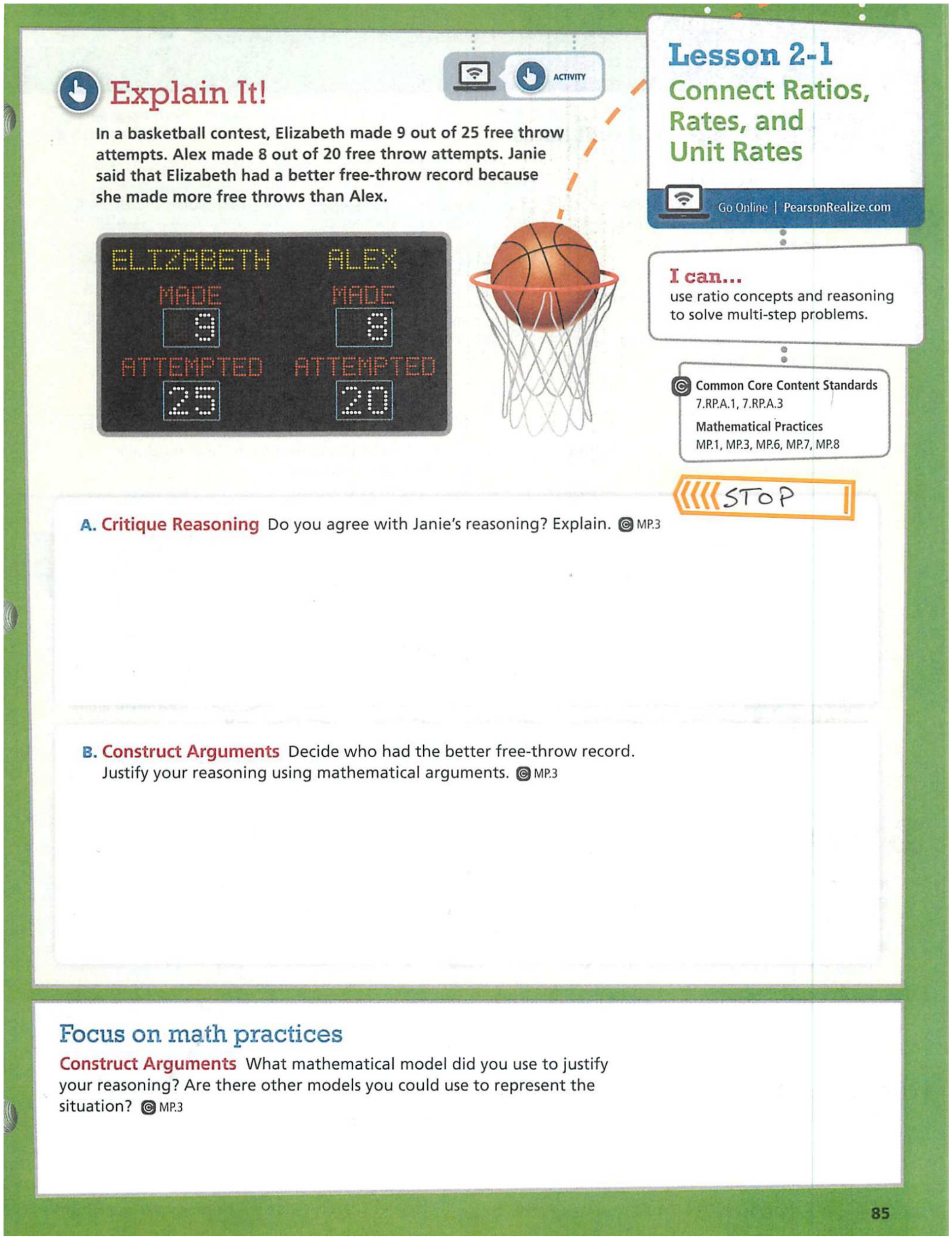
- Strategy 25 - Create a Mental Image
    - Participant suggests an effort to picture something related to the problem at hand.

APPENDIX B

3-page Reading Passage

Figure B-1

Page 1



ACTIVITY

Explain It!

In a basketball contest, Elizabeth made 9 out of 25 free throw attempts. Alex made 8 out of 20 free throw attempts. Janie said that Elizabeth had a better free-throw record because she made more free throws than Alex.

ELIZABETH	ALEX
MADE	MADE
9	8
ATTEMPTED	ATTEMPTED
25	20

Lesson 2-1

Connect Ratios, Rates, and Unit Rates

Go Online | PearsonRealize.com

I can...
use ratio concepts and reasoning to solve multi-step problems.

Common Core Content Standards
7.RP.A.1, 7.RP.A.3

Mathematical Practices
MP.1, MP.3, MP.6, MP.7, MP.8

STOP

A. Critique Reasoning Do you agree with Janie's reasoning? Explain. MP.3

B. Construct Arguments Decide who had the better free-throw record. Justify your reasoning using mathematical arguments. MP.3

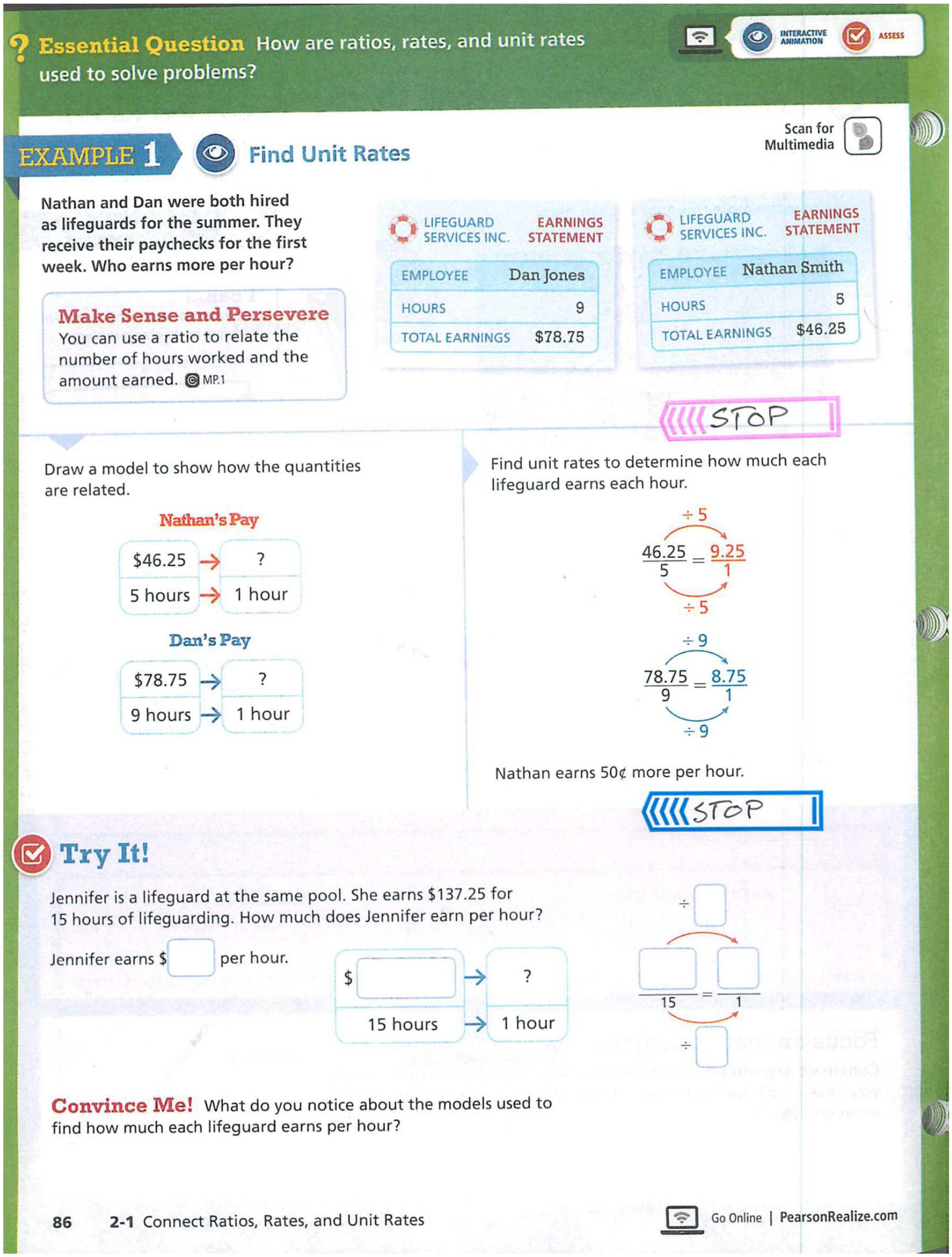
Focus on math practices

Construct Arguments What mathematical model did you use to justify your reasoning? Are there other models you could use to represent the situation? MP.3

85

Figure B-2

Page 2



? Essential Question How are ratios, rates, and unit rates used to solve problems?

INTERACTIVE ANIMATION ASSESS

Scan for Multimedia

EXAMPLE 1 Find Unit Rates

Nathan and Dan were both hired as lifeguards for the summer. They receive their paychecks for the first week. Who earns more per hour?

Make Sense and Persevere You can use a ratio to relate the number of hours worked and the amount earned. MP.1

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Dan Jones
HOURS	9
TOTAL EARNINGS	$78.75

LIFEGUARD SERVICES INC. EARNINGS STATEMENT

EMPLOYEE	Nathan Smith
HOURS	5
TOTAL EARNINGS	$46.25

STOP

Draw a model to show how the quantities are related.

Nathan's Pay

$46.25 → ?
5 hours → 1 hour

Dan's Pay

$78.75 → ?
9 hours → 1 hour

Find unit rates to determine how much each lifeguard earns each hour.

$$\frac{46.25}{5} \underset{\div 5}{\overset{\div 5}{=}} \frac{9.25}{1}$$

$$\frac{78.75}{9} \underset{\div 9}{\overset{\div 9}{=}} \frac{8.75}{1}$$

Nathan earns 50¢ more per hour.

STOP

Try It!

Jennifer is a lifeguard at the same pool. She earns $137.25 for 15 hours of lifeguarding. How much does Jennifer earn per hour?

Jennifer earns $☐ per hour.

$☐ → ?
15 hours → 1 hour

$$\frac{\square}{15} \underset{\div \square}{\overset{\div \square}{=}} \frac{\square}{1}$$

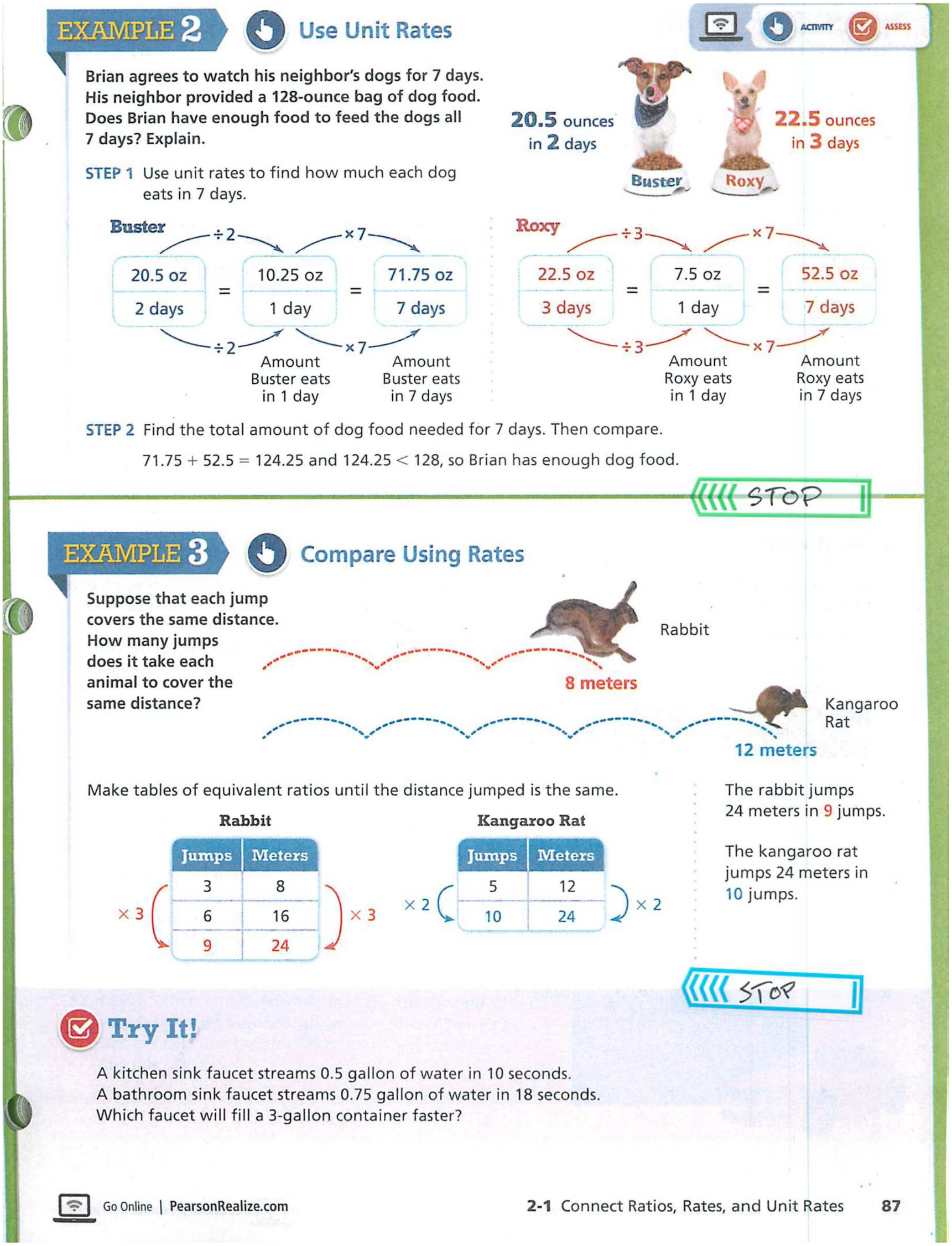
Convince Me! What do you notice about the models used to find how much each lifeguard earns per hour?

86 **2-1** Connect Ratios, Rates, and Unit Rates

Go Online | PearsonRealize.com

Figure B-3

Page 3



EXAMPLE 2 Use Unit Rates

ACTIVITY ASSESS

Brian agrees to watch his neighbor's dogs for 7 days. His neighbor provided a 128-ounce bag of dog food. Does Brian have enough food to feed the dogs all 7 days? Explain.

20.5 ounces in **2** days

Buster

Roxy

22.5 ounces in **3** days

STEP 1 Use unit rates to find how much each dog eats in 7 days.

Buster

$$\frac{20.5 \text{ oz}}{2 \text{ days}} \xrightarrow{\div 2} \frac{10.25 \text{ oz}}{1 \text{ day}} \xrightarrow{\times 7} \frac{71.75 \text{ oz}}{7 \text{ days}}$$

$$\frac{20.5 \text{ oz}}{2 \text{ days}} = \frac{10.25 \text{ oz}}{1 \text{ day}} = \frac{71.75 \text{ oz}}{7 \text{ days}}$$

Amount Buster eats in 1 day

Amount Buster eats in 7 days

Roxy

$$\frac{22.5 \text{ oz}}{3 \text{ days}} \xrightarrow{\div 3} \frac{7.5 \text{ oz}}{1 \text{ day}} \xrightarrow{\times 7} \frac{52.5 \text{ oz}}{7 \text{ days}}$$

$$\frac{22.5 \text{ oz}}{3 \text{ days}} = \frac{7.5 \text{ oz}}{1 \text{ day}} = \frac{52.5 \text{ oz}}{7 \text{ days}}$$

Amount Roxy eats in 1 day

Amount Roxy eats in 7 days

STEP 2 Find the total amount of dog food needed for 7 days. Then compare.

71.75 + 52.5 = 124.25 and 124.25 < 128, so Brian has enough dog food.

STOP

EXAMPLE 3 Compare Using Rates

Suppose that each jump covers the same distance. How many jumps does it take each animal to cover the same distance?

Rabbit

8 meters

Kangaroo Rat

12 meters

Make tables of equivalent ratios until the distance jumped is the same.

Rabbit (× 3)

Jumps	Meters
3	8
6	16
9	24

Kangaroo Rat (× 2)

Jumps	Meters
5	12
10	24

The rabbit jumps 24 meters in 9 jumps.

The kangaroo rat jumps 24 meters in 10 jumps.

STOP

Try It!

A kitchen sink faucet streams 0.5 gallon of water in 10 seconds.
A bathroom sink faucet streams 0.75 gallon of water in 18 seconds.
Which faucet will fill a 3-gallon container faster?

Go Online | PearsonRealize.com

2-1 Connect Ratios, Rates, and Unit Rates 87

APPENDIX C

Example of Coding

Table C-1

Example of Coding

Time mm:ss	Speaker	Statement	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
16:16	Researcher:	Okay. Okay. Um, great. So we're going to look at that. Try it!																									
16:26	Sadie:	Okay. "A kitchen sink faucet streams 0.5 gallons of water in 10 seconds. A bathroom sink faucet streams, zero point 75 gallons of water and 18 seconds, which faucet will fill a three-gallon container faster?" Do you want me to do it?				1										1											
16:44	Researcher:	Sure. Yeah.																									

Table C-1 (cont'd)

Time mm:ss	Speaker	Statement	S01	S02	S03	S04	S05	S06	S07	S08	S09	S10	S11	S12	S13	S14	S15	S16	S17	S18	S19	S20	S21	S22	S23	S24	S25
16:45	PAUSE:	[pause: 02 min, 13 sec]															1										
18:58	Sadie:	Um, the faucet that's running 0.75 gallons of water in 18 seconds will fill up the three-gallon bucket faster.																									
19:10	Researcher:	Okay. Can you talk about how you figured that out?																									
19:14	Sadie:	I found um, 10 and eighteens common denominator, which was 90 and then I just multiplied the numerator by nine. Oh no, I should have done it by five. I did that wrong. [pause] Wait, no. Zero point five should've been done by nine and 18 should have been done by five					1	1									1									1	

Note: Table represents roughly 3 minutes of one student's work presented as an example of coding

Cells S01 to S25 represent the 25 reading strategies explored throughout this study
Text color and cell shading are color-coded to show code tied to student utterance

APPENDIX D

Reading Strategy Usage by Task

Table D-1

Reading Strategy Usage by Task

	Total	Count	Task 1	Count	Task 2	Count	Task 3	Count	Task 4	Count	Task 5	Count	Task 6	Count
S01	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S02	18	1	10	1	1	1	5	1	1	1	1	1	0	0
S03	18	1	5	1	2	1	1	1	1	1	2	1	1	1
S04	104	1	18	1	18	1	17	1	18	1	17	1	15	1
S05	97	1	19	1	21	1	13	1	11	1	9	1	16	1
S06	43	1	12	1	5	1	2	1	3	1	5	1	13	1
S07	7	1	2	1	0	0	0	0	1	1	0	0	1	1
S08	86	1	20	1	13	1	13	1	12	1	13	1	12	1
S09	14	1	0	0	1	1	1	1	1	1	7	1	0	0
S10	11	1	3	1	3	1	1	1	2	1	0	0	0	0
S11	26	1	0	0	13	1	9	1	1	1	1	1	2	1
S12	2	1	1	1	0	0	0	0	0	0	1	1	0	0
S13	17	1	0	0	6	1	0	0	8	1	3	1	0	0
S14	36	1	11	1	7	1	5	1	3	1	2	1	4	1
S15	89	1	17	1	10	1	17	1	9	1	10	1	16	1
S16	10	1	2	1	1	1	0	0	0	0	0	0	0	0
S17	34	1	6	1	5	1	3	1	3	1	0	0	2	1
S18	38	1	8	1	6	1	1	1	3	1	1	1	0	0

Table D-1 (cont'd)

	Total	Count	Task 1	Count	Task 2	Count	Task 3	Count	Task 4	Count	Task 5	Count	Task 6	Count
S19	7	1	0	0	0	0	1	1	0	0	0	0	0	0
S20	31	1	2	1	4	1	3	1	3	1	3	1	1	1
S21	0	0	0	0	0	0	0	0	0	0	0	0	0	0
S22	53	1	7	1	10	1	4	1	7	1	8	1	3	1
S23	29	1	2	1	2	1	10	1	3	1	2	1	1	1
S24	77	1	13	1	13	1	12	1	10	1	7	1	14	1
S25	4	1	0	0	0	0	0	0	1	1	1	1	0	0
n	22		22		22		22		22		22		21	
sum	851	23	158	18	141	20	118	18	101	20	93	18	101	14
m	38.68		7.18		6.41		5.36		4.59		4.23		4.81	

Note: S01 to S25 represent the 25 strategies outlined for this study
Green cells within the S01 to S25 rows represent the most used strategies for given task
Red cells within the S01 to S25 rows represent the least used strategies for given task
"n" represents the number of participants completing that task
"sum" represents the number of Tasks within which a strategy was utilized by the 22 students
"m" the average number of strategies used within the task

APPENDIX E

Unique Strategies Used Per Participant

Table E-1

Unique Strategies Used Per Participant

	S 01	S 02	S 03	S 04	S 05	S 06	S 07	S 08	S 09	S 10	S 11	S 12	S 13	S 14	S 15	S 16	S 17	S 18	S 19	S 20	S 21	S 22	S 23	S 24	S 25	Total
Tim		1		1	1	1		1		1	1			1	1		1	1		1		1		1		14
Jace			1	1	1			1	1	1	1				1		1	1		1		1	1	1		14
Fred		1	1	1	1	1		1	1		1		1	1	1	1	1	1	1	1		1	1	1		19
Joe		1		1	1	1	1	1			1		1	1	1	1	1	1	1	1		1		1		17
Bob				1	1	1		1	1		1		1	1	1	1	1	1		1		1	1	1		16
Alex		1		1	1	1		1		1	1	1	1	1	1		1	1				1		1		15
Figglestein		1		1	1	1		1						1	1	1	1	1				1		1		12
Charlotte			1	1	1	1	1	1	1		1			1	1		1	1	1	1		1	1	1		17
Jeff					1			1							1		1	1	1				1	1		8
Sadie		1	1	1	1	1		1		1	1		1	1	1			1		1		1	1	1		16
Morgan		1		1	1	1		1	1		1		1		1	1	1	1		1		1	1	1	1	17
Dave		1	1	1	1	1		1	1	1	1		1	1	1		1	1		1		1	1	1		18
Hannah			1	1	1	1	1	1			1		1	1	1		1	1		1		1	1	1		16
Eggy		1		1	1	1		1	1	1	1		1		1		1	1				1	1	1		15

Table E-1 (cont'd)

Students	S 01	S 02	S 03	S 04	S 05	S 06	S 07	S 08	S 09	S 10	S 11	S 12	S 13	S 14	S 15	S 16	S 17	S 18	S 19	S 20	S 21	S 22	S 23	S 24	S 25	Total
Carolynn				1	1	1		1		1					1			1				1		1		9
Maya				1	1	1		1					1	1	1		1	1		1			1	1		12
Johnny				1	1	1	1	1			1			1	1		1	1	1	1		1	1	1	1	16
Henrick				1	1	1	1	1	1		1			1	1	1	1	1				1		1		14
Geoff		1	1		1	1		1	1					1	1	1	1	1		1			1	1		14
Gracie			1		1	1	1	1				1				1	1	1	1	1		1	1			13
Henry		1	1	1	1	1		1					1		1		1	1		1		1		1		13
Rachel		1			1	1		1			1				1			1	1	1		1		1		11
Totals	0	12	9	18	22	20	6	22	9	7	15	2	11	14	21	8	19	22	7	16	0	19	14	21	2	14.36

Note: Cells S01 to S25 represent the 25 reading strategies explored throughout this study
The row total has a maximum of 25 (for the 25 strategies explored)
The column total has a maximum of 22 (for the total number of participants)
The bottom right cell represents the total number of strategies used at least once (316) divided by the 22 participants
"1" means student was observed using strategy at least one time
" " means student was not observed using strategy

APPENDIX F

Total Reading Strategies Implemented, by Student and Strategy

Table F-1

Total Reading Strategies Implemented, by Student and Strategy

	S 01	S 02	S 03	S 04	S 05	S 06	S 07	S 08	S 09	S 10	S 11	S 12	S 13	S 14	S 15	S 16	S 17	S 18	S 19	S 20	S 21	S 22	S 23	S 24	S 25	Total
Tim		1		5	4	1		5		1	1			1	4		1	1		1		4		7		37
Jace			1	6	5			4	2	1	2				3		2	2		3		2	1	3		37
Fred		2	3	6	4	2		3	2		2		1	3	5	1	2	1	1	1		2	1	3		45
Joe		2		6	6	3	1	3			2		3	2	3	1	1	1	1	1		1		1		38
Bob				6	2	1		4	1		1		1	3	5	1	1	1		1		1	1	2		32
Alex		1		6	4	1		3		1	2	1	2	4	5		1	2				1		4		38
Figglestein		1		5	6	1		3						5	6	2	1	4				3		6		43
Charlotte			1	5	4	1	2	2	1		1			3	6		1	1	1	1		2	4	4		40
Jeff					3			2							7		1	1	1				1	1		17
Sadie		1	1	6	2	1		5		1	1		1	3	6			2		2		1	2	2		37
Morgan		1		5	6	1		4	1		1		1		5	2	2	1		1		2	1	4	1	39
Dave		4	3	6	4	2		5	1	1	1		2	4	6		3	3		1		5	3	5		59
Hannah			1	6	4	2	1	4			4		1	1	4		2	3		3		3	3	3		45
Eggy		1		6	4	2		4	3	4	3		1		5		1	2				6	2	6		50

Table F-1 (cont'd)

	S 01	S 02	S 03	S 04	S 05	S 06	S 07	S 08	S 09	S 10	S 11	S 12	S 13	S 14	S 15	S 16	S 17	S 18	S 19	S 20	S 21	S 22	S 23	S 24	S 25	Total
Carolynn				5	3	2		3		2					4			2				2		3		26
Maya				7	5	5		7					1	3	3		3	1		2			1	6		44
Johnny				6	7	2	1	4			2			2	3		1	3	1	2		2	2	5	3	46
Henrick				6	2	2	1	3	2		2			1	5	1	2	1				3		2		33
Geoff		2	2		6	4		5	1					1	2	1	4	1		2			3	3		37
Gracie			4		4	3	1	6				1				1	3	1	1	2		5	4			36
Henry		1	2	6	5	3		1					3		1		2	2		7		7		3		43
Rachel		1			7	4		6			1				1			2	1	1		1		4		29
Totals	0	18	18	104	97	43	7	86	14	11	26	2	17	36	89	10	34	38	7	31	0	53	29	77	4	38.68

Note: Cells S01 to S25 represent the 25 reading strategies explored throughout this study

Number within cell represents total number of tasks in which strategy was observed (max = 7)

Blank cells show participant did not utilize that strategy in any of the seven tasks

The row total has a maximum of 175 (25 strategies times max of 7 tasks)

The column total has a maximum of 154 (22 participants times max of 7 tasks)

The bottom right cell represents mean strategies used per interview

APPENDIX G

Page 1 Interview Transcript – Eggy

<Begin Explain It!>

00:00 Researcher: So I'd like you to go ahead and read until that stop point.
00:04 Eggy: Explain it! In a basketball contest, Elizabeth made nine out of 25 free throw attempts. Alex made eight out of 20 free throw attempts. Janie said that Elizabeth had a better free throw record because she made more free throws than Alex. Lesson one: connect ratios, rates and unit rates. Go online, Pearsonrealize dot com. I can use ratios, ratio concepts and reasoning to solve multi step problems. [pause] Um, common core standards, blah, blah, blah. Umm--
00:37 Researcher: Uh, what are you thinking about?
00:39 Eggy: Um, do I agree with Janie's reasoning? [pause] Janie said that Elizabeth had a better free throw record because she made more free throws and Alex. It could be, [pause] it could be that Elizabeth was lucky to this time. You'd need more to make like an actual ratio. It depends really, because they were really close and it could be like, [pause] I mean Alex, for one, um, Elizabeth attempted 25 times, made nine of them, and then Alex attempted 20 times, made eight of them. So if we were putting, putting those in fractions, then nine over 25 and eight over 20, well not even, not good ratios to compare to. So-
01:32 Researcher: sure.
01:33 Eggy: You'd have to try- You'd have to, not transform, you'd have to convert those.
01:38 Researcher: Okay. Okay. Um, are you thinking anything else about those problems or about that? This text up here.
01:48 Eggy: Um no.
01:48 Researcher: Okay. So what I'd like to do is look at questions A and B and I'd like to hear you think aloud through both of those.
01:56 Eggy: Oh, okay.
01:56 Researcher: You're welcome to write on it as well. I just, I'd like you to verbalize what you're doing.
02:01 Eggy: Oh yes. Okay. Um, so do I just answer these questions? Okay. Uh, a do I agree with Janie's reasoning? Explain.
02:11 Eggy: So Janie says that Elizabeth had done better free throw record because she made more free throws in Alex. I disagree because one-- you need more data to see if it's actually, like, if it's actually consistent and if it's there consistency. And another thing is that Elizabeth made nine, uh, made nine shots out of 25 attempted ones. Alex made eight shots out of 20 attempted ones. So therefore, if we were to convert the fractions, um, eight over 20 can be simplified to, uh, two over five.
03:00 Researcher: That's interesting.
03:11 Eggy: Um, two out of five, so eight over 20 equals two over five. Um, now five times five is 25, so we'd multiply both the numerator and the denominator by five to get two times five, 10, five times five, 20. I mean 5 times five, 25. Whoops. Um, and if that were the case,

then assuming that she would get, she's very consistent. Um, Elizabeth got more than, um, Alex. Sorry, Alex got more than Elizabeth.

03:49 Researcher: Okay. So answering this question, you would say what,

03:55 Eggy: um, Janie's- Janie's statement is incorrect. Um, because if Elizabeth were to stay-- if Alex were to stay consistent and you converted the-- if Alex were to stay consistent and Alex took 25 attempted shots, Alex would make 10 sho- Um, 10, 10 free throws or 10 free throw attempts

04:27 Researcher: Okay.

04:28 Eggy: Yeah.

04:28 Researcher: Cool.

04:29 Eggy: Um, and the construct arguments decide who had a better free throw record. Just file your reasoning using mathematical arguments. Oh, well, I accidentally did that ahead of time.

04:39 Researcher: Not a problem.

04:41 Eggy: Um, so with mathematical decide who had a better free throw record, if I was to go back, look looking back at my math, I saw that if they were to be equal fractions and assuming that they both were consistent. And when I think fractions, I mean the numerator is how many free throw attempt shots were made and denominator, how many were attempted if they were to stay, um, consistent.

05:09 Researcher: Sure.

05:09 Eggy: Then nine over-, then Alex would made more and Elizabeth, um, Alex would've made one more than Elizabeth.

05:19 Researcher: Sure. Excellent. Okay. Um, we're going to flip to the next page and I'd like you to read the first part up to the pink stop sign.

<End Explain It!>

APPENDIX H

Page 2 Interview Transcript – Morgan

<Begin Example 1>

06:40 Morgan: Okay. “Nathan and Dan were both hired as lifeguards for the summer. They received their paychecks for the first week. Who earns more per hour?” So “make sense in persevere. You can use a ratio to relate the number of hours worked and the amount earned.” So they've got services inc. So, um, his total earnings and-- okay. Yeah
07:12 Researcher: Okay. So what are you thinking about?
07:16 Morgan: I'm thinking about, like, dividing and trying to, like, divide them even they again, like, but just as mainly when I'm thinking about.
07:27 Researcher: sure, sure [pause] is division [inaudible] what would you divide? What do you think?
07:34 Morgan: Maybe three for Dan Jones. And I was thinking, cause it's kinda hard for that one, I was thinking, like, five maybe.
07:45 Researcher: Cool. So, um, okay, so this is the textbook and in an example they usually explain the whole thing. Um, so we read the first part, let's see how they solved it.
07:58 Morgan: So draw model to show how the quantities are related. So then and Nathen's pay. So they, they did per hour, so they divided it per hour. So they just divided it per hour basically.
08:21 Researcher: Okay. So what are you thinking- oh, you see -
08:28 Morgan: Yeah, so like, just dividing that by five and finding that in my nine to make it [pause] that ratio.
08:42 Researcher: Sure.
08:43 Morgan: That's basically what I think and I'm going through in my mind.
08:49 Researcher: Okay. Okay. How similar is this to what you wanted to do with the basketball?
08:56 Morgan: At first I was thinking about doing this, but then I got like a different way be- when I re read it like the first time we went through it, I thought this way. But yeah.
09:12 Researcher: All right. Great. Is there anything else that you're thinking about in that middle section between the pink and blue stop signs?
09:20 Morgan: I was just thinking about like the way they arranged it and like, instead of like flip-flopped
09:26 Researcher: okay.
09:28 Morgan: Yeah.

<End Example 1, Begin Try It! (the first)>

09:30 Researcher: Okay. Um, so we're going to look at the, try it now.
09:33 Morgan: Okay.
09:34 Researcher: And I'd like you to work through that.

09:37 Morgan: Okay. So Jennifer has a lifeguard at the same pool. She earns 137 and tw- dollars and 25 cents for 15 hours of lifeguarding. How much does Jennifer earn per hour? So it'd be, for this one, it'd be 137 dollars and 25 cents. You'd have to find that by doing 137 point 25 divide that by 15

10:14 PAUSE: [pause: 02 min, 10 sec]

12:23 Morgan: so [pause] not sure. [pause]

12:34 Researcher: so can you remind me what the question is asking?

12:36 Morgan: So it's asking how much does Jennifer earn per hour if she earns 137 dollars and 25 cents for 15 hours of lifeguarding.

12:49 Researcher: Okay. And can you walk me through your solution?

12:52 Morgan: So what I did is, cause I know 15 divided by 15 is one so I'd have to divide one 37 point 25 by 15 and I got nine Oh nine I'm point 483 cents basically said it's 9 dollars eighty- and 483 cents I got if you did dollars. But-- so, my answer, then, would be 900.483, okay. With the line above it.

13:35 Researcher: Okay. What's the line represent?

13:37 Morgan: Like it repeats.

13:38 Researcher: Okay, cool. Uh, is there anything else that you're thinking about?

13:48 Morgan: I mean, I'm kinda like questioning it. Like having it repeat, it's like I'm just questioning that it's all like, I'm thinking about.

13:58 Researcher: Okay. Why, why does that feel uncomfortable?

14:01 Morgan: I mean like with like dollars and stuff. Like it doesn't, it feels like it isn't like repeat, like, over and over and over again.

14:11 Researcher: Sure. Yeah, that makes sense. Okay. Is there anything else you were thinking about?

14:20 Morgan: Not really.

14:21 Researcher: Okay, great. Again, we'll revisit all of these things. Anything that you want to look at at the end, we'll revisit those.

14:27 Morgan: Okay

Try It! (the first)

<End Try it! (the first)>

APPENDIX I

Page 3 Interview Transcript – Henry

<Begin Example 2>

9:40:00 Researcher: Awesome. So we're going to flip the page and um, I'd like you to look at example two.
9:47:00 Henry: Okay.
9:47:00 Researcher: [pause: 00 min, 10 sec].
9:57:00 Henry: Brian agrees to watch his neighbor's dog dogs for seven days. His neighbors provided 128-ounce bag of their dog food. Does Brian have enough food to feed all seven days? And it's so it says there's two dogs. Well actually it does not say how many dogs there are. There are.
10:17:00 Researcher: Okay.
10:18:00 Henry: We can figure this out by right here though where it says the dogs and it has good information like when it says 20 point five ounces in two days, which can help us determine how much one dog we eat. Um, and then the other one is 22 point five ounces in three days, which can also tell us how much the dog eats and you can find- like right here. And that's also very good. You can, it goes down to one day. the other side too. So it goes down to one day and you can see 10 point two five ounces and 7 point five ounces per day, and you can add those up together to get 17 point seven five ounces per day and multiply that by seven, which you would get, let's say you'd get 123 point two five.
11:14:00 Researcher: okay.
11:20:00 Henry: 123 point two five and he has 128 just which means he has enough, Oh, 124 point five point two five but shows that he has enough.
11:30:00 Researcher: Okay.
11:32:00 Henry: So I think that's a good, that's a good question. And it's pretty, it's pretty clear because the questions as he, he agrees to do is, and does he have enough food and there's enough information here to w- if we don't have all this, I think there need to be a little bit more information out there. Like there's two dogs and how much they eat. But if you're still gonna have the pictures, then it's good.
11:54:00 Researcher: Okay. Okay. Do you have any other thoughts about example two?
11:58:00 Henry: example two.
12:02:00 Researcher: That's the dog problem.
12:03:00 Henry: Yeah. Yeah. It's, it's good. I think it's a good question. And it, it shows that they have enough, how much they have and how much he needs. So it's, it's good.

<End Example 2, Begin Example 3>

12:15:00 Researcher: Okay. Uh, do you want to read example three?
12:18:00 Henry: Yup. All right. So suppose that each jump covers the same distance. How much, how many jumps does it take each animal to cover the same distance? All right. And then it has tables down here and it says, and it shows where they will intersect kind of. And

so it's a rabbit makes three jumps every eight meters. And the kangaroo rat ju- ha- makes five jumps every 12 meters. And you multiply, you can multiply those by like equivalent ratios, which will be three and two. So kangaroo rat jump, like if you multiply by two, you can see, well, you have to find their common denominators. And so 12 times two equals 24 and eight times three equals eight times three equals 24. So that's a common denominator. You can see how much, how many jumps it takes them to reach the same. Um, what do you say? Distance. And so for the kangaroo rat, it takes ten jumps to get to 24 meters and it takes nine jumps for the rabbit to get to 24, which is good. So shows, and also right here, this is very good too. It shows the, it shows like the distance per hop, which is just with this be divided by three with this be divided by five, which is good. It's a good thing. And if you wanted to go more, go more, you could see how much they jump. um, how- the distance for every jump they take. But that's not what they're asking. So I think this is a good question and it's, it's, it's a good, it has a lot of information that, that shows,

14:00:00 Researcher: sure.

14:00:00 Henry: That shows that it's good.

14:02:00 Researcher: Okay. Any, do you have any other thoughts?

14:05:00 Henry: Um, no, I actually really liked the way it's, um, laid out. Like I was talking about the, like the jumps. That's a very good, and I think that it's, I think that's a good, it's a good, it's has great layout and it's a great, um problem,

<End Example 3, Begin Try It! (the second>

14:22:00 Researcher: awesome. Um, how about the Try It! at the bottom? Yeah.

14:26:00 Henry: A kitchen sink faucet streams, zero point th- five gallons of water in ten seconds. A bathroom sink faucet streams zero point seven five gallons in 18 seconds. Which faucet will feel a three gallon container faster? Alright, so this one you don't really need all the diagrams and stuff because it does have enough information in the problem.

14:51:00 Researcher: Okay.

14:52:00 Henry: So that's very good. You can, it's, you can tell how much you just have to, you know, multiply this by, so it's all like, so to get this to three, you can divide it well, you can divide three gallons by zero point five, whidh'd you six. And then you multiply six by 10 which would get you 60.

15:18:00 Researcher: Okay.

15:18:00 Henry: And then for this one it's zero point seven five which you can, you divide that by three gallons, which will get you four.

15:27:00 Researcher: Okay.

15:27:00 Henry: And then you multiply 18 seconds by four to get you 72 seconds. So you can, so the kitchen sink faucet will fill gallons container, like, fill the three gallon container faster, which I like. I like how I like how it's set up because it has enough information.

15:48:00 Researcher: Sure.

15:49:00 Henry: It's not like going all out. You can, you can tell, you can tell what they're, you can tell what they're saying. It's good.

15:57:00 Researcher: Cool. Awesome. Any other, sorry. No, that's fine. Any other thoughts?

16:03:00 Henry: Um, well I personally like diagrams and stuff, but this was like maybe the information, this information is really, like, needed, but maybe you can put like a picture of each faucet, like saying that it drips 18 per second. That's not really needed cause you ha- you already, like, have it in there. But me, I like seeing sti- things, like, visually.

<End Try It! (the second)>

_

REFERENCES

REFERENCES

Acuna, K. (2019, November 5). *'The Incredibles' actually takes place decades ago — Here's the moment that proves it*. Insider. https://www.insider.com/what-year-does-the-incredibles-take-place-2018-2

Adams, A. E., Pegg, J., & Case, M. (2015). Anticipation guides: Reading for mathematics understanding. *Mathematics Teacher, 108*(7), 498-504.

Adams, T. L. (2003). Reading mathematics: More than words can say. *The Reading Teacher, 56*(8), 786-795.

Afflerbach, P., Pearson, P. D., & Paris, S. G. (2008). Clarifying differences between reading skills and reading strategies. *The Reading Teacher, 61*(5), 364-373.

Armstrong, A., Ming, K., & Helf, S. (2018). Content area literacy in the mathematics classroom. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 91*(2), 85-95. doi:10.1080/00098655.2017.1411131

Associated Press. (1992, October 21). *Company news: Mattel says it erred; Teen Talk Barbie turns silent on math*. The New York Times. https://www.nytimes.com/1992/10/21/business/company-news-mattel-says-it-erred-teen-talk-barbie-turns-silent-on-math.html

Axelrod, J. & Moret, M. (2020, March 29). *Schools scramble to provide meals to hungry students amid COVID-19 pandemic*. Pittsburgh Post-Gazette. https://www.post-gazette.com/news/education/2020/03/29/school-lunches-pittsburgh-grab-and-go-meals-student-COVID-19-pandemic-Wilkinsburg-PPS-food-bank/stories/202003190072

Beaudine, G. (2018). From mathematical reading to mathematical literacy. *Mathematics Teaching in the Middle School, 23*(6), 318-323.

Beaudine, G. (2019a). Reading and math go side-by-side: Students explore the complexities between two core disciplines. In S. Otten, A. G. Candela, Z. de Araujo, C. Haines, & C. Munter, (Eds.). *Proceedings of the forty-first annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 1541-1545). St Louis, MO: University of Missouri.

Beaudine, G. (2019b). *"Those have decimals ... I'll do this one": Student approaches to mathematical reading*. [Unpublished manuscript]. Department of Teacher Education, Michigan State University, East Lansing, MI.

Beaudine, G., Osibodu, O., & Beavers, A. (2017). Disney's metaphorical exploration of racism and stereotypes: A review of Zootopia. *Comparative Education Review, 61*(1), 227-234. doi:10.1086/690061

Bergeson, K. T., & Rosheim, K. (2018). Literacy, Equity, and the Employment of iPads in the Classroom: A Comparison of Secure and Developing Readers. *International Journal of Education in Mathematics, Science and Technology, 6*(2), 173-181.

Bird, B. (Director). (2004). Incredibles [Film]. Pixar Animation Studios, Walt Disney Pictures.

Bird, B. (Director). (2018). Incredibles 2 [Film]. Pixar Animation Studios, Walt Disney Pictures.

Brozo, W. G. & Crain, S. (2018). Writing in math: A disciplinary literacy approach. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 91*(1), 7-13. doi:10.1080/00098655.2017.1342435

Brozo, W. G., Moorman, G., Meyer, C., & Stewart, T. (2013). Content area reading and disciplinary literacy: A case for the radical center. *Journal of Adolescent & Adult Literacy, 56*(5), 353-357.

Bumiller, E. (2006, February 3). *Bush urges study of math and science*. The New York Times. https://www.nytimes.com/2006/02/04/politics/bush-urges-study-of-math-and-science.html

Cameron, C. E., Grimm, K. J., Steele, J. S., Castro-Schilo, L., & Grissmer, D. W. (2015). Nonlinear Gompertz curve models of achievement gaps in mathematics and reading. *Journal of Educational Psychology, 107*(3), 789.

Carr, P. G. (2016a). *NCES statement on TIMSS 2015*. National Center for Education Statistics. https://nces.ed.gov/whatsnew/commissioner/remarks2016/11_29_2016.asp

Carr, P. G. (2016b). *NCES statement on PISA 2015 results*. National Center for Education Statistics. https://nces.ed.gov/whatsnew/commissioner/remarks2016/12_06_2016.asp

Carter, T. A., & Dean, E. O. (2006). Mathematics intervention for grades 5-11: Teaching mathematics, reading, or both? *Reading Psychology, 27*(2-3), 127-146. doi:10.1080/02702710600640248

Chen, C. & Chiu, C. (2016). Collaboration scripts for enhancing metacognitive self-regulation and mathematics literacy. *International Journal of Science and Mathematics Education, 14*(2), 263-280. doi:10.1007/s10763-015-9681-y

Colbert, S. (2006, February 7). *Math Is Hard*. The Colbert Report. http://www.cc.com/video-clips/nzdokc/the-colbert-report-math-is-hard

Common Core State Standards Initiative. (2020). Common core state standards initiative: Preparing America’s students for success. corestandards.org.

Croce, K., & K. McCormick, M. (2020). Developing disciplinary literacy in mathematics: Learning from professionals who use mathematics in their jobs. *Journal of Adolescent & Adult Literacy, 63*(4), 415-423. doi:10.1002/jaal.1013

DeVos, B. [@BetsyDeVosEd]. (2020, April 2). *Great to see this solution-oriented innovation happening in South Carolina!* [Tweet]. Twitter.com. https://twitter.com/BetsyDeVosED/status/1245805309744033800

Dietiker, L. (2013). Mathematical texts as narrative: Rethinking curriculum. *For the Learning of Mathematics, 33*(3), 14-19.

Dixon, F. A., Yssel, N., McConnell, J. M., & Hardin, T. (2014). Differentiated instruction, professional development, and teacher efficacy. *Journal for the Education of the Gifted, 37*(2), 111-127. doi:10.1177/0162353214529042

Doerr, H. M., & Temple, C. (2016). It’s a different kind of reading: Two middle-grade teachers’ evolving perspectives on reading in mathematics. *Journal of Literacy Research, 48*(1), 5-38.

Dostal, H. M., & Robinson, R. (2018). Doing mathematics with purpose: Mathematical text types. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 91*(1), 21-28. doi:10.1080/00098655.2017.1357409

Firdaus, F. M., Wahyudin, & Herman, T. (2017). Improving Primary Students' Mathematical Literacy through Problem Based Learning and Direct Instruction. *Educational Research and Reviews, 12*(4), 212-219.

Florida Legislature. (2016). The 2016 Florida statutes (1008.25). Tallahassee, FL. http://www.leg.state.fl.us/Statutes/index.cfm?App_mode=Display_Statute&URL=1000-1099/1008/Sections/1008.25.html

Forseman, S. (2017). *Envisionmath 2.0, student edition, grade 7, volume 1*. Pearson Publishing.

Fuchs, L. S., Fuchs, D., Hosp, M. K., & Jenkins, J. R. (2001). Oral reading fluency as an indicator of reading competence: A theoretical, empirical, and historical analysis. *Scientific Studies of Reading, 5*(3), 239-256. doi:10.1207/S1532799XSSR0503_3

Fuentes, P. (1998). Reading comprehension in mathematics. *The Clearing House: A Journal of Educational Strategies, Issues and Ideas, 72*(2), 81-88. doi:10.1080/00098659809599602

Gewertz, C. (2017, April 28). *PARCC and Smarter Balanced choose new management*. Education Week. https://blogs.edweek.org/edweek/high_school_and_beyond/2017/04/parcc_smarter_balanced_choose_new_management.html

Gibbons, P. (2009). *English learners, academic literacy, and thinking: Learning in the challenge zone*. Portsmouth, NH: Heinemann.

Ginsburg, H. (1997). *Entering the child's mind: The clinical interview in psychological research and practice*. Cambridge University Press.

Goldberg, D. (2020, March 17). *'It's going to disappear': Trump's changing tone on coronavirus*. Politico.com. https://www.politico.com/news/2020/03/17/how-trump-shifted-his-tone-on-coronavirus-134246

Goldstein, D., Popescu, A., & Hannah-Jones, N. (2020, April 6). *As school moves online, many students stay logged out*. The New York Times. https://www.nytimes.com/2020/04/06/us/coronavirus-schools-attendance-absent.html

Griffo, V. B., Madda, C. L., Pearson, P. D., & Raphael, T. E. (2015). Current issues and best practices in literacy instruction. In L. M. Morrow & L. B. Gambrell (Eds), *Best practices in literacy instruction (5th ed.)* (pp. 37-60). New York: Guilford Press.

Gunn, J. (2018, September 18). *For teachers: The lifelong impact of illiteracy.* Room 241 (Blog). https://education.cu-portland.edu/blog/classroom-resources/illiteracy-impacts/

Harkness, S. S., & Brass, A. (2017). How preservice teachers make meaning of mathematics methods texts. *International Journal for the Scholarship of Teaching and Learning, 11*(2) doi:10.20429/ijsotl.2017.110217

Herbel-Eisenmann, B., Johnson, K. R., Otten, S., Cirillo, M., & Steele, M. D. (2015). Mapping talk about the mathematics register in a secondary mathematics teacher study group. *Journal of Mathematical Behavior, 40*, 29-42.

Herbel-Eisenmann, B., & Wagner, D. (2007). A framework for uncovering the way a textbook may position the mathematics learner. *For the Learning of Mathematics, 27*(2), 8-14.

Hilden, K. & Pressley, M. (2011). Verbal protocols of reading. In N. K. Duke & M. H. Mallette (Eds.), *Literacy research methodologies, second edition* (pp. 427-440). Guilford.

Hildreth, G. H. (1947). *Learning the three R's (2nd ed.)*. Minneapolis: Educational Publishers.

Hillman, A. M. (2014). A literature review on disciplinary literacy. *Journal of Adolescent & Adult Literacy, 57*(5), 397-406.

Kirby, S. N. (2003). *Developing an R&D Program to Improve Reading Comprehension*. RAND Corporation. https://www.rand.org/pubs/research_briefs/RB8024.html

Knudson, K. (2015, September 9). *The Common Core is today's New Math – which is actually a good thing*. The Conversation. https://theconversation.com/the-common-core-is-todays-new-math-which-is-actually-a-good-thing-46585

Lent, R. C. (2015). *This is disciplinary literacy: Reading, writing, thinking, and doing... content area by content area*. Corwin Press.

Livingston, M. (2018, March 9). *Barbie may struggle with math. But Mattel has had bigger problems with its line of dolls*. Los Angeles Times. https://www.latimes.com/nation/la-na-barbies-sidebar-20180309-story.html

Martin, C., & Polly, D. (2016). Examining the impact of writing and literacy connections on mathematics learning. *Investigations in Mathematics Learning, 8*(3), 59-74. doi:10.1080/24727466.2016.11790354

Massey, D., & Riley, L. (2013). Reading math textbooks: An algebra teacher's pattern of thinking. *Journal of Adolescent & Adult Literacy, 56*(7), 577-586.

Mattel (2020). Our family of brands. Retrieved from: https://www.mattel.com/en-us/brands-page

Michigan Education Agency. (2017). Learn what's in the new third grade reading law. https://mea.org/learn-whats-in-the-new-third-grade-reading-law/

Moje, E. B. (2008). Foregrounding the disciplines in secondary literacy teaching and learning: A call for change. *Journal of Adolescent & Adult Literacy, 52*(2), 96-107. doi:10.1598/JAAL.52.2.1

Molina, C. (2012). *The problem with math is English: A language-focused approach to helping all students develop a deeper understanding of mathematics*. John Wiley & Sons.

Mongillo, M. B. (2017). Creating mathematicians and scientists: Disciplinary literacy in the early childhood classroom. *Early Child Development and Care, 187*(3-4), 331-341. doi:10.1080/03004430.2016.1236090

Moschkovich, J., & Zahner, W. (2018). Using the academic literacy in mathematics framework to uncover multiple aspects of activity during peer mathematical discussions. *Zdm, 50*(6).

Nagel, D. (2020, May 1). *Updated list of statewide school closures with closure dates 2020*. The Journal: Transforming Education Through Technology. https://thejournal.com/articles/2020/03/17/list-of-states-shutting-down-all-their-schools-grows-to-36.aspx

National Council of Teachers of English. (2005, November 17). Position statements: Multimodal literacies. NCTE.org. https://ncte.org/statement/multimodalliteracies/

National Council of Teachers of English. (2020). *Literacies and languages for all: Whole language beliefs*. NCTE.org. https://ncte.org/groups/lla/beliefs/

National Education Agency. (2019). *Punitive Policies – Third Grade Retention*. NEA.org http://www.nea.org/tools/punitive-policies-third-grade-retention.html

New London Group. (1996). A pedagogy of multiliteracies: Designing social futures. *Harvard Educational Review, 66*(1), 60-92.

Nguyen, T. L. (2020, March 24). *School districts hand out tablets, laptops to students so they can learn at home*. WTSP-TV. https://www.wtsp.com/article/news/health/coronavirus/school-districts-hand-out-tablets-and-laptops-to-students-so-they-can-learn-at-home/67-9af9a29a-1af1-4be3-9aa1-d9e17909d4f9

Norwood, C. (2020, April 27). *With schools shuttered, districts struggle to feed students and communities*. PBS NewsHour. https://www.pbs.org/newshour/nation/with-schools-shuttered-districts-struggle-to-feed-students-and-communities

Organisation for Economic Co-operation and Development. (2013). *PISA 2012 assessment and analytical framework: Mathematics, reading, science, problem solving and financial literacy.* Organisation for Economic Co-operation and Development.

Pearson Publishing. (2003). *College board selects Pearson to provide SAT scoring*. Pearson.com. https://www.pearson.com/corporate/news/media/news-announcements/2003/07/college-board-selects-pearson-to-provide-sat-scoring.html

Pearson Publishing. (2019). *About us*. Pearson.com. Retrieved from: https://www.pearson.com/us/about.html

Peikes, K. (2020, March 16). *Schools providing grab-and-go meals during COVID-19 closures*. Iowa Public Radio. https://www.iowapublicradio.org/post/schools-providing-grab-and-go-meals-during-covid-19-closures#stream/0

Penn State Learning. (2020). *Test taking tips*. PennState Undergraduate Education. https://pennstatelearning.psu.edu/test-taking-tips

Pimm, D. (1987). *Speaking mathematically: Communication in mathematics classrooms.* Routledge & K. Paul in association with Methuen.

Piraro, D. (2020). About Bizarro. https://www.bizarro.com/about

Proliteracy.org. (2020). *U.S. adult literacy facts*. Proliteracy.org. https://proliteracy.org/Portals/0/pdf/PL_AdultLitFacts_US_flyer.pdf?ver=2016-05-06-145137-067

Pressley, M., & Afflerbach, P. (1995). *Verbal protocols of reading: The nature of constructively responsive reading*. Hillsdale, N.J: Lawrence Erlbaum Associates.

Purcell-Gates, V., Duke, N. K., & Martineau, J. A. (2007). Learning to read and write genre-specific text: Roles of authentic experience and explicit teaching. *Reading Research Quarterly, 42*(1), 8-45. doi:10.1598/RRQ.42.1.1

Rauf, D. S. (2020, April 1). *Coronavirus Squeezes Supply of Chromebooks, iPads, and Other Digital Learning Devices*. Education Week. https://www.edweek.org/ew/articles/2020/04/01/coronavirus-squeezes-supply-of-chromebooks-ipads-and.html

Shanahan, C., & Shanahan, T. (2014). Does disciplinary literacy have a place in elementary school?. *The Reading Teacher, 67*(8), 636-639. doi:10.1002/trtr.1257

Shanahan, C., Shanahan, T., & Misischia, C. (2011). Analysis of expert readers in three disciplines: History, mathematics, and chemistry. *Journal of Literacy Research, 43*(4), 393-429.

Shanahan, T., & Shanahan, C. (2008). Teaching disciplinary literacy to adolescents: Rethinking content- area literacy. *Harvard Educational Review, 78*(1), 40-59. doi:10.17763/haer.78.1.v62444321p602101

Shanahan, T., & Shanahan, C. (2012). What is disciplinary literacy and why does it matter? *Topics in Language Disorders, 32*(1), 7-18.

Shanahan, T., & Shanahan, C. R. (2015). Disciplinary literacy comes to middle school. *Voices from the Middle, 22*(3), 10.

Shepherd, M. D., & van de Sande, C. C. (2014). Reading mathematics for understanding—From novice to expert. *The Journal of Mathematical Behavior, 35*, 74-86.

Sheppard, Z. [@zwsheppard] (2020, March 30). *@OconeeSC_school bus Wi-Fi locations getting up and running today.* [Tweet]. Twitter.com https://twitter.com/zwsheppard/status/1244678930382159875

Sigley, R., & Wilkinson, L. C. (2015). Ariel's cycles of problem solving: An adolescent acquires the mathematics register. *Journal of Mathematical Behavior, 40*, 75-87.

Snow, C. (2002). *Reading for Understanding: Toward an R&D Program in Reading Comprehension*. RAND Corporation. https://www.rand.org/pubs/monograph_reports/MR1465.html

Solomon, Y., & O'Neill, J. (1998). Mathematics and narrative. *Language and Education, 12*(3), 210-221. doi:10.1080/09500789808666749

Temple, C., & Doerr, H. M. (2018). How do teachers develop and enact a disciplinary view of literacy in mathematics?. *Journal of Adolescent & Adult Literacy, 61*(5), 483-488. doi:10.1002/jaal.664

Texas Education Agency. (2016). Grade-level retention in Texas public schools, 2014-15 (Document No. GE17 601 05). Austin TX. http://tea.texas.gov/acctres/retention_student_performance_2014-15.pdf

Thomas, J. R. (2020, March 23). *Laptops headed to students in struggling school districts so learning can resume*. The CT Mirror. https://ctmirror.org/2020/03/23/laptops-headed-to-students-in-struggling-school-districts-so-learning-can-resume/

Todd, J. (2020, February 20). *Sadlier's math blog: Math test-taking strategies for elementary students*. Sadlier School. https://www.sadlier.com/school/sadlier-math-blog/math-test-taking-strategies-for-elementary-students-math-multiple-choice-test-taking-strategies

Tomlinson, C. A. & Jarvis, J. (2009). Differentiation: Making curriculum work for all students through responsive planning and instruction. In Renzulli, J. S., Gubbins, E. J., McMillen, K. S., Eckert, R. D., Little, C. A. (Eds.), *Systems and models for developing programs for the gifted and talented* (pp. 599-628). Storrs, CT: Creative Learning Press.

U.S. Department of Education. (n.d.a). No Child Left Behind: Elementary and Secondary Education Act (ESEA). https://www2.ed.gov/nclb/landing.jhtml

U. S. Department of Education. (n.d.b). Every Student Succeeds Act (ESSA). https://www.ed.gov/essa?src=rn

U. S. Department of Education. (2016). Programs: Race to the top funds. https://www2.ed.gov/programs/racetothetop/index.html

Watkins, A. E. (1979). The symbols and grammatical structures of mathematical english and the reading comprehension of college students. *Journal for Research in Mathematics Education, 10*(3), 216-218. doi:10.2307/748810

Wayno. & Piraro, D. (2012, May 3). Bizarro. https://www.comicskingdom.com/bizarro/2012-05-03

Wyatt, D., Pressley, M., El-Dinary, P. B., Stein, S., Evans, P., & Brown, R. (1993). Comprehension strategies, worth and credibility monitoring, and evaluations: Cold and hot cognition when experts read professional articles that are important to them. *Learning and Individual Differences, 5*(1), 49-72.

Zheng, B., Arada, K., Niiya, M., & Warschauer, M. (2014). One-to-one laptops in K-12 classrooms: Voices of students. *Pedagogies: An International Journal, 9*(4), 279-299.