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THESIS
A STRESS ANALYSIS
OF SILO CONSTRUCTION

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1915

THESIS

Strains + stresses
Wind-pressure
Title: Silo construction

SUPPLEMENTARY
MATERIAL
IN BACK OF BOOK

This thesis was contributed by

B. Giffels

~~R. F. Giffels~~

under the date indicated by the department stamp, to
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A Stress Analysis of Silo Construction


A Thesis Submitted to

The Faculty of

MICHIGAN AGRICULTURAL COLLEGE

By


R.F. Giffels


B. Giffels

Candidates for the Degree of

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THESIS

COPIES OF
MATERIAL
IN BACK OF BOOK

SCOPE OF THESIS.

In this thesis it has been our purpose to first determine how, in general, stresses are developed in silos, due to all the destroying agencies that may act upon them. We next determined what the usual practice in silo design is, by analyzing several of the modern types of silos. Analyses were made of wood stave, reinforced concrete, metal, and vitrified tile silos. Our results enabled us to decide as to the rationality or irrationality of the present accepted theory of ensilage pressures, and to recommend changes in the assumed pressures of the ensilage. We were also enabled to recommend a method of determining by experiments what stresses actually occur in the silos, and from these results to determine the most probable theory for the amount and direction of the internal forces.

FOREWORD.

The original intention in taking up this thesis was to determine the amount of the internal forces acting in a silo, and the manner in which they act. The reason for doing this was the belief that no very complete and accurate theory for these forces had ever been worked out.

Our intention was to apply Jansen's Theory for the pressures in tall bins to this field. However a thorough study of this theory showed us that it could not be satisfactorily applied. Two reasons for this became apparent. In the first place, Jansen's Theory was worked out for granular substances, such as grain, coal, sand, and similar materials. Ensilage is a fibrous material, entirely unlike those just named. Hence the theory could not be used with any assurance that correct values would be determined. Secondly, if Jansen's theory were to be used, extensive experiments would be required to work out the constants involved; namely, the ratio between the vertical and horizontal pressures in the substance, and its angle of repose. For these reasons this theory was abandoned.

We next attempted to investigate the values for silo pressures developed experimentally by the late Prof. F.H.King, of the Wisconsin Agricultural Experiment Station. Considerable time was spent in trying to determine the methods used by Prof. King in working out his experiments. In this we were unsuccessful (See letter from Mr. F.M. White, Agricultural Engineering Dept, Wisconsin University, in insert.) Prof. King determined

the horizontal pressure increment to be eleven lbs. per square foot per foot in depth. This seems to be quite generally accepted among the agricultural colleges, as it is used by Wisconsin Agricultural College, Iowa Agricultural College (Bulletin 141, Iowa Agr. Exp. Station), and has been used in design by Prof. H.H. Musselman of the Farm Mech. Department of The Michigan Agricultural College.

For investigation we selected representative silos of each common type. In making this selection we took those for which we had the most complete details of construction. The silos investigated were: The Ross Wood Stave Silo; the Ross In-De-Str-Uct-O galvanized metal silo, both made by The E.W. Ross Co., Springfield, Ohio; the Zyro galvanized metal silo, made by the Canton Culvert & Silo Co., Canton, Ohio; The Guernsey vitrified glazed hollow tile silo, made by The Guernsey Clay Co., Indianapolis, Indiana; and the Hy-Rib Concrete silo, made by the Trussed Concrete Steel Co., Youngstown, Ohio.

This thesis has been made possible only by the generosity of several manufacturers of silos. We especially wish to thank The E.W. Ross Co., The Canton Culvert & Silo Co., The Guernsey Clay Co., and The Trussed Concrete Steel Co. We also wish to thank Prof. C.A. Melick, and Prof. H.H. Musselman for advice and suggestions. We are indebted to "Walls, Bins, and Grain Elevators", by Ketchum, and "Modern Silo Construction", Bulletin 141, Iowa Experiment Station. Values for wind pressures were taken from the "American Civil Engineer's Pocketbook".

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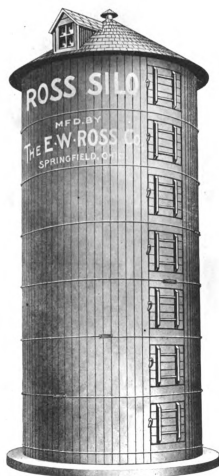
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ROSS Thoroughly Manufactured WOOD STAVE SILO



Licensed under Harter Pat. No. 621131

Furnished in "LONGLEAF" YELLOW PINE, "OREGON" FIR
OR OTHER SUITABLE MATERIALS

ROSS WOOD STAVE SILO.

ANALYSIS OF ROSS WOOD STAVE SILO.

This silo is 40 ft. high and 24 ft. in inside diameter. As shown by the photograph on the preceding page, the construction consists of 2"x 4"^{staves} set vertically and bound together by round steel hoops threaded at each end, and malleable iron lugs.

On page 9 of the manufacturers catalog included in the pocket is given a table of the number and size of hoops for each size silo. This 40ft. silo has 6 $\frac{3}{4}$ " hoops at the bottom and 10 $\frac{5}{8}$ " hoops above. We assumed the hoops spaced 31" apart, leaving a distance of $7\frac{1}{2}$ " from the top hoop to the top of the silo, and from the bottom hoop to the bottom of the silo.

In figuring the pressure that comes on one hoop we assumed that the pressure that comes on that length of stave from a point half the distance to the next lower hoop, or to the bottom of the silo in the case of the lowest hoop, to a point half the distance to the next higher hoop, is carried by each hoop.

The numbering of the hoops in the following table of stresses is from the top down, the top hoop being number 1. Areas of hoops given are areas at the root of a standard thread, as taken from the American Civil Engineer's Pocket-book.

The unit stress necessary in all hoops to prevent

deformation by shear between staves when empty is computed, rather than full, because of the additional resistance offered by the ensilage when the silo is full.

TABLE OF HOOP STRESSES DUE TO INTERNAL PRESSURE.

Hoop. #	Stave length transmitting head. pressure. "	Mean head. '	Area of hoop sq.in.	Tension in hoop. #	Unit Stress. lbs/sq.in.
1	23	11.5	.202	241.5	1195
2	31	38.5	.202	1094.0	5410
3	31	69.5	.202	1975	9770
4	31	100.5	.202	2860	14200
5	31	131.5	.202	3740	18500
6	31	162.5	.202	4620	22850
7	31	193.5	.202	5500	27200
8	31	224.5	.202	6380	31600
9	31	255.5	.202	7250	35900
10	31	286.5	.202	8150	40300
11	31	317.5	.302	9020	29900
12	31	348.5	.302	9920	32850
13	31	379.5	.302	10800	35800
14	31	410.5	.302	11700	38800
15	31	441.5	.302	12550	41600
16	23	468.5	.302	9880	32700

WIND STRESSES WHEN EMPTY.

On cylindrical surfaces a wind velocity of 120 mi/hr. gives a pressure of 20 lbs. per square foot of section normal to the direction of the wind. (A.C.E. Handbook, page 493)

$$\text{Moment of wind on staves} = 24 \times 40 \times 20' \times 20' = 384000 \text{ ft.lbs.}$$

Assuming an additional height of 4 ft. for roof,

$$\text{Moment of wind on roof} = 4 \times 24 \times 20' \times 42' = 80600 \text{ ft.lbs.}$$

$$\text{Total moment} = 384000 + 80600 = 464600 \text{ ft. lbs.}$$

This moment produces shear between the different staves,

this shear being greatest in a plane through the center of the silo, perpendicular to the direction of the wind.

Let S = Total shear in section through the center.

" M = Moment in the silo walls

(Treating whole structure as a beam
fixed at one end)

Let \bar{Y} = Distance from centre of stress in half-section to neutral axis of section.

Assuming unit stress at unit distance from neutral axis: $M = f(y^2)$ or $M = \sum(dl \cdot y^2) = \sum dl (R \sin \theta)^2$ (See Fig)

Let F = Total stress in half-section.

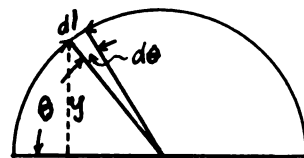
$$F = \sum(y \, dl) = \sum(dl \cdot R \sin \theta)$$

$$\bar{Y} = M/F$$

$$= \frac{\int_0^{\pi/2} \frac{1}{2} R^3 (1 - \cos 2\theta) \, d\theta}{\int_0^{\pi/2} \frac{1}{2} R^2 \sin \theta \, d\theta}$$

$$(dl = R \, d\theta)$$

$$= \left[\frac{\frac{1}{4} R (\theta - \frac{1}{2} \sin 2\theta)}{-\cos \theta} \right]_0^{\pi/2} = \pi R/4 = .7854 R$$



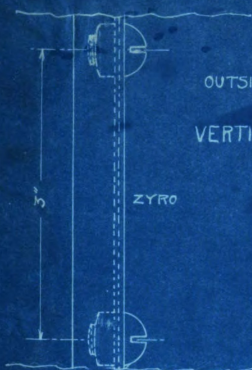
$$\text{But } S = M/\bar{Y} = \frac{464600}{.7854 \times 24} = 24600 \text{ lbs.}$$

Assuming coefficient of friction of wood on wood to be .25,

Total stress necessary in hoops = $\frac{24600}{2 \times .25} = 49200$ (lbs)

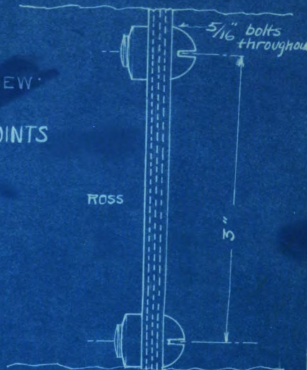
Total area of hoops = 3.832 sq. in.

Unit stress in hoops = $49200 \div 3.832 = 12800$ lbs. per sq.in.



OUTSIDE VIEW
OF
VERTICAL JOINTS

ZYRO

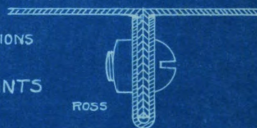


ROSS



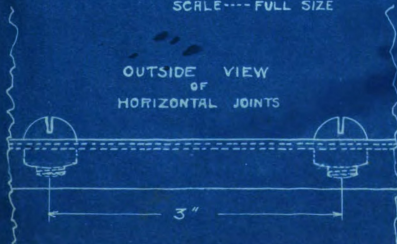
CROSS-SECTIONS
OF
VERTICAL JOINTS

ZYRO



ROSS

DETAILS OF JOINTS
OF
ZYRO AND ROSS
METAL SILOS
SCALE----FULL SIZE



OUTSIDE VIEW
OF
HORIZONTAL JOINTS

SECTION
OF
HORIZONTAL JOINTS



ANALYSIS OF METAL SILOS.

Both metal silos tested are made of sheets of galvanized iron of varying thicknesses, according to the position in the silo at which they are used. The ends of the curved sheets are fastened together with bolts thru the vertical-outstanding flanges, as shown by the blue-print of the joint, to form a circular ring. The rings are bolted together thru horizontal outstanding flanges.

The difference in the construction of the vertical joint caused us to analyze both silos.

The stress in any one ring of the silo is figured for the head that acts on a point three inches above the bottom of that ring, except in the case of the bottom ring, which is set in concrete, in which case it is figured for one foot above the bottom of the silo.

The unit stress at the root of the thread of the bolts is figured. The horizontal tension and the combined vertical compressive stresses in the body of the walls were figured, and then combined according to the method given on page 225, American Civil Engineer's Handbook. For the ratio between vertical pressure and the increase in horizontal tension caused by that pressure, we used 3 : 1.

Due to the special construction of the vertical flanges, a method of computing the stress in each bolt had to be developed

Development Of Formula for Stress per Bolt in Vertical Joint Due to Unit Horizontal Tension.

Assume the bolts in the vertical flange to be drawn up snugly so that that portion of the flange outside of the gauge line will remain fixed in position, even though that portion inside the gauge line is deformed by a horizontal tension in the body wall. Assume that there is the same bending in the section through the corner of the angle that there is in the section along the gauge line for the bolt holes. If this assumption is true the bending moments in the two sections mentioned are proportional to their sectional areas.

(See blue-print of flanges)

Let a = distance from back of angle to gauge line.

" A' = section area of corner of angle for a length equal to spacing of bolt holes.

Let A'' = Section area of metal between two adjacent bolt holes along the gauge line.

Let M' = Moment in A' due to P

" M'' = Moment in A'' due to P

$$Pa = M' + M'' \quad \text{and} \quad M' : M'' = A' : A''$$

$$M' = A'M''/A''$$

$$Pa = M''(1 + A'/A'') \quad \text{and} \quad M'' = \frac{A''Pa}{A'' + A'}$$

Let b = distance from gauge line to the center of pressure of the pressure existing between the outer halves of the two flanges. Let R = The amount of this pressure. In order that the summation of moments about the gauge line shall equal 0, $R \cdot b$ must equal M'' .

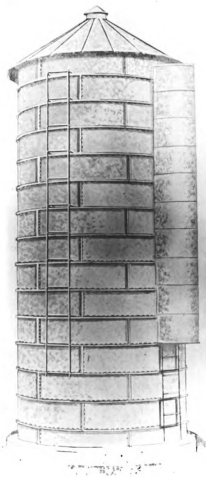
$$R b = M''$$

$$\text{Then } R = M''/b = \frac{A'' \cdot P \cdot a}{(A'' + A')b}$$

Total pressure on a bolt = $R + P$

$$T = P \left(1 + \frac{A'' \cdot a}{(A'' + A')b} \right)$$

ROSS IN-DE-STR-UCT-O SILO
Galvanized METAL



TRADE MARK  GUARANTEED BY METAL

ROSS METAL SILO.

Ross In-De-Str-Uct-O Met-1 Silo.

It will be seen from the blue-print on the preceding page and the cuts in the manufacturers catalog that an apron flange is used for the horizontal joints, but in the vertical joints a U-shaped reinforcing strip is put over the flanges and the bolts put through four thicknesses of metal.

As noted on page 23 of the catalog, the spacing of the bolts is 3 inches and their diameter is $5/16"$.

On page 44 of the catalog is a table of the sizes, weights, capacities, and the number of circles of the different gauges of material used.

For the silo, 46' high and 23' $10\frac{1}{2}"$ inside diameter, and capacity of 429 tons^{wt}, 14,565 lbs; there are 7 circles of 14 gauge, 13 circles of 16 gauge, and 3 circles of 18 gauge.

Thickness of 14 gauge metal is 0.078125", of 16 gauge is 0.0625", of 18 gauge is 0.05" (Page 394, Am. C.E. Handbook)

As we were unable to obtain sufficient data from the manufacturer regarding dimensions of the vertical joints, (See letter in insert) we were obliged to make assumptions as to the position of the gauge line on the flange. We therefore assumed that the portion of the flange outside the gauge line is the same width as the distance from the gauge line to the back of the angle. The assumption that the pressure between the backs of the outer portions of the flanges varies directly as the distance from the gauge line

gives the following formulas:

$$b = \frac{2}{3} a \quad T = P \left(1 + \frac{A'' a}{(A'' + A') b} \right)$$

Assuming thickness of reinforcing strip to be same as thickness of body wall, (Thickness = t)

$$A' = 3 \cdot t \text{ sq. in.} \quad A'' = 2 \frac{9}{16} (2t) = 5 \frac{1}{8} t$$

$$T = 1.946 P$$

The area at the root of the thread of a 5/16" bolt is .045 sq.in.

In the following table of circles and stresses the circles are numbered consecutively from the bottom, the lowest circle being No. 1.

TABLE OF MAX STRESSES COMING IN THE LOWEST CIRCLE OF EACH GAUGE OF METAL. 23' 10½" x 46' Silo.

No. of rim	Thick- ness. (Inches)	Eff. head (ft.)	Tens. per 3" of ht. (lbs)	Tension in bolt (lbs)	Unit stress in bolt (#/sq.in)	Unit Tens. in body wall (#sq.in.)
1	.078125	45.00	1343	2618	58100	5740
8	.0625	31.75	949	1843	41000	5060
21	.05	5.75	172	334	7420	1145

COMPRESSIVE STRESSES

Assuming a coefficient of friction of 0.35 for ensilage on galvanized iron, the total downward force exerted by the ensilage on the silo wall for any height H on a linear foot of horizontal section is equal to $\frac{1}{2} H^2 \times 0.35 = 1.925 H^2$

Since weight of silo equals 14565# and height is 46', assume weight per foot equal to 317#. Effective perimeter is 3.1416 x 23' 10½" = 75'. Average weight per square foot is 317#/75 = 4.225lbs.

Compressive Stress Due to Wind.

Max. vertical compressive stress at any section, due to a moment M , equals $Mc/I = \frac{Mr}{\pi r^2(2/\pi)} = M/4R^2t$
 where t = thickness of metal in inches, R = radius of silo in inches, and M is in inch pounds.

Assume effective height of roof to be 4 ft.

Then moment in any ~~any~~ section in inch pounds is equal to:

$$\begin{aligned} & (\text{Hd. of silage} + 4')^2 \cdot \frac{1}{2} \cdot 20(23.875 \times 12) \\ & = 2860(\text{Eff. Hd.} + 4')^2 \end{aligned}$$

TABLE OF COMPRESSIVE STRESSES DUE TO WEIGHT OF METAL AND FRICTION OF ENSILAGE.

No. of rim.	Thickness of metal. (inches)	Effective head. (feet)	Comp. stress due to wt. of metal (#/sq. in.)	Comp. stress due to friction (#/sq. in.)
1	.078125	45	228	4160
8	.0625	31.75	179	2580
21	.05	5.75		106

TABLE OF COMPRESSIVE STRESSES DUE TO WIND.

No. of rim	Thickness of metal (inches)	Effective head. (feet)	Wind moment in section (inch lbs)	Comp. stress due to wind (#/sq. in.)
1	.078125	45	6,860,000	1070
8	.0625	31.75	3,650,000	6215
21	.05	5.75	272,000	1202

TABLE OF COMBINED STRESSES.

No. of rim.	Direct tensile stress (#/sq.in.)	Total compressive stress. (#/sq.in.)	Combined tensile stress (Due to direct tension and compression) (#/sq. in.)
1	5740	5458	7560
8	5060	3469	6215
21	1145	171	1202



ZYRO METAL SILO.

THE ZERO NETTAL SILO.

From the blue-print shown preceding the discussion of the Ross silo, it will be seen that the horizontal joint of the Zyro silo is the same as that of the Ross. The vertical joint, however, is different, and in the Zyro silo consists of an "L" flange and an apron flange.

Standard 5/16" bolts spaced 3" apart, center to center, are used in the outstanding flanges. (See page 16 of catalog)

For table of weights and sizes of silos see pages 22 and 23 of catalog. In the 19'-9" x 50'-0" silo (The size analyzed) ^{wt. is} 16070 lbs. without the roof and 14299 lbs. ^{wt. is} without the roof. There are three rims of 12 gauge, 8 rims of 14 gauge, and 13 rims of 16 gauge.

In the analysis of the flanged vertical joint it is necessary that one should have certain dimensions. When we wrote to the manufacturer for this data we received no reply. Hence we assumed that the width of that portion of the flange outside of the gauge line is the same as that portion inside of it. Then assuming, as in the analysis of the Ross silo, that the pressure between the backs of the outstanding flanges varies as the distance from the gauge line, we derive the following formulae:

$$b = \frac{2}{3} a \quad T = P \left(1 + \frac{A'' \cdot a}{(A'' + A') b} \right)$$

$$A' = 3 t \text{ sq. in.}$$

$$A'' = 2 \frac{9}{16} t \text{ sq. in.} = 2.562 \text{ sq. in.}$$

$$T = P \left(1 + \frac{2.562 \text{ ta}}{t(2.562 + 3)(2/3 a)} \right)$$

$$T = 1.692 P$$

(For derivation of above formulae and meaning of letters used see discussion of Ross metal silo.)

In the following table of rims and stresses the rims are numbered consecutively from the bottom, the lowest rim being Number 1.

No. of rim.	Thickness (inches)	Eff. head (ft)	Tens. per 3" of ht. (lbs)	Tension in bolt (lbs)	Unit stress in bolt at root of thd (lbs/sq.in.)	Unit tens in wall. (lbs/sq.in)
1	.109375	49	1329	2246	50,000	4,055
4	.078125	43.5	1182	2000	44,500	5,040
12	.050000	26.8	954	1610	35,800	6,360

Compressive Stresses.

As developed in the discussion of the Ross metal silo, the compressive stress, due to friction of ensilage, in one foot of horizontal section is $1.925 H^2$, where H is the effective head of ensilage on the section.

The total weight of the silo, with the roof, is 16070 lbs. (See catalog.) Wt. of silo without roof is 14299 lbs. Wt. of roof is 1761 lbs. If the weight were evenly distributed over the whole silo the average weight per square-foot of surface of wall would be equal to:

$$14299 / (50 \times \pi \times 1975) = 4.61 \text{ lbs. per sq. in.}$$

As the weight of the roof is carried by (3.1416 X 19.75)ft the additional stress on each foot of horizontal section is $1761/(3.1416 \times 19.75) = 28.4$ (lbs. per foot)

Wind Stresses.

Assuming the effective diameter of silo, including 40" for depth of feeding chute, is 23', and the effective height of the roof is 3', the moment in any section due to a wind load of 20 lbs. per sq. ft. of cross section is equal to M. (M is in inch pounds)

$$M = \frac{1}{2} \cdot 20 (23 \times 12) (\text{Eff. hd.} + 3)^2 = 2760 (\text{Eff. hd.} + 3)^2$$

Stress due to M is S.

$S = M/(4 R^2 t)$ where R is radius of silo and t is thickness of wall.

TABLE OF COMPRESSIVE STRESSES DUE TO WEIGHT OF METAL AND FRICTION OF ENSILAGE.

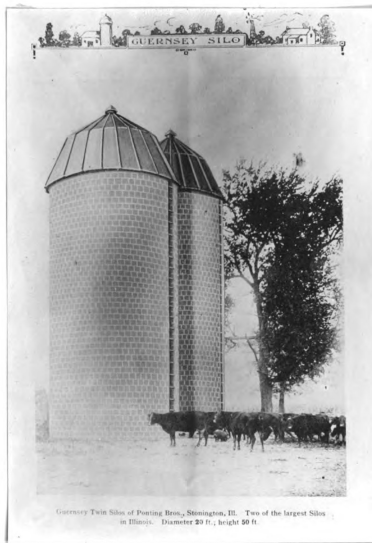
No. of rim	Thickness of metal (inches)	Eff. head (ft)	Comp. stress due to wt. of metal. (#/sq. in.)	Comp. stress due to friction. (#/sq. in.)
1	.109375	49.0	194	3580
4	.078125	43.5	244	3280
12	.050000	26.8	630	2405

TABLE OF COMPRESSIVE STRESSES DUE TO WIND.

No. of rim	Thickness of metal (inches)	Eff. head (ft.)	Wind moment in section (inch lbs)	Comp. stress due to wind (lbs/sq.in.)
1	.109375	49.0	7,465,000	1210
4	.078125	43.5	5,974,000	1353
12	.050000	26.8	2,450,000	868

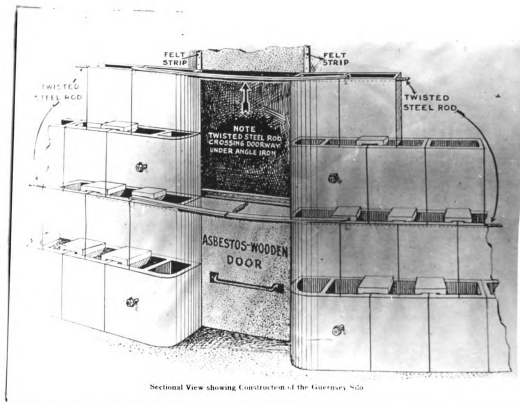
TABLE OF COMBINED STRESSES.

No. of rim.	Direct tensile stress	Total compressive stress	Combined tensile stress (Due to dir- ect tension and effect of compress.)
	(lbs./sq.in)	(lbs./sq.in.)	(lbs./sq.in.)
1	4055	4984	- 6336
4	5040	4877	+ 6666
12	6360	3893	+ 7658



Guernsey Twin Silos of Ponting Bros., Stonington, Ill. Two of the largest Silos in Illinois. Diameter 20 ft., height 50 ft.

GUERNSEY VITRIFIED TILE SILO.



SECTIONAL VIEW
OF
GUERNSEY VITRIFIED TILE SILO.

ANALYSIS OF GUERNSEY VITRIFIED TILE SILO.

This silo is 60' high and 20' in inside diameter. The photographs on the preceding page show the construction. The walls are made of hollow vitrified tile, one foot in height, fastened together with mortar joints and tile clamps, and reinforced with 7/16" square steel bands, twisted cold. (See catalog in pocket, pages 4, 10, 13, and 15.) In this size silo the bands are laid in every course of tile for the first ten courses, and every second course from the tenth to the top of the silo. (See letter in pocket) In the first ten courses every second band crosses the door opening, under the ladder step, which is a steel angle. This angle also carries tension, and thus makes this part of the circumference as strong as the rest. The entire bursting pressure was assumed to be taken by the bands, though the tile clamps and mortar joints really take part of the tension.

Since the same size bands are used throughout, it was only necessary to compute the stresses in the lowest band, and in the lowest one of the bands which are spaced two courses apart. That is, the stresses were figured in the No. 1 band and in the No. 11 band, numbering from the bottom upward.

We also tested this silo for overturning, due to wind load, when empty. The weight of the silo was approximately determined by weighing a 15" by 15"

tile, made by another manufacturer, and assuming that the weight per unit of surface was the same for this silo.

TABLE OF STRESSES IN REINFORCING BANDS.

Guernsey Vitrified Tile Silo. 20' x 60'.

No. of band.	Area of band. (sq. in.)	Spacing of bands. (ft)	Head. Pressure (ft) (#/sq.ft.)	Tension (lbs)	Unit Stress in band in band. (#/sq.in)
1	.1907	1	59	649	33000
11	.1907	2	48	528	55900

Overturning Moment Due to Wind. 12' x 50' Silo.

In testing for overturning a different size silo was used than in testing for bursting pressure. In each case the aim was to test that size silo which was most likely to fail from the action of the forces for which it was tested.

Allowing 6" for thickness of wall, area of cross-section is $13 \times 60 + (6 \times 13) = 728$ sq. ft.

(Assuming 6' additional height for roof)

Moment of wind $M' = 728 \times 20 \times 28 = 408000$ ft. lbs.

Weight of 15"x 15" tile = 50lbs.

Therefore assume weight of wall per square foot of surface equal to :

$$50\left(\frac{12}{15}\right)^2 = 32 \text{ lbs.}$$

Weight of total surface

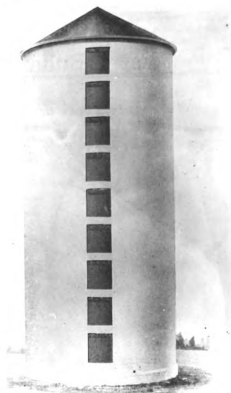
$$= 12 \times 3.1416 \times 50 \times 32 = 60,400 \text{ lbs.}$$

Using an effective lever arm of 5 feet, the resisting

moment M'' , of this weight $= 5 \times 60400 = 302000$ ft. lbs.

This would not resist a wind pressure of 30 lbs. per square foot on a flat surface, or 20 pounds on a cylindrical surface, but would just resist a pressure of 22.2 pounds per square foot on a flat surface, which would be produced by a wind velocity of 86 mi. per hour.

($p = a v^2$, when $a = .003$. See A.C.E. Handbook.)



HY-RIB CONCRETE SILO.

HY-RIB CONCRETE SILO.

ANALYSIS OF HY-RIB CONCRETE SILO.

A silo 50' high and 20' in inside diameter, of this make, was analysed. The walls are $3\frac{1}{2}$ " thick in the lower half of the silo, and 3" thick in the upper half. The walls are made by plastering concrete on Hy-Rib, which is expanded steel. The construction is shown by the accompanying blue-prints and the catalog in the pocket.

The cross-sectional areas of the strips of Hy-Rib were determined from a Hy-Rib handbook. We assumed that 22 gauge Hy-Rib was used for the lower one-third of the walls, 24 gauge for the next one-third, and 26 gauge for the top one-third. (See letter in pocket.) The steel was assumed to carry all the bursting pressure.

The compressive stress in the concrete was computed, taking into consideration the weight of the structure, the weight of the ensilage, and the wind stresses. The unit weight of concrete was taken to be 155 lbs. per cubic foot. The coefficient of friction of the ensilage on the silo walls was taken to be .40. The wind pressure was taken, as with the other silos, to be 20 lbs. per sq. ft. of cross section.

The bands referred to in the tabulations of stresses are numbered from the top downward. Each band is a strip of Hy-Rib 2 feet wide, except between the doors, where the strips are 1'-5" wide, and are reinforced with $\frac{3}{8}$ " Rib bars 10' long.

TENSION IN HY-RIB. 20' x 50' SILO.

No. of band.	Mean Head on band.	Mean Head between doors	Total Tens. in band.	Tot. Tens. between Do.
	ft.	ft.	lbs.	lbs.
1	1		220	
2	5		660	
3	5	4.71	1100	2070
4	7		1540	
5	9	8.71	1980	3830
6	11		2420	
7	13	12.71	2860	5600
8	15		3031	
9	17	16.71	3740	7360
10	19		4180	
11	21	20.71	4620	9110
12	23		5060	
13	25	24.71	5500	10870
14	27		5940	
15	29	28.71	6380	12630
16	31		6820	
17	33	32.71	7260	14400
18	35		7700	
19	37	36.71	8140	16150
20	39		8580	
21	41	40.71	9020	17900
22	43		9460	
23	45	44.71	9900	19600
24	47		10340	
25	49		10780	

AREAS AND STRESSES IN BANDS BETWEEN DOORS.

Band No.	No. of Rib bars	Area Gauge Rib of bars.Hy-Rib.	Area of Hy-Rib.	Total area band.	Total stress lbs.	Unit stress #/sq.in.
		sq.in.	sq.in.	sq.in.		
3	1	.1406 28	.246	.3866	2070	5350
5	1	.1406 28	.246	.3866	3830	9900
7	1	.1406 28	.246	.3866	5600	14500
9	1	.1406 28	.246	.3866	7360	19000
11	1	.1406 24	.328	.4686	9110	19500
13	2	.2812 24	.328	.6092	10870	17850
15	2	.2812 24	.328	.6092	12630	20800
17	3	.4218 24	.328	.7498	14400	19200
19	3	.4218 22	.387	.8088	16150	20000
21	3	.4218 22	.387	.8088	17900	22200
23	3	.4218 22	.387	.8088	19600	24300

AREAS AND STRESSES IN BANDS.

(In vertical sections which do not pass through doors)

No. of band.	Gauge.	Area. sq.in.	Total stress. lbs.	Unit stress. lbs./sq.in.
1	26	.328	220	670
2	26	.328	660	2010
3	26	.328	1100	3750
4	26	.328	1540	4700
5	26	.328	1980	6040
6	26	.328	2420	7400
7	26	.328	2860	8600
8	26	.328	3031	9250
9	26	.328	3740	11400
10	24	.438	4180	9340
11	24	.438	4620	10550
12	24	.438	5060	11550
13	24	.438	5500	12550
14	24	.438	5940	13550
15	24	.438	6380	14550
16	24	.438	6820	15550
17	24	.438	7260	16000
18	24	.438	7700	14100
19	22	.546	8140	14900
20	22	.546	8580	15700
21	22	.546	9020	16500
22	22	.546	9460	17300
23	22	.546	9900	18100
24	22	.546	10340	18900
25	22	.546	10780	19700

COMPRESSIVE ~~WIND~~ STRESSES.

The moment of inertia of a section of a thin cylinder = $I = \frac{1}{2} A \cdot R^2$, where A = area of section and R = radius of section.

$$I = \frac{1}{2} \times 3.1416 \times 243\frac{1}{2} \times 3\frac{1}{2} \times 122^2$$

$$\text{Unit stress } S = \frac{M \cdot c}{I} \quad (\text{Due to moment of wind, } M)$$

$$= \frac{52 \times 20 \times 20 \times 26 \times 12 \times 120}{\frac{1}{2} \times 3.1416 \times 243\frac{1}{2} \times 3\frac{1}{2} \times 122^2} = 39.1 \text{ lbs./sq.in.}$$

Assume weight of concrete = 155 lbs. per cu. ft.

Volume of roof per inch of circumference = V'

$$V' = 12\frac{1}{2} \times \frac{1}{2} \times 12 \times 3\frac{1}{2} = 262.5 \text{ cu. in.}$$

Volume of wall per inch of circumference = V''

$$V'' = (12 \times 25 \times 3\frac{1}{2}) + (12 \times 25 \times 3) = 1950 \text{ cu. in.}$$

$$262.5 + 1950 = 2212.5 \text{ cu. in.} = 1.28 \text{ cu. ft.}$$

$1.28 \times 155 = 198.5 =$ Weight of wall and roof in pounds
per inch of circumference.

$198.5 / 3\frac{1}{2} = 56.7 =$ Unit stress at base of wall due to
weight of silo. (lbs. per sq. in.)

Mean head on wall = 25 feet.

$25 \times 11 = 275 =$ Mean pressure on wall in lbs/sq.ft.

Assume coefficient of friction of ensilage on concrete
equals 40%.

40% of 275 = 110 = Mean vertical pressure in lbs. per
square foot of wall.

$$\frac{110 \times 50 \times 1/12}{3\frac{1}{2}} = 131 = \text{Unit stress in lbs. per sq. in.}$$

due to friction of ensilage on walls.

$131 + 39.1 + 56.7 = 206.8 =$ Total unit compressive
stress in walls at base of silo, in lbs. per sq. in.

Resistance to Overturning.

It will be seen from the computation of compressive stresses that the tensile stress at the bottom of the walls due to wind load is 39.1 lbs./sq. in. But the weight of the silo causes a compressive stress of 56.7 lbs. per sq. in. Hence at no time is there any tension developed in the vertical rods in the walls. For the same reason the silo will not overturn from wind load.

CONCLUSION.

It will be noticed that in all the silos analyzed very high unit stresses are found, by using Prof. King's value of eleven lbs. for the horizontal pressure increment per foot in depth. Tensile stresses as high as 58000 lbs. per sq. in. were found in wrought iron bolts. In several cases the tensile stress in steel reached a point above the elastic limit of the metal.

Since none of these silos are failures, the only logical conclusion is that the pressure increment of eleven lbs. per sq. ft. per foot in depth is much too high. Since the elastic limit of wrought iron is about 25000 lbs. per sq. inch, the actual unit tension in the W.I. bolts of the metal silos probably did not exceed this value, instead of being 58000 lbs. per sq. in. , as we found it to be. Therefore the horizontal pressure increment did not exceed $25/58$ of eleven, or 4.75 lbs. per sq. ft. per foot of depth. This value of 4.75 then seems to be a conservative value to use in design.

The horizontal pressure on the walls probably does not vary directly as the depth. The ratio between the horizontal pressures is probably smaller at the bottom than at the top of the silo. Also the ensilage would seem to be held up by friction to a greater degree than at the top. However, a study of the

capacities, in tons, of various heights of silos of the same diameter, shows us that the density of the ensilage is much greater at the bottom than at the top of the silo. (See page 22 of Zyro catalog in pocket.) This increase in density is offset by the decrease in the ratio of the horizontal to the vertical pressures. Therefore a reasonable working theory is that the horizontal pressures vary directly as the depth of the ensilage.

Probably the best method of determining these pressures would be to take extensometer measurements on the hoops of a stave silo while the silo was being filled, and after the ensilage had had time to settle. In doing this it would be necessary to have the hoops loose enough at all times so that no stress would be developed due to the swelling of the wood staves. In this way the actual tension in the hoops due to the internal pressure could be found, and from these values the internal pressures at all heights could be computed.

Another thing which will be noticed is that some of the silos examined would not withstand a wind pressure of 20 lbs per sq. ft. of cross-section. However this pressure is given by a wind velocity of 120 mi. per hour. This velocity is attained only about once in a life-time, and it would hardly be economy to design

for this velocity with such small structures. This is especially true of the wood silo, the life of which is only about fifteen or twenty years.

Pocket has: 1 Suppl.

109
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"long. 10-20"

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