THESIS

Analysis of stresses in
Wash. Ave. Bridge Lansing, Mich.

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1913

This thesis was contributed by

Joseph Alexander Macdonald

under the date indicated by the department stamp, to replace the original which was destroyed in the fire of March 5, 1916.

Civil engineering Bridges & [illegible]

- THESIS -

An Analysis of the Stresses in the Washington Avenue Concrete Arch Bridge Over the Grand River at the City of Lansing, Michigan.

B. S

- by -

J. A. Macdonald, E. H. Meyer.

June - 1913.

THESIS

The object of this thesis is to theoretically investigate the stability of and determine the maximum stresses in the arch ring of the reinforced concrete bridge over the Grand River at Washington Avenue, Lansing, Michigan.

The arch ring is of the no hinged Melan type having a clear span of 120 feet, a rise of 23 feet, and a total width of 54 feet the width of roadway being 36 feet 10 inches. The arch ring is composed of 1 : 2: 4 Portland Cement Concrete with reinforcement consisting of two 3" x 3" x 3/8" angles, top and bottom laced. The reinforcing ribs are placed 3 feet c. to c. The arch ring is supported by abutments consisting of 1 : 3 : 7 Portland Cement Concrete mixture resting on rock bottom.

The dead load to be carried by the arch ring consists of the arch ring, the earth backfill, and a four inch brick pavement having a six inch concrete foundations. At the time of construction the span of this bridge was one of the longest of its type, and we find the dead load to be excessive, as is shown by Table G. A double track of the Michigan United Traction Company's line passes over the bridge, which carries the ordinary city cars as well as the heavy interurban cars of the Lansing - Jackson division.

Being unable to obtain specifications of the Michigan United Traction Company's cars for the live wheel loads we selected a standard car, as given by the Electric Railway Journal April 22, 1911. This car has a total weight of 80,000 # supported by two trucks. The trucks are spaced 35 feet c. to c. and have a wheel base of six feet six inches. A standard uniform live load of 100# per square foot was considered as covering the bridge except for the space occupied by the car, and here considered as a strip 10 feet wide.

In analyzing the arch we selected a section 3 feet wide and computed the maximum stresses due to dead and live loads. In our analysis we learned that the live load stresses due to the car were much greater than the stresses due to a uniform load of 100# per square foot covering the same area, and for this reason we applied the test to a section directly underneath the car track. The distribution of the live axle loads to the arch ring through the earth filling, is a problem not very well understood and its discussion seems to be avoided by the majority of technical writers. In our analysis we assumed that the axle load was transmitted over a width of tem feet on the arch giving a concentrated load of 6000 pounds at each axle of the interurban car for an arch ring section three feet wide.

The analytical theory as developed by Mr. C. A. Melick was the method employed in determining the stresses in the arch ring The fundamental formulae for the elastic theory method are described and derived on Plate A, the same being a copy of treatment developed by Mr. Melick.

The arch ring was divided into twenty radial sections, the co-ordinates for the centre of the sections being given in Table A, and a combination of the properties are found in Tables B and C. The results obtained from the tables mentioned above are then used to obtain the values of V_L, H_L, M_L, and M_r, which are tabulated in Table D, and the vlaues given in this table are for a vertical load of 1000# concentrated at the section under consideration, and also neglecting the effect of axial thrust. From the calculations shown on Plate B we find that the effect of axial thrust is very small and for this reason it has not been taken into consideration.

On Plate 3 the values of H_L, V_L, M_L, and M_r were plotted to scale and from these the force polygons were constructed. The reactions given in the force polygons were then resolved into their respective components, thus giving the thrusts and radial shears at each of the sections for a moving load of 1000#. The values for the thrusts, shears, and moments were then scaled and tabulated in Table E, and influence lines for the same were drawn. The results from Table E were used to determine the top and bottom fibre stresses, which are given in Table F, due to a moving load of 1000#, and influence lines for these values were also drawn. In Table G we have given the top and bottom fibre stresses at each section for the actual dead loads, on each arch section, considered as concentrated loads. These stresses were then summed up for each section for the dead load covering the entire bridge giving the values as found on Plate 9.

The position of the live load for producing maximum fibre stress was determined by means of a slider, which was constructed by spacing the wheels to the same scale as the horizontal scale of the influence lines, and this position was obtained by placing the slider on the influence lines so that the sum of the products of the wheel loads, in units of thousands of pounds, by their respective stresses as given by the ordinate of the influence line, produced a maximum value, and the result thus obtained was the value of the fibre stress for the section under consideration. These values we find tabulated on Plate 9.

The live and dead load unit fibre stresses were then combined giving the maximum fibre stresses as shown by Plate 9.

The analysis for finding the shear at each section was exactly the same as for finding the fibre stresses. On Plate 10 we have tabulated the shear due to the dead load concentration, as assumed, and in Table J we have given the summation of the shears for each section produced by the dead load, covering the entire bridge. The value of maximum shear due to the live axle load was determined with the slider and the influence lines for radial shear in the same manner as described for the fibre stresses. The combined maximum dead and live load shears are tabulated on Plate 11.

The stresses in the steel were determined by the graphical method as shown on Plate 12. The coexistenttop and bottom unit fibre stresses of the concrete arch were plotted to scale, as shown by the horizontal lines, and the distance between them being equal to the depth of the arch ring. The unit stress in the concrete at the steel was scaled from the horizontal line whose distance from the concrete stresses corresponds to the distance of the steel from the extreme concrete fibre and the value of the stress in the steel is equal to fifteen (the assumed value of the "modulus ratio") by the stress scaled from the diagram. The value of maximum unit stresses in the steel is tabulated in Table 9.

Conclusion.

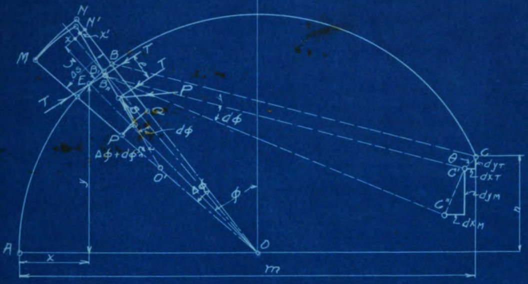
By referring to Plate 9 it will be seen that the maximum unit stresses produced by thrust and bending are high, but that the results obtained are within the allowable limits except at the points (0) and (1) where we find a tension of 150# per square inch on the bottom fibre. The allowable unit stress is 50# per square inch.

From the results obtained in shear as is shown by Plate 11 it is found that the maximum unit shearing stresses are excessive, but it is found that the shear bars in the arch ring bring the unit shearing stresses well within the allowable limit. The allowable unit shear for concrete alone in 40 # per square inch.

NOTE: Since the completion of this thesis we have obtained the weight and wheel spacing of the heaviest car that passes over the bridge, from C. B. Mosgan, General Superintendent of the M.W.T., and they are as follows. The car has a total weight of 84,930 pounds supported by two trucks spaced 36 feet 6 inches c. to c.

Since the live load is but a small part of the load on the bridge, and since the above car is practically the same as the one we selected in working our thesis, we conclude that the results shown in our thesis would be changed very slightly by using the above car.

Theoretical Analysis of the Elastic Arch.



Fundamental Formulae:–

Let ADBC represent a portion of the linear arch, and let MNPQ be any section of the arch ring. Let E on the linear arch be the center of this section with Co-ordinates (x.y) measured from A. Let the central angle $\Delta\phi$ be subtended by the Arc DB of length Δs.

This section of the Arch Ring will be acted upon by some Resultant Arch Pressure P, which may be resolved into a Radial Shear S, a Normal Axial Thrust T, and a Bending Moment T.e. = M. These Shears are small in Ordinary Arches and Arch Theories neglect them in the Theory of Stresses, just as is done in the Common Theory of Flecture for Beams. This leaves then, for consideration, the two agents Thrust & Bending acting on the section: these both vary with the different Sections of the arch ring. T shortens the section DB uniformly an amount $\frac{T\Delta s}{A.E}$. A Rise of Temperature lengthens the section DB uniformly an amount $e.t.\Delta s$ The Resultant lengthening of the section $DB = -BB' = e.t.\Delta s - \frac{T\Delta s}{A.E}$. A negative Value of M will cause the section DB to deform in the direction DB'' changing $\Delta\phi$ by an amount $d\phi$. ϕ is measured from the Vertical thru the Crown – Positive to the left, Negative to the Right., hence this change $d\phi$ is a negative one. At xx' then $\frac{3\Delta s}{E} = \frac{M3}{EI}$. But $xx' = 3d\phi$ hence $d\phi = \frac{M\Delta s}{EI}$ and since when M is negative $d\phi$ is negative, the Equation stands correct for sign. Now imagine that, for any arch loading, starting At A, each section is taken in order and its effect on the arch found separately and in turn. Then MP may be regarded as firmly fixed to the last deformed section preceeding and the effect of the deformation of MNPQ on the portion of the arch to the Right determined. Let us find the effect of this deformation on the movement of the point C whose coordinates are (m.n). The change in Δs, BB', due to Thrust & Temperature will produce an equal change CC' at C. Also The change in $\Delta\phi$ due to M will produce a change at C of C'C'' such that $C'B''C'' = d\phi$ The increase in span m – due to Thrust & Temperature on this section alone $= dx_T = -CC'\cos\phi = -BB'\cos\phi = -BB'\frac{\Delta x}{\Delta s} = e.t.\Delta x - \frac{T.\Delta x}{A.E}$.

The increase in Rise - n - due to Thrust & Temperature on this section alone $= -dX_T = -CC' \sin\phi = -BB'\frac{\Delta y}{\Delta s} = et\,\Delta y. - \frac{T\Delta y}{A.E.}$ The increase in span - m - due to a Positive Bending Moment M will be dx_M. But $\frac{dx_M}{C'C''} = \frac{y-n}{B''C''} = \frac{dx_M}{B''C''d\phi}$ Hence $dx_M = (y-n)\,d\phi = \frac{My\Delta s}{E.I.} - \frac{nM\Delta s}{E.I.}$. The increase in Rise - n - due to a positive Bending Moment M will be dy_M. But $\frac{dy_M}{C'C''} = \frac{dy_M}{B''C''d\phi} = \frac{m-x}{B''C''}$ or $dy_M = (m-x)\,d\phi = \frac{M.x.\Delta s}{E.I} + \frac{m\,M.\Delta s}{E.I.}$

The total increase in span - m - due to a Thrust T, a Rise in temperature of t° F, & a Positive Bending Moment M, acting on this section alone will be $dx = et\,\Delta x - \frac{T\Delta x}{A.E.} + \frac{My\Delta s}{E.I} - \frac{nM\Delta s}{E.I}$. The total increase in Rise - n - due to a Thrust T, a Rise in Temperature of t° F, & a Postive Bending Moment M acting on this section alone will be $dy = e.t.\Delta y - \frac{T.\Delta y}{A.E.} - \frac{Mx.\Delta s}{E.I} + \frac{m\,M\Delta s}{E.I.}$

In a fixed Arch - Symmetrical - the following Conditions are Assumed -

1. The Radial Lines at the Springing Remain Unchanged in Direction - thus, summing up $d\phi$ for all sections between Springing Points. $\sum_0^L d\phi = \sum_0^L \frac{M.\Delta s}{E.I} = 0$

2. The Supports are Fixed an Immoveable Distance apart - i.e. are Rigid - Thus $\sum_0^L dx = \sum_0^L e.t.\Delta x - \sum_0^L \frac{T.\Delta x}{A.E.} + \sum_0^L \frac{M.y.\Delta s}{E.I} = 0$ since $\sum_0^L \frac{n.M.\Delta s}{E.I}$ becomes 0 $\sum_0^L \frac{M.\Delta s}{E.I} = 0$

3. The Supports are Relatively Immoveable in Elevation - Then $\sum_0^L dy = -\sum_0^L et\Delta y + \sum_0^L \frac{T.\Delta y}{A.E} - \sum_0^L \frac{Mx\Delta s}{E.I} = 0$ since the term $\sum_0^L \frac{m\,M\Delta s}{E.I}$ becomes $m\sum_0^L \frac{M\Delta s}{E.I}$ and since $\sum_0^L \frac{M.\Delta s}{E.I} = 0$

From Figure below. $M = M_L + V_L x - H_L y - W_V(x-a)\ x>a\ - W_H(y-b)\ x>a$

$$T = H_L \cos\phi + V_L \sin\phi + W_H \cos\phi\ x>a - W_V \sin\phi\ x>a.$$

Substituting these values in Equations 1. 2 & 3 they becoome

1. $M_L\sum_0^L \frac{\Delta s}{I} + V_L\sum_0^L \frac{x\Delta s}{I} - H_L\sum_0^L \frac{y.\Delta s}{I} - W_V\sum_a^L \frac{(x-a)\Delta s}{I} - W_H\sum_a^L \frac{(y-b)\Delta s}{I} = 0$

2. $E.e.t.\sum_0^L \Delta x - H_L\sum_0^L \frac{\Delta x.\cos\phi}{A} - V_L\sum_0^L \frac{\Delta x\sin\phi}{A} - W_H\sum_a^L \frac{\Delta x\cos\phi}{A} + W_V\sum_a^L \frac{\Delta x\sin\phi}{A} + M_L\sum_0^L \frac{y\Delta s}{I} + V_L\sum_0^L \frac{xy\Delta s}{I} - H_L\sum_0^L \frac{y^2\Delta s}{I} - W_V\sum_a^L \frac{(x-a)y\Delta s}{I} - W_H\sum_a^L \frac{(y-b)y\Delta s}{I} = 0$

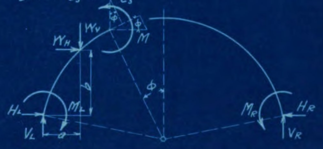
3. $-E.e.t.\sum_0^L \Delta y + H_L\sum_0^L \frac{\Delta y\cos\phi}{A} + V_L\sum_0^L \frac{\Delta y\sin\phi}{A} + W_H\sum_a^L \frac{\Delta y\cos\phi}{A} - W_V\sum_a^L \frac{\Delta y\sin\phi}{A} - M_L\sum_0^L \frac{x\Delta s}{I} - V_L\sum_0^L \frac{x^2\Delta s}{I} + H_L\sum_0^L \frac{xy\Delta s}{I} + W_V\sum_a^L \frac{(x-a)x\Delta s}{I} + W_H\sum_a^L \frac{(y-b)x\Delta s}{I} = 0$

Rewriting the above equations they become

1. $a_1 M_L + b_1 V_L + c_1 H_L + d_1 = 0$ or $H_L = -\frac{a_1}{c_1}M_L - \frac{b_1}{c_1}V_L - \frac{d_1}{c_1}$

2. $a_2 M_L + b_2 V_L + c_2 H_L + d_2 = 0$ or $H_L = -\frac{a_2}{c_2}M_L - \frac{b_2}{c_2}V_L - \frac{d_2}{c_2}$

3. $a_3 M_L + b_3 V_L + c_3 H_L + d_3 = 0$ or $H_L = -\frac{a^3}{c_3}M_L - \frac{b^3}{c_3}V_L - \frac{d^3}{c_3}$



Substituting 3 in 1 & 2 $-M_L\left(\frac{a_1}{c_1}-\frac{a_3}{c_3}\right)=-V_L\left(\frac{b_1}{c_1}-\frac{b_3}{c_3}\right)-\left(\frac{d_1}{c_1}-\frac{d_3}{c_3}\right)$

or $A_1 M_L = B_1 V_L + C_1$ or $M_L = \frac{B_1}{A_1} V_L + \frac{C_1}{A_1}$

$M_L\left(\frac{a_2}{c_2}-\frac{a_3}{c_3}\right)=-V_L\left(\frac{b_2}{c_2}-\frac{b_3}{c_3}\right)-\left(\frac{d_2}{c_2}-\frac{d_3}{c_3}\right)$ or $A_2 M_L = B_2 V_L + C_2$ or $M_L = \frac{B_2}{A_2} V_L + \frac{C_2}{A_2}$

Solving the last two equations gives $\left(\frac{B_1}{A_1}-\frac{B_2}{A_2}\right)V_L = -\left(\frac{C_1}{A_1}-\frac{C_2}{A_2}\right)$ or $D_1 V_L = E_1$ or $V_L = \frac{E_1}{D_1}$

But for Symmetrical Arches $A = 0$ hence $V_L = -\frac{C_1}{B_1}$

The following terms are constants for any given arch and are independent of the loading.

$a_1, a_2, a_3, b_1, b_2, b_3, c_1, c_2, c_3, A_2, B_1, B_2$ Their values are from above.

$a_1 = \sum_0^L \frac{\Delta s}{I}$ $b_1 = \sum_0^L \frac{x\Delta s}{I}$ $c_1 = -\sum_0^L \frac{y\Delta s}{I}$

$a_2 = \sum_0^L \frac{y\Delta s}{I}$ $b_2 = \sum_0^L \frac{xy\Delta s}{I} - \sum_0^L \frac{\Delta x \sin\phi}{A}$ $c_2 = -\sum_0^L \frac{y^2\Delta s}{I} - \sum_0^L \frac{\Delta x . \cos\phi}{A}$

$a_3 = -\sum_0^L \frac{x\Delta s}{I}$ $b_3 = \sum_0^L \frac{x^2\Delta s}{I} - \sum_0^L \frac{\Delta y \sin\phi}{A}$ $c_3 = \sum_0^L \frac{xy . \Delta s}{I} + \sum_0^L \frac{\Delta y \cos\phi}{A}$

$A_1 = 0$ $B_1 = \frac{b_3}{c_3} - \frac{b_1}{c_1}$

$A_2 = \frac{a_2}{c_2} - \frac{a_3}{c_3}$ $B_2 = \frac{b_3}{c_3} - \frac{b_2}{c_2}$

The following terms are dependent on the loading

d_1, d_2, d_3, C_1 & C_2 Their values are from above -

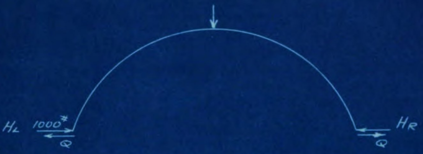
$$d_1 = -W_V \sum_a^L \frac{(x-a)\Delta s}{I} - W_H \sum_a^L \frac{(y-b)\Delta s}{I}$$

$$d_2 = E.e.t. \sum_0^L \Delta x + W_V\left[\sum_a^L \frac{(x-a)y\Delta s}{I} - \sum_a^L \frac{\Delta x \sin\phi}{A}\right] - W_H\left[\sum_a^L \frac{(y-b)y\Delta s}{I} + \sum_a^L \frac{\Delta x \cos\phi}{A}\right]$$

$$d_3 = -Eet \sum_0^L \Delta y + W_V\left[\sum_a^L \frac{(x-a)x\Delta s}{I} - \sum_a^L \frac{\Delta y \sin\phi}{A}\right] + W_H\left[\sum_a^L \frac{(y-b)x\Delta s}{I} + \sum_a^L \frac{\Delta y \cos\phi}{A}\right]$$

$C_1 = \frac{d_3}{c_3} - \frac{d_1}{c_1}$ $C_2 = \frac{d_3}{c_3} - \frac{d_2}{c_2}$

Consideration of Axial Thrust.



Disregarding thrust, H_L due to Positive Bending Moment = $\sum_0^L \frac{My\Delta s}{E.I.}$

Substituting 1000y for M we have H_L for 1000#

Then $H_L = \sum_0^L \frac{1000 y^2 \Delta s}{EI}$

But due to the Axial thrust acting, H_L is reduced an amount Q proportional to 1000# where $H_L = 1000$#

change in span due to thrust is given by $-\sum_0^L \frac{T\Delta x}{E.A.} = \frac{T}{E}\sum_0^L \frac{\Delta x}{A}$

$$\text{then} \quad \frac{1000}{Q} = -\frac{\frac{1000}{E}\sum_0^L \frac{y^2\Delta s}{I}}{\frac{T}{E}\sum_0^L \frac{\Delta x}{A}}$$

$$\therefore \quad Q = -\frac{\frac{1000}{E}T\sum_0^L \frac{\Delta x}{A}}{\frac{1000}{E}\sum_0^L \frac{y^2\Delta s}{I}} = -\frac{T\sum_0^L \frac{\Delta x}{A}}{\sum_0^L \frac{y^2\Delta s}{I}}$$

Actual $H = H_L - Q$

Load at	Q per 1000#	Q for H_L
10	3.837#	9.1#
9'	3.468	7.66
8'	2.996	5.65
7'	2.356	3.45
6'	1.701	1.76
5'	1.137	.74
4'	0.701	.25
3'	0.413	.06
2'	0.220	.001

$\frac{9.1}{2502} = 0.4\%$ This being greatest value for ratio of Q to H, we neglected the effect of Thrust in our calculations.

Points			
	x	y	ϕ
1.	3.1215	3.1508	42°-49'-5
2	9.668	8.3124	33°-40'-16.
3	16.196	12.0133	25°-25'-40
4	22.638	14.5664	17°-48'-58
5	28.975	16.2406	12°-58'-38.
6	35.120	17.5272	10°-40'-46.
7	41.370	18.5779	8°-21'-33.
8	47.872	19.3914	5°-57'-37.
9	54.374	19.9329	3°-34'-19.
10	64.125	20.2381	0°-00'-00"
9'	73.876	19.9329	3°-34'-19.
8'	80.378	19.3914	5°-57'-37.
7'	86.879	18.5779	8°-21'-33.
6'	93.129	17.5272	10°-40'-46.
5'	99.275	16.2406	12°-58'-38.
4'	105.611	14.5664	17°-48'-58"
3'	112.054	12.0133	25°-25'-40"
2'	118.581	8.3124	33°-40'-16.
1'	125.128	3.1508	42°-49'-54.

	Load at 10	
	$\frac{(x-a)\Delta s}{I}$	$\frac{(x-a)x\Delta s}{I}$
9'	22.145609	1636.0334
8'	33.68345	2707.4082
7'	43.22637	3755.5025
6'	46.94766	4372.2215
5'	55.29839	5489.7308
4'	37.25147	3934.1797
3'	17.01281	1906.3429
2'	9.56251	1133.9364
1'	6.02031	753.3128
Σ	271.14857	25688.6682

	Load at 5'	
	$\frac{(x-a)\Delta s}{I}$	$\frac{(x-a)x\Delta s}{I}$
5'		
4'	5.68985	600.9129
3'	4.53596	508.2704
2'	3.39025	402.0215
1'	2.55146	319.2603
Σ	16.16753	1830.4652

a_1	$+\sum_0^L \frac{\Delta s}{I}$	+27.186297
a_2	$+\sum_0^L \frac{y\Delta s}{I}$	+494.05646
a_3	$-\sum_0^L \frac{x\Delta s}{I}$	-1743.32129
b_1	$+\sum_0^L \frac{x\Delta s}{I}$	+1743.32129
b_2	$+\sum_0^L \frac{xy\Delta s}{I}$	+31681.6726
b_3	$-\sum_0^L \frac{x^2\Delta s}{I}$	-128387.402
c_1	$-\sum_0^L \frac{y\Delta s}{I}$	-494.05646
c_2	$-\sum_0^L \frac{y^2\Delta s}{I}$	-9163.8589
c_3	$+\sum_0^L \frac{xy\Delta s}{I}$	+31681.6726
	$\frac{a_2}{c_2}$	-.05391358
	$-\frac{a_3}{c_3}$	+.05502617
A_2	Σ	+.00111259
	$\frac{b_3}{c_3}$	-4.0524186
	$-\frac{b_1}{c_1}$	+3.528587
B_1	Σ	-.5238316
	$\frac{b_3}{c_3}$	-4.0524186
	$-\frac{b_2}{c_2}$	+3.457241
B_2	Σ	-.5951776

Load at	d_1	d_2	d_3	$+\frac{d_3}{c_3}$	$-\frac{d_1}{c_1}$	C_1	$+\frac{d_3}{c_3}$	$-\frac{d_2}{c_2}$	C_2
10	-271148.568	-4464046.	+25688668.	+810.8368	-548.8210	+262.0158	+810.8368	-487.1372	+323.6996
9'	-164254.173	-2574436.	+16190061.	+511.0229	-332.4603	+178.5626	+511.0229	-280.9336	+230.0894
8'	-107746.291	-1608830.	+10947506.	+345.5469	-218.0849	+127.4619	+345.5469	-175.5625	+169.9844
7'	-64712.372	-904506.	+6767969.	+213.6554	-130.9817	+82.6737	+213.6554	-98.7036	+114.9518
6'	-35219.456	-448056.	+3821189.	+120.6119	-71.2869	+49.3257	+120.6119	-48.8938	+71.7182
5'	-16167.529	-173592.	+1830465.	+57.7768	-32.7240	+25.0527	+57.7768	-18.9432	+38.8336
4'	-6490.312	-52471.	+767313.	+24.2195	-13.1368	+11.0827	+24.2195	-5.7259	+18.4936
3'	-2404.987	-13331.	+293635.	+9.2683	-4.8678	+4.4004	+9.2683	-1.4547	+7.8136
2'	-646.110	-2036.	+80847.	+2.5518	-1.3077	+1.2441	+2.5518	-.2221	+2.3297

TABLE C

$d_1 = -1000 \sum_a^L \frac{(x-a)\Delta s}{I}$

$d_2 = -1000 \sum_a^L \frac{(x-a)y\Delta s}{I}$

$d_3 = +1000 \sum_a^L \frac{(x-a)x\Delta s}{I}$

$C_1 = \frac{d_3}{c_3} - \frac{d_1}{c_1}$

$C_2 = \frac{d_3}{c_3} - \frac{d_2}{c_2}$

$V_L = -\frac{C_1}{B_1}$

$M_L = \frac{B_2}{A_2} \cdot V_L + \frac{C_2}{A_2}$

$H_L = H_R = -\frac{a_1}{c_1} M_L - \frac{b_1}{c_1} V_L - \frac{d_1}{c_1}$

$M_R = M_L + V_L L - 1000(L-a)$.

PLATE 2

TABLE D

J.A.M.

Load at	V_L	$\frac{B_2}{A_2} V_L$	$\frac{C_2}{A_2}$	M_L	$-\frac{a_1}{c_1} M_L$	$-\frac{b_1}{c_1} V_L$	$-\frac{d_1}{c_1}$	H_L	M_R
10	+500.1909	-267576.08	+290942.45	+23366.37	+1285.7742	+1764.9675	-548.8210	+2501.92	+23390.86
9'	+340.8779	-182351.87	+206805.17	+24453.29	+1346.5841	+1202.8173	-332.4603	+2215.94	+13797.09
8'	+243.3262	-130166.84	+152782.63	+22615.79	+1244.4726	+858.5978	-218.0849	+1884.98	+5950.35
7'	+157.8249	-84428.08	+103319.08	+18891.00	+1039.5095	+556.8989	-130.9817	+1465.42	-2238.00
6'	+94.1633	-50372.43	+64460.57	+14088.14	+775.2238	+332.2633	-71.2863	+1036.20	-8955.0
5'	+47.8259	-25584.38	+34903.77	+9319.39	+512.8153	+168.7579	-32.7240	+648.85	-13522.0
4'	+21.1569	-11317.87	+16622.12	+5304.25	+291.8753	+74.6541	-13.1368	+353.39	-14621.0
3'	+8.4005	-4493.84	+7022.86	+2529.03	+139.1642	+29.6419	-4.8678	+163.94	-12590
2'	+2.3749	-1270.48	+2093.94	+823.46	+45.3123	+8.3803	-1.3078	+52.38	-8540

...ears, & Bending Moments.

...s 10-2".

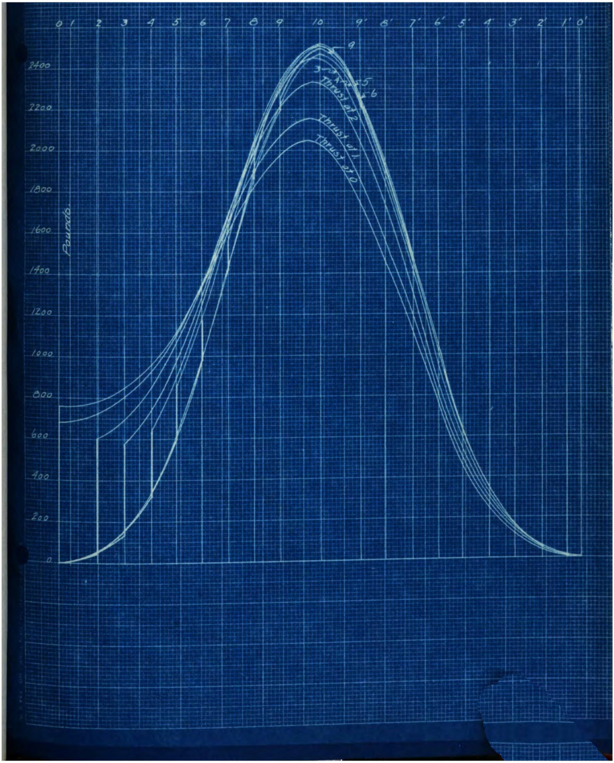
Lansing, Mich.

...3" Rise 23'-0"

	Load at 10			…oad at 3'		Load at 2'		
	T.	S	M	S	M	T	S	M
0	2030	-1497	+4.5 23500	-115	+15.9 2640	35	-40	+16.4 886
1	2148	-1330	+6.6 16350	-107	+12.5 2070	40	-38	+13.0 723
2	2336	-460	+3. 7425	-85	+7.9 1310	45	-29	+8.5 459
3	2453	-607	+0.5 1230	-64	+4.7 780	50	-20	+5.2 281
4	2512	-270	-.9 2230	-43	+2.5 415	54	-15	+3.1 162
5	2525	-75	-1.4 3470	-30	+1.3 216	55	-12	+1.9 102
6	2525	-28	-1.4 3470	-30	+.4 66	56	-10	+.8 43
7	2522	+122	-1. 2476	-20	-.2 33	55	-8	0 000
8	2515	+230	-.5 1230	-10	-.6 99	55	-5	-.5 27
9	2504	+340	+.4 990	0	-.9 149	56	0	-.7 38
10	2476 2476	+500 -500	+2. 495	+10	-.3 50	56	+5	-.2 11
9'		–		+15	+.6 99	55	+6	+.6 32
8'				+25	+1.4 232	55	+7	+1.4 76
7'				+30	+2.8 465	55	+10	+2.8 151
6'				+38	+4.0 664	54	+12	+4.0 216
5'				+40	+5.5 913	54	+15	+5.5 297
4'				+60	+8.0 1330	54	+20	+8.0 432
3'				+80 -825	+11.0 1826	54	+25	+10.9 589
2'				-734	-23.0 3815	47 600	+30 -800	-14.6 788
1'				-610	-58.5 9710	725	-690	-104. 5620
0'				-535	-77.0 12780	782	-624	-163 8800

Note:- Stresses

Thrusts &



0
1
2
3
4
5
6
7
8
9
10
9'
8'
7'
6'
5'
4'
3'
2'
1'
0'
2400
2200
2000
1800
1600
1400
1200
1000
800
600
400
200
0
Pounds.
9
5
6
Thrust at 2
Thrust at 1
Thrust at 0

PLATE 5

TABLE F

	Load at 10				Load at 9'				Load at 8'			
	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{I}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$
0	61	+388	+450	-326	53	+403	+456	-350	44.	+377	+421	-333
1	90	+599	+609	-509	78	+546	+624	-468	65	+626	+691	-561
2	186	+444	+630	-258	162	+531	+693	-369	16	+545	+681	-409
3	238	+125	+363	+113	209	+291	+500	-82	176	+384	+561	-207
4	277	-442	-165	+719	245	000	+245	+245	208	+75	+282	+133
5	317	-1003	-686	+1320	281	+643	-362	+424	239	-384	-46	+623
6	325	-1122	-797	+1447	289	-813	-524	+1102	247	-639	-392	+886
7	334	-801	-467	+1135	297	-1006	-709	+1303	253	-615	-362	+868
8	342	-426	-84	+768	306	-1147	-841	+1453	260	-719	-450	+979
9	351	+363	+714	-12	314	-1055	-741	+1369	268	-835	-567	+1103
10	363 363	+200	+563	+163	325	+450	+774	-124	279	-154	+125	+433
9'	351	+363	+714	-12	308 317	+1708	+2016 +2025	-1400 -1408	263	+835	+1098	-573
8'	342	-426	-84	+768	310	+611	+921	-301	254 268	+1700	+1954 +1967	-1446 -1432
7'	334	-801	-467	+1135	304	-359	-55	+663	263	+615	+878	-352
6'	325	-1122	-797	+1447	297	-747	-450	+1044	259	-348	-89	+607
5'	317	-1003	-686	+1320	290	-1093	-802	+1384	253	-934	-680	+1188
4'	277	-442	-165	+719	256	-836	-580	+1092	225	-941	-716	+1166
3'	238	+125	+363	+113	222	-291	-69	+513	198	-480	-282	+678
2'	186	+444	+630	-258	176	+66	+242	+110	159	-159	000	+318
1'	90	+599	+609	-509	87	+325	+412	-238	80	+90	+170	-10
0'	61	+388	+450	-326	60	+227	+287	-168	56	+97	+153	-41

PLATE 6

TABLE-F

	Load at 7'				Load at 6'				Load at 5'			
	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$
0	33	+315	+348	-281	23	+230	+254	-208	14	+152	+166	-138
1	49	+531	+580	-481	34	+392	+426	-358	21	+262	+283	-241
2	104	+482	+586	-378	72	+384	+456	-312	44	+260	+304	-216
3	135	+371	+506	-236	93	+319	+412	-226	58	+233	+291	-175
4	159	+232	+391	-73	110	+243	+353	-133	69	+203	+272	-134
5	184	-169	+15	+353	127	000	+127	+127	80	+93	+173	-13
6	189	-448	-258	+637	131	-187	+56	+318	82	-78	+4	+160
7	194	-568	-374	+762	135	-362	-227	+497	84	-207	-123	+291
8	200	-755	-555	+955	139	-526	-387	+665	87	-286	-199	+373
9	206	-751	-545	+957	143	-486	-343	+629	90	-305	-215	+395
10	215	-534	-319	+748	149	-371	-222	+520	93	-207	-213	+301
9'	203	+643	+847	-440	142	+336	+478	-194	89	+188	+277	-99
8'	193	+1208	+1400	-1015	137	+597	+734	-460	86	+330	+416	-244
7'	188 207	+1799	+1987	-1611	132	+1022	+1155	-891	83	+600	+683	-517
6'	205	+536	+732	-331	126 150	+1528	+1644 +1678	-1402 -1378	79	+880	+979	-801
5'	203	-508	-305	+710	150	+294	+444	-144	77 105	+1165	+1242 +1270	-1088 -1060
4'	182	-871	-688	+1053	138	-606	-468	+744	100	-190	-90	+290
3'	163	-606	-443	+769	127	-629	-502	+756	96	-556	-461	+651
2'	134	-359	-225	+493	107	-488	-381	+595	84	-536	-452	+620
1'	69	-145	-76	+214	57	-328	-271	+385	47	-445	-398	+492
0'	49	-36	+13	+85	41	-146	-106	+188	34	-220	-186	+254

PLATE 7

TABLE-F

	Load at 4'				Load at 3'				Load at 2'			
	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$	$\frac{T}{A}$	$\frac{Mc}{I}$	$\frac{T}{A}+\frac{Mc}{I}$	$\frac{T}{A}-\frac{Mc}{I}$
0	18	+90	+108	-72	4	+44	+48	-40	1.	+15	+16	-15
1	11	+154	+165	-143	5	+76	+81	-71	2.	+26	+28	-24
2	24	+153	+177	-129	11	+78	+89	-67	3	+27	+30	-24
3	31	+156	+187	-125	15	+79	+94	-64	5	+28	+33	-23
4	37	+153	+190	-116	18	+82	+100	-64	6	+32	+38	-26
5	43	+101	+144	-68	20	+62	+82	-42	7	+29	+36	-22
6	45	000	+45	+45	21	+20	+41	+1	7	+13	+20	-6
7	46	-57	-11	+103	22	-10	+11	+33	7	000	+7	+7
8	48	-108	-60	+156	22	-34	-12	+56	7	-9	-2	+16
9	49	-128	-79	+177	23	-55	-32	+78	8	-14	-6	+22
10	51	-71	-20	+121	24	-20	+4	+44	8	-4	+4	+12
9'	48	+103	+151	-55	22	+36	+58	-14	8	+12	+20	-4
8'	47	+181	+228	-134	22	+80	+102	-58	7	+26	+33	-19
7'	46	+317	+363	-271	22	+150	+172	-128	7	+49	+56	-42
6'	44	+428	+472	-384	21	+203	+224	-182	7	+66	+73	-59
5'	42	+607	+649	-565	20	+264	+284	-244	7	+86	+93	-79
4'	36 70	+569	+605 +639	-534 -500	17	+264	+281	-247	6	+86	+92	-80
3'	71	-266	-195	+337	14 56	+185	+200 +241	-170 -129	5	+60	+65	-55
2'	66	-449	-383	+515	55	-228	-173	+283	3 47	+47	+51 +94	-43 000
1'	39	-455	-416	+494	34	-356	-322	+390	30	+206	+236	-176
0'	29	-240	-211	+269	26	-211	-185	+237	24	+145	+169	-121

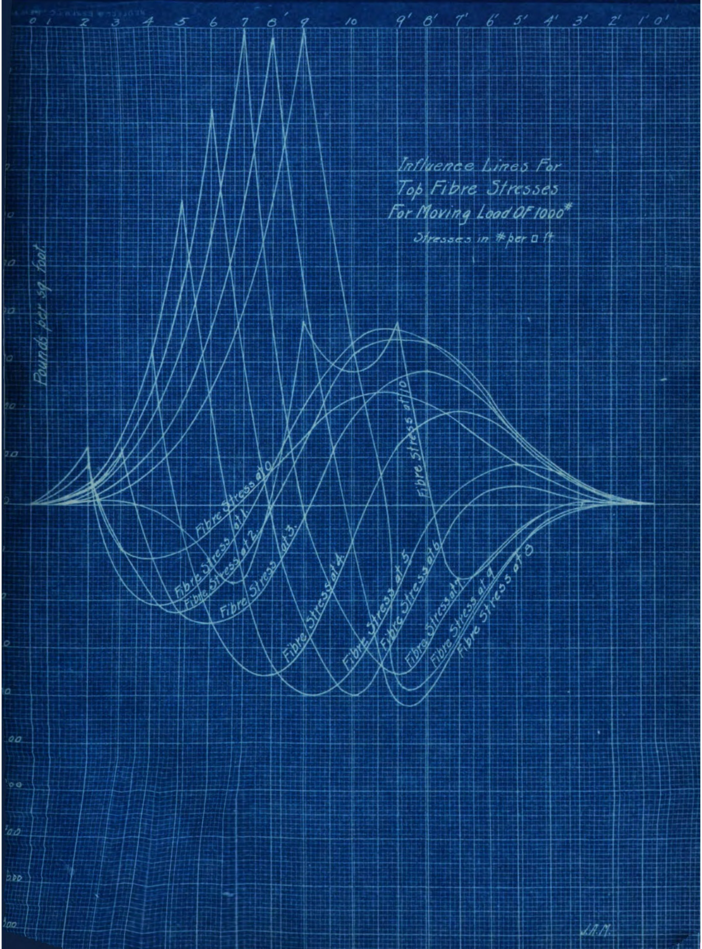
	Load at 10		Load at 9'	
	Top	Bottom	Top	Botto
0	+11140	-8070	+5877	-399
1	+17060	-12600	+8040	-608
2	+15600	-6387	+8925	-47
3	+7980	+2797	+6445	-105
4	-4080	+17800	+3158	+31
5	-16900	+32680	-4660	+119
6	-19700	+35870	-6750	+142
7	-11560	+28100	-9140	+167
8	-2079	+19010	-1083	+187
9	+17670	-297	-9540	+176
10	+13900	+4035	+9980	-15
9'	+17670	-297	+26050	-181
8'	-2079	+19010	+11890	-38
7'	-11560	+28100	-708	+85
6'	-19700	+35870	-5800	+13
5'	-16900	+32680	-10340	+17
4'	-4080	+17800	-7470	+14
3'	+7980	+2797	-889	+6
2'	+15600	-6387	+3120	+14
1'	+17060	-12600	+5310	-3
0'	+11140	-8070	+3699	-21
	24.755		12.888	

Note:- Stresses in above

EARTH BACKING
BRICK PAVEMEN
CONCRETE -
REINFORCING R

Pt.	Earth Fill	Pave
1	31700*	24
2	25400	24
3	19100	24
4	13900	2
5	10550	2
6	7750	2
7	6620	2
8	4500	2
9	4100	2
10	7800	4

0'



0 1 2 3 4 5 6 7 8 9 10 9' 8' 7' 6' 5' 4' 3' 2' 1' 0'
Influence Lines For
Top Fibre Stresses
For Moving Load Of 1000#
Stresses in # per □ ft.
Pounds per sq. foot
Fibre Stress at 0
Fibre Stress at 1
Fibre Stress at 2
Fibre Stress at 3
Fibre Stress at 4
Fibre Stress at 5
Fibre Stress at 6
Fibre Stress at 7
Fibre Stress at 8
Fibre Stress at 9
Fibre Stress at 10
J.A.M.

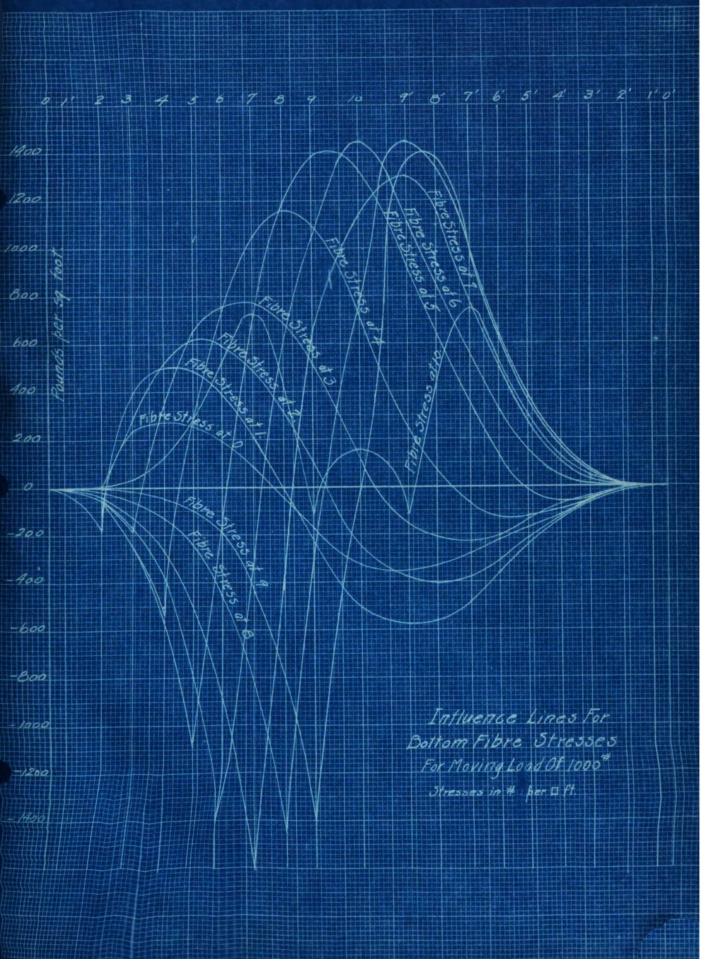

Influence Lines For
Bottom Fibre Stresses
For Moving Load Of 1000#
Stresses in #. per □ ft.
Pounds per sq foot
Fibre Stress at 0
Fibre Stress at 1
Fibre Stress at 2
Fibre Stress at 3
Fibre Stress at 4
Fibre Stress at 5
Fibre Stress at 6
Fibre Stress at 7
Fibre Stress at 8
Fibre Stress at 9
Fibre Stress at 10
0 1' 2 3 4 5 6 7 8 9 10 9' 8' 7' 6' 5' 4' 3' 2' 1' 0'
1400
1200
1000
800
600
400
200
0
-200
-400
-600
-800
-1000
-1200
-1400

Plate 9.

Note:- All stresses are in Pounds per Sq. Inch; Plus denotes Comp., Minus - Tension

	Concrete Stresses						Steel Str.	
	Dead Load		Live Load		Max. Combine Stresses		Max. Steel Stresses	
	Top Fibre	Bottom	Top	Bottom	Top	Bottom	Top	Bottom
0	+262	-100	+50	-42	+312	-142		
1	+333	-114	+80	-62	+413	-176	+4280	-1410
2	+259	+142	+75	+50	+334	+192	+4575	+3300
3	+202	+255	+54	+62	+256	+317	+4650	+4725
4	+170	+337	+33	+98	+203	+435	+3375	+6300
5	+238	+289	+71	+135	+309	+360	+4740	+5325
6	+334	+258	+97	+132	+431	+390	+6600	+7300
7	+407	+176	+118	+116	+525	+292	+7410	+4800
8	+435	+228	+129	+127	+564	+355	+8000	+5500
9	+375	+243	+96	+119	+471	+352	+6825	+5500
10	+70	+544	+49	+55	+119	+599	+2700	+8175

Position of Loading For Max. Top Fibre Stresses

Position of Loading For Max. Bottom Fibre Stresses.

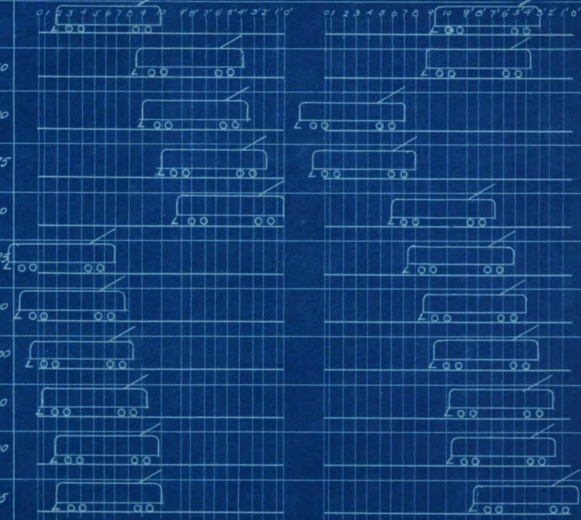


PLATE 10

TABLE H.

Total - Shears In Pounds.

Pt.	Load at 10	Load at 9'	Load at 8'	Load at 7'	Load at 6'	Load at 5'	Load at 4'	Load at 3'	Load at 2'
0	-37060	-18260	-23560	-15540	-12509	-8820	-6733	-4142	-1955
1	-32930	-16370	-15180	-14060	-11330	-8018	-6096	-3855	-1857
2	-23760	-12220	-11540	-10800	-8757	-6254	-4793	-3062	-1417
3	-15030	-8288	-7980	-7644	-6273	-4570	-3602	-2305	-978
4	-6684	-4420	-4558	-4539	-3898	-2887	-2272	-1549	-733
5	-1857	-2191	-2570	-2787	-2448	-1904	-1579	-1081	-586
6	-693	-1031	-1515	-1784	-1704	-1403	-1247	-1081	-489
7	+3020	0	-541	-924	-1052	-862	-831	-720	-391
8	+5794	+1288	+541	+796	-217	-361	-471	-360	-244
9	+8417	+2526	+1623	+1115	+544	+200	+138	0	0
10	+1238 -1238	+4382	+3313	+2548	+1740	+1002	+693	+360	+244
9'	+8417	-6186 +6702	+3313	+2548	+1740	+902	+775	+540	+293
8'	+5794	-5619	-5883 +7628	+4857	+3626	+2305	+1579	+901	+342
7'	+3020	-4317	-6424	+5956 -9874	+4351	+2806	+2022	+1081	+489
6'	-693	-3029	-5234	-8760	+5167 -12680	+3408	+2438	+1369	+586
5'	-1857	-1804	-4125	-7803	-11780	+4010 -15640	+2771	+1441	+733
4'	-6684	+773	-1758	-5478	-9880	-14170	+3604 -22770	+2161	+978
3'	-15030	+4640	+1826	-2070	-6890	-11730	-20360	+2882 -29720	+1222
2'	-23760	+8828	+5748	+1815	-3445	-8820	-17180	-26440	+1466 -39100
1'	-32930	+13400	+10150	+6163	+634	-5152	-13160	-21970	-33730
0'	-37060	+15720	+12370	+8520	+2864	-3147	-10800	-19270	-30500

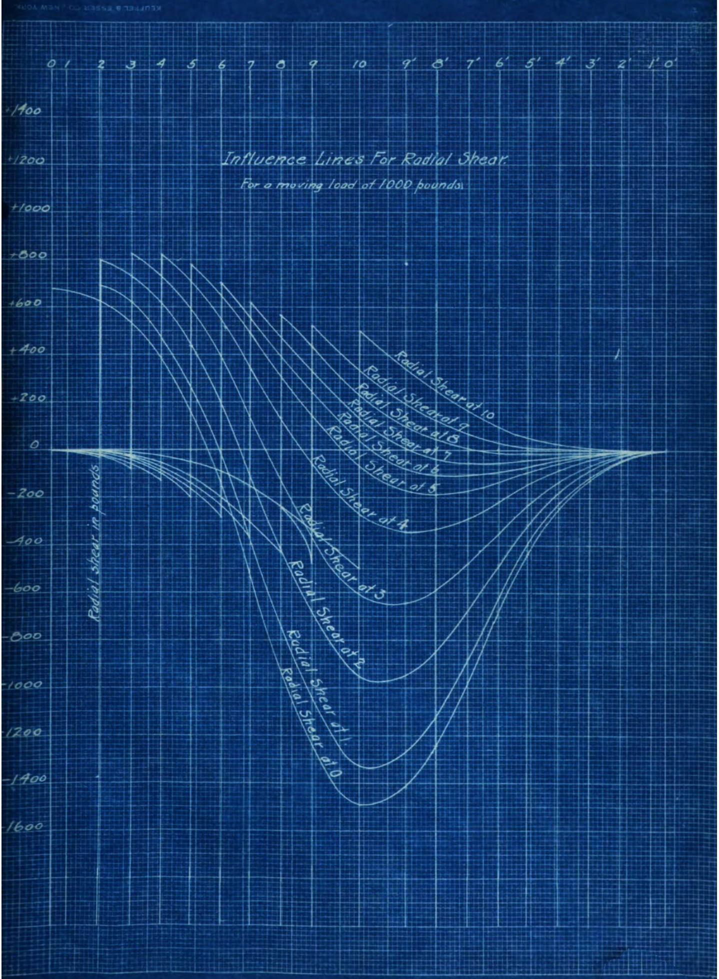

Influence Lines For Radial Shear.
For a moving load of 1000 pounds.
0 1 2 3 4 5 6 7 8 9 10 9' 8' 7' 6' 5' 4' 3' 2' 1' 0'
+1400
+1200
+1000
+800
+600
+400
+200
0
-200
-400
-600
-800
-1000
-1200
-1400
-1600
Radial shear in pounds
Radial Shear at 10
Radial Shear at 9
Radial Shear at 8
Radial Shear at 7
Radial Shear at 6
Radial Shear at 5
Radial Shear at 4
Radial Shear at 3
Radial Shear at 2
Radial Shear at 1
Radial Shear at 0

Plate II.

Shears in Pds./Sq. In.			
	Dead	Live	Max.
0	-32	-5	-37
1	-45	-6	-41
2	-88	-9	-97
3	-78.5	-6.8	-85.3
4	-60	-3.8	-63.8
5	-43	-2.2	-45.2
6	-35.7	-1	-36.7
7	-5.7	-.55	-6.25
8	+15.5	+5.8	+21.3
9	+24.5	+5.8	+30.3
10	+29	+6.0	+35.

Position of Load for Max. Shears.

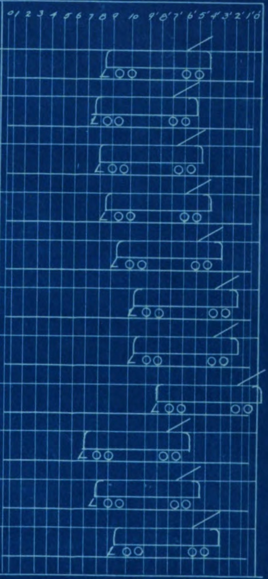
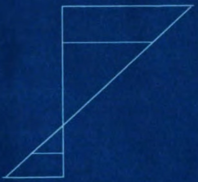


Plate 12.

DIAGRAMS OF FIBRE STRESSES FOR DETERMINING STRESSES IN STEEL.

Vertical Scale-1" = 4' Horizontal Scale-1" = 300#

Section No. 1.



Section No 2.

Section No 3

Section No. 4.

Section No. 5.

Section No 6.

Section No. 7.

Section No 8.

Section No. 9.

Section No. 10.

USE ONLY