

THESIS

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THE ANALYSIS OF A CONCRETE ARCH  
OVER THORNAPPLE RIVER NEAR HASTINGS

ALLIE L. HATOVSKY

1922



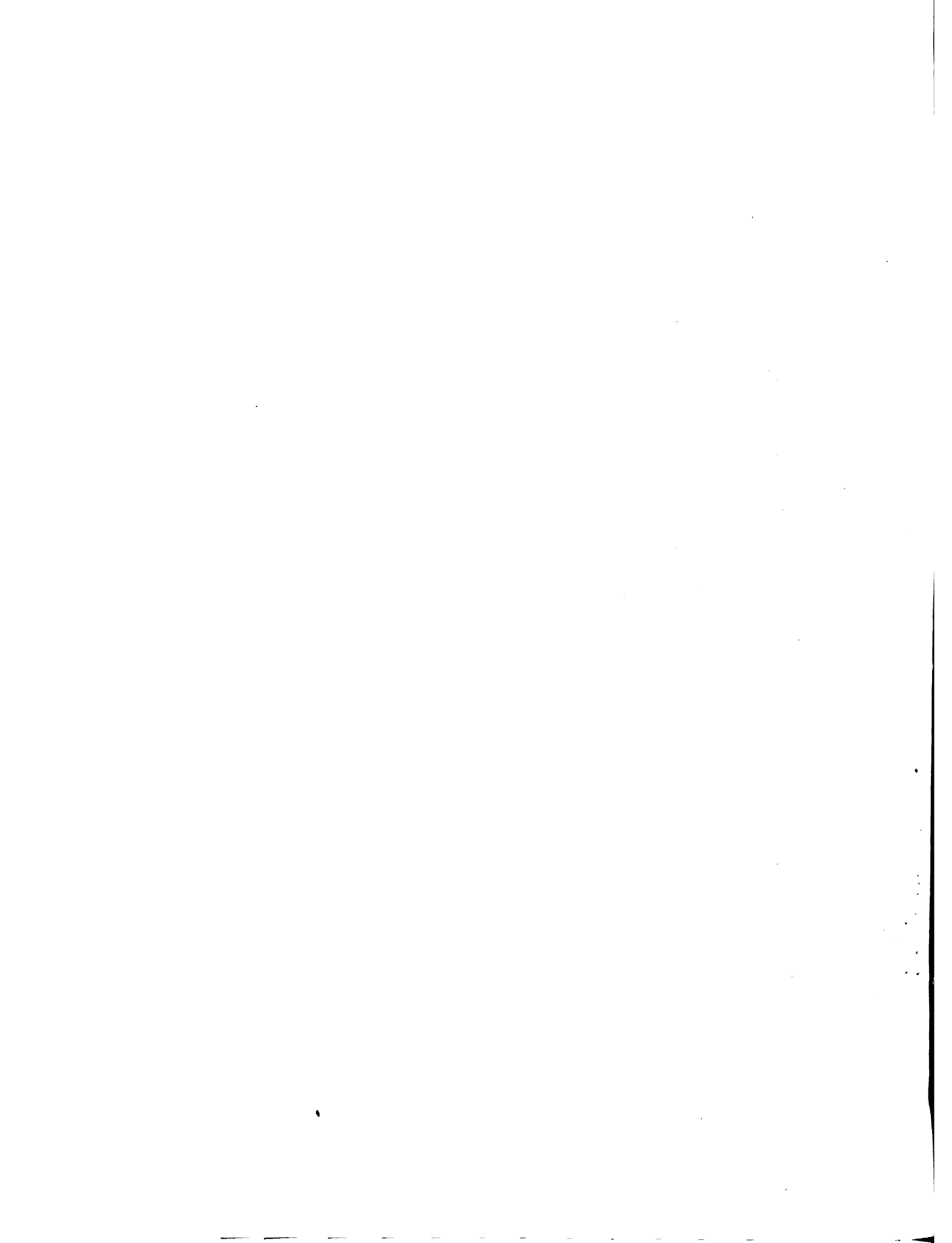
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**THE ANALYSIS OF A CONCRETE ARCH  
OVER THORNAPPLE RIVER NEAR HASTINGS**

**A THESIS**

**Submitted to the Faculty of  
THE MICHIGAN AGRICULTURAL COLLEGE**

**BY**

**ALLIE L. HATOVSKY**

**For the Degree of  
BACHELOR OF SCIENCE**

**JUNE 1922**

THESIS

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## PREFACE

In this volume the author has set forth in detail what he considers to be the most practical method for the analysis of a concrete arch. The primary reason for compiling this volume was to become more fully acquainted with the design and analysis of concrete arches.

The author wishes to acknowledge his indebtedness to Heel and Johnson for the diagrams reproduced from their book "Concrete Engineers' Handbook"; to Mr. C.A.Melick, Bridge Engineer of the State Highway, for his kind assistance in furnishing plans of the structure and helpful suggestions; to Prof. Allen for his valuable instruction and guidance in the various courses in concrete design; and to Prof. Vedder for his aidful criticism and suggestions.

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## DESCRIPTION OF THE STRUCTURE

This bridge is being built by the State Highway Department across Thornapple River just north the city of Hastings. It replaces a steel bowstring bridge which has been condemned as inadequate for the present traffic loads. It is a single span, reinforced concrete bridge designed by the Bridge Department of the State Highway in accordance with their specifications and requirements. The contract for its construction was let to W.G. Crebo, of Grand Rapids, during July 1921. It was to be completed by the end of last year; but, due to the fact that quick sand was encountered when excavating for the abutments, only the abutments and the wings have been poured at the present time.

The superstructure consists of ten panels of 10'-10" over the arch and three panels of 10'-3" over each abutment, making a total length of 172'-4". The roadway is 24' wide, which is quite an increase over that of the old bridge.

The arch consists of two arch rings, each 5'-4" wide, connected by a cross brace of reinforced concrete 22' on each side of the crown. They are 2'-0" thick at the crown and 6'-0" thick at the springing line. They are three centered circular arch rings with a clear span of 100' and a rise of 18'. The springing line elevation is 1.7' above mean water level which is sufficient since the river is only 4' deep.

The superstructure consists of a 10" floor slab with curbs and railings made up of spindles and pilasters as shown by the accompanying blue print. The superstructure



above the arch rests upon reinforced cross beams which are supported on the arch rings by nine pair of spandrel columns.

These columns are interconnected by small arches.

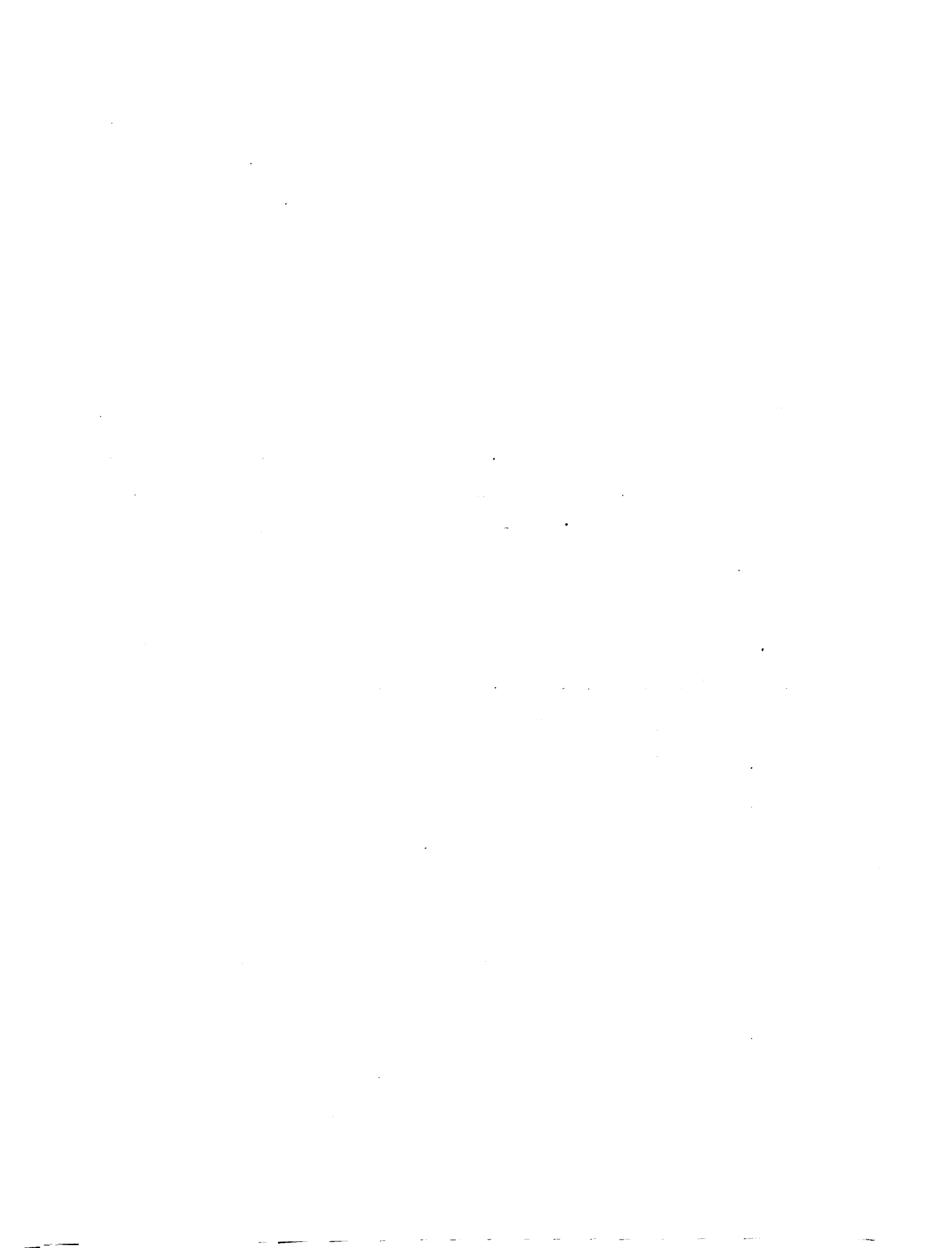
The reinforcing is of steel bars thru-out, laid on the principle of one-way reinforcing with just enough transverse to prevent checking or cracking.

### METHODS OF ANALYSIS

There are a number of both graphical and analytical methods, or a combination of the two, which have been used in the past in arch analysis. The two main theories upon which they have been based are the Line-of-Thrust Theory and the Elastic Theory.

Since the dead load usually controls the shape of the arch ring, an approximate graphical analysis based on the Line-of-Thrust Theory is first made. The object of this analysis is to determine whether the line of thrust falls within the middle third, which indicates that there is no tension in the concrete. Since the elastic properties of concrete is not taken into consideration in this theory, it is useful only for preliminary investigations as to the proper shape of the arch ring.

All of the methods of design and analysis based upon the elastic theory have been derived by making numerous assumptions. Also many uncertain factors enter, some of which are the following: the approximate character of the flexure formulas; the uncertainty of the tensile stresses in concrete; the variation in the live load; the effect of temperature



variations; the effect of the shrinkage of concrete; and the effect of slight movement or distortion of the abutments. Thus conditions justify the use of Cochrane's Formulas and Diagrams in the analysis of a concrete arch ring. These formulas and diagrams were compiled by Mr. Cochrane from thorough investigations of a great number of arch designs found in technical literature. He also constructed a curve giving the ratio of the thickness of the arch ring at any point to that at the crown for various ratios of the thickness of the arch ring at the springing line to its thickness at the crown. He also determined that if an arch ring were designed in accordance to this curve, the maximum stress occurred at the springing line or at the crown. Thus only these two points have to be investigated to determine the safety of the arch ring.

This arch ring was not laid out by Cochrane's curves; but, since it is almost exactly thru-out as specified by them, the use of Cochrane's formulas and diagrams are applicable. Also the stresses at only the springing line and at the crown will have to be investigated.

OUTLINE OF  
APPROXIMATE GRAPHICAL ANALYSIS OF A CONCRETE ARCH

by the

LINE - OF - THRUST THEORY

1. Draw one-half of the arch ring to as large a scale as convenient.
2. Divide the semi-arch ring into five sectional divisions with the dividing sections at the spandrel columns.
3. Locate the centers of gravity of the trapezoidal sections.
  - a. Extend DE until UN = RS
  - b. Extend BC in the opposite direction until PM = TU.
  - c. The intersection of MN and the median OP locates the center of gravity.
4. Compute the weight of each section.
5. Compute the weight of the spandrel columns and dead load supported.
6. Draw in the lines of force.
7. Lay off the load line.
8. Select a convenient pole O on a horizontal thru K and draw in the rays of the force polygon.
9. Construct the corresponding equilibrium polygon starting at the center of the springing line.
10. Extend the first and last rays, ea and ek, until they intersect at W. This locates the center of forces.
11. Draw the resultant of forces vertically thru the point W.
12. Extend a horizontal line from Z until it intersects the resultant at Y, which is the point of intersection of the first and last rays of the required equilibrium polygon.
13. Draw XY.
14. Draw the corresponding rays of the force polygon parallel to XY from point A, thus locating point O at its intersection with the horizontal from K.
15. Construct the corresponding equilibrium polygon parallel to the rays of the force polygon drawn from the pole O.
16. Draw in the arch axis.

DEAD LOADS

	<u>Left</u>	<u>Height</u> <u>Right</u>	<u>Av.</u>	<u>l</u>	<u>b</u>	<u>D.L.</u>	<u>Force</u>
	<u>ft.</u>	<u>ft.</u>	<u>ft.</u>	<u>ft.</u>	<u>ft.</u>	<u>lbs.</u>	<u>lbs.</u>
1	6.80	4.90	5.85	7.7	5.33	36,200	AB
						30,030	BC
2	4.90	3.03	3.965	10.83	"	34,400	CD
						33,875	DE
3	3.03	2.28	2.655	"	"	25,000	EF
						40,440	FG
4	2.28	2.05	2.165	"	"	18,780	GH
						32,570	HI
5	2.05	2.00	2.025	"	"	17,580	IJ
						16,210	JK
<b>Total D.L. ---</b>						<b>283,085</b>	

## NOTATIONS

- $l$  = span of arch axis.
- $h$  = rise of arch axis.
- $r$  = ratio of the rise to the span of the arch axis.
- $t_o$  = thickness of the arch ring at the crown.
- $t_e$  = thickness of the arch ring at the springing line.
- $V_s$  = ratio of  $t_e$  to  $t_o$  .
- $E_s$  = modulus of elasticity of steel.
- $E_c$  = modulus of elasticity of concrete.
- $n$  = modulus ratio of steel and concrete.
- $k$  = coefficient of expansion of concrete.
- $w$  = weight per cubic yard of concrete.
- $w_c$  = dead load per linear foot at the crown.
- $T_c$  or  $H_c$  = horizontal thrust at the crown.
- $V_c$  = vertical thrust at the crown.
- $M_c$  = bending moment at the crown.
- $T_s$  = horizontal thrust at the springing line.
- $V_s$  = vertical thrust at the springing line.
- $M_s$  = bending moment at the springing line.
- $A_s$  = area of steel.
- $A_o$  = equivalent area of concrete at the crown.
- $A_e$  = equivalent area of concrete at the springing line.
- $I_o$  = equivalent moment of inertia at the crown.
- $I_e$  = equivalent moment of inertia at the springing line.
- $f_a$  = average stress.
- $t$  = variation of temperature in degrees.

ANALYSIS BY THE ELASTIC THEORY  
 With the Use of  
COCHRANE'S FORMULAS AND DIAGRAMS

DATA

l = 103.4'      t = 2.0'      E = 30,000,000  
 h = 16.5'      t = 6.0'      E = 2,000,000      n = 15  
 r = .160      U = 3.0      k = 0.0000055

LIVE LOAD

Specifications: 100 lbs. per sq.ft. plus 25% for impact.

$$w = 125 \times b/2 = 125 \times 12 = 1500 \text{ lbs./lin.ft. per arch ring.}$$

$$wl = 1500 \times 103.4 = 155,100 \text{ lbs.}$$

$$wl^2 = 1500 \times 103.4^2 = 16,030,000 \text{ ft.lbs.}$$

Dead LOAD

NOTE: The volume of the whole center span section was used in determining the dead load per linear foot per arch at the crown.

1. Floor Slab.

$$v = bdl = 13'-3" \times 10" \times 10'-10"$$

$$= (13.25 \times 10 \times 130) / 144 = \text{-----} 119.55 \text{ cu.ft.}$$

2. Floor Beam at the Crown.

$$v = bdl = 1'-0" \times 28.5" \times 11'-2"$$

$$= (1 \times 28.5 \times 134) / 144 = \text{-----} 26.55 \text{ " "}$$

$$= (4" \times 4" \times 11'-2")$$

$$= (16 \times 134) / 1732 = \text{-----} 1.24 \text{ " "}$$

3. Curb and Brackets.

$2'-6" \times 6" \times 10'-10" = 30 \times 6 \times 130 = 23400$  cu.in.  
 $3" \times 6" \times 2'-3" = 3 \times 6 \times 27 = 486$  " "  
 $1'-3" \times 4" \times 2'-0" = 15 \times 4 \times 24 = 1440$  " "  
 $9.5" \times 4" \times 1'-10" = 9.5 \times 4 \times 22 = 836$  " "  
 $7" \times 2" \times 1'-8" = 7 \times 2 \times 20 = 280$  " "  
 $2'-2" \times 9" \times 1'-6" = 26 \times 9 \times 18 = 4215$  " "

Total = ----- 30650 cu.in.  
 = 165.04 cu.ft.

4. Pilasters over Crown.

$2" \times 1'-9" \times 1'-9" = 2 \times 21 \times 21 = 882$  cu.in.  
 $6" \times 1'-9" \times 1'-6" = 6 \times 21 \times 18 = 3024$  " "  
 $2'-7" \times 1'-6" \times 1'-6" = 31 \times 18 \times 18 = 10040$  " "  
 $6" \times 1'-9" \times 1'-9" = 6 \times 21 \times 21 = 2646$  " "

Total = ----- 16592 cu.in.  
 = 9.57 " "

5. Spindles ( without upper or lower railings )

$2" \times 9" \times 11" = 2 \times 9 \times 11 = 198$  cu.in.  
 $2'-3" \times 7" \times 9" = 27 \times 7 \times 9 = 1701$  " "  
 $2" \times 9" \times 11" = 2 \times 9 \times 11 = 198$  " "

2097 cu.in.  
 = 8.50 " "

6. Railings

$2" \times 1'-3" \times 6" \times 10'-10" = \frac{2 \times 15 \times 6 \times 130}{1732} = 13.51$  " "

7. Spandrel Column.

$2'-11" \times 1'-6" \times 1'-6" = 35 \times 18 \times 18 = 11330$  cu.in.  
 $10" \times 1'-9" \times 1'-9" = 10 \times 21 \times 21 = 4410$  " "

15740 cu.in.  
 = 9.08 " "



**8. Arch Between Spandrel Columns.**

$$\begin{aligned}
 &1'-6" (9'-4" \times 1'-10") = .6818 \times 9'-4" \times 1'-7" \\
 &= 18 (112 \times 22) = 18 (.6818 \times 112 \times 19) \\
 &= 18 (2464 \times 1454) = 18 \times 1010 = 18180 \text{ cu.in.} = 10.49 \text{ cu.ft.}
 \end{aligned}$$

-----  
 Total vol. of superstructure of center span = 216.16 cu.ft.

Vol. per lin. ft. =  $216.16 \times 150/12 = \text{----} 19.98 \text{ cu.ft.}$

Vol. of arch per lin. ft. at crown  
 =  $2 \times 5.333 = \text{-----} 10.67 \text{ " "}$

Total Vol. per lin. ft. at Crown = 30.65 cu.ft.

Dead Load per lin. ft. at Crown  
 =  $30.65 \times 150 = 4598 \text{ lbs.}$

Dead load not found in center span.

**9. Rib Braces.**

$$\begin{aligned}
 &8'-1" \times 1'-6" \times 1'-6" = 97/12 \times 1.5 \times 1.5 = 18.20 \text{ cu.ft.} \\
 &2' \times 2' \times 1'-6" = 2 \times 2 \times 1.5 = \text{-----} 6.00 \text{ " "}
 \end{aligned}$$

**10. Extra Weight at the Rib Brace.**

$$\begin{aligned}
 &3" \times 28.5" \times 11'-2" = 3 \times 28.5 \times 134 \\
 &= 11470 \text{ cu.in.} \\
 &(48"-15") \times 7.5" \times 118-2" = \\
 &33 \times 7.5 \times 134 = \text{----} 34190 \text{ " "} \\
 &45660 \text{ cu.in.} \\
 &= 26.33 \text{ " "}
 \end{aligned}$$

Total vol. of extra load = 50.53 cu.ft.

Since this load is concentrated near the quarter point it is reasonable to consider it as uniformly distributed.

Thus,  $w = (50.53 \times 150)/51.7 = 149 \text{ lbs/sq ft.}$

and  $w_c = 4598 + 149 = 4747 \text{ lbs.}$



$$w_1 = 4747 \times 103.4 = 491,300 \text{ lbs.}$$

$$w_1^2 = 4747 \times 103.4^2 = 50,780,000 \text{ ft.lbs.}$$

### THRUSTS AND MOMENTS

#### Dead Load - Diagram #1

$$T_c \text{ Or } H_c = Cw_c l = +0.876 \times 491,300 = 430,000 \text{ lbs.}$$

$$V_s = Cw_c l = 0.630 \times 491,300 = 309,800 \text{ lbs.}$$

$$T_s = Cw_c l = 1.085 \times 491,300 = 533,000 \text{ lbs.}$$

#### Live Load, Max. + Mom. at Crown.- Diagram #2

$$T_c = Cw_c l = 0.405 \times 155,100 = 62,800 \text{ lbs.}$$

$$M_c = Cw_c l^2 = 0.00420 \times 16,030,000 = 67,300 \text{ ft.lbs.}$$

#### Live Load, Max. - Mom. at Crown.- Diagram #2

$$T_c = Cw_c l = 0.407 \times 155,100 = 63,100 \text{ lbs.}$$

$$M_c = Cw_c l^2 = -0.00357 \times 16,030,000 = -57,250 \text{ ft.lbs.}$$

#### Live Load, Max. + Mom. at Springing Line. Diagram #3

$$T_s = Cw_c l = 0.570 \times 155,100 = 88,400 \text{ lbs.}$$

$$T_c = Cw_c l = 0.590 \times 155,100 = 91,500 \text{ lbs.}$$

$$M_s = Cw_c l^2 = 0.0317 \times 16,030,000 \text{ ft.lbs.}$$

#### Live Load, Max. - Mom. at Springing Line. Diagram #3

$$T_s = Cw_c l = 0.387 \times 155,100 = 60,160 \text{ lbs.}$$

$$T_c = Cw_c l = 0.223 \times 155,100 = 34,580 \text{ lbs.}$$

$$M_s = Cw_c l^2 = -0.0256 \times 16,030,000 = -410,500 \text{ ft.lbs.}$$

Fall of Temperature of 40 - Diagram #6

$$t_c t_p E = 0.0000055 \times 40 \times 288,000,000 = 63,400 \text{ lbs./sq.in.}$$

$$T_c = \frac{-\alpha t_c t_p E I_p}{h^2} = \frac{-39.2 \times 63,400 \times 5.296}{16.50^2} = -48,350 \text{ lbs.}$$

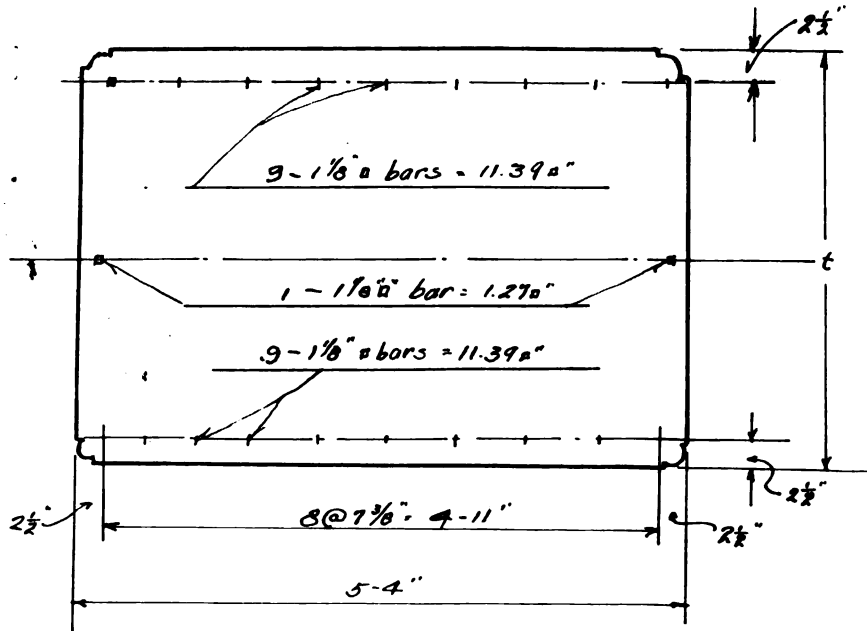
$$M_c = -C_2 \frac{h T_c}{100} = \frac{-16.8 \times 16.5 \times -48,350}{100} = 134,100 \text{ ft.lbs.}$$

$$\begin{aligned} T_s &= (1.09 - 1.75r) T_c = (1.09 - .280) T_c \\ &= .81 \times -48,350 = -39,200 \text{ lbs.} \end{aligned}$$

$$\begin{aligned} M_s &= M_c + h T_c = 134,100 - (16.50 \times 48,350) \\ &= 134,100 - 788,000 = -653,900 \text{ ft.lbs.} \end{aligned}$$



Cross-Section of the Arch Ring.



MOMENTS OF INERTIA AND EQUIVALENT AREAS.

At the Crown.

$$\begin{aligned}
 I_o &= \frac{bd^3}{12} + \frac{rAn}{144} + \frac{2A_s n}{12} \\
 &= \frac{5.33 \times 2^3}{12} + \frac{9.5^2 \times 22.78 \times 15}{144 \times 144} + \frac{2 \times .565^4 \times 15}{12} \\
 &= 3.558 + 1.487 + .251 = 5.296 \text{ in.}^4
 \end{aligned}$$

$$\begin{aligned}
 A_o &= bd + 15A_s = (5.333 \times 2) + (15 \times 25.32)/144 \\
 &= 10.67 + 2.64 = 13.31 \text{ sq. ft.}
 \end{aligned}$$

At the Springing Line.

$$\begin{aligned}
 I &= \frac{5.33 \times 6^3}{12} + \frac{33.5^2 \times 22.78 \times 15}{144 \times 144} + 0.251 \\
 &= 96.00 + 18.480 + .251 = 114.731 \text{ in.}^4 \\
 A_e &= 5.333 \times 6 + (15 \times 25.32)/144 = 32.00 + 2.64 = 34.64 \text{ sq. ft.}
 \end{aligned}$$



## AVERAGE STRESSES

### Dead Load

$$f_a = C(T_c/A_o) = (0.846 \times 430,000)/13.31 = 27,500 \text{ lbs.}$$

### L. L. Producing Max. + Mom. at Crown

$$f_a = C(T_c/A_o) = (.821 \times 62,800)/13.31 = 3,870 \text{ lbs.}$$

### L. L. Producing Max. Mom. ( - ) at Crown.

$$f_a = C(T_c/A_o) = (.860 \times 63,100)/13.31 = 4,075 \text{ lbs.}$$

### L. L. Producing Max. + Mom. at Springing Line.

$$f_a = C(T_c/A_o) = (.828 \times 91,500)/13.31 = 5,690 \text{ lbs.}$$

### L.L. Producinging . - Mom. at Springing Line

$$f_a = C(T_c/A_o) = (.855 \times 34,580)/13.31 = 2,220 \text{ lbs.}$$

### Temperature Drop of 40°.

$$f_a = C(T_c/A_o) = (.759 \times -46,800)/13.31 = -2,670 \text{ lbs.}$$

### Arch-Shortening.

For each combination of loading, the arch-shortening thrusts and moments bear the same ratio to the thrusts and moments due to a fall of 40° in temperature, as does the total average stress to the stress  $t_c t_d E$ .



SUMMARY FOR MAX. + MOM. AT CROWN

(a) D.L., L.L., & Arch Shortening.

	<u>THRUST</u>	<u>MOMENT</u>	<u>AV. STRESSES</u>
Dead Load	+450,000		+27,500
Live Load	+ 62,800	+ 67,500	+ 3,870
Arch Short.	- 22,990	+ 63,700	- 1,270
<b>Total</b>	<b>+469,810</b>	<b>+131,000</b>	<b>+30,100</b>

(b) D.L., L.L., Temp. Var. & Arch S.

	<u>THRUST</u>	<u>MOMENT</u>	<u>AV. STRESSES</u>
D.L. & L.L.	492,800	67,500	31,370
Temp. Var.	- 48,350	134,100	- 2,670
Arch. S.	- 21,000	58,200	- 1,160
<b>Totals</b>	<b>423,450</b>	<b>259,600</b>	<b>27,540</b>

Computation of  $f_a$ ,  $T_c$ , &  $M_c$  for Arch S.

Let  $f_a$  due to Arch S. =  $x$

$$\text{Then } \frac{x}{-2,670} = \frac{31,370 - x}{63,400}$$

$$x = \frac{-2670 \times 31,370}{66,070} = -1,270 \text{ lbs.}$$

$$x = \frac{-2670 \times 28,700}{66,070} = -1,160 \text{ lbs.}$$

Let  $T_c = y$

$$\text{Then } \frac{y}{-48,350} = \frac{30,100}{63,400}$$

$$y = \frac{-48,350 \times 30,100}{66,070} = -22,990 \text{ lbs.}$$

$$y = \frac{-48350 \times 27,540}{66,070} = -21,000 \text{ lbs.}$$

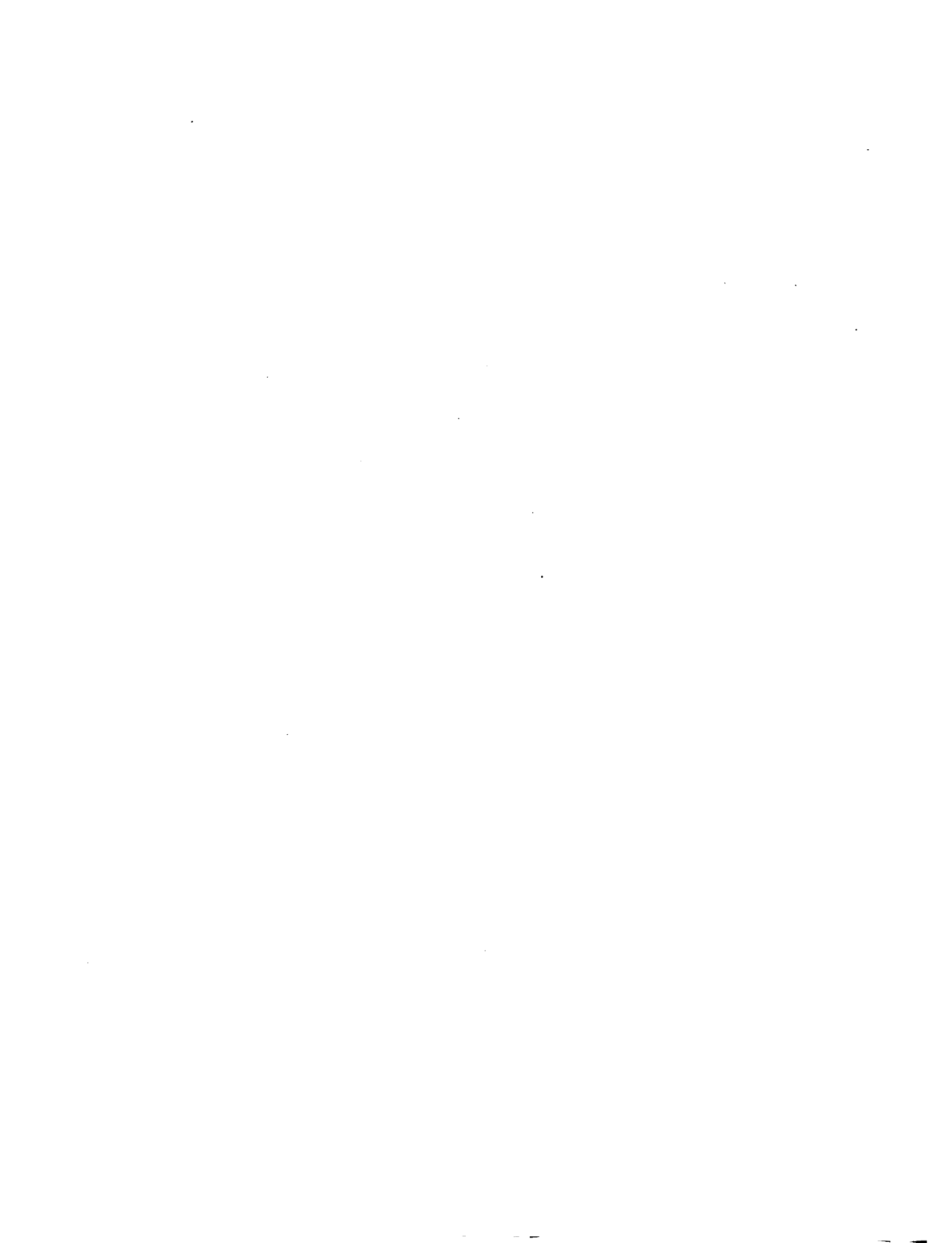


Let  $M_c$  due to Arch-S. =  $x$

$$\text{Then } \frac{x}{134,100} = \frac{30,000}{63,400}$$

$$\text{and } x = \frac{134,100 \times 30,100}{63,400} = 63,700 \text{ ft. lbs.}$$

$$x = \frac{134,100 \times 27,540}{63,400} = 58,200 \text{ ft. lbs.}$$



SUMMARY FOR MAX.  $\sigma$  AT CROWN

(a) D.L., L.L., & Arch-Shortening.

	<u>THRUST</u>	<u>MOMENT</u>	<u>AV. STRESSES</u>
Dead Load	430,000		27,500
Live Load	63,100	- 57,250	4,075
Arch-S.	- 23,130	64,000	- 1,280
	<hr/>	<hr/>	<hr/>
Total	479,970	6,750	30,295

(b) D.L., L.L., Temp. Var. & Arch-S.

	<u>THRUST</u>	<u>MOMENT</u>	<u>AV. STRESSES</u>
Dead L. & L.L.	493,100	- 57,250	31,575
Temp. Var.	- 48,350	134,100	- 2,670
Arch-S.	- 21,150	58,550	- 1,170
	<hr/>	<hr/>	<hr/>
Total	423,700	127,200	27,735

Above summary shows that there is no negative moment at the crown.



SUMMARY FOR MAX.  $\frac{1}{2}$  MOM. AT SPRINGING LINE.

(a) D.L., L.L., & Arch- Shortening.

	<u>THRUST</u> lbs.	<u>MOMENT</u> ft.lbs.	<u>AV. STRESSES</u> lbs.
Dead Load	533,000		27,500
Live Load	88,400	508,200	5,690
Arch-S.	- 19,700	- 328,300	- 1,340
	<hr/>	<hr/>	<hr/>
Total	601,700	179,900	31,850

(b) D.L., L.L., Temp. Var. & Arch-S.

	<u>THRUST</u> lbs.	<u>MOMENT</u> ft.lbs.	<u>AV. STRESSES</u> lbs.
Dead L. & L.L.	621,400	508,200	33,190
Temp. Var.	39,200	653,9000	2,670
Arch-S.	- 21,280	- 354,800	- 1,450
	<hr/>	<hr/>	<hr/>
Total	639,320	807,300	34,410





SUMMARY FOR MAX. - MOM. AT SPRINGING LINE.

(a) D.L., L.L., & Arch-Shortening.

	<u>THRUST</u> lbs.	<u>MOMENT</u> ft.lbs.	<u>AV. STRESSES</u> lbs.
Dead Load	533,000		27,500
Live Load	60,160	-410,500	2,220
Arch-S.	- 17,620	-294,000	- 1,200
	<hr/>	<hr/>	<hr/>
Total	575,540	-604,500	28,520

(b) D.L., L.L., Temp. Var. & Arch-S.

	<u>THRUST</u> lbs.	<u>MOMENT</u> ft.lbs.	<u>AV. STRESSES</u> lbs.
D.L., L.L.,	593,160	- 410,500	29,720
Temp. Var.	- 39,200	- 653,900	- 2,670
Arch-S.	- 16,030	- 267,700	- 1,090
	<hr/>	<hr/>	<hr/>
Total	537,930	-1,332,100	25,960



APPROXIMATE MAXIMUM FIBER STRESSES

Max. - Mom. at Crown.

$$f_c = \frac{P}{A} \pm \frac{Mc}{I} = \frac{T_c}{A_c} \pm \frac{M_c t_c}{2I_c} \quad (\text{lbs. per sq. ft.})$$

$$(a) \quad f_c = \frac{469,810}{13.31} + \frac{131,000}{5,296} = 31,800 + 24,730 = 59,980 \\ = 59,980 / 144 = 417 \#/\text{sq.in.}$$

$$(b) \quad f_c = \frac{423,450}{13.31} + \frac{259,600}{5,296} = 31,800 + 49,000 = 80,800 \\ = 80,800 / 144 = 561 \#/\text{sq.in.}$$

Max. - Mom. at Crown.

No negative moment at the crown.

Max. - Mom. at the Springing Line

$$f_c = \frac{T_c}{A_c} \pm \frac{M_c t_c}{2I_c} \quad (\text{lbs. per sq. ft.})$$

$$(a) \quad f_c = \frac{575,540}{34.64} + \frac{604,500 \times 6}{2 \times 114.73} = 16,610 + 15,800 = 32,410 \# \\ = 32,410 / 144 = 225 \#/\text{sq.in.}$$

$$(b) \quad f_c = \frac{537,930}{34.64} - \frac{1,332,100 \times 6}{2 \times 114.73} = 15,520 - 34,820 = -50,340 \# \\ = 50,340 / 144 = 350 \#/\text{sq.in.}$$

Max. + Mom. at the Springing Line.

$$(a) \quad f_c = 153 \#/\text{sq.in.}$$

$$(b) \quad f_c = 282 \#/\text{sq.in.}$$



## CORRECTION FOR MAX. FIBRE STRESSES

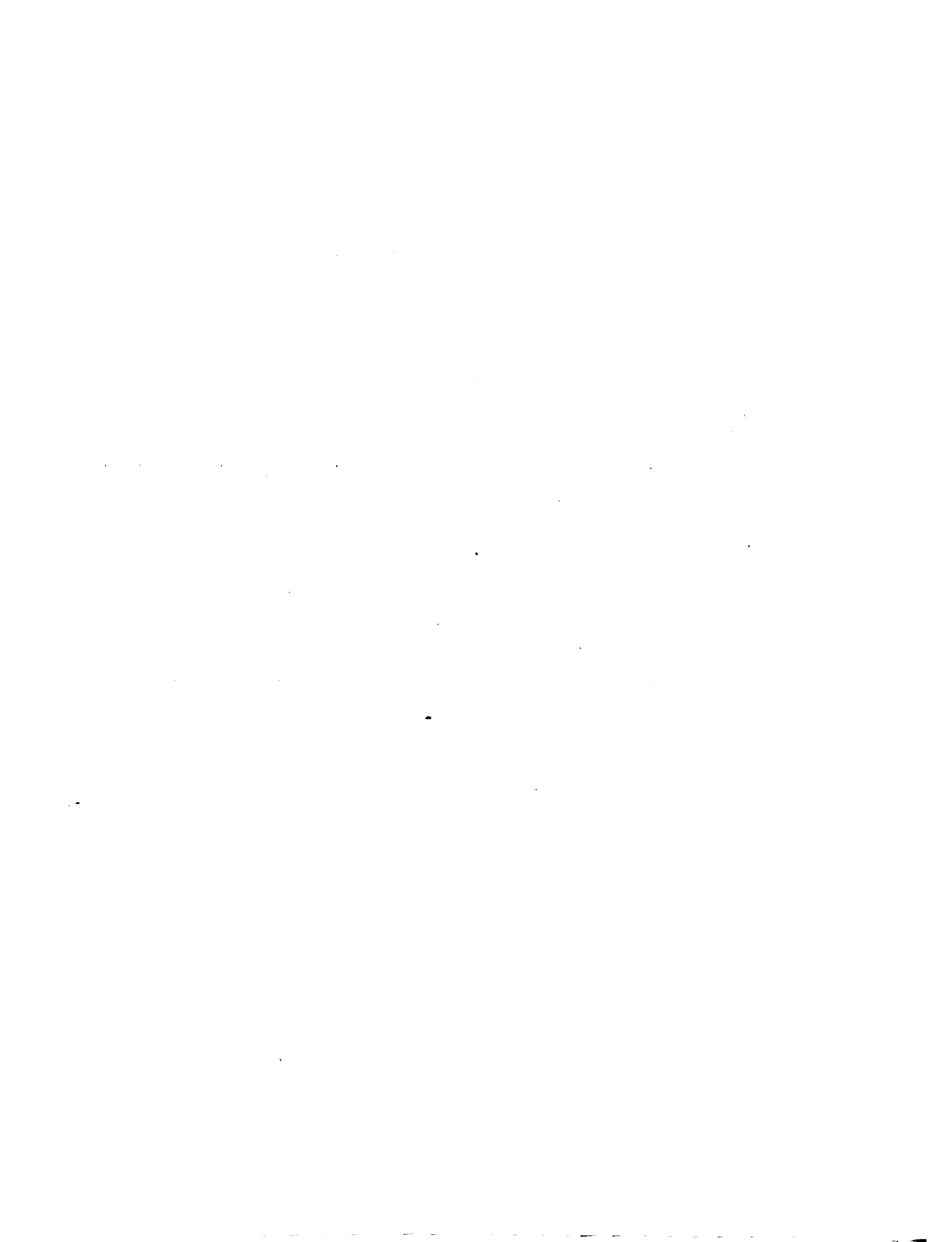
Note; The results obtained by Cochrane's are a little low, so a correction is made by considering the thrust of the dead load as having an eccentricity of one-fortieth of the arch section.

### Correction for Stresses at Crown

$$\begin{aligned}\text{Corr.} &= (d.l.T_c \times .025 \times t_c \times e) / 144 I_c \\ &= (430,000 \times .025 \times 2) / (144 \times 5.296) = 28.2 \text{ #/sq.in.}\end{aligned}$$

### Correction for stresses at Springing Line.

$$\begin{aligned}\text{Corr.} &= (d.l.T_s \times .025 \times t_s \times e) / 144 I_s \\ &= (533,000 \times .025 \times 6 \times 3) / (144 \times 13.31) = 125.2 \text{ #/sq.in.}\end{aligned}$$



MAXIMUM FIBER STRESSES

Condition of Loading

	Max. - Mom. at crown <u>#/sq.in.</u>	Max. + Mom. at spring. <u>#/sq.in.</u>	Max. - Mom. at spring. <u>#/sq.in.</u>
(a)	417	153	225
(b)	517	282	350
COrrrection	28	125	125
Total (a)	<u>445</u>	<u>278</u>	<u>355</u>
Total (b)	589	407	475

FINAL SUMMARY AND COMPARISION

	By Cochrane's Method <u>#/sq.in.</u>	By Melick's Method <u>#/sq.in.</u>	Allowable <u>#/sq.in.</u>
Crown -----	589	696	650
Springing Line	475	497	650

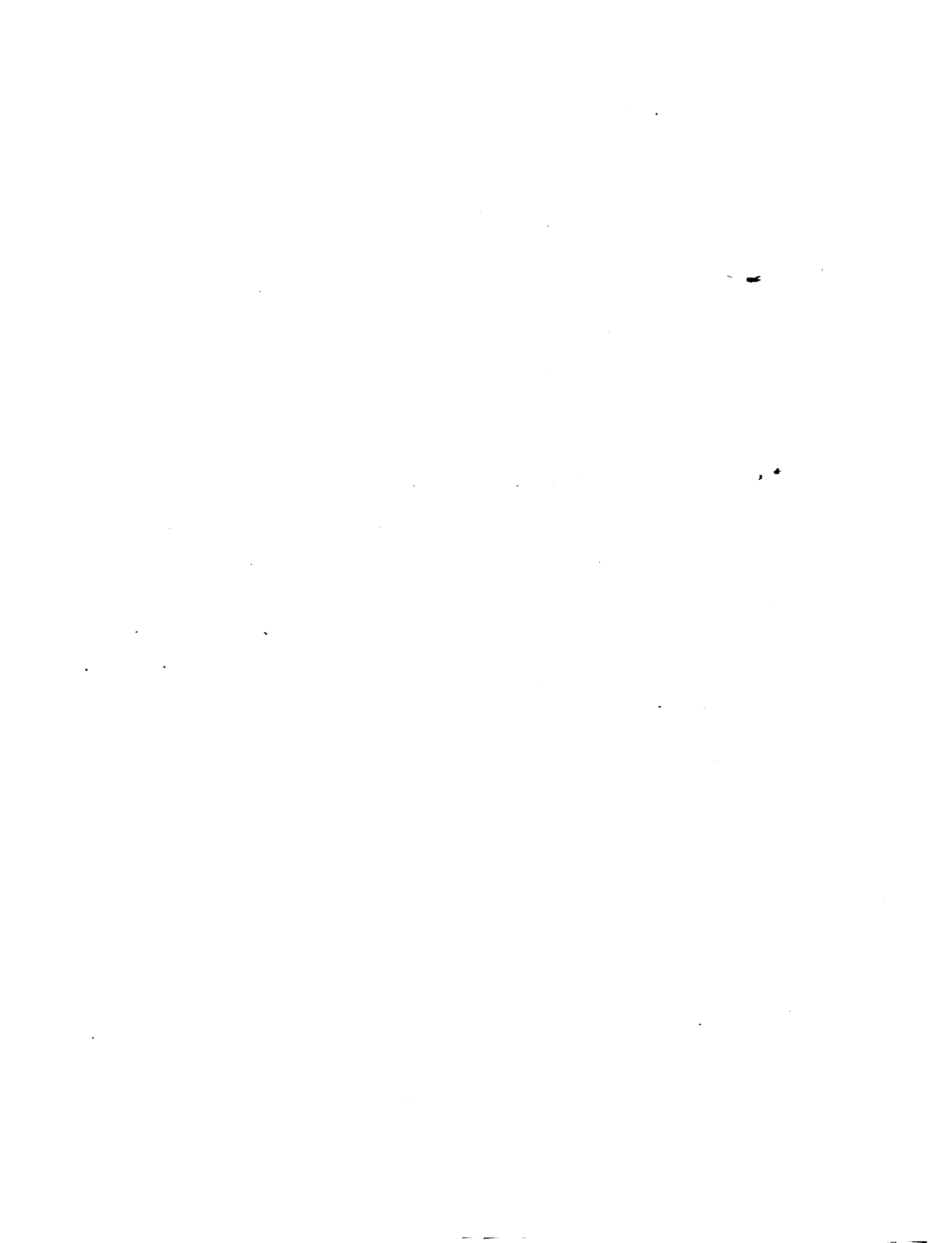
## CONCLUSION

In the graphical analysis, the line of thrust followed very closely the axis of the arch ring, which shows that the arch ring is acceptable for further analysis. Since the line of thrust falls above the arch axis, it indicates that the arch ring is a trifle heavier than necessary.

The analysis by Cochrane's method shows that the maximum fiber stress at the crown is 9.4% below the allowable, and the maximum fiber stress at the springing line is 26.9% below the allowable. These results check very closely with those obtained at the State Highway office by the use of Mr. Melick's method. Their maximum stress at the crown was 7.1% above the allowable, and the maximum fiber stress at the springing line was 17.4% below the allowable.

Thus, from the preceding results, it is evident that the arch is safe and well designed. The only correction that might be suggested is that the thickness be reduced slightly from the haunch to the springing line; but the change is not advisable for it would introduce a fractional U which would be very bothersome and undesirable in analysis by Cochrane's method.



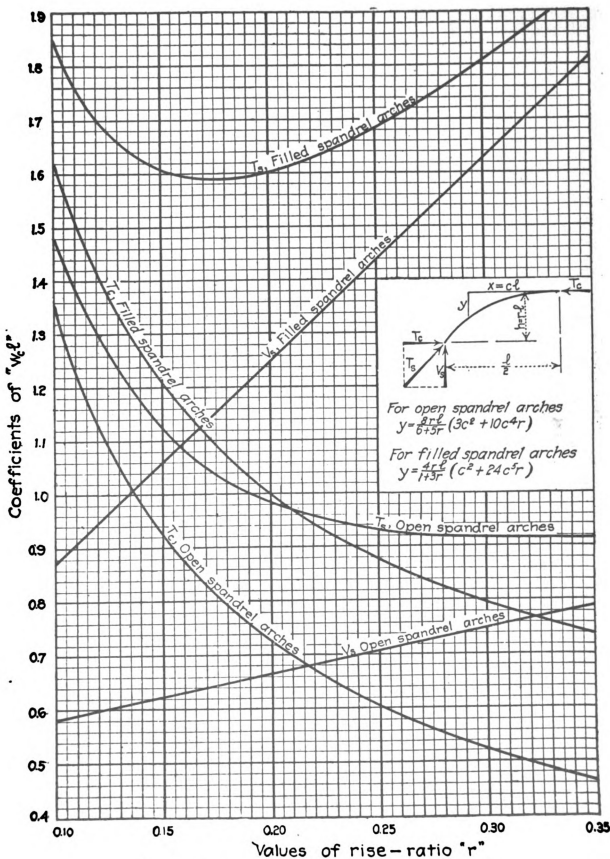


**DIAGRAMS**

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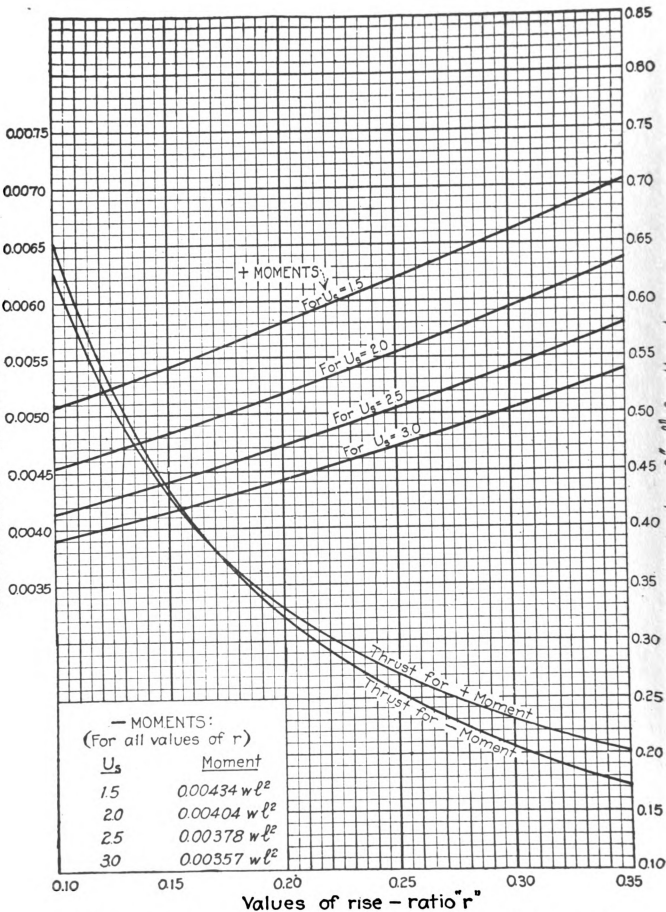
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Coefficients of "wL²" for moments

Approximate dead-load thrusts and moments.

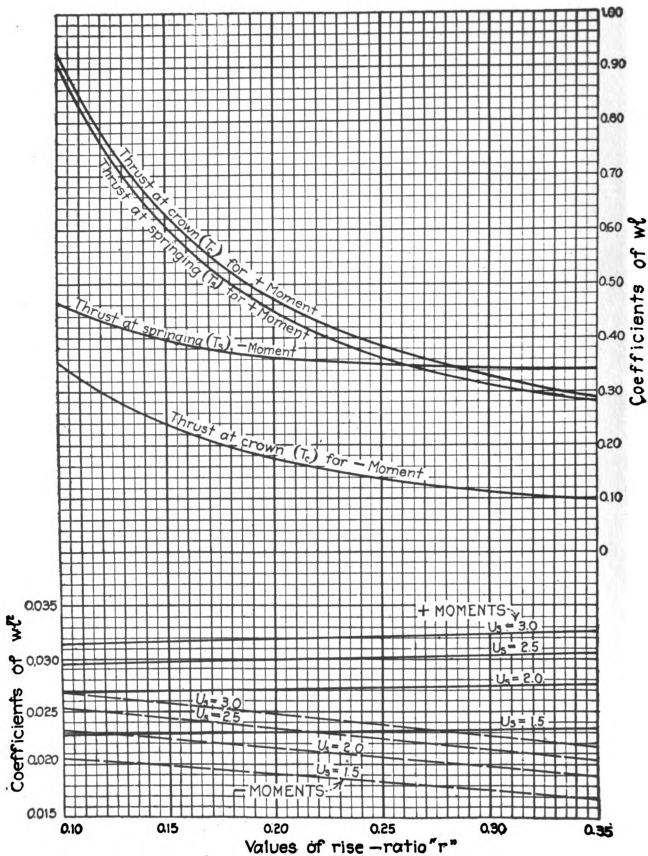
Coefficients of " $wl^2$ " for moments



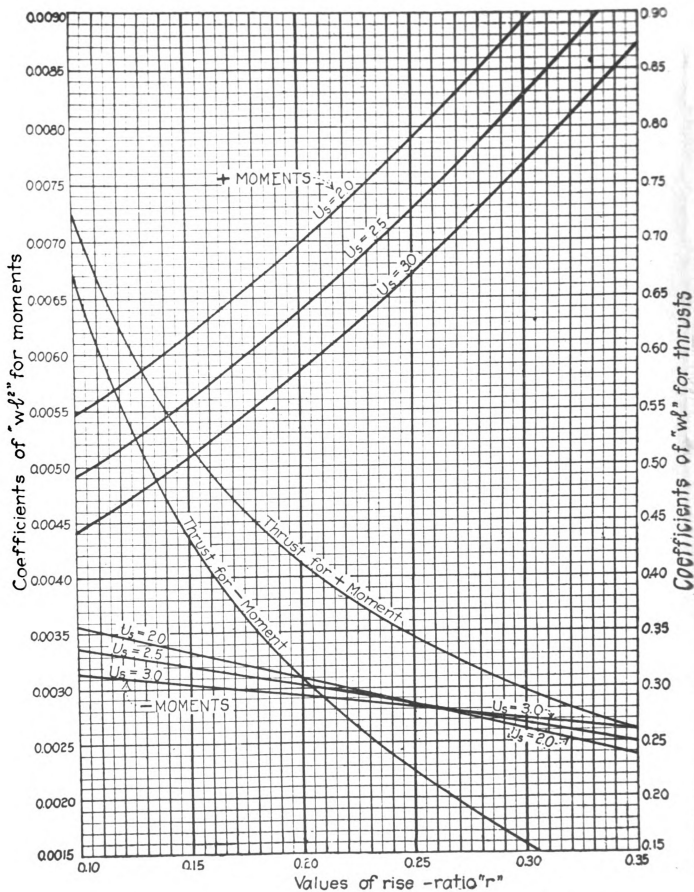
Coefficients of " $wl$ " for thrusts

Live-load thrusts and moments at crown; open spandrel arches.

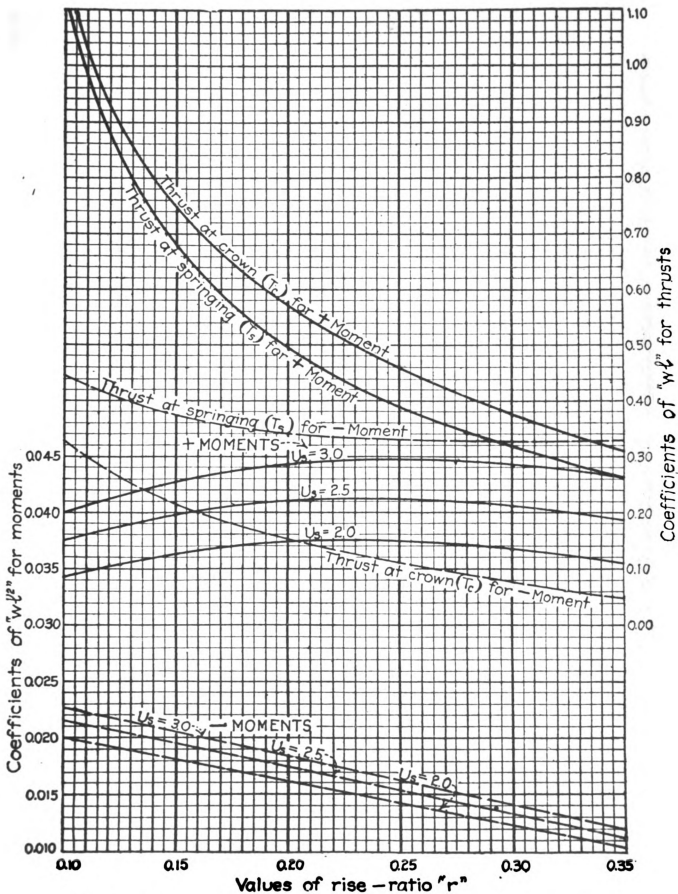




Live-load thrusts and moments at springing; open spandrel arches.



Live-load thrusts and moments at crown; filled spandrel arches.



Live-load thrusts and moments at springing; filled spandrel arches.

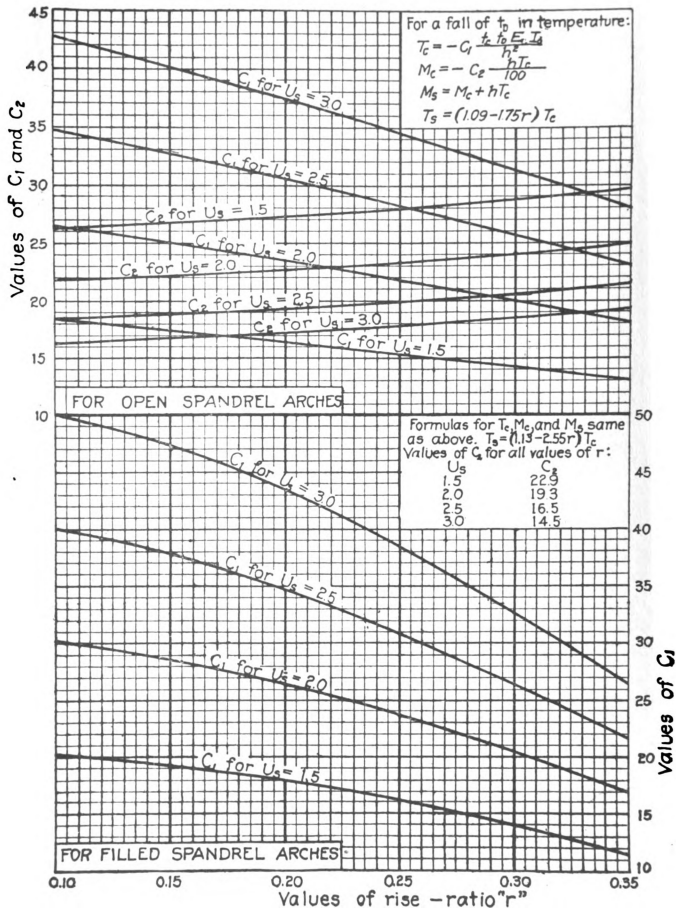


Values of  $C_1$  and  $C_2$

45

40

35



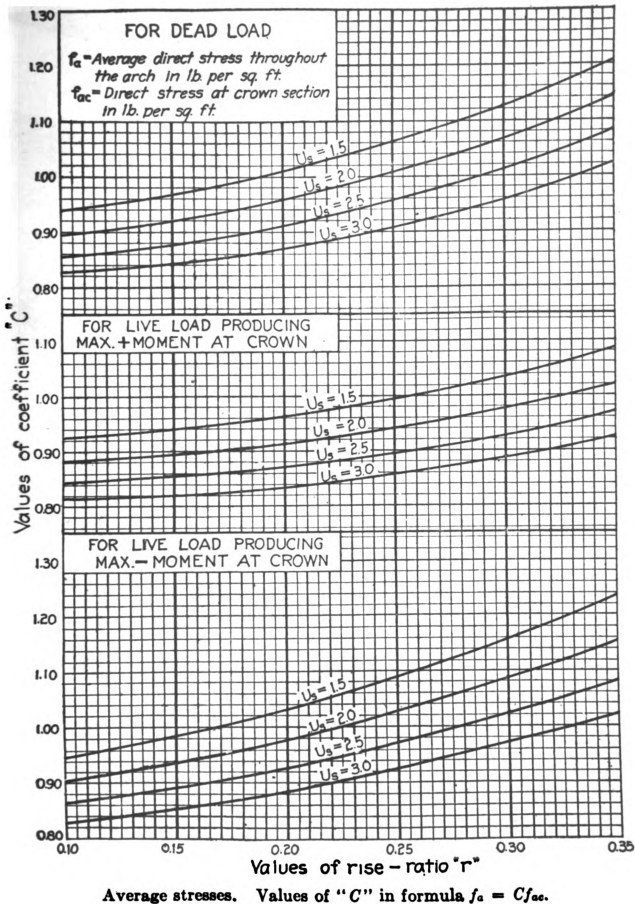
Temperature and rib shortening.

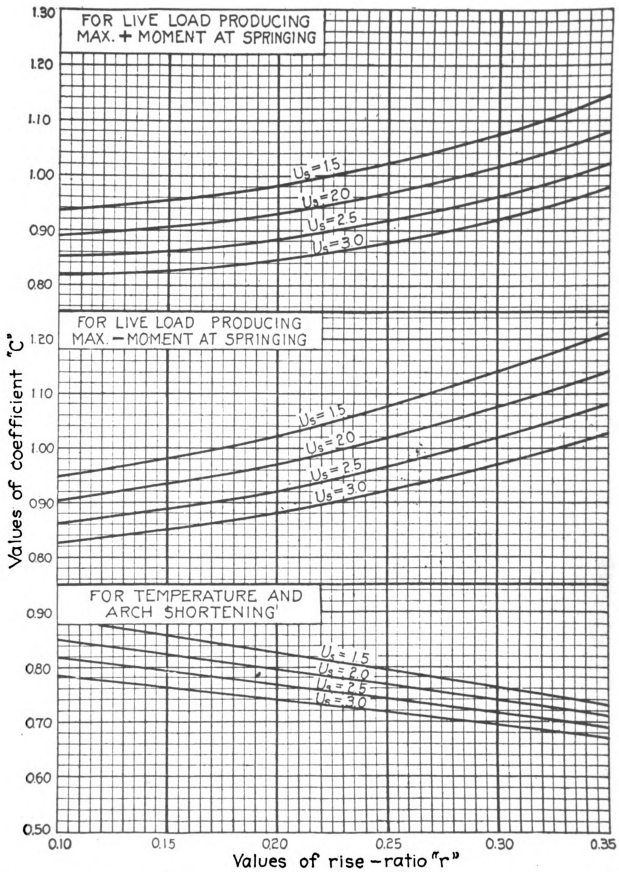


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