THESIS

MOMENT OF INERTIA OF IRREGULAR SOLIDS

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A THESIS ON A

COMPARISON OF METHODS FOR DETERMINING THE MOMENT OF INERTIA

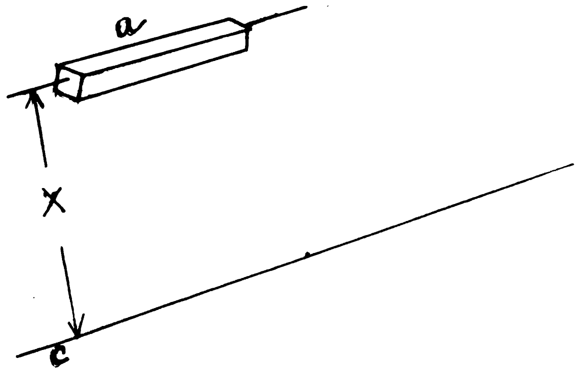
OF IRREGULAR BODIES.

Geo. [illegible]. [illegible] and J.C. [illegible], Experimenters.

AGRICULTURAL COLLEGE:
Aug.4,189[illegible].

The three most used methods have been those of the torsion [illegible] physical pendulum, [illegible] deriving I from the center of oscillation as determined by [illegible] by [illegible].

The proposed method which so far has never been used, to the writer's knowledge, is outlined as follows: The moment of inertia of a solid about any given axis is the summation of the moments of inertia of all the differential volumes composing the solid about their own gravity axis parallel to the axis of reference transferred to this axis of reference. To illustrate, let a be such a differential volume, for simplicity considered a [illegible] parallel piped, [illegible] at x distance from the axis of reference c.



Then the I_g of a about its gravity axis parallel to c is $1/12ma^2$, and its I_c equals $1/12ma^2$ plus mx^2. It will be observed that the quantity I_g is small compared with mx^2, and the essence of the proposed method is that it may be neglected without appreciable error.

In this simplification, it only remains to hit upon some expedient method of determining the masses, or the weights , and the location of these differential volumes with respect to the axis in question.

Method I. The solid used in the comparison of this last method and a graphical method was a connecting rod from the [illegible] Engine Works, a drawing of which is shown in plate 1.

The rod was suspended by a differential pulley and a pointer attached to it indicated readings on the scale erected at [illegible] the dipping tank. The overflow caused by each lowering of the rod was caught in a glass which measured each differential volume in cubic centimeters. All of these points are illustrated in the accompanying photograph.

The results of the experiments are tabulated in table 1. The first column contains the readings indicated on the scale, the readings being taken just as the rod touched the water. Column two contains the thickness of each section in inches. Column three contains the distance of the center of gravity of each section from the gravity axis parallel to the crank pin. The determination of the position of this axis will be hereinafter explained. The gravity axis of each section was considered as midway of its thickness. Column four contains the square of the distance of the center of the section from the axis [illegible]. Column five contains the number of cubic centimeters in each differential volume. Column six contains the number of cubic inches in each differential volume. Column seven contains the number of pounds in each differential volume. An appreciable error enters here on account of the failure to take into account the difference in location between trunk and [illegible] inertia of each differential volume with respect to the axis of reference.

The position of the gravity axis of the rod was determined [illegible], and by a method of weighing each end of the rod, knowing the distance between [illegible], and also by a graphical method of German origin, de[illegible] [illegible] H. [illegible] class '06, of Lehigh University, [illegible] Lehigh [illegible].

The method [illegible] was so susceptible of error that the results could not be relied upon. The method by weights located the axis at [illegible] inches from the extreme wrist-pin end of the rod, and the method from the German located it at 21.33 inches from the same end.

1.	2.	3.	4.	5.	6.	7.	8.
In ft.							
5.000	0	23.44	0	0	0	0	0
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	13.47	4.011	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.400	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	3.001	[illegible]
[illegible]	[illegible]	23.50	[illegible]	[illegible]	21.57	3.000	[illegible]
[illegible]	[illegible]	35.00	690.510	300	13.52	5.433	3440.30
5.068	1.41	37.00	[illegible]	400	[illegible]	3.003	[illegible]
[illegible]	[illegible]	33.133	703.100	300	13.30	5.131	3577.00
[illegible]	1.713	20.050	420.000	430	23.03	7.357	3451.75
[illegible]	1.53	13.82	572.300	[illegible]	23.73	3.001	[illegible]
[illegible]	1.75	[illegible]	[illegible]	310	13.81	3.537	1135.23
4.430	[illegible]	[illegible]	[illegible]	[illegible]	20.74	5.307	1334.70
4.30	3.44	10.30	181.30	310	14.31	4.030	330.14
[illegible]	1.573	10.044	100.50	220	13.42	3.755	[illegible]
3.707	3.334	7.113	55.143	430	21.33	7.171	397.75
3.437	3.343	4.131	17.314	440	23.36	7.515	130.43
3.325	2.04	1.615	2.3030	300	13.30	5.134	11.30
[illegible]	1.33	.432.	.3121	[illegible]	13.31	5.305	1.13
[illegible]	[illegible]	[illegible]	3.3303	450	37.45	7.303	33.32
3.313	[illegible]	[illegible]	[illegible]	330	23.33	3.053	345.72
[illegible]	3.373	9.334	43.03	350	21.35	5.073	533.43
[illegible]	[illegible]	13.433	153.00	340	23.74	5.307	[illegible]
1.33	2.715	15.030	553.330	300	13.30	5.134	1154.45
1.75	1.30	17.333	503.34	100	10.03	3.074	301.03
[illegible]	2.730	[illegible]	[illegible]	[illegible]	17.03	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	471.37	[illegible]	13.30	5.101	3415.30
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	37.03	10.530	[illegible]
[illegible]	1.30	33.053	570.130	430	[illegible]	3.103	5537.45.
[illegible]	2.15	31.053	[illegible]	310	[illegible]	10.413	[illegible]
[illegible]	1.53	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	5103.05
[illegible]	[illegible]	[illegible]	[illegible]	340	14.34	4.033	4103.70
[illegible]	[illegible]	12.462	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
	[illegible]			[illegible]	[illegible]	[illegible]	[illegible]

Total length of rod equals [illegible] inches.

Total volume in cubic inches equals [illegible].

Total weight in pounds (derived by calculation) equals [illegible]

Total weight in pounds (by weighing) equals [illegible].

This value, [illegible], expresses the I_p of the rod in terms of weight in pounds, and linear dimensions in inches.

Method II. This is the Culmann method described by Heck. The method is primarily applicable to irregular plane figures, but may be applied to the connecting rod in the following described manner: The rod was divided into fifty-seven sections, the center lines of which are shown in plate I. which shows the rod drawn to one half scale. The positions of these center lines of section are again shown on the line MO in AB, plate III. The area of each section as found by the planimeter from plate II is laid off to a scale of 1 inch equals 1 square inch, on ordinates erected at its own section center. Column 2 of the table hereinafter to be described shows the area of the respective sections as drawn, i.e. to a half scale. Through the ends of these ordinates the curve MLO is drawn. Now if we consider this curve and base OM to bound a plate of unit thickness and having a volume equal to that of the rod and of equivalent distribution, we can treat this as a plane figure and locate the position of its centroid or the center of gravity and determine the moment of inertia about this line. The construction is as follows: Take any ordinate, as LK, produce to the point N on the reference axis DD', and find the point P where the line joining N and [illegible] meets the ordinate LK. Project P on OO' at S and join [illegible] KS. R is a point on the first and T of the second derived curve; in this manner the points of the curves [illegible] and [illegible] are found.

S_1 equals area of the figure [illegible].

S_2 equals the area of the figure [illegible].

S_3 equals the area of the figure [illegible].

The following relations are true: AG equals S_1 / S_3 X, where AG is the distance from AA' of the center of gravity, and X is the distance AB.

I_a equals $S_3 X^2$.

I_g equals $S_3 X^2 - S_1 \overline{AG}^2$.

Taking [illegible] measurements for the scale of the drawings.

S_1 equals [illegible].48

S_3 equals [illegible]

S_2 equals [illegible].24

X equals [illegible] inches.

Making the substitutions in the above equations, I_g equals [illegible] expressed in volume and inches.

Table II, column 1 contains the number of the section. Column 2 contains the area of the section in square inches. Column 3 contains the thickness of the section in inches. Column 4 contains the volume of the section in cubic inches. Column 5 contains the weight of the section in pounds. Columns 4 and 5 were calculated and entered as a check to the the first method. [illegible] the volume of the rod to be [illegible] cubic inches, and the weight to be 100.4[illegible] pounds.

1.	2.	3.	4.	5.
1	3.13	.13	[illegible]	1.374
2	3.27	.215	3.623	1.013
3	[illegible]	.12	[illegible]	[illegible]
4	4.05	.175	5.370	1.500
5	4.33	.13	[illegible]	1.511
6	[illegible]	[illegible]	[illegible]	[illegible]
7	3.33	.13	[illegible]	.303
8	2.12	.21	[illegible]	[illegible]
9	1.72	[illegible]	3.501	[illegible]
10	1.50	.13	1.533	.427
11	1.56	.11	1.407	[illegible]
12	1.74	.04	3.341	.366
13	3.13	.33	3.740	1.040
14	[illegible]	[illegible]	[illegible]	[illegible]
15	3.34	.04	1.135	.830
16	4.31	.20	6.000	1.061
17	[illegible]	.03	[illegible]	.781
18	3.60	.130	4.105	1.175
19	4.00	.33	3.702	2.450
20	4.22	.74	[illegible]	[illegible]
21	[illegible]	.218	[illegible]	1.763
22	4.01	.48	13.474	3.773
23	3.31	.301	[illegible]	1.004
24	2.71	[illegible]	[illegible]	[illegible]
25	[illegible]	[illegible]	[illegible]	[illegible]
26	[illegible]	.41	[illegible]	[illegible]
27	[illegible]	.21	[illegible]	[illegible]
28	[illegible]	.10	[illegible]	.737
29	[illegible]	.13	[illegible]	[illegible]
30	[illegible]	.10	[illegible]	[illegible]
31	[illegible]	.11	[illegible]	[illegible]
32	[illegible]	.23	3.070	1.104
33	[illegible]	[illegible]	[illegible]	[illegible]
34	[illegible]	[illegible]	3.363	[illegible]
35	1.44	[illegible]	11.63	[illegible]
36	[illegible]	[illegible]	[illegible]	[illegible]
37	[illegible]	[illegible]	[illegible]	[illegible]
38	1.74	[illegible]	[illegible]	[illegible]
39	[illegible]	[illegible]	[illegible]	[illegible]
40	[illegible]	[illegible]	[illegible]	[illegible]
41	[illegible]	[illegible]	16.33	[illegible]
42	[illegible]	[illegible]	[illegible]	[illegible]
43	[illegible]	[illegible]	[illegible]	[illegible]
44	3.10	[illegible]	[illegible]	[illegible]
45	[illegible]	[illegible]	[illegible]	[illegible]

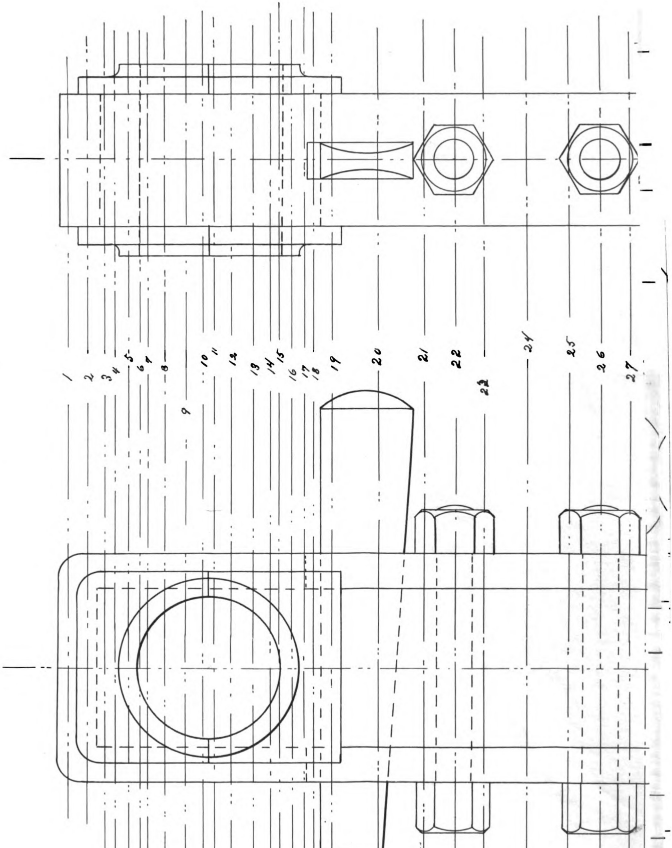
1.	2.	3.	4.	5.
46	3.10	1.0[illegible]	13.0[illegible]	[illegible]
[illegible]	[illegible]	1.0[illegible]	1[illegible].84	4.7[illegible]
[illegible]	1.[illegible]	1.00	1[illegible].3[illegible]	4.8[illegible]
[illegible]	1.7[illegible]	1.[illegible]	1[illegible]	3.[illegible]7
50	1.[illegible]	1.0[illegible]	18.44	3.7[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible].4[illegible]
[illegible]	.1[illegible]	1.00	11.[illegible]	3.3[illegible]
53	1.33	.13	1.7[illegible]	.47[illegible]
5[illegible]	1.4[illegible]	.1[illegible]	[illegible]	[illegible]
55	1.[illegible]	.[illegible]	7.120	[illegible]
56	2.50	.13	2.457	.[illegible]
[illegible]	[illegible]	.1[illegible]	[illegible]	.[illegible]
58	[illegible].[illegible]	.44	13.[illegible]	3.0[illegible]7
[illegible]	4.[illegible]	.2[illegible]	[illegible].4[illegible]	1.214
60	4.40	.42	15.[illegible]	4.[illegible]
[illegible]	4.[illegible]	.[illegible]	[illegible]	1.[illegible]4
[illegible]	[illegible].70	.53	1[illegible]	5.51[illegible]
[illegible]	4.0[illegible]	.21	6.[illegible]1	1.[illegible]
[illegible]	[illegible]	.[illegible]	1[illegible]	4.[illegible]
65	4.[illegible]	.2[illegible]	6.4[illegible]	1.[illegible]
[illegible]	4.27	.5[illegible]	1[illegible].[illegible]13	5.[illegible]40
67	[illegible].47	.17	[illegible]	1.[illegible]
[illegible]	[illegible].11	.[illegible]	13.[illegible]40	3.[illegible]
[illegible]	[illegible]	.1[illegible]	[illegible]	.[illegible]
70	[illegible].31	.[illegible]	[illegible]	.[illegible]7
7[illegible]	5.[illegible]	.2[illegible]	11.[illegible]	[illegible]
[illegible]	[illegible].05	.0[illegible]	[illegible]	.5[illegible]
[illegible]	[illegible].01	.[illegible]	[illegible]	1.[illegible]4[illegible]
[illegible]	[illegible].01	.[illegible]	[illegible].1[illegible]	1.[illegible]
[illegible]	1.70	.13	1.[illegible]	.457
[illegible]	1.0[illegible]	.14	[illegible]	.7[illegible]5
[illegible]	1.[illegible]	.1[illegible]	1.[illegible]4	.4[illegible]
[illegible]	1.70	.1[illegible]	1.[illegible]3	.457
[illegible]	[illegible].01	.[illegible]	[illegible]	1.441
[illegible]	[illegible].34	.[illegible]	[illegible]	1.[illegible]
[illegible]	4.05	.1[illegible]	[illegible].[illegible]40	.[illegible]7
[illegible]	[illegible]	.[illegible]	1[illegible]	[illegible].1[illegible]
[illegible]	[illegible]	.0[illegible]	1.[illegible]	.418
[illegible]	4.[illegible]1	.10	[illegible]	1.[illegible]
[illegible]	[illegible].01	.14	[illegible].1[illegible]	1.[illegible]
[illegible]	[illegible]	.18	[illegible]	1.371
[illegible]	[illegible]	.[illegible]	[illegible].5[illegible]	.7[illegible]

In conclusion we have only to compare the results of the [illegible] methods. [illegible] we find that th[illegible] [illegible] less than [illegible] graphic [illegible] method.

[illegible] more accurate [illegible] by dipping. [illegible] would [illegible] reduced to a minimum. Since, [illegible] of [illegible] would have [illegible] would have approached more [illegible] a [illegible] inspection.

It is [illegible] possible that with more accurate apparatus for [illegible] the [illegible] results would be [illegible] to [illegible] more closely.

The simplicity of the apparatus, ease of manipulation, and [illegible] accuracy of the results [illegible] the practicability of the meth[illegible] [illegible] of the [illegible].



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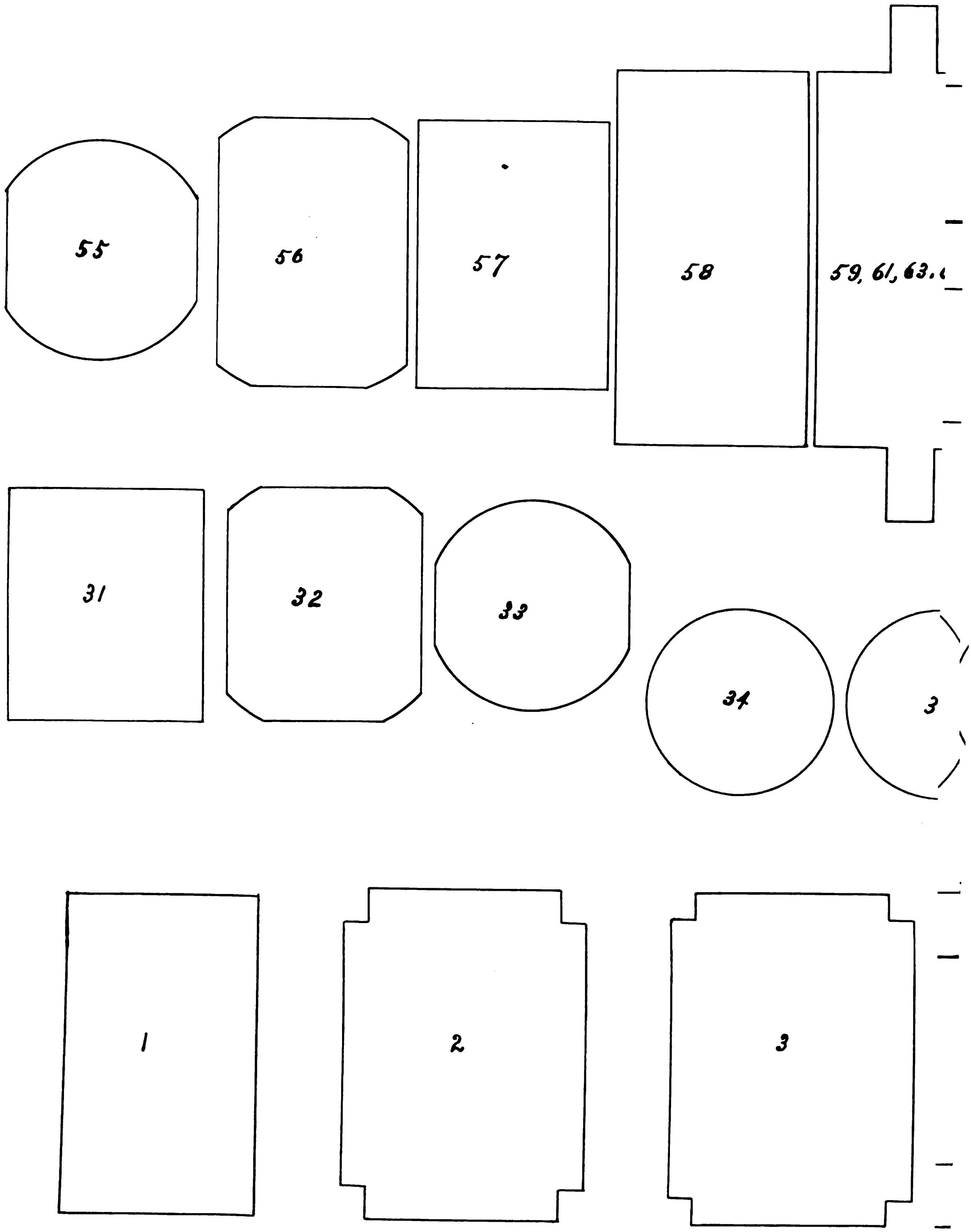
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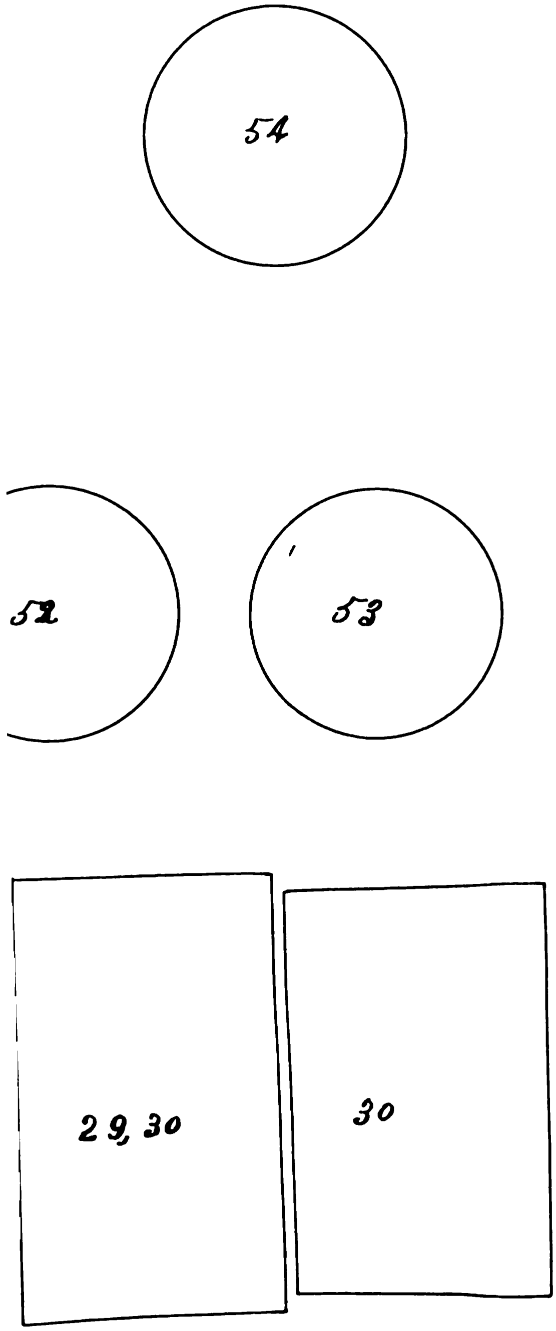
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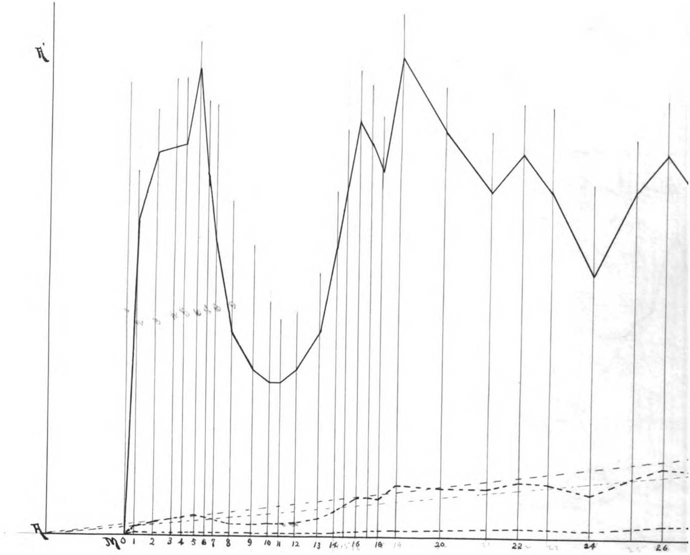
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[redacted]

Simmons

Comparison of methods for determining the moment of inertia of irregular bodies.

Bill Jones

Simmons

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