ELEMENTS OF FIELD GUN CARRIAGE DESIGN

THESIS FOR DEGREE OF M. E.

DAYTON A. GURNEY

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Ordnance

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FOREWORD.

This paper, a compilation of the formulae now in use in the Ordnance Office, United States Army, is intended as a guide for future calculations. The formulae have previously been scattered throughout many sets of calculations on various carriages. They, together with their derivation and method of application, are here collected for the first time.

D. A. G.

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I. VELOCITY OF PROJECTILE IN THE BORE.

The formula used for the calculation of the curve of velocity of the projectile in the bore as a function of space is the binomial formula of Col. Ingalls published in the Journal of the United States Artillery of November-December, 1903. While these formulae probably do not as accurately represent the true action of the powder gases as do the trinomial formulae published by the same author at a later date, they are sufficiently accurate for calculating throttling areas in gun carriages and possess the advantages of giving smooth pressure and velocity curves and of facility in use.

The following elements must be known in order to calculate the pressure and velocity curves:

$P_{max.}$ = Normal maximum pressure in bore in pounds per square inch.

$V_{muz.}$ = Muzzle velocity of projectile in feet per second.

u = Travel of projectile in inches.

C = Capacity of powder chamber in cubic inches.

δ = Specific gravity of powder grains (about 1.56).

w = Weight of projectile in pounds.

ϖ = Weight of charge in pounds.

c = Caliber in feet.

From the above the following secondary constants can be calculated.

$$\Delta = [1.44218]\frac{\varpi}{C} = \frac{\text{weight of charge}}{\text{weight of water required to fill chamber}}$$

called "density of loading."

$$a^3 = \frac{1}{\Delta} - \frac{1}{\delta} = \frac{\Delta - \delta}{\Delta\,\delta}$$

$$Z_o = \left(C - \frac{\varpi}{\delta}\frac{1728}{62.5}\right) \frac{1}{\frac{\pi}{4} c^2 \cdot 1728} = [\bar{2}.30955] \frac{a^2 \bar{\omega}}{c^2} =$$

reduced length of initial air space in chamber in feet.

$$\alpha = \frac{M_1}{M} = \frac{2w}{144 \pi g\ c^2\ Z_o} = [\bar{3}.82866] \frac{w}{a^2\ \bar{\omega}}$$

$$\mathcal{E}_{muz.} = \frac{u}{12\ Z_o} = \text{number of expansions.}$$

The fundamental formulae for pressure and velocity are

(1) $\underline{P} = M_1 X_2 (1 - NX_3)$

(2) $\underline{V}^2 = MX_1\ (1 - NX_o)$

Col. Ingalls states that the pressure curve has a maximum ordinate when $\mathcal{E} = 0.4524$. A large number of calculations with these formulae have shown that this is not strictly true for howitzers and field guns. The average of several calculations made gives the maximum pressure to occur at $\mathcal{E} = .65$ for field guns and at $\mathcal{E} = .6$ for howitzers. The method of arriving at the true position of the maximum pressure is as follows:

From the fundamental equations above we may write

$$\frac{\underline{V}^2_{muz.}}{\underline{P}_{max.}} = \frac{MX_1\ (1 - NX_o)}{M_1 X_2\ (1 - NX_3)} = \frac{MX_1\ (1 - NX_o)}{\alpha M\ X_2 (1 - NX_3)} = \frac{X_1\ (1 - NX_o)}{\alpha X_2\ (1 - NX_3)}$$

$$\underline{V}^2 \alpha X_2 - \underline{V}^2 \alpha X_2 N X_3 = \underline{P} X_1 - \underline{P} N X_o X_1$$

$$\underline{P} N X_o X_1 - \underline{V}^2 \alpha X_2 N X_3 = \underline{P} X_1 - \underline{V}^2 \alpha X_2$$

(3) $$N = \frac{\underline{P} X_1 - \underline{V}^2 \alpha X_2}{\underline{P} X_o X_1 - \underline{V}^2 \alpha X_2 X_3}$$

Take out X_o and X_1 corresponding to $\mathcal{E}_{muz.}$ from the tables of these functions.

Assume $\mathcal{E}$ for maximum pressure in accordance with some similar gun previously calculated. In the absence of any indication as to what this should be it should be assumed at about .6. Take out X_2 and X_3 corresponding to this assumed value of $\mathcal{E}$.

Calculate N from the formula derived above.

Calculate M from the following formula

$$(4) \quad M = \frac{\underline{V}^2}{X_1 - NX_oN_1}$$

To check: Calculate $\underline{P}$ from the formula $\underline{P} = M_1X_2 \; (1 - NX_3)$. It should agree with the given maximum pressure.

Now calculate $\underline{P}$ for the higher and lower tabular values of $\mathcal{E}$ next to that assumed. If this $\underline{P}$ is less in each case than $\underline{P}_{max.}$, the point assumed is the correct one for the maximum $\underline{P}$. If either is greater, calculate $\underline{P}$ for successive tabular values of $\mathcal{E}$ until $\underline{P}$ passes through a maximum, assume the value of $\mathcal{E}$ corresponding to this calculated $\underline{P}_{max.}$ as a new trial value and recalculate N, M, M_1, and $\underline{P}$ as before. Proceed in this way until the true position of the maximum is obtained. The values of N and M thus derived are the true ones to be used in the equation

$\underline{V}^2 = MX_o - MNX_oX_1$ for plotting the velocity curve.

The pressure curve may be calculated by the formula

$\underline{P} = M_1X_2 - M_1NX_2X_3$, if desired, but this curve is used in designing guns rather than carriages and is not required for our purpose.

To plot the curve of velocity of projectile as a function of space. (Fig. 1).

Having obtained M and N, $\underline{V}^2$ and consequently $\underline{V}$ may be obtained for any value of $\mathcal{E}$ or u by substitution in the formula. The points may be taken rather far apart especially after passing the maximum pressure. The points may be plotted to any convenient scale and a smooth curve should be drawn through all of them. If any point is off the curve it should be recalculated.

To calculate the curve of velocity of projectile as a function of time (Fig. 3).

A method of calculating this curve is given by Captain (now General) Crozier in Ordnance Construction Note No. 57. It consists in plotting the curve of reciprocals of the velocity as a function of time$\left[\frac{1}{v} = f(t)\right]$ and of measuring the areas under successive portions of this curve. This area is $\int \frac{1}{v}\, ds = \int \frac{dt}{ds}\, ds = \int dt = t.$

However, it is not necessary to plot the curve of reciprocals, as the curve of velocity as a function of time may be obtained directly from the curve of velocity as a function of space.

The expression for the area under any portion of this curve is (Fig. 1)

$$A = \Delta x V_m = \Delta x \frac{\Delta x}{\Delta t} = \frac{\overline{\Delta x}^2}{\Delta t}$$

$$(5) \quad \therefore \Delta t = \frac{\overline{\Delta x}^2}{A}$$

The area under successive portions of the curve may be obtained by planimeter or by any other convenient method, and the successive times thus be calculated. The accuracy

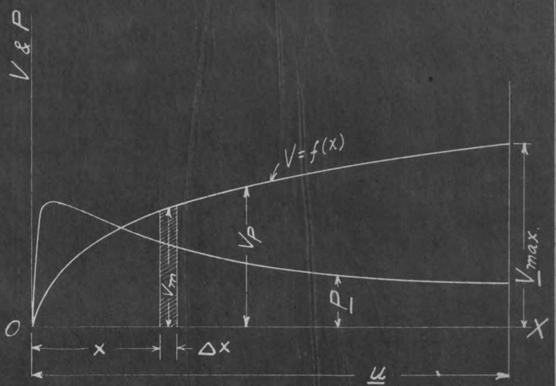


FIG. 1.

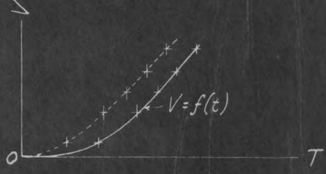


FIG. 2.

of this method increases, the smaller the increments of x become.

Both of these methods of calculation fail near the origin. The first method fails because the Y-axis becomes tangent to the reciprocal curve at infinity and it is impossible to measure the area correctly or to plot the curve with any degree of accuracy through points near the Y-axis. The second method fails because the units of x and A become too small to be measured with accuracy near the origin.

However, the true time for this portion of the curve may be approximated to any required degree of accuracy.

Having calculated the time corresponding to a number of spaces as near the origin of motion as practicable, plot the curve of velocity as a function of time to a large scale, preferably on cross section paper (See Fig. 2). One of the properties of this curve is that the area beneath any portion thereof represents the distance traveled over during that time. Therefore, if the area under the curve up to any point is unequal to the corresponding space traversed, the curve is incorrect and should be adjusted accordingly.

In Fig. 2 the dotted line represents the preliminary or calculated curve while the full line represents the adjusted or true curve.

The whole of the curve of velocity of the projectile as a function of time may now be plotted to a convenient scale. The maximum abscissa of this curve is called t'.

II. VELOCITY OF FREE RECOIL (Fig. 3).

The velocity of free recoil is the velocity which the recoiling parts would have if they were acted on only by the powder pressure and not retarded in any way. It attains its maximum at the time the powder gases cease to act, i. e., when the bore pressure becomes atmospheric pressure.

V_p = velocity of projectile in bore (previously calculated).

$\breve{\omega}$ = weight of charge.

w = weight of projectile.

W_r = weight of recoiling parts.

V_f = velocity of free recoil.

Assuming, as seems rational, that the average velocity of the powder is one-half that of the projectile.

$$W_r\ V_f = \left(w + \frac{\breve{\omega}}{2}\right)\ V_p$$

$$(6) \quad V_f = V_p \frac{w + \frac{\breve{\omega}}{2}}{W_r}$$

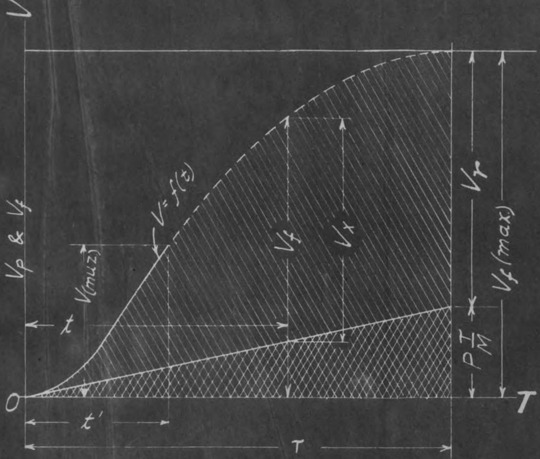
The average velocity of the powder gases as they emerge from the bore after the projectile has left it is generally taken as 4700 ft./sec. (Rausenberger gives 2.5 V_p).

$$W_r\ V_{f\ max.} = W_p V_{muz.} + 4700\,\breve{\omega}\ .$$

$$(7) \quad V_{f\ max.} = \frac{W_p V_{muz.} + 4700\,\breve{\omega}}{W_r}$$

The curve of velocity of projectile as a function of time can be used for velocity of free recoil as a function of time by simply changing the vertical scale in the ratio of $\frac{w + \frac{\breve{\omega}}{2}}{W_r}$. Then draw a horizontal line parallel to the axis

of t at a distance of $V_{f\ max.}$ therefrom and connect this line



TOTAL SHADED AREA $= \xi$

CROSS SHADED AREA $= P\dfrac{T^2}{2M}$

SINGLE SHADED AREA $= \xi - P\dfrac{T^2}{2M}$

UNSHADED AREA $= V_f T - \xi$

FIG. 3.

with the curve by means of a smooth curve. The exact position of the point of tangency of the curve and horizontal line cannot be accurately determined, but it makes very little difference so far as the results of the calculations are concerned. The time to the point of tangency is termed $\mathcal{T}$. It is equal approximately to 3t' for gun carriages and to 2t' for howitzer carriages. It will be generally sufficiently accurate to take it arbitrarily at some even thousandth of a second. t' is the time elapsed when the projectile leaves the muzzle and $\mathcal{T}$ is the time elapsed when the powder gases cease to act.

The area under this curve represents the distance traveled over in free recoil up to any time t for:

$$\text{Area} = \int V dt = \int \frac{dx}{dt}\, dt = \int dx = x$$

The total area up to time $\mathcal{T}$ is the distance traveled in free recoil up to that time and is called ξ. This area should be measured by any convenient method, preferably by planimeter, and determined. As a partial check measure the area under the curve up to time t'. This area to the scale used for velocity of projectile in the bore should give the length of travel u.

III. STABILITY.

The stability of a carriage is that force which applied through the center of gravity of the recoiling parts in a direction parallel to the piston rod pull and having a lever arm to the bearing point of the spade is just sufficient to overturn the carriage.

It is denoted by R when in battery and by r when at full recoil (Fig. 4).

W_t = total weight

W_r = weight of recoiling parts

$Rz = W_t x$

$rz = W_t x - W_r b \cos \phi$

$Rz - rz = \cancel{W_t x} - \cancel{W_t x} + W_r b \cos \phi = W_r b \cos \phi.$

(8) $$R - r = \frac{W_r b \cos \phi}{z}$$

For $\phi = o$; $\cos \phi = 1$; and $z = h$

(9) $$R - r = \frac{W_r b}{h}$$

R - r is known as the "loss of stability during recoil." It is used in deriving an expression for total resistance or "total pull."

From figure 4 it is evident that as ϕ increases z passes through 0 and becomes negative. R and r therefore, become very large and after the line of force passes through the bearing point of the spade no question of stability can enter. This increase in R with the increase in ϕ enables us to increase the piston rod pull and to shorten the recoil, as the gun is elevated, without endangering stability.

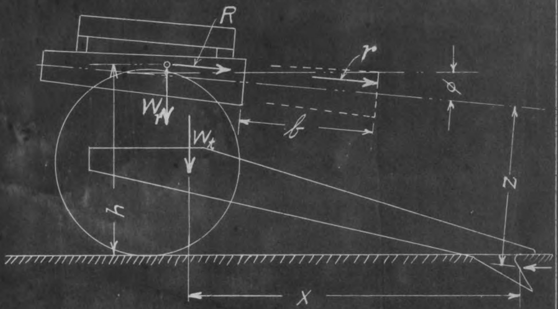


FIG. 4.

IV. TOTAL PULL (Fig. 5).

We are now prepared to derive an expression for the total resistance tending to retard the recoiling parts and to bring them to rest; this is called the "total pull." For convenience the characters used are here again set down and defined.

R = stability at beginning of recoil in lbs.

r = stability at end of recoil in lbs.

P = total pull at beginning of recoil in lbs.

p = total pull at end of recoil in lbs.

b = length of recoil in feet.

ξ = length of free recoil at time τ.

τ = time elapsed when powder gases cease to act.

M = mass of recoiling parts $= \frac{W_r}{g}$

For convenience in making the calculations it is assumed

1st that P is constant up to time τ.

2nd that the slope of curve of total pull from P to p shall be parallel to that from R to r. This gives the greatest possible stability consistent with the 1st assumption. Very little would be gained by letting the slope of the curve from P to p extend over the entire length of b.

The retarding effect of P on the velocity of free recoil is

$$V = \int a\,dt = \int \frac{P}{M}\,dt = \frac{P}{M}\int dt = \frac{Pt}{M}$$

for $t = \tau$

the retardation is $P\frac{\tau}{M}$

The velocity of recoil at this time is therefore

$$V_{f\ max.} - P\frac{\tau}{M} = V_r$$

The effect of P on the distance traveled is

$$S = \int v dt = \int \frac{P}{M}\, t dt = \frac{P}{M}\int t dt = \frac{Pt^2}{2M}$$

for $t = \tau$ the effect is $\frac{P\tau^2}{2M}$

The distance recoiled at time τ is therefore

$$\xi - \frac{P\tau^2}{2M}$$

From the diagram (Fig. 5) we may now write

$$\frac{P - p}{b - \xi + P\frac{\tau^2}{2M}} = \frac{R - r}{b}$$

$$(10) \quad p = P - \frac{R - r}{b}\left(b - \xi + P\frac{\tau^2}{2M}\right)$$

$$\frac{P + p}{2} = P - \frac{R - r}{2b}\left(b - \xi + P\frac{\tau^2}{2M}\right)$$

$$\frac{P + p}{2}\left(b - \xi + \frac{P\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - \frac{P\tau}{M}\right)^2$$

$$\left[P - \frac{(R - r)\left(b - \xi + P\frac{\tau^2}{2M}\right)}{2b}\right]\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

$$P(b - \xi) + \frac{P^2\tau^2}{2M} - \frac{(R - r)}{2b}\left(b - \xi + P\frac{\tau^2}{2M}\right)^2 = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

$$P(b - \xi) + \cancel{\frac{P^2\tau^2}{2M}} - \frac{(R - r)(b - \xi)^2}{2b} - \frac{(R - r)}{2b}\, 2(b - \xi)\, P\frac{\tau^2}{2M} -$$

$$\frac{R - r}{2b}\;\frac{P^2\tau^4}{4M^2} = \frac{MV_f^2}{2} - PV_f\tau + \cancel{\frac{P^2\tau^2}{2M}}$$

The term $\frac{R - r}{2b}\;\frac{P^2\tau^4}{4M^2}$ contains τ^4 and may be neglected

in comparison with the other terms.

$$(11)\quad P\left[b - \xi + V_f\tau - \frac{R-r}{b}(b-\xi)\frac{\tau^2}{2M}\right] = \frac{MV_f^2}{2} + \frac{R-r}{2b}(b-\xi)^2$$

This is readily solved for P.

For the condition of theoretical stability at 0° elevation

$$P_{max.} = r = R - \frac{W_r}{h}b = P - \frac{W_r}{h}\left(b - \xi + P\frac{\tau^2}{2M}\right)$$

$$R - \cancel{\frac{W_r}{h}b} = P - \cancel{\frac{W_r b}{h}} + \frac{W_r\xi}{h} - \frac{W_r}{h}P\frac{\tau^2}{2M}$$

$$(12)\quad P_{max.} = \frac{R - \frac{W_r}{h}\xi}{1 - \frac{W_r}{h}\cdot\frac{\tau^2}{2M}}$$

If therefore P as **derived** from equation (11) is greater than $P_{max.}$, stability will not obtain.

p is obtained from the equation

$$(10)\quad p = P - \frac{R-r}{b}\left(b - \xi + P\frac{\tau^2}{2M}\right)$$

Check:

$$(13)\quad \frac{P+p}{2}\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{M\left(V_f - P\frac{\tau}{M}\right)^2}{2}$$

For uniform pull P is constant and $P = p < r$

$$P\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

$$P\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{MV_f^2}{2} - PV_f\tau + \frac{P^2\tau^2}{2M}$$

$$(14)\quad P = \frac{MV_f^2}{2}\cdot\frac{1}{b - \xi + V_f\tau}$$

This might have been written by inspection from equation (11) by placing $R - r = o$.

This formula is useful in obtaining a preliminary estimate of P; for $\frac{MV_f^2}{2}$ is easily calculated; b may be assumed and ξ and $V_f\tau$ estimated from similar carriages.

Check:

$$(15)\quad P\left(b - \varepsilon + P\frac{\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

For carriages in which variable recoil is used it is customary to use the formula for long recoil (11) for elevations up to about 5°. For elevations above the line where the over-turning force passes through the bearing point of the spade, 17° approximately, the formula for short recoil (14) should be used.

If we attempt to use the formula for long recoil (11) for all intermediate elevations it is found that R - r and consequently P - p becomesvery large for the higher elevations, i. e. as h becomes small in the formula $R - r = \frac{W_r b}{h}$. The slope of the curve of pull increases very rapidly therefore with the elevation. As the slope of the curve of pull for short recoil is 0, the transition between the two formulae is irregular.

Since stability at these intermediate elevations is generally ample it is customary to ignore equation (11) and to make the slope of the curve of pull vary uniformly with the elevation from that calculated at 5° (approximately) to zero at the 1st position where the short recoil formula (14) is used.

This slope is $\left(\frac{P - p}{b - \varepsilon + \frac{P\tau^2}{2M}}\right)$.

b, ε, τ^2, and M are known for all elevations.

P can be estimated roughly as being inversely proportional to b. We can then make a table like the following:

Elev.	$b - \xi + P\frac{\tau^2}{2M}$	Slope	$P - p$
0°	2.677	117	313
5°	2.512	151	380
10°	2.08	100	208
15°	1.923	50	96
20°	1.600	0	0

Instead of formula (11) we derive the following:

let $P - p = C'$

$$p = P - C'$$

$$\frac{P + p}{2} = P - \frac{C'}{2}$$

$$\left(P - \frac{C'}{2}\right)\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

$$P(b - \xi) + \cancel{P^2\frac{\tau^2}{2M}} - \frac{C'}{2}(b - \xi) - P\frac{\tau^2}{2M}\,\frac{C'}{2} = \frac{MV_f^2}{2} - PV_f\tau + \cancel{P^2\frac{\tau^2}{2M}}$$

(16) $$P\left(b - \xi + V_f\tau - \frac{\tau^2 C'}{4M}\right) = \frac{MV_f^2}{2} + \frac{C'}{2}(b - \xi)$$

from which we get P and therefore p.

Check:

(13) $$\frac{P + p}{2}\left(b - \xi + P\frac{\tau^2}{2M}\right) = \frac{M}{2}\left(V_f - P\frac{\tau}{M}\right)^2$$

V. HYDRAULIC RESISTANCE (Fig. 5).

The hydraulic resistance is the net resistance to motion to be taken up in the hydraulic cylinder after subtracting all other forms of resistance from the total pull and allowing for the component of the weight of the recoiling parts parallel to the piston rod. It is denoted by R_h.

Let S = load on the springs at any length of recoil.

F = friction on recoil guides.

B = stuffing box friction.

P_x = total pull at any distance x from origin.

f = coefficient of friction.

(17) Then $R_h = P_x + W_r \sin. \phi - S - F - B.$

In general R_h can be expressed a a linear function of x for;

$P_x = f(x)$ from diagram. It is, of course, a different function before and after time τ, but linear in both cases.

$W_r \sin. \phi$ is constant for any elevation. (Note that this term is negative when the gun is in depression).

$S = f(x)$ (linear).

$F = W_r f \cos \phi$ is accurate enough in general, although the friction due to the pinching action on the recoil guides is neglected.

f should be taken small, .1, for this calculation.

B = 100 lbs. per in. of diameter of piston rod, a constant.

Let A = effective area of piston

then $\frac{R_h}{A}$ = pressure in recoil cylinder.

VI. VELOCITY OF RETARDED RECOIL (Figs. 3 and 6).

The velocity of retarded recoil is the actual velocity of the recoiling parts with respect to the carriage.

For times prior to τ it is solved for as follows:

$$V_x = V_f - P\frac{t}{M} \qquad\qquad x = \xi - \frac{Pt^2}{2M}$$

Draw ordinates of the curve of velocity as a function of time about .001 to .002 sec. apart and measure the areas beneath the curve between these successive ordinates, then construct a table like the following:

t	$P\frac{t}{M}$	V_f	V_x	$\frac{Pt^2}{2M}$	ξ	x
.001						
.002						
.003						
.005						
.007						
.009						
.013						
.017						
.019 = τ						

Slide rule results for $P\frac{t}{M}$ and $\frac{Pt^2}{2M}$ are sufficiently accurate. V_f and ξ are obtained from the curve.

As a check we may plot V_x as a function of x. The result should be a smooth curve. The point of $V_{x\,max.}$ will occur somewhat before V_r and it is at this point that the maximum opening of the throttling orifice occurs.

For times after τ the following equation holds

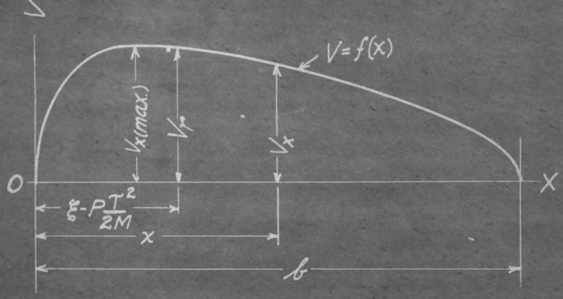


FIG. 6.

$$\left(\frac{P + P_x}{2}\right)\left[x - \left(\xi - P\frac{\tau^2}{2M}\right)\right] = \frac{M}{2}\left(V_r^2 - V_x^2\right)$$

As P_x is linear it may be expressed as $P_x = c - dx$ where c and d are numerical and known (see page 17).

$$\frac{P + c - dx}{\cancel{2}} \cdot \left(x - \xi + P\frac{\tau^2}{2M}\right) = \frac{M}{\cancel{2}}\left(V_r^2 - V_x^2\right)$$

$$(P + c)\,x - (P + c)\left(\xi - P\frac{\tau^2}{2M}\right) - dx^2 + dx\left(\xi - P\frac{\tau^2}{2M}\right) =$$

$$M\left(V_r^2 - V_x^2\right)$$

$$\left[P + c + d\left(\xi - \frac{P\tau^2}{2M}\right)\right]x - dx^2 - (P + c)\left(\xi - \frac{P\tau^2}{2M}\right) =$$

$$MV_r^2 - MV_x^2$$

$$(18)\quad V_x^2 = \frac{MV_r^2 + (P + c)\left(\xi - \frac{P\tau^2}{2M}\right)}{M} - \left[\frac{P + c + d\left(\xi - \frac{P\tau^2}{2M}\right)}{M}\right]$$

$$x + \frac{d}{M}x^2$$

Check; Place $x = b$

V_x^2 must $= 0$.

The equation is of the form

$$(19)\quad V_x^2 = A - B\,x + Cx^2$$

As a large number of points are necessary to be found on the curve it is less laborious to solve it by the method of differences. On account of the form of the equation, the second difference is constant. We write;

$$V_{x1}^2 = A - Bx_1 + Cx_1^2$$

$$V_{x2}^2 = A - Bx_2 + Cx_2^2$$

Let $x_2 = x_1 + a$ where $a = \Delta x$

$$V_{x2}^2 = A - Bx_1 - Ba + Cx_1^2 + 2Cx_1a + Ca^2$$

$$V_{x1}^2 = A - Bx_1 + Cx_1^2$$

$$V_{x2}^2 - V_{x1}^2 = Ca^2 - Ba + 2Cax_1$$

Similarly $V_{x3}^2 - V_{x2}^2 = Ca^2 - Ba + 2Cax_2$

$$V_{x3}^2 - V_{x2}^2 = Ca^2 - Ba + 2Cax_1 + 2Ca^2$$

$\therefore$ 2nd difference $= 2Ca^2$

at origin $x = x_1 = 0.$

$\therefore$ ~~1st~~ first difference $= -(Ba - Ca^2)$

Having selected "a" and made the other necessary substitutions prepare a table of 1st differences. Subtract these successively from A. The result is V_x^2 from which we can obtain V_x.

<u>For the condition of uniform pull</u> $P_x = P$, a constant, and the formula becomes

$$P(x - \xi + \frac{PT^2}{2M}) = \frac{M}{2}(V_r^2 - V_x^2)$$

$$Px - P(\xi - \frac{PT^2}{2M}) = \frac{M}{2} V_r^2 - \frac{M}{2} V_x^2$$

$$V_x^2 = \frac{MV_r^2 + 2P(\xi - \frac{PT^2}{2M})}{M} - \frac{2Px}{M}$$

This is the equation of a simple parabola, in which the first difference is constant. It may be solved directly or, more easily, by differences.

Check; for $x = b$; V_x^2 must $= 0$.

VII. THROTTLING AREAS.

(a) for straight throttling through an orifice between the piston and cylinder.

The theoretical velocity of efflux from any orifice is $v(\text{theoretical}) = \sqrt{2gh}$.

Denoting the pressure in pounds per square inch by p and the weight of a cubic foot of the liquid by δ we may write $p = \frac{h\delta}{144}$ or $h = \frac{144\,p}{\delta}$

$$v(\text{theoretical}) = \sqrt{\frac{288g}{\delta}\,p}$$

For hydroline, $\delta = 53$, and substituting this value and also the numerical value of g we have

$$(21)\quad v(\text{theoretical}) = \sqrt{\frac{288 \cdot 32.2}{53}\,p} = 13.22\sqrt{p}$$

Let $v(\text{actual}) = \frac{v(\text{theoretical}) - C}{K}$ where C and K are constants to be determined experimentally.

If a is the area of orifice, V the velocity of the piston, (the V_x previously calculated) and A the effective piston area.

$$VA = va = \frac{v(\text{theoretical})\,a - Ca}{K}$$

$$a = \frac{KVA}{v(\text{theoretical}) - C} = \frac{KVA}{13.22\sqrt{p} - C}$$

$$p = \frac{R_h}{A}$$

$$a = \frac{KVA^{3/2}}{13.22\sqrt{R_h} - C}$$

For purposes of analysis we can neglect C, it being small in comparison with the other term of the denoninator.

Then $a = \frac{KVA^{3/2}}{13.22\sqrt{R_h}}$

From which we see that "a" varies directly as the velocity of recoil and inversely as the square root of the hydraulic pull. If the latter is constant, let $Q = \frac{P}{R_h}$

$$V_x^2 = QR_h(b-x)$$

$$\frac{V_x^2}{R_h} = Q(b-x) \qquad \frac{V_x}{R_h} = Q^{\frac{1}{2}}\sqrt{b-x}$$

$$a = \frac{KA^{3/2}}{13.22} Q^{\frac{1}{2}}\sqrt{(b-x)}$$

$$a^2 = \text{constant } (b-x)$$

This is the equation of a parabola and under these conditions "a" is independent of the velocity and depends only on x. This is very useful property as it means that with a properly constructed throttling orifice we get substantially correct recoil no matter what the initial velocity. As R_h is not quite constant due to springs, piston clearance, etc., the actual recoil will vary slightly under certain conditions but the variation is not sufficient to cause serious mechanical difficulty.

For straight throttling through an orifice between the piston and cylinder the values determined for the constants in the area formula are K = 2.22; C = - 96.

In order to avoid the danger of over cutting the bars, K should be taken about 90% of 2.22 or = 2. for the initial trial value. If firings show it to be too small the area can then be enlarged.

For convenience in handling, it is sometimes desirable to have the coefficients of contraction expressed directly in terms of p.

Place v(actual) $= \dfrac{v(\text{theoretical})}{C_1 + pC_2}$

$$VA = va = \frac{v(\text{theoretical})\ a}{C_1 + pC_2}$$

$$(23)\quad a = \frac{(C_1 + pC_2)\ VA}{13.22\ \sqrt{p}}$$

By making proper substitutions we obtain $C_1 = 1.697$; $C_2 = .0001081$.

$$(23a)\quad a = \frac{(1.697 + .0001081\ p)\ VA}{13.22\ \sqrt{p}}$$

or using the safe value of 90% "a" as before

$$(23b)\quad a = \frac{(1.527 + .0000973\ p)\ VA}{13.22\ \sqrt{p}}$$

From the value of "a" derived from either formula 22 or formula 23 it is necessary to subtract the clearance area between the piston and cylinder, a_o. The amount of area to remove from the throttling bars is therefore $a - a_o$.

(b) For the condition of throttling through an orifice from the front of the piston into a bypass and from the bypass through another orifice into the space behind the piston. We will first derive a method for determining the coefficients of contraction under this peculiar condition.

$$a = \frac{KVA}{13.22\ \sqrt{p} - C} \quad \text{as before.}$$

The area of orifice is made up of three parts

a_o = clearance, a constant

a_1 = area in front of piston (toward which it is moving)

a_2 = area in rear of piston (opposite to a_1)

Let K_o = total coefficient of contraction corresponding to a_o; $(C = 0)$

Let K_1 = total coefficient of contraction corresponding to a_1 and a_2.

p = pressure in front of piston.

p_1 = pressure in bypass.

Pressure in rear of piston = 0.

V_o = velocity of oil through a_o.

V_1 = " " " " a_1.

V_2 = " " " " a_2.

$R_h = pA$; $R_1 = (p - p_1)A$; $R_2 = p_1 A$

$$VA = V_o a_o + V_1 a_1 = V_o a_o + V_2 a_2$$

(21) $v(\text{theoretical}) = 13.22\sqrt{p}$

$$V_o = \frac{13.22}{K_o}\sqrt{p} \;;\quad V_1 = \frac{13.22}{K_1}\sqrt{p - p_1};\quad V_2 = \frac{13.22}{K_1}\sqrt{p_1}$$

$$V_1 a_1 = V_2 a_2$$

$$\frac{\cancel{13.22}}{\cancel{K_1}} a_1 \sqrt{p - p_1} = \frac{\cancel{13.22}}{\cancel{K_1}} a_2 \sqrt{p_1}$$

$$a_1^2 (p - p_1) = a_2^2 p_1$$

$$p_1 = \frac{a_1^2}{a_1^2 + a_2^2} p$$

$$a_1 = \frac{KVA}{13.22\sqrt{p_1} - C}$$

$$13.22 \sqrt{p_1} - C = KV \frac{A}{a_1}$$

$$C = 13.22 \sqrt{p_1} - KV \frac{A}{a_1}$$

$$C = 13.22 \sqrt{\frac{a_1^2}{a_1^2 + a_2^2} \quad p} - KV \frac{A}{a_1}$$

In any round for which both the pressures and velocities have been measured instrumentally V, p, a_1, and a_2 are known at any point. Consequently a straight line representing the equation $C = f(K)$ can be plotted for any point of the curve. Plot several such curves. The average of the intersections should give the correct values of K and C. From a certain round for 3.8-inch Howitzer Carriage so plotted these values were found to be $K = 1.75$; $C = - 50$.

These values have since been tried on a number of carriages and have given fairly good results on all.

<u>To calculate the throttling area knowing the constants K and C.</u>

On account of the complexity of the formula we will neglect the small amount of oil which passes through a_o in our calculations. Then when the areas have been computed we will deduct a_o therefrom.

$$a_2 = \frac{1.75\ VA}{13.22\sqrt{\frac{R_2}{A}} + 50}$$

$$\sqrt{\frac{R_2}{A}} = \frac{1.75\ VA - 50\ a_2}{13.22\ a_2}$$

$$\frac{R_h}{A} - \frac{R_2}{A} = \frac{R_1}{A}$$

$$\frac{R_1}{A} = \frac{R_h}{A} - \left(\frac{1.75\ VA - 50\ a_2}{13.22\ a_2}\right)^2$$

$$a_1 = \frac{1.75\ VA}{13.22\sqrt{\frac{R_1}{A}} + 50}$$

$$a_1 = \frac{1.75\ VA}{13.22\left[\frac{R_h}{A} - \left(\frac{1.75\ VA - 50\ a_2}{13.22\ a_2}\right)^2\right]^{\frac{1}{2}} + 50}$$

$$a_1 = \frac{1.75\ VA}{\frac{13.22}{\sqrt{A}}\sqrt{R_h - \left(\frac{1.75\ VA^{3/2} - 50\ A^{\frac{1}{2}}a_2}{13.22\ a_2}\right)^2} + 50}$$

(24)

$$a_1 = \frac{1.75\ VA}{\frac{13.22}{\sqrt{A}}\sqrt{R_h - \left(\frac{1.75\ A^{3/2}}{13.22}\ \frac{V}{a_2} - \frac{50A^{\frac{1}{2}}}{13.22}\right)^2} + 50}$$

Check; Substitute $a_{max.}$ for a_2, $V_{max.}$ and the corresponding R_h. a_1 should $= a_2$. At the point of maximum velocity the area also must be maximum.

$\therefore$ $a_1 = a_2$, and $p_1 = \frac{p}{2}$; $\frac{R_2}{A} = \frac{R_h}{2A}$

(25)

$$\therefore\ a_{max.} = \frac{1.75\ VA}{13.22\sqrt{\frac{R_h}{2A}} + 50}$$

The method of using the formulae for throttling areas is as follows:

1st; Determine V_x and R_h for values of x whose distance apart is equal to the thickness of the piston. (For points prior to maximum velocity it is better that they should be separated by only one-half or one-quarter this amount.)

2nd; Solve for $a_{max.}$ (formula 25).

3rd; Substituting $a_{max.}$ for a_2 in formulae (24) and the proper values of R_h and V_x solve for a_1, and area for the point next nearest the maximum

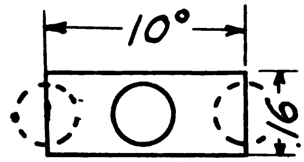
Now ($a_{max.}$ - this a_1) is the area covered by the piston in moving from the 2nd position to the 1st, ∴ it must be added to $a_{max.}$ to get the new a_2. Solve for each successive value of a_1 in a similar manner.

In formula (24), a_1 and a_2 may be interchanged and the same equation be used for solving for a_2 to get the points on the curve after the maximum in a similar manner.

Having solved for area on both sides of the piston at all necessary points subtract from each the clearance area a_o.

The greater portion of the area is generally made up of drilled holes through the recoil valve. Having determined the size of hole which is to be used find its area. The number of holes at any point of recoil is then $N = \frac{a - a_o}{\text{area hole}}$. It is customary to drill the holes for trial one or two twist drill sizes smaller than calculated, and to gradually enlarge them in order to get correct recoil and good pressure and velocity curves.

Length of ports in the liner.

If the rows of holes in the valve are a certain number of degrees apart it would at first appear that to have row No. 2 begin to open at the same time that row No. 1 begins to close, the ports in the liner should be the same number of degrees long, and that to get the correct theoretical opening at elevations intermediate to those for which the rows of holes in the valve were calculated the ports in the liner should have square ends i. e., they should be this shape: 

Experience has shown, however, that with ports of this shape too much oil escapes through the throttling orifices at the intermediate elevations and too great recoil and too high end pressures are obtained. Consequently it has been found necessary not only to provide the ports with circular ends, but also to make them slightly shorter than 10°. No entirely satisfactory explanation of this phenommenon has been advanced. The following empirical formula for length of port is based on experiments performed on 4 different calibers, viz: 3", 3.8", 4.7" and 6", and may be taken as correct within these limits only.

Length = .0811·(Inside diameter of liner).

VIII. FORCES.

The esternal forces on a mobile carriage are:

(a) Powder pressure.

(b) Force on rifling.

(c) Weight.

(d) Vertical reaction under wheels.

(e) Vertical reaction on float.

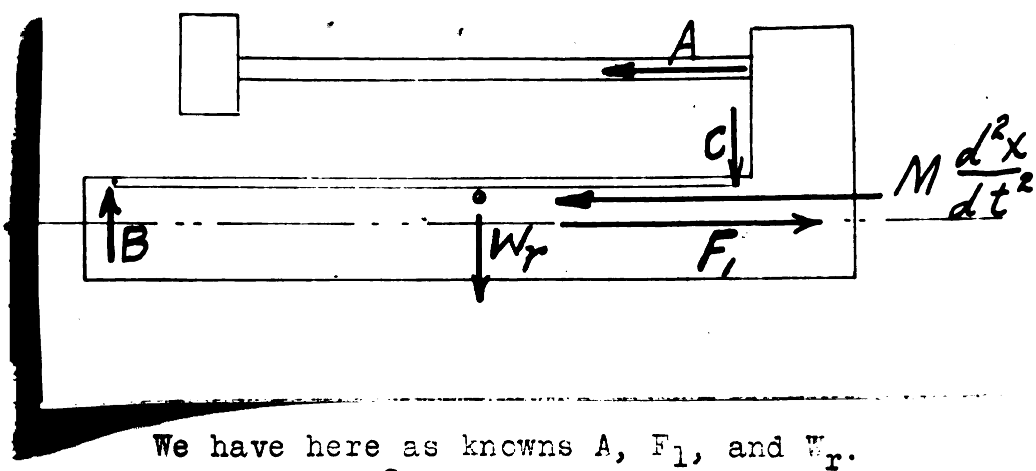
(f) Horizontal reaction on spade.

Knowing the first 3, it is possible to obtain the remainder from the equations.

$\Sigma X = 0; \quad \Sigma Y = 0; \quad \Sigma Pa = 0.$

It is, however, necessary to obtain also the forces at all connections throughout the carriage in order to determine the strength of the various parts. We will take as typical the case of a howitzer in battery at 0° elevation with powder gases acting.

Forces on recoiling parts.



We have here as knowns A, F_1, and W_r.

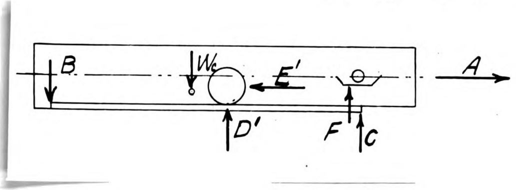
Unknowns $M\frac{d^2x}{dt^2}$, B, and C.

The unknowns may readily be determined from the three fundamental equations.

$\Sigma X = 0; \quad \Sigma Y = 0; \quad \Sigma Pa = 0.$

A, B, and C are transmitted to the cradle.

Forces on tipping parts (except rocker)

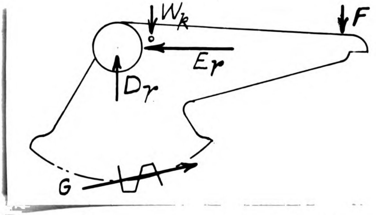


Knowns	Unknowns
A, B, C, W_c	D', E', F

Transmitted to rocker F.

Transmitted to top carriage D', E'.

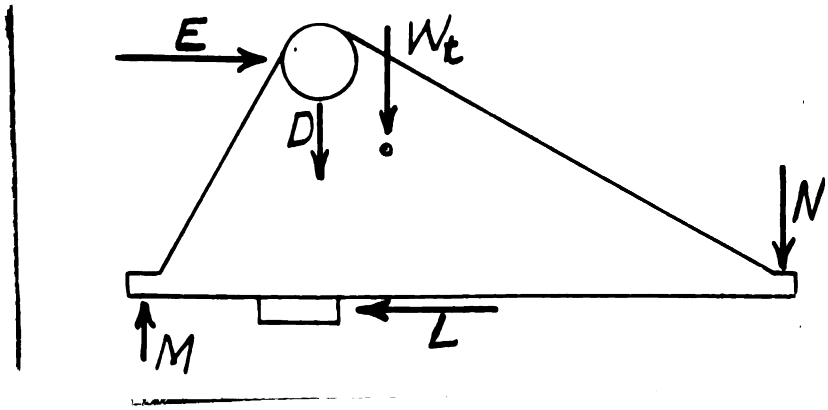
Forces on rocker



Knowns	Unknowns
F, W_k	D_r, E_r, G

Transmitted to top carriage D_r, E_r, G

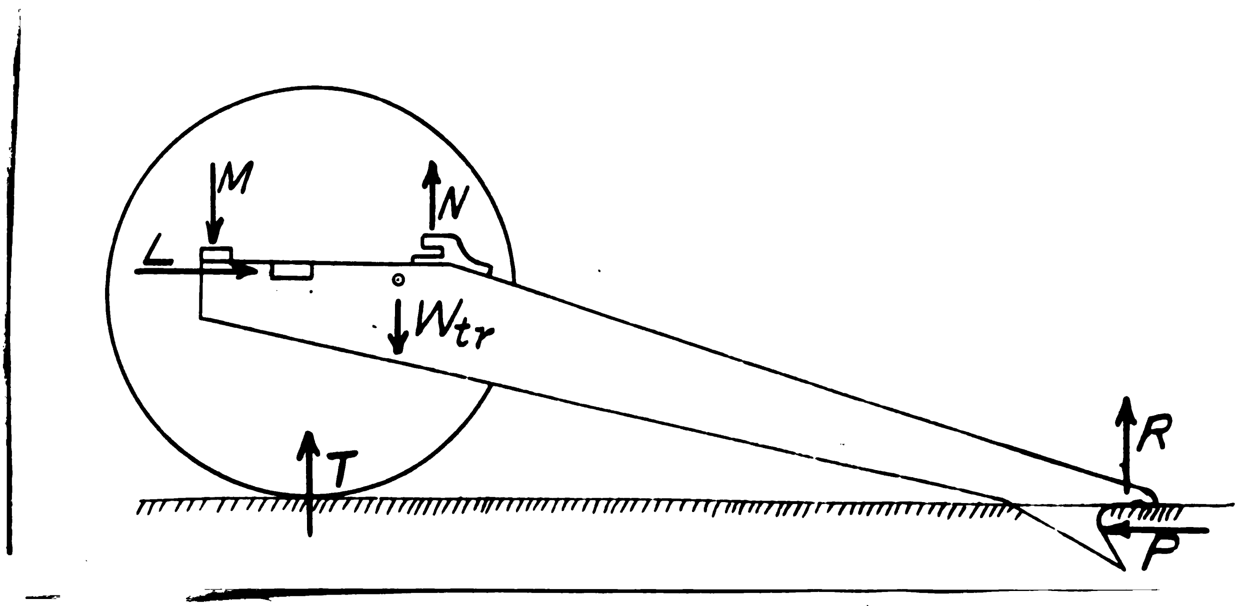
Forces on top carriage.



Knowns	Unknowns
$D = D_l + D_r$ (algebraic)	L
$E = E_l + E_r$ (algebraic)	M
W_t	N

Transmitted to trail L, M, N.

Forces on trail



Knowns	Unknowns
L, M, N, W_{tr}	T, P, R

If $T < 0$ the carriage is not stable, i. e., it will jump on firing. Carriages are generally calculated to be stable with gun at 0° elevation and powder gases not acting. This should hold both for the "in battery" and "from battery" conditions.

The major forces enumerated above should be calculated for a carriage having variable recoil for each of the seven conditions.

(1) In battery 0° elevation, powder gases acting.

(2) In battery 0° elevation, powder gases not acting.

(3) From battery 0° elevation.

(4) Start of counterrecoil 0° elevation.

(5) In battery maximum elevation, powder gases acting.

(6) In battery maximum elevation, powder gases not acting.

(7) From battery maximum elevation.

(1) has been sufficiently explained.

In (2) $F_1 = 0$ and $M\frac{d^2x}{dt^2}$ is reversed.

In (3) the base $\overline{BC}$ is the shortest possible.

(4) is like (3), but piston rod pull drops out of A.

(5), (6), and (7) correspond to (1), (2), and (3) respectively.

Minor forces.

It is not the purpose of this article to lay down general rules for calculating all the individual forces which may come upon various components of a carriage, but only to indicate what forces are most likely to cause trouble and what should be calculated. In addition to the major forces it is necessary sometimes to calculate the effect due to rifling, stability when fired at maximum traverse, the tendency of the components of the recoiling parts to separate, the force on traversing gear due (a) to effect of rifling at maximum elevation, (b) to the center of gravity of the recoiling parts lying at a certain distance, horizontally, from the center of the bore.

In connection with the calculation of forces on the recoiling parts it may be noted that the internal force $M \frac{d^2x}{dt^2}$ acting through the center of gravity thereof should never be omitted. All forces tending to produce rotation of the recoiling parts either vertically or horizontally act with an arm to the center of gravity of the recoiling parts. A great advantage is gained in reducing the turning moments on the carriage if the center of gravity of the recoiling parts can be kept at or near the axis of the bore. No advantage is gained by placing the trunnions near this center of gravity. They should be located as nearly as practicable on the center of gravity of the tipping parts and should give a breech preponderance loaded and a muzzle preponderance with the empty cartridge case in the chamber.

IX. STRESSES.

Having determined the forces, the stresses are calculated by the ordinary formulae of mechanics. The bending moment diagram for the trail is a straight line and can be easily plotted by calculating the moment at two points. To the compressive stress due to bending must be added that due to straight compression. From the tensile stress due to bending should be subtracted the compressive stress due to straight compression.

The tangential stress in the recoil cylinder, it being a cylinder with closed ends should be calculated by the formula of Claverino.

$$\theta = \frac{3(D_1^2 - D_o^2)\ P_o}{4D_1^2 + D_o^2}$$

The tangential stress in the valve and liner, they being cylinders with open ends, should be calculated by Birnie's formula

$$\theta = \frac{3(D_1^2 - D_o^2)\ P_o}{4D_1^2 + 2D_o^2}$$

As the metal of both valve and liner are cut away by holes and ports it is necessary to make allowance for this in claculating stresses.

For instance in case the liner has a row of ports .16 wide, spaced .333 apart, the stress is that calculated by the above formula of Birnie divided by $\frac{.333 - .16}{.333}$ or by 52%.

The internal pressure P_o to be used in all these formulae is $\frac{R_{h\ max.}}{A}$

The cylinders are tested with twice the working pressure and the corresponding test stress should not be greater than

7/8 the elastic limit. Consequently, the working stress should not be greater than 7/16 the elastic limit.

X. COUNTER-RECOIL SPRINGS.

The counter-recoil springs must be strong enough to return the piece to battery at all angles of elevation. The forces to be overcome by the springs are the stuffing box friction, the component of the weight acting parallel to the axis of the cylinder and the friction on the guides.

If P_a = load on springs at assembled height

and ϕ = max. angle of elevation

$P_a = F_s + W_r \sin \phi + W_r \cos \phi \, f.$

Take F_s (stuffing box friction) = 100 lbs. per in. of diameter

W_r = total load springs have to return to battery.

f = (coefficient of friction) = .2.

For a stirrup spring P_a for the outer column should be somewhat greater than for the inner column since the outer column has to lift all that the inner one does and in addition the component of the weight of the inner spring and stirrup.

P_a for carriages having ϕ max. = 40° should be somewhat in excess of the total weight of the recoiling parts.

The ratio of $\frac{\text{load solid}}{\text{load assembled}} = \frac{P}{P_a}$ should be 2/1, for the condition of minimum weight of springs to do a given amount of work. A lesser value is, however, advantageous in some instances. Bars of square or rectangular section give the best results for these spraings as those of round section have a tendency to deform when compressed solid dynamically as probably happens at the beginning of recoil.

Spring formulae.

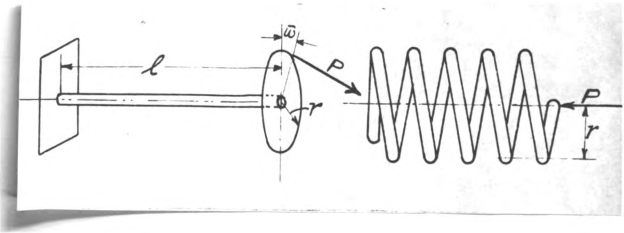
The formulae in most common use for calculating springs are those of Reuleaux and Hütte. Hütte's formulae have heretofore been used in all recent Ordnance Office calculations. These formulae when applied to actual measured data on howitzer carriage springs recently purchased, give values of torsional modulus of elasticity much lower than those given in any handbook. The handbook values vary from 10,000,000 lbs. per sq. in. to 14,000,000 lbs. per sq. in., while those determined from measurements on springs from the Hütte formulae are sometimes as low as 6,000,000 lbs. per sq. in.

There has recently come to my attention a third set of formulae which should give somewhat greater values of this modulus and which are probably more nearly correct. They were published by C. Bach in a work "Die Machinen Elemente."

Below is given the derivation of a set of formulae based on theory, and in parallel columns the corresponding formulae of Reuleaux, Hütte, and Bach.

Torsional Spring Formulae

Comparison of the formulae of Bach, Hütte, and Reuleaux with the theoretical formulae:



Torsional Spring Formulae

Nomenclature:

G = torsional modulus of elasticity.

P = load.

S = fiber stress due to load P.

ϖ = angular distortion due to load P.

f = deflection due to load P.

r = arm on which P acts (for helical spring, r = mean radius of coil)

l = length of bar.

n = number of coils.

I = polar moment of inertia of bar.

e = distance of most strained fiber from neutral axis.

$\frac{I}{e}$ = section modulus.

V = volume of spring.

W = work done on spring by application of load P.

General (true for any form of bar)

$$G = \frac{\text{stress}}{\text{strain}} = \frac{S}{\varpi \frac{e}{l}} = \frac{lS}{\varpi e}$$

$$\varpi = \frac{l}{e} \cdot \frac{S}{G} \qquad f = r\varpi = \frac{rl}{e}\frac{S}{G}$$

$$Pr = S\frac{I}{e} \qquad P = \frac{I}{er} S$$

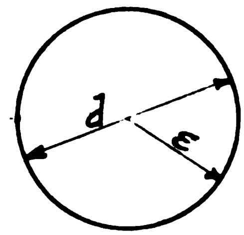
$$S = \frac{Per}{I} = \frac{feG}{rl} \qquad f = \frac{r^2 l}{I} \frac{P}{G}$$

For helical spring $l = 2\pi rn$

$$f = \frac{2\pi r^2 n}{e} \frac{S}{G} \; ; \qquad f = \frac{2\pi r^3 n}{I} \frac{P}{G}$$

Torsional Spring Formulae

Case I. Cylindrical Bar.



d = diameter of rod.

$$e = \frac{d}{2}$$

$$I = \frac{\pi d^4}{32}$$

$$\frac{I}{e} = \frac{d^3}{16}$$

Theoretical, Hütte, Reuleaux, and Bach.

$$f = \frac{2rl}{d} \cdot \frac{S}{G}$$

For helical spring

$$l = 2\pi rn$$

$$f = \frac{4\pi r^2 n}{d} \quad \frac{S}{G}$$

$$P = \frac{\pi d^3}{16r} S$$

$$d^3 = \frac{16r}{\pi} \quad \frac{P}{S}$$

$$f = \frac{32r^2 l}{\pi d^4} \quad \frac{P}{G}$$

$$f = \frac{64r^3 n}{d^4} \quad \frac{P}{G}$$

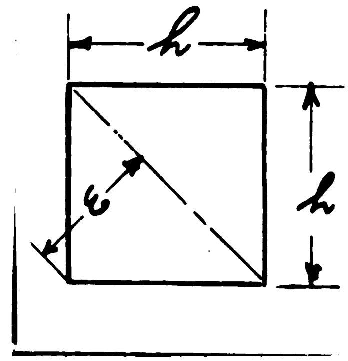
$$V = \frac{\pi d^2}{4} l$$

$$W = \frac{Pf}{2} = \frac{1}{4} V \frac{S^2}{G}$$

$$V = 4W \frac{G}{S^2}$$

Torsional Spring Formulae

Case II. Square bar.



h = side of square

$$e = \frac{h}{2}\sqrt{2} = \frac{h}{\sqrt{2}}$$

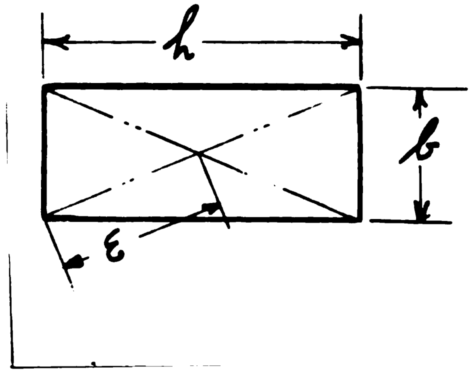
$$I = \frac{h^4}{6}$$

$$\frac{I}{e} = \frac{h^3}{3\sqrt{2}}$$

Theoretical and Reuleaux	Hütte	Bach
$f = \frac{\sqrt{2}\, rl}{h}\frac{S}{G} = \frac{1.414\, rl}{h}\frac{S}{G}$	$f = \frac{1.6\, rl}{h}\frac{S}{G}$	$f = \frac{1.79\, rl}{h}\frac{S}{G}$
For helical spring $l = 2\pi rn$.		
$f = \frac{2\sqrt{2}\pi rn}{h}\frac{S}{G} = \frac{2.828\,\pi r^2 n}{h}\cdot\frac{S}{G}$	$f = \frac{3.2\,\pi r^2 n}{h}\frac{S}{G}$	$f = \frac{3.58\,\pi r^2 n}{h}\frac{S}{G}$
$P = \frac{h^3}{3\sqrt{2}r} S = .235\frac{h^3}{r} S$	$P = \frac{2}{9}\frac{h^3}{r} S$	$P = \frac{2}{9}\frac{h^3}{r} S$
$h^3 = 3\sqrt{2}r\frac{P}{S} = 4.242r\frac{P}{S}$	$h^3 = 4.5r\frac{P}{S}$	$h^3 = 4.5r\frac{P}{S}$
$f = \frac{6r^2 l}{h^4}\frac{P}{G}$	$f = \frac{7.2r^2 l}{h^4}\frac{P}{G}$	$f = \frac{8.05r^2 l}{h^4}\frac{P}{G}$
$f = \frac{12\pi r^3 n}{h^4}\frac{P}{G}$	$f = \frac{14.4\pi r^3 n}{h^4}\frac{P}{G}$	$f = \frac{16.1\pi r^3 n}{h^4}\frac{P}{G}$
$V = h^2 l$		
$W = \frac{Pf}{2} = \frac{1}{6}V\frac{S^2}{G}$		
$V = 6W\frac{G}{S^2}$	$V = 5.625W\frac{G}{S^2}$	$V = 5.03W\frac{G}{S^2}$

Torsional Spring Formulae.

Case III. Rectangular Bar.



h = large dimension

b = small dimension

$\xi = \frac{b}{h} < 1$

$e = \frac{1}{2}\sqrt{b^2 + h^2}$

$I = \frac{bh}{12}(b^2 + h^2)$

$\frac{I}{e} =$

f =

for

f =

P =

$h^3 =$

$f = \frac{1}{b}$

$f = \frac{2}{b}$

V = b

$W = \frac{P}{2}$

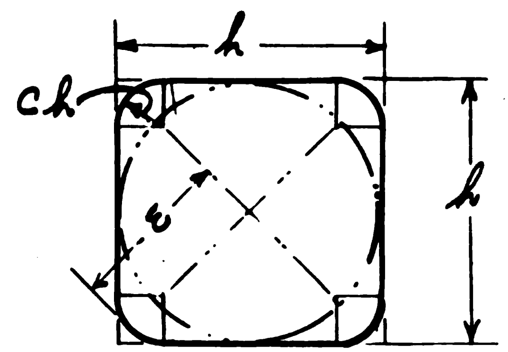
V = 6W

Torsional Spring Formulae.

Case IV. Square bar with rounded corners.

The fact that a perfectly sharp corner on a square bar is undesirable, especially if it must work with a comparatively small clearance, has necessitated the derivation of a set of formulae for a square bar with corners rounded. Below is given the derivation of the formulae (a) for theoretical polar moment of inertia (b) for the Hütte formulae, (c) for the Bach formulae.

(a) Theoretical formula



h = side of square.

ch = radius of corner.

$$e = \frac{h}{2}\sqrt{2} - ch\sqrt{2} + ch$$

$$e = \frac{h}{\sqrt{2}}\,(1 - 2c + c\sqrt{2}) = \frac{h}{\sqrt{2}}\left[1 - c(2 - \sqrt{2})\right].$$

$$e = \frac{h}{\sqrt{2}}\,(1 - .5858c).$$

$$I_p = I_1 - 4I_2 + 4I_3.$$

$$I_1 = \frac{h^4}{6}$$

$$I_2 = \frac{c^4h^4}{6} + c^2h^2\left[\left(\frac{h}{2} - \frac{ch}{2}\right)\sqrt{2}\right]^2$$

$$I_2 = \frac{c^4h^4}{6} + c^2h^4\left(\frac{1-c}{\sqrt{2}}\right)^2 = \frac{c^4h^4}{6} + \frac{c^2h^4}{2}(1-c)^2$$

$$I_3 = 2\left[\frac{\pi}{16}\,c^4h^4 + \frac{\pi c^2h^2}{4}\,(.25 - .5756c + .1512c^2)\,h^2\right]$$

$$I = \frac{h^4}{6} - \frac{4c^4h^4}{6} - (1 - 2c + c^2)\,\frac{12c^2h^4}{6} + \frac{3\pi c^4h^4}{6} + \frac{12\pi c^2h^4}{6}$$

$$(.25 - .5756c + .1512c^2).$$

$$I = \frac{h^4}{6}\,(1 - 4c^4 - 12c^2 + 24c^3 - 12c^4 + 3\pi c^4 + 3\pi c^2 - 6.907\pi c^3 +$$

$$1.815\pi c^4)$$

$$I = \frac{h^4}{6}(1 - 2.575c^2 + 2.301c^3 - .875c^4)$$

Let $A = \left(\frac{1}{1 - .5858c}\right)$

Let $B = (1 - 2.575c^2 + 2.301c^3 - .875c^4)$

$$e = \frac{h}{\sqrt{2}}\ \frac{1}{A}$$

$$I = \frac{h^4}{6} B \qquad \frac{I}{e} = \frac{h^3}{3\sqrt{2}} AB$$

(b) Hütte formulae.

As it is known from experience that the theoretical values of moment of inertia and section modulus are not correct for square and rectangular bars, it is necessary to modify the above derived theoretical formulae to suit those of Hütte and of Bach. It is necessary that for the condition $c = .5$; e and I should agree with the formulae for round bar and that for $c = 0$, e and I should agree with the corresponding formulae for square bar.

In the Hütte formulae e must $= \frac{h}{1.6}$ for square bar and $\frac{h}{2}$ for round.

The first condition will be satisfied if we make $e = \frac{h}{1.6}(1 - qc)$ this equation being of the same form as the theoretical equation, but with different numerical coefficients.

If $q = .4$, the condition $e = \frac{h}{2}$ for round bar will also be satisfied.

Therefore we take $e = \frac{h}{1.6}(1 - .4c)$.

In the same set of formulae I must equal $\frac{h^4}{7.2}$ for square bar and $\frac{h^4\pi}{32}$ for round.

The first condition will be satisfied if we make $I = \frac{h^4}{7.2}(1 - xc^2 + yc^3 - zc^4)$, this equation being of the same form

as the theoretical, but with different numerical coefficients.

If we assume that x : y : z in the same ratio as the original theoretical coefficients, viz: 2.575 : 2.301 : .875 we have $y = .895x$; $z = .340x$

$$\therefore \quad I = \frac{h^4}{7.2}(1 - xc^2 + .895c^3x - .34c^4x)$$

If $x = 1.641$, the condition $I = h^4 \frac{\pi}{32}$ will also be satisfied. Therefore, we take

$$I = \frac{h^4}{7.2}(1 - 1.844c^2 + 1.65c^3 - .627c^4)$$

let $A' = \frac{1}{1 - .4c}$; let $B' = 1 - 1.844c^2 + 1.65c^3 - .627c^4$

$$e = \frac{h}{1.6}\ \frac{1}{A'}\ ; \quad I = \frac{h^4}{7.2}\ B'; \quad \frac{I}{e} = \frac{2}{9}\ h^3A'B'$$

(c) Bach Formulae.

By a similar method of reasoning we find, using the Bach formulae

$$e = \frac{h}{1.79}(1 - .21c)$$

$$I = \frac{h^4}{8.05}(1 - 1.313c^2 + 1.175c^3 - .447c^4)$$

Let $\frac{1}{1 - .21c} = A''$

$1 - 1.313c^2 + 1.175c^3 - .447c^4 = B''$

$$e = \frac{h}{1.79}\ \frac{1}{A''}\ ; \quad I = \frac{h^4}{8.05}\ B''$$

$$\frac{I}{e} = \frac{2}{9}\ h^3\ A''\ B''$$

Now arrange the formulae in parallel colums as before:

Case IV. (Square bar with round

(a) Theoretical

$$f = \frac{\sqrt{2}rl}{h} A \frac{S}{G} = \frac{2\sqrt{2}\pi r^2 n}{h} A \frac{S}{G}$$

$$P = \frac{h^3}{3\sqrt{2}r} ABS = .235 \frac{h^3}{r} ABS$$

$$h^3 = 3\sqrt{2}r \frac{1}{AB} \frac{P}{S} = 4.242 \frac{r}{AB} \frac{P}{S}$$

$$f = \frac{6r^2 l}{h^4} \frac{1}{B} \frac{P}{G}$$

$$f = \frac{12\pi r^3 n}{h^4} \frac{1}{B} \frac{P}{G}$$

$$V = h^2 l(1 - .858c^2)$$

$$W = \frac{1}{6} V \frac{S^2}{G} \left(\frac{A^2B}{1 - .858c^2}\right)$$

$$\text{Let } C = \left(\frac{A^2B}{1 - .858c^2}\right)$$

$$V = 6W \frac{G}{S^2} \frac{1}{C}$$

An analysis of the above formulae discloses the fact that

(1) Fiber stress being equal, f varies as A, A', or A".

(2) Fiber stress being equal, P varies as AB, A'B' or A"B"

(3) Load being equal, f varies as $\frac{1}{B}$, $\frac{1}{B'}$ or $\frac{1}{B''}$.

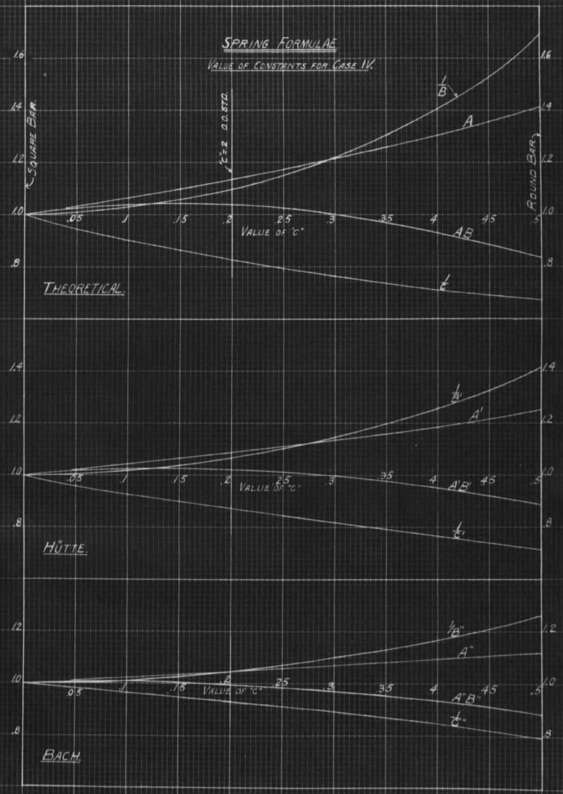
(4) Fiber stress being equal, V (∴ weight of spring) varies as $\frac{1}{C}$, $\frac{1}{C'}$, or $\frac{1}{C''}$.

A, AB, $\frac{1}{B}$, and $\frac{1}{C}$ are tabulated below for each of the three sets of formulae given above.

	Theoretical				Hütte			
c	A	AB	$\frac{1}{B}$	$\frac{1}{C}$	A'	A'B'	$\frac{1}{B'}$	$\frac{1}{C'}$
0.	1.	1.	1.	1.	1.	1.	1.	1.
.05	1.030	1.024	1.006	.947	1.020	1.014	1.003	.966
.1	1.062	1.036	1.025	.902	1.042	1.025	1.016	.928
.15	1.096	1.041	1.053	.861	1.064	1.025	1.037	.900
.2	1.133	1.036	1.094	.823	1.086	1.019	1.064	.873
.25	1.172	1.022	1.147	.791	1.111	1.010	1.099	.843
.3	1.213	.999	1.215	.762	1.137	.994	1.143	.818
.4	1.306	.931	1.403	.710	1.191	.950	1.254	.764
.5	1.414	.833	1.698	.667	1.250	.884	1.414	.712

	Bach			
c	A"	A"B"	$\frac{1}{B''}$	$\frac{1}{C''}$
0.	1.	1.	1.	1.
.05	1.011	1.007	1.003	.980
.1	1.021	1.007	1.012	.966
.15	1.032	1.005	1.028	.947
.2	1.043	.996	1.047	.931
.25	1.056	.986	1.071	.911
.3	1.067	.970	1.099	.895
.4	1.092	.934	1.172	.846
.5	1.118	.884	1.26	.795

These constants are plotted to scale on the following page.


SPRING FORMULAE.
VALUE OF CONSTANTS FOR CASE IV.
"C"=.2 D.0.57D.
SQUARE BAR
ROUND BAR
1/B
A
AB
1/C
VALUE OF "C"
THEORETICAL.
1/B'
A'
A'B'
1/C'
HÜTTE.
1/B"
A"
A"B"
1/C"
BACH.

Upon examining the curves and tables we find that, irrespective of the formulae used, as the ratio of radius of corner to side of bar, c, increases, the safe load first increases, then diminishes and finally becomes equal to that for round bar, while at the same time the deflection increases and the weight of spring decreases.

For instance, if the Hütte formulae are used, we find that A'B' has a maximum value of 1.03 at c = .125, and that consequently P for this value of c may be 3% greater than for a square bar, while, at the same time, the deflection per coil would be 5% greater, since $f \propto A'$, and the weight would be only 91% of that for a square bar. At c = .275, A'B' = 1 and ∴ P would be the same as for a square bar, the deflection per coil being increased 12% and the weight decreased 17%.

From this it is evident that the sharp corners of a square bar in torsion are a worse than useless burden since they add useless weight and metal which is first overstressed, thus reducing the allowable load.

For Ordnance Office practice it is customary to take c = .2.

XI. GENERAL PROCEDURE.

In taking up the design of a carriage, it is desirable to first lay down a set of specifications which should be adhered to as closely as possible. The following is a sample set of specifications for a carriage and represents the information which should be available before proceeding with the design.

Specifications for 4.7-inch Howitzer Carriage.

(1) Ballistics of howitzer:

Weight of projectile 60 lbs.

Weight of powder charge 1.75 lbs. (for 900 feet/sec.)

Muzzle velocity (3rd zone) 900 feet/sec.

Maximum powder pressure 24000 lbs./sq. in.

(2) Limits of elevation:

5° depression to 40° elevation.

(3) Limits of azimuth:

each
3°/side of ℄ of carriage.

(4) Weight of carriage with howitzer, tools, etc., ready for firing not more than 4800 lbs.:

(5) Stability to obtain at all elevations from 0° up.

(6) A variable recoil will be obtained by use of the valve similar to that on the 3-inch Mountain Howitzer carriage and the 3.8-inch Howitzer Carriage; the method of operating the valve to be the same as for the 3.8-inch Howitzer Carriage.

(7) Counter-recoil springs to be helical; the cross-section of the bar to be square with corners rounded to a radius = 20% of the side. The springs to be concentric with the recoil cylinder. Maximum allowable fiber stress at solid height 150,000 lbs. per sq. in. Load at assembled height to be suf-

ficient to return howitzer to battery at maximum elevation under adverse conditions.

(8) Elevating gear to be worm and rack (Hindley system) of usual type. Handwheel on left side of carriage only.

(9) Rocker to be made detachable from cradle and to have an arm to carry sight.

(10) A quick return mechanism to be provided to permit the return of the Howitzer from high elevation to zero to permit loading without disturbing the operation of sighting. This mechanism to be operated from the right side of the carriage.

(11) Traversing to be accomplished by rotating a top carriage about a pintle by means of a screw and nut operated on the left side.

(12) The sight will have three scales to accommodate the three zones of fire; a zone shutter and an adjustable level will be provided.

(13) An azimuth scale will be provided in a position to be easily read by the gunner in laying the piece.

(14) For transportation, the carriage will be provided with a traveling lock to relieve the traversing and elevating gears.

(15) A shield .15 inch thick will be provided.

(16) Axle seats for two gunners to be provided.

(17) Carriage to have a road brake of lever type.

(18) Carriage to have a hollow steel axle with offset axle arm. Axle to be removable.

(19) Standard 58-inch wheels will be used.

(20) Trail seats for two gunners will be provided, one on each side.

(21) Track will be 60-inch center to center of tires.

(22) Compartments will be provided in the trail, one for tools and one for packing away the sight. The panoramic sight will be carried in a spring supported case on the shield. Sponge and rammer to be carried in fastenings on side of trail.

(23) Area of spade to be sufficient to resist greatest horizontal component of pull at 60 lbs. per sq. in.

(24) Spade to be fixed and not to limit turning angle of limber. Float to extend to rear of spade.

(25) Standard lunette to be used.

(26) Arrangement of parts in cradle will permit removal of cylinder without removing springs, and removal of springs without removing cylinder.

(27) A firing mechanism will be provided which will have the customary shaft return.

(28) All working surfaces to be protected so far as possible from dust and rust, and provided with means for proper lubrication. All oil holes to be closed with handy oilers.

(29) Crown nuts with split pins to be used in preference to any other method of locking nuts. Taper pins to be avoided.

(30) All parts to be designed with a permissible stress of 1/2 the elastic limit.

(31) Counter recoil to be controlled by the customary counter-recoil buffer.

(32) Maximum permissable stress in recoil cylinder 2750 lbs. per sq. in.

(33) No cast steel will be used for pieces having thin or widely varying sections.

(34) Standard material to be used throughout the construction whenever practicable.

Preliminary Calculations.

(1) To get an idea of the piston rod pull, length of recoil, stability, etc., estimate the weights of the various portions of the carriage such as shield, wheels, axle, sights, gun, trail, tipping parts, etc.

(2) Compute the longitudinal center of gravity from these figures and calculate stability from equation 9, page 11, assuming the location of the center of gravity of the recoiling parts to be a reasonable distance above the ground.

(3) Estimate V_f from equation 7, page 9.

(4) Estimate total pull from equation 14, page 14.

(5) If pull is greater than stability, calculate by (2) above, assume a new value for length of recoil and recalculate pull.

(6) Make a preliminary layout to see if the proposed construction looks feasible.

Final Calculations.

(1) Calculate weight of recoiling parts accurately from detail drawings.

(2) Calculate springs.

(3) Calculate total pull at maximum and minimum recoil.

(4) Calculate forces on carriage and stresses on individual parts.

(5) Calculate velocity of projectile in bore of gun.

(6) Calculate and plot curve of velocity of free recoil.

(7) Calculate velocity of retarded recoil.

(8) Calculate hydraulic resistance.

(9) Calculate throttling areas.